Wind Plant Aerodynamics- A Spectral Analysis for Energy Entrainment

What is the key role of turbulence in wind energy & what are critical scales for energy supply to the wind farm?

How can this knowledge be used to design more efficient wind farms?

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Presentation Overview

1) Motivation & Challenges

2) Objectives

3) Equations of Motions: RANS, MKE

4) Experimental Research- Scaled down wind farm

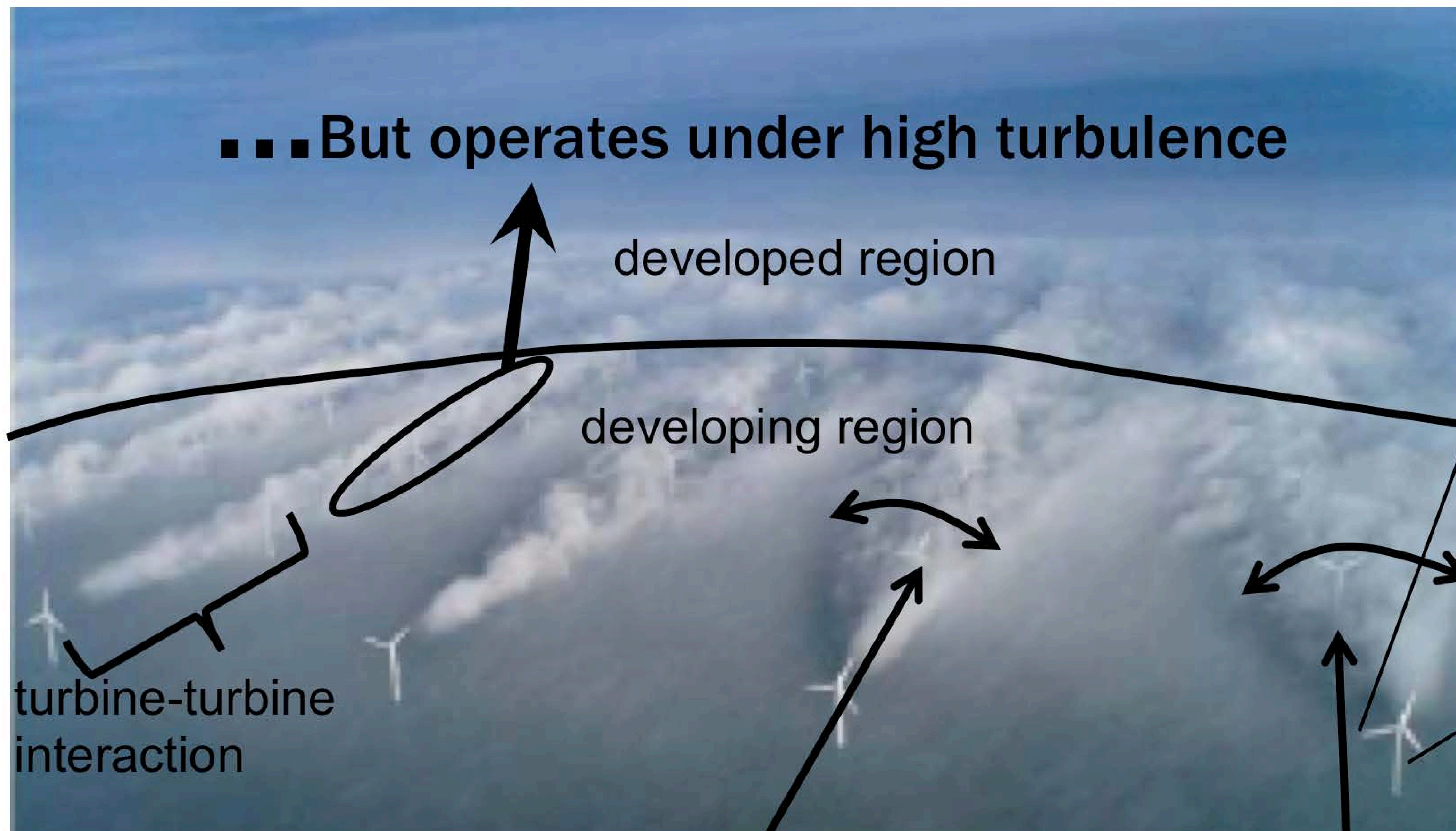
5) Results: a) Developing WTBL, c) Energy Budget
b.) Low Dimensional Analysis

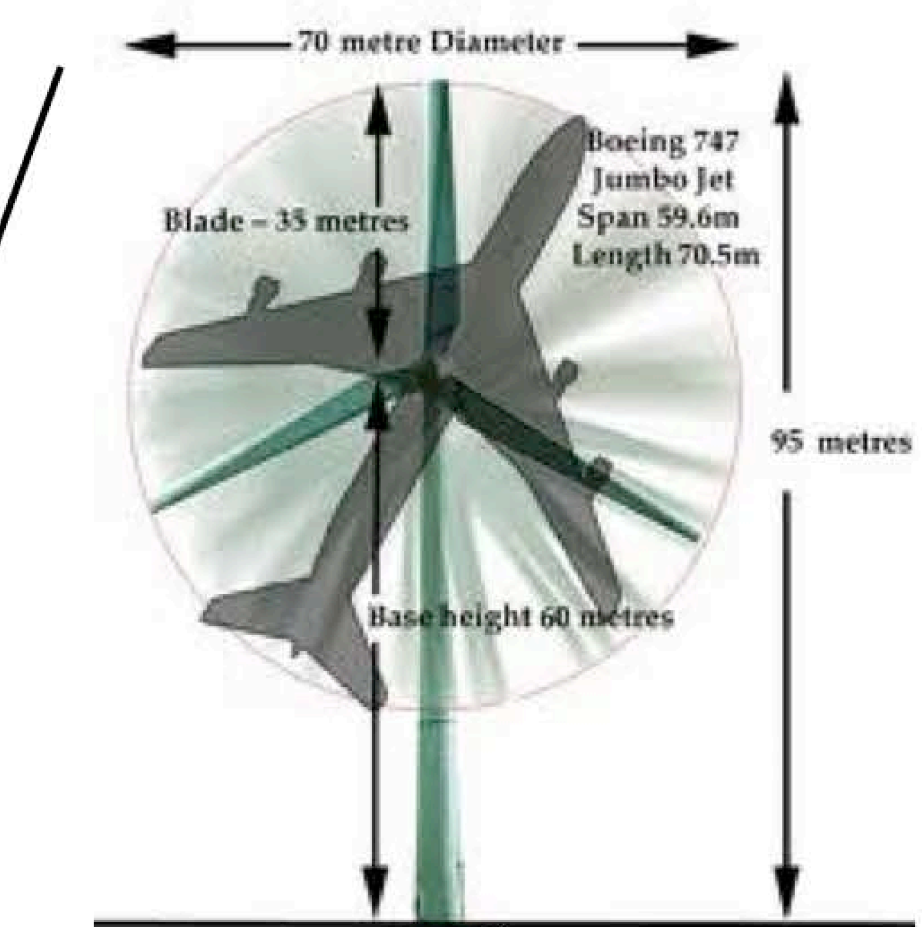
6) Conclusions

Motivation

Horns Rev 1 owned by Vattenfall. Photographer Christian Steiness

complicated maintenance due to large sizes





Black and Veatch 2001

varying inflow conditions

1) What is the Role of Turbulence in MKE Balance and Its Interaction with ABL?

2) What is the Role of Turbulence in the Characterization of Developing of Wind Arrays?

Challenges

Wake loss is the most uncertain cost of wind farms (~ 20%)

Cost

Wind turbines are designed to last 20 years; however, breakdowns appear as early as 7 years (cracked blades, broken gearboxes and generators).

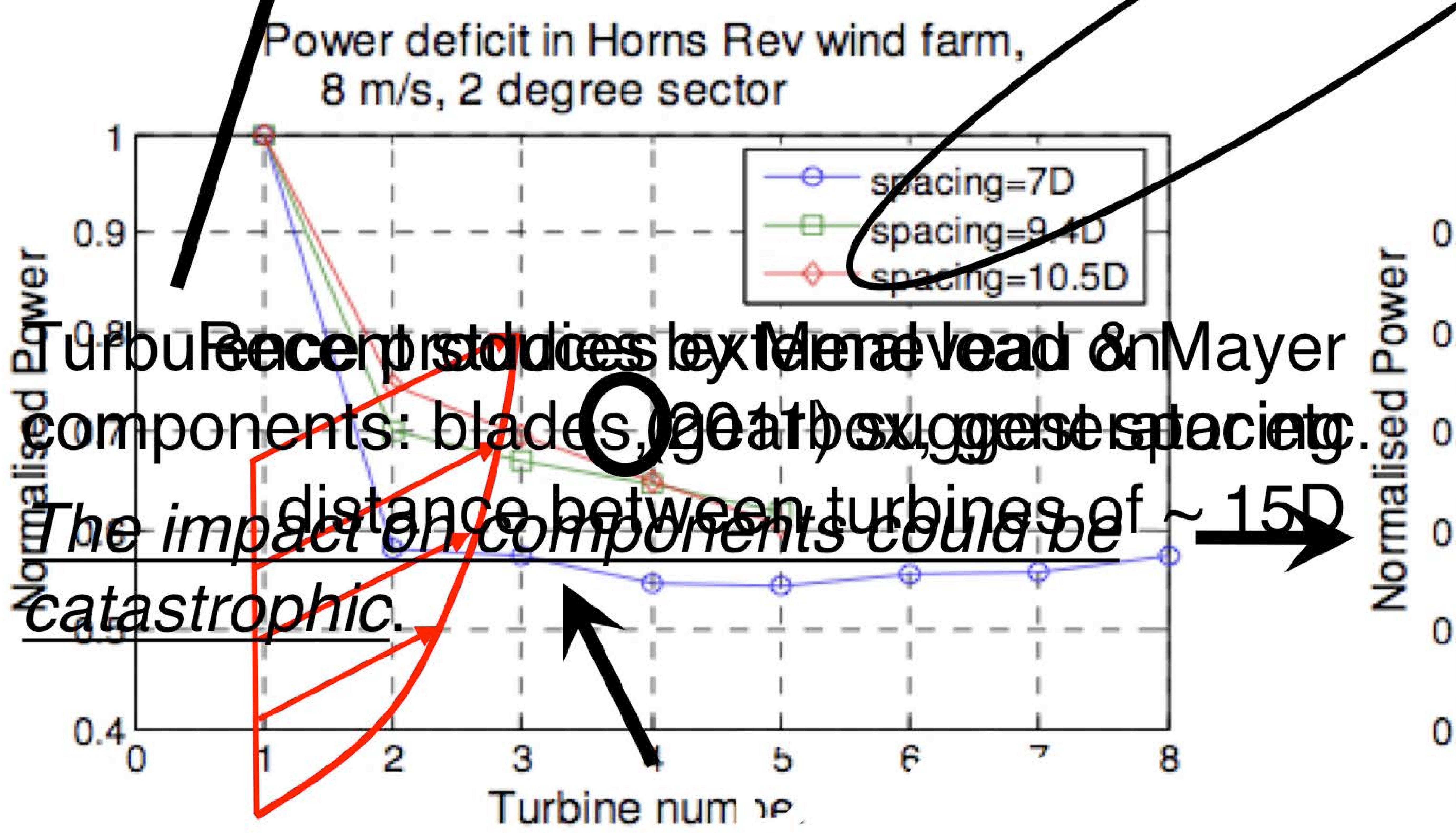
Costly repairs that increase pay-back time and cost of energy (COE).

Efficiency

Blade pitch is difficult to control in a highly turbulent environment. Therefore, efficiency is not always at its highest.

Power loss due to wakes of upstream turbines of about 15%

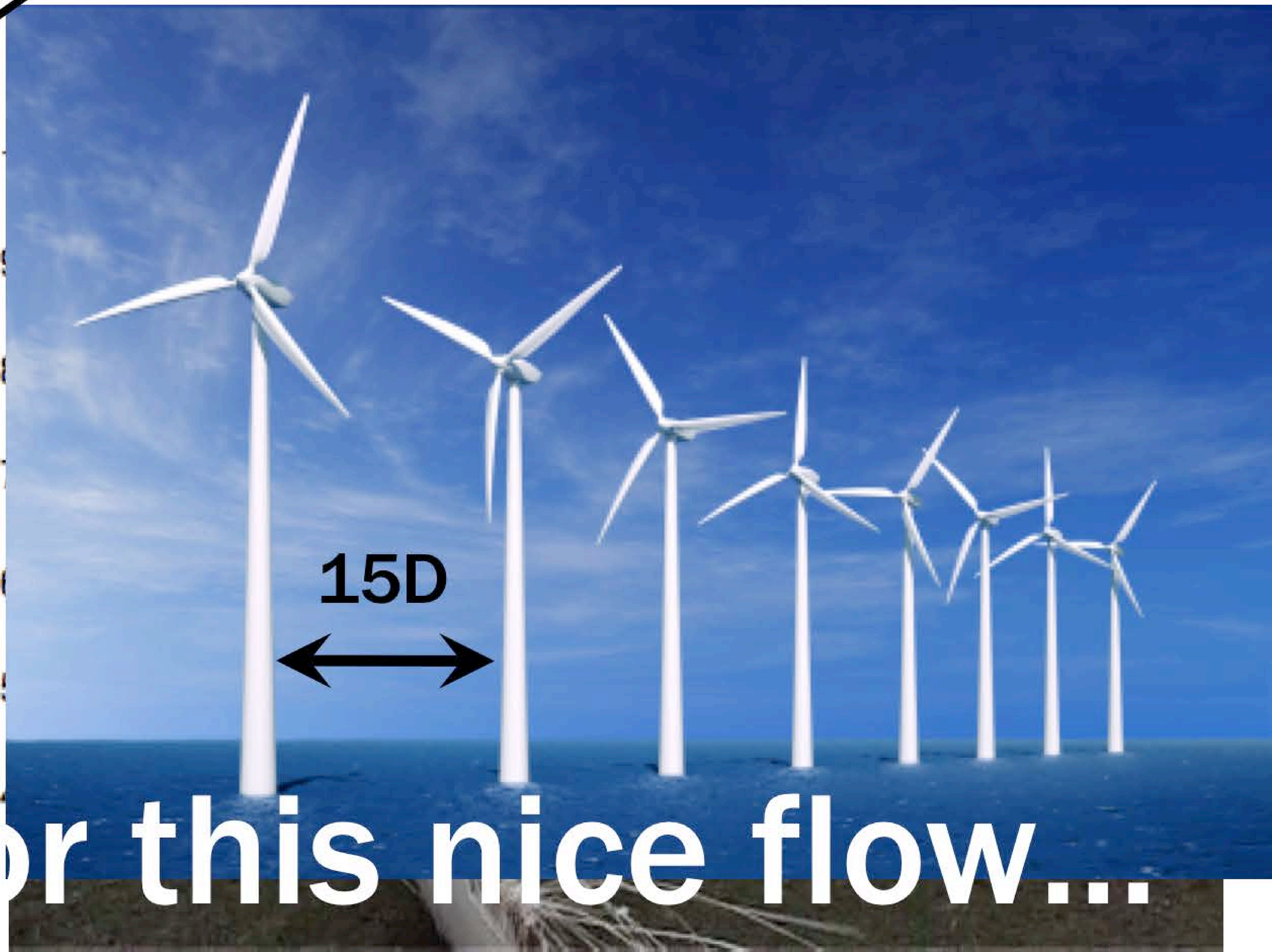
Wake losses are important (as much as 40% loss, Barthelmie *et al.* 2007)



Turbulence produces extreme load on components: blades, gearbox, generator, etc.

