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CLASSIFICATION OF END PLATE CONNECTIONS WITH APPLICATION TO GABLE FRAMES

by

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(ABSTRACT)

In this study, connection classification system is developed on the basis of previous classifications. Further, flexible connections are modeled by matrix displacement method. The effect of flexible connections are studied on gable frames.

Firstly, flush end-plate connections with single row of bolts at the tension flange, are classified. The classification system was developed in this study. End plate connections whose moment-rotation curves are known are classified on the basis of moments as FR (fully restrained) and PR (partially restrained) connections. Further, the connections are also classified by entering a plot with coordinates- Ratio of Moment at the connection and plastic moment and ratio of corresponding rotation and rotation at plastic moment. Depending on the location the connection can be classified.

Secondly, for connections, the rotational stiffness is determined from the moment rotation curves and used in the computer code to implement flexible connections. The effect is studied on gable frames. For the loading and frame used there is not much variation in moments at the flexible joints due to connection flexibility and hence flush end plate connections can be used in gable frames effectively.

Acknowledgements

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NOMENCLATURE

A_B = area of beam section

b_p = width of end plate

d_b = nominal bolt diameter

E = Youngs modulus of elasticity

F_{by} = yield stress of beam material

F_{yb} = yield stress of bolt material

g_b = width of equivalent rectangular bolt area

M_y = yield moment of the beam cross section

M_p = plastic moment of beam cross section

p_f = pitch of bolt (top of flange to cl of bolt row

t_f = beam flange thickness

t_p = thickness of end plate

t_w = web thickness

L = length of the beam

δ = end plate separation

θ = connection rotation

Chapter I

INTRODUCTION

1.1 INTRODUCTION

The purpose of this study is to develop a classification method for flush moment end-plate connections and to study the moment variation and deflection of partially restrained connections in gable frames. Hence, the types of connections presently used in practice will be discussed, followed by a description of end plate connections. Connection behavior and classification as per AISC specification will be discussed next. Later, an introduction to gable frame and classification of connections as per the AISC LRFD specification, (1986), will also be discussed.

Unlike reinforced concrete or prestressed concrete structures, metal structures consist of an assemblage of members and joints. It has been observed that the member behavior could be explained satisfactorily by existing structural analysis methods such as slope deflection method, principle of virtual work, or by generalized matrix methods, in spite of complicated structures and varied loading arrangement. However, the theories established for the members have been found inapplicable for the joints or connections. The connections have been found to behave vastly different due to the connection material and joint detailing. Local restraints have been found to affect the joint deflection and the resulting stresses. Hence, connection design has been mainly based on experimentation and empirical relations.

The ideas motivating study of PR connections are multifarious. It has been observed that as a common practice in buildings, the connections are designed as fully flexible connections as far as gravity loads are concerned. Further, the connections are designed to resist the moments generated due to lateral loads like wind or seismic loads, etc. The above is necessitated in order to attain overall stability of the frame. Now, although the connections were not designed as moment connections for gravity loads they will possess certain amount of restraint. It is a matter of conjecture whether connections designed thus are safe. Another school of thought may be that, since certain amount of restraint will have to be provided for the lateral load, the resistance may be considered for gravity loads also and save on beam and column sizes. Moreover, partially restrained connections give us insight into an important aspect of structural behavior.

Fully restrained connections are heavy and are more expensive in order to maintain the original angle between intersecting members. At the same time fully flexible connections are difficult to achieve. As an example, we may consider a beam with fully restrained ends. Considering elastic behavior beam end moments would be $WL^2/12$ and the midspan moment $WL^2/24$. With fully flexible connections the midspan moment would be $WL^2/8$. However, if semi rigid connection is intended, the end and midspan moment could be balanced at $WL^2/16$ after allowing adequate rotation at supports.Thus, it can be concluded that the study of PR type connections may be found to be quite productive.

1.1.1 END PLATE CONNECTIONS

This category of connection consists of a plate shop welded to the end of a beam. Rows of high tensioned bolts are then used to fix the beam to a column or another beam at site. This type of connection is economic, easy to erect and fabricate. They are used as moment resisting connections. There are two basic types of moment end plate configurations which are as noted below.

A. Flush End Plate Connections: This is of the type in which the plate at the end of the beam is flush with the beam flanges. They can be further subdivided into the following categories with self evident definitions.

i) Two-Bolt Unstiffened Flush End Plate Connection.

ii) Four-Bolt Unstiffened Flush End Plate Connection.

iii) Four-Bolt Stiffened Flush End Plate Connection with web gusset plate between tension row of bolts.

iv) Four-Bolt Stiffened End Plate Connection with web gusset plate outside the tension row of bolts.

The above connections are shown in Figures 1(a), (b), (c), and (d).

B. Extended End Plate Connections: This is of the type in which the welded plate at the end of the beam projects beyond the flanges of the beam. The various configurations are as noted below.

i) Four-Bolt Unstiffened Extended End PLate Connection.

ii) Four-Bolt Stiffened Extended End Plate Connection.

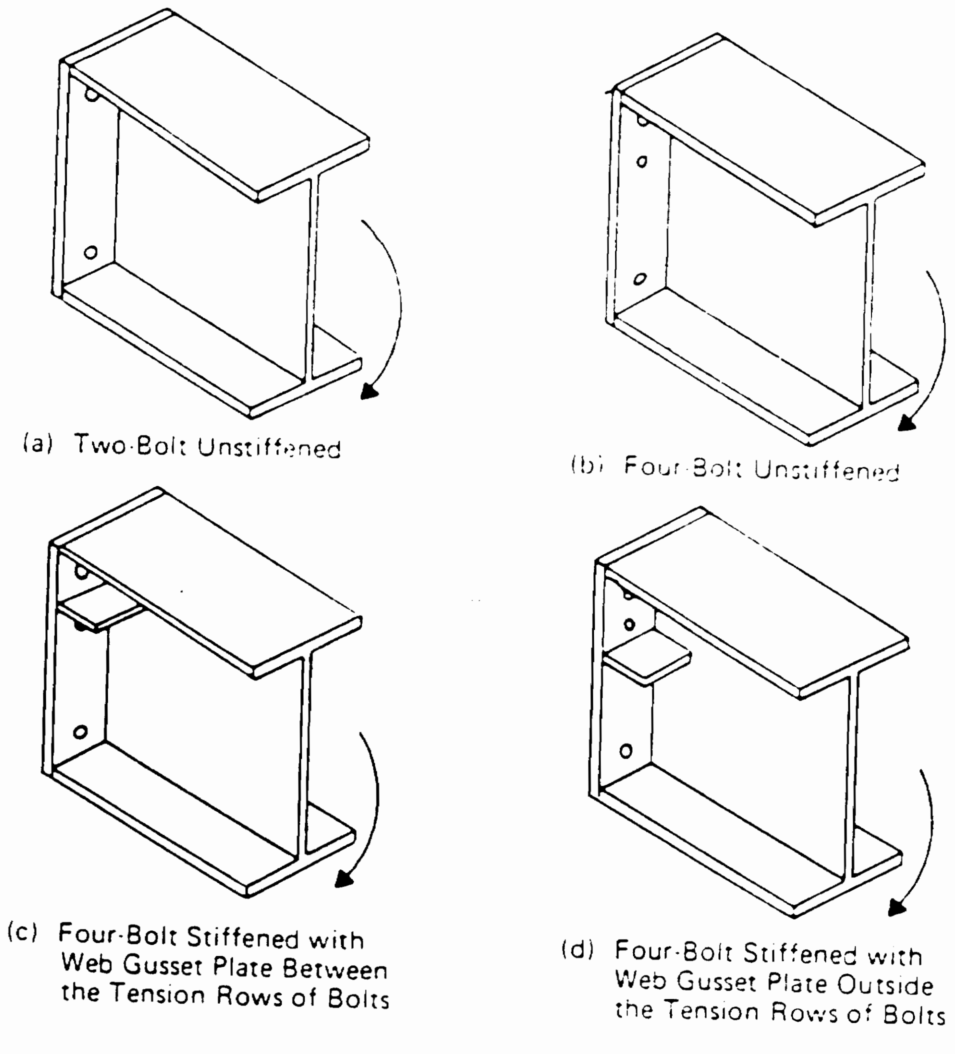


Figure 1. Flush End Plate Connections: (a) Two bolt unstiffened (b) Four bolt unstiffened (c) Four bolt stiffened with web gusset plate between the tension row of bolts (d) Four bolt stiffened with web gusset plate outside the tension row of bolts

iii) Four-Bolt Wide Unstiffened End Plate Connection.

iv) Eight-Bolts Stiffened Extended End Plate Connection.

The connections are shown in Figures 2(a), (b), (c), and (d).

1.1.2 CONNECTION BEHAVIOR AND AISC CLASSIFICATION

The complexities of connections have been mentioned earlier. However, the type of connection as a whole has been found to govern the interaction of the members. The overall stability of a frame depends on the connection type. Hence it is imperative that a clear insight is obtained into the behavior of the joints. Any connection designed to resist one type of load may develop secondary effects rendering it unsafe or unacceptable by the serviceability requirements. For example, introduction of a flexible connection in a portal frame may reduce the joint moments due to gravity load. Concurrently it may increase the lateral sway due to wind or seismic load rendering the frame unstable.

As per Section 1.2 of AISC(1978) for working stress design, three types of connections are specified depending upon the type of construction and related design assumptions. The AISC definitions are:

> i) TYPE-1: This is commonly referred to as Rigid Frame or Continuous Frame connection. Full continuity is achieved in this type so sufficient rigidity is there in the connection to maintain virtually the original angle between the intersecting members. The rotational restraint available should be around 90 percent or more of that required to prevent any angle change.
>
> ii) TYPE-2: This is commonly referred to as Simple Framing, unrestrained or free ended connection. The underlying assumption is that for gravity loading the ends of the girder transmits the end shears only and are virtually free to rotate under gravity loads. The framing may be considered simple if the initial angle between the intersecting members may

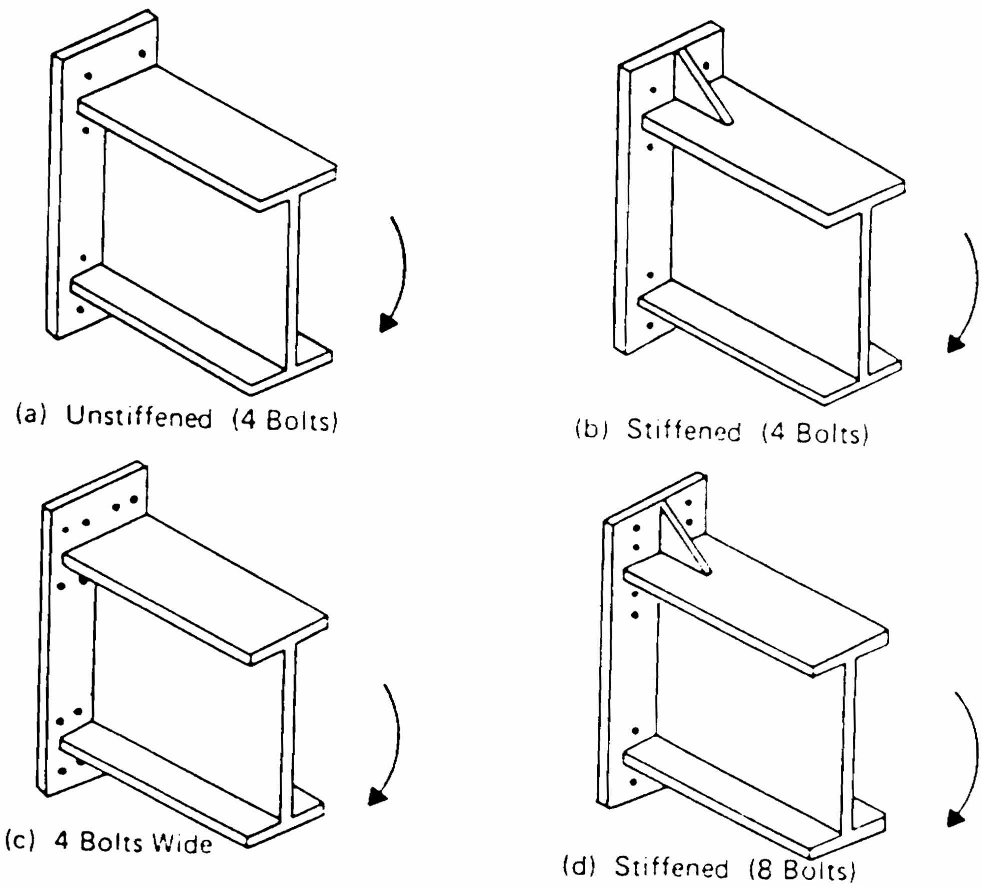


Figure 2. Extended End Plate Configuration: (a) Unstiffened four bolts (b) Stiffened four bolts (c) Four bolts wide unstiffened (d) Four bolts wide stiffened

change upto 80 percent of the rotation which would result if a frictionless hinge is provided at the said connection. Type-2 connections are used for design of simple beams in working stress design. In plastic design Type-2 connection is not used in the direction of the main member in which the plastic strength is to be developed.

iii) TYPE-3: This is commonly referred to as Semi-rigid framing or partially restrained connection. The rotational restraint of the connection should be between 20 and 90 percent of that aforementioned Type-1 connections i.e. the joint prevents relative change in angle between the intersecting members. Hence the moment transmitted across the connection is neither negligible nor is it as high as in rigid connection. AISC, however, stipulates that Type-3 connections may be provided when the connection between the girders and beams possess a "Dependable and known" moment capacity intermediate in degree between the rigidity of Type-1 connection and flexibility of Type-2 connection.

Previously three types of connections were classified by AISC. Later, the connections were classified by LRFD(1986). As per the LRFD specification, there are two types of connections. One of the connection is fully restrained, Type FR, and the other one is partially restrained, Type PR, as explained below:

i) TYPE FR: This type of connection is fully restrained and is commonly referred to as rigid frame or continuous frame. The underlying assumption is that the connections are sufficiently rigid so as to maintain the original angle between the intersecting members virtually unchanged.

ii) TYPE PR: This type of connection is partially restrained. The assumption here is that the connections of beams and girders are not sufficiently rigid to maintain the original angle between the intersecting members, virtually unchanged.

The design of connection Type FR is allowed unconditionally by the LRFD specification but certain restrictions exist for Type PR connections. There should be sufficient evidence that a PR connection will be able to develop the predictable proportion of the full end restraint. The fixity of the connection may be ignored, but in this case the connection of beam and girder ends are such that for gravity loads the connections transmit end shear only and are free to rotate. The connections in this case should have the following characteristics:

i) The members and the connections should have sufficient capacity to resist the factored gravity loads, specified in Section A4.1 of LRFD (1986), specification, as simple beams.

ii) The members and the connections should have sufficient capacity to resist the factored lateral loads.

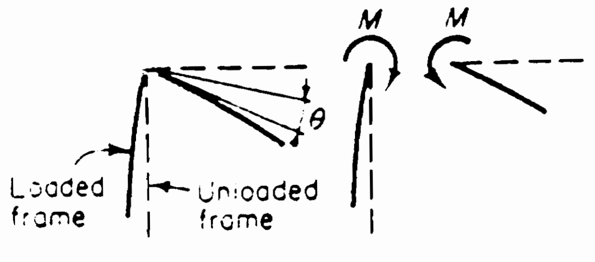
iii) The connections should have sufficient inelastic rotational capacity under the combined effect of factored gravity and lateral load to prevent over stressing of the bolts, rivets or welds.

If the rotational restraint of the connection is used either for the stability of the structure as a whole or for the design of the connected members, the capacity of the connection to develop the desired rotational restraint should be established by analytical or empirical methods. Due to the use of PR connection some inelastic but self limiting deformation of the structural steel part may take place.

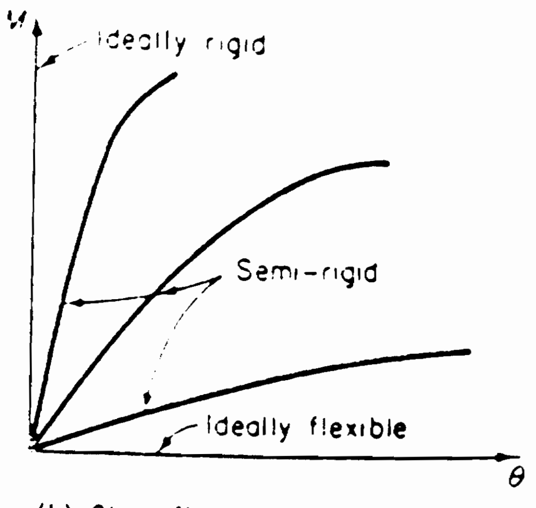
Moreover, stability should be provided to the structure as a whole and also for individual compression members. Further, the deflected shape of the individual members and the lateral load resisting system may give rise to load effects and they should also be considered.

In view of the above classifications, it is evident that the main characteristic of connections is the moment-rotation relationship. This is a curve representing the moment resisted by a connection, related to the relative rotation θ of the intersecting members at the point of intersection. Their relative rotation is shown in Figures 3(a), and (b). The underlying assumption is that the connection has no length and the members meet at a point. The straight vertical line in Figure 3(b) represents an ideally rigid connection in which there is no deflection during transmittal of moment. The horizontal line represents the ideal simple connection. This means that whatever relative rotation is exerted between the members, the connection will not develop any moment. In actual practice, however, the ideal rigid connection and the ideal flexible connection are never realized . Real structures have rigid joints and flexible joints which do not satisfy the idealized connections defined above.

In actual practice, each joint has a finite length contrary to the assumption of no length. The connecting materials, bolts deform considerably and the splice angles and plates deform also. The main member in the connection region may deform marginally. For example let us consider the connection of a beam to a column with a groove weld and stiffened column as shown in Figure 4. This is equivalent to metal inserted between the connecting members and is considered to be nearest to a rigid connection. But in a bending test, this connection will also show some rotation between the beam and the column. In simple connections there is rotation due to connection and also due to



(a) Definition of M and θ



(b) Classification of connections

Figure 3. Connection Characteristics: (a) Definition of M and θ (b) Classification of Connections

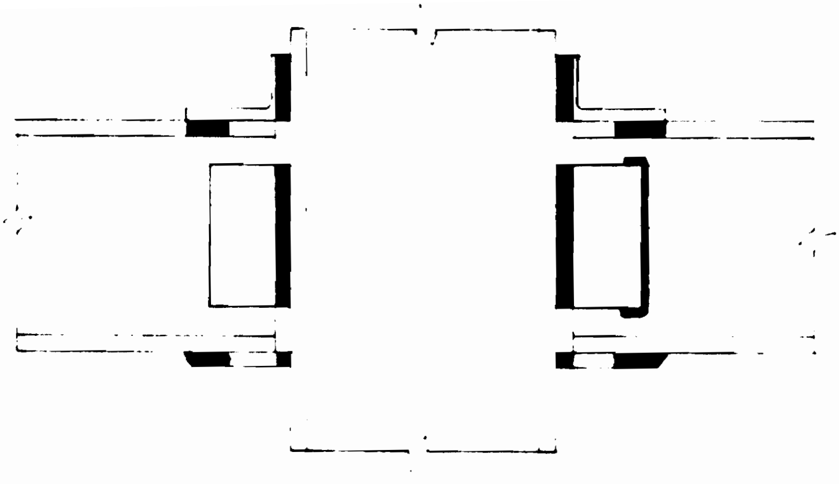


Figure 4. Welded Moment Connection

deformation of members in the vicinity of the connection. However, the connection deforms so much that the deformations of the members are negligible compared to joints. While considering rigidity of fixed connections however, the deformations of the members are comparable to that of the connection. Hence, the interpretation of the AISC stipulations for connections and a general agreement is that, rigid connection when joined to fully fixed supports will attain 90 percent or more of the fixed end moment in the supported beam. On the same lines, simple connections will develop upto 20 percent of the fixed end moment in the supported member.

A considerable number of moment-rotation (M-θ) curves have been obtained after conducting tests on various types of connections. In many of the cases they have been categorized as the three different types of connections described above. For analytical purposes, the non-linearity of the connection M-θ relationship poses the maximum problem. Most of the connections have been observed to behave inelastically. The flexible connections behave inelastically from the moment, the loads are applied. The rigid connections also become inelastic but at higher loads depending upon the effective rigidity. Studies have been conducted and moment-rotation relationship predicted in some specific type of connections but the data is still not sufficient to generalize the prediction. Kukreti, Murray, Abolmali, (1987) have conducted analytical as well as experimental research into the "End Plate Connection Moment-Rotation Relationship" and a prediction equation for the moment-rotation relationship has been developed.

In their study of "Behavior of Type-2 Steel Frames" Ackroyd, and Gerstle, (1982), have classified the different types of connections. The connections have been classified on the basis of connection flexibility after expressing it as a function of the ratio of the rotational beam stiffness and the connection modulus. The above classification is applicable to all connections.

In this study, the moment-rotation relationship will be utilized to classify end plate connection as rigid, simple or flexible connection as per AISC specification. In view of the LRFD specification, it will be termed as fully restrained or partially restrained connection. Further, the above classification will be modified as per the studies conducted by Bjorhovde, Brozzetti, and Colson, (1987). The classification is categorized in a nondimensional form.

1.1.3 GABLE FRAMES

In this section, the gable frame will be discussed briefly and the application of flexible connection or partially restrained connection to the gable frame will be discussed. The main consideration will be the application of end plate connections.

The gable frame is one of the most common structural systems. It is usually single bay and single storied. It is generally used for godowns, factory sheds , hangers etc. The two supports could both be fixed or one of the supports could be fixed and the other roller. Usually both the supports are pinned rendering the structure indeterminate to the first degree. The different types of gable frames are shown in Figure 5. The above support condition is achieved by using a pinned base or by providing an ordinary base plate with two bolts as shown in Figure 6.

Further, to maintain the support conditions the following systems are followed. The supports are tied together by cables or tie rods below the floor level as shown in Figure 6. Another way to keep the relative distance constant between supports is to found the footing on firm soil or on rock, and pile foundation in case of loose soil. The horizontal shear of the column is transferred to the tie through the base friction or better still is through anchor bolts. When tie rods are provided, they should take the whole horizontal thrust except the horizontal shear due to wind load.

