Development and Analysis of the Lumped Parameter Model of a PiezoHydraulic Actuator

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To my father, Maurice K. Nasser, and my mother, Lucienne Nasser

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Abstract

Hybrid actuation is an expanding field in which several systems, such as a mechanical, electrical, hydraulic, pneumatic, and/or thermal, among others, are integrated in order to combine certain aspects of each system, and achieve a better and more efficient performance under certain operating conditions.

The concept of piezohydraulic actuation takes advantage of the high force capabilities that piezoceramics have and combines it with the operation at high frequencies, in order to achieve the hydraulic actuation of a system under a specified stroke and force. High frequency rectification translates the low stroke of a piezoelectric stack into a desired amount of stroke per unit time. Thus, the low displacement, oscillatory motion of the piezoelectric device (coupled with a high frequency operation) is translated into a unidirectional motion of a hydraulic cylinder.

As part of this research, a benchtop piezohydraulic unit has been developed and the concept of piezohydraulic actuation has been demonstrated. The effective bidirectional displacement of a hydraulic cylinder through the actuation of a piezoelectric stack has been achieved. A lumped parameter model is developed in order to simulate the dynamics of the hydraulic system and of the entire piezohydraulic unit. The model did approximate the response of the piezohydraulic unit under a one-sided operation. Time response analysis is performed through the frequency spectrum comparison of the measured and the simulated data. Then a two-stage cycle simulation is used to model the pumping operation of the unit. Discrepancies were obtained between the model and the actual system for the single-ended piezohydraulic unit, nonetheless, a good approximation has been achieved for the pumping operation of the double-ended unit under certain conditions.

Furthermore, several factors have been identified that may limit the operation of the piezohydraulic unit. First, the need of high displacement piezoelectric actuators often comes with the requirement of high voltage operation along with high current consumptions. Thus, the amplifier becomes the first limitation to overcome. Second, is the response of the controlled valves. The highest valve operating frequency and their time response will set the limit on the piezohydraulic unit. And finally, once these limitations are overcome, the unit is eventually limited by the dynamics of the fluid and the hydraulic system itself. Attenuation in the frequency response, or the operation near resonance and the possibility of cavitation, are some of the aspects that eventually will limit the operation of the piezohydraulic unit.

A custom made, high displacement stack is used along with a custom made switching amplifier. The current system is being limited by the second factor, the solenoid valves. Nonethelss the analysis performed has addressed the relevant issues required for the design and use of another set of controlled valves. Finally, the eventual limitation from the hydraulic system has been determined through the analysis of the fluid dynamics of the system. The analysis does not account for potential cavitation, and future operation at high frequencies should take it into account.

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Chapter 1

Introduction

A bench-top, test setup unit of a piezohydraulic actuator has been designed and constructed, with the effort and contributions from Julio Lodetti, Nikola Vujic, Antoine Latapie and Esteve Simon. The following chapters are related to the development and analysis of the lumped parameter model used to simulate the experimental results and to predict the operation of the piezohydraulic unit under different conditions.

1.1 Motivation

Hydraulics have long been used due to their reliability and the wide range of forces and stroke actuation that can be achieved. Control surfaces of airplanes and robot arms are examples of systems the use hydraulics for actuation. But the implementation of hydraulics requires the use of pumps, pressure lines, return lines, reservoirs, and the actuator cylinders (as shown in Figure 1.1). Moreover, a critical issue is that pipes must connect the location of the input to the output device (cylinder). This implies more hardware, which in turn translates into more weight and maintenance, key aspects for many applications. As the servo-motor technology has become more precise, reliable, and of reduced weight, many of the hydraulic operated systems that required forces and strokes achievable by their electrical counterparts, are being replaced.

The piezohydraulic concept combines the use of piezoelectric stacks and their capabilities with those corresponding to hydraulic systems. It is an attempt to fill the gap existent between the force, displacement and power characteristics of hydraulic devices, and those corresponding to the servomotor technology (note that for many applications the



Figure 1.1: Doebelin (1972).

comparison has to be based on a an equivalent size and weight).

Piezoelectric stacks are composed of piezoceramic wafers that are stacked physically in series but are connected electrically in parallel. Piezoceramic wafers are elements that can be stressed electrically. When a voltage is applied, their dimensions change and a resulting force is exerted (by the piezoceramic). In the same manner, if the piezoceramic is stressed by an external force, then it generates a charge, and a voltage that is associated with it. Thus, a piezoceramic can be used as a sensor or as an actuator, or both. A general introduction to piezoceramic elements can be found in the catalogs of piezoceramic manufacturers such as Piezo Systems (1998) and Physik Instrumente (1999). A more detailed analysis of piezoelectric materials and the fundamentals of piezoelectricity is performed by Ikeda (1990). Furthermore, for standards, and the general constitutive equations for a piezoceramic material, refer to the IEEE standard on piezoelectricity (1987).

The development, study and analysis of piezoceramics is a fast growing field, as well as the areas to which they are applied. A piezoceramic couples electrical and mechanical systems as a sensor, an actuator, or even both. As an actuator, advantages include the possible operation at high frequencies (up to the kHz region), the output of high pushing forces (several kN), high stiffnesses (in the order of kN/mm and higher), no wear and tear, fast responses (sub-millisecond) and an accuracy in the micrometer and even in the nanometer scale. On the other hand, stacks of piezoceramics may exert high pushing forces but they exhibit low pulling force capability. This is due to the eventual separation of the piezoceramic layers in a stack. Thus, piezoelectric stacks under dynamic operation are usually protected with a mechanical preload that compensates for eventual pulling forces and prevents the ceramic stack from being damaged. Another disadvantage experienced in many applications, is the fact that generally piezoceramics by themselves exhibit very low displacements, with strains in the order of 0.001 units (0.10 percent). Thus, usually piezoceramics are integrated with other elements or designed in such a way that their displacement is amplified while keeping the force within the desired limits. Figure 1.2 shows some commercially available piezoelectric (PZT) stacks obtained from the catalogs of their respective manufacturers.



Figure 1.2: PZT Stacks: a) Piezo Systems (1998), b)Physik Instrumente (1999).

1.1.1 The PiezoHydraulic Concept

The concept of piezohydraulic actuation takes advantage of the high force capabilities that piezoceramics have and combines it with the operation at high frequencies. The piezoelectric stack is used as a piston type of pump that is connected in a closed circuit with the hydraulic actuator (output cylinder). The output of the piston-pump arrangement is regulated with a pair of controlled valves, and therefore the direction of motion of the output cylinder is controlled. Furthermore, even though the driver of the piston pump -the piezoelectric stackexhibits low displacements, the high frequency rectification performed with the controlled valves, translates the low stroke of the piezoelectric stack into a desirable amount of stroke (per unit time) for the output cylinder. In other words, the low displacement, oscillatory motion of the piezoelectric device (coupled with a high frequency operation) is translated into a unidirectional motion of the hydraulic cylinder.



Figure 1.3: Generic: a) Hydraulic System, b)Piezohydraulic System.

Also, as shown in the figure, the piezoelectric stack replaces the hydraulic pump with the advantage of no wear or need for maintenance, and higher response times. In addition, the entire system of hydraulic lines used to connect the input to the output are eliminated, since the piezoelectric device and the actuator are incorporated into one unit, the *piezohydraulic unit*. The input to this unit is achieved through electrical wires which are lightweight and easy to install and distribute.

1.2 Literature Review

1.2.1 Piezoelectric Hybrid Actuators

As mentioned earlier, piezoelectric actuated applications is a fast growing field. Piezoelectrics are often used in products for micropositioning, for control and suppression of vibrations, and for dynamic actuation, among others. In the field of dynamic operation, piezoelectric hybrid actuators are the trend.

One of the groups is the electromechanical-piezoelectric hybrid actuators, where inchworm type of motors are under extensive research and development. These devices, rely on a set of piezoelectric actuators that work simultaneously in order to move and clamp the rod of a cylinder. Advantages include high clamping forces and displacements at considerable speeds, that are only limited by the size of the rod. Disadvantages include the use of friction as the primary source of actuation, which results in energy losses and eventual wear in the system. The concept is explained and illustrated in the article "Inchworm Actuator" from NASA's technical briefings [NASA (1994)]. Additional documents that deal with the design, modeling, analysis and performance of inchworm motors are Lee and Esashi (1995), Bexell et al. (1994) and Frank et al. (1999).

Another group is the field of piezohydraulic devices, such as piezoelectric pumps. The literature review performed in this field resulted in various documents and articles that relate to piezoelectric pumps, specifically *micro* pumps. Some of these articles include the study and development of micropumps, such as in Gerlach and Wurmus (1995), and Koch et al. (1998); and also the combination of these micropumps with micro-valves or valveless arrangements, such as in Smits (1990), and Ullmann (1998). Nonetheless, of all the literature review only one article referred to the development of a piezohydraulic unit that uses the actuation of a piezoelectric stack along with a hydraulic system and an output cylinder. The article "Piezoelectric Hydraulic Pump", by Mauck and Lynch (1999), presents the results of the operation of a piezohydraulic unit composed of a piezoelectric stack, a set of check valves, a closed hydraulic circuit that includes a reservoir, and a four way valve to control the operation of the output cylinder.

The experimental test setup unit that was developed under this research, eliminates the use of the four way valve with the replacement of the check valves with controlled valves. It is the timing of these controlled valves that specifies the direction of movement of the hydraulic cylinder. Moreover, the installation of controlled valves permits the manipulation of the cycle under which the piezoelectric stack operates, optimizing the process. And finally there is no need for a reservoir and the hydraulic system is reduced to a simple, closed circuit pipeline.

With respect to the modeling of a piezohydraulic unit, the only document found was the article titled "Electromechanical Modeling of Hybrid PiezoHydraulic Actuator System for Active Vibration Control" by Tang et al. (1997). Starting with the equations for a piezoceramic crystal, the transfer function for a piezoelectric actuator is derived. Then, the analysis of the hydraulic system is performed through the transfer function matrix measurement method. Thus the model of the entire piezohydraulic actuator is obtained by coupling the transfer function derived for the piezoelectric actuator with the transfer function measured for the hydraulic system.

1.2.2 Lumped Models for Fluid System Analysis

Lumped parameter models for fluid pipelines or components have long been used. One early reference is the "Handbook of Fluid Dynamics" by Streeter (1961), where lumped parameter models are used to study pressure transients in hydraulic pipelines that demonstrate both inertial and elastic effects. A good introduction and explanation of the fluid elements of resistance, capacitance and inductance is found in "System Dynamics, modeling and response" by Doebelin (1972). Furthermore, the application of the lumped parameter model to a fluid pipeline and the comparison of the results with those corresponding to a distributed model, has been found in Doebelin (1980). The analysis in both cases, follows a transfer function approach. For a more recent reference, there is an entire section for "lumped models for hydraulic systems" in the publication "Understanding Dynamic Systems" by Dorny (1993). Concerning journal articles, in Wang and TAN (1997) the "Coupled analysis of fluid transients and structural dynamic responses of a pipeline system" is performed. The analysis starts with the extended one dimensional fluid filled water hammer equations in a pipe system, couples it with the structural equations of the pipe, and then the resulting differential equations are solved using the Galerkin's method, which ends up expressing the set of equations in terms of a mass, damping, stiffness and force coefficient matrices. In Wolf and Paronesso (1992) a lumped parameter model is used for the time domain analysis of a semi-infinite uniform fluid channel.



Figure 1.4: a) Pietrabissa et al. (1996), b) Doebelin (1972).

The most representative example of the use of a lumped parameter model for the study of fluid pipelines, has been found in Pietrabissa et al. (1996). In this article, a lumped parameter model for different coronary bypasses is developed in order to evaluate the fluid dynamics. Figure 1.4 shows some diagrams of lumped parameter models, obtained from the references cited.

1.3 Overview of Thesis

1.3.1 Research Objectives

The objective of the first year of this research effort was to build a benchtop that would demonstrate the concept and provide the tools necessary to obtain experimental data about the performance of the piezohydraulic unit. Most of the components used are standard off-the-shelf parts, due to their conventional and easy installation, but mainly because of their availability and relatively fast shipment times. The reasoning behind it was that once the test setup is built and analyzed, the limitations outlined, and the possible improvements obtained, then it is possible to set the next set of objectives or goals. Furthermore, it is possible to acquire specific components or custom build them in accordance to the new set of specifications, for the rearrangement or even the complete redesign of the entire unit. For example, latest technology advances have enabled companies like TRS Ceramics to develop single crystal actuators "with strain levels in excess of 1% and exhibit five times the strain

energy density of conventional piezoceramics. Unlike piezoceramic actuators that employ strain magnification schemes, single crystal actuators can thus deliver higher strain levels without sacrificing generative force" [TRS Ceramics (2000)]. Although stacked actuator performance improvements are yet to come, they have under development actuators that operate at 1000V and 500V with strain levels greater than 0.2%. Another area is the research and development of piezoelectric driven valves since they exhibit fast responses and high operating frequencies. As these and other products become available, then the design of piezohydraulic units can be updated and their performance enhanced.

1.3.2 Contribution

In this thesis a lumped parameter model is developed in order to determine the response of the fluid in a hybrid-actuator. The lumped parameter model developed can be used to study the excitation of any fluid under any method as long as the assumptions made are satisfied. The experimental work of this thesis involves the use of a piezo-electric stack, a pair of solenoid valves and two cylinders that along with the connectors and the piping, constituted most of the piezo-hydraulic system. One last important component is the power amplifier used to drive the stack.

From research of previous work, and from the experience of the current work, it is possible to identify the following set of limitations involved with a piezohydraulic unit. First, the need of high displacement actuators often comes with the requirement of high voltage operation (from 150V to 1000V) along with high current consumptions (up to hundreds of mA). Many amplifiers in the market only offer peak current capabilities of 100mA to 200mA. Therefore, the amplifier to be used limits the type of piezoelectric stack employed. Second, is the response of the controlled valves. The highest operating frequency will set the limit on the piezohydraulic unit. And finally, once these limitations are overcome, the unit is eventually limited by the dynamics of the fluid and the hydraulic system itself. The frequency response, the operation near resonance and the presence of cavitation, are some of the aspects that would limit the entire unit.

Also, early test setup arrangements and procedures revealed the importance of eliminating to the greatest extent, the amount of air entrainment in the system. The great influence of parameters like these, prompted to the modeling and the understanding of the dynamics of the system. The analysis of the system involved the study of the excitation of the hydraulic fluid at various frequencies, and the corresponding time response. For these purposes, a lumped parameter approach was used to model the fluid. The model is obtained by developing electrical components that are analogous to a fluid system under certain conditions. Then these electrical components are used to develop the model of a lump of fluid in a pipeline. Once the electrical model is obtained, an equivalent mechanical system is developed (for one lump) and used to model the entire hydraulic network. Finally the model is integrated with the rest of the systems that form part of the piezohydraulic unit.

The entire model for the piezohydraulic system was developed in a modular fashion, in order to simplify its coding in Matlab, and to facilitate changes within the components of the system itself. The model is eventually defined in state space form, in order to take advantage of the simple simulation tools that are provided by Matlab. Thus, it is possible to simulate the response of the actuator to the stack's excitations and quantify the importance and the tradeoffs between several design parameters. Some include: the type of stack and the type of fluid used, the type of excitation and its frequency, the location and magnitude of resonance, the percentage of air entrained in the system, and the magnitude at which the system is pressurized, among others. Also, the state-space formulation in Matlab, enables the future development of pole-placement controllers that would operate over the entire system.

1.3.3 Approach

Chapter 2 is an introduction to the piezohydraulic system. The electrical system, and the mechanical system are presented and modeled. The hydraulic system is also discussed, specifically on how the model is used. The extensive derivation of the model itself is performed in the following chapter. Furthermore, the electrical and mechanical systems are coupled through the electro-mechanical equations for a piezoelectric stack. The resulting electro-mechanical system is coupled with the hydraulic system through the introduction of constraints with the variational approach described by Hamilton's principle. Characteristics, limitations, modeling and derivations are specifically developed with respect to the components used in the systems of the piezohydraulic unit developed under this research.

Chapter 3 introduces the lumped parameter model to analyze the fluid system. It links the lumped mass type of analysis used to obtain a lumped parameter model of a fluid, to the governing equations of a fluid system that are applied to a control volume. The result is the definition of the fluid elements of resistance, capacitance (compliance) and inductance (inertance). Then they are used to develop a model analogous to a lump of fluid in a pipeline.

Chapter 4 uses the models and equations of each of the systems presented in Chapter 2. It also uses the model of the hydraulic system discussed in Chapter 3, and it combines all the information in order to develop the model for the entire piezohydraulic system.

Chapter 5 is devoted to the experimental and the simulated results under one-sided operation (oscillation) and two-sided operation (pumping) with both, a single ended and a double ended cylinder. The results are analyzed and tradeoffs are outlined.

Finally, *Chapter* 6 summarizes conclusions, proposes future work and formulates the corresponding recommendations.

Chapter 2

The PiezoHydraulic Unit

2.1 Introduction

This chapter is an introduction to the piezohydraulic system. The electrical system, and the mechanical system are presented and modeled. The hydraulic system is also discussed, specifically on how the model is used. The extensive derivation of the model itself is performed in the following chapter. Furthermore, the electrical and mechanical systems are coupled through the electro-mechanical equations for a piezoelectric stack. The resulting electro-mechanical system is coupled with the hydraulic system through the introduction of constraints with the variational approach described by Hamilton's principle.Characteristics, limitations, modeling and derivations are specifically developed with respect to the components used in the systems of the piezohydraulic unit developed under this research.

Thus, the topics discussed in this chapter are:

- The electrical system: composed of the power supply and the amplifier.
- The electro-mechanical coupling: performed by the piezoelectric stack.
- The mechanical system: composed of the piston, rods and links involved.
- The mechanical-hydraulic coupling: through the introduction of constraints.
- The hydraulic system: analyzed with two separate models.
- And, the controlled valve dynamics: operation and effects.

Figure 2.1 shows the actual version of the single-ended piezohydraulic unit that has been assembled as part of this research, with the contribution of research assistants Nikola Vujic and Julio Lodetti.



Figure 2.1: Test setup of the single-ended piezohydraulic unit developed.

Figure 2.2 is a simplified diagram of the test unit developed. Components are not drawn to scale. The electrical system is composed by the power supply and the amplifier, while the mechanical system is composed of an input component (the coupler-force gage-rod-piston) and an output component (the piston and rod of the actuator). The hydraulic system can be divided in two sides, and as shown in the figure below, they are labeled as Side A and Side B. The definition and the modeling of these sides depends on the operation and the timing of the solenoid valves. Furthermore, testing and modeling was not only



Figure 2.2: Diagram of a single-ended piezohydraulic unit.

performed with a single-ended cylinder (Figure 2.1), but also with a double-ended cylinder (as shown in Figure 2.3). The test setup for both, the single-ended and the double-ended unit are basically the same. In addition to the hydraulic cylinder, the only difference lies on the length of the pipes, as well as the adaptors required to connect the pipes to the respective hydraulic cylinder.

Also, note that a vacuum pump is used to evacuate the air in the system before filling it with fluid. This is done to ensure that the amount of entrained air is practically eliminated once the unit is filled with fluid. The fluid is supplied with a pressurized reservoir, that it is also used to control the initial pressure on the hydraulic system. Both of these components are isolated with manually controlled valves. Furthermore, their connection to the unit is only needed during first time fluid filling procedures, or re-fills. Afterwards they are easily removed thanks to the installation of a quick connect/disconnect hookup, which is highlighted in red in the figure below.



Figure 2.3: Setup of the double-ended piezohydraulic unit.

In the next sections, the various systems that form part of the piezo-hydraulic unit are presented and discussed. Characteristics, limitations, modeling and derivations are specifically developed with respect to the components used in the piezohydraulic unit developed in this research.

2.2 The Electrical System

The main components of the electrical system are the power supply and the amplifier. The requirements and the output characteristics of the amplifier depend on the piezo-electric stack (PZT) used, and in the same manner, the operating characteristics of the piezo stack are determined by the system's overall requirements. Piezo-electric stacks are often classified into a "High Voltage PZT" group and a "Low Voltage PZT" group. Each group has different current consumption requirements, which also depend on whether the device is used under static or dynamic operation. Voltage, current and frequency of operation often determine the type of amplifier needed. Piezo-ceramic manufacturer catalogs such as Piezo Systems (1998) and Physik Instrumente (1999) provide several types of amplifiers suitable for different applications.

For this research, Dynamic Structures and Materials (DSM), a Nashville based company, has developed a switching amplifier that delivers the input required by the piezoelectric stack while reducing the amount of power dissipated in the process. The prototype is shown in Figure 2.4. It is a three-channel unit, 2 channels targeted for PZT operating valves (400V) and 1 channel for the piezoelectric stack actuator (150V). The maximum current rating is 1.55 Amps and it supplies a total power output of 270 Watts. For additional information refer to the appendix.

AMPLIFIER INPUTS	AMPLIFIER OUTPUTS
	F1 FP 0 0 0 0 0 01 02 03 P
0 II I2 I3A I3B I4 OFF	$\begin{array}{cccccccccccccccccccccccccccccccccccc$

Front Panel



Figure 2.4: Three channel, recirculating PZT driver developed by DSM.

The amplifier current controls the PZT stack with a switching signal. The result is a triangular voltage waveform across the capacitive load (PZT stack). Furthermore, and as shown in the next section, piezoelectric stacks display a fairly linear relationship between the voltage applied and their free displacement. Thus, the excitation of the mechanical-hydraulic system through the stack is also triangular. An ideal representation of the relationship between the current, charge, voltage and displacement of the PZT stack/amplifier system is shown in Figure 2.5. It is ideal, because the amplifier's current signal is assumed to be a "clean" step wave (with no noise), and the piezoelectric stack is assumed to be unloaded and therefore modeled as a capacitive load. The second assumption is valid under free operation, and can be obtained from the final constitutive equations for the PZT stack, derived in the following section. The result of the simplified system is a current source driving a capacitor.



Figure 2.5: Ideal, unloaded representation of a piezoelectric stack.

Finally, recall that the force and displacement of the piezoelectric stack are exerted through the mechanical system into the hydraulic system. This discontinuous type of excitation, as opposed to a "smooth" signal such as a sine wave, may have several effects on the fluid system. These will be discussed in Chapter 5, along with the measured and simulated results. Nonetheless it is clear that a tradeoff is to be considered between the frequency type of excitation of the fluid (and the resulting implications) versus the type of amplifier used.

2.3 The Electro-mechanical Coupling: Model of the Piezoelectric Stack

In many cases, a piezoelectric (PZT) stack couples electrical and mechanical systems as a sensor, an actuator, or even both. In our piezohydraulic system, the stack serves as an actuator that couples the electrical and mechanical systems. This section is devoted to the analysis of the properties, electro-mechanical equations, and performance of a piezoelectric stack. A general introduction to piezoceramic elements can be found in the catalogs of piezoceramic manufacturers such as Piezo Systems (1998) and Physik Instrumente (1999). A more detailed analysis of piezoelectric materials and the fundamentals of piezoelectricity is performed by Ikeda (1990). Furthermore, for standards, and the general constitutive equations for a piezoceramic material, refer to the IEEE standard on piezoelectricity (1987). The objective of this section is to merge the information given in the references mentioned previously, to obtain the electro-mechanical equations specifically for a piezoelectric stack. As a contribution of this thesis, this section relates the various notations used and summarizes the relevant information such that a clear and detailed derivation is developed for the electro-mechanical equations of a piezoelectric stack.

A piezoelectric stack consists of a stack of thin piezoceramic elements. Various piezoelectric designs can be found in the "*PZT Fundamentals*" subsection of Physik Instrumente (1999). When a voltage is applied, the piezoceramic element is stressed electrically and its dimensions change. In the same manner, "if it is stressed mechanically by a force, then it generates an electric charge. If the electrodes are not short-circuited, a voltage associated with the charge appears" [Piezo Systems (1998)]. Because of this, a piezoceramic can be used as a sensor or as an actuator, or both.

One important aspect of a piezoceramic is that in addition to its piezoelectric properties and its geometry, its response also depends on the direction of the mechanical and electrical excitation. Therefore, usually a piezoceramic and its properties are labeled with respect to the axes in Figure 2.6, where P is the polarization vector. Also, the 3^{rd} axis is defined as parallel to the direction of polarization of the ceramic. This direction is determined during the manufacturing process and it is a result of a high DC voltage applied between a pair of electroded faces.



Figure 2.6: Axes and polarization of a piezoceramic element.

In the study of piezoelectricity, properties have subscripts, superscripts, or both. A single subscript gives the direction or the axis of interest. "Piezoelectric coefficients with double subscripts link electrical and mechanical quantities. The first subscript gives the direction of the electrical field associated with the voltage applied, or the charge produced. The second subscript gives the direction of the mechanical stress or strain" Piezo Systems (1998). As shown in Figure 2.7, the induced electrical field (from the applied voltage or charge) for the piezoelectric stack is in the 3^{rd} direction, as well as the force applied and the displacement. Thus, a coefficient X will be denoted as X_{33} . Furthermore, superscripts specify either a mechanical or an electrical boundary condition. Following the standard notation given in Piezo Systems (1998), the following superscripts are used:

> T = constant stress = mechanically free E = constant electrical field = short circuit D = constant electrical displacement = open circuitS = constant strain = mechanically clamped



Figure 2.7: Piezoelectric stack: a) Voltage controlled, b) Current/Charge controlled.

Generally, a piezoelectric stack is assumed to strain in only one direction. Thus, the constitutive equations for a one-dimensional excitation and deformation of a piezoceramic element are [ANSI/IEEE Standard 176 (1987)]:

$$D = \epsilon^T E + d T \tag{2.1}$$

$$S = dE + s^E T \tag{2.2}$$

where:	D = Electric Density or Flux Density	$\left[\frac{C}{m^2}\right]$
	E = Electric Field	$\left[\frac{V}{m}\right]$
	T = Mechanical Stress	$\left[\frac{N}{m^2}\right]$
	S = Mechanical Strain	$\left[\frac{m}{m}\right]$
	ϵ = Dielectric Permittivity of the Material	$\left[\frac{F}{m} = \frac{C^2}{Nm^2}\right]$
	d = Piezoelectric d-constant	$\left[\frac{m}{V} = \frac{C}{N}\right]$
	s = Mechanical Compliance	$\left[\frac{m^2}{N}\right]$

and all of these parameters are either related to, or a function of the direction of the mechanical or electrical excitation. For example, the value of the d-constant, may change as it is expressed as $d_{33}, d_{31}, d_{15}...$ Then, following the use of subscripts and superscripts in piezoelectricity, the equations (2.1) and (2.2) defined for a piezoelectric stack as shown in Figure 2.7, are expressed more specifically as:

$$D_3 = \epsilon_3^T E_3 + d_{33} T_3 \tag{2.3}$$

$$S_3 = d_{33} E_3 + s_3^E T_3 \tag{2.4}$$

In order to simplify the following expressions, the subscripts and superscripts will not be included, granted that it is already known that the properties are those linked to both the mechanical and electrical excitations in the direction of the 3^{rd} axis. Furthermore, since D_3 represents a charge per area, then the electric density will be referred to as the surface charge density, which in Serway (1994) is defined as:

$$\sigma_q = \frac{Q}{A} \tag{2.5}$$

where a uniformly distributed charge Q, on a surface of area A has been assumed, and the "q" subscript has been added to distinguish it from the mechanical stress. Also, the
dielectric permittivity of the material, ϵ , is related to the constant of permittivity of free space, ϵ_o , by the dielectric constant [Ikeda (1990)]:

$$k = \frac{\epsilon}{\epsilon_o} \tag{2.6}$$

By using equations (2.5) and (2.6), and by adopting the conventional notation for the mechanical stress (σ_m) and the mechanical strain (ϵ_m), then the equations for a piezoelectric stack (2.3 and 2.4) are written as:

$$\sigma_q = (k\epsilon_o) E + d \sigma_m \tag{2.7}$$

$$\epsilon_m = dE + s\sigma_m \tag{2.8}$$

Following the notation shown in Figure 2.7, the mechanical strain is defined as:

$$\epsilon_m = +\frac{\Delta L}{L} \tag{2.9}$$

Positive because for the sign convention used, an elongation represents a positive strain. Furthermore, the mechanical stress is defined as:

$$\sigma_m = \frac{F}{A} = \frac{-F_{pzt}}{A} \tag{2.10}$$

and it is negative because F_{pzt} is a compressive force. Note that F_{pzt} represents the force developed or exerted by the piezoelectric stack on the element it is acting on. A breakup of the stack and the actuated element, along with a free body diagram will result in the presence of a force as depicted in Figure 2.7. Thus, by substituting equations (2.9) and (2.10) into equations (2.7) and (2.8), then the electro-mechanical equations for a piezoelectric stack become:

$$\frac{Q_{pzt}}{A} = k \epsilon_o E - d \frac{F_{pzt}}{A}$$
(2.11)

$$\frac{\Delta L}{L} = dE - s \frac{F_{pzt}}{A}$$
(2.12)

From physics, the electric potential difference between two points, is related to an induced electric field in the direction of the movement of a positive charge from point A to B by the equation [Serway (1994)]:

$$V_B - V_A = -\int_A^B E \, ds \tag{2.13}$$

where given the sign convention used, point B is at a lower electric potential than point A $(V_A > V_B)$, and ds denotes the differential distance between point A and B. Thus, for the piezoelectric stack shown in Figure 2.7, the upper surface (shaded in gray) has a potential V_A while the lower surface has a potential V_B , and the voltage or the electric potential across the piezoelectric stack can be expressed as $V_{pzt} = V_A - V_B$. Therefore, equation (2.13) can be written as:

$$V_{pzt} = \int_{A}^{B} E \, d\ell \tag{2.14}$$

Figure 2.8 illustrates the relationship expressed in equations (2.13 and 2.14). In this figure, an induced electric field in the direction shown causes a positive charge q_o , to move from point A to a lower electric potential at point B (assuming zero initial conditions for the charge). Since the electric field is constant, then equation (2.13) reduces to $V_B - V_A = -E d$. Figure 2.8b is an ideal representation of the piezoelectric stack shown in Figure 2.7. By



a) Positive Test Charge [Serway (1994)]

b) Uniform Surface Charge Distribution



assuming equipotential surfaces (continuous distribution of points having the same potential) or similarly, by assuming a uniform surface charge distribution, the piezoelectric stack can be represented as a two-plate capacitor. By neglecting the effect of the sides of each plate, then the electric field can be described as uniform across the distance, L, between both plates (and therefore, constant within $d\ell$). As a result, then the potential difference in equation (2.14), and the electric field, can be expressed as:

$$V_{pzt} = EL \qquad \Longrightarrow \qquad E = \frac{V_{pzt}}{L} \tag{2.15}$$

Substituting the previous expression for the electric field into the equations of the piezoelectric stack (equations 2.11 and 2.12) yields to the following equations:

$$\frac{Q_{pzt}}{A} = k \epsilon_o \frac{V_{pzt}}{L} - d \frac{F_{pzt}}{A}$$
(2.16)

$$\frac{\Delta L}{L} = d \, \frac{V_{pzt}}{L} - s \, \frac{F_{pzt}}{A} \tag{2.17}$$

Furthermore, both expressions can be rearranged in the form:

$$Q_{pzt} = \left(k \frac{\epsilon_o A}{L}\right) V_{pzt} - (d) F_{pzt}$$
(2.18)

$$\Delta L = (d) V_{pzt} - \left(\frac{sL}{A}\right) F_{pzt}$$
(2.19)

Recall that these equations are valid for a PZT stack when: the area A is the surface perpendicular to the polar or 3^{rd} axis, the length L is parallel to it, and the rest of the properties (with exception to the constant of permittivity of free space) are those that correspond to the excitation and response along the 3^{rd} axis. Also note that the piezoelectric d-constant appears as the coefficient of the force in the first equation, and as the coefficient of the voltage in the second equation. Thus, the d-constant (as some other piezoelectric constants) can be expressed in two different ways, that as expected, should be equal to one another. As presented in Piezo Systems (1998), the d-constant is sometimes expressed as the ratio:

$$d = \frac{short\ circuit\ charge\ density}{applied\ mechanical\ stress} \left[\frac{C/m^2}{N/m^2}\right]$$
(2.20)

which is used to define the coefficient of the force in equation (2.18). In addition, the d-constant is also expressed as the ratio:

$$d = \frac{strain \ developed}{applied \ electric \ field} \left[\frac{m/m}{V/m}\right]$$
(2.21)

which in turn, is used to define the coefficient of the voltage in equation (2.19). Both d-constant representations have the same units $\left(\begin{bmatrix} C\\N \end{bmatrix} = \begin{bmatrix} m\\V \end{bmatrix}\right)$ and are equivalent to one another. Furthermore, in Leo (1999), the coefficient of the voltage is also referred to as the free displacement per unit voltage and it is denoted as x_o . This is because under free

operation (no load), the force in equation (2.19) is not present and the expression reduces to $\Delta L = (d) V_{pzt}$. Thus, for the model of the piezoelectric stack, the d-constant represents the stress-free extension per unit volt ($d = x_o$).

