THE EFFECT OF LENGTH ON TENSILE STRENGTH PARALLEL-TO-GRAIN IN STRUCTURAL LUMBER

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(ABSTRACT)

Two sizes (2x4 and 2x10) and two grades (2250f-1.9E and No. 2 KD15) of Southern Pine lumber having three different test span lengths of 30, 90 and 120 inches were tested in tension parallel-to-grain. Results obtained from the tests indicated that the tensile strengths of the 30-inch test specimens were significantly higher than the tensile strengths of the 90- and 120-inch test specimens.

A tensile strength-length effect model was developed for generating tensile strength values of lumber taking the length effect into consideration. The model generates tensile strength values for lumber longer than 30 inches in multiples of 30 inches, ie. 60-, 90- 120-inch lengths. The two sizes and two grades of Southern Pine lumber formed the data base for developing the model.

The tensile strength-length effect model utilized an MOE variability model which generated serially correlated MOE's along 30-inch segments for a piece of lumber using a second-order Markov model. The segment MOE values were then used

in a first-order Markov model to generate serially correlated tensile strength residuals for each 30-inch segment. The segment MOE values and the segment tensile strength residuals were then inputted into a weighted least squares regression to obtain the tensile strength parallel-to-grain for each 30-inch segment. The tensile strength of the generated piece of lumber was then determined using the weakest-link concept; the minimum segment tensile strength value was selected as the tensile strength of the generated piece of lumber.

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CHAPTER I. INTRODUCTION

By current design practice (National Forest Products Association, 1982), trusses of all lengths have the same implied safety. Failure experience, however, indicates that long-span trusses have a higher incidence of failure than short-span trusses. While there are many factors involved, it is believed that a decreasing tensile strength with an increase in the lower chord length may be one factor. Because of the poor performance, many manufacturers will not design or produce trusses longer than 60 to 80 feet. An empirical evaluation of the effect of length on tensile strength will provide the basis for a model that can be used for safety adjustments in long-span truss design.

A tensile length model will be useful for several additional applications:

1. During a recent cooperative research project between Virginia Tech and the U. S. Forest Products Laboratory (FPL) (Bender, 1980), a computer-based model was developed for the reliability of glued-laminated (glulam) beams at room temperature and when exposed to fire. In the process of calibrating the room temperature portion of the developed model with some existing glulam beam data, it was discovered that adding a length adjustment significantly improved the strength comparisons between the model and available data.

The length adjustment was included to account for the difference in the length of the tensile specimens used as data input for the model, and the length of the various test beams subject to maximum moment.

Unpublished test results from Trus Joist Corp. indicate a signicant reduction in tensile strength with increasing lengths of MICRO=LAM (Trus Joist Corp., 1979). It was hypothesized that lumber, with larger defects, should be more sensitive to length. Since no tensile strength length adjustment was known for structural lumber, it was assumed that lumber behaved as a weakest link type structure when stressed in tension parallel to grain (Bender, 1980).

Bohannan (1966 a,b) studied the effect of volume on the bending strength of laminated timber beams. The volume effect is based on the "weakest-link theory." In applying weakest link theory to wood beams the volume is replaced by the beam aspect area (Liu, 1981). To obtain bending strength results for beams of different lengths and depths, the stress function is integrated over the area of the beam. An alternate approach to integrating a stress function over an area is to evaluate the bending strength of the laminated beam by Monte Carlo simulation (Bender, 1980). The stress in each piece of lumber between end joints is calculated and compared to a strength value. Hence tensile strength values for pieces of lumber of various lengths are needed.

2. In the Canadian glulam computer simulation model (Foschi, 1980), 6-inch-long finite elements are used: hence tension information on 6-inch lengths of lumber is needed for that model. Six-inch lengths of full-size lumber are difficult, if not impossible to test in tension parallel-to-grain. As a compromise the length model can be used to predict the 30-inch tensile strength.

3. The In-Grade testing program provides data on limited lengths. If a length tensile model is verified, it can be used to make adjustments to In-Grade tensile data. Tensile strength data from various studies have been collected on specimens of different lengths. Data to adjust for different gage lengths would be useful in comparing tensile strength results.

The objectives of this study were to:

1. Determine if there is a length effect on tensile strength parallel-to-grain in two sizes (2x4 and 2x10) and two grades (2250f-1.9E and No. 2 KD15) of Southern Pine lumber.
2. Develop a length effect model for tensile strength parallel-to-grain for the two sizes and two grades of Southern Pine lumber.
3. Demonstrate the application of the tensile strength-length effect model.

CHAPTER II. REVIEW OF LITERATURE

In current design practice, the stiffness of a piece of lumber indicates some average stiffness value for the whole piece and the strength indicates the stress capacity of the whole piece of lumber. Because a piece of lumber usually contains defect areas such as knots and grain deviations along the piece, presumably a more accurate representation of lumber strength or stiffness would include variability in strength or stiffness with size. Several researchers have documented a variability of strength of a wood member with size.

Buchanan (1983) reports that the strength of wood flexural members decreases as the size of the test specimen increases. Bohannan (1966) also confirms that bending strength of wood beams decreases as the size of the beam increases. Bohannan's results for clear, straight-grained, Douglas-fir beams having sizes ranging from 1 inch deep by 14 inches long to 31-1/2 inches deep by 48 feet long shows a decrease in the average modulus of rupture with increasing length and depth.

Recently, Kunesh and Johnson (1974) carried out axial tension tests on clear Douglas-fir lumber and identified a pronounced size effect, with the average strength of 2x10 lumber being only 81% of the strength of 2x4 lumber. Since all specimens were 12 feet long, no conclusions regarding a length effect were possible. Buchanan (1983) also cites other investigations where a trend of increasing tensile strength with decreasing size was identified.

A size effect on shear strength of wood beams was noted by Liu (1980) during a study on glued-laminated Douglas-fir beams. Liu's test results showed that the mean failure shear stress decreased with increasing volume.

Barrett (1974) reports that a similar size effect exists when testing tensile strength perpendicular-to-grain of Douglas-fir. Barrett's report consists of tensile strength results for uniformly loaded glued-laminated Douglas-fir blocks of commercial material for clear Douglas-fir blocks loaded perpendicular-to-grain. In both cases, the average strength of the material decreases with increasing volume. More specifically, the results also show a decrease in the average tensile strength when only the length is increased.

The following three sections describe models which can be used to aid in the explanation of the size effect on strength of lumber specimens. The first section discusses Weibull's weakest link theory. Several applications of the weakest link theory are provided. The second section describes the use of stochastic models to model the variability of strength and stiffness in a lumber specimen. The final section discusses a regression approach used by (Woeste et al, 1979) to model and generate by the computer a compatible set of strength and stiffness values.

2.1 WEAKEST-LINK APPLICATIONS IN LUMBER

The relationship between the size and strength of a wood member has been the subject of research for many years. It is apparent that as size increases in a wood member the strength of the member decreases. Assuming that wood is a perfectly brittle material, that is, a material in which total failure occurs when fracture occurs at the weakest point, the size effect can be explained by the statistical theory of material strength, or, the "weakest-link theory" (Weibull, 1939).

According to Weibull (1939), the probability that a chain of n links will have strength greater than or equal to X is given by:

$$1 - F_n(x) = [1 - F(x)]^n = S_n \qquad (2.1)$$

where $F_n(x)$ is the probability distribution of strength for chains of n links, and S_n is the survival probability. Taking logarithms:

$$\ln[1 - F_n(x)] = -B = n*\ln[1 - F(x)] \qquad (2.2)$$

Generally, the stress distribution within a body varies with position. In this case, the value of B becomes

$$B = -\int_V \eta(x)dv \qquad (2.3)$$

where $\eta(x) = \ln[1-F(x)]$ from Equation 2.2. The form of the function, η, must fit the cumulative distribution of strength and describe the strength property being investigated. Bohannan (1966) used the material function

$$\eta(x) = kx^m \qquad (2.4)$$

in which k and m are material constants, in his study of the size effect on bending strength. Barrett (1974) described the material function to be

$$\eta(x) = [(x - x_1)/x_0]^m \qquad (2.5)$$

where x_1 is an arbitrarily lower limit or minimum strength, m and x_0 are material properties, m being a dimensionless "shape parameter" and x_0 being a "scale parameter" with units of stress. Assuming $x_1 = 0$ (Barrett, 1974), let $(1/x_0)^m = k$ and Equation 2.5 is the same as Equation 2.4.

Substitution of Equations 2.3 and 2.5 into Equation 2.2 gives the cumulative distribution function (CDF) of strength to be

$$F(x) = 1 - \exp(-B) = 1 - \exp[-\int_V (x/x_0)^m dv] \qquad (2.6)$$

If the stress distribution is assumed to be uniform then,

$$F(x) = 1 - \exp(-B) = 1 - \exp[-V(x/x_0)^m] \qquad (2.7)$$

which is the two-parameter Weibull distribution.

The "weakest-link theory" has been used in many applications to explain the size effect on strength of lumber, mainly in the form of the two-parameter Weibull distribution (Equation 2.7). The following sections discuss the application of the "weakest-link theory" on different strength properties.

2.1.1 BENDING STRENGTH

Bohannan (1966) used the statistical theory of strength of materials to explain the relationship between size and bending strength of wood members. Using the linear stress theory to determine the stress distribution of failure, Bohannan found the CDF for a beam with uniform volume under two-point loading. The theoretical CDF was compared to an observed CDF of three sets of data of clear straight-grained Douglas-fir. The comparison showed some disagreement between theory and the observed data.

A modified statistical strength theory was then derived to better explain the size effect on bending strength in wood members with uniform volume (Bohannan, 1966). By rationalizing that the size effect on modulus of rupture is independent of the beam width, Bohannan developed an expression for the CDF of bending strength dependent only on length and depth of the beam. Comparing the theoretical and experimental modulus of rupture using the same data showed reasonably good agreement between theory and observed data (Bohannan, 1966).

According to Liu (1981), the "weakest-link theory" can also be used to analyze the size effect on bending strength of tapered wood beams under arbitrary loading conditions. Like Bohannan (1966), Liu adopted the linear bending stress distribution to determine the CDF of bending strength. Liu also developed the analysis of the size-strength relationship of wood beams by considering only the aspect area as an effect on the bending strength.

2.1.2 TENSILE STRENGTH PERPENDICULAR-TO-GRAIN

The weakest-link concept has been applied to predict the relationship between specimen volume and load-carrying capacity for Douglas-fir specimens loaded in uniform tension perpendicular-to-grain (Barrett, 1974). The theoretical model is a linear log-volume to log-strength relationship derived from the two-parameter Weibull CDF. The relationship between strength and volume is given in the following equation:

$$\log(x) = a - (1/m)\log(V) \tag{2.8}$$

where:

$$a = (1/m)\log[-\ln(1-F(x))] + \log(x_0) - (1/m)\log\Psi \tag{2.9}$$

Ψ = constant depending on stress distribution and shape parameter, m

The validity of the weakest link model was tested using the results of tests on uniformly loaded Douglas-fir specimens. The hypothesis that the weakest-link concept applies was accepted on the basis of the high coefficients of determination obtained by least-squares regression relating log-volume to log-strength (Barrett, 1974).

2.1.3 SHEAR STRENGTH

Liu (1980) developed a model to describe the size effect on shear strength in wood members by applying Weibull's statistical theory (1939). Assuming that the shear stresses are uniformly distributed over a uniform volume, Equation 2.3 becomes

$$B = \beta V(x_0/W_0)^m \tag{2.10}$$

in which

$$W_0 = \tfrac{1}{2}k(\pi)^{\frac{1}{2}}[\Gamma(m + 1)/\Gamma(m + 3/2)]^{-1/m}$$

where Γ = the Gamma function; β is based on the shear force diagram

Substituting Equation 2.10 into Equation 2.6 gives the CDF

$$F(x) = 1 - \exp[-\beta V(x/W_0)^m] \tag{2.11}$$

The data set used to test and compare to the weakest-link model was shear failure results of 5 samples of glued-laminated Douglas-fir beams. A comparison of the theoretical model to the observed data provided adequate evidence that the weakest-link analysis interprets the size effect in shear strength (Liu, 1980).

2.1.4 TENSILE STRENGTH PARALLEL-TO-GRAIN

Poutanen (1984) discusses the possibility of a length effect on tensile strength parallel-to-grain in lumber. Although he does not have data to verify the length effect, he theoretically claims that an increased beam length results in a decrease in tensile strength. Assuming that a defect has cumulative strength distribution function F, and the wood member has n defects, Poutanen describes the probability for survival at stress level σ_1 as:

$$P_1 = (1 - F(\sigma_1))^n \tag{2.12}$$

If size is increased k times, the number of defects increases r times. Assuming k = r when only the length is increased, the probability of survival becomes

$$P_2 = (1 - F(\sigma_2))^{kn} \tag{2.13}$$

at stress level σ_2. Since all sizes fail with equal probability, $P_2 = P_1$, which implies $F(\sigma_2)<F(\sigma_1)$. Subsequently,

if $F(\sigma_2)<F(\sigma_1)$, then $\sigma_2<\sigma_1$ which implies that the member with increased length is weaker in tension parallel-to-grain (Poutanen, 1984).

2.2 STOCHASTIC MODELS

It is reasonable to assume that lumber exhibits serial correlation; ie, that the strength or stiffness in one segment is correlated to the strength or stiffness at the previous segment. Lag-k serial correlation, ρ_k, is the correlation between an observation at one interval length and an observation at k previous intervals. A Markov process is one stochastic model that generates values for each segment while preserving the significant serial correlation between segments.

2.2.1 FIRST ORDER MARKOV PROCESS

A first order Markov process can be used to model a stochastic series if the serial correlation for lags greater than one are not important. Haan (1977) defines a first order Markov process by the equation

$$x_{i+1} = \mu_x - \rho_1(x_i - \mu_x) + t_{i+1}\sigma_x(1 - \rho_1^2)^{1/2} \qquad (2.14)$$

where: x_i = the value of the process at segment i

μ_x = the mean of X

σ_x = the standard deviation of X

ρ_1 = the first-order serial correlation

t_{i+1} = standard normal deviate, N(0,1)

This model assumes X to be from a normal distribution with mean μ_x, variance σ_x^2, denoted by $N(\mu_x, \sigma_x^2)$, and first-order serial correlation ρ_1. It is also assumed that t_{t+1} is independent of X_i. Under these assumptions, this model generates synthetic events that preserve the mean, standard deviation, and first-order serial correlation.

When the first-order Markov model cannot assume that the process is stationary in its first three moments, it is possible to generalize the model to account for variation between segments. If the mean, variance or serial correlation varies between segments the first-order Markov model becomes

$$x_{i+1} = \mu_{x,i+1} + (\rho_1 \sigma_{x,i+1}/\sigma_{x,i})(x_i - \mu_{x,i}) + t_{i+1}\sigma_{x,i+1}(1 - \rho_1^2)^{1/2} \qquad (2.15)$$

Again, this model assumes X to be from a normal distribution (Haan, 1977).

2.2.2 HIGHER ORDER MARKOV MODEL

If the serial correlation for lags greater than one are important the model given by Equation 2.14 can be generalized to include the effects of higher order serial correlation. Haan (1977) describes this higher order Markov model as

$$x_{i+1} = \beta_0 + \beta_1 x_i + \beta_2 x_{i-1} + \ldots + \beta_m x_{m-1} + \varepsilon_{i+1} \quad (2.16)$$

where the X_i's represent the observed data values and the β's are multiple regression coefficients. If normality in the data is assumed, the random element becomes

$$\varepsilon_{i+1} = \sigma_x t(1 - R^2)^{\frac{1}{2}} \quad (2.17)$$

where σ_x^2 is the variance of X, R^2 is the multiple coefficient of determination between X_{i+1} and X_i, X_{i-1}, ..., X_{i-m+1}, and t is a random observation from a standard normal distribution, N(0,1).

Kline et al (1985) used a second-order Markov process to model the lengthwise variability of modulus of elasticity (MOE) along a piece of lumber. By using a second order Markov process, Kline was able to generate 30-inch segment MOE values while preserving the lag-1 and lag-2 serial correlation between segments.

The parameters used to develop the model were obtained from four data sets of four lumber grade and size groups. The groups are 2x4 and 2x10 2250f-1.9E machine stress rated (MSR) and 2x4 and 2x10 No. 2 KD15 visually graded Southern Pine. There were approximately 50 lumber specimens in each of the four data groups. A flatwise static MOE was measured on four 30-inch segments in each specimen.

The second-order Markov model was fitted to the MOE data. The first term in Equation 2.16, β_0, is reduced to zero if X is constructed so that its expected value is zero and thus the second-order Markov process is simplified by

$$x_{i+1} = \beta_1 x_i + \beta_2 x_{i-1} + \varepsilon_{i+1} \tag{2.18}$$

The lag-1 and lag-2 serial correlations are both preserved with the second-order Markov model when

$$\beta_1 = (\rho_1 + \rho_1\rho_2)/(1 - \rho_1^2) \tag{2.19}$$

and

$$\beta_2 = (\rho_2 - \rho_1^2)/(1 - \rho_1^2) \tag{2.20}$$

where ρ_1 and ρ_2 are estimated by the lag-1 and lag-2 serial correlations r_1 and r_2 for each grade and size (Yevjevich, 1972).

Using Equation 2.18, a specified number of serially correlated MOE 30-inch segment values are generated. Since β_0 has been reduced to zero, the generated values have an expected value of zero. Kline used the following procedure to convert his model generated values to lengthwise 30-inch segment MOE values. First, the average segment MOE value in the model is added to each of the generated values. Then, the average of the segment MOE values is calculated and each segment MOE is divided by the piece-average MOE to obtain MOE indexes. Next, a random-piece MOE is generated from a prescribed probability distribution of the desired size and grade of lumber. The MOE indexes are multiplied by the random-piece MOE observation to obtain the lengthwise segment MOE values.

2.3 WEIGHTED LEAST SQUARES REGRESSION MODEL

The positive correlation between MOE and lumber strength properties such as modulus of rupture, tensile strength parallel-to-grain, and compressive strength parallel-to-grain has been successfully modeled by (Woeste et al, 1979) using a weighted least squares regression model. The weighted least squares regression model is of the form

$$Y = \beta_1 X + \beta_0 + \varepsilon \tag{2.21}$$

where Y = the strength property to be generated

X = the independent variable, MOE

and ε is assumed to be normally distributed with a mean of zero and residual variance equal to K times X. Parameters β_1, β_0 and K are estimated by

$$b_1 = \frac{\Sigma 1/X \Sigma Y - n \Sigma Y/X}{\Sigma 1/X \Sigma X - n^2} \tag{2.22}$$

$$b_0 = \frac{\bar{X}\Sigma Y/X - \Sigma Y}{\bar{X}\Sigma 1/X - n} \qquad (2.23)$$

$$K = \frac{b_0^2 s_x^2 (1 - r^2)}{\bar{X} r^2} \qquad (2.24)$$

where r is the estimated linear correlation coefficient, s_x^2 is the estimated variance of X, and the summation over 1 to n is implied.

In some cases, the weighted least squares regression model, Equation 2.21, exhibits a lack of fit near the lower left corner of the scattergram of strength versus stiffness. Then, "it is highly probable that a logarithmic transformation on the dependent variable (strength property) will greatly improve the relationship" (Woeste et al, 1979). The weighted least squares regression model becomes

$$\ln(Y) = \beta_0 + \beta_1 X + \varepsilon \qquad (2.25)$$

where Var(E) = KX and the parameters β_0, β_1 and K are calculated by Equations 2.22, 2.23 and 2.24, respectively, by replacing Y with ln(Y).

CHAPTER III. EXPERIMENTAL DESIGN

3.1 DESCRIPTION OF MATERIAL

One thousand pieces of 16-foot nominal 2-inch dimension Southern Pine lumber of two sizes and two grades were obtained by competitive bid on the open market. Two sizes and two grades were chosen so as to define the length effect for a wide quality range to broaden the application of results. The sizes and grades and the numbers of pieces of each were as follows:

Number	Size	Grade
250	2x4	No. 2 KD15
250	2x4	2250f-1.9E
250	2x10	No. 2 KD15
250	2x10	2250f-1.9E

The actual number of usable pieces varied slightly from the above numbers. The No. 2 KD15 is a visual-stress grade denoted by VG and the 2250f-1.9E is machine stress-rated (MSR).

The grades and sizes were chosen to cover the range of common truss spans found in practice. A short span roof truss would typically be designed with 2x4 lumber with the lowest grade used being No. 2 KD15 Southern Pine. For longer spans, 2x10 machine stress-rated lumber would be common. The grade 2250f-1.9E Southern Pine was chosen since it is the highest grade likely to be used for roof truss construction.

In the Southern Pine structural lumber market, material grade-stamped No. 2 most commonly includes material No. 2 and better in quality, as was the case here. A quality supervisor from the Northern Hardwood and Pine Manufacturing Association was contracted to visually regrade the No. 2 KD15 material. The result of that regrade is tabulated below:

	Number of Pieces 2x4	Number of Pieces 2x10
Select Structural Dense (SSD)[1]	35	31
Select Structural (SS)	4	5
No. 1 Dense	77	61
No. 1	11	42
No. 2 Dense	79	58
No. 2	38	59
Total	244	256

[1]KD is dropped from the grade name for convenience and is hereafter implied.

The "regraded" lumber was combined into four groups based on similarities of allowable bending stress values, F_b, specified in the National Forest Products Association (1982) for the individual grades. The grade or grades in each group, the F_b values for the individual grades, and the numbers of specimens resulting from the combinations are shown in Table 3.1.

3.1 CONDITIONING

The lumber was conditioned to equilibrium moisture content in a room controlled at 75°F and 68 percent relative

TABLE 3.1. The "regraded" No. 2 lumber groups, their allowable bending stresses and the number of specimens in each group.

GROUP	GRADE	2X4 F_b (psi)	2X4 NO. OF SPECIMENS	2X10 F_b (psi)	2X10 NO. OF SPECIMENS
1	SSD	2250	35	2200	31
2	SS	2150	81	1850	66
	No. 1 D	2150		1850	
3	No. 1	1850	90	1600	101
	No. 2 D	1800		1650	
4	No. 2	1550	39	1300	59
	TOTAL		245		257

humidity (≈ 12% EMC). A capacitance moisture meter was used to monitor the conditioning progress.

3.1.2 ASSIGNMENT TO TREATMENT GROUPS

The lumber was assigned to the three test lengths such that their distributions of strength would be as equivalent as possible. A full-span modulus of elasticity (MOE) was determined on all pieces by the vibration method. For the MSR lumber, all pieces were ranked by MOE (2x4 and 2x10 independently). The five pieces having the lowest five MOE values were randomly assigned to the 30-, 90- and 120-inch test groups: one to the 30-inch groups and two each to the 90- and 120-inch groups. Only one was assigned to the 30-inch group because each 16-foot piece was long enough to yield two specimens for the 30-inch test. The five pieces with the next five lowest MOE values were randomly assigned, and so on, until the three test groups were established with approximately 100 specimens per test length.

The "regraded" visually graded lumber was ranked by MOE separately for each "grade" group. They were then assigned to the three test lengths in the same way as the MSR lumber. Tables 3.2 and 3.3 list the assigned treatment groups and number of specimens of the "regraded" visually graded lumber.

TABLE 3.2. The number of specimens in each grade group and treatment group of visually graded 2x4 are shown.

GROUP	TREATMENT GROUP 30-inch	90-inch	120-inch
1	14	14	14
2	34	32	32
3	36	36	36
4	14	16	16
Totals	98	98	98

TABLE 3.3. The number of specimens in each grade group and treatment group of visually graded 2x10 are shown.

GROUP	TREATMENT GROUP		
	30-inch	90-inch	120-inch
1	14	12	12
2	26	26	27
3	40	41	40
4	24	25	22
Totals	104	104	101

3.1.3 MEASUREMENT OF MOE

A flatwise static MOE was determined on four 30-inch long segments in each specimen designated for the 30-inch tension test, two on each side of the center line of the specimens (Figure 3.1). Dead loads (a pre-load and a final load) were applied to the third-points of a 90-inch span with an air operated ram. The loads were 25 and 100 pounds for the 2x4's and 75 and 275 pounds for the 2x10's. On several specimens with low stiffness, the system "bottomed out" and the pre- and the final loads had to be reduced accordingly. When testing the various segments, an upward force was applied on the opposite end at the center of the overhang to counter the weight of the overhang and eliminate significant reverse bending moments.

Deflections were measured (at pre-load and the final load) between the load points with a LVDT mounted on a yoke and suspended from the specimen at the load points. This test arrangement permitted calculation of a shear-free MOE.

A rocker was provided at one support, at one end of the deflection yoke to account for twist in some members. A roller support at the end opposite the rocker support allowed for the lengthing of the specimen as it was loaded. The deflection yoke was suspended from 1/4-inch dowels laying crosswise on top of the specimen. The small diameter dowels

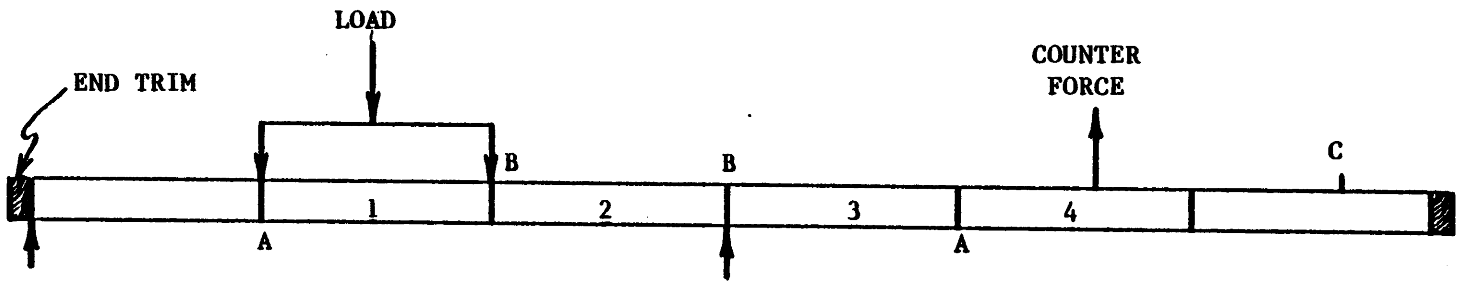

Figure 3.1 Location of four 30-inch segments for MOE measurements and the loading configuration for Segment "1". For Segment "2", the supports are at points "A", the load at points "B" and the counter force at point "C". To test segments "3" and "4", the specimen was turned end for end in the equipment. The specimens were cut in two at the center (Point "B" on the right) for tension tests of segments "1" and "4".

were flexible enough to "seat" on specimens rather than rock on those that were cupped (convex side up).

Width and thickness were measured to the nearest 0.01 inch at the center of each 30-inch segment which were used to calculate MOE for individual segments.

Three repetitions were performed on each segment and the MOE values reported are the averages of the measurements.

3.1.4 TENSION TESTING

In preparation for testing, the pieces designated for the 30-inch length were cut in two pieces between MOE-segments 2 and 3 and then trimmed on the opposite end to a 90-inch length. The specimens designated for the 90-inch and 120-inch tests were trimmed equally on both ends to total lengths of 150 and 180 inches, respectively. Table 3.4 shows the required arrangement of tensile specimens in the tensile grips. Width and thickness were measured to the nearest 0.01 inch at the center of the test zone. Specimens were centered between grips and loaded to failure in the U. S. Forest Products Laboratory tension machine at a rate of 360 lbs/sec.

The specimens were tested in groups of 25, systematically varying the test length. This procedure permitted testing 100 specimens without changing the machine settings, e.g. 25 specimens each of 2x4 VG, 2x10 VG, 2x4 MSR and 2x10

Table 3.4. Required arrangement of tensile specimens in the tensile grips.

Treatment No.	Length between Grips	Length in Grips	Test Specimen Length
A	30	60	90[1]
B	90	120	150[2]
C	120	150	180

[1]Cut 2 from center of 16-foot piece
[2]Cut from center of 16-foot piece

MSR for one length test, then switching to another length, etc.

Defects associated with the failures were mapped and a description of the failure recorded. The moisture content and specific gravity were measured on a 1-inch disc cut from as near the failure as possible.

3.2 DATA ANALYSIS

The measured axial tensile force was used to calculate the tensile strength parallel-to-grain for each specimen. Tables 3.5, 3.6 and 3.7 show the mean tensile strength for each group of lumber. The mean tensile strength decreases with an increase in length for every group from the 30-inch specimen to the 90-inch specimen. Then, the mean tensile strength either decreases or increases from the 90-inch to 120-inch specimen in the visually graded lumber. The mean tensile strength continues to decrease from the 90-inch specimen to the 120-inch specimen in the MSR lumber groups. Figures 3.2 through 3.5 show the decrease in mean tensile strength in every group from the 30-inch specimen to the 120-inch specimen.

Table 3.5. Mean tensile strength in psi of visually graded 2x4 lumber sample.

Visually Graded 2x4	30 in.	90 in.	120 in.
Group 1	9856	8517	8186
Group 2	6166	4719	4472
Group 3	4702	4528	3946
Group 4	3312	2740	3197

Table 3.6. Mean tensile strength in psi of visually graded 2x10 lumber sample.

Visually Graded 2x10	30 in.	90 in.	120 in.
Group 1	9685	6572	6596
Group 2	5603	4268	4119
Group 3	4644	4028	3412
Group 4	3035	2631	2084

Table 3.7. Mean tensile strength in psi of 2250f-1.9E lumber.

