Chapter 8

State Complexity

In the discussion of the comparison of logical complexity to algorithmic complexity, we concluded that logical complexity represents a relaxed interpretation of algorithmic complexity because it ignores patterns which can not be ignored in a rigorous application of algorithmic complexity. Logical complexity accounts for state changes, but does not differentiate between the actual states and therefore can not recognize patterns. State complexity does identify the number of distinct system states and represents a simple form of pattern recognition that is therefore closer to the definition of algorithmic complexity. However, we theorized that a complexity measure in line with a strict interpretation of algorithmic complexity may not be beneficial in optimal searches, based on the shape of the qualitative complexity/performance relationship in Figure 7.52. For conventional elevator (transportation) systems with bi-directional, dedicated pathways, evolutions with all carriages in operation with the fewest interactions are optimal with respect to performance. These evolutions have a constant number of phase lags and therefore the greatest number of state changes per system cycle and therefore the greatest logical complexity. However, the number of distinct states entered by these evolutions can vary over a wide range despite few carriage interactions, depending on the variety of carriage destinations. The algorithmic complexity of an evolution with a high throughput may therefore be little different than the algorithmic complexity of an evolution with a low throughput. A definition of complexity that results in a wide range of throughputs for a given complexity and a wide range of complexities for a given throughput has limited value in searches for optimal configurations.

8.1 Simple Systems

To determine how closely state complexity approaches the definition of algorithmic complexity and the usefulness of state complexity in optimization and establishing correlations between complexity and throughput, we again begin with an analysis of the simplest conventional system. As defined, a carriage enters four states during a cycle, which correspond to loading, travel to a magazine, unloading, and return to a queue. An additional state also exists which describes a carriage returning to the level of the queues, but without a queue selected. Since there is only one queue, magazine, and one path in the simplest system, the carriage ordinarily enters only four distinct states throughout the course of the evolution. For the deterministic cycle times used, each system cycle corresponds to 258 time steps. The state complexity is therefore the ratio of the number of distinct states entered to the product of the number of items carried and the system cycle time. Since the number of items in a queue at the start of an evolution is always 25, the minimum theoretical state complexity is $\frac{4}{25 \cdot 258} = 0.000620$. However, the actual state complexity for the simple system is 0.000784. The discrepancy results from two factors related to the initial and terminal conditions. All carriages are assumed to begin unloaded at the top of their respective shafts. A system cycle therefore requires a round trip between a queue and a magazine and, to complete an integer number of cycles, a carriage must return to its starting position at the end of the evolution. However, since evolutions end when the last item is delivered, the carriage is located outside of a magazine and the last system cycle is incomplete. The actual temporal evolution length is therefore less than the theoretical value, which increases the state complexity. As the carriage unloads the last item, it becomes unsure of its destination queue because there are no items left in the queue. Although the carriage is only in this state for one time step, it still represents a distinct state and increases the total number of distinct states to five, further increasing the state complexity.

Analysis of the discrepancy between the theoretical and actual state complexities for the simplest evolution illustrates the sensitivity of the state complexity to the effects of transients, particularly those from additional states. Even if a state is entered only once in the course of an evolution, the state complexity can be affected significantly. In the simple case considered, the system is in one of the four cycle states for 6341 of the 6342 time steps (99.9842%) which corresponds to a state complexity of 0.000631. The addition of the single state results in an increase of the state complexity of 24.3%. However, the disproportionate increase in the state complexity from the addition of this one state agrees with the definition of algorithmic complexity. Changes in the complexity from additional cycles involving the four most frequent states are negligible - only the bits describing the number of cycles changes, which is small relative to the bits required to describe the cycle. An additional state (or pattern), regardless of the frequency of occurrence, requires a complete description which involves significantly more information.

As defined, the state complexity is a function of the evolution length. In this regard, state complexity strays from algorithmic complexity because additional cycles (more items) have a negligible impact on algorithmic complexity but decreases the state complexity. In the limit of the number of items, the state complexity approaches zero asymptotically. However, the rate of the change of the state complexity with respect to the temporal evolution length and therefore the number of items involved is inversely proportional to the square of the number of items. For sufficiently large quantities of items, the effect of the number of cycles is therefore negligible, bringing state complexity closer to agreement with the definition of algorithmic complexity. For the simple system, if the number of items were increased from 25 to 26, the state complexity decreases 3.8% (assuming an integer number of cycles).

The impact of the temporal evolution length on the state complexity suggests that only comparisons between evolutions involving the same number of items are valid. While the minimum and maximum theoretical throughputs are unaffected by the number of items delivered, a decrease in the number of items delivered tends to decrease the temporal evolution length and bias the state complexity towards larger values. Since the number of items delivered is the only dynamic evolution measure common to all evolutions of a particular system size with respect to state complexity, halting evolutions can not validly be compared to non-halting evolutions. For a set of complete evolutions of the same system size, the relative distribution of state complexities should therefore be constant regardless of the number of items transported.

To perform an analysis consistent with that for logical complexity, we next explore the set of 2-2-2 evolutions, a set small enough to explore exhaustively but large enough to exhibit a range of behaviors and performance. Figure 8.1 shows the two-dimensional distribution of complete 2-2-2 evolutions. Since the absolute values of the state complexities for different system sizes can be significantly different, the scale of the state complexity is set to half of the maximum state complexity encountered in all systems considered to provide suitable detail. Although comparisons between complete and incomplete evolutions are invalid and the two-dimensional distribution of non-halting evolutions is not presented, it is interesting to note that the complete and non-halting evolutions have approximately the same boundaries, at least for 2-2-2 size systems. The shape of the distribution is not significantly different from the shape of the logical complexity distribution. A maximum throughput is evident, which is known to exist from the analysis of logical complexity. For the set of 2-2-2 evolutions however, the maximum throughput boundary is more clearly defined. Although the possible minimum and maximum state complexity boundaries are less apparent, the distribution suggests a triangular boundary with one vertex at the minimum state complexity/minimum throughput combination corresponding to a mimic.

Larger system sizes always contain a mimic of a q-1-1 system (where q is the number of queues in the system), which always share approximately the same logical complexity despite differences in the number of items transported because the number of items, the number of logical steps, and the number of temporal steps tend to increase proportionally. So doubling the number of items will not change the logical complexity despite the doubling of the temporal evolution length because it also doubles the number of logical steps. For state complexity however, the relative number of temporal steps and the actual states visited are important. To be a mimic of a q-1-1 system and be complete, an evolution must transport all items from *all* queues to the magazine corresponding to the lone item type. With δ additional queues with a constant c items per queue, the carriage must transfer an additional δc items, resulting in δ times the temporal evolution length. Since δ additional queues are visited, the system experiences δ additional distinct system patterns. However, the number

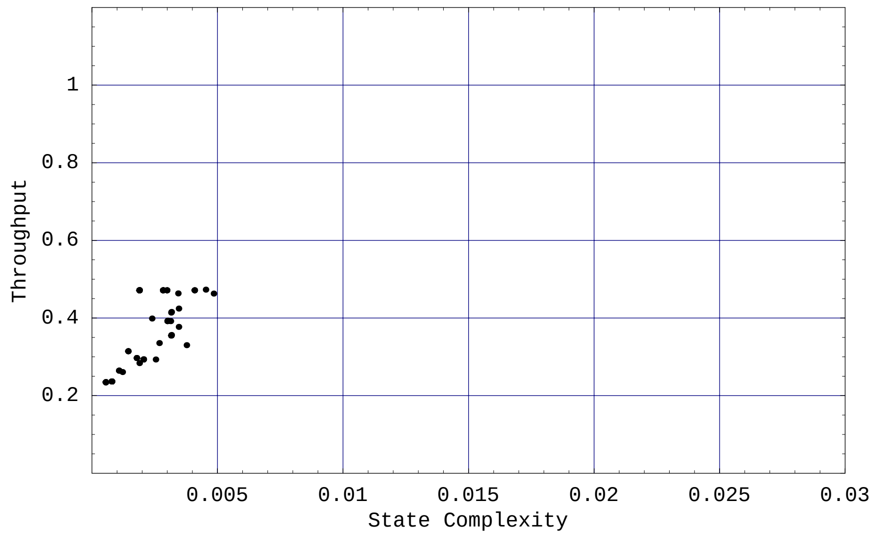


Figure 8.1: The two-dimensional distribution of complete 2-2-2 evolutions with respect to state complexity. The shape of the distribution appears similar to the shape of the distribution of logical complexities.

of distinct states does not increase δ times. States within each distinct pattern describing the carriage traveling to the magazine and unloading in the magazine are common to all patterns because, in a q-1-1 mimic, all items go to a single magazine. The only distinct states in each pattern are those describing travel to a queue and loading in a queue. For mimics of the simplest system, we might therefore think of the number of states as in Equation 8.1, where two distinct states are added with every queue involved to a constant two states describing travel to the magazine and unloading in the single magazine.

$$C_S = \frac{2\,(q+1)+2}{\tau c\,(q+1)} \tag{8.1}$$

For large numbers of queues, the common states related to magazines are negligible with respect to the total numbers of states and the state complexity approaches a lower limit of $\frac{2}{\tau c}$, the ratio of additional states per additional queue to the product of the number of temporal steps per additional item, τ, and the number of items in each additional queue, c. The rate that the state complexity changes with respect to the number of additional queues is described in Equation 8.2 and shows that the difference in the state complexities of large system sizes that mimic q-1-1 systems is not significant and the largest difference from any single additional queue exists between the state complexity of the actual 1-1-1 evolution and the 2-2-2 mimic of the 1-1-1 evolution.

$$\frac{dC_S}{dq} = -\frac{2}{\tau c\,(q+1)} \tag{8.2}$$

Mimicry does not result when queue distributions are comprised of a single item type if the proper physical connectivity is present. Evolutions with the minimum state complexity at the maximum throughput reflect this condition and correspond to configurations with both carriages connected to the magazine corresponding to the lone item type. Each shaft is effectively connected to a unique queue, which is not served by any other shaft. Since carriages run between a single queue and magazine, each carriage has only one pattern throughout the evolution. Individually, the carriages have a state complexity identical to that of the simple 1-1-1 evolution. However, state complexity is based on the number of *system* states and therefore the combination of individual carriage states. Since carriages share a common magazine, a phase lag is introduced in the evolution, resulting in eight combinations of carriage states per system cycle, twice the number of states per cycle for an individual carriage. Although the carriages share a common magazine and therefore exist in the identical state, there is no reduction in the number of distinct states per system cycle as with mimics because the sequencing of the combinations of states result in distinct system states.

Theoretically, two carriages with equal numbers of distinct individual states could correspond to an evolution with the same number of system states if the carriages change states simultaneously. This could only occur if the system cycle times meet particular conditions or if the carriages effectively act independently. Independent carriages are only possible when decision logic describing default or priority destinations is removed or carriages have more information about the state of the entire system and if there are equal numbers of item types - conditions not considered in the model used. Carriage independence is also possible when independent queue-shaft-magazine subsystems exist, which we have seen always results in incomplete evolutions with only a fraction of items equal to $\frac{1}{s}$ delivered, where s is the number of shafts.

The maximum theoretical number of distinct system states possible from two carriages with equal numbers of distinct individual states is always twice the number of individual carriage states, regardless of the individual cycle times of each carriage state. The actual sequencing of states for the deterministic, two carriage evolution with a phase lag is presented in Figure 8.2. Because the maximum theoretical number of system states is independent from the cycle times and is only dependent on the sequencing, the maximum value is always attained as long as the sums of the individual cycle times are equal except when individual carriage states change simultaneously. The same results apply to sequences involving additional carriages.

Since there is only one item type and carriages pull from respective queues, there is only one sequence of system states (ignoring transients) and therefore the minimum number of states for any evolution involving both carriages operating for the entire evolution. With the minimum number of states and carriages always in operation, the state complexity is at a minimum and the throughput is near maximum.

The phase lag created by the delays associated with resource sharing result in additional distinct states at the end of the evolution. Since the carriages are phase lagged, one carriage

First Carriage	Second Carriage
Loading	Travel to a queue
Travel to a magazine	Travel to a queue
Travel to a magazine	Loading
Travel to a magazine	Travel to a magazine
Unloading	Travel to a magazine
Travel to a queue	Travel to a magazine
Travel to a queue	Unloading
Travel to a queue	Travel to a queue

Figure 8.2: The dominant sequence of carriage states in a two carriage system with a constant phase lag. The queue and magazine identification numbers are omitted because, with a constant phase lag, the sequence of the same types of states is identical.

ends before the other and adopts the 'unsure of queue' state while the remaining operational carriage continues to transport its items. The additional number of distinct system states is dependent on the state of the operational carriage when the first carriage finishes but will never be greater than the number of distinct *carriage* states of the operational carriage including the 'unsure of queue' state (or combinations of states of multiple operational carriages including those created by carriage termination). For the 2-2-2 systems considered, there are three additional system states resulting from the transient period at the end of the evolution corresponding to the combination of the unsure first carriage with the second carriage traveling to a magazine, unloading, and becoming unsure of its own destination queue.

Transients also occur at the start of these evolutions as a result of the interaction of the carriages. The delay of the second carriage that results in the phase lag is not introduced until the carriages arrive at the same magazine in the first system cycle, meaning the system experiences the distinct state of both carriages loading simultaneously in their respective queues - a state not experienced at any other point in the evolution. More distinct states are possible in the period between the start of the evolution and the introduction of the phase lag. However, the state of both carriages moving simultaneously to the magazine occurs in the repetitive pattern as well. If additional detail were used in their definition (carriage level, for instance), these states would be distinct as carriages are in identical positions in the transient period but offset during operation in the dominant pattern.

Although the simplest complete evolution with maximum throughput consists of a single primary pattern, it has a greater state complexity than the complete 2-2-2 mimic of the 2-1-1 evolution that contains two patterns. According to the simplest interpretation of algorithmic complexity, the complexity of the mimic should be greater than that for the two-carriage, single pattern evolution. However, the information required to define the states in each pattern is important. Two carriages creating one pattern require the definition of eight states. Each pattern of the single carriage evolution nominally requires four states (so the states common to both patterns arguably result in a slight underestimate of complexity), making the number of states approximately equal. However, any measure of complexity must account for *all* system states. The four states resulting from transients at the start and end of the two-carriage, single pattern evolution, which represent $\frac{1}{3}$ of the total number of distinct states, significantly increases the complexity. The single state resulting from the transient at the end of the evolution of the mimic also has a significant impact on the total number of distinct states, but at 25%, the effect is less than that for the two-carriage system. Once again, we see that transients have a significant impact on the state complexity and can not be ignored, despite the fact that they typically represent a small fraction of the entire temporal evolution length. The effects of transients are therefore in accordance with the definition of algorithmic complexity. The information required for their description is on the same order of magnitude as the information required to define the patterns that dominate the temporal evolution because most of the information in the description of a pattern is related to the states and the number of repetitions is highly compressible.

In addition to the additional states resulting from transients, the state complexity corresponding to the two-carriage, single pattern evolution is also greater than the state complexity of the mimic because the temporal evolution length of the mimic is approximately twice that of the two carriage system. Normalization with respect to the temporal evolution length does not reflect any aspect of algorithmic complexity and state complexity therefore presents a skewed view of algorithmic complexity. However, as discussed with respect to logical complexity, a strict application of algorithmic complexity may not be desirable in either the characterization of a complexity-performance relationship or in creating a tool towards system optimization.

The absence of all but one item type is a necessary but insufficient condition for creating the simplest patterns and lowest state complexity for complete evolutions. For configurations with complete shaft-magazine connectivity with respect to the magazine corresponding to the single present item type and complete shaft-queue connectivity (like configurations 247 and 255), both carriages visit both queues, so each carriage has two distinct patterns. With two distinct patterns for each carriage, each comprised of four states, four combinations of patterns are possible, each with a theoretical maximum of eight state combinations, for a total of 32 possible state combinations (ignoring transients).

When states common to each pattern are considered, the possible number of distinct states is 18, assuming the phase lag is constant and the sequencing of carriage states is identical to the two-carriage, single pattern evolution. This number can also be determined by calculating the number of unique combinations of states at each stage in the sequence defined in Figure 8.2. For instance, in the first state, where the first carriage is loading and the second carriage is traveling to a queue, the first carriage can be loading an item bound for the only magazine in either the first or second queue and the second carriage can be either traveling to the first or second queue, for a total of four combinations. In the second state, the first carriage can only travel to the single magazine while the second carriage still has two possible queues to travel to, resulting in two possible combinations for this state. These examples illustrate how the numbers of physical locations and the number of carriages, through their effect on the sequencing of events, impact the state complexity.

Assuming a constant number of states resulting from transients, it is therefore possible for the state complexity to be significantly greater than the measured value. However, because of logic dictating that a carriage select the first available queue it finds, and because queues are evaluated for availability in order (by their nominal assigned identification), lower numbered queues (or those considered earlier) have a de facto priority. Both carriages therefore empty the first queue before simultaneously switching to the second queue, and only two of the possible four combinations of carriage patterns are utilized, resulting in 14 distinct states when repeated states are removed. With the proper ratio of item types in each queue to the number of carriages, or in the presence of additional logic specifying how resources are shared, additional system patterns are possible that result in greater state complexity.

With mixed queue distributions, the de facto priority logic that is applied as a result of greater physical connectivity is combined with logic specifying that the most abundant type in any queue be selected if possible to create the greatest mix of individual carriage patterns and therefore the greatest number of distinct states achieved. Figure 8.3 shows the compressed system state history for the completely connected 255 configuration with a 40-60 queue distribution. The evolution is nearly identical to the 255 evolution with a 60-40 queue distribution discussed with respect to logical complexity, with carriages first removing the most abundant item from the first queue until the numbers of each item type are equal, then removing both item types at identical rates. When the first queue is empty, the same process occurs in the second queue. The evolution therefore experiences four phases, each with a distinct pattern consisting of eight states. When states common to multiple patterns and additional states from transients are taken into account, the evolution contains a total of 26 system states.

The number of distinct states in this evolution is approximately half the maximum number of states possible for the same configuration. After states common to multiple patterns are taken into account, the theoretical maximum number of distinct states is 40. These states are determined from the sequence of state changes for two carriages in Figure 8.2 and from the possible number of each state. There are four possible loading states, two distinct states with respect to travel to a magazine, two possible loading states (one for each magazine), and two states describing travel to the queues.

8.2 Theoretical Boundaries

Based on the number of queues, shafts, and magazines and the sequence of state changes for the corresponding number of shafts (carriages), which may be dependent on the operational logic, the variety of carriage states illustrates that it is possible to calculate a maximum theoretical state complexity. The example evolutions also illustrate that it is possible to

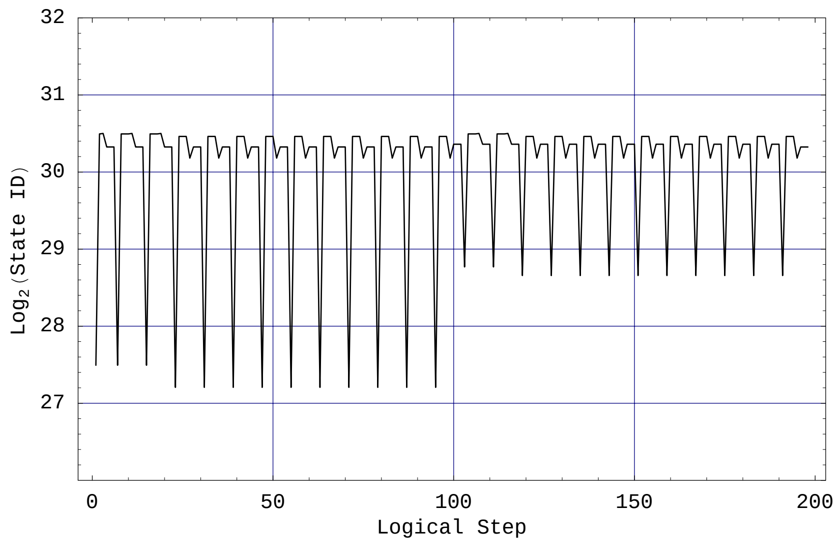


Figure 8.3: The compressed state history for evolution 255 2-2-2 (40-60). Four system patterns result because of the variety of item types in the queues, the complete physical connectivity, and the system logic.

calculate a maximum theoretical state complexity corresponding to a specific queue distribution - queue distributions with a single item type effectively limit the number of carriage destinations and decrease the possible state complexity. It is also possible to account for the physical connectivity by adjusting the combinations of carriage states to reflect achievable states.

The maximum theoretical state complexity as described is only applicable at the maximum throughput, where all carriages are operational for the entire evolution and carriages only enter the patterns defined. When halting occurs, calculation of the maximum theoretical state complexity must account for distinct states entered by different numbers of operational carriages. For instance, in a configuration with three carriages, it is necessary to determine the state combinations of all three carriages, state combinations of two carriages, and individual carriage states. The number of states possible with halting is therefore much greater than for an evolution with fully operational carriages and, for a comparable temporal evolution length (with only a single two-carriage cycle and a single one-carriage cycle for instance), the state complexity is also much greater. Since the actual fractions of an evolution corresponding to the number of carriages operating simultaneously is unknown without explicit simulation, it is impossible to determine the actual number of distinct states, since some of the fractions may be zero. The best estimate of the maximum number of states therefore must include all possible states for all possible numbers of operational carriages. The maximum theoretical state complexity at any given throughput is calculated by assuming the number of items transported is constant. At the minimum throughput, the number of operational carriages is again known, since the minimum throughput always corresponds to a mimic of a system with a single carriage.

Calculation of the minimum theoretical state complexity for evolutions with either one or all carriages operational is also different than for evolutions with carriages that halt partially through the evolution. The minimum number of states in a complete evolution will always correspond to the queue distribution with the lowest diversity - one containing a single item type, because it effectively eliminates the possible number of destinations. The minimum theoretical state complexity at the maximum possible throughput should therefore always correspond to an evolution in which all carriages are connected to respective queues and the magazine corresponding to the lone item type. The minimum number of phase lags occurs, but there is at least one for the evolution to be complete, since all carriages must share a common magazine. At the minimum throughput, a single carriage delivers all items from all queues (to be complete) and the state complexity is described by Equation 8.1. For evolutions with halting carriages, all possible system states created from the possible sequences of all combinations of the number of operational carriages are assumed, but the number of magazines is effectively reduced to one.

A comparison of the definitions of the minimum and maximum state complexities reveals that, at the minimum throughput, the minimum and maximum state complexities are not necessarily equal. The maximum state complexity corresponds to an evolution that has almost the greatest variety in the queue distribution and therefore involves all possible magazines but one (if all magazines were involved, then a valid configuration could never emulate a one carriage system). The minimum state complexity however, involves only a single magazine, keeping the possible magazine destinations at a minimum. The state complexity distribution therefore contains a non-singular boundary corresponding to the minimum throughput, although it may be short in comparison to the range of the maximum throughput boundary. Evolutions can exist between the minimum and maximum, depending on the number of item types absent in the queue distribution. For 2-2-2 size systems however, there can be at most only a single item type absent, so there is only a single point defining the minimum throughput.

The theoretical boundaries for the state complexity for 2-2-2 size systems are presented in Figure 8.4. The boundary lines reflect the minimum and maximum state complexities for evolutions with various numbers of operational carriages through the range of possible throughputs. The points 'A' and 'B' respectively represent the minimum and maximum theoretical state complexities at the maximum theoretical throughput, when the number of operational carriages and therefore the number of possible states are known. Similarly, point 'C' corresponds to the minimum/maximum state complexity at the minimum theoretical throughput, when only a single carriage is in operation. For system sizes with three or more magazines, a point 'D' would also exist that would represent the maximum state complexity at the minimum throughput.

Figure 8.4 reveals discrepancies between the actual distribution of evolutions, the theoretical state complexities at the minimum and maximum theoretical throughputs, and the theoret-

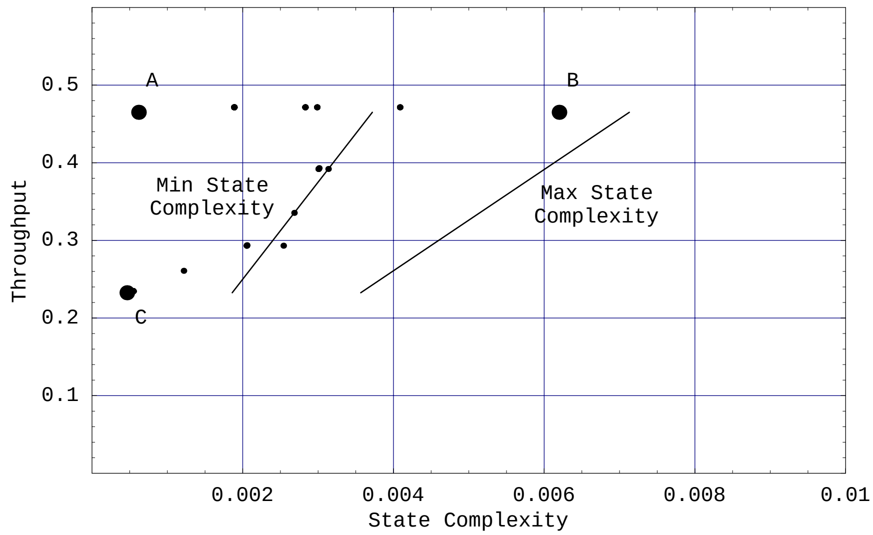


Figure 8.4: The relationship between the minimum/maximum theoretical state complexities and the actual distribution of evolutions. The evolutions are not necessarily within the boundaries because of states resulting from transient periods and because not all possible states are visited. The differences between the minimum/maximum state complexity points at the minimum and maximum throughputs and the minimum/maximum state complexity boundary lines are a result of the lack of knowledge of the actual number of states used in evolutions with carriages that halt.

ical state complexity boundary lines across the range of throughputs.

The differences between the actual and theoretical minimum and maximum state complexities occur for two reasons. First, the theoretical state complexities consider all states that are possible given the variety of the queue distribution to account for the lack of information regarding the actual numbers of operational carriages. The lack of information also explains the differences between the minimum and maximum state complexities at the minimum and maximum theoretical throughputs, where the number of operational carriages is known throughout the entire evolution. Because of logic that effectively limits the physical connectivity and therefore the variety of combinations of states visited, the actual number of states visited is typically much lower than the maximum, which has the effect of decreasing the state complexity. Second, the minimum and maximum state complexities consider only the possible steady state cycles created when a phase lag is introduced and therefore ignore the effect of transients on the state complexity, which we have seen can significantly increase the state complexity.

Since the theoretical state complexity boundaries account for all possible carriage state combinations that are feasible within the constraints of model definitions, the differences between the actual and theoretical state complexities may be thought of as being equivalent to the result of the introduction of randomness or variability into the model. This variability could be a function of non-deterministic cycle times, the inclusion of actual physical dimensions, or the elimination of the assumption of identical distances between each queue and magazine.

Variability tends to both shift and stretch the complexity boundaries (at least the maximum boundaries) towards greater complexity values. The increase in complexity is simply a reflection of the effects of variations, and is not a function of the underlying relationships between complexity and performance that may be applicable towards optimization. In other words, while the presence of greater variability creates a more realistic representation of the naval weapons elevator system and would appear to decrease the usefulness of behavior as part of an optimization or search tool because variability increases the range of complexity values at a particular throughput, variability obscures the reasons why correlations may be present between complexity and performance (or robustness) by inducing variability in the distributions of complexity and performance values. The potential strength of relationships is missed for the same reason. And since our task is to identify the presence, strength, and causes of relationships, we ignore non-determinism in our analyses in order to capture the fundamental reasons why relationships between behavior, performance, and robustness may exist, while recognzing that the usefulness of behavior as part of an optimization tool may be limited depending on the amount of system variability.

The relative effects of the inclusion of all potential states and the omission of transient states are apparent at the minimum theoretical state complexities corresponding to the minimum and maximum throughputs, because of the additional information known regarding the evolutions. At point 'A' in Figure 8.4, the possible number of states (in steady state cycles) is equal to the actual number of states visited in cycles so the difference between the minimum actual state complexity at the maximum throughput and the minimum theoretical state complexity at 'A' results only from additional states from transients. As long as there are some transients, point 'A' will therefore always be an underestimate of the actual minimum state complexity at the maximum throughput. At point 'C', the same condition applies since the theoretical number of possible states again matches the actual number of visited states. However, the actual state complexity at the minimum throughput is closer to the theoretical value at 'C' than at 'A', indicating that fewer transients exist when a single carriage is operational.

