A Game Theoretic-based Transactive Energy Framework for Distributed Energy Resources

Bilal Ahmad Bhatti

Dissertation submitted to the faculty of the Virginia Polytechnic Institute and State University in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

In

Electrical Engineering

Robert P. Broadwater, Chair

Virgilio A. Centeno

Vassilis Kekatos

A. Lynn Abbott

Christopher Beattie

December 9, 2020

Blacksburg, VA

Keywords: Transactive energy, Co-simulation, Game Theory, Microgrids, Distributed Energy Resources

A Game Theoretic-based Transactive Energy Framework for Distributed Energy Resources

Bilal Ahmad Bhatti

ABSTRACT (academic)

Power systems have evolved significantly during the last two decades with the advent of Distributed Energy Resources (DERs) like solar PV. Traditionally, large power plants were considered as the sole source of energy in the power systems. However, DERs connected to the transmission and the distribution systems are creating a paradigm shift from a centralized generation to a distributed one. Though the variable power output from these DERs poses challenges to the reliable operation of the grid, it also presents opportunities to design control and coordination approaches to improve system efficiency and operational reliability. Moreover, building new transmission lines to meet ever-increasing load demand is not always viable. Thus, the industry is leaning towards developing non-wires alternatives. Considering the existing limitations of the transmission system, line congestions, and logistic/economic constraints associated with its capacity expansion, leveraging DERs to supply distribution system loads is attractive and thus capturing the attention of researchers and the electric power industry.

The primary objective of this dissertation is to develop a framework that enables DERs to supply local area load by co-simulating the power system and transactive system representations of the network. To realize this objective, a novel distributed optimization and game theory-based network representation is developed that optimally computes the power output of the Home Microgrids/DER aggregators. Moreover, the optimum operational schedules of the DERs within these Home Microgrids/DER aggregators are also computed. The novel electrical-transactive co-simulation ensures that the solution is optimum in the context of power systems i.e. power flow constraints are not violated while the payoffs are maximized for the Home Microgrids/DER aggregators. The transactive mechanism involves two-way iterative signaling. The signaling is modeled as an infinite

strategy, multiplayer, non-cooperative game, and a novel theory is developed for the game model.

The dissertation also introduces a novel concept of ranking the Home Microgrids/DER aggregators according to their historic performance, thus leading to fairness, higher participation, and transparency. Significant advantages offered by the framework include consumption of local generation, transmission upgrade deferral, mitigation of line congestions in peak periods, and reduced transmission systems losses.

A Game Theoretic-based Transactive Energy Framework for Distributed Energy Resources

Bilal Ahmad Bhatti

GENERAL AUDIENCE ABSTRACT

In past, electricity was primarily produced by the large fossil fuel-based and nuclear power plants, usually located farther away from the populated areas where the bulk of the electricity consumption occurs. The electricity from the power plants is carried by the transmission lines to the populated areas where it is distributed to end-users via a distribution network. However, during the last two decades, issues like global warming and depleting fossil fuels have led to the development and increased adoption of renewable energy resources like solar photovoltaics (PV), wind turbines, etc. These resources are commonly known as Distributed Energy Resources (DERs), and they are connected to both the transmission and the distribution systems. Initially, they were mainly used to supply the load within the facility in which they are installed. However, the electric load (demand) continues to grow while adding new fossil fuel-based plants and transmission lines are becoming logistically/economically challenging. Thus, researchers are working on developing techniques that can enable DERs to supply the loads in the distribution system to which they are connected.

This dissertation develops a method to use DERs for load support in the distribution systems. Specifically, the buildings that house the DERs can use the energy generated by the DERs to supply the local load (building load), and once the total generation exceeds the load demand, the building can inject the power into the distribution system to support the local area load. The proposed framework considers the electric network constraints like limits of lines supplying the power and limits of the transformers. The proposed work also develops a new method to maximize the benefit (in terms of profit) for the DER owners. A ranking system is introduced for the DER owners that enhances the transparency and fairness of the process.

The key benefits offered by the proposed work include reduced losses in the transmission system, more energy consumed closer to the point of generation, and avoidance of transmission line and large central generation additions.

DEDICATION

This dissertation is dedicated to my parents, Ali Muhammad Bhatti and Nabila Tabassum. Without their constant support, untiring efforts, and continuous encouragement, I could not have achieved this feat. They always backed me to reach higher grounds and I cannot thank them enough for what they have done for me throughout my life. I would also like to thank my younger sister Mahnoor Ali for all her love and support. She along with my brother Zubair Ahmad Bhatti has been a source of constant motivation over the years.

ACKNOWLEDGEMENT

Words are not enough to acknowledge the continuous support, encouragement, and guidance from my Ph.D. advisor Dr. Robert Broadwater. He has been a constant source of motivation in this long journey and aside from his professional expertise, he is really fun to work with. He keeps the learning process enjoyable and I had a great time at his lab. I would also like to thank Dr. Virgilio Centeno, Dr. Vassilis Kekatos, Dr. Lynn Abbott, and Dr. Christopher Beattie for serving on my Ph.D. committee. They always extended their full support and guidance in different matters throughout my Ph.D.

I would also like to acknowledge the role of my parents and siblings in keeping me motivated and constantly encouraging me to reach the finish line.

Table of Contents

List of Figures

List of Tables

List of Abbreviations

DER	Distributed Energy Resource
DSO	Distribution System Operator
DG	Diesel Generator
DGU	Dispatchable Generation Unit
ES	Energy Storage
ES+/ ES-	Energy Storage during discharging/charging mode
GT	Gas Turbine
HEMS	Home Energy Management System
HMG	Home Micro-Grid
CA	Coordinating Authority
PRI	Player Reputation Index
NDU	Non-Dispatchable Unit
PV	Photo-Voltaic
RTOI	Real Time Operating Interval
SOC	State of Charge
UTR	Utilities or Retailers
WT	Wind Turbine
NIRA	Nikaido-Isoda and Relaxation Algorithms

Chapter 1: Introduction

1.1.Electricity Demand, Renewable Adoption, and Associated Challenges

Electricity demand is steadily growing, driven by increasing population, economic growth, and rising urbanization. The U.S. Energy Information Administration (EIA) projects an annual growth rate of 1 % in U.S. electricity demand [1] through 2050 as shown in Figure 1. On a global scale, the International Energy Agency (IEA) projects an annual electricity demand growth rate of 2.1 % to 2040 [2]. It projects a strong demand increase in developing economies due to industrialization, urbanization, and rising incomes, as shown in Figure 2. In advanced economies, increasing demand is offset to a certain extent by government incentives to adopt green energy and energy efficiency improvements. Thus, per capita electricity consumption is increasing every year since global electricity demand continues to increase faster than the world population [3].

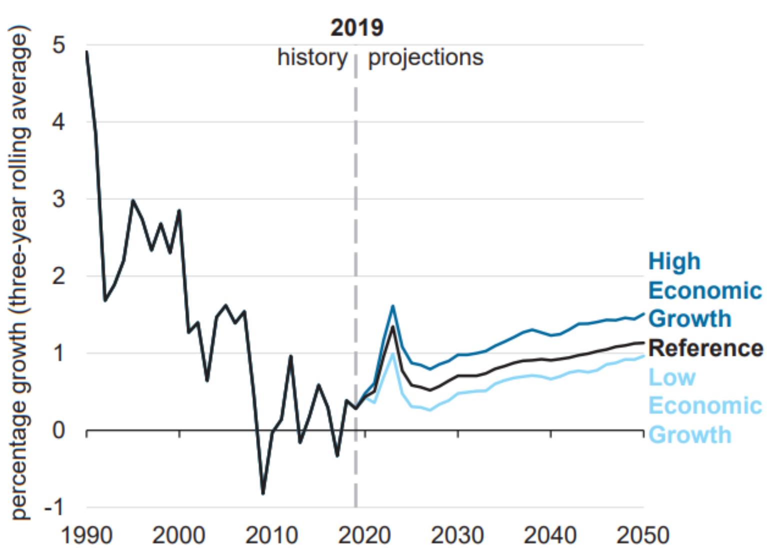

Figure 1. U.S. Annual Electricity demand growth rate projected till 2050 [1]

Traditionally, increasing electricity demand has been addressed by building new generation capacity. However, building new power plants to meet peak demand is highly inefficient. This is highly relevant in the scenario when peak demand increases faster than the base demand. In such a scenario, a bulk of generation capacity remains idle during non-peak hours of a day. Building new generation facilities also requires expansion of the existing transmission capacity. However, building new transmission lines is logistically, economically, and environmentally non-viable in many cases. Moreover, authorities around the globe discourage building new fossil fuel-based plants in an attempt to reduce the carbon footprint and adverse environmental impacts. This has led to increased adoption of renewable energy resources in the last two decades.

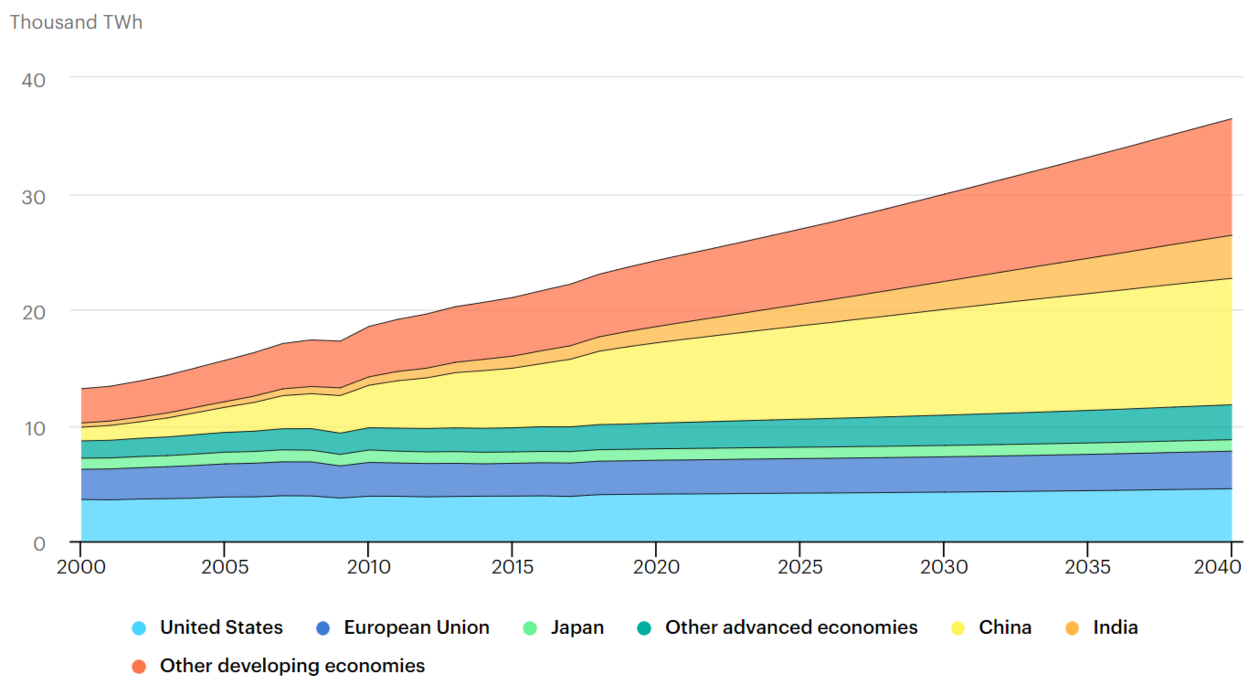

Figure 2. Global Electricity demand increase till 2040 as projected by the International Energy Agency (IEA) [2]

In 2019, the U.S. EIA projected that renewable generation will account for roughly half of the global electricity generation by 2050 [4]. Solar energy is expected to dominate the renewable energy space followed by wind energy. Figure 3 shows the U.S. projected renewable electricity generation until 2050. Such growth is enabled by clean energy programs and targets set forth by authorities. One such initiative is the U.S. Department of Energy's Sunshot program that aims to significantly reduce the Levelized Cost of Energy (LCOE) of solar energy [5]. Many states around the U.S. offer incentives for adopting renewable energy and developing enabling technologies, such as advanced inverters. The states of California and Hawaii have been the leaders within the U.S. in this regard. Around the globe, countries such as Germany, United Kingdom, China, India, Denmark, and Australia are leading a future towards clean energy.

The increasing penetrations of renewables and their grid integration pose several challenges to the grid operation. Due to the inherently variable nature of renewable generation resources like solar and wind, ensuring supply-demand balance in the short and long term is not straightforward. Significant reserves must be online to handle the intermittent nature of renewable generation [6], [7]. Though fast ramping resources like battery energy storage can be deployed to dampen this issue, storage penetrations are not sufficient to tackle this problem alone. Using traditional operational reserves to offset the variability of renewables is highly inefficient since keeping conventional generation online to provide reserves defeats the purpose of adopting renewables i.e. reduced carbon footprint. Moreover, it is economically non-viable to use the bulk of conventional generation to provide reserves.

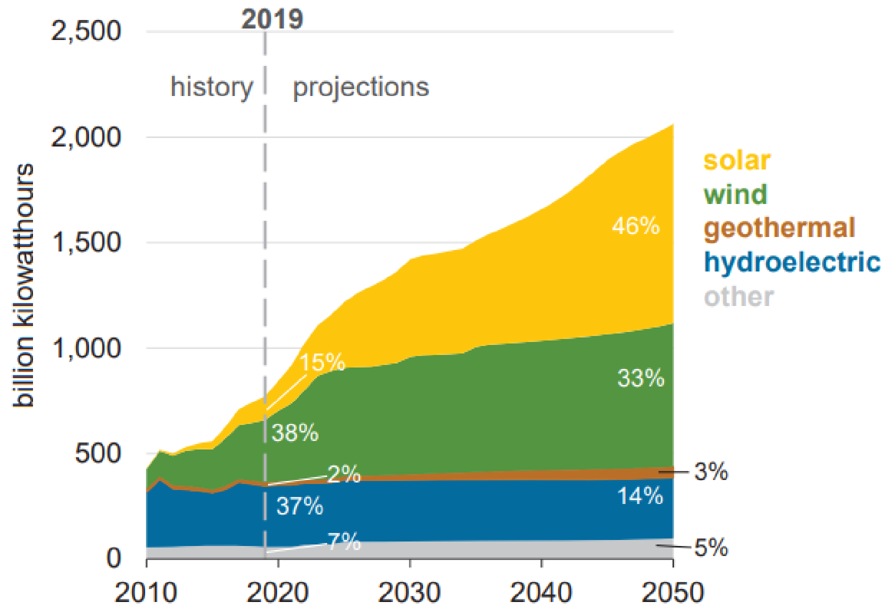

Figure 3. U.S. projected renewable generation projected by EIA [1]

Thus, traditional supply-side management techniques are not sufficient and novel control/coordination schemes (on demand side) must be designed to coordinate the operation of Distributed Energy Resources (DERs).

1.2.Demand Side Control and Energy Management Approaches

There are four generic topologies of energy management approaches in distribution systems [8] as shown in Figure 4. The nature of communication and the degree of autonomy determines the nature of the approach. The simplest form is top-down switching, also referred to as the Direct Load Control (DLC), in which the communication is unidirectional from utilities to customers and decisions on local issues are made centrally i.e. the utility or load-serving entity controls the customer load based on prior financial agreement. This approach does not consider the device state and user preference or comfort. It also compromises the privacy and autonomy of the customer by directly controlling loads using a unidirectional signal. Traditionally, this approach has been used to provide peak shaving and load shifting services [9]–[11]. Newer iterations of this control approach attempt to operate flexible loads rather than simply turning it ON or OFF. One such example is the control of thermostatic loads such as Heat Ventilation and Air Conditioning (HVAC) [12]–[15]. Recently, this methodology has been extended to control plug-in electric vehicles and storage [16], [17] to provide ancillary services, such as frequency regulation.