There are certain structural advantages in using a gable frame. The rigid frame effect reduces the mid span moment. Further, the geometry helps in sense that by using sloped rafters the horizontal

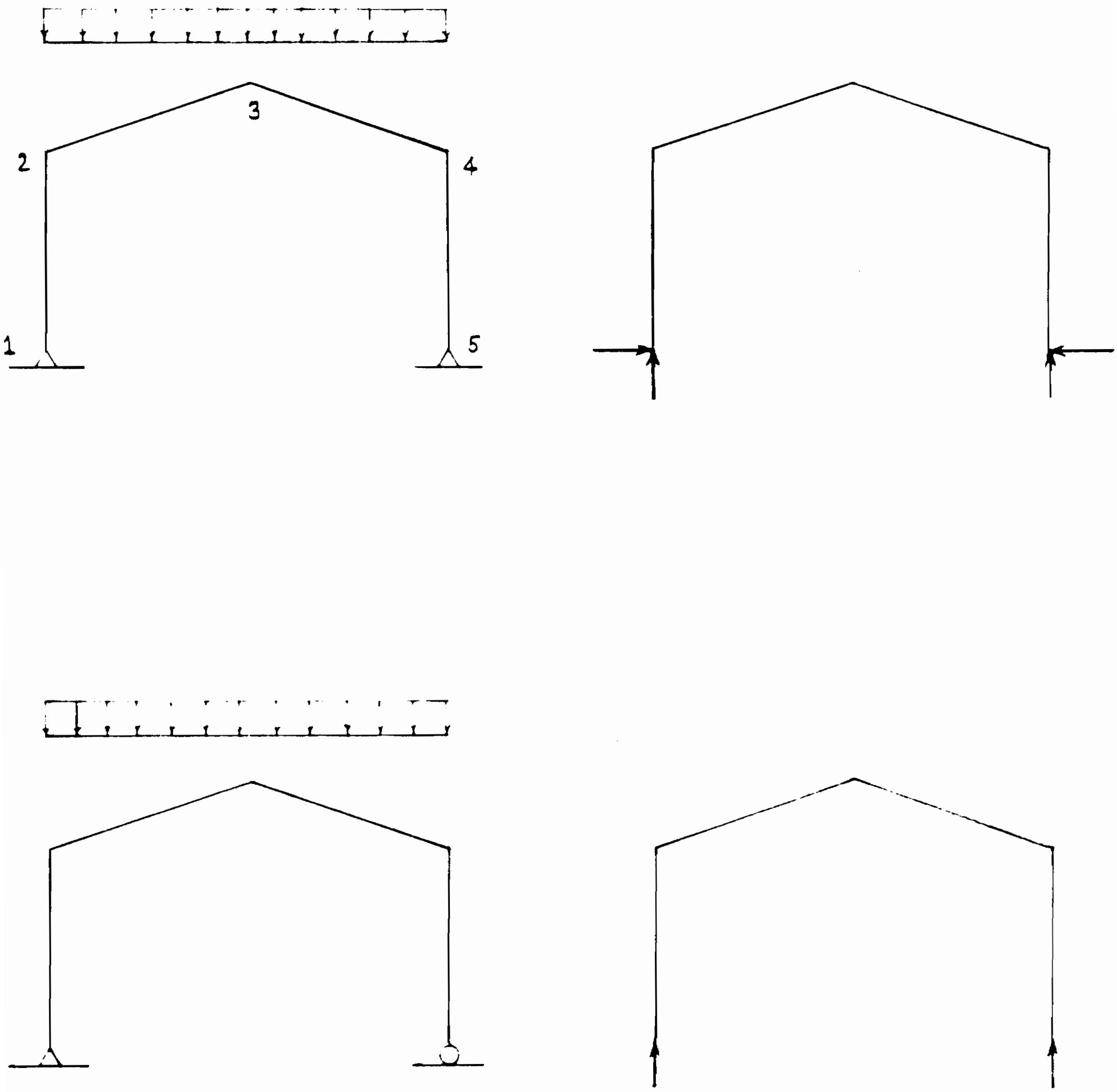


Figure 5. Rigid Gable Frame: (a) Pin-pin frame (b) Pin-roller frame

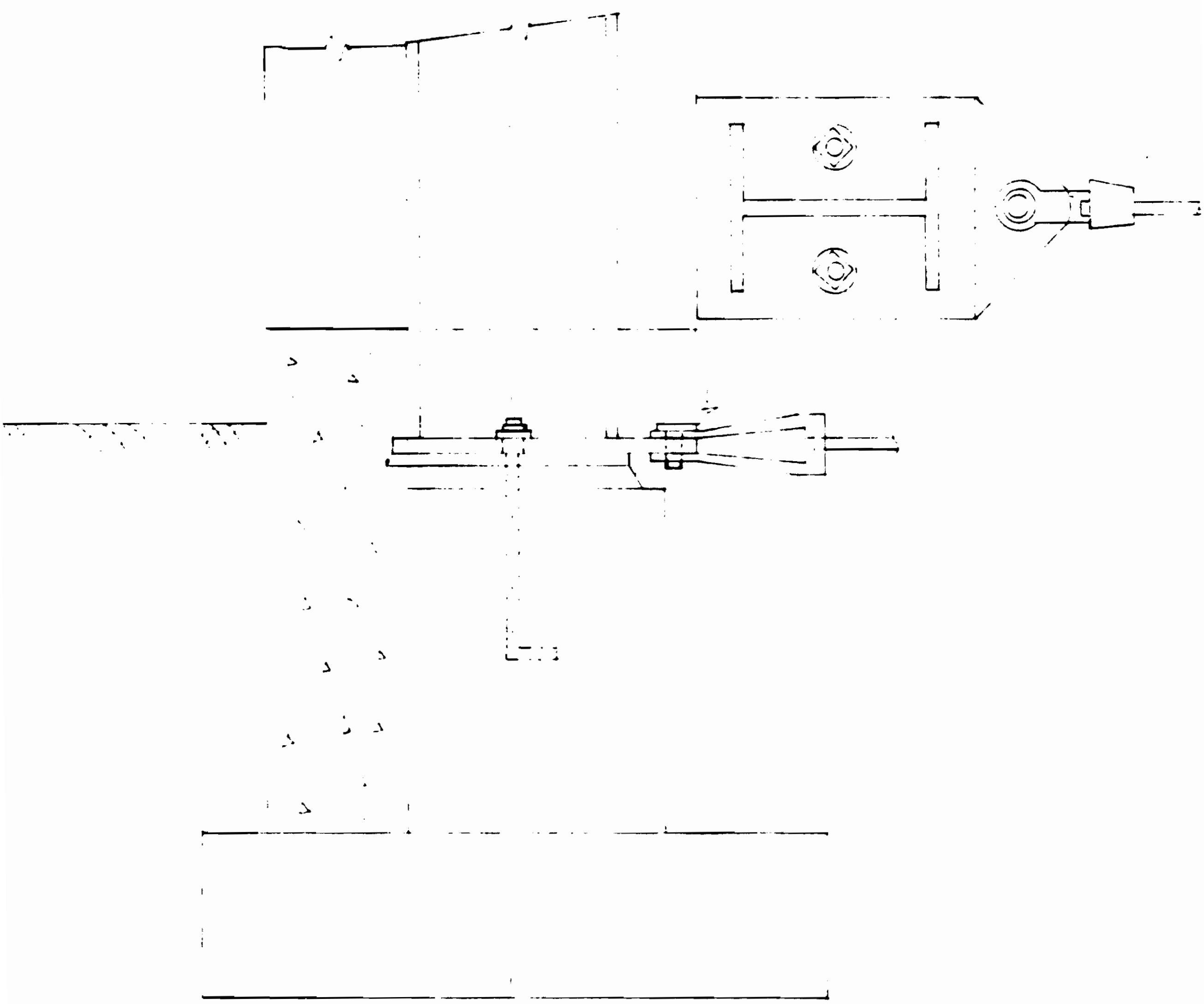


Figure 6. Rigid Gable Frame Foundation Details

shear is increased. This further reduces the mid span moment. Hence it can be observed that a portal frame with pinned bases will have higher midspan moment and end connection moment than a gable frame of comparable span. The connections in the above discussion is considered rigid or fixed with no relative rotation. The intention here is to study the effect on the gable frame if the connections are flexible.

1.2 LITERATURE REVIEW

1.2.1 LITERATURE REVIEW ON CONNECTION CLASSIFICATION

Ackroyd, and Gerstle, (1982) in their study, have considered connection flexibility as a ratio of rotational beam stiffness, EI/L and the connection modulus, k. Connection modulus, k, is the characteristic of the connection and is the ratio of the differential moment to the differential rotation. Depending upon the value of EI/kL the connection has been classified as Type-1, rigid connection, Type-2, flexible, or Type-3, semi rigid connection,as previously discussed.

Bijlard, and Zoetemeijer, (1986) suggested a classification similar to the one stated above. For common loading conditions, the relationship between moment, loads and deflection have been represented with a rigid connection, multiplied by a reduction factor R. R is a function of k, the ratio between the rotational spring stiffness and the bending stiffness of the beam. Thus $k = cl/EI$, where 'c' is the calculated value of the rotational spring stiffness of the connection. Depending on the value of k the connection is classified as rigid, hinge, or semi rigid.

Bjorhovde, Brozzeti, and Colson, (1987) have suggested a classification system in a conceptual form only. According to their study the connections are classified on the basis of the ratio between the beam moment and the fully developed plastic moment of the beam, M_p; the ratio of the connection rotation and the beam rotation at the moment M_p. It has been suggested that the system of classification may be useful for the connections whose moment-rotation curves are not available at present.

In this study it is intended to establish a more rational classification system on the basis of previous classifications, then corelate the classification with AISC and LRFD specifications. Further, a criteria will be developed on the basis of suggestions by Bjorhovde, Brozzeti, Colson, (1988).

1.2.2 LITERATURE REVIEW ON CONNECTION FLEXIBILITY

A considerable amount of research has been conducted in semi rigid connections. Nearly all the factors affecting the behavior of the connection has been defined. Some of the research papers have dealt with a pure analytical approach. Others have tried a more practical approach to facilitate the connection design in design offices. Some of the works are discussed below.

Socrates, and Tarpy,Jr., (1979) have conducted some research into analysis of frames with semi rigid beam to column end plate connections. The Moment-Rotation curves developed through an experimental program were used and a finite element model utilized. The semi-rigid connections were modeled using the stiffness method. Frames with end-plate connection and unstiffened column flanges were then analysed. In the models for beam and column, axial deformation was neglected. The gravity girder end moments for the semi-rigidly connected multistoried frame reduced with respect to rigid frame depending upon the location of the member. For wind loading the moments in exterior girders reduced on all floors. For wind loading, moments in the interior girders decreased at the lower floors and increased at the top floor. The gravity column moments were lower for semi rigidly connected frame than for rigidly connected ones and the wind moments were higher. The combined column end moments for gravity and wind loading were lower and higher depending upon the floor of the concerned column.. Deflection of the frame was more for wind loading. Girder deflection were higher for semi rigid connections but within the acceptable design standards.

Piotr, Moncarz and Gerstle, (1981) have used an analytical approach to study the behavior of PR multistoried frame. The matrix displacement method was used. The analysis conducted was non-linear and the structure stiffness varies as a function of load and load history. The element stiffness matrix had to be modified from each load to the next load, through an iterative process. Axial shortening of the beam model was not considered. It was concluded that from the study that field bolted and lightly-welded connections should not be considered as rigid. Simple connections in frames may be uneconomical and of unknown strength. Flexible connections assumed to have linear response in moment-rotation relationship show good corelation to the assumption. Sequential load application does not have considerable effect on sways of frame.

Socrates (1987), has presented a method of analysis using the stiffness analysis of structures. A typical beam element (frame) with six degrees of freedom is utilized with six degree of freedom column element, using the P-Delta effect. The connection flexibility modeled by a two degree of freedom spring element whose stiffness is the secant stiffness at a particular load level for a given connection. The non linearity of the connection is accounted for by iterative process.

Gerstle, and Cook, Jr., (1987) have modeled flexible connection behavior as elastic-perfectly plastic to facilitate application in design offices. It was intended to show that the frame behaves as linearly elastic at working loads and at ultimate stage is sufficiently ductile. It was concluded that elastic-plastic analysis agrees appreciably with the non linear analysis. Connection rotations are within the linear range at working loads. At ultimate loads plastic hinge method gives acceptable results.

1.3 OBJECTIVES AND SCOPE OF RESEARCH

The objectives of the present research can be divided into two parts. The first objective is to classify the flush end-plate connections on the basis of studies conducted by Kukreti, Murray and Abolmali (1987), predicting the moment-rotation relationship. Secondly, the objective is to model flexible connections in gable frames and study the moment and deflection characteristics of gable frames.

The first part of the presentation commences with a brief description of the existing system of classification studied by Ackroyd and Gerstle (1982), which is related to the AISC ASD Specification (1978). Then the moment rotation relationship developed by Kukreti, Murray, and Abolmali (1987), will be used and the classification and moment-rotation relationship corelated to obtain the proposed classification system. The previous classification is defined with respect to the stiffness of the beam. In the proposed classification once we have the moment-rotation relationship it can be observed that the classification of the connection will depend on the moment resisted by the beam or girder. Finally the conceptual presentation regarding classification by Bjorhovde, Brozzetti, and Colson (1987), will be studied for flush-moment-end-plate connections.

The second part of the study consists of implementing flexibility at the junction of the beam and column. A brief description of the different methods by which the flexibility can be implemented is presented. This study deals with flexibility being modeled as an infinitesimal rotational spring. The rotational spring can transmit moment proportional to the joint rotation but does not resist any shear or axial forces. The beam and column is modeled as frame elements and the stiffness matrix for them remain unchanged.

Chapter II

CLASSIFICATION

2.1 INTRODUCTION

The fabrication of end plate connection is quite simple and the erection process is less time consuming. So the end plate connections are being used increasingly for buildings and industrial structures. The flexibility of its use as a beam to beam connection (splice plate connection) or a beam to column connection encourages a study into its behavior. The previous study predicting the moment-rotation relationship by Kukreti, Murray, Abolmali, (1987), was related to flush end plate connections. Hence, in this study the classifications will be specified for flush end-plate connections.

The classification study is based on the studies by Ackroyd, and Gerstle, (1982), as per the AISC ASD Specification, (1978). The present study of the classification is related to the AISC LRFD specification, (1986). To correlate the classification systems they are summarized here. As per the ASD specification - Type-I is rigid, Type-II is fully flexible or pinned, and Type-III is semi rigid connection which is between the above two types of connection. The LRFD specification states - FR as rigid frame and PR as partially restrained or flexible connection.

The types of connections have been discussed in Chapter I. The connections have been classified based on their stiffness. However in the analysis the main concern is moment, shear etc. Hence it would be more convenient if the connections are classified on the basis of moments. Depending upon the moment rotation curve, the connection may behave as rigid upto a certain moment. But, as the moment increases, the connection becomes flexible and the rotation increases. With further increase in moment the connection attains plastic moment and the connection behaves as a pin connection incapable of resisting any further moment. Thus beyond a certain moment, the connection behaves as semi rigid, and then as simple connection.

Another point of view should be considered in favor of classification on the basis of moments. It has been observed that the moment-rotation curve is non linear and for the design purpose stiffness is considered for classification which usually is the initial stiffness. But, there can be some connections with initial stiffness high for lower moments i.e., for low moments the moment-rotation curve is very steep, but for a small increase in moment, the rotation may increase drastically and the moment-rotation curve may be flat. These connections may behave as rigid at low moments but in the practical utility range they are flexible. An idea of the moment, beyond which the connection becomes flexible, is necessary.

Thus, it is more reasonable to consider the behavior of the connections on the basis of moments. For a particular connection there will be a range of moments between which it would be classified as TypeI, II, or III.

2.2 PRESENT CLASSIFICATION

Ackroyd, and Gerstle, (1982). developed a relationship between the connection stiffness and the ultimate strength, M_u, of the connection. Experimental curves for different types of connections are shown in Figure 7. The design moment of the beam or girder is multiplied by a factor to obtain the ultimate moment and the curve is entered by calculating M_ud/ F_y where d is the expected girder depth and F_y , is the yield strength of the material. Now, depending on the connection type (top and seat angles to the column flange etc.) the design connection stiffness can be found out and the connection can be classified. Studies by Ackroyd and Gerstle, (1982), have established a generalized criteria for the type of construction to be considered for unbraced multistoried steel building frames. For the connection flexibility analysis, the flexibility of the connected girder is modeled as a beam element to which rotational spring elements of appropriate moduli, representing the connections, are added to each end, as shown in Figure 8.

The effects of connection flexibility are expressed as a function of the ratio of the rotational beam stiffness, EI/L and the connection modulus, k : EI/kL. The ratio of the rotational and translational stiffness coefficients, and of fixed end moments under uniform load, for the beam with flexible connections to the same beam constants for the rigidly connected prismatic beam, versus the log of the stiffness ratio EI/kL, is plotted as shown in Figure 9 The nondimensionalized beam constants vary from zero, for very soft connections, to unity for rigid connections. Referring to Figure 9 it may be noted that for values of EI/kL less than 0.05, the values of beam constant will be within 20 percent of those for rigid joints, and the flexibility of connection can be reasonably neglected. When EI/kL is greater than 2.0 the values of beam constants will be sufficiently equal to zero and may be neglected. Flexible connection shall be considered when value of EI/kL lies between .05 and 2. Thus the classification can be summarized as-

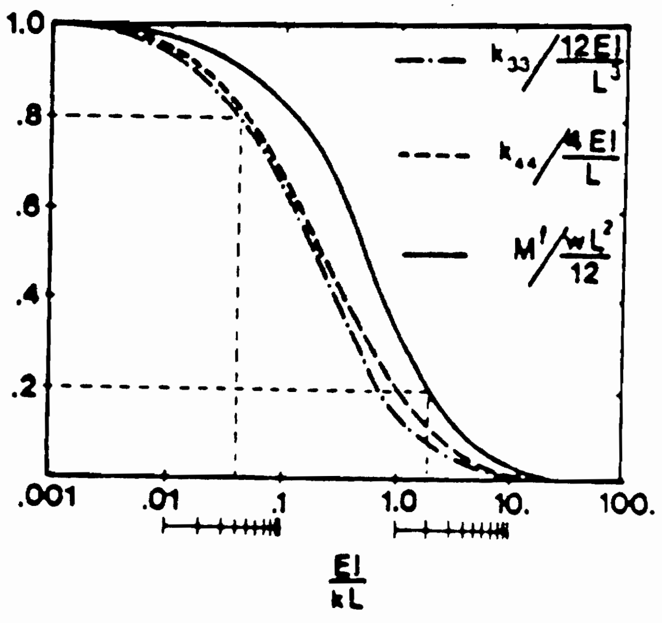


Figure 7. Strength versus Stiffness for Beam-Column connections

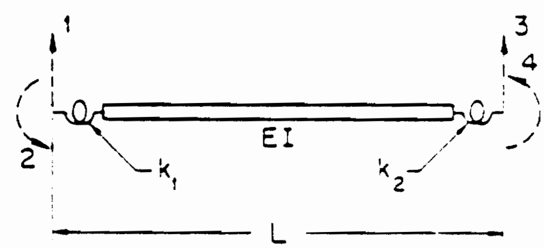


Figure 8. Spring Elements Representing Connections

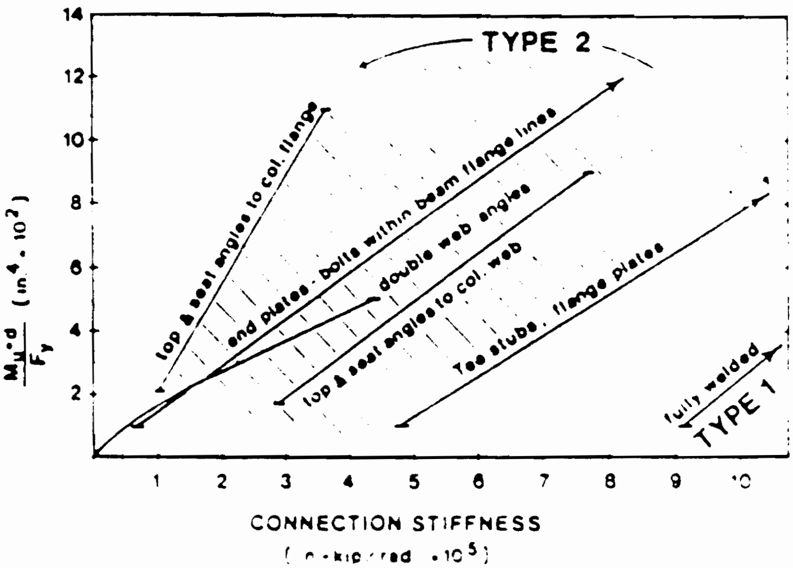


Figure 9. Beam Constants versus Nondimensional Connection Stiffness

Type I $EI/kL < 0.05$

Type II $EI/kL > 2.0$

Type III $0.05 < EI/kL < 2.0$ [2.1]

Bijlard, and Zoetemeijer, (1986), presented a classification system similar to the system described above. The relationship between the moments, loads and deflection, for common loading conditions, are represented with rigid connection multiplied by a reduction factor R, which is a function of k. The factor k is the ratio between the rotational spring stiffness and the bending stiffness of the beam. Thus $k = cL/EI$ where, c is the calculated value of rotational stiffness of the connection.

