

Investigation of Dynamics in Turbulent Swirling Flows Aided by Linear Stability Analysis

by

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ABSTRACT

Turbulent swirling flows are important in many applications including gas turbines, furnaces and cyclone dust separators among others. Although the mean flow fields have been relatively well studied, a complete understanding of the flow field including its dynamics has not been achieved. The work contained in this dissertation attempts to shed further light on the behavior of turbulent swirling flows, especially focused on the dynamic behavior of a turbulent swirling flow encountering a sudden expansion. Experiments were performed in a new isothermal turbulent swirling flow test facility. Two geometrical nozzle configurations were studied. The center-body nozzle configuration exhibits a cylindrical center-body in the center of the nozzle. The free vortex nozzle configuration is obtained when the cylindrical center-body is removed. Detailed laser velocimeter measurements were performed to map out the flow field near the sudden expansion of the 2.9" (ID) nozzle leading to the 7.4" (ID) downstream section.

In addition to presenting detailed flow profiles for both nozzle and downstream flow fields, representative frequency spectra of the flow dynamics are presented. Along with the flow time histories and histograms, the wide variety of dynamic behavior was thus described in great detail. The dynamics observed in the experiment can be classified into three main categories: coherent and large scale motion, intermittent

motion and coherent periodic motion. Free vortex geometry flows, in the parameter space of the experiments (Swirl number = 0 - 0.21), exhibited mostly coherent and large scale motion. The spectra in these cases were broadband with very light concentration of spectral energy observed in some specific cases. Center-body geometry flows exhibited all three categories of flows as swirl strength was increased from zero. Flows with little or no swirl exhibited broad-band spectra similar to those for the free vortex geometry. Intermediate swirl levels resulted in a large amount of low frequency energy which, with the aid of the time histories, was identified as a large scale intermittence associated with radial movement of the annular jet as it enters the sudden expansion. Large swirl levels resulted in high magnitude coherent oscillations concentrated largely just downstream of the sudden expansion.

Linear stability analysis was used to help in the interpretation of the observed dynamics. Although, as implemented here (using the parallel flow assumption), the analysis was not successful in quantitatively matching the experimentally observed dynamics, significant insight into the physical mechanisms of the observed dynamics was obtained from the analysis. Specifically, the coherent oscillations observed for larger swirl levels were able to be described in terms of the interaction between the inner and outer shear layers of the flow field.

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To my family

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Nomenclature

a_1, a_2, \dots	generic quantity / variable
a_M	constant in ring jet profile that helps determine location of maximum axial velocity (Equation 6.31)
a_q	coefficient for non-linear mapping of physical domain into Chebychev domain
\mathbf{a}_k	vector of coefficients for the k-th Chebychev polynomial
A	area
A_M	swirl strength parameter in ring jet with swirl profile (Equation 6.31)
b_1, b_2, \dots	generic quantity / variable
b_M	constant in ring jet swirl profile that helps determine location of maximum swirl velocity (Equation 6.31)
c	ω/α , the complex wave phase speed
c_t	proportionality constant in the determination of turbulent viscosity from k and l_m
Cal_k	calibration transfer function of microphone k, with B&K microphone in first channel. Includes B&K microphone sensitivity (0.0473 V/Pa)
ch_d	inter-channel delay, 5 μsec
ch_k	channel number k
COH_{12}	ordinary coherence between signals 1 and 2
$CP_{s_{12}}$	cross spectrum between signals 1 and 2
D	diameter

D_n	nozzle diameter, 2.9"
D_h	hydraulic diameter of the nozzle with center-body, 1.9"
DD	matrix of constant coefficients for the unknowns \mathbf{Q} not involving the eigenvalue (α or ω)
d_b	beam separation distance at the focusing lens
d_f	fringe spacing for the laser interference pattern
DFT	direct Fourier transform of a signal
dS	differential surface element
dt	differential amount of time
E	similar to DD except constants are multipliers of the eigenvalue
$f(x,y,t)$	perturbation function
$F(y)$	eigenfunction, relative distribution of the disturbance in y
$F(r)$	radial velocity eigenfunction
f_d	frequency estimate of Doppler burst
f	cyclical frequency (Hz)
f_l	focal length of beam focusing lens
f_m	mixing frequency used to electronically down shift the Doppler frequency
f_s	optical shift frequency applied by the Bragg cell
f_{sam}	mean sampling frequency
g	generic function
G	azimuthal (swirl) velocity eigenfunction, $G(r)$
H	axial velocity eigenfunction, $H(r)$
H_{12}	actual transfer function between P_1 and P_2
H_{12}^m	calibration and delay corrected transfer function between P_1 and P_2
H_{12}^v	raw voltage transfer function between P_1 and P_2 sensors
i	$\sqrt{-1}$
k	turbulent kinetic energy
kl_m	model constant representing the product of a mean length scale with a mean turbulent kinetic energy.
l_m	turbulent length scale

m	azimuthal mode number, $\in I$, the set of integers
N	number of data points
P	pressure disturbance eigenfunction, $P(r)$
P_1, P_2	Fourier transformed pressures of microphones 1 and 2
p	pressure (non-dimensionalized)
p_1, p_2, \dots	profile fitting parameters
Ps	power spectrum
Psd	power spectral density
Q	volumetric flow-rate
\mathbf{Q}	solution vector including all coefficients of the Chebychev polynomials
r	radial coordinate
R_p	complex pressure reflection coefficient
Re	Reynolds number = $U_{ref} D_{ref} / \nu$
s	variable $s(r, \theta, t)$
S	swirl number
S_t	turbulent swirl number
t	time
t_t	turbulent time scale
T	measurement period
TF_{12}	transfer function between signal 1 and 2
T_k	k-th Chebychev polynomial
U,u	non-dimensionalized axial flow profile function, ($U(y)$ or $U(r)$)
U_{oM}	back-flow parameter of ring jet profile with swirl (Equation 6.31)
\vec{u}	acoustic velocity vector
\vec{V}	velocity vector (U, V_r, W)
V_r, v_r	non-dimensionalized radial flow profile function, $V(r)$
W,w	non-dimensionalized azimuthal (swirl) flow profile function, $W(r)$
W_T	transmitted acoustic power
x	axial coordinate
y	direction of flow profile function

v_d	velocity corresponding to measured Doppler frequency
α	axial wavenumber
Γ	circulation, $W \cdot r$
Δr	separation distance of microphones
θ	azimuthal coordinate
θ_{sl}	shear layer thickness
∇	gradient operator
λ_f	Frobenius exponent in power expansion
λ_w	Wasow power expansion coefficients
Λ	matrix whose diagonal entries are eigenvalues
ν	kinematic viscosity
ν_t	turbulent viscosity
ξ	Chebychev domain coordinate $\xi \in [-1, 1]$
ρ	density
τ	stress tensor components
Ψ	disturbance stream function
ω	temporal frequency (radians/sec)
$\omega_{\alpha\alpha}$	coefficient to fit dispersion relation: $(\omega - \omega_o) = (\omega_{\alpha\alpha}/2) * (\alpha - \alpha_o)$
Ω	angular velocity, V/r
o	indicates the wavenumber and frequency of the mode with zero group velocity
a^*	indicates complex conjugate of quantity a
\tilde{a}	indicates the fluctuating component of the quantity a
a_{rms}	indicates the RMS component of the quantity a
a_{un}	uncertainty of quantity a
a_w	quantity evaluated at the wall
\overline{ab}	indicates the time average of the product of a and b

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