

# **An Economic Theory of Leadership**

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(ABSTRACT)

This dissertation develops an economic theory of leadership based on assignment of information. Common theories assume that organizations exist to reduce transaction costs by replacing imperfect markets with incomplete long term contracts that give managers the power to command subordinates. This view reverses all of these premises: I study an organization in which it is costless to transmit and process information, contracts exist in the background if at all, and agents are not bound to the organization. The organization is held together by economies of scale in generating information and by the advantages of controlling access to that information. The minimalist model of organizations produces a minimalist theory of leadership: leaders have no special talent but are leaders simply because they are given exclusive access to certain information. A single leader induces a first best outcome if his incentives are aligned with his subordinates. If a single leader is not credible, then diluting the power of leadership by appointing multiple informed leaders can ensure credibility and improve efficiency but can not produce the first best. If agents are differentiated by their costs of cooperation the most cooperative player is not necessarily the best leader. In this scenario, the ability of the group to sustain fully cooperative outcomes may depend on the player with the least propensity to cooperate. Therefore, to maximize efficiency (i.e., to maximize the range of circumstances in which efficient cooperation is sustainable), the group should sometimes promote less cooperative people. Here, "less cooperative" means lazy or busy rather than disagreeable. This dissertation also applies the idea of leadership (endorsement) to voluntary provision of public goods. I show that when the leader is unable to fully reveal his information expected contributions, ex-ante, are unambiguously higher in the leader-follower setting. That is partial revelation of information induces more contribution compared to full revelation or complete information. I also show that if the utility functions are linear then ex-ante welfare is unambiguously higher in the presence of an informed endorser.

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I dedicate my dissertation to my family, Tooran, Naser and Raha Komai, for their love and support all through my life.

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# Chapter 1

## Literature Review

### 1.1 Introduction

Leadership has been long studied by political theorists and social scientists. It has, however, generally been neglected by economists. Common theories of organizations have focused on formal and incomplete long-term contracts that give managers the power to control their subordinates (e.g. Hart (1995) and Grossman and Hart (1986)). Such theories have produced important insights, but miss the point that leadership is distinct from formal authority. The reason is that many of the incentives for cooperation are difficult to modify by contracts. Therefore, to induce cooperation managers must encourage and motivate their subordinates. This is where leadership comes into the picture.

I distinguish leadership from formal authority. This dissertation is organized in three chapters. The first chapter gathers the literature on leadership in management and economics. Leadership has been widely studied in management in various contexts and from many different aspects. This variety along with the informal style of management scholars makes this literature confusing and hard to understand from an economist's perspective. Unlike in management, leadership has not been highly recognized in economics. The purpose of chapter one is to show the potential for economic research on leadership. It is time for economists to take action toward formalizing this concept.

In the second chapter, I develop a minimalist model of leadership based on assignment of information. The model, while representing only one dimension of organizations, can replicate

several stylized facts about organizational forms and behavior. My model does not fall into the contract design literature mentioned above. Contracts are external to the model and appear only implicitly in the exogenous payoffs that agents receive. The model does raise interesting contracts design issues, which I do not answer. In my model, an organization is held together only by economies of scale in generating information and by the advantages of restricting access to that information. Leaders have no special talent but are leaders simply because they are given exclusive access to certain information. I consider an organization in which leaders are exogenously informed about the quality of the project undertaken by the organization and decide whether to participate in the project. Leaders' participation is observable by their followers and serves as a signal which partially transmits their information. The binary nature of the decision does not allow the leaders to fully reveal their information even if they want to. Followers rely on this partial information to make their decision.

My model turns on the observation that leaders are able to use their informational advantage to solve cooperation and coordination problems by sending a vague signal which partly reveals their information to their followers. Uninformed followers are more cooperative, because they do not know when their cooperative actions actually produce high personal pay-off. They rely on the imperfect inferences that they draw from an informed leader's actions. Leaders are also more cooperative in the presence of ill informed agents, because their effort transmits part of their information to the others.

In a homogeneous setting, I show that a single leader often induces cooperation, coordination and the unconstrained first best if he can not credibly reveal all of his information. In contrast the standard free-riding and coordination problems usually preclude first-best efficiency if all agents are informed.

To induce cooperation and coordination, a leader should convince his followers that he is not misleading them by transmitting incorrect information. A powerless leader will be recognized and won't be followed by his followers. In this case, diluting the power of leadership by appointing multiple informed leaders can improve efficiency but cannot achieve the first best.

I also consider a scenario where agents are differentiated by their cost of cooperation. Relaxing the homogeneity assumption is not only more realistic but also allows us to address the issue of leadership selection. In the new setting we can focus on players' characteristics to identify the

optimal leader. I argue that If agents are differentiated by their costs of cooperation, then the ability of the group to sustain cooperative outcomes may depend on the follower whose behavior is least cooperative. I show that it is never optimal to promote the most cooperative player to the leadership position. To maximize efficiency (i.e., to maximize the range of circumstances in which efficient cooperation is sustainable), the group should often choose an average player. If the average player is not powerful, it improves efficiency to choose the leaders from among less cooperative (i.e. lazier or busier) players.

Chapter 3 applies the idea of leadership to the private provision of public goods. In this chapter I develop a model in which the leader is exogenously informed about the value of the public project. The leader makes two separate decisions: whether to endorse the project and how much to participate. The leader's endorsement (commitment) is costly and observable by the other players. His amount of contribution, however, can not be verified by the others. As in the former models, the leader is unable to credibly transmit all of his information, for his amount of contribution is not verifiable.

The theoretical setting in this chapter is different and somewhat complementary to that of chapter 2. One important distinction is that the new setting restricts the payoff structure to a quasi-linear form but generalizes the participation decision by making it continuous rather than binary. Another distinction is that unlike chapter 2, in the model of chapter 3 information transmission is costly and the leader's signal is unproductive.

I show that partial revelation of information by the leader increases the overall contributions to lower return projects which can not be sufficiently financed under complete information. The ex-post results in this chapter are weaker than those of chapter 2 because participation decisions are continuous and the leader's signal is not welfare increasing. That is, in some cases the ex-post contributions to the higher return projects may be larger under complete information. I show, however, that the leader increases the overall amount of contribution ex-ante. That is the leader is able to increase the amount of contribution on average. Finally it is shown that when followers' payoff functions are linear and the return to the project is uniformly distributed, the ex-ante welfare is higher under partial revelation than that of full revelation or complete information. I also specify the conditions under which the leader increases the welfare level ex-post.



## **1.2 Leadership in Business and Management**

Leadership may be defined as the process (act) of influencing the activities of an organized group in its efforts toward goal setting and goal achievement. (Stogdill, 1950, p. 3)

Leadership is a process of influence between a leader and those who are followers. (Hollander, 1978, p. 1)

Leadership is the behavior of an individual when he is directing the activities of a group toward a shared goal. (Hemphill and Coons, 1957, p. 7)

The statement, ‘ a leader tries to influence other people in a given direction is relatively simple, but it seems to capture the essence of what we mean by leadership’...(Korman, 1971, p. 115).

As it can be seen from the above definitions, a leader is interpreted as someone who sets direction in an effort and influences people to follow that direction.

Leadership has been long studied by business scholars but it has almost been neglected by economists. Many of business scholars are seeking to understand and write about the concept of leadership today. It can be confusing to try to understand leadership in business literature, for different scholars analyze leadership in many different contexts and from many different perspectives. There are numerous theories about leadership in business. There are also numerous theories about leadership styles and how leaders carry out their roles.

To explore the territory of leadership in business and to investigate the potential for research in economics, I briefly scan some of the major studies in the business literature and economics in this chapter.

The major approaches to leadership in business and management can be classified into four groups: Classical approaches, Contemporary approaches, Alternative approaches and the New wave approaches.

### **1.2.1 The Classical Approaches**

Ohio State Model, Contingency Model and Participative leadership are the most known classical approaches in leadership literature in business and management. These theories have been introduced initially by Schriesheim, Cogliser and Neider; Ayman, Chemers and Fiedler; and

Vroom and Jago, respectively. These theories are instantly recognizable by any student in social sciences because of their resilience and longevity during the last 35 years.

### **Ohio State Model**

The title of this section refers to a highly influential series of studies conducted by an interdisciplinary team of researchers. Its main impact derives from its relatively early development of precise operational definitions of what people in leadership position do. The research team was genuinely interdisciplinary, in that although psychologists predominated, a sociologist, an economist, and an educationalist were also members. The focus was on the activities of leaders.

The group actually reduced this accumulation to 130 questionnaire items. This research instrument was named the Leader Behavior Description Questionnaire (LBDQ), and was supposed to reflect eight theoretical aspects of leader behavior.

A central development in the Ohio programme was a factor analysis of the results of the administration of the LBDQ to see members of air crews, who were asked to describe the behavior of their leaders in terms of the 130 questionnaire descriptions.

The analysis revealed that four factors predominated in the depictions of the leader behavior. The first factor was called consideration, which denoted natural trust, liking and respect in the relationship between leaders and their subordinates. The second factor was named initially structure. Leaders, whose behavioral descriptions result in their receiving high scores on this dimension tend to organize work tightly, to structure the work context, to provide clear-cut definitions of role responsibility, and generally play a very active part in getting the work at hand fully scheduled. The third factor is called production emphasis, which is regarded as indicative of motivating the crew to greater activity by emphasizing the mission or job to be done. The fourth factor is named sensitivity, which refers to the awareness of social interrelationship and pressures existing both inside and outside the subordinates.

The first two factors were the central ingredients of the measurement of leader's behavior. The other two factors, however, were recognized as being theoretically interesting, but not capable of specification with the same degree of certainty.

The Ohio state questionnaire instruments have been developed during the time but the general methodology of the Ohio State studies are the same. The approach is to focus upon

putative leaders in an organization. The questionnaires are administered to their subordinates in order to gauge the behavior of the leaders. The questionnaires invariably comprise additional scales to examine various aspects of the climate or morale of the work group attached to the leaders. The scores of the respondents in each group are then aggregated to discern that leader's leadership profile and the general climate or moral by the group.

The cluster of the methodological procedures reflects three important characteristics, which underpin much of the research in Ohio State Studies. First, the work group is the level of analysis. Second, The focus is upon what have been called designated or putative leaders, that is on individuals in supervisory positions within a broader hierarchy. Third, The Ohio State studies seeks to relate descriptions of leadership to measures of outcome.

It is for the development of rigorous measures of leadership behavior and their catchy descriptions of such behavior that the Ohio State researchers will be remembered.

### **Contingency Model**

The contingency model of leadership effectiveness was presented in its most complete form in Fiedler (1967) and Fiedler and Chemers (1974). The evolution of the model and the development of its constructs covers three decades of research. The model predicts that a leader's effectiveness is based on two main factors: A leader's attributes, referred to as task or relationship motivational orientation (formerly referred to as style), and a leader's situational control (formerly referred to as situational favorability). The model predicts that leaders who have a task motivational orientation compared to those, who have a relationship orientation or motivation will be more successful in high and low control situations (Fiedler, 1987).

The model is, by design, multi-level and multi-source. That is, measures of the leader's motivational orientation are based on the leader's responses (individual level); Characteristics of the situation have been measured by the leader's report and/or that of subordinates and experimenters (multi-level and multi-source), and outcomes have been assessed at the group level, primarily group performance (Fiedler, 1978) as determined by objective measures, supervisor ratings and average follower satisfaction (Rice, 1981).

The model has been the target of numerous criticism through its evolution and has been an impetus for over 200 empirical studies. The result have supported the model .

## **Participative Leadership**

How much influence a leader should permit his subordinates over matters relating to their jobs is an important decision that all leaders face. This is the fundamental issue, which underpins the widespread interest in how participative a leader should be, that is, how far he should allow subordinates to determine how the work, for which he is responsible should be done; how closely he should supervise them; and how far he should take their views into account.

An example of an approach to the study of the leadership behavior, which focuses successively on participative leadership is the influential framework adopted by Tannenbaum and Schmidt (1958). The title of their work was "How to choose a leadership pattern". In fact the framework is about how to choose a participative leadership pattern, as the article is effectively about what degree of participation ought to be allowed to subordinates and under what circumstances. They conceptualize the range of leadership behaviors as a continuum with boss-centered leadership and subordinate-centered leadership as the two poles.

Participative leadership, involves a shift away from authoritarian, highly directive forms of leadership towards a broader range of individuals being allowed and encouraged to play a part in decision-making. The idea that greater participation must be encouraged in contemporary organizations is widespread in the literature. Social scientists have always been at the forefront of the advocacy of participative management. Mulder (1971, p. 31) calls it 'the most vital organizational problem of our time', while Preston and Post (1974) refer to participative management as the 'third managerial revolution'. Anthony (1978, pp. 27-9) cites eight possible advantages of participatory leadership: greater readiness to accept change, more peaceful manager-subordinate relations, greater trust in management, increased employee commitment to the organization, greater ease in the management of the subordinates, improved quality of management decisions, improved upward communication and improved teamwork.

Other benefits of participation are often mentioned in the literature: for example, under participative management people identify more with a decision and often see themselves as having an investment in it; participation leads to a better understanding of the organization; it enhances people's self development; and so on.

Potential disadvantages are often mentioned in the literature on participation: it may bring conflict to the open to such a degree that the organization falters; it may lead to time consuming

decisions and possibly ones, which are based too much on compromise; managers may be riddled with anxiety if they are faced with being responsible for large numbers of decisions with which they have little agreement; and so on.

One of the difficulties inherent in reviewing the literature on participation is simply that researchers differ in what they mean by it. In consequence it is necessary to adopt a loose definition of what is in any case an imprecise concept.

The research can be grouped into three main headings: experimental studies in non-organizational settings, field experimental studies in organizations, and correlational studies in organizations.

The reviews of relevant literature and studies show disparate findings. While, there is a fair amount of evidence to suggest that participation contributes to satisfaction, as well as performance, there is too much unsupporting evidence to be too confident. We can name two factors that account for these discrepancies. Firstly, the studies reviewed cover a variety of different types of participation, so that a failure to take into account the diversity of forms of participation and their relative effects has contributed to a field in which comparisons between different investigations are not always entirely sensible. Secondly, it has been recognized that participative leadership outcome relationships are likely to be situationally contingent. The failure to take such contextual factors into account may on occasions have led to the various discrepancies. Because of the disparate findings, and the fuzzy implications of many of the studies, the area of participation is a confusing one.

### **1.2.2 The Contemporary Approaches**

The contemporary approaches have a more explicit focus on both of the leaders and the development of their followers. This subtle shift can be seen in the three leading papers in this area. Klein's and House's charismatic leadership, Avolio's and Bass's transformational leadership and LMX approach of Graen and Uhl-Bien are examples the contemporary approaches.

#### **Charismatic Leadership**

Charismatic leadership has its root in political science and sociology but it can apply to organizational charisma. Charisma is a fire that ignites followers' energy, commitment and per-

formance. Charisma resides not in the leader, nor a follower, but in the relationship between a leader, who has charismatic qualities and a follower who is open to charisma, within a charisma-conductive environment.

Charismatic leadership theory and research considers charisma to be the product of three elements. A leader who has charismatic qualities, followers, who are open and susceptible to charisma and an environment conducive to charisma.

#### **(a) Leaders**

Charismatic leadership theory has identified a number of personal characteristics and behaviors that distinguish leaders who have the potential to ignite a fire of charisma within their subordinates. These personal characteristics include for example, prosocial assertiveness, self confidence, need for social influence, moral conviction, and concern for the moral exercise of power (e.g., Bass, 1988; Conger & Kanungo, 1988; House et al., 1991; House et al., 1994). The charismatic behaviors also include articulation of distal ideological goals, communication of high expectations and confidence in followers, emphasis on symbolic and expressive aspects of the task, articulation of a visionary mission that is discrepant from the status quo, references to the collective and the collective identity, and assumption of personal risk and sacrifices (e.g., Bass, 1985; Conger & Kanungo, 1987; House et al., 1994; Shamir et al., 1993).

It is common, within the literature on charismatic leadership, to suggest that the personal characteristics and behaviors listed above make a leader charismatic or distinguish charismatic leaders. But as mentioned before, the leaders characteristics and behaviors are necessary but not sufficient to ignite charisma within subordinates. Followers and the environment also are other important factors that are necessary to form charismatic relationships.

#### **(b) Followers**

To form charismatic relationships, subordinates also have to be open or susceptible to charisma. Some literature suggest that the followers who are most open to charisma are vulnerable and/ or looking for direction or psychological meaning in life (Conger & Kanungo, 1988).

Another view suggests that the followers in charismatic relationships are not weak but are instead compatible and comfortable with their leader's vision and style (Shamir et al., 1993).

The third view suggests, implicitly, that followers in charismatic relationships do not differ

significantly from followers involved in other, non charismatic relationships. The tacit assumption appears to be the leaders with charismatic qualities are so compelling and persuasive that all followers, regardless of their personal characteristics, readily fall under these leaders' influence. The characteristics of followers within charismatic relationships have not been investigated empirically and therefore is ripe for empirical study.

### **(c) The Charisma-Conductive Environment**

Some theorists argue that charismatic relationships are facilitated in the time of crises (e.g., Bass, 1985; Burns, 1978, Conger & Kanungo, 1988; House 1977; Weber, 1947). In crises, individuals are uncertain and stressed and, thus, open to the influence of persuasive leaders who offer an inspiring vision of the crisis resolved.

Charismatic leadership scholars also suggest that a variety of environmental conditions, which simply arouse uncertainty but do not constitute real crises, may help the development of charismatic leadership. Shamir and his colleagues (1993), argued that the emergence of charismatic relationship is facilitated in work settings in which performance goals can not be easily specified and measured, extrinsic rewards can not be made clearly contingent on individual performance, and/ or there are few situational cues, constrains and reinforcers to guide behavior and provide incentives for specific performance.

The environmental conditions conducive to charisma have received more theoretical attention than the follower characteristics, but no greater empirical attention.

As we have emphasized, according to the charismatic leadership theory, charisma resides in the relationship of a follower and a leader and is the product of the leader, the follower and the situation. This conceptualization is the core of charismatic leadership theory and has been used as a stepping stone to new insights in charismatic leadership literature. A dynamic theory of charismatic leadership is yet to be developed and Issues such as socialized and personalized charismatic leaders, susceptible followers, Homogeneity of charisma, and its determinants and consequences await further theoretical and empirical developments.

## **Transformational Leadership**

The concept of transformational leadership was introduced by Bass (1985) and Burns (1978) as distinct from other categories of leadership in that transformational leaders empower their

followers and encourage them to do more than they originally expected to do. Transformational leaders were further described as having vision and as inspiring trust and respect in subordinates. For the past decade, this view of leadership has become a central notion in the study of leadership and has revitalized leadership research.

Bass (1985) examined transformational leadership through his development of the multi factor leadership questionnaire (MLQ). The first version of MLQ included three factors, whose joint influence was described as creating the special effect of the transformational leader: (1)Charisma; (2) individual consideration; (3) intellectual stimulation. In a later version of MLQ (Bass and Avolio), the factor of charisma was divided into two parts: idealized influence and inspirational motivation. The revised MLQ thus includes four factors of transformational leadership that measure the following influences of the transformational leader: (1) idealized influence is defined with respect to both the leader's behavior and the followers attributions about the leader. Idealized leaders consider the needs of the others before their own personal needs, avoid the use of power for personal gain, demonstrate high moral standards, and set the challenging goals for their followers. Jointly, these behaviors set the leaders as role models for their followers; (2) inspirational motivation refers to the ways by which transformational leaders motivate and inspire those around them, mostly by providing meaning and challenge. Specifically they do so by displaying enthusiasm and optimism, by involving the followers in envisioning attractive future status, by communicating high expectations, and by demonstrating commitment to the shared goals; (3) Individualized consideration represents the leader's consistent effort to treat each individual as a special person and to act as a mentor, who develops his or her followers' potential; (4) intellectual stimulation represents the leader's effort to stimulate the followers to be innovative and creative as well as the leader's effort to encourage followers to question assumptions and to reframe problems and approach them in new ways.

A large body of research portrays the transformational leader as different from, and sometimes obtaining superior outcomes to, those of other leadership styles. The focus of transformational leadership has created renewed interest in the question of what predisposes a person to become a transformational leader and has motivated a large number of personality configuration literature.



## **Leader-Member Exchange**

The leader-member exchange approach focuses solely on (leader-follower) relationships or linkages. This is an extremely important clarification of the LMX approach. The leader per se and the followers per se are not of interest in this approach. The primary variable (LMX) in Graen's and Uhl-Bien's papers is defined as involving mutual trust, reciprocal trust, and mutual obligation.

This new definition helps explain why the LMX approach, unlike the other approaches, is not specific about the appropriate level of analysis for followers or leaders.

An essential premise of leader-member exchange theory is that leaders and supervisors have limited amounts of personal, social, and organizational resources (e.g., time, energy, role, discretion and positional power) and thus distribute such resources among their subordinates selectively (e.g., Dansereau, Graen & Haga, 1975; Graen & Scandura, 1987; Graen & Uhl-Bien, 1995). Leaders do not interact with all subordinates equally, which, overtime results in the formation of LMXs that vary in quality. Interactions in higher-quality LMXs are characterized by increased levels of information exchange, mutual support, informal influence, trust, and greater negotiating latitude and input in decisions. Lower-quality LMXs are characterized by more formal supervision, less support, and less trust and attention from the leader.

The LMX theory has enhanced our understanding of the leadership communication process between superiors and subordinates. Some research explain how the quality of LMX affect subordinates' and superior's communication in areas such as discourse patterns, upward influence, communication expectations, cooperative communication, perceived organizational justice and decision-making practices (e.g., Fairhurst, 1993; Fairhurst & Chandler, 1989; Jablin, 1987; Krone, 1992; Lee, 1997,2001). LMX theory, however, is still open to questions and further developments.

### **1.2.3 Alternative Approaches**

The ideas of information processing, the substitutes for leadership approach and romance of leadership, are grouped separated from classic or contemporary views in the alternative category.

## Information processing

During the past four decades there has been a trend for management practitioners and I/O psychologists to apply information processing principles to develop theory and improve management or personnel practice. Much information processing research can be characterized in terms of one of four general models, which provide a guiding, often implicit framework for research. These models are labeled rational, limited capacity, expert, and cybernetic models of information processing and behavior. Although these models are not distinguished clearly, it is useful to view these models as different information processing perspective or metatheories.

Rational models assume that people thoroughly process all relevant information in order to maximize relevant outcome. Traditionally, rational models dominated management science and economic theory. The general evaluation of rational information processing models is that they are strong in two respects. They have stimulated very explicit theories in a wide range of areas, and their analytic basis and extensive use of information makes them prescriptively useful. The two main deficiencies are that they usually are not descriptively accurate, nor do they generate applications that people can apply easily. Instead, applications consistent with rational models often require explicit instruction and the use of formal procedures or informational aids such as computers. Thus, although rational models are appropriate in some situations and they can be followed by people, they do not provide a very general explanation of human behavior.

In contrast to rational models, limited capacity models focus on how people simplify information processing while still generating adequate but not optimal behaviors. These explanations of human behavior require only limited amounts and limited processing of information. Interest in these models stems from recognition of human information processing limitations. Many studies document that human choices and behaviors do not agree with predictions from rational models. One of the simplest and most influential limited capacity models is the satisfying model proposed by Simon (1955). Rather than assuming exhaustive processing, this model assumes that processing stops when the first acceptable alternative is identified. Management science theorists similarly have characterized organizations as adaptively or boundedly rational (Cyert & March, 1963; March & Simon, 1958). Limited capacity models do not require extensive knowledge or omniscience, as do rational models. Instead, people work within a very limited conceptualization of problems, considering only a few of all possible alternatives. Thus, limited

capacity models are more congruent with short-term memory capacities than rational models because they require the use of less information at one time and simpler evaluation procedures.

Limited capacity models emphasize the role of cognitive heuristics and simplifying knowledge structures in reducing information processing demands. Unlike the conscious use of algorithms characteristic of rational models, heuristics are used informally and often without awareness.

Limited capacity models apply to leadership behavior ratings (Mitchell, Larson & Green , 1977), Leadership perceptions (Lord, Foti & De Vader, 1984), categorization in performance appraisal (Feldman, 1981), biases in social perceptions (Nisbett & Ross, 1980), and strategic decision making (Brief & Downey, 1983; Dutton & Jackson, 1987). This evaluation of limited capacity models shows that they have almost opposite strengths and Weaknesses of rational models. Thus, limited capacity and rational models are often seen as contrasting perspectives. The strengths of the limited capacity models are that they describe typical information processing in many situations, provide a cognitively simple model for applications, and stimulate new theories in several areas. However, the perspective value is weak because the model recognizes that typical human information processing does not always lead to optimal decisions or solutions.

The recognition that expertise supplements simplified information processing defines a set of models, which are labeled expert information processing . The key assumption underlying these models is that people rely on already developed knowledge structures to supplement simplified means of processing information. However, these knowledge structures pertain only to a specific content domain. Thus, an expert is defined as someone with a large knowledge base in a particular context. For example, chess masters have approximately 50,000 chunks, or familiar chess patterns, in memory (Simon, 1987).

