

# **An Integrated Framework and Smart Algorithm for Vehicle Localization in Intelligent Transportation Systems**

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## Abstract

Intelligent Transportation Systems (ITS) have emerged to use different technologies to promote safety, convenience, and efficiency of transportation networks. Many applications of ITS depend on the availability of the real-time positioning of the vehicles in the network. In this research, the two open challenges in the field of vehicle localization for ITS are introduced and addressed.

First, in order to have safe and efficient transportation systems, the locations of the vehicles need to be available everywhere in a network. Conventional localization techniques mostly rely on Global Positioning System (GPS) technology which cannot meet the accuracy requirements for all applications in all situations. This work advances the study of vehicle positioning in ITS by introducing an integrated positioning framework which uses several resources including GPS, vehicle-to-infrastructure and vehicle-to-vehicle communications, radio-frequency identification, and dead reckoning. These technologies are used to provide more reliable and accurate location information. The suggested framework fills the gap between the accuracy of the current vehicle localization techniques and the required one for many ITS applications.

Second, different ITS applications have different localization accuracy and latency requirements. A smart positioning algorithm is proposed which enable us to change the positioning accuracy delivered by the algorithm based on different applications. The algorithm utilizes only the most effective resources to achieve the required accuracy, even if more resources are available. In this way, the complexity of the system and the running time decrease while the desired accuracy is obtained. The adjective *Smart* is selected because the algorithm smartly selects the most effective connection which has the most contribution to vehicle positioning when a connection needs to be added. On the other hand, when a connection should be removed, the algorithm smartly selects the least effective one which has the least contribution to the position estimation.

This study also provides an overview about the positioning requirements for different ITS applications. A close-to-real-world scenario has been developed and simulated in MATLAB to evaluate the performance of the proposed algorithms. The simulation results show that the vehicle can acquire accurate location in different environments using the suggested Integrated framework. Moreover, the advantages of the proposed Smart algorithm in terms of accuracy and running time are presented through a series of comprehensive simulations.

# Dedication

I would like to dedicate my thesis to Reza and to my beloved family for their love, endless support and encouragement

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I would like to express my gratitude to my advisor, Prof. Jesus M. de la Garza for his constant encouragement and invaluable suggestions. I would not have been able to complete this journey without his aid and supports. His guidance helped me throughout this research and writing of this thesis. Besides my advisor, I would like to express my appreciations to my committee members Dr. John E. Taylor and Dr. Sunil Sinha for their valuable comments and attention to details over the course of this research.

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# List of Abbreviations

CP	Cooperative Positioning
CRLB	Cramer-Rao Lower Bound
DSRC	Dedicated short-range communications
GPS	Global Positioning System
HOV	High-Occupancy Vehicle
MAP	Maximum a posteriori
ML	Maximum Likelihood
ITS	Intelligent Transportation Systems
RFID	Radio-Frequency Identification
RSS	Received Signal Strength
TDOA	Time-difference-of-arrival
TOA	Time-of-arrival
USDOT	United State Department of Transportation
VANET	Vehicular ad hoc network
V2I	Vehicle-to-Infrastructure
V2V	Vehicle-to-Vehicle
WSN	Wireless Sensor Network

# 1 Introduction

## 1.1 Overview and Problem Statement

The significant growth of population and vehicles demands more reliable and efficient transportation networks everywhere, from large and small cities to suburb and rural areas, especially in the United States where most of the families have several vehicles and everyone uses a personal vehicle. The performance of the current transportation systems can be enhanced by using information technology and making them more intelligent. Therefore, Intelligent Transportation Systems (ITS) have emerged to improve safety, efficiency, and navigation quality of the transportation networks by exploiting different types of technologies. ITS have wide range applications from safety to traveler information. Some applications of ITS such as collision warning messaging need to have very accurate location information from the vehicles, while some others such as data dissemination do not need any vehicles location information to operate, but they can provide better services if location information is incorporated. Safety is one of the most important applications of ITS and benefits a lot from location information. Each year many people injure and die due to accidents in highways. As an example, in 2010, there were an estimated 5,419,000 crashes, killing 32,885 and injuring 2,239,000 in the United States (NHTSA 2012). Although the trend is decreasing over the past decades, the number of fatalities is still significant. Motor vehicle accidents not only affect the people involved, but also they have impacts on the economy. According to a study released by the Centers for Disease Control and Prevention, under \$100 billion is spent for medical care and injury-related productivity losses in the United States each year which includes \$3.6 billion annually toward injuries to children (Naumann RB 2010). That is why researchers are mostly concern about the safety of transportation networks. As will be described later in detail, many applications of ITS require location information from the elements everywhere in a network. Therefore, positioning and location information play an important role in the functionality of ITS.

Current vehicle positioning techniques highly rely on the Global Positioning System (GPS). However, GPS cannot provide reliable location information for all applications under all situations. GPS do not work at all in indoor environments such as parking garage and tunnels and its performance suffers from multipath in dense environments such forest and commercial areas. Therefore, there is a special need for an alternative positioning technique which could provide the accuracy requirements of all ITS applications under different geographical environments.

Another major problem in this field is that each application of ITS requires a specific localization accuracy at a specific latency. For example, collision warning message is a safety application of ITS which requires highly accurate location information from the vehicles at the least amount of latency. The reason is that a small error in the location information of the vehicles may cause a tragic accident. Moreover, the message warning should be processed as fast as possible to enable the drivers to decide accordingly and prevent the accident. On the other hand, traveler information is another application of ITS which does not require accurate location information. In this case, once the vehicle enters in a specific area, useful information is provided for the driver. Therefore, accurate location of the vehicle is not required. Moreover, this information is not required to be received by the driver very quickly and high latency is acceptable. Therefore, I believe there is a need for a Smart localization technique which is able to deliver the required localization accuracy at the required latency depending on the different applications.

## **1.2 Objectives and Scopes**

The main objective of this study is to develop an Integrated and Smart vehicle localization algorithm for ITS which can deliver good localization accuracy under different geographical environments.

Although ITS include any transportation systems such as rail, water, and air transportation systems, in this work I only limit my work to the road transportation systems for which the ITS were originally developed. ITS were initially launched to solve the problem of traffic congestion. However, safety has become the priority in ITS since many people lost their

lives because of lack of sufficient preventive technologies. With so many people of all ages on the road, implementing traffic safety is a must. Data and communication are the fundamentals of the safety in ITS and location of each elements in a transportation system is one of the most important information needed to be available. The scope of this research is limited to study the different technologies which enable us to determine the locations of the elements in a transportation system. The reliability of a safety program is dependent on how accurate the location information is. Several localization techniques have been proposed for ITS, each of them has their own specific strengths and limitations. Therefore, none of these techniques can provide us with reliable location information in all situations. One of the main objectives of this research is to introduce and develop an Integrated localization framework for ITS. Exploiting different vehicle localization techniques, the proposed framework should provide significantly higher positioning accuracy than any individual techniques in all situations. Another main objective of this research is to develop a Smart localization algorithm which delivers the required localization accuracy at the least amount of complexity. A series of computer simulations are conducted to evaluate the performance of the proposed algorithms. The list below includes the main tasks that will be accomplished in this research:

1. Review and study of different applications of ITS;
2. Comparison and evaluation of the different localization techniques for ITS;
3. Introduction and development of an Integrated vehicle positioning framework;
4. Introduction and development of a Smart vehicle positioning technique;
5. Evaluation and comparison of the proposed framework using computer simulations

### **1.3 Contributions to the Body of Knowledge**

In this research, vehicle localization which is an important aspect of any intelligent transportation networks is studied. The main objectives of this research are fully described in the previous section. I believe there are two main challenges in this field which have not been addressed properly before. First, a localization technique is required to

provide accurate location information of the vehicles everywhere in a network. Second, there is a need for having a localization technique which is able to deliver the required localization accuracy along with the required latency based on different applications. Therefore, the main contributions of this research to the field are:

1. Introduction of an Integrated vehicle localization technique
2. Development of an Integrated vehicle localization algorithm
3. Development of a Smart vehicle localization algorithm

I hope this thesis is able to address the two important mentioned challenges and provide a clear path for future research in this field.

## **1.4 Thesis Organization**

An introduction about ITS and their applications is provided in Chapter 2. I also described the different localization techniques that have been proposed for vehicle localization in this chapter. In Chapter 3, I will provide a comprehensive literature review in the field of localization. I will review the important studies in this field and evaluate their strengths and weaknesses. Chapter 4 provides the details of the methodology to the mentioned problem. An introduction about estimation theory is also included in this chapter which helps the reader to better understand the content of the chapter. The performance of the proposed localization is evaluated through a series of MATLAB computer simulations in Chapter 5. Finally, Chapter 6 concludes the thesis and discusses future research.

# 2 Intelligent Transportation Systems Applications

In this chapter, first an introduction about ITS is provided. The idea of emergence of ITS, their applications, and their advantages are described. It will be shown how important the aspect of localization is for different ITS applications. Then, to provide the reader with an overview about positioning and localization, different localization methods that are being used to find the location of a vehicle in a network are described.

## 2.1 Overview

Intelligent transportation systems (ITS) use different kinds of technologies to enhance the performance of a transportation network. The main outcome of the ITS is to save lives, time, money, energy, and the environment (Ezell 2010; Miles 2010; Pina 2013). Information and communication systems build the fundamentals of ITS. The first step in the ITS program is to collect data and information from the elements in a transportation system. What I mean by elements is everything that plays a role in a transportation network such as vehicles, drivers, infrastructures, users, roads, road signs, cameras, etc. Then, the information is processed and the result is sent to the surrounding elements.

## 2.2 Applications

In the beginning, ITS were proposed to help the problem of traffic congestion. However, ITS will be used for many other purposes such as safety (Chen et al. 2010; Figueiredo et al. 2001; Rumar et al. 1999), security (Hamza-Lup et al. 2004), weather information (Boselly III et al. 1993), and traffic management (der van Heijden and Marchau 2002). Although as depicted in FIGURE 1, ITS have many applications, here I will only mention several of them that have recently received more attention in the literature (Miles 2010). Since the main scope of this work is the importance of vehicle positioning in ITS applications, the minimum location information requirement for each application is also described.

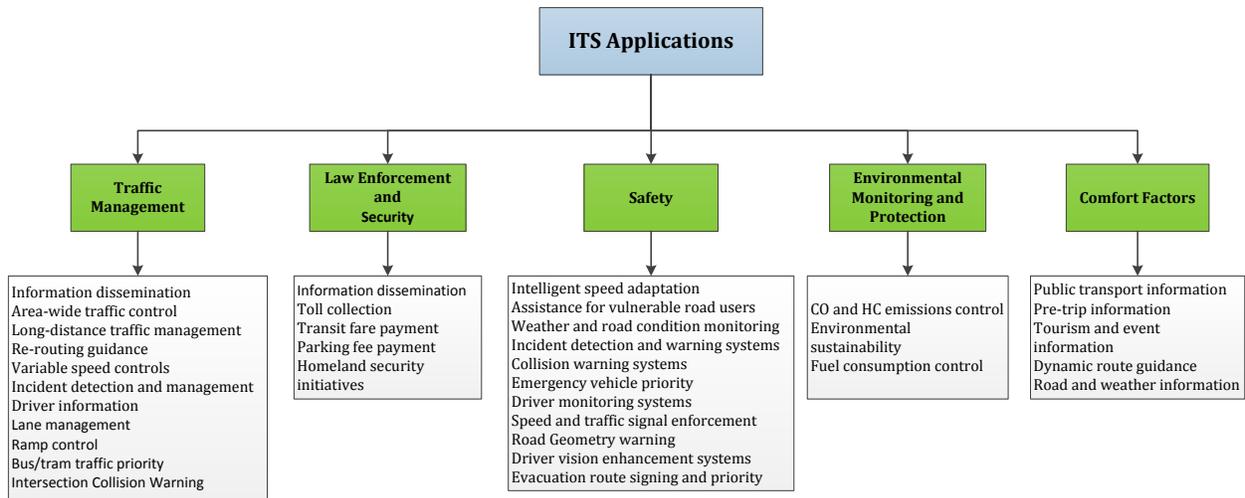


FIGURE 1 - The applications of ITS adapted from (Miles 2010)

### 2.2.1 Traffic Management

As mentioned before, traffic management was the first application for ITS. The demography of the United States has changed recently. The population has been growing rapidly during the past century. Many rural populations have migrated to urban and suburban areas. Moreover, more people use personal vehicles in the United States than in any other country. Therefore, traffic congestion is the main problem in most of the US cities. ITS applications try to enhance the performance of the networks by employing real-time management and monitoring such as online control systems, managing demands, encouraging off-peak travel, and dynamic traffic light sequence. For example, dynamic traffic light sequence exploits RFID technology using proper algorithms and databases to effectively manage the time of a traffic light for multi vehicles, multi lanes and multi road junction areas (der van Heijden and Marchau 2002). Most application in this category demand low to medium localization accuracy. For instance, the traffic flow in an intersection can be estimated using a technique with medium accuracy and therefore the

traffic light sequences can be managed. In this case knowing the absolute location of the vehicles is not significant and the overall traffic flow would be enough.

Inter vehicle communications can be utilized for grouping the vehicles into platoons (i.e., platooning) to increase the capacity of the roadways. In this method, a group of vehicles move very close to each other and increase and decrease their speeds simultaneously. Using this method, the vehicles do not need to pass other vehicles or change lanes. Another example would be automated sensing which assists the driver to pass and change lanes and reduces the risk of maneuvering in the highways. This system is currently installed in several automobiles such as Cadillac XTS and ATS. This kind of application requires accurate location. However, an accuracy range of 1 to 5 meters is within an acceptable range.

#### ***2.2.1.1 Information Dissemination***

This application is highly important and is shared among many other applications of ITS. Sending information about the condition of roads, accidents, and congestion is highly vital in many applications of ITS (Myr 2003; Nadeem et al. 2004). Using data dissemination, the elements within a transportation network are able to convey messages to the desired destinations. Data and information is used to improve the quality and efficiency of a transportation system and to make it more convenient for the users. Because of these important functions, data dissemination has received significant attention in the literature in recent years. For this application, there is no need for having a very accurate localization. A range of 10-30 meters is an acceptable range for this specific application. For example, assume that there is congestion within a road. In this situation, it would be sufficient for the user to be informed about this congestion so that they can use other routes. Thus, it is not very significant for the drivers to be informed about the exact location of the congestion (Chen et al. 2011). Another example is the situation where the driver is lost in an area or part of a city and the goal is to inform him about the approximate location. It should be mentioned that these applications have some overlap with traffic management applications in which more localization accuracy is demanded.

### **2.2.2 Law Enforcement and Security**

ITS can be used to detect and identify the vehicles which are not following the laws in the road. Previously, it had been done by monitoring the roads via cameras recording vehicles' movements. An operator was required to watch and monitor constantly and detect the violating vehicles. However recently, several methods have been proposed where the process is completely automated. For instance, speed cameras which estimate the speed of the vehicles using radar can detect high speed vehicles automatically and capture their plate's numbers. Another example of ITS application in automatic road enforcement is railroad crossing (i.e., an intersection where a railway line crosses a road at the same level) where detectors are used to identify the vehicles that pass railways illegally. High technology cameras exploiting image processing techniques are used to detect the number of passengers in a vehicle passing through a High-Occupancy Vehicle lane (HOV). There are many other examples I can mention for using high technology devices which enforce laws in the roads (Cao et al. 2003). It should be noted that there is a direct relationship between this application of ITS and safety. As more sophisticated and precise equipment is used for law enforcement in the roads, more people obey the laws. Therefore, we would have safer transportation networks (de Fuentes et al. 2012). Applications in this category require low to high localization accuracy. For instance, for toll collection, only detecting the presence of the vehicle at a specific highway or road is sufficiency and accurate location information is not required. However, some applications in which national security is involved higher accurate location information from a suspected vehicle is required.

### **2.2.3 Safety**

Safety is the first concern of each transportation system. Currently more than 32,000 Americans between ages 4 and 35 lose their lives in vehicle crashes every year (NHTSA 2012). Therefore, the most important application of ITS is to make roads safer and more secure. Different technologies have been proposed for safety of a transportation network through ITS. As depicted in FIGURE 1, there are many safety applications associated with ITS. Intersection message warning is an important safety application of ITS. At intersections, vehicles running the traffic light when it turns to red are the most common cause of accidents which can result in serious injuries and losses. Using vehicular

communications, a preceding vehicle informs other following vehicles of a possible accident by transmitting a collision warning signal (Yin et al. 2004). Emergency electronic brake lights, slow vehicle warning, intersection collision warning, and pre-crash sensing are examples of other safety applications for ITS (Papadimitratos et al. 2009). A large portion of accidents in highway is motorcycles crashes. Most of the time, the reason of these accidents is not seeing motorcycles. To have a safe transportation network the motorcycles need to share information about their location to other vehicles which gives the drivers warning about the likelihood of accident occurrences (Bayly et al. 2006). Safety applications of ITS typically require medium to high localization accuracy. For instance, for collision warning message and emergency message highly accurate location information of the vehicles involved in an accident is required (Shladover and Tan 2006).

#### **2.2.4 Environmental Monitoring and Protection**

As an effect of population and vehicles growth, our environment is now more vulnerable to air and noise pollution. The increase of carbon dioxide and nitrogen oxides emitted from vehicle can cause serious damages to people in the large cities. ITS can provide many solutions to this problem. Continuous monitoring of air and noise pollution can be done through the automated detectors and the traffic flow can be managed timely. For instance, if in one area the level of carbon dioxide exceeds the dangerous level, an automated traffic management system can guide vehicles to the alternative roads or other directions. Another application of ITS in environmental monitoring and protection is to automatically detect vehicles that do not have emission standards and then suspend the vehicle's registration (Mehta et al. 2003). Applications in this category typically require low localization accuracy. For the mentioned application above, the accuracy of 30 meters or even larger is sufficient because the goal is to monitor and report the condition of an area within a city. Therefore an approximation to that area would be helpful for further decision making for addressing the problem.

#### **2.2.5 Comfort Factors**

In this category, the applications are designed to provide users with more pleasant and comfortable travels. V2V and V2I technologies can be very useful for these applications.

Traveler information, public transportation information, and automatic parking payment are examples of this category, all of them use both V2V and V2I technologies (Sugiura and Dermawan 2005). Applications in this category require low to high localization accuracy. For instance, one of the most useful applications of this category is public transportation information. In this application, the users are informed of the arrival and departure times of buses at a bus station. The location and velocity of a specific bus is used to estimate its arrival time. Therefore, accuracy location information from the bus is required for this application. On the other hand, automatic parking payment application does not require an exact location of the vehicle in a parking garage, as it is only necessary to detect the presence of the vehicle in the parking and charge it the parking fee.

#### ***2.2.5.1 Traveler information***

Traveler information is categorized as a comfort application. Useful information or advertisement can be sent to vehicles using special local vehicles or infrastructure and downloaded by approaching vehicles. For instance, once the vehicle enters a street, the cellphone of the driver informs him of the surrounding shops, restaurants, and gas stations (Adler and Blue 1998). This system is known as traveler information system. This application can be implemented based on the cellular communications where many users share their information with others such their reviews and experience with the neighboring shops and restaurants. Moreover, other warning information such as the existence of ice or wild animals within the road can be provided to the vehicles. This specific application requires low to medium localization accuracy, since typically the exact location of the vehicle is not required and the presence of the vehicle at a specific area should be detected.

#### **2.2.6 Concluding Remark**

The localization accuracy and latency requirements of different ITS applications are summarized in TABLE 1. The green, orange, and red colors represent low, medium, and high, respectively. For instance, traffic light management requires medium localization accuracy at medium latency, while collision warning application requires very high localization accuracy at low latency.

TABLE 1 - Localization accuracy and latency requirements for ITS applications

Application	Accuracy	Latency
<b>Traffic Management</b>		
Traffic light Management	Medium	Medium
Variable Speed Limits	Low	High
Data Dissemination	Low	Medium
<b>Law Enforcement</b>		
Traffic Signal Violation	Medium	High
Electronic Toll Collect	Low	High
Homeland Security	High	Medium
<b>Safety</b>		
Collision Warning	High	Low
Lane Change Warning	High	Low
Detour Warning	Medium	Medium
Emergency Request	High	High
<b>Environmental Monitoring</b>		
CO Warning	Low	High
<b>Comfort Applications</b>		
Map Download/Update	Medium	High
Traveler Information	Low	High
Public Transportation Info.	Medium	Medium
Automatic Parking	High	High

## 2.3 Terminology

In this section, I would like to describe some terms which are widely used in the ITS literature. This section helps the reader to better understand the contents of this thesis.

### 2.3.1 Connected Vehicle

The increasing need for mobility leads to congestions, accidents, and emergency situations which results in many economic, social, and environmental impacts. The concept of vehicular communications which is based on information exchange among the vehicles is one of the effective ways that has come to address these problems by increasing the safety of transportation networks and improving traffic flow on roads and highways (Hartenstein and Laberteaux 2010; Papadimitratos et al. 2009). The communication between vehicles in the network is sometimes referred to as Connected Vehicle in the literature. Connected

vehicle technology is divided into two groups: Vehicle-to-Vehicle (V2V) and Vehicle-to-Infrastructure (V2I).

V2V Communications is one of the recent topics in the field of ITS safety (Boukerche et al. 2008). V2V is basically composed of a series of dynamic data exchange among neighboring vehicles. Having some important data, each vehicle can transfer its information to a neighboring vehicle using a wireless channel. These information may include (but not limited to) speed, location, distance to the neighbors, longitude, latitude, angle, etc. In this system, each vehicle is equipped with a central processor which analyzes the data and transfers important messages to the neighboring vehicles. For instance, in the time of an accident, the central processor senses the danger based on available data from the vehicle itself and its neighbor, informs the driver of the danger, and transfers the warning message to other vehicles to avoid accident. The main goal, therefore, is to equip all vehicles in the roads (cars, trucks, buses, and motorcycles) with a V2V technology in order to enable them to communicate with each other and exchange information. As a result, this technology helps drivers to avoid accidents which can significantly increase the safety in a transportation network and decrease fatalities by 76% (Schagrin 2013). FIGURE 2 shows an example of V2V communication. Encountering a dangerous situation, the blue car communicates with the green car and informs it of the situation. This will prevent the green car from accident and decrease the further damages on the blue car. Although several studies have been done in this area (Sichitiu and Kihl 2008; Tsugawa 2002; Yang et al. 2004), still many questions have been left without answer. For example, how much the infrastructure and the device in each vehicle would cost? Do different vehicles need different types of technologies (trucks might need a different V2V device than cars)?

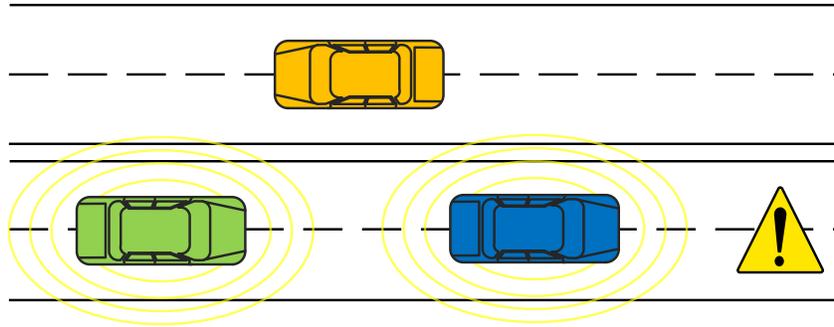


FIGURE 2 - Vehicle-to-vehicle (V2V) communications.

The communication between vehicles is sometimes called inter-vehicle communications (IVC). IVC systems typically do not rely on infrastructures. In these systems vehicles are communicating and exchanging information using onboard units (OBU) installed on vehicles. IVC is categorized into the two main categories (Sichitiu and Kihl 2008):

1. **Single-hop inter-vehicle communications (SIVC):** In this system each vehicle can only communicate with its neighboring vehicles. In other words, each vehicle can send a message to the vehicles that are located within its communication (transmission) range. These systems are useful for the conditions where only short range communications are needed. In these situations, it is sufficient for each vehicle to be informed of a situation by its neighboring vehicle (e.g., being informed about the lane merging by its succeeding vehicle.)
2. **Multi-hop inter-vehicle communications (MIVC):** In this system, using an intermediate vehicle, a vehicle can transmit information not only to its neighboring vehicles but also to the other vehicles in the network. This method is mostly useful where longer communication ranges are required such as traffic monitoring. Compared to SIVC, MIVC are more complex as multi-hop routing is required.

Vehicle-to-Infrastructure (V2I) Communications is another topic in the field of connected vehicles. In this case, vehicles exchange information with the infrastructure using almost the same technologies as V2V. Therefore, the overall objective is enabling the vehicles to communicate not only with each other but also with the infrastructure. Since infrastructure does not have the limitations that typically a vehicle does (e.g., space and cost constraint), it can be equipped with a much faster and more sophisticated central processor which can

analyze data more effectively and transfer it to the vehicles. Moreover, infrastructure is usually equipped with communication systems that have a higher range and a lower latency. The communication between the vehicle and infrastructure is called Roadside to vehicle communication (RVC), since the communications are taking place between OBU in the vehicle and road side units (RSUs). Based on the application, RVC technology can be divided into two: sparse RVC (SRVC) and ubiquitous RVC (URVC) systems. In the former, only important spots (hot spots) such as traffic lights, gas stations, and airports are covered. In the latter, large areas such as all the highways and roads are equipped with RVC technology. These systems can be deployed in special places where communication to the infrastructure is essential for providing the drivers with useful information or arranging systems for a more efficient traffic flow (e.g., the number of available slots in a parking lot and traffic light scheduling based on the traffic flow). On the other hand, equipping all roads with high speed communications makes the transportation networks significantly safer and offers new applications that are unavailable with current systems. However, establishing this system requires a considerable amount of investment to cover all existing roadways(Ikawa et al. 2010).

Detecting high-risk situations in advance, the infrastructure can alert drivers and prevent accidents. Some studies show that another 12% of road accidents can be decreased by V2I communications where V2V communications are not able to handle them (Schagrin 2013). As obvious, V2I communications for safety is a complement of V2V communications and should be done collaboratively. In conclusion, the main aim of V2I communications is to maximize the safety and efficiency of a transportation network with minimum number of required infrastructures (since they are generally expensive to build) and use V2I for the cases which V2V communications are not able to tackle. FIGURE 3 shows an example of V2I communication. The cars do not have a direct view of each other and the probability of an accident is very high. Because of the obstructions, V2V communications may be interrupted. However, using V2I communications, the infrastructure can alarm both cars and guide them to find a safe way to pass each other.

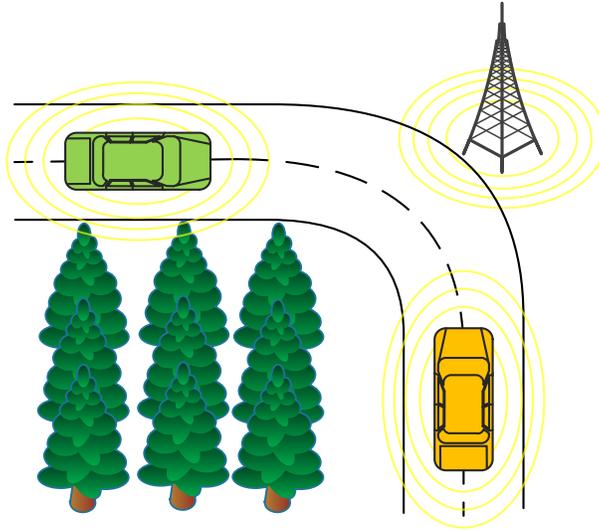


FIGURE 3 - Vehicle-to-infrastructure (V2I) communications.