In addition to the *d*-constant, there are two more coefficients left in the set of equations (2.18) and (2.19). By inspection, the expression $\left(k \frac{\epsilon_o A}{L}\right)$ represents the capacitance of the PZT stack:

$$C_{pzt} = k \frac{\epsilon_o A}{L} \qquad \left[F \ , \ \frac{C^2}{N \ m} \right] \tag{2.22}$$

which follows from the definition of capacitance [Serway (1994)]. Moreover, the expression $\left(\frac{sL}{A}\right)$ from equation (2.19), is related through "s" to the mechanical compliance. Since the compliance is inversely proportional to the stiffness of an element, and a deformation is related to the force by $F = k_s x$ (where k_s is the stiffness of the spring element), then it is possible to relate the term $\left(\frac{sL}{A}\right)$ to the stiffness of the PZT stack actuator, k_a , through the expression:

$$k_a = \frac{A}{sL} \qquad \left[\frac{N}{m}\right] \tag{2.23}$$

By substituting equations (2.22),(2.23), $x_o = d$, and x_{pzt} for the extension or displacement of the stack, ΔL , into equations (2.18) and (2.19), then the result is the following constitutive equations for the piezoelectric stack:

$$Q_{pzt} = C_{pzt} V_{pzt} - x_o F_{pzt}$$

$$(2.24)$$

$$x_{pzt} = x_o V_{pzt} - \frac{1}{k_a} F_{pzt}$$
 (2.25)

Under free operation, or no load, the stack does not exert any force and equation (2.25) reduces to $x_{pzt} = x_o V_{pzt}$. Thus, x_{pzt} becomes the free displacement of the stack, it varies linearly with voltage, and it will be denoted as x_{free} . In the same manner, if the stack operates under a very large load, its output displacement x_{pzt} is zero and equation (2.25) reduces to $F_{pzt} = k_a x_o V_{pzt}$. This is the blocked force of the PZT stack and it will be denoted as F_{blkd} . Furthermore, for the operation of the stack under the maximum voltage allowed, V_{max} , equation (2.25) can be rearranged in the form:

$$F_{pzt} = -k_a x_{pzt} + k_a x_o V_{max}$$

$$(2.26)$$

while the free displacement and the blocked force are expressed as:

$$x_{free} = x_o V_{pzt} \tag{2.27}$$

$$F_{blkd} = k_a \, x_o \, V_{pzt} \tag{2.28}$$

Equation (2.26) can be plotted as shown in Figure 2.9, and it represents the characteristic curve for a piezoelectric stack. The characteristic curve defines the operating point of a PZT stack, given the loading conditions it is operating under.



Figure 2.9: Force-displacement characteristic curve for a stack.



Figure 2.10: Charge-voltage characteristic curve for a stack.

Figure 2.9 represents the force-displacement characteristics for a piezoelectric stack. It results from Equation (2.26) and it describes the mechanical performance of the PZT stack. In the same manner, Equation (2.24) yields to the charge-voltage characteristic shown in Figure 2.10, and it is related to the electrical performance of the PZT stack.

In the piezohydraulic unit, the piezoelectric stack actuates under different conditions through the different stages of the operating cycle. The operating point in the forcedisplacement and charge-voltage characteristic curves depends on the nature of the stage, i.e. the conditions under which the stack is operating. Thus, cycle curves can be obtained, and an example of these are shown in Figure 2.11.



Figure 2.11: Stack under pump cycle operation with an incompressible fluid.

The figure represents one cycle of a piezoelectric stack within a constant low pressure reservoir and a high pressure side, and as shown in the diagram in Figure 2.12, both sides are restrained with a pair of check valves. The fluid is assumed to be essentially incompressible.

The replacement of the check values with controlled (solenoid) values affects the location of the four points that define the cycles shown in the Figure 2.11. Furthermore, the elimination of an ideal constant low pressure and high pressure side along with the addition of hydraulic components, would affect the shape of the curves between each of the points. A complete and detailed study of these cycles is performed and presented in Leo and Nasser (2000), where mechanical and electrical efficiencies are defined and related to the characteristic curves and the operating cycle of the piezoelectric stack.



Figure 2.12: Diagram of a check valve restrained piezohydraulic unit.

Below, Figure 2.13 shows the piezoelectric stack used in the piezo-hydraulic unit, along with a list of its properties.



Stiffness	37 N / mm
Blocked Force	3500 N (estimated)
Max Free Displacement	110 m m
Capacitance	39 m F
Input Voltage	-20 to 150 V (150 V_{pp})

Figure 2.13: Pre-loaded piezoelectric stack.

The piezoelectric stack is a custom made unit developed by Dynamic Structures and Materials (DSM), and as mentioned previously in the *Electrical System* section, it is operated under a triangular charge/voltage waveform that results from a square wave current excitation. The free displacement and the rated blocked force are comparable to commercial stacks that usually require 1000 Volts.

2.3.1 Operation under Load

In Figure 2.13, the wires attached to the case of the unit represent the mechanical preload for the piezoelectric stack. Recall that under dynamic operation, a mechanical preload is used to compensate for pulling forces and therefore, protect the ceramics of a stack from damage. The wires used are pre-stretched and composed of Nitinol. Nitinol is a super-elastic binary nickel titanium alloy with optimal properties, including a low permanent set, high loading and unloading plateau stresses, and excellent kink-resistant characteristics. The high loading and unloading plateau stresses can be observed in Figure 2.14 (which has been provided by Dynamic Structures and Materials). The importance of this characteristic is that if the stretched nitinol is set to operate in the flat region of the curve, then regardless of the strain, the piezoelectric stack will be loaded with a constant stress. This is important since the type of preload in a piezoelectric stack may affect its performance.



Figure 2.14: Typical stress-strain curve for super-elastic nitinol.

The remainder of the section is a brief explanation on the effects of the type of preload on a piezoelectric stack.

As described in Physik Instrumente (1999), a piezoelectric actuator is an elastic body with a given stiffness, and from a mechanical standpoint it will be represented with a spring of stiffness k_a . Furthermore, it can be operated under two different types of loading.

The first case, is when the load remains constant during the expansion process. This is represented in Figure 2.15, where a mass exerts a constant force on the piezoelectric stack. The force of the mass compresses the piezoelectric stack until equilibrium is reached. Thus, the initial position of the stack changes by the amount of $\Delta x = F/k_a$. However, and as represented in the figure, the constant force and the nonzero initial condition does not affect the stack's free displacement capability.



Figure 2.15: Effect of a constant force pre-load on a piezoelectric stack.

The second case occurs when the load on the stack changes during the expansion process. In Figure 2.16 the stack is loaded with a spring of stiffness k_s , that is coupled in parallel to it. Therefore, the free displacement of the unloaded stack is $x_{free_A} = F_{pzt}/k_a$ (case A), while the free displacement of the spring loaded stack (shown as case B) becomes $x_{free_B} = F_{pzt}/(k_a + k_s)$.



Figure 2.16: Effect of a spring pre-load on a piezoelectric stack.

Thus, a spring load does affect the free displacement capability of the piezoelectric stack, reducing the free displacement by Δx where:

$$\Delta x = x_{free_A} - x_{free_B} = x_{free_A} \left(1 - \frac{x_{free_B}}{x_{free_A}}\right)$$
$$= x_{free_A} \left(1 - \frac{F_{pzt}/(k_a + k_s)}{F_{pzt}/k_a}\right)$$
$$= x_{free_A} \left(1 - \frac{k_a}{k_a + k_s}\right)$$
(2.29)

Furthermore, the free displacement of the loaded case can be also expressed as a function of the original free displacement:

$$x_{free_B} = \frac{F_{pzt}}{(k_a + k_s)} = \frac{F_{pzt}}{(k_a + k_s)} \frac{k_a}{F_{pzt}} \frac{F_{pzt}}{k_a} = \frac{k_a}{(k_a + k_s)} x_{free_A}$$
(2.30)

and by substituting the expression above and the equivalent stiffness of the stack into equation (2.25), then the resulting constitutive equation for a piezoelectric stack becomes:

$$\begin{aligned}
x_{pzt} &= x_{free} - \frac{1}{k_a} F_{pzt} \\
&= \frac{k_a}{(k_a + k_s)} x_{free_A} - \frac{1}{k_a + k_s} F_{pzt} \\
&= \frac{k_a}{(k_a + k_s)} x_o V - \frac{1}{k_a + k_s} F_{pzt}
\end{aligned} \tag{2.31}$$

Finally note that for the blocked case $(x_{pzt=0})$, the blocked force capability of the piezoelectric stack is still $F_{blkd} = k_a x_o V_{pzt}$ (equation 2.28).

2.3.2 Current Controlled Operation

The rearranged constitutive equation (2.26) relates the force exerted by the piezo stack and its displacement, to the voltage applied on it. Then equation (2.24) relates these parameters to the resulting charge. In fact, these equations represent a voltage controlled stack, meaning that it is controlled with a voltage input. It is, the most general form of these expressions. Nonetheless, for a current controlled or a charge controlled system it is necessary to express equation (2.26) in terms of the charge across the piezo stack, Q_{pzt} . In order to do so, it is useful to solve equation (2.24) for the voltage across the piezoelectric stack:

$$V_{pzt} = \frac{Q_{pzt} - x_o F_{pzt}}{C_{pzt}} \tag{2.32}$$

Then by the substituting this expression into equation (2.26):

$$F_{pzt} = k_a x_o \left(\frac{Q_{pzt}}{C_{pzt}} + \frac{x_o F_{pzt}}{C_{pzt}} \right) - k_a x_{pzt}$$
$$= \frac{k_a x_o}{C_{pzt}} Q_{pzt} + \frac{k_a x_o^2}{C_{pzt}} F_{pzt} - k_a x_{pzt}$$
(2.33)

and further manipulation yields to:

$$\begin{pmatrix} 1 - \frac{k_a x_o^2}{C_{pzt}} \end{pmatrix} F_{pzt} = \frac{k_a x_o}{C_{pzt}} Q_{pzt} - k_a x_{pzt}$$

$$F_{pzt} = \begin{pmatrix} \frac{k_a x_o}{C_{pzt}} \\ \frac{C_{pzt} - k_a x_o^2}{C_{pzt}} \end{pmatrix} Q_{pzt} - \begin{pmatrix} \frac{k_a}{C_{pzt} - k_a x_o^2} \\ \frac{C_{pzt} - k_a x_o^2}{C_{pzt}} \end{pmatrix} x_{pzt}$$

$$F_{pzt} = \begin{pmatrix} \frac{k_a x_o}{C_{pzt} - k_a x_o^2} \end{pmatrix} Q_{pzt} - \begin{pmatrix} \frac{k_a C_{pzt}}{C_{pzt} - k_a x_o^2} \end{pmatrix} x_{pzt}$$

$$(2.34)$$

where the term $C_{pzt} - k_a x_o^2$ is also known as the blocked capacitance of the piezoelectric stack, or C_{blkd} . Also, further substitution of the coefficients F_1 and F_2 for the coefficients of the previous equation, will reduce the expression to:

$$F_{pzt} = F_1 Q_{pzt} - F_2 x_{pzt} (2.35)$$

where

$$F_1 = \frac{k_a x_o}{C_{pzt} - k_a x_o^2} \quad , \quad \left[\frac{N}{C}\right]$$
(2.36)

$$F_2 = \frac{k_a C_{pzt}}{C_{pzt} - k_a x_o^2} \quad , \quad \left[\frac{N}{m}\right]$$

$$(2.37)$$

Thus, equations (2.35) and (2.32) represent the set of constitutive equations for a charge controlled piezoelectric stack. Expressing the set of constitutive equations in terms of the force and the input variable (as done in equations (2.24) and (2.25)) then the charge controlled equations for a piezoelectric stack become:

$$V_{pzt} = \left(\frac{1}{C_{pzt}}\right) Q_{pzt} - \left(\frac{x_o}{C_{pzt}}\right) F_{pzt}$$
(2.38)

$$x_{pzt} = \left(\frac{x_o}{C_{pzt}}\right) Q_{pzt} - \left(\frac{C_{pzt} - k_a x_o^2}{k_a C_{pzt}}\right) F_{pzt}$$
(2.39)

As mentioned in Section 2.2, the amplifier used in the experimental setup current controls the piezoelectric actuator. Therefore, these equations will become useful during the assembly of the entire model for the piezohydraulic unit, in *Chapter 4*.

2.4 The Mechanical System

As discussed previously, the mechanical system consists of two elements: an input component and the output component. The input component transmits the force and displacement exerted by the stack to the hydraulic fluid. It is composed of various elements, as shown in Figure 2.17, and it is modeled in a lumped parameter fashion. Masses M_r and M_{p1} account for the entire mass of the component while the stiffness K_r is the equivalent stiffness of all the elements that form part of the input component.



Figure 2.17: Lumped model of the mechanical input component.

The output component is basically the piston and the rod of the output cylinder. It is modeled simply as a mass that lies between the two sides of the hydraulic system. Figure 2.18



Figure 2.18: Lumped model of the mechanical output component.

displays that notation that will be used in model. The spring load K_{il} is an additional element that is used only when an asymmetric (one sided) output cylinder is employed. It accounts for the force needed to reach equilibrium once the system is pressurized. From Figure 2.19, it can be observed that when the entire system is pressurized at a level P_i , then a net force F_{il} will result from the difference in areas on both sides of the piston.

$$F_{net} = F_B - F_A = Pi(A_B - A_A) = Pi\frac{\pi}{4}(D_A^2 - D_B^2)$$
(2.40)

The cylinder then moves in that direction until it reaches equilibrium and it stops. At this point the net resulting force is being counteracted by the force of the expanded/contracted fluid.



Figure 2.19: A single-ended hydraulic cylinder under equal pressure.

The net effect is modeled with a compressed spring of stiffness K_{il} . The distance traveled, x_{il} , depends on the magnitude of the force and therefore also on the pressure. For Pi = 100 psi, x_{il} is usually in the order of 10^3 microns. Once this distance is measured, then the stiffness can be obtained, through the equation:

$$K_{il} = \frac{F_{net}}{x_{il}} = \frac{Pi \pi}{4 x_{il}} (D_A^2 - D_B^2)$$
(2.41)

Finally note that for both, the input and the output components, the dynamic friction of the mechanical elements has been neglected. This is because the nonlinear behavior has been assumed to have no considerable effect on the linear model within its operating conditions.

2.5 Mechanical-hydraulic Coupling

2.5.1 System Coupling

Following the diagram of the piezohydraulic unit in Figure 2.2, and by assuming that one solenoid valve is open while the other one is closed, then it is possible to arrange the various systems that integrate it in the following manner:



Figure 2.20: Coupling within the systems of the piezohydraulic unit.

Each block represents a system that is analyzed and modeled separately from the others. This is done in order to simplify the modeling and to facilitate changes in the actual properties or the components of a system. Afterwards, it is necessary to integrate all the systems together for the development of the entire model of the piezohydraulic unit. In many cases, systems may be defined in different coordinates and a coordinate transformation is required. In our case, all the systems are eventually defined in the same coordinate system (the mechanical system) but there is still a need to couple them together. This is usually performed with the introduction of constraints.

2.5.2 Coordinate Transformations - Introducing Constraints

Hamilton's Principle:

As stated in Meirovitch (1967), Hamilton's principle is the most famous and advanced variational principle of mechanics. It considers the entire motion of a system between two times t_1 and t_2 and the variation δ of the path between these two instants. This variation is sometimes referred to as a virtual displacement, since they are not true displacements and therefore there is no time change associated with them. "The symbol δ was introduced by Lagrange to emphasize the virtual character of the variations as opposed to the symbol d, which designates differentials" [Meirovitch (1967)]. Furthermore, Hamilton's principle "is an integral principle, and it reduces the problems of dynamics to the investigation of a scalar definite integral. This formulation has the remarkable advantage of being invariant to the coordinate system used to express the integrand" [Meirovitch (1970)].

Dynamic systems can be defined with various sets of coordinates, but usually the minimum number of coordinates required is employed, and these constitute the set of n generalized coordinates. The generalized coordinates are all independent of one another and they represent the number of degrees of freedom a system has. Figure 2.21 represents a system that is defined with n = 3 generalized coordinates, x, y, and z.



Figure 2.21: True and varied path in the configuration space.

Therefore, points are defined in a dimensional space (in this case it is 3 dimensional) known as the *configuration space*. As the configuration of the system changes with time, a path is traced in the configuration space. This path is shown in bold in the figure below, and it is referred to as the *true path*. The *varied path* is also shown, and it represents small variations δ in the position (x, y, z) with no associated time change. However, the principle stipulates that at the two time instants t_1 and t_2 , the variation δ is zero and both the true and the varied path coincide.

Hamilton's principle states that the variation of the integral of the energy in a system with respect to time is zero. In the mathematical form, it translates to the expression [Meirovitch (1967)]:

$$\delta \int_{t_1}^{t_2} (\mathcal{L}) \, dt = 0 \tag{2.42}$$

where $\mathcal{L} = T - V$. The symbol \mathcal{L} represents the Lagrangian, which is usually defined in terms of the kinetic energy, T, and the potential energy, V, that is present in the system.



Figure 2.22: Second order mass-spring system.

In order to associate Hamilton's principle with the introduction of constraints, lets consider the simple mass-spring system shown in Figure 2.22. The mass is a source of kinetic energy while the spring "carries" the potential energy. By definition:

$$\partial V = F \partial x = (k x) \partial x$$
$$V = \frac{1}{2} k x^2$$
(2.43)

and

$$T = \frac{1}{2}mv^2 = \frac{1}{2}m\dot{x}^2 \tag{2.44}$$

Then by substituting these expressions into equation (2.42) the result is:

$$\delta \int_{t_1}^{t_2} \left(\frac{1}{2} m \, \dot{x}^2 - \frac{1}{2} k \, x^2 \right) \, dt = 0 \tag{2.45}$$

where the variation δ is an operator that follows the rules of calculus. It operates over the generalized coordinates and it can be moved inside or outside the integral as long as the system is *holonomic*. This is the designation given to a system in which the constraints are a function of the coordinates or the coordinates and time [Meirovitch (1967)].

$$\int_{t_1}^{t_2} \delta\left[\left(\frac{1}{2}m\,\dot{x}^2\right) - \left(\frac{1}{2}k\,x^2\right)\right]\,dt = 0 \tag{2.46}$$

Then, since $\delta(\dot{x}^2) = 2 \dot{x} \, \delta(\dot{x})$ and $\delta(x^2) = 2 \, x \, \delta(x)$ the previous equation becomes:

$$\int_{t_1}^{t_2} m \,\dot{x} \,\delta\left(\dot{x}\right) \,dt - \int_{t_1}^{t_2} k \,x \,\delta\left(x\right) \,dt = 0 \tag{2.47}$$

Furthermore, the first integral can be simplified through integration by parts, which is usually expressed as $\int u v' dt = u v - \int v u' dt$. By using the following expressions

$$u = m \dot{x} \quad v' = \delta(\dot{x})$$
$$u' = m \ddot{x} \quad v = \delta(x)$$

it is possible to simplify the first integral in equation (2.47) and express the entire equation in the form:

$$\left(m\,\dot{x}\,\delta(x)\,\left|_{t_{1}}^{t_{2}}\,-\,\int_{t_{1}}^{t_{2}}\,\delta(x)\,m\,\ddot{x}\,dt\right)-\,\int_{t_{1}}^{t_{2}}\,\delta(x)\,k\,x\,dt=0\tag{2.48}$$

In addition, by using the stipulation that at the time instants t_1 and t_2 the variation δ is zero, then the first term of equation (2.48) is equal to zero and the expression reduces to:

$$-\int_{t_1}^{t_2} \delta(x) \, (m \, \ddot{x} + k \, x) \, dt = 0 \tag{2.49}$$

Moreover, the integral is equal to zero when the integrand is also equal to zero,

$$\delta(x) \left[m \ddot{x} + k x \right] = 0 \tag{2.50}$$

and since the variation $\delta(x)$ is arbitrary, then it follows that

$$m\ddot{x} + kx = 0 \tag{2.51}$$

This is, the equation of motion of the mass-spring system in the Newtonian form. But, what is important from this derivation, is that if a coordinate transformation is required, then it is necessary to perform the operation not only in the equation of motion, but also in the variation itself. In other words, if the variables x and \dot{x} are transformed to the coordinates \hat{x} and \hat{x} , then the change of variables has to be done in both terms of equation (2.50), the variation and the equation of motion. Perhaps, the following example with a second order system, will illustrate this issue.

Introducing Constraints in a Multi-Degree of Freedom System:

In this section, the introduction of constraints in a three degree of freedom system is covered. The example serves as the basis of the modeling performed for the entire piezohydraulic system done in Chapter 4, and it will be referred to often. This is done to simplify the analysis in Chapter 4, which involves a system of a high degree of freedom. If each component of the pipeline in the experimental setup is modeled with only one lump, then the lumped parameter model of the system would exhibit 30 degrees of freedom which would require the use of 60 states. Therefore, this example is used, not only to further explain the introduction of constraints, but to also layout the same procedure involved in the analysis of the model in Chapter 4.



Figure 2.23: Second order forced mass-spring-damper system.

Figure 2.23 represents a three degree of freedom system, but the dynamics of the first mass are uncoupled to the rest of the system. Both parts of the system can be coupled by introducing a constraint. For example, by forcing the second displacement, x_2 , to be a constant, A_{ctt} , times the first displacement, x_1 , then the original set of coordinates are transformed and the original system becomes coupled. Once the system is coupled, then it would reduce to two degrees of freedom. It is the result of the introduction of the constraint.

From the free body diagram of each of the masses, it is possible to derive the equation of motion for each one of them, to express them in matrix form, and then include them in equation (2.50). The result is:

$$\delta \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}^T \left(\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{bmatrix} + \begin{bmatrix} b_1 & 0 & 0 \\ 0 & b_2 & -b_2 \\ 0 & -b_2 & b_2 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} + \begin{bmatrix} k_1 & 0 & 0 \\ 0 & k_2 & -k_2 \\ 0 & -k_2 & k_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} F = 0$$
(2.52)

By designating M, B, K, and fm to the constant coefficient matrices, then the set of differential equations can be written in matrix form as:

$$\delta(\underline{x}^T) \left[M \, \underline{\ddot{x}} + B \, \underline{\dot{x}} + K \, \underline{x} - fm \, F \right] = 0 \tag{2.53}$$

and note that the variation operates over the transpose of the set of coordinates. Now, as mentioned previously, lets couple the first and the second coordinates by introducing the constraint: $x_2 = A_{ctt} x_1$. Expressed in matrix form:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ A_{ctt} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_3 \end{bmatrix}$$
(2.54)
$$\underline{x} = C \qquad \underline{\hat{x}}$$

Thus, the vector \underline{x} represents the original set of coordinates while the vector $\underline{\hat{x}}$ represents the new set of coordinates, obtained through the transformation or the constraint matrix C. From the previous equation $\underline{x} = C \underline{\hat{x}}$ it will also follow that:

$$\frac{\dot{x}}{\dot{x}} = C \, \hat{\underline{x}}$$

$$\frac{\ddot{x}}{\dot{x}} = C \, \hat{\underline{x}}$$

$$\delta(\underline{x}^{T}) = \delta(\underline{\hat{x}}^{T} \, C^{T})$$
(2.55)

With these relations, equation (2.53) can be expressed in terms of the transformed coordinates as it follows:

$$\delta(\underline{\hat{x}}^T C^T) \left[M C \, \underline{\hat{x}} + B C \, \underline{\hat{x}} + K C \, \underline{\hat{x}} - fm \, F \right] = 0 \tag{2.56}$$

Since the transpose of the constraint matrix C is also a constant, then it is possible to remove it from the variation operation and include it in the equation as

$$\delta(\underline{\hat{x}}^T) \left[C^T M C \,\underline{\hat{x}} + C^T B C \,\underline{\hat{x}} + C^T K C \,\underline{\hat{x}} - C^T f m \, F \right] = 0 \tag{2.57}$$

Again, the variation of $\underline{\hat{x}}^T$ is arbitrary (nonzero) and the previous equation can be reduced to the form:

$$[C^T M C] \underline{\hat{x}} + [C^T B C] \underline{\hat{x}} + [C^T K C] \underline{\hat{x}} = [C^T f m] F$$
(2.58)

Equation (2.53) represents the set of transformed equations of motion expressed in the Newtonian form. Furthermore, if the transformed, constant coefficient matrices of mass, damper, spring, and force input are expressed as M_C , B_C , K_C , and fm_C , then the previous equation becomes:

$$[M_C]\,\underline{\hat{x}} = -[B_C]\,\underline{\hat{x}} - [K_C]\,\underline{\hat{x}} + [fm_C]\,F \tag{2.59}$$

If both sides is multiplied by the inverse of M_C , then the result is

$$\hat{\underline{x}} = -[M_C]^{-1}[B_C]\hat{\underline{x}} - [M_C]^{-1}[K_C]\hat{\underline{x}} + [M_C]^{-1}[fm_C]F$$
(2.60)

And by following the notation used in the model, and in the Matlab code, the constant matrix coefficients are defined as

$$Akm_{C} = -[M_{C}]^{-1}[K_{C}]$$

$$Abm_{C} = -[M_{C}]^{-1}[B_{C}]$$

$$Blower_{C} = +[M_{C}]^{-1}[fm_{C}]$$
(2.61)

Then Equation (2.60) reduces to

$$\underline{\hat{x}} = [Abm_C] \underline{\hat{x}} + [AkmC] \underline{\hat{x}} + [Blower_C] F$$
(2.62)

and note that the minus signs have been already accounted for.

Finally, the matrix equation (2.62) can be rearranged in the form:

$$\begin{bmatrix} \frac{\hat{x}}{\hat{x}} \\ \frac{\hat{x}}{\hat{x}} \end{bmatrix} = \begin{bmatrix} 0 & I \\ Akm_C & Abm_C \end{bmatrix} \begin{bmatrix} \frac{\hat{x}}{\hat{x}} \\ \frac{\hat{x}}{\hat{x}} \end{bmatrix} + \begin{bmatrix} 0 \\ Blower_C \end{bmatrix} F$$
(2.63)

$$\underline{\dot{x}} = A \qquad \underline{x} + B \qquad \underline{u} \qquad (2.64)$$

This matrix equation (equation 2.63), is expressed in what is known as the state space formulation (equation 2.64). The vectors \underline{x} and $\underline{\dot{x}}$, represent the states and their corresponding derivatives, the A matrix contains all the information about the dynamics of the system, while the B matrix specifies the input and its coupling on the system. In this example, the states are the transformed coordinates of the system, and be aware that the notation of $\underline{\dot{x}}$, $\underline{x}, \underline{u}, A$, and B, used in the state space formulation above, does not represent or match any of the variables used in the derivation of the equations of the mass-spring-damper system in study.

The model of the entire piezohydraulic unit will be covered in Chapter 4. As it will become clear by then, the entire system will be modeled with a set of masses, dampers and springs. Furthermore, the entire derivation is performed so that the equations of motion are expressed in state space form. This is, because state space techniques and their simple implementation in Matlab, are employed to analyze the system and to run the simulations. An introduction to state-space methods can be found in Friedland (1986).

2.6 The Hydraulic System

For the analysis of the fluid in the piezohydraulic unit, the hydraulic system is modeled with two different sets of models, one for each side of the unit. As shown in Figure 2.2 these sides have been labeled as A or B, and even if the output cylinder is symmetric (double ended) both sides have to be considered in order to take into account for the difference in geometry (pipe lengths, or different components).

The excitation and the response of the fluid in the piezohydraulic unit is representative of a nonlinear, time variant system. In the lumped parameter model developed, the nonlinear aspect is approximated with a linear model that is valid for small perturbations around an operating point. Furthermore, the pumping process is broken into two steps or stages. Within each stage, the linear lumped parameter model uses constant (time invariant) coefficients, that in order to approximate the time variance aspect, are updated (modified) after each cycle.

The algorithm followed in the two-stage cycle is shown in Figures 2.24 and 2.25. These figures were developed under the assumption that the rod of the output cylinder moves in the direction towards side A (see Figure 2.2). The direction of the displacement of the output cylinder rod is controlled by the timing of the solenoid valves with respect to the operation of the stack. During the first stage, the piezoelectric stack is under positive displacement (pushing or forward stroke) while valve B is open and valve A is closed. Then, during the second stage, the piezoelectric stack is under negative displacement (pulling or backward stroke) while valve A is open and valve B is closed. One way of understanding



Figure 2.24: Model B used during the first stage.

these figures, is to relate them to the open path of fluid the stack "sees" during each stage. Also note that if the same coordinate is used for the output, then the direction for positive output displacement in the second stage, is as shown in the next figure.



Figure 2.25: Model A used during the second stage.

Once the end of the second stage is achieved then one complete cycle has been performed. At this moment, the coefficients of the state space model for the hydraulic system are updated with respect to the final state of the system. To be more specific, it is the coefficients that are related to the output cylinder the ones that are updated. Further detail is covered in *Chapter* 4.

2.7 Controlled Valve Dynamics: Operation and Effects

One important aspect that is not part of the hydraulic system but has a considerable effect on its input, is the dynamic characteristics of the solenoid valves along with the way they are employed. Although this section focuses on the solenoid valves used in the experimental setup, it is also valid for most electrically controlled valves that are coupled to a mechanical system. These usually involve a time constant to reach steady state. The solenoid valves used consist of a piston actuated valve that is within a coil or an inductor, and therefore, its response characteristics depend on the mass of the piston and the counteracting spring, the inductor, and the load to which is subjected. The important aspect for this discussion, is not the constitution of the valve, but the fact that depending on its constitution, there are two main aspects that are associated with it. The first one is the time response, which determines how fast the valves respond. The second aspect is the frequency response, or frequency range under which the valve operates effectively.

The solenoid valves used in the benchtop test setup unit are normally closed, and therefore a voltage is required to keep them open. Figure 2.26 relates the voltage input to the solenoid valve with its output, or actual state. The output figure represents a generic curve. The displacement of the piston in the valve may not be linear, but the point is to illustrate that there is a period of time needed to move from the closed position to the open one, and vice versa.



Figure 2.26: Generic representation of the time response of a solenoid valve.

The magnitude of the *transition time* (t_{tr}) is related to the time response of the solenoid valve. The implications of having a *transition time* in the complete execution of an opening or a closing operation will become relevant when the unit is under operation, and both valves are set under different timing patterns. These aspects are discussed in *Chapter* 4, under the two-stage cycle model.

Furthermore, note that for a given transition time, the frequency of operation may be increased up to the point were the piston is open or closed only for an instant and the resulting output curve reduces to a triangular waveform. This is, assuming that the piston moves in a linear fashion from one of the positions to the other. But again, the point is that the operation of the valves at this frequency will not ensure an effective response.



Figure 2.27: Effect of an increased operating frequency on the time response.

Further increase of the operating frequency will result in a similar triangular waveform except that the peak of the output curve, will never achieve the fully open condition. The conclusion is that valves operated under these frequencies will always be "somewhat" open. Operation near the vicinity of a period that is twice the magnitude of the transition time will not ensure a proper response from the valve. Thus, a critical frequency can be defined, and it is related to the transition time in the following manner:

$$F_{valvecr} = \frac{1}{2t_{tr}} \tag{2.65}$$

This critical frequency is related to the frequency response of the valve, and therefore it provides an approximate range of frequency to which the operation of the valve is effective.

The solenoid valves currently used in the experimental setup were tested (under no load) and the results suggested that a cyclic full opening and closing was not achievable at a frequency of 10Hz or higher. Therefore the corresponding transition time expected is of 0.05 seconds. Nonetheless, the critical frequency may reduce as the valve operates under loaded conditions (in the presence of pressurized fluid) and therefore the corresponding transition time may increase. Measured data confirmed the ineffective operation of the valves at frequencies close to 10 Hz. Thus, the frequency response of the valves used became a limitation for the operation of the piezohydraulic unit. Although the transition time is related to the frequency response, additional effects are to be considered when both valves

are used in the operation of the unit, with different phasing or timing patterns. These aspects are covered in Chapter 4.

2.8 Summary

The basic systems that compose the piezohydraulic unit have been introduced, along with their corresponding models. In summary, the most important aspects and some final observations are outlined.

The power amplifier should be matched with the piezoelectric stack in order to prevent operating limitations. The piezoelectric stack serves as the piston pump actuator that couples the electrical and mechanical systems. Its output capabilities (force and displacement) along with the operation of the solenoid valves will affect the speed of response of the output hydraulic cylinder.

The mechanical and hydraulic systems are modeled in a lumped parameter fashion. The introduction of constraints through the use of Hamilton's principle, is performed in order to couple both systems. Furthermore, the analysis of the hydraulic system follows a two-stage cycle algorithm, with two different models used for each stage.

And finally the controlled valve dynamics are introduced. Their operation and their effect on the input to the hydraulic system is analyzed. The following chapter will explain the development of the model used to represent the hydraulic system of the piezohydraulic unit.

Chapter 3

Lumped Parameter Fluid System

3.1 Introduction

The purpose of this chapter is to link the lumped mass type of analysis used to obtain a lumped parameter model of a fluid, to the governing equations of a fluid system that are applied to a control volume. In a lumped parameter model, the fluid system is divided into lumps, with a lumped mass and average parameters such as velocity and pressure. Then, the system elements are obtained by applying conservation of mass and Newton's Law to the lump of fluid. This type of analysis is the approach taken by Doebelin (1972) and in similar derivations found in other texts, and it yields to the definition of the fluid system elements of fluid resistance, capacitance (compliance) and inductance (inertance). It is of interest to link these system elements to the governing equations of a fluid system, and to clearly know and understand the assumptions made, so then each system element can be used to construct a lumped parameter model of a fluid element under the respective circumstances.

3.1.1 Analogous Systems

Before proceeding with the details of the lumped parameter model for the fluid, it is necessary to discuss the topic of analogous systems, and how it constitutes a tool to obtain an approximate electrical lumped parameter model of the fluid system, and then used to convert it or transform it into an analogous mechanical system. In many cases electrical, mechanical and fluid systems can be described with equivalent differential equations and equivalent or analogous variables. The result is that analogous systems have similar solutions and it is an additional tool that can be used to extend the solution of one particular system to all the analogous systems. This is done by using the same differential equations along with the corresponding analogous variables. However, there are two types of analogous systems. The force-current analogy "relates the analogous through- and across-variables of the electrical and mechanical systems", as described in Dorf and Bishop (1995). The second type of analogy is known as the force-voltage analogy, and it relates the velocity and current variables of a system.

