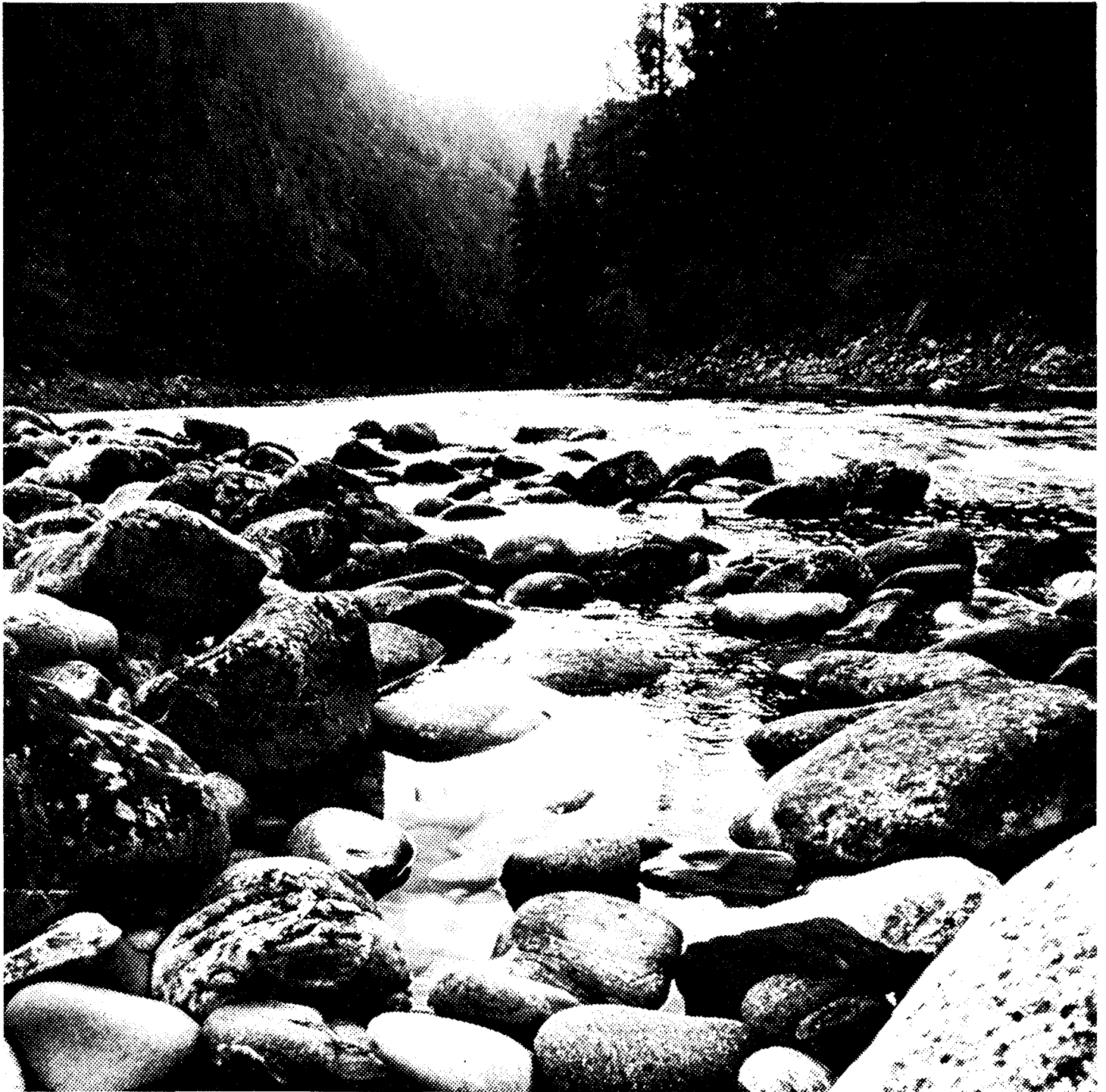# AIDS FOR UNIT PLANNING
ON THE APPALACHIAN NATIONAL FORESTS

AIDS FOR UNIT PLANNING ON THE

APPALACHIAN NATIONAL FORESTS

Harold E. Burkhart, William A. Leuschner,
R. Dean Stuck, John R. Porter, and Marion R. Reynolds

Preface

This report summarizes the results of studies conducted in
response to a cooperative agreement between the Southern Region,
U.S. Forest Service and the Department of Forestry and Forest
Products, Virginia Polytechnic Institute and State University.
The objective of the agreement was to improve National Forest manage-
ment planning techniques. The agreement covered the period July 1,
1973 to June 30, 1975. Literature citations are given for those
who desire additional detail.