Referring to Figure 10, it can be observed that for cases 1, 3, 4 for $k = 1000$, R approaches 1. Moreover for k greater than 25 reduction factor R is greater than or equal to 80. For k less than 0.5, R approaches zero. Thus it can be represented as-

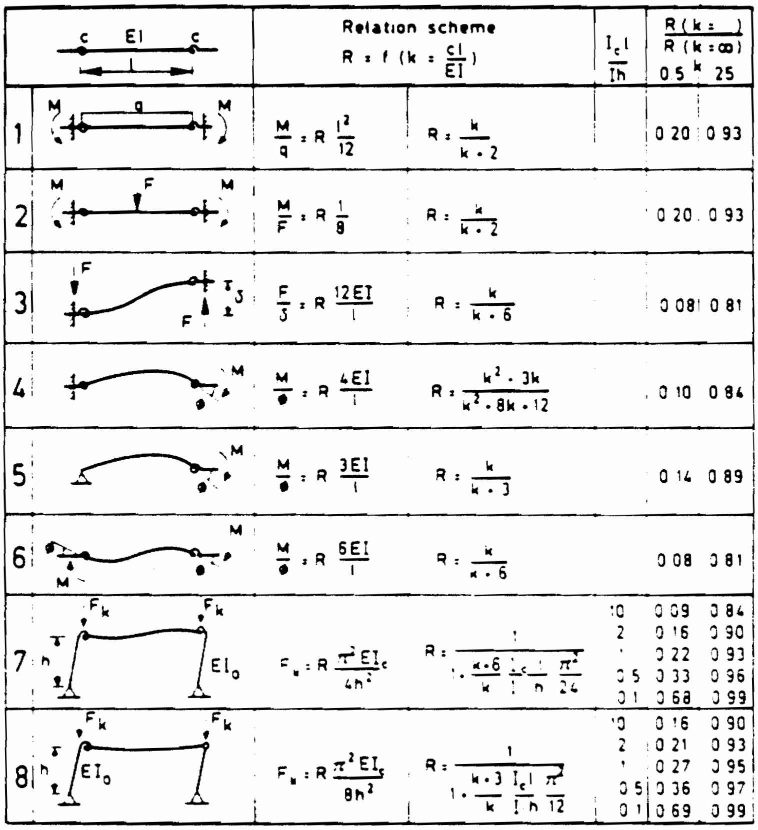
Type I $k > 25$

Type II $k < 0.5$

Type III $0.5 < k > 25$ [2.2]

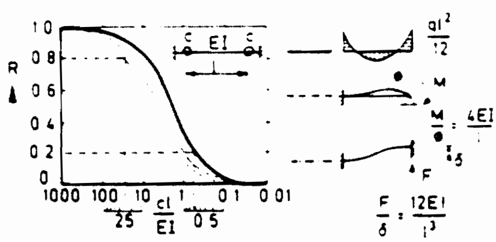
2.3 PROPOSED CLASSIFICATION

The relative merits in considering the moment as a basis for classification compared to the stiffness of the connection has been discussed in the introduction. Hence, it was considered to utilize the studies conducted by Kukreti, Murray, and Abolmali, (1987), regarding moment-rotation relationship for flush end-plate connection with single row of bolts at the tension flange. The following relationship was established in the referenced study:



	Relation scheme $R = f(k = \frac{cl}{EI})$		$\frac{I_c l}{I_h}$	$\frac{R(k=\ )}{R(k=\infty)}$ 0.5	k 25
1	$\frac{M}{q} = R\,\frac{l^2}{12}$	$R = \frac{k}{k+2}$		0.20	0.93
2	$\frac{M}{F} = R\,\frac{l}{8}$	$R = \frac{k}{k+2}$		0.20	0.93
3	$\frac{F}{\delta} = R\,\frac{12EI}{l}$	$R = \frac{k}{k+6}$		0.08	0.81
4	$\frac{M}{\phi} = R\,\frac{4EI}{l}$	$R = \frac{k^2+3k}{k^2+8k+12}$		0.10	0.84
5	$\frac{M}{\phi} = R\,\frac{3EI}{l}$	$R = \frac{k}{k+3}$		0.14	0.89
6	$\frac{M}{\phi} = R\,\frac{6EI}{l}$	$R = \frac{k}{k+6}$		0.08	0.81
7	$F_k = R\,\frac{\pi^2 EI_c}{4h^2}$	$R = \frac{1}{1+\frac{k+6}{k}\,\frac{I_c l}{I h}\,\frac{\pi^2}{24}}$	10 2 1 0.5 0.1	0.09 0.16 0.22 0.33 0.68	0.84 0.90 0.93 0.96 0.99
8	$F_k = R\,\frac{\pi^2 EI_c}{8h^2}$	$R = \frac{1}{1+\frac{k+3}{k}\,\frac{I_c l}{I h}\,\frac{\pi^2}{12}}$	10 2 1 0.5 0.1	0.16 0.21 0.27 0.36 0.69	0.90 0.93 0.95 0.97 0.99

(a)



(b)

Figure 10. Relation Scheme for the Reduction Factor R for forces and deflections as a function of the relative connection stiffness.

$$\theta = CM^{\beta} \qquad [2.3]$$

where

M = moment applied to the connection

θ = rotation of the flush end plate connection

β = constant, for single bolt row flush end plate is equal to 1.356

C = coefficient depending on end plate thickness, beam dimensions, bolt diameter, and pitch of the tension bolt

Also

$$\theta = \delta / h \qquad [2.4]$$

where

δ = plate separation from column

h = depth of the beam or girder

$$C = 359 \times 10^{-6} \cdot \frac{(p_f)^{2.227}}{(h)^{2.616} \times (t_w)^{0.501} \times (t_f)^{0.038} \times (d_b)^{0.849} \times (g_b)^{0.519} \times (b_p)^{0.218} \times (t_p)^{1.539}} \qquad [2.5]$$

where

$$g_b = \left[\frac{1}{3} \right] \left[\frac{F_{by}}{F_{yb}} \right] \left[\frac{A_B}{d_b} \right] \qquad [2.6]$$

In the equations (2.3) to (2.6) the lengths are expressed in inches, and moments in kip-ft. From the moment-rotation curve, the idea is to establish a criteria for the type of connection. It is intended to determine a threshold moment below which a connection would be classified as FR as per LRFD or Type I of ASD specification and above which it is to be PR connection or Type II or III of ASD specification. The findings of Ackroyd,and Gerstle, (1982), and the moment-rotation relationship of Kukreti, Murray, and Abolmali, (1987), will be used to develop the proposed classification system. From Equation (2.3)

$$\theta = CM^{\beta}$$

Differentiating Equation (2.3)

$$\frac{d\theta}{dM} = C\beta M^{\beta-1} \qquad [2.7]$$

From the Moment-rotation curves

$$k = \frac{dM}{d\theta}$$

Substituting $\frac{d\theta}{dM}$ from Equation [1.6]

$$\frac{1}{k} = C\beta M^{\beta-1}$$

$$M = \left[\frac{1}{C\beta k}\right]^{\frac{1}{\beta-1}} \qquad [2.8]$$

In the above expression β is known, C depends on the connection parameters and k can be obtained from the Moment-rotation curve. However, to obtain a moment based on the classification done earlier, k is determined from the fixity criteria of Ackroyd, and Gerstle, (1982). Hence for a particular moment-rotation curve the moment can be determined by substituting the above values in Equation 2.8. Thus, a range of moment can be specified for a particular connection to classify it as per ASD or LRFD specifications.

2.4 EXAMPLE

Calculations showing the classification for three connections from tests by Kukreti, Murray, Abolmali, (1987), with respective test designations for beams of depth 10in., 16in., 24in., are as follows:

1.0 Test designation $F-\frac{5}{8}-\frac{3}{8}-10$

where,

$$\text{bolt dia} = \frac{5}{8}\text{in.}$$

$$\text{endplate thickness} = \frac{3}{8}\text{in.}$$

$$\text{beam depth} = 10 \text{ in.}$$

$$\text{span, l} = 10 \text{ ft.}$$

$$\frac{EI}{kL} = 0.05 \quad \text{connection Type-I from Ackroyd (1982)}$$

$$E = 29 \times 10^3 \text{ ksi}$$

$$I = 2(5\times\frac{1}{4})(4.75)^2 + \frac{1}{4}\frac{(9.5)^3}{12} = 56.406 + 17.86 = 74.268 in^4$$

$$A = 2(5\times\frac{1}{4}) + 9.5\times\frac{1}{4} = 4.875 in^2$$

$$k = 29\times 10^3 \times \frac{74.268}{0.05} \times 10 \times 12 = 3.59 \times 10^5 \text{ in-k/rad}$$

$$k = 0.299 \times 10^5 \text{ in-k/rad}$$

$$g_b = \left[\frac{1}{3}\right]\left[\frac{F_{by}}{F_{yb}}\right]\left[\frac{A_B}{d_b}\right]$$

$$g_b = \frac{1\times 50\times 4.875}{3\times 81\times 0.625} = 1.605$$

$$C = 359\times 10^{-6}\frac{(1.25)^{2.227}}{(10)^{2.616}(.25)^{0.501}(.25)^{.038}(.625)^{.849}(1.605)^{.519}(5)^{.212}(.375)^{1.539}}$$

$$C = 1.131 \times 10^{-5}$$

$$M = \left[\frac{1}{C\beta k}\right]^{\frac{1}{\beta-1}}$$

$$= \left[\frac{1}{1.131\times 10^{-5}\times 1.356\times .299\times 10^5}\right]^{\frac{1}{0.356}}$$

$$M = 8.94 \text{ ft-kips}$$

for the 10in. beam with given connection details as above and span 10ft. we can conclude that it is of type FR up to a moment of 8.94 ft-kips and of Type PR above that.

The plot of the above classification is shown in Figure 11.

2.0 Test designation $F-\frac{3}{4}-\frac{1}{2}-16$

where

bolt dia $= \frac{3}{4}$in.

endplate thickness $= \frac{1}{2}$in.

beam depth $= 16$ in.

span, l $= 20$ ft

$$\frac{EI}{kL} = 0.05$$

$$E = 29 \times 10^3 \text{ ksi}$$

$$I = 2(6 \times \frac{1}{4})(7.875)^2 + \frac{1}{4}\frac{(15.5)^3}{12} = 186.046 + 155.16 = 341.2 in^4$$

$$A_B = 2(6 \times \frac{1}{4}) + 15.5 \times \frac{1}{4} = 6.875 in^2$$

$$k = 29 \times 10^3 \times \frac{341.2}{0.05} \times 20 \times 12 = 8.246 \times 10^5 \text{ "k/rad}$$

$$k = 0.687 \times 10^5 \text{ in-k/rad}$$

$$g_b = \left[\frac{1}{3}\right]\left[\frac{F_{by}}{F_{yb}}\right]\left[\frac{A_B}{d_b}\right]$$

$$g_b = \frac{1 \times 50 \times 6.875}{3 \times 81 \times 0.75} = 1.886$$

$$C = 359 \times 10^{-6}\frac{(1.5)^{2.227}}{(16)^{2.616}(.25)^{0.501}(.25)^{.038}(.75)^{.849}(1.886)^{.519}(6)^{.212}(.5)^{1.539}}$$

$$C = 2.39 \times 10^{-6}$$

$$M = \left[\frac{1}{C\beta k}\right]^{\frac{1}{\beta-1}}$$

$$= \left[\frac{1}{2.39 \times 10^{-6} \times 1.356 \times 0.687 \times 10^5}\right]^{\frac{1}{0.356}}$$

$$M = 68.00 \text{ ft-kips}$$

for the 16in. beam with given connection details as above and span 20ft. we can conclude that it is of type FR up to a moment of 68.0 ft-kips and of type PR above that, as shown in Figure 12.

3.0 Test designation $F-\frac{3}{4}-\frac{1}{2}-24A$

where

bolt dia $= \frac{3}{4}$in.

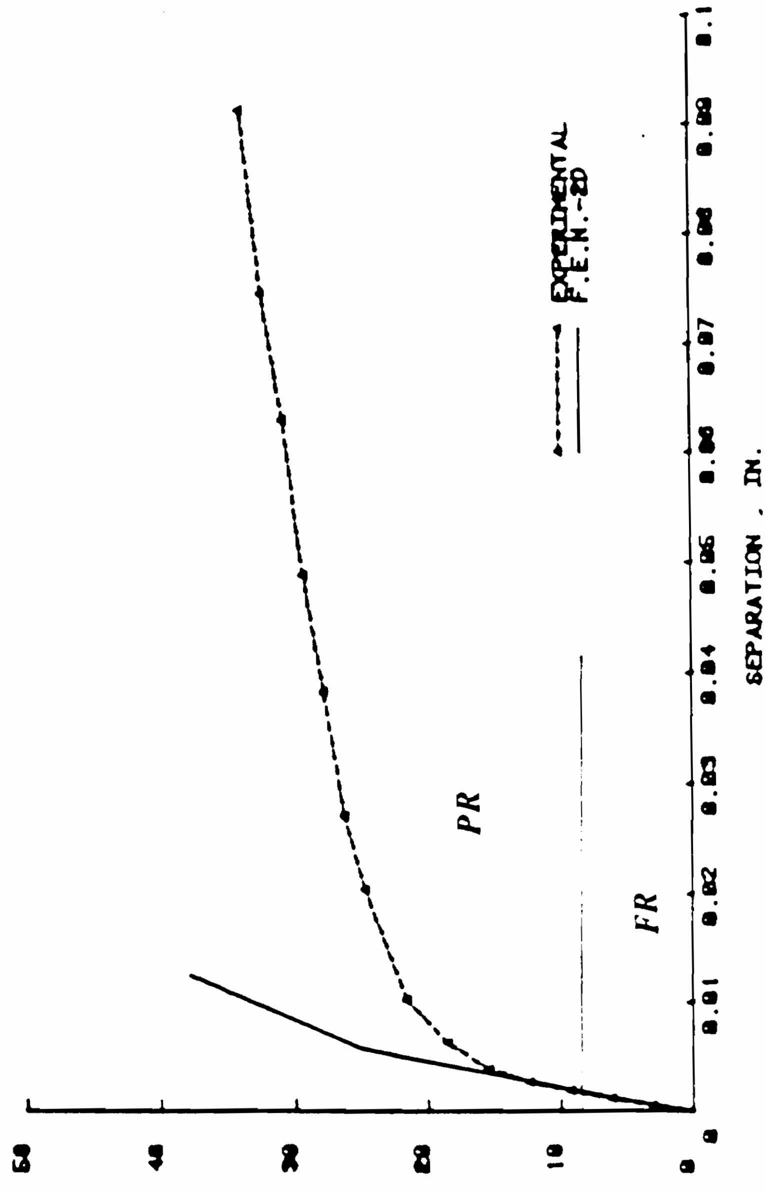


Figure 11. Moment versus Plate Separation Test-1

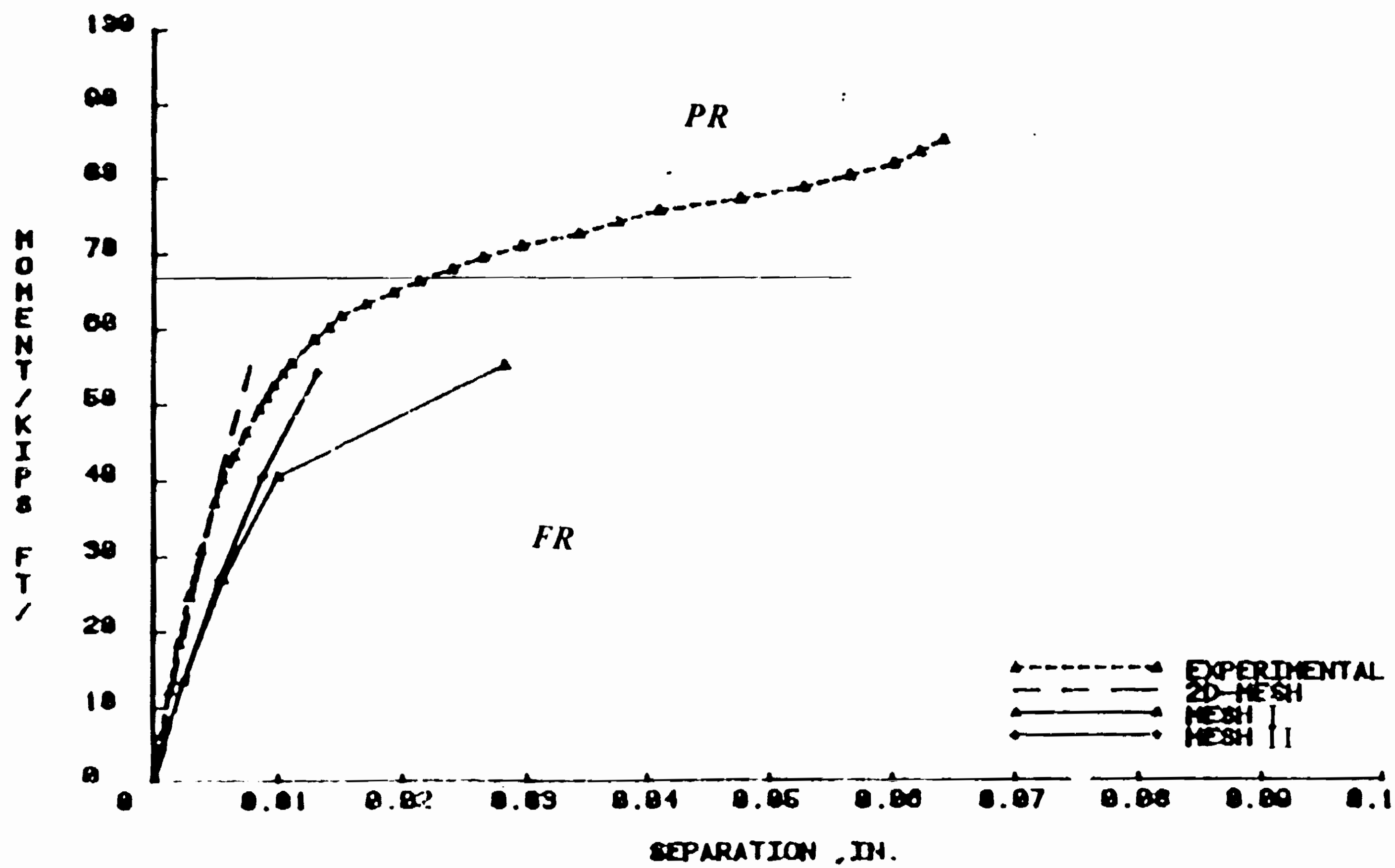


Figure 12. Moment versus Plate Separation Test-2

endplate thickness $= \frac{1}{2}$ in.

beam depth $= 24$ in.

span, l $= 20$ ft.

$$\frac{EI}{kL} = 0.05$$

$$E = 29 \times 10^3 \text{ ksi}$$

$$I = 2(6 \times \frac{1}{4})(11.875)^2 + \frac{1}{4}\frac{(23.5)^3}{12} = 423.047 + 270.372 = 693.42 in^4$$

$$A_B = 2(6 \times \frac{1}{4}) + 23.5 \times \frac{1}{4} = 8.875 in^2$$

$$k = 29 \times 10^3 \times \frac{693.42}{0.05} \times 20 \times 12 = 16.758 \times 10^5 \text{ "k/rad}$$

$$k = 1.396 \times 10^5 \text{ in-k/rad}$$

$$g_b = \left[\frac{1}{3}\right]\left[\frac{F_{by}}{F_{yb}}\right]\left[\frac{A_B}{d_b}\right]$$

$$g_b = \frac{1 \times 50 \times 8.875}{3 \times 81 \times 0.75} = 2.435$$

$$C = 359 \times 10^{-6} \frac{(1.75)^{2.227}}{(24)^{2.616}(.25)^{0.501}(.25)^{.038}(.75)^{.849}(2.435)^{.519}(6)^{.212}(.5)^{1.539}}$$

$$C = 1.0217 \times 10^{-6}$$

$$M = \left[\frac{1}{C\beta k}\right]^{\frac{1}{\beta-1}}$$

$$= \left[\frac{1}{1.0217 \times 10^{-6} \times 1.356 \times 1.396 \times 10^5}\right]^{\frac{1}{0.356}}$$

$$M = 101 \text{ ft-kips}$$

for the 24in. beam with given connection details as above and span 20ft. we can conclude that it is of Type FR up to a moment of 101.0 ft-kips and of Type PR above that, as shown in Figure 13.

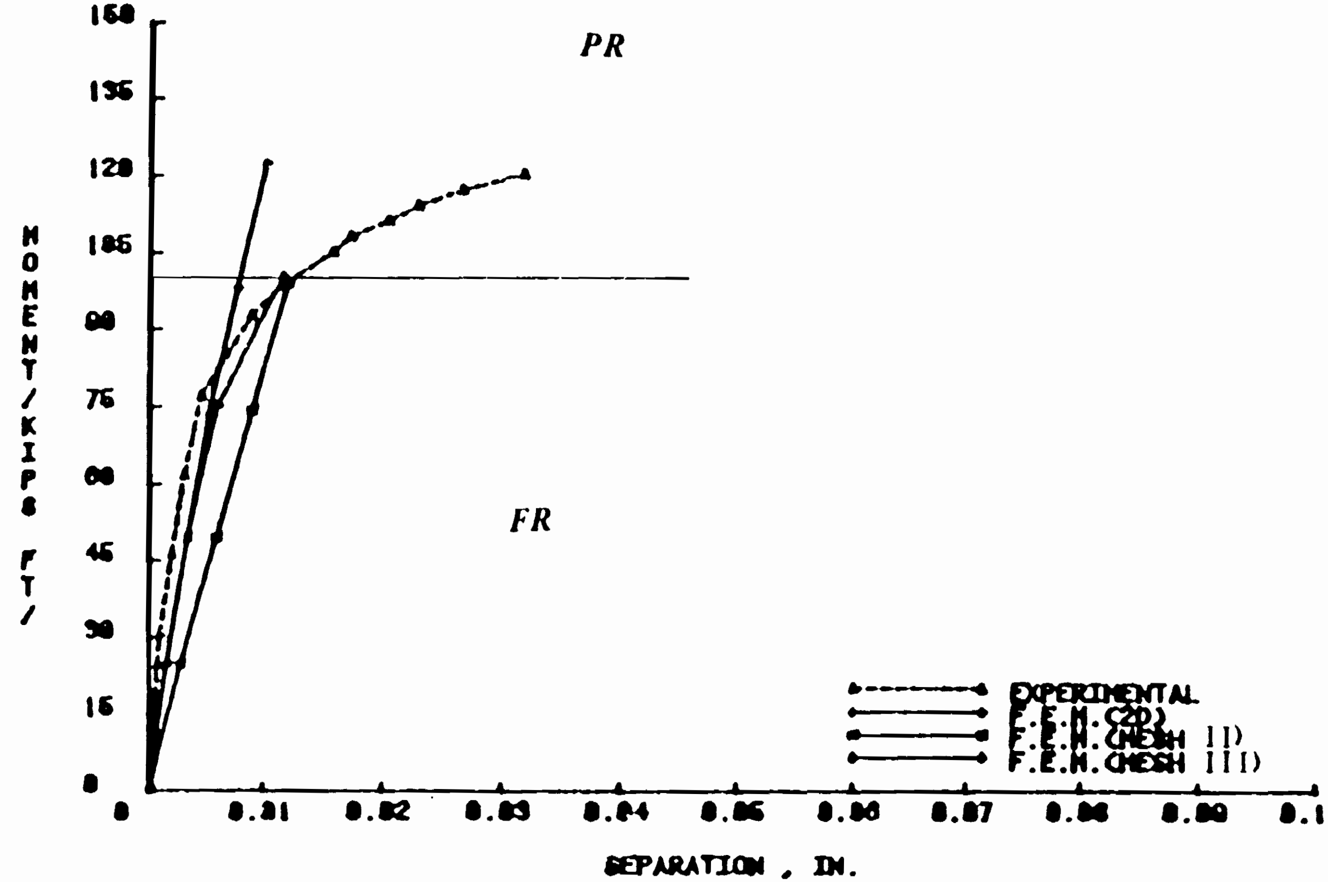


Figure 13. Moment versus Plate Separation Test-3

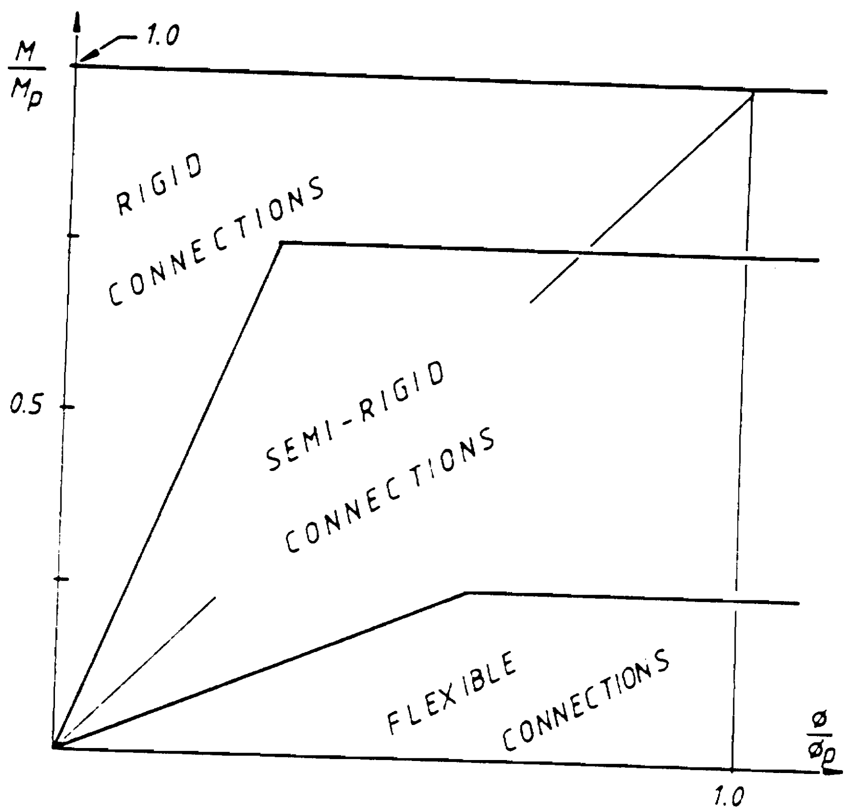


Figure 14. Non-Dimensional Connection Classification Diagram System

2.5 MODIFIED CLASSIFICATION

Further studies have been done by Bjorhovde, Brozzetti, Colson, (1987), in which a simpler method is suggested. A schematic non dimensional connection M-ϕ classification diagram is shown in Figure 14 in its conceptual form. In this type of non dimensional classification, connections where the moment-rotation curves are not available can also be classified.