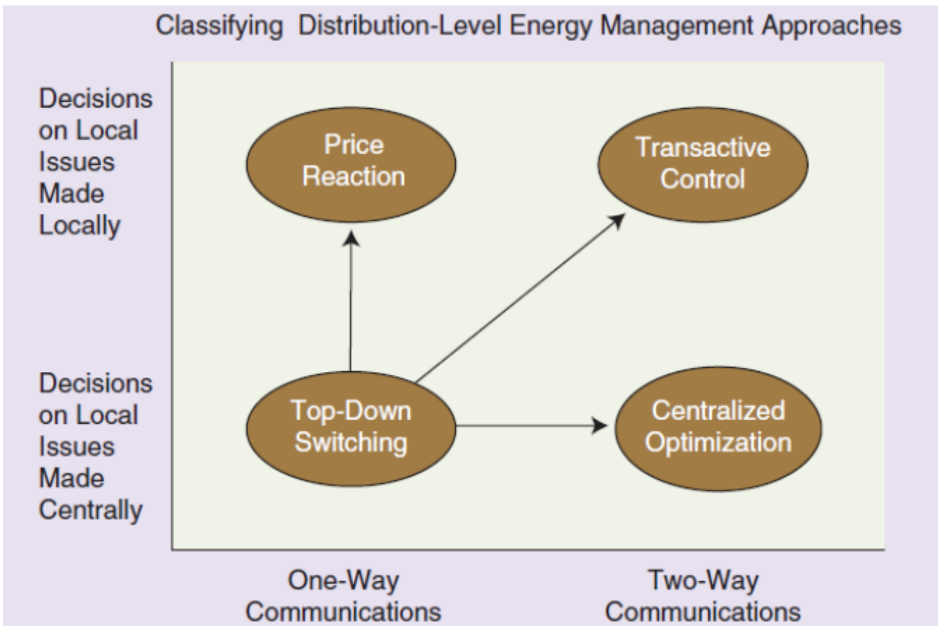

Figure 4. Taxonomy of energy management approaches in distribution systems (Demand-side management) [8]

The autonomy of a customer is preserved in Price Reaction Control (PRC) since decisions are taken locally at the customer premises, although the communication remains unidirectional. Price incentives or economic signals are sent from a load-serving entity (or utility) to the customer where an intelligent agent reacts to price signals and optimally schedules the operation of the device, such as HVAC, electric vehicle, battery storage, water heater, etc. Since decisions are taken locally so user preferences and device state are considered while scheduling the operation of the device. However, one-way communication and lack of feedback remain a significant drawback of this control paradigm since the load-serving entity can never be sure of the availability of resources at the customer premises. Common implementations of this control paradigm include Time-of-use pricing (TOU), Real-time pricing (RTP), and on-peak prices [18]–[21].

Centralized Optimization approaches incorporate customer feedback since they are built on bidirectional communication methods. However, local decisions are taken centrally at the load-serving entity or utility and communicated to the customer for execution. The two-way communication ensures that customer preferences and device states are taken into account while computing the optimal solution for the entire system. However, privacy/security concerns arise due to data sharing from the customer. Also, as the system size grows, the computation time to find the optimal solution for the entire system grows non-linearly and is highly dependent on communication speed.

Finally, the fourth approach on the two-axis graph of Figure 4 is transactive control and coordination. This is the most comprehensive control paradigm built on two-way communication. Decisions on local issues are made locally giving customer full autonomy and data security. Transactive energy systems are defined by GridWise® architectural council as [22]:

"A system of economic and control mechanisms that allows the dynamic balance of supply and demand across the entire electrical infrastructure using value as a key operational parameter."

Transactive energy systems are based on distributed control and optimization approaches. Contrary to price reactive control where wholesale price (or some altered form of the wholesale price) is passed on to the customer, an optimal price is discovered in a transactive energy system. A systematic framework is constructed where selling and buying offers are accepted and system solution is reached in an iterative manner. Once converged to a solution, optimal prices and quantities (also known as cleared quantities) are communicated to the market participants. Being a comprehensive approach, this paradigm requires a significant theoretical foundation, analysis of convergence properties, and stability analysis of convergence. Such distributed computation relies on intelligent devices or agents negotiating on behalf of participating resources. A number of pilot projects in the U.S. have already demonstrated the potential of transactive energy systems. The Olympic Peninsula Pilot project was the first project that proved this concept [23], [24]. It was followed up by American Electric Power (AEP) gridSMART® field demonstration [25], [26]. Like its predecessor, it demonstrated the use of the double-auction negotiations to establish optimal prices and quantities for market participants. Other projects include the Pacific Northwest Smart Grid Demonstration (PNWSGD) [27], [28], and PowerMatching City [29] which demonstrated the potential of transactive energy systems.

1.3.Problem Statement

The work presented in this dissertation belongs to the transactive energy topology of DER control and coordination. A game theory-based framework is constructed at the distribution level enabling DER aggregators or Home Microgrids (HMGs) to supply the load in the distribution system. The two-way iterative negotiations lead to an optimal solution of the system. Theoretical foundations of the proposed work are based on game theory, distributed control, and optimization. Following are some of the functional and design specifications of the proposed work:

- Creation of electrical and transactive representations of the distribution systems.
- Co-simulation of electrical and transactive representations. This ensures that the two systems can be modeled and simulated separately in domain-specific tools, yet they are time-synchronized and exchange boundary conditions.

- Finding an optimal solution for every participating entity. The solution should be electrically, mathematically, and economically optimal, otherwise there is no incentive for entities to participate.
- The concept of fairness and transparency should be enforced. Participating entities should be incentivized to act fairly towards their competitors and the rest of the system.
- Consideration of privacy issues. There should be ideally no information sharing between participating entities. Information sharing between participating entities and the Coordinating Authority (CA) should be minimized.
- There should be no limit to the number of entities that can supply the distribution system.
- Electrical system and supply-side reliability should be considered.
- Consumption of the local generation should be encouraged to maximize the benefit of DER deployment.
- Implementation should be generic and interoperable. This would enable multi-infrastructure simulation like Transmission-Distribution-Transactive (TDT) co-simulation.

1.4.Dissertation Organization

The rest of the dissertation is organized as follows. Chapter 2 presents a comprehensive literature review, points out the gaps in the existing literature, and lists the contributions of this dissertation. Chapter 3 discusses the prerequisites and defines the language used in this dissertation. A high-level overview of the proposed framework and its components is provided in chapter 4. It paints a bigger picture and discusses the interaction of different components with each other. Chapter 5 takes a deep dive into the framework while restricting itself to a certain class of problems, whereas a more generic class is discussed in chapter 6. Detailed case studies are presented in chapter 7 where results are discussed in depth along with the implementation details. Chapter 8 concludes the dissertation with conclusions and a list of future topics.

Chapter 2: Literature Review

Given the importance of coordinating DER operation and leveraging their response for the benefit of the grid, there have been many efforts in the literature to develop energy management approaches. Given the scope of work, the literature review in this section is focused on approaches that are based on two-way communication and decentralized decision making i.e. transactive class of energy management systems as discussed in chapter 1.

Leveraging DER response at the distribution level is usually regarded as a multi-agent system where each participating agent or player is solely interested in their profit. The participating players have usually no regard for the system. Hence, many efforts in the literature have modeled this process as a competitive, infinite strategy, multiplayer game. In [30] the authors proposed a retail transactive energy framework named as energy internet. It consists of residential customers (energy cells) with high penetration of distributed energy resources. Together with electricity suppliers/utilities (utility cells), energy cells participate in the energy internet modeled using game theory. Global and local optimum solutions are calculated using a class of solvers known as Nikaido-Isoda and Relaxation Algorithms (NIRA). The solution is based on an iterative algorithm and the theoretical results are validated on a 13-node test system. Using a similar modeling approach, [31] proposed a distribution system framework consisting of residential prosumers while considering group coalition formations. It identifies the new roles for residential prosumers and utility suppliers to support such transactive frameworks. Like [30], it utilizes NIRA based methods to find an optimal solution of the system iteratively. However, it explores novel scenarios where residential prosumers may cooperate to form a coalition rather than just competing all the time. Some group coalitions are found to be more beneficial for certain players as compared to others.

The work also discussed a fair profit allocation under a group coalition formation. Numerically, it validated results on a 13-node test system to draw the conclusions.

Authors in [32] proposed a game-theoretic based framework to study the interactions between residential prosumers while considering some distribution system constraints, such as line congestions. The role of customers in determining network reconfigurability is investigated. The concept of Distributed Locational Marginal Price (DLMP) is also discussed and the system is solved using NIRA. The economic operation of the system is claimed to be improved by the incorporation of various system-level and device-level constraints. Similarly, a transactive energy distribution system is discussed in [33] where a novel modeling approach is adopted to reflect the dynamic behaviors of residential prosumers. Under the proposed framework, utilities provide ancillary services in addition to supplying electricity. Residential prosumers manage their local generators and loads while participating in the transactive framework. It uses DLMP to investigate the impact of prosumers on a temporal and spatial scale on the electric grid. Results are validated on a 13-node test system and it is concluded that prosumers can improve their payoffs by satisfying the stated economic dispatch. In [34] authors proposed a distribution energy sharing scheme for three types of players i.e. consumers, generators, and retailers. The process is modeled as a multiplayer, non-cooperative game and solved using NIRA methods. It provides a statistical analysis by considering the uncertainties in demand and supply. It also considers the participation of multiple retailers. The work demonstrated how the proposed framework can lower the optimum prices (availability of cheaper energy), encourage consumption of local generation, and incentivize players to support the load in the distribution system. It also enforced various device-level constraints on each player to ensure local optimality.

A Cournot model-based electricity market is proposed in [35] where the dynamic behavior of market participants is analyzed. A dynamic Cournot game model is proposed and stability of equilibrium is analyzed. In [36], a distributed optimization-based game-theoretical model is proposed and solved. Buyers play a game to procure cheap energy from a pool of available sellers. The proposed approach decreases the individual energy bills of the buyers and increases the profits of the sellers. For comparison, a centralized optimization-based approach is also presented. It is shown that the distributed optimization results in higher net profits for the participating players. However, it is assumed that each player knows the pool of available sellers and their capacities before the game starts. Each player also knows the offer price of the other players. A game-theory based scheme is proposed in [37] where DER aggregators (termed as multi-Microgrids) try to minimize their operational cost. The scheme is administrated centrally by a Distribution Network Operator (DNO) whose objective is profit maximization. The optimal solution of the system is obtained using Karush-Kuhn-Tucker (KKT) methods. It is shown that the KKT methods can effectively balance (simultaneously optimize) the objectives of DNO and multi-Microgrids. The KKT methods, like the NIRA and Cournot model, assume that a given player is knowledgeable about her competitors/peers. This knowledge can be the strategy set of other players, their offer prices, or the functional form of the payoff model. Hence, these methods cannot be used to solve for the optimal solution of a game theory-based problem in the setting of imperfect or incomplete information.

Several studies in recent years have investigated incomplete information games. A repeated game-theoretic energy sharing for demand response aggregators is proposed in [38]. The players, i.e. demand response aggregators, do not know the cost or payoff functions of the other players. Each player tries to maximize profit by selling the energy stored in electrochemical cells to the other players. A dynamic economic dispatch is performed to update the DLMPs. The game is modeled as a Stackelberg game and solved using a novel dynamic economic dispatch algorithm. The framework considers two types of dynamic pricing schemes, i.e. Real-Time Pricing (RTP) and Time-of-Use (TOU) pricing. It is validated in a 24-node test system and claims to lower the electricity costs of the participating players. A two-stage iterative Stackelberg game is proposed in [39] where sellers (named as distributed energy stations) lead the game by deciding the unit prices of electricity. The consumers, i.e. energy users, act as followers in this hierarchal game model. The multi-leader, multi-follower Stackelberg game is solved using the best response algorithm which guarantees convergence to a Stackelberg equilibrium. Similarly, a two-stage Stackelberg game is proposed in [40] which considers three types of players i.e. grid operator, service providers (sellers), and customers. The approach is incentive-based where the grid operator publishes an incentive to multiple service providers who then engage enrolled customers. This leads to iterative negotiations between service providers and customers regarding demand reduction. The existence of a unique Stackelberg equilibrium is demonstrated and convergence is achieved via a novel distributed algorithm.

A reverse auction game-theoretic model is proposed in [41]. The methodology is tested on a smart grid test system with a synchronous generator, inverter-based DERs, and loads. A multi-agent algorithm is used to solve the reverse auction day-ahead energy sharing, i.e. hour-ahead schedule, for 24 hours of a day. The platform schedules DER unit-commitment for each hour of the day and is tested on a real-world system. A reinforcement learning-based solution for incomplete information, competitive games is introduced in [42]. Each participating player uses the learning scheme to generate a strategy set and associated probabilities of playing a particular strategy. The private information of each player is not shared with other players, thus constituting an incomplete information game. The uniqueness and convergence to game equilibrium are guaranteed for the proposed double-auction scheme upon convergence. A novel two-stage stochastic game model proposed in [43] aims to reduce the risk of not meeting the demand from DER based Microgrids (suppliers). Each DER aggregator employs a conditional value at risk (CVAR) function to reduce the risk associated with uncertainty in supply from DER Microgrids. A Cournot pricing mechanism is adopted for the game model and a solution is obtained using a Sample Average Approximation (SAA) technique. The work also investigates the uniqueness of convergence to a game solution. The proposed technique is validated by simulations using real-world data.

A two-stage, non-cooperative Stackelberg game is proposed in [44] for demand-side management in residential area microgrids. It considers community energy storage as a leader in the game since it sets the price and attempts revenue maximization. The followers in this game are the customers with solar photovoltaic-based DERs who play a competitive repeated game. The objective of the game followers is to minimize electricity costs. An iterative algorithm is demonstrated to solve for a unique Stackelberg equilibrium of the stated game. Similarly, a two-stage Stackelberg game is proposed in [45] for energy management and sharing. The microgrid operator acts as the leader in this game and PV-based prosumers act as the followers. The objective of the microgrid operator is to maximize its profit while coordinating the energy sharing among prosumers. On the other hand, prosumers also act to maximize their payoffs. The game is solved using a non-linear programming method and convergence to a unique Stackelberg equilibrium is discussed. Moreover, the methodology is validated using data from real-world PV systems. It also demonstrates that the equilibrium converged price has a positive impact on the net energy profile of the system. Though several studies have investigated incomplete information games, the notion of enforcing fairness and transparency remains a gap. In such frameworks, incentives are not enough to make players act in a beneficial way towards each other. Thus, reliability forcing methods and indices must be designed to leverage the maximum benefits of such frameworks.

Some of the efforts in the literature that address behaviors of participating players are based on developing a psychological motivational framework, while the rest are concerned with fair profit allocation among the players forming a coalition in the setting of cooperative games. A motivational phycology framework is presented in [46] to increase user participation. It develops motivational models that encourage players to participate in energy trading. However, this energy trading is peer-to-peer, i.e. without any central coordinating or enforcing authority. The work also proposes a competitive, game-theoretic model to enable peer to peer energy trading. It studies group coalition formations and claims to reduce the cost of electricity for participating players. A price discrimination scheme is discussed in [47] where smart grid energy users decide on the price to charge a central coordinating authority. The game is modeled as a cake cutting, cooperative game, and a socially optimal solution is achieved using a distributed optimization algorithm. The objective function is designed to maximize the net benefit of all the participating players (termed as energy users). The work also analyzes price sensitivity and formulates a benefit function for price discrimination. Moreover, fairness is enforced in form of price discrimination, where each player gets their due payoff at the solution convergence. A non-cooperative, Stackelberg game model in [48] considers energy sharing among prosumers (named as residential units) and a central coordinating authority (names as shared facility controller). Specifically, the study focuses on energy storage capacity being committed to serving the load. It proposes a modified auction scheme between prosumers and a coordinating authority to solve the Stackelberg game model. The study demonstrates incentive capability in the form of the current price that each player gets at solution convergence.

For cooperative games where players may form a group coalition, a fair profit allocation can be determined using the Shapley value method. One such study is [31] where group coalition formations are discussed. In [49] the Shapely value method is compared with three other profit allocation schemes for cooperative games. They include the Nucleolus method, DP equivalent method, and the Nash-Harsanyi method. A mixed-integer linear programming (MILP) method is used to solve the proposed cooperative game. The authors also discuss two indices to compare the quantitative value of using each of the fair profit allocation schemes since each scheme may result in a different profit allocation. The study also analyzes the stability of each scheme and validates results using data from real-world buildings. Similarly, [50] discusses the Shapley value method from a transactive reliability perspective. A reliability-centered maintenance approach is proposed to analyze the system at various loading conditions and a new optimization framework is used to solve the system. A risk-averse cooperative game in [51] proposed the participation of Virtual Power Plants (VPPs) in day-ahead and real-time markets. The game is solved using a novel stochastic programming approach while a fair profit allocation among VPPs is ensured by using Nucleolus and Shapley value methods. A conditional value at risk (CVAR) is computed for each player to account for uncertainties in electricity price, electricity demand, and DER generation.