### ***2.3.1.1 ITS Safety Pilot***

USDOT is currently undertaking a Connected Vehicle Safety Pilot Program which is a scientific research program for implementing connected vehicle technology and smart roads in real-world scenarios. The program includes performance evaluation, data collection, policies and process observation, determining human and usability factors, etc. Several government agencies, car manufactures, public agencies, and universities are involved in this program. The main vision of the program is to demonstrate the safety advantages and applications of connected vehicle technology and also reveal other related non-safety applications. Basically, the USDOT pursue the following four goals in this program:

1. Determination the effectiveness of connected vehicle technology for safety in real-world scenarios
2. Collection of data and information from ordinary drivers who use connected vehicle technologies
3. Identification of the potential safety advantages of connected vehicle technology
4. Determination the options to accelerate the implementation of connected vehicle

This program has two critical initiatives test: Pilot Driver Clinics and Safety Pilot Model Deployment. In the first test, connected vehicle technology is tested in a real-world scenario. The applications of connected vehicle are evaluated for a variety of light-weight vehicles and under different test conditions. Moreover, the behaviors and responses of drivers to this new technology are evaluated. Volunteers participate in several designed tests in the driver clinics which have various test tracks and parking facilities. There are six clinics throughout the United States. There are also some clinics specific for heavy-weight vehicles such as trucks. The test was conducted from August 2011 to January 2012.

The second test is developed by University of Michigan Transportation Research Institute (UMTRI). In the second test, connected vehicle technology is tested under wider and denser conditions in which about 3,000 vehicles equipped with V2V technology are involved. A highly dense environment is created in which the vehicles continuously talk to each other. Highly accurate data is collected to support the capabilities of connected vehicles for a mixture of different vehicles including trucks, cars, and transit vehicles and the drivers with three different age groups; younger, middle age, and older.

### **2.3.2 VANET**

A Vehicular Ad Hoc Networks (VANET) refers to a series of mobile vehicles in a transportation network which communicate wirelessly over a short range of 100 to 300 meters, but in total create a large network with a wide range (Hartenstein and Laberteaux 2010). VANET takes advantages of both V2V and V2I connections. Since most of these applications mentioned above use wireless channel to transfer data and information, many applications of ITS can be classified as VANET applications as well. Although the concept of using wireless technologies in vehicular communications has emerged in the 1980's, in the last few years VANET has been a topic of interest among researchers and developers. The major goals of VANET are to enhance the vehicular safety and efficiency, and to reduce the harmful effects of transportation networks on the environment. VANET is, in fact, an example of mobile ad hoc network (MANET), since vehicles are mobile and also create an ad hoc network. Ad hoc refers to a network where nodes can operate without infrastructure. Each node can participate in data transmission with other further nodes using a series of intermediate nodes. Nodes in ad hoc network typically requires lower

communication range and lower power consumption, as they only require to communicate with neighboring nodes. If a node needs to be connected to another node over longer ranges, the communication can be done through intermediate nodes(Hartenstein and Laberteaux 2010).

### **2.3.3 DSRC**

Dedicated short-range communications (DSRC) is a two-way wireless communication channel specially allocated for vehicle and automotive applications. Any wireless technology that operates in DSRC bandwidth is classified as a DSRC technology. In the US, 75MHz of spectrum in the 5.9GHz band is allocated for DSRC. Since the bandwidth is licensed, secure communication without significant inference can be easily done (few non-transportation applications operating in this range). Operating in a high frequency bandwidth, DSRC technologies can deliver high data rate and low latency data transmission. Current DSRC standards allow device to offer communication rates up to 27 Mbps with a communication range up to 1000 m, which enable the development of high data rate real-time ITS applications, long range measurement for localization along with opportunities for high-speed in-vehicle internet services(Iqbal and Yukimatsu 2011; Liu et al. 2005).

## 3 Literature Review

Positioning plays a crucial role in any intelligent transportation systems. A transportation network is composed of several elements including vehicles, motorcycles, bicycles, pedestrians, roads, infrastructures, etc. To have a convenient and reliable transportation system, we are interested in knowing the location of the elements with a reasonable accuracy. There are several types of localization systems that are being used for ITS applications and each application calls for particular system or systems based on its requirements.

### 3.1 Overview

In this research, I mostly concentrate on the importance of vehicle positioning for ITS applications. One of the most important information transferred among the vehicles or between vehicles and infrastructures is the *Here I Am* data message(Parker and Valaee 2006). Therefore, a reliable intelligent transportation network requires that the locations of most elements (especially vehicles) in the network to be available. The location of the infrastructure is typically available, since they are built at fixed, specific locations. Finding the location of vehicles is a difficult task because they are mobile and experience different weather and geographical situations. It should be noted that the terms localization, positioning, and navigation are used interchangeably which all means determining (estimating) the location.

In this section, an overview about several techniques used for vehicle positioning is provided. A summary of different localization techniques that will be described in this section can be found in FIGURE 4. The Global Positioning System (GPS) is one of the most popular and oldest localization techniques and it is widely used for vehicle positioning (Spiker 1996; Sun et al. 2000). Wireless sensor network (WSN) has recently emerged and found its way in localization (Losilla et al. 2011; Tubaishat et al. 2009). Although the accuracy of RFID localization is not as accurate as WSN and GPS, it can be helpful in some harsh environments(Ali and Hassanein 2009). Having different features from other mentioned methods, the vision-based technique is also significantly practical for vehicle

localization(Atev et al. 2005; Buch et al. 2011). Each localization technique will be described in more detail in the next sections. But before that, some other terms should be defined. The localization techniques are divided into several groups. FIGURE 4 shows the classification of the localization algorithms. In next sections, I will describe each group in detail.

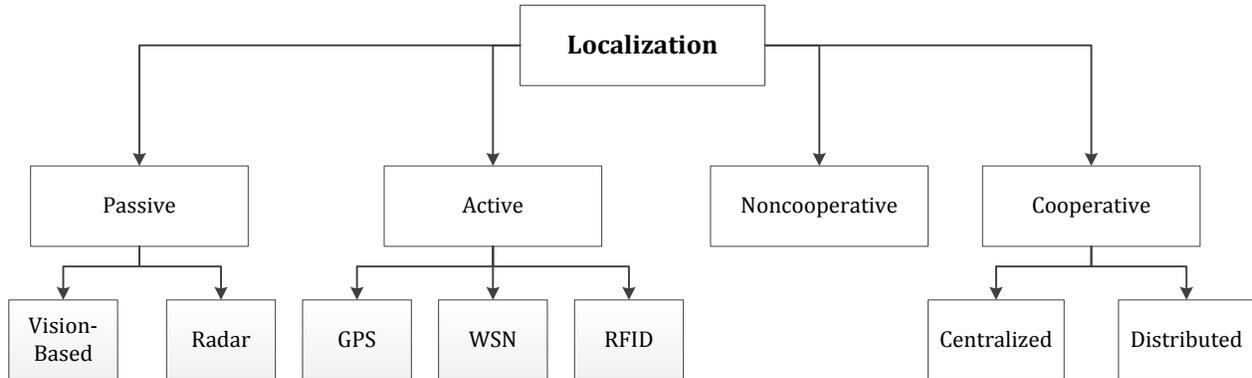


FIGURE 4 - Classification of the localization techniques.

### 3.1.1 Cooperative and Noncooperative

Localization algorithms are typically divided into two major groups: Cooperative and Noncooperative. In noncooperative networks, vehicles are only able to communicate with infrastructure. Therefore, if two vehicles need to communicate with each other, it has to be done through the infrastructure. Cellular systems are an example of noncooperative networks. In noncooperative systems, every communication has to be performed through a base station. Two neighboring vehicles cannot connect directly to each other. Therefore, infrastructure is required to cover the entire network. Although to transfer data only one infrastructure is required, the vehicle needs to be connected to at least three infrastructures to be localized uniquely. Therefore, either longer communication range or dense infrastructure is required in noncooperative networks to ensure that the vehicle is connected to enough number of infrastructures. The former increases the interference in the network significantly. The latter is not energy and cost effective.

In cooperative networks, vehicles can communicate not only to the infrastructure but also to each other. Ad hoc systems are an example of cooperative networks. Based on the definitions, in terms of vehicle communications, noncooperative localization only uses V2I communications, while cooperative localization uses both V2V and V2I communications. Cooperation among the vehicles has several advantages. Using cooperative networks eliminates the need for dense infrastructure and long communication range. Cooperative networks are highly cost and energy efficient. The interference in the network decreases, as vehicles do not need to communicate with infrastructure located far away from them. Since in VANET the vehicles can communicate with each other without aid of any infrastructure it is categorized as a cooperative network.

FIGURE 5 shows an example of cooperative and noncooperative vehicle networks. As can be seen, in the noncooperative network, the vehicles do not have sufficient connections, because the communication range is limited and they can only communicate with infrastructures. On the other hand, in the cooperative network, the vehicles have at least three connections either with infrastructures or other vehicles, even if the communication range is limited. In a cooperative network, all vehicles can be localized, while in a noncooperative network, only one vehicle might be localized.

Cooperative localization itself is divided into Centralized and Distributed algorithms. In the former, all information is sent to a central processor and then processed. The locations of all elements are determined simultaneously in the central processor and sent back to them. In the latter, the location of each element is processed inside itself and shared with neighboring elements(Patwari et al. 2005).

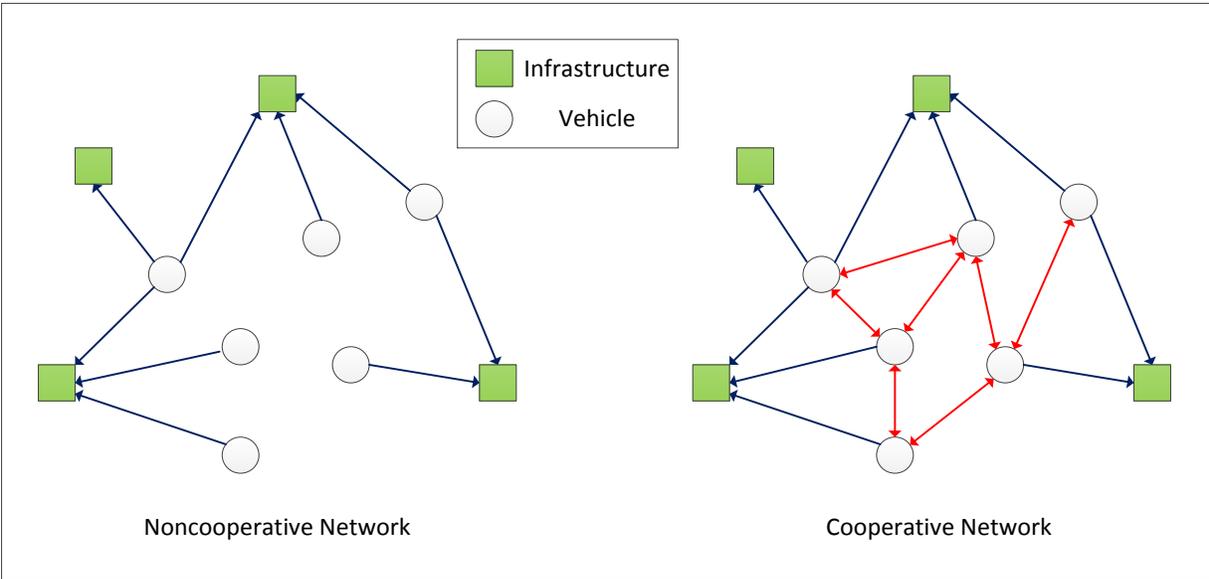


FIGURE 5 - Cooperative and noncooperative vehicle networks.

### 3.1.2 Active and Passive

The localization techniques are also divided into two groups of active and passive. In active systems, an electronic device or tag is installed on the vehicle to be localized. The majority of the localization techniques are classified as active systems. For instance, the Global Positioning System (GPS) is an active localization technique in which a GPS receiver should be carried on the vehicle. On the other hand, in the passive systems, the vehicle is localized without a need for any physical devices. Vision-based techniques are an example of passive localization technique, where only a special camera is required for localizing the vehicle. Radar systems can also be used for passive localization, as no device is required on the object to be localized. Both active and passive localization techniques can be used for noncooperative and cooperative networks.

### 3.1.3 Relative and Absolute

There are two types of location systems: absolute and relative. The former system uses a shared coordinates of all elements' locations. GPS is an example of absolute location system where all satellites are using the same coordination system and the location of the vehicle is reported based on these coordinates. In the relative location system, each element uses

its own coordinates. For instance, a rescue team searching for lost victims uses its own hand device to find the location of the victims. In case of several rescue teams, each team finds the locations of the victim relative to itself. Determining whether a location is absolute or relative depends on what information is available and how the system uses it (Hightower and Borriello 2001).

## **3.2 GPS**

Global navigation satellite system (GNSS) refers to any satellite navigation system with global coverage. GPS is in fact an example of GNSS which was first created by the U.S. Department of Defense. Russian GLONASS is another global operational GNSS. In this context, when we talk about GPS, we mean any satellite-based navigation systems and are not limited to the United States NAVSTAR GPS. The first and the most accessible navigation technique is GPS (Hofmann-Wellenhof 1997; Spiker 1996). GPS is a satellite-based navigation system which can be used to find the vehicles' location. The GPS project was developed in 1973 for the first time by the U.S. Department of Defense (DoD). Nowadays, it is used for many civil, commercial, industrial, and military applications. Communicating with several satellites above the earth, the GPS receiver is a piece of equipment that is placed inside a vehicle to determine its location.

Currently, there are 32 GPS satellites at the height of 20,180 km covering all parts of the earth. Each satellite travels its orbit in about 12 hours (Ahmed 2002). Satellites are equipped with four atomic clocks which are very accurate. However, they are still synchronized from the earth. The GPS receiver uses time-of-arrival (TOA) measurements to determine its location. In TOA method, the travel time of the signal between the receiver and satellites are measured. The synchronization between the satellites and the receiver is a very difficult task. Although the clocks of the satellites are very accurate, the receiver should take the time error into account. Only 1  $\mu$ s can produce an estimation error of 300 m (Ahmed 2002). Therefore, as will be later shown, four parameters should be estimated from a series of TOA measurements: the coordinates of the receiver, and the clock error. For two-dimensional and three-dimensional localization, the receiver needs to be connected to at least three and four satellites respectively. The satellites communicate with

the receivers at two specific frequencies: L1 (1575.42 MHz) and L2 (1227.60 MHz). Civilian receivers use L1 frequency, while military receivers use both L1 and L2 frequencies. Satellites frequently transmit the navigation message to the receiver, which is required for the determination of the current location of the satellite as well as signal travel time. Consisting of 1500 bits, each navigation message includes important information: satellite clock and health data of the satellite, satellite orbital data (ephemeris), and orbital data for all other satellites (almanac). The receiver is required to demodulate the message and extract the information. The ephemeris is updated every 2 hours, while the almanac is updated typically every 24 hours (Ahmed 2002).

Once the GPS receiver receives the message, it uses the received time and the transmitted time to find the travel time of the signal between the satellite and the receiver. Then the travel time is used to estimate the range between the receiver and the satellite. Afterwards, the receiver uses the estimated ranges from several satellites to determine its position. Let  $\mathbf{x} = [x \ y \ z]$  be the location of the GPS receiver and  $\mathbf{y}_j = [x_{s,j} \ y_{s,j} \ z_{s,j}]$  be the location of the  $j$ th satellite. Let  $t_j$  be the received time of the signal at the receiver from the  $j$ th satellite. Therefore, it can be modeled as

$$t_j = \frac{d_j}{c} + t_o + n_j \quad (1)$$

where  $t_o$  is the transmission time,  $c$  is the speed of light and  $d_i$  is the true distance between the satellite and receiver:

$$d_j = \sqrt{(x - x_{s,j})^2 + (y - y_{s,j})^2 + (z - z_{s,j})^2} \quad (2)$$

and finally  $n_j$  refers to the measurement error which is typically modeled as a zero-mean Gaussian random variable with variance of  $\sigma_{s,j}^2$ . To design a reliable GPS receiver, it is necessary that the parameters causing the errors be taken into account.

TABLE 2 - A Summary of the sources of error in GPS.

<b>Source</b>	<b>Effect (m)</b>
Signal arrival (C/A)	3.0
Ephemeris errors	2.1
Satellite clock errors	2.1
Multipath distortion	1.4
Ionosphere effects	4.0
Tropospheric effects	0.7
$\sigma_R(C/A)$	6.7

The effect of each source of error is expressed as the user equivalent range error (UERE). UERE determines the amount of error on the range measurement in meters.

The receiver hardware, itself, is not perfect which leads to an error on the measurement noise. The experimental results show that this error would be around 3.0 m for civilian receivers and 0.3 m for military receivers. Since military receivers typically use more sophisticated hardware, this error is considerably lower in military receivers than the civilian ones.

The receiver extracts the satellites locations from ephemeris data. This data should be updated and sent to the receiver regularly. However, sometimes the receiver uses up to 1-hour old data and predicts the future location of the satellite. Therefore, the receiver might not have access to the true location of the satellites which leads to an error on the range measurement. Error on the ephemeris data is anticipated to produce 2.0-2.5 m error on the range measurements.

Although the satellites are equipped with highly accurate atomic internal clock, they are not perfect and still subject to error which will lead to inaccuracy in the range measurement. The error made by imperfect satellite clock is expected to be around 2.1 m.

Multipath has huge impact on the accuracy of the range measurement, especially in dense environments where several objects such as walls and buildings cause the GPS signal to arrive at the receiver from different paths. Multipath distortion produces approximately 1.4 m error on the range measurement.

Other sources of error in the range measurement of the receiver come from the atmosphere. The atmosphere can be the source of two types of errors: ionospheric and tropospheric delay. Ionospheric region composes a part of the atmosphere including several layers with different altitudes and thickness which varies over time due to changes in the temperature and the amount of sun radiation. This region changes the propagation velocity of the GPS signal which itself causes errors in the range estimation. The troposphere is the lowest region of the atmosphere. Similar to ionosphere, the properties of this region changes over time with several parameters such as humidity, pressure, and temperature. The tropospheric region also affects the propagation velocity of the signal making error in the range measurement.

The overall error on the range measurement can be obtained by taking root sum square of above mentioned individual errors as follows:

$$\sigma_R = \sqrt{3.0^2 + 2.1^2 + 2.1^2 + 1.4^2 + 4.0^2 + 0.7^2} = 6.7\text{m} \quad (3)$$

Typically, a GPS receiver uses four or more satellites to achieve a reasonable positioning accuracy. Generally speaking, the more the satellites to be used, the higher the accuracy to be achieved(Ochieng and Sauer 2002). However, as the number of used satellites increases, the complexity of the localization algorithm increases which can lead to significant latency in positioning. Although GPS receivers are used to determine a 3-dimensional location estimation, in several cases a 2-dimensional location can be used, for instance in an aircraft or a ship where the third dimension (height) is generally known or can be measured by an alternative device (e.g., altimeter). In very rare conditions, a GPS receiver can determine the location with fewer than three available satellites. In this case, the GPS may exploit some information from previous available locations which is called dead reckoning(Aono et al. 1998). However, in this case, localization would not be as accurate as using more satellites. Another technique for enhancing the accuracy of navigation when there are not enough visible satellites is Map Matching (Jagadeesh et al. 2004). Map matching algorithms using several databases including all the maps and information where the vehicular network is located to improve the navigation accuracy by constraining the estimation based on available road maps. As described here, the number of visible satellites to the GPS

receiver is very important to achieve a reliable accuracy. FIGURE 6 shows a vehicle being localized by four visible satellites and a GPS receiver placed inside the vehicle.

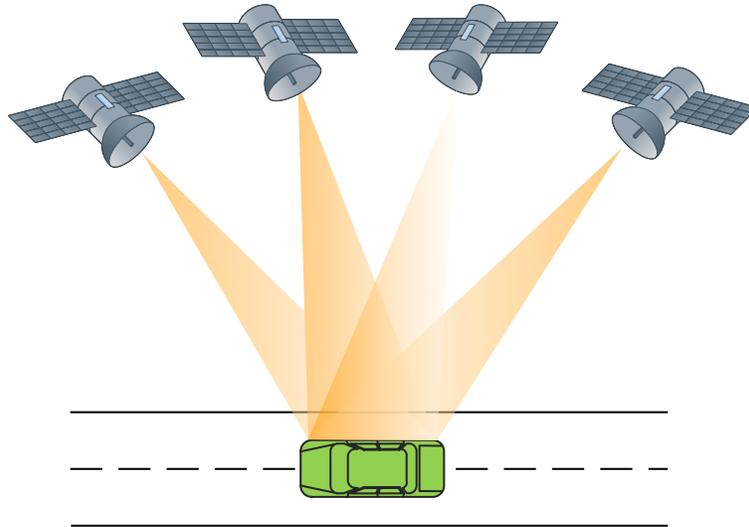


FIGURE 6 - The Global Positioning System (GPS).

Most studies in the field of ITS indicate that GPS, by itself, cannot provide accurate position information in all situations and should be assisted by extra resources (Ahmed 2002; Tsui 2005). The GPS receiver needs to have a line-of-sight connection with several satellites. However, this requirement cannot be satisfied in dense and indoor environments where most connections are not line-of-sight. In undercover areas such as tunnels and parking garages, the satellite signals attenuate deeply and cannot reach to the receiver. In dense environments, GPS signals suffer from multipath effects, meaning that the signals travel further to get to the receiver than they should have which results in the GPS miscalculating the position. The reason is that the signals travel on different paths rather than a direct line-of-sight. Multipath has a large impact on the accuracy of GPS. Due to shortcoming of GPS systems in these environments, alternative positioning techniques have been introduced (Feng and Law 2002).

### 3.2.1 DGPS

Some studies in the field of positioning are focusing on using Differential Global Positioning System (DGPS) to improve the accuracy of localization obtained by a GPS receiver. DGPS is

a promising enhancement to the classic GPS that can result in more accurate position estimation in the most cases. In this technique, by using a reference point or a network of reference stations with known locations the difference between the estimated GPS pseudoranges and the internally computed pseudoranges using the reference stations can be computed and broadcasted to the receiver station. Then the receiver station can correct them accordingly. In fact, this technique can be used to reduce the GPS error sources. The amount of improvement that can be achieved by this technique is dependent on the location of the reference stations and the architecture of the network. Thus, many studies have focused on different algorithms for the design of reference stations to achieve a better accuracy. DGPS was first used for maritime navigation systems. For this application, the reference stations are installed near the coast line. Using the information obtained by them, the GPS error is delivered to the ships and they can correct their coordinates. Since there is no obstacle (e.g., buildings) distorting GPS signal in the seas, the calculated GPS error is the same for all the ships passing near a particular reference station. Therefore, this technique is very suitable for ship navigation applications. The main drawback of this system is that it completely relies on GPS and, therefore, cannot operate in case of GPS outage. Moreover, in a transportation network, there can be many tall buildings around the road which block the signals and make the signals to be distorted and the GPS error will be different even for the vehicles that are passing within a small area near a reference point. Another drawback of using DGPS sensors inside the vehicles is the cost associated with installing the sensors in each vehicle.

### **3.3 WSN**

The second navigation technique that can be applied to localize the vehicle is to use a wireless sensor network (WSN) (Patwari et al. 2005; Sayed et al. 2005). A WSN consists of several sensors distributed in an area which can communicate with each other through a wireless channel and transfer useful information. Having different features and sizes, sensors can be used for many applications including control, monitoring, and tracking. A sensor is composed of several parts including radio transceiver (a device which is able to transmit and receive through a wireless channel), an internal antenna, a microcontroller,

electrical circuits, and an energy source (e.g., a battery). Depending on the energy and cost constraints of the application, sensors come with different sizes and computational capabilities. Moreover, sensors are able to work in both noncooperative (single-hop) and cooperative (multi-hop) networks (Patwari et al. 2005). Besides the ability to transfer information among each other and with infrastructures, sensors can be used for navigation. Sensor localization includes several nodes called anchor nodes whose locations are known and several nodes called source nodes whose locations are unknown and should be determined. Anchor nodes can be placed in a fixed location such as on an infrastructure or traffic light, while source nodes are located inside vehicles in order to help them to be localized. It should be noted that a single source node inside a vehicle can be used for both data transfer and sensor localization simultaneously. Localization among sensors is performed through noisy measurements. These measurements include, but not limited to, time-of-arrival (TOA), received signal strength (RSS), and angle of arrival (AOA). TOA-based sensor localization follows the same technique that is used in GPS in which the travel time of the signal is measured, except anchor nodes in the earth play the role of satellites. There are several advantages for sensor localization over GPS which make it more interesting. First, a sensor is typically cheaper and less bulky than a GPS receiver. Second, a GPS receiver can be only used to determine the location. However, the location would be valuable if it could be transmitted to the neighboring vehicles or the infrastructure. Therefore, we still need an external device to transmit the location information to the surrounding area through a wireless channel. On the other hand, a sensor is able to transmit data and localize itself simultaneously without the aid of an extra device, since sensors are basically built to do so. Third, not every place in a transportation network can be covered with satellites. As mentioned earlier, to achieve a reasonable accuracy, a GPS receiver needs to have access to at least four visible satellites. This requirement cannot be fulfilled in all situations. For instance, when a vehicle is inside a tunnel or a multi-story parking garage, its GPS receiver no longer works, as satellite signals attenuate sharply in indoor environments. This gap can be covered by using wireless sensor network localization in these areas. In FIGURE 7, the location of the vehicle equipped with a source node is determined by anchor nodes using wireless sensor network localization.

### 3.3.1 Measurement Methods

In this section, I will describe the measurement methods typically used for sensor localization. In all methods, let  $\mathbf{x} = [x \ y \ z]$  be the location of the source node and  $\mathbf{y}_j = [x_{a,j} \ y_{a,j} \ z_{a,j}]$  be the location of the  $j$ th anchor node.

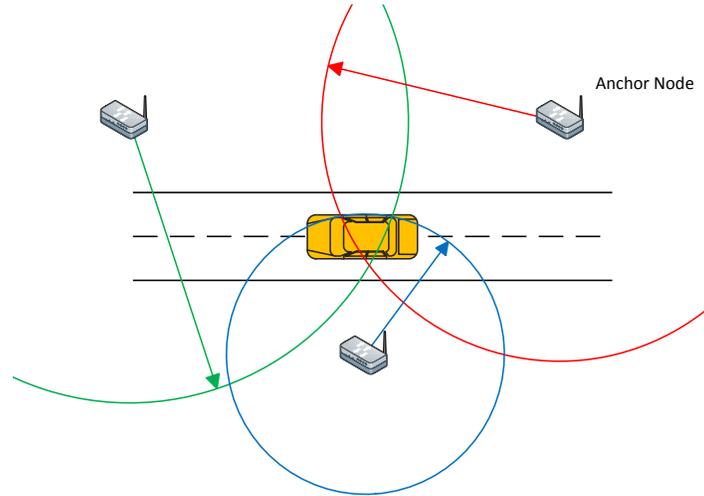


FIGURE 7 - Wireless sensor network localization.