By using the connection curves obtained from Kukreti, Murray, and Abolmali, (1987), the plastic moment is determined as per the studies done by Zandonini, and Zanon, (1987). The plastic moment M_p is defined as the moment at which a significant increase of the plastic deformation of connection becomes apparent. The M_p can be determined as shown in Figure 15.

Calculations for classification of connections from the tests by Kukreti, Murray, and Abolmali, (1987), shown in Example 2.4 are as follows:

1. Beam h = 10in. $\quad F-\frac{5}{8}-\frac{3}{8}-10$

FR

$M_p = 23.5$ ft-kips $\qquad \phi_p = 4.37 \times \frac{10^{-3}}{10}$

M = 8.94 ft-kips $\qquad \phi = 1.33 \times \frac{10^{-3}}{10}$

$M/M_p = 0.38 \qquad \phi/\phi_p = 0.30$

1.a Beam h = 10in. $\quad F-\frac{5}{8}-\frac{3}{8}-10$

FR

$M_p = 23.50$ ft-kips $\qquad \phi_p = 4.37 \times \frac{10^{-3}}{10}$

M = 5.0 ft-kips $\qquad \phi = 3.0 \times \frac{10^{-3}}{10}$

$M/M_p = 0.21 \qquad \phi/\phi_p = 0.22$

2. Beam h = 16in. $\quad F-\frac{3}{4}-\frac{1}{2}-16$

FR

$M_p = 69.0$ ft-kips $\qquad \phi_p = 8.23 \times \frac{10^{-3}}{16}$

M = 68.0 ft-kips $\qquad \phi = 8.0 \times \frac{10^{-3}}{10}$

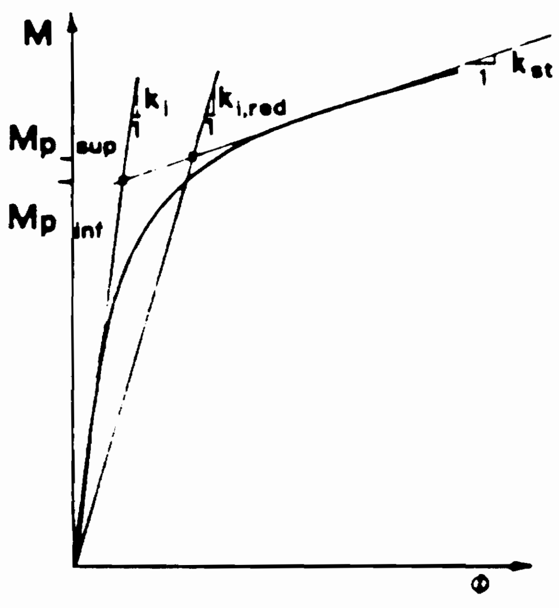


Figure 15. Determination of Plastic Moment

$M/M_p = 0.985$ $\phi/\phi_p = 0.97$

2.a Beam h = 16in. $F - \frac{3}{4} - \frac{1}{2} - 16$

FR

$M_p = 69.0$ ft-kips $\phi_p = 8.23 \times \frac{10^{-3}}{16}$

M = 50.0 ft-kips $\phi = 6.6 \times \frac{10^{-3}}{10}$

$M/M_p = 0.72$ $\phi/\phi_p = 0.80$

2.b Beam h = 16in. $F - \frac{3}{4} - \frac{1}{2} - 16$

FR

$M_p = 69.0$ ft-kips $\phi_p = 8.23 \times \frac{10^{-3}}{16}$

M = 25.0 ft-kips $\phi = 2.4 \times \frac{10^{-3}}{10}$

$M/M_p = 0.36$ $\phi/\phi_p = 0.29$

3. Beam h = 24in. $F - \frac{3}{4} - \frac{1}{2} - 24$

FR

$M_p = 111.0$ ft-kips $\phi_p = 6.33 \times \frac{10^{-3}}{24}$

M = 101.0 ft-kips $\phi = 6.0 \times \frac{10^{-3}}{10}$

$M/M_p = 0.91$ $\phi/\phi_p = 0.95$

3.a Beam h = 24in. $F - \frac{3}{4} - \frac{1}{2} - 24$

FR

$M_p = 111.0$ ft-kips $\phi_p = 6.33 \times \frac{10^{-3}}{24}$

M = 75.0 ft-kips $\phi = 3.33 \times \frac{10^{-3}}{10}$

$M/M_p = 0.68$ $\phi/\phi_p = 0.53$

3.b Beam h = 24in. $F - \frac{3}{4} - \frac{1}{2} - 24$

FR

$M_p = 111.0$ ft-kips $\phi_p = 6.33 \times \frac{10^{-3}}{24}$

M = 50.0 ft-kips $\phi = 2.0 \times \frac{10^{-3}}{10}$

$M/M_p = 0.45$ $\phi/\phi_p = 0.32$

The plot for M/M_p and ϕ/ϕ_p is shown in Figure 16. We also know that the connections selected are all type FR upto the moments determined in Section 2.4. Hence we can draw a demarcation

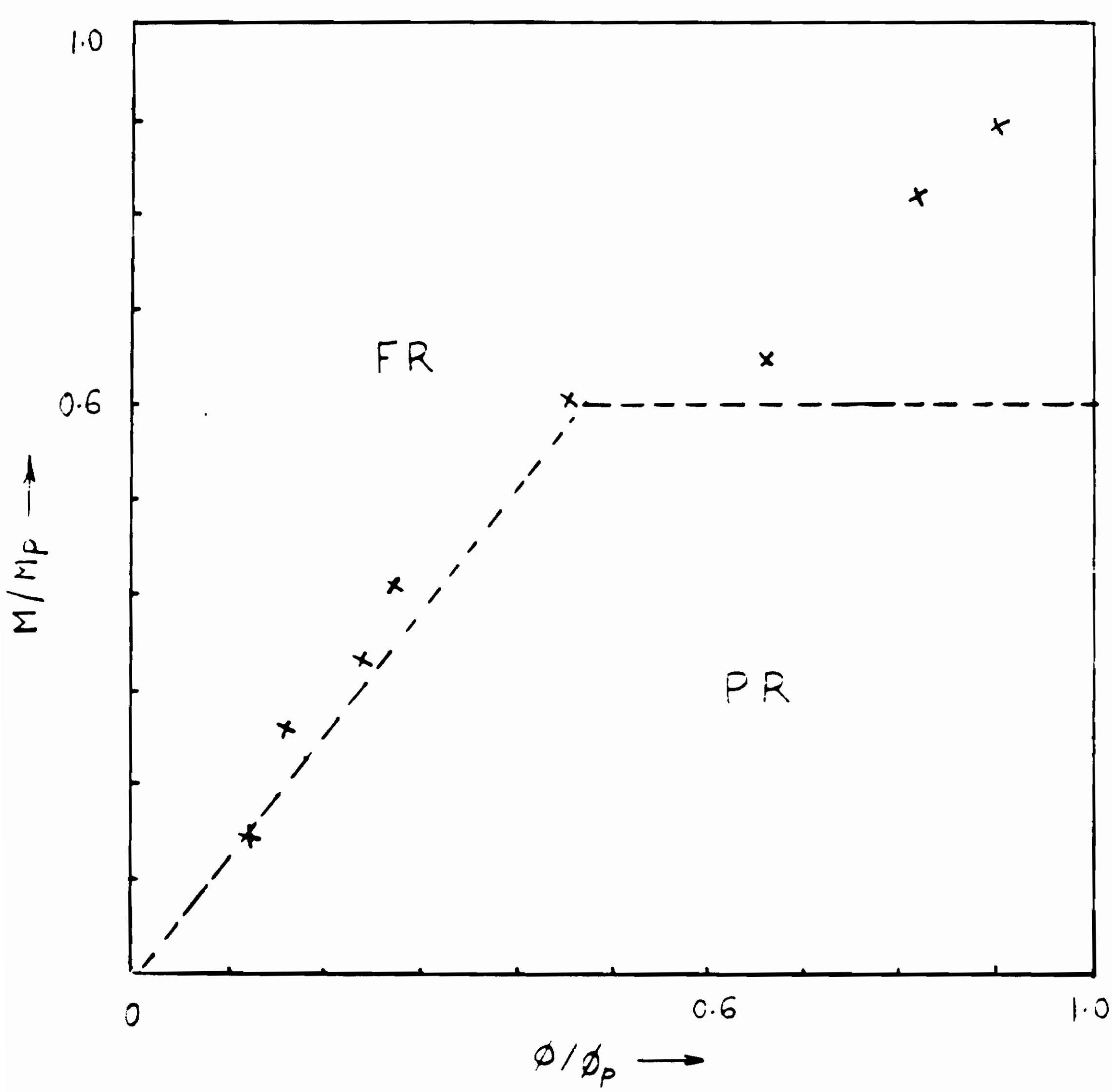


Figure 16. Connection Classification

line shown in figure 16 such that the coordinates of M/M_p and ϕ/ϕ_p above the line will be FR connection. The coordinates below the line will suggest PR connection.

2.6 CONCLUSION

It is observed that the classification of connection depends on the span of the beam as discussed in Section 2.2. We can define the connection stiffness, k when we know the span of the beam. The initial slope of the Moment-rotation curve depends on the span of the beam. The same is noted by Bjorhovde, Brozzeti, and Colson, (1987), as shown in Figure 17. It can be observed from the relationship TypeI of Equation 2.1 EI/kL = 0.05, that for larger span L, connection modulus k will be lower. If we insert k in the relationship of Equation 2.8, the connection will be classified as FR for larger moments, than if the connection is classified for shorter span. Hence in the proposed classification of section 2.3 the length should be specified along with moment for classification.

The modified nondimensional classification system seems to be more rational than the previous system. Once the moment applied to a connection and the resulting rotation is known, the plastic moment of the connection and the corresponding rotation is known, the connection can be classified as FR or PR connection.

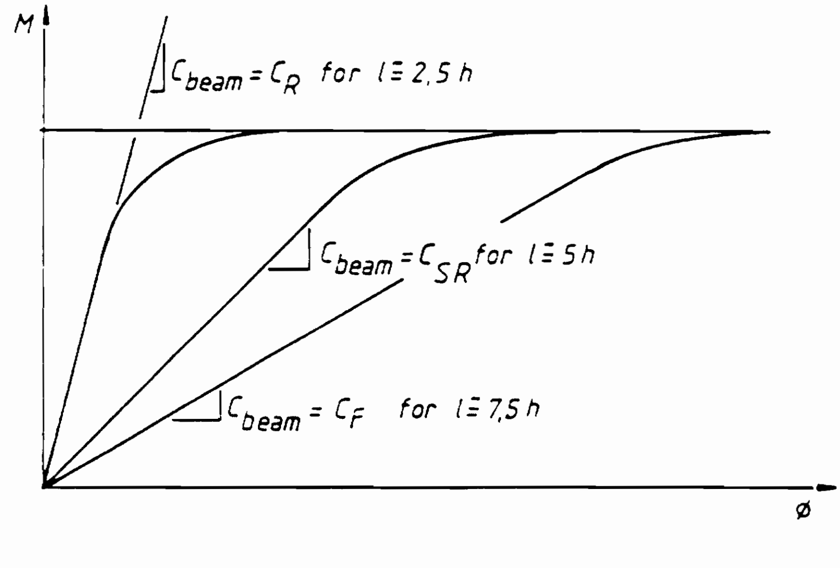


Figure 17. Beam M- ϕ Curves as a Function of Beam Length

Chapter III

CONNECTION FLEXIBILITY

3.1 INTRODUCTION

Flexibility has been modeled previously using the matrix displacement method by Gere & Weaver,(1965), Livesley,(1964,1975), and Holzer,(1985), among others. Although the basic principle of considering the connection as a rotational spring remains the same, the treatment is different in each of the methods. Some of the methods are discussed briefly and a suitable model selected for actual use.

3.2 FLEXIBLE CONNECTIONS AND INTERNAL RELEASE

3.2.1 FLEXIBLE CONNECTION BY SEGMENTED MEMBERS

Livesley,(1964,1975), extended the matrix analysis for segmented members to incorporate flexible connections. The length of the member from a plane frame is L and flexural rigidity EI. The moment at end 1 is considered EIk_1/L and at end 2 is EIk_1/L for unit difference in rotation. The member is shown in figure 18. The lengths of the connections are considered negligible. Hence, although the rotations may vary, the translations at the ends of the members are same as that of corresponding joints. The origin is located at the mid point and the direction of end 1 to end 2 is positive. The element equilibrium matrices are given by

$$H_1 = \begin{bmatrix} 1 & L/2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad H_2 = \begin{bmatrix} 1 & -L/2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad [3.1]$$

Elastic properties of structural elements are represented by

$$r = Ke \qquad [3.2]$$

$$e = Fr \qquad [3.3]$$

where,

r = stress resultant vector

e = deformation vector

K = stiffness

F = flexibility

The component t of the stress resultant vector r produces axial force, which is related to extension e_x by

$$t = \frac{EA}{L} e_x \qquad [3.4]$$

The component q gives the deformed shape of the beam as in Figure 19. The relationship of q with e_y is

$$q = 12 \frac{EI}{L^3} e_y \qquad [3.5]$$

The relationship of m with ϕ is given by

$$m = \frac{EI}{L} \phi \qquad [3.6]$$

Thus, the above three equations can be combined together to have

$$\begin{bmatrix} t \\ q \\ m \end{bmatrix} = \begin{bmatrix} EA/L & 0 & 0 \\ 0 & 12EI/L^3 & 0 \\ 0 & 0 & EI/L \end{bmatrix} \begin{bmatrix} e_x \\ e_y \\ \phi \end{bmatrix} \quad \mathrm{v}r = Ke \qquad [3.7]$$

Inversion of K produces

$$\begin{bmatrix} e_x \\ e_y \\ \phi \end{bmatrix} = \begin{bmatrix} L/EA & 0 & 0 \\ 0 & L^3/12EI & 0 \\ 0 & 0 & L/EI \end{bmatrix} \begin{bmatrix} t \\ q \\ m \end{bmatrix} \quad \mathrm{v}e = Fr \qquad [3.8]$$

Thus the equilibrium equation is

$$p = H\,r \qquad [3.9]$$

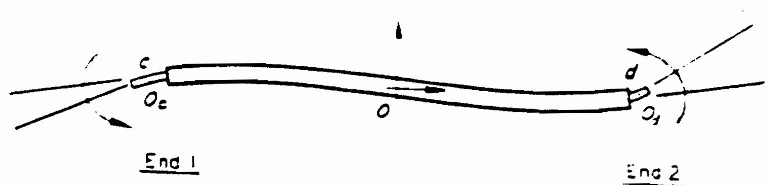


Figure 18. Member With Flexible End Connections

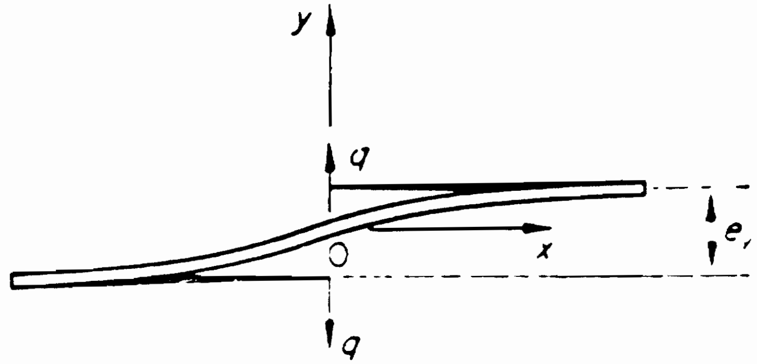


Figure 19. Deformation of Straight Uniform Member Associated With q

$$\begin{bmatrix} p_{x_1} \\ p_{y_1} \\ m_1 \end{bmatrix} = - \begin{bmatrix} 1 & x_1 & 0 \\ 0 & -1 & 0 \\ y_1 & 0 & 1 \end{bmatrix} \begin{bmatrix} t \\ q \\ m \end{bmatrix} \quad \vee p_1 = H_1 r \tag{3.10}$$

$$\begin{bmatrix} p_{x_2} \\ p_{y_2} \\ m_2 \end{bmatrix} = \begin{bmatrix} 1 & x_1 & 0 \\ 0 & -1 & 0 \\ y_2 & 0 & 1 \end{bmatrix} \begin{bmatrix} t \\ q \\ m \end{bmatrix} \quad \vee p_2 = H_2 r \tag{3.11}$$

$$e = H_1^t\, d_1 + H_2^t\, d_2 \tag{3.12}$$

Combining Equations (3.9), (3.10), (3.11), and (3.6) results in

$$p_1 = H_1 r = H_1 K e = H_1 K H_1^t d_1 + H_1 K H_2^t d_2$$

$$p_2 = H_2 r = H_2 K e = H_2 K H_1^t d_1 + H_2 K H_2^t d_2 \tag{3.13}$$

$$p_1 = K_{11} d_1 + K_{12} d_2$$

$$p_2 = K_{21} d_1 + K_{22} d_2 \tag{3.14}$$

Indicating subscript c and d for connections and m for member. Considering Equations (3.1) and (3.8) for segmented member, the flexibility matrix and equilibrium matrix of segment and member are given by

$$F_c = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & L/EIk_1 \end{bmatrix} \qquad H_c = \begin{bmatrix} 1 & L/2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$F_d = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & L/EIk_2 \end{bmatrix} \qquad H_d = \begin{bmatrix} 1 & -L/2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$F_m = \begin{bmatrix} L/EA & 0 & 0 \\ 0 & L^3/12EI & 0 \\ 0 & 0 & L/EI \end{bmatrix} \qquad H_m = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The coordinate axes of the segment and member are same. Hence the flexibility matrix is given by

$$F = H_c^t F_c H_c + F_m + H_d^t F_d H_d$$

Solving the above equation

$$F = \begin{bmatrix} L/EA & (L^2/2EI)(1/k_1 - 1/k_2) & 0 \\ 0 & (L^3/12EI)(1 + 3/k_1 + 3/k_2) & (L^2/2EI)(1/k_1 - 1/k_2) \\ 0 & 0 & (L/EI)(1 + 1/k_1 + 1/k_2) \end{bmatrix}$$

Thus inversion of the above gives

$$K = \begin{bmatrix} EA/L & 6EI/L^2k(k_1 - k_2) & 0 \\ 0 & 12EI/L^3k(k_1k_2 + k_1 + k_2) & 6EI/L^2k(k_1 - k_2)(k_1k_2 + 3k_1 + 3k_2) \\ 0 & 0 & EI/Lk \end{bmatrix} \qquad [3.15]$$

where,

$$k = k_1k_2 + 4(k_1 + k_2) + 12$$

$$K_{ij} = H_i K H_j^t$$

Thus K_{11}, $K_{12} = K_{21}^t$, K_{22} can be obtained and nodal loads can be determined from Equation (3.14)

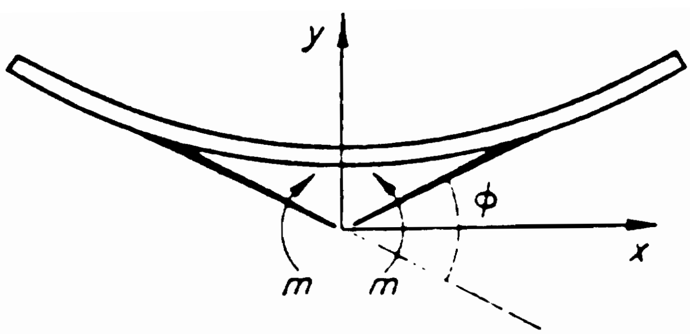


Figure 20. **Deformation of Straight Uniform Member Associated With m**

3.2.2 FLEXIBLE CONNECTION BY INTERNAL RELEASE

Another method of modeling the flexible connection is by internal release. By release in a structure it is meant that the continuity constraint at a joint is removed. The method presented by Holzer, (1985), is given here. Internal releases are of two types, joint release and element release.

