Allowable Bending Strength Enhancement of 2 by 4 Lumber by Tension and Compression Proofloading

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(ABSTRACT)

Research has established that correlations exist between bending and tension, and bending and compression strength of lumber. Because of this correlation, improvement in bending strength may be realized from proofloading in tension or compression.

The data utilized in a reliability analysis was from Galligan et al. (1986) that characterized the properties of 2-inch softwood dimension lumber with regressions and probability distributions. Randomly selected groups of 2 by 4 1650f-1.5E Hem-fir and No.2 KD Southern Pine were evaluated for bending strength. One group from each species was selected as a control and tested in bending. The other groups were proofloaded in tension and compression at two stress levels and the survivors were tested in bending to failure.

Based on the concept of equal reliability and utilizing the load distributions from Thurmond (1986), the tensile and compressive proofloaded strength distributions were compared to the control. The probability of failure for the control group is found, then with an iterative approach, the bending strength values of the proofloaded sample distribution are artificially altered by a factor K until the probabilities of failure for the proofloaded and control groups are similar. The K is a shift factor relating the amount the proofloaded strength distribution must be shifted on the x-axis to give the same reliability as the control.

Simple 5th percentile comparisons, the advanced first order second moment (AFOSM) and numerical integration analysis methods were used to evaluate increases in allowable bending strength from proofloading in tension and compression. Proofloading in tension or in compression both produced significant increases in allowable bending strength for the Hem-fir grade. Proofloading in tension to a target 15 percent breakage level, or 2,838. psi, yielded for the survivors an increase of 72 percent in allowable bending strength. The allowable bending strength increased 60 percent due to a compressive proofloading to a target 15 percent breakage level.

The allowable bending strength increased as the proofloading level increased for both tension and compression proofloading with the Hem-fir grade. The southern pine visual stress grade did not show a consistent trend between proofloading level and improvement in allowable bending strength. The lack of a trend between proofloading level and allowable bending strength was attributed to possible sampling error.

The fifth percentile analysis method, the AFOSM method and numerical integration method were compared. For lumber strength comparisons, a simple fifth percentile analysis was not the preferred method. The AFOSM method and the numerical integration method provided identical results in terms of their application in adjusting allowable bending stresses. It was not possible to show that the approximate AFOSM method can be used exclusively in lieu of the numerical integration method for reliability calculations.

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INTRODUCTION

Lumber, like other building materials, has a great degree of variability in its strength. Knots and associated grain deviations, general grain slope, end splits, seasoning checks, and moisture content are just a few of the characteristics which cause strength variability. Lumber has a statistical distribution associated with its ultimate strength in bending, another for tension parallel-to-grain, and yet another for compression parallel-to-grain. It is these three strength distributions which are of major concern for structural design and the fifth percentiles are used to derive the allowable design strength values.

The main mechanical properties of lumber for structural design purposes are modulus of elasticity in bending (MOE), bending strength or modulus of rupture (MOR), tensile strength parallel-to-grain, shear strength, and compressive strength parallel and perpendicular to the grain. The stiffness property of MOE is positively correlated to all the other strengths in a positive way. Research has established correlations between bending and tension, and bending and compression strengths. The purpose of this research is not to determine the magnitude of these correlations but to utilize the fact that they exist.

Proof testing or proofloading is a process of applying a load to a specimen with the purpose of ensuring the strength is greater than a desired level. Any specimen, not of the pre-set strength level will be broken and therefore excluded from the sample. By excluding the weaker specimens, the strength distribution of the remaining specimens will be shifted to the right and have less variance than the strength distribution for the original group. The objective of the research is to determine the increase in allowable bending strength of lumber when proof tested at various levels in tension and compression.

The research objective can be accomplished by using differential reliability and the appropriate mathematical and statistical methods. A probability of failure for the proofloaded and the control distributions can be found and compared. When calculating probability of failure, the lower tail of the strength distribution is of major concern. The impact of the variability of lumber strength on the probability of failure makes it important to utilize the best statistical tools available for the analysis of lumber properties.

Reliability analysis as defined by Suddarth et al. (1978) consists of comparing mathematically associated load and resistance distributions to produce a probability of failure. The advantage of such an approach is that precision of the estimated probability of failure is limited only by data and not by mathematical procedures. Confidence in the probability of failure is greater when more is known about the true load and true resistance distributions, thus emphasizing the importance of good data bases for both load and resistances.

These analyses quantify any improvement in bending strength due to proof testing at various levels in tension and compression. The probability of failure for a control sample is found, then with an iterative approach, the bending strength values of the proofloaded sample distribution are artificially altered by a factor K until the probabilities of failure for the proofloaded and control lumber are similar. This K is a shift factor demonstrating the amount the proofloaded lumber strength values must be shifted to give the same probability of failure as the control. The loads used in this study to determine the probability of failure are the recommended snow and floor live load distributions from Thurmond et al. (1986). Using the three load cases derived by Thurmond et al. (1986), K factors are found that quantify the effect of proofloading in one stress mode on bending strength of the proof tested survivors. The K factor chosen is the one which provides the most conservative improvement of bending strength due to the proofloading treatment.

LITERATURE REVIEW

2.1 Engineering and Probability

A basic goal of structural engineering is to develop an optimum economic balance between economy and safety. There are many uncertainties in design for both loads and resistances. The variability of material properties provides uncertainties concerning the strength or resistance of a structure. Magnitude and location of loads and assumptions in stress analysis produce uncertainties in determining the loads on a structure. Theory used for modeling loading and performing structural analysis also adds a dimension of uncertainty. Uncertainties can be divided into two types, statistical as exemplified by the variability of wind or snow loads and nonstatistical such as intuitive judgement and simplifying assumptions.

The purpose of most engineering analyses is to provide information for the purpose of decision making. Because uncertainties exist in almost all engineering decision making, the contribution of probability theory provides a unified framework for the modeling of these uncertainties, and for their systematic analysis and updating in terms that are meaningful and suitable for the quantitative evaluation of risk. Proof testing to determine if a product meets a desired level of strength is one way to decrease risk by eliminating any of the product which does not meet a desired strength level.

2.2 Proofloading Applications

Proofloading has applications in many fields. Johnson (1980) discusses examples found in literature such as the glass for Skylab and the steel fabrication industry. Within the wood science area, finger-joints for glulam timber have been proofloaded to ensure strength. Process variables such as joint cutting, adhesive application, joint assembly and curing are all critical in beam strength for glulam lumber. Eby (1981) states that proofloading end-joints is an effective method to assure that beam strength meets an assigned value. He also describes the development of a machine for proofloading glulam lumber in bending and getting building code acceptance for proof testing by the glulam industry.

Australia began to use commercial proof grading of structural timber as early as 1981 with the use of continuous proof testing machines described by Wooster (1981). These machines have the capacity of 1000-2000 pieces a day. A constant bending moment is applied to the lumber as it passes through the machine. Because of the relatively low cost of the machine and its ease of use, many mills are using it along with visual grading to control variability in lumber grades. Leicester (1982) discusses how the difficulties associated with visual grading due to slope of grain and species identification can be compensated for with the proof testing procedure.

Optimum cost-benefit proof levels for commercial in-line production were proposed by Bechtel (1983). He found the optimum proofloading level may break as much as 10 percent of the lumber tested. His method required the knowledge of the lower tail of the strength distribution a priori or from test data to determine the optimum test load value. He also noted that the more tightly dispersed the strength distribution the fewer pieces will be broken at the optimum level. For machine stress rated (MSR) graded lumber the modulus of elasticity (MOE) can be used to set the proofloading level. The correlation between MOE and strength properties allows the optimum proofloading level to be set before proofloading begins.

2.3 Damage From Proofloading

The concern of damage due to proofloading is a valid one. The added cost from broken pieces and the possible damage to the remaining pieces have the potential of making proofloading undesirable. Added cost from pieces that are broken and any possible damage must be offset by the increase in the strength distribution of the surviving pieces. In a publication by Freas (1949), he discussed the use of high proofloads on specimens of ladder stock. His conclusions made many skeptical of proofloading and caused a long delay in the research of proofloading to ensure strength levels. More recent research experiments shed valuable insight into the theory of why and when damage occurs.

Studies concerned with the theoretical aspects of damage during loading yielded results consistent with experimental studies. Schaffer (1973) investigated Douglas-fir to determine bond rupture as a function of time during tensile loading at various temperatures. When tests on 1 by 1/8 inch strips of clear Douglas-fir were performed at room temperature, failure of all bonds occurred in approximately 7.7 seconds; at 7.5 seconds, 98 percent of the bonds were unbroken, and at 7.0 seconds no bonds were broken. From these results he concluded that unless the load is very close to the failure load, little or no damage occurred.

Gerhards (1979) proposed a linear cumulative damage theory that relates to proofloading. The theoretical relation indicates that loads that do not cause failure may have little effect on residual strength. From his theory a very small percentage may be weakened, and the remaining pieces will have a residual strength equal to their original strength.

More recent research on modeling wood damage accumulation from stochastic loads was conducted by Corotis and Sheehan (1986). Little significant damage accumulation occurred when using realistic structural designs and load models. In research by Gerhards and Link (1987) a ramp bending load was applied to a constant level to specimens of 2 by 4 Douglas-fir for a duration of 4.65, 33.9 and 220 days. Surviving specimens were loaded to failure in bending. All failed at ramp loads higher than the constant load, except for one specimen having a long, moderate slope-of-grain split. The long load duration could be considered to be equivalent to a proofloading of a very long duration. It was concluded that a long load duration test does not cause damage which will result in failure below the level of the original constant load.

In an experiment using reverse proofloading in bending of lumber by Marin and Woeste(1981), 2 by 4 No. 2 Dense KD Southern Pine did not show any significant damage due to proofloading at the approximate fifth percentile, or 3366 psi. In another experiment by McLain and Woeste (1986) 2 by 6 Dense Select Structural, No. 1 Dense, and No. 2 Dense KD15 Southern Pine were proofloaded in tension at 1.6 to 2.0 times the published rate. That research resulted in the conclusion that damage in surviving lumber due to the proof load used was nonexistent or at worst minimal.

Woeste et al. (1987) conducted a research experiment with 2 by 4 No. 2 Dense KD Southern Pine using both single and reverse bending proofloads. No damage due to the proofloading was detected.

Only relatively low levels of proofloading are of interest for commercial applications. Based on published research it is valid to assume there is no appreciable damage to surviving lumber due to proof loading in tension or bending at these low load levels.

2.4 Loading Rates For Proof Testing

The rate of loading for proof testing is a critical issue for commercial applications. Using the ASTM D 198-84 rates for loading each specimen, the cost associated with proofloading would be prohibitive due to the amount of time required. Spencer (1979) published work on the rate of loading effect in bending for 2 by 6 No. 2 and better grade Douglas-fir lumber. He found the rate of loading effect was dependent on the strength of the lumber. DeBonis et al. (1980) tested the rate of loading effect in bending on 2 by 4 No. 2 KD Southern Pine. Both Spencer (1979) and DeBonis et al. (1980) reported that the inherently stronger specimens showed an increasing strength for increased rates of stressing. For the weaker specimens the differences were not great and in a few cases strength decreased as the loading rate was increased.

Gerhards et al. (1984) found that for 2 by 6 Douglas-fir meeting the visual requirements of tension lamination for glulam beams when tested in tension at rates 10 to 25 times the ASTM rate, the weaker strength lumber appeared lowered in strength and the stronger lumber appeared to gain strength. They concluded that caution should be used when writing standards for faster rates of loading in tension. McLain and Woeste (1986) used 2 by 6 No. 2 KD Southern Pine proofloaded in tension to establish a rate of loading adjustment factor for a rate 20 times the ASTM standard. They defined a rate adjustment curve for testing at the faster rate showing an increase in strength as the specimen strength increased. A load rate increase applied to all the lumber strengths as opposed to the average and stronger pieces of the Gerhards et al. (1984) experiment using Douglas-fir.

Bender et al. (1987) studied 2 by 10 No. 2 KD Southern Pine in tension. He proposed a rate adjustment curve to relate stress levels between the faster rate and the ASTM load rate. These adjustments will allow commercial applications using proofloading in tension to have confidence with the results at rates faster than the ASTM-D198 rate.

2.5 Correlations of Lumber Strength Properties

The major structural properties of lumber are modulus of elasticity in bending (MOE), bending strength or modulus of rupture (MOR), tensile strength parallel-to-grain, shear strength, and compressive strength parallel and perpendicular to the grain. The stiffness property of MOE is correlated to tensile strength, bending strength, and compressive strengths in a positive way. Hoyle (1966) conducted research on the correlation between MOE and strength properties such as bending strength and tension strength parallel-to-grain. His research led the way for machine stress grading of lumber. Research publications on the correlation of MOE with bending, tension and compression since Hoyle (1966) are too numerous to survey. Recently Galligan et al. (1986) published comprehensive regressions of bending, tension and compression strength on E for various sizes and grades of southern pine, Hem-fir and Douglas-fir. Machine stress rated (MSR) lumber has become widely used and the process is based on these correlations in important commercial species.

The effect of the correlation between bending and tension or between bending and compression is very important because of their implications in truss design. For example the loads causing bending stress may simultaneously cause tension stress. Research by Suddarth et al. (1979) on a single element member subject to combined bending and tensile stress suggest that the degree of correlation between bending and tension strength has the effect of making the load capacity more variable.

The degree of correlation is difficult to determine when both properties are measured by destructive means. When a specimen is broken in bending to determine bending strength, the tensile strength parallel-to-grain can not be found for that specimen. In other words, the specimen cannot be broken twice. A non-destructive means of estimating correlations of these strength properties is by the use of statistical and mathematical methods. Johnson and Galligan (1983) used a basic approach which depends upon identifying the correlation between the residuals in two regressions used to predict two strength properties from the same prediction variable MOE. This approach based on the correlation of residuals also utilized information gained from proofloading. The correlation between the residuals of bending and tension can be thought of as conditional correlation between bending and tension. Because bending and tension cannot be observed on a single specimen, large samples of 2 by 4 1.5E-1650f MSR Hem-fir and No. 2 KD Southern Pine lumber were randomly subdivided so they could be tested in various failure modes and proofload levels. One set each was tested exclusively in bending, tension and compression and the remaining sets were proofloaded in tension or compression with survivors failed in bending. Target proofload levels were set at 5 and 15 percent of the bending failure level.

Evans et al. (1984) used a simulation procedure involving proofloading a specimen in one failure mode and then failing the survivors in another mode to determine the effect of the error and variance of the correlation estimate of strength properties as a function of proofloading levels. The correlation between tension and bending is necessary for using an increased design value in bending derived from proofloading in tension.

2.6 Probability Distributions of Lumber Properties

Information about a lumber population is found through lumber sampling. The probability distribution that most closely fits a sampling is an estimate of the distribution for the population sampled. As the sample size increases, confidence that the distribution estimate represents the population is increased. Galligan et al. (1986) suggests that a sample size of at least 80 specimens is needed to determine a distribution that closely resembles the population distribution.

At the present, ASTM D 2915-84 provides a standard method for evaluating allowable properties for grades of structural lumber which uses either nonparametric or parametric methods. Haberman and Ethington (1975) and Warren (1974) discuss nonparametric methods for establishing lumber allowable stresses. They found that, in addition to being conservative, the results are dramatically influenced by the true underlying strength distribution. Due to these limitations resulting from using a nonparametric approach, parametric methods are the better choice. Also the calculations needed for reliability-based design methods can be readily made from parametric distributions. Because reliability is strongly influenced by the distribution extremes it is important that an appropriate distribution is selected to model the data.

Lumber properties data are commonly modeled by normal, lognormal, or 3-parameter Weibull distributions. The normal is symmetrical and has well defined properties allowing calculation of confidence intervals, but it extends to negative values which are not representative of strength data. The lognormal is nonnegative and lends itself to easy calculation of various statistics. It is positive skewed and has limited flexibility. The 3-parameter Weibull is very flexible, accounting for symmetricalness and positive or negative skewness. It can be fitted to many diverse distributions through selection of its three parameters: location, scale, and shape. Its disadvantage is its complexity; however, a value for any percentile of the distribution can be found with hand calculations.

To determine if a distribution fits the sample data, a number of tests can be performed such as the Chi-square test, the Kolmogorov-Smirnov test and others described by Law and Kelton (1982). The Chi-square test is essentially a comparison of the data histogram with a fitted density function and can be used on any type distribution. Another way to distinguish between distributions is by using the maximum log-likelihood values calculated for the distributions of interest. Dumonceaux et al. (1973) derived a likelihood ratio test for discriminating between models with unknown location and scale parameters. Dumonceaux and Antle (1973) used this ratio test to construct a hypothesis test to discriminate between the lognormal and Weibull distributions.

Galligan et al. (1986) used the Chi-square goodness-of-fit test and visual assessment to select the distribution best describing the data. In cases where the Chi-square and visual assessment did not provide an obvious best choice, the distribution with the maximum log-likelihood value was chosen.

2.7 Probability Distributions For Loads

Uncertainties in loads can come from inherent randomness, transformations of loads into load effects, and in the representation of a 3-dimensional structure by a series of members and connections. Roof loads and live loads in residential housing were analyzed by Thurmond (1982). He determined a method by which the loads could be used with lumber strengths to compare contrasting lumber data sets based on the concept of equal reliability. All loads were maximum lifetime loads with the design life assumed to be fifty years.

Snow loads are often the major load considered in the design of roofs. Thurmond et al. (1984) found that load parameters for the maximum lifetime roof load for numerous locations follow a lognormal distribution with a ratio of mean snow load to nominal snow load of 0.69 and a coefficient of variation equal to 0.44. The nominal snow load was the design snow load for the roof. The nominal dead load for the roof was the design dead load. The dead load distribution was assumed to be lognormal with a coefficient of variation of 0.1.

Thurmond et al. (1984) analyzed past research to determine the distribution which best models the live loads in residential housing. He found that live loads in residential housing follow the Extreme Value Type I distribution with ratios of the mean live load to nominal live load of 0.94 and 0.73 with respective coefficients of variation of 0.21 and 0.19. The reason for two load cases, both of which are based on the research by Chalk and Corotis (1980), was due to different methods of combining load parameters. Thurmond et al. (1986) suggest that the two cases used for reliability analysis purposes will account for the shortcomings due to limited load survey information and knowledge about the actual load combinations.

The dead load for residential housing is defined as the weight of the structure and all permanently fixed equipment. The mean dead load depends on the type of structure and its use. Thurmond et al. (1984) recommended that the mean dead load be calculated for each application to determine the ratio of mean dead load to nominal dead load.

To determine the probability of failure for a given load and strength distribution, the distributions must be compatible and have the same units. Thurmond et al. (1984) employed a process to create compatibility. The design loads were assumed equal to the allowable design strength at a design point. The load distribution was therefore positioned relative to the design point by the mean load to nominal load ratio. The dead load and live load must be represented in the correct proportion to the total load. The equations used to determine the parameters of the total load distribution for a bending member are:

$$\mu_T = \frac{D_n}{T_n}(\overline{D}/D_n)\, F_b\,(LDF) + \frac{L_n}{T_n}(\overline{L}/L_n)\, F_b\,(LDF) \qquad [1]$$

$$\Omega_T = \frac{\sqrt{(\mu_D\,\Omega_D)^2 + (\mu_L\,\Omega_L)^2}}{\mu_T} \qquad [2]$$

where:

μ_T = mean total lifetime load, psf

Ω_T = coefficient of variation of the total lifetime load

D_n = nominal dead load, psf, design value

L_n = nominal live load, psf, design value

T_n = total nominal load ($D_n + L_n$) , psf

$\overline{D}/D_n$ = normalized mean of the dead load distribution

$\overline{L}/L_n$ = normalized mean of the maximum lifetime live load distribution

μ_D = mean dead load

Ω_D = coefficient of variation of the dead load

μ_L = mean maximum lifetime live load

Ω_L = coefficient of variation of the live load

F_b = the adjusted allowable design value

LDF = load duration factor

The total load is the sum of the dead and live loads. Because the live load has significantly more variability than the dead load, Thurmond (1982) showed the total load can also be described by the distribution for the live load. Thus, for residential construction, the total load is approximately a Extreme Value Type I distribution for floors and a lognormal distribution for roofs.

2.8 Reliability Concepts and Analyses

Since engineering designs are formulated under varying degrees of uncertainty, safety factors based on past experience and judgement have been used to manage uncertainty under a deterministic format. The deterministic format has a number of limitations. It does not result in consistent risk, uncertainties cannot be quantified, and the actual level of risk is undefined. Early design methods used the fifth percentile of the strength distribution to maintain an acceptable level of reliability. "Normal duration" lumber design values are based on a fifth percentile with a general adjustment factor of 2.1 for bending and tension, and 1.9 for compression. A load duration factor of 1.33 for wind, 1.15 for snow, 1.0 for live and 0.9 for permanent loads allows for variations in end-use conditions. Deterministic methods use adjustment factors to account for varying distributions which impact on reliability.

Probabilistic methods for wood design were introduced by Bonnicksen and Suddarth (1966) with significant additional contributions by Zahn (1977). With the probabilistic approach, loads and resistances are treated as random variables, distributions are obtained from available data, and the design can be based on "acceptable" risk or probability of failure. The probability of failure is defined as an event were the resistance or strength (R) is less than the load (S).

$$p_f = \mathrm{Prob}\,(R < S) \qquad [3]$$

For load and strength distributions which are continuous and mutually independent, the probability of failure equation is:

$$p_f = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{s} f_R(r)dr \right] f_S(s)ds \qquad [4]$$

where:

f_R (r) is the probability density function of the resistance distribution.

f_S (s) is the probability density function of the load distribution.