Table 3.1: Force-Voltage Analogy.

Mechanical		Electrical		Fluid or Hydraulic	
M	(mass)	L	(inductance)	I_f	(inductance)
b	(damping)	R	(resistance)	\dot{R}_{f}	(resistance)
K	(stiffness)	1/C	(1/capacitance)	$1/C_f$	(1/compliance)
x	(displacement)	Q	(charge)	A	(volume)
ż,	(velocity)	i	(current)	q	(flowrate)
F	(force)	ΔV	(voltage)	ΔP	(pressure difference)

In this chapter, the elements of fluid resistance, fluid inertance or inductance, and fluid compliance or capacitance are derived. Then they are used to develop an equivalent electrical circuit to a lump of fluid. This equivalency is based on the force-voltage analysis, and the relationship between the analogous variables is shown in Table 3.1.

Table 3.2: System Analogy

	Resistance/	Capacitance/	Inductance/
System	Damping	Stiffness	Mass
Mechanical	$F = C_b \dot{\mathbf{x}}$	F = k x	$F = M \frac{d\dot{\mathbf{X}}}{dt}$
Electrical	$\Delta V = R i$	$\Delta V = \frac{1}{C} \int i dt$	$\Delta V = L \frac{di}{dt}$
Hydraulic	$\Delta P = R_f q$	$P = \frac{1}{C_f} \int q dt$	$\Delta P = I_f \frac{dq}{dt}$

The analogous variables can then be used to construct equivalent differential equations among the analogous systems. Starting with the known components of the mechanical elements of damping, stiffness and inertia (mass), it is possible to obtain equivalent equations in the electrical and fluid system with the use of the analogous variables. The result is shown in Table 3.2. Finally, the system analogy is used to obtain a mechanical system that is analogous to the electrical representation of a lump of fluid.

3.2 Fluid Model: Description and Assumptions

Consider the flow of a fluid in a constant area pipe as in Figure 3.1. Due to viscous effects, a fluid must satisfy the no slip condition at the wall, and therefore the velocity profile is not uniform over the cross sectional area, A. But, even though the velocity, V, and the pressure, P, do vary from point to point over the flow cross section in a real fluid, the lumped parameter model is based on a one dimensional flow model in which the velocity and pressure are uniform over the area. Thus, the average velocity and pressure correspond to the values at any point in the cross section. In a lumped parameter analysis, the pipeline is broken into segments. Within each segment or lump, pressure and velocity may vary with time, but are also assumed to be uniform over the volume of the lump. In the same manner, the density ρ , is assumed to be uniform over the entire lump. Doebelin (1972) uses these assumptions and considers the behavior of one typical lump (the n^{th}) to obtain the definitions of the basic fluid elements.



Figure 3.1: Lumped model of a fluid pipeline [Doebelin (1972)].

Some derivation and the definition of the fluid elements of resistance, capacitance and inductance, can be found in several texts, such as in Dorny (1993), Lindsay and Katz (1978) and in Doebelin (1972). Nonetheless, in this chapter, these fluid elements are rigorously derived from the governing equations of a fluid system. In order to do so, and in a slightly different approach, we will consider a lump as our control volume. It is like a local control volume type of analysis. As shown in Figure 3.2, X_{n-1} are the parameters entering the n^{th} control volume and X_n represent the parameters leaving the control volume. By using a lumped model approach, some properties such as density, are assumed to be uniform within the lump. They are also denoted as X_n , where X is the property of the n^{th} lump. Changes of the n^{th} parameter X_n are denoted as dX_n . Changes between the n^{th} and the previous lump n-1 are represented by dX. Finally, for the definition, explanation and further detail on the governing equations of a fluid system refer to Munson et al. (1998).



Figure 3.2: The n^{th} lump as a control volume.

3.3 System Elements

As discussed in the previous chapter, there are three basic elements for a hydraulic system. The fluid's compliance, friction and inertia have electrical analogies of capacitance, resistance and inductance, respectively. In the next subsections, the fluid elements are obtained through the use of the continuity equation, the energy equation and the equation for the conservation of momentum. These form part of the governing equations of a fluid flow and they are stated in any fluids book. The text used and a good reference is Munson et al. (1998). By applying a certain set of assumptions for each case, each governing equation can be reduced to the form of the set of hydraulic equations shown in Table 3.2, where a constant is a function of pressure (P) and either the flowrate (q), its integral $(\int q dt)$, or its derivative (dq/dt). These constants are then defined as either the fluid capacitance, fluid

resistance or the fluid inductance. It is the constant capacitance, resistance and inertance what makes a lumped parameter model a linear one. It approximates the behavior of the fluid around an operating point. Otherwise, the analysis of a fluid under general circumstances and not around an operating point would require a model to incorporate a varying capacitance, resistance and/or inductance.

3.3.1 Fluid Capacitance

The fluid capacitance is related to the compliance of the liquid. Real fluids, including liquids, are to some point compressible. This compressibility shows as a mass storage or a mass release and becomes the difference between the amount of mass in and mass out of the control volume. Equation (3.1) is the general continuity equation for a fluid in a control volume. The first term represents the time rate of change of mass inside the control volume (unsteady term). The second term represents the mass flowrate flux through the boundaries of the control volume (control surfaces).

$$\frac{\partial}{\partial t} \int_{cv} \rho \, d\forall \, + \, \int_{cs} \rho(\vec{V} \cdot \vec{n}) \, dA = 0 \tag{3.1}$$

As mentioned previously, for a lumped parameter model, the properties of a corresponding lump are assumed to be uniform throughout the entire volume. Thus, by assuming density to be uniform through the entire volume, the first integral reduces to a mass rate of change. For an incompressible fluid, the first term would not manifest itself and the mass flowrates would be equal to one another.

$$\left(\frac{dM}{dt}\right)_{cv} + \sum \left(\rho AV\right)_{out} - \sum \left(\rho AV\right)_{in} = 0$$
(3.2)

By adopting the notation established for the control volume of the n^{th} fluid lump shown in Figure 3.2, equation (3.2) can be written in the form:

$$(\rho_n A V_n - \rho_{n-1} A V_{n-1}) dt = -dM$$
(3.3)

where dM can be written as $\rho_n d\forall$, since it is a change of mass in the n^{th} lump and its density, ρ_n , is assumed to be uniform. But to continue with our linear lumped model approach, it is now necessary to assume that the density change $(d\rho = \rho_n - \rho_{n-1})$ is small around some operating point. This assumption will be referred to as the small density change assumption (SDC). Therefore, $\rho_{n-1} = \rho_n = \rho$ and equation (3.3) becomes:

$$(A V_n - A V_{n-1}) \rho dt = -\rho d \forall$$

$$(3.4)$$

The term $d\forall$ is related to the compressibility of the fluid, which in turn is described by the bulk modulus. Also referred to as the bulk modulus of elasticity, it states the differential pressure change dP needed to create a differential change in volume, $d\forall$, of a volume \forall , for that particular fluid. It is a measure then, of how easily can the volume (and therefore the density) of a certain mass of fluid change with a change in pressure. It is a property that can be estimated experimentally, and it is defined as [Munson et al. (1998)]:

$$B = -\frac{dP}{d\forall / \forall} \tag{3.5}$$

The bulk modulus has units of pressure and a large value indicates that a fluid is relatively incompressible. Using the flowrate definition, q = V A, and substituting the expression for $d\forall$ (obtained from the definition of the bulk modulus) into equation (3.4),

$$(q_n - q_{n-1}) dt = \frac{\forall dP_n}{B}$$
(3.6)

Furthermore, since the volume of a lump and the bulk modulus of a fluid remain constant with time, it is possible to solve for dP, integrate both sides, and obtain the expression:

$$\int dP_n = \int \frac{1}{\frac{\forall}{B}} (q_n - q_{n-1}) dt$$
(3.7)

which can be simplified into:

$$P_n = \frac{1}{\frac{\forall}{B}} \int q \, dt \tag{3.8}$$

where P_n is the pressure across the n^{th} lump, and $q = q_n - q_{n-1}$ represents the amount of flow that is either stored or released at that particular pressure. By comparing equation (3.8) with the corresponding expression in Table 3.2, the fluid capacitance is defined as:

$$C_f = \frac{A l}{B} \quad , \quad \left[\frac{m^2 m}{Pa} = \frac{m^5}{N}\right] \tag{3.9}$$

where A is the cross-sectional area of the lump, and l, its length. The fluid capacitance defined is valid for a uniform, one dimensional flow, with small density changes around an operating point.

3.3.2 Fluid Resistance

The fluid resistance is related to the viscous effects of a fluid flow. It is associated to the energy dissipation due to frictional losses and other minor losses (due to the geometry of the fluid flow). The effect of these losses is a pressure drop in the fluid flow, that is also related to the amount of fluid flow or flowrate. Equation (3.10) is the energy equation for a fluid in a control volume, and it is derived from the 1^{st} Law of Thermodynamics.

$$\frac{\partial}{\partial t} \int_{cv} e \rho \, d\forall + \int_{cs} e \, \rho(\vec{V} \cdot \vec{n}) \, dA = \dot{\mathbf{Q}}_{net_{in}} + \dot{\mathbf{W}}_{shaft_{in}} \tag{3.10}$$

Assuming a steady , one dimensional, and compressible fluid flow, the energy equation can be expressed as:

$$\dot{\mathbf{m}}\left[\Delta u + \Delta(P/\rho) + \Delta V^2/2 + g\Delta z\right] = \dot{\mathbf{Q}}_{net_{in}} + \dot{\mathbf{W}}_{shaft_{in}}$$
(3.11)

where Δu is the change in internal energy for the fluid. The rest of the terms in brackets account for the total change in energy. For a fluid flow in a pipe, the rate of work is zero $(\dot{W} = 0)$ and for our analysis, the amount of heat transfer as well as the gravitational effects are neglected ($\dot{Q} = 0, \Delta z = 0$). Then, equation (3.11) reduces to:

$$\frac{u_{out} - u_{in}}{g} + \frac{P_{out}}{\rho_{out} g} - \frac{P_{in}}{\rho_{in} g} + \frac{V_{out}^2 - V_{in}^2}{2g} = 0$$
(3.12)

Equation (3.12) is in head form, and it has units of length. $\Delta(P/\rho)/g$ is known as the pressure head, while $\Delta u/g$ is defined as the head loss, h_L . It is the head loss then, the term that accounts for losses due to friction and to changes in the geometry of the flow. Thus, the definition of head loss can be substituted into equation (3.12). Also, note that a steady process has been assumed (when deriving equation 3.11). Therefore, for a constant area pipe, along with the steady and SDC assumption, the continuity equation (3.2) reduces to $V_{in} = V_{out}$ and $\rho_{in} = \rho_{out}$; and equation (3.12) reduces to:

$$h_L = \frac{P_{in} - P_{out}}{\rho g} \tag{3.13}$$

By dividing both sides by the flowrate, and by using the notation in Figure 3.2, the previous equation becomes:

$$\frac{\rho g h_L}{V_n A} = \frac{P_{n-1} - P_n}{q_n}$$
(3.14)

and rearranging:

$$\Delta P = \left(\frac{\rho g h_L}{V_n A}\right) q_n \tag{3.15}$$

By comparing equation (3.15) with the corresponding expression in Table 3.2, the fluid resistance is defined as:

$$R_f = \frac{\rho g h_L}{V A} \tag{3.16}$$

where the head loss coefficient, h_L , is obtained with experimental parameters and equation (3.17). The first term represents the frictional losses while the second term represents a summation of the minor losses in the system. Minor losses account for changes in the pipeline geometry and changes in the cross-sectional area (bends, valves, tees, etc).

$$h_L = f \frac{l}{D} \frac{V^2}{2g} + \sum K_L \frac{V^2}{2g}$$
(3.17)

For the first term, the frictional effects are given by the friction factor, f. It is a function of the nature of the fluid and the properties of the flow (determined by the Reynolds number), and the roughness (ε) of the pipe relative to its size. The Moody chart shows the functional dependence of f on Re and ε/D and it can be found in any fluids book. For laminar flow (Re < 2100) the friction factor has a linear relationship and it is only a function of the Reynolds number:

$$f = \frac{64}{R_e} \tag{3.18}$$

where the Reynolds number, Re, is a nondimensional parameter that relates inertial effects to viscous effects, and it is defined as:

$$Re = \frac{\rho V D}{\mu} \tag{3.19}$$

In the case of turbulent flow, the *Colebrook formula* or other equations can be used. But for our analysis, a laminar flow will be assumed. For further details refer to Munson et al. (1998).

For the second term of equation (3.17), the minor loss coefficient (K_L) is strongly dependent on the geometry of the component considered as well as on the Reynolds number.

For very large Reynolds number, inertial effects dominate and the head loss coefficient becomes only a function of geometry. Thus, for most practical cases, the loss coefficient is determined by experimental means (where the flow has a high Re) and it is tabulated with the corresponding geometry. For a low Reynolds number flow, it is desirable to include its effect (Re) on the loss coefficient (K_L). This is achieved by expressing the loss coefficient indirectly in terms of an equivalent length to diameter ratio of a straight pipe, such that:

$$K_L = f\left(\frac{l}{d}\right) eq \tag{3.20}$$

Substituting equation (3.20) into the expression for the head loss coefficient (equation 3.17), and then into equation (3.16),

$$R_f = \frac{\rho g}{VA} \left(f \frac{l}{D} \frac{V^2}{2g} + f \sum \left(\frac{l}{d} \right)_{eq} \frac{V^2}{2g} \right)$$
(3.21)

Furthermore, by substituting equations (3.18) and (3.19) into the equation above, and by assuming that $A = \frac{\pi}{4} D^2$, then the resulting expression for the resistance of a given lump of fluid becomes:

$$R_f = \frac{128\,\mu\,l}{\pi\,D^4} + \frac{128\,\mu}{\pi\,D^3} \sum \left(\frac{l}{d}\right)_{eq} \quad , \quad \left[\frac{N\,s}{m^2}\,\frac{m}{m^4} = \frac{Kg}{m^4\,s}\right] \tag{3.22}$$

which it can be also expressed as

$$R_f = \frac{128\,\mu}{\pi\,D^4} \left(l + D\,\sum \left(l/d\right)_{eq} \right) \quad , \quad \left[\frac{Kg}{m^4\,s}\right] \tag{3.23}$$

Finally, it is important to recall that equation (3.23) is valid for a steady, one dimensional, essentially incompressible, adiabatic, laminar flow.

3.3.3 Fluid Inductance

The fluid inductance is also referred to as fluid inertance, since it is associated with the inertial effects of a lump of fluid. In Doebelin (1972) the inertia element is derived by applying Newton's 2^{nd} Law to a mass lump of fluid. Following our local control volume type of approach, we start with the general conservation of momentum equation:

$$\sum \vec{F}_{cv} = \frac{\partial}{\partial t} \int_{cv} \vec{V} \rho \, d\forall \, + \, \int_{cs} \vec{V} \rho(\vec{V} \cdot \vec{n}) \, dA \tag{3.24}$$

where the first integral represents the time rate of change of momentum within the control volume, and the second term reflects the momentum flux through the boundaries of the control volume. In accordance with our lumped type of control volume model, we assume that the density and velocity are uniform within the lump itself, and equation (3.24) becomes:

$$\sum \vec{F}_{cv} = \frac{d}{dt} (\vec{V} \rho \,\forall) + \sum (\dot{\mathbf{m}} \vec{V})_{out} - \sum (\dot{\mathbf{m}} \vec{V})_{in}$$
(3.25)

By assuming that density changes with time are small around an operating point, and by applying the equation in the x-direction (one dimensional flow):

$$\sum F_x = \rho \forall \, \frac{dV}{dt} \tag{3.26}$$

Note that the momentum flux terms do not manifest themselves anymore. This is because for an "essentially" constant density the continuity equation yields to equal mass flowrates and velocities in and out of the control volume. Applying the previous equation to the n^{th} lump shown in Figure 3.2:

$$AP_{n-1} - AP_n - F_f = \rho A l \frac{dV_n}{dt}$$
(3.27)

where F_f is the friction force the pipe exerts on the fluid element, and it is obtained by multiplying equation (3.15) by the cross-sectional area of the lump, A. Substitution in the equation yields to:

$$A(P_{n-1} - P_n) - R_f A q_n = \rho l \frac{dq_n}{dt}$$
(3.28)

If the resistance (friction) is neglected, then equation (3.28) reduces to:

$$P_{n-1} - P_n = \Delta P = \frac{\rho l}{A} \frac{dq_n}{dt}$$
(3.29)

By comparing equation (3.29) with the corresponding expression in Table 3.2, the fluid inductance is defined as:

$$I_f = \frac{\rho l}{A} \quad , \quad \left[\frac{\frac{Kg}{m^3}m}{m^2} = \frac{Kg}{m^4}\right] \tag{3.30}$$

Furthermore, it is valid for the same assumptions under which the capacitance was derived. That is, one dimensional flow and small density changes (w.r.t. time) around an operating point. Finally, note that equation (3.28) contains both the resistance (friction) and the inductance (inertia). If the inductance is neglected, then the expression for resistance (equation 3.15) is obtained.

3.4 Additional Considerations

3.4.1 Effective Bulk Modulus

As discussed previously, in the derivation of the fluid capacitance, the bulk modulus relates the effects of a pressure change to the volume change of a given volume of fluid. It is a measure of the 'stiffness' of the fluid. And a small amount of entrained air can result in a great reduction of the stiffness of a hydraulic fluid. Also, depending on their geometry and the internal pressure, metal pipes and other elements within a hydraulic system may also deform slightly and therefore exhibit some compliance. Furthermore, accumulators are specifically designed to introduce high compliance or capacitance (and therefore, lower stiffness) in a desired location of the hydraulic line. They become fluid energy storage devices that are used as a short term fluid supply, to damp out transients, and to assist the pump, among other things. For our analysis, accumulators will not be considered. Their analysis can be found in Chapter 4 of Doebelin (1972).

Equation (3.5) can be arranged in the form of $\Delta \forall = -\frac{\forall}{B} \Delta P$. The negative sign indicates that for an increase in pressure by ΔP , there will be a decrease in volume by $\Delta \forall$. Accounting for entrained air, as an x percent of the total volume \forall , then the effective volume change can be defined as:

$$\Delta \forall_e = -\left[\frac{(1-x)\forall}{B_f} + \frac{x\forall}{B_g}\right] \Delta P \tag{3.31}$$

In addition, it is possible to include the flexibility of the pipes. Thus, lets consider briefly the strain and stresses in pipes carrying pressurized fluids and relate them to the effective change of volume.



Figure 3.3: Stresses in a pressurized cylindrical vessel.

The stress of a pipe is related to its strain, or percentage of deformation, through the modulus of elasticity (E). Assuming that the deformation or change of volume of the
fluid is equal to that of the pipe, then the strain can be written as $\epsilon = -\Delta \forall / \forall$, where \forall and $\Delta \forall$ correspond to the fluid. The negative sign corresponds to our definition of $\Delta \forall$ as a negative change in volume. Thus, by substituting this expression into Hooke's Law,

$$\sigma = E \epsilon \tag{3.32}$$

and solving for the change in volume, then

$$\Delta \forall = -\frac{\forall}{E} \, \sigma \tag{3.33}$$

For a cylindrical pressure vessel of internal radius a, external radius b and internal pressure P, the maximum stresses occur at the inner surface, where r = a [Shigley and Mitchell (1993)]. Their magnitudes are

$$\sigma_t = P \frac{b^2 + a^2}{b^2 - a^2} \tag{3.34}$$

$$\sigma_r = -P \tag{3.35}$$

where the longitudinal stress has been neglected and σ_t and σ_r represent the tangential and radial stress, respectively. Equation (3.34) can be written in terms of the internal diameter D, and the pipe's thickness t; by substituting a = D and b = D + 2t. Further manipulation will yield to:

$$\sigma_t = P \frac{2D^2 + 4tD + 4t^2}{4tD + 4t^2} \tag{3.36}$$

Both, tangential and radial stresses become a function of ΔP for deformations that correspond to a change in the internal pressure P. By using equations (3.36),(3.35) and (3.33), then the total change in volume of a fluid within a cylindrical vessel becomes:

$$\Delta \forall = -\left[\frac{1}{E} \left(\frac{2D^2 + 4tD + 4t^2}{4tD + 4t^2} - 1\right)\right] \forall \Delta P$$
(3.37)

Note that as the thickness increases, the amount of deformation (represented by the change in volume) reduces, up to the point where the terms $4tD + 4t^2$ become much greater than $2D^2$ and the change in volume becomes $\Delta \forall \approx 0$. On the other hand, as the thickness decreases, the amount of change in volume increases, up to the point where the thickness of a pipe is about 5%, or less, of its radius. "Then the radial stress which results from pressurizing the vessel is quite small compared to the tangential stress. Under these conditions the tangential stress can be assumed to be uniformly distributed across the wall thickness. When this assumption is made, the vessel is called a thin-walled pressure vessel" [Shigley and Mitchell (1993)]. Since for our analysis the pipes are connected to other elements (they are not closed at the ends), then our assumption for the longitudinal stress still holds and we only consider the tangential stress. Known also as the hoop stress, it is defined as

$$\sigma_t = \frac{PD}{2t} \tag{3.38}$$

Then, similar to the previous approach for a pressurized cylinder, the change in volume of a fluid within a thin walled vessel is

$$\Delta \forall = -\left[\frac{D}{2\,t\,E}\right] \forall \Delta P \quad , \quad t \le 0.05D \tag{3.39}$$

Thus, in general, the volume change inside a pressurized vessel can be written as:

$$\Delta \forall = - [C_{pv}] \ \Delta P \tag{3.40}$$

where C_{pv} is either one of the constants within the brackets of equations (3.37) and (3.39). Then adding to the total effective volume change $\Delta \forall_e$ of equation (3.31) the effect of the compliance in a pressurized vessel, the total effective volume becomes:

$$\Delta \forall_e = -\left[\frac{(1-x)}{B_f} + \frac{x}{B_g} + C_{pv}\right] \forall \Delta P \tag{3.41}$$

Finally, by using the definition of the bulk modulus (equation 3.5) then the *effective bulk* modulus is defined as:

$$B_e = \left[\frac{(1-x)}{B_f} + \frac{x}{B_g} + C_{pv}\right]^{-1}$$
(3.42)

where B_e accounts for the stiffness of the fluid (B_f) , the percentage of air entrained in the system (x) along with its stiffness B_g , and the compliance of the pressurized vessel (C_{pv}) . Note that the percentage of air will dramatically affect the effective bulk modulus. On the other hand, the contribution of the pipe's compliance is relatively small and its inclusion depends on the nature of the analysis.

3.4.2 Equivalent Fluid Mass

As mentioned in Section 3.2, the lumped parameter model assumes a uniform velocity profile across the cross section of the fluid flow. But in reality, the flow of a fluid exhibits various profiles, depending on the viscosity of the fluid, the nature of the flow (laminar or turbulent), and the frequency of the excitation. Taken from Doebelin (1972), Figure 3.4 shows the velocity profile for various flow conditions. In various applications, the basic equations of a fluid flow are altered so that they appear in terms of average velocities. This type of analysis is illustrated in Streeter (1961), where a constant is added in the equations of energy and conservation of momentum. In our case, the set of assumptions made in the derivation of the fluid system elements make the inductance, the only element that needs to be corrected. But from equation (3.30), it can be observed that the inductance is related through the density, to the mass of the lump of fluid. The inductance is related to the inertial effects of a lump of fluid, and the correction is performed so that the kinetic energy in both, the steady laminar flow case and the model's uniform case, is conserved.

As pictured for the steady laminar flow, the velocity profile is parabolic, and in Munson et al. (1998) it is written as:

$$V(r) = \left(\frac{\Delta P D^2}{16\mu l}\right) \left[1 - \left(\frac{2r}{D}\right)^2\right]$$
(3.43)

where ΔP corresponds to the pressure loss in a length l of the pipe due to viscous forces (friction). Furthermore, the pressure difference and the viscous forces are related through the wall shear stress, and the previous equation can be written as:

$$V(r) = \frac{\tau_w D}{4\mu} \left[1 - \left(\frac{r}{R}\right)^2 \right] = V_c \left[1 - \left(\frac{r}{R}\right)^2 \right]$$
(3.44)

where V_c is the maximum velocity of the profile and it is at the centerline (where r = 0). For the lumped parameter model, the velocity profile is uniform and it is based on the average velocity,

$$V_{avg} = \frac{\int \rho(\vec{V} \cdot \vec{n}) \, dA}{\rho \, A} \tag{3.45}$$



Figure 3.4: Velocity profiles for various pipe flow conditions [Doebelin (1972)].

and for a pipe flow, $V(r) = (\vec{V} \cdot \vec{n})$ and $dA = 2\pi r \, dr$. Then for the steady laminar flow case, the average velocity is

$$V_{avg} = \frac{\int_0^R \rho \, V_c \left(1 - \frac{r^2}{R^2}\right) \, (2\pi r \, dr)}{\rho \, \pi R^2} = \frac{V_c}{2} \tag{3.46}$$

Now it is possible to calculate the kinetic energy for both cases. In the steady laminar case, it is a function of the parabolic velocity,

$$KE = \frac{m[V(r)]^2}{2} = \frac{\rho Al}{2} [V(r)]^2$$
(3.47)

Realizing that the velocity varies across the cross sectional area, A, then the following integral is defined,

$$KE = \int \frac{\rho l}{2} [V(r)]^2 dA = \int_0^R \frac{\rho A l}{2} V_c^2 \left(1 - \frac{r^2}{R^2}\right)^2 (2\pi r) dr$$
(3.48)

and solved to obtain the kinetic energy in terms of the actual mass of the lump and the maximum velocity:

$$KE = \frac{1}{6}\rho Al V_c^2 = \frac{1}{6}mV_c^2$$
(3.49)

Similarly, for a uniform velocity profile that uses the average velocity of $V_c/2$, the kinetic energy is:

$$KE_m = \frac{m_m (V_c/2)^2}{2} = \frac{1}{8} m_m V_c^2$$
(3.50)

By equating both, the actual and the model's kinetic energy, the model's mass is found to be one third higher than the actual mass,

$$m_m = \frac{4}{3}m\tag{3.51}$$

Furthermore, from equation (3.30), the fluid inductance can be written as:

$$I_f = \frac{\rho l}{A} = \frac{m}{A l} \frac{l}{A} = \frac{m}{A^2}$$
(3.52)

Then, by using equations (3.51) and (3.52), the relationship between the model's inductance and the actual inductance of a fluid lump is also:

$$(I_f)_m = \frac{4}{3} I_f \tag{3.53}$$

Therefore for a steady laminar flow, the fluid element of inductance needs to be corrected so it is one third higher than the actual value.

3.5 Oscillating Fluid Flow

3.5.1 Operating Frequency

The results found in the previous section (equations 3.51 and 3.53) can be also applied in unsteady or oscillating flows (but steady in the mean) when the frequency is sufficiently low. "As frequency increases (Figure 3.4) the velocity profile becomes more square and the correct mass approaches the *physical* mass $\rho A l$. The inertance for laminar flow is thus always between $(4/3)\rho l/A$ and $\rho l/A$, the midpoint $(7/6)\rho l/A$ occurring at about $w = 50\mu/R^2\rho$ " [Doebelin (1972)]. Thus, the midpoint frequency in a sinusoidal laminar flow, within a pipe of radius R, can be expressed as:

$$f_c = \frac{25\mu}{\pi\rho R^2} \tag{3.54}$$

where f_c is denoted as the critical frequency. For sinusoidal laminar flow with frequencies below f_c , the inertance can be approximately corrected by 4/3 of the actual value. On the other hand, for frequencies higher than f_c , the inertance is close enough to the actual value and no correction is needed. Also note from equation (3.54), that the larger the area of the pipe, the faster the inertance of an increasing frequency sinusoidal laminar flow will approach the physical mass of $\rho A l$ and therefore the actual inertance of $\rho l/A$.

But in addition to f_c , another critical frequency needs to be defined and it will be denoted as $f_{c_{Re}}$. This term, $f_{c_{Re}}$, will represent the highest frequency for which an oscillating fluid flow will be laminar. This is useful, since our previous discussion of inductance correction has been developed under the assumption of laminar flow. Moreover, the same assumption applied for the derivation of the fluid resistance in Section 3.3.2. Under this section, and as shown in the Moody chart (found in Munson et al. (1998)), it has been established that laminar flows occur at a Reynolds number below of 2100. Thus from equation (3.19), laminar flow exists when

$$Re = \frac{\rho \, V D}{\mu} < 2100 \tag{3.55}$$

For our derivation of $f_{c_{Re}}$, an oscillating fluid flow can be obtained through the sinusoidal motion of a piston in a constant area pipe, as depicted in Figure 3.5



Figure 3.5: Oscillating fluid flow.

Denoting A_x as the magnitude of an oscillating displacement x, of frequency w, then the velocity is:

$$V = \frac{dx}{dt} = A_x w \sin(wt) = (2\pi A_x f) \sin(wt)$$
(3.56)

Then, by substituting the magnitude of the velocity into equation (3.19):

$$R_e = \frac{\rho \left(2\pi A_x f\right) D}{\mu} \tag{3.57}$$

The critical frequency $f_{c_{RE}}$ corresponds to the highest Reynolds number for which there is laminar flow, and if expressed in terms of the radius, R, then the previous equation becomes

$$\frac{\rho \left(2\pi A_x f_{c_{RE}}\right)(R/2)}{\mu} = 2100 \tag{3.58}$$

Finally, the critical frequency can be defined as

$$f_{c_{RE}} < \frac{2100\mu}{\pi A_x \rho R} \tag{3.59}$$

This result applies for a sinusoidal fluid flow, and below the value of $f_{c_{RE}}$ the flow is considered to be sinusoidal. Thus, the assumptions made in sections 3.3.2 and 3.4.2 require that the frequency of an oscillating fluid flow be below its corresponding critical value.

3.5.2 The Fluid Elements as a Function of Frequency

The fluid elements of capacitance, resistance and inductance were derived as an analogy to their electrical counterparts. As in any electrical circuit, the presence of a capacitor or an inductor adds phase information to the dynamics of the system, and the operating frequency becomes important. Following there is a brief discussion on the effect of the fluid elements with respect to the range of the operating frequency.

First, let's consider the capacitive effect. Equation (3.9) defines the capacitance as a function of (inversely proportional to) the bulk modulus of the fluid. Further on, Equation (3.42)defines the effective bulk modulus. Recall that the percentage of air will dramatically affect the effective bulk modulus. On the other hand, the contribution of the pipe's compliance is relatively small. Hence, the pipe's capacitance might seem of no importance. "To the contrary, however, the rapid shutoff of flow in a pipe causes a pressure surge owing to the interaction between the momentum of the fluid and the minute capacitance of the pipe and fluid. These pressure surges are observed as fluid hammer" [Dorny (1993)]. As described by Streeter (1961), the term fluid hammer or water hammer "is commonly used to cover all pressure transients in hydraulic pipelines that demonstrate both inertial and elastic effects". As a synonym for fluid transients, fluid hammer might not be relevant for a particular system, but the pipe's capacitance may still be important. This is because the general fluid element of capacitance is also analogous to its electrical counterpart, in the sense that the capacitive effect increases as the frequency of the network decreases $(Z_C = 1/-jwC)$. Thus, for a hydraulic system where the percentage of entrained air is relatively small and the frequency of excitation is low, then the capacitive effect of a pipe does become relevant. The capacitance of a thin walled vessel is the first one to consider, and whether to include or neglect the effect corresponding to a regular pressurized cylinder depends on the nature of the analysis.

Also, and in the same manner as with the fluid capacitance, the inertia of a fluid is analogous to the electrical inductance, in the sense that the inductive effect increases as the frequency is raised $(Z_L = jwL)$. Therefore, a corrected mass for the fluid inductance, does become relevant when from a fluids standpoint, you are operating below the critical frequency f_c , but from an electrical network standpoint, it is high enough such that the inductive effect can't be neglected.

Thus, the lumped parameter model analogous to an electrical network will be largely dependent upon the range of frequencies under consideration. Often, models exposed to low frequency excitations include only resistive and capacitive elements, but also include and inductive element when operating under high frequencies [Foster and Parker (1970)].

3.6 Fluid Capacitance, Resistance and Inductance Network

3.6.1 Single Lump of Fluid Model

In this section, many variables are represented with the same letter. Therefore, the notation used in Table 3.1 is used to avoid any confusion. Note that a variable, x, does not represent the same parameter as X.

Derived from the general conservation of momentum equation (in section 3.3.3), equation (3.28) can be written as:

$$P_{n-1} - P_n = R_f q_n + \frac{\rho l}{A} \frac{dq_n}{dt}$$

$$(3.60)$$

Using the definition of inductance (equation 3.53) and dropping the subscripts for the n^{th} lump, then the previous equation can be stated as:

$$\Delta P = R_f \, q + I_f \, \frac{dq}{dt} \tag{3.61}$$

Furthermore, by using the force-voltage analogy displayed in Table 3.1, the flowrate q is equivalent to a current i, and the pressure difference ΔP is analogous to a voltage drop ΔV . Then the previous equation becomes the voltage drop across both, a resistor and an inductor in series (as shown in Figure 3.6):

$$\Delta V = R_f \, i + I_f \, \frac{di}{dt} \tag{3.62}$$



Figure 3.6: Model of a lump of fluid (non-capacitive).