Size	30 in.	90 in.	120 in.
2x4	9436	8107	8100
2x10	9270	8139	7976

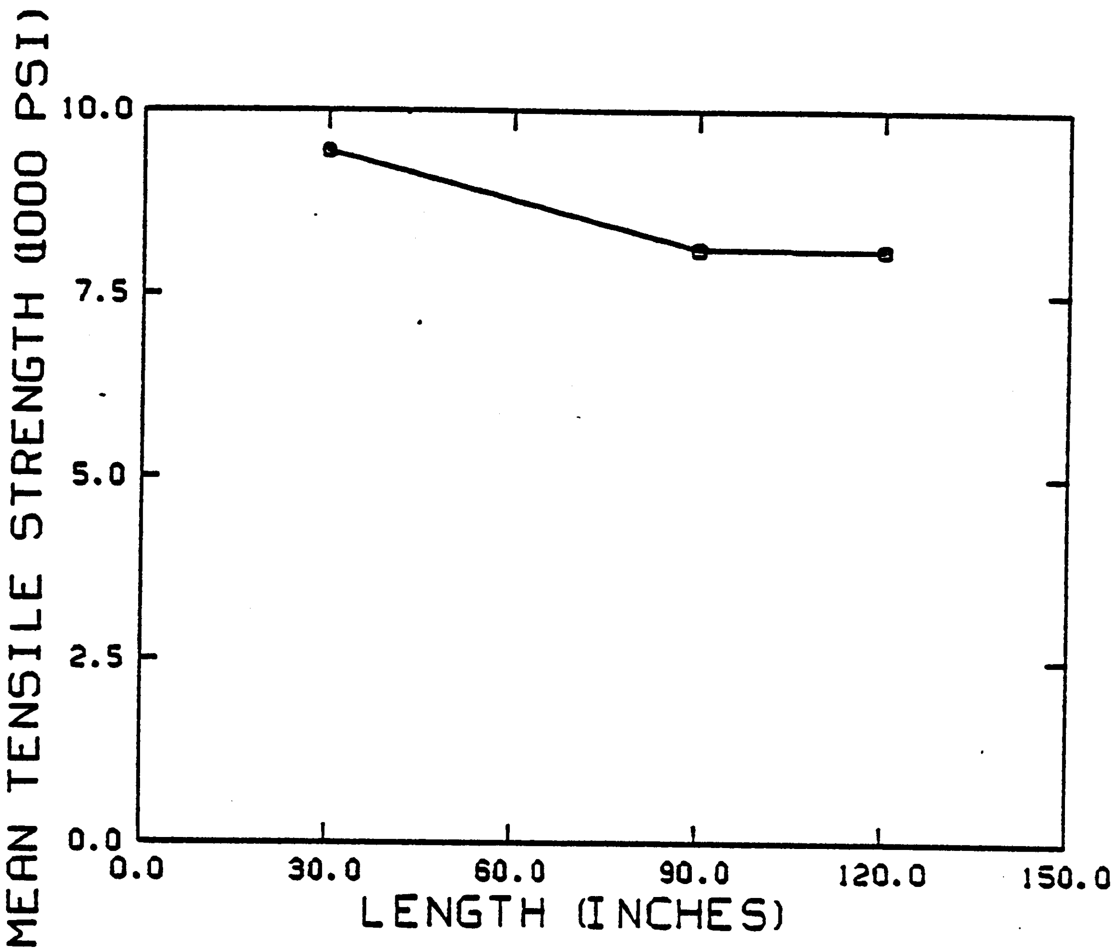

Figure 3.2 Mean tensile strength of the 2x4 2250f-1.9E MSR Southern Pine versus length of the test span. Tension specimens were tested at 30-, 90- and 120-inch test spans.

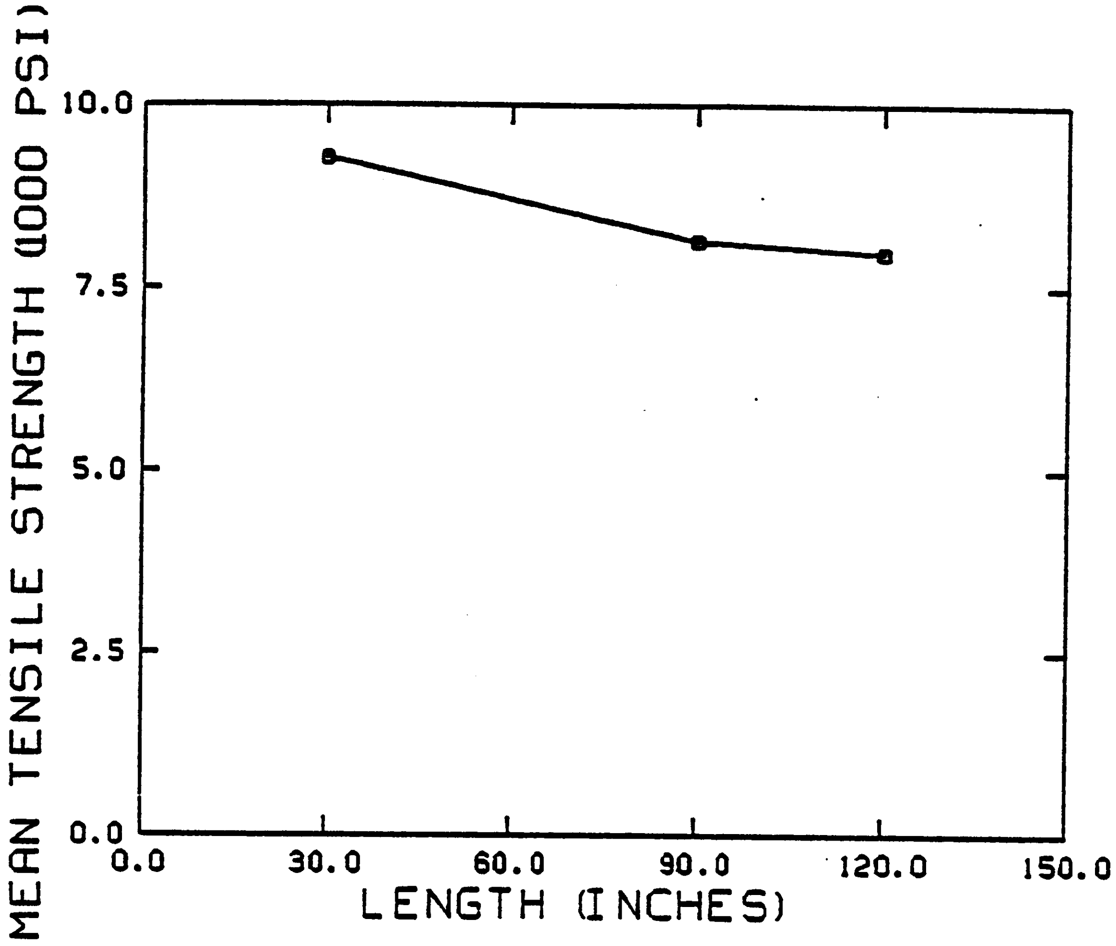

Figure 3.3 Mean tensile strength of the 2x10 2250f-1.9E MSR Southern Pine versus length of the test span. Tension specimens were tested at 30-, 90-inch and 120-inch test spans.

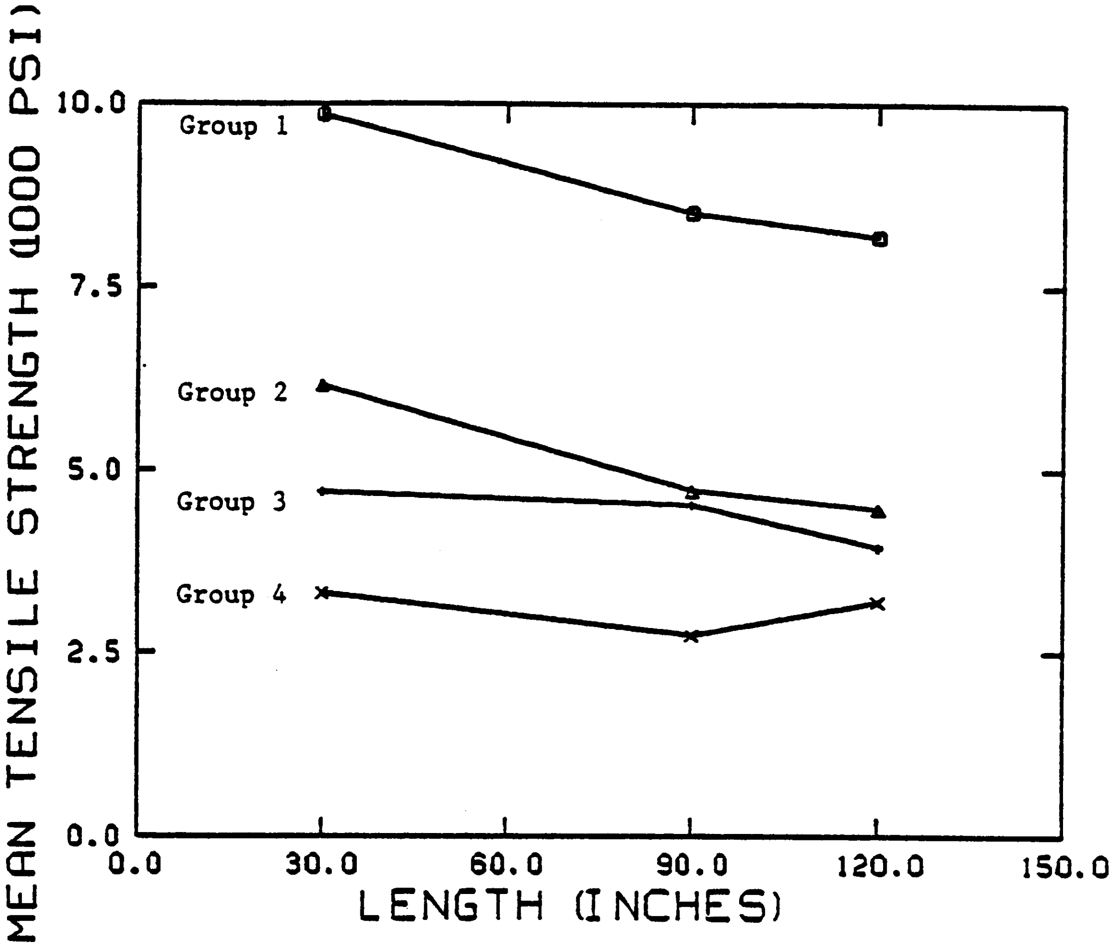

Figure 3.4 Mean tensile strength of the 2x4 No. 2 KD15 Southern Pine versus length of the test span. Tension specimens were tested at 30-, 90- and 120-inch test spans.

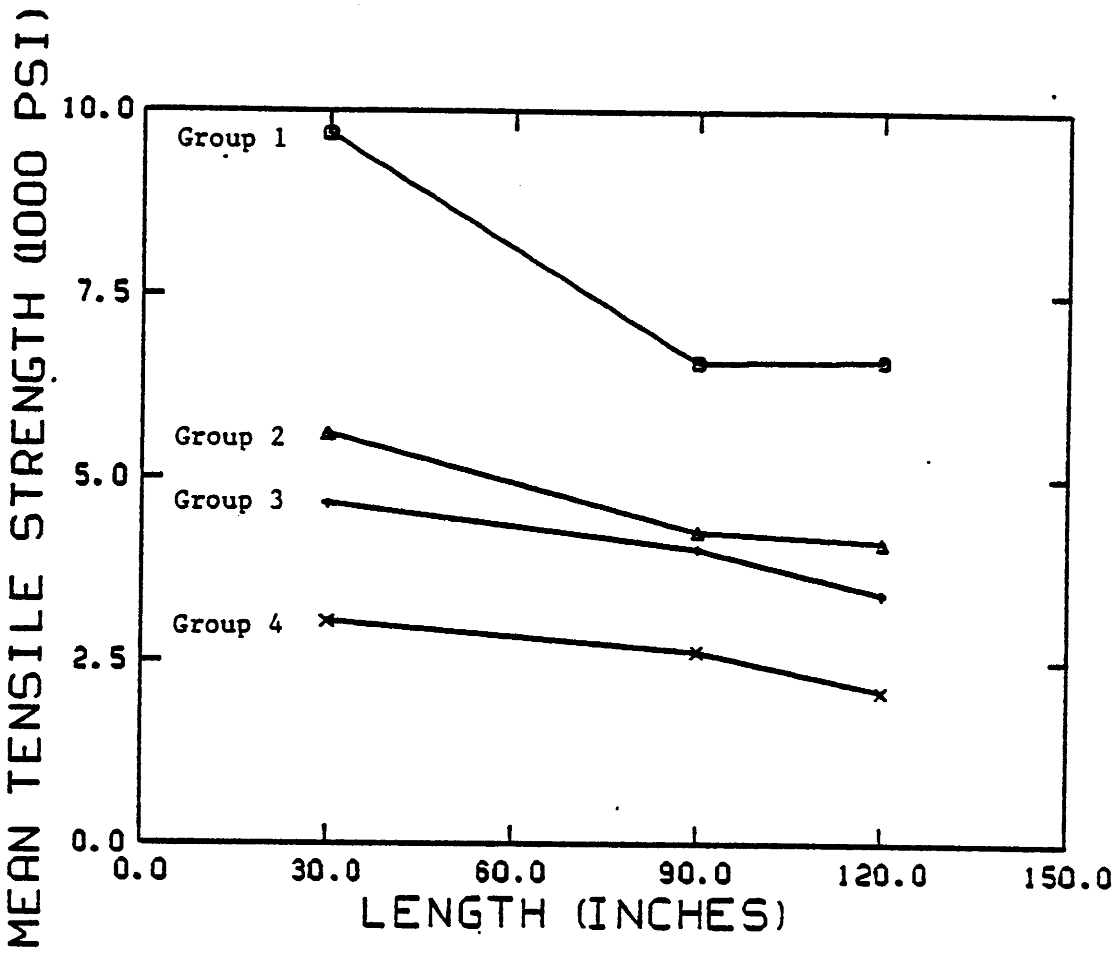

Figure 3.5 Mean tensile strength of the 2x10 No. 2 KD15 Southern Pine versus length of the test span. Tension specimens were tested at 30-, 90- and 120-inch test spans.

Tables 3.8, 3.9 and 3.10 list the coefficients of variation, COV, of the tensile strength. The COV generally increases with a decrease in grade quality. The only exception is an increase in COV from Group 3 to Group 4 in the 120-inch specimen of 2x10 visually graded lumber.

Next, the fifth percentile tensile strength was calculated for each group of lumber. A lognormal distribution was assumed and a fifth percentile value was calculated from the distribution fit. Again, a decrease in tensile strength with an increase in specimen length can be observed (Figures 3.6 - 3.9).

The correlation structure of the tensile strength was considered along the piece of lumber. Since two 30-inch segments were taken from a single piece of lumber, the correlation coefficient between segments "1" and "4" was determined for each specimen of the 30-inch treatment group. The likelihood that the tensile strength is correlated between segments "1" and "4" was tested using the t distribution (Haan, 1977). The hypothesis $H_0 = \rho_{1,4} = 0$ was tested, $\rho_{1,4}$ being the population correlation coefficient of the tensile strength between segments "1" and "4" of the 30-inch specimens. If $\rho = 0$, then the quantity

$$t = r[(n-2)/(1-r^2)]^{1/2} \qquad (3.1)$$

r = sample estimate population correlation coefficient

Table 3.8. Coefficient of variation of the tensile stength in visually graded 2x4 lumber sample.

Visually Graded 2x4	30 in.	90 in.	120 in.
Group 1	0.250	0.215	0.270
Group 2	0.285	0.364	0.310
Group 3	0.428	0.405	0.506
Group 4	0.430	0.452	0.541

Table 3.9. Coefficient of variation of the tensile stength in visually graded 2x10 lumber sample.

Visually Graded 2x10	30 in.	90 in.	120 in.
Group 1	0.266	0.316	0.367
Group 2	0.429	0.413	0.589
Group 3	0.527	0.699	0.645
Group 4	0.562	0.862	0.525

Table 3.10. Coefficient of variation of the tensile strength of the 2250f-1.9E lumber.

Size	30 in.	90 in.	120 in.
2x4	0.305	0.286	0.289
2x10	0.251	0.252	0.262

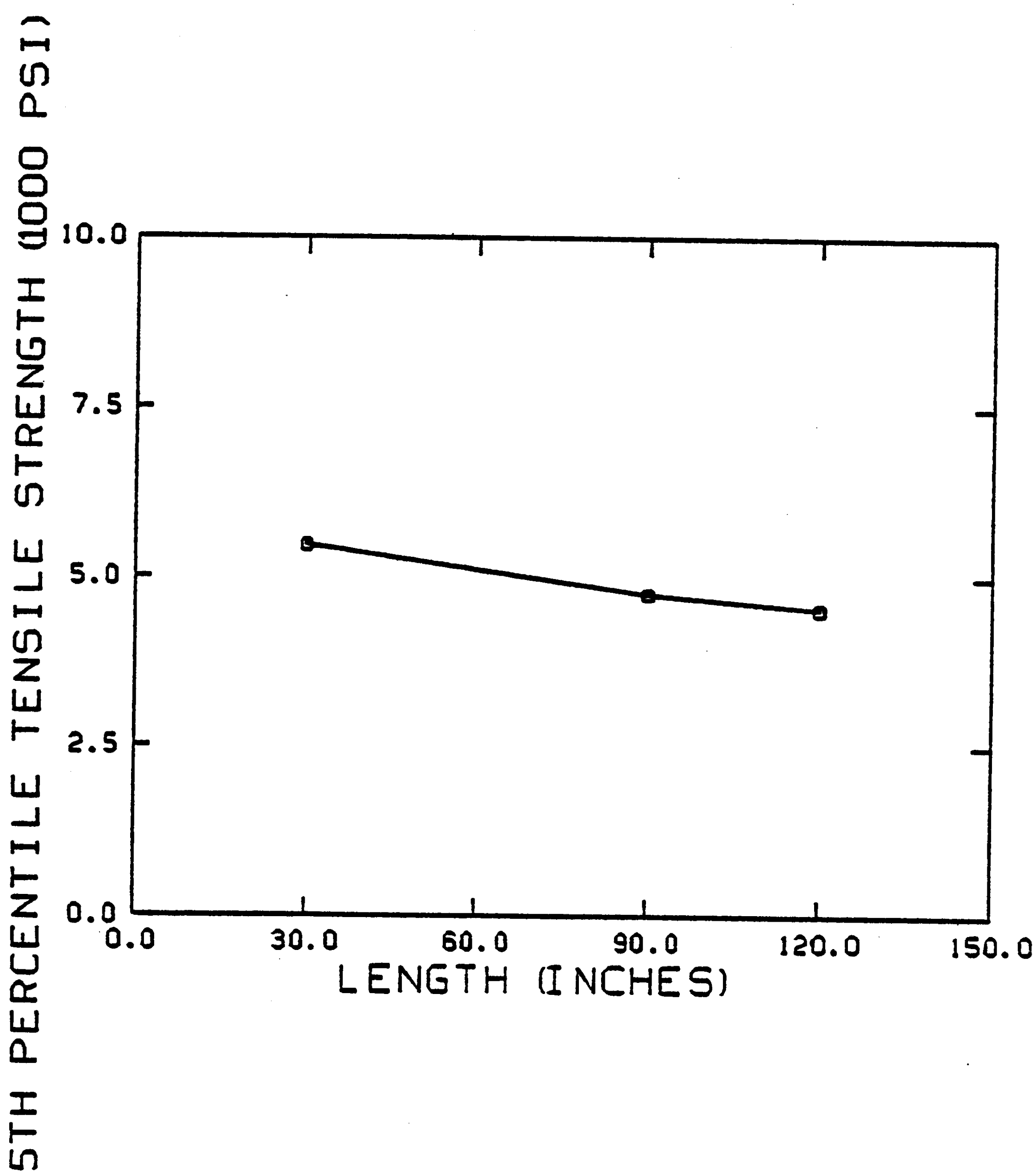

Figure 3.6 5th percentile tensile strength of the 2x4 2250f-1.9E MSR Southern Pine versus length of the test span. Tension specimens were tested at 30-, 90- and 120-inch test spans.

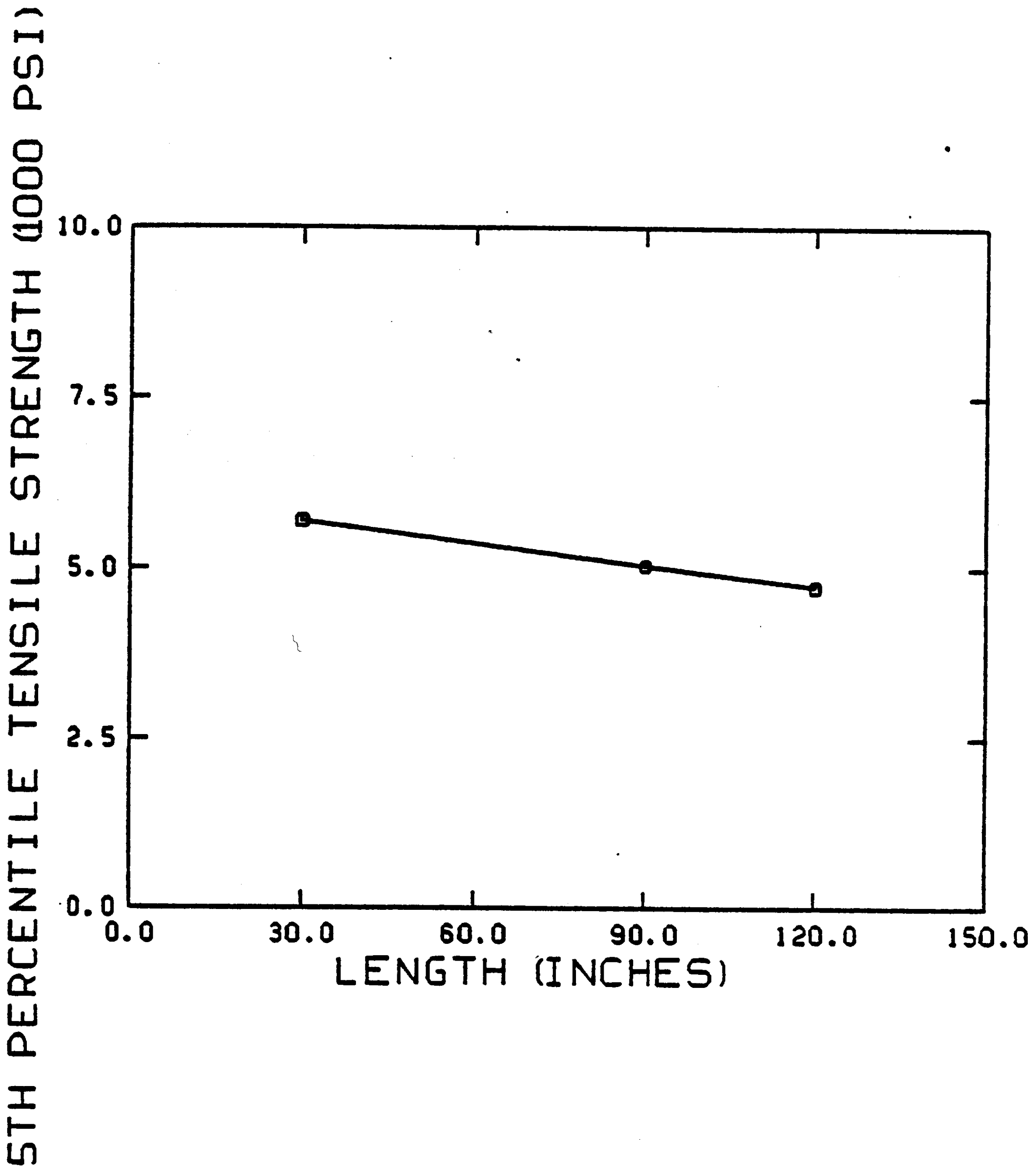

Figure 3.7 5th percentile tensile strength of the 2x10 2250f-1.9E MSR Southern Pine versus length of the test span. Tension specimens were tested at 30-, 90- and 120-inch test spans.

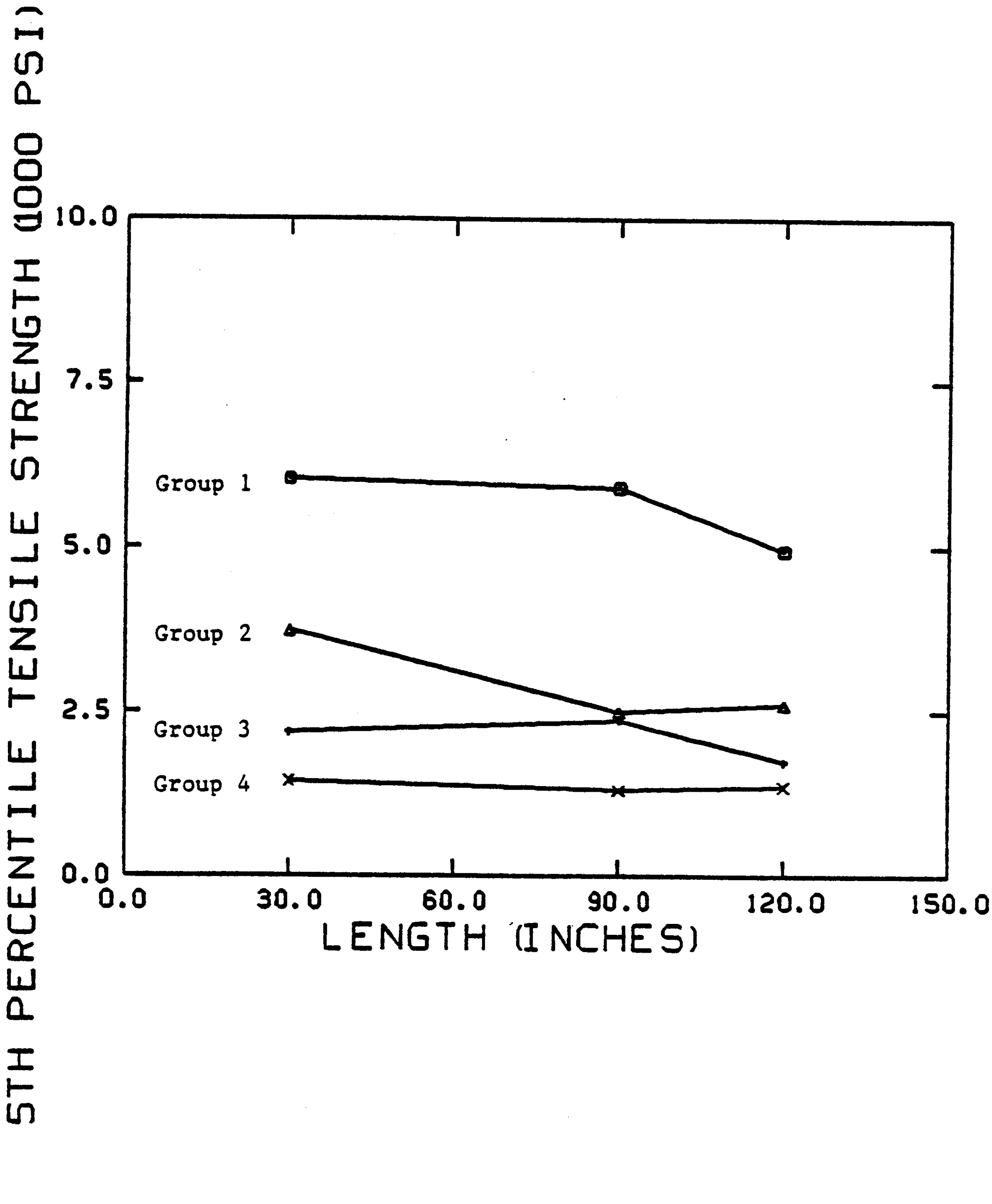

Figure 3.8 5th percentile tensile strength of the 2x4 No. 2 KD15 Southern Pine versus length of the test span. Tension specimens were tested at 30-, 90- and 120-inch test spans.

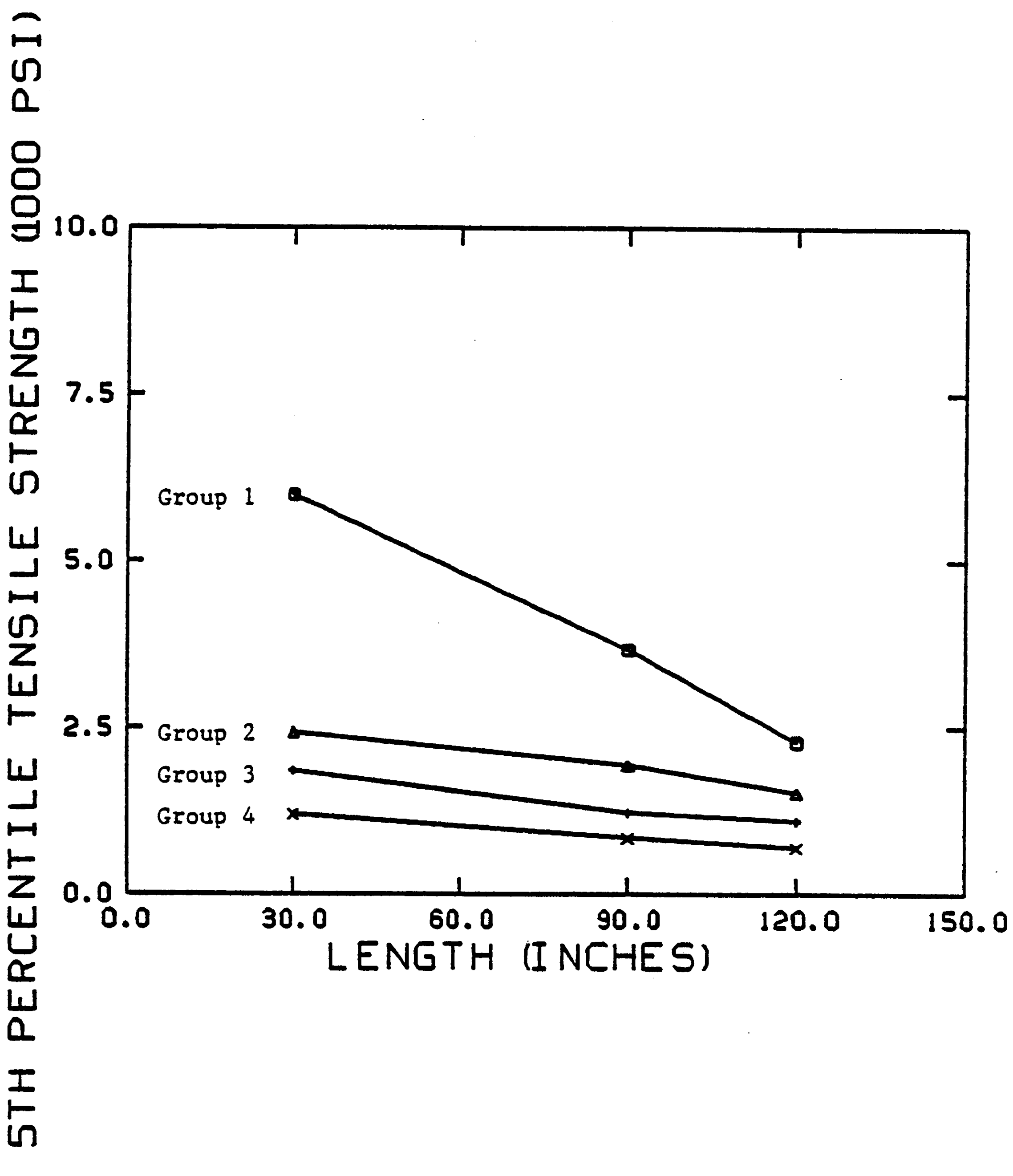

Figure 3.9 5th percentile tensile strength of the 2x10 No. 2 KD15 Southern Pine versus length of the test span. Tension specimens were tested at 30-, 90- and 120-inch test spans.

has a t distribution with n-2 degrees of freedom where n is the sample size. To test the hypothesis $H_0: \rho_{1,4} = 0$, the test statistic was calculated from equation 3.1 and H_0 was rejected when $|t| > t_{1-\alpha/2,n-2}$. The percentile value, $t_{1-\alpha/2,n-2}$ was chosen at the 0.05 level of significance.

