APPLICATION OF CONTROL CHARTS TO SMALL LOT ACCEPTANCE

by

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PREFACE

The writer has been engaged in the design and development of explosive power sources for Naval aircraft systems while employed by the U. S. Naval Weapons Laboratory, Dahlgren, Virginia, since 1958. Increased applications of these power sources to aircraft systems compounded with more stringent performance requirements has emphasized quality control. Although the reader may get the impression that the thesis is primarily concerned with a specific case study, it must be reminded that these explosive power sources, referred to as power cartridges, have wide application in aircraft, ships, missiles, weapons, space programs, and undersea exploration programs by the Navy, Air Force, Army, and National Space Agency. Hence, the approaches discussed herein can be applied to many cases where small, infrequent production lots of power cartridges and destructive testing presents a challenging application for statistical quality control techniques. Further, the sampling scheme discussed has general application in analyzing a supplier's process capability as well as lot quality for any product.

The thesis begins with a discussion of the problems involved in the development and production of power cartridges for Naval aircraft systems. An illustrative example points out the need for SQC analyses in the production of power cartridges where performance limits crowd dual specification limits. A production acceptance sampling scheme is proposed which will evaluate the process capability as well as the lot quality.

Such a scheme provides a basis for partial lot acceptance and aids in the determination of causes for rejected lots. The scheme, though somewhat novel, is not any technological breakthrough; but rather a logical reasoning approach, utilizing basic statistical techniques, applied to a real world problem. In applying the scheme, it is assumed that temperature is the only operational environment that produces a significant effect on cartridge performance and that there is a definite temperature-performance relationship which may or may not be adequately defined. The advantages of defining this relationship by a predictive mathematical model is discussed with the use of an illustrative example.

Company names are not included because of the difficulties in obtaining clearance.

The writer is much indebted to the U. S. Naval Weapons Laboratory for allowing the writer to conduct a study concerning the reliability of power cartridges for aircraft systems. All of the material contained in this thesis came from a part of this study.

Very special thanks are due to Mrs. James Brown and Miss Virginia Cummings who assisted the writer in the study. Mrs. Brown, a mathematician and computer programmer, debugged the writer's computer program and was responsible for the incorporation of a plotting subroutine into the program. She also performed the calculations of Appendix VI and VII. Miss Cummings, a mathematician, was responsible for coordinating the cartridge firing tests of Chapter 2. In addition, she helped in numerous calculations, particularly control chart calculations to verify computer outputs. Miss Cummings is the colleague referred to in Chapter 5.

Thanks are also in order for Miss Diane Newton and Miss Barbara Stevens for their capable typing of the manuscript. Their patience and understanding were appreciated.

TABLE OF CONTENTS

TABLE OF CONTENTS (Continued)

TABLE OF CONTENTS (Continued)

FIGURES

TABLES

CHAPTER 1

THE PROBLEM

1.1 POWER CARTRIDGES

There are numerous mechanical functions requiring large energy inputs of short duration that must be performed in aircraft systems only once in a mission profile. Cartridges are energy sources which can be ideally applied to perform these functions. These power packages transform the energy of burning propellant into useful work. Typical applications of cartridges in aircraft are: personnel escape systems, bomb ejector systems, engine igniter systems, fire extinguisher systems, refueling systems, hoist systems, and emergency flotation systems. A typical Naval aircraft utilizes approximately 30 cartridges in these various systems. Cartridges can supply energies ranging from 60 ft-lbs to 70,000 ft-lbs and are very responsive (one to five milliseconds) in supplying this power. Their small size to energy, weight to energy, and cost to energy ratios make them preferable over other power sources. They can be adapted to electrical or mechanical systems and are quite versatile in the functions they can perform.

The basic cartridge components - an initiator, booster charge, and main propellant charge - are hermetically sealed in a metal case. Inert metal parts such as sleeves, cups, retainers, closure discs, etc. are used in the assembly of the basic components. Cartridges may be fired mechanically or electrically in which case the initiator will be a percussion primer or an ignition element respectively. An ignition element consists of an electric bridgewire buttered with sensitive explosive and surrounded by a booster charge. Upon the application of electrical current, the bridgewire heats up rapidly and ignites the buttered explosive which in turn ignites the booster charge. Normally, an ignition element suffices as both the initiator and booster charge in a cartridge. The booster charge ignites the main propellant charge which ruptures the cartridge case and releases hot propellant gases. These gases produce pressure to act on pistons which do work. Cartridges may be required to function:

1. instantly (within five milliseconds). These cartridges utilize fast burning propellant of fine granulation.
2. instantly (within five milliseconds) after a time delay (0.40 to 6.0 seconds). These cartridges incorporate a chemical composition between the initiator and main charge which burns at a known uniform rate.
3. over a period of seconds. These cartridges utilize a solid propellant grain whose burning rate can be controlled.

1.2 QUALITY CHARACTERISTICS

Cartridges are classified as impulse or delay. Output is the main quality characteristic of impulse cartridges. Both delay time and output are the main quality characteristics of delay cartridges. Output is measured in such parameters as pressure, impulse (area under pressure-time curve), ft-lbs, and velocity. Both characteristics normally have dual specification limits. Time delays are required to allow proper sequencing of events in a system. If an event occurs too soon or too late, the system will fail. The need for a minimum output specification limit is quite obvious. The maximum output specification limit is needed to prevent damage to the system. This limit is sometimes excluded when the strength safety factor of the system is relatively high.

1.3 INHERENT PROBLEMS

The major problem area in the lot acceptance of cartridges is, unfortunately, an inherent one. It can be simply stated as insuring a high degree of reliability in the production of small, infrequent lots where the critical quality characteristics can only be inspected by destructive testing. This problem is magnified by a recent case. A production lot of 167 cartridges was manufactured at an approximate cost of $135 per cartridge. A high degree of reliability is required for this cartridge because a malfunction could result in the loss of a helicopter and its crew members. According to Table I of reference (3), to be 95 percent confident that 99 percent of the cartridges would function properly, a minimum sample size of over 117 would be required for destructive testing. Such a sample size is unrealistic and prohibitive.

Lot sizes normally vary between 300 and 20,000. Unfortunately, the cartridges requiring the highest reliability are associated with the smaller lots (300 to 1300). Lack of standardization and relatively short service life of cartridges are responsible for small and infrequent procurement. For instance, one particular cartridge, with a three-year service life, may be designed for only one particular aircraft. The fact that only 200 of this type aircraft may be operational requires the procurement of only 200 cartridges (plus test samples) every three years. Small and infrequent lots are not lucrative enough for big businesses. Small companies, without automation or quality control staffs, are normally the recipients of government contracts for the manufacture of cartridges. The Government's policy of competitive bidding in contract administration encourages multi-source procurement which compounds the problem. Another problem deals with the product itself. Explosives by their very nature are somewhat unpredictable as to their consistency of performance. The fact that destructive testing is involved and the lack of sophisticated instrumentation restricts the study of the explosive phenomena.

1.4 PROCESS CONTROL IN DEVELOPMENT

It cannot be sufficiently emphasized that reliability must be built into an item. It is impossible to test reliability into an end item. A production lot acceptance test serves to estimate the reliability that an item actually has. If the test indicates that the reliability is too low, one must back up to find the problem. The problem could be related to the design of the item or the process which produced it.

In the design and development of power cartridges, the process is overlooked. Current policies and procedures do not focus attention on the process. One notable example of this is the fact that the developer, in many cases, is not the manufacturer of production lots. The majority of the development engineers do not have production experience and therefore, concentrate their attention only on design. Consequently, the developer neglects the importance of the process and the manufacturer, who is not familiar with the design or application of the cartridge, is ignorant of the effects the process may have on performance. Destructive testing adds to the problem because it dictates the use of minimum prototypes in development, thereby restricting possible process data collection.

The fate of the cartridge design depends on the process used in the manufacture of development prototypes. Excessive care in the manufacture of prototypes by skilled modelmakers may result in the acceptance of a poor production design. Conversely, lack of quality control by the model-makers could result in the rejection of a good design. The former is perhaps more frequently the case.

Since most cartridges are low production items, the type of process is relatively unimportant as long as it is thoroughly documented in development so that performance reproducibility in production can be obtained. A recent case points this out. The drawing of a metal part used in a percussion detonator did not specify the process to be used in its manufacture. It specified only the physical dimensions and the material. In development prototypes, the part was stamped out and the tests of the detonator proved to be satisfactory. A contract was let for the production of 500 detonators. The contractor machined the part which met the requirements of the drawing. However, the shearing characteristics of the machined part were significantly different from the stamped part to give unsatisfactory detonator performance. Since the contractor had fulfilled the requirements of the contract drawing, the Navy had no alternative but to pay for the bad lot.

If the cartridge is to be a high production item, then the type of process becomes rather important from a cost point of view. Value engineering should be emphasized early in development. High production techniques should be thoroughly evaluated in development since design performance can be greatly influenced by the manner of manufacture. Development costs will be high because of tooling cost. However, process problems could be resolved by the designer and manufacturer prior to the release of the cartridge to service and the tooling would be available for immediate production. However, this optimum approach will meet resistance at the higher echelons. Unfortunately the Navy budget for cartridges is divided between two groups, R&D and procurement, which are diametrically opposed in policy. The R&D group does not want to bear the cost of production tooling, a procurement function. On the other hand, the procurement group does not want to release procurement funds until the design is released and fully documented for competitive bidding. Even if the R&D group did finance the tooling cost, under the competitive policy of the procurement group, the manufacturer who helped the designer may not be awarded the subsequent contract for production, thus production problems must be solved again. The Navy is, however, putting new emphasis in the developer-first producer policy as permitted by the Armed Services

Procurement Regulation. Captain Howard, Deputy Chief of Naval Material (Procurement), addressing the Tenth Annual Seapower Symposium recently in Washington, D. C., indicated that the Navy considers quality control to be one of its most critical concerns today. He said, while convinced of the importance of competition, the Navy also recognizes that "in some of our programs, we can do ourselves a disservice if we go into competition prematurely."

The previous discussion has emphasized the importance of studying the process during the development of cartridges. There are five distinct phases from the paper design to the production of cartridges. They are:

1. Preliminary development (experimental models)
2. Advanced development (prototypes)
3. Design qualification (advanced prototypes)
4. Preproduction
5. Production

The developer is responsible for the first three phases, the manufacturer for the last two. In the first phase, the designer experiments with models to determine whether his paper design is feasible. Limited tests are conducted and appropriate design changes are made. Ten to 50 models may be fabricated. If the design appears feasible, the designer proceeds to the next phase. In the advanced development phase, environmental effects and component interactions are studied in prototype tests. Fifty to 100 prototypes may be fabricated. The writer contends that the designer should consider the process effects in this phase, for instance, the effect of machining a component as opposed to stamping, so that the methods to be used in the manufacture of the end item can be established. Burr (2) indicates control charts may be particularly helpful in the exploratory phases of research where it is not yet known which factors are most influential on the variable under study.

The qualification phase is the final development phase. Tests are conducted in accordance with MIL-D-21625D. It is during this phase that the reliability of the design under operational conditions is estimated. If the test results are satisfactory, the design is approved and final drawings and specifications are prepared. Approximately 200 advanced prototypes are tested. A typical request for the manufacture of these prototypes contains the statement, "Manufacturing methods used in production should be as close to those anticipated for procurement on a production basis as time limitation and quantities required justify."

Yet, the process used is not studied, and consequently, is not adequately documented. Normally the manufacture of these prototypes is done by another government activity or a contractor without the presence of the design engineer. Because of insufficient information feedback from the manufacture, the designer may be unaware of serious production problems.

It is imperative that the design engineer monitor the manufacture of these prototypes. The use of statistical control charts during this phase could be extremely helpful in analyzing the process capability.

The remaining two phases, preproduction and production, will be discussed in the next section.

1.5 PROCESS CONTROL IN PRODUCTION

Once the final cartridge specifications and drawings have been released to the procurement group, the developer relinquishes process control responsibility to the manufacturer. Specifications require the submission of a preproduction lot (less than 100 cartridges) by the contractor prior to entering production to ascertain that he has the technical capability to produce acceptable items. Process studies would appear to be essential in this phase; yet to this writer's knowledge, it is not done. The use of control charts would aid in analyzing possible production problems so that procedures could be formulated for the subsequent production run. In the production run, control charts could be used to maintain process control. There are several justifiable reasons why the contractor does not utilize statistical methods in controlling the process:

1. they are not required to by contract, nor would it be feasible to do so,
2. being generally small companies, they lack the quality control staffing,
3. they lack the test equipment and instrumentation,
4. and they are not familiar enough with design to know what process effects will affect performance.

Hence, the contractor produces the best he knows how, hoping the lot quality will be accepted.

Production lot acceptance sampling is in accordance with MIL-STD-105D or MIL-STD-414. Using these acceptance procedures alone, the cause for rejection, whether it be in the design or process, cannot be ascertained. A process study is presented in the next chapter to illustrate how control charts could be used to analyze a cartridge production process.

CHAPTER 2

A PROCESS STUDY

2.1 INTRODUCTION

Since 1959, 24 production lots of the MARK 5 delay cartridge have been manufactured by four sources of supply. Thirteen of these lots failed to meet the specification performance requirements. Lot size varied from 600 to 5,000. The high rejection rate could be caused by the lack of quality control in production, noncompatability of the process capability with specification limits, or design deficiencies. If statistical quality control had been utilized, perhaps the cause for rejection could have been ascertained.

Permission was given by the U. S. Naval Weapons Laboratory, Dahlgren, Virginia, and the ABC Company for the writer to conduct a process capability study of the manufacture of 3,000 MARK 5 type delay cartridges. This study was conducted independently of lot acceptance tests.

2.2 CARTRIDGE DESCRIPTION

The MARK 5 delay cartridge is used to open a parachute automatically two seconds after the pilot or crew member has been ejected from an aircraft in an emergency escape. The cartridge employs a standard .38 caliber pistol cartridge case, with the open end crimped over a closure cup (Figure 2-1). The cartridge is center-fired with an M42 percussion primer. In the assembly a delay insert is placed in the cartridge body. The order of assembly of the delay insert is: porous disc (to diffuse primer blast), ignition mix (to ignite the delay column),

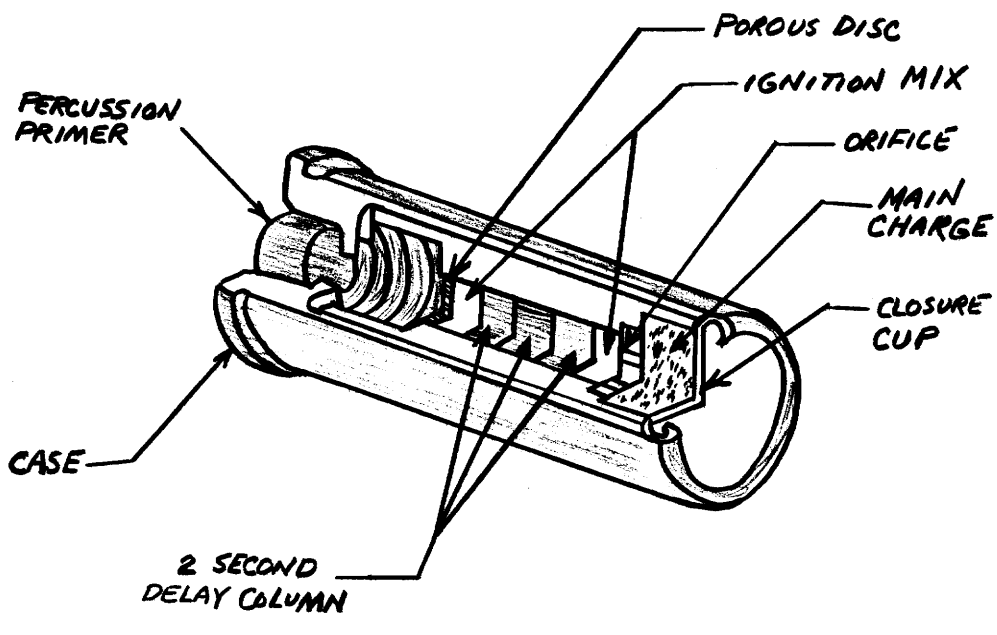


FIGURE 2-1. MARK 5 DELAY CARTRIDGE

delay column (a chemical composition which burns at a known uniform rate), ignition mix (to ignite main charge), orifice, and main charge. A closure cup is crimped over the delay insert. The cartridge is hermetically sealed. Length of the cartridge is approximately 1.1 inches.

The cartridge is inserted in the breech of the parachute opener attached to the parachute pack. As the seat is ejected from the cockpit, a lanyard is pulled tight, and withdraws a sear pin from the parachute release mechanism. This action releases a firing pin which strikes and ignites the cartridge. After a two-second delay, the parachute is opened automatically.

Two critical performance quality characteristics are involved - delay time and energy output. The ejection sequence requires that the parachute be opened during the time span of 1.62 to 2.53 seconds after initial ejection, otherwise the pilot or crew member will be fatally or critically injured during low altitude ejections. The cartridge must also provide a minimum of 48 ft-lbs of energy to open the parachute. There is no upper limit of energy output since it is physically impossible for the cartridge to contain the minimum amount of propellant that would damage the system. The cartridge must function within the above limits over the temperature range of -65°F to +160°F.

2.3 OBJECTIVES

The objectives of the process capability study were:

1. to determine if the process was in statistical control,
2. to determine probable causes if there was a lack of statistical control,

3. to determine if the estimated process quality could meet specification limits,
4. to determine if the effect of test temperature on cartridge performance is significant,
5. and to compare the results of the process analysis with the results of the independently conducted lot acceptance tests with regard to lot quality.

2.4 THE MANUFACTURER

ABC is a small company employing approximately 100 employees. The company specializes in the development and manufacture of explosive items for the petroleum industry. It has developed and manufactured several missile destruct systems for various government agencies. Prior to the manufacture of this lot, ABC had manufactured nine lots (beginning in June 1964) of the MARK 5 and MARK 6 delay cartridges. All lots failed to meet specifications but were passed on waivers. The company project engineer responsible for the manufacture of these cartridges had been formerly associated with another manufacturer of the MARK 5 delay cartridge. The company had a Quality Assurance Department whose main function was inspection. No formal quality control programs were administered. Most of the employees of the company were technicians and production workers. The professional staff was capable but meager. Many of the key personnel were ex-military with ordnance experience and had experience with other companies in the explosive field.

2.5 THE PROCESS

The writer visited ABC during the period of 11 - 13 July 1966 to observe the manufacturing process and to outline procedures to be used in the process study. The process was essentially an assembly operation with the components being procured from outside sources. The components were 100 percent inspected by ABC prior to assembly. The assembly line consisted of five women. The assembly operation was essentially a hand operation. The only equipment utilized were presses and weighing scales. The assembly process was broken down into a written step-by-step detailed procedure, including inspection. Critical dimensions and weighings were checked by two persons. The daily production rate was approximately 200 cartridges. Although no formal statistical quality control procedures were administered, the project engineer did take samples from the line sporadically and tested them for delay times at -65°F and 160°F. Data on these tests were not made available.

2.6 THE PROCEDURE

All cartridges manufactured were serially numbered by order of production. This requirement had not been specified in the past. Two subgroups of five samples were taken daily, one in the morning (approximately 10:00 a.m.) and one in the afternoon (approximately 2:00 p.m.). The decision to take two subgroups per day was based on the anticipated daily production rate of 200 and the minimum number of subgroups recommended by Grant (5) for the establishment of trial control limits. The subgroup size of five was originally selected on the basis that five seems to be the most commonly used size in industry and in this particular case, the cost of testing for this particular process study was not a significant factor. Subgroups were selected in a manner to give the maximum chance for the quality characteristic measurements in each subgroup to be alike and the maximum chance for the subgroups to differ one from the other; i.e., samples in each subgroup were selected consecutively from production and morning to afternoon, day to day variations were allowed from subgroup to subgroup. Originally it was planned to test all the samples at 70°F. However, it was later decided that more information could be obtained relative to specification limits if tests were conducted at the specification temperature extremes of -65°F and 160°F since it was known from past history that temperature will affect performance. To eliminate inconvenience and to minimize further sampling cost, the already selected subgroups of five were divided into subgroups of three for -65°F tests and subgroups of two for the 160°F tests. Selection of the larger sample size for the -65°F tests was made on the basis of past history indicating that -65°F temperature produced more erratic effects on performance than 160°F temperature. Although the originally planned subgroup size of five would have been better on statistical grounds, subgroup sizes of two or three are perhaps more practical in the manufacture of explosive items due to the cost of destructive testing.