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INTRODUCTION

The National Forests of the United States are managed so that they produce a variety of goods and services. In passing the multiple-use, sustained yield act, the U.S. Congress (1960) defined multiple-use management as: ". . . the management of all the various renewable surface resources . . . so that they are utilized in the combination that will best meet the needs of the American people; . . . and harmonious and coordinated management of the various resources, each with the other, without impairment of the productivity of the land, with consideration being given to the relative values of the various resources, and not necessarily the combinations of uses that will give the greatest dollar return or the greatest unit output."

However, as pointed out by Thompson and Richards (1970), there are some basic difficulties involved in putting the Congressional definition into practice. First, there is no direct way to evaluate the relative value of various product mixes because products obtained from National Forests, other than timber and forage, are not exchanged in a market. Also, in many (if not most) cases sufficient input-output data for specific products and data on the technological relationships between products are not available. For example, the results of wildlife habitat improvement in terms of man-days of recreation, or even in changes in animal population, are generally unknown. The changes in timber or water or forage output due to competing wildlife habitat practices are similarly unknown.

All National Forests have these problems in common but the National Forests in the Appalachians are, for two reasons, particularly suitable for study. First, because they are relatively close to the nation's population centers and get, therefore, heavy human use with its resulting pressures on the forest environment. Second, because

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the Appalachian National Forests have substituted Unit Plans for
the former Ranger District Multiple-Use Plan, (U.S.D.A., Forest
Service, 1971). The unit plan is "a total management plan for all
the social, economic, natural resource, and other environmental
situations found within a unit."

Unit planning is a new approach and the Appalachian National
Forests are using it with optimism. However, such a new and broad-
based approach is almost certain to bring about discovery of problem
areas where knowledge is limited or nonexistant and where analytical
capabilities not generally available in an administrative organization
are needed.

Several investigators (e.g. Halterman, 1972; Navon, 1971) have
suggested mathematical programming techniques, particularly linear
programming, as an aid in forest management planning. A major focus
of this study was to develop a model and demonstrate the use of
linear programming as an aid to unit planning. Since the validity
of the answers derived from any decision process, including linear
programming, is dependent upon the validity of the input data, this
study also dealt with the data collected for unit planning: both
with what data to collect and with data collection efficiency.

The High Knob Unit on the Clinch Ranger District of the Jefferson
National Forest was chosen for this study because it encompasses all
of the major activities that must be considered when developing a
Unit Plan. This area is particularly well suited for timber produc-
tion, mineral extraction, and recreation; consequently, there were
many potential conflicts requiring analysis.

A LINEAR PROGRAMMING MODEL
FOR UNIT PLANNING

Linear programming (LP) is a useful aid to unit planning because
it can allow rapid examination of many management alternatives, because
an LP formulation provides an "optimal" solution, and because LP
algorithms are generally available and well documented. The LP model
designed for this study yields a solution to the maximum timber volume
that can be harvested while still meeting all constraints and multiple-
use production objectives. It used existing data and information
systems. No new field data were needed and planning was restricted
to small acreages which could be identified by the District Ranger.

A linear program minimizes or maximizes a linear objective
function subject to linear constraints. The unit planning model has
four parts: the objective function; the constraints placed upon the

objective function in order to produce the desired multiple-use products; the constraints placed upon the entire system by budget restrictions; and cutting and technical constraints. The following discussion emphasizes the derivation of variables used in the LP model, which are usually supplied by the planning team, and the results of the LP solution. More detailed explanations of the model and variables can be found in Leuschner, et al. (1975) and Porter (1974).

Objective Function

The objective function to be maximized in this application of LP was the sum of the acres cut during the planning period multiplied by the average yield per acre cut (Table 1, eq. 1). The maximized variable, T, comes from the LP solution and will be discussed later.

The LP model requires that the planning unit be divided into grid cells for which basic inventory information is available. For example, the High Knob Unit's grid cells were 21.6 acres and a version of the Harvard Grid Program (Sinton and Steinitz, 1971) was used to compile maps from the inventory data. Contiguous grid cells were aggregated into homogeneous tracts having the same Ecological Management Unit, Forest Type, Stand Condition Class, and Land Class codes (Porter, 1974, pp. 10-15). The LP solution indicates the number of acres to cut (X) in each individual aggregate (the jth aggregate).

Average timber yield per acre cut in the jth aggregate (C_j) must be estimated by the planning team. Our model used separate estimates for conventional clearcuts, cable logging clearcuts, and partial cuts for commercial thinning. Yield values were determined by the timber specialist's judgment with specific consideration of the stand condition classes and site indices in each aggregate (Porter, 1974, pp. 15-18). There can be more than one C_j for an aggregate and the C_j values are the estimated average timber yield per acre cut over the 10 year planning period.