A growing body of literature indicates that experts and novices differ in the way that schema are structured. Several studies illustrate that experts and novices also differ in the way information is processed. Experts recall of information is less biased, and they focus more on inconsistencies in the stimulus material than novices (Fiske, Kinder, & Larter, 1983). Generalizing this model of the human information processor, this literature suggests that experts store and retrieve information from long term memory differently than novices (Glaser, 1982). Experts knowledge structures in long term memory are larger and more easily accessed from

short term memory. In this sense, extensive knowledge substitutes for limits processing capacity in short term memory. Put concretely, experts often recognize immediately what novices require great effort to discover. However, it should be stressed that experts are not superior information processors in a general sense; rather they perform better only within their specific domain of expertise.

One primary difference between experts and novices is the greater knowledge base that experts acquire through experience in a specific domain (Glaser, 1984). Abelson and Black (1986) suggested that individuals with experience in familiar contexts have different knowledge structures than those unfamiliar with the context. Therefore, they can apply different problem solving strategies. Experts recognize relevant categories more quickly than do novices, and these categories are linked more strongly to appropriate actions. Novices tend to organize knowledge around literal objects and surface features explicit in a given problem statement. Experts on the other hand, possess schema derived from knowledge of the subject matter. These schema are organized around principles that subsume the literal objects (Glaser, 1984, 1988). Moreover, when experts identify a principle, it is connected in memory to applications of the principle. Thus as Glaser (1988) noted, the organization of experts' knowledge structures efficiently translates problem information into problem solutions. In this sense, heuristic processing under expert information processing models is something to be developed, not overcome, as it is in limited capacity models.

Chi, Glaser, and Farr (1988) emphasized the interaction between knowledge structures and the processes of reasoning and problem solving. They explained that experts achieve a high level of competence because their organized body of conceptual and procedural knowledge can be accessed and integrated with superior monitoring and self regulatory skills.

Being relatively new, expert models have not generated extensive theory in the management area and the perspective value of such models is unexplored in the management literature. However, recent development in using expert systems suggest that it may be high. Moreover, work in artificial intelligence indicates that many problems, which can not be solved merely by extensive computer processing can be solved quickly by incorporating the content specific knowledge of experts. Work in artificial intelligence has emphasized knowledge based strategies for the last decade. The major draw back of these models is that becoming an expert often

takes years of intensive study or experience. Thus, the descriptive value of expert models are limited to career-oriented jobs, skilled trades, jobs with low turnover, or social situations that are very common. Similarly, applications based on expert models may be very difficult for non-expert to use due to their lack of necessary knowledge structures in long-term memory. It is not yet known whether the acquisition of knowledge structures can be accelerated by specific training techniques, but the issue is certainly important for the training area.

The final set of information processing models is more dynamic than the previous three. Hogarth (1981) criticized much of the judgement and decision-making literature for focusing on discrete events, particularly since much human judgment involves continuous adaptation to complex and changing environments. Similarly, Ostrom argued that action is integral to cognitive processes (1984, p. 26). Such criticism supports the use of cybernetic models, which represent information processing and actions as being interspersed over time.

Cybernetic information processing models are dynamic, whereas all three previous models are static. In cybernetic models, behavior, learning, and the nature of cognitive processes themselves may be altered by feedback. Also cybernetic models have future, present and retrospective orientations. The interpretation of past task or social information is intermixed over time with planning future activities and executing current behaviors. The prior models adopt a single time perspective, which may be either prospective or retrospective.

Like rational models, cybernetic models may be optimized in the long run, but they do this by learning and adaptation, rather than by sophisticated processing before choice or behavior. Further, as noted by Hogarth, many of the heuristic processes, which seem suboptimal for discrete choices may be functional in a continuous environment. Hogarth (1981) argued that a dynamic perspective provides an alternative definition of rationality based on an evolutionary adaptation rather than the stable optimization procedures of rational economic models discussed earlier.

Kleinmuntz and Thomas (1987) compared action oriented (cybernetic) to judgment oriented (rational) decision strategies in a dynamic medical decision making task. They found that most subjects used a judgement oriented strategy, which was enhanced by the provision of a rational decision making aid. However, they also found that under conditions of low risk, the best judgement oriented subjects barely reached the performance levels of a random, action oriented

benchmark procedure. Thus, decision procedures, which seem inefficient in static environments may be very effective when choices can be repeated.

Another advantage of cybernetic models is that they are as applicable to learning as they are to the generation of behavior. Cybernetic models, which can be traced to the early work of Ashby (1956) and Wiener (1954), have been applied in several areas. Cybernetic information processing characterizes models of problem recognition in which information about performance levels is compared to some relevant standard as a means to identify problems. Cybernetic models also apply to social perceptions in general (Hastie & Park, 1986), leadership perceptions (Lord & Mahr, 1989b), and performance appraisal (DeNisi & Williams, 1988). Such models posit that social perceptions are periodically reformulated by updating past perceptions with current behavioral information. When such updating occurs, general impressions are stored in long-term memory, but specific behavioral information is lost. Finally, cybernetic information processing is reflected in control theory models of behavior (Carver & Scheier, 1982; Powers, 1973) and motivation (Lord & Hanges, 1987).

The current evaluation of the cybernetic information processing models places them high in terms of stimulating theory and in terms of both descriptive and prescriptive value. However, it should be remembered that cybernetic models can not be used effectively when feedback is slow or when courses of actions are costly to reverse. In terms of generating applications that fit typical information processing, such applications require accurate, fast, and frequent feedback. If these conditions are present, people should find applications based on cybernetic models easy to use.

### **Substitutes for Leadership**

The idea of substitutes for leadership was formulated most explicitly by Kerr and Jermier (1978) to draw attention to the possibility that many of the situational factors, which moderate leadership-outcome relationships do so by tending to negate the leader's ability to either improve or impair subordinate satisfaction or performance. This can occur because particular situational factors neutralize the effects of leadership style. Kerr and Jermier distinguish between two general types of leader behavioral: Relationship-oriented/supportive/people-centered leadership and task oriented/instrumental/job-centered leadership.

According to Kerr and Jermier, substitutes for leadership can best be grouped into three headings: subordinate, task, and organizational characteristics. They argue that different substitutes tend to neutralize the effects of different leadership styles. They report a study of city police in which the relative effects of the substitutes and both instrumental and supportive leader behavior were examined to discern their relative impact on police officers' organizational commitment and role ambiguity. They found that the impact of leader behavior on these two outcome variables was small. They also find that few of the substitutes had a strong effect on the outcomes, which renders an interpretation of the lack of influence of the leader behaviors somewhat indeterminate.

A very similar study was conducted on hospital employees by Howell and Dorfman (1981) using the same leader behavior and substitutes scales. The two outcome variables investigated were organizational commitment and job satisfaction. They find out that a leader's supportive behavior will make a difference to organizational commitment and job satisfaction and substitutes do not negate such leader behavior.

Finally, Podsakoff et al. (1984) also investigated the impact of the substitutes for leadership scales on the effect of different reward and punishment behaviors. In this investigation, very little evidence was found for the substitutes for leadership approach.

Kerr and Jermier's approach is interesting but it seems that the first tranche of results is not too encouraging. It may be that a concentration of a wider range of leader behaviors and substitutes in relation to each other will bear fruit.

### **Romance of leadership**

The romance of leadership notion (introduced by Meindle, Ehrlich & Dukerich, 1985) refers to the prominence of leaders and leadership in the way organizational actors and observers address organizational issues and problems, revealing a potential bias or false assumption-making regarding the relative importance of leadership factors to the functioning of groups and organizations.

The romance of leadership emphasizes leadership as a social construction. Attention is focused on development of theory and hypothesis regarding the features, outcomes, and implications of the social construction process, as it occurs among followers and as it is affected

by the contexts in which they are embedded. It seeks to understand the existence of general and more situation-specific concepts of leaders and how they are conceptualized and otherwise constructed by actors and observers.

Although there are currently many available perspectives that highlight the thoughts and phenomenology of the leader, the romance of the leadership is about the thoughts of followers: how leaders are constructed and represented in their thought systems. The romance of leadership perspective focuses on the linkage between leaders and followers as constructed in the minds of followers. Rather than assuming leaders and followers are linked in a substantially casual way, it assumes that the relationship between leaders and followers is primarily a constructed one, heavily influenced by interfollower factors and relationships. The behavioral linkages between the leader and follower are seen as a derivative of the constructions made by followers. The behavior of followers is assumed to be much less under the control and influence of forces that govern the social construction process itself.

One aspect of a leader-centric perspective is a focus on the persona of the leader. The romance of leadership perspective moves a researcher away from the personality of the leader as a significant substantive and casual force on the thoughts and actions of followers. It instead places more weight on the images of leaders that followers construct for one another. It assumes that followers react to, and are more influenced by, their constructions of the leader's personality than they are by the true personality of the leader. It is the personalities of leaders as imagined or constructed by followers that become the object of study, not actual or clinical personalities per se.

Similarly, this approach does not explain or deal with the behavior of the leader and the direct impact of that behavior on followers. In other words, direct effects of the actions and activities of the leader, independent of and unmoderated or unmediated by social construction processes, are not addressed. Thus, leadership is assumed to be revealed not in the actions or exertions of the leader but as part of the way actors experience organizational processes. In essence, leadership is very much in the eyes of the beholder: followers not the leader define it. From this perspective, the idea that leadership can not and does not occur without followers is taken literally to be true.

The leader centric perspective favors the rather direct control of followers by engaging in



so called leadership behaviors. The present approach would emphasize more indirect and less tightly controlled effects on followers. Manipulations of contexts and constructions, rather than of leader behaviors, would in a sense, constitute the practice of leadership. Rather than searching for the right personality, one would search for the opportunity to create the right impression. Reputations would be more significant than actions. Rather than being concerned about engaging in the right practices, one would be concerned about creating the right spin. The creation and sustenance of the interpretive dominance regarding leadership would have the highest priority.

Those who have aspirations for an objective theory of leadership will find great difficulty with the inherently subjectivistic, social constructionist view being advanced here. A subjective definition and a social construction view of leadership does not imply that it can not be studied through normal scientific processes of inquiry. Meindle shows it is possible to use the romance of leadership notion, with its constructionistic, followership-centric bent to formulate testable hypotheses. There are likely many existing possibilities for research at all levels of analysis, the only real limitation being the creativity and the interest of the researcher.

#### **1.2.4 The New Wave Approaches**

Self leadership, Multiple-Linkage Model, Multi level theory and Individualized leadership can be categorized in the new wave approaches in leadership literature.

##### **Self Leadership**

Self leadership literature pays attention to a neglected aspect of organizational behavior- the influence organization members exert over themselves. This managerial focus has emerged primarily from the social learning theory literature (Bandura, 1969; Cautela, 1969; Goldfried and Merbaum, 1973; Kanfer, 1970; Manhoney and Arnkoff, 1978, 1979; Manhoney & Thoresen, 1974). In the organization literature, this process generally has been referred to as self management (Andrasik & Heimberg, 1982; Luthans & Davis, 1979; Manz and Sims, 1980; Marx, 1982; Mills, 1983).

Organizations impose multiple controls of varying character on employees. One view suggests that the control process involves applying rational, manageable, central mechanisms to

influence employees through external means to assure that the organization achieves its goals.

An alternative view, however, shifts the perspective of the control system-controllee interface significantly. Self leadership perspective, views each person as possessing an internal self control system (Manz, 1979). Organizational control systems in their most basic form provide performance standards, evaluation mechanisms, and systems of reward and punishment. Similarly, individuals possess self generated personal standards, engage in self evaluation processes and self administrative rewards and punishments in managing their daily activities (Bandura, 1977a, Mahoney and Thoresen, 1974; Manz and Sims, 1980). Even though these mechanisms take place frequently in an almost automatic manner, this makes them no less powerful.

Further more, while organizations provide employees with certain values and beliefs packaged into cultures, corporate visions and so forth, people too possess their own systems of values, beliefs, and visions for their future. In addition, the counterparts of organization rules, policies, and operating procedures are represented internally in the form of behavioral and psychological scripts or programs held at various levels of abstraction.

The point is organizations provide organizational control systems that influence people but these systems do not access individual actions directly. Rather, the impact of organizational control mechanisms is determined by the way they influence, in intended as well as unintended ways, the self control systems within organization members.

While this perspective is not new, an analysis of theory and research in the field reveals that it has not been well integrated into organizational management. The literature does include cognitive mediation of external stimuli (e.g. social learning theory views- Davis & Luthans, 1980; Manz & Sims, 1980) and attributed causes to observed physical actions (Feldman, 1981; Green & Mitchell, 1979; Mitchell & Larson & Green, 1977; Staw, 1975) but does not adequately recognize the self-influence system as a focal point for enhanced understanding and practice of organizational management. This perspective, on the other hand, suggests that the self influence system is the ultimate system of control. In addition, it suggests that this internal control system must receive significant attention in its own right before maximum benefits for the organization and employee are realized.

Recent work on cybernetic (control) theory provides a useful perspective for making concrete the nature of employee self-regulating systems (Carver & Scheier, 1981, 1982). Carver and

Scheier (1981) present an insightful view of the self regulating process involving : (a) input perceptions of existing conditions, (b) comparison of the perception with an existing reference value, (c) output behaviors to reduce discrepancies from the standard, and (d) a consequent impact on the environment.

From this view, an employee attempting to achieve a given production standard would operate within a closed loop of control aimed at minimizing deviations from standards in existing performance. Unless an environmental disturbance of some kind occurred, this self regulating process theoretically could occur indefinitely.

Carver and Scheier (1981, 1982) further speculated, based on the work of powers (1973a, 1973b), that standards emerge from a hierarchical organization of control systems. That is, standards for a particular control system loop derive from superordinate systems of control. Thus, an employee working to achieve a minimum deviation from a production standard at one level may serve higher level systems aimed at higher level standards.

From an organizational perspective, recognizing and facilitating employee self regulating systems pose a viable and more realistic view of control than views centered entirely on external influence. In addition, overreliance on external controls can lead to number of dysfunctional employee behaviors: Rigid bureaucratic behavior, inputting of invalid information into management information systems and so forth.

More relevant treatments of self management to date focused on strategies designed to facilitate behaviors targeted for change (Andrasik & Heimberg, 1982; Luthans & Davis, 1979; Manz & Sims, 1980). This work generally reflects the view that behaviors are not performed for their intrinsic value but because of their necessity or because of what the performer will receive for his performance. A widely recognized definition of self-control, one that illustrates this view, is: A person displays self control when in the relative absence of immediate external constraints he engages in behavior, whose previous probability has been less than that of alternatively available behaviors (a less attractive behavior but one that is implied to be more desirable).

Several specific self management strategies can be identified. Mahoney and Arnkoff (1978, 1979) provided a useful array of strategies that were applied in clinical contexts. These include, self observation, self goal setting, cueing strategies, self reinforcement, self punishment, and rehearsal. Much of the employee self management literature has centered on adaptations of

these self control strategies for addressing management problems (Andrasik & Heimberg, 1982; Luthans & Davis, 1979; Manz & Sims, 1980, 1981).

Luthans and Davis, (1979) provided descriptions of cueing strategy interventions across a variety of work contexts. Physical cues such as a wall graph to chart progress on target behaviors and a magnetic message board were used to self induce desired behavioral change in specific cases. Manz and Sims (1980) explicated the relevance of the broader range of self control strategies, especially as substitutes for formal organizational leadership. Self observation, cueing strategies, self goal setting, self reward, self punishment, and the rehearsal were each discussed in terms of their applicability to organizational contexts. Andrasik and Heimberg (1982) developed a behavioral self management program for individualized self modification of targeted work behaviors. Their approach involved pinpointing a specific behavior for change, observing the behavior over time, developing a behavioral change plan involving self reward or some other self influence strategy, and adjusting the plan based on self awareness of a need for change.

In terms of cybernetic self regulating systems (Carver & Scheier, 1981, 1982), these employee self management perspectives can be viewed as providing a set of strategies that facilitate behaviors that serve to reduce deviations from higher level reference values that the employee may or may not have helped establish. That is, the governing standards at higher levels of abstraction can remain largely externally defined even though lower level standards to reach the goals may be personally created. Mills (1983) argued that factors such as the normative system and professional norms can exercise just as much control over the individual as a mechanistic situation in which the performance process is manipulated directly. This view is consistent with arguments that employee self control is perhaps more an illusion than a reality (Dunbar, 1981) and that self managed individuals are far from loosely supervised or controlled (Mills, 1983). In addition, it has been argued that self management strategies themselves are behaviors that require reinforcement in order to be maintained (Kerr & Slocum, 1981; Manz & Sims, 1980; Thoresen & Mahoney, 1974). Because of this dependence on external reinforcement, it could be argued that the self management approach violates Thoresen and Mahoney's (1974) definition cited earlier, in the long run. That is, while immediate external constraints or supports may not be required, longer-term reinforcement is.

The considerable attention devoted to individual self influence processes in organizations

has been focusing primarily on self management that facilitates behaviors that are not naturally motivating and that meet externally anchored standards. Manz (1986), proposed a more comprehensive approach to more fully address the higher-level standards/ reasons that employee self influence is performed and to suggest self influence strategies that allow the intrinsic value of work to help enhance individual performance.

Further research and theoretical development is needed to address several central elements of self influence- for example, the derivation of personal standards at multiple hierarchical levels, human thought patterns, self influence strategies that build motivation into target behaviors- that have been neglected in the employee self management literature.

### **Multiple Linkage Model**

Most leadership theories deal with a few selected behaviors rather than a wide range of leadership behaviors. An exception is Yukl's (1989, 1994) multiple linkage model, which identifies categories of leadership behavior that are relevant for most types of managerial positions. The multiple linkage model builds on earlier theories of effective leadership behavior and theories of effective groups. According to the model, the effects of leader behavior on work unit performance are mediated by individual level intervening variables (subordinate effort, role clarity, and ability) and by group level intervening variables (work organization, teamwork, resources for doing the work and external coordination). Some of the leadership behaviors (clarifying, delegating, developing, recognizing, supporting) are used primarily to influence the individual level intervening variables. Other leadership behaviors (planning, problem solving, monitoring, team building) are used primarily to improve group-level intervening variables. However, there is no simple one to one correspondence between leader behaviors and the intervening variables. Rather, effective leadership depends on the overall pattern of leader behavior and its relevance to the situation. Situational variables (the nature of the task, the characteristics of subordinates, and the external environment) influence the intervening variables and determine which leadership behaviors are relevant to a particular manager. The multiple linkage model and most prior research on leadership behavior deal with leadership effectiveness rather than advancement.

There has been little empirical research to test the model. Although hundreds of studies

have been conducted in the past four decades to investigate the behavior associated with effective leadership, most of this research has examined broad categories of task oriented behavior that are difficult to relate to the demands and challenges faced by managers in different situations. The number of studies on specific behaviors is still small, and different researchers have examined different subsets of behaviors, making it difficult to compare results across studies.

### **Multi-level Theory**

Some years ago, Dublin (1979) contrasted the terms leadership of organizations and leadership in organizations in an insightful decision. Leadership of organizations is similar to what some now term strategic leadership, it involves human actors in interaction with the organization as an entity. Leadership in organizations involves the kind of lower organizational level, face to face interactions that comprise more than 90 percent of the current leadership literature (Hunt, 1991; Phillips & Hunt, 1992).

Hunt's model, which examines leadership up and down the organizational hierarchy, involves the complex relationships of both kinds of leadership operating together. Hunt's (1991) model argues, first, that there are critical tasks that must be performed by leaders if an organization is to perform effectively. Because of an assumed increasingly complex setting as one moves higher in an organization, these critical tasks become increasingly complex and qualitatively different. The extended model assumes that the critical tasks can be divided by organizational levels within three domains. The bottom domain is labeled direct or production, The middle domain is called organizational, and the top domain is labeled systems or strategic.

The number of levels encompassing critical tasks within the domains is argued to vary as a function of the organization's size, the time span, and the requirement that each level add value to both its higher and its next lower level. The model argues that generally, even for the largest organizations, the number of levels probably should not exceed seven, from the employee level to the very top. For a large complex organization, then, there would be: (1) an employee and two leadership levels in the production domain; (2) two leadership levels in the organizational domain; and (3) two leadership levels in the strategic domain.

Hunt's extended model also assumes that accompanying the increasing task complexity by organizational level, there must be an increasing level of the leader cognitive capacity. Con-

sistent with requisite variety notions, there should be a rough match between leader cognitive capacity and critical task complexity at each organizational level. The extended level also assumes that there is an accompanying leader behavioral complexity notion comprised of leader behaviors or skills. Hunt's model also includes leader background, predisposition, value preferences, organizational culture or subculture, and various aspects of organizational and subunit effectiveness. Finally, the organization is considered to be embedded within external environment and societal culture aspects.

### **Individualized leadership**

The individualized leadership is introduced in a paper by Dansereau et al. This approach views people as forming relationships with one individual totally independent of the relationships they form with other individuals. There need to be no consistency on the part of an individual in forming relationships with multiple individuals. That is, an individual may treat a group of people the same way or all differently; it depends on how he or she views the other individuals. According to this view, formal as well as informal relationships between a focal individual or a superior and other individual (e.g. a subordinate) tell us nothing about that focal individual's relationship with any other individual.

In this new approach to leadership, leaders first provide support for the sense of self-worth of followers as unique individuals, who are independent of other individuals they interact. Second, in exchange, followers then perform in ways that satisfy the leader. Third, as a result, leaders and followers link in dyads, where there is consistency and agreement, yet differences between, these independent dyads.

This theory is tested in a number of studies, and nearly identical effects were found in all studies. This approach would be enhanced by considering some of the features of other new wave approaches. An increase in the number of variables of interest in future research seems appropriate.

### 1.3 Leadership in Economics

The concept of leadership has not received as much attention in economics as it has in business and management. There are some interesting economic contributions to the leadership theory literature, however, which analyze leadership as a concept distinguished from formal authority, namely Kreps (1990a), Rotemberg and Saloner (1993), Hermalin (1998), Vesterlund (2000), and Andreoni (2004). Kreps, Rotemberg and Saloner begin from an incomplete contract setting, Hermalin takes his starting point in the Holmstrom (1982) complete contracts team model and Vesterlund and Andreoni address leadership in a charitable fund-raising context.

Strictly speaking Kreps's paper is not about leadership per se, but rather about corporate culture and how reputation that a corporate culture may help build provides an important part of the explanation of firms' organization. Nevertheless, it certainly makes provision for leadership. Kreps's paper is an explorative discussion aimed at convincing organizational economists of the possibility of alternative routes of research. Starting from property rights/incomplete contracts theory (Grossman and Hart, 1986; Hart, 1995), Kreps argues that incompleteness of contracts may produce a need for implicit contracts. However, in the face of unforeseen contingencies it is not clear how implicit contracts should be administered; in particular it is not clear how well standard reputation arguments work with unforeseen contingencies. The possible role of leadership in this setting is to provide general principles that instruct employees and suppliers about how unforeseen contingencies will be handled in the future by management.

Another notable leadership theory is Julio J. Rotemberg and Garth Saloner (1993), which studies the question of leadership styles. Rotemberg and Saloner are more taken up with how leadership styles are influenced by environmental contingencies. However, the same basic insights as in Kreps, namely that the provision of incentives is not straight forward under incomplete contracting, plays a key role in their paper. Rotemberg and Saloner provide an economic model in which leadership style has an important effect on firms' profitability. They show that senior management's style can alter the incentives that can be provided for subordinates to ferret out profitable opportunities for the firm. Leadership style is modeled as the degree to which the leader empathizes with followers (formally, the weight the leader's utility function assigns to the followers' utility).

Rotemberg and Saloner argue that the personality of the leader affects both the management



style and the ease with which this incentive problem is overcome. Specifically, they study the relationship between a firm's environment and its optimal leadership style inside a setting, where contracts between the firm and subordinates are incomplete so that providing incentives to subordinates is not straightforward. Leadership style, whether based on organizational culture or the personality of the leader, then affects the incentive contracts that can be offered to subordinates.

Rotemberg and Saloner show that in an incomplete contracting environment, empathy can serve as a commitment device and, therefore, be valuable. They argue that leaders who empathize with their employees adopt a participatory style and that the shareholders gain from appointing such leaders when the firm has the potential for exploiting numerous innovative ideas. By contrast, when the environment is poor in new ideas, shareholders benefit from hiring a more selfish (i.e., more profit maximizing) leader whose style is more autocratic.

Hermalin offers a finely honed theory about leadership behavior. He defines a leader as someone with followers and argues that following is inherently a voluntary activity and, therefore, a central question in understanding leadership is how does a leader induce others to follow him. As an economist he presumes that followers follow because it is in their interest to do so. Hermalin argues that followers believe that leader has better information about what they should do than they have. He believes that leadership is about transmitting information to the followers and convincing them that he is transmitting the correct information.

Hermalin suggests two ways in which a leader can convince his followers to put in more effort in the organizational activities. One is leader's sacrifice: The leader offers gifts to the followers. The followers respond not because they want the gifts themselves, but because the leader's sacrifice convinces them that she must truly consider this to be a worthwhile activity. The other way to convince the followers in Hermalin's opinion is leading by example: The leader himself puts in long hours on the activity, thereby convincing followers that she indeed considers it worthwhile.

He studies incentive problems in the context of the team model of Holmstrom (1982). He argues that Bengt Holmstrom's team model is well suited for studying leadership. First, because the leader shares in the team's output, she has an incentive to exaggerate the value of effort devoted to the common activity. Second, because the information structure limits the leader's

ability to coerce followers, she must induce their voluntary compliance with his wishes. As Holmstrom showed, since each person gets only a fraction of the overall return to his effort, he expends less than the first-best level of effort on the common endeavor. In other words, he fails to internalize the positive externality his effort has for the firm. This team problem is thus simply an example for free riding problem endemic to the allocation of public goods.