Because of the limitation of time and money and due to the lack of test equipment by the contractor, sample testing was conducted at the

U. S. Naval Weapons Laboratory after the entire lot had been manufactured, thereby eliminating process control during production.

2.7 COMPUTER PROGRAM

A program for an IBM 7030 digital computer was developed for control charting. Such a program will be invaluable should the proposed sampling scheme of Chapter 3 be accepted. Appendix I contains a flow chart of the program. Appendix II contains a card listing of the program. Comment cards are included in the listing as well as a variables glossary, Appendix III, to facilitate program interpretation. Test data is tabularized in Appendix IV. Figures 1 through 8 of Appendix V are $\overline{X}$ and R charts for delay times and output for -65°F tests and 160°F tests. The charts are facsimiles of the outputs from the computer plotter.

2.8 CONTROL CHART ANALYSIS

2.8.1 Temperature Effect

Before going into the detailed analysis of the control charts, some comments in regard to test temperature are in order. Before conducting the tests, it was expected that test temperature would have a significant effect on the means of the quality characteristics. The burning rate of the delay composition and energy output increases as temperature increases. Some effect, though not as great, was expected in the ranges of the quality characteristics. Generally, low temperatures give more erratic performance. Since charts were maintained for both temperatures individually, these known effects would be self compensating in regard to control limits. However, if an excessive number of points fell out of control for one test temperature when compared to the other temperature, then one could suspect temperature as an assignable cause of variation in addition to the known effects.

2.8.2 Subgroup Size

In comparing control charts of the quality characteristics by test temperature the reader must take into account the difference in subgroup size which undoubtedly produced some error.

2.8.3 Points Out Of Control

Six of the eight control charts had points falling out of control. A summary of points falling out of control, by charts and subgroups (SG), is given in Table 2-1.

TABLE 2-1

POINTS FALLING OUT OF CONTROL

Test Temp.	Quality Characteristic			
	Delay Time		Output	
	$\bar{X}$	R	$\bar{X}$	R
-65°F	2 (SG #8, 32)	1 (SG #8)	1 (SG #10)	1 (SG #27)
160°F	0	1 (GS #8)	0	1 (SG #14)

The fact that the range charts for both quality characteristics in -65°F and 160°F tests had points falling outside the control limits indicates the process variability is out of statistical control. Further, since one point fell out of control for both quality characteristics for both temperatures, one must exclude temperature as an assignable cause of variation. The fact that the means were in control for both quality charactersitics in the 160°F tests, but out of control in the -65°F tests might lead one to conclude that temperature might be an assignable cause of variability of the process mean. However, the fact that six of the seven points falling out of control were associated with subgroups taken in the afternoon (three points alone were associated with Subgroup #8) may be more significant. The effect of morning versus afternoon production will be investigated later.

2.8.4 $\overline{X}$ Charts

In observing the $\overline{X}$ chart for delay times in -65°F tests (Figure 1, Appendix V), a gradual shift in the mean during the entire production can be noted. This shift is not as predominate in the 160°F tests (Figure 5, Appendix V). However, if Subgroups #1 through #5 and #15 through #18 were neglected in the latter chart, a shift might be detected. Also in regard to the latter chart, a short run of six points (beginning with Subgroup #12) can be noted. In observing the $\overline{X}$ chart for output in the -65°F tests (Figure 3, Appendix V), a shift in the mean (with a short run of five points beginning with Subgroup #7, and a long run of 10 out of 11 points beginning with Subgroup #22) can be noted. Though not as pronounced and without runs, this shift can also be noted in the 160°F test (Figure 7, Appendix V).

2.8.5 R Charts

The difference in subgroup size for the temperature tests must be considered a significant factor in comparing the variability of the quality characteristics by temperature. However, the fact that the majority of the peaks on the R chart for delay time in -65°F tests (Figure 2, Appendix V), but not for the 160°F tests (Figure 6, Appendix V), were associated with afternoon subgroups now compounds the morning versus afternoon effect with a possible temperature effect. This will be discussed next.

2.9 MORNING VERSUS AFTERNOON PRODUCTION

To test the hypothesis that variances of the morning and afternoon production for the quality characteristics at each test temperature were equal, Bartlett's tests of (8) were conducted on the control chart data. It was assumed that the samples were taken at random from normal populations. Results of these statistical tests (Appendix VI) at .05 level of significance are tabulated in Table 2-2.

TABLE 2-2

H_0: VARIANCES OF MORNING AND AFTERNOON PRODUCTION ARE EQUAL

Test Temp.	Delay	Output
-65°F	Reject H_0	Cannot Reject H_0
160°F	Cannot Reject H_0	Cannot Reject H_0

Further, to test the hypothesis that the means of the morning and afternoon production for the quality characteristics at each test temperature, analysis of variance tests of (8) were conducted on the same data. Results of these statistical tests (Appendix VII) at .05 level of significance are tabulated in Table 2-3.

TABLE 2-3

H_0: MEANS OF MORNING AND AFTERNOON PRODUCTION ARE EQUAL

Test Temp.	Delay	Output
-65°F	No test, σ^2 not equal	Cannot Reject H_0
160°F	Cannot Reject H_0	Cannot Reject H_0

The results of both of these statistical tests and the analysis of the control charts indicates to the writer that there was a problem in afternoon production in regard to delay time variability which was sensitive only to the -65°F tests. The lack of humidity control during production was concieved by the writer as one possible explanation. The relative humidity is normally higher in the afternoon than in the morning. In addition, by afternoon the delay composition, laid out for the day's production, would have more "soaking" time. If moisture was absorbed by the afternoon production, it is conjectured that the moisture could have been solidified in the delay composition during the -65°F tests while vaporized in the +160°F tests, thereby producing a significant effect in one case and not in the other.

2.10 PROCESS IN CONTROL

The data from the subgroups which had points on the control charts out of statistical control were discarded and new control limits were calculated to bring the process under control (see Appendix VIII). This procedure is recommended by (5) to estimate the process capability. The estimated process capability was needed for comparison with specification limits and the results of the lot acceptance tests. It must be emphasized here that the validity of the test procedures used in these SQC tests and lot acceptance tests in the past is questionable. However, until this problem is thoroughly investigated, it is assumed that the test data is valid and can be used to estimate the process capability. The fact that temperature has an effect on both quality characteristics, and that specification limits must be met over a temperature range, required analyzing the process capability for the quality characteristic at each temperature extreme. Process natural tolerance limits (NTL) for both quality characteristics for each temperature are calculated in Appendix VIII.

Figure 2-2 graphically depicts the process capability for this lot in relation to the specification limits. The range of the process natural tolerance limits for both quality characteristics over the temperature range of -65°F to 160°F can be calculated as follows:

DELAY TIME IN SECONDS

Range = upper NTL for -65°F - lower NTL for 160°F

= 2.575 - 1.776

= .799

OUTPUT IN FT-LBS

Range = upper NTL for 160°F - lower NTL for -65°F

= 98.88 - 55.00

= 43.88

The process range of 0.799 seconds for delay time over the temperature range is compatible with the specification range of 0.910 seconds. However, the process is not centered with regard to the specification limits. The lot will be 0.69 percent defective in regard to the upper specification limit at -65°F and less than .017 percent defective in regard to the lower specification limit at 160°F. Since output has only a single specification limit, the process range is unimportant. The process is capable of meeting the minimum output specification over the temperature range.

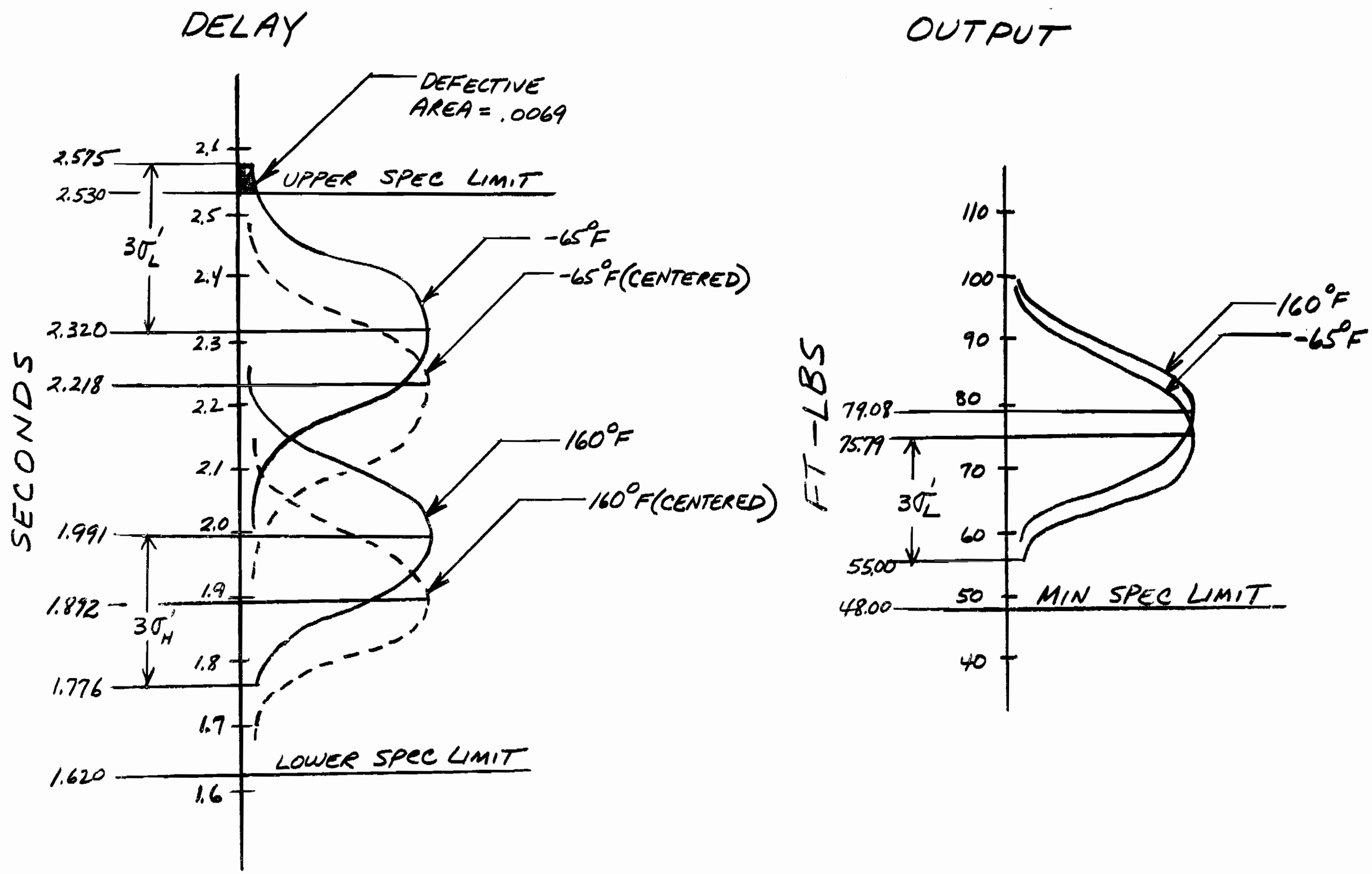


FIGURE 3-2. ABC COMPANY PROCESS CAPABILITY FOR MANUFACTURING MARK 5 DELAY CARTRIDGES

2.11 CENTERING THE PROCESS

If the ABC Company, neglecting the possible morning versus afternoon effect, could maintain statistical control for their process, the process means should be carefully centered with respect to the delay time specification limits. Recommended process means are calculated.

Let: σ'_L = estimated standard deviation of low temperature tests (-65°F)

σ'_H = estimated standard deviation of high temperature tests (160°F)

$\bar{X}_L = \bar{\bar{X}}$ of low temperature tests

$\bar{X}_H = \bar{\bar{X}}$ of high temperature tests

Assume: Definite temperature-performance relationship so that $\bar{X}_L - \bar{X}_H$ can be considered constant.

Then:

$$\text{MAX } \bar{X}_L = USL - 3\sigma'_L = 2.530-3\ (.0851) = 2.275$$

$$\text{MIN } \bar{X}_L = LSL + 3\sigma'_H + (\bar{X}_L - \bar{X}_H) = 1.620 + 3\ (.0716) + (2.320 - 1.994) = 2.161$$

$$\text{MAX } \bar{X}_H = USL - 3\sigma'_L - (\bar{X}_L - \bar{X}_H) = 2.530-3\ (.0851) - (2.320 - 1.994) = 1.949$$

$$\text{MIN } \bar{X}_L = LSL + 3\sigma'_H = 1.620 + 3\ (.0716) = 1.835$$

$$\text{Recommended } \bar{X}_L = \frac{\text{MIN } \bar{X}_L + \text{MAX } \bar{X}_L}{2} = \frac{2.275 + 2.161}{2} = 2.218$$

$$\text{Recommended } \bar{X}_H = \frac{\text{MIN } \bar{X}_H + \text{MAX } \bar{X}_H}{2} = \frac{1.949 + 1.835}{2} = 1.892$$

Figure 2-2 graphically depicts the centered process.

2.12 COMPARISON WITH LOT ACCEPTANCE TESTS

The results of the control chart analysis were in favorable agreement with the lot acceptance tests. All of the cartridges tested in the lot acceptance tests met specification limits, although several cartridges produced delay times at -65°F just under the upper specification limit - which was predicted in the control chart analysis. The estimated lot means of the quality characteristics for each temperature were comparable. The estimated standard deviation for the lot acceptance tests were somewhat higher for the quality characteristics at each temperature. This is reasonable since shifts in the means for both quality characteristics at each temperature were noted in the control chart analysis. Such shifts would not be detected in the lot acceptance tests where the samples were randomly selected.

2.13 CONCLUSIONS

From the process study, the following conclusions are made:

1. The process is out of statistical control.
2. Morning versus afternoon production may be an assignable cause of process variation.
3. The estimated process capability is compatible with specification limits.
4. Test temperature, by itself, does not appear to be an assignable cause of process variation.
5. The estimated process capability did favorably agree with lot acceptance tests. It must also be pointed our here that the

psychological factor of knowing that the process was under study no doubt influenced the quality of the lot which was much better than the previous lots produced by ABC. Further investigations should be made to determine cuases of shifts and the morning versus afternoon production variation. Unfortunately, many months have passed since the production of this lot and such investigations would probably be fruitless. In the future, however, the implementation of process control in the production of cartridges would aid in the detection of causes of variation.

CHAPTER 3

LOT ACCEPTANCE AND CONTROL CHARTS

3.1 A LOGICAL STARTING POINT

The value of control charts in the development and production of power cartridges has been discussed in Chapter 1. The writer has hopefully presented the need for control charts. The next step is to implement the use of such a tool. The acceptance of the use of control charts cannot be gained readily on the basis of one case as presented in Chapter 2. Like any new policy, it must be given a trial run to prove its usefulness. Theoretically, one would think that control charts should be applied first in the developmental phase. There are several valid objections to this. They are:

1. Presently, design engineers are somewhat inexperienced in production techniques.
2. The limited manufacture of prototypes, as dictated by current policies, does not allow for a thorough process study.
3. The process used in the manufacture of prototypes may not be indicative of the production process.
4. The destructive tests for control charting would be in addition to required environmental tests and therefore would appreciably increase developmental costs.

These objections could be overruled if we applied control charts first to production lots by:

1. having the design engineer monitor the contractor's production to study actual production techniques being used. This will give the engineer valuable experience in production techniques as well as aiding him in the control chart analysis.
2. combining control chart tests with production lot acceptance tests to eliminate increased cost.

Another important advantage may be derived by applying control charts first to production. Many cartridges have already been developed and are in production. Production problems seem to occur rather frequently. By using control charts as a trouble shooting mechanism, perhaps the sources of the problems can be found. No doubt, the information gained will be extremely useful in implementing process studies in the development phase.

Having justified production as a starting point, control charting must be introduced in such a manner that it will be readily accepted on a trial basis. This essentially depends on whether or not control charting can be combined with production lot acceptance testing. By doing so, the application of control charts can be evaluated at no added cost.

3.2 A REQUIREMENT

Sampling procedures for production lot acceptance of cartridges must be in accordance with MIL-STD-105D, Sampling Procedures and Tables for Inspection by Attributes, or MIL-STD-414, Sampling Procedures and Tables for Inspection by Variables for Percent Defective, according to current policy.

3.3 A DISADVANTAGE OF CONTROL CHARTS IN LOT ACCEPTANCE

Overlooking the previously stated requirement, there is another reason why control charts could not be the sole basis for production lot acceptance. Although not done in the case presented in the previous chapter; random sampling, a requirement for acceptance sampling, could be achieved in the selection of rational subgroups for control charting according to reference (5). However, if the process were deemed out of statistical control, there would be no basis for determining the lot quality. The lot quality may be acceptable even though the process may be out of statistical control.

3.4 AN ACCEPTANCE TEAM

If a sampling scheme could satisfy the requirements of MIL-STD-105D or MIL-STD-414 and control charting methods, it would have the following advantages:

1. would not disrupt the present acceptance procedures to any degree,
2. valuable information could be obtained about the process in addition to just the function of accepting or rejecting lots,
3. two functions, process study and lot acceptance, would be performed for the price of one,
4. and would permit acceptance of part of a lot,

With regard to production acceptance, the latter advantage has particular merit. Presently, no record of cartridge production is made; i.e., cartridges are not serialized or identified by order or date of production. The lot is assumed homogeneous and if an unacceptable number of defective cartridges is found in the acceptance sample, the lot is rejected. However, homogeneity may not be the case. A sudden shift in the mean may have occurred during the latter part of production and the majority of the defective samples may have come from this phase of production. Control charts would have detected this shift and if the defective samples did come from this phase of production; then one could justify accepting those cartridges produced up to the shift in the mean and rejecting those after the shift providing that the cartridges could be properly identified. Should this justification be questioned, another sample could be taken. Both the Navy and the contractor would benefit by partial lot acceptance.

3.5 RANDOM SAMPLE?

Though a fine technicality, the question of randomness of the samples is the only factor which can be argued in the proposed sampling scheme depending upon one's interpretation of MIL-STD-105D and MIL-STD-414. To be truly random, for a total sample size of n, each cartridge in the lot has n chances of being selected. In order to incorporate control charting into the sampling procedure, however, if we divide the total sample size n into k subgroups of size m where m samples are randomly selected over k periods of time; then each cartridge in the lot has only m chances of being selected. There are pros and cons for both cases. For example, one could argue that during one period of production, a number of defective cartridges could be produced and in the latter case, each defective cartridge would only have m chances of being selected. Conversely, one could argue that all the cartridges produced during one period of time could be defective and in the truly random case, there is a chance that no sample would be drawn from this period. In terms of probability, however, neither argument is valid. The fact that paragraph 7.3 of MIL-STD-105D implies the acceptance of periodic sampling, as would be the case in control chart sampling, allows the writer to assume that his proposed sampling scheme will meet the requirements of MIL-STD-105D or MIL-STD-414 with regard to randomness. Further, each cartridge has the chance to be selected without regard to its quality.

3.6 SAMPLE SIZE

Sample size is the most important criterion in the use of the combination sampling scheme. In keeping with the objective of not increasing testing cost, the sample size dictated by the acceptance procedure presently used, which is dependent on the Military Standard specified, lot size, and AQL; must be compatible with the minimum sampling required for a valid control chart analysis. The minimum sample size required for a valid control chart analysis has been determined by the writer as 75. This is based on 25 subgroups, the minimum recommended by reference (5), and a subgroup size of three.

Subgroup size of three was chosen by the writer on the basis of minimizing the number of destructive tests, yet allowing an estimate of R in a subgroup when a measurement of one sample is lost due to instrumentation failure. The factors influencing the acceptance sample size are worthy of discussion since they have a significant bearing on the applicability of the proposed scheme.