A coalitional game-theory based energy trading between networked microgrids is introduced in [52]. A novel auction-based matching method is proposed to compute the utility of each coalition between the microgrids. For fair coalition utility allocation, the Shapley value method is used. A local power exchange algorithm is used to find the game solution. The authors also introduce a novel technique to find the most stable and optimal coalitions in terms of maximizing utility. The study demonstrates an average increase of 16% in the utility of each microgrid. In [53], a cooperative game model is proposed for a community microgrid to facilitate peer-to-peer energy trading. Fairness among prosumers is ensured by the Shapley value method and it is compared with algorithms like bill sharing, supply-demand ratio, and mid-market rate. A constrained, non-linear programming method is used to solve the game. The research effort attempts to capture the conflicting interests of prosumers in peer-to-peer energy trading. Specifically, the work is focused on the dispatch of battery energy storage in community microgrids. Hence, for cooperative games, the Shapley value method has been used predominantly for fair profit allocation along with other techniques like cake cutting methods. For non-cooperative schemes, fairness has been enforced in the form of the motivational phycology framework or the optimum price that a player gets. This is similar to Independent System Operator (ISO) operation where generators get penalized for deviating from their commitments in the form of financial penalties incurred.

A hierarchical decision-making scheme is introduced in [54] for microgrids in a competitive setting. The process is modeled as a two-stage, multi-leader, multi-follower Stackelberg game. The game is led by the seller microgrids by deciding the amount of energy for sale. They are followed by the buyer microgrids by submitting price bids to the sellers. Buyers procure energy in proportion to submitted bids whereas the sellers get the revenue in proportion to sales. However, the study disregards conventional suppliers or retailers. The uniqueness and convergence properties of a Stackelberg equilibrium are also discussed. An aperiodic, event-driven, energy sharing scheme is introduced in [55]. The concept of consumer reward is introduced to incentivize sharing. In contrast to time-based periodic schemes, trading is initiated when a buyer requests energy from the grid operator and posts a reward for supplying. The sellers, reacting to the posted reward, submit their energy bids to the grid operator. Once converged, the posted reward is allocated to the sellers in proportion to their bids. This is a two-stage, non-cooperative, Stackelberg game where buyers act as leaders and sellers act as followers. The optimal trading algorithm is shown to converge to a unique Stackelberg equilibrium. However, the aperiodic market is problematic in the sense that ISO markets operate periodically. Thus, coordinating operations with the ISO market requires a periodic microgrid energy market. A peer-to-peer energy trading platform demonstrated in [56] establishes a four-layered architecture for energy trading in microgrids. The trading is directly between peers or prosumers without the intervention of a central entity. The game is modeled as the finite strategy game where the strategy set of each player is limited to binary space, i.e. flexible demand OFF or ON. The solution is obtained via optimization and results are validated on a low voltage microgrid network.

Table 1. Review of existing techniques to model energy sharing in distribution systems using game-theoretic approaches

Reference #	**Solution method**	**Game type**
[30]	NIRA	Complete information, non-cooperative
[31]	NIRA	Complete information, non-cooperative while investigating group coalition formations
[32]	NIRA	Complete information, non-cooperative
[33]	NIRA	Complete information, non-cooperative

[34]	NIRA	Complete information, non-cooperative
[35]	Cournot model	Complete information, Cournot game, non-cooperative
[36]	Distributed Optimization algorithm	Complete information, non-cooperative
[37]	Karush-Kuhn-Tucker (KKT) method	Complete information, non-cooperative
[38]	Dynamic Economic Dispatch	Incomplete information, stackelberg game
[39]	Best response algorithm	Incomplete information, non-cooperative two-stage stackelberg game
[40]	Novel Distributed algorithm	Incomplete information, non-cooperative two-stage stackelberg game
[41]	Multi-agent algorithm	Incomplete information, Reverse auction game model

[42]	Adaptive reinforcement learning algorithm	Incomplete information, non-cooperative, double auction game model
[43]	Sample Average Approximation (SAA)	Incomplete information, two-stage, stochastic game model
[44]	Novel iterative algorithm	Incomplete information, non-cooperative two-stage stackelberg game
[45]	Non-linear programming	Incomplete information, non-cooperative two-stage stackelberg game
[46]	Peer-peer, system equilibrium not required	Incomplete information, non-cooperative game with group coalition formations
[47]	Distributed optimization algorithm	Cooperative game, cake cutting game model
[48]	Modified auction approach	Non-cooperative stackelberg game
[49]	Mixed-integer linear programming (MILP)	Cooperative game model, Shapley value method, the

		Nucleolus, DP equivalent method, Nash-Harsanyi
[50]	Novel optimization framework	Cooperative game model, Shapley value method
[51]	Novel stochastic programming approach	Cooperative game model, Shapley value method, Nucleolus method
[52]	Novel local power exchange algorithm	Cooperative game model, auction-based matching method, Shapley value method
[53]	Constrained nonlinear programming (CNLP)	Cooperative game model, Shapley value method
[54]	Hierarchal decision making scheme	Non-cooperative game, Multi-leader multi-follower Stackelberg game
[55]	Novel optimal trading algorithm	Non-cooperative game, Stackelberg game model
[56]	Novel platform named as Elecbay	Non-cooperative game, four-layered architecture

2.1.Gaps in the existing literature

Based on the literature review, a list of gaps in the current state of the art is compiled as follows:

- Assuming perfect knowledge – Several existing studies assume that each player is aware of the strategies played by the other players. This is undesirable since it violates the privacy of a customer.
- Using model-based approaches for the solution – When it is assumed that a player knows the game payoff function and the game model, then the solution can be obtained by using model-based approaches like NIRA. This is not realistic since this game information (strategy to payoff mapping) is not usually shared with the players.
- Lack of player reputation indices – The behavior of a player is critical in determining the social welfare of energy management approaches. If a player behaves well, i.e. sticks to commitments, then the player should get a higher reward in form of a higher payoff from the coordinating authority compared to players who deviate from their commitments. Existing literature penalizes players by imposing penalties or reducing the clearing price. However, a mechanism or index that keeps track of a player's historical performance is lacking.
- Lack of reliability indices in the game model – Having a reliability index embedded in the game model is desirable. However, it is lacking in the current state of the art. Such an index, when integrated into the game model, ensures that more energy is procured from reliable resources as compared to unreliable resources, and thus providing an overall improvement of reliability.
- Quadratic DER cost functions – In current literature, DER cost functions are approximated as linear or quadratic functions. However, the higher-order cost function can capture cost

dynamics more accurately (discussed in chapter 6). Thus, this remains a gap in the existing work.

2.2. Contributions of the proposed work

Following are the novel contributions of the work presented in this dissertation:

- A novel electrical-transactive co-simulation framework is proposed to supply distribution system load from DERs and their aggregators.
- A novel game model is proposed with an integrated player reputation index. The game model is infinite strategy-based, non-cooperative, and rewards players who behave in a beneficial way towards the system.
- The concept of the Player Reputation Index (PRI) is proposed. It provides three key functions of tracking the historical performance of each player, rewarding players according to their behavior, and reliability.
- A distributed, gradient-based scheme of extremum seeking is demonstrated to solve for the unique solution, i.e. Nash equilibrium of the proposed framework.
- Games with non-quadratic payoff functions are considered i.e. games inclusive of players with non-quadratic DER cost functions.
- A generic, dynamic mapping of the strategy set to payoff functions is discussed. The game model is revised accordingly to accommodate dynamic mapping.
- For generic, non-quadratic games, the existence of multiple Nash equilibria is considered. Convergence properties of stable Nash equilibrium are studied.
- It is demonstrated that when extremum seeking is used to model Nash seeking behavior of the players, the system converges to one of the stable Nash equilibrium points.

Chapter 3: Discussion on Pre-requisites

This chapter reviews the pre-requisites that assist in developing an understanding of the proposed work. First, the core concept of game theory is briefly reviewed. Then, transactive energy agents are introduced followed by a discussion on communication technologies that enable smart grid communication.

3.1.Game Theory: A brief review

The core concept of the work presented in this dissertation stems from a branch of applied mathematics, namely the modern game theory. It can be defined as:

"Game theory provides the framework and language to model the strategic interactions between two or more agents in a situation containing set rules and outcomes"

Thus, it has proven to be a strong tool in modeling multi-agent systems. Particularly, it pertains to mathematical modeling of conflict and cooperation among rational, intelligent, agents. The first formal discussion on game theory was presented in a paper titled 'On the theory of Games of Strategy' authored by a Hungarian-American mathematician, John von Neumann in 1928 [57]. He followed it up with more foundational work in this area and published his book titled 'Theory of Games and Economic Behavior' with co-author Oskar Morgenstern in 1944 [58]. Some of the most significant contributions to modern game theory came from American mathematician John Forbes Nash Jr., who formulated the concept of Nash equilibrium. In the 1950s, he proved that every finite n-player, non-zero-sum non-cooperative game has a solution that was named after him as Nash equilibrium.

Modern game theory serves as a strong applied tool used in a wide array of scientific and technological disciplines. Common application areas include fields such as computer science, engineering, systems science, and economics. Game Theory is regarded as one of the seven sub-areas of complex systems [59] as shown in Figure 5. Some definitions from the language of game theory are discussed below. They are used extensively throughout this dissertation.

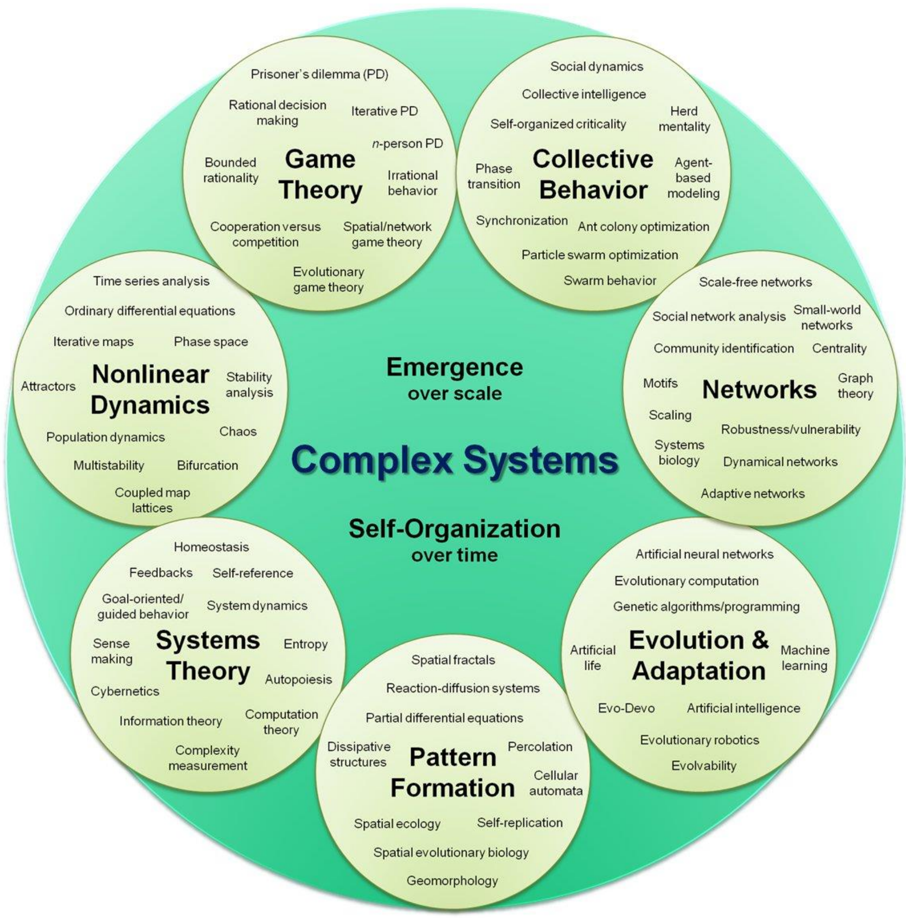

Figure 5. Classification of complex systems [59]. Game theory is one of the seven sub-areas defined in the paradigm of complex systems

Game

Any interaction or set of circumstances whose outcome is dependent on the actions of two or more decision-makers. The utility of each decision-maker is affected by its actions as well as the actions (decisions taken) by the other decision-makers.

Players

The decision-makers participating in the game, such as individuals, business entities, firms, negotiating brokers, etc.

Strategy

A complete plan of action that a player will execute depending on the set of circumstances that may arise in the game.

Payoff

Pertains to the value attributed with the outcome of a game corresponding to a strategy played by the player. This value is usually in some quantifiable form such as dollars.

Zero-Sum games

Games in which a gain for one player translates into a loss for another player and vice versa. The net change in utility of the game remains zero.

Non-Zero-Sum games

In such games, outcomes can drive the game's utility up or down. The total gains and losses from all players do not add to zero.

Cooperative games

The games in which the players cooperate for a higher mutual payoff. The group that they form is known as a coalition.

Non-Cooperative games

The games in which the players compete with each other for a higher payoff. Their interests are in conflict and they are solely interested in maximizing their payoffs. These are the most common types of games.

Nash Equilibrium

The optimal outcome of a game. Once the game converges to a Nash equilibrium, no player has any incentive to change her strategy since there is no incremental benefit in deviating from Nash optimal strategy for any player.

Shapley Value

The average marginal contribution of a player's utility across all possible coalitions. This is used mainly in cooperative games.

Complete Information games

Games in which each player knows the strategies played by the other players, their payoff functions, and their types. This common knowledge is shared among all the players.

Incomplete Information games

Games in which a given player may or may not be aware of the strategy set of other players, their payoff functions, and their types.

Imperfect Information games

Games in which the players are only unaware of the strategies played by the other players. However, the rest of the information, such as payoff functions, is a shared knowledge.

Figure 6 shows the five common types of games.

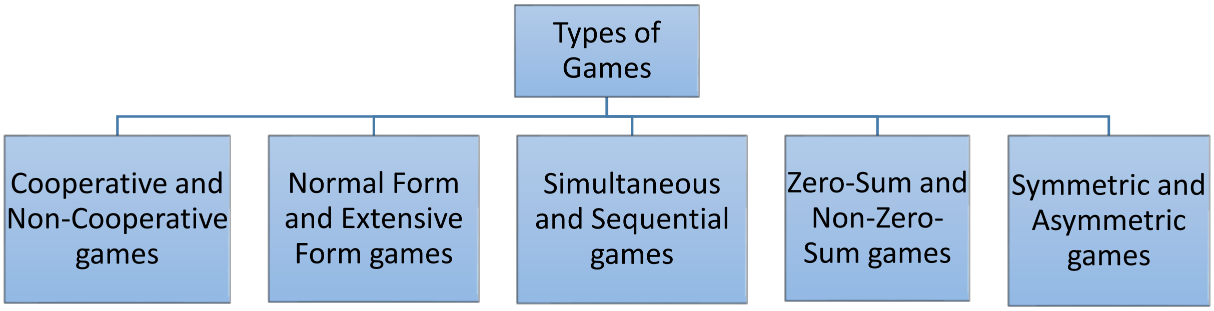

Figure 6. Different types of games

3.2.Transactive Energy Agents

Transactive energy agents refer to a family of intelligent devices that enable the seamless grid and transactive integration of the DERs. Formally, a transactive energy agent can be defined as an intelligent controller, or a decision-maker, that sits on top of a DER like PV, battery, EV, etc. Considering DER state, user preferences, price forecasts, and other DER specific parameters like EV vehicle availability, a transactive agent formulates the bids for a DER. These bids can be sent to the CA, however, usually, they are first sent to a DER aggregator like an energy management system. Transactive agents are very generic and their implementation can range from simple optimization to complex machine learning or fuzzy logic-based implementation. The implementation type is usually dictated by the nature of inputs, i.e. deterministic or stochastic and computational complexity available. However, the underlying goal for any agent is to maximize profit while considering all applicable constraints.