Joint release can be moment release, shear release or axial force release as shown in Figure 21. Two element ends may be considered as shown in Figure 22. If the joints are rigid the continuity constraints are

$$D_1^I = D_1^J = q_1 \qquad [3.16a]$$

$$D_2^I = D_2^J = q_2 \qquad [3.16b]$$

$$D_3^I = D_3^J = q_3 \qquad [3.16c]$$

Slopes and deflections are shown in Figure 22(c). If a hinge is introduced then the joint will have four degrees of freedom as in Figure 21(a) and Equation 3.16(c) is modified to

$$D_3^I = q_3 \;\; ; \;\; D_3{}^J = q_4 \qquad [3.17]$$

The slope discontinuity is shown in Figure 21(b). The other releases, shear release and axial force release are shown in Figure 21(c) and 21(e), respectively, and the corresponding discontinuities in Figure 21(d) and 21(f), respectively.

The conditions of compatibility of joints are as follows:

Shear Release:

$$D_1^I = D_1^J = q_1 \qquad [3.18a]$$

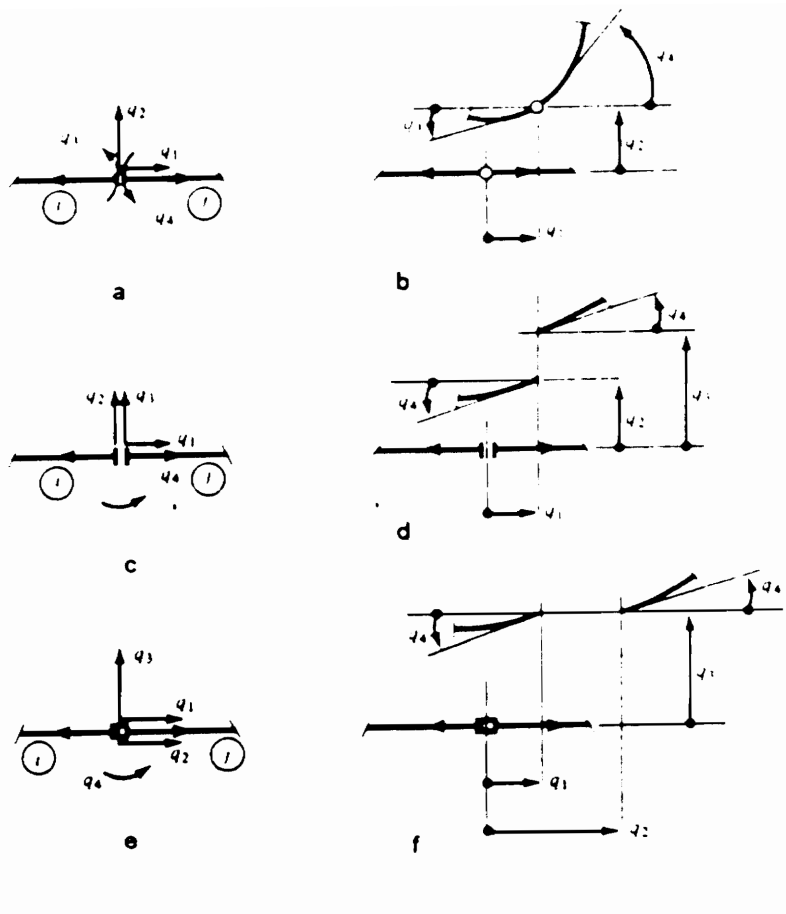


Figure 21. Joint Releases: (a,b) Moment Release (c,d) Shear release (e,f) Axial Force Release.

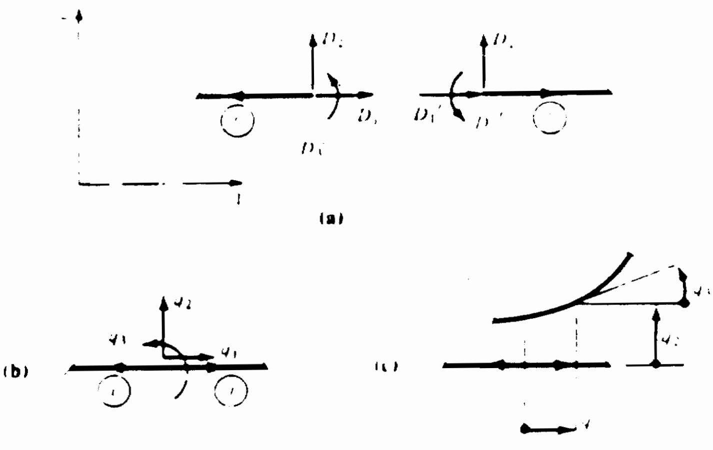


Figure 22. Fixed Joints: (a) Element ends (b,c) Rigid Joints

$$D_2^i = q_2 \quad , \quad D_2^j = q_3 \tag{3.18b}$$

$$D_3^i = D_3^j = q_4 \tag{3.18c}$$

Axial force release:

$$D_1^i = q_1 \quad , \quad D_1^j = q_2 \tag{3.19a}$$

$$D_2^i = D_2^j = q_3 \tag{3.19b}$$

$$D_3^i = D_3^j = q_4 \tag{3.19c}$$

The advantage of using joint release is that the element models remain the same. However there is a disadvantage also and that is, it increases the degrees of freedom of the system model. Although for small structures increase in degrees of freedom will not create a problem but it may become crucial in larger structures.

3.2.3 FLEXIBLE CONNECTION BY CONDENSATION

Another method is the method of element release. This is commonly known as condensation and substructuring. The substructures which are known as complex elements or super elements, are assembled from simple elements like beam or truss elements. The complex elements have been studied by Livesley, (1964,1975), McGuire and Gallagher, (1979); Tong and Rossettos, (1977); Ziekiewicz, (1977). The method presented by Holzer, (1985), is used here. The internal degrees of freedom of the substructure elements are not shared with the other elements. Hence these degrees of freedoms are known as dependent variables and they are removed from the element model. Thus the complex model is condensed to a model containing the external degrees of freedom only. The flexible joint can now be assembled in a complex element. Then, eliminating the internal release by condensation, the complex element can be treated as a standard frame element.

The condensation operation can be represented as shown below. The complex element can be written as

$$\begin{bmatrix} \bar{f}_e \\ f_i \end{bmatrix} = \begin{bmatrix} k_{ee} & k_{ii} \\ k_{ie} & k_{ei} \end{bmatrix} \begin{bmatrix} d_e \\ d_i \end{bmatrix} \tag{3.20}$$

In the above equation, subscript e represents the external displacement and forces, and i represents the internal displacements and forces. The Equation (4.19) can be expanded as

$$\bar{f}_e = k_{ee} d_e + k_{ei} d_i \tag{3.21a}$$

$$f_i = k_{ie} d_e + k_{ii} d_i \tag{3.21b}$$

From Equation 4.21(b)

$$d_i = -k_{ii}^{-1} k_{ie} d_e + k_{ii}^{-1} f_i \tag{3.22}$$

Substituting d_i in Equation 4.21(a)

$$\bar{f}_e = k_{ee} d_e - k_{ei} k_{ii}^{-1} k_{ie} d_e + k_{ei} k_{ii}^{-1} f_i$$

$$\bar{f}_e = [k_{ee} - k_{ei} k_{ii}^{-1} k_{ie}] d_e + k_{ei} k_{ii}^{-1} f_i$$

$$\bar{f}_e = f_e + f_e^{*} \tag{3.23}$$

where,

$$f_e = k_e d_e \tag{3.24}$$

$$k_e = k_{ee} - k_{ei} k_{ii}^{-1} k_{ie} \tag{3.25}$$

$$f_e^* = k_{ei}k_{ii}^{-1}f_i \qquad [3.26]$$

From Equation (3.23) and (3.24) it can be seen that

$\bar{f}_e = f_e^*$ when $d_e = 0$. From Equations (3.23) and (3.26) it can be seen that $\bar{f}_e = f_e$ when $f_i = 0$ Thus it is concluded that f_e^* is the external force vector generated by the element loads f_i when there are no external displacements. As an example, a beam spring assembly can be considered as shown in Figure 23. The f_e^* represents the fixed end force vector. This case is similar to the fixed end force vector $\hat{f}$ due to element actions. The fixed end force vector $\hat{f}$ is due to element actions in general but f^* is caused specifically by element loads f^i The external displacements are determined by the matrix displacement method and consequently the internal displacements can be computed by Equation (3.21).

The method of formulation and condensation element is useful when a structure contains a large number of identical complex elements. Also, it is useful if the structure is too large to be solved as a single system , with the available computer. The structure may be divided into two or more substructures and then solved.

3.3 APPLIED FLEXIBLE FRAME MODEL

It is intended to study the moment and deflection characteristics of a gable frame with partially restrained connection or flexible connection. The idea is to adopt a simple system so that the basic frame element model need not be modified. The structure is not large enough to consider complex model using condensation and substructures.

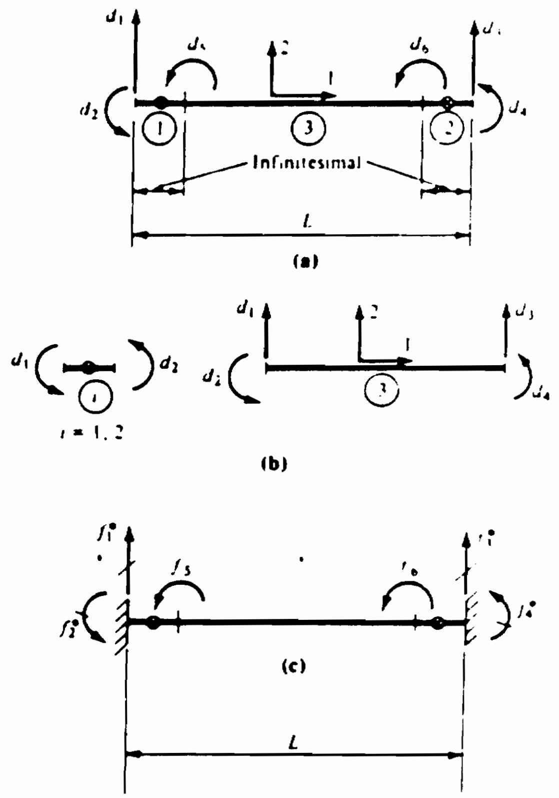


Figure 23. **Complex Element (Holzer 1985, Figure 4.18):** (a) Beam-spring Assembly (b) Elements (c) fixed-end forces

The flexibility of the joint was modeled by incorporating a rotational spring model at the connection of two frame elements. Livesley, (1964,1975), Holzer, (1985), explained flexible connections as joints, which transmits a moment proportional to the relative rotation of the joint. The rotational spring model is shown in Figure 24 along with a frame element model. The stiffness matrix for a frame element model in local coordinates is

$$K = \alpha \begin{bmatrix} \beta & 6L & 0 & 0 & 0 & 4L^2 \\ 0 & -12 & 6L & 0 & -12 & -6L \\ 0 & 0 & 4L^2 & \beta & -6L & 0 \\ -\beta & 6L & 0 & 0 & 0 & 2L^2 \\ 0 & 12 & -6L & 0 & 12 & 6L \\ 0 & 0 & 2L^2 & -\beta & -6L & 0 \end{bmatrix} \qquad [3.27]$$

where,

$$\alpha = \frac{EI}{L^3}$$

$$\beta = A\,\frac{L^2}{I} \qquad [3.28]$$

The moment-rotation relationship of the rotational spring is given by

$$M = \gamma(\theta_R - \theta_L) \qquad [3.29]$$

where, γ = rotational spring stiffness , moment per unit rotation

From Figure 24(b) and 24(c)

$$-f_1 = M = f_2 \ , \ \ d_1 = \theta_L \ , \ \ d_2 = \theta_R \qquad [3.30]$$

Thus, the spring model from Equation (3.29)

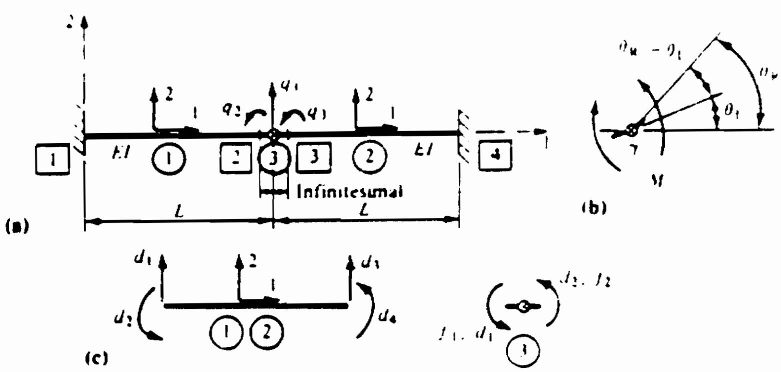


Figure 24. **Beam-spring Assembly:** (a) Rotational Spring (b) elements

$$\begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \gamma \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} \qquad [3.31]$$

The joint forces which are applied and reactive external forces of a joint are calculated from conditions of joint equilibrium.

$$P_j = \sum_{l=1}^{NE} F_j^l \quad where, \quad j = 1, 2, ...NJ \qquad [3.32]$$

and

$P_j =$ force vector at joint j

NE = Number of element

NJ = Number of joint

$F_j^i = F_a^i$ if the a end of element i is incident to joint j;

$= F_b^i$ if the b end of element i is incident to joint j;

$=$ 0 otherwise.

In the spring element model considered for the flexible connection, shear forces are not transmitted. Only the moments are transmitted as shown in Equation (3.29). But in order to use the equilibrium Equation (3.30) to compute the joint forces there should be one to one correspondence between the element forces and joint forces. Thus, the spring element model can be represented as

$$f = kd$$

In matrix form, the above equation can be represented as

$$
\begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \end{bmatrix} = \gamma \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix} \qquad [3.33]
$$

Thus a model is ready for the frame element and also for the rotational spring. The models may now be used in conjunction to represent a frame with flexible connection.

3.4 COMPUTER CODE

A computer code is developed based on a program developed in CE 4002(Holzer, 1988) using Matrix Displacement method for analysis of framed structures. The members are considered connected by rotational springs at the joints. The members are modeled as frame elements and the joints as rotational springs. Details of the computer code are presented in Apendix A. The new models developed in this thesis are identified in the tree chart of the complete program in Apendix A.

Chapter IV

FLEXIBLE FRAME MODEL APPLICATION

4.1 INTRODUCTION

In the previous chapter the computer code implementation for the flexibility of connections was discussed. A computer code utilizing the underlying principles was also developed. The effect of flexibility will now be studied in gable frames. An end plate connection was selected from studies conducted by Kukreti, Murray, Abolmali, (1987). The stiffness was computed from the relevant plots and introduced into the data for running the computer program. The stiffness is then reduced by a known amount and the effect on moments and displacements are observed. The gable frames were also analyzed with fixed joints. The slopes of the rafters were varied and different spans were selected and the gable frames analyzed. The results are summarized and discussed in the following sections.

4.2 TEST PROBLEMS

Four frames were chosen to run the test problems. The same member properties were provided for all the members. The typical frames are 25 ft apart. The first two frames have a rafter slope of 1 in 12. Frames 3 and 4 have a rafter slope of 1 in 3. The knee height of frames 1 and 2 was fixed at 20 ft and for 3 and 4 at 15 ft. The vertical uniformly distributed load of 25 psf. resulted in a load of 0.625 k/ft. Another load 50 psf was applied which is equivalent to 1.25 k/ft. The lateral load (wind load) of 20 psf. resulted in 0.5 k/ft and another load of 40 psf was applied which is equal to 1.0 k/ft. The test result from Kukreti, Murray, Abolmali, (1987), F-3/4-1/2-24A was used and the spring stiffness calculated as $k_1 = 60.48X10^5$ k-in/rad. Two other values of spring stiffnesses were used in the computer analysis of the frame. They are $k_2 = 18.34X10^5$ k-in/rad and $k_3 = 6.05X10^5$ k-in/rad which are 30 percent and 10 percent of of k_1 respectively.

The computer test results are tabulated in Table 4.1 and 4.2, for vertical loads and Table 4.3 and 4.4, for lateral loads.

4.3 OBSERVATIONS

The variation in moments between fixed and flexible connections are tabulated in Tables 5, and 6 for vertical load and Table 7, and 8 for lateral load. Variation in moments due to the introduction of flexible connections is calculated from moments when the joints are fixed. The observations of the computer test results are summarized as follows:

4.3.1 TEST FRAME TP-1

In case of the vertical load of 0.625 k/ft for frame TP-1 and for the connection stiffness selected, $k_1 = 60.48X10^5$ k-in/rad., the moments at joints 2 and 4 of the gable frame are reduced by 0.519 percent and at joint 3, the apex , the moment was increased by 0.537 percent with respect to the moments with joints 2, 3, and 4 fixed.

The connection stiffness at joints 2, 3, and 4 were reduced from an initial stiffness k_1 to .30 k_1 and the frame was analyzed for the same vertical load. The moments at the joints 2 and 4 reduced by 1.59 percent and at joint 3, the moment increased by 1.76 percent.

The connection stiffness of joints 2, 3, and 4, were further reduced to 10 percent of k_1 and the frame was loaded as above. The reduction of moment at joints 2 and 4 was 4.32 percent and the increase in moment at joint 3 was 4.92 percent.

The vertical load was increased to 1.25 k/ft and the frame was analyzed with joints fixed and also for different spring stiffnesses. The variation in moments was found to be negligible compared to the corresponding test results with the previous loading.

The frame TP-1 was then analyzed for a lateral loading of 0.5 k/ft and the connection stiffnesses were varied as before. It was observed that for k_1 the moment increased by 0.084 percent at joint 2, decreased by 1.38 percent at joint 3, and by 0.11 percent at joint 4. For reduced stiffness the moment increased by 0.336 percent at joint 2, decreased by 4.99 percent at joint 3 and decreased by 0.348 percent at joint 4. With further reduction in joint stiffness the variation is increased by 0.97 percent at joint 2, decreased by 13.85 percent at joint 3 and decreased by 0..898 percent at joint 4.

The lateral load was increased to 1.0 k/ft and the above frame was analyzed for fixed connection and for connections with different spring stiffnesses as before. For k_1, the moment at joint 2 increased by 0.095 percent and at joint 3 reduced by 1.52 percent and at joint 4 reduced by .11 percent. For k_2 joint 2, the moment increased by 0.346, at joint 3, it decreased by 5.12 percent and at joint 4 reduced by 0.33 percent.

4.3.2 TEST FRAME TP-2

Then the frame TP-2 was tested for the different loads and joint stiffnesses. The knee height of the frame was retained at 20 ft. The span was increased to 60 ft. The same slope of 1 in 12 was maintained for the frame rafters. For the joint spring stiffness k_1, the reduction of moment at joint 2 was 0.416 percent and at joint 3 was increased by 0.599 percent. The variation thus was higher than the similar frame of lesser span.

With increased vertical loading the variations in moments at different joints remained nearly similar to that with lesser load.

For the lateral loading of 0.5 k/ft, the moment at joints 2, and 4 reduced, and at joint 3 increased compared to the similar frame of lesser span. As the connection stiffness was reduced the variation became more pronounced although the pattern remained the same.

As the lateral loading was increased the moment at joint 2 was increased to 0.063 percent and joint 4 to 0.098 percent and joint 3 decreased to 7.11 percent. For further decrease in joint spring stiffnesses the variations continue as above. There is no appreciable percentage variation of moments due to the increase in loading.

4.3.3 TEST FRAME TP-3

The next set of analysis was done with the slope increased to 1 in 3, the span and connection stiffness remaining as in previous test cases. The frame was named as TP-3 in the variation tables 5-3 and 5-4.

For vertical loading of 0.625 k/ft and k_1, the moment at joint2 decreased by 0.522 percent and joint 3 increased by 1.39 percent. There was an appreciable increase in percentage variation from TP-1 for the same loading.

As the stiffness of the connection is reduced to k_1 the moment in joint 2 is reduced by 4.36 percent whereas at joint 3 the moment is increased 12.27 percent. Increase in loading to 1.25 k/ft did not introduce an appreciable percentage variation of moments from the previous loading.

For a lateral load of 0.5 k/ft and k_1 the moment at joint 2 increased by 0.153 percent and joint 3 reduced by 1.99 percent and joint 4 reduced by 0.151 percent. An appreciable increase in moment was observed for joint 2 and decrease in moment for joint 3 and 4. However for increased lateral load of 1.0 k/ft percentage variation reduced marginally compared to that of lower loads.

4.3.4 TEST FRAME TP-4

The span of the frame TP-4 is 60 ft and the slope is 1 in 3 as in the previous case. For vertical loading of 0.625 k/ft the decrease in moment for joint 2 was 0.346 percent and the increase in joint 3 was 1.75 percent. Comparing the results to the frame TP-2, it can be observed that the percentage variation is less for corresponding joint 2 than in corresponding joint 3.

The same trend as above was observed for reduced connection stiffness. For increased vertical load the variation in moments were similar to lesser load.

The frame was tested for a lateral load of 0.5 k/ft and the increase in moment at joint 2 was 0.164 percent and the decrease in joint 3 was 2.84 percent and the decrease in joint 4 was 0.12 percent. There is a considerable increase in moments compared to the same span frame TP-2 with a lesser slope. For increased lateral load there was no appreciable variation in moments.

4.4 CONCLUSION

From the above analysis results, it can be concluded that for the same span and slope of rafters, increase in vertical or lateral loading does not effect an appreciable percentage variation in moments.

For increased span of gable frames with the same slope, increased variation in moment was observed in case of vertical load. For lateral loading the variation of moment at the joint 3 increases whereas for joint 2 and 4 it reduces.

In case of frames with the same spans but different rafter slopes, the variation in moment at joint 3 increases with raise in slope.

On application of lateral loads to frames of the same span but increased slope, the variation in moment was found to increase for joint 2, 3, and 4 as rafter slope is increased.