3.6.1 AQL

Present policy establishes an AQL of .04 percent for life saving cartridges, i.e. cartridges used in aircraft personnel escape systems, and AQLs of .10 percent and .25 percent for other cartridges depending on the consequence of a malfunction. When MIL-STD-414 is used, Inspection Level IV, normal inspection is specified. However, some trade-off is given relative to the sample size when MIL-STD-105D is used. Inspection Level II, normal inspection, and AQLs of .10 percent and .25 percent are used for life saving and other cartridges respectively to give more practical sample sizes for small lots. One might conclude that these AQLs are rather stringent considering the cartridge application and the probabilities involved. This can be illustrated by a realistic example. Assume a production lot of 1,000 MARK 5 delay cartridges of Chapter 2 was .10 percent defective with respect to delay time. This would mean that there was only one defective cartridge in the entire lot. With aircraft being quite reliable, the probability of this defective cartridge being in an emergency situation at low altitude would be very low. However, other aspects must be considered. The OC curves of the sampling plans are based on continuous production with an average outgoing quality level. The manufacture of cartridges cannot be regarded as such. Hence, with a lot size of 1,000, an AQL of .10 percent, and normal inspection under MIL-STD-105D; there is a 50-50 chance that a lot can be accepted which is six times as defective as the AQL. Another important aspect is a psychological one. The fact that a defective cartridge may result in the loss of a pilot's life takes preference over any probability theory. The value of human life is impossible to equate to any probability theory. Although stringent, AQLs of .04 percent to .25 percent appear to be within the realm of cartridge production. Needless to say, these low AQLs help increase sample size and enhance the applicability of the proposed scheme.

3.6.2 Lot Size

As previously stated in Chapter 1, production lots of cartridges vary between 300 to 20,000 cartridges. It must be pointed out that the cartridge application has a direct relationship with lot size and AQL which enhances the compatibility of lot acceptance sampling and control chart sampling. Life saving cartridges, thanks to high reliability of aircraft, are of low usage; yet demand greater reliability. Hence, these cartridges are normally associated with the smaller lot sizes (300 to 1300) and the more stringent AQLs (.04 percent to .10 percent).

Other cartridges, such as bomb ejector cartridges, are of high usage and demand less reliability. These cartridges are associated with the higher AQL (.25 percent). This relationship of cartridge application with lot size and AQL maximizes sample size.

3.6.3 Military Standard

The most significant factor in regard to sample size is the military standard used. Compare the sample sizes of MIL-STD-105D and MIL-STD-414 in Tables 3-1 and 3-2 in accordance with present policies.

TABLE 3-1

SAMPLE SIZE FOR LIFE SAVING CARTRIDGES

Lot Size	Sample Size	
	MIL-STD-105D Inspection Level II Normal Inspection .10 Percent AQL	MIL-STD-414 Inspection Level IV Normal Inspection Variability Unknown Standard Deviation Method .04 Percent AQL
301- 500	125	25
501- 800	125	30
801-1200	125	35
1201-1300	125	35

TABLE 3-2

SAMPLE SIZE FOR HIGH USAGE CARTRIDGES

Lot Size	Sample Size	
	MIL-STD-105D Inspection Level II Normal Inspection .25 Percent AQL	MIL-STD-414 Inspection Level IV Normal Inspection Variability Unknown Standard Deviation Method .10 - .25 Percent AQL
1301- 3200	200	40
3201- 8000	200	50
8001-10000	200	75
10001-20000	315	75

From this comparison, it can be seen that sample sizes of MIL-STD-105D are compatible in all cases with the minimum control chart sampling while those of MIL-STD-414 are not with the exception of lots over 8,000. Presently, MIL-STD-105D is used somewhat more than MIL-STD-414. In section 3.7, the writer justifies the nonuse of MIL-STD-414 in production lot acceptance of cartridges.

3.7 MISUSE OF MIL-STD-414

It is generally agreed that for a given quality protection, smaller samples may be used with variables criteria than with attributes. There is a misconception that MIL-STD-414 should be specified for low usage cartridges on the basis that sample size is minimized by the use of variables and that MIL-STD-414 is the only generally accepted variables sampling procedure. MIL-STD-414 is based on the assumption that measurements are selected at random from a normal distribution. Many frequency distributions of a product's quality characteristics are roughly normal when the product comes from a single source and is produced within a short period of time, reference (5). However, with relatively tight specification limits, how valid is the assumption of roughly normal. If the acceptance plan is based on normality, and the assumption is false, then an incorrect level of protection will be indicated. Therefore, it is best to test for normality before selecting a sampling plan. Sampling plans for cartridges are selected during the final development phase. Present policies do not provide for sufficient data in development to conduct normality tests. Even if normality could be ascertained in development, with multi-source and infrequent procurement, normality cannot be guaranteed in production. One solution to this dilemma would be to specify normality tests for every production lot in the cartridge specification. If normality was indicated, MIL-STD-414 could be applied. If not, the acceptance sampling procedures of reference (6), using variables with non-normal distributions, could be applied.

The above arguments against the use of MIL-STD-414 are certainly valid; however, they are of secondary importance. The prime reason against the use of this document concerns itself with the product. Consider the cartridge as a system. Even though the end performance parameter of the system can be a variable, there are components within the system whose quality characteristic can only be observed as an attribute. For instance, consider the delay column and ignition mix of Figure 2-1. The assembly of these components is perhaps the most critical operation in manufacture of the MARK 5 delay cartridge. The function of the first ignition mix is to ignite the delay column and can only be observed as an attribute. The delay column has two functions; to provide a time delay and to ignite the second ignition mix. Here, the former function can be observed as a variable, the latter only as an attribute. In order to be assured of a given quality protection, the sample size should be based on attributes criteria.

3.8 VARIABLES DATA

Use of MIL-STD-105D has been justified. Normally fraction defective charts are applied to process studies where attributes are the criteria. However, with destructive testing, the cost would be prohibitive due to the large sample size required. Variables data can be used with MIL-STD-105D; in fact, it is a presently accepted practice in cartridge production lot acceptance. Using variables data, the quality protection is higher than the AQL specified because of the qualitative data. This somewhat justifies the previously mentioned policy of specifying higher AQLs with MIL-STD-105D. The cartridge is classified as simply a defective or nondefective according to whether or not the measured quality characteristic is within specification limits. With variables data, $\bar{X}$ and R control charts can be maintained.

3.9 TRADE OFF

Control charting could be readily incorporated into the current production lot acceptance procedures where MIL-STD-105D is employed with variables data. Although random sampling is permissible in the selection of rational subgroups for control charts, some sensitivity in process shifts is traded for the lot acceptance function.

CHAPTER 4

A SAMPLING SCHEME

4.1 GENERAL DESCRIPTION

The proposed cartridge production acceptance sampling scheme applies only to cases where MIL-STD-105D and variables data are specified. Sample sizes will be dictated by MIL-STD-105D and divided into subgroups according to order of production. Samples within subgroups will be randomly selected from cartridges produced during the subgroup interval.

Production lot acceptance will be based on the acceptance criteria of MIL-STD-105D and lot performance. Control charts will be maintained to study the process. If the lot is rejected, control charts may indicate the cause and provide a basis for partial lot acceptance.

4.2 SUBGROUP INTERVAL

Subgroup interval can be based on unit or time periods of production. The latter is recommended so that morning versus afternoon, day to day, week to week effects can be investigated. However, the basis is dependent upon lot size and daily production rate. For instance, if the daily production was high compared to the lot size, the unit basis may be more appropriate due to the relatively short period of time span. The decision as to the basis of subgrouping should be deferred until the lot has been manufactured and production records submitted.

4.3 PRODUCTION RECORDS

Cartridge specifications or procurement contracts should specify the serialization of cartridges by order of production and the maintenance of production records. It is recommended that a periodic production log be maintained of cartridge serial numbers. The starting and stopping of the production line should be recorded by time and cartridge serial number.

4.4 PROCEDURE

After the lot has been produced and the production log submitted, the process analyzer must decide on the number of subgroups and the subgroup size. The imposed constraints are:

$$k \geq 25$$

$$n \geq 3$$

$$kn \leq t$$

where k = number of subgroups

n = subgroup size

t = sample size as dictated by MIL-STD-105D

From Chapter 3, the reader will recall that kn_{min} will always be compatible with t. The basis of subgrouping will be heavily dependent on the constraint imposed upon k. For instance, if the process analyzer wanted to study the process on a daily basis (subgroup interval = one day) and there were only 15 days of production; he could not do so. However, he still could base his subgrouping on a time period of production by reducing the time span to half day intervals (k = 30). The constraints do allow for flexibility. Consider the following two examples.

EXAMPLE 1: lot size = 5,000
daily production rate = 200
t = 200

In this example, it would seem more appropriate to base the subgroup interval upon time period of production. The subgroup interval could be based on day or half day intervals with k being 25 and 50 respectively which would dictate n being 8 and 4 respectively. In fact, if some sampling basis were discounted, both schemes could be employed from the same data provided the random sampling was based on k = 50, n = 4.

EXAMPLE 2: lot size = 5,000
daily production rate = 1,000
t = 200

In this example, it would be desirable to base the subgroup interval upon unit period of production. The process analyzer has a choice of k being 25, 28, 33, 40, 50, and 66 with n being 8, 7, 6, 5, 4, and 3 respectively.

Once k and n have been decided upon, samples can be randomly selected within the subgroup interval and test fired. In the case where $kn < t$, the extra samples should be randomly selected from the entire lot and tested for production lot acceptance only.

4.5 TESTS

Grant (7) points out that the ambient test is only a substitute, dictated by economy, for an environmental test. Ambient tests are usually used for production testing largely because of their simplicity and economy.

Since cartridge performance is relatively sensitive to temperature, ambient temperature tests cannot be specified for production tests. Presently, normal temperature tests of 70°F ± 5°F are specified which require temperature conditioning equipment. The writer contends that production tests of cartridges should be conducted at design temperature extremes rather than 70°F on the basis of:

1. present undefined temperature-performance relationships. Specification limits are normally established for a temperature operating range of -65°F to 160°F. Unless a temperature-performance relationship has been defined, performance at -65°F or 160°F cannot be predicted from 70°F tests for comparison with specification limits.
2. the slight increase in test costs

Since temperature conditioning equipment is required even for 70°F tests, the additional cost of conditioning cartridges to -65°F or 160°F would be negligible. Admittedly, there would be some testing difficulties. Presently, for 70°F test, the test vehicle is not temperature conditioned due to the fact that experience has indicated that the heat transfer between the cartridge and test vehicle is negligible if the cartridge is fired within five minutes after removal from the conditioning chamber. For -65°F or 160°F cartridge temperature conditioning, there may be a heat transfer problem. However, it is envisioned that such a problem could be solved relatively easy and inexpensive through the use of insulation by "heat or cold sinks."

4.6 THREE CASES

Historically, there is a definite relationship between temperature and cartridge performance which may or may not be defined. For testing, three cases must be considered. They are:

1. unknown temperature-performance relationship and the single specification limit,
2. unknown temperature-performance relationship and the double specification limit, and
3. known temperature-performance relationship and the single or double specification limit(s).

Single specification limits are normally associated with a lower specification limit and apply only to impulse cartridges. Since energy outputs decrease as temperatures decrease, -65°F tests should be conducted in Case 1. Double specification limits always apply to delay cartridges, and in most instances to impulse cartridges. Hence for Case 2, tests should be conducted at both -65°F and 160°F. This would require doubling the sample sizes of MIL-STD-105D. If the temperature-performance relationship were known as in Case 3, tests could be conducted at any temperature and performance predicted for any other temperature. For Case 2, which is the most frequent, attempts should be made to define the temperature-performance relationship to reduce testing cost. In the next chapter, such an attempt is made for the MARK 5 delay cartridge.

CHAPTER 5

A PREDICTIVE MODEL

5.1 APPROACH

A short investigation by a colleague, reference (4), greatly influenced the approach used by the writer in the development of a predictive mathematical model for the performance of the MARK 5 delay cartridge over the temperature range of -65°F to 160°F. This investigation consisted of a computerized statistical analysis of the production test data of the MARK 4 delay cartridge. The MARK 4 delay cartridge is identical to the MARK 5 delay cartridge except for the delay time range. The significant conclusions from this investigation were:

1. the variances of the delay time characteristic can be considered constant from -65°F to 160°F within each lot.
2. the relationship between delay time and temperature can be considered linear within each lot.
3. the hypothesis that the linear regression lines for delay time of all lots are equal must be rejected.
4. the manufacturer may be the significant factor for the rejection of the above hypothesis.

In formulating a predictive model for production acceptance of the MARK 5 delay cartridge, the writer utilized the following approach:

1. conduct acceptance tests at -65°F and predict performance at 160°F. By doing so, the need for an output predictive model is eliminated.

2. consider two variables in the delay time model, temperature and process dispersion. Only temperature was considered in the investigation of reference (4).
3. use the parameter of range at -65°F for process dispersion. Assume range of dispersion to be constant from -65°F to 160°F.
4. use the parameters of minimum delay time at -65°F versus minimum delay time at 160°F among lots for the temperature effect variable rather than the delay time versus temperature parameters within each lot as in (4).
5. analyze the relationship of delay time and temperature by manufacturer.
6. select a manufacturer, and by multiple linear regression analysis, develop a predictive model for delay time.
7. determine accuracy of model.

5.2 TESTS AND SPECIFICATION LIMITS

As the reader may recall from Chapter 2, the delay time characteristic of the MARK 5 delay cartridge has dual specification limits while the output characteristic has only a lower specification limit. Additionally, one must recall that temperature has a pronounced effect on both characteristics. Output decreases as temperature decreases. Conversely, delay time increases as temperature decreases. Hence, if production tests were conducted at -65°F, cartridge performance in regard to upper specification limit for delay time and the lower specification limit for output could be verified directly. A model for predicting delay time at -160°F would indirectly verify cartridge performance with respect to the lower specification limit for delay time. Conducting tests at any other temperature in the -65°F to 160°F operating range would require a predictive model for output, and the double use of the model for delay time. Cartridge performance with respect to the three specification limits would be totally predicted.

In production lot acceptance tests of MARK 5 delay cartridges in accordance with MIL-STD-105D, emphasis is on individual cartridge performance rather than lot means and dispersion. Every cartridge performance should be within specification limits. Due to the lot size, inspection level, and AQL, rejection number will always be one. Therefore, the model developed for predicting delay time at 160°F was based on the indirect temperature relationship of the minimum delay time recorded of the -65°F test with that of the minimum delay time recorded of the 160°F test of past production lots. Assuming a valid model, any predicted value at 160°F below the lower specification limit for delay time would cause rejection of a lot.

5.3 TEMPERATURE AND PROCESS DISPERSION RELATIONSHIP

Historically, temperature has produced a significant effect on delay times of the MARK 5 delay cartridge. Assume the linear relationship of (4) is valid. Logically then, process dispersion will have an effect on the location and possibly the slope of this linear regression line with respect to predicting the minimum delay time at 160°F. However, due to the parameters selected to determine temperature relationship among past lots (minimum delay time at -65°F versus minimum delay time at 160°F), initial thoughts were that the process dispersion effects would already be compensated for within these parameters themselves. Nevertheless, both temperature, indirectly, and process dispersion were considered in the model.

5.4 MANUFACTURER EFFECT

Production data from 23 lots of the MARK 5 delay cartridge were available. Figure 5-1 is a scatter diagram of delay time means at -65°F and 160°F for all lots. From this figure, one must agree that a linear correlation between means of the two temperatures appears remote. Yet, if one were to differentiate the lots by the four manufacturers, Figures 5-2 through 5-5, a different conclusion would be drawn. Since all manufacturers in essence produced the cartridges from the same drawings, one must conclude that the process has a significant effect on cartridge performance. Again the need for process studies regarding the manufacture of power cartridges is emphasized.

5.5 THE MODEL

Manufacturer B was selected for the development of a delay time predictive model for his process. The basis of selection was the larger number of lots submitted, and the greater range of delay time. A multiple linear regression analysis of the production lot data of Manufacturer B is contained in Appendix IX. It is concluded that there is a linear trend at the .05 level of significance of the minimum delay