At throughputs between the theoretical minimum and maximum that correspond to evolutions with halting carriages, less information is known about the actual states visited because the halting sequence is unknown. Therefore, while Figure 8.4 indicates the minimum and maximum theoretical state complexities consistently overestimate the actual state complexities, it is impossible to determine the relative effects of the overestimation due to the inclusion of all possible states and the underestimation due to the omission of states from transients because it is impossible to separate the states from cycles and transients from the state complexity value alone. The relative effects can only be determined by explicitly analyzing the states visited in each evolution.

Figure 8.4 suggests that the state complexity can be maximal at less than maximal throughputs because the potential number of distinct states, created from the combinations of carriage states for various numbers of operational carriages, is greater. This relationship follows the qualitative complexity/throughput relationship from Figure 7.52 based on the application of algorithmic complexity - halting carriages increase the number of distinct patterns requiring description, but imply that throughput is less than optimal. However, whether or not an evolution with sub-optimal throughput corresponds to the maximum state complexity is dependent on the relative effects of halting creating additional evolution states and the increase in the temporal evolution length resulting from the loss of operational carriages.

Normalization of the number of distinct states with respect to the temporal evolution length has a correlating effect between state complexity and throughput and implies that, to achieve the maximum state complexity at a less than optimal throughput, halting must occur late in the evolution. Since more information is required to describe the patterns than to describe the number of each pattern, evolutions dominated by patterns involving all carriages, but containing patterns constructed from all possible numbers of operational carriages should correspond to the maximum state complexity, ignoring the effects of transients. As carriages halt earlier in the evolution, the number of states may be greater than the number of states for an evolution involving all carriages throughout the entire evolution, but the increase in the temporal evolution length caused by disproportionate carriage utilization will result in lower state complexity.

To achieve the maximum number of states *per pattern*, an evolution must also have the maximum number of phase lags possible because the number of combinations of individual carriage states is greater with phase lags. However, the maximum number of phase lags does not necessarily result in the maximum number of *system* states. We can imagine an evolution involving a single carriage with no phase lags that transports multiple item types from multiple queues. This evolution creates more patterns and distinct states than an evolution (like 103 2-2-2 (0-100)) of the same system size that involves carriages transporting a single item type from respective queues to a common magazine, introducing the maximum number of phase lags but resulting in one system pattern (ignoring transients). The effects of the number of phase lags on state complexity through the number of combinations of individual carriage states must therefore be determined in the context of the number of system patterns.

8.3 Qualitative Characterizations

The boundaries in Figure 8.4 are created assuming a constant, maximum number of phase lags. If the states created from various phase lags are included, the maximum state complexity boundary shifts to the right, approaching the maximum state complexities possible for non-deterministic systems, where all combinations of individual carriage states are possible. Phase lags are therefore related to the state complexity, but do not characterize the state complexity in the same way as with logical complexity.

To determine how important phase lags are to state complexity and to indirectly determine the relationship between the number of phase lags and the number of distinct states, we plot the distribution of evolutions with respect to logical complexity, which is characterized by the number of phase lags, in the state complexity space. Figure 8.5 presents a mapping of logical complexity values into state complexity space, where the darkness of a point indicates the value of the evolution's logical complexity (darker values indicate greater logical complexity). The state complexity and throughput scales are modified from the typical ranges used to provide suitable detail. Since the average number of phase lags is directly related to the logical complexity, the mapping therefore indicates the effects of the number of phase lags on the state complexity. If the number of phase lags were a direct indicator of state complexity, then the darkness should increase as the state complexity increases across any line of constant throughput and points of constant darkness should be in line. Figure 8.5 shows that the state complexity does not necessarily increase with the average number of phase lags. Across any line of constant throughput, there may be various numbers of phase lags with no general trend, or similar numbers of phase lags distributed across a range of state complexity values. This mapping indicates that the number of phase lags does not necessarily result in more system patterns and, despite the additional combinations of states associated with larger numbers of phase lags, state complexity does not necessarily increase. Figure 8.5 also illustrates the differences between the definitions of logical complexity and state complexity. Since logical complexity ignores the actual values of system states and only identifies state changes, evolutions with various state complexities share the maximum value of logical complexity.

Without the number of phase lags as a suitable parameter, a possible, but not particularly useful characterization of the state complexity space is based on the queue distribution. With only a single item type, all carriages are forced to the same magazine, which results in time delays and phase lags in complete evolutions. When more carriages are involved, the state complexity increases, but not significantly because only a small number of patterns are used. However, increasing the variety - the diversity of item types present - in the queue distribution will tend to increase the state complexity for the same throughput because variety in the item types increases the variety of locations visited and therefore the number of states entered. Variety in the queue distribution (with logic dictating destination priority) also increases the possibility for greater carriage interaction. Because the state complexity ignores the number of repetitions of any pattern, the relative distributions of distinct item types is unimportant. It is only important that variety be present. A (1-99) queue distribution will therefore result in a state complexity equivalent to that for a (50-50) queue distribution, assuming the temporal evolution lengths are identical, and both of these queue distributions will have greater state complexity than for a (0-100) or (100-0) queue distribution. Since the relative item types are unimportant, it is only possible to distinguish between queue distributions with variety and no variety (and different degrees of variety for systems with greater numbers of magazines where various numbers of item types may be absent).

It is still possible to characterize throughput by the average number of operational carriages,

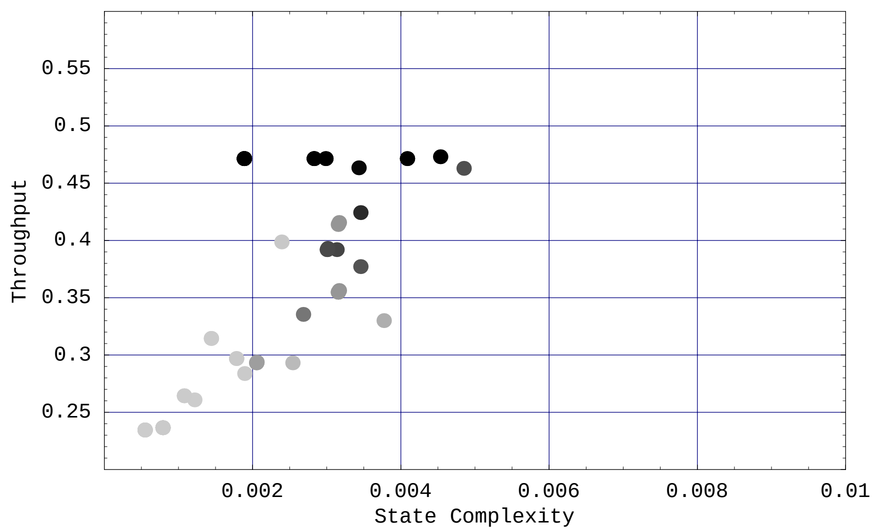


Figure 8.5: The mapping of logical complexity values in state complexity space. Darker points correspond to greater logical complexity and therefore a greater equivalent number of phase lags. There is no particular relationship between the number of phase lags and the state complexity. At the maximum throughput, all evolutions have the maximum number of phase lags, but the same is not true across other lines of constant throughput.

which is dependent on when carriages halt in the course of an evolution, although halting affects the state complexity through the temporal evolution length and is therefore not strictly associated with the throughput axis.

The resultant qualitative characterization of state complexity with respect to variety in the queue distribution and the onset of carriage haltings is presented in Figure 8.6. While for systems with greater than two magazines there are varying degrees of queue distribution variety beyond “variety” and “no variety”, and these generally correspond to the relative state complexity, ultimately, the state complexity is a function of the queue distribution taken in the context of the physical connectivity. There is therefore no distinct line corresponding to queue distribution variety, limiting the characterization of the state complexity/throughput space to a qualitative one.

8.4 State Complexity and Throughput

For logical complexity, temporal evolution length is less of an issue because logical evolution length typically increases proportionally so comparisons of incomplete and complete evolutions are arguably valid. However, for state complexity, the number of distinct states and the number of patterns are not scalable with temporal evolution length and comparisons of incomplete and complete evolutions can be quite misleading. As a result, we only consider

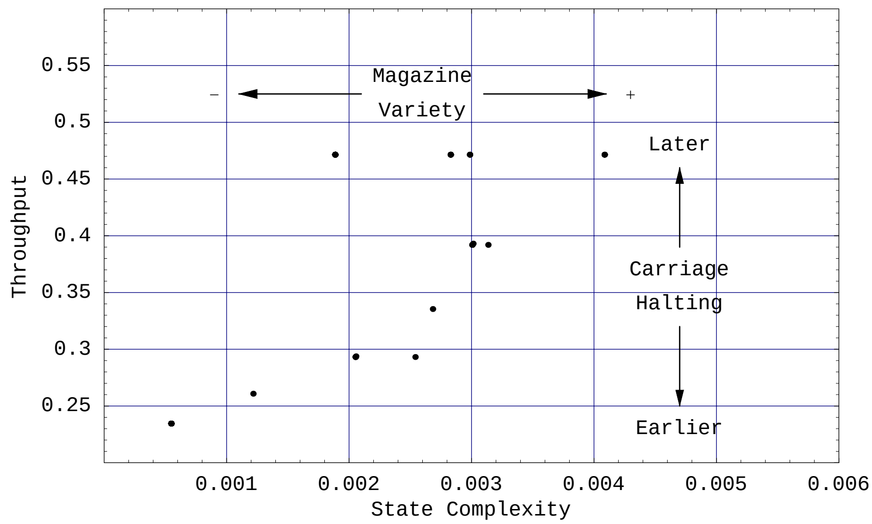


Figure 8.6: The qualitative characterization of the state complexity space with respect to halting carriages and variety in the queue distribution. Variety in the queue distribution permits greater exploration of possible states, increasing the state complexity. Carriages that halt later in an evolution result in lower temporal evolution lengths and therefore greater throughput and greater state complexity.

complete evolutions and their subsets in establishing correlations between state complexity and throughput, although comparisons with non-halting evolution sets are used to illustrate the differences between these evolutions and complete evolutions.

The summary of mean values for the state complexity for non-unique and unique evolution subsets for 2-2-2 size systems is presented in Table 8.1. For 2-2-2 size systems, the mean state complexity of incomplete evolutions is less than the mean for the entire set of non-halting evolutions, resulting in a greater mean state complexity for complete evolutions. While a similar increase for logical complexity suggests a correlation between complexity and adaptability in the presence of uncontrollable queues, the same can not be said with respect to state complexity. Because state complexity is based on a normalization with respect to temporal evolution length, it is invalid to make comparisons of evolutions that transport different quantities of items for a system of a particular size, since the item quantities delivered are the only dynamic attribute common to all evolutions. To properly evaluate the effects of incomplete evolutions on the mean state complexity, an analysis of additional system sizes is required.

Comparisons of complete evolution sets for different system sizes remain valid and show that the mean state complexity for the evolutions corresponding to robust configurations (those *configurations* that result in complete transfer for all queues considered) is greater by approximately six percent. This difference again implies, as it did with respect to logical complexity, that as a group, the most adaptable configurations correspond to the most

Table 8.1: Summary of the mean state complexity and throughput for 2-2-2 evolution subsets. As the set is refined, the mean complexity increases, indicating a correlation between adaptability and complexity. Increasing throughput also suggests correlations between adaptability and performance as well as complexity and performance. For 2-2-2 size systems, the mean state complexity of incomplete evolutions is 0.002281, less than the mean for non-halting evolutions. This difference is not always the case, since the state complexity is sensitive to temporal evolution length and comparisons of incomplete and complete evolutions are misleading.

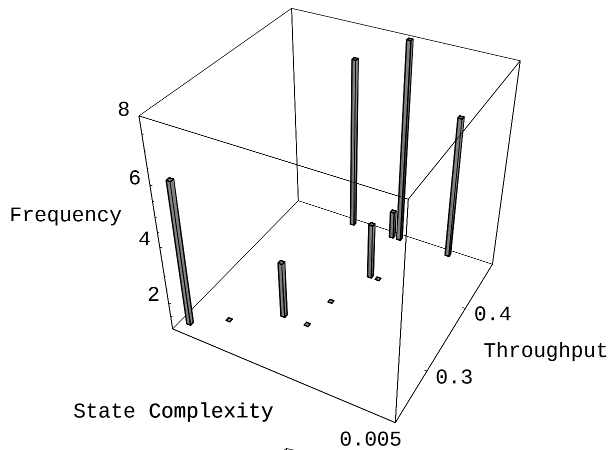
	mean C_S	mean R
Non-halting	0.002386	0.3748
Complete	0.002453	0.4015
Robust	0.002591	0.4069
Unique Non-halting	0.002767	0.4058
Unique Complete	0.002790	0.4310
Unique Robust	0.002969	0.4715
Mimics	0.000668	0.2355

complex evolutions. Since the throughputs remain unchanged regardless of the complexity measure used, the greater mean throughput associated with the evolutions of robust configurations also implies a direct relationship between complexity and performance and therefore between adaptability and performance.

Although the set of evolutions corresponding to the most robust configurations have the greatest mean state complexity and throughput of any set achieved, the mean values are not equal to the maximum values achieved, although some evolutions in the set may correspond to maximum values. Figures 8.7 and 8.8 illustrate the distribution of values for complete and robust evolution sets with three-dimensional frequency landscapes and cross-sectional histograms. The maximum values for the scales of all figures are based on the maximum frequencies for the set of non-halting evolutions to facilitate comparisons. Because the same scale is used, it is evident that the maximum frequencies remain relatively unchanged for all evolution sets. The similarities exist because the evolutions located at the maximum throughput for a range of state complexities are common to all sets and helps explain the trend toward increased throughput and state complexity with set refinement.

These clusters of evolutions are also evident in Figures 8.9 and 8.10, showing the three-dimensional frequency landscapes and histograms with respect to state complexity for unique sets of complete and robust evolutions. The common evolutions essentially define the unique, robust evolution set and again help explain the increases in the mean state complexity for these sets evident in Table 8.1. The greater mean values of these unique sets relative to their corresponding non-unique equivalents again indicates a relationship between adaptability,

(a)



(b)

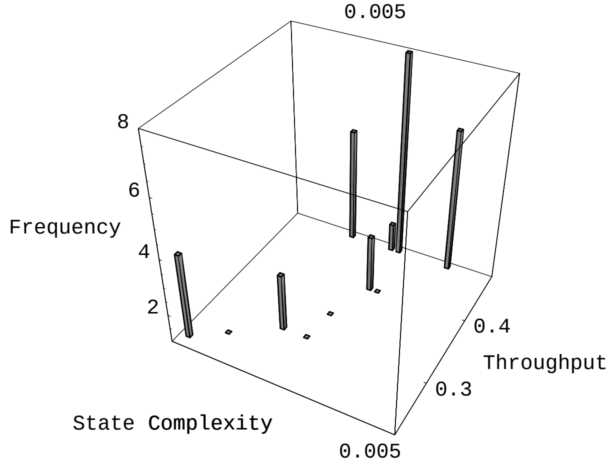
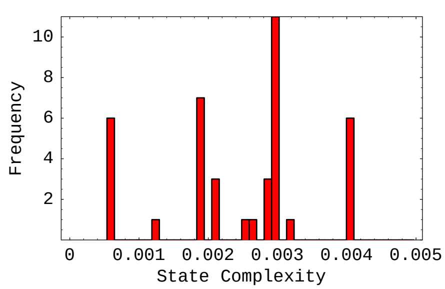
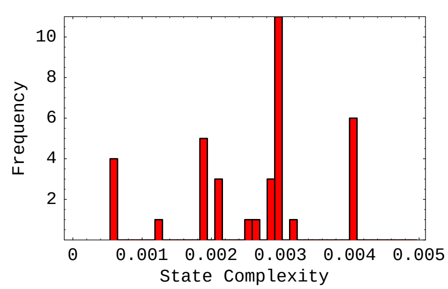


Figure 8.7: The three-dimensional state complexity/throughput frequency landscapes for (a) complete evolutions and (b) robust evolutions for 2-2-2 size systems. The maximum frequencies are similar in both distributions because the most robust configurations that exist at the maximum throughput are common to both sets.



(a)



(b)

Figure 8.8: The cross-sectional histograms with respect to state complexity for (a) complete evolutions, (b) robust evolutions for 2-2-2 size systems.

complexity and throughput. It also suggests that system size is related to state complexity and throughput. Unique evolutions represent the largest size configurations possible in the set and are able to support greater state complexity and throughput. This relationship is intuitive - more carriages means larger state spaces and greater potential for more states visited and less time to deliver a set of items.

(a)

(b)

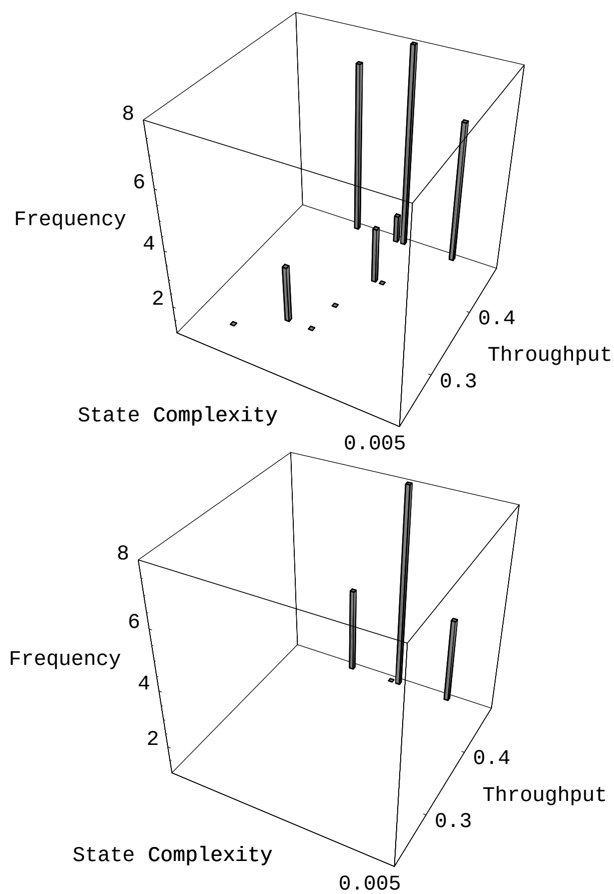
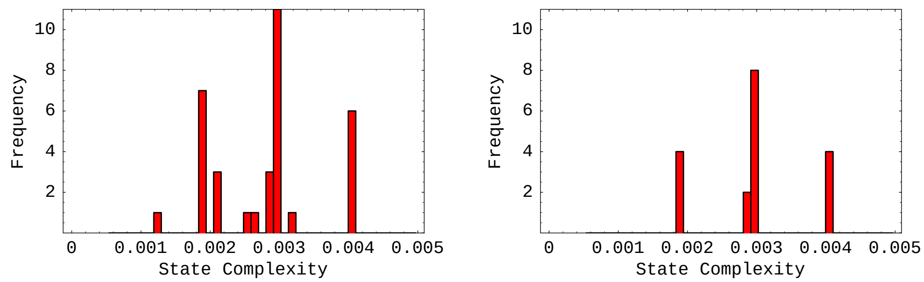


Figure 8.9: The three-dimensional state complexity/throughput frequency landscapes for (a) unique complete evolutions and (b) unique robust evolutions for 2-2-2 size systems. The maximum frequency is identical to those for all complete evolutions and all robust evolutions, because the evolutions at the maximum throughput are again common to all sets.

As with logical complexity, it is difficult to draw any conclusions regarding the relationship between adaptability, state complexity, and throughput based on a single system size, especially when the system size contains relatively few evolutions.



(a) (b)

Figure 8.10: The cross-sectional histograms with respect to state complexity for (a) unique complete evolutions and (b) unique robust evolutions for 2-2-2 size systems. While the mean state complexities are greater for unique and robust evolutions, the histograms indicate that uniqueness and robustness are no guarantee of maximum throughput of complexity.

8.5 More Complex Systems

The analysis of other size systems, particularly larger systems containing significantly more evolutions, provides supporting (or contradictory) evidence for the preliminary conclusions regarding state complexity drawn from observations of 2-2-2 evolution sets. Besides the primary goal of identifying possible correlations between state complexity and throughput, analysis of different system sizes illustrates the effects of changing the absolute and relative numbers of queues, shafts, and magazines in the context of the logic used. The distributions of other system sizes further demonstrate the practical application of the definition of algorithmic complexity and also illustrate the effects of determinism on state complexity.

The mean state complexities for the various evolution subsets of non-trivial system sizes are listed in Table 8.2. The mean values in Table 8.2 illustrate why comparisons between incomplete and complete evolutions are invalid. The analysis of 2-2-2 size systems showed that the mean state complexity of all non-halting evolutions is less than the mean state complexities of the complete and robust evolution sets. The same is true with respect to the mean state complexities of unique evolution sets. This same relationship for logical complexity led to the conclusion that a relationship exists between robustness with respect to uncontrollable queues, logical complexity, and throughput. However, for many of the evolution sets considered in Table 8.2, the mean state complexities of non-halting evolutions are greater than the mean state complexities of respective complete evolutions. The reason for the difference is a result of the normalization of the number of states with the temporal evolution length. Since the number of patterns does not affect the number of states entered, but does affect the temporal evolution length, an incomplete evolution that enters the same number of distinct states as a complete evolution has a greater state complexity because halting (evolution halting, not individual carriage halting) cuts the evolution length short. Even if the number of distinct states entered in an incomplete evolution is less than the number of distinct states entered in a complete evolution, the state complexity may still be greater if halting occurs early enough in the evolution. The differences in the mean values therefore reflects the number of incomplete evolutions, the numbers of distinct states entered by each, and the point in each evolution at which halting occurs.

For all system sizes listed in Table 8.2, the mean state complexity of the evolutions corresponding to the most robust configurations, those configurations that result in complete evolutions for all queue distributions considered, are greater than the mean state complexity of all complete evolutions of the same system size. This relationship exists for both non-unique and unique evolution sets. It also exists regardless of the number of queue distributions considered, which can vary considerably depending on the number of magazines. The number of queue distributions is also dependent on the increment used to describe the fraction of item types. For 20 percent increments, there are six queue distributions for a system with two magazines, 21 queue distributions for systems with three magazines, and 56 queue distributions for systems with four magazines.

8.6 State Complexity and Robustness

In the analysis of logical complexity, we saw that the mean throughput of the most robust configurations is also always greater than the mean throughput for the set of complete evolutions. The increase of both the state complexity and throughput indicates a correlation between the values, and since the increases correspond to the most robust configurations, a correlation between these values and adaptability.

However, while the mean state complexity and throughput corresponding to the most robust configurations are greater than the mean values for all complete evolutions, they are never equal to the maximum values achieved in the set, although evolutions existing at one or both of the maximum values (state complexity and throughput) may belong to the complete set. Of course, if some robust evolutions correspond to the maximum state complexity or throughput, then other robust evolutions must exist with state complexities or throughputs less than the mean values for the robust set, indicating that all of the most robust evolutions do not necessarily have state complexities and throughputs greater than the mean values corresponding to all complete evolutions.

The mean state complexities (and throughputs) corresponding to the most robust configurations are also not necessarily the maximum mean values for any set of evolutions with various levels of robustness. Figure 8.11(a) shows the mean values of state complexity as a function of the level of robustness, using the complete evolution set of 2-3-4 system size as an example. The level of robustness is defined as the number of queue distributions that result in complete evolutions for a particular configuration. Since there are four magazines in the 2-3-4 example system, there are 56 possible queue distributions and therefore 56 levels of robustness. All configurations with a common level of robustness are therefore capable of the same number of complete evolutions, but not necessarily with the same queue distribu-

Table 8.2: The mean state complexity for evolution subsets of different system sizes (N = non-halting, C = complete, R = robust, UN = unique and non-halting, UC = unique and complete, UR = unique and robust, M = mimics). The mean state complexity of robust evolutions is always greater than the mean state complexity for the corresponding complete evolutions for all system sizes, implying a correlation between adaptability and state complexity. This relationship is true for non-unique and unique evolution sets. The mean state complexities for unique evolution sets are also always greater than the corresponding non-unique evolution sets. This trend along with the lower than average state complexities of mimics suggest that larger systems, with respect to the number of shafts, support greater complexity. The inconsistent relationship between non-halting and complete evolution sets illustrate the misleading effects of incomplete evolutions due to the normalization by the temporal evolution length.

System	N	C	R	UN	UC	UR	M
1-2-2	0.003649	0.003649	0.003649	0.004221	0.004221	0.004724	0.000788
1-2-3	0.004212	0.004212	0.004212	0.004921	0.004921	0.005985	0.001035
1-2-4	0.004651	0.004651	0.004651	0.005396	0.005396	0.006767	0.001223
1-3-2	0.008441	0.008441	0.008441	0.010109	0.010109	0.012195	0.002606
1-3-3	0.008612	0.008612	0.008612	0.010858	0.010858	0.014131	0.003313
1-3-4	0.009440	0.009440	0.009440	0.011946	0.011946	0.016131	0.003940
1-4-2	0.014686	0.014686	0.014686	0.017853	0.017853	0.021516	0.005184
1-4-3	0.015008	0.015008	0.015008	0.019555	0.019555	0.024709	0.007309
1-4-4	0.015842	0.015842	0.015842	0.021012	0.021012	0.024709	0.008367
2-2-2	0.002386	0.002453	0.002591	0.002767	0.002790	0.002969	0.000668
2-2-3	0.002874	0.002843	0.003059	0.003344	0.003322	0.003712	0.000873
2-3-2	0.004914	0.004891	0.005171	0.005921	0.005995	0.007109	0.001703
2-3-3	0.005507	0.005431	0.006017	0.006955	0.006877	0.008962	0.002345
2-3-4	0.006245	0.006071	0.006914	0.007870	0.007660	0.010203	0.002845
2-4-2	0.007732	0.007269	0.007609	0.009391	0.008957	0.010914	0.003382
2-4-3	0.008471	0.008203	0.008924	0.010956	0.010553	0.012724	0.004743
3-2-2	0.001876	0.001935	0.002076	0.002170	0.002155	0.002271	0.000578
3-2-3	0.002318	0.002289	0.002527	0.002694	0.002635	0.002893	0.000751
3-2-4	0.002691	0.002574	0.002867	0.003104	0.002991	0.003316	0.000884
3-3-2	0.004076	0.004156	0.004585	0.004969	0.005191	0.006635	0.001331
3-3-3	0.004664	0.004723	0.005422	0.005947	0.006104	0.008803	0.001928
3-3-4	0.005347	0.005340	0.006200	0.006791	0.006862	0.010266	0.010413
3-4-2	0.006358	0.006192	0.006689	0.007827	0.007908	0.010413	0.002566
4-2-2	0.001545	0.001537	0.001668	0.001779	0.001684	0.001772	0.000523
4-2-3	0.001948	0.001798	0.001991	0.002260	0.002043	0.002206	0.000678
4-2-4	0.002287	0.002014	0.002211	0.002638	0.002320	0.002517	0.000796
4-3-2	0.003508	0.003628	0.004124	0.004285	0.004502	0.005629	0.001122
4-3-3	0.004072	0.004138	0.004964	0.005205	0.005346	0.007361	0.001667
4-4-2	0.005640	0.005779	0.006420	0.006976	0.007502	0.010867	0.002158

tions. Not every level of robustness has corresponding configurations - the mean values for these levels are omitted from Figure 8.11 - although the sum of the number of evolutions corresponding to each level is equal to the total number of complete evolutions.

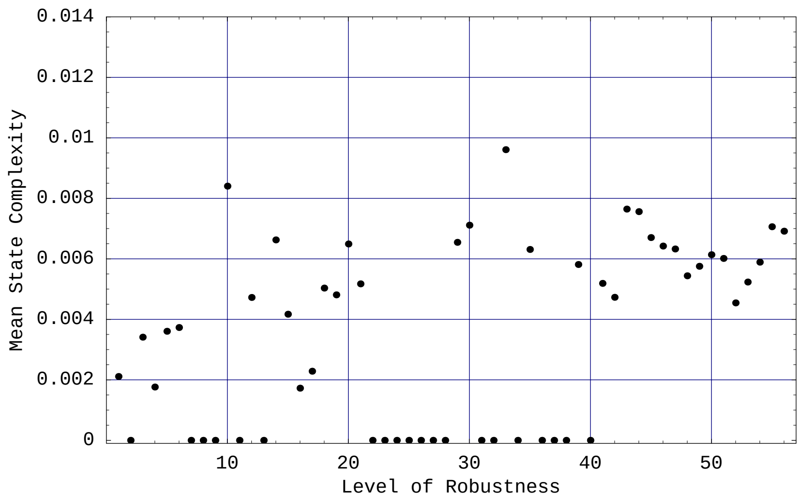
Figure 8.11(a) reveals no apparent relationship between the level of robustness and the mean state complexity. The maximum mean state complexity corresponds to configurations that complete delivery of only 33 queue distributions. However, at a mean state complexity value of 0.0096, the maximum mean state complexity is still significantly less than the maximum state complexity achieved by a 2-3-4 configuration of 0.02489. There is no apparent relationship between mean throughput and the level of robustness as well, as illustrated in Figure 8.11(b).