The theoretical limits of integration are negative to positive infinity, however, the strength distribution can not be nonnegative so the lower limit of that integration would be zero.

A calculated probability of failure, although not an exact representation of structural safety, can be used as a benchmark for comparing various designs or loads and resistance combinations. The systematic comparison of the probability of failure of one design situation to the probability of failure of a second design was named differential reliability by Suddarth et al. (1978). Thurmond et al. (1986) used a K factor which provides a quantitative comparison of the strength distributions. By changing the parameters of a distribution, strength values can be shifted up or down the x-axis without changing the distribution shape. The amount of shift in strength on the x-axis which yields approximately the same probability of failure for a comparison case as the benchmark provides the K factor.

Reliability methods can accommodate design procedures at varying levels. Thoft-Christensen and Baker (1982) describe three levels. Level-1 is a semi probabilistic method where partial safety factors are used. Load and resistance factors are obtained from reliability analyses but the overall format is deterministic. Level-2 is an approximate probabilistic method. Safety checks are performed at selected points of the failure boundary. It is assumed that the distributions are approximate, and the resistance and load are independent. Two methods of this approximate probabilistic method are the first order second moment (FOSM) and the advanced first order second moment (AFOSM) methods. Ellingwood et al. (1980) used approximate probabilistic methods to establish minimum design loads for building code requirements. Level-3 is an exact numerical method for evaluation of the actual distributions. By using numerical integration on the resistance and load distributions a probability of failure can be found.

The approximate methods use the first and second moment of the governing function, which represents the mean value and variance that are used to characterize uncertainty of the variables and the linearizations performed for the analysis. A limit state equation, which is continuous and linearized at some point is used.

$$Z = g\,(X_1^{\times}, X_2^{\times}, \dots, X_n^{\times}) \;/\; \sum (X_i - X_i^{\times}) \frac{dg}{dX_i} X_i^{\times} \qquad [5]$$

where:

$g\,(X_1^{\times}, X_2^{\times}, \dots, X_n^{\times})$ is the governing function and

$(X_1^{\times}, X_2^{\times}, \dots, X_n^{\times})$ identifies the point of linearization.

The mean value of the random variable is used as the linearization point and the reliability index is computed independent of probability distributions in the FOSM method. Therefore, the index is dependent only on the central tendency and dispersion of the limit state function. The accuracy for this mean value method is not as good as when the actual distributions are taken into consideration. Two shortcomings of the FOSM method are that errors are introduced when $g\,(X_1^{\times}, X_2^{\times}, \dots, X_n^{\times})$ is nonlinear and the mean point is some distance from $g\,(X_1^{\times}, X_2^{\times}, \dots, X_n^{\times}) = 0$. Also the FOSM method is not invariant, that is beta is dependent on the algebraic formulation of $g\,(X_1^{\times}, X_2^{\times}, \dots, X_n^{\times})$. For example, given $g(A,P,R) = 0$, the axial stress equation, $P/A - R = 0$, will give different results than the force equation, $P - R \times A = 0$.

The advanced first order second moment (AFOSM) method linearizes the limit state function at a point on the failure surface, and it also takes into account the distribution of the variable. The problem of invariance is circumvented because the first-order approximations are evaluated at a design point on the failure surface (Ang and Tang, 1984). Hasofer and Lind (1974) transform the X_i variables to reduced variables x_i with zero mean and unit standard deviation by the equation

$$x_i = \frac{X_i - \overline{X}_i}{\sigma_{xi}} \qquad [6]$$

This transformation yields a limit state equation of

$$g_1(x_1, x_2, \ldots, x_n) = 0 \qquad [7]$$

The reliability index, β, is defined as the shortest distance between the $g_1(x_1, x_2, \ldots, x_n) = 0$ surface and the origin. The point on the surface where the shortest distance occurs is called the design point. The design point is found by solving the following equations;

$$\alpha_i = \frac{\frac{dg_1}{dx_i}}{\sqrt{\Delta}} \qquad [8]$$

$$x_i^{\times} = -\alpha_i \ \beta \qquad [9]$$

$$g_1(x_1^{\times}, x_2^{\times}, \ldots, x_n^{\times}) = 0 \qquad [10]$$

where:

$\Delta = \sum(\frac{dg_1}{dx_i})^2$

α_i 's are the direction cosines which are adjusted to minimize beta.

β is a relative measure of reliability.

An advantage of this AFOSM method is that nonnormal variables can be transformed into equivalent normal variables thru the design point x_i on the failure surface. Rackwitz and Fiessler

(1976) used the first and second moment of the normal variable such that at point x_i, the probability density function (pdf) and cumulative density function (cdf) of the original and approximating normal variable are the same. The formulas for the standard deviation and the mean of the approximating normal distribution are shown here.

$$\sigma_i^N = \frac{\phi\,[\Phi^{-1}\{F_i(x_i^{\times})\}]}{f_i(x_i^{\times})} \tag{11}$$

$$\bar{x}_i^N = x_i^{\times} - \Phi^{-1}\{F_i(x_i^{\times})\}\,\sigma_i^N \tag{12}$$

where:

σ_i^N = the standard deviation of the approximating normal distribution

$\bar{x}_i^N$ = the mean of the approximating normal distribution

f_i = the non-normal pdf of x_i

F_i = the non-normal cdf of x_i

ϕ = the standard normal density function

Φ = the standard normal cumulative density function

This transformation to normal equivalents is done by approximating the distribution of x_i by a normal distribution at the design point x_i on the failure surface. Using these normalized parameters for the resistance and load distributions, an approximation of the true probability of failure is found.

ANALYTICAL PROCEDURE

3.1 Lumber Used

This reliability analysis uses data from a report by Galligan et al. (1986) that characterizes the properties of 2-inch softwood dimension lumber with regressions and probability distributions. A machine stress rated (MSR) grade of 2 by 4 1.5E-1650f Hem-fir a visual grade of 2 by 4 No. 2 KD Southern Pine were chosen because they are common in house trusses. The Hem-fir was collected at a production mill in the Cascade range of Washington and the southern pine in Oklahoma. The testing was done at Washington State University after moisture content stabilized to 12 plus or minus 2 percent. For each species, selection was made at random to make uniform groups for predetermined testing. For each species, groups of 80 pieces were broken in bending, tension, and compression. Two groups of 120 pieces each for both species were proofloaded in tension with target breakages of 5 and 15 percent. Likewise, two groups of 120 pieces each for both species were proofloaded in compression with target breakages of 5 and 15 percent. The groups of 120 pieces were failed in bending after proofloading to estimate the correlation of lumber strength properties.

The same ten groups of lumber as described for strength correlation determination were used as part of this reliability analysis. The groups from each species where 80 pieces were broken in bending were used to establish the bending strength distributions of the controls. This control group and the four proofloaded groups of each species were used to determine the effect of proofloading on bending strength.

The data files were identified as follows:

CNBH - The CoNtrol Bending Hem-fir.

T5H - Proofloading in Tension at the target 5 percent breakage Hem-fir.

T15H - Proofloading in Tension at the target 15 percent breakage Hem-fir.

C5H - Proofloading in Compression at the target 5 percent breakage Hem-fir.

C15H - Proofloading in Compression at the target 15 percent breakage Hem-fir.

CNBS - The CoNtrol Bending Southern Pine.

T5S - Proofloading in Tension at the target 5 percent breakage Southern Pine.

T15S - Proofloading in Tension at the target 15 percent breakage Southern Pine.

C5S - Proofloading in Compression at the target 5 percent breakage Southern Pine.

C15S - Proofloading in Compression at the target 15 percent breakage Southern Pine.

Table 3.1 is a summary of the lumber groups used in the reliability analysis showing the total pieces in the group, the number and percent broken by proofloading, and the number of remaining pieces broken in bending.

3.2 Distribution Selection

Lognormal and 3-parameter Weibull distributions were fitted to each data set to determine which distribution best describes the bending strength for each of the 10 groups of lumber. The Chi-square goodness-of-fit test at the 5 percent level was first conducted. If neither distribution

Table 3.1 Summary of the lumber groups used in the reliability analysis listing the sample size, number and percent broken by proofloading and the number of remaining pieces broken in bending.

2 by 4 1650f-1.5E Hem-fir

	T5H	T15H	C5H	C15H	CNBH*
Sample size	120	120	120	120	80
Pieces broken in proofloading	9	16	10	29	-
Percent broken in proofloading	7.5	13.3	8.3	24.2	-
Remaining pieces broken in bending	111	104	110	91	-

2 by 4 No. 2 KD Southern Pine

	T5S	T15S	C5S	C15S	CNBS*
Sample size	120	120	120	120	80
Pieces broken in proofloading	8	18	6	16	-
Percent broken in proofloading	6.7	15.0	5.0	13.3	-
Remaining pieces broken in bending	112	102	114	104	-

* The bending controls were not proof tested.

was rejected, then the maximum log-likelihood value was used to choose between the lognormal and 3-parameter Weibull distributions, as suggested by Galligan et al. (1986). An overlay of the fitted distribution on the histogram of the data was also used for a visual assessment of the fit. Visual assessment is discussed and recommended by Law and Kelton (1982). A summary of the Chi-square goodness-of-fit test and maximum log-likelihood values for each group is shown in Tables 3.2 and 3.3. The distribution chosen as best describing the data is the 3-parameter Weibull distribution for each of the Hem-fir groups and the lognormal distribution for each of the southern pine groups. Tables 3.2 and 3.3 also gives the parameters of the selected distributions. Figures 1 through 10 are the plotted histogram with the best fitting distribution superimposed on each of the 10 data sets.

The maximum log-likelihood calculations do not take into account the number of distribution parameters, therefore, for equal log-likelihood values, the lognormal distribution is preferred because it has one less parameter than the 3-parameter Weibull distribution. For the two cases, tension proofloading at 5 percent and compression proofloading at 5 percent, the Chi-square goodness-of-fit test rejected the 3-parameter Weibull distribution even with the maximum log-likelihood value showed a 1 or 2 point advantage. The southern pine data from the control and tension proofloading at 15 percent both fit the lognormal and 3-parameter Weibull distributions equally well based on the Chi-square test, visual assessment, and nearly equal log-likelihood values. Therefore, the lognormal distribution was chosen for these two groups because for all other groups of southern pine, the lognormal distribution was selected.

3.3 Fifth Percentile Analysis

The fifth percentile value for each group of lumber was calculated from the distribution best representing the data. The following formulas yield the fifth percentile value for the 3-parameter Weibull and lognormal distributions:

Table 3.2 Summary of the Chi-square goodness-of-fit test and the maximum log-likelihood values for the Hem-fir groups. Also listed are the distributions selected and their paramenters.

2 by 4 1650f-1.5E Hem-fir

	T5H	T15H	C5H	C15H	CNBH
Sample size	111	104	110	91	80
Chi-square α = (.05)					
Weibull	FTR*	FTR	FTR	FTR	FTR
Lognormal	FTR	FTR	FTR	FTR	FTR
Log-likelihood					
Weibull	-207	-198	-224	-171	-164
Lognormal	-210	-201	-227	-174	-169
Distribution selected	Weibull	Weibull	Weibull	Weibull	Weibull
Parameters					
Location, μ (ksi)	2.539	3.660	2.207	2.285	0.375
Scale, σ (ksi)	4.282	3.640	4.851	5.050	6.388
Shape, η	2.590	1.975	2.421	3.114	3.284

* Failed to reject, FTR.

Table 3.3 Summary of the Chi-square goodness-of-fit test and the maximum log-likelihood values for the southern pine groups. Also listed are the distributions selected and their paramenters.

2 by 4 No. 2 KD Southern Pine	T5S	T15S	C5S	C15S	CNBS
Sample size	112	102	114	104	80
Chi-square α = (.05)					
Weibull	Rejected	FTR	Rejected	Rejected	FTR
Lognormal, LN	FTR*	FTR	FTR	FTR	FTR
Log-likelihood					
Weibull	-190	-170	-205	-184	-144
Lognormal, LN	-192	-171	-206	-181	-144
Distribution selected	LN	LN	LN	LN	LN
Parameters					
Scale, λ (ksi)	1.233	1.392	1.459	1.412	1.353
Shape, ζ	0.394	0.323	0.345	0.339	0.379

* Failed to reject, FTR.

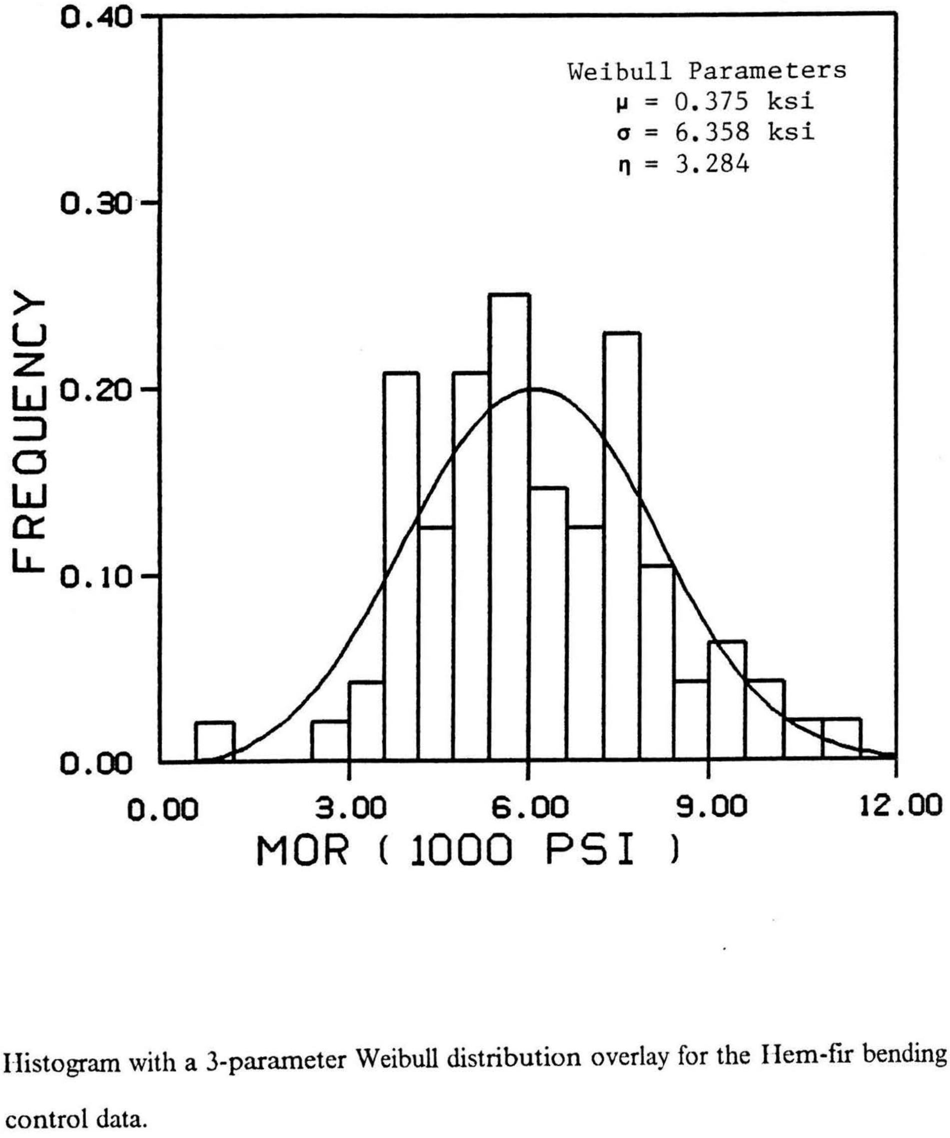

Figure 1. Histogram with a 3-parameter Weibull distribution overlay for the Hem-fir bending control data.

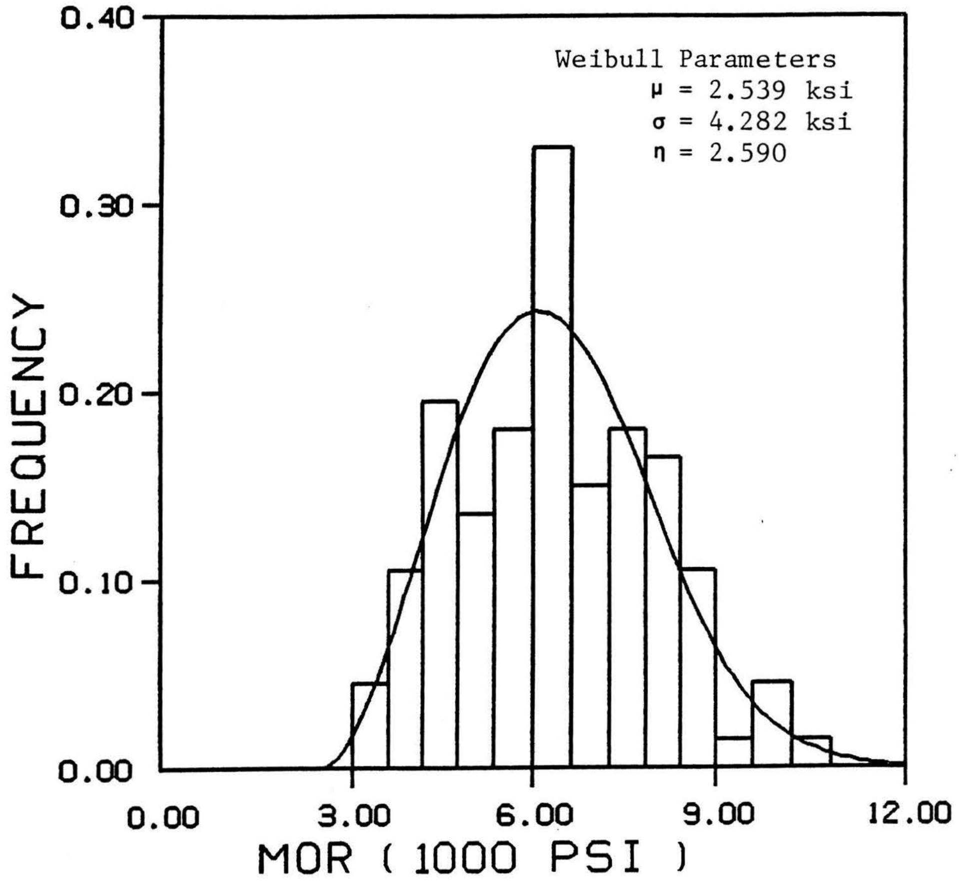

Figure 2. Histogram with a 3-parameter Weibull distribution overlay for the tension proofloading Hem-fir at a 5 percent target level.

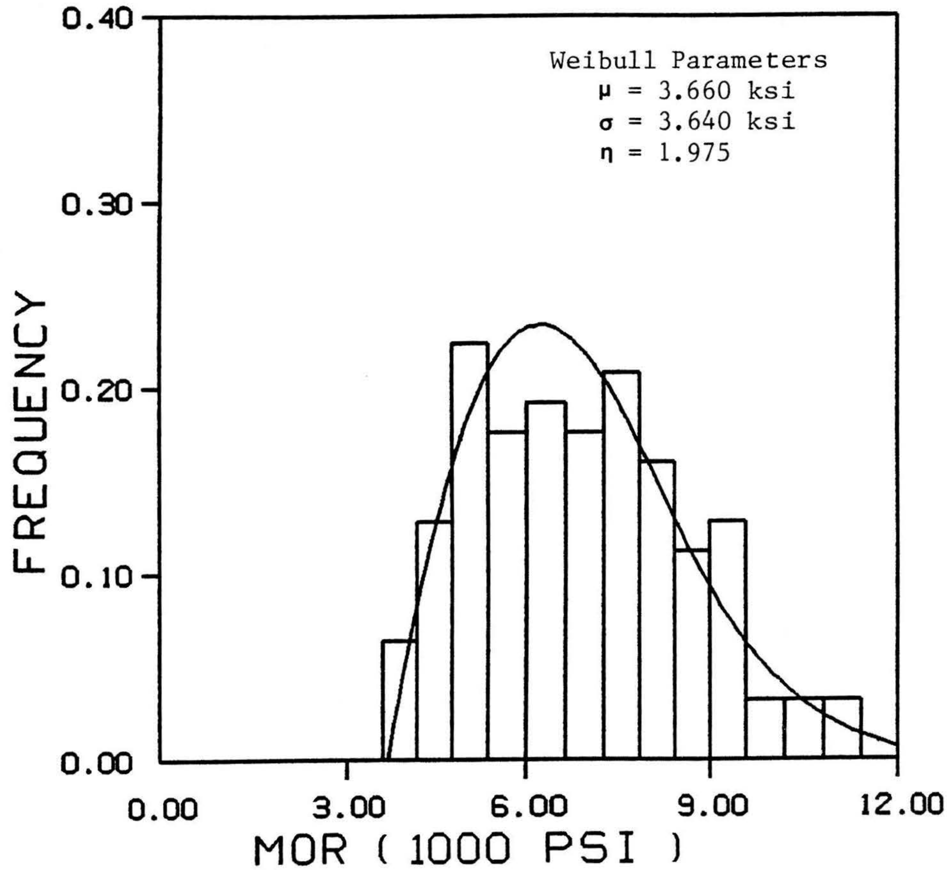

Figure 3. Histogram with a 3-parameter Weibull distribution overlay for the tension proofloading Hem-fir at a 15 percent target level.