Multiple-Use Production Constraints

The production constraints are formulated so that for each multiple-use product the existing capacity, minus any losses that might occur from cutting and plus the gains from management activities, must be the same or larger than the ten year multiple-use production objective (Table 1, eq. 2). The planning team must, therefore, estimate the existing capacity (H_i), the loss of the multiple-use

Table 1. Mathematical formulation of the general linear programming model for the High Knob Unit, Jefferson National Forest.

$$\text{Maximize } T = \sum_{j=1}^{n} C_j X_j \tag{1}$$

Subject to:

$$H_i - \left(\sum_{j=1}^{n} a_{ij} X_j\right) + \left(\sum_{j=1}^{n} \sum_{k=1}^{p} b_{ijk} Y_{ijk}\right) \geq R_i \tag{2}$$

$$\sum_{j=1}^{n} \sum_{k=1}^{p} d_{qjk} X_j + \sum_{j=1}^{n} \sum_{k=1}^{p} e_{qjk} Y_{ijk} \leq E_q \tag{3}$$

$$\sum_{j=1}^{n} X_j \geq F \tag{4}$$

$$X_j \leq G_j \tag{5}$$

where:

T = the total timber volume cut for the ten year period.

C_j = the estimated average yield per unit of area cut in the jth cell.

X_j = the area cut in the jth cell--the choice variable.

n = the number of cells or geographical subunits.

H_i = the existing capacity on the unit to provide the ith multiple-use product.

a_{ij} = the loss per unit of area cut of the ith multiple-use product in the jth cell.

b_{ijk} = the average contribution per unit of the kth management activity to the ith multiple-use product in the jth cell.

Y_{ijk} = the number of units of the kth management activity contributing to the ith multiple-use product in the jth cell.

R_i = the production objective for the ith multiple-use product.

n = the number of cells.

p = the number of management activities.

d_{qjk} = the cost per unit of area cut for the kth management activity in the jth cell chargeable to the qth budget.

e_{qjk} = the cost per unit of management activity of the kth management activity in the jth cell chargeable to the qth budget spent on the ith non-timber multiple-use product.

E_q = the budget constraint for the qth budget for the planning period.

F = the area clearcutting objective for the unit during the planning period.

G_j = the total number of acres which may be cut in the jth cell.

activity per acre cut (a_{ij}), the gain per unit of management activity (b_{ijk}), choose the units in which the management activity is measured (Y_{ijk}), and set the multiple-use production objective (R_i). The method of calculating these variables depends on whether the production objectives are hunting or non-hunting (Table 2). The number of acres to cut in each aggregate (X_j) and the number of units of each management activity to implement in each aggregate (Y_{ijk}) are provided in the LP solution.

Non-Hunting Activities. All non-hunting activity variables were expressed in visitor days; however, any convenient unit of measurement can be used (Porter, 1974, pp. 22-32). The existing capacity (H_i) was estimated by:[2]/

$$H_i = \text{visitor days/turnover X turnovers/day X days used/season X 10} \quad (6)$$

where:

$$\text{visitor days/turnover} = \text{people at one time X average length of stay(hrs)/} \atop \text{12 hours} \quad (7)$$

These formulae are less complicated than they appear. For example, the visitor days per turnover (eq. 7) is simply the average length of stay in terms of a 12-hour visitor day multiplied by the capacity of the recreation facility. Turnovers/day is an estimate of the number of different parties which can use a facility during one day and can be any number greater than zero. The days used per season and the constant "10" simply convert the day-use to annual and then to 10-year estimates. Estimates for all terms in equations 6 and 7 can be based on field studies, expert opinion, or some combination of the two. Obviously, the better the estimates the better the final results.

The loss per acre cut (a_{ij}) for picnicking and camping was calculated by first estimating the total number of acres in picnicking or camping sites and then dividing the total visitor days use in a base year (1972) by this acreage. This provides an estimate of the visitor days use per acre per year which is then multiplied by 10 if one assumes the loss will occur over all 10 years or by 5 if one assumes a mean. The loss per acre cut for a specific aggregate is obtained by multiplying the number of picnicking or camping acres in that aggregate by the preceding figure. These constants are assigned only to aggregates with existing or potential picnic and camp sites. All others have a coefficient value of zero.

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[2]/The notation in Leuschner, et al. (1975) differs from that in Porter, (1974). For example, Porter uses A instead of H_i. This paper uses the notation in Leuschner, et al. (1975).

Table 2. Activities for which multiple-use production objectives
          were set.

Non-Hunting Activities   Hunting Activities

Camping   Turkey hunting
  Developed   Squirrel hunting
  Undeveloped   Grouse hunting
Picnicking   Deer hunting
Viewing outstanding scenery
Pleasure Driving
  Auto
  Motorcycle and Scooter
General pleasure boating
Swimming
Horseback riding
Fishing
  Warm water
  Cold water
Unguided tours
Bicycling
Hiking

The losses per acre cut for hiking and horseback riding were obtained similarly. The capacity per acre was obtained by dividing the estimated capacity, H_i, by the total acres in the planning unit where hiking and horseback riding could occur. The Forest Service recreation specialist then estimated the proportionate change in use per acre which would occur if clearcutting took place. This estimate was made for each aggregate, multiplied by the capacity per acre, and used as the estimate of a_{ij}.