Hermalin assumes that only one leader has information about the return to effort allocated to the common endeavor. Given asymmetric information he considers two possible ways that the leader can credibly communicate all of his information. First he considers a mechanism design setting. Hermalin shows that a mechanism that makes side payments among team members a function of the leader's announcement about his information can duplicate the symmetric-information second-best outcome. In the second setting Hermalin allows the leader to lead by example, that is, he expends effort before the other workers. Based on the leader's effort, the other workers form beliefs about the leader's information. He shows that leading by example yields an outcome that is superior to the symmetric-information outcome. The reason for this conclusion is that the hidden information problem counteracts the team problem: The need to convince the other workers increases the leader's incentives, so he works harder.

Hermalin, then proceeds to derive what the optimal contract (shares) should be when the leader leads by example. He finds that in a small team, he has the smallest share, but in a large team he has the largest share. Hermalin argues that under certain conditions, leading by example dominates symmetric information even when attention is restricted to equal shares.

Hermalin focuses on what the leader does to induce a following. He does not consider the questions of how the leader is chosen, why people want to be leaders or who is the best choice for the leadership position. Hermalin's theory also falls into the contract economics approach as of Kreps's, Rotemberg's and Saloner's works.

Vesterlund's model investigates sequential fund-raising and the role of the fund-raiser in fund-raising games under incomplete information. The hypothesis of her paper is that when the information about the quality of the charity is unknown, an announcement strategy for a high type charity is successful because it helps reveal the information about the quality of the charity. Vesterlund's paper assumes that the first contributor obtains costly information about the charity's quality. She argues that when the information cost is sufficiently low, a first mover

who is informed with a high signal fully reveals the value of the charity through a large initial contribution. This increase in contribution decreases the donation of the second contributor but the overall contribution level exceeds the complete information scenario.

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Andreoni develops a model of leadership giving in charitable fund-raising. His aim is to provide a positive economic model for the observation that charitable fund-raisers often rely on leadership givers, who are typically wealthy individuals who give exceptionally large gifts to the charity. He argues that gifts can be a signal of the quality of the charitable good. Since the person sending the signal would rather all followers think the quality is high, the leader must give an exceptionally large gift for the signal of quality to be credible. He argues that the game of providing the signal then reduces to a familiar war of attrition game where the person with the lowest cost-benefit ratio is the one who provides the good immediately who is the wealthiest person assuming identical preferences.

The above modelling efforts are neat, logical and produce interesting and sometimes counter-intuitive conclusions. The basic thrust of this literature is to conceptualize virtually any issue related to the economics of organizations in terms of solving incentive conflicts. Thus the essence of the above contributions is that leaders exist because they resolve incentive conflicts, albeit sophisticated and non-standard ones.

The role of leadership, however, is not limited to resolving incentive conflicts. The view that all, or most, organizational phenomena are reducible to problems of aligning incentives is one that is implicitly contradicted by contributions to organization studies (Thompson, 1967), The executive and leadership literature (Barnard, 1948; Carlsson, 1951; Selznick 1957; Kotter,

1996), political science (Calvert,1992,1995), sociology (coleman, 1990) and in some quarters of economics of organization (Milgrom and Roberts, 1992, Ch. 4; Camerer and Knez, 1994, 1996, 1997; Langlois, 1998; Weber, 1998; Langlois and Foss, 1999). Many of these contributions are directly related to the issue of leadership.

For example, Coleman (1990) observes that charismatic authority may be a response to coordination problems that do not necessarily turn on misaligned incentives. Camerer and Knez (1994, 1996, 1997) and Calvert (1992, 1995) argue that attention should be shifted to coordination games (rather than cooperation games) in seeking a foundation for the understanding of organizational phenomena.

Nicolai J. Foss (2000) provides an explorative discussion somewhat in the style of Kreps, aimed to emphasize the importance of leadership as a way of solving the coordination problems. He argues that all the emphasis has been on cooperation games, i.e. games where the payoff space of the game is such that the efficient outcomes are not supportable as equilibria at least in one-shot play. The key problem that such a game leads one to ponder is how to avoid the Pareto-inferior outcome. Indeed the basic hold-up situation has a prisoners' dilemma structure and this is also the case with the team production problem and other problems with information externalities and moral hazard.

Foss criticizes the lack of interest in the interaction problems that may be represented by coordination games, which is in his opinion a somewhat surprising neglect given the increasing emphasis on such problems in other areas of economics, such as standards, conventions, learning behavior and macroeconomics.

He mostly focuses on shared interest coordination games distinct from coordination games with mixed interests<sup>1</sup>. Foss argues that classical game theory solves the coordination problem by defining it away, that is, by assuming that agents by means of pure ratiocination can reason their way to equilibrium. Moreover, sometimes in classical game theory literature it is argued that suppressing coordination problems is justified because it allows concentration on essentials. Foss, argues that coordination games are non-trivial and quite fundamental and claims that leadership may, in certain situations, be a low cost device to solve the problem of coordination.

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<sup>1</sup>In the shared interest coordination games players' preferences over equilibria coincide. In contrast, coordination games with mixed interests will also exhibit multiple equilibria, but these equilibria are ranked differently by the players.

Foss defines leadership as the taking of actions that coordinate the complimentary actions of many people through the creation of belief conditions that substitute for common knowledge and where these actions characteristically consist of some act of communication directed at those being led. He distinguishes between coordination problems in which common knowledge about payoffs and strategies obtains initially and those in which it does not. He also makes a distinction between games where agents can communicate by exchanging cheap talk at no or low cost and those in which they can't (or communication is very costly). However, the leader will be privileged by being the only player who can always communicate if he so chooses. Foss introduces four cases following this distinction, in which leadership plays different roles.

Case 1) This represents the case where leadership is least likely to play a role, since knowledge is common and agents may communicate at low or no cost.

Case 2) This represents the case, where agents can not engage in communication but the common knowledge assumption is maintained. In this case, based on substantial empirical evidence players may not choose the efficient equilibrium. In this situation the leader may, by playing the efficient equilibrium and making this common knowledge, induce the other players to play the efficient equilibrium.

Case 3) This refers to the situation, where knowledge is not held in common but agents may communicate at no or low cost. In this case if communication costs are zero, one could expect common knowledge conditions to be established instantaneously and coordination follow in the same split second. There may be a role for leadership if communication costs are positive.

Case 4) This represents the situation where knowledge is not held in common and agents can not communicate. This case is the most realistic of the four cases. In this situation players have incomplete information (or non at all) about other players, available strategies, previous plays, etc. and games have to be redefined and played anew. In this situation there is unlikely to be an exact correspondence between players, strategies and outcomes of the game. There will be likely to be multiple equilibria. Foss argues that in such a situation leadership may be conceptualized as picking one equilibrium out of a multiplicity, for establishing belief conditions that approximate common knowledge.

Foss states that leadership may be thought of in terms of remedying: (i) problems of coordinating on an equilibrium when agents are initially outside the equilibrium; (ii) problems of

coordinating on one equilibrium out of a multitude; (iii) problems of moving from an inferior equilibrium to the efficient equilibrium by influencing the beliefs that agents hold.

Leadership as an institutional solution for coordination problems, has also been investigated in experimental economics literature. Wilson and Rhodes (1997) begin their research with one component of leadership- its coordinating role- and disentangle how leadership matters for followers. They proceed their analysis as a simple one-sided signalling game from leaders to followers and investigate when a leader's signals are credible. The empirical analysis is based on a series of laboratories experiments in which groups of four actors were involved in a series of one stage coordination games. They show that although leadership is crucial for coordinating followers, the introduction of uncertainty about the type of leader markedly decreases the ameliorating impact of leadership.

Wilson and Rhodes, concentrate on three types of n-player coordination games. The first is a pure coordination game with no leader. The second is a pure coordination game with a leader, who produces cheap talk signals for followers. The third type is a coordination game with an uncertain type leader. Their game has a symmetric payoff structure and therefore there is no Pareto ranking over the Nash equilibria. In the absence of a Pareto superior alternative, each of the equilibria is equivalent. The problem for players is to coordinate on a single choice, a task that is not as simple as it seems.

They show that in a pure coordination game with no leader, subjects fail to coordinate as predicted. In their second setup, full coordination is not automatic. The existence of a credible leader, however, considerably increases the overall coordination rates. In the third stage, Wilson and Rhodes introduce uncertainty about the types of the leaders. In other words they induce some uncertainty about the commonality of interests between leader and follower. From the followers standpoint a good leader's incentives are aligned with theirs. A bad leader's interests, however, diverge from those of the followers. A good leader has strong incentives to send a coordinating signal to the followers. On the other hand, a bad leader always has an incentive to send misleading signals to his followers. In this setup the leaders are informed about their type but followers only have probabilistic information about which kind of leader they are facing. The results show that in non of the trials under this condition did followers fully coordinate. They also show that in assessing leader's suggestion, subjects are sensitive

not only to the presence of uncertainty but to the degree of that uncertainty. Therefore they conclude that leadership is clearly important for resolving coordination problems. Leadership can serve as a focal point, helping followers choose one equilibrium from among several. They argue, however, that the only presence of leadership is no guarantee that coordination problems will be solved. If followers are uncertain about their leader's incentives, then they can easily ignore leadership.

Another interesting experimental economics contribution to the leadership literature is Vesterlund, Potters, and Sefton (2004). They examine endogenous sequencing in voluntary contribution games when some donors do not know the true value of the good. They not only study whether an informed leader can use her contribution to convince others of the quality of the public good, but also whether the sequential ordering may arise endogenously. Their experimental study shows that the vast majority of subjects prefer that the informed contributor gives first and the contribution of the informed donor be revealed to the uninformed. They find out that the resulting contributions and earnings in these endogenously generated sequential games are much larger than those found when subjects make donations simultaneously. When the informed player's donation is announced, the uninformed player mimics her behavior and the informed player correctly anticipates this response. They also compare the endogenous determination of contribution orderings to the case where the ordering is set exogenously. This result suggests that the gain from announcements is smaller when the sequence of play is determined by an outside party.

This dissertation is different from the current leadership literature in economics in several ways. First, unlike Kreps, Rotemberg, Saloner and Hermalin my model does not fall into the contract design literature<sup>2</sup>. My purpose is not to disregard other important aspects of organizations but to show that it is possible to address important questions of organizational design while suppressing all issues of monitoring, bargaining and contracts. I also argue that leadership is not necessarily a very complicated concept. Leaders sometimes play an important role in achieving cooperation and coordination even if they have no special talent. In my model the leader is simply an average player with exclusive access to information. I model an environment in which an organization is held together simply by the advantages of controlling

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<sup>2</sup>There is no question that payoffs may be affected by contracts, but this issue remains in the background.

access to information.

Second, In my model as of Hermalin's and Vesterlund's, a leader is simply an average player who has exclusive information about the value of the project undertaken by the organization but unlike their model the leader's participation in the project only partially transfers his information to his subordinates. Therefore, in my model the leader increases efficiency not by revealing all of his information but by sending a vague signal which partially reveals the value of the project. In my model, the first best is achieved not only because the leader works harder to set an example for the others, but also because followers do not know exactly when their participation yields high personal payoffs. Therefore, the employees participate to socially efficient projects which they refuse to participate in if the value of the project is fully revealed<sup>3</sup>.

Third, while the current literature on leadership addresses incentive conflicts and coordination problems separately, I define leadership as an institutional solution to both cooperation and coordination problems. That is, in this model a leader not only eliminates incentive conflicts but also coordinates the group members at the same time.

The fourth contribution of this paper is to address the power of leadership theoretically. I argue that leadership is not just about information transmission. A leader should convince his followers that he is transmitting correct information. A leader who is unable to convince his followers is not able to induce a following and is considered to be a powerless leader. I show that appointing multiple leaders improves the power of leadership. An organization with multiple informed leaders can not achieve the first best but is more efficient than an organization in which the value of the project is common knowledge. I also specify the optimal number of leaders that maximizes efficiency.

This dissertation also addresses the issue of leadership selection by considering a heterogeneous structure in which the employees are differentiated by their cost of participation. I show that it is never optimal to promote the most cooperative player to the leadership position. To maximize efficiency, the group should often choose an average player. If the average player is not powerful enough to induce a following it improves efficiency to choose the leaders from among less cooperative players.

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<sup>3</sup>The issue of private and social value of information and its role in designing efficient social/economic policies and structures have also been addressed in information economics literature (Hirshleifer 1971, Stephen 2002, and Arthur 1992)



The next chapter introduces the first four models and presents the results in more detail.

## Chapter 2

# An Economic Theory of Leadership Based on Assignment of Information

In this chapter, I develop a minimalist theory of leadership in which leadership is formed only based on assignment of information. This chapter introduces four models representing four different organizational structures. First I explain the basic set up which is common in all four models. Then as a benchmark, I consider a homogeneous organization in which all the members are equally informed about the value of the project undertaken by the organization. The second model represents a different organizational structure in which only one player is informed about the value of the project and is appointed to be the only leader of the organization. The rest of the players are uninformed followers. In the third model, I consider an organization with multiple informed leaders who lead symmetric groups of followers. The fourth model considers a single leader organization but relaxes the assumption of homogeneous population.

As mentioned before, this section investigates how leadership which is formed based on information improves participation and efficiency within an organization. This analysis can be applied to a setting where the adoption of new methods can increase a firm's profitability. Adoption of new methods may be everything from simple changes in the production process to the introduction of completely new products. All of these require that employees first think about ways to change the firm's operations and later cooperate to implement the change. Most large scale changes in a firm go through several stages of this kind.

From the firm's point of view the generation of proposals is extremely valuable and needs to be encouraged. One can consider a setting, where employees are supposed to start investment projects that might lead to a change. It is not possible, however, to ensure that employees work hard at generating viable proposals. This is an appropriate assumption whenever the activity of proposal generation can not itself be structured and monitored. We can argue that the outside appearance of a proposal need not bear a close relationship to the amount of effort that went into developing it because it is hard to monitor and measure intellectual activity. This problem arises not only in the research laboratory but also where a large number of employees is expected to develop ideas for continuous improvement. In conclusion when a firm wants to generate proposals for change they confront a difficult incentive problem.

Other potential applications include cooperation to win a contest or contract or political campaigns.

The following section introduces the basic setup of our model.

## 2.1 The Basic Setup

In this section I construct the basic setup, which is common in all four models. I consider  $m$  identical players,  $i \in I = \{1, 2, \dots, m\}$ , who constitute an organization or otherwise benefit from collective action. Each player makes a discrete decision whether to join a project undertaken by the organization. Therefore, each player's action set is  $A = \{P, N\}$ , where,  $P$  denotes participation and  $N$  stands for not participating in the project.

The quality index of the project is the random variable  $x$ , which is distributed on the interval  $[0, \bar{x}]$  with the density function  $\phi(x)$ . Participating in the project is costly and the cost of participation is fixed at  $c > 0$  for all players. Let  $a_i \in A$  denote player  $i$ 's action and  $c(a_i)$  his cost of participation. Thus  $c(N) = 0$  and  $c(P) = c$ .

At the beginning of the game, nature chooses  $x$ . Then some or all players observe  $x$  (according to the model considered) and each player chooses an action  $a_i \in A$ . I assume that uninformed players do not participate in the project.

Player  $i$ 's payoff is  $\gamma(a_i, q; x) - c(a_i)$ , where  $q$  is the fraction of the  $m$  players who participate in the project. Assume that the function  $\gamma : A \times [0, 1] \times \mathfrak{R}_+ \rightarrow \mathfrak{R}$  is  $C^2$ ; it is economically

inconsequential but convenient to extend the domain of  $\gamma$  for  $x > \bar{x}$  and to  $q \in [0, 1]$  rather than just multiples of  $\frac{1}{m}$ . The payoff  $\gamma$  may include purely personal and noncontractible payoffs as well as payoffs resulting from contracts. By taking  $\gamma$  to be exogenous, I leave issues of contract design in the background. As I mentioned before, the leaders and followers have equal claims to resources in the sense that payoffs  $\gamma$  are symmetric; they are distinguished only by who has access to information.

I impose several conditions on  $\gamma$ , listed below. If everyone else participates, then Assumption (1b) says that free riding is optimal ex ante but participating is optimal for high quality projects ( $x$  near  $\bar{x}$ ). Assumption (1c) says that it is never optimal to participate by oneself. While it is a dominant strategy not to participate in low quality projects, (1b) and (1c) imply that, for high quality projects, the players are engaged in a coordination game for which  $q = 0$  and  $q = 1$  are both Nash equilibria. Assumption (1d) means that there are increasing returns to participation, for subgroups of any size  $k$ ; this assumption helps to rule out equilibria in which some players participate and others do not. Assumption (1e) states that participation has positive spillovers for nonparticipants. Assumptions (1f) and (1g) imply that higher project quality ( $x$ ) benefits participants more than nonparticipants and weakly increases the spillovers that participants get from other participants. Finally, assumption (1h) implies nondecreasing returns to participation at the level of the entire group. (The term in brackets is the sum of players' benefits, ignoring participation costs.)

$$\gamma(a_i, q; 0) = \gamma(N, 0; x) = 0 \quad (1a)$$

$$E_x \left[ \gamma(P, 1; x) - \gamma(N, 1 - \frac{1}{m}; x) \right] < c ; \quad \gamma(P, 1; \bar{x}) - \gamma(N, 1 - \frac{1}{m}; \bar{x}) > c \quad (1b)$$

$$\gamma(P, \frac{1}{m}; x) < c \text{ for } x \leq \bar{x} \quad (1c)$$

$$\frac{\partial \left[ \gamma(P, z + \frac{k}{m}; x) - \gamma(N, z; x) \right]}{\partial z} > 0 \text{ for } k = 1, 2, .. \quad (1d)$$

$$\frac{\partial \gamma}{\partial q}(a_i, q; x) > 0 \text{ for } x > 0 \quad (1e)$$

$$\frac{\partial \gamma}{\partial x}(P, q; x) > \frac{\partial \gamma}{\partial x}(N, q; x) \geq 0 \text{ for } q > 0 \quad (1f)$$

$$\frac{\partial^2 \gamma}{\partial x \partial q}(P, q; x) \geq 0 \quad (1g)$$

$$\frac{\partial^2 [qm\gamma(P, q; x) + (1-q)m\gamma(N, q; x)]}{\partial q^2} \geq 0 \quad (1h)$$

To develop this analysis it is useful to reconstruct players' old payoff structure,  $\gamma(a_i, q; x)$ . I assume that player's gain from each project consists of two components: a common component and a private gain. Players earn a common gain regardless of their contribution and a private gain from participating to the project. The common and private components are shown by  $\alpha(q_{-i}; x, m)$  and  $\beta(q_{-i}; x, m)$  where  $q_{-i}$  is the fraction of the population who participate in the project other than player  $i$  himself (note that  $q = \frac{(m-1)q_{-i} + 1}{m}$  or  $q = \frac{(m-1)q_{-i}}{m}$ ). Therefore, each player  $i$  gets a payoff of  $\alpha(q_{-i}; x, m) + \beta(q_{-i}; x, m) - c$  if he joins the project or a payoff of  $\alpha(q_{-i}; x, m)$  if he does not participate. The components  $\alpha$  and  $\beta$  can be directly derived from  $\gamma$  as follows:

$$\alpha(q_{-i}; x, m) = \gamma(N, \frac{(m-1)q_{-i}}{m}; x)$$

$$\beta(q_{-i}; x, m) = \gamma(P, \frac{(m-1)q_{-i} + 1}{m}; x) - \alpha(q_{-i}; x, m)$$

Our previous assumptions about  $\gamma$ , imply that  $\alpha$  is strictly increasing with respect to  $x$  and  $q_{-i}$  ( $\frac{\partial \alpha(q_{-i}; x, m)}{\partial x} > 0$ ,  $\frac{\partial \alpha(q_{-i}; x, m)}{\partial q_{-i}} > 0$ ) and  $\alpha(q_{-i}; 0, m) = \beta(q_{-i}; 0, m) = 0$ .

**Lemma 1** (a)  $\frac{\partial \beta(q_{-i}; x, m)}{\partial x} > 0$  and  $\frac{\partial \beta(q_{-i}; x, m)}{\partial q_{-i}} > 0$ , for all  $x, q_{-i}$  and  $m$ .

*See the appendix for the proof.*

In the next section I introduce a model which represents a homogeneous organization in which all members have complete information and show how under complete information an organization is unable to achieve the first best.

## 2.2 Model 1: Complete Information Scenario; Homogeneous Population

As a benchmark, I first consider the case that all players observe the project quality  $x$  and then make simultaneous participation decisions. A player's strategy thus takes the form  $s : [0, \bar{x}] \rightarrow A$ . Let  $\#P$  and  $\#N$  denote the numbers assigned to participation and non-participation respectively. let  $\#P \equiv 1$  and  $\#N \equiv 0$  and let

$$\Pi_m(s_i, s_{-i}; x) \equiv \gamma \left( s_i(x), \frac{\sum_{j \in I} \#s_j(x)}{m}; x \right) - c(s_i(x))$$

denote player  $i$ 's payoff from adopting strategy  $s_i$  given  $x$  and other player's strategy  $s_{-i}$ . A Nash equilibrium of the game is the strategy profile  $S^* = (s_i^*, i \in I)$  such that:

$$s_i^*(x) = \underset{s_i}{\text{Arg max}} \Pi_m(s_i, s_{-i}^*; x) \quad \forall i \in I, x \in [0, \bar{x}]$$

For simplicity, I restrict my attention only to symmetric equilibria such that each player adopts a threshold strategy. A threshold strategy is  $x^*$  such that players play  $P$  iff  $x > x^*$ .

Allowing other equilibria introduces no new phenomena of interest and just some additional complex equilibria.

**Theorem 2** *Define the unique threshold  $\tau_m \in (0, \bar{x})$  such that*

$$h_m(\tau_m) = \beta(1; \tau_m, m) - c = 0$$

*The threshold strategy  $x^*$  constitutes a Nash equilibrium iff  $\tau_m \leq x^*$*

*See the appendix for the proof.*

Theorem (2) states that in the complete information scenario, players decide not to participate in the project for all values of  $x$  less than  $\tau_m$ . For any value of  $x$  between  $\tau_m$  and  $\bar{x}$

the game is a coordination game. In the sense that each player has the option to choose any threshold strategy between  $\tau_m$  and  $\bar{x}$ . As explained before, I have restricted my attention to the equilibria at which all players choose the same threshold strategy. There is, however, no guarantee that all players coordinate on the same strategy. The threshold  $\tau_m$  is the players' lowest participation threshold, which shows the lowest value of  $x$  at which each player is willing to participate if everybody else participates in the project.

Let  $W(q; x) \equiv mq\gamma(P, q; x) + m(1 - q)\gamma(N, q; x) - mcq$  be the total surplus produced by the organization. Assumption (1h) implies that  $\frac{\partial^2 W(q, x)}{\partial q^2} > 0$ , which implies that, for any  $x$ , the first best obtains at  $q = 0$  or  $q = 1$ . Therefore, if  $W(1, x) > 0$  for some value of  $x$  smaller than  $\tau_m$ , it is efficient for the players to participate in the project. According to the results from Theorem (2), however, the lowest value of  $x$  at which players may all participate in the project is  $\tau_m$ , which represents the standard free-riding problem.

A second problem with the continuum of equilibria described by Theorem 2 is that their multiplicity may make it difficult to coordinate on a common threshold if  $x \in [\tau_m, \bar{x}]$ . This may increase the likelihood that individual players adopt high thresholds (act uncooperative).

This means that in the complete information scenario, the organization does not achieve the first best, for players are not willing to cooperate (incentive conflicts problem) or are unable to coordinate (coordination problem) on a socially efficient project. The following example illustrates both obstacles to efficiency, and I return to it as I consider alternative information structures.

**Example 1:** Consider an organization with 8 identical players ( $m = 8$ ), who are all informed about the value of  $x$  and simultaneously decide whether to participate in the project. Let  $\gamma(P, q; x) = 3x + 11xq$ ,  $\gamma(N, q; x) = 6xq$  and  $c = 14$ . Assume that  $x$  is uniformly distributed over the interval  $[0, 3]$ . Then the set of equilibrium participation thresholds is:  $[\tau_m, \bar{x}] = [1.6, 3]$ . The threshold  $\tau_m = 1.6$  is the player's lowest participation threshold, which shows the lowest value of  $x$  at which each player is willing to participate if everybody else participates in the project. For all values of  $x$  less than 1.6 no player is willing to participate in the project. For  $x \in [1.6, 3]$ , the game is a coordination game.

The value of  $\gamma(P, q; x) - c = 3x + 11xq - 14$ , however, is positive for all values of  $x$  greater than 1 if all players participate in the project. Since the payoff structure satisfies increasing

returns to participation at the level of the entire group, the efficient outcome is for all players to participate in the project for all values of  $x \in (1, 3]$ , which is not supported by any of the equilibria in the game. The conclusion is: in the complete information scenario, the organization does not achieve the first best.

In the next section I show that the standard coordination and cooperation problems can be solved by appointing a single leader, who has exclusive access to information and is unable to fully transmit his information to the others.

### **2.3 Model 2: Incomplete Information Scenario with a Single Leader; Homogeneous Population**

This section pursues the idea that, even if it is costless, as in model 1, to make information about the project quality available to all players, it may improve efficiency to select one player arbitrarily and give her exclusive access to that information (e.g., the output of the organization's research department). In other words, it may improve efficiency to reduce player's access to information artificially. It is implicit in this approach that the excluded players cannot discover  $x$  for themselves, perhaps because that would be too costly for an individual.