#### 3.3.1.1 TOA

TOA method is also used for GPS. In this method, the time that the signal travels between the source node and the anchor node is measured (Patwari et al. 2005):

$$t_j = \frac{d_j}{c} + n_j \quad (4)$$

The distance between the nodes can be easily computed by multiplying the measured time by the propagation velocity:

$$r_j = ct_j = d_j + cn_j \quad (5)$$

Again  $n_j$  defines the measurement error. However, in this case, the sources of error are different from the GPS errors. The generalized cross-correlator is typically used to find the received time at the source node (Patwari et al. 2005). The accuracy of TOA measurement is dependent on the signal bandwidth,  $B$ , and the signal-to-noise ratio (SNR). It can be shown that the accuracy of TOA measurement for a given system with fixed bandwidth and SNR is lower bounded by (Patwari et al. 2005):

$$\text{Var}[\text{TOA}] \geq \frac{1}{(2\pi B)^2 \text{SNR}} \quad (6)$$

From (6) it can be seen that as the SNR increases the accuracy of TOA measurements increases. However, the wireless systems have limitations on the transmit power and it is not possible to increase the transmit power unlimitedly. Alternatively, high accuracy in TOA measurements can be obtained by increasing the bandwidth of the signal. Moreover, as can be seen from (6), the bandwidth has higher impact on the accuracy of TOA than the received SNR. That is why ultra wideband (UWB) systems are highly popular in radiolocation applications (Correal et al. 2003). An UWB system uses a narrow pulse with a short duration which causes its bandwidth to be spread over the entire spectrum. A signal is considered to be UWB if its fractional bandwidth (the ratio of its bandwidth to its center frequency) is larger than 0.2. The very high bandwidth of UWB signals makes it ideal for high-precision positioning applications. Multipath propagation also affects the accuracy of TOA measurement. Arriving several copies of the signal at the receiver makes the measurement of the TOA difficult and decreases its resolution. This problem can be also mitigated by using UWB signals in which the multipath components can be separated more easily. Similar to GPS, the TOA measurement requires perfect calibration between the anchor nodes and the source node. If the nodes are not perfectly synchronized the effect of clock can affect the measurement model, similar to the model of the GPS. Alternatively, time-difference-of-arrival (TDOA) method can be used (Vaghefi and Buehrer 2013). In TDOA technique, an anchor node is selected as a reference (typically the one that is closer to the source node) and its TOA measurement is subtracted from the TOA of the other anchor nodes. By this way, the dependency of the measurement model on the imperfect synchronization is eliminated. Another way to deal with the lack of synchronization between the source node and the anchor node is to use two-way (or round-trip) TOA method. In this method, the source node transmits a signal and the anchor node replies back once it receives the signal. Therefore, the source node measure the round trip travel time of its own signal rather than the travel time between itself and anchor node.

### 3.3.1.2 RSS

In RSS method, the amount of the signal attenuation from the source node to the anchor node is measured. When a source node transmits a signal to an anchor node, its power degrades as it travels. The average measured power received at the anchor node is defined as the received signal strength (RSS). These measurements are dependent on the distance between the anchor and the source nodes and are modeled as (Patwari et al. 2003):

$$P_j = P_0 + 10\beta \log \frac{d_j}{d_0} + n_j \quad (7)$$

where  $P_j$  is the received power at the anchor node and  $P_0$  is the received power at the reference distance  $d_0$ .  $\beta$  is the path-loss exponent defining the rate at which the power of the signal attenuates as it propagates through space. The value of the path-loss exponent varies from 2 in free space and 4 in dense and harsh environments.  $n_j$  refers to the shadowing which is modeled as a zero-mean Gaussian random variable. The standard deviation of the shadowing is expressed in dBm and is irrelevant to distance (unlike TOA method). Fading and shadowing are the two sources of error in RSS method. Fading may be due to multipath effects (i.e., when an object blocks the clear view of the signal between the source node and anchor node which results in receiving the signal from more than one path). The other source of error is shadowing from obstacles which affects the wave propagation. The effect of fading can be mitigated by averaging over time and the effect of shadowing is considered in  $n_j$  (Patwari et al. 2005). The RSS measurements are used for localization in two ways. The location of source nodes can be directly estimated from the RSS measurements. However, the distance between the source node and anchor nodes can be first extracted from the RSS measurements and then the location of the source nodes is estimated from the obtained distance measurements (similar to TOA method). The distance between the source node and the anchor node is estimated as (Patwari et al. 2005):

$$d_j \approx d_0 10^{\frac{P_j - P_0}{10\beta}} \quad (8)$$

Similar to TOA, the RSS method requires a calibration among the nodes. The source node needs to know the transmit power in order to calculate the distance. If the transmit powers

are unknown, differential RSS (similar to TDOA) method can be used and the dependency of the model on the transmit power is removed.

### 3.3.1.3 AOA

Unlike the previous methods, in this method, the distance between the anchor node and the source node is not measured. Instead, in AOA method, the direction between them is measured. Two ways are commonly used in sensors to measure AOA. One can measure the AOA by using the RSS ratio between two (or more) directional antennas located on the sensor. Another method is to use a sensor array at the receiver and employ array signal processing techniques. The AOA measurements are modeled as (Patwari et al. 2005)

$$\alpha_j \approx \tan^{-1} \frac{y_{a,j} - y}{x_{a,j} - x} + n_j \quad (9)$$

Note that in this method the model is not a function of distance. Moreover, the AOA measurements are typically used for 2-D localization. An extension of AOA method for 3-D localization is more complicated and rarely used in practice.  $n_j$  refers to the measurement noise and depends on the hardware implementation and the network environment. Multipath propagation affects the accuracy of AOA method. Experimental results show that the average error on the AOA measurements is about 2 to 6 degrees. AOA method is less popular than the TOA and RSS methods, mainly because of expensive hardware implementation.

### 3.3.1.4 Hybrid

In some applications, two or more types of the measurements are available. As using hybrid localization techniques, the accuracy of localization can be significantly improved over regular methods, this technique has become very popular recently. Several hybrid techniques have been proposed in the literature, such as RSS/TOA, RSS/TDOA, TOA/AOA, etc. However, it should be noted that hybrid systems typically have higher complexity than the regular systems.

### 3.3.2 V2I

Several studies are conducted on the applications of WSN for ITS and vehicle detection (Katiyar et al. 2011; Losilla et al. 2011). A WSN is composed of autonomous sensors that

convey data within their networks. In WSN localization, sensors with known location (anchors) are used to localize the sensors with unknown location (Patwari et al. 2005). Since the anchors are typically installed on infrastructures, this method is usually referred to as V2I communications. (Tewolde 2012) has studied the advantages of using V2I technology in terms of reducing cost and power consumption along with increasing the efficiency of the network. Using both V2V and V2I communications, (Katiyar et al. 2011) has presented an algorithm which can increase the safety in roadways. They have proposed a WSN architecture for improving the safety and efficiency of a transportation network. In their system, they take advantage of WSN and Bluetooth technology. In the proposed architecture, some powerful sensors known as base stations are installed on important and critical areas and transfer information to the vehicles passing them. Moreover, the base stations are able to communicate with each other in cases where there is no base station near the vehicles. They highlighted the advantages of the communication among vehicles to prevent accidents. They assumed that each vehicle knows its location and can transfer it to the neighboring vehicles. However, they have not elaborated on how the vehicles obtain their locations. (Khanafer et al. 2009) have studied V2I architecture for intelligent transportation systems. The main objectives of that study are to determine the requirements for having efficient wireless sensor architecture for different types of applications and cover the advantages and drawbacks of each method. They propose a network in which several sensors are used to monitor physical parameters or environmental conditions, such as temperature, sound, and vibration. Architecture for supporting a navigation system using wireless sensor is also introduced. One application of this system is that it can provide the driver with an optimal route (i.e., the route that is more economical in terms of gasoline consumption with the least travel time). The proposed system is based on infrastructure-less V2V protocols.

### **3.4 V2V**

Cooperation among vehicles which is also referred to as V2V communications is one approach to improve the accuracy of localization obtained by GPS (Graettinger et al. 2001; Kukshya et al. 2005; Oh et al. 2010; Tsugawa 2002). Cooperative positioning refers to a

situation where a vehicle communicates to other nearby vehicles and uses their information to improve its location estimation (Tsugawa 2002). It has been shown that cooperation among the vehicles improves vehicular positioning accuracy considerably (Yao et al. 2011). A simulation study has been conducted in (Yao et al. 2011) to examine the performance of cooperative systems in VANET environment. Since the performance of cooperative positioning (CP) in VANET highly depends on the number of measurements obtained from available neighboring resources, they investigated the usefulness of CP under several communication factors in VANET. Typically, the more vehicles or sources of information exist around a vehicle, the more accurate location information can be obtained. However, increasing the number of neighboring vehicles may increase the dedicated short range communications (DSRC) wireless channel congestion which results in more transmission collisions and network channel deterioration. Therefore, they studied and characterized the performance of CP under various DSRC communication scenarios (i.e., traffic load, repeated broadcasts, and transmission range). Moreover, the impact of localization constraints on CP accuracy has been studied. More specifically, the comparison of non-cooperative and cooperative positioning systems has been provided (Patwari et al. 2005). It has been shown that CP is more effective when the GPS accuracy is poor. In addition, the effect of ranging accuracy on CP performance has been investigated (Yao et al. 2011). Based on this study, to have a better CP accuracy, more accurate ranging techniques should be utilized.

Using V2V communications, (Tsugawa 2002; Yang et al. 2004) NHTSA 2012 have proposed cooperative algorithms for several ITS applications or collision warning and indicate that they can provide a higher positioning accuracy (Tsugawa 2002) and lower latency in delivering emergency warning (Yang et al. 2004). Using V2V communication, the vehicles can acquire their more precise locations by having the GPS coordinates of the neighboring vehicles. In case of GPS outage, the vehicles can only estimate their relative location by communication with other vehicles. Exchanging the GPS data with the neighboring vehicles, the vehicle is able not only to estimate more precisely its location but also to disseminate information and emergency warning messages. (Kukshya et al. 2005) have addressed some challenges in the field of cooperative collision warning, one of which is its

dependence on real-time GPS information. They have studied three different scenarios of V2V communications application for location estimation. In the first scenario, it is assumed that all vehicles are equipped with GPS and dead reckoning (DR) systems. In DR technique, previous information (location and velocity) is used to predict the next time-step's information (Aono et al. 1998; King et al. 2005). In this way, the vehicles can obtain their location using GPS and DR information and transfer the data to the neighboring vehicles. In case of not having access to GPS signals from enough number of satellites (i.e., four or more), the location is determined by using DR. In the second scenario, it is assumed that GPS systems have been installed in some of the vehicles. The vehicles without GPS and DR can take advantage of the provided information by other vehicles to determine their own location information. In cases where a vehicle equipped with GPS and DR system does not have access to GPS signals, the vehicles will be grouped and listed as the vehicles without GPS systems. In the third scenario which is the main contribution of this study, it is assumed that none of the vehicles is equipped with GPS system and DR (e.g., persistence GPS outage). Studying cooperative collision warning based on V2V communications, they have proposed a solution to achieve reasonable accuracy in these situations. It is indicated that their system reduces the dependency of the cooperative collision warning on real-time GPS while performing vehicle positioning. However in their algorithm, in case of GPS outage, the vehicles can only have their relative positions not their absolute ones. Taking advantage of other resources such as RFID systems and V2I can be more beneficial in this case.

### **3.5 RFID**

A new technology that uses a wireless communication system with radio-frequency electromagnetic fields to transfer data from a tag to a reader is called radio-frequency identification (RFID) (Chawla and Ha 2007). Similar to WSN, RFID can be used for both localization and data transfer simultaneously (Zhou and Shi 2009). An RFID system includes several RFID tags with unknown locations and several RFID readers with known locations. WSN and RFID share many similar characteristics. However, there are several tangible differences. First, RFID tags are typically smaller and much cheaper than sensors.

They usually consume less power (some RFID tags do not even require a battery, since they receive their energy through electromagnetic fields). Second, RFID tags typically have lower ranges and lower data rate than sensors. Third, unlike sensors, RFID tags are not able to measure ranges from an RFID receiver or vice versa. Therefore, a reader is only able to tell whether a specific tag is inside its communication range or not. This property of RFID systems makes them less popular for critical situations where a highly accurate positioning is required. However, their other properties such as low prices and small sizes still make them useful for localization in some situations. In FIGURE 8, an RFID localization system is depicted. The vehicle is equipped with an RFID tag. Once the vehicle comes inside the communication range of the RFID reader, its presence is detected. However, the reader cannot determine the exact location of the vehicle. Let  $\mathbf{y}_j = [x_{r,j} \ y_{r,j} \ z_{r,j}]$  be the location of the  $j$ th RFID reader. The measurement obtained from the RFID tag (installed on the vehicle) is modeled as

$$d_j \leq r_j \tag{10}$$

$$\sqrt{(x_{r,j} - x)^2 + (y_{r,j} - y)^2 + (z_{r,j} - z)^2} \leq r_j$$

By using several RFID readers, the area where the presence of RFID tag is more probable can be constrained. This technique is also widely used for toll collecting and vehicle detection (Blythe 1999).

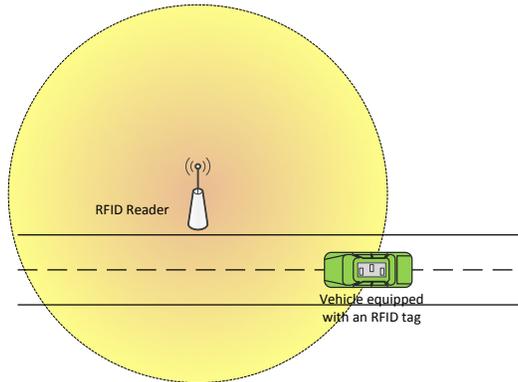


FIGURE 8 - Radio-frequency identification (RFID) localization.

Several studies suggest incorporating RFID systems to improve the GPS accuracy. An RFID-aided localization system has been proposed in (Lee et al. 2012). It is assumed that all the vehicles are equipped with RFID system. However, in their system, some of the vehicles have the GPS system installed on them. DSRC technologies are used to broadcast the data to the neighboring vehicles. In this research, it is assumed that passive RFID tags are attached to the road surface or the vehicles and RFID readers are installed on the vehicle for reading the data from RFID tags. In addition, a reference vehicle that has a GPS installed on that will obtain data and information from the RFID tags. In fact, this reference vehicle plays the role of reference stations in DGPS systems. The difference is that in this system the reference vehicle is a moving object. In this system, when the vehicle passes a stationary RFID tag, the absolute coordination is calculated and the GPS error is determined accordingly based on the estimated position given by GPS. This calculated error will be broadcasted to the other neighboring vehicles so that they can correct their own GPS coordination. Therefore, in this system, two different data will be provided for the vehicle: first, the coordinates from its own GPS device and second, the estimated location information from RFID sensors. Using this information, the vehicle computes the GPS error and shares it with other neighboring vehicles. Hence, neighboring vehicles can refine their GPS location estimation based on shared information. Although this algorithm results in a more accurate localization, it highly relies on GPS and cannot be used in cases where GPS outages can happen. An active RFID system in which all vehicles are equipped with RFID readers has been studied in (Zhang et al. 2011). RFID tags are also installed along the lane to help the vehicle localize itself more accurately. It is indicated that this system can improve the positioning accuracy and can be used in places where GPS access is limited. However, a stand-alone RFID system cannot provide a promising accuracy in indoor places such as parking garages and tunnels, or within dense environments such as forests and canyons. In these places, using other supplemental resources such as V2I communications is recommended. Although positioning only based on RFID systems can be cheap and fast (Zhou and Shi 2009), its performance would not be comparable with GPS (when it has good reception) and other positioning systems which use V2I connections.

### 3.6 Map Matching

Another concept that has received great attention in the literature is Map Matching for improving the locations of the vehicles in a network. The information provided by DR sensors along with the road data are used to determine the spatial reference of vehicle location using a map matching process which are then used to improve the accuracy of localization. The main objective of the map matching technique is to use map data stored in the device to match with the location obtained from an external resource such as GPS and apply necessary corrections in order to increase the accuracy of localization. In such techniques, a primary positioning technique is required and map matching plays a role of an auxiliary algorithm just to enhance the accuracy. Therefore, the map matching techniques would be useless when the primary positioning systems fail to provide a preliminary location of the vehicle. Moreover, these techniques require large amount of data to be stored in the device located in the vehicle and the maps and data should be updated frequently. The processing time of location estimation using these techniques is typically high which makes them less practical. However, recently several map matching algorithms have been proposed in the literature to address these problems (Jagadeesh et al. 2004; Quddus et al. 2007). There are different ways to perform map matching in a transportation network. It can be done by a simple search technique (e.g., point to point and point to curve matching) (Kim 1996) or by using more advanced and complicated techniques such as the modified extended Kalman filter, fuzzy logic, etc. The performance and speed of the different algorithms are dependent on the technique that is used for them. It has been mentioned that fuzzy logic can provide a better performance in comparison with other techniques. In (Jagadeesh et al. 2004), a map matching algorithm is developed and examined through computer simulations. In this work, the GPS signal behavior is analyzed and an algorithm is developed for determining the vehicle location when GPS is the sole mean of positioning. The algorithm tries to evaluate the likelihood of the vehicle moving on each path around the current location of the vehicle given by GPS. Therefore, as the vehicle moves on a road, the previous roads behind the vehicle will be removed from the candidate road list and the successor roads are added to that list and therefore it helps increasing the location estimation accuracy obtained by GPS.

### **3.7 Dead Reckoning**

Dead reckoning (DR) is another technique that can be used to improve the performance of positioning. In this technique, the previous location information of the vehicle is used to increase the accuracy of future locations of the vehicle. Several underlying dynamic models are used to predict the next location of the vehicle which is then combined with the data obtained from an external resource such as GPS to estimate the location of the vehicle. In (Lahrech et al. 2005), the authors proposed an algorithm which uses DR along with map matching to improve the accuracy of the positioning obtained by a GPS receiver. When GPS satellites are available, the algorithm corrects the location using DR and the road map database to obtain better accuracy. If the GPS data is not available, the algorithm predicts the location of the vehicle based on the dynamics equations of DR. Then, the algorithm uses the map matching technique and corrects the current estimation with extra information from road map data, if available. DR is highly beneficial when an external resource of positioning such as GPS is available. In the situations that GPS is unavailable, DR only provides a good estimation for a short period of time and within a short distance. The estimation error of DR techniques can be very large when DR is being used for a long period of time in case of GPS outage, as DR estimates the location of the vehicle merely based on the prediction and previous estimations and all the estimation errors are accumulated.

### **3.8 Inertial Navigation System**

Another way for improving the accuracy of localization obtained from a GPS receiver is to use an inertial navigation system (INS). This system is composed of a central processor along with several sensors such as motion sensors (accelerometers) or rotation sensors (gyroscopes). An INS is installed in the vehicle to provide it with more information which can be combined with the data obtained from the GPS and used to find more accurate location of the vehicle. Using these sensors, the central processor calculates the direction and the velocity of a vehicle to estimate the location of the vehicle. In (Davidson et al. 2009), it is shown that the GPS data can be incorporated with INS sensors using an

extended Kalman filter in order to obtain the desired accuracy in dense environments. Calibrated INS sensors can be used for vehicle positioning when no GPS signal is available or when the signals suffer from multipath effects. The main advantage of this system is that the vehicle does not need any external resource and therefore all the location information of the vehicle can be determined by information provided from the sensors and a known starting point. However, in this situation, the accuracy of these systems degrades over time, as no external resource is used to help the INS sensors. Similar to DR techniques, if INS sensors are used for a long time without any external resource, they make large estimation errors. Therefore, GPS is typically used along with INS sensor to ensure that a desired accuracy can be delivered for a long period of time (Farrell 2008; Farrell et al. 2000).

### **3.9 Cellular**

Cellular localization is another alternative for vehicle positioning. In this system, the current infrastructure of cellular systems can be used. In order to operate properly, the mobile stations need to connect with the base stations. For data transmission, the mobile station is required to connect to only one base station. However, for localization at least three base stations are required. Different measurement techniques (RSS, TOA, TDOA, and AOA) described for WSN can be also used for cellular localization (Caffery and Stuber 1998; Mensing and Plass 2006; Song 1994). However, RSS is the most common measurement technique typically used for cellular localization. There are some advantages and disadvantages associated with using cellular systems for vehicle localization. One of the most important advantages of cellular systems is that their communication ranges are significantly larger than WSNs (Chen et al. 2006). The range of a cellular system can reach up to 35 km, about 350 times longer than a WSN using Wi-Fi communications with maximum range of 100 m. Moreover, a cellular network is more organized and steady than a WSN. Most of the base stations that are already installed and in operation can be used for localization. However, using WSN for vehicle localization requires additional infrastructures and equipment to be installed on the roads. Cellular systems operate in licensed bands (e.g., 900 MHz) in which fewer devices are operating and result in less interference. However, most WSNs operate in unlicensed bands in which many devices are

allowed to operate such as Bluetooth headsets and wireless routers. One of the disadvantages of cellular systems is that they cannot operate in a cooperative ways, meaning that the mobile stations cannot communicate directly to each other and any communication has to be done through an intermediate base station (Caffery and Stuber 1998). Although recently several approaches have been suggested to make cooperation among mobile stations possible, significant efforts are still required to design and create the required standards and policies. Despite the fact that the cellular systems are available in most areas, with the current cellular technologies their localization performance is less accurate than GPS (Boukerche et al. 2008).

### **3.10 Vision-Based**

This method is significantly different from the previous mentioned ones. In the vision-based localization method, it is not required to equip the vehicle with any kind of devices. However, an external camera is required to capture a movement video of the vehicle. Then, the recorded data is sent to a central processor either online or offline (Chapuis et al. 2002; Tan 2000). The central processor localizes the location of the vehicle using image processing techniques. The advantage of this technique is that we no longer need to have a device in the vehicle. On the other hand, for this method, since all information should be recorded in video formats, a large amount of data needs to be stored. Moreover, since the system needs to process a large amount of data, its latency is typically longer than other methods. Although recently some highly sophisticated algorithms have been proposed, the performance of this method is affected by different weather and lighting conditions. Since most of the current methods are based on pattern recognition and several objects in a scene may share a similar pattern, it would be very difficult for the system to correctly detect and track vehicles whenever and wherever there are many vehicles presented in a scene. Currently, this method is widely used for toll collection and HOV lane vehicle identification (Anagnostopoulos et al. 2006).

## 4 Methodology

In this chapter, the proposed Integrated and Smart algorithm is described. Before that, I need to describe some terms which will help the reader to understand the materials mentioned in the chapter more easily.

### 4.1 Estimation Theory

Estimation theory is a branch of signal processing and statistics which basically deals with the estimation of an unknown parameter form a series of noisy measurements. The main goal of the estimation theory is to come up with an estimator. An estimator extracts the unknown parameters of interest form noisy observations. The first step in finding an estimator is to model the measurements (observations). Assume we are interested in estimating  $M$  unknown parameters defined as (Kay 1998):

$$\boldsymbol{\theta} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_M \end{bmatrix} \quad (11)$$

The goal is to find an estimator which estimates  $\boldsymbol{\theta}$  from  $N$  noisy measurements defined as:

$$\mathbf{x} = \begin{bmatrix} x[1] \\ x[2] \\ \vdots \\ x[N] \end{bmatrix} \quad (12)$$

To clarify the concept, I would like to introduce a simple example. One of the oldest and most practical examples of the estimation theory is estimating a direct current (DC) level  $\theta$  from an additive white Gaussian noise (AWGN). The measurements are modeled as:

$$x[n] = \theta + w[n], \quad n = 1, \dots, N \quad (13)$$

where  $w[n]$  is a zero-mean Gaussian random variable with variance of  $\sigma^2$ . Intuitively, taking the mean from the measurements gives us a reasonable estimator:

$$\hat{\theta} = \frac{1}{N} \sum_{n=1}^N x[n] \quad (14)$$

Several questions will be raised here:

- 1: How accurate is this estimation?
- 2: Is there any other estimators?

These questions will be answered in the content of estimation theory. As can be seen from (14), the estimator  $\hat{\theta}$  is a random variable, since it is a function of random variables. Now, I want to define an important term in the estimation theory. An estimator is *unbiased*, if the expected value of the estimation reaches to the true value of the unknown parameter to be estimated. Let's find the expected value of the estimator in (14):

$$\begin{aligned} E[\hat{\theta}] &= E \left[ \frac{1}{N} \sum_{n=1}^N x[n] \right] \\ &= E \left[ \frac{1}{N} \sum_{n=1}^N \theta + w[n] \right] \\ &= E \left[ \frac{1}{N} \sum_{n=1}^N \theta + \frac{1}{N} \sum_{n=1}^N w[n] \right] \\ &= \theta + E \left[ \frac{1}{N} \sum_{n=1}^N w[n] \right] \\ &= \theta \end{aligned} \quad (15)$$

where the following facts have been used:

- 1: Since  $\theta$  is deterministic parameter, its expected value is equal to  $\theta$ .
- 2:  $w[n]$  is zero-mean random variable, therefore its expected value is equal to 0.

Therefore, from (15), we can see that the estimator in (14) is unbiased. Unbiased estimators have some important features which I will mention later. The variance of an estimator also gives us important information. The variance of an estimator is defined as:

$$\text{Var}[\hat{\theta}] = E \left[ (\hat{\theta} - E[\hat{\theta}])^2 \right] \quad (16)$$

If the estimator is unbiased:

$$\begin{aligned}\text{Var}[\hat{\theta}] &= \text{E}[(\hat{\theta} - \theta)^2] \\ &= e\end{aligned}\tag{17}$$

Therefore, for an unbiased estimator the variance of the estimator gives us the estimation error which will be defined later in the following section.

