

# **A Comprehensive Dynamic Model of the Column Flotation Unit Operation**

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## **(ABSTRACT)**

The core of this project was the development of a column flotation dynamic model that can reasonably predict the changes in the concentrations of all solid and bubble species, along the full column height. A dynamic model of a process is normally composed of a set of partial or ordinary differential equations that describe the state of the process at any given time or position inside the system volume. Such equations can be obtained from fundamental material and/or energy balances, or from phenomenological derivations based on knowledge about the behavior of the system. A phenomenological approach referred to as population balance modeling was employed here.

Initially, a two-phase model was formulated, which represents the behavior of the gas phase in a frother solution. The column was viewed as consisting of three main regions: a collection region, a stabilized froth and a draining froth. Experiments were carried out, based on conductivity techniques, for obtaining empirical data for model validation and parameter estimation. After testing the two-phase model, the equations for the solid species were derived. Consideration of the effects of bubble loading, slurry density and slurry viscosity on bubble rise velocity and, therefore, on air fraction is included in the model. Bubble coalescence in the froth is represented as a rate phenomenon characterized by a series of coalescence efficiency rate parameters. Auxiliary equations that help describe the settling of free particles, the buoyancy of air bubbles, and the processes of attachment and detachment, were also developed and incorporated into the model. The detachment of solids from the bubbles in the froth zones was attributed to coalescence, and it was assumed to be proportional to the net loss of bubble surface area.

Almost all parameters needed to solve the model equations are readily available. The set of differential equations that comprise the model can be solved numerically by applying finite difference approximation techniques. An iteration has to be performed, which involves calculating the product flowrate at steady state, modifying the tailings rate and solving the model again until a mass balance is satisfied.

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# Table of Contents

	<b>Page</b>
<b>Abstract</b> .....	ii
<b>Acknowledgements</b> .....	iii
<b>Table of Contents</b> .....	iv
<b>List of Tables</b> .....	viii
<b>List of Figures</b> .....	x
<b>Chapter 1: Introduction</b> .....	1
1.1. General Information on Column Flotation .....	2
1.2. Objectives .....	5
1.3. Organization .....	5
<b>Chapter 2: A Review of the Literature on Column Flotation Modeling</b> ..	7
2.1 Introduction .....	7
2.2 Background .....	8
2.3 Two-Phase Models .....	9
2.3.1 Column Hydrodynamics .....	9
2.3.2 Froth Modeling .....	13
- Cellular Foams .....	15
- Expanded Beds .....	18
2.3.3 Bubble Coalescence .....	24
2.4 Three-Phase Models .....	26
2.4.1 Bubble-Particle Interactions in the Collection Zone ..	26
- Bubble-Particle Collisions .....	28
- Bubble-Particle Attachment .....	29
- Detachment .....	31
- Fundamental Particle Collection Models .....	31
2.4.2 Mineralized Froth .....	32
- Froth Stability .....	36
2.4.3 Mixing .....	37
2.4.4 Wash Water Effects .....	43
2.4.5 Column Flotation Kinetics .....	44
2.4.6 Steady-State Column Models .....	47
2.4.7 Dynamic Models .....	49
- Phenomenological Models .....	50
- Empirical Models .....	53

2.5	Summary .....	53
2.6	Nomenclature .....	55
<b>Chapter 3: Estimation of Air Fraction Profiles in Column Stabilized Froth and Froth Regions Using Conductivity Probes .....</b>		<b>57</b>
3.1	Introduction .....	57
3.2	Experimental Setup .....	58
3.3	Two-Phase System .....	59
	- Interface and Stabilized Froth .....	62
	- Draining Froth .....	74
3.4	Three-Phase System .....	83
	- Interface and Stabilized Froth .....	83
	- Draining Froth .....	94
3.5	Conclusions .....	105
<b>Chapter 4: Column Flotation Two-Phase Dynamic Model .....</b>		<b>106</b>
4.1	Introduction .....	106
4.2	Background .....	106
4.3	Model Development .....	114
4.3.1	Pulp Region .....	116
	-Model Assumptions .....	116
	-Air-Phase Dynamic Equations .....	116
4.3.2	Froth Regions .....	117
	-Model Assumptions .....	117
	-Coalescence Representation .....	118
	-Quasi-Steady-State Approach for Representing Process Dynamics .....	130
	-Truly Dynamic Solution to Froth Equations .....	139
	-Air-Phase Dynamic Equations .....	140
	-Interface .....	140
	-Stabilized Froth .....	141
	-Wash Water Addition Zone .....	141
	-Draining Froth .....	141
	- Estimation of Coalescence Efficiency Rate Parameters .....	142
	-Two-Phase .....	143
	-Stabilized Froth .....	143
	-Draining Froth .....	146
	-Three-Phase .....	148
	-Stabilized Froth .....	148
	-Draining Froth .....	152
4.4	Model Solving .....	153
4.5	Simulations .....	154