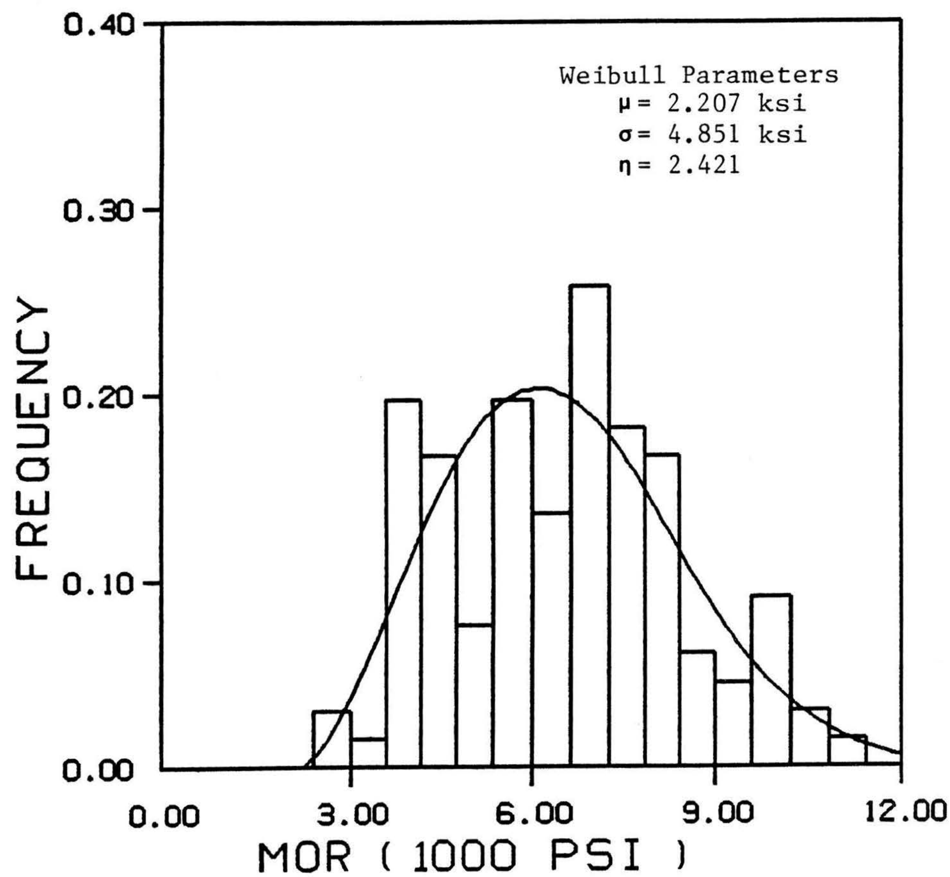

Figure 4. Histogram with a 3-parameter Weibull distribution overlay for the compressive proofloading Hem-fir at a 5 percent target level.

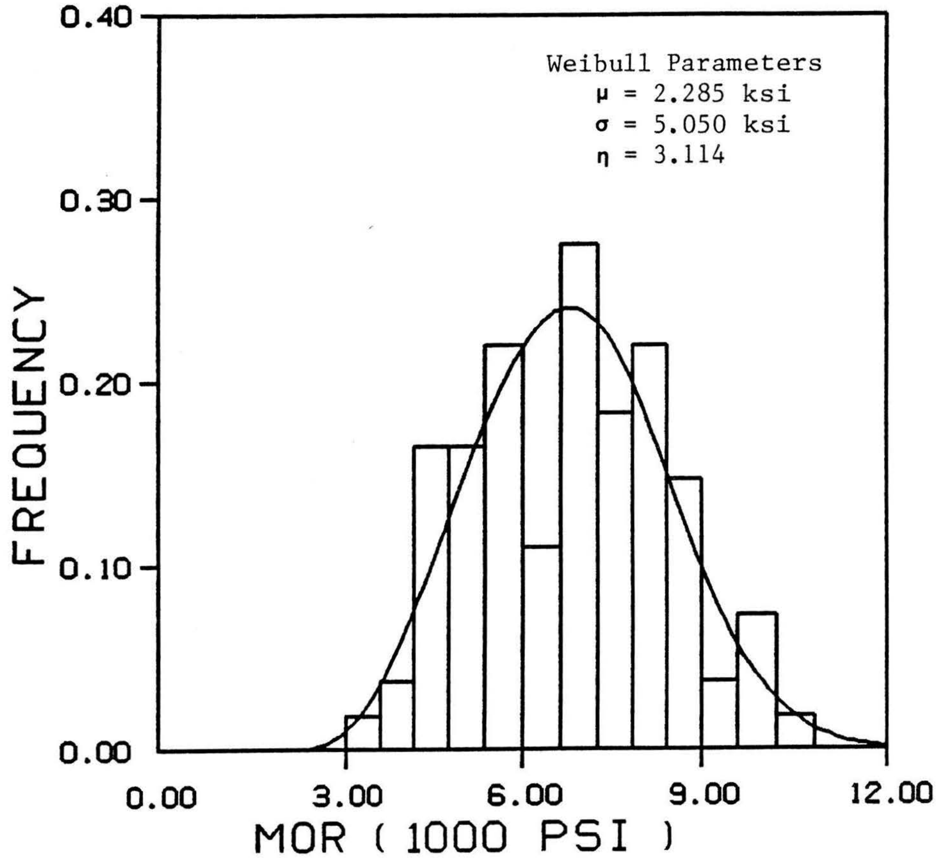

Figure 5. Histogram with a 3-parameter Weibull distribution overlay for the compressive proofloading Hem-fir at a 15 percent target level.

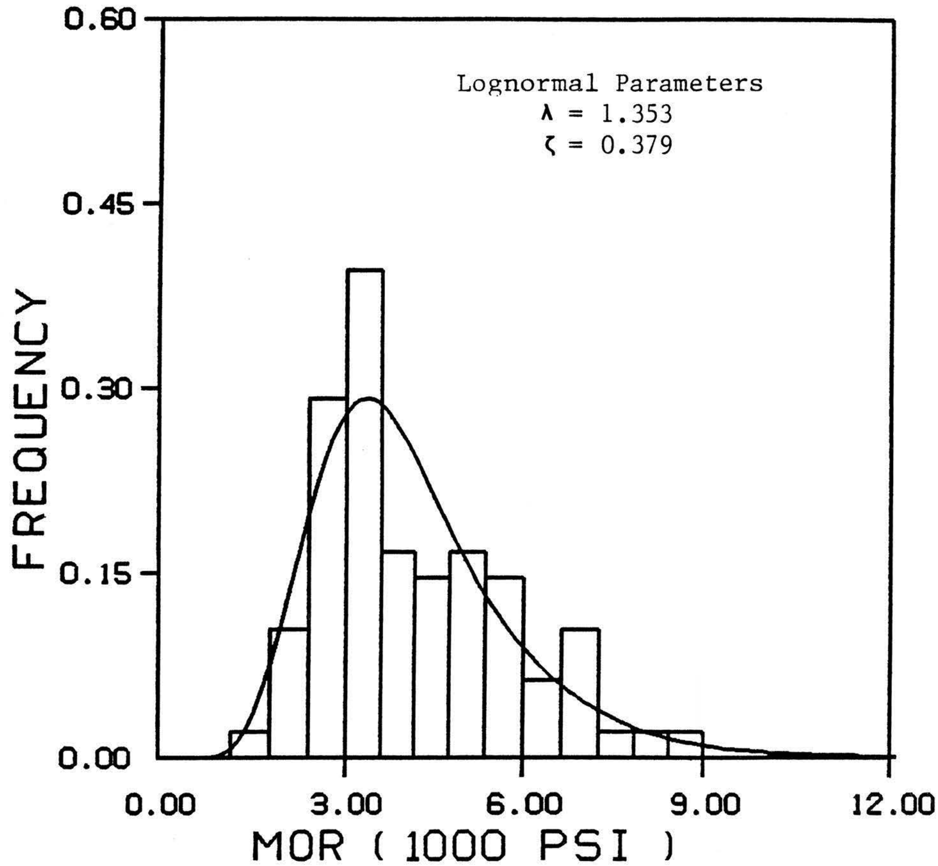

Figure 6. Histogram with a lognormal distribution overlay for the southern pine bending control data.

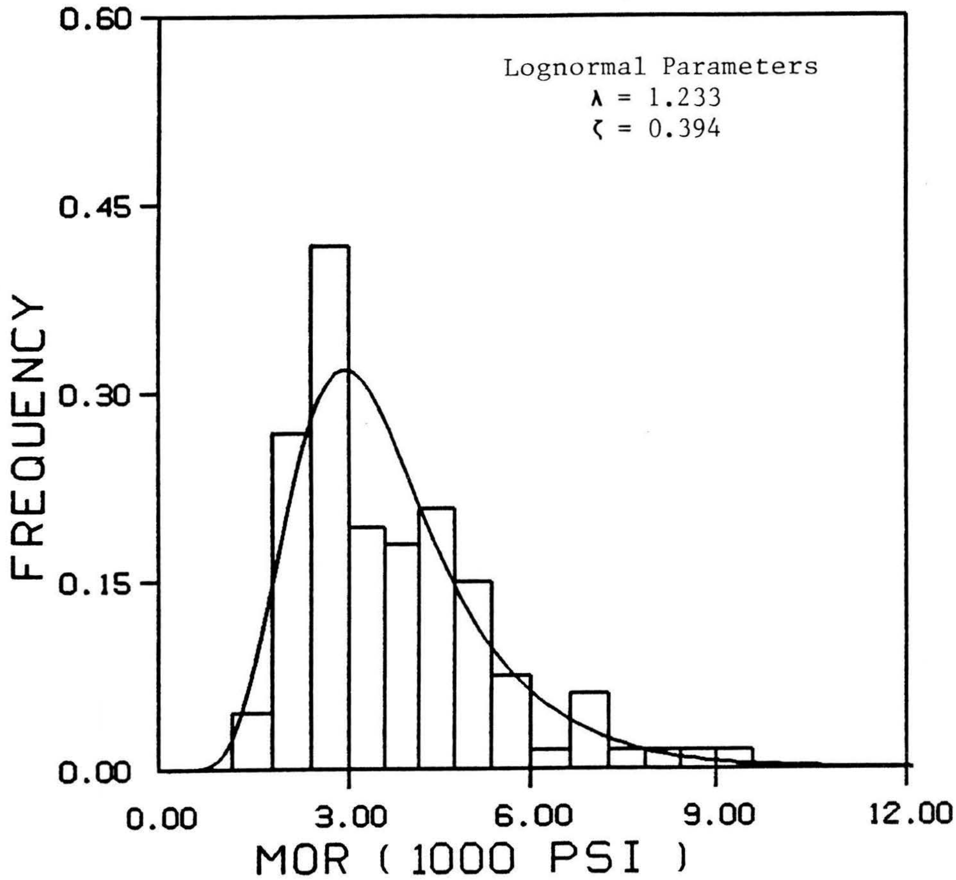

Figure 7. Histogram with a lognormal distribution overlay for the tension proofloading southern pine at a 5 percent target level.

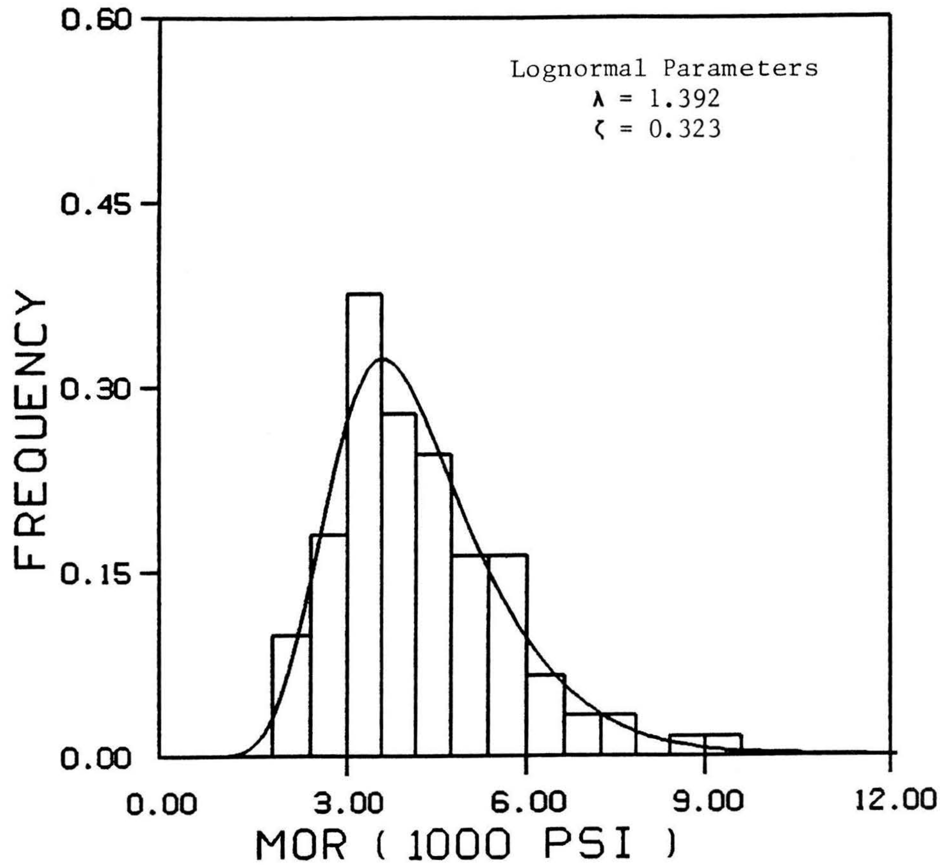

Figure 8. Histogram with a lognormal distribution overlay for the tension proofloading southern pine at a 15 percent target level.

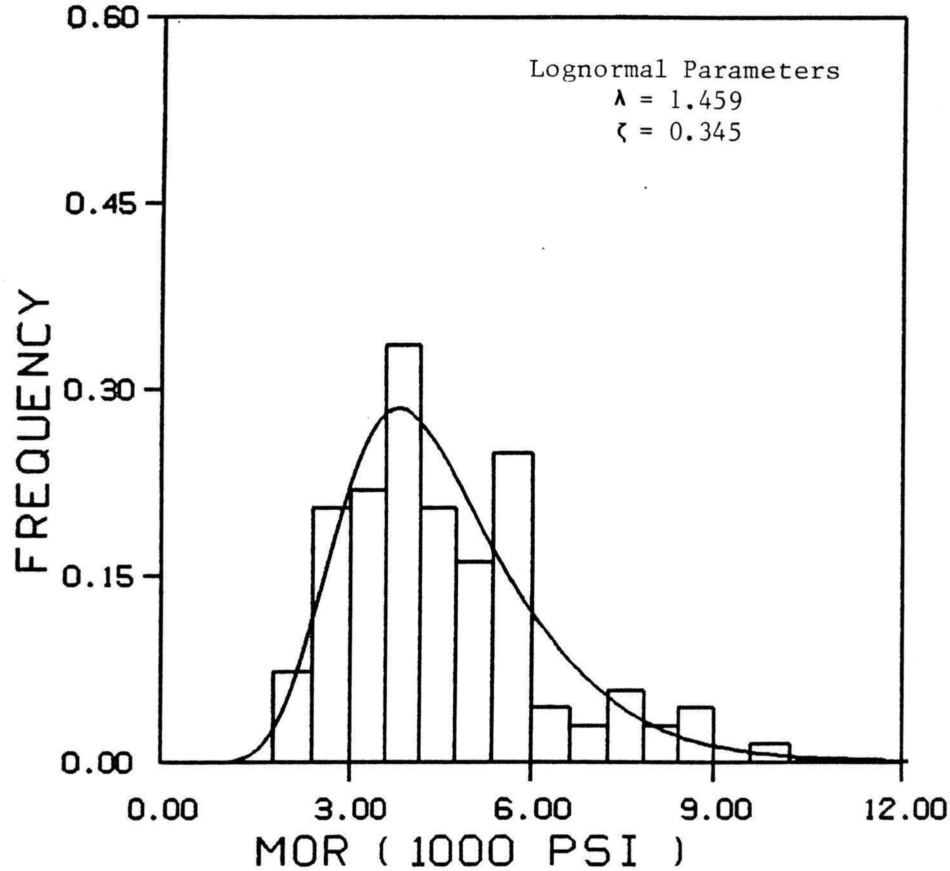

Figure 9. Histogram with a lognormal distribution overlay for the compressive proofloading southern pine at a 5 percent target level.

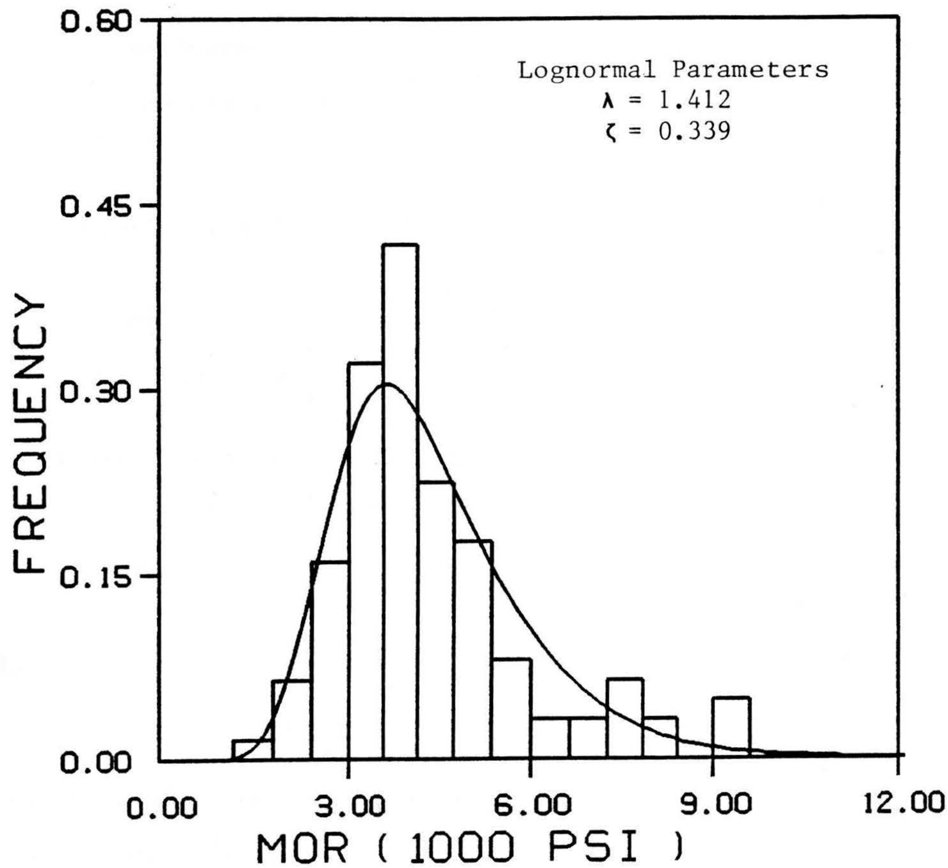

Figure 10. Histogram with a lognormal distribution overlay for the compressive proofloading southern pine at a 15 percent target level.

3-parameter Weibull:

$$x_{.05} = \mu + \sigma \ (\ln .05)^{\frac{1}{\eta}} \qquad [13]$$

where:

μ = location parameter

σ = scale parameter

η = shape parameter

Lognormal:

$$x_{.05} = \exp \ [\lambda - 1.645 \zeta] \qquad [14]$$

where:

λ = scale parameter

ζ = shape parameter

Table 3.4 lists the fifth percentile values for each group.

3.4 Loads

To determine probability of failure, appropriate load distributions must be calculated to be used with the resistance or strength distributions. Because the data base is from 2 by 4 lumber, which is routinely used in residential truss fabrication, residential housing loads are selected to use with roof and floor truss analyses.

Thurmond et al. (1986) recommended using three load cases. The following equations were used to determine the parameters of the distributions for the recommended load cases.

Table 3.4 Summary of the fifth percentile results for both Hem-fir and southern pine.

2 by 4 1650f-1.5E Hem-fir

	T5H	T15H	C5H	C15H	CNBH
Fifth percentile, ksi.	3.90	4.47	3.63	4.23	2.96

2 by 4 No. 2 KD Southern Pine

	T5S	T15S	C5S	C15S	CNBS
Fifth percentile, ksi.	1.79	2.36	2.43	2.35	2.07

$$\mu_T = \frac{D_n}{T_n} (\overline{D}/D_n)\, F_b\, (LDF) + \frac{L_n}{T_n} (\overline{L}/L_n)\, F_b\, (LDF) \qquad [15]$$

$$\Omega_T = \frac{\sqrt{(\mu_D \Omega_D)^2 + (\mu_L \Omega_L)^2}}{\mu_T} \qquad [16]$$

where:

μ_T = mean total lifetime load, psf

Ω_T = coefficient of variation of the total lifetime load

D_n = nominal dead load, psf, design value

L_n = nominal live load, psf, design value

T_n = total nominal load ($D_n + L_n$) , psf

$\overline{D}/D_n$ = normalized mean of the dead load distribution

$\overline{L}/L_n$ = normalized mean of the maximum lifetime live load distribution

μ_D = mean dead load

Ω_D = coefficient of variation of the dead load

μ_L = mean maximum lifetime live load

Ω_L = coefficient of variation of the live load

F_b = allowable normal duration bending stress

LDF = load duration factor

To derive F_b the fifth percentile of the data from a ten minute test was divided by 1.6 to convert to a ten year normal duration and 1.3 to provide for safety. The product of these adjustments is the more familiar 2.1 adjustment factor which can be found in ASTM D245-81 (ASTM, 1987).