The impact on components could be catastrophic.

Recent studies by Meneveau & Mayer (2011) suggest spacing distance between turbines of ~ 15D



Objectives

Wind Tunnel Experiments:

1/850 scale 3 x5 turbine array

- Understand the role of the turbulence in the scaled down wind farm.
- Determine whether or not the **wind farm is fully-developed or not?**
    - power measurements
    - budget analysis of MKE
    - Mean profiles between turbines
- Understand the **role of large scales of turbulence** in wind farm performance.

The fully developed WTBL

Fundamental Questions:

1) How to characterized a fully developed or developing wind farm?

2) What is the WTBL Profile?

3) At what rates do the various turbulence statistics develop?

The Wake Profile & Financial Risk:
Layout Optimization and Energy Output

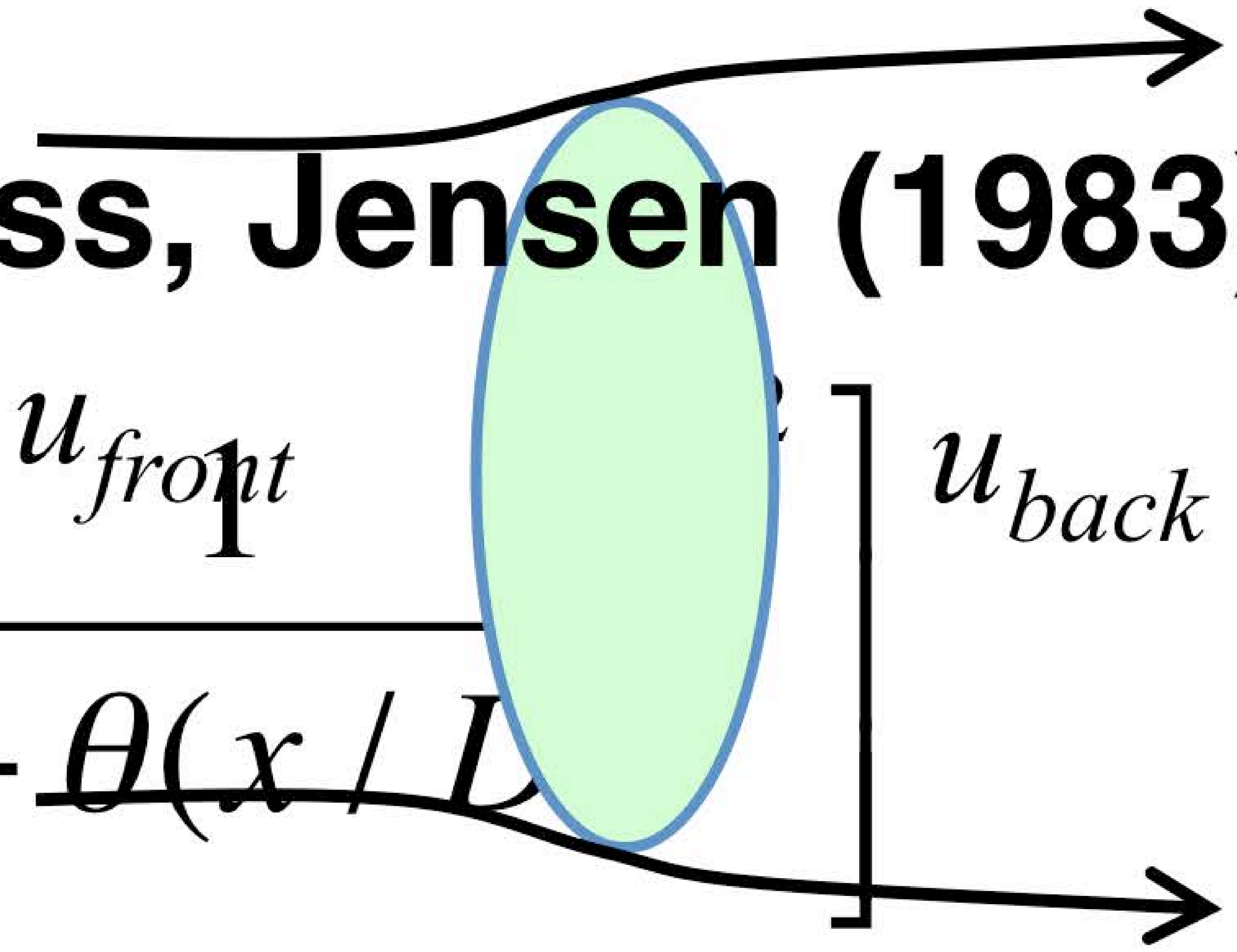
1) Induction factor for wind turbines

2) Wake Models

PARK model for wake loss, Jensen (1983)

$$a = \frac{1}{2}\left(1 - \frac{u_{back}}{u_{front}}\right)$$

$$U(x) = U_0\left[1 - a\left(\frac{1}{1+\theta(x/D)}\right)^2\right]$$



Assumptions:

Used wake models

velocity inside wake ➔ **assumed to be axi-symmetric and uniform**

free stream velocity ➔ **assumed to be uniform**

entrainment parameter, θ, ➔ **empirical value assigned**

Θ and k both depend on turbulence structure and are input parameters!

Output power prediction/ Maximize energy extraction

Wind turbine placement (layout) optimization

Spatial Average

Time and space (x, z) averaging:

A 3-D spatio-temporal field

$$u(x, y, z, t)$$

Becomes a 1-D steady field

$$\left\langle \overline{U}(y) \right\rangle_{xz}$$

Momentum Theory: RANS

Apply to x component of Navier-Stokes:

$$\langle \overline{u} \rangle_{xz} \frac{\partial \langle \overline{u} \rangle_{xz}}{\partial x} + \langle \overline{v} \rangle_{xz} \frac{\partial \langle \overline{u} \rangle_{xz}}{\partial x} = -\frac{1}{\rho}\frac{dp_\infty}{dx} + \frac{d}{dy}\left(-\langle \overline{u'v'} \rangle_{xz} - \langle \overline{u}''\overline{v}'' \rangle_{xz} \right) + \langle \overline{f_x} \rangle_{xz}$$

Reynolds shear stresses, due to RAD

Correlations between mean velocity deviations from their spatial mean "dispersive stress"
(Raupach et al. Appl Mech Rev 44, 1991)

averaged thrust force

MKE Equation

Multiplying the momentum by the mean ve… …gy equation:

$$-\frac{\partial}{\partial y}\left(\langle\overline{u'v'}\rangle\langle\overline{u}\rangle+\langle\overline{u}''\overline{v}''\rangle\langle\overline{u}\rangle\right)+\langle\overline{u'v'}\rangle\frac{\partial\langle\overline{u}\rangle}{\partial y}+\langle\overline{u}''\overline{v}''\rangle\frac{\partial\langle\overline{u}\rangle}{\partial y}-\mathcal{P}(y)\approx 0$$

Kinetic energy flux

Dispersive flux (due to spatial average

Turbulent Dissipation

Dispersive Dissipation

Product of the spatially averaged velocity and the averaged thrust force

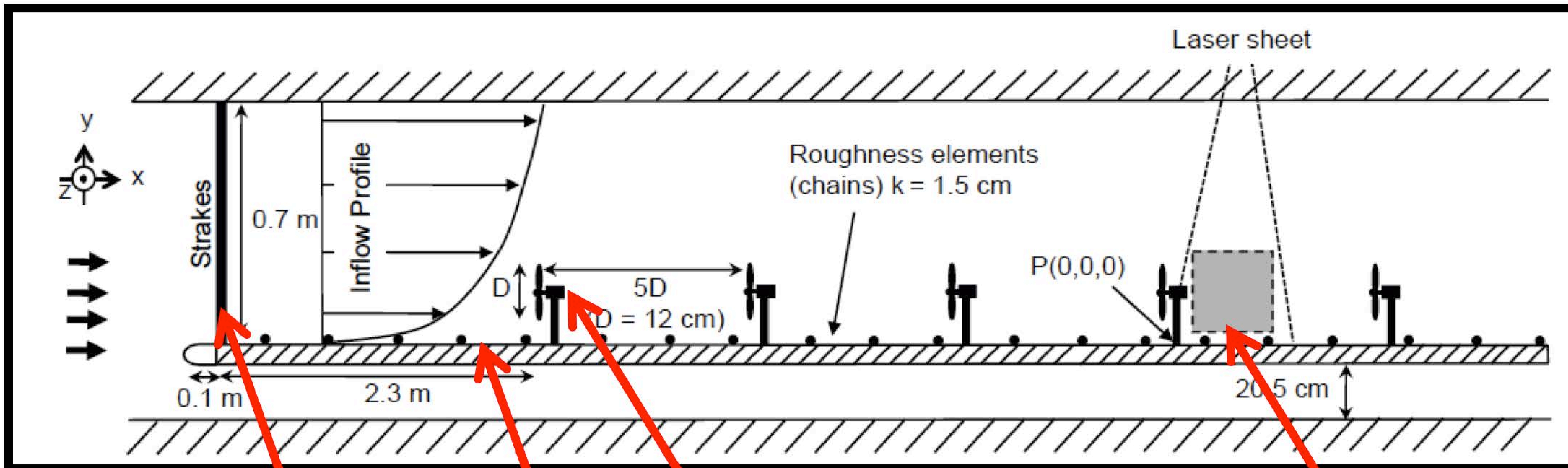
$$\langle\overline{u}\rangle\frac{\partial\frac{1}{2}\langle\overline{u}\rangle^2}{\partial x}+\langle\overline{v}\rangle\frac{\partial\frac{1}{2}\langle\overline{u}\rangle^2}{\partial y}=$$

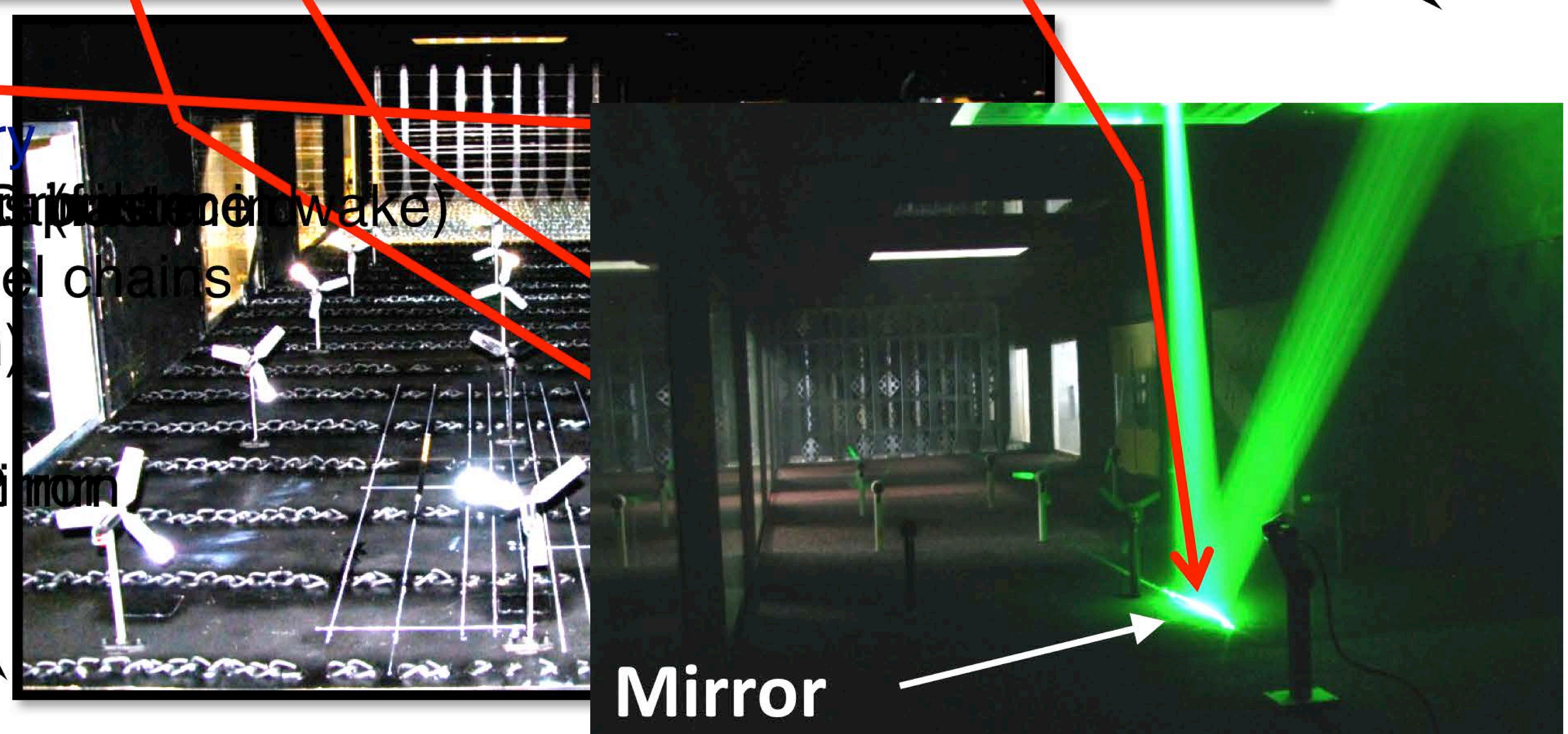
$$-\frac{1}{\rho}\langle\overline{u}\rangle\frac{dp_\infty}{dx}-\frac{\partial}{\partial y}\left(\langle\overline{u'v'}\rangle\langle\overline{u}\rangle+\langle\overline{u}''\overline{v}''\rangle\langle\overline{u}\rangle\right)+\langle\overline{u'v'}\rangle\frac{\partial\langle\overline{u}\rangle}{\partial y}+\langle\overline{u}''\overline{v}''\rangle\frac{\partial\langle\overline{u}\rangle}{\partial y}-\mathcal{P}(y)$$

These terms are <u>dominant</u> in the inner region

What is the role of turbulent momentum & KE flux in the energy entrainment?

