# The Measurement and Evaluation of Performance of Urban Transit Systems: The Case of Bus Routes 

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#### Abstract

(ABSTRACT)

The issues of performance measurement and efficiency analyses for transit industries have been gaining significance due to severe operating conditions and financial constraints in which these transit agencies provide service.

In this research, we present an approach to measure the performance of Urban Transit Networks, specifically, bus routes that comprise the network. We propose a math programming model that evaluates the efficiencies of bus routes taking into consideration, the service providers, the users and the societal perspectives. This model is based on Data Envelopment Analysis (DEA) methodology and derives from Network Theory, Network Modeling in DEA, Goal Programming \& Goal-DEA and 'Environmental' Variables.

This approach enables the decision maker to determine the performance of its units of operations ('bus routes' in our case), optimally allocate scarce resources and achieve target levels for 'externality ${ }^{1}$, variables for these bus routes and for the whole network. We further recommend modifications to the model, for adaptation to other modes of transportation as well as extend its applicability to other applications/scenarios.


[^0]
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## Chapter 1

## Introduction

### 1.1 Motivation

The problem of measuring productive or operational performance has always been of concern in transportation, especially in case of Public Transit Agencies (Fielding (1985)). In most countries, transit services are provided by public, private or mixed ${ }^{2}$ companies. These transit companies provide services to passengers within cities and their suburbs through different modes such as buses, trams, trolleys, trains etc. Although these transit agencies are major contributors to any economy, they operate under severe operating constraints and it would seem reasonable to believe that this can be attributed to inefficient use of resources, reduced ridership and decreasing level of assistance from the government (Nolan (1996)).

Continuous evaluation of its own performance aids a transit agency to identify inefficiencies and their sources. Once these deficiencies are detected, then implementing changes for improvement or reengineering the process of provision of transportation services becomes the next step. Performance evaluation will enable an organization to reduce its dependence on government assistance due to efficient use of its resources. Thus it is imperative for these agencies to measure and monitor their performance.

Previously, productive performance of Public Transit Agencies has been analyzed and evaluated using (a) index number approaches (Parkan (1997)) (b) econometric approaches (Sheth (1999)). These approaches have not captured the provider-user perspectives ${ }^{3}$, neither have they addressed societal variables like accidents, noise pollution or air quality levels (societal perspective). Also, existing operating conditions such as transfer points on that route or residential or commercial complexes on and around that route or traffic congestion, impact of parking space or the accessibility of bus

[^1]stops are not addressed. These traffic operating conditions need to be considered as they may severely affect performance of transit agencies in certain area or zones. Besides, analyzing the productive performance of transit systems with respect to all three perspectives is preferable to partial evaluation which is based on single perspective.

### 1.2 Research Purpose

The ability to improve transit performance relies on the ability to measure it. There has been research in the past where performance of transit agencies has been evaluated and the transit agency is compared with similar transit agencies (Boile (2001)). The literature also refers to public transit research wherein DEA has been used as an efficiency/performance evaluation tool. In this research, the performance of transit systems is assessed addressing the issues discussed above (Section 1.1).

A bus transit network typically is comprised of individual bus routes. These transit networks affect the transportation of the public in the area they serve through these routes. This bus route establishes a link between the provider, the user and the society. The performance measurement is achieved by evaluating the efficiency and effectiveness of individual bus routes in providing the regional service.

This research effort differs from the work already done in the literature as below,
Transit agencies offering bus services have not been evaluated based on bus route being the unit of analyses

Transit agencies have not been evaluated from provider, user and the societal perspectives

Data Envelopment Analysis (DEA) has not been used to evaluate bus routes before

### 1.3 Research Objectives

The research has the following objectives.
a) To propose framework to measure performance of urban transit networks that are characterized by deterministic, incommensurate inputs (resources) and outputs (services), multiple goals (such as efficiency, local societal targets and global societal
targets) and multiple perspectives (such as provider, user and societal) This framework will also provide information to the service provider i.e. government authorities or transit agencies for decision-making with respect to the evaluation of technical efficiency, allocation of scarce resources and the achievement of targets for individual routes as well as the whole network with respect to societal variables such as accidents, noise pollution, emissions and resource degraded. These societal variables or externalities are, namely, noise pollution, safety, air quality and conservation of natural resources and concern the society in and around the transit network.
b) To generalize the proposed model to so that it is applicable to other modes of transportation as well as in intra-modal cases, where a route is characterized by different modes of transportation (bus route-train route) and also to other applications
c) To demonstrate the approach as well as the model by evaluating public transit agency that provides bus service using Data Envelopment Analysis (DEA)

### 1.4 Overview of the Research Methodology

In this research, performance evaluation is accomplished by using a combination of Data Envelopment Analysis (DEA), (Charnes, Cooper, Rhodes, (1978)), network theory, (Färe (1991)), goal programming, (Athanassopoulos (1995)), and multiple regression (Ruggiero (1998), Ray (1997)).

The DEA approach is based on mathematical programming, to deal with evaluation of efficiency of public transportation services. Data Envelopment Analysis (DEA) is an Operations Research approach that measures the performance of units consuming multiple inputs and producing multiple, incommensurate outputs. It has been widely used in the past either individually or with other techniques like regression, Tobit analysis, etc. to evaluate the productive performance in various areas of transportation such as Air transportation (Gillen and Lall (1998)), Bus operations (Boile (2001)) etc. It is based on a non-parametric, deterministic reference technology, which assesses the relative performance of units with similar goals or objectives.

Individual, isolated bus routes of an urban transit network are considered as Decision Making Units (DMU) and the proposed research evaluates the relative efficiency of these routes. The study considers both the provider and the passenger viewpoints. The provider provides service, which is consumed by the passenger. Thus, there are outputs from provider that form the inputs for the passenger or the user of the system. This is captured using the Network model in DEA as developed by Färe (1991).

The provider impacts the user as well as society when providing for service by generating societal impacts or externalities. In practice, acceptable target levels are set for these societal variables for each unit as well as for the whole system. It is intended to achieve these targets and this is accomplished by using Goal-Programming and Go-DEA as developed by Athanassopoulos (1995).

The provider delivers the transit service on these routes which differ from each other based on different traffic operating conditions. These factors greatly influence the efficiency score of the routes as well as determine the set of routes with which the route that is being assessed is compared with. The operating conditions are referred as environmental factors and are considered in the proposed model deriving from the approach outlined by Ruggiero (1998) and Ray (1997).

### 1.5 Organization of the Document

This chapter introduces the research undertaken, the objectives that the research aims to achieve, the motivation for research and an overview of general methodology adopted.

Chapter 2 reviews the literature, relevant to the research. It introduces the issues of importance when measuring the performance of Public Transit Networks. It presents concepts of efficiency, productivity and performance measurement with and without the use of Data Envelopment Analysis (DEA). It introduces Data Envelopment Analysis (DEA) as a multi-attribute efficiency measurement technique. It also explains Network theory, Goal Programming as used in DEA (Färe (1991), Athanassopoulos (1995)).

The models proposed are explained in Chapter 3 with notations. Chapter 4 discusses the datasets, the results and inferences obtained from the modeling approach as outlined in Chapter 3. Chapter 5 concludes and discusses future research.

## Chapter 2

## Literature Review

The literature in performance measurement of Public Transit Systems is reviewed in this chapter, specifically speaking the case of bus transit networks. This chapter, first, introduces Data Envelopment Analysis (DEA), which is used as a technique for the measurement of bus transit network performance. This chapter also presents some of the methods and approaches that have been used to evaluate efficiency in the field of transportation. It further focuses on conceptual models of Goal Programming and Network Theory within the DEA methodology.

This chapter is organized as follows. The first section, (Section 2.1), focuses on the definitions, concepts of efficiency and performance measurement. The second section, (Section 2.2), discusses the basic fundamental constructs of DEA. The third section, (Section 2.3), presents the issues of concern when measuring performance of Public Transit Systems, the approaches used, the motivation behind using them, their advantages and disadvantages. The fourth section, (Section 2.4), reviews the DEA research for transportation systems and agencies. Finally, the fifth section, (Section 2.5), discusses Goal Programming and Network models used within the DEA framework.

Thus the chapter is broadly divided as follows,
Performance Measurement Concepts (Section 2.1)
Data Envelopment Analysis (Section 2.2)
Public Transit System (Section 2.3)
DEA in Transportation Systems (Section 2.4)
Multiple Objective Modeling \& Goal Programming, Network Modeling in DEA (Section 2.5)
Following section presents some definitions and concepts that provide insights in the field of performance measurement.

### 2.1 Performance Measurement Concepts

There are different performance measurement methods and approaches in the literature. These methods may differ from each other depending upon the field they are applied to or on the basis of the metrics or the data (e.g. price data or input-output quantity data) they use or the approach they employ, or even on their basic underlying assumptions (e.g. cost minimization or profit maximization).

Performance is a broad term because it depends upon how the organization defines it. For some organizations, performance refers to profitability, for some it means reach, for others it may translate to customer service and satisfaction and yet, for others it can mean reputation and credibility. Any organization strives hard to improve its performance. This helps the organization to achieve its goals and objectives. The ability to improve performance lies in the ability to measure it. Performance is a relative concept, i.e., it can be measured for an organization over time, or within geographical areas, or as compared to similar private or public sector organizations, or it can be used to compare the sub-units within the organization as per Coelli et al. (1998). In an organization that converts inputs to outputs, efficiency or productivity is a commonly used measure for performance evaluation.
'Efficiency' is defined as the ratio of output to input.

$$
\begin{equation*}
\text { Efficiency }(.)=\frac{\text { output }}{\text { input }} \tag{2.1}
\end{equation*}
$$

'Productivity' is also defined on similar lines, as a ratio of output to input. The difference between the two, according to Charnes, Cooper, Rhodes (1978) is that 'Productivity' is generally used to evaluate employee performance (e.g., output per manhour, output per number of workers).

$$
\begin{equation*}
\text { Productivity }=\frac{\text { output }}{\text { input }(\text { Labor })} \tag{2.2}
\end{equation*}
$$

An organization may have more than one input and output. If the efficiency is not expressed as a ratio of all the outputs to all of the inputs, then there is a chance that gains attributable to a factor are imputed through some other factor. Such a measure is referred to as, 'Partial factor productivity'. Research in productivity and performance
measurement has led to the development of other measures that include all the input factors simultaneously. These inputs (and outputs) are combined to obtain a single output to input ratio. This measure is referred to as, 'Total factor productivity'. This single output to single input ratio can be obtained by weighing inputs and or outputs preferentially. But assigning weights to the inputs and outputs is itself an important challenge. It is quite possible that these inputs and outputs are incommensurate and then how to combine them again needs consideration.

The following sub-sections introduce concepts and terminology used when measuring performance.

### 2.1.1 Production Technology



Figure 2.1 Production Technology

A 'production technology' is defined as a set of all feasible inputs and outputs. For example, the set $(\bar{X}, \bar{Y})$ such that input vector represented by $\bar{X}$, is transformed to the output vector, represented by $\bar{Y}$, is a 'production technology' (Refer Figure 2.1). A 'production technology' can be associated with private or public sector firms producing goods where it is generally referred to as the production process. It can be applied to service industries like travel agencies or restaurants and even the non-profit organizations like schools or hospitals.

### 2.1.2 Production Function

In a production technology inputs get transformed to outputs. A 'production function' describes the technical relationship between the outputs and inputs of the technology. Coelli et al. (1998) suggests that a production function defines the maximum outputs attainable from a given vector of inputs and is represented by equation (2.3).

$$
\begin{equation*}
f(\bar{X})=\bar{Y} \tag{2.3}
\end{equation*}
$$

### 2.1.3 Returns to Scale



Figure 2.2 Returns To Scale
'Returns to Scale' reflects the degree to which there is a change in the outputs with change in inputs. 'Returns to Scale' can be constant or variable. If the change in inputs leads to a proportionate change in outputs then this is termed as 'Constant Returns to Scale' otherwise it is called 'Variable Returns to Scale' (See Figure 2.2). 'Variable Returns to Scale' is further classified as 'Increasing Returns to Scale' and 'Decreasing Returns to Scale'. If the output levels increase at different rate than that of inputs, then it is termed as 'Increasing Returns to Scale' whereas if the output levels decrease at a rate different than that of the input levels, then this is termed as 'Decreasing Returns to Scale'.

### 2.1.4 Concept of Technical Efficiency

The discussion in this section provides a brief introduction to modern efficiency measurement concept with regards to Data Envelopment Analysis.

Generally speaking, efficiency is defined as a ratio of outputs to inputs.

$$
\begin{equation*}
\text { Efficiency (. ) }=\frac{\text { output }}{\text { input }} \tag{2.4}
\end{equation*}
$$

With regards to Data Envelopment Analysis, efficiency is referred as Overall Technical Efficiency or Total Economic Efficiency. Kerstens and Vanden Eeckaut (1998) propose that this Overall Technical Efficiency can be expressed as:

$$
\begin{equation*}
O T E=(T E) \cdot(S T E) \cdot(S C E) \cdot(A E) \tag{2.5}
\end{equation*}
$$

-Overall Technical Efficiency (OTE): Represents the Overall Technical Efficiency of the observation
-Technical Efficiency (TE): An observation is technically inefficient if it lies in the interior of the Production Possibility set.
-Structural Efficiency (STE): A technically efficient observation is structurally inefficient if some inputs yield negative marginal products. An observation is structurally inefficient when the inputs (outputs) are increased (decreased), then the output levels (input levels) decrease (increase).
-Scale Efficiency (SCE): SCE denotes the extent to which a DMU is or is not scale efficient. An observation is scale efficient if the scale of production remains same over different 'production technology' (see 2.1.2).
-Allocative efficiency (AE) is efficiency of attaining a specified goal, for example cost minimization. Consider an observation that is technically efficient. This observation (DMU) will lie on a face of the production frontier. If this facet of the frontier is parallel to one of the axes that represents the inputs (outputs) then it quite possible that there is an input-output mix for this observation (DMU) such that it is technically efficient and mix efficient too, in the sense that it is has
minimum cost among other such points on that face of the frontier. This efficiency is mix or allocative efficiency.

Although all these efficiencies form a part of the Overall Technical Efficiency (OTE), in this research, technical efficiency is considered.

The theoretical concept of 'technical efficiency' has existed in economic literature since Debreu (1951) and Koopmans (1951). 'Technical efficiency' was defined as a feasible input/output vector where it is technologically impossible to increase any output (and/or reduce any input) without simultaneously reducing another output (and/or increasing any other input) (Koopmans (1951)).

Farrell (1957) proposed that the efficiency of a firm consists of two components: 'technical efficiency, which reflects the ability of a firm to obtain maximal output from a given set of inputs, and 'allocative efficiency' or 'mix efficiency', which reflects the ability of a firm to use the inputs in optimal (cost minimization) proportions, given the production technology and the input prices. These two measures then in combination provide a measure of 'total economic efficiency'.

In case of frontier reference technologies ${ }^{4}$ like DEA, technical efficiency measurement involves comparing input consumption or output production of an observation (DMU) with respect to the corresponding observation (DMU) or a virtual DMU on the frontier (Fried et al. (1993), Färe et al. (1994), Charnes et al. (1994)). The next section introduces Data Envelopment Analysis (DEA) as a methodology used for the performance measurement of transit systems.

### 2.2. Data Envelopment Analysis (DEA)

Performance measurement approaches in the literature can be broadly classified into three categories (a) index number approaches ((Parkan (1997) e.g. partial factor productivity indices, total factor productivity indices, financial and operational ratios etc.), (b) econometric approaches (e.g. the statistical estimation of Cobb-Douglas, Translog, Leontief type functions etc.) (Sheth (1999)), (c) mathematical-programming approaches (e.g. Data Envelopment Analysis (DEA)). These performance measurement

[^2]approaches involve comparisons over time or comparisons can be within geographical areas or comparisons of sub units within the organization or with other organizations as a whole.

This research effort uses DEA for performance measurement because of its ability to address qualitative benefits and subjective data such as 'happiness', 'pleasantness' etc., and issues like level of service, customer satisfaction and multiple, incommensurate inputs and outputs. Additionally, DEA is based on mathematical programming. It can handle a large number of variables and constraints. Moreover the extensive literature from mathematical programming can be used to guide analyses and interpretation.

Data Envelopment Analysis (DEA) was conceptualized by Charnes Cooper and Rhodes (1978) as a procedure for measuring the relative efficiency of a set of public and/or non-profit organizations. It is a relatively new area in the field of operations research and management science. DEA as defined by Giokas (1997) is as follows:
"DEA measures relative efficiency (of DMUs) by estimating an empirical production function, which represents the highest values of outputs/benefits that could be generated by inputs/resources as given by a range of observed input/output measures during a common time period."

In DEA, the organization under study is called a 'Decision Making Unit' (DMU). Thus a DMU is an 'entity that converts inputs to outputs and whose performance is to be evaluated' (Charnes et al. (2000)). Hence the definition of DMU depends upon the application. For example, in managerial applications, a DMU can be a bank, departmental store, in engineering applications; it can be a manufacturing facility or a machine and so forth. DEA can be used to compare a group of public as well as private sector organizational units. While econometric methods like regression analysis employ "average observations", DEA uses the "reference frontier technology" or "best practice observation" (Charnes et al. (1994)). Thus there is a need to establish a standard or a benchmark. This standard represents the level of efficiency that is achieved with (a) the least amount of inputs with a constant output level (b) or the maximum output level with a constant input level.

DMUs with minimum input levels for constant output levels or with maximum output levels for the constant input levels in the group describe the best practice
technology and are termed as efficient DMUs. The less efficient ones are compared with the best practice to determine the sources of their inefficiency, i.e., the amount of input quantities to be reduced and/or the output quantities to increase.

The efficiency of a DMU is computed as the ratio of the aggregate output to the aggregate input. The aggregate output value and the aggregate input value is obtained by weighing the outputs and inputs respectively. These weights are determined directly from the data. These weights are variable and represent the best set of weights for the DMU such that the efficiency of the DMU is maximized relative to other DMUs.

As DEA is based on the concept of best practice, in a plot of observations (DMUs), the locus of all the efficient observations is called the 'production frontier'. It is obtained by joining all the DMUs that produce maximum output levels with minimum input levels. All the inefficient observations lie on one side of the frontier. The side of the frontier depends on whether one considers an input orientation or an output orientation (See Figure 2.3). As the production frontier envelops the other data points, this analysis is called Data Envelopment Analysis (DEA). The set of all the feasible input-output combinations i.e. the efficient ones on the frontier as well as the inefficient ones, form the area that indicates possibility of production and is called 'Production Possibility Set $(P)$ '.


Figure 2.3 Input and Output Orientation

The efficiency of a DMU can be improved either by increasing its output levels, keeping the input levels fixed or by reducing the input levels and keeping the output levels fixed or by increasing the output levels and reducing the input levels simultaneously. When the output levels are increased, keeping the input levels constant, then the efficiency measure is called 'output-oriented efficiency measure'. The observations on the frontier or the technically efficient observations are the ones that have higher output levels. The relative efficiency of inefficient DMUs is evaluated with respect to these observations. Similarly, when the input levels are reduced, keeping the output levels constant, then the efficiency measure is called 'input-oriented efficiency measure'. The observations on the frontier or the technically efficient observations are the ones that have minimum input levels. The relative efficiency of inefficient DMUs is evaluated with respect to these observations.


Figure 2.4 Input-Output Measures

For DMU 'A'
Input oriented efficiency measure $=\frac{O P}{O A} \ldots \ldots \ldots \ldots \ldots \ldots$ from Figure 2.4
Output oriented efficiency measure $=\frac{O A}{O P}$

Another issue that needs to be considered in case of Congestion ${ }^{5}$ or in case of undesirable outputs or while measuring mixed or allocative efficiency is of Disposability of inputs or outputs. Disposability of inputs and outputs can either be strong or weak. 'Strong disposability of inputs' suggests that whenever any input is increased (or decreased) with other input levels constant then it increases (or decreases) the output levels. Similarly strong disposability of outputs suggests that whenever an output level is decreased (or increased), the input level decreases (or increases). On the other hand, 'weak disposability of output' indicates that when an output level is decreased, then input level may increase. Similarly weak disposability of input suggests that when input levels are increased then output level may decrease (See Figure 2.5).


Figure 2.5 Disposability

In the following sub-sections basic DEA models like CCR, BCC are presented. It also provides their primal and dual formulations, their significance and the information they provide while evaluating efficiencies.

### 2.2.1 The Charnes, Cooper, Rhodes (1978) Model (CCR model - Input Orientation)

The CCR-DEA model was originally proposed by Charnes, Cooper and Rhodes in 1978 and hence this model is called the CCR model. This model assumes Constant

[^3]Returns to Scale. The CCR model is an extension of Farrell (1957) work on technical efficiency.

Mathematically the CCR model can be expressed as follows. Consider a set of DMUs from 1 to N , with any DMU ' j ' having a production technology ( $\mathrm{X}_{\mathrm{j}}, \mathrm{Y}_{\mathrm{j}}$ ). Let $\mathrm{X}_{\mathrm{j}}$ represent the set of ' $m$ ' inputs for $D M U$ ' $j$ ', i.e., $X_{j}=\left(x_{1 j}, x_{2 j}, \ldots, x_{m j}\right)$ and $Y_{j}$ represent the set of ' $s$ ' outputs for DMU ' $\mathfrak{j}$ ', i.e., $Y_{j}=\left(y_{1 j}, y_{2 j}, \ldots . y_{s j}\right)$. Let $U=\left(u_{1}, u_{2}, \ldots ., u_{s}\right)$ be the weight vector for the output set and $V=\left(v_{1}, v_{2}, \ldots, v_{m}\right)$ be the set of input weights.

The CCR model is represented as:
M1 (Fractional Program)
$\operatorname{Max} \frac{u_{1} y_{1 o}+u_{2} y_{2 o}+\cdots+u_{1} y_{s o}}{v_{1} y_{1 o}+v_{2} x_{2 o}+\cdots+v_{1} x_{m o}}$
Subject to

$$
\begin{align*}
& \frac{u_{1} y_{1 j}+u_{2} y_{2 j}+\cdots+u_{1} y_{s j}}{v_{1} x_{1 j}+v_{2} x_{2 j}+\cdots+v_{1} x_{m j}}=1(j=1 \cdots N)  \tag{2.9}\\
& v_{1}, v_{2,} \cdots v_{m}=0  \tag{2.10}\\
& u_{1}, u_{2,} \cdots u_{s}=0
\end{align*}
$$

The above model is a fractional linear program for DMU ' $\mathrm{j}_{0}$ '. Equation (2.8) represents the efficiency score as a weighted sum of the outputs to the weighted sum of the inputs. Constraint (2.9) indicates the selection of 'best' weights for DMU ' $\mathrm{j}_{\mathrm{o}}$ ' such that with these weights the efficiency score for all other DMUs remains feasible (less than or equal to 1 ). Constraint (2.10) indicates that these weights should be non-negative. This formulation remains the same for all DMUs.

The above fractional program can be converted to a linear program. The linear CCR model is as follows:

M2
$\operatorname{Max} u_{1} y_{1 o}+u_{2} y_{2 o}+\cdots+u_{1} y_{s o}$
Subject to

$$
\begin{align*}
& v_{1} y_{1 o}+v_{2} x_{2 o}+\cdots+v_{1} x_{m o}=1  \tag{2.12}\\
& u_{1} y_{1 j}+u_{2} y_{2 j}+\cdots+u_{1} y_{s j}=v_{1} y_{1 j}+v_{2} x_{2 j}+\cdots+v_{1} x_{m j} \quad(j=1 \cdots N)  \tag{2.13}\\
& v_{1}, v_{2}, \cdots v_{m}=0 \\
& u_{1}, u_{2}, \cdots u_{s}=0 \tag{2.14}
\end{align*}
$$

This model represents the primal form. It is equivalent to the fractional program (Cooper, Seiford, Tone (1995)). This model is solved for each DMU from $\mathrm{j}=1 \ldots \mathrm{~N}$ and the relative efficiency is computed. The DMUs on the frontier have ' 1 ' as their efficiency score indicating that these units are technically efficient and the inefficient units have the efficiency score less that 1 (input oriented case). In output oriented model, the inefficient units have an efficiency score greater than 1 but the efficient units have ' 1 ' as their score.

Generally the number of DMUs is greater than the number of inputs and outputs. In the dual representation of the linear CCR model, the inputs and outputs would form the constraints (Cooper, Seiford, Tone (1995)). For computational convenience and better inference, the dual representation is generally preferred and evaluated. This dual representation of the CCR model is as follows.

M3
Min.
Subject to

$$
\begin{array}{ll}
\cdot{ }_{c} x_{i o}-\sum_{j=1}^{J} \cdot{ }_{j} x_{i j}=0 & i=(1 \ldots m) \\
\sum_{j=1}^{J} \cdot{ }_{j} y_{r j}=y_{r o} & r=(1 \ldots s) \\
\cdot{ }_{j}=0 & j=(1 \ldots J) \tag{2.18}
\end{array}
$$

where
$\cdot{ }_{c}$ : is a real variable $\left(0<{ }_{c}<1\right)$ and represents the radial (equi-proportionate change) input efficiency measure
$\cdot_{j}$ : represents the contribution of efficient DMU ' $\mathfrak{j}$ ' towards virtual input or output.
$\mathrm{x}_{\mathrm{ij}}$ : the $\mathrm{i}^{\text {th }}$ input of $\mathrm{j}^{\text {th }}$ DMU
$y_{\mathrm{r} j}$ : the $\mathrm{r}^{\text {th }}$ output of $\mathrm{j}^{\text {th }}$ DMU
Equation (2.15) represents the minimum value the inputs of the DMU ' $\mathrm{j}_{\mathrm{o}}$ ' can be reduced to. Constraints (2.16) and (2.17) represent the set of efficient DMUs, which contribute towards the virtual input and output of the DMU ' $\mathrm{j}_{\mathrm{o}}$ ' respectively. (2.18) suggests that contribution of efficient DMUs should be nonnegative.

The input excesses and output shortfalls can be determined by solving the following model (M4) after obtaining the value of '. c' from M3

M4
Max. $=\left(\sum_{i=1}^{m} s_{i}^{-}+\sum_{i=1}^{s} s_{r}^{+}\right)$
Subject to

$$
\begin{array}{ll}
\cdot{ }_{c} x_{i o}-\sum_{j=1}^{J} \cdot{ }_{j} x_{i j}+s_{i}^{-}=0 & i=(1 \ldots m) \\
\sum_{j=1}^{J} \cdot{ }_{j} y_{r j}-s_{r}^{+}=y_{r o} & r=(1 \ldots s) \\
\cdot{ }_{j}=0, s_{i}{ }^{-}=0, s_{r}{ }^{+}=0, \cdot{ }_{c}=0 & \cdot j, i, r \tag{2.21}
\end{array}
$$

where

$$
\begin{aligned}
& s_{i}^{-} \cdot R^{m}=\text { input excess for input ' } \mathrm{i} \text { ' } \\
& s_{r}^{+} \cdot R^{n}=\text { output shortfall for output ' } \mathrm{r} \text { ' }
\end{aligned}
$$

The CCR model, combining formulations (M3) and (M4) is expressed as
M5
Min. $-\varepsilon\left(\sum_{i=1}^{m} s_{i}^{-}+\sum_{i=1}^{s} s_{r}^{+}\right)$
Subject to

$$
\begin{array}{ll}
\cdot{ }_{c} x_{i o}-\sum_{j=1}^{J} \cdot{ }_{j} x_{i j}+s_{i}^{-}=0 & i=(1 \ldots m) \\
\sum_{j=1}^{J} \cdot{ }_{j} y_{r j}-s_{r}^{+}=y_{r o} & r=(1 \ldots s) \\
\cdot{ }_{j}=0, s_{i}^{-}=0, s_{r}^{+}=0, \cdot{ }_{c}=0 & \cdot j, i, r \tag{2.25}
\end{array}
$$

where
$s_{i}{ }^{-} . R^{m}=$ input excess for input ' i '
$s_{r}{ }^{+} . R^{n}=$ output shortfall for output ' $r$ '
.${ }_{c}$ : is a real variable $\left(0<.{ }_{c}<1\right)$ and represents the radial input efficiency measure
. : represents the contribution of efficient DMU ' j ' towards virtual input or output.
$\varepsilon$ : small non Archimedean number

### 2.2.2 The Banker, Charnes, Cooper Model (1984) (BCC model - Input Orientation)

The CCR model is based on the Constant Returns to Scale assumption. When this assumption is relaxed it leads to Variable Returns to Scale characterizations with (a) increasing returns to scale, (b) decreasing returns to scale and (c) constant returns to scale. This model is called BCC model. It was developed by Banker, Charnes and Cooper (1984) to accommodate different scale of operations of the DMUs.

The convex hull of the efficient DMUs forms the production frontier in BCC model. It has piecewise linear and concave characteristics (Cooper et al. (2000)).

Adopting the same notations as in the CCR model (Section (2.2.1)), the following linear program represents the envelopment form of the input-oriented BCC model.

M6
Min ${ }_{B}$

## Subject to

$$
\begin{array}{ll}
\cdot{ }_{B} x_{i o}-\sum_{j=1}^{J} \cdot{ }_{j} x_{i j}=0 & i=(1 \ldots m) \\
\sum_{j=1}^{J} \cdot{ }_{j} y_{r j}=y_{r o} & r=(1 \ldots s) \\
\sum_{j=1}^{J} \cdot{ }_{j}=1 & \\
\cdot{ }_{j}=0 & j=(1 \ldots J) \tag{2.30}
\end{array}
$$

where

- ${ }_{B}$ : is a real variable $\left(0<{ }_{B}<1\right)$ and represents the radial input efficiency measure . ${ }_{j}$ : represents the contribution of efficient DMU ' j ' towards virtual input or output.
$\mathrm{x}_{\mathrm{ij}}:$ : $^{\text {th }}$ input of $\mathrm{j}^{\text {th }}$ DMU
$\mathrm{y}_{\mathrm{r}}: \mathrm{r}^{\text {th }}$ output of $\mathrm{j}^{\text {th }}$ DMU

The above equations are similar to that in the CCR model except for (2.29). This constraint is the Variable Returns to Scale constraint. This constraint normalizes the DMUs so that different scale of operations of the DMUs can be accounted for.

The input excesses and output shortfalls can be determined by solving the following model (M7) after obtaining the value of ${ }^{‘} .{ }_{B}{ }^{\prime}$ from M6

M7
$\operatorname{Max} .=\left(\sum_{i=1}^{m} s_{i}^{-}+\sum_{i=1}^{s} s_{r}^{+}\right)$
Subject to

$$
\begin{equation*}
\cdot{ }_{B} x_{i o}-\sum_{j=1}^{J} \cdot{ }_{j} x_{i j}+s_{i}^{-}=0 \quad i=(1 \ldots m) \tag{2.32}
\end{equation*}
$$

$$
\begin{array}{ll}
\sum_{j=1}^{J} \cdot{ }_{j} y_{r j}-s_{r}^{+}=y_{r o} & r=(1 \ldots s) \\
\sum_{j=1}^{J} \cdot{ }_{j}=1 & \\
\cdot_{j}=0, s_{i}^{-}=0, s_{r}^{+}=0,{ }_{B}=0 & . j, i, r \tag{2.35}
\end{array}
$$

where
$s_{i}{ }^{-} . R^{m}=$ input excess for input ' i '
$s_{r}{ }^{+} . R^{n}=$ output shortfall for output ' $r$ '

The BCC model, combining formulations (M6) and (M7) is expressed as M8
$\operatorname{Min} .{ }_{B}-\varepsilon\left(\sum_{i=1}^{m} s_{i}^{-}+\sum_{i=1}^{s} s_{r}^{+}\right)$
Subject to

$$
\begin{array}{ll}
{ }_{B} x_{i o}-\sum_{j=1}^{J} \cdot{ }_{j} x_{i j}+s_{i}^{-}=0 & i=(1 \ldots m) \\
\sum_{j=1}^{J} \cdot{ }_{j} y_{r j}-s_{r}^{+}=y_{r o} & r=(1 \ldots s) \\
\sum_{j=1}^{J} \cdot{ }_{j}=1 & \\
\cdot{ }_{j}=0, s_{i}{ }^{-}=0, s_{r}^{+}=0, .{ }_{c}=0 & \cdot j, i, r \tag{2.40}
\end{array}
$$

where
$s_{i}{ }^{-} . R^{m}=$ input excess for input ' i '
$s_{r}{ }^{+} . R^{n}=$ output shortfall for output ' $r$ '
. ${ }_{B}$ : is a real variable $\left(0<,{ }_{\mathrm{B}}<1\right)$ and represents the radial input efficiency measure
. : represents the contribution of efficient DMU ' j ' towards virtual input or output.
$\varepsilon$ : small non Archimedean number
The CCR and the BCC models are the basic DEA models. There are other models like the Additive model, the Multiplicative model, the Free Disposal Hull (Cooper,

Seiford, Tone (1995)) Go-DEA (Athanassopoulos (1995)), and the Network model (Färe (1991)) that have been developed to address different issues like improving slacks and surpluses, target setting, intermediate products and so on. Many a times the original DEA formulations are modified to include non-discretionary variables (inputs/outputs) (Banker, Morey (1986), Ruggiero (1998)) or categorical variables (Banker, Morey (1986)) or environmental variables (Ruggiero (1986)). The compatibility of DEA with other disciplines and its flexibility to be applied in different scenarios makes it, an appropriate, accurate and many a times, the only approach for performance measurement.

The next section reviews literature on the development of performance measures for Public Transit Systems.

### 2.3 Public Transit Systems

The federal and state governments provide transit systems for public transportation in urban areas, directly or through agencies. The modes of transportation vary from buses, trolley, trams, trains etc. The authorities spend billions of dollars on the construction and maintenance of these transit systems and networks. Besides controlling congestion, noise pollution, air quality levels and restoration of other natural resources within the urban area, the government, through effective and efficient transit services, seeks to promote smooth movement of traffic and optimal utilization of space.

Transit systems of most urban area operate under severe financial constraints in spite of being an important part of the economy. The revenue generated through the users is insufficient and the transit service providers need assistance from the government. In order for these transit agencies to reduce dependencies on government assistance, they need to monitor their performances. To achieve this extensive and detailed productivity studies need to be carried out. This has aided identification of a performance metrics. This performance metric enables performance evaluation and further analytic studies on productivity, efficiency and quality of services of transportation systems.

In the field of transportation, there has been research in productivity and performance measurement but this has been limited capital and labor investment. As productivity studies have been common in case of manufacturing as well as service sectors, the approaches and inferences can be derived from them. However there are
several aspects that make performance measurement in case of urban transportation systems different (Tomazinis (1975)). Some of them are as follows:
(a) The transit systems are generally multi-modal and integrated in functional services
(b) These services are monopolistic.
(c) They are obligatory for the system as well as the users. This is because of the system's inability to withdraw its service. In case of users; they have a limited option of services to choose from.
(d) Since the output in this case is service, the level of quality of service also needs to be addressed.
'Speed' or 'Vehicle Operating Speed' has mainly been the performance criterion used in the literature. This indicates the speed of the service vehicle and hence the trips made or the number of passengers transported. Although this is an important parameter when measuring transit performance, it should not be the only criterion (Ewing (1995)). "Speed", as the only criterion is insufficient because the objective of transit agencies is changing from providing service to improving the quality of service provided. Thus there is a need to include issues of customer satisfaction and growth management, i.e. energy conservation, neighborhood protection and downtown revitalization. Level-of-service measures can be used in conjunction with speed to measure performance for this purpose, but these level-of-service measures should be carefully developed as otherwise they act as proxies of speed (Ewing (1995)). Pratt and Lomax (1996) propose that the performance measures should also be functions of time-based measures along with level of service measures in order to represent the full-range of passenger needs.

Ewing (1995) also suggests that performance measures should address the following four notions,

Mobility: It is defined as the ease with which individuals can move about. It depends upon speed, auto ownership, parking availability, transit route density \& sidewalk connectivity. It leads to freely moving the population, moderating costs of travel, and providing numerous travel options and fuel-efficient traffic.

Accessibility: It is defined as how convenient is it to get to different points in the area under consideration. It leads to fuel-efficient land use.

Livability focuses on calmed traffic (with respect to noise, volume \& speed) and aesthetics of the area.

Sustainability deals with the ability to meet the needs of the present without affecting the means to achieve the needs of the future in terms of environmental pollution and fuel consumption.
Pratt and Lomax (1996) suggest that measurement of transportation performance generally revolves around the provision of (a) adequate and (b) quality transportation services where, (a) Adequacy is readily described by congestion measures of (i) mobility and (ii) accessibility. (i) Mobility is the ability to be mobile, i.e., to travel freely. It is a strong indicator of efficiency, in the sense that it measures volume and speed of transporting goods and people to their destinations. (ii) Accessibility is the ability to access a travel area or a zone within a reasonable time. Accessibility measures the absolute time of travel and can be used for the acceptable congestion measure.
(b) Quality of transportation service is considered as a complement of congestion, and hence it can be described by quantifying congestion. Hence congestion can be used to measure the transportation performance and the effects of congestion namely travel time, rate, delays can also be used for analysis (Pratt and Lomax (1996)).

Codd et al., (1996) suggests that performance measures should address criteria like mobility, connectivity, accessibility, cost-effectiveness, energy efficiency, air quality, natural resources impacts, noise, safety, neighborhood impact employment effects and economic impact. They also state that designing of a set of appropriate performance measures requires identifying (a) the underlying purpose, (b) the use of these measures, and (c) the knowledge base of the audience who will be using this information. Some features that need to be considered while developing the performance measures are as follows,

As the outcomes of efficiency and effectiveness analysis depend upon the developed performance measures, these performance indicators should have a strong correlation to the goals and the objectives of the study.
The performance measurement framework should include all points of view, i.e., the user, the government/provider of the service, operator and society (Tomazinis (1975)).

The performance measurement framework should be broad enough to be applicable to different modes of transportation.

They should represent different states of the system correctly.
They should be sensitive to changes and improvements done to the system and should communicate the desired information to the user.

Performance measures should account for transfers and connections points.
In transit systems, performance can be measured by quantifying varied goals like smooth vehicular traffic flow, satisfaction of reasonable access to jobs, goods, services, achieving maximum throughput. Ewing (1986) suggests two performance measures Vehicle Miles Traveled (VMT), Vehicle Hours Traveled (VHT). Both VHT and VMT account for accessibility, mobility, sustainability and livability, but VHT accounts for congestion as well and is better than VMT for measures related to accessibility and sustainability, as it is time based measure.

Pratt and Lomax (1996) suggest the following variables to be used for performance measurement,
a) Travel time: time required to traverse to the destination
b) Desired travel time: the expected time to reach the destination
c) Trip length: the distance associated with the trip
d) Avg. travel speed: Speed of travel, averaged over a relevant parameter
e) Actual travel rate: reciprocal of the travel time
f) Desired travel rate: reciprocal of the desired travel time
g) Vehicle volume: the seating and standing capacity of the vehicle
h) Actual volume/ person volume: actual average number of passengers
i) Avg. waiting time window: the waiting time averaged over the network
j) Headway: the time difference between two consecutive service vehicles
k) Service coverage: The duration of the availability of the service

1) Service frequency: the reciprocal of Headway
m) Delay rate: time lost due to congestion per mile or per passenger mile
n) Total delay: Delay for the entire route
o) Signals and other such unavoidable interruptions:

### 2.3.1 Analytical Approaches for Performance Measurement in Transit Systems

In the recent years the paradigm in transit systems has moved from provision of service to improving the quality of service, profit making and competitiveness. This is because the government has sought to privatize transportation services in urban areas in order to control costs and reduce the need for public assistance. As public transit agencies are offering their services under severe operating conditions characterized by the decrease in financial assistance from the government and reduced rider ship, it is necessary that for reliable operations, these agencies use their resources efficiently. Thus it has become imperative for public transit organizations to rationalize the operating costs and to improve the quality of offered services. This can be brought about by benchmarking, shedding of unproductive services and optimizing the allocation of scarce resources.

This has led to the modification of overall strategies and the revision of business policies. In order to improve performance, it is necessary to measure it. In transportation, productivity has been used as a measure of performance. It has been mainly associated with labor and capital. This conventional definition of productivity is incomplete and cannot be used for performance measurement (Carotenuto et al. (2001)). Performance measurement in the transportation field, previously, has been approached through different techniques some of which are presented below.

Fielding et al., (1985) combine the performance measures from Section 15 database (1980) into indicators of system efficiency ${ }^{6}$ and indicators of system effectiveness ${ }^{7}$ and use factor analysis to identify the factors affecting the performance of bus transit.

Polus and Tomecki (1986) present a cost/benefit approach for evaluating transportation systems. All the benefits are evaluated in monetary terms and then compared to the cost of the project and the outcome is a single ratio. This approach has a disadvantage when the benefits are subjective and cannot be expressed in monetary

[^4]terms. Hence it becomes a challenge to express all the benefits or measure of effectiveness in terms of costs.

Obeng and Addus (1986) introduce the concept of total factor productivity in public transit. This can be used as a unified measure of transit performance as the analysis then includes all the factors that affect the performance of public transit systems. This approach is parametric and hence needs other tools to identify the parameter of the production function. They use a three stage least squares approach to estimate parameters of the total factor productivity model.

Gathon (1989) evaluates the efficiency of urban transit firms based on the two models, Ordinary Least Squares and the Free Disposal Hull. The efficiency of 60 firms with number of seat-kilometers traveled as the output and labor/ hours of work as the input is evaluated using the data from the UITP Handbook of Public transport Statistics, 1985-1986.

Chu et al. (1992) suggest the use of effectiveness along with efficiency to completely represent productivity in case of transit systems. The term efficiency defines the relationship between input and output, which in transit systems, refers to the use of inputs to produce service as outputs. Effectiveness on the other hand refers to consumption of service or in more general terms, achieving of pre-determined objectives by way of the outputs produced. Both efficiency and effectiveness together, can address productivity as well as issues relating to quality of service, customer satisfaction, growth management in case of performance measurement of transit systems. Efficiency and Effectiveness can be captured by the DEA methodology.

Ndoh and Ashford (1994) suggest the use of fuzzy sets to describe user's viewpoint. Fuzzy Sets can ideally capture uncertainty. Service provided can be expressed qualitatively by the user and this can be expressed conveniently using Fuzzy Sets.

Ryus et al., (2000) introduce the Transit Level-of-Service (TLOS) indicator that Florida's DoT uses for measurement of transit performance. This indicator is a measure of transit availability. They propose that transit quality of service is made up of transit availability and comfort and convenience from passenger's viewpoint. It includes service
coverage, frequency, duration, and the availability and quality of pedestrian routes to the stops, population and job density and is used to measure the level of service provided.

TLOS of a system makes use of the inputs like transit routes and stop locations, street networks with pedestrian pathways, population and employment data for transit area zones, transit vehicle arrival times. TLOS for a system can be obtained by averaging over all locations in the area, weighting the TLOS accordingly depending upon the parameters decided for different zones in the area.

### 2.4 Data Envelopment Analysis in Transit Systems

DEA was conceptualized by Charnes, Cooper and Rhodes in 1978 to measure public school efficiency. It is a deterministic, reference technology. It does not require Cobb-Douglas form of production function as in case of other technologies. This nonparametric method is based on piecewise linear frontier that forms the reference technology. DEA uses math-programming techniques are used to estimate the frontier. In DEA, the best practice forms the frontier and this frontier envelops the remaining data points, subject to convexity and linearity. Due to loose assumptions of linearity and convexity, Kerstens (1996) suggests that the technical efficiency measured by nonparametric, inner bound references technologies like DEA is always greater if not equal to that measured by parametric approaches.

DEA has been extensively applied in transportation research. It has been used to measure technical efficiencies for different services offered by a department, (Husain et al. (2000)) or to the compare different transit companies within the same area, or over different geographical areas, ((Kerstens (1996)). DEA has been used in air traffic engineering for achieving resource allocation, crew scheduling, flight scheduling, and deciding of optimal flight networks through efficiency evaluations (Adler (1999)). It has been used to prioritize accident sites on highways (ref) to compute efficiency of ports ref (Tongzon (2001)). This research effort utilizes DEA to (a) achieve optimal allocation of public resources to different services provided by a public transit organization, and (b) to attain equilibrium between mobility demands and public transit supply.

Some of the works done in the field of performance measurement of transit systems using DEA are summarized as below.

The Charnes Cooper and Rhodes (CCR) model (1978) has also been used to evaluate the bus enterprises under public managements by Tone and Sawada (1991). The authors have classified the activities that convert inputs to outputs based upon the resources it uses, e.g., financial resources (cost), production resources (employee/labor, bus/service vehicles), service resources (services offered), utilized resources (utilized services). As a result of this, the efficiency is classified as service efficiency, cost efficiency, income efficiency and public service efficiency and these are evaluated using the following variables,

|  | Input | Output |
| :---: | :---: | :---: |
| Service efficiency | number of cars, employees | distance covered (service coverage) |
| Cost efficiency | operating expenses | number of cars and employees |
| Income efficiency | number of cars and <br> employees | operating income (annual) |
| Public service | number of cars and <br> efficiency | density of service (average daily <br> service frequency) |

Table 2.1 Efficiency Classification

Chu et al. (1992) uses the CCR relative efficiency model to compare transit agencies operating in the US. These transit agencies are divided into two groups, one serving large metropolitan areas and other group serving small cities and large towns based on a study by Fielding, Brenner and Faust (1985). The data from the National Urban Mass transportation statistics, 1986 section 15 annual report (US DoT) and Socio economic data from the transportation planning data for urbanized areas (US DoT, 1985) is used. These agencies were divided on the basis of the data of the public transit agencies into 12 peer groups according to their size, speed and ratio of services offered during peak hours to services offered during regular base hours (peak to base service ratio) using cluster analysis. Out of these 12 groups, two smaller groups were combined and eleven
groups were identified. The CCR model is used with outputs as revenue vehicle hours of service (annual) and annual unlinked passenger trips and inputs as the weighted sum of annual revenue vehicle hours, urbanized area population density, proportion of households without automobile and annual financial assistance per passenger and weighted sum of expenses (annual vehicle operating expenses, annual maintenance expenses, annual general/administrative expenses and annual other expenses)

Nolan (1996) uses the data from Section 15 US DoT database 1989-1993, and the CCR model to evaluate the efficiency of mid-sized transit agencies in US.