This electrical circuit is valid for some fluid systems, where there is a minor compliance that can be neglected and therefore no storage or release of mass occurs. For example, the flow of fluid through a rigid pipe in a system where transients or minor changes from an average output does not matter. Also remembering that the current is analogous to the flowrate, then it can be seen that the flowrate in, is equal to the flowrate out. This agrees with the continuity equation (3.1) first introduced in section 3.3.1:

$$\frac{\partial}{\partial t} \int_{cv} \rho \, d\forall \, + \, \int_{cs} \rho(\vec{V} \cdot \vec{n}) \, dA = 0 \tag{3.63}$$

In the absence of compliance in the system, there is no mass storage or release and the time rate of change of mass is equal to zero. This implies that mass is constant, and therefore density is constant (for a control volume). Thus the second term of equation (3.63) is left, with density, as a constant that does not manifest itself. Then, the flowrate balance is obtained by substituting the definition of flowrate (q = V A).

$$\rho \int_{cs} (\vec{V} \cdot \vec{n}) dA = 0$$

$$\sum (AV)_{out} - \sum (AV)_{in} = 0$$

$$\sum (q)_{out} - \sum (q)_{in} = 0$$
(3.64)

In order to account for the compliance in a system, it is necessary to use a capacitor in the way it is shown in Figure 3.7. This type of configuration has been used to model a lump of several fluid pipeline systems, such as water pipes, oil ducts, and arteries, among others. References are covered in the literature review presented in the *Introduction*.



Figure 3.7: Model of a lump of fluid.

As shown in the figure above, the current out, i_2 , is not necessarily the same as the current in, i_1 . It could be lower or even higher, depending on the charging or discharging effect of the capacitor. Thus, in the presence of compliance (modeled with a capacitor) there is some energy storage and/or release, and therefore the flowrates in and out of the lump are not equal. Nonetheless, it is very important to realize that this model (Figure 3.7) is different than the first one (Figure 3.6) in a very important aspect. The first model is derived from the equation of conservation of momentum and satisfies the continuity equation. The second model results from the addition of the capacitive effect in order to simulate the compliance represented by the first term of the continuity equation. In addition, recall from previous sections that the derivation of the fluid element of capacitance had a different set of assumptions than those used to obtain the fluid resistance and inductance. Thus, the result is a model that *approximates* the lump of fluid and the fluid dynamics in a pipeline.

3.6.2 Model of a Fluid Pipeline

Following the model of a lump of fluid (Figure 3.7) then a hydraulic pipeline can be modeled as shown in Figure 3.8. The current source represents a flow source, and it is a good analogy to the case shown in Figure 3.5 (as an alternating current source), where an oscillating piston defines the flowrate through the pipeline.



Figure 3.8: Analogous electrical model of a fluid pipeline.

The easiest way to analyze this circuit (for programming purposes) is to apply Kirchoff's voltage law (KVL). The principle behind it is that no energy is lost or created in an electric circuit and therefore the net voltage around a closed circuit is zero [Rizzoni (1996)]. By applying Kirchoff's voltage law to each loop, and by expressing each voltage drop in terms of the charge, Q, then the following set of equations is obtained:

$$L_1 \frac{d^2 Q_1}{dt^2} + \frac{1}{C_1} (Q_1 - Q_2) + R_1 \frac{dQ_1}{dt} = 0$$
(3.65)

$$L_2 \frac{d^2 Q_2}{dt^2} + \frac{1}{C_2} (Q_2 - Q_3) + \frac{1}{C_1} (Q_2 - Q_1) + R_2 \frac{dQ_2}{dt} = 0$$
(3.66)

$$L_3 \frac{d^2 Q_3}{dt^2} + \frac{1}{C_3} (Q_3 - Q_4) + \frac{1}{C_2} (Q_3 - Q_2) + R_3 \frac{dQ_3}{dt} = 0$$
(3.67)

Note that the current source, which represents the flow source, appears in the equation as the term dQ_1/dt . By using the force-voltage analogy shown in Table 3.1 it is possible to write an equivalent set of mechanical equations in terms of the elements of mass, spring and damping:

$$m_1 \ddot{\mathbf{x}}_1 + k_1 \left(x_1 - x_2 \right) + b_1 \dot{\mathbf{x}}_1 = 0 \tag{3.68}$$

$$m_2 \ddot{\mathbf{x}}_2 + k_2 (x_2 - x_3) + k_1 (x_2 - x_1) + b_2 \dot{\mathbf{x}}_2 = 0$$
(3.69)

$$m_3 \ddot{\mathbf{x}}_3 + k_3 \left(x_3 - x_4 \right) + k_2 \left(x_3 - x_2 \right) + b_3 \dot{\mathbf{x}}_3 = 0 \tag{3.70}$$

Then, by inspection of these equations, it is possible to construct the following mechanical system as the analogous system to the electrical circuit model shown in Figure 3.8.



Figure 3.9: Analogous mechanical model of a fluid pipeline.

Note that the input to the system, which represents the flow source, is \dot{x}_1 , the velocity of the first lump (from the mechanical system standpoint). Recall that from the forcevoltage analogy, a given flowrate is analogous to a given velocity in a mechanical system. Nonetheless, once the mechanical system is defined, then the input to the system may also be expressed in terms of a displacement, such as x_1 , instead of the velocity, \dot{x}_1 . Also note that if further lumps are to be added, then the next mass would be coupled to the stiffness k_3 . Otherwise, the spring corresponding to the last lump (in this case the 3^{rd} lump) would be also coupled to ground.

Thus Figure 3.9 represents a mechanical lumped parameter model of a fluid pipeline, with the elements of stiffness, damping and mass related to the electrical analogies of capacitance, resistance and inductance, respectively. Recall from equations (3.9), (3.23) and (3.30), and from the force-voltage analogy shown in Table 3.1, that:

$$C_f = \frac{A l}{B} \sim \frac{1}{k} \qquad , \qquad \left[\frac{m^2 m}{Pa} = \frac{m^5}{N}\right] \tag{3.71}$$

$$R_f = \frac{128\,\mu\,l}{\pi\,D^4} \left(1 + D\,\sum\,(l/d)_{eq} \right) \,\sim\, b \qquad, \qquad \left[\frac{N\,s}{m^2}\,\frac{m}{m^4} \,=\,\frac{Kg}{m^4\,s} \right] \tag{3.72}$$

$$I_f = \frac{\rho l}{A} \sim m \qquad , \qquad \left\lfloor \frac{\frac{Kg}{m^3} m}{m^2} = \frac{Kg}{m^4} \right\rfloor \tag{3.73}$$

Then the analogous stiffness, damping and mass for each lump of fluid, is defined as:

$$k = A^2 \frac{1}{C_f} = \frac{AB}{l} \qquad , \qquad \left[\frac{N}{m}\right] \tag{3.74}$$

$$b = A^2 R_f = 8 \pi \mu \left(l + D \sum_{eq} (l/d)_{eq} \right) , \qquad \left[\frac{Kg}{s} \right]$$
(3.75)

$$m = A^2 I_f = \rho A l \qquad , \qquad [Kg] \qquad (3.76)$$

Note that for equations (3.72) and (3.75) an area of $A = \frac{\pi}{4} D^2$ has been assumed.

In summary, the definitions stated in the set of equations (3.74), (3.75) and (3.76), along with the set of differential equations (3.68), (3.69) and (3.70), that describe the mechanical system shown in Figure 3.9, represent a three-lump model of the fluid in the following system:



Figure 3.10: Fluid-mechanical oscillating system.

The next step is to verify that the lumped parameter model converges as the number of lumps for a given fluid pipeline is increased. This is performed in the following section. Finally, once the convergence of the model is obtained, then it is possible to couple the lumped parameter model of the fluid pipeline to the dynamics of the stack and the piston used as the flow source, to the dynamics of the output cylinder (the unit's output actuator), and ultimately, also to the dynamics of the valves. This is covered in the next chapter, *Model of the PiezoHydraulic Unit*.

3.6.3 Lumped Model Convergence

A lumped parameter model of a fluid pipeline is similar to a finite element model of a beam, and in general, it is desired that as the number of lumps or elements is increased, the results of the model differ less and less and eventually, a further increase in the number of lumps would have no effect on the output or the results. This is known as the convergence of a lumped or finite element model, which is often the point at which the model best matches the exact theoretical solution or the corresponding experimental results. Usually the extent to which a model is required to match the exact solution depends on the nature of the problem. As the number of lumps or elements is increased, the dynamics of the modeled system and the actual system begin to match, starting with the slowest pole, and then following the higher frequency or faster poles. This is illustrated in Figure 3.11, where a third-order system is depicted. The first pole, p_1 , is the slowest or the lowest frequency pole, and eventually it converges to a given value. Faster poles p_2 and p_3 will also converge, but with a greater number of lumps used to model the system.



Figure 3.11: Pole location, and corresponding convergence (generic curves).

The convergence of the lumped parameter model for a fluid pipeline was performed on the system shown in Figure 3.10. The fluid pipeline is modeled with a mechanical system as shown in Figure 3.9 but initially with one lump, then with two lumps, and further on. Note that a one element system represents a second order system, and therefore it has two poles related to it. For a two lump system, there would be four poles, and so on.

Figure 3.12 is a representation of the algorithm followed for the convergence analysis. First, the fluid pipeline is modeled with one lump, and the values used for the stiffness (k_o) , damping (b_o) , and mass (m_o) elements are displayed in Table 3.3. Each part or component of the hydraulic pipeline (such as a tee, a bend or a pipe) is modeled with at least one lump. Then, by considering the setup of the piezohydraulic unit, the lowest stiffness of a part in the system was used as k_o . Furthermore, the addition of all the damping elements was used as b_o , and in the same manner, the total mass of fluid of the system, was used as m_o . The objective, is to start with an approximate one lump, second order model of the system in order to perform the convergence analysis. For now, it is not an attempt to obtain the number of lumps needed for the convergence of the hydraulic system in the piezohydraulic unit, but to verify that the model itself does converge. Once the lumped parameter model is proved to converge, then the convergence analysis of the hydraulic system as it is (currently it is not a one-part system but a multi-part system) can be performed.



Figure 3.12: Addition of lumps for the convergence analysis.

Note that an additional spring has been added at the input. The stiffness k_r , represents the rod and the piston elements through which the input to the hydraulic system is applied. For the purpose of this analysis, the input is considered to be the displacement x_{in} . Also note that as a function of the number of lumps, n, then the equivalent elements of stiffness, damping and mass are obtained through the expression shown in Table 3.3. This

Table 3.3: Equivalent stiffness, damping and mass elements.

One-lump equivalent system:	For two lumps $n = 2, 1 < i < n$:	For 'n' lumps, 1 < i < n :
	$k_i = k_o * 2$ $b_i = b_o / 2 \cdots$ $m_i = m_o / 2$	$\begin{array}{l} k_{i} \ = k_{o} \ ^{*} n \\ b_{i} \ = b_{o} \ / \ n \\ m_{i} \ = m_{o} \ / \ n \end{array}$

expression is obtained from the set of equations (3.74), (3.75) and (3.76), where the stiffness is inversely proportional to the length while the damping and the mass are proportional to the length. Thus, for a pipe of a definite length l, then the length of lump depends on the number of lumps used (l/n), and the stiffness element of each lump then becomes a proportional function of n. In the same manner, the damping and mass elements for each lump become inversely proportional to n, as shown in Table 3.3.

The analysis of the one-lump model up to the n^{th} lump model was performed using the state-space formulation. Thus, the *n* differential equations of an n^{th} lump system, can be formulated in the form of the matrix equation (2.52), used to describe the second-order system introduced in *Chapter* 2. Then the set of equations can be expressed in the state space form (equation 2.64), where the dynamics of the system are related to the eigenvalues of the "A" matrix. The eigenvalues represent the poles of the characteristic equation, and it is used to analyze the convergence of a system, as explained previously and shown in Figure 3.11.

In the figure below, the frequency magnitude of the slowest pole (the smallest in magnitude of the entire set of poles) is plotted versus the number of lumps used to define the system.



Figure 3.13: Variation of the slowest pole with respect to the number of lumps used.

The next figure shows the percent change in the magnitude of the frequency of the slowest pole as the result of the increase of the number of lumps by one unit. After the 11^{th} lump, the percent change in magnitude is less than 1%. Thus, 11 lumps become the

minimum number of elements required to at least model the low frequency dynamics of the system. Furthermore, a divergence in these results would indicate that there is an error in the model.



Figure 3.14: Percent change of the slowest pole versus the number of lumps used.

The effect of using additional lumps in the input-output relationship of the system has been also analyzed. This relationship is usually presented as a transfer function of the output y, to the input x, and expressed using Laplace transforms, as Y(s)/X(s), in order to relate the ratio to the frequency domain. In the convergence analysis, the input is the displacement x_{in} , while the output is the displacement of the last lump of fluid, x_n . But due to the set of differential equations involved in a multi-degree of freedom system, then the state space formulation for the input-output relationship has been used, defined as [Friedland (1986)]:

$$\frac{Y(s)}{X(s)} = C \ (sI - A)^{-1} B + D \tag{3.77}$$

were Y(s)/X(s) is known as the transfer function and it represents the frequency response of the system, with the assumption of zero initial conditions.

The next figure, shows the effect of additional lumps in the frequency response of the system. For these simulations, a sinusoidal displacement input is assumed. Looking at Figure 3.12, the input operates through an input stiffness (that represents the stiffness of the piston) and into the mass of the first lump. The output is assumed to be the displacement of the last lump.



Figure 3.15: Frequency response as a function of the number of lumps.

The resulting frequency response resembles that of a low pass filter. The dark blue curve represents a system modeled with one lump, the green curve corresponds to two lumps, the red one for three lumps, and so on. Note that as the number of lumps is increased, the DC component of the frequency response shifts down, the break frequency and the bandwidth increases, as well as the magnitude or the order of the rollover. Eventually, the frequency response curves converge and a further increase in the number of lumps has no considerable effect on the dynamics of the modeled system. For this system, the frequency response curves were plotted up to a number of lumps n = 12. From the figure, it is easy to observe that the system has *approximately* converged towards the last set of curves. Furthermore, knowing that less than a 1% change in the slowest pole is achieved after 11 lumps, and that further faster poles would require additional lumps, then it becomes clear that the low frequency dynamics dominate the system.

One last note, is that the stiffness, damping and mass used in these simulations are in the order of: $k \sim 10^4$, $b \sim 10^1$, and $m \sim 10^{-3}$, which corresponds to an approximate and equivalent second order system of the experimental setup of the piezohydraulic unit. Further simulations indicate that only significant changes in the order of these parameters would affect the minimum number of lumps required to model the low frequency dynamics of the system. The Matlab m.file *thesis_convergence.m* was developed and used for these simulations, and for further reference, it has been included in the appendix.

3.7 Summary

Following the system analogy presented at the beginning of the chapter, the fluid elements of resistance, capacitance and inductance have been derived from the governing equations used for a control volume and applied to the fluid in a pipeline. The lumped parameter model is based on a one-dimensional flow that divides the pipeline in lumps of volume with uniform properties, such as pressure and velocity. Once the model of a single lump of fluid has been obtained, then a mechanical system that model the entire fluid pipeline has been derived. Finally, the convergence of the model has been verified, and the results discussed.

Chapter 4

Model of the PiezoHydraulic Unit

4.1 Introduction

This chapter uses the models and equations of each of the systems presented in *Chapter* 2. It also uses the model of the hydraulic system that was discussed in *Chapter* 3, and it combines all the information in order to develop the model for the entire piezohydraulic system. The analysis of the hydraulic system, and therefore the analysis of the entire unit, will follow a two-stage cycle algorithm (as introduced in the *Hydraulic System* section of *Chapter* 2).

Also, the following terminology will be used:

- One-sided operation: operation of the unit with one valve open and the other one closed. The result is an oscillating motion of the hydraulic cylinder. Therefore, this type of operation is also referred to as *oscillating operation*, or under *oscillation*.

- Two-stage operation: operation of the unit with both valves opening and closing with a specified phasing or timing pattern. The result is a directed fluid flow through the hydraulic system, and therefore this type of operation is also denoted as the *pumping operation*.

- Single-ended cylinder: asymmetric hydraulic cylinder, since there is only one rod attached to one of the sides of the piston.

- Double-ended cylinder: symmetric hydraulic cylinder, with rods extending from both sides of the piston.

- First side: as it will become clear further on, at any given moment there will always be one valve closed and one valve open (regardless of a one-sided or a two-stage operation). The result, is that the diagram of the piezohydraulic unit in Figure 2.2 can then be represented as shown in Figure 4.1. The hydraulic system that is connected to the open valve, and therefore to the piezoelectric stack, is denoted as the *first side*, or the *oscillating side*.

- Other side: the hydraulic system that is to the other side of the piston (of the output cylinder) and ends at the closed valve. Furthermore, the corresponding elements of fluid stiffness, damping and mass are distinguished with those of the first side, by using the subscript "OS".

Finally, the terms *output cylinder* and *hydraulic actuator* or *hydraulic cylinder* are used interchangeably.



Figure 4.1: Representation of a double-ended piezohydraulic unit under one-sided operation.

Figure 4.1 is the one-sided representation of the piezohydraulic unit. A model of this representation is used to simulate the one-sided operation of the piezohydraulic unit.



Figure 4.2: Simulation of a pumping operation with a two-stage cycle model of the a double-ended piezohydraulic unit.

Furthermore, the model is derived in Section 4.2. Then, two different one-sided models are used in order to simulate the pumping operation of the unit, with the two-stage cycle process shown in Figure 4.2. Thus, one model is excited during the pushing stroke of the stack while the other one, during the pulling stroke. Following the same notation introduced in *Chapter 2*, *Model B* is used during the first stage since it is *Side B* the side with direct contact with the piezoelectric actuator. In the same manner, *Model A* is used during the second stage. A switch of models, *Model A* during the first (pushing) stage while *Model B* for the second (pulling) stage results in the opposite (downward) movement of the rod of the hydraulic cylinder. The two-stage cycle operation is discussed in Section 4.3.

4.2 One Sided Operation

Following the previous two-stage cycle example, for the purpose of this section, solenoid valve B will be assumed to be open while solenoid valve A is closed (*Model B*). Figure 4.3 shows the corresponding mechanical model of the entire piezohydraulic unit (as shown in



Figure 4.3: Lumped parameter model of the piezohydraulic unit.

Figure 4.1). The input to this system is the force exerted by the piezoelectric stack, F_{pzt} , which is a function of either the voltage or the charge across the stack itself. The output is the displacement of the piston of the output cylinder, x_c . The variable l_p refers to the number of fluid lumps used to define the first side, which in this stage is Side B. Similarly, l_{p-os} is the number of lumps used to define the other side, in this case, Side A. The unconnected links between some of the masses in the model are due to a separate model development (in the Matlab code) of some of the systems within the unit. And as discussed in *Chapter* 2 these are coupled through the introduction of constraints that would set both displacements across an unconnected link as equal to one another. For the model used and shown in Figure 4.3, three constraints will be required.

First it is necessary to derive the equations of motion that describe this system. The analysis of the free body diagram of the first mass, m_r , yields to the following equation:

$$m_r \ddot{\mathbf{x}}_r = F_{pzt} - k_r \, x_r + k_r \, x_{p_1} \tag{4.1}$$

For a piezoelectric stack that is voltage-controlled, equation (2.26) is used and its substitution into equation (4.1) results into:

$$\ddot{\mathbf{x}}_r = \left(\frac{-k_a - k_r}{m_r}\right) x_r + \left(\frac{k_r}{m_r}\right) x_{p_1} + \left(\frac{k_a x_o}{M_r}\right) V_{pzt}$$
(4.2)

Similarly, for a charge-controlled piezoelectric stack, equation (2.35) is used and its substitution into equation (4.1) results into:

$$m_r \ddot{\mathbf{x}}_r = F_1 Q_{pzt} - F_2 x_r - k_r x_r + k_r x_{p_1}$$
(4.3)

$$\ddot{\mathbf{x}}_r = \left(\frac{-F_2 - k_r}{m_r}\right) x_r + \left(\frac{k_r}{m_r}\right) x_{p_1} + \left(\frac{F_1}{m_r}\right) Q_{pzt}$$
(4.4)

The analysis of the free body diagram of the rest of the masses, will yield to the following set of pairs of equations. The first equation of a pair is the equation of motion following Newton's Law. The second equation expresses the equation of motion in a state space format. Also note that the notation used is consistent with the Matlab code developed. For the input cylinder piston's mass:

$$m_{p_1} \ddot{\mathbf{x}}_{p_1} = k_r \, x_{p_1} - k_r \, x_1 \tag{4.5}$$

$$\ddot{\mathbf{x}}_{p1} = \left(\frac{-k_r}{m_{p_1}}\right) x_{p_1} + \left(\frac{k_r}{m_{p_1}}\right) x_r \tag{4.6}$$

For the first mass of Side B, m_1 ,

$$m_1 \ddot{\mathbf{x}}_1 = -b_1 \dot{\mathbf{x}}_1 - k_1 \left(x_1 - x_2 \right) \tag{4.7}$$

$$\ddot{\mathbf{x}}_{1} = \left(\frac{-b_{1}}{m_{1}}\right)\dot{\mathbf{x}}_{1} + \left(\frac{-k_{1}}{m_{1}}\right)x_{1} + \left(\frac{k_{1}}{m_{1}}\right)x_{2}$$
(4.8)

For the second mass, m_2 ,

$$m_2 \ddot{\mathbf{x}}_2 = -b_2 \dot{\mathbf{x}}_2 - k_2 (x_2 - x_3) - k_1 (x_2 - x_1)$$
(4.9)

$$\ddot{\mathbf{x}}_{2} = \left(\frac{-b_{2}}{m_{2}}\right)\dot{\mathbf{x}}_{2} + \left(\frac{-k_{2}-k_{1}}{m_{2}}\right)x_{2} + \left(\frac{k_{2}}{m_{2}}\right)x_{3} + \left(\frac{k_{1}}{m_{2}}\right)x_{1}$$
(4.10)

Furthermore, after the second mass the corresponding equations for the rest of the masses follow the same pattern, up until the last lump for the side. In other words, for $2 < n < l_p$ the pairs of equations can be expressed as:

$$m_n \ddot{\mathbf{x}}_n = -b_n \dot{\mathbf{x}}_n - k_n \left(x_n - x_{n+1} \right) - k_{n-1} \left(x_n - x_{n-1} \right)$$
(4.11)

$$\ddot{\mathbf{x}}_n = \left(\frac{-b_n}{m_n}\right)\dot{\mathbf{x}}_n + \left(\frac{-k_n - k_{n-1}}{m_n}\right)x_n + \left(\frac{k_n}{m_n}\right)x_{n+1} + \left(\frac{k_{n-1}}{m_n}\right)x_{n-1} \quad (4.12)$$

For the last lump and mass (when n = lp) the subscript lp is used. Also, for the additional mass, the subscript fi is used. It represents a negligible 1% of the mass of the last lump, m_{lp} , but it has been introduced in order to have a second order equation related to the displacement at the end of the last spring for this side, k_{lp} . This displacement is shown as x_{fi} and it will be later eliminated as the model of Side B gets coupled with the piston's mass, M_c . Further on, following this notation, equation (4.12) can be written for the last lump as:

$$\ddot{\mathbf{x}}_{lp} = \left(\frac{-b_{lp}}{m_{lp}}\right) \dot{\mathbf{x}}_{lp} + \left(\frac{-k_{lp} - k_{lp-1}}{m_{lp}}\right) x_{lp} + \left(\frac{k_{lp}}{m_{lp}}\right) x_{fi} + \left(\frac{k_{lp-1}}{m_{lp}}\right) x_{lp-1}$$
(4.13)

Furthermore, the analysis of the free body diagram of the additional mass result in the equations:

$$m_{fi} \ddot{\mathbf{x}}_{fi} = k_{lp} \left(x_{lp} - x_{fi} \right) \tag{4.14}$$

$$\ddot{\mathbf{x}}_{fi} = \left(\frac{-k_{lp}}{m_{fi}}\right) x_{fi} + \left(\frac{k_{lp}}{m_{fi}}\right) x_{lp} \tag{4.15}$$

Continuing with the analysis of the piston of the output cylinder,

$$m_c \ddot{\mathbf{x}}_c = (-b_L) \dot{\mathbf{x}}_c - (k_L + k_{il}) x_c$$
 (4.16)

$$\ddot{\mathbf{x}}_c = \left(\frac{-b_L}{m_c}\right)\dot{\mathbf{x}}_c + \left(\frac{-k_L - k_{il}}{m_c}\right)x_c \tag{4.17}$$

The terms k_L and b_L represent static spring and damping loads that have been added, as shown in Figure 4.3. They are identified as static loads since they are attached to ground, or a constant reference point. For the simulations performed in the next chapter, these variables are set equal to zero. Nonetheless, they have been added $(k_L \text{ and } b_L)$ in the derivations, in order to enable the study of the effects of a load in future research. Also recall that the stiffness k_{il} is only used when a single ended cylinder is under operation. Therefore, the equation for a double sided cylinder under no load would reduce to only the mass or the inertia of the piston.

In the same manner as for the first side (Side B), for the other side (Side A) the corresponding equations of motion use the subscript "os", and the equations for the first mass, m_{os_2} , are expressed as:

$$m_{os1}\ddot{\mathbf{x}}_{os1} = -b_{os1}\dot{\mathbf{x}}_{os1} - k_{os1}\left(x_{os1} - x_{os2}\right)$$
(4.18)

$$\ddot{\mathbf{x}}_{os1} = \left(\frac{-b_{os1}}{m_{os1}}\right) \dot{\mathbf{x}}_{os1} + \left(\frac{-k_{os1}}{m_{os1}}\right) x_{os1} + \left(\frac{k_{os1}}{m_{os1}}\right) x_{os2}$$
(4.19)

For the second mass, m_{os_2} :

$$\ddot{\mathbf{x}}_{os_2} = \left(\frac{-b_{os_2}}{m_{os_2}}\right) \dot{\mathbf{x}}_{os_2} + \left(\frac{-k_{os_2} - k_{os_1}}{m_{os_2}}\right) x_{os_2} + \left(\frac{k_{os_2}}{m_{os_2}}\right) x_{os_3} + \left(\frac{k_{os_1}}{m_{os_2}}\right) x_{os_1}$$
(4.20)

From this point up to the lump before the last mass, the equations follow the same pattern as in equation (4.12). Thus, for $2 < n < (lp_{os} - 1)$ the equations are expressed as:

$$\ddot{\mathbf{x}}_{os_n} = \left(\frac{-b_{os_n}}{m_{os_n}}\right) \dot{\mathbf{x}}_{os_n} + \left(\frac{-k_{os_n} - k_{os_{n-1}}}{m_{os_n}}\right) x_{os_n} + \left(\frac{k_{os_n}}{m_{os_n}}\right) x_{os_{n+1}} + \left(\frac{k_{os_{n-1}}}{m_{os_n}}\right) x_{os_{n-1}}$$
(4.21)

Finally, for the last lump (when $n = lp_{os}$), the subscript lp_{os} is used and its corresponding equation is expressed as:

$$\ddot{\mathbf{x}}_{lp_{os}} = \left(\frac{-b_{lp_{os}}}{m_{lp_{os}}}\right) \dot{\mathbf{x}}_{lp_{os}} + \left(\frac{-k_{lp_{os}} - k_{lp_{os}-1}}{m_{lp_{os}}}\right) x_{lp_{os}} + \left(\frac{k_{lp_{os}-1}}{m_{lp_{os}}}\right) x_{lp_{os}-1}$$
(4.22)

Once all these equations are defined, they can be grouped in a matrix type of formulation just like equation (2.52), which describes the system shown in Figure 2.23, used as an example in Section 2.5.2. Actually, in the Matlab code developed, the model shown in Figure 4.3 is initially expressed in state space form. Then it is transformed into the Newtonian formulation. Once this formulation is obtained, then it is possible to introduce the following constraints:

$$\begin{array}{rcl}
x_1 &=& x_{p1} \\
x_{fi} &=& x_c \\
x_{os1} &=& x_c
\end{array}$$
(4.23)

Therefore the constraint matrix C, introduced in Section 2.5.2, is defined as shown in the following page. Again, note that by the introduction of the three constraints in the matrix equation, three degrees of freedom have been eliminated. Thus, variables x_1, x_{fi} , and x_{os1} do not manifest themselves in the new set of coordinates \hat{x} .

x_r			1	0	0	0	••	0	0	0	••	0	0			
x_{pi}			0	1	0	0		0	0	0		0	0		- س -	1
x_1			0	1	0	0		0	0	0	••	0	0		x_r	
x_2			0	0	1	0		0	0	0		0	0		x_{pi}	
x_3			0	0	0	1		0	0	0		0	0		x_2	
:			:			:		:		:		:	0		x_3	
x_{lp}			0	0	0	0		1	0	0		0	0		:	(, , ,)
x_{fi}		=	0	0	0	0		0	1	0		0	0		x_{lp}	(4.24)
x_{c}			0	0	0	0		0	1	0		0	0		x_c	
x_{os1}			0	0	0	1		0	1	0		0	0		x_{os2}	
r?				0	0	-		0	0	1		0	0		:	
				0	0				0		••		0		x_{lpos-1}	
•				0	0	•		•	0	•		•	0		x_{lp_os}	$\left \begin{array}{c} (ld-3) x \end{array} \right $
x_{lp_os-1}				0	0	1	••	0	0	0	••	1	0			. ,
x_{lp_os}	$ _{ldx 1}$		ΓU	0	0	T	••	U	0	U	••	T	0 -	d ld x (ld-3)		
\underline{x}		=						C							\hat{x}	

Furthermore, the set of equations derived in this section and expressed in a Newtonian matrix form such as in equation (2.52), can be transformed with the constraint matrix into the coupled set of equations:

$$\delta(\underline{\hat{x}}^T C^T) \left[M C \,\underline{\hat{\hat{x}}} + B C \,\underline{\hat{\hat{x}}} + K C \,\underline{\hat{x}} - fm \,Q_{pzt} \right] = 0 \tag{4.25}$$

Note that the matrix equation is the same as equation (2.56), except that the matrices M, B, K, and fm now correspond to the set of equations developed in this section. Also note that the input for the current/charge controlled system becomes the charge, Q_{pzt} .

Finally, following the same notation and the same procedure outlined in Section 2.5.2, the coupled set of equations for the model of the first stage (equation 4.25) can be then expressed in the state space form:

$$\begin{bmatrix} \frac{\hat{x}}{\hat{x}} \\ \frac{\hat{x}}{\hat{x}} \end{bmatrix} = \begin{bmatrix} 0 & I \\ Akm_C & Abm_C \end{bmatrix} \begin{bmatrix} \frac{\hat{x}}{\hat{x}} \\ \frac{\hat{x}}{\hat{x}} \end{bmatrix} + \begin{bmatrix} 0 \\ Blower_C \end{bmatrix} Q_{pzt}$$
(4.26)
$$\frac{\hat{x}}{\hat{x}} = A \qquad \frac{\hat{x}}{\hat{x}} + B \qquad \underline{u}$$

It is important to remember that this model in state space form represents the operation of the piezohydraulic unit with one valve open and one valve closed (one-sided model). Thus, if an oscillatory excitation Q_{pzt} is induced, and this model is used, then an oscillatory output, x_c , is expected. This section developed the model with the assumption that solenoid valve B is open while valve A is closed. The reverse scenario can be also modeled (where *Side A* becomes the *first side* and *Side B* the *other side*). It is just a matter of assigning the variables m_n , k_n , and b_n the values corresponding to the lumps created from *Side A*, and in the same manner, assigning the variables m_{osn} , k_{osn} , and b_{osn} the values corresponding to the lumps created from *Side B*. Finally, and as mentioned previously, the one-sided operation -where one valve is closed and one is open- is referred to as *oscillation*. The *oscillating* mode was used to gather experimental data and compare it with the predicted data from the simulation of the model. It is a measure for validating the model before using it to predict the operation of the piezohydraulic unit under the two-stage cycle operation.

4.3 Two-Stage Cycle Operation

The two-stage cycle operation results in the flow of fluid through the hydraulic system, and therefore it will be denoted as the *pumping* operation. Also, for the purpose of the following discussion, it will be assumed that the displacement of the stack exhibits a fairly linear relationship with the input charge. Furthermore, a triangular-wave charge input will be assumed (as in Figure 4.4) and therefore, the stroke of the piezoelectric actuator will be similar in nature.

The two-stage cycle operation uses two different one sided models in order to simulate the operation of the piezohydraulic unit. Assuming that for the first stage valve B is open while valve A is closed, then the corresponding one sided model (*Model B*) is developed and excited. Afterwards, a second one sided model is developed for the second stage (*Model A*), where valve A is open and valve B is closed, and it is excited. Notice that the models used



Figure 4.4: Input signal.

in each stage have the same structure but they differ from one other. Thus, the set of states in the vector \underline{x} for the first stage model is different than those for the second stage model. By representing each model with the notation and the diagrams used in Section 2.6, then the input signal can be also considered in two stages, as shown in the figure below.



Figure 4.5: Input signal for: model B (top), and model A (bottom).

Notice that during the first stage (from 0 to T/2) the stack is under its forward stroke (pushing) and *Model B* is excited. Therefore, the piston is pushed towards the direction of *Side A*. In the same manner, during the second stage (from T/2 to T) the stack is under its backward stroke (pulling) and *Model A* is excited. As a result, the piston is pulled towards the direction of *Side A*. The net effect after one cycle (two stages), is the movement of the piston towards *Side A*, by an amount Δx . In summary, by exciting *Side B* during the first stage, the result is a piston movement towards *Side A*, which will be denoted as a *BA pumping operation*. Then similarly, an *AB pumping operation* results from the excitation of Side A during the first stage, and a piston movement towards Side B.

The BA type of pumping operation shown in Figure 4.5 is achieved through the 50% onset timing pattern of the valves, which is defined as:

It is important to note that Figure 4.5 assumes an instantaneous opening and closing of the solenoid values at the every half of a period (at 0, T/2, T...). Nonetheless, controlled values in general require a specified amount of time to fully open or fully close once the input value signal is received (*Chapter 2*). This *transition time* can not be neglected and its effect on the two-stage model of the hydraulic system is shown in the figure below.