For example, a generic transactive agent for battery energy storage is shown in Figure 7. The inputs for the battery agent include hourly price forecasts, customer preferences, and battery specifications. If the agent is not able to generate the price forecasts, they are communicated to the agent by the energy management system. User preferences for battery storage may impose constraints, such as ceasing battery discharge beyond a certain depth of discharge. Similarly, a user may impose a time-based restriction on the discharge of the battery to have sufficient reserves for certain hours of the day. Battery state variables, such as the current state of charge, charge/discharge rates, charging/discharging efficiency, and losses are also considered by the agent while generating the bids. The output of the agent depends on the resolution of the transactive framework for which the bids are generated, i.e. hourly for an hour-ahead scenario. Moreover, bids can be quantity only (also called Q-bids) or price-quantity bids (PQ-bids). In PQ-bids, a price is placed on the quantity that is negotiated between the agent and the CA whereas Q-bids have no price on quantities. This generic implementation can be modeled as a linear or non-linear optimization problem depending on the type of constraints and objective function. In addition to arbitrage, a battery agent can also optimize a grid friendly objective function such as loss minimization and suppression of voltage deviations [60].

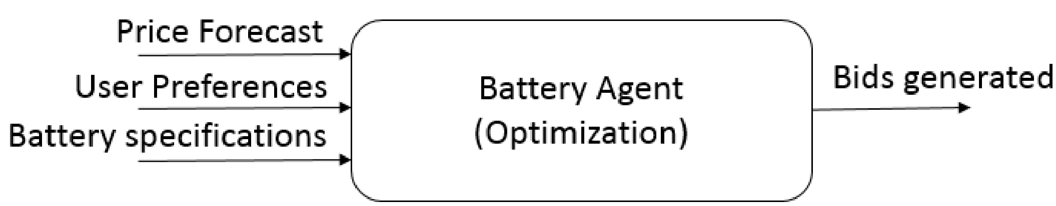

Figure 7. Structure of a generic transactive agent for the battery energy storage

Like the battery agent, an electric vehicle (EV) agent uses inputs such as price forecasts and battery specifications as shown in Figure 8. However, the availability of a vehicle for grid services is dependent on the driving patterns and routine of the customer. These behaviors are not deterministic, and hence the agent implementation needs to consider the uncertainty associated with the availability of the vehicle. Thus, such an agent may be modeled and implemented as a stochastic optimization problem.

Similarly, other DERs and smart devices at homes may have their transactive energy agents. They include PV, water heaters, HVACs, microwaves, ovens, etc. The first comprehensive transactive energy simulation platform was implemented by Pacific Northwest National Laboratory in 2016-2017 as part of their transactive energy program. Formulating these agents and optimally implementing them remains an active area of research.

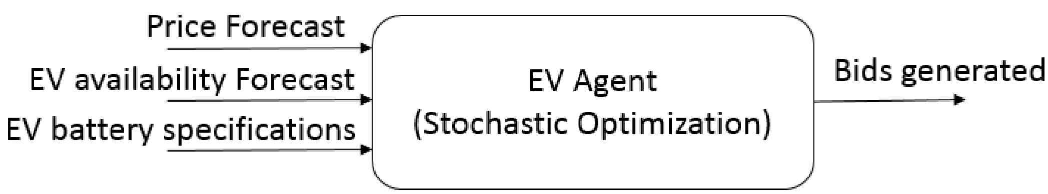

Figure 8. Structure of a generic transactive agent for an electric vehicle

3.3.Communication Methods and Enabling Technologies

In addition to intelligent devices, interoperability and communication are essential for enabling any transactive energy framework. Hence, it is useful to discuss enabling technologies for the implementation of transactive energy frameworks. Noticeably, VOLTTRON is an open-source software platform for distributed sensing, control, and communication of smart grid devices [61]. Supported by the U.S. Department of Energy, this lightweight package can run on portable, inexpensive, small-scale processors. Figure 9 shows the VOLTRON powered Raspberry Pi controller developed at Pacific Northwest National Laboratory. This platform enables application development for smart grid devices and provides a secure message bus for connectivity among different modules. It supports the agent-based implementation of software modules that perform the desired functions, such as battery agent bidding for the battery, electric vehicle agent bidding for the electric vehicle, and so on. Due to its higher interoperability, it enables the installation of custom drivers and interfaces for other IoT-based devices. Most distinguishing features of VOLTRON include cost-effectiveness, scalability, interoperability, and security.

The 2018 version of the IEEE 1547 standard also listed several communication protocols for communication. They include IEEE Std. 2030.5 (SEP2), IEEE Std. 1815 (DNP3), and SunSpec Modbus. However, it dropped IEC 61850 which is commonly used as the standard communication protocol for intelligent electronic devices at the substations.

Figure 9. A Raspberry Pi based VOLTTRON controller

Chapter 4: High-level System Overview

This chapter presents a high-level overview of the proposed work as shown in Figure 10. The next few chapters take a deeper dive into the individual sections of this framework.

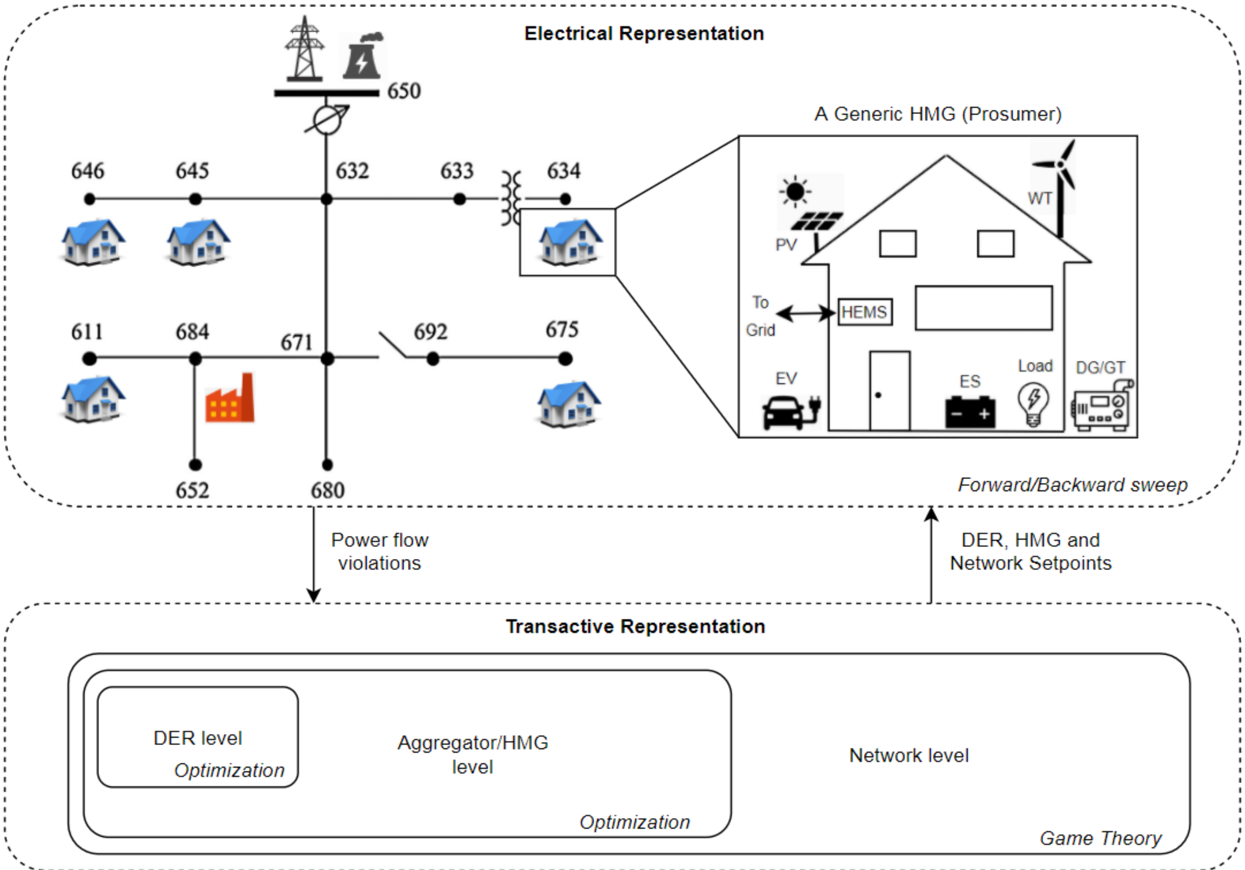

Figure 10. A High-level overview of the proposed work

4.1.System Characteristics

As Figure 10 shows, each component in the system has an electrical and a transactive representation. The two representations are required to be tightly-coupled, i.e. synchronized in time and exchanging data with each other. Some of the important features of the electrical representation include:

- Detailed model of an electrical feeder. For example, Figure 10 shows a modified version of the IEEE 13 bus distribution feeder. The loads in this feeder are realized using generic prosumers termed as a Home-Microgrids (HMGs). The source is modeled as a standard Utility or Retailer (UTR)
- A generic HMG may contain all or some of the components shown in Figure 10. Due to the presence of DERs, a given HMG at any given time may act as a load or a generator.
- A UTR always acts as a source/generator
- Due to the radial nature of distribution feeders, the electrical model is solved using a forward/backward sweep method.

Each component in the electrical system has a corresponding transactive system representation. The transactive representation is layered-structured with the following characteristics:

- At the DER or device level, an optimization agent model generates an optimum operational schedule for a DER. For example, a battery agent creating a charge/discharge schedule based on battery specifications, expected load profile, forecasted prices, and user preferences. Similarly, an electric vehicle agent generating an optimum charge/discharge schedule for EV battery considering EV battery specifications, vehicle availability forecast, price forecast, load forecast, and user preferences.
- At the DER aggregator level or HMG level, an economic dispatch model aims to supply all the local demand within HMG from the cheapest generation resources available. Since an energy management utility, such as a Home Energy Management System (HEMS), within HMG has knowledge and control of all DERs installed, hence this is modeled as an optimization problem.

- At the Network level, optimum outputs from all the prosumers and generators are required to be determined. At this level, since every entity, i.e. prosumers and generators, are interested in maximizing their utility, and there is no shared knowledge, hence network optimization cannot be used to solve the transactive system. This multi-agent system is modeled as a game theory problem to determine an optimum operating point for all the prosumers, i.e. HMGs and the generators, i.e. UTRs.

The two representations, though modeled separately, cannot be solved independently as a solution that is optimum in transactive representation may violate a power flow constraint in electrical representation, and vice versa. Thus, the two representations are required to be tightly coupled. They exchange time-synchronized data with each other. This includes:

- Power flow results and violations from electrical representation to the transactive representation. The common violations include line congestions, transformer overloads, voltage violations, etc.
- DER operational schedules, HMG outputs, and UTR set points.

4.2.Implementation Specifications

The modeling approach shown in Figure 10 can be implemented using a co-simulation architecture. This is shown in Figure 11 where a distribution system model is co-simulated with the corresponding transactive model using Hierarchical Engine for Large-scale Infrastructure Co-Simulation (HELICS) [62]. HELICS is an open-source, light-weight package that is used to co-simulate multi-domain systems modeled in their respective tools. Here, the electrical model, i.e. the distribution system is modeled in GridLab-D, whereas the transactive model is realized in MATLAB. HELICS coordinates the data exchange between these systems and keeps them in a tightly-coupled, time-synchronized state. Though Figure 11 shows a transmission system model, it is beyond the scope of the work presented in this dissertation as it is part of the future work discussed later.

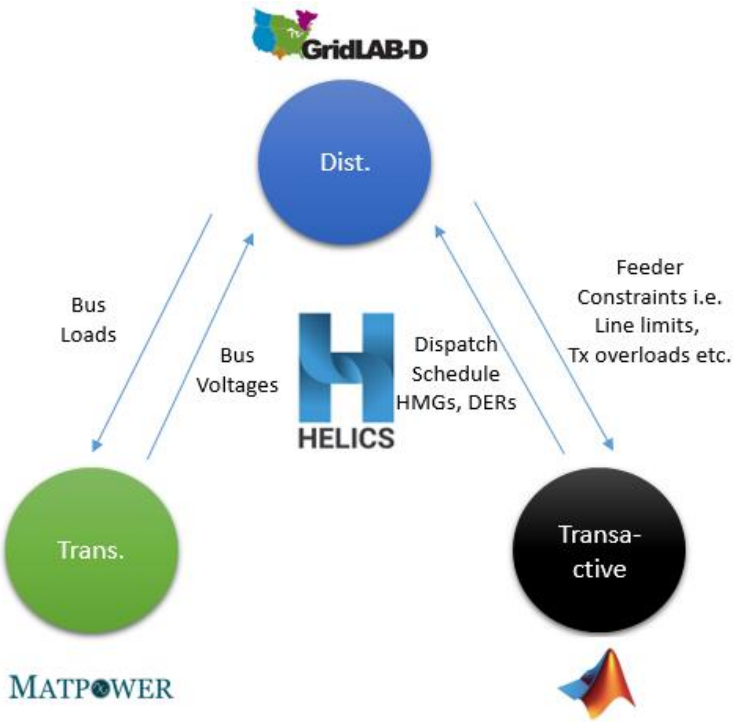

Figure 11. A HELICS co-simulation enabling data exchange between different modeling paradigms

As electrical and transactive representations of the system exchange data with each other, the solution is declared optimum only when:

- Power flow converges and there is no power flow constraint violation or system violation such as line overload etc.
- The transactive system converges with optimum outputs for HMGs and UTRs. The optimization agents at DER and HMG level must converge before the network convergence occurs.

The detailed drill-down of individual models and solution methodologies are discussed in the next chapters.

Chapter 5: Transactive Modeling Based on Quadratic Game theory

5.1.Introduction

This chapter discusses a comprehensive transactive model for the distribution systems where the payoff functions of DERs are modeled as the quadratic functions. Once the theory is built for this simplified case, it can be extended to a more general class of games as discussed in Chapter 6. The proposed framework is modeled as a non-cooperative, infinite strategy, multiplayer game. A novel game model is proposed with an embedded notion of the reputation of each player. The concept of a Player Reputation Index (PRI) is also proposed to track each player's historical performance and reward her accordingly upon convergence. Moreover, Nash seeking behavior of each player is modeled using the extremum seeking method. The chapter also discusses the solution to the problem and analyzes the convergence characteristics of these quadratic games.

A list of set indices, constants, and variable definitions used throughout this chapter are listed below:

Indices and Sets

$\Delta t =$ RTOI, minutes

$\Delta t' =$ HEMS refresh interval, seconds

$N, N'', N' =$ Number of Total/Buyer/Seller HMGs

$M =$ Number of UTRs

$\alpha^{(.),k}, \beta^{(.),k}, \gamma^{(.),k} =$ Coefficients for production cost function of resource $(.)$ in kth HMG, $\forall k \in \{1,2, \dots., N\}$

$a^k, \omega^k, \emptyset^k =$ Extremum seeking parameters for kth HMG, $a^k, \omega^k, \emptyset^k > 0 \quad \forall k \in \{1,2, \dots., N\}$

Constants

$P_{max}^{(.),k}, P_{min}^{(.),k} =$ Maximum/Minimum power output (kW) of resource $(.)$ in kth HMG, $k \epsilon \{1,2, \dots, N\}$

$SOC_{max}^{k}, SOC_{min}^{k} =$ Maximum/Minimum allowable SOC (%) for ES in kth HMG, $k \epsilon \{1,2, \dots, N\}$

$\varepsilon_{ES}^{k} =$ Round trip cycle efficiency of ES in kth HMG, $k \epsilon \{1,2, \dots, N\}$

$ES_{cap}^{k} =$ Total capacity (kWh) of ES in kth HMG, $k \epsilon \{1,2, \dots, N\}$

$S^{HMG,k} =$ Fixed amount $(\$/h)$ paid by kth HMG to grid owner as the service fee, $k \epsilon \{1,2, \dots, N\}$

$\rho^{k} =$ Weighting factor for computation of PRI for kth HMG, $k \epsilon \{1,2, \dots, N\}$

Functions and Variables

$J_t^{HMG,k}, R_t^{HMG,k}, C_t^{HMG,k} =$ Payoff/Revenue/Cost function $(\$/h)$ for kth HMG at time t, $k \epsilon \{1,2, \dots, N\}$

$J_t^{UTR,i} =$ Payoff function $(\$/h)$ for ith UTR at time t, $i \epsilon \{1,2, \dots, M\}$

$\pi_t^{(.),k} =$ Production cost $(\$/h)$ of generation from resource $(.)$ in kth HMG at time t, $k \epsilon \{1,2, \dots, N\}$

$P_t^{(.),k}, \hat{P}_t^{(.),k} =$ Actual/Predicted power output (kW) from resource $(.)$ in kth HMG at time t, $k \epsilon \{1,2, \dots, N\}$

$P_t^{L,k}, \hat{P}_t^{L,k} =$ Actual/Predicted total load (kW) (flexible and non-flexible) of kth HMG at time t, $k \epsilon \{1,2, \dots, N\}$

SOC_t^k = State of charge (%) of ES in kth HMG at time t, $k \in \{1,2, \dots, N\}$

σ_t^k = PRI (%) for kth HMG at time t, $k \in \{1,2, \dots, N\}$

$f_{sell,t}^{HMG,k}, f_{buy,t}^{HMG,k}$ = Optimum Selling/buying price ($\$/kWh$) for kth HMG at time t, $k \in \{1,2, \dots, N\}$

$P_{sell,t}^{HMG,k}, P_{buy,t}^{HMG,k}$ = Optimum Selling/buying quantity (kW) for kth HMG at time t, $k \in \{1,2, \dots, N\}$

$f_{sell,t}^{UTR,i}$ = Optimum Selling price ($\$/kWh$) for ith UTR at time t, $i \in \{1,2, \dots, M\}$

$P_{sell,t}^{UTR,i}$ = Optimum Selling quantity (kW) for ith UTR at time t, $i \in \{1,2, \dots, M\}$

S_t^i = Total service fee from all subscribers of ith UTR at time t, $i \in \{1,2, \dots, M\}$

Specifically, this chapter discusses the following contributions [63] of this dissertation:

- A novel game model is proposed with an integrated player reputation index. The game model is infinite strategy-based, non-cooperative, and rewarding to players who behave in a beneficial way towards the system.
- The concept of the Player Reputation Index (PRI) is proposed. It provides three key functions of tracking the historical performance of each player, rewarding players according to their behavior, and reliability improvement.
- A distributed gradient-based scheme of extremum seeking is demonstrated to solve for the unique solution, i.e. Nash equilibrium of the proposed framework.