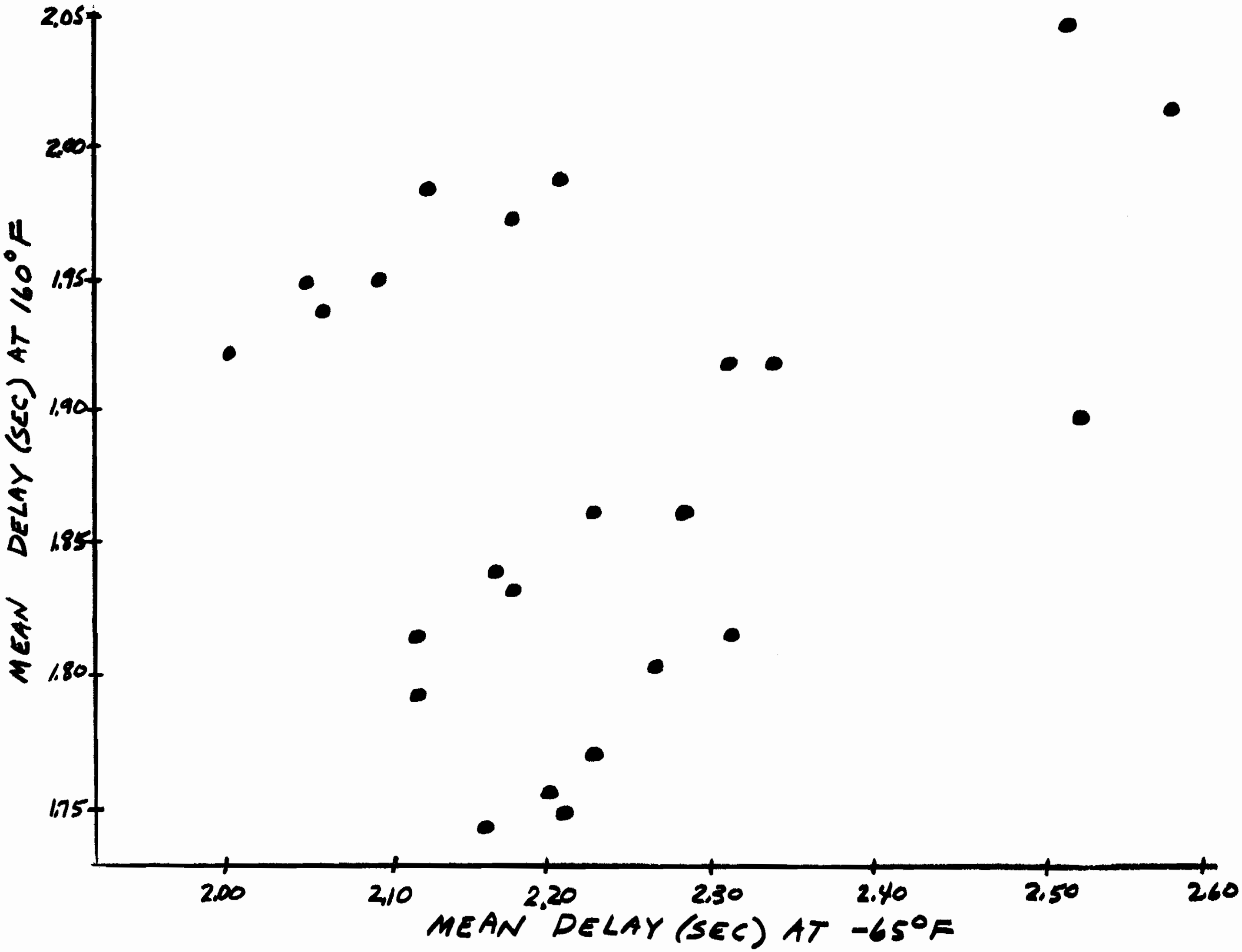


FIGURE 5-1. SCATTER DIAGRAM OF DELAY MEANS, ALL LOTS

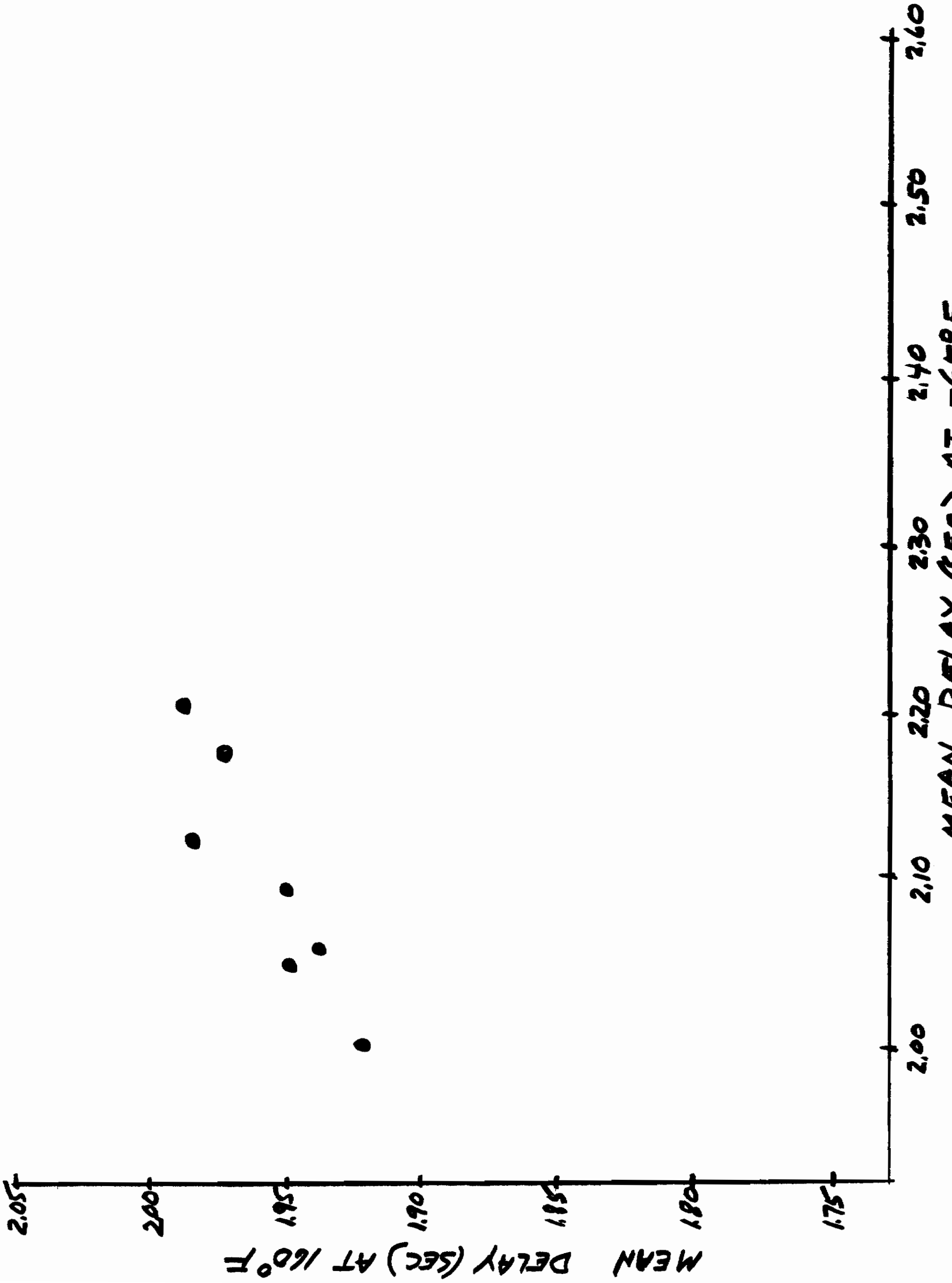


FIGURE 5-2. SCATTER DIAGRAM OF DELAY MEANS, MFG. A

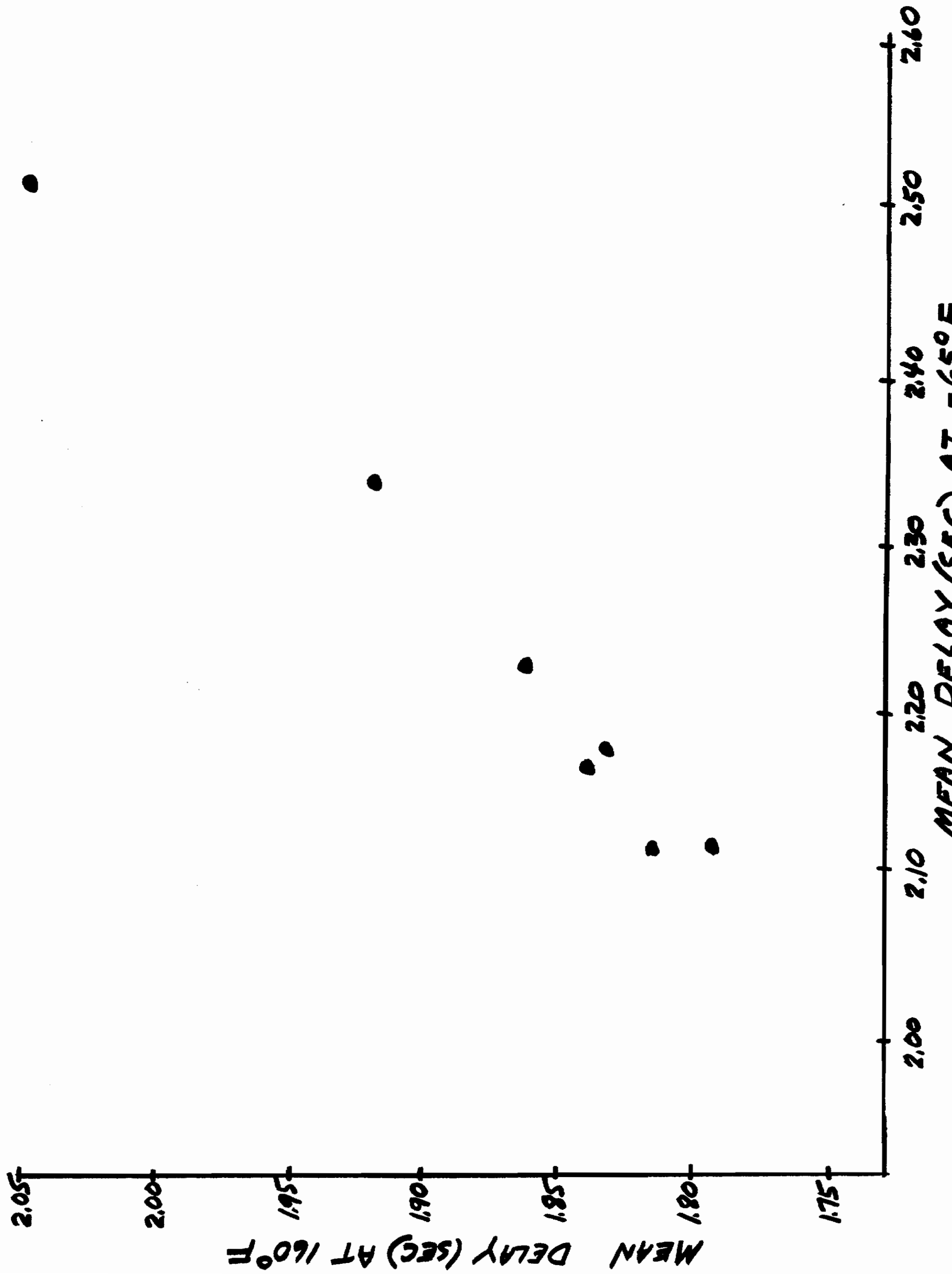


FIGURE 5-3. SCATTER DIAGRAM OF DELAY MEANS, MFG. B

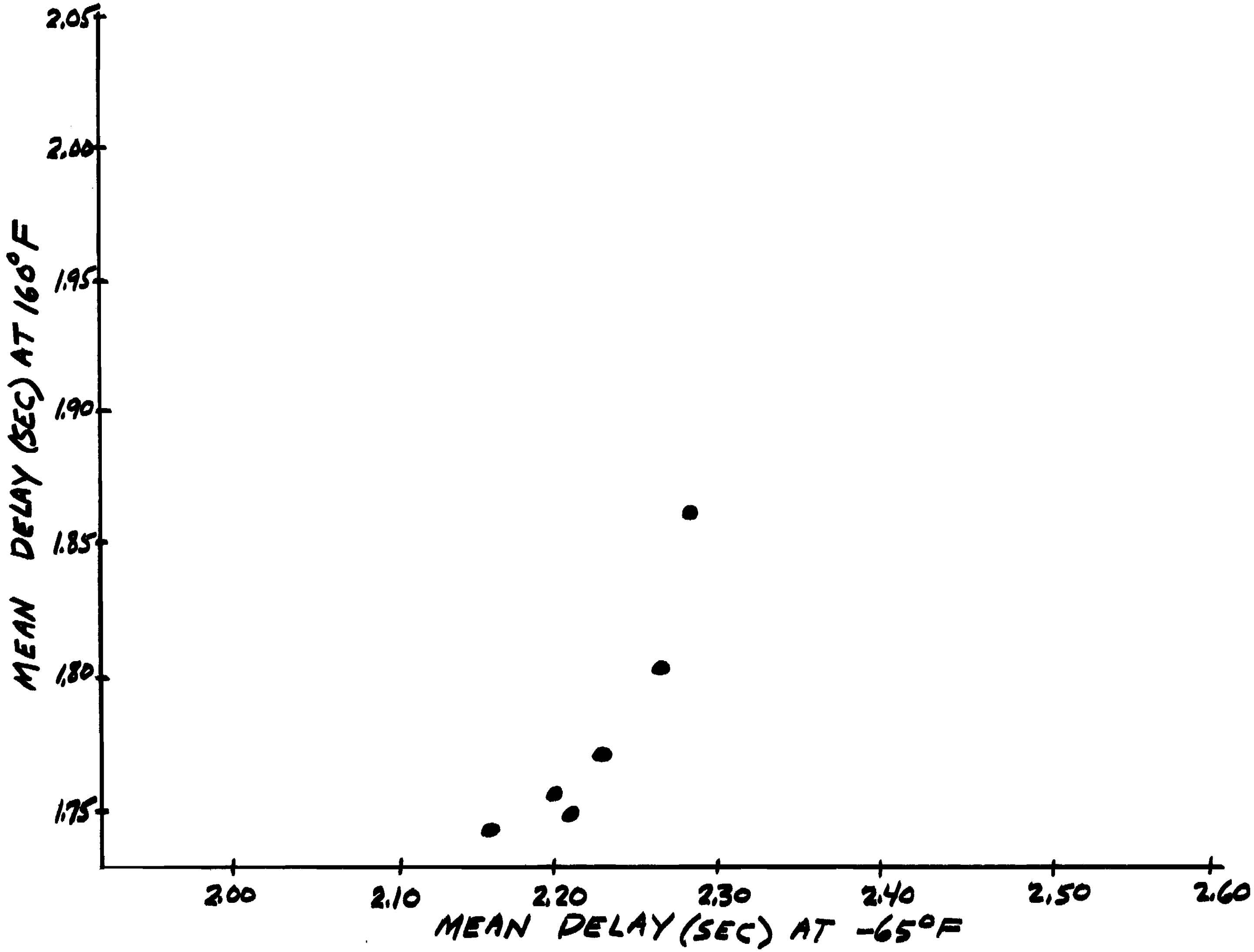


FIGURE 5-4. SCATTER DIAGRAM OF DELAY MEANS, MFG. C

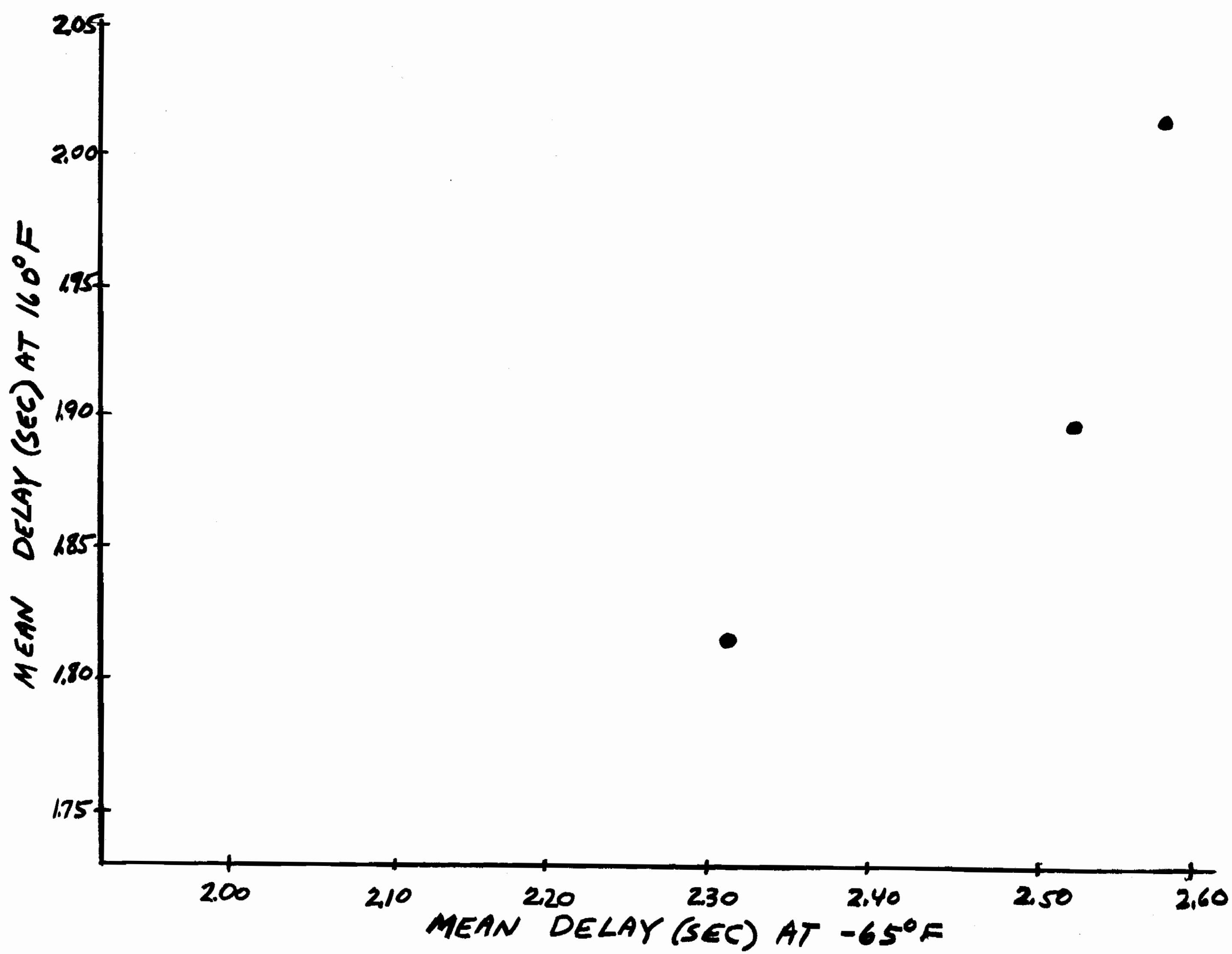


FIGURE 5-5 SCATTER DIAGRAM OF DELAY MEANS, MFG. D

time recorded at 160°F which depends on the minimum delay time recorded at -65°F or the process dispersion of delay time at -65°F or both. The estimated regression of y on X_1 and X_2 is:

$$y = -.059691 + .872169X_1 + .000564X_2$$

where y = minimum delay time at 160°F

X_1 = minimum delay time at -65°F

X_2 = range of delay time at -65°F

In trial computations, it was found that y is very insensitive to X_2. Either the premonition that the process dispersion effect would be already incorporated into the temperature effect parameters was true or the process dispersion effect was insignificant when compared to the temperature effect. Regardless of the case, the regression equation can be reduced to:

$$y = -.059691 + .872169X_1$$

The model was applied to the production lot data. Table 5-1 compares the predictive results of the model with the observed results.

TABLE 5-1

MODEL RESULTS VERSUS OBSERVED RESULTS

Lot	Minimum Delay Time at 160°F	
	Model	Observed
1	1.751	1.775
2	1.925	1.925
3	1.859	1.850
4	1.802	1.800
5	1.772	1.750
6	1.772	1.825
7	1.728	1.675

The average error was .0219 second, or 1.22 percent. Maximum error was 3.07 percent.

5.6 LIMITATIONS

It is known that temperature has a significant relationship upon cartridge performance. Perhaps this relationship is linear in most cases and can be expressed in simple mathematical form. However, the fact that the production process has a significant bearing on the validity of the model certainly limits the use of such a model in production acceptance from several standpoints. Firstly, a certain amount of production lot data must be acquired before a model can be developed. This means that such a model could not be incorporated into cartridge performance specifications for initial production. Secondly, even if from a single source of supply, infrequent procurement certainly gives a definite lack of guarantee that the process will be the same from lot to lot. Thirdly, the competitive procurement policy would dictate the development of models for each manufacturer which would be impractical. Nevertheless, the use of mathematical models should not be neglected. Before McNamara's policy of closing down Government production plants, the bulk of cartridge production was done by one Government plant. There is a present trend toward reinstating the production of cartridges by a Government plant. If this materializes, then greater control of the process can be affected. With single source, process controlled production, mathematical models for predicting cartridge performance will have important application in production acceptance.

CHAPTER 6

SUMMARY

6.1 CONTROL CHARTS

Advances in air technology have resulted in increased cartridge performance requirements which are crowding the present "state of the art" design performance. Process control, which would seem mandatory, is not implemented by contractors for several justifiable reasons. Further, because production lots are small and infrequent, and because destructive testing is involved; the problem of confidence level and minimum sample size becomes paramount in acceptance tests. The use of control charts in production acceptance tests by the Government can provide:

1. additional confidence in lot acceptance because additional information is available.
2. valuable information feedback to the design engineer and contractor for the resolution of design and production problems.
3. partial lot acceptance.

Control charting could be readily incorporated into current cartridge production lot acceptance procedures where MIL-STD-105D is employed with variables data. Cartridge serialization and production records would be the only additional requirements.

6.2 PROCESS EFFECTS

The development of mathematical models for predicting cartridge performance could substantially reduce the cost of production lot acceptance tests. However, the use of such models will be limited due to the significant effect of the process which presently cannot be controlled from manufacturer to manufacturer. This conclusion from Chapter 5 and the results of the case study of Chapter 2 has indicated to the writer the importance of the process. Presently, considerable research is being devoted to finding better materials and techniques to improve cartridge performance. Although the writer is not advocating the censure of research, the diversion of some of the research manpower and money to process studies of the production of current designs may produce startling results. Besides, even design improvements as a result of research cannot guarantee any increased reliability if process control is not maintained in production. Process studies are, therefore, recommended to improve lot quality and affect consistency of cartridge performance from one manufacturer to another.

6.3 GENERAL APPLICABILITY

Although the writer has applied a sampling scheme, which allows for process study and lot acceptance independently with the same samples, to explosive power sources; the scheme has more general applicability. It could be applied by any commerical or governmental agency to verify the product quality level submitted by vendors and the variability of that quality level in conjunction with the agency's lot acceptance procedures. Such information would be useful in the selection of a vendor. Grant (5) points out the use of control charts by a purchaser to help a vendor improve his process. With the proposed scheme, this could also be done in conjunction with the purchaser's lot acceptance procedures.

The use of linear regression analysis in predicting environment-performance relationships should have general application in production acceptance tests where:

1. there is sufficient historical data to verify linearity.
2. the effect of the process from one manufacturer to another is insignificant.

Many environment-performance relationships have been found to be approximately linear, particularly when temperature is the environment involved and the temperature range not too severe.

6.4 RECOMMENDED STUDIES

The contents of this thesis have been concerned with process study and production lot acceptance of Navy explosive power sources. These are only a few of the areas which should be investigated in regard to the reliability of cartridge performance. Studies are recommended in the following areas:

1. Correlation of performance between production test fixtures and actual systems. The objective of this study would be to validate test fixtures and procedures. Some work in this area has been accomplished. Two approaches are recommended, theoretical and emphirical. The theoretical approach would be the optimum but the most difficult due to the many variables that would have to be defined. The known effects of these variables would be helpful in establishing future design parameters.

2. Determination of the effects of environmental sequential tests on performance. Limited study by this writer has indicated that these environmental tests, which are currently neglected in development, have adverse effects on cartridge performance. Even though the magnitude of the individual environments used in this limited study may have been too severe, thus producing unrealistic results; the operational environment of cartridges is constantly changing and these cyclic environments should be simulated to verify cartridge reliability.
3. Determination of realistic AQLs. The writer has always questioned the tight AQLs imposed upon cartridges, particularly when small lots and destructive testing are involved. Though not to be meant as a contradiction to what has already been said, a study of the theoretical probabilities of cartridge malfunctions in service might be highly informative, particularly when compared to actual percentage of malfunctions.

REFERENCES

1. Douglas H. W. Allan, Statistical Quality Control, Reinhold Publishing Corporation, New York, 1959.

2. Irwing W. Burr, Engineering Statistics and Quality Control, McGraw-Hill Book Company, New York, 1953.

3. "Tolerance Limits for Small Lots," G. L. Burrows and C. Silber, Industrial Quality Control, Vol XIX, No. 8, February 1963.

