COMPRESSIVE CRIPPLING OF STRUCTURAL SECTIONS

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ABSTRACT

A method is presented for the calculation of the crippling stress of structural sections as a function of material properties and the proportions of the section. The presence of formed or anisotropic material is accounted for by the use of an effective stress-strain curve in defining the material properties. The method applies to many sections for which a procedure for calculating crippling was not previously available.

INTRODUCTION

An important part of structural design is the determination of the allowable compressive loads carried by the columns and stringers that are part of the basic aircraft structure. If a member is relatively long or of compact cross section, the stress at failure can be adequately predicted by Euler's column formula using the tangent modulus for inelastic stresses. However, for shorter lengths, many sections composed of plate elements buckle locally before the Euler load is reached, which causes a reduction in column stiffness and a corresponding reduction in the ultimate load. As length is decreased, a point is reached where failure is primarily a function of the cross-sectional proportions, and any further decrease in length will cause no appreciable change in stress. For this portion of the column curve, where strength is relatively independent of length, the stress at failure will be referred to as the crippling stress. The crippling stress is then the upper limit to the column curve and in general provides an index to the capacity of a section to carry compressive load.

Theoretical determination of crippling stresses has proved to be so difficult that only a few simple cases have been analyzed. Accordingly, information necessary for design has been obtained experimentally by testing various sections which were representative of the shapes, proportions, and materials in current use. However, recent developments in high-speed flight have presented new problems for the structural designer. Aerodynamic heating of the airframe caused by supersonic speeds has brought about the use of more heat resistant materials such as titanium and the high strength steels. Introduction of these "new" materials plus the addition of temperature as a variable requires many additional tests if crippling is to be determined by experiment alone. To eliminate the need for such an extensive testing program, several investigators have developed crippling strength equations, based on the available data, relating crippling stress to the proportions of a section and the properties of the material. These equations are useful for evaluating a structural material at elevated temperature, and when necessary a few tests can be made to confirm the calculations. For sections which are formed of flat sheet, the method of reference 1 is shown to be applicable for a large number of shapes and proportions. In reference 2, a method is presented for predicting the crippling strength of formed zee and channel sections and extruded zee, channel, and H sections having webs with relatively small width-thickness ratios. Tests made in the present investigation indicate that the equations presented in reference 2 cannot be extended to sections for which the width of the web is large with respect to its thickness, and therefore probably cannot be considered to apply to extrusions in general. For extruded zee, channel, and H sections with webs of large width-thickness ratios, plus other shaped extrusions, no method is readily available for predicting the crippling stress. The purpose of the present paper is to present a method for predicting crippling stresses which will include the ranges not covered by existing methods as well as applying to sections for which adequate methods are available.

NOMENCLATURE

b	width of plate element
a	length of plate element
t	thickness of plate element
A	area of plate element
b_{eff}	effective width of plate element
b'	developed width of a section
P	load
σ	average stress
σ_{cy}	0.2 percent offset compressive yield stress
$\bar{\sigma}_f$	crippling stress of a section
σ_{cr}	buckling stress
σ_{el}	elastic buckling stress
σ_e	edge stress
ϵ_e	edge strain
E	modulus of elasticity
E_S	secant modulus of elasticity
E_T	tangent modulus of elasticity
σ_n	stress at which $E_T = \frac{1}{n} E_S$
E_{Sn}	secant modulus at σ_n
E_2	$(E_{S2}\sigma_2)^{1/2}$ when $\sigma_W \leq \sigma_2$
	$(E_S\sigma_2)^{1/2}$ when $\sigma_W > \sigma_2$

E_3 $(E_{S3}\sigma_3^2)^{1/3}$ when $\sigma_F \leq \sigma_3$

$(E_S\sigma_3^2)^{1/3}$ when $\sigma_F > \sigma_3$

μ Poisson's ratio

D plate flexural stiffness per unit width $\dfrac{Et^3}{12(1 - \mu^2)}$

k buckling stress coefficient

η plasticity reduction factor

I moment of inertia

$EI/bD - A\sigma_{cr}a^2/bDm^2\pi^2$

m, n integers

C, K constants

Subscripts:

W applies to web

F applies to flange

METHOD OF ANALYSIS

Crippling failures can be divided into two categories; one in which local buckling is initiated in the high stress region (stresses greater than about 3/4 of the yield stress) with failure occurring at little or no increase in load, and one in which local buckling is elastic with a definite margin between buckling and failure. Calculations of the crippling stress for the first case is in essence a calculation of the plastic buckling stress. To compute plastic buckling, the method of reference 3 has been shown to compare favorably with experiment. By this procedure, the actual buckling stress is related, by the coefficient η, to the stress at which buckling would occur assuming perfect elasticity, where η is a function of the stress level. η is also a function of the boundary conditions, but, for plates which are supported along the edges, η is almost independent of the rotational restraint. Therefore, for sections composed of plate elements with varying degrees of rotational restraint at the joints, the value of η for a simply supported plate is sufficiently accurate to calculate the buckling stress. The buckling stress is then given by the method of reference 3 as

$$\sigma_{cr} = \eta\sigma_{el} \tag{1}$$

where for a simply supported plate

$$\eta = \frac{E_S}{E}\left(\frac{1}{2} + \frac{1}{2}\sqrt{\frac{1}{4} + \frac{3}{4}\frac{E_T}{E_S}}\right)$$

The problem that remains then is the calculation of the crippling stress of sections which buckle elastically and then sustain further load before failure.

One of the earliest methods used for predicting crippling stresses was to calculate the buckling stress for each plate element and average these stresses according to area. The method is conservative in most cases because of the ability of plate elements to support load after buckling. However, the concept that the crippling stress may be obtained by averaging stresses that are characteristic of the individual elements, can still be used to formulate an empirical crippling analysis if expressions can be found which more accurately reflect the stresses carried by each element. When a section is divided into plate elements, some will be supported along both edges (webs) and the rest will be supported along only one edge (flanges). If a stress σ_W is associated with each web element, and a stress σ_F is associated with each flange element, the crippling stress will be given by

$$\bar{\sigma}_f = \frac{\sum A_W \sigma_W + \sum A_F \sigma_F}{\sum A_W + \sum A_F} \qquad (2)$$

The validity of separating the crippling load into the loads carried by the individual parts, as indicated in equation (2), will be illustrated in the following sections and appropriate expressions for σ_W and σ_F will be developed.

