RIDE QUALITY ASSESSMENT USING PROBE VEHICLE ACCELERATION MEASUREMENTS

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ABSTRACT

New vehicle technology is leading to efficient methods for assessing the condition of the national highway system. Utilizing simple sensors installed in vehicles, such as accelerometers, could provide a cost effective way to assess ride quality for pavement management. This paper builds on a pilot study that compared data gathered from accelerometers to the current state of the art practices for measuring ride quality. After promising results with preliminary acceleration data, robust data collection was performed on the Virginia Smart Road under various operational conditions and using two vehicles: a Volvo truck and a Ford Fusion using the DAS system developed by the Virginia Tech Transportation Institute. Profile measurements were also obtained for comparison using an inertial laser profiler. Tests were performed at 40, 50, and 65 mph (65, 80, and 105 km/h). A GPS device was used to accurately calculate vehicle position and speed. Repeatability of acceleration and profile measurements were calculated. Effect of vehicle type and testing speed on the acceleration profile was estimated. Results show that under controlled testing conditions, roadway roughness can be accurately estimated using probe vehicle acceleration data. This suggests that instrumented probe vehicles might be a viable and effective way of implementing a pavement condition assessment program in the near future.

1. INTRODUCTION

Pavement asset management, which encompasses the monitoring of primary roads, secondary roads, bridges, and interstate highways, relies heavily on robust and accurate data collection. Advances in vehicle technology, including computing power, advanced sensors, imaging, spatial referencing, and distributed databases have led transportation agencies to develop new ways to collect, store, and analyze data [1]. In the past, these innovations in transportation data collection and processing have taken the form of task-specific vehicles. For example, state-of-the-art practices in pavement smoothness collection data involve high-speed laser profilers, which are designed and tailored explicitly for that purpose. With the recent advances in technology on board production vehicles, it is possible that using information from passenger cars and trucks could provide transportation agencies with a new method for collecting data: probe vehicles.

The idea of using probe vehicles as useful traffic and infrastructure tools has been explored for several years. By measuring with built-in, inexpensive vehicle sensors and transmitting data via an available wireless connection, a large amount of data can be collected at a relatively low cost. In a fully integrated network, the passenger cars would essentially become the data collection devices. In the United States, the U.S. Department of Transportation (DOT) and Federal Highway Administration (FHWA) launched a nationwide initiative in 2003 known as the Vehicle Infrastructure Integration (VII). The VII program supervised multiple research projects aimed at using wireless communication between vehicles and infrastructure to improve safety and mobility [2]. Probe vehicles naturally played a major role in a majority of the research and pavement condition monitoring continues to be a major component of the research involved [2].

Flintsch et al [3] explored a potential application of probe vehicles. The paper examined the use of global positioning systems (GPS), speedometers, and vertical accelerometers installed in instrumented probe vehicles to measure ride quality, or pavement smoothness. The acceleration measurements taken from an instrumented vehicle were compared to road profile measurements taken by a high-speed laser profiler on the Virginia Smart Road. The results showed promising visual and statistical similarity between the two types of measurements, indicating that acceleration could possibly be used to describe ride quality in the future [3]. This paper builds on that preliminary study by further examining acceleration measurements from two different instrumented vehicles and under various operational conditions on the Virginia Smart Road.

2. OBJECTIVE

The objective of this paper is to show some interesting findings regarding the estimation of ride quality of existing roads from instrumented vehicles used during naturalistic driving studies. To accomplish this task, the study:

- ✓ Assessed the repeatability of acceleration measurements
- ✓ Compared acceleration and smoothness profile measurements
- ✓ Evaluated the effect of vehicle type and vehicle speed on acceleration measurements; and
- ✓ Implemented a threshold to isolate areas of poor pavement condition spatial

3. BACKGROUND

Ride quality, or smoothness, is used by state agencies to prioritize maintenance, repair, and rehabilitation work for their road networks. It is currently measured by high-speed laser profilers. Profilers use a combination of lasers (one in each wheelpath) to measure vertical deviations in the pavement surface. The combination of measurements makes up the roadway’s longitudinal profile. The longitudinal profile is then used to compute the International Roughness Index (IRI) which was developed by the NCHRP and the World Bank [4]. The IRI represents the accumulation of vertical differences along an otherwise smooth surface (typically expressed in in/mi). The procedure for determining the IRI is described in ASTM E1926. The smoothness of the road that is quantified by the IRI has a direct impact on roadway users. Many of the costs associated with driving, including gas consumption, tire wear, and durability are reduced with a smoother road surface. A road with more bumps will have a negative impact on the performance of the vehicle. In addition, drivers have a much keener perception of road smoothness than other condition indicators such as friction or structural capacity. Therefore, it is important for a road to have an acceptable level of smoothness [5].