As it can be observed in Figure 4.6, due to the transition time, one valve is in the process of fully closing while the other is in the process of fully opening. This brief period of time, will be referred to as a *valve overlap*. During a period of *valve overlap*, both sides of the system are no longer isolated and therefore there is no longer a pushing or a pulling effect on either side. Because of this, the input signal 'seen' by the hydraulic system is assumed to be constant during this period, meaning that the piezoelectric stack is not moving and therefore the fluid is not being excited. This is illustrated in Figure 4.7.



Figure 4.7: Actual input signal used for Model A and Model B.

Although this constant signal approach is a good approximation, it is not by any means, an exact model of the system. Thus, while transients may occur in this brief period of time they are not expected to be predicted by the simulation of the current model. In order to include the dynamics involved in this short period of time, then it is necessary to define an additional stage, where the entire hydraulic system is excited but it is no longer separated in two sides. The complexity added (not only to the model but mainly to the Matlab code) to predict these transients is, at this point, not worth of achieving.

4.4 Transition Between Stages and Cycles

Time-Variance Implications:

Since each stage consists of two different models, and therefore a different set of states, then the initial condition response has been neglected and thus, only the forced response is taken into account. This approach does affect the correct prediction of the outcome of the pumping operation of the piezohydraulic unit with a single-ended cylinder. It does not, however, affect the correct prediction of the pumping operation with a double-sided cylinder. Other considerations include the initial condition response versus the forced response of the system in study.

System-Element Update:

After two-stages, one cycle is completed. By this time, the piston of the output cylinder has moved and amount Δx , and therefore the volume of fluid contained in each side of the cylinder changes in proportion to this amount. Thus, after one cycle, the geometry of the lumps of fluid contained within the cylinder is updated and then the elements of mass, stiffness and damping are calculated again. This updating could be done after every stage, but it was not implemented in order to avoid further complexity in the program. Furthermore, the analysis of the dynamics of the entire system at both extreme cases, when the piston is at one end of the cylinder and then at the other end, revealed no considerable change. The poles, which represent the roots of the characteristic equation or the eigenvalues of the A matrix, are a good indication of the dynamics of the system. When analyzed for both of the cases mentioned previously, their change in magnitude was less than 1%. This indicates that even though the dynamics associated with the cylinder itself do change (due to the displacement of the piston), the overall effect of the entire system coupled together is small and depending on the nature of the analysis, it could be neglected.

4.5 Summary

The lumped parameter model of the entire piezohydraulic unit has been integrated and the resulting equations derived for further use in state space form. The concept of one-sided operations has been introduced, along with the two-stage cycle simulation of the pumping operation of the piezohydraulic unit. The input to the hydraulic system has been analyzed for the valve timing case where opening and closing occurs at every half of a period. The effect of the transition time has been introduced, and the resulting input obtained, as a preview of the analysis that is performed on the measured data in *Chapter5*. Finally, the assumptions and implications of the two-stage cycle model are discussed. The most important one, being the zero initial condition that is used at the beginning of each stage due to the nature of having two different models. Thus, the simulated response of the model only considers the forced response output of the system and is not expected to predict the output of an initial condition response dominated system.

Chapter 5

Measurements and Simulations

5.1 Introduction

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This chapter presents the data measured from the benchtop piezohydraulic unit under operation, and the corresponding data from the simulation of the model developed. The information has been organized in the following main sections:

- Experimental Setup (Section 5.2)
- Simulation Parameters (Section 5.3)
- Single-ended Cylinder (Section 5.4)

One-sided Operation:	Input measurements			
	Time response measurements			
	Time response simulations and analysis			
	Frequency response simulation			
Two-stage Cycle Operation:	Measured and simulated results			
Double-ended Cylinder (Section 5.5)				

One-sided Operation:	Time response measurements,				
	simulations and analysis				
	Frequency response simulation				
Two-stage Cycle Operation:	Measured and simulated results				

Finally, a summary of the most important aspects and the corresponding conclusions are included.

5.2 Experimental Setup

The figure below is a representation of the test setup used to obtain the experimental data. Furthermore, for a more detailed diagram of the piezohydraulic unit, refer to Figure 2.2. Also, note that the same notation for the hydraulic sides is used. The distinction between sides A and B is important, specially for a single-ended cylinder, where the geometry involved in the modeling of the fluid is asymmetric.



Figure 5.1: Diagram of the test setup used to obtain data.

As shown in the figure, the input to the piezohydraulic unit is software controlled and it is transmitted through a terminal connected to the computer in use. The frequency of operation and the timing of the piezoelectric stack along with the set of solenoid valves is specified through *Matlab* *.m files, that are used in *Simulink*, and finally interfaced with the digital signal processor through the use of *DS pace*. The digital signal processor then outputs the signals to the piezohydraulic unit. In the same manner, output measurements from the unit itself are captured by the digital signal processor, which is shown in *DS pace* and recorded as *.mat files that are usable in *Matlab*.

Since the amplifier current controls the piezoelectric stack, then a current measurement is performed in order to provide data about the input to the system. This current signal is then integrated in order to obtain the charge input required by the charge-controlled model of the system. In the same manner, a laser vibrometer is used to determine the displacement of the piston of the hydraulic cylinder. The data, provides information about the output of the system, and its measurement provides the output/input relationship needed when comparing the simulated output of the model with the measured output of the system.



Figure 5.2: Display of the modular-benchtop-test setup piezohydraulic unit.

Figure 5.2 shows once again the experimental test setup of the piezohydraulic unit. The figure displays several of the components of the benchtop unit. These components have been integrated in a modular design, in order to simplify the addition, replacement or removal of components. Furthermore, it makes easier the location and troubleshooting of certain problems that may arise.

5.3 Simulation Parameters

The predicted results for the operation of the piezohydraulic unit have been obtained through the simulation of the state space model of the system, derived and defined in *Chapter* 4. Furthermore, all the simulations were performed with the following design parameters:

- Initial pressure, P_i , at 100 psi; since previous to all of the following experiments, the initial pressure of the hydraulic system was set at 100 psi.

- The percentage of air entrained, ρ_{air} , at 0.001%. The percentage of air entrained in the test setup is a quantity that is not easy to determine, but as discussed in *Chapter* 3, it is a parameter that should not be neglected. For simulation purposes, it was assumed that the entrained air is distributed uniformly over the entire hydraulic system, and that its presence is almost zero. Thus, the comparison of the measured and the simulated data is performed under the assumption that there is almost no air in the system. Then, the effects of an increasing percentage of air is analyzed and discussed along the trends and tradeoffs in Section 5.6.

- For the single-ended cylinder, the initial displacement x_{il} (defined in *Chapter 2*), is set at 2500 μ m; which was approximately the average in all of the experiments performed.

5.4 Single-ended Cylinder

The use of a single-ended cylinder in the piezohydraulic unit yielded to unwanted results, which will become clear further on. Nonetheless, the data captured and the analysis performed is presented in order to introduce several aspects and trends that will be discussed in more detail under the double-ended cylinder configuration.

5.4.1 One-sided Operation

Input Measurements:

The input to the piezohydraulic unit from the current-controlled amplifier is measured through a shunt resistance. Since a switching, current control amplifier is used, and as discussed in *Chapter2*, then a step-wave current signal is expected. Figures 5.3 and 5.4 show the current measured during the operation of the piezohydraulic unit at a frequency of 10Hz and 50Hz respectively.



Figure 5.3: Measured current under operation at 10Hz.


Figure 5.4: Measured current under operation at 50Hz.

Note that the magnitude of the noise is relatively the same, but as the frequency of operation is increased, the amplitude of the current drawn by the piezoelectric stack also increases. Then, the presence of noise at lower frequencies becomes more "dominant", in the sense that its magnitude becomes much larger than that of the actual signal. Nonetheless, once the current signal is integrated to obtain charge, the noise is reduced. But the high content of noise in the current signal may yield to a varying DC component per period of time, and the result is a charge waveform as shown below. Thus, even though the noise is reduced through integration, it is its effect on the DC component of the signal what causes trouble.



Figure 5.5: Charge obtained from the measured current at a frequency of 10Hz.

Figure 5.5 shows the charge waveform that results from the integration of the measured current signal (with noise content) at a frequency of 10Hz. The result is a charge signal that may not be used as the input to the model. This is because the time simulations will result in an output signal with the same varying DC component pattern, which does not represent the pattern of the actual (measured) output signal. In other words, the simulated output will also exhibit a decaying component (as shown in Figure 5.5) while the measured output exhibits no decay and a constant DC component per period of time. The ideal case is to have an input charge signal with a constant DC component per period of time, as shown in the following figure.



Figure 5.6: Charge obtained from a noise-reduced current at a frequency of 50Hz.

The charge signal shown in Figure 5.6 has been obtained through the integration of a noise-reduced current signal. The noise-reduction required the use of several filtering procedures and DC correction techniques. The important issue is not the type of noise reducing process used, but the noise reduction of the signal without any additional and unwanted filtering that would result in the distortion of the actual signal.

Time Response Measurements:

The following set of figures represent the time response data for the single-ended piezohydraulic unit under a one-sided operation. In other words, the figures show the measured displacement of the piston of the hydraulic cylinder, x_c , versus time.



Figure 5.7: Time response measured under one-sided operation, for five different operating frequencies. Initial pressure, $P_i = 100$ psi.

Recall that under one-sided operation the piezohydraulic unit operates with one valve open and the other one closed. Therefore the excitation of the hydraulic system (through the piezoelectric stack) with an oscillating signal, will result in an oscillating output. As mentioned previously, the input to the stack is a triangular charge waveform (as shown in Figure 5.6), and as discussed in *Chapter* 2, the resulting voltage and displacement of the piezoelectric stack is similar in nature. Thus, for a triangular input excitation under a one-sided operation, a similar waveform is expected. Furthermore, the data captured and shown in Figure 5.7 correspond to the operation of the piezohydraulic unit with valve Bopen and valve A closed. Thus, *Side* B is excited directly by the piezoelectric stack while *Side* A remains as a closed system at the other side of the piston.

Referring again to the set of time responses shown in Figure 5.7, note that for a frequency of 10Hz, the output displacement is similar to the triangular input waveform, but as the frequency of operation (and therefore of excitation) is increased, the shape of the output displacement waveform slowly changes to resemble more the shape of a sine wave. Also, while the shape of the wave changes as the frequency increases, note that the amplitude remains fairly constant. These observations suggest that the dynamics of the piezohydraulic unit, which in turn, reflect the dynamics of the hydraulic system, are similar to the dynamics of a low pass filter. This is because a triangular waveform has frequency content at the fundamental frequency along with a number of harmonic (by using Fourier transforms to express the triangular wave). The high frequency harmonic content seems to be filtered and as the frequency of operation increases, eventually the resulting signal has most of its frequency content at one frequency, the fundamental frequency. More detail will be covered as the analysis is performed. For more information on complex relations, Fourier transforms, fundamental and harmonic frequencies, and frequency spectrum analysis of waveforms refer to Beckwith et al. (1995).

Time Response Simulations and Analysis:

The set of simulated time responses are shown in the next page. These time simulations used the data from the measured current to determine the input to the model. Thus, Figure 5.8 corresponds to the simulation of the operation measured and shown in Figure 5.7. Note that from the comparison of the magnitude of the curves and the transition from a triangular waveform to a sinusoidal curve, both figures seem to correlate well with one another.



Figure 5.8: Time response simulated with the corresponding measured charge input.

The comparison between the time curves at frequencies of 10Hz, 50Hz and 90Hz is shown below. From these set of curves, it seems that the "transition" towards a sine-wave curve occurs at a lower frequency in the modeled system. Nonetheless, a visual comparison of the time response curves does not provide enough information to draw conclusions on the correlation between the measured response and the simulated one.



Figure 5.9: Measured and simulated time response comparison.

In order to have an effective comparison of these curves, a Fast Fourier Transform (FFT) has been performed to obtain the frequency spectrum of each signal. The comparison of the frequency content between the measured and the simulated data does indeed, provide a good indication on the correlation between them.

The frequency content comparison in Figure 5.10 is performed for the data obtained at a frequency of 10Hz. Thus, for this triangular waveform the fundamental frequency is 10Hz, and harmonic content is expected at 30Hz, 50Hz, 70Hz and so on (as shown in the figure below). Furthermore, the magnitude of the fundamental frequency component is slightly higher in the simulated data. Nonetheless, the frequency content at the fundamental frequency and at the corresponding harmonics, is well matched by the simulated data.



Figure 5.10: Frequency spectrum of the simulated (top) and the measured (bottom) output data, with the operating frequency at 10 Hz.

The frequency spectrum comparison for the data obtained at the frequencies of 50Hz and 90Hz is shown in the following page. For the 50Hz case, the simulated data seems to match the fundamental harmonic along with the first and the second harmonic (150Hz and 250Hz). The magnitude of the frequency component is, as in the 10Hz case, higher than the measured data. On the other hand, in the 90Hz case the frequency content of the second harmonic of the simulated data is considerably smaller than the one in the measured data. Thus, the fundamental frequency is more dominant in the simulated case, which explains why the simulated response at 90Hz is closer to a sine wave than the corresponding measured response.



Figure 5.11: Frequency spectrum of the simulated (top) and the measured (bottom) output data, with the operating frequency at 50 Hz.



Figure 5.12: Frequency spectrum of the simulated (top) and the measured (bottom) output data, with the operating frequency at 90 Hz.

Frequency Response Simulation and Analysis:

The frequency response represents the steady-state response of a system to a sinusoidal input. Thus, for the frequency response of $|x_c(s)/Q_{in}(s)|$ in Figure 5.13, the magnitude and phase of a sinusoidal output x_c , is given per unit of magnitude of a sinusoidal input Q_{in} .



Figure 5.13: Simulated frequency response of a single-ended piezohydraulic unit.

From both of these figures, the following observations are made:

- Resonance occurs at ~ 158 Hz with a magnitude of $\sim 17 \ \mu m/m$ C.
- A rolloff of \sim 45dB/decade after resonance.
- A bandwidth of approximately 240Hz.
- The magnitude of the DC response is approximately 5.5 μ m/mC.

- And a phase lag of ~ 5.5 degrees at low frequencies. The lag starts to increase at about 22 Hz (-14 degrees) and at 100Hz it is approximately -40 degrees.

From Figure 5.6, the magnitude of the measured charge at a frequency of 50Hz is approximately $6.5*10^{-3}$ Coulombs, or 6.5mC. Furthermore, at 50 Hz, the frequency response shows that $|x_c(s)/Q_{in}(s)| \approx 6.0 \ \mu \text{m/mC}$. Thus the sinusoidal output x_c , is expected to be of 39 microns of magnitude (6μ m/mC * 6.5 mC). But the input to the piezohydraulic unit is a triangular charge waveform. In addition, the frequency spectrum of a sinusoidal signal has only content at its frequency "w", while the frequency spectrum of a triangular waveform has frequency content at its frequency "w", and at every odd multiple of it, 3w, 5w, 7w.... This explains the change in shape in the output curves shown in Figure 5.7. At a frequency of 10 Hz, the triangular input signal has also frequency content at 30, 50, 70, 90, 110 Hz.... Thus, only the sixth harmonic component and higher (110 Hz and up) are attenuated. The result is an almost triangular output with a magnitude close to what is specified by the frequency response plot. Meanwhile, at a frequency of 50 Hz, the triangular input signal has also frequency content at 150, 250, 350 Hz. and therefore only the content at the fundamental frequency (50Hz) remains unchanged. All the following harmonics are attenuated and the result is a signal that has most of its content at the fundamental frequency, thus, looking more like a sine wave. Closer to the resonant frequency, the time response at 90 Hz looks more like a sine wave rather than a triangular wave. Thus, and as mentioned previously, the frequency response of the hydraulic system resembles that of a second order low pass filter, with a corner or passband edge frequency around 240 Hz. In other words, the fluid serves as a low pass filter, where frequencies below the resonance are not affected, while those signals with fundamental frequencies or harmonics higher than resonance get attenuated.

Furthermore, the analysis of the simulated frequency response along with the frequency spectrum plots (figures 5.10, 5.11 and 5.12) suggest that both, the modeled and the actual system have very similar dynamics. But the difference existent between the magnitude and the frequency content of the measured and the simulated data suggests a frequency response comparison as shown below.



Figure 5.14: Generic frequency response comparison of two systems with similar dynamics, but shifted in the frequency domain. Curve 1 represents the modeled system while curve 2 represents the actual system.

Figure 5.14 is a generic comparison between two systems that have a similar frequency response, with the exception that they are "shifted" from one another. This "shift" is related to the overall equivalent stiffness of the system, k, and the entire mass, m. Changes in these quantities do affect the location of the resonant frequency, w_r , since it is a linear function of the natural frequency, w_n , and:

$$w_n = \sqrt{\frac{k}{m}} \tag{5.1}$$

Furthermore, considering the hydraulic system , then the stiffness can be replaced with the expression k = A B/l (equation 3.74), and the mass as $m = \rho A l$ (equation 3.76). Then equation 5.1 becomes:

$$w_n = \sqrt{\frac{B}{\rho \, l^2}} \tag{5.2}$$

where the bulk modulus, B, is a function of the percentage of air entrained in the system (as discussed in *Chapter 3*). Furthermore, the difficult measurement of the quantity of air present in the system makes this parameter an uncertainty. Also, for some hydraulic components (such as a hydraulic piston cylinder) the determination of the length of the hydraulic fluid within results from an estimate. Therefore, approximations of the bulk modulus and of the geometry of the hydraulic system add a margin of uncertainty in the simulated frequency response. In addition, major approximations are made while obtaining the value of the damping terms. This is because they are a function of loss coefficients or equivalent lengths (equation 3.75). These quantities are usually obtained through experimental data and tabulated. The values vary from reference to reference, depending on the specific conditions under which the tests are performed (such as Reynolds number). Therfore the use of these quantities is an approximation and it adds uncertainty.

Thus, since the stiffness, damping and mass elements do affect the response of a system, then it is necessary to be aware of the uncertainties present in the parameters used to define these elements, particularly when performing comparisons such as the one between the frequency responses.

5.4.2 Two-stage Cycle Operation:

Considering the notation used in Figures 5.2 and 2.2, and the types of operation outlined in the two-stage cycle operation section of *Chapter* 4, then, for a single-ended cylinder it becomes imperative to draw the distinction between an AB and a BA pumping operation. This will become clear during the following discussion.



Figure 5.15: BA pumping operation at a frequency of 3Hz. $P_i = 100$ psi.

Figure 5.15, shows the time response of the single-ended piezohydraulic unit under

the *BA* pumping configuration. Recall that under this type of operation the piston of the cylinder moves towards *Side A*. Thus, after one cycle, the piston moves by an amount Δx and the volume of fluid displaced in *Side A* of the hydraulic cylinder is pumped back to *Side B*. Nonetheless, the difference of cross-sectional areas between both sides of the piston, implies the movement of a certain mass of fluid in a given volume at *Side A*, to a larger volume in *Side B*. This causes a continuous pressure drop at *Side B* that eventually affects the actuation of the piezoelectric stack and decreases the output displacement up to the point were pressure distributions prevent the further movement of the piston.

Also, tests performed under the BA pumping configuration were not very consistent as of the rate of decay or decrease of the speed of the output piston. Nonetheless, a very consistent trend is shown in Figure 5.16, where the rate of decrease in the speed



Figure 5.16: BA pumping operation at various frequencies. $P_i = 100$ psi.

of the output increased as the frequency increased. In other words, at higher frequencies the system converged faster to a steady time response. Also, the amount of displacement after a complete cycle, Δx , decreased drastically at frequencies higher than 5Hz. These aspects are attributed to the combined effect of a limited operating frequency range of the solenoid valves (as discussed in *Chapter 2*), along with the use of a 50% duty cycle for their timing with respect to the stack (as covered in *Chapter 4*). More detail about these limitations

and their relationship with the measured data, is done with the double-ended cylinder case.

On the other hand, for tests under the AB pumping configuration, the piston of the cylinder is expected to move towards Side A. Nonetheless, experimental data revealed no movement at all, regardless of the frequency of operation. This is again, due to the asymmetry in the cylinder. After one cycle, the piston moves by an amount Δx and the volume of fluid displaced in Side B of the hydraulic cylinder is pumped back to Side A. But the difference of cross-sectional areas between both sides of the piston, implies the movement of a certain mass of fluid in given volume at Side B, to a smaller volume in Side A. Basically, the operation at this configuration requires the compression "at large" of the hydraulic fluid. Therefore, the piston does not move and no pumping operation is accomplished.

Two-stage cycle comparison:

Figure 5.17 is a comparison of the measured and simulated output results during the twostage cycle operation of a single-ended piezohydraulic unit, at a frequency of 3Hz. Detailed analysis of the ability of the model to simulate the two-stage cycle operation of the pump is left for the double-ended cylinder case. What is evident from the figure below is that the model matches the initial slope of the curve and then continues with a constant slope curve.



Figure 5.17: Comparison of the measured and simulated output results during the two-stage cycle operation of a single-ended piezohydraulic unit, at a frequency of 3Hz.

This is due to the fact that the response obtained from the simulation of the model represents the forced response of the system while assuming zero initial conditions between each stage and cycle (as explained in Chapter 4). The implication is that the state variables in the vector $\underline{\hat{x}}$, are assumed to be equal to zero at the beginning of each simulation. These states, which are displacements and velocities, are analogous to volumes and flow rates in the fluid system. By assuming a zero initial condition at a new stage then the final value or condition of a state in the previous stage is not considered. In other words, the volume or flow of fluid from the previous stage is not considered when simulating the model for the next stage. This is quite significant for the operation under the single-ended cylinder, since as discussed earlier, the asymmetry causes a different amount of volume displacement at each of the sides (A & B). Therefore the states of each of the models (A & B) should reflect that at the beginning and the ending of each simulation at each stage. The failure to do so results in the constant slope simulation, as re-iterated in the figure below.



Figure 5.18: Generic representation of the curves for a simulated and a measured two-stage cycle (pumping) operation.

Thus, the simulation only represents the forced-response of the system and it neglects the initial condition response by assuming zero initial conditions. This is, the difference between both of the curves shown in Figure 5.18. On the other hand, the forced-response does dominate during the operation of a double-ended cylinder, as it will be discussed in the following section.

5.5 Double-ended Cylinder

The unidirectional response of a one-sided cylinder (only under the BA pumping configuration), along with the decaying nature of the response, prompted the use of a double-ended cylinder. The geometric symmetry of the hydraulic cylinder suggests that the unidirectional, decaying response, should be replaced by a directional, linear response.

5.5.1 One-sided Operation

Time Response Measurements, Simulation and Analysis:

The measured and simulated data is presented in the same format and organization as for the one-sided cylinder. First, the measured time response data is shown and then compared to the simulated data. A good correlation between these sets of data is an indication of the correct modeling of the dynamics of the system under *oscillation*. Then it is possible to use the model under certain assumptions, in order to simulate the two-stage cycle operation of the system.

The measured time response is shown in Figure 5.19, the corresponding simulated response is shown in Figure 5.20, while the time comparison of both is shown in Figure 5.21. These figures are displayed in the following three pages.

The measured time response follows a similar transition pattern to a sine wave, as to the single-ended cylinder case. The magnitude of the response seems to increase slightly as the frequency is increased. The simulated time response follows the same pattern as the measured data, while its magnitude remains fairly constant throughout the captured frequency range. The comparison of both curves shows close approximation by the model to the actual response of the system. One noticeable pattern is that at low frequencies, the simulated response shows a larger magnitude and a phase lag with respect to the measured response. The best correlation of magnitude and phase is achieved at the frequency of 50Hz. Then for higher frequencies, there is not much of a difference in magnitude, but a phase lead is exhibited by the simulated response with respect to the its measured counterpart.

Nonetheless, and as performed for the single-ended cylinder case, a frequency spectrum analysis is performed on the time response data in order to establish the correlation between the simulated and the measured response.



Figure 5.19: Time response measured under one-sided operation, for five different operating frequencies. Initial pressure, $P_i = 100$ psi, Valve A = Closed, Valve B = Open.



Figure 5.20: Time response simulated with the corresponding measured charge input.



Figure 5.21: Measured and simulated time response comparison.

The frequency spectrum of the simulated and the measured time data for the frequency of 10 Hz is shown in the figure below.



Figure 5.22: Frequency spectrum of the simulated (top) and the measured (bottom) output data, with the operating frequency at 10 Hz.

Since the operating frequency is at 10Hz, then as mentioned earlier, frequency content is expected at odd multiples of the fundamental frequency, f. That is, at 3f, 5f, 7f..., or as it is usually expressed, 3w, 5w, 7w...(circular frequency). Thus, the first harmonic content is at 30Hz, the second harmonic content is at 50Hz, and so on.

The frequency spectrum of the simulated and the measured time response for the remaining data, captured at the operating frequencies of 30Hz, 50Hz, 70Hz and 90Hz, is shown in the following set of figures. For the operating frequency of 30Hz, note that the magnitude of the frequency content in the simulated data is higher than in the measured one. For the rest of the operating frequencies, the magnitudes are fairly equal to one another. Also note that an unexpected frequency content shows in the measured data and it is clearly noticeable at the operating frequencies higher than 50Hz. In the 50Hz case, the unexpected frequency content appears at 100Hz, it becomes more prominent in the 70Hz case, showing at 140Hz, while it is not as noticeable in the 90Hz case, but it still shows at 180Hz. Note that in all the cases, the unexpected frequency appears at the first even multiple of the fundamental frequency, that is 2f or 2w. The same trend is exhibited in



Figure 5.23: Frequency spectrum of the simulated (top) and the measured (bottom) output data, with the operating frequency at 30 Hz.



Figure 5.24: Frequency spectrum of the simulated (top) and the measured (bottom) output data, with the operating frequency at 50 Hz.



Figure 5.25: Frequency spectrum of the simulated (top) and the measured (bottom) output data, with the operating frequency at 70 Hz.



Figure 5.26: Frequency spectrum of the simulated (top) and the measured (bottom) output data, with the operating frequency at 90 Hz.

the frequency spectrum figures shown previously, for the one-sided cylinder case. The frequency content at 2f or 2w is unexpected since a triangular waveform is used, with frequency content at f, 3f, 5f,... Its presence is attributed to the measured charge signal, which as shown in Figure 5.6, it is not a 100% clean signal. Also, the frequency response of the actual signal may differ slightly, specially at the region after resonance. More on the frequency response analysis is covered next.

Table 5.1 is a comparison of the frequency content between the measured and the simulated data, and it shows that the best correlation between the actual and the modeled system occurs at the operating frequency of 50Hz. The magnitude comparison is performed as the percentage difference of the simulated data with respect to the measured data (simulated/measured).

Fundamental Content		Expected Harmonics				Frequency Content	
		First		Second		at "2f" or "2w"	
Freq [Hz]	Mag [%]	Freq [Hz]	Mag [%]	Freq [Hz]	Mag [%]	Freq [Hz]	Mag [%]
10	+ 30	30	+ 30	50	+ 27	20	- 60
30	+ 30	90	+ 32	150	+ 12	30	- 78
50	0	150	- 17	300	0	100	- 66
70	+ 13	210	0	350	-	140	- 57
90	+ 1	270	- 14	450	-	180	- 50

Table 5.1: Correlation of the measured and simulated time response through the comparison of the magnitude of the frequency content.

Dashes are used for frequency content that has almost a zero magnitude. For the frequencies of 70 and 90Hz, the second harmonic barely manifests itself due to the attenuation experienced after the resonant frequency. Finally, the data tabulated above indicates that the model performs a better simulation of the actual system at operating frequencies higher than 50Hz, where at least one or more frequency component of the simulated data, differs by 1% or less with respect to the measured data.

Table 5.2 compares the peak to peak magnitude of the measured and the simulated time response data. A good correlation of the harmonic content ensures the correct prediction of most of the dynamics of the system. Nonetheless, the magnitude of every harmonic or frequency content does affect the total magnitude of the signal, and therefore, if some harmonic content is predicted with a much higher value, then it could be offset by another harmonic predicted with a much lower magnitude. Thus, the overall effect on the prediction of the magnitude of the actual signal is not clear in Table 5.1, and therefore, the time comparison of the signal has been performed and it is shown below.

Frequency	Peak-to-peak mag	Pct difference, [+/- 1%]	
[Hz]	Measured data	Simulated data	simulated / measured
10	44.91	58.95	31 %
30	43.86	52.98	21 %
50	52.16	53.80	3 %
70	49.83	52.81	6 %
90	56.37	56.14	0.5 %

Table 5.2: Comparison of the peak to peak magnitude of the measured and the simulated time response.

The tabulated data confirms the previous conclusion in that the simulation of the model yields better predictions at higher frequencies.

Finally note that the simulated system overestimates the actual response in all cases. The comparison however, has to be made with the fact that the simulations have been performed on a very stiff system, with an amount of entrained air of 0.001% while the amount of entrained air in the actual system is unknown. These results suggest that the actual system is indeed, very stiff, and that the amount of entrained air is minimal.

Frequency Response Simulation:

The simulated frequency response of Side B of the double-ended piezohydraulic unit is shown in Figure 5.27, where the output of the system is the displacement of the piston of the hydraulic cylinder, x_c , while the input is the charge that results from the current through the piezoelectric stack, Q_{in} .



Figure 5.27: Simulated frequency response of a double-ended piezohydraulic unit.

From both of these figures, the following observations are made:

- A resonant peak does not show in the simulation. The flat "peak" suggests that resonance -thus highly damped- occurs between 119Hz and 143 Hz with a magnitude of ~ 10.7 μ m/mC.

- A rolloff of ~ 50dB/decade after resonance.

- A bandwidth of approximately 214 Hz.

- The magnitude of the DC response is approximately 7.5 μ m/mC.

And a phase lag slowly increases right from the beginning. At 10Hz the phase is approximately -4 degrees, at 20Hz it is about -7 degrees, and at 100Hz it is around -50 degrees.

The comparison of this frequency response with the one for the single-ended case, suggests that for the double-ended unit, the system is more damped, stiffer and with more mass. Although both stiffness and mass seem to be larger, the equivalent stiffness to mass ratio has to be smaller in order to have a lowering effect on the resonant frequency.

Figure 5.28 is a comparison of the simulated frequency response for Model A and Model B. Recall that for Model A, Side A is excited first, while for Model B, it is Side B.



Figure 5.28: Simulated frequency response comparison between model A and B.

The dynamics of both models are very similar, differing only at very high frequencies. Recall that for the double-ended unit, the hydraulic cylinder is symmetrical while minor differences may exist on both sides due to pipe lengths or adaptors used. Thus the time response data for both sides is expected to be close to being identical.

5.5.2 Two-stage Cycle Operation:

As mentioned earlier, the use of a double-ended cylinder enabled the bidirectional pumping operation of the piezohydraulic unit. Furthermore, and as expected due to the symmetry, the rates of displacement for the AB pumping configuration and the BA pumping configuration are almost the same. This is illustrated in Figure 5.29, where a positive displacement corresponds to the BA pumping configuration and a negative displacement, to the AB counterpart. This relationship between the displacement and the corresponding pumping configuration is maintained throughout the rest of the figures in this chapter, being just a



Figure 5.29: Measured time response under bi-directional pumping operation at 3Hz.

function of the orientation of the unit with respect to the laser vibrometer (as shown in Figure 2.2). Figure 5.29 represents the time data that has been captured starting from a zero -reference- displacement. A BA pumping operation was induced through the timing of the valves, then a closed-valve period is shown, followed by another BA pumping oper-

ation. The valves were closed again, and then their timing was switched to induce the AB pumping operation. Also note that once both valves are closed, there is no transient and the displacement of the unit remains steady with no change.

Furthermore, the linear relationship of the pumping operation remained constant and did not change, as in the single-ended case. This is shown in Figure 5.30 where the pumping operation of the unit is captured with the displacement of the piston/rod of the cylinder starting at one end and culminating almost at the other end. The sudden drops from about $2500\mu m$ to $-2500\mu m$ do not represent the response of the system, but an outof-range condition from the laser vibrometer. Once its range limit is achieved, then the measurement device resets back to the lower limit. Therefore, the data shown corresponds to a total displacement of about 20mm, which represents an approximate 79% of the oneinch stroke of the hydraulic cylinder.



Figure 5.30: Measured time response under the BA pumping operation (at 3Hz) between both ends of the cylinder.

The two previous figures have been obtained through an operating frequency of 3Hz and a 25% of fset timing of the valves. Moreover, the pumping operation and the data measurement has been performed for two types of valve timing. They are denoted as a 50% onset timing and a 25% of fset timing. The definition of these timing patterns is illustrated in the following figure.



Figure 5.31: Valve and amplifier/stack control voltage curves for the: 50% onset timing case (top), and the 25% of fset timing case (bottom). Operating frequency = 5Hz.

The blue and green curves represent the voltage across the valves. The solenoid valves used are normally closed and therefore a voltage is required to keep them open. Thus, a high (6V) represents an open solenoid and a low (-1.5V) represents a closed valve. Although no voltage is required to keep the valves closed, the negative voltage is applied to induce a faster response from the open position to the closed one.

The red curve on each figure represents the digital TTL input voltage signal to the amplifier. The digital input signal is used to set the operating frequency of the amplifier. Furthermore, it is a representation of the step-current signal supplied by the switching amplifier. It is a not-to-scale representation since the red curve shown represents an input voltage, but the output current of the amplifier is still similar in shape. Thus, following the discussion in Section 2.2 and the result in Figure 2.5, then the red curve in both plots of Figure 5.31 can be integrated to obtain a non-scaled representation of the ideal charge, voltage and displacement of the piezoelectric stack. This is performed for the 50% onset timing case and the result is shown in Figure 5.32.



Figure 5.32: Relating the displacement of the stack (black) to the current signal through it (red) and the valve timing pattern (blue and green); F = 5Hz.