Determination of σ_W and σ_F

The problem of evaluating expressions for σ_W and σ_F is that of satisfying equation (2) for a variety of proportions and a number of materials. The results of reference 4 can be used to establish a relationship between σ_W and $\frac{b_W}{t_W}$. Here a large number of plates of various materials were tested in compression in V-groove edge fixture in order to determine a suitable parameter to define material properties which effect plate compressive strength. One such parameter was obtained from an effective width analysis using this expression for effective width

$$\frac{b_{eff}}{b} = \frac{\sigma}{\sigma_e} = \left(\frac{\varepsilon_{cr}}{\varepsilon_e}\right)^{1/n} \tag{3}$$

The average stress corresponding to any edge strain is then

$$\sigma = \frac{\sigma_e}{(\varepsilon_e)^{1/n}}(\varepsilon_{cr})^{1/n} \tag{4}$$

This may also be written as

$$\sigma = K\,\frac{\sigma_e}{(\varepsilon_e)^{1/n}}\left(\frac{t}{b}\right)^{2/n} \tag{5}$$

Equation (5) then represents a curve of average stress against unit shortening. $\bar{\sigma}_f$, which is the maximum value of σ, can be obtained by equating to zero the derivative of σ with respect to σ_e which gives

$$\bar{\sigma}_f = K(E_{Sn}\sigma_n{}^{n-1})^{1/n}\left(\frac{t}{b}\right)^{2/n} \tag{6}$$

where σ_n is stress at which $E_T = \frac{1}{n} E_S$. The experimental results indicate n should be taken as two to obtain correlation for web elements. For stresses greater than σ_2, correlation can be effected in the plastic range by using the parameter $(E_S\sigma_2)^{1/2}$. The results of these tests are shown in figure 1 where $\frac{\sigma_W}{E_2}$ is plotted against $\frac{b_W}{t_W}$.
($E_2 = (E_{S2}\sigma_2)^{1/2}$ when $\sigma_W \leq \sigma_2$ and $E_2 = (E_S\sigma_2)^{1/2}$ when $\sigma_W > \sigma_2$.)
The points lie very close to a single curve indicating the suitability of E_2 as a material correlation parameter. The data can be approximated by the equation.

$$\sigma_W = 1.75E_2 \frac{t_W}{b_W} \tag{7}$$

Equation (7) is also shown to be applicable to square tubes and to web elements of more complicated structures such as sheet-stringer panels.

The determination of an expression for σ_F is not as simple and a more indirect approach was used. If σ_W is assumed to be given by equation (7), values of σ_F can be obtained experimentally from a crippling test on a zee or channel cross section. A large number of

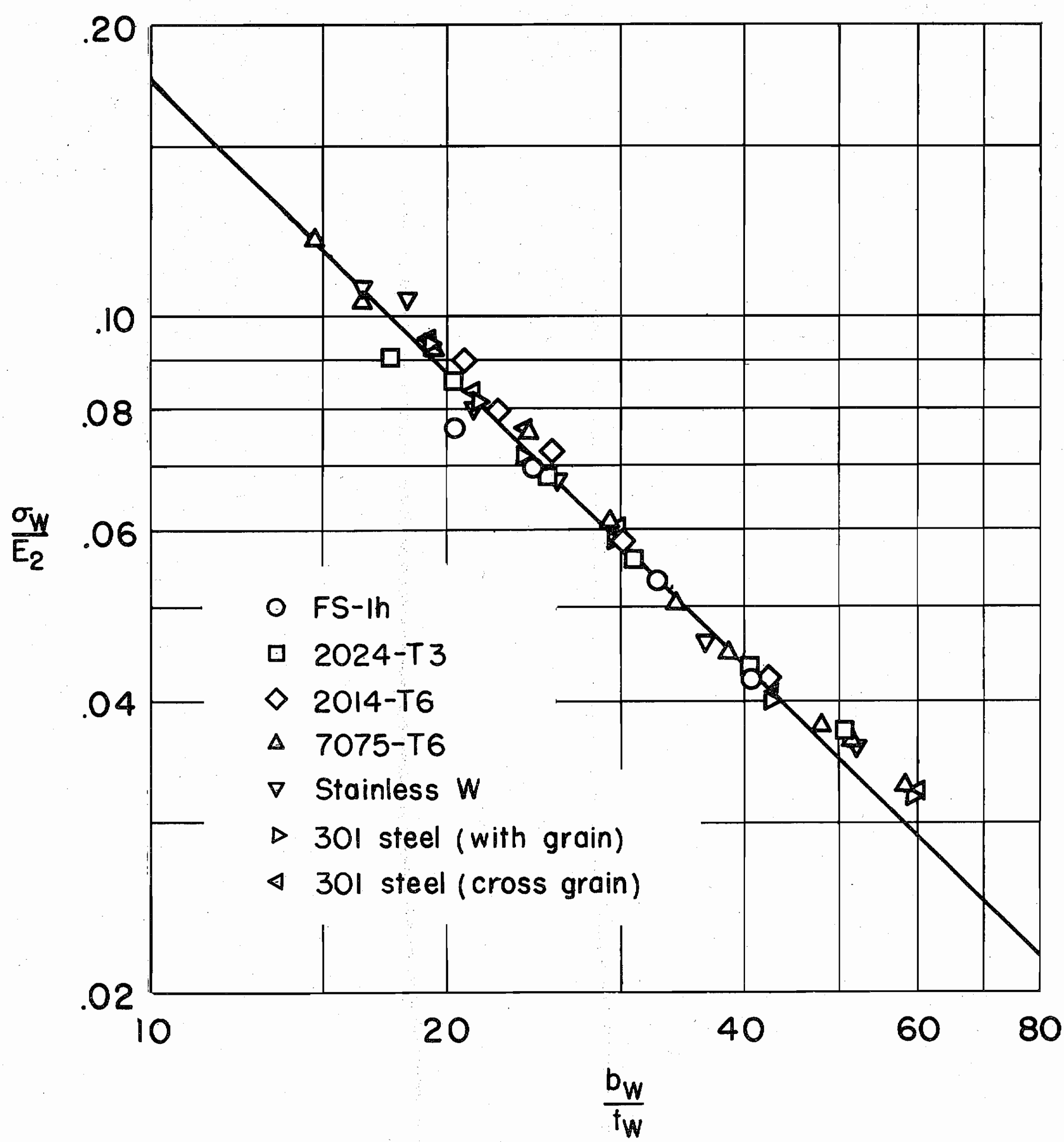

Figure 1. - Average stress developed in webs.

tests on these sections have been made and the results are reported in references 5 through 12. The stress carried in the flanges of these sections can be defined from equation (2) as

$$\sigma_F = \bar{\sigma}_f + \frac{\bar{\sigma}_f - \sigma_W}{2\ b_F/b_W} \tag{8}$$

Using equation (8), σ_F was computed for the zee and channel sections which buckled at less than 3/4 of the yield stress. The results of these calculations indicated that σ_F was related to $\frac{t_F}{b_F}$ by an equation of the form

$$\sigma_F = K\left(\frac{t_F}{b_F}\right)^{2/3} \tag{9}$$

Since this expression is similar to equation (6) with n equal to three, correlation among materials was attempted by the use of the parameter E_3 in the same manner as E_2 was used to correlate web stresses among different materials. Accordingly $\frac{\sigma_F}{E_3}$ was plotted aginst $\frac{b_F}{t_F}$ as shown in figure 2. In all cases, the dimensions b_F and b_W were measured between the centerlines of the plate elements as indicated in the figure. The material parameters for the formed section were calculated from an effective stress-strain curve which is described in the next section. Deviations of the data from a simple curve may be due to (1) experimental scatter, (2) variation of the web stress from that given by equation (7),