The recommended snow and live loads, based on research by Thurmond et al.(1986) are shown in Table 3.5. In addition to these loads, the ratio of mean dead load to nominal dead load must

Table 3.5 Recommended snow and floor live load distributions for the reliability analyses of lumber properties data from Thurmond et al. (1986).

Load	Distribution	$\overline{X}/X_n$	Ω_x
Snow	Lognormal	0.69	0.44
Load A	Extreme Value Type I	0.94	0.21
Load D	Extreme Value Type I	0.73	0.19

be determined for each application. The coefficient of variation of the dead load was assumed to be 0.10 as suggested by Thurmond et al. (1986).

The total lifetime load is equal to the sum of two random variables, live plus dead. Thurmond et al. (1986) found for reliability comparisons the resulting distribution to be approximated by the lognormal distribution due to the lognormal snow load having a large coefficient of variation and the dead load having a relatively small coefficient of variation. Likewise, for Extreme Value Type I live loads and lognormal dead loads the resulting distribution was found to be approximated by the Extreme Value Type I distribution due to the Extreme Value Type I live load having a large coefficient of variation and the dead load having a relatively small coefficient of variation.

For these analyses, the mean dead loads are calculated using a "typical" residential construction. The results are shown in Table 3.6, where the mean dead load for the top and bottom chords is calculated as 1/2 the total truss weight. The pad and carpet weight was calculated for a 12 oz. pad and a carpet with total weight of 45 oz. per square yard. Publications by Lumbermate Company, Anon. (1983) and Anon. (1986) along with the American Institute of Timber Construction (1985) were used for most weight estimates.

Timber Truss Housing Systems, Inc. of Roanoke, Virginia provided seven single story residential floor plans ranging from 950 to 2600 square feet for use in determination of the dead weight due to interior walls. The average of the seven residences contained 96 linear feet of wall per 1000 square feet of space. For interior walls with 24" o.c. studs weighing 1.3 psf., plus gypsum board weighting 2.0 psf. on each side, the total wall weight is calculated at 5.3 psf. Dead load for the interior walls is calculated by multiplying the weight of the wall surface by wall height and length per square foot. This results in a dead load of 4.1 psf. for the floor area in "typical" residential housing due to interior walls.

The reliability analyses use the strength data from each species group with the load data for the roof top and bottom chords. Likewise, the floor truss top chord with Load A and D from Thurmond (1986) and the floor truss bottom chord are also used with the strength data.

The control fifth percentile for Hem-fir and 20-10-10 loading for the roof were used to determine μ_T, the mean total lifetime load in ksi. The following values were used in equations 15 and 16 for calculating μ_T and Ω_T for the roof truss top chord.

F_b = 2.96/2.1 = 1.41 ksi

D_n = 10 psf, S_n = 20 psf, T_n = 30 psf

$\overline{D}/D_n$ = 5.3/10 = 0.53 from Table 3.6

$\overline{L}/L_n$ = 0.69 from Table 3.5 for snow loads

LDF = 1.15 for snow loads

The appropriate load for the roof 4/12 W-truss top chord is lognormal with parameters, μ_T equal to 1.033 ksi and Ω_T equal to 0.319.

For calculating μ_T for the roof truss bottom chord there is no snow or live load portion; therefore, the following equation and values were used for calculating μ_T and Ω_T for the roof truss bottom chord.

$\mu_T = \mu_D = (\overline{D}/D_n)\, F_b\, (LDF)$

$\Omega_T = 0.10$, the assumed dead load coefficient of variation from Thurmond et al. (1986)

F_b = 2.96/2.1 = 1.41 ksi

$\overline{D}/D_n$ = 5.1/10 = 0.51 from Table 3.6

LDF = 1.15 for snow loads

The appropriate load distribution for the roof truss bottom chord was lognormal with parameters, μ_T equal to 0.828 ksi and Ω_T equal to 0.10.

The floor truss was evaluated under 40-10-5 loading using Load A and Load D from Thurmond et al. (1986). Calculation of the parameters of the total lifetime load distribution associated with Load A requires using equation 15 and 16 with the following values:

$F_b = 2.96/2.1 = 1.41$ ksi

$\overline{L}/L_n = 0.94$ from Table 3.5

$\overline{D}/D_n = 8.1/10 = 0.81$ from Table 3.6

$D_n = 10$ psf, $L_n = 40$ psf, $T_n = 50$ psf

LDF = 1.0 for live loads

For Load A the floor truss top chord total lifetime load distribution was Extreme Value Type I with parameters, μ_T equal to 1.290 ksi. and Ω_T equal to 0.174.

The total lifetime load distribution parameters associated with Load D were calculated in the same way except $\overline{L}/L_n$ equals 0.73 from Table 3.5. For Load D the floor truss top chord load distribution was Extreme Value Type I with parameters, μ_T equal to 1.053 and Ω_T equal to 0.15.

The floor truss bottom chord total lifetime load distribution parameters were calculated in the same way as the roof truss bottom chord using $\overline{D}/D_n$ equal to 0.66 from Table 3.6. Table 3.7 is a summary of load distributions, their mean and coefficient of variation for each loading case for Hem-fir.

The values for southern pine were computed in the same way as those for Hem-fir, only the fifth percentile value was 2.07 ksi and thus F_b equals 0.986 ksi. A summary of the total lifetime load

Table 3.6 The mean dead loads assumed for a residential roof and floor system.

Roof truss top chord			
	2 X 4 top chord -	1.3 psf	
	1/2 inch plywood -	1.5 psf	
	235# asbestos shingles -	2.5 psf	
	Total -	5.3 psf	Nominal = 10 psf
Roof truss bottom chord			
	2 X 4 bottom chord -	1.3 psf	
	1/2 inch gypsum board -	2.0 psf	
	6 inch glass wool insulation -	1.8 psf	
	Total -	5.1 psf	Nominal = 10 psf
Floor truss top chord			
	2 X 4 top chord -	1.3 psf	
	3/4 inch T&G plywood -	2.3 psf	
	Pad and carpet -	0.4 psf	
	Interior walls -	4.1psf	
	Total -	8.1 psf	Nominal = 10 psf
Floor truss bottom chord			
	2 X 4 bottom chord -	1.3 psf	
	1/2 inch gypsum board -	2.0 psf	
	Total -	3.3 psf	Nominal = 5.0 psf

Table 3.7 Summary of load distributions, their mean and coefficient of variation for each loading case used with the Hem-fir data.

Roof truss	Load distribution	μ (ksi)	Ω
Top chord	Lognormal	1.033	0.319
Bottom chord	Lognormal	0.828	0.100

Floor truss	Load distribution	μ (ksi)	Ω
Top chord			
Load A	Extreme Value Type I	1.290	0.174
Load D	Extreme Value Type I	1.053	0.150
Bottom chord	Lognormal	0.932	0.100

distributions, their mean and coefficient of variation for each loading case using southern pine are found in Table 3.8.

These load distributions were used with the control bending strength distributions to determine a benchmark probability of failure for each species. The proofloaded bending strength distributions were then used with the same load distributions to compare with the benchmark probability of failure.

When analyzing a truss chord a combination of stresses will be present. For example bending and tension stresses are present in the lower chord of a roof or floor truss under gravity loads. The question here is how will the combined stresses affect reliability analyses of lumber properties.

The bending stress contribution to the combined stress index (CSI) in the chords of a floor truss can be found using research from Suddarth et al. (1981). A "typical" parallel floor truss, shown in Suddarth et at. (1981), was used to determine the percent of the CSI value attributed by bending stress. The equivalent column length to depth ratio in the chords of the truss did not exceed that for a short column in a highly stressed member; therefore, J equals 0 and Equation 18 was used to determine the percent of the total load effect resulting from the bending stress, f_b.

$$CSI = \frac{f_b}{F_b' - (J \times f_c)} + \frac{f_c}{F_c'} \qquad [18]$$

where:

CSI = combined stress index

f_b = M/S = stress at extreme fiber in bending, psi

M = applied moment, in-lb

S = section modulus, in^3

F_b' = design value for extreme fiber in bending adjusted for slenderness, psi

J = factor using l_e/d, the slenderness ratio

Table 3.8 Summary of load distributions, their mean and coefficient of variation for each loading case used with southern pine data.

Roof truss	Load distribution	μ (ksi)	Ω
Top chord	Lognormal	0.722	0.319
Bottom chord	Lognormal	0.578	0.100

Floor truss	Load distribution	μ (ksi)	Ω
Top chord			
Load A	Extreme Value Type I	0.901	0.174
Load D	Extreme Value Type I	0.735	0.150
Bottom chord	Lognormal	0.651	0.100

f_c = stress in compression parallel to grain, psi

F_c' = design value for compression parallel to grain adjusted for l_e/d, psi

When CSI is known and J equals 0 the percent contribution of bending stress to stress interaction can be found using f_b and F_b' because CSI is the sum of bending and compressive ratios. Under a 40-10-5 psf loading, the floor truss 2 by 4 top chord analyzed had the following values:

$f_b = M/S = 323/1.31 = 247$ psi

$F_b' = 1850$ psi

$J = 0$

Using equation 18 the bending stress contribution was 0.133 or 15.0 percent of the CSI value for the top chord. Likewise, under 40-10-5 psi loading the floor truss bottom chord had a CSI of 1.057 and a moment 179 in-lb. All other values were the same as those for the top chord. Therefore, for the floor truss bottom chord the bending stress contribution was 0.074 or 7.0 percent of the CSI value.

In a typical 4/12 W-truss under 20-10-10 psf loading, 55 percent of the stress interaction, CSI, will be contributed by bending stress for the top chord. This fact may not convert directly to a percentage of allowable bending stress used for a particular design because of the term J times f_c subtracting from F_b in the CSI equation. Yet often the panel point CSI controls where J equals 0. Thus it was a good assumption for these reliability analyses to use 55 percent of the load as a typical amount of bending stress in the top chord. Using the same approach, 50 percent of the total stress interaction was attributed to bending stress in the bottom chord. Table 3.9 lists the percent of load due to bending stress in each chord.

The partial loads will be used in the reliability analyses to determine a new set of K factors. The K factors from the partial loads are likely to be smaller than those were full bending loads were

Table 3.9 The percent of bending stress contribution to the CSI value for both both top and bottom chords in roof and floor trusses.

Roof truss	Percent of bending stress contribution
Top chord	55
Bottom chord	50

Floor truss	Percent of bending stress contribution
Top chord	15
Bottom chord	7

used. The larger K factor from full bending loads result in a smaller shift on the x-axis. When evaluating the increase of bending strength due to proofloading, the largest K factor was used in each reliability analyses to provide the most conservative case.

3.5 AFOSM Method

The AFOSM method was used with the selected distributions and design point to find a reliability index, β, for each control group. Beta is calculated from the AFOSM method so the corresponding probability of failure can be found in a standard normal distribution table. The original load and resistance distributions are "normalized" at the design point to find a normal distribution which approximates the original distributions' tail. To do this a normal distribution is found having the same probability density function (pdf) and cumulative density function (cdf) at the design point as the distribution being approximated. The normal distribution has two parameters and, because there are two equations, it is possible to find the parameters of a unique normal distribution which approximates the tail of the actual distribution. The parameters for the approximating normal distribution can be found using the following steps when either a 3-parameter Weibull or Extreme Value Type I distribution is being approximated.

1) Calculate ψ the pdf value at the design point.
2) Calculate Ψ the cdf value at the design point.
3) Calculate Φ^{-1}, the standard inverse normal, at Ψ.

4) $$\sigma^N = \frac{1.0}{\psi\sqrt{2.0\,\pi}} \exp\left(\frac{-[\Phi^{-1}(\Psi)]^2}{2.0}\right)$$

5) $$\mu^N = \text{Design Point} - \sigma^N[\Phi^{-1}(\Psi)]$$

When the distribution is lognormal finding the normal parameters is more direct.

$$\sigma^N = \zeta * \text{Design Point}$$

$$\mu^N = \text{Design Point} * \{ 1.0 - \ln(\text{Design Point}) + \lambda \}$$

where:

σ^N = the standard deviation for the normal curve that approximates the tail of the given distribution.

μ^N = the mean for the normal curve that approximates the tail of the given distribution.

ζ = the shape parameter for the log for the lognormal curve to be approximated by a normal curve.

λ = the scale parameter for the lognormal curve to be approximated by a normal curve.

There are two ways for making lumber comparisons by the AFOSM method. One approach would be to calculate a minimum β for both the control case and proofloading case, thereby allowing the design point to be determined by β. In general, the two design points reached both having a minimum β will not be equal. For this approach the normal approximation of the load for the control case will not be the same as for the proofloading case. Simply stated for one case, the total floor load having an Extreme Value Type I distribution will have an approximating normal distribution for the control different from the proofloaded case because of the two different design points being used.

Another approach for making lumber comparisons with the AFOSM method is to use the same design point for both the control and the proofloaded case. The desirable feature of this approach is that the approximated normal load distributions in both cases, control and proofloading treatment, are identical. The disadvantage of a single design point for lumber comparisons is that minimum β values are not compared. In this research the choice was made to use identical load distributions in the comparisons.

The design points used in this lumber reliability analysis were found by taking the fifth percentile of the control bending strength distribution, dividing it by the general adjustment factor of 2.1 and multiplying by a load duration factor of 1.15 for total roof snow load and 1.0 for total floor load. The design points for roof snow load were 1.621 ksi for Hem-fir and 1.134 ksi for the southern pine, 1.410 ksi and 0.986 ksi for total floor load respectively. These design points were used in the AFOSM method to determine a β value for calculating probability of failure.

Figures 11, 12, 13, and 14 graphically illustrate the AFOSM method, where the tails of two distributions are approximated by the normal curves. Computer programs were used to calculate β for any load and resistance combination with loads following lognormal or Extreme Value Type I distributions and resistances following lognormal or 3-parameter Weibull distributions. Computer programs are given in the Appendix. The design point, defined in the preceding paragraph, was used by the AFOSM method for the calculation of β. β was calculated from the normal approximations and a corresponding probability of failure found in a standard normal table. To compare the proofloaded cases to the control, K factors were used. Using the β for the control group, the parameters of the proofloaded group were adjusted such that the distribution was shifted to the right or left until a near equal β was found.

The transformation needed to shift the distribution on the x-axis is of the form $Y = KX$. The statistics of Y, given X is lognormal, follows by using the log transformation and expectation

$$\ln Y = \ln X + \ln K \qquad [19]$$

$$E(\ln Y) = E(\ln X) + E(\ln K) \qquad [20]$$

Thus

$$\lambda_Y = \lambda_X + \ln K \qquad [21]$$

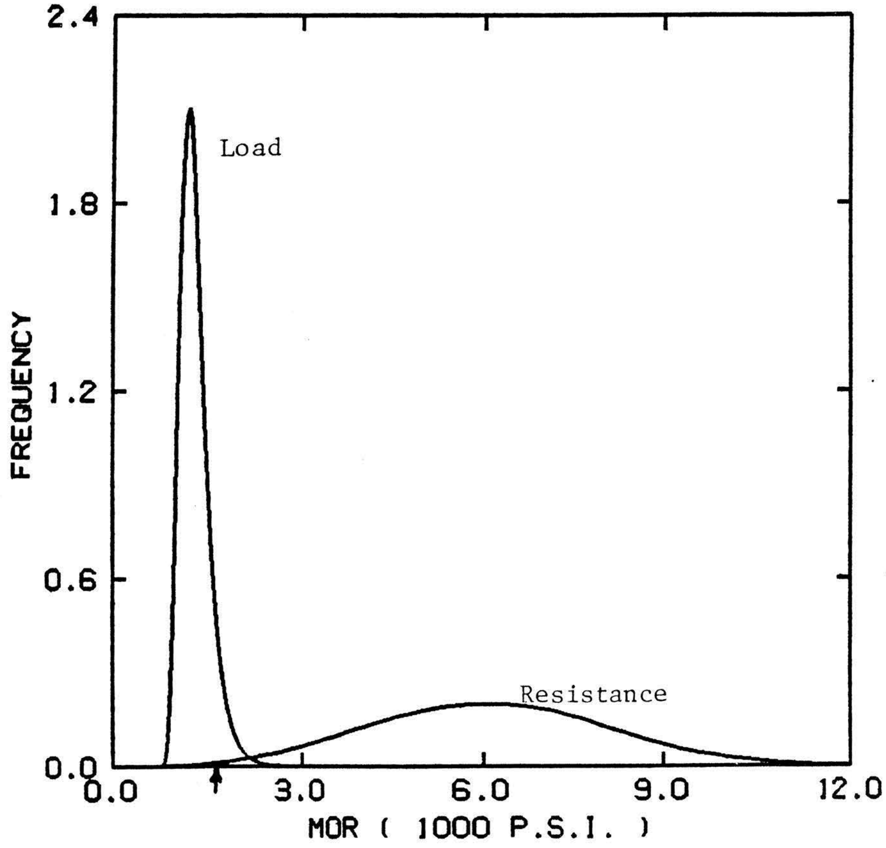

Figure 11. Extreme Value Type I load distribution and 3-parameter Weibull distribution used to show details of the AFOSM method.

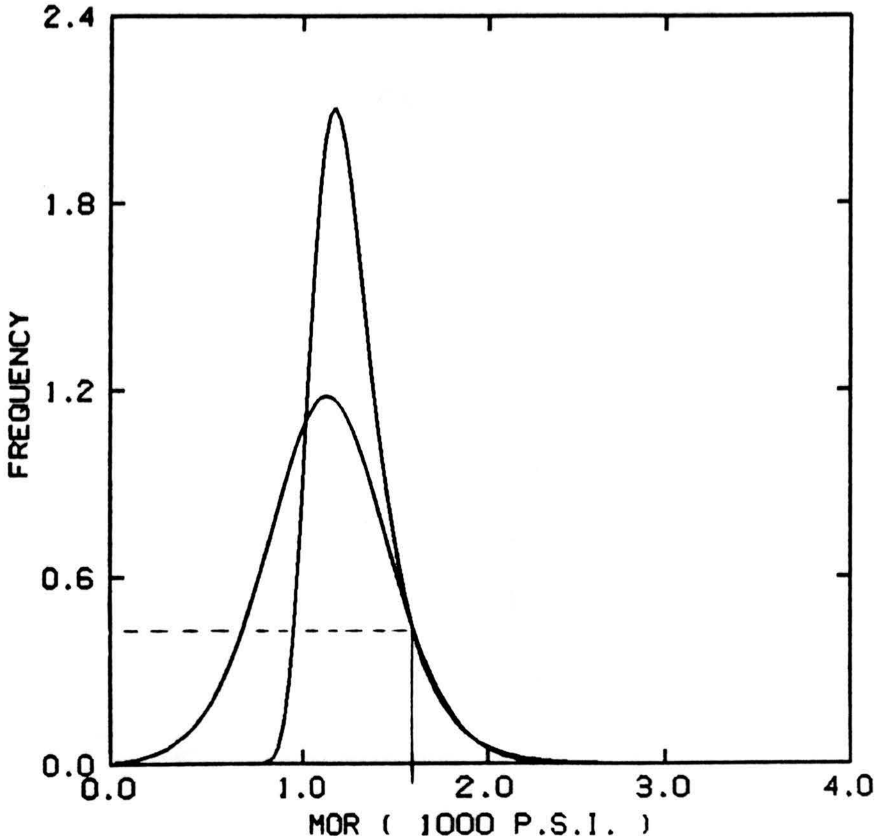

Figure 12. The Extreme Value Type I load distribution and the normal distribution with a tail to the right of the design point which approximates the Extreme Value Type I distribution tail.

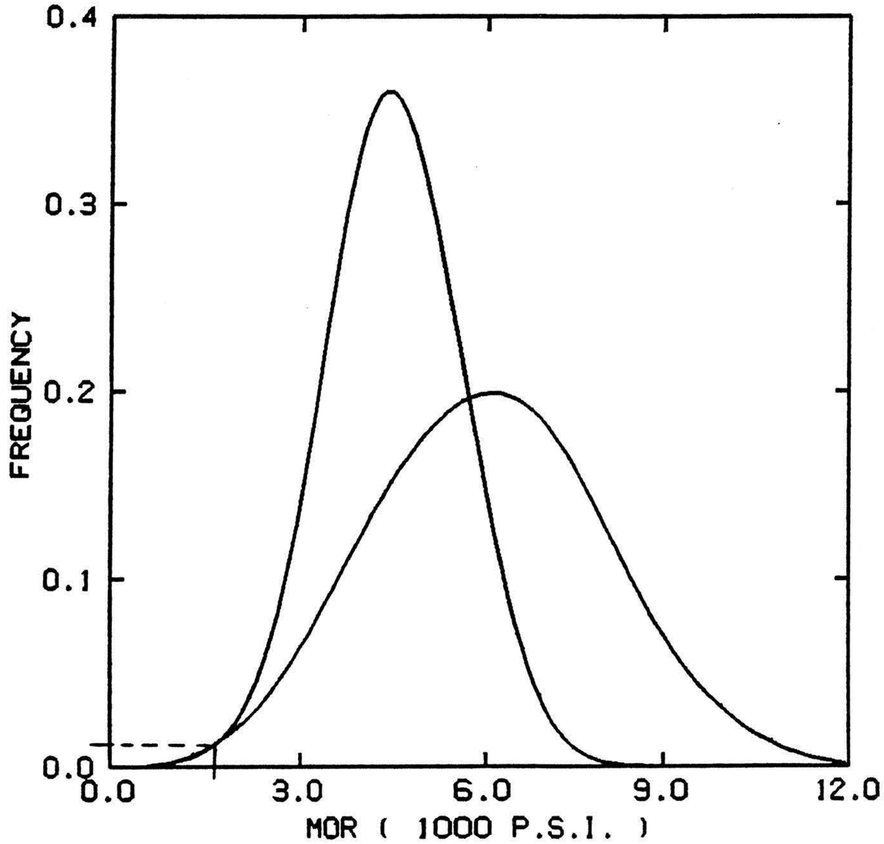

Figure 13. The 3-parameter Weibull resistance distribution and the normal distribution with a tail to the left of the design point which approximates the 3-parameter Weibull distribution tail.