4. "Report on Second Phase of Rotational Training Program," V. A. Cummings, Memorandum, U. S. Naval Weapons Laboratory, Dahlgren, Va., January 1967.

5. Eugene L. Grant, Statistical Quality Control, McGraw-Hill Book Company, New York, 1964.

6. "Acceptance Sampling by Variables with Non-Normal Distributions," D. J. Guarino, Graduate Term Paper, V. P. I., Blacksburg, Virginia, December 1965.

7. W. Grant Ireson, Reliability Handbook, McGraw-Hill Book Company, New York, 1966.

8. R. Lowell Wine, Statistics for Scientists and Engineers, Prentice Hall, Inc., Englewood Cliffs, N. J., 1964.

9. MIL-STD-105D, Sampling Procedures and Tables for Inspection by Attributes, 29 April 1963.

10. MIL-STD-414, Sampling Procedures and Tables for Inspection by Variables for Percent Defective, 11 June 1957.

11. MIL-D-21625D, Design and Evaluation of Cartridges for Cartridge Actuated Devices, 12 April 1966.

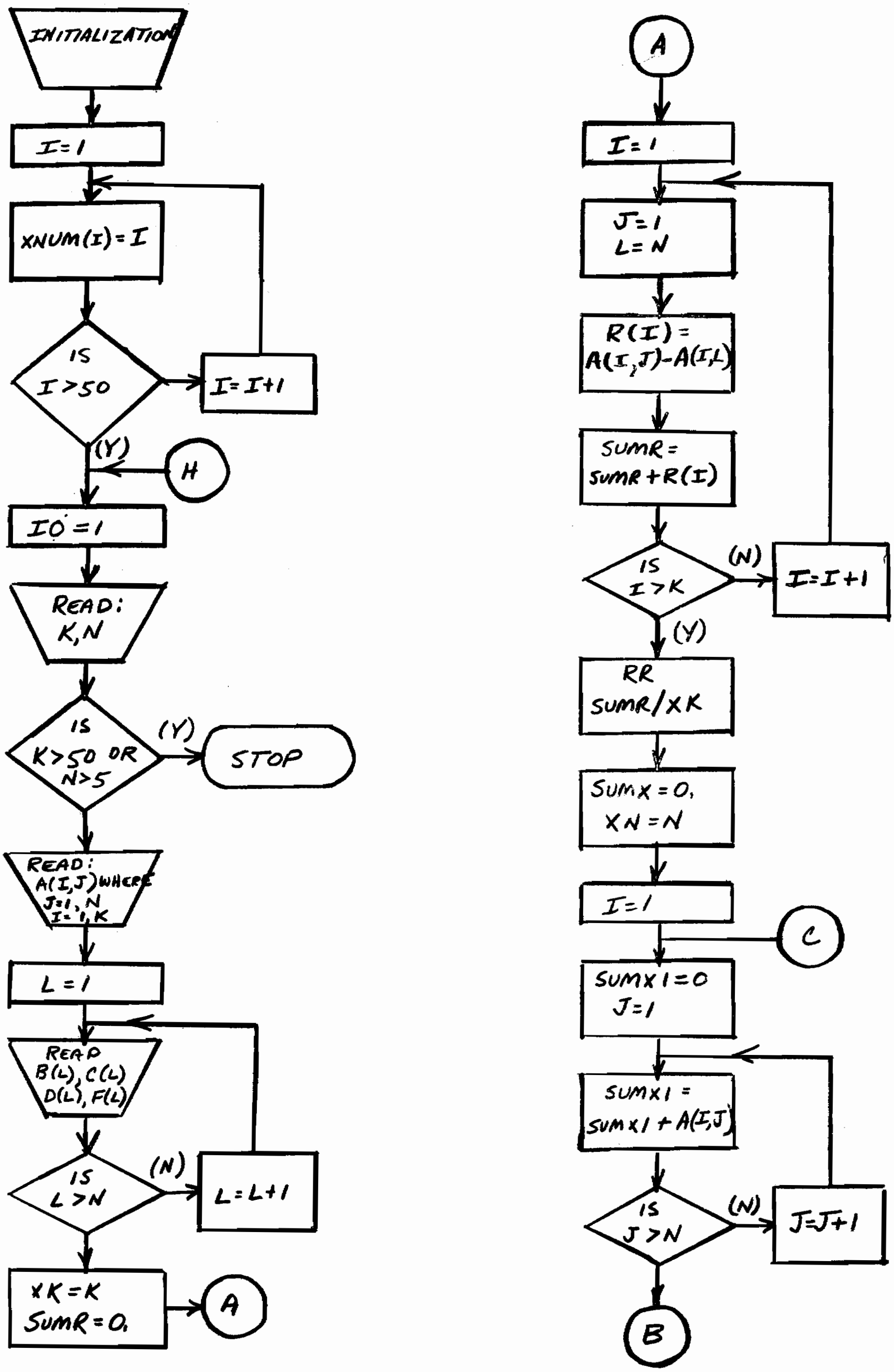
VITA

Bobby C. Layman was born and raised in the Shenandoah Valley of Virginia. He received his secondary education in the public schools of Frederick County, Virginia. After receiving his B.S. degree in Industrial Engineering at Virginia Polytechnic Institute in 1958, he attended extensions of George Washington University and American University as a part-time student. He fulfilled his military obligation in the U. S. Army Ready Reserves having reached the rank of Captain. With the exception of a tour of active duty in the Army, he has been employed by the U. S. Naval Weapons Laboratory since 1958. In 1965, he was selected by the Navy for the full-time advanced study program. He presently holds the title of Supervisory Mechanical Engineer and is engaged in directing research and development programs. He is a co-inventor of an underwater ordnance device. He and his wife, daughter and son reside in Dahlgren, Virginia.

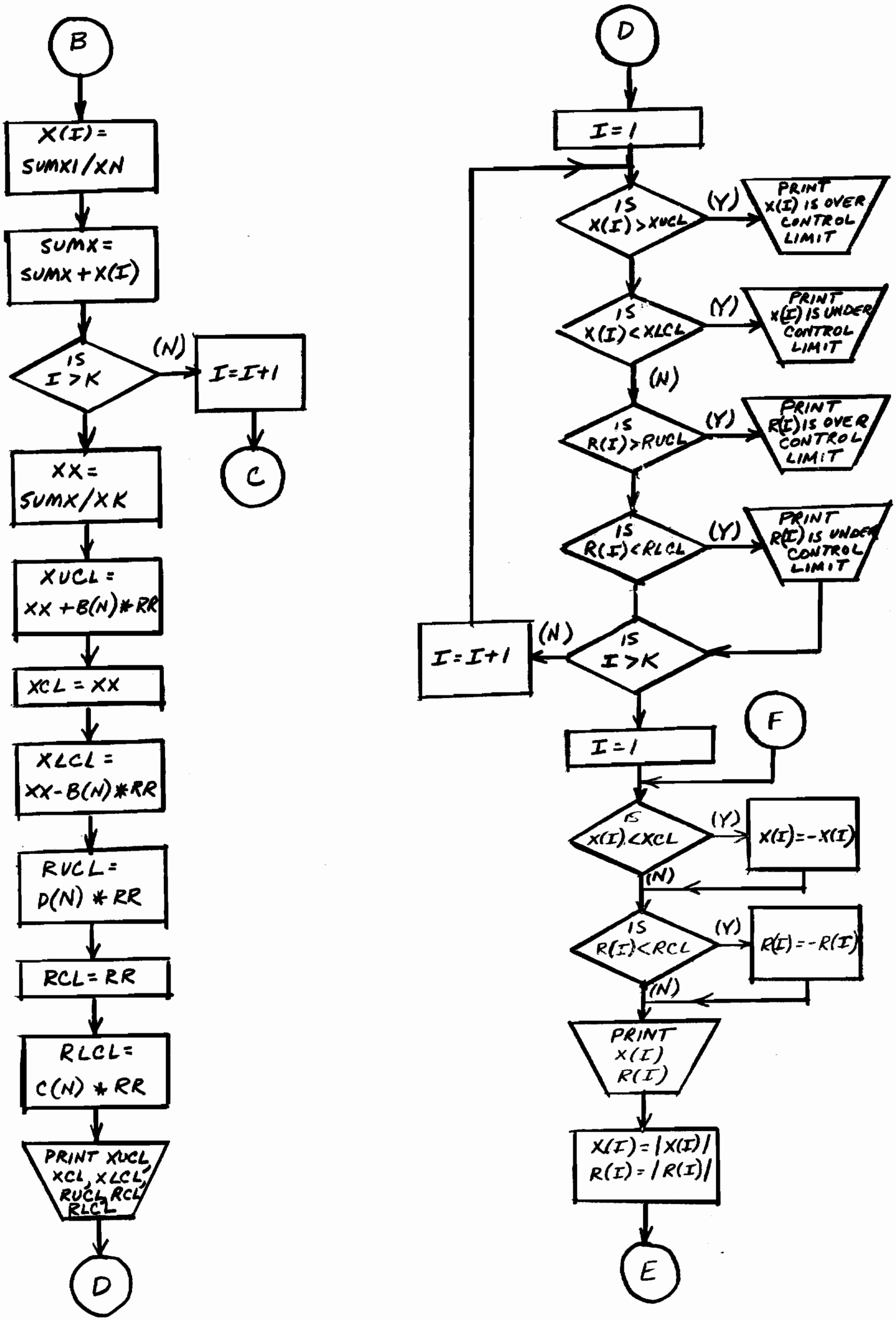
Bobby C. Layman

APPENDIX I

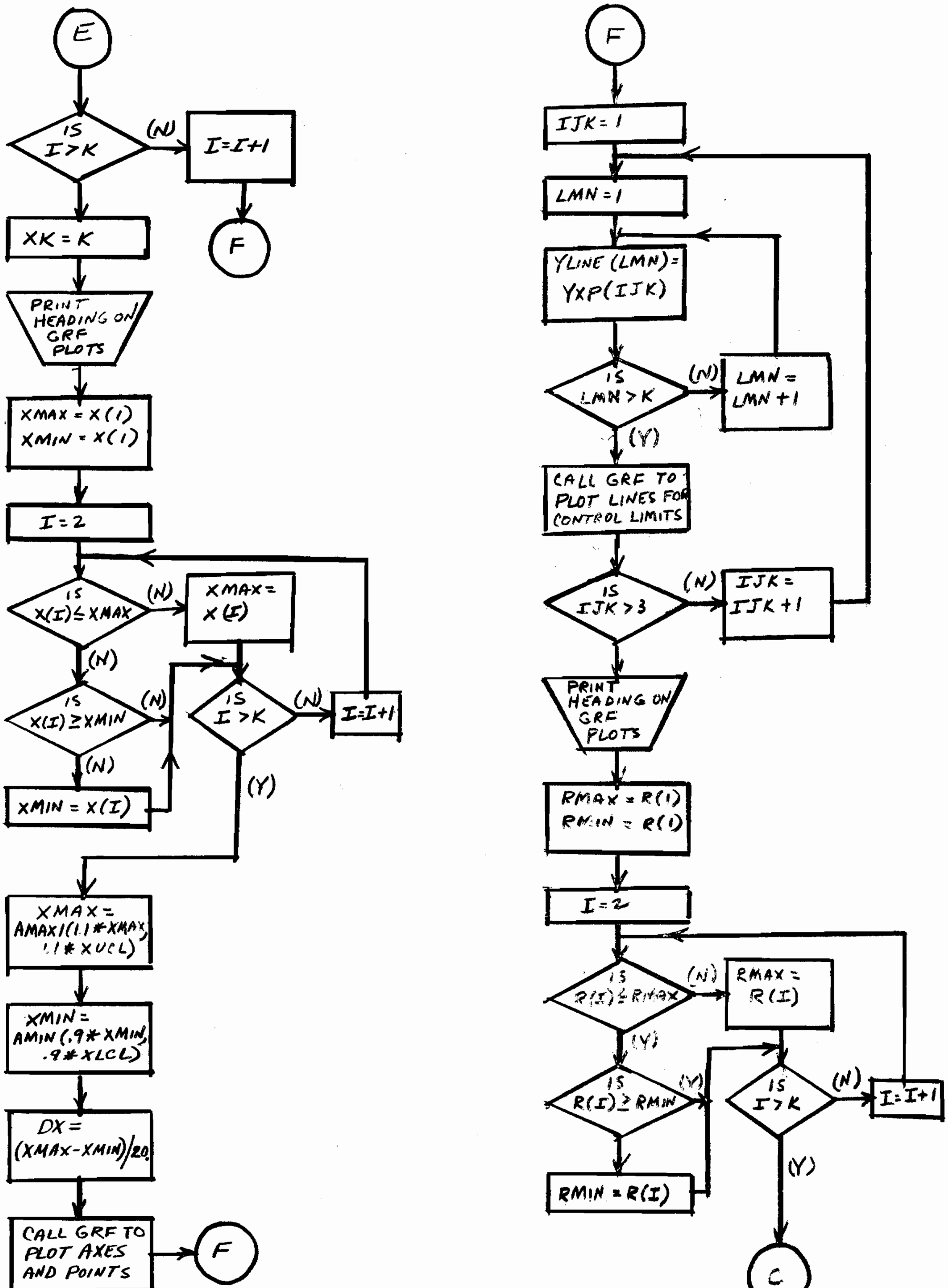
FLOW CHART FOR CONTROL CHART COMPUTER PROGRAM



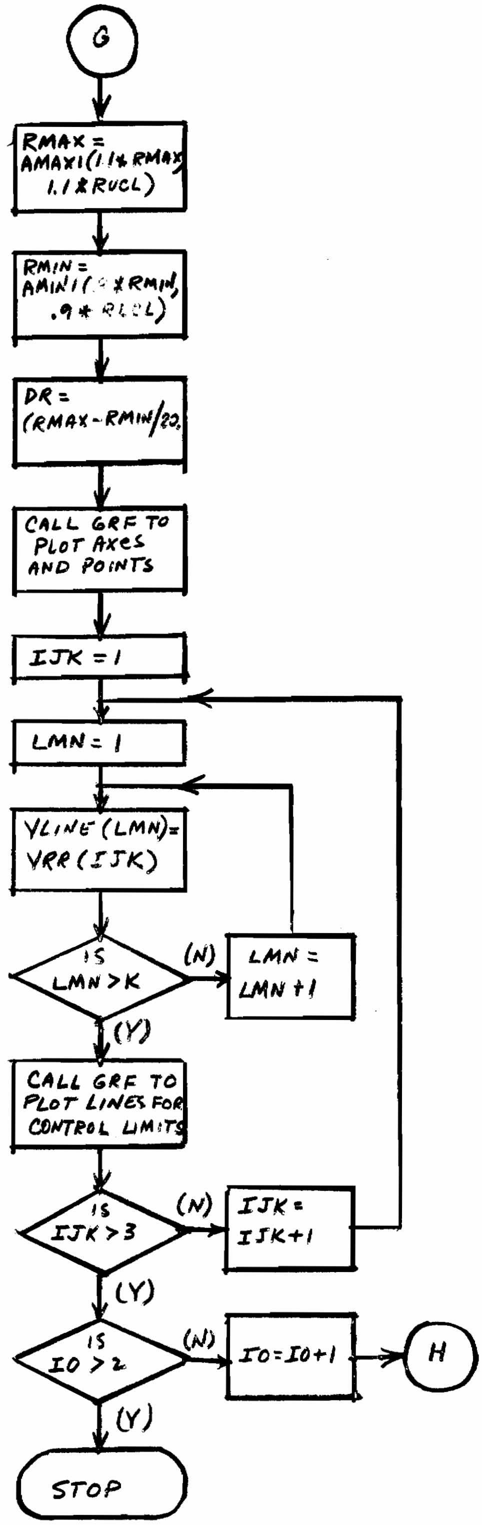
APPENDIX I (CONTINUED)



APPENDIX I (CONTINUED)



APPENDIX I (CONTINUED)



APPENDIX II

CARD LISTING OF CONTROL CHART COMPUTER PROGRAM

```
      DIMENSIØN A(50,5),B(5),C(5),D(5),X(50),R(50),F(5),F2(1),FMT(5)
      DIMENSIØN XNUM (50),YXP(3),YRP(3),YLINE(50)
      CALL SETIT
      CALL CRTID (4HQC51)
      EQUIVALENCE (YXP(1),XCL),(YXP(2),XLCL),(YXP(3),XUCL),
     1           (YRP(1),RCL),(YRP(2),RLCL),(YRP(3),RUCL)
      DØ 2 I=1,50
    2 XNUM (I)=1
      DØ 100 1Ø=1,2
      DATA (FMT(I),I=1,5)(6H(F7,3),7H(2F7,3),7H(3F7,3),7H(4F7,3),
     1     7H(5F7,3))
      READ 1,K,N
    1 FORMAT (212)
      1F(K,GT,50,ØR,N,GT,5) STØP
      F2(1)=FMT(N)
      READ F2,((A(I,J),J=1,N),I=1,K)
      DØ 3 L=1,N
    3 READ 4, B(L),C(L),D(L),F(L)
    4 FØRMAT (4F7,3)
C     CØMPUTE RANGE ØF EACH SUBGRØUP ØF DATA
      XK=K
      SUMR=0
      DØ 5  I=1,K
      J=1
      L=N
      R(I) = A(I,J)-A(I,L)
C     SUM RANGE VALUES
    5 SUMR = SUMR+R(I)
C     CØMPUTE RANGE MEDIAN
      RR = SUMR/XK
C     CØMPUTE MEDIAN ØF EACH SUBGRØUP ØF DATA
      SUMX = 0
      XN=N
      DØ 9 I=1,K
      SUMX1 =0,
      DØ 8 J=1,N
    8 SUMX1 =SUMX1 +A(I,J)
      X(I) = SUMX1/XN
C     SUM ØF MEDIAN VALUES
    9 SUMX = SUMX +X(I)
C     CØMPUTE AVERAGE MEDIAN
      XX=SUMX/XK
C     CALL SUBRØUTINE TØ CØMPUTE RANGE AND MEDIAN CØNTRØL LIMITS
   10 CALL CØNTR (XX,B,RR,D,C,XUCL,XCL,XLCL,RUCL,RCL,RLCL,N)
      IF (1Ø,EQ,2) GØ TØ 111
```

APPENDIX II (Continued)

```
      PRINT11,XUCL,XCL,XLCL,RUCL,RCL,RLCL
   11 FØRMAT (1H220X,6HØUTPUT//1H 5HXUCL=,F7,3,5X,4HXCL=F7,3,
     1         5X,5HXLCL=,F7,3/1H 5HRUCL=,F7,3,5X,4HRCL=,F7,3,5X,
     2         5HRLCL=,F7,3/)
      GØ TØ 113
  111 PRINT 112,XUCL,XCL,XLCL,RUCL,RCL,RLCL
  112 FØRMAT (1H220X,5HDELAY//1H 5HXUCL=,F7,3,5X,4HXCL=,F7,3,5X,
     1            5HXLCL=,F7.3/1H 5HRUCL=,F7,3,5X,4HRCL=,F7,3,5X,
     2            5HRLCL=,F7,3/)
  113 DØ 12 I=1,K
      IF (X(I),GT,XUCL) PRINT 131,I
      IF (X(I),LT,XLCL) PRINT 132,I
      IF (R(I),GT,RUCL) PRINT 141,I
      IF (R(I),LT,RLCL) PRINT 142,I
   12 CØNTINUE
  131 FØRMAT (1H 1HX,12,22H IS ØVER CØNTRØL LIMIT)
  132 FØRMAT (1H 1HX,12,23H IS BELØW CØNTRØL LIMIT)
  141 FØRMAT (1H 1HR,12,22H IS ØVER CØNTRØL LIMIT)
  142 FØRMAT (1H 1HR,12,23H IS BELØW CØNTRØL LIMIT)
      DØ 18 I=1,K
      IF (X(I),LT,XCL) X(I)=-X(I)
      IF (R(I),LT,RCL) R(I)=-R(I)
      PRINT 17,I,X(I),I,R(I)
   17 FØRMAT (1H 4X,1HX,12,1H=,F7,3,16X,1HR,12,1H=,F7,3)
      X(I)=ABS(X(I))
   18 R(I) = ABS(R(I))
      XK=K
      IF (IØ,EQ,1) GØ TØ 611
      PRINT 6,XUCL,XCL,XLCL
    6 FØRMAT (2HS2,BOX,20HMEDIAN CØNTRØL CHART/2HSO,28X,5HEX 56,
     12X,5HMOD 2,2X,9H160 DELAY/2HSO,19X,7HXUCL = ,F7,3,3X,
     26HXCL = ,F7,3,3X,7HXLCL = ,F7,3)
      GØ TØ 62
  611 PRINT 61,XUCL,XCL,XLCL
   61 FØRMAT (2HS2,30X,20HMEDIAN CØNTRØL CHART/2HSO,28X,5HEX 56,
     12X,5HMØD 2,2X,10H160 ØUTPUT/2HSO,19X,7HXUCL = ,F7,3,3X,
     26HXCL = ,F7,3,3X,7HXLCL = ,F7,3)
   62 XMAX=X(1)
      XMIN=X(1)
      DØ 200 I=2,K
      IF (X(I),LE,XMAX) GØ TØ 1200
      XMAX=X(I)
      GØ TØ 200
 1200 IF(X(I),GE,XMIN) GØ TØ 200
      XMIN=X(I)
  200 CØNTINUE
      XMAX=AMAX1(1,1*XMAX,1,1*XUCL)
      XMIN=AMIN1(,9*XMIN, 9*XLCL)
```