- It is demonstrated that when extremum seeking is used to model Nash seeking behavior of the players, the system converges to one of the stable Nash equilibrium.

Section 5.2 of this chapter introduces the high-level concept of the proposed framework and discusses the two types of players, whereas sections 5.3 and 5.4 take a deeper dive into the transactive representation, theory for Nash seeking, and the game model.

5.2.Types of Players

There are two types of players i.e. Home Microgrids (HMGs) and Utilities or Retailers (UTRs). They are formally defined below:

5.2.1. HMGs

As shown in Figure 12, the definition of HMGs applies to modern (commercial or residential) green buildings that may have all or any of the following with respective transactive energy agent:

- Photovoltaics (PV)
- Small-scale wind turbines (WT)
- Diesel generator (DG) or a gas turbine (GT)
- Energy Storage (ES)
- Electric Vehicle (EV) – V2G type
- Flexible and non-flexible load

Supplying its load is the primary motivation for any HMG owner to deploy these generation services (both non-dispatchable (NDU) and Dispatchable (DGU)). However, when their generation exceeds load, HMGs will behave as generators for the network, and thus surplus power is available to be sold. HMGs act as loads for other intervals, when generation is not sufficient to meet local loads. It is assumed that each HMG has a Home Energy Management System (HEMS) installed. The key functions of the HEMS include:

- Predicting renewable energy generation (i.e. from PV and WT)
- Predicting the local load to be supplied (at least for the duration of the next RTOI)
- Estimate the optimum schedule of local generation, i.e. the optimal resource commitment within an HMG
- Communication with the CA for the selling or purchase of power from other HMGs or UTRs

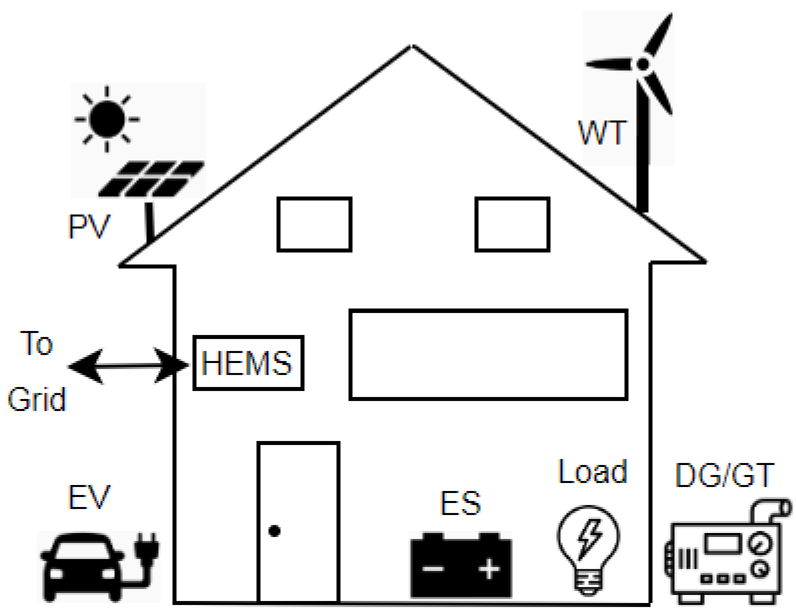

Figure 12. A general HMG with installed DERs

The pay-off function or net-profit for the *kth* HMG can be computed as

$$J_t^{HMG,k} = R_t^{HMG,k} - C_t^{HMG,k} \quad (1)$$

The revenue $R_t^{HMG,k}$ can be formulated as

$$R_t^{HMG,k} = \begin{cases} f_{sell,t}^{HMG,k} * P_t^{net,k}, & if \;\; P_t^{net,k} > 0 \\ f_{buy,t}^{HMG,k} * P_t^{net,k}, & if \;\; P_t^{net,k} < 0 \end{cases} \tag{2}$$

where

$$P_t^{net,k} = P_t^{WT,k} + P_t^{PV,k} + P_t^{DG/GT,k} \pm P_t^{ES\pm,k} - P_t^{L,k} \tag{3}$$

The cost function $C_t^{HMG,k}$ can be written as

$$C_t^{HMG,k} = \pi_t^{WT,k} + \pi_t^{PV,k} + \pi_t^{DG/GT,k} + \pi_t^{ES+,k} + S^{HMG,k} \tag{4}$$

,where the service fee $S^{HMG,k}$ is an additional amount paid by the HMG owner for power bought from the UTR. It is assumed that this fee is constant for all the HMGs.

Traditionally, production costs have been modeled as a quadratic function for synchronous generators, such as DG and GT [64], [65]. The constant term in the quadratic function accounts for the fixed costs, first-order terms account for operations and maintenance costs, and second-order terms may account for fuel costs (in combination with first-order terms). Similar quadratic functions have recently been used in the literature for energy storage dependent DERs [64], [66]. The fuel cost is almost zero for non-conventional generators, such as PV and WT, since air and solar irradiance is effectively free, so a quadratic function with a very small second-order term can be used [67], [68]. While some literature has modeled the cost of PV output using higher-order functions, terms greater than second-order terms are ignored in this work. All production costs here are modeled as quadratic functions with only a negligible loss of precision, taking into account the latest literature.

Thus, the production costs in (4) can be formulated as

$$\pi_t^{DG/GT,k} = \alpha^{DG/GT,k} * \left(P_t^{DG/GT,k}\right)^2 + \beta^{DG/GT,k} * P_t^{DG/GT,k} + \gamma^{DG/GT,k} \tag{5}$$

$$\pi_t^{PV,k} = \alpha^{PV,k} * \left(P_t^{PV,k}\right)^2 + \beta^{PV,k} * P_t^{PV,k} + \gamma^{PV,k} \tag{6}$$

$$\pi_t^{WT,k} = \alpha^{WT,k} * \left(P_t^{WT,k}\right)^2 + \beta^{WT,k} * P_t^{WT,k} + \gamma^{WT,k} \tag{7}$$

$$\pi_t^{ES,k} = \alpha^{ES,k} * \left(P_t^{ES,k}\right)^2 + \beta^{ES,k} * P_t^{ES,k} + \gamma^{ES,k} \tag{8}$$

The coefficients $\alpha^{DG/GT,k}$ and $\beta^{DG/GT,k}$ in (5) represent the running costs, such as fuel and maintenance, associated with diesel generators and/or gas turbines, whereas $\gamma^{DG/GT,k}$ represents the installation or fixed capital cost. Similarly, in (6), (7), and (8) the first and second-order coefficients represent running costs, like maintenance/cleaning of PV panels, inverters, and WT or ES, while constant terms account for fixed capital costs (i-e. onetime upfront costs).

5.2.2. UTRs

Under the proposed framework, the role of conventional utilities or retailers, i.e. UTRs, changes. Since HMGs can partially support the load of their peers, hence primary functions of UTRs may include:

- Providing maintenance support to the grid and operating the grid. They may also form a coalition with DSOs and have some stake in supporting the CA.
- Supplying power to HMGs when the combined output of HMGs is not sufficient to supply the load in an area.

Under this modified operational specification, UTRs may generate their revenue from selling power to HMGs when needed, charging HMGs a service fee for using their grid infrastructure, and from their stake in the CA. Thus, going forward the role of conventional utilities will evolve and their revenue streams may look very different from the current state of operation.

The payoff for the ith UTR may be formulated as

$$J_t^{UTR,i} = f_{sell,t}^{UTR,i} * P_{sell,t}^{UTR,i} + S_t^i \quad (9)$$

Figure 13 shows the interaction of players, i.e. HMGs and UTRs, with the CA. In addition to the conventional power lines, communication infrastructure is required to implement the proposed framework. The CA, HMGs, and UTRs connect to this shared communication bus supporting an iterative and bi-directional communication. Bid price/quantities are sent by the players to the CA, whereas optimum price/quantities are sent by the CA to the players. During iterative negotiations, instantaneous price/quantities are communicated to the players who adjust their subsequent bids. The operating interval closes once convergence to a solution occurs. This will be described later in section 5.4.

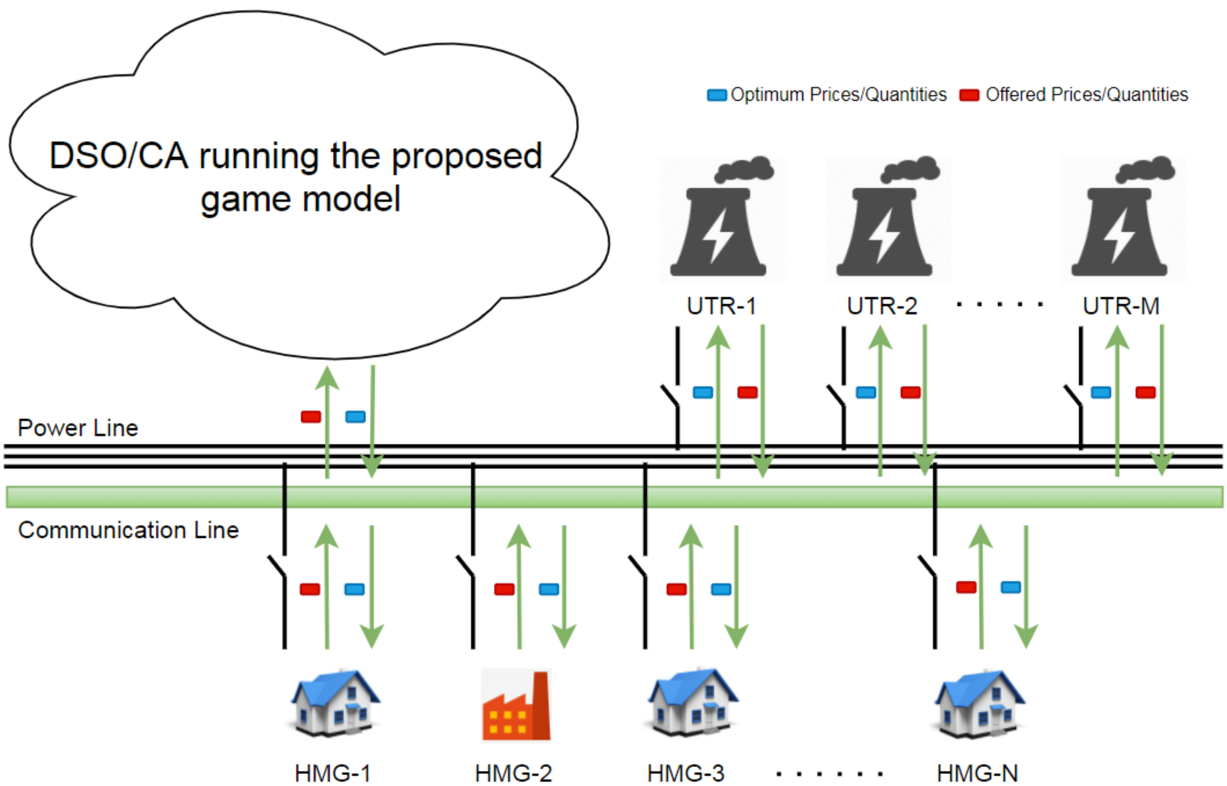

Figure 13. Interactions among HMGs, UTRs and CA (same as MO discussed in [63])

It should also be noted that (1)-(9) are subject to following constraints for each time step *t*:

$$P_{min}^{DG/GT,k} < P_t^{DG/GT,k} < P_{max}^{DG/GT,k}, \quad \forall k \tag{10}$$

$$0 < P_t^{WT,k} < P_{max}^{WT,k}, \quad \forall k \tag{11}$$

$$0 < P_t^{PV,k} < P_{max}^{PV,k}, \quad \forall k \tag{12}$$

$$0 < P_t^{ES\pm,k} < P_{max}^{ES\pm,k}, \quad \forall k \tag{13}$$

$$SOC_{min}^{k} < SOC_t^{k} < SOC_{max}^{k}, \quad \forall k \tag{14}$$

$$SOC_{t+1}^{k} - SOC_t^{k} = \frac{\left(P_{\Delta t}^{ES-,k} - P_{\Delta t}^{ES+,k}\right) * \Delta t}{\varepsilon_{ES}^{k} * ES_{cap}^{k} * 60}, \quad \forall k \tag{15}$$

$$\sum_{k=1}^{N} (P_t^{WT,k} + P_t^{PV,k} + P_t^{DG/GT,k} + P_t^{ES+,k}) + \sum_{i=1}^{M} P_t^{UTR,i} = \sum_{k=1}^{N} (P_t^{L,k} + P_t^{ES-,k}), \quad \forall k \tag{16}$$

$$PF(.) \leq 0 \tag{17}$$

, where constraints (10)-(13) enforce minimum and maximum output limits on DG/GT, WT, PV, and ES respectively. Constraint (14) provides a bound on the state of charge (SOC) for the ES. The coulomb counting-based SOC keeping of ES is implemented by (15). The SOC of the ES is updated considering current SOC, round trip cycle efficiency, charge/discharge power, and time resolution. The supply-demand balance at each time step is enforced by (16), whereas (17) implicitly defines the power flow constraints coming from electrical representation.

5.3.Steps to the Solution

Figure 14 shows the flow chart of the proposed transactive representation. Following are the steps to reaching a single optimum solution:

1. Using the predicted power output from the renewables (i.e. PV and WT), each HEMS first computes the local estimated load (within each HMG) and the maximum generation. These forecasts are updated after every HEMS refresh period (i-e. $\Delta t'$). Using these forecasts, an optimal resource allocation problem that ensures maximum consumption of local generation, while minimizing production costs, is solved. Since the HMG owner owns all the generation resources, this is set up as a normal optimization problem. At a time step t_0, the objective function in (18) is minimized by the HEMS installed in HMG $k_0 \epsilon \{1,2, \dots., N\}$ to allocate resources for the next HEMS time interval i.e. $\Delta t' = t_1 - t_0$

$$\min \ (\pi_{\Delta t'}^{WT,k_0} + \pi_{\Delta t'}^{PV,k_0} + \pi_{\Delta t'}^{DG/GT,k_0} + \pi_{\Delta t'}^{ES+,k_0}) \tag{18}$$

Subject to:

$$P_{\Delta t'}^{WT,k_0} + P_{\Delta t'}^{PV,k_0} + P_{\Delta t'}^{DG/GT,k_0} + P_{\Delta t'}^{ES+,k_0} = P_{\Delta t'}^{ES-,k_0} + P_{\Delta t'}^{L,k_0} \tag{19}$$

(10) - (15)

The decision variables in this optimization problem are the power outputs of DG/GT and ES. The power outputs of non-dispatchable sources, i.e. PV and WT, are assumed to be equal to their forecasted values for maximum efficiency and return of investment.