While a longitudinal profile is one way of quantifying smoothness, the basic concept relating vertical acceleration to ride quality is also valid. It can be argued from a physical perspective that road profile (i.e., a measure in the pavement profile of vertical distance variations experienced by a vehicle) has a direct influence on the vertical acceleration of a vehicle traveling along the roadway. The vehicle travel speed (horizontal speed) influences the speed at which it traces this profile (vertical speed) and the changes in speed determine the acceleration. By measuring the vertical acceleration in a moving vehicle, smoothness can be quantified in a manner that is directly related to the actual discomfort associated with traversing a pavement surface. In this way, the collected acceleration can be used to provide ride quality data. This concept, which was validated in the preliminary study, provides the basis for the testing and analysis performed in the paper.

Unlike longitudinal profile, which requires sophisticated lasers, vertical acceleration can be measured by sensors found in many of today's production vehicles. Most vehicles with a higher center of gravity, including small trucks and sport utility vehicles, are equipped with rollover stability control, which include vertical accelerometers. Newer passenger cars are also outfitted with accelerometers for electronic stability control. These devices establish an inertial reference for the vehicle. This can then be translated into an indicator of smoothness as the vehicle travels the road surface. The setup of an accelerometer on a profiling vehicle is outlined in the ASTM Standard E 950-98.

During testing, VTTI's naturalistic driving studies (NDS) instrumented vehicles were used as the study's probe vehicles. Naturalistic data collection is the collection of driver behavior and performance data in a natural environment. This includes vehicle data including position (GPS), speed, orientation, acceleration, braking characteristics, and environment data including roadway type, lane count, traffic density, time of day, and weather. In effect, the vehicle becomes the data collection device, which relates back to the main concept of probe vehicles [6]. Focus is placed on the use of vehicle position, orientation, speed, and acceleration for the calculation of pavement smoothness and associated global positioning. These instrumented vehicles are operated in a unique testing environment at the Virginia Smart Road. The Virginia Smart Road, located at VTTI, provides an opportunity to gather pavement condition data in a closed circuit while maintaining the characteristics of public roads. The sections of the Smart Road are composed of varying pavement types. That enables the identification of different ride quality characteristics using probe vehicles. It is an ideal way to gather and assess pavement condition data without the need for testing on public roads. All of the tests performed in this study occurred on the Virginia Smart Road using the instrument vehicle system described above.

4. EXPERIMENTAL SETUP

4.1. Testing Vehicles

Three test vehicles were used for data collection. The first two vehicles were instrumented with the Virginia Tech Transportation Institute Data Acquisition System (DAS). These vehicles were used for the collection of acceleration data. GPS positioning, speed, and distance covered supplemented the acceleration measurements. The first DAS instrumented vehicle was a small sedan, namely a Ford Fusion. The Fusion was the primary probe vehicle during testing. The head unit and IMU were installed below the vehicle's rear view mirror. The collected data was displayed on a laptop that is constantly on while the car is running. The Ford Fusion was used on all three days of testing. In order to expand the study to examine the performance of commercial vehicles, a 1997 Volvo Tractor Trailer was introduced as the second DAS instrumented vehicle. The tractor trailer was only used on the second day of testing. Each vehicle collected the same information, which was recorded on a hard drive and uploaded to a VTTI database.

The third vehicle represented the control of the experiment, a Virginia Department of Transportation (VDOT) high speed laser-profiler. This type of vehicle is the state-of-the-art practice for evaluating ride quality. The van measures the pavement profile using two lasers (one in each wheel path), which record the distance from the laser to the pavement; an accelerometer is used to correct for vehicle vertical movement. This information is then used to compute the International Roughness Index (IRI), which quantities the smoothness of the road. The profiler was only used on the second day of testing.

4.2. Testing Environment and Process

The collection of acceleration and smoothness data took place over three days on the Virginia Smart Road. All test runs were performed strictly uphill in the right lane. The test section began 150 feet before the beginning of Section 1 (just after the bridge) and ended 50 feet after the end of Section 5 (just before the exit ramp). Both start and end locations were marked with a bump strip and cone. In addition, a third bump strip was placed at the beginning of the Section 2 (grooved concrete). For the DAS vehicles, the runs were triggered by a button press at the beginning and ending cone. A special cone was used by the profiler to trigger its runs. Ten runs were made at 50 mph by all vehicles. An additional 2 runs each were made at 40 mph (65 km/h) and 65 mph (105 km/h) by all vehicles. The Fusion then completed 20 additional runs, all at 50 mph (80 km/h), using the same experimental setup.