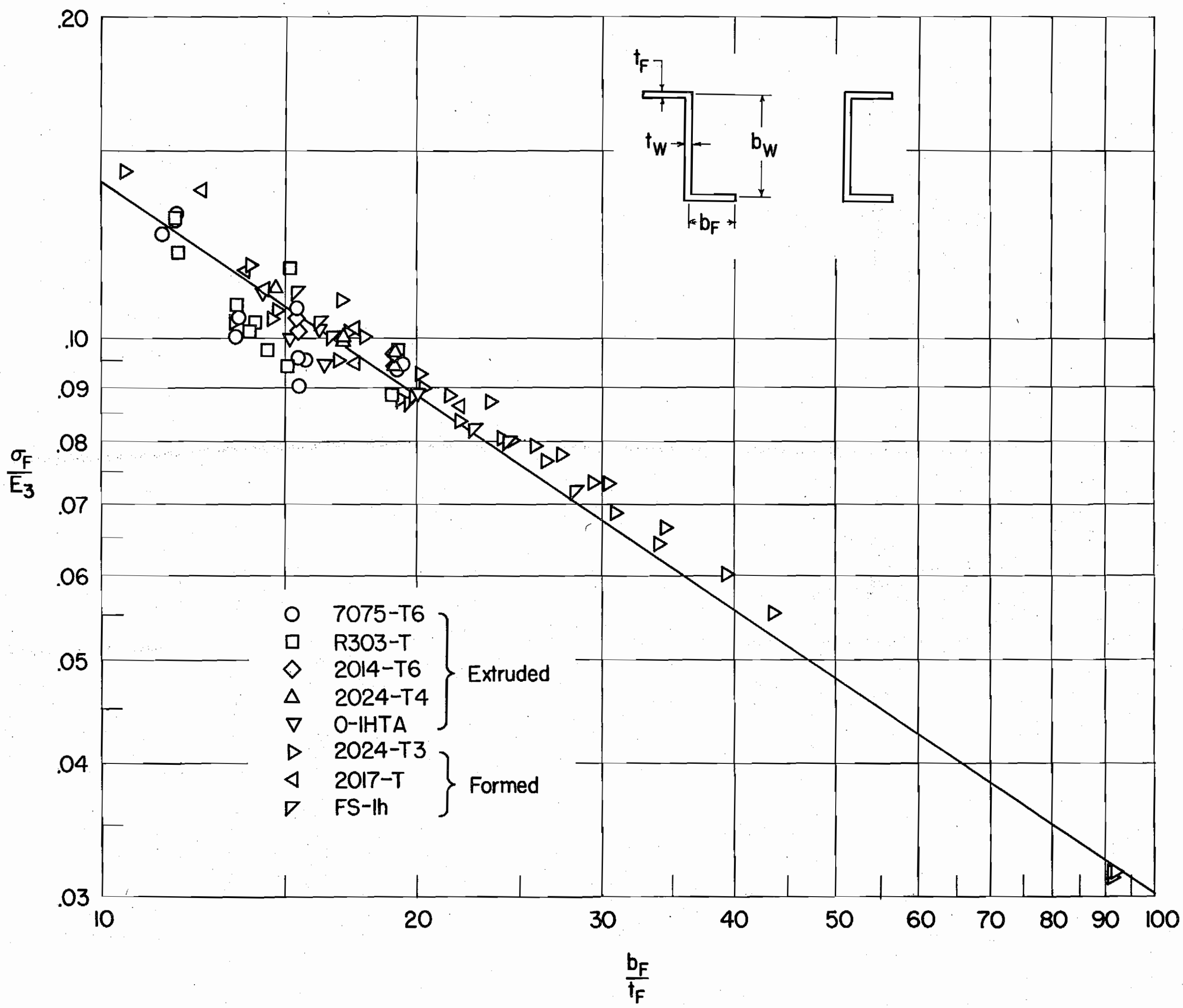

Figure 2.- Average stress developed in flanges of zee and channel sections.

and (3) variation of the flange stress due to interaction with other plate elements. On a plot such as figure 2, errors from all three sources are attributed to the flange which represents only a portion of the total cross-sectional area. The low points around $\frac{b_F}{t_F}$ equal to 15 correspond to tests where the length was long enough that column action may have been present. Taking these possible sources of error into account the scatter is not great, and on a logarithmic plot the test points tend to lie on a straight line given by the equation

$$\sigma_F = 0.65\, E_3 \left(\frac{t_F}{b_F}\right)^{2/3} \tag{10}$$

where

$$E_3 = (E_{S3}\sigma_3^2)^{1/3} \qquad \sigma_F \leq \sigma_3$$

$$E_3 = (E_S\sigma_3^2)^{1/3} \qquad \sigma_F > \sigma_3$$

The line was drawn to give the best fit to the data for all the different materials.

Equations (7) and (10) were found to give good results when applied to other sections, with the exception that the method is conservative for sections which have more than two elements meeting at a flange joint, such as an H section. Observation of crippling tests has shown that as load increases after buckling, distortions grow until the corners themselves buckle, resulting in failure. The resistance to this corner buckling is much greater if more than two elements meet at a joint. To evaluate this affect, the flange stresses were computed for the H sections tested in reference 5 through 9 using equation (7) for the web stresses. These results are shown in figure 3. An equation of the same form as equation (10) may be fitted to these data but the coefficient is now 0.75 instead of 0.65.

An empirical equation for the average stress at failure of a flange element may then be written as

$$\sigma_F = C_F E_S \left(\frac{t_F}{b_F}\right)^{2/3} \tag{11}$$

where

$C_F = 0.65$ two elements at a flange joint

$C_F = 0.75$ more than two elements at a flange joint.

Material Properties and the Effective Stress-Strain Curve

The equations for σ_W and σ_F have a term which represents the effect of geometry on crippling, and another term which is a measure of material properties. For sections which have uniform properties, the material parameters can be calculated directly from the compressive stress-strain curve. If a material is anisotropic or has a variation in material properties due to forming, the stress-strain curve can be modified to account for these variations. Reference 4 presents a simple empirical method of constructing this modified or effective stress-strain