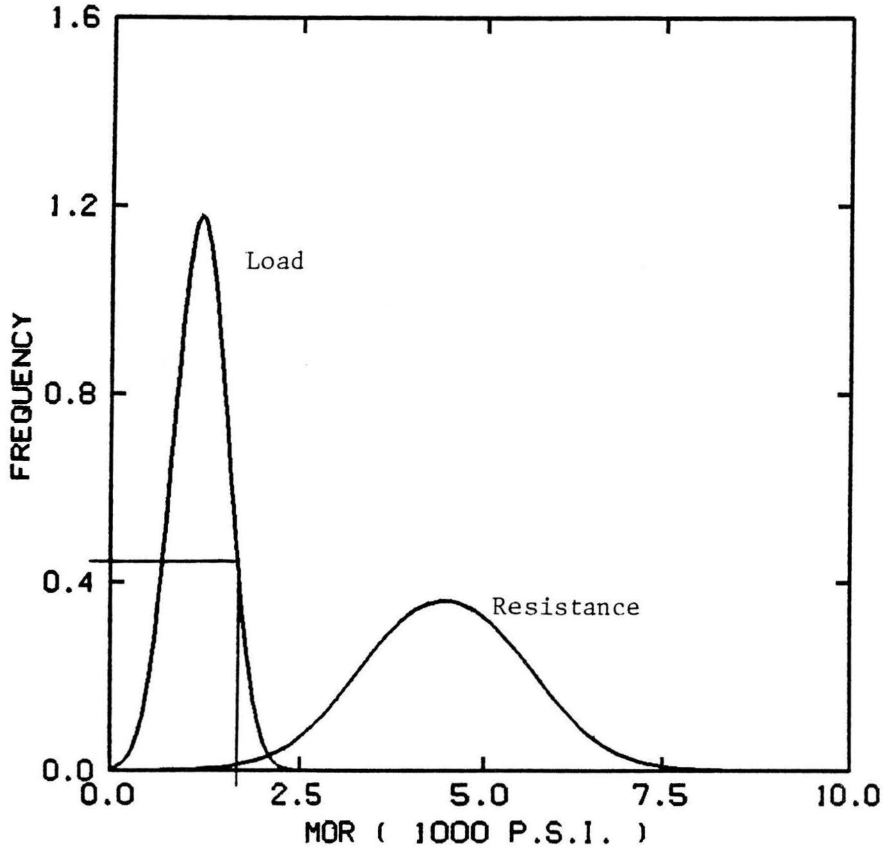

Figure 14. The normal distributions used to calculate beta. They both have overlapping tails which approximate the original Extreme Value Type I and 3-parameter Weibull distributions.

To shift the lognormal curve, λ_X was changed by the amount ln K and ζ was held constant since the shape of the distribution is not changed. To show the shape doesn't change, equation [19] can be used with the rule for the variance of a linear function as

$$\text{var}(\ln Y) = \text{var}(\ln X) + \text{var}(\ln K) \qquad [22]$$

$$\zeta_Y^2 = \zeta_X^2 + 0 \qquad [23]$$

Shifting the 3-parameter Weibull distribution up or down the x-axis, does not change the shape parameter of the distribution and it can be shown the parameters μ and σ are altered by the product of K. Therefore, the 3-parameter Weibull distribution was shifted by changing μ by μ times K, σ by σ times K, and η the shape parameter was left unchanged. The Appendix contains the computer programs used for calculating K-factors given the desired β, the design point, and the parameters of the load and resistance distributions.

3.6 Numerical Integration Method

The probability of failure can be defined mathematically by a double integral when the load and strength distributions are continuous and mutually independent as

$$p_f = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{s} f_R(r)dr \right] f_S(s)ds \qquad [24]$$

where:

$f_R(r)$ = the probability density function of the resistance distribution

$f_S(s)$ = the probability density function of the load distribution

When the double integral of f_R (r) and f_S(s) cannot be solved using standard integration methods Equation 24 can be evaluated using numerical techniques. Differential reliability described by Suddarth et al. (1978), was used to determine a probability of failure for each group. After finding the probability of failure for the control with a predetermined load, the parameters of the proofloaded groups were adjusted by a K factor until a similar probability of failure was found. Computer programs used to solve the double integral and calculate K factors for the numerical integration method were from Thurmond (1982). The K factors found with this method were compared with the K factors from the AFOSM method. The results from both the AFOSM and numerical integration method were also compared to the fifth percentile analysis and differences noted in the next section.

RESULTS AND DISCUSSION

4.1 Fifth Percentile Results

The fifth percentile value for each proofloaded case was compared to the control fifth percentile value. Table 4.1 is a summary of the fifth percentile values for both Hem-fir and southern pine. Since allowable bending stresses are based on the fifth percentile, these results imply the NDS specified bending strength value for 1650f-1.5E Hem-fir could be increased 32 percent provided the lumber was proof tested to 2,611. psi in tension as was the T5H case. It must be noted the 32 percent increase is subject to sampling error since it was found from the ratios of two fifth percentile estimates.

All proofloading cases for Hem-fir showed an increase in bending strength over the control. As both tension and compression proofloading levels were increased, the fifth percentile of the bending strength also increased. The largest bending strength increase for Hem-fir was a 51 percent increase due to tension proofloading for the T15H case. The smallest bending strength increase was a 22 percent increase due to compression proofloading for the C5H case.

Table 4.1 Summary of the fifth percentile results for both Hem-fir and southern pine using the control as a benchmark.

2 by 4 1650f-1.5E Hem-fir

	T5H	T15H	C5H	C15H	CNBH
Fifth percentile, ksi.	3.90	4.47	3.63	4.23	2.96
Ratio of the fifth percentile to control	1.32	1.51	1.23	1.43	

2 by 4 No. 2 KD Southern Pine

	T5S	T15S	C5S	C15S	CNBS
Fifth percentile, ksi.	1.79	2.36	2.43	2.35	2.07
Ratio of the fifth percentile to control	.86	1.14	1.17	1.13	

The No. 2 KD Southern Pine results were not as consistent, in that increasing the proofload level in either tension or compression did not always increase the bending fifth percentile. For example, the control sample CNBS had a fifth percentile value greater than the fifth percentile value for the T5S case. Also, the C15S case had a fifth percentile value less than the fifth percentile value for the C5S case, the reverse of what would be expected.

The lack of a uniform trend between the proofloading and the resulting fifth percentile value could be due to the greater variability of southern pine population sampled. The bending strength coefficient of variation for the control No. 2 KD Southern Pine was 0.406 versus 0.310 for the 1650f-1.5E MSR Hem-fir. The variability of the fifth percentile estimate generally increases with increasing variance of the underlying population. Thus for the southern pine the sampling error of the fifth percentile could have dominated over the proofloading effect on bending strength.

4.2 Hem-fir Integration Method Results

The numerical integration method of analysis was used for each control and proofloaded case. Figure 15 graphically compares the distributions from the Hem-fir data control and tension proofloading treatments. The left tails of the tension proofloaded cases were shifted to the right of the control with the greatest shift associated with the largest proofload level. Likewise, Figure 16 shows the Hem-fir distributions for the control and compression proofloading cases. The left tails of the compression proofloads were also shifted to the right of the control similar to the tension proofload cases. Figures 17 and 18 graphically compare the distributions of the southern pine control case along with the resulting distributions from the tension and compression proofloading treatments, respectively. The left tails of the tension proofloaded cases are shifted to the right of the control for the larger proofload level. However, the distribution curve for the smaller proofload level lies to the left of the control curve, indicating most likely a large sampling error. In the compression proofload cases both were shifted to the right of the control, however the smaller

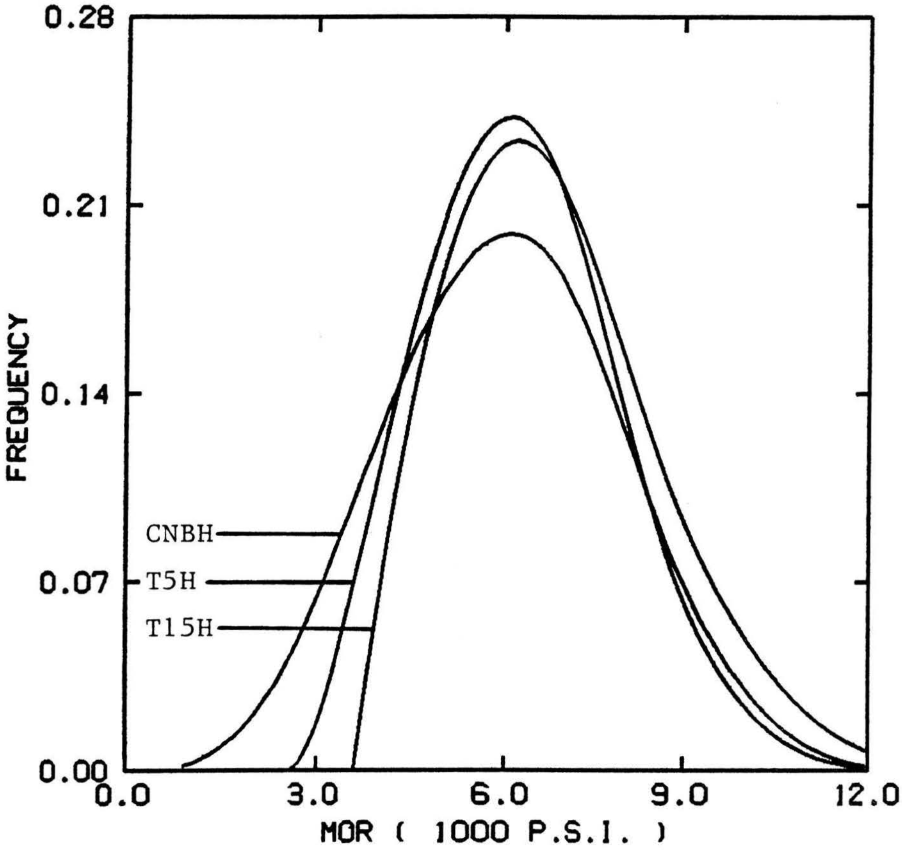

Figure 15. Hem-fir bending strength distributions for control and tension proofloading at the 5 and 15 percent levels.

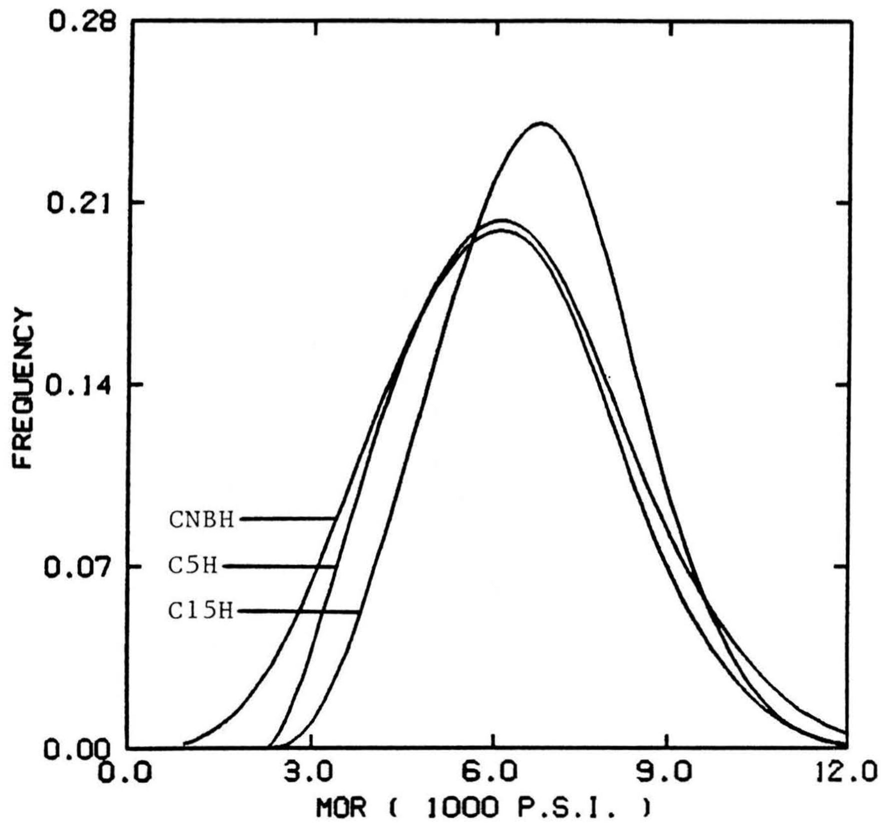

Figure 16. Hem-fir bending strength distributions for control and compressive proofloading at the 5 and 15 percent levels.

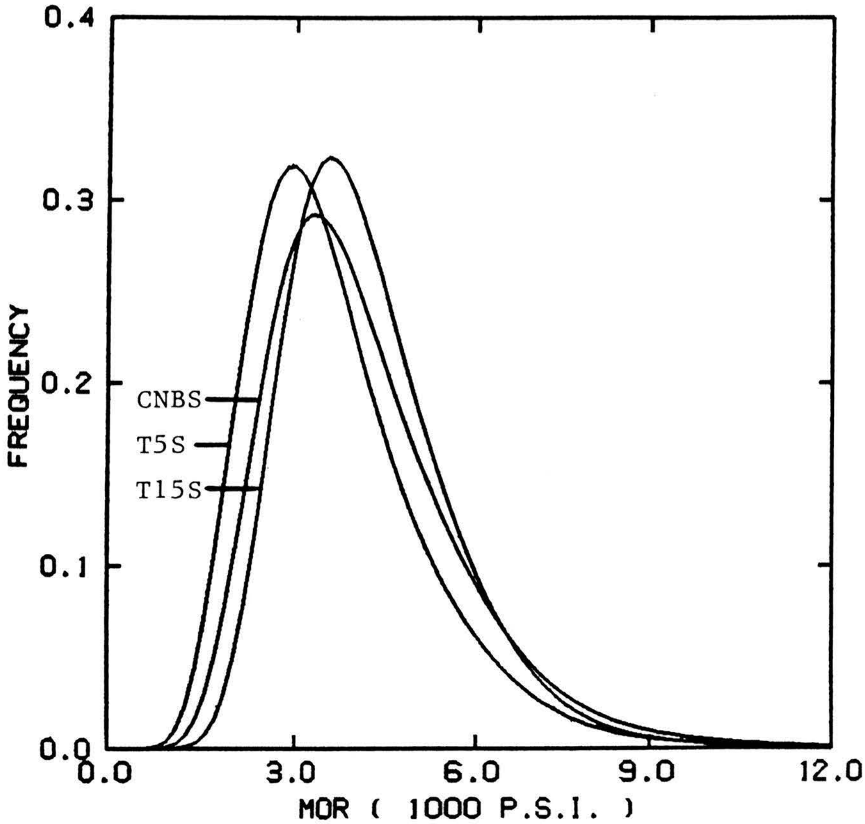

Figure 17. Southern pine bending strength distributions for control and tension proofloading at the 5 and 15 percent levels.

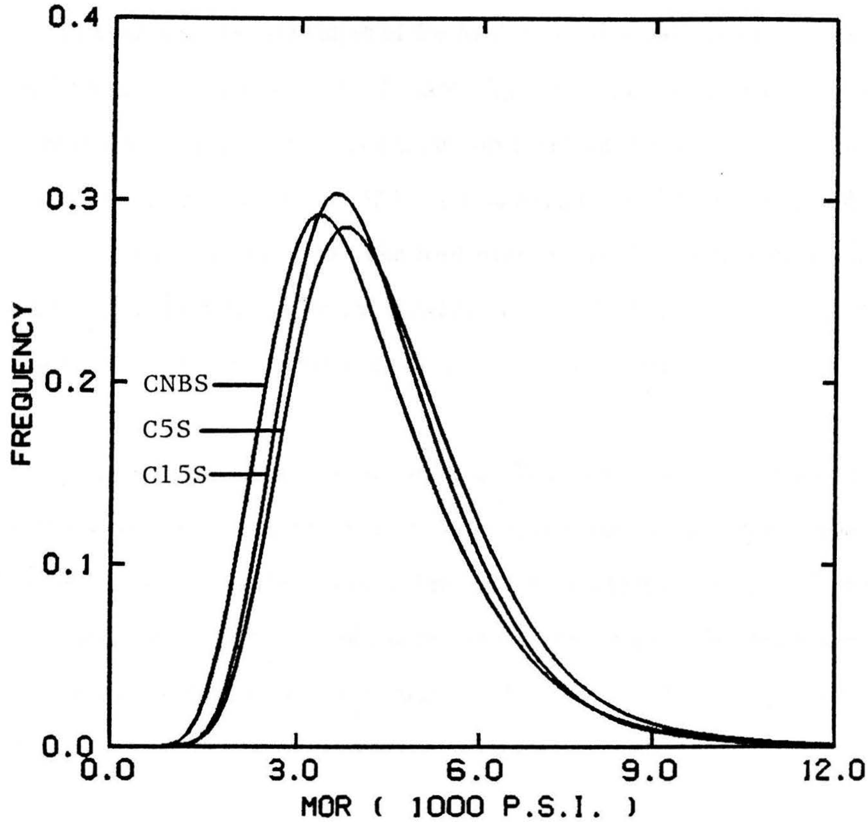

Figure 18. Southern pine bending strength distributions for control and compressive proofloading at the 5 and 15 percent levels.

proofloading level was shifted more to the right than the larger proofloading level. Again, this mixed outcome suggests a large sampling error.

Probabilities of failure and K factors were found using a full bending stress load where the loads were lognormal and Extreme Value Type I shown in Table 3.5. The bending strength data from the control and proofloaded samples were adjusted for load duration by dividing the test machine data by 1.6 to convert to a normal duration of 10 years. The results are summarized in Table 4.2. The benchmark probability of failure for the roof truss top chord was found using the lognormal load distribution having parameters μ equal 1.032 ksi and Ω equal 0.319. The resulting probability of failure was 0.501×10^{-2} found in Table 4.2. The roof truss top chord K factors are found using the same load distribution parameters as for the benchmark case. Each proofloaded distribution is shifted on the x-axis by a K factor to yield a similar probability of failure.

K factors can be used to shift either loads or resistances. To demonstrate this fact the floor load A for the T5H case was shifted by the reciprocal of K instead of the strength distribution being shifted by the K factor. The same probability of failure resulted. Using the example T5H with the live load A for the floor truss top chord, the allowable bending stress was multiplied by the reciprocal of K equal to 1.493 and used in equation 15 and 16 which provided the Extreme Value Type I load parameters,

$$\mu_T = \frac{10}{50}(.81)\frac{2.964}{2.1}(1.0)\,(1.493) \;+\; \frac{40}{50}(.94)\frac{2.964}{2.1}(1.0)\,(1.493)$$

$$\mu_T = 0.341 + 1.585 = 1.926 \text{ ksi}$$

and

$$\Omega_T = \frac{\sqrt{(0.341 \times 0.1)^2 + (1.585 \times 0.21)^2}}{1.926} = 0.174$$

Table 4.2 Integration method results for Hem-fir. Reference probability of failure for the control group and K-factors for the proofloaded groups are listed for each loading case.

2 by 4 1650f-1.5E Hem-fir					
	T5H	T15H	C5H	C15H	CNBH
Load case	K-factor	K-factor	K-factor	K-factor	p_f
Roof top chord μ = 1.032 Ω=0.319	0.645	0.560	0.695	0.600	0.501 X 10^{-2}
Roof bottom chord μ =0.827 Ω=0.10	0.435	0.345	0.495	0.430	0.106 X 10^{-2}
Floor load A μ = 1.288 Ω=0.174	0.670	0.580	0.725	0.625	0.148 X 10^{-1}
Floor load D μ = 1.051 Ω=0.15	0.590	0.500	0.650	0.560	0.626 X 10^{-2}
Floor bottom chord μ =0.930 Ω=0.10	0.520	0.420	0.585	0.505	0.344 X 10^{-2}
Minimum 1/K value	1.49	1.72	1.38	1.60	

This load was used with the following T5H data parameters adjusted by 1/1.6 for conversion from a 10 minute test value to a 10 year duration.

$$\mu = 2.539/1.6 = 1.587$$

$$\sigma = 4.282/1.6 = 2.676$$

$$\eta = 2.590$$

The resulting probability of failure from the shifted load distribution was 0.1487×10^{-1} which was the same as found when the resistance distribution was shifted on the x-axis to find the K factor. Thus the minimum allowable increase for bending stress, F_b, is the reciprocal of K shown in Table 4.2.

The MSR 1650f-1.5E Hem-fir results show that as proofloading levels were increased in tension or compression the allowable bending strength also increased. For MSR Hem-fir lumber it appears as few as 80 specimens can be used to define the increase in allowable bending strength due to tension or compression proofloading. However, 200 specimens would be a more appropriate sample size since 200 is generally recognized by the research community as necessary for allowable stress or fifth percentile determination.