5. VERIFICATION RESULTS

5.1. Acceleration and Smoothness Comparison

Although the relationship between acceleration measurements and road profile was validated in the preliminary analysis of this study, it was again examined after data collection. The comparison is presented in Figure 1. Both plots display the same trends that were validated in the pilot study. Green lines indicate the separation of three main sections of the Smart Road testing area. The red circles indicate spikes in the measurements that can be observed in both measurement types at matching locations.

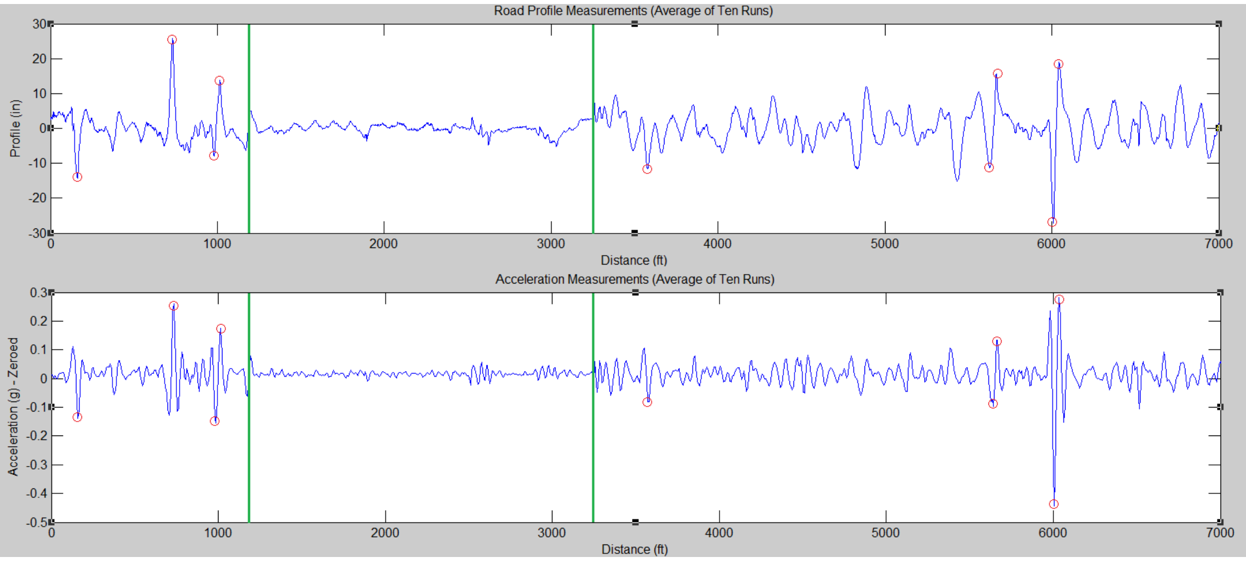

FIGURE 1 Comparison of profile measurements and acceleration measurements.

5.2. Repeatability of Acceleration Measurements

The pilot study concluded that the acceleration measurements demonstrated an acceptable level of repeatability. However, the results warranted further inspection since only four test runs were used in the pilot study. With 30 test runs performed during the instrumented vehicle data collection, a more accurate and statistically relevant representation of repeatability could be realized. The test runs, taken at 50mph (80 km/h), were used in the analysis. All of the measurements were taken by the Ford Fusion, travelling uphill in the right lane of the Virginia Smart Road. A plot of ten of the 30 test runs can be found in Figure 2. From visual inspection, it is evident that the runs are very repeatable, since little deviation from the general trend is evident.