APPENDIX II (Continued)

```
      DX=(XMAX-XMIN)/20,
      CALL GRF(0,XMIN,XK,XMAX,0,0,1,DX,0,2,0,K,XNUM,X,1,1,2H67,1,0)
      DØ 210 IJK=1,3
      DØ 201 LMN=1,K
  201 YLINE(LMN)=YXP(IJK)
      CALL GRF(0,0,0,0,0,0,0,0,0,0,1,K,XNUM,YLINE,1,1,2H54,1,0)
  210 CØNTINUE
      CALL INTVL (XTIME)
      IF (1Ø,EQ,1) GØ TØ 711
      PRINT 7, RUCL,RCL,RLCL
    7 FØRMAT (2HS2,30X,19HRANGE CØNTRØL CHART/2HS0,28X,5HEX 56,2X,
     15HMØD 2,2X,9H160 DELAY/2HS0,19X,7HRUCL = ,F7,3,3X,6HRCL = ,F7,3,
     23X,7HRLCL = ,F7,3)
      GØ TØ 72
  711 PRINT 71,RUCL,RCL,RLCL
   71 FØRMAT (2HS2,30X,19HRANGE CØNTRØL CHART/2HS0,28X,5HEX 56,2X,
     15HMØD 2,2X,10H160 ØUTPUT/2HS0,19X,7HRUCL = ,F7,3,3X,6HRCL = ,F7,3,
     23X,7HRLCL = ,F7,3)
   72 RMAX=R(1)
      RMIN= R(1)
      DØ 300 1=2,K
      IF (R(I),LE,RMAX) GØ TØ 1300
      RMAX=R(I)
      GØ TØ 300
 1300 IF(R(I),GE,RMIN) GØ TØ 300
      RMIN=R(I)
  300 CØNTINUE
      RMAX=AMAX1(1,1*RMAX,1,1*RUCL)
      RMIN=AMIN1(,9*RMIN, 9*RLCL)
      DR=(RMAX-RMIN)/20,
      CALL GRF (0,RMIN,XK,RMAX,0,0,1,DR,0,2,0,K,XNUM,R,1,1,2H67,1,0)
      DØ 220 IJK=1,3
      DØ 211 LMN=1,K
  211 YLINE(LMN)=YRP(IJK)
      CALL GRF(0,0,0,0,0,0,0,0,0,0,1,K,XNUM,YLINE,1,1,2H54,1,0)
  220 CØNTINUE
  100 CØNTINUE
      CALL TIME(TØTAL)
      PRINT 1000, TØTAL
 1000 FORMAT (27H RUNNING TIME IN SECØNDS = ,F7,3)
      STØP
      END
```

APPENDIX III

VARIABLES GLOSSARY FOR CONTROL CHART COMPUTER PROGRAM

Variable Name	Description
XNUM	the number of points to be plotted by the GRF plot subroutine
K	number of subgroups
N	number of samples in each subgroup
B(N), C(N), D(N)	tabulated 3 sigma limit factors
SUMR	sum of range values
R(I)	range of subgroup I
RR	range mean
SUMX	sum of mean values
SUMXI	sum of sample values in each subgroup
X(I)	mean of subgroup I
XX	mean of entire sample; mean of subgroup means
XUCL	mean upper control limit
XCL	mean center line; equivalent to XX
XLCL	mean lower control limit
RUCL	range upper control limit
RCL	range center line; equivalent to RR
RLCL	range lower control limit
XMAX	maximum point to be plotted by GRF subroutine on the mean control chart

APPENDIX III (Continued)

Variable Name	Description
XMIN	minimum point to be plotted by GRF subroutine on the mean control chart
DX	distance between tick marks on Y axis of mean control chart
RMAX	maximum point to be plotted by GRF subroutine on the range control chart
RMIN	minimum point to be plotted by GRF subroutine on the range control chart
DR	distance between tick marks on Y axis of range control chart

APPENDIX IV

CONTROL CHART DATA

		-65°F Tests										+160°F Tests							
Subgroup	Date Mfg.	Recorded Delay Times			$\bar{X}$	R	Recorded Output			$\bar{X}$	R	Recorded Delay Times		$\bar{X}$	R	Recorded Output		$\bar{X}$	R
1	7/8 AM	2.482	2.349	2.412	2.414	.133	77.55	77.55	56.20	70.43	21.35	1.930	2.012	1.971	.082	82.27	79.90	81.08	2.37
2	PM	2.296	2.492	2.305	2.364	.196	82.27	77.55	77.55	80.44	4.72	1.961	1.987	1.974	.026	82.27	81.07	81.67	1.20
3	7/11 AM	2.477	2.367	2.313	2.386	.164	78.73	68.17	79.90	75.60	11.73	1.962	1.919	1.941	.043	82.27	75.20	78.74	7.07
4	PM	2.243	2.339	2.343	2.308	.100	65.80	77.55	51.70	65.02	25.85	1.959	1.939	1.949	.020	72.87	68.17	70.52	4.70
5	7/12 AM	2.378	2.524	2.383	2.428	.146	72.87	75.20	61.10	69.73	14.10	1.976	1.969	1.972	.007	61.10	75.20	68.15	14.10
6	PM	2.488	2.193	2.178	2.286	.310	81.07	82.27	79.90	81.08	2.37	2.043	2.003	2.023	.040	69.33	91.67	80.50	22.34
7	7/13 AM	2.403	2.343	2.354	2.367	.060	77.55	68.17	77.55	74.43	9.38	1.924	2.126	2.025	.202	69.33	75.20	72.26	5.87
8	PM	2.301	2.693	2.419	2.471	.392	79.90	58.77	82.27	73.65	23.50	1.948	2.366	2.157	.418	82.27	75.20	78.74	7.07
9	7/14 AM	2.239	2.377	2.336	2.317	.141	65.80	61.10	66.97	64.62	5.87	1.828	1.948	1.888	.120	84.60	82.27	83.44	2.33
10	PM	2.654	2.363	2.361	2.459	.293	47.00	*	75.20	61.10	28.20	2.176	1.983	2.079	.193	82.27	81.07	81.64	1.20
11	7/15 AM	2.328	2.337	2.382	2.349	.054	65.80	79.90	65.80	70.50	14.10	2.141	2.191	2.166	.050	69.33	82.27	75.80	12.94
12	PM	2.373	2.371	2.450	2.398	.079	84.60	79.90	72.37	78.96	12.23	1.911	1.974	1.942	.063	77.55	82.27	79.91	4.72
13	7/18 AM	2.267	2.358	2.226	2.284	.132	75.20	70.50	79.90	75.20	9.40	2.060	1.774	1.917	.286	91.67	63.47	77.57	28.20
14	PM	2.125	2.387	2.304	2.272	.262	86.97	79.90	63.47	76.78	23.50	1.951	1.926	1.938	.025	94.00	61.10	77.55	32.90
15	7/19 AM	2.351	2.332	2.375	2.353	.043	84.60	75.20	75.20	78.33	9.40	1.932	1.971	1.951	.039	61.10	70.50	65.80	9.40
16	PM	2.391	2.212	2.402	2.335	.190	75.20	70.50	77.55	74.42	7.05	1.990	1.909	1.950	.081	75.20	77.55	76.38	2.35
17	7/20 AM	2.330	2.309	2.328	2.322	.021	79.90	81.07	75.20	78.72	5.87	1.925	2.021	1.973	.096	84.60	77.55	81.08	7.05
18	PM	2.281	2.407	2.244	2.311	.163	70.90	77.55	58.77	72.07	21.13	2.186	1.999	2.093	.187	70.50	77.55	74.02	7.05
19	7/21 AM	2.374	2.378	2.321	2.358	.057	70.50	79.90	82.27	77.55	11.77	2.055	1.994	2.025	.061	82.27	75.20	78.74	7.07
20	PM	2.273	2.310	2.347	2.310	.074	77.55	77.55	68.17	74.42	9.38	1.998	2.100	2.049	.102	79.90	84.60	82.25	4.70
21	7/22 AM	2.352	2.238	2.239	2.276	.114	65.80	72.87	54.07	64.24	18.80	1.984	1.979	1.981	.005	79.90	79.90	79.90	0
22	PM	2.306	2.247	2.164	2.239	.142	72.87	79.90	82.27	78.35	9.40	1.943	*	1.943	--	85.77	77.55	81.66	8.22
23	7/25 AM	2.285	2.313	2.415	2.338	.130	86.97	72.87	70.50	76.78	16.47	2.017	2.029	2.023	.012	84.60	65.80	75.20	18.80
24	PM	2.361	2.230	2.180	2.257	.181	68.17	79.90	96.37	81.48	28.20	2.183	2.041	2.112	.142	83.43	79.90	81.66	3.53
25	7/27 AM	2.255	2.472	2.318	2.348	.217	84.60	84.60	79.90	83.03	4.70	1.990	2.041	2.002	.024	71.67	89.30	80.48	17.63
26	PM	2.144	2.229	2.147	2.173	.085	82.27	77.55	77.55	79.12	4.72	1.894	1.962	1.928	.068	94.00	86.97	90.48	7.03
27	7/28 AM	2.279	2.175	2.263	2.239	.104	56.40	96.37	77.55	76.77	39.97	2.042	1.966	2.004	.076	94.00	75.20	84.60	18.80
28	PM	2.68	2.141	2.336	2.248	.195	75.20	75.20	75.20	75.20	0	1.937	2.192	2.064	.255	77.55	75.20	76.38	2.35
29	7/29 AM	2.282	2.464	2.249	2.332	.215	72.87	79.90	75.20	75.92	7.03	1.915	1.918	1.916	.003	94.00	86.97	90.48	7.03
30	PM	2.207	2.207	2.136	2.183	.071	77.55	86.97	75.20	79.91	11.77	2.003	2.042	2.022	.039	86.97	82.27	84.62	4.70
31	8/1 AM	2.160	2.402	2.167	2.243	.242	84.60	78.73	82.27	81.87	5.87	1.966	1.985	1.975	.019	79.90	82.27	81.08	2.37
32	PM	2.120	2.120	2.108	2.119	.021	86.97	84.60	86.97	86.18	2.37	1.930	2.113	2.021	.183	79.90	70.50	75.20	9.40
TOTALS					74.087	4.727				2411.60	420.20			63.974	2.967			2527.58	284.49

*Denotes Instrumentation Failure

APPENDIX V
CONTROL CHARTS

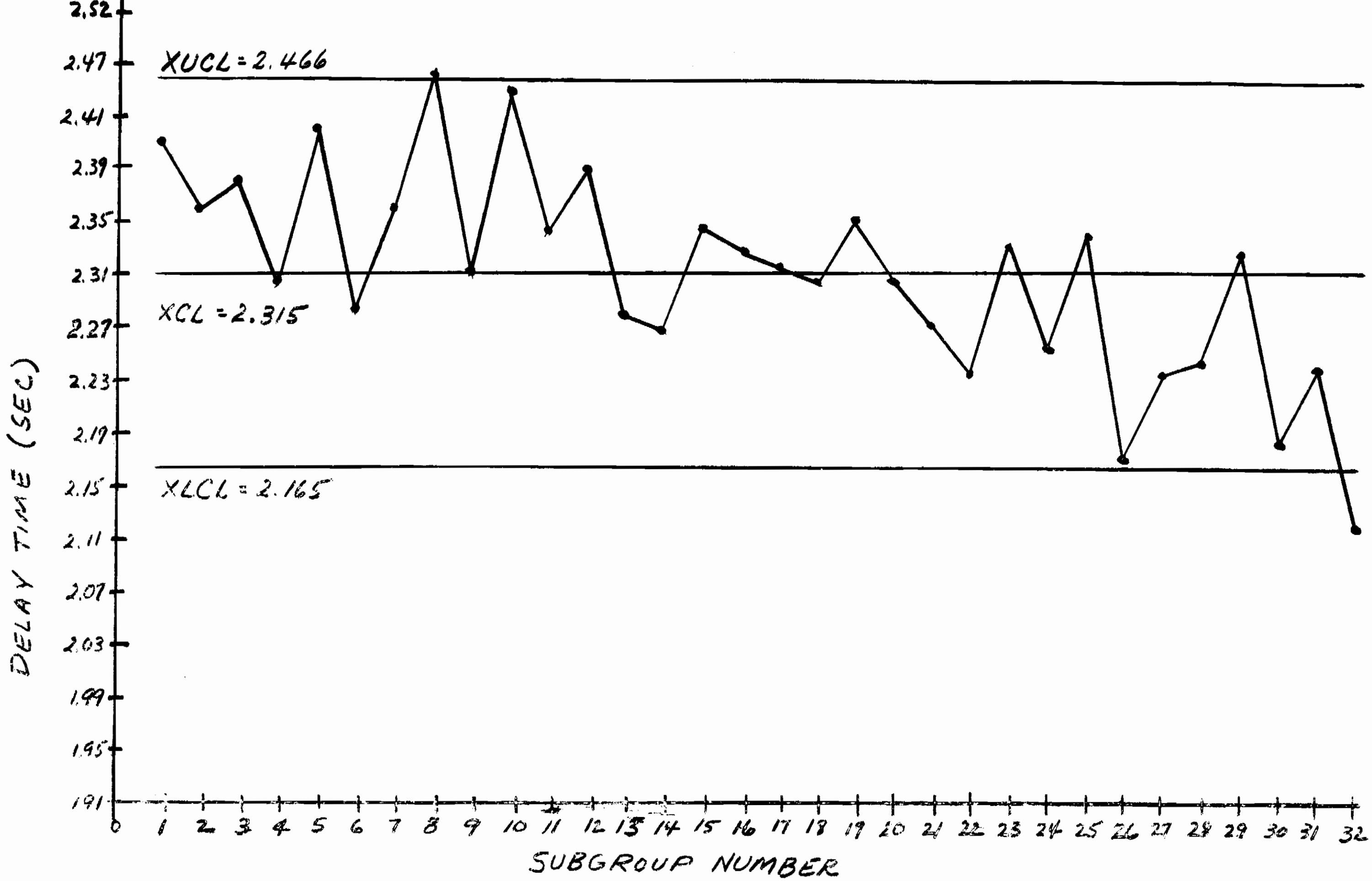


FIGURE V-1. $\bar{X}$ CHART FOR -65°F DELAY

APPENDIX V (CONTINUED)

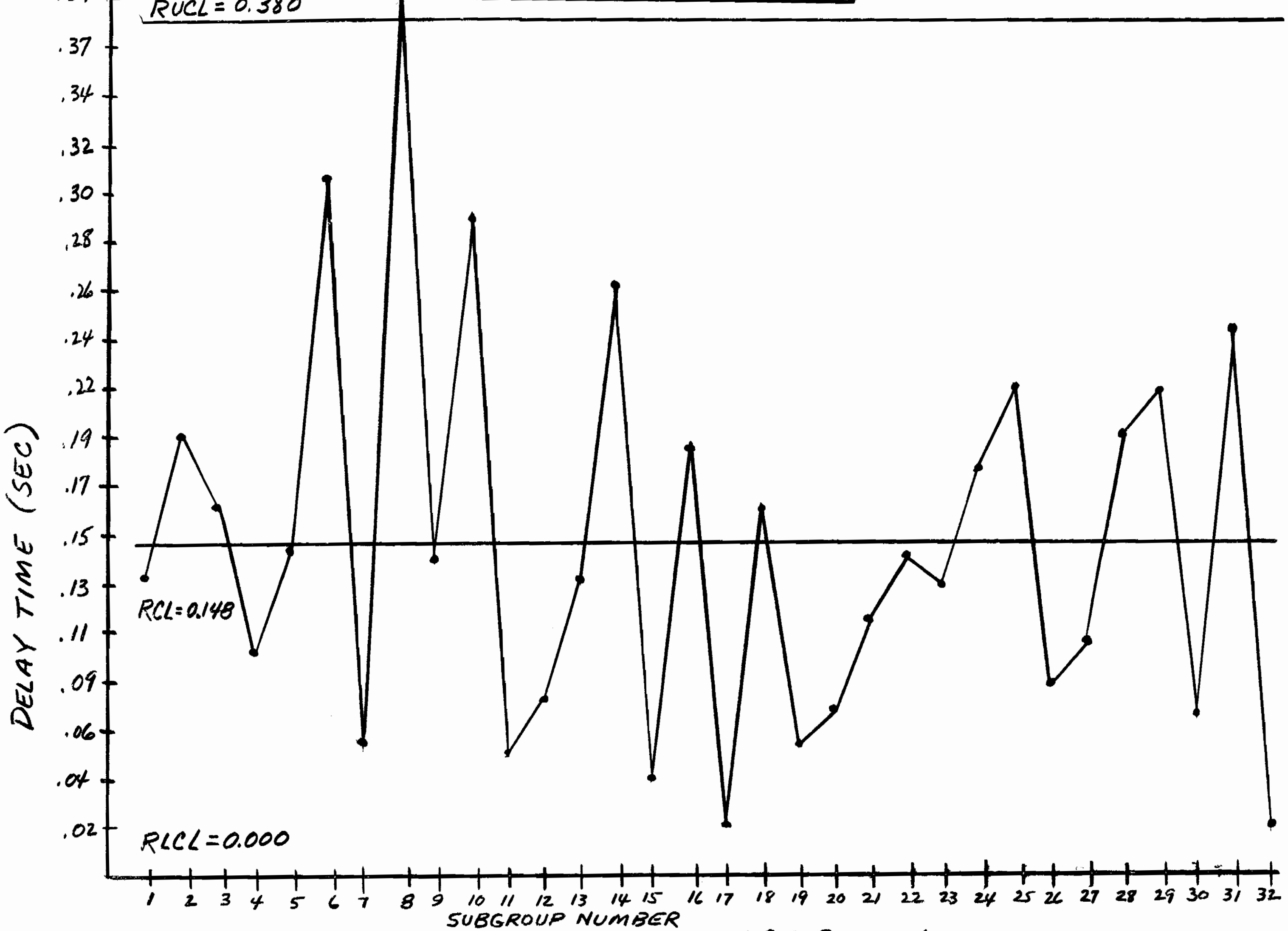


FIGURE V-2. R CHART FOR -65°F DELAY

APPENDIX V (CONTINUED)