	-Simulation No. 1 .....	154
	-Simulation No. 2 .....	158
	-Simulation No. 3 .....	161
	-Simulation No. 4 .....	162
4.6	Model Validation .....	165
4.7	Solution using Quasi-Steady-State Technique for the Draining Froth .....	175
4.8	Summary .....	177
<b>Chapter 5: Three-Phase Model of the Column Flotation Process .....</b>		<b>178</b>
5.1	Introduction .....	178
5.2	Background .....	179
5.3	Model Development .....	179
5.3.1	Model Assumptions .....	182
	- Collection Region .....	182
	- Stabilized Froth .....	183
	- Draining Froth .....	184
5.3.2	Collection Zone Modeling .....	185
	- Air Phase Equations .....	185
	- Free Solids Equations .....	188
	- Attached Solids Equations .....	192
5.3.3	Stabilized Froth Modeling .....	195
	- Model Equations .....	199
	- Interface .....	199
	-Air Phase .....	199
	-Attached Solids .....	200
	-Free Solids .....	200
	- Stabilized Froth .....	200
	-Air Phase .....	200
	-Attached Solids .....	200
	-Free Solids .....	200
5.3.4	Wash Water Zone and Draining Froth Equations .....	201
	-Air Phase .....	203
	-Attached Solids .....	203
	-Free Solids .....	203
5.3.5	Recovery Calculations .....	203
5.4	Simulations .....	206
5.4.1	Simulation No. 1 .....	206
5.4.2	Simulation No. 2 .....	231
5.4.3	Simulation No. 3 .....	246
5.5	Model Validation .....	250
5.6	Summary .....	256

<b>Chapter 6: Conclusions</b> .....	258
<b>Chapter 7: Recommendations for Future Work</b> .....	261
<b>References</b> .....	262
<b>Appendix A: MATLAB Program Listings</b> .....	277
<b>Vita</b> .....	309

## List of Tables

		<b>Page</b>
Table 3.1	Experimental Conditions During Air-Holdup Profile Measurements in Stabilized Froth .....	62
Table 3.2	Comparison between Experimental and Calculated Pulp Air Fractions .....	68
Table 3.3	Experimental Conditions During Air-Holdup Profile Measurements in Draining Froth .....	74
Table 3.4	Experimental Conditions During Air-Holdup Profile Measurements in Three-Phase Stabilized Froth .....	85
Table 3.5	Measurements Obtained in the Stabilized Froth during Test No.1 for Each of the Eleven Electrodes (with Solids) .....	85
Table 3.6	Measurements Obtained in the Stabilized Froth during Test No.2 for Each of the Eleven Electrodes (with Solids) .....	88
Table 3.7	Measurements Obtained in the Stabilized Froth during Test No.3 for Each of the Eleven Electrodes (with Solids) .....	91
Table 3.8	Experimental Conditions During Air-Holdup Profile Measurements in Three-Phase Draining Froth .....	95
Table 4.1	Estimated Coalescence Rate Parameters in a Two-Phase Stabilized Froth Using Several Functions Relating the Rate Parameter to Average Bubble Diameter .....	145
Table 4.2	Estimated Coalescence Rate Parameters in a Two-Phase Draining Froth Using Several Functions Relating the Rate Parameter to Average Bubble Diameter .....	147
Table 4.3	Estimated Coalescence Rate Efficiency Parameters for Each of the Measured Air Fraction Profiles in the Three-Phase Stabilized Froth .....	151
Table 4.4	Estimated Coalescence Rate Efficiency Parameters Corresponding to Draining Froth Profiles Obtained with Three Different Feed Rates .....	152