The distribution of mean state complexities and throughputs with respect to levels of robustness appears to contradict the conclusions reached from the observations of the mean state complexities and throughputs of the most robust configurations relative to the mean state complexities and throughputs of all complete evolutions - the more robust a configuration, the greater the mean complexity and performance. However, the mean state complexities and throughputs of the most robust configurations are consistently greater than the respective mean values for all complete evolutions. In the search for optimally adaptive configurations, the relationship between the most robust configurations, state complexity, and throughput is more relevant than the relationship between state complexity and throughput at lower levels of robustness, unless we are willing to accept less robust configurations.

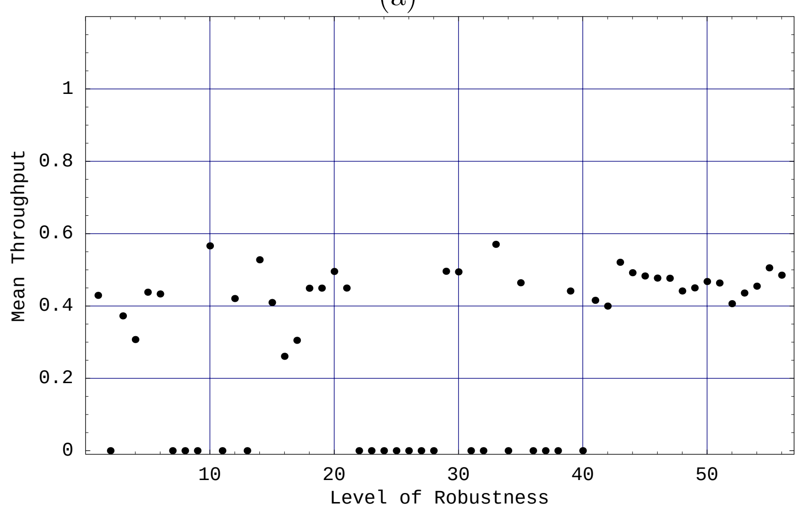
While Figures 8.11(a) and (b) do not indicate any individual correlations of state complexity or throughput with the level of robustness, they do further illustrate the correlation between state complexity and throughput in the context of the sets of evolutions associated with the various levels of robustness. When the mean state complexities at all levels of robustness are plotted against the corresponding mean throughputs, as in Figure 8.12(a) for complete 2-3-4 evolutions, a relationship is evident, although the points are out of order with respect to the levels of robustness. The resulting relationship is distinct from the relationship observed in the set of all complete evolutions in Figure 8.12(b) because the averaging of values for the different levels of robustness "dampens" the effects of outliers. Averaging results in a correlation value for the mean values at various levels of robustness of 0.857, which is significantly greater than the correlation value of 0.768 for all evolutions. Because it is comprised of averages, the distribution of mean values is within the bounds defined by the distribution of individual evolutions.

8.7 Absolute and Relative System Size

The distribution of average values represents a method for portraying frequency data in two dimensions, although the explicit bounds of the distribution are lost. Since the entire evolution set can be averaged in different ways, the method of averaging puts the distribution



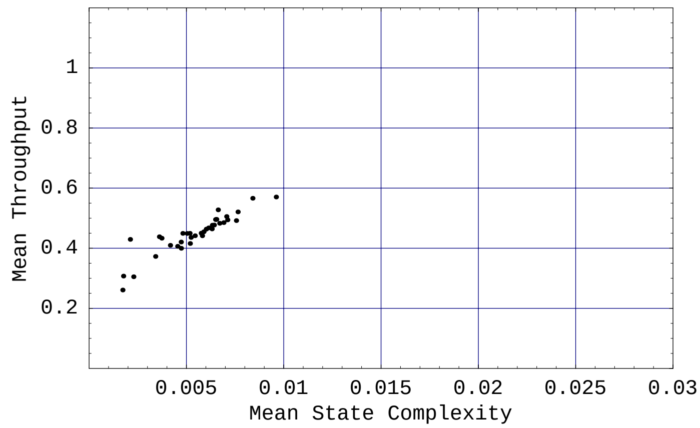
(a)



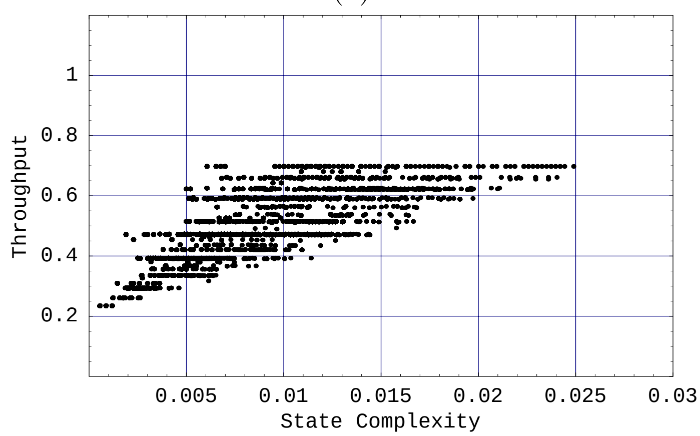
(b)

Figure 8.11: The (a) mean state complexities and (b) throughputs of evolutions with different levels of robustness for 2-3-4 complete evolutions. With four magazines, there are 56 levels of robustness - configurations with complete evolutions for all 56 queue distributions are the most robust and those with a single complete evolution are the least robust. There is no apparent relationship between the level of robustness and either the mean state complexity or the mean throughput, although the mean state complexities and throughputs for the most robust evolutions are consistently greater than the mean values for the entire set of complete evolutions.

in a specific context.



(a)



(b)

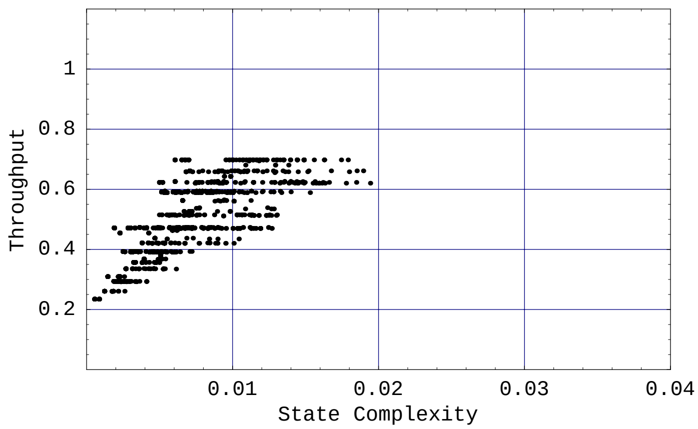
Figure 8.12: The distributions of (a) mean state complexities and corresponding mean throughputs at different levels of robustness and (b) all complete evolutions of 2-3-4 system size. The relationship of mean values does not correspond to the levels of robustness, but still indicate a relationship between state complexity and throughput. The mean values in (a) are within the space occupied by evolutions in (b) and provide a rough indication of the frequency of state complexity/throughput combinations.

In addition to showing that the mean state complexities of robust evolutions are greater than those for complete evolutions for all system sizes, indicating a correlation between adaptability, state complexity, and throughput, Table 8.2 also indicates that the mean state complexities of unique complete and unique robust sets are greater than their respective non-unique sets for all system sizes. As for logical complexity, this relationship appears to indicate a correlation between system size, in terms of the number of carriages, and the ability to support greater complexity because the mimics forming the complement of the non-unique and unique sets represent smaller system sizes. For logical complexity, more carriages mean, on average, more logical steps per system cycle because more phase lags are possible. Throughput increases with more carriages because additional carriages will, on average, result in shorter temporal evolution lengths for the same number of items. Further evidence of the relationship between logical complexity (and throughput) and system size is based on the comparison of unique evolution sets of different size systems. Increasing the number of queues or magazines has negligible impact on the mean state complexity, but increasing the system size through the number of carriages results in increases in logical complexity.

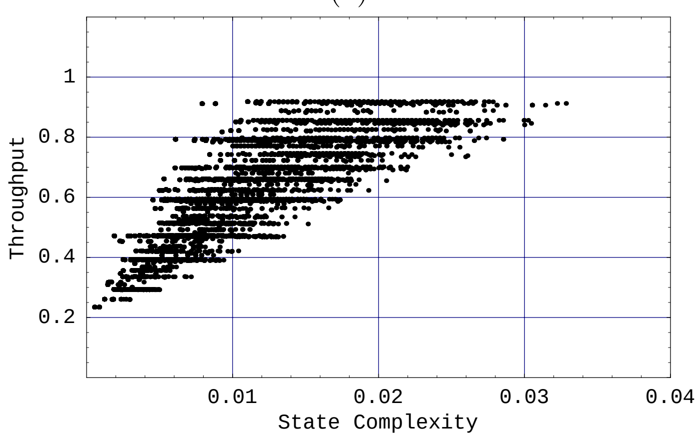
However, while the state complexities of unique evolution sets are consistently greater than the state complexities of corresponding non-unique sets for all system sizes, the absolute and relative numbers of queues and magazines in addition to the numbers of carriages can have a significant impact on the mean state complexity, and the relationship between the state complexity and system size based solely on the number of carriages is inconsistent.

As for logical complexity, the state complexity increases with additional carriages, but not for the same reasons. Increasing the number of carriages increases the potential number of system states because more phase lags and various numbers of operational carriages are possible. Also, the decrease in the temporal evolution length resulting from additional carriages (that increased the throughput) tends to also increase the state complexity. The relationship between the number of carriages and the state complexity is evident in the comparison of non-unique and unique evolution sets since mimics are defined with respect to the number of equivalent carriages. The relationship is also evident in a comparison of different size systems with queues and magazines held constant. Figure 8.13 illustrates how the effects of increasing the number of carriages while keeping the number of queues and magazines constant, using complete 2-3-3 and 2-4-3 evolution sets as examples. The state complexity scales have been adjusted from earlier distributions to account for all 2-4-3 evolutions. The evolution corresponding to the maximum state complexity at the maximum throughput for 2-3-3 size systems has 82 states and a temporal evolution length of 4557 steps. The 2-4-3 evolution corresponding to the maximum state complexity has 108 states and 3288 steps. Both the number of distinct states entered and the temporal evolution lengths change approximately proportionally with the number of carriages, although the number of distinct states entered is not necessarily always close to the proportional amount.

Table 8.2 indicates that, as the numbers of shafts and queues are held constant and the number of magazines is increased, the mean state complexity increases. The addition of magazines increases the system state space but does not significantly change temporal evolution length because the same number of items are transported, just to a greater variety of locations. Since the temporal evolution length is relatively unaffected, throughput is essentially independent of the number of magazines. The effect of increasing the number of magazines is essentially equivalent to the effects of variety of item types in queue distributions in the binary characterization of state complexity for 2-2-2 evolutions. Figure 8.14 shows the effects of changing the number of magazines, using the normal state complexity scale. With two queues, the complete evolutions for both sets contain 50 items. As described



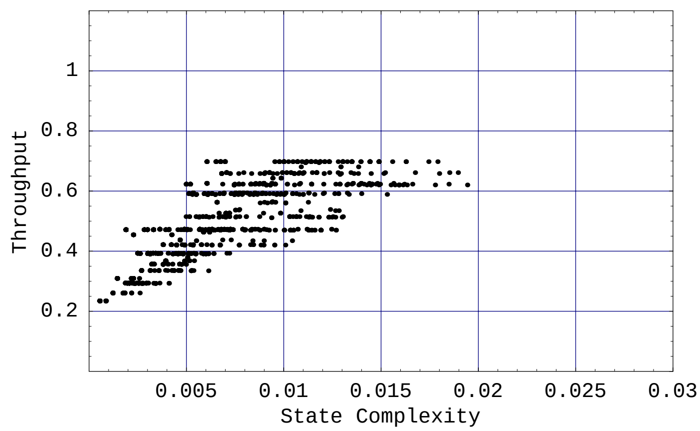
(a)



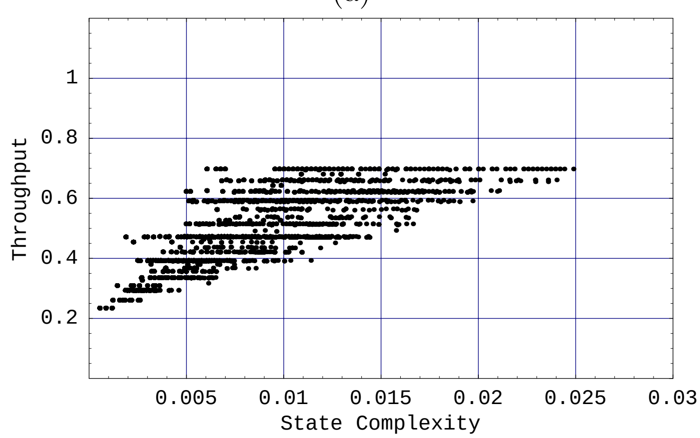
(b)

Figure 8.13: A comparison of the effects of additional carriages on state complexity. The maximum state complexity at the maximum throughput for the (a) 2-3-3 evolutions contains 82 distinct states and 4557 steps. The 2-4-3 evolution with the maximum state complexity from (b) enters 108 distinct states in 3288 steps. Additional carriages tend to increase the number of distinct states and decrease the temporal evolution length, resulting in greater state complexities.

previously, the evolution with the maximum state complexity at the maximum throughput from the 2-3-3 evolution set has 82 distinct states and a temporal evolution length of 4557 steps, resulting in a state complexity of 0.01799. However, the 2-3-4 evolution with the maximum state complexity enters 107 distinct states in 4299 steps, slightly less than the steps required for the 2-3-3 evolution. With the same number of items and similar temporal evolution length for both systems, the throughputs are approximately identical, differing by only 5.7 percent. The number of distinct states however, differ again by approximately the same proportion as the number of carriages.



(a)



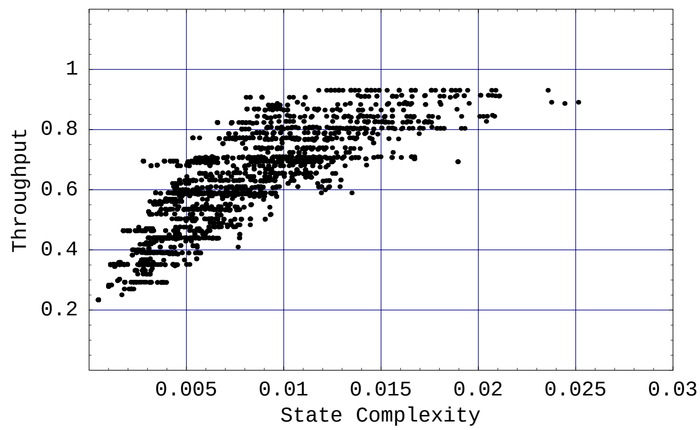
(b)

Figure 8.14: Increasing the number of magazines with the number of queues and shafts constant increases the state complexity and keeps the throughput approximately constant. The ratio of the state complexity of the evolution with the maximum state complexity at the maximum throughput for 2-3-3 evolutions in (a) to the state complexity of the evolution corresponding to the maximum state complexity for 2-3-4 systems in (b) is approximately equal to the ratio of the number of magazines.

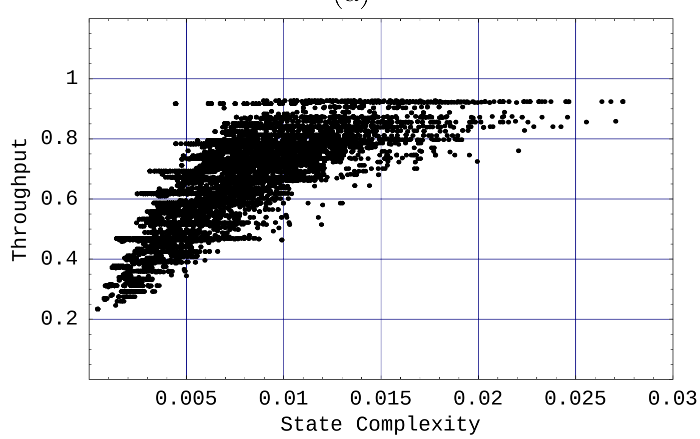
Increasing the number of queues also increases the number of possible system states and the potential for greater state complexity. However, since the number of items in each queue is identical at the start of an evolution, increasing the number of queues also increases the total number of items to be delivered. For the number of items assumed in each queue in an evolution, the additional temporal evolution length resulting from delivering additional items effectively balances the additional states created from individual carriage state combinations involving the additional queues, although increasing the number of queues has the most significant impact on state complexity when the number of carriages is low. Figure 8.15 shows the distributions of (a) complete 3-4-2 evolutions and (b) complete 4-4-2 evolutions. Despite the difference in the number of queues, the shapes and values of the distributions are quite similar. Since both distributions contain evolutions with high state complexity at less than maximal throughput, but both do not have evolutions with maximum state complexity and maximum throughput, the less than maximal throughput evolutions are used for comparison. At approximately the same relative location in the distribution, the 3-4-2 evolution enters 127 distinct states and has a temporal evolution length of 5052 steps and the 4-4-2 evolution enters 189 states within 6988 evolution steps. Increasing the number of items in an evolution guarantees that the temporal evolution length will increase approximately proportionally to the number of additional queues and the ratio of temporal evolution lengths of 0.72 is approximately equal to the ratio of the number of queues. However, increasing the number of queues only increases the *potential* for additional states - the fraction of potential states entered is dependent on the evolution dynamics and the simulation logic, such as the de facto priority rules, and typically only a small fraction of possible states are entered. Since the fraction of potential states entered is largely independent of the number of items assumed in queues, the proportional increase of the number of distinct states and temporal evolution length with an increase in the number of queues indicates the assumed number of items in queues is a balance point.

The relationship between the change in the state complexity from additional queues and the number of items in the queues implies that it is theoretically possible to achieve greater state complexity from the addition of queues. The increase in the number of new states must be greater relative to the increase in the temporal evolution length, which only occurs when the number of items per queue is sufficiently low.

The effect of additional queues on state complexity also suggests that comparisons between systems with different numbers of queues with respect to state complexity are invalid. Because of the normalization by the temporal evolution length, the number of items in the system must be identical to keep the temporal evolution length approximately constant and provide a common means of comparison. Keeping the number of items in all systems constant, but dividing them equally across the queues scales the state complexity to permit comparisons of different system sizes, and leaves other complexity measures, like logical complexity and the number of distinct states entered largely unaffected. To fully utilize the state complexity as an optimization tool, characterization of the space with respect to a known number of items is required. With different or unknown numbers of items, we are limited to internal comparisons of a given system size and comparisons of systems with an



(a)



(b)

Figure 8.15: For the assumed number of items per queue, changing the number of queues has little effect on the distribution of evolutions in state complexity space. The 3-4-2 evolutions in (a) have a very similar distribution to the 4-4-2 distributions in (b) (If we looked at 2-4-2 evolutions, the distribution of evolutions would also be similar). The evolutions all share similar mean state complexities.

equal number of queues.

For state complexity, increasing the number of queues has little effect (for systems with small numbers of carriages, there can be a slight decrease in complexity) while increasing the number of shafts or magazines increases state complexity. The amount of change of the state complexity is not only dependent on the relative values of queues, shafts, and magazines, but also the absolute values, since system logic is dependent on absolutes (like the maximum number of carriages that can reserve access to a queue or magazine or the number of carriages that can occupy a space or share resources). These absolute values found in the logic can affect the evolution dynamics, so that state complexities and throughputs are not necessarily scalable.

The increase in the number of carriages does not proportionally increase the state complexity, but not just because it affects both the number of states and the temporal evolution length. The number of combinations of states resulting from sequencing of state changes, demonstrated by Figure 8.2 for 2-2-2 evolutions with the maximum phase lag, is not necessarily proportional to the number of carriages. Since each carriage enters four distinct states in a typical pattern, the maximum number of combinations of states is four times the number of carriages. The number of states forming a sequence can be less however, if individual carriage states change simultaneously. For instance, the sequence of state changes corresponding to the maximum number of phase lags for an example three carriage system involves ten states, listed in Figure 8.16. Ten states, not twelve, comprise the sequence because at steps three and nine, two carriages change simultaneously. Since a smaller fraction of the state space is explored (for this number of phase lags involving a single queue and magazine), the state complexity is less than an equivalent system in which state changes of carriages occur at different times.

State	First Carriage	Second Carriage	Third Carriage
1	Loading in Q2	Travel to Q2	Travel to M1
2	Loading in Q2	Travel to Q2	Unloading in M1
3	Travel to M1	Loading in Q2	Unloading in M1
4	Travel to M1	Loading in Q2	Travel to Q2
5	Travel to M1	Travel to M1	Travel to Q2
6	Unloading in M1	Travel to M1	Travel to Q2
7	Unloading in M1	Travel to M1	Loading in Q2
8	Travel to Q2	Travel to M1	Loading in Q2
9	Travel to Q2	Unloading in M1	Travel to M1
10	Travel to Q2	Travel to Q2	Travel to M1

Figure 8.16: While twelve state changes are possible in a deterministic sequence involving three carriages, each with four distinct states, only ten occur in this sequence because two of the three carriages change states simultaneously in states three and nine. The sequence of states impacts the state complexity.

Assuming steady state behavior in a deterministic system, the effect of sequencing on state complexity can be taken to an (unrealistic) limit if the number of carriages is sufficiently large enough or the operational cycle times are sufficiently low enough. In these cases, the first carriage "catches up" to the last carriage, introducing delays that propagate through the carriages, and it is possible that no first order pattern exists (a repetitive pattern where the number of items delivered is equal to the number of carriages). Higher order patterns are possible, which repeat over a number of individual carriage cycles. The effect on state complexity is dependent on the relative effects of greater exploration of the state space and longer temporal evolution lengths resulting from additional delays.

While the number of carriages determines the number of states in a sequence, the actual number of states is dependent on the number of queues and magazines, so carriages, queues, and magazines collectively determine the potential state space of a configuration. In the analysis of 2-2-2 evolutions, we saw the number of state combinations is determined by summing the number of possible states at each step in the sequence, which is non-linearly dependent on the number of queues and magazines. Sequences involving more states that require a description of magazines or queues correspond to a greater number of state combinations.

The maximum number of states per sequence always corresponds to a sequence with the maximum number of phase lags. Because it yields the maximum number of distinct states, the maximum number of phase lags for the number of operational carriages possible are assumed in determining the maximum theoretical state complexities at all ranges of throughputs and therefore all equivalent numbers of operational carriages. However, the relative numbers of carriages, queues, and magazines, along with the system dynamics can result in any number of equivalent phase lags, regardless of the number of operational carriages. In the course of an evolution, it is therefore possible and quite common, for a variety of equivalent phase lags to occur, even though the number of operational carriages remains constant. For instance, in an evolution involving three carriages that remain operational for the entire evolution, all carriages can change states simultaneously resulting in no phase lags, any two carriages can change states simultaneously while a third has a single phase lag, or all three carriages can change states at different times for the maximum number of phase lags. The maximum theoretical state complexities in Figure 8.4 are therefore an underestimate. To obtain the maximum number of states, we must consider the combinations of states for various phase lags for all numbers of operational carriages.

The maximum theoretical state complexity boundary is not necessarily simple to calculate because the sequences corresponding to each phase lag must be determined from the evolution dynamics. However, since the possible sequences of carriage states are based on deterministic cycle times, the maximum deterministic state complexity is bounded by the maximum deterministic state complexity assuming the maximum number of phase lags possible for a given number of operational carriages (the boundary defined as in Figure 8.4) and the maximum state complexity corresponding to non-deterministic evolutions. For non-deterministic cycle times, the entire evolution can essentially be treated as a transient because no local or global patterns may emerge. The possible system states will therefore be all possible combinations of individual carriage states that are possible within the framework of the operational logic, including halting states which account for different numbers of operational carriages. For instance, if the logic specifies that there are only enough resources in a queue to load a single carriage at a time, then any system state describing simultaneous loading of two or more carriages in the same queue does not belong to the state space. Ignoring system states removed by logical restrictions, a rough estimate of the maximum theoretical number of states in a non-deterministic evolution is described by Equation 8.3, where the first term corresponds to the possible halting states of carriages, the second term describes what item types a carriage can load from what queue, the third term describes what magazines a loaded carriage can travel to, the fourth term defines what magazines in which unloading can occur, and the last term indicates the queues to which unloaded carriages can move.

$$((q+m)+(qm)+(m)+(m)+(q))^s = (qm+3m+2q)^s \tag{8.3}$$

The maximum theoretical number of states for non-deterministic cycle times is a gross overestimate of the actual number of states entered and, assuming temporal evolution length remains approximately constant, corresponds to a gross overestimate of the state complexity. In part, this difference illustrates the effects of determinism on system patterns and the resultant state complexity.

Regardless of whether all possible phase lags (with all numbers of operational carriages) or simply the maximum number of phase lags are considered in determining the maximum theoretical state complexity, the maximum theoretical state complexity at a sub-maximal throughput is always greater than the maximum theoretical state complexity at the maximum theoretical throughput because only the maximum number of operational carriages is possible at the maximum throughput, which limits the potential state space. The actual distributions of evolutions often match the theoretical distributions at these locations and the differences are evident in most of the distributions illustrated in Figures 8.13, 8.14, and 8.15. However, the ratio of the actual state complexities at the maximal and sub-maximal throughputs is typically greater than the ratio for theoretical values. Since the temporal evolution lengths are approximately equal at this point, the greater ratio for actual state complexities implies that evolutions at sub-maximal throughput enter a smaller fraction of the potential state space than evolutions with maximal throughput.

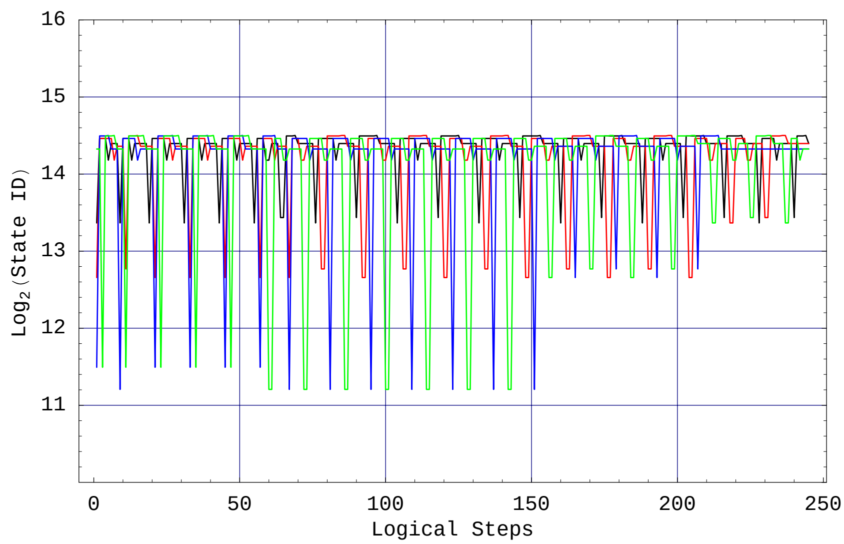
A comparison of actual evolutions corresponding to these points illustrates the effects of the number of operational carriages on the state complexity. Figure 8.17 shows the superposed compressed carriage histories for the evolutions corresponding to the maximum throughput (sub-maximal state complexity) and to the maximum state complexity (sub-maximal throughput) for 3-4-2 size systems. Both evolutions experience patterns involving all carriages with one, two, and three equivalent phase lags and enter a variety of state combinations. In fact, the evolutions are *identical* for the first 215 logical steps. However, Figure 8.18, an illustration of the temporal evolutions of individual carriage states for the evolution of configuration 194559, the evolution corresponding to maximum state complexity, reveals that carriages two and three halt just prior to the end of the evolution, and not simultaneously, whereas the evolution of 196607, the evolution corresponding to maximum throughput, involves all carriages that, at the end of the evolution, follow patterns and enter states previously entered. This enables 194559 to explore a greater number of distinct states, with only a minor effect on the temporal evolution length to result in a greater state complexity.

The discrepancy between state complexities at maximal and sub-maximal throughput is also evident at the minimum state complexity boundary for some distributions for the same reasons. However, the differences are less obvious at the minimum state complexity boundary because the minimum state complexity boundary does not actually reflect the minimum number of states, but the minimum *potential* number of states given various numbers of operational carriages. The actual number of states entered is typically less than the minimum potential, placing the actual boundary left of the 'minimum' boundary.