Experimental Setup

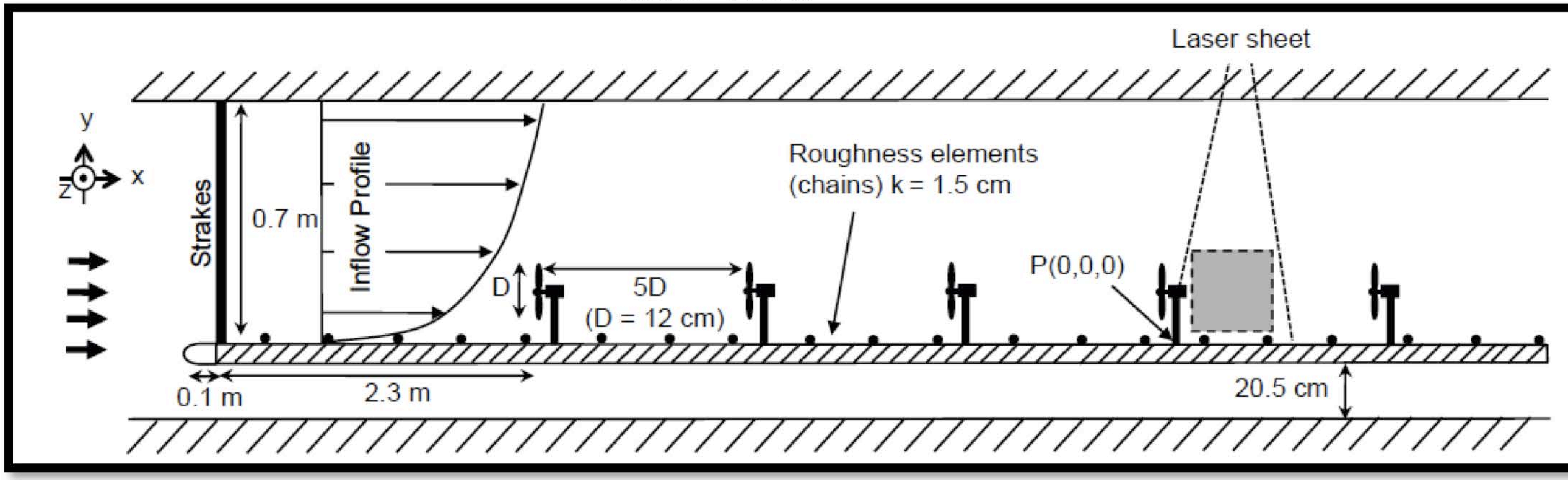




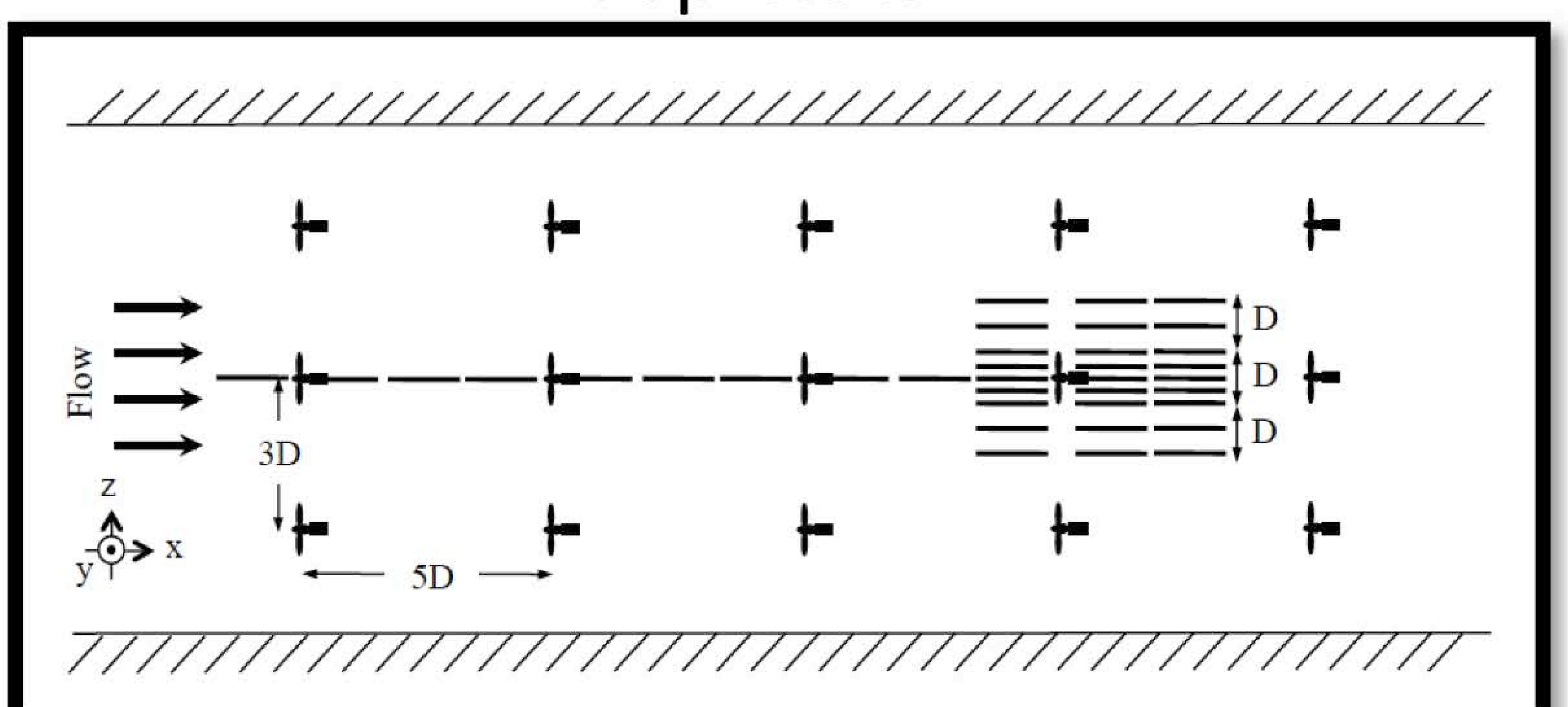
- s_x = 5D and s_z = 3D
- Tip-speed ratio = 4

PIV Measurement Locations

Side View



Top View



Particle Image Velocimetry

Δt setup to 80 μs to 150μs (faster in wake)

FOV 23 cm x 23 cm

Mirror

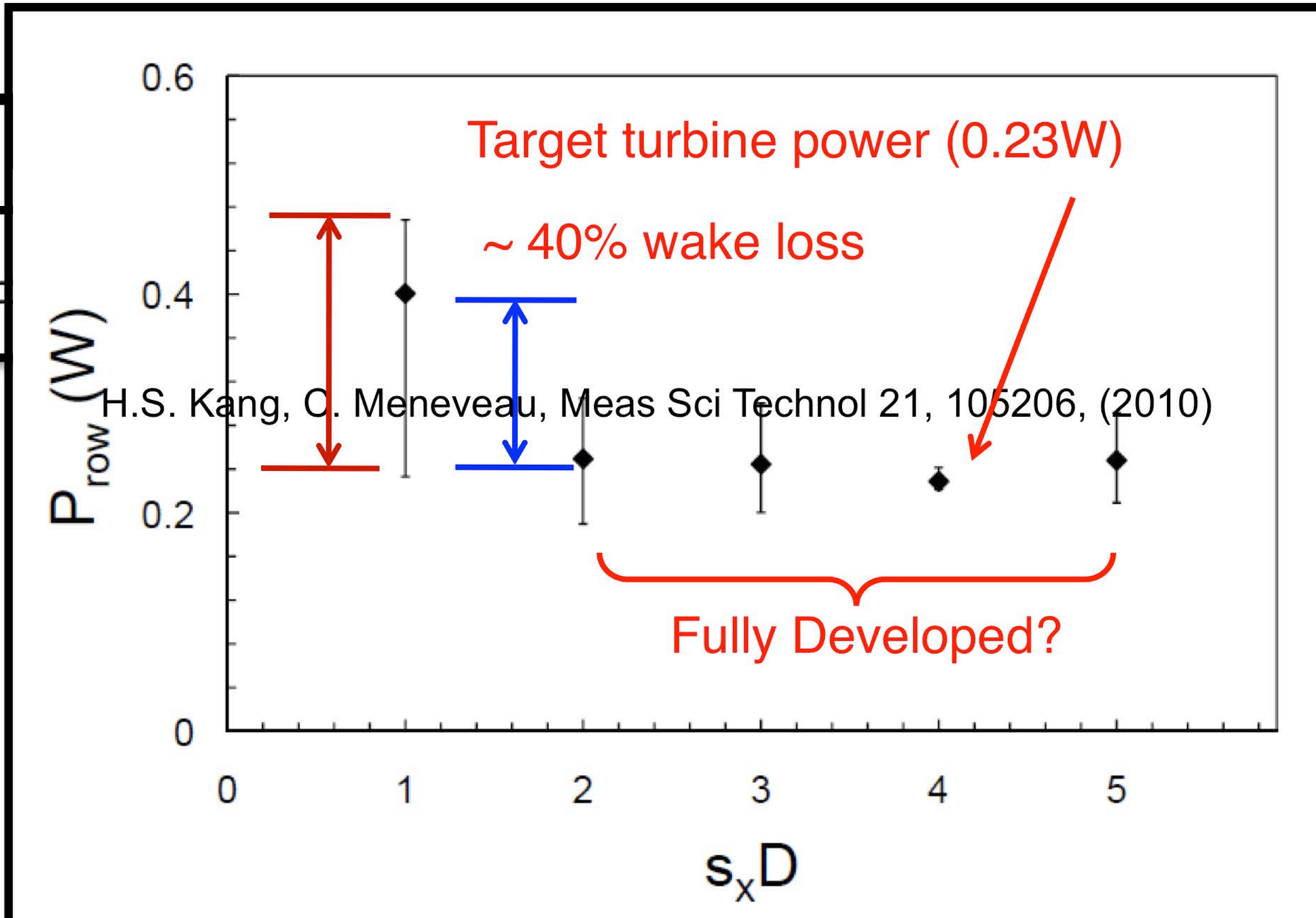
3,000 samples at 7 Hz

Laser sheet thickness 1.2 mm

Wind Turbine Models

- Rotors - water jet cut + 3D print mold
- DC motor
- 3 by 5 Array: $s_x = 5D$ and $s_z = 3D$
- Tip-speed ratio = 4