Table 3.11 shows the sample estimate correlation coefficient $r_{1,4}$ and the result of the hypothesis test for each grade and size. The hypothesis that the tensile strength in segments "1" and "4" are not correlated was rejected for both the 2x4 and 2x10 2250f-1.9E lumber. The hypothesis was also rejected for Groups 2 and 3 of the 2x4 lumber, Group 3 of the 2x10 lumber and for both the 2x4 and 2x10 grade-stamped No. 2 lumber.

Table 3.11. Summary of hypothesis test illustrating the correlation of the ultimate tensile stress in segments "1" and "4" of the 30-inch groups at the 0.05 significance level.

Size	Grade Group	$r_{1,4}$	$H_0: p_{1,4} = 0$
2x4	2250f-1.9E	0.444	reject
2x4	Group 1	0.380	cannot reject
2x4	Group 2	0.827	reject
2x4	Group 3	0.776	reject
2x4	Group 4	0.649	cannot reject
2x10	2250f-1.9E	0.402	reject
2x10	Group 1	-0.120	cannot reject
2x10	Group 2	0.299	cannot reject
2x10	Group 3	0.555	reject
2x10	Group 4	-0.096	cannot reject
2x4	No. 2 KD15 "as graded"	0.831	reject
2x10	No. 2 KD15 "as graded"	0.589	reject

CHAPTER IV. MODEL DEVELOPMENT

The purpose of this study was to develop a length effect model for tensile strength parallel-to-grain. The data from the 30-inch treatment groups were used to derive the parameters for the model. Because the number of specimens in each treatment group of visually graded lumber was small, it was decided, for purposes of demonstrating the method of modelling, to recombine the re-graded groups. In order to determine the success of the model, probability distributions of tensile strength were simulated for a 90-inch length and a 120-inch length and then independently verified against the test data from the 90-inch and 120-inch treatment groups.

The first approach used to derive the probability distributions of tensile strength parallel-to-grain was to use Weibull's "weakest-link theory" (1939). Using Weibull's theory, the lengthwise segments of a lumber specimen are assumed to be non-correlated. The second model developed takes into account the possible lengthwise correlations of tensile strength. A detailed description of each model is included in the following sections of this chapter.

4.1 DEVELOPMENT OF A MODEL ASSUMING INDEPENDENT SEGMENTS

Using Weibull's "weakest-link theory" (1939), the ultimate tensile strength in a 120-inch specimen of lumber can be defined as the tensile strength in the weakest 30-inch segment of the four 30-inch segments forming the specimen. Likewise, the tensile strength in the weakest segment of the three 30-inch segments forming a 90-inch specimen is the tensile strength of that specimen. According to Ang and Tang (1984), if the segments forming a specimen of n segments are assumed to be statistically independent and identically distributed, the CDF of the tensile strength of a specimen with n segments, is

$$F_{Y_1}(y) = 1 - [1 - F_X(y)]^n \qquad (4.1)$$

It follows that the probability density function (PDF) becomes

$$f_{Y_1}(y) = n[1 - F_X(y)]^{n-1} f_X(y) \qquad (4.2)$$

where: y = tensile strength parallel-to-grain

n = number of 30-inch segments

$f_X(y)$ = PDF of tensile strength of a 30-inch segment

$F_X(y)$ = CDF of tensile strength of a 30-inch segment

The first step in the development of the model was to determine the PDF's and CDF's for each of the 30-inch treatment groups. Figures 4.1 through 4.4 show histograms of tensile strength parallel-to-grain from each of the 30-inch treatment groups. A visual inspection of Figures 4.1 through 4.4 suggested that the lognormal distribution might provide a good fit. Accordingly, the lognormal distribution was overlayed on the tensile strength histograms shown in Figures 4.1 through 4.4. Visual inspection of the distribution indicated a good fit.

A weakest-link model was formed using Equation 4.2. So, $f_{Y_1}(y)$ equals the distribution of the 90-inch or 120-inch data, assuming the segments are independent; n equal 3 for the 90-inch data and n equal 4 for the 120-inch data; and $f_X(y)$ and $F_X(y)$ are the lognormal distribution for the 30-inch data and its probability distribution, respectively. Figures 4.5 through 4.8 show relative frequency histograms of the tensile strength of 90-inch treatment groups. Superimposed onto the histograms are the appropriate probability functions, $f_{Y_1}(y)$, with n equal 3. Figures 4.9 through 4.12 show relative frequency histograms of the tensile strength 120-inch treatment groups and $f_{Y_1}(y)$ with n equal 4 superimposed onto the histograms.

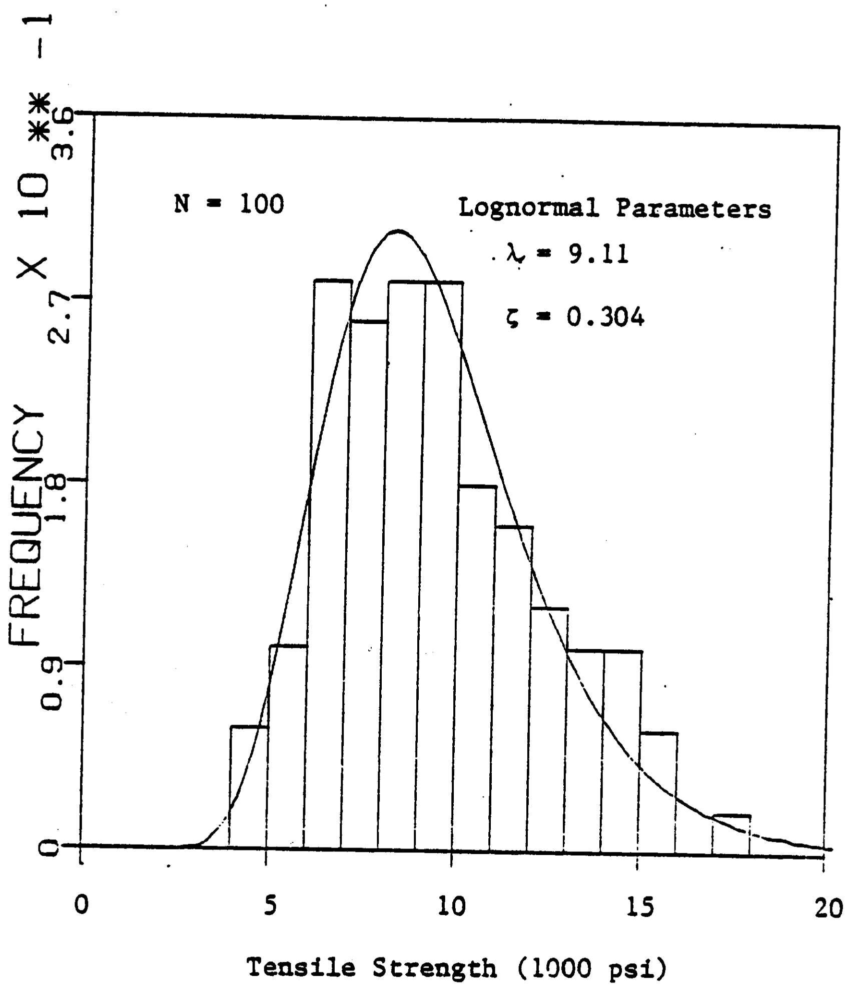

Figure 4.1 A lognormal distribution is superimposed on the histogram of the tensile strength of the 2x4 2250f-1.9E MSR 30" treatment group.

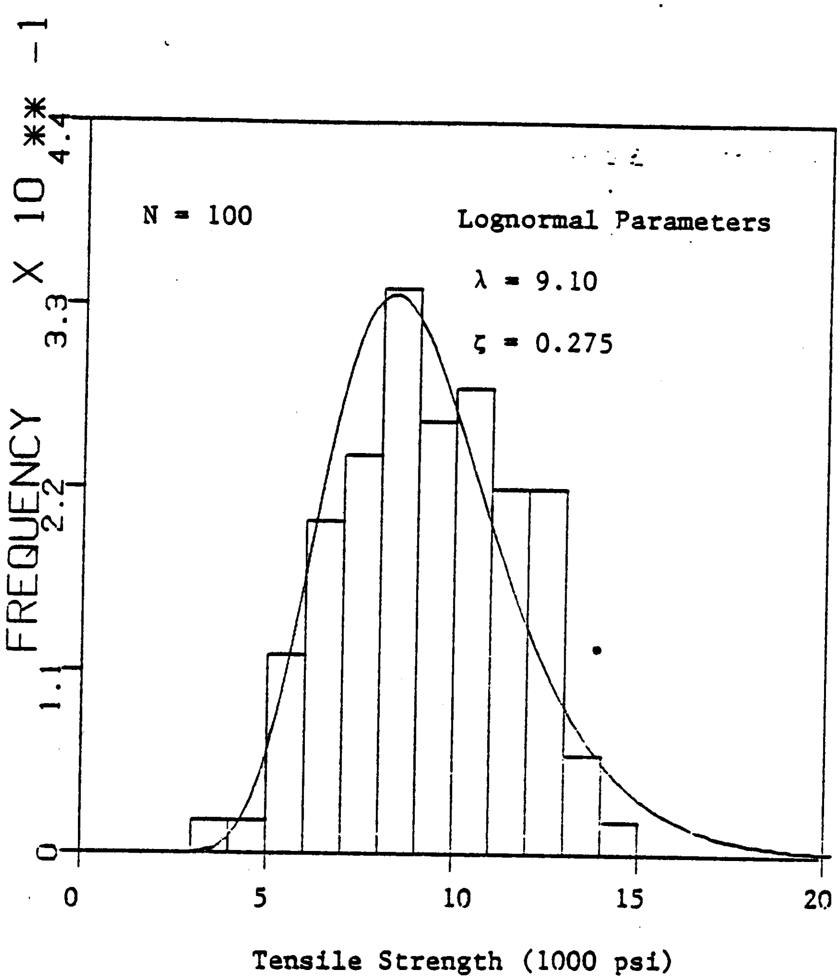

Figure 4.2 A lognormal distribution is superimposed on the histogram of the tensile strength of the 2x10 2250f-1.9E MSR 30" treatment group.

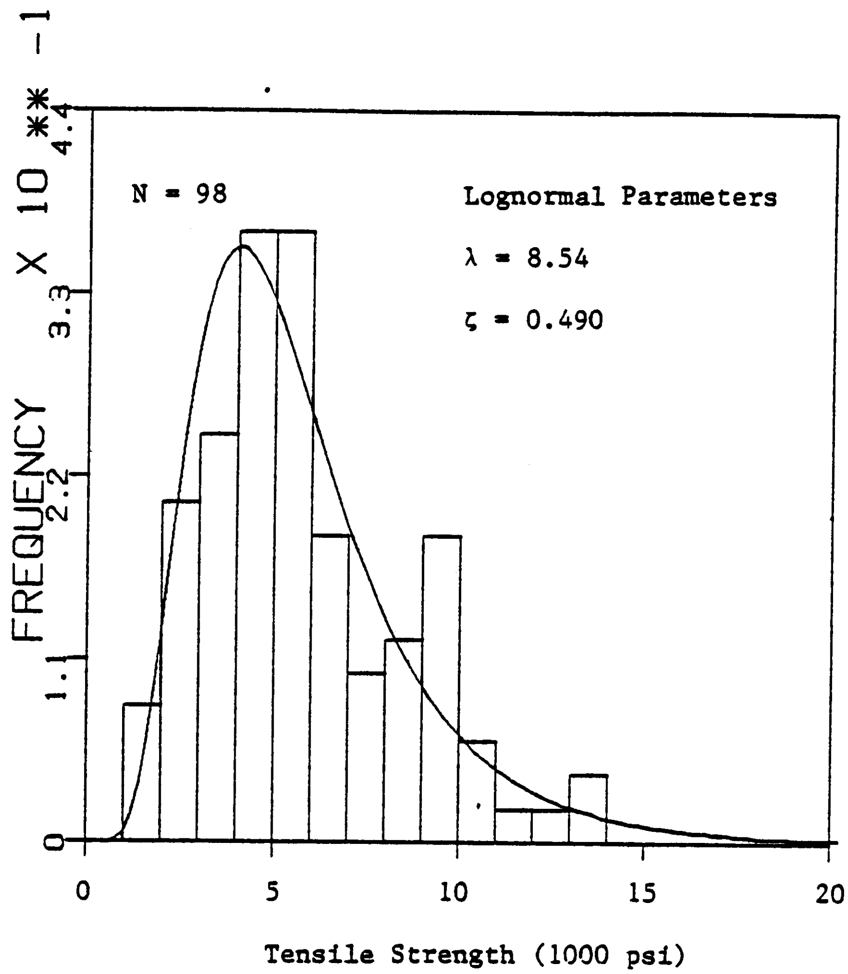

Figure 4.3 A lognormal distribution is superimposed on the histogram of the tensile strength of the 2x4 No. 2 KD15 30" treatment group.

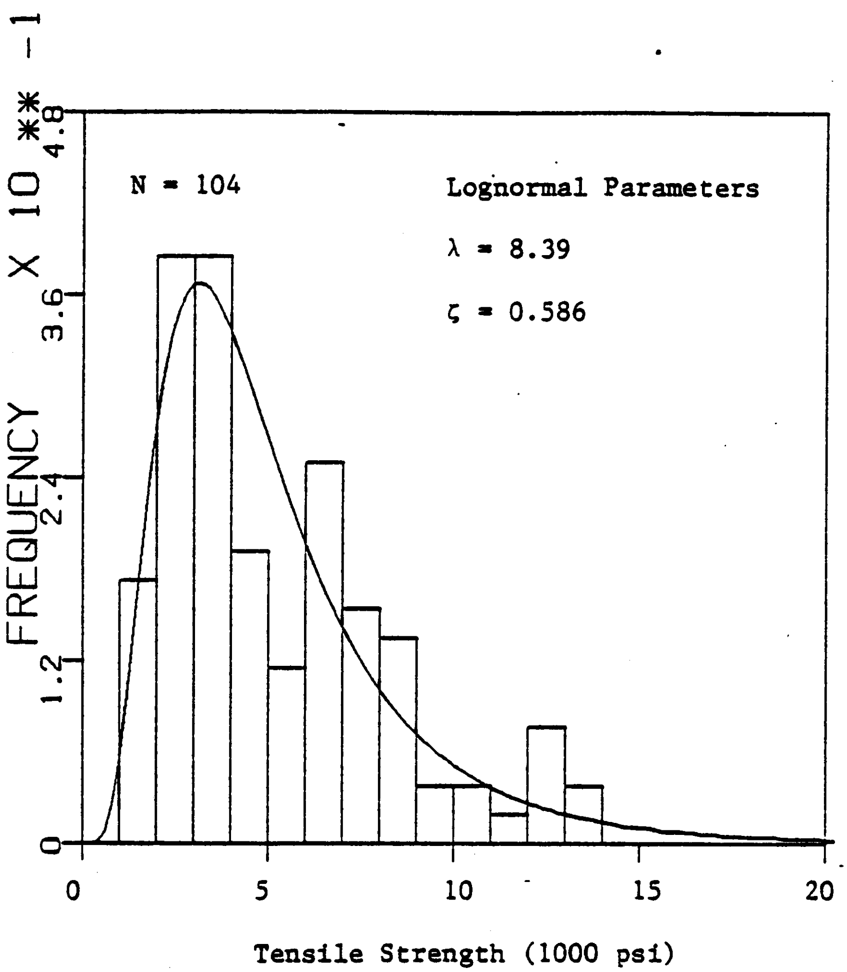

Figure 4.4 A lognormal distribution is superimposed on the histogram of the tensile strength of the 2x10 No. 2 KD15 30" treatment group.

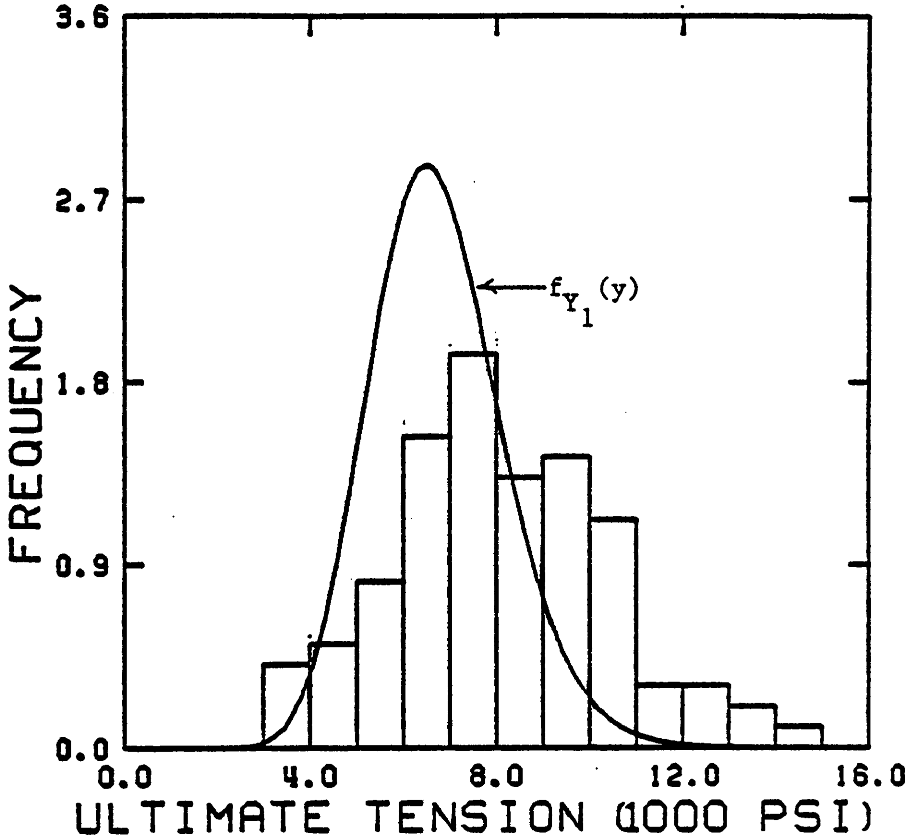

Figure 4.5 The histogram of the ultimate tension of the 2x4 2250f-1.9E MSR 90" treatment group is shown. The probability distribution, $f_{Y_1}(y)$, with n=3 is superimposed onto the histogram.

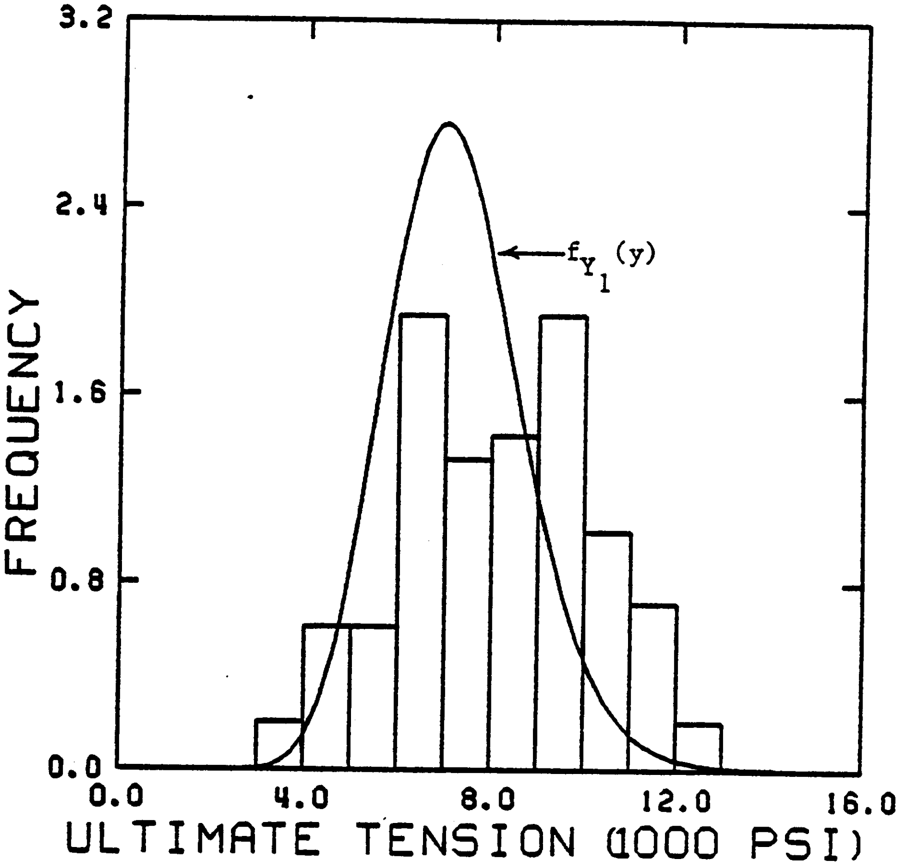

Figure 4.6 The histogram of the ultimate tension of the 2x10 2250f-1.9E MSR 90" treatment group is shown. The probability distribution, f_{Y_1}, with n=3 is superposed onto the histogram.

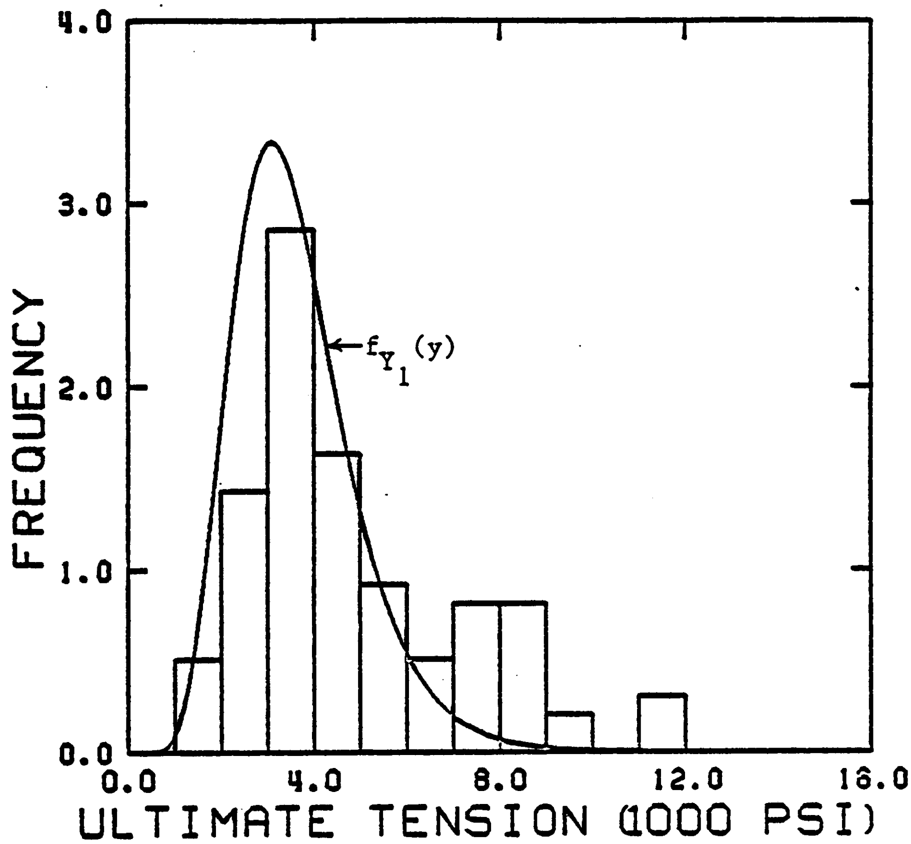

Figure 4.7 The histogram of the ultimate tension of the 2x4 No. 2 KD15 90" treatment group is shown. The probability distribution, f_{Y_1}, with n=3 is superimposed onto the histogram.

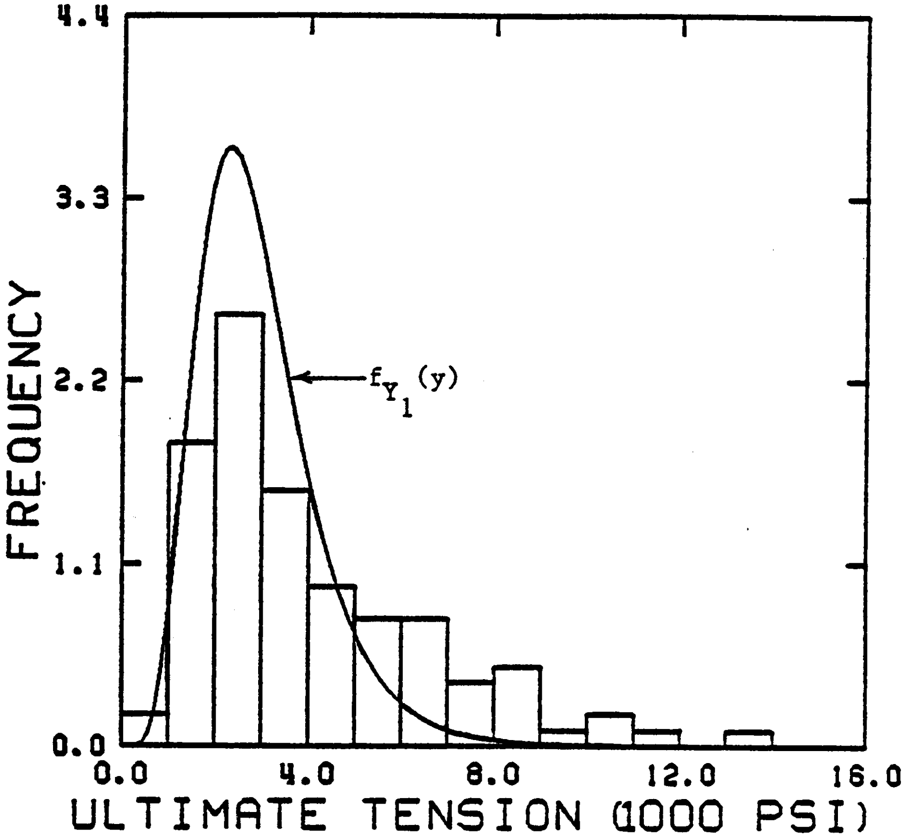

Figure 4.8 The histogram of the ultimate tension of the 2x10 No. 2 KD15 90" treatment group is shown. The probability distribution, f_{Y_1}, with n=3 is superimposed onto the histogram.

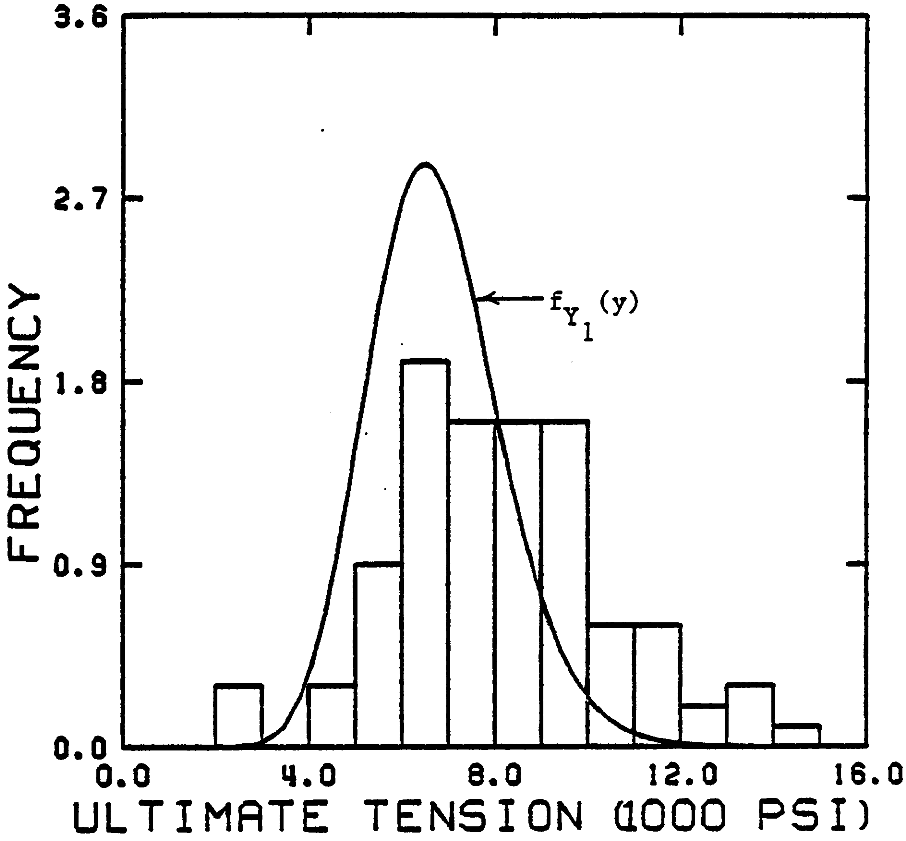

Figure 4.9 The histogram of the ultimate tension of the 2x4 2250f-1.9E 120" treatment group is shown. The probability distribution, f_{Y_1}, with n=4 is superimposed onto the histogram.