I revise the model by changing (only) the timing and the information structure. I assume that exactly one player  $l \in I$  observes  $x$  (the leader) and that the leader acts first choosing  $a_l \in A$ . Then the remaining followers observe the leader's action and act simultaneously. Let  $F \equiv I \setminus \{l\}$  denote the set of followers.

In this extensive game, the leader's strategy takes the form  $s_l : [0, \bar{x}] \rightarrow A$  and each follower's strategy takes the form  $s_f : A \rightarrow A$ . Note that the leader cannot report  $x$  to the followers, even if she wishes to. The interpretation is that the leader typically benefits by reporting a false (e.g., high) value of  $x$ , and the followers are unable to verify the leader's report. Each follower's four possible strategies can be abbreviated  $S_f = \{PP, NN, C, R\}$ , where the strategy  $PP$  means that follower  $f$  always participates,  $NN$  means that follower  $f$  never participates,  $C$  means that the follower always copies the leader's action and  $R$  means always reject the leader's



actions. Let

$$\Pi_1(s_l, s_{-l}; x) \equiv \gamma \left( s_l(x), \frac{\#s_l(x) + \sum_{i \in F} \#s_i(s_l(x))}{m}; x \right) - c(s_l(x))$$

denote the leader's payoff as a function of all players' strategies, and similarly for each follower  $f \in F$ :

$$\Pi_1(s_f, s_{-f}; x) \equiv \gamma \left( s_f(s_l(x)), \frac{\#s_l(x) + \sum_{i \in F} \#s_i(s_l(x))}{m}; x \right) - c(s_f(s_l(x)))$$

Let  $\chi$  be a measurable subset of  $[0, \bar{x}]$  and  $\mu(\chi) = \text{prob}(x \in \chi)$ . Given  $s_l(x)$ , define  $\chi_P = \{x \in [0, \bar{x}] : s_l(x) = P\}$  and  $\chi_N = \{x \in [0, \bar{x}] : s_l(x) = N\}$ . A Perfect Bayesian equilibrium of the game is the strategy profile  $S^* = (s_l^*, s_f^*)$  and the posterior beliefs  $\mu(\chi | \chi_P)$  and  $\mu(\chi | \chi_N)$  such that:

$$\begin{aligned} s_l^*(x) &= \underset{s_l}{\text{Arg max}} \Pi_1(s_l, s_{-l}^*; x) && \forall x \in [0, \bar{x}] \\ s_f^*(a_l) &= \underset{s_f}{\text{Arg max}} E_x[\Pi_1(s_f, s_{-f}^*; x) | s_l^*(x) = a_l] \\ \forall a_l \in A &\text{ such that } a_l \text{ occurs with positive probability given } s_l^*, \forall f \in F. \\ \mu(\chi | \chi_P) &= \frac{\mu(\chi)\mu(\chi_P | \chi)}{\mu(\chi_P)} \\ \mu(\chi | \chi_N) &= \frac{\mu(\chi)\mu(\chi_N | \chi)}{\mu(\chi_N)} \end{aligned}$$

The three following lemmas and theorem help us characterize the equilibrium. Lemma 3 simplifies the followers' gain from participation. Lemma 4 shows that all followers copy the leader in any nontrivial equilibrium which implies that the leader leads by example. Finally, Theorem 5 characterizes the equilibria.

**Lemma 3** *Followers' expected gain from participation conditional on the leader's action, depends only on their conditional expected private gain,  $E[\beta(q_{-i}; x, m) | a_l, s_l]$ .*

*See the appendix for the proof.*

**Lemma 4** *If  $(s_l^*, s_f^*, f \in F)$  is an equilibrium strategy profile such that the leader participates with positive probability, then  $s_f^* = C$  for all  $f \in F$ .*

*See the appendix for the proof.*

The idea behind Lemma 4 is that the equilibrium participation of any one leader or follower implies, by increasing returns to participation (assumption (1d)), that others should also participate. Followers should not participate unconditionally, however, because the ex ante returns are too low (assumption (1b)). The only remaining possibility is for followers to copy the leader.

If the leader adopts a threshold strategy  $x^*$ , then the followers learn from his action either that  $x \leq x^*$  or  $x > x^*$ . If  $x^*$  is too small, then the leader asks too much and the followers become unwilling to follow. In this case, we say that the leader has no power. The following theorem shows the existence of an equilibrium in which the leader is powerful and is able to induce a following.

**Theorem 5** *Define the unique thresholds  $\tau_1 \in (0, \bar{x})$ ,  $\tau^F \in (0, \bar{x})$ , such that*

$$h_1(\tau_1) = \alpha(1; \tau_1, m) + \beta(1; \tau_1, m) - c = 0$$

$$\hat{r}(\tau^F) = E_x[\beta(1; x, m) \mid x \geq \tau^F] - c = 0$$

*Then, the pair  $(x^*, s_f^*, f \in F)$  is a positive participation equilibrium iff  $x^* = \tau_1$  ( $\tau^F < \tau_1$ ) and  $s_f^* = C$  for all  $f \in F$ .*

*See the appendix for the proof.*

The presence of a single informed leader greatly reduces the number of equilibria. The above theorem characterizes those equilibria. There always exists a trivial no-participation equilibrium, with the leader choosing the threshold strategy  $x^* = \bar{x}$  (i.e., she never participates) and the followers all choosing  $NN$ . Aside from such trivial equilibria, there exists at most one positive-participation equilibrium, meaning that the probability that at least one player participates exceeds zero.

Lets explain the positive-participation equilibrium in more detail. The threshold  $\tau_1$  represents the leader's optimal strategy given that all followers play  $C$ . If  $\tau^1 < \tau^F$ , then the signal conveyed by his participation decision (i.e.,  $x > x^*$ ) is too weak to convince the followers to participate: they would expect her to participate too often in circumstances that they would not find individually advantageous. Therefore, equilibrium participation requires satisfaction

of the condition  $\tau_1 \geq \tau^F$ . If this holds, then the leader can persuade everyone to participate whenever  $x > \tau_1$ . In proposition 6, I show that because  $\tau_1 < \tau_m$ , this is more participation than is attainable in the case of complete information. The intuition is that the leader's inability to reveal fully his information allows him to persuade the followers into participating for low values of  $x$  such that they would be unwilling to participate if they were fully informed. The reader shall see, however, that all of the followers benefit, both ex ante and ex post, from participation.

**Proposition 6** *The single leader's participation threshold is smaller than players' lowest participation threshold under complete information ( $\tau_1 < \tau_m$ ).*

*See the appendix for the proof.*

Another important implication of Theorem 5 is that the game with a single leader has at most *one* nontrivial equilibrium. The presence of the leader thus not only increases participation but also largely solves the coordination problem evident in Theorem 2. Leadership, artificially created by restricting access to information, can thus solve both incentive conflicts and coordination problems.

In the next section, I show how the organization can achieve the first best.

## 2.4 Efficiency and the single leader

Recall that  $W(q; x) \equiv mq\gamma(P, q; x) + m(1-q)\gamma(N, q; x) - mcq$  is the total surplus produced by the organization. The next theorem shows that a single powerful leader always maximizes  $W(q; x)$ , ex post, in any non-trivial equilibrium. That is, a single leader induces the unconstrained first best if he is powerful. The intuition is that a powerful leader, anticipating that every follower will copy his behavior, acts as a representative agent on behalf of the group.

**Theorem 7** *If  $\tau_1 \geq \tau^F$ , then the unique positive-participation equilibrium achieves the first best (i.e., maximizes  $W(q; x)$  over  $q$ , given any  $x$ ).*

**Proof.** Assumption (1h) implies that  $\frac{\partial^2 W(q; x)}{\partial q^2} > 0$ , which implies that, for any  $x$ , the first best obtains at  $q = 0$  or  $q = 1$ . Let  $\Delta W(1; x) = m\gamma(P, 1; x) - mc$  denote the change in the total surplus from full participation compare to no participation.

By assumption  $\frac{\partial W(1; x)}{\partial x} = \frac{\partial \gamma(P, 1; x)}{\partial x} > 0$ .

By definition of  $\tau_1$  and since  $\frac{\partial W(1; x)}{\partial x} > 0$  we can conclude that  $\Delta W(1; x) > 0$  for all  $x > \tau_1$  and  $\Delta W(1; x) < 0$  for all  $x < \tau_1$ . Therefore, full participation ( $q = 1$ ) is efficient for all  $x > \tau_1$  and  $q = 0$  is efficient for all  $x < \tau_1$ .

Therefore, since  $\tau_1 > \tau^F$ , the existence of the positive participation equilibrium from Theorem (6) implies that a single leader can induce a first best outcome. ■

**Example 2:** Consider the organization introduced in Example 1. Now, however, assume that exactly one player is promoted to the leadership position and allowed to observe  $x$ . The leader is powerful because  $\tau_1 = 1$  exceeds  $\tau^F = 0.2$ . Therefore, a positive participation equilibrium exists. In this equilibrium, the leader participates only for  $x \geq 1$  and is followed by everyone else.

If the leader is not powerful because the condition ( $\tau_1 > \tau^F$ ) is not satisfied, he won't be able to induce a first best outcome simply because he will not be followed by his subordinates.

There are several potential ways to increase the power of the leader (short of redesigning the payoff structure  $\gamma$ ). One way is to relax the assumption that the population is homogeneous. Considering a heterogeneous organization is not only more realistic, but also allows us to focus on the players' characteristics to choose the most appropriate person for the leadership position, who is powerful and able to create the highest surplus (This scenario will be discussed in model 4). It will be shown that if some players have high participation costs, then appointing a lazy or busy (high cost) player to the leadership position can increase the power of leadership.

Another way to increase the power of leadership is to appoint multiple informed leaders in the organizational structure. This increases each leader's power by reducing each leader's influence. I show that multiple leadership can restore the power of leaders by increasing their participation threshold above  $\tau^F$ .

It will be shown that the total surplus in the multi-leader scenario is higher than the complete information scenario, but multiple leadership can not induce the unconstrained first best. The multi-leader scenario is interesting but it raises the question of how leaders can coordinate on participation. This creates an incentive for future research. Considering a hierarchical organization and appointing a top leader followed by subleaders may solve the coordination problem among the subleaders. This idea is undeveloped at this stage but it may bear fruit in

the future.

Next section analyzes how multiple leadership affects the power of leadership and the equilibrium outcome of the game. It also addresses the question of what is the optimal number of leaders that maximizes the total surplus produced by the organization.

## 2.5 Model 3: Incomplete Information Scenario with Multiple Leaders; Homogeneous Population

This section considers the same model of leadership, with one difference. I now allow multiple players to lead identically sized groups of followers. Let  $L \subset I$  denote the set of leaders and  $F(l) \subset I$  the set of followers of leader  $l \in L$ . I assume that each follower has only one leader and no followers;  $L$  and  $F(l)_{l \in L}$  thus constitute a partition of  $I$ . Let  $n \equiv \#L \in N(m)$  denote the number of leaders, where  $N(m) \equiv \{1, \dots, m\}$  denotes the integer factors of  $m$ , and let  $r_n \equiv \frac{m}{n} - 1$  denote the number of followers per leader. It is economically reasonable to allow  $n$  to take integer values  $n \notin N(m)$ , but my formal analysis does not accommodate fractional followers.

I show that multiple leaders can be powerful and induce participation in circumstances such that a single leader could not persuade anyone to participate. Unlike a single leader, multiple leaders generally cannot induce first best outcomes ex ante, but I derive the number of leaders needed to support the constrained optimum.

The model of multiple leaders may be appropriate for political or other contexts in which numerous groups work together toward a common objective. The leader of a single large group might have too many followers to be powerful (because he gets a large payoff from his followers' participation and therefore he has a greater incentive to exaggerate the value of the project), but smaller groups can cooperate effectively if their leaders can solve the coordination problem among themselves.

Consider the following extensive game, similar to the game of the previous section. Nature initially chooses  $x$ , the leaders observe  $x$ , and then each leader  $l \in L$  chooses an action  $a_l \in A$ . After the leaders act simultaneously, each follower  $f \in F(l)$  observes leader  $l$ 's action and chooses an action  $a_f \in A$ . The followers act simultaneously. As before, a leader's strategy is

a function  $s_l : [0, \bar{x}] \rightarrow A$ , and a follower's strategy is a function  $s_f : A \rightarrow A$ . The followers' strategies can again be represented by the set  $S_F = \{PP, NN, C, R\}$ . Let

$$\Pi_n(s_i, s_{-i}; x) \equiv \pi \left( s_i(s_l(x)), \frac{\sum_{l \in L} [\#s_l(x) + \sum_{f \in F(l)} \#s_f(s_l(x))]}{m}; x \right) - c(s_i(s_l(x)))$$

denote the payoff to follower  $i \in F(l)$ , as a function of all players' strategies. If player  $i$  is the leader, then the formula is the same except that  $s_i(x)$  replaces  $s_i(s_l(x))$ .

To simplify matters, I consider only group symmetric equilibria, meaning that each group adopts a strategy profile identical to that of each other group. If the followers adopt diverse strategies, then the proportion of followers who adopt a given strategy is the same in each group.

Recall that  $\chi$  is a measurable subset of  $[0, \bar{x}]$  and  $\mu(\chi) = \text{prob}(x \in \chi)$ . Given  $s_l(x)$ , I also defined  $\chi_P = \{x \in [0, \bar{x}] : s_l(x) = P\}$  and  $\chi_N = \{x \in [0, \bar{x}] : s_l(x) = N\}$ . A Perfect Bayesian equilibrium of the game is the strategy profile  $S^* = (s_l^*; s_f^*, f \in F(l))$  and the posterior beliefs  $\mu(\chi | \chi_P)$  and  $\mu(\chi | \chi_N)$  such that:

$$\begin{aligned} s_l^*(x) &= \underset{s_l}{\text{Arg max}} \Pi_n(s_l, s_{-l}^*; x) && \forall x \in [0, \bar{x}] \\ s_f^*(a_l) &= \underset{s_f}{\text{Arg max}} E_x[\Pi_n(s_f, s_{-f}^*; x) | s_l^*(x) = a_l] \\ \forall a_l \in A &\text{ such that } a_l \text{ occurs with positive probability given } s_l^*, \quad \forall f \in F(l) \\ \mu(\chi | \chi_P) &= \frac{\mu(\chi)\mu(\chi_P | \chi)}{\mu(\chi_P)} \\ \mu(\chi | \chi_N) &= \frac{\mu(\chi)\mu(\chi_N | \chi)}{\mu(\chi_N)} \end{aligned}$$

As for the other models, assumption (1c) implies that the multi-leader case always supports a no-participation equilibrium (e.g. leaders never participate and every follower plays  $NN$ ). If leaders participate with positive probability, however, then I can show, as in the case of a single leader, that followers must copy their leaders in equilibrium.

**Lemma 8** *If  $(s_l^*; s_f^*, f \in F(l))$  is a group symmetric equilibrium such that leaders participate with positive probability, then  $s_f^* = C$  for all  $f \in F(l)$ .*

*See the appendix for the proof.*

As in the complete information setting of Model 1, but not the single leader setting of Model 2, the presence of multiple informed players introduces coordination problems that support complicated equilibria incorporating non-threshold strategies. To simplify matters somewhat, I follow the pattern of Model 1 by restricting attention to threshold strategies for the informed players. The following theorem, generalizes Theorems 2 and 5 from the extreme cases  $n = m$  (i.e. complete information) and  $n = 1$  to intermediate numbers of leaders.

**Theorem 9** *Define the unique thresholds  $\tau_n^A \in (0, \bar{x})$ ,  $\tau_n^B > \tau_n^A$ ,  $\tau^F \in (0, \bar{x})$ , such that*

$$h_n^A(\tau_n^A) = \alpha(1; \tau_n^A, m) + \beta(1; \tau_n^A, m) - \alpha\left(\frac{m\left(1 - \frac{r_n}{m}\right) - 1}{m - 1}; \tau_n^A, m\right) - c = 0$$

$$h_n^B(\tau_n^B) = \alpha\left(\frac{r_n}{m - 1}; \tau_n^B, m\right) + \beta\left(\frac{r_n}{m - 1}; \tau_n^B, m\right) - c = 0$$

$$\hat{r}(\tau^F) = E_x[\beta(1; x, m) \mid x \geq \tau^F] - c = 0$$

*Then, the pair  $(x^*; s_f^*, f \in F(l))$  is a positive participation group symmetric equilibrium iff  $\text{Max}\{\tau_n^A, \tau^F\} \leq x^* \leq \tau_n^B$  and  $s_f^* = C$ .*

*See the appendix for the proof.*

Theorem 9 shows that the presence of multiple leaders complicates the determination of equilibrium, because the leaders' participation threshold is not unique; the leaders play a coordination game among themselves, in which the (symmetric) equilibrium participation threshold can fall anywhere in the interval  $[\tau_n^A, \tau_n^B]$ . The threshold  $\tau_n^A$  is the lowest value of  $x$  at which a leader is willing to participate if all other leaders are also willing to participate. The threshold  $\tau_n^B$  is the lowest value of  $x$  for which a leader is willing to participate even if no other leader participates. For  $n = 1$  the interval  $[\tau_n^A, \tau_n^B]$  collapses to the point  $\tau_1$ .

Theorem 9 shows that the power constraint for  $n$  leaders, analogous to the  $\tau_1 \geq \tau^F$  condition for one leader, is  $\tau_n^B \geq \tau^F$ . Proposition 10 shows that increasing the number of leaders relaxes this constraint.

**Proposition 10** *If  $n' > n$  then  $\tau_{n'}^A > \tau_n^A$  and  $\tau_{n'}^B > \tau_n^B$ .*

See the appendix for the proof.

As mentioned before, the above proposition shows that the leaders' participation threshold increases with the number of leaders. Therefore appointing more leaders can improve leaders' power.

As the reader can see, the participation interval in the multi-leader case ( $Max \{ \tau_n^A, \tau^F \} \leq x^* \leq \tau_n^B$ ) captures smaller values of  $x$ , relative to the complete information scenario ( $\tau_m \leq x^* \leq \bar{x}$ ), because  $\tau_m > \tau_n^A$ . This means that the total surplus is higher in the multi-leader case.

### 2.5.1 Optimal Number of Leaders

Theorem 9 and Proposition 10 suggest that the optimal number of leaders is whatever value of  $n \in N(m)$  that is just large enough to create a powerful leader ( $\tau_n^B \geq \tau^F$ ). Fewer leaders cannot induce participation because they are powerful, but it is suboptimal to introduce more leaders because their increased incentive to free ride off each other causes unnecessary efficiency losses. The next theorem makes this conjecture precise.

One complication is that the multiplicity of positive-participation equilibria for sufficiently large  $n$  introduces ambiguity into the determination of the optimal  $n$ . I will consider two welfare measures, an optimistic measure  $\overline{V}(n)$ , which assumes that the leaders coordinate on the most efficient equilibrium, and a pessimistic measure  $\underline{V}(n)$ .

To develop these measures, I first summarize the results concerning equilibria for various numbers of leaders  $n$ . Every positive-participation equilibrium described by Theorems 2, 5, and 9 is characterized by a single parameter  $x^* \in [0, x^*]$ , the participation threshold of informed players. If  $x > x^*$ , then everyone participates; if  $x \leq x^*$ , then no one participates. In every case,  $x^*$  must satisfy the constraint  $x^* \geq \tau^F (> 0)$  and the leaders' equilibrium condition  $x^* \in X^*(n)$ , where  $X^*(1) = \{\tau_1\}$ ,  $X^*(m) = [\tau_m, \bar{x}]$ , and  $X^*(n) = [\tau_n^A, \tau_n^B]$  for intermediate values of  $n$ . Expected per capita surplus, as a function of equilibrium  $x^*$ , is:

$$\widehat{W}(x^*) \equiv \int_{x^*}^{\bar{x}} [\gamma(P, 1; x) - c] \phi(x) dx$$

Let  $\overline{V}(n)$  and  $\underline{V}(n)$  denote, respectively, the maximized and minimized values of  $\widehat{W}(x^*)$  subject to  $x^* \in [\tau^F, \bar{x}] \cap X^*(n)$  and  $n \in N(m)$ , or zero if the constraint set is empty (i.e., there exist



no positive-participation equilibria).

Theorem 11 shows that the problems of maximizing  $\underline{V}(n)$  and  $\overline{V}(n)$  have the same solution.

**Theorem 11** *Let  $n^*$  denote the smallest element of  $N(m)$  that satisfies  $\tau^F \leq \tau_n^B$ ; then  $n^*$  maximizes  $\underline{V}(n)$  and  $\overline{V}(n)$  among  $n \in N(m)$ .*

*See the appendix for the proof.*

The following example illustrates how the number of leaders affects the leaders' power and efficiency and why  $n^*$  is the optimal number of leaders.

**Example 3:** Consider an organization with 8 identical players as in Example 1, who all observe the value of  $x$  and simultaneously decide whether to participate in the project. Let  $\gamma(P, q; x) = 7x(8q+1)$ ,  $\gamma(N, q; x) = 48xq$ , and  $c = 42$ . Assume that  $x$  is uniformly distributed on the interval  $[0, 3]$ . Under complete information, the set of equilibrium participation thresholds, from Theorem 1, is:  $[\tau_m, \bar{x}] = [2, 3]$ . For  $x \in [2, 3]$ , the game is a coordination game and it is efficient for everyone to participate.

The first best outcome, however, is for all players to participate for  $x > 0.66$ . Appointing a single leader cannot support this outcome, because the condition  $\tau_1 \geq \tau^F$  fails, with  $\tau_1 = 0.66$  while  $\tau^F = 1$ . One way to establish the power of leadership is to appoint multiple leaders. Each leader then has fewer followers, less impact on total participation, and less incentive to participate for small values of  $x$ .

Specifically, two is the smallest number of leaders that satisfies the condition  $\tau_n^B \geq \tau^F$  and supports a positive-participation;  $n=2$  leaders can coordinate on any participation threshold  $x^* \in [\tau_2^A, \tau_2^B] \approx [1.08, 1.20]$ . The most efficient equilibrium has two leaders, each participating for  $x \geq 1.08$ . This equilibrium fails to achieve the first-best, however, because it does not support full participation ex post, if  $x \in (.66, 1.08)$ .

I leave for future study the important question of how multiple leaders coordinate among themselves, but a hierarchical organization with a top leader and partially informed subleaders appears to support better cooperation and coordination than are available from the two-tier hierarchy considered here.

As I mentioned before, another potential way to address the power of leadership is to relax the assumption that the population is homogeneous. The main purpose of this variation is

to focus on player's characteristics to appoint a powerful leader. Considering a heterogeneous population raises the question of who is the best choice for the leadership position.

In the next section, I consider a scenario where players are differentiated by their participation costs. I show that it never improves efficiency to promote the most cooperative player to the leadership position even if they are powerful. I also show that an average player is the best choice for the leadership position if he is powerful. If an average player is not powerful enough to induce a following then promoting less cooperative (lazier or busier) players establishes a powerful leadership and therefore produces the highest possible surplus.

## 2.6 Model 4: Incomplete Information Scenario with a Single Leader; Heterogeneous Population

In this section I adopt the same basic structure as in previous models but allow players to be differentiated by their cost of participation. To do so, I consider

$$C = \{c_i(P) \mid c_{i+1}(P) = c_i(p) + r, r > 0, i \in I\}$$

to be the set of players' participation costs.

I show that in the heterogeneous population model, complete information structure is inefficient as it was in the homogeneous population scenario, for fully informed players refuse to participate in cases where full participation is efficient.

In previous sections, I introduced leadership as a solution to this problem. I showed that an informed single leader who is powerful can produce a first best outcome in cases where the first best can not be obtained under complete information. The main purpose of the present variation is to consider the question: which player should serve as a leader. I start the analysis with the following example.

**Example 4:** Consider the same organization as example 1 with 8 players who are differentiated by their cost of participation.  $c = \{c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8\} = \{10, 11, 12, 13, 14, 15, 16, 17\}$  be the set of players' participation costs. As in example 1, I assume that  $\gamma(P, q; x) = 3x + 11xq$  and  $\gamma(N, q; x) = 6xq$ . I also assume that  $x$  is uniformly distributed over the interval  $[0, 3.2]$ .

If we restrict our attention to the threshold strategies we can see that all players are willing

to participate in the project if  $x > 2$ . For  $x \in [1, 2]$ , the game is a coordination game for all or some of the players. For all values of  $x$  less than 1, no player is willing to participate in the project.

The total surplus, however, is positive for all values of  $x$  greater than 0.84 if all players participate in the project. Since the players' cost of participation does not vary significantly across the players and the payoff structure satisfies increasing returns to participation at the level of the entire group, the efficient outcome is for all players to fully participate in the project for all values of  $x \in (0.84, 3.2]$ . As the reader can see, however, for  $x \in (0.84, 1)$ , full participation is not supported by any equilibrium in the game because the dominant strategy for all players is not to participate in the project. Also, for  $x \in [1, 2]$  full cooperation is not necessarily the equilibrium outcome because all or some of the players play a coordination game among themselves. The conclusion is: in the complete information scenario, the organization can not achieve the first best.

Lets consider an incomplete information scenario in which only one player (player  $l$ ) is informed about the quality of the project and is appointed to be the only leader of the organization.