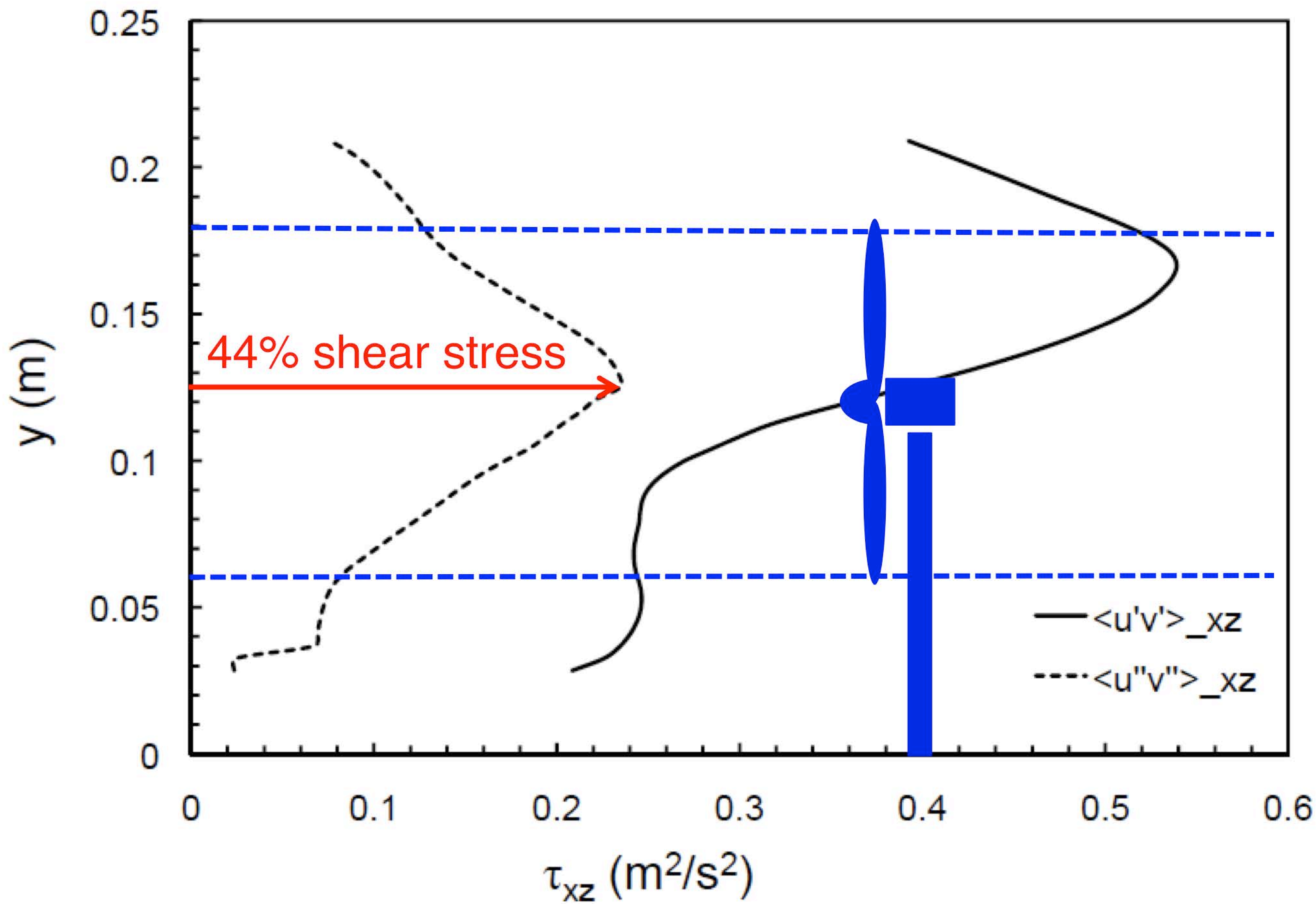
$\overline{P}_{WT} =$ … $05682\overline{I})$



H.S. Kang, C. Meneveau, Meas Sci Technol 21, 105206, (2010)

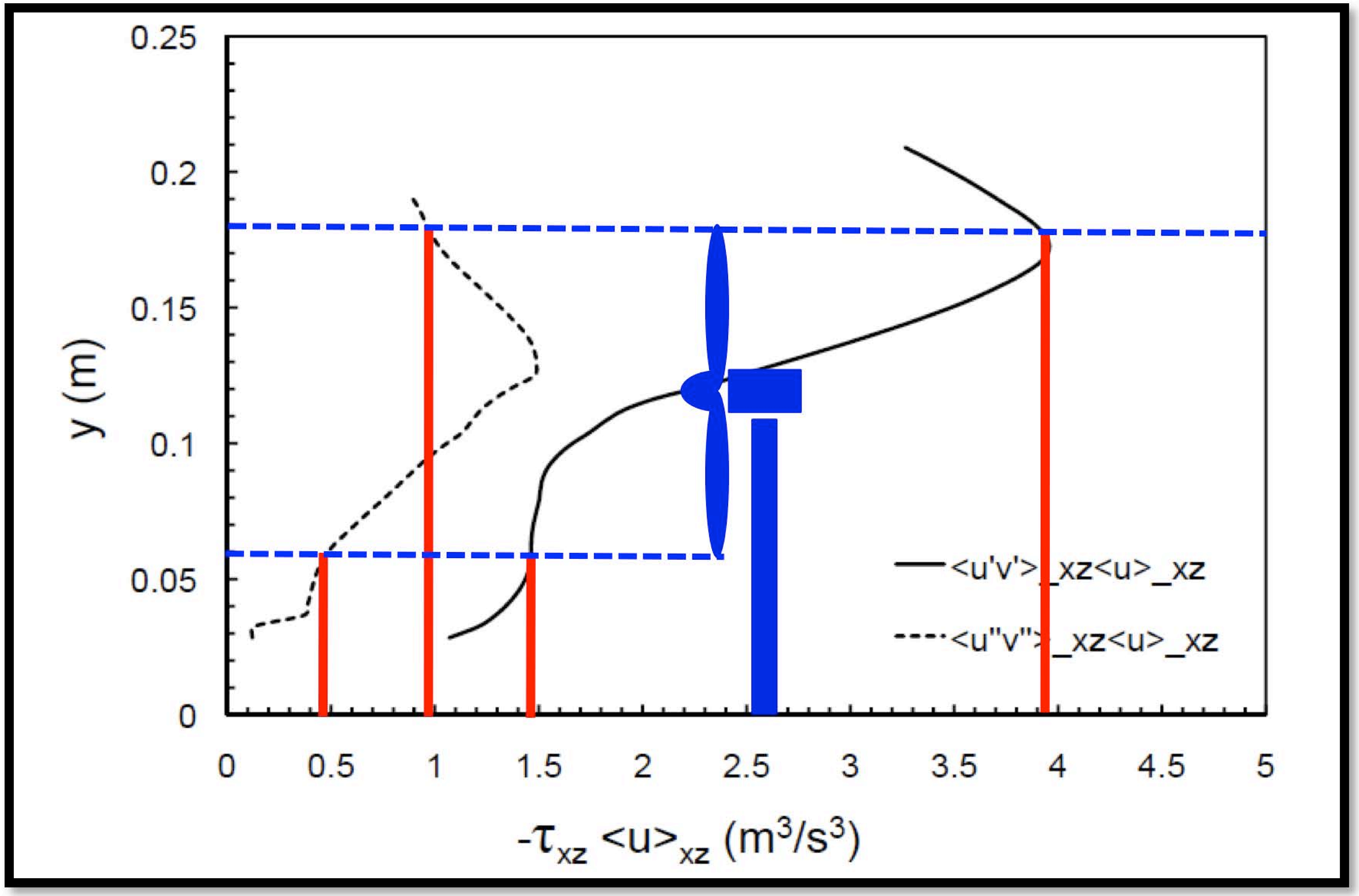
- Averaged over the duration of the experiments and over each of the 5 rows of turbines.
- Wake loss consistent with field experiments by Van Leuven (1992) and Barthelmie *et al.* (2007)

Horizontally Averaged:
Reynolds Shear & Dispersive Stresses



Fluxes of MKE due to

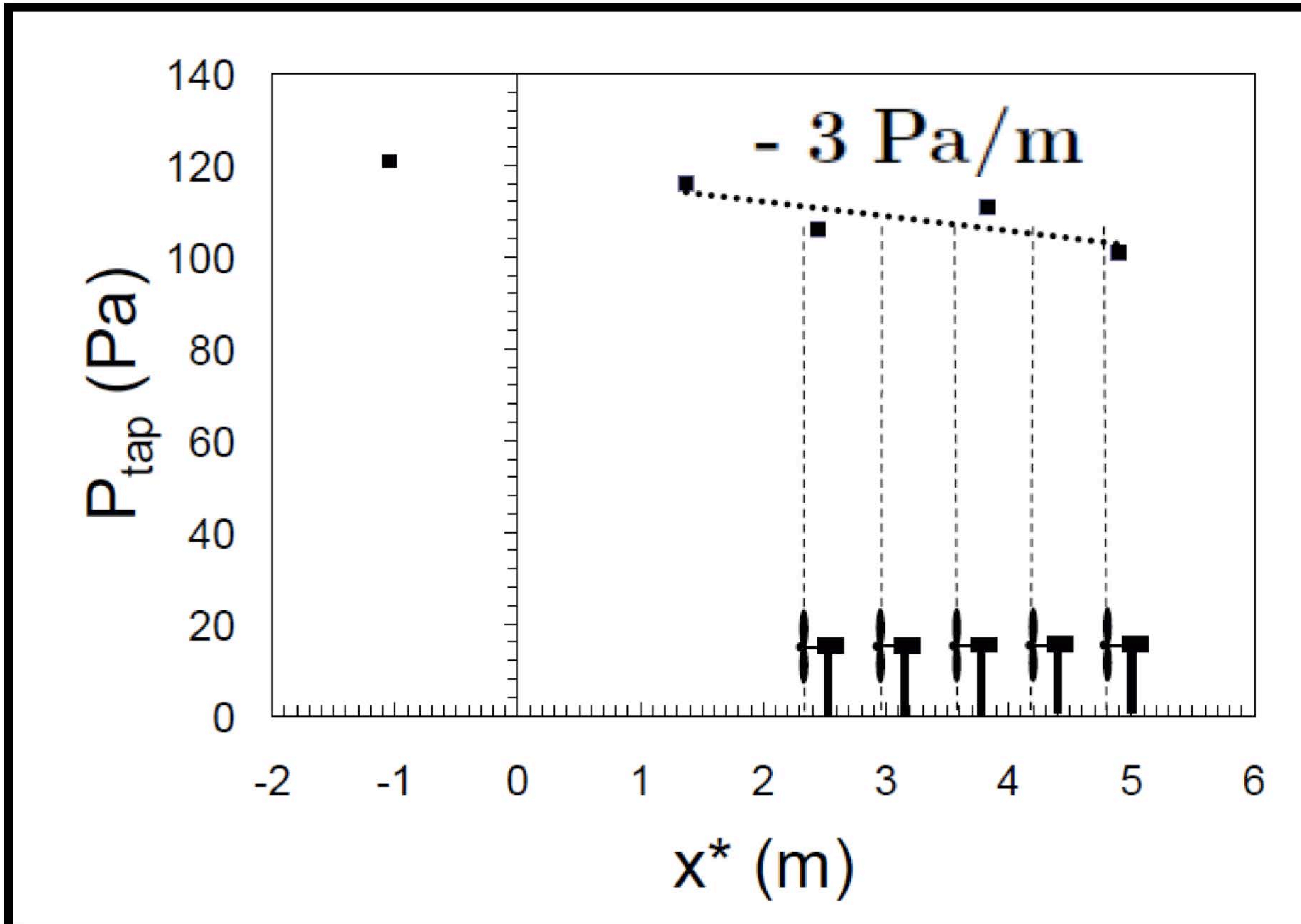
Reynolds Shear & Dispersive Stresses



$$P_{flux-shear} = \rho\,(s_x s_z D^2)\left[\langle\overline{u'v'}\rangle_{xz}\langle\overline{u}\rangle_{xz}(y_{hi}) - \langle\overline{u'v'}\rangle_{xz}\langle\overline{u}\rangle_{xz}(y_{lo})\right] = \underline{0.62\text{ W}}$$

$$P_{flux-disp} = \rho\,(s_x s_z D^2)\left[\langle\overline{u''v''}\rangle_{xz}\langle\overline{u}\rangle_{xz}(y_{hi}) - \langle\overline{u''v''}\rangle_{xz}\langle\overline{u}\rangle_{xz}(y_{lo})\right] = \underline{0.13\text{ W}}$$

MKE Contribution of the Pressure Gradient



$$P_{press} = \langle \overline{u} \rangle_{xz} \frac{-dp_\infty}{dx} s_x s_z D^3 \approx \underline{0.54W},$$

Balance of MKE

$$P_{wt} = P_{flux-shear} + P_{flux-disp} - P_{loss-shear} - P_{loss-disp} + P_{press}$$

	Flux due to	Power (W)
	$\overline{P}_{wt}$	-0.23
$P_{flux-shear}$	$< u'v' >_{xz} < u >_{xz}$	0.62
$P_{flux-disp}$	$< u''v'' >_{xz} < u >_{xz}$	0.13
$P_{loss-shear}$	$< u'v' >_{xz} \frac{d < u >_{xz}}{dx}$	-0.12
$P_{loss-disp}$	$< u''v'' >_{xz} \frac{d < u >_{xz}}{dx}$	-0.06
P_{press}	$< u >_{xz} \frac{dp_\infty}{dx}$	0.54

All terms considered are of importance, including those associated with the dispersive stress.