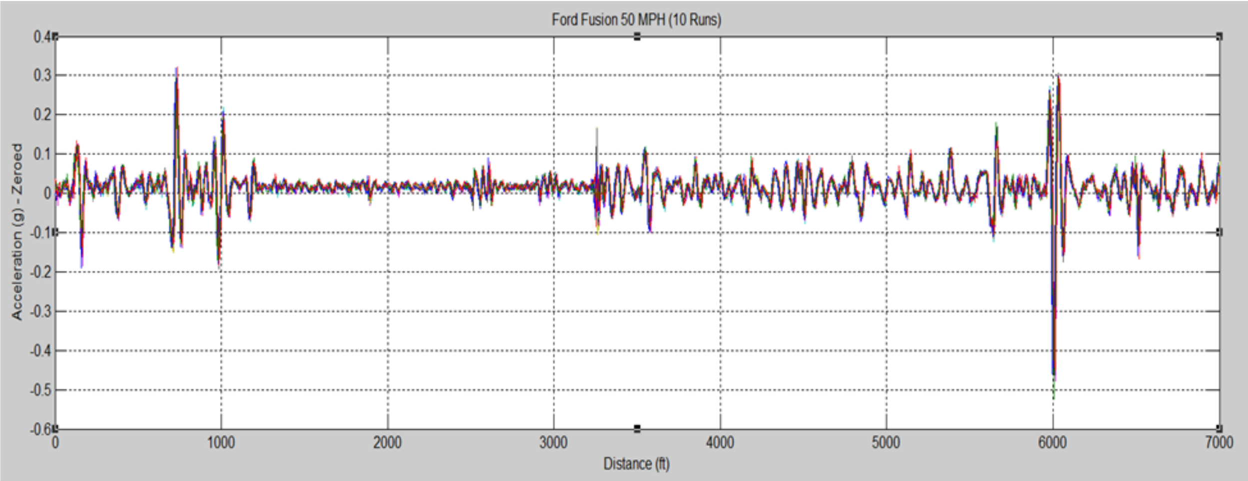

FIGURE 2 Acceleration measurements taken by instrumented Ford Fusion

One of the signal processing methods that can be used to determine the accuracy of acceleration measurements is cross-correlation. This technique has been previously successfully implemented to determine the repeatability and reproducibility of the profiler measurements [7]. Cross-correlation is a measure used to verify the similarity of two waveforms. It is defined as follows in Equation 1 [8]:

$$\varphi_{xy}(\tau) = E[x(t)y(t+\tau)] = \lim_{L\to\infty} \frac{1}{L}\int_0^L x(t)y(t+\tau)dt;\ \tau \geq 0 \qquad (1)$$

where,
E[·] = expected value
τ = shift factor
x(t), y(t) = two waveforms defined in the range of t = [0 , ∞)

Since the acceleration measurements are discrete, the cross-correlation function can be estimated by:

$$\varphi_{xy}(m) = E[x_n y_{n+m}] = \lim_{L\to\infty} \frac{1}{L}\sum_{n=0}^{L-1} x_n y_{n+m};\quad m \geq 0 \qquad (2)$$

where,
m = shift between the measurements

Equation 2 can be normalized by dividing it by the standard deviation of the two waveforms (measurements). To make the computations more efficient, the waveform measurements can be shifted to have a mean of zero [8]. Cross-correlation can then be used to find how much one waveform needs to be shifted to obtain the best match with

another waveform. The amount of shifting that provides the highest cross-correlation is selected. After shifting the signal, the integral of the product of both signals is calculated based on Equation 1. The integral is maximized when the signals perfectly match. This procedure can be used to determine the optimum shift to synchronize the measurements.
The cross-correlation between two acceleration measurement sets was calculated using the MATLAB cross-correlation function. The test measurements all exhibited either a zero or one measurement shift, indicating that they were aligned very well prior to the cross-correlation calculation. Therefore, it was determined that no shift needed to be applied, and that the repeatability would be optimal if the runs were left unshifted. Figure 3 demonstrates this concept.