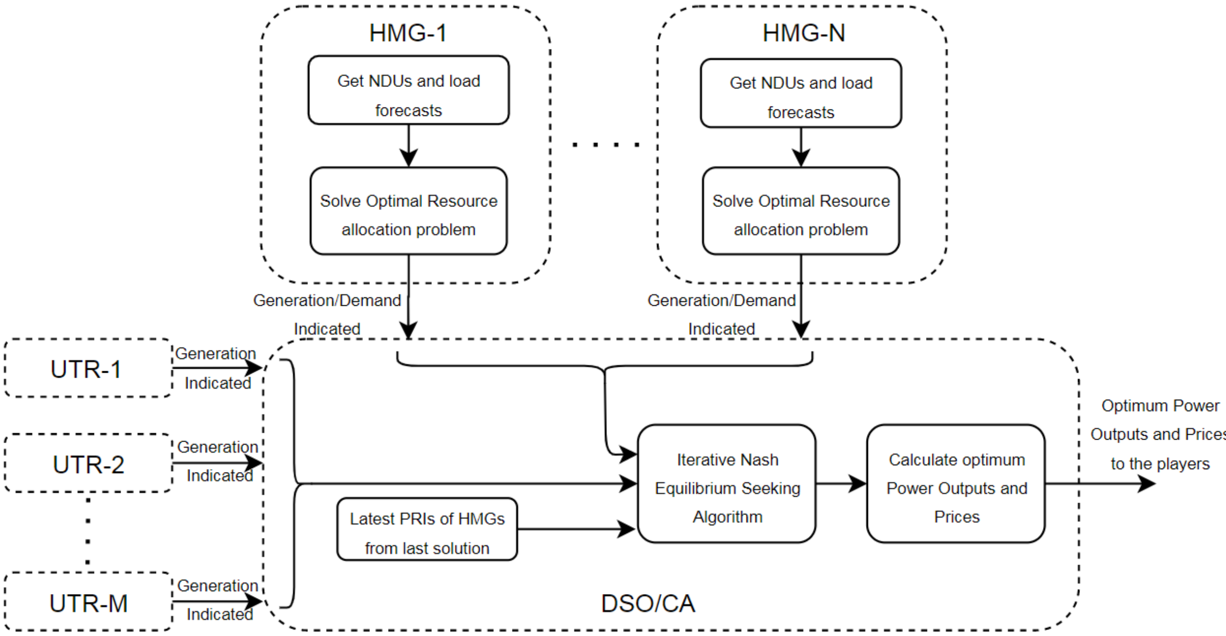

Figure 14. Steps in reaching the solution of the proposed framework

2. HEMS decides, using forecasts and local commitment decisions, whether it wants to engage as a seller or a buyer in the RTOI. If the HEMS is left with surplus production after meeting the local load, it signals the CA that it will participate as a seller for the next operating cycle (only indicating the maximum expected surplus generation at an arbitrary offer price). The HEMS indicates its position as a buyer in the event of a power shortage by sending a quantity only demand bid to the CA.

3. Once the CA receives the bids from the players, an iterative negotiating process starts which is modeled as a non-cooperative, multiplayer game. The strategy set of this game consists of the prices ($\$/kWh$) of the power ($kW$) that the players are willing to sell in the next operating period. Whereas their payoff is the net profit ($). The game model and Nash seeking process is explained in the next section. Notably, the strategies played by the players are private, i.e. they are only known by the CA, thus not compromising the privacy of the players. Once the

negotiations converge to a solution, optimum prices and quantities are established, supply and demand are balanced, and the operation ceases till the next period.

4. All the optimum parameters are regarded as binding once convergence occurs. Therefore, before re-opening the process at the end of the operating cycle, the CA compares the actual power outputs with the ones committed and updates the PRIs of all players (explained in section 5.5) accordingly. In addition to this, financial settlements are made, reflecting the financial gain that each player gains from supplying the load.

5.4. Theory – Game model and Nash seeking process

Expanding the payoff function of a general HMG as given by (1) yields:

$$J_t^{HMG,k} = R_t^{HMG,k} - C_t^{HMG,k}$$

$$= f_{sell,t}^{HMG,k} * \left(P_t^{WT,k} + P_t^{PV,k} + P_t^{DG/GT,k} \pm P_t^{ES\pm,k} - P_t^{L,k}\right) -$$

$$(\pi_t^{WT,k} + \pi_t^{PV,k} + \pi_t^{DG/GT,k} + \pi_t^{ES+,k} + S^{HMG,k})$$

Approximating the production cost from all generators by an equivalent quadratic function $\pi_t^{net,k}$ accounting for the net generation $P_t^{net,k}$ such that

$$\pi_t^{net,k} = \alpha^{net,k} * \left(P_t^{net,k}\right)^2 + \beta^{net,k} * P_t^{net,k} + \gamma^{net,k} \qquad (20)$$

Using (20), $J_t^{HMG,k}$ becomes

$$J_t^{HMG,k} = f_{sell,t}^{HMG,k} * \left(P_t^{net,k}\right) - \left(\pi_t^{net,k} + S^{HMG,k}\right)$$

$$= f_{sell,t}^{HMG,k} * \left(P_t^{net,k}\right) - \left(\alpha^{net,k} * \left(P_t^{net,k}\right)^2 + \beta^{net,k} * P_t^{net,k} + \gamma^{net,k} + S^k\right)$$

$$= -\alpha^{net,k} * \left(P_t^{net,k}\right)^2 - (\beta^{net,k} - f_{sell,t}^{HMG,k}) * P_t^{net,k} - (\gamma^{net,k} + S^k) \qquad (21)$$

Where the assumption in (20) is valid since bids from all DERs belonging to an HMG are aggregated by the HEMS. Equation (21) represents a quadratic mapping between payoff and the net generation for a player. It will be shown later that net generation $P_t^{net,k}$ is linearly related with the strategy of the player, i.e. $f_{sell,t}^{HMG,k}$. Thus, there exists a quadratic relationship between strategy set and payoff function. Such games with quadratic payoffs have a unique Nash equilibrium [69]. Thus, the players play a strategy, get a resulting payoff from the CA, and then play a new strategy till they converge to an optimum solution.

Figure 15 shows the modeling of the Nash seeking framework. For a certain operating interval denoted as Y under consideration, suppose there are N' seller HMGs and N'' buyer HMGs. Where $N' \cup N'' = N$ i.e. total number of HMGs. Additionally, there are also M UTRs.

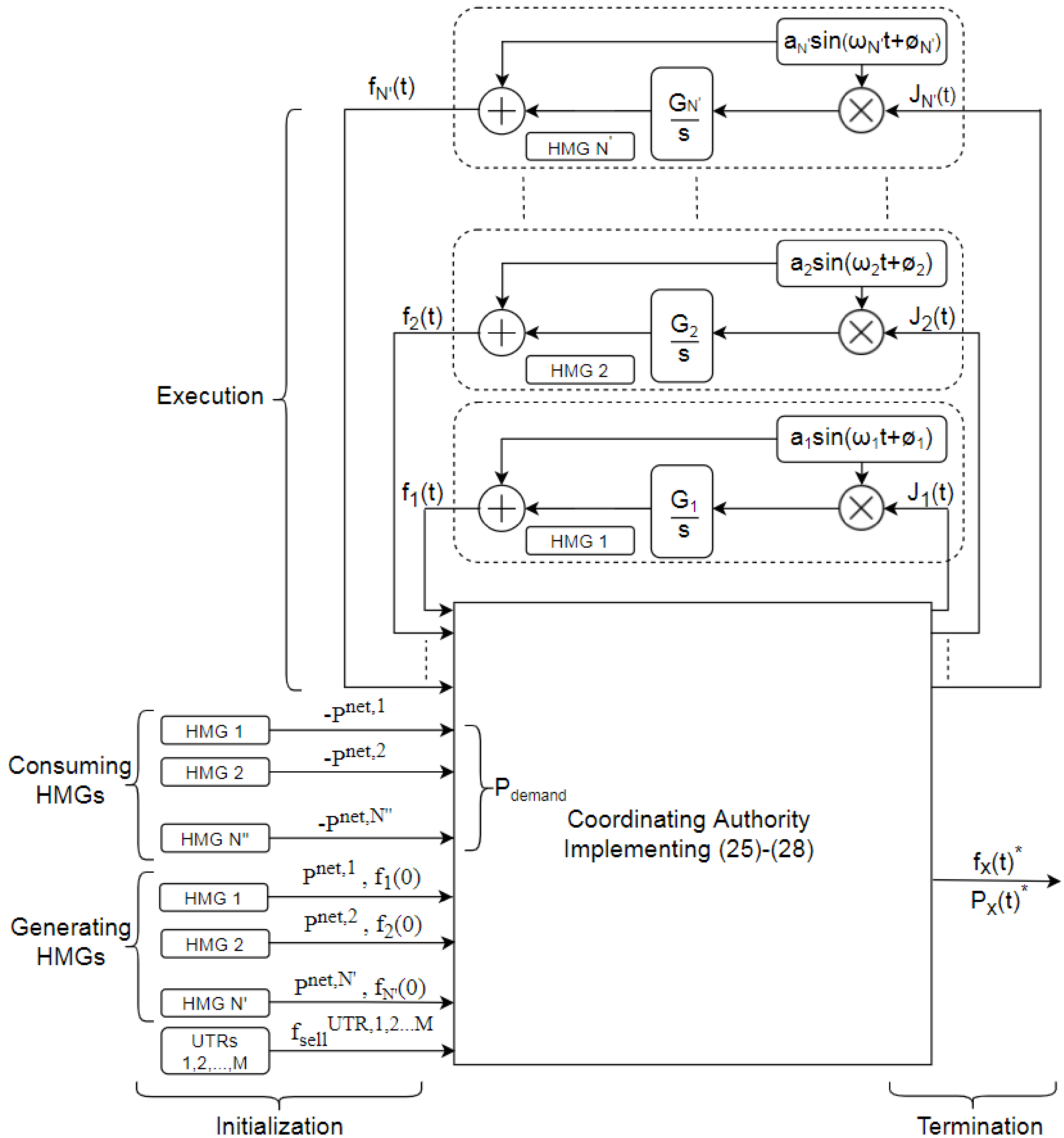

Figure 15. Modeling of the Nash equilibrium seeking behavior of the players - Interaction of the players with the game model

The three steps in a single convergence, i.e. initialization, execution, and termination, are explained below:

5.4.1. Initialization

As mentioned earlier, during the initialization phase, all buyer HMGs send a demand or quantity bid to the CA. The sum of all these demands establishes the total demand for the upcoming RTOI:

$$P_{demand} = \sum_{k=1}^{N''} P^{net,k} \tag{22}$$

Also, the seller HMGs indicate their available power outputs to the CA. They indicate the quantity they are willing to sell at an initial offer price. This price serves as the initial condition for the iterative negotiation phase. All the UTRs also indicate their initial offer prices.

5.4.2. Execution

After the initialization phase, an iterative negotiation phase starts where players play a strategy, i.e. a price placed on the quantity, they are willing to inject into the network. Once CA receives these prices, it translates them into payoffs for each player using the game model. Once players receive a new payoff, they adjust their strategies and send them to CA again. This process repeats till convergence to Nash equilibrium occurs. It is assumed that players are trying to maximize their utility, i.e. payoff, and thus seeking a Nash equilibrium. As shown in Figure 15, each player applies a sinusoidal perturbation to its strategy and measures the resulting payoff. The following equations explain the gradient-based Nash seeking behavior of the players

$$f_x(t) = F_x(t) + \frac{G_x}{s}[F_x(t)J_x(t)] \tag{23}$$

, where

$$F_x(t) = a_x \sin(\omega_x t + \emptyset_x) \tag{24}$$

and $x \in \{1,2, \dots, N'\}$. Moreover from (21), we have

$$J_x(t) = -\alpha^{net,x} * \left(P_x(t)\right)^2 - (\beta^{net,x} - f_x(t)) * P_x(t) - (\gamma^{net,x} + S^{HMG,x}) \qquad (25)$$

At the CA level strategies (prices) are translated into an instantaneous payoff for each player using a game model as proposed in Figure 16. It can be seen that the quantity sold, i.e. $P_x(t)$, is a function of the strategy played (i.e. the selling price $f_x(t)$), the Player Reputation Index (PRI) of each player μ_t^x, and the total demand P_{demand}.

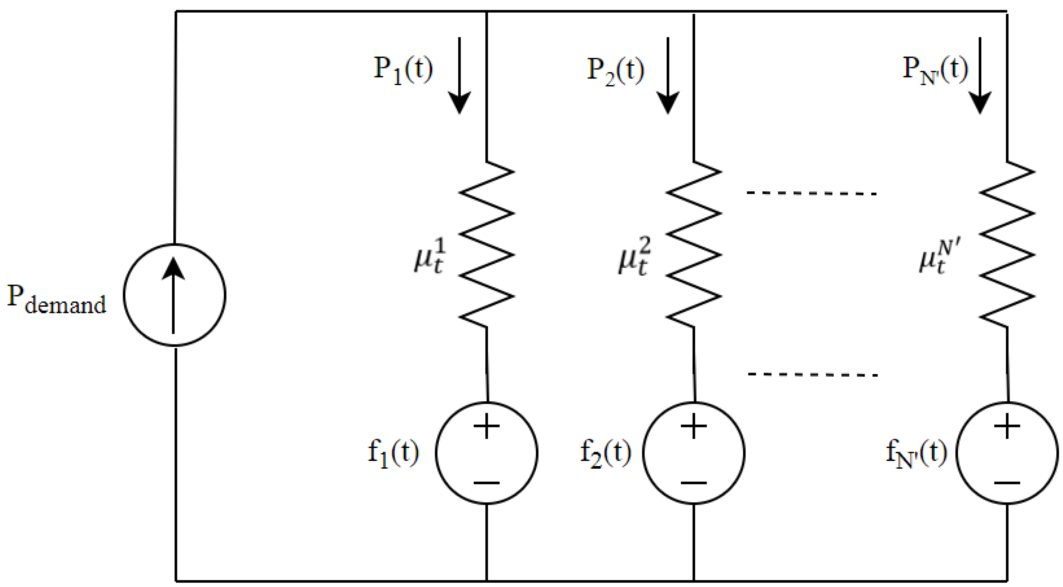

Figure 16. Game model at the CA level

The game model of Figure 16 is expressed in closed form as:

$$P_x(t) = \frac{\mu_{Y-1,||}^x}{\mu_{Y-1}^x}\left[P_{demand} - \frac{f_x(t)}{\bar{\mu}_{Y-1}^x} + \sum_{y \neq x}^{N'} \frac{f_y(t)}{\mu_{Y-1}^y}\right] \qquad (26)$$

, where

$$\frac{1}{\mu_{Y-1,||}^{x}} = \sum_{x=1}^{N'} \frac{1}{\mu_{Y-1}^{x}} \quad (27)$$

$$\frac{1}{\bar{\mu}_{Y-1}^{x}} = \sum_{y \neq x}^{N'} \frac{1}{\mu_{Y-1}^{y}} \quad (28)$$

$$\mu_t^{N'} = 1 - \frac{\sigma_t^x}{100} \quad (29)$$

,where (26) represents the quantity sold as a function of the selling price and is derived by applying the superposition principle to the model of Figure 16. Equation (27) represents the equivalent PRIs of all the players, whereas (28) represents the PRIs of all the players except the player x. Equation (29) is the complement of the PRIs, and represents the resistive element in the game model. Since payoff is directly related to optimum quantity (25), hence in conjunction with (26), it shows that the payoff is a quadratic function of the strategy set, i.e. the selling prices. Hence, this again confirms that the game under consideration is a quadratic game. From (26), here is how the relationship between the PRIs, optimum quantities, and payoffs play out in different scenarios:

- If the PRI σ_t^x of a player x decreases, and the PRIs of the other players remain unchanged, the complement of PRI, i.e. μ_t^x increases, resulting in a decreased optimum quantity $P_x(t)$ and payoff $J_x(t)$ for the player x
- If the PRI σ_t^x of a player x increases, and the PRIs of the other players remain unchanged, the complement of PRI, i.e. μ_t^x decreases, resulting in a higher optimum quantity $P_x(t)$ and payoff $J_x(t)$ for the player x

- If the PRI σ_t^x of a player x remains unchanged, and the equivalent PRI of the other players improve, the equivalent complement $\bar{\mu}_t^x$ decreases. Thus, resulting in a decreased optimum quantity $P_x(t)$ and payoff $J_x(t)$ for the player x
- If the PRI σ_t^x of a player x remains unchanged, and the equivalent PRI of the other players decrease, the equivalent complement $\bar{\mu}_t^x$ improves. Thus, resulting in a higher optimum quantity $P_x(t)$ and payoff $J_x(t)$ for the player x
- If the bid price, i.e. $f_x(t)$, for a player x decreases, the optimum quantity $P_x(t)$ and payoff $J_x(t)$ increase for player x
- If the bid price, i.e. $f_x(t)$, for a player x increases, the optimum quantity $P_x(t)$ and payoff $J_x(t)$ decrease for player x
- If the bid price, i.e. $f_x(t)$, remains unchanged for the player x, and the other players decrease their bid price $f_y(t)$, the optimum quantity $P_x(t)$ and payoff $J_x(t)$ decrease for player x
- If the bid price, i.e. $f_x(t)$, remains unchanged for the player x, and the other players increase their bid price $f_y(t)$, the optimum quantity $P_x(t)$ and payoff $J_x(t)$ increase for player x

Let us briefly examine the gradient-based Nash seeking behavior of the players before discussing the termination stage. Figure 17 shows the quadratic relationship between the payoff and the strategy, i.e. the selling price for each player, as established by (25) and (26). A player can start either on the left or right side of the optimal strategy, which lies in the middle. At the optimal strategy the utility is maximized. Here, the utility is the payoff, hence when an optimal strategy is played, payoff is maximized.