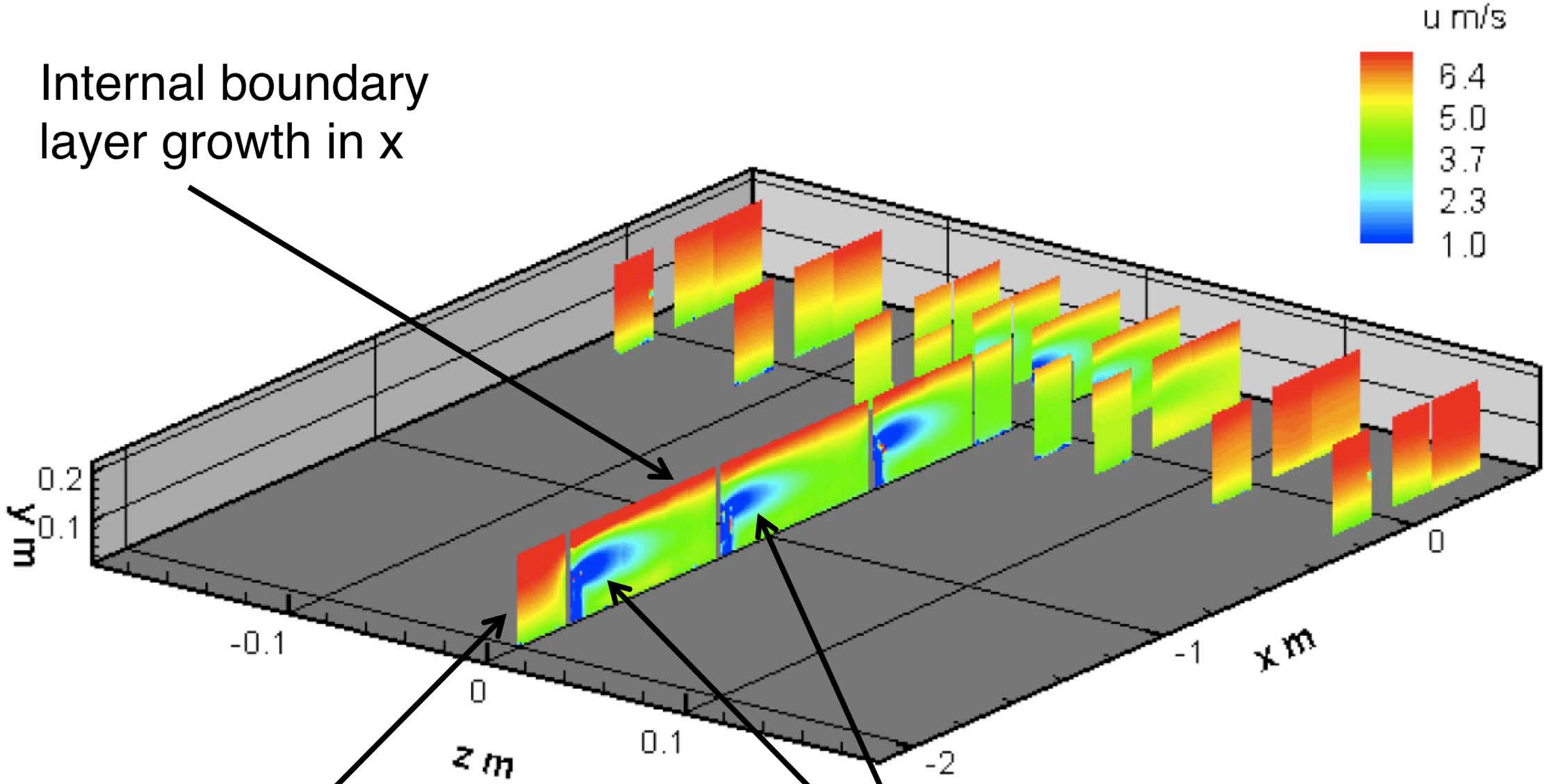
*Budget is not balanced. **Array is too small to be fully developed***

Significant contribution of advection terms is expected

Lebron, Meneveau, Castillo (JoT 2013)

Mean Streamwise Velocity

Internal boundary layer growth in x

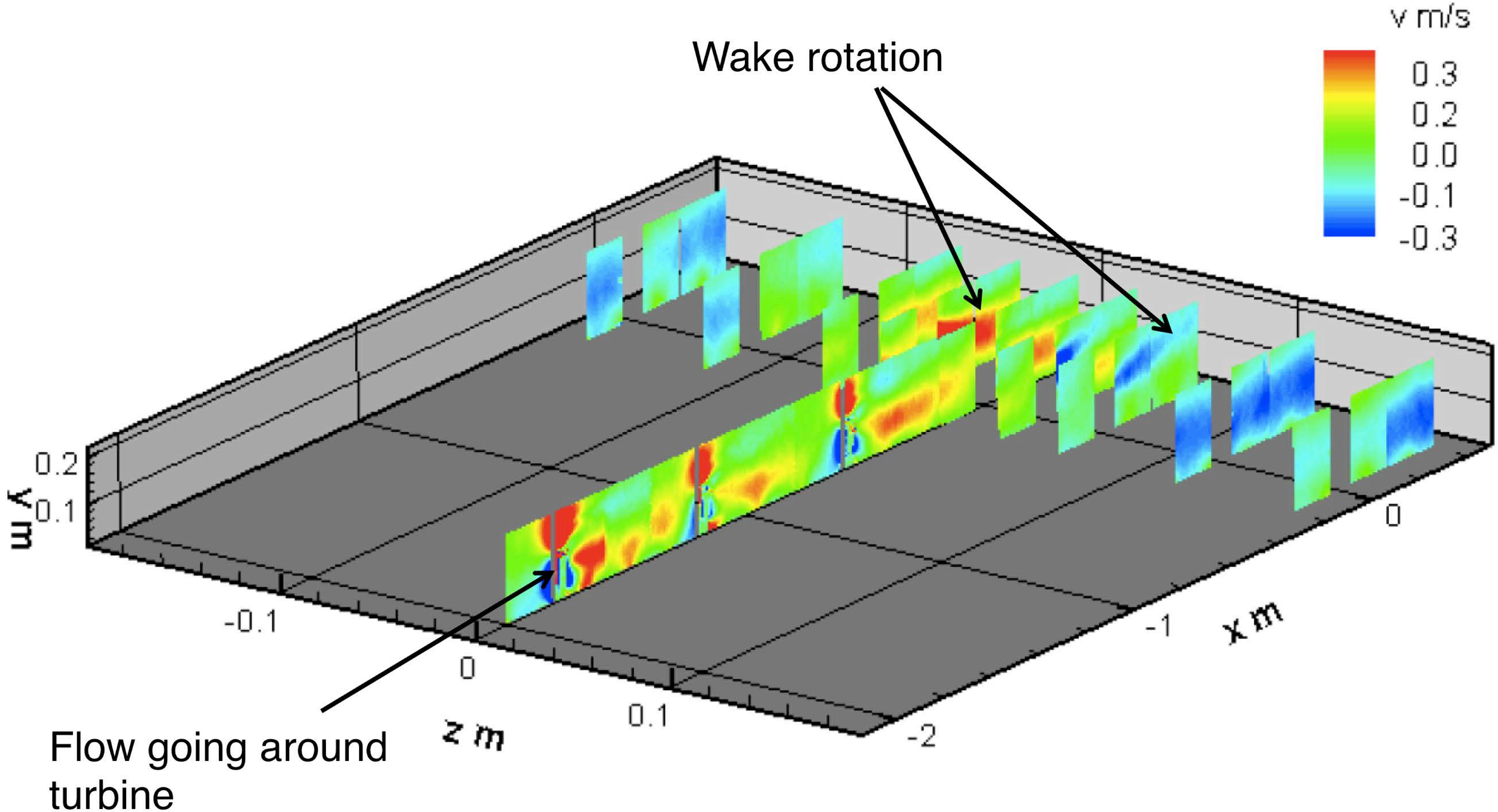


Slowdown due to turbine

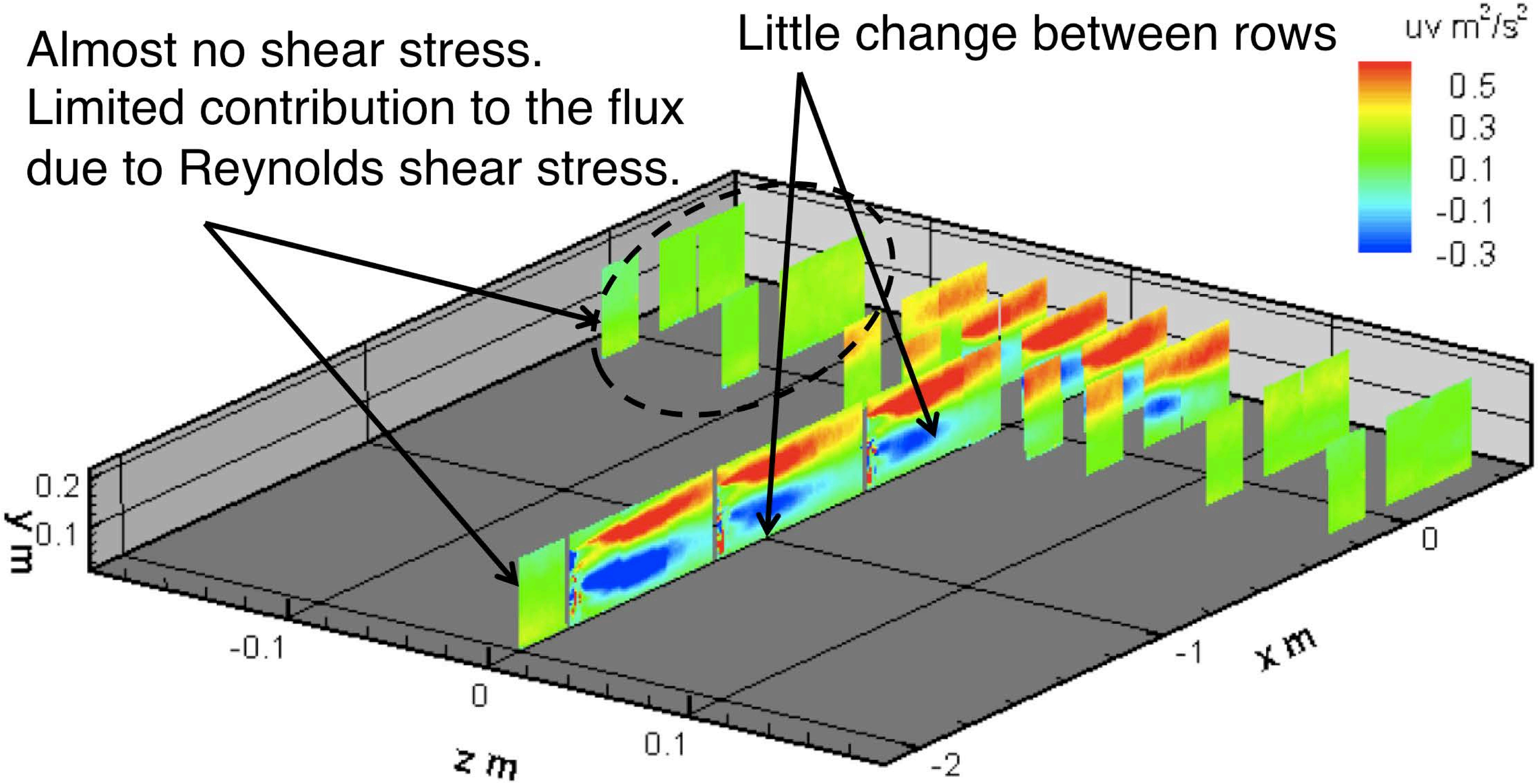
"Faster" wake recovery of downstream turbines

Contour stretched in the z-direction

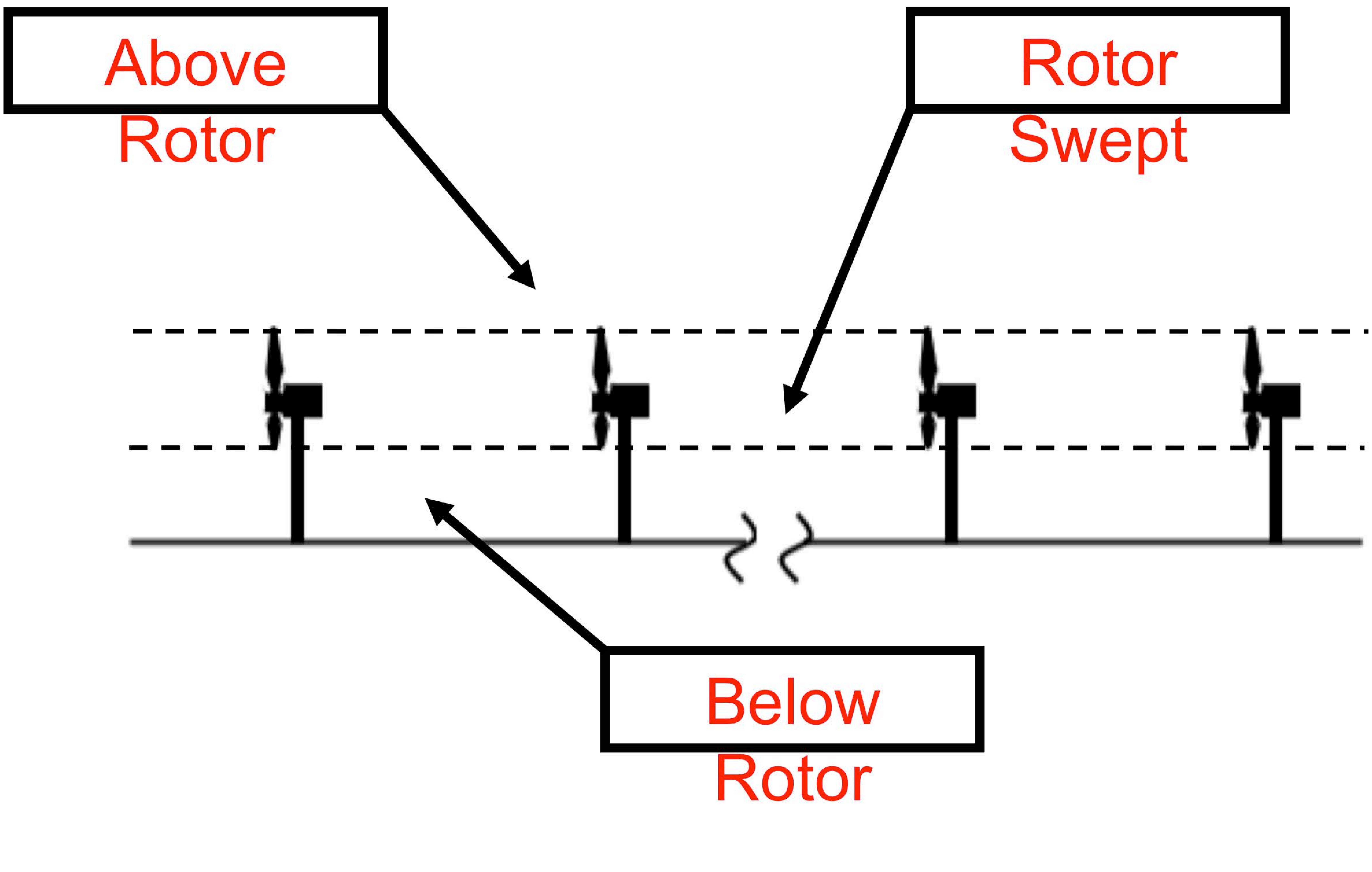
Mean Vertical Velocity



Reynolds Shear Stresses



Analysis: 3 Layer Approach



Analysis: 3 Layer Approach

Decreasing y

3 Layer Approach: Mean Streamwise Velocity

(a) U, above rotor

(c) U, rotor swept

(e) U, below rotor

Decreasing y

The dashed lines represent the envelope of the minimum and maximum values across the four inter-turbine regions.