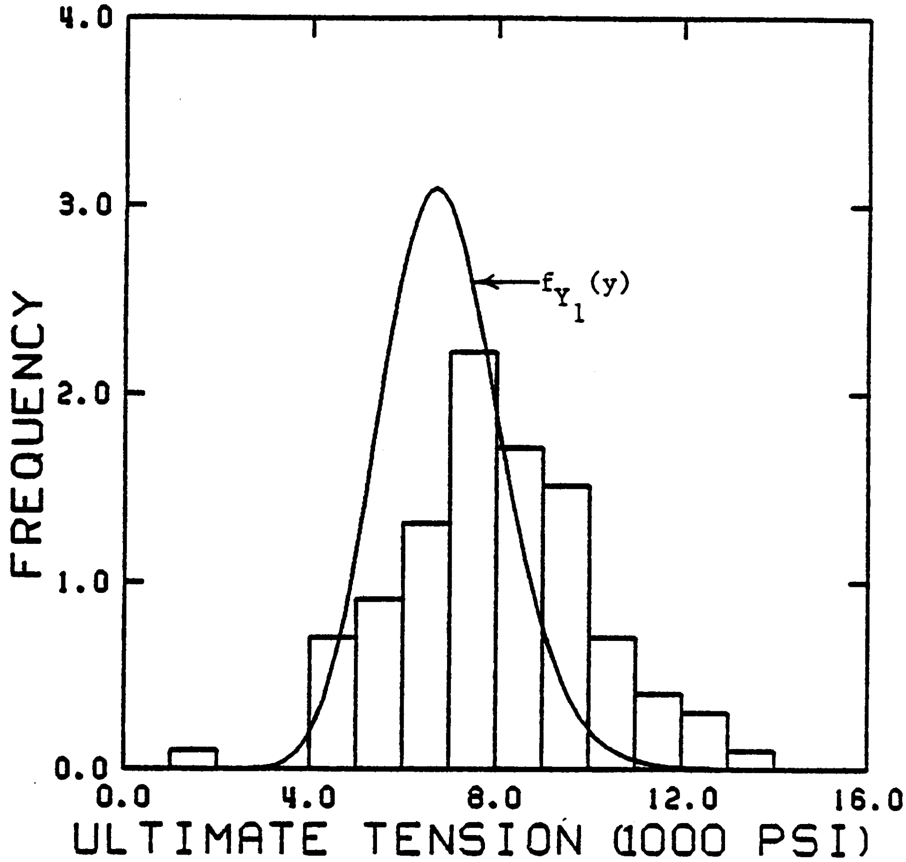

Figure 4.10 The histogram of the ultimate tension of the 2x10 2250f-1.9E 120" treatment group is shown. The probability distribution, f_{Y_1}, with n=4 is superposed onto the histogram.

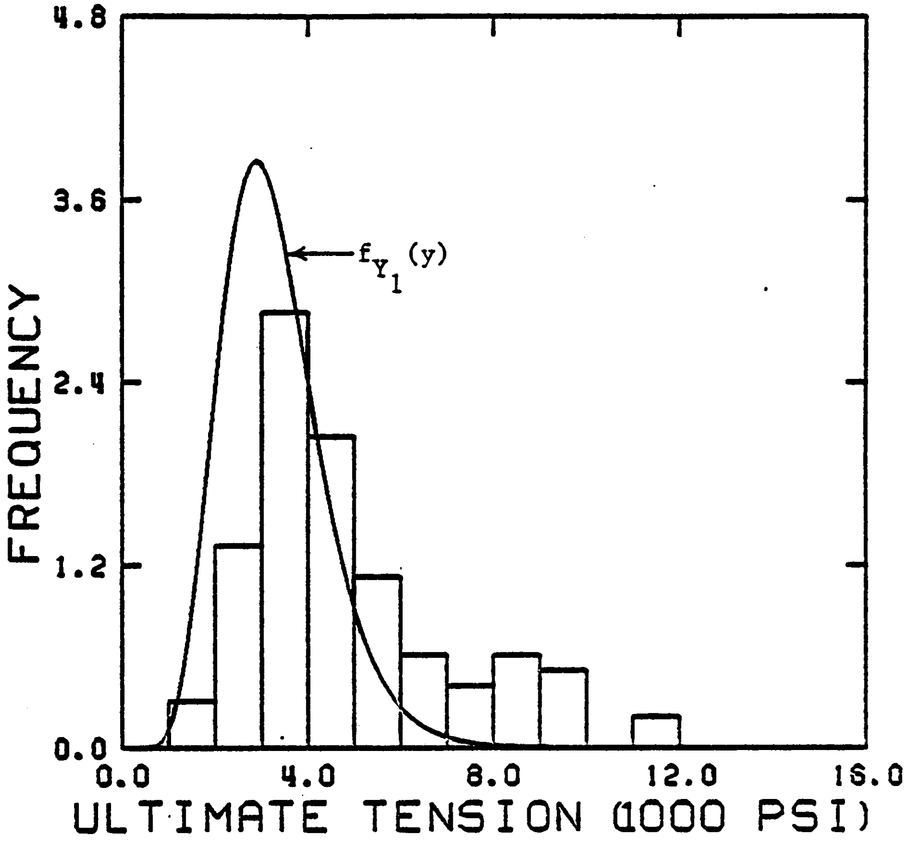

Figure 4.11 The histogram of the ultimate tension of the 2x4 No. 2 KD15 120" treatment group is shown. The probability distribution, f_{Y_1}, with n=4 is superimposed onto the histogram.

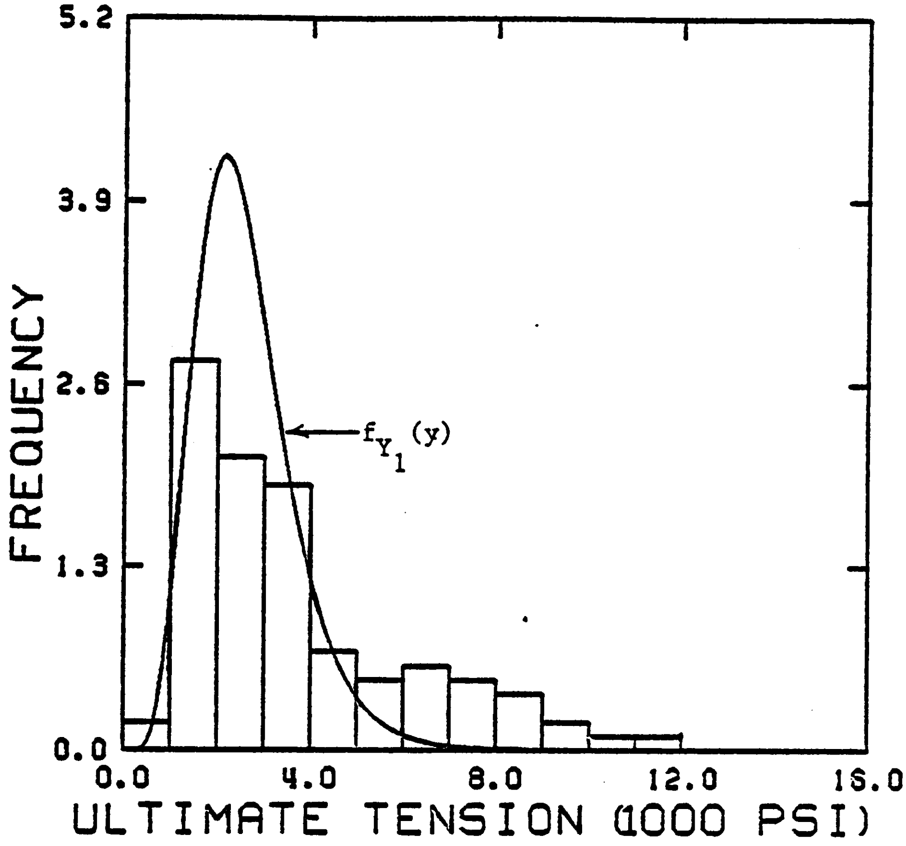

Figure 4.12 The histogram of the ultimate tension of the 2x10 No. 2 KD15 120" treatment group is shown. The probability distribution, f_{Y_1}, with n=4 is superimposed onto the histogram.

4.2 DEVELOPMENT OF A MODEL OF CORRELATED SEGMENTS

Table 3.11 shows that there is a significant correlation between the tensile strength of the first and fourth 30-inch segments in each of the four grade and size groups. Therefore, it is reasonable to assume that lumber exhibits lag-1 serial correlation; ie, that the tensile strength parallel-to-grain is correlated to the tensile strength of the previous segment. A tensile strength - length effect model was developed taking into account the serial correlation of tensile strength between segments. In the discussion that follows, the method used to develop the model will be described. This method involves the following steps.

1.) Determine 30-inch segment MOE's for a lumber specimen using a MOE variability model (Kline et al, 1985).

2.) Input segment MOE's into an appropriate weighted least squares regression model to obtain tensile strength parallel-to-grain for each of the 30-inch segments of the lumber specimen.

3.) Using a weakest-link theory, the minimum segment tensile strength is defined as the tensile strength of the lumber specimen.

4.2.1 GENERATION OF SEGMENT MOE'S

A MOE variability model (Kline et al, 1985) generated serially correlated MOE's along 30-inch segments for a piece of lumber. The type of lumber, number of segments and a random observation from a distributon of average MOE values were inputted into the model to obtain the segment MOE's.

In order to eliminate the generation of unrealistic segment MOE values, the MOE variability model (Kline et al, 1985) was modified. A minimum segment MOE, EMIN, and a maximum segment MOE, EMAX, were inputted into the model. If a generated segment MOE was not in the specified range as indicated above, another series of segment MOE's was generated in its place. The minimum and maximum segment MOE's selected for the modified MOE variability model were the minimum and maximum segment MOE's from the four data sets of 2X4 and 2X10 2250f-1.9E and NO.2 KD15 Southern Pine used in the development of the MOE variability model (Kline et al, 1985). Table 4.1 lists the minimum and maximum segment MOE values used in the modified MOE variability models. Appendix A contains a program listing of the modified model.

The distributions of MOE to be inputted into the four MOE variability models were determined by using MOE data from the 30-inch treatment groups. A lognormal distribution was fitted to the 30-inch MOE values for both groups of MSR lumber. Figures 4.13 and 4.14 show the histograms and the fitted lognormal density curves. A 3-parameter Weibull distribution was fitted to the VG 30-inch lumber groups. Figures 4.15 and 4.16 show the histograms and the fitted lognormal density curves. The fitted density curves and

TABLE 4.1. The minimum and maximum 30-inch segment MOE values for each grade and size group used in the development of the MOE variability model (Kline et al, 1985).

SIZE	GRADE	EMIN ($\times 10^6$ psi)	EMAX ($\times 10^6$ psi)
2x4	2250f-1.9E	1.579	3.623
2x4	Grade-stamped No. 2	0.461	3.245
2x10	2250f-1.9E	1.679	3.630
2x10	Grade-stamped No. 2	0.528	3.236

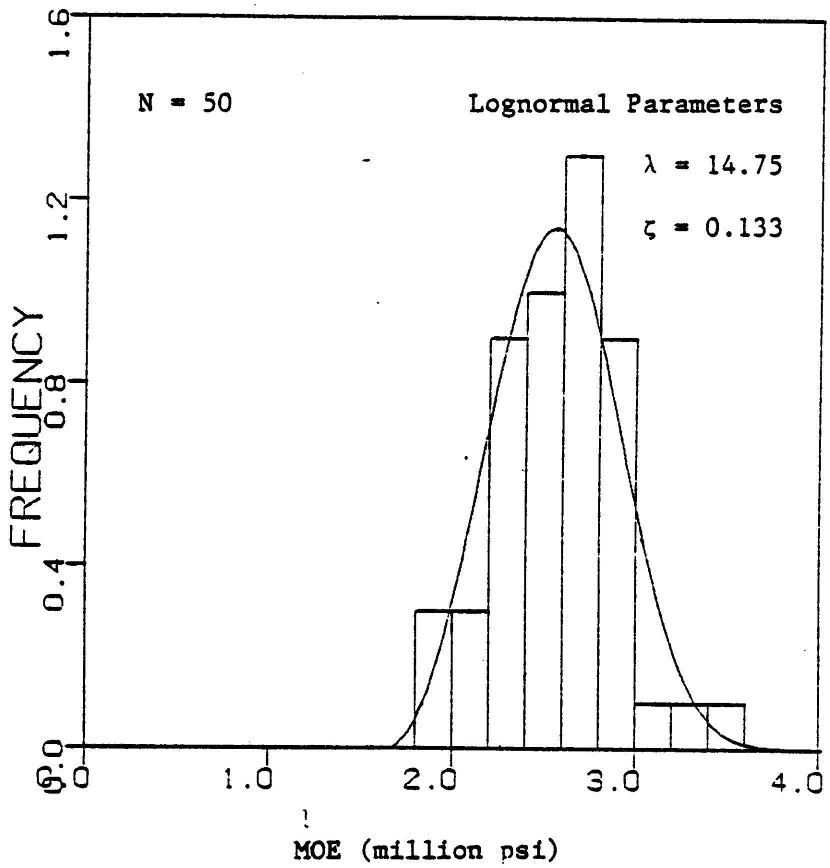

Figure 4.13 30-inch segment MOE values measured from the 2x4 2250f-1.9E MSR 30" treatment group were used to form the histogram. A lognormal distribution fit is superimposed onto the histogram.

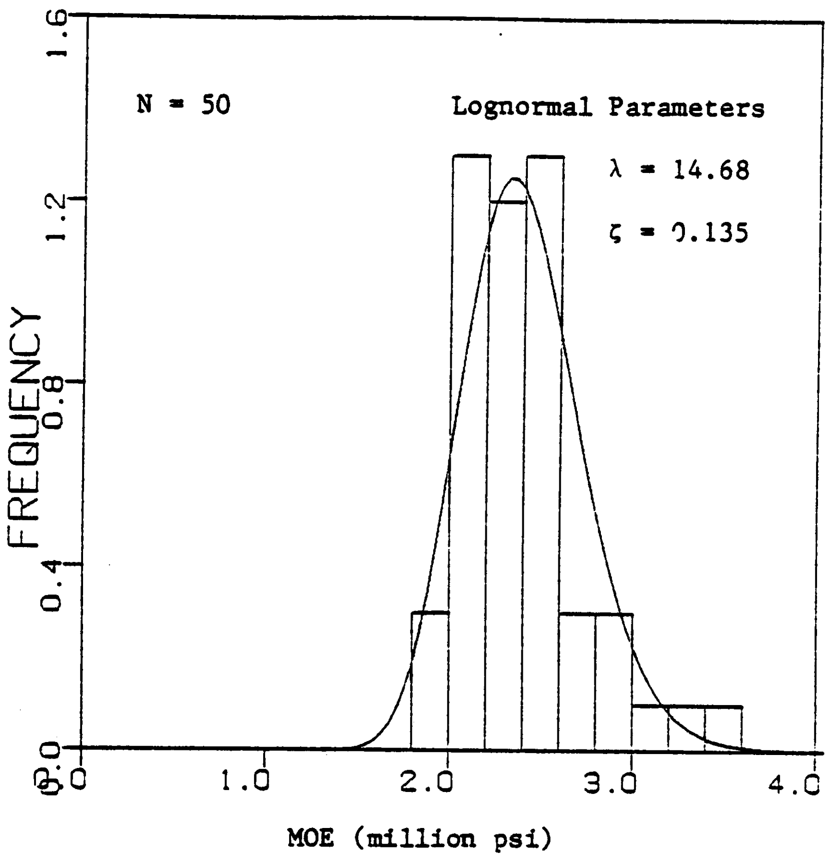

Figure 4.14 30-inch segment MOE values measured from the 2x10 2250f-1.9E MSR 30" treatment group were used to form the histogram. A lognormal distribution fit is superimposed onto the histogram.

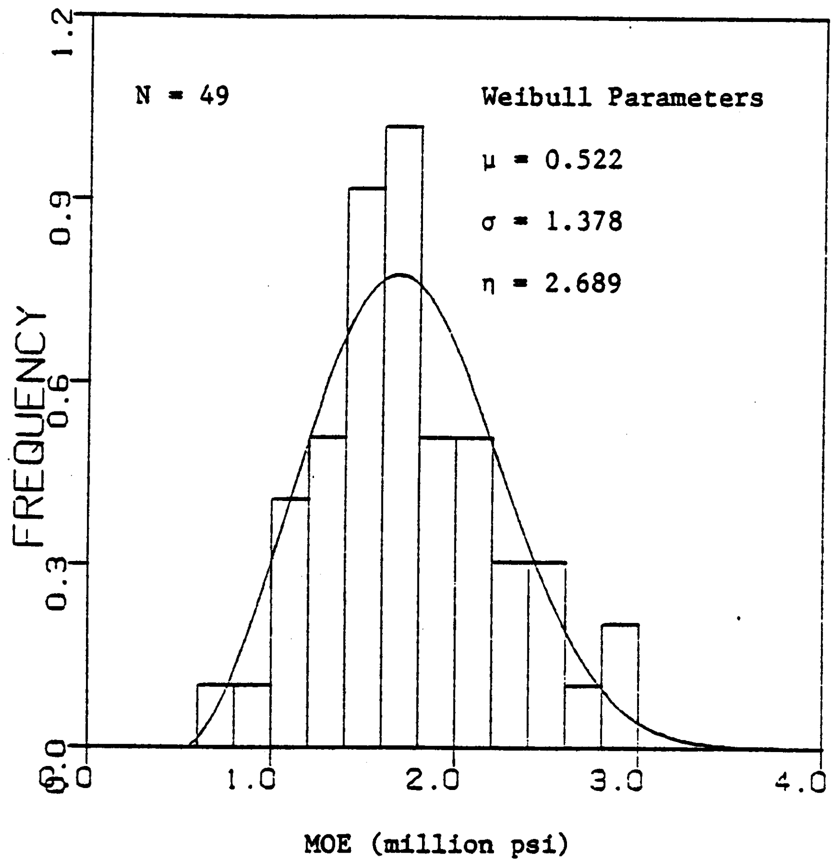

Figure 4.15 30-inch segment MOE values measured from the 2x4 No. 2 KD15 30" treatment group were used to form the histogram. A 3-parameter Weibull distribution fit is superimposed onto the histogram.

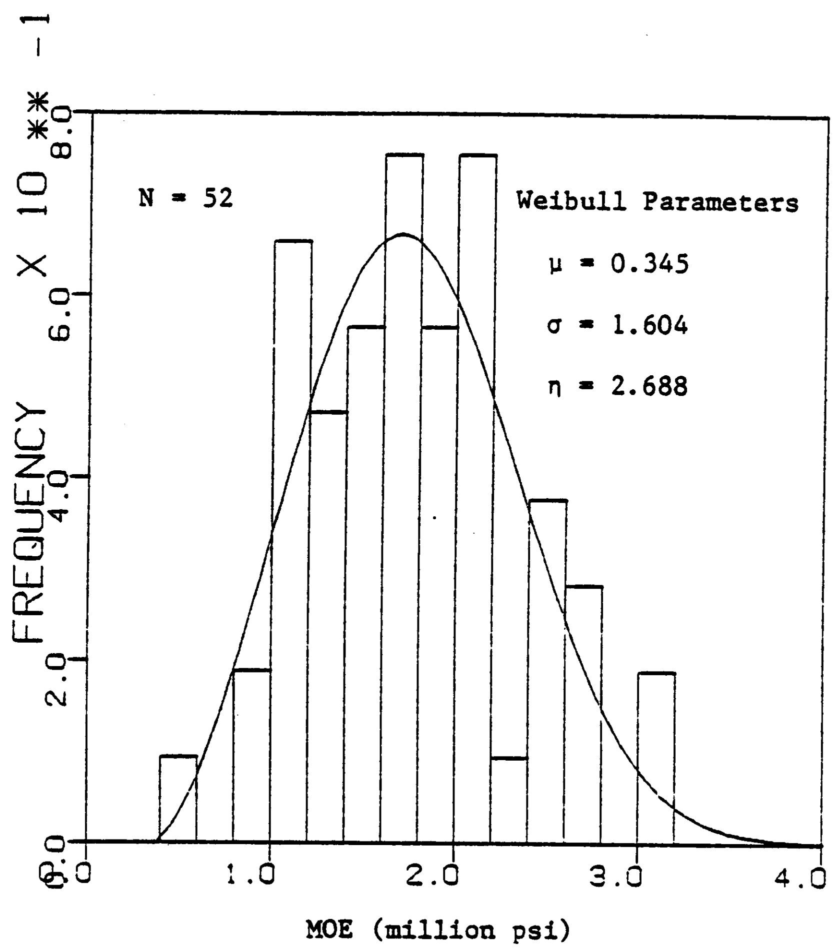

Figure 4.16 30-inch segment MOE values measured from the 2x10 No. 2 KD15 30" treatment group were used to form the histogram. A 3-parameter Weibull distribution fit is superimposed onto the histogram.

histograms were visually inspected for conformance of the applied fit to the data. Visual inspection of the distributions indicated a good fit.

Limits were assigned to prevent the generation of unrealistic piece-average MOE values. A minimum average MOE, ECMIN, and a maximum average MOE, ECMAX, were inputted into the tensile strength - length effect model. If a MOE value was generated which exceeded these limits a new value was generated. This procedure results in selecting random observations of average MOE values from a truncated distribution. The truncation points selected for the models were,

$$ECMIN = EMIN + 0.3x10^6 \text{ psi} \tag{4.3}$$

and

$$ECMAX = EMAX - 0.3x10^6 \text{ psi} \tag{4.4}$$

where EMIN and EMAX are listed in Table 4.1 for each grade and size group.

4.2.2 WEIGHTED LEAST SQUARES REGRESSION

A weighted least squares regression model developed by (Woeste et al, 1979) was used to generate tensile strength values parallel-to-grain for the 30-inch segments of a lumber specimen when the MOE of the segments was given. Equation 2.21 was used with Y, the dependent variable equal to tensile strength parallel-to-grain. The estimated parameters b_0, b_1

and K were calculated for each of the grade and size groups using the MOE and tensile strength data from the 30-inch treatment groups. Figures 4.17 through 4.20 show scattergrams of the actual MOE-tensile strength pairs with overlays of the regression lines and curves which bound 99 percent of the residuals under the assumptions of the model. The data points should lie symmetrically about the regression line if the correct model is used. However, each of the four groups exhibits a lack of fit near the bottom 99 percent boundaries. Also, the MSR 2X4, VSR 2X4 and VSR 2X10 groups exhibit a lack of fit since data points exceed the upper boundary.

Woeste et al (1979) found that when data exhibit this type of lack of fit, it is likely that a logarithmic transformation on the dependent variable, tensile strength in this case, will greatly improve the relationship. Hence, the new regression model is given by Equation 2.25.

Figures 4.21 through 4.24 show plots of the transformed regression lines along with the 99 percent boundaries overlayed on scattergrams of the actual data. These models displayed no obvious lack of fit and, therefore, were used to simulate subsequent tensile strength values. These models are also practical since they are unlikely to simulate values of near zero strength, a characteristic that is assumed to be rare.

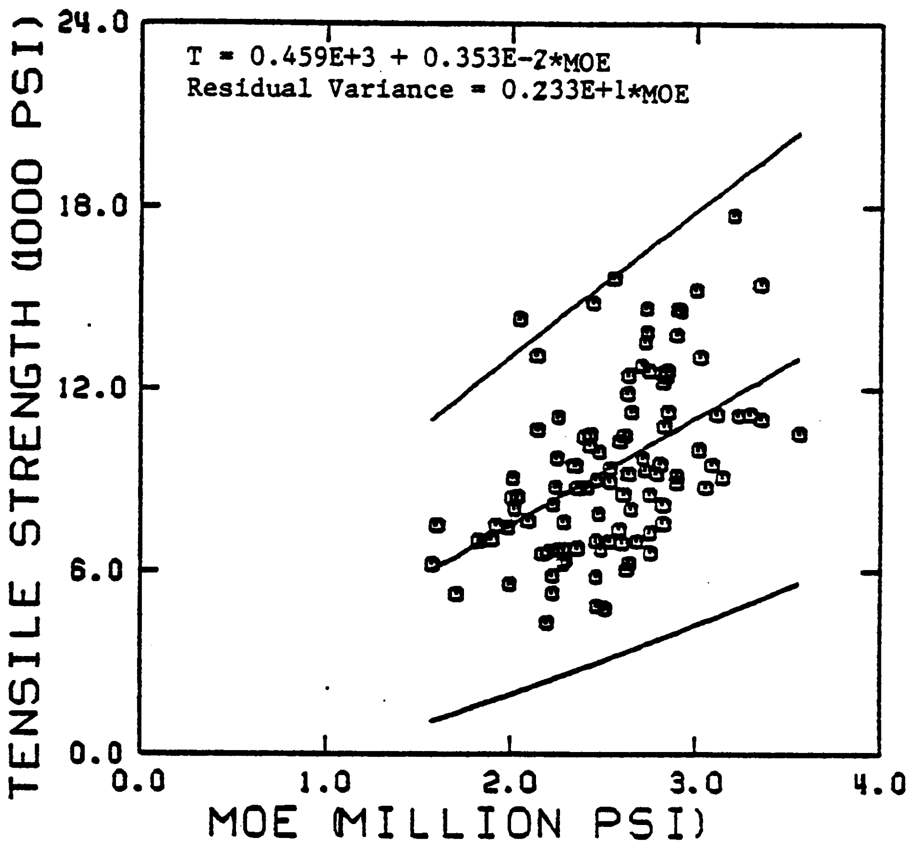

Figure 4.17 Results of weighted least squares regression analysis on the 2x4 2250f-1.9E 30" treatment group of the tensile strength - MOE relationship. The results are shown superimposed on the scattergram.

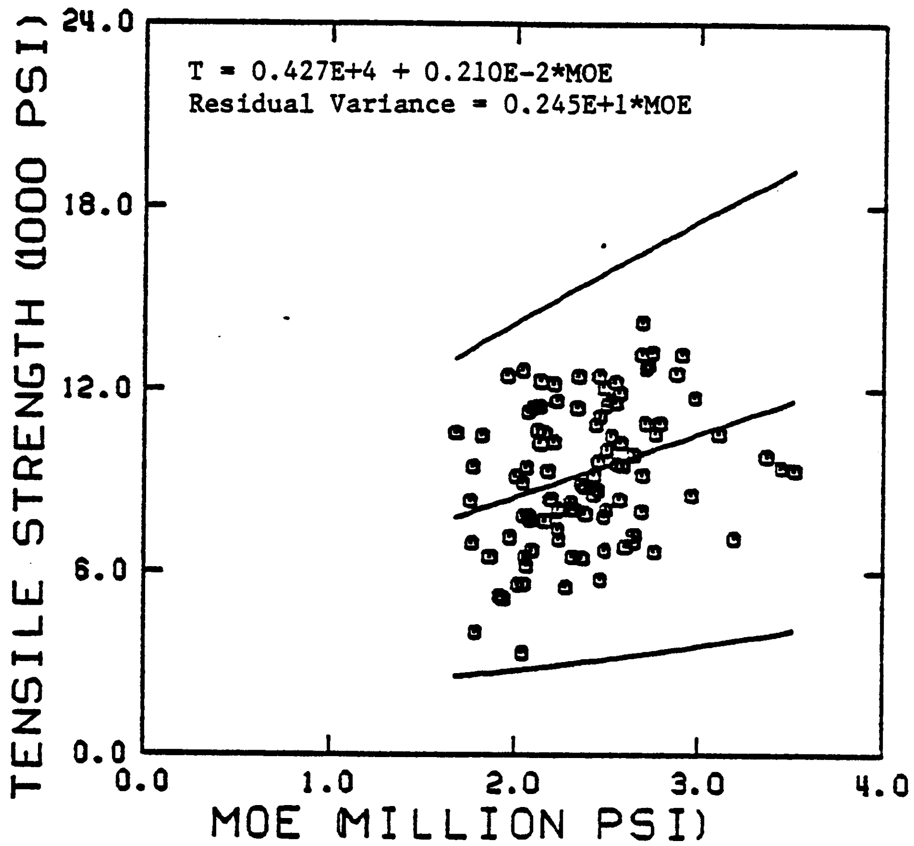

Figure 4.18 Results of weighted least squares regression analysis on the 2x10 2250f-1.9E 30" treatment group of the tensile strength - MOE relationship. The results are shown superimposed on the scattergram.

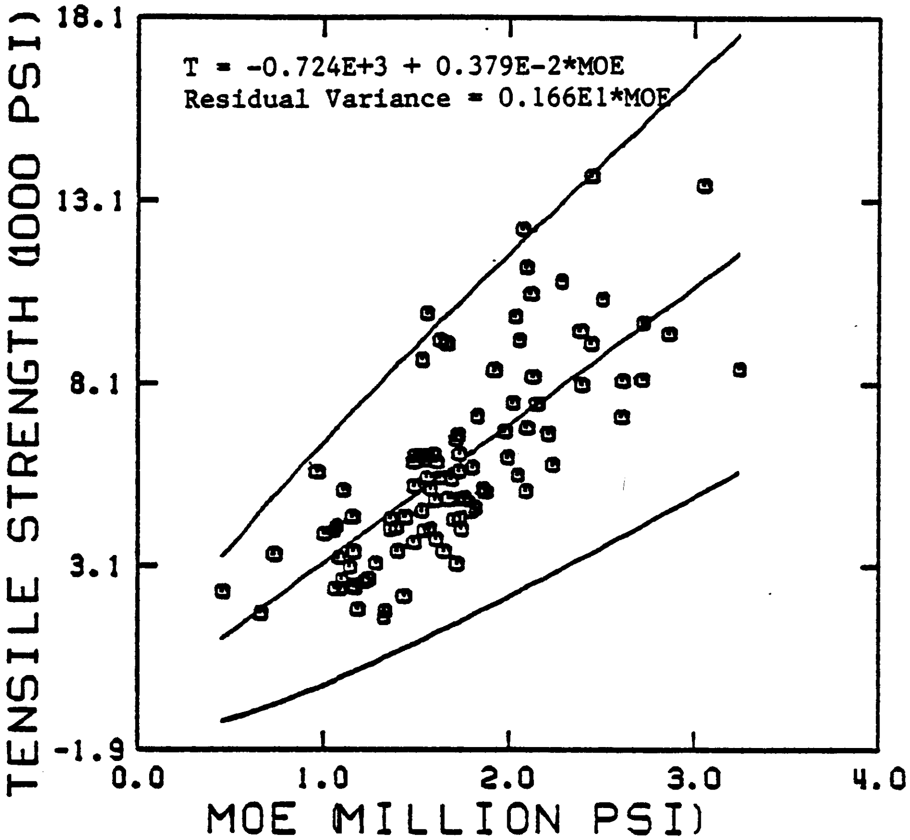

Figure 4.19 Results of weighted least squares regression analysis on the 2x4 No. 2 30" treatment group of the tensile strength - MOE relationship. The results are shown superimposed on the scattergram.