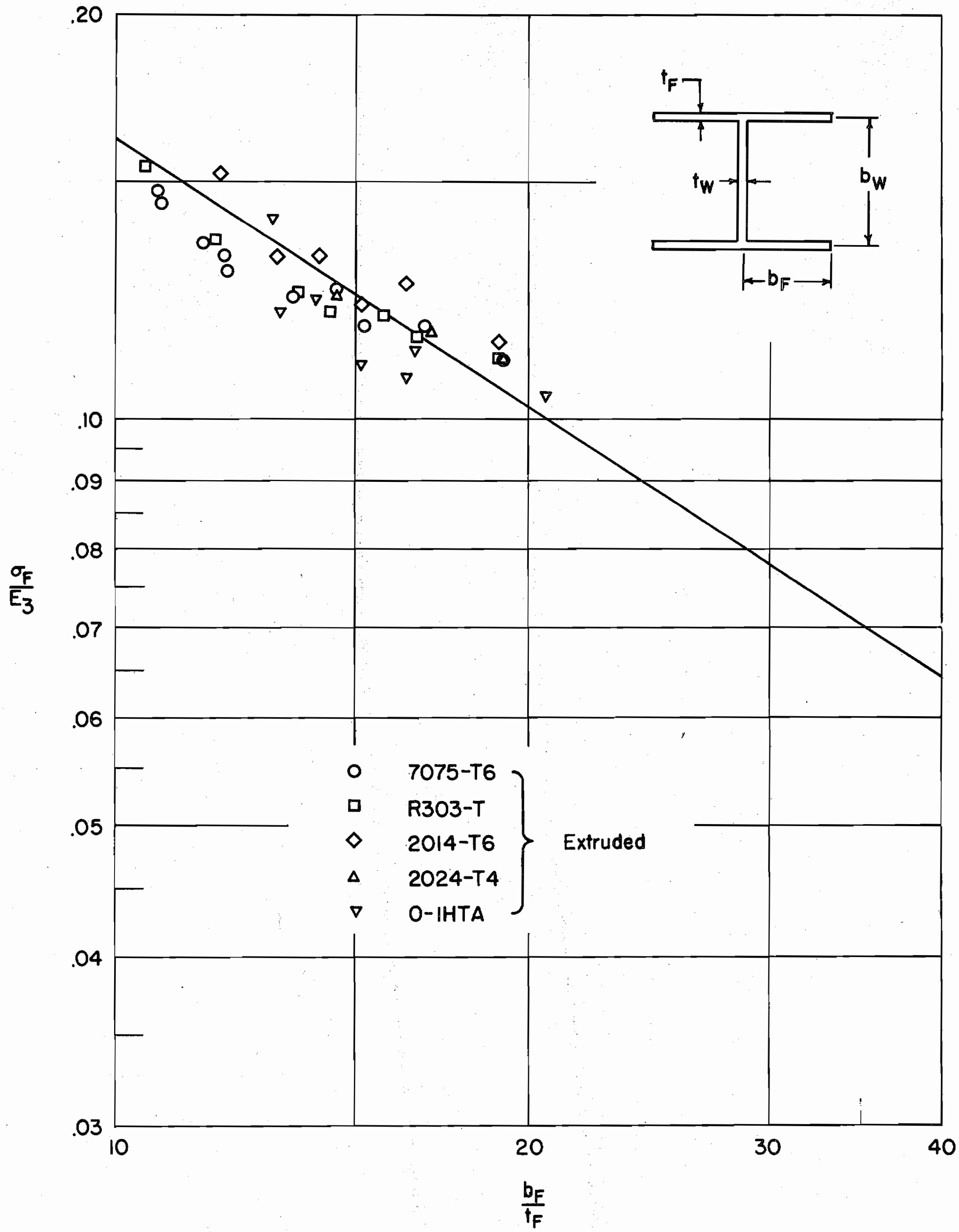

Figure 3.- Average stress developed in flanges of H sections.

curve for anisotropic material. In this case, the effective stress-strain curve was computed as a weighted average of the stress-strain curve in the loading direction and the stress-strain curve in the transverse direction, where the stress-strain curve in the loading direction was weighted twice as much as the stress-strain curve in the transverse direction. Correlation that can be achieved by this type of correction is illustrated in figure 1 where test points for 301 steel, which is highly anisotropic, lie on the same curve as those for the isotropic materials. For most materials this correction is negligible, but is quite important in a material such as stainless steel.

When a section is formed, the yield stress in the vicinity of the formed corner is generally increased over that of the flat sheet. This increase in yield stress results in a higher crippling stress than for an identical section with uniform properties corresponding to the flat sheet material. Here again it is possible to define an effective stress-strain curve which will give satisfactory correlation when used to calculate the material parameters necessary to predict the crippling stress. Several methods of combining the properties of the formed and unformed sheet were tried, and the one that appeared to work best was also one of the simplest. For formed sections it was found that the effective stress-strain curve can be taken as the average of the stress-strain curves obtained from the formed corner material and the flat sheet material. Even though the formed material may be a small percentage of total area, it is necessary to weight its stress-strain curve as strongly as that for the flat sheet material, because crippling is in reality failure of the corner; hence will be influenced most by the material properties in the vicinity of the formed corner. When the effective stress-strain curve is used, test points for formed sections intermingle with those for extrusions as shown in figure 1. The formulas then imply that the geometric effect of the curved corner on crippling is negligible, but the effect of increased yield stress due to forming is important. A similar situation exists regarding the effect of the curved corner on buckling stresses. For elastic buckling the conventional formulas can be used without considering the effect of the curved corner (forming in general does not change E), but in the plastic range the formulas are conservative which may be attributed to the increase in yield stress. These conclusions are based on the tests reported in reference 10 through 12, where the sections were formed to a bend radius of about three to four times the sheet thickness.

The effective stress-strain curve can also be used, in conjunction with equation (1), to calculate the buckling stress for sections of anisotropic material or for sections which have a variation in material properties due to forming. This is illustrated in figure 4, where experimental buckling stresses for formed zee and channel sections tested in reference 10, are compared with computations based on the effective stress-strain curve. σ_{el} is computed from the conventional plate buckling formula

$$\sigma_{el} = \frac{k_W \pi^2 E}{12(1-\mu^2)} \left(\frac{t_W}{b_W} \right)^2 \qquad (12)$$