3 Layers: Mean Wall Normal Velocity

Analysis: 3 Layer Approach

Decreasing y

Analysis: 3 Layer Approach

(b) V, above rotor

(d) V, rotor swept

(f) V, below rotor

The dashed lines represent the envelope of the minimum and maximum values across the four inter-turbine regions.

Development of Layers

$$\frac{\sqrt{\dfrac{1}{A_{12}}\iint |U_1(x,y) - U_2(x,y)|^2 \, dxdy}}{\sqrt{\dfrac{1}{A_0}\iint |U_0(x,y)|^2 \, dxdy}}$$

Analysis: 3 Layer Approach

parameter		difference norm between turbines		
		Turbines 1-2	Turbines 2-3	Turbines 3-4
U	AR	0.063	0.027	0.04
	RS	0.049	0.032	0.041
	BR	0.094	0.045	0.065
V	AR	0.297	0.550	0.485
	RS	0.377	0.385	0.256
	BR	0.494	0.590	0.246
$\langle u'u' \rangle$	AR	1.57	0.794	0.292
	RS	0.232	0.286	0.138
	BR	0.156	0.128	0.093
$\langle u'v' \rangle$	AR	2.55	2.05	0.977
	RS	0.678	0.412	0.272
	BR	0.256	0.179	0.155
$\langle v'v' \rangle$	AR	0.642	0.931	0.344
	RS	0.805	0.615	0.287
	BR	0.457	0.2247	0.146
$U \langle u'v' \rangle$	AR	2.25	1.58	0.679
	RS	0.466	0.311	0.188
	BR	0.251	0.157	0.129

increasing x

increasing y

increasing x

decreasing y

increasing x

decreasing y

Summary

- ***The importance of fluxes by the Reynolds shear stress*** **has been reinforced in the energy entrainment.**
- **Residual of the budget of mean kinetic energy fluxes is not zero.**
  - ***Array is not fully developed.***
- The *mean streamwise velocity approaches a fully developed state the most rapidly*.
  - the mean streamwise velocity develops quickly in the above-rotor region.
  - more gradually below the rotor region, most likely due to contributions to the development from both the wall and the turbines.
- Low development is more gradual for ***the second-order statistics***.

Low Dimensional Analysis

Apply **POD Analysis** to scaled wind turbine data

↓

Assess modal contributions to **MKE entrainment**

↓

Derive **characteristic length scale** for each mode

↓

Determine **MKE contribution from the large scales**

Low Dimensional: POD

- **Mean Kinetic Energy Equation**:

$$\iiint E\,dV + \iint \left(\langle U_i \rangle E + \underline{\langle U_j \rangle \langle u'_i u'_j \rangle} + \langle U_i \rangle \frac{\langle p \rangle}{\rho} - 2\nu \langle U_j \rangle S_{ij} \right) n_i\,dA - \iiint (P + \varepsilon)$$

- Following Cal *et. al*. (2009) we examine the **vertical flux of kinetic energy** into the array:

$$-\iint \langle U_1 \rangle \langle u'_1 u'_2 \rangle + \langle U_2 \rangle \langle u'_2 u'_2 \rangle\, dA$$

2nd order terms

Low Dimensional

- $-\langle U_1 \rangle \langle u'_1 u'_2 \rangle$ is the term we presently analyze in detail.
- Using **2D PIV data** from a scaled wind turbine array, we have this term as a function of x and y.

POD of MKE Fluxes

- Newman *et al. (2012):* Time shifted PIV data *allows for* ***modal expansions of the Reynolds Stresses****, but not velocity fields.*

<u>Key point</u>:

There exists a ***modal expansion for the Reynolds Stresses, applicable for non-homogenous flow****.*

A methodology will be introduced ***to determine modal length scales, based on Coherence Energy Transfer.***

Low Dimensional: POD

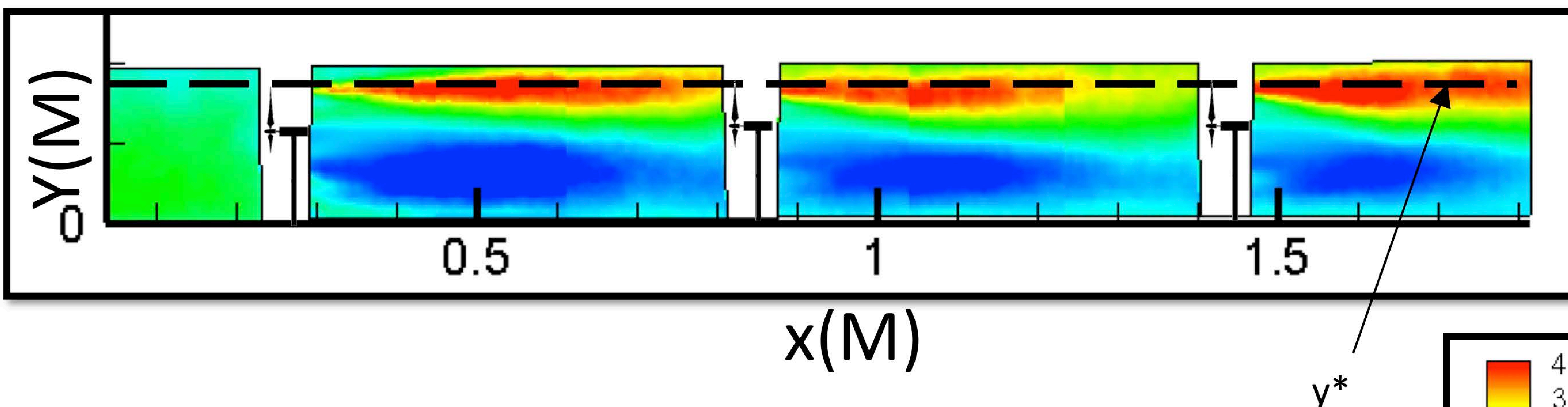
- **Reynolds stresses modal expansion**:

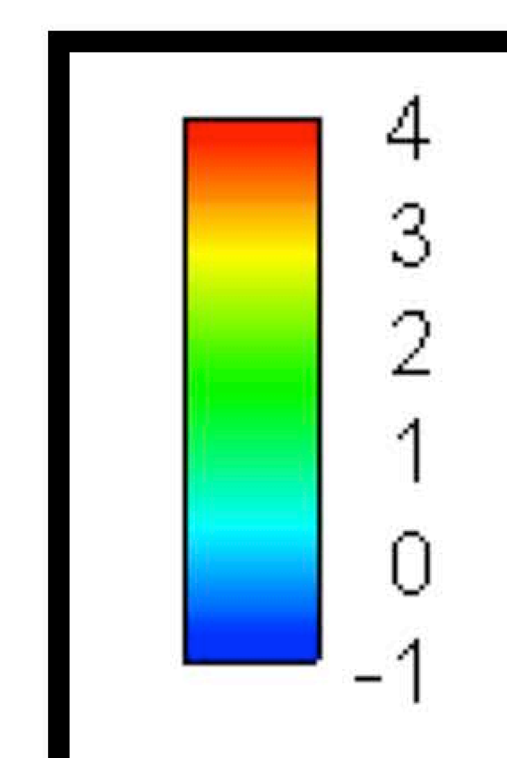
$$\left\langle u_i\left(\vec{x},t\right)u_j\left(\vec{x},t\right)\right\rangle = \sum_{n=1}^{N_t} \lambda^n \phi_i^n\left(\vec{x}\right)\phi_j^{n*}\left(\vec{x}\right)$$

Eigenfunctions computed from PIV data

Results: MKE Fluxes (Full Field)

- **Flux contributions**: integrate modes in the streamwise direction at the highest vertical point of a turbine blade tip, y*





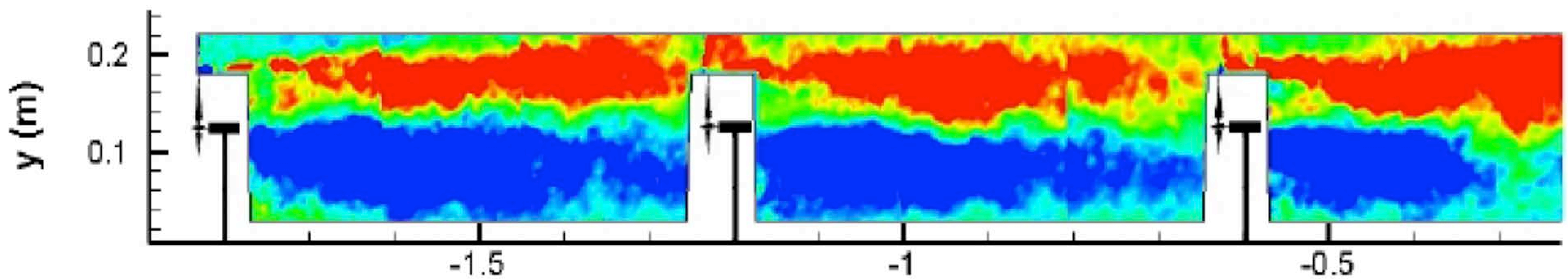
-<U><u’v’> Full Field

3,000 time steps used in averaging

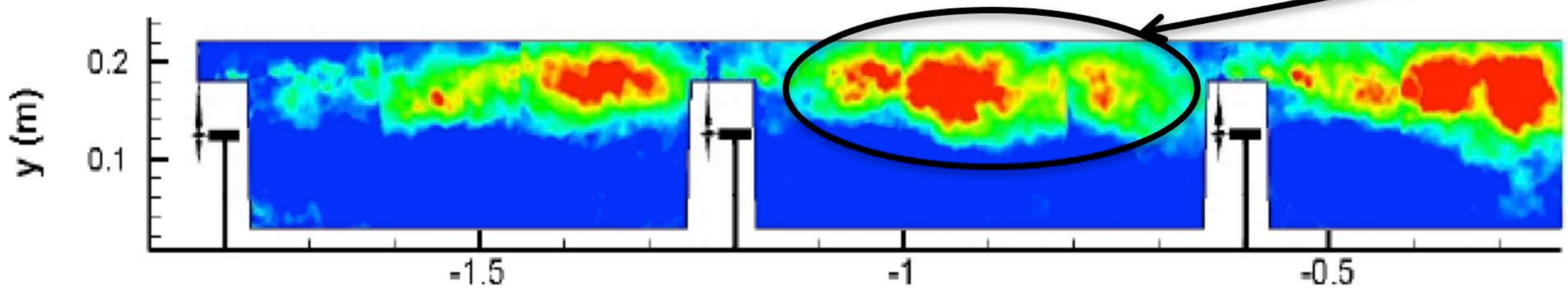
Newman, Drew, Castillo (2013, submitted)

Modal Visualizations:

Reynolds Shear Stress

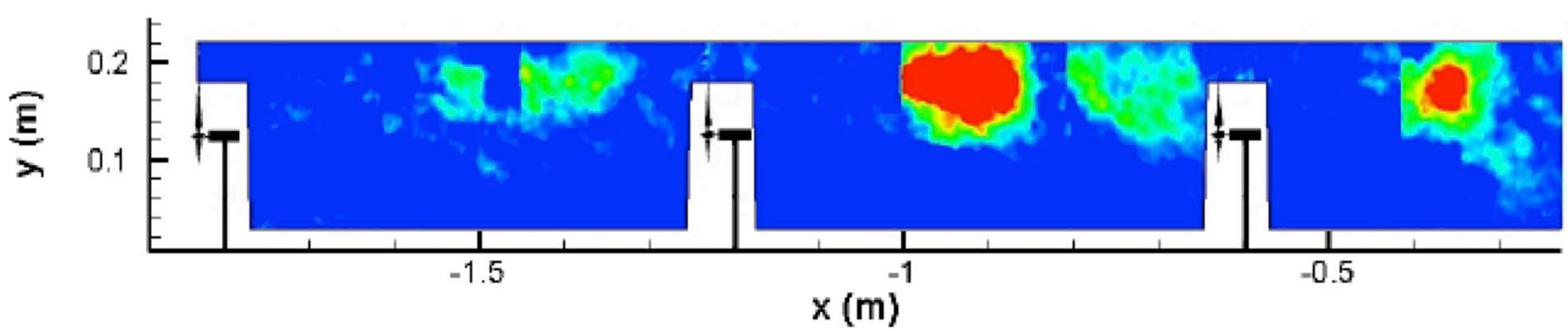


Full $-\langle u'v' \rangle$ field



13 mode reconstruction of $-\langle u'v' \rangle$ field

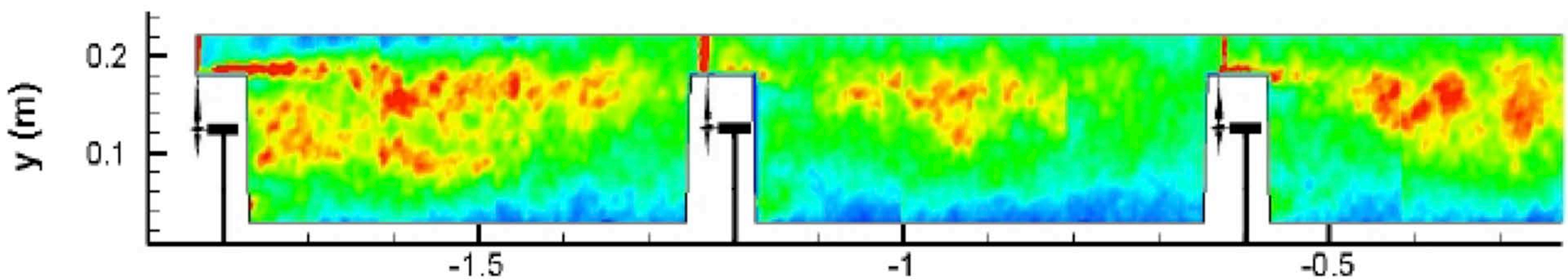
Regions of intense Reynolds shear stress with large stream-wise dimension