As in model 2, I assume that other players are uninformed followers. I also assume that  $F(l)$  is the set of followers who follow leader  $l$ .

One important distinction between this model and model 2 is that the leader in model 2 is a representative player with the same characteristics of the others. In the current model, however, the leader is different from his followers, for players are differentiated. Therefore, players' characteristics become crucial in selecting the appropriate leader.

As in model 2, I consider the following sequential-move game. At the beginning of the game, nature chooses  $x$ . The realized value of  $x$  is only observed by the leader. The distribution of  $x$ , however, is common knowledge. In the first stage of the game leader  $l$  observes  $x$  and chooses an action  $a_l \in A$ . Leader  $l$ 's strategy can therefore be described by  $s_l = [0, \bar{x}] \rightarrow A$  and his possible strategy set can be shown by  $S_l = \{P, N\}$ . In the second stage of the game each follower observes the leader's action, updates his beliefs about the value of the project and chooses an action  $a_f \in A$ . Thus the followers' strategy and their possible strategy set can be shown by  $s_f : A \rightarrow A$  and  $S_f = \{PP, NN, C, R\}$  respectively.

As in model 2 and 3, I assume that an uninformed follower does not participate in the project.

Let

$$\Pi_{1l}(s_l, s_{-l}; x) \equiv \gamma \left( s_l(x), \frac{\#s_l(x) + \sum_{i \in F} \#s_i(s_l(x))}{m}; x \right) - c_l(s_l(x))$$

denote the leader's payoff as a function of all players' strategies, and similarly for each follower  $f \in F$ :

$$\Pi_{1f}(s_f, s_{-f}; x) \equiv \gamma \left( s_f(s_l(x)), \frac{\#s_l(x) + \sum_{i \in F} \#s_i(s_l(x))}{m}; x \right) - c_f(s_f(s_l(x)))$$

A Perfect Bayesian equilibrium of the game is the strategy profile  $S^* = (s_l^*; s_f^*, f \in F(l))$  and the posterior beliefs  $\mu(\chi | \chi_P)$  and  $\mu(\chi | \chi_N)$  such that:

$$\begin{aligned} s_l^*(x) &= \underset{s_l}{\text{Arg max}} \Pi_1(s_l, s_{-l}; x) && \forall x \in [0, \bar{x}] \\ s_f^*(a_l) &= \underset{s_f}{\text{Arg max}} E_x[\Pi_{f1}(s_f, s_{-f}; x) | s_l^*(x) = a_l] \\ \forall a_l &\in A \text{ such that } a_l \text{ occurs with positive probability given } s_l^*, \forall f \in F. \\ \mu(\chi | \chi_P) &= \frac{\mu(\chi)\mu(\chi_P | \chi)}{\mu(\chi_P)} \\ \mu(\chi | \chi_N) &= \frac{\mu(\chi)\mu(\chi_N | \chi)}{\mu(\chi_N)} \end{aligned}$$

To avoid complexity at this point I restrict my attention to the follower symmetric equilibria where either all followers participate in the project or nobody does at all.

The following theorem shows the existence of an equilibrium in which the leader is powerful and is able to induce a following.

**Theorem 12** Define the unique thresholds  $\tau_{l1} \in (0, \bar{x})$ ,  $\bar{\tau}^F \in (0, \bar{x})$ , such that

$$h_{l1}(\tau_{l1}) = \alpha(1; \tau_{l1}, m) + \beta(1; \tau_{l1}, m) - c_l = 0$$

$$\hat{r}(\bar{\tau}^F) = E_x[\beta(1; x, m) | x \geq \bar{\tau}^F] - \bar{c} = 0$$

where  $\bar{c}$  is the highest cost of participation. Then, the pair  $(x^*, s_f^*)_{f \in F(l)}$  is a positive participation follower symmetric equilibrium iff  $x^* = \tau_{l1}$  ( $\tau_{l1} \geq \bar{\tau}^F$ ) and  $s_f^* = C$ .

See the appendix for the proof.

As in Model 2, presence of a single informed leader greatly reduces the number of equilibria. The above theorem characterizes the follower symmetric equilibria. There always exists a trivial no-participation equilibrium, with the leader choosing the threshold strategy  $x^* = \bar{x}$  (i.e., she never participates) and the followers all choosing  $NN$ . Aside from such trivial equilibria, there exists at most one positive-participation follower symmetric equilibrium.

Lets explain the positive-participation symmetric equilibrium in more detail. The threshold  $\tau_{l1}$  represents the leader's optimal strategy given that all followers play  $C$ . If  $\tau_{l1} < \bar{\tau}^F$ , then the signal conveyed by his participation decision (i.e.,  $x > x^*$ ) is too weak to convince all the followers to participate: some or all followers would expect her to participate too often in circumstances that they would not find individually advantageous. Therefore, the follower symmetric equilibrium participation requires satisfaction of the condition  $\tau_{l1} \geq \bar{\tau}^F$ . If this holds, then the leader can persuade everyone to participate whenever  $x > \tau_{l1}$ .

**Example 5:** In example 4, I considered an organizational structure with 8 players who are differentiated by their cost of participation. I showed that players are not willing to participate or fail to coordinate on an efficient outcome if they are all equally informed about the value of the project.

In this example, I consider a scenario where player 1 ( $c_1 = 10$ ) is the leader and the rest of the players are uninformed followers. The existence of the positive participation equilibrium in Theorem 12 implies that if the leader is convincing to all his subordinates (i.e. if  $\tau_{11} \geq \bar{\tau}^F$ ) he will decide to participate in the project for all values of  $x$  greater than  $\tau_{11}$  and will be followed by his subordinates.

In this example, the leader is powerful, for  $\tau_{11} = 0.62$  is larger than  $\bar{\tau}^F = 0.1$ . Therefore, the leader will contribute for all values of  $x$  greater than .62 and will be followed by his subordinates. In the complete information scenario, however, full participation is not necessarily the equilibrium outcome for all values of  $x$  smaller than 2.

Up to this point I have only focused on how and under what circumstances a single leader induces full participation. One important consideration, however, is that whether a single leader is able to improve efficiency by inducing full contribution.

In Model 2, I showed that a single leader can induce a first best outcome. The current scenario, however, is different from Model 2, for the fact that now players are differentiated by their cost of participation. Relaxing the homogeneity assumption of Model 2 changes the efficiency results and raises interesting questions such as who is the best choice for the leadership position.

In the next section, I derive the efficiency results and address the problem of leadership selection.

### 2.6.1 Efficiency and the Choice of the Leader

In the homogeneous population scenario, I showed that a powerful single leader can induce the unconstrained first best by inducing cooperation and coordination in situations where informed players are not willing to cooperate or are unable to coordinate on an efficient outcome.

This results from the assumption that players are homogeneous and the leader is just a representative player with exclusive information about the value of the project. Therefore, the leader's payoff function is exactly the same as the others and his incentives are aligned with his followers. Thus, if the leader gains a positive payoff from participating to the project, so do the other players.

The story, however, is different in the heterogeneous scenario. Since players are differentiated by their cost of participation, the leader's payoff is different from his subordinates. Therefore the leader's incentives is not always aligned with his followers and a single leader does not necessarily induce an efficient outcome.

To see this consider example 5. As one can see from example 5, player 1's cost of participation is the smallest among the players and therefore the projects that yield him a positive payoff do not necessarily produce a positive payoff for the other players. Therefore, promoting him to the leadership position creates a negative ex-post surplus for  $x \in [0.62, 0.84]$ , because he participates in the project for  $x > 0.62$  and is followed by the rest since he is powerful ( $\tau_{11} = 0.62 > \bar{\tau}^F = 0.1$ ).

The question is: which player should serve as the leader. The next four propositions and theorems address this question.

Recall that  $C = \{c_i(P) \mid c_{i+1}(P) = c_i(p) + r, r > 0, i \in I\}$  is the set of players' participation costs. Lets sort the population by their cost of participation and let  $w(q; x) = mq\gamma(q; x, P) + m(1-q)\gamma(q; x, N) - rc_1q - \frac{rmq(mq-1)}{2}$  be the total surplus produced by the first  $q$  fraction of the population. The following lemma shows that if participation costs do not vary significantly across the players, the efficient outcome obtains at  $q = 0$  or  $q = 1$ . That is, if participation costs are not significantly different, then partial contribution is never optimal. Theorem 14 addresses the issue of leadership selection. It shows that an average leader is the best choice for the leadership position if he is powerful. Proposition 15 is used to prove Theorem 14 and 16.

**Lemma 13** *Let  $\frac{\partial^2 [q\gamma(q; x, P) + (1-q)\gamma(q; x, N)]}{m\partial q^2} = A$ . If  $r < A$  then the efficient outcome obtains at  $q = 0$  and  $q = 1$ .*

*See the appendix for the proof.*

**Theorem 14** *Let  $c_a$  denote the average realized cost of participation and define the unique threshold  $\tau_{a1}$  such that*

$$h_{a1}(\tau_{a1}) = \alpha(1; \tau_{a1}, m) + \beta(1; \tau_{a1}, m) - c_a = 0$$

*If  $r < A$  and  $\tau_{a1} \geq \bar{\tau}_f$ , then choosing a leader with  $c_l = c_a$  induces first best.*

*See the appendix for the proof.*

The intuition behind Theorem 14 is that leaders with participation costs smaller than average are willing to participate for low return projects and therefore induce a negative total surplus. Therefore, they are unable to improve efficiency even if they are powerful. In the case where the average player is powerful, appointing leaders with higher participation costs is also suboptimal because their lack of incentive for participation causes unnecessary efficiency loss.

The story, however, is different if the average player is not powerful enough to induce a following. In this case, Proposition 15 and Theorem 16 shows that players with larger participation costs are more powerful.

**Proposition 15** *If  $c_i > c_k$  then  $\tau_i > \tau_k$ .*

*See the appendix for the proof.*

**Theorem 16** *If the average leader is not powerful, then define  $C^c = \{c_i > c_a \text{ which satisfies } \tau_{i1} > \bar{\tau}_f\}$ . Choosing a leader with  $c_l = \text{Min}C^c$  maximizes the total surplus obtainable from a follower symmetric equilibrium.*

*See the appendix for the proof.*

Theorem 16 shows that when the average leader is not powerful, it improves efficiency to appoint a lazier or busier player (lazy or busy enough to satisfy the leadership's power constraint). The intuition is that less cooperative players are less likely to participate for low value projects. Therefore, the participation of a high cost leader is a more convincing signal for his followers.

So far I have restricted my attention to follower symmetric equilibria in which all followers follow the same strategy. The cost differentiation, however, introduces some complexities. First, there are equilibria in which the leader is convincing for some followers but not convincing for the others. Therefore leader's participation is followed by some but not by the others. Second, if participation costs are significantly different, then partial participation may become optimal. I leave the partial participation equilibria for future research and conclude this chapter in the next section.

## 2.7 Conclusion

I developed a theory of leadership unrelated to questions of voting and contract design, in which contracts are external to the model. In my theory, an organization is held together only by the substantial fixed costs of generating information and by the advantages of restricting access to that information. Leaders have no special talent but are leaders simply because they are given exclusive access to information. I show that such a minimal leader can simultaneously resolve otherwise serious failures of cooperation and coordination. Indeed the leader can often induce the unconstrained first best, even though every player has incentive to free ride.

These results turn some traditional ideas in organizational design on their head. Instead of designing an organization to mitigate problems resulting from costly information transmission and processing, I assume that information transmission and processing are costless and demonstrate the advantages of keeping subordinates (or voters) uninformed.



A key assumption is that subordinates are unable to verify the costless claims that leaders may make about the information to which they have exclusive access. This prevents full revelation and consequently causes rational subordinates to be more cooperative.

In the sparse formal theory of leadership in economics, the nearest antecedent to my work is Hermalin (1998) model of a leader who (like my model) is characterized primarily by having superior information. Hermalin's leader, however, fully reveals his information in equilibrium, and the efficiency gains resulting are qualitatively smaller than those obtaining in my model.

My model is related to the idea of information cascades, but unlike the many studies focusing on the inefficiencies induced by cascades, I use the leader-follower relationship to improve efficiency.

To induce cooperation a leader must convince the followers that she is transmitting correct information and not misleading them. Therefore, a single leader may not produce first-best outcomes if his actions are not convincing because his leadership role gives him too much influence over collective payoffs, at too little cost to himself. In this case, I show that diluting the power of the leadership by appointing multiple informed leaders can improve efficiency but cannot achieve the first best.

If agents are differentiated by their costs of cooperation, then I show that appointing uncooperative (i.e. 'busy' or 'lazy') leaders can produce more efficiency than appointing a 'representative' leader, if the representative leader would not be powerful. In contrast, it is never optimal to appoint an especially cooperative leader, unless one wishes to account for positive externalities that cooperation within the modelled group may confer on unmodelled outsiders.

I leave for future research analyzing the complications of the multi-leadership and heterogeneous population models. The heterogeneous organization introduces the existence of partial participation equilibria and the possible optimality of partial participation. The multi-leader organization on the other hand, introduces complications such as how the leaders can coordinate among themselves. A hierarchical structure comprising a top leader and multiple subleaders may be a way to construct an organizational structure that supports better cooperation and coordination. The latter will be addressed in a subsequent paper.

## 2.8 Appendix

**proof of Lemma 1:**

*Part a:*

*By definition we have:*

$$\beta(q_{-i}; x, m) = \gamma\left(P, \frac{(m-1)q_{-i} + 1}{m}; x\right) - \gamma\left(N, \frac{(m-1)q_{-i}}{m}; x\right)$$

*Assumptions (1f) and (1g) imply that:*

$$\frac{\partial \beta(q_{-i}; x, M)}{\partial x} = \frac{\partial \gamma\left(P, \frac{(m-1)q_{-i} + 1}{m}; x\right)}{\partial x} - \frac{\partial \gamma\left(N, \frac{(m-1)q_{-i}}{m}; x\right)}{\partial x} > 0$$

*Summarizing:*

$$\frac{\partial \beta(q_{-i}; x, M)}{\partial x} > 0$$

*Part b:*

*Assumption (1d) for  $k = 1$  implies that:*

$$\frac{\partial \left[ \gamma\left(P, z + \frac{1}{m}; x\right) - \gamma\left(N, z; x\right) \right]}{\partial z} > 0$$

*For  $z = \frac{(m-1)q_{-i}}{m}$  we have:*

$$\frac{\partial \left[ \gamma\left(P, \frac{(m-1)q_{-i} + 1}{m}; x\right) - \gamma\left(N, \frac{(m-1)q_{-i}}{m}; x\right) \right]}{\partial z} > 0$$

$$\frac{\partial \left[ \gamma\left(P, \frac{(m-1)q_{-i} + 1}{m}; x\right) - \gamma\left(N, \frac{(m-1)q_{-i}}{m}; x\right) \right]}{\partial q_{-i}} \times \frac{\partial q_{-i}}{\partial z} > 0$$

Therefore,

$$\frac{\partial \beta(q_{-i}; x, M)}{\partial q_{-i}} = \frac{\partial \left[ \gamma(P, \frac{(m-1)q_{-i} + 1}{m}; x) - \gamma(N, \frac{(m-1)q_{-i}}{m}; x) \right]}{\partial q_{-i}} > 0$$

Summarizing:

$$\frac{\partial \beta(q_{-i}; x, M)}{\partial q_{-i}} > 0$$

**Proof of Theorem 2:** See the proof of theorem 9, which is the general version of theorem 2.

**Proof of Lemma 3:**

Let

$$r(a_l, q_{-f}) = E_x \left[ \gamma(P, \frac{(m-1)q_{-f} + 1}{m}; x) - c - \gamma(N, \frac{(m-1)q_{-f}}{m}; x) \mid a_l, s_l \right]$$

denote the followers' expected marginal gain from participation given  $q_{-f}$  and leaders' action  $a_l$ , and  $f \in F(l)$ .

by definition of  $\gamma, \alpha$  and  $\beta$  we have:

$$r(q_{-f}, a_l) = E_x \left[ \alpha(x, q_{-f}; m) + \beta(x, q_{-f}; m) - \alpha(x, q_{-f}; m) - c \mid a_l, s_l \right]$$

Therefore,

$$r(q_{-f}, a_l) = E_x \left[ \beta(x, q_{-f}; m) - c \mid a_l, s_l \right]$$

**Proof of Lemma 4:** See the proof of lemma 8, which is the general version of lemma 4.

**Proof of Theorem 5:**

See the proof of theorem 9, which is the general version of this theorem.

**Proof of Proposition 6:**

See the proof of proposition 10, which is the general version of this proposition.

**Proof of Lemma 8:**

Because the result is vacuous for  $n = m$ , assume  $n < m$ . The proof comprises four steps.

For the given equilibrium:

- (i) All followers choose the same strategy.
- (ii) That strategy cannot be  $NN$ .
- (iii) That strategy cannot be  $R$ .
- (iv) That strategy cannot be  $PP$ .

For  $a \in A$ , let  $X(a) \equiv \{x \in [0, \bar{x}] \mid s_l^*(x) = P\}$ , the inverse image of any leader  $l$ 's action  $a$ .

By assumption,  $X(P)$  occurs with positive probability.

*Proof of (i).* The idea of the proof is that increasing returns to participation imply that if any follower chooses to participate, then others should also. Fix  $l \in L$  and  $a \in A$ . It is sufficient to show that  $s_f^*(a)$  takes the same value for all  $f \in F(l)$ . Suppose that this does not happen. Let  $\{F^N, F^P\}$  denote the partition of  $F(l)$ ,  $F^N$  and  $F^P$  non-empty, such that  $s_f^*(a) = N$  for all  $f \in F^N$  and  $s_f^*(a) = P$  for all  $f \in F^P$ . Let  $\psi(q_{-f})$  denote  $f$ 's (private) gain from participating after observing  $a$ , as a function of the participation rate of everyone else:<sup>1</sup>

$$\psi(q_{-f}) \equiv E_x[\beta(q_{-f}; x) \mid x \in X(a)] = \int_{x \in X(a)} \beta(q_{-f}; x) \phi(x \mid x \in X(a)) dx$$

Because  $\partial\beta/\partial q_{-f} > 0$ :  $\psi^l(q_{-f}) > 0$ . Choose  $f \in F^N$  and  $f' \in F^P$ . Then  $q_{-f} > q_{-f'}$ , implying  $\psi(q_{-f}) > \psi(q_{-f'})$ , but this contradicts the premise that follower  $f'$  chooses to participate while  $f$  does not. Therefore, every  $f \in F(l)$  adopts the same response to  $a$ .

*Proof of (ii).* The idea is that a leader without followers faces incentives similar to those of a follower, and if such a leader ever chooses to participate, then increasing returns to participation imply that followers should also. Suppose that all followers choose  $NN$ . Because no one

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<sup>1</sup>The  $\psi$  function differs from the  $h$  function of Lemma 3 because  $\psi$  describes a follower's gain from participating as a function of others' participation, holding his leader's action fixed, instead of as a function of the leader's strategy, holding participation fixed.

follows leader  $l$ , her gain from participating given  $x \in X(P)$  is  $\beta(\frac{n-1}{m-1}; x) \geq 0$ , and  $\partial\beta/\partial q_{-i} > 0$  then implies  $\beta(\frac{n}{m-1}; x) > 0$ , but that equals a follower's gain from participating given  $x \in X(P)$ . Therefore, a follower's gain from participating, after observing  $P$ , is  $E_x[\beta(\frac{n}{m-1}; x) | x \in X(P)] > 0$ , contradicting the optimality of  $NN$ .

*Proof of (iii).* The idea is similar to part (ii), except that a leader with rebellious followers has even smaller incentives to participate, and if she nevertheless participates then her followers should also. Suppose that all followers choose  $R$ . Leader  $l$ 's gain from participating given  $x \in X(P)$  is  $\pi(P, \frac{n}{m}; x) - \pi(N, \frac{n-1}{m} + r_n; x) - c \geq 0$ , and (1e) implies that this is weakly smaller than  $\pi(P, \frac{n}{m}; x) - \pi(N, \frac{n-1}{m}; x) - c = \beta(\frac{n-1}{m-1}; x)$ . Therefore,  $\beta(\frac{n-1}{m-1}; x) > 0$ , and the rest of the proof follows part (ii).

*Proof of (iv).* A follower who plays  $PP$  earns expected payoff  $E_x[\beta(q_{-i}; x)]$  given  $q_{-i}$ , but (A2) implies that this is strictly negative regardless of other players' strategies, implying that  $PP$  earns a strictly negative payoff ex ante. Because (1e) implies that  $NN$  earns a non-negative ex ante payoff,  $PP$  cannot be optimal.

### **Proof of Theorem 9:**

To prove this theorem I need to show the following:

(i) *There exists a unique threshold  $\tau_n^A \in (0, \bar{x})$  such that  $h_n^A(\tau_n^A) = 0$ .*

(ii) *There exists a unique threshold  $\tau_n^B > \tau_n^A$  such that  $h_n^B(\tau_n^B) = 0$ .*

(iii) *There exists a unique threshold  $\tau^F \in (0, \bar{x})$  such that  $\hat{r}(\tau^F) = 0$ .*

(iv) *The strategy profile  $(x^*, s_f^*, f \in F(l))$ , where  $\text{Max}\{\tau_n^A, \tau^F\} \leq x^* \leq \tau_n^B$  and  $s_f^* = C$ , is a group symmetric equilibrium.*

(v) *If  $(x^*, s_f^*, f \in F(l))$  is a group symmetric equilibrium then  $\text{Max}\{\tau_n^A, \tau^F\} \leq x^* \leq \tau_n^B$  and  $s_f^* = C$ .*

Proof of (i):

The threshold  $\tau_n^A$  is defined by:

$$h_n^A(\tau_n^A) = \alpha(1; \tau_n^A, m) + \beta(1; \tau_n^A, m) - \alpha\left(\frac{m(1 - \frac{r_n}{m}) - 1}{m - 1}; \tau_n^A, m\right) - c$$

By definition of  $\gamma$ ,  $\alpha$  and  $\beta$  we have:

$$h_n^A(\tau_n^A) = \gamma(P, 1; \tau_n^A) - \gamma(N, 1 - \frac{r_n}{m} - \frac{1}{m}; \tau_n^A) - c$$

Clearly,

$$h_n^A(0) < 0. \quad (i - a)$$

Also from assumption (1b) we have:

$$h_n^A(\bar{x}) = \gamma(P, 1; \bar{x}) - \gamma(N, 1 - \frac{r_n}{m} - \frac{1}{m}; \bar{x}) - c \quad (i - b)$$

Since  $h_n^A(x)$  is continuous and increasing in  $x$ , the inequalities (i - a) and (i - b) imply that, there exists a unique  $\tau_n^A \in (0, \bar{x})$  such that  $h_n^A(\tau_n^A) = 0$ .

Proof of (ii):

The threshold  $\tau_n^B$  is defined by:

$$h_n^B(\tau_n^B) = \alpha(\frac{r_n}{m-1}; \tau_n^B, m) + \beta(\frac{r_n}{m-1}; \tau_n^B, m) - c = \gamma(P, \frac{r_n+1}{m}; \tau_n^B) - c$$

If there is only one leader,  $r_n = m - 1$ . Then,  $h_n^A(x) = h_n^B(x)$  and therefore  $\tau_n^A = \tau_n^B$  is the unique value such that  $h_n^A(\tau_n^A) = h_n^B(\tau_n^B) = 0$ .

Suppose  $n > 1$ . Clearly,  $h_n^B(0) < 0$ . Our assumption of convenience implies that  $h_n^B(x) > 0$  for  $x$  sufficiently large enough. Since  $h_n^B(x)$  is continuous, our assumption of convenience implies that there exists a unique value  $\tau_n^B$  such that  $h_n^B(\tau_n^B) = 0$ .

Proof of (iii):

According to lemma 2,  $r(P, 1) = E_x[\beta(1; x, m) \mid P, s_l] - c$  denotes followers' expected marginal gain from participating in the project given  $q_{-l} = 1$  and  $a_l = P$ . Since we restrict our attention to threshold strategies for the leaders,  $r(P, 1)$  can be redefined as:

$$\hat{r}(\tau) = E_x[\beta(1; x, m) - c \mid x \geq \tau, s_l]$$

,where  $\tau$  is the leaders' threshold strategy.