Kerstens (1996) compares a sample of French Urban Transit Companies, using DEA and considers the following 3 cases, (a) VRS with strong disposability in inputs and outputs (b) VRS with weak disposability in both inputs and outputs (the author selects the weak disposable model to explain all patterns of congestion of inputs and outputs) (c) the FDH technology. The author uses the data published by the Ministry of transport for French Urban Transit companies operating within a specific perimeter. These are single mode, single operator, and bus operating companies. Total fuel consumption, number of vehicles and the average number of employees are used as inputs. The author evaluates the efficiency of these urban transit companies with two alternative outputs vehiclekilometers and seat-kilometers. The author concludes from his analyses that technical efficiency score of an observation as well as the mean technical efficiency in FDH is greater than DEA score assuming weak disposability of inputs for his data set. He is of the opinion that as congestion exists for these urban transit companies, a weakly disposable model needs to be used. He also suggests a two-stage approach where the technical efficiency is measured at the first stage and then Tobit analyses are carried out at the second stage.

Viton (1996) studies the efficiency of U.S. multi-mode bus transit systems using DEA to document an effective use of its input resources. The author uses the FY1990 Section 15 data, published by FTA for systems that offer motorbus and demand responsive bus services. The author considers two input oriented models, the Russell measure and the VRS DEA measure once with weak disposability and then with strong disposability. This is to investigate the technical efficiency and the congestion effect. The
outputs used are the quantity of vehicle-miles and unlinked passenger trips. The inputs are selected from three sources:
a.Environmental variables: includes average speed (as a proxy for terrain), stopspacing and levels of congestion.
b.Conventional inputs: includes fleet size, gallons of fuel used and number of person hours.
c. Cost inputs: include all those inputs which do not have a physical measure and can be expressed in terms of dollars.

Carotenuto et al. (2001) carries out the analysis on both public and private transit agencies, using CRS and VRS, based on the seminal work by Farrell (1957) to obtain a measure of pure technical efficiency, scale efficiency and the overall efficiency. The authors use Passenger freight as output and number of buses, number of workers and total fuel consumed as inputs for the analysis.

Husain et al., (2000) present a study that evaluates the performance of the services of a public sector organization in Selangor, Malaysia. The study evaluates performance of 46 service units in the Road Transport Department (RTD) services in Malaysia by measuring its multiple inputs \& outputs service activities, weighted by cost indices. The CCR model uses labor, cost of labor, services as inputs and total production from services, total revenue as outputs for performance measurement. The authors compare these RTD service units to identify relatively inefficient units, and then measure the magnitude of the inefficiencies. They then compare the inefficient and efficient services and suggest the ways to reduce the inefficiencies. The data used is from 1998, Road Transport Dept., Selangor, Malaysia.

Boile (2001) compares a group of transit systems, obtained from National Transit Database (1998) of FTA with respect to multiple inputs and multiple outputs. She uses DEA to identify efficient and inefficient systems in terms of allocating resources to provide services. She uses the CCR and BCC DEA models with annual vehicle operating, maintenance, administration and non-vehicle expenses as the input and annual vehicle revenue hour, annual passenger trips to measure technical and scale efficiencies for 23 transit systems. These 23 transit systems are obtained after clustering to form a meaningful peer group.

This research effort derives from Network Theory, goal Programming and Data Envelopment Analysis for development of a model to evaluate the efficiency of bus routes of a bus transit network. This is to address provider-user perspectives, target setting and resource allocation.

The next section reviews the interface between Goal Programming and DEA as developed by Athanassopoulos (1995). It also presents the non-parametric productivity measure that accounts for intermediate products as conceptualized by Färe (1991). Both of these concepts are used in this research to account for targets and capturing multiple perspectives and goals

### 2.5 Multi-objective Programming

The case of multiple objectives is treated extensively in the literature (Athanassopoulos (1995)). In multi-level, multi-unit organizations, planning generally involves decision-making at various levels. Many a times a system comprises many subsystems. Generally speaking, many of these sub-systems have conflicting goals either with other subsystems or with the parent system. So, in order to optimize the whole system, it becomes necessary to optimize these sub-systems. This requires (a) maximizing the global achievements of the system, (b) maximizing the contribution of individual units to global targets and (c) maximizing the shares of each individual unit (Athanassopoulos (1995)). Thus, these local goals that are associated with the subsystems and the global goals that are associated with the whole system are to be addressed in conjunction with each other. This is difficult with conflicting objectives and requires effective planning and co-ordination. In such a scenario, i.e., when there are conflicting objectives, the representation of these objectives is generally achieved by weighing these objectives depending upon their degree of importance. This in itself is a challenge because the selection of the weight structure then becomes subjective and varies with the decision-maker.

### 2.5.1 Goal Programming and Go-DEA

A special case in multiple objective programming is when the target levels or goals of sub-systems and of the complete system are known. The problem then boils
down to reducing the deviation of the levels obtained from the predetermined target levels. A preferential weight structure can be introduced to attach the degree of importance to certain deviations subject to a set of constraints. This approach is better known as Goal Programming and is widely used when target levels are known. This is because it enables the representation of conflicting goals and with preferences attached to these multiple objectives by the way of weights. Many a times target levels or goals for sub-systems and the complete system are unknown. In such cases the, the targets are obtained in the solution of the problem.

In certain applications where DEA is used for efficiency analysis local and/or global targets for input and output variables are known. This information can be used to measure performance. Golany (1988) first spoke of measuring efficiency in terms of achieving targets. Charnes et al. (1978) estimate the relative efficiency of a DMU based on the maximum pro rata adjustment of all its inputs or outputs that would render the DMU efficient (CCR model). Banker and Morey (1986) developed DEA models where a unit was rendered efficient with maximum pro rata adjustment to its input or output, while keeping all the other variables at constant level. This is because there is a possibility that a DMU may have different priorities from other units for one or more inputs or outputs. Thus the DMU might have different target levels for these inputs and/or outputs and hence it is projected onto a different facet of the frontier.

These scenarios have led to the extension of the basic DEA (BCC, CCR) models. Thanassoulis and Dyson (1992) extended these DEA models to estimate alternative input and output target levels that will render inefficient units efficient. These models incorporated user's preferences for reduction of certain inputs (or production of certain outputs) at a greater rate than other inputs (outputs). This was achieved by estimating the efficiency and the peers after scaling of a particular input (output) or a subset of inputs (outputs). In cases where it is required to scale a subset of inputs or outputs differently then these inputs (outputs) can be weighed preferentially. The relative efficiency score is a weighted combination of the factors by which the corresponding input or output levels are sought to be adjusted and these scores are different than those generated by the CCR or BCC models. The preferential weights are selected based upon priorities or simply by running the model iteratively till the best targets are achieved. These models were used
to evaluate 22 programs offered by a College in Texas where the inputs used are contact hours, FTE instructors, facilities space and instruction expenses and outputs are revenue generated, students employed and employer satisfaction.

Although this representation by Thanassoulis and Dyson (1992) is ideal to represent scaling (preferentially) a subset of all inputs and outputs but it is not sufficient to address the problem when all DMUs are to be considered simultaneously. This can be further explained with the concept of global and local goals. Consider a system with subsystems. Let these sub-systems be the DMUs. Now if it is required to improve the input (output) level of one of the DMU then this is a local goal as it relates only to that specific DMU. This can be easily represented by Thanassoulis and Dyson (1992) formulations. But when it is required to achieve improvement in input or output levels with respect to the whole system, i.e., for all DMUs (global goal), then the Thanassoulis and Dyson (1992) formulation is not sufficient.

Athanassopoulos (1995) proposed the Goal-DEA model to accommodate for local and global efficiency. Goal-DEA is an interface between Goal Programming and DEA that integrates target setting and resource allocation for multi-level planning problems. The model minimizes the deviation from local and global level targets subject to three groups of constraints. The first set of constraints represents individual DMUs. The second set represents effectiveness and achievement of global targets and third set represents the consideration of obtaining a budget balance. This budget balance constraints link local and global goals. The author suggests that in those cases where it is needed to estimate global targets, the concept of structural efficiency (Section 2.1.4) can be used.

The Goal- DEA formulation as proposed by Athanassopoulos (1995) is as follows:
$\operatorname{Min}\left\{\sum_{i . I_{v}} W_{i}^{g} \frac{d_{i}^{+}}{G X_{i}}+\sum_{i . I_{v}} W_{r}^{g} \frac{d_{r}^{-}}{G Y_{r}}, \sum_{j=1}^{N} \sum_{i . I_{c}}\left(w_{i}^{n} \frac{n_{i}^{j}}{x_{i}^{j}}+w_{i}^{p} \frac{p_{i}^{j}}{x_{i}^{j}}\right)+\sum_{j=1}^{N} \sum_{r . O_{c}}\left(w_{r}^{n} \frac{n_{r}^{j}}{y_{r}^{j}}+w_{r}^{p} \frac{p_{r}^{j}}{y_{r}^{j}}\right)\right\}$

## Subject to

Individual DMUs:
$\sum_{j=1}^{N} \delta_{j}^{j_{o}} y_{r j}-p_{r}^{j_{o}}+n_{r}^{j_{o}}=y_{r}^{j_{o}}, \cdots r . O_{c}, . j_{o}$
$\sum_{j=1}^{N} \delta_{j}^{j_{o}} x_{i j}-p_{i}^{j_{o}}+n_{i}^{j_{o}}=x_{i}^{j_{o}}, \cdots i . \quad I_{c}, . j_{o}$
$\sum_{j=1}^{N} \delta_{j}^{j_{o}} y_{r j}=y_{r}^{j_{0}}, \cdots r . O_{f}, . j_{o}$
$\sum_{j=1}^{N} \delta_{j}^{j_{o}} x_{i j}=x_{i}^{j_{0}}, \cdots i . I_{f}, j_{o}$
Global targets:
$\sum_{j=1}^{N} \delta_{j}^{1} x_{i j}+\sum_{j=1}^{N} \delta_{j}^{2} x_{i j}+\ldots+\sum_{j=1}^{N} \delta_{j}^{N} x_{i j}-d_{i}^{+}=G X_{i}, \cdots . i . I_{v}$

Balance constraints:
$\sum_{i . I_{B} \neq 1}^{N} \sum_{j}^{N}\left(\delta_{j}^{1}+\ldots+\delta_{j}^{N}\right) x_{i j}-\sum_{r . O_{B} \neq 1}^{N}\left(\boldsymbol{\delta}_{j}^{1}+\ldots+\delta_{j}^{N}\right) y_{r j}=B$
.i. $I_{B}$ and. r. $O_{B}$,
$\delta_{j}^{j_{o}}, n_{i}^{j}, n_{r}^{j}, p_{i}^{j}, p_{r}^{j}, d_{i}, d_{r}=0$
$V X_{i}=0 \ldots . i . I_{v}$
$V Y_{r}=0 \ldots r . O_{v}$
where:
$d_{i}, d_{r}$ : deviations in inputs and outputs from the Global targets $p_{i}^{j}, p_{r}^{j}$ : positive deviations in input and output of DMU ' j ’ $n_{i}^{j}, n_{r}^{j}$ : negative deviation in input and output of DMU ' j '
$x_{i j}, y_{r j}$ : input and output of DMU ' j '
$V X_{i}, V Y_{r}$ : unknown global targets for inputs and outputs
$I_{v}, O_{v}$ : set of input and output whose global targets are unknown
$I_{f}, O_{f}$ : set of fixed input and output
$I_{B}, O_{B}$ : set of commensurable inputs and outputs
$I_{c}, O_{c}$ : set of controllable inputs and outputs
B: constant; represents the balance between commensurable inputs and outputs $G X_{i}, G Y_{r}$ : Global targets for inputs and outputs
$W_{i}^{g} W_{r}^{g}$ : weights for deviations in global targets for inputs and outputs
$w_{u}^{n}, w_{r}^{n}$ : weights for the positive deviation of local targets in inputs and outputs $w_{i}^{p}, w_{r}^{p}$ : weights for the negative deviation of local targets in inputs and outputs.

Equation (2.41) represents the objective function. It indicates preferential minimization of deviations from their targets. (2.42), (2.43), (2.44) and (2.45) represent the constraints for the individual DMU on its controllable output set, controllable input set, the non-controllable output set and the non-controllable input set. As these relate to individual DMUs they can be considered as local goals. (2.46) and (2.47) represent constraints for global targets when these targets are known whereas (2.48) and (2.49) represents global target constraints when the goals are unknown and are determined from the model. Equation (2.50) is known as the budget balance constraint. This relates the global and local goals and can be modified depending upon the application where GoDEA is used.

Athanassopoulos (1995) applies this Go-DEA model to Greek local authorities to aid decision making for resource allocation and target setting. The author considers socio-demographic variables like built up area, heavy industrial size, actual households, general area, fiscal capacity, total size of house area and general area. The inputs used are labor costs, service expenses, maintenance costs and loans and outputs used are central grants, fees \& charges, investments and rate of charges. The concept of Goal-DEA has been utilized with modification in some cases and without modification in other cases.

Hoopes, Triantis, and Patrangel (2000) present the goal programming data envelopment analysis approach to assess plant level and process level performance in case of a circuit board manufacturing facility with serial production stages. The authors represent the 3 groups of constraints (Athanassopoulos (1995)) as three interrelated objectives of operational efficiency, effectiveness and production line balance between consecutive stages as important for planning and control in a manufacturing facility. The model determines efficiency scores for the same assembly line at different time periods. Thus these time periods are the decision-making units. The model achieves minimum deviation in its three goals, plant level targets, process level targets and production balance targets subject to a set of constraints in the DEA framework. The model uses the output-oriented approach with labor and materials as the input and number of boards as the output.

In this research effort, there are societal variables like number of accidents, noise pollution, air quality and resources degraded. The authorities generally have certain safe threshold levels fixed for these variables in a certain area. Even though it is required that these variables be as low as possible, in general case, it suffices if these variables are just below the upper permissible threshold level. This is conveniently represented in the DEA model through a Goal Programming framework with the objective function being to minimize the deviations from the target levels subject to the set of constraints.

### 2.5.2 Network Activity Analysis Model

The efficiency of a production technology comprising of a single unit that consumes inputs and produces outputs can be easily determined by non-parametric approaches. When the production technology comprises more than one subsystem then there are intermediate products formed. As in the general case, the efficiency of a system is evaluated on the basis of inputs it consumes and outputs it produces, without consideration of the intermediate products formed. If these intermediate products are consumed partially or that they act as final outputs, then it becomes important to account for them. This scenario in DEA can be conveniently expressed with the Network Activity Analysis model developed by Färe (1991). The Network Activity Analysis model is
generally used to represent production that is often characterized by multiple inputs and multiple outputs of multiple production stages. Färe's (1991) model measures efficiency of firms with intermediate inputs using DEA. This model is based network theory and efficiency evaluating on DEA.

The model also represents the system as a process with nodes and links as in case of a network. The nodes represent sub-processing within the system and links represent the inputs and outputs flowing in and out of nodes.

Färe's (1991) Network Activity Analysis model for a network with 4 nodes, as shown below is as follows (See Figure 2.6)


Figure 2.6 Network for the Model

The overall process for DMU ' $\mathrm{j}_{\mathrm{o}}$ ' can be represented as:
Max .
Subject to
$\sum_{j=1}^{N} \cdot{ }_{j 0}^{1} x_{j}={ }_{0}^{1} x_{j_{o}} \quad . i=1 \ldots S$
$\sum_{j=1}^{N} \cdot\left({ }_{j}{ }_{1}^{2} y_{j}+{ }_{1}^{3} y_{j}\right)={ }_{1}^{2} y_{j_{o}}+.{ }^{3} y_{j_{o}} \quad . r=1 \ldots m$
$\sum_{j=1}^{N} \propto_{j}{ }^{2} x_{j}={ }_{0}^{2} x_{j_{o}}$
$\sum_{j=1}^{N} \alpha_{j 1}^{2} y_{j}={ }_{1}^{2} y_{j_{o}} \quad . i=1 \ldots S$

$$
\begin{array}{ll}
\sum_{j=1}^{N} \alpha_{j}{ }_{2}^{3} y_{j}=\cdot{ }_{2}^{3} y_{j_{o}} & . r=1 \ldots m \\
., ., \propto=0 & . j \tag{2.60}
\end{array}
$$

where
x :input from 1...i...s
y : output from 1...r...m
. : output efficiency measure
., $\propto$ : intensity parameters
Equation (2.54) is the objective function which represents the output efficiency measure. It suggests maximizing the final output subject to other constraints. (2.56) and (2.57) represent the constraints on the input set and output set respectively for Node 1. (2.58), (2.59) and (2.60) refer to Node 2 and are constraints on the inputs directly to Node 2 , inputs from Node 1 and the final output set respectively.

The above formulation uses the output oriented approach i.e. obtain a set of maximum outputs such that the constraints with regards to all the nodes in the network hold true. The model assumes constant returns to scale and strong disposability in both inputs (as represented by the distribution of input constraint) and outputs. Since, Overall Efficiency $=($ Technical Efficiency $) *($ Allocative Efficiency $)$ Overall Efficiency = (revenue) $/($ cost $)$

Färe suggests that the model can be used to determine the revenue generated if the output price vector is known. This can be used to determine overall efficiency and hence allocative efficiency, if cost vector is known.

Färe and Whitaker (1995) use the Network Activity Analysis model to analyze the agricultural production efficiency. The study compares the efficiency scores obtained from network model and from a model that does not consider intermediate products. The analysis is carried out on 137 farms the data for which is obtained from the 1989 Dairy version of the Farm Cost and Return Survey. The inputs used are fertilizer, seed, cropland, labor, equipment, machinery, tools, transportation expenses, depreciation, fuel, commercial feed, nutrition supply, pasture, veterinary, dairy supplies expenses and purchased hay. The outputs are corn for grain, corn for silage, soybeans, wheat, oats, hay and milk. The authors compare results from the model when considering intermediate
products and from the model without considering intermediate products using bootstrapping and kernel density estimation and conclude that in case of the model without intermediate products, due to high dimensionality, more observations are on the frontier than in the model with intermediate products.

Färe et al (1995) uses the Network Model to measure productivity of Swedish pharmacies by incorporating attributes into the technology. These attributes are used to measure the service quality of these pharmacies. The authors apply an input oriented model that permits strong disposability in outputs to 1990-1991 data on 257 pharmacies. Hours of pharmacists' service, hours of technical staff services, value of other services, labor variables and costs incurred are used as inputs. Number of prescriptions, dose package deliveries, drug deliveries, special articles for handicapped, special food for handicapped, over the counter goods and information on drugs are used as outputs. The three attributes used are number of hours open for business per week, waiting time and percent of prescriptions, which can be filled within one day.

Golany et al (2002) use the concept underlying in the Network Activity Analysis model to develop an efficiency measurement framework for flow line systems that simultaneously compute the efficiency of aggregate system at each stage and along the line. The authors compare the 3 models, Constant Returns to Scale (CRS), Disaggregate (DA) variant of CRS and a Complete Transferability (CT) variant of the Färe and Grosskopf (1996) model. The authors develop an efficiency evaluation method that computes the efficiencies of stages from the aggregate CT efficiency measure.

The research involves measuring the performance of bus route considering both, the providers and the user's perspective. The provider provides service by consuming inputs. This service can be quantified by certain outputs such as schedule reliability, trip time and vehicle-seat miles and is consumed by the user who can be considered to produce passenger-miles. Thus this service expressed in terms of outputs acts as input for the user and hence intermediate output for the provider-user system. The network activity analysis model developed by Färe (1991) is used to capture this provider-user scenario.

The model to evaluate the efficiency of bus routes is based on the concepts discussed in the literature. The model is developed and presented in Chapter 3.

## Chapter 3

## Research Methodology \& Model

### 3.1 Introduction

This research aims at developing a methodology to measure and compare the efficiency and effectiveness of individual bus routes. The first chapter of this thesis provided the general overview of research methodology. It introduced the problem and also presented the motivation, the purpose and the objectives of the research. The second chapter introduced the concepts in Data Envelopment Analysis, Network Theory, Goal Programming and Public Transit performance measurement. This chapter explores the possibility of combining the above techniques and approaches to:

Deal with issues of optimal allocation of scarce resources for the service provider.

Enable the decision-maker (transit service provider, e.g., government or transit agencies) to achieve global as well as local performance targets and efficiency levels.

Specifically evaluate amounts of inputs to be consumed and outputs to be produced for each DMU (the bus route) to become efficient.

The model is proposed in this chapter. The model uses Data Envelopment Analysis (DEA), (Charnes, Cooper and Rhodes (1978)), the Network model (Färe (1991)) and GoDEA (Athanassopoulos (1995)) to compute the efficiencies of bus routes. Additionally, this chapter discusses the objectives that are achieved, the assumptions made, the corresponding variables used and their definitions, the notation and terminology adopted and the rationale for the particular approach taken.

### 3.2 Rationale for the Approach

The research problem has three distinguishing features,
(i) Multiple Perspectives
(ii) Local \& Global targets
(iii) Environmental Factors

These three features are addressed utilizing three unique approaches from the DEA literature. This section illustrates the rationale of these adopted approaches to address the characteristics of the research problem.

### 3.2.1 Multiple Perspectives

While computing efficiency of a bus route, it is necessary to include both, i.e., the provider's as well as the consumer's viewpoints. This is because both these viewpoints define an 'efficient' route in a different, if not contrasting manner. According to the provider's viewpoint, an efficient route is the one that will provide adequate service whereas for the customer, an efficient route is the one that has the most quality attributes such as the shortest travel time or the highest level of seating comfort.

Now consider a process in which one can meaningfully define its subsystems. In such a process, there may be intermediate products, which may or may not form the final output set. As these intermediate products are indicative of the process that transforms inputs (resources) into outputs (products/services), it is, many a times, important to represent these intermediate products as they help capture the essence of the whole transformation process.

In this research, the concept of intermediate product (e.g., vehicle-miles, schedule reliability and avg. traveling time) is utilized to link the provider and the passenger perspectives. This is because the provider provides transit service through bus routes and passengers consume this service since they use these corresponding routes to commute. In order to consider both (the provider and the passenger) viewpoints, an inter-linked network of nodes is used to represent a bus route. As the bus route is represented as a network of interlinked nodes, the computation of the efficiency score of a bus route can be equivalently viewed as the computation of the efficiency score of this network of nodes. In this research multiple perspectives are addressed by utilizing the network model framework in DEA developed by Färe (1991).

### 3.2.2 Local \& Global Targets

In order to accommodate the achievement of multiple societal considerations, the model also utilizes GoDEA formulation developed by Athanassopoulos (1995). The

GoDEA model permits projection of decision-making units (DMUs) onto any point on the frontier rather than onto the point defined by the Debreu-Farrell measure. The decision-maker defines the performance targets that become the projected points on the frontier. This research requires the achievement of local (for individual bus-routes) output targets as well as global (for the entire bus-route network) output targets with regards to externalities or societal variables such as air quality, noise pollution, natural resources, and safety and the logic and structure of the GoDEA framework is utilized to achieve this.

### 3.2.3 Environmental Factors

The environment or the operating conditions within which the DMU, i.e., the bus route provides service varies significantly. According to Banker and Morey (1986) and Ruggiero (1998), a decision-making unit (DMU) should be compared only with DMUs operating in similar or harsher environments. The variables that define the operating environment in DEA are referred to in this research as environmental variables. These variables determine the appropriate reference set for the DMU, whose relative efficiency is being evaluated. This is accounted for in the proposed model by adopting the Ruggiero (1998) and Ray (1997) approaches.

Hence the proposed model is a result of a combination of the network model approach developed by Färe (1991), the GoDEA approach developed by Athanassopoulos (1995), and the three-stage approach that considers environmental effects as proposed by Ruggiero (1998) and Ray (1997).

### 3.3 Objectives and Variable Definitions

In this section, objectives important to the Provider as well as the Passenger are identified. Based on these objectives, variables (inputs and outputs) for the respective Nodes are defined. The variables describing the operating environment or the traffic operating conditions are also defined. Lastly, societal perspective through societal variables is also illustrated.

### 3.3.1 Variables

The provider of the transit service, bus service in this case, can be the Government or a private organization that is contracted to provide the transit service. For a service provider, an 'efficient' route will be one that seeks to achieve objectives such as maximizing the average passengers transported, the revenue generated or minimizing the total operating costs. The variables that correspond to these objectives are provided below. They are further classified into inputs and outputs with respect to the provider. The inputs are as follows.
a) Headway: It is the time difference between two consecutive service vehicles on a route. Its reciprocal is a measure of the service frequency of the route. This variable is averaged over the entire route to accommodate different headways on different segments of the route.
Average Headway $=\frac{\left\{\sum(\text { headway })(\text { length of segment })\right\}}{\sum \text { length of the route }}$
For example, if for a route length of 10 miles the headway is $10 \mathrm{~min}, 5 \mathrm{~min}, 12$ min for 5,2 and 3 miles respectively, then the average headway is $=9.6 \mathrm{~min}$

In the dataset, the inverse of Headway is used for each bus routes as this variable is an input variable and inputs for efficient DMUs have lower values than for inefficient DMUs.
b) Service duration: It is the total time during a day for which the service is provided.

For example, if service is provided for 10 hrs during the day then the value for Service Duration for that route is 10 . If the service duration differs over different days of the week for the route, then their average is taken as the value for this variable.
c) Congestion: Volume/Capacity (V/C) ratio is used as a measure of congestion. It is the ratio of the volume of a route to its capacity. It measures the actual volume of traffic on the route, excluding the volume generated by the service vehicles to the total capacity of the route for a certain time period. Congestion is averaged over the entire route.

For example, if there are five lanes, then the capacity is 5 and if, actually 4 vehicles pass, then

$$
\begin{equation*}
V / C=0.8 \tag{3.2}
\end{equation*}
$$

Congestion is not used directly in the model but is used to determine Average Traveling Time and Schedule Reliability which form the intermediate output.
d) Costs: This variable represents the total cost involved in offering services on the route per day. It includes infrastructure, maintenance, operating and other miscellaneous costs for all the service vehicles operating on that route.
e) Number of intersections: This represents the total number of intersections (including signalized) on a bus route. If a bus route has more intersections, then this leads to greater delays and hence decreases the efficiency. As 'Number of intersections' forms an input, its reciprocal is used in the model to be in accordance with the general assumptions involving input representation in DEA.
f) Number of Priority Lanes: This represents the lanes on the bus route that are accessible only to buses. If the route has more priority lanes for buses, it increases their efficiency.

The provider of the transit service provides the following outputs.
a) Vehicle-seat miles: This is a measure of the number of trips. It is a product of seat capacity of each service vehicle times the total number of miles traveled by all the vehicles operating on that route.

Vehicle-seat miles $=($ total seat capacity of the service vehicle) (total miles traveled by all the service vehicles on the route during the day)
b) Schedule Reliability: This variable is the ratio of number of on-time arrivals to the total arrivals. It is a measure of the reliability of the scheduled arrivals at the bus stops on the route. Its reciprocal measures number of delays on the route.
(total number of on-time arrivals of all service vehciles
Schedule Reliability $=\frac{\text { at all the bus stops on the route) }}{\text { (total number of arrivals of all service vehicles }}$
at all the bus stops on the route)
c) Average Traveling Time: This variable measures the time taken for one complete trip. It is average of the time taken by each vehicle for one trip.

Average Traveling Time $=$ Average Time taken by a service vehicle from its origin to destination point

The consumer of transit service is the passenger or the user of the system. For a passenger, an efficient route will be the one that has minimum trip and waiting time, minimum fares, higher interconnectivity, greater service coverage, higher safety and higher comfort level.

In the model, the Passenger node represents this perspective. The variables are further classified into inputs and outputs. Considering the outputs from the provider, namely vehicle-miles, schedule reliability or delay and average trip time are common to the passenger node as well. The only difference is that they form the outputs in case of the provider while they form inputs in case of passenger. Thus, vehicle-miles, average trip-time and schedule reliability or delay form the input set for the Passenger node.
a) Passenger-miles: The output at Passenger node is passenger-miles as its output, which is a product of the total number of passengers traveling on that route and the total miles traveled on the corresponding route. It is a measure of the effectiveness of the consumption of transit services

### 3.3.2 Environmental Factors

There are four environmental variables that define the operating environment or traffic operating conditions in which the transit services are provided. These variables are as follows,
a) Accessibility factor: This factor is a measure of the population within 400 meters (Tomazinis (1975)) of the bus stops on a specific route. Although the bus route passes through an area, only people who have access to the bus stops use it. This variable represents the passenger traffic and hence needs to be taken into account while computing the efficiency of the route.
b) Parking space availability factor: One of the factors for which people use transit services is the availability of parking space at their destinations. This variable is a measure of parking space availability on and around the route. An urban area is generally
associated with unavailability of parking space. If a bus route operates through an area that has adequate parking space will reduce the use of public transit system for commuting.
c) Population density factor: A bus route that passes through an area, which has a higher number of offices, schools, and residences as compared to other routes, has greater passenger traffic. This needs to be considered in the efficiency evaluation of the bus route. The population density factor accounts for this.
d) Connectivity factor: This factor is a measure of the interconnectivity of the route with other routes of same mode as well as different modes such as train routes or trolley routes etc. A bus route is more user-friendly for commuters if it permits transfer to other routes of the same mode and other modes as well. As this leads to higher passenger traffic and it needs to be considered when evaluating performance.
e) Comfort Standards factor: This factor indicates the level of comfort for the passenger when using the route. In this research, it basically refers to a loading factor for the service vehicle.

Loading factor $=\frac{\text { Number of seats occupied }+ \text { standees }}{\text { Total capacity }}$
f) Infrastructure factor: This factor is used in the variations of the models to illustrate the effect of an additional environmental variable. It represents the infrastructure, like bus stops, priority lanes and intersections on the bus route.

Based on these variables, the operating environment of a bus route can be identified. They are included in the model so that the efficiency of bus route is computed with respect to only those bus routes that operate in similar or harsher environments so that the efficiency of the bus route is with consideration of the traffic operating conditions.

### 3.3.3 Societal Variables

The people who do not use the transit service, i.e., the society in general, are also affected by the transit system and hence this needs consideration. Correspondingly the objectives important from the societal perspective are that of safety, noise pollution, air quality and resources degraded.

The variables that correspond to these concepts are as follows.
a) Number of accidents: It measures the total number of accidents and incidents on the route. Accidents and incidents occur on bus routes and they affect the transit service provision. It is important that these numbers are kept to a minimum or within a certain target as predetermined by the society.
b) Emissions: This variable measures air quality. As service vehicles operate on the route, the air quality deteriorates due to emissions. It is important for the people that live on and around the route that the emissions are well within the levels decided by the government or environmental authorities.
c) Noise Pollution: This variable measures the noise pollution due to service vehicles on the route.
d) Resources degraded: This variable measures the dollar value of resources degraded due to the provision of service. These resources are natural resources such trees, fertile land, water etc. degraded to lay the route and offer services.

### 3.4 Model Description

A bus route is depicted by means of a network with the entities (e.g. provider, passenger etc.) as nodes and links of the network representing the inputs and outputs. Measuring the efficiency of the bus route is equivalent to the measurement of efficiency of this network. Figure 3.1, is a graphical illustration of the network with inputs and outputs connecting the relevant nodes.

The nodes are as follows:

| Node | Description |
| :---: | :---: |
| 0 | Collection of Inputs |
| 1 | Provider of Service, (Government) |
| 2 | Passenger |
| 3 | Collection of Outputs |

Table 3.1 The Description of the Nodes


Figure 3.1 Graphical Representation of the Bus Route Network

The inputs at Node 1, i.e., the Provider Node, are 'headway', 'cost' and 'service duration'. 'Number of priority lanes', and 'Number of intersections' form the infrastructure variables (fixed variables; relating to infrastructure) at Provider Node. The outputs from this node, 'Vehicle-Miles', 'Average Traveling Time' and 'Schedule Reliability’ serve as inputs to Node 2 (Passenger Node). Node 2, in addition, is evaluated using environmental variables like 'Accessibility factor', 'Connectivity factor', 'Population density', 'Parking space availability factor', 'Comfort Standard factor' and 'Infrastructure factor'. Node 3 represents collection of outputs from Node 2 and Node 1. These outputs are 'Passenger-Miles' and the externalities like 'Number of accidents', 'Noise Pollution', 'Emission' and 'Resource degraded'.

### 3.5 Assumptions

This section describes a set of assumptions on which the model is based. They are as follows,
(a) The bus routes are isolated entities, possess similar activities and produce comparable products, i.e., services.
(b) The interlinked network of nodes represents the bus route and hence the efficiency of the network is equivalent to that of the route.
(c) The service vehicles are similar in size in terms of seating capacity and consume the same quantity and type of fuel per route length.

### 3.6 Notation

The notation used in the model is presented as follows,

## Inputs: - x

The index used to represent the inputs is ' i ' where $\mathrm{i}=$ headway, congestion, cost, service duration, number of intersections and priority lane. These inputs, for the ' j ' , DMU, are represented individually as:
$\mathrm{X}_{\mathrm{jhd}}$ :- headway
$\mathrm{X}_{\mathrm{jcost}}$ - - cost
$\mathrm{x}_{\mathrm{jservd}}$ : - service duration
$\mathrm{x}_{\mathrm{jint}}$ :- number of intersections
$\mathrm{x}_{\mathrm{jprIn}}$ :- priority lanes

## Outputs \& Intermediate Outputs: - y

The index used for representing outputs and intermediate outputs is ' $r$ ' where $r=$ average trip time, vehicle-miles, schedule reliability and passenger-miles. These outputs for the ' j 'th DMU are represented in the model as
$y_{j t i m}$ : - average traveling time
$\mathrm{y}_{\mathrm{jvm}}$ :- vehicle-miles
$\mathrm{y}_{\text {jsrel }}$ :- schedule reliability
$\mathrm{y}_{\mathrm{jpm}}$ :- passenger-miles

## Externalities: - y

The externalities or the societal variables also form part of the output set and are represented by ' $y$ ' although the index used is ' $f$ ' where $f=$ number of accidents, noise pollution, emissions and the dollar value of resources degraded. These variables for the $\mathrm{j}^{\text {th }}$ DMU are as follows,
$\mathrm{y}_{\mathrm{jacc}}$ :- number of accidents
$y_{j e m i}$ :- emissions
$y_{\text {jnp }}$ :- noise pollution
y $_{\mathrm{jres}}$ :- resources degraded (\$ value)
The number of bus routes (DMUs) are ' N ', i.e., $\mathrm{j}=1 \ldots \mathrm{j} . . \mathrm{N}$. The formulation will evaluate the efficiency of ' $\mathrm{j}_{0}$ th, bus route (DMU).
Intensity Variables: $-. \mathrm{j}, \delta_{\mathrm{j}}, \propto_{\mathrm{j}}$ and. j for the ${ }^{\mathrm{f}} \mathrm{j}$ 'th, DMU and $\Downarrow$ (from regression)

## Environmental Variables: - Z

The index used is ' $q$ ' where $q=$ accessibility factor, parking space availability factor, population density factor and connectivity factor. These environmental variables for the $\mathrm{j}^{\text {th }} \mathrm{DMU}$ are as follows,
$\mathrm{z}_{\mathrm{jacsb}}$ : - accessibility factor
$\mathrm{z}_{\mathrm{jpsaf}}$ : - parking space availability factor
$\mathrm{z}_{\text {jpop }}$ : - population density factor
$\mathrm{z}_{\mathrm{j} \text { conn: }}$ - connectivity factor
$\mathrm{z}_{\mathrm{jcstd}}$ :- comfort standard factor
$\mathrm{z}_{\mathrm{jcomb}}$ : - combination factor

## Efficiency Measures: - ${ }_{j},{ }_{j}$,

## Deviations:-

$\mathrm{p}_{\mathrm{f}}$ : - positive deviation for the ' f ' , externality from the local target for the individual bus route.
$n_{f}$ : - negative deviation for the ' $f$ 't, externality from the local target for individual bus route.
$d_{f}^{+}$: - positive deviation for the ' f ' , externality from the Global target for the complete bus network.
$d_{f}^{-}$: - negative deviation for the ' f ', externality from the Global target for the complete bus network.

In case of the societal variables such as number of accidents, emissions, noise pollution and resources degraded, their levels need to be below the target level. Hence in the model only positive deviations are considered and the objective is to minimize these deviations from the targets. The positive deviations represent the output levels exceeding
the societal variable target levels from above by the bus route and they need to be kept at a minimum.

Targets:
$\mathrm{Ly}_{\mathrm{jf}}$ :- known output target (local) for $f^{\text {th }}$ externality variable of the $\mathrm{f}^{\text {th }}$ DMU
$l y_{\mathrm{jf}}$ :- when the local target is unknown for the ' $f$ 'h, externality
$\mathrm{Gy}_{\mathrm{f}}$ : - when the global target ${ }^{8}$ is known for the ' $f$ 'h, externality
$\mathrm{gy}_{\mathrm{f}}$ : - when the global target is unknown for the ' $f$ ' ${ }^{\text {th }}$, externality

### 3.7 The Model

The model is split into two phases in order to bridge the gap between the available data and the dataset needed for computation purposes. In this section, the model is presented.

### 3.7.1 Phase 1

In the model, accessibility factor, parking space availability, population density of the route, connectivity factor, comfort standard factor (infrastructure factor) are the environmental variables considered. Preliminary calculations on these environmental factors are carried out on in the first phase. There is quite a possibility that because of these environmental variables, many bus routes might be projected as efficient although they might not be. When a DEA model is run on a dataset with consideration of an environmental variable, then the set of DMUs form smaller subsets based on the value of this environmental variable. When there are more environmental variables in a model, then there is a possibility of many subsets of DMUs having only itself in the subset. This occurs if a DMU has a very harsh value for one of the environmental variable. As a result there are greater chances of a DMU being efficient though it might not be actually. The Ruggiero (1998) and Ray (1997) approaches are adopted to account for the above effect.

[^5]
### 3.7.1.1 Phase 1 - Stage 1

The first phase is further split into two stages. In stage 1, the response variable or the efficiency score of the bus route, ' .', is evaluated i.e., efficiency score of the DMU is determined without considering any environmental factors in Stage 1 of Phase 1. This efficiency score inherits the effects of the operating environment. These effects are factored out in the stage 2 of Phase 2 using Simple Linear Regression.

Deriving from the complete network representation of DMU, Ph1_Stl will evaluate DMUs such that the representation is as below,


Figure 3.2 Graphical Representation of the Bus Route Network as per Stage 1, Phase 1

Mathematically,
Phase 1 - Stage 1
$\operatorname{Max} \gamma_{j}$
Subject to:
Output Constraint (Passenger Node)

$$
\begin{equation*}
\sum_{j=1}^{N} \delta_{j_{2}}{ }_{2} y_{j_{p m}}=\gamma_{j_{o}}{ }_{2}{ }^{3} y_{j p m} \tag{3.8}
\end{equation*}
$$

Input Constraint (Passenger Node)

$$
\begin{align*}
& \sum_{j=1}^{N} \delta_{j_{1}}^{2} y_{j s r e l}={ }_{1}^{2} y_{j o s r e l}  \tag{3.9}\\
& \sum_{j=1}^{N} \delta_{j_{1}}^{2} y_{j v m}={ }_{1}^{2} y_{j o v m}  \tag{3.10}\\
& \sum_{j=1}^{N} \delta_{j_{1}}^{2} y_{j t i m}={ }_{1}^{2} y_{j o t i m} \tag{3.11}
\end{align*}
$$

Input Constraint (Provider Node)

$$
\begin{align*}
& \sum_{j=1}^{N} \rho_{j_{0}}{ }^{\prime} x_{j h d}={ }_{0} x_{j o h d}  \tag{3.12}\\
& \sum_{j=1}^{N} \rho_{j_{0}}{ }^{1} x_{j \mathrm{cost}}={ }_{0}{ }^{1} x_{j o \mathrm{cost}}  \tag{3.13}\\
& \sum_{j=1}^{N} \rho_{j}{ }_{0}{ }^{1} x_{j s e r v d}={ }_{0}{ }^{1} x_{j o s e r v d}  \tag{3.14}\\
& \sum_{j=1}^{N} \rho_{j}{ }_{0}{ }^{1} x_{j \text { int }}={ }_{0}^{1} x_{j \text { o int }}  \tag{3.15}\\
& \sum_{j=1}^{N} \rho_{j_{0}}{ }^{1} x_{j \mathrm{prln}}={ }_{0}{ }^{1} x_{j o \mathrm{prln}} \tag{3.16}
\end{align*}
$$

Output (Intermediate) Constraint (Provider Node)

$$
\begin{align*}
& \sum_{j=1}^{N} \rho_{j_{1}}^{2} y_{j s r e l}={ }_{1}^{2} y_{j s s e l}  \tag{3.17}\\
& \sum_{j=1}^{N} \rho_{j_{1}}^{2} y_{j v m}={ }_{1}^{2} y_{j o v m}  \tag{3.18}\\
& \sum_{j=1}^{N} \rho_{j_{1}}{ }^{2} y_{j t i m}={ }_{1}^{2} y_{j o t i m} \tag{3.19}
\end{align*}
$$

Variable Returns to Scale Constraint

$$
\begin{align*}
& \sum_{j=1}^{N} \delta_{j}=1, \sum_{j=1}^{N} \rho_{j}=1  \tag{3.20}\\
& \delta_{j}, \rho_{j}=0 \tag{3.21}
\end{align*}
$$

The output efficiency measure $\left(._{j_{o}}\right)$ for the ${ }^{\prime} \mathrm{jo}_{\mathrm{o}}{ }^{\text {th }}$, DMU is evaluated (Equation 3.7) subject to input and output constraints for the Passenger Node ((3.9), (3.10), (3.11) and (3.8) respectively) and for the Provider Node ((3.12), (3.13), (3.14), (3.15), (3.16) and (3.17), (3.18), (3.19) respectively) disregarding the environmental effects. Variable Returns to Scale (VRS) is represented by equation (3.20). (Although the VRS constraint is expressed here, the results are evaluated for both Constant Returns to Scale as well as Variable Returns to Scale)

### 3.7.1.2 Phase 1 - Stage 2

As stated earlier, in the second stage of this phase (Phase 1), output efficiency measure $\left(._{j_{o}}\right)$ for the ${ }^{\prime} \mathrm{j}_{0}{ }^{\text {th }}$, DMU is regressed against the environmental variables to factor out their effects. The parameters of this Simple Linear Regression are used to estimate a single environmental variable for each bus route. This combined variable is a weighted combination of the individual environmental variables. This combined continuous environmental variable is used in Phase 2, which represents the main model. Mathematically,

Phase 1 - Stage 2:
$\cdot j_{o}=\alpha+\Downarrow_{a c s b} z_{j_{0} a c s b}+\psi_{p s a f} z_{j_{p} p a f}+\psi_{\text {pop }} z_{j_{p} p o p}+\psi_{\text {conn }} z_{j_{o} \text { conn }}+\psi_{c s t d} z_{j_{o} c s t d}+\psi_{\text {inf }} z_{j_{o} \text { inf }}$

$$
\begin{equation*}
z_{j o c o m b}=\sum \Downarrow_{z_{j o q}} \tag{3.22}
\end{equation*}
$$

where ' $q$ ' represents the environmental factors, i.e., accessibility, parking space availability, population density, connectivity, comfort standard and infrastructure factors

Equation (3.22) represents the equation used for regression and equation (3.23) represents the combined environmental variable.

### 3.7.2 Phase 2

In Phase 2, there are three objective functions to optimize, namely
(i) Maximizing the efficiency scaling factor for the final output, passenger-miles, from the Passenger Node, (3.24)
(ii) Minimizing the sum of weighted, normalized deviations of the outputs from their target values for individual bus-routes, (3.25)
(iii) Minimizing the sum of weighted normalized deviations of outputs from their target values for the entire network, (3.26).

Hence a Goal Programming formulation is adopted. One of the fundamental advantages of the GoDEA approach is that the decision maker can state explicit preferences among the various objectives. These explicit preferences are usually denoted by weights. Furthermore, we can solve the problem sequentially. For example, the problem is solved according to the first goal and then this becomes a constraint when solving the problem for the second goal, etc. The objective functions are optimized subject to the constraints for both the nodes simultaneously.

Mathematically,

$$
\begin{align*}
& \text { Max: . jo }  \tag{3.24}\\
& \text { Min: } \quad \sum_{j=1}^{N} \sum_{f} w_{f}^{l}\left(\frac{p_{f}}{L y_{j o f}}\right)  \tag{3.25}\\
& \text { Min: } \quad \sum_{f} w_{f}^{g}\left(\frac{d_{f}^{+}}{G y_{j_{0} f}}\right) \tag{3.26}
\end{align*}
$$

The weights in 3.25 and 3.26 represent the relative preferences for each deviation. However, we can have weights associated with each goal.

The constraints for each bus route (network) are as below.