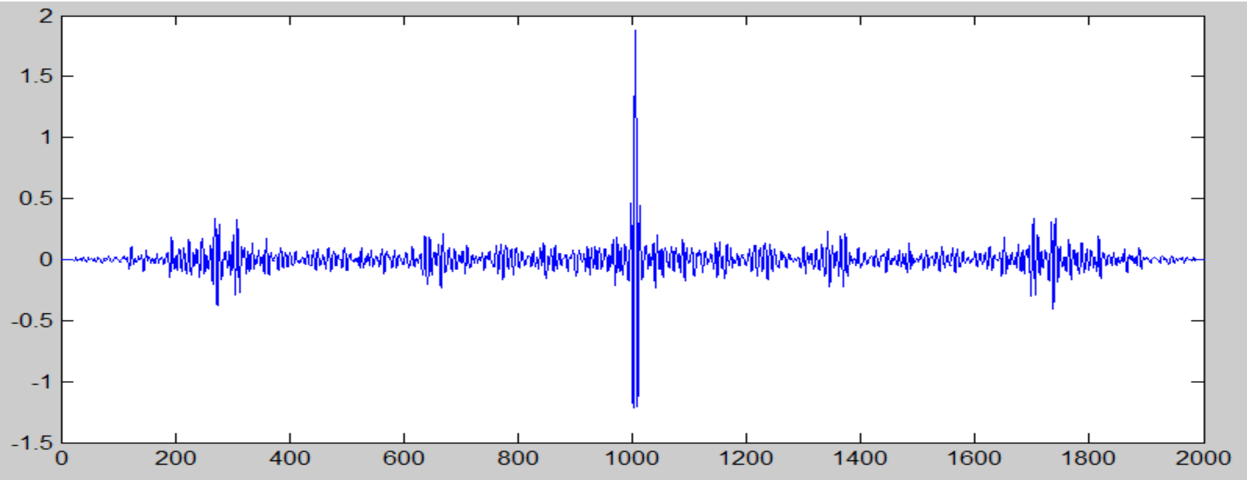

FIGURE 3 Cross-correlation between acceleration test runs 1 and 10.

The normalized cross correlation values can also be examined to determine how similar each signal is to the other. A maximum cross correlation value that is close to 1 (or 100%) demonstrates that the acceleration measurements are very repeatable and will retain their similarity over multiple runs. The normalized maximum cross correlation values are shown in Table 1 as percentages. All of the runs displayed acceptable correlation with values over 90%, while the majority of the values displayed exceptional correlation with values over 97%.

TABLE 1 Normalized Cross Correlation Percentages for Acceleration Measurements

Test Runs Compared	Correlation Value (%)
Runs 1 and 10	90.46
Runs 2 and 10	97.02
Runs 3 and 10	97.75
Runs 4 and 10	97.38
Runs 5 and 10	97.52
Runs 6 and 10	97.33
Runs 7 and 10	97.55
Runs 8 and 10	90.33
Runs 9 and 10	96.55

To evaluate repeatability, the standard deviation of the differences between the reference run and the other runs was evaluated. These results, along with the variance for each test run, are shown in Table 2. Only the first ten comparisons are shown below, although each of the 30 runs were used in the calculations. The square root of the average variance describes the total average standard deviation, which was calculated for all thirty test runs.

This was determined to be 0.0664. These values can be considered relatively low (ideal repeatability) across the acceleration measurements.

TABLE 2 Acceleration Standard Deviation and Variance of Differences

Measurement Type	**Test Runs Compared**	**Value**
Standard Deviation of	Runs 1 and 10	0.0199
Standard Deviation of	Runs 2 and 10	0.0398
Standard Deviation of	Runs 3 and 10	0.0416
Standard Deviation of	Runs 4 and 10	0.0444
Standard Deviation of	Runs 5 and 10	0.0376
Standard Deviation of	Runs 6 and 10	0.0350
Standard Deviation of	Runs 7 and 10	0.0101
Standard Deviation of	Runs 8 and 10	0.0200
Standard Deviation of	Runs 9 and 10	0.0441
Variance of Differences	Runs 1 and 10	0.0004
Variance of Differences	Runs 2 and 10	0.0016
Variance of Differences	Runs 3 and 10	0.0017
Variance of Differences	Runs 4 and 10	0.0020
Variance of Differences	Runs 5 and 10	0.0014
Variance of Differences	Runs 6 and 10	0.0012
Variance of Differences	Runs 7 and 10	0.0001
Variance of Differences	Runs 8 and 10	0.0004
Variance of Differences	Runs 9 and 10	0.0019

To compare the repeatability of acceleration measurements to road profile measurements (current practice), a signal-to-noise ratio for each measurement type was computed. The signal-to-noise ratio can accurately compare the repeatability of two measurement sets that use different units. The signal-to-noise ratio was found by dividing the average standard deviation of all measurement sets by the total average standard deviation of measurement differences (shown above as 0.0664 for acceleration measurements).

The calculated signal-to-noise ratio for the acceleration measurements was 1.73. The same process was used to evaluate the ten runs performed by the Volvo Tractor Trailer. The results showed promising repeatability, but not to the extent of the Fusion. This could be due to the dynamics of the larger vehicle, which experiences more acceleration variability on the road. The average standard deviation of differences for the truck was calculated to be 0.0712. The signal-to-noise ratio for the truck was determined to be 0.73. The calculated signal-to-noise ratio for the profile measurements was 1.16. The higher the signal-to-noise ratio, the more repeatable the measurements are. This shows that the accelerometer measurements with the sedan are at least as repeatable, if not more repeatable, as the road profile measurements collected by high speed laser profilers. Meanwhile, the repeatability of the tractor trailer measurements may merit further investigation.