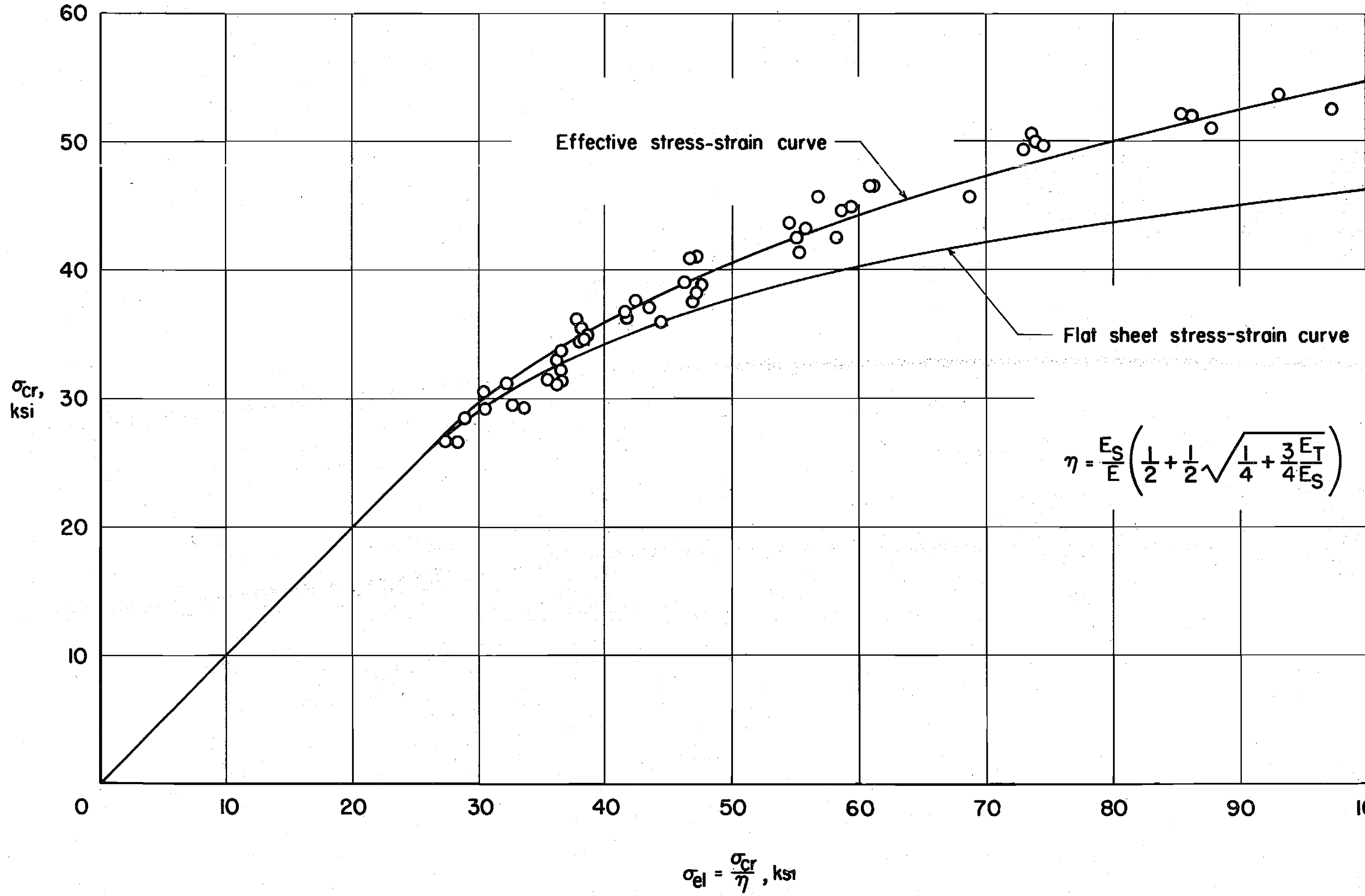

Figure 4.– Buckling stress of 2024-T3 aluminum-alloy zee and channel sections.

Values of k_w can be found in reference 13. If the effect of forming is neglected, the calculated buckling stresses are quite conservative as indicated by the lower curve.

Therefore, the procedures and methods developed in the previous section can be used for formed sections or anisotropic material by the introduction of the effective stress-strain curve in the calculation of buckling as well as crippling stresses.

Size of Flange Necessary to Effectively Support a Web

Observation of crippling tests of flanged sections show that one type of failure occurs when the flange width is large and another type when the width is small. In the first case, the joints remain straight until failure, while in the second case, the flange joint is translated upon buckling and failure occurs at a reduced load. The equations developed in the previous section apply when the flange is large, hence it is desirable to know the minimum width flange that will force a node along a joint.

In order to establish a criterion the flange is considered an elastic beam providing deflectional restraint to a web. From the solution for the buckling stress of a plate restrained by elastic beams the significant parameters defining the restraint offered by the flange may be determined. In this solution, given by Timoshenko in reference 14, it is seen that the buckling stress is a function of a stiffness parameter θ, where for a flange supporting a web

$$\theta = \frac{EI_F}{b_W D_W} - \frac{A_F \sigma_{cr} a^2}{b_W D_W m^2 \pi^2} \qquad (13)$$

and as θ approaches infinity the buckling stress approaches that of a simply supported plate. However, if θ is of the order of 100 or greater, the buckling curve differs little from the simply supported buckling curve and for these proportions the crippling equations are valid. It is to be expected, as θ is reduced, a value will be reached where the buckling stress is sufficiently below that of a simply supported plate that the crippling equations will no longer be valid. In other words, above a certain value of θ, the crippling stress can be predicted by the method presented in the previous section, while below this value the procedure is unconservative. (Actually there probably is no clear dividing line between the two types of failure, but rather a transition from one to the other over a range of values of θ.) To determine a transition value of θ, the crippling stress of several sections with relatively small flanges was computed and compared with experiment. For these proportions, the second term of equation (8) was small compared to the first, so θ was taken as $EI_F/b_W D_W$ which may be written as

$K\left(\frac{b_F}{b_W}\right)^3\left(\frac{b_W}{t_W}\right)^2\left(\frac{t_F}{t_W}\right)$. It was found that if $\left(\frac{b_F}{b_W}\right)^3\left(\frac{b_W}{t_W}\right)^2\left(\frac{t_F}{t_W}\right)$ was greater than 20, the predicted stress was generally in agreement with experiment, while if this quantity was less than 20, the predictions were often unconservative. Most flange proportions encountered in practice are more than adequate to effectively support the webs.

In some cases, however, sections such as zees or hats may have an additional small flange or lip which is not large enough to satisfy the above criterion. This is illustrated in figure 5, where the crippling stresses of lipped zees are plotted against the size of lip. When the width of the lip is large, corresponding to the criterion

$$\left(\frac{b_F}{b_W}\right)^3\left(\frac{b_W}{t_W}\right)^2\left(\frac{t_F}{t_W}\right) > 20 \tag{14}$$

$\bar{\sigma}_f$ can be predicted by the method of the present investigation. Also, when the lip width is zero, $\bar{\sigma}_f$ can be computed as for a plain zee section. By drawing a straight line from $\bar{\sigma}_f$ corresponding to the smallest value of $\frac{b_F}{b_W}$ that will satisfy equations (9), to $\bar{\sigma}_f$ representing the crippling stress of a plain zee, the intermediate test points can be adequately predicted.

DISCUSSION

The empirical crippling strength equations presented give the average stress carried by each of the individual plate elements which comprise a structural section. The loads carried by each element may be obtained by multiplying equations (7) and (10) by the appropriate area to give