Subject to:

## Constraints for Passenger Node (Node 2)

Output Constraint

$$
\begin{equation*}
\sum_{j=1}^{N} \alpha_{j}{ }_{2}^{3} y_{j p m}=\cdot{ }_{j_{o}}{ }^{3} y_{j_{o} p m} \tag{3.27}
\end{equation*}
$$

Input Constraints

$$
\begin{equation*}
\sum_{j=1}^{N} \propto_{j}^{2} y_{j v m}={ }_{1}^{2} y_{j_{o} v m} \tag{3.28}
\end{equation*}
$$

$$
\begin{align*}
& \sum_{j=1}^{N} \propto_{j}^{2} y_{j s \text { rel }}={ }_{1}^{2} y_{j_{o} \text { srel }}  \tag{3.29}\\
& \sum_{j=1}^{N} \propto_{j}^{2} y_{j \text { tim }}={ }_{1}^{2} y_{j_{o} \text { tim }} \tag{3.30}
\end{align*}
$$

Variable Returns to Scale for the Passenger Node

$$
\begin{align*}
& \sum_{j=1}^{N} \alpha_{j}=1  \tag{3.31}\\
& \alpha_{j}=0 \quad \text { if } \quad z_{j c o m b}>z_{j_{o} \text { comb }}  \tag{3.32}\\
& \alpha_{j}=0 \quad . \quad \text { j if } \quad z_{j \text { comb }}=z_{j_{o} c o m b} \tag{3.33}
\end{align*}
$$

Consider the Passenger node, i.e., Node2. Constraint (3.27) represents a composite bus route, which is a convex combination of bus routes on the frontier, such that efficiency score for the ' $\mathrm{j}_{0}$ th , bus route and hence the passenger-miles is maximum. The bus routes that contribute to the composite bus route form the reference set for the corresponding bus route. The constraints (3.28), (3.29) and (3.30) represent vehiclemiles, schedule reliability and average traveling time which form the input to the Passenger Node. These are outputs from Node 1 but form the input set to Node 2 and hence the sub-script and super-script on the left side of ' $y$ ' in the constraints. They indicate that the composite bus route which forms the benchmark for the ' $\mathrm{j}_{\mathrm{o}}{ }^{\text {th }}$, bus would have greater or equal value for passenger-miles with lesser or equal values for schedule reliability vehicle-miles and for average traveling time. Equation (3.31), (3.32) and (3.33) represents Variable Returns to Scale and selection of bus routes for the composite bus route only if they operate in similar or harsher environment than the ' $\mathrm{j}_{0}$ th, bus route.

## Constraints for Provider Node (Node 1)

Constraints for inputs

$$
\begin{align*}
& \sum_{j=1}^{N} \cdot{ }_{j}{ }_{0}^{1} x_{j h d}={ }_{0}^{1} x_{j h d}  \tag{3.34}\\
& \sum_{j=1}^{N} \cdot{ }_{j}{ }_{0}^{1} x_{j \cos t}={ }_{0}^{1} x_{j o \cos t} \tag{3.35}
\end{align*}
$$

$$
\begin{align*}
& \sum_{j=1}^{N} \cdot{ }_{j}^{1} x_{0}^{1} x_{j s e r v d}={ }_{0}^{1} x_{j_{j s e r v d}}  \tag{3.36}\\
& \sum_{j=1}^{N} \cdot{ }_{j}^{1} x_{0} x_{j \mathrm{int}}={ }_{0}^{1} x_{j o \mathrm{int}}  \tag{3.37}\\
& \sum_{j=1}^{N} \cdot{ }_{j 0}^{1} x_{j p r \mathrm{ln}}={ }_{0}^{1} x_{j o p r \mathrm{ln}} \tag{3.38}
\end{align*}
$$

Constraints for Outputs

$$
\begin{align*}
& \sum_{j=1}^{N} \cdot{ }_{j}^{2} y_{j v m}={ }_{1}^{2} y_{j o v m}  \tag{3.39}\\
& \sum_{j=1}^{N} \cdot{ }_{j}^{2} y_{j s r e l}={ }_{1}^{2} y_{j o s r e l}  \tag{3.40}\\
& \sum_{j=1}^{N} \cdot{ }_{j} y_{j t i m}=y_{j j_{0} t i m} \tag{3.41}
\end{align*}
$$

Variable Returns to Scale

$$
\begin{align*}
& \sum_{j=1}^{N} \cdot{ }_{j}=1  \tag{3.42}\\
& \cdot{ }_{j}=0, \quad z_{j c o m b}>z_{j c o m b}  \tag{3.43}\\
& \cdot_{j}>0 \quad . \quad \mathrm{j} \tag{3.44}
\end{align*}
$$

Consider the Provider Node, Node 1. Constraints (3.34), (3.35), (3.36) and (3.37) represent restrictions on inputs namely headway, cost service duration and number of intersection. They suggest the selection of peers such that they have a lower, if not equal, value for these inputs than the ' $\mathrm{j}_{\mathrm{o}}$ th' bus route. Equations, (3.38) represent constraint on number of priority lanes. Constraints (3.39), (3.40) and (3.41) represent the outputs, vehicle-miles, schedule reliability and average traveling time, from Node 1 which form the inputs for Node 2. These constraints indicate that the composite bus route should have greater than or equal value for vehicle-miles, schedule reliability and trip time for the composite route of ' $\mathrm{j}_{\mathrm{o}}$ th, bus route Equations (3.42), (3.43), (3.44) are the Variable Returns to Scale constraints.

## Constraints for Local targets and Global targets

Local target for externalities for individual bus-route

$$
\begin{equation*}
\sum_{j=1}^{N} \cdot{ }_{j 1}^{3} y_{i f}-p_{f}+n_{f}={ }_{1}^{3} L y_{j o f} \tag{3.45}
\end{equation*}
$$

. f (externalities, whose targets are known)

$$
\begin{equation*}
\sum_{j=1}^{N} \cdot{ }_{j=1}^{3} y_{j f \ni}={ }_{1}^{3} l y_{j o f \ni} \tag{3.46}
\end{equation*}
$$

- fЭ(externalities, whose targets are not known)

Global target for externalities for the entire network
$\sum_{i=1}^{N} \sum_{j=1}^{N} .{ }_{j 1}^{3} y^{i}{ }_{j f}-d_{f}^{+}+d_{f}^{-}=G y_{f}$

- f (externalities, whose targets are known)

$$
\begin{equation*}
\sum_{i=1}^{N} \sum_{j=1}^{N} \cdot{ }_{j 1}^{3} y^{i}{ }_{i f \ni}=g y_{f \ni} \tag{3.48}
\end{equation*}
$$

- $\mathfrak{f}$ (externalities, whose targets are not known)

$$
\begin{equation*}
p_{f}, n_{f}, d_{f}^{+}, d_{f}^{-}, g y_{f}, l y_{f}=0 \cdot f \tag{3.49}
\end{equation*}
$$

Constraint (3.45) represents externalities like number of accidents, noise pollution, emission and dollar value of resource degraded flowing from Node 1 to Node 3. It suggests the selection of peers such that the negative or the positive deviation from the known targets for an externality is least. Equation (3.46) is used if the local target for certain externality of a bus route is not known. Equations (3.47) and (3.48) are used to represent the achievement of Global Targets if known and if not known respectively.

According to Athanassopoulos (1995), the global targets for externalities can be determined as follows,

$$
\begin{equation*}
G y_{f}=. \quad{ }_{f}\left(\sum_{j=1}^{N}{ }_{1}^{3} y_{j f}\right) \tag{3.50}
\end{equation*}
$$

where '. $\quad$ 'is obtained by solving the following linear programs as many times as many targets are sought.
Min .

Subject to

$$
\begin{equation*}
\sum_{j=1}^{N} \cdot{ }_{j 1}^{3} \mathcal{Y}_{\hat{j f}}=. \quad \sum_{j=1}^{N}{ }_{1}^{3} y_{j \hat{f}} \tag{3.52}
\end{equation*}
$$

For other externalities,

$$
\begin{equation*}
\sum_{j=1}^{N} \cdot{ }_{j 1}^{3} y_{i f}=. \quad \sum_{j=1}^{N}{ }_{1}^{3} y_{j f} \tag{3.53}
\end{equation*}
$$

For controllable outputs,

$$
\begin{equation*}
\sum_{j=1}^{N} \cdot{ }_{j}^{2} y_{j}=\sum_{j=1}^{N}{ }_{1}^{2} y_{j} \tag{3.54}
\end{equation*}
$$

For non-controllable outputs,

$$
\begin{equation*}
\sum_{j=1}^{N} \cdot{ }_{j 1}^{2} y_{j}=\sum_{j=1}^{N}{ }_{1}^{2} y_{j} \tag{3.55}
\end{equation*}
$$

For controllable inputs,

$$
\begin{equation*}
\sum_{j=1}^{N} \cdot{ }_{j}{ }_{0}^{1} x_{j}=\sum_{j=1}^{N}{ }_{0}^{1} x_{j} \tag{3.56}
\end{equation*}
$$

For non-controllable inputs

$$
\begin{equation*}
\sum_{j=1}^{N} \cdot{ }_{j}{ }_{0}^{1} x_{j}=\sum_{j=1}^{N}{ }_{0}^{1} x_{j} \tag{3.57}
\end{equation*}
$$

$$
\begin{equation*}
. \geq 0 \tag{3.58}
\end{equation*}
$$

Additional notations used:

- $\quad$ : factor to scale each externality to estimate target value
- : intensity

Constraint (3.52) represents the selection of DMUs such that the externality ' $\hat{f}$ ' is maximized. Equations (3.53), (3.54), (3.55), (3.56) and (3.57) represent constraints on other externalities, controllable outputs, non-controllable outputs, controllable inputs and non-controllable inputs respectively.

The above model had been used to generate results. These results are presented in the next Chapter (Chapter 4). In this chapter, the datasets utilized and the data generation methods are also presented.

## Chapter 4 <br> Data Generation \& Results

This research effort utilizes the concepts of Data Envelopment Analysis (DEA) (Charnes, Cooper \& Rhodes (1978)), Network Model in DEA (Färe (1991)) and GoalDEA (Athanassopoulos (1995)) to evaluate the efficiency of Urban Transit System, specifically, bus routes. This chapter discusses the results obtained from the model proposed in Chapter 3 (Section 3.7). This chapter is broadly divided into four sections. The first section (4.1) presents the datasets used in the model. The second section (Section 4.2) discusses the variations considered in the model. The software used for the purpose of evaluation of bus routes is discussed in the third section (4.3). The fourth section (4.4) describes the code developed for evaluating the datasets. The results obtained are summarized in the fifth section (Section 4.5).

### 4.1 Data Generation Framework

Data forms an important part of any research effort. This is because it aids testing of the model, for the validation and verification purposes, to determine the robustness of the model of any model. It is also very important that the model and data do not influence each other, or else the results will be biased. Due to this the model might bear some limitations such as data sensitivity (the model result may vary with respect to different datasets) and this may lead to insignificant conclusions.

In this research effort, two data sets are considered, namely, (i) 100 bus routes that are simulated and is referred to as the 100 Simulated Dataset and (ii) 8 bus routes collected from the Blacksburg Transit Authority and will be referred to as the 8 BT Dataset. This section describes these datasets as well as the logic used to simulate data for the variables used in the model (Description for these variables is provided in Chapter 3, Section (3.3)).

### 4.1.1 Types of Datasets

## (a) The 100 Simulated Dataset

The 100 Simulated Dataset is a simulated dataset for a hypothetical transit agency operating 100 bus routes. Of these 100 bus routes, certain provide service within an urban area, certain operate between urban areas and the suburbs and the remaining operate in the suburban areas. For this research, of these 100 routes, urban, suburban and sub_urban (bus routes that operate between urban and suburban areas) are determined by assigning a Uniform Random Variable (RV) between 0 and 1 for these 100 DMUs. All the DMUs with Uniform RV below 0.33 are classified as suburban, with Random Variable between 0.331 and 0.66 are classified as suburban to urban and all the remaining are classified as urban.

## (b) 8 BT Dataset

This dataset is based on the data collected for the 8 routes on which the Blacksburg Transit Agency (BT) provides service. The data is available for the following variables.
(i) Route length
(ii) Number of buses per day
(iii) Average Traveling Time
(iv) Service Duration
(v) Headway
(vi) Number of intersections
(vii) Vehicle-miles
(viii) Number of Accidents
(ix) Cost
(x) Passenger-miles

Accordingly, two sub-datasets are considered. The first sub-dataset represents only the available data and the second sub-dataset has data from the first subdataset and has simulated data for the variables for which there is no data available. The data is simulated for the following variables based on general observation and adopting the same procedure as for the 100 bus routes as explained above:
(xi) Environmental or Traffic Operating factors, namely,
a. Accessibility Factor
b. Parking Space Availability Factor
c. Connectivity Factor
d. Population Density Factor
e. Comfort Standing Factor
f. Infrastructure Factor
(xii) Schedule Reliability
(xiii) Emissions
(xiv) Noise Pollution

### 4.1.2 Assumptions

The following assumptions form the basis for the data generation methods employed in this research effort. The data for all the variables is simulated based on the following assumptions.

The bus routes are isolated entities, have similar activities and comparable products
Although this research effort deals with an Urban Transit Network, bus routes offering services in the suburbs of this urban area are also considered as they belong to the same transit agency that provides services in the urban area. The assumption is that bus routes offering services in the suburbs form a part of the transit system network.

An effort to isolate independent variables is made and their functional relationship with other required (dependent) variables is utilized to generate data. This approach is adopted as it assumes technological or organizational dependencies between variables.

Service vehicles operate continuously, i.e., the trips are made immediately although the trip times are inflated by 10 percent to accommodate the break time before starting a new trip.
The trip in either way is considered as different routes even if they are symmetrical. The total number of service vehicles operated by the Transit

Agency is much less than the sum of service vehicles on each route. This is achieved by scheduling the same bus on different routes at different times during the week.

### 4.1.3 Data Generation for the Variables

The procedure utilized to generate data for independent and dependant variables is as follows,

### 4.1.3.1 Variables

## (i) Headway

The 'headway' for these 100 bus routes is generated by using random number generation methods. This is accomplished using 'Randbetween' function in Excel. This worksheet function generates uniform random integers between the limits specified inclusive of the limits.

For example: Randbetween $(4,9)$ will generate integers from the set $\{4,5,6,7,8$, 9\} using Uniform Random Number Distribution.

Observing the distribution of the 'headway' for a local transit agency (Blacksburg Transit Agency) offering services as well as discussion in group meetings, the following ranges were decided for the hypothetical routes, (The assumption here is that the hypothetical agency with 100 routes has the same characteristics as BT)

| Type of Route | Range | Worksheet Function |
| :--- | :--- | :--- |
| Urban | $8-12 \mathrm{~min}$ | $2 *$ randbetween $(4,6)$ |
| Sub_Urban | $10-25 \mathrm{~min}$ | $5 *$ randbetween $(2,5)$ |
| Suburban | $20-60 \mathrm{~min}$ | $5 *$ randbetween $(4,12)$ |

Table 4.1 Range for Headway

Consider the range of 'Headway' for the Urban Routes. 'Randbetween (8, 12)' will generate Uniformly Distributed Random Integers from the set $\{8,9,10,11,12\}$ As it is assumed that 'Headway' can be 8 minutes or 10 minutes or 12 minutes, a factor 2 is
multiplied to the random numbers generated by the 'randbetween $(4,6)$ ' function. Due to this factor, the value of the 'Headway' will not be 9 or 11. Similarly for the 'Headway' ranges for Suburban and Sub_Urban routes.

As 'Headway' is an input, in DEA representation in the model, the reciprocal of 'Headway' is used so that input variable for the DMU is lesser than the corresponding input variable for the virtual DMU. This is in accordance with the assumptions of DEA.

## (ii) Service Duration

The 'service duration' for these 100 bus routes is generated similarly by using random number generation methods in Excel. Observing the distribution of the 'service duration' for a local transit agency (Blacksburg Transit Agency) offering services, the following ranges were decided for these 100 hypothetical routes,

## Type of Route Range Worksheet Function

| Urban | $16-19$ hours | randbetween $(16,19)$ |
| :--- | :--- | :--- |
| Sub_Urban | $16-19$ hours | randbetween $(16,19)$ |
| Suburban | $16-19$ hours | randbetween $(16,19)$ |

Table 4.2 Range for Service Duration

The 'Randbetween $(16,19)$ ' generates uniformly distributed random integers from the set $\{16,17,18,19\}$.

## (iii) Costs

The 'Costs' variable indicates the total costs to provide service on the route. It is generated for all, i.e., suburban, sub_urban and urban routes, using the following expression

Total Costs $=($ Costs per hour of service provided $) *($ Avg. Service Duration $)$
Cost per hour of service provided is assumed to be a random number following a Uniform distribution, lying between $\$ 40$ and $\$ 60$ for all the three types of bus routes, namely, Urban, Suburban and Sub_Urban. These ranges were selected based on the data

Blacksburg Transit Authority data and group meetings. The costs per hour for 100 bus routes are generated by 'randbetween' function in the Excel worksheet.

## (iv) Route Length

Although the 'route length' is not directly used in the model, it is used for generating data for other variables that are a function of the 'route length'. This variable is generated by using random number generation methods. Observing the distribution of the 'route length' for a local transit agency offering services (Blacksburg Transit Agency), the following ranges were decided for these hypothetical routes,

## Type of Route Range Worksheet Function

| Urban | $6-10$ miles | risktriang $(6,8,10)$ |
| :--- | :--- | :--- |
| Sub_Urban | $8-15$ miles | risktriang $(8,12,15)$ |
| Suburban | $10-25$ miles | risktriang $(10,18,25)$ |

Table 4.3 Range for Route Length

The ranges for 'Route Length' for all three (Urban, Suburban, Sub_Urban) routes are based on the distribution of Blacksburg Transit Authority data, National Transit Database (2001) and group meetings. It was decided to use Normal distribution to generate this data but as the minimum, maximum and median values for the ranges of each type of bus route are known, triangular distribution is utilized. The 'Risktriang' function generates random numbers following Triangular distribution. The parameters for this function are in the parentheses. The parameters represent the minimum, median and maximum of the range for the distribution. These parameters for the routes differ for each sub category as the ranges differ for each category.

The 'Risktriang' function is not a part of the standard Excel package. A free trial version can be downloaded from www.palisade.com.

## (v) Operating Speed

Although the 'operating speed' is not directly used in the model, it is used for generating data for other variables that are a function of operating speed. This variable is
generated by using random number generation methods. Observing the distribution of the 'operating speed' for a local transit agency offering services (Blacksburg Transit Agency) and group meetings, the following ranges were decided for these hypothetical routes,

| Type of Route | Range | Worksheet Function |
| :--- | :--- | :--- |
| Urban | $10-15 \mathrm{mph}$ | risktriang $(10,12,15)$ |
| Sub_Urban | $15-30 \mathrm{mph}$ | risktriang(15,20,30) |
| Suburban | $20-35 \mathrm{mph}$ | risktriang(20,30,35) |

Table 4.4 Range for Operating Speed

It was decided to use Normal distribution to generate this data but as the minimum, maximum and median values for the ranges of each type of bus route are known, triangular distribution is utilized. The 'Risktriang' function is used as it generates random numbers following Triangular distribution. The parameters for this function are in the parentheses. The parameters represent the minimum, median and maximum of the range for the distribution. These parameters for the routes differ for each sub category as the ranges differ for each category.

## (vi) Average Traveling Time

'Average Traveling Time' represents the average of the total time for a service vehicle to traverse the complete route. The following expressions are used to generate data for 'Average Traveling Time',

Traveling Time' $=$ Route Length/Operating Speed
Traveling Time $=$ 1.1 Traveling Time'
Avg. Traveling Time $=\frac{\text { Avg. of Traveling times of all the service vehicles on the route }}{\text { Congestion }}$

Note: An assumption that the initial time (break time etc.) to be 10 percent of Trip Time is made.
'Congestion' is used in the above expression as it affects the traveling time. Intuitively, greater the value for 'Congestion', greater is the traveling time. 'Congestion' is generated as a Uniform Random Number, lying between ' 0 ' and ' 1 ' with ' 1 ' being maximum 'Congestion'. As a result, the 'Avg. Traveling Time' is divided by the value of 'Congestion'.

For these 100 bus routes, 'Congestion' is generated by using Triangular Distribution. This is because the minimum, maximum and median of the range for 'Congestion' is known. The parameters for this function are in the parentheses. The parameters represent the minimum, median and maximum of the range for the distribution.

These range parameters are based on the data Blacksburg Transit Agency data and group meetings,

| Type of Route | Range | Worksheet Function |
| :--- | :--- | :--- |
| Urban | $0.25-0.55$ | risktriang $(0.25,0.45,0.55)$ |
| Sub_Urban | $0.35-0.7$ | risktriang $(0.35,0.55,0.7)$ |
| Suburban | $0.6-0.9$ | risktriang $(0.6,0.82,0.9)$ |

Table 4.5 Range for Congestion

## (vii) Number of intersections

This variable represents the total number of signalized and unsignalized intersections. This variable is generated by dividing the route length with the avg. distance between successive intersections on the corresponding routes. No. of intersections $=$ Route length/Avg. Distance between successive intersections

The range for the number of intersections in Urban, Sub_Urban and Suburban routes are generated using random number generation functions in Excel based on the distribution of the intersection (signalized and unsignalized) for an urban area and its suburbs Blacksburg Transit Agency and Fairfax Connector, the following ranges were decided for these hypothetical routes,

| Type of Route | Range | Worksheet Function |
| :---: | :---: | :---: |
| Urban | 100-150 | ROUND (1000*Route Length*1.6/X,0) |
|  | meters | Where ' $X$ ' = randbetween (100,150) |
| Sub_Urban | 200-400 | ROUND (1000*Route Length*1.6/X,0) |
|  | meters | Where ' X ' = randbetween $(200,400)$ |
| Suburban | 250-500 | ROUND (1000*Route Length*1.6/X,0) |
|  | meters | Where ' X ' = randbetween $(250,500)$ |

Table 4.6 Range for Number of Intersections

The above function generates a Random Variable in the specified range. The worksheet function has a factor of 1.6 to convert miles to kilometers and a factor of 1000 Route length in miles to kilometers and 1000 to convert kilometers to meters.

As the 'Number of intersections' is an integer, worksheet function 'Round' will round the value obtained from the above function to the nearest integer.

Inverse of 'Number of intersections' is used in the model to keep in accordance with the assumptions of DEA. 'Number of intersections' is an input and input variable for the DMU whose efficiency score is evaluated is lesser than the corresponding input variable for the virtual DMU. Hence in the DEA representation in the model, the reciprocal of 'Headway' is used.

## (viii) Number of priority lanes

The 'Number of priority lanes' for these 100 bus routes is generated by using random number generation methods in Excel. The following ranges were decided for these hypothetical routes based on discussions in group meetings,

| Type of Route | Range | Worksheet Functio |
| :--- | :--- | :--- |
| Urban | 0,1 | randbewteen $(0,1)$ |
| Sub_Urban | 0,2 | randbetween $(0,2)$ |
| Suburban | 0,2 | randbetween $(0,2)$ |

Table 4.7 Range for Priority Lanes

## (ix) Vehicle-miles

In this model, 'Vehicle-miles' is an intermediate product. It forms the output set for the Provider Node and input set for the Passenger Node. This variable is generated using the following relation,

Vehicle-miles per day $=$ Route length*Number of Trips per day
'Number of trips' on a route is evaluated using the following expression,
Number of Trips per day = Service Duration/Headway
As this expression may yield non-integral values, Excel worksheet function, Round((Service Duration)/(Headway),0) is used to round the value of 'Number of Trips' to the nearest integer.
'Route Length' generation is described in Section 4.1.3, (iv).

## (x) Schedule Reliability

'Schedule Reliability' measures the reliability of the transit schedule. It captures the delay due to 'Congestion'.

In this research, the data for this variable is generated such that it measures the number of times the service vehicle arrives on time at the bus stop in a day. The average over all the service vehicles for that route over the service duration is obtained. The following formula is used in the worksheet,

$$
=\text { Round }\{(1-\text { congestion }) * \text { Number of bus stops })\}
$$

'Congestion' is determined as described in Section 4.1.3 (vi). This product yields number of times the service vehicle arrives on time at the designated stops. 'Round' function rounds the number to the nearest integral value.

## (xi) Passenger-miles

'Passenger-miles' form the final output, i.e., the output at Passenger Node. It is a measure of the total number of passengers that use that route and the miles traveled during the day. This variable is simulated using the following expression

Passenger-miles $=$ vehicle-miles*avg. actual capacity
'Avg. Actual Capacity' represents the seats taken on an average during a day. The data for this variable for these three types of routes is generated using the following ranges as discussed in group meetings,

## Type of Route

Urban
1-45
Sub_Urban
1-50
1-55
Suburban

Range Worksheet Function
randbewteen $(1,45)$
randbetween $(1,50)$
randbetween $(1,55)$

Table 4.8 Range for Passenger-miles

### 4.1.3.2 Environmental factors

The following is the description of the procedure utilized to generate Environmental factors, namely, 'Accessibility factor', 'Connectivity factor', 'Parking space availability factor', 'Population density factor', 'Comfort Standard factor' and 'Infrastructure factor'. The data for these Environmental variables is generated using 'Randbetween' function. These factors are represented by numbers ranging from 1 to 5 with 1 representing the 'best' environmental conditions and ' 5 ' representing the harshest environmental conditions.

## (i) Accessibility factor

The 'Accessibility factor' is a Uniform Random integer lying between 1 and 5 (both inclusive). The ranges within which this factor would lie for the Urban, Sub_Urban and Suburb routes were decided in group meetings and are as follows,

Type of Route

| Urban | $1-3$ | randbetween $(1,3)$ |
| :--- | :--- | :--- |
| Sub_Urban | $2-4$ | randbetween $(2,4)$ |
| Suburban | $3-5$ | randbetween $(3,5)$ |

Table 4.9 Range for Accessibility Factor

## (ii) Connectivity Factor

This factor depends upon the number of bus stops on the route length and distribution of population along the route length. The 'number of bus stops' needed on the route can be determined by the following expression,

No. of bus stops $=$ Route Length/determined distance between bus stops

- where distance between stops is generated using Uniform distribution.

After determining the 'Number of bus stops on a route' the values for 'Connectivity Factor' are determined as below. The ranges for the three types of routes are determined though group meetings.

## Type of Worksheet Function

## Route

Urban

Sub_Urban
IF ((Number of bus stops $<=16$ ), randbetween (3,5), (IF((Number of bus stops $>=18)$, randbetween $(1,3)$, randbetween $(2,4))$ )) IF ((Number of bus stops < = 23), randbetween (3,5), (IF((Number of bus stops $>=28)$, randbetween $(1,3)$, randbetween $(2,4))$ )) IF ((Number of bus stops $<=36$ ), randbetween (3, 5), (IF((Number
Suburban of bus stops $>=41$ ), randbetween $(1,3)$, randbetween $(2,4))$ ))

Table 4.10 Range for Connectivity Factor

Consider the function for Sub_Urban routes,
This worksheet function would assign a uniform random number lying between 3 and 5 if the number of bus stops (determined as explained earlier) are less than or equal to 36 , else it would assign a uniform random number lying between 1 and 3 is the number of bus stops on the route are more than or equal to 41 and it would assign a random number lying between 2 and 4 for the number of bus stops between 36 and 41 .

## (iii) Population Density Factor:

'Congestion' on and around the route is a fair indicator of the population density on the route or the popularity of the route. This is utilized to generate 'Population density factor' for these 100 routes.

Depending upon the range in which 'Congestion' Section 4.1.3 (vi) lies a random number if allotted to the route. The worksheet function is a conditional formula similar to the one used to generate 'Connectivity Factor' as depicted below. The ranges for the three types of routes is determined during group meetings

## Type of Worksheet Function

## Route

Urban

Sub_Urban
IF((Congestion $<0.75)$,randbetween(3,5), IF((Congestion $>=0.8)$, randbetween $(1,3)$,randbetween $(2,4))$ ))
IF ((Congestion $<0.49$ ), randbetween $(3,5)$, (IF((Congestion $>=$
$0.58)$, randbetween $(1,3)$, randbetween $(2,4)))$ )
IF((Congestion $<0.38)$, randbetween $(3,5)$, (IF((Congestion $>=0.45)$,
Suburban
randbetween $(1,3)$, randbetween $(2,4))$ ))

Table 4.11 Range for Population Density Factor

## (iv) Comfort Standard factor

This variable is associated to the relative loading of the service vehicle in other words, the availability of seats on the service vehicles. As a result, a loading factor is
allotted to these 100 routes with 1 being excellent and 5 being very poor. The worksheet formula is generated similarly to 'Connectivity factor' in sense that it is a conditional function and is as follows. The range of 'Comfort Standard Factor' for the three types of routes is determined during group meetings.

```
Type of Worksheet Function
Route
    IF ((Loading Factor \(<=0.38\) ), randbetween(1,3), (IF((Loading
Urban
    Factor \(>=0.58\) ), randbetween \((3,5)\), randbetween \((2,4)\) )))
    IF(( Loading Factor \(<=0.5\) ), randbetween(1,3), (IF ((Loading Factor
Sub_Urban
    \(>=0.6)\), randbetween \((3,5)\), randbetween \((2,4))\) ))
    IF ((Loading Factor \(<=0.33\) ), randbetween(1,3), (IF)(Loading
Suburban
    Factor \(>=0.67\) ), randbetween \((3,5)\), randbetween \((2,4)\) )))
```

Table 4.12 Range for Comfort Standard Factor

## (v) Infrastructure factor

This variable relates to the infrastructure on the route, mainly bus stops and priority lanes. It is used for a variation of the model that does not have a constraint for 'Number of priority lanes' and 'Number of intersections'. It is generated based on ranges decided during group meetings. They are as follows,

## Type of Route

Urban 3-5
Sub_Urban 2-4
Suburban
1-4

## Worksheet Function

randbetween $(3,5)$
randbetween $(2,4)$
randbetween $(1,4)$

Table 4. 13 Range for Infrastructure Factor

### 4.1.3.3 Societal Variables

The procedure used to simulate data for the societal variables, namely, ' 'Number of Accidents’, 'Resources Degraded', 'Noise Pollution' and 'Emissions' is as below,

## (i) Number of Accidents

This variable includes number of accidents as well as incidents that occur on a route with regards to all the service vehicles operating on that route for the entire period during which the service is offered on that route per day.

The data for this variable is generated using 'Randbetween' function. The ranges were decided during group meetings and with reference to Blacksburg Transit Authority data.

| Type of Route | Range | Worksheet Function |
| :--- | :--- | :--- |
| Urban | $0-12$ | randbetween $(0,12)$ |
| Sub_Urban | $0-17$ | randbetween $(0,17)$ |
| Suburban | $0-15$ | randbetween $(0,15)$ |

Table 4.14 Range for Number of Accidents

## (ii) Resources Degraded

This variable represents all the resources that are degraded due to the provision of transit services. In this research, the data for this variable is generated as a dollar value that corresponds to the resources that are degraded,

The data is generated from a Uniform Distribution with the following ranges. These ranges were decided in group meetings.

|  | Range | Worksheet Function |
| :--- | :--- | :--- |
| Type of Route |  |  |
| Urban | $900-1250$ | randbetween $(900,1250)$ |
| Sub_Urban | $700-900$ | randbetween $(700,900)$ |
| Suburban | $500-750$ | randbetween $(500,750)$ |

Table 4.15 Range for Resources Degraded

## (iii) Emissions

This variable represents the emissions from the service vehicles. It is used to measure air quality and the contribution of the service vehicles to pollution levels as a result of provision of transit service. Generally the pollution levels in the air are measured as 'parts per million' (ppm). This represents the parts of the pollutant per million parts of air (www.epa.com). For the purpose of this research, the data for this variable is generated using the following expression.

$$
\begin{equation*}
\text { Emission }=(\text { Number of vehicles }) *(\text { trip time }) *(X) \tag{4.10}
\end{equation*}
$$

It is assumed that ' X ' which is Uniform Random Number between 0 and 1 represents the contribution of pollutants by a service vehicle operating on a route during a day. Thus the expression measures 'emissions' by all the service vehicles on the route considering their actual operating time.

## (iv) Noise Pollution

This variable measures the noise pollution contributed by the service vehicles as a result of transit services. The data for 'Noise Pollution' along the route is generated using the following expression,

$$
\begin{equation*}
\text { Noise pollution for that route }=\text { Noise Pollution (in db)/headway } \tag{4.11}
\end{equation*}
$$

This variable would measure the 'Noise Pollution' in $\mathrm{db} / \mathrm{min}$. The ranges for Noise Pollution for the three types of routes were decided based on the following two sources and group meetings discussions,
http://home.earthlink.net/~wphyilla/iplparkx.html
http://www.geocities.com/trolley_coalition/noise.html

Type of Route

| Urban | $80-85 \mathrm{db}$ | randbetween $(80,85)$ |
| :--- | :--- | :--- |
| Sub_Urban | $78-82 \mathrm{db}$ | randbetween $(78,82)$ |
| Suburban | $75-80 \mathrm{db}$ | randbetween $(75,80)$ |

Table 4.16 Range for Noise Pollution

The complete datasets are provided in the Appendix B (1.1, 1.2, 2.1 and 2.2). Please refer Appendix B 2.1 and B 2.2 for 'The 100 Simulated Dataset' and Appendix B 1.1 and 1.2 for 'BT Dataset'.

### 4.2 Model Variations



Figure 4.1 is a graphical representation of a 'Bus Route'.

The math model to evaluate the efficiency of Public Transit Systems is conceptualized in Chapter 3. It has 2 stages with the first stage having two phases. In the first phase of the first stage, the efficiency of the Passenger Node (Node 2 of Figure 4.1) is computed. This efficiency score includes the environmental effects. In the second
phase, the efficiency score of each DMU is regressed against the environmental variables and a single combined environmental variable or traffic operating factor is computed. In the second stage, the efficiency of each bus route, i.e., of each network representation of the bus route is evaluated with the consideration of the operating environment that is characterized by the combined environmental factor.

In the second phase, the objective function consists of three goals or sub-objective functions. For the first objective function, the efficiency score of each bus route is maximized. For the second objective function, the local deviations of the societal variables for each bus route are minimized. In these computations, the efficiency score (evaluated as a result of the first objective function) for the bus route is taken as a known value. Similarly for the third objective function, i.e., minimizing global societal deviations, the efficiency scores of the bus routes are substituted from the first objective function. These formulations are evaluated for Constant Returns to Scale as well as Variable Returns to Scale assumptions.

This approach can be classified as hierarchal. It is also possible to have a simultaneous approach wherein, the model is modified to have a single objective function. This is achieved by weighing and normalizing the sub-objective functions. The choice of weights can be determined by the user. These weights indicate the relative importance of the sub-objective functions. 'Unitizing' aids to add up the different quantities, namely the efficiency score with local deviations and global deviations. In this case, the objective function would not have physical significance as the sub-objective functions measure different quantities (deviations and efficiency score). It would still be possible to compare the DMUs. This is because it would be equivalent for all the DMUs, it.

In this research, the variations of the models that are considered are presented in the Table 4.17.

Sr. No. Types of Dataset

1. 100 Simulated Dataset
2. 

BT Dataset

Types of Model
Infrastructure 1

Infrastructure 2 CRS
VRS
. ,
Real Dataset CRS

VRS
Simulated Dataset CRS
VRS

Table 4. 17 Variations in the Model

## (a) The 100 Simulated Dataset

For the 100 Simulated Dataset, Infrastructure 1 and Infrastructure 2 are the two model variations that are considered. The purpose of these variations is to study the differences in the results obtained when a certain quantity is used as a variable and when it is represented in the model as an environmental variable.
(i) Infrastructure_1: In this variation, two infrastructure variables namely, 'Number of intersections' and 'Number of priority lanes' are used in the model. Constant Returns to Scale (CRS) and Variable Returns to Scale (VRS) assumptions are considered.
(ii) Infrastructure_2: In this variation, an environmental infrastructure variable is used instead of the two infrastructure variables ('Number of intersections' and 'Number of priority lanes') are used in the model. Constant Returns to Scale (CRS) and Variable Returns to Scale (VRS) assumptions are considered.

## (b) BT Dataset

BT Dataset is broadly classified as BT_Real and BT_Simulated. BT_Real uses the data obtained from Blacksburg Transit Authority. In case of BT_Simulated, due to unavailability of data for all the variables defined in the model, data was simulated for the remaining variables. Both Constant Returns to Scale and Variable Returns to Scale assumptions are considered for each of the BT model variation.

### 4.3 Software

Visual Basic for Applications (VBA) and Premium Solver in Excel are used as the software to develop the code to evaluate the efficiency of the bus routes, compute target levels for societal variables such as number of accidents, emissions, noise pollution and resources degraded for these bus routes and over the entire network of bus routes. VBA used as a programming language enables control over the flow of computations, iterations, the generating and storing of results and the generating charts and graphs. Premium Solver is used for computations. Excel Analysis Tool Pack is used for the regression.

### 4.4 Code

This section describes the code (Please Refer to Appendix A (1.1, 1.2, 2.1 and 2.2)) that has been developed for the illustration of the model. The model variations are discussed in Section 4.2. These models differ form each other by (inclusion or exclusion) of certain variable(s) or constraint(s). In spite of these differences, the code for all these models is similar.

The program is divided into smaller modules for computational and coding convenience. The modules and their description are described as follows,
(a) Clear

This module clears all the values from the previous run. In other words, it resets the model and restores the initial values of all the variables. This enables the Solver to begin its computation from a set of initial values than the existing values.
(b) Solve_1_2

This module solves the first stage of the first phase. It evaluates the efficiency of the network or the bus route without considering the effects of the environmental or the traffic operating factors (namely; the accessibility, the connectivity, the parking space availability, and popularity factors). The output of this module provides the efficiency scores for each bus route. These are used in the next module, i.e., Reg_1_1.
(c) Reg_1_1

This represents the second stage of the first phase of the model. The efficiency scores of the bus routes are regressed against all the traffic operating and environmental factors to consider the effects of all these factors. The output of this module is a single environmental variable that represents a weighted sum of other environmental factors for each bus route.
(d) Sort_Env_1_2

This macro sorts all the bus routes in descending order of the values of the combined environmental factor. This macro is used for ease of computational ease only. It is required that a DMU be compared with only with those DMUs that operate in similar or harsher environment. As a result, sorting of the DMUs according to the descending order of the environmental factor aids the development of an iterative loop such that the DMU is compared with other DMUs operating in similar or harsher environment.
(e) Solve_2_Efficiency

This module forms the first part of the main model. It evaluates the efficiency of the bus routes with the consideration of the environment within which the bus routes offer service. In this module, the bus routes are compared with only those bus routes operating within similar or harsher environments (worse conditions), i.e., with those bus routes with a higher or equal value of the combined environmental factor.

## (f) Solve_2_Local Targets

This module evaluates the deviations for each bus route with respect to their societal variables (i.e., accidents, emissions, noise pollution, and resources degraded). These deviations are computed from the target levels set for the societal variables by the concerned authorities.
(g) Solve_3_Global Targets

This module evaluates the deviations of the same societal variables for the whole network. Again, the global target levels are pre-set by concerned authorities.

These modules are represented in a flowchart that follows

| MODULES | ACTION | $\begin{gathered} \text { OUTPUT } \\ \text { GENERATED } \end{gathered}$ | PURPOSE |
| :---: | :---: | :---: | :---: |
| CLEAR | Clears the output from the previous iteration. <br> Resets the values of all variables to zero. | ' 0 ' for all variable cell and Blank cells for the regression and results range. | Restrict Excel from starting a program run from the values of the previous iteration. |
| SOLVE_1 | Evaluates the Phase 1 Efficiency score (Ph_1) for each DMUs. | Ph_1 Efficiency scores for all DMUs. | Determine the efficiency score without considering the environmental factors. |
| REG_1_1 | Regresses the Ph1_efficiency score of each of the DMU with the corresponding values of their environmental factor. | Combined <br> Environmental factor. | Factors out the effect of individual environmental factor in order to determine the Combined Environmental factor. |
| SORT_1_2 | Sorts DMUs in descending order of intensity of the combined environmental factor. | DMUs arranged according to decreasing degree of harshness. | Compute the efficiency score of DMUs with respect to DMUs operating in similar or harsher environments. |
| SOLVE EFFICIENC Y | Evaluates the Phase 2 Efficiency score (Ph_2) for each DMU. | Ph_2 Efficiency scores for all DMUs. | Determine the efficiency score with regards to the combined environmental factors. |
| $\begin{gathered} \hline \text { SOLVE } \\ \text { LOCAL } \\ \text { TARGETS } \\ \hline \end{gathered}$ | Evaluates the local deviation of societal variables from their predetermined targets. | Deviations for bus routes for societal variables. | Determine the weighted sum of deviations for societal variables of bus routes from the set target levels. |
| SOLVE GLOBAL TARGETS | Evaluates the global deviation of societal variables from their global predetermined targets. | Deviations for the entire network for global societal variables. | Determine the weighted sum of deviations for societal variables of the entire network from the set global target levels. |

Figure 4.2 Flowchart representing the Code Structure

### 4.5 Results

For BT model, real and simulated datasets for both the VRS and CRS cases are considered. The datasets for each model variation are evaluated using Excel Solver and the results are generated. This section presents the results for these different models and datasets.

For 100 hypothetical bus route the results are generated for the following models, Infrastructure 1

Infrastructure 2
The datasets for each model variation are evaluated using Excel Solver and the results are generated. This section presents the results for these different models and datasets.

### 4.5.1 The BT Dataset

This section illustrates the results for CRS and VRS models for (i) The Real Data (ii) Partially Simulated Data BT Datasets.

### 4.5.1.1 BT Real (Original) Dataset

This dataset has the following variables for the 8 bus routes on which BT provides service,

1) Headway (represented as Service Frequency)
2) Total costs
3) Service duration
4) Number of intersections
5) Average Traveling Time
6) Vehicle-miles
7) Passenger-miles
8) Number of Accidents

The following table (Table 4.18) provides the summary of the results for the BT real dataset.
(a) CRS BT Real Results

| DMU | Bus Route | Efficiency | Peer | Contribution | Local <br> Deviations | Global <br> Deviations |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Main Street | 1.566751 | 4 | 1 | 0.0157923 | 0.0157923 |
| 2 | Hethwood <br> /Windsor <br> Hill | 1.303437 | 4 | 1 | 0.0137429 | 0.0137429 |
| 3 | Toms <br> Creek A | 2.551272 | 4 | 1 | 0.0067182 | 0.0067182 |
| 4 | Toms <br> Creek B | 1 | 4 | 1 | 0.0019069 | 0.0019069 |
| 5 | Two Town <br> Trolley BB | 9.88341 | 4 | 0.4 | 0.0027869 | 0.0027869 |
| 6 | Two Town <br> Trolley CB | 23.57614 | 4 | 0.3 | 0.0004645 | 0.0004645 |
| 7 | Hokie <br> Express | 8.578152 | 4 | 0.8 | 0.0018579 | 0.0018579 |
| 8 | Ramble <br> Road | 34.95218 | 4 | 0.8 | 0.000929 | 0.000929 |

Table 4. 18 Summary of Results for BT Real Data (CRS case)

The third column, 'Efficiency', represents the relative efficiency scores for each route. As this is an output increasing model, efficient routes have efficiency score equal to 1 and inefficient routes have efficiency score greater than 1 (or equal to 1 with nonzero slacks). It can be seen from the table that only bus route 'Toms Creek B' is efficient. It forms the peer for each of the remaining 7 inefficient routes. The contributions or intensities of the efficient DMU ('Toms Creek B') for the other inefficient ones are also presented in the table. The column, 'Local Deviations', represents the deviation for the societal variable (number of accidents) for each of the route.

As there is only one societal variable, the Global Deviations for the whole network equals the Local Deviation. These deviations for the bus routes are minimized using their corresponding efficiency scores as known variable.
The results are presented graphically as below in Figures (4.3) and (4.4).


Figure 4.3 Efficiency Scores of the BT Bus Routes (Constant Returns to Scale)


Figure 4.4 Local Deviations for No. of Accidents for BT Routes (Constant Returns to Scale)

The details of the slacks in the variables for inefficient DMUs are provided in the Table 4.19 (next page). It can be observed that the slacks for all the variables for the efficient DMU, Toms Creek B, are zero.