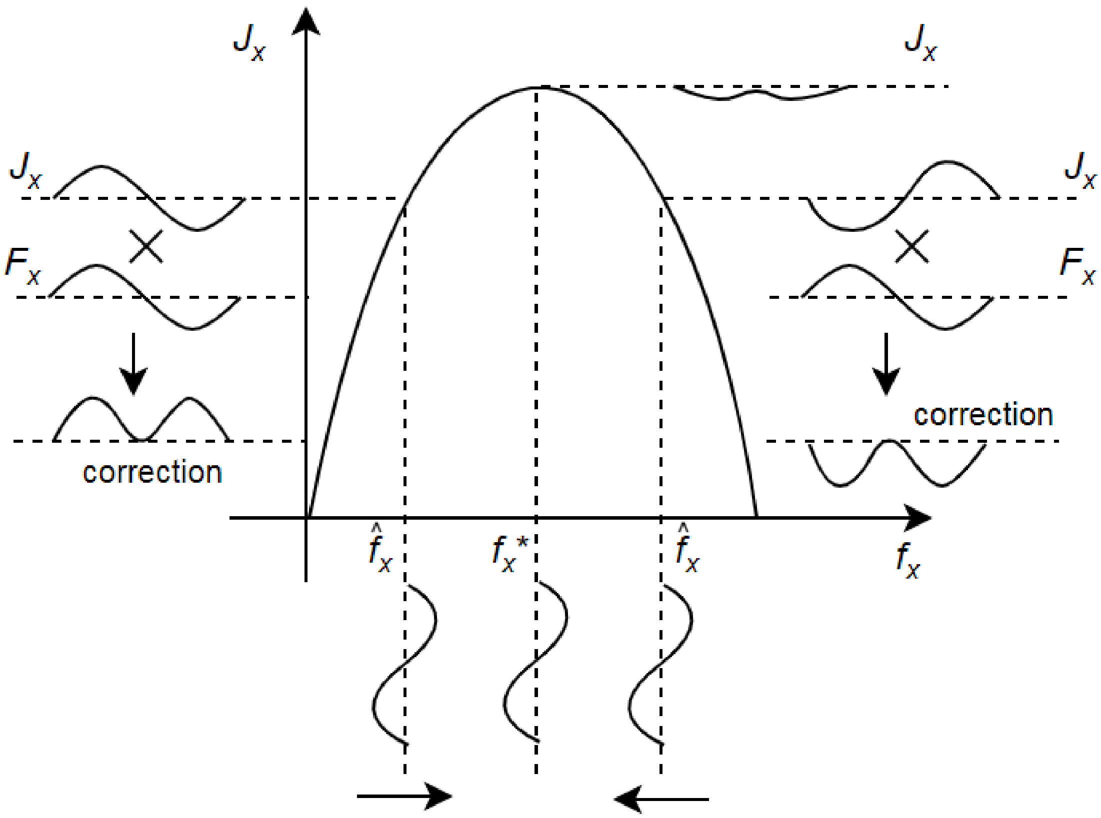

Figure 17. Gradient-based Nash seeking behavior of the players

Suppose the initial offer price of a player is less than the optimal price. This means that the player starts on the left side of the optimal price. Now, to estimate the gradient of the payoff, the player probes by applying the sinusoidal perturbations to its strategy and observes the payoff. These perturbations are translated into corresponding perturbations in the payoff returned to the player. By multiplying instantaneous payoff with originally applied perturbations, a correction factor is obtained. In this case, a positive correction is generated, indicating that the optimum lies to the right of the current strategy. Thus, the player adds this correction to its current strategy and repeats the process till it converges to a narrow band around the optimal strategy. Similarly, for a player starting on the right side of the optimum strategy, a negative correction is generated. Once the players seek Nash equilibrium using this gradient search-based approach (23)-(24) and the CA implements equations (25)-(28), the system converges to a Nash equilibrium [69].

5.4.3. Termination

Once convergence is reached, optimum power outputs and prices are established as

$$f_{sell,Y}^{HMG,x} = f_x(t)^* \qquad \forall\, x \,\epsilon\, \{1,2, \dots N'\} \tag{30}$$

$$P_{sell,Y}^{HMG,x} = P_x(t)^* \qquad \forall\, x \,\epsilon\, \{1,2, \dots N'\} \tag{31}$$

$$P_{buy,Y}^{HMG,y} = P_t^{net,y} \qquad \forall\, y \,\epsilon\, \{1,2, \dots N''\} \tag{32}$$

$$f_{buy,Y}^{HMG,y} = \left(\sum_{x=1}^{N'} f_{sell,Y}^{HMG,x}\right) * \left(\frac{P_{buy,Y}^{HMG,y}}{P_{demand}}\right) + f_{loss,Y}^{HMG,y} \qquad \forall\, y \,\epsilon\, \{1,2, \dots N''\} \tag{33}$$

where,

Y = current operating period

$(\,.\,)^*$ = value at convergence

$f_{loss,Y}^{HMG,y}$ = distribution system losses

5.5.Player Reputation Index (PRI)

The concept of PRI is introduced to ensure that the players behave in a just and transparent way towards the system and the other players. It is same as the Market Reputation Index (MRI) stated in [63]. This is a per-unit number which can be expressed in percentage as well. By retrospectively looking at the commitments made, and any deviations from commitments, CA updates this index at the end of an operating period at time *t*. Formally, it is defined as:

$$\sigma_t^x = \begin{cases} \sigma_{t-1}^x - \rho^k * \dfrac{\left|\hat{P}_{sell,t}^{HMG,x} - P_{sell,t}^{HMG,x}\right|}{P_{sell,t}^{HMG,x}}, & if \ \hat{P}_{sell,t}^{HMG,x} \neq P_{sell,t}^{HMG,x} \\ \sigma_{t-1}^x * (1 + \rho^k), & Else \end{cases} \quad (34)$$

,where ρ^k represents a constant penalizing weighting factor.

A player with a higher PRI, i.e. closer to 100 %, is ranked as a good player. As shown in the previous section, PRIs are embedded in the game model. Hence, a good rating of a player helps the player in getting a higher payoff from the CA as compared to a player with a bad rating. To improve PRIs, the players must stick to their commitments and install better equipment to enable improved forecasting of renewables and loads. Thus, this rating system also promotes technological advancements in addition to fairness.

Solutions of quadratic games can be obtained using the theoretical foundations established in this chapter. They are validated in Chapter 7 where case studies are discussed. The next chapter extends the concept introduced in this chapter to a class of generic games, i.e. non-quadratic games.

Chapter 6: Transactive Modeling Based on Non-quadratic Game Theory

6.1.Introduction

The previous chapter introduced a game theoretic-based method to optimally compute power injections (quantities sold) and compensations (payoffs) for HMGs. It focused on quadratic cost functions for the DERs, thus resulting in a quadratic game. This chapter extends the concept to a more generic class of non-quadratic games. This extension is important since the cost functions of DERs in HMGs can be non-quadratic. As the literature reports [70], [71], cost functions can be non-quadratic along with a non-linear demand. Hence, it is necessary to extend the previously discussed concepts to a general class of non-quadratic games.

The contributions of the dissertation discussed in this chapter include [72]:

- Games with non-quadratic payoff functions are discussed, i.e. games inclusive of players with non-quadratic DER cost functions.
- A generic dynamic mapping of the strategy set to payoff functions is discussed. The game model is revised accordingly to accommodate dynamic mapping.
- For generic non-quadratic games, the existence of multiple Nash equilibria is discussed. Convergence properties of stable Nash equilibrium are studied.
- It is demonstrated that when extremum seeking is used to model Nash seeking behavior of the players, the system converges to one of the stable Nash equilibrium.

Contrary to quadratic games where Nash equilibrium is unique, multiple equilibria can exist for non-quadratic games as discussed later in this chapter. Some of these multiple equilibria can be unstable [69]. In game theory, an equilibrium is classified as a stable equilibrium [73] if a small change in strategy for one player at equilibrium leads to a condition where the following are satisfied:

- “The player who did not change has no better strategy in the new circumstance”
- “The player who did change is now playing with a strictly worse strategy”

Hence, when dealing with non-quadratic games, it becomes critical to justify that the proposed Nash seeking method converges only to a stable Nash equilibrium. It is shown later in this chapter that the gradient search-based method converges only to a stable Nash equilibrium.

This chapter is organized as follows. Section 6.2 discusses non-quadratic cost functions followed by a discussion on the role of CA in Section 6.3. The modified dynamic transactive game model is discussed in Section 6.4. Solution stability and convergence characteristics are examined in Section 6.5.

6.2.Non-quadratic polynomial cost functions

The following non-quadratic polynomials are used to model the cost functions of the DERs within an HMG:

$$\pi_t^{DG/GT,k} = \alpha_n^{DG/GT,k} * \left(P_t^{DG/GT,k}\right)^n + \alpha_{n-1}^{DG/GT,k} * \left(P_t^{DG/GT,k}\right)^{n-1} + \cdots + \alpha_1^{DG/GT,k} * P_t^{DG/GT,k} + \alpha_0^{DG/GT,k} \tag{35}$$

$$\pi_t^{PV,k} = \alpha_n^{PV,k} * \left(P_t^{PV,k}\right)^n + \alpha_{n-1}^{PV,k} * \left(P_t^{PV,k}\right)^{n-1} + \cdots + \alpha_1^{PV,k} * P_t^{PV,k} + \alpha_0^{PV,k} \tag{36}$$

$$\pi_t^{WT,k} = \alpha_n^{WT,k} * \left(P_t^{WT,k}\right)^n + \alpha_{n-1}^{WT,k} * \left(P_t^{WT,k}\right)^{n-1} + \cdots + \alpha_1^{WT,k} * P_t^{WT,k} + \alpha_0^{WT,k} \tag{37}$$

$$\pi_t^{ES,k} = \alpha_n^{ES,k} * \left(P_t^{ES,k}\right)^n + \alpha_{n-1}^{ES,k} * \left(P_t^{ES,k}\right)^{n-1} + \cdots + \alpha_1^{ES,k} * P_t^{ES,k} + \alpha_0^{ES,k} \tag{38}$$

where,

$\alpha_n^{(.),k} =$ nth-order coefficient for production cost function of resource $(.)$ in kth HMG, $\forall k \epsilon \{1,2, \ldots., N\}$

These non-quadratic polynomials replace the quadratic functions (5)-(8) used previously for the quadratic case. Equations (1)-(4) remain valid for the HMGs in this chapter.

Similarly (9) for UTRs and constraints (10)-(17) remain applicable in this chapter.

6.3.Role of the Coordinating Authority

The primary responsibilities of the CA include:

- Initiate the operating cycle by inviting bids from the players
- Run the iterative execution
- Maintain a PRI for each player
- Once a stable convergence is reached, communicate the optimum prices and quantities to the players.

The stages of initialization, iterative execution, and termination are shown in Figure 18. Though it looks similar, this process is slightly different from the quadratic case discussed earlier. Specifically, it differs in the following aspects:

- Non-quadratic cost functions result in non-quadratic payoff functions for HMGs. These generic cost characteristics are accommodated for the DERs within HMGs
- A different game model, i.e. a dynamic game model, is implemented at the CA level as explained in the next section

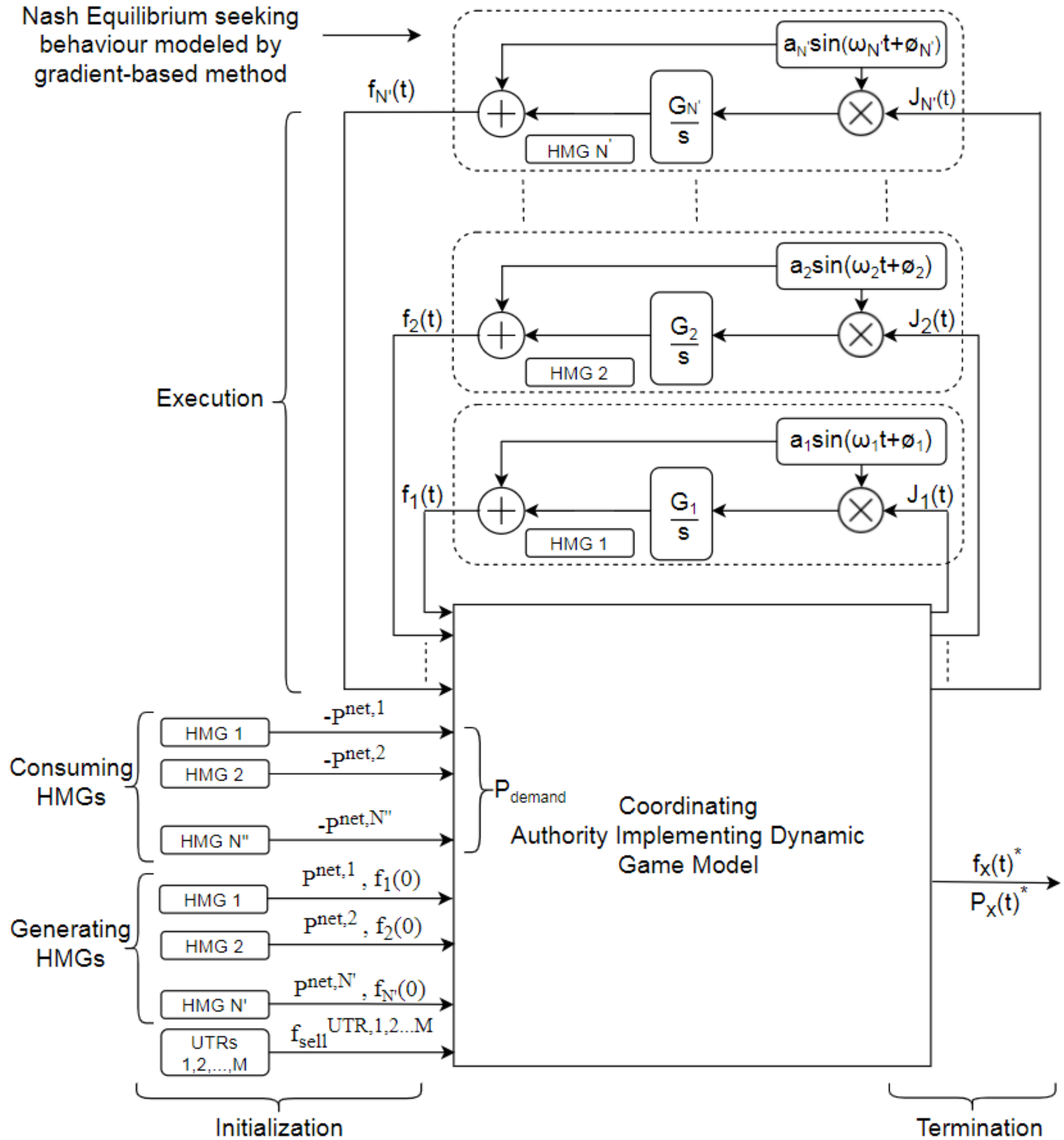

Figure 18. Iterative execution and modeling the Nash seeking behavior of the players

6.4.Dynamic Game Model Formulation

Figure 19 shows a revised version of the game model discussed previously. It includes an inductance of value 1 to model the process dynamics as explained below.