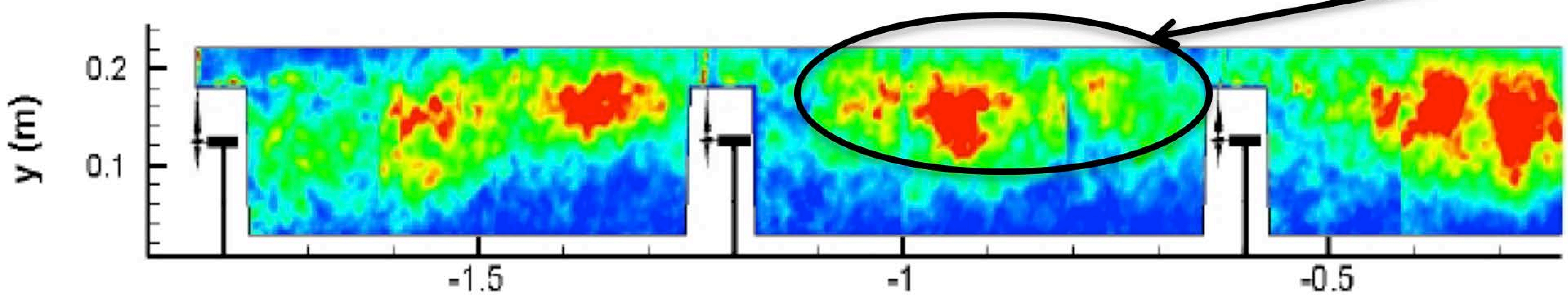
$-\langle u'v' \rangle$ mode 1

100 time steps used in averaging

Modal Visualizations:
Reynolds Wall-normal Stress

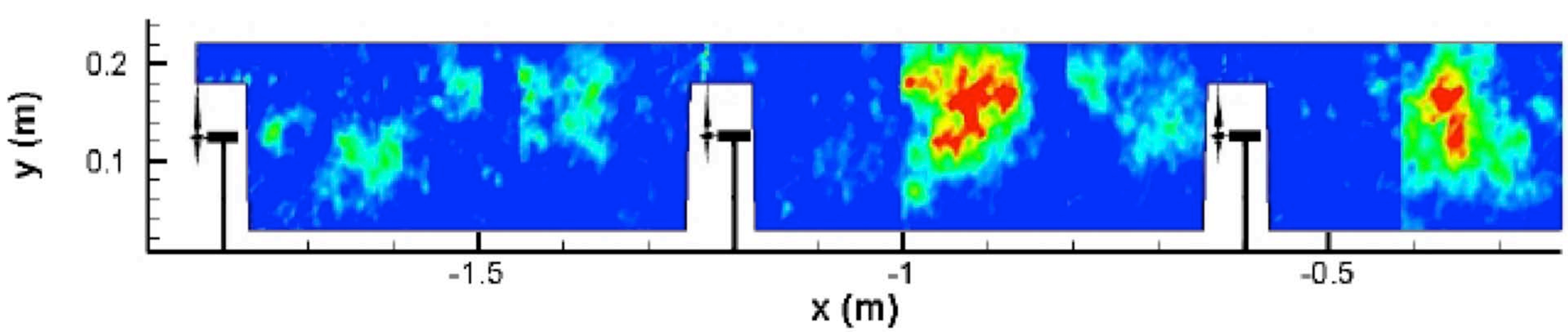


Full $\langle v'v' \rangle$ field



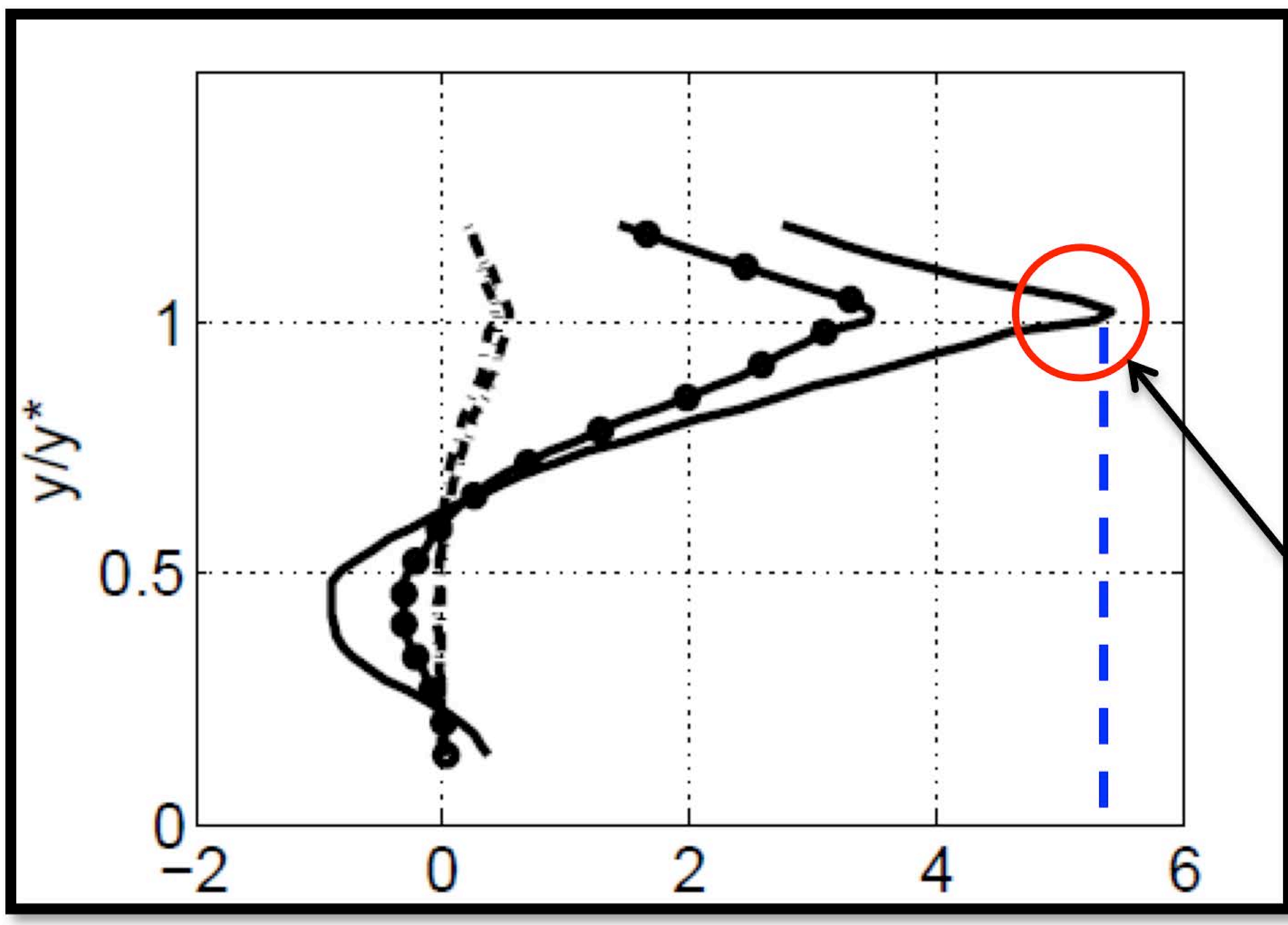
13 mode reconstruction of $\langle v'v' \rangle$ field

Regions of intense Reynolds wall normal stress with large stream-wise dimension

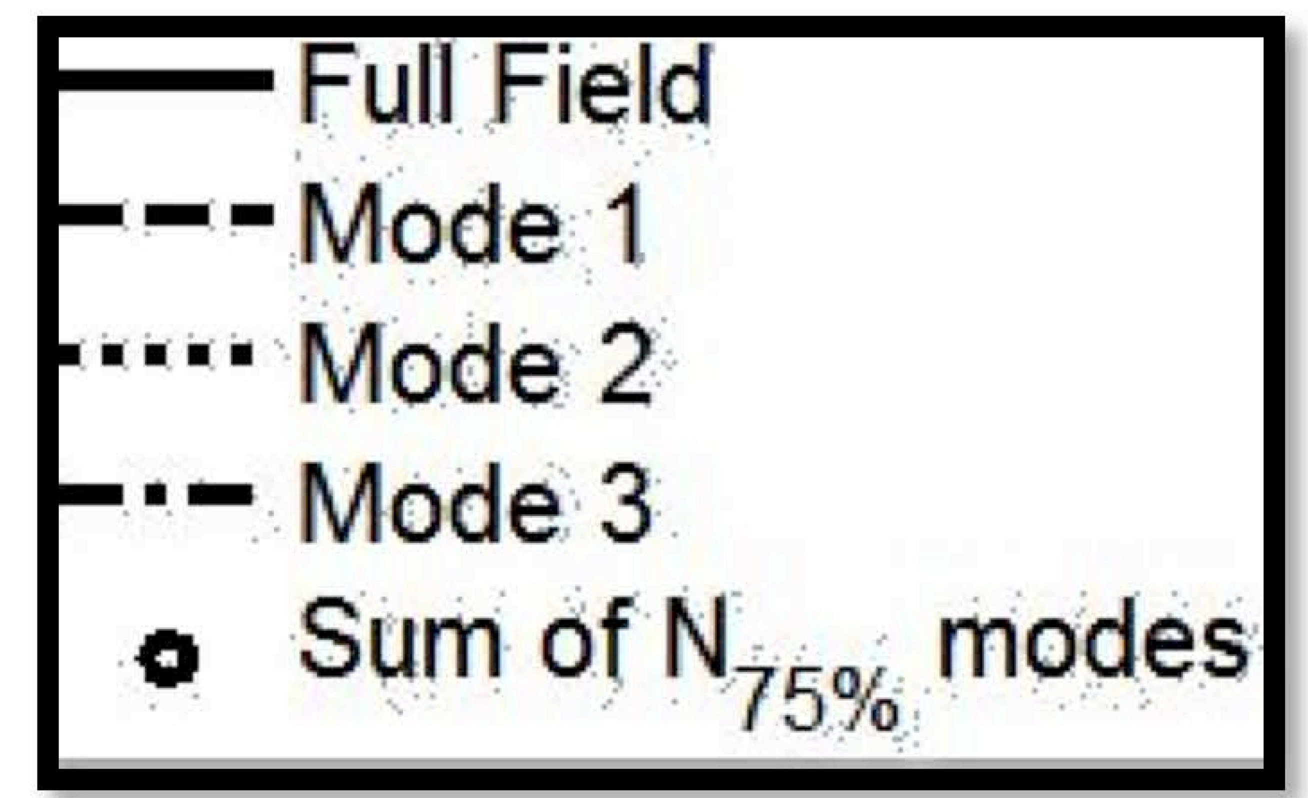


$\langle v'v' \rangle$ mode 1

Entrainment or Extraction of MKE



$$-\langle U\rangle\langle u'v'\rangle$$



The shear term is positive at y*
 - entrainment

Modal Entrainment

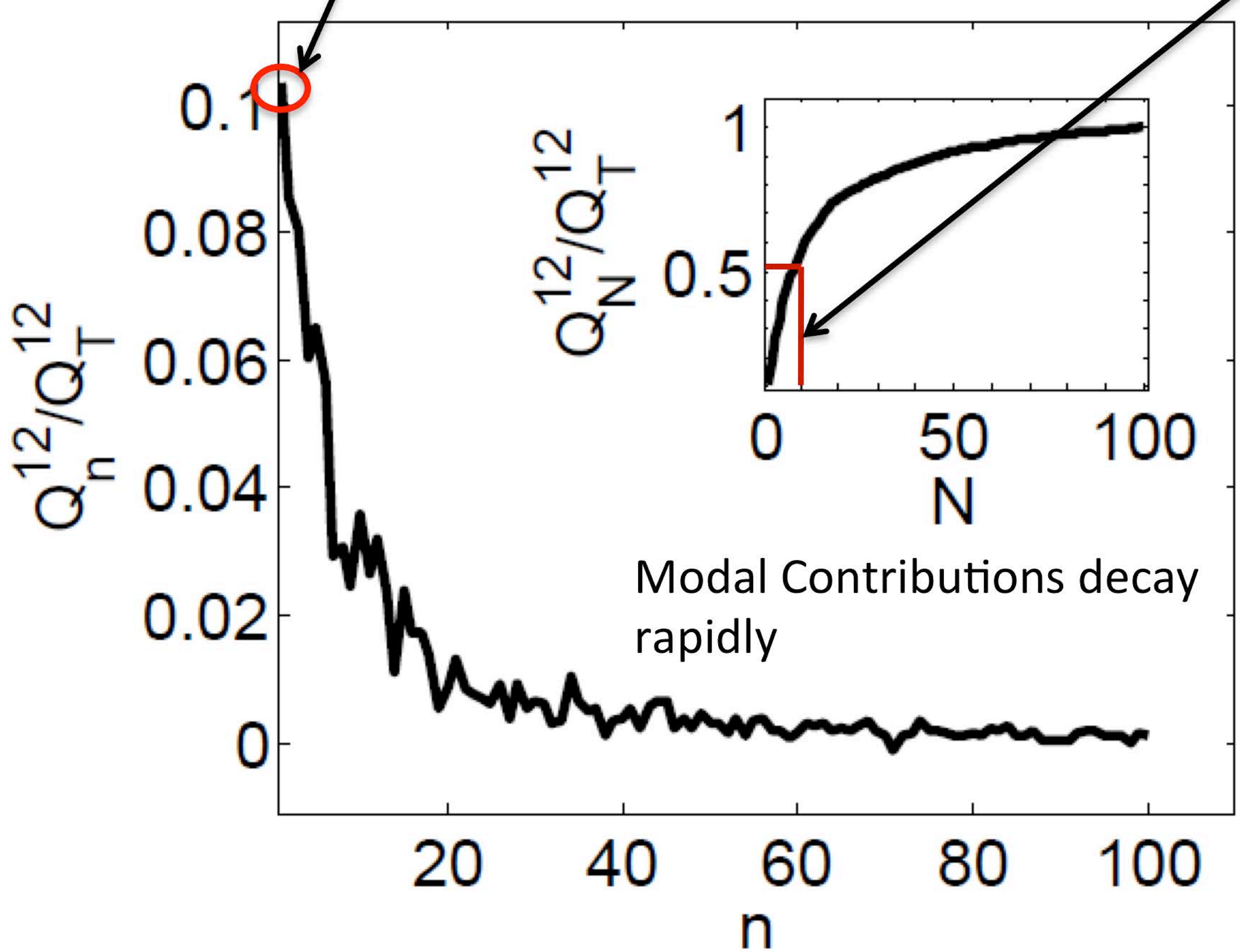
$$Q_{12}^{n} = \int_{L_x} \langle U(x, y^*, t) \rangle \lambda^{n} \phi_1^{n}(x, y^*) \phi_2^{n}(x, y^*) dx$$

$$Q_T^{N} = \int_{L_x} \langle U(x, y^*, t) \rangle \langle u'(x, y^*, t) v'(x, y^*, t) \rangle dx$$

$$Q_{12}^{n} / Q_{12}^{T}$$

Modal MKE Entrainment (Shear)