6. ACCELERATION VARIABILITY

6.1. Effect of Speed on Acceleration Measurements

Unlike laser-based profile measurements, acceleration measurements are sensitive to variations in the probe vehicle being used. The variability of different aspects of acceleration testing was analyzed. First, the effect of speed was tested, since the speed of the vehicle affects the intensity of acceleration measurements. For this analysis, runs at 40, 50, and 65 mph were performed on the Virginia Smart Road. A plot of one run at each speed is shown in Figure 5 below for the Ford Fusion.

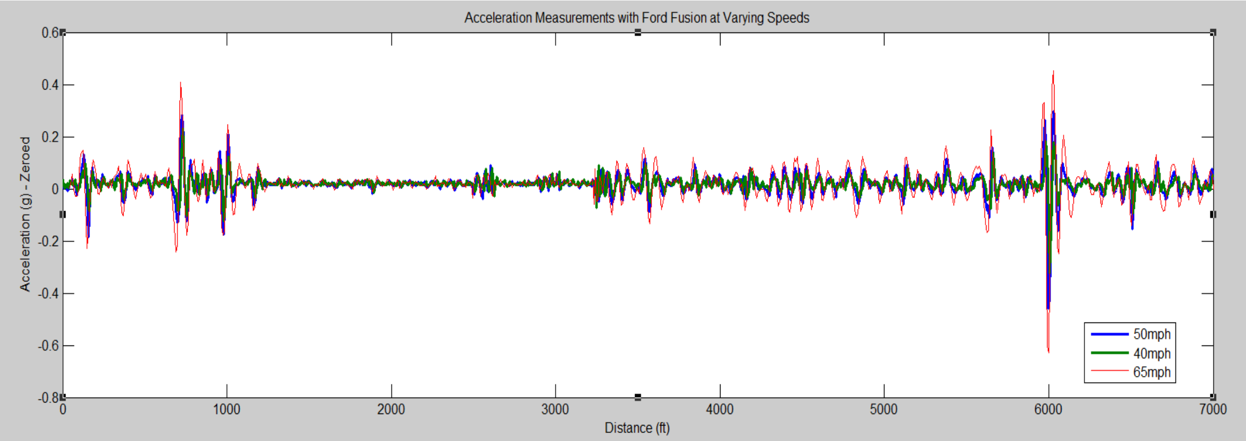

FIGURE 5 Acceleration measurements with Ford Fusion at varying speeds.

From visual inspection, it is clear that measurements taken at 65 mph (105 km/h) result in higher measured acceleration than those taken at lower speeds. To quantify the difference between speeds, the L1 norm, which is the sum of the absolute values of the acceleration measurements over the entire testing section length, was calculated. The larger the total absolute value, the more acceleration the probe vehicle experiences. One run at each speed was analyzed in this manner. Table 3 summarizes these results.

TABLE 3 Sum of Acceleration Absolute Value for Entire Testing Section.

Speed	**L1 norm of Acceleration Measurements**
40 mph (65 km/h)	25.97 g
50 mph (80 km/h)	29.73 g
65 mph (105 km/h)	48.04 g

It does not appear that the increase in magnitude is linear, which is to be expected because vehicle dynamics dictate that acceleration greatly increases at high speeds. The 65 mph (105 km/h) measurements experienced a 61% increase in magnitude when compared to the 50 mph (80 km/h) measurements. Meanwhile, the 50 mph (80 km/h) measurements experienced only a 14% increase in magnitude when compared to the 40 mph (65 km/h) measurements. It can be concluded that as speed increases, so does the magnitude of the acceleration. The general ride quality trend still remains intact as each run exhibits similar peaks and rough sections.

6.2. Effect of Vehicle Type on Acceleration Measurements

The type of vehicle also has an effect on the magnitude of acceleration measurements. This is due to variations in vehicle parameters, including suspension type and tire size. To illustrate this difference, two probe vehicles were used during testing: a small passenger car and a large tractor trailer. A plot of one run with each vehicle at 50 mph (80 km/h) is shown in Figure 6 below.

