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THE DIFFICULTIES INVOLVED IN CALCULATING $\delta\rho$.

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ABSTRACT

We discuss the difficulties that arise when one tries to calculate $\delta\rho$ using dispersion relations.

1. Introduction

Recently, several authors have attempted to make use of dispersion relations (DR's) to calculate the contribution of non–relativistic (NR) bound states and threshold enhancements to the ρ parameter.[1,2] The idea was to write the ρ parameter as a dispersion integral over the imaginary parts of the vacuum polarization functions, and then to use the Schrödinger equation to calculate the contributions of NR intermediate states to these spectral functions.

In Ref. 1, the contribution of possible 4th generation fermion–antifermion bound states formed from Higgs exchange was calculated, while Ref. 2 calculated the contribution of the $t\bar{t}$ threshold enhancement due to QCD binding effects.

In this short note, we would like to point out that these calculations suffer from a fundamental flaw: the sign of the contribution of NR states to the ρ parameter is actually indeterminable. This is due to the fact that it is possible to write two different DR's for $\delta\rho$ and the contribution of NR states changes sign depending on which DR is used.

In the following, we will give a brief discussion on how this comes about.

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2. The Dispersion Relations

Following Ref. 2 we introduce the following notation for current–current correlation functions:

$$\begin{aligned} \Pi_{\mu\nu}(q) &= -i \int d^4x e^{iq\cdot x} \langle 0 | T^* \left[J_\mu(x) J_\nu(0) \right] | 0 \rangle \\ &= g_{\mu\nu} \Pi(s) + q_\mu q_\nu \lambda(s) \\ &= \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \Pi(s) + \left(\frac{q_\mu q_\nu}{q^2} \right) \Delta(s) \end{aligned} \tag{1}$$

where $s = q^2$. Note that

$$\Delta(s) = \Pi(s) + s\lambda(s). \tag{2}$$

Therefore,

$$\Delta(0) = \Pi(0) \tag{3}$$

unless $\lambda(s)$ has a pole at $s = 0$. Now the shift of the ρ parameter away from its tree level value of $\rho = 1$ is usually expressed as the difference between the *transverse* parts of the charged and neutral isospin current correlators:

$$\delta\rho = \frac{1}{v^2} \left[\Pi_{+-}(0) - \Pi_{33}(0) \right], \tag{4}$$

where $v \approx 246\text{GeV}$ is the Higgs VEV. However, if we are considering the contribution of the 3rd or 4th generation fermions to $\delta\rho$ as in Refs. 1 and 2, one can also express $\delta\rho$ as a difference between the *longitudinal* parts since the λ–functions are free of poles at $s = 0$. Hence,

$$\delta\rho = \frac{1}{v^2} \left[\Delta_{+-}(0) - \Delta_{33}(0) \right]. \tag{5}$$

Applying Cauchy's theorem to Eq. 4, we find the DR

$$\delta\rho = \delta\rho^{\mathrm{T}}(\Lambda^2) + \delta R^{\mathrm{T}}(\Lambda^2), \tag{6}$$

where

$$\begin{aligned} \delta\rho^{\mathrm{T}}(\Lambda^2) &\equiv \frac{1}{v^2} \left[\frac{1}{\pi} \int^{\Lambda^2} \frac{ds}{s} \left\{ \mathrm{Im}\Pi_{+-}(s) - \mathrm{Im}\Pi_{33}(s) \right\} \right], \\ \delta R^{\mathrm{T}}(\Lambda^2) &\equiv \frac{1}{v^2} \left[\frac{1}{2\pi i} \oint_{|s|=\Lambda^2} \left\{ \Pi_{+-}(s) - \Pi_{33}(s) \right\} \right]. \end{aligned} \tag{7}$$

This was the DR used in Ref. 1. Alternatively, we can apply Cauchy's theorem to Eq. 5 and obtain

$$\delta\rho = \delta\rho^{\mathrm{L}}(\Lambda^2) + \delta R^{\mathrm{L}}(\Lambda^2), \tag{8}$$

where

$$\delta\rho^{\mathrm{L}}(\Lambda^2) \equiv \frac{1}{v^2} \left[\frac{1}{\pi} \int^{\Lambda^2} \frac{ds}{s} \left\{ \mathrm{Im}\Delta_{+-}(s) - \mathrm{Im}\Delta_{33}(s) \right\} \right],$$

$$\delta R^{\mathrm{L}}(\Lambda^2) \equiv \frac{1}{v^2}\left[\frac{1}{2\pi i}\oint_{|s|=\Lambda^2}\{\Delta_{+-}(s)-\Delta_{33}(s)\}\right]. \quad (9)$$

This was the DR used in Ref. 2. We keep the radius of the integration contour Λ^2 finite in our expressions since $\delta R^{\mathrm{T}}(\Lambda^2)$ and $\delta R^{\mathrm{L}}(\Lambda^2)$ may not necessarily converge to zero as $\Lambda^2 \to \infty$.[3]

3. Contribution of Non–Relativistic States

Now let us consider the contribution of NR states to $\delta\rho^{\mathrm{T}}(\Lambda^2)$ and $\delta\rho^{\mathrm{L}}(\Lambda^2)$. In the NR limit, the imaginary parts of $\Pi(s)$ and $\Delta(s)$ can be obtained by solving for the Green's function of the Schrödinger equation.[4] In particular, the bound state contributions are proportional to the value of the NR wavefunction evaluated at the origin squared. Spin 1 states contribute to $-\mathrm{Im}\Pi(s)$ while spin 0 states contribute to $\mathrm{Im}\Delta(s)$. Since the NR potential has no spin dependent term, the spin 1 vector and spin 0 pseudoscalar states will have the exact same wavefunction. Therefore, the contribution of NR states to the spectral functions satisfies

$$-\mathrm{Im}\Pi^{\mathrm{NR}}_{+-}(s) = \mathrm{Im}\Delta^{\mathrm{NR}}_{+-}(s), \qquad -\mathrm{Im}\Pi^{\mathrm{NR}}_{33}(s) = \mathrm{Im}\Delta^{\mathrm{NR}}_{33}(s), \quad (10)$$

which implies

$$\delta\rho^{\mathrm{T}}_{\mathrm{NR}} = -\delta\rho^{\mathrm{L}}_{\mathrm{NR}}. \quad (11)$$

This shows that the sign of the contribution of NR states to the ρ parameter depends on the DR used. Since both Eqs. 6 and 8 must give the same result, the contribution of the relativistic states away from the threshold and the possible contribution from the integral around the circle at $|s| = \Lambda^2$ must account for the difference.

4. Conclusion

We have shown that the sign of the contribution of NR states to $\delta\rho$ depends on the DR used and is therefore indeterminable. One must include the contribution from the entire integration contour to get a meaningful result.

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