#### 4.1.1 Cramér–Rao lower bound (CRLB)

The CRLB expresses a lower bound on the variance of any unbiased estimators. The CRLB is used as a benchmark to compare the performance of unbiased estimators. In other words, it tells us how accurate the estimator is and how far its performance is from the lower bound. An estimator is optimal if its variance reaches the CRLB. Let's consider a more general case of AWGN model given in (13)(Kay 1998):

$$x[n] = f_n(\boldsymbol{\theta}) + w[n], \quad n = 1, \dots, N\tag{18}$$

where  $f_n(\boldsymbol{\theta})$  is a function of the unknown parameters. For the example in (13), the function,  $f_n(\boldsymbol{\theta}) = \theta$ , is just a linear transformation. To progress, the probability distribution function of the measurements should be determined. Since  $w[n]$  is a zero-mean Gaussian random variable with variance of  $\sigma_n^2$ :

$$p(w[n]) = \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-\frac{w^2[n]}{2\sigma_n^2}}\tag{19}$$

The distribution of  $x[n]$  is also Gaussian with mean of  $f_n(\boldsymbol{\theta})$  and variance of  $\sigma_n^2$ . Therefore, the distribution of the vector  $\mathbf{x}$  will be:

$$\begin{aligned}p(\mathbf{x}|\boldsymbol{\theta}) &= \prod_{n=1}^N p(x[n]|\boldsymbol{\theta}) \\ p(\mathbf{x}|\boldsymbol{\theta}) &= \frac{1}{\sqrt{(2\pi)^N \prod_{n=1}^N \sigma_n^2}} e^{-\sum_{n=1}^N \frac{(x[n]-f_n(\boldsymbol{\theta}))^2}{2\sigma_n^2}}\end{aligned}\tag{20}$$

The CRLB is obtained from the Fisher information matrix. Suppose  $p(\mathbf{x}|\boldsymbol{\theta})$  is the distribution of the measurements, the Fisher information matrix is obtained by:

$$[\mathbf{I}(\boldsymbol{\theta})]_{ij} = -\mathbb{E} \left[ \frac{\partial^2 \log p(\mathbf{x}|\boldsymbol{\theta})}{\partial \theta_i \partial \theta_j} \right], \quad i, j = 1, \dots, M \quad (21)$$

The PDF of the measurements should satisfy the regularity condition. However, this concept is outside of the scope of this work. Gaussian distributions with which I mostly deal in this research always satisfy the regularity condition. If the measurements have the Gaussian distribution, they will satisfy the regularity condition and the Fisher information matrix is simplified as follows:

$$\mathbf{I}_{i,j} = \frac{\partial \boldsymbol{\mu}^T}{\partial \theta_i} \mathbf{C}^{-1} \frac{\partial \boldsymbol{\mu}}{\partial \theta_j} + \frac{1}{2} \text{trace} \left( \mathbf{C}^{-1} \frac{\mathbf{C}}{\partial \theta_i} \mathbf{C}^{-1} \frac{\mathbf{C}}{\partial \theta_j} \right) \quad (22)$$

where

$$\boldsymbol{\mu} = \begin{bmatrix} f_1(\boldsymbol{\theta}) \\ f_2(\boldsymbol{\theta}) \\ \vdots \\ f_N(\boldsymbol{\theta}) \end{bmatrix} \quad (23)$$

If the covariance matrix of the measurements is not dependent on the unknown parameters, the Fisher information matrix is simplified as:

$$\mathbf{I}(\boldsymbol{\theta}) = \frac{\partial \boldsymbol{\mu}^T}{\partial \boldsymbol{\theta}} \mathbf{C}^{-1} \frac{\partial \boldsymbol{\mu}}{\partial \boldsymbol{\theta}} \quad (24)$$

Where  $\partial \boldsymbol{\mu} / \partial \boldsymbol{\theta}$  is the Jacobian matrix defined as:

$$\frac{\partial \boldsymbol{\mu}}{\partial \boldsymbol{\theta}} = \begin{bmatrix} \frac{\partial f_1(\boldsymbol{\theta})}{\partial \theta_1} & \frac{\partial f_1(\boldsymbol{\theta})}{\partial \theta_2} & \dots & \frac{\partial f_1(\boldsymbol{\theta})}{\partial \theta_M} \\ \frac{\partial f_2(\boldsymbol{\theta})}{\partial \theta_1} & \frac{\partial f_2(\boldsymbol{\theta})}{\partial \theta_2} & \dots & \frac{\partial f_2(\boldsymbol{\theta})}{\partial \theta_M} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_N(\boldsymbol{\theta})}{\partial \theta_1} & \frac{\partial f_N(\boldsymbol{\theta})}{\partial \theta_2} & \dots & \frac{\partial f_N(\boldsymbol{\theta})}{\partial \theta_M} \end{bmatrix} \quad (25)$$

Once the Fisher information matrix is determined, the CRLB is defined as:

$$\text{Var}[\hat{\theta}_i] \geq [\mathbf{I}(\boldsymbol{\theta})^{-1}]_{ii} \quad (26)$$

Now, let us derive the CRLB of the model in (13). The first thing we need to is to determine the Jacobian matrix. In this case, there is only one unknown parameter and there are  $N$  measurements. Therefore, the Jacobian matrix is in fact  $N \times 1$  vector

$$\frac{\partial \boldsymbol{\mu}}{\partial \theta} = \begin{bmatrix} \frac{\partial f_1(\theta)}{\partial \theta} \\ \frac{\partial f_2(\theta)}{\partial \theta} \\ \vdots \\ \frac{\partial f_N(\theta)}{\partial \theta} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \quad (27)$$

since  $\partial f_1(\theta)/\partial \theta = 1$ . Now, we need to calculate the Fisher information matrix:

$$\mathbf{I} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}^T \begin{bmatrix} \sigma^{-2} & 0 & \dots & 0 \\ 0 & \sigma^{-2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sigma^{-2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} = \frac{N}{\sigma^2} \quad (28)$$

Therefore, the CRLB of any unbiased estimator for the model in (13)

$$\text{Var}[\hat{\theta}] \geq \frac{\sigma^2}{N} \quad (29)$$

From (29), the following conclusions can be drawn

- 1: The CRLB of the model in (13) has a direct relationship to  $\sigma^2$ . This means that if the variance of the measurement errors increases, the performance of the unbiased estimator decreases.
- 2: The CRLB of the model in (13) has a reverse relationship with  $N$ . This means that as the number of measurements increases, the CRLB of the model decreases and the performance of the unbiased estimator increases.
- 3: The error of the estimator in (14) is not lower than  $\sigma^2/N$ , since the estimator in (14) is unbiased and cannot provide better performance than the CRLB.

### 4.1.2 Maximum likelihood

The maximum likelihood (ML) estimation is a method of estimating unknown parameters. The ML estimator is a popular estimator because of its several important features. The ML estimator is asymptotically normal, meaning that when the number of measurements tends infinity the distribution of the ML estimator reaches to a normal distribution with mean

equal to the true parameter to be estimated (i.e., unbiased) and with a variance equal to the CRLB (i.e., efficient). However, the ML estimator is not easy to derive for all models. Assume  $p(\mathbf{x}|\boldsymbol{\theta})$  be the distribution (also called likelihood function) of the measurements; the ML estimator is obtained by:

$$\hat{\boldsymbol{\theta}}_{\text{ML}} = \max_{\boldsymbol{\theta}} p(\mathbf{x}|\boldsymbol{\theta}) \quad (30)$$

Therefore, the ML estimator is looking for the value of  $\boldsymbol{\theta}$  that maximizes the likelihood function of the measurements. Since the logarithm is a non-decreasing function, it is sometimes easier to find the maximum of the logarithm of the likelihood function instead:

$$\begin{aligned} \hat{\boldsymbol{\theta}}_{\text{ML}} &= \max_{\boldsymbol{\theta}} \log p(\mathbf{x}|\boldsymbol{\theta}) \\ &= \max_{\boldsymbol{\theta}} \log \prod_{n=1}^N p(x[n]|\boldsymbol{\theta}) \\ &= \max_{\boldsymbol{\theta}} \sum_{n=1}^N \log p(x[n]|\boldsymbol{\theta}) \end{aligned} \quad (31)$$

If the distribution of the measurements is Gaussian, the ML estimation can be further simplified. The likelihood function of the unknown parameters for Gaussian distribution is given in (20). Plugging (20) in (31), the ML estimator for the Gaussian measurements would be:

$$\begin{aligned} \hat{\boldsymbol{\theta}}_{\text{ML}} &= \max_{\boldsymbol{\theta}} \log p(\mathbf{x}|\boldsymbol{\theta}) \\ &= \max_{\boldsymbol{\theta}} \log \frac{1}{\sqrt{(2\pi\sigma^2)^N}} e^{-\sum_{n=1}^N \frac{w^2[n]}{2\sigma_i^2}} \\ &= \max_{\boldsymbol{\theta}} \log \frac{1}{\sqrt{(2\pi\sigma^2)^N}} - \sum_{n=1}^N \frac{w^2[n]}{2\sigma_i^2} \\ &= \max_{\boldsymbol{\theta}} - \sum_{n=1}^N \frac{w^2[n]}{2\sigma_i^2} \end{aligned} \quad (32)$$

Maximizing a function over its argument is equivalent to minimizing that function over the same argument with a sign change. Therefore, the ML estimator for the model in (18) with Gaussian distribution can be obtained by:

$$\begin{aligned}\hat{\theta}_{\text{ML}} &= \min_{\theta} \sum_{n=1}^N \frac{w^2[n]}{\sigma_i^2} \\ &= \min_{\theta} \sum_{n=1}^N \frac{1}{\sigma_i^2} (x[n] - f_n(\theta))^2\end{aligned}\quad (33)$$

Therefore, the ML estimator of the models with Gaussian distribution is simply calculated by minimizing the difference between the measurements and the true model. Now, let us derive the ML estimator for the example model in (13):

$$\hat{\theta}_{\text{ML}} = \min_{\theta} \sum_{n=1}^N (x[n] - A)^2 \quad (34)$$

To find the minimum of the cost function in (34), we can take the first derivative and then set it to zero

$$\begin{aligned}\frac{\partial}{\partial A} \sum_{n=0}^{N-1} (x[n] - A)^2 &= \sum_{n=1}^N 2(x[n] - A) \\ &= 2 \sum_{n=1}^N x[n] - NA = 0\end{aligned}\quad (35)$$

Therefore, the ML estimator would be

$$\hat{A} = \frac{1}{N} \sum_{n=1}^N x[n] \quad (36)$$

As can be seen, the estimator we obtained intuitively in (14) was in fact the ML estimator. The ML estimator in (36) is optimal and its accuracy achieves the CRLB for sufficiently large data sample.

Here, I have run a series of simulations to show the optimality of the ML estimator for the model in (13). The MSE performance of the ML estimator in (36) as a function of number of measurements is depicted in FIGURE 9. The CRLB of the model in (13) which is derived in (29) is also shown in FIGURE 9. Both the MSE of the ML estimator and the CRLB decrease as the number of measurements increases. However, the performance of the ML estimator reaches the CRLB by increasing the number of measurements, showing that the ML estimator is asymptotically optimal.

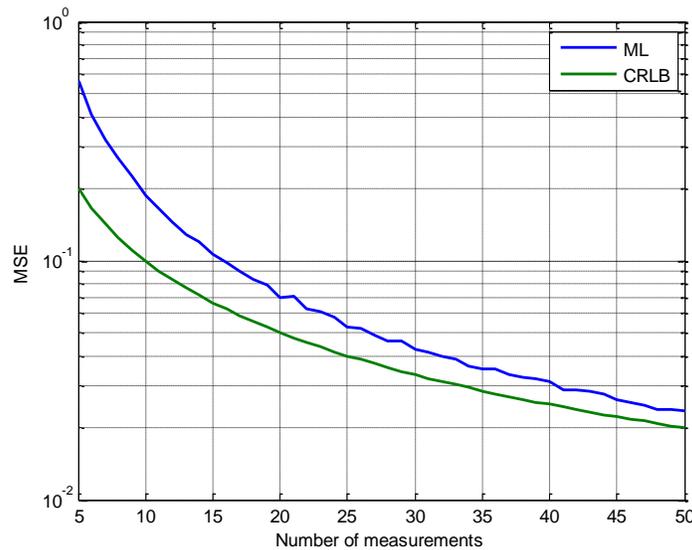


FIGURE 9 - The performance of the ML estimator.

Let us consider another example which is a general case of the example in (13). Suppose we have a model in which the measurements are a linear function of the unknown parameters. The model is defined as follows:

$$\mathbf{x} = \mathbf{H}\boldsymbol{\theta} + \mathbf{w} \quad (37)$$

where  $\mathbf{x}$  is the vector of the measurements.  $\mathbf{H}$  is a known matrix of dimension  $N \times M$ , where  $N$  and  $M$  are the number of the measurements and unknown variables, respectively. If the distribution of the measurements error,  $\mathbf{w}$ , is Gaussian, the ML estimator can be obtained from (33), hence:

$$\begin{aligned}
\hat{\theta}_{\text{ML}} &= \min_{\theta} \sum_{n=1}^N \frac{1}{\sigma_i^2} (x[n] - f_n(\theta))^2 \\
&= \min_{\theta} (\mathbf{x} - \mathbf{H}\theta)^T \mathbf{C}^{-1} (\mathbf{x} - \mathbf{H}\theta)
\end{aligned} \tag{38}$$

where  $\mathbf{C} = \mathbb{E}[\mathbf{w}\mathbf{w}^T]$  is the covariance matrix of the measurement noises. It should be noted that in (38), matrix multiplications are used instead of summation. To find the ML estimator, we need to determine the minimum of the cost function. This can be done by taking the first derivative of the cost function with respect to the unknown parameters and then set it to zero:

$$\begin{aligned}
\frac{\partial}{\partial \theta} (\mathbf{x} - \mathbf{H}\theta)^T \mathbf{C}^{-1} (\mathbf{x} - \mathbf{H}\theta) &= 2 \frac{\partial (\mathbf{H}\theta)^T}{\partial \theta} \mathbf{C}^{-1} (\mathbf{x} - \mathbf{H}\theta) \\
&= 2\mathbf{H}^T \mathbf{C}^{-1} (\mathbf{x} - \mathbf{H}\theta) \\
&= 2\mathbf{H}^T \mathbf{C}^{-1} \mathbf{x} - 2\mathbf{H}^T \mathbf{C}^{-1} \mathbf{H}\theta = \mathbf{0}
\end{aligned} \tag{39}$$

Therefore, the ML estimator would be

$$\hat{\theta} = (\mathbf{H}^T \mathbf{C}^{-1} \mathbf{H})^{-1} \mathbf{H}^T \mathbf{C}^{-1} \mathbf{x} \tag{40}$$

The ML estimator obtained for the model of (13) which is found in (36) can be verified with the above expression by setting  $\mathbf{H} = \mathbf{1}$  and  $\mathbf{C} = \mathbf{I}$ :

$$\begin{aligned}
\hat{\theta} &= (\mathbf{1}^T \mathbf{I} \mathbf{1})^{-1} \mathbf{1}^T \mathbf{I} \mathbf{x} \\
&= \frac{\mathbf{1}^T \mathbf{x}}{\mathbf{1}^T \mathbf{1}} = \frac{\mathbf{1}^T \mathbf{x}}{N} = \frac{\sum_{n=1}^N x[n]}{N}
\end{aligned} \tag{41}$$

which is the same as what we found in (36).

The models in (13) and (37) include a linear function of unknown parameters, hence, the ML estimator can be written in a closed-form solution. However, in many estimation problems, the model is a nonlinear function of the unknown parameters. In these cases, the ML estimator cannot be derived easily as in (36). To clarify that, let us mention another example. Now, we consider a more complex example where we try to estimate the frequency of a sinusoids signal from a series of noisy measurements. Therefore, the model would be:

$$x[n] = \cos(2\pi f_0 n) + w[n], \quad n = -N, \dots, 0, \dots, N \quad (42)$$

In the above model  $x[n]$  is the measurement and  $f_0$  is the unknown parameter to be estimated. Since the noise term,  $w[n]$ , is Gaussian, the ML estimator can be obtained from (33) (assuming a fixed variance for noises):

$$\hat{\theta}_{\text{ML}} = \min_{\theta} \sum_{n=-N}^N (x[n] - \cos(2\pi f_0 n))^2 \quad (43)$$

Similarly, the minimum of the cost function in (43) can be found by taking the first derivative and then set it to zero:

$$\begin{aligned} \frac{\partial}{\partial f_0} \sum_{n=-N}^N (x[n] - \cos(2\pi f_0 n))^2 \\ = 4\pi n \sin(2\pi f_0 n) (x[n] - a \cos(2\pi f_0 n)) \\ = 0 \end{aligned} \quad (44)$$

There is no Closed-form solution for (44). One way to solve the nonlinear ML estimator in (43) is to use the Gauss-Newton (GN) method. The GN method is obtained by applying the first order Taylor series on the model and converting approximately the nonlinear model into a linear one. In this way, the GN method can iteratively find the minimum of the cost function. Now, consider the general case of the measurement in (18), where  $f_n(\theta)$  is a nonlinear function of the unknown parameters  $\theta$ . The ML estimator of the model in (18) can be found approximately using the GN method as follows(Kay 1998):

$$\hat{\theta}^{k+1} = \hat{\theta}^k + \left( \mathbf{H}^T(\hat{\theta}^k) \mathbf{H}(\hat{\theta}^k) \right)^{-1} \mathbf{H}^T(\hat{\theta}^k) [\mathbf{x} - \mathbf{F}(\hat{\theta}^k)] \quad (45)$$

Where  $\mathbf{H}(\hat{\theta}^k)$  is in fact the Jacobian matrix defined in (25) evaluated at the  $\hat{\theta}^k$ :

$$\mathbf{H}(\hat{\theta}^k) = \left. \frac{\partial \boldsymbol{\mu}}{\partial \boldsymbol{\theta}} \right|_{\boldsymbol{\theta}=\hat{\theta}^k} \quad (46)$$

The iteration might stop when the change in  $\hat{\theta}^k$  from one iteration to the next one is less than a certain value

$$\left| \left( \mathbf{H}^T(\hat{\boldsymbol{\theta}}^k) \mathbf{H}(\hat{\boldsymbol{\theta}}^k) \right)^{-1} \mathbf{H}^T(\hat{\boldsymbol{\theta}}^k) [\mathbf{x} - \mathbf{F}(\hat{\boldsymbol{\theta}}^k)] \right| < \epsilon \quad (47)$$

The value of initial point  $\hat{\boldsymbol{\theta}}^0$  can be either determined from prior information about the unknown parameter or generated randomly. Since the original model is nonlinear, there are several points that satisfy (47). In fact, every point whose derivative is zero satisfies (47). These points are called critical points which may include local minima, local maxima, and saddle points. However, for the ML estimator, we are interested in the point which is the global minimum and its value is lower than any other points. Therefore, initialization is very important in solving nonlinear models. To make the above statement clearer, I provide an example in FIGURE 10 which shows a nonlinear cost function. The point we are interested in is around 3 which is associated with the global minimum of the cost function. If the GN method is initialized in some point around 9, the solver returns 10.7 as a solution, because at this point the derivative is zero and then the iterations stop. The point 10.7 is a local minimum and is not the solution of the problem. Therefore, we will have a large estimation error. On the other hand, if the solver is initialized at some point around 2, the solver hopefully ends up at 3 which is the global minimum. As can be seen, the initial point has a large impact on the accuracy of the GN method. Therefore, a suitable initial point should be selected, if the GN method is used to solve the ML estimator of a nonlinear model.

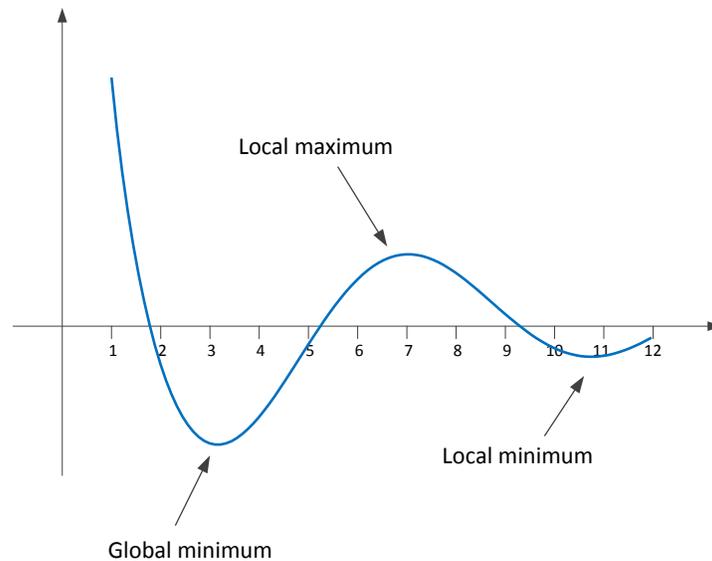


FIGURE 10 - Nonlinear Problem

Let us come back to our example in (42). Now, I want to apply the GN method to the model. First, I need to calculate the Jacobian matrix:

$$\mathbf{H} = \frac{\partial \boldsymbol{\mu}}{\partial \boldsymbol{\theta}} = \begin{bmatrix} -N2\pi \sin(-2\pi f_0 N) \\ -(N-1)2\pi \sin(-2\pi f_0 (N-1)) \\ \vdots \\ N2\pi \sin(2\pi f_0 N) \end{bmatrix} \quad (48)$$

and

$$\begin{aligned} \mathbf{H}^T \mathbf{H} &= 4\pi^2 \sum_{n=-N}^N n^2 \sin^2(2\pi f_0 n) \\ &\approx \frac{4\pi^2}{3} N(N+1)(2N+1) \end{aligned} \quad (49)$$

Therefore, the ML estimation of the model in (42) using the GN method is obtained by

$$\hat{f}_0^{k+1} = \hat{f}_0^k - \frac{3}{4\pi N^3} \sum_{n=-N}^N n \sin(2\pi f_0 n) (x[n] - \cos(2\pi f_0 n)) \quad (50)$$

### 4.1.3 Bayesian Estimation

So far, we assumed that we do not have any side information about the unknown parameter  $\boldsymbol{\theta}$  and we try to estimate it from a series of noisy measurements. We also assumed that the unknown parameter  $\boldsymbol{\theta}$  is deterministic. However, in some estimation problems, the unknown parameter is a random variable and has a known prior distribution. The classic estimation cannot address these types of problems. Instead, the Bayesian estimation can be used to estimate the unknown parameter. Remember that in the classic estimation, the ML estimator is optimal and can achieve the CRLB accuracy. In the Bayesian estimation, we are interested in a maximum a posteriori (MAP) estimator. However, the MAP estimator is not optimal in all situations. If the distribution of the measurements is Gaussian, the MAP estimator is optimal. The MAP estimator is obtained as(Kay 1998):

$$\hat{\boldsymbol{\theta}}_{\text{MAP}} = \max_{\boldsymbol{\theta}} p(\boldsymbol{\theta}|\mathbf{x}) \quad (51)$$

$$\begin{aligned}
&= \max_{\boldsymbol{\theta}} p(\mathbf{x}|\boldsymbol{\theta})p(\boldsymbol{\theta}) \\
&= \max_{\boldsymbol{\theta}} \log p(\mathbf{x}|\boldsymbol{\theta})p(\boldsymbol{\theta}) \\
&= \max_{\boldsymbol{\theta}} \{\log p(\mathbf{x}|\boldsymbol{\theta}) + \log p(\boldsymbol{\theta})\}
\end{aligned}$$

where  $p(\boldsymbol{\theta}|\mathbf{x})$  is the posterior pdf we are trying to maximize,  $p(\mathbf{x}|\boldsymbol{\theta})$  is the distribution of the measurements, and  $p(\boldsymbol{\theta})$  is the distribution of the unknown parameters. Now, let us consider a case where the unknown parameter has a Gaussian distribution with mean of  $\boldsymbol{\mu}_{\boldsymbol{\theta}}$  and variance of  $\mathbf{C}_{\boldsymbol{\theta}}$ , hence we have:

$$\begin{aligned}
\mathbf{x} &= \mathbf{f}(\boldsymbol{\theta}) + \mathbf{w} \\
\boldsymbol{\theta} &\sim \mathcal{N}(\boldsymbol{\mu}_{\boldsymbol{\theta}}, \mathbf{C}_{\boldsymbol{\theta}})
\end{aligned} \tag{52}$$

It can be shown that the corresponding MAP estimator of the model in (52) is obtained by:

$$\hat{\boldsymbol{\theta}}_{\text{MAP}} = \min_{\boldsymbol{\theta}} \{(\mathbf{x} - \mathbf{f}(\boldsymbol{\theta}))^T \mathbf{C}^{-1}(\mathbf{x} - \mathbf{f}(\boldsymbol{\theta})) + (\boldsymbol{\theta} - \boldsymbol{\mu}_{\boldsymbol{\theta}})^T \mathbf{C}_{\boldsymbol{\theta}}^{-1}(\boldsymbol{\theta} - \boldsymbol{\mu}_{\boldsymbol{\theta}})\} \tag{53}$$

Now, assume a simpler case, where the measurement model is linear, i.e.:

$$\begin{aligned}
\mathbf{x} &= \mathbf{H}\boldsymbol{\theta} + \mathbf{w} \\
\boldsymbol{\theta} &\sim \mathcal{N}(\boldsymbol{\mu}_{\boldsymbol{\theta}}, \mathbf{C}_{\boldsymbol{\theta}})
\end{aligned} \tag{54}$$

Then, the MAP estimator is:

$$\hat{\boldsymbol{\theta}}_{\text{MAP}} = \min_{\boldsymbol{\theta}} \{(\mathbf{x} - \mathbf{H}\boldsymbol{\theta})^T \mathbf{C}^{-1}(\mathbf{x} - \mathbf{H}\boldsymbol{\theta}) + (\boldsymbol{\theta} - \boldsymbol{\mu}_{\boldsymbol{\theta}})^T \mathbf{C}_{\boldsymbol{\theta}}^{-1}(\boldsymbol{\theta} - \boldsymbol{\mu}_{\boldsymbol{\theta}})\} \tag{55}$$

To find the minimum of the cost function, we need to take the first derivative of the cost function with respect to the unknown parameter and then set it to zero:

$$\begin{aligned}
& \frac{\partial}{\partial \boldsymbol{\theta}} \{(\mathbf{x} - \mathbf{H}\boldsymbol{\theta})^T \mathbf{C}^{-1}(\mathbf{x} - \mathbf{H}\boldsymbol{\theta}) + (\boldsymbol{\theta} - \boldsymbol{\mu}_\theta)^T \mathbf{C}_\theta^{-1}(\boldsymbol{\theta} - \boldsymbol{\mu}_\theta)\} \\
&= \frac{\partial}{\partial \boldsymbol{\theta}} (\mathbf{x} - \mathbf{H}\boldsymbol{\theta})^T \mathbf{C}^{-1}(\mathbf{x} - \mathbf{H}\boldsymbol{\theta}) + \frac{\partial}{\partial \boldsymbol{\theta}} (\boldsymbol{\theta} - \boldsymbol{\mu}_\theta)^T \mathbf{C}_\theta^{-1}(\boldsymbol{\theta} - \boldsymbol{\mu}_\theta) \\
&= 2 \frac{\partial (\mathbf{H}\boldsymbol{\theta})^T}{\partial \boldsymbol{\theta}} \mathbf{C}^{-1}(\mathbf{x} - \mathbf{H}\boldsymbol{\theta}) + 2 \frac{\partial (\boldsymbol{\theta})^T}{\partial \boldsymbol{\theta}} \mathbf{C}_\theta^{-1}(\boldsymbol{\theta} - \boldsymbol{\mu}_\theta) \tag{56} \\
&= 2\mathbf{H}^T \mathbf{C}^{-1}(\mathbf{x} - \mathbf{H}\boldsymbol{\theta}) + 2\mathbf{C}_\theta^{-1}(\boldsymbol{\theta} - \boldsymbol{\mu}_\theta) \\
&= 2(\mathbf{H}\mathbf{C}^{-1}\mathbf{x} - \mathbf{H}^T \mathbf{C}^{-1}\mathbf{H}\boldsymbol{\theta} + \mathbf{C}_\theta^{-1}\boldsymbol{\theta} - \mathbf{C}_\theta^{-1}\boldsymbol{\mu}_\theta) = \mathbf{0}
\end{aligned}$$

After some calculations and manipulations, the MAP estimator would be:

$$\hat{\boldsymbol{\theta}}_{\text{MAP}} = \boldsymbol{\mu}_\theta + (\mathbf{C}_\theta^{-1} + \mathbf{H}^T \mathbf{C}^{-1} \mathbf{H})^{-1} \mathbf{H}^T \mathbf{Q}^{-1}(\mathbf{x} - \mathbf{H}\boldsymbol{\mu}_\theta) \tag{57}$$

It can be shown that the MAP estimator provides better estimation than the ML estimator if prior information about the unknown parameter is available.