Jefferson National Forest personnel maintained that any clear-cutting would be so executed that there would be no loss in capacity for viewing scenery, auto driving, pleasure boating, swimming, warm and cold water fishing, unguided tours, and bicycling. There-fore, the loss per acre cut for these activities was zero although the model allows non-zero values.

The possible management activities (Y_{ijk}) must first be identi-fied. For example, construction of a picnic site in the jth aggre-gate, construction of hiking trails in several aggregates, or man-hours of a specialist's time spent on the whole unit. Each manage-ment activity's contribution to capacity (b_{ijk}) must then be esti-mated. This may not be too difficult because existing construction plans may contain capacity estimates or there may be capacity guide-lines in various handbooks. The planning team can include any activities they deem feasible and locations of the management activities may or may not be tied to specific aggregates or groups of aggregates.

Multiple-use production objectives (R_i) can be estimated how-ever the planning team and Forest Supervisor deem appropriate. Initial estimates for the High Knob unit were made judgmentally by adding anticipated percentage increases to 1972 RIM use estimates and by incorporating public listening session results.

Hunting Activities. The acres of habitat for each game species were estimated from the featured species map. Turkey-squirrel and deer-grouse were assumed to occupy the same habitat. Slightly different capacity estimates (H_i) were made for each species; each will be discussed separately (Porter, 1974, pp. 32-40). All hunt-ing was measured in hunter hours.

The U.S.FS. Wildlife Handbook states there must be at least 10 turkeys per square mile for the species to be featured. The Jefferson National Forest wildlife specialist estimated that four and a half percent of the population was harvested and that 48 hunter hours were expended per bird bagged, counting both successful and unsuccessful hunters. The Unit capacity was then estimated by:

H_i = Sq. miles of habitat X no. birds/sq. miles X proportion of
population harvested X hunter hours/birds bagged X 10   (8)
seasons

The U.S.F.S. Wildlife Handbook also states that 100 acres of
squirrel habitat can support seven hunting trips per season. The
wildlife specialist then estimated that a squirrel hunting trip
took about 4 hours and capacity was estimated by:

H_i = 100's of acres of habitat X 7 trips/100 acres X 4 hours/trip
X 10 seasons   (9)

The wildlife specialist also estimated that there was one hunter
day a season for each 15 acres of grouse habitat, that there were
four hours in a hunter day, and that there were 88 days in a season.
The grouse capacity was then estimated by:

H_i = Acres of habitat X 1 hunter day/15 acres X 4 hours/hunter day
X 88 days/season X 10 seasons   (10)

Deer capacity was derived by using the U.S.F.S. Wildlife
Handbook estimate that 10,000 acres of deer habitat could support
660 mandays of hunting. This converted to 3,960 hunter hours (at
6 hrs/day) and a simple proportion was taken where:

H_i = 3,960 hunter hours/10,000 acres of habitat X Acres of deer
habitat   (11)

The losses per acre of timber cut (a_{ij}) were estimated the same
way for all species. First, the capacity was divided by the acres
of habitat to obtain the hunter hours per acre. Next, the wildlife
specialist estimated the change in hunting due to clearcutting and
partial cutting. This change was negative for squirrel, positive
for the other species. The hunter hours per acre were then multi-
plied by the proportion change to estimate a_{ij}. In formula form:

a_{ij} = H_i/acres of habitat X proportion change due to cut   (12)

An increase per unit of management activity (b_{ijk}) was esti-
mated only for squirrel hunting because cutting had a positive
effect on all other species and was considered the most effective

and least costly management activity. The wildlife specialist again used his judgment and estimated that squirrel hunting would increase an average of five hours for the planning period for every manhour spent marking den trees. Hunting production goals (R_i) were set by increasing the capacity estimates five percent.

Budget Constraints

The budget constraints were formulated so that the sum of the dollars spent per unit of management activity multiplied by the number of units of that activity must be the same or less than the amount budgeted (Table 1, eq. 3). The planning team estimated the cost per acre harvested (d_{qjk}) and the cost per unit of other management activities (e_{qjk}). As before, the number of acres cut (X_j) and the number of units of each management activity (Y_{ijk}) were given in the LP solution. The team also set a maximum value for the various budgets (E_q) (Porter, 1974, pp. 39, 41-46).

Timber sales and administration, and reforestation were the two timber budgets (d_{qjk}) used. Sales and administration costs per acre were estimated using past costs and considering the type of cut and the distance of the aggregate from the Ranger Station. Reforestation costs were based on the same considerations except, obviously, they were zero for partial cuts.

The other management activities (e_{qjk}) were construction of scenic overviews, picnic areas, and additional miles of horseback and hiking trails. The average cost per mile of each type of trail was estimated from past records and the remaining construction activities had unique cost estimates based on their plans. Operating costs could have been incorporated easily; for example, maintenance cost per visitor day for picnic areas and campgrounds and annual maintenance cost per mile of trail.