The Global Deviation for the entire network ( 8 bus routes) of routes for Blacksburg Transit is ' 0 '.

| DMU | Bus Route | Output | Intermediate Outputs |  | Controllable Inputs |  |  | Infrastructure Inputs | Intermediate Outputs |  | Societal variables and Targets |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $y_{\text {jpm }}$ | $\mathrm{y}_{\mathrm{jvm}}$ | $\mathrm{y}_{\mathrm{jtim}}$ | $\mathrm{X}_{\text {jhd }}$ | $\mathrm{X}_{\mathrm{j} \text { cost }}$ | $\mathrm{X}_{\mathrm{j} \text { servd }}$ | $\mathrm{X}_{\mathrm{jint}}$ | $\mathrm{y}_{\mathrm{jvm}}$ | $y_{\text {jitim }}$ | $\mathrm{y}_{\text {jacc }}$ | Ly ${ }_{\text {jacc }}$ |
| 1 | Main Street | 0 | 73 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | Hethwood/Windsor Hill | 0 | 62.3 | 0 | 0 | 225 | 0 | 0.02 | 0 | 0 | 0 | 0 |
| 3 | Toms Creek A | 0 | 25.7 | 0 | 0.01 | 1585 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | Toms Creek B | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | Two Town Trolley BB | 0 | 0 | 0 | 0.02 | 1351 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | Two Town Trolley CB | 0 | 0 | 0.05 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | Hokie Express | 0 | 0 | 0.01 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8 | Ramble Road | 0 | 32.5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Table 4.19 Summary of Slacks for 8 BT Bus Routes for all the variables (CRS case)

## (b)VRS BT Real Results

| DMU | Bus Route | Efficiency | Peers - <br> PRSsenger <br> Node | Deviations |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Main Street | 1.566751 | 4 | 0.01579 |
| 2 | Hethwood/Windsor <br> Hill | 1.303437 | 4 | 0.01424 |
| 3 | Toms Creek A | 2.551272 | 4 | 0.00889 |
| 4 | Toms Creek B | 1 | 4 | 0.00515 |
| 5 | Two Town Trolley <br> BB | 5.220691 | 5 | 0.00279 |
| 6 | Two Town Trolley <br> CB | 1 | 6 | 0.00279 |
| 7 | Hokie Express | 7.648846 | 4,6 | 0.00186 |
| 8 | Ramble Road | 30.92689 | 5 | 0.00093 |

Table 4.20 Summary of Results for BT real data (VRS case)

The table above (Table 4.20) presents a summary of results for VRS consideration for the BT real dataset under variable returns to scales. It can be inferred that DMUs Toms Creek B, Two Town Trolley BB and Two Town Trolley CB are the efficient DMUs whereas other DMUs are inefficient.

Figures (4.5) and (4.6) provide a representation of the efficiency scores and the deviation from the targets.


Figure 4.5 Efficiency Scores for BT Bus Routes (Variable Returns to Scale)


Figure 4.6 Local Deviations for No. of Accidents for BT Routes (Variable Returns to Scale)

The details of the slacks in the variables for inefficient DMUs are provided in the Table 4.21 (next page). It can be observed that the slacks for all the variables for the efficient DMUs, Toms Creek B and Two Town Trolley CB are zero.

The Global Deviation for the entire network (8 bus routes) of routes for Blacksburg Transit is ' 0 '.

| DMU | Bus Route | Output | Intermediate Outputs |  | Controllable Inputs |  |  | Infrastructure Inputs | Intermediate Outputs |  | Societal variables and Targets |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\mathrm{y}_{\mathrm{jpm}}$ | $\mathrm{y}_{\text {jvm }}$ | $\mathbf{y}_{\mathrm{jtim}}$ | $\mathbf{x}_{\text {jhd }}$ | $\mathbf{x}_{\text {jcost }}$ | $\mathbf{X}_{\text {jservd }}$ | $\mathbf{x}_{\text {jint }}$ | $\mathbf{y}_{\text {jvm }}$ | $\mathrm{y}_{\mathrm{jtim}}$ | $\mathbf{y}_{\text {jace }}$ | $\mathbf{L y ~}_{\mathbf{j a c c}}$ |
| 1 | Main Street | 0 | 73 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | Hethwood/Windsor Hill | 0 | 62.333 | 0 | 0 | 184 | 0.51 | 0.0169 | 0 | 0 | 0 | 0 |
| 3 | Toms Creek A | 0 | 25.667 | 0 | 0.015 | 1409 | 2.29 | 0 | 0 | 0 | 0 | 0 |
| 4 | Toms Creek B | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | Two Town Trolley BB | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | Two Town Trolley CB | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | Hokie Express | 0 | 0 | 0 | 0.019 | 1063 | 3.73 | 0 | 0 | 0 | 0 | 0 |
| 8 | Ramble Road | 0 | 39.667 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Table 4.21 Summary of Slacks for 8 BT Bus Routes for all the variables (VRS case)

Figure (4.7) compares CRS and VRS Efficiency scores for the 8 BT routes


Figure 4.7 Plot comparing the CRS and VRS efficiency for BT Bus Routes

It can be inferred from the above plot that CRS efficiency is greater (if not equal) than VRS efficiency for all 8 bus routes, thus in accordance with the postulates of DEA.

### 4.5.1.2 BT Simulated Dataset

This dataset uses the available data for the 8 BT routes and has simulated data for remaining variables, namely; the environmental variables, the societal variables other than number of accidents and their global as well as local targets
The results are generated such that the DMUs are compared with only those DMUs that operate in similar or harsher environments.

Table 4.22 provides the summary of the results.
(a) CRS BT Simulated Results

| Sr. No. | Bus Route | Efficiency | Peers | Contribution | Local <br> Deviations |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Two Town Trolley <br> CB | 1 | 1 | 0 | 0.002787 |
| 2 | Ramble Road | 1 | 2 | 0 | 0.067596 |
| 3 | Hethwood/Windsor <br> Hills | 1.41155923 | 2 | 0.721 | 0.430514 |
| 4 | Two Town Trolley <br> BB | 1 | 4 | 0 | 0.002787 |
| 5 | Hokie Express | 5.03232234 | 2 | 0.293 | 1.636779 |
| 6 | Toms Creek A | 1 | 6 |  | 1.265631 |
| 7 | Toms Creek B | 4.36772575 | 2 | 0.514 | 1.259128 |
| 8 | Main Street | 1 | 8 | 0 | 4.616301 |

Table 4.22 Summary of Results for BT Simulated Dataset (CRS case)

It can be observed from the above table that Hethwood/Windsor Hills, Hokie Express and Toms Creek B are inefficient. All three of them have Ramble Road as their peer.

The weighted local deviation for the bus routes is a weighted sum of deviations of societal variables. These deviations represent deviation of societal variable for the bus route from its predetermined target. It can be seen that Main Street has the highest value for Local Deviation. This indicates a higher value for societal variables and hence it is necessary to take steps to reduce the value of these variables.

Similarly Global Deviation for a societal variable is the deviation for the whole network of bus routes from its preset global target for that variable. The weighted Global Deviation represents the weighted sum of Global Deviations for all the societal variables. The DMUs are evaluated based on the intensity of their combined environmental factor. With successive iterations, a DMU is added to the set of DMUs for the evaluation of its
efficiency score and the weighted Global Deviation represents weighted sum of deviations of the societal variables with those many DMUs forming the network.

The following Figures (4.8) and (4.9) provide plots of Efficiency scores, Local Deviations of DMUs respectively.


Figure 4.8 Efficiency Scores of BT Bus Routes (Constant Returns to Scale - Simulated)


Figure 4.9 Local Deviations of BT Bus Routes (Constant Returns to Scale - Simulated)

The details of the slacks in the variables for inefficient DMUs are provided in the Table 4.23 (next page). There are two parts in the table for Phase 1 and Phase 2. Phase 1 indicates that three DMUs, namely Toms Creek B, Hethwood/Windsor Hills and Main Street are efficient. Phase 2 indicates that besides Hokie Express, Toms Creek B and

Hethwood/Windsor Hills, all the other bus routes are efficient. It can be observed that the slacks for all the variables for the efficient DMUs are zero.

The Global Deviation for the entire network ( 8 bus routes) of routes for Blacksburg Transit is ' 0 '.

| Phase 1 |  | Output <br> $\mathbf{y}_{\text {jpm }}$ | Intermediate Outputs |  |  | Controllable Inputs |  |  | Infrastructure Inputs <br> $\mathbf{x}_{\text {jint }}$ | Intermediate Outputs |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Sr. <br> No. | DMU |  | $\mathbf{y}_{\text {jsrel }}$ | $\mathbf{y}_{\text {jvm }}$ | $\mathrm{y}_{\text {jtim }}$ | $\mathbf{x}_{\text {jhd }}$ | $\mathbf{x}_{\text {jcost }}$ | $\mathbf{x}_{\text {jservd }}$ |  | $\mathbf{y}_{\text {jsrel }}$ | $\mathbf{y}_{\text {jvm }}$ | $\mathrm{y}_{\mathrm{jtim}}$ |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 3 | 0 | 0 | 1.35 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 5 | 0 | 0 | 0 | 0.85 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 6 | 0 | 0.159 | 0 | 0.98 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | 7 | 0 | 0 | 0 | 1.62 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8 | 8 | 0 | 0 | 0 | 1.51 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |


| Phase 2 |  | Output | Intermediate Outputs |  |  | Controllable Inputs |  |  | Infrastructure Inputs | Intermediate Outputs |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Sr. <br> No. | DMU | $\mathrm{y}_{\mathrm{jpm}}$ | $\mathbf{y}_{\text {jsrel }}$ | $\mathrm{y}_{\mathrm{ivm}}$ | $\mathrm{y}_{\text {jtim }}$ | $\mathbf{x}_{\text {jhd }}$ | $\mathbf{x}_{\text {jcost }}$ | $\mathbf{x}_{\text {jservd }}$ | $\mathrm{X}_{\mathrm{jint}}$ | $\mathbf{y}_{\text {jsrel }}$ | $\mathbf{y}_{\mathbf{j v m}}$ | $\mathbf{y}_{\text {jtim }}$ |
| 1 | 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 5 | 0 | 0.71 | 0 | 0.84 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 7 | 0 | 0.49 | 0 | 1.61 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 3 | 0 | 2.28 | 15 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |


(b) VRS BT Simulated Results

| Sr. No. | Bus Route | Efficiency | Peers | Contribution | Local <br> Deviations |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Main Street | 1 |  |  | 4.6163013 |
| 2 | Hethwood/Windsor <br> Hill | 1 |  |  | 4.7638477 |
| 3 | Toms Creek A | 1.1600239 | 1 | 0.706 | 4.2656311 |
|  |  | 6 | 0.293 | 4.2591284 |  |
| 4 | Toms Creek B | 1 |  |  | 0.3361202 |
| 5 | Two Town Trolley <br> BB | 1 |  |  | 0.3361202 |
| 6 | Two Town Trolley <br> CB | 1 |  |  | 3.3034452 |
| 7 | Hokie Express | 1 |  |  | 0.400929 |
| 8 | Ramble Road | 1 |  |  |  |

Table 4. 24 Summary of Slacks for 8 BT Bus Routes for all the variables (VRS case)

It can be observed from the above table (Table 4.24) that Tom's Creek A is inefficient. It has Main Street and Two Town Trolley CB as its peers.

The weighted local deviation for the bus routes is a weighted sum of deviations of societal variables. These deviations represent deviation of societal variable for the bus route from its predetermined target. It can be seen that Main Street, Hethwood/Windsor Hills, Toms Creek A and Toms Creek B have the higher value for Local Deviation. This indicates a higher value for societal variables and hence it is necessary to take steps to reduce the value of these variables.

Similarly Global Deviation for a societal variable is the deviation for the whole network of bus routes from its preset global target for that variable. The weighted Global Deviation represents the weighted sum of Global Deviations for all the societal variables. The DMUs are evaluated based on the intensity of their combined environmental factor.

With successive iterations, a DMU is added to the set of DMUs for the evaluation of its efficiency score and the weighted Global Deviation represents weighted sum of deviations of the societal variables with those many DMUs forming the network.
The following Figures (4.10) and (4.11) provide plots of Efficiency scores, Local Deviations of DMUs respectively.


Figure 4.10 Efficiency Scores of BT Bus Routes (Variable Returns to Scale - Simulated)


Figure 4.11 Local Deviations for BT Bus Routes (Variable Returns to Scale - Simulated)

Figure (4.12) compares the CRS and VRS efficiency for the 8 bus routessimulated dataset. It can be observed that CRS efficiency is greater than (if not equal) to VRS efficiency.


Figure 4.12 Plot Comparing CRS and VRS Efficiency Scores for BT Bus Routes (Simulated)

The details of the slacks in the variables for inefficient DMUs are provided in the Table 4.25 (next page). There are two parts in the table for Phase 1 and Phase 2. Phase 1 indicates that three DMUs, namely, Toms Creek A, Ramble Road and Hokie Express are inefficient. Phase 2 indicates that besides Toms Creek A, all the other bus routes are efficient. It can be observed that the slacks for all the variables for the efficient DMUs are zero.

The Global Deviation for the entire network ( 8 bus routes) of routes for Blacksburg Transit is ' 0 '.

| Phase 1 |  | Output | Intermediate Outputs |  |  | Controllable Inputs |  |  | Infrastructure Inputs | Intermediate Outputs |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. | DMU | $\mathrm{y}_{\mathrm{ipm}}$ | $\mathrm{y}_{\text {jsrel }}$ | $\mathrm{y}_{\mathrm{ivm}}$ | $\mathrm{y}_{\mathrm{jtim}}$ | $\mathbf{x}_{\text {jhd }}$ | $\mathbf{x}_{\text {jcost }}$ | $\mathbf{x}_{\text {jservd }}$ | $\mathbf{x}_{\text {jint }}$ | $\mathrm{y}_{\text {jsrel }}$ | $\mathrm{y}_{\mathrm{ivm}}$ | $\mathrm{y}_{\mathrm{jtim}}$ |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 3 | 0 | 0 | 25.7 | 0.16 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | 7 | 0 | 0 | 0 | 1.03 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8 | 8 | 0 | 0 | 0 | 0.91 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |


| Phase 1 |  | Output | Intermediate Outputs |  |  | Controllable Inputs |  |  | Infrastructure Inputs | Intermediate Outputs |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. | DMU | $\mathrm{y}_{\mathrm{jpm}}$ | $\mathbf{y}_{\text {jsrel }}$ | $\mathrm{y}_{\mathrm{jvm}}$ | $\mathrm{y}_{\text {jtim }}$ | $\mathbf{x}_{\text {jhd }}$ | $\mathbf{x}_{\text {jcost }}$ | $\mathbf{x}_{\text {jservd }}$ | $\mathbf{x}_{\text {jint }}$ | $\mathbf{y}_{\text {jsrel }}$ | $\mathrm{y}_{\mathrm{jvm}}$ | $\mathrm{y}_{\text {jtim }}$ |
| 1 | 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 3 | 0 | 2 | 0 | 0.1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |



### 4.5.2 100 Bus Routes

This section presents the results for the 100 hypothetical bus routes. The dataset for these routes was populated using simulation and Random Numbers distribution as described in Section (4.1). The results are generated for two basic models,
(i) Infrastructure 1
(ii) Infrastructure 2

### 4.5.2.1 Infrastructure 1

In this variation, two infrastructure variables, 'Number of intersections' and 'Number of priority' lanes are used. For the purpose of illustration, both CRS and VRS assumptions are considered.

## (a) CRS Infrastructure 1

In this section, results and charts for CRS Infrastructure 1 model are presented. The results table (Table 4.26) is a snapshot of the complete table (Refer Appendix C 1). For illustration purposes, the Table 4.26 contains only 20 of the 100 bus routes.

| DMU | Ph2 Efficiency | ZLcomb | Local <br> Deviation | Ph1 Efficiency |
| ---: | ---: | :--- | :--- | ---: |
| 1 | 1.26979 | 3.588429 | 0.613333 | 2.25 |
| 2 | 1.50115 | 3.113795 | 0.4 | 1.741935 |
| 3 | 1.609417 | 2.915917 | 1.6 | 2 |
| 4 | 1.203224 | 2.956255 | 0 | 1.35 |
| 5 | 1 | 3.645598 | 0 | 1.741935 |
| 6 | 1.538165 | 2.397578 | 2.8 | 1.741935 |
| 7 | 2.185144 | 2.258902 | 1 | 2.454545 |
| 8 | 6.770684 | 4.951616 | 0 | 18 |
| 9 | 1.710069 | 2.184651 | 0 | 1.928571 |
| 10 | 1.025515 | 4.683104 | 1.032848 | 4.153846 |
| 11 | 2.090515 | 2.473611 | 0 | 2.335037 |
| 12 | 1.338555 | 1.947461 | 0 | 1.459459 |
| 13 | 5.216044 | 2.257895 | 0 | 6 |
| 14 | 1.928571 | 1.365473 | 0 | 1.928571 |
| 15 | 2.346013 | 2.215973 | 0.972956 | 2.7 |
| 16 |  | 1 | 3.361839 | 0.598403 |

Table 4. 26 Summary of Results for CRS 100 (20 DMUs)

The first column represents the bus routes as numbered in the order they were generated. 'Ph2 Efficiency' in column 2 represents efficiency scores for DMUs, considering the effect of the combined environmental factor. Column 3 represents the combined environmental factor, 'Zjcomb' for each route. This represents the degree of harshness of the environment. 'Ph 1 Efficiency' in column 4 represents the efficiency
score of the DMUs without consideration of 'Zjcomb' in the analysis. Column 5 represents the weighted sum of deviations of societal variables for each bus route.
The following, Figure 4.13 and Figure 4.14 presents the Efficiency Scores (Ph 2 Efficiency) and the Local Deviations for all the 100 DMUs respectively,


Figure 4.13 Efficiency Scores of 100 Bus Routes (Constant Returns to Scale - Infrastructure 1)


Figure 4.14 Local Deviations of 100 Bus Routes (Constant Returns to Scale - Infrastructure 1)

Figure 4.15 below is a plot of the combined Environmental factor and the Efficiency score of bus route. This indicates the variation in the efficiency scores of the Bus Routes with the variation in the value of the combined environmental factor, ' $Z \mathrm{Zjcomb}$ '.


Figure 4.15 Plot of Zjcomb and Efficiency of the 100 Bus Routes (Constant Returns to Scale Infrastructure 1)


Figure 4.16 Plot of Ph 1and Ph2 Efficiency Scores of 100 Bus Routes (Constant Returns to Scale)

Figure 4.16 is a plot of the Ph1 Efficiency score and Ph 2 Efficiency Score of each bus route.

The slacks for the first 20 bus routes are presented in Table 4.27 (next page). From the values of the Slacks, it can be inferred that as DMUs 5,16 have zero slacks. As they have zero slacks and efficiency score 1 they form the set of 'efficient' bus routes among the first 20 bus routes.

|  | Output | Intermediate Outputs (Passenger) |  |  | Controllable Inputs |  |  | Infrastructure Inputs |  | Intermediate Outputs (Provider) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| DMU | $\mathbf{y}_{\text {jpm }}$ | $\mathbf{y}_{\text {jsrel }}$ | $\mathbf{y}_{\text {jvm }}$ | $\mathrm{y}_{\mathrm{jtim}}$ | $\mathbf{x}_{\text {jhd }}$ | $\mathbf{X}_{\text {jcost }}$ | $\mathbf{x}_{\text {jservd }}$ | $\mathbf{x}_{\text {jint }}$ | $\mathbf{x}_{\text {jprln }}$ | $\mathrm{y}_{\text {jsrel }}$ | $\mathrm{y}_{\mathrm{jvm}}$ | $\mathrm{y}_{\mathrm{jtim}}$ |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 2.0927 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 16.882 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 3.7379 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0 | 19.255 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | 0 | 2.6478 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 61.97 | 0.02305 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 9 | 0 | 6.8504 | 0 | 0 | 0 | 0 | 0.494 | 0 | 0 | 0 | 0 | 0 |
| 10 | 0 | 6.2702 | 122.2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 11 | 0 | 13.057 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 12 | 0 | 8.0132 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 13 | 0 | 15.942 | 0 | 0 | 0.0021 | 0 | 1.493 | 0 | 0 | 0 | 0 | 0 |
| 14 | 0 | 11.459 | 0 | 0.27471 | 0.0017 | 0 | 0.545 | 0 | 0 | 0 | 0 | 0.11817 |
| 15 | 0 | 16.236 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 16 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 17 | 0 | 13.855 | 0 | 0 | 0.0221 | 0 | 1.885 | 0 | 0 | 0 | 0 | 0 |
| 18 | 0 | 11.779 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 19 | 0 | 13.006 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 20 | 0 | 14.24 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Table 4.27 Slacks for the first 20 Bus Routes for all variables (CRS Case)

## (b) VRS Infrastructure 1

In this section, results and charts for VRS Infrastructure 1 model are presented. The results table (Table 4.28) is a snapshot of the complete table (Refer Appendix C 2). For illustration purposes, the Table 4.28 contains only 20 of the 100 bus routes.

| DMU | Ph2 <br> Efficiency | $\mathrm{z}_{\text {jomb }}$ | Local <br> Deviation | Ph1 <br> Efficiency |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 4.478637 | 2.264 | 1.676368 |
| 2 | 1.381097 | 2.620492 | 0.8 | 1.417465 |
| 3 | 1.584996 | 2.405777 | 3.2 | 1.615587 |
| 4 | 1.196687 | 2.375896 | 0 | 1.200138 |
| 5 | 1 | 2.931079 | 0 | 1.433711 |
| 6 | 1.43729 | 2.324905 | 5.6 | 1.483374 |
| 7 | 1.826378 | 1.955795 | 2 | 1.837915 |
| 8 | 2.623724 | 3.912794 | 4.161508 | 14.49512 |
| 9 | 1.410957 | 1.75353 | 0 | 1.410957 |
| 10 | 1.327755 | 3.663078 | 0 | 3.38206 |
| 11 | 1.680287 | 2.130401 | 0 | 1.680287 |
| 12 | 1.305885 | 1.660897 | 0 | 1.305885 |
| 13 | 4.873115 | 2.132401 | 0.153843 | 4.975106 |
| 14 | 1.497879 | 1.193017 | 0 | 1.497879 |
| 15 | 2.201191 | 2.035337 | 1.372956 | 2.26011 |
| 16 | 1 | 2.723693 | 0.598403 | 1 |
| 17 | 2.133401 | 2.252484 | 1.181721 | 2.239058 |
| 18 | 1.326101 | 2.527859 | 0.013333 | 1.326101 |
| 19 | 1 | 3.41384 | 2.40369 | 3.274463 |
| 20 | 1 | 2.955033 | 2.797259 | 1.376445 |

Table 4.28 Summary of Results for VRS 100 (20 DMUs)

The first column in Table 4.28 represents the bus routes as numbered in the order they were generated. 'Ph2 Efficiency' in column 2 represents efficiency scores for

DMUs, considering the effect of the combined environmental factor. Column 3 represents the combined environmental factor, 'Zjcomb' for each route. This represents the degree of harshness of the environment. 'Ph 1 Efficiency' in column 4 represents the efficiency score of the DMUs without consideration of 'Zjcomb' in the analysis. Column 5 represents the weighted sum of deviations of societal variables for each bus route.
The following, Figure 4.17 and Figure 4.18 presents the Efficiency Scores (Ph 2 Efficiency) and the Local Deviations for all the 100 DMUs respectively,


Figure 4.17 Efficiency Scores of 100 Bus Routes (Variable Returns to Scale - Infrastructure 1)


Figure 4.18 Local Deviations of 100 Bus Routes (Variable Returns to Scale - Infrastructure 1)

The plot of 'Zjcomb' vs. Efficiency score as depicted in Figure 4.19 represents the variation of the Efficiency score with Zjcomb.


Figure 4.19 Plot of 'Zjcomb' and the Efficiency score of the 100 Bus Routes (Variable Returns to Scale - Infrastructure 1)

The plot of Ph1 Efficiency vs. Ph 2 Efficiency for the 100 Bus Routes is provided in Figure 4.20. It indicates the influence of the environmental factor on the efficiency score.


Figure 4.20 Plot of Ph 1 Efficiency Score and Ph 2 Efficiency Score (Infrastructure 1)

The slacks for the first 20 bus routes are presented in Table 4.29 (next page). The DMUs $1,5,16,19$ and 20 have efficiency scores as 1 (Refer Table 4.28). But as only DMUs 1 and 19 have slacks for all the variables as zero (Refer Table 4.29), they form the set of efficient DMUs among these first 20 bus routes.


Table 4.29 Slacks for the first 20 Bus Routes for all variables (VRS Case)

### 4.5.2.2 Infrastructure 2

In this variation an environmental variable, 'Zjinf' is used instead of the two infrastructure variables, 'Number of intersections' and 'Number of priority' lanes are used. For the purpose of illustration, both CRS and VRS assumptions are considered.

## (a) CRS Infrastructure 2

In this section, results and charts for CRS Infrastructure 2 model are presented. The results table (Table 4.30) is a snapshot of the complete table (Refer Appendix C 3). For illustration purposes, the Table 4.30 contains only 20 of the 100 bus routes.

| DMU | Ph2_Efficiency | zicomb | Local <br> Deviation | Ph1_Efficiency |
| ---: | ---: | :--- | :--- | ---: |
| 1 | 2.007332 | 3.032547 | 0 | 2.25 |
| 2 | 1.234275 | 3.523395 | 0.4 | 1.741935 |
| 3 | 1.091514 | 3.549617 | 1.6 | 2 |
| 4 | 1.235081 | 3.227719 | 0 | 1.35 |
| 5 | 1 | 4.046059 | 0 | 1.741935 |
| 6 | 1.538165 | 2.572971 | 2.8 | 1.741935 |
| 7 | 2.185144 | 2.428276 | 0 | 2.454545 |
| 8 | 6.770684 | 4.588256 | 0 | 18 |
| 9 | 1.710069 | 2.584323 | 0 | 1.928571 |
| 10 | 1.025515 | 4.950787 | 1.032848 | 4.153846 |
| 11 | 2.090515 | 3.140239 | 0 | 2.335037 |
| 12 | 1.338555 | 1.839728 | 0 | 1.459459 |
| 13 | 5.216044 | 2.818602 | 0 | 6 |
| 14 | 1.928571 | 1.37737 | 0 | 1.928571 |
| 15 | 2.346013 | 2.074423 | 0 | 2.7 |
| 16 | 1.105273 | 3.243419 | 0 | 1.285714 |
| 17 | 2.271554 | 2.369423 | 0 | 2.571429 |
| 18 | 1.456994 | 3.077413 | 0.013333 | 1.588235 |
| 19 | 2.532745 | 3.788814 | 0 | 4.5 |
| 20 | 1.346252 | 3.328977 | 2.397259 | 1.636364 |

Table 4.30 Summary of Results for CRS 100 (20 DMUs)

The first column represents the bus routes as numbered in the order they were generated. 'Ph2 Efficiency' in column 2 represents efficiency scores for DMUs,
considering the effect of the combined environmental factor. Column 3 represents the combined environmental factor, 'Zjcomb' for each route. This represents the degree of harshness of the environment. 'Ph 1 Efficiency' in column 4 represents the efficiency score of the DMUs without consideration of 'Zjcomb' in the analysis. Column 5 represents the weighted sum of deviations of societal variables for each bus route.
The following, Figure 4.21 and Figure 4.22 presents the Efficiency Scores (Ph 2 Efficiency) and the Local Deviations for all the 100 DMUs respectively,


Figure 4.21 Efficiency Scores of 100 Bus Routes (Constant Returns to Scale - Infrastructure 2)


Figure 4.22 Local Deviations of 100 Bus Routes (Constant Returns to Scale - Infrastructure 2)

The plot of 'Zjcomb' vs. Efficiency score as depicted in Figure 4.23 represents the variation of the efficiency score with Zjcomb.


Figure 4.23 Plot of ‘Zjcomb’ and the Efficiency score of the 100 Bus Routes (Constant Returns to Scale - Infrastructure 2)

The plot of Ph1 Efficiency vs. Ph 2 Efficiency for the 100 Bus Routes is provided in Figure 4.24. It indicates the influence of the environmental factor on the efficiency score.


Figure 4.24 Plot of Ph 1 Efficiency Score and Ph 2 Efficiency Score (Constant Returns to Scale Infrastructure 2)

The slacks for the first 20 bus routes are presented in Table 4.31 (next page). The DMU 5 is an efficient DMU as it has efficiency scores 1 (Refer Table 4.30) and has slacks for all the variables as zero (Refer Table 4.31).


Table 4.31 Slacks for the first 20 Bus Routes for all variables (CRS Case)

## (b) VRS Infrastructure 2

In this section, results and charts for VRS Infrastructure 2 model are presented. The results table (Table 4.32) is a snapshot of the complete table (Refer Appendix C 4). For illustration purposes, the Table 4.32 contains only 20 of the 100 bus routes.

| DMU | Ph2 <br> Efficiency | zjcomb | Local Deviation | Ph1 <br> Efficiency |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 1.676368 | 2.561329 | 0.06854 | 5.15366 |
| 2 | 1.151315 | 2.944513 | 0.4 | 1.376445 |
| 3 | 1.051204 | 2.907076 | 0.4 | 1.108934 |
| 4 | 1.196687 | 2.590642 | 0.4 | 1.094423 |
| 5 | 1 | 3.24787 | 0 | 4.975106 |
| 6 | 1.456074 | 2.463653 | 2.8 | 1.968585 |
| 7 | 1.800125 | 2.089782 | 0.242468 | 1.075281 |
| 8 | 5.690376 | 3.795737 | 0 | 1.200138 |
| 9 | 1.410957 | 2.069696 | 0.214196 | 1.322818 |
| 10 | 1 | 3.920493 | 1.032848 | 1.417465 |
| 11 | 1.036289 | 2.657748 | 1.6 | 2.591244 |
| 12 | 1.305885 | 1.575673 | 2.6257 | 1.710606 |
| 13 | 4.790795 | 2.575958 | 0 | 3.52022 |
| 14 | 1.497879 | 1.202428 | 0.251894 | 1 |
| 15 | 2.201191 | 1.923362 | 0.674087 | 1.655145 |
| 16 | 1 | 2.630015 | 0 | 2.469349 |
| 17 | 2.172395 | 2.201162 | 0.760374 | 1.693065 |
| 18 | 1.237486 | 2.686712 | 0.715243 | 2.106788 |
| 19 | 1.327755 | 3.294877 | 1.094742 | 1.305885 |
| 20 | 1 | 2.962557 | 0 | 1.326101 |

Table 4.32 Summary of Results for VRS 100 (20 DMUs)

The first column in Table 4.32 represents the bus routes as numbered in the order they were generated. ' Ph 2 Efficiency' in column 2 represents efficiency scores for

DMUs, considering the effect of the combined environmental factor. Column 3 represents the combined environmental factor, 'Zjcomb' for each route. This represents the degree of harshness of the environment. 'Ph 1 Efficiency' in column 4 represents the efficiency score of the DMUs without consideration of 'Zjcomb' in the analysis. Column 5 represents the weighted sum of deviations of societal variables for each bus route. The following, Figure 4.25 and Figure 4.26 presents the Efficiency Scores (Ph 2 Efficiency) and the Local Deviations for all the 100 DMUs respectively,


Figure 4.25 Efficiency Scores of 100 Bus Routes (Variable Returns to Scale - Infrastructure 2)


Figure 4.26 Local Deviations of 100 Bus Routes (Variable Returns to Scale - Infrastructure 2)

The plot of 'Zjcomb' vs. Efficiency score as depicted in Figure 4.27 represents the variation of the efficiency score with Zjcomb.


Figure 4.27 Plot of 'Zjcomb' and the Efficiency score of the 100 Bus Routes (Variable Returns to Scale - Infrastructure 2)

The plot of Ph1 Efficiency vs. Ph 2 Efficiency for the 100 Bus Routes is provided in Figure 4.28. It indicates the influence of the environmental factor on the efficiency score.


Figure 4.28 Plot of Ph 1 Efficiency Score and Ph 2 Efficiency Score (Variable Returns to Scale Infrastructure 2)

The slacks for the first 20 bus routes are presented in Table 4.33 (next page). The DMUs 5, 10, 16 and 20 have efficiency scores as 1 (Refer Table 4.32). But as only DMUs 5, 10 and 20 have slacks for all the variables as zero (Refer Table 4.33), they form the set of efficient DMUs from these 20 DMUs.



### 4.5.2.3 Comparison of Infrastructure 1 and Infrastructure 2 Models

In this section, results from the models Infrastructure 1 and Infrastructure 2 for '100 Simulated Routes' are compared with each other. Both CRS as well as VRS variations are considered.

Figure 4.29 compares the CRS and VRS efficiency for Infrastructure 1. It can be observed that the CRS efficiency for the bus routes is equal or greater than the VRS efficiency.


Figure 4.29 CRS vs. VRS Efficiency Comparison for Infrastructure 1

Figure 4.30 compares the CRS and VRS efficiency for Infrastructure 2. It can be observed that the CRS efficiency for the bus routes is equal or greater than the VRS efficiency.


Figure 4.30 CRS vs. VRS Efficiency Comparison for Infrastructure 2

Figure 4.31 represents the CRS efficiency comparison for Infrastructure 1 and Infrastructure 2.


Figure 4.31 CRS Efficiency Score Comparisons for Infrastructure 1 and Infrastructure 2 models

From the Figure 4.31, it can be inferred that the efficiency scores for the two models has similar trends and it does not differ significantly from each other.

Figure 4.32 represents the VRS efficiency comparison for Infrastructure 1 and Infrastructure 2


Figure 4.32 VRS Efficiency Score Comparisons for Infrastructure 1 and Infrastructure 2 models

From the Figure 4.32, it can be inferred that the efficiency scores for the two models has similar trends and it does not differ significantly from each other.

In the next Chapter, (Chapter 5) Conclusions and Future Scope of Research are discussed and the research effort is summarized.

## Chapter 5

## Summary \& Conclusions

This chapter presents the conclusions and contributions of this research effort. It is divided into three sections. The first section summarizes the research effort. The second section describes the major contribution of this research. Finally, the third section provides the concluding comments and some recommendations for future research.

### 5.1 Summary

This research effort presents an approach to measure the performance of Urban Transit Systems, specifically, bus transit networks. This evaluation is brought about by the measurement of the efficiency and effectiveness of individual bus route that comprise the network. The efficiency analysis is primarily based on a math programming technique, Data Envelopment Analysis (DEA) (Charnes, Cooper and Rhodes (1978)).

The parameters for measuring performance of bus routes differ depending upon whether the evaluation considers service provision or service consumption as the objective. In this research effort, both, service provision and consumption are addressed and hence both concepts of efficiency and effectiveness are considered.

In this research effort, the performance measurement framework encompasses the provider's, users and societal perspectives. The provider provides transit services through the bus route, which is consumed by the passenger or the user who use that bus route. While providing services, the provider also impacts the society through these bus routes. As the bus route relates to the provider, the user and to society as well, it is used as the unit of analysis to measure performance of the Transit agency taking into account these three perspectives.

In this research, a network of interlinked nodes is used to represent a bus route and this network represents the provider, passenger and societal perspectives. Performance (efficiency and effectiveness) is then evaluated for this network. . This is achieved by using the concept of Network Model in DEA as developed by Färe (1991).

It is assumed that there are acceptable known thresholds for the societal variables. The impact on the society due to the provision of service is captured by measuring the
deviations from these targets or thresholds using Goal programming and Go-DEA approach as developed by Athanassopoulos (1995).

The research also considers the environment in which the bus route operates, i.e., provides service. These conditions are considered and represented environmental variables, deriving from the DEA terminology. Ruggiero (1998) and Ray (1997) approaches are used to address these environmental effects.

### 5.2 Research Contributions \& Concluding Comments

The purpose of this research effort was to propose a modeling approach, to evaluate the Transit agencies offering bus services with the bus route being the unit of analysis;
to evaluate the Transit agency from the provider's, user's and societal perspectives;
to use DEA as a measurement tool for Transit agency performance evaluation; to extend the DEA methodology by combining the network model and Go-DEA approaches with a consideration of the impact of environmental variables.

This research effort has extended the DEA methodology in general besides answering research questions such as:

How does the decision maker monitor and improve the performance of Urban Transit Systems taking into account the different perspectives simultaneously? How can one consider customer satisfaction and growth management (environment and societal sustainability) in the performance analysis?

Which variables represent performance for each perspective (user, provider and society) considered?

What should the unit of analysis be when evaluating urban transit systems?
What are the targets and peers that can be used for comparisons?
How does the decision maker allocate scarce resources to inefficient units thus improving their performance and consequently of the performance of transit agencies?

Summarizing, this research has contributed to the literature by,
Enabling the measurement of performance of urban transit networks that are characterized by deterministic, incommensurate inputs (resources) and outputs (services), multiple goals (such as efficiency, local societal targets and global societal targets) and multiple perspectives (such as the provider, user and society ) Providing information to the service provider, i.e., government authorities or transit agencies for decision-making with respect to the evaluation of technical efficiency, allocation of scarce resources and the achievement of targets for individual routes as well as the whole network with respect to societal variables such as accidents, noise pollution, emissions and resource degraded that concern society.

Demonstrating a generic performance measurement approach using Data Envelopment Analysis (DEA) by evaluating the case of bus routes. This approach can be modified and altered so that it is applicable to other modes of transportation as well as in intra-modal cases, where a route is characterized by different modes of transportation, for example, bus route-train route.

### 5.3 Scope for Future Research \& Concluding Comments

In this section, the scope for future research and application of this modeling approach are discussed.

## Adapting the Model to Address Different Transportation Modes

The model proposed is applied to evaluate the performance of bus routes, thus indirectly addressing transit agencies that operate bus services. The model can be modified in terms of its objective function, variables and constraints to address other modes of transportation systems.

For example, the model can be altered to evaluate transit rail routes (networks), freight route networks, airline networks or shipping networks. Depending upon the perspectives to address the model can be revised with consideration of relevant variables to the unit of analysis and perspectives.

## Adapting the Model to Address the Intra-modal Case

The model can be modified in terms of its variables to evaluate the efficiencies of an intra-modal route, thus measure the performance of transit agencies operating intra-modal transit services.

For example, consider a scenario where in the unit of analysis involves two modes (e.g. bus-transit rail). The variables can be modified accordingly and the model can be altered.

## Using Other Variables

Depending upon the objective and unit of analysis, set of variables can be chosen accordingly. As a result, the variable list used in the presented model can be enhanced by utilizing other variables or by altering the definitions of the current variable list or adding more variables. This is in consideration to the fact that there are alternative ways to collect data and that there might be other variables which might define 'efficiency' more appropriately.

## Other Perspectives

The model can be altered to include/exclude one or more perspectives. Depending on the objective to be achieved (e.g. maximize profit, minimize loss) the perspectives can be defined and modeled accordingly with corresponding changes to the variables and hence the model.

## Undesirable Variables

The model can be modified to incorporate undesirable outputs/outcomes. In this regard, the model can be checked for validity and accordance with the DEA postulates. In this research,

Headway is considered in the model as Service Frequency.
The reciprocal value is used for Number of Intersections.
This is done because both these variables form the input set and in DEA representation of inputs the peer of the virtual DMU consumes less than or equal to the input consumed by the DMU that is being evaluated. It may not be possible to take the
reciprocal or express the variable in some other terms or to subtract the value of the variable from a large number. This may be because convexity may not hold true, or there may be disposability issues (negative marginal products) or the variables modified that are to be utilized may not have any physical significance. Hence there is a need to delve into these aspects of the model.

## Different Modeling Applications

In this research, the model represents a DEA math program for a bus route so as to capture different perspectives. This approach can be adopted or modified to address other scenarios and application (for e.g. any networks with targets or goals) or models, involving the combination of goal programming, the network model, and the inclusion of environmental variables in Data Envelopment Analysis (DEA).

The proposed model addresses two important hypotheses specifically,

To evaluate the feasibility of modeling Network DEA model and Goal-DEA simultaneously and also considering environmental variables

To develop a methodology to evaluate bus transit network with respect to different perspectives, namely, passenger, provider and society

The attempt to resolve these hypotheses increases the utility of this research effort towards the development of a modeling approach that derives from three (Network DEA, Goal DEA and Environmental Factors) areas in the DEA literature as well as proposes a unique, comprehensive approach to evaluate the efficiency of transit networks.