$$\frac{P_W}{t_W^2} = 1.75\ E_2 \tag{15}$$

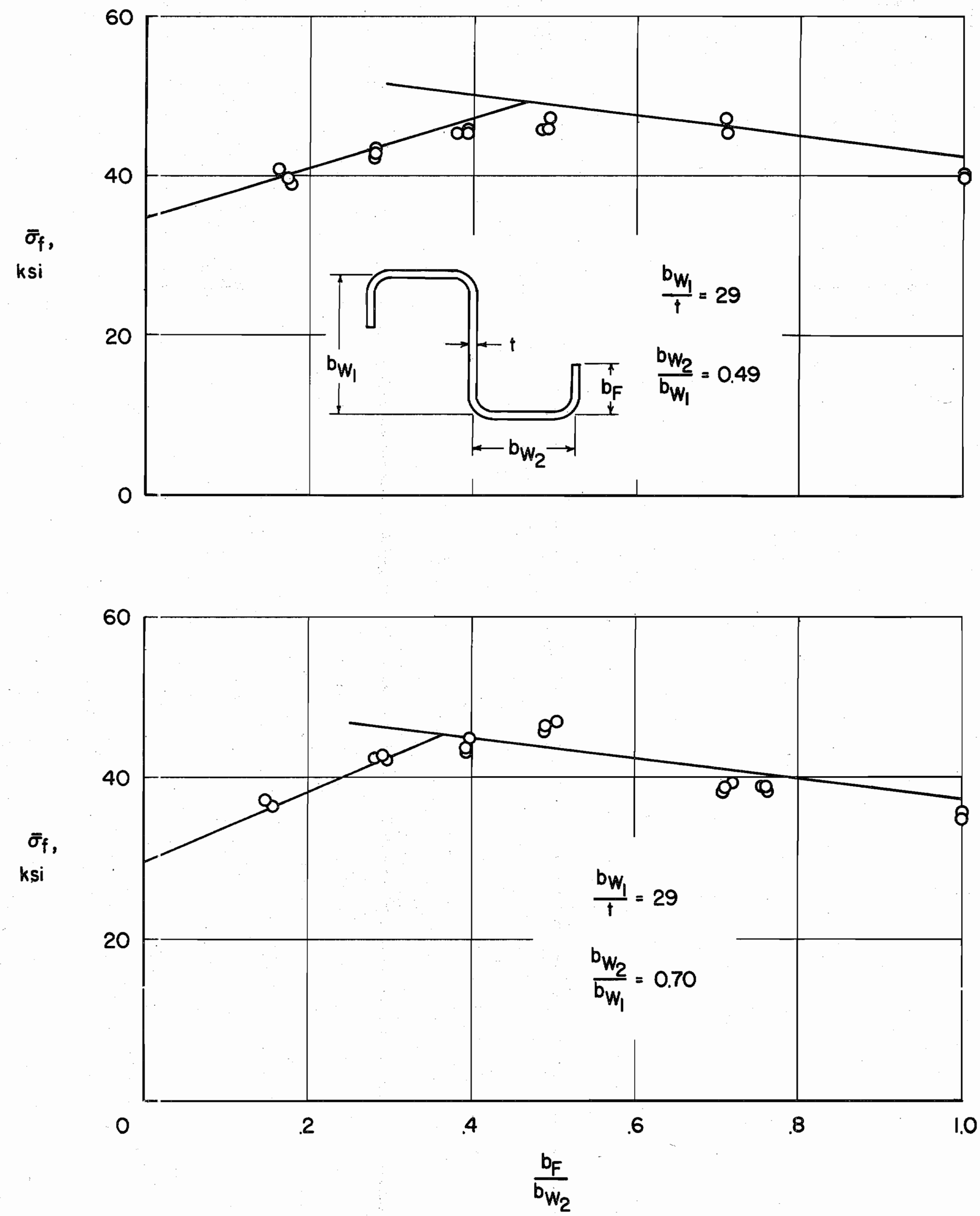

Figure 5.- Crippling stress of 2024-T3 aluminum-alloy lipped zee sections.

for a web, and

$$\frac{P_F}{t_F^2} = C_F E_3 \left(\frac{b_F}{t_F}\right)^{1/3} \tag{16}$$

for a flange. It is seen that for a given thickness, the load carried by a web is constant for all values of $\frac{b_W}{t_W}$ except when $\frac{b_W}{t_W}$ is small enough that the crippling stress is in the plastic range, while a flange will support a load that increases with $\frac{b_F}{t_F}$. Using equations (15) and (16), conclusions can be quickly drawn to determine the optimum distribution of material for certain shapes. For example in a zee or channel section, the optimum proportions for any thickness will be such that σ_W is close to σ_2. Since at this value of stress an increase in web width will not increase the web load, the rest of the material should be apportioned to the flanges where an increase in flange width is beneficial. Calculations show that $\bar{\sigma}_f$ for a zee or channel section may be reduced up to 10 or 15 percent by not making the optimum distribution of material between flanges and webs. Some investigations, such as references 1 and 11, have ignored this difference and have presented crippling strength equation for zee and channel sections that give $\bar{\sigma}_f$ as a function of $\frac{b'}{t}$ ($b' = b_W + 2b_F$, the developed width of the section). No consideration was given to the effect of the distribution of material between the flanges and webs. In order to give some idea of the effect of the distribution of material between webs and flanges on the strength of zee and channel sections, the data for channel sections of reference 10 have been replotted in figure 6. Values of $\bar{\sigma}_f$ are plotted against $\frac{b_F}{b_W}$ for constant values of $\frac{b'}{t}$ by cross plotting the data as given in figure 21 of reference 10. The variation of $\bar{\sigma}_f$ for any given value of $\frac{b'}{t}$ is large enough in some cases that considerable error would be introduced assuming no variation. To further study the effect of material distribution on the strength of sections, a series of 7075-T6 zee sections, with essentially constant values of $\frac{b'}{t}$, were tested. These results are tabulated in Table 1 and plotted in figure 7. Buckling stresses were computed from equation (12) directly as η equals one in all cases. A variation in $\bar{\sigma}_f$ is present (about 10 percent) and the method of the present paper adequately predicts the crippling stress throughout the range of proportions.

As stated previously, the value of $\frac{b_F}{b_W}$ at which $\bar{\sigma}_f$ is a maximum for a zee or channel section will be such that σ_W approximately equals σ_2. For the zee sections of figure 7 having $\frac{b'}{t} = 60$, this will occur at $\frac{b_F}{b_W} \approx 0.90$. However, the maximum value of the buckling stress is at $\frac{b_F}{b_W} = 0.4$, which means that for a certain range, $\bar{\sigma}_f$ was increasing while σ_{cr} was decreasing. In other words if a section is so proportioned to maximize the buckling stress these proportions will not necessarily be optimum for crippling.