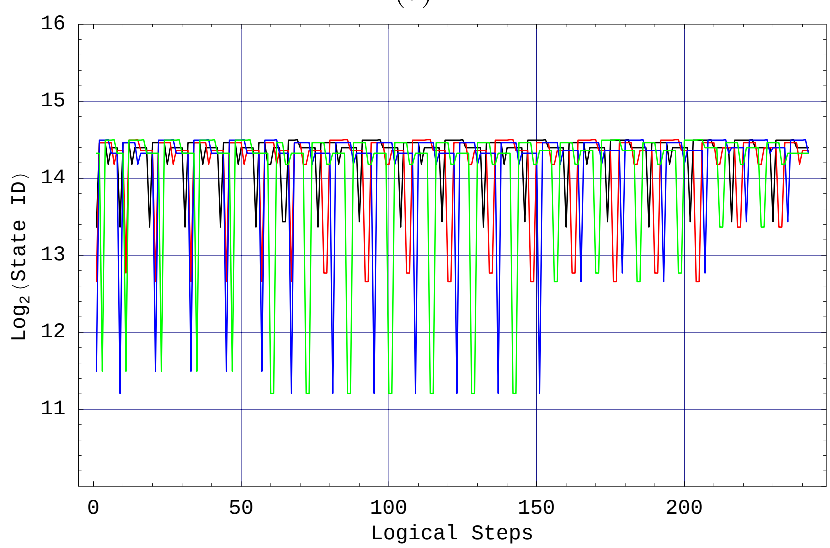
The association of the maximum state complexity with the less than maximum throughput in 3-4-2, and other system sizes, further demonstrates the similarity of state complexity to the strict interpretation of algorithmic complexity. Evolutions with optimal throughput should correspond to evolutions with no delays and with all carriages operational, which equates to simpler behaviors - at most there is diversity in the patterns and phase lags. As a result of their simplicity, these evolutions are highly compressible and therefore have low algorithmic complexity. Evolutions that experience halting of carriages can enter the same state space entered by simpler evolutions (given sufficient time) and can also enter a larger state space formed from various combinations of operational carriages and phase lags. If these evolutions explore these spaces, more states/patterns require definition for a complete description, resulting in less compression and greater algorithmic complexity. But halting carriages, while increasing complexity, also result in lower throughput.

Because algorithmic complexity is largely unaffected by the number of repetitions of a pattern, the point at which carriage halting occurs in an evolution does not affect the algorithmic complexity, but can affect the throughput. Evolutions with a wide range of performance can therefore have equivalent algorithmic complexity, making a strict interpretation of algorithmic complexity useless for establishing correlations between behavior and performance and therefore as part of an optimization tool. However, normalization of the 'algorithmic complexity component' - the number of distinct states - by the temporal evolution length is the state complexity's saving grace. This normalization separates algorithmically equivalent evolutions by the point in the evolution at which carriages halt (equivalent number of carriages halt) and introduces a correlation between algorithmic complexity and performance. At the maximum state complexity, halting occurs late enough in the evolution to allow the system to explore the potential state space, but not at too great an expense of throughput (as for 194559 in Figure 8.17(b)). At this point, state complexity matches the strict definition of algorithmic complexity, but as halting occurs earlier in the evolution, the definitions deviate.

An analysis of measuring complexity simply using the number of distinct states entered -



(a)



(b)

Figure 8.17: The compressed carriage state histories for (a) 194559 3-4-2 (60-40), the evolution corresponding to maximum state complexity, and for (b) 196607 3-4-2 (60-40), the evolution corresponding to maximum throughput. The evolutions are identical, except in the final stages, where the evolution of 194559 enters additional states corresponding to three, then two operational carriages.

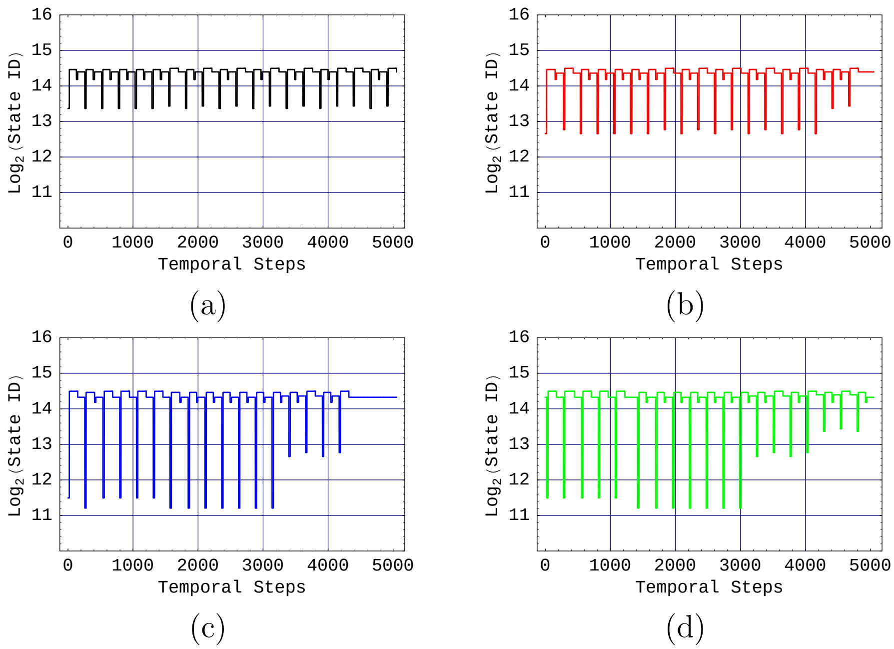


Figure 8.18: The individual temporal carriage state histories for evolution 194559 3-4-2 (60-40) ((a) is the first carriage, (b) is the second carriage, (c) is the third carriage, and (d) is the fourth carriage). Carriages two and three halt just prior to the end of the evolution, allowing the evolution to explore the spaces created by two, three, and four operational carriages. Since halting occurs late in the evolution, temporal evolution length is marginally greater than for 196607, so state complexity is greater.

the closest we come to in our definitions of complexity to algorithmic complexity without identifying patterns of various orders - illustrates the effect of normalization in creating a useful complexity measure using the concept of algorithmic complexity. However, prior to looking at the number of distinct states as a quantitative measure, we first look at the compressed state complexity, which is similar to the state complexity, except it involves normalization with respect to a different evolution attribute.

Chapter 9

Compressed State Complexity

By definition, the compressed state complexity and the state complexity are related by the logical complexity - specifically, the compressed state complexity is the ratio of the state complexity to the logical complexity, making the compressed state complexity the ratio of the number of unique states entered to the number of logical steps in an evolution. The direct relationship between the compressed state complexity and the state complexity is evident in Figure 9.1, the mapping of the state complexities of evolutions in the compressed state complexity/throughput space. The mapping uses 3-3-3 evolutions rather than 2-2-2 evolutions simply because relationships are clearer with more evolutions in the set. Additionally, only the set of complete evolutions are used for the same reasons described for state complexity. The mapping illustrates the similarity between the compressed state complexity and state complexity distributions and, since the measures are both normalizations of the number of unique states occurring during an evolution, the differences between the distributions are a function of the ratio of logical to temporal evolution lengths - the logical complexity.

9.1 Theoretical Boundaries

By the definition of compressed state complexity, we should expect an inverse relationship between compressed state complexity and logical complexity. However, the mapping of logical complexities into compressed state complexity space, presented in Figure 9.2, does not show a definitive inverse relationship. Logical complexities are distributed throughput the compressed state complexity space along any line of constant throughput.

The near direct relationship between state complexity and compressed state complexity and the wide range of logical complexities possible for a wide range of compressed state complexities initially appears to create a contradiction. Since the state complexity and the compressed state complexity are related by the logical complexity and the state complexity maps nearly directly into the compressed state complexity space, how can the logical complexity not map

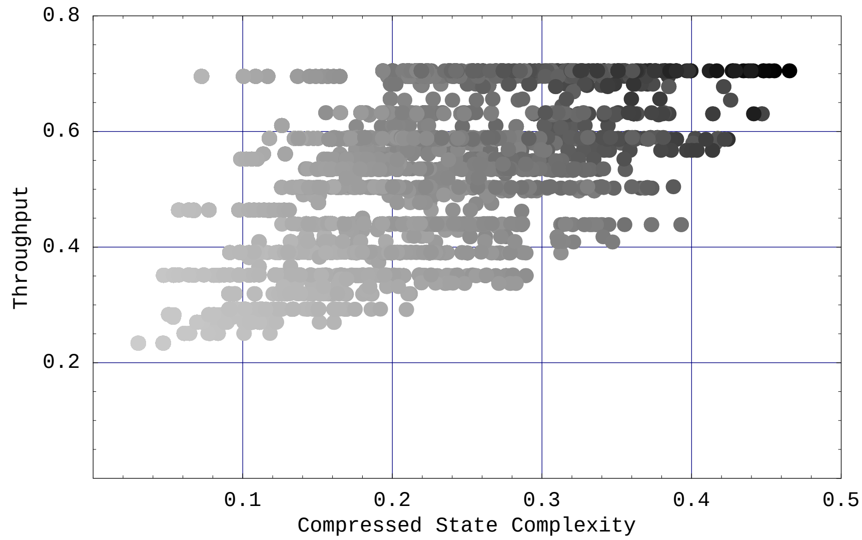


Figure 9.1: The mapping of state complexity values in compressed state complexity/throughput space for 3-3-3 evolutions. Darker colors indicate evolutions with greater state complexity. The mapping indicates that state complexity and compressed state complexity share similar distributions and, since the two measures are based on the number of unique states, any differences are attributable to the logical complexity.

equally well? We will see later that the explanation is related to the relative sensitivities of the compressed state complexity to the state and logical complexities.

Theoretically, the minimum compressed state complexity corresponds to an evolution with the minimum state complexity and the maximum logical complexity at a given throughput. Given what we know about the characterizations of the state and logical complexity spaces, the minimum compressed state complexity should therefore correspond to an evolution with low exploration of the state space (low queue distribution variety and the simplest repetitive patterns involving all carriages) and the most phase lags possible. The theoretical maximum corresponds to an evolution that explores a great deal of its state space (and therefore has variety in the queue distributions and use of de facto priority logic) but has a minimum number of phase lags, meaning the carriages are essentially independent. While the conditions for the theoretical minimum are possible (e.g. a completely connected configuration with a single item type like 255 2-2-2 (0-100)), the conditions for the maximum theoretical compressed state complexity are unlikely for the model used. In the discussion of logical complexity, we saw that carriage independence throughout the entire evolution only occurs in incomplete evolutions.

To obtain the theoretical minimum and maximum, we have assumed that it is possible for the evolutions with a given state complexity to correspond to any logical complexity possible for a given throughput. However, because of system dynamics, this assumption is invalid and practical limitations on the range of one complexity measure exist from the value of another

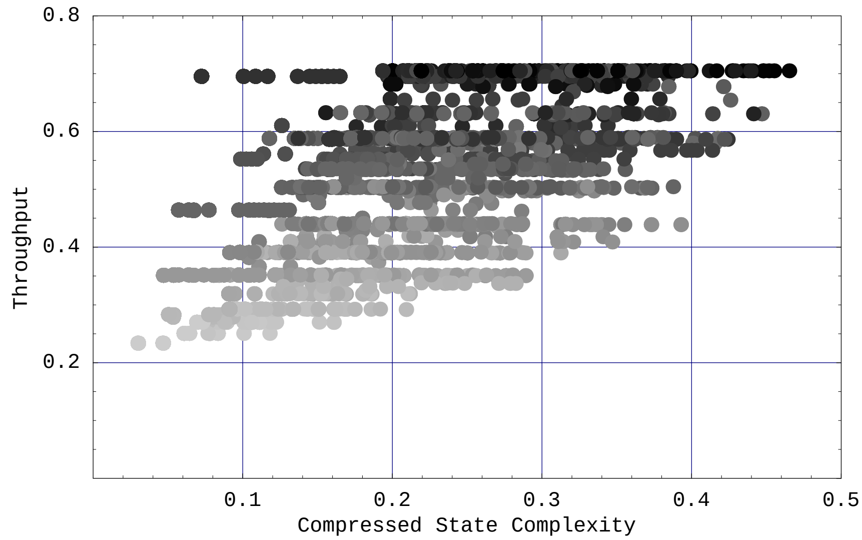


Figure 9.2: The mapping of logical complexity values in compressed state complexity/throughput space for 3-3-3 evolutions. Darker colors indicate evolutions with greater logical complexity. The logical complexity should be inversely related to the compressed state complexity and colors should get lighter as compressed state complexity increases along a line of constant throughput. However, logical complexity values are seemingly distributed across a range of compressed state complexity values.

complexity measure and is evident in Figure 9.2. Figure 9.3 provides a greater detail of the mapping at the maximum throughput (the color scale is reset so the lightest color corresponds to the minimum logical complexity of the evolutions with maximum throughput), and shows that minimum and maximum values can not be found throughout the entire distribution.

Figure 9.4 presents possibly the clearest illustration of the practical bounds on the relationship between variables. Figure 9.4 is a direct comparison of the compressed state complexity and state complexity and supports the correlation evident in the mapping of state complexities into compressed state complexity space in Figure 9.1. To completely illustrate the relationship between compressed state complexity and state complexity, the three-dimensional frequency distribution of Figure 9.4 is required. However, the correlation of 0.939 supports the general trend in the two-dimensional representation of the relationship in Figure 9.4. The most interesting feature of Figure 9.4 is not the general relationship between the complexity measures, but the lines of various slopes, that correspond to evolutions with constant logical complexity. Since each line extends over a limited range of the compressed state complexity/state complexity space, we see directly that there are effective limits on the values of logical complexity with respect to both the state and compressed state complexities. We would also see these limits in direct comparisons of the compressed state complexity and state complexity with logical complexity.

A more subtle interpretation of the lines of constant logical complexity illustrates the effective

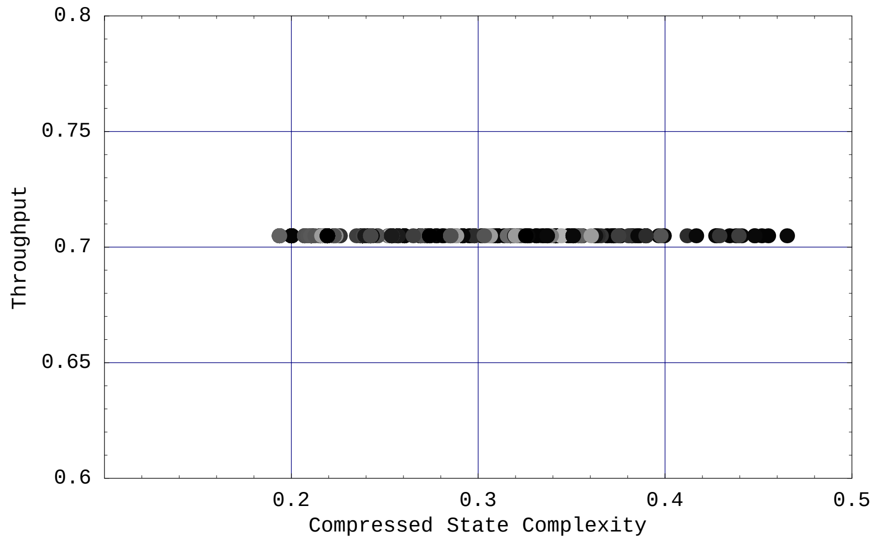


Figure 9.3: The mapping of logical complexity values into the compressed state complexity space for evolutions with the maximum throughput. The detail shows no apparent pattern in the distribution, although logical complexity values can not be found throughout the range of compressed state complexities.

bounds on the number of distinct states entered in an evolution to the state, compressed state, and logical complexities. Since the state and compressed state complexities are both based on the number of distinct states entered in the course of an evolution, the number of states must increase along a line of constant logical complexity as the state and compressed state complexities are simultaneously increased. Knowing the absolute values of the state and compressed state complexities for a given logical complexity, we can determine the actual number of distinct states without necessarily knowing the absolute number of logical or temporal steps since the logical complexity is the ratio of the state complexity to the compressed state complexity. Qualitatively, the number of states increases along the line of constant logical complexity as the state and compressed state complexities increase. The fact that lines of constant logical complexity do not extend throughout the entire space and have upper and lower limits of state and compressed state complexity implies that the number of distinct states entered is effectively bounded by the state and compressed state complexities as well as the logical complexity. These effective boundaries illustrate that all combinations of complexity values are not necessarily possible and specifically, the maximum theoretical values of the compressed state complexity, relating the maximum state complexity with the minimum logical complexity, are unachievable.

The mixture of logical and state complexity combinations shows that we do not know the actual position of the combination of state and logical complexity combinations resulting in the maximum compressed state complexity. But the distribution of evolutions in the compressed state complexity/throughput space demonstrates that the combinations are ap-

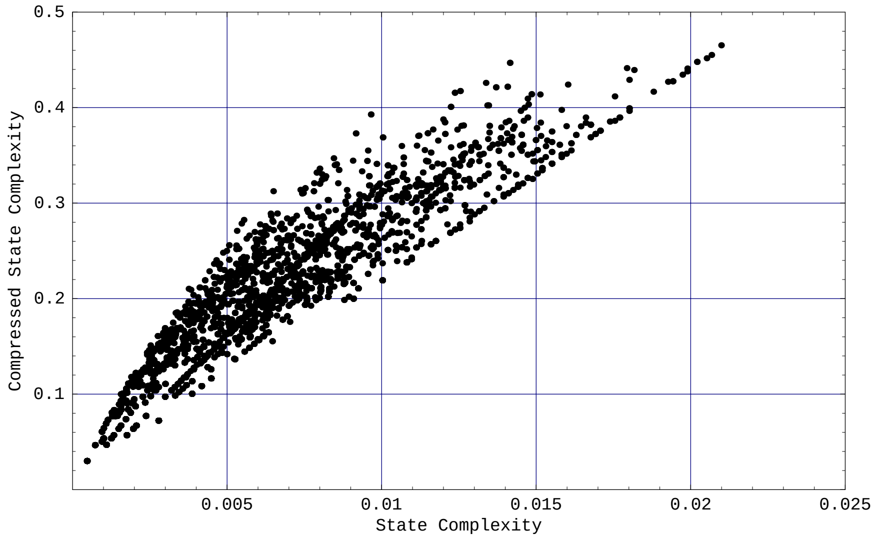


Figure 9.4: A comparison of the compressed state complexity to the state complexity for the set of complete 3-3-3 evolutionsshows not only a general direct relationship between the measures, but also lines of constant slope that correspond to lines of constant logical complexity. The endpoints of the lines illustrate effective limitations of values of logical complexity with respect to state and compressed state complexities. The lines can also be interpreted to indicate an effective limit on the number of states possible for a given logical, state, and compressed state complexity.

proximately linear, or at least what the minimum and maximum boundaries of the state and logical complexities are, assuming the minimum and maximum theoretical boundaries of the state and logical complexities are approximately linear with respect to throughput.

The ratio of two linear equations with a common dependent variable always results in a non-linear relationship with respect to the dependent variable (with one exception). The analogy to the elevator system is the relationship of the compressed state complexity to the throughput, which is the ratio of the state complexity and the logical complexity relationships to throughput. The ratio of two linear equations at a common dependent value, y, is given by Equation 9.1, where r equals the ratio of the independent variables in each of the linear equations.

$$r = \frac{x_1}{x_2} = \frac{m_2}{m_1}\left(\frac{y - b_1}{y - b_2}\right) \tag{9.1}$$

Rearranging Equation 9.1 in terms of r to express y as the independent variable results in Equation 9.2. The first and second derivatives of the relationship between the dependent variable, y, and the ratio of independent variables, r, are presented in Equations 9.3 and 9.4.

$$y = \frac{rb_2 - \frac{m_2}{m_1} b_1}{r - \frac{m_2}{m_1}} \tag{9.2}$$

$$\frac{dy}{dr} = \frac{(b_1 - b_2)\frac{m_2}{m_1}}{\left(r - \frac{m_2}{m_1}\right)^2} \tag{9.3}$$

$$\frac{d^2y}{dr^2} = \frac{(b_2 - b_1)\frac{m_2}{m_1}}{\left(r - \frac{m_2}{m_1}\right)^3} \tag{9.4}$$

All of the equations reveal a discontinuity at $r = \frac{m_2}{m_1}$ and that, for large ratios (positive and negative), the value of y asymptotically approaches b_2. The direction of the approach is based on the relative values of the intercepts, b_1 and b_2, which control the slope and curvature of the relationships on either side of the discontinuity. When $b_1 > b_2$, the slope is always positive, regardless of the value of r. Conversely, when $b_1 < b_2$, the slope is always negative. For a curve that always has a positive slope, asymptotically approaches a constant value at $\pm\infty$, and has a discontinuity at a finite value, $\frac{m_2}{m_1}$, the curvature must be positive for values of r less than $\frac{m_2}{m_1}$ and negative for values of r greater than $\frac{m_2}{m_1}$. The reverse is true for a curve with a negative slope for all values of r, and is validated by looking at the second derivative.

In the context of compressed state complexity, where compressed state complexity $= r$ and throughput $= y$, the subscript 1 denotes state complexity values and 2 denotes logical complexity values, we are only interested in the region where throughput is less than $+\infty$ and greater than the maximum of b_1 and b_2 (corresponding to positive state and logical complexities). In this region, the curve always has positive curvature, since the limits of Equation 9.2 are a finite value, b_2. However, the slope is dependent on the relative values of the intercepts of the linear equations. For $b_1 > b_2$, the slope is positive and the compressed state complexity is less than $\frac{m_2}{m_1}$. If $b_2 > b_1$, the slope is negative and compressed state complexity is greater than $\frac{m_2}{m_1}$. The one exception that yields a linear relationship from the ratio of two linear equations, referred to earlier, occurs when $b_1 = b_2$, which results in a constant throughput equal to the intercept.

Since the boundaries of the compressed state complexity asymptotically approach a limiting value, we can get an idea of the ratio of the slopes of the actual distribution of corresponding evolutions in the state and logical spaces and we can bound one variable based on another. That is, what linear relationship between logical complexity and throughput results in a non-linear compressed state complexity distribution, considering all evolutions lying on a certain linear state complexity-throughput distribution. While the limiting values of the compressed state complexity do not indicate the absolute values of the slopes of corresponding state and logical complexity relationships to throughput, we can obtain the theoretical boundaries by comparing the actual minimum and maximum boundaries of state and logical complexity.

The logical and state complexity distributions with respect to throughput for the set of complete evolutions are presented along with a line defining the assumed minimum and maximum complexity boundaries in Figure 9.5. The minimum logical complexity boundary is defined as the theoretical boundary encompassing (nearly) all points and does not correspond to a fixed number of equivalent phase lags. It therefore includes a few points that correspond to evolutions in which carriages act independently for some fraction of the evolution, resulting in no phase lags, but all but one carriage halts prior to the end of the evolution. If the line were redrawn to reflect a line of constant equivalent phase lags, the effect on the maximum compressed state complexity/throughput boundary would occur primarily at lower throughputs.

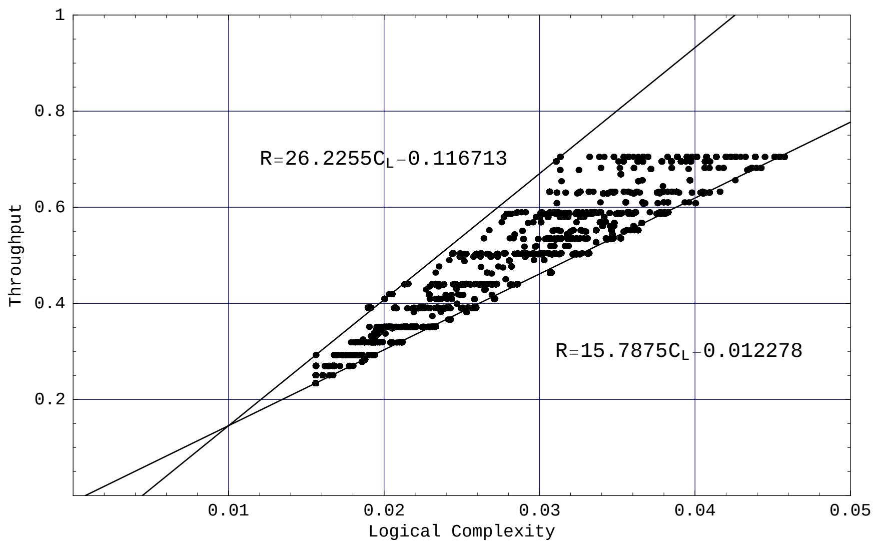
For evolutions of 3-3-3 size systems, the maximum state complexity at the maximum throughput lies along the same assumed line that includes maximum state complexities at lower throughputs, which is described by Equation 9.8. However, we have seen that the maximum state complexity can occur at less than maximum throughputs. Additionally, the maximum state complexity boundary is not necessarily linear. Therefore, any conclusions based on results using the maximum state complexity must be made in the context of the *assumed* linearization of the boundary. Like the minimum logical complexity boundary, the assumed maximum state complexity boundary encompasses all evolutions rather than connecting the maximum state complexities at the minimum and maximum throughputs and affects the maximum compressed state complexity, primarily at lower throughputs.

The equations of the linear boundaries assumed in Figure 9.5 are given in Equations 9.5 to 9.8. The equations show that the intercepts of logical complexity equations are always less than those for the state complexity equations ($b_1 > b_2$), so the slopes of the equations defining the ratios are always positive and therefore have positive curvature in the region of interest.

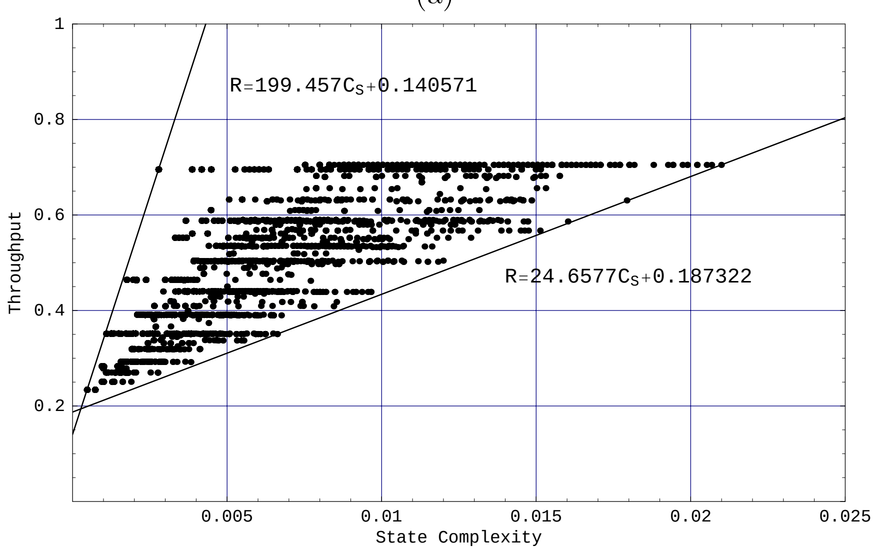
$$\begin{aligned}
Minimum\ Logical\ Complexity: &\quad R = 26.2255C_L - 0.116713 && (9.5)\\
Maximum\ Logical\ Complexity: &\quad R = 15.7875C_L - 0.012278 && (9.6)\\
Minimum\ State\ Complexity: &\quad R = 199.457C_S + 0.140571 && (9.7)\\
Maximum\ State\ Complexity: &\quad R = 24.6577C_S + 0.187322 && (9.8)
\end{aligned}$$

The equations for the minimum and maximum theoretical boundaries of compressed state complexity, based on the assumed actual boundaries of the logical and state complexities are described in Equations 9.9 and 9.10. The minimum boundary corresponds to a comparison of the maximum state complexity and the minimum logical complexity boundaries and the minimum boundary is created from the ratio of the minimum state and maximum logical complexity boundaries.

$$Min\ Compressed\ State\ Complexity: \quad R = \frac{-0.012278C_C - 0.0111265}{C_C - 0.0791524} \tag{9.9}$$



(a)



(b)

Figure 9.5: The two-dimensional distributions of 3-3-3 evolutions in (a) logical complexity/throughput space and (b) state complexity/throughput space. The distributions include the assumed linearizations of the minimum and maximum complexity boundaries.

$$Max\ Compressed\ State\ Complexity: \quad R = \frac{-0.116713C_C - 0.199233}{C_C - 1.06358} \tag{9.10}$$

These lines, along with the distribution of 3-3-3 evolutions in compressed state complexity/throughput space, are plotted in Figure 9.6 (the dashed line indicates the line corresponding to the ratio of the maximum state complexity and the minimum logical complexity boundary defined by a constant equivalent phase lag). Equations 9.9 and 9.10 indicate that the limiting values of the compressed state complexity for 3-3-3 evolutions are 0.079 for the minimum boundary and 1.064 for the maximum boundary.