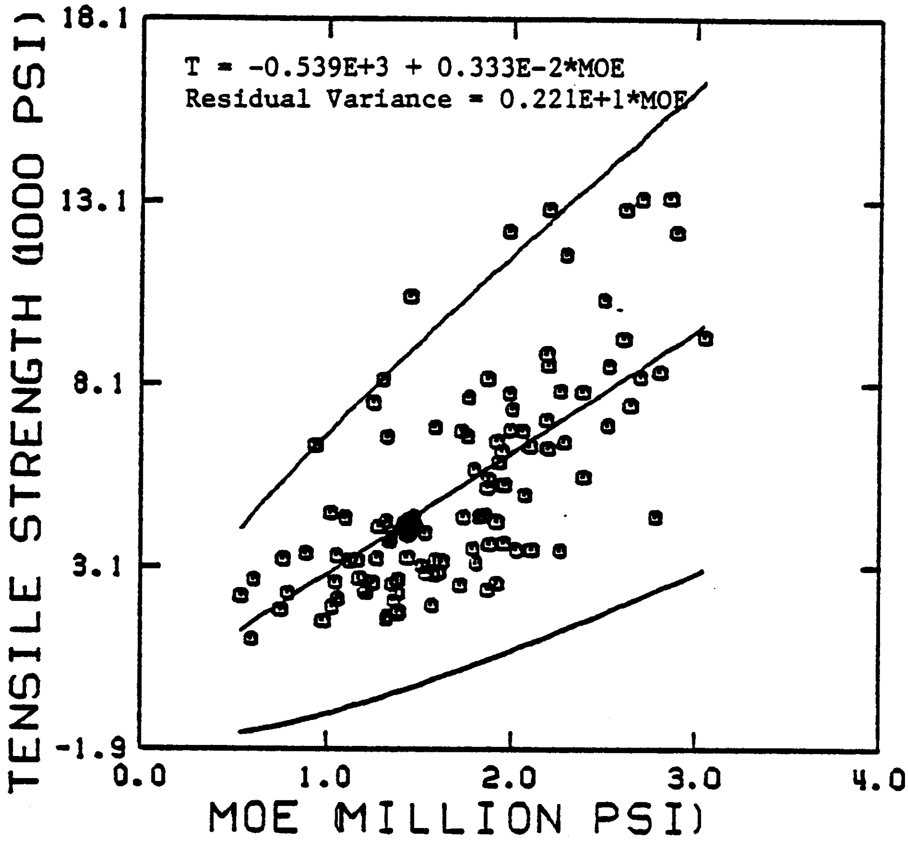

Figure 4.20 Results of weighted least squares regression analysis on the 2x10 No. 2 30" treatment group of the tensile strength - MOE relationship. The results are shown superimposed on the scattergram.

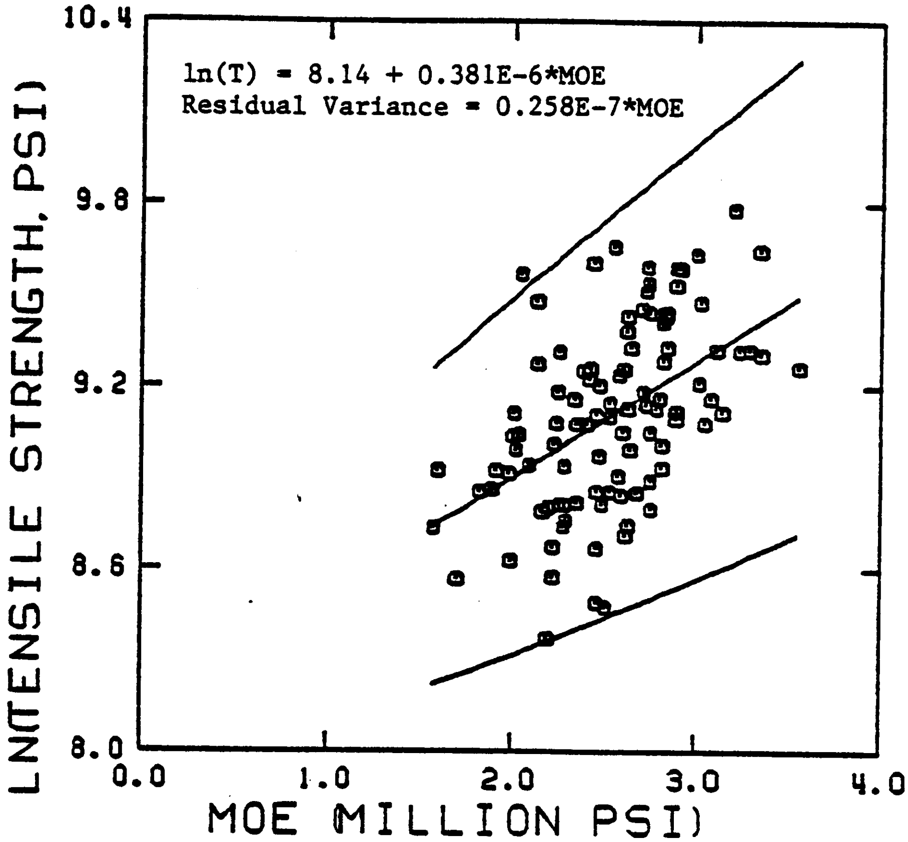

Figure 4.21 A weighted least squares regression model was used on the transformed data of the 2x4 2250f-1.9E 30" treatment group and is graphically illustrated with the scattergram.

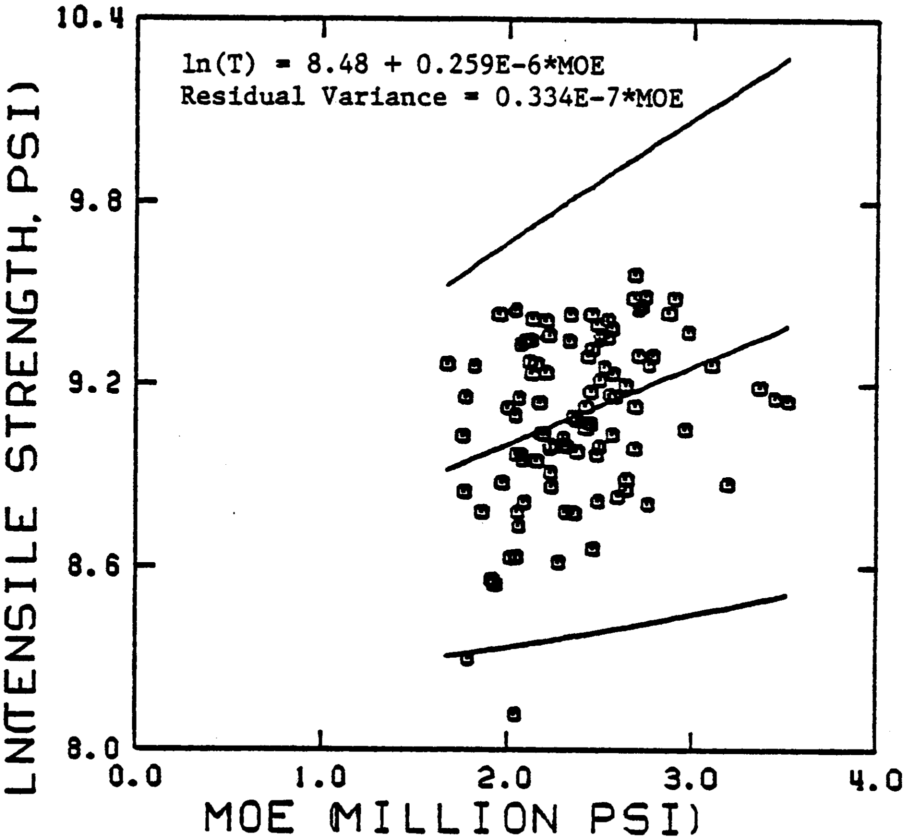

Figure 4.22 A weighted least squares regression model was used on the transformed data of the 2x10 2250f-1.9E 30" treatment group and is graphically illustrated with the scattergram.

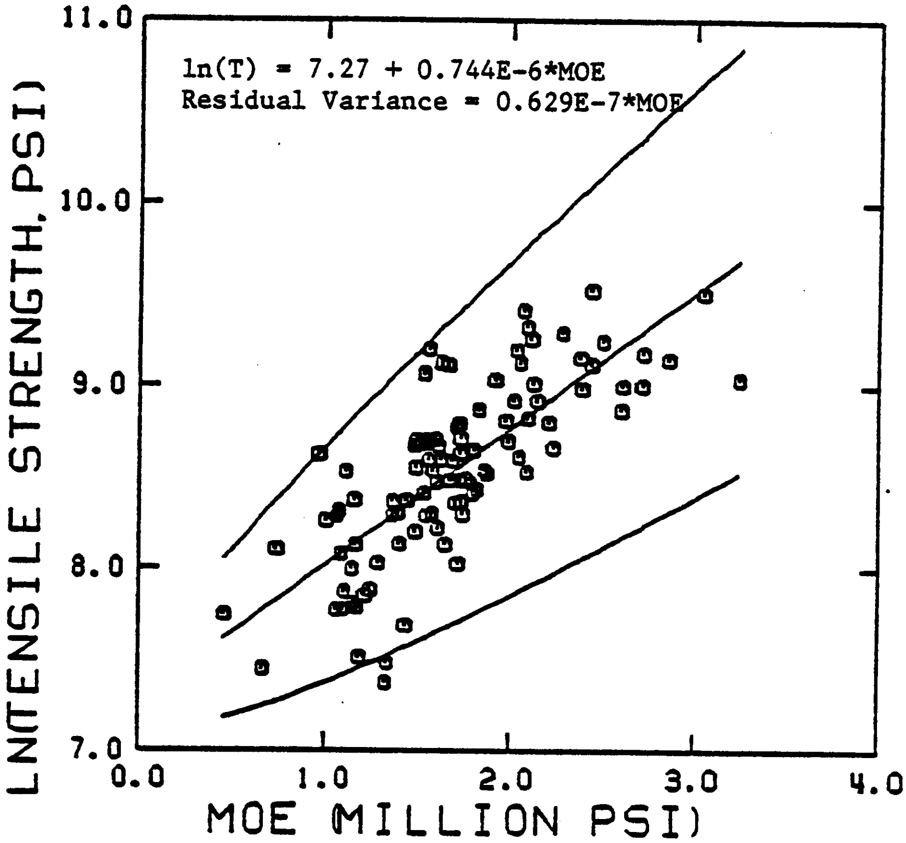

Figure 4.23 A weighted least squares regression model was used on the transformed data of the 2x4 No. 2 30" treatment group and is graphically illustrated with the scattergram.

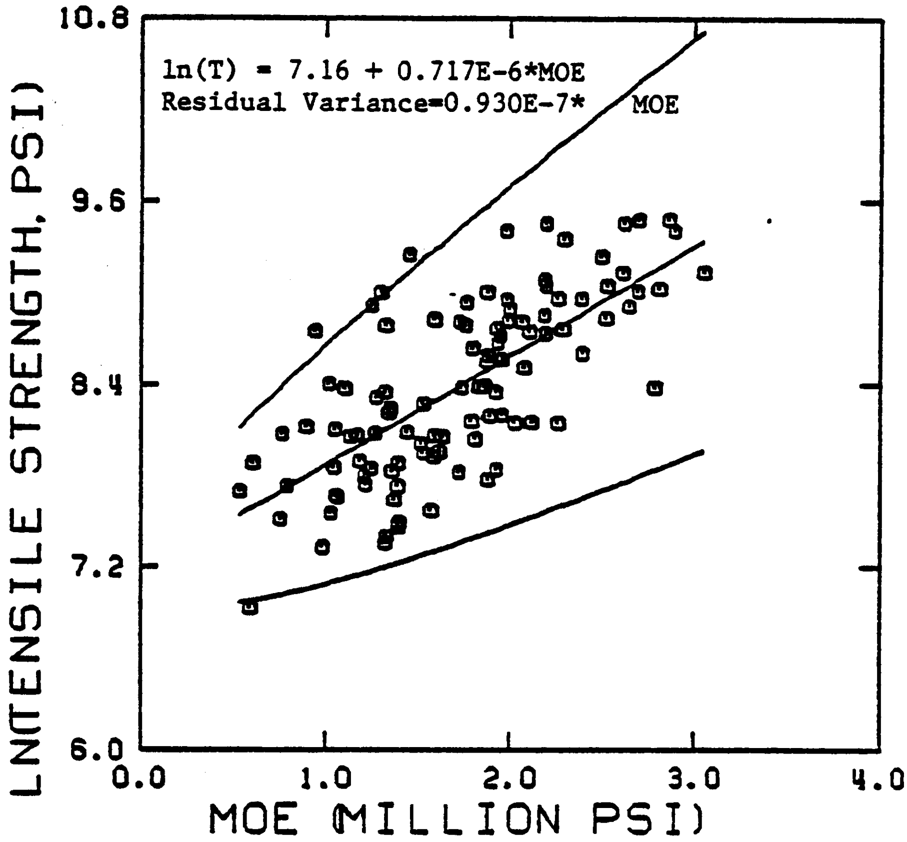

Figure 4.24 A weighted least squares regression model was used on the transformed data of the 2x10 No. 2 30" treatment group and is graphically illustrated with the scattergram.

4.2.3 SPECIMEN TENSILE STRENGTH

The final step was to generate values of tensile strength for each of the 30-inch segments of a lumber specimen. Using a weakest-link theory analagous to Weibull's "weakest-link theory" (1939), the minimum 30-inch segment tensile strength is the tensile strength of the lumber specimen.

It was felt that a maximum tensile strength value should be inputted into the model to prevent the generation of unrealistic tensile strength values of the lumber specimen. For this study, the maximum tensile strength of the 30-inch segments of the four data sets of 2X4 and 2X10 NO.2 KD15 and 2250f-1.9E MSR Southern Pine, 17737.0 psi, was chosen as the best estimate of the maximum tensile strength. If a specimen tensile strength value greater than 17737.0 psi was generated, the generation procedure would generate a new set of segment tensile strength values. Thus, a new tensile strength value of the lumber specimen was determined. As stated before, the maximum tensile strength value for this study was selected from the available test data. However, users of the model can input their own maximum tensile strength value.

4.3 PRELIMINARY VALIDATION RESULTS

Since the tensile strength - length effect models were developed using the data from the 30-inch treatment groups, the models were independently verified against the test data from the 90-inch and 120-inch treatment groups. Both models described in sections 4.1 and 4.2 failed to predict the actual tensile behavior of the 90-inch and 120-inch treatment groups. However,the correlated segment model of section 4.2 provided a better fit than the independent segments model, which suggested that perhaps the correlated segment model could be refined to successfully model the tensile strength behavior in a lumber specimen.

4.3.1 INDEPENDENT SEGMENTS MODEL

A visual appraisal of Figures 4.5 through 4.12 suggests that the tensile strength - length effect model which assumes non-correlated segments underpredicts the tensile strength of a lumber specimen. In addition to the visual test, a Kolmogorov-Smirnov goodness of fit test was conducted for each of the eight models. All of the models were rejected at the 5 percent significance level. This result is not counter-intuitive since Table 3.11 shows that there is a significant correlation between segments 1 and 4 in the

30-inch treatment groups, which indicates that there is a significant correlation between the 30-inch segments.

4.3.2 CORRELATED SEGMENTS MODEL

Tensile strength - length effect models were developed for 2X4 visually graded and MSR 90-inch and 120-inch pieces of lumber using the procedure which assumes the segments are correlated. Two-thousand average MOE values were generated and inputted into the modified MOE variability model for each of the four groups. The segment MOE values were then used in the weighted least squares regression to obtain the tensile strength parallel-to-grain for each 30-inch segment. The tensile strength values of the generated lumber specimens were then determined using a weakest-link theory.

Next, the PDF's for each of the tensile strength models were determined in order to compare them to the actual test data. A visual inspection indicated that the lognormal distribution best fit the generated tensile strength values of each model. Figures 4.25 through 4.28 show relative frequency histograms of the tensile strength of the 90-inch and 120-inch treatment groups of both the 2X4 visually graded and MSR lumber. The appropriate correlated segment models are superimposed onto the histograms. Also, visual inspection indicated that the models do not adequately describe the data. The four models were rejected using a Kolmogorov-

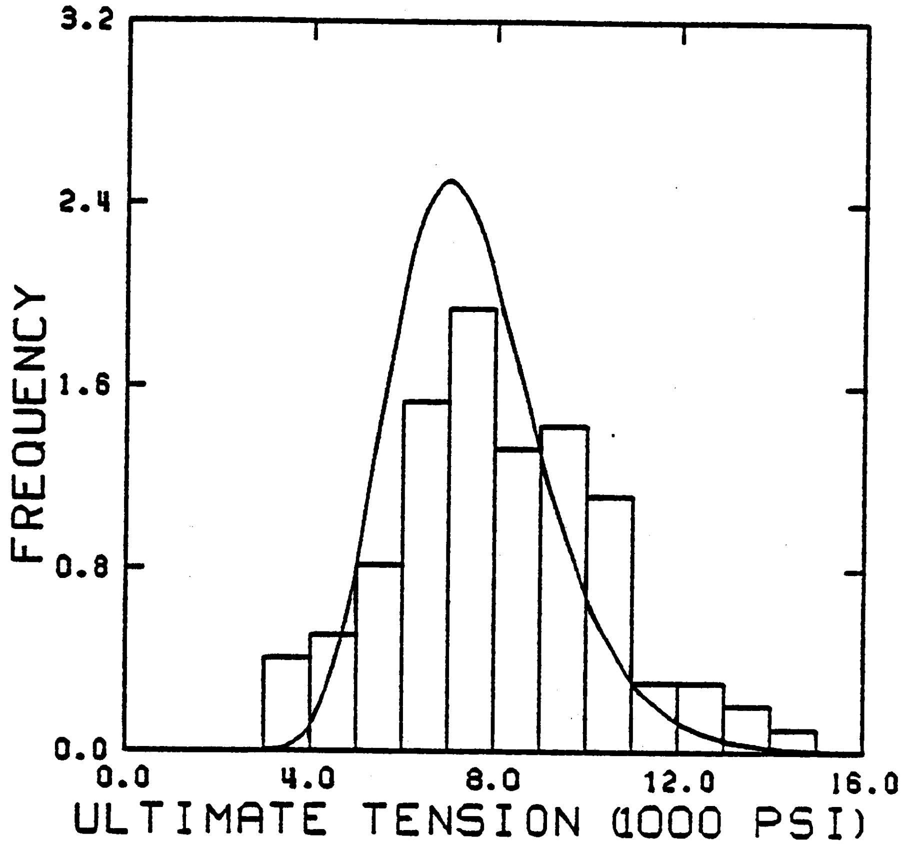

Figure 4.25 The histogram of the ultimate tension of the 2x4 2250f-1.9E 90" treatment group is shown with the correlated segment model probability curve superimposed onto the histogram.

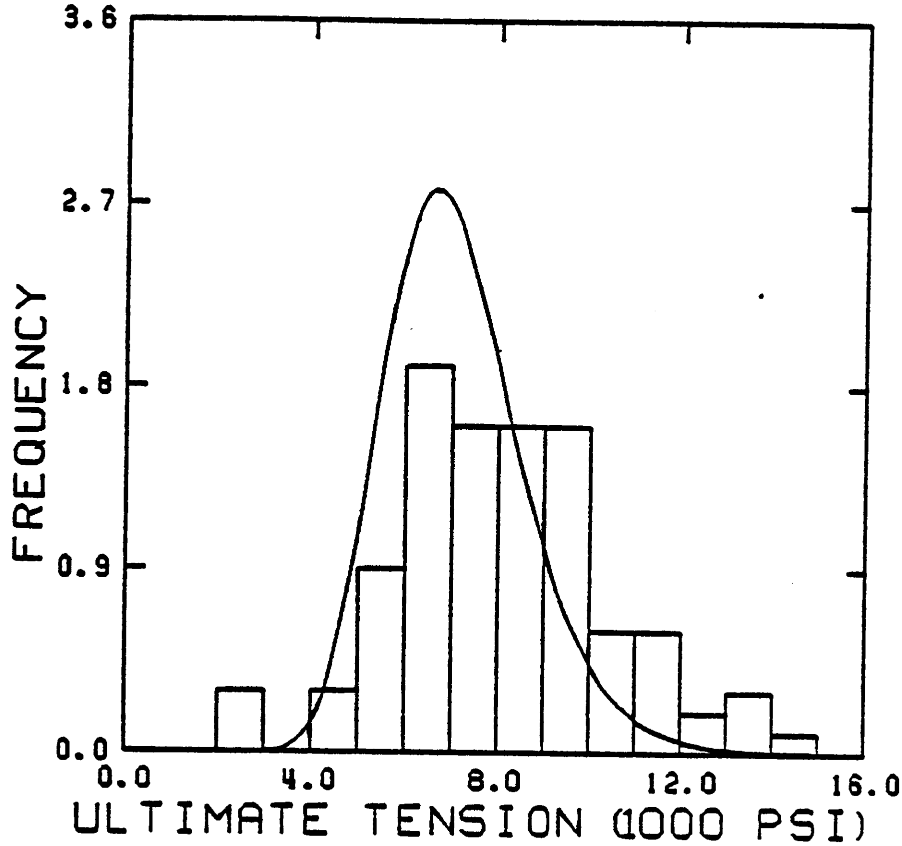

Figure 4.26 The histogram of the ultimate tension of the 2x4 2250f-1.9E 120" treatment group is shown with the correlated segment model probability curve superimposed onto the histogram.

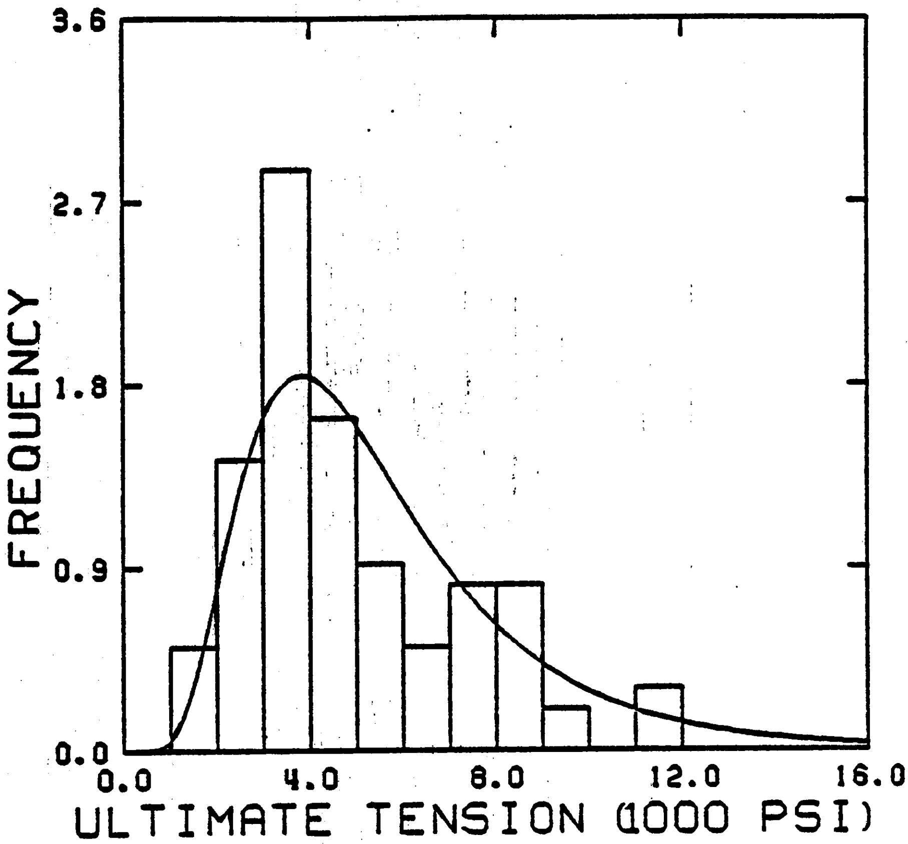

Figure 4.27 The histogram of the ultimate tension of the 2x4 No. 2 90" treatment group is shown with the correlated segment model probability curve superimposed onto the histogram.

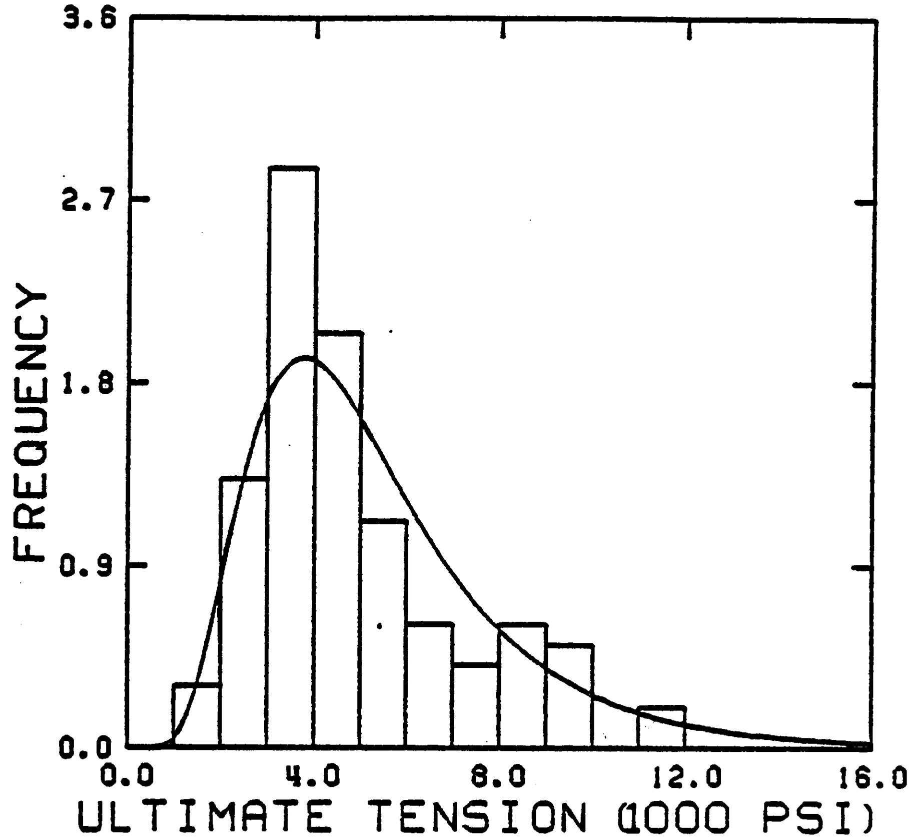

Figure 4.28 The histogram of the ultimate tension of the 2x4 No. 2 120" treatment group is shown with the correlated segment model probability curve superimposed onto the histogram.

Smirnov goodness-of-fit test at the 5 percent significance level. However, the correlated segments models appeared to better predict the behavior of the tensile strength data than did the independent segments models. These findings suggested that perhaps the correlated segments model could be altered to successfully predict the behavior of the tensile strength parallel-to-grain in a lumber specimen.

CHAPTER V. MODEL REFINEMENTS

Initial validation results indicated that both the independent segment model and the correlated segment model inadequately predicted tensile strength parallel-to-grain based on test data from the 90-inch and 120-inch treatment groups. Hence, model refinements were incorporated into the tensile strength-length effect model which assumed correlation between segments. Tensile strength predictions from the refined model were then checked against the available test data. The predicted and actual tensile strength values were in good agreement.

5.1 MODELING OF THE RESIDUAL

When generating the tensile strength values for each segment in a piece of lumber in the previous model of section 4.2, the only correlation between segments resulted from the correlation of MOE. However, it is possible that the tensile strength residuals of neighboring segments are also correlated. This tensile strength residual correlation is illustrated in Figure 5.1. Figure 5.1(a) shows a simulated scattergram of the ln(tensile strength) vs. MOE with the regression equation superimposed. It can be expected that if the residual of segment 1 is positive and large, as shown

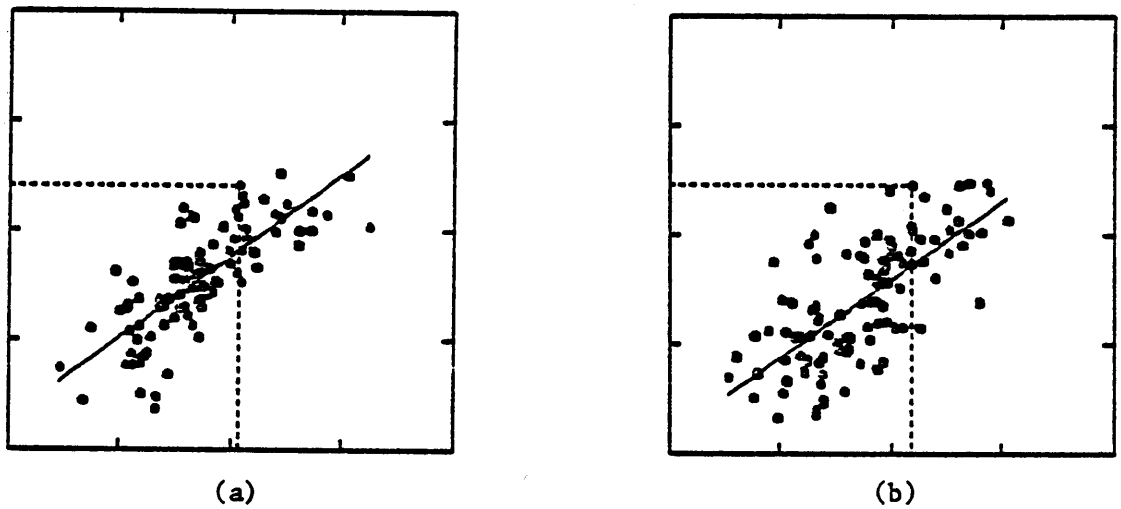

Figure 5.1 A simulated scattergram of the tensile strength - MOE relationship. The regression line is included.
(a) MOE and tensile strength of segment 1.
(b) MOE and tensile strength of segment 2.

in Figure 5.1(a), then the residual of segment 2 is likely to also be positive (Figure 5.1(b)). The basis for this reasoning is primarily due to the extensive research on the correlations of bending and tension, and bending and compressive strength in the same piece of lumber (Evans et al, 1984; Green et al, 1984).

One way of modeling this residual correlation is by assuming the residuals in the log space follow a first-order Markov normal process. A first-order Markov model generates a series of values from a normal distribution while preserving the first-order, or lag-1, serial correlation. The first-order Markov model also generates serial correlations of any lag k by the theoretical model

$$\rho_k = \rho_1{}^k \tag{5.1}$$

where ρ_1 is estimated by r_1, the estimated lag-1 serial correlation (Haan, 1977).