The percentages of bending stress contribution to the CSI value for the top and bottom chords given in Table 3.9 were used to calculate a percentage of the total load to be used with each proofload case. When using a percentage of the load the K factors all decreased as shown in Table 4.3. In each case a lower K factor or greater shift on the x-axis was required to yield the same probability of failure as the control.

The design process requires a conservative choice for design values when more than one outcome is present. For purposes of defining a permissible increase in allowable bending stresses due to a tension or compression proofloading treatment, the larger K factor from each proofloading case

Table 4.3 Integration method results for Hem-fir. Reference probability of failure for the control group and K-factors for the proofloaded groups are listed. The loads were calculated from the total load using the percent of bending stress contribution to the CSI value.

2 by 4 1650f-1.5E Hem-fir					
	T5H	T15H	C5H	C15H	CNBH
Load case	K-factor	K-factor	K-factor	K-factor	p_f
Roof top chord $\mu = 0.568$ $\Omega = 0.319$	0.495	0.420	0.545	0.465	0.353×10^{-3}
Roof bottom chord $\mu = 0.414$ $\Omega = 0.10$	0.270	0.200	0.300	0.275	0.153×10^{-4}
Floor load A $\mu = 0.193$ $\Omega = 0.174$	0.265	0.210	0.305	0.265	0.687×10^{-7}
Floor load D $\mu = 0.158$ $\Omega = 0.15$	0.240	0.185	0.270	0.240	0.173×10^{-8}
Floor bottom chord $\mu = 0.065$ $\Omega = 0.10$	**	**	**	**	0
Minimum 1/K value	2.02	2.38	1.83	2.17	

** Indicates no comparison made because the extremely small loads used resulted in a nearly zero probability of failure.

must be chosen since the reciprocal of K is the amount by which the F_b value for the tested lumber can be increased and subsequently used in design.

4.3 Southern Pine Integration Method Results

Table 4.4 is a summary for 2 by 4 No. 2 KD Southern Pine showing the benchmark probability of failures for the control and the K factors for the proofloaded cases under roof and floor loads. The K factor for the T5S case is larger than 1.0 indicating the the allowable bending strength for the T5S case was less than the control. The C15S case had a K factor greater than the C5S case. Thus, the increased compression proofloading level for the C15S case did not produce bending strength improvements above the C5S case. The lack of a trend between the proofloading level and K factors could indicate the sampling error is dominate over the strength benefiting effect of proofloading in both tension and compression. Thus, based on the visual grade of 2 by 4 No. 2 KD Southern Pine, 80 specimens was clearly inadequate. A sample size of 200 may be adequate, however, there is no assurance of useful results even with 200.

Using a percent of the load equal to the percent of bending stress contribution to the CSI value the probability of failure and K factors were calculated for each loading condition. The resulting K factors shown in Table 4.5 are lower for the smaller loads than when the full load was used, except in the case when the original K factor was greater than 1.0. Thus, if an increase in allowable bending strength was present, the full loading cases provided the most conservative increases in allowable bending strength to be used in design.

Table 4.4 Integration method results for southern pine. Reference probability of failure for the control group and K-factors for the proofloaded groups are listed for each loading case.

2 by 4 No. 2 KD Southern Pine

	T5S	T15S	C5S	C15S	CNBS
Load case	K-factor	K-factor	K-factor	K-factor	p_f
Roof top chord $\mu = 0.722$ $\Omega = 0.319$	1.165	0.855	0.835	0.865	0.219×10^{-2}
Roof bottom chord $\mu = 0.578$ $\Omega = 0.10$	1.195	0.775	0.790	0.805	0.289×10^{-4}
Floor load A $\mu = 0.901$ $\Omega = 0.174$	1.170	0.860	0.840	0.870	0.822×10^{-2}
Floor load D $\mu = 0.735$ $\Omega = 0.15$	1.175	0.835	0.825	0.850	0.171×10^{-2}
Floor bottom chord $\mu = 0.651$ $\Omega = 0.10$	1.180	0.800	0.805	0.830	0.388×10^{-3}
Minimum 1/K value	.84	1.16	1.19	1.15	

Table 4.5 Integration method results for the southern pine. Reference probability of failure for the control group and K-factors for the proofloaded groups are listed. The loads were calculated from the total load using the percent of bending stress contribution to the CSI value.

2 by 4 No. 2 KD Southern Pine					
	T5S	T15S	C5S	C15S	CNBS
Load case	K-factor	K-factor	K-factor	K-factor	p_f
Roof top chord μ = 0.397 Ω = 0.319	1.180	0.810	0.810	0.835	0.237×10^{-4}
Roof bottom chord μ = 0.289 Ω = 0.10	1.220	0.705	0.740	0.755	0.354×10^{-8}
Floor load A μ = 0.135 Ω = 0.174	0.915	0.595	0.605	0.620	0.109×10^{-10}
Floor load D μ = 0.110 Ω = 0.15	0.800	0.520	0.530	0.540	0.124×10^{-12}
Floor bottom chord μ = 0.046 Ω = 0.10	**	**	**	**	0
Minimum 1/K value	0.82	1.23	1.23	1.20	

** Indicates no comparison made because the extremely small loads used resulted in a nearly zero probability of failure.

4.4 Design Values

Design values based on these reliability analyses show that the Hem-fir and southern pine do not provide similar results. The effect of the correlation between tension and bending, and compression and bending found in the tail of the distribution require a much larger sample size for the southern pine to yield consistent results.

For discussion purposes assume the sampling errors to be acceptable. The amount of increase in allowable bending strength due to proofloading was the reciprocal of K. Also, assuming the experimental lumber was representative of the population of 2 by 4 1650f-1.5E Hem-fir, lumber proofloaded in tension to 2,611. psi could claim an allowable bending stress, F_b, increased by the factor 1.49 or from 1,650. to 2,458. psi. Likewise, the same type lumber proofloaded to 2,857. psi could claim an allowable bending stress increase by the factor 1.72 or from 1,650. to 2,838. psi. Figure 19 graphically shows the two proofloading levels used in this study and the allowable bending strength increases for the Hem-fir grade assuming a linear relationship. The tension proofload levels below 2,611. psi, no increase in F_b would be recommended. For tension proofload levels above 2,857. psi, a fixed increase in F_b equal to 1.72 would be recommended. Figure 19 and the increase in F_b values shown are only for demonstration purposes, the values reported are not reliable estimates for the population of 1650f-1.5E Hem-fir.

The tension proofloading levels of interest for future research are those levels lying between the lowest level for economic feasibility and the maximum stress level in the lumber producing forces at the joints beyond which can not be transferred using toothed metal truss plates. By using these maximum and minimum proofloading levels along with at least one other intermediate proofloading level, a nonlinear trend, if present, can be defined for allowable bending strength increases over the range tested.

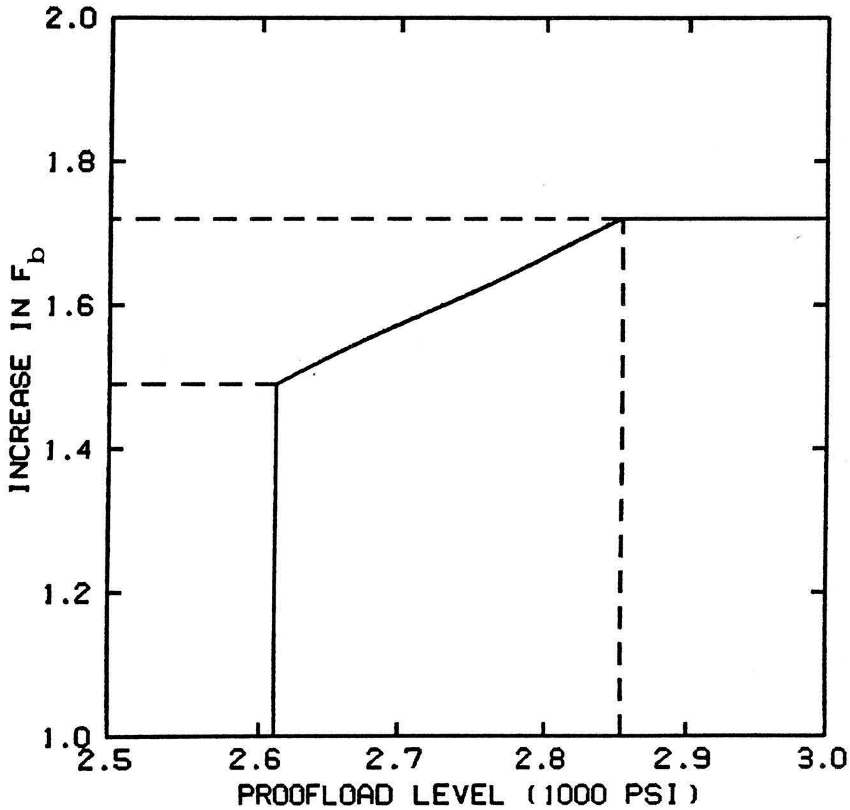

Figure 19. A demonstration of allowable bending strength increases for a 2 by 4 1650f-1.5E Hem-fir sample corresponding to various tension proofloading levels in psi. Values reported are not reliable estimates for the population of 1650f-1.5E Hem-fir.

4.5 AFOSM Method Results

As discussed earlier a design point is needed at which to perform the calculations necessary to determine a beta value using the AFOSM method. The design points used were 1.621 ksi for Hem-fir and 1.134 ksi for the southern pine. These design points do not provide a minimum beta, but they are appropriate points for comparing the AFOSM method with the numerical integration method used.

The AFOSM method requires calculation of the probability density function (pdf) and the cumulative density function (cdf) at the design point. The 3-parameter Weibull distribution provided the best fit for all groups of 1650f-1.5E Hem-fir data. For the design point, 1.621 ksi, the 3-parameter Weibull fits of the proofloaded cases had location parameters above the design point; therefore, no normal distribution could be approximated for that point. Thus, no K factors were found for comparison with the integration method.

The lognormal distribution provided the best fit for all groups of No. 2 KD Southern Pine data groups. The lognormal distribution range is from zero to infinity so each case could be evaluated by the AFOSM method using the design point of 1.134 ksi. A beta value was first found for the control, and then a K factor was used to shift the proofloaded cases on the x-axis to find a nearly equal beta value. The results shown in Table 4.6 show that tension proofloading for the T5S case did not provide an increase in allowable bending strength over the control. However, the T15S case had an increase of 19 percent in allowable bending strength over the control. The compression proofloaded groups show a minimum increase in allowable bending strength for the C15S case of 15 percent and a 19 percent increase in allowable bending strength for the C5S case.

The minimum reciprocal of K values of Table 4.6 were identical to those found using the numerical integration method in Table 4.4 to two significant figures. Table 4.7 lists the probability

Table 4.6 AFOSM method results for southern pine. Reference beta value for the control group and K factors for proofloaded groups are listed for each loading case.

2 by 4 No. 2 KD Southern Pine					
	T5S	T15S	C5S	C15S	CNBS
Load case	K-factor	K-factor	K-factor	K-factor	β
Roof top chord $\mu = 0.722$ $\Omega = 0.319$	1.170	0.855	0.840	0.865	2.849
Roof bottom chord $\mu = 0.578$ $\Omega = 0.10$	1.200	0.775	0.790	0.810	4.022
Floor load A $\mu = 0.901$ $\Omega = 0.174$	1.165	0.860	0.840	0.870	2.428
Floor load D $\mu = 0.735$ $\Omega = 0.15$	1.175	0.835	0.825	0.855	3.003
Floor bottom chord $\mu = 0.651$ $\Omega = 0.10$	1.185	0.805	0.810	0.830	3.361
Minimum 1/K value	.83	1.16	1.19	1.15	

Table 4.7 A comparison of the probability of failures from the AFOSM and the numerical integration method.

Hem-fir	P_f using AFOSM	P_f using integration
Roof top chord	0.474×10^{-2}	0.510×10^{-2}
Roof bottom chord	0.853×10^{-3}	0.106×10^{-2}
Floor Load A	0.128×10^{-1}	0.148×10^{-1}
Floor Load D	0.424×10^{-2}	0.626×10^{-2}
Floor bottom chord	0.279×10^{-2}	0.344×10^{-2}

Southern pine	P_f using AFOSM	P_f using integration
Roof top chord	0.219×10^{-2}	0.219×10^{-2}
Roof bottom chord	0.289×10^{-4}	0.289×10^{-4}
Floor load A	0.759×10^{-2}	0.822×10^{-2}
Floor load D	0.135×10^{-2}	0.171×10^{-2}
Floor bottom chord	0.388×10^{-3}	0.388×10^{-3}

of failure from each load condition for both the AFOSM method and the mathematical integration method. Although the probability of failures for each load case were not always equal to 3 significant figures in each method, they were of the same magnitude. For example, comparing the probability of failure for the two methods of the floor load D case, a ratio of 1.27 was found. However, the K factors that resulted from using the two methods are identical in terms of their application in allowable stress adjustment. In fact, for all cases, either method provided similar K factors. Therefore, for this particular lumber reliability analysis, the AFOSM and numerical integration methods are equivalent. It would be desirable to generalize and recommend the AFOSM method for use in future lumber analyses due to computational ease. However, due to a limited number of comparisons involving different loadings and lumber strength data, more research comparisons using the AFOSM method and integration method are necessary before the AFOSM method can be used alone and become a standard method.

SUMMARY AND CONCLUSIONS

The reliability study utilized the data from a report by Galligan et al. (1986) that characterizes the properties of 2-inch softwood dimension lumber with regressions and probability distributions. Ten randomized groups of both 2 by 4 1650f-1.5E Hem-fir and No. 2 KD Southern Pine were formed. One group from each species was broken in bending, with the remaining groups broken in bending after proofloading in tension or compression.

The purpose of this study using the Galligan data was to identify an improvement in bending strength from tension or compression proofloading, due to the correlations that exist between tension and bending, and between compression and bending. Based on the concept of equal reliability, and utilizing the load distributions from Thurmond et al. (1986), the tension and compression proofloaded strength distributions were compared to the control. The control strength distributions for each species where all pieces were tested in bending were used to establish a benchmark probability of failure. Using a K factor adjustment the strength distributions of the proofloaded groups were shifted on the x-axis until a probability of failure approximately equal to the benchmark probability of failure resulted.

A fifth percentile, numerical integration and the advanced first order second moment (AFOSM) methods were used to analyze the data. The fifth percentile values were calculated for the the controls and each proofloading case. The Hem-fir fifth percentile values for the proofloading cases were greater than the control fifth percentile and the difference between the proofloading cases and control increased as the proofloading levels increased. The southern pine fifth percentile of bending strength for the tension proofloading at the 5 percent target breakage level was less than the control fifth percentile value. Also, the compression proofloading at the target 15 percent breakage case had a bending strength fifth percentile value less than bending strength fifth percentile for compression proofloading at the target 5 percent level. These mixed results for southern pine indicate a sampling problem.

For the numerical integration method utilized with the three load distributions recommended by Thurmond et al. (1986), the Hem-fir grade showed a significant increase in allowable bending strength due to the proofloading in both tension and compression, with additional increases as the level of proofloading was increased. Using the most conservative loading case from an implementation standpoint, proofloading in tension at the target 5 percent level of breakage resulted in a 49 percent increase in allowable bending strength. At the target 15 percent breakage level, a 72 percent increase in allowable bending strength resulted. Compressive proofloading at the target 5 percent breakage level provided a 38 percent increase in allowable bending strength, and at the target 15 percent breakage level a 60 percent increase in allowable bending strength.

The southern pine results were more variable in that increasing the proofload level in either tension or compression did not always increase the allowable bending strength. For the case of tension proofloading at a target 5 percent breakage, the allowable bending strength was calculated to be 16 percent below the control which was identified as a sampling problem. However, tension proofloading at a target 15 percent breakage showed an increase of 16 percent in allowable bending strength. The compression proofloading at a target 5 percent breakage resulted in a 19 percent increase in allowable bending strength. The compressive proofloading at a target 15 percent breakage resulted in a 15 percent increase in allowable bending strength, or 4 percent less than the increase from the lower proofloading level. It is believed the sampling error for southern pine dominated over the effect of proofloading on the bending strength causing the mixed results. Because of the mixed results with the visually graded southern pine, much larger sample sizes are recommended for future research.

The AFOSM method was used on all the control data resulting in probabilities of failure of the same magnitude as those obtained using the integration method. Using the AFOSM method on the southern pine resulted in K factors, equal to two significant figures, to those found using the numerical integration method. It was concluded that both methods provided identical results in terms of their application to allowable stress adjustment.

The AFOSM method is desirable over the numerical integration method due to its computational ease. Since, a limited number of comparisons involving different loadings and lumber strength data were conducted, more research comparisons are necessary before making a recommendation for using only the AFOSM method in reliability analyses. The fifth percentile analysis does not take into account the interaction of the load and resistance distributions. Therefore, the results are less accurate than the AFOSM and numerical integration methods in terms of providing equal safety levels upon the use of the resulting stress adjustment factor.

From this study there is good evidence that bending strength and tension strength of the respective distribution tails are correlated, as well as bending and compression distribution tails. The sample sizes in this study were inadequate to quantify the amount of an allowable bending strength increase due to prooftesting. However, the results from the limited sample sizes used provide justification for a comprehensive study using much larger sample sizes.

In the event the truss fabrication industry implements tension proofloading, allowable bending strength benefits due to the tension proofloading could be used in truss design. The cost associated with the tension proofloading could therefore be offset, not only by the increase in the allowable tension stress, F_t, but also by the increase in the allowable bending strength, F_b.