Since,  $\frac{\partial \beta(1; x, m)}{\partial x} > 0$ , we have:

$$\begin{aligned}
\frac{\partial \hat{r}(\tau)}{\partial \tau} &= \frac{\frac{\partial E_x[\beta(1; x, m) - c \mid x \geq \tau, s_l]}{\partial \tau}}{\partial \tau} \\
&= \frac{f(\tau)}{1 - F(\tau)} \times \int_{\tau}^{\bar{x}} \frac{\beta(1; x, m) f(x) dx}{1 - F(\tau)} - \frac{\beta(1; x, m) f(\tau)}{1 - F(\tau)} \\
&> \frac{f(\tau)}{1 - F(\tau)} \times \int_{\tau}^{\bar{x}} \frac{\beta(1; x, m) f(x) dx}{1 - F(\tau)} - \frac{\beta(1; x, m) f(\tau)}{1 - F(\tau)} \\
&= \frac{f(\tau) \beta(1; x, m)}{1 - F(\tau)} \times \left[ \int_{\tau}^{\bar{x}} \frac{f(x) dx}{1 - F(\tau)} - 1 \right] = \frac{f(\tau) \beta(1; x, m)}{1 - F(\tau)} \times 0 = 0
\end{aligned}$$

Therefore,

$$\frac{\partial \hat{r}(\tau)}{\partial \tau} > 0$$

The threshold  $\tau^F$  is defined by:

$$\hat{r}(\tau) = E_x[\beta(1; x, m) \mid x \geq \tau, s_l] - c = 0$$

Clearly,

$$\hat{r}(0) < 0 \quad (iii - a)$$

We also have:

$$x^* = \bar{x} \text{ and } s_f^* = NN \text{ or } E_x[\beta(1; x, m) \mid x \geq \tau, s_l] - c > 0 \quad (iii - b)$$

Since  $\hat{r}(\tau)$  is continuous and increasing in  $\tau$ , the inequalities (iii - a) and (iii - b) imply that there exists a unique  $\tau^F \in (0, \bar{x})$  such that  $\hat{r}(\tau^F) = 0$ .

proof of (v):

To prove part (iv), we have to prove that:

(iv-a) The threshold strategy  $Max \{ \tau_n^A, \tau^F \} \leq x^* \leq \tau_n^B$  is leader  $l$ 's best response to

followers playing  $C$  and all the other leaders playing the threshold strategy  $Max \{ \tau_n^A, \tau^F \} \leq x^* \leq \tau_n^B$ .

(iv-b) The strategy  $C$  is follower  $f$ 's best response to all the other followers playing  $C$  and leaders playing the threshold strategy  $Max \{ \tau_n^A, \tau^F \} \leq x^* \leq \tau_n^B$ .

Proof of (iv-a):

Leader  $l$ 's marginal gain from participation for all  $x > x^*$ , when followers choose to play  $C$  and other leaders choose the threshold  $x^*$  is:

$$h_n^A(x) = \alpha(1; x, m) + \beta(1; x, m) - \alpha\left(\frac{m(1 - \frac{r_n}{m}) - 1}{m - 1}; x, m\right) - c$$

Clearly  $h_n^A(x) > 0$  implies that  $s_l^*(x) = P$  for  $l \in L$ .

Since  $\frac{\partial h_n^A(x)}{\partial x} > 0$ , we have:

$h_n^A(x) > h_n^A(x^*) \geq h_n^A(\tau_n^A) = 0$  for all  $x > x^*$ . Therefore,  $s_l^*(x) = P$  for all  $l \in L$  and  $x > x^*$ .

Leader  $l$ 's marginal gain from participation for all  $x < x^*$  when followers play  $C$  and other leader's choose the threshold strategy  $Max \{ \tau_n^B, \tau^F \} \leq x^* \leq \tau_n^B$  is:

$$h_n^B(x) = \alpha\left(\frac{r_n}{m - 1}; x, m\right) + \beta\left(\frac{r_n}{m - 1}; x, m\right) - c$$

Clearly,  $h_n^B(x) < 0$  implies that  $s_l^*(x) = N$  for  $l \in L$ .

Since  $\frac{\partial h_n^B(x)}{\partial x} > 0$ , we have:

$$h_n^B(x) < h_n^B(x^*) \leq h_n^B(\tau_n^B) = 0 \text{ for all } x < x^*$$

Therefore,  $s_l^*(x) = N$  for  $l \in L$  and  $x < x^*$ .

Summarizing:

$s_l^*(x) = P$  for all  $l \in L$  and  $x > x^*$ .

$s_l^*(x) = N$  for all  $l \in L$  and  $x < x^*$ .

Therefore,  $Max \{ \tau_n^A, \tau^F \} \leq x^* \leq \tau_n^B$  is leader  $i$ 's best response to followers playing  $C$  and all the other leaders playing the threshold strategy  $Max \{ \tau_n^A, \tau^F \} \leq x^* \leq \tau_n^B$ .

Proof of (iv-b):



If follower  $f$  knows that other followers play  $C$  and leaders play the threshold strategy  $Max \{ \tau_n^A, \tau^F \} \leq x^* \leq \tau_n^B$ , his expected gain from participation in the project when he observes  $a_l = P$  is:

$$\hat{r}(x^*) = E_x[\beta(1; x, m) \mid x \geq x^*, s_l] - c$$

Clearly  $\hat{r}(x) > 0$  implies that  $s_f^*(x) = P$  for all  $f \in F(l)_{l \in L}$ .

Since  $\frac{\partial \hat{r}(x)}{\partial x} > 0$ , we have:

$$\hat{r}(x^*) \geq \hat{r}(\tau^F) = 0$$

This implies that  $s_f^*(x) = P$  is optimal for all  $f \in F(l)_{l \in L}$ .

Follower  $f$ 's expected gain from participation in the project when he observes  $a_l = N$  is:

$$\hat{r}(x^*) = E_x[\beta(0; x, m) \mid x < x^*, s_l] - c < 0$$

This implies that  $s_f^*(N) = N$  is optimal for all  $f \in F(l)_{l \in L}$ .

Summarizing:  $s_f^*(P) = P$  and  $s_f^*(N) = N$  is optimal for all  $f \in F(l)_{l \in L}$ .

Therefore,  $C$  is follower's best response to all the other followers playing  $C$  and leaders playing the threshold strategy  $Max \{ \tau_n^A, \tau^F \} \leq x^* \leq \tau_n^B$ .

Proof of (v):

*To prove part (v) we have to prove that:*

*(v-a) There is no equilibrium, where leaders participate with probability one.*

*(v-b) Any equilibrium at which leaders participate with probability zero must be of the kind described in part a.*

*It remains the equilibrium where leaders play  $P$  with probability strictly between zero and one. In this case based on lemma 3 followers play  $C$ .*

*(v-c) The threshold  $x^*$  can not be less than  $\tau_n^A$ .*

*(v-d) The threshold  $x^*$  can not be less than  $\tau^F$ .*

*(v-e) The threshold  $x^*$  can not be greater than  $\tau_n^B$ .*

Proof of (v-a):

By assumption  $\gamma(P, q; x)$  is a continuous function of  $x$  and  $\gamma(P, q; 0) = 0$  for all  $q \in [0, 1]$ . This implies that  $\gamma(P, q; x) - c < 0$  for all  $q \in [0, 1]$  and  $x \in [0, \varepsilon)$  for some  $\varepsilon > 0$ . Therefore, the dominant strategy for values of  $x \in [0, \varepsilon)$  is not to participate. Since  $\text{prob}(\varepsilon) > 0$ , there is no equilibrium where leaders participate with probability one.

Proof of (v-b):

Consider an equilibrium such that leaders participate with probability zero. Under these circumstances followers are effectively uninformed and therefore choose not to participate according to assumption (\*). Therefore, such an equilibrium must be of the kind described in part a.

Proof of (v-c):

Consider  $x^* < \tau_n^A$ . Since  $\frac{\partial h_n^A(x)}{\partial x} > 0$ , there exists an  $x' \in (x^*, \tau_n^A)$  such that  $h_n^A(x^*) < h_n^A(x') < h_n^A(\tau_n^A) = 0$ . Therefore,  $h_n^A(x') < 0$  for  $x' \in (x^*, \tau_n^A)$ , which implies that  $s_l(x') = N$  for the leader. This contradicts the definition of  $x^*$ . Thus  $x^* < \tau_n^A$  is not the leader's best response to the strategy  $C$  played by the followers.

Proof of (v-d):

If  $\tau^F < \tau_n^A$ , then  $x^*$  can not be less than  $\tau^F$  by proof of part (v-c). Otherwise  $\frac{\partial \hat{r}(x)}{\partial x} > 0$ , implies that  $\hat{r}(x) < \hat{r}(\tau^F) = 0$  for all  $x \in (x^*, \tau^F)$ .

Therefore,  $s_f(P) = N$  for all  $x \in (x^*, \tau^F)$  and all  $f \in F(l)_{l \in L}$ .

Proof of (v-e):

Consider  $x^* > \tau_n^B$ . There exists an  $x' \in (\tau_n^B, x^*)$  such that  $h_n^B(x') > h_n^B(\tau_n^B) = 0$ . This implies that  $s_l(x') = P$  for all  $l \in L$ , which contradicts the definition of  $x^*$ .

### Proof of Proposition 10:

Recall that by definition:

$$\gamma(P, 1; \tau_n^A) - \gamma(N, 1 - \frac{r_f}{m} - \frac{1}{m}; \tau_n^A) - c = 0$$

$$\gamma(P, \frac{r_f + 1}{m}; \tau_n^B) - c = 0$$

Since,  $r_f = \frac{m}{n} - 1$  we have:

$$\gamma(P, 1; \tau_n^A) - \gamma(N, 1 - \frac{1}{n}; \tau_n^A) - c = 0$$

$$\gamma(P, \frac{1}{n}; \tau_n^B) - c = 0$$

Now suppose that  $n' > n$  but  $\tau_{n'}^A < \tau_n^A$ . Then (1f) and (1e) implies that:

$$\gamma(P, 1; \tau_n^A) - \gamma(N, 1 - \frac{1}{n}; \tau_n^A) - c > \gamma(P, 1; \tau_n^A) - \gamma(N, 1 - \frac{1}{n'}; \tau_n^A) - c$$

$$\gamma(P, 1; \tau_{n'}^A) - \gamma(P, 1 - \frac{1}{n'}; \tau_{n'}^A) - c = 0$$

, which is a contradiction. Therefore,  $\tau_{n'}^A > \tau_n^A$  if  $n' > n$ .

Now suppose  $n' > n$  but  $\tau_{n'}^B < \tau_n^B$ . Then  $\frac{\partial \gamma(a_i, q, x)}{\partial x} > 0$  and  $\frac{\partial \gamma(a_i, q, x)}{\partial q} > 0$  imply that:

$$\gamma(P, \frac{1}{n}; \tau_n^B) - c > \gamma(P, \frac{1}{n}; \tau_{n'}^B) - c > \gamma(P, \frac{1}{n'}; \tau_{n'}^B) - c = 0$$

, which is a contradiction. Therefore,  $\tau_{n'}^B > \tau_n^B$  if  $n' > n$ .

### Proof of Theorem 11:

The definition of  $\tau_1$  implies that  $\pi(P, 1; \tau_1) = c$ . Given (1f), this implies:

$$\widehat{W}'(x^*) = -[\pi(P, 1; x^*) - c] \phi(x^*) \leq 0 \quad \text{for } x^* \geq \tau_1$$

Proposition 8 shows that  $x^* \geq \tau_1$  for all  $x^* \in X^*(n)$ , any  $n$ , and Theorem 5 shows that  $x^* = \tau_1$  maximizes  $\widehat{W}(x^*)$ . Therefore,  $\widehat{W}'(x^*) \leq 0$  is for  $x^* \in [\tau^F, \bar{x}] \cap X^*(n)$ , any  $n$ . Theorem 7 and Proposition 8 imply that the interval  $X^*(n)$  always owns a point smaller than  $\bar{x}$ , and Theorem 1 and Lemma 3 imply that  $\tau^F < \bar{x}$ , implying that  $[\tau^F, \bar{x}] \cap X^*(n) = \emptyset$  iff  $x < \tau^F$  for all  $x \in X^*(n)$ . Because  $\tau_1^B = \tau_1$  and  $\tau_m^B > \bar{x}$  (from Lemma 3), this condition holds iff  $\tau^F > \tau_m^B$ , and Proposition 8 implies that the last condition holds iff  $n < n^*$ . Therefore,  $\overline{V}(n) = \underline{V}(n) = 0$

for  $n < n^*$ . Clearly  $\overline{V}(n) \geq 0$  and  $\underline{V}(n) \geq 0$  for all  $n$ . For  $n \geq n^*$ ,  $[\tau^F, \overline{x}] \cap X^*(n) \neq \emptyset$  and Proposition 8 shows that its minimal and maximal elements are increasing in  $n$  for  $n \geq n^*$ . Because  $\widehat{W}'(x^*) \leq 0$  on that domain,  $\overline{V}(n)$  and  $\underline{V}(n)$  are both decreasing in  $n$  for  $n \geq n^*$ . Therefore,  $\overline{V}(n)$  and  $\underline{V}(n)$  are both maximized at  $n = n^*$ .

**Proof of Theorem 12:**

To prove this theorem we need to show the following:

(i) *There exists a unique threshold  $\tau_{l1} \in (0, \overline{x})$  such that  $h_{l1}(\tau_{l1}) = 0$ .*

(ii) *There exists a unique threshold  $\overline{\tau}^F \in (0, \overline{x})$  such that  $\widehat{r}(\overline{\tau}^F) = 0$ .*

(iii) *The strategy profile  $(x^*, s_f^*)_{f \in F(l)}$ , where  $x^* = \tau_{l1}$  and  $\tau_{l1} \geq \overline{\tau}^F$  and  $s_f^* = C$ , is a symmetric equilibrium.*

(iv) *If  $(x^*, s_f^*)_{f \in F(l)}$  is a group symmetric equilibrium then it satisfies (a) or (b).*

Proof of (i):

See the proof of part (i) from theorem 9.

Proof of (ii):

See the proof of part (ii) from theorem 9.

Proof of (iii):

To prove part (iii), we have to prove that:

(iii-a) The threshold strategy  $x^* = \tau_{l1}(\tau_{l1} \geq \overline{\tau}^F)$  is leader i's best response to followers playing  $C$ .

(iii-b) The strategy  $C$  is follower j's best response to all the other followers playing  $C$  and the leader playing the threshold strategy  $x^* = \tau_{l1}(\tau_{l1} \geq \overline{\tau}^F)$ .

Proof of (iii-a):

Leader  $l$ 's marginal gain from participation when followers choose to play  $C$  is:

$$h_{l1}(x) = \alpha(1; x, m) + \beta(1; x, m) - c_l$$

Clearly  $h_{l1}(x) > 0$  implies that  $s_l^*(x) = P$  for  $l \in L$  and  $h_{l1}(x) < 0$  implies that  $s_l^*(x) = N$  for  $l \in L$ .

Since  $\frac{\partial h_{l1}(x)}{\partial x} > 0$ , we have:

$h_{l1}(x) > h_{l1}(x^*) = h_{l1}(\tau_{l1}) = 0$  for all  $x > x^*$  and  $h_{l1}(x) < h_{l1}(x^*) = h_{l1}(\tau_{l1}) = 0$  for all  $x < x^*$ . Therefore,  $s_l^*(x) = P$  for all  $l \in L$  and  $x > x^*$ , and  $s_l^*(x) = N$  for all  $l \in L$  and  $x < x^*$ .

Proof of (iii-b):

If follower  $j$  knows that other followers play  $C$  and the leader plays the threshold strategy  $x^* = \tau_{l1}(\tau_{l1} \geq \bar{\tau}^F)$ , his expected gain from participation in the project when he observes  $a_l = P$  is:

$$\hat{r}(x^*) = E_x[\beta(1; x, m) \mid x \geq x^*, s_l] - c$$

Clearly  $\hat{r}(x) > 0$  implies that  $s_f^*(x) = P$  for all  $f \in F(l)_{l \in L}$ .

Since  $\frac{\partial \hat{r}(x)}{\partial x} > 0$ , we have:

$$\hat{r}(x^*) \geq \hat{r}(\bar{\tau}^F) = 0$$

This implies that  $s_f^*(x) = P$  is optimal for all  $f \in F(l)_{l \in L}$ .

Follower  $j$ 's expected gain from participation in the project when he observes  $a_l = N$  is:

$$E_x[\beta(\frac{1}{m}; x, m) \mid x < x^*, s_l] - c_f < 0$$

This implies that  $s_f^*(N) = N$  is optimal for all  $f \in F(l)_{l \in L}$ .

Proof of (iv):

To prove part (iv) we have to prove that:

(iv-a) *There is no equilibrium, where leaders participate with probability one.*

(iv-b) *Any equilibrium at which leaders participate with probability zero must be of the kind described in part a.*

(iv-c) *It remains the equilibrium where leaders play  $P$  with probability strictly between zero and one. In this case we can show that there is no equilibrium where all followers play  $PP$ ,  $R$  or  $NN$ .*

(iv-d) *The threshold  $x^*$  can not be less or greater than  $\tau_{l1}$ .*

(iv-e) *The threshold  $x^*$  can not be greater than  $\bar{\tau}^F$ .*

Proof of (iv-a):

By assumption  $\gamma(P, q; x)$  is a continuous function of  $x$  and  $\gamma(P, q; 0) = 0$  for all  $q \in [0, 1]$ . This implies that  $\gamma(P, q; x) - c_i < 0$  for all  $q \in [0, 1]$  and  $x \in [0, \varepsilon)$  for some  $\varepsilon > 0$ . Therefore, the dominant strategy for values of  $x \in [0, \varepsilon)$  is not to participate. Since  $\text{prob}(\varepsilon) > 0$ , there is no equilibrium where leaders participate with probability one.

Proof of (iv-b):

See the proof of part (v-b) in theorem 9.

Proof of (iv-c):

(iv-c-1) Followers do not choose to play  $PP$  at equilibrium (see lemma 10 for the proof).

(iv-c-2) There is no equilibrium where all followers play  $R$ . Suppose that  $s_f^* = R$  for all  $f \in F(l)$ . Let  $\chi = \{x : a_l(x) = P\}$  for any leader  $l \in L$ . By assumption  $x \in \chi$  occurs with positive probability. If  $x \in \chi$  occurs then optimization for leader  $l$  implies that:

$$\gamma(P, \frac{1}{m}; x) - c_l \geq \gamma(N, \frac{m-1}{m}; x)$$

Assumption (1e) implies that:

$$\gamma(P, \frac{2}{m}; x) > \gamma(P, \frac{1}{m}; x)$$

$$\gamma(N, \frac{m-1}{m}; x) > \gamma(N, \frac{1}{m}; x)$$

The above inequalities all together imply that:

$$\gamma(P, \frac{2}{m}; x) - \gamma(N, \frac{1}{m}; x) - c_l > 0$$

or

$$\beta(\frac{1}{m-1}; x, m) - c_l > 0$$

Taking conditional expectations from both sides if the inequality we have:

$$\frac{E}{x} \left[ \beta(\frac{1}{m-1}; x, m) \mid P, s_l \right] - c_l > 0$$

Therefore,

$$E_x \left[ \beta\left(\frac{1}{m-1}; x, m\right) \mid P, s_l \right] - c_f > 0 \text{ for all } c_f < c_l$$

Then Lemma 3 immediately implies that  $s_f^*(P) = P$  for all followers with  $c_f < c_l$ , which is a contradiction. Therefore there is no equilibrium where all followers play  $R$ .

(iv-c-3) There is no equilibrium where all followers play  $NN$ .

Suppose  $s_f^* = NN$  for all  $f \in F(l)$ . If  $x \in \chi$  occurs, then the optimization for any leader requires that:

$$\gamma\left(P, \frac{1}{m}; x\right) - c_l = \beta(0; x, m) - c_l \geq 0$$

Since  $\frac{\partial \beta(q_{-i}; x, m)}{\partial q_{-i}} > 0$  we have:

$$\beta\left(\frac{1}{m}; x, m\right) - c_l > 0$$

By taking expectations we have:

$$E_x \left[ \beta\left(\frac{1}{m-1}; x, m\right) \mid P, s_l \right] - c_l > 0$$

Therefore,

$$E_x \left[ \beta\left(\frac{1}{m-1}; x, m\right) \mid P, s_l \right] - c_f > 0 \text{ for all } c_f < c_l$$

Then Lemma 3 immediately implies that  $s_f^*(P) = P$  for all followers with  $c_f < c_l$ , which is a contradiction. Therefore there is no equilibrium where all followers play  $NN$ .

Proof of (iv-d):

Consider  $x^* < \tau_{l1}$ . Since  $\frac{\partial h_{l1}(x)}{\partial x} > 0$ , there exists an  $x' \in (x^*, \tau_{l1})$  such that if  $h_{l1}(x^*) < h_{l1}(x') < h_{l1}(\tau_{l1}) = 0$ . Therefore,  $s_l^*(x') = N$  for leader  $l$ . This contradicts the definition of  $x^*$ . Thus  $x^* < \tau_{l1}$  is not leader  $l$ 's best response to the strategy  $C$  played by the followers.

Now consider  $x^* \geq \tau_{l1}$ . Then, there exists an  $x' \in (\tau_{l1}, x^*)$  such that  $h_{l1}(x') > h_{l1}(\tau_{l1}) = 0$ . This implies that  $s_l(x') = P$  for leader  $l$ , which contradicts the definition of  $x^*$ .

Proof of (iv-e):

In the equilibrium where all followers follow the same strategy,  $x^*$  can not be less than  $\bar{\tau}^F$ . Suppose  $x^* <^F$ , then  $\frac{\partial \hat{r}(x)}{\partial x} > 0$ , implies that  $\hat{r}(x) < \hat{r}(\bar{\tau}^F) = 0$  for all  $x \in (x^*, \bar{\tau}^F)$ . Thus,  $s_f(P) = N$  for  $f \in F(l)$  with  $c_f \in (c_l, \bar{c})$ .

**Proof of Lemma 13:**

If  $r < A$ , then  $\frac{\partial^2 [mq\gamma(P, q; x) + m(1 - q)\gamma(N, q; x)]}{m\partial q^2} - rm^2 > 0$ . Therefore,  $\frac{\partial^2 w(q, x)}{m\partial q^2} > 0$  which implies that the efficient outcome obtains at  $q = 0$  or  $q = 1$ .

**Proof of Theorem 14**

Let  $w(1, x) = m\alpha(1; x, m) + m\beta(1; x, m) - \sum_{i \in I} c_i$  denote the total surplus produced by full participation. Since  $c_a = \frac{\sum_{i \in I} c_i}{m}$ , we have:  $w(1, x) = m[\alpha(1; x, m) + \beta(1; x, m) - c_a]$ .

By definition we know that  $w(1, \tau_{a1}) = m[\alpha(1; \tau_{a1}, m) + \beta(1; \tau_{a1}, m) - c_a] = 0$ . Moreover,  $\frac{\partial \alpha(1; x, m)}{\partial x} > 0$  and  $\frac{\partial \beta(1; x, m)}{\partial x} > 0$  imply that  $\frac{\partial w(q, x)}{\partial x} > 0$ . Thus,  $w(q, x) > 0$  for all  $x > \tau_{a1}$  and  $w(q, x) < 0$  for all  $x < \tau_{a1}$ . Since  $r < A$ , Lemma 15 implies that the efficient outcome obtains at  $q = 1$  and  $q = 0$  for  $x > \tau_{a1}$  and  $x < \tau_{a1}$  respectively. Choose  $c_l = c_a$ , then  $\tau_{l1} = \tau_{a1}$ . Because  $\bar{\tau}_f \leq \tau_{a1}$ , the existence of type (b) equilibrium from Theorem 12 implies that the leader can induce a first best outcome.

Claim: Leader  $l$ 's cost of participation can not be smaller than  $c_a$ . To prove this consider a scenario where,  $c_l < c_a$ . Then, Proposition 15 implies that  $\tau_{l1} < \tau_{a1}$ . Since  $\frac{\partial w(q, x)}{\partial x} > 0$  we have  $w(1, \tau_{l1}) < w(1, x) < w(1, \tau_{a1}) = 0$  for  $x \in (\tau_{l1}, \tau_{a1})$ . Under these circumstances the existence of type (b) equilibrium induces a negative surplus which is an inefficient outcome.

Leader  $l$ 's cost of participation can not be larger than  $c_a$ . To prove this consider a scenario where,  $c_l > c_a$ . Then, proposition 15 implies that  $\tau_{l1} > \tau_{a1}$ . Since  $\frac{\partial w(q, x)}{\partial x} > 0$  we have  $w(1, \tau_{l1}) > w(1, x) > w(1, \tau_{a1}) = 0$  for  $x \in (\tau_{a1}, \tau_{l1})$ . Therefore, the efficient outcome is for leader  $l$  to participate in the project for  $x \in (\tau_{a1}, \tau_{l1})$ . Leader  $l$ , however, refuses to participate in the project which results to no participation and zero surplus.

**Proof of Proposition 15:**

Let  $c_l > c_k$  but  $\tau_l \leq \tau_k$ . Then  $\frac{\partial \alpha(q; x, m)}{\partial x} > 0$  and  $\frac{\partial \beta(q; x, m)}{\partial x} > 0$  imply that:  $\alpha(1; \tau_{l1}, m) + \beta(1; \tau_{l1}, m) - c_l < \alpha(1; \tau_{k1}, m) + \beta(1; \tau_{k1}, m) - c_k = 0$  which contradicts the definition of  $\tau_1$  in Theorem 12.



**Proof of Theorem 16:**

If  $c_l < MinC^c$ , then according to proposition 15,  $\tau_{l1} > \bar{\tau}_f$ . Therefore, the leader is not powerful. In this case, nobody participates in the project and  $w(0, x) = 0$ .

If  $c_l < MinC^c$ , then according to lemma 15,  $\tau_{l1} > \tau_{Min1}$ . Since by assumption  $\frac{\partial w(q, x)}{\partial x} > 0$  and  $c_a < MinC^c$  we have:  $w(1, \tau_{l1}) > w(1, x) > w(1, \tau_{Min1}) > 0$  for  $x \in (\tau_{Min1}, \tau_{l1})$ . Therefore, full participation is efficient for  $x \in (\tau_{Min1}, \tau_{l1})$ . Leader  $l$ , however, refuses to participate; Thus the surplus obtained from a symmetric equilibrium will be zero for  $x \in (\tau_{Min1}, \tau_{l1})$ .