Even though it is physically impossible to determine the first-order serial correlation, the lag-3 serial correlation can be estimated from the data from the 30-inch treatment groups. Then by taking the inverse of Equation 5.1, the lag-1 serial correlation can be estimated to be inputted into the Markov model.

As mentioned previously, the MOE and tensile strength data were collected from the first and fourth 30-inch segments of a piece of lumber. Therefore, the lag-3 serial correlation can be calculated for each grade and size group by estimating the residuals using Equation 2.25 in the following form

$$\varepsilon_1 = \ln(Y_1) - b_0 - b_1X_1 \tag{5.2}$$

$$\varepsilon_4 = \ln(Y_4) - b_0 - b_1X_4$$

where $Var(\varepsilon_i) = KX_1$ and the estimated parameters b_0, b_1 and K are given in the model development. Table 5.1 lists the estimated lag-3 serial correlation, r_3, and r_1 calculated from Equation 5.1 for each grade and size group.

Since it is assumed that the residuals are normally distributed with mean equal to zero and residual variance equal to K times MOE, the first-order Markov model in the form of Equation 2.15 was used to account for the variability in the residual variance of the segment. Equation 2.15 is simplified by

$$X_{i+1} = r_1(K^*(MOE)_{i+1})/(K^*(MOE)_i)X_i + t_{i+1}K^*(MOE)_{i+1}(1 - r^2)^{\frac{1}{2}} \tag{5.3}$$

where the mean of X, μ_X, is zero, the variance is K*MOE and the lag-1 serial correlation is estimated by r_1.

5.2 GENERATION OF RESIDUALS

A model was developed which generated residuals in the log space along a piece of lumber. A different residual was

TABLE 5.1. The estimated lag-3 serial correlation and the calculated lag-1 serial correlation for each size and grade group.

SIZE	GRADE	r_3[1]	r_1[2]
2x4	2250f-1.9E	0.444	0.763
2x4	Grade-stamped No. 2	0.831	0.940
2x10	2250f-1.9E	0.402	0.738
2x10	Grade-stamped No. 2	0.589	0.838

[1]estimated from test data
[2]calculated from r_3

generated every 30 inches along the length of a piece of lumber. These residuals are dependent on the generated MOE values along the piece because the variance of the residual equals K times MOE. In summary, two processes are occuring: 1) MOE varies within each piece of lumber, and 2) the tensile strength residual in the log space varies within each piece with the variance equal to K times MOE. The 30-inch segment MOE variability within each piece of lumber was shown to be modeled by a second-order Markov process (Kline et al, 1985). The 30-inch segment tensile strength residual within each piece was modeled by a first-order Markov process.

To start the generation process, X_1 in Equation 5.3 was arbitrarily set equal to zero and 10 values were generated and discarded. This action is required to eliminate bias in the first residual generated. Similarily, 10 MOE values were also generated from the MOE variability model and discarded to provide the residual generation procedure with the appropriate segment MOE values. The 11th residual and MOE values were assigned to the first segment generated, the 12th values were assigned to the second 30-inch segment and so on for each generated specimen. The segment residuals and MOE values were then inputted into the weighted least squares regression model (Equation 2.2) to obtain the tensile strength of each segment.

In summary, the length effect model was refined by using a first-order Markov model to generate serially correlated tensile strength residuals in the log space. In so doing, an additional 10 MOE values were generated by the MOE variability model for input into the first-order Markov model. Tensile strength values were generated for each 30-inch segment of a piece of lumber and a weakest-link theory was used to determine the tensile strength of the piece of lumber. Appendix B contains a program listing of the tensile strength-length effect model.

CHAPTER VI. RESULTS

The refined tensile strength-length effect models were used to generate tensile strength values to be independently verified by the test data from the 90-inch and 120-inch treatment groups. Two thousand piece-average MOE values were generated and inputted into the modified MOE variability model for each of the eight groups cited in Table 6.1. The segment MOE values were then used in the first-order Markov model to generate segment residuals. The segment MOE values and the segment residuals were then inputted into the weighted least squares regression to obtain the tensile strength parallel-to-grain for each 30-inch segment. The tensile strength values of the generated lumber specimens were then determined using a weakest-link theory.

The PDF's for each of the tensile strength models were determined in order to compare them to the actual test data. Figures 6.1 through 6.8 show histograms of the generated tensile strength values from each of the eight models. A visual appraisal of the histograms of Figures 6.1 through 6.8 suggested that the lognormal distribution might provide a good statistical fit. Accordingly, the lognormal distribution was overlayed on the tensile strength histograms shown in Figures 6.1 through 6.8. Visual inspection indicated that

TABLE 6.1. Listing of the eight groups of tensile strength values simulated by the tensile strength-length effect models.

SIZE	GRADE	LENGTH
2x4	2250f-1.9E	120"
2x4	2250f-1.9E	90"
2x4	Grade-stamped No. 2	120"
2x4	Grade-stamped No. 2	90"
2x10	2250f-1.9E	120"
2x10	2250f-1.9E	90"
2x10	Grade-stamped No. 2	120"
2x10	Grade-stamped No. 2	90"

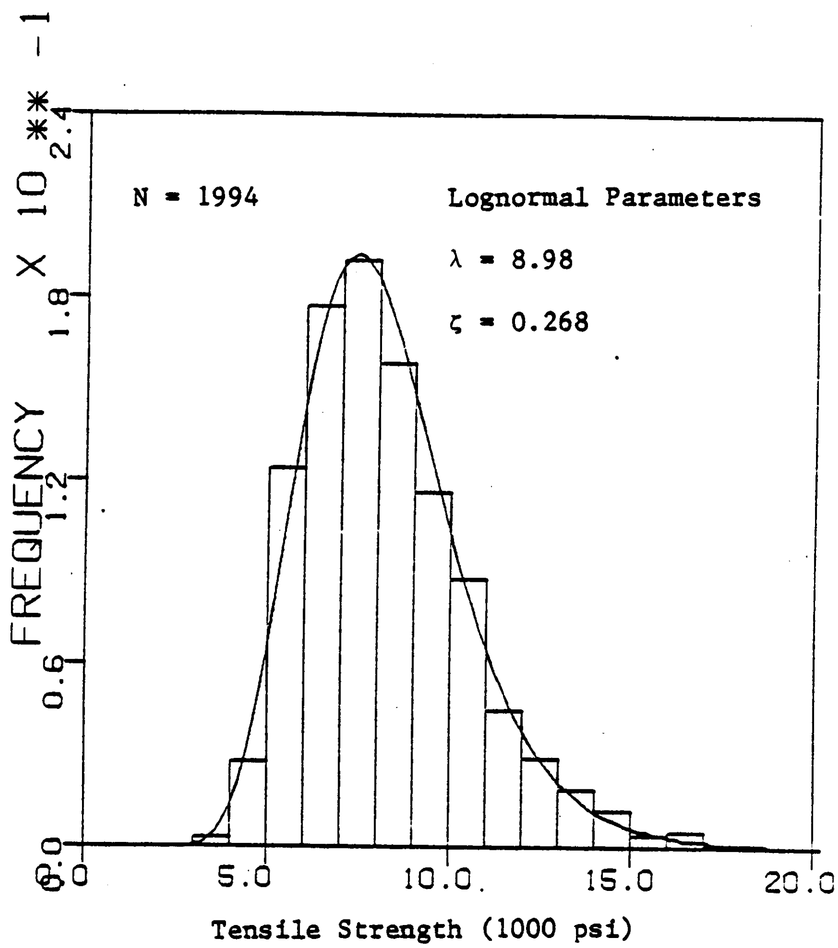

Figure 6.1 A lognormal distribution is superimposed onto the histogram of the generated tensile strength values from the length effect model of the 90" 2x4 2250f-1.9E MSR Southern Pine.

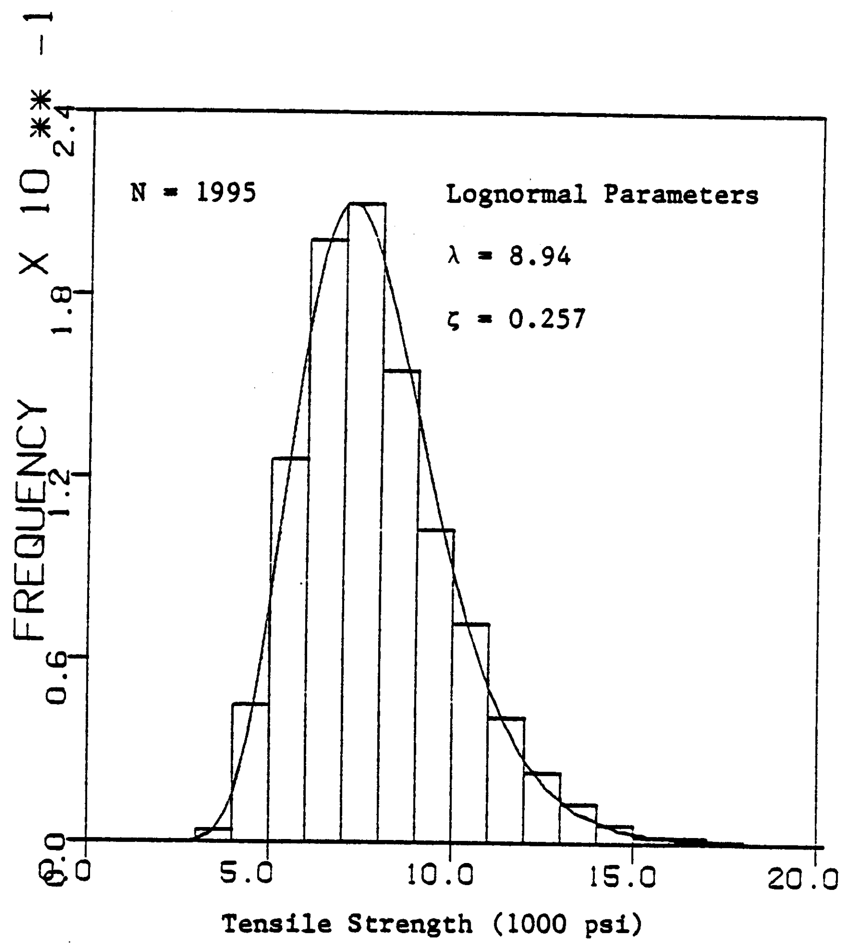

Figure 6.2 A lognormal distribution is superimposed onto the histogram of the generated tensile strength values from the length effect model of the 120" 2x4 2250f-1.9E MSR Southern Pine.

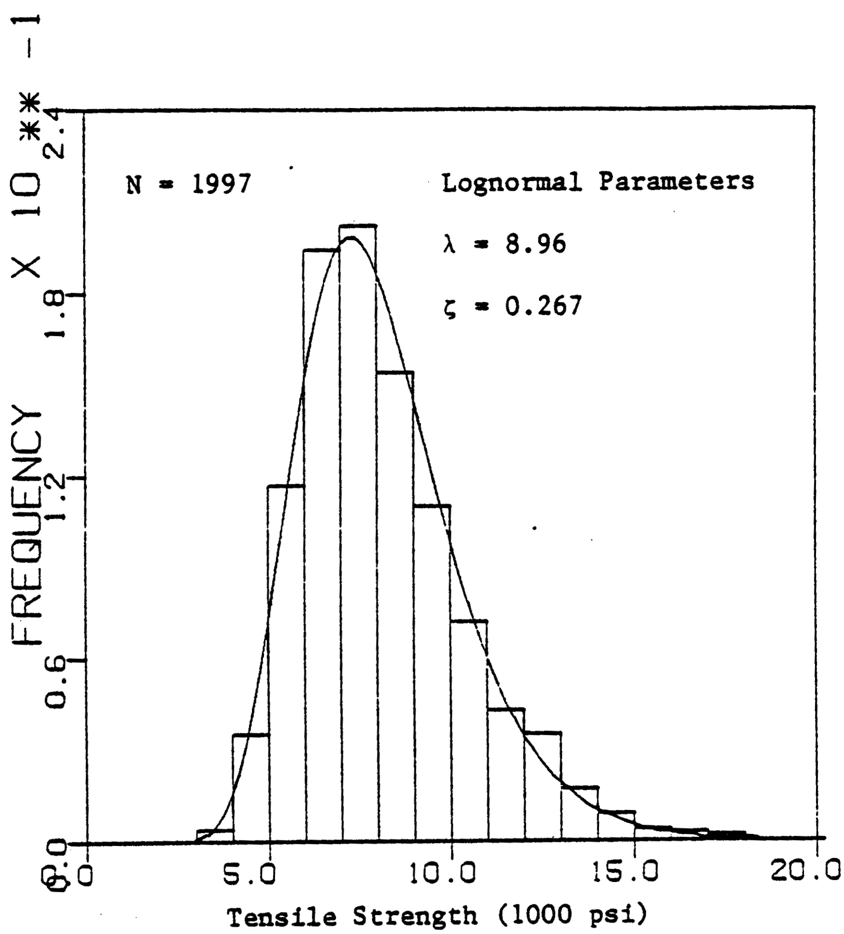

Figure 6.3 A lognormal distribution is superimposed onto the histogram of the generated tensile strength values from the length effect model of the 90" 2x10 2250f-1.9E MSR Southern Pine.

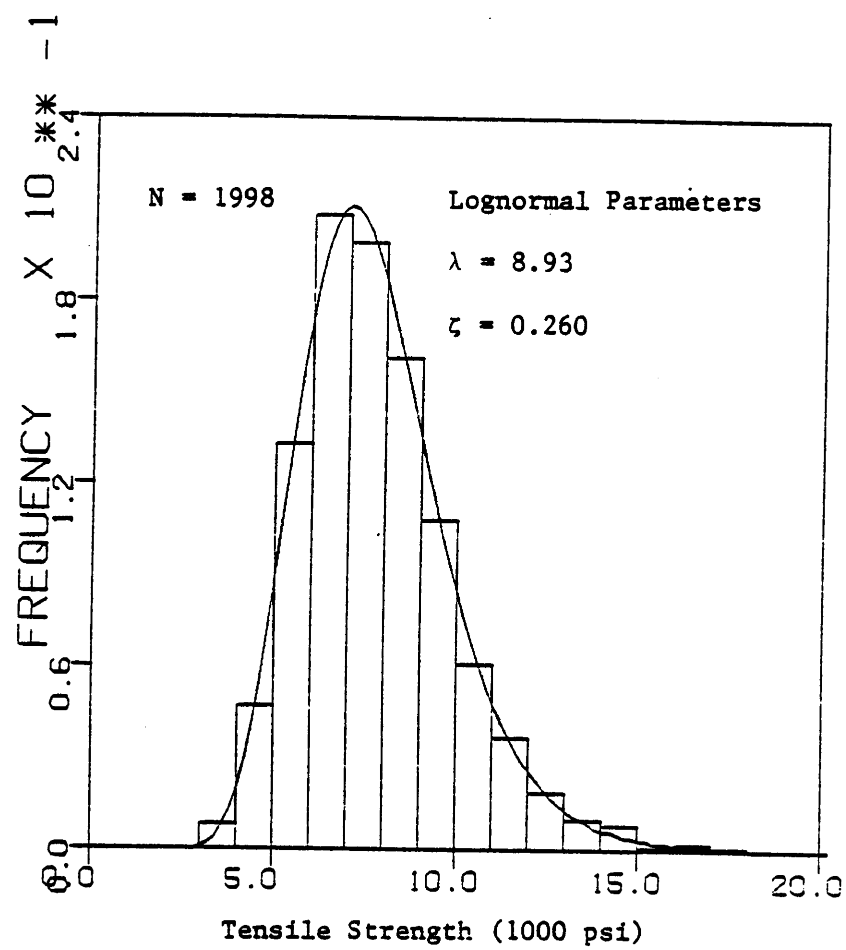

Figure 6.4 A lognormal distribution is superimposed onto the histogram of the generated tensile strength values from the length effect model of the 120" 2x10 2250f-1.9E MSR Southern Pine.

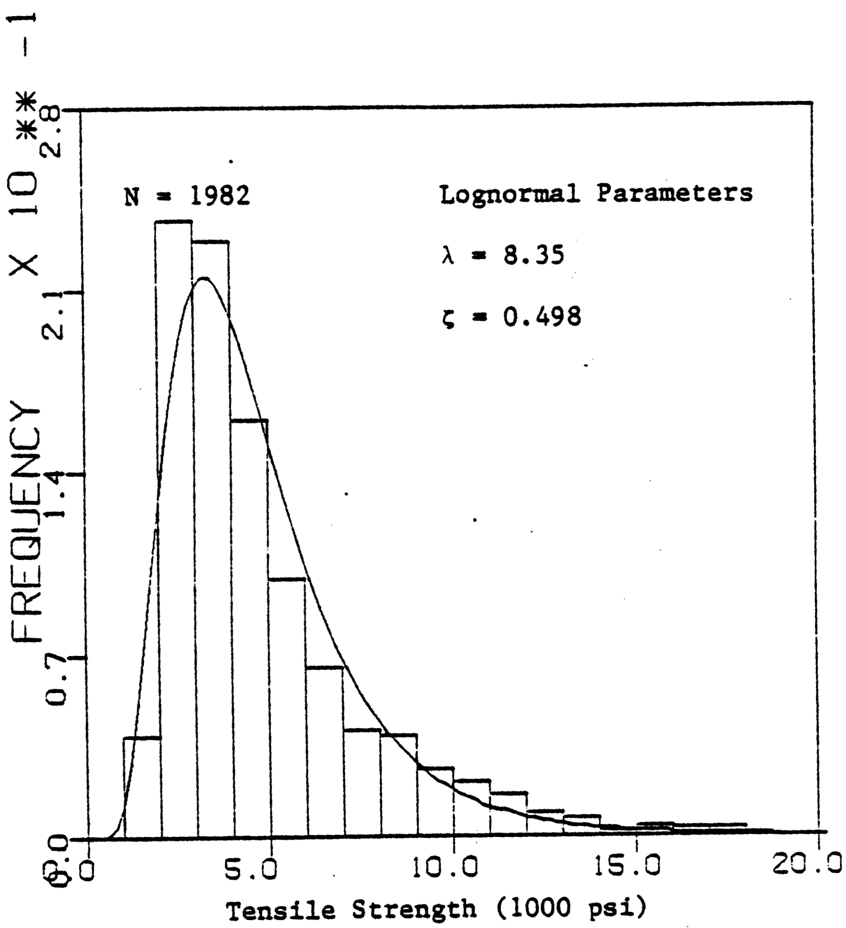

Figure 6.5 A lognormal distribution is superimposed onto the histogram of the generated tensile strength values from the length effect model of the 90" 2x4 No. 2 KD15 Southern Pine.

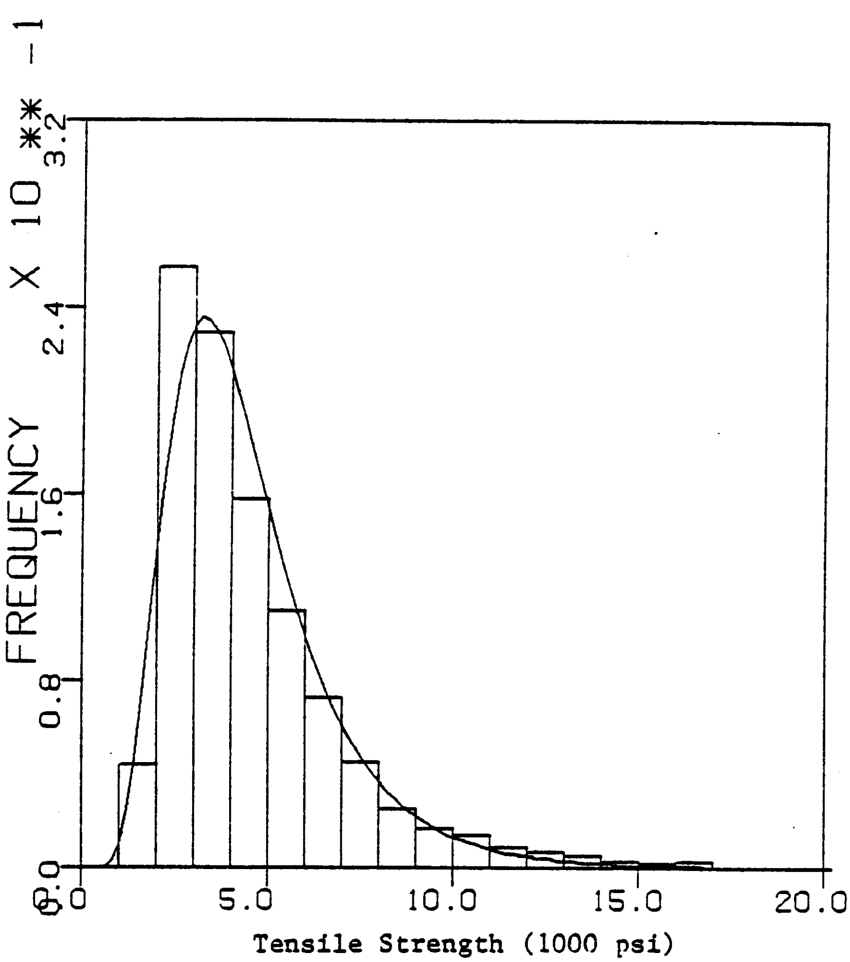

Figure 6.6 A lognormal distribution is superimposed onto the histogram of the generated tensile strength values from the length effect model of the 120" 2x4 No. 2 KD15 Southern Pine.

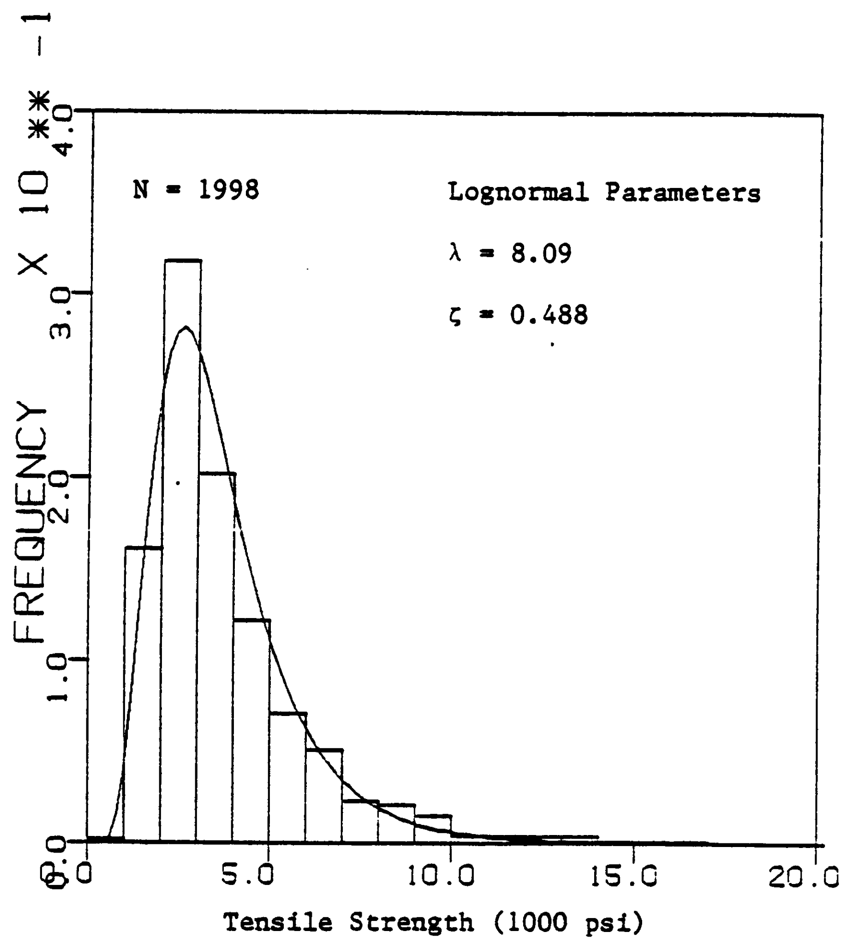

Figure 6.7 A lognormal distribution is superimposed onto the histogram of the generated tensile strength values from the length effect model of the 90" 2x10 No. 2 KD15 Southern Pine.

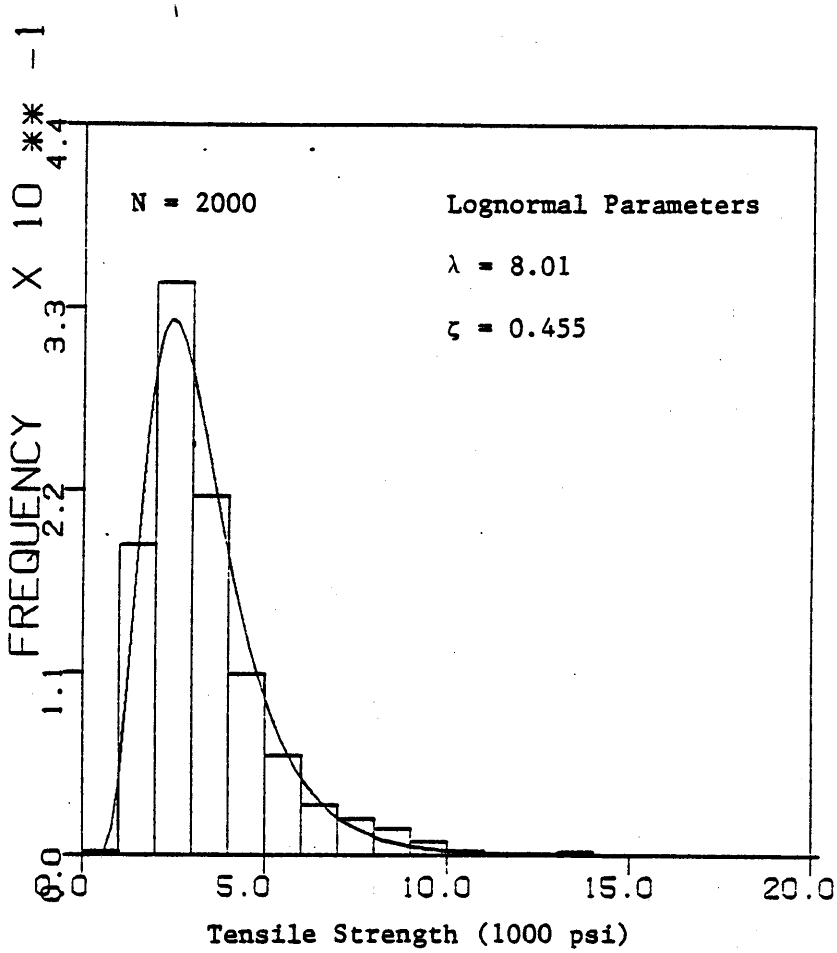

Figure 6.8 A lognormal distribution is superimposed onto the histogram of the generated tensile strength values from the length effect model of the 120" 2x10 No. 2 KD15 Southern Pine.

the lognormal distribution fit the generated tensile strength values for each of the eight cases under study.

Figures 6.9 through 6.16 show relative frequency histograms of the tensile strength of the 90-inch and 120-inch groups. The model generated tensile strength-length effect probability curves are superimposed onto histograms of the actual test data. A visual appraisal indicated that the models adequately describe the data. A Kolmogorov-Smirnov goodness-of-fit test was conducted on the eight sets of test data and model generated probability curves, and the only case to be rejected at the 5 percent significance level was the case of the 2x10 MSR 120-inch lumber (Figure 6.12).

Using a goodness-of-fit test at the 5 percent significance level indicates that a true hypothesis is rejected, on the average, 5 out of 100 tests, or, 1 in about 20 tests. In this study, 1 out of 8 cases was rejected (2x10 MSR 120-inch). Therefore, it is very possible that the Kolmogorov-Smirnov goodness-of-fit test in this case rejected a true hypothesis.

Table 6.2 lists a comparison of the lag-3 serial correlation estimated from 30-inch segments 1 and 4 tested and the lag-3 serial correlation calculated from the model generated 120-inch pieces of lumber. Tensile strength values generated for the first and fourth segments of the 120-inch piece of lumber were used to determine the lag-3 serial correlation

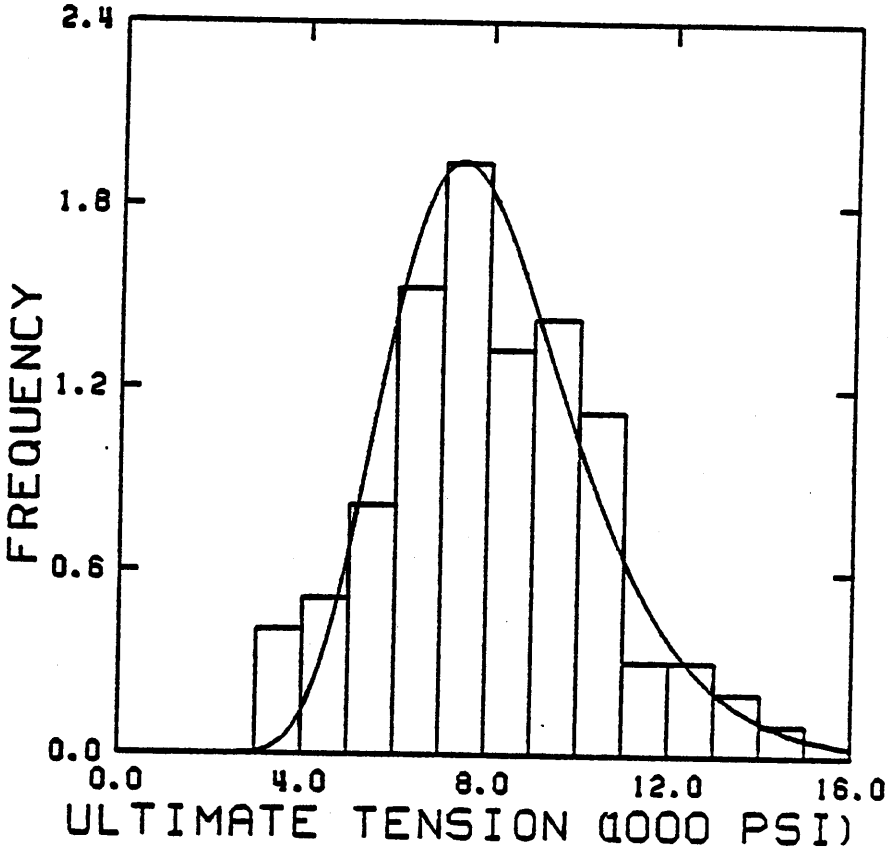

Figure 6.9 The histogram of the ultimate tension of the 2x4 2250f-1.9E MSR 90" treatment group is shown. The model generated tensile strength - length effect probability curve is superimposed onto the histogram.