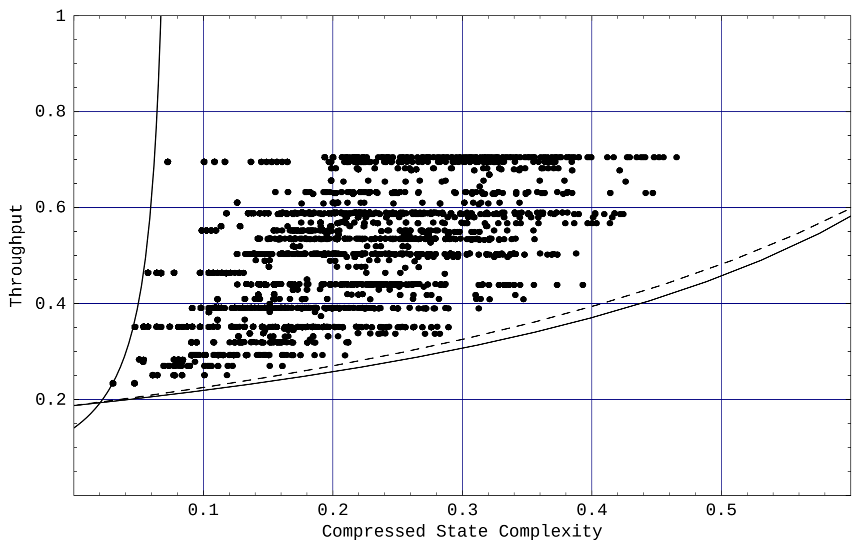


Figure 9.6: The distribution of 3-3-3 evolutions with respect to compressed state complexity and throughput along with the extreme boundaries defined by assumed linearizations of the minimum and maximum boundaries of the state and logical complexities. Evolutions exist near the minimum boundary, indicating that evolutions with minimum state complexity can also have maximum logical complexity. The large difference between the actual distribution and the maximum boundary indicate that evolutions with the maximum state complexity do not have the minimum logical complexity.

While the throughputs are too low to correspond to the limiting compressed state complexities, the minimum compressed state complexity boundary is close to the actual distribution, indicating that evolutions exist with minimum state complexity and maximum logical complexity. The actual distributions never lie on the maximum compressed state complexity boundary, but are closer at lower throughputs. However, we assume linear boundaries for state complexity distributions that are constructed to encompass all evolutions. The choice of the assumed linear relationship affects how actual distributions relate to the theoretical compressed state complexity boundaries. Ideally, the actual, non-linear relationships should be used to define all boundaries to indicate how state complexities and logical complexities correspond to each other.

Although the linearized representation of boundaries is an estimation, one of the points used for all boundaries corresponds to an evolution at the maximum throughput, meaning the assumed boundaries pass through their respective maximum throughput points and the minimum and maximum relationships are exact at the maximum throughput, regardless of the slope and intercept of the linearizations. The large difference between the actual maximum compressed state complexity and the maximum boundary at the maximum throughput, which can be extrapolated to lower throughputs, indicates that evolutions with maximum state complexity never have minimal logical complexity, at least at higher throughputs.

To identify what logical complexities are possible when state complexity is maximal, and what state complexities are possible when logical complexity is maximal, we compare the maximum state complexity and maximum logical complexity and compare the minimum state complexity and the minimum logical complexity. The curves describing these ratios are presented in Figure 9.7 along with the minimum and maximum compressed state complexity boundaries.

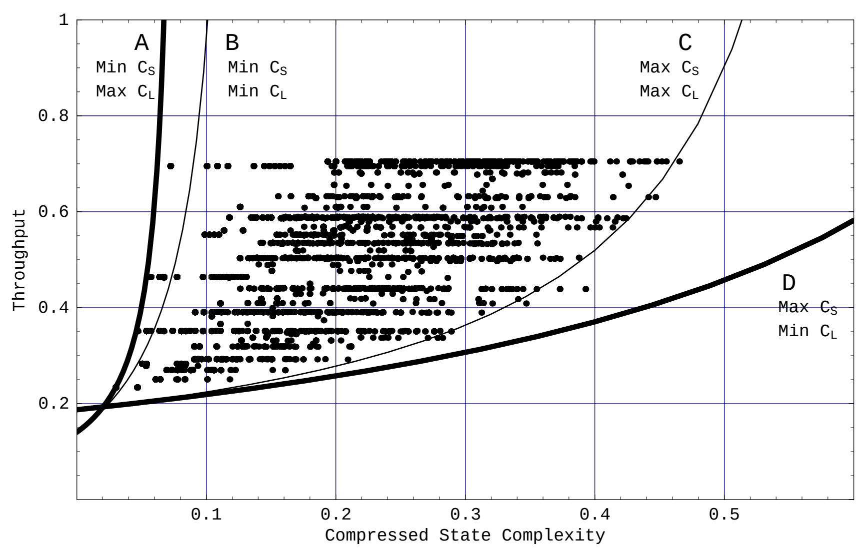


Figure 9.7: The distribution of 3-3-3 evolutions with the (A) minimum and (D) maximum compressed state complexity boundaries along with the curves describing (B) the ratio of minimum state complexities to minimum logical complexities and (C) the ratio of maximum state complexities to minimum logical complexities. These curves collectively indicate some limits on corresponding values of state complexity and logical complexity.

When an evolution has maximum state complexity, the compressed state complexity can only be in the region between curves C and D. For compressed state complexities less than those defined by curve C, the state complexity must therefore be less than maximal. It is certainly possible that evolutions with less than the maximum state complexity exist in this region if the logical complexity is low enough, except on the maximum compressed state complexity curve D. However, the absence of evolutions in this region (with the exception of some points at moderate throughputs) indicates that the points along curve C must correspond to evolutions with maximum state complexity and that these evolutions must have maximum logical complexity. Therefore, for the set of complete evolutions, the majority (if not all) of evolutions with maximum state complexity also have maximum logical complexity.

At lower compressed state complexities, Figure 9.7 shows points lying along curve A. Compressed state complexities along this curve are only possible for an evolution with minimum state complexity and maximum logical complexity. Between curves A and B however, we can not definitively say anything regarding the absolute values of state and logical complexities, only the relative values, because an evolution with state complexity greater than the minimum value can have a compressed state complexity in this region with the appropriate logical complexity. *If* the state complexity is minimal however, then the compressed state complexity will lie in this region regardless of the logical complexity. For example, the point between curves A and B with the maximum throughput corresponds to 206 evolutions with the minimum state complexity of 0.00278. The logical complexity is 0.0385, 54 percent of the difference between the minimum and maximum logical complexities at the corresponding throughput of 0.695. Similarly, *if* we know that the logical complexity is maximal, then the compressed state complexity will lie somewhere between curves A and C and *if* we know that the logical complexity is minimal, then the compressed state complexity will lie between curves B and D.

Just as in the region between A and B, the absolute values of the state and logical complexities are ambiguous in all regions. However, the curves do indicate limits on minimum and maximum values of state and logical complexities. We have already seen that the state complexity must be less than the maximum value at compressed state complexities less than those defined by curve C at a given throughput. Similarly, for compressed state complexities greater than curve B, the state complexity must be greater then the minimum because there is no logical complexity low enough to result in a higher compressed state complexity with the minimum state complexity. Limits associated with these curves are also related to the logical complexity. At compressed state complexities less than those defined by curve B, the logical complexity is always greater than the minimum value, and the logical complexity must be less than the maximum for compressed state complexities less than those of curve C.

Figure 9.7 shows that the distances between the minimum and maximum logical complexities for a constant state complexity are significantly smaller than the distances between the minimum and maximum state complexities for a constant logical complexity. That is, the differences in the compressed state complexities from A to B and from C to D are smaller than from B to D and from A to C. This explains why the state complexity maps nearly directly into the compressed state complexity/throughput space in Figure 9.1 but the logical complexity maps throughout the space in Figure 9.2. This difference implies that the compressed state complexity can correspond to a limited range of state complexities and compressed state complexity is insensitive to changes in logical complexity, which can correspond to a wide range of compressed state complexities.

9.2 Algorithmic Complexity

Since the compressed state complexity is always bounded by the ratio of the slope of the logical complexity to the state complexity as the throughput increases, assuming the slope of the boundary defining the logical complexity is never ∞ and the boundary defining the state complexity is never 0, the compressed state complexity is distinctly different than the state and logical complexities, which can grow without bound in complexity with, for example, the addition of more shafts/carriages. Given this fundamental difference between bounded and unbounded complexity measures, the existence of bounds needs to be investigated in the context of algorithmic complexity, the basis for all of our measures. For a given size system, a bound on the algorithmic complexity certainly exists and is equal to the maximum randomness, or lack of any patterns. If we define the lack of any patterns as the maximum distinct states that can logically be entered, then for even modest size systems, the algorithmic complexity may be extremely large, but still finite and tractable. But changes in the system size changes the limits on the bounds. Additional patterns (to the point of no identifiable global patterns) equate to more information required for a complete description so, increases in the number of queues, shafts, or magazines, which create more states/patterns therefore results in greater algorithmic complexity.

The compressed state complexity however, is largely unaffected by increases in system size, as illustrated by Figure 9.8. Only changes in the number of magazines significantly affects the compressed state complexity boundary because additional magazines result in more states entered but do not change the range of logical steps per cycle because the number of carriages and items in the systems are unaffected.

So, while the algorithmic complexity has a finite limit for a given system size, the bound is significantly greater than that for the compressed state complexity. Additionally, the bounds on algorithmic complexity increase with increases in system size while the compressed state complexity bounds remain relatively unchanged. The compressed state complexity therefore does not match the definition of algorithmic complexity with respect to boundaries.

The algorithmic complexity and the compressed state complexity also differ with respect to the simplest evolution. For algorithmic complexity, the evolution with the greatest throughput is also the simplest (involving all carriages throughout the evolution). For the greatest compression, the evolution has the fewest patterns *and* the number of states defining each pattern is minimal. This type of evolution corresponds to the minimum state complexity (minimum number of states entered) and the minimum logical complexity (fewest phase lags and therefore fewest unique states per pattern). However, the minimum compressed state complexity at the maximum throughput occurs for an evolution with the fewest patterns, but the greatest number of states defining each pattern (number of phase lags).

The algorithmic complexity and the compressed state complexity do match at the greatest complexity/performance combination. The most algorithmically complex point (non-chaotic, involving patterns) has a number of patterns and a number of states defining each pattern.

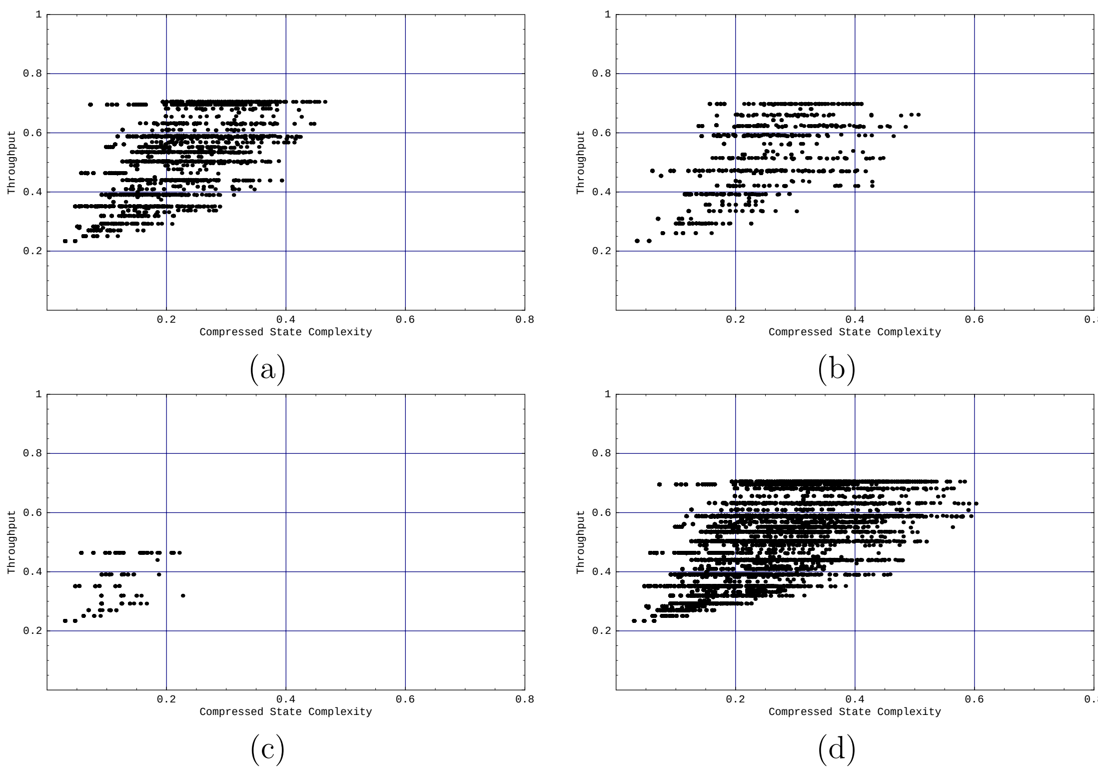


Figure 9.8: The two-dimensional distributions with respect to compressed state complexity for complete (a) 3-3-3 evolutions, (b) 2-3-3 evolutions, (c) 3-2-3 evolutions, and (d) 3-3-4 evolutions. The compressed state complexity boundaries are largely unaffected by changes in system size, except for changes in the number of magazines. Additional magazines result in more possible states, but similar (logical) evolution lengths because the number of items remains constant as does the number of carriages to transport them.

In the definition of boundaries in Figure 9.7, we saw that evolutions with the maximum number of patterns only have maximum or near maximum states per pattern (phase lags) and the maximum theoretical compressed state complexity, corresponding to the maximum state complexity and minimum logical complexity, is not achieved. Additionally, this maximum point can correspond to a less than maximum throughput, since halting of carriages increases the potential state space and increases the algorithmic complexity through additional required information and compressed state complexity through the number of states entered.

At lower throughputs, the compressed state complexity also matches the algorithmic complexity at both the minimum and maximum complexity boundaries. The information required to describe the number of repetitions of each pattern in an evolution is small compared to the information required to define the states in the patterns as well as their sequencing. The time at which carriages halt (if carriages halt) in an evolution therefore has little impact on the algorithmic complexity, although, if carriages halt earlier, there is less potential to explore the larger possible state space resulting from greater numbers of carriages operating simultaneously. The minimum and maximum algorithmic complexity boundaries therefore remain relatively constant over a range of throughputs, but might have a tendency to decrease as carriages halt earlier in the evolution, which leads to lower throughput. At the lowest throughput, only a single carriage is involved, which always has the fewest patterns composed of the fewest states (for the given number of queues and magazines) and the algorithmic complexity is always less than the algorithmic complexity of complete evolutions where all carriages are involved. This description matches the distribution of evolutions with respect to compressed state complexity very closely, although it is unclear if the slopes of the boundaries conform with the definition of algorithmic complexity.

9.3 Compressed State Complexity, Throughput, and Robustness

Although the compressed state complexity has its roots in algorithmic complexity, we must recall that algorithmic complexity may be an accurate measure of complexity, but not necessarily one that shares a relationship with performance and therefore useful in terms of use as an optimization tool. The salient differences between algorithmic complexity and compressed state complexity may result in a useful relationship between complexity and performance with respect to optimization, which is identifiable partly through the comparison of mean values of compressed state complexity and throughput for different system sizes.

Table 9.1 presents the mean compressed state complexity values for system sizes with relative numbers of queues, shafts, and magazines and/or consisting of sufficient numbers of evolutions to make them of interest. The table also includes the mean values for evolutions that mimic smaller systems with respect to the number of shafts. As with state complexity, it is misleading to include incomplete evolutions because of the effects of evolution length - in this case, logical evolution length - on compressed state complexity. However, they are included to indicate how incomplete evolutions can raise the mean complexity. An incomplete evolution ends sooner than an "equivalent" complete evolution, but may or may not enter the same number, or more, states. Assuming the number of unique states remains constant, the shorter logical evolution length results in a greater compressed state complexity. If enough incomplete evolutions terminate early enough, compared to the complete evolutions in the remainder of the same set, then the mean compressed state complexity for a non-halting evolution set can be greater than the mean compressed state complexity for the set of corresponding complete evolutions, as is the case for all of the system sizes listed in Table 9.1[1].

Since incomplete evolutions misrepresent the mean compressed state complexity, we only consider complete evolution sets and evolutions of the most robust configurations. Table 9.1 indicates that the mean compressed state complexity of all evolutions corresponding to the most robust configurations is greater than the mean for the set of all complete evolutions for all system sizes involving more than one queue. This result is expressed graphically in Figure 9.9, which compares the absolute mean values for the complete and robust evolution sets. The figure also offers an initial view of the effects of changes in the relative number of queues, shafts, and magazines on the compressed state complexity. As the number of queues increases, the mean compressed state complexity falls slightly. Increases in both the number of shafts and magazines results in increases in the mean compressed state complexity, although shafts have a more significant impact.

While the greater mean compressed state complexities of the most robust evolutions appear to indicate a correlation between complexity and adaptability on face value, especially because of the consistency in the trend across the complete range of system sizes considered, the mean values provide no indication of the numbers of evolutions in the most robust set and if it is significant enough to reach a conclusion regarding robustness and complexity or how values are distributed about the mean. Nor can we tell from the mean value what makes a configuration robust, or restricts its robustness. This lack of information also prohibits any conclusions regarding a relationship between complexity, robustness, and throughput, which we have seen is also consistently greater for the most robust evolutions compared to all complete evolutions. In order to reach any conclusions about the relationship between complexity, robustness, and throughput, we must examine the relationships at all levels of robustness - the number of queue distributions considered that result in complete evolutions - and determine what is required of an evolution for a certain level of robustness. In doing so, we will also revisit the effects of the relative numbers of queues, shafts, and magazines.

[1]For sets involving a single queue (or single shaft or single magazine), there are no incomplete evolutions because all carriages must be connected to at least one queue and at least one magazine and each queue and each magazine must be connected to at least one shaft by definition of a valid configuration. The evolutions of these configurations will always be complete because a path is always present to the lone entity, but not necessarily unique because not all shafts are required.

Table 9.1: The mean compressed state complexity for evolution subsets of different system sizes (N = non-halting, C = complete, R = robust, UN = unique and non-halting, UC = unique and complete, UR = unique and robust, M = mimics). Mean values for sets containing non-halting evolutions are greater than the mean values for complete evolutions because halting evolutions terminate earlier, but contain approximately the same number of distinct states. The mean values for the most robust non-unique evolutions are always greater than the complete set of complete evolutions, but the same is not true for unique evolutions. To see how complexity is related to robustness and adaptability, we have to look at the mean values of evolutions at each level of robustness.

System	N	C	R	UN	UC	UR	M
1-2-2	0.135	0.135	0.135	0.152	0.152	0.1533	0.05
1-2-3	0.16008	0.16008	0.16008	0.18117	0.18117	0.19429	0.06565
1-2-4	0.17805	0.17805	0.17805	0.19989	0.19989	0.21964	0.0776
1-3-2	0.22594	0.22594	0.22594	0.26371	0.26371	0.28533	0.09375
1-3-3	0.23631	0.23631	0.23631	0.28311	0.28311	0.32146	0.12587
1-3-4	0.25912	0.25912	0.25912	0.30849	0.30849	0.36246	0.15074
1-4-2	0.32251	0.32251	0.32251	0.38041	0.38041	0.39501	0.14882
1-4-3	0.33263	0.33263	0.33263	0.40782	0.40782	0.44056	0.20531
1-4-4	0.34868	0.34868	0.34868	0.42914	0.42914	0.47275	0.23237
2-2-2	0.09994	0.09014	0.09484	0.11270	0.09987	0.09540	0.04250
2-2-3	0.12488	0.10915	0.11630	0.14119	0.12344	0.11929	0.05554
2-2-4	0.14508	0.12399	0.13232	0.16293	0.14048	0.13516	0.06550
2-3-2	0.16296	0.15935	0.16785	0.19374	0.19316	0.21647	0.06483
2-3-3	0.18890	0.18015	0.19710	0.22979	0.22287	0.27064	0.09959
2-3-4	0.21610	0.20247	0.22566	0.26076	0.24831	0.30780	0.12261
2-4-2	0.22065	0.20416	0.21283	0.26351	0.24598	0.25595	0.10827
2-4-3	0.24603	0.23440	0.25069	0.30419	0.28839	0.29417	0.15878
3-2-2	0.08369	0.07571	0.08060	0.09429	0.08257	0.08259	0.03688
3-2-3	0.10573	0.09196	0.09999	0.11961	0.10290	0.10475	0.04796
3-2-4	0.12352	0.10483	0.11438	0.13887	0.11811	0.11997	0.05643
3-3-2	0.13812	0.13090	0.14400	0.16543	0.15947	0.17783	0.05416
3-3-3	0.16559	0.15446	0.17488	0.20284	0.19300	0.23372	0.08615
3-3-4	0.19189	0.17643	0.20071	0.23304	0.21867	0.26975	0.10730
3-4-2	0.18892	0.17802	0.19231	0.23008	0.22445	0.25560	0.08268
4-2-2	0.07193	0.06297	0.06726	0.08075	0.06770	0.06801	0.03346
4-2-3	0.09233	0.07623	0.08277	0.10437	0.08425	0.08475	0.04336
4-2-4	0.10872	0.08706	0.09398	0.12233	0.09780	0.09672	0.05092
4-3-2	0.12237	0.11678	0.13204	0.14700	0.14203	0.16247	0.04682
4-3-3	0.14760	0.13713	0.16140	0.18137	0.17212	0.21288	0.07591
4-4-2	0.16730	0.15927	0.17677	0.20403	0.20168	0.23941	0.07159

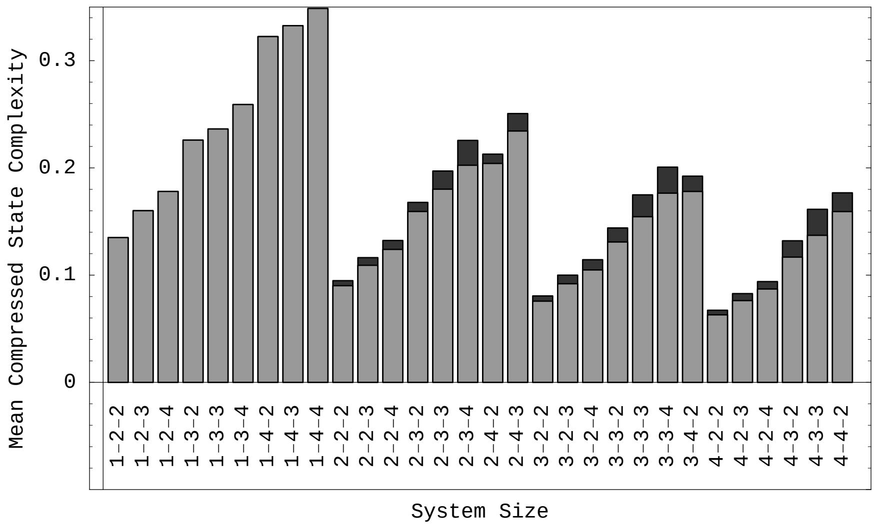


Figure 9.9: A comparison of the mean compressed state complexity values for all complete evolutions (light) and for evolutions corresponding to the most robust configurations (dark). For smaller systems with a single queue, there are no incomplete evolutions by definition of a valid configuration. For larger systems, the mean compressed state complexity for the most robust evolutions is always greater than the mean for the set of all complete evolutions. As the number of shafts and magazines increases, the mean compressed state complexity tends to increase. However, increases in the number of queues tends to decrease the mean compressed state complexity.

We begin by looking at the mean values at all levels of robustness for all complete 2-3-4 evolutions, which is the same system size investigated in the discussion of robustness with respect to state complexity. In that discussion, we saw little apparent correlation between the level of robustness and the state complexity or throughput, and both the maximum mean values of the state complexity and throughput do not correspond to the most robust configurations. Figure 9.10 indicates the same results with respect to compressed state complexity. However, the format has been changed slightly to illustrate additional concepts. The mean values have all been normalized by their respective maximum mean values across all levels of robustness to permit comparisons of different complexity measures as well as the throughput. For the 2-3-4 evolutions the maximum mean values for all attributes occur at a level of robustness of 33, and all normalized mean values therefore have a value of unity at this level of robustness.

While all levels of robustness are not represented by evolutions in this set and there is no such concept of a non-integer level of robustness, the mean values are joined to create robustness 'curves'. Joining the normalized mean values is done only as a graphical device to facilitate comparisons between complexity measures and throughput with respect to robustness and is not meant to imply any general function of complexity or throughput based on

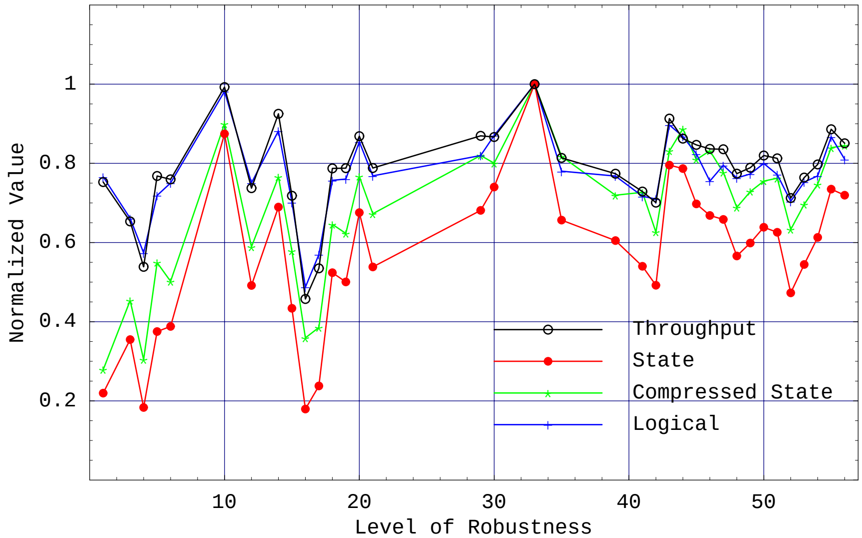


Figure 9.10: A comparison of normalized mean values for throughput, compressed state complexity, state complexity, and logical complexity for all complete 2-3-4 evolutions at all levels of robustness. All attributes show little apparent correlation with robustness, but the "curves" share similar shapes, indicating some relationship between attributes.

robustness. Additionally, to make comparisons clearer, the levels of robustness not represented by evolutions are omitted, so lines are drawn only between levels of robustness with non-zero normalized mean values. Again, this is a graphical device and does not imply that normalized mean values greater than 0 exist at levels of robustness with no corresponding evolutions.

Figure 9.10 shows the same absence of any apparent correlation between robustness and either complexity or throughput. However, it does reveal similarities in the shapes of the various robustness "curves". These similarities occur in nearly all complete evolution sets for many measures, with relative peaks and valleys occurring at identical levels of robustness. If the 'number of states' complexity measure were included, it would be very similar to the state and compressed state complexity curves and, we will see later that, the curves of potential measures also share similarities.

The similarity in the shapes of the curves implies that correlations are present not only between each complexity measure and throughput, but also between the various complexity measures. However, correlations are dependent on both the number and distribution of values defining the mean at each level of robustness. Figure 9.10 provides no information about the number of evolutions or configurations that share a certain level of robustness. Since a configuration often evolves in a similar manner for a range of input streams, resulting in similar complexity and performance values, a level of robustness defined by a few or only one configuration may not represent the possible range of complexities and performance as well as a level of robustness defined by a large fraction of the set of all complete evolutions, although they are treated identically in Figure 9.10 and in any correlation calculation. Figure 9.11 shows the normalized mean curves for throughput and compressed state complexity from Figure 9.10, superposed with a curve describing the number of evolutions (not configurations) at each level of robustness. The number of evolutions is normalized by the maximum number of evolutions at any level of robustness to allow comparison with the complexity and throughput curves so the absolute values of the numbers of evolutions are lost. However, unlike the curves describing mean values of complexity and throughput, points at all levels of robustness are joined, not just those with non-zero values because the absence of evolutions at a level of robustness is significant. For the 2-3-4 evolution set, the maximum number of evolutions is 10080, which corresponds to the highest level of robustness.