Table 4.5	Operating Conditions Set During the Measurement of the Conductivity Profiles Employed for Model Verification .....	165
Table 5.1	Experimental Conditions During Study of Column Dynamic Responses .....	251

## List of Figures

		<b>Page</b>
Figure 1.1	Flotation Column Layout .....	4
Figure 2.1	Column Hydrodynamic Zones .....	14
Figure 2.2	Cellular Foam Structure .....	15
Figure 2.3	Plateau Border .....	16
Figure 2.4	Mineralized Bubble With Slurry Layer, as Assumed by Moys (1978) .....	33
Figure 2.5	Film Rupture Caused By Particle (from Dippenaar, 1982a) ..	37
Figure 2.6	Plug-Flow-With-Recycle Model .....	40
Figure 2.7	Tanks-In-Series Model with Backflow .....	41
Figure 2.8	Unidirectional Tanks-In-Series Model .....	41
Figure 3.1	Ring Conductivity Probe Used to Study the Stabilized Froth .....	60
Figure 3.2	Conductivity Cell Used in the Study of the Draining Froth ..	61
Figure 3.3	Conductance Ratio Profile obtained from Test No. 1 (Two-Phase System) .....	63
Figure 3.4	Conductance Ratio Profile obtained from Test No. 2 (Two-Phase System) .....	64
Figure 3.5	Conductance Ratio Profile obtained from Test No. 3 (Two-Phase System) .....	65
Figure 3.6	Conductance Ratio Profile obtained from Test No. 4 (Two-Phase System) .....	66
Figure 3.7	Conductance Ratio Profile obtained from Test No. 5 (Two-Phase System) .....	67

Figure 3.8	Calculated Air Fraction Profile Obtained From Conductivity Test No.1 (Two-Phase System) .....	69
Figure 3.9	Calculated Air Fraction Profile Obtained From Conductivity Test No.2 (Two-Phase System) .....	70
Figure 3.10	Calculated Air Fraction Profile Obtained From Conductivity Test No.3 (Two-Phase System) .....	71
Figure 3.11	Calculated Air Fraction Profile Obtained From Conductivity Test No.2 (Two-Phase System) .....	72
Figure 3.12	Calculated Air Fraction Profile Obtained From Conductivity Test No.2 (Two-Phase System) .....	73
Figure 3.13a	Conductance Ratio Profiles in Two-Phase Draining Froth (Depth = 4 cm) .....	75
Figure 3.13b	Air Fraction Profiles in Two-Phase Draining Froth (Depth = 4 cm) .....	76
Figure 3.14a	Conductance Ratio Profiles in Two-Phase Draining Froth (Depth = 6 cm) .....	77
Figure 3.14b	Air Fraction Profiles in Two-Phase Draining Froth (Depth = 6 cm) .....	78
Figure 3.15a	Conductance Ratio Profiles in Two-Phase Draining Froth (Depth = 8 cm) .....	79
Figure 3.15b	Air Fraction Profiles in Two-Phase Draining Froth (Depth = 8 cm) .....	80
Figure 3.16a	Conductance Ratio Profiles in Two-Phase Draining Froth (Depth = 8 cm, Higher Frother Rate) .....	81
Figure 3.16b	Air Fraction Profiles in Two-Phase Draining Froth (Depth = 8 cm , Higher Frother Rate) .....	82
Figure 3.17	Experimental Setup for Determining Air Fraction Profile in Column Stabilized Froth Using a Conductivity Probe .....	84
Figure 3.18	Conductance Ratios along the Stabilized Froth Corresponding to Test No.1 (with Solids) .....	86