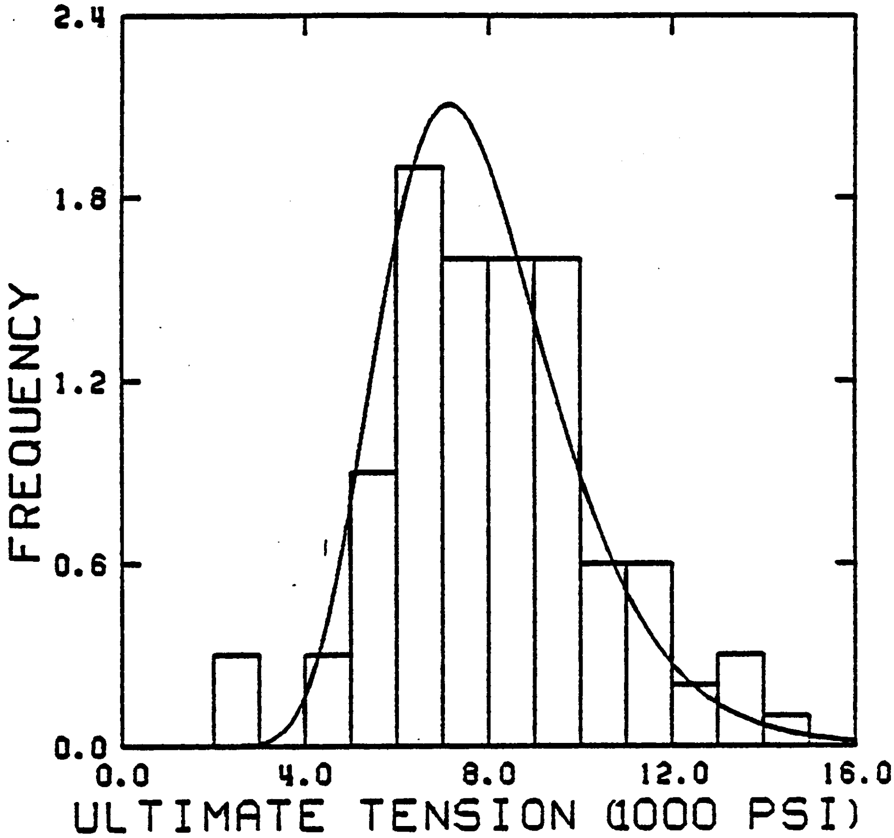

Figure 6.10 The histogram of the ultimate tension of the 2x4 2250f-1.9E MSR 120" treatment group is shown. The model generated tensile strength - length effect probability curve is superimposed onto the histogram.

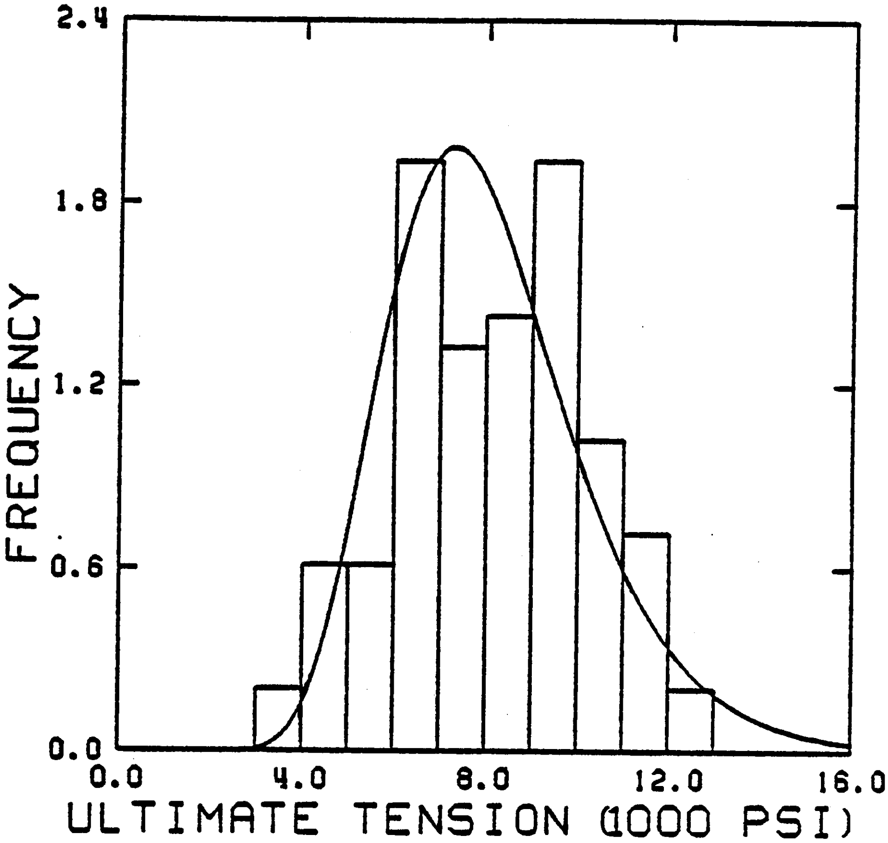

Figure 6.11 The histogram of the ultimate tension of the 2x10 2250f-1.9E MSR 90" treatment group is shown. The model generated tensile strength - length effect probability curve is superimposed onto the histogram.

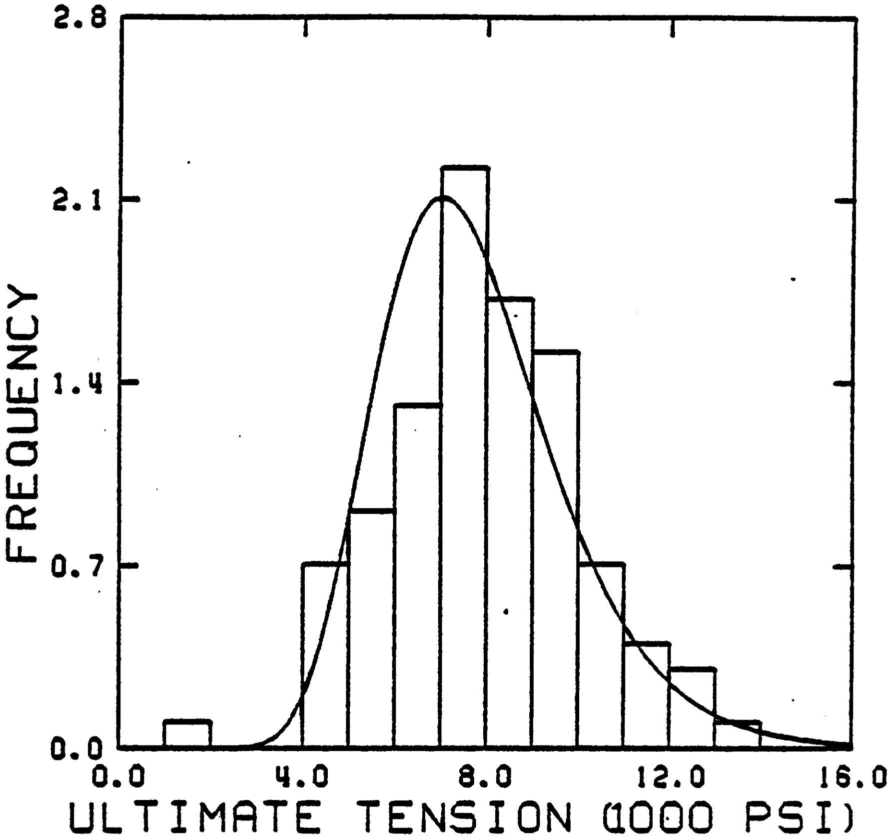

Figure 6.12 The histogram of the ultimate tension of the 2x10 2250f-1.9E MSR 120" treatment group is shown. The model generated tensile strength - length effect probability curve is superimposed onto the histogram.

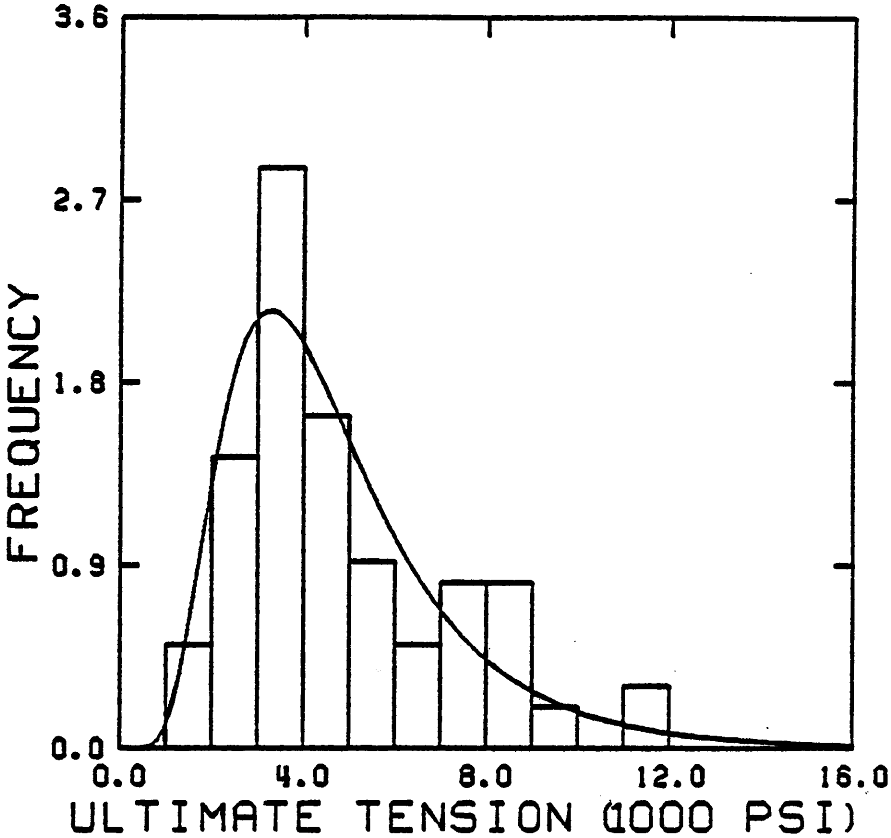

Figure 6.13 The histogram of the ultimate tension of the 2x4 No. 2 90" treatment group is shown. The model generated tensile strength - length effect probability curve is superimposed onto the histogram.

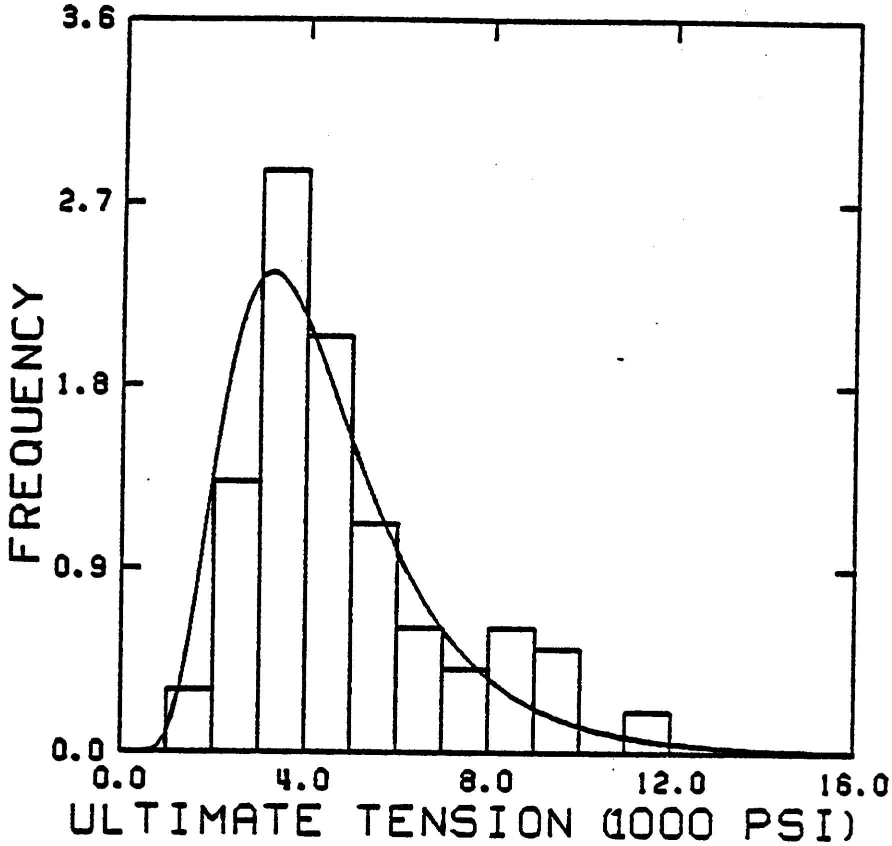

Figure 6.14 The histogram of the ultimate tension of the 2x4 No. 2 120" treatment group is shown. The model generated tensile strength - length effect probability curve is superimposed onto the histogram.

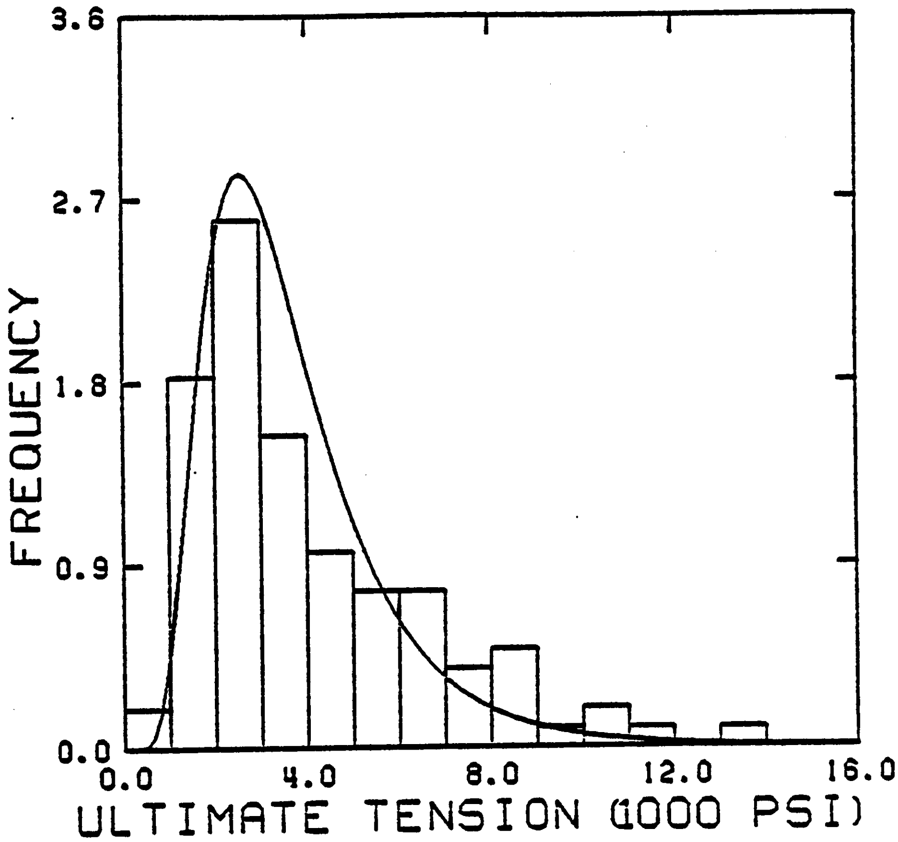

Figure 6.15 The histogram of the ultimate tension of the 2x10 No. 2 90" treatment group is shown. The model generated tensile strength - length effect probability curve is superimposed onto the histogram.

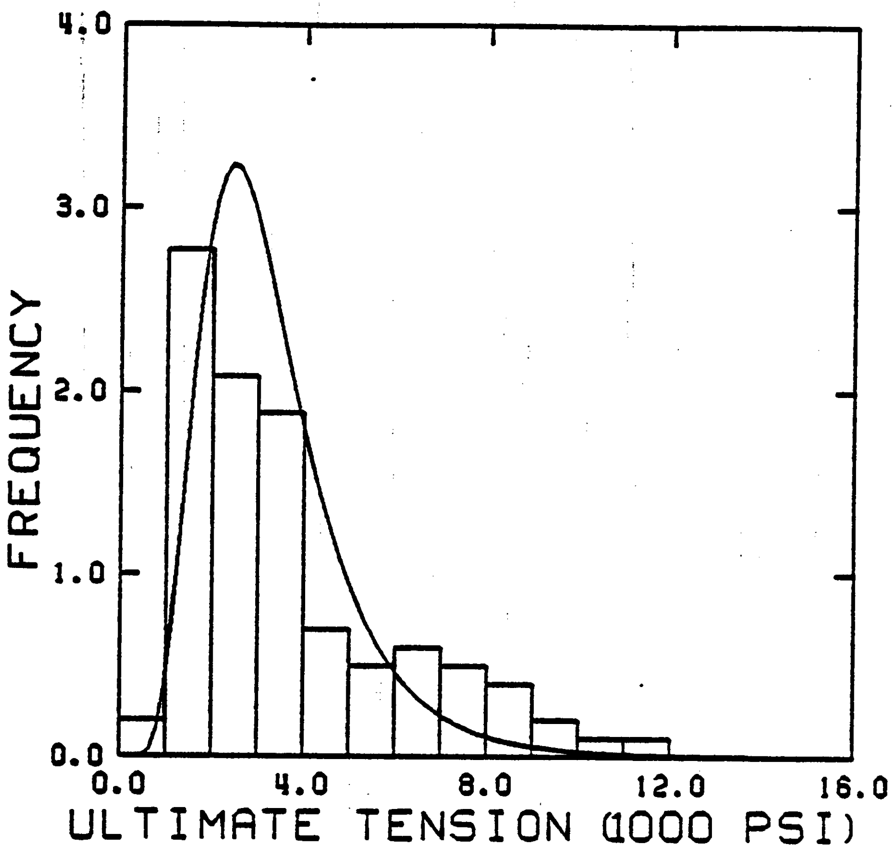

Figure 6.16 The histogram of the ultimate tension of the 2x10 No. 2 120" treatment group is shown. The model generated tensile strength - length effect probability curve is superimposed onto the histogram.

TABLE 6.2. Lag-3 serial correlation of tensile strength by size and grade.

SIZE	GRADE	ACTUAL	MODEL
2x4	2250f-1.9E	0.444	0.453
2x4	Grade-stamped No. 2	0.831	0.631
2x10	2250f-1.9E	0.402	0.362
2x10	Grade-stamped No. 2	0.589	0.399

of the model. No glaring differences can be found in the comparison of the model and the actual test data.

The fifth percentile values of tensile strength were also compared between the model and the actual data. The fifth percentile values were calculated assuming a lognormal distribution. Table 6.3 lists the fifth percentile values for the model and the tensile test data. Again, the values generated from the model appear to agree well with the actual data, especially if one considers the small sample sizes used to estimate the test data based fifth percentile.

TABLE 6.3. Fifth percentile values of tensile strength in psi by size and grade.

SIZE	GRADE	LENGTH	ACTUAL[1]	MODEL
2x4	2250f-1.9E	120"	4533	4982
2x4	2250f-1.9E	90"	4749	5102
2x4	Grade-stamped No. 2	120"	2103	1866
2x4	Grade-stamped No. 2	90"	1995	1855
2x10	2250f-1.9E	120"	4733	4902
2x10	2250f-1.9E	90"	5039	5031
2x10	Grade-stamped No. 2	120"	1025	1421
2x10	Grade-stamped No. 2	90"	1230	1456

[1]Assuming log-normal distribution

CHAPTER VII. APPLICATION OF THE LENGTH EFFECT MODEL

The following discussion relating to the "In-Grade" program gives one immediate application of the tensile length effect research. There are other applications in the areas of truss design and glued-laminated beam research. However, these latter applications require improvements in load models and are generally of a more complex nature requiring structural analysis.

7.1 DEVELOPMENT OF A TENSILE STRENGTH LENGTH ADJUSTMENT

The grading agencies participating in the "In-Grade" testing program have been testing the ultimate tensile strength of lumber at an 8-foot gage length and a 12-foot gage length. It has been shown that length of a tensile test span has significant effect on the ultimate tensile strength of structural lumber. The results obtained by the grading agencies are therefore influenced by the test length and cannot be compared directly. It may be useful to adjust the ultimate tension value at the gage length tested to the corresponding ultimate tension if the piece was tested at a standard length in order to compare and compile results. A tensile strength length adjustment model was developed to adjust the 5th percentile tensile strength at any gage length to the corresponding 5th percentile tensile strength at a standard length for two grades and two sizes of Southern Pine lumber. The 5th percentile strength was chosen because of its use in the derivation of allowable design stresses.

The data base used to develop the tensile length adjustment model included 2x4 and 2x10 2250f-1.9E and 2x4 and 2x10 No. 2 KD15 Southern Pine. These four groups each include three test length groups, a 30-inch gage length, a 90-inch gage length and a 120-inch gage length. A lognormal distribution was seen to be the best fit for each of the four test groups and the 5th percentile tensile strength was estimated for subsequent use in developing the tensile length adjustment model.

A standard length of 120 inches was selected since it was recommended in the In-Grade planning documents and the data base contained 5th percentile strength values at a 120-inch gage length. In addition, the maximum effective buckling length-to-depth ratio for tension members in truss design shall not exceed 80 (Truss Plate Institute, 1978). All lumber used in the In-Grade testing program and in most truss design has a minimum nominal dimension of 2 inches, resulting in a maximum allowable length of 120 inches. Also, foreseeing that a length adjustment might be applied to all species in the In-Grade tension testing program, a standard length of 120 inches is between the 8-foot and 12-foot gage lengths currently being used.

Since the longest test gage length was 120 inches, it was felt that more data of some form was needed to avoid an unreasonable extrapolation for longer lengths. A 5th percentile tensile strength at 300 inches was chosen as a desired upper limit of length in order to make the model applicable for length adjustments for all lengths of solid-sawn Southern Pine lumber. The 5th percentile tensile strength of a 300 inch specimen was predicted from the tensile strength-length effect model which generates ultimate tensile strength of a piece of Southern Pine lumber based on the strength of a 30-inch piece.

The data base now includes four data points for each of the four grade and size groups, three from actual data and one predicted value for a 300-inch specimen. These points were converted to length adjustment factors by dividing each 5th percentile tensile strength by the corresponding 120-inch 5th percentile tensile strength. The length adjustment is denoted by Equation 7.1.

$$Y = T(.)/T(120") \qquad (7.1)$$

where $T(.)$ = 5th percentile tensile strength at any length

$T(120")$ = 5th percentile tensile strength at 120 inches

The sample sizes, 5th percentile tensile strengths and length adjustments for each grade and size group are given in Tables 7.1 through 7.4.

Tables 7.3 and 7.4 list model generated 5th percentile tensile strength values for the 300 inch length which are significantly higher than the tensile strength values at the 120 inch length calculated from the test data for the 2x4 and 2x10 No. 2 KD15 Southern Pine lumber groups. However, it has been determined that tensile strength decreases with increasing length. This anomoly of an increase in tensile strength from 120 inches to 300 inches may be attributed to an overprediction by the tensile strength-length effect model, or the test data may not be representing the actual behavior of tensile strength at 120 inches due to sampling error. Also, increase in tensile strength from 120 inches to 300 inches may be attributed to both an overprediction of the model and a misrepresentation of the test data. Therefore, the data obtained for the 2x4 and 2x10 No. 2 KD15 Southern Pine lumber groups will be used to develop a tensile length adjustment model since they are the best estimates of the tensile strength behavior available.

The length data exhibit a non-linear decrease in the length adjustment with increasing length. This would be expected according to the asymptotic theory of statistical extremes (Ang and Tang, 1984). As illustrated in Ang and Tang (1984), the distribution of values of the smallest value in

Table 7.1. 5th percentile tensile strength and length adjustment, Y, for 2x4 2250f-1.9E Southern Pine.

Length (inches)	Sample Size	5th percentile Tensile Strength (psi)	Y
30	100	5470	1.207
90	98	4749	1.048
120	100	4533	1.000
300	NA	4547[1]	1.003

[1]Model generated.

Table 7.2. 5th percentile tensile strength and length adjustment, Y, for 2x10 2250f-1.9E Southern Pine.

Length (inches)	Sample Size	5th percentile Tensile Strength (psi)	Y
30	100	5692	1.203
90	98	5039	1.065
120	99	4733	1.000
300	NA	4476[1]	0.946

[1]Model generated.

Table 7.3. 5th percentile tensile strength and length adjustment, Y, for 2x4 No. 2 KD15 Southern Pine.

Length (inches)	Sample Size	5th percentile Tensile Strength (psi)	Y
30	98	2285	1.086
90	98	1995	0.949
120	98	2103	1.000
300	NA	1685[1]	0.801

[1]Model generated.

Table 7.4. 5th percentile tensile strength and length adjustment, Y, for 2x10 No. 2 KD15 Southern Pine.

Length (inches)	Sample Size	5th percentile Tensile Strength (psi)	Y
30	104	1679	1.641
90	104	1230	1.202
120	101	1023	1.000
300	NA	1230[1]	1.202

[1]Model generated.

a sample of size n decreases as n increases. However, the shift in distribution from one sample size to a larger sample size becomes less dramatic as n increases. In a general sense, the asymptotic theory of statistical extremes explains the asymptotic behavior of the data as length increases.

The tensile length adjustment model that best describes this length effect is the non-linear regression model given by Equation 7.2.

$$Y = A + B \times C^X \tag{7.2}$$

where X = gage length in inches

Y = length adjustment for 5th percentile tensile strength

The least-square estimates of the parameters A, B and C were computed using the data base for each grade and size group. The resulting curve was then vertically translated by adjusting A such that Y = 1 for X = 120". In this manner, lumber tested in tension with a 10-foot gage length will not be subject to any length adjustment. Table 7.5 shows the estimated parameters for each grade and size group. A graphical interpretation of the tensile strength length adjustment model is given in Figures 7.1 through 7.4 along with the actual data used to develop the model.

Table 7.5. Estimated parameters A, B and C of equation 2 for the tensile length adjustment model

Lumber Group	A	B	C
2x4 2250f-1.9E	0.980	0.470	0.974
2x4 No. 2 KD15	0.765	0.429	0.995
2x10 2250f-1.9E	0.939	0.424	0.984
2x10 No. 2 KD15	0.995	2.463	0.949

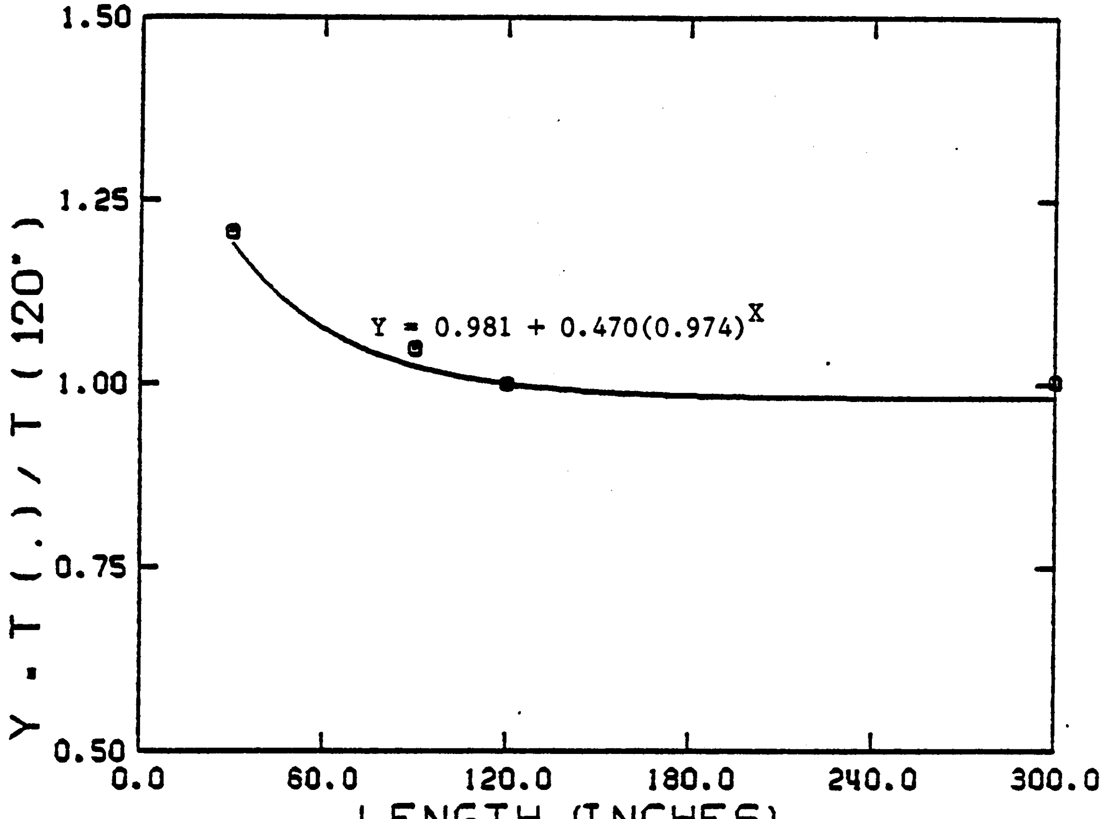

Figure 7.1 Length adjustment, Y, for 2x4 2250f-1.9E Southern Pine. A "least-squares" curve was determined and then forced through the point Y equals 1.0 for length equal 120 inches. The three leftmost points are from tensile test data while the rightmost point was simulated by a tensile strength model.