#### 4.1.4 Estimation Error

The performance of an estimator is evaluated by calculating the estimation error. Let  $\theta$  be the true value of the parameter to be estimated and  $\hat{\theta}$  be the estimate of  $\theta$  obtained by an estimator. Therefore, the estimation error is obtained by:

$$e = \hat{\theta} - \theta \tag{58}$$

Recall that the estimate  $\hat{\theta}$  is a random variable, because it is a function of the measurements which are random. The performance of the estimator cannot be obtained only from one error, since the measurement noises are random and sometimes they are small and the estimation error would be small and vice versa. Therefore, to have a more tangible metric to evaluate the performance of an estimator, we need to repeat the same experiment multiple times and calculate the error. Now suppose, we repeat the considered experiment  $N$  times, then let  $\hat{\theta}_i$  be the estimate of the  $i$ th realization, then we have:

$$e_i = \hat{\theta}_i - \theta \tag{59}$$

Now, I want to introduce a metric called mean square error (MSE) which is calculated as:

$$\begin{aligned}
\text{MSE}(\hat{\theta}) &= \mathbb{E}\{e^2\} \\
&= \mathbb{E}\{(\hat{\theta} - \theta)^2\}
\end{aligned} \tag{60}$$

MSE is in fact the expected value of square errors. For sufficiently large  $N$ , the expected value can be approximated by the average operation as follows:

$$\begin{aligned}
\text{MSE}(\hat{\theta}) &= \mathbb{E}(e^2) \\
&= \frac{1}{N} \sum_{i=1}^N e_i^2 \\
&= \frac{1}{N} \sum_{i=1}^N (\hat{\theta}_i - \theta)^2
\end{aligned} \tag{61}$$

MSE is used to compare the performances of the different estimators. For instance, if the MSE of the estimator A is smaller than that of the estimator B, the estimator A performs better than the estimator B. Note that the unit of MSE is the square of the unit of  $\hat{\theta}$ . For instance, if the unit of  $\hat{\theta}$  is meter (m), then the unit of  $\text{MSE}(\hat{\theta})$  is square meter ( $m^2$ ). Another important metric is root mean square error (RMSE) which is calculated as:

$$\begin{aligned}
\text{RMSE}(\hat{\theta}) &= \sqrt{\mathbb{E}(e^2)} \\
&= \sqrt{\text{MSE}(\hat{\theta})} \\
&= \sqrt{\frac{1}{N} \sum_{i=1}^N (\hat{\theta}_i - \theta)^2}
\end{aligned} \tag{62}$$

The advantage of RMSE is that it has the same unit as  $\hat{\theta}$ . Note that if the estimator is unbiased, we have  $\mathbb{E}(e) = 0$  and therefore the variance and MSE of the estimator are the same:

$$\begin{aligned}
\text{Var}(\hat{\theta}) &= \mathbb{E} \left[ (\hat{\theta} - \mathbb{E}(\hat{\theta}))^2 \right] \\
&= \mathbb{E} \left[ (\hat{\theta} - \theta)^2 \right] \\
&= \text{MSE}(\hat{\theta})
\end{aligned} \tag{63}$$

From (26), we can conclude that the MSE of an unbiased estimator should be larger than the CRLB:

$$\text{MSE}(\hat{\theta}) \geq \text{CRLB}(\hat{\theta}) \tag{64}$$

It should be noted that the above expression indicates that the expected value of the square errors should be larger than the CRLB. However, it is possible that for an individual realization, the error  $e_i$  falls below the CRLB. Another important thing is that we approximate the expected value with the average operation in order to calculate the MSE, hence one must ensure that the number of realizations,  $N$ , is sufficiently large. Otherwise the inequality in (64) might be not held.

The cumulative distribution function (CDF) is another metric used to evaluate the performance of an estimator. As shown in (61), the MSE only returns one value. Sometimes one of the errors might be very large and makes the MSE larger than expected. This situation frequently occurs in the estimators that are sensitive to large measurement noises. In this case, the MSE might not be a proper metric to evaluate the performance the estimator and CDF of the errors should be used instead. In this metric, the experiment is repeated multiple times and the error for each realization is calculated and then an empirical CDF of the errors is plotted. FIGURE 11 shows the comparison between the error CDFs of two estimators. At 80% of the time, the estimator B has lower error than the estimator A. However, 20% of the time, the estimator A performs better than the estimator B which produces very large errors (as depicted in the long tail of the red curve). Note that the estimator B has a larger MSE than the estimator A, although estimator B performs better than the estimator A about 80% of the time.

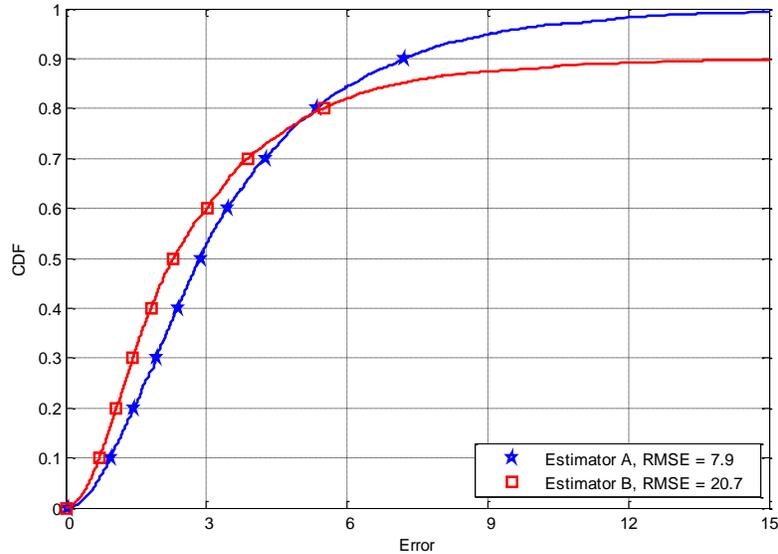


FIGURE 11 - Comparison between two the CDFs of two estimators

Therefore, based on the requirements, an appropriate metric should be applied to compare the performances of the estimators. Typically, CDF conveys the most amount of information about an estimator. However, CDF is not suitable when some parameters are changed during the experiments and the behavior of the estimator for different parameters is desired.

#### 4.1.5 Concluding Remark

FIGURE 12 represents a summary of the concepts discussed in the previous sections. The highlighted boxes are the concepts and methods that are applied for this research. ML estimator is used to estimate deterministic unknown parameters, while a MAP estimator is used when unknown parameters are random. Both ML and MAP estimators are optimal and can achieve the CRLB accuracy asymptotically. In this research, the locations of the vehicles are assumed to be random; therefore a MAP estimator is used. If the model is linear, the ML and MAP estimators have closed-form solutions. However, if the model is nonlinear, either GN method or a MATLAB routine can be used to solve them. The performance of an estimator can be compared with the CRLB to evaluate how accurate is this estimator. In addition, MSE, RMSE, and error CDF of an estimator can be compared to those of other estimators. In my specific problem, I am interested in estimating the location of the vehicle from a series of noisy measurements. As will be seen, the measurement

model is nonlinear and from this model, a MAP estimator will be derived. Since the model is nonlinear, the MAP estimator will be solved with a MATLAB routine. Then in the simulation results, RMSE and CDF are used to evaluate the performance of the proposed algorithm. Moreover, in the Smart algorithm, the CRLB of each connection is used to determine its effectiveness when one should be added or removed.

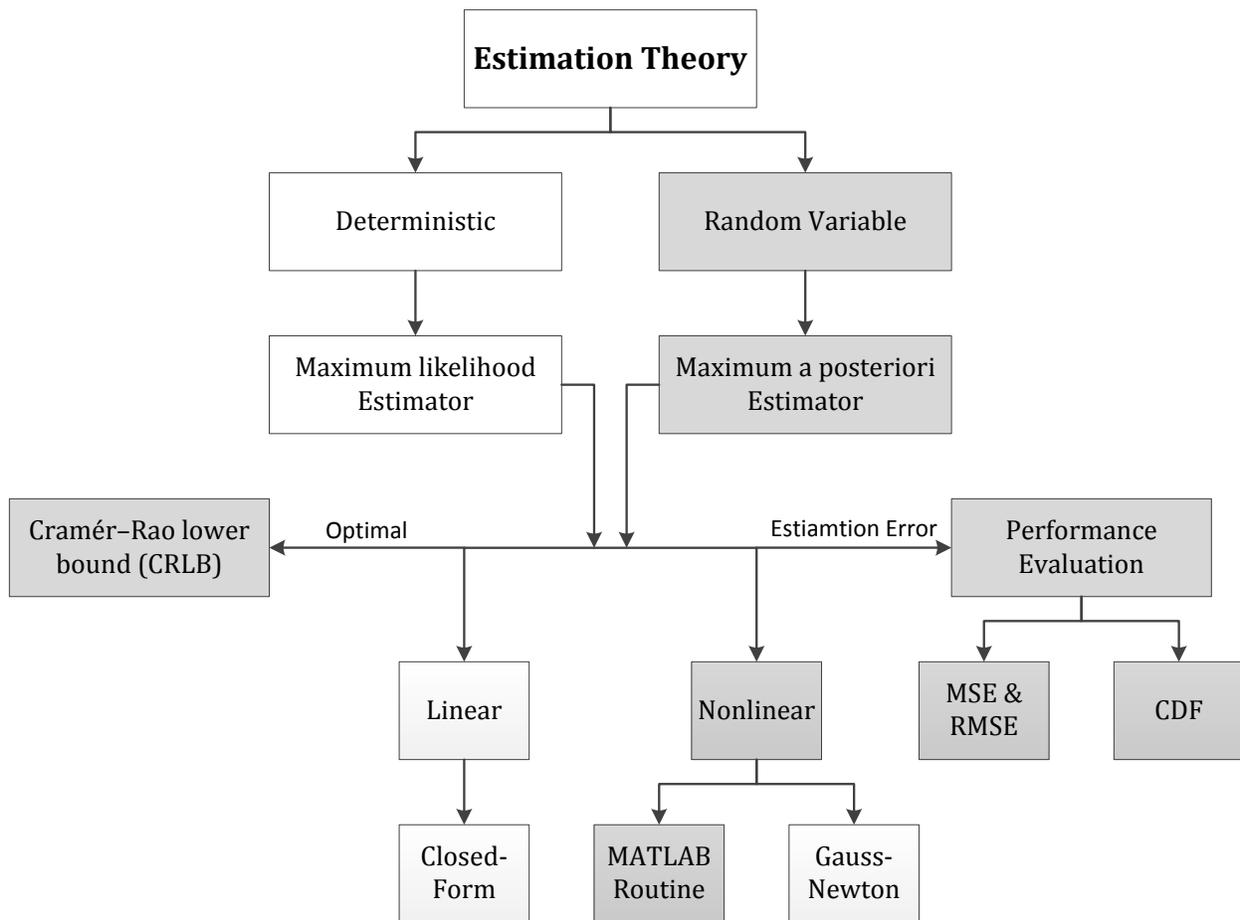


FIGURE 12 - Estimation Theory

## 4.2 Integrated Algorithm

In this section, the Integrated algorithm which is used to estimate the vehicle's location is described. The main idea is to estimate the locations of the vehicles over time from a series of noisy measurements which may include ranging obtained from GPS satellite, V2I, and V2V connections. Moreover, the algorithm is able to use RFID technology in which RFID

readers can inform us of the presence of the vehicles within their communication ranges. Before describing the algorithm, it is required to define several variables. Let  $\mathbf{x}_i[k] = [x_{v,i}^k \ y_{v,i}^k \ z_{v,i}^k]$  be the location of the  $i$ th vehicle at the  $k$ th time-step and  $\mathbf{y}_j[k] = [x_{u,j}^k \ y_{u,j}^k \ z_{u,j}^k]$  be the location of  $j$ th unit at the  $k$ th time-step. Units refer to GPS satellites, wireless anchors, or RFID readers. There are  $N$  vehicles and  $M$  units which include  $N_s$  satellites,  $N_w$  wireless anchors, and  $N_r$  RFID readers in the network. Therefore, the  $j$ th unit is defined based on the following rules:

$$\begin{aligned} 1 \leq j \leq N_s &\quad \Rightarrow \text{Satellites} \\ N_s + 1 \leq j \leq N_s + N_w &\quad \Rightarrow \text{Anchors} \\ N_s + N_w + 1 \leq j \leq M &\quad \Rightarrow \text{RFID readers} \end{aligned} \quad (65)$$

At the  $k$ th time-step, the  $i$ th vehicle is only connected to the neighboring units or other vehicle. To show the connectivity of a vehicle, four sets are defined as follows:

$$\begin{aligned} \mathcal{S}_i^k &= \{j | 1 \leq j \leq N_s, \text{ } j\text{th satellite is connected to } i\text{th vehicle}\} \\ \mathcal{W}_i^k &= \{j | N_s + 1 \leq j \leq N_s + N_w, \text{ } j\text{th anchor is connected to } i\text{th vehicle}\} \\ \mathcal{D}_i^k &= \{j | N_s + N_w + 1 \leq j \leq M, \text{ } j\text{th RFID is connected to } i\text{th vehicle}\} \\ \mathcal{V}_i^k &= \{l | 1 \leq l \leq N, \text{ } l\text{th vehicle is connected to } i\text{th vehicle}\} \end{aligned} \quad (66)$$

Now, I will introduce the measurement model of the localization problem. Let  $r_{ij}[k]$  be the range measurement between the  $i$ th vehicle and the  $j$ th unit (except RFID readers) at the  $k$ th time-step. Therefore, it can be modeled as (Patwari et al. 2005; Tsui 2005; Wymeersch et al. 2009):

$$\begin{aligned} r_{ij}[k] &= d_{ij}[k] + n_{ij}[k], \quad i = 1, 2, \dots, N, \quad j \in \mathcal{S}_i^k \cup \mathcal{W}_i^k \\ r_{il}[k] &= d_{il}[k] + n_{il}[k], \quad i = 1, 2, \dots, N, \quad l \in \mathcal{V}_i^k \end{aligned} \quad (67)$$

where  $d_{ij}[k]$  and  $d_{il}[k]$  are the true distances defined as:

$$\begin{aligned} d_{ij}[k] &= \sqrt{(x_{v,i}^k - x_{u,j}^k)^2 + (y_{v,i}^k - y_{u,j}^k)^2 + (z_{v,i}^k - z_{u,j}^k)^2}, \quad j \in \mathcal{S}_i^k \cup \mathcal{W}_i^k \\ d_{il}[k] &= \sqrt{(x_{v,i}^k - x_{v,l}^k)^2 + (y_{v,i}^k - y_{v,l}^k)^2 + (z_{v,i}^k - z_{v,l}^k)^2}, \quad l \in \mathcal{V}_i^k \end{aligned} \quad (68)$$

The terms  $n_{ij}[k]$  and  $n_{il}[k]$  are the measurement noises. In most studies, noises are modeled as Gaussian random variables with variance  $\sigma_{ij}^2$  (Patwari et al. 2005). Although

several studies consider other statistical distributions for noises such as Uniform or Laplace, Gaussian is the most common one and the closest one to real measurements (Guvenc and Chong 2009).

Now, we are interested in obtaining the ML estimator of the model in **(1)**, since it is an optimal estimator and it can attain the CRLB asymptotically (i.e., when the number of measurements tends to infinity). Since the distribution of the measurements is Gaussian, from **(33)**, the ML estimator can be simply found by the following optimization problem(Kay 1998):

$$\begin{aligned} \min_{\mathbf{x}_i^{[k]}} \sum_{j \in \mathcal{S}_i^k \cup \mathcal{W}_i^k} \sigma_{ij}^{-2} \left( r_{ij} - \sqrt{(x_{v,i}^k - x_{u,j}^k)^2 + (y_{v,i}^k - y_{u,j}^k)^2 + (z_{v,i}^k - z_{u,j}^k)^2} \right)^2 \\ + \sum_{l \in \mathcal{V}_i^k} \sigma_{il}^{-2} \left( r_{il} - \sqrt{(x_{v,i}^k - x_{v,l}^k)^2 + (y_{v,i}^k - y_{v,l}^k)^2 + (z_{v,i}^k - z_{v,l}^k)^2} \right)^2 \end{aligned} \quad (69)$$

The problem in **(69)** is nonlinear and it does not have a closed-form solution. The solution of **(69)** can be approximately found with iterative methods such as GN algorithm(Kay 1998). Alternatively, the problem in **(69)** can be solved with the MATLAB routine *fminunc* which is in fact an iterative optimization solver. The algorithm used in *fminunc* uses the same approach as the GN method. However, it exploits some other properties to make it more accurate than the GN method.

Typically, the error due to the lack of synchronization between the receiver and the GPS satellite (Tsui 2005; Vaghefi and Buehrer 2013) is compensated by adding an extra parameter (clock offset) to **(69)**. For the sake of simplicity, the effect of clock error is neglected here. However, it does not change the relative performance of the algorithms.

So far, the algorithm uses the range measurements obtained from the GPS satellites, V2I, and V2V connections. I also need to include the measurements obtained from RFID readers in the estimation. Unlike GPS, V2I, V2V connections, an RFID reader only tells us whether the vehicle is present in its communication range or not. The information obtained from an RFID reader cannot be expressed as the model in **(1)**. However, their information can be modeled as(Zhang et al. 2011):

$$d_{ij} \leq r_{ij}, \quad j \in \mathcal{D}_i^k \quad (70)$$

where  $r_{ij}$  for RFID readers are defined by their communication ranges. The expression in (70) can serve as a constraint on the solution of (69), meaning that the information from an RFID reader tells us whether or not the location of the vehicle is inside its communication range. Therefore, given the model for RFID readers in (70), the problem in (69) can be modified as:

$$\begin{aligned} \min_{\mathbf{x}_i[k]} \quad & \sum_{j \in \mathcal{S}_i^k \cup \mathcal{W}_i^k} \sigma_{ij}^{-2} \left( r_{ij} - \sqrt{(x_{v,i}^k - x_{u,j}^k)^2 + (y_{v,i}^k - y_{u,j}^k)^2 + (z_{v,i}^k - z_{u,j}^k)^2} \right)^2 \\ & + \sum_{l \in \mathcal{V}_i^k} \sigma_{il}^{-2} \left( r_{il} - \sqrt{(x_{v,i}^k - x_{v,l}^k)^2 + (y_{v,i}^k - y_{v,l}^k)^2 + (z_{v,i}^k - z_{v,l}^k)^2} \right)^2 \\ \text{s. t.} \quad & \sqrt{(x_{v,i}^k - x_{u,j}^k)^2 + (y_{v,i}^k - y_{u,j}^k)^2 + (z_{v,i}^k - z_{u,j}^k)^2} \leq r_{ij}, \quad j \in \mathcal{D}_i^k \end{aligned} \quad (71)$$

Similar to (69), this problem is non-linear and does not have a closed-form solution. Since this problem also has a non-linear constraint, more complex methods are required to solve it. This is the cost that should be paid to exploit RFID connections. The MATLAB routine *fmincon* can be used to solve (71). Now, all information available for the vehicles is used to estimate their location in (71).

In some situations, no external information is available to the vehicle. Therefore, the vehicle needs to find its location only based on the previous estimation, which is called dead reckoning (DR). To include the DR in our model, we need to estimate the velocity of the vehicle, in addition to its location. Let  $\mathbf{v}_i[k] = [v_{x,i}^k \quad v_{y,i}^k \quad v_{z,i}^k]$  be the velocity of the  $i$ th vehicle at the  $k$ th time-step. The relationship between the previous and current location of the vehicle can be modeled as (Kay 1998):

$$\boldsymbol{\theta}_i[k] = \mathbf{A}\boldsymbol{\theta}_i[k-1] + \mathbf{w}_i[k] \quad (72)$$

where  $\boldsymbol{\theta}_i[k] = [\mathbf{x}_i[k], \mathbf{v}_i[k]]$  and (Kay 1998):

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & \Delta & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$\mathbf{w}_i[k]$  defines the prediction error and is typically modeled as a Gaussian random variable with variance  $\mathbf{Q}_{w,i}[k]$ .  $\Delta$  is the time-step between two sets of measurements. Now assume that at the time step  $k - 1$ , we have an estimate from the location and velocity of the vehicle,  $\hat{\boldsymbol{\theta}}_i[k - 1]$ . Using the model in (72), we can predict the location and velocity of the vehicle for the next time step  $k$  which has a Gaussian distribution with the following parameters:

$$\begin{aligned} \mathbb{E}[\boldsymbol{\theta}_i[k]] &= \hat{\boldsymbol{\theta}}_i[k|k - 1] = \mathbf{A}\hat{\boldsymbol{\theta}}_i[k - 1|k - 1] \\ \text{Var}[\boldsymbol{\theta}_i[k]] &= \mathbf{P}_i[k|k - 1] = \mathbf{A}\mathbf{P}_i[k - 1|k - 1]\mathbf{A}^T + \mathbf{Q}_{w,i}[k] \end{aligned} \quad (73)$$

Therefore, the prediction of the  $\boldsymbol{\theta}_i[k]$  can be modeled as a Gaussian random variable:

$$\boldsymbol{\theta}_i[k] \sim \mathcal{N}(\hat{\boldsymbol{\theta}}_i[k|k - 1], \mathbf{P}_i[k|k - 1]) \quad (74)$$

To include DR in our estimation, we need to use the Bayesian estimation rather than the classic estimation, because at each time step, we not only have some measurements but also some prior information of the parameter of the interest obtained from the DR. Therefore, as mentioned earlier, the Bayesian estimation in this case can provide better accuracy than the classic estimation. Including both DR and measurements, (71) can be modified as follows:

$$\begin{aligned} \min_{\mathbf{x}_i[k]} \sum_{j \in \mathcal{S}_i^k \cup \mathcal{W}_i^k} \sigma_{ij}^{-2} \left( r_{ij} - \sqrt{(x_{v,i}^k - x_{u,j}^k)^2 + (y_{v,i}^k - y_{u,j}^k)^2 + (z_{v,i}^k - z_{u,j}^k)^2} \right)^2 \\ + \sum_{l \in \mathcal{V}_i^k} \sigma_{il}^{-2} \left( r_{il} - \sqrt{(x_{v,i}^k - x_{v,l}^k)^2 + (y_{v,i}^k - y_{v,l}^k)^2 + (z_{v,i}^k - z_{v,l}^k)^2} \right)^2 \\ + (\boldsymbol{\theta}_i[k] - \hat{\boldsymbol{\theta}}_i[k|k - 1])^T \mathbf{P}_i[k|k - 1]^{-1} (\boldsymbol{\theta}_i[k] - \hat{\boldsymbol{\theta}}_i[k|k - 1]) \end{aligned} \quad (75)$$

$$\text{s. t.} \quad \sqrt{(x_{v,i}^k - x_{u,j}^k)^2 + (y_{v,i}^k - y_{u,j}^k)^2 + (z_{v,i}^k - z_{u,j}^k)^2} \leq r_{ij}, \quad j \in \mathcal{D}_i^k$$

The estimate of the vehicle location and velocity at the  $k$ th time-step,  $\hat{\boldsymbol{\theta}}_i[k|k]$ , is in fact the solution of (75). Now, we also need to determine the variance of the estimate, i.e.,  $\mathbf{P}_i[k|k]$ , since it will be used for the prediction of the next time step, as shown in (73). The variance of the estimate is obtained as:

$$\begin{aligned} \mathbf{P}_i[k|k] &= (\mathbf{I} - \mathbf{K}_i[k]\mathbf{H}_i[k])\mathbf{P}_i[k|k-1] \\ \mathbf{K}_i[k] &= \mathbf{P}_i[k|k-1]\mathbf{H}_i^T[k](\mathbf{H}_i[k]\mathbf{P}_i[k|k-1]\mathbf{H}_i^T[k] + \mathbf{Q}_{r,i}[k])^{-1} \end{aligned} \quad (76)$$

where  $\mathbf{Q}_{r,i}[k]$  is the covariance matrix associated with the variance of the measurements  $\sigma_{ij}^2$  and  $\mathbf{H}_i[k]$  is the Jacobian matrix defined as follows:

$$\mathbf{H}_i[k] = \begin{bmatrix} \sum_{j \in \mathcal{A}_i^k} \frac{(x_{v,i} - x_{u,i})^2}{\sigma_{ij}^2 d_{ij}^2} & \sum_{j \in \mathcal{A}_i^k} \frac{(y_{v,i} - y_{u,i})(x_{v,i} - x_{u,i})}{\sigma_{ij}^2 d_{ij}^2} & \sum_{j \in \mathcal{A}_i^k} \frac{(z_{v,i} - z_{u,i})(x_{v,i} - x_{u,i})}{\sigma_{ij}^2 d_{ij}^2} \\ \sum_{j \in \mathcal{A}_i^k} \frac{(x_{v,i} - x_{u,i})(y_{v,i} - y_{u,i})}{\sigma_{ij}^2 d_{ij}^2} & \sum_{j \in \mathcal{A}_i^k} \frac{(y_{v,i} - y_{u,i})^2}{\sigma_{ij}^2 d_{ij}^2} & \sum_{j \in \mathcal{A}_i^k} \frac{(z_{v,i} - z_{u,i})(y_{v,i} - y_{u,i})}{\sigma_{ij}^2 d_{ij}^2} \\ \sum_{j \in \mathcal{A}_i^k} \frac{(x_{v,i} - x_{u,i})(z_{v,i} - z_{u,i})}{\sigma_{ij}^2 d_{ij}^2} & \sum_{j \in \mathcal{A}_i^k} \frac{(y_{v,i} - y_{u,i})(z_{v,i} - z_{u,i})}{\sigma_{ij}^2 d_{ij}^2} & \sum_{j \in \mathcal{A}_i^k} \frac{(z_{v,i} - z_{u,i})^2}{\sigma_{ij}^2 d_{ij}^2} \end{bmatrix} \quad (77)$$

where  $\mathcal{A}_i^k = \mathcal{S}_i^k \cup \mathcal{W}_i^k \cup \mathcal{V}_i^k$ . It should be noted that the measurements from the RFID readers cannot be included in the calculation of the CRLB, since their information is not continuous.

In the problem in (75), the first, second, and third terms refer to vehicle-unit measurements, vehicle-vehicle measurements, and internal prediction, respectively. Moreover, the constraint in (75) refers to vehicle-RFID connection. Unlike the problem in (71), in this case, it is necessary to estimate the vehicle's velocity jointly with its location. The main reason is that the estimated velocity is used in predicting of the vehicle's future location. In the simulations, (75) is solved with the MATLAB routine *fmincon*. In signal processing literature, the problems in (71) and (75) are called localization and tracking, respectively (Kay 1998). The latter is usually solved with a method called extended Kalman

filter (EKF). EKF is a Bayesian estimator based on the GN method. However, (75) cannot be solved with EKF because it includes a constraint. This is the main reason that MATLAB routine *fmincon* is applied which is a more complex algorithm than EKF. The problem in (75) can be solved in two ways: distributed and centralized. In the former, the location of all vehicles is estimated simultaneously (Patwari et al. 2005). In the latter, the location of each vehicle is estimated individually. Therefore in this case, the location of the desired vehicle is estimated by replacing the unknown locations of other vehicles with their predicted ones in (75). Although the centralized technique provides higher accuracy, its complexity grows exponentially as the number of vehicles increases (Wymeersch et al. 2009). Hence, the distributed technique is employed here.