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## Appendix A 1.1

## Code for BT Real Dataset, Constant Returns To Scale Model

```
Sub Clear()
    Sheets("(Ph1_St1)").Select
        Range("C22:C38").Value = 0
    Sheets("(Ph1_St2)").Select
        'Range("B3:V12").ClearContents
        Range("S17:AE39").ClearContents
    Sheets("(Ph2)").Select
        Range("C20:D27").Value = 0
        Range("F19").Value = 0
    Sheets("Local").Select
        Range("C20:D27").Value = 0
        Range("C34:D37").Value = 0
        Range("F33").Value = 0
    Sheets("Global").Select
        Range("C20:D27").Value = 0
        Range("C34:D37").Value = 0
        Range("F33").Value = 0
        Range("C40:D43").Value = 0
        Range("G40:G43").Value = 0
        Range("F39").Value = 0
        'Sheets("Output").Select
        ' Range("C3:W12").ClearContents
        ' Range("D17:X26").ClearContents
        ' Range("D30:X39").ClearContents
    ' Sheets("Slacks").Select
        ' Range("C3:L12").ClearContents
    ' Range("C17:M26").ClearContents
    ' Range("o17:026").ClearContents
    ' Range("q17:q26").ClearContents
    ' Range("s17:s26").ClearContents
    ' Range("C31:M40").ClearContents
    ' Range("S31:S40").ClearContents
    ' Range("O31:O40").ClearContents
    ' Range("Q31:Q40").ClearContents
    ' Range("N42:Q42").ClearContents
```

End Sub

```
Sub Solve_Efficiency()
' solves Macro
' solves and stores output for stage 1, phase 1
'
' Keyboard Shortcut: Ctrl+t
    Application.ScreenUpdating = False
    Sheets("Output").Select
    Range("B3").Select
    'Range("B3").Select
    Sheets("Slacks").Select
    Range("C3").Select
    'Optimize
    Sheets("Efficiency").Select
    Dim n As Integer
    For n=1 To 8
    Sheets("Efficiency").Select
    Sheets("Efficiency").Range("A17").Value = n
    SolverOk SetCell:="$H$21", MaxMinVal:=1, ValueOf:="0", ByChange:="$C$22:$C$38"
    SolverSolve (True)
    Sheets("Efficiency").Select
    Range("C22:C38").Copy
    Sheets("Output").Select
    ActiveCell.PasteSpecial Paste:=xlValue, Operation:=xlNone, SkipBlanks:=
        False, Transpose:=True
    ActiveCell.Offset(1, 0).Select
    Sheets("Efficiency").Select
    Range("L24:L32").Copy
    Sheets("Slacks").Select
    ActiveCell.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:= _
        False, Transpose:=True
    ActiveCell.Offset(1, 0).Select
```

    Next n
    End Sub

```
Sub Reg_1_20
' Reg_2_1 Macro
' Does Regression for Stage 2 of Phase 1
' Keyboard Shortcut: Ctrl+u
    Sheets("(Ph1_St2)").Select
    Application.Run "ATPVBAEN.XLA!Regress", ActiveSheet.Range("$B$16:$B$23"),
        ActiveSheet.Range("$C$16:$H$23"), True, False, , ActiveSheet.Range( 
            "$S$17:$AB$40"), False, False, False, False, , False
    Sheets("(Ph1_St2)").Select
    Range("T34:T39").Select
    Selection.Copy
    Range("C14").Select
    Selection.PasteSpecial Paste:=xlValue, Operation:=xlNone, SkipBlanks:=
        False, Transpose:=True
End Sub
Sub Sort_Env_1_2()
'
' Sort_Env_1_2 Macro
' Ranks the bus routes as per their degree of harshness
'
' Keyboard Shortcut: Ctrl+i
    Sheets("(Ph1_St2)").Select
    Range("A15:I23").Select
    Selection.Copy
    Range("B25").Select
    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
        :=False, Transpose:=False
    'Range("G35").Select
    'Application.CutCopyMode = False
    Range("B26:J33").Sort Key1:=Range("J30"), Order 1:=x1Descending, Header:=
        xlGuess, OrderCustom:=1, MatchCase:=False, Orientation:=xlTopToBottom,
        DataOption1:=xlSortNormal
    'Range("A25:J33").Copy
    'Sheets("Data(Ph2)").Select
    'Range("A2").PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _
        :=False, Transpose:=False
    End Sub
Sub Solve_LocalTargets()
Dim n As Integer
Dim i As Integer
Dim j As Integer
```

$$
\text { Application.ScreenUpdating }=\text { False }
$$

Sheets("Output").Select
Range("C15").Select
Sheets("Slacks").Select
Range("C15").Select
For $\mathrm{n}=1$ To 8
Sheets("Local").Select
Sheets("Local").Range("A16").Value = n
$\mathrm{i}=\mathrm{n}+19$
$\mathrm{j}=\mathrm{n}+2$
'Sheets("Local").Range("J22").Select
' ActiveCell.FormulaR1C1 = "= sumproduct(R20C3:R" \& i \& "C3,R3C12:R" \& j \& "C12)"
' ActiveCell.Offset(1, 0).Select
' ActiveCell.FormulaR1C1 = "= sumproduct(R20C3:R" \& i \& "C3,R3C11:R" \& j \& "C11)"
' ActiveCell.Offset( 1,0 ).Select
' ActiveCell.FormulaR1C1 = "= sumproduct(R20C3:R" \& i \& "C3,R3C10:R" \& j \& "C10)"
' ActiveCell.Offset( 1,0 ).Select
,
' ActiveCell.FormulaR1C1 = "= sumproduct(R20C3:R" \& i \& "C3,R3C9:R" \& j \& "C9)"
' ActiveCell.Offset(1, 0).Select
' ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C4:R" \& j \& "C4)" ' ActiveCell.Offset( 1,0 ).Select
' ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C5:R" \& j \& "C5)"
${ }^{\prime}$ ActiveCell.Offset(1, 0).Select
' ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C6:R" \& j \& "C6)"
' ActiveCell.Offset(1, 0).Select
' ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C7:R" \& j \& "C7)"
' ActiveCell.Offset(1, 0).Select
' ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C8:R" \& j \& "C8)"
' ActiveCell.Offset(1, 0).Select
'ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C9:R" \& j \& "C9)" 'ActiveCell.Offset(1, 0).Select

```
    ' ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" & i & "C4,R3C11:R" & j &
"C11)"
    ' ActiveCell.Offset(1, 0).Select
    ' ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" & i & "C4,R3C10:R" & j &
"C10)"
    ' ActiveCell.Offset(1, 0).Select
    ' ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" & i & "C4,R3C9:R" & j & "C9)"
    ' ActiveCell.Offset(1,0).Select
    ' ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" & i & "C4,R3C11:R" & j &
"C11)"
    ' ActiveCell.Offset(1, 0).Select
    ' ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" & i & "C4,R3C10:R" & j &
"C10)"
    ' ActiveCell.Offset(1, 0).Select
    ' ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" & i & "C4,R3C9:R" & j & "C9)"
    ' ActiveCell.Offset(1,0).Select
```

```
    ' ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" & i & "C4,R3C13:R" & j &
```

    ' ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" & i & "C4,R3C13:R" & j &
    "C13)- R36C3 + R36C4"
"C13)- R36C3 + R36C4"
' ActiveCell.Offset(1, 0).Select
' ActiveCell.Offset(1, 0).Select
' ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C15:R" \& j \&
' ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C15:R" \& j \&
' ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C15:R" \& j \&
"C15)- R37C3 + R37C4"
"C15)- R37C3 + R37C4"
"C15)- R37C3 + R37C4"
' ActiveCell.Offset(1, 0).Select
' ActiveCell.Offset(1, 0).Select
' ActiveCell.Offset(1, 0).Select
' ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C17:R" \& j \&
' ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C17:R" \& j \&
' ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C17:R" \& j \&
"C17)- R38C3 + R38C4"
"C17)- R38C3 + R38C4"
"C17)- R38C3 + R38C4"
' ActiveCell.Offset(1, 0).Select
' ActiveCell.Offset(1, 0).Select
' ActiveCell.Offset(1, 0).Select
'ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C20:R" \& j \&
'ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C20:R" \& j \&
'ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C20:R" \& j \&
"C20)- R37C3 + R37C4"
"C20)- R37C3 + R37C4"
"C20)- R37C3 + R37C4"
'ActiveCell.Offset(1, 0).Select
'ActiveCell.Offset(1, 0).Select
'ActiveCell.Offset(1, 0).Select
SolverOk SetCell:="\$K$19", MaxMinVal:=2, ValueOf:="0",
    SolverOk SetCell:="$K$19", MaxMinVal:=2, ValueOf:="0",
ByChange:="$C$20:$D$27,$e$33:$\$33"
ByChange:="$C$20:$D$27,$e$33:$\$33"
SolverSolve (True)
Sheets("Local").Select
Sheets("Local").Range("C20:C27").Copy
Sheets("Output").Select
ActiveCell.PasteSpecial Paste:=xlPasteAll, Operation:=xlNone, SkipBlanks:=
False, Transpose:=True

```

ActiveCell.Offset( 0,8 ).Select
Sheets("Local").Select
Range("D20:D27").Copy
Sheets("Output").Select
ActiveCell.PasteSpecial Paste:=xlPasteAll, Operation:=xlNone, SkipBlanks:= False, Transpose:=True
ActiveCell.Offset(0, 8).Select

Sheets("Local").Select
Sheets("Local").Range("K19").Copy
Sheets("Output").Select
ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=False
ActiveCell.Offset(1, -16).Select
Sheets("Local").Select
Range("M22:M30").Copy
Sheets("Slacks").Select
ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=True
ActiveCell.Offset( 0,9 ).Select
Sheets("Local").Select
Range("M33").Copy
Sheets("Slacks").Select
ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=False
ActiveCell.Offset(1, -9).Select
' Sheets("Local").Select
' Range("M37").Copy
' Sheets("Slacks").Select
' ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _
' :=False, Transpose:=False
' ActiveCell.Offset(0, 2).Select
' Sheets("Local").Select
' Range("M38").Copy
'Sheets("Slacks").Select
'ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _ ':=False, Transpose:=False
'ActiveCell.Offset(0, 2).Select
```

    'Sheets("Local").Select
    'Range("M37").Copy
    ' Sheets("Slacks").Select
' ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _
' :=False, Transpose:=False
' ActiveCell.Offset(1, -16).Select

```

Next \(n\)

\author{
End Sub \\ Sub Solve_GlobalTargets() \\ ' Solve_2A Macro \\ ' solving second phase
}

Dim n As Integer
Dim i As Integer
Dim j As Integer

Application.ScreenUpdating \(=\) False
Sheets("Output").Select
Range("C26").Select
Range("C26").Select
Sheets("Slacks").Select
Range("C27").Select
\[
\text { For } \mathrm{n}=1 \text { To } 8
\]

Sheets("Global").Select
Sheets("Global").Range("A16").Value = n
' \(\mathrm{i}=\mathrm{n}+19\)
' \(\mathrm{j}=\mathrm{n}+2\)
1
' Sheets("Global").Range("J22").Select
' ActiveCell.FormulaR1C1 = "= sumproduct(R20C3:R" \& i \& "C3,R3C12:R" \& j \&
' ActiveCell.Offset(1, 0).Select
" ActiveCell.FormulaR1C1 = "= sumproduct(R20C3:R" \& i \& "C3,R3C11:R" \& j \& "C11)" ActiveCell.Offset(1, 0).Select

ActiveCell.FormulaR1C1 = "= sumproduct(R20C3:R" \& i \& "C3,R3C10:R" \& j \&
ActiveCell.Offset(1, 0).Select
ActiveCell.FormulaR1C1 = "= sumproduct(R20C3:R" \& i \& "C3,R3C9:R" \& j \&
ActiveCell.Offset(1, 0).Select
ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C4:R" \& j \&
ActiveCell.Offset(1, 0).Select
ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C5:R" \& j \&
ActiveCell.Offset(1, 0).Select
ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C6:R" \& j \&
ActiveCell.Offset(1, 0).Select
ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C7:R" \& j \& "C7)"

ActiveCell.Offset(1, 0).Select
1
" ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C8:R" \& j \& "C8)"

ActiveCell.Offset( 1,0 ).Select
'ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C9:R" \& j \& "C9)" 'ActiveCell.Offset( 3,0 ).Select
```

    ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" & i & "C4,R3C11:R" & j &
    "C11)"
ActiveCell.Offset(1, 0).Select
ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C10:R" \& j \&
"C10)"
ActiveCell.Offset(1, 0).Select
ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C9:R" \& j \&
"C9)"
ActiveCell.Offset(3, 0).Select

```

ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C13:R" \& j \& "C13)- R36C3 + R36C4"

ActiveCell.Offset(1, 0).Select
```

                            ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" & i & "C4,R3C15:R" & j &
    "C15)- R37C3 + R37C4"
ActiveCell.Offset(1, 0).Select
ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C17:R" \& j \&
"C17)- R38C3 + R38C4"
ActiveCell.Offset(3, 0).Select
'ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C20:R" \& j \&
"C20)- R37C3 + R37C4"
'ActiveCell.Offset(3, 0).Select
ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C13:R" \& j \&
"C13)- R41C3 + R41C4 + R41C7"
ActiveCell.Copy
Range("G41").Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=False
ActiveCell.Offset(1, 3).Select
ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C15:R" \& j \&
"C15)- R42C3 + R42C4 + R42C7"
ActiveCell.Copy
Range("G42").Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=False
ActiveCell.Offset(1, 3).Select
' ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C17:R" \& j \&
"C17)- R43C3 + R43C4 + R43C7"
ActiveCell.Copy
Range("G43").Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=False
ActiveCell.Offset(1, 3).Select
'ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C20:R" \& j \&
"C20)- R43C3 + R43C4 + R43C7"
'ActiveCell.Copy
'Range("G43").Select
'Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
':=False, Transpose:=False
'ActiveCell.Offset(1, 3).Select
SolverOk SetCell:="\$M$19", MaxMinVal:=2, ValueOf:="0",
ByChange:="$C$20:$D$27,$d$33:$e$33,$d$36:$e\$36"
SolverSolve (True)
Sheets("Global").Select
Sheets("Global").Range("C20:C27").Copy

```
```

    Sheets("Output").Select
    ActiveCell.PasteSpecial Paste:=xlPasteAll, Operation:=xlNone, SkipBlanks:=
    False, Transpose:=True
    ActiveCell.Offset(0, 8).Select
    Sheets("Global").Select
    Range("D20:D27").Copy
    Sheets("Output").Select
    ActiveCell.PasteSpecial Paste:=xlPasteAll, Operation:=xlNone, SkipBlanks:=
    False, Transpose:=True
    ActiveCell.Offset(0, 8).Select
    Sheets("Global").Select
    Sheets("Global").Range("M19").Copy
    Sheets("Output").Select
    ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
    :=False, Transpose:=False
    ActiveCell.Offset(1, -16).Select
    Sheets("Global").Select
    Range("M22:M31").Copy
    Sheets("Slacks").Select
    ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
    :=False, Transpose:=True
    ActiveCell.Offset(0, 9).Select
    Sheets("Global").Select
Range("M33").Copy
Sheets("Slacks").Select
ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=False
ActiveCell.Offset(1,-9).Select
' Sheets("Global").Select
' Range("M37").Copy
'
' Sheets("Slacks").Select
' ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
' :=False, Transpose:=False
' ActiveCell.Offset(0, 2).Select
'
' Sheets("Global").Select
' Range("M38").Copy
' Sheets("Slacks").Select

```
' ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=False, Transpose:=False
' ActiveCell.Offset(1, -16).Select
'Sheets("Global").Select
'Range("M37").Copy
'Sheets("Slacks").Select
'ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks ':=False, Transpose:=False
'ActiveCell.Offset(1, -16).Select

\section*{Next n}

Sheets("Global").Select
Range("M36").Copy
Sheets("Slacks").Select
Range("N41").PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=True

End Sub

\section*{Appendix A 1.2}

\section*{Code for BT Simulated Dataset, Variable Returns To Scale Model}
```

Sub Clear()
Sheets("(Ph1_St1)").Select
Range("C22:C38").Value = 0
Sheets("(Ph1_St2)").Select
'Range("B3:V12").ClearContents
Range("S17:AE39").ClearContents
Sheets("(Ph2)").Select
Range("C20:D27").Value = 0
Range("F19").Value = 0
Sheets("Local").Select
Range("C20:D27").Value = 0
Range("C34:D37").Value = 0
Range("F33").Value = 0
Sheets("Global").Select
Range("C20:D27").Value = 0
Range("C34:D37").Value = 0
Range("F33").Value = 0
Range("C40:D43").Value = 0
Range("G40:G43").Value = 0
Range("F39").Value = 0
'Sheets("Output").Select
' Range("C3:W12").ClearContents
' Range("D17:X26").ClearContents
' Range("D30:X39").ClearContents
' Sheets("Slacks").Select
' Range("C3:L12").ClearContents
' Range("C17:M26").ClearContents
' Range("o17:026").ClearContents
' Range("q17:q26").ClearContents
' Range("s17:s26").ClearContents
' Range("C31:M40").ClearContents
' Range("S31:S40").ClearContents
' Range("O31:O40").ClearContents
' Range("Q31:Q40").ClearContents
' Range("N42:Q42").ClearContents
End Sub
Sub Solve_1_1()

```
```

' solve_1_1 Macro
' solves and stores output for stage 1, phase 1
'
' Keyboard Shortcut: Ctrl+t
Application.ScreenUpdating = False
' Selecting the cell to start on the Ph1_St2 sheet
Sheets("(Ph1_St2)").Select
Range("B3").Select
Sheets("Slacks_2").Select
Range("B2").Select
'Optimize
Dim n As Integer
For n=1 To 8
Sheets("(Ph1_St1)").Select
Sheets("(Ph1_St1)").Range("A17").Value = n
SolverOk SetCell:="\$G$21", MaxMinVal:=1, ValueOf:="0", ByChange:="$C$22:$C\$38"
SolverSolve (True)
Sheets("(Ph1 St1)").Select
Range("C22:C38").Copy
Sheets("(Ph1_St2)").Select
ActiveCell.PasteSpecial Paste:=xlValue, Operation:=xlNone, SkipBlanks:=
False, Transpose:=True
ActiveCell.Offset(1, 0).Select
Sheets("(Ph1_St1)").Select
Range("K24:K34").Copy
Sheets("Slacks_2").Select
ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks:=
False, Transpose:=True
ActiveCell.Offset(1, 0).Select
Next n
Sheets("(Ph1_St2)").Select
Range("A1").Select
Sheets("(Ph1_St1)").Select
Range("A1").Select
End Sub
Sub Reg_1_2()
' Reg_2_1 Macro
' Does Regression for Stage 2 of Phase 1

```
' Keyboard Shortcut: Ctrl+u
```

    Sheets("(Ph1_St2)").Select
    Application.Run "ATPVBAEN.XLA!Regress", ActiveSheet.Range("$B$16:$B$23"),
        ActiveSheet.Range("$C$16:$H$23"), True, False, , ActiveSheet.Range(
        "$S$17:$AB$40"), False, False, False, False, , False
    Sheets("(Ph1_St2)").Select
    Range("T34:T39").Select
    Selection.Copy
    Range("C14").Select
    Selection.PasteSpecial Paste:=xlValue, Operation:=xlNone, SkipBlanks:=
        False, Transpose:=True
    End Sub
Sub Sort_Env_1_2()
' Sort Env_1_2 Macro
' Ranks the bus routes as per their degree of harshness
'
' Keyboard Shortcut: Ctrl+i
Sheets("(Ph1 St2)").Select
Range("A15:I23").Select
Selection.Copy
Range("B25").Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=False
'Range("G35").Select
'Application.CutCopyMode = False
Range("B26:J33").Sort Key1:=Range("J30"), Order 1:=xlDescending, Header:=
xlGuess, OrderCustom:=1, MatchCase:=False, Orientation:=xlTopToBottom,
DataOption1:=xlSortNormal
'Range("A25:J33").Copy
'Sheets("Data(Ph2)").Select
'Range("A2").PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _
:=False, Transpose:=False
End Sub
Sub Solve_2_Efficiency()
' Solve_2A Macro
' solving second phase
Dim n As Integer
Dim i As Integer
Dim j As Integer

```

Application.ScreenUpdating \(=\) False
Sheets("Output").Select
Range("D3").Select
Sheets("Slacks").Select
Range("C3").Select
For \(\mathrm{n}=1\) To 8
Sheets("(Ph2)").Select
Sheets("(Ph2)").Range("A16").Value = n
\(\mathrm{i}=\mathrm{n}+19\)
\(\mathrm{j}=\mathrm{n}+2\)
Sheets("(Ph2)").Range("J22").Select
ActiveCell.FormulaR1C1 = "= sumproduct(R20C3:R" \& i \& "C3,R3C12:R" \& j \& "C12)"

ActiveCell.Offset( 1,0 ).Select
ActiveCell.FormulaR1C1 = "= sumproduct(R20C3:R" \& i \& "C3,R3C11:R" \& j \& "C11)"

ActiveCell.Offset( 1,0 ).Select
ActiveCell.FormulaR1C1 = "= sumproduct(R20C3:R" \& i \& "C3,R3C10:R" \& j \& "C10)"

ActiveCell.Offset(1, 0).Select
ActiveCell.FormulaR1C1 = "= sumproduct(R20C3:R" \& i \& "C3,R3C9:R" \& j \& "C9)" ActiveCell.Offset( 1,0 ).Select

ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C4:R" \& j \& "C4)" ActiveCell.Offset( 1,0 ).Select

ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C5:R" \& j \& "C5)" ActiveCell.Offset( 1,0 ).Select

ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C6:R" \& j \& "C6)" ActiveCell.Offset( 1,0 ).Select

ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C7:R" \& j \& "C7)" ActiveCell.Offset(1, 0).Select
'ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C8:R" \& j \& "C8)"
'ActiveCell.Offset(1, 0).Select
ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C11:R" \& j \& "C11)"

ActiveCell.Offset(1, 0).Select

ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C10:R" \& j \& "C10)"

ActiveCell.Offset(1, 0).Select
ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C9:R" \& j \& "C9)" ActiveCell.Offset( 1,0 ).Select

ActiveCell.FormulaR1C1 = "= sum(R20C3:R" \& i \& "C3)"
ActiveCell.Offset( 1,0 ).Select
ActiveCell.FormulaR1C1 = "= sum(R20C4:R" \& i \& "C4)"
ActiveCell.Offset(1, 0).Select
'ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C9:R" \& j \& "C9)" 'ActiveCell.Offset(3, 0).Select
'ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C14:R" \& j \& "C14)- R34C3 + R34C4"
'ActiveCell.Offset( 1,0 ).Select
'ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C16:R" \& j \& "C16)- R35C3 + R35C4"
'ActiveCell.Offset( 1,0 ).Select
'ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C18:R" \& j \& "C18)- R36C3 + R36C4"
'ActiveCell.Offset( 1,0 ).Select
'ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C20:R" \& j \& "C20)- R37C20 + R37C4"
'ActiveCell.Offset( 1,0 ).Select
SolverOk SetCell:="\$I\$19", MaxMinVal:=1, ValueOf:="0", ByChange:="\$C\$20:\$D\$" \& i \& ",\$F\$19"

SolverSolve (True)
Sheets("(Ph2)").Select
Sheets("(Ph2)").Range("C20:C33").Copy
Sheets("Output").Select
ActiveCell.PasteSpecial Paste:=xlPasteAll, Operation:=xlNone, SkipBlanks:=
False, Transpose:=True
ActiveCell.Offset( 0,8 ).Select
Sheets("(Ph2)").Select
Range("D20:D27").Copy
Sheets("Output").Select
ActiveCell.PasteSpecial Paste:=xlPasteAll, Operation:=xlNone, SkipBlanks:=
False, Transpose:=True
ActiveCell.Offset( \(0,-9\) ).Select

Sheets("(Ph2)").Select
Sheets("(Ph2)").Range("F19").Copy
Sheets("Output").Select
ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=False
ActiveCell.Offset(1, 1).Select
Sheets("(Ph2)").Select
Range("M22:M32").Copy
Sheets("Slacks").Select
ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=True
ActiveCell.Offset(1, 0).Select
'Sheets("(Ph2)").Select
'Range("M34").Copy
'Sheets("Slacks").Select
'ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=False
'ActiveCell.Offset( 0,2 ).Select
'Sheets("(Ph2)").Select
'Range("M35").Copy
'Sheets("Slacks").Select
'ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _ :=False, Transpose:=False
'ActiveCell.Offset( 0,2 ).Select
'Sheets("(Ph2)").Select
'Range("M36").Copy
'Sheets("Slacks").Select
'ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _ :=False, Transpose:=False
'ActiveCell.Offset( 0,2 ).Select
'Sheets("(Ph2)").Select
'Range("M37").Copy
'Sheets("Slacks").Select
'ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _ ':=False, Transpose:=False
'ActiveCell.Offset(1, -16).Select

Next \(n\)

End Sub
Sub Solve_2_LocalTargets()

Dim n As Integer
Dim i As Integer
Dim j As Integer

Application.ScreenUpdating \(=\) False
Sheets("Output"). Select
Range("D15").Select

Sheets("Slacks").Select
Range("C15").Select

For \(n=1\) To 8

Sheets("Local").Select
Sheets("Local").Range("A16").Value = n
\[
\begin{aligned}
& \mathrm{i}=\mathrm{n}+19 \\
& \mathrm{j}=\mathrm{n}+2
\end{aligned}
\]

Sheets("Local").Range("J22").Select
ActiveCell.FormulaR1C1 = "= sumproduct(R20C3:R" \& i \& "C3,R3C12:R" \& j \& "C12)"

ActiveCell.Offset(1, 0). Select
ActiveCell.FormulaR1C1 = "= sumproduct(R20C3:R" \& i \& "C3,R3C11:R" \& j \& "C11)"

ActiveCell.Offset(1, 0).Select
ActiveCell.FormulaR1C1 = "= sumproduct(R20C3:R" \& i \& "C3,R3C10:R" \& j \& "C10)"

ActiveCell.Offset(1, 0).Select
ActiveCell.FormulaR1C1 = "= sumproduct(R20C3:R" \& i \& "C3,R3C9:R" \& j \& "C9)" ActiveCell.Offset(1, 0).Select

ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C4:R" \& j \& "C4)" ActiveCell.Offset(1, 0).Select

ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C5:R" \& j \& "C5)" ActiveCell.Offset(1, 0).Select

ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C6:R" \& j \& "C6)" ActiveCell.Offset( 1,0 ).Select

ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C7:R" \& j \& "C7)" ActiveCell.Offset( 1,0 ).Select
'ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C8:R" \& j \& "C8)" 'ActiveCell.Offset( 1,0 ).Select
'ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C9:R" \& j \& "C9)"
'ActiveCell.Offset( 1,0 ).Select
ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C11:R" \& j \& "C11)"

ActiveCell.Offset(1, 0).Select
ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C10:R" \& j \& "C10)"

ActiveCell.Offset( 1,0 ).Select
ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C9:R" \& j \& "C9)" ActiveCell.Offset( 3,0 ).Select

ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C13:R" \& j \& "C13)R35C3 + R35C4"

ActiveCell.Offset(1, 0).Select
ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C15:R" \& j \& "C15)R36C3 + R36C4"

ActiveCell.Offset \((1,0)\).Select
ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C17:R" \& j \& "C17)R37C3 + R37C4"

ActiveCell.Offset(3, 0).Select
'ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C20:R" \& j \& "C20)- R37C3 + R37C4"
'ActiveCell.Offset( 1,0 ).Select
ActiveCell.FormulaR1C1 = "=sum(R20C3:R" \& i \& "C3)"
ActiveCell.Offset \((1,0)\).Select
ActiveCell.FormulaR1C1 = "=sum(R20C4:R" \& i \& "C4)"
ActiveCell.Offset(1, 0).Select

SolverOk SetCell:="\$K\$19", MaxMinVal:=2, ValueOf:="0", ByChange:="\$C\$20:\$D\$" \& i \& ",\$C\$35:\$D\$37"
```

SolverSolve (True)

```

Sheets("Local").Select
Sheets("Local").Range("C20:C27").Copy
Sheets("Output").Select
ActiveCell.PasteSpecial Paste:=xlPasteAll, Operation:=xlNone, SkipBlanks:=
False, Transpose:=True
ActiveCell.Offset( 0,8 ).Select
Sheets("Local").Select
Range("D20:D27").Copy
Sheets("Output").Select
ActiveCell.PasteSpecial Paste:=xlPasteAll, Operation:=xlNone, SkipBlanks:= False, Transpose:=True
ActiveCell.Offset( 0,8 ).Select

Sheets("Local").Select
Sheets("Local").Range("K19").Copy
Sheets("Output").Select
ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=False
ActiveCell.Offset(1, -16).Select
Sheets("Local").Select
Range("M22:M32").Copy
Sheets("Slacks").Select
ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=True
ActiveCell.Offset \((0,11)\).Select
Sheets("Local").Select
Range("M35").Copy
Sheets("Slacks").Select
ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=False, Transpose:=False
ActiveCell.Offset( 0,2 ).Select
Sheets("Local").Select
Range("M36").Copy
Sheets("Slacks").Select
ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=False
ActiveCell.Offset( 0,2 ).Select
```

Sheets("Local").Select
Range("M37").Copy
'Sheets("Slacks").Select
'ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _
':=False, Transpose:=False
'ActiveCell.Offset(0, 2).Select
'Sheets("Local").Select
'Range("M37").Copy
Sheets("Slacks").Select
ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=False
ActiveCell.Offset(1, -15).Select

```

Next \(n\)

\section*{End Sub}

Sub Solve_2_GlobalTargets()
'
' Solve_2A Macro
' solving second phase
,

Dim n As Integer
Dim i As Integer
Dim j As Integer

Application.ScreenUpdating \(=\) False
Sheets("Output").Select
Range("D26").Select
Sheets("Slacks").Select
Range("C27").Select
For \(\mathrm{n}=1\) To 8
Sheets("Global").Select
Sheets("Global").Range("A16").Value = n
\[
\begin{aligned}
& \mathrm{i}=\mathrm{n}+19 \\
& \mathrm{j}=\mathrm{n}+2
\end{aligned}
\]
\begin{tabular}{|c|c|}
\hline \multicolumn{2}{|r|}{\multirow[t]{2}{*}{\begin{tabular}{l}
Sheets("Global").Range("J22").Select \\
ActiveCell.FormulaR1C1 = "= sumproduct(R20C3:R" \& i \& "C3,R3C12:R" \& j \&
\end{tabular}}} \\
\hline & \\
\hline \multicolumn{2}{|l|}{"C12)"} \\
\hline & ActiveCell.Offset( 1,0\()\).Select \\
\hline & ActiveCell.FormulaR1C1 = "= sumproduct(R20C3:R" \& i \& "C3,R3C11:R" \& j \& \\
\hline \multirow[t]{2}{*}{"C11)"} & \\
\hline & ActiveCell.Offset( 1,0\()\).Select \\
\hline \multirow{3}{*}{"C10)"} & ActiveCell.FormulaR1C1 = "= sumproduct(R20C3:R" \& i \& "C3,R3C10:R" \& j \& \\
\hline & \\
\hline & ActiveCell.Offset(1, 0).Select \\
\hline \multirow{3}{*}{"C9)"} & ActiveCell.FormulaR1C1 = "= sumproduct(R20C3:R" \& i \& "C3,R3C9:R" \& j \& \\
\hline & \\
\hline & ActiveCell.Offset( 1,0\()\).Select \\
\hline \multirow{3}{*}{"C4)"} & ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C4:R" \& j \& \\
\hline & \\
\hline & ActiveCell.Offset(1, 0).Select \\
\hline \multirow{3}{*}{"C5)"} & ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C5:R" \& j \& \\
\hline & \\
\hline & ActiveCell.Offset( 1,0 ).Select \\
\hline \multirow[b]{2}{*}{"C6)"} & ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C6:R" \& j \& \\
\hline & ActiveCell.Offset(1, 0).Select \\
\hline \multirow{3}{*}{"C7)"} & ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C7:R" \& j \& \\
\hline & \\
\hline & ActiveCell.Offset( 1,0 ).Select \\
\hline \multirow{3}{*}{"C8)"} & 'ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C8:R" \& j \& \\
\hline & \\
\hline & 'ActiveCell.Offset( 1,0 ).Select \\
\hline \multirow{3}{*}{"C9)"} & 'ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C9:R" \& j \& \\
\hline & \\
\hline & 'ActiveCell.Offset(3, 0).Select \\
\hline \multirow{3}{*}{"C11)"} & ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C11:R" \& j \& \\
\hline & \\
\hline & ActiveCell.Offset(1, 0).Select \\
\hline \multirow{3}{*}{"C10)"} & ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C10:R" \& j \& \\
\hline & \\
\hline & ActiveCell.Offset(1, 0).Select \\
\hline
\end{tabular}

ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C9:R" \& j \& "C9)"

ActiveCell.Offset(3, 0).Select

ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C13:R" \& j \& "C13)- R35C3 + R35C4"

ActiveCell.Offset \((1,0)\).Select
ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C15:R" \& j \& "C15)- R36C3 + R36C4"

ActiveCell.Offset \((1,0)\).Select
ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C17:R" \& j \& "C17)- R37C3 + R37C4"

ActiveCell.Offset(3, 0).Select
'ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C20:R" \& j \& "C20)- R37C3 + R37C4"
'ActiveCell.Offset( 3,0 ).Select
ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C13:R" \& j \& "C13)- R40C3 + R40C4 + R40C7"

ActiveCell.Copy
Range("G40").Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=False
ActiveCell.Offset(1, 3).Select
ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C15:R" \& j \& "C15)- R41C3 + R41C4 + R41C7"

ActiveCell.Copy
Range("G41").Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=False
ActiveCell.Offset(1, 3).Select

ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C17:R" \& j \& "C17)- R42C3 + R42C4 + R42C7"

ActiveCell.Copy
Range("G42").Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _
:=False, Transpose:=False
ActiveCell.Offset(3, 3).Select
ActiveCell.FormulaR1C1 = "=sum(R20C3:R" \& i \& "C3)"
ActiveCell.Offset( 1,0 ).Select
ActiveCell.FormulaR1C1 = "=sum(R20C4:R" \& i \& "C4)"
ActiveCell.Offset( 1,0 ).Select
```

    'ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" & i & "C4,R3C20:R" & j &
    "C20)- R43C3 + R43C4 + R43C7"
'ActiveCell.Copy
'Range("G43").Select
'Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
':=False, Transpose:=False
'ActiveCell.Offset(1, 3).Select
SolverOk SetCell:="\$M$19", MaxMinVal:=2, ValueOf:="0", ByChange:="$C$20:$D$"
& i & ",$C$35:$D$37,$C$40:$D\$42"
SolverSolve (True)
Sheets("Global").Select
Sheets("Global").Range("C20:C27").Copy
Sheets("Output").Select
ActiveCell.PasteSpecial Paste:=xlPasteAll, Operation:=xlNone, SkipBlanks:=

```
\(\qquad\)
``` False, Transpose:=True
ActiveCell.Offset( 0,8 ).Select
Sheets("Global").Select
Range("D20:D27").Copy
Sheets("Output").Select
ActiveCell.PasteSpecial Paste:=xlPasteAll, Operation:=xlNone, SkipBlanks:=
False, Transpose:=True
ActiveCell.Offset( 0,8 ).Select
Sheets("Global").Select
Sheets("Global").Range("M19").Copy
Sheets("Output").Select
ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=False, Transpose:=False
ActiveCell.Offset(1, -16).Select
Sheets("Global").Select
Range("M22:M32").Copy
Sheets("Slacks").Select
ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=False, Transpose:=True
ActiveCell.Offset( 0,11 ).Select
Sheets("Global").Select
Range("M35").Copy
Sheets("Slacks").Select
ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
```

:=False, Transpose:=False
ActiveCell.Offset(0, 2).Select
Sheets("Global").Select
Range("M36").Copy
Sheets("Slacks").Select
ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=False, Transpose:=False
ActiveCell.Offset(0, 2).Select
Sheets("Global").Select
Range("M37").Copy
Sheets("Slacks").Select
ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=False
ActiveCell.Offset(1, -15).Select
'Sheets("Global").Select
'Range("M37").Copy
'Sheets("Slacks").Select
'ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
':=False, Transpose:=False
'ActiveCell.Offset(1, -16).Select

Next $n$
Sheets("Global").Select
Range("M40:M42").Copy
Sheets("Slacks").Select
Range("L41").PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks $:=$ False, Transpose:=True

End Sub

## Appendix A 2.1

## Code for Infrastructure 1, Constant Returns To Scale

Sub Clear()
' This module clears the the variables values and regression range left over from
' previous data set
' 12/10/02, Chintan Sheth
Sheets("(Ph1_St1)").Select
'Range("A4").Value = 1
Range("B10:C109").Value = 0
Range("E9").Value = 0
' The first phase - first stage
Sheets("(Ph1_St2)").Select
'Range("B3:V12").ClearContents
Range("W17:AE40").ClearContents
' The first phase - second stage
Sheets("(Ph2)").Select
Range("C8:C107").Value = 0
Range("F7").Value = 0
' The second phase - Efficiency
Sheets("Local").Select
Range("C8:D107").Value = 0
Range("p25:q28").Value = 0
' The second phase - Local Targets
Sheets("Global").Select
Range("C8:D107").Value = 0
Range("p25:q28").Value = 0
Range("P31:Q34").Value = 0
Range("G31:G34").Value = 0
'The second phase - Global Targets
'Sheets("Output").Select
' Range("C3:W12").ClearContents
' Range("D17:X26").ClearContents
' Range("D30:X39").ClearContents
,
'Sheets("Slacks").Select
' Range("C3:L12").ClearContents
' Range("C17:M26").ClearContents
' Range("o17:026").ClearContents
'Range("q17:q26").ClearContents
'Range("s17:s26").ClearContents

'Range("C31:M40").ClearContents<br>'Range("S31:S40").ClearContents<br>'Range("O31:O40").ClearContents<br>'Range("Q31:Q40").ClearContents<br>'Range("N42:Q42").ClearContents

End Sub
Sub Solve_1_1()
' CRS 100 GP
' solve_1_1 Macro
' solves and stores output for stage 1, phase 1. It does not consider the effects of
' environmental/traffic operating factors
' for Constant Returns to Scale assumption, with no modifications
' 12/10/02, Chintan Sheth
' Keyboard Shortcut: Ctrl+t
Application.ScreenUpdating = False ' Prevents Updating of the Sheet with every iteration
Dim x As Integer 'Declaring a variable
' Creates a template for the storing output values; namely, the peers and the slcaks
Sheets("Peers_1_1").Select
Range("A3").Select
For $\mathrm{x}=1$ To 100
Sheets("Peers_1_1").Select
ActiveCell.Value $=x$
ActiveCell.Offset( 1,0 ).Select
Next x
Sheets("Peers_1_1").Select
Range("A $10 \overline{0} 6$ ").Select
For $\mathrm{x}=1$ To 100
Sheets("Peers_1_1").Select
ActiveCell. Value $=x$
ActiveCell.Offset( 1,0 ).Select
Next x
Sheets("Peers_1_1").Select
Range(" $\overline{\mathrm{C}} 2 \overline{\text { " }}$ ). Select
For $\mathrm{x}=1$ To 100
Sheets("Peers_1_1").Select
ActiveCell. Value $=x$
ActiveCell.Offset( 0,1 ).Select
Next x
Sheets("Peers_1_1").Select

Range("C105").Select
For $\mathrm{x}=1$ To 100
Sheets("Peers_1_1").Select
ActiveCell.Value $=x$
ActiveCell.Offset( 0,1 ).Select
Next x
Sheets("Slacks_1_1").Select
Range("C3").Select
' Selects the cell on the sheet for Peers of output phase 1 and stage 1 to be used as ActiveCell
' for the purpose of looping
Sheets("Peers_1_1").Select
Range("B3").Select

## 'Main loop

Dim n As Integer
' $\mathrm{n}=$ number of DMUs(bus routes), acts as a counter in the table from which Solver
' picks up the DMU to evaluate
For $\mathrm{n}=1$ To 100
Sheets("(Ph1_St1)").Select
Sheets("(Ph1_St1)").Range("A4").Value = n
'Setting Solver parameters
SolverOk SetCell:="\$G\$8", MaxMinVal:=1, ValueOf:="0", ByChange:="\$B\$10:\$C\$109,\$E\$9"

SolverSolve (True)
'Storing of Results
Sheets("(Ph1_St1)").Select
Range("\$E\$9").Copy
Sheets("Peers_1_1").Select
ActiveCell.PasteSpecial Paste:=xlValue, Operation:=xlNone, SkipBlanks:= False, Transpose:=False
ActiveCell.Offset( 0,1 ).Select
Sheets("(Ph1_St1)").Select
Range("\$B\$10:\$B\$109").Copy
Sheets("Peers_1_1").Select
ActiveCell.PasteSpecial Paste:=xlValue, Operation:=xlNone, SkipBlanks:=
False, Transpose:=True
ActiveCell.Offset(103, 0).Select
Sheets("(Ph1_St1)").Select

Range("\$C\$10:\$C\$109").Copy
Sheets("Peers_1_1").Select
ActiveCell.PasteSpecial Paste:=xlValue, Operation:=xlNone, SkipBlanks:=
False, Transpose:=True
ActiveCell.Offset(-102, -1).Select
Sheets("(Ph1_St1)").Select
Range("\$K\$11:\$K\$22").Copy
Sheets("Slacks_1_1").Select
ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks:= False, Transpose:=True
ActiveCell.Offset(1, 0).Select

## Next $n$

Sheets("(Ph1_St2)").Select
Range("A1").Select
Sheets("(Ph1_St1)").Select
Range("A1").Select
End Sub
Sub Reg_1_1()
'
' Reg_2_1 Macro
' Does Regression for Stage 2 of Phase 1 to obtain the effect/importance of each environmental
' factor towards the efficiency of stage 1 of phase 1
,
' Keyboard Shortcut: Ctrl+u
Dim x As Integer
' Sheets("(Ph1_St2)").Select
' Range("A5").Select
'For $\mathrm{x}=1$ To 100
' Sheets("(Ph1_St2)").Select

- ActiveCell. Value $=x$
' ActiveCell.Offset( 1,0 ).Select
' Seeting up Regression parameters
Sheets("(Ph1_St2)").Select
Application.Run "ATPVBAEN.XLA!Regress", ActiveSheet.Range("\$B\$5:\$B\$104"), ActiveSheet.Range("\$C\$5:\$G\$104"), True, False, , ActiveSheet.Range(
"\$W\$17:\$AE\$39"), False, False, False, False, , False
' The following evaluates a combined environmental factor as a weighted sum of the
' individual environmental factors where the weights are obtained from the Regression
Sheets("(Ph1_St2)").Select
Range("X34:X38").Select
Selection.Copy

```
    Range("C3").Select
    Selection.PasteSpecial Paste:=xlValue, Operation:=xlNone, SkipBlanks:=
        False, Transpose:=True
End Sub
Sub Sort_Env_1_2()
,
' Sort_Env_1_2 Macro
' Ranks the DMU's (bus routes) as per the degree of harshness of environmental factors, i.e.,
' the macro sorts the regresses list in descending order, where the first DMU has the harshest
' environment
' Keyboard Shortcut: Ctrl+i
Dim x As Integer
    Sheets("(Ph1_St2)").Select
        Range("K5").Select
        For x = 1 To 100
        Sheets("(Ph1_St2)").Select
            ActiveCell.Value = x
        ActiveCell.Offset(1, 0).Select
        Next x
    Range("A4:H104").Select
    Selection.Copy
    Range("L4").Select
    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
        :=False, Transpose:=False
    'Range("G35").Select
    'Application.CutCopyMode = False
    Range("L5:S104").Sort Key1:=Range("S35"), Order1:=xlDescending, Header:=
        xlGuess, OrderCustom:=1, MatchCase:=False, Orientation:=x1TopToBottom,
        DataOption1:=xlSortNormal
End Sub
Sub Solve_2_Efficiency()
' Solve_2A Macro
' solving second phase
' This macro is for the second stage, i.e., the main model. It evaluates the efficiency of the
' network/DMU/bus route by comparing it with only those DMU's operating in similar or harsher
' environments
'
    Dim n As Integer
    Dim i As Integer
    Dim j As Integer
    Dim x As Integer
    ' Declaring variables, these variables act as counters
```

    Application.ScreenUpdating \(=\) False
    Sheets("Peers_2_1").Select
    ```
Range("A3:B102").Copy
Range("A106").Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=False
Range("B3:B102").Copy
Range("D2").Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=True
Range("B3:B102").Copy
Range("D105").Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=True
```

Sheets("Peers_2_1").Select
Range("C3").Select
Sheets("Slacks_2_1").Select
Range("C3").Select
For $\mathrm{n}=1$ To 100
Sheets("(Ph2)").Select
Sheets("(Ph2)").Range("A4").Value = n
' Initializing the values of i and j
' These act as counters for the selection of cells values without referencing the ' cells directly

$$
\begin{aligned}
& \mathrm{i}=\mathrm{n}+7 \\
& \mathrm{j}=\mathrm{n}+3
\end{aligned}
$$

' Selection of Cell, for the first constraint
Sheets("(Ph2)").Range("J10").Select
' Constraint for Passenger-miles
ActiveCell.FormulaR1C1 = "= sumproduct(R8C3:R" \& i \& "C3,R4C35:R" \& j \& "C35)" ActiveCell.Offset(1, 0).Select
' Constraint for Schedule Reliability
ActiveCell.FormulaR1C1 = "= sumproduct(R8C3:R" \& i \& "C3,R4C34:R" \& j \& "C34)" ActiveCell.Offset(1, 0).Select
' Constraint for Vehicle-miles
ActiveCell.FormulaR1C1 = "= sumproduct(R8C3:R" \& i \& "C3,R4C33:R" \& j \& "C33)" ActiveCell.Offset(1, 0).Select
' Constraint for Trip Time
ActiveCell.FormulaR1C1 = "= sumproduct(R8C3:R" \& i \& "C3,R4C32:R" \& j \& "C32)"

ActiveCell.Offset( 1,0 ).Select
' Constraint for Headway
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C26:R" \& j \& "C26)" ActiveCell.Offset( 1,0 ).Select
' Constraint for Cost
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C27:R" \& j \& "C27)" ActiveCell.Offset(1, 0).Select
' Constraint Service Duration
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C28:R" \& j \& "C28)" ActiveCell.Offset(1, 0).Select
' Constraint for Number of Intersection
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C29:R" \& j \& "C29)" ActiveCell.Offset(1, 0).Select
' Constraint for Number of Priority Lanes
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C30:R" \& j \& "C30)" ActiveCell.Offset( 1,0 ).Select
' Constraint for Congestion
'ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C31:R" \& j \& "C31)" 'ActiveCell.Offset( 1,0 ).Select
' Constraint for Schedule Reliability
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C34:R" \& j \& "C34)" ActiveCell.Offset( 1,0 ).Select
' Constraint for Vehicle-miles
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C33:R" \& j \& "C33)" ActiveCell.Offset( 1,0 ).Select
' Constraint for Trip Time
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C32:R" \& j \& "C32)"
ActiveCell.Offset( 1,0 ).Select
'ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C14:R" \& j \& "C14)- R34C3 + R34C4"
'ActiveCell.Offset(1, 0).Select
'ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C16:R" \& j \& "C16)- R35C3 + R35C4"
'ActiveCell.Offset(1, 0).Select
'ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C18:R" \& j \& "C18)- R36C3 + R36C4"
'ActiveCell.Offset( 1,0 ).Select
'ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C20:R" \& j \& "C20)- R37C20 + R37C4"
'ActiveCell.Offset(1, 0).Select
'ActiveCell.FormulaR1C1 = "= sum(R8C3:R" \& i \& "C3)"
'ActiveCell.Offset(1, 0).Select
'ActiveCell.FormulaR1C1 = "= sum(R8C4:R" \& i \& "C4)"
'ActiveCell.Offset( 1,0 ).Select

## ' Setting up Solver parameters

SolverOk SetCell:="\$I\$7", MaxMinVal:=1, ValueOf:="0", ByChange:="\$C\$8:\$D\$" \& i \& ",\$F\$7"

SolverSolve (True)
' Storing of Peers, Slack for each DMU
Sheets("(Ph2)").Select
Sheets("(Ph2)").Range("F7").Copy
Sheets("Peers_2_1").Select
ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=False
ActiveCell.Offset( 0,1 ).Select
Sheets("(Ph2)").Select
Sheets("(Ph2)").Range("C8:C107").Copy
Sheets("Peers_2_1").Select
ActiveCell.PasteSpecial Paste:=xlPasteAll, Operation:=xlNone, SkipBlanks:=
False, Transpose:=True
ActiveCell.Offset(103, 0).Select
Sheets("(Ph2)").Select
Range("D8:D107").Copy
Sheets("Peers_2_1").Select
ActiveCell.PasteSpecial Paste:=xlPasteAll, Operation:=xlNone, SkipBlanks:=
False, Transpose:=True
ActiveCell.Offset(-102, -1).Select
'Sheets("(Ph2)").Select
'Sheets("(Ph2)").Range("F19").Copy
'Sheets("Output").Select
'ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _
':=False, Transpose:=False
'ActiveCell.Offset(1, 1).Select

```
Sheets("(Ph2)").Select
Range("M10:M22").Copy
Sheets("Slacks_2_1").Select
ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=True
ActiveCell.Offset(1, 0).Select
'Sheets("(Ph2)").Select
'Range("M34").Copy
'Sheets("Slacks").Select
'ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _
:=False, Transpose:=False
'ActiveCell.Offset(0, 2).Select
'Sheets("(Ph2)").Select
'Range("M35").Copy
'Sheets("Slacks").Select
'ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=False
'ActiveCell.Offset(0, 2).Select
'Sheets("(Ph2)").Select
'Range("M36").Copy
'Sheets("Slacks").Select
'ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=False
'ActiveCell.Offset(0, 2).Select
'Sheets("(Ph2)").Select
'Range("M37").Copy
'Sheets("Slacks").Select
'ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _
':=False, Transpose:=False
'ActiveCell.Offset(1, -16).Select
```


## Next $n$

Sheets("Peers_2_1").Select
Range("C3:C102").Copy

Range("C106").PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=False, Transpose:=False

## End Sub

Sub Solve_2_LocalTargets()
' Solves for the achieving of the targets of each DMU with respect to Societal Variables.
' Uses the Efficiency of the DMU as known in this module.
Dim n As Integer
Dim i As Integer
Dim j As Integer
Dim x As Integer

Application.ScreenUpdating $=$ False
Sheets("Peers_2_2").Select
Range("B3:B102").Copy
Range("E2").PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=True
Sheets("Peers_2_2").Select
Range("A3:D102").Copy
Range("A106").PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=False
Range("B3:B102").Copy
Range("E105").PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=True
Sheets("Peers_2_2").Select
Range("D3").Select
Sheets("Slacks_2_2").Select
Range("C3").Select

For $\mathrm{n}=1$ To 100
Sheets("Local").Select
Sheets("Local").Range("A4").Value = n

$$
\begin{aligned}
& \mathrm{i}=\mathrm{n}+7 \\
& \mathrm{j}=\mathrm{n}+3
\end{aligned}
$$

Sheets("Local").Range("J10").Select
' Constraint for Passenger-miles
ActiveCell.FormulaR1C1 = "= sumproduct(R8C3:R" \& i \& "C3,R4C35:R" \& j \& "C35)" ActiveCell.Offset(1, 0).Select
' Constraint for Schedule Reliability
ActiveCell.FormulaR1C1 = "= sumproduct(R8C3:R" \& i \& "C3,R4C34:R" \& j \& "C34)" ActiveCell.Offset( 1,0 ).Select
' Constraint for Vehicle-miles
ActiveCell.FormulaR1C1 = "= sumproduct(R8C3:R" \& i \& "C3,R4C33:R" \& j \& "C33)" ActiveCell.Offset(1, 0).Select
' Constraint for Trip Time
ActiveCell.FormulaR1C1 = "= sumproduct(R8C3:R" \& i \& "C3,R4C32:R" \& j \& "C32)" ActiveCell.Offset(1, 0).Select
' Constraint for Headway
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C26:R" \& j \& "C26)" ActiveCell.Offset(1, 0).Select
' Constraint for Cost
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C27:R" \& j \& "C27)" ActiveCell.Offset( 1,0 ).Select
' Constraint Service Duration
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C28:R" \& j \& "C28)" ActiveCell.Offset( 1,0 ).Select
' Constraint for Number of Intersections
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C29:R" \& j \& "C29)" ActiveCell.Offset $(1,0)$.Select
' Constraint for Number of Priority Lanes
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C30:R" \& j \& "C30)" ActiveCell.Offset( 1,0 ).Select
' Constraint for Congestion
'ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C31:R" \& j \& "C31)" 'ActiveCell.Offset( 1,0 ).Select
' Constraint for Schedule Reliability
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C34:R" \& j \& "C34)"
ActiveCell.Offset( 1,0 ).Select
' Constraint for Vehicle-miles
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C33:R" \& j \& "C33)" ActiveCell.Offset( 1,0 ).Select
' Constraint for Trip Time
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C32:R" \& j \& "C32)" ActiveCell.Offset( 3,0 ).Select
'Local Targets
'Accidents
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C36:R" \& j \& "C36)R24C16 + R24C17"