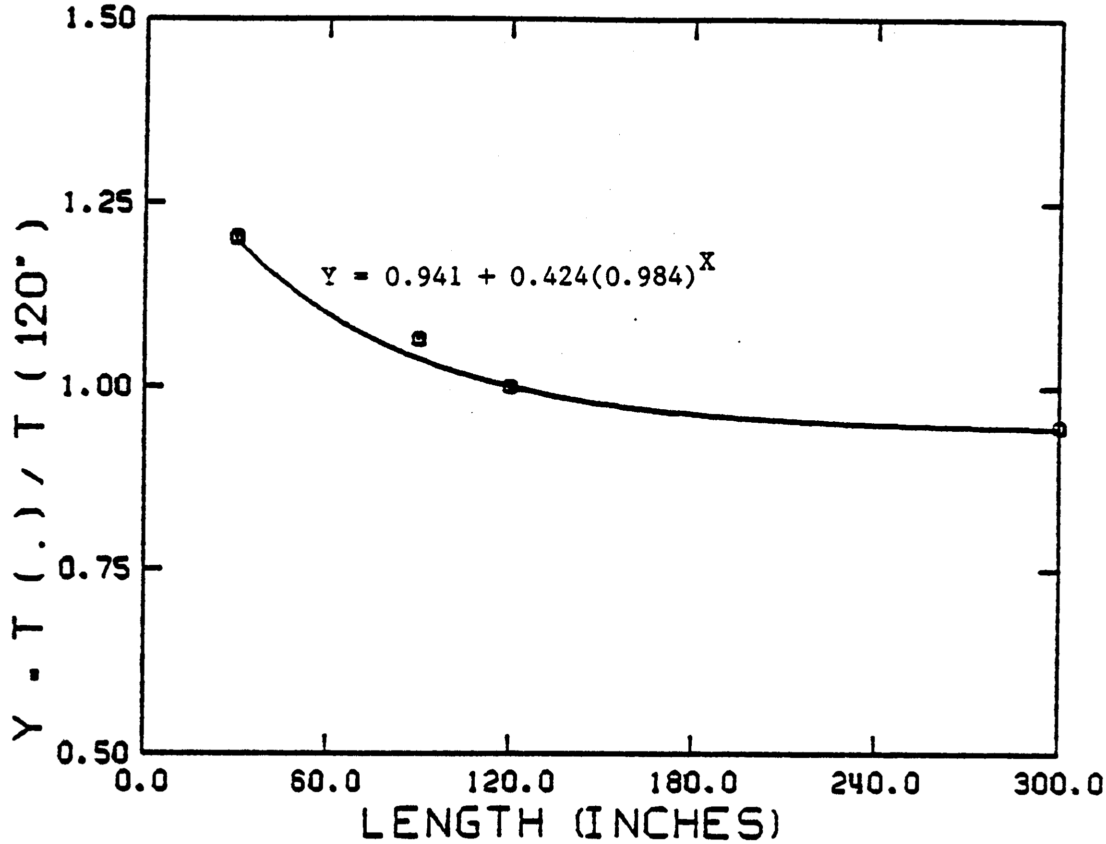

Figure 7.2 Length adjustment, Y, for 2x10 2250f-1.9E Southern Pine. A "least-squares" curve was determined and then forced through the point Y equals 1.0 for length equal 120 inches. The three leftmost points are from tensile test data while the rightmost point was simulated by a tensile strength model.

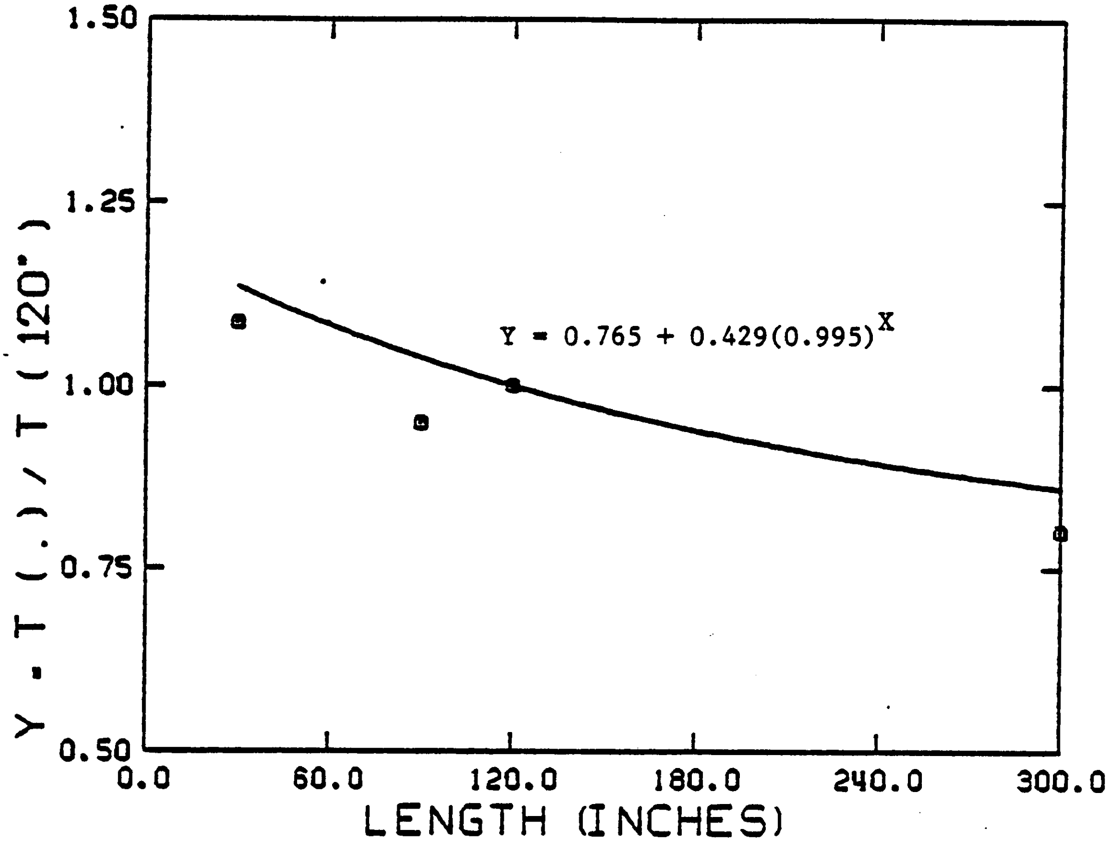

Figure 7.3 Length adjustment, Y, for 2x4 No. 2 KD15 Southern Pine. A "least-squares" curve was determined and then forced through the point Y equals 1.0 for length equal 120 inches. The three leftmost points are from tensile test data while the rightmost point was simulated by a tensile strength model.

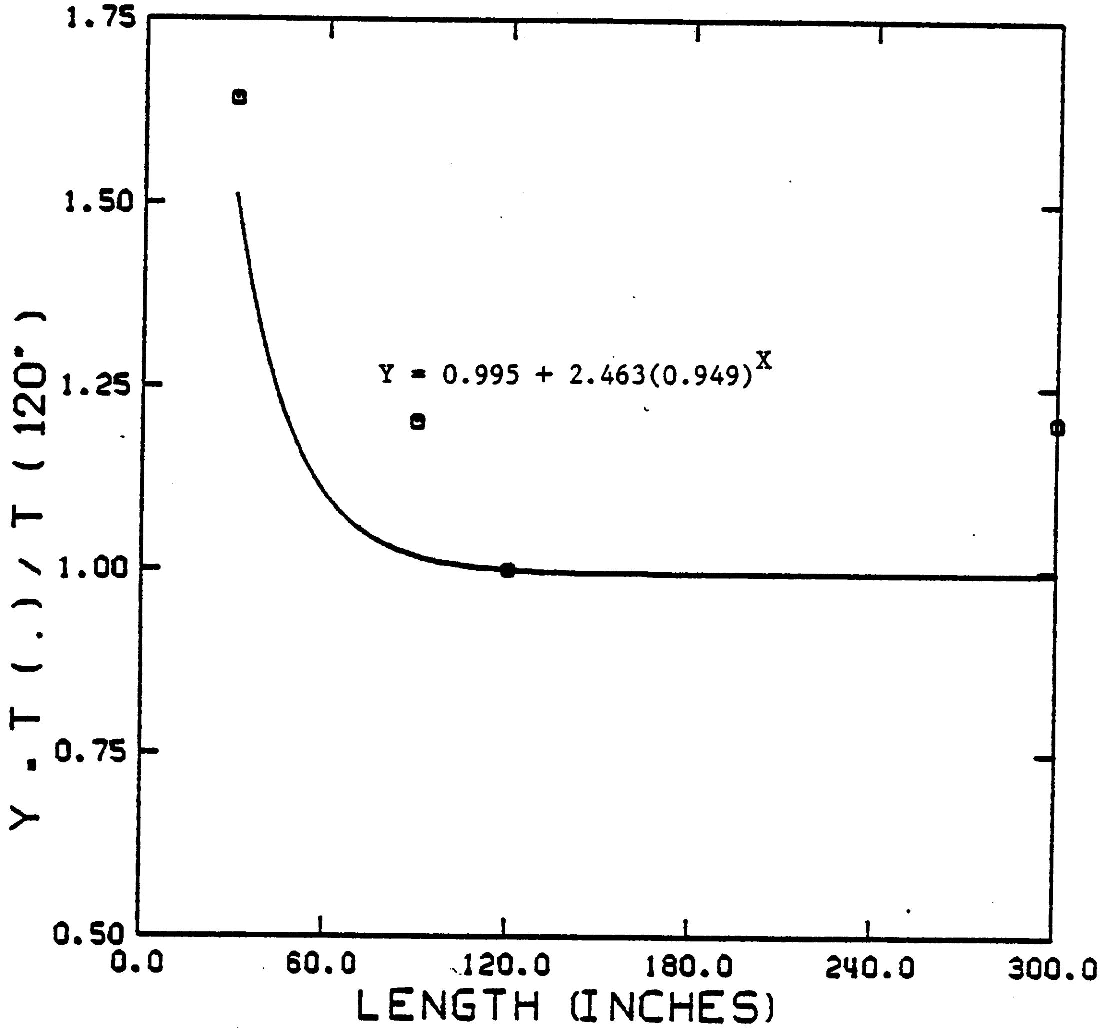

Figure 7.4 Length adjustment, Y, for 2x10 No. 2 KD15 Southern Pine. A "least-squares" curve was determined and then forced through the point Y equals 1.0 for length equal 120 inches. The three leftmost points are from tensile test data while the rightmost point was simulated by a tensile strength model.

7.2 APPLICATIONS OF THE TENSILE STRENGTH LENGTH ADJUSTMENT

The tensile strength length adjustment model was developed to transform the 5th percentile tensile strength of a piece of Southern Pine lumber to the equivalent 5th percentile tensile strength at a standard length of 120 inches. Application of the model as it might be used in the In-Grade testing program is demonstrated using the following example.

A test is conducted to determine the ultimate tensile strength of 2x4 No. 2 KD15 Southern Pine lumber at a gage length of 12 feet or 144 inches. A distribution is fitted to the tensile strength data and the 5th percentile tensile strength is estimated. From Figure 7.3, the length adjustment at 144 inches equals 0.97. To find the equivalent 5th percentile tensile strength at 120 inches, the 144-inch 5th percentile tensile strength is divided by the length adjustment. Therefore, the 5th percentile tensile strength as if it were tested at the standard length would be the calculated 5th percentile strength divided by the length adjustment.

T(120") = 5th percentile strength at 120 inches
= T(144")/Y

where T(144") = 5th percentile strength at 144 inches

Y = length adjustment at 144 inches
= 0.97

The model can also be used to adjust the design value of tension parallel-to-grain in a wood member with respect to the length of the member. This adjustment is accomplished simply by multiplying the length adjustment, Y, by the design value of tensile strength at the standard length, in this case 120 inches. For example, a standard panel in floor truss design is 30 inches and the truss chord is a 2x4 2250f-1.9E Southern Pine. From Figure 7.1, Y = 1.19 at 30 inches. This means that the 5th percentile tensile strength at 30 inches is 19 percent stronger than the 5th percentile tensile strength at the standard test length of 120 inches. In a floor truss the center panel is the most highly stressed in tension; therefore, the impact on design could be significant. If all the panels across the bottom chord had an equal level of high tensile stress, the previous statement would not be valid since, in effect, the lumber subject to the high tensile stress would be 28 - 30 feet.

In summary, a tensile length adjustment model was developed for possible use in the In-Grade tension testing program to adjust the tensile strength tested at different gage lengths to a single tensile strength if the test had been conducted at a standard length of 120 inches. The model determines length adjustments for two grades and two sizes of Southern Pine lumber, 2x4 and 2x10 2250f-1.9E and 2x4 and 2x10 No. 2 KD15. Also, an example was given pointing to the application of the model in wood truss design where the length of the member is taken into account when designing tension members.

CHAPTER VIII. SUMMARY AND CONCLUSIONS

8.1 SUMMARY

This study was designed to investigate the effect of length on tensile strength parallel-to-grain in lumber. In addition, a model was developed to generate tensile strength values of lumber taking into consideration the length of the piece of lumber. These objectives were accomplished by testing two sizes (2x4 and 2x10) and two grades (2250f-1.9E and No. 2 KD15) of Southern Pine lumber having three different test span lengths of 30, 90 and 120 inches. A sample of approximately 100 specimens in each treatment group was tested which was adequate for model development. The tensile strengths in the 30-inch treatment groups were significantly higher than the tensile strengths in the 90-inch and 120-inch treatment groups for each grade and size.

Tensile strength and MOE data from the 30-inch treatment groups were used to develop tensile strength-length effect models for the four grade and size groups. An MOE variability model (Kline et al, 1985) was modified and used to generate 30-inch segment MOE values for a piece of lumber. The segment MOE values were inputted into a weighted least squares regression model in which the residuals were assumed to follow a first-order Markov process. The weighted least squares regression model generated 30-inch segment tensile strength values of the piece of lumber. Using the weakest-link concept, the minimum segment tensile strength value was selected as the tensile strength of the generated piece of lumber.

When generating segment tensile strength values, the probability of generating lower tensile strength values increases as the number of segments, or the length of the lumber, increases. Thus, the tensile strength-length effect model predicts a lower tensile strength for a longer piece of lumber.

The data base and resulting model were used to define a length adjustment factor for tensile tests having different gage lengths. For practical reasons a standard tension test gage length of 10 feet was chosen.

8.2 CONCLUSIONS

For two sizes and two grades of Southern Pine lumber, MOE and tensile strength of segments along the length of a board were found to follow a parallel Markov process. A modified version of the MOE variability model (Kline et al, 1985) was necessary to model segment MOE. The MOE variability model is a second-order Markov process. Segment tensile strength was found to follow a parallel first-order Markov process. The two Markov processes were linked by a weighted least squares regression model involving segment tensile strength as the dependent variable and segment MOE as the independent variable. This study showed that the tensile strength distribution of lumber 90 and 120 inches long can be adequately modeled from a statistically based knowledge of the MOE and tensile strength characteristics of 30 inch lumber segments.

It was concluded that the tensile strength-length effect cannot be modeled by assuming the lengthwise segments of a lumber specimen are non-correlated. A model using Weibull's "weakest-link theory" (Weibull, 1939) in which the segments are assumed to be independent was found to inadequately describe the tensile behavior of a lumber specimen. An independent segment model underpredicts the tensile strength of a lumber specimen.

The tensile strength-length effect model which was developed by allowing a regression of tensile strength on MOE to account for the serial correlation in MOE and its resulting impact on tensile strength was also rejected because of its inability to adequately describe the test data. In this correlated segment model, the only correlation between 30-inch segments in predicting tension was derived from the correlation of MOE between the segments. From the satisfactory results obtained from the refined model, the serial correlation in segment tensile strength parallel-to-grain is higher than that predicted from the model using only serial correlation in MOE.

The data base for the study was limited in scope and size. Larger samples, other commercial grades and species groups should be studied to verify the tensile strength-length effect as being a parallel Markov process and to improve on the estimates of the model parameters.

APPENDIX A. PROGRAM LISTING OF THE MODIFIED MOE VARIABILITY MODEL.

```
      SUBROUTINE LENMOE(DSEED,E,NS,N,LC,VMOE,IR)
C
C**********************************************************************
C*****                                                           *****
C*****    SUBROUTINE LENMOE TAKES AN INPUT ARRAY OF THE AVERAGE   *****
C*****    MOE FOR A LUMBER SPECIMEN AND DETERMINES THE LENGTHWISE *****
C*****    VARIABILITY IN MOE.  THE MOE FOR 30-INCH SEGMENTS ARE   *****
C*****    FOUND FOR ANY SPECIMEN LENGTH.  4 DIFFERENT SIZES AND   *****
C*****    GRADES OF LUMBER CAN BE CONSIDERED:  A) 2X4 2250F-1.9E  *****
C*****    MSR LUMBER B) 2X4 NO. 2 KD15 SOUTHERN PINE LUMBER       *****
C*****    C) 2X10 2250F-1.9E MSR LUMBER AND D) 2X10 NO. 2 KD15    *****
C*****    SOUTHERN PINE LUMBER.                                   *****
C*****                                                           *****
C**********************************************************************
C
C         THE STANDARD NORMAL RANDOM NUMBER GENERATOR GGNML FROM THE
C                 IMSL ROUTINE LIBRARY (1982) IS USED.
C
C         DSEED = INPUT/OUTPUT DOUBLE PRECISION VARIABLE ASSIGNED A
C                 RANDOM INTEGER VALUE IN THE EXCLUSIVE RANGE
C                 (1.D0,2147483647.D0). DSEED IS REPLACED BY A NEW
C                 VALUE TO BE USED IN A SUBSEQUENT CALL.
C
C         E     = AN INPUT VECTOR OF DIMENSION NS OF RANDOM OBSERVATIONS
C                 OF MOE FROM THE APPROPRIATE DISTRIBUTION.
C
C         NS    = THE NUMBER OF SPECIMENS (INPUT).
C
C         N     = THE NUMBER OF 30-INCH SEGMENTS PER SPECIMEN (INPUT).
C
C         LC    = IF INPUT = 1  -  OUTPUT = MOE OF 2X4 2250F-1.9E MSR
C                                          LUMBER
C                 IF INPUT = 2  -  OUTPUT = MOE OF 2X4 NO. 2 KD15
C                                          SOUTHERN PINE LUMBER
C                 IF INPUT = 3  -  OUTPUT = MOE OF 2X10 2250F-1.9E MSR
C                                          LUMBER
C                 IF INPUT = 4  -  OUTPUT = MOE OF 2X10 NO. 2 KD15
C                                          SOUTHERN PINE LUMBER
C
C         VMOE  = THE OUTPUT MATRIX OF VARIABILITY IN MOE FOR EACH
C                 SPECIMEN WITH DIMENSION NS BY N.
C
C         IR    = THE NUMBER OF SPECIMENS REJECTED DUE TO SPECIFIED
C                 MAXIMUM AND MINIMUM MOE VALUES (OUTPUT).
C**********************************************************************
C
C      A MINIMUM SEGMENT MOE, EMIN, AND A MAXIMUM SEGMENT MOE, EMAX,
C      ARE GIVEN TO AVOID THE GENERATION OF UNREALISTIC SEGMENT MOE
C      VALUES USING THE MARKOV MODEL.  EMIN AND EMAX ARE THE MINIMUM
C      AND MAXIMUM SEGMENT MOES FOR THE FOUR DATA SETS OF 2X4 AND 2X10
C      2250F-1.9E MSR AND NO. 2 KD15 SOUTHERN PINE.
C
C      IF A GENERATED SEGMENT MOE OF A SPECIMEN IS NOT IN THE
C      SPECIFIED RANGE AS INDICATED ABOVE, ANOTHER SERIES OF SEGMENT
C      MOE VALUES IS GENERATED FOR THAT SPECIMEN.
C
C**********************************************************************
C
      DIMENSION X(110),E(NS),VMOE(NS,N)
      DOUBLE PRECISION DSEED
C
C     ALL PARAMETERS ARE SPECIFIED
C
      GO TO (1,2,3,4),LC
C
C     THE PARAMETERS FOR 2X4 2250F-1.9E MSR
C
```

```
    1 RESD =    0.3775E+06
      GBAR =    0.2571E+07
      B1   =    0.630
      B2   =    0.190
      R2   =    0.619
      EMIN =    0.1579E+07
      EMAX =    0.3623E+07
      GO TO 5
C
C   THE PARAMETERS FOR 2X4 NO. 2 KD15
C
    2 RESD =    0.5075E+06
      GBAR =    0.1749E+07
      B1   =    0.606
      B2   =    0.332
      R2   =    0.843
      EMIN =    0.4610E+06
      EMAX =    0.3245E+07
      GO TO 5
C
C   THE PARAMETERS FOR 2X10 2250F-1.9E MSR
C
    3 RESD =    0.3601E+06
      GBAR =    0.2402E+07
      B1   =    0.817
      B2   =    0.095
      R2   =    0.818
      EMIN =    0.1679E+07
      EMAX =    0.3630E+07
      GO TO 5
C
C   THE PARAMETERS FOR 2X10 NO. 2 KD15
C
    4 RESD =    0.6006E+06
      GBAR =    0.1770E+07
      B1   =    0.816
      B2   =    0.117
      R2   =    0.856
      EMIN =    0.5280E+06
      EMAX =    0.3236E+07
C
C   THE SECOND ORDER MARKOV MODEL GENERATES LENGTHWISE MOE
C
    5 M = N + 10
      IR = 0
      DO 100 J = 1,NS
    9     X(1) =   0.0
          X(2) =   0.0
         DO 10 I = 1,M
            CALL GGNML (DSEED,1,T)
            XRES  = RESD * T * SQRT(1.0 - R2)
            XMARK = B2 * X(I) + B1 * X(I+1)
            X(I+2)=  XMARK + XRES
   10    CONTINUE
          SUM = 0.0
         DO 11 I = 11,M
            X(I-10) = X(I) + GBAR
            SUM = SUM + X(I-10)
   11    CONTINUE
          PCBAR = SUM/FLOAT(N)
         DO 12 I = 1,N
          VMOE(J,I) = X(I)/PCBAR * E(J)
          IF (VMOE(J,I).GE.EMIN.AND.VMOE(J,I).LE.EMAX) GOTO 12
          IR = IR + 1
          GOTO 9
   12    CONTINUE
  100 CONTINUE
      RETURN
```

END

APPENDIX B. PROGRAM LISTING OF THE TENSILE STRENGTH-LENGTH EFFECT MODEL.

```
C************************************************************************
C******                                                            ******
C******   THIS ROUTINE GENERATES SETS OF LENGTHWISE 30-INCH TENSILE ******
C******   STRESS VALUES GIVEN THE DISTRIBUTION OF THE AVERAGE MOE   ******
C******   OF A LUMBER SPECIMEN.  THE ULTIMATE TENSILE STRESS OF THE ******
C******   SPECIMEN IS THEN DETERMINED USING A WEAKEST LINK THEORY   ******
C******   WHERE THE LINK TENSILE VALUES ARE SERIALLY CORRELATED.    ******
C******                                                            ******
C******   USES SUBROUTINE LENMOE AND THE ROUTINES GGNML AND GGUBS   ******
C******   FROM THE IMSL ROUTINE LIBRARY (1982).                     ******
C******                                                            ******
C************************************************************************
C
C          ECMAX = EMAX - 300000.0 PSI, WHERE EMAX IS THE MAXIMUM
C                  SEGMENT MOE SPECIFIED IN SUBROUTINE LENMOE.
C
C          ECMIN = EMIN + 300000.0 PSI, WHERE EMIN IS THE MINIMUM
C                  SEGMENT MOE SPECIFIED IN SUBROUTINE LENMOE.
C
C       CAUTION:   IN ANY CASE ECMIN MUST BE GREATER THAN EMIN SPECIFIED
C                  IN SUBROUTINE LENMOE AND ECMAX MUST BE LESS THAN EMAX
C                  SPECIFIED IN SUBROUTINE LENMOE.
C
C          EPR    = A VECTOR OF DIMENSION NR OF RANDOM OBSERVATIONS
C                   OF PIECE-AVERAGE MOE FROM THE APPROPRIATE DISTRIBUTION.
C
C          T      = A MATRIX OF 30-INCH TENSILE STRENGTH VALUES FOR NR
C                   SPECIMENS WITH DIMENSION NR BY N.  N IS THE NUMBER OF
C                   DESIRED 30-INCH SEGMENTS PER PIECE OF LUMBER.
C
C          TS     = A VECTOR OF DIMENSION NR OF ULTIMATE TENSILE
C                   STRENGTH OF EACH SPECIMEN.
C
C************************************************************************
      DOUBLE PRECISION DSEED
      DIMENSION EPR(2000),VMOE(2000,20),T(2000,10),VRES(2000,20),SC(9),
     CTS(2000)
C
C     NUMBER OF SPECIMENS
      NR = 2000
C
C     NUMBER OF 30-INCH SEGMENTS DESIRED.
      N = 4
C
      M = N + 10
      DSEED = 950348.D0
C
C     SELECTION OF SIZE AND GRADE OF LUMBER
      LT = 4
C
      GOTO (1,2,3,4) LT
C
C     THE PARAMETERS FOR THE 2X4 2250F-1.9E MSR
C
C         MOE PARAMETERS
    1 ECMIN = 1.879E+06
      ECMAX = 3.323E+06
      XLAMDA = 14.75
      ZETA = 0.13298
C         REGRESSION PARAMETERS
      XK = 0.257586024E-07
      B0 = 0.813790131E+01
      B1 = 0.381411610E-06
C         RESIDUAL CORRELATION
      RO3 = 0.39737
      GO TO 5
C
C     THE PARAMETERS FOR THE 2X4 NO. 2 KD15
```

```
C
C          MOE PARAMETERS
    2 ECMIN = 0.761E+06
      ECMAX = 2.945E+06
      EPS = 0.5221550E+06
      SIG = 1.3784885E+06
      NU  = 2.6894493
C          REGRESSION PARAMETERS
      XK = 0.629349302E-07
      B0 = 0.727360916E+01
      B1 = 0.744034367E-06
C          RESIDUAL CORRELATION
      RO3 = 0.56002
      GO TO 6
C
C     THE PARAMETERS FOR THE 2X10 2250F-1.9E MSR
C
C          MOE PARAMETERS
    3 ECMIN = 1.979E+06
      ECMAX = 3.330E+06
      XLAMDA = 14.68
      ZETA = 0.13502
C          REGRESSION PARAMETERS
      XK = 0.333599424E-07
      B0 = 0.848429012E+01
      B1 = 0.258614875E-06
C          RESIDUAL CORRELATION
      RO3 = 0.32111
      GO TO 5
C
C     THE PARAMETERS FOR THE 2X10 NO. 2 KD15
C
C          MOE PARAMETERS
    4 ECMIN = 0.828E+06
      ECMAX = 2.936E+06
      EPS = 0.3449451E+06
      SIG = 1.6037426E+06
      NU  = 2.6883287
C          REGRESSION PARAMETERS
      XK = 0.930342026E-07
      B0 = 0.715514374E+01
      B1 = 0.716911018E-06
C          RESIDUAL CORRELATION
      RO3 = 0.028605
      GO TO 6
C
C
C     THE AVERAGE MOE VALUES ARE GENERATED AND A NEW MOE VALUE IS
C     GENERATED IF THE MOE EXCEEDS ECMIN OR ECMAX.
C
5     DO 10 I=1,NR
7     CALL GGNML(DSEED,1,R)
      EPR(I) = EXP(XLAMDA + R*ZETA)
      IF (EPR(I).LT.ECMIN.OR.EPR(I).GT.ECMAX) GO TO 7
10    CONTINUE
      GO TO 16
6     DO 15 I = 1,NR
11    CALL GGUBS(DSEED,1,R)
      EPR(I) = SIG*(-ALOG(R))**(1./NU) + EPS
      IF (EPR(I).LT.ECMIN.OR.EPR(I).GT.ECMAX) GO TO 11
15    CONTINUE
16    CALL LENMOE(DSEED,EPR,NR,M,LT,VMOE,IR)
C
C     GENERATE T USING WEIGHTED LEAST SQUARES REGRESSION.
C
      RO1 = RO3**(1./3.)
      DO 30 I=1,NR
      TS(I) = 100000.0
```

```
      VRES(I,1) = 0.0
          DO 40 J = 2,M
              CALL GGNML(DSEED,1,R)
              SIGI = SQRT(XK * VMOE(I,J))
              SIGK = SQRT(XK * VMOE(I,J-1))
              EP = RO1 * SIGI/SIGK * VRES(I,J-1)
              PS = R * SIGI * SQRT(1.0 - RO1 * RO1)
              VRES(I,J) = EP + PS
   40     CONTINUE
      DO 60 J=11,M
         T(I,J-10) = EXP(B0 + B1*VMOE(I,J) + VRES(I,J))
C
C     WEAKEST LINK THEORY
C
         IF (T(I,J-10).LT.TS(I)) TS(I) = T(I,J-10)
60    CONTINUE
C
C     TMAX IS THE MAXIMUM TENSILE STRENGTH FOR THE 30 INCH SEGMENTS
C     FOR THE FOUR DATA SETS OF 2X4 AND 2X10 2250F-1.9E AND NO. 2
C     KD15 SOUTHERN PINE.  A TENSILE STRESS VALUE IS REJECTED IF IT
C     EXCEEDS TMAX.
C
      TMAX = 17737.0
      IF (TS(I).GT.TMAX) GO TO 30
      WRITE (6,200) TS(I)
30    CONTINUE
      MM = N - 1
C
C     THE SERIAL CORRELATION IS CALCULATED FOR LAGS 1 THRU MM
C
      DO 70 K=1,MM
         CALL SERCOR (T,TS,TMAX,K,N,NR,SC(K))
70    CONTINUE
      WRITE (7,300) (SC(K),K=1,MM)
200   FORMAT(10X,F7.0)
300   FORMAT(3F15.5)
      WRITE(8,400) IR
400   FORMAT(I6,' BOARDS WERE REJECTED DUE TO SPECIFIED MINIMUM AND MAXI
     CMUM MOE VALUES.')
      STOP
      END
C
C   SUBROUTINE SERCOR CALCULATES THE SERIAL CORRELATIONS AT A
C   SPECIFIED LAG FOR EACH LUMBER SPECIMEN WITH ULTIMATE TENSILE
C   STRENGTH LESS THAN OR EQUAL TO XMAX.
C
      SUBROUTINE SERCOR(X,XX,XMAX,K,N,NX,RK)
      DIMENSION X(NX,N),XX(NX)
      T1=0.
      T2=0.
      T3=0.
      T4=0.
      T5=0.
      L=N-K
      M=NX*L
      DO 10 I=1,NX
      IF (XX(I).GT.XMAX) GO TO 10
      DO 20 J=1,L
      T1=T1+X(I,J)*X(I,J+K)
      T2=T2+X(I,J)
      T3=T3+X(I,J+K)
      T4=T4+X(I,J)**2
      T5=T5+X(I,J+K)**2
   20 CONTINUE
   10 CONTINUE
      T6=(T1-T2*T3/FLOAT(M))
      T7=(T4-T2**2/FLOAT(M))**0.5
      T8=(T5-T3**2/FLOAT(M))**0.5
```

```
RK=T6/(T7*T8)
RETURN
END
```

REFERENCES

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