### 4.3 Smart Algorithm

Generally speaking, the more connections the vehicle has, the higher the accuracy of the localization will be. On the other hand, as the number of connections increases, the complexity of the algorithm intensifies. Therefore, a tradeoff between the two should be maintained. This is the main problem we need to solve in this section. In the problem in (75), the vehicle is using all connections to estimate its location. However, sometimes the vehicle is connected to several units and not all of them are necessarily useful. These connections slow down the estimation process and do not provide significant improvement. The Smart algorithm derived here filters out the redundant connections and keeps those connections that provide the desired accuracy. The proposed Smart algorithm processes the available connections and reports the useful ones to the Integrated algorithm in (75).

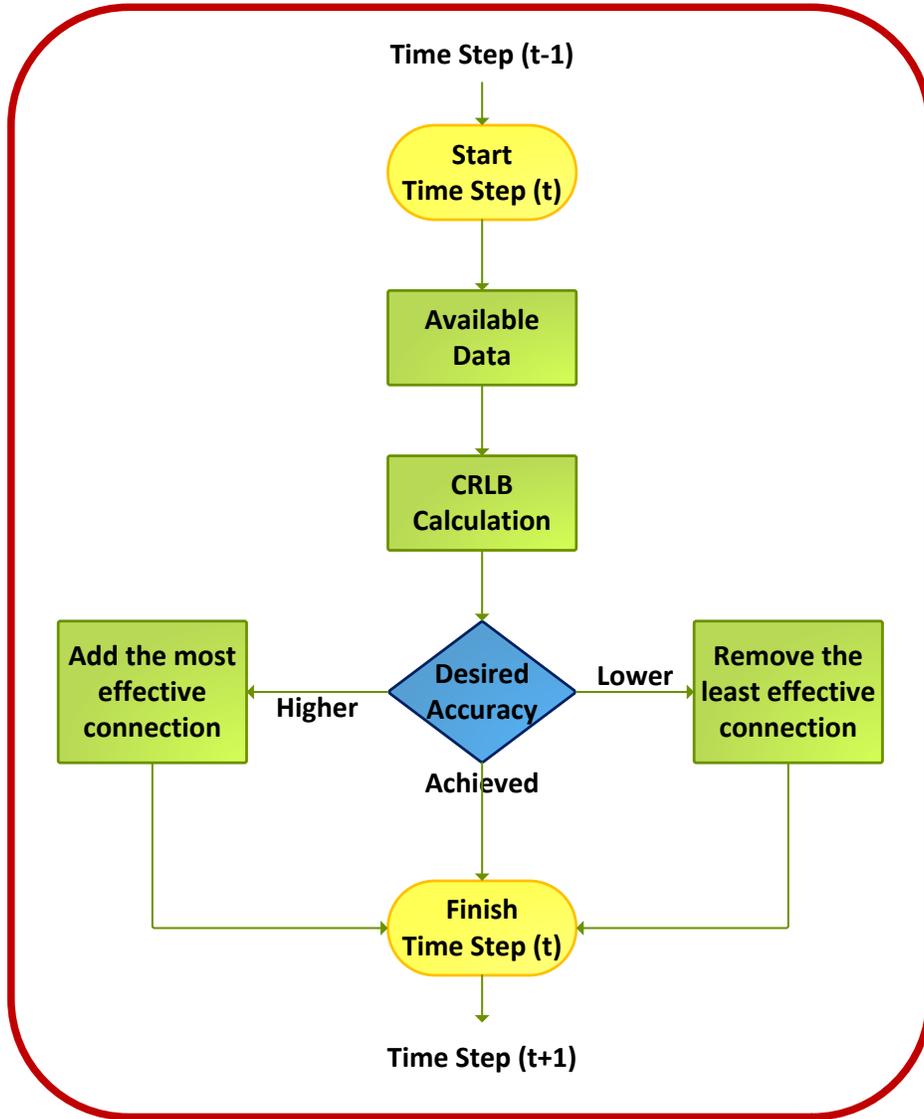


FIGURE 13 - The flow chart of the proposed Smart Algorithm

The main idea of the algorithm is summarized in FIGURE 13. The Algorithm at each time step checks whether or not the previous selected connections can provide the required accuracy. Since the CRLB provides a lower bound on the accuracy of the estimation, it will be used as a benchmark to check whether or not the estimator can achieve the required accuracy. If the desired accuracy is achieved by the selected connections the algorithm continues to the next step. However, if the CRLB is lower than the desired accuracy, it means that there are more than enough connections. Therefore, the algorithm removes the least effective connections from the set. On the other hand, if the CRLB is higher than the

desired accuracy, it means that the selected connections are not enough. Hence, the algorithm adds the most effective available connections to the set. The proposed Smart algorithm is mathematically described in Algorithm 1. The line by line detailed description of the proposed Smart algorithm is provided as follows:

**Line 1:** The inputs of the Smart algorithm are provided:

$\mathcal{A}_i^k$ , a set including all connections of vehicle  $i$  at time step  $k$

$\mathcal{C}_i^{k-1}$ , a set of used connections provided by the Smart algorithm at time step  $k - 1$

$\hat{\boldsymbol{\theta}}_i[k - 1|k - 1]$ , the estimates of the vehicles locations at time step  $k - 1$

$\mathbf{y}_j[k]$ , the location of the unit at time step  $k$

$\epsilon$ , the required accuracy

**Line 2:** The vehicles locations at time step  $k$  are predicted.

**Line 3:** The intersection of the  $\mathcal{C}_i^{k-1}$  and  $\mathcal{A}_i^k$  is obtained as stored in  $\hat{\mathcal{C}}_i^k$  (called predicted set). I want to know which units are still connected to the vehicle at the current time step.

**Line 4:** If the size of the predicted set is larger than 4, I calculate the FIM given the connections defined in  $\hat{\mathcal{C}}_i^k$  and store in  $\mathbf{I}$ . The Fisher information matrix of the connections is calculated using the function *Fisher* which is defined in (79).

**Line 5:** If the number of connections is smaller than 4, the Fisher Information Matrix (FIM) cannot be determined and is set to a small value to ensure that the algorithm adds more connections.

**Line 6:** The predicted accuracy,  $\delta$ , using the CRLB (Trace of the inverse of the FIM) is compared with the desired accuracy,  $\epsilon$ . I also consider a 15% tolerance.

**Line 7-14:** If the predicted accuracy is better than required accuracy,  $\delta < -15\%\epsilon$ , the algorithm needs to remove a connection. However, I need to determine which connection should be removed from the set. Intuitively, it is better select the connection whose removal has the least impact on the predicted accuracy. One way to calculate the effect of removing or adding a connection on the predicted accuracy is to calculate the CRLB for the new set of connections from scratch. It typically takes a lot of processing time, especially when the number of connections is high. However, the Smart algorithm is only useful when the running time of the connection selection process plus location estimation based on those connections is less than that of the fully-Integrated algorithm using all connections. Alternatively, an approximation is introduced (line 15 in Algorithm 1) which can be

calculated significantly faster. This approximation comes from the following mathematical expression (Hazewinkel 1993):

$$(\mathbf{I} + \varepsilon \mathbf{Z})^{-1} = \mathbf{I}^{-1} - \varepsilon \mathbf{I}^{-1} \mathbf{Z} \mathbf{I}^{-1} + O(\varepsilon^2). \quad (78)$$

where  $\mathbf{Z}$  is the Fisher information matrix of the connection to be added or removed from the current set. Since matrix inversion is a complex process for large matrices, the impact of a new connection can be simply calculated by using the above approximation  $(\mathbf{I}^{-1} \mathbf{Z} \mathbf{I}^{-1})$  which can be determined significantly faster than calculating and inverting a new FIM. Therefore, adding or removing the considered connection changes the CRLB by  $\mathbf{I}^{-1} \mathbf{Z} \mathbf{I}^{-1}$ . In this case, I am interested in removing a connection which has the least effect on the accuracy. Therefore, a connection which has the lowest value of  $\text{Trace}\{\mathbf{I}^{-1} \mathbf{Z} \mathbf{I}^{-1}\}$  is removed from the set. It should be noted that in this case, if there are RFID connections in the set, the algorithm removes them for the next time step. The reason is that in these situations, RFIDs will not provide the localization with a noticeable improvement.

**Line 15-25:** On the other hand, if the predicted accuracy is worse than the required accuracy,  $\delta > 15\% \epsilon$ , the algorithm needs to add a connection to the set to compensate the lack of sufficient accuracy. Similar to the previous case, selection of a connection is performed based on the CRLB. However, in this case, I am interested in a connection which delivers the highest accuracy improvement. Again, I start calculating  $\mathbf{I}^{-1} \mathbf{Z} \mathbf{I}^{-1}$  for all available connections and select the one that has the highest value of  $\text{Trace}\{\mathbf{I}^{-1} \mathbf{Z} \mathbf{I}^{-1}\}$ . Therefore, we anticipate that this connection delivers the best improvement and add it to the set. In the situations when there is no extra resources other than RFID readers to be added in the set, all available RFID connections will be included in the set (line 17).

The Fisher information matrix (FIM) which is used to calculate the CRLB is given as follows:

$$\begin{aligned}
 & \text{Fisher}(\mathbf{Y} \mid j \in \mathcal{A}_i, \mathbf{x}_i, \sigma_{ij}^2) \\
 &= \begin{bmatrix} \sum_{j \in \mathcal{A}_i} \frac{(x_{v,i} - x_{u,i})^2}{\sigma_{ij}^2 d_{ij}^2} & \sum_{j \in \mathcal{A}_i} \frac{(y_{v,i} - y_{u,i})(x_{v,i} - x_{u,i})}{\sigma_{ij}^2 d_{ij}^2} & \sum_{j \in \mathcal{A}_i} \frac{(z_{v,i} - z_{u,i})(x_{v,i} - x_{u,i})}{\sigma_{ij}^2 d_{ij}^2} \\ \sum_{j \in \mathcal{A}_i} \frac{(x_{v,i} - x_{u,i})(y_{v,i} - y_{u,i})}{\sigma_{ij}^2 d_{ij}^2} & \sum_{j \in \mathcal{A}_i} \frac{(y_{v,i} - y_{u,i})^2}{\sigma_{ij}^2 d_{ij}^2} & \sum_{j \in \mathcal{A}_i} \frac{(z_{v,i} - z_{u,i})(y_{v,i} - y_{u,i})}{\sigma_{ij}^2 d_{ij}^2} \\ \sum_{j \in \mathcal{A}_i} \frac{(x_{v,i} - x_{u,i})(z_{v,i} - z_{u,i})}{\sigma_{ij}^2 d_{ij}^2} & \sum_{j \in \mathcal{A}_i} \frac{(y_{v,i} - y_{u,i})(z_{v,i} - z_{u,i})}{\sigma_{ij}^2 d_{ij}^2} & \sum_{j \in \mathcal{A}_i} \frac{(z_{v,i} - z_{u,i})^2}{\sigma_{ij}^2 d_{ij}^2} \end{bmatrix} \quad (79)
 \end{aligned}$$

**Line 26-28:** If the predicted accuracy falls between  $(0.85\epsilon, 1.15\epsilon)$ , the algorithm proceeds without any changes. This means that we expect that the current connections can provide the desired accuracy and no change in connections is required.

This procedure is repeated in each time step and the algorithm changes the set of connections based on the desired accuracy and the predicted one.

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**ALGORITHM 1 Smart Vehicle Localization**