The maximum dollars available in a budget during the planning period (E_q) were usually estimated with the supervisor and based on past budgets and the supervisor's estimate of future prospects. Varying this estimate can demonstrate how multiple-use production changes with budget cuts or increases.

Cutting and Technical Constraints

The national forests have cutting objectives which are usually stated in number of acres to clearcut per year. These objectives are allowed for in the model by stating that the sum of all acres

cut in all the aggregates must be equal to or greater than the objective (Table 1, eq. 4).$\underline{3/}$ The cutting objective (F) was set by multiplying the annual cutting objectives by the number of years in the planning period. The number of acres cut (X_j) comes from the program solution. Other cutting objectives could be set; for example, acres of cable logging or partial cutting.

It is also necessary to constrain the LP not to designate more acres for cutting in an aggregate than are physically possible to cut (Table 1, eq. 5). An aggregate is seldom completely harvestable because of lack of timber, timber age class, special reservations, or other restrictions. Therefore, the planning team must calculate the number of acres in each aggregate which can be harvested during the planning period (G). Estimates are needed for each harvesting system used in an aggregate. For example, four equations are needed for an aggregate having three possible harvesting systems, one for each system and one for the total acres.

Special Features

There were several special features used on the High Knob LP application. These were a decision guideline for building a lake (Porter, 1974, pp. 30,32), an erosion control technique (Porter, 1974, p. 19), and a detailed explanation of the cell aggregation procedure (Porter, 1974, pp. 10-15). These features are not discussed because of the need to maintain generality and to save space. Interested parties are urged to read Porter.

Solution Results

The LP solution specifies the number of acres to cut in each aggregate by each harvesting system in order to produce the most timber while still meeting the multiple-use production objectives. However, the District Ranger must locate the actual stand in the aggregate, decide when during the ten-year period the timber will be harvested, and designate it for cutting. The solution also indicates the different management activities to be performed; for example, the construction of 6.25 miles of trails and two picnic areas. These activities can be location-specific if appropriately entered into the model.

$\underline{3/}$We used an inequality because the acreage that could be clear-cut was less than F but a strict equality could be used. We lowered F below the original objective, used an "equal to or greater than" sign, and let the LP solve for the maximum acres which could be cut while providing the other multiple-use products.

The solution also indicates the amount of each multiple-use product which will be supplied during the planning period, for example, million board feet of timber and thousand visitor days of horseback riding. The amount of the budget spent is also shown. Additional analytical statistics, such as how much production of one item must be sacrificed in order to produce an additional unit of another, are also available.

Discussion

The usefulness of the preceding results for management guidance are obvious. In addition, the planning team can make sensitivity analyses by varying the multiple-use production and budget constraints and examining additional LP solutions. This will allow them to better understand how much production of one product must be traded-off to increase the production of another and also the effects of absolute resource constraints, such as limited budgets. The ready availability of LP algorithms and the relatively low cost of computer runs should allow teams to examine many alternatives.

Variable estimation was emphasized because this is likely to be the first job undertaken by the planning team. However, it will probably be necessary for one or more planning team members to be trained in the execution of linear programming algorithms and the interpretation of LP results if the analytic methods discussed are to be used effectively.

The variable estimation techniques described have some obvious weaknesses and are not necessarily the only or the best way to make estimates. They are methods that V.P.I. & S.U. and Jefferson National Forest personnel devised to fit the data and time available. We present them as suggestions and heartily encourage future practitioners to improve upon them and to develop new methods. The methodological weaknesses also indicate areas needing further study. For example, the hunting variables were highly dependent upon expert opinion. It may be desirable to supplement these opinions with studies indicating the number of hours per bird bagged or per hunting trip.

The most important study results are not the methods used to estimate the variables but rather the analytical framework provided by the LP model. Estimation of variables will change as inventory procedures change over geographical area and time. However, the model provides one way of examining the complex interactions of forest management.

DATA INPUT FOR UNIT PLANNING

Functional inventories currently being conducted were described in the inventory phase of the study. Next, the minimum number of variables needed for the information now being collected was analyzed. A framework for optimal allocation of inventory resources for multiple-use planning was developed and recommendations for modifications in data handling and planning procedures on the Jefferson National Forest were made. For a more detailed description of this phase of the study see Stuck (1976).

Functional Inventories

Currently, each functional staff in the Forest Supervisor's office is responsible for gathering certain input information for unit planning. A brief description of the data gathered by each is given below.

Engineering. The engineering staff conducts no field inventory but is a support unit for other functional areas. Their job includes surveys, supervision of construction of roads, trails and other improvements, and maintenance operations.

Fire and Law Enforcement. This functional staff is responsible for guiding enforcement of the laws and regulations on the forest and protecting forest property. No field inventories are conducted, but information is kept on roads, location of fire fighting equipment, and other related matters.