ActiveCell.Offset(1, 0).Select
'Emissions
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C38:R" \& j \& "C38)R25C16 + R25C17"

ActiveCell.Offset(1, 0).Select
'Noise Pollution
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C40:R" \& j \& "C40)R26C16 + R26C17"

ActiveCell.Offset(1, 0).Select
'Resources Degraded
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C42:R" \& j \& "C42)R27C16 + R27C17"

ActiveCell.Offset(3, 0).Select
'ActiveCell.FormulaR1C1 = "= sum(R8C3:R" \& i \& "C3)"
'ActiveCell.Offset( 1,0 ).Select
'ActiveCell.FormulaR1C1 = "= sum(R8C4:R" \& i \& "C4)"
'ActiveCell.Offset( 1,0 ).Select

[^6]Sheets("Peers_2_2").Select
ActiveCell.PasteSpecial Paste:=xlPasteAll, Operation:=xlNone, SkipBlanks:= False, Transpose:=True
ActiveCell.Offset(-102, -1).Select

Sheets("Local").Select
Range("M10:M21").Copy

Sheets("Slacks_2_2").Select
ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=True
ActiveCell.Offset(0, 12). Select

Sheets("Local").Select
Range("P24").Copy
Sheets("Slacks_2_2").Select
ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=False
ActiveCell.Offset(0, 2).Select

Sheets("Local").Select
Range("P25").Copy
Sheets("Slacks_2_2").Select
ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=False
ActiveCell.Offset(0, 2). Select
Sheets("Local").Select
Range("P26").Copy
Sheets("Slacks_2_2").Select
ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=False, Transpose:=False
ActiveCell.Offset(0, 2). Select
Sheets("Local"). Select
Range("P27").Copy
Sheets("Slacks_2_2").Select
ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=False
ActiveCell.Offset(1, -18).Select

Next $n$
Sheets("Peers_2_2").Select
Range("D3:D102").Copy
Range("D106").PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=False, Transpose:=False

## End Sub

Sub Solve_2_GlobalTargets()
' Solve_2A Macro
' solving second phase
' Solves for Global targets for Societal Variables.
' Uses the Efficiency of the DMUs as known in this module

Dim n As Integer
Dim i As Integer
Dim j As Integer

Application.ScreenUpdating $=$ False
Sheets("Peers_2_3").Select
Range("B3:B102").Copy
Range("E2").PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks $:=$ False, Transpose:=True

Sheets("Peers_2_3").Select
Range("A3:C102").Copy
Range("A106").PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _ $:=$ False, Transpose:=False

Range("B3:B102").Copy
Range("E105").PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _ :=False, Transpose:=True

Sheets("Peers_2_3").Select
Range("D3").Select
Sheets("Slacks_2_3").Select
Range("C3").Select
For $n=1$ To 100

|  |  |
| :--- | :--- |
|  | Sheets("Global").Select |
| Sheets("Global").Range("A4").Value $=\mathrm{n}$ |  |
|  | $\mathrm{i}=\mathrm{n}+7$ |
|  | $\mathrm{j}=\mathrm{n}+3$ |

' Constraint for Number of Priority Lanes
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C30:R" \& j \&
"C30)"
ActiveCell.Offset( 1,0 ).Select
' Constraint for Congestion
'ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C31:R" \& j \&
'ActiveCell.Offset( 1,0 ).Select
' Constraint for Schedule Reliability
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C34:R" \& j \&
ActiveCell.Offset( 1,0 ).Select
' Constraint for Vehicle-miles
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C33:R" \& j \&
ActiveCell.Offset( 1,0 ).Select
' Constraint for Trip Time
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C32:R" \& j \& "C32)"

ActiveCell.Offset(3, 0).Select
'Local Targets
'Accidents
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C36:R" \& j \& "C36)- R24C16 + R24C17"

ActiveCell.Offset( 1,0 ).Select
'Emissions
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C38:R" \& j \&
"C38)- R25C16 + R25C17"
ActiveCell.Offset( 1,0 ).Select
'Noise Pollution
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C40:R" \& j \&
"C40)- R26C16 + R26C17"
ActiveCell.Offset( 1,0 ).Select
'Resources Degraded
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C42:R" \& j \&
"C42)- R27C17 + R27C17"
ActiveCell.Offset(3, 0).Select
'Global Targets
'Accidents

ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C36:R" \& j \& "C36)- R30C16 + R30C17 + R30C7"

ActiveCell.Copy
Range("G30"). Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
$:=$ False, Transpose: $=$ False
ActiveCell.Offset(1, 3).Select
'Emissions
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C38:R" \& j \&
"C38)- R31C16 + R31C17 + R31C7"
ActiveCell.Copy
Range("G31").Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=False
ActiveCell.Offset(1, 3).Select
'Noise Pollution
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C40:R" \& j \&
"C40)- R32C16 + R32C17 + R32C7"
ActiveCell.Copy
Range("G32").Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
$:=$ False, Transpose: $=$ False
ActiveCell.Offset(1, 3).Select
'Resources Degraded
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C42:R" \& j \& "C42)- R33C16 + R33C17 + R33C7"

ActiveCell.Copy
Range("G33").Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=False, Transpose:=False
ActiveCell.Offset(3, 3). Select
'ActiveCell.FormulaR1C1 = "= sum(R8C3:R" \& i \& "C3)"
'ActiveCell.Offset(1, 0). Select
'ActiveCell.FormulaR1C1 = "= sum(R8C4:R" \& i \& "C4)"
'ActiveCell.Offset(1, 0).Select
'Setting up Solver parameters
SolverOk SetCell:="\$M\$7", MaxMinVal:=2, ValueOf:="0", ByChange:="\$C\$8:\$D\$" \& i
\& ",\$p\$24:\$q\$27,\$p\$30:\$q\$33"
SolverSolve (True)
'Storing of Results
Sheets("Global"). Select
Sheets("Global").Range("M7").Copy

Sheets("Peers_2_3").Select
ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=False, Transpose:=False
ActiveCell.Offset( 0,1 ).Select
Sheets("Global").Select
Sheets("Global").Range("C8:C107").Copy
Sheets("Peers_2_3").Select
ActiveCell.PasteSpecial Paste:=xlPasteAll, Operation:=xlNone, SkipBlanks:=
False, Transpose:=True
ActiveCell.Offset(103, 0).Select
Sheets("Global").Select
Range("D8:D107").Copy
Sheets("Peers_2_3").Select
ActiveCell.PasteSpecial Paste:=xlPasteAll, Operation:=xlNone, SkipBlanks:=
False, Transpose: $=$ True
ActiveCell.Offset(-102, -1).Select

Sheets("Global").Select
Range("M10:M21").Copy
Sheets("Slacks_2_3").Select
ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=True
ActiveCell.Offset( 0,12 ).Select
Sheets("Global").Select
Range("P24").Copy
Sheets("Slacks_2_3").Select
ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=False, Transpose:=False
ActiveCell.Offset( 0,2 ).Select
Sheets("Global").Select
Range("P25").Copy
Sheets("Slacks_2_3").Select
ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=False, Transpose:=False
ActiveCell.Offset( 0,2 ).Select
Sheets("Global").Select
Range("P26").Copy

Sheets("Slacks_2_3").Select
ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _ :=False, Transpose:=False
ActiveCell.Offset( 0,2 ).Select
Sheets("Global").Select
Range("P27").Copy
Sheets("Slacks_2_3").Select
ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=False, Transpose:=False
ActiveCell.Offset(1, -18).Select

## Next $n$

Sheets("Global").Select
Range("P30:P33").Copy
Sheets("Slacks_2_3").Select
Range("W3").PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _ :=False, Transpose:=True

Sheets("Peers_2_3").Select
Range("D3:D102").Copy
Range("D106").PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=False, Transpose:=False

End Sub

## Appendix A 2.2

## Code for Infrastructure 2, Variable Returns To Scale

```
Sub Clear()
    Sheets("(Ph1_St1)").Select
        Range("B10:C109").Value = 0
        Range("E9").Value = 0
    Sheets("(Ph1_St2)").Select
        'Range("B3:V12").ClearContents
        Range("W17:AE40").ClearContents
    Sheets("(Ph2)").Select
        Range("C8:D107").Value = 0
        Range("F7").Value = 0
    Sheets("Local").Select
        Range("C8:D107").Value = 0
        Range("p22:q25").Value = 0
        Range("S21").Value = 0
    Sheets("Global").Select
        Range("C8:D107").Value = 0
        Range("p22:Q25").Value = 0
        Range("p28:q31").Value = 0
        Range("G28:G31").Value = 0
    'Sheets("Output").Select
    ' Range("C3:W12").ClearContents
    ' Range("D17:X26").ClearContents
        ' Range("D30:X39").ClearContents
    'Sheets("Slacks").Select
    ' Range("C3:L12").ClearContents
    ' Range("C17:M26").ClearContents
    ' Range("o17:026").ClearContents
        'Range("q17:q26").ClearContents
        'Range("s17:s26").ClearContents
        'Range("C31:M40").ClearContents
        'Range("S31:S40").ClearContents
        'Range("O31:O40").ClearContents
        'Range("Q31:Q40").ClearContents
        'Range("N42:Q42").ClearContents
End Sub
```

Sub Solve_1_1()
' VRS100_GP

```
' solve_1_1 Macro
```

' solves and stores output for stage 1, phase 1
' This macro evaluates the model for Variable Returns to Scale.
' Keyboard Shortcut: Ctrl+t
,
Application.ScreenUpdating = False
'Prevents updating of the screen with every iteration

## Dim x As Integer

' Setting up the sheet for storing Slacks and Peers
Sheets("Peers_1_1").Select
Range("A3").Select
For $\mathrm{x}=1$ To 100
Sheets("Peers_1_1").Select
ActiveCell.Value $=x$
ActiveCell.Offset(1, 0).Select
Next x
Sheets("Peers_1_1").Select
Range("A106").Select
For $\mathrm{x}=1$ To 100
Sheets("Peers_1_1").Select
ActiveCell. Value $=x$
ActiveCell.Offset( 1,0 ).Select
Next x
Sheets("Peers_1_1").Select
Range("C2").Select
For $\mathrm{x}=1$ To 100
Sheets("Peers_1_1").Select
ActiveCell. Value $=x$
ActiveCell.Offset( $(0,1)$.Select
Next x
Sheets("Peers_1_1").Select
Range("C105").Select
For $\mathrm{x}=1$ To 100
Sheets("Peers_1_1").Select
ActiveCell.Value $=x$
ActiveCell.Offset( $(0,1)$.Select
Next x
Sheets("Peers_1_1").Select
Range("B3").Select

Sheets("Slacks_1_1").Select

Range("C3").Select

## 'Main Model

## Dim n As Integer

For $\mathrm{n}=1$ To 100
Sheets("(Ph1_St1)").Select
Sheets("(Ph1_St1)").Range("A4").Value = n
'Setting up Solver parameters
SolverOk SetCell:="\$G\$8", MaxMinVal:=1, ValueOf:="0", ByChange:="\$B\$10:\$C\$109,\$E\$9"

SolverSolve (True)
' Storing output for Slacks and Peers
Sheets("(Ph1_St1)").Select
Range("\$E\$9").Copy
Sheets("Peers_1_1").Select
ActiveCell.PasteSpecial Paste:=xlValue, Operation:=xlNone, SkipBlanks:=
False, Transpose:=False
ActiveCell.Offset $(0,1)$.Select
Sheets("(Ph1_St1)").Select
Range("\$B\$10:\$B\$109").Copy
Sheets("Peers_1_1").Select
ActiveCell.PasteSpecial Paste:=xlValue, Operation:=xlNone, SkipBlanks:= _
False, Transpose:=True
ActiveCell.Offset(103, 0).Select
Sheets("(Ph1_St1)").Select
Range("\$C\$10:\$C\$109").Copy
Sheets("Peers_1_1").Select
ActiveCell.PasteSpecial Paste:=xlValue, Operation:=xlNone, SkipBlanks:=
False, Transpose:=True
ActiveCell.Offset(-102, -1).Select
Sheets("(Ph1_St1)").Select
Range("\$K\$11:\$k\$22").Copy
Sheets("Slacks_1_1").Select
ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks:=

False, Transpose:=True
ActiveCell.Offset(1, 0).Select
Next $n$
Sheets("Peers_1_1").Select
Range("B3:B10 $\overline{2}$ ").Copy

Range("B106").PasteSpecial Paste:=xlValue, Operation:=xlNone, SkipBlanks:= _
False, Transpose:=False

```
End Sub
Sub Reg_1_1()
```

' Reg_2_1 Macro
' Does Regression for Stage 2 of Phase 1
' Keyboard Shortcut: Ctrl+u
Dim x As Integer
' Sheets("(Ph1_St2)").Select
' Range("A5").Select
'For $\mathrm{x}=1$ To 100
' Sheets("(Ph1_St2)").Select

- ActiveCell.Value $=x$
- ActiveCell.Offset( 1,0 ).Select

Sheets("(Ph1_St2)").Select
Application.Run "ATPVBAEN.XLA!Regress", ActiveSheet.Range("\$B\$5:\$B\$104"), ActiveSheet.Range("\$C\$5:\$G\$104"), True, False, , ActiveSheet.Range( "\$W\$17:\$AE\$42"), False, False, False, False, , False

Sheets("(Ph1_St2)").Select
Range("X34:X38").Select
Selection.Copy
Range("C3").Select
Selection.PasteSpecial Paste:=xlValue, Operation:=xlNone, SkipBlanks:= False, Transpose:=True
End Sub
Sub Sort_Env_1_2()
'
' Sort_Env_1_2 Macro
' Ranks the bus routes as per their degree of harshness
,
' Keyboard Shortcut: Ctrl+i
Dim x As Integer
Sheets("(Ph1_St2)").Select
Range("K5").Select
For $\mathrm{x}=1$ To 100

Sheets("(Ph1_St2)").Select
ActiveCell. Value $=x$
ActiveCell.Offset(1, 0). Select
Next x
Range("A5:H104").Select
Selection.Copy
Range("L5").Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks $:=$ False, Transpose:=False
'Range("G35").Select
'Application.CutCopyMode = False
Range("L5:S104").Sort Key1:=Range("S255"), Order1:=xlDescending, Header:= _ xlGuess, OrderCustom:=1, MatchCase:=False, Orientation:=xlTopToBottom, DataOptionl:=xlSortNormal
End Sub

## Sub Solve_2_Efficiency()

'
' Solve_2A Macro
' solving second phase
' This macro forms the main part of the model. It evaluates the efficiency of the bus routes,
' now considering a combined environmental variable.

Dim n As Integer
Dim i As Integer
Dim j As Integer
Dim x As Integer

Application.ScreenUpdating $=$ False
'Prevents updating of the screen with every iteration
'Setting up the sheet for Slacks and Peers
Sheets("Peers_2_1").Select
Range("A3:B102").Copy
Range("A106").Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _
$:=$ False, Transpose:=False
Range("B3:B102").Copy
Range("D2").Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _
:=False, Transpose:=True
Range("B3:B102").Copy
Range("D105").Select

Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=True
Sheets("Peers_2_1").Select
Range("C3").Select
Sheets("Slacks_2_1").Select
Range("C3").Select

## 'Main Loop

For $\mathrm{n}=1$ To 100
Sheets("(Ph2)").Select
Sheets("(Ph2)").Range("A4").Value = n
$\mathrm{i}=\mathrm{n}+7$
$\mathrm{j}=\mathrm{n}+3$
Sheets("(Ph2)").Range("J10").Select
'Constraint for Passenger-miles
ActiveCell.FormulaR1C1 = "= sumproduct(R8C3:R" \& i \& "C3,R4C35:R" \& j \& "C35)"
ActiveCell.Offset( 1,0 ).Select
' Constraint for Schedule Reliability
ActiveCell.FormulaR1C1 = "= sumproduct(R8C3:R" \& i \& "C3,R4C34:R" \& j \& "C34)" ActiveCell.Offset( 1,0 ).Select
'Constraint for Vehicle-miles
ActiveCell.FormulaR1C1 = "= sumproduct(R8C3:R" \& i \& "C3,R4C33:R" \& j \& "C33)" ActiveCell.Offset( 1,0 ).Select
'Constraint for Trip time
ActiveCell.FormulaR1C1 = "= sumproduct(R8C3:R" \& i \& "C3,R4C32:R" \& j \& "C32)" ActiveCell.Offset( 1,0 ).Select
'Constraint for Headway
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C26:R" \& j \& "C26)" ActiveCell.Offset(1, 0).Select
'Constraint for Cost
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C27:R" \& j \& "C27)" ActiveCell.Offset(1, 0).Select
'Constraint for Service Duration
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C28:R" \& j \& "C28)" ActiveCell.Offset(1, 0).Select
'Constraint for Number of intersections

ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C29:R" \& j \& "C29)" ActiveCell.Offset(1, 0).Select
'Constraint for Number of Priority Lanes
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C30:R" \& j \& "C30)" ActiveCell.Offset(1, 0).Select
'Constraint for Congestion
'ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C31:R" \& j \& "C31)" 'ActiveCell.Offset(1, 0).Select
' Constraint for Schedule Reliability
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C34:R" \& j \& "C34)"
ActiveCell.Offset(1, 0).Select
'Constraint for Vehicle-miles
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C33:R" \& j \& "C33)" ActiveCell.Offset( 1,0 ).Select
'Constraint for Trip time
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C32:R" \& j \& "C32)" ActiveCell.Offset( 1,0 ).Select

```
'ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C14:R" \& j \& "C14)- R34C3 + R34C4"
'ActiveCell.Offset( 1,0 ).Select
'ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C16:R" \& j \& "C16)- R35C3 + R35C4"
'ActiveCell.Offset( 1,0 ).Select
```

```
    'ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" & i & "C4,R3C18:R" & j &
```

    'ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" & i & "C4,R3C18:R" & j &
    "C18)- R36C3 + R36C4"
"C18)- R36C3 + R36C4"
'ActiveCell.Offset(1, 0).Select
'ActiveCell.Offset(1, 0).Select
'ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C20:R" \& j \&
'ActiveCell.FormulaR1C1 = "= sumproduct(R20C4:R" \& i \& "C4,R3C20:R" \& j \&
"C20)- R37C20 + R37C4"
"C20)- R37C20 + R37C4"
'ActiveCell.Offset(1, 0).Select
'ActiveCell.Offset(1, 0).Select
'Variable Returns to Scale
'Variable Returns to Scale
ActiveCell.FormulaR1C1 = "= sum(R8C3:R" \& i \& "C3)"
ActiveCell.FormulaR1C1 = "= sum(R8C3:R" \& i \& "C3)"
ActiveCell.Offset(1, 0).Select
ActiveCell.Offset(1, 0).Select
ActiveCell.FormulaR1C1 = "= sum(R8C4:R" \& i \& "C4)"
ActiveCell.FormulaR1C1 = "= sum(R8C4:R" \& i \& "C4)"
ActiveCell.Offset(1, 0).Select
ActiveCell.Offset(1, 0).Select
'Setting up Solver parameters
'Setting up Solver parameters
SolverOk SetCell:="\$I$7", MaxMinVal:=1, ValueOf:="0", ByChange:="$C$8:$D$" & i &
    SolverOk SetCell:="$I$7", MaxMinVal:=1, ValueOf:="0", ByChange:="$C$8:$D$" & i &
",$F\$7"

```
",$F$7"
```


## SolverSolve (True)

'Storing ouput values for Slacks and Peers
Sheets("(Ph2)").Select
Sheets("(Ph2)").Range("F7").Copy
Sheets("Peers_2_1").Select
ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=False
ActiveCell.Offset( 0,1 ).Select
Sheets("(Ph2)").Select
Sheets("(Ph2)").Range("C8:C107").Copy
Sheets("Peers_2_1").Select
ActiveCell.PasteSpecial Paste:=xlPasteAll, Operation:=xlNone, SkipBlanks:= False, Transpose:=True
ActiveCell.Offset(103, 0).Select
Sheets("(Ph2)").Select
Range("D8:D107").Copy
Sheets("Peers_2 1").Select
ActiveCell.PasteSpecial Paste:=xlPasteAll, Operation:=xlNone, SkipBlanks:= False, Transpose:=True
ActiveCell.Offset(-102, -1).Select
'Sheets("(Ph2)").Select
'Sheets("(Ph2)").Range("F19").Copy
'Sheets("Output").Select
'ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _ ':=False, Transpose:=False
'ActiveCell.Offset(1, 1).Select
Sheets("(Ph2)").Select
Range("M10:M122").Copy
Sheets("Slacks_2_1").Select
ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=False, Transpose:=True
ActiveCell.Offset(1, 0).Select
'Sheets("(Ph2)").Select
'Range("M34").Copy
'Sheets("Slacks").Select
'ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _ :=False, Transpose:=False

```
'ActiveCell.Offset(0, 2).Select
'Sheets("(Ph2)").Select
'Range("M35").Copy
'Sheets("Slacks").Select
'ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _
:=False, Transpose:=False
'ActiveCell.Offset(0, 2).Select
'Sheets("(Ph2)").Select
'Range("M36").Copy
'Sheets("Slacks").Select
'ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _
:=False, Transpose:=False
'ActiveCell.Offset(0, 2).Select
'Sheets("(Ph2)").Select
'Range("M37").Copy
'Sheets("Slacks").Select
'ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
':=False, Transpose:=False
'ActiveCell.Offset(1, -16).Select
```

Next $n$
Sheets("Peers_2_1").Select
Range("C3:C102").Copy
Range("C106").PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=False, Transpose:=False

## End Sub

Sub Solve_2_LocalTargets()
' This macro evaluates the deviation of the societal variables for each bus route with respect ' to the already determined targets

Dim n As Integer
Dim i As Integer
Dim j As Integer
Dim x As Integer

Application.ScreenUpdating = False
'Prevent screen updating
'Setting up the sheets for storing of Slacks and Peers

Sheets("Peers_2_2").Select
Range("B3:B102").Copy
Range("E2").PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=False, Transpose:=True

Sheets("Peers_2_2").Select
Range("A3:C102").Copy
Range("A106").PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=False
Range("B3:B102").Copy
Range("E105").PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=True
Sheets("Peers_2_2").Select
Range("D3").Select
Sheets("Slacks_2_2").Select
Range("C3").Select
'Main Loop
For $\mathrm{n}=1$ To 100
Sheets("Local").Select
Sheets("Local").Range("A4").Value = n

$$
\begin{aligned}
& \mathrm{i}=\mathrm{n}+7 \\
& \mathrm{j}=\mathrm{n}+3
\end{aligned}
$$

Sheets("Local").Range("J10").Select
'Constraint for Passenger-miles
ActiveCell.FormulaR1C1 = "= sumproduct(R8C3:R" \& i \& "C3,R4C35:R" \& j \& "C35)"
ActiveCell.Offset(1, 0).Select
'Constraint for Schedule Reliabilty
ActiveCell.FormulaR1C1 = "= sumproduct(R8C3:R" \& i \& "C3,R4C34:R" \& j \& "C34)" ActiveCell.Offset(1, 0).Select
'Constraint for Vehicle-miles
ActiveCell.FormulaR1C1 = "= sumproduct(R8C3:R" \& i \& "C3,R4C33:R" \& j \& "C33)" ActiveCell.Offset( 1,0 ).Select
'Constraint for Trip Time
ActiveCell.FormulaR1C1 = "= sumproduct(R8C3:R" \& i \& "C3,R4C32:R" \& j \& "C32)" ActiveCell.Offset(1, 0).Select
'Constraint for Headway
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C26:R" \& j \& "C26)" ActiveCell.Offset(1, 0).Select
'Constraint for Cost
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C27:R" \& j \& "C27)" ActiveCell.Offset( 1,0 ).Select
'Constraint for Service Duration
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C28:R" \& j \& "C28)" ActiveCell.Offset(1, 0).Select
'Constraint for Intersection
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C29:R" \& j \& "C29)" ActiveCell.Offset( 1,0 ).Select
'Constraint for Priority Lanes
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C30:R" \& j \& "C30)" ActiveCell.Offset( 1,0 ).Select
'Constraint for Congestion
'ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C31:R" \& j \& "C31)" 'ActiveCell.Offset( 1,0 ).Select
'Constraint for Schedule Reliabilty
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C34:R" \& j \& "C34)" ActiveCell.Offset $(1,0)$.Select
'Constraint for Vehicle-miles
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C33:R" \& j \& "C33)"
ActiveCell.Offset $(1,0)$.Select
'Constraint for Trip Time
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C32:R" \& j \& "C32)" ActiveCell.Offset(3, 0).Select
'Local Targets
'Accidents
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C36:R" \& j \& "C36)R24C16 + R24C17"

ActiveCell.Offset(1, 0).Select
'Emissions
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C38:R" \& j \& "C38)R25C16 + R25C17"

ActiveCell.Offset( 1,0 ).Select
'Noise Pollution

ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C40:R" \& j \& "C40)R26C16 + R26C17"

ActiveCell.Offset(1, 0).Select
'Resources Degraded
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C42:R" \& j \& "C42)R27C16 + R27C17"

ActiveCell.Offset(3, 0).Select
'Variable Returns to Scale
ActiveCell.FormulaR1C1 = "= sum(R8C3:R" \& i \& "C3)"
ActiveCell.Offset(1, 0).Select
ActiveCell.FormulaR1C1 = "= sum(R8C4:R" \& i \& "C4)"
ActiveCell.Offset(1, 0).Select
'Setting up Solver parameters
SolverOk SetCell:="\$K\$7", MaxMinVal:=2, ValueOf:="0", ByChange:="\$C\$8:\$D\$" \& i \& ",\$p\$24:\$q\$27"

SolverSolve (True)
'Storing values for Slacks and Peers of each bus route
Sheets("Local").Select
Sheets("Local").Range("K7").Copy
Sheets("Peers_2_2").Select
ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=False
ActiveCell.Offset $(0,1)$.Select
Sheets("Local").Select
Sheets("Local").Range("C8:C107").Copy
Sheets("Peers_2_2").Select
ActiveCell.PasteSpecial Paste:=xlPasteAll, Operation:=xlNone, SkipBlanks:= _
False, Transpose:=True
ActiveCell.Offset(103, 0).Select
Sheets("Local").Select
Range("D8:D107").Copy
Sheets("Peers_2_2").Select
ActiveCell.PasteSpecial Paste:=xlPasteAll, Operation:=xlNone, SkipBlanks:=
False, Transpose:=True
ActiveCell.Offset(-102, -1).Select

Sheets("Local").Select

Range("M10:M21").Copy
Sheets("Slacks_2_2").Select
ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=False, Transpose:=True
ActiveCell.Offset(0, 12).Select
Sheets("Local").Select
Range("P24").Copy
Sheets("Slacks_2_2").Select
ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=False
ActiveCell.Offset(0, 2).Select
Sheets("Local").Select
Range("P25").Copy
Sheets("Slacks_2_2").Select
ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=False
ActiveCell.Offset(0, 2).Select
Sheets("Local").Select
Range("P26").Copy
Sheets("Slacks_2_2").Select
ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=False
ActiveCell.Offset( 0,2 ).Select
Sheets("Local").Select
Range("P27").Copy
Sheets("Slacks_2_2").Select
ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=False, Transpose:=False
ActiveCell.Offset(1, -18).Select

## Next $n$

Sheets("Peers_2_2").Select
Range("D3:D102").Copy
Range("D106").PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=False, Transpose:=False

End Sub

Sub Solve_2_GlobalTargets()
' This macro evaluates the deviations of the societal variables for the whole network.
' Solve_2A Macro
' solving second phase
,

Dim n As Integer
Dim i As Integer
Dim j As Integer

Application.ScreenUpdating = False
'Prevents updating of sheet with every iteration
'Setting up the sheet for Slacks and Peers
Sheets("Peers_2_3").Select
Range("B3:B102").Copy
Range("E2").PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=True
Sheets("Peers_2_3").Select
Range("A3:C102").Copy
Range("A106").PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=False, Transpose:=False

Range("B3:B102").Copy
Range("E105").PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=False, Transpose:=True

Sheets("Peers_2_3").Select
Range("D3").Select
Sheets("Slacks_2_3").Select
Range("C3").Select
'Main Loop
For $\mathrm{n}=1$ To 100
Sheets("Global").Select
Sheets("Global").Range("A4").Value = n

$$
\begin{aligned}
& \mathrm{i}=\mathrm{n}+7 \\
& \mathrm{j}=\mathrm{n}+3
\end{aligned}
$$

Sheets("Global").Range("J10").Select
'Constraint for Passenger-miles

| "C35)" |  |
| :---: | :---: |
|  | ActiveCell.Offset( 1,0 ).Select |
|  | 'Constraint for Schedule Reliability |
|  |  |
| "C34)" | ActiveCell Offset(1, 0) Select |
|  | 'Constraint for Vehicle-miles |
|  |  |
| "C33)" |  |
|  | ActiveCell.Offset(1, 0).Select |
|  | 'Constraint for Trip Time |
|  |  |
| "C32)" | ActiveCell.Offset(1, 0).Select |
|  | 'Constraint for Headway |
|  |  |
| "C26)" |  |
|  | ActiveCell.Offset(1, 0).Select |
|  | 'Constraint for Costs |
|  |  |
| "C27)" | ActiveCell.Offset(1, 0).Select |
|  | 'COnstraint for Service Duration |
|  |  |
| "C28)" |  |
|  | ActiveCell.Offset(1, 0).Select |
|  | 'Constraint for Number of intersections |
|  |  |
| "C29)" |  |
|  | ActiveCell.Offset(1, 0).Select |
|  | 'Constraint for Priority lanes |
|  |  |
| "C30)" |  |
|  | ActiveCell.Offset(1, 0).Select |
|  | 'Constraint for Congestion |
|  |  |
| "C31)" | 'ActiveCell.Offset(1, 0).Select |
|  | 'Constraint for Schedule Reliability |



# Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks 

:=False, Transpose:=False
ActiveCell.Offset(1, 3).Select
'Noise Pollution
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C40:R" \& j \&
"C40)- R32C16 + R32C17 + R32C7"
ActiveCell.Copy
Range("G32").Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _
:=False, Transpose:=False
ActiveCell.Offset(1, 3).Select
'Resources Degraded
ActiveCell.FormulaR1C1 = "= sumproduct(R8C4:R" \& i \& "C4,R4C42:R" \& j \&
"C42)- R33C16 + R33C17 + R33C7"
ActiveCell.Copy
Range("G33").Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=False
ActiveCell.Offset(3, 3).Select
'Variable Returns to Scale
ActiveCell.FormulaR1C1 = "= sum(R8C3:R" \& i \& "C3)"
ActiveCell.Offset(1, 0).Select
ActiveCell.FormulaR1C1 = "= sum(R8C4:R" \& i \& "C4)"
ActiveCell.Offset(1, 0). Select
'Setting up Solver parameters
SolverOk SetCell:="\$M\$7", MaxMinVal:=2, ValueOf:="0", ByChange:="\$C\$8:\$D\$" \& i
\& ",\$p\$24:\$q\$27,\$p\$30:\$q\$33"
SolverSolve (True)
'Storing of Slacks and Peers for each bus route
Sheets("Global").Select
Sheets("Global").Range("M7").Copy
Sheets("Peers_2_3").Select
ActiveCell.Paste-Special Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _
$:=$ False, Transpose:=False
ActiveCell.Offset(0, 1).Select
Sheets("Global").Select
Sheets("Global").Range("C8:C107").Copy
Sheets("Peers_2_3").Select
ActiveCell.PasteSpecial Paste:=xlPasteAll, Operation:=xlNone, SkipBlanks:=
False, Transpose:=True
ActiveCell.Offset(103, 0).Select

Sheets("Global").Select
Range("D8:D107").Copy
Sheets("Peers_2_3").Select
ActiveCell.PasteSpecial Paste:=xlPasteAll, Operation:=xlNone, SkipBlanks:= False, Transpose:=True
ActiveCell.Offset(-102, -1).Select

Sheets("Global").Select
Range("M10:M21").Copy
Sheets("Slacks_2_3").Select
ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=True
ActiveCell.Offset( 0,12 ).Select
Sheets("Global").Select
Range("P24").Copy
Sheets("Slacks_2_3").Select
ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
:=False, Transpose:=False
ActiveCell.Offset(0, 2).Select
Sheets("Global").Select
Range("P25").Copy
Sheets("Slacks_2_3").Select
ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=False, Transpose:=False
ActiveCell.Offset( 0,2 ).Select
Sheets("Global").Select
Range("P26").Copy
Sheets("Slacks_2_3").Select
ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _ :=False, Transpose:=False
ActiveCell.Offset( 0,2 ).Select
Sheets("Global").Select
Range("P27").Copy
Sheets("Slacks_2_3").Select
ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=False, Transpose:=False
ActiveCell.Offset(1, -18).Select

## Next $n$

Sheets("Peers_2_3").Select
Range("D3:D102").Copy
Range("D106").PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks :=False, Transpose:=False

Sheets("Global").Select
Range("P30:P33").Copy
Sheets("Slacks_2_3").Select
Range("W3").PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _ :=False, Transpose:=True

End Sub
Appendix B 1.1

| DMU | Bus Route | Controllable Inputs |  |  | Infrastructure Inputs xjint | Intermediate Outputs |  | Output <br> yjpm | Societal variables and Targets |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | xjhd | xjcost | xjservd |  | yjtim | yjvm |  | yjace | Lyjace |
| 1 | Main Street | 0.038 | 1210 | 18.5 | 0.04347826 | 0.07 | 196 | 1985 | 0.016 | 0 |
| 2 | Hethwood/Windsor Hills | 0.043 | 1281 | 18.5 | 0.07692308 | 0.07 | 185.3 | 2386 | 0.017 | 0 |
| 3 | Toms Creek A | 0.07 | 2135 | 18.5 | 0.11111111 | 0.07 | 148.7 | 1219 | 0.028 | 0 |
| 4 | Toms Creek B | 0.07 | 1637 | 18.5 | 0.11111111 | 0.07 | 123 | 3110 | 0.021 | 0 |
| 5 | Two Town Trolley BB | 0.017 | 213.5 | 12 | 0.14285714 | 0.03 | 54.33 | 139 | 0.003 | 0 |
| 6 | Two Town Trolley CB | 0.017 | 213.5 | 9.5 | 0.05555556 | 0.07 | 34.5 | 37 | 0.003 | 0 |
| 7 | Hokie Express | 0.079 | 142.3 | 13.7 | 0.2 | 0.07 | 95.33 | 281 | 0.002 | 0 |
| 8 | Ramble Road | 0.033 | 71.15 | 14.7 | 0.05 | 0.03 | 94 | 68 | 0.0009 | 0 |

Appendix B 1.2
BT Simulated Dataset

|  | ${ }^{\text {du }}$［ $\mathcal{S}_{T}$ | $\stackrel{n}{\sim}$ | $\stackrel{\sim}{\sim}$ | $\stackrel{n}{i}$ | $\stackrel{\sim}{\sim}$ | $\stackrel{\sim}{i}$ | $\stackrel{\sim}{i}$ | $\stackrel{\sim}{i}$ | $\stackrel{n}{i}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | du ${ }^{\text {¢ }}$ K | $\stackrel{\infty}{\stackrel{\infty}{m}}$ | $\begin{gathered} \underset{\sim}{n} \\ \underset{\sim}{n} \end{gathered}$ | $\begin{aligned} & \underset{\sim}{n} \\ & i \end{aligned}$ | $\begin{aligned} & \underset{i}{i} \\ & i \end{aligned}$ | $\stackrel{+}{-}$ | $\stackrel{\substack{\infty}}{\sim}$ | $\begin{aligned} & \hat{\infty} \\ & \hat{n} \\ & \hline \end{aligned}$ | － |
|  |  | O. | Ô | O. | $\begin{aligned} & \text { Nat } \\ & \text { O. } \end{aligned}$ | O. | O. | 合 | O |
|  | ！ | $\frac{m}{0}$ | $\stackrel{m}{0}$ | $\stackrel{\square}{0}$ | $\stackrel{\square}{\circ}$ | $\stackrel{\ominus}{\circ}$ | ${ }_{\circ}$ | $\stackrel{5}{0}$ | $\stackrel{\bigcirc}{0}$ |
|  |  | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | － |
|  | งัセโК | $\stackrel{0}{0}$ | O | $\begin{aligned} & \text { od } \\ & \text { O} \end{aligned}$ | $\stackrel{\rightharpoonup}{0}$ | ờ | O̊ | Ö. | ＋ |
| pndino | ud！$¢$ | $\stackrel{\otimes}{\Omega}$ | $\stackrel{\otimes}{\stackrel{\infty}{\sim}}$ | $\stackrel{\rightharpoonup}{\mathrm{I}}$ | $\stackrel{\ominus}{m}$ | ¢ | n | $\stackrel{\rightharpoonup}{\sim}$ | $\stackrel{\circ}{\circ}$ |
| s²ndıno әұе！рәш．әұиІ | ［2．ss！$\kappa$ | － | － | $m$ | m | － | － | － |  |
|  | ш＾！${ }^{\text {¢ }}$ | $\stackrel{\text { ® }}{ }$ | $\begin{aligned} & \underset{\infty}{\infty} \\ & \stackrel{\infty}{\infty} \end{aligned}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{0} \\ & \substack{\dot{d}} \end{aligned}$ | ヘิ | $\begin{aligned} & \text { m} \\ & \text { ๙̀ } \end{aligned}$ | $\stackrel{n}{m}$ | $\underset{\substack{\infty \\ \underset{\sim}{n}}}{ }$ | す |
|  | ш！̣̂ | $\stackrel{\sim}{n}$ | $\frac{\cong}{i}$ | $\stackrel{\bar{\infty}}{\stackrel{n}{n}}$ | $\underset{\underset{F}{F}}{F}$ | $\stackrel{\bar{\infty}}{\stackrel{+}{+}}$ | $\stackrel{\bar{\sim}}{\sim}$ | $\underset{\text { ì }}{\underset{\sim}{n}}$ | ＋ |
| stnduI จ．mpon．ңse．．уиі | \％u！${ }^{\text {¢ }}$ |  | $\begin{aligned} & \text { N} \\ & \text { O. } \\ & \text { O. } \end{aligned}$ | $\begin{aligned} & \Xi \\ & \vdots \\ & \hline \end{aligned}$ | $\underset{\vdots}{\Xi}$ | $\begin{aligned} & \stackrel{0}{0} \\ & \underset{\substack{c}}{6} \end{aligned}$ |  | O゙ | \％ |
| słnduI әqвाі．घиод | pa．zs！${ }^{\text {d }}$ | $\stackrel{n}{\infty}$ | $\stackrel{n}{\infty}$ | $\underset{\sim}{\infty}$ | $\stackrel{n}{\infty}$ | $\simeq$ | $\because$ | $\underset{\underset{\sim}{i}}{ }$ | $\stackrel{Y}{ \pm}$ |
|  | 7800 ${ }^{\text {x }}$ | $\begin{aligned} & \mathrm{O} \\ & \mathrm{I} \end{aligned}$ | $\stackrel{\rightharpoonup}{\mathbf{N}}$ | $\stackrel{n}{\sim}$ | $\widehat{\widehat{0}}$ | $\stackrel{m}{n}$ | $\stackrel{m}{\sim}$ | I | $\stackrel{\sim}{~}$ |
|  | pu！${ }^{\text {c }}$ | $\bigcirc$ | $\bigcirc$ | $\vec{O}$ | $\bar{O}$ | $\bigcirc$ | $\bigcirc$ | $\vec{O}$ | $\bigcirc$ |
|  | Ju！${ }^{\text {ctz }}$ | m | m | $\sim$ | $\sim$ | m | m | － | $\sim$ |
|  | pıso！z | $\sim$ | $\sim$ | ＋ | $\checkmark$ | ナ | m | m | $m$ |
|  | pdod！${ }^{\text {z }}$ | $\sim$ | － | － | － | m | － | － | m |
|  | uиoo！z | m | m | － | － | ＋ | n | － | m |
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|  | qssetz | m | $\sim$ | － | － | $\checkmark$ | ＋ | － | m |
| әпnoy sng |  |  |  |  | $\begin{aligned} & \text { n } \\ & \stackrel{y}{0} \\ & \stackrel{0}{0} \\ & 0 \\ & \vdots \\ & \vdots \end{aligned}$ |  |  |  |  |
| nud |  | － | $\sim$ | m | ＋ | n | $\bigcirc$ | $\checkmark$ | $\infty$ |