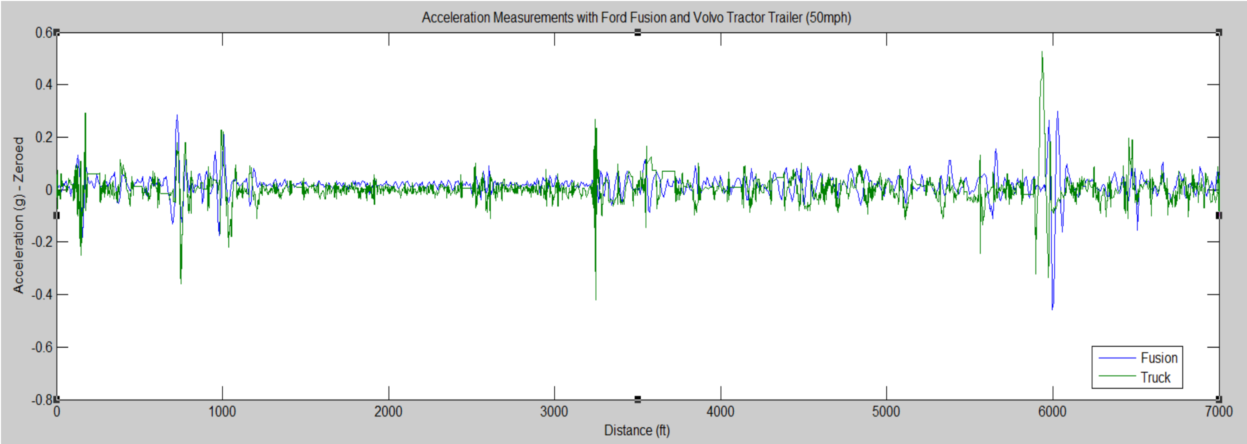

FIGURE 6 Acceleration measurements with Ford Fusion and Volvo Tractor Trailer.

From visual inspection, it is clear that acceleration measurements taken with the tractor trailer are higher in magnitude than those taken with the sedan. The vehicle dynamics and suspension of the larger, heavier truck cause the vehicle to experience more acceleration as a result of road roughness. To quantify the difference between vehicle types, the L1 norm of the acceleration measurements over the entire testing section length was calculated. One run using each vehicle (50 mph [80 m/h]) was analyzed in this manner. Table 4 summarizes these results.

TABLE 4 Sum of Acceleration Absolute Value for Entire Testing Section by Vehicle.

Vehicle	**L1 norm of Acceleration Measurements**
Ford Fusion	29.73 g
Tractor Trailer	40.97 g

Therefore, it can be concluded that larger vehicles, operating under the same driving conditions, will experience larger acceleration due to varying vehicle dynamics and suspension. It is likely that the magnitude of acceleration measurements will be slightly different for all vehicles, but that they can be grouped into classes that have similar properties. For a probe vehicle system to be implemented, these groups would have to be determined prior to data collection. The proof of concept remains intact because the same general trend is followed for both vehicles. Only the magnitudes of the acceleration measurements differ.

7. ROUGH PAVEMENT DETECTION ANALYSIS

One possible application for the use of probe vehicle acceleration data is identification of rough pavement sections. By using the data to highlight areas of poor pavement condition, where the pavement smoothness may not be adequate, maintenance agencies can prioritize projects in a more efficient manner. A possible method for this process is establishing an acceleration threshold value. If a specific section exceeds that acceleration threshold, it could alert maintenance personnel that the pavement is not meeting its functional performance requirements. A threshold value could also be used with individual acceleration measurements in order to identify spot locations of poor condition. This could be useful in identifying potholes, large cracks, and uneven joints.

This strategy was employed with the collected data from the Ford Fusion using a Matlab script file. The script file implemented both a section acceleration threshold (0.04 g) and a single location threshold (0.2 g). Any sections or locations that exceeded these values were reported. The test section of the Virginia Smart Road was split into sections of 30 data points, each approximately 225 feet (70 m) in length. If employed on highways, it may be beneficial to increase the length of each section to tenth-mile or half-mile distances. Figure 7, shown below, identifies the “poor” conditioned areas of the Virginia Smart Road based on the threshold values established above. Since the Smart Road is relatively well conditioned, the high acceleration measurements are mostly due to bridge joints. The large spike on the right of the graph is the final joint on the small bridge near the end of the test section. The yellow sections indicate the segments of the Smart Road that exceeded an average of 0.04 g. The red circles indicate specific measurements that exceeded 0.2 g. These selected thresholds were specific to the type of car used in the experiment (Ford Fusion). Different thresholds would have to be used for different types of vehicles. For this purpose, thresholds could be based on a more rigorous analysis such as the measured acceleration standard deviation.