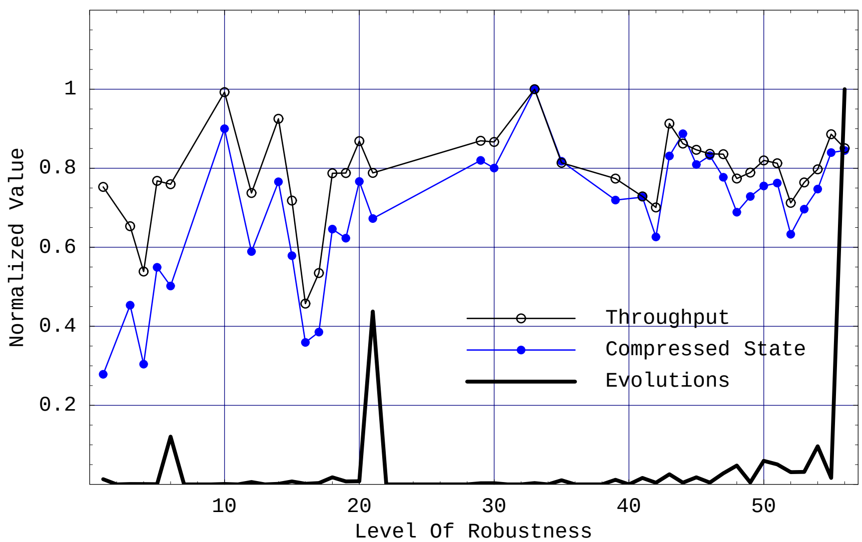


Figure 9.11: The throughput and compressed state complexity level of robustness curves with the normalized number of evolutions at each level of robustness. The majority of evolutions occur at the 6th, 21st and 56th levels of robustness, but correlations using the mean values treat all levels of robustness equally, regardless of the number of evolutions present.

Figure 9.11 shows that the majority of evolutions (74%) define only three levels of robustness: 6, 21, and 56. Of these evolutions, 64% correspond to the most robust configurations. The shape of the curves in Figure 9.10 are then arguably misleading - levels of robustness composed of a small number of evolutions have little averaging and may alter our perception of the relationship between robustness and complexity/throughput. For instance, there is only one configuration each that result in 33 and 16 evolutions, which correspond respectively to the maximum and minimum mean values. At the same time, the set of valid configurations is determined in an exhaustive search so no configuration is omitted and the evolutions are deterministic. Figure 9.10 then does represent an accurate and complete characterization of the relationship between the level of robustness and complexity/throughput and arguably, no configurations should be omitted because a relatively small number of them exist at a certain level of robustness. However, the relationship between mean values and the level of robustness should always be viewed in the context of the number of configurations or evolutions, as well as the distribution of values involved at each level because the mean alone by no means indicates trends or correlations.

To give us an idea of the distribution of values for complexity measures and throughput, we use the set of evolutions of the most robust configurations for complete 2-3-4 evolutions as an example. Figure 9.12 presents the compressed state complexity value for each of the 10080 evolutions comprising the most robust set. Because the evolutions are listed according to their configuration code number and the queue distribution used, little order is evident. However, when the evolutions are sorted according to their compressed state complexity value, we get an idea of the distribution. Figure 9.13 shows the distribution of sorted values relative to the mean compressed state complexity for the most robust configurations. In this interpretation of the distribution, flat areas indicate clusters of same-valued evolutions and the slope of the line indicates the uniformity of the distribution. Figure 9.13 shows a fairly uniform slope with pockets of clusters at lower compressed state complexity values. At higher values of compressed state complexity, the slope increases rapidly, indicating a tail in the distribution.

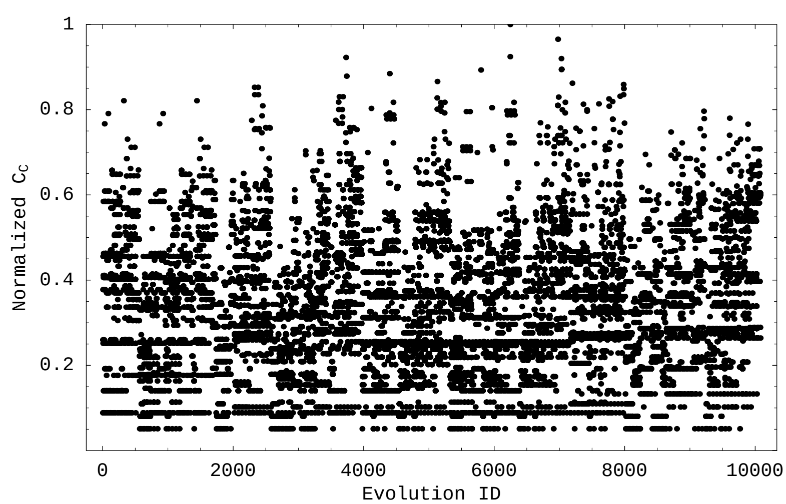


Figure 9.12: The distribution of compressed state complexity values for the 10080 evolutions corresponding to the most robust 2-3-4 configurations. No relationship is apparent in the distribution, but the order of the evolutions is based on the configuration code numbers and queue distributions.

Figures 9.14, 9.15, and 9.16 present the distributions of the throughput, state complexity, and logical complexity in the same format. Figure 9.15 shows a similar distribution as that for the compressed state complexity, with the characteristic tail at higher values, although clusters exist at moderately low state complexity values, indicating more of a skewed normal distribution. While the distributions alone are not sufficient to establish a correlation between these measures because there is no information describing a mapping of values, similar

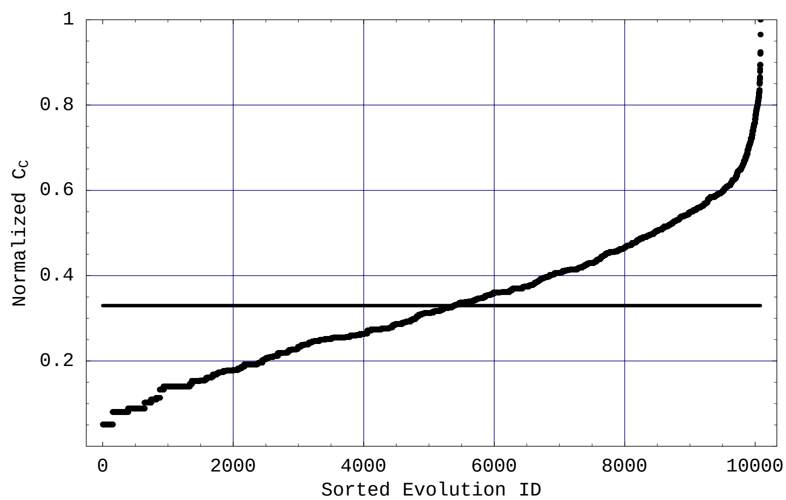


Figure 9.13: The distribution of compressed state complexity values for the most robust 2-3-4 evolutions, sorted according to their compressed state complexity values. The shape of the curve indicates a uniform distribution at lower compressed state complexity values, with some peaks at the lowest values. A tail exists at higher values of compressed state complexity.

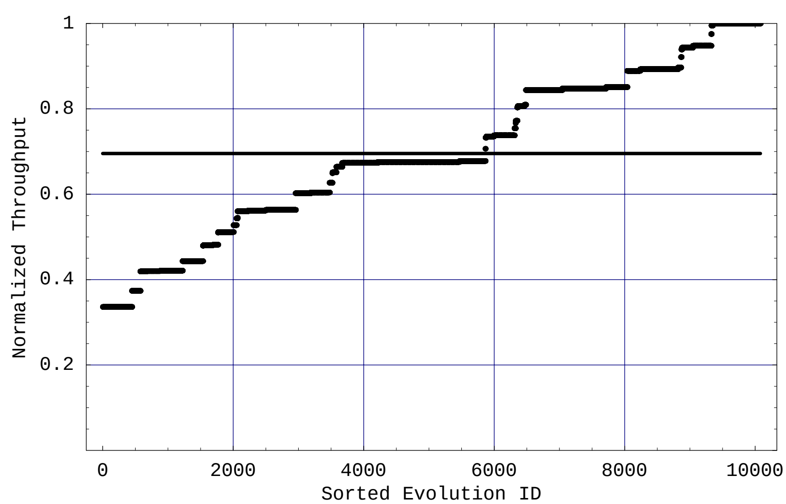


Figure 9.14: The distribution of sorted normalized throughput values for the most robust 2-3-4 evolutions. The distribution is much more discretized than the distribution for compressed state complexity, with clusters at various throughput levels.

distributions are definitely a necessary condition for a strong correlation.

The differences between these distributions and the distributions for the logical complexity and throughput therefore help explain the weaker correlations between these attributes observed in Figure 9.10. The logical complexity shows a jagged distribution with little uni-

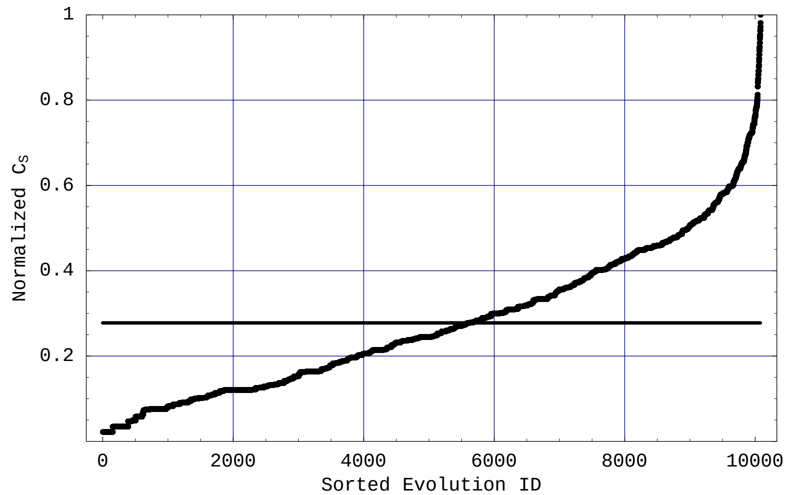


Figure 9.15: The distribution of sorted normalized state complexity values for the most robust 2-3-4 evolutions relative to the normalized mean value. The distribution is similar to that for compressed state complexity, but clusters occur at moderately low values.

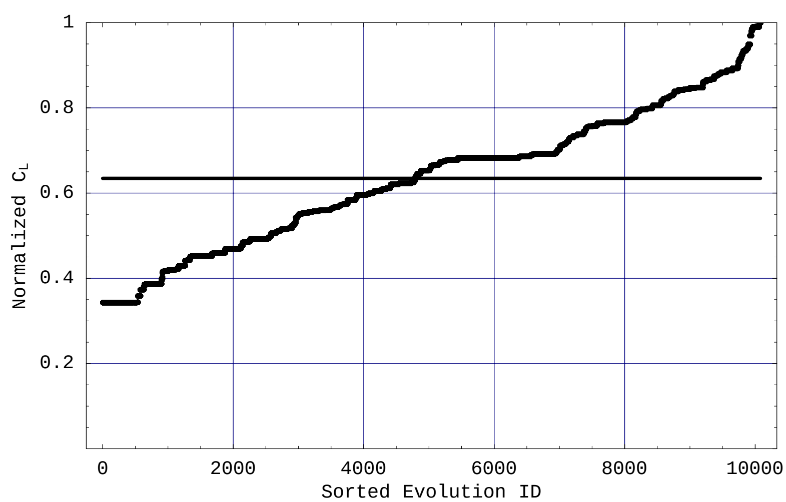


Figure 9.16: The distribution of sorted normalized logical complexity values for the most robust 2-3-4 evolutions. The logical complexity is distributed similarly to the throughput, with a discrete distribution.

formity, a large cluster at a value approximately equal to the mean, and a slight tail at higher logical complexities. The distribution of the throughput values also shows a jagged distribution with significant clustering at various throughput levels. The clusters contain a variety of numbers of evolutions, indicating a very discrete, non-uniform distribution.

But the representations of the distributions in Figures 9.13 to 9.16 only provide information at a single level of robustness. To visualize the complete distribution of values at all levels of robustness, we turn to a three-dimensional frequency distribution much like the one used to illustrate the densities of attribute relationships. The frequency distribution offers a global view of the relationship between robustness and an attribute, but at the slight expense of the detail offered by distributions at a single level of robustness. Figures 9.17 through 9.20 present the three-dimensional distributions of complexity measures and throughput at each level of robustness for all complete 2-3-4 evolutions. Each attribute is normalized with respect to the maximum value in the set. While the figures do not provide any conclusive information regarding correlations between various complexity measures and throughput, they represent a form of the relationship between each attribute in the context of the number and distribution of evolutions.

The distribution of evolutions with respect to throughput in Figure 9.17 reveals a discrete distribution of values at each level of robustness, just as for the most robust evolutions in Figure 9.14, with a similar range of throughputs. A dominant cluster of evolutions occurs at a normalized throughput value of approximately 0.675 at many levels of robustness, particularly those corresponding to the greatest numbers of evolutions (this corresponds to the flat part of the curve in Figure 9.14), although the mean throughput at these levels is always greater than the dominant value. Since the distributions are fairly consistent, there appears to be little correlation between throughput and robustness and, for complete 2-3-4 evolutions, the correlation value is 0.310.

The distribution of logical complexity values for all levels of robustness is similar to the distribution of throughput values, with the existence of a dominant cluster - again at a normalized value of 0.675 - and a fairly consistent range of logical complexity values at any level of robustness. Distributions typically also have tails at higher values of logical complexity and a secondary peak at the minimum logical complexity, the number of which is consistently about 47% of the number of evolutions found in the primary peak. With a tail at higher logical complexities and a significant secondary peak at lower logical complexities, the mean logical complexity is consistently lower than the value corresponding to the dominant peak. Since the logical complexity values are distributed similarly at all levels of robustness, there is little correlation between them. The similarity between the relationships of mean values and robustness for logical complexity and throughput, the high correlation value between logical complexity and throughput, and the low correlation value between mean logical complexities and robustness of 0.26 support this conclusion.

The relationships between state complexity and robustness, shown in Figure 9.19, are distinctly different than those for logical complexity or throughput. At each level of robustness,

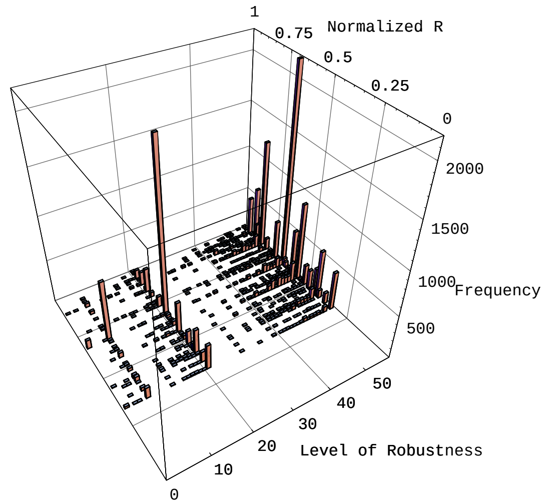


Figure 9.17: The three-dimensional frequency distribution of throughput values for all complete 2-3-4 evolutions. Many levels of robustness, particularly those composed of large numbers of evolutions, have a dominant cluster at a normalized throughput of 0.675. When put in the context of the number of evolutions, the distributions at most levels of robustness appear similar, with secondary clusters at similar throughput values, meaning that little correlation is present between throughput and robustness.

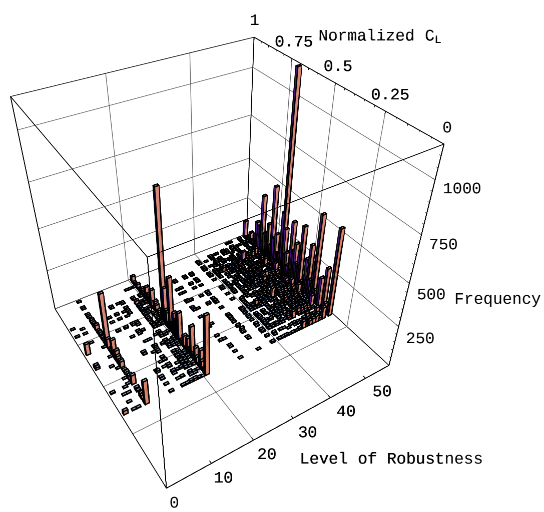


Figure 9.18: The frequency landscape for logical complexities for all complete 2-3-4 evolutions is similar in its discrete nature to that for the throughput. However, many levels of robustness have a secondary cluster at the minimum normalized value and a tail at higher logical complexity values.

the state complexity has a skewed normal distribution, with a peak cluster at a state complexity value lower than the mean and a tail at higher state complexity values. Additionally, Figure 9.19 shows that the range of state complexity values is dependent on the level of robustness, with a common normalized minimum value of 0 and a normalized maximum that tends to decrease with decreases in the robustness. However, the normalized maximum tends to jump back up at more heavily populated levels of robustness - the 6th and 21st levels - although not to the maximum level, creating what appears to be two superposed rates of decay in maximum state complexity with respect to robustness. Although the normalized value of the peak cluster remains approximately constant for distributions at all levels of robustness, the similar shape, but decreased range of the distributions results in a lower mean state complexity at lower levels of robustness, evident in Figure 9.10 at levels of robustness with significant populations. However, the correlation value between the mean state complexity and level of robustness remains low - at 0.521 - because of the mean values at some sparsely populated levels of robustness.

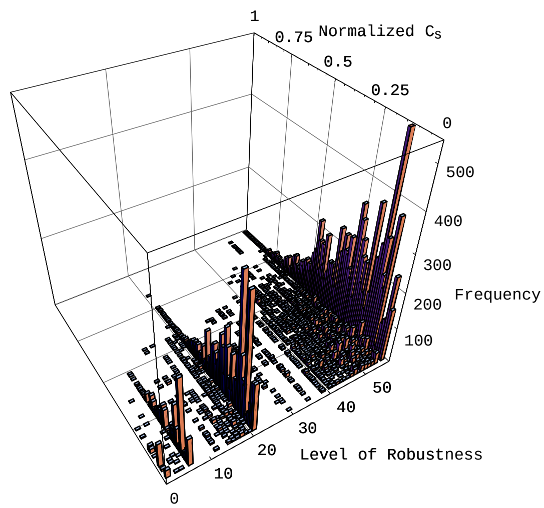


Figure 9.19: The distributions of state complexity values for all complete 2-3-4 evolutions at all levels of robustness share a skewed normal shape. Because the dominant cluster occurs at approximately the same state complexity, but the maximum range of values decreases with decreases in robustness, the correlation between the mean state complexity and robustness levels is stronger than that for throughput or logical complexity.

The relationship between the compressed state complexity and robustness is similar to that for the state complexity. Figure 9.20 shows the same peak values at lower values of compressed state complexity and tails at higher compressed state complexities. The range of values is also dependent on the level of robustness, with an approximately identical minimum value for all levels of robustness and a maximum value that increases with increased robustness. The peak cluster of values that occurs at approximately the same absolute compressed state complexity value for all levels of robustness and the greater maximum values at levels of robustness involving greater numbers of evolutions tend to counter the correlating effects of the increased range of values, resulting in a correlation value of 0.619.

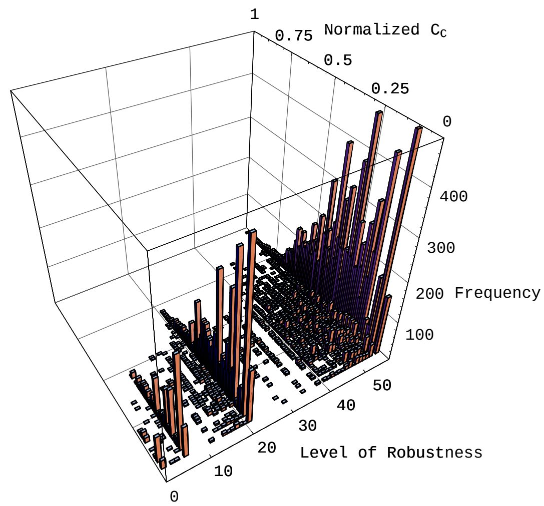


Figure 9.20: The distributions of compressed state complexities at all levels of robustness for all complete 2-3-4 evolutions are similar to the distributions for state complexities. Apparent in both sets of distributions are the superposed decreases in the maximum values evident at the 6th, 21st, and 56th levels of robustness - the levels containing the majority of evolutions.

As mentioned in the discussion of Figure 9.11 and evident in the three dimensional frequency distributions for throughput and each complexity measure, the majority of evolutions occur at the 6th, 21st, and 56th levels of robustness. But, by the definition of the level of robustness, the number of evolutions corresponding to a single configuration is equal to the level of robustness, so there are more evolutions per configuration at higher levels of robustness. Figure 9.21 presents the same relationship between throughput, compressed state complexity, and populations at all levels of robustness for all complete 2-3-4 evolutions, but the populations are defined in terms of the number of *configurations*. Description of the number of configurations at each level of robustness reveals that most configurations are at the 6th, 21st, and 56th levels of robustness, which we saw from the numbers of evolutions at each level of robustness. But a significant number of configurations are capable of only one complete evolution out of the possible input streams considered, an observation not apparent when populations are defined in terms of the number of evolutions. And when the number of configurations are considered, the populations at higher levels of robustness (immediately lower than the 56th) become less significant because they are simply composed of a small number of configurations with high robustness and therefore more evolutions, although this fact does not affect the importance of averaging on putting the mean values in the context of populations. Also apparent in Figure 9.21 is the exponential decrease in the number of evolutions in the regions between the levels of robustness containing the majority of configurations. For example, as the level of robustness decreases from 56 to 21, the number of configurations drops rapidly to sparsely populated levels of robustness - at levels of robustness immediately greater than the 21st level, there are no evolutions. The same characteristics exist between the 6th and 21st levels of robustness and between the 1st and 6th levels of robustness and is reminiscent of the decrease in the maximum state and compressed state complexities.

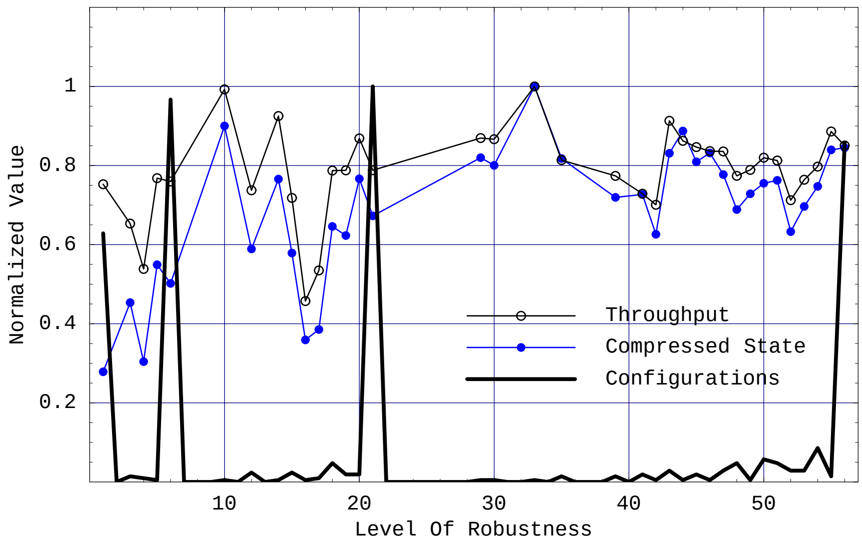


Figure 9.21: The number of configurations at each level of robustness for all complete 2-3-4 evolutions, compared with the mean throughput and compressed state complexity. Because of how the level of robustness is defined, the large number of configurations with only one complete evolution is not apparent when only the number of evolutions is considered. Most configurations result in either 1, 6, 21, or 56 complete evolutions for the 56 queue distributions considered for systems with 4 magazines.

The trend in the population of levels of robustness is not specific to the 2-3-4 complete evolution set - it is shared by all system sizes, although differences in the relative numbers of evolutions/configurations at the dominant levels of robustness and at intermediate levels of robustness help explain the effects of relative numbers of queues, shafts, and magazines. Figures 9.22 to 9.23 present the number of configurations at each level of robustness, normalized with respect to the maximum number of configurations at any level of robustness, for a range of systems with both two and three shafts. The figures also include the mean throughput and compressed state complexity values for levels of robustness (not connected) at levels of robustness that are populated by configurations. Once again, the points indicating the normalized number of configurations are joined to create a curve despite the fact that the relationship between the number of configurations and robustness is not continuous, simply to make comparisons between populations and mean attribute values and to illustrate the similarities between system sizes.

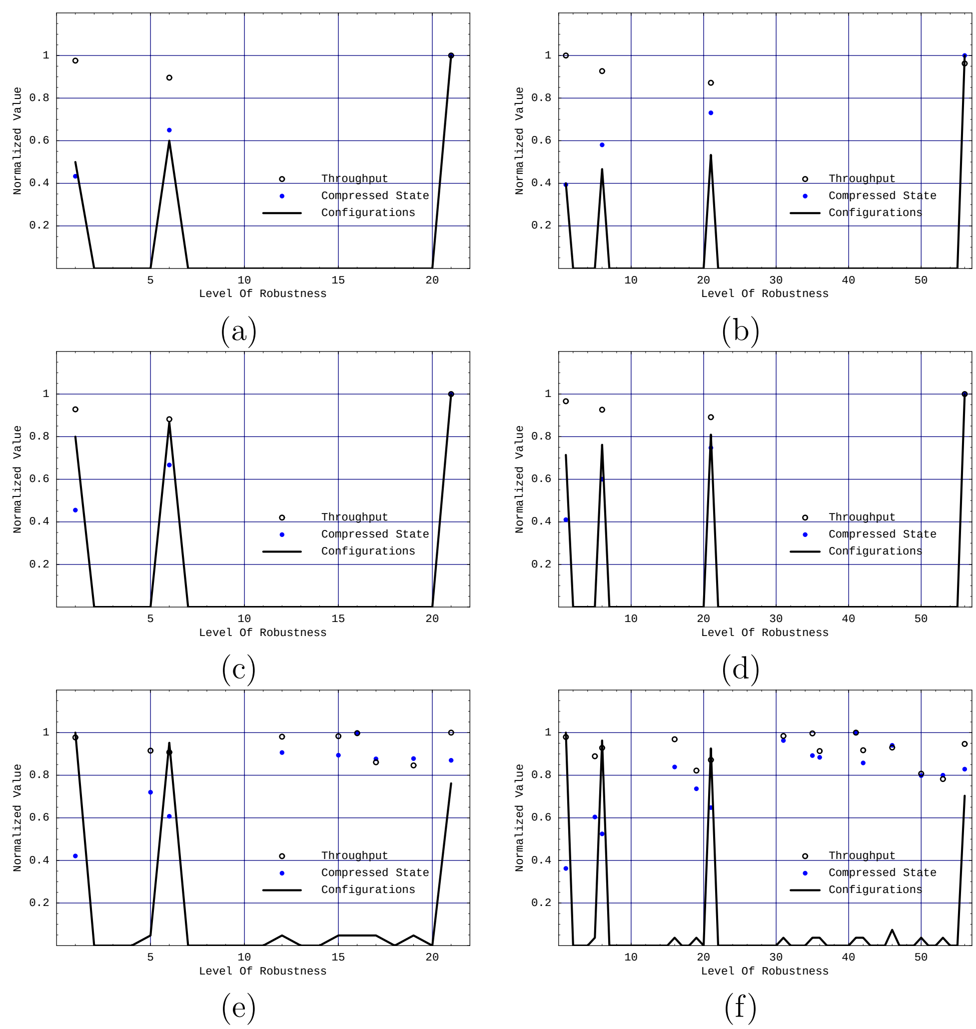


Figure 9.22: The mean throughputs and mean compressed state complexities compared to the ‘curves’ of the number of configurations at all levels of robustness for all complete (a) 2-2-3, (b) 2-2-4, (c) 3-2-3, (d) 3-2-4, (e) 4-2-3, and (f) 4-2-4 evolutions. For all system sizes, the majority of configurations exist at the 1st, 6th, 21st, and 56th levels of robustness (systems with three magazines have a maximum of 21 levels of robustness).