Table 1. Moments(k-ft) in gable frame for vertical load-1

FRAME	SPAN	SLOPE	JT.NO.	.625 k/ft			
				FIXED	k_1	k_2	k_3
			TT.NO.	100	101	102	103
TP-1	40 ft	1 in 12	2	61.61	61.29	60.63	58.95
			3	57.71	58.02	58.73	60.55
			TT.NO.	200	201	202	203
TP-2	60 ft	1 in 12	2	148.90	148.28	146.96	143.46
			3	111.87	112.54	114.03	117.96
			TT.NO.	300	301	302	303
TP-3	40 ft	1 in 3	2	57.4	57.1	56.5	54.9
			3	28.77	29.17	30.10	32.30
			TT.NO.	350	351	352	353
TP-4	60 ft	1 in 3	2	126.91	126.47	125.52	123.0
			3	40.03	40.73	42.31	46.49

Table 2. Moments(k-ft) in gable frame for vertical load-2

FRAME	SPAN	SLOPE	JT.NO.	1.25 k/ft			
				FIXED	k_1	k_2	k_3
			TT.NO.	110	111	112	113
TP-1	40 ft	1 in 12	2	123.2	122.58	121.26	117.9
			3	115.41	116.05	117.47	121.11
			TT.NO.	210	211	212	213
TP-2	60 ft	1 in 12	2	297.8	296.56	293.92	286.9
			3	223.7	225.07	228.5	235.9
			TT.NO.	310	311	312	313
TP-3	40 ft	1 in 3	2	114.8	114.2	112.98	109.8
			3	57.5	58.33	60.10	64.6
			TT.NO.	360	361	362	363
TP-4	60 ft	1 in 3	2	253.82	252.93	251.03	246.0
			3	80.06	81.47	84.61	92.97

Table 3. Moments(k-ft) in gable frame for lateral load-1

FRAME	SPAN	SLOPE	JT.NO.	.5 k/ft			
				FIXED	k_1	k_2	k_3
			TT.NO.	120	121	122	123
TP-1	40 ft	1 in 12	2	47.59	47.63	47.75	48.05
			3	3.61	3.56	3.43	3.11
			4	54.55	54.49	54.36	54.06
			TT.NO.	220	221	222	223
TP-2	60 ft	1 in 12	2	46.96	46.98	47.07	47.3
			3	1.97	1.92	1.82	1.57
			4	55.93	55.89	55.80	55.58
			TT.NO.	320	321	322	323
TP-3	40 ft	1 in 3	2	32.61	32.66	32.76	33.02
			3	3.51	3.44	3.29	2.92
			4	39.77	39.71	39.61	39.35
			TT.NO.	370	371	372	373
TP-4	60 ft	1 in 3	2	30.35	30.40	30.53	30.86
			3	3.16	3.07	2.86	2.30
			4	49.87	49.81	49.68	49.35

Table 4. Moments(k-ft)in gable frame for lateral load-2

FRAME	SPAN	SLOPE	JT.NO.	1.0 k/ft			
				FIXED	k_1	k_2	k_3
			TT.NO.	130	131	132	133
TP-1	40 ft	1 in 12	2	95.18	95.27	95.51	96.10
			3	7.23	7.12	6.86	6.21
			4	109.1	108.98	108.74	108.13
			TT.NO.	230	231	232	233
TP-2	60 ft	1 in 12	2	93.91	93.97	94.10	94.60
			3	3.94	3.85	3.66	3.14
			4	111.88	111.77	111.6	111.2
			TT.NO.	330	331	332	333
TP-3	40 ft	1 in 3	2	65.22	65.31	65.52	66.03
			3	7.01	6.88	6.58	5.83
			4	79.53	79.43	79.22	78.70
			TT.NO.	380	381	382	383
TP-4	60 ft	1 in 3	2	60.69	60.80	61.05	61.72
			3	6.32	6.13	5.71	4.61
			4	99.74	99.61	99.36	98.76

Table 5. Moment variation in percent for vertical load-1

FRAME	SPAN	SLOPE	JT.NO.	.625 k/ft			
				FIXED	k_1	k_2	k_3
			TT.NO.	100	101	102	103
TP-1	40 ft	1 in 12	2		-.519	-1.59	-4.32
			3		.537	1.76	4.925
			TT.NO.	200	201	202	203
TP-2	60 ft	1 in 12	2		-.416	-1.30	-3.65
			3		.599	1.93	5.44
			TT.NO.	300	301	302	303
TP-3	40 ft	1 in 3	2		-.522	-1.57	-4.36
			3		1.39	4.62	12.27
			TT.NO.	350	351	352	353
TP-4	60 ft	1 in 3	2		-.346	-1.09	-3.08
			3		1.75	5.70	16.13

Table 6. Moment variation in percent for vertical load-2

				1.25 k/ft			
FRAME	SPAN	SLOPE	JT.NO.	FIXED	k_1	k_2	k_3
			TT.NO.	110	111	112	113
TP-1	40 ft	1 in 12	2		-.503	-1.57	-4.30
			3		.545	1.78	4.93
			TT.NO.	210	211	212	213
TP-2	60 ft	1 in 12	2		-.416	-1.30	-3.66
			3		.612	2.15	5.45
			TT.NO.	310	311	312	313
TP-3	40 ft	1 in 3	2		-.523	-1.59	-4.36
			3		1.44	4.52	12.34
			TT.NO.	360	361	362	363
TP-4	60 ft	1 in 3	2		-.35	-1.10	-3.08
			3		1.76	5.68	16.13

Table 7. Moment variation in percent for lateral load-1

FRAME	SPAN	SLOPE	JT.NO.	.5 k/ft			
				FIXED	k_1	k_2	k_3
			TT.NO.	120	121	122	123
TP-1	40 ft	1 in 12	2		.084	.336	.97
			3		-1.38	-4.99	-13.85
			4		-.110	-.348	-.898
			TT.NO.	220	221	222	223
TP-2	60 ft	1 in 12	2		.042	.234	.724
			3		-2.5	-7.6	-20.3
			4		-.072	-.232	-.625
			TT.NO.	320	321	322	323
TP-3	40 ft	1 in 3	2		.153	.460	1.26
			3		-1.99	-6.27	-16.81
			4		-.151	-.402	-1.06
			TT.NO.	370	371	372	373
TP-4	60 ft	1 in 3	2		.164	.593	1.68
			3		-2.84	-9.49	-27.2
			4		-.12	-.38	-1.04

\

Table 8. Moment variation in percent for lateral load-2

FRAME	SPAN	SLOPE	JT.NO.	1.0 k/ft			
				FIXED	k_1	k_2	k_3
			TT.NO.	130	131	132	133
TP-1	40 ft	1 in 12	2		.095	.346	.967
			3		-1.52	-5.12	-14.11
			4		-.11	-.33	-.889
			TT.NO.	230	231	232	233
TP-2	60 ft	1 in 12	2		.063	.202	.735
			3		-2.28	-7.11	-20.3
			4		-.098	-.25	-.608
			TT.NO.	330	331	332	333
TP-3	40 ft	1 in 3	2		.138	.460	1.24
			3		-1.85	-6.13	-16.83
			4		-.126	-.39	-1.04
			TT.NO.	380	381	382	383
TP-4	60 ft	1 in 3	2		.181	.593	1.5
			3		-3.01	-9.65	-27.06
			4		-.13	-.381	-.983

Chapter V

SUMMARY AND FUTURE DEVELOPMENTS

5.1 SUMMARY

In this study the first part dealt with the classification of the flush end-plate connection with a single row of bolts at the tension region. The moment-rotation curves were obtained from the studies conducted by Kukreti, Murray, Abolmali, (1987). The basis of classification were the studies done by Ackroyd and Gerstle, (1982). The connections could now be classified on the basis of moment at the connection. The classification was further modified by the studies conducted by Bjorhovde, Brozzetti, and Colson, (1987). A graph was plotted between M / M_p and ϕ/ϕ_p. The zone for fully restrained, FR and partially restrained, PR is marked on the graph to classify the connection type.

In the next part of the study, the partially restrained connections are applied to gable frames with pinned ends, and the moment and deflection characteristics are studied. Rotational springs of known stiffnesses are introduced at the joints. The stiffnesses were calculated from the tests conducted by Kukreti, Murray, Abolmali, (1987). The joints are modeled using matrix displacement method and the computer code developed for it. Different spans and rafter slopes of gable frames were chosen and different loads were considered. The program was used to analyze the gable frames. First the end displacements are determined and then the moments are calculated.

5.2 CONCLUSIONS

Concurrent with the scope of research the conclusion is divided into two parts. The first part deals with the classification of connections as type PR or FR, as per LRFD specification, (1986). The second part deals with the effect of specified rotational stiffnesses on pin-pin Gable frames.

5.2.1 CONNECTION CLASSIFICATION

In this part of the study, the flush end plate connection with a single row of bolt at the tension region, studied by Kukreti, Murray, Abolmali, (1987), was classified. The classification was based on the classification system of Ackroyd and Gerstle, (1982). The moment rotation relationship of Kukreti, Murray, Abolmali, (1987), was used in the above classification. Depending on the moment a connection can be classified as FR or PR connection. However, since Classification of Connection was based on the stiffness of the beam, the length is also a variable for classifying the connection. Hence for a particular length of beam, a specific moment is determined. The connection will be FR below the specified moment and PR above that. Connection classification is also determined on the basis of suggestions by Bjorhovde, Brozzeti, Colson, (1987). The connection examples from Kukreti, Murray, Abolmali, (1987), are found to be FR and the plot is shown in Figure 16 demarcating the FR & PR connections.

5.2.2 CONNECTION FLEXIBILITY

It was observed from the above study that for a particular frame, percentage variation of moments at a particular joint does not vary appreciably with the loading intensity. For vertical loading, as the span was increased, the percentage variation of moment at a particular joint increased. For lateral loading the percentage variation increases in the windward side of the frame but reduces at the other joints. With the increase in slope the moment at the apex increases. With the level of loading used, the connections considered from the tests by Kukreti, Murray, Abolmali, (1987), are classified as FR connections. After reducing the stiffness of the connection to thirty percent of the initial stiffness k_1, the variation in moments are not found to be appreciable to require a change in connection design. Hence it can be concluded that flush end plate connections can be used efficiently and economically in gable frames.

5.3 FUTURE RECOMMENDATIONS

The scope of work was limited to flush end-plate connections only. The method could be extended to the other types of end plate connections provided the corresponding moment-rotation curve is available. Effect of cyclic loading and dynamic loading on the flexible joint frame would be useful. It is considered that the panel effect reduces the flexibility of the connection. Hence, a study towards panel effect in conjunction with flexibility will be worthwhile.