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1:  Inputs:  $\mathcal{A}_i^k, \mathcal{C}_i^{k-1}, \hat{\boldsymbol{\theta}}_i[k-1|k-1], \mathbf{y}_j[k], \epsilon$ 
2:   $\hat{\mathbf{x}}_i[k] = \mathbf{A}\hat{\boldsymbol{\theta}}_i[k|k-1]$ , the first three elements
3:   $\hat{\mathcal{C}}_i^k = \mathcal{C}_i^{k-1} \cap \mathcal{A}_i^k$ 
4:  if  $|\hat{\mathcal{C}}_i^k| \geq 4$  then  $\mathbf{I} = \text{Fisher}(\mathbf{y}[k] \ j \in \hat{\mathcal{C}}_i^k, \hat{\mathbf{x}}_i[k], \sigma_{ij}^2)$ 
5:  else  $\mathbf{I} = 0.25\epsilon^{-2}$  * end if
6:   $\delta = \sqrt{\text{Trace}\{\mathbf{I}^{-1}\}} - \epsilon$ 
7:  if  $\delta < -15\%\epsilon$  then
8:       $\hat{\mathcal{C}}_i^k = \hat{\mathcal{C}}_i^k - D_i^k$ 
9:      for  $j \in \hat{\mathcal{C}}_i^k$  do
10:          $\mathbf{Z} = \text{Fisher}(\mathbf{y}_j[k], \hat{\mathbf{x}}_i[k], \sigma_{ij}^2)$ 
11:          $[\mathbf{b}]_j = \text{Trace}\{\mathbf{I}^{-1}\mathbf{Z}\mathbf{I}^{-1}\}$ 
12:      end for
13:       $j_{min} = \{j \in \hat{\mathcal{C}}_i^k \mid [\mathbf{b}]_j = \{\min(\mathbf{b})\}\}$ 
14:       $\mathcal{C}_i^k = \hat{\mathcal{C}}_i^k - j_{min}$ 
15:  elseif  $\delta > 15\%\epsilon$  then
16:      if  $\mathcal{C}_i^k \cap \mathcal{A}_i^k - D_i^k = \emptyset$  then
17:          $\mathcal{C}_i^k = \hat{\mathcal{C}}_i^k \cup D_i^k$ 
18:      else
19:         for  $j \in \hat{\mathcal{C}}_i^k \cap \mathcal{A}_i^k - D_i^k$  do
20:             $\mathbf{Z} = \text{Fisher}(\mathbf{y}_j[k], \hat{\mathbf{x}}_i[k], \sigma_{ij}^2)$ 
21:             $[\mathbf{b}]_j = \text{Trace}\{\mathbf{I}^{-1}\mathbf{Z}\mathbf{I}^{-1}\}$ 
22:         end for
23:          $j_{max} = \{j \in \hat{\mathcal{C}}_i^k \mid [\mathbf{b}]_j = \{\max(\mathbf{b})\}\}$ 
24:          $\hat{\mathcal{C}}_i^k = \hat{\mathcal{C}}_i^k + j_{max}$ 
25:      end if
26:  else
27:       $\mathcal{C}_i^k = \hat{\mathcal{C}}_i^k$ 
28:  end if

```

---

$\mathcal{A}_i^k$ : All available connections at the  $k$ th time-step ( $\mathcal{S}_k \cup \mathcal{W}_k \cup \mathcal{D}_k \cup \mathcal{V}_k$ )

$\mathcal{C}_i^k$ : Connections using the Smart algorithm ( $\mathcal{C}_i^k \subseteq \mathcal{A}_i^k$ )

$\hat{\boldsymbol{\theta}}_i[k|k-1]$ : The predicted location and velocity of the  $i$ th vehicle at the  $k$ th time-step

$\epsilon$ : The required accuracy

\* To ensure that the number of connections is always more than 4, otherwise a connection should be added

# 5 Simulation Results and Discussion

In this section, the performance of the proposed Integrated and Smart algorithm is evaluated through computer simulations. Computer simulations enable us to analyze and compare the performance of the localization algorithms easily under different conditions. Computer simulations are widely used in different areas such as physics, economics, social science, and engineering(Vaghefi et al. 2013; Wymeersch et al. 2009).

## 5.1 Network Setup

### 5.1.1 GPS

In the simulations, the true locations of the GPS satellites have been used. Each GPS satellite has a unique orbit and its location is defined by ephemerides. The Cartesian locations of the satellites are extracted from the ephemeris data using MATLAB script EASY17 (Borre 2003). The recent ephemeris information of the current 31 satellites is obtained from the National Geodetic Survey (NGS) database. EASY 17 provides us with MATLAB functions *satpos* and *get\_eph* whose inputs are the ephemeris data obtained from the NGS database and the elapsed time from the reference time. These two functions are provided as appendices at the end of this document. For these functions, the outputs are the Cartesian locations of the satellites at that specific time. FIGURE 14 shows the simulation of the satellite locations for a period of 48 hours using EASY17. To have a clearer graph, I only plotted 9 satellites in FIGURE 14. However, for the simulation all 33 satellites are simulated.

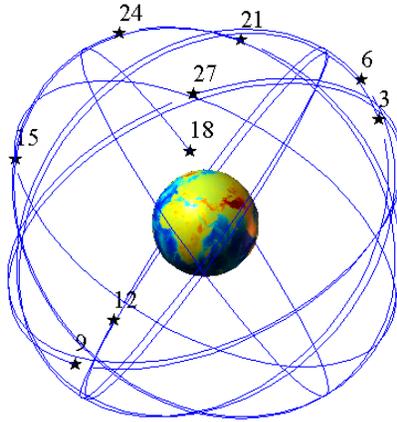


FIGURE 14 - The example of the simulated locations of the GPS satellites by EASY17.

The next step in simulating GPS is to determine the noise associated with GPS measurements which is a critical task. As mentioned earlier, the accuracy of the GPS range measurements is dependent on several parameters such as ionospheric effects, ephemeris errors, satellite clock errors, multipath distortion, and tropospheric effects (Tsui 2005). The effect of each parameter is indicated in terms of User Equivalent Range Errors (Tsui 2005). The overall error on the range measurement of GPS positioning can be obtained by taking the root sum square of the above mentioned individual errors (Tsui 2005).

Quantifying the sources of errors for GPS is a difficult task. Alternatively, as suggested in (Biskaduros and Buehrer 2012), experimental results show that the average error on the measurements of a GPS receiver depends on the elevation of the satellite and the environment where the receiver is located. Therefore, in the simulations, the surrounding areas are divided into top view and side view based on the satellite elevation. Five different environments are also considered: clear view, commercial, residential, forest, and indoor. In Table 2, the accuracy of the range measurement under different satellite elevations and environments is summarized. For example, from TABLE 3, if the receiver is located inside a commercial area and the satellite elevation is around  $\pi/3$ , the corresponding accuracy of the range measurement for this link would be set to 15 m. For indoor environments, I assume that GPS does not work and no parameter is considered for them in TABLE 3. An example of different views of the satellites is depicted in FIGURE 15.

TABLE 3 - GPS parameters for simulations

Environment	Top View [m]	Side View [m]	Off View		
Clear View	$0 \leq \theta \leq \pi/4$	4	$\pi/4 < \theta \leq \pi/2$	5	$\pi/2 < \theta \leq \pi$
Residential	$0 \leq \theta \leq \pi/6$	8	$\pi/6 \leq \theta \leq \pi/3$	13	$\pi/3 < \theta \leq \pi$
Commercial	$0 \leq \theta \leq \pi/6$	10	$\pi/6 \leq \theta \leq \pi/3$	15	$\pi/3 < \theta \leq \pi$
Forest	$0 \leq \theta \leq \pi/6$	18	$\pi/6 \leq \theta \leq \pi/3$	24	$\pi/3 < \theta \leq \pi$
Indoor	-	-	-	-	-

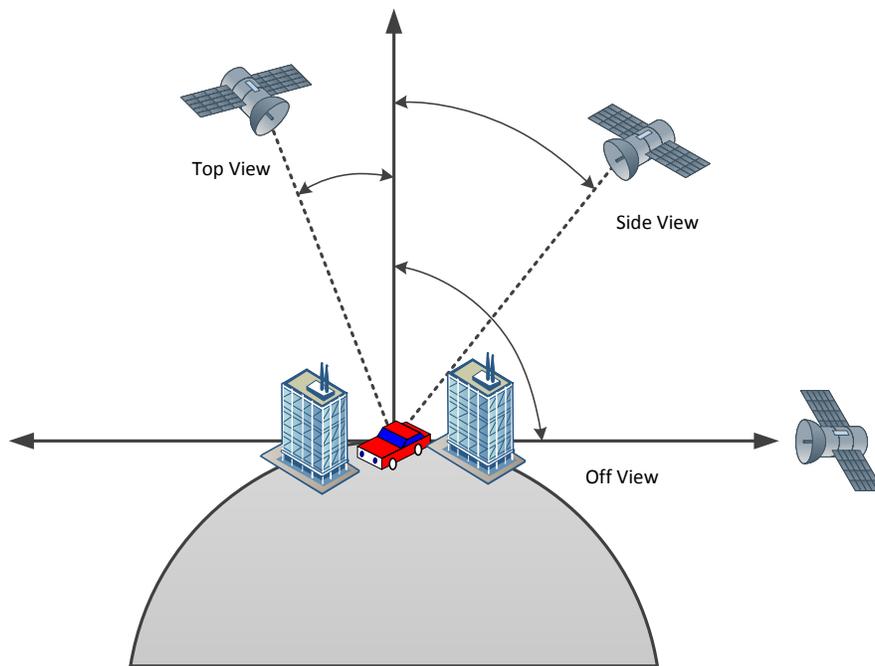


FIGURE 15 - An example of different satellite elevations

### 5.1.2 V2I

The accuracy of the range measurement in V2I depends on the selected method of ranging (Sayed et al. 2005). In this work, TOA ranging method was considered. The accuracy of ranging in TOA method is dependent on the environment and the distance between the anchors and the vehicle. It has been shown that the variance on the range measurements can be modeled as (Jia and Buehrer 2011):

$$\sigma^2 = \alpha d^\beta \quad (80)$$

where  $\alpha$  defines the correlation between the distance and the variance and depends on the transmit power and hardware implementation (Jia and Buehrer 2011). It is assumed that all anchors in the network have the same transmit power and hardware specifications.  $\beta$  is the path-loss-exponent (PLE) defining the rate at which the power of the signal attenuates as it propagates through the space. The value of the PLE varies from 2 in free space and 4 in dense and harsh environments (Jia and Buehrer 2011; Vaghefi et al. 2013). The model in (80) implies that in dense environments the accuracy of the range measurements tends to be lower, as  $\beta$  gets larger. In Table 2, the values of  $\alpha$  and  $\beta$  for the simulations are provided. Another parameter that should be considered for V2I is the communication range. In TOA ranging, the anchor nodes transmit the signals and the receiver measures the travel time between the anchor and itself. However, if the transmitted signal attenuates such that its power falls down below the receiver noise level, the signal cannot be detected by the receiver. The communication range is, in fact, the distance at which the signal power is so low that the receiver cannot detect it. Since in V2I communications, radio-frequency electromagnetic fields are used, the log-distance path-loss model is used to determine their communication ranges. Log-distance path-loss model, which is provided in (7), is a function of PLE. Therefore, the larger the value of the PLE is, the higher the attenuation is and the shorter the communication range will be. Therefore, based on the environment and the value of  $\beta$ , different communication ranges are considered for anchors, as shown TABLE 4.

### 5.1.3 RFID

Unlike GPS and V2I, no range measurement is performed among RFID components (readers and tags). Therefore, no measurement error is considered for RFID networks. However, the communication range of RFID tags would be different depending on the environment where they are located. Similar to V2I, RFID components use radio-frequency electromagnetic fields; hence, the log-distance path-loss model is used to determine their communication ranges. Therefore, in harsh environment where the value of path-loss exponent is greater, shorter communication ranges are anticipated. TABLE 4 summarizes the communication ranges of the RFID readers for the proposed network under different environments.

### 5.1.4 V2V

Each vehicle in the network is equipped with a GPS receiver, a sensor, and an RFID tag which are used to collect satellite, V2I, and RFID reader signals, respectively. However, the sensor placed in the vehicle can be also used for V2V communications. The same approach as described in V2I section can be applied to find the accuracy of the range measurement between vehicles using V2V communications. Since smaller sensors with lower transmit power are usually placed in the vehicles, their communication range is typically shorter than the anchors. As mentioned before, the value of  $\beta$  depends on the environment, hence in TABLE 4, the same  $\beta$  is considered for both V2V and V2I connections, while different  $\beta$  is selected for different environments (Jia and Buehrer 2011). On the other hand, the value of  $\alpha$  mostly depends on hardware implementation of the receiver. Since in both V2V and V2I connections, the vehicle is the receiver, the same  $\alpha$  is considered for them. TABLE 2 also summarizes the parameters used for the variance of V2V links and their communication ranges under different conditions.

TABLE 4 - A Summary of the Parameters Used in the Computer Simulations.

Environment	V2I			RFID	V2V		
	$\alpha$	$\beta$	Range [m]	Range [m]	$\alpha$	$\beta$	Range [m]
Clear View	2.0E-3	2	300	8	2.0E-3	2	120
Residential	2.0E-3	3	200	6	2.0E-3	3	110
Commercial	2.0E-3	3	150	6	2.0E-3	3	90
Forest	2.0E-3	4	110	5	2.0E-3	4	70
Indoor	2.0E-3	4	100	4	2.0E-3	4	60

## 5.2 Simulation Steps

The steps of the computer simulations to evaluate the performance of the proposed algorithm are depicted in FIGURE 16. In the first step, we need to design a network. The network is intuitively drawn in the Microsoft Visio. I tried to include different geographical areas in the simulated network. The detailed description of the simulated network will be presented in the next section.

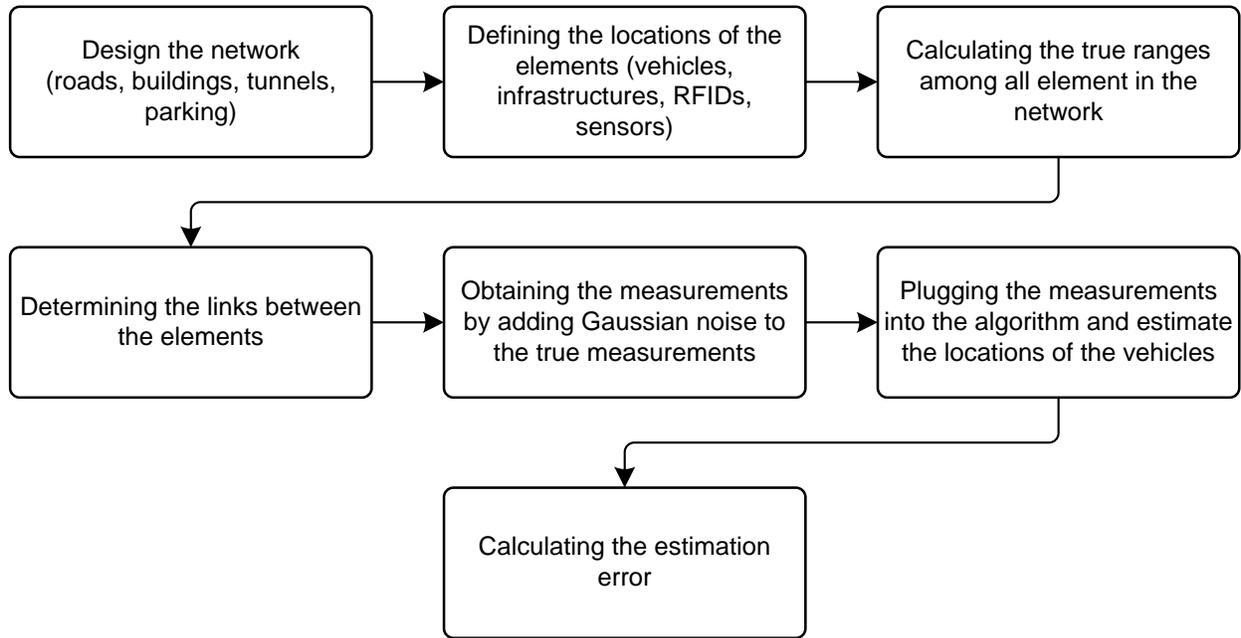


FIGURE 16 - Simulation steps

In the next step, I need to define the locations of the vehicles and other elements such as satellites, infrastructures, RFIDs, sensors, etc. Since the locations of the vehicles and satellites change over time, their locations should be defined for each time step. However, other elements are fixed over time and their locations should be defined once.

Then, I need to calculate the true ranges among the elements of the network. To do that, the MATLAB routine *dist* is used. Let  $\mathbf{x} \in \mathfrak{R}^{3 \times M}$  and  $\mathbf{y} \in \mathfrak{R}^{3 \times N}$  be the matrices of the locations of the  $M$  vehicles and  $N$  satellites, the *dist* command in the MATLAB creates a  $M \times N$  matrix whose  $m$ th and  $n$ th element is the true distance between the  $m$ th vehicle and  $n$ th satellite. I need to repeat this for all time steps. I also need to do the same process for calculating the distance between the vehicle and other units including sensors, RFID readers and other vehicles.

Once the ranges between the elements of the network are calculated, I need to determine the connectivity. So far I have the ranges between all elements with each other. However, in real cases not all elements are connected, but only the neighboring elements can communicate with each other. To take the connectivity into account, I define a matrix of 0 and 1 (called  $\mathbf{A}$ ) whose components determine whether two elements are connected or not.

If the  $i$ th and  $j$ th elements are connected, the  $(i, j)$ th component of the matrix is 1 and if the  $i$ th and  $j$ th elements are not connected, the  $(i, j)$ th component of the matrix is 0. To determine the components of the matrix, the communication ranges of elements in the network is used. For instance, from TABLE 4, the communication range of the RFID reader in clear environments is 8 meters. If the calculated distance between an RFID reader and a vehicle in the previous step exceeds 8 meter, I set their corresponding value in the matrix  $\mathbf{A}$  to zero. The connectivity for the rest of network is calculated similarly. However, the connectivity for GPS satellites should be defined in a different way, since the distance does not play a role in determining the connectivity (as all satellites almost have the same distance from the vehicles in the earth). Satellites elevations are used to determine the connectivity. TABLE 3 shows the elevation in which the satellites do not have a connection with the vehicles for different environments. It should be noted that based on the assumptions, the network is located in the northern hemisphere, hence in all environments the satellites are not connected to the vehicle for the elevations larger than  $\pi/2$ .

In the next step, I need to obtain the range measurements. To do that, the measurement model given in the previous section is used. Based on the measurement model, the measured ranges are in fact the true ranges corrupted by an additive Gaussian noise. In the MATLAB environment, a Gaussian random variable with mean of  $\mu$  and variance of  $\sigma^2$  is generated as:

$$\mathbf{N} = \mu + \sigma \times \text{rand}(m, n) \quad (81)$$

where the MATLAB routine *rand* generates independent and identically disturbed Gaussian random variable with mean of zero and variance of 1. Therefore, matrix  $\mathbf{N}$  is a  $m$  by  $n$  matrix whose elements are Gaussian random variables with mean of  $\mu$  and variance of  $\sigma^2$ . Finally, the measured ranges are obtained by adding the generated noise to the true ranges obtained in the previous step.

In the next step, I need to determine the locations of the vehicles from noisy measurements obtained from the previous step. Therefore, the range measurements along with other parameters, such as the locations of units (satellites, infrastructures, and RFID readers), the connectivity matrix, etc. are plugged into the algorithm. The output of the algorithm is the

estimation of the specific vehicle at the specific time. Therefore, the algorithm should be run for each vehicle at each time step.

In the next step, the performance of the estimation should be evaluated. To do that, the estimation error needs to be calculated. Different methods for evaluating the performance of an estimator are described in detail in the previous chapter. CDF, MSE, or RMSE can be used in this step.

### 5.2.1 Flowchart

The flowchart of the proposed Integrated algorithm is provided in FIGURE 17. In the first block, some variables are defined such as the locations of units and vehicles, communication ranges of units, and noise parameters. In this block,  $M$ ,  $N$ , and  $T$  are the number of units, vehicles, and time steps, respectively.

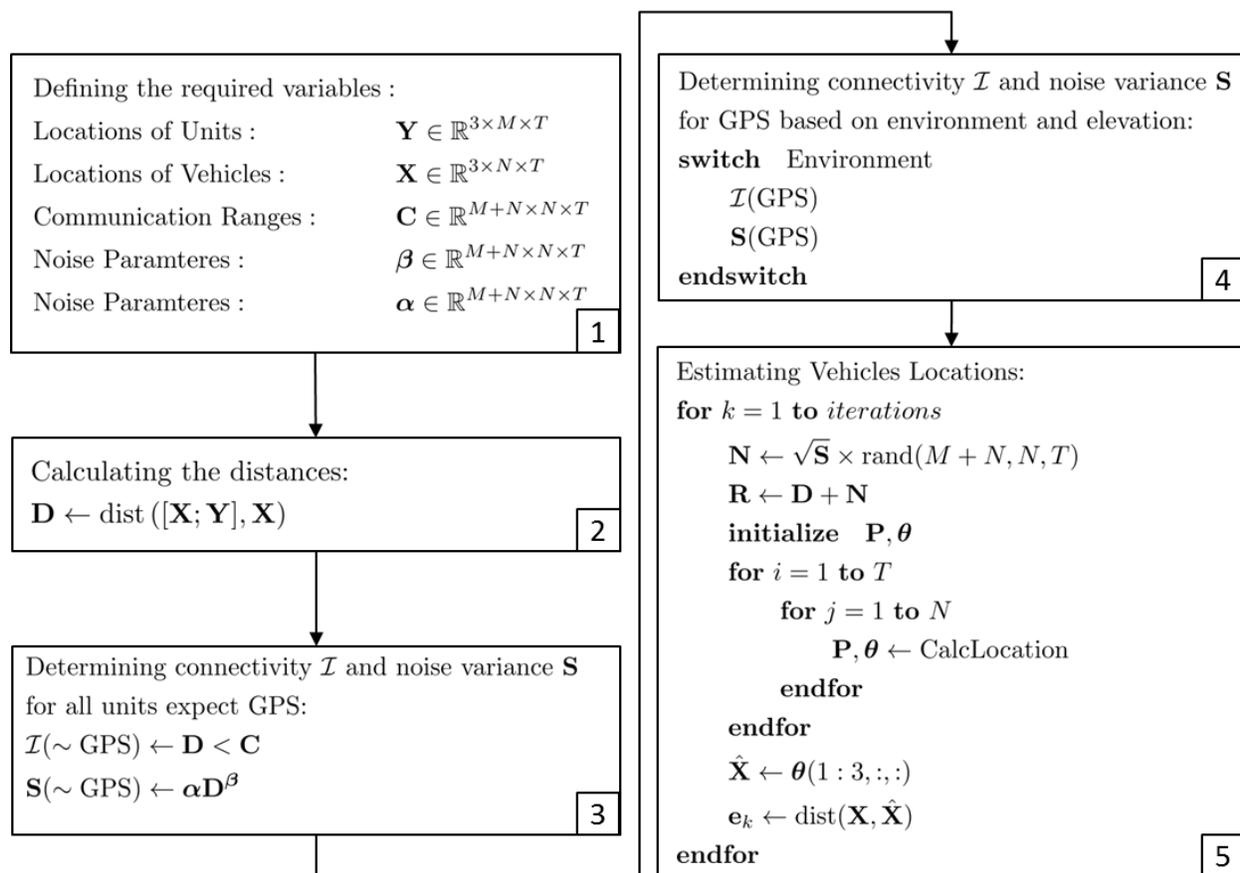


FIGURE 17 - Flowchart of simulation setup

In the second block, the true distances between the vehicles and units are calculated by the MATLAB routine *dist*. In the third block, the connectivity and noise variance of the measurement for the units except GPS are determined. In this case, the connectivity is a function of the distances and the communications ranges. In the fourth block, the connectivity and noise variance of the measurement for GPS connections are determined. The connectivity is a function of environment and satellites elevation. In the last block, the locations of the vehicles are estimated. The scenario is repeated several times, as depicted by the index  $k$ . In each iteration, the new noise matrix is generated and new measurements are obtained. Then, the location of each vehicle is estimated at each time step, represented by indices  $i$  and  $j$ . The *ClacLocation* function is used to estimate the vehicles location whose flowchart is depicted in FIGURE 18.

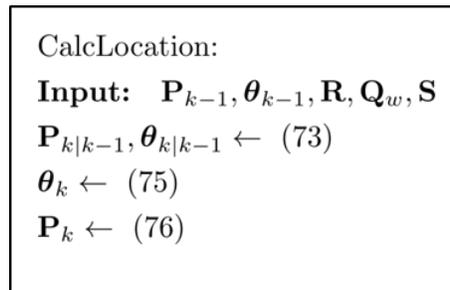


FIGURE 18 - Flowchart of *ClacLocation* function

$\boldsymbol{\theta} \in \mathcal{R}^{6 \times N}$  is the estimates of the locations and velocities of the vehicles and  $\mathbf{P} \in \mathcal{R}^{6 \times 6 \times N}$  is the covariance matrix of the estimates. These parameters are described in detail in section 4.2. Once the estimates of the locations are obtained, the estimation error is calculated for each iteration. Then, the estimation errors are either plotted in a CDF or RMSE figure.

### 5.3 Simulated Transportation Network

The proposed transportation network and the traveling path of the desired vehicle are depicted in FIGURE 19. The traveling path of the desired vehicle is illustrated with a red curve. Different conditions are included in the simulated network. The traveling path can be divided into 7 different segments.

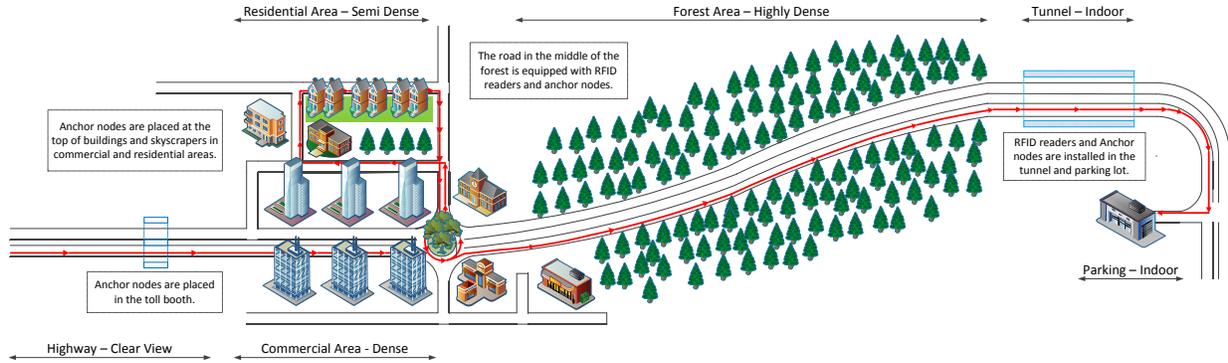


FIGURE 19 - The plot of the proposed transportation network

Several anchors and RFID readers are installed around the road. Moreover, the vehicle can take advantage of V2V connections when they are available. There are 15 other vehicles in the network. The number of V2V connections varies from 0 to 6. In the first segment, the desired vehicle travels along a highway. This environment is classified as Clear View, since there is no object surrounding the road. The vehicle is connected to several GPS satellites. Moreover, there are some sensors located in the toll booth. In the second segment, the vehicle travels within a commercial area where the road is surrounded by tall buildings and skyscrapers. The GPS reception in this area is weak. However, some anchors are placed on top of the buildings to help the vehicle find its location more accurately. The vehicle can also utilize V2V communications with surrounding vehicles. Then, the vehicle travels in a residential area. Here, the condition is almost similar to the previous segment, except GPS reception is more powerful in this area as the buildings are typically shorter. Several anchors and other vehicles are also placed in this area. Afterwards, the vehicle goes into a forest area where the GPS reception is very weak due to the highly-dense environments. However, the road is equipped with several anchors and RFID readers. Then, the vehicle enters a tunnel which is classified as an indoor environment where no GPS reception is available. Here, the vehicle tries to find its location merely with the help of anchors and RFID readers located inside the tunnel and neighboring vehicles. In the sixth segment, the vehicle again passes a clear area and enters a parking garage which is, again, classified as an indoor environment where the vehicle can take advantage of RFID technology, V2V and V2I communications. The vehicle moves around a circle in the parking and finally stops.

## 5.4 Integrated Algorithm Performance

As mentioned before, the Integrated algorithm uses all the existing resources to improve the accuracy of localization. In this section, the performance of the proposed Integrated algorithm is evaluated. In FIGURE 20, the CDF of the localization error is depicted for different cases. To provide a smoother plot, we need to repeat the whole simulated network several times. In this way we will have more points on the CDF plot. FIGURE 20 shows the result of performing 10 repetitions for the desired vehicle.

The performance of GPS is almost satisfactory in all regions except for indoor (i.e., tunnel and parking garage) and very dense (i.e., forest) environments. On the other hand, the estimated location of the vehicle using the Integrated algorithm provides remarkable performance in all regions, especially in highly dense and indoor environments where GPS reception is very weak. When the vehicle uses only GPS connections, the accuracy of the localization is very poor in about 15% of the time especially when the vehicle is moving in indoor areas. In addition, the integrated positioning provides considerably better performance not only when the vehicle is in indoor regions but also other regions where GPS reception is relatively good. The reason is that the integrated positioning exploits other resources which enhance the positioning accuracy.

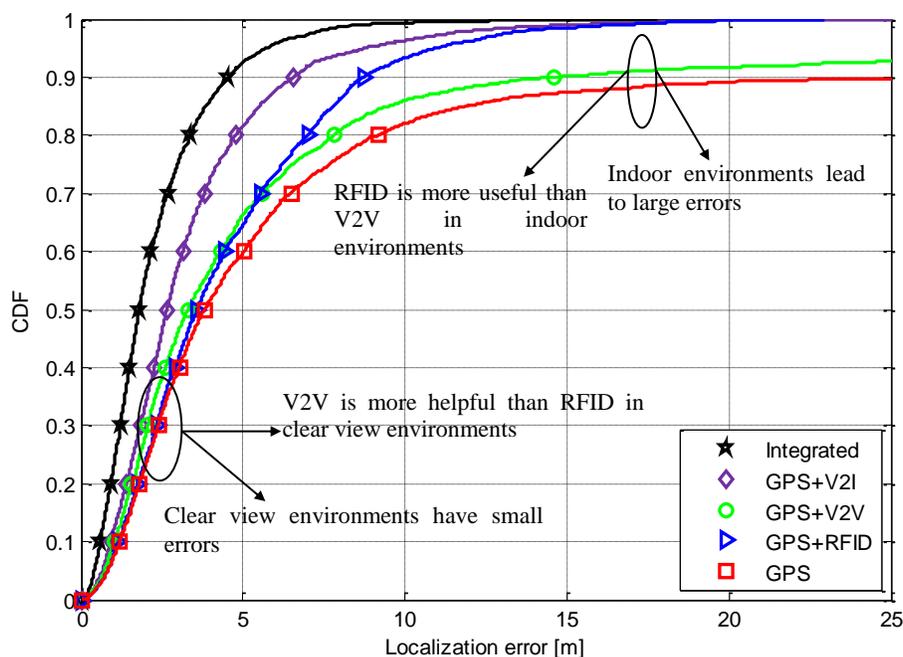


FIGURE 20 - The CDF of the localization error.

As depicted in the blue curve in FIGURE 20 , RFID technology can improve the location performance, especially when the vehicle is inside the tunnel and parking garage. However, in other regions where the GPS reception is almost sufficient, RFID technology cannot help the algorithm in terms of accuracy.

The behavior of GPS+V2V is almost opposite to GPS+RFID. In other words, V2V technology can slightly help the vehicle in the clear view, commercial and residential regions because it provides the vehicle with more useful connections. However, V2V cannot improve the performance in indoor regions considerably because in indoor environments V2V technology uses other vehicles information which are also inside the tunnel (or parking garage) and do not have enough connections due to GPS outage and as a result their location information is not as reliable as other resources such as RFID readers and V2I.

As depicted in the violet curve, among GPS-aided techniques (RFID, V2V, and V2I), V2I provides considerably better accuracy. The reason is that unlike RFID, V2I is associated with the range measurement which is more useful than presence detection for localization. On the other hand, V2I has more valuable information than V2V because the source of information in V2I is an anchor with a fixed and known location, while the source of information in V2V is another vehicle whose location is not accurate.

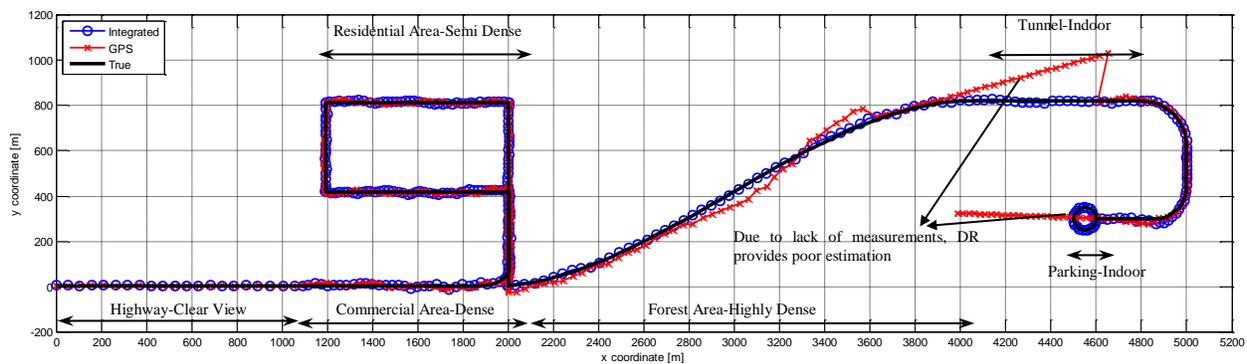


FIGURE 21 - The true and estimated locations by Integrated and GPS only algorithms.

In FIGURE 21, I plotted the true and estimated location of the vehicle obtained from the Integrated algorithm and GPS only positioning. As can be seen, there is no significant difference between the estimates obtained from the Integrated and GPS algorithms in clear view environments, since in these areas, the GPS receiver has enough connections with

direct view with the satellites and can provide highly accurate estimate. Therefore, adding more localization resources would not be useful. In commercial and residential environments, GPS algorithm still can provide satisfactory estimate. However, in forest environments, GPS fails to provide a good accuracy, mainly because the vehicle does not have sufficient connections to the satellites, while the Integrated algorithm perform significantly better, as it exploits other resources in this area such as RFID readers and V2I connections. In indoor environments (tunnel and parking garage), GPS algorithm estimates the location of vehicle completely wrong, since the GPS receiver does not have any connections to the satellites. On the other hand, the Integrated algorithm can still provide a reasonable accuracy in these areas.

The RMSE of localization at each time-step for different algorithms is also depicted in FIGURE 22. The whole network is simulated 10 times and RMSE is calculated over 10 realizations at each time step. As can be seen, the Integrated algorithm performs better than other algorithms in all situations. GPS has very large errors when the vehicle is inside the tunnel and parking areas. As mentioned earlier, GPS-V2V performs better than GPS-RFID in most areas. However, GPS-V2V performs worse than GPS-RFID in indoor environments, since no resource other than GPS is available and vehicles have only a relative location from each other. GPS-RFID does not provide significant improvement to GPS algorithm in clear, dense and semi-dense environments where standalone GPS has already enough accuracy. However, in indoor environments where GPS does not work at all, GPS-RFID provides reasonable accuracy. GPS-V2I is highly beneficial in most areas, especially in indoor environments where no other resources can perform better than V2I.

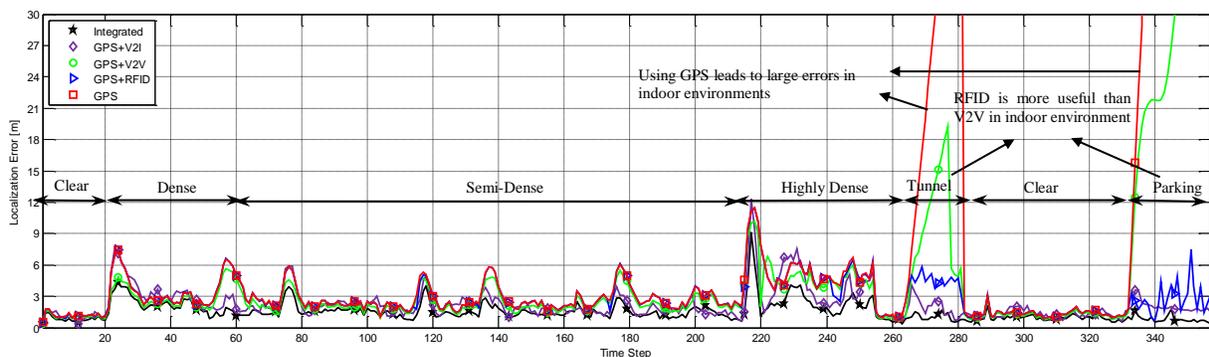


FIGURE 22 - The RMSE of localization at each time-step.

In all previous cases, the algorithms use the internal DR. Evaluating the effect of the internal DR sensor, FIGURE 23 shows that using DR is highly beneficial for both integrated positioning and GPS positioning. However, DR is not useful for GPS positioning when the vehicle is inside indoor environments. In DR technique, the previous estimate is used to predict the next vehicle's location. If no measurement is available and if the vehicle changes its velocity often, the prediction and the true location of the vehicle get farther and farther apart which generates significantly large errors. Therefore, using DR without having enough measurements and sources does not necessarily lead to performance improvement. This conclusion is also clearly demonstrated in FIGURE 21 where DR is not useful anymore when the vehicle enters the tunnel and parking garage, as DR predicts the wrong direction for the vehicle in the absence of measurements.

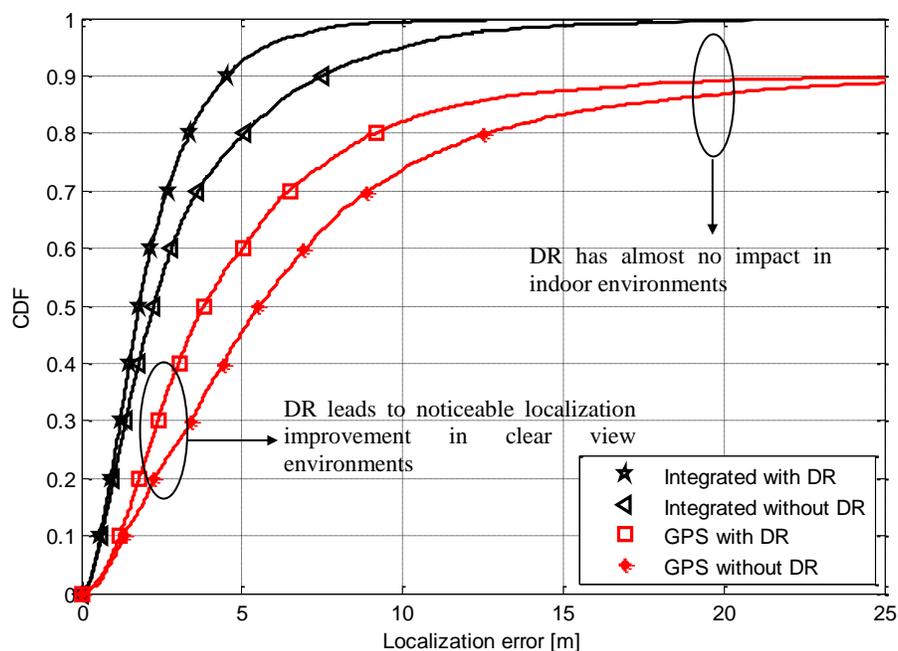


FIGURE 23 - The CDF of the localization error of the algorithms with and without DR.

In FIGURE 24, I compare the RMSE of localization for Integrated and GPS algorithms with and without DR. When the vehicle is in indoor environments, DR cannot improve the accuracy of the standalone GPS, mainly because there is no other resource available to the vehicle and DR would be useless in this case. However, the DR provides improvement for

the Integrated algorithm when the vehicle is in indoor environments, because some resources other than DR are available to the vehicle. In other areas, the DR improves the performances of both Integrated and standalone GPS algorithms.

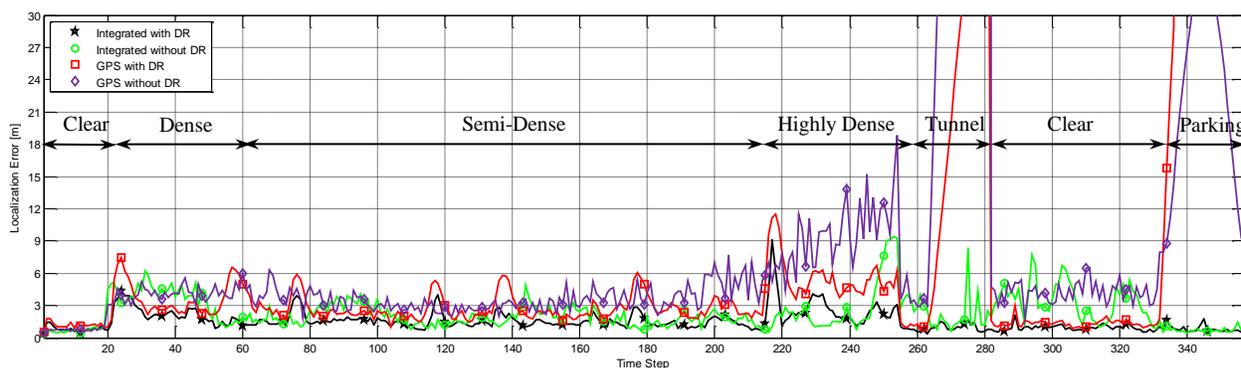


FIGURE 24 - Comparing the RMSE of localization for algorithms with and without DR.

## 5.5 Smart Algorithm Performance

As mentioned earlier, the Integrated positioning improves the performance of the localization in most conditions. However, in some situations, there is no need for the vehicle to use all its available connections, since using all information does not necessarily lead to a noticeable higher accuracy and it increases the localization complexity considerably. For example, in a situation where a vehicle has five V2V and three V2I connections, it is not necessary to use RFID or GPS connections as the vehicle can localize itself using these eight connections with sufficient accuracy. In addition, in some situations it is not very critical to have a very precise accuracy or it would be preferred for the location to be estimated in a least amount of time with minimum accuracy requirement. For example, in a situation when a driver is lost in a part of the transportation network, approximate location information with the accuracy of 20 m would be sufficient.

FIGURE 25 shows the simulation results of the proposed Smart algorithm. As mentioned earlier, the Smart algorithm is designed to use enough number of resources for the vehicle localization to achieve the required accuracy. In FIGURE 25, the CDF of the localization error of the Smart algorithm for different levels of accuracy is depicted. The label “Smart-x”

stands for the Smart algorithm with a required accuracy of  $x$  meter(s). As the accuracy constraint increases, the CDF curve of the Smart algorithm tends to the CDF curve of the fully-Integrated algorithm.

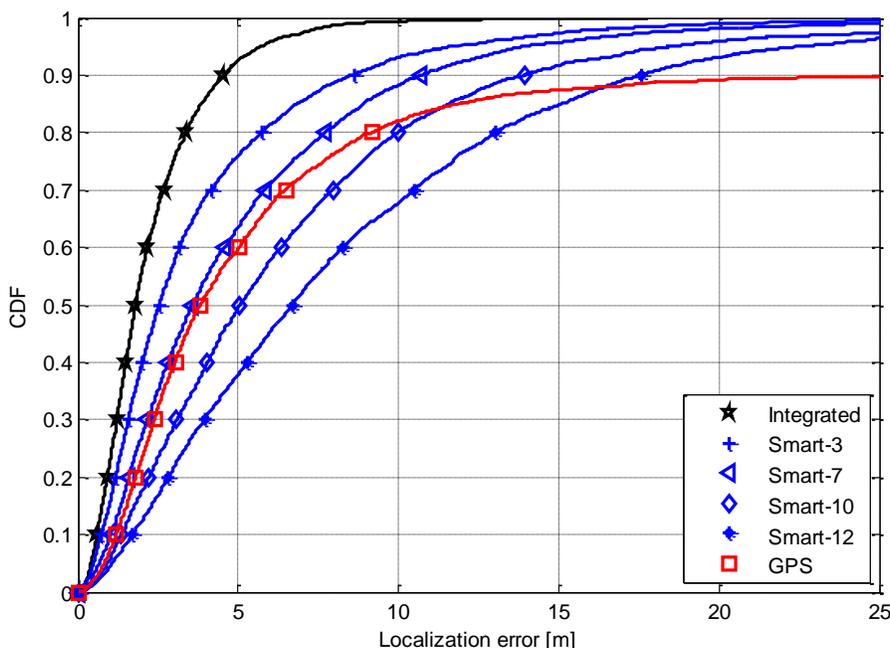


FIGURE 25 - The CDF of the localization error for the Smart algorithm

FIGURE 25 shows that the user can regulate the accuracy of the Smart algorithm based on the desired accuracy. The smart algorithm tries to use the most useful connections to achieve the desired accuracy at the least amount of processing time. In other words, among all the resources, the algorithm chooses the one that has increases the accuracy the most. In this research, an approximation to CRLB has been used as a benchmark for determining the most efficient resources to be added. In TABLE 5, I compare the RMSE and running time of the different Smart- $x$  algorithms. Consider Smart-7 algorithm. In this algorithm, the desired accuracy is set to 7 meters. As can be seen from FIGURE 25, about 80% of the time, it almost has the same performance as the standalone GPS. However, about 20% of the time fails to provide a good accuracy. As shown earlier, the large error of the GPS algorithm is due to its poor reception in indoor environments. TABLE 5 shows that Smart-7 not only has about 9%  $(\{89 - 81\}/89)$  less running time than GPS but also provides 85%  $(\{47.7 - 7.1\}/47.7)$

lower average localization error. Large RMSE of GPS positioning is due to its poor performance in indoor environments. In other regions, Smart-7 and GPS provide roughly the same performance, given that Smart-7 uses an average fewer connections and it leads to lower complexity.

Now consider Smart-12 algorithm. Although Smart-12 is worse than GPS in about 88% of the situations, it still has significantly lower RMSE than GPS. It is because Smart-12 has noticeable less error in indoor and dense areas.

TABLE 5 - The RMSE and running time of the Smart algorithm.

Algorithm	Average number of connections	Running time WRT integrated (%)	RMSE(m)
Integrated	13.7	100	2.9
Smart-3	11.7	90	3.8
Smart-7	8.5	81	7.1
Smart-10	5	76	10.1
Smart-12	4.1	72	11.8
GPS	11	89	47.7

FIGURE 25 also shows that in comparison with GPS, Smart-3 algorithm improves the performance of the localization by about 38% ( $\{6.5 - 4.0\}/6.5$ ) at 70% CDF of error and about 92% ( $\{47.7 - 3.8\}/47.7$ ) in the average error. However, Smart-3 only spends about 1% ( $\{90 - 89\}/89$ ) more running time than GPS.

TABLE 5 shows that there is a difference between the required accuracy and the achieved RMSE for the Smart algorithm. For Smart-3, Smart-7, and Smart-10, the achieved RMSE is larger than the required accuracy. The reason is that the Smart algorithm uses the CRLB to predict the error and the CRLB is a lower bound and typically lower than the actual error. Hence, achieving the CRLB is too optimistic. Despite the fact that the difference is less than 10% in these cases, the user can set the required accuracy more conservatively. This means that if the user would like to achieve the RMSE of 3m, they can set the accuracy for the Smart algorithm to 2m.

On the other hand, the RMSE of Smart-12 is slightly lower than 12m. The reason is that in some situations, only four connections can provide better accuracy than 12m and the

algorithm cannot remove any connections because at least four connections are required to have a unique solution (Patwari et al. 2005).

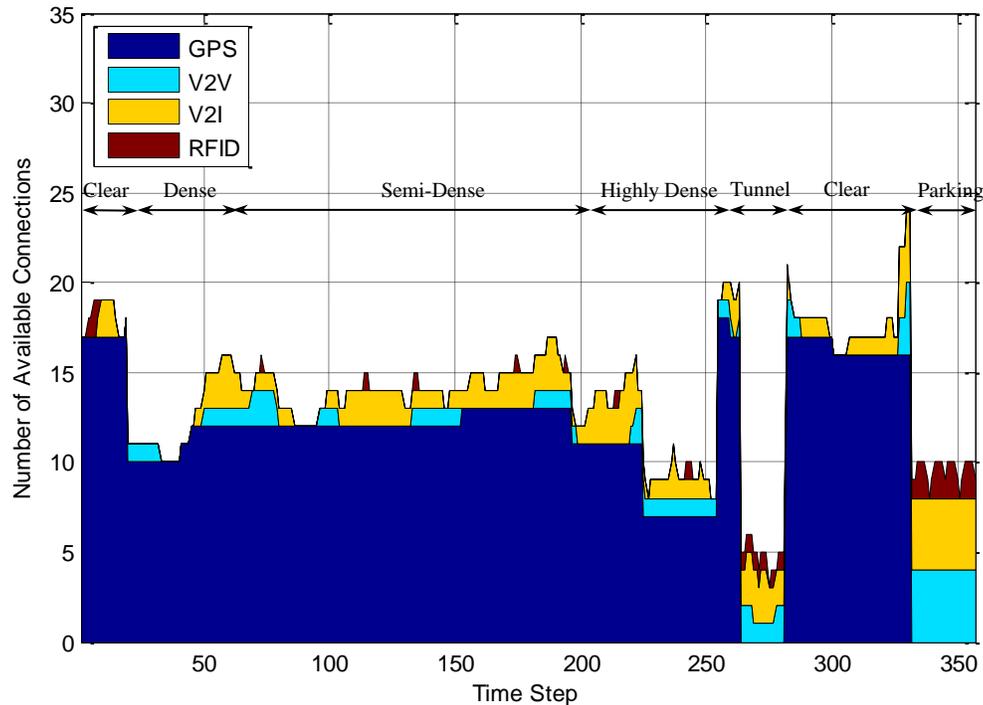


FIGURE 26 - Available connections at each time-step.

FIGURE 26 shows the number of available connections at each time step. As can be seen, in time step 0-30 where the vehicle is moving in the clear view environment, it has more than 15 connections to GPS satellites. Once the vehicle enters to the commercial area, the number of available GPS connections falls below 10. In the residential area, the vehicle not only has more than 10 GPS connections but also has several V2V and V2I connections. At time step 200 to 250, where the vehicle is in forest environment, the number of GPS connections is limited (e.g., less than 8) but V2I connections are available to it. At time step 260-280 and 330-350, the vehicle is in indoor environment and the vehicle does not have access to any GPS satellites, although other resources including RFID readers, V2V and V2I connections are provided.

FIGURE 27 shows the number of connections used by different algorithms at each time step. It is clear that stand alone GPS algorithm uses only GPS connections, while the Integrated algorithm makes use of all available connections at each time step. FIGURE 27

also shows how the Smart algorithm changes the number of the connections based on the environment and connection availability. For instance, the Smart algorithm decreases the number of connections in clear view, commercial and residential regions. However, in indoor environments the desired accuracy cannot be achieved and the Smart algorithm needs to use all available connections.

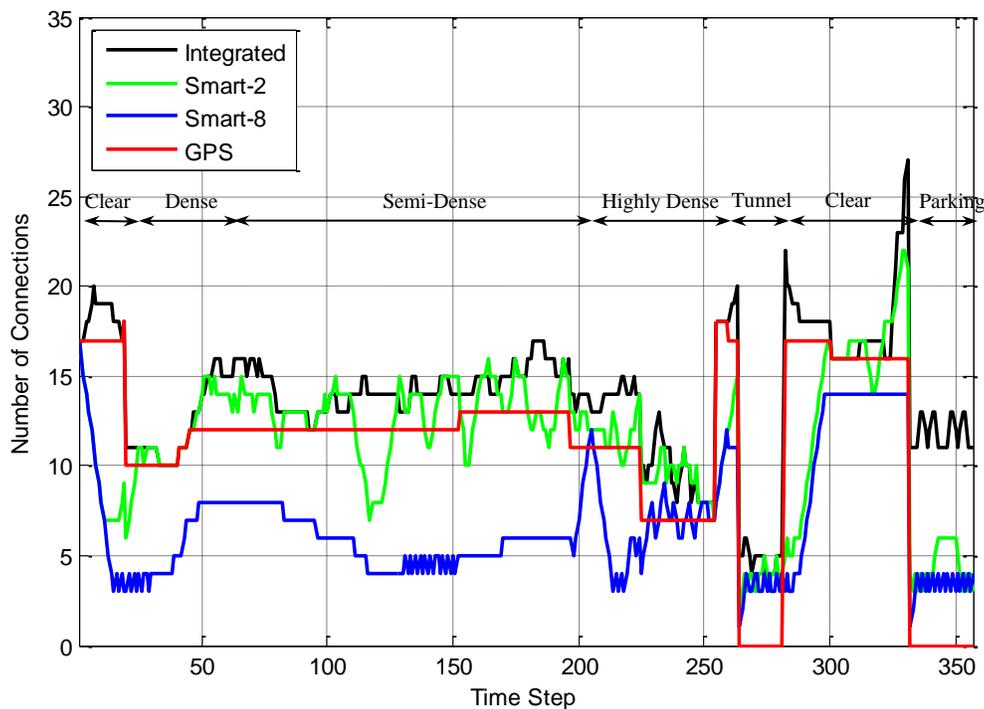


FIGURE 27 - Total number of connections at each time-step

## 6 Limitations

This chapter discusses any elements of this study that could be considered limitations in achieving a completely accurate analysis.

Computer simulations were used to evaluate the performance of the proposed systems. Computer simulations are widely used in many science and engineering topics. Simulations enable us to repeat several scenarios and optimize algorithms accordingly. Using computer simulations, many algorithms' problems can be solved easily before hardware implementation is performed. Therefore, it would be very useful to examine the algorithms via simulations because once an algorithm is implemented in hardware, it will be difficult to modify it. In other words, simulations can save time and cost in many aspects. However, it is sometimes difficult to consider every parameters and variables of an actual scenario in a simulated one which might impact the results obtained by simulations. Here, I provided a list of limitations I had during my simulations:

### 6.1 Vehicles

In the simulated network, I only considered 17 vehicles. However, adding more vehicles make the simulated network much closer to a real scenario. The problem is that for each vehicle, I needed to consider a path and generate its location for each time step. The travel paths of vehicles should be confined in the designed roads and cannot be generated randomly. Therefore, a lot of time was required for generating the locations of the vehicles inside the network. Since it was assumed that the locations of vehicles were unknown, their locations needed to be estimated during simulations. Although I have only shown the localization performance for the desired vehicle, the algorithms should be run for all other 16 vehicles to determine their locations. Therefore, in terms of performance evaluation, the process is very time consuming. This constraint might affect the performance of those algorithms that are using V2V technology because this is the only situation that other vehicles are aiding the localization of the desired vehicle. In this condition, having more measurements from more vehicles, the vehicle can be localized more accurately.

## 6.2 Noise Variance

For the simulation, it is assumed that the variances of the measurements are known. The variances of the measurements are used for determining the accuracy of each link. The variances are used in both Integrated and Smart algorithms. For the Integrated algorithm, different weights are assigned to different links in accordance with their variances. In other words, a link with a higher variance should be less effective in vehicle positioning than a link with lower variance. The reason is that, the measurement obtained from the former is less accurate than the measurement obtained from the latter. For the Smart algorithm, the variances are used for calculating the CRLB that determines which links are more useful than the others. In classic GPS, the algorithm used for positioning does not consider the weighting. The reason is that the variances of all satellite links are approximately the same. However, when other resources are used along with GPS, the variances of the measurements should be taken into account, as different resources have different measurement accuracies. The main limitation here is that the variances of the measurements cannot be determined accurately in all scenarios. Uncertainties in the variances of the measurements might affect the results obtained from the simulations. However, I anticipate that this will affect all algorithms and their relative performances remain the same.

## 6.3 NLOS Propagation

Another limitation of the simulation part of this study is the effect of Non-line-of-sight (NLOS) links. In my simulation, I assumed that all links are line-of-sight (LOS). However, this assumption is not valid in real scenarios. In clear view environments, we expect that most of links are LOS, as there are few obstructions between the units and vehicles. In dense and semi-dense environments, there are several NLOS links between the unit and vehicle, as the direct view between them is sometimes blocked by an object. NLOS measurements are significantly larger than their true ranges and have a great impact on the accuracy of localization. Simulating NLOS links is a very difficult task. A NLOS link is typically modeled as follows:

$$r_i = d_i + n_i + b_i \quad (82)$$

which is the same as LOS link except an unknown large value  $b_i$  is added to the measurement.  $b_i$  is called NLOS bias and makes the measurement significantly larger than its true value. The best way to simulate a NLOS link is to use ray tracing techniques. In this technique, the location of every object in the network should be available. If the direct view of the unit and vehicle is blocked by an object, the algorithm tries to find every other way that a signal can travel by bouncing to other objects and reaches to the receiver. It requires that all different combinations of paths between the unit and vehicle are determined. Therefore,  $b_i$  is calculated by the length of the shortest path between the unit and vehicle. Obtaining the NLOS biases is outside the scope of this study because the location and size and 3D coordinates of every element in the network should be considered. Even if this information was available, ray tracing technique would be very time consuming, as all different combinations of paths between the unit and vehicle should be determined. This process has to be repeated for each unit and each vehicle at each time step which takes large amount of memory and processing time. Another way of simulating NLOS biases is to generate them randomly, similar to what I did for the measurement noises. Uniform or exponential distributions are typically used to generate NLOS biases. In my simulations, I used different variances for measurements based on different environments. For instance, variances are larger in the commercial areas than the clear view areas. In this way, I accommodated a part of NLOS effects in the measurements. However, the performances of all algorithms might be affected by NLOS propagation in real scenario. As will be discussed later, studying the effect of NLOS propagation on the accuracy of localization is one of our future works.

## 6.4 Noise Distribution

Another limitation would be different distributions for noises. In my simulations, Gaussian distribution was selected for generating noises associated with the measurement errors. However, in real scenarios, the measurement noises might have different distributions. NLOS propagation and several other parameters such as antennas and electronic circuits used in devices might change the distribution of the measurement noises. Having different

noise distributions other than the one assumed in the simulation will affect the performance of localization. However, Gaussian is widely used in simulations as it represents most characteristics of the measurement noises.

## 7 Conclusions

The rapid growth of population and vehicle transportations demands more reliable and efficient transportation networks. Intelligent transport systems are developed to achieve this goal by exploiting different technologies and applications. In this research, the importance of vehicle positioning for ITS applications was studied. It was also shown that different applications of ITS require different location accuracy requirements. Some applications of ITS such as warning message dissemination require highly accurate location information, while some others such as environmental monitoring might not require any location information but can benefit substantially from it.

Most current vehicle positioning techniques rely on the Global Positioning System (GPS) which is the most accessible and easiest way of vehicle localization. It was shown that the GPS cannot meet the accuracy requirements of many ITS applications in all situations. GPS receivers do not operate accurately in indoor and dense environments. In the former, the GPS satellites attenuate sharply and cannot be received by the receiver. In the latter, several obstructions and objects create multipath propagation which hurt the performance of GPS receivers.

Therefore, we concluded that there are two open challenges which still need to be addressed in this field. First, to have a safer and more efficient transportation networks, the locations of the elements, especially vehicles, should be available everywhere in the network under different geographical conditions. Second, based on different applications, different localization accuracies are required. Some applications require highly accurate location information but can tolerate longer latency, while some others do not require very accurate localization but the information should be available instantly. Therefore, a system which can deliver location information from the vehicles at a specific accuracy in all situations is strictly necessary. To address these important problems, I proposed the following solutions:

- ❖ I developed an Integrated positioning framework to address the first problem. In this framework, the system is able to exploit several positioning techniques and does not only rely on GPS connections. In the proposed algorithm, besides GPS, other resources

including radio-frequency identification (RFID), vehicle-to-vehicle (V2V) communications, vehicle-to-infrastructure (V2I) communications, and dead-reckoning (DR) were used. A closed-to-real scenario was developed in MATLAB and the performance of the proposed Integrated algorithm was evaluated. Simulation results showed that a reliable transportation network should not rely on stand-alone GPS technology for vehicle localization, since in some situations, GPS is not able to provide the required localization accuracy. Different positioning techniques under different geographical environments were evaluated. It was shown that although in dense environments an aided system such as DR or V2V can help the vehicle to improve the location information obtained by a GPS receiver, in indoor environments any positioning systems which merely depend on GPS fail to deliver the required accuracy. For the specific scenario simulated here, GPS positioning failed to provide reasonable accuracy about 15% of the time, especially when the vehicle was in indoor environments or in highly dense areas. The proposed fully Integrated algorithm provided significantly better performance in indoor environments and more than 50% improvement in other regions. Among considered resources (V2I, V2V, RFID) incorporated with GPS, V2I was more helpful. Comparing GPS+V2V and GPS+RFID positioning, the former and the latter add more improvement to stand-alone GPS accuracy in clear and indoor environments, respectively.

- ❖ For the second problem, I proposed a Smart positioning algorithm. As shown in the simulation results, higher localization accuracy demands for higher complexity and processing time (which results in latency). Therefore, a Smart algorithm was proposed which regulates the number of resources used for localization in order to achieve the desired accuracy at a reasonable complexity. In other words, the proposed Smart algorithm considering the connections availability and geographical conditions tries to deliver the minimum positioning requirements of a given ITS application at the least amount of time. Simulation results showed that The Smart algorithm was able to achieve the desired accuracy at the least amount of complexity. More specifically, for the considered transportation network, the Smart algorithm provided about 90% higher accuracy than the stand-alone GPS with the same level of complexity and processing time.

## 8 Future Work

The possible future works are listed as follows

1. Incorporating other positioning techniques such as vision-based method, passive radar, and map matching in our simulations is a future work. This requires several modifications to the introduced Integrated algorithm. Although additional information from other resources might improve the positioning accuracy, it also increases the complexity of the system. Therefore, extra efforts are required to modify the proposed Smart algorithm to include these techniques.
2. In the proposed algorithms, it was assumed that time-of-arrival-based ranging method is used by all localization techniques. However, some current devices use different measurements method such as received signal strength (RSS) and angle-of-arrival (AOA). Developing the Integrated and Smart algorithm to incorporate these methods is a future work.
3. One of the most important issues in vehicle localization is non-line-of-sight propagation. When the direct view between the unit and vehicle is blocked by an object, a NLOS link occurs. Not including NLOS propagation was one of main limitations of this research because, in this research, it is assumed that all links are line-of-sight. Since it is not a valid assumption in real scenario, simulating NLOS links in the network and analyzing the sensitivity of the proposed algorithm to NLOS propagation would be a possible future work which can significantly validate the creditability of the algorithm in real scenarios.
4. In this work, the ML estimator is used to determine the unknown parameters. The ML estimator is optimal and its performance is superior to any unbiased estimators. However, solving the ML estimator is computationally complex, as the ML estimator has no closed-form solution. Using different estimators such as linear least squares estimators and convex estimators can be a possible future work. Linear estimators have a closed-form solution and have lower complexity than the ML estimator, however, their performance is not as good as the ML estimator. The convex estimators have lower complexity and in some situations perform as well as the ML

estimator. However, formulating the problem as a convex estimator is a difficult task. Using neural networks and machine learning could be another way to estimate the location of the vehicle. They are useful in situations when the measurements do not follow the Gaussian or any other specific distribution.

- Although computer simulations are widely used for evaluating the performance of the different algorithms in many engineering and science studies, these are the real world experiences that can actually validate the credibility of an algorithm, as they can reveal many unpredictable deficiencies, challenges, and problems. Therefore, examining the suggested Integrated framework and the proposed Smart algorithm in a test-bed is also a possible future work. Moreover, the proposed algorithm will be implemented in a “real time” environment for a rear-signaling warning application. Many types of rear-end crashes which result in higher-than-usual rates of fatalities and injuries can be prevented by using a proper rear-signaling warning technology. However, current systems mostly rely on GPS which is not reliable in all situations. The proposed algorithm will be useful to address this problem.

FIGURE 28 shows the flowchart for the steps accomplished and some future works. The blue blocks in FIGURE 28 depict the work already accomplished. The black blocks depict the proposed future research.

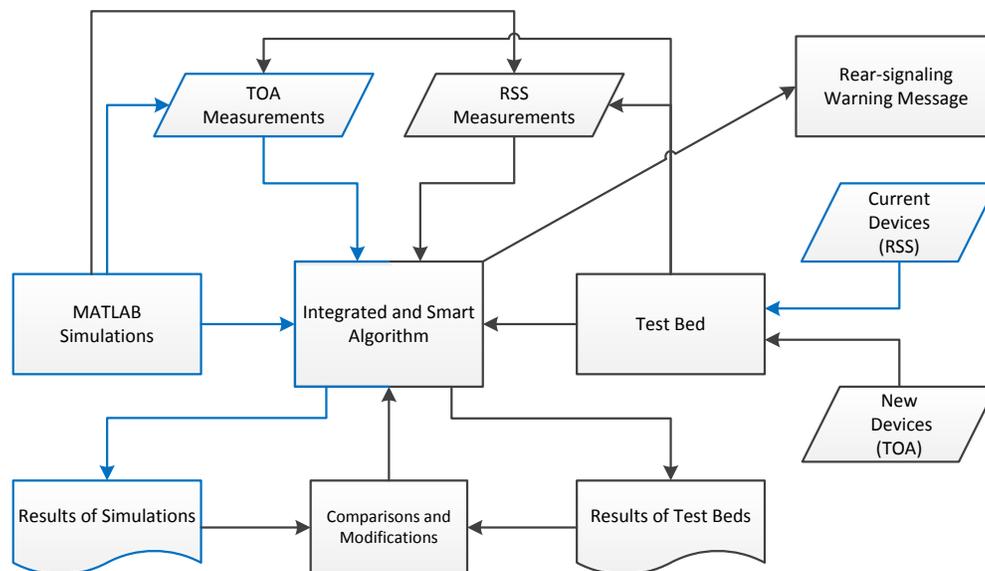


FIGURE 28 - The flowchart of some future works

6. Another area of research would be developing a technology adoption curve. This curve explains how the use of new technologies spreads through a society, and why they are adopted over old methods. For instance, by developing a V2V technology adoption curve we can determine and compare the time it is going to take to achieve certain performance expectations, like reduced number of accidents.

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# Appendices

The MATLAB functions which are used for determining the locations of the satellites are provided here (Borre 2003; Borre 2009).

## A.1 get\_eph