First mode accounts for about 13% of the total entrainment



Modal Contributions decay rapidly

Sum of 1st 9 modes accounts for 54% of the total entrainment

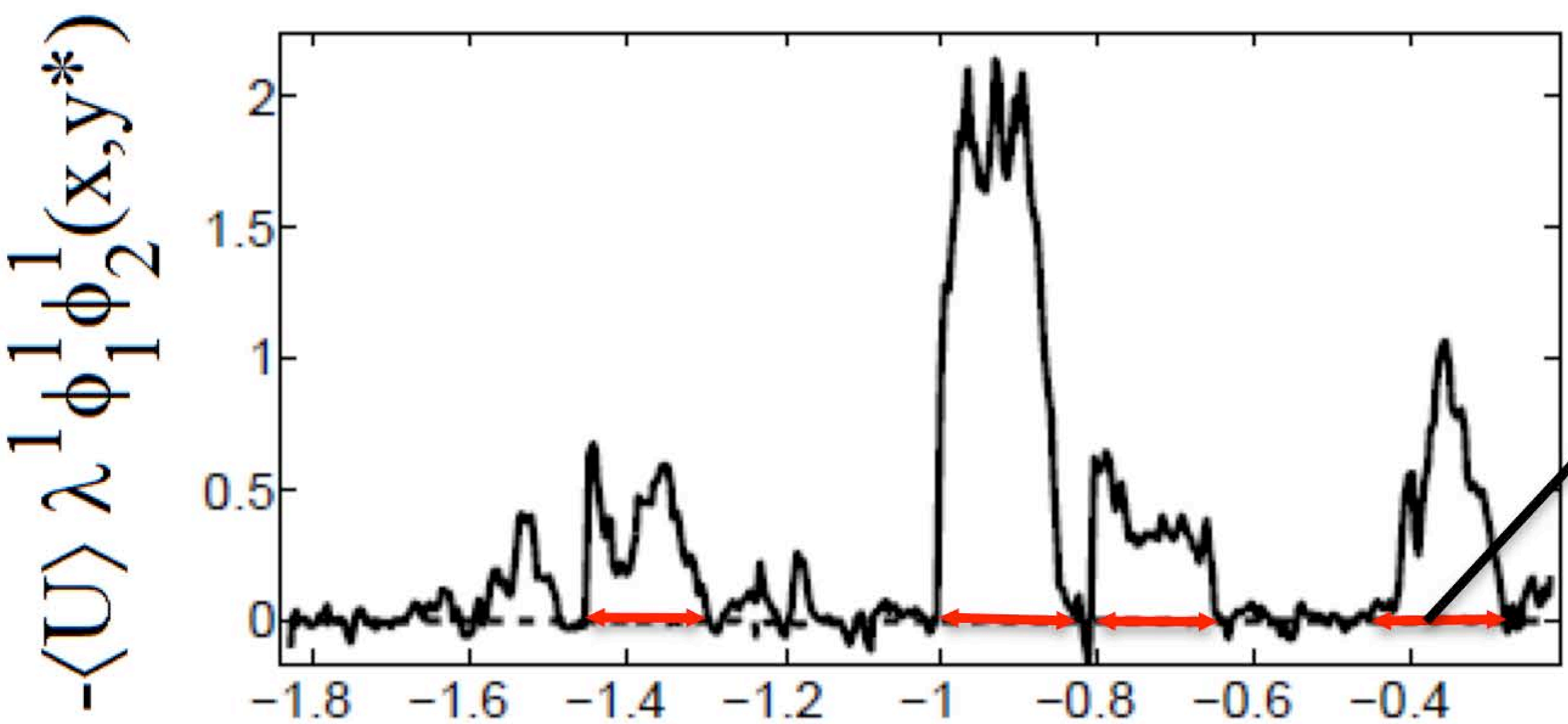
n is individual mode number

N is sum of first N modes

Modal Length Scales:

Coherent Energy Transfer

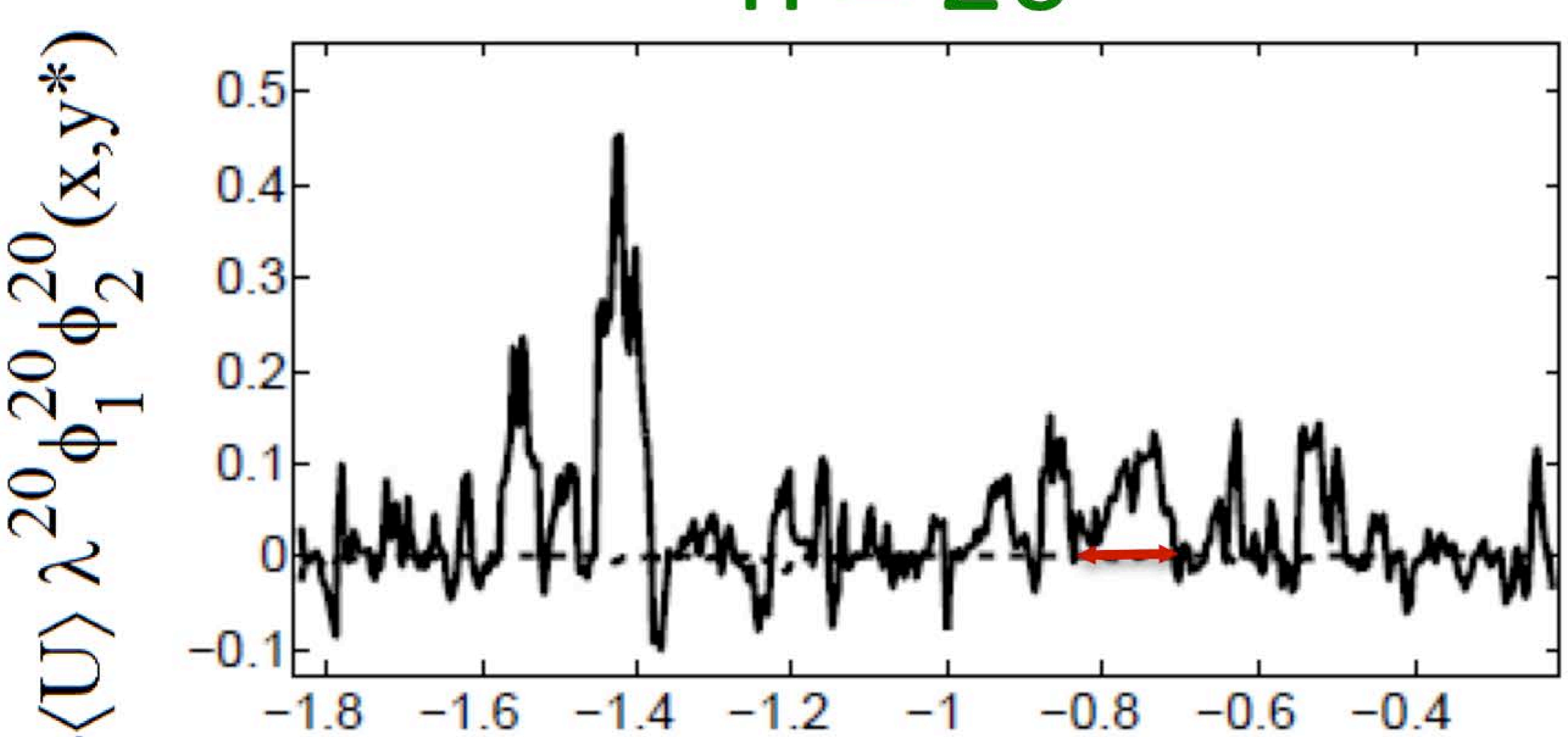
n = 1



x (m)

Coherent Energy Transfer
- Choose Largest as **characteristic length**

n = 20

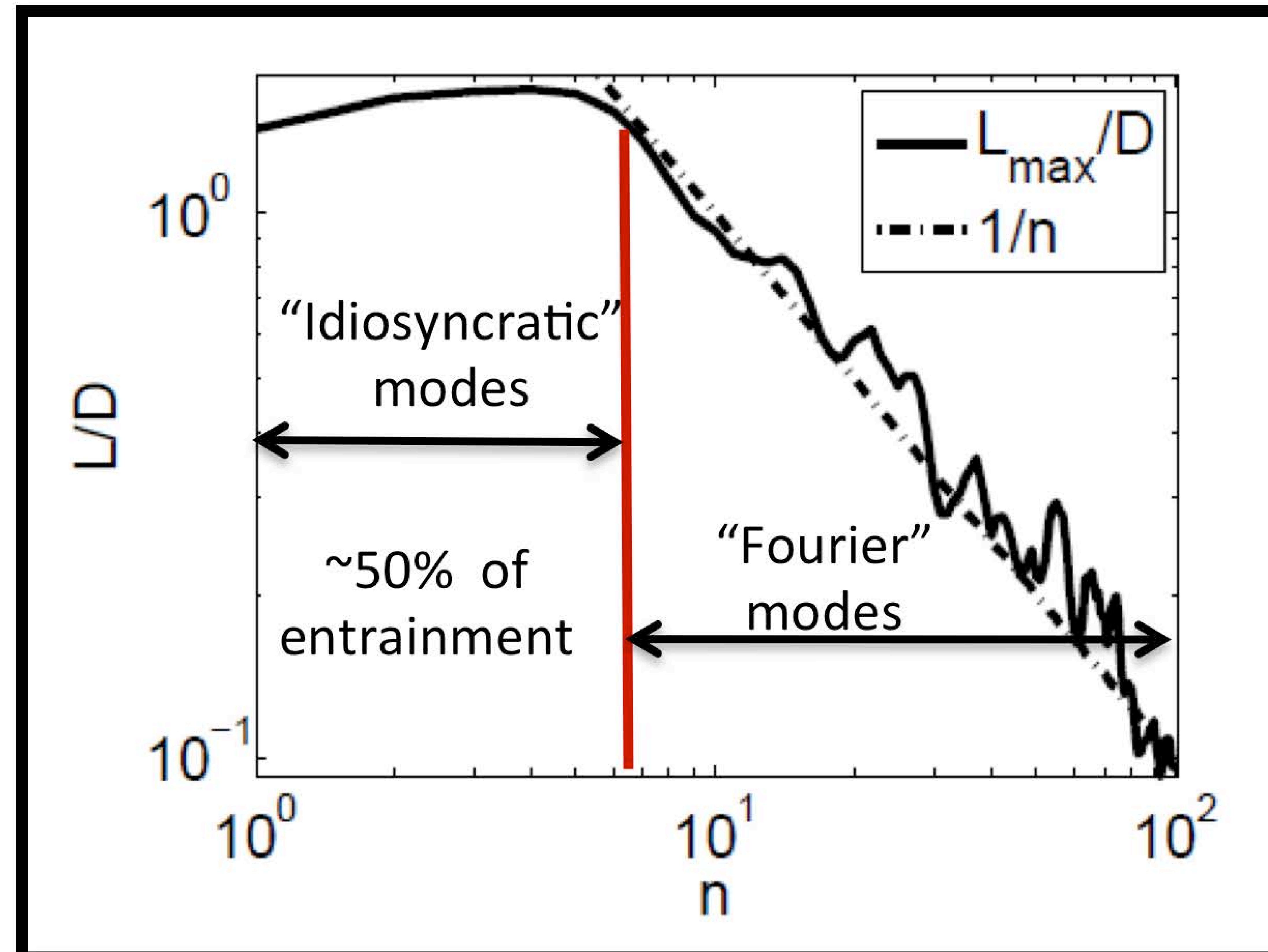


x (m)

Modal Length Scales

Large Scales: In-homogeneous, and contain most of the energy for the wind farm

Small Scales: Homogeneous, decay as 1/n, contribute to little energy



Higher modes have approximately same modal length decay as Fourier modes.

Also observed by Baltzer & Adrian (2011), for different length scale definition

Baltzer, J., Adrian, R., "Structure, scaling and synthesis of proper orthogonal decomposition modes of inhomogeneous turbulence", Physics of Fluids 2011; **23**

Modal Length Scales

$$\sum_{n=1}^{9} \lambda^n \phi_i^n(\vec{x}) \phi_j^{n*}(\vec{x})$$

L = 13D (max length in exp. Data)

54% of entrainment

Conclusions

Major contributions to the ***MKE entrainment are achieved by large scale*** motions associated with sums of Re shear stress (idiosyncratic) modes

Sum of first 9 modes: 54% Entrainment, L = 13D

further

These modes had L > D and can be physically interpreted to represent the large scale motions.

Conclusions (cont.)

The Wind Farm is not fully developed

By demonstrating ***the importance of the Reynolds Stresses*** ***and*** then showing these are associated with larger scales motions

Future research should focus on manipulation of these large scales as opposed to small scale manipulation