Appendix B 2.1


| $\stackrel{\square}{\wedge}$ | $\stackrel{\square}{2}$ | $\stackrel{\text { ® }}{ }$ | 8 | $\bigcirc$ | 8 | \％ | 8 | 8 | $\stackrel{8}{2}$ | ， | R | ¢ | R | § |  |  | 8 | 2 |  |  | $\stackrel{\text { ® }}{ }$ | $8$ | $\therefore$ | 8 | $\circ$ | $\stackrel{\square}{2}$ | $\stackrel{\rightharpoonup}{\circ}$ | $\bigcirc$ | $\stackrel{\circ}{8}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\stackrel{\circ}{\sim}$ | O | $\stackrel{0}{n}$ | $\stackrel{0}{n}$ | \％ | © | bin | $\frac{n}{n}$ | $\stackrel{\infty}{i}$ | 엉 | $8$ |  | $0$ | 역 |  |  |  | f |  |  |  | of | $8$ | $i n$ | $8$ | 읏 | $\underset{\infty}{\circ}$ | $0$ | － | 8 |
| － | － | － | n | － | － | $\sim$ | i | － | － | $\sim$ | N | $\stackrel{\sim}{n}$ | i |  |  |  | $0$ | $i \stackrel{n}{i}$ |  |  | 入 | d | i | $\left\lvert\, \begin{aligned} & n \\ & i \end{aligned}\right.$ | $\stackrel{n}{n}$ | へ | － | $\stackrel{\sim}{3}$ | $\stackrel{\sim}{n}$ |
| $\stackrel{\sim}{\sim}$ | － | $\rightarrow$ | $\cdots$ | $-$ | － |  |  | － | N |  | m | $-$ | － |  |  |  | $\dot{m}$ |  |  |  | $\overline{\mathrm{m}}$ |  | $\bigcirc$ | － | F | N | 子 | $\cdots$ | $\infty$ |
| $\stackrel{\infty}{0}$ | $\begin{array}{\|c\|} \hline \infty \\ 0 \\ \hline \end{array}$ | $\stackrel{\infty}{\infty}$ | $\stackrel{\infty}{\circ}$ | $\begin{array}{l\|l\|} \hline \infty \\ 0 \\ \hline \end{array}$ | $0_{0}^{\infty}$ | $0$ | $\bullet$ | $\begin{array}{\|l\|} \infty \\ \infty \end{array}$ | $0$ | $0^{\circ}$ | $\dot{0}$ | $\stackrel{\infty}{\infty}$ | $0$ |  |  |  | $\underset{\circ}{\infty}$ |  |  |  | $0_{0}^{\infty}$ | $\stackrel{\infty}{\infty}$ | $\stackrel{\infty}{0}$ | $\stackrel{\infty}{\infty}$ | $\stackrel{\infty}{\infty}$ | $\stackrel{\infty}{\infty}$ | $\circ$ | $\stackrel{\infty}{\infty}$ | $\bigcirc$ |
| $\|\underset{O}{0}\|$ | $\begin{aligned} & o f \\ & 0 \\ & \hline \end{aligned}$ | S | O | $\stackrel{\sim}{9}$ | $\stackrel{\infty}{̣}$ | $0^{\circ}$ |  | $01$ | $\begin{aligned} & 6 \\ & 0 \\ & \hline \end{aligned}$ |  |  | $\stackrel{\infty}{\substack{0}}$ | $\underset{O}{0}$ |  |  |  | $\stackrel{N}{N}$ |  |  |  | $0$ | $\xlongequal{7}$ | $\mathfrak{o}$ | $\stackrel{O}{0}$ | $\stackrel{\sim}{\circ}$ | $\frac{0}{0}$ | $\circ$ | $\stackrel{n}{\square}$ | $\stackrel{\sim}{3}$ |
| in | n | n | n | n | in | n |  | in | $\checkmark$ |  |  |  | n |  |  |  | in |  |  |  | n | n |  | n |  | n |  |  |  |
| － | in | － | $\sim$ | $\checkmark$ | n | $\bigcirc$ | $\bigcirc$ | $\sim$ | － |  |  | च | $\infty$ |  |  |  | $\sim$ |  |  |  |  | $\cdots$ | $\checkmark$ | a |  | $\bigcirc$ |  | $\bigcirc$ | $\bigcirc$ |
| $\left\|\begin{array}{l} 0 \\ \dot{f} \\ \underset{\sim}{2} \end{array}\right\|$ | $\left\lvert\, \begin{aligned} & \infty \\ & \underset{\sim}{n} \\ & \underset{N}{N} \\ & \\ & \hline \end{aligned}\right.$ | $\begin{gathered} 1 \\ 0 \\ 0 \\ 0 \\ o \\ o \\ 0 \end{gathered}$ | $\begin{aligned} & \text { in } \\ & \text { í } \\ & \text { in } \end{aligned}$ | $\begin{gathered} \underset{\sim}{n} \\ \underset{\sim}{n} \\ \hline \end{gathered}$ |  |  | $\mathfrak{\infty}$ | $\begin{array}{\|c} \underset{j}{j} \\ \underset{j}{y} \\ \underset{y}{2} \end{array}$ |  |  | $\begin{aligned} & \dot{+} \\ & \dot{~} \\ & \hline \end{aligned}$ | $\begin{aligned} & \tilde{n} \\ & \underset{y}{c} \\ & \underset{\sim}{7} \end{aligned}$ | $\left\lvert\, \begin{aligned} & n \\ & \dot{\sim} \\ & 0 \\ & \vdots \\ & \vdots \end{aligned}\right.$ |  |  |  | $\begin{aligned} & \text { oे } \\ & \text { - } \\ & \text { o } \end{aligned}$ |  |  |  | $\begin{aligned} & \text { a } \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{gathered} 0 \\ \stackrel{0}{0} \\ \dot{\infty} \\ \infty \end{gathered}$ |  | $\begin{aligned} & \vec{i} \\ & \dot{n} \\ & \underset{\sim}{2} \end{aligned}$ |  |  | $\begin{aligned} & 0 \\ & \hat{c} \\ & \underset{\sim}{n} \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { d } \\ & \text { ç } \\ & \text { O} \\ & \text { d } \end{aligned}$ | N1 ¢ O $=$ |
| ＜ | へ | ส | $\bigcirc$ | त | N | $\stackrel{1}{\circ}$ | $\stackrel{\sim}{\sim}$ | $\cdots$ | へ |  |  | $\stackrel{\sim}{\sim}$ | － |  |  |  | N | － |  |  | $\sim$ |  | ন | N | $\bigcirc$ | N | $\bigcirc$ | $\infty$ | $=$ |
| $\begin{aligned} & \exists \\ & \vdots \end{aligned}$ | $\left\|\begin{array}{l} \dot{6} \\ \dot{8} \end{array}\right\|$ | $\left\lvert\, \begin{aligned} & \dot{f} \\ & 寸 \end{aligned}\right.$ | $\left.\begin{gathered} 7 \\ \dot{i} \\ i n \end{gathered} \right\rvert\,$ | $\stackrel{\infty}{\dot{f}}$ | $\stackrel{\rightharpoonup}{n}$ | $\dot{子}$ | ذ | $\left\|\begin{array}{c} m \\ i n \end{array}\right\|$ | $\stackrel{2}{i}$ | $\sqrt{6}$ |  |  | $\stackrel{\infty}{+}$ |  |  |  | $\begin{aligned} & 6 \\ & \underset{子}{9} \end{aligned}$ |  |  |  | $9$ | $\begin{aligned} & n \\ & \infty \\ & \dot{子} \end{aligned}$ | $\dot{\sim}$ | $\underset{\sim}{\stackrel{\rightharpoonup}{\mathrm{N}}}$ | $\begin{aligned} & 1 \\ & 0 \\ & 8 \end{aligned}$ | $\begin{aligned} & n \\ & \underset{\sim}{n} \end{aligned}$ | $\underset{\sim}{\infty}$ | $\underset{\sim}{\grave{N}}$ | $\stackrel{\square}{\text { m }}$ |
| $\left\lvert\, \begin{aligned} & 0 \\ & 0 \end{aligned}\right.$ | $\stackrel{n}{\square}$ | $8$ | $0$ | $0$ | $\underset{0}{\hat{0}}$ | $\hat{o}$ | $\underset{o}{i}$ | $\underset{-}{\wedge}$ | $3$ | $\stackrel{\rightharpoonup}{0}$ | $\underset{0}{6}$ | $\exists$ | $0$ |  |  |  | － |  |  |  | － | $0$ | $0$ | $\stackrel{0}{0}$ | $\stackrel{\infty}{\infty}$ | $\pm$ | $\hat{o}$ | $\stackrel{n}{i}$ | $\stackrel{\sim}{0}$ |
| － | $\sim$ | － | － | $\sim$ | $\sim$ |  | － |  | － |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $\sim$ |  | － |  |
| $\bigcirc$ | － | － | － | － | － |  |  |  | － |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\bigcirc$ | $\bigcirc$ | $\stackrel{\sim}{\sim}$ | $\neg$ | $\wedge$ | $\infty$ | $\bigcirc$ |  | $\cdots$ | － | N |  | へ | $\cdots$ |  |  |  | 二 |  |  |  | $\bullet$ | $\bigcirc$ | 工 | $\stackrel{\sim}{\infty}$ | $\infty$ | $\bigcirc$ | こ | $\stackrel{\square}{-}$ | $\stackrel{\sim}{\sim}$ |
| II | $\|8\|$ | N | 응 | $\dot{i}$ | $\stackrel{\infty}{0}$ |  | 국 | $\stackrel{N}{N}$ | $\stackrel{\circ}{2}$ | $26$ | $\approx$ | ふ | $\infty$ |  |  |  | $\infty$ |  |  |  | वু | $\underset{6}{2}$ | $\infty$ | $\underset{\sim}{\infty}$ | $\circlearrowright$ | $\underset{\infty}{\infty}$ | 잉 | $\underset{O}{\mathrm{~N}}$ | － |
| $\bigcirc$ | $\bigcirc$ | － | $\bigcirc$ | － | － | O | $\overrightarrow{0}$ | $\bigcirc$ | 0 | 0 |  |  | 0 |  |  |  | － |  |  |  | － |  |  | $\bigcirc$ | $\overline{0}$ | $\bigcirc$ | $\overrightarrow{0}$ | $=$ | $\stackrel{\square}{0}$ |
| $\sim$ | － | m | $m$ | － | $\cdots$ | $\cdots$ | n | $\cdots$ | $\cdots$ | $\checkmark$ | N | $\sim$ | n |  |  |  | － |  |  |  | in |  |  |  |  | m |  |  | $\checkmark$ |
| $m$ | $m$ | ＋ | $\sim$ | － |  | $\cdots$ | ＋ | ＋ | n | $\checkmark$ |  | m | $\sim$ |  |  | J | ＋ |  |  |  | $\sim$ | ナ | $\sim$ | m | $m$ | $\cdots$ | m | － | N |
| － | $m$ | m | － | － | n | － | $\checkmark$ | ＊ | $\sim$ | $\checkmark$ |  | in | － |  |  |  | ＋ |  |  |  |  |  | － |  |  |  |  | m | － |
| $\sim$ | m | － | － | $\sim$ | $\cdots$ | $\cdots$ | － | $\sim$ | $\cdots$ | $\cdots$ |  | $\sim$ | － |  |  |  | － | $\sim$ |  |  | $m$ | － | $n$ | － | $m$ | n | $\checkmark$ | $\sim$ | － |
| $n$ | m | m | m | m | n | $\cdots$ | $\cdots$ | $\cdots$ | n |  |  | in | － | － |  |  | m | い |  |  | m | $\cdots$ | $\checkmark$ | $\dagger$ | $\sim$ | $\cdots$ |  | m | － |
|  | $\sim$ | $\cdots$ | $\pm$ | $\sim$ | $\bigcirc$ | ， | 二 | $\stackrel{\infty}{\sim}$ | 9 | O | 1 | ন | N | N |  | N | へ | $\stackrel{\sim}{2}$ |  | ， | $\stackrel{\sim}{\sim}$ | N | － | m | m | m | m | m | ¢ |


| $8$ | $8$ | $8$ | $8$ | $8$ | $\stackrel{8}{2}$ | $\stackrel{8}{2}$ | $\stackrel{8}{2}$ | $\begin{array}{\|l\|} \hline 8 \\ \hline \end{array}$ | $\stackrel{8}{\mathrm{P}}$ | $8$ | $8$ | $8$ | $8$ | $8$ | $\stackrel{8}{2}$ | $8$ | $8$ | $8$ | $8$ | $8$ | $8$ | $8$ | $8$ | $8$ | $\stackrel{8}{1}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| প | － | 어N | $\underset{\infty}{\text { O}}$ | $\stackrel{e}{n}$ | $\begin{aligned} & 0 \\ & \infty \\ & \infty \end{aligned}$ | $\stackrel{0}{\infty}$ | $\left\|\begin{array}{l} 0 \\ \infty \\ \infty \end{array}\right\|$ | $\frac{2}{2}$ | $\stackrel{\text { N}}{N}$ | $\stackrel{8}{2}$ | $\stackrel{\text { ㅅNN }}{ }$ | $\stackrel{\rightharpoonup}{N}$ | $\stackrel{\ominus}{\circ}$ | $8$ | $8$ | $8$ | $\underset{\sim}{0}$ | $\underset{\sim}{0}$ | $8$ | $8$ | $\underset{\infty}{\infty}$ | $\stackrel{\circ}{2}$ | 은 | $\stackrel{\text { ㅅNㅅ }}{ }$ | $\stackrel{\bigcirc}{\infty}$ |
| i | $\cdots$ | ～ | $\stackrel{n}{i}$ | $\begin{aligned} & n \\ & i \end{aligned}$ | $\stackrel{n}{i}$ | $\stackrel{n}{i}$ | $\stackrel{n}{i}$ | $\left\|\begin{array}{l} n \\ i \end{array}\right\|$ | $\stackrel{n}{n}$ | $\stackrel{n}{n}$ | $\stackrel{n}{n}$ | $\stackrel{n}{i}$ | $\left.\begin{aligned} & n \\ & i \end{aligned} \right\rvert\,$ | $\begin{aligned} & n \\ & i \end{aligned}$ | $\stackrel{n}{n}$ | $\stackrel{n}{i}$ | $\stackrel{n}{n}$ | $\stackrel{n}{n}$ | $\stackrel{n}{n}$ | $\stackrel{n}{i}$ | $\stackrel{n}{n}$ | $\stackrel{n}{n}$ | $\stackrel{n}{n}$ | $\stackrel{n}{i}$ | $\stackrel{n}{n}$ |
| $\stackrel{N}{\text { N }}$ | $\stackrel{\rightharpoonup}{\infty}$ | $\vec{\square}$ | $\underset{\infty}{N}$ | $n$ | $\begin{aligned} & n \\ & n \end{aligned}$ | $\stackrel{\infty}{\sim}$ | n | $\begin{gathered} n \\ i n \end{gathered}$ | F | $\overrightarrow{\mathrm{m}}$ | $\stackrel{\underset{\sim}{i}}{i}$ | $\stackrel{\infty}{\sim}$ | $\begin{aligned} & n \\ & n \end{aligned}$ | $\stackrel{N}{n}$ | $\begin{aligned} & N \\ & \infty \\ & \infty \end{aligned}$ | $\vec{m}$ | $\stackrel{m}{n}$ | $\begin{aligned} & n \\ & n \end{aligned}$ | $\begin{aligned} & n \\ & i n \end{aligned}$ | $\vec{\infty}$ | $\checkmark$ | $\stackrel{N}{\infty}$ | $\stackrel{\rightharpoonup}{\infty}$ | $\bar{m}$ | $\cdots$ |
| $\begin{aligned} & \infty \\ & 0 \\ & 0 \end{aligned}$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\begin{aligned} & \infty \\ & 0 \\ & \hline \end{aligned}$ | $\infty$ | $\begin{aligned} & \infty \\ & 0 \\ & \hline \end{aligned}$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\stackrel{\infty}{0}$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\stackrel{\infty}{0}$ | $\infty$ | $\cdots$ |
| $\bar{m}$ | $\bigcirc$ | $\overrightarrow{\mathrm{i}}$ | $\overrightarrow{0}$ | $\frac{m}{0}$ | $\stackrel{m}{\sim}$ | $\begin{aligned} & \infty \\ & 0 \\ & 0 \\ & \hline \end{aligned}$ | $\left\|\begin{array}{c} \underset{\sim}{\dot{0}} \end{array}\right\|$ | $\frac{N}{0}$ | $\underset{\sim}{\underset{O}{*}}$ | oi | $\begin{aligned} & \mathbf{~} \\ & \mathbf{0} \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & i \end{aligned}$ | $\underset{\sim}{\mathrm{N}}$ | $\begin{aligned} & \pm \\ & 0 \\ & 0 \end{aligned}$ | $0$ | $\overline{\widehat{o}}$ | $\begin{aligned} & \bar{n} \\ & 0 \end{aligned}$ | $\begin{aligned} & 0 \\ & ? \\ & 0 \end{aligned}$ | $\stackrel{2}{0}$ | $\begin{aligned} & 0 \\ & 0 \end{aligned}$ | $\stackrel{\rightharpoonup}{\infty}$ | $\begin{aligned} & \infty \\ & \infty \\ & 0 \end{aligned}$ | $\begin{aligned} & \infty \\ & \infty \\ & 0 \\ & \hline \end{aligned}$ | $\stackrel{\rightharpoonup}{*}$ | $\stackrel{\square}{\square}$ |
| in | n | in | in | in | in | in | n | n | in | n | in | n | in | in | n | in | n | in | in | $n$ | in | n | n | n | in |
| $\stackrel{\sim}{\mathrm{N}}$ | $\bigcirc$ | $\bigcirc$ | $\sim$ | ナ | n | $m$ | $\sim$ | $\bigcirc$ | N | N | $\sim$ | m | $\sim$ | $\bigcirc$ | $\bigcirc$ | $\cdots$ | n | $\sim$ | $\infty$ | $\sim$ | $\pm$ | 으 | 응 | こ | 0 |
| $\begin{aligned} & n \\ & \tilde{n} \\ & 0 \\ & \dot{0} \\ & \hat{6} \end{aligned}$ | $\begin{aligned} & \underset{N}{2} \\ & \vdots \\ & \underset{N}{j} \\ & \underset{N}{\prime} \end{aligned}$ | $\begin{aligned} & \mathbb{N} \\ & \underset{\sim}{\infty} \\ & \underset{\sim}{\theta} \end{aligned}$ | $\begin{aligned} & \hat{0} \\ & \infty \\ & 0 \\ & \vdots \\ & \text { N} \\ & \text { N} \end{aligned}$ | $\begin{aligned} & \hat{N} \\ & i n \\ & \underset{\sim}{t} \end{aligned}$ | $\begin{aligned} & o \\ & \dot{0} \\ & \dot{i} \\ & \vec{j} \end{aligned}$ | $\begin{aligned} & 0 \\ & \underset{\sim}{n} \\ & \underset{\sim}{\lambda} \\ & \vec{j} \end{aligned}$ | $\left\|\begin{array}{c} \infty \\ \vdots \\ 0 \\ i \\ i \\ 0 \\ \infty \end{array}\right\|$ | $\begin{aligned} & n \\ & n \\ & n \\ & n \\ & n \\ & \eta \end{aligned}$ | $\begin{aligned} & \vec{G} \\ & \infty \\ & \dot{\infty} \\ & \dot{2} \\ & \dot{\infty} \end{aligned}$ | $\begin{aligned} & n \\ & = \\ & \underset{y}{n} \\ & n \\ & n \end{aligned}$ | $\begin{aligned} & \hat{n} \\ & n \\ & \underset{n}{1} \\ & n \end{aligned}$ | $\begin{gathered} N \\ \underset{N}{N} \\ \underset{N}{n} \end{gathered}$ | $\begin{aligned} & 0 \\ & \text { i } \\ & \text { N} \\ & \text { N } \end{aligned}$ | $\begin{aligned} & 0 \\ & 2 \\ & 0 \\ & 0 \\ & 0 \\ & \vdots \\ & \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & n \\ & 0 \\ & \dot{n} \\ & \dot{n} \\ & \hline \end{aligned}$ | $\begin{aligned} & \hat{a} \\ & \hat{n} \\ & \hat{2} \\ & \hat{0} \end{aligned}$ |  | $\begin{gathered} \infty \\ n \\ \infty \\ \infty \\ \infty \\ \infty \\ \infty \\ \hline \end{gathered}$ |  |  | $\begin{gathered} 0 \\ 0 \\ \vdots \\ \vdots \\ \infty \\ \underset{\sigma}{\alpha} \end{gathered}$ |  | $\begin{aligned} & n \\ & \underset{2}{n} \\ & \hat{n} \\ & \underset{i}{n} \\ & n \end{aligned}$ |  |
| $\cdots$ | $\cdots$ | N | 工 | $\bigcirc$ | $\sim$ | $\cdots$ | ㄱ | $\exists$ | $\wedge$ | $\cdots$ | $\pm$ | $\cdots$ | $\bigcirc$ | N | ～ | $\pm$ | N | $\bigcirc$ | च | － | $\cdots$ | $\cdots$ | 二 | の | の |
| $\stackrel{\infty}{\substack{\infty \\ \underset{\sim}{j}}}$ | $\stackrel{n}{\sim}$ | $\stackrel{\rightharpoonup}{\lambda}$ |  | $\begin{gathered} \stackrel{0}{\circ} \\ \stackrel{\rightharpoonup}{m} \end{gathered}$ | $\begin{array}{\|c} n \\ \end{array}$ | $\begin{aligned} & n \\ & n \\ & i \end{aligned}$ | $\left\lvert\, \begin{gathered} 0 \\ \underset{\sim}{N} \\ \text { m } \end{gathered}\right.$ | $\overrightarrow{\mathrm{N}}$ | $\begin{aligned} & \hat{0} \\ & \underset{\sim}{n} \\ & \text { w} \end{aligned}$ | $\frac{\mathfrak{i}}{\underset{\sim}{7}}$ | $\begin{aligned} & \underset{\sim}{7} \\ & \underset{\sim}{\sim} \end{aligned}$ | $\stackrel{\underset{\sim}{\mathrm{N}}}{\substack{2}}$ | $\begin{aligned} & \text { N } \\ & \underset{\sim}{7} \end{aligned}$ | $\underset{\sim}{\underset{\gamma}{\gamma}}$ | $\begin{aligned} & \underset{\sim}{*} \\ & \underset{\sim}{\sim} \\ & \underset{\sim}{2} \end{aligned}$ | $\begin{aligned} & n \\ & 0 \\ & 0 \\ & \hline \end{aligned}$ | $\stackrel{n}{\infty}$ | $\begin{aligned} & n \\ & \dot{0} \\ & \dot{\gamma} \end{aligned}$ | $\begin{aligned} & n \\ & \underset{\sim}{n} \\ & \underset{\sim}{2} \end{aligned}$ | $\frac{n}{\vdots}$ | $\begin{aligned} & \infty \\ & 0 \\ & 0 \\ & \dot{\gamma} \end{aligned}$ | $\begin{aligned} & 0 \\ & \dot{O} \\ & \dot{\gamma} \end{aligned}$ | $\frac{n}{n}$ | $\begin{gathered} 0 \\ i \\ i \\ \infty \end{gathered}$ | m $i$ $n$ $n$ $n$ |
| $\underset{\sim}{?}$ | $9$ | $\stackrel{\sim}{\underset{O}{0}}$ | $\exists$ | $$ | $\underset{0}{*}$ | $\begin{aligned} & \infty \\ & \infty \\ & 0 \\ & \hline \end{aligned}$ | $\hat{i}$ | － | $\stackrel{\rightharpoonup}{*}$ | $\begin{aligned} & \ddagger \\ & \infty \\ & 0 \end{aligned}$ | $\xlongequal{3}$ | $\begin{aligned} & \infty \\ & \infty \\ & 0 \end{aligned}$ | $\stackrel{n}{n}$ | $\stackrel{n}{\pi}$ | $\begin{aligned} & \infty \\ & n \\ & 0 \end{aligned}$ | $\begin{aligned} & \infty \\ & \infty \\ & 0 \end{aligned}$ | $\stackrel{n}{?}$ | $\stackrel{n}{n}$ | $\begin{aligned} & \infty \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \hat{0} \\ & 0 \end{aligned}$ | $\hat{\infty}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \end{aligned}$ | － | $\stackrel{n}{n}$ | $\infty$ |
| － | $\bigcirc$ | $\sim$ | － | － | N | N | － | － | － | $\bigcirc$ | － | $\sim$ | － | － | 0 | － | N | N | － | $\sim$ | N | － | N | $\bigcirc$ | 0 |
| $\bigcirc$ | 0 | $\bigcirc$ | $\bigcirc$ | O | 0 | 0 | － | － | 0 | 0 | － | － | $\bigcirc$ | 0 | 0 | 0 | 0 | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | 0 | 0 | 0 | 0 | 0 |
| N | へ | $\bigcirc$ | $\bigcirc$ | N | $\bigcirc$ | $\infty$ | － | $\bigcirc$ | － | の | $\infty$ | $\bigcirc$ | $\infty$ | 9 | 9 | 9 | $\stackrel{\sim}{\sim}$ | 工 | $\infty$ | $\infty$ | $\infty$ | 9 | $\bigcirc$ | 9 | $\infty$ |
| $\underset{\infty}{+\infty}$ | o | $\underset{\widehat{V}}{N}$ | $\begin{aligned} & 0 \\ & \text { O} \\ & \text { S } \end{aligned}$ | $\begin{aligned} & n \\ & n \\ & \hline \end{aligned}$ | $\begin{aligned} & \infty \\ & + \\ & \hline \end{aligned}$ | $\begin{aligned} & \pm \\ & \infty \\ & \hline \end{aligned}$ | $\vec{\sigma}$ | $\left.\begin{aligned} & 0 \\ & 0 \\ & 0 \end{aligned} \right\rvert\,$ | © | $0_{0}^{\infty}$ | $\begin{aligned} & \text { U } \\ & \infty \\ & \hline \end{aligned}$ | $\frac{0}{\infty}$ | $\underset{N}{ \pm}$ | $\begin{aligned} & 0 \\ & \text { O} \\ & \hline \end{aligned}$ | $\bar{n}$ | ob | $\begin{aligned} & \mathrm{N} \\ & 0 \end{aligned}$ | 핫 | $\begin{aligned} & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\frac{\infty}{\sigma}$ | ${ }_{0}^{0}$ | $\stackrel{\infty}{\infty}$ | $$ | $\begin{aligned} & t \\ & 0 \\ & \hline \end{aligned}$ | 2 |
| $\bigcirc$ | $\overrightarrow{0}$ | $\overrightarrow{0}$ | $\overrightarrow{0}$ | $\bigcirc$ | $\overrightarrow{0}$ | $\overline{0}$ | $\overline{0}$ | $\overrightarrow{0} \mid$ | $\stackrel{\rightharpoonup}{0}$ | $\bigcirc$ | $\overline{0}$ | $\overrightarrow{0}$ | $\overrightarrow{0}$ | $\bigcirc$ | $\overrightarrow{0}$ | $\bigcirc$ | $\bigcirc$ | $\overline{0}$ | $\stackrel{\rightharpoonup}{0}$ | $\overrightarrow{0}$ | $\stackrel{\rightharpoonup}{0}$ | $\overline{0}$ | $\stackrel{\rightharpoonup}{0}$ | $\bigcirc$ | $\overrightarrow{0}$ |
| $m$ | N | － | m | $n$ | － | N | － | $\cdots$ | N | n | － | $\cdots$ | $m$ | ナ | N | $\checkmark$ | $m$ | m | N | $m$ | in | m | ナ | $\cdots$ | N |
| n | in | $m$ | ナ | $\nabla$ | N | ナ | $\checkmark$ | $\checkmark$ | － | $m$ | n | $\checkmark$ | $\checkmark$ | － | $\cdots$ | ナ | in | $\sim$ | $\sim$ | $\sim$ | ナ | － | m | $m$ | N |
| $\checkmark$ | $m$ | － | N | N | ナ | $\sim$ | in | $m$ | $m$ | $m$ | $m$ | $m$ | in | $m$ | m | ナ | n | $m$ | $\checkmark$ | m | $m$ | N | $\cdots$ | ＊ | ナ |
| $\sim$ | $\sim$ | $\sim$ | N | m | $m$ | $m$ | － | $\nabla$ | m | $\sim$ | $m$ | $\checkmark$ | $\checkmark$ | m | N | ナ | $\cdots$ | $\sim$ | $\sim$ | $\checkmark$ | ナ | － | N | $m$ | ナ |
| $m$ | $\checkmark$ | $m$ | $m$ | $\checkmark$ | ナ | $m$ | $m$ | ナ | $\checkmark$ | $m$ | N | N | $\sim$ | $\checkmark$ | ナ | － | $\checkmark$ | $\checkmark$ | N | m | $\cdots$ | $\cdots$ | $m$ | N | N |
| n | $\cdots$ | ल | ¢ | $\ni$ | $\stackrel{\text { N }}{ }$ | $\cdots$ | ナ | か | $\bigcirc$ | $\stackrel{\text { ¢ }}{ }$ | $\stackrel{\infty}{+}$ | ช | i | $\stackrel{\rightharpoonup}{n}$ | N | n | 岕 | in | $\cdots$ | in | $\cdots$ | is | 8 | $\bar{\square}$ | N |


| 63 | 3 | 3 | 3 | 2 | 5 | 0.1 | 848 | 16 | 0 | 1 | 0.79 | 307.5 | 9 | 11377.265 | 6 | 5 | 1.02 | 0.8 | 3.9 | 2.5 | 700 | 700 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 64 | 2 | 2 | 4 | 2 | 3 | 0.1 | 640 | 16 | 0 | 0 | 0.97 | 371.5 | 8 | 4086.9028 | 5 | 5 | 0.39 | 0.8 | 4.1 | 2.5 | 890 | 700 |
| 65 | 4 | 2 | 2 | 4 | 4 | 0.1 | 782 | 17 | 0 | 1 | 1.08 | 361.1 | 14 | 11193.351 | 4 | 5 | 0.57 | 0.8 | 5.3 | 2.5 | 900 | 700 |
| 66 | 2 | 3 | 1 | 2 | 2 | 0.1 | 884 | 17 | 0 | 1 | 0.79 | 404.9 | 13 | 8097.138 | 6 | 5 | 0.62 | 0.8 | 3.9 | 2.5 | 760 | 700 |
| 67 | 2 | 3 | 2 | 5 | 2 | 0 | 810 | 18 | 0 | 1 | 0.97 | 401.4 | 14 | 12042.924 | 9 | 5 | 0.42 | 0.8 | 3.2 | 2.5 | 750 | 700 |
| 68 | 2 | 3 | 2 | 3 | 3 | 0 | 752 | 16 | 0 | 2 | 0.64 | 296 | 16 | 6808.4888 | 15 | 5 | 1.51 | 0.8 | 3.3 | 2.5 | 800 | 700 |
| 69 | 3 | 4 | 5 | 1 | 4 | 0.1 | 931 | 19 | 0 | 0 | 0.27 | 197.8 | 3 | 4549.7068 | 7 | 5 | 1.26 | 0.8 | 10 | 2.5 | 970 | 700 |
| 70 | 3 | 4 | 2 | 2 | 1 | 0.1 | 1044 | 18 | 0 | 1 | 0.16 | 199.2 | 3 | 3386.3092 | 8 | 5 | 4.91 | 0.8 | 8.3 | 2.5 | 930 | 700 |
| 71 | 2 | 5 | 1 | 2 | 4 | 0.1 | 954 | 18 | 0 | 1 | 0.31 | 225.6 | 4 | 8796.7304 | 11 | 5 | 0.91 | 0.8 | 8.4 | 2.5 | 1150 | 700 |
| 72 | 3 | 5 | 3 | 3 | 2 | 0.1 | 1020 | 17 | 0 | 0 | 0.18 | 193.7 | 2 | 4454.5926 | 8 | 5 | 0.25 | 0.8 | 7 | 2.5 | 980 | 700 |
| 73 | 2 | 4 | 1 | 1 | 4 | 0.1 | 1140 | 19 | 0 | 1 | 0.18 | 196.3 | 3 | 4711.9853 | 1 | 5 | 3.86 | 0.8 | 11 | 2.5 | 1080 | 700 |
| 74 | 1 | 4 | 5 | 1 | 2 | 0.1 | 792 | 18 | 0 | 1 | 0.34 | 235.5 | 3 | 4475.432 | 9 | 5 | 0.72 | 0.8 | 10 | 2.5 | 940 | 700 |
| 75 | 1 | 5 | 3 | 1 | 1 | 0.1 | 736 | 16 | 0 | 1 | 0.23 | 170.5 | 3 | 1875.7444 | 2 | 5 | 2.06 | 0.8 | 6.9 | 2.5 | 920 | 700 |
| 76 | 1 | 5 | 1 | 2 | 1 | 0.1 | 760 | 19 | 0 | 0 | 0.29 | 249.6 | 3 | 3993.2067 | 3 | 5 | 1.79 | 0.8 | 8.4 | 2.5 | 1180 | 700 |
| 77 | 1 | 4 | 2 | 1 | 2 | 0.1 | 1008 | 18 | 0 | 1 | 0.21 | 181.9 | 3 | 909.3579 | 6 | 5 | 0.11 | 0.8 | 10 | 2.5 | 1170 | 700 |
| 78 | 3 | 4 | 3 | 3 | 2 | 0.1 | 792 | 18 | 0 | 0 | 0.35 | 194.4 | 4 | 5637.8584 | 4 | 5 | 2.28 | 0.8 | 8.4 | 2.5 | 910 | 700 |
| 79 | 3 | 5 | 3 | 3 | 4 | 0.1 | 952 | 17 | 0 | 1 | 0.2 | 176.1 | 4 | 8101.8926 | 4 | 5 | 6.04 | 0.8 | 11 | 2.5 | 1140 | 700 |
| 80 | 2 | 5 | 5 | 1 | 5 | 0.1 | 896 | 16 | 0 | 0 | 0.28 | 176 | 3 | 7038.3976 | 7 | 5 | 1.05 | 0.8 | 11 | 2.5 | 1190 | 700 |
| 81 | 2 | 4 | 3 | 2 | 2 | 0.1 | 800 | 16 | 0 | 1 | 0.22 | 165.3 | 4 | 2810.509 | 3 | 5 | 1.12 | 0.8 | 6.8 | 2.5 | 1000 | 700 |
| 82 | 3 | 4 | 3 | 3 | 4 | 0.1 | 912 | 16 | 0 | 0 | 0.29 | 178.6 | 4 | 4465.4335 | 1 | 5 | 2.53 | 0.8 | 11 | 2.5 | 1150 | 700 |
| 83 | 1 | 4 | 3 | 2 | 4 | 0.1 | 756 | 18 | 0 | 1 | 0.19 | 193.4 | 3 | 6768.9468 | 4 | 5 | 3.39 | 0.8 | 8.4 | 2.5 | 1010 | 700 |
| 84 | 1 | 5 | 2 | 1 | 3 | 0.1 | 855 | 19 | 0 | 1 | 0.31 | 246.6 | 3 | 12577.8 | 0 | 5 | 0.06 | 0.8 | 6.7 | 2.5 | 1210 | 700 |
| 85 | 3 | 5 | 3 | 3 | 3 | 0.1 | 779 | 19 | 0 | 1 | 0.17 | 190.4 | 3 | 3045.7224 | 10 | 5 | 1.73 | 0.8 | 8.3 | 2.5 | 1230 | 700 |
| 86 | 2 | 4 | 2 | 3 | 2 | 0.1 | 969 | 19 | 0 | 0 | 0.23 | 225.1 | 2 | 4277.5593 | 2 | 5 | 2.1 | 0.8 | 11 | 2.5 | 1250 | 700 |
| 87 | 3 | 4 | 4 | 3 | 3 | 0.1 | 864 | 16 | 0 | 1 | 0.26 | 167.4 | 3 | 1171.692 | 5 | 5 | 0.48 | 0.8 | 6.8 | 2.5 | 980 | 700 |
| 88 | 2 | 4 | 1 | 1 | 1 | 0.1 | 1083 | 19 | 0 | 0 | 0.19 | 224.5 | 3 | 4040.4631 | 4 | 5 | 0.24 | 0.8 | 10 | 2.5 | 930 | 700 |


| 89 | 1 | 4 | 3 | 1 | 4 | 0.1 | 784 | 16 | 0 | 0 | 0.31 | 178.8 | 3 | 7690.406 | 7 | 5 | 0.85 | 0.8 | 6.9 | 2.5 | 970 | 700 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 90 | 3 | 4 | 3 | 1 | 2 | 0.1 | 1003 | 17 | 0 | 1 | 0.2 | 186 | 3 | 4837.1695 | 9 | 5 | 0.81 | 0.8 | 10 | 2.5 | 1090 | 700 |
| 91 | 2 | 5 | 3 | 3 | 3 | 0.1 | 988 | 19 | 0 | 0 | 0.26 | 197.3 | 4 | 8483.3518 | 6 | 5 | 0.34 | 0.8 | 8.4 | 2.5 | 990 | 700 |
| 92 | 1 | 4 | 2 | 2 | 1 | 0.1 | 816 | 16 | 0 | 1 | 0.28 | 200.4 | 3 | 1803.2825 | 8 | 5 | 2.22 | 0.8 | 8.2 | 2.5 | 1100 | 700 |
| 93 | 3 | 4 | 3 | 3 | 1 | 0.1 | 855 | 19 | 0 | 0 | 0.22 | 219.3 | 3 | 3069.7376 | 10 | 5 | 2.94 | 0.8 | 10 | 2.5 | 970 | 700 |
| 94 | 1 | 4 | 1 | 3 | 2 | 0.1 | 901 | 17 | 0 | 1 | 0.28 | 175.8 | 4 | 2636.2908 | 3 | 5 | 1.28 | 0.8 | 8.4 | 2.5 | 930 | 700 |
| 95 | 2 | 5 | 5 | 2 | 4 | 0.1 | 874 | 19 | 0 | 0 | 0.41 | 247.7 | 3 | 7431.717 | 12 | 5 | 0.54 | 0.8 | 8 | 2.5 | 950 | 700 |
| 96 | 2 | 5 | 2 | 1 | 4 | 0.1 | 828 | 18 | 0 | 0 | 0.24 | 192.7 | 3 | 5974.94 | 4 | 5 | 2.44 | 0.8 | 6.8 | 2.5 | 1230 | 700 |
| 97 | 2 | 4 | 4 | 2 | 4 | 0.1 | 918 | 18 | 0 | 0 | 0.28 | 222.5 | 3 | 12016.675 | 3 | 5 | 2 | 0.8 | 10 | 2.5 | 920 | 700 |
| 98 | 1 | 4 | 4 | 1 | 5 | 0.1 | 656 | 16 | 0 | 1 | 0.28 | 177.4 | 3 | 6917.3762 | 8 | 5 | 0.78 | 0.8 | 6.8 | 2.5 | 1100 | 700 |
| 99 | 3 | 4 | 3 | 3 | 3 | 0.1 | 714 | 17 | 0 | 1 | 0.19 | 192.1 | 3 | 5955.3418 | 7 | 5 | 1.58 | 0.8 | 7.1 | 2.5 | 980 | 700 |
| 100 | 1 | 5 | 3 | 3 | 3 | 0.1 | 810 | 18 | 0 | 0 | 0.16 | 218.9 | 2 | 9852.1056 | 9 | 5 | 2.18 | 0.8 | 8.2 | 2.5 | 1110 | 700 |

Table 1 Dataset for 100 hypothetical bus routes for Societal Variables and their Targets
Appendix B 2.2
Simulated Dataset for Infrastructure 2, 100 Hypothetical Bus Routes


| $\stackrel{\text { ® }}{\text { ¢ }}$ | $\stackrel{\text { ® }}{\text { ® }}$ | $\stackrel{9}{\mathrm{R}}$ | $\stackrel{9}{\mathrm{R}}$ | $\stackrel{9}{\mathrm{R}}$ | $\stackrel{\mathrm{Q}}{\mathrm{R}}$ | $\stackrel{\ominus}{\mathrm{Q}}$ | $\stackrel{\mathrm{R}}{\mathrm{R}}$ | $\stackrel{9}{\mathrm{R}}$ | $\stackrel{9}{2}$ | $\stackrel{\mathrm{Q}}{\mathrm{R}}$ | $\stackrel{9}{\mathrm{R}}$ | $\stackrel{\ominus}{\mathrm{R}}$ | $\stackrel{\mathrm{Q}}{\mathrm{R}}$ | $\stackrel{\mathrm{R}}{\mathrm{R}}$ | $\stackrel{8}{\mathrm{R}}$ | $\stackrel{\mathrm{R}}{\mathrm{R}}$ | $\stackrel{9}{2}$ | $\stackrel{8}{\mathrm{R}}$ | $\stackrel{9}{\mathrm{R}}$ | $\stackrel{9}{\mathrm{R}}$ | $\stackrel{9}{\mathrm{R}}$ | $\stackrel{\mathrm{P}}{\mathrm{R}}$ | $\stackrel{\mathrm{R}}{\mathrm{R}}$ | $\stackrel{9}{2}$ | $\bigcirc$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{array}{\|l} \hline \underset{\sim}{\circ} \\ \stackrel{1}{2} \end{array}$ |  | $\begin{aligned} & 0 \\ & i n \end{aligned}$ | $\begin{aligned} & 0 \\ & i n \end{aligned}$ | in | $8$ | $\begin{aligned} & \hline \stackrel{y}{n} \\ & i \end{aligned}$ | $\begin{aligned} & \hline \underset{\infty}{\infty} \\ & i \end{aligned}$ | 엉 | $\begin{aligned} & \hline 8 \\ & i n \end{aligned}$ | প্ত্ত | $\underset{\sim}{~}$ | $\begin{aligned} & 0 \\ & n \\ & n \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \end{aligned}$ | io | o্ত | io | $\begin{gathered} \mathrm{o} \\ \text { i } \end{gathered}$ | O | $\begin{aligned} & 0 \\ & n \\ & n \end{aligned}$ | O | $\underset{N}{\mathrm{~N}}$ | $\underset{\infty}{\underset{\infty}{\prime}}$ | $\begin{aligned} & \hline \stackrel{n}{\infty} \\ & \infty \end{aligned}$ | $\begin{aligned} & i \stackrel{n}{\infty} \\ & \infty \end{aligned}$ | পু |
| $\begin{aligned} & \stackrel{n}{n} \\ & \end{aligned}$ | $\stackrel{n}{i}$ | $\stackrel{n}{i}$ | $\stackrel{n}{i}$ | $\stackrel{n}{i}$ | $\stackrel{n}{i}$ | $\begin{aligned} & n \\ & n \\ & n \end{aligned}$ | $\stackrel{n}{n}$ | $\stackrel{n}{i}$ | $\stackrel{n}{i}$ | $\stackrel{n}{n}$ | $\stackrel{n}{i}$ | $\stackrel{n}{i}$ | $\stackrel{n}{n}$ | $\stackrel{n}{n}$ | $\mathfrak{n}$ | $\stackrel{n}{n}$ | $\stackrel{n}{i}$ | $\underset{\sim}{n}$ | $\stackrel{n}{\mathrm{i}}$ | $\stackrel{n}{i}$ | $\stackrel{n}{i}$ | $\stackrel{n}{n}$ | $\stackrel{n}{n}$ | $\stackrel{n}{i}$ | $\stackrel{\sim}{n}$ |
| $?$ | $\begin{aligned} & \infty \\ & \\ & \hline \end{aligned}$ | $\stackrel{+}{n}$ | $\underset{\sim}{N}$ | $\stackrel{-}{-}$ | $\bigcirc$ | ন | $\begin{aligned} & n \\ & \underset{\sim}{n} \end{aligned}$ | $\begin{aligned} & \pm \\ & \underset{i}{ } \end{aligned}$ | $\underset{\mathrm{N}}{\mathrm{~N}}$ | $\begin{aligned} & \sim \\ & n \\ & \hdashline \end{aligned}$ | $\begin{aligned} & \pm \\ & \underset{i}{ } \end{aligned}$ | $\xrightarrow[n]{n}$ | $\begin{aligned} & \underset{\sim}{m} \\ & \stackrel{1}{2} \end{aligned}$ | $\begin{aligned} & \stackrel{o}{n} \\ & \underset{m}{2} \end{aligned}$ | $\begin{aligned} & \sim \\ & \sim \end{aligned}$ | $\begin{aligned} & 0 \\ & \stackrel{\rightharpoonup}{2} \end{aligned}$ | $\underset{\mathrm{N}}{\mathrm{~N}}$ | m | $\begin{aligned} & \infty \\ & \infty \\ & -i \end{aligned}$ | $\begin{aligned} & \hat{b} \\ & i \end{aligned}$ | $\cdots$ | $\begin{aligned} & \underset{\sim}{n} \\ & \dot{m} \end{aligned}$ | $\cdots$ | $\begin{aligned} & \stackrel{\rightharpoonup}{f} \\ & i \end{aligned}$ | $\infty$ |
| $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | ${ }_{0}^{\infty}$ | ${ }_{0}^{\infty}$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | ${ }_{0}^{\infty}$ | ${ }_{0}^{\infty}$ | ${ }_{0}^{\infty}$ | ${ }_{0}^{\infty}$ | $\infty$ | ${ }_{0}^{\infty}$ | ${ }_{0}^{\infty}$ | $\infty$ | $\infty$ | $\infty$ | ${ }_{0}^{\infty}$ | $\infty$ | $\stackrel{\infty}{\circ}$ |
| $\begin{aligned} & n \\ & 0 \end{aligned}$ | $\underset{\circ}{+}$ | $0$ | $\bigcirc$ | $\cdots$ | $\cdots$ | ò | $0$ | $\underset{0}{6}$ | $\underset{\sim}{N}$ | on | $\underset{0}{2}$ | $\underset{o}{N}$ | $\underset{o}{\circ}$ | $0$ | $\overline{0}$ | $\overrightarrow{0}$ | － | $\xrightarrow{\sim}$ | $\underset{o}{\mathrm{~N}}$ | $\bigcirc$ | $?$ | $\underset{0}{N}$ | $0$ | $\xrightarrow{\sim}$ | $\cdots$ |
| $\cdots$ | n | n | n | n | n | $\cdots$ | $\cdots$ | $\cdots$ | n | $\cdots$ | n | n | $\cdots$ | n | $\cdots$ | $\cdots$ | n | $\cdots$ | $\cdots$ | n | n | $\cdots$ | n | n | $n$ |
| － | n | $\bigcirc$ | $\cdots$ | $\cdots$ | m | $\bigcirc$ | N | $\checkmark$ | N | च | $\infty$ | $\bigcirc$ | $\bigcirc$ | $\cdots$ | $\bigcirc$ | $\bigcirc$ | m | $\cdots$ | － | の | m | $\bigcirc$ | の | $\bigcirc$ | 0 |
| $$ | $\begin{aligned} & \text { Ǹ } \\ & \underset{N}{n} \\ & \hline \end{aligned}$ | $\begin{aligned} & \infty \\ & \underset{\sim}{\infty} \\ & \underset{子}{2} \end{aligned}$ | $\begin{aligned} & \text { No } \\ & \text { O} \\ & \stackrel{2}{0} \end{aligned}$ | $\begin{aligned} & N \\ & \underset{N}{N} \\ & \infty \end{aligned}$ | $\begin{aligned} & \overline{0} \\ & \cdots \\ & \hat{N} \end{aligned}$ | $\begin{aligned} & \hat{\infty} \\ & \substack{\infty \\ \hline} \end{aligned}$ | $$ | $\frac{n}{2}$ | $\begin{aligned} & \text { g } \\ & \underset{~}{寸} \end{aligned}$ | $\frac{\mathrm{N}}{\mathrm{~N}}$ | $\begin{aligned} & \hline 0 \\ & \text { o } \\ & \stackrel{y}{c} \\ & \underset{N}{2} \end{aligned}$ | $\begin{aligned} & \hat{O} \\ & \hat{\sigma} \end{aligned}$ | $\begin{aligned} & \infty \\ & \hline 0 \\ & \hline 0 \\ & \hline \end{aligned}$ | $\begin{aligned} & \hat{6} \\ & \text { 子 } \end{aligned}$ | $\begin{aligned} & \hat{\infty} \\ & \stackrel{\infty}{2} \end{aligned}$ | $\begin{aligned} & \underset{\sim}{n} \\ & \text { n } \\ & \text { N} \end{aligned}$ | $\begin{aligned} & 9 \\ & 6 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \infty \\ & \infty \\ & \infty \\ & \hline \end{aligned}$ | \|o | $\begin{aligned} & n \\ & \underset{m}{n} \\ & \hline \end{aligned}$ | $\begin{gathered} \stackrel{0}{\text { N }} \\ \underset{\sim}{n} \end{gathered}$ | $\begin{aligned} & n \\ & \substack{n \\ i \\ \infty} \end{aligned}$ | $\begin{aligned} & n \\ & \cdots \\ & n \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & \hline \end{aligned}$ | $\begin{aligned} & \stackrel{0}{2} \\ & \underset{\sim}{Z} \end{aligned}$ |
| 악 | $\bigcirc$ | N | 9 | N | 안 | 어 | $\cdots$ | $\cdots$ | 入 | $\infty$ | － | $\pm$ | ก | N | $\cdots$ | 9 | $\cdots$ | へ | ন | N | $\bigcirc$ | N | の | $\infty$ | こ |
| $\begin{aligned} & \vec{\exists} \\ & \vec{\sigma} \end{aligned}$ | $\begin{aligned} & n \\ & n \\ & 0 \\ & \vdots \\ & i \end{aligned}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{\infty} \\ & \underset{寸}{+} \\ & \underset{寸}{\prime} \end{aligned}$ | $\begin{aligned} & \underset{+}{+} \\ & \underset{\sim}{n} \\ & i n \end{aligned}$ | $\begin{aligned} & \hline \stackrel{0}{\infty} \\ & \underset{\sim}{\circ} \\ & \underset{寸}{ } \end{aligned}$ | $\begin{aligned} & \overline{0} \\ & i \end{aligned}$ | $\vec{~}$ | $\begin{aligned} & \stackrel{r}{n} \\ & \stackrel{n}{n} \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & \dot{n} \end{aligned}$ | $\begin{aligned} & 9 \\ & 0 \\ & \dot{f} \end{aligned}$ | $\begin{aligned} & \underset{i}{2} \\ & \underset{i}{n} \end{aligned}$ | $\begin{aligned} & \underset{\sim}{\sim} \\ & \underset{\sim}{\infty} \\ & \underset{\sim}{2} \end{aligned}$ | ল্రి | $\begin{aligned} & n \\ & n \\ & n \\ & n \end{aligned}$ | $\begin{aligned} & \stackrel{o}{i} \\ & \stackrel{\sim}{\sigma} \end{aligned}$ | $\begin{aligned} & \underset{\sim}{~} \\ & \underset{N}{n} \end{aligned}$ | $\begin{aligned} & \underset{\sim}{\mathrm{i}} \\ & \underset{\sim}{n} \end{aligned}$ | $\begin{aligned} & \underset{\sim}{9} \\ & \underset{\gamma}{7} \end{aligned}$ | $\begin{aligned} & \hline \Omega \\ & \infty \\ & \underset{子}{\circ} \end{aligned}$ | $\underset{\underset{\sim}{*}}{\stackrel{\rightharpoonup}{*}}$ |  | $\begin{aligned} & \mathrm{N} \\ & \stackrel{y}{\circ} \\ & \stackrel{y}{4} \end{aligned}$ | $\begin{aligned} & \stackrel{\sim}{i} \\ & \underset{\sim}{\mathrm{~N}} \end{aligned}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{+} \\ & \dot{\infty} \\ & \underset{\sim}{\infty} \end{aligned}$ | $\begin{aligned} & \text { ò } \\ & \text { Ǹ } \end{aligned}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{\dot{7}} \\ & \underset{\sim}{ \pm} \end{aligned}$ |
| $\stackrel{\underset{\sigma}{\circ}}{ }$ | $\xrightarrow{+}$ | $\stackrel{\Im}{0}$ | － | $\stackrel{0}{0}$ | $\stackrel{\imath}{0}$ | $\stackrel{\imath}{0}$ | $\cdots$ | － | $\stackrel{\Im}{0}$ | 三 | － | $\stackrel{\sigma}{-}$ | － | $\square$ | $\stackrel{+}{-}$ | － | $\xrightarrow{\text { ¹ }}$ | $\stackrel{\imath}{\circ}$ | $\stackrel{\Im}{\circ}$ | $\stackrel{0}{0}$ | $9$ | $\overline{=}$ | $0$ | $\underset{0}{\circ}$ | $\stackrel{\sim}{0}$ |
| $\begin{array}{\|l} \hline \infty \\ n \\ \tilde{0} \end{array}$ | $\begin{aligned} & 2 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \underset{\sim}{\sim} \\ & \underset{0}{0} \end{aligned}$ | $\begin{aligned} & 2 \\ & \underset{\sim}{2} \\ & 0 \end{aligned}$ | $\underset{\sim}{\underset{F}{f}}$ | $\because$ | $\begin{aligned} & \hline 0 \\ & \underset{\sim}{+} \\ & 0 \end{aligned}$ | $\begin{aligned} & 0 \\ & \stackrel{y}{2} \\ & 0 \end{aligned}$ | $\begin{aligned} & N \\ & \text { N} \\ & 0 \end{aligned}$ | $\begin{aligned} & n \\ & n \\ & n \\ & 0 \end{aligned}$ | $\begin{aligned} & \pm \\ & \underset{0}{*} \\ & \hline \end{aligned}$ |  | $$ | $$ | $\begin{aligned} & \infty \\ & \stackrel{y}{n} \\ & \stackrel{\rightharpoonup}{0} \end{aligned}$ | $\begin{aligned} & \underset{\sim}{J} \\ & \underset{0}{2} \end{aligned}$ | $\begin{gathered} \infty \\ \infty \\ \end{gathered}$ | $\stackrel{N}{7}$ | $\begin{aligned} & \infty \\ & \underset{\sim}{2} \\ & 0 \end{aligned}$ | $\underset{\underset{\sim}{\infty}}{\stackrel{\infty}{*}}$ |  | $\begin{aligned} & 12 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \infty \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & n \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \infty \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\stackrel{n}{n}$ |
| 2 | $\bigcirc$ | $\infty$ | O | N | $\infty$ | $\bigcirc$ | $\bigcirc$ | O | N | － | $\bigcirc$ | 2 | 9 | 工 | $\bigcirc$ | － | $\bigcirc$ | $\bigcirc$ | － | $\infty$ | $\infty$ | 2 | N | $\bigcirc$ | $\infty$ |
| $\overline{\mathrm{I}}$ | $\underset{\infty}{\infty}$ | $\underset{\text { N }}{\mathrm{N}}$ | $\frac{18}{2}$ | $\begin{array}{\|c} N \\ \underset{\sim}{n} \end{array}$ | $\begin{aligned} & \hline 8 \\ & \hline 0 \end{aligned}$ | $\begin{aligned} & \hline 0 \\ & \cdots \end{aligned}$ | $\underset{N}{\text { N}}$ | $\frac{8}{2}$ | ñ | $\underset{n}{n}$ | $8$ | $\stackrel{8}{2}$ | $\underset{\sim}{\infty}$ | $\hat{\infty}$ | $\underset{\sim}{\text { So }}$ | $\bar{n}$ | $\underset{\alpha}{\infty}$ | $\underset{N}{N}$ | $\widehat{\infty}$ | $\underset{\sim}{\infty}$ | $\begin{aligned} & \mathrm{N} \\ & \hline \mathrm{O} \\ & \hline \end{aligned}$ | $\begin{aligned} & \infty \\ & \infty \\ & \infty \end{aligned}$ | 잉 | $\underset{\mathrm{N}}{\mathrm{~N}}$ | $\begin{aligned} & 8 \\ & 0 \\ & 0 \end{aligned}$ |
| $$ | $\begin{aligned} & \mathrm{N} \\ & \text { Ò } \\ & 0 \end{aligned}$ | O | Ò | O | $\begin{aligned} & \mathrm{N} \\ & \mathrm{O} \\ & 0 \\ & \hline \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & n \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | Ò | $0$ | O | Ò | $\frac{1}{0}$ | $\begin{aligned} & \hline \\ & 0 \\ & 0 \end{aligned}$ | $0$ | $\begin{aligned} & 1 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \text { İ } \\ & \text { Ó } \\ & 0 \end{aligned}$ | $0$ | $0$ | $\begin{aligned} & 2 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & n \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & 9 \\ & 0 \\ & \hline \end{aligned}$ | $\begin{aligned} & 6 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \hat{e} \\ & 0 \\ & 0 \end{aligned}$ | $\overrightarrow{0}$ |
| $n$ | m | n | m | m | $\checkmark$ | m | $\checkmark$ | n | ナ | m | － | n | m | $\cdots$ | $\cdots$ | － | n | $\checkmark$ | $\checkmark$ | n | ナ | $\checkmark$ | m | － | $\checkmark$ |
| $\sim$ | $\checkmark$ | m | $\cdots$ | $\checkmark$ | m | $\cdots$ | m | m | N | $\sim$ | in | $\sim$ | － | － | － | m | n | － | － | m | N | $\cdots$ | － | $\checkmark$ | $\checkmark$ |
| m | m | $\checkmark$ | N | $\checkmark$ | m | － | $\checkmark$ | $n$ | $\checkmark$ | m | N | N | $\checkmark$ | $\checkmark$ | N | m | N | $\checkmark$ | $\sim$ | m | m | m | m | － | $\sim$ |
| － | m | m | － | $\checkmark$ | n | － | $\checkmark$ | N | － | $\cdots$ | － | in | $\cdots$ | － | $\cdots$ | － | n | N | － | N | N | m | $\checkmark$ | m | － |
| $\sim$ | m | － | － | N | m | － | N | m | － | $\sim$ | － | $\sim$ | m | － | N | － | m | － | $\cdots$ | － | m | $\cdots$ | $\checkmark$ | N | － |
| m | m | m | m | m | n | m | m | $\cdots$ | n | $n$ | $\checkmark$ | m | $n$ | m | $\cdots$ | $n$ | m | m | $\checkmark$ | $\checkmark$ | N | m | N | m | $\checkmark$ |
| च | N | $\cdots$ | $\pm$ | $\cdots$ | $\bigcirc$ | へ | $\infty$ | 2 | $\stackrel{\text { ® }}{\text { N }}$ | ন | ヘ | $\cdots$ | $\stackrel{ \pm}{\sim}$ | へ | $\underset{\sim}{\infty}$ | $\underset{N}{N}$ | $\underset{\sim}{\infty}$ | $\underset{\lambda}{2}$ | ¢ | m | N | $\bar{m}$ | － | n | － |