Figure 3.19	Estimated Air Fraction Profile in the Stabilized Froth for Test No.1 (with Solids) .....	87
Figure 3.20	Conductance Ratios along the Stabilized Froth Corresponding to Test No.2 (with Solids) .....	89
Figure 3.21	Estimated Air Fraction Profile in the Stabilized Froth for Test No.2 (with Solids) .....	90
Figure 3.22	Conductance Ratios along the Stabilized Froth Corresponding to Test No.3 (with Solids) .....	92
Figure 3.23	Estimated Air Fraction Profile in the Stabilized Froth for Test No.3 (with Solids) .....	93
Figure 3.24	Experimental Setup for Determining Air Fraction Profile in Column Draining Froth Using a Conductivity Probe .....	94
Figure 3.25	Air Fraction Profiles in the Three-Phase Draining Froth Determined with a Conductivity Technique for Various Frother Addition Rates .....	96
Figure 3.26	Air Fraction Profiles in a Three-Phase Draining Froth for Two Distinct Frother Addition Rates .....	97
Figure 3.27	Air Fraction Profiles in a Two-Phase Draining Froth for Two Distinct Frother Addition Rates .....	98
Figure 3.28	Air Fraction Profiles in the Three-Phase Draining Froth Determined with a Conductivity Technique for Various Feed Rates .....	99
Figure 3.29	Air Fraction Profiles for a Two-Phase Draining Froth, and for a Three-Phase Froth with a Column Superficial Feed Velocity = 0.16 cm/sec .....	100
Figure 3.30	Air Fraction Profiles for a Two-Phase Draining Froth and for a Three-Phase Froth with a Column Superficial Feed Velocity = 0.08 cm/sec .....	101
Figure 3.31	Air Fraction Profiles for a Two-Phase Draining Froth and for Two Other Three-Phase Froths with Different Column Feed Rates .....	102

Figure 3.32	Air Fraction Profiles in a Three-Phase Draining Froth for Two Distinct Air Rates .....	103
Figure 3.33	Air Fraction Profiles in a Two-Phase Draining Froth for Two Distinct Air Rates .....	104
Figure 4.1	Comparison of the Bubble Slip Velocity Predicted by Several Expressions (Eq.1: Richardson-Zaki equation; Eq.2: Marrucci's; Eq.3: Pal and Masliyah's; Eq. 4: Lockett and Kirkpatrick's; Eq. 5: iterative solution with Richardson-Zaki relationship) .....	109
Figure 4.2	Solutions of the Drift-Flux Model Corresponding to the Pulp and Froth Phases (Interception Points) .....	110
Figure 4.3	Solutions of the Drift Flux Model for Different Gas Superficial Velocities Expressed in cm/sec .....	111
Figure 4.4	Bubble Slip Velocity-Air Fraction Relationships for Expanded Bubble Beds .....	113
Figure 4.5	Schematic Representation of the Column Regions During Two-Phase (Air-Water) Operation .....	115
Figure 4.6	Air Fraction Profile in the Stabilized Froth Calculated with the Set of Simultaneous Equations [40]-[43] ( $\lambda_{11}=0.05/\text{sec}$ , $\lambda_{12}=0.10/\text{sec}$ , $\lambda_{13}=0.20/\text{sec}$ ) .....	123
Figure 4.7	Air Fraction Profile in the Stabilized Froth Calculated with the Simultaneous Equations [40]-[43] ( $\lambda_{11}=0.70/\text{sec}$ , $\lambda_{12}=0.50/\text{sec}$ , $\lambda_{13}=0.15/\text{sec}$ ) .....	124
Figure 4.8	Air Fraction Profile in the Stabilized Froth Calculated with the Taylor Series Approximation (Equation [48]) ( $\lambda_{11}=0.75/\text{sec}$ , $\lambda_{12}=0.075/\text{sec}$ ) .....	125
Figure 4.9	Comparison of the Air Fraction Profiles in the Stabilized Froth Calculated with the Taylor Series Approximation for Two Different Gas Velocities ( $V_g$ in cm/sec) ( $\lambda_{11}=0.70/\text{sec}$ , $\lambda_{12}=0.075/\text{sec}$ ) .....	126
Figure 4.10	Comparison of Air Fraction Profiles in Stabilized Froth Calculated with a Taylor Series Approximation for Two Different Sets of Coalescence Parameters (From $\lambda_{11}=0.7/\text{sec}$ and $\lambda_{12}=0.075/\text{sec}$ to $\lambda_{11}=0.95/\text{sec}$ and $\lambda_{12}=0.095/\text{sec}$ ..	127