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Appendix A

TREE CHART-COMPUTER CODE

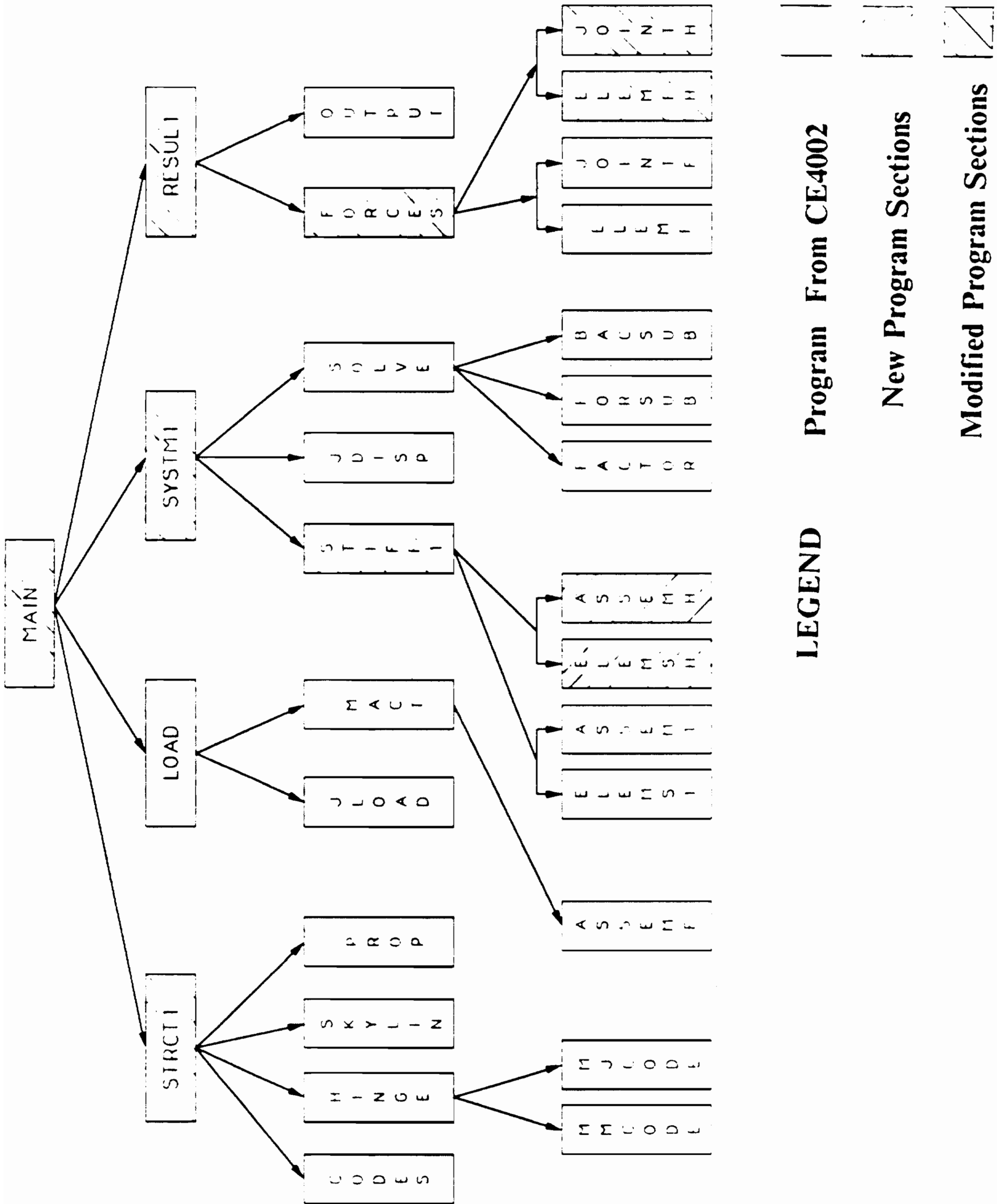

Figure 25. Tree Chart-Computer Code

Appendix B

COMPUTER CODE

Program modifications CE4002 shown in tree chart

```
C***************************************************************
C*                    PROGRAM FUNCTION                          *
C***************************************************************
C     MATRIX DISPLACEMENT ANALYSIS (FIG. 3.9) OF PLANE FRAMES
C
C     CAPABILITIES AND LIMITATIONS:
C       PRISMATIC MEMBERS
C       INTERNAL HINGES
C       MULTIPLE LOAD CONDITIONS:
C         JOINT LOADS
C         COMBINATIONS OF THE FOLLOWING MEMBER ACTIONS:
C           CONCENTRATED LOAD (FIG. A.9, P. 376)
C           DISTRIBUTED LOAD (FIG. A.10, P. 377)
C           TEMPERATURE CHANGE (EQ. 4.238 WITH DELTA T = 0, P. 224)
C         PRESCRIBED JOINT DISPLACEMENTS
C***************************************************************
C*                     INPUT DATA                               *
C***************************************************************
C     LIST-DIRECTED INPUT:
C     INPUT UNITS: KIP, INCH, RADIAN, FAHRENHEIT
C
C     1. ENTER DATE IN THE FORM  12/9/87  (IN MAIN)
C          DATE
C
C     2. ENTER NUMBER OF ELEMENTS, NUMBER OF JOINTS,
C        AND NUMBER OF LOAD CONDITIONS (IN MAIN)
C          NE, NJ, NLC
C
C     3. ENTER MEMBER INCIDENCES (IN STRUCT)
C          MINC(1,I), MINC(2,I)  I=1,NE
C
C     4. ENTER FOR EACH JOINT CONSTRAINT (IN STRUCT)
C          JNUM, JDIR
C          AFTER LAST JOINT CONSTRAINT ENTER
C          0, 0
C
C     5. IF THERE ARE INTERNAL HINGES ENTER (IN HINGE)
```

```
C          MN, ME
C          AFTER LAST HINGE ENTER
C          0, 0
C        ELSE ENTER
C          0, 0
C        END IF
C
C     6. ENTER JOINT COORDINATES (IN PROP)
C          X(1,J), X(2,J)  J = 1,NJ
C
C     7. ENTER MEMBER PROPERTIES (IN PROP)
C          AREA(I), ZI(I), EMOD(I), CTE(I)   I = 1,NE
C
C     8. DO FOR EACH LOAD CONDITION
C
C          IF THERE ARE JOINT LOADS ENTER (IN JLOAD)
C            JNUM, JDIR, FORCE
C            AFTER LAST JOINT LOAD ENTER
C            0, 0, 0
C          ELSE ENTER
C            0, 0, 0
C          END IF
C
C          IF THERE ARE MEMBER ACTIONS ENTER (IN MACT)
C            MN, MAT, ACT, DIST
C            AFTER LAST MEMBER ACTION ENTER
C            0, 0, 0, 0
C          ELSE ENTER
C            0, 0, 0, 0
C          END IF
C
C          IF THERE ARE PRESCRIBED JOINT DISPLACEMENTS ENTER (IN JDISP)
C            JNUM, JDIR, DISP
C            AFTER LAST PRESCRIBED JOINT DISPLACEMENT ENTER
C            0, 0, 0
C          ELSE ENTER
C            0, 0, 0
C          END IF
C
C        END DO
C*****************************************************************
C*                    MAIN PROGRAM                              *
C*                                        MARCH 21, 1989        *
C*****************************************************************
C
C--- MODIFICATIONS OF CE4002 PROGRAM:
C       DYNAMIC ARRAY DIMENSIONING
C       SKYLINE EQUATION SOLVER
C       INTERNAL HINGES
C       PRESCRIBED JOINT DISPLACEMENTS
C
C
C--- NEW PARAMETERS AND VARIABLES
C
C     A(LIM)       MEMORY BLOCK
C
C     DISP         PRESCRIBED JOINT DISPLACEMENT
C
C     KHT(3*NJ)    COLUMN HEIGHT VECTOR (TEMPORARY VECTOR NEEDED TO
C                  GENERATE MAXA IN SUBROUTINE SKYLIN)
C                  KHT(I) = HALF BANDWIDTH OF ITH COLUMN OF K (HOLZER
C                       PP. 282 - 285, BATHE P. 703)
C
C     LIM          NUMBER OF MEMORY LOCATIONS ALLOCATED
C
C     LSS          LENGTH OF SS
C
C     MAXA(NEQ+1)  STORES ADDRESSES OF DIAGONAL ELEMENTS OF K
C                  MAXA(J) = LOCATION OF K(J,J) IN SYSTEM STIFFNESS
C                       VECTOR SS (FIG. 6.9, P.285)
C     ME           MEMBER END
C                  ME = 1 FOR A-END
```

```
C              ME = 2 FOR B-END
C
C    PFAC        PENALTY FACTOR
C
C    PNUM         PENALTY NUMBER
C              PNUM = PNFAC*MAX(STIFFNESS COEFFICIENT)
C
C    SS(LSS)      SYSTEM STIFFNESS VECTOR (VECTOR A, FIG. 6.4D P. 285)
C
C— CONVENTION FOR POINTERS
C
C    NNAME        LOCATION OF FIRST ELEMENT OF ARRAY NAME IN A(LIM)
C
C— DELETED PARAMETERS AND VARIABLES
C
C    MX, MXNEQ, MBD
C
C——————————————————————————————————————
C
C
      CHARACTER*(*) TITLE, UNITS, DATE*8, FNAME*12
      PARAMETER (LIM = 4000, TITLE = 'PLANE FRAME ANALYSIS',
     $          UNITS = 'UNITS: KIP, INCH, RADIAN, FAHRENHEIT')
      INTEGER A
      DIMENSION A(LIM)
C
C    RESERVE MEMORY; READ AND ECHO NE, NJ, NLC; SET POINTERS FOR DATA
C    ARRAYS; IF MEMORY IS ADEQUATE CALL STRUCT, ELSE SEND MESSAGE AND
C    STOP; SET POINTERS FOR SOLUTION ARRAYS; IF MEMORY IS ADEQUATE,
C    CALL FOR EACH LOAD CONDITION  LOAD, SYSTEM, AND RESULT, ELSE
C    SEND MESSAGE AND STOP.
C
C——————————————————————————
C    OPEN DATA FILES: ********* FOR PC ONLY*******
*     WRITE(*,'("0",A\)') 'INPUT DATA FILE: '
*     READ(*,'(A)') FNAME
*     OPEN(5,FILE = FNAME)
*     OPEN(6,FILE = 'FRAME.OUT', STATUS = 'NEW')
C——————————————————————————
C
C    ASSIGN FNAME FOR MAIN FRAME ONLY
      FNAME = 'ONLY FOR PC'
C
      READ(5,'(A)') DATE
      WRITE(6,10) TITLE,'(CE 4002 1988)',
     $'DATA FILE: ',FNAME,'DATE: ',DATE,UNITS
   10 FORMAT('1',T5,73('*')/T5,'*',T32,A,T77,'*'/T5,'*',T35,A,T77,'*'/
     $        T5,73('*')//T5,2(A),T64,2(A)///T5,A)
      READ(5,*) NE, NJ, NLC
      WRITE(6,20) 'CONTROL VARIABLES',
     $          'Number of elements',NE,'Number of joints',NJ,
     $          'Number of load conditions',NLC
   20 FORMAT(///T5,A/3(T5,A,T30,I4/))
C
C— SET POINTERS FOR DATA ARRAYS (IN STRUCT): IF SPRINGS ARE USED
C    FOR ALL SUPPORTS (POSSIBLE FUTURE EXTENSION), P(3*NJ) IS NOT
C    REQUIRED BECAUSE REACTIVE FORCES ARE EQUAL TO SPRING FORCES.
C
      NP = 1
      NAREA = NP + 3*NJ
      NZI = NAREA + NE
      NEMOD = NZI + NE
      NGM = NEMOD + NE
      NCTE = NGM + NE
      NELENG = NCTE + NE
      NC1 = NELENG + NE
      NC2 = NC1 + NE
      NMT = NC2 + NE
      NMCODE = NMT + NE
      NJCODE = NMCODE + 6*NE
      NMINC = NJCODE + 3*NJ
```

```
      NMAXA = NMINC + 2*NE
C      TEMPORARILY LET NEQ = 3*NJ TO SET POINTERS FOR NKHT AND MAX
      NEQ = 3*NJ
      NKHT = NMAXA + (NEQ + 1)
      MAX = NKHT + NEQ-1
C
C— IF MEMORY IS ADEQUATE CALL STRUCT, ELSE SEND MESSAGE AND STOP.
C
      IF(MAX .LE. LIM) THEN
        CALL STRCT1(A(NP),A(NAREA),A(NZI),A(NEMOD),A(NGM),A(NCTE),
     $          A(NELENG),A(NC1),A(NC2),A(NMT),A(NMAXA),A(NKHT),
     $          A(NMCODE),A(NJCODE),A(NMINC),NE,NJ,NEQ,LSS)
C
C—— SET POINTERS FOR SOLUTION ARRAYS (IN LOAD, SYSTEM, RESULT)
C
 89     FORMAT( 6(I10))
        NF = NMAXA + (NEQ + 1)
        NSS = NF + 6*NE
        NQ = NSS + LSS
        NNA = NQ + NEQ
        MAX = NNA + NE-1
C
C—— IF MEMORY IS ADEQUATE CALL FOR EACH LOAD CONDITION LOAD,
C        SYSTEM, AND RESULT.
C
        IF(MAX .LE. LIM) THEN
          DO 30 LC = 1, NLC
            CALL LOAD(A(NF),A(NQ),A(NAREA),A(NEMOD),A(NCTE),A(NELENG)
     $          ,A(NC1),A(NC2),A(NMCODE),A(NJCODE),A(NNA),NE,
     $          NEQ,LC)
            CALL SYSTM1(A(NSS),A(NQ),A(NAREA),A(NZI),A(NEMOD),A(NGM)
     $          ,A(NELENG),A(NC1),A(NC2),A(NMT),
     $           A(NMAXA),A(NMCODE),A(NJCODE),NE,NEQ,LSS,LC)
            CALL RESUL1(A(NF),A(NP),A(NQ),A(NAREA),A(NZI),A(NEMOD),
     $           A(NGM),A(NELENG),A(NC1),A(NC2),A(NMT),
     $           A(NMCODE),A(NJCODE),A(NMINC),NE,NJ)
 30       CONTINUE
        ELSE
          WRITE(6,'(T10,A/)') 'ERROR MESSAGE: INCREASE MEMORY'
        END IF
      ELSE
        WRITE(6,'(T10,A/)') 'ERROR MESSAGE: INCREASE MEMORY'
      ENDIF
C
      STOP
      END
```

```
C*********************************************************************
C*                      STRCT1                              *
C*********************************************************************
      SUBROUTINE STRCT1(X,AREA,ZI,EMOD,GM,CTE,ELENG,C1,C2,MT,
     $                   MAXA,KHT,MCODE,JCODE, MINC,NE,NJ,NEQ,LSS)
      DIMENSION X(3,*),AREA(*),ZI(*),EMOD(*),GM(*),CTE(*),ELENG(*),
     $ C1(*),C2(*),MT(*),MAXA(*),KHT(*),MCODE(6,*),JCODE(3,*),MINC(2,*)
C
C     READ AND ECHO THE MEMBER INCIDENCES, MINC(L,I)
C     MEMBER TYPE, MT(I) ; INITIALIZE THE  ELEMENTS
C     OF THE JOINT CODE MATRIX, JCODE, TO UNITY, READ AND
C     ECHO FOR EACH JOINT CONSTRAINT THE JOINT NUMBER, JNUM, AND
C     JOINT DIRECTION, JDIR, AND STORE A ZERO IN THE CORRESPONDING
C     LOCATION OF JCODE (END OF DATA MARKER JNUM = 0); CALL CODES,
C     HINGE, SKYLIN, AND PROP.
C
      WRITE(6,10) 'MEMBER INCIDENCES','MEMBER TYPE',
     $'MEMBER','A-END','B-END'
   10 FORMAT(///T10,A,T28,A/T10,A,T17,A,T23,A)
      DO 30 I = 1,NE
        READ(5,*) MINC(1,I),MINC(2,I),MT(I)
        WRITE(6,20) I,MINC(1,I),MINC(2,I),MT(I)
   20   FORMAT(T11,I3,T18,I3,T24,I3,T30,I3)
   30 CONTINUE
C
C
      WRITE(6,40) 'JOINT CONSTRAINTS','JOINT','DIRECTION'
   40 FORMAT(///T10,A/T10,A,T17,A)
      READ(5,*) JNUM,JDIR
   50 IF (JNUM.NE.0) THEN
        WRITE(6,60) JNUM,JDIR
   60   FORMAT(T11,I3,T19,I3)
        READ(5,*) JNUM,JDIR
        GO TO 50
      END IF
C
      CALL CODES(MCODE,JCODE,MINC,NE,NJ,NEQ)
C
      CALL HINGE(MCODE,JCODE,NE,NJ,NEQ)
C
      CALL SKYLIN(KHT,MAXA,MCODE,NE,NEQ,LSS)
C
      CALL PROP(X,AREA,ZI,EMOD,GM,CTE,ELENG,C1,C2,MINC,NE,NJ)
C
      RETURN
      END
C*********************************************************************
C*                      CODES                               *
C*********************************************************************
      SUBROUTINE CODES(MCODE,JCODE,MINC,NE,NJ,NEQ)
      DIMENSION MCODE(6,*),JCODE(3,*),MINC(2,*)
C
C     GENERATE THE JOINT CODE, JCODE, BY ASSIGNING INTEGERS IN SEQUENCE,
C     BY COLUMNS, TO ALL NONZERO ELEMENTS OF JCODE FROM 1 TO NEQ;
C     GENERATE THE MEMBER CODE, MCODE, BY TRANSFERRING VIA MINC COLUMNS
C     OF JCODE INTO COLUMNS OF MCODE.
C     — READ AND ECHO JCODE —
C
      DO 20 J = 1,NJ
        DO 10 L = 1,3
          READ(5,*) JCODE(L,J)
   10   CONTINUE
   20 CONTINUE
      WRITE(6,30)'JCODE'
   30 FORMAT(///T10,A)
      DO 40 L = 1,3
        WRITE(6,*)(JCODE(L,I),I = 1,NJ)
   40 CONTINUE
C
      NEQ = 0
      DO 60 J = 1,NJ
        DO 50 L = 1,3
```

```
          IF(JCODE(L,J) .GT. NEQ) THEN
             NEQ = JCODE(L,J)
          END IF
   50   CONTINUE
   60 CONTINUE
      WRITE(6,*)'NEQ = ', NEQ
C
C     -- READ AND ECHO MCODE --
C
      DO 80 I = 1,NE
        DO 70 L = 1,6
          READ(5,*) MCODE(L,I)
   70   CONTINUE
   80 CONTINUE
      WRITE(6,90)'MCODE'
   90 FORMAT(///T10,A)
      DO 100 L = 1,6
        WRITE(6,*)(MCODE(L,I),I = 1,NE)
  100 CONTINUE
      RETURN
      END
C*******************************************************************
C                         HINGE                            *
C*******************************************************************
      SUBROUTINE HINGE(MCODE,JCODE,NE,NJ,NEQ)
      DIMENSION MCODE(6,*), JCODE(3,*)
C
C     MODIFY MCODE AMD JCODE TO ACCOUNT FOR INTERNAL HINGES
C
      READ(5,*) MN,ME
C
      IF (MN .NE. 0) THEN
         WRITE(6,'(//T10,A/)') 'INTERNAL HINGES'
         WRITE(6,'(T10,A,3X,A/)') 'MEMBER', 'HINGE LOCATION'
 10      IF (MN .NE. 0) THEN
            NEQ = NEQ + 1
            WRITE(6,'(T10,I3,10X,I2/)') MN,ME
            CALL MMCODE(MCODE,NE,MN,ME,JDOF)
            CALL MJCODE(JCODE,NJ,JDOF)
            READ(5,*) MN,ME
            GO TO 10
         ENDIF
      ELSE
         WRITE (6,'(//T10,A/)') 'NO INTERNAL HINGES'
      ENDIF
C
      RETURN
      END
C*******************************************************************
C                         MMCODE                           *
C*******************************************************************
      SUBROUTINE MMCODE(MCODE,NE,MN,ME,JDOF)
      DIMENSION MCODE(6,*)
C
C     MODIFY MCODE
C
      IF (ME .EQ. 1) THEN
         JDOF = MCODE(3,MN)
      ELSE
         JDOF = MCODE(6,MN)
      ENDIF
C
      DO 20 J = 1, NE
         DO 10 L = 1, 6
            IF (MCODE(L,J) .GT. JDOF) THEN
               MCODE(L,J) = MCODE(L,J) + 1
            ENDIF
   10    CONTINUE
   20 CONTINUE
      IF (ME .EQ. 1) THEN
         MCODE(3,MN) = JDOF + 1
      ELSE
```

```
         MCODE(6,MN) = JDOF + 1
      ENDIF
C
      RETURN
      END
C*******************************************************************
C                       MJCODE                              *
C*******************************************************************
      SUBROUTINE MJCODE(JCODE,NJ,JDOF)
      DIMENSION JCODE(3,*)
C
C      MODIFY JCODE
C
      DO 20 J = 1, NJ
         DO 10 L = 1, 3
            IF (JCODE(L,J) .GT. JDOF) THEN
               JCODE(L,J) = JCODE(L,J) + 1
            ENDIF
   10    CONTINUE
   20 CONTINUE
C
      RETURN
      END
C*******************************************************************
C*                      SKYLIN                              *
C*******************************************************************
C
C     SKYLIN DETERMINES KHT USING MCODE, AND DETERMINES MAXA FROM KHT.
C
      SUBROUTINE SKYLIN(KHT,MAXA,MCODE,NE,NEQ,LSS)
      DIMENSION KHT(*),MAXA(*),MCODE(6,*)
C
      DO 10 I=1,NEQ
        KHT(I)=0
   10 CONTINUE
C
C     GENERATE KHT
C
      DO 30 I=1,NE
        J=1
   15   IF(MCODE(J,I) .EQ. 0 .AND. J .LT. 6) THEN
          J=J+1
          GO TO 15
        ENDIF
        MIN=MCODE(J,I)
        J=J+1
        DO 20 L=J,6
          K=MCODE(L,I)
          IF(K.NE.0) THEN
            KHT(K)=MAX0(KHT(K),(K-MIN))
          ENDIF
   20   CONTINUE
   30 CONTINUE
      WRITE(6,100)
  100 FORMAT(//'-',11X,'I',10X,'KHT(I)',10X,'MAXA(I)')
C
C     GENERATE MAXA
C
      MAXA(1)=1
      DO 40 I=1,NEQ
        WRITE(6,200) I,KHT(I),MAXA(I)
  200   FORMAT('0',7X,I5,9X,I5,11X,I5)
        MAXA(I+1)=MAXA(I)+KHT(I)+1
   40 CONTINUE
      LSS=MAXA(NEQ+1)-1
      I=NEQ+1
      WRITE(6,300) I,MAXA(I),LSS
  300 FORMAT('0',7X,I5,25X,I5//7X,2X,'LSS = ',I5)
      RETURN
      END
C*******************************************************************
```

```
C*                       PROP                              *
C**********************************************************************
      SUBROUTINE PROP(X,AREA,ZI,EMOD,GM,CTE,ELENG,C1,C2,MINC,NE,NJ)
      DIMENSION X(3,*),AREA(*),ZI(*),EMOD(*),GM(*),CTE(*),ELENG(*),
     $                        C1(*),C2(*),MINC(2,*)
C
C     READ AND ECHO THE JOINT COORDINATES, X(L,J); COMPUTE FOR EACH
C     ELEMENTS BY EQS. (C.21) THE LENGTH, ELENG(I), AND THE DIRECTION
C     COSINES, C1(I),C2(I); READ FOR EACH ELEMENT THE CROSS-SECTIONAL
C     AREA, MODULUS OF ELASTICITY, EMOD(I), AND THE COEFFICIENT OF
C     THERMAL EXPANSION, CTE(I): PRINT ELEMENT PROPERTIES.
C
      READ (5,*)(X(1,J),X(2,J),J = 1,NJ)
      WRITE(6,10)'JOINT COORDINATES'
 10   FORMAT(///T10,A)
      DO 30 J = 1,NJ
        WRITE(6,20)J,X(1,J),X(2,J)
 20     FORMAT(T10,I3,T18,F7.3,T30,F7.3)
 30   CONTINUE
      WRITE(6,40)'ELEMENT PROPERTIES'
 40   FORMAT(//T10,A)
      WRITE(6,50)'MOMENT OF','MODULUS OF','ROTATION','COEFF. OF','AREA'
     $ ,'INERTIA','ELASTICITY','STIFFNESS','TH. EXPN.','LENGTH'
 50   FORMAT(/T15,A,T25,A,T37,A,T47,A
     $       /T5,A,T15,A,T25,A,T37,A,T47,A,T57,A)
      DO 70 I = 1,NE
        J = MINC(1,I)
        K = MINC(2,I)
        EL1 = X(1,K)-X(1,J)
        EL2 = X(2,K)-X(2,J)
        ELENG(I) = SQRT(EL1**2 + EL2**2)
        C1(I) = EL1/ELENG(I)
        C2(I) = EL2/ELENG(I)
        READ(5,*) AREA(I),ZI(I),EMOD(I),GM(I),CTE(I)
        WRITE(6,60) AREA(I),ZI(I),EMOD(I),GM(I),CTE(I),ELENG(I)
 60     FORMAT(/T5,E8.2,T15,E8.2,T25,E8.2,T37,E8.2,T47,E8.2,T57,F7.3)
 70   CONTINUE
      RETURN
      END
```

```
C*********************************************************************
C*                         LOAD                              *
C*********************************************************************
      SUBROUTINE LOAD(F,Q,AREA,EMOD,CTE,ELENG,C1,C2,
     $           MCODE,JCODE,NA,NE,NEQ,LC)
      DIMENSION F(6,*),Q(*),AREA(*),EMOD(*),CTE(*),ELENG(*),C1(*),C2(*),
     $        MCODE(6,*),JCODE(3,*),NA(*)
C
C     INITIALIZE TO ZERO THE JOINT LOAD VECTOR, Q, THE LOCAL ELEMENT
C     (MEMBER) FORCE VECTOR, F, AND THE NUMBER OF ACTIONS VECTOR, NA;
C     CALL JLOAD AND MACT.
C
      WRITE(6,10) 'LOAD CONDITION',LC
   10 FORMAT('1',5(/),T10,A,T25,I2/'+',T10,17('_'))
C
      DO 20 K = 1,NEQ
         Q(K) = 0.0
   20 CONTINUE
C
      DO 40 I = 1,NE
        NA(I) = 0
        DO 30 L = 1,6
          F(L,I) = 0.0
   30   CONTINUE
   40 CONTINUE
C
      CALL JLOAD(Q,JCODE,NEQ)
      CALL MACT(F,Q,AREA,EMOD,CTE,ELENG,C1,C2,MCODE,NA,NE)
C
      RETURN
      END
C*********************************************************************
C*                         JLOAD                             *
C*********************************************************************
      SUBROUTINE JLOAD(Q,JCODE,NEQ)
      DIMENSION Q(*),JCODE(3,*)
C
C     READ THE JOINT NUMBER, JNUM, THE JOINT DIRECTION, JDIR, AND THE
C     APPLIED FORCE, FORCE; WHILE JNUM IS NOT EQUAL TO ZERO, PRINT JNUM,
C     JDIR, FORCE,STORE FORCE IN Q, AND READ JNUM, JDIR, FORCE.
C
      READ(5,*) JNUM,JDIR,FORCE
      IF ( JNUM .NE. 0 ) THEN
        WRITE(6,10)'JOINT LOADS'
 10     FORMAT(///T10,A)
        WRITE(6,20) 'JOINT','JOINT','NUMBER','DIRECTION','FORCE'
 20     FORMAT(//T10,A,T18,A/T10,A,T18,A,T29,A)
 30     IF ( JNUM .NE. 0 ) THEN
           WRITE(6,40) JNUM,JDIR,FORCE
 40         FORMAT(/T10,I4,T19,I4,T28,F7.2)
           K = JCODE(JDIR,JNUM)
           Q(K) = FORCE
           READ(5,*) JNUM,JDIR,FORCE
           GO TO 30
        ENDIF
      ELSE
        WRITE(6,50)'NO JOINT LOADS'
 50     FORMAT(///T10,A)
      ENDIF
*       WRITE(6,60)'Q VALUES'
*60     FORMAT(//T10,A)
*       DO 80 K = 1,NEQ
*       WRITE(6,70)'Q(',K,')  =',Q(K)
*70     FORMAT(/T10,A,T12,I1,T13,A,T19,F7.2)
*80     CONTINUE
      RETURN
      END
C*********************************************************************
C*                         MACT                              *
C*********************************************************************
      SUBROUTINE MACT(F,Q,AREA,EMOD,CTE,ELENG,C1,C2,MCODE,NA,NE)
```

```
      DIMENSION F(6,*),Q(*),AREA(*),EMOD(*),CTE(*),ELENG(*),NA(*)
C
C     READ THE MEMBER NUMBER, MN, THE MEMBER ACTION TYPE, MAT, THE
C     ACTION, ACT, AND THE DISTANCE, DIST; WHILE MN IS NOT EQUAL TO
C     ZERO, PRINT MN, MAT, ACT, DIST, INCREMENT THE ACTION COUNTER,
C     COMPUTE AND ACCUMULATE THE FIXED-END FORCES, AND READ MN, MAT,
C     ACT, DIST; CALL ASSEMF.
C
      READ(5,*) MN, MAT, ACT, DIST
      IF(MN.NE.0) THEN
        WRITE(6,10)'MEMBER ACTIONS','MEMBER','TYPE','ACTION','DISTANCE'
   10   FORMAT(///T10,A/T10,A,T17,A,T26,A,T33,A)
   20   IF (MN.NE.0) THEN
          WRITE(6,30) MN,MAT,ACT,DIST
   30     FORMAT(T11,I3,T19,I1,T22,F10.3,T33,F8.3)
C
          NA(MN) = NA(MN) + 1
          IF (MAT.EQ.1) THEN
C             CONCENTRATED LOAD
            EL = ELENG(MN)
            A = DIST/EL
            F(2,MN) = F(2,MN)-ACT*(1.0 + A**2*(2.0*A-3.0))
            F(3,MN) = F(3,MN)-ACT*EL*A*(1.0-A)**2
            F(5,MN) = F(5,MN) + ACT*A**2*(2.0*A-3.0)
            F(6,MN) = F(6,MN) + ACT*EL*A**2*(1.0-A)
          ELSE IF (MAT.EQ.2) THEN
C             DISTRIBUTED LOAD
            EL = ELENG(MN)
            A = DIST/EL
            P = ACT*EL
            F(2,MN) = F(2,MN)-P*(1.0-A)**3*(1.0 + A)/2.0
            F(3,MN) = F(3,MN)-P*EL*(1.0-A)**3*(1.0 + 3.0*A)/12.0
            F(5,MN) = F(5,MN)-P*(1.0 + A**3*(A-2.0))/2.0
            F(6,MN) = F(6,MN) + P*EL*(1.0 + A**3*(3.0*A-4.0))/12.0