| 89 | 1 | 4 | 3 | 1 | 4 | 1 | 0.083 | 784 | 16 | 0.812 | 0.3 | 178.8 | 3 | 7690 | 7 | 5 | 0.8 | 0.8 | 6.92 | 2.5 | 970 | 700 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 90 | 3 | 4 | 3 | 1 | 2 | 4 | 0.125 | 1003 | 17 | 0.845 | 0.2 | 186 | 3 | 4837 | 9 | 5 | 0.8 | 0.8 | 10.4 | 2.5 | 1090 | 700 |
| 91 | 2 | 5 | 3 | 3 | 3 | 4 | 0.1 | 988 | 19 | 0.801 | 0.3 | 197.3 | 4 | 8483 | 6 | 5 | 0.3 | 0.8 | 8.4 | 2.5 | 990 | 700 |
| 92 | 1 | 4 | 2 | 2 | 1 | 1 | 0.1 | 816 | 16 | 0.817 | 0.3 | 200.4 | 3 | 1803 | 8 | 5 | 2.2 | 0.8 | 8.2 | 2.5 | 1100 | 700 |
| 93 | 3 | 4 | 3 | 3 | 1 | 2 | 0.125 | 855 | 19 | 0.853 | 0.2 | 219.3 | 3 | 3070 | 10 | 5 | 2.9 | 0.8 | 10.3 | 2.5 | 970 | 700 |
| 94 | 1 | 4 | 1 | 3 | 2 | 4 | 0.1 | 901 | 17 | 0.787 | 0.3 | 175.8 | 4 | 2636 | 3 | 5 | 1.3 | 0.8 | 8.4 | 2.5 | 930 | 700 |
| 95 | 2 | 5 | 5 | 2 | 4 | 3 | 0.1 | 874 | 19 | 0.792 | 0.4 | 247.7 | 3 | 7432 | 12 | 5 | 0.5 | 0.8 | 8 | 2.5 | 950 | 700 |
| 96 | 2 | 5 | 2 | 1 | 4 | 3 | 0.083 | 828 | 18 | 0.829 | 0.2 | 192.7 | 3 | 5975 | 4 | 5 | 2.4 | 0.8 | 6.75 | 2.5 | 1230 | 700 |
| 97 | 2 | 4 | 4 | 2 | 4 | 3 | 0.125 | 918 | 18 | 0.837 | 0.3 | 222.5 | 3 | 12017 | 3 | 5 | 2 | 0.8 | 10 | 2.5 | 920 | 700 |
| 98 | 1 | 4 | 4 | 1 | 5 | 2 | 0.083 | 656 | 16 | 0.828 | 0.3 | 177.4 | 3 | 6917 | 8 | 5 | 0.8 | 0.8 | 6.75 | 2.5 | 1100 | 700 |
| 99 | 3 | 4 | 3 | 3 | 3 | 3 | 0.083 | 714 | 17 | 0.838 | 0.2 | 192.1 | 3 | 5955 | 7 | 5 | 1.6 | 0.8 | 7.08 | 2.5 | 980 | 700 |
| 100 | 1 | 5 | 3 | 3 | 3 | 4 | 0.1 | 810 | 18 | 0.886 | 0.2 | 218.9 | 2 | 9852 | 9 | 5 | 2.2 | 0.8 | 8.2 | 2.5 | 1110 | 700 |

Table 1 Dataset for 100 hypothetical bus routes for Societal Variables and their Targets

## Appendix C 1

Results for CRS Infrastructure 1

| DMU | Ph2 <br> Efficiency | zjcomb | Local <br> Deviation | Ph1 <br> Efficiency |
| ---: | ---: | :--- | ---: | ---: |
| 1 | 1.26979 | 3.588429 | 0.613333 | 2.25 |
| 2 | 1.50115 | 3.113795 | 0.4 | 1.741935 |
| 3 | 1.609417 | 2.915917 | 1.6 | 2 |
| 4 | 1.203224 | 2.956255 | 0 | 1.35 |
| 5 | 1 | 3.645598 | 0 | 1.741935 |
| 6 | 1.538165 | 2.397578 | 2.8 | 1.741935 |
| 7 | 2.185144 | 2.258902 | 1 | 2.454545 |
| 8 | 6.770684 | 4.951616 | 0 | 18 |
| 9 | 1.710069 | 2.184651 | 0 | 1.928571 |
| 10 | 1.025515 | 4.683104 | 1.032848 | 4.153846 |
| 11 | 2.090515 | 2.473611 | 0 | 2.335037 |
| 12 | 1.338555 | 1.947461 | 0 | 1.459459 |
| 13 | 5.216044 | 2.257895 | 0 | 6 |
| 14 | 1.928571 | 1.365473 | 0 | 1.928571 |
| 15 | 2.346013 | 2.215973 | 0.972956 | 2.7 |
| 16 | 1 | 3.361839 | 0.598403 | 1.285714 |
| 17 | 2.271554 | 2.434299 | 0 | 2.571429 |
| 18 | 1.422674 | 2.876605 | 0.013333 | 1.588235 |
| 19 | 1.424976 | 4.254262 | 0 | 4.5 |
| 20 | 1.327675 | 3.319465 | 2.397259 | 1.636364 |
| 21 | 2.017244 | 3.932974 | 1.2 | 3.857143 |
| 22 | 1.285714 | 0.421081 | 0.657143 | 1.285714 |
| 23 | 2.149674 | 2.644815 | 0 | 2.347826 |
| 24 | 1 | 5.570326 | 1.264 | 3 |
| 25 | 2.734572 | 3.755564 | 1.864 | 4.909091 |
| 26 | 1.432381 | 4.059193 | 1 | 3.176471 |
| 27 | 1.081472 | 2.653632 | 1 | 1.2 |
| 28 | 1.317073 | 1.105224 | 0 | 1.317073 |
| 29 | 3.792835 | 3.402755 | 2.264098 | 6.75 |
| 30 | 1.531339 | 3.419009 | 0 | 2.842105 |
| 31 | 1.346414 | 1.923951 | 0.331573 | 1.542857 |
| 32 | 1.775468 | 2.715448 | 0.007752 | 2 |
| 33 | 2.640474 | 2.608093 | 0.195363 | 3 |
|  |  |  |  |  |
| 10 |  |  |  |  |


| 34 | 1 | 4.171194 | 1.82185 | 2.16 |
| :---: | :---: | :---: | :---: | :---: |
| 35 | 1.08 | 0.436329 | 0 | 1.08 |
| 36 | 1.390499 | 2.408631 | 0 | 1.542857 |
| 37 | 2.55 | 3.411018 | 0 | 2.7 |
| 38 | 1.034037 | 4.272119 | 3.282857 | 2.571429 |
| 39 | 12.30867 | 3.134243 | 2.490042 | 13.5 |
| 40 | 1.026781 | 2.523797 | 0 | 1.125 |
| 41 | 1.063787 | 2.021711 | 0 | 1.148936 |
| 42 | 2.027587 | 3.948221 | 2.547455 | 3.6 |
| 43 | 3.322197 | 3.626734 | 2.348571 | 6 |
| 44 | 1 | 5.258885 | 3.335238 | 2.347826 |
| 45 | 1 | 3.961685 | 0 | 1.741935 |
| 46 | 1.875122 | 2.576772 | 0 | 2.076923 |
| 47 | 1.5 | 0.844523 | 0 | 1.5 |
| 48 | 2.810514 | 4.621311 | 2.761508 | 6.75 |
| 49 | 4.646297 | 3.207939 | 3.723433 | 5.4 |
| 50 | 1 | 3.560748 | 2.586667 | 1.058824 |
| 51 | 1.421053 | 1.255507 | 0 | 1.421053 |
| 52 | 1.470868 | 3.203293 | 0 | 2.076923 |
| 53 | 1.246534 | 3.477458 | 0 | 1.636364 |
| 54 | 1 | 4.406602 | 0.354857 | 2.454545 |
| 55 | 4.636364 | 2.008247 | 2.629524 | 4.909091 |
| 56 | 3.002291 | 2.091537 | 0 | 3.375 |
| 57 | 2.636675 | 2.515985 | 5.985796 | 3 |
| 58 | 1.081076 | 2.263548 | 0.173655 | 1.2 |
| 59 | 1.471964 | 3.408408 | 0.005133 | 2.25 |
| 60 | 1.384615 | 1.505155 | 0.418301 | 1.384615 |
| 61 | 2.517058 | 2.407624 | 0 | 2.842105 |
| 62 | 1.39237 | 2.976149 | 0 | 1.588235 |
| 63 | 1.459459 | 0.752415 | 0 | 1.459459 |
| 64 | 4.909091 | 1.430905 | 0 | 4.909091 |
| 65 | 1.640462 | 2.240038 | 0 | 1.741935 |
| 66 | 2.408477 | 2.00463 | 0 | 2.7 |
| 67 | 1 | 3.784275 | 0 | 1.8 |
| 68 | 2.117537 | 2.054816 | 3.338915 | 2.347826 |
| 69 | 2.029276 | 1.673748 | 0 | 2.347826 |
| 70 | 1.193275 | 3.660846 | 8.383725 | 2.746676 |
| 71 | 1.384615 | 1.567977 | 0 | 1.384615 |
| 72 | 1 | 4.153337 | 2.8 | 2.063781 |


|  | 2.004941 | 0.591258 | 5.796077 | 2.004941 |
| ---: | ---: | ---: | ---: | ---: |
| 74 | 2.45279 | 2.241266 | 0 | 2.760901 |
| 75 | 3.442581 | 2.991396 | 2.223851 | 4.909091 |
| 76 | 1.64241 | 3.173001 | 4.288511 | 3.18593 |
| 77 | 9.241386 | 1.712054 | 2.440415 | 10.55127 |
| 78 | 1 | 3.711031 | 0 | 1.862069 |
| 79 | 1 | 2.832072 | 10.38116 | 1.136857 |
| 80 | 1.35 | 1.078549 | 0 | 1.35 |
| 81 | 2.735526 | 2.799744 | 2.564131 | 3.176471 |
| 82 | 1.893058 | 2.389767 | 0 | 2.16 |
| 83 | 1.402876 | 1.101607 | 4.078656 | 1.402876 |
| 84 | 1.005349 | 1.493727 | 0 | 1.005349 |
| 85 | 1.428612 | 3.492705 | 5.240041 | 3.024313 |
| 86 | 1.048124 | 3.157754 | 5.665901 | 2.303207 |
| 87 | 4.839073 | 3.226803 | 1.694638 | 7.714286 |
| 88 | 2.518325 | 2.573155 | 3.090951 | 2.617764 |
| 89 | 1.255814 | 0.567194 | 0 | 1.255814 |
| 90 | 1.766785 | 2.642204 | 3.1034 | 1.962223 |
| 91 | 1 | 3.115831 | 0 | 1.255814 |
| 92 | 3.653722 | 2.907099 | 0.891634 | 6 |
| 93 | 1 | 4.371663 | 7.155021 | 3.574881 |
| 94 | 3.152662 | 2.604476 | 0.757698 | 3.6 |
| 95 | 1.544146 | 2.273594 | 0 | 1.705364 |
| 96 | 1.738635 | 1.209968 | 0 | 1.738635 |
| 97 | 1 | 1.654884 | 0 | 1 |
| 98 | 1.384615 | 0.082966 | 0 | 1.384615 |
| 99 | 1.029776 | 3.050399 | 3.603924 | 1.592844 |
| 100 | 1 | 2.738958 | 5.395611 | 1 |
|  |  |  |  |  |

## Appendix C 2

Results for VRS Infrastructure 1

| DMU | Ph2_Efficiency | zjcomb | Local <br> Deviation | Ph1_Efficiency |
| ---: | ---: | :--- | ---: | ---: |
| 1 | 1 | 4.478637 | 2.264 | 1.676368 |
| 2 | 1.381097 | 2.620492 | 0.8 | 1.417465 |
| 3 | 1.584996 | 2.405777 | 3.2 | 1.615587 |
| 4 | 1.196687 | 2.375896 | 0 | 1.200138 |
| 5 | 1 | 2.931079 | 0 | 1.433711 |
| 6 | 1.43729 | 2.324905 | 5.6 | 1.483374 |
| 7 | 1.826378 | 1.955795 | 2 | 1.837915 |
| 8 | 2.623724 | 3.912794 | 4.161508 | 14.49512 |
| 9 | 1.410957 | 1.75353 | 0 | 1.410957 |
| 10 | 1.327755 | 3.663078 | 0 | 3.38206 |
| 11 | 1.680287 | 2.130401 | 0 | 1.680287 |
| 12 | 1.305885 | 1.660897 | 0 | 1.305885 |
| 13 | 4.873115 | 2.132401 | 0.153843 | 4.975106 |
| 14 | 1.497879 | 1.193017 | 0 | 1.497879 |
| 15 | 2.201191 | 2.035337 | 1.372956 | 2.26011 |
| 16 | 1 | 2.723693 | 0.598403 |  |
| 17 | 2.133401 | 2.252484 | 1.181721 | 2.239058 |
| 18 | 1.326101 | 2.527859 | 0.013333 | 1.326101 |
| 19 | 1 | 3.41384 | 2.40369 | 3.274463 |
| 20 | 1 | 2.955033 | 2.797259 | 1.376445 |
| 21 | 1.324228 | 3.143794 | 2 | 3.142561 |
| 22 | 1.108934 | 0.493109 | 1.257143 | 1.108934 |
| 23 | 1.56197 | 2.080998 | 0 | 1.56197 |
| 24 | 1 | 4.183739 | 5.335238 | 2.338846 |
| 25 | 1 | 3.196693 | 3.4 | 4.411313 |
| 26 | 1.519644 | 3.057593 | 2.095142 | 2.548909 |
| 27 | 1.03237 | 2.293024 | 2 | 1.051488 |
| 28 | 1.247085 | 0.878806 | 0.610422 | 1.247085 |
| 29 | 2.514909 | 2.997364 | 3.864098 | 5.478994 |
| 30 | 2.165946 | 2.653703 | 0 | 2.311963 |
| 31 | 1.279011 | 1.767722 | 2.416418 | 1.323468 |
| 32 | 1.483604 | 2.240722 | 0 | 1.579707 |
| 33 | 1.853778 | 2.153419 | 0.449973 | 2.149124 |
| 34 | 1 | 3.243666 | 1.661905 | 2.106788 |
|  |  |  |  |  |
|  |  |  |  | 0 |


| 35 | 1.050663 | 0.326054 | 4.083774 | 1.050663 |
| :---: | :---: | :---: | :---: | :---: |
| 36 | 1.379471 | 1.811759 | 0.351707 | 1.442328 |
| 37 | 2.712214 | 3.055593 | 2.348571 | 2.591244 |
| 38 | 1 | 3.708738 | 1.632848 | 2.469349 |
| 39 | 10.17847 | 2.622923 | 2.690042 | 13.20277 |
| 40 | 1 | 2.287694 | 0.406207 | 1 |
| 41 | 1.06369 | 1.863162 | 0.246273 | 1.094423 |
| 42 | 1 | 3.00107 | 1.213333 | 3.52022 |
| 43 | 1.869359 | 3.01395 | 2.547455 | 5.15366 |
| 44 | 5.690376 | 4.083179 | 0 | 2.241236 |
| 45 | 2.424541 | 3.237529 | 3.464 | 1.661672 |
| 46 | 1.870865 | 1.871612 | 1.325493 | 1.968585 |
| 47 | 1.30731 | 0.892998 | 0.333714 | 1.30731 |
| 48 | 1 | 3.738188 | 0.354857 | 6.29448 |
| 49 | 4.403666 | 2.673391 | 3.723433 | 4.870251 |
| 50 | 1 | 2.913557 | 3.986667 | 1 |
| 51 | 1.141176 | 0.886567 | 0 | 1.141176 |
| 52 | 1.457518 | 2.655703 | 2.105112 | 1.693065 |
| 53 | 1.253252 | 2.871226 | 0 | 1.518719 |
| 54 | 1 | 3.71517 | 4.282857 | 2.122681 |
| 55 | 4.609959 | 1.633447 | 4.029524 | 4.660609 |
| 56 | 2.843346 | 1.675778 | 0 | 2.883631 |
| 57 | 2.518402 | 1.899062 | 0.561841 | 2.595662 |
| 58 | 1.054375 | 1.973483 | 0.114605 | 1.075281 |
| 59 | 1.427298 | 2.838446 | 3.553507 | 1.970637 |
| 60 | 1.284426 | 1.385521 | 0.15434 | 1.322818 |
| 61 | 2.516798 | 1.988365 | 0 | 2.732346 |
| 62 | 1.393872 | 2.22653 | 0 | 1.482617 |
| 63 | 1.414362 | 0.638641 | 0 | 1.414362 |
| 64 | 4.656714 | 1.183255 | 0.891429 | 4.656714 |
| 65 | 1.621172 | 2.080309 | 0 | 1.667189 |
| 66 | 2.329589 | 1.590906 | 0.22716 | 2.385546 |
| 67 | 1 | 3.248786 | 2.62185 | 1.655145 |
| 68 | 2.116648 | 1.7482 | 5.338915 | 2.282642 |
| 69 | 1.980616 | 1.116971 | 0.312766 | 2.072901 |
| 70 | 1 | 2.764024 | 2.534661 | 1 |
| 71 | 1.382538 | 1.156613 | 0 | 1.382538 |
| 72 | 1.624742 | 3.181005 | 2.4 | 1 |
| 73 | 1.549654 | 0.351503 | 0 | 1.549654 |


| 74 | 2.430059 | 1.531742 | 0 | 2.752778 |
| :---: | :---: | :---: | :---: | :---: |
| 75 | 1 | 2.059475 | 1.796891 | 1 |
| 76 | 1.660507 | 2.349043 | 2.382362 | 3.05235 |
| 77 | 5.932308 | 1.171494 | 0 | 6.830637 |
| 78 | 1 | 2.921318 | 2.998512 | 1.710606 |
| 79 | 1 | 2.211648 | 10.38116 | 1 |
| 80 | 1 | 0.614687 | 0.534747 | 1 |
| 81 | 1 | 2.106447 | 2.564131 | 1 |
| 82 | 1.842196 | 1.936273 | 6.054054 | 1.861386 |
| 83 | 1.19571 | 0.836265 | 0.256555 | 1.207666 |
| 84 | 1 | 0.954347 | 0 | 1 |
| 85 | 1 | 2.704171 | 6.240041 | 1 |
| 86 | 1.041387 | 2.516098 | 0.619844 | 2.303207 |
| 87 | 1 | 2.548878 | 1.384706 | 1 |
| 88 | 2.518256 | 1.829071 | 0.385275 | 2.594073 |
| 89 | 1 | 0.306531 | 0.534747 | 1 |
| 90 | 1.348366 | 1.861851 | 0.247171 | 1.419242 |
| 91 | 1 | 2.419034 | 0.509317 | 1.165457 |
| 92 | 3.610265 | 2.19375 | 0.533718 | 5.391087 |
| 93 | 1 | 3.300189 | 1.967429 | 3.561623 |
| 94 | 2.960357 | 2.110878 | 0 | 3.00348 |
| 95 | 1.539303 | 1.636943 | 0 | 1.692449 |
| 96 | 1.331057 | 0.746962 | 0 | 1.468683 |
| 97 | 1 | 1.241485 | 0.448542 | 1 |
| 98 | 1.048566 | -0.06591 | 1.531913 | 1.048566 |
| 99 | 1 | 2.428795 | 0.053552 | 1.33092 |
| 100 | 1 | 2.133897 | 6.195611 | 1 |

## Appendix C 3

Results for CRS Infrastructure 2

| DMU | Ph2_Efficiency | zjcomb | Local <br> Deviation | Ph1_Efficiency |
| ---: | ---: | :--- | ---: | ---: |
| 1 | 2.007332 | 3.032547 | 0 | 2.25 |
| 2 | 1.234275 | 3.523395 | 0.4 | 1.741935 |
| 3 | 1.091514 | 3.549617 | 1.6 | 2 |
| 4 | 1.235081 | 3.227719 | 0 | 1.35 |
| 5 | 1 | 4.046059 | 0 | 1.741935 |
| 6 | 1.538165 | 2.572971 | 2.8 | 1.741935 |
| 7 | 2.185144 | 2.428276 | 0 | 2.454545 |
| 8 | 6.770684 | 4.588256 | 0 | 18 |
| 9 | 1.710069 | 2.584323 | 0 | 1.928571 |
| 10 | 1.025515 | 4.950787 | 1.032848 | 4.153846 |
| 11 | 2.090515 | 3.140239 | 0 | 2.335037 |
| 12 | 1.338555 | 1.839728 | 0 | 1.459459 |
| 13 | 5.216044 | 2.818602 | 0 | 6 |
| 14 | 1.928571 | 1.37737 | 0 | 1.928571 |
| 15 | 2.346013 | 2.074423 | 0 | 2.7 |
| 16 | 1.105273 | 3.243419 | 0 | 1.285714 |
| 17 | 2.271554 | 2.369423 | 0 | 2.571429 |
| 18 | 1.456994 | 3.077413 | 0.013333 | 1.588235 |
| 19 | 2.532745 | 3.788814 | 0 | 4.5 |
| 20 | 1.346252 | 3.328977 | 2.397259 | 1.636364 |
| 21 | 2.732658 | 3.535898 | 0.246316 | 3.857143 |
| 22 | 1.285714 | 0.467578 | 0 | 1.285714 |
| 23 | 2.149674 | 3.236396 | 0 | 2.347826 |
| 24 |  | 1 | 5.14568 | 1.264 |


| 35 | 1.08 | 0.73691 | 0 | 1.08 |
| :---: | :---: | :---: | :---: | :---: |
| 36 | 1.390499 | 2.400088 | 0 | 1.542857 |
| 37 | 2.55 | 2.976215 | 0 | 2.7 |
| 38 | 1.032519 | 4.330122 | 0.112284 | 2.571429 |
| 39 | 11.59822 | 3.546004 | 0 | 13.5 |
| 40 | 1.026781 | 2.184097 | 0 | 1.125 |
| 41 | 1.148936 | 1.683682 | 0 | 1.148936 |
| 42 | 2.972054 | 3.506618 | 0 | 3.6 |
| 43 | 4.028539 | 3.596464 | 0 | 6 |
| 44 | 1 | 4.855744 | 3.335238 | 2.347826 |
| 45 | 1.591877 | 3.351248 | 0 | 1.741935 |
| 46 | 1.875122 | 2.755393 | 0 | 2.076923 |
| 47 | 1.5 | 1.024586 | 0 | 1.5 |
| 48 | 2.810514 | 4.445951 | 2.761508 | 6.75 |
| 49 | 3.915939 | 3.547772 | 0 | 5.4 |
| 50 | 1 | 3.843863 | 0.476452 | 1.058824 |
| 51 | 1.421053 | 1.048025 | 0 | 1.421053 |
| 52 | 1.661657 | 3.338069 | 0 | 2.076923 |
| 53 | 1.487186 | 2.794916 | 0 | 1.636364 |
| 54 | 1 | 4.331213 | 0.354857 | 2.454545 |
| 55 | 4.636364 | 1.839052 | 0 | 4.909091 |
| 56 | 3.002291 | 2.589387 | 0 | 3.375 |
| 57 | 2.636675 | 2.756069 | 0 | 3 |
| 58 | 1.081076 | 2.040755 | 0 | 1.2 |
| 59 | 1.896117 | 3.301464 | 0 | 2.25 |
| 60 | 1.384615 | 1.131738 | 0 | 1.384615 |
| 61 | 2.517058 | 2.193188 | 0 | 2.842105 |
| 62 | 1.44014 | 2.810917 | 0 | 1.588235 |
| 63 | 1.459459 | 0.937936 | 0 | 1.459459 |
| 64 | 4.909091 | 1.586397 | 0 | 4.909091 |
| 65 | 1.640462 | 1.978682 | 0 | 1.741935 |
| 66 | 2.408477 | 2.554627 | 0 | 2.7 |
| 67 | 1.426856 | 3.593528 | 0 | 1.8 |
| 68 | 2.117537 | 1.897097 | 0 | 2.347826 |
| 69 | 2.347826 | 1.254531 | 0 | 2.347826 |
| 70 | 1.193275 | 3.718166 | 0 | 2.746676 |
| 71 | 1.384615 | 1.367402 | 0 | 1.384615 |
| 72 | 1 | 3.768625 | 0 | 2.063781 |
| 73 | 2.004941 | 1.059223 | 0 | 2.004941 |


| 74 | 2.45279 | 2.262585 | 0 | 2.760901 |
| ---: | ---: | ---: | ---: | ---: |
| 75 | 3.442581 | 3.378861 | 0 | 4.909091 |
| 76 | 2.883662 | 2.981572 | 0 | 3.18593 |
| 77 | 10.00895 | 1.818448 | 0 | 10.55127 |
| 78 | 1.198359 | 3.657861 | 0 | 1.862069 |
| 79 | 1.002939 | 2.061258 | 0 | 1.136857 |
| 80 | 1.35 | 1.057793 | 0 | 1.35 |
| 81 | 2.742678 | 2.364258 | 0 | 3.176471 |
| 82 | 1.893058 | 2.547718 | 0 | 2.16 |
| 83 | 1.402876 | 1.352378 | 0 | 1.402876 |
| 84 | 1 | 1.822061 | 0 | 1.005349 |
| 85 | 2.845459 | 2.765635 | 0 | 3.024313 |
| 86 | 1.048124 | 3.608077 | 0 | 2.303207 |
| 87 | 5.565735 | 3.400141 | 0 | 7.714286 |
| 88 | 2.518325 | 2.57513 | 0 | 2.617764 |
| 89 | 1.255814 | 0.259126 | 0 | 1.255814 |
| 90 | 1.766785 | 2.96442 | 0 | 1.962223 |
| 91 | 1 | 3.461123 | 0 | 1.255814 |
| 92 | 5.189601 | 2.72024 | 0 | 6 |
| 93 | 1 | 4.063626 | 0 | 3.574881 |
| 94 | 3.152662 | 3.259681 | 0 | 3.6 |
| 95 | 1.544146 | 2.258197 | 0 | 1.705364 |
| 96 | 1.738635 | 1.318033 | 0 | 1.738635 |
| 97 | 1 | 1.700774 | 0 | 1 |
| 98 | 1.384615 | 0.001407 | 0 | 1.384615 |
| 99 | 1.473589 | 2.953483 | 0 | 1.592844 |
| 100 | 1 | 3.260772 | 0 | 1 |
|  |  |  |  |  |

## Appendix C 4

## Results for VRS Infrastructure 2

| DMU | Ph2_Efficiency | zjcomb | Local <br> Deviation | Ph1_Efficiency |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 1.676368 | 2.561329 | 0.06854 | 5.15366 |
| 2 | 1.151315 | 2.944513 | 0.4 | 1.376445 |
|  | 1.051204 | 2.907076 | 0.4 | 1.108934 |
| 4 | 1.196687 | 2.590642 | 0.4 | 1.094423 |
| 5 | 1 | 3.24787 | 0 | 4.975106 |
| 6 | 1.456074 | 2.463653 | 2.8 | 1.968585 |
| 7 | 1.800125 | 2.089782 | 0.242468 | 1.075281 |
| 8 | 5.690376 | 3.795737 | 0 | 1.200138 |
| 9 | 1.410957 | 2.069696 | 0.214196 | 1.322818 |
| 10 | 1 | 3.920493 | 1.032848 | 1.417465 |
| 11 | 1.036289 | 2.657748 | 1.6 | 2.591244 |
| 12 | 1.305885 | 1.575673 | 2.6257 | 1.710606 |
| 13 | 4.790795 | 2.575958 | 0 | 3.52022 |
| 14 | 1.497879 | 1.202428 | 0.251894 | 1 |
| 15 | 2.201191 | 1.923362 | 0.674087 | 1.655145 |
| 16 | 1 | 2.630015 | 0 | 2.469349 |
| 17 | 2.172395 | 2.201162 | 0.760374 | 1.693065 |
| 18 | 1.237486 | 2.686712 | 0.715243 | 2.106788 |
| 19 | 1.327755 | 3.294877 | 1.094742 | 1.305885 |
| 20 | 1 | 2.962557 | 0 | 1.326101 |
| 21 | 2.62522 | 2.866891 | 0.71 | 2.548909 |
| 22 | 1.108934 | 0.52989 | 2.119951 | 1.048566 |
| 23 | 1.56197 | 2.548978 | 0 | 2.241236 |
| 24 | 1 | 4.142714 | 1.264 | 1.676368 |
| 25 | 1.856362 | 3.727864 | 1.864 | 1.837915 |
| 26 | 1.180961 | 3.367097 | 1 | 1.410957 |
| 27 | 1.039886 | 2.229009 | 0.8032 | 1.141176 |
| 28 | 1.247085 | 0.744476 | 0.59617 | 3.561623 |
| 29 | 1.752828 | 3.296343 | 2.264098 | 1.680287 |
| 30 | 1.452666 | 2.84411 | 0.980318 | 1.051488 |
| 31 | 1.27668 | 2.124438 | 0.300838 | 4.660609 |
| 32 | 1.450958 | 2.623007 | 0.129433 | 1 |
| 33 | 2.091771 | 2.339023 | 0.277742 | 4.870251 |
| 34 | 1 | 3.358538 | 0.455268 | 3.38206 |
| 35 | 1.050663 | 0.563834 | 0.627758 | 1 |
| 36 | 1.379471 | 1.805 | 0.415818 | 1.382538 |
| 37 | 2.315754 | 2.713634 | 0.118352 | 1.579707 |
| 38 | 1 | 3.761054 | 3.282857 | 1.483374 |
| 39 | 7.328266 | 2.948654 | 2.264098 | 3.274463 |
| 40 | 1 | 2.018969 | 0 | 1.667189 |
| 41 | 1.06369 | 1.595758 | 0.280774 | 6.830637 |
| 42 | 3.145074 | 2.664612 | 0.603699 | 1.442328 |


| 43 | 3.682306 | 3.031647 | 0.77393 | 2.239058 |
| :---: | :---: | :---: | :---: | :---: |
| 44 | 1 | 3.864827 | 1.06684 | 1.615587 |
| 45 | 1.493908 | 2.760768 | 0.829842 | 2.311963 |
| 46 | 1.870865 | 2.012913 | 1.92023 | 2.385546 |
| 47 | 1.30731 | 1.03544 | 0.298653 | 1 |
| 48 | 2.623724 | 3.774073 | 1.751492 | 1.433711 |
| 49 | 3.720763 | 2.942222 | 0.96456 | 3.142561 |
| 50 | 1 | 3.13752 | 0 | 1 |
| 51 | 1.141176 | 0.722434 | 0.298653 | 3.00348 |
| 52 | 1.012376 | 2.76232 | 0.134344 | 5.478994 |
| 53 | 1.469589 | 2.331289 | 0.17199 | 1 |
| 54 | 1 | 3.678551 | 0.225524 | 14.49512 |
| 55 | 4.609959 | 1.499602 | 0.763599 | 1 |
| 56 | 2.813548 | 2.069611 | 0.287823 | 2.732346 |
| 57 | 2.512511 | 2.088984 | 0.524655 | 1.970637 |
| 58 | 1.054375 | 1.797239 | 0.415818 | 1 |
| 59 | 1.427298 | 2.753846 | 0.134344 | 1.323468 |
| 60 | 1.322818 | 1.090123 | 0.430424 | 2.303207 |
| 61 | 2.509339 | 1.818732 | 0.299271 | 1 |
| 62 | 1.426613 | 2.09582 | 0.287823 | 2.595662 |
| 63 | 1.414362 | 0.7854 | 0.534747 | 1.165457 |
| 64 | 4.533459 | 1.30626 | 1.9 | 1 |
| 65 | 1.602517 | 1.873559 | 1.428848 | 2.282642 |
| 66 | 2.321641 | 2.02599 | 0.468922 | 4.656714 |
| 67 | 1 | 3.149295 | 0 | 2.26011 |
| 68 | 2.091066 | 1.623433 | 0.646722 | 1 |
| 69 | 1.980616 | 0.785342 | 0.415818 | 5.391087 |
| 70 | 1 | 2.809368 | 0 | 1.247085 |
| 71 | 1.382538 | 0.997944 | 0.415818 | 1 |
| 72 | 1 | 2.892361 | 0 | 1.56197 |
| 73 | 1.549654 | 0.721695 | 0 | 1.692449 |
| 74 | 2.430059 | 1.548606 | 0.350857 | 1 |
| 75 | 1 | 2.365986 | 0 | 1.30731 |
| 76 | 2.835802 | 2.19761 | 1.215849 | 1.518719 |
| 77 | 5.932308 | 1.255659 | 0.415818 | 1.207666 |
| 78 | 1 | 2.879256 | 0 | 2.338846 |
| 79 | 1 | 1.601882 | 0 | 3.05235 |
| 80 | 1 | 0.598268 | 0 | 1.468683 |
| 81 | 1 | 1.761948 | 0 | 1.549654 |
| 82 | 1 | 2.061223 | 0 | 1.482617 |
| 83 | 1.19571 | 1.034642 | 0.222248 | 2.594073 |
| 84 | 1 | 1.214081 | 0 | 1 |
| 85 | 1 | 2.12901 | 0 | 2.122681 |
| 86 | 1.041387 | 2.872334 | 0.134344 | 4.411313 |
| 87 | 1 | 2.686 | 0 | 1.050663 |
| 88 | 2.516278 | 1.830634 | 0.415818 | 2.072901 |
| 89 | 1 | 0.06283 | 0 | 1.33092 |
| 90 | 1 | 2.116745 | 0 | 2.883631 |


| 91 | 1 | 2.692182 | 0 | 2.149124 |
| ---: | ---: | ---: | ---: | ---: |
| 92 | 4.250542 | 2.045932 | 0.572164 | 1.414362 |
| 93 | 1 | 3.170162 | 0 | 1.497879 |
| 94 | 1.220591 | 2.629189 | 0.954323 | 13.20277 |
| 95 | 1.539303 | 1.624763 | 0.415818 | 2.752778 |
| 96 | 1.331057 | 0.832449 | 0.265847 | 1.419242 |
| 97 | 1 | 1.277786 | 0 | 1.861386 |
| 98 | 1.048566 | -0.13043 | 0.616952 | 1 |
| 99 | 1 | 2.352128 | 0 | 6.29448 |
| 100 | 1 | 2.546686 | 0 | 1.661672 |

## Vita

## Chintan H. Sheth

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The Washington Post, Springfield, Virginia - Summer Intern
Virginia Tech, Virginia - Graduate Research Assistant - Systems Performance Laboratory,

American Red Cross, Fairfax, Virginia - Presidential Intern
Crompton Greaves Limited, Bombay, India - Research Associate EngineerTrainee

Jyoti Engineers, Bombay, India - Engineer-Trainee


[^0]:    ${ }^{1}$ Externality variables: Variables affecting the society (Discussed in the Document)

[^1]:    ${ }^{2}$ Mixed transit agency is a combination of a private and public transit agency.
    ${ }^{3}$ Provider-user perspective: Addressing issues from the point view of the provider of transit services, i.e., the government or a private transit agency as well as from the point of view of the passengers, who consume the service.

[^2]:    ${ }^{4}$ Frontier reference: the frontier represents the best practice which forms the reference scale and as DEA is based on the concept of benchmarking, hence frontier reference technology

[^3]:    ${ }^{5}$ Congestion: This is with respect to DEA; implies availability of inputs (outputs) corresponding to outputs (inputs)

[^4]:    ${ }^{6}$ System efficiency is defined as the relationship between resource inputs and produced outputs. It includes factors of overall cost efficiency, labor utilization and vehicle utilization.
    ${ }^{7}$ System effectiveness is defined as the ability of transit organization to meet certain goals, which includes service utilization, service quality and accessibility.

[^5]:    ${ }^{8}$ Global Target: The targets associated with the whole network are represented as Global Targets.

[^6]:    ' Setting up Solver parameters
    SolverOk SetCell:="\$k\$7", MaxMinVal:=2, ValueOf:="0", ByChange:="\$C\$8:\$D\$" \& i \& ",\$p\$24:\$q\$27"

    SolverSolve (True)
    ' Storing of Slacks and Peers
    Sheets("Local").Select
    Sheets("Local").Range("K7").Copy
    Sheets("Peers_2_2").Select
    ActiveCell.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks
    :=False, Transpose:=False
    ActiveCell.Offset( $(0,1)$.Select
    Sheets("Local").Select
    Sheets("Local").Range("C8:C107").Copy
    Sheets("Peers_2_2").Select
    ActiveCell.PasteSpecial Paste:=xlPasteAll, Operation:=xlNone, SkipBlanks:=
    False, Transpose:=True
    ActiveCell.Offset(103, 0).Select
    Sheets("Local").Select
    Range("D8:D107").Copy