Figure 4.11	Comparison of the Air Fraction Profiles in the Stabilized Froth Calculated with the Taylor Series Approximation, with $\lambda_{11}=0.70/\text{sec}$ and $\lambda_{12}=0.075/\text{sec}$ , for Two Different Boundary Conditions: (Pulp Air Fraction Components) .....	128
Figure 4.12	Predicted Air Fraction Profiles in the Froth Corresponding to Each Bubble Size Class, Calculated with Taylor-Series Approximation and Iterating on the Froth Height ( $\lambda_{11}=0.025/\text{sec}$ , $\lambda_{12}=0.075/\text{sec}$ ) .....	129
Figure 4.13	Dynamic Changes in the Pulp Air Fraction Predicted by the Dynamic Pulp Equation (Equation [51] ) with $\lambda_{11}=0.025/\text{sec}$ , $\lambda_{12}=0.075/\text{sec}$ .....	131
Figure 4.14	Predicted Change in the Overall Air Fraction Profile in the Froth Corresponding to the Dynamic Change in Pulp Air Fraction Depicted in Figure 4.13 (Quasi-Steady-State Technique) ..	132
Figure 4.15	Predicted Change in the Overall Air Fraction Profile in the Froth After an Increase in the Liquid Velocity $V_l$ from 0.1 cm/sec at $t=0$ to 0.2 cm/sec at $t=140$ sec (Quasi-Steady-State Technique) .....	133
Figure 4.16	Time Variation in Froth Height for an Increase in Liquid Velocity from $V_l=0.1$ to $V_l=0.2$ cm/sec (Quasi-Steady-State Technique) .....	134
Figure 4.17	Predicted Change in the Overall Air Fraction Profile in the Froth After an Increase in the Superficial Gas Velocity $V_g$ (Quasi-Steady-State Technique) .....	135
Figure 4.18	Predicted Variation in Froth Height for an Increase in Gas Superficial Velocity from $V_g=1.0$ cm/sec to $V_g=1.5$ cm/sec (Quasi-Steady-State Technique) .....	136
Figure 4.19	Predicted Change in the Overall Air Fraction Profile in the Froth After a Decrease in the Coalescence Rate Parameters $\lambda$ from $\lambda_{11}=0.025/\text{sec}$ , $\lambda_{12}=0.075/\text{sec}$ to $\lambda_{11}=0.015/\text{sec}$ , $\lambda_{12}=0.030/\text{sec}$ at Constant Liquid and Gas Velocities (Quasi-Steady-State Technique) .....	137
Figure 4.20	Change in the Froth Height Required to Maintain Same Overflow Rate after a Decrease in the Values of the Coalescence Rate Parameters in Equation [48] (Quasi-Steady-State Technique) .....	138

Figure 4.21	Functions Used for Fitting the Coalescence Parameter to Two-Phase Experimental Profiles in the Stabilized Froth .....	144
Figure 4.22	Comparison of the Backcalculated and Empirical Air Fraction Profiles in a Two-Phase Stabilized Froth .....	146
Figure 4.23	Backcalculated and Empirical Air Fraction Profile of a Two-Phase Draining Froth .....	148
Figure 4.24	Backcalculated and Experimental Air Fraction Profile in Stabilized Froth with Solids (Test No.1) .....	149
Figure 4.25	Backcalculated and Experimental Air Fraction Profile in Stabilized Froth with Solids (Test No.2) .....	150
Figure 4.26	Backcalculated and Experimental Air Fraction Profile in Stabilized Froth with Solids (Test No.3) .....	151
Figure 4.27	Backcalculated and Experimental Air Fraction Profiles in Draining Froth for Three Distinct Feed Rates .....	153
Figure 4.28	Dynamic Solution to Two-Phase Model for Each of the Column Zones ( $V_g=1.0$ cm/sec; $V_b=0.281$ cm/sec) .....	155
Figure 4.29	Predicted Air Fraction Profile Along the Full Column Length for Each Iteration ( $V_g=1.0$ cm/sec; $V_b=0.281$ cm/sec) ..	156
Figure 4.30	Close View of the Predicted Air Fraction Profile Along the Froth for Each Iteration ( $V_g=1.0$ cm/sec; $V_b=0.281$ cm/sec) .....	157
Figure 4.31	Predicted Air Fraction Profile Along the Whole Column Length ( $V_g=1.2$ cm/sec; $V_b=0.225$ cm/sec) .....	158
Figure 4.32	Close View of the Predicted Air Fraction Profile Along the Froth ( $V_g=1.2$ cm/sec; $V_b=0.225$ cm/sec) .....	159
Figure 4.33	Dynamic Solution to Two-Phase Model for Each of the Column Zones ( $V_g=1.2$ cm/sec; $V_b=0.225$ cm/sec) .....	160
Figure 4.34	Predicted Air Fraction Profile Along the Column Length ( <i>Smaller <math>D_b</math> ave</i> ; $V_g=1.0$ cm/sec; $V_b=0.275$ cm/sec) .....	161
Figure 4.35	Close View of the Predicted Air Fraction Profile Along the Froth ( <i>Smaller <math>D_b</math> ave</i> ; $V_g=1.0$ cm/sec; $V_b=0.275$ cm/sec) ..	162