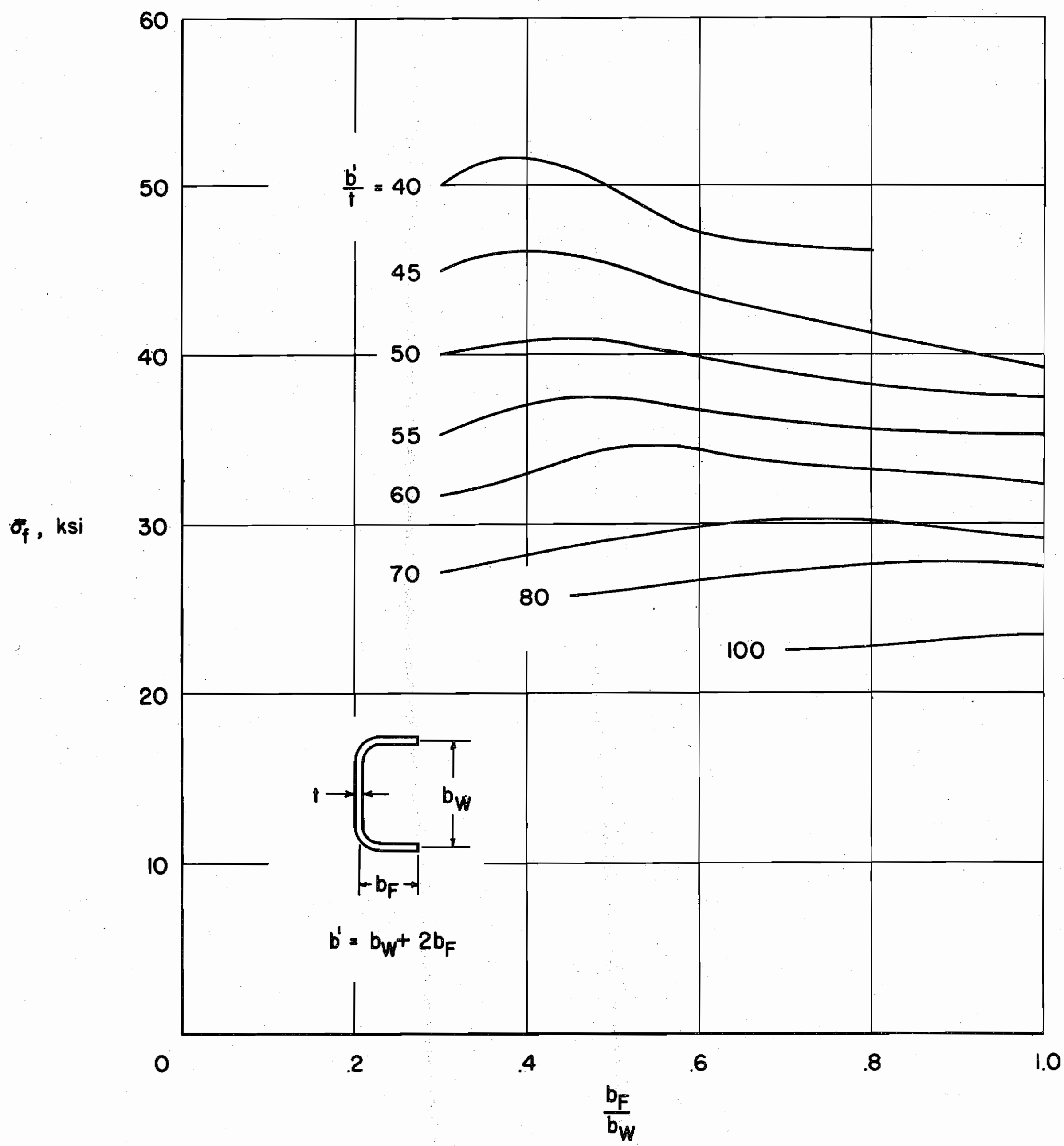

Figure 6.- Crippling stress of 2024-T3 aluminum-alloy channel sections.

TABLE I

DIMENSIONS AND TEST RESULTS FOR FORMED 7075-T6

ALUMINUM ALLOY ZEE SECTIONS[a]

$\frac{b_W}{t_W}$	$\frac{b_F}{b_W}$	$\frac{a}{b_W}$	Area	σ_{cr}	$\bar{\sigma}_f$
42.0	0.213	3.94	0.589	24.8	33.1[b]
41.7	.214	2.55	.593	27.0	41.0
29.9	.491	4.82	.594	34.6	43.5
30.1	.490	4.82	.595	29.4	43.1
21.6	.870	6.27	.595	22.4	45.0
22.0	.863	6.21	.590	21.2	44.9
15.2	1.46	8.89	.599	20.2	41.6
14.7	1.51	9.11	.600	19.0	41.5

[a]Thickness of sheet was nominally 0.102 with $\frac{b'}{t} = 60$

[b]Column failure

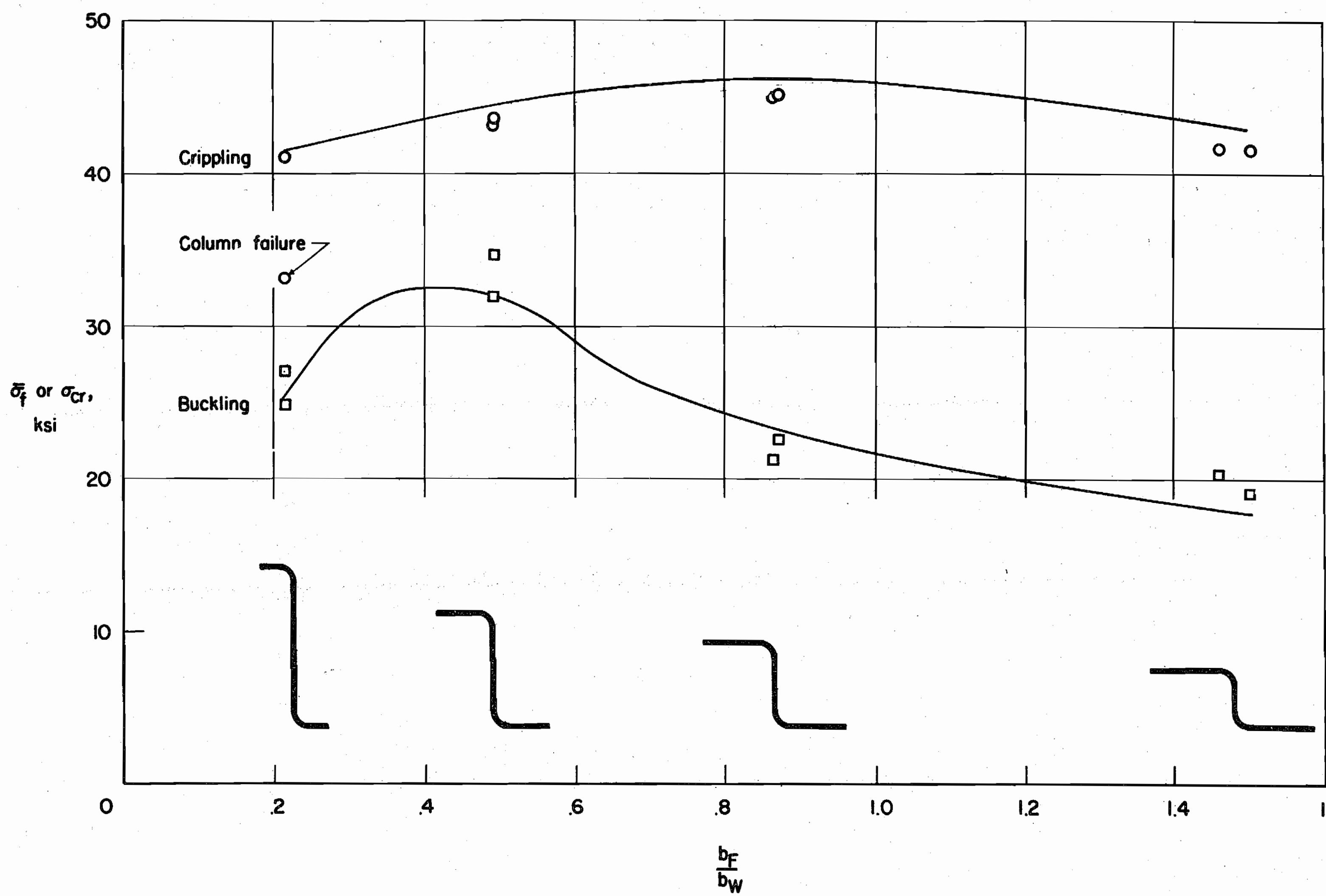

Figure 7.- Buckling and crippling stresses of 7075-T6 aluminum alloy zee sections having $\frac{b'}{t}$ = 60.

Earlier investigators discovered an apparent discrepancy between crippling tests of formed and extruded sections that could not be readily explained. When $\bar{\sigma}_f$ was plotted against σ_{cr}, one curve resulted for extrusions, while a family of curves applied to formed sections. (See, for example, figure 12 of reference 2.) The difference was thought to be caused by the presence of the curved corner and the increase in yield stress due to forming, but it was not accounted for quantitatively. These conclusions, however, were drawn from tests on extrusions with $\frac{b_W}{t_W} < 25$. In this range of proportions, a critical examination of the data shows that extrusions and formed sections have essentially the same behavior; that is crippling results plot as one curve of $\bar{\sigma}_f$ against σ_{cr}. It could be expected then if information were made available for extrusions with $\frac{b_W}{t_W} > 25$, a family of curves of $\bar{\sigma}_f$ against σ_{cr} could be plotted. Accordingly, as part of the present investigation tests were made of extruded zee sections with $\frac{b_W}{t_W}$ equal to 30, 35, and 40. The failing stresses, which are tabulated in Table II, are seen to be definitely lower than the predictions of reference 2, but are in agreement with the method of the present paper. Since each shape and proportion has a separate relationship of σ_{cr} to $\bar{\sigma}_f$, it can be concluded that in general one cannot predict the crippling stress of a structure using an empirical relationship between σ_{cr} and $\bar{\sigma}_f$ found for another shape or proportion.