Lands. The lands staff is responsible for settling land claims, acquiring rights-of-way and administering land-use permits; no regular field inventories are conducted.

Landscape Architecture. This staff establishes distance zones with respect to distances from prominent viewing points on the planning unit and describes sensitivity levels. The staff is a support unit for other functional areas, especially for timber harvest layouts and recreational developments.

Recreation. Information on recreation use is available through the Recreation Information Management (RIM) system. RIM data are used for recreation projections. Also a variety of inventories are kept for future recreational development.

Soils. The soils specialist conducts a "phase I" survey to determine Ecological Management Units (EMU) on a planning unit. A "phase II" inventory is conducted when a more intensive soil description is needed such as when constructing a recreational area.

Timber. The timber staff inventories the timber resources of
the forest for the purpose of developing a stand prescription.
An inventory of the area is made to estimate the merchantable wood
volume when timber is to be sold.

Wildlife. The wildlife staff conducts a wildlife habitat survey
on different survey units within a planning unit. Present wildlife
population levels and trends, timber overstory, wildlife food, key
wildlife areas, access, limiting factors for various wildlife
species, soils, and other data are obtained on each survey unit.

Analysis of Inventory
Data Currently Being Collected

A correlation matrix (which shows the degree of linear associa-
tion between variables) was computed to determine if there are
redundancies in current data collection. If the correlation is
strong between two variables, perhaps one can be eliminated from
observation since both are providing essentially the same information.
Using the grid-cell coded values for 11 variables that were suitable
for correlation analysis, a correlation matrix (Table 3) was formed.
Simple linear correlation coefficients can range in value from −1
to +1 with values near −1 or +1 indicating close linear association,
and values near zero indicating no linear relationship between the
variables. The values in Table 3 indicate that none of the 11
variables are closely associated, and, assuming that all are impor-
tant for Unit Planning, none can be eliminated. Additional analyses
using multiple linear regression techniques and factor analysis were
performed and the same conclusions were reached.

A Framework for Optimal
Allocation of Inventory Resources
for Multiple-Use Planning

A typical inventory is designed to be efficient from a sampling
standpoint, the data are collected and presented to the manager to
use for decision making. A more logical approach is possible when
practical decisions are to be made with the aid of sample observa-
tions. First, the manager should list the decisions that must be
made. Next a list of information that will aid in making these
decisions should be compiled. Some of the input information will,
in most cases, be obtained by sampling. The decisions will pre-
sumably be more soundly based if the sample estimates have low
errors. Thus we may be able to calculate, in monetary terms,
the loss that will be incurred in a decision through an error in
the estimate. The frequency distribution of the errors for a
specified sampling method will depend on the sample size. As the
sample size increases, the expected loss will decrease, but the cost
of sampling will increase. A reasonable criterion for choosing the
sample size is to minimize the sum of the sampling cost plus the
expected loss. This cost-plus-loss minimization is recommended as

Table 3. Correlation matrix of the inventoried data on the High Knob Unit, Jefferson National Forest.

	Land Forms	Soil Source	Soil Texture	Water Regime	Soil Modifiers	Slope
Land Forms	1.0000	0.1111	0.0310	0.2966	-0.0877	0.0698
Soil Source		1.0000	-0.3975	0.2155	0.0451	0.2123
Soil Texture			1.0000	-0.0609	-0.0597	-0.1663
Water Regime				1.0000	0.2665	0.0264
Soil Modifiers					1.0000	0.1035
Slope						1.0000
Geology						
Site Index						
Stand Condition						
Forest Type						
Wildlife Species						

	Geology	Site Index	Stand Condition	Forest Type	Wildlife Species
Land Forms	0.1614	-0.0368	0.0327	-0.0192	-0.0397
Soil Source	0.1253	0.1025	-0.0867	0.0347	-0.1854
Soil Texture	-0.0820	-0.0140	0.0022	0.0812	0.2067
Water Regime	0.1189	0.0287	-0.0149	-0.0189	0.0747
Soil Modifiers	-0.1247	0.0209	0.0013	-0.0447	-0.0059
Slope	-0.0302	0.1912	-0.1333	0.0801	-0.0089
Geology	1.0000	-0.0453	0.1130	0.0536	-0.3272
Site Index		1.0000	-0.1192	0.0714	0.1010
Stand Condition			1.0000	-0.0623	-0.1105
Forest Type				1.0000	0.1351
Wildlife Species					1.0000

a framework for the allocation of sampling resources when data are collected for Unit Planning.