Since the red curve also represents the current across the stack, then the black curve is associated with its displacement. Furthermore, the positive slope side of the black curve represents the pushing stroke of the stack, while the negative slope side represents the pulling stroke. Also, note that valve B is set to open right at the beginning of the pushing stroke of the stack and it is set to close at the end of the stroke. At this point, valve A is set to open as the pulling stroke begins, and it is set to close at the end of the stroke. Thus, the valves are set to open and close at the endpoint of every stroke, and that is what the term *onset timing* refers to. Furthermore, the valves remain open or closed for half of a period, or 50% of the time. Therefore this type of valve timing is referred to as the 50% *onset timing* case, and it is, the valve timing pattern for which a time delay effect was analyzed in *Chapter* 4.

In the same manner, the term 25% of fset timing is used since the valves remain open only for 25% of the period, T_{st} (which is the period of the stack). Therefore, the period of time the valve is open is $T_{vo} = 0.25 T_{st}$, while the equivalent period of the pulse is $T_v = 0.5 T_{st}$. They have an offset timing since they neither open or close at the endpoints of each of the strokes of the stack (at 0, $T_{st}/2$, T_{st} ...). For this 25% of fset timing in particular, valve A has been set to open at a period of time after the pushing stroke begins and to close slightly before it ends. The same is done for valve B in relation to the pulling stroke. Thus, with the open period of the values set at $T_{vo} = 0.25 T_{st}$, then the timing used for the 25% of fset timing in Figure 5.31 is defined by the equations:

Valve B opens @	t_1	=	$0 + 0.2 T_{st}$	$= 0.2 T_{st}$	
Valve B closes @	t_2	=	$t_1 + T_{vo}$	$= 0.45 T_{st}$	
Valve A opens @	t_3	=	$T_{st}/2 + 0.2T_{st}$	$= 0.7 T_{st}$	
Valve A closes @	t_4	=	$t_3 + T_{vo}$	$= 0.95 T_{st}$	(5.3)

By taking into account the instants at which each valve opens and closes, then it is possible to determine the input 'seen' by the hydraulic system, as shown in Figure 5.33



Figure 5.33: Valve and amplifier/stack control voltage curves for the: 50% onset timing case (top), and the 25% of fset timing case (bottom). Operating frequency = 5Hz.

Thus, the type of valve timing used is expected to have an impact on the output of the system. These effects are reflected in the nature of the time response under the pumping operation, and they are presented in the following section.

Time measurements:

Pumping operation under the 50% onset timing pattern:

Figures 5.34, 5.35, 5.36, and 5.37, show the measured time response under the pumping operation for the frequencies of 3Hz, 5Hz, 7Hz, and 9Hz, respectively. Both pumping directions are included for each case.



Figure 5.34: Measured time response under the pumping operation at 3Hz, for: the AB pumping configuration (left), and the BA pumping configuration (right).



Figure 5.35: Measured time response under the pumping operation at 5Hz, for: the AB pumping configuration (left), and the BA pumping configuration (right).



Figure 5.36: Measured time response under the pumping operation at 7Hz, for: the AB pumping configuration (left), and the BA pumping configuration (right).



Figure 5.37: Measured time response under the pumping operation at 9Hz, for: the AB pumping configuration (left), and the BA pumping configuration (right).

From these set of figures, several observations can be made. They hold for both pumping directions, but for ease of appreciation, consider the BA pumping configuration (figures to the right). First, after each increase in displacement, there is a corresponding "return" or decrease in displacement that is smaller in magnitude. Also notice that this "return" displacement increases in magnitude as the frequency of operation increases. Furthermore, it will be referred to as the spring effect, for reasons that will become obvious later. Finally, after the return, a constant position (no displacement) is observed before

the beginning of a new cycle.

Figure 5.38 is one of a set of data that was captured under the same conditions as for Figures 5.34, 5.35, 5.36, and 5.37, but with a lower resolution in order to capture a greater range in the time domain.



Figure 5.38: Obtaining the speed of response under the pumping operation at 3Hz, for: the *AB* pumping configuration (left), and the *BA* pumping configuration (right).

Then, they are used to obtain an average change in displacement per unit of time. Thus, the speed of operation as a function of frequency and the displacement achieved per cycle versus the operating frequency are determined, and the result is shown in Figure 5.39.



Figure 5.39: Measured performance under the 50% onset timing pattern, for both, the AB and the BA pumping operations.

Pumping operation under the 25% offset timing pattern:

Time response measurements have been also performed with the operation of the piezohydraulic unit under the 25% offset timing case. The results are shown in Figures 5.40, 5.41, 5.42, 5.43, 5.44, and 5.45. Both pumping directions are included for each case.



Figure 5.40: Measured time response under the pumping operation at 3Hz, for: the AB pumping configuration (left), and the BA pumping configuration (right).



Figure 5.41: Measured time response under the pumping operation at 4Hz, for: the AB pumping configuration (left), and the BA pumping configuration (right).



Figure 5.42: Measured time response under the pumping operation at 5Hz, for: the AB pumping configuration (left), and the BA pumping configuration (right).



Figure 5.43: Measured time response under the pumping operation at 6Hz, for: the AB pumping configuration (left), and the BA pumping configuration (right).

Note that the measurements for the first pair of frequencies (3Hz and 4Hz) do not exhibit the *spring effect* present in the measurements for the 50% onset timing case. A small *return* is first noticeable at the operating frequency of 5Hz, and it increases as the frequency increases, up to the point where the *return* becomes of the same magnitude as the previous increase in displacement. This is the case for Figure 5.45, where the operating frequency is at 8Hz.


Figure 5.44: Measured time response under the pumping operation at 7Hz, for: the AB pumping configuration (left), and the BA pumping configuration (right).



Figure 5.45: Measured time response under the pumping operation at 8Hz, for: the AB pumping configuration (left), and the BA pumping configuration (right).

The same condition occurs under the 50% onset timing case but at a higher frequency. Note that the *return* is almost of the same magnitude as the increase in displacement stroke, in Figure 5.37, but there is still an average change in displacement while the frequency is at 9Hz. On the other hand, there is an average change in displacement under the 25% of fsetset timing case at 7Hz (Figure 5.44) but no considerable change in the average position is achieved at 8Hz (Figure 5.45).

In addition to these differences, and appreciation of the different pumping performance between both types of valve timing, is possible through the comparison of Figure 5.39 (for the 50% onset timing) and Figure 5.46 (for the 25% of fset timing).



Figure 5.46: Measured performance under the 25% of fset timing pattern, for both, the AB and the BA pumping operations.

Pumping operation performance analysis:

From the comparison of Figures 5.39 and 5.46 it is possible to observe that under the 50% onset timing pattern the piezohydraulic unit has a higher frequency range of operation, but with a 25% offset timing pattern, higher speeds and displacements per cycle are achieved. This is due to the spring effect that is present in all the measurements for the 50% onset timing case, while it only shows at higher frequencies under the 25% offset timing case. Thus, the return in the spring effect reduces the performance of the unit, and therefore it is an undesired effect. The main cause of this effect is related to the transition time of the valves and the possible valve overlap (discussed in detail in the upcoming transition time and valve overlap section).

Also, it is necessary to state that all the measured results for the pumping operation of the double-ended piezohydraulic unit represent slightly over half the speed values expected. This is because only one side is being excited effectively during the two-stage cycle (pumping) operation, according to the following analysis of the data measured. A close analysis of the pumping curves shown in Figures 5.34 to 5.38 and 5.40 to 5.45 reveals that a significant change in position, Δx , occurs only once per cycle. And as presented in *Chapter* 4, during the pumping operation a displacement is expected in the first (pushing) stage while another displacement is expected at the second (pulling) stage.



Figure 5.47: Time response of the BA pumping configuration of Figure 5.35 with the corresponding control signals for the valves and the stack.

Figure 5.47 shows the time response under the BA pumping operation (Figure 5.42) with the respective control signals for the valves and the stack. Note that when valve

B is open, there is a noticeable displacement, while with valve A open, a much smaller displacement is barely noticed. Thus, these results suggest that there is a problem with the operating condition of valve A. Furthermore, all time response simulations for the double-ended unit under the one-sided operation (oscillation) captured and shown in Figure 5.19 correspond to the direct excitation of *Side B*, with valve A closed and valve B open. As the reverse condition was attempted (valve A open, valve B closed), no significant oscillation was achieved with the direct excitation of *Side A*. This, confirmed the possibility of a problem with valve A. Then, the fluid in the system was evacuated and the unit was dismantled. During the process, a small string of Teflon was found at the small inlet of solenoid valve A (0.8mm Dia) and therefore causing a partial block to the movement of fluid. The partial block explains the performance obtained for the oscillating and pumping conditions. Note that a total block, would reduce the pumping operation performance to that of an oscillating procedure.

Finally, given these conditions, the set of data presented previously has been used since a very stiff system was achieved (negligible or no amount of entrained air) and a correlation of the measured and the simulated data can be achieved with a reduced concern with respect to the uncertainty of an unknown amount of entrained air. Recall that even though valve A has been found to be "clogged", it does not affect the time response under the one-sided operation (oscillation) with valve B open and valve A closed. The pumping data, proved to be useful to introduce the trends between both timing patterns, and to show how the analysis of the control signals may be useful to identify and troubleshoot problems.

A second set of data (case II), for each timing pattern, has been captured. The objective, to obtain data for the pumping operation of the unit with both valves unblocked and working properly, has been achieved. A displacement is captured with the excitation of each side. Nonetheless, extensive and repeated efforts were unable to obtain a "stiff" system, as before. Most possibly an unidentified leakage has been preventing the "re-fill" of the piezohydraulic unit with reduced or no amount of entrained air. Although it is not an ideal set of measured data, the measured response is a good representation of the effects of having entrained air in the system.

Time measurements (case II):

Pumping operation under the 50% onset timing pattern (case II):

Figures 5.48, 5.49, 5.50, and 5.51, show the measured time response under the pumping operation for the frequencies of 2Hz, 4Hz, 6Hz, and 8Hz, respectively. Both pumping directions are included for each case.



Figure 5.48: Measured time response under the pumping operation at 2Hz, for: the AB pumping configuration (left), and the BA pumping configuration (right).



Figure 5.49: Measured time response under the pumping operation at 4Hz, for: the AB pumping configuration (left), and the BA pumping configuration (right).

Comparison of the low frequency time response between case I (Figure 5.34, 3Hz) and case II (Figures 5.48, 2Hz, and 5.48, 4Hz) yields to several observations. Furthermore, consider again the *BA* pumping configuration (figures to the right). First, as mentioned previously, in case I there is one increase in position per cycle, while two increases can be observed in case II. Also note that the *return* in both cases is of about the same magnitude. The magnitude of the displacements are however, considerably different. The only increase in position per cycle in Figure 5.34 is of about $30\mu m$, while the magnitude of both



Figure 5.50: Measured time response under the pumping operation at 6Hz, for: the AB pumping configuration (left), and the BA pumping configuration (right).



Figure 5.51: Measured time response under the pumping operation at 8Hz, for: the AB pumping configuration (left), and the BA pumping configuration (right).

displacements per cycle in Figure 5.48 is lower. Also note that each of the two increases per cycle is of different magnitude. In Figure 5.48 for instance, as one side is excited, a displacement of $20\mu m$ is achieved, while exciting the other side, a $10\mu m$ displacement is achieved. The lower displacements are a result of the presence of entrained air in the system, which reduces the performance of the unit. The uneven displacement per stage within a pumping cycle (in case II) might be attributed to an uneven air percentage distribution within the system.

Figure 5.52 shows the time response data of the unit under a one-sided operation. Note that for the excitation of side A (valve A open, valve B closed), the magnitude of oscillation is of about $14\mu m$. On the other hand, a $\sim 20\mu m$ oscillation is achieved with the excitation of side B. The uneven oscillatory magnitudes are reflected during the pumping operation of the unit, and corresponding this oscillation example is Figure 5.51, where one displacement is slightly higher than the other one within a pumping period or cycle.



Figure 5.52: One-sided operation (oscillation) at 8Hz of (case II): side A (left), and side B (right).

Figure 5.53 shows the performance of the system. From the comparison of Figure 5.53 (case II) with Figure 5.39 (case I), note that the range of frequency for the captured data in case II is from 1Hz to 9Hz, while in case I, it is from 3Hz to 9Hz. As mentioned previously, the presence of only one increase in position per cycle in case I, versus two displacements per cycle in case II, should result in roughly half the speed and displacement for case I with respect to case II. Nonetheless, the performance curves for case II are much lower than those of case I, mainly due to the presence of entrained air.



Figure 5.53: Measured performance under the 50% inset timing pattern, for both, the AB and the BA pumping operations (case II).

Pumping operation under the 25% offset timing pattern (case II):

Figures 5.54, 5.55, 5.56, and 5.57, show the measured time response under the pumping operation for the frequencies of 3Hz, 5Hz, 7Hz, and 8Hz, respectively. Both pumping directions are included for each case.



Figure 5.54: Measured time response under the pumping operation at 3Hz, for: the AB pumping configuration (left), and the BA pumping configuration (right).

Similar to case II of the 50% onset timing pattern, the 25% of fset timing pattern (case II) holds the same difference from its counterpart in case I, in the sense that two displacements are achieved per unit cycle. The uneven displacement magnitude, and all the aspects discussed previously apply. Also the same observations performed between both timing patterns in case I apply here, between both timing patterns in case II.



Figure 5.55: Measured time response under the pumping operation at 5Hz, for: the AB pumping configuration (left), and the BA pumping configuration (right).



Figure 5.56: Measured time response under the pumping operation at 7Hz, for: the AB pumping configuration (left), and the BA pumping configuration (right).



Figure 5.57: Measured time response under the pumping operation at 8Hz, for: the AB pumping configuration (left), and the BA pumping configuration (right).

Figure 5.58 shows the performance of the piezohydraulic unit under the 25% of fset timing pattern and the conditions of case II. The speed and displacement per cycle versus frequency is considerably higher than the corresponding valve timing pattern in case I. Nonetheless, performance values for case II are still lower than double the values of case I, as expected. Again, it is the result of the entrained air in the system. Furthermore, the performance curves for case II show considerable drop after 7Hz, which is due to the effect of the *transition time* and the possible *valve overlap*. The drop is not as large in case I,



Figure 5.58: Measured performance under the 25% of fset timing pattern, for both, the AB and the BA pumping operations (case II).

since the speed values are not as large and the *valve overlap* effect is attenuated due to the partial block of the valve.

5.5.3 Analysis of the two-stage cycle operation:

Time delay and valve overlap:

The comparison of Figures 5.39 and 5.46 for case I, and of Figures 5.53 and 5.58 for case II, shows that under the 50% onset timing pattern the piezohydraulic unit has a higher frequency range of operation, but with a 25% offset timing pattern, higher speeds and displacements per cycle are achieved. This is due to the spring effect that is present in all the measurements for the 50% onset timing case, while it only shows at higher frequencies under the 25% offset timing case. Thus, the return in the spring effect reduces the performance of the unit, and therefore it is an undesired effect. The main cause of this effect is related to the transition time of the valves and the possible valve overlap.

The effects of the transition time of the valves and the resulting valve overlap have been introduced with the 50% onset timing case and analyzed in Chapter 4. The following discussion applies the same concepts to the 25% offset timing case, and compares both timing patterns.

Figure 5.59 shows the valve timing patterns along with the effects of the transition time. Recall that the 50% onset timing pattern follows a valve timing as specified in equation (4.27), while the 25% of fset timing case follows that of equation (5.3), with a valve period of $T_{vo} = 0.25 T_{st}$. Furthermore, the purple lines show the extent of the transition time, which has been set as 0.05sec. Thin, solid black lines represent the input (displacement of the stack) 'seen' by the hydraulic system. If, during an open valve period there is both, a forward and a backward stroke (as in the bottom case of Figure 5.59), then thick, solid black lines are used to represent the *net total* input 'seen' by the hydraulic system. Finally, the dashed lines are used for two conditions at the same time. The first, dashed lines cover regions where neither valve is open, and the time delay effect is no longer present. Thus the input is not seen by the hydraulic system. Second, dashed lines are also used to cover regions were both valves happen to be open at the same time. As discussed in *Chapter* 4, this is modeled as if the hydraulic system does not 'see' any input, and therefore it is the same as if both valves were closed: a constant position (no displacement). Figure 5.59 shows the valve and stack control signals for an operating frequency of 3Hz, Figure 5.60 corresponds to the 5Hz operation, and Figure 5.61 to 7Hz. Note that under the 50% onset timing case (top figures), the opening of one valve and the closing of the other is set for the same instant. Therefore, the transition time of the closing valve and the opening valve overlap, and are shown with one purple curve per cycle. Note that during this period, both valves are open and therefore, due to the previous excitation of one side, the system experiences the *spring effect* in order to return to equilibrium. This is the reason why regardless of the frequency of operation, the 50% onset timing case will always display a *return* in its response. Finally, as the frequency increases, the *valve overlap* due to the transition time becomes more significant. For a time delay of 0.05, an operating frequency of 10Hz results in a constantly open system, that is, both valves are always somewhat open, within the process of fully closing and fully opening. Therefore, operation of the system close to this frequency or higher, results in no considerable output or response.



Figure 5.59: The effect of the transition time and the resulting value overlap for the: 50% onset timing case (top), and the 25% of fset timing case (bottom). Operating frequency = 3Hz, time delay = 0.05 sec.

The analysis of the input signal 'seen' by the hydraulic system under the 25% of fset timing case is not as simple. In order to keep it as simple as possible, only the transition time related to a closing process is shown in Figures 5.59 and 5.60. Note that when a signal rises from -1.5V to +6.5V the corresponding valve is set to open and the transition time associated with it is not shown. When the signal falls back to -1.5V, the valve is set to close and the corresponding valve transition is shown in purple. Note that when a valve is set to close, it is not assumed to be closed until the transition time has elapsed. During that period of time, it is considered to be partially open. Finally, note that for the 3Hz and 5Hz operation under the 25% offset timing case, there is no valve overlap, even when the transition time is taken into account. Nonetheless, during the period of time where a valve is open, the hydraulic system will 'see' most of the input from a stroke (forward stroke, for the blue curve) while also experiencing some of the reverse stroke (backward,



Figure 5.60: The effect of the transition time and the resulting value overlap for the: 50% onset timing case (top), and the 25% of fset timing case (bottom). Operating frequency = 5Hz, time delay = 0.05sec.

for the blue curve) as the valve fully closes. The result is a small *return* that is not due to *valve overlap*, and it can be observed in the measured data for the pumping operation (presented earlier) up to 5 Hz.

As the frequency increases the opening transition time overlaps with the closing transition time, as shown in the 25% of fset timing case for 7Hz, displayed below (bottom figure). The opening valve transition time is only shown for the first two pulses. The closing valve transition time is shown during the entire range. With no need for detail, note that major valve overlap occurs and therefore the input 'seen' by the hydraulic system is drastically reduced. This explains the reason why the performance of the piezohydraulic unit under the 25% offset timing case (Figure 5.58) exhibited a sharp drop at a frequency higher than 7Hz. Furthermore this is an indication that the transition time of the actual valves is indeed, close to the value of 0.05sec.



Figure 5.61: The effect of the transition time and the resulting value overlap for the: 50% onset timing case (top), and the 25% of fset timing case (bottom). Operating frequency = 7Hz, time delay = 0.05sec.

5.6 Simulation Results:

The simulation of the pumping operation uses two models in a two-stage cycle algorithm, and as mentioned in *Chapter* 4, the result is a model that predicts the forced response of the system, while neglecting the natural response with the use of zero initial conditions. The initial condition of the system becomes important when the timing pattern employed and the operating frequency used result in a valve overlap condition. Thus, the simulations are only expected to predict the output under operating conditions with no valve overlap.

Following the notation and the direction of displacement shown in Figure 4.2, along with a valve timing as shown in Figure 5.32, consider the following example. During the first stage, valve B is open and valve A is closed, and therefore Side B is excited directly and moves a given distance while Side A is being 'compressed'. Then at the same instant, valve B closes and valve A opens, and the previously 'compressed' Side A is pulled by the stack, while Side B is 'expanded' (second stage). At the end of the cycle, the output cylinder moves by an amount Δx and the cycle starts again. Now, instead of assuming an instantaneous opening and closing of the valves, consider the presence of a transition time, as shown in Figure 5.59 (top figure). Then, at the end of the first stage, as Side B is moved and Side A is 'compressed' the valve overlap occurs, and for a brief period of time both sides are connected. Therefore, the 'compressed' Side A acts like a pre-loaded spring that expands as the load is reduced in order to achieve equilibrium. The effect, has been denoted as the spring effect and the reverse displacement as the return. But as discussed in Chapter 4 the two-stage cycle model assumes that both sides are separated from one another at each stage, and therfore the dynamics during a valve overlap are not simulated. Furthermore, periods of valve overlap are modeled in the same manner as periods of no excitation, and therefore no displacements. That is, similar to periods of time when both valves are closed.

The result is shown in Figure 5.62, where the measured time response of the piezohydraulic unit under the 50% onset timing pattern (case II) is compared with a simulated response at the corresponding frequency (5Hz) and a valve transition time of 0.05 sec. As it is shown, the measured data exhibits a *return* that it is not accounted for in the simulations. The average of the magnitude of the return is approximately $11\mu m$ and therefore, $66\mu m$ are lost due to the total of six *returns*, two per cycle, for the three cycles shown. In addition, the simulation assumes a stiff hydraulic system, while the measured data is from the set of responses captured under case II. The amount of entrained air not only reduces drastically the performance of the system, but an uneven distribution of its presence may cause an uneven response as the one measured. Uneven, because the displacement during one stage is slightly larger than the displacement at the other stage.



Figure 5.62: Left: measured pumping operation under the 50% onset timing pattern (case II) at a frequency of 5Hz. Right: corresponding simulated response with a transition time of 0.05, and a percentage of air of 0.001%.

Thus, in addition to the valve overlap, the percentage of air entrained in the hydraulic system is another important parameter in the simulation process. In the lumped parameter model, the amount of entrained air is specified as a percentage of the total volume of the lump, affecting the equivalent bulk modulus (equation 3.42), and therefore, the stiffness element (equation 3.74) of each lump. Also, a considerable amount of air would affect the total mass of a lump.

Furthermore, the effect of the entrained air on the frequency response of the system has been analyzed. Figure 5.63 shows the frequency response of *model* B as function of the amount of entrained air in the system. Recall that *model* B is the one-sided model that assumes a valve B open and a valve A closed condition. In addition, an important assumption is the uniform distribution of the entrained air throughout the entire hydraulic system.



Figure 5.63: Effect of a uniform percentage of air entrained in the system on the frequency response of model B.

Note that the presence of entrained air affects first, the higher frequency dynamics. Then, as the percentage of entrained air is increased, the decay in the frequency response becomes more critical for lower frequency responses. Thus, for the operation of the unit at 5Hz, the uniform distribution of 5% of entrained air within the system reduces the magnitude of the response and increases the phase lag with respect to the input. Therefore, if a valve timing analysis is performed with an output signal with considerable phase lag, then the black curves used in the previous section (*Transition time and valve overlap*) to represent the input to the system, have to be shifted accordingly.

Moreover, Figure 5.64 shows the effect of an uneven distribution of entrained air within the system. It is the time simulation of the pumping operation at 5Hz, under the 50% onset timing pattern, with a valve transition time of 0.05sec and a 4% air distribution throughout the entire system, except for the piping in side A, which has been set with an air presence of 8%. By comparing this figure, with the simulated response in Figure 5.62 (right figure) it can be easily seen that the total displacement, and therefore the speed of response, reduces drastically. In addition, the uneven distribution of air causes an uneven displacement per stage within a cycle. In the figure, the first stage corresponds to the



Figure 5.64: Simulated pumping operation under the 50% onset timing pattern, a transition time of 0.05 and an operating frequency of 5Hz. The percentage of air used is 4%, except for the piping of Side A, which has been set at 8%.

excitation of *model* B and shows a greater displacement than in the second stage, where *model* A is excited. But again, the model does not take into account the *spring effect* that results from the *valve overlap* in this timing pattern, and therefore, the resulting simulation will have a much higher total displacement and speed of response.

The transition time and valve overlap analysis on the 25% of fset timing pattern showed that under the assumption of a transition time of 0.05sec, there is no valve overlap with operating frequencies up to 5Hz. Figure 5.65 compares the measured response under the 25% of fset timing pattern (case II) at a frequency of 3Hz with the corresponding simulation. Again, for the simulated response, a transition time of 0.05sec has been assumed, and also, an uneven percentage of air distribution. After several iterations, a 0.9% of air in the entire system has been used, with 2.5 times that quantity in the piping of Side A. Thus, only the component for the pipe in Side A has been set with a 2.25% of entrained air, while the rest of the system remained with a uniform distribution of 0.9%. With these parameters, a close approximation of the measured data has been achieved. Note that the displacement at each is stage is not only different in magnitude, but also in slope. Recall that the measured response corresponds to a *BA* pumping configuration. Therefore, Side *B* is excited directly in the first stage (valve B open, valve A closed) while Side A is excited in the second stage. With the uneven air percentage distribution, Side A becomes 'softer' or less stiff than Side B. The result is that during the first stage, a higher displacement is achieved than the one in the second stage. Remember, that each stage can be 'seen' as a one-side model as introduced in *Chapter* 4 (Figure 4.1).



Figure 5.65: Left: measured pumping operation under the 25% offset timing pattern (case II) at a frequency of 3Hz. Right: corresponding simulated response with a transition time of 0.05, and an uneven percentage of air distribution: 2.25% for the piping of Side A, and 0.9% for the rest of the system.

Further simulations indicated that small changes in this uneven distribution of entrained air result in considerable changes in the magnitude and the shape of the response of the system. Also, the effect of this uneven distribution of air does depend on the part being affected. Changes in the piping (as done in the previous example) have a different effect on the performance than the result from changes in the hydraulic cylinder, for example. The conclusion is, that the amount of entrained air in a system has a great influence on the response of the system. The nature of this response, is also very susceptible to uneven air distributions. Finally, the proven reduction in performance, and the resulting implications, make the elimination of entrained air within the system a top priority.

5.6.1 Simulated performance:

Simulations have been obtained for the 50% onset timing case. Further analysis of these simulations will lead to series of observations that apply to the 25% offset timing and that may be used to obtain an initial estimate.

Figure 5.66 shows the simulated performance of the piezohydraulic unit under the 50% onset timing pattern. The transition time used is of 0.05sec and the system has been assumed to be almost as stiff as it can be, by setting the amount of entrained air to 0.001%. The performance of the simulated system is much higher than the actual system (by comparison with Figure 5.53) due to two differences: the model does not account for losses due to the spring effect (which become larger as the frequency increases), and the actual system contained some entrained air. Therefore, Figure 5.66 is not a good representation of the actual performance of the system, nonetheless, the general trend of a reduced performance with an increased valve transition time, still holds for the actual system.



Figure 5.66: Simulated performance of the pumping operation under the 50% onset timing for various valve transition time magnitudes. Percentage of air = 0.001%.

Figures 5.67 and 5.68 show the simulated performance of the piezohydraulic unit

under the 25% of f set timing pattern, with a varying valve transition time. Also, the system has been assumed to be almost as stiff as it can be, by setting the amount of entrained air to 0.001%. The difference between both plots is the way the input 'seen' by the hydraulic system is defined. The first figure, uses the definition used in the *Transition time and valve overlap* section, as shown in Figure 5.59 (bottom figure). Figure 5.68 uses a different method to determine the input 'seen' by the system and therefore the resulting performance is different, as it is shown. Comparison of these performance curves with the measured data in Figure 5.58 shows similar trends up to a frequency higher than 7Hz. The drop in speed and displacement per cycle in the actual system has been related to the resulting valve overlap, which is not accounted for in the model. The valve overlap is related to the transition time of the valves, and as defined in *Chapter* 2, the highest frequency of operation for a valve without the generation of valve overlap is:

$$F_{valvecr} = \frac{1}{2t_{tr}} \tag{5.4}$$

Expressing the equation in terms of the equivalent period of the values, T_v , and rearranging:

$$\frac{1}{T_v} = \frac{1}{2t_{tr}}$$

$$\frac{1}{2T_{vo}} = \frac{1}{2t_{tr}}$$

$$T_{vo} = t_{tr}$$
(5.5)

Then, with the 25% of fset timing pattern used, $T_{vo} = 0.25 T_{st}$ can be substituted into the expression, and the resulting equation expressed in terms of the period or the frequency of the stack:

$$0.25 T_{st} = t_{tr}$$

$$T_{st} = 4 t_{tr}$$
(5.6)

$$F_{st} = \frac{1}{4t_{tr}} \tag{5.7}$$

Thus, F_{st} is the highest frequency of operation that can be used without generating any valve overlap. This frequency is determined for every valve transition time used in Figures 5.67 and 5.68 and the result is shown in Table 5.3. Furthermore, since the model does



Figure 5.67: Simulated performance of the pumping operation under the 25% of fset timing for various transition time magnitudes. Percentage of air = 0.001\%.



Figure 5.68: Simulated performance of the pumping operation under the 25% of fset timing for various transition time magnitudes. Percentage of air = 0.001\%.

not account for the *return* during a valve overlap, then the performance curves for the 25% of fset timing pattern shown previously, are only valid up to the frequency shown in Table 5.3.

25 % offset timing pattern								
t _{tr} [sec]	T _{min} [sec]	F _{max} [Hz]						
0.01	0.04	25.0						
0.03	0.12	~ 8.3						
0.05	0.20	5.0						
0.07	0.28	~ 3.6						
0.09	0.36	~ 2.8						

Table 5.3: Maximum operating frequency with no valve overlap.

Finally, recall that although a good estimate may be obtained for the valve transition time, both parameters, the transition time and the amount of entrained air, are simulation parameters that can't be measured. Furthermore, the percentage of air present becomes the greatest uncertainty in the simulated results, since its value in the actual system is hard to determine while it has a great impact on the performance of the system. Therefore, the additional unknown in a 25% of fset timing pattern, which is how to model the input 'seen' by the hydraulic system, should be determined with a set of measured data with almost no entrained air content, in order to reduce the effect of air presence and the underlying uncertainty of not knowing the actual quantity. Once the modeled input is calibrated to this data, then the performance curves shown in Figures 5.67 and 5.68 can be used to predict the performance of the actual system, up to the frequencies permitted by the transition time of the valves. The trends in general, remain the same.

5.7 Summary

Time response measurements and simulations have presented and analyzed. The comparison of the measured and the simulated responses under the one-sided operation demonstrate that the lumped parameter model for the hydraulic system is a good approximation and it does predict the dynamics of the fluid. Analysis on the data corresponding to the pumping operation, showed that the two-stage cycle model is a good approximation of the actual system only under operating conditions were the valve timing pattern, and the transition time delay of the valves do not "create" an *overlap* condition. In addition, it has been demonstrated that the *overlap* condition is responsible for the *spring effect* which in turn, reduces and limits the output capacity of the piezohydraulic unit. Thus, even though the model does not approximate the operation under an *overlap* condition, the limited output implies that this type of operation is not desired in the first place. Finally, the *overlap* condition can be avoided with a valve timing such as the 25% offset timing pattern used, but as analyzed and shown for the 25% offset timing case, the *overlap* – free region of operation is achieved up to a given frequency, that in turn, depends on the nature of the pattern and the valve transition time.

Chapter 6

Conclusions

A benchtop piezohydraulic unit has been developed and the concept of piezohydraulic actuation has been demonstrated. From the analysis performed in this research, the following conclusions are made:

- The lumped parameter model (linear model) is a good approximation of the dynamics of the fluid pipeline in the hydraulic system (nonlinear system).
- The model (time invariant) simulates the pumping operation (time variant) through a two-stage cycle algorithm that is valid under regions of no valve overlap.
- The time response of the valves (transition time) and their operating range of frequency eventually will limit the operation of the unit (under a given valve timing pattern).
- The performance of the system is highly dependent on the valve timing chosen.

Furthermore, in a brief summary of the most important aspects it can be stated that:

• The separate analysis of each of the systems that compose a piezohydraulic unit enables the determination of the effects of each system on the entire unit. Thus, the electrical system, the mechanical system, and the hydraulic system have been modeled separately. Furthermore, the electrical and mechanical systems are coupled through the electro-mechanical equations for a piezoelectric stack. The resulting electro-mechanical system is coupled with the hydraulic system through the introduction of constraints with the variational approach described by Hamilton's principle.

- Following the force-voltage system analogy introduced, the fluid elements of resistance, capacitance and inductance have been derived from the governing equations used for a control volume and applied to the fluid in a pipeline. The lumped parameter model is based on a one-dimensional flow that divides the pipeline in lumps of volume with uniform properties, such as pressure and velocity. Once the model of a single lump of fluid has been obtained, then a mechanical system that models the entire fluid pipeline has been derived. Furthermore, the convergence of the model has been verified, and the results discussed.
- The analysis of the hydraulic system involves two different models that are used in a two step algorithm. Thus, a two-stage cycle simulation is performed in order to predict the output of the piezohydraulic unit under the pumping operation. Moreover, even though they are not part of the hydraulic system as a lump of fluid, the valves have an important role. Their dynamics and their timing with respect to the piezoelectric stack determine the input seen by the hydraulic system and therefore, have a great effect on the output and the expected performance of the unit.
- The effective bidirectional displacement of a hydraulic cylinder through the actuation of a piezoelectric stack has been achieved. Data from the one-sided operation of the piezohydraulic unit has been captured and used to validate the model of the actual system. Time response analysis is performed through the frequency spectrum comparison of the measured and the simulated data. Then a two-stage cycle simulation is used to model the pumping operation of the unit.
- The simulated response obtained from the two-stage cycle model, represents the forced response of the system and assumes zero initial conditions. Thus, after one stage, states have a definite value and the zero initial condition assumption for the next stage, neglects the initial condition response. The initial condition response does manifest itself in the total response of the one-sided piezohydraulic unit. Therefore, the simulated response was unable to predict correctly the response of the actual system. On the other hand, analysis of the operation of the double-ended cylinder concluded that the forced response is dominant and that the initial condition response can be neglected (only in regions of no valve overlap). Therefore, the two-stage cycle lumped parameter model does predict the pumping operation of the double-ended

unit under regions of no valve overlap. Furthermore, the good correlation obtained for the one-sided time response simulations suggest that the lumped parameter model was good enough to predict the dynamics of the hydraulic system for both, the singleended and the double-ended unit.