Figure 4.36	Predicted Air Fraction Profile Along the Column Length ( <i>Froth Depth = 30 cm; Vg=1.0 cm/sec; Vb=0.260 cm/sec</i> ) ..	163
Figure 4.37	Close View of the Predicted Air Fraction Profile Along the Froth ( <i>Froth Depth=30 cm; Vg=1.0 cm/sec; Vb=0.260 cm/sec</i> ) ..	164
Figure 4.38	Experimental Air Fraction Profiles Obtained Through Conductivity Measurements at Several Gas Velocities and Used for Model Verification .....	166
Figure 4.39	Backcalculated and Experimental Profile in the Stabilized Froth Corresponding to <i>Vg=1.35 cm/sec</i> .....	167
Figure 4.40	Predicted Profile (solid line) Versus Experimental Air Fractions for <i>Vg=1.5 cm/sec</i> .....	168
Figure 4.41	Backcalculated and Experimental Profile in the Stabilized Froth Corresponding to <i>Vg=1.5 cm/sec</i> .....	169
Figure 4.42	Predicted Profile (solid line) Versus Experimental Air Fractions for <i>Vg=1.65 cm/sec</i> .....	170
Figure 4.43	Measured and Predicted Steady-State Profiles for an Increase in Gas Rate from <i>Vg=1.35 cm/sec</i> to <i>Vg=1.5 cm/sec</i> .....	171
Figure 4.44	Measured and Predicted Steady-State Profiles for an Increase in Gas Rate from <i>Vg=1.5 cm/sec</i> to <i>Vg=1.65 cm/sec</i> .....	172
Figure 4.45	Comparison of Measured and Simulated Time Responses for the Pulp Air Fraction .....	173
Figure 4.46	Comparison of Measured and Simulated Time Responses for Air Fraction in the Stabilized Froth .....	174
Figure 4.47	Comparison of the Solutions Provided by the Quasi-Steady- State Approximation and the Dynamic Equations at the Top Column Zone .....	176
Figure 5.1	Flotation Column Regions .....	181
Figure 5.2	Flows of Free Particles Around a Perfectly Mixed Zone in the Collection Region .....	191
Figure 5.3	Flows of Attached Particles Around a Perfectly Mixed Zone in the Collection Region .....	193

Figure 5.4	Flowchart of the Procedure Followed for Solving the Model Equations .....	202
Figure 5.5	Column Zone Recoveries .....	205
Figure 5.6	Dynamic Changes in Air Fraction in Several Zones Along the Column (Simulation No. 1) .....	209
Figure 5.7	Final Air Fraction Profile Along Column Height (Simulation No. 1) .....	210
Figure 5.8	Dynamic Changes in the Concentration of Attached Solids in Several Zones Along the Column (Simulation No. 1) .....	211
Figure 5.9	Final Mass Concentration of Attached Solids Along Column Height (Simulation No. 1) .....	212
Figure 5.10	Dynamic Changes in the Concentration of Free Solids in Several Zones Along the Column (Simulation No. 1) .....	213
Figure 5.11	Final Mass Concentration of Free Solids Along Column Height (Simulation No. 1) .....	214
Figure 5.12	Total Concentration of Solids Along Column Height at Steady-State (Simulation No. 1) .....	215
Figure 5.13	Total Solid Concentration Profiles Along the Froth of a Flotation Column - Data After Ross and vanDeventer (1988), and Falutsu and Dobby (1992) .....	216
Figure 5.14	Total Solid Concentration Profiles Along the Collection Region of a Flotation Column, after Dobby and Finch (1986) .....	217
Figure 5.15	Rates of Each Composition Class in the Feed Stream at Each Time Step (Simulation No. 1) .....	219
Figure 5.16	Rates of Each Composition Class Entering the Froth by Flotation at Each Time Step (Simulation No. 1) .....	220
Figure 5.17	Rates of Each Composition Class Entering the Froth by Entrainment at Each Time Step (Simulation No. 1) .....	221