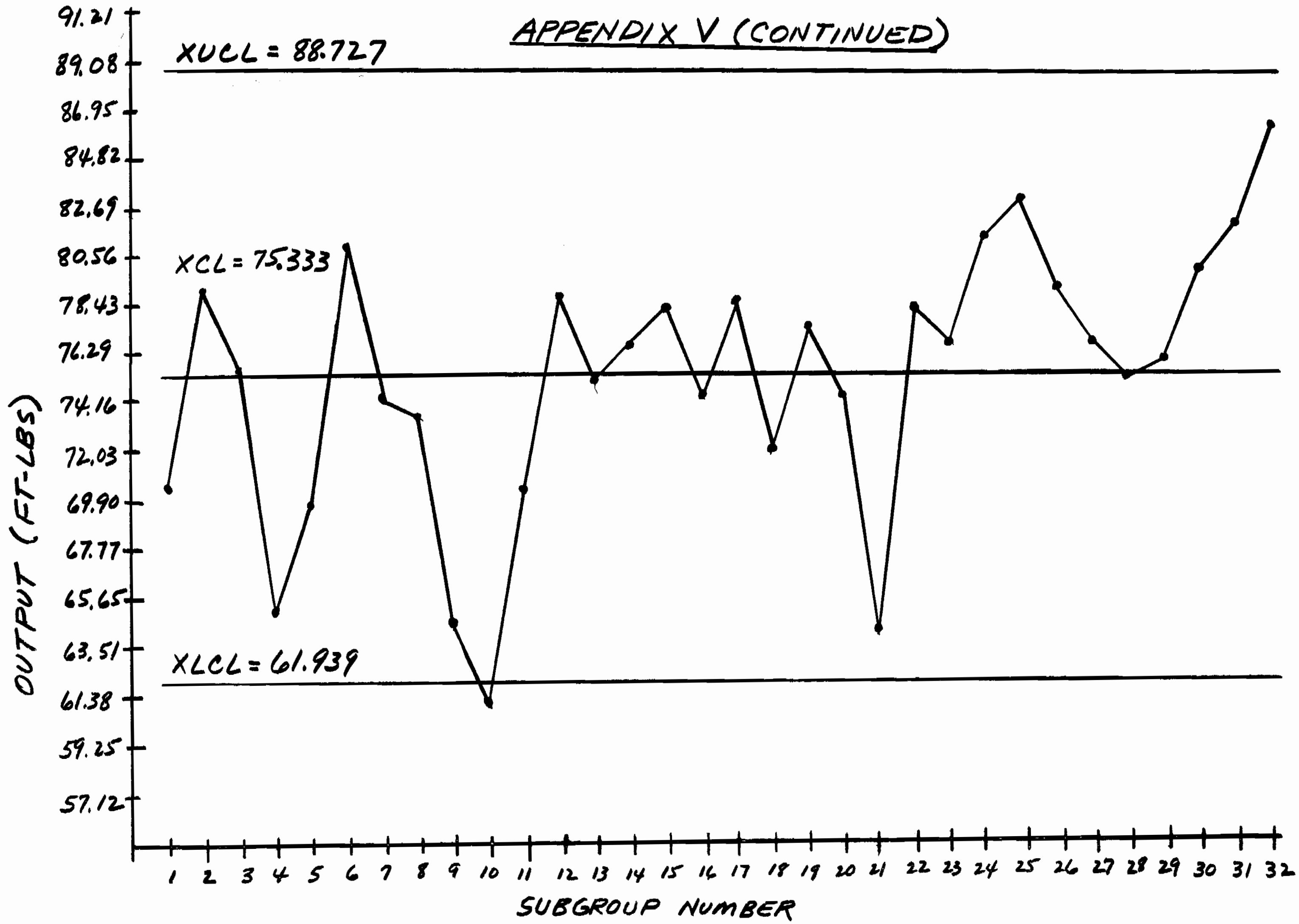


FIGURE V-3. $\bar{X}$ CHART FOR -65°F OUTPUT

APPENDIX V (CONTINUED)

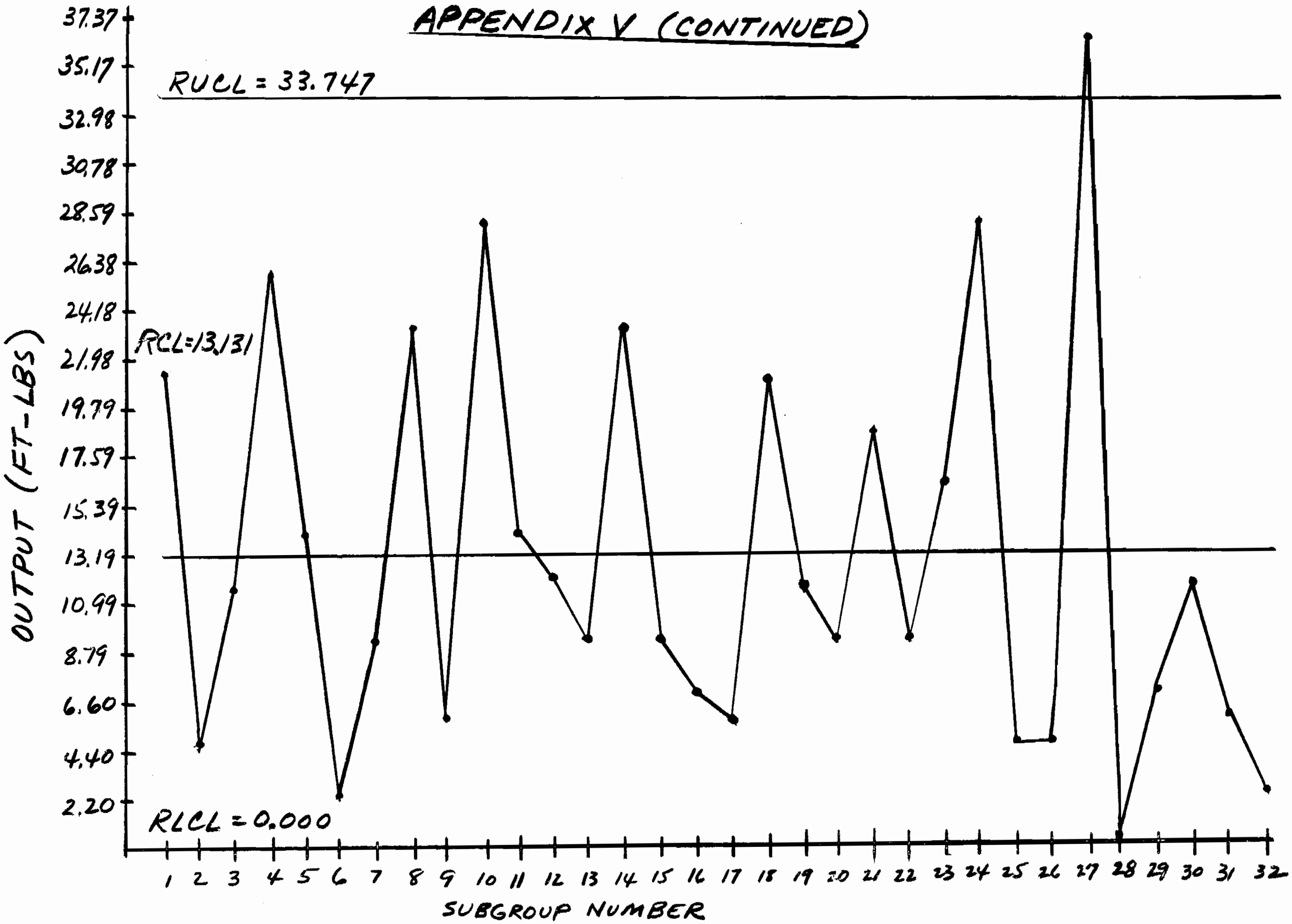


FIGURE V-4. R CHART FOR -65°F OUTPUT

APPENDIX V (CONTINUED)

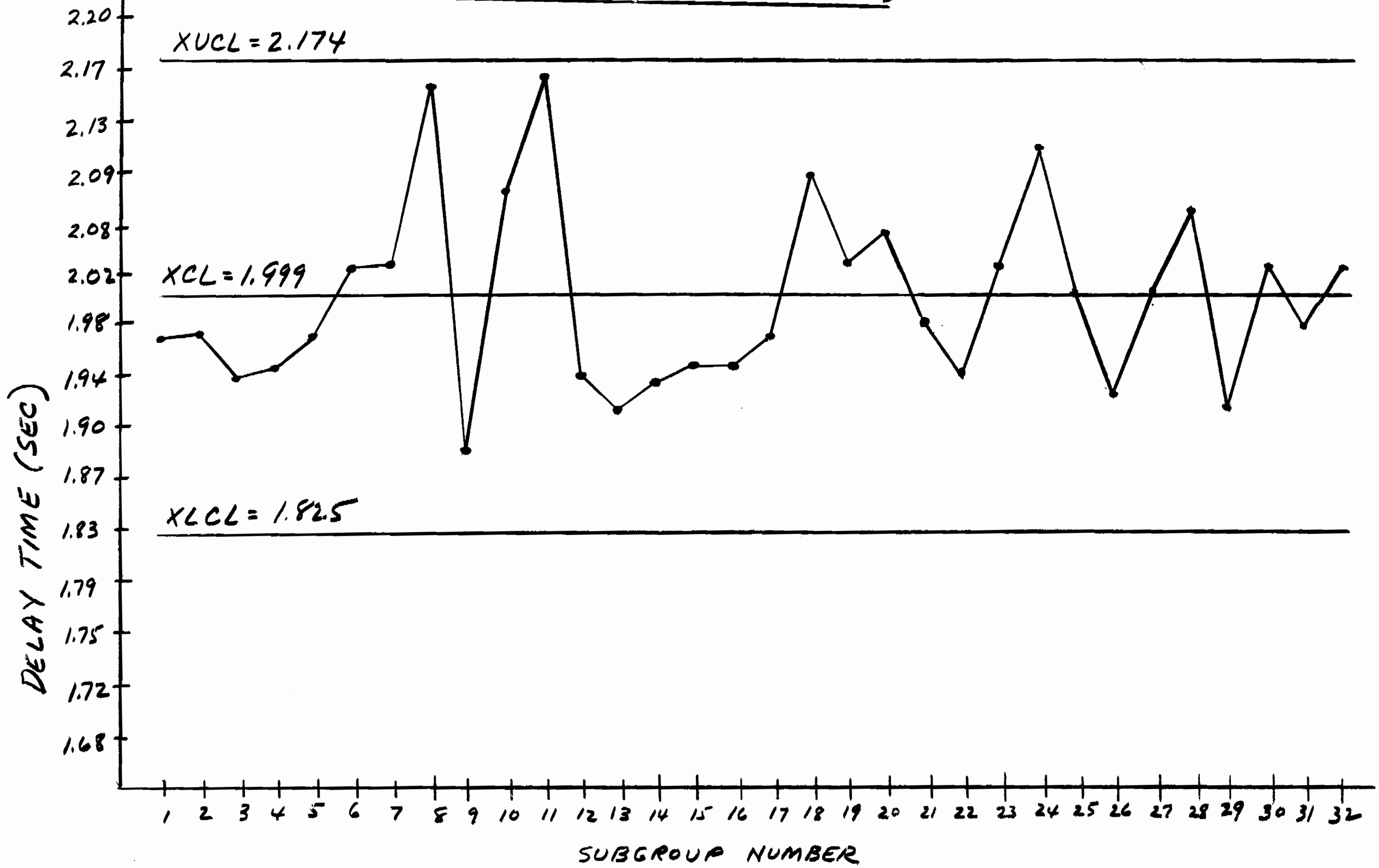


FIGURE V-5. $\bar{X}$ CHART FOR 160°F DELAY

APPENDIX V (CONTINUED)

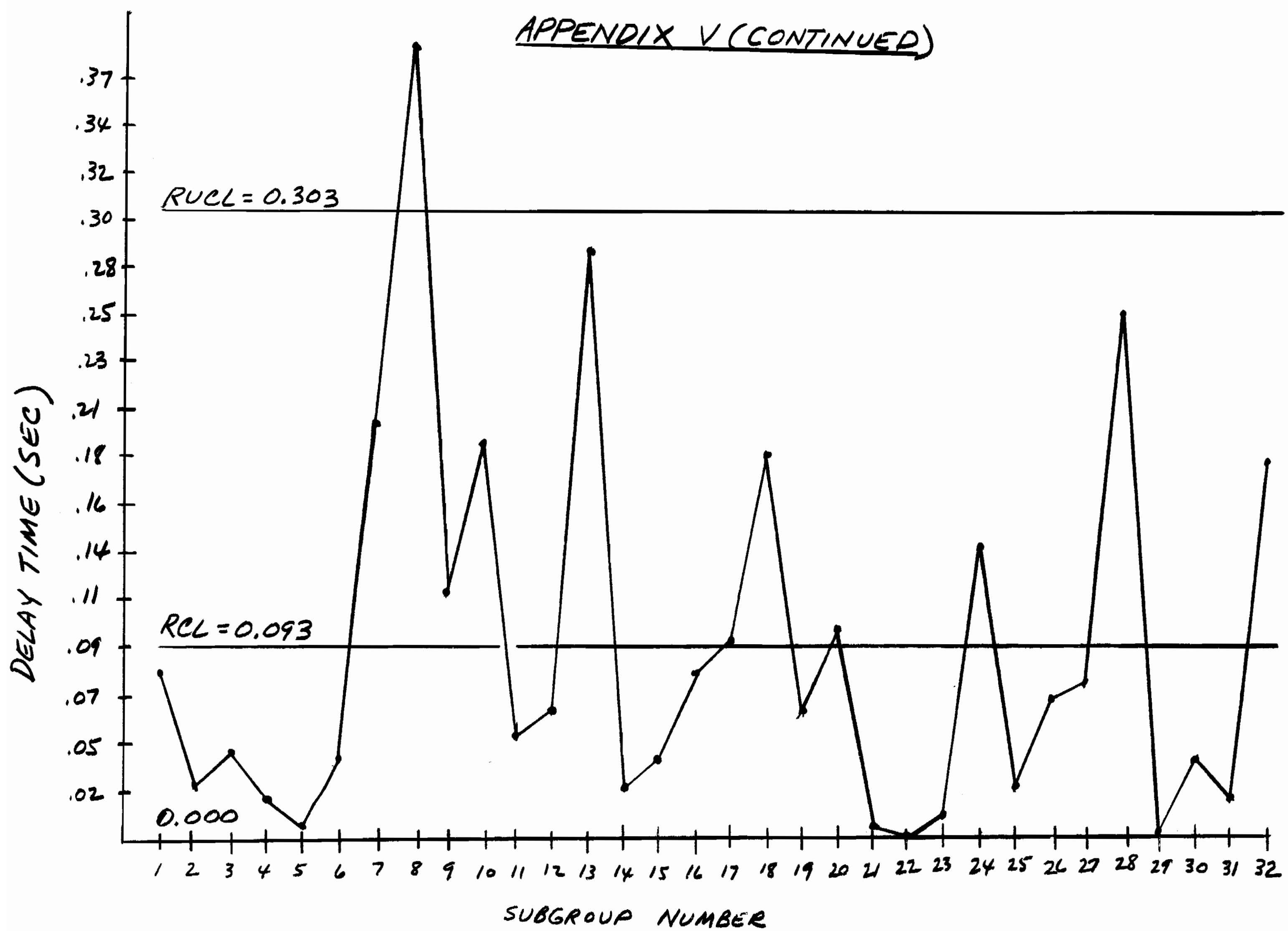


FIGURE V-6. R CHART FOR 160°F DELAY

APPENDIX V (CONTINUED)

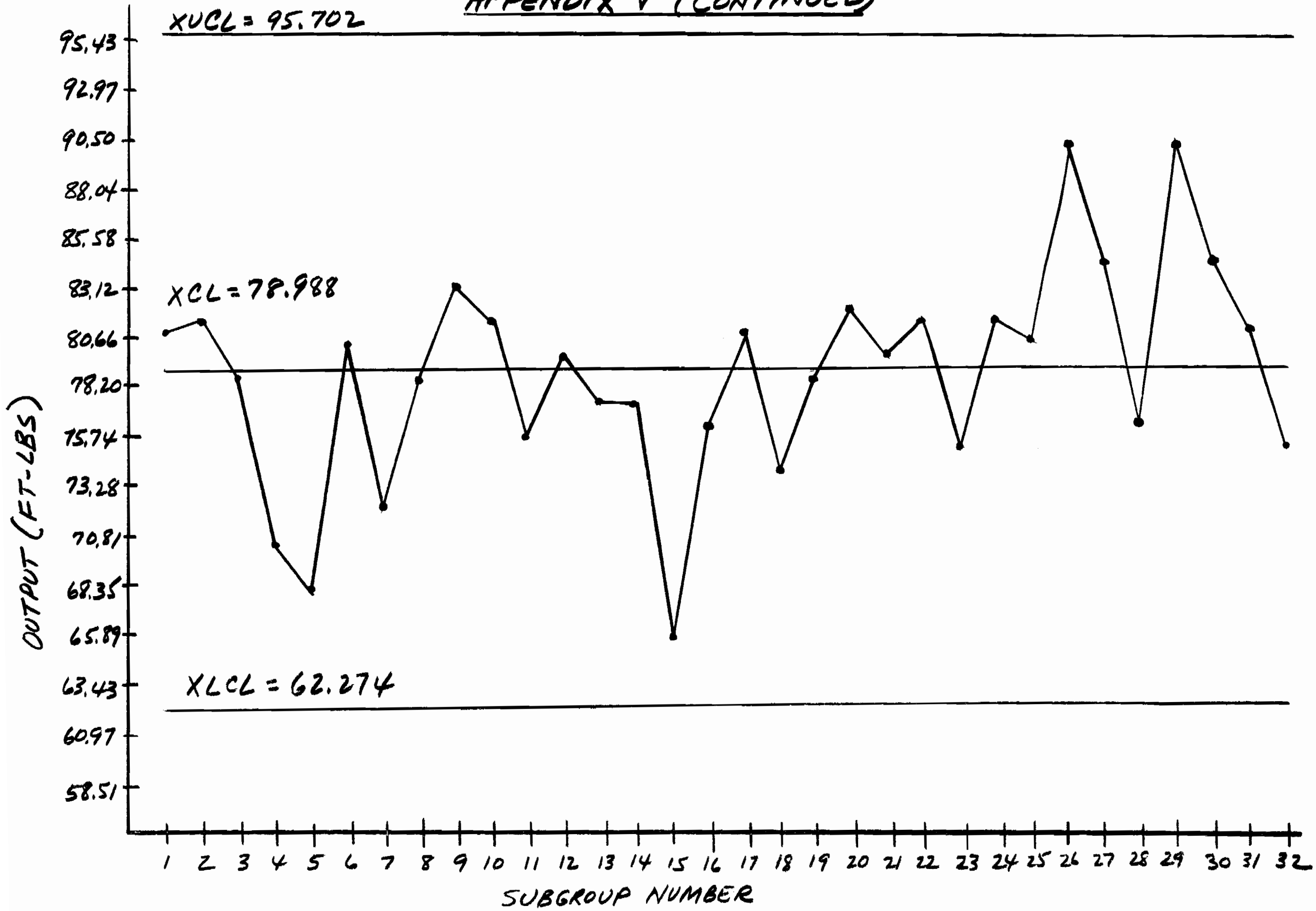


FIGURE V-7. $\bar{X}$ CHART FOR 160°F OUTPUT

APPENDIX V (CONTINUED)