As an illustration of the application of the cost-plus-loss minimization procedure for allocating sampling resources, consider the following simplified example. The manager has a single decision to make: when to cut a given timber stand. The harvesting decision for this approximately even-aged stand will be based on the average stand age which is estimated on the basis of a sample of tree measurements. The decision of when to cut will be more soundly based if the sample estimate of age has low error. In order to calculate, in monetary terms, the loss that will be incurred in a decision through an error in the estimate, the frequency distribution of the errors must be specified. The loss that will be incurred in a decision through an error of amount z in the estimate is denoted by $l(z)$. Although the actual value of z is not predictable in advance, we can find the frequency distribution $f(z, n)$ of z which for a specified sampling method will depend on the sample size n. The expected loss for a given size of sample is

$$L(n) = \int l(z) f(z,n) dz.$$

The purpose of taking the sample is to diminish this loss.

As one increases the sample size to decrease the expected loss, the cost of sampling increases. A logical choice for sample size is one which minimizes cost-plus-expected-loss. If $C(n)$ is the cost of a sample of size n, then one would minimize

$$C(n) + L(n)$$

since this is the total expected cost involved in taking the sample and in making decisions from its results.

A simple application occurs when the loss function, $l(z)$, is λz^2, where λ is a constant. If z is defined as

$$\hat{Y} - Y$$

where for this example, $\hat{Y}$ is the sample estimate of average stand age and Y is the true average stand age, then

$$L(n) = \int \lambda z^2 f(z,n) dz.$$

The expected loss may also be written as

$$L(n) = \lambda E(z^2)$$

$$= \lambda E(\hat{Y} - Y)^2$$

$$= \lambda V(\hat{Y})$$

where $V(\hat{Y})$ is the variance of the estimate $\hat{Y}$.

For simple random sampling (without replacement)

$$V(\hat{Y}) = S^2/n - S^2/N$$

thus

$$L(n) = \lambda S^2/n - \lambda S^2/N.$$

Suppose the cost function for the sample can be written

$$C(n) = C_0 + C_1 n$$

where C_0 is the overhead cost and C_1 is the cost per sample. The cost-plus-loss function which is to be minimized is

$$C_0 + C_1 n + \lambda S^2/n - \lambda S^2/N.$$

Using methods of differential calculus, the sample size n which will minimize the sampling cost plus the expected loss is

$$n = \sqrt{\lambda S^2 / C_1}.$$

Suppose that data on timber growth rates, harvesting costs, and other factors enable us to determine a value for λ in the loss function. This value reflects the loss that will be incurred due to harvesting timber that is not mature or that is overmature. Further suppose that λ was found to equal 10 (the units of λ are dollars/years^2). From prior sampling experience in stands of the type being considered, we have an estimate of the variance (S^2) among age determinations which is equal to 100 (the units of S^2 are years^2). Prior experience also provides information on the cost of sampling and C_1 is found to be $10 per sample observation. Thus the sample size which will minimize the cost of taking the sample and in making decisions from its results is

$$n = \sqrt{\lambda S^2 / C_1} = \sqrt{10(100)/10} = 10.$$

Figure 1 shows graphically the relationship of total cost, sampling cost, and expected loss to sample size. To construct the curves, overhead cost (C_0) was arbitrarily set equal to $100 and the population size (N) was assumed to be very large (thus the term $\lambda S^2/N$ is close to zero). It can be seen, by visual inspection of Figure 1, that the minimum total expected cost is in fact achieved when sample size is equal to 10.

Applying the Cost-Plus-Loss Minimization Procedure

The cost-plus-loss minimization procedure was applied to data from the High Knob Planning Unit of the Jefferson National Forest. Major steps involved in the application were: (1) to list the decisions necessary for unit planning, (2) to determine the input information needed to make these decisions, and (3) to compute the sampling intensities that will minimize the total cost of obtaining data plus expected cost involved in using the data to make decisions.

Resource management decisions for unit plans were grouped into three production categories — timber, wildlife, and recreation. All remaining functional areas in the Supervisor's Office act as support for these three production areas. Inventories from both production and support function groups are, however, necessary to obtain the required information to make unit planning decisions. Many decisions are not based on field inventories; some are policy decisions while others are made from existing information sources. Listed below are examples of decisions that were required for the High Knob Planning Unit and the inventory information that is needed to more soundly base the decision.

1. What timber stands within the unit have timber products that could be scheduled for harvest?

    a. stand age
    b. forest type
    c. stand condition class
    d. site index
    e. operability