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APPENDIX

Computer Programs to Calculate Beta Values

```
C*****************************************************************
C
C   BLNLN              BETA CALCULATION
C
C
C            ADVANCED FIRST ORDER SECOND MOMENT METHOD
C
C        USING LOGNORMAL LOAD AND RESISTANCE DISTRIBUTIONS
C
C
C*****************************************************************
C
C
C   THIS PROGRAM CALCULATES THE RELIABILITY INDEX, BETA, WHEN THE
C   LOAD FOLLOWS A LOGNORMAL DISTRIBUTION AND THE RESISTANCE
C   DISTRIBUTION IS ALSO LOGNORMAL. INPUT PARAMETERS ARE THE MEAN
C   AND C.O.V. FOR THE LOAD AND THE MEAN AND STANDARD DEVIATION OF
C   THE LOGS FOR THE RESISTANCE DISTRIBUTION.
C
C
C     CARD 1 -- SPECIFY DESIGN POINT TO USE ...
C          (F10.4)  -- 'DESIGN POINT' IS THE VALUE ON THE X-AXIS
C                      USED FOR THE BETA CALCULATIONS (K.S.I.)
C
C     CARD 2 -- SPECIFY LOAD MEAN, COV, AND STRENGTH STATISTICS
C          (5F10.4) -- 'MEAN' IS THE MEAN OF THE LOAD DISTRIBUTION
C                      (K.S.I.)
C                   -- 'COV' IS THE COEFFICIENT OF VARIATION FOR
C                      THE LOAD
C                   -- 'RMU' IS THE MEAN OF THE LOGS FOR LOGNOR-
C                      MAL CURVES (K.S.I.)
C                   -- 'RSIG' IS THE STD. DEV. OF THE LOGS FOR
C                      THE RESISTANCE DISTRIBUTION (K.S.I.)
C
```

```
C***********************************************************************
C
      REAL MEAN
C
C  READ IN THE INPUT PARAMETERS AS LISTED IN THE USER'S GUIDE
C
      READ(1,12)DPT
   12 FORMAT(F10.4)
      READ(1,13)MEAN,COV,RMU,RSIG
   13 FORMAT(4F10.4)
      WRITE(2,1)
    1 FORMAT(/,15X,'LOAD DISTRIBUTION -- LOGNORMAL',/)
      WRITE(2,4)
    4 FORMAT(12X,'RESISTANCE DISTRIBUTION -- LOGNORMAL',///)
C
C  CALL SUBROUTINE TO FIND THE  PARAMETERS OF THE LOAD
C
      CALL PARLN(MEAN,COV,ALAM,VARLN)
      WRITE(2,5) DPT
    5 FORMAT(5X,'I. DESIGN POINT ....... =',F8.4,//)
      SQVAR=VARLN**.5
      WRITE(2,8) MEAN,ALAM,COV,SQVAR,RMU,RSIG
    8 FORMAT(4X,'II. LOADING STATISTICS ... MEAN LOAD =',F10.4,5X,
     1 'LAMBDA =',F8.4,/,
     2      4X,'                 LOAD C.O.V. =',F8.4,5X,
     3 '  ZETA =',F8.4,///,
     4      3X,'III. RESISTANCE STATISTICS ... LAMBDA =',F8.4,/,
     5      3X,'                               ZETA =',F8.4,//)
C
C   CALCULATE THE STANDARD DEVIATION FOR NORMALLY DISTRIBUTED
C
      RSDN = DPT*RSIG
      QSDN = DPT*SQVAR
C
C   CALCULATE THE MEAN FOR A NORMALLY DISTRIBUTED VARIABLE
C
      RMN = DPT*(1.0-ALOG(DPT)+RMU)
      QMN = DPT*(1.0-ALOG(DPT)+ALAM)
C
C   CALCULATE THE DELTA AND ALPHAS
C
      DELTA=(RSDN**2+QSDN**2)**.5
      RALF = -(RSDN/DELTA)
      QALF = QSDN/DELTA
C
C   CALCULATE BETA
C
      BETA=(QMN-RMN)/(RALF*RSDN-QALF*QSDN)
C
C
C
      WRITE(2,9)BETA
    9 FORMAT(4X,'IV. RESULTING BETA  .......... =',F12.8)
```

```
      STOP
      END
C***********************************************************************
C
C     SUBROUTINE PARLN(MEAN,COV,MNLN,VARLN)
C
C***********************************************************************
      SUBROUTINE PARLN(MEAN,COV,MNLN,VARLN)
      DOUBLE PRECISION MEAN,COV,MNLN,VARLN
      MNLN = 0.5*DLOG((MEAN**2)/((COV**2) + 1.))
      VARLN = DLOG((COV**2) + 1.)
      RETURN
      END
```

```
C*********************************************************************
C
C    BE1LN            BETA CALCULATION
C
C
C          ADVANCED FIRST ORDER SECOND MOMENT METHOD
C
C    EXTREME TYPE I LOAD AND LOGNORMAL RESISTANCE DISTRIBUTIONS
C
C
C*********************************************************************
C
C
C   THIS PROGRAM CALCULATES THE RELIABILITY INDEX, BETA, WHEN THE
C   LOAD FOLLOWS AN EXTREME TYPE I DISTRIBUTION AND THE RESISTANCE
C   DISTRIBUTION IS LOGNORMAL. INPUT PARAMETERS ARE THE MEAN AND
C   C.O.V. OF THE LOAD AND THE MEAN AND STANDARD DEVIATION OF THE
C   LOGS FOR THE RESISTANCE DISTRIBUTION.
C
C
C     CARD 1 -- SPECIFY DESIGN POINT TO USE ...
C            (F10.4)  -- 'DESIGN POINT' IS THE VALUE ON THE X-AXIS
C                        USED FOR THE BETA CALCULATIONS (K.S.I.)
C
C     CARD 2 -- SPECIFY LOAD MEAN, COV, AND STRENGTH STATISTICS
C            (5F10.4) -- 'MEAN' IS THE MEAN OF THE LOAD DISTRIBUTION
C                        (K.S.I.)
C                     -- 'COV' IS THE COEFFICIENT OF VARIATION FOR
C                        THE LOAD
C                     -- 'RMU' IS THE MEAN OF THE LOGS FOR LOGNOR-
C                        MAL CURVES (K.S.I.)
C                     -- 'RSIG' IS THE STD. DEV. OF THE LOGS FOR
C                        THE RESISTANCE DISTRIBUTION (K.S.I.)
C
C*********************************************************************
C
      REAL MEAN,COV,A,B
C
C   READ IN THE INPUT PARAMETERS AS LISTED IN THE USER'S GUIDE
C
      READ(1,12)DPT
   12 FORMAT(F10.4)
      READ(1,13)MEAN,COV,RMU,RSIG
   13 FORMAT(4F10.4)
      WRITE(2,1)
    1 FORMAT(/,15X,'LOAD DISTRIBUTION -- EXTREME TYPE I',/)
      WRITE(2,4)
    4 FORMAT(15X,'RESISTANCE DISTRIBUTION -- LOGNORMAL',///)
C
C   CALCULATE THE  PARAMETERS OF THE LOAD
C
      STD=COV*MEAN
      A=STD/1.283
```

```
      B=MEAN-0.577*A
      PI=3.1415927
      WRITE(2,5) DPT
    5 FORMAT(5X,'I. DESIGN POINT ....... =',F8.4,//)
      WRITE(2,8) MEAN,A,COV,B,RMU,RSIG
    8 FORMAT(4X,'II. LOADING STATISTICS ... MEAN LOAD =',F8.4,5X,
     1 'ALPHA =',F8.4,/,
     2      4X,'                  LOAD C.O.V. =',F8.4,5X,
     3 ' BETA =',F8.4,///,
     4      3X,'III. RESISTANCE STATISTICS ... LAMBDA =',F8.4,/,
     5      3X,'                             ZETA =',F8.4,//)
C
C     CALCULATE VALUES FOR CDF AND PDF AT THE DESIGN POINT
C
      FDQ =EXP(-((DPT-B)/A)-EXP(-((DPT-B)/A)))/A
C
      FCQ =EXP(-EXP(-(DPT-B)/A))
C
C     CALCULATE THE INVERVE NORMAL FUNCTION AT FCQ
C
      CALL MDNRIS(FCQ,Y,IER)
      IF (IER.EQ.0) GO TO 20
      WRITE(2,59)IER
   59 FORMAT(5X,'ERROR CODE FROM IMSL IS ',I4/)
C
C     CALCULATE THE NORMAL DENSITY FUNCTION PARAMETERS
C
   20 QSIGN=(1.0/(FDQ*(2.0*PI)**.5))*EXP(-(Y**2/2.0))
      QMUN=DPT-QSIGN*Y
      FCQND=(1.0/(QSIGN*(2*PI)**.5))*EXP(-((DPT-QMUN)/QSIGN)**2/2.0)
C
C     CALCULATE THE STANDARD DEVIATION FOR NORMALLY DISTRIBUTED
C
      RSDN = DPT*RSIG
      QSDN = QSIGN
C
C     CALCULATE THE MEAN FOR A NORMALLY DISTRIBUTED VARIABLE
C
      RMN = DPT*(1-ALOG(DPT)+RMU)
      QMN = QMUN
C
C     CALCULATE THE DELTA AND ALPHAS
C
      DELTA=(RSDN**2+QSDN**2)**.5
      RALF = -(RSDN/DELTA)
      QALF = QSDN/DELTA
C
C     CALCULATE BETA
C
      BETA=(QMN-RMN)/(RALF*RSDN-QALF*QSDN)
C
C
C
```

```
      WRITE(2,9)BETA
  9   FORMAT(4X,'IV. RESULTING BETA  .......... =',F12.8)
      STOP
      END
```

```
C***********************************************************************
C
C  BLNW             BETA CALCULATION
C
C
C          ADVANCED FIRST ORDER SECOND MOMENT METHOD
C
C      USING LOGNORMAL LOAD AND WEIBULL RESISTANCE DISTRIBUTIONS
C
C
C***********************************************************************
C
C
C  THIS PROGRAM CALCULATES THE RELIABILITY INDEX, BETA, WHEN THE
C  LOAD FOLLOWS A LOGNORMAL DISTRIBUTION AND THE RESISTANCE
C  DISTRIBUTION IS WEIBULL. INPUT PARAMETERS ARE THE MEAN AND
C  C.O.V. OF THE LOAD AND THE LOCATION, SCALE, AND SHAPE
C  RESISTANCE PARAMETERS.
C
C
C     CARD 1 -- SPECIFY DESIGN POINT TO USE ...
C          (F10.4)  -- 'DESIGN POINT' IS THE VALUE ON THE X-AXIS
C                      USED FOR THE BETA CALCULATIONS (K.S.I.)
C
C     CARD 2 -- SPECIFY LOAD MEAN, COV, AND STRENGTH STATISTICS
C          (5F10.4) -- 'MEAN' IS THE MEAN OF THE LOAD DISTRIBUTION
C                      (K.S.I.)
C                   -- 'COV' IS THE COEFFICIENT OF VARIATION FOR
C                      THE LOAD
C                   -- 'RMU' IS THE LOCATION PARAMETER FOR THE
C                      WEIBULL CURVE (K.S.I.)
C                   -- 'RSIG' IS THE SCALE PARAMETER FOR THE
C                      WEIBULL RESISTANCE DISTRIBUTION (K.S.I.)
C                   -- 'RETA, IS THE SHAPE PARAMETER FOR THE
C                      WEIBULL RESISTANCE DISTRIBUTION (K.S.I.)
C
C***********************************************************************
C
      REAL MEAN
C
C  READ IN THE INPUT PARAMETERS AS LISTED IN THE USER'S GUIDE
C
      READ(1,12)DPT
   12 FORMAT(F10.4)
      READ(1,13)MEAN,COV,RMU,RSIG,RETA
   13 FORMAT(5F10.4)
      WRITE(2,1)
    1 FORMAT(/,15X,'LOAD DISTRIBUTION -- LOGNORMAL',/)
      WRITE(2,4)
    4 FORMAT(15X,'RESISTANCE DISTRIBUTION -- WEIBULL',//)
C
C  CALL SUBROUTINE TO FIND THE  PARAMETERS OF THE LOAD
C
```

```
      CALL PARLN(MEAN,COV,ALAM,VARLN)
      PI=3.1415927
      WRITE(2,5) DPT
    5 FORMAT(5X,'I. DESIGN POINT ....... =',F8.4,/)
      SQVAR=VARLN**.5
      WRITE(2,8) MEAN,ALAM,COV,SQVAR,RMU,RSIG,RETA
    8 FORMAT(4X,'II. LOADING STATISTICS ... MEAN LOAD =',F8.4,5X,
     1 'LAMBDA =',F8.4,/,
     2      4X,'                    LOAD C.O.V. =',F8.4,5X,
     3 ' ZETA =',F8.4,///,
     4      3X,'III. RESISTANCE STATISTICS ... LOCATION =',F8.4,/,
     5      3X,'                          SCALE =',F8.4,/,
     6      3X,'                          SHAPE =',F8.4,//)
C
C     CALCULATE PDF AND CDF AT THE DESIGN POINT FOR THE
C          RESISTANCE DISTRIBUTION
C
      FDR=(RETA/RSIG)*((DPT-RMU)/RSIG)**(RETA-1.0)*EXP(-((DPT-RMU)/RSIG
     &)**RETA)
C
      FCR = 1-EXP(-((DPT-RMU)/RSIG)**RETA)
C
C     STEP 3
C
      CALL MDNRIS(FCR,Y,IER)
      IF (IER.EQ.0) GO TO 20
      WRITE(2,153)IER
  153 FORMAT(5X,'ERROR CODE FROM IMSL IS ',I4/)
C
C     DETERMINE PARAMETERS FOR A NORMAL CURVE
C
   20 RSIGN=(1.0/(FDR*(2*PI)**.5))*EXP(-(Y**2)/2)
      RMUN=DPT-RSIGN*Y
      FCRND=(1/(RSIGN*(2.0*PI)**.5))*EXP(-((DPT-RMUN)/RSIGN)**2/2.0)
C
C     CALCULATE THE STANDARD DEVIATION FOR NORMALLY DISTRIBUTED
C
      RSDN = RSIGN
      QSDN = DPT*SQVAR
C
C     CALCULATE THE MEAN FOR A NORMALLY DISTRIBUTED VARIABLE
C
      RMN = RMUN
      QMN = DPT*(1.0-ALOG(DPT)+ALAM)
C
C     CALCULATE THE DELTA AND ALPHAS
C
      DELTA=(RSDN**2+QSDN**2)**.5
      RALF = -(RSDN/DELTA)
      QALF = QSDN/DELTA
C
C     CALCULATE BETA
C
```

```
      BETA=(QMN-RMN)/(RALF*RSDN-QALF*QSDN)
C
C
C
      WRITE(2,9)BETA
    9 FORMAT(4X,'IV. RESULTING BETA  .......... =',F12.8)
      STOP
      END
C***********************************************************************
C
C      SUBROUTINE PARLN(MEAN,COV,MNLN,VARLN)
C
C***********************************************************************
      SUBROUTINE PARLN(MEAN,COV,MNLN,VARLN)
      DOUBLE PRECISION MEAN,COV,MNLN,VARLN
      MNLN=0.5*DLOG((MEAN**2)/((COV**2)+1.))
      VARLN=DLOG((COV**2)+1.)
      RETURN
      END
```

```
C*****************************************************************
C
C   BE1W              BETA CALCULATION
C
C
C        ADVANCED FIRST ORDER SECOND MOMENT METHOD
C
C    EXTREME TYPE I LOAD AND WEIBULL RESISTANCE DISTRIBUTIONS
C
C
C*****************************************************************
C
C
C  THIS PROGRAM CALCULATES THE RELIABILITY INDEX, BETA, WHEN THE
C  LOAD FOLLOWS AN EXTREME TYPE I DISTRIBUTION AND THE RESISTANCE
C  DISTRIBUTION IS LOGNORMAL.  INPUT PARAMETERS ARE THE MEAN AND
C  C.O.V. OF THE LOAD AND THE LOCATION, SCALE, AND SHAPE PARAMETERS
C  OF THE RESISTANCE DISTRIBUTION.
C
C
C    CARD 1 -- SPECIFY DESIGN POINT TO USE ...
C          (F10.4)  -- 'DESIGN POINT' IS THE VALUE ON THE X-AXIS
C                      USED FOR THE BETA CALCULATIONS (K.S.I.)
C
C    CARD 2 -- SPECIFY LOAD MEAN, COV, AND STRENGTH STATISTICS
C          (5F10.4) -- 'MEAN' IS THE MEAN OF THE LOAD DISTRIBUTION
C                      (K.S.I.)
C                   -- 'COV' IS THE COEFFICIENT OF VARIATION FOR
C                      THE LOAD
C                   -- 'RMU' IS THE LOCATION PARAMETER OF THE
C                      RESISTANCE DISTRIBUTION (K.S.I.)
C                   -- 'RSIG' IS THE SCALE PARAMETER OF THE
C                      RESISTANCE DISTRIBUTION (K.S.I.)
C                   -- 'RETA' IS THE SHAPE PARAMETER OF THE
C                      RESISTANCE DISTRIBUTION (K.S.I.)
C
C*****************************************************************
C
      REAL MEAN,COV,A,B
C
C  READ IN THE INPUT PARAMETERS AS LISTED IN THE USER'S GUIDE
C
      READ(1,12)DPT
   12 FORMAT(F10.5)
      READ(1,13)MEAN,COV,RMU,RSIG,RETA
   13 FORMAT(5F10.5)
      WRITE(2,111)
  111 FORMAT(/,15X,'LOAD DISTRIBUTION -- EXTREME TYPE I',/)
      WRITE(2,114)
  114 FORMAT(15X,'RESISTANCE DISTRIBUTION -- WEIBULL',///)
C
C  CALCULATE THE  PARAMETERS OF THE LOAD
C
```

```
      STD=COV*MEAN
      A=STD/1.283
      B=MEAN-0.577*A
      PI=3.1415927
      WRITE(2,5) DPT
    5 FORMAT(5X,'I. DESIGN POINT ....... =',F8.4,//)
      WRITE(2,8) MEAN,A,COV,B,RMU,RSIG,RETA
    8 FORMAT(4X,'II. LOADING STATISTICS ... MEAN LOAD =',F8.4,5X,
     1 'ALPHA =',F8.4,/,
     2       4X,'                   LOAD C.O.V. =',F8.4,5X,
     3 ' BETA =',F8.4,///,
     4       3X,'III. RESISTANCE STATISTICS ... LOCATION =',F8.4,/,
     5       3X,'                           SCALE =',F8.4,/,
     6       3X,'                           SHAPE =',F8.4,///)
C
C     CALCULATE VALUES FOR THE CDF'S AND PDF'S AT THE DESIGN POINT
C
      FDQ =EXP(-((DPT-B)/A)-EXP(-((DPT-B)/A)))/A
      FDR =(RETA/RSIG)*(((DPT-RMU)/RSIG)**(RETA-1.0))*EXP(-(((DPT-RMU)
     &/RSIG)**RETA))
C
      FCQ =EXP(-EXP(-(DPT-B)/A))
      FCR = 1-EXP(-((DPT-RMU)/RSIG)**RETA)
      WRITE(2,67)FDR,FCR
   67 FORMAT(5X,'FDR =',F10.5,' FCR =',F10.5/)
C
C     CALCULATE THE INVERVE NORMAL FUNCTION AT FCQ
C
      CALL MDNRIS(FCR,YR,IER)
      IF (IER.EQ.0) GO TO 20
      WRITE(2,59)IER
   20 CALL MDNRIS(FCQ,YQ,IER)
      IF (IER.EQ.0) GO TO 30
      WRITE(2,59)IER
   59 FORMAT(5X,'ERROR CODE FROM IMSL IS ',I4)
C
C     CALCULATE THE NORMAL DENSITY FUNCTION PARAMETERS MU AND SIGMA
C
   30 RSIGN=(1.0/(FDR*(2.0*PI)**.5))*EXP(-(YR**2/2.0))
      RMUN=DPT-RSIGN*YR
      QSIGN=(1.0/(FDQ*(2.0*PI)**.5))*EXP(-(YQ**2/2.0))
      QMUN=DPT-QSIGN*YQ
      FCRND=(1.0/(RSIGN*(2*PI)**.5))*EXP(-((DPT-RMUN)/RSIGN)**2/2.0)
      WRITE(2,68)YR,FCRND
   68 FORMAT(5X,' YR =',F10.6,' FCRND =',F10.6/)
C
C     CALCULATE THE STANDARD DEVIATION FOR NORMALLY DISTRIBUTED
C
      RSDN = RSIGN
      QSDN = QSIGN
      WRITE(2,230)RSIGN,QSIGN
  230 FORMAT(5X,'RSIGN =',F10.5,' QSIGN =',F10.5/)
C
```

```
C   CALCULATE THE MEAN FOR A NORMALLY DISTRIBUTED VARIABLE
C
    RMN = RMUN
    QMN = QMUN
    WRITE(2,240)RMUN,QMUN
240 FORMAT(5X,' RMUN =',F10.5,' QMUN =',F10.5/)
C
C   CALCULATE THE DELTA AND ALPHAS
C
    DELTA=(RSDN**2+QSDN**2)**.5
    RALF = -(RSDN/DELTA)
    QALF = QSDN/DELTA
C
C   CALCULATE BETA
C
    BETA=(QMN-RMN)/(RALF*RSDN-QALF*QSDN)
C
C
C
    WRITE(2,9)BETA
  9 FORMAT(4X,'IV. RESULTING BETA .......... =',F12.8)
    STOP
    END
```