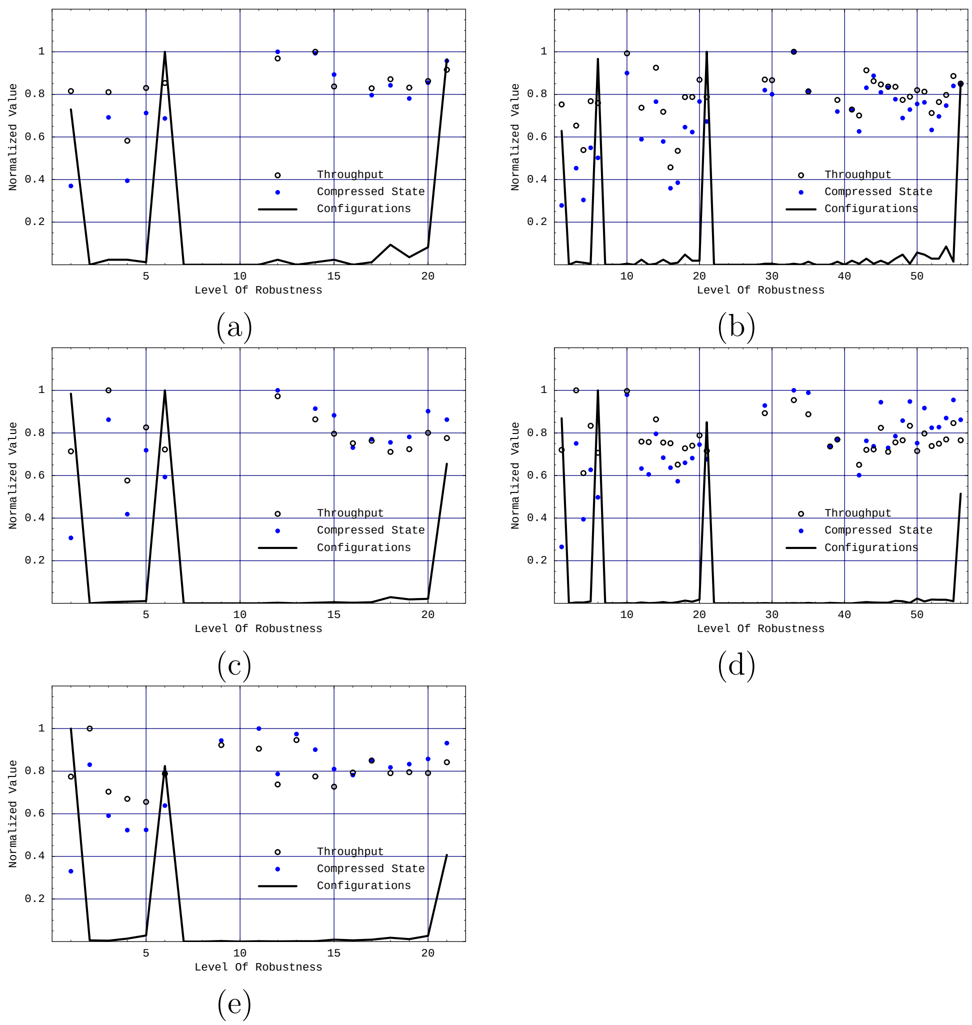


Figure 9.23: The mean throughputs and mean compressed state complexities compared to the ‘curves’ of the number of configurations at all levels of robustness for all complete (a) 2-3-3, (b) 2-3-4, (c) 3-3-3, (d) 3-3-4, and (e) 4-3-3 evolutions. These are the same system sizes as in the previous figure, but with three shafts, rather than two. With respect to the curves describing the number of configurations, the number of queues, shafts, and magazines has little impact on the dominance of the 1st, 6th, 21st, and 56th levels of robustness.

All figures have peak populations at the 1st, 6th, and 21st levels of robustness. For systems with four magazines, a peak also exists at the 56th level of robustness (for systems with three magazines (three item types), there is a maximum of 21 queue distributions and therefore 21 levels of robustness). As the number of queues is increased, the relative number of configurations/evolutions corresponding to lower levels of robustness increases. This trend is evident in the comparison of any column in Figure 9.22 or 9.23. The addition of shafts also results in the presence of non-peak populations for any size system, although these non-peak populations are a function of the number of each type of location. It is difficult to make comparisons between distributions involving different numbers of magazines since the levels of robustness are dependent on the number of magazines. However, in the regions common to systems with both three and four magazines - up to the 21st level of robustness - the relative numbers of configurations are strikingly similar.

9.4 Mimicry, Uniqueness, and Physical Connectivity

While the effects of the change of each variable are consistent, measurement of the effects requires definition of changes in the context of all other variables because the queues, shafts, and magazines are interdependent. However, quantification of the effects of changes in the number of queues and shafts (and magazines) on the relative number of configurations at each level of robustness is difficult without explicitly looking at the configurations in each system (or measuring the relative numbers) because there is no explicit relationship between the number of queues, shafts, and magazines and the number of valid configurations or, more specifically, the number of specific types of configurations.

The types of configurations are critical to understanding the relative effects of queues, shafts, and magazines and explain the clusters of configurations with either 1, 6, 21, or 56 complete evolutions. When the physical connectivities are such that no shaft connects a queue with a magazine, then any input stream that includes item types bound for this sparsely connected magazine can not result in a complete evolution since all queues are assumed to carry identical item loads. The number of undelivered items is equal to the product of the number of absent connections for a magazine with respect to queues and the number of each item type in each of the queues corresponding to the absent connection. For example, in the configuration shown in Figure 9.24, there is no shaft that connects the second queue and the first magazine and any items carried in the second queue bound for the first magazine are therefore undeliverable. The configuration is therefore incapable of complete delivery for any queue distribution involving items bound for the first magazine, regardless of the percentage. It does however result in complete evolutions for all queue distributions composed of item types corresponding to the last three magazines and no item types for the first magazine (queue distributions of the form ((0-x-y-z), where $x + y + z = 100\%$). With only three item types present, there are only 21 possible queue distributions (for the queue item type fractional incremental value of 20% considered), the same number of complete evolutions defining the most robust three magazine systems. So the level of robustness partially serves as an indicator of mimicry with respect to the number of magazines, more specifically, the number of magazines with at least one connection to every queue. We might also therefore expect configurations that mimic the most robust one and two magazine systems that are complete for one and six evolutions, respectively, corresponding to the number of possible queue distributions with one and two item types.

$$(SQ) = \begin{pmatrix} 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad (SM) = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix} \qquad (QM) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{pmatrix}$$

Figure 9.24: The incidence matrices for configuration 90479 2-3-4 show that there is no path between the second queue and the first magazine. Therefore, any queue distribution involving items bound for the first magazine will go undelivered from the first queue, resulting in an incomplete evolution. As a result, this configuration is only complete for the 21 queue distributions for which item types corresponding to the first magazine are absent and the configuration mimics a system with three magazines.

But because there is no closed form solution to determine the number of valid configurations with non-zero QM incidence matrix columns, let alone a closed form solution specifying the number of valid configurations, it is impossible to predict the distribution of evolutions/configurations at various levels of robustness. So more queues and shafts (and magazines) means more configurations with 'absent' connectivity with respect to queues and magazines, which result in more configurations with lower levels of robustness, but also more configurations with 'present' connectivity for all magazines which result higher levels of robustness and the potential for complete delivery of all queue distributions, or maximum robustness. We define a configuration with absent connectivity as a configuration for which there is no connection between a magazine and a queue, denoted by a 0 in the QM matrix. Absent connectivity differs from sparse connectivity, which also describes the lack of connections between entities, because absent connectivity refers to the QM matrix in which values can range from 0 to the maximum number of shafts while entries in the SQ and SM matrices are binary. So absent connectivity in the QM matrix refers to a specific condition of no connections while sparse connectivity in the QM matrix refers to the general condition of low numbers of connections between queues and magazines resulting from sparse connectivity in the SQ and SM matrices. Conversely, we define present connectivity for a magazine as the existence of at least one connection to all queues, or no 0's in the corresponding column of the QM matrix. Complete connectivity with respect to queues and magazines is a special case involving present connectivity, where all entries in the QM matrix are equal to the number of shafts in the configuration and only results when complete connectivity exists between all shafts and queues and between all shafts and magazines.

Because the number of configurations with absent and present connectivity is unpredictable based on the system size, it is impossible to determine what effects changes in the numbers of queues, shafts, and magazines have on the distribution of configurations at various levels of robustness. However, knowing that certain connectivity characteristics are required for complete evolutions for any input stream enables the design of physical structures with adaptive properties.

The explanation of the distribution of evolutions with respect to robustness in terms of their physical connectivity and mimicry of systems with fewer magazines raises the question of how configurations can have levels of robustness other than 1, 6, 21, or 56 - the numbers of combinations for the assumed queue distributions for systems with up to four magazines. The answer lies in the fact that the set of complete evolutions may contain configurations that, for some queue distributions, result in halting evolutions (that jam) because of their physical structure and the control logic used. For instance, a configuration with four magazines that halts for a single queue distribution, but results in complete evolutions for the remaining 55 queue distributions results in a level of robustness of 55. The trailing off of the number of configurations between levels of maximum robustness with respect to the number of magazines mimicked (1, 6, 21, and 56) in Figures 9.22 and 9.23 therefore implies that the number of configurations with physical connectivities resulting in some amount of halting evolutions is significantly lower than the number of configurations that result in complete evolutions for all queue distributions or for the number of queue distributions possible for the number of magazines with present connectivity.

Evidence that the levels of robustness not corresponding to all complete evolutions or simply to the most robust mimics is composed of configurations with halting evolutions is provided in Figure 9.25, which shows the number of configurations at each level of robustness, normalized by the maximum number of configurations at any level of robustness, for the set of 2-3-4 configurations for which no evolution halts or becomes hung-up. The number of the most robust configurations remains constant, since no evolutions could possibly halt and be complete. The numbers of configurations with maximum robustness for the number of mimicked magazines are approximately identical to the numbers for the set of all complete evolutions with any difference being the result of configurations that are not mimics, or mimic a greater number of magazines, but halt in enough evolutions to lower their robustness to levels equal to robust, mimicked levels. However, there are no configurations at intermediate levels of robustness, meaning that robustness is specified entirely by the connectivity of the QM matrix. For all magazines with at least one connection to all queues, from now on referred to as 'relevant' magazines, all corresponding items are deliverable, although not necessarily with the use of all shafts. So all queue distributions composed of items corresponding solely to magazines with present connectivity, or relevant magazines, can not be anything but complete, although the evolutions may not be unique, and configurations *for the set of complete evolutions* can only exist at levels of robustness corresponding to the most robust levels for the actual and mimicked number of magazines (ignoring configurations that halt for some queue distributions).

The set of all complete evolutions, and even the set of complete evolutions resulting from configurations for which no evolutions halt, while showing the effects of mimicry with respect to magazines, do not indicate anything regarding mimicry in the usual sense used throughout

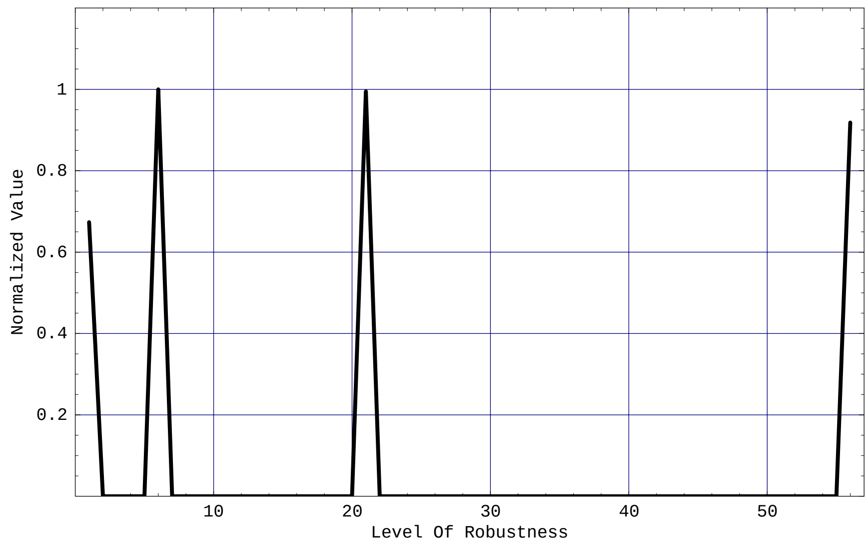


Figure 9.25: The normalized number of configurations at all levels of robustness for the set of complete evolutions created from configurations that never result in a halting evolution for any queue distribution considered. No configurations exist at levels of robustness that do not correspond to maximum levels with various numbers of magazines because only configurations that experience halting exist at intermediate levels of robustness.

this work - mimicry with respect to the number of carriages/shafts. By definition, a complete evolution need not involve all carriages and, as a result, a configuration that is complete for all input streams may mimic a smaller system for some input streams. In accounting for mimicry with respect to shafts, the distributions of the number of configurations or evolutions across the range of robustness levels may therefore be distinctly different than the distribution for all complete evolutions. These differences not only characterize how the properties of uniqueness relate to levels of robustness, but also show the effects of relative numbers of queues, shafts, and magazines and indicate correlations between certain complexity measures and robustness.

Figure 9.26 presents the distribution of the normalized number of configurations across the range of robustness levels for unique, complete 2-3-4 evolutions. The distribution is distinctly different from the distribution of all complete evolutions in Figure 9.21, with clusters of configurations at intermediate levels of robustness not equal to maximum levels for various numbers of magazines, implying that many of the sets of configurations are composed of mimics with respect to the number of shafts.

An apparent feature of the distribution of configurations with unique complete evolutions is the lack of a peak at the most robust level. The smaller the relative (and absolute) number of configurations with unique evolutions at the highest level of robustness is a result of the connectivity between shafts and magazines. To be capable of complete evolutions for all queue distributions, sufficient connectivity must exist between the queues and magazines.

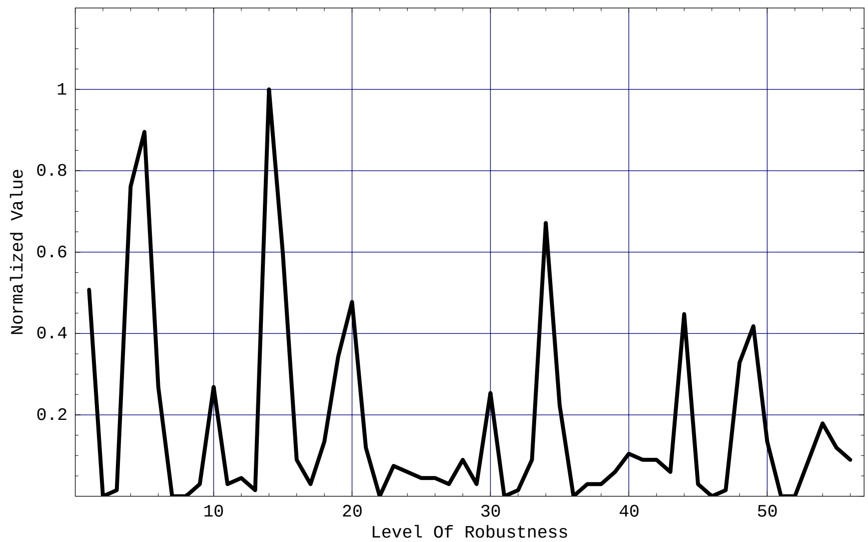


Figure 9.26: The normalized number of configurations at each level of robustness for the set of unique complete 2-3-4 evolutions. Unlike for all complete evolutions, configurations have intermediate levels of robustness because incomplete connectivity between shafts and magazines results in non-unique evolutions for some queue distributions.

However, for the involvement of all carriages required for uniqueness, complete connectivity between shafts and magazines is required. If not, then any queue distribution involving item types corresponding only to magazines lacking a connection to a given shaft put that shaft out of action and result in mimicry of system without that shaft, although the connectivity may still result in a complete evolution. For instance, for configuration 260095 2-3-4, shown in Figure 9.27, there are multiple paths between every queue and magazine, so all queue distributions result in complete evolutions. However, when the queues contain only items bound for the first magazine, the first shaft is not utilized because it is not connected to the first magazine, resulting in mimicry of a configuration with two shafts. The queue distribution containing only items for the first magazine is the only one that results in mimicry and a non-unique evolution, and configuration 260095 has a level of robustness of 55 when we consider uniqueness.

$$(SQ) = \begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{pmatrix} \qquad (SM) = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \qquad (QM) = \begin{pmatrix} 2 & 3 & 3 & 3 \\ 2 & 3 & 3 & 3 \end{pmatrix}$$

Figure 9.27: The incidence matrices for configuration 260095 2-3-4 show that the first shaft is not connected to the first magazine. When the queue distribution is (100-0-0-0), the first shaft is not utilized, despite complete connectivity to all queues and the evolution mimics a two shaft configuration.

For configurations with complete connectivity between shafts and magazines, the connectivity between shafts and queues is irrelevant with respect to evolution completeness and uniqueness because every queue can reach every magazine through some shaft and all shafts are always involved because all queues are assumed to contain identical items and there are therefore always items that each shaft can carry to any magazine. The number of configurations corresponding to the highest level of robustness is therefore equal to the number of valid SQ incidence matrices that are possible with complete shaft-magazine connectivity.

At levels of robustness less than the maximum, completeness and uniqueness remain strongly related to the connectivity between shafts and magazines, but at these levels, SM connectivity alone does not determine robustness and the SQ incidence matrix becomes relevant. Decreasing both the SM and SQ connectivity have the effect of reducing robustness. But the manner in which connectivity is decreased has an important effect on the robustness of the configurations. The number of queue distributions that result in non-unique evolutions is based on the union of combinations exclusively formed by those item types corresponding to unconnected magazines for each queue. So configuration 259967 2-3-4 in Figure 9.28 is equivalent to configuration 260095 2-3-4 in Figure 9.27 with respect to robustness even though connectivity in the SM matrix is sparser because there is still only one queue distribution, (100-0-0-0) that results in a non-unique distribution, although configuration 259967 mimics a single carriage system for this queue distribution while configuration 260095 mimics a system with two carriages. Configuration 260031 2-3-4, shown in Figure 9.29, has the same number of connections between shafts and magazines as configuration 259967 2-3-4, but the first and second queues have absent connections to different magazines. Therefore, configuration 259967 is not unique for the queue distribution with all items bound for the first magazine because the first shaft is not utilized and for the queue distribution with all items bound for the second magazine because the second shaft is not utilized, resulting in a level of robustness of 54, two less than the maximum level. Configuration 259071 2-3-4, shown in Figure 9.30 also has two missing connections, but for the same shaft. As a result, any queue distribution composed of item types solely bound for the first *or* second magazine will result in mimicry of a two shaft system. There are six queue distributions containing items corresponding only to the first and second magazines, including the queue distributions with all item types bound for the first and second magazines ((100-0-0-0) and (0-100-0-0)).

$$(SQ) = \begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{pmatrix} \quad (SM) = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \quad (QM) = \begin{pmatrix} 1 & 3 & 3 & 3 \\ 1 & 3 & 3 & 3 \end{pmatrix}$$

Figure 9.28: Configuration 259967 2-3-4 is identical to configuration 260095 2-3-4 except that the second shaft is not connected to the same magazine as the first shaft. Although performance is affected by sparser connectivity, configuration 259967 is equivalent to 260095 with respect to robustness - both result in non-unique evolutions only when all items are bound for the first magazine.

Although connectivity between shafts and queues is irrelevant with respect to uniqueness and completeness (but not performance) for systems with complete connectivity between

$$(SQ) = \begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{pmatrix} \quad (SM) = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \quad (QM) = \begin{pmatrix} 2 & 2 & 3 & 3 \\ 2 & 2 & 3 & 3 \end{pmatrix}$$

Figure 9.29: The first two shafts in configuration 260031 2-3-4, as in 259967, are each missing a single connection to a magazine. However, the magazines are different, so there are two queue distributions that result in non-unique evolutions: (100-0-0-0) and (0-100-0-0). With two non-unique evolutions, configuration 260031 has a level of robustness of 54.

$$(SQ) = \begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{pmatrix} \quad (SM) = \begin{pmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \quad (QM) = \begin{pmatrix} 2 & 2 & 3 & 3 \\ 2 & 2 & 3 & 3 \end{pmatrix}$$

Figure 9.30: Configuration 259071 2-3-4 has the same number of connections between queues and magazines as configuration 260031 2-3-4 and also two missing connections in the SM matrix. However, both connections are for the same shaft, so no queue distributions with any combination of item types solely for the first and second magazines of the form (x-y-0-0) result in a unique evolution, where $x + y = 100$.

shafts and magazines, all valid shaft-queue combinations are possible. However, as SM connectivity is decreased, SQ connectivity must increase in order to maintain the same present connectivity between queues and magazines. A limit exists on the minimal SM connectivity possible to still result in a QM matrix free of absent connections. Below this limit, the SM matrix again decreases from near complete levels and the SQ matrices are all combinations that result in absent connectivity for a single magazine and mimicry of a system with one less magazine. The SM matrix decreases from near complete, but not totally complete levels because complete connectivity always results in maximum robustness. However, the SM sub-matrix defined by the relevant magazines is complete. This process repeats, with SM connectivity dropping and the limits of SQ connectivity increasing to maintain the level of mimicry with respect to magazines until mimicry of a system with a single, completely connected magazine occurs and all other magazines have a single connection to a common shaft. The SQ connectivity results in QM connectivity with only one relevant magazine and a single connection to a common queue for all other magazines, meaning only one queue distribution consisting of all items bound for the completely connected magazine results in a unique evolution. For the 2-3-4 configurations with four magazines, the first limit encountered as SM connectivity decreases therefore corresponds to the 21^{st} level of robustness, where a single column with absent connectivity occurs in the QM matrix results in mimics of a system with three magazines.

While more on the relationship between physical connectivities and robustness is addressed later in the section regarding potential complexity measures, the observations of unique, complete 2-3-4 evolutions indicate that knowing the relationship between physical connectivity and robustness and utilization enables the a priori design of configurations in the face of unpredictable environments, while accounting for practical limitations in arrangements.

The relationships between connectivity and robustness are not limited to 2-3-4 evolutions, although the shape of the distributions of the number of configurations at different levels of robustness may change for different system sizes. Figures 9.31 and 9.32 present the distributions of the normalized number of configurations across the range of levels of robustness for the same system sizes presented in Figures 9.22 and 9.23. The differences in the distributions reflect the robustness of complete evolutions that mimic systems with fewer shafts.

As configurations at intermediate levels of robustness indicate the fraction of configurations resulting in a halting evolution for the set of all complete evolutions, intermediate levels of robustness for the set of unique, complete evolutions indicate the fraction of configurations with a given amount of halting evolutions and non-unique evolutions. For instance, in the distributions in Figure 9.32 for systems with three magazines, we see that a significant number of configurations have a total of seven non-unique and halting evolutions (for the $21 - 7 = 14$th level of robustness). Comparison of distributions indicates that the relative number of configurations with non-unique and halting evolutions increases as the number of queues increases, although the predominant absolute total number of halting and non-unique evolutions remains constant. These trends are also apparent in comparisons of distributions in Figure 9.31 for systems with two shafts. However, with two shafts, the number of halting and non-unique evolutions for a configuration is less than for systems with three shafts, primarily because proportionally fewer non-unique evolutions are possible in systems with fewer shafts.

As with the set of all complete evolutions, relationships between the complexity measures/throughput and robustness must be made in the context of the relative number of configurations/evolutions at each level of robustness. However, for complete characterization of the relationships, no values at any level of robustness can be ignored, regardless of the number of configurations present.

The mean throughput at each level of robustness for unique, complete 2-3-4 evolutions are presented in Figure 9.33. Just as for the set of all complete evolutions, the figure reveals no apparent relationship between mean throughput and robustness, which is supported by a low correlation value of 0.370. The reasons for a weak relationship become clear when put in the context of what we know about the connections between physical connectivities and robustness. In this context, we also see that correlations between throughput and robustness are present for configurations with certain physical properties.

At each level of robustness corresponding to the maximum level of robustness for different numbers of magazines (1, 6, 21, and 56), the SM sub-matrices created from the columns corresponding to relevant magazines always have complete connectivity. By definition, we will call these configurations robust mimics. With complete connectivity in the SM (sub)matrix, there is no queue distribution comprised solely of items corresponding to the completely connected magazines that results in an incomplete evolution, regardless of the connectivity between shafts and queues. As a result, a complete range of connectivities in the SQ matrix

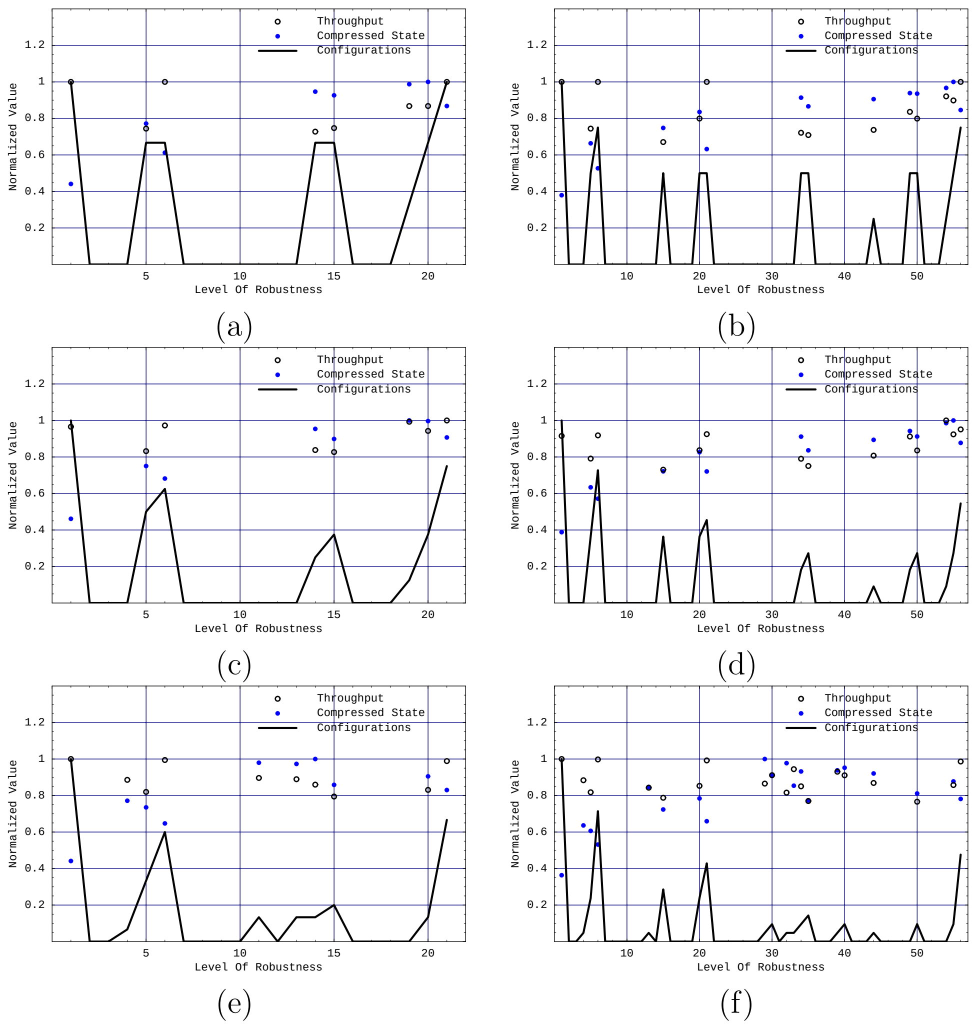
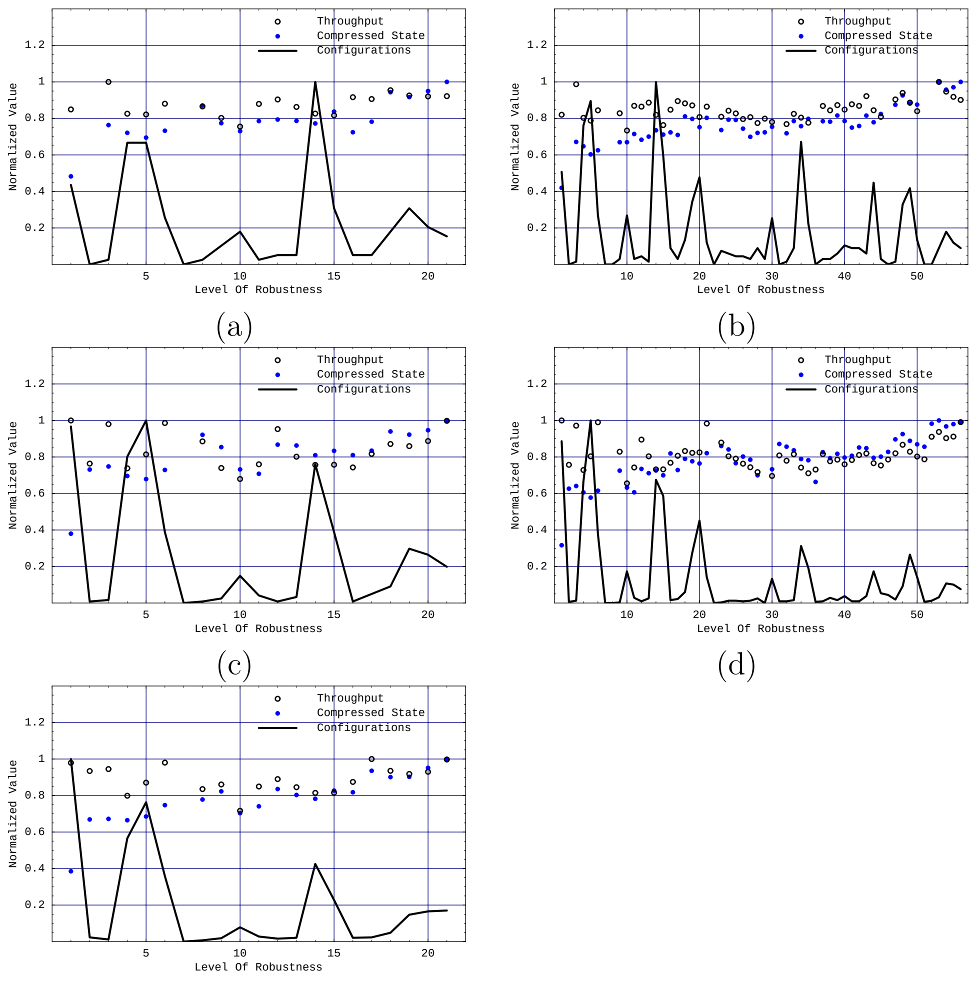


Figure 9.31: The mean throughputs and mean compressed state complexities compared to the ‘curves’ of the number of configurations at all levels of robustness for unique complete (a) 2-2-3, (b) 2-2-4, (c) 3-2-3, (d) 3-2-4, (e) 4-2-3, and (f) 4-2-4 evolutions. Configurations populate levels of robustness other than the most robust levels for different numbers of magazines, reflecting the number of evolutions that mimic systems with fewer shafts.