2. Are there visual restrictions that dictate the type of timber harvest?

    a. distance zones, sensitivity levels

3. Are there soil restrictions that dictate the type of timber harvest?
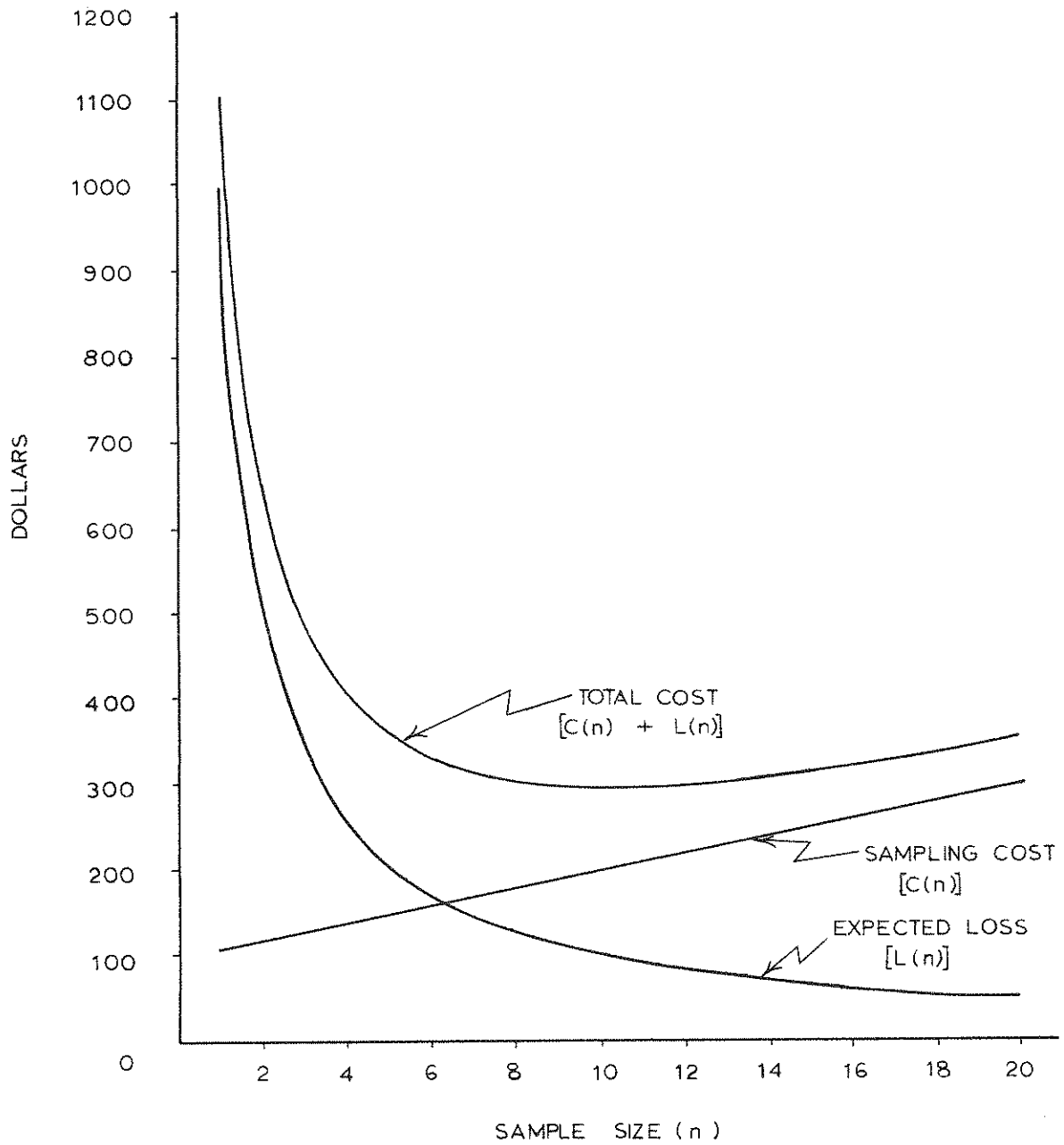
    a. soil modifiers

Figure 1.   Relationship of total cost, sampling cost, and
expected loss to sample size.

4. What wildlife species are to be featured on the unit?

    a. stand age
    b. forest type

5. Which streams are to be stocked with trout?

    a. trout reproduction

6. Are there areas where public use should be restricted?

    a. soil modifiers

The data needed to make the above decisions are obtained from inventories by four functional staffs. As part of the timber inventory, stand age, stand condition class, forest type, site index, and operability are determined. The wildlife inventory identifies trout reproduction. Soil modifiers are inventoried by the soils specialist. The landscape architect inventories the distance zones and sensitivity levels.

After listing the decisions and the input information needed for each decision, optimal sample sizes (subject to simplifying assumptions) were computed. The additional data required to compute a minimum cost-plus-loss were: (1) estimates of the variance of variables to be observed, (2) cost of sampling, (3) λ values for the loss functions, (4) weights to be applied to the variables for each decision, and (5) number of times each decision must be made when compiling the unit plan. All of the above-listed information was obtained by consulting the staff of the Jefferson National Forest. Some values were derived from information on file, while other values were subjective estimates.

Although there might be adequate data on the cost per sample unit for gathering various types of information, it is very difficult to develop loss functions. However, with the best approximations available, use of the cost-plus-loss procedure should provide an objective guide to the allocation of sampling resources. Stuck (1976) discusses, in more detail, the cost-plus-loss concept and its application to unit planning.

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