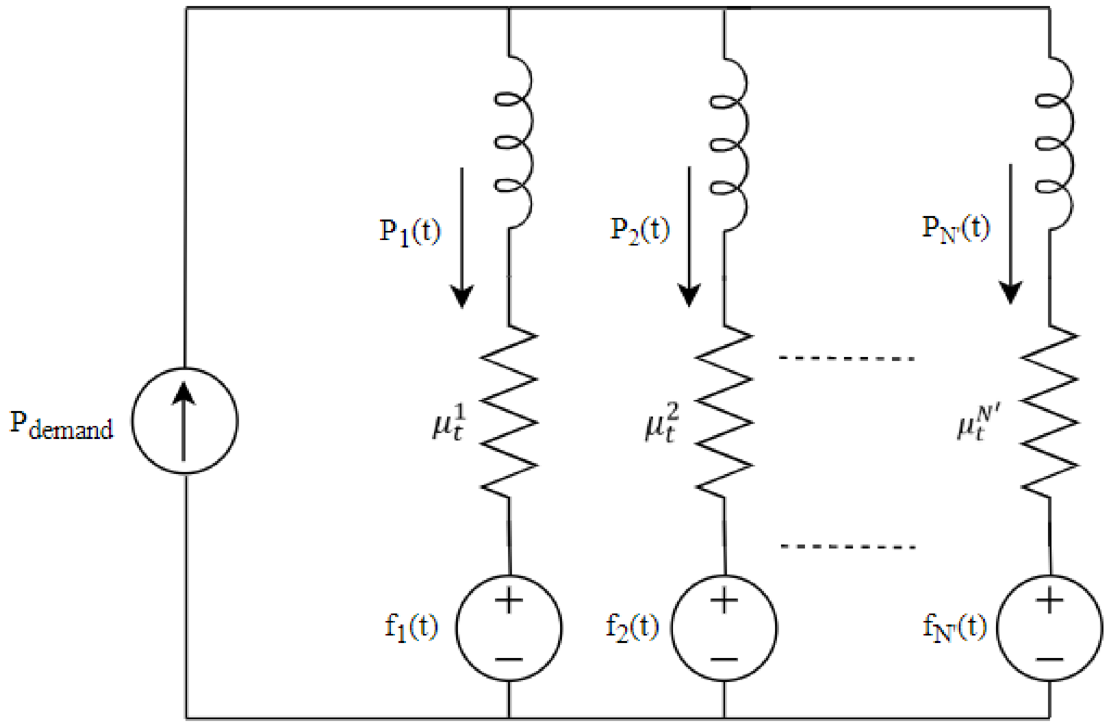

Figure 19. Dynamic game model implemented at the CA level

Considering the model of Figure 19, the following dynamic system represents a generic non-quadratic game consisting of *N*' seller HMGs:

$$\dot{P}_1(t) = -\frac{\mu_{Y-1}^1}{\mu_{Y-1,||}} P_1(t) - \frac{1}{\bar{\mu}_{Y-1}^1} f_1(t) + \left\{ \frac{1}{\mu_{Y-1}^2} f_2(t) + \frac{1}{\mu_{Y-1}^3} f_3(t) + \cdots + \frac{1}{\mu_{Y-1}^{N'}} f_{N'}(t) \right\} + P_{demand} \quad (39)$$

$$\dot{P}_2(t) = -\frac{\mu_{Y-1}^2}{\mu_{Y-1,||}} P_2(t) - \frac{1}{\bar{\mu}_{Y-1}^2} f_2(t) + \left\{ \frac{1}{\mu_{Y-1}^1} f_1(t) + \frac{1}{\mu_{Y-1}^3} f_3(t) + \cdots + \frac{1}{\mu_{Y-1}^{N'}} f_{N'}(t) \right\} + P_{demand} \quad (40)$$

$$\vdots$$

$$\dot{P}_{N'}(t) = -\frac{\mu_{Y-1}^{N'}}{\mu_{Y-1,||}} P_{N'}(t) - \frac{1}{\bar{\mu}_{Y-1}^{N'}} f_{N'}(t) + \left\{ \frac{1}{\mu_{Y-1}^1} f_1(t) + \frac{1}{\mu_{Y-1}^2} f_2(t) + \cdots + \frac{1}{\mu_{Y-1}^{N'-1}} f_{N'-1}(t) \right\} + P_{demand} \quad (41)$$

$$J_1(t) = a_{11} P_1(t)^n + a_{12} P_1(t)^{n-1} + \cdots + a_{1n} P_1(t) + a_{00} P_1(t)^{k_1} P_2(t)^{k_2} \ldots . P_{N'}(t)^{k_{N'}} + \left(\alpha_0^{net,1} + S^{HMG,1}\right) \quad (42)$$

$$J_2(t) = a_{21} P_2(t)^n + a_{22} P_2(t)^{n-1} + \cdots + a_{2n} P_2(t) + a_{00} P_1(t)^{k_1} P_2(t)^{k_2} \ldots . P_{N'}(t)^{k_{N'}} + \left(\alpha_0^{net,2} + S^{HMG,2}\right) \quad (43)$$

$$\vdots$$

$$J_{N'}(t) = a_{N'1}P_{N'}(t)^n + a_{N'2}P_{N'}(t)^{n-1} + \cdots + a_{N'n}P_{N'}(t) + a_{00}P_1(t)^{k_1}P_2(t)^{k_2} \dots . P_{N'}(t)^{k_{N'}} + \left(\alpha_0^{net,N'} + S^{HMG,N'}\right) \tag{44}$$

, where

$$\frac{1}{\mu_{Y-1,||}^{x}} = \sum_{x=1}^{N'} \frac{1}{\mu_{Y-1}^{x}} \tag{45}$$

$$\frac{1}{\bar{\mu}_{Y-1}^{x}} = \sum_{y \neq x}^{N'} \frac{1}{\mu_{Y-1}^{y}} \tag{46}$$

$$\mu_t^{N'} = 1 - \frac{\sigma_t^x}{100} \tag{47}$$

$$\sigma_t^x = \begin{cases} \sigma_{t-1}^x - \rho^k * \frac{\left|\hat{P}_{sell,t}^{HMG,x} - P_{sell,t}^{HMG,x}\right|}{P_{sell,t}^{HMG,x}}, & if \;\; \hat{P}_{sell,t}^{HMG,x} \neq P_{sell,t}^{HMG,x} \\ \sigma_{t-1}^x * (1 + \rho^k), & Else \end{cases} \tag{48}$$

, where

ρ^k = a constant weighting penalization factor

n = degree of polynomial for payoff functions

$P_x(t)\ \forall\ x \in \{1,2, \dots N'\}$ = the state variable of the dynamic system

The *N*' differential equations (39)-(41) mathematically represent the game model of Figure 19. Moreover, the *N*' payoff equations (42)-(44) represent the non-quadratic payoff functions of the players.

To illustrate how multiple equilibria exist for non-quadratic games, by setting $\dot{P}_x(t) = 0 \; \forall \, x \in \{1,2,\ldots,N'\}$ the equilibrium state of the dynamic system is obtained as:

$$-\frac{\mu_{Y-1}^{1}}{\mu_{Y-1,||}}P_1(t) - \frac{1}{\bar{\mu}_{Y-1}^{1}}f_1(t) + \left\{\frac{1}{\mu_{Y-1}^{2}}f_2(t) + \frac{1}{\mu_{Y-1}^{3}}f_3(t) + \cdots + \frac{1}{\mu_{Y-1}^{N'}}f_{N'}(t)\right\} + P_{demand} = 0$$

$$-\frac{\mu_{Y-1}^{2}}{\mu_{Y-1,||}}P_2(t) - \frac{1}{\bar{\mu}_{Y-1}^{2}}f_2(t) + \left\{\frac{1}{\mu_{Y-1}^{1}}f_1(t) + \frac{1}{\mu_{Y-1}^{3}}f_3(t) + \cdots + \frac{1}{\mu_{Y-1}^{N'}}f_{N'}(t)\right\} + P_{demand} = 0$$

$$\vdots$$

$$-\frac{\mu_{Y-1}^{N'}}{\mu_{Y-1,||}}P_{N'}(t) - \frac{1}{\bar{\mu}_{Y-1}^{N'}}f_{N'}(t) + \left\{\frac{1}{\mu_{Y-1}^{1}}f_1(t) + \frac{1}{\mu_{Y-1}^{2}}f_2(t) + \cdots + \frac{1}{\mu_{Y-1}^{N'-1}}f_{N'-1}(t)\right\} + P_{demand} = 0$$

This leads to the state variable vector at equilibrium as:

$$\begin{bmatrix} \bar{P}_1(t) \\ \bar{P}_2(t) \\ \vdots \\ \bar{P}_{N'}(t) \end{bmatrix} = \begin{bmatrix} \frac{\mu_{Y-1,||}}{\mu_{Y-1}^{1}}\left\{P_{demand} - \frac{1}{\bar{\mu}_{Y-1}^{1}}f_1(t) + \left(\frac{1}{\mu_{Y-1}^{2}}f_2(t) + \frac{1}{\mu_{Y-1}^{3}}f_3(t) + \cdots + \frac{1}{\mu_{Y-1}^{N'}}f_{N'}(t)\right)\right\} \\ \frac{\mu_{Y-1,||}}{\mu_{Y-1}^{2}}\left\{P_{demand} - \frac{1}{\bar{\mu}_{Y-1}^{2}}f_2(t) + \left(\frac{1}{\mu_{Y-1}^{1}}f_1(t) + \frac{1}{\mu_{Y-1}^{3}}f_3(t) + \cdots + \frac{1}{\mu_{Y-1}^{N'}}f_{N'}(t)\right)\right\} \\ \vdots \\ \frac{\mu_{Y-1,||}}{\mu_{Y-1}^{N'}}\left\{P_{demand} - \frac{1}{\bar{\mu}_{Y-1}^{N'}}f_{N'}(t) + \left(\frac{1}{\mu_{Y-1}^{1}}f_1(t) + \frac{1}{\mu_{Y-1}^{2}}f_2(t) + \cdots + \frac{1}{\mu_{Y-1}^{N'-1}}f_{N'-1}(t)\right)\right\} \end{bmatrix} \tag{49}$$

From (42)-(44), the payoff functions at equilibrium are:

$$J_1(t) = a_{11}\bar{P}_1(t)^n + a_{12}\bar{P}_1(t)^{n-1} + \cdots + a_{1n}\bar{P}_1(t) + a_{00}\bar{P}_1(t)^{k_1}\bar{P}_2(t)^{k_2} \ldots . \bar{P}_{N'}(t)^{k_{N'}} + (\alpha_0^{net,1} + S^{HMG,1})$$

$$J_2(t) = a_{21}\bar{P}_2(t)^n + a_{22}\bar{P}_2(t)^{n-1} + \cdots + a_{2n}\bar{P}_2(t) + a_{00}\bar{P}_1(t)^{k_1}\bar{P}_2(t)^{k_2} \ldots . \bar{P}_{N'}(t)^{k_{N'}} + (\alpha_0^{net,2} + S^{HMG,2})$$

$$\vdots$$

$$J_{N'}(t) = a_{N'1}\bar{P}_{N'}(t)^n + a_{N'2}\bar{P}_{N'}(t)^{n-1} + \cdots + a_{N'n}\bar{P}_{N'}(t) + a_{00}\bar{P}_1(t)^{k_1}\bar{P}_2(t)^{k_2} \ldots . \bar{P}_{N'}(t)^{k_{N'}} + (\alpha_0^{net,N'} + S^{HMG,N'})$$

Since state variable at equilibrium is a function of PRIs and strategies as shown by (49) i-e. $\bar{P}_x(t) = g_x\big(f_1(t), f_2(t), \dots, f_{N'}(t), \mu^1_{Y-1}, \mu^2_{Y-1}, \dots, \mu^{N'}_{Y-1}\big)\ \forall\ x\ \in \{1,2,\dots,N'\}$, hence

$$J_x(t) = h_x\big(f_1(t), f_2(t), \dots, f_{N'}(t), \mu^1_{Y-1}, \mu^2_{Y-1}, \dots, \mu^{N'}_{Y-1}\big) \forall\ x\ \in \{1,2,\dots,N'\}$$

In expanded form, this becomes

$$J_1(t) = h_1\big(f_1(t), f_2(t), \dots, f_{N'}(t), \mu^1_{Y-1}, \mu^2_{Y-1}, \dots, \mu^{N'}_{Y-1}\big)$$

$$J_2(t) = h_2\big(f_1(t), f_2(t), \dots, f_{N'}(t), \mu^1_{Y-1}, \mu^2_{Y-1}, \dots, \mu^{N'}_{Y-1}\big)$$

$$\vdots$$

$$J_{N'}(t) = h_{N'}\big(f_1(t), f_2(t), \dots, f_{N'}(t), \mu^1_{Y-1}, \mu^2_{Y-1}, \dots, \mu^{N'}_{Y-1}\big)$$

When players seek Nash equilibrium, their objective is to maximize their payoffs with respect to their strategies. The quantity to be maximized is $\frac{\partial J_x(t)}{\partial f_x(t)}\ \forall\ x\ \in \{1,2,\dots,N'\}$,

$$\frac{\partial J_1(t)}{\partial f_1(t)} = \frac{\partial h_1\big(f_1(t), f_2(t), \dots, f_{N'}(t), \mu^1_{Y-1}, \mu^2_{Y-1}, \dots, \mu^{N'}_{Y-1}\big)}{\partial f_1(t)} = q_1(t)$$

$$\frac{\partial J_2(t)}{\partial f_2(t)} = \frac{\partial h_2\big(f_1(t), f_2(t), \dots, f_{N'}(t), \mu^1_{Y-1}, \mu^2_{Y-1}, \dots, \mu^{N'}_{Y-1}\big)}{\partial f_2(t)} = q_2(t)$$

$$\vdots$$

$$\frac{\partial J_{N'}(t)}{\partial f_{N'}(t)} = \frac{\partial h_{N'}\big(f_1(t), f_2(t), \dots, f_{N'}(t), \mu^1_{Y-1}, \mu^2_{Y-1}, \dots, \mu^{N'}_{Y-1}\big)}{\partial f_{N'}(t)} = q_{N'}(t)$$

This leads to *N*’ equations in *N*’ variables and the *N*’ solutions of these equations represent the multiple Nash equilibria to which the system may converge. This is by definition, at Nash equilibrium, the payoff becomes constant irrespective of the strategy played, i.e. partial derivative $\frac{\partial J_x(t)}{\partial f_x(t)}$ goes to zero. Thus, there exist N' possible Nash equilibria for this dynamic system.

Setting $\frac{\partial J_x(t)}{\partial f_x(t)} = 0 \ \forall x \in \{1,2,\dots,N^{'}\}$,

$$q_1(t) = \frac{\partial h_1\left(f_1(t), f_2(t), \dots, f_{N'}(t), \mu_{Y-1}^1, \mu_{Y-1}^2, \dots, \mu_{Y-1}^{N'}\right)}{\partial f_1(t)} = 0 \tag{50}$$

$$q_2(t) = \frac{\partial h_2\left(f_1(t), f_2(t), \dots, f_{N'}(t), \mu_{Y-1}^1, \mu_{Y-1}^2, \dots, \mu_{Y-1}^{N'}\right)}{\partial f_2(t)} = 0 \tag{51}$$

$$\vdots$$

$$q_{N'}(t) = \frac{\partial h_{N'}\left(f_1(t), f_2(t), \dots, f_{N'}(t), \mu_{Y-1}^1, \mu_{Y-1}^2, \dots, \mu_{Y-1}^{N'}\right)}{\partial f_{N'}(t)} = 0 \tag{52}$$

The solution of these N' equations yields multiple Nash equilibria. Hence, for non-quadratic games, multiple Nash equilibria exist and the system may converge to one of the stable equilibrium, depending on the initial condition. However, not all equilibria among N' equilibria are necessarily stable.

Once the convergence occurs for iteration Y, optimum prices and quantities are established as:

$$f_{sell,Y}^{HMG,x} = f_x(t)^* \qquad \