(a) (b) (c) (d) (e)

Figure 9.32: The mean throughputs and mean compressed state complexities compared to the ‘curves’ of the number of configurations at all levels of robustness for unique complete (a) 2-3-3, (b) 2-3-4, (c) 3-3-3, (d) 3-3-4, and (e) 4-3-3 evolutions. At levels of robustness corresponding to the most robust for different numbers of mimicked magazines, the numbers of configurations are relatively low.

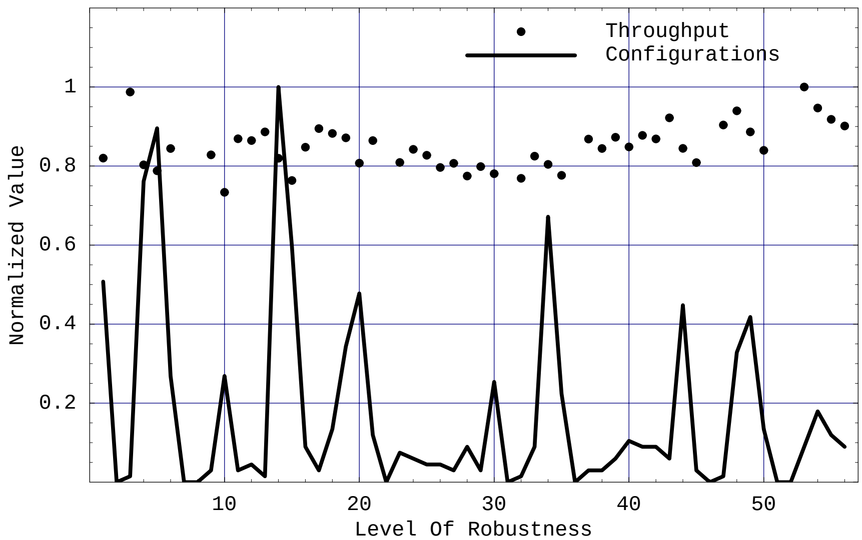


Figure 9.33: The mean throughput values for all levels of robustness for the set of *unique* complete 2-3-4 evolutions in the context of the number of configurations comprising each level. Although there are more configurations at intermediate levels of robustness, there is still no apparent relationship between throughput and robustness.

is possible, as long as the configuration remains valid, and the number of paths between queues and completely connected magazines also runs over a wide range. As the number of mimicked magazines decreases along with the level of robustness, there are more possible configurations that result in QM sub-matrices with present connectivity because the relationship between the QM sub-matrix of magazines with present connectivity and the SQ and SM matrices is not one-to-one. That is, as the QM sub-matrix decreases in size, there are multiple ways to obtain it when we are free to have any values in the remainder of the matrix. However, this effect is limited by the framework of the definition of valid configurations. For the unique complete 2-3-4 evolution set, there are 34 SQ/SM combinations that mimic the most robust configurations with a single magazine, 18 combinations that mimic the most robust two magazine configurations, 8 combinations that mimic the most robust three magazine configurations and 6 combinations that result in the most robust configurations involving all four magazines.

With complete connectivity in the SM (sub)matrices, the number of connections to each magazine for any queue is constant - that is, the entries in each row of the QM (sub)matrix of relevant magazines are identical. The value of the entries is dependent on the SQ connectivity, with a maximum equal to the number of shafts. Since the entries are always equal for configurations that represent the most robust configurations or mimic the most robust configurations with fewer magazines, the possible QM combinations are always equal, regardless of the number of relevant magazines.

While there are S^Q combinations of identical rows in the QM (sub)matrix possible, typically less than this number is observed because of the way we define valid configurations by a lowest energy state so that, with some exceptions again related to how valid configurations are defined, the number of connections of an 'earlier numbered' queue is typically less than or equal to the number of connections of 'later numbered' queues. For the example 2-3-4 unique, complete evolutions, the combinations of queue-magazine connections are identical for the configurations at the levels of robustness corresponding to robust mimics. The only exception occurs for the system with complete connectivity between all queues and magazines, a special case that results from complete connectivity between shafts and queues. Figure 9.34 shows some example configurations that mimic systems with three magazines that have different combinations of identical rows for the relevant magazines.

Configuration 104319:

$$(SQ) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (SM) = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \quad (QM) = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 \end{pmatrix}$$

Configuration 219135:

$$(SQ) = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 1 \end{pmatrix} \quad (SM) = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \quad (QM) = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 2 & 3 & 3 & 3 \end{pmatrix}$$

Configuration 108543:

$$(SQ) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 0 \end{pmatrix} \quad (SM) = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \quad (QM) = \begin{pmatrix} 2 & 2 & 2 & 2 \\ 0 & 1 & 1 & 1 \end{pmatrix}$$

Configuration 124799:

$$(SQ) = \begin{pmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 0 \end{pmatrix} \quad (SM) = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \quad (QM) = \begin{pmatrix} 1 & 2 & 2 & 2 \\ 0 & 2 & 2 & 2 \end{pmatrix}$$

Configuration 251775:

$$(SQ) = \begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{pmatrix} \quad (SM) = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \quad (QM) = \begin{pmatrix} 0 & 2 & 2 & 2 \\ 1 & 3 & 3 & 3 \end{pmatrix}$$

Figure 9.34: The incidence matrices for a selection of 2-3-4 configurations that have three relevant magazines and complete connectivity for the SM sub-matrix corresponding to these magazines. With complete connectivity in the SM sub-matrix, the configurations mimic the most robust unique complete evolutions of systems with three magazines and therefore have a level of robustness of 21. For the last three columns in each of the QM matrices, the entries in each column for each row are identical. Possible combinations of numbers of connections for each queue vary and are dependent on the SQ and SM matrices and how valid configurations are defined. The number of configurations with each combination affects how robustness is related to throughput because lower values in the QM sub-matrix equate to lower performance.

Configurations in which each queue is connected by the same number of shafts to every magazine with complete connectivity to shafts have identical performance, with the exception of the possible application of de facto priority rules. Regardless of the complete connectivity present in the SM matrix, differences in SQ matrices that still result in identical QM matrices can change the timing of events and introduce resource sharing and phase lags. However, the evolutions corresponding to queue distributions with no items bound for magazines without complete connectivity to shafts are therefore identical to those evolutions with the same queue distributions, but for configurations with complete connectivity for the unused magazines. For example, the SM matrices for the configurations in Figure 9.35 are identical with respect to the 21 queue distributions lacking any item types corresponding to the first magazine (0-x-y-z) if the SQ matrices are identical. The performance and behavior of the evolutions are therefore also identical.

$$(SM) = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \qquad (SM) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

Figure 9.35: The first system has complete connectivity for only three of the four magazines while the second system has complete connectivity for all magazines. If the two systems have identical SQ matrices, then the systems have identical behavior and performance for the 21 queue distributions with item types corresponding only to the three magazines with complete connectivity in the first system. For these queue distributions, the first magazine is not involved in either system.

Although configurations with the same SQ connectivity but different numbers of magazines with completely connectivity to shafts have identical evolutions for the queue distributions composed entirely of item types corresponding to common magazines with complete connectivity, the mean throughput at the levels of robustness corresponding to the robust mimics is dependent on the relative number of configurations with various combinations of connectivity between each queue and the magazines with present connectivity and the unique shaft-queue connectivities that are possible for each QM combination for the number of relevant magazines.

If the number of configurations with different combinations of entries in QM (constant for each row for all relevant magazines) were the same and the same variations in the shaft-queue matrices occurred for all sets of configurations with different numbers of relevant magazines and complete connectivity in the SM matrix, then the mean throughput would be identical at all levels of robustness corresponding to robust mimics. However, when all magazines are not relevant, there can be multiple SM matrices with the same SQ matrix that result in the same QM connectivity for relevant magazines. While the complete connectivity of the relevant magazines remains unchanged, the remainder of the magazines can have a range of connectivities within the bounds of the definition of valid configurations, depending on the SQ matrix. And while variations in the SM matrix do not affect throughput directly because evolutions involving incompletely connected magazines do not belong to the set of unique, complete evolutions, the existence of multiple identical configurations with respect to the relevant magazines means these configurations and their resulting throughputs carry greater weight towards the mean throughput. Variations in the SM matrix that result in identical configurations with respect to the relevant magazines are more common in configurations with fewer shafts connecting queues and relevant magazines because of the way valid configurations are defined. Typically, the lowest number of connections between any queue and relevant magazines and therefore the minimum number of shaft connections of any queue, since the relevant magazines are completely connected to shafts, dictate the throughput. For example, for the two QM matrices presented in Figure 9.36, the configuration corresponding to the QM matrix in (a) will have lower throughput for many queue distributions than the configuration in (b) because only one shaft (the third) delivers all items from the first queue to all magazines in (a), but the load is shared in (b) between at least two shafts (assuming one does not arrive at a queue with no items because of delayed information about queue inventories). Since there are relatively more configurations with fewer connections for systems with fewer relevant magazines because there are more variations in SM possible with greater numbers of 'irrelevant' magazines, and configurations with fewer connections typically have lower throughput, the mean throughput should decrease as the number of relevant magazines decreases.

$$\text{(a)} \quad (QM) = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 3 & 3 & 3 \end{pmatrix}$$
$$\text{(b)} \quad (QM) = \begin{pmatrix} 0 & 2 & 2 & 2 \\ 1 & 2 & 2 & 2 \end{pmatrix}$$

Figure 9.36: For the two systems in (a) and (b), only the last three magazines are relevant to unique complete evolutions. Even though the average connectivity to queues is the same for these magazines, the configuration with the QM matrix in (b) will have higher throughput for many queue distributions because loads are shared between at least two shafts throughout the evolution while there is only one shaft connecting the first queue to all magazines in (a) that must handle all items in the first queue.

But as the number of completely connected magazines decreases and more variation in the SM matrix for irrelevant magazines is possible, there is also more variation in the SQ matrices possible because of the definition of valid configurations in terms of lowest energy states. Variations in SQ can result in identical QM sub-matrices of relevant matrices, but not necessarily identical performance because of de facto priority rules. Furthermore, the SQ variants are often not found in any configuration with more relevant magazines, meaning that new behaviors and performance are possible in configurations with fewer relevant magazines, which can increase or decrease the mean values. As an example, consider the configurations shown in Figure 9.37 involving three relevant magazines. Because of the incompletely connected first magazine, two SQ variants are possible that result in the same combination of connectivities for relevant magazines in the QM matrix, but only one SQ matrix matches the SQ matrix for a configuration with four relevant magazines and the same combination of connectivities in the QM matrix. The SQ matrix for configuration 104319 is not valid in a configuration with complete connectivity between shafts and all magazines because the second and third rows are not in a lowest energy state. This matrix is valid for the configuration with three relevant magazines because the second and third rows in the SM matrix for configuration 104319 are not identical, so the rows do not have to be in a lowest energy state.

Configuration 92031:

$$(SQ) = \begin{pmatrix} 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \quad (SM) = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \quad (QM) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \end{pmatrix}$$

Configuration 104319:

$$(SQ) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (SM) = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \quad (QM) = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 \end{pmatrix}$$

Figure 9.37: The incidence matrices for configurations 92031 2-3-4 and 104319 2-3-4. The two configurations have identical QM matrices for the three relevant magazines, but only the SQ matrix for configuration 92031 corresponds to a valid SQ matrix for a configuration with four relevant magazines and the same combination of entries in the QM matrix for relevant magazines. Despite the identical QM matrices, variations in the SQ matrix can result in differences in behavior and performance. The existence of variations and the differences in behavior and performance can result in differences in the mean values of complexity and throughput for sets of configurations with different numbers of relevant magazines.

There are typically fewer SQ variants that result in QM matrices with greater connectivity for relevant magazines because the SQ matrices require greater connectivity, meaning there is less opportunity to obtain a magazine with absent connectivity to queues and therefore the proper number of relevant magazines. Again, the relative number of QM matrices with lower connectivity increases as the number of completely connected magazines and the level of robustness decrease, meaning the mean throughput should also decrease with the level of robustness.

Since configurations with less connectivity between queues and relevant magazines tend to dominate any of the sets of configurations with different numbers of relevant magazines and since configurations with the same SQ matrices and the same QM connectivity for relevant magazines share identical evolutions for some set of queue distributions, the mean throughputs for the respective sets of resulting evolutions are typically very similar, if not identical, despite variations in SQ and SM in systems with fewer relevant magazines that tend to decrease throughput. However, if a difference is present, the mean throughput will be lower at lower levels of robustness because of the effects of variants. Figure 9.38 shows the mean throughputs for the complete 2-3-4 evolutions at different levels of robustness again, but with a highlight of the mean values at levels of robustness that correspond to the most robust levels for systems with different numbers of magazines. These mean values are nearly identical, despite the large variations in the mean values at intermediate levels of robustness and regardless of the number of configurations that define the mean, although there is a slight increase in the mean values with increases in robustness. Similar mean throughput values at the levels of robustness corresponding to robust mimics are apparent in all system sizes which can be seen in Figures 9.31 and 9.32. The similarities in the mean values help explain the lack of any definitive correlation between robustness and throughput, not completely by themselves, but in combination with trends in the mean throughput at intermediate levels of robustness.

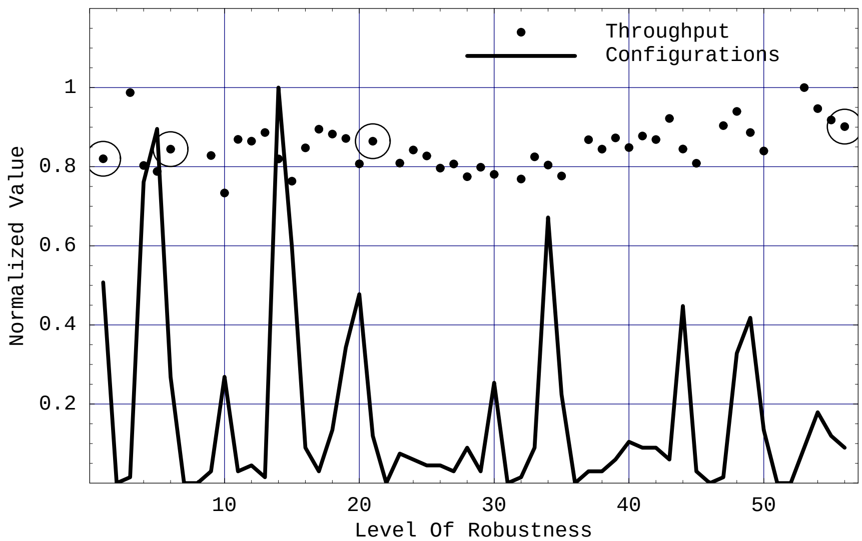


Figure 9.38: The normalized mean throughput values for all levels of robustness for unique complete 2-3-4 evolutions with highlights at the 1st, 6th, 21st, and 56th levels of robustness - the levels corresponding to the most robust configurations with 1, 2, 3, and 4 magazines. The mean throughputs at these levels of robustness are very similar because configurations with the same SQ matrices resulting in the same QM connectivity for relevant magazines have identical evolutions for some queue distributions. The mean values increase slightly with robustness because of the effects of variations in SQ and SM possible in systems that mimic fewer completely connected magazines.

The same similarities in normalized values at levels of robustness corresponding to the various levels of robust mimicry occur for logical complexity, which also partly explain the low correlation of 0.310 between robustness and logical complexity for unique complete evolutions. However, the state and compressed state complexities do not have similar normalized mean values at these levels of robustness, which is evident in Figure 9.39 for 2-3-4 systems and in Figures 9.31 and 9.32 for a more complete set of system sizes for compressed state complexity. Figure 9.39 shows an exponential decrease in the state and compressed state complexities with decreasing robustness for the most robust levels. Imagine two configurations, A and B, where A has one fewer relevant magazine than B and the configurations have identical SQ connectivity and identical SM and QM connectivity for the common relevant magazines. While the set of evolutions corresponding to queue distributions composed solely of items corresponding to the relevant magazines common to A and B are identical for both configurations, the remaining evolutions for B corresponding to queue distributions that include item types corresponding to the additional relevant magazine in B have additional states. With all else equal, like throughput and temporal evolution length, these evolutions involving additional states corresponding to the additional relevant magazine have greater state and compressed state complexities, and raise the mean value, depending on the difference in the number of relevant magazines. Since the differences in the mean values is dependent on mimicry with respect to magazines, the exponential decrease in state and compressed state complexity is apparent in both complete and unique complete evolution sets.

The trends in throughput and complexity measures at levels of robustness corresponding to the most robust levels for different numbers of magazines provide the basis for trends across all levels of robustness, but do not provide a complete picture. At intermediate levels of robustness, where the connectivity between shafts and relevant magazines is not complete, the maximum number of possible shafts connecting queues to magazines is less than that possible with complete SM connectivity for relevant magazines. The number of connections between queues and magazines decreases with decreased connectivity in SM and therefore with decreased robustness. But fewer paths between queues and magazines means disproportionate utilization of shafts is possible - a shaft that represents the only path to a magazine from a queue must transport all items bound to that magazine from that queue alone and represents a limiting factor. Additionally, if this shaft services additional magazines and queues or represents the only connection between multiple queues and magazines, it is likely to carry a disproportionate number of items, increasing the evolution length and decreasing the throughput. Configurations with less connectivity in the SM sub-matrix defined by relevant magazines are therefore not only capable of fewer complete evolutions because of fewer paths between queues and magazines, but also result in lower throughput because of the disproportionate utilization of shafts.

The effect of SM connectivity on throughput is apparent when we consider only those configurations that have the same number of relevant magazines because, as we have seen, the mean throughputs of robust mimics are essentially equivalent and, because of the sharing of loads that arises from more complete SM connectivity, are greater than the mean throughputs of more robust configurations with more relevant magazines, but fewer connections between queues and these magazines. Figure 9.40 presents the relationship between the mean throughput and robustness for those configurations that have present connectivity for all magazines and levels of robustness greater than the level corresponding to the robust mimic of a three magazine system. The correlation between the mean throughput and robustness is much stronger at 0.767, compared to 0.370 for the mean throughput values at all levels of robustness which includes mimics and is very close to the value of 0.310 for all complete evolutions. When all configurations with present connectivity for all magazines are considered, the correlation value of 0.561 is still significantly higher. The difference in

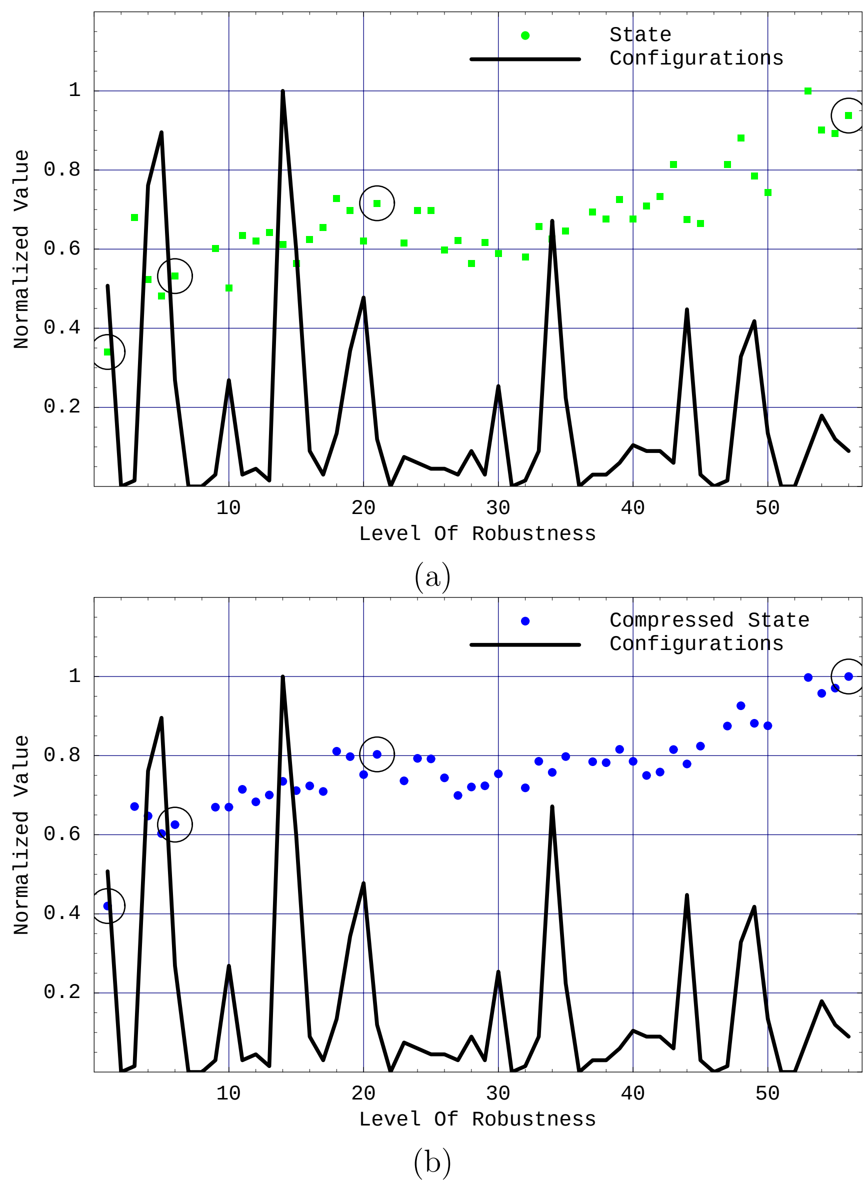


Figure 9.39: The (a) normalized mean state complexity and (b) normalized mean compressed state complexity values in the context of the number of configurations for all levels of robustness for 2-3-4 evolutions. The mean values at the levels corresponding to the most robust levels for different numbers of magazines decrease exponentially with decreasing robustness because the numbers of magazines involved in evolutions with fewer magazines with complete connectivity is lower.

the correlation values results from the existence of configurations with four magazines with present connectivity with mean throughputs greater than the minimum mean value that correspond to levels of robustness less than that for a robust mimic of a three magazine system (the 21st level). The existence of these configurations raises an apparent contradiction in the relationship between connectivity and throughput. However, these configurations result in halting evolutions for some queue distributions, reducing the potential number of complete evolutions possible and lowering the level of robustness, although their connectivity may be greater than that for some configurations at higher levels of robustness. Accounting for these configurations, the stronger correlation values for the subset of configurations with present connectivity for all magazines illustrate the effects of connectivity on throughput and demonstrate the existence of correlations between throughput and robustness in the context of connectivity. Furthermore, because the mean throughputs of the most robust configurations and the robust mimics are essentially equal and mean throughput decreases as connectivity decreases between the shafts and the respective relevant magazines, the relationship between throughput and robustness is cyclic, decreasing with decreasing robustness until the level of robustness corresponding to the robust mimics of configurations with one less magazine, then decreasing again as SM connectivity again decreases.

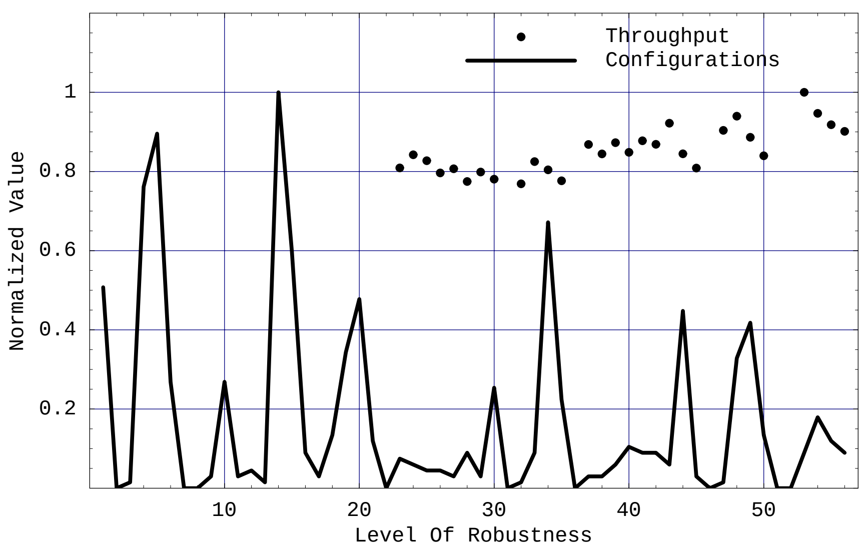


Figure 9.40: The normalized mean throughput values for unique complete 2-3-4 evolutions of configurations with four magazines with present connectivity and levels of robustness greater than the level corresponding to robust mimics of three magazine systems. The correlation value between these mean values and robustness is 0.767, significantly higher than the value of 0.370 for mean throughput values at all levels of robustness which includes mimics.

The correlation between logical complexity and robustness also changes in the context of connectivity. For configurations with present connectivity for all magazines and levels of robustness greater than the level corresponding to robust mimicry of a three magazine system, the correlation value is 0.655, which is significantly greater than the correlation value of 0.310 for mean logical complexity values for configurations at all levels of robustness. However, the correlation values between mean compressed state complexity and robustness and between mean state complexity and robustness do not change significantly relative to the connectivity considered. The correlation value between the compressed state complexity and robustness is 0.850 for configurations at all levels of robustness and 0.851 for configurations with present connectivity for four magazines and levels of robustness greater than 21. Between the 6th and 21st levels of robustness, which correspond to the levels for robust mimics of two and three magazine systems, the correlation value is also 0.851. Similarly, the correlation value between the state complexity and robustness is 0.787 for all configurations and 0.836 for configurations with present connectivity for all four magazines. Correlation values remain relatively unchanged for these complexity measures because fewer states are possible in evolutions with fewer numbers of active magazines. Therefore the cyclic relationship between robustness and mean values evident for throughput and logical complexity as the level of robustness passes through a level corresponding to a robust mimic is not present for the state and compressed state complexities.

Chapter 10

Number of Unique States Used

10.1 Algorithmic Complexity

In the discussion of state complexity, we saw that normalization by the temporal evolution length introduced a correlation between state complexity and throughput and resulted in a divergence from a strict interpretation of algorithmic complexity.

At the minimum and maximum throughputs, the number of operational carriages is known. However, intermediate throughput values do not explicitly indicate the number of operational carriages and therefore the sequence of carriage haltings is unknown. The maximum potential number of state combinations is therefore assumed for all intermediate throughputs. The throughput does indicate some information regarding the *timing* of carriage haltings, which has a direct influence on temporal evolution length. Since both the throughput and state complexity are both inversely proportional to the temporal evolution length, a linear relationship between state complexity and throughput is inherent.

According to the definition of algorithmic complexity, the information required to describe the number of repetitions of states/patterns is negligible with respect to the information required to describe the states comprising system patterns, the number of distinct patterns, and their global sequencing. Normalization by the temporal evolution length therefore results in a divergence from a strict definition of algorithmic complexity and implies that description of complexity solely by the number of states is a closer representation of algorithmic complexity.

However, under the assumption of a constant minimum/maximum theoretical number of states, removal of normalization, while bringing complexity conceptually closer to algorithmic complexity, removes any correlation between complexity and throughput. The theoretical distribution is presented in Figure 10.1(a) for intermediate throughputs under this assumption and indicates that the information required to describe an evolution at a lower