- Analysis of the dynamics of the system revealed that the location of the piston within the hydraulic cylinder has no considerable effect. As the piston is moved from one end of the cylinder, to the other, the slowest pole of the entire system changes by less than 1%. In the code, the varying stiffness, damping and mass elements are only updated after each cycle. Nonetheless, if the simulation is performed for a doubleended cylinder, where only the forced response is enough to model the system, then the update process is unnecessary since it has no effect on the output of the system.
- Finally, from the experimental side of the research, it is possible to identify the following set of limitations involved with a piezohydraulic unit. First, the need of high displacement piezoelectric actuators often comes with the requirement of high voltage operation along with high current consumptions. Thus, the amplifier becomes the first limitation to overcome. Second, is the response of the controlled valves. The highest valve operating frequency will set the limit on the piezohydraulic unit. And finally, once these limitations are overcome, the unit is eventually limited by the dynamics of the fluid and the hydraulic system itself. Attenuation in the frequency response, or the operation near resonance and the possibility of cavitation, are some of the aspects that eventually will limit the operation of the piezohydraulic unit.

6.1 Recommendations and Future Work

From the experience of this research, the following recommendations on future work are suggested:

• In order to minimize or eliminate entrained air in the system, the use of a vacuum pump is highly recommended for at least one hour (with new oil). Vacuum pump use is only needed for first time hydraulic fill or re-fills. Nonetheless, change the vacuum pump oil right after each evacuation, while the oil in the pump is hot. This ensures faster, higher evacuation and longer pump life. Re-used oil seems to require much

longer periods to achieve the same evacuation rate.

- The performance of the system has been affected several times due to particles present in the fluid that eventually block the passage through the valves. The use of Teflon as a sealant is common and suggested only in components that may require it. Special attention is recommended while connecting or disconnecting a component, since a small and hard to notice Teflon particle may end up inside the fluid pipeline.
- Furthermore, application of torque in the installation of pipe connectors should be carefully applied, in order to prevent torsional loads that could cause a minor and un-noticeable leakage in the system. Also, in order to prevent unwanted particles that may be introduced in a re-fill process, it is recommended to install a filter after the pressurized cylinder, and before the quick connect link to the piezohydraulic unit.
- The solenoid values are off-the-shelf components the were used due to their availability, fast delivery, and standard installation. Now that the important aspects and resulting limitations of the values have been identified, then the next step is to experiment with higher frequency response values that may also have much faster time responses. Perhaps the use of PZT actuated values that may withstand the pressure at which the final system will operated are recommended.
- The lumped parameter model developed does not account for the eventual appearance of cavitation and its effect on the performance of the system. Cavitation is more likely to occur during the pulling strokes of the piezoelectric stack, and the operation with a pre-pressurized system helps to prevent it. Nonetheless, once the limiting effect of the valves is overcome, then it is necessary to include the effects of a high frequency operation on the dynamics of the lumped parameter model. One option is to keep track of the forces of each lump of fluid. Then these can be translated into the pressures acting on each lump. The objective is to ensure that these pressure values are higher than the vapor pressure of the fluid. If the pressure of one lump within the hydraulic system falls close or below the vapor pressure of the fluid, then cavitation is likely to occur, and the formation of vapor bubbles will indeed reduce drastically the effectiveness of the unit (as it has been shown with the effect of an increased percentage of air entrained in the system) or even prevent its operation at

all. Furthermore, damage may be done to the components present, such as the piston in the input hydraulic cylinder.

- Since cavitation is related to the pressures in the system, and these are associated with the forces acting on each lump, then it is intuitive to think that the operation of the system close to or under resonance may induce cavitation. This is because, for a mass spring system, for example, once the frequency of operation becomes close to resonance, the amount of force required to excite the system exhibits a drastic drop. The analogy to a hydraulic system suggests that the excitation of a fluid pipeline close to resonance, may induce high pressure drops and eventual cavitation. Further literature review and analysis of previous research on this topic is highly recommended.
- If resonance and cavitation do become a limiting factor on the operating frequency, then the analysis and possible development of the following piezohydraulic system is recommended. With the configuration shown in Figure 6.1, at any given stage there is always a stack pushing the fluid. Then the stack under a pulling stroke is prevented from lowering the pressure on the fluid due to pulling forces. The second stack,



Figure 6.1: Double-ended piezohydraulic unit operating under the actuation of a pair of synchronized piezoelectric stacks.

basically serves as an accumulator by providing additional force (and pressure) on the system. Furthermore, this configuration may enhance the force capability of the output cylinder. This is because under the current operating configuration (shown in Figure 4.2), during the pushing stage, one side is pushed while the other side is being compressed. In the same manner, during a pulling stage, one side is pulled while the other side is being expanded. In this scenario, pressure increases or decreases occur at both sides.

- Although two different types of timing patterns have been analyzed, it is recommended to perform a series of experiments to quantity the effects of several different timing patterns on the performance of the piezohydraulic unit. Effects on displacement have been simulated with the model and compared with experimental data. Nonetheless the effect on the output force of the hydraulic cylinder has yet to be analyzed. By output force, it is referred to the amount of force that the output cylinder may exert on a load or vice versa, at a given rate of displacement or at no displacement (clamped force).
- Study the effects of a constant spring and damper load on the output cylinder. The model developed offers the possibility of including a constant spring and damper load. Thus simulations can be performed and correlated with some measured data. One suggestion is to express the stiffness of the load as a percentage of the softest stiffness in the hydraulic system, and to involve in a load performance analysis, the use of various timing patterns. Both, the stiffness of the load (and therefore the force it is exerting) and the valve timing, are two of the factors considered to affect the performance of the piezohydraulic unit under load.
- The lumped parameter model seemed to simulate well the dynamics of the fluid system up to a frequency of 100 Hz. Nonetheless, the assumptions and the nature of the lumped parameter model suggest that a good correlation will start to decrease at some point, and continue to decrease as the frequency is increased. Then, a distributed type of model is suggested, and a good example can be found in Doebelin (1980), where a comparison between the distributed and the lumped parameter model is made.
- Finally, it is necessary to define and to determine the characteristics of the unit, such as efficency parameters, power densities, energy consumption, work output, etc... in order to establish advantages and disadvantages with current hybrid actuators.

Bibliography

ANSI/IEEE Standard 176, *IEEE Standard on Piezoelectricity*, Standards Committee of the IEEE Ultrasonics, Ferroelectrics, and Frequency Control Society, 1987.

Beckwith, T., Marangoni, R., and Lienhard, J., Mechanical Measurements, Fifth Edition, Addison-Wesley Publishing Company, Reading, Massachusetts, 1995.

Bexell, A., Tiensuu, A., Schweitz, J., and Soederkvist, J., *Characterization of an inchworm prototype motor.*, Sensors and Actuators., Vol. A43, No. 1/3, pp. 322, 1994.

Doebelin, E., **System Dynamics Modeling and Response**, Bell & Howell Company, Columbus, Ohio, 1972.

Doebelin, E., System Modeling and Response, Theoretical and Experimental Approaches, John Wiley & Sons, New York, 1980.

Dorf, R. and Bishop, R., Modern Control Systems, Seventh Edition, Addison-Wesley Publishing Company, Reading, Massachusetts, 1995.

Dorny, C., Understanding Dynamic Systems, Prentice-Hall, New Jersey, 1993.

Foster, K. and Parker, G., Fluidics, Components and Circuits, John Wiley&Sons, London, 1970.

Frank, J., Koopmann, G., Chen, W., and Lesieutre., G., *Design and performance of a high-force piezoelectric inchworm motor.*, **SPIE Proceedings.**, Vol. 3668, pp. 717–723, 1999.

Friedland, B., Control System Design, MacGraw-Hill, Boston, 1986.

Gerlach, T. and Wurmus, H., Working principle and performance of the dynamic micropump., Sensors and Actuators., Vol. A, No. 50, pp. 135–140, 1995. Ikeda, T., Fundamentals of Piezoelectricity, Oxford Science Publications, Oxford, 1990.

Koch, M., Harris, N., Evans, A., White, N., and Brunnschweiler, A., A novel micromachined pump based on thick-film piezoelectric actuation., Sensors and Actuators., Vol. A, No. 70, pp. 98–103, 1998.

Lee, S. and Esashi, M., Design of the electrostatic linear microactuator based on the inchworm motion., Mechatronics: mechanics, electronics, control., Vol. 5, No. 8, pp. 963–972, 1995.

Leo, D., Energy Analysis of Piezoelectric-Actuated Structure Driven by Linear Amplifiers, Journal of Intelligent Material Systems and Structure, Vol. 10, No. 1, pp. 36–45, 1999.

Leo, D. and Nasser, K., Efficiency of frequency-rectified piezohydraulic and piezopneumatic actuation, Proceedings of the ASME Adaptive Structures and Materials Symposium, Vol. AD-60, pp. 485–497, 2000.

Lindsay, J. and Katz, S., Dynamics of Physical Circuits and Systems, Matrix Publishers, Illinois, 1978.

Mauck, L. and Lynch, C., *Piezoelectric Hydraulic Pump.*, **SPIE Proceedings.**, Vol. 3668, 1999.

Meirovitch, L., Analytical Methods in Vibrations, MacMillan, New York, 1967.

Meirovitch, L., Methods of Analytical Dynamics, MacGraw-Hill, New York, 1970.

Munson, B., Young, D., and Okiishi, T., Fundamentals of Fluid Mechanics, Second Edition, John Wiley & Sons, New York, 1998.

NASA, Inchworm Actuator., NASA tech briefs., Vol. 18, No. 4, pp. 70, 1994.

Physik Instrumente, *PZT Fundamentals*, Catalog: Products for Micropositioning, 1999.

Pietrabissa, R., Mantero, S., Marotta, T., and Menicanti, L., A lumped parameter model to evaluate the fluid dynamics of different coronary bypasses., Med. Eng. Phys., Vol. 18, No. 6, pp. 477–484, 1996. Piezo Systems, Introduction to Piezoelectricity, Catalog # 3, 1998.

Rizzoni, G., **Principles and Applications of Electrical Engineering**, Second Edition, Irwin, Chicago, 1996.

Serway, R., Principles of Physics, Harcourt Brace Publishers, Fort Worth, 1994.

Shigley, J. and Mitchell, L., Mechanical Engineering Design, Fourth Edition, McGraw-Hill, New York, 1993.

Smits, J., *Piezoelectric Micropump with Three Valves Working Peristaltically.*, Sensors and Actuators., Vol. A, No. 21-23, pp. 203–206, 1990.

Streeter, V., Handbook of Fluid Dynamics, First Edition, McGraw-Hill, New York, 1961.

Tang, P., Palazzolo, A., Kascak, A., and Montague, G., *Electromechanical Modeling of Hybrid Piezohydraulic Actuator System for Active Vibration and Control.*, **Transactions** of the ASME., Vol. 119, 1997.

TRS Ceramics, www.trsceramics.com, 2000.

Ullmann, A., The Piezoelectric valve-less pump – performance enhancement analysis., Sensors and Actuators., Vol. A, No. 69, pp. 97–105, 1998.

Wang, Z. and TAN, S., Coupled analysis of fluid transients and structural dynamic responses of a pipeline system., Journal of Hydraulic Research., Vol. 35, No. 1, pp. 119– 132, 1997.

Wolf, J. and Paronesso, A., Lumped-parameter model and recursive evaluation of interaction forces of semi-infinite uniform fluid channel for time-domain dam-reservoir analysis., Earthquake Engineering and Structural Dynamics., Vol. 21, pp. 811–831, 1992.



Appendix A

Switching Amplifier Specifications (provided by Dynamic Structures and Materials)

The front panel is shown in the Figure A.1.

- I1 & I2: Digital input voltage (TTL signal, 0 to 5V), used to determine the operating frequency for ports 1 and 2.

- I3A: Digital input voltage (TTL signal, 0 to 5V), used to determine the operating frequency for port 3.

- I3B: Analog input voltage (0 to 3V), used to determine the current supply for port 3.

- I4: Not used.

- IP: Input power voltage (up to 80 VDC).

- O1 & O2: Output ports 1 and 2. Square voltage (0-400V) for capacitive loads of up to 400nF.

- O3: Output port 3. Triangular voltage waveform (0-150V) for capacitive loads of up to $40\mu F$.

- F1: Fuse for port 1, 2 Amps.

- F2: Fuse for port 2, 2 Amps.
- F3: Fuse for port 3, 3 Amps.
- FP: Fuse for input power port 2, 5 Amps.

AMPLIFIER INPUTS				AMPLIFIER OUTPUTS							
ON						0	0	F 1	0	F P	0
	$oldsymbol{ightarrow}$	ullet	$oldsymbol{ightarrow}$	ullet	ullet	01	O 2		03		IP
V OFF	I 1	I 2	I 3A	I 3B	I 4	0	0	\bigcirc	0	\bigcirc	0
011								F 2		F 3	

Figure A.1: Front Panel of the Amplifier (Built by Dynamic Structures and Materials).

A simplified version of the circuit of the switching amplifier for the port used to power the piezoelectric stack (port 3) is shown below.



Figure A.2: Simplified I/O Circuit for the Amplifier (provided by Carlos, in DSM).
Appendix B

Additional Specifications

Additional specifications on relevant components used in the experimental benchtop piezohydraulic unit have been included.

Figure B.1 shows a custom designed part used to connect the input hydraulic cylinder to the solenoid valves.



Figure B.1: Designed connecting tee drawings.

Figure B.2 shows the information provided by the manufacturer of the solenoid valves.



Figure B.2: Information on the solenoid valves used.

Figure B.3 shows the circuit developed with Nikola Vujic and Julio Lodetti, in order to power and drive the solenoid valves.



Figure B.3: Circuit used to drive the solenoid valves along with DSpace.



Figure B.4 is a drawing of the double ended cylinder used (provided by Bimba).

Figure B.4: Specifications on the double-ended cylinder used.

The following two pages contain the information provided by Mobil about the hydraulic fluid used (type HFA).



maximum or minimum are typical and may vary slightly.

Health and Safety

Based on available toxicological information, it has been determined that this product poses no significant health risk when used and handled property. Information on use and handling, as well as health and safety information, can be found in the Material Safety Data Sheet which can be obtained from your local distributor; or by calling 1-800-662-4525 and selecting propmpt 2.

For additional technical information or to identify the nearest U.S. Mobil supply source, call 1-800-662-4525.

Mobil Aero	HFA	HF
Product Number	490110	49012-8
Gravity, API	33	30
Specific Gravity	.8601	.8762
Pour Point, (°C(°F)	-59(- 75)	-59(-75)
Flash Point, COC, °C(°F), ASTM D 92	96(205)	82(180)
Flash Point, COC, "C("F), ASTM D 93		100 (212)
Viscosity, cSt at -40°C (-40°F) at 40°C (104°F) at 100C (212°F) Viscosity Index	429 12.6 5.1	500 14 4.9
Color	Red	Red
Tan	0.03	0.008
Evaporation Loss, %	Passes	Passes
Storage Stability, 12 Mo. at 24°C (75°F)	Passes	Passes
Low Temperature Stability, 72 h at -54°C (-65°F)	Passes	Passes
Rubber swell, 168 (at 70°C (158°F), Total	25.1	Passes

Mobil Oil

Corporation TECHNICAL PUBLICATIONS 3225 Gallows Road, Fairfax, Virginia 22037-0001 Due to continual product research and develop-

http://www.exxon.mobil.com/mobil_productdata/aviation/aeroHF.html

2/16/2000

Appendix C

Convergence of the Lumped Parameter Model

The following pages show the Matlab code used to develop the convergence simulations presented in *Chapter 3*. The following plots can be obtained:

- Variation of the slowest pole with respect to the number of lumps used.
- Percent change of the slowest pole versus the number of lumps used.
- Frequency response of the system as a function of the number of lumps.

```
&_____%
% thesis_convergence.m
                                                         2
% DESCRIPTION: Testing the convergence of the lumped parameter model. %
% September, 2000
                                                         2
                                                         8
% Khalil Nasser
%------%
clc
clear all
close all
% Defining Parameters:
% Input stiffness (it's the same as Kr)
Kin = 10^{4};
% for p_PA
% Ko = 1.3806*10^4;
% bo = 3.3969*10^0;
% Mo = 5.3118*10^-4;
% for an estimate of entire side A
% Ko = 1.3806*10^4;
% bo = 3.11*10^1;
% Mo = 5.36*10^-3;
% for an ESTIMATE of the entire system
Ko = 1.26*10^4; %lowest stiffness
bo = 4.89*10^1; %all damping added
Mo = 8.74*10^-3; %all masses added
% For the Frequency Response:
w = logspace(-1,6,1000)*2*pi;
                              % Freq in rad/sec
n = input('Range, Number of Lumps from 1 to ');
lf=1;
for c=1:n
  clear A A_lower A_lower_l A_lower_r
for i=1:lf
  eval(sprintf('M%d = Mo/lf;',i));
  eval(sprintf('K%d = Ko*lf;',i));
  eval(sprintf('b%d = bo/lf;',i ));
end
8------8
%------%
% Defining the coefficients of the A matrix,
% which represents the dynamics of the system.
if lf==1
     a_{11} = (-K1-Kin)/M1;
     a_1d = -b1/M1;
end
```

```
a_{21} = K1/M2;
    a 22 = (-K2-K1)/M2;
    a_2d = -b2/M2;
end
if lf>2
    a_{11} = (-K1-Kin)/M1;
    a_{12} = K1/M1;
    a_1d = -b1/M1;
  for i=2:lf-1
      eval(sprintf('a_%d1 = K%d/M%d;',i,i-1,i));
       eval(sprintf('a_%d2 = (-K%d-K%d)/M%d;',i,i,i,i-1,i));
      eval(sprintf('a %d3 = K%d/M%d;',i,i,i));
      eval(sprintf('a_%dd = -b%d/M%d;',i,i,i));
  end
  for i=lf
       eval(sprintf('a_%d1 = K%d/M%d;',i,i-1,i));
       eval(sprintf('a_%d2 = (-K%d-K%d)/M%d;',i,i,i,i-1,i));
       eval(sprintf('a %dd = -b%d/M%d;',i,i,i));
  end
end
<u>%______%</u>
%______%
% Defining the coefficient of the B matrix:
Bin = Kin/M1;
<u>%______%</u>
<u>%______%</u>
% Constructing the A,B,C,D matrices:
% NOTE: Size of A is nxn where n = # of states = 2* # of lumps = 2*lf
if lf==1
  A = [ 0 1
               ;
     a_11 a_1d ];
  B = [0; Bin];
  C = [1 0];
  D = [0];
end
olo______of
if lf==2
   A = \begin{bmatrix} 0 & 0 & 1 & 0 & i \\ 0 & 0 & 0 & 1 & i \end{bmatrix} 
                        ;
      a_11 a_12 a_1d 0 ;
a_21 a_22 0 a_2d ];
  B = [0; 0; 0; Bin; 0];
  C = [0 1 0 0];
  D = [0];
end
%_____
         if lf>2
  A_lower(1,:)=[ a_11 a_12 zeros(1,lf-2) a_1d zeros(1,lf-1) ];
  if lf==3
    A_lower(2,:)=[ a_21 a_22 a_23
                                      0 a 2d
                                                 0
1;
    A_1ower(3,:) = [0 a_31 a_32]
                                      0 0 a_3d
];
```

```
eval(sprintf('A_lower_r(%d,:)=[ zeros(1,%d) a_%dd
zeros(1,%d) ];',i,i-1,i,lf-1-(i-1) ));
         eval(sprintf('A_lower(%d,:)=[ A_lower_l(%d,:) A_lower_r(%d,:)
];',i,i,i ));
      end
      for i=lf
         eval(sprintf('A_lower_1(%d,:)=[ zeros(1,%d) a_%d1 a_%d2
];',i,i-2,i,i ) );
        eval(sprintf('A lower r(%d,:)=[ zeros(1,%d) a %dd ];',i,i-1,i
));
        eval(sprintf('A_lower(%d,:)=[ A_lower_l(%d,:) A_lower_r(%d,:)
];',i,i,i ));
     end
   end
   A = [zeros(lf) eye(lf);
             A_lower ];
   B = [ zeros(lf,1) ; Bin ; zeros(lf-1,1) ];
   C = [zeros(1,lf-1) \ 1 \ zeros(1,lf)];
   D = [0];
end
<u>%______%</u>
% Obtaining the slowest pole -lowest frequency- of the system
% and developing convergence plots
%А
Wn = damp(A);
Fn = Wn/(2*pi);
Fn_min = min(Fn);
Freq_min(lf) = Fn_min;
n_lump(lf) = lf;
if lf>1
   pct_change(lf) = abs((Freq_min(lf-1) -
Freq_min(lf))*100/Freq_min(lf-1));
   if pct change(lf)<1</pre>
     n lump(lf)
     pct_change(lf)
   end
end
% Assigning the A,B,C,D system corresponding to the number of lumps lf,
% and computing the corresponding frequency response
eval(sprintf('A%d = A;',lf));
eval(sprintf('B%d = B;',lf));
eval(sprintf('C%d = C;',lf));
eval(sprintf('D%d = D;',lf));
lf = lf+1;
end
% Output Plots:
figure(1)
plot(n lump,Freq min, '*-')
xlabel('Number of Lumps')
ylabel('Frequency, [Hz]')
title('Slowest or Lowest Frequency Pole')
```

```
c=1;
fprintf(' \n ')
figure(3)
while c==1
in1 = input(' Frequency Response Number, from 1 to 1f, 0 = quit [0] ');
in2 = isempty(in1); if in2==1 in1=0; end
if in1>0
  w = logspace(-1,6,1000)*2*pi; % Freq in rad/sec
   eval(sprintf('[mag,phase] = bode(A%d,B%d,C%d,D%d,1,w);',in1,in1,in1,in1));
   loglog(w/2/pi,mag)
   title(sprintf('Frequency Response with %d lumps',in1))
   xlabel('Frequency [Hz]');
  ylabel(sprintf('X%d(s)/Xin(s)',in1));
   % Grouping all the output for one plot
   eval(sprintf('mag_t(:,%d) = mag;',in1));
   eval(sprintf('phase_t(:,%d) = phase;',in1));
end
hold on
if in1==0 c=0; end
end
figure(3)
qrid
hold off
figure(4)
loglog(w/2/pi,mag_t)
title(sprintf('Frequency Response as a Function of the Number of Lumps', in1))
xlabel('Frequency [Hz]');
ylabel('Xout(s)/Xin(s)');
legend('1','2','3','4','5','6','7','8','9','10','11','12')
gtext('No. Lumps')
grid
```

Appendix D

Partial Matlab code Developed for the Simulations

The Matlab code has been developed in a modular fashion. The *thesis_main01.m* file is shown in the following two pages, and it is the main file of the entire code. It calls the rest of the .m files and updates .mat files containing data from variables, or entire operations. Through this file, measured oscillating data can be viewed and compared with a simulated set of data that has been generated with the corresponding measured input. Also, it is possible to generate simulations with theoretical input signals, that is, input signals that are defined by the user rather than using measured data.

One notorious branch or division, is the *thesis_pumping.m* file. It is just like the current main file, but it is solely devoted for the organization and execution of the files needed to simulate a pumping operation.

Finally, the .m file $thesis_fluidparam02.m$ has been included, as a reference of the values used to define the lumps of the various parts that compose the fluid system. The rest of the .m files have not been included due to their extensive size. For further information about the Matlab code, contact the author.

```
8------8
% thesis main01.m
                                                      %
% DESCRIPTION: MAIN Program - calls all the other 'm' files
                                                      %
% May-June-September, 2000
                                                       %
% Khalil Nasser
                                                       %
8-----%
clc
clear all
close all
%______%
%-----%
thesis_mechparam
save thesis_data_mechparam
clear all
load thesis_data_mechparam
thesis input01
if choice==2 choice2=0;end
if choice2==2
  % Theoretical - Pumping
  thesis_pumping
else
thesis_fluidparam02
save thesis_data_fluidparam p_PA p_PB p_CPC p_PC p_CC p_PCY p_SV
p_SVc p_SVc1 p_SVW p_AAC p_ACA p_ACB p_ACBc p_ACBW Pi pair p_SPA p_SPB
p_SCA p_SCB p_ELB p_TEE
pause
clear all
load thesis_data_fluidparam
thesis parts01
load thesis_data_mechparam
load thesis data initial
thesis coeff
save thesis_data_coeff
thesis_matrices
save thesis_data_matrices
save thesis_data_matrices01 A B1 B2 C D lp lp_os ld nc Pi
clear all
load thesis_data_input
load thesis_data_matrices01
load thesis_choice
if choice==1
  % Theoretical - Oscillation
```

```
thesis_simulation_1
else
 % Experimental - Oscillation
 thesis_simulation_2
end
load thesis_choice
if choice==1
 thesis_tfr01
end
load thesis_choice
if choice==2
 thesis_fft
end
save thesis_data_fft
end
clear all
8-----%
fprintf(' n n')
******** END OF PROGRAM ******** \n')
fprintf('
fprintf(' n n')
8-----
```

```
format short e
```