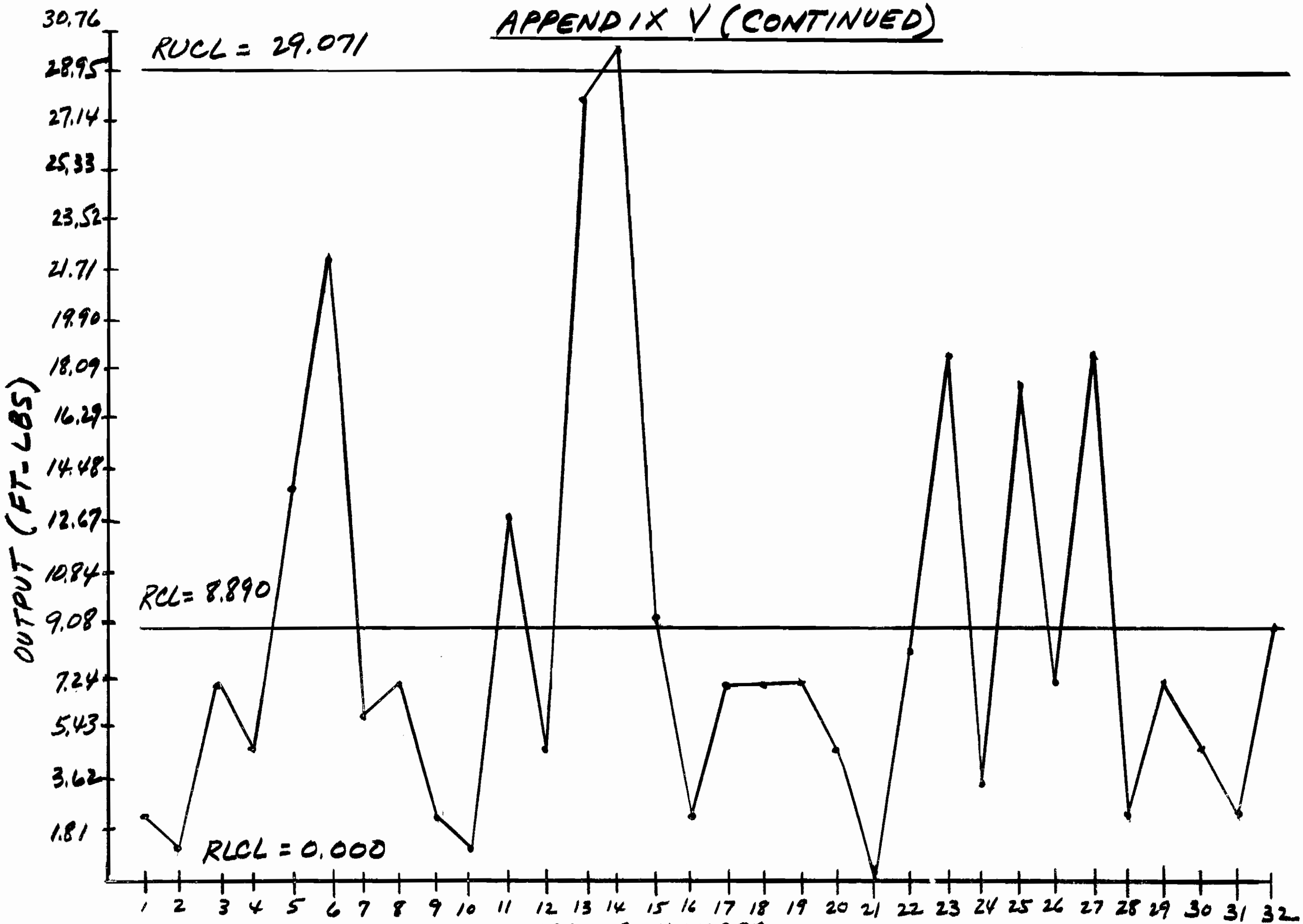


FIGURE V-8. R CHART FOR 160°F OUTPUT

APPENDIX VI

BARTLETT'S TEST

1. FORMULAS (reference (8)

H_o: $\sigma_1^2 = \sigma_2^2 = \ldots = \sigma_k^2$

When $n_1 = n_2 = \ldots = n_k = n$

B statistic becomes

$$B = (n-1)(\ln S^2 - \sum_{i=1}^{K} \ln S_i^2)/C$$

where

$$C = 1 + (K+1)/3K(n-1)$$

$$S^2 = \sum_{i=1}^{K} S_i^2/K$$

$$S_i^2 = \sum_{i=1}^{N} (X_i - \overline{X}_i)K(n-1)$$

n_i = number of values in each treatment

K = number of treatments

$$n = \sum_{i=1}^{K} n_i$$

When $n_1 \neq n_2 \neq \ldots \neq n_k$

B statistic becomes

$$B = M/C$$

where

$$M = V \ln S^2 - \sum V_i \ln S_i^2$$

APPENDIX VI (Continued)

$$V = \sum_{i=1}^{K} V_i$$

$$S^2 = \sum_{i=1}^{K} i\ S_i^2/V$$

$$C = 1 + (\sum \frac{1}{V_i} - \frac{1}{V})3(K-1)$$

$V_i = N_i - 1$

n_i = number of values in each treatment

K = number of treatments

$$S_i^2 = \sum_{i=1}^{N} (X_i - \overline{X}_i)/(n-1)$$

$$n = \sum_{i=1}^{K} n_i$$

2. APPLICATION OF BARTLETT'S TEST TO SQC DATA

Bartlett's Test was applied to the SQC data of Appendix IV. The treatments were morning production and afternoon production, where i=1 and i=2 respectively.

A. -65°F Delay Time

$n_1 = n_2$

$S_1^2 = .006316$ $\qquad \overline{X}_1 = 2.335$

$S_2^2 = .016769$ $\qquad \overline{X}_2 = 2.296$

$S^2 = (.006316 + .016769)/2 = .011543$

$\ln S_1^2 = -5.06468$

$\ln S_2^2 = -4.09414$

APPENDIX VI (Continued)

$\ln S^2 = 4.46194$

$M = 47[2(-4.46194) - (-9.15882)] = 11.04218$

$C = 1 + 3/6(47) = 1.0106$

$B = 11.04218/1.0106 = 10.9$

$\chi^2_{.05(1)} = 3.84$

$B > \chi^2_{.05(1)}$, therefore reject H_0.

B. -65°F Output

$n_1 \neq n_2$

$V = 47 + 46 = 93$ $\bar{X}_1 = 74.61$

$S_1^2 = 72.902$ $\bar{X}_2 = 76.37$

$S_2^2 = 83.836$

$S^2 = [72.902(47) + 83.836(46)]/93 = 78.31$

$\ln S_1^2 = 4.28910$

$\ln S_2^2 = 4.42892$

$\ln S^2 = 4.36069$

$M = 93(4.36069)-[47(4.28910) + 46(4.42892)] = .22615$

$C = 1 + \left(\frac{1}{47} + \frac{1}{46} - \frac{1}{93}\right)/3 = 1.01076$

$B = M/C = .22615/1.01076 = .223$

$\chi^2_{.05(1)} = 3.84$

$B < \chi^2_{.05(1)}$, therefore cannot reject H_0.

APPENDIX VI (Continued)

C. 160°F Delay Time

$n_1 \neq n_2$

$V = 31 + 30 = 61$

$S_1^2 = .006630$ $\qquad \bar{X}_1 = 1.983$

$S_2^2 = .011599$ $\qquad \bar{X}_2 = 2.018$

$S^2 = [.006630(31) + .011599(30)]/61 = .009074$

$\ln S_1^2 = -5.01617$

$\ln S_2^2 = -4.46541$

$\ln S^2 = -4.70233$

$M = 61(-4.70235) - [31(-5.01617) + 30(-4.46541)] = 2.62022$

$C = 1 + \left(\frac{1}{31} + \frac{1}{30} - \frac{1}{61}\right)/3 = 1.04925$

$B = 2.62022/1.04925 = 2.49$

$X^2_{.05}(1) = 3.84$

$B < X^2_{.05}(1)$, therefore cannot reject H_0.

D. 160°F Output

$n_1 = n_2$ $\qquad \bar{X}_1 = 78.40$

$S_1^2 = 76.569$ $\qquad \bar{X}_2 = 79.58$

$S_2^2 = 53.399$

$S^2 = (76.569 + 53.399)/2 = 64.984$

$\ln S_1^2 = 4.26914$

$\ln S_2^2 = 3.97777$

$\ln S^2 = 4.17409$

APPENDIX VI (Continued)

$M = 31[2(4.17409) - (4.26914 + 3.97777)] = 3.13937$

$C = 1 + 3/(3(2)(31)) = 1.01613$

$B = 3.13937/1.01613 = 3.09$

$X^2_{.05}(1) = 3.84$

$B < X^2_{.05}(1)$, therefore cannot reject H_0.

APPENDIX VII

ANALYSIS OF VARIANCE TESTS

1. FORMULAS (reference (8))

$H_0: \mu_1 = \mu_2 = \ldots = \mu_k$

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	Test Statistic
Among Means	ASS	K-1	S_2^2 = ASS/(K-1)	$F_c = S_2^2/S_p^2$
Within	WSS	n.-K	S_p^2 = WSS/(n.-K)	
Total	TSS	n.-1		

TSS (total sum of squares) =

$$\sum_{i}^{K} \sum_{j}^{n_i} X_{ij}^2 - T_{..}^2/n.$$

ASS (among means sum of squares) =

$$\sum_{i=1}^{K} (T_{i.}^2/n_i) - T_{..}^2/n.$$

WSS = TSS - ASS

where

K = number of treatments

n_i = number of values in each treatment

$$T_{..} = \sum_{i=1}^{K} \sum_{j=1}^{n_i} X_{ij}$$

$$T_i = \sum_{j=1}^{n_i} X_{ij}$$

APPENDIX VII (Continued)

$$n. = \sum_{i=1}^{K} n_i$$

2. APPLICATION OF ANALYSIS OF VARIANCE TO SQC DATA

The analysis of variance was conducted on the SQC data of Appendix IV. Treatments were morning production and afternoon production, where i-1 and i=2 respectively.

A. -65°F Output

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	Calculated Test Statistic
Among Means	73.4406	1	73.4406	.94
Within	7288.1830	93	78.3676	
Total	7361.6236	94		

X_1 = 3,581.42 X_2 = 3,589.46

$(X_1)^2$ = 12,826,569.2164 $(X_2)^2$ = 12,884,223.0916

X_{ij}^2 = 548,640.7812 N. = 95

T.. = 7,170.88

$(T..)^2$ = 51,421,519.9744

TSS = 548,640.7812 - 51,421,519.9744/95 = 7,361.6236

ASS = (12,826,569.2164/48 + 12,884,223.0916/47) -

51,421,519,9744/95 = 73.4406

WSS = 7,361.6236 - 73.4406 = 7288.1830

$F_{.05}(1,93)$ = 3.96

Calculated test statistic $<$ $F_{.05}(1,93)$, therefore cannot reject H_0.

APPENDIX VII (Continued)

B. 160°F Delay Time

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Squares	Calculated Test Statistic
Among Means	.01873	1	.01873	2.06
Within	.55350	61	.00909	
Total	.57223	62		

$X_1 = 63.463$ $X_2 = 62.549$

$(X_1)^2 = 4,027.55237$ $(X_2)^2 = 3,912.37740$

$X_{12}^2 = 252.62023$ $N. = 63$

$T.. = 126.012$

$(T..)^2 = 15,879.02414$

$TSS = 252.62023 - 15,879.02414/63 = .57223$

$ASS = 4,027.55237/32 + 3,912.37740/31 - 15,879.02414/63 =$

$.01873$

$WSS = .57223 - .01873 = .55350$

$F_{.05}(1,61) = 4.00$

Calculated test statistic $< F_{.05}(1,61)$, therefore cannot reject H_0.

APPENDIX VII (Continued)

C. 160°F Output

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Squares	Calculated Test Statistic
Among Means	22.1017	1	22.1017	.34
Within	4029.0163	62	64.9840	
Total	4051.1180	63		

$X_1 = 2{,}508.81$ $\qquad$ $X_2 = 2{,}546.42$

$(X_1)^2 = 6{,}294{,}127.6161$ $\qquad$ $(X_2)^2 = 6{,}484{,}254.8164$

$X_{12}^2 = 403{,}353.4673$

$T.. = 5{,}055.23$ $\qquad$ $N. = 64$

$(T..)^2 = 25{,}555{,}350.3529$

$TSS = 403{,}353.4673 - 25{,}555{,}350.3529/64 = 4051.1180$

$ASS = 6{,}294{,}127.6161/32 + 6{,}484{,}254.8164/32$

$\qquad - 25{,}555{,}350.3529/64 = 22.1017$

$WSS = 4051.1180 - 22.1017 = 4029.0163$

$F_{.05}(1,62) = 3.996$

Calculated Test statistic $< F_{.05}(1,62)$, therefore cannot reject H_o.

APPENDIX VIII

CALCULATIONS OF PROCESS NATURAL TOLERANCE LIMITS

1. -65°F DELAY TIME

From Appendix IV: $\sum \overline{X} = 74.087, \sum R = 4.727$, k = 32

Excluding SG #8 and 32: $\sum \overline{X} = 69.601, \sum R = 4.314$, k = 30

$$\overline{\overline{X}} = \frac{\overline{X}}{k} = \frac{69.601}{30} = 2.320$$

$$\overline{R} = \frac{R}{k} = \frac{4.314}{30} = .144$$

$\overline{X}LCL = 2.173$, $\overline{X}UCL = 2.467$ (means now in control)

RLCL = 0, RUCL = .370 (ranges now in control)

$$\sigma' = \frac{\overline{R}}{d_2}$$ (see Table B, reference (5))

$$\sigma' = \frac{.144}{1.693} = .0851$$

$$NTL = \overline{\overline{X}} \pm 3 = 2.320 \pm 3(.0851) = \begin{matrix} 2.575 \\ 2.065 \end{matrix}$$

2. -65°F OUTPUT

From Appendix IV: $\sum \overline{X} = 2411.60, \sum R = 420.20$, k = 32

Excluding SG #10 and 27: $\sum \overline{X} = 2273.73, \sum R = 352.03$, k = 30

$$\overline{\overline{X}} = \frac{2273.73}{30} = 75.79$$

$$\overline{R} = \frac{352.03}{30} = 11.73$$

$\overline{X}LCL = 63.83$, $\overline{X}UCL = 87.75$ (means now in control)

RLCL = 0, RUCL = 30.15 (ranges now in control)

APPENDIX VIII (Continued)

$$\sigma' = \frac{11.73}{1.693} = 6.93$$

$$NTL = 75.79 \pm 3\ (6.93) = \begin{matrix} 96.58 \\ 55.00 \end{matrix}$$

3. 160°F DELAY TIME

From Appendix IV: $\sum \overline{X} = 63.974$ (k = 32), $\sum R = 2.967$ (k = 31)

Excluding SG #8, $\sum \overline{X} = 61.817$ (k = 31), $\sum R = 2.549$ (k = 30)

$$\overline{\overline{X}} = \frac{61.817}{31} = 1.994$$

$$\overline{R} = \frac{2.549}{30} = .0850$$

$\overline{X}LCL = 1.834$, $\overline{X}UCL = 2.154$ (mean of SG #11 now out of control)

RLCL = 0, RUCL = .278 (range of SG #13 now out of control)

Excluding SG #11 and 13: $\sum \overline{X} = 57.730$ (k = 29), $\sum R = 2.263$ (k = 28)

$$\overline{\overline{X}} = \frac{57.730}{29} = 1.991$$

$$\overline{R} = \frac{2.263}{28} = .0808$$

$\overline{X}LCL = 1.839$, $\overline{X}UCL = 2.143$ (means now in control)

RLCL = 0, RUCL = .264 (Ranges now in control)

$$\sigma' = \frac{.0808}{1.128} = .0716$$

$$NTL = 1.991 \pm 3(.0716) = \begin{matrix} 2.206 \\ 1.776 \end{matrix}$$

4. 160°F OUTPUT

From Appendix IV: $\sum \overline{X} = 2527.58$, $\sum R = 284.49$, k = 32

Excluding SG #14: $\sum \overline{X} = 2450.03$, $\sum R = 251.59$, k = 31

$$\overline{\overline{X}} = \frac{2450.03}{31} = 79.03$$

APPENDIX VIII (Continued)

$\overline{R} = \frac{251.59}{31} = 8.12$

$\overline{X}LCL = 63.76$, $\overline{X}UCL = 94.30$ (means now in control)

RLCL = 0, RUCL = 26.55 (range of SG #32 now out of control)

Excluding SG #13: $\sum \overline{X} = 2372.46$, $\sum R = 223.39$, k = 30

$\overline{\overline{X}} = \frac{2372.46}{30} = 79.08$

$\overline{R} = \frac{223.39}{30} = 7.45$

$\overline{X}LCL = 65.08$, $\overline{X}UCL = 93.05$ (means now in control)

RLCL = 0, RUCL = 24.36 (ranges now in control)

$\sigma' = \frac{7.45}{1.128} = 6.60$

$NTL = 79.08 \pm 3(6.60) = \begin{matrix} 98.88 \\ 59.28 \end{matrix}$

APPENDIX IX

MULTIPLE LINEAR REGRESSION ANALYSIS

Formulas used in the analysis were in accordance with reference (8).

X_1 = minimum delay time recorded in -65°F tests

X_2 = delay time range of -65°F tests

Y = minimum delay time recorded in 160°F tests

$$Y = a + b_1X_1 + b_2X_2$$

$$a = \bar{Y} - \bar{X}_1 b_1 - \bar{X}_2 b_2$$

$$\bar{Y} = \frac{\sum Y_u}{n}, \quad \bar{X}_1 = \frac{\sum X_{1u}}{n}, \quad \bar{X}_2 = \frac{\sum X_{2u}}{n}$$

$$b_1 = \frac{\Delta_1}{\Delta}, \quad b_2 = \frac{\Delta_2}{\Delta}$$

$$\Delta_1 = \begin{vmatrix} g_1 & a_{12} \\ g_2 & a_{22} \end{vmatrix}, \quad \Delta_2 = \begin{vmatrix} a_{11} & g_1 \\ a_{12} & g_2 \end{vmatrix}, \quad \Delta = \begin{vmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{vmatrix}$$

$$g_i = \sum X_{iu} Y_u - \frac{(X_{iu})(Y_u)}{n}$$

$$a_{ii} = \sum X_{iu}^2 - \frac{(X_{iu})^2}{n}$$

$$a_{12} = X_{1u} X_{2u} - \frac{(X_{1u})(X_{2u})}{n}$$

Applying these formulas to Manufacturer B's MARK 5 delay cartridge production lot data:

APPENDIX IX (Continued)

Lot No.	X_1	X_2	Y	X_1^2
1	2.075	.175	1.775	4.305625
2	2.275	.300	1.925	5.175625
3	2.200	.250	1.850	4.840000
4	2.125	.075	1.800	4.515625
5	2.100	.050	1.750	4.410000
6	2.100	.225	1.825	4.410000
7	2.050	.200	1.675	4.202500
	X_{1u} = 14.925	X_{2u} = 1.275	Y_u = 12.6000	X_{1u}^2 = 31.859375

Lot No.	X_2^2	X_1X_2	X_1Y	X_2Y
1	.030625	.363125	3.683125	.310625
2	.090000	.682500	4.379375	.577500
3	.062500	.550000	4.070000	.462500
4	.005625	.159375	3.825000	.135000
5	.002500	.105000	3.675000	.08750
6	.050625	.472500	3.832500	.410625
7	.040000	.410000	3.433750	.335000
	X_{2u}^2 = .281875	X_1X_2 = 2.74250	X_1Y_u = 26.898750	X_2Y_u = 2.378750

APPENDIX IX (Continued)

$$a_{11} = 31.859375 - \frac{(14.925)^2}{7} = .037143$$

$$a_{12} = 2.74250 - \frac{(14.925)(1.275)}{7} = .02402$$

$$a_{22} = .281875 - \frac{(1.275)^2}{7} = .049643$$

$$g_1 = 26.898750 - \frac{(14.925)(12.60)}{7} = .03375$$

$$g_2 = 2.31875 - \frac{(1.275)(12.60)}{7} = .02375$$

$$\Delta_1 = \begin{vmatrix} .03375 & .02402 \\ .02375 & .049643 \end{vmatrix} = .00110497625$$

$$\Delta_2 = \begin{vmatrix} .037143 & .03375 \\ .02402 & .02375 \end{vmatrix} = .00007147125$$

$$\Delta = \begin{vmatrix} .037143 & .02402 \\ .02402 & .049643 \end{vmatrix} = .0012669295$$

$$b_1 = \frac{.00110497625}{.0012669295} = .8721686$$

$$b_2 = \frac{.00007147125}{.0012669295} = .00056412965$$

$$a = \frac{12.6}{7} - \frac{14.925}{7}(.8721686) - \frac{1.275}{7}(.00056412965) = -.059691$$

$$Y = -0.059691 + .872169\, X_1 + .000564\, X_2$$

To test the hypothesis

$$H_0: \beta_1 = \beta_2 = 0$$

the test statistic is

$$\frac{\frac{SS\ reg}{2}}{\frac{SS\ res}{4}} = \frac{\frac{.029449}{2}}{\frac{.00805}{4}} = 7.33$$

APPENDIX IX (Continued)

where SS reg = $g_i\ b_i$ = .029449

SS res = $(Y_u - \overline{Y})^2$ - SS reg = .00805

Since 7.33 $>$ $F_{.05}(2,4)$ of 6.94, H_o is rejected and a linear trend is concluded.

Abstract
of
a Thesis for a M. S. in I. E.
Entitled
APPLICATION OF CONTROL CHARTS TO SMALL LOT ACCEPTANCE

By
Bobby C. Layman

Small, infrequent production lots and destructive testing present a challenging application for statistical quality control techniques. A sampling scheme is devised which allows for process study and lot acceptance independently with the same samples. The scheme can be incorporated into existing acceptance procedures, providing variables data is utilized and the sample size is 75 or greater. Advantages, limitations, and trade-offs of the scheme are discussed. Multiple regression analysis is applied to the development of empirical mathematical models for predicting product performance at design temperature extremes to minimize cost of acceptance tests. The sampling scheme and the development of performance models are related to explosive power sources used in Naval aircraft systems.