Summarizing: Choosing a leader with  $c_l < MinC^c$  or  $c_l > MinC^c$  reduces the total surplus obtainable by a symmetric equilibrium and therefore  $c_i = MinC^c$ .

## Chapter 3

# Leadership in Public Good Projects

The private provision of public goods is an area of much study in public economics. Several researchers have examined the private provision of public goods in simultaneous-move Nash games in which all agents choose their contribution levels without knowledge of others' contribution decisions. A standard result of this theoretical research on public goods is that pure public goods are under-provided by voluntary contributions of private individuals. For details see Bergstrom, Blume and Varian (1985), Warr (1983), Bergstrom (1986) and Corns and Sandler (1984,85).

Varian (1992) addresses the role of leadership in public projects by considering a Stackelberg contribution game in which players sequentially contribute to a public project with an additive nature. In his model, the leader's move is observable to the followers and therefore the leader can credibly commit to his contribution in a way that it is not possible in a simultaneous move game. The main finding of Varian is that the total contribution in sequential-move games is at most as large as that of a simultaneous-move game. The reason is that if donations are observed and the leader commits to a one time contribution, then he can effectively free ride on his follower by committing to a low initial donation. Varian first shows this result with a quasi-linear utility function and then proves it for any differentiable, strictly concave utility function.

Leaders, however, increase the overall contribution in some public projects. Sung (1998)

shows that Varian's result is reversed for weakest link public goods<sup>1</sup> and does not necessarily hold for best shot public good projects<sup>2</sup>. Romano and Yildirim (1998) show that sequential-move contributions lead to larger donations if the utility function is increasing in the donations and the followers' best response function is increasing in the contribution of the leader.

The closest work to my model is Hermalin (1998), and Vesterlund (2000). Hermalin's work is explained in the first chapter. His work is in the spirit of Holmstrom complete contracts team model (1982). He investigates a team production problem using a sequential-move game in which a player (the leader) is exogenously informed about the marginal return to effort. He suggests two ways in which a leader can convince his followers to put in more effort in organizational activities. One is leader's sacrifice: The leader offers gifts to the followers. Followers respond not because they want the gifts themselves, but because the leader's sacrifice convinces them that she must truly consider this to be a worthwhile activity. Another way to convince followers in Hermalin's opinion is leading by example: the leader himself puts in long hours on the activity, thereby convincing the followers that she indeed considers it worthwhile. In both scenarios the leader's action fully reveals his information about the rate of return to effort and increases the overall contribution above the contribution level of complete information scenario.

Vesterlund's model investigates sequential fund-raising and the role of the fund-raiser in fund-raising games under incomplete information. The hypothesis of her paper is that when the information about the quality of the charity is unknown, an announcement strategy for a high type charity is successful because it helps reveal the information about the quality of the charity. Vesterlund's paper assumes that the first contributor obtains costly information about the charity's quality. She argues that when the information cost is sufficiently low, a first mover who is informed with a high signal fully reveals the value of the charity through a large initial contribution. This increase in contribution decreases the donation of the second contributor but the overall contribution level exceeds the complete information scenario.

The above modelling efforts by Vesterlund and Hermalin are logical and interesting and

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<sup>1</sup>The weakest-link public good has its aggregate supply level defined as the minimum of all individual contribution levels.

<sup>2</sup>The best-shot public good, has a social composition level being equal to the maximum of all individual contributions. In other words, only the best counts.

produce somewhat counter-intuitive conclusions. The basic thrust of both papers is to show how the leader is able to induce higher participation by fully transmitting his information to the other players via a welfare increasing signal.

The model in this chapter changes the above theme in the following way: I argue that ill-informed followers tend to be more cooperative, when the leader is unable to credibly reveal all of his information. The reason is that they do not know when their cooperative actions actually produce high personal payoffs.

To show this, I consider a setup where the leader has two strategies. First, he decides whether to make a costly commitment to the public project and then he decides how much to contribute. A key assumption is that followers only observe the leaders' commitment signal but are unable to verify the leaders' exact amount of contribution. This prevents full revelation and allows the leader to persuade his followers into participating in cases where they would be unwilling to participate if they were fully informed. As in chapter 2 we will see that followers benefit from participation. Partial revelation of information therefore, is more efficient than full revelation or complete information. The idea of this chapter is similar to that of chapter 2 but my new model is different and somewhat complementary to the previous model. One main distinction is that the participation decision in the new model is continuous rather than discrete. That is this chapter generalizes the contribution decision. The payoff structure in this model is, however, more restrictive; that is, in this model I restrict my attention to quasi-linear payoff functions.

This change in structure changes the ex-post contributions and welfare results: the ex-post results are ambiguous in some cases; the ex-ante contribution and welfare results, however, still support the idea that partial revelation of information by a powerful leader is more efficient than full revelation or complete information.

My model does not explain leading by example as in Vesterlund's or Hermalin's papers, for the leader's contribution level is not observable by the followers. Also, in this model the leader sends an unproductive signal, which is not the case in Vesterlund's and Hermalin's models.

This chapter is organized as the following. Section 3.1, introduces a game theoretic model of public goods provision under complete information. I consider a public good game in which information about the marginal return to the public project is common knowledge and players

simultaneously decide how much to contribute to the project. I show that the standard incentive conflicts and free-riding problem lead to under-provision of the public good in cases, where participation is the efficient outcome. Section 3.2 introduces an alternative scenario in which only one player (the leader) is exogenously informed about the value of the project and is followed by uninformed followers. I show that a powerful leader is able to affect other players' behavior by affecting their posterior beliefs via a costly signal. Section 3.3 shows that partial revelation of information by a powerful leader induces full cooperation among the players and increases the overall contribution ex-ante. I also show that the leader increases the ex-post contributions in some cases where cooperation is not supported under complete information. Section 3.4 analyzes a scenario in which partial revelation of information improves efficiency (the efficiency result in this model is weaker than that of chapter 2 due to the continuity in the participation decision; that is, the leader is not able to induce the unconstrained first best). Section 3.5 concludes the results and investigates possible extensions.

### 3.1 Model 1: Provision of Public Goods under Complete Information

In this section, as a benchmark, a simple model is developed in which a homogeneous population of players contribute to a public project. I analyze a complete information scenario in which all agents are equally informed about the marginal return to the public project and simultaneously decide how much to contribute.

Consider  $m + 1$  identical players  $i \in I = \{0, 1, 2, \dots, m\}$ . Each player divides his endowment  $w$  between consumption of a private good,  $y_i \geq 0$ , and contribution to a public project,  $x_i \geq 0$ . Therefore  $x_i = w - y_i$ .

The utility function of each player has the following form:

$$V(x_0, x_1, \dots, x_n, y_i) = \alpha \sum_{j=0}^m x_j + U(y_i)$$

where  $\alpha$  is the marginal return to the aggregate contribution to the public good<sup>3</sup>. I assume

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<sup>3</sup>I could have considered a concave utility function which is more general. Considering a concave utility

that  $\alpha$  is distributed on the interval  $[0, \bar{\alpha}]$  with the density function  $f(\alpha)$ . I also assume that  $U : \Re_+ \rightarrow \Re$  is a  $C^3$  function, which is strictly concave ( $U'' < 0$ ) and strictly monotone increasing ( $U' > 0$ ) over the interval  $[0, w]$ . Furthermore, I assume that  $\lim_{y \rightarrow 0} U'(y) = +\infty$  and  $\lim_{y \rightarrow \infty} U'(y) = 0$  and players' absolute risk aversion is non increasing, implying  $U''' > 0$ <sup>4</sup>.

Substituting for  $y_i$ , players' utility function can be written as:

$$\widehat{V}(x_0, x_1, \dots, x_n) = \alpha \sum_{j=0}^m x_j + U(w - x_i)$$

I consider a simultaneous move game in which  $\alpha$  is determined by the nature. All players observe the realized value of  $\alpha$  and simultaneously decide how much to contribute to the public project.

A Nash equilibrium of the game is a vector of contributions  $(\tilde{X}(\alpha))$  such that

$$\tilde{X}(\alpha) = \text{Arg max}_{x_i} \alpha \sum_{j=0}^m x_j + U(w - x_i) \quad (2a)$$

Consider the maximization problem of a representative player. Each player decides to make a zero or positive contribution. If the marginal return to the public project is small, such that the player's marginal utility of money exceeds the marginal return to the project, then he won't have any incentive to contribute to the project. For players to make a positive contribution, the marginal return to the public project has to be at least as large as their marginal utility of money. The following proposition represents this fact. Proposition 17 introduces players' Contribution Threshold under complete information: the smallest value of  $\alpha$  above which players are willing to make a positive contribution to the project. Most proofs including that of Proposition 17 are collected in the appendix.

**Proposition 17** *There exists a threshold  $\tilde{\alpha}$  such that*

$$X(\alpha) = 0 \quad \text{if } \alpha < \tilde{\alpha}$$

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function, however, introduces more complexities but no new phenomenon of interest.

<sup>4</sup>Player  $i$ 's absolute risk aversion is non increasing if  $\frac{-U'''U' + U''^2}{U'^2} \leq 0$ . Since  $u' > 0$  by assumption,  $U'''$  should be positive for the inequality to be held.

$$X(\alpha) = 1 \quad \text{if } \alpha > \tilde{\alpha}$$

*See the appendix for the proof.*

The threshold  $\tilde{\alpha}$  represents the smallest value of  $\alpha$  above which players are willing to make a positive contribution. For  $\alpha > \tilde{\alpha}$ , players' optimal contribution level  $\tilde{X}(\alpha)$  is positive and results from the following first order condition:

$$\alpha = \frac{\partial U}{\partial x_i}$$

This condition states that player  $i$ 's optimal contribution level must set his marginal utility of money equal to the marginal return to the public project. Since  $U'' < 0$ ,  $\lim_{y \rightarrow 0} U'(y) = \infty$  and  $\lim_{y \rightarrow \infty} U'(y) = 0$ , for  $\alpha > \tilde{\alpha}$  an interior solution exists and the above first order condition is necessary and sufficient for the existence of a unique maximum. The following lemma shows that once player  $i$  decides to participate to the public project, his contribution is a concave function of  $\alpha$ . This lemma is used in future proofs.

**Lemma 18** *For any  $\alpha > \tilde{\alpha}$  player  $i$ 's optimal contribution  $\tilde{X}(\alpha)$  is a concave function of  $\alpha$ .*

*See the appendix for the proof.*

For  $\alpha < \tilde{\alpha}$ , the dominant strategy is not to contribute to the project. One motivation for my work is the observation (shown below) that players may refuse to make a positive contribution even when it is efficient to do so.

To see this define

$$\Delta W(\alpha) = (m + 1) [\alpha(m + 1)X(\alpha) + U(w - X(\alpha)) - U(w)]$$

to be the welfare gain obtained by the group for the positive contribution  $X(\alpha)$ . If  $\Delta W(\alpha) > 0$  for some  $\alpha < \tilde{\alpha}$ , then the efficient outcome is to contribute to the public project. Positive contribution, however, is not supported at the equilibrium by the implication of Lemma 1. This illustrates the standard free-riding problem that leads to underprovision of public goods under complete information.

In the next section I show that the standard cooperation problem in this public project can be solved by a leader who is given exclusive information about the value of  $\alpha$  but is unable to fully transmit this information to the others.

### 3.2 Model 2: Provision of Public Goods with a Single Leader

This section pursues the idea that a leader can increase efficiency via partial revelation of information rather than full revelation. That is, a leader can improve cooperation and efficiency by sending a vague rather than a precise signal.

One important consideration as also mentioned in chapter 2, is the power of the leader. The power of the leader has extensively been addressed in the previous chapter. Creating information asymmetry does not improve efficiency if the leader is not powerful (convincing).

I consider the model from section 3.1 but revise the timing and information structure of the game. Now I consider a sequential-move game under asymmetric information. At the beginning of the game, nature determines the value of the public project  $\alpha$ . The realized value of  $\alpha$  is observed only by one player (the leader). The distribution of  $\alpha$  is assumed to be common knowledge.

In the first stage of the game, the leader decides whether to commit and contribute to the project. The leader's commitment strategy is shown by  $C : [0, \bar{\alpha}] \rightarrow \{0, 1\}$ . The value of  $C(\alpha)$  is equal to 1 if the leader makes a commitment to the public project and 0 if he does not. The leader's contribution strategy is  $X_0 : [0, \bar{\alpha}] \rightarrow \mathfrak{R}_+$ . The value of  $X_0(\alpha)$  is 0 if the leader does not contribute to the project and is a positive number if he does. The leader's commitment is assumed to be costly (One might think of it as the reputation that the leader loses if he makes a commitment to a low return project). The commitment cost  $R : [0, \bar{\alpha}] \rightarrow \mathfrak{R}_+$  is specified as:

$$R(\alpha) = m\theta C(\alpha)r(\alpha)$$

As the reader can see,  $R(\alpha)$  depends on four components: The number of followers,  $m$ , an exogenous scaling factor,  $\theta > 0$ , the leader's commitment choice,  $C(\alpha)$ , and a reputation loss function  $r : [0, \bar{\alpha}] \rightarrow \mathfrak{R}_+$ .

I assume that the reputation loss function is decreasing in  $\alpha$  ( $r' < 0$ ), meaning that a leader



who commits to a higher quality project is less likely to lose his reputation. I also assume that the reputation loss  $r(\alpha)$ , is a convex function of  $\alpha$  for all  $\alpha < \tilde{\alpha}$  and takes the value of zero for  $\alpha > \tilde{\alpha}$ . That is, the leader does not lose his reputation for committing to high value projects to which individuals are willing to contribute under complete information.

At the end of the first stage followers observe the leader's commitment but are unable to observe his amount of contribution. Note that the leader can not credibly reveal  $\alpha$  to his followers even if he wishes to.

In the second stage of the game, having observed the leader's actions, followers update their beliefs about the value of  $\alpha$  and simultaneously decide how much to contribute to the public project. The followers' strategy is  $X : \{0, 1\} \longrightarrow \mathfrak{R}_+$ .

The leader's and followers' utility functions are

$$V_l(x_0, x_1, \dots, x_n) = \alpha \sum_{j=0}^m x_j + U(w - x_l) - R(\alpha)$$

and

$$V_f(x_0, x_1, \dots, x_n) = \alpha \sum_{j=0}^m x_j + U(w - x_f)$$

respectively.

Let  $A$  be a measurable subset of  $[0, \bar{\alpha}]$  and  $\mu(A) = \text{prob}(\alpha \in A)$ . Given  $C(\alpha)$ , define  $A_0 = \{\alpha \in [0, \bar{\alpha}] : C(\alpha) = 0\}$  and  $A_1 = \{\alpha \in [0, \bar{\alpha}] : C(\alpha) = 1\}$ .

A Pure Strategy Perfect Bayesian Equilibrium of the game is the strategy profile

$(X_l^*(\alpha), X_f^*(C^*(\alpha)), C^*(\alpha))$  and the posterior beliefs  $\mu(A | A_0)$  and  $\mu(A | A_1)$  such that:

$$X_l^*(\alpha) = \underset{X_l}{\text{Arg max}} \alpha [X_l(\alpha) + mX_f^*(C^*(\alpha))] + U(w - X_l(\alpha)) \text{ for all } \alpha \quad (2b)$$

$$\begin{aligned} X_f^*(C^*(\alpha)) &= \underset{X_f}{\text{Arg max}} E(\alpha | C^*(\alpha)) \left[ X_l^*(\alpha) + \sum_{j=1}^m X_j(C^*(\alpha)) \right] + U(w - X_f(C^*(\alpha))) \\ &\text{For all } C^*(\alpha) \end{aligned} \quad (2c)$$

$$\begin{aligned}
C^*(\alpha) &= 1 & \text{if} & & \alpha m [X_f^*(1) - X_f^*(0)] - m\theta r(\alpha) > 0 \\
C^*(\alpha) &= 0 & \text{if} & & \alpha m [X_f^*(1) - X_f^*(0)] - m\theta r(\alpha) < 0
\end{aligned} \tag{2d}$$

*for all  $\alpha$*

$$\mu(A | A_0) = \frac{\mu(A)\mu(A_0 | A)}{\mu(A_0)} \quad \text{for all measurable } A \in [0, \bar{\alpha}] \tag{2e}$$

$$\mu(A | A_1) = \frac{\mu(A)\mu(A_1 | A)}{\mu(A_1)} \quad \text{for all measurable } A \in [0, \bar{\alpha}] \tag{2f}$$

Note: I restrict my attention to perfect Bayesian equilibria such that the events  $A_0$  and  $A_1$  happen with positive probability.

Equation 2b says that the leader's optimal contribution level  $X_l^*(\alpha)$  maximizes his utility for all possible values of  $\alpha$ . Equations 2c and 2d are the perfection conditions. Equation 2c determines the follower's optimal contribution level  $X_f^*(C^*(\alpha))$ . It states that followers react optimally to the leader's action given their posterior beliefs about the value of  $\alpha$ . The leader's optimal commitment strategy  $C^*(\alpha)$  is determined by 2d, which states that the leader takes into account the effect of his commitment action on his followers' contribution levels. Finally equations 2e and 2f correspond to the application of Bayes' rule.

To help build intuition about the equilibrium conditions I analyze the game in some detail. Recall that in the first stage of the game, only the leader observes the exact value of  $\alpha$ , while the distribution of  $\alpha$  is common knowledge. After observing  $\alpha$  the leader decides whether to commit and contribute to the public project taking into account the followers' optimal reaction. The leader's commitment decision can be observed by the followers, while his amount of contribution can not be observed. Since the leader's contribution is not verifiable by his followers, his commitment decision is the only signal that transmits his information to his follower. Therefore, followers update their prior beliefs and react optimally to the leader's commitment choice given their posterior beliefs about the value of  $\alpha$ .

The leader sets his contribution level,  $X_l^*(\alpha)$ , as explained in the complete information scenario. For his commitment strategy, he takes into account the effect of his commitment

action on his followers' contribution level. The leader makes a costly commitment to the project if his gain from the followers' contribution is more than his commitment cost. The following proposition represents this fact by specifying the Leader's Commitment Threshold: the smallest value of  $\alpha$  above which the leader is willing to make a commitment to the public project.

**Proposition 19** *In any equilibrium,  $(X_l^*(\alpha), X_f^*(C^*(\alpha)), C^*(\alpha))$ , there exists a threshold  $\alpha^* \in [0, \bar{\alpha}]$ , such that*

$$C^*(\alpha) = 1 \quad \text{for all } \alpha > \alpha^*$$

$$C^*(\alpha) = 0 \quad \text{for all } \alpha < \alpha^*$$

*See the appendix for the proof.*

Proposition 19 simplifies the leader's commitment signal. The leader's commitment choice reveals whether the realized value of  $\alpha$  is higher or lower than  $\alpha^*$ . As you can see this signal partially reveals the leaders information but does not reveal the exact value of  $\alpha$ .

The followers' optimal reaction to the leader's commitment choice can be analyzed as follows:

Follower  $f$ , who receives a no commitment signal ( $C(\alpha) = 0$ ), infers that  $\alpha < \alpha^*$ . In the sequel I show that followers who receive a no commitment signal choose not to contribute.

Follower  $f$ , who receives a commitment signal ( $C(\alpha) = 1$ ), will infer that  $\alpha > \alpha^*$ . Therefore, his conditional expected utility given the leader's commitment is:

$$\begin{aligned} E(\widehat{V}(x_0, x_1, \dots, x_n) \mid C(\alpha) = 1) &= E(\alpha \sum_{j=0}^n x_j \mid C(\alpha) = 1) + U(w - x_f) \\ &= \int_{\alpha^*}^{\bar{\alpha}} (\alpha \sum_{j=0}^n x_j) f(\alpha \mid \alpha > \alpha^*) d\alpha + U(w - x_i) \end{aligned}$$

Follower  $f$  decides to contribute to the project if his conditional expected gain from partic-

icipation is positive given the leader's action, that is,

$$E(\alpha \sum_{j=0}^n x_j | C(\alpha)) + U(w - x_i) > E(\alpha \sum_{j \neq i}^n x_j | C(\alpha)) + U(w) \text{ for some } x_i > 0$$

For a follower who decides to make a positive contribution the first order condition implies that  $E(\alpha | C(\alpha)) = \frac{\partial U}{\partial x_i}$ . Given the assumptions about the utility function once follower  $f$  decides to contribute to the public project an interior solution exists and the above first order condition is necessary and sufficient for the existence of a unique maximum  $X_f^*(1)$ .

As one can see from the above analysis, the contribution decision of an uninformed follower depends on  $E(\alpha | C(\alpha))$  rather than  $\alpha$  itself. This enables a powerful leader to increase the overall participation by manipulating his followers' expectations via a vague signal. The next section investigates this in detail.

### 3.3 Overall Contributions and the Single Leader

In this section I analyze three main issues. First, even if it is costless to inform everybody about the quality of a project, it may improve cooperation to inform only one player. Second, cooperation improves if the informed player (the leader) is unable to credibly reveal all of his information. The third issue is about the leader's power. The leader has the incentive to exaggerate the value of the project, for he gets a large benefit from his followers' participation. Followers recognize such a leader and refuse to follow him. Appointing a single informed leader will not improve efficiency unless we choose a player who is able to convince the rest. I will specify the conditions sufficient to create a powerful leader.

This section is organized as the following. First, Proposition 21 shows that under incomplete information followers base their contribution decision on  $E(\alpha | C(\alpha))$  rather than  $\alpha$ . Proposition 22 addresses the power of leadership and specifies the condition under which the leader is powerful enough to induce a following. I show that a powerful leader can increase ex-post contributions to some low return but efficient projects. For cases where the ex-post result is ambiguous, Theorem 23 states that a powerful leader increases the level of contribution ex-ante (on average) by partial revelation of his private information.

**Lemma 20** Recall that  $\tilde{\alpha}$  is the players' complete information contribution threshold and  $g(\alpha) = \alpha\tilde{X}(\alpha) + U(w - \tilde{X}(\alpha)) - U(w)$  denotes player's equilibrium utility gain from his contribution to the public project under complete information. Then,

$$g(\alpha) = 0 \quad \text{for all } \alpha < \tilde{\alpha}$$

$$g(\alpha) > 0 \quad \text{for all } \alpha > \tilde{\alpha}$$

See the appendix for the proof.

**Proposition 21** Consider the incomplete information scenario. Recall that follower  $f$ 's expected utility function after observing a commitment decision is :

$$E(\hat{V}(x_1, x_2, \dots, x_n) \mid C(\alpha)) = E(\alpha \sum_{j=1}^n x_j \mid C(\alpha)) + U(w - x_f)$$

Because  $X_f^*(C(\alpha))$  is optimal,  $E(\alpha \mid C(\alpha)) > \tilde{\alpha}$  implies  $X_f^*(C(\alpha)) > 0$  and  $E(\alpha \mid C(\alpha)) < \tilde{\alpha}$  implies  $X_f^*(C(\alpha)) = 0$ .

See the appendix for the proof.

Proposition 21 states that under incomplete information followers decide to make a positive contribution if  $E(\alpha \mid C(\alpha))$  is large enough to exceed  $\tilde{\alpha}$ . That is, they contribute if the leader's commitment signal convinces them that the project yields a large personal payoff; large enough to encourage participation even under complete information.

It can be shown that the leader's commitment threshold  $\alpha^*$  is always below the complete information contribution threshold  $\tilde{\alpha}$ <sup>5</sup>. This means that, followers who observe a no commitment signal decide not to contribute to the project ( $X_f^*(0) = 0$ ).

Followers who observe a commitment signal choose to contribute if the leader's commitment is convincing them that  $\alpha$  is high enough. A leader should not be willing to contribute for very

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<sup>5</sup>By assumption  $r(\alpha) = 0$  for all  $\alpha > \tilde{\alpha}$ . Therefore  $C(\alpha) = 1$  for all  $\alpha > \tilde{\alpha}$ . This implies that  $\alpha > \alpha^*$  for all  $\alpha > \tilde{\alpha}$ . Therefore  $\tilde{\alpha} \geq \alpha^*$

small values of  $\alpha$  because he will not be convincing and therefore will not be followed by the other players. The following proposition specifies the condition that a powerful leader should satisfy.

**Proposition 22** *Consider the incomplete information scenario. Recall that  $\alpha^*$  is the leader's commitment threshold. There exists a threshold  $\alpha_c^* \in [0, \tilde{\alpha}]$ , such that:*

$$E(\alpha \mid C(\alpha) = 1) > \tilde{\alpha} \quad \text{if } \alpha^* > \alpha_c^*$$

$$E(\alpha \mid C(\alpha) = 1) < \tilde{\alpha} \quad \text{if } \alpha^* < \alpha_c^*$$

*See the appendix for the proof.*

The threshold  $\alpha_c^*$  specified in Proposition 22, is called the Critical Threshold. A leader is powerful if his commitment threshold exceeds  $\alpha_c^*$ . Lets assume that a powerful leader exists and consider the possible scenarios that can occur from the ex-post point of view:

**Case 1)**  $\alpha^* < \alpha \leq \tilde{\alpha}$

**Case 2)**  $\alpha < \alpha^* \leq \tilde{\alpha}$

**Case 3)**  $\alpha^* \leq \tilde{\alpha} < \alpha$

In Case 1, players refuse to contribute in the complete information scenario. Under incomplete information, however, the leader commits to the public good project ( $C(\alpha) = 1$ ) and his commitment is followed by the others. Therefore, followers' ex-post contribution under incomplete information will exceed that of complete information.