          ELSE
C             TEMPERATURE CHANGE
            P = AREA(MN)*EMOD(MN)*CTE(MN)*ACT
            F(1,MN) = F(1,MN) + P
            F(4,MN) = F(4,MN)-P
          END IF
          READ(5,*) MN, MAT, ACT, DIST
          GO TO 20
        END IF
        CALL ASSEMF(F,Q,C1,C2,MCODE,NA,NE)
      ELSE
        WRITE(6,40) 'NO MEMBER ACTIONS'
   40   FORMAT(///T10,A)
      END IF
C
      RETURN
      END
C*****************************************************************
C*                       ASSEMF                                  *
C*****************************************************************
      SUBROUTINE ASSEMF(F,Q,C1,C2,MCODE,NA,NE)
      DIMENSION F(6,*),Q(*),C1(*),C2(*),MCODE(6,*),NA(*)
C
C     TRANSFORM AND ASSEMBLE THE LOCAL FIXED-END FORCES, F(L,I), TO
C     PRODUCE THE EQUIVALENT JOINT LOAD VECTOR, Q, BY EQS. 2.34, 2.37,
C     3.92-3.94 AND THE FORCE TRANSFORMATION OF SECTION 2.4.
C
C     STATEMENT FUNCTION
      FG(C1I,C2I,FLX,FLY) = C1I * FLX + C2I * FLY
C
      DO 20 I = 1, NE
        IF (NA(I) .NE. 0) THEN
          DO 10 L = 1, 6
            K = MCODE(L,I)
            IF (K .NE. 0) THEN
              IF (L .EQ. 1) THEN
                Q(K) = Q(K) - FG(C1(I),-C2(I),F(1,I),F(2,I))
              ELSE IF (L .EQ. 2) THEN
```

```
            Q(K) = Q(K) - FG(C2(I), C1(I),F(1,I),F(2,I))
          ELSE IF (L .EQ. 3) THEN
            Q(K) = Q(K) - F(3,I)
          ELSE IF (L .EQ. 4) THEN
            Q(K) = Q(K) - FG(C1(I),-C2(I),F(4,I),F(5,I))
          ELSE IF (L .EQ. 5) THEN
            Q(K) = Q(K) - FG(C2(I), C1(I),F(4,I),F(5,I))
          ELSE
            Q(K) = Q(K) - F(6,I)
          END IF
        END IF
10      CONTINUE
      END IF
20 CONTINUE
   RETURN
   END
```

```
C*********************************************************************
C*                        SYSTM1                              *
C*********************************************************************
      SUBROUTINE SYSTM1(SS,Q,AREA,ZI,EMOD,GM,ELENG,C1,C2,MT,
     $                MAXA,MCODE,JCODE,NE,NEQ,LSS,LC)
C
      DIMENSION SS(*),Q(*),AREA(*),ZI(*),EMOD(*),GM(*),ELENG(*),
     $        C1(*),C2(*),MT(*),MAXA(*),MCODE(6,*),JCODE(3,*)
C
C     FOR THE FIRST LOAD CONDITION, LC = 1, CALL STIFF JDISP AND SOLVE;
C     FOR SUBSEQUENT LOAD CONDITIONS, LC > 1, CALL JDISP AND SOLVE.
C     NOTE : THE INPUT ARGUMENT, Q, STORES THE JOINT LOADS; THE OUTPUT
C           ARGUMENT, Q, STORES THE JOINT DISPLACEMENTS.
C
      IF ( LC .EQ. 1 ) THEN
        CALL STIFF1(SS,AREA,ZI,EMOD,GM,ELENG,C1,C2,MT,
     $            MAXA,MCODE,NE,LSS)
      END IF
        CALL JDISP(SS,Q,MAXA,JCODE,NEQ)
        CALL SOLVE(SS,Q,MAXA,NEQ,LC)
      RETURN
      END
C*********************************************************************
C*                        STIFF1                              *
C*********************************************************************
      SUBROUTINE STIFF1(SS,AREA,ZI,EMOD,GM,ELENG,C1,C2,MT,
     $               MAXA,MCODE,NE,LSS)
      DIMENSION SS(*),AREA(*),ZI(*),EMOD(*),GM(*),ELENG(*),
     $        C1(*),C2(*),MT(*),G(7),MAXA(*),MCODE(6,*)
C
C     INITIALIZE THE SYSTEM STIFFNESS MATRIX, SS, TO ZERO; FOR
C     EACH ELEMENT IF MT = 1 CALL ELEMS1 AND ASSEM1, IF MT = 2 HINGE,
C     CALL ELEMSH AND ASSEMH.
C
      DO 10 L = 1,LSS
        SS(L) = 0.0
   10 CONTINUE
      DO 20 N = 1,NE
      EL = MT(N)
        IF (EL .EQ. 1) THEN
          CALL ELEMS1(AREA,ZI,EMOD,ELENG,C1,C2,G,N)
          CALL ASSEM1(SS,G,MCODE,MAXA,N)
        ELSE
          CALL ELEMSH(AREA,ZI,EMOD,GM,ELENG,C1,C2,N)
          CALL ASSEMH(SS,GM,MCODE,MAXA,N)
        END IF
   20 CONTINUE
      RETURN
      END
C*********************************************************************
C*                        ELEMS1                              *
C*********************************************************************
      SUBROUTINE ELEMS1(AREA,ZI,EMOD,ELENG,C1,C2,G,N)
      DIMENSION AREA(*),ZI(*),EMOD(*),ELENG(*),C1(*),C2(*),G(7)
C
C     FOR ELEMENT N, COMPUTE THE GLOBAL STIFFNESS COEFFIENTS, G(7),
C     DEFINED IN EQS. 3.64.
C
C
      ALP = EMOD(N)*ZI(N)/ELENG(N)**3
      BET = AREA(N)*ELENG(N)**2/ZI(N)
      G(1) = ALP*(BET*C1(N)**2 + 12.0*C2(N)**2)
      G(2) = ALP*C1(N)*C2(N)*(BET-12.0)
      G(3) = ALP*(BET*C2(N)**2 + 12.0*C1(N)**2)
      G(4) = -ALP*6.0*ELENG(N)*C2(N)
      G(5) = ALP*6.0*ELENG(N)*C1(N)
      G(6) = ALP*4.0*ELENG(N)**2
      G(7) = ALP*2.0*ELENG(N)**2
      RETURN
      END
C*********************************************************************
```

```
C*                       ASSEM1                          *
C*********************************************************************
      SUBROUTINE ASSEM1(SS,G,MCODE,MAXA,N)
      DIMENSION SS(*),G(*),MCODE(6,*),MAXA(*)
      INTEGER INDEX(6,6)/1,2,4,-1,-2,4,  2,3,5,-2,-3,5,  4,5,6,-4,-5,7,
     $              -1,-2,-4,1,2,-4, -2,-3,-5,2,3,-5, 4,5,7,-4,-5,6/
C
C     INITIALIZE INDEX BY EQ.7.4; ASSIGN STIFFNESS COEFFICIENTS, G(L),
C     OF ELEMENT N TO THE SYSTEM STIFFNESS MATRIX, SS, BY INDEX, MCODE,
C     AND MAXA (EQ. 6.9).
C
      DO 20 JE = 1, 6
        J = MCODE(JE,N)
        IF (J .NE. 0) THEN
          DO 10 IE = 1, JE
            I = MCODE(IE,N)
            IF (I .NE. 0) THEN
              K = MAXA(J) + J - I
              L = INDEX(IE,JE)
              IF (L .GT. 0) THEN
                SS(K) = SS(K) + G(L)
              ELSE
                SS(K) = SS(K) - G(-L)
              END IF
            END IF
   10     CONTINUE
        END IF
   20 CONTINUE
      RETURN
      END
C*********************************************************************
C*                        ELEMSH                         *
C*********************************************************************
      SUBROUTINE ELEMSH(AREA,ZI,EMOD,GM,ELENG,C1,C2,N)
*          CALL ELEMSH(AREA,ZI,EMOD,GAM,ELENG,C1,C2,G,N)
      DIMENSION AREA(*),ZI(*),EMOD(*),GM(1),ELENG(*),C1(*),C2(*)
C
C     FOR ELEMENT N, COMPUTE THE GLOBAL STIFFNESS COEFFIENTS, GM(1),
C     DEFINED IN EQS. 3.64.
C
C
      GM(1)=GM(N)
      WRITE(6,10)'MEMBER NO.','ROTATION STIFFNESS'
   10 FORMAT(///T10,A,T20,A)
      WRITE(6,20) N,GM(1)
   20 FORMAT(/T13,I3,T26,E8.2)
C
      RETURN
      END
C*********************************************************************
C*                       ASSEMH                          *
C*********************************************************************
      SUBROUTINE ASSEMH(SS,GM,MCODE,MAXA,N)
      DIMENSION SS(*),GM(*),MCODE(6,*),MAXA(*)
      INTEGER INDEX(6,6)/0,0,0,0,0,0,  0,0,0,0,0,0,  0,0,1,0,0,-1,
     $              0,0,0,0,0,0,  0,0,0,0,0,0,  0,0,-1,0,0,1/
C
C     INITIALIZE INDEX BY EQ.7.4; ASSIGN STIFFNESS COEFFICIENTS, G(L),
C     OF ELEMENT N TO THE SYSTEM STIFFNESS MATRIX, SS, BY INDEX, MCODE,
C     AND MAXA (EQ. 6.9).
C
      DO 20 JE = 1, 6
        J = MCODE(JE,N)
        IF (J .NE. 0) THEN
          DO 10 IE = 1, JE
            I = MCODE(IE,N)
            IF (I .NE. 0) THEN
              K = MAXA(J) + J - I
              L = INDEX(IE,JE)
              IF (L .GT. 0) THEN
                SS(K) = SS(K) + GM(L)
              ELSE
```

```
            SS(K) = SS(K) - GM(-L)
          END IF
        END IF
 10     CONTINUE
      END IF
 20 CONTINUE
    RETURN
    END
C*****************************************************************
C*                    JDISP                          *
C*****************************************************************
    SUBROUTINE JDISP(SS,Q,MAXA,JCODE,NEQ)
    DIMENSION SS(*),Q(*),MAXA(*),JCODE(3,*)
C
C    IMPOSE PRESCRIBED JOINT DISPLACEMENTS AT THE SYSTEM LEVEL BY MODI-
C    FYING SS AND Q ACCORDING TO PENALTY METHOD (BATHE PP.111,140,141)
C
    PFAC = 10000.0
    READ(5,*) JNUM,JDIR,DISP
    IF(JNUM .NE. 0) THEN
      WRITE(6,'(//T10,A/)') 'PRESCRIBED JOINT DISPLACEMENTS'
      WRITE(6,'(T10,A,2X,A,2X,A/)') 'JOINT', 'DIRECTION',
   $                       'DISPLACEMENT'
      PNUM = 0.0
      DO 10 J = 1,NEQ
        IF(SS(MAXA(J)) .GT. PNUM) THEN
          PNUM = SS(MAXA(J))
        ENDIF
10     CONTINUE
        PNUM = PNUM*PFAC
C
20      IF(JNUM .NE. 0) THEN
        WRITE(6,'(T10,I3,6X,I3,8X,F7.3/)') JNUM,JDIR,DISP
        J = JCODE(JDIR,JNUM)
        SS(MAXA(J)) = PNUM
        Q(J) = PNUM*DISP
        READ(5,*) JNUM,JDIR,DISP
        GO TO 20
      ENDIF
    ELSE
      WRITE(6,'(//T10,A/)') 'NO PRESCRIBED JOINT DISPLACEMENTS'
    ENDIF
C
    RETURN
    END
C*****************************************************************
C*                    SOLVE                          *
C*****************************************************************
    SUBROUTINE SOLVE(SS,Q,MAXA,NEQ,LC)
    DIMENSION SS(*),Q(*),MAXA(*)
C
C    SOLVE DETERMINES THE SOLUTION TO THE SYSTEM EQUATIONS BY COMPACT
C    GAUSSIAN ELIMINATION (HOLZER, PP. 290, 296, 307) BASED ON THE
C    SUBROUTINE COLSOL (BATHE P. 721) AND THE MODIFICATION BY MICHAEL
C    BUTLER (MS 1984): FOR THE FIRST LOAD CONDITION, LC = 1, CALL
C    FACTOR,FORSUB,AND BACSUB; FOR SUBSEQUENT LOAD CONDITIONS, LC > 1,
C    CALL FORSUB AND BACSUB.
C
    IF (LC.EQ.1) THEN
       CALL FACTOR(SS,MAXA,NEQ)
    END IF
       CALL FORSUB(SS,Q,MAXA,NEQ)
       CALL BACSUB(SS,Q,MAXA,NEQ)
    RETURN
    END
C*****************************************************************
C*                    FACTOR                         *
C*****************************************************************
C
C    FACTOR PERFORMS THE LDU FACTORIZATION OF THE STIFFNESS MATRIX.
C
    SUBROUTINE FACTOR(SS,MAXA,NEQ)
```

```
      DIMENSION SS(*),MAXA(*)
C
      DO 80 N = 1,NEQ
        KN = MAXA(N)
        KL = KN + 1
        KU = MAXA(N + 1)-1
        KH = KU-KL
        IF(KH) 70,50,10
   10   K = N-KH
        IC = 0
        KLT = KU
        DO 40 J = 1,KH
          IC = IC + 1
          KLT = KLT-1
          KI = MAXA(K)
          ND = MAXA(K + 1)-KI-1
          IF(ND) 40,40,20
   20     KK = MIN0(IC,ND)
          C = 0.00
          DO 30 L = 1,KK
   30     C = C + SS(KI + L)*SS(KLT + L)
          SS(KLT) = SS(KLT)-C
   40   K = K + 1
   50   K = N
        B = 0.00
        DO 60 KK = KL,KU
          K = K-1
          KI = MAXA(K)
          C = SS(KK)/SS(KI)
          B = B + C*SS(KK)
   60   SS(KK) = C
        SS(KN) = SS(KN)-B
C
C     STOP EXECUTION IF A ZERO PIVOT IS DETECTED
C
   70   IF(SS(KN).EQ.0.00) THEN
          PRINT 75,N,SS(KN)
   75     FORMAT('-STIFFNESS MATRIX IS NOT POSITIVE DEFINITE'/'0PIVOT IS
     $    ZERO FOR D.O.F. ',I4/'0PIVOT  =  ',E15.8)
          STOP
        END IF
C
   80 CONTINUE
      RETURN
      END
C
C*******************************************************************
C*                          FORSUB                                 *
C*******************************************************************
C
C     FORSUB PERFORMS THE FORWARD SUBSTITION.
C
      SUBROUTINE FORSUB(SS,Q,MAXA,NEQ)
      DIMENSION SS(*),Q(*),MAXA(*)
C
      DO 20 N = 1,NEQ
        KL = MAXA(N) + 1
        KU = MAXA(N + 1)-1
        KH = KU-KL
        IF(KH.GE.0) THEN
          K = N
          C = 0.00
          DO 10 KK = KL,KU
            K = K-1
            C = C + SS(KK)*Q(K)
   10     CONTINUE
          Q(N) = Q(N)-C
        END IF
   20 CONTINUE
      RETURN
      END
C
```

```
C**********************************************************************
C*                        BACSUB                              *
C**********************************************************************
C
C     BACSUB PERFORMS BACK-SUBSTITUTION TO OBTAIN THE SOLUTION.
C
      SUBROUTINE BACSUB(SS,Q,MAXA,NEQ)
      DIMENSION SS(*),Q(*),MAXA(*)
C
      DO 10 N = 1,NEQ
        K = MAXA(N)
        Q(N) = Q(N)/SS(K)
   10 CONTINUE
      IF(NEQ.EQ.1) RETURN
      N = NEQ
      DO 30 L = 2,NEQ
        KL = MAXA(N) + 1
        KU = MAXA(N + 1)-1
        KH = KU-KL
        IF(KH.GE.0)THEN
          K = N
          DO 20 KK = KL,KU
            K = K-1
            Q(K) = Q(K)-SS(KK)*Q(N)
   20     CONTINUE
        END IF
        N = N-1
   30 CONTINUE
      RETURN
      END
```

```
C*******************************************************************
C*                     RESUL1                          *
C*******************************************************************
      SUBROUTINE RESUL1(F,P,Q,AREA,ZI,EMOD,GM,ELENG,C1,C2,MT,
     $           MCODE,JCODE,MINC,NE,NJ)
      DIMENSION F(6,*),P(3,*),Q(*),AREA(*),ZI(*),EMOD(*),GM(*),ELENG(*)
     $      ,C1(*),C2(*),MT(*),MCODE(6,*),JCODE(3,*),MINC(2,*)
C
C     INITIALIZE THE JOINT FORCE MATRIX, P, TO ZERO ; CALL
C     FORCES AND OUTPUT.
C
      DO 20 J = 1,NJ
        DO 10 L = 1,3
          P(L,J) = 0.0
   10   CONTINUE
   20 CONTINUE
C
      CALL FORCES(F,P,Q,AREA,ZI,EMOD,GM,ELENG,C1,C2,MCODE,MINC,NE,MT)
      CALL OUTPUT(F,P,Q,JCODE,NE,NJ)
      RETURN
      END
C*******************************************************************
C*                     FORCES                          *
C*******************************************************************
      SUBROUTINE FORCES(F,P,Q,AREA,ZI,EMOD,GM,ELENG,C1,C2,
     $  MCODE,MINC,NE,MT)
      DIMENSION F(6,*),P(3,*),Q(*),AREA(*),ZI(*),EMOD(*),GM(*),ELENG(*)
     $      ,C1(*),C2(*),MT(*),MCODE(6,*),MINC(2,*),D(6)
C
C     FOR EACH ELEMENT I, CALL ELEMF AND JOINTF.
C
      DO 10 I = 1, NE
      EL = MT(I)
        IF(EL .EQ. 1) THEN
            CALL ELEMF(F,Q,AREA,ZI,EMOD,ELENG,C1,C2,MCODE,I)
            CALL JOINTF(F,P,C1,C2,MINC,I)
        ELSE
            CALL ELEMFH(F,Q,AREA,ZI,EMOD,GM,ELENG,C1,C2,MCODE,I)
            CALL JOINTH(F,P,C1,C2,MINC,I,GM)
        END IF
   10 CONTINUE
      RETURN
      END
C*******************************************************************
C*                     ELEMF                          *
C*******************************************************************
      SUBROUTINE ELEMF(F,Q,AREA,ZI,EMOD,ELENG,C1,C2,MCODE,I)
      DIMENSION F(6,*),Q(*),AREA(*),ZI(*),EMOD(*),ELENG(*),C1(*),C2(*),
     $       D(6),MCODE(6,*)
C
C     COMPUTE THE LOCAL FORCES IN ELEMENT I, F(6,I) : DETERMINE THE
C     GLOBAL ELEMENT DISPLACEMENTS, D(6), FROM THE JOINT DISPLACEMENT
C     VECTOR, Q, VIA MCODE ; COMPUTE THE LOCAL FORCES AT THE A-END
C     OF THE ELEMENT BY EQS. 2.34, 2.36 & 3.59 ; USE EQUILIBRIUM TO
C     COMPUTE THE LOCAL FORCES AT THE B-END OF THE ELEMENT AND
C     EQ. 3.95 TO COMPUTE THE ACTUAL ELEMENT FORCES.
C
      DO 10 L = 1,6
        K = MCODE(L,I)
        IF(K.EQ.0)THEN
            D(L) = 0.0
        ELSE
        WRITE (6,*) ' ELEMF, K  =', K
            D(L) = Q(K)
        ENDIF
   10 CONTINUE
C
      D1 = C1(I)*D(1) + C2(I)*D(2)
      D2 = -C2(I)*D(1) + C1(I)*D(2)
      D3 = D(3)
      D4 = C1(I)*D(4) + C2(I)*D(5)
      D5 = -C2(I)*D(4) + C1(I)*D(5)
```

```
      D6 = D(6)
C
      ALP = EMOD(I)*ZI(I)/ELENG(I)**3
      BET = AREA(I)*ELENG(I)**2/ZI(I)
      F1 = ALP*BET*(D1-D4)
      F2 = ALP*(12*D2 + 6*ELENG(I)*D3-12*D5 + 6*ELENG(I)*D6)
      F3 = ALP*(6*ELENG(I)*D2 + 4*(ELENG(I)**2)*D3-6*ELENG(I)*D5
     S + 2*(ELENG(I)**2)*D6)
C
C     ACTUAL ELEMENT-END FORCES
C
      F(1,I) = F(1,I) + F1
      F(2,I) = F(2,I) + F2
      F(3,I) = F(3,I) + F3
      F(4,I) = F(4,I)-F1
      F(5,I) = F(5,I)-F2
      F(6,I) = F(6,I) + ELENG(I)*F2-F3
      RETURN
      END
C*******************************************************************
C*                        JOINTF                                   *
C*******************************************************************
      SUBROUTINE JOINTF(F,P,C1,C2,MINC,I)
      DIMENSION F(6,*),P(3,*),C1(*),C2(*),MINC(2,*)
C
C     TRANSFORM THE LOCAL FORCES OF ELEMENT I, F(6,I), TO GLOBAL
C     FORCES AND ASSIGN THEM TO THE JOINT FORCE MATRIX, P, BY
C     EQS. 2.29, 2.34, 2.37, AND MINC.
C
C     STATEMENT FUNCTION
      FG(C1I,C2I,FLX,FLY) = C1I*FLX + C2I*FLY
C
      J = MINC(1,I)
      K = MINC(2,I)
C
      P(1,J) = P(1,J) + FG(C1(I),-C2(I),F(1,I),F(2,I))
      P(2,J) = P(2,J) + FG(C2(I), C1(I),F(1,I),F(2,I))
      P(3,J) = P(3,J) + F(3,I)
C
      P(1,K) = P(1,K) + FG(C1(I),-C2(I),F(4,I),F(5,I))
      P(2,K) = P(2,K) + FG(C2(I), C1(I),F(4,I),F(5,I))
      P(3,K) = P(3,K) + F(6,I)
      RETURN
      END
C*******************************************************************
C*                        ELEMFH                                   *
C*******************************************************************
      SUBROUTINE ELEMFH(F,Q,AREA,ZI,EMOD,GM,ELENG,C1,C2,MCODE,I)
      DIMENSION F(6,*),Q(*),AREA(*),ZI(*),EMOD(*),GM(*),ELENG(*),
     S          C1(*),C2(*),D(6),MCODE(6,*)
C
C     COMPUTE THE LOCAL FORCES IN ELEMENT I, F(6,I) : DETERMINE THE
C     GLOBAL ELEMENT DISPLACEMENTS, D(6), FROM THE JOINT DISPLACEMENT
C     VECTOR, Q, VIA MCODE ; COMPUTE THE LOCAL FORCES AT THE A-END
C     OF THE ELEMENT BY EQS. 2.34, 2.36 & 3.59 ; USE EQUILIBRIUM TO
C     COMPUTE THE LOCAL FORCES AT THE B-END OF THE ELEMENT AND
C     EQ. 3.95 TO COMPUTE THE ACTUAL ELEMENT FORCES.
C
      DO 10 L = 1,6
        K = MCODE(L,I)
        IF(K.EQ.0)THEN
            D(L) = 0.0
        ELSE
        WRITE (6,*) 'SUB ELEMFH, K  =', K
            D(L) = Q(K)
        ENDIF
   10 CONTINUE
C
      D1 = C1(I)*D(1) + C2(I)*D(2)
      D2 = -C2(I)*D(1) + C1(I)*D(2)
      D3 = D(3)
      D4 = C1(I)*D(4) + C2(I)*D(5)
```

```
      D5 = -C2(I)*D(4) + C1(I)*D(5)
      D6 = D(6)
C
C
      F3 = GM(I)*(D3-D6)
C
C      ACTUAL ELEMENT-END FORCES
C
      F(1,I) = F(1,I)
      F(2,I) = F(2,I)
      F(3,I) = F(3,I) + F3
      F(4,I) = F(4,I)
      F(5,I) = F(5,I)
      F(6,I) = F(6,I)-F3
C
      RETURN
      END
C*******************************************************************
C*                          JOINTH                                  *
C*******************************************************************
      SUBROUTINE JOINTH(F,P,C1,C2,MINC,I)
      DIMENSION F(6,*),P(3,*),C1(*),C2(*),MINC(2,*)
C
C      TRANSFORM THE LOCAL FORCES OF ELEMENT I, F(6,I), TO GLOBAL
C      FORCES AND ASSIGN THEM TO THE JOINT FORCE MATRIX, P, BY
C      EQS. 2.29, 2.34, 2.37, AND MINC.
C
      J = MINC(1,I)
      K = MINC(2,I)
C
      P(1,J) = P(1,J)
      P(2,J) = P(2,J)
      P(3,J) = P(3,J) + F(3,I)
      P(1,K) = P(1,K)
      P(2,K) = P(2,K)
      P(3,K) = P(3,K) + F(6,I)
      RETURN
      END
C*******************************************************************
C*                          OUTPUT                                  *
C*******************************************************************
      SUBROUTINE OUTPUT(F,P,Q,JCODE,NE,NJ)
      DIMENSION F(6,*),P(3,*),Q(*),JCODE(3,*)
C
      WRITE(6,10)'JOINT DISPLACEMENT AND FORCES'
 10   FORMAT(///T10,A)
      WRITE(6,20)'JOINT','DIRECTION','DISPLACEMENT','FORCE'
 20   FORMAT(//T10,A,T18,A,T30,A,T47,A)
      DO 60 J = 1,NJ
        DO 50 L = 1,3
            K = JCODE(L,J)
            IF (K .EQ. 0) THEN
                DISP = 0
            ELSE
                DISP = Q(K)
            END IF
            IF(L.EQ.1)THEN
                WRITE(6,30)J,L,DISP,P(L,J)
 30              FORMAT(/T10,I3,T18,I3,T30,E10.4,T47,E10.4)
            ELSE
                WRITE(6,40)L,DISP,P(L,J)
 40              FORMAT(/T18,I3,T30,E10.4,T47,E10.4)
            ENDIF
 50     CONTINUE
 60   CONTINUE
      WRITE(6,70)'LOCAL ELEMENT FORCES'
 70   FORMAT(///T10,A)
      DO 120 I = 1,NE
        WRITE(6,80)'ELEMENT NO.',I
 80     FORMAT(//T10,A,T18,I3)
        WRITE(6,90)'END - A','END - B'
 90     FORMAT(/T10,A,T20,A)
```

```
      DO 110 J = 1,3
          WRITE(6,100)F(J,I),F(J + 3,I)
100          FORMAT(T10,F10.2,T20,F10.2)
110     CONTINUE
120  CONTINUE
     RETURN
     END
```

Vita

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