TABLE II

CRIPPLING STRESS OF EXTRUDED 7075-T6 ALUMINUM-ALLOY ZEE SECTIONS

WITH $\frac{b_W}{t_W} > 25.$ $\sigma_{cy} = 78.6$ ksi E = 10,500 ksi

$\frac{b_W}{t_W}$	σ_{cr} (ksi)	$\bar{\sigma}_f$ (ksi)	Predicted results (ksi)	
			Author	Reference 2*
30.4	39.2	47.1	47.6	52.8
30.2	39.4	47.3	47.8	53.0
30.4	39.3	47.0	47.6	52.9
35.4	25.5	40.3	42.7	47.4
35.4	26.5	41.4	44.0	47.9
41.0	24.4	37.2	38.7	46.8
39.8	23.8	38.5	39.4	46.6
39.5	25.1	38.9	39.7	47.2

$*\bar{\sigma}_f = 0.80(\sigma_{cr})^{1/4}(\sigma_{cy})^{3/4}$

Although the equations developed are based on tests where buckling was less than 3/4 of the yield stress, it was found that in many cases they could be extended without serious loss in accuracy. Calculation appearing in figures 5 and 8 in many cases involve stresses which are much greater than 3/4 of the yield stress yet are in good agreement with experiment. For high plastic stresses, where buckling and failure are practically identical, the formulas may be conservative, but generally are not more than 15 percent in error. Thus it is possible to use these crippling equations as a conservative estimate of the plastic buckling stress (or failure stress) of sections for which a buckling analysis is not readily available.

In the case of a built-up structure, such as a panel with stringers riveted to a sheet, the crippling stress equations can be applied, provided the riveted attachment is adequate. The problem of riveting has been studied extensively in reference 15 where the conditions necessary to achieve this adequate riveting are presented.

Since the crippling stress is given as a function of material parameters calculated from the stress-strain curve, it is expected that the equations will apply to tests at elevated temperatures if the appropriate elevated temperature stress-strain curve is used. This is shown in figure 8 where the crippling strength equations are compared with the results for H section tested at elevated temperature in reference 16.

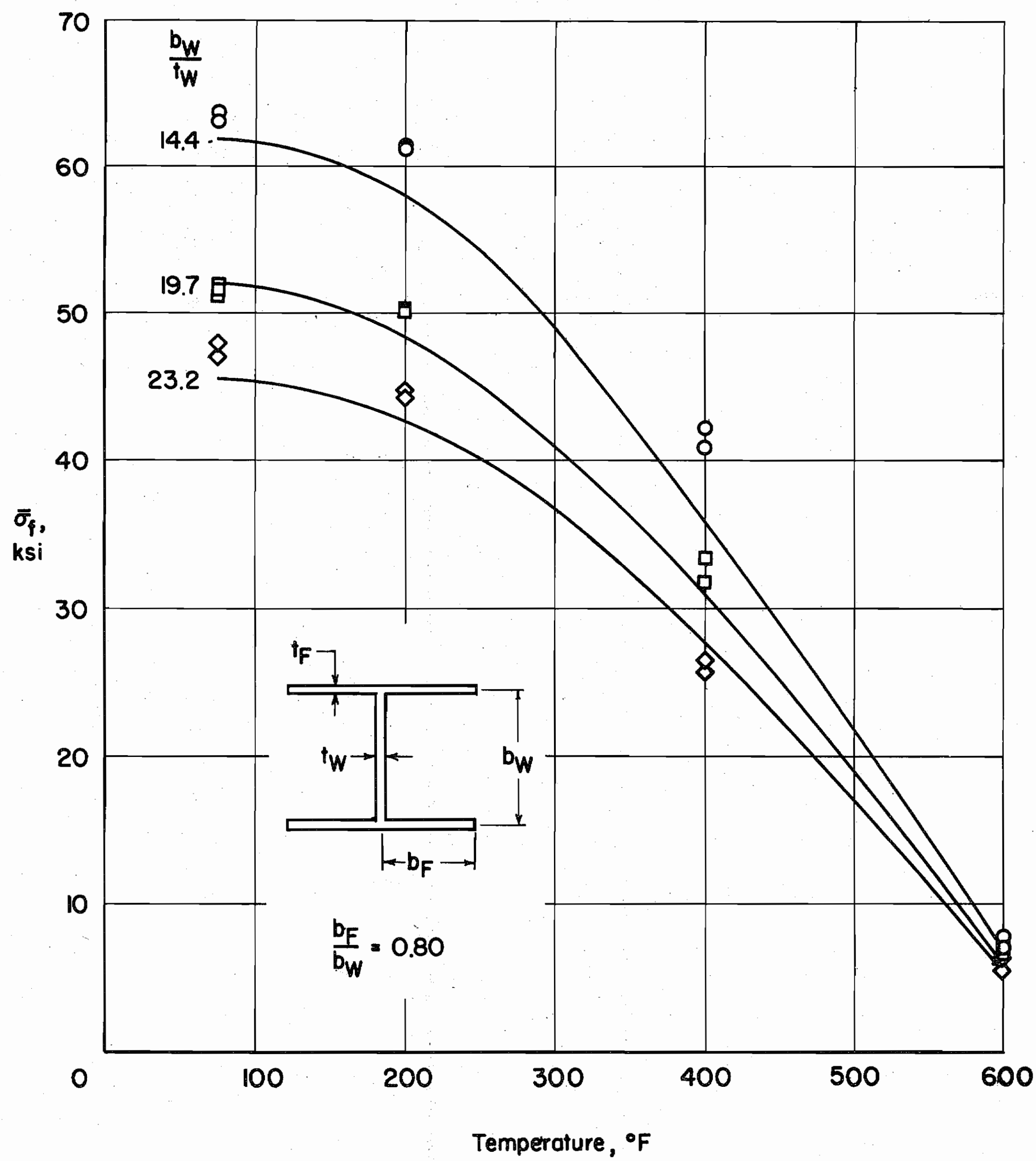

Figure 8. - Crippling stress of 7075-T6 aluminum-alloy H sections at elevated temperatures.

CONCLUSIONS

An empirical analysis of the crippling strength of structural sections has been presented. The following conclusions can be made from the analysis.

1. To satisfy the purpose of a strength analysis, the crippling load for an integral cross section may be taken as the sum of a set of defined crippling loads which are characteristic of the individual plate elements.

2. Correlation of crippling stresses among material may be effected by the use of parameters which can be easily calculated from the compressive stress-strain curve.

3. The presence of formed or anisotropic material can be accounted for by using an effective stress-strain curve in calculating the material parameters.

4. A simple criterion defines the minimum size of flange necessary to maintain nodes at a joint after buckling. For sections with flanges smaller than this minimum size, an extension of the basic procedure can be used to calculate the crippling stress.

5. Proportioning a section to maximize the buckling stress will not necessarily maximize the crippling stress.

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