Figure 5.18	Rates of Each Composition Class Entering the Froth by Entrainment at Each Time Step - Bigger Scale (Simulation No. 1) ..	222
Figure 5.19	Rates of Each Composition Class Leaving with the Tailings Stream at Each Time Step (Simulation No. 1) .....	223
Figure 5.20	Fractional Recoveries in the Concentrate of Each Composition Class at Each Time Step (Simulation No. 1) .....	224
Figure 5.21	Fractional Recoveries in the Tailings of Each Composition Class at Each Time Step (Simulation No. 1) .....	225
Figure 5.22	Fractional Content of Each Composition Class in the Concentrate at Each Time Step (Simulation No. 1) .....	226
Figure 5.23	Predicted and Measured Dynamic Responses in Concentrate for a Feed Velocity of <i>0.4 cm/sec</i> .....	227
Figure 5.24	Predicted and Measured Dynamic Responses in Tailings for a Feed Velocity of <i>0.4 cm/sec</i> .....	228
Figure 5.25	Predicted and Measured Dynamic Responses in Concentrate for a Feed Velocity of <i>1.0 cm/sec</i> .....	229
Figure 5.26	Predicted and Measured Dynamic Responses in Tailings for a Feed Velocity of <i>1.0 cm/sec</i> .....	230
Figure 5.27	Steady-State Air Fraction Profile at <i>t=14 mins</i> (Simulation No.2) .....	232
Figure 5.28	Steady-State Profiles of All Air Fraction Component at <i>t=14 mins</i> ( <i>e1</i> :Smallest Size Class, <i>e4</i> : Largest Size Class) (Simulation No.2) .....	233
Figure 5.29	Dynamic Responses of the Total Attached Solid Concentration in Various Column Zones (Simulation No.2) .....	234
Figure 5.30	Steady-State Profile of the Concentration of Attached Solids at <i>t=14 mins</i> (Simulation No.2) .....	235
Figure 5.31	Dynamic Responses of the Total Free Solid Concentration in Various Column Zones (Simulation No.2) .....	236
Figure 5.32	Steady-State Profile of the Concentration of Free Solids at <i>t=14 mins</i> (Simulation No.2) .....	237

Figure 5.33	Steady-State Profile of the Total Solid Concentration at $t=14$ mins (Simulation No.2) .....	238
Figure 5.34	Dynamic Changes in the Rate of Solids Carried by the Bubbles from the Pulp to the Froth for Each Solid Species (Simulation No.2) .....	240
Figure 5.35	Dynamic Changes in the Net Rate of Solids Transported by the Slurry from the Pulp to the Froth for Each Solid Species (Simulation No.2) .....	241
Figure 5.36	Dynamic Changes in Tailings Solid Rates for Each Solid Species (Simulation No.2) .....	242
Figure 5.37	Dynamic Changes in the Fractional Recovery in the Concentrate for Each Solid Species (Simulation No.2) .....	243
Figure 5.38	Dynamic Changes in the Fractional Recovery in the Tailings Stream for Each Solid Species (Simulation No.2) .....	244
Figure 5.39	Fractional Content of Each Composition Class in the Concentrate at Each Time Step (Simulation No. 2) .....	245
Figure 5.40	Steady-State Profile of the Concentration of Attached Solids at $t=15$ mins (Simulation No.3) .....	248
Figure 5.41	Steady-State Profile of the Concentration of Free Solids at $t=15$ mins (Simulation No.3) .....	249
Figure 5.42	Steady-State Profile of the Total Solid Concentration at $t=15$ mins (Simulation No.3) .....	250
Figure 5.43	Predicted and Experimental Concentrate Ash Values After an Increase in Aeration Rate .....	252
Figure 5.44	Predicted and Experimental Concentrate Ash Recoveries After an Increase in Aeration Rate .....	253
Figure 5.45	Predicted and Experimental Concentrate Ash Values After an Increase in Frother Rate (Smaller Average Bubble Size) .....	254
Figure 5.46	Predicted and Experimental Concentrate Ash Content After an Increase in Feed Rate .....	255