Computer Programs to Calculate K Factors

```
C************************************************************************
C
C  KBLNLN    K-FACTOR CALCULATION COMPARING BETAS FROM THE
C
C
C           ADVANCED FIRST ORDER SECOND MOMENT METHOD
C
C         USING LOGNORMAL LOAD AND RESISTANCE DISTRIBUTIONS
C
C
C************************************************************************
C
C
C  THIS PROGRAM CALCULATES THE K-FACTOR COMPARING BETA WHEN THE
C  LOAD FOLLOWS A LOGNORMAL DISTRIBUTION AND THE RESISTANCE
C  DISTRIBUTION IS ALSO LOGNORMAL.  INPUT PARAMETERS ARE THE MEAN
C  AND C.O.V. OF THE LOAD AND THE MEAN AND STD. DEV. OF THE LOGS
C  FOR THE RESISTANCE PARAMETERS.
C
C
C    CARD 1 -- SPECIFY BETA AND DESIGN POINT AND INITIAL K-FACTOR...
C          (3F10.6) -- 'BETAK' VALUE USED FOR COMPARISION
C                   -- 'DESIGN POINT' IS THE VALUE ON THE X-AXIS
C                      USED FOR THE BETA CALCULATIONS (K.S.I.)
C                   -- 'FK' INITIAL K-FACTOR
C
C    CARD 2 -- SPECIFY LOAD MEAN, COV, AND STRENGTH STATISTICS
C          (5F10.4) -- 'MEAN' IS THE MEAN OF THE LOAD DISTRIBUTION
C                      (K.S.I.)
C                   -- 'COV' IS THE COEFFICIENT OF VARIATION FOR
C                      THE LOAD
C                   -- 'RMU' IS THE MEAN OF THE LOGS FOR THE
C                      LOGNORMAL CURVE (K.S.I.)
C                   -- 'RSIG' IS THE STD. DEV. OF THE LOGS FOR
C                      THE RESISTANCE DISTRIBUTION (K.S.I.)
C
C************************************************************************
C
      REAL MEAN
C
C  READ IN THE INPUT PARAMETERS AS LISTED IN THE USER'S GUIDE
C
      READ(1,12)BETAK,DPT,FK
   12 FORMAT(3F10.6)
      READ(1,13)MEAN,COV,RMU,RSIG
   13 FORMAT(4F10.4)
      WRITE(2,1)
    1 FORMAT(/,15X,'LOAD DISTRIBUTION -- LOGNORMAL',/)
      WRITE(2,4)
```

```
  4 FORMAT(12X,'RESISTANCE DISTRIBUTION -- LOGNORMAL',///)
C
C   FIND THE  PARAMETERS OF THE LOAD
C
    ALAM=0.5*LOG((MEAN**2)/((COV**2)+1.0))
    VARLN=LOG((COV**2)+1.0)
    SQVAR=VARLN**.5
    PI=3.1415927
    WRITE(2,5) DPT,BETAK,FK
  5 FORMAT(5X,'I. INPUT VALUES ......DESIGN POINT =',F10.4,/,
   1     5X,'                   TARGET BETA =',F10.4,/,
   2     5X,'                    INITIAL K  =',F10.4,//)
    WRITE(2,8) MEAN,ALAM,COV,SQVAR,RMU,RSIG
  8 FORMAT(4X,'II. LOADING STATISTICS ... MEAN LOAD =',F10.4,5X,
   1 'LAMBDA =',F8.4,/,
   2     4X,'                    LOAD C.O.V.  =',F8.4,5X,
   3 '  ZETA =',F8.4,///,
   4     3X,'III. RESISTANCE STATISTICS ... LAMBDA =',F8.4,/,
   5     3X,'                          ZETA =',F8.4,//)
    WRITE(2,9)
  9 FORMAT(/,8X,'_______________________________________',//
   1      ,8X,'  K-FACTOR        BETA              ',/
   2      ,8X,'______________________________________',/)
C
C   INITILIZE THE STARTING PARAMETERS
C
    NPASS=1
    GO TO 30
 20  NPASS = NPASS + 1
    FK=FK+.005
C
C   INCREMENT RESISTANCE PARAMETETERS
C
 30  RKMU=RMU+LOG(FK)
C
C   FIND BETA WITH CHANGED PARAMETERS
C
C   CALCULATE THE STANDARD DEVIATION FOR NORMALLY DISTRIBUTED
C
    RSDN = DPT*RSIG
    QSDN = DPT*SQVAR
C
C   CALCULATE THE MEAN FOR A NORMALLY DISTRIBUTED VARIABLE
C
    RMN = DPT*(1.0-ALOG(DPT)+RKMU)
    QMN = DPT*(1.0-ALOG(DPT)+ALAM)
C
C   CALCULATE THE DELTA AND ALPHAS
C
    DELTA=(RSDN**2+QSDN**2)**.5
    RALF = -(RSDN/DELTA)
    QALF = QSDN/DELTA
C
```

```
C    CALCULATE BETA
C
     BETA=(QMN-RMN)/(RALF*RSDN-QALF*QSDN)
C
C    CHECK TO SEE IF BETAK IS GREATER THAN BETA
C
     IF (BETAK.LT.BETA) GO TO 50
     WRITE(2,33)FK,BETA
  33 FORMAT(10X,F6.3,5X,F10.7)
     GO TO 20
  50 IF (NPASS.EQ.1) GO TO 60
C
     WRITE(2,33)FK,BETA
     GO TO 70
C
  60 WRITE(2,43)
  43 FORMAT(10X,'INITIAL K-FACTOR TOO LARGE')
  70 CONTINUE
     STOP
     END
```

```
C*********************************************************************
C
C  KBE1LN    K-FACTOR CALCULATION COMPARING BETAS FROM THE
C
C
C         ADVANCED FIRST ORDER SECOND MOMENT METHOD
C
C   USING EXTREME TYPE I LOAD AND LOGNORMAL RESISTANCE DISTRIBUTIONS
C
C
C*********************************************************************
C
C
C  THIS PROGRAM CALCULATES THE K-FACTOR COMPARING BETA WHEN THE
C  LOAD IS A EXTREME TYPE I DISTRIBUTION AND THE RESISTANCE
C  DISTRIBUTION IS LOGNORMAL.  INPUT PARAMETERS ARE THE MEAN
C  AND C.O.V. OF THE LOAD AND THE MEAN AND STD. DEV. OF THE LOGS
C  FOR THE RESISTANCE PARAMETERS.
C
C
C    CARD 1 -- SPECIFY BETA AND DESIGN POINT AND INITIAL K-FACTOR...
C          (3F10.6) -- 'BETAK' VALUE USED FOR COMPARISION
C                   -- 'DESIGN POINT' IS THE VALUE ON THE X-AXIS
C                      USED FOR THE BETA CALCULATIONS (K.S.I.)
C                   -- 'FK' INITIAL K-FACTOR
C
C    CARD 2 -- SPECIFY LOAD MEAN, COV, AND STRENGTH STATISTICS
C          (5F10.4) -- 'MEAN' IS THE MEAN OF THE LOAD DISTRIBUTION
C                      (K.S.I.)
C                   -- 'COV' IS THE COEFFICIENT OF VARIATION FOR
C                      THE LOAD
C                   -- 'RMU' IS THE MEAN OF THE LOGS FOR THE
C                      LOGNORMAL CURVE (K.S.I.)
C                   -- 'RSIG' IS THE STD. DEV. OF THE LOGS FOR
C                      THE RESISTANCE DISTRIBUTION (K.S.I.)
C
C*********************************************************************
C
      REAL MEAN
C
C  READ IN THE INPUT PARAMETERS AS LISTED IN THE USER'S GUIDE
C
      READ(1,12)BETAK,DPT,FK
   12 FORMAT(3F10.6)
      READ(1,13)MEAN,COV,RMU,RSIG
   13 FORMAT(4F10.4)
      WRITE(2,1)
    1 FORMAT(/,12X,'LOAD DISTRIBUTION -- EXTREME TYPE I',/)
      WRITE(2,4)
    4 FORMAT(12X,'RESISTANCE DISTRIBUTION -- LOGNORMAL',///)
C
C   FIND THE  PARAMETERS OF THE LOAD
C
```

```
      STD=COV*MEAN
      A=STD/1.283
      B=MEAN-0.577*A
      PI=3.1415927
      WRITE(2,5) DPT,BETAK,FK
    5 FORMAT(5X,'I. INPUT VALUES ......DESIGN POINT =',F10.4,/,
     1      5X,'                    TARGET BETA =',F10.4,/,
     2      5X,'                      INITIAL K =',F10.4,//)
      WRITE(2,8) MEAN,A,COV,B,RMU,RSIG
    8 FORMAT(4X,'II. LOADING STATISTICS ... MEAN LOAD =',F10.4,5X,
     1 'ALPHA =',F8.4,/,
     2      4X,'                      LOAD C.O.V. =',F8.4,5X,
     3 ' BETA =',F8.4,///,
     4      3X,'III. RESISTANCE STATISTICS ... LAMBDA =',F8.4,/,
     5      3X,'                             ZETA =',F8.4,//)
      WRITE(2,9)
    9 FORMAT(/,8X,'________________________________',//
     1      ,8X,' K-FACTOR         BETA          ',/
     2      ,8X,'________________________________',/)
C
C     INITILIZE THE STARTING PARAMETERS
C
      NPASS=1
      GO TO 30
   20 NPASS = NPASS + 1
      FK=FK+.005
C
C     INCREMENT RESISTANCE PARAMETETERS
C
   30 RKMU=RMU+LOG(FK)
C
C     FIND BETA WITH CHANGED PARAMETERS
C
C     CALCULATE VALUES FOR CDF AND PDF AT THE DESIGN POINT
C
      FDQ =EXP(-((DPT-B)/A)-EXP(-((DPT-B)/A)))/A
C
      FCQ =EXP(-EXP(-(DPT-B)/A))
C
C     CALCULATE THE INVERVE NORMAL FUNCTION AT FCQ
C
      CALL MDNRIS(FCQ,Y,IER)
      IF (IER.EQ.0) GO TO 200
      WRITE(2,59)IER
   59 FORMAT(5X,'ERROR CODE FROM IMSL IS ',I4/)
C
C     CALCULATE THE NORMAL DENSITY FUNCTION PARAMETERS
C
  200 QSIGN=(1.0/(FDQ*(2.0*PI)**.5))*EXP(-(Y**2/2.0))
      QMUN=DPT-QSIGN*Y
C
C     CALCULATE THE STANDARD DEVIATION FOR NORMALLY DISTRIBUTED
C
```

```
      RSDN = DPT*RSIG
      QSDN = QSIGN
C
C     CALCULATE THE MEAN FOR A NORMALLY DISTRIBUTED VARIABLE
C
      RMN = DPT*(1-ALOG(DPT)+RKMU)
      QMN = QMUN
C
C     CALCULATE THE DELTA AND ALPHAS
C
      DELTA=(RSDN**2+QSDN**2)**.5
      RALF = -(RSDN/DELTA)
      QALF = QSDN/DELTA
C
C     CALCULATE BETA
C
      BETA=(QMN-RMN)/(RALF*RSDN-QALF*QSDN)
C
C     CHECK TO SEE IF BETAK IS GREATER THAN BETA
C
      IF (BETAK.LT.BETA) GO TO 50
      WRITE(2,33)FK,BETA
   33 FORMAT(10X,F6.3,10X,F10.7)
      GO TO 20
   50 IF (NPASS.EQ.1) GO TO 60
C
      WRITE(2,33)FK,BETA
      GO TO 70
C
   60 WRITE(2,43)
   43 FORMAT(10X,'INITIAL K-FACTOR TOO LARGE')
   70 CONTINUE
      STOP
      END
```

```
C***********************************************************************
C
C  KBLNW     K-FACTOR CALCULATION COMPARING BETAS FROM THE
C
C
C          ADVANCED FIRST ORDER SECOND MOMENT METHOD
C
C      USING LOGNORMAL LOAD AND WEIBULL RESISTANCE DISTRIBUTIONS
C
C
C***********************************************************************
C
C
C  THIS PROGRAM CALCULATES THE K-FACTOR COMPARING BETA WHEN THE
C  LOAD FOLLOWS A LOGNORMAL DISTRIBUTION AND THE RESISTANCE
C  DISTRIBUTION IS WEIBULL.  INPUT PARAMETERS AND THE MEAN
C  AND C.O.V.  OF THE LOAD AND THE LOCATION, SCALE, AND
C  SHAPE RESISTANCE PARAMETERS.
C
C
C    CARD 1 -- SPECIFY BETA AND DESIGN POINT AND INITIAL K-FACTOR...
C          (3F10.6) -- 'BETAK' VALUE USED FOR COMPARISION
C                   -- 'DESIGN POINT' IS THE VALUE ON THE X-AXIS
C                      USED FOR THE BETA CALCULATIONS (K.S.I.)
C                   -- 'FK' INITIAL K-FACTOR
C
C    CARD 2 -- SPECIFY LOAD MEAN, COV, AND STRENGTH STATISTICS
C          (5F10.4) -- 'MEAN' IS THE MEAN OF THE LOAD DISTRIBUTION
C                      (K.S.I.)
C                   -- 'COV' IS THE COEFFICIENT OF VARIATION FOR
C                      THE LOAD
C                   -- 'RMU' IS THE LOCATION PARAMETER FOR THE
C                      WEIBULL CURVE (K.S.I.)
C                   -- 'RSIG' IS THE SCALE PARAMETER FOR THE
C                      WEIBULL RESISTANCE DISTRIBUTION (K.S.I.)
C                   -- 'RETA, IS THE SHAPE PARAMETER FOR THE
C                      WEIBULL RESISTANCE DISTRIBUTION (K.S.I.)
C
C***********************************************************************
C
      REAL MEAN
C
C  READ IN THE INPUT PARAMETERS AS LISTED IN THE USER'S GUIDE
C
      READ(1,12)BETAK,DPT,FK
   12 FORMAT(3F10.6)
      READ(1,13)MEAN,COV,RMU,RSIG,RETA
   13 FORMAT(5F10.4)
      WRITE(2,1)
    1 FORMAT(/,14X,'LOAD DISTRIBUTION -- LOGNORMAL',/)
      WRITE(2,4)
    4 FORMAT(12X,'RESISTANCE DISTRIBUTION -- WEIBULL',///)
C
```

```
C   FIND THE  PARAMETERS OF THE LOAD
C
      ALAM=0.5*LOG((MEAN**2)/((COV**2)+1.0))
      VARLN=LOG((COV**2)+1.0)
      SQVAR=VARLN**.5
      PI=3.1415927
      WRITE(2,5) DPT,BETAK,FK
    5 FORMAT(5X,'I. INITIAL VALUES ....... DESIGN POINT  =',F10.6,/,
     1      5X,'                        TARGET BETA  =',F10.6,/,
     2      5X,'                    INITIAL K VALUE  =',F6.3//)
      WRITE(2,8) MEAN,ALAM,COV,SQVAR,RMU,RSIG,RETA
    8 FORMAT(4X,'II. LOADING STATISTICS ... MEAN LOAD  =',F10.4,5X,
     1 'LAMBDA  =',F8.4,/,
     2      4X,'                    LOAD C.O.V.  =',F8.4,5X,
     3 '  ZETA  =',F8.4,///,
     4      3X,'III. RESISTANCE STATISTICS ... LOCATION  =',F8.4,/,
     5      3X,'                            SCALE  =',F8.4,/,
     6      3X,'                            SHAPE  =',F8.4,//)
      WRITE(2,23)
   23 FORMAT(/,8X,'____________________________________________',//
     2      ,8X,'  K-FACTOR          BETA           ',/
     3      ,8X,'____________________________________________',/)
C
C   INITILIZE THE STARTING PARAMETERS
C
      NPASS=1
      GO TO 30
   20 NPASS = NPASS + 1
      FK=FK+.005
C
C   INCREMENT RESISTANCE PARAMETETERS
C
   30 RKSIG=RSIG*FK
      RKMU=RMU*FK
C
C   FIND BETA WITH CHANGED PARAMETERS
C
C   CALCULATE PDF AND CDF AT THE DESIGN POINT FOR THE
C       RESISTANCE DISTRIBUTION
C
      FDR=(RETA/RKSIG)*((DPT-RKMU)/RKSIG)**(RETA-1.0)*EXP(-((DPT-RKMU)/
     &RKSIG)**RETA)
C
      FCR = 1-EXP(-((DPT-RKMU)/RKSIG)**RETA)
C
C  STEP 3
C
      CALL MDNRIS(FCR,Y,IER)
      IF (IER.EQ.0) GO TO 200
      WRITE(2,153)IER
  153 FORMAT(5X,'ERROR CODE FROM IMSL IS ',I4/)
C
```

```
C   DETERMINE PARAMETERS FOR A NORMAL CURVE
C
 200 RSIGN=(1.0/(FDR*(2*PI)**.5))*EXP(-(Y**2)/2)
     RMUN=DPT-RSIGN*Y
     FCRND=(1/(RSIGN*(2.0*PI)**.5))*EXP(-((DPT-RMUN)/RSIGN)**2/2.0)
C
C   CALCULATE THE STANDARD DEVIATION FOR NORMALLY DISTRIBUTED
C
     RSDN = RSIGN
     QSDN = DPT*SQVAR
C
C   CALCULATE THE MEAN FOR A NORMALLY DISTRIBUTED VARIABLE
C
     RMN = RMUN
     QMN = DPT*(1.0-ALOG(DPT)+ALAM)
C
C   CALCULATE THE DELTA AND ALPHAS
C
     DELTA=(RSDN**2+QSDN**2)**.5
     RALF = -(RSDN/DELTA)
     QALF = QSDN/DELTA
C
C   CALCULATE BETA
C
     BETA=(QMN-RMN)/(RALF*RSDN-QALF*QSDN)
C
C
C
C   CHECK TO SEE IF BETAK IS GREATER THAN BETA
C
     IF (BETA.GT.BETAK) GO TO 50
     WRITE(2,33)FK,BETA
  33 FORMAT(10X,F6.3,12X,F10.7)
     GO TO 20
C
C  CHECK TO SEE IF INITIAL K IS TOO LARGE
C
  50 IF (NPASS.EQ.1) GO TO 60
C
     WRITE(2,33)FK,BETA
     GO TO 70
C
  60 WRITE(2,43)
  43 FORMAT(10X,'INITIAL K-FACTOR TOO LARGE')
  70 CONTINUE
     STOP
     END
```

```
C*********************************************************************
C
C  KBE1W    K-FACTOR CALCULATION COMPARING BETAS FROM THE
C
C
C        ADVANCED FIRST ORDER SECOND MOMENT METHOD
C
C    USING EXTREME TYPE I LOAD AND WEIBULL RESISTANCE DISTRIBUTIONS
C
C
C*********************************************************************
C
C
C  THIS PROGRAM CALCULATES THE K-FACTOR COMPARING BETA WHEN THE
C  LOAD IS EXTREME TYPE I DISTRIBUTION AND THE RESISTANCE
C  DISTRIBUTION IS WEIBULL.  INPUT PARAMETERS AND THE MEAN
C  AND C.O.V. OF THE LOAD AND THE LOCATION, SCALE, AND SHAPE
C  RESISTANCE PARAMETERS.
C
C
C    CARD 1 -- SPECIFY BETA AND DESIGN POINT AND INITIAL K-FACTOR...
C          (3F10.6) -- 'BETAK' VALUE USED FOR COMPARISION
C                   -- 'DESIGN POINT' IS THE VALUE ON THE X-AXIS
C                      USED FOR THE BETA CALCULATIONS (K.S.I.)
C                   -- 'FK' INITIAL K-FACTOR
C
C    CARD 2 -- SPECIFY LOAD MEAN, COV, AND STRENGTH STATISTICS
C          (5F10.4) -- 'MEAN' IS THE MEAN OF THE LOAD DISTRIBUTION
C                      (K.S.I.)
C                   -- 'COV' IS THE COEFFICIENT OF VARIATION FOR
C                      THE LOAD
C                   -- 'RMU' IS THE LOCATION PARAMETER FOR THE
C                      WEIBULL CURVE (K.S.I.)
C                   -- 'RSIG' IS THE SCALE PARAMETER FOR THE
C                      WEIBULL RESISTANCE DISTRIBUTION (K.S.I.)
C                   -- 'RETA, IS THE SHAPE PARAMETER FOR THE
C                      WEIBULL RESISTANCE DISTRIBUTION (K.S.I.)
C
C*********************************************************************
C
      REAL MEAN
C
C  READ IN THE INPUT PARAMETERS AS LISTED IN THE USER'S GUIDE
C
      READ(1,12)BETAK,DPT,FK
   12 FORMAT(3F10.6)
      READ(1,13)MEAN,COV,RMU,RSIG,RETA
   13 FORMAT(5F10.4)
      WRITE(2,1)
    1 FORMAT(/,12X,'LOAD DISTRIBUTION -- EXTREME TYPE I',/)
      WRITE(2,4)
    4 FORMAT(12X,'RESISTANCE DISTRIBUTION -- WEIBULL',///)
C
```

```
C    FIND THE  PARAMETERS OF THE LOAD
C
      STD=COV*MEAN
      A=STD/1.283
      B=MEAN-0.577*A
      PI=3.1415927
      WRITE(2,5) DPT,BETAK,FK
    5 FORMAT(5X,'I. INITIAL VALUES ....... DESIGN POINT  =',F10.6,/,
     1     5X,'                          TARGET BETA  =',F10.6,/,
     2     5X,'                        INITIAL K VALUE  =',F6.3//)
      WRITE(2,8) MEAN,A,COV,B,RMU,RSIG,RETA
    8 FORMAT(4X,'II. LOADING STATISTICS ... MEAN LOAD  =',F10.4,5X,
     1 'LAMBDA  =',F8.4,/,
     2     4X,'                       LOAD C.O.V.  =',F8.4,5X,
     3 '  ZETA  =',F8.4,///,
     4     3X,'III. RESISTANCE STATISTICS ... LOCATION  =',F8.4,/,
     5     3X,'                             SCALE  =',F8.4,/,
     6     3X,'                             SHAPE  =',F8.4,//)
      WRITE(2,23)
   23 FORMAT(/,8X,'____________________________________________',//
     2      ,8X,'  K-FACTOR          BETA          ',/
     3      ,8X,'____________________________________________',/)
C
C    INITILIZE THE STARTING PARAMETERS
C
      NPASS=1
      GO TO 30
   20 NPASS = NPASS + 1
      FK=FK+.005
C
C    INCREMENT RESISTANCE PARAMETETERS
C
   30 RKSIG=RSIG*FK
      RKMU=RMU*FK
C
C    FIND BETA WITH CHANGED PARAMETERS
C
C    CALCULATE VALUES FOR THE CDF'S AND PDF'S AT THE DESIGN POINT
C
      FDQ =EXP(-((DPT-B)/A)-EXP(-((DPT-B)/A)))/A
      FDR =(RETA/RKSIG)*((DPT-RKMU)/RKSIG)**(RETA-1.0)*EXP(-((DPT-RKMU)/
     &RKSIG)**RETA)
C
      FCQ =EXP(-EXP(-(DPT-B)/A))
      FCR =1-EXP(-((DPT-RKMU)/RKSIG)**RETA)
C
C    CALCULATE THE INVERVE NORMAL FUNCTION AT FCQ
C
      CALL MDNRIS(FCR,YR,IER)
      IF (IER.EQ.0) GO TO 200
      WRITE(2,59)IER
  200 CALL MDNRIS(FCQ,YQ,IER)
      IF (IER.EQ.0) GO TO 300
```

```
      WRITE(2,59)IER
   59 FORMAT(5X,'ERROR CODE FROM IMSL IS ',I4)
C
C     CALCULATE THE NORMAL DENSITY FUNCTION PARAMETERS MU AND SIGMA
C
  300 RSIGN=(1.0/(FDR*(2.0*PI)**.5))*EXP(-(YR**2/2.0))
      RMUN=DPT-RSIGN*YR
      QSIGN=(1.0/(FDQ*(2.0*PI)**.5))*EXP(-(YQ**2/2.0))
      QMUN=DPT-QSIGN*YQ
      FCQND=(1.0/(QSIGN*(2*PI)**.5))*EXP(-((DPT-QMUN)/QSIGN)**2/2.0)
C
C     CALCULATE THE STANDARD DEVIATION FOR NORMALLY DISTRIBUTED
C
      RSDN = RSIGN
      QSDN = QSIGN
C
C     CALCULATE THE MEAN FOR A NORMALLY DISTRIBUTED VARIABLE
C
      RMN = RMUN
      QMN = QMUN
C
C     CALCULATE THE DELTA AND ALPHAS
C
      DELTA=(RSDN**2+QSDN**2)**.5
      RALF = -(RSDN/DELTA)
      QALF = QSDN/DELTA
C
C     CALCULATE BETA
C
      BETA=(QMN-RMN)/(RALF*RSDN-QALF*QSDN)
C
C     CHECK TO SEE IF BETAK IS GREATER THAN BETA
C
      IF (BETA.GT.BETAK) GO TO 50
      WRITE(2,33)FK,BETA
   33 FORMAT(10X,F6.3,12X,F10.7)
      GO TO 20
C
C    CHECK TO SEE IF INITIAL K IS TOO LARGE
C
   50 IF (NPASS.EQ.1) GO TO 60
C
      WRITE(2,33)FK,BETA
      GO TO 70
C
   60 WRITE(2,43)
   43 FORMAT(10X,'INITIAL K-FACTOR TOO LARGE')
   70 CONTINUE
      STOP
      END
```

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