% END

```
%
% thesis fluidparam02.m
% DESCRIPTION: Defining the fluid's lumped parameters.
                                                              %
                                                             $
             Each component is broken into geometrical lumpes that are divided further in order to ensure the
%
2
                                                             %
             convergence of the simulations, with a finer grid.
%
                                                             %
% May-June, 2000
                                                              %
% Khalil Nasser
                                                              8
%_____%
% Data used for an individual run of the file:
% clear all
% pair=0.1;
% Pi=100;
% Dcyl=(0.5*2.54/100);
% Dch=(5/8*2.54/100);
% FLUID's Properties:
% Density is assumed to have small variations around its original
% value (operating point).
density = 860.1;
                                  % HFA MOBIL Fluid [Kq/m^3]
                                  % HF Dynamic Visc. [N*s/m^2]
viscosity = 3.8*10^-1;
Bpsi = 220000;
                                  % HF Bulk Modulus [Psi]
                                   % HF Bulk Modulus [Pa]
Bf = 6895*Bpsi;
% Re # used to calculate (1/d)equiv from Kl factors from MUNSON fluids
% book. IT IS NOT the Re# of the system (thus, f=64/Re, laminar flow).
reb = 1800;
% AIR's Properties:
% Air's pressure can be approximated to the initial pressure (for bulk
% modulus calculations) in the oscillating operation (Good for small
% variations around Pi). It is NOT the case for the pumping config.
Bg = 6895*1.4*Pi;
                                 % Air's Bulk Modulus [Pa]
% LUMP's EXTERNAL BODY (LEB) Properties (pipes..conectors..):
% Modulus of Elasticity of the following materials:
E_aluminum = 71*10^9; % [Pa]
E_brass = 106*10^9;
E_carbonsteel = 207*10^9;
                                 % [Pa]
                                 % [Pa]
E castiron = 100*10^9;
                                 % [Pa]
E_stainless = 190*10^9;
                                 % [Pa]
8------8
% Part: PIPE A w/ bends (PA)
xpa = pair/100;
                                 % Fraction of Air
                                 % Pipe's thickness [m]
tpa = .7112e-3;
                                 % Pipe's Elasticity [Pa]
% Internal Diameter [m]
Epa = E_stainless;
Dpa = 1.7526e-3;
                                 % Area
                                                    [m^2]
Apa = pi*(Dpa^2)/4;
Lpa = 256e-3;
                                  % Length
                                                     [m]
% The equivalent Bulk Modulus of the Lump of Fluid [Pa]
Bpa = ( Dpa/(tpa*Epa) + (1-xpa)/Bf + xpa/Bg )^-1;
% Pipe's minor losses:
1d_1 = 20;
                                  % Long rad 90 deg elbow
1d_2 = 5.625;
                                   % Long rad 45 deg elbow
```

```
1d_3 = 16;
                                  % Regular 45 deg elbow
%ld_3 = 11.25;
                                  % Regular 45 deg elbow
                                   % Long rad 90 deg elbow
1d \ 4 = 20;
ld_eq = ld_1+ld_2+ld_3+ld_4;
Kpa = Apa*Bpa/Lpa;
                                  % Capacitance (stiff)[N/m]
Kpa= Apa^Bpa/Lpa;% Capacitance (stiff)[N/mMpa= density*Apa*Lpa;% Inductance (mass) [Kg]
bpa = 8*pi*viscosity*(Lpa + Dpa*ld_eq);% Resistance (damp.) [Kg/s]
8-----%
% Part: PIPE B w/ bends (PB)
xpb = pair/100;
tpb = .7112e-3;
Epb = E_stainless;
                                  % Fraction of Air
xpb = pair/100;
                                % Pipe's thickness [m]
% Pipe's Elasticity [Pa]
% Internal Diameter [m]
Dpb = 1.7526e-3;
                                 % Area
% Length
Apb = pi*(Dpb^2)/4;
                                                    [m^2]
Lpb = 280e-3;
                                                    [m]
% The equivalent Bulk Modulus of the Lump of Fluid [Pa]
Bpb = (Dpb/(tpb*Epb) + (1-xpb)/Bf + xpb/Bg)^{-1};
% Pipe's minor losses:
                                 % Std 90 degree elbow
1d_1 = 30;
1d_2 = 30;
                                  % Std 90 degree elbow
                                  % Std 90 degree elbow
1d_3 = 30;
ld_eq = ld_1 + ld_2 + ld_3;
Kpb = Apb*Bpb/Lpb;% Capacitance (stiff)[N/m]Mpb = density*Apb*Lpb;% Inductance (mass) [Kg]
bpb = 8*pi*viscosity*(Lpb + Dpb*ld_eq);% Resistance (damp.) [Kg/s]
8------8
oo______o
% Part: Customized Brass Pipe Conectors (CPC)
                                 % Fraction of Air
xcpc = pair/100;
                                 % CPC's thickness [m]
% CPC's Elasticity [Pa]
tcpc = 2.25e-3;
Ecpc = E_brass;
Dcpc = 1.7e-3;
                                 % Internal Diameter [m]
Acpc = pi*(Dcpc^2)/4; % Area
Lcpc = 25.5e-3; % Length
                                 % Area [m^2]
                                                     [m]
% The equivalent Bulk Modulus of the Lump of Fluid [Pa]
%Bcpc = ( Dcpc/(tcpc*Ecpc) + (1-xcpc)/Bf + xcpc/Bg )^-1;
Bcpc = ((1-xcpc)/Bf + xcpc/Bg)^{-1};
% CPC's minor losses:
ld_1 = (.5)/(64/reb);
                                 % ld = K/f for one side
ld_1 = (.5)/(64/reb);
ld_2 = (.98)/(64/reb);
                                 % ld = K/f for other side
ld eq = ld 1+ld 2;
Kcpc = Acpc*Bcpc/Lcpc; % Capacitance (stiff)[N/m]
Mcpc = density*Acpc*Lcpc; % Inductance (mass) [Kg]
bcpc= 8*pi*viscosity*(Lcpc+Dcpc*ld_eq);% Resistance (damp.) [Kg/s]
8------8
%______%
% Part: Pipe Conectors (PC)
% PC model is good for any direction. Minor losses differnces due to
% direction operation is averaged and compensated by the use of PC
% in pairs (2 per pipe)
xpc = pair/100;
                                  % Fraction of Air
                                  % PC's thickness [m]
tpc = 2.25e-3;
                                  % PC's Elasticity [Pa]
Epc = E_brass;
```

```
Dpc = [ 1.7526e-3 2.32e-3 ]; % Internal Diameter [m]
Apc = pi*(Dpc.^2)./4; % Area [m^2]
Lpc = [ (17e-3)/2 ]; % Length [m]
% The equivalent Bulk Modulus of the Lump of Fluid [Pa]
%Bpc = ( Dpc/(tpc*Epc) + (1-xpc)/Bf + xpc/Bg ).^-1;
Bpc = ( (1-xpc)/Bf + xpc/Bg ).^{-1};
% PC's minor losses:
bpc = 8*pi*viscosity*(Lpc+Dpc*ld_eq); % Resistance (damp.) [Kg/s]
%bpc = ( 1/b(1) + 1/b(2) )^-1; % Equiv. Damping [Kg/s]
<u>%______%</u>
% Part: Customized Chamber (CC)
% Due to symmetry assumptions of charge and discharge area changes,
% the CC model is valid for both directions of operation.
xcc = pair/100; % Fraction of Air
tcc = [ 2.5e-3 6.71e-3 1.71e-3 ] ; % CC's thicknessEcc = E_carbonsteel;% CC's Elasticity
                                                       [m]
                                                       [Pa]
Dcc = [ 10e-3 1.5875e-3 1.5875e-3 ];% Internal Diameter [m]
Acc = pi.*(Dcc.^2)./4; % Area
Lcc = [ 3.5e-3 8e-3 35e-3 ]; % Length
                                                      [m^2]
                                                       [m]
% The equivalent Bulk Modulus of the Lump of Fluid [Pa]
%Bcc = ( Dcc./(tcc.*Ecc) + (1-xcc)/Bf + xcc/Bg ).^-1;
Bcc = ((1-xcc)/Bf + xcc/Bg).^{-1};
% Chamber's minor losses:
ld_1 = (.5)/(64/reb);
                                  % ld = K/f for one side
1d 2 = 30;
                                  % Std 90 degree elbow
ld_3 = (.98)/(64/reb);
                                   % ld = K/f for other side
ld_eq = [ ld_1 ld_2 ld_3];
Kcc = Acc.*Bcc./Lcc;
                                  % Capacitance (stiff)[N/m]
%Kcc = ( 1/K(1) + 1/K(2) + 1/K(3) )^-1;% Equiv. Stiffness [N/m]

        Mcc
        = density*Acc.*Lcc;
        % Inductance (mass)
        [Kg]

        %Mcc
        = M(1) + M(2) + M(3);
        % Equiv. Mass
        [Kg]

bcc = 8*pi*viscosity*(Lcc +Dcc.*ld_eq);% Resistance (damp.) [Kg/s]
bcc = (1/b(1) + 1/b(2) + 1/b(3))^{-1}; Equiv. Damping [Kq/s]
%------%
%------%
% Part: Pumping Cylinder (PCY)
xpcy = pair/100; % Fraction of Air
tpcy = [ 10e-3 10e-3 ]; % PCY's thickness [m]
Epcy = E_brass; % PCY's Elasticity [Pa]
Dpcy = [ Dch 10e-3 ]; % Internal Diameter [m]
Apcy = pi.*(Dpcy.^2)./4; % Area [m^2]
Lpcy = [ 2.5e-3 4e-3 ]; % Length [m]
%Bpcy = ( Dpcy./(tpcy.*Epcy) + (1-xpcy)/Bf + xpcy/Bg ).^-1;
```

```
Bpcy = ((1-xpcy)/Bf + xpcy/Bg).^{-1};
% Chamber's minor losses:
ld_1 = (.325)/(64/reb);
                                         % ld = K/f for change in A
ld_eq = ld_1;
Kpcy = Apcy.*Bpcy./Lpcy;
                                                 % Capacitance (stiff)[N/m]

      Kpcy
      = Apcy.*Bpcy./Lpcy;
      % Capacitance (stiff)[N/m]

      %Kpcy = (1/K(1) + 1/K(2))^-1;
      % Equiv. Stiffness [N/m]

      Mpcy
      = density*Apcy.*Lpcy;
      % Inductance (mass) [Kg]

      %Mpcy = M(1) + M(2);
      % Equiv. Mass [Kg]

bpcy = 8*pi*viscosity*(Lpcy+Dpcy*ld_eq);% Resistance (damp.) [Kg/s]
%bpcy = ( 1/b(1) + 1/b(2) )^-1; % Equiv. Damping [Kg/s]
8-----%
%______%
% Part: Adaptor for the Actuator Cylinder (AAC)
% The equivalent Bulk Modulus of the Lump of Fluid [Pa]
%Baac = ( Daac./(taac.*Eaac) + (1-xaac)/Bf + xaac/Bg ).^-1;
Baac = ((1-xaac)/Bf + xaac/Bg).^{-1};
% AAC's minor losses:

      ld_a = (.49)/(64/reb);
      % ld = K/f - dA in part

      ld_b = (.85)/(64/reb);
      % ld = K/f - dA into cyl

ld eq = [ld a ld b];

      Kaac
      = Aaac.*Baac./Laac;
      % Capacitance (stiff)[N/m]

      %Kaac = (1/K(1) + 1/K(2))^-1;
      % Equiv. Stiffness [N/m]

      Maac
      = density*Aaac.*Laac;
      % Inductance (mass) [Kg]

      %Maac = M(1) + M(2);
      % Equiv. Mass [Kg]

baac = 8*pi*viscosity*(Laac+Daac.*ld_eq);% Resistance (damp.) [Kg/s]
%baac = (1/b(1) + 1/b(2))^-1; % Equiv. Damping [Kg/s]
%-----%
%______%
% Part: Solenoid Valves (SV)

      xsv = pair/100;
      % Fraction of Air

      tsv = [5e-3 15e-3 5e-3];
      % SV's thickness [m]

      Esv = E_carbonsteel;
      % SV's Elasticity [Pa]

      Dsv = [8.8e-3 1.5e-3 8.8e-3];
      % Internal Diameter [m]

Asv = pi.*(Dsv.^2)./4; % Area
Lsv = [ 4.8e-3 5e-3 11.45e-3 ]; % Length
                                                                          [m^2]
                                                                           [m]
% The equivalent Bulk Modulus of the Lump of Fluid [Pa]
%Bsv = ( Dsv./(tsv.*Esv) + (1-xsv)/Bf + xsv/Bg ).^-1;
Bsv = ((1-xsv)/Bf + xsv/Bg).^{-1};
% SV's minor losses:
ld_b = (1.11)/(64/reb)+30+30; % Side B
ld m = 30; % Std 90 degree elbow
ld_a = (1.12)/(64/reb)+30+30; % Side A
ld_eq = [ ld_b ld_m ld_a ];
Ksv = Asv.*Bsv./Lsv;
                                               % Capacitance (stiff)[N/m]
```

```
Msv= density*Asv.*Lsv;% Inductance (mass)[Kg]%Msv= M(1) + M(2);% Equiv. Mass[Kg]
bsv = 8*pi*viscosity*(Lsv+Dsv.*ld_eq);% Resistance (damp.) [Kg/s]
%bsv = ( 1/b(1) + 1/b(2) )^-1; % Equiv. Damping [Kg/s]
<u>%______%</u>
% Part: Closed Solenoid Valve (SVc)
% Pipe Conected to Side A
Bsvc = Bsv(1);
Bsvc = Bsv(2);
Ksvc = Ksv(2);
                                % Capacitance (stiff)[N/m]
Msvc = 0.25 Msv(2);
                                % Inductance (mass) [Kg]
Msvc = 0.5*Msv(2);
                                  % Inductance (mass) [Kg]
bsvc = bsv(2);
                                 % Resistance (damp.) [Kg/s]
% Taking the wall as the valve's piston (w)
Asvw = pi/4*(15e-3)^2;
Lsvw = 5e-3;
Ksvw = Asvw*E_stainless/Lsvw; % Capacitance (stiff)[N/m]
Msvw = 0.75 Msv(2);
Msvw = 0.5*Msv(2);
bsvw = 0;
Bsvw = 0; % no bulk mod for a solid
% Part: Closed Solenoid Valve (SVc1)
% Pipe Conected to Side A of valve
% Taking the wall as the valve's piston (w)
Asvw = pi/4*(15e-3)^2;
Lsvw = 5e-3;
Ksvw = Asvw*E_stainless/Lsvw; % Capacitance (stiff)[N/m]
Bsvc1 = [Bsv(1)];
Ksvc1 = [Ksv(1) * Ksvw/(Ksv(1) + Ksvw)];
Msvc1 = [Msv(1)];
bsvc1 = [ bsv(1) ];
oc______o
% FOR THE ACTUATOR-CYLINDER
% Initial position of the piston (from 0 to 1 in)
dp = (0.5)*(2.54/100); % [m]
&_____%
% Part: Actuator Cylinder Side B (ACB)
                               % Fraction of Air
xacb = pair/100;
%tacb = [ 5e-3 2.6e-3 ] ;
tacb = [ 0 2025
                               % ACB's thickness [m]
tacb = [ 0.3937*Dcyl 0.2047*Dcyl ];
Eacb = [E_aluminum E_brass]; % ACB's Elasticity [Pa]
%Dacb = [ 9.53e-3 12.7e-3 ];
                               % Internal Diameter [m]
Dacb = [ 0.7504*Dcyl Dcyl ];
                               % Area
Aacb = pi.*(Dacb.^2)./4;
                                                  [m^2]
Lacb = [ 19.05e-3 (25.4e-3 -dp) ]; % Length
                                                 [m]
% The equivalent Bulk Modulus of the Lump of Fluid [Pa]
%Bacb = ( Dacb./(tacb.*Eacb) + (1-xacb)/Bf + xacb/Bg ).^-1;
Bacb = ((1-xacb)/Bf + xacb/Bg).^{-1};
% ACB's minor losses:
ld b = (.22)/(64/\text{reb});
                        % Side B reduction in A
ld_eq = ld_b;
```

```
Kacb= Aacb.*Bacb./Lacb;% Capacitance (Sec.)%Kacb = (1/K(1) + 1/K(2))^-1;% Equiv. Stiffness [N/m]Macb= density*Aacb.*Lacb;% Inductance (mass) [Kg]%(1) + M(2);% Equiv. Mass [Kg]
                                      % Capacitance (stiff)[N/m]
bacb = 8*pi*viscosity*(Lacb+Dacb*ld_eq);% Resistance (damp.) [Kg/s]
bacb = (1/b(1) + 1/b(2))^{-1}; & Equiv. Damping [Kg/s]
% Part: Actuator Cylinder Side B -closed- (ACBc) -last lump
Bacbc = Bacb;
Kacbc = Kacb;
                                    % Capacitance (stiff)[N/m]
%Macbc = [ Macb(1) 0.25*Macb(2) ]; % Inductance (mass) [Kg]
Macbc = [ Macb(1) 0.5*Macb(2) ]; % Inductance (mass) [Kg]
bacbc = bacb; % Resistance (damp.) [F
                                 % Resistance (damp.) [Kg/s]
% Taking the wall as the cylinder's end (ACBW)
Aacbw = pi/4*(Dcyl)^2;
Lacbw = Dcyl/2;
Kacbw = Aacbw*E_aluminum/Lacbw; % Capacitance (stiff)[N/m]
Macbw = 0.75 Macb(2);
Macbw = 0.5*Macb(2);
bacbw = 0;
Bacbw = 0; % no bulk mod for a solid
<u>%______%</u>
% Part: Actuator Cylinder Side A (ACA)
xaca = pair/100;
                                    % Fraction of Air
taca = 2.6e-3;
                                    % ACB's thickness [m]
taca = 0.2047*Dcyl;
Eaca = E brass;
                                  % ACB's Elasticity [Pa]
                                   %
%Dpist= 12.7e-3;
                                                        [m]
Dpist = Dcyl;
%Drod = 6.35e-3;
                                   %
                                                        [m]
%Drod = 0.5*Dcyl;
%Drod = 0.5 Degr;
%Daca = sqrt(Dpist^2 - Drod^2); % Equiv Diameter
Aaca = pi.*(Dpist^2 - Drod^2)./4; % Area
Laca = dp + 19.05e-3; % Length
                                                        [m]
                                                        [m^2]
                                                        [m]
% The equivalent Bulk Modulus of the Lump of Fluid [Pa]
%Baca = ( Daca/(taca*Eaca) + (1-xaca)/Bf + xaca/Bg )^-1;
Baca = ((1-xaca)/Bf + xaca/Bg)^{-1};
Kaca = Aaca*Baca/Laca;% Capacitance (stiff)[N/m]Maca = density*Aaca*Laca;% Inductance (mass) [Kg]baca = 8*pi*viscosity*(Laca);% Resistance (damp.) [Kg/s]
%______%
%______%
<u>%______%</u>
% Part: PIPE A w/ bends (SPA)
xspa = 2*pair/100;
                                      % Fraction of Air
                                 % Flaction cl 1
% Pipe's thickness [m]
% Pipe's Elasticity [Pa]
% Internal Diameter [m]
[m^2
tspa = .7112e-3;
Espa = E_stainless;
Dspa = 1.7526e-3;
                                                        [m^2]
Aspa = pi*(Dspa^2)/4;
                                   % Area
Lspa = 140e-3;
                                    % Length
                                                        [m]
% The equivalent Bulk Modulus of the Lump of Fluid [Pa]
Bspa = ( Dspa/(tspa*Epa) + (1-xspa)/Bf + xspa/Bg )^-1;
% Pipe's minor losses:
ld_1 = 5.625;
                                    % Long rad 45 deg elbow
```

```
1d_2 = 2;
                                        % Long rad 15 deg elbow
1d \ 3 = 20;
                                          % Long rad 90 deg elbow
ld_eq = ld_1+ld_2+ld_3;
Kspa = Aspa*Bspa/Lspa;% Capacitance (stiff)[N/m]Mspa = density*Aspa*Lspa;% Inductance (mass) [Kg]
bspa = 8*pi*viscosity*(Lspa + Dspa*ld_eq);% Resistance (damp.)
 [Kg/s]
<u>%______%</u>
<u>%______%</u>
% Part: PIPE B w/ bends (SPB)
xspb = pair/100;
                                          % Fraction of Air
                                      % Pipe's thickness [m]
% Pipe's Elasticity [Pa]
% Internal Diameter [m]
tspb = .7112e-3;
Espb = E_stainless;
Dspb = 1.7526e-3;
Dspp = 1.75200-57
Aspb = pi*(Dspb^2)/4; % Area
Lspb = 1500-3; % Length
                                                                [m^2]
                                                               [ m ]
% The equivalent Bulk Modulus of the Lump of Fluid [Pa]
Bspb = (Dspb/(tspb*Espb) + (1-xspb)/Bf + xspb/Bq)^{-1};
% Pipe's minor losses:
                                        % Long rad 45 deg elbow
ld_1 = 5.625;
1d_2 = 20;
                                        % Long rad 90 deg elbow
1d \ 3 = 20;
                                          % Long rad 90 deg elbow
ld eq = ld 1+ld 2+ld 3;
Kspb = Aspb*Bspb/Lspb;
                                             % Capacitance (stiff)[N/m]
Kspb = Aspb*Bspb/Lspb;% Capacitance (stiff)[N/1Mspb = density*Aspb*Lspb;% Inductance (mass) [Kg]
bspb = 8*pi*viscosity*(Lspb + Dspb*ld_eq);% Resistance (damp.)
  [Kq/s]
%______%
<u>%______%</u>
% Part: one side of TEE (TEE)

      xtee = pair/100;
      % Fraction of Air

      Dtee = [ 4.75e-3 4.75e-3 ];
      % Internal Diameter [m]

      Atee = pi.*(Dtee.^2)./4;
      % Area [m^2]

      Ltee = [ 4.5e-3 10.5e-3 ];
      % Length [m]

% The equivalent Bulk Modulus of the Lump of Fluid [Pa]
Btee = ( (1-xtee)/Bf + xtee/Bg ).^-1;
% Minor losses:
ld_1 = ((.42+.58)/2)/(64/reb); % ld = K/f for change in A
ld_2 = 30+((.42+.58)/2)/(64/reb); % Std 90 deg elbow + change in A
ld_eq = [ ld_1 ld_2 ];
Ktee= Atee.*Btee./Ltee;% Capacitance (stiff)[N/m]Mtee= density*Atee.*Ltee;% Inductance (mass) [Kg]
btee = 8*pi*viscosity*(Ltee+Dtee.*ld_eq); % Resistance (damp.)
  [Kg/s]
%______%
%______%

      xelb = pair/100;
      % Fraction of Air

      Delb = [ 2.32e-3 4.75e-3 ];
      % Internal Diameter [m]

      Aelb = pi.*(Delb.^2)./4;
      % Area [m^2]

      Lelb = [ 5.4e-3 10.5e-3 ];
      % Length [m]

% The equivalent Bulk Modulus of the Lump of Fluid [Pa]
Belb = ((1-xelb)/Bf + xelb/Bg).^{-1};
```

```
% Minor losses:

      ld_1 = ((.42+.58)/2)/(64/reb);
      % ld = K/f for change in A

      ld 2 = 30;
      % Std 90 deg elbow

ld_eq = [ ld_1 ld_2 ];
Kelb = Aelb.*Belb./Lelb; % Capacitance (stiff)[N/m]
Melb = density*Aelb.*Lelb; % Inductance (mass) [Kg]
belb = 8*pi*viscosity*(Lelb+Delb.*ld_eq); % Resistance (damp.)
  [Kg/s]
&_____&
<u>%______%</u>
% Part: Symmetric Cylinder Side B (SCB) -from port to bore-
%pair scb = 5*pair
scb = (pair scb)/100;
                                        % Fraction of Air
xscb = pair/100;
                                        % Fraction of Air
Dscb = [ 14e-3 19.05e-3 ]; % Internal Diameter [m]
Drs = 0.25*25.4*10^-3; % Rod's Diameter [m]

      DIS
      = 0.25.25.4*10 -5;
      % Rod's Diameter

      Ascb
      = pi.*(Dscb.^2 - Drs^2)./4;
      % Area

      Lscb
      = [ 17e-3 (18.375e-3 +dp) ];
      % Length

                                                               [m^2]
                                                              [m]
% The equivalent Bulk Modulus of the Lump of Fluid [Pa]
Bscb = ((1-xscb)/Bf + xscb/Bg).^{-1};
ld_1 = 30+((.82+.5)/2)/(64/reb); % Std 90 deg elbow + change in A
ld_2 = (0.2)/(64/reb); % Change in A
ld_eq = [ ld_1 ld_2 ];
Kscb = Ascb.*Bscb./Lscb; % Capacitance (stiff)[N/m]
Mscb = density*Ascb.*Lscb; % Inductance (mass) [Kg]
bscb = 8*pi*viscosity*(Lscb+Dscb.*ld_eq); % Resistance (damp.) [Kg/s]
8------8
<u>%______%</u>
% Part: Symmetric Cylinder Side A (SCA) -from bore to port-
%pair_sca = 3*pair
                                        % Fraction of Air
%xsca = (pair_sca)/100;
xsca = pair/100;
                                       % Fraction of Air

      Dsca = [ 19.05e-3 14e-3 ];
      % Internal Diameter [m]

      Drs = 0.25*25.4*10^-3;
      % Rod's Diameter [m]

Asca = pi.*(Dsca.^2 - Drs^2)./4; % Area
Lsca = [ (18.375e-3 -dp) 17e-3 ]; % Length
                                                               [m^2]
                                                               [m]
% The equivalent Bulk Modulus of the Lump of Fluid [Pa]
Bsca = ((1-xsca)/Bf + xsca/Bg)^{-1};
ld_1 = (0.2)/(64/reb);
ld_1 = (0.2)/(64/reb); % Change in A
ld_2 = 30+((.82+.5)/2)/(64/reb); % Std 90 deg elbow + change in A
ld_eq = [ ld_1 ld_2 ];
Ksca = Asca.*Bsca./Lsca;
Ksca = Asca.*Bsca./Lsca;% Capacitance (stiff)[N/m]Msca = density*Asca.*Lsca;% Inductance (mass) [Kg]
bsca = 8*pi*viscosity*(Lsca+Dsca.*ld_eq); % Resistance (damp.) [Kg/s]
٩_____٩
%______%
% FORMATTED OUTPUT:
fprintf('\n\n Press Any Key to Continue')
pause
fprintf(' n n')
```

fprintf('OUTPUT FROM thesis_fluidparam.m') fprintf(' n n')fprintf(' Element\t Stiffness [N/m]\t Damping [Kg/s]\t Mass [Kg]') fprintf('\t Bulk Modulus [Pa] \n') fprintf(' PA\t %1.4d\t %1.4d\t %1.4d\t %1.4d \n',Kpa,bpa,Mpa,Bpa) %1.4d\t %1.4d\t %1.4d\t %1.4d fprintf(' PB∖t $\langle n', Kpb, bpb, Mpb, Bpb \rangle$ fprintf(' CPC\t %1.4d\t %1.4d\t %1.4d\t %1.4d \n',Kcpc,bcpc,Mcpc,Bcpc) PC 1\t %1.4d\t %1.4d\t %1.4d\t %1.4d fprintf(' n', Kpc(1), bpc(1), Mpc(1), Bpc(1)fprintf(' PC 2 t%1.4d\t %1.4d\t %1.4d\t %1.4d $\langle n', Kpc(2), bpc(2), Mpc(2), Bpc(1) \rangle$ fprintf(' CC 1\t %1.4d\t %1.4d\t %1.4d\t %1.4d n', Kcc(1), bcc(1), Mcc(1), Bcc(1)fprintf(' CC 2\t %1.4d\t %1.4d\t %1.4d\t %1.4d n', Kcc(2), bcc(2), Mcc(2), Bcc(1)fprintf(' CC 3\t %1.4d\t %1.4d\t %1.4d\t %1.4d n', Kcc(3), bcc(3), Mcc(3), Bcc(1)fprintf(' PCY 1\t %1.4d\t %1.4d\t %1.4d\t %1.4d n', Kpcy(1), bpcy(1), Mpcy(1), Bpcy(1)fprintf(' PCY 2\t %1.4d\t %1.4d\t %1.4d\t %1.4d n', Kpcy(2), bpcy(2), Mpcy(2), Bpcy(1)) %1.4d\t %1.4d\t %1.4d\t %1.4d fprintf(' SV 1∖t n', Ksv(1), bsv(1), Msv(1), Bsv(1)fprintf(' SV 2 t%1.4d\t %1.4d\t %1.4d\t %1.4d n', Ksv(2), bsv(2), Msv(2), Bsv(1)fprintf(' SVc∖t %1.4d\t %1.4d\t %1.4d\t %1.4d \n',Ksvc,bsvc,Msvc,Bsvc) fprintf(' AAC 1\t %1.4d\t %1.4d\t %1.4d\t %1.4d n', Kaac(1), baac(1), Maac(1), Baac(1)) fprintf(' AAC 2\t %1.4d\t %1.4d\t %1.4d\t %1.4d n', Kaac(2), baac(2), Maac(2), Baac(1)) fprintf(' ACA\t %1.4d\t %1.4d\t %1.4d\t %1.4d \n',Kaca,baca,Maca,Baca) %1.4d\t %1.4d\t %1.4d\t %1.4d fprintf(' ACB 1 tn', Kacb(1), bacb(1), Macb(1), Bacb(1)fprintf(' ACB 2\t %1.4d\t %1.4d\t %1.4d\t %1.4d n', Kacb(2), bacb(2), Macb(2), Bacb(1)%1.4d\t %1.4d\t %1.4d\t %1.4d fprintf(' ACBc 1\t n', Kacbc(1), bacbc(1), Macbc(1), Bacbc(1)) fprintf(' ACBc 2\t %1.4d\t %1.4d\t %1.4d\t %1.4d n', Kacbc(2), bacbc(2), Macbc(2), Bacbc(1)) %1.4d\t %1.4d\t\t %1.4d \n',Ksvw,bsvw,Msvw) fprintf(' SVW\t fprintf(' ACBW\t %1.4d\t %1.4d\t\t %1.4d \n',Kacbw,bacbw,Macbw) fprintf(' n n')fprintf('\t Stiff. \t Damping Mass Bulk Modulus $\langle t' \rangle$

```
fprintf('Wn \t z-damp. coeff. \n')
fprintf('\t [N/micron] [Kg/s]
                                                                                        [q]
                                                                                                               [Psi] \ \t[Hz]
n'
K = [
Kpa;Kpb;Kcpc;Kpc(1);Kcc(2);Kcc(2);Kcc(3);Kpcy(1);Kpcy(2);Ksv(1);
Ksv(2);Ksvc;Kaac(1);Kaac(2);Kaca;Kacb(1);Kacb(2);Kacbc(1);Kacbc(2);Ksvw
;Kacbw ];
                                       % [N/m]
Ku = K.*10^{-6};
                                                                                                                                                            %
[N/um]
b = [
bpa;bpb;bcpc;bpc(1);bcc(1);bcc(1);bcc(1);bpcy(1);bpcy(1);bsv(1);
bsv(1);bsvc;baac(1);bacc(1);bacb(1);bacbc(1);bacbc(1);bsvw
;bacbw ];
                                        % [Kq/s]
M = [
Mpa; Mpb; Mcpc; Mpc(1); Mpc(2); Mcc(1); Mcc(2); Mcc(3); Mpcy(1); Mpcy(2); Msv(1);
Msv(2);Msvc;Maac(1);Maac(2);Maca;Macb(1);Macb(2);Macbc(1);Macbc(2);Msvw
;Macbw ];
                                       % [Kq]
Mu = M.*10^{3};
                                                                                                                                                            %
[q]
B = [
Bpa; Bpb; Bcpc; Bpc(1); Bcc(1); Bcc(1); Bcc(1); Bcc(1); Bpcy(1); Bpcy(1);
Bsv(1);Bsvc;Baac(1);Baac(1);Baca;Bacb(1);Bacb(1);Bacbc(1);Bacbc(1);Bsvw
;Bacbw ]; % [Pa]
%B = [
Bpa; Bpb; Bcpc; Bpc(1); Bcc(2); Bcc(2); Bcc(3); Bpcy(1); Bpcy(2); Bsv(1);
Bsv(2);Bsvc;Baac(1);Baac(2);Baca;Bacb(1);Bacb(2);Bacbc(1);Bacbc(2);Bsvw
;Bacbw ]; % [Pa]
Bu = B./6895;
                                                                                                                                                            %
[Psi]
Wn = sqrt(K./M);
                                                                                                                                                            %
[rad/sec]
                                                                                                                                                            %
Fn = Wn./2./pi;
[Hz]
z = (b.*Wn)./(2*K);
c=1;
for n=1:length(K)
       if n \ge length(K) - 1
              fprintf('%d\t %4.2f \t %2.4f \t %2.4f %5.2f
%9.2f
                                %3.4f \n', c, Ku(n), b(n), Mu(n), Bu(n), Fn(n), z(n))
       else
              %if n==1;fprintf(' ');end
                                                                     %2.4f \t
              fprintf('%d\t %2.5f \t
                                                                                                      %2.4f
                                                                                                                              %5.2f
%9.2f
                                  3.4f \ (n',c,Ku(n),b(n),Mu(n),Bu(n),Fn(n),z(n))
       end
       c=c+1;
end
%fprintf('\t %4.1f \t %2.4f \t %2.4f \t %5.2f \n',Kw*10^-
6, bw, Mw*10^3, Bw, )
fprintf(' n n')
  clear K b M B
%_____%
```

```
% ADDING MORE LUMPS PER COMPONENT (except for wall elements):
% if Mo,Ko,bo are original quantities for a lump within a component,
% then M, K, b are the new values of the broken up component into lf
lumps
% K = Ko*lf
b = bo/lf
% M = Mo/lf
clear n
% p_PA
n = 4;
for i=1:n; K_pa(i) = n*Kpa; b_pa(i) = bpa/n; M_pa(i) = Mpa/n; end
% p PB
n = 4;
for i=1:n; K_pb(i) = n*Kpb; b_pb(i) = bpb/n; M_pb(i) = Mpb/n; end
% p CPC
n = 2;
for i=1:n; K cpc(i) = n*Kcpc; b cpc(i) = bcpc/n; M cpc(i) = Mcpc/n; end
% p_PC
n = 2;
for i=1:n; K_pc(i) = n*Kpc(1); b_pc(i) = bpc(1)/n; M_pc(i) = Mpc(1)/n;
end
n2 = 2;
for i=n+1:n+n2; K_pc(i) = n2*Kpc(2); b_pc(i) = bpc(2)/n2; M_pc(i) =
Mpc(2)/n2; end
% p_CC
n = 2;
for i=1:n; K_cc(i) = n*Kcc(1); b_cc(i) = bcc(1)/n; M_cc(i) = Mcc(1)/n;
end
n2 = 2;
for i=n+1:n+n2; K_cc(i) = n2*Kcc(2); b_cc(i) = bcc(2)/n2; M_cc(i) =
Mcc(2)/n2; end
n3 = 2;
for i=n+n2+1:n+n2+n3; K_cc(i) = n3*Kcc(3); b_cc(i) = bcc(3)/n3; M_cc(i)
= Mcc(3)/n3; end
% p PCY
n = 2;
for i=1:n; K_pcy(i) = n*Kpcy(1); b_pcy(i) = bpcy(1)/n; M_pcy(i) =
Mpcy(1)/n; end
n2 = 2;
for i=n+1:n+n2; K_pcy(i) = n2*Kpcy(2); b_pcy(i) = bpcy(2)/n2; M_pcy(i)
= Mpcy(2)/n2; end
% p_SV
n = 2;
for i=1:n; K_sv(i) = n*Ksv(1); b_sv(i) = bsv(1)/n; M_sv(i) = Msv(1)/n;
end
n2 = 2;
for i=n+1:n+n2; K sv(i) = n2*Ksv(2); b sv(i) = bsv(2)/n2; M sv(i) =
Msv(2)/n2; end
n3 = 2;
```

```
for i=n+n2+1:n+n2+n3; K_sv(i) = n3*Ksv(3); b_sv(i) = bsv(3)/n3; M_sv(i)
= Msv(3)/n3; end
% p_SVc
n = 2;
for i=1:n; K_svc(i) = n*Ksvc; b_svc(i) = bsvc/n; M_svc(i) = Msvc/n; end
% p_SVc1
n = 2;
for i=1:n; K_svcl(i) = n*Ksvcl(1); b_svcl(i) = bsvcl(1)/n; M_svcl(i) =
Msvcl(1)/n; end
% p AAC
n = 2;
for i=1:n; K_aac(i) = n*Kaac(1); b_aac(i) = baac(1)/n; M_aac(i) =
Maac(1)/n; end
n2 = 2i
for i=n+1:n+n2; K_aac(i) = n2*Kaac(2); b_aac(i) = baac(2)/n2; M_aac(i)
= Maac(2)/n2; end
% p_ACA
n = 4;
for i=1:n; K_aca(i) = n*Kaca; b_aca(i) = baca/n; M_aca(i) = Maca/n; end
% p_ACB
n = 2;
for i=1:n; K_acb(i) = n*Kacb(1); b_acb(i) = bacb(1)/n; M_acb(i) =
Macb(1)/n; end
n^2 = 2;
for i=n+1:n+n2; K_acb(i) = n2*Kacb(2); b_acb(i) = bacb(2)/n2; M_acb(i)
= Macb(2)/n2; end
% p_ACBc
n = 2;
for i=1:n; K_acbc(i) = n*Kacbc(1); b_acbc(i) = bacbc(1)/n; M_acbc(i) =
Macbc(1)/n; end
n2 = 2i
for i=n+1:n+n2; K_acbc(i) = n2*Kacbc(2); b_acbc(i) = bacbc(2)/n2;
M_acbc(i) = Macbc(2)/n2; end
                                   _____8
§_____
% For the symmetric system:
% p_SPA
n = 4;
for i=1:n; K spa(i) = n*Kspa; b spa(i) = bspa/n; M spa(i) = Mspa/n; end
% p SPB
n = 4;
for i=1:n; K_spb(i) = n*Kspb; b_spb(i) = bspb/n; M_spb(i) = Mspb/n; end
% p_SCA
n = 2;
for i=1:n; K_sca(i) = n*Ksca(1); b_sca(i) = bsca(1)/n; M_sca(i) =
Msca(1)/n; end
n2 = 2;
for i=n+1:n+n2; K_sca(i) = n2*Ksca(2); b_sca(i) = bsca(2)/n2; M_sca(i)
= Msca(2)/n2; end
```

```
% p_SCB
```

```
n = 2;
for i=1:n; K_scb(i) = n*Kscb(1); b_scb(i) = bscb(1)/n; M_scb(i) = Mscb(1)/n;
end
n2 = 2;
for i=n+1:n+n2; K_scb(i) = n2*Kscb(2); b_scb(i) = bscb(2)/n2; M_scb(i) =
Mscb(2)/n2; end
% p_ELB
n = 2;
for i=1:n; K_elb(i) = n*Kelb(1); b_elb(i) = belb(1)/n; M_elb(i) = Melb(1)/n;
end
n2 = 2;
for i=n+1:n+n2; K_elb(i) = n2*Kelb(2); b_elb(i) = belb(2)/n2; M_elb(i) =
Melb(2)/n2; end
% p_TEE
n = 2;
for i=1:n; K_tee(i) = n*Ktee(1); b_tee(i) = btee(1)/n; M_tee(i) = Mtee(1)/n;
end
n2 = 2;
for i=n+1:n+n2; K tee(i) = n2*Ktee(2); b tee(i) = btee(2)/n2; M tee(i) =
Mtee(2)/n2; end
%_____%
% GROUPING DATA FOR FURTHER USE IN thesis coeff
p_PA = [ K_pa ; b_pa ; M_pa ];
p_PB = [ K_pb ; b_pb ; M_pb ];
p_CPC = [ K_cpc ; b_cpc ; M_cpc ];
p_PC = [ K_pc ; b_pc ; M_pc ];
p_CC = [ K_cc ; b_cc ; M_cc ];
p_PCY = [ K_pcy ; b_pcy ; M_pcy ];
p_SV = [ K_sv ; b_sv ; M_sv ];
p_SVc = [ K_svc ; b_svc ; M_svc ];
p_SVc1 = [ K_svc1 ; b_svc1 ; M_svc1 ];
p_SVW = [ Ksvw ; bsvw ; Msvw ];
p_AAC = [ K_aac ; b_aac ; M_aac ];
p_ACA = [ K_aca ; b_aca ; M_aca ];
p_ACB = [ K_acb ; b_acb ; M_acb ];
p_ACBc = [K_acbc; b_acbc; M_acbc];
p_ACBW = [Kacbw; bacbw; Macbw];
p SPA = [K spa; b spa; M spa];
p_SPB = [ K_spb ; b_spb ; M_spb ];
p SCA = [K sca; b sca; M sca];
p SCB = [K scb; b scb; M scb];
p ELB = [ K elb ; b elb ; M elb ];
p_TEE = [ K_tee ; b_tee ; M_tee ];
```

Vita

Khalil Maurice Nasser, son of Maurice and Lucy Nasser, was born in November 24, 1976, in Caracas, Venezuela. He attended a bilingual school and graduated from Madison High School in July 1994. He enrolled in Virginia Tech later that August, and graduated in May 1999 with a B.S. in Mechanical Engineering, *Summa Cum Laude*. Staying at Virginia Tech another year and a half, he pursued a Master of Science in Mechanical Engineering with the Center for Intelligent Material Systems and Structures (CIMSS), concentrating on piezoelectrics, hydraulics and controls.

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This thesis was typeset with $\operatorname{LATE} X 2_{\varepsilon}^{1}$ by the author.

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