In the second case, players refuse to participate in the public project under complete information. Under incomplete information the leader also refuses to commit to the project. Therefore, nobody cooperates under either scenario.

In case three, everyone contributes in both complete and incomplete information scenarios, but the size of the contributions may be different in each case. Whether followers contribute more under complete or incomplete information depends on whether  $E(\alpha \mid C(\alpha))$  is higher or lower than the true value of  $\alpha$ .

In case three, there exists a range of  $\alpha$ , ( $\alpha \in (\alpha^*, \hat{\alpha})$ ,  $\tilde{\alpha} < \hat{\alpha} < \bar{\alpha}$ ), for which followers'

contributions under incomplete information exceeds those under complete information ex-post. But ex-post contributions may be larger under complete information for higher values of  $\alpha$ .

From the ex-ante point of view the incomplete information scenario seems less ambiguous. Theorem 23 states that a powerful leader can increase follower's expected contribution ex-ante.

**Theorem 23** *Given  $\alpha^* > \alpha_c^*$ ,  $E_\alpha [X_f^*(1)] > E_\alpha [\tilde{X}(\alpha)]$*

*See the appendix for the proof.*

The above theorem states that followers who follow a powerful leader contribute more on average.

A powerful leader can increase the average level of contribution above the complete information scenario. An important issue, however, is how the leader's exclusive access to information affects the total surplus generated by the group. In the next section, I derive the efficiency results for a special case with linear utility functions. In the future, I will develop the efficiency analysis beyond the linear case presented in the next section.

### 3.4 Efficiency Improvements and the Single Leader

To analyze the welfare effects of the leader's information advantage, I consider the following scenario with linear utility functions.

Suppose that players' utility functions are linear in the overall return to the public project and consumption of the private good:

$$\widehat{V}(x_0, x_1, \dots, x_n) = \alpha \sum_{j=1}^n x_j + w - x_i$$

As in the previous model,  $x_i \geq 0$  is the amount of contribution and  $\alpha$  is the marginal return to the aggregate contribution to the public project.

In the complete information scenario, players' optimal contribution level is the result of the following first order condition:

$$\frac{\partial \widehat{V}(x_0, x_1, \dots, x_n)}{\partial x_i} = \alpha - 1 = 0$$

Therefore,

$$\begin{aligned}\tilde{X}(\alpha) &= w, \text{ for } \alpha > 1 \\ \tilde{X}(\alpha) &= 0, \text{ for } \alpha < 1 \\ \tilde{X}(\alpha) &\in [0, w], \text{ for } \alpha = 1\end{aligned}$$

That is under complete information, players are willing to contribute all their endowment if  $\alpha$  is greater than their complete information contribution threshold  $\tilde{\alpha} = 1$  and they have no incentive to contribute if  $\alpha$  is less than  $\tilde{\alpha} = 1$ .

In the incomplete information scenario, however, the story is different. In this example as in the previous model, the leader moves first, deciding whether to commit and how much to contribute to the project with only his commitment decision being observable by the followers. As in the previous models, I assume that the leader's commitment to the public project is costly and the cost of commitment is  $R(\alpha) = n\theta C(\alpha)r(\alpha)$ , with  $r' < 0$  and  $r'' > 0$ . Followers extract information from the leader's commitment decision and decide how much to contribute to the project.

Under such circumstances, there exists an equilibrium in which the leader commits to the project and is followed by the others if the following conditions hold:

$$\alpha > \alpha^* = \frac{\theta r(\alpha^*)c}{w} \tag{i}$$

$$E(\alpha \mid \alpha \geq \alpha^*) > 1 \tag{ii}$$

Condition (i) says that the realized value of  $\alpha$  should be higher than the leader's commitment threshold  $\alpha^*$ ; that is it should be worthwhile for the leader to commit to the public project. Condition (ii) states the condition under which the leader's signal is powerful (convincing). This condition states that followers' expected value of  $\alpha$  after observing the leader's commitment, should be greater than their complete information contribution threshold  $\tilde{\alpha} = 1$ , for it should be worthwhile for them to contribute.

In the above equilibrium, for any  $\alpha > 1$ , the followers' contribution under incomplete in-



formation is the same as that of complete information. For all values of  $\alpha \in (\alpha^*, 1)$ , however, incomplete information leads to higher contributions, for the followers make no contribution under complete information while they make a positive contribution in the incomplete information scenario. Therefore, in this example partial revelation of the quality of the public project helps the leader to increase the followers' contribution level in the incomplete information scenario. An important issue, however, is how the surplus generated by the group in the single leader scenario is different from the complete information case.

The welfare levels in the complete and incomplete information scenarios, can be compared as follows:

For any  $\alpha > \tilde{\alpha}$ , the ex-post welfare gain from the follower's contribution as a function of  $\alpha$ , equals to  $\Delta W(\alpha) = n[(n+1)\alpha - 1]w$ , which is positive and equal in both cases of complete and incomplete information for all  $\alpha > \tilde{\alpha}$ . For all  $\alpha \in (\alpha^*, \tilde{\alpha})$  followers do not make a positive contribution in the complete information scenario. The ex-post welfare gain from the followers' contribution under incomplete information equals  $\Delta W_I(\alpha) = n[(n+1)\alpha - 1]w - R(\alpha)$  and it will be positive for all values of  $\alpha$  that satisfy the inequality  $\alpha > \frac{R(\alpha)}{n(n+1)w} + \frac{1}{n+1}$ . The following proposition specifies the sufficient condition for achieving a positive ex-post welfare gain under incomplete information.

**Proposition 24** *For any  $\alpha \in (\alpha^*, \tilde{\alpha})$ ,  $r(\alpha^*) \geq \frac{w}{n\theta}$  implies  $\Delta W_I(\alpha) > 0$ .*

*See the appendix for the proof.*

The above proposition states that the ex-post welfare level under incomplete information scenario is higher than that of complete information if the leader's commitment cost is large enough to prevent him from committing to a socially worthless project.

Let us suppose that the leader's commitment cost is not large enough and therefore the inequality  $r(\alpha^*) \geq \frac{w}{n\theta}$  is no longer satisfied. Then, we can still show that the expected welfare level under incomplete information is higher than under complete information.

**Proposition 25** *Let  $E(\Delta W_I(\alpha)) = \int_{\alpha^*}^{\tilde{\alpha}} \{n[(n+1)\alpha - 1]w - n\theta r(\alpha)\} \frac{1}{\alpha} d\alpha$  denote the ex-ante expected welfare gain under incomplete information. Then,  $E(\Delta W_I(\alpha)) > 0$ .*

*See the appendix for the proof.*

Proposition 25 states that the welfare level on average is higher under incomplete information than that of complete information. This shows that the leader's exclusive access to information can improve efficiency ex-ante by partially transmitting his information to the others.

The following section concludes this chapter and discusses the further extensions.

### 3.5 Conclusion

A standard result of the theory of the public goods is that public goods are under-provided by voluntary contributions.

I introduce leadership as an institutional solution to incentive conflicts and free-riding problems in public good games. By my definition, a leader is a person who has exclusive information about the value of the public project.

I introduce a game theoretic model of public good provision under two different scenarios: the complete information scenario and the incomplete information scenario.

In the single leader scenario, the leader is exogenously informed about the return to the public project and is able to credibly transmit part of his information to the others by making a costly commitment. Comparing these two cases, I draw the following conclusions:

I show that under incomplete information a powerful leader who has exclusive access to the information can increase the overall contribution and the welfare level if he is unable to transmit all of his information to the others.

I argue that under fairly general assumptions the leader can increase his followers' ex-post contributions. In cases where the ex-post result is ambiguous, I show that the ex-ante contributions are larger in the presence of a powerful leader than those of complete information. In other words, a powerful leader can increase the followers' average contributions under incomplete information.

A leader can not induce a following unless he is able to convince his followers that he is transmitting the correct information. I specify the condition under which the leader is powerful enough to induce a following.

I also consider a scenario with linear utility functions and show that a powerful leader can increase the ex-post social surplus if his commitment cost is large enough to prevent him

from committing to a socially worthless project. In cases where the ex-post welfare result is ambiguous, I show that the ex-ante welfare in a single leader scenario is higher than that under complete information, meaning that a powerful leader can increase the average welfare level. In the future I seek to derive the welfare results for the more general utility function described in the model.

### 3.6 Appendix

#### Proof of Proposition 17:

Let  $g(\alpha) = \alpha\tilde{X}(\alpha) + U(w - \tilde{X}(\alpha)) - U(w)$  denote the equilibrium utility gain from player  $i$ 's contribution to the public project. Clearly  $g(\alpha) > 0$  implies  $\tilde{X}(\alpha) > 0$ .

If  $\tilde{X}(\alpha) = 0$  for all  $\alpha \in [0, \bar{\alpha}]$ , then  $\tilde{\alpha} = \bar{\alpha}$ .

Suppose  $\tilde{X}(\hat{\alpha}) > 0$  for some  $\hat{\alpha} \in [0, \bar{\alpha}]$ . Then  $g(\hat{\alpha}) \geq 0$  and we have:  $g(\alpha) \geq \alpha\tilde{X}(\hat{\alpha}) + U(w - \tilde{X}(\hat{\alpha})) - U(w) > g(\hat{\alpha}) \geq 0$  for all  $\alpha > \hat{\alpha}$ . This implies that  $\tilde{X}(\alpha) > 0$  for all  $\alpha > \hat{\alpha}$ .

Summarizing:  $\tilde{X}(\hat{\alpha}) > 0$  for some  $\hat{\alpha} \in [0, \bar{\alpha}]$  implies that  $\tilde{X}(\alpha) > 0$  for all  $\alpha > \hat{\alpha}$ . Pick  $\tilde{\alpha} = \text{Inf} \{ \alpha : \tilde{X}(\alpha) > 0 \}$ .

#### Proof of Lemma 18:

For any  $\alpha > \tilde{\alpha}$ , player  $i$ 's optimization problem with respect to the private good  $y_i$ , takes the following form:

$$\underset{y_i}{\text{Max}} \quad \tilde{V}(y_i) = \alpha \sum_{j=1}^n (w - y_j) + U(y_i)$$

The first order condition implies that:

$$U'(y_i) = \alpha$$

Since  $U$  is strictly concave  $U'$  is invertible and:

$$[U']^{-1}(\alpha) = y_i$$

Therefore, player i's contribution as a function of  $\alpha$  is the following:

$$\tilde{X}(\alpha) = w - [U']^{-1}(\alpha)$$

Let  $[U']^{-1}(\alpha) = g(\alpha)$  and  $f(y_i) = g^{-1}(y_i) = U'(y_i)$ . According to our assumptions  $f'(y_i)$  is well defined and non-zero. Therefore, Inverse Function Theorem implies that:

$$f'(y_i) = \frac{1}{g'[f(y_i)]} \quad \text{for all } y_i > 0$$

Taking the derivative from both sides of the above equality we have:

$$f''(y_i) = \frac{-f'(y_i) \cdot g''[f(y_i)]}{[g'[f(y_i)]]^2} \quad \text{for all } y_i > 0$$

or

$$U'''(y_i) = \frac{-U''(y_i) \cdot g''[f(y_i)]}{[g'[f(y_i)]]^2} \quad \text{for all } y_i > 0$$

Since  $U'''(y_i) > 0$  and  $U''(y_i) < 0$  by assumption, then  $g''[f(y_i)] > 0$  for all  $y_i > 0$ , implying that  $g(\alpha) = [U']^{-1}(\alpha)$  is convex in  $\alpha$ . This implies that  $\tilde{X}(\alpha)$  is concave in  $\alpha$ .

**Proof of Proposition 19:**

Let  $h(\alpha) = \alpha n (X^*(1) - X^*(0)) - n\theta r(\alpha)$  denote the leader's utility gain from the followers' contribution if he commits to the project. Clearly,  $h(\alpha) > 0$  implies  $C(\alpha) = 1$  and  $h(\alpha) < 0$  implies  $C(\alpha) = 0$ .

If  $C(\alpha) = 0$  for all  $\alpha \in [0, \bar{\alpha}]$ , then  $\alpha^* = \bar{\alpha}$ .

Suppose  $C(\hat{\alpha}) = 1$  for some  $\hat{\alpha} \in [0, \bar{\alpha}]$ . Then  $h(\hat{\alpha}) \geq 0$ , implying that  $(X^*(1) - X^*(0)) > 0$ . We also know by assumption that  $r'(\alpha) < 0$ . Thus we can conclude that  $h'(\alpha) > 0$  for all  $\alpha > \hat{\alpha}$ . This implies that  $h(\alpha) > 0$  for all  $\alpha > \hat{\alpha}$ . Therefore  $C(\alpha) = 1$  for all  $\alpha > \hat{\alpha}$ .

Summarizing:  $C(\hat{\alpha}) = 1$  for some  $\hat{\alpha} \in [0, \bar{\alpha}]$ , implies  $C(\alpha) = 1$  for all  $\alpha > \hat{\alpha}$ . Pick  $\alpha^* = \text{Inf} \{ \alpha : C(\alpha) = 1 \}$ .

**Proof of Lemma 20:**

For all  $\alpha < \tilde{\alpha}$ , lemma 1 implies that  $\tilde{X}(\alpha) = 0$ . Thus  $g(\alpha) = 0$  for all  $\alpha < \tilde{\alpha}$ . For all  $\alpha > \tilde{\alpha}$ , lemma 1 implies that  $\tilde{X}(\alpha) > 0$ . According to Lemma 18,  $\tilde{X}(\alpha)$  is a concave function. Thus application of the envelope theorem implies that  $\frac{dg(\alpha)}{d\alpha} = \frac{\partial g(\alpha)}{\partial \alpha} = \tilde{X}(\alpha) > 0$ . Therefore  $g(\alpha) > g(\tilde{\alpha}) \geq 0$  for all  $\alpha > \tilde{\alpha}$ .

Summarizing:  $g(\alpha) = 0$  for all  $\alpha < \tilde{\alpha}$  and  $g(\alpha) > 0$  for all  $\alpha > \tilde{\alpha}$ .

**Proof of Proposition 21:**

Let  $B(c(\alpha)) = E(\alpha | C(\alpha))X^*(C(\alpha)) + U(w - X^*(C(\alpha))) - U(w)$ , denote followers' expected utility gain from participation.

Suppose  $E(\alpha | C(\alpha)) < \tilde{\alpha}$ . If  $X^*(C(\alpha)) > 0$ , then

$E(\alpha | C(\alpha))X^*(C(\alpha)) + U(w - X^*(C(\alpha))) - U(w) < \tilde{\alpha}X^*(C(\alpha)) + U(w - X^*(C(\alpha))) - U(w) \leq 0$ , contradicting optimality of  $X^*(C(\alpha))$ . Therefore, we can conclude that  $X^*(C(\alpha)) = 0$ .

Suppose  $E(\alpha | C(\alpha)) > \tilde{\alpha}$ , then the optimality of  $X^*(C(\alpha))$  along with lemma 4 implies that:

$E(\alpha | C(\alpha))X^*(C(\alpha)) + U(w - X^*(C(\alpha))) - U(w) \geq E(\alpha | C(\alpha))\tilde{X}(E(\alpha | C(\alpha))) + U(w - \tilde{X}(E(\alpha | C(\alpha)))) - U(w) = g[E(\alpha | C(\alpha))] > 0$ . Therefore,  $X^*(C(\alpha)) > 0$ .

Summarizing:  $E(\alpha | C(\alpha)) < \tilde{\alpha}$  implies  $X^*(C(\alpha)) = 0$  and  $E(\alpha | C(\alpha)) > \tilde{\alpha}$  implies  $X^*(C(\alpha)) > 0$ .

**Proof of Proposition 22:**

For this proof, let  $\alpha^*$  denote the leader's choice of contribution threshold without requiring that this be an equilibrium choice.

To prove this lemma we first prove fact (1):

$$\frac{\partial E(\alpha | C(\alpha) = 1)}{\partial \alpha^*} > 0$$

Proof of fact (1):

$$\begin{aligned} E(\alpha | C(\alpha) = 1) &= E(\alpha | \alpha \geq \alpha^*) = \int_{\alpha^*}^{\bar{\alpha}} \alpha f(\alpha | \alpha \geq \alpha^*) d\alpha \\ &= \int_{\alpha^*}^{\bar{\alpha}} \frac{\alpha f(\alpha)}{1 - F(\alpha^*)} d\alpha = \frac{1}{1 - F(\alpha^*)} \int_{\alpha^*}^{\bar{\alpha}} \alpha f(\alpha) d\alpha \end{aligned}$$

Taking the derivative of  $E(\alpha \mid \alpha \geq \alpha^*)$  with respect to  $\alpha^*$  we have:

$$\begin{aligned}
\frac{\partial E(\alpha \mid \alpha \geq \alpha^*)}{\partial \alpha^*} &= \frac{f(\alpha^*)}{[1 - F(\alpha^*)]^2} \int_{\alpha^*}^{\bar{\alpha}} \alpha f(\alpha) d\alpha - \frac{\alpha^* f(\alpha^*)}{1 - F(\alpha^*)} \\
&= \frac{f(\alpha^*)}{1 - F(\alpha^*)} \int_{\alpha^*}^{\bar{\alpha}} \frac{\alpha f(\alpha)}{1 - F(\alpha^*)} d\alpha - \frac{\alpha^* f(\alpha^*)}{1 - F(\alpha^*)} \\
&> \frac{f(\alpha^*)}{1 - F(\alpha^*)} \int_{\alpha^*}^{\bar{\alpha}} \frac{\alpha^* f(\alpha)}{1 - F(\alpha^*)} d\alpha - \frac{\alpha^* f(\alpha^*)}{1 - F(\alpha^*)} \\
&= \frac{\alpha^* f(\alpha^*)}{1 - F(\alpha^*)} \left[ \int_{\alpha^*}^{\bar{\alpha}} \frac{f(\alpha)}{1 - F(\alpha^*)} d\alpha - 1 \right] = \frac{\alpha^* f(\alpha^*)}{1 - F(\alpha^*)} \times 0 = 0
\end{aligned}$$

(Proof of fact 1 completed.)

Considering fact one,

If  $E(\alpha \mid C(\alpha) = 1) < \tilde{\alpha}$  for all  $\alpha^* \in [0, \tilde{\alpha}]$ , then  $\alpha_c^* = \tilde{\alpha}$ .

If  $E(\alpha \mid C(\alpha) = 1) > \tilde{\alpha}$  for all  $\alpha^* \in [0, \tilde{\alpha}]$ , then  $\alpha_c^* = 0$ .

If  $E(\alpha \mid C(\alpha) = 1) = \tilde{\alpha}$ , let  $\alpha_c^* = \alpha^*$ .

### Proof of Theorem 23:

By definition  $\tilde{X}(\alpha)$  is the amount of player  $i$ 's contribution in the complete information scenario that maximizes:

$$\hat{V}(x_1, x_2, \dots, x_n) = \alpha \sum_{j=1}^n x_j + U(w - x_i)$$

If  $\alpha > \tilde{\alpha}$  (or if an interior solution exists), the first order condition for follower  $i$  implies that

$$\alpha = U'(w - \tilde{X}(\alpha)) \tag{i}$$

By definition  $X^*(1)$  is the amount of follower  $i$ 's contribution under incomplete information

that maximizes:

$$E \left[ \widehat{V}(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n) | C(\alpha) = 1 \right] = E(\alpha | C(\alpha) = 1) \left[ X_0(\alpha) + \sum_{i=1}^n X_i(C(\alpha) = 1) \right] + U(w - X(1))$$

If  $X^*(1) > 0$  (or if an interior solution exists), the first order condition for followers implies that:

$$E(\alpha | C(\alpha) = 1) = U'(w - X^*(1)) \quad (\text{ii})$$

Since  $U''' > 0$ , Jensen's inequality implies that:  $U'[E(Z)] < E[U'(Z)]$  for any random variable  $Z$ . Therefore,

$$U' \left[ E \left( w - \tilde{X}(\alpha) | C(\alpha) = 1 \right) \right] < E \left[ U'(w - \tilde{X}(\alpha)) | C(\alpha) = 1 \right] \quad (\text{iii})$$

$\alpha^* \leq \tilde{\alpha}$  implies that:

$$E \left[ \tilde{X}(\alpha) | C(\alpha) = 0 \right] = 0$$

Thus:

$$\begin{aligned} E \left[ \tilde{X}(\alpha) \right] &= \Pr(\alpha < \alpha^*) \cdot E \left[ \tilde{X}(\alpha) | C(\alpha) = 0 \right] + \Pr(\alpha \geq \alpha^*) \cdot E \left[ \tilde{X}(\alpha) | C(\alpha) = 1 \right] \quad (\text{iv}) \\ &= \Pr(\alpha \geq \alpha^*) \cdot E \left[ \tilde{X}(\alpha) | C(\alpha) = 1 \right] \end{aligned}$$

(i) and (iii) imply that:

$$U' \left[ E \left( w - \tilde{X}(\alpha) | C(\alpha) = 1 \right) \right] < E(\alpha | C(\alpha) = 1)$$

Using (ii) we have:

$$U' \left[ E \left( w - \tilde{X}(\alpha) | C(\alpha) = 1 \right) \right] < U'(w - X^*(1))$$

Because  $U'' < 0$  it follows that:

$$w - E \left( \tilde{X}(\alpha) | C(\alpha) = 1 \right) > w - X^*(1)$$

or

$$E \left( \tilde{X}(\alpha) | C(\alpha) = 1 \right) < X^*(1)$$

Thus,

$$\Pr(\alpha \geq \alpha^*) \cdot E \left( \tilde{X}(\alpha) | C(\alpha) = 1 \right) < \Pr(\alpha \geq \alpha^*) \cdot X^*(1)$$

Therefore from (iv) we have:

$$E[X^*(1)] > E[\tilde{X}(\alpha)]$$

**Proof of Proposition 24:**

The inequality  $r(\alpha^*) \geq \frac{w}{n\theta}$  implies that  $n\theta r(\alpha^*) \geq w$ . Adding  $\theta r(\alpha^*)$  to both sides of the inequality I have:

$$(n+1)\theta r(\alpha^*) - w - \theta r(\alpha^*) \geq 0$$

If I multiply and divide the first term by  $w$  I get:

$$\frac{w(n+1)\theta r(\alpha^*)}{w} - w - \theta r(\alpha^*) \geq 0$$



Since according to equation i,  $\alpha^* = \frac{\theta r(\alpha^*)}{w}$ , I can rewrite the above inequality as:

$$(n+1)w\alpha^* - w - \theta r(\alpha^*) \geq 0$$

Multiplying both sides by  $n$  I get:

$$n[(n+1)\alpha^* - 1]w - n\theta r(\alpha^*) \geq 0$$

The above inequality simply shows that  $\Delta W_I(\alpha) \geq 0$  at  $\alpha = \alpha^*$ . Since  $r' < 0$ ,  $\Delta W_I(\alpha)$  is an increasing function of  $\alpha$ . Therefore  $\Delta W_I(\alpha) \geq 0$  at  $\alpha = \alpha^*$ , implies that  $\Delta W_I(\alpha) \geq 0$  for any  $\alpha \in (\alpha^*, \tilde{\alpha})$ .

Summarizing:  $r(\alpha^*) \geq \frac{w}{n\theta}$  implies  $\Delta W_I(\alpha) \geq 0$  for any  $\alpha \in (\alpha^*, \tilde{\alpha})$ .

**Proof of Proposition 25:**

Choose  $\theta' > 0$  such that  $r(\alpha^*) = \theta'(\tilde{\alpha} - \alpha^*)$  and define

$$\begin{aligned} A &= \int_{\alpha^*}^{\tilde{\alpha}} \{n[(n+1)\alpha - 1]w - n\theta\theta'(\tilde{\alpha} - \alpha)\} \frac{1}{\alpha} d\alpha \\ A &= \int_{\alpha^*}^{\tilde{\alpha}} [n(n+1)\alpha w - nw - n\theta\theta'\tilde{\alpha} + n\theta\theta'\alpha] \frac{1}{\alpha} d\alpha \\ A &= (n^2w + nw + n\theta\theta') \int_{\alpha^*}^{\tilde{\alpha}} \alpha \frac{1}{\alpha} d\alpha - (nw + n\theta\theta'\tilde{\alpha}) \int_{\alpha^*}^{\tilde{\alpha}} \frac{1}{\alpha} d\alpha \\ A &= (n^2w + nw + n\theta\theta') \left[ \frac{\alpha^2}{2\alpha} \right]_{\alpha^*}^{\tilde{\alpha}} - (nw + n\theta\theta'\tilde{\alpha}) \left[ \frac{\alpha}{\alpha} \right]_{\alpha^*}^{\tilde{\alpha}} \\ A &= (n^2w + nw + n\theta\theta') \left[ \frac{w^2 + 2w\theta\theta'}{2\alpha(w^2 + \theta^2\theta'^2 + 2w\theta\theta')} \right] - \left( \frac{nw}{\alpha} \right) \\ A &= \frac{n^2w^3 + 2n^2w^2\theta\theta' - nw^3 - nw^2\theta\theta'}{2\alpha(w^2 + \theta^2\theta'^2 + 2w\theta\theta')} \end{aligned}$$

As it can be seen,  $A$  is positive for  $n > 0$ . By assumption  $r(\alpha)$  is a convex function. This implies that  $E(\Delta W_I(\alpha))$  is a concave function of  $\alpha$ . Therefore,  $E(\Delta W_I(\alpha)) > A > 0$ .

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