```
function Eph = get_eph(ephemeridesfile)
%GET_EPH The ephemerides contained in ephemeridesfile
% are reshaped into a matrix with 21 rows and
% as many columns as there are ephemerides.
% Typical call eph = get_eph('rinex_n.dat')
%Kai Borre 10-10-96
%Copyright (c) by Kai Borre
%$Revision 1.0 $ $Date: 1997/09/23 $

fide = fopen(ephemeridesfile);
[eph, count] = fread(fide,inf,'double');
noeph = count/21;
Eph = reshape(eph, 21, noeph);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

## A.2 satpos

```
function satp = satpos(t,eph);
%SATPOS Calculation of X,Y,Z coordinates at time t
% for given ephemeris eph
%Kai Borre 04-09-96
%Copyright (c) by Kai Borre
%$Revision: 1.1 $ $Date: 2004/02/09 $

GM = 3.986005e14; % earth's universal gravitational
% parameter m^3/s^2
Omegae_dot = 7.2921151467e-5; % earth rotation rate, rad/s

% Units are either seconds, meters, or radians
% Assigning the local variables to eph
svprn = eph(1);
af2 = eph(2);
M0 = eph(3);
roota = eph(4);
```

```

deltan = eph(5);
ecc = eph(6);
omega = eph(7);
cuc = eph(8);
cus = eph(9);
crc = eph(10);
crs = eph(11);
i0 = eph(12);
idot = eph(13);
cic = eph(14);
cis = eph(15);
Omega0 = eph(16);
Omegadot= eph(17);
toe = eph(18);
af0 = eph(19);
af1 = eph(20);
toc = eph(21);

```

#### % Procedure for coordinate calculation

```

A = roota*roota;
tk = check_t(t-toe);
n0 = sqrt(GM/A^3);
n = n0+deltan;
M = M0+n*tk;
M = rem(M+2*pi,2*pi);
E = M;
for i = 1:10
    E_old = E;
    E = M+ecc*sin(E);
    dE = rem(E-E_old,2*pi);
    if abs(dE) < 1.e-12
        break;
    end
end
E = rem(E+2*pi,2*pi);
v = atan2(sqrt(1-ecc^2)*sin(E), cos(E)-ecc);
phi = v+omega;
phi = rem(phi,2*pi);
u = phi + cuc*cos(2*phi)+cus*sin(2*phi);
r = A*(1-ecc*cos(E)) + crc*cos(2*phi)+crs*sin(2*phi);
i = i0+idot*tk + cic*cos(2*phi)+cis*sin(2*phi);
Omega = Omega0+(Omegadot-Omegae_dot)*tk-Omegae_dot*toe;
Omega = rem(Omega+2*pi,2*pi);
x1 = cos(u)*r;
y1 = sin(u)*r;
satp(1,1) = x1*cos(Omega)-y1*cos(i)*sin(Omega);

```

```
satp(2,1) = x1*sin(Omega)+y1*cos(i)*cos(Omega);  
satp(3,1) = y1*sin(i);  
%%%%%%%%%%%% end satpos.m %%%%%%%%%%
```