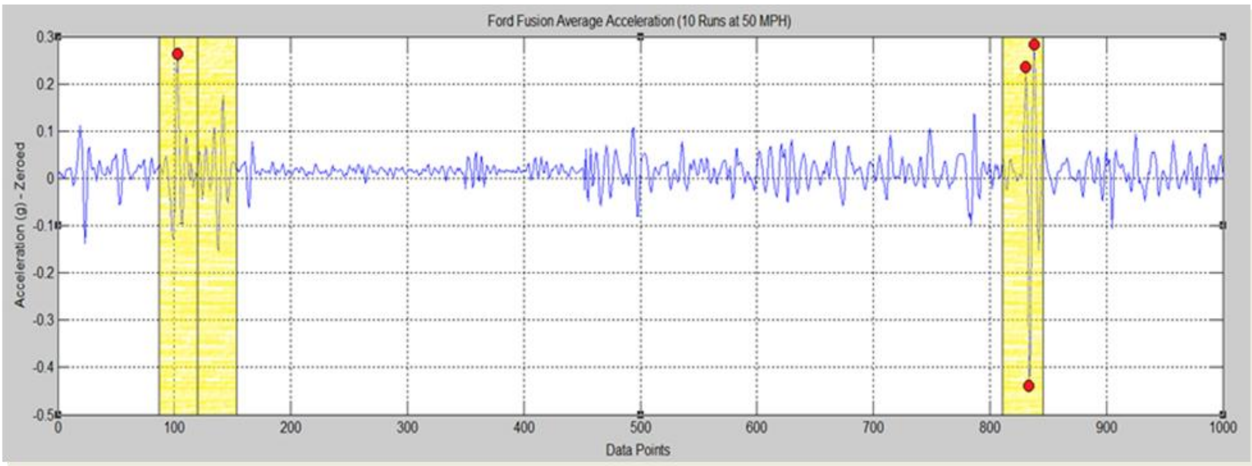

FIGURE 7 Acceleration measurements with Ford Fusion for rough pavement detection.

8. FINDINGS AND CONCLUSIONS

This paper reviews the results from a three day data collection of acceleration measurements using instrumented vehicles on the Virginia Smart Road. The following identify the conclusions of the study:

- ✓ Based on the standard deviation of measurement differences, it is confirmed that the acceleration runs are very repeatable (average standard deviation of 0.0664) with the instrumented sedan. The acceleration measurements yielded a higher signal-to-noise ratio than the profile measurements. Furthermore, the repeatability of acceleration measurements remains at an acceptable level even with the addition of more test runs (30 total were completed). More investigation is required using the instrumented tractor trailer, as it proved to be not as repeatable as the sedan.
- ✓ Preliminary analysis suggested that the acceleration profiles and smoothness measurements were very similar. This was further validated with the collection of more data. The measurements follow the same trends through the various pavement sections of the Virginia Smart Road. Most of the large peaks in each waveband can be seen in the same locations. This confirms the theory that the vehicle experiences larger vertical accelerations when traveling across rough pavement and allows for further assessment of acceleration as a proper tool for describing ride quality.
- ✓ Acceleration measurements are sensitive to both speed and vehicle type. Faster speeds result in acceleration measurements of higher magnitude. This was

confirmed by comparing results from test runs at 40, 50, and 65mph (65, 80, and 105 km/h). In addition, due to varying suspension and dynamics of vehicles, acceleration measurements differ in magnitude by type of vehicle. The tractor trailer, a heavier vehicle with different suspension characteristics, exhibited higher acceleration signatures. A scaling factor would need to be implemented to account for both variation types, however, it should be noted that the general trend was maintained throughout all tests.

- ✓ An analysis of acceleration measurements was performed by implementing an acceleration threshold. Sections or locations that exceed a given threshold may indicate an area of poorly conditioned pavement. For the data taken on the Virginia Smart Road, three sections of roughly 225 feet (70 m) were identified as “poor” using an average section acceleration threshold of 0.04 g. In addition, 4 specific point locations were identified as “poor” using a single acceleration threshold of 0.2 g. This could be useful for maintenance agencies that must prioritize road projects by providing some insight into what sections of road are exhibiting rough characteristics.

The results of this data collection and analysis show that acceleration measurements can be useful in describing road ride quality. Due to new advances in on-board vehicle technology, the collection of acceleration data can be performed at low cost with extremely high and frequent coverage if implemented in an effective system. This could provide transportation agencies with a new method for evaluating the functional condition of its pavement assets by identifying areas of poor ride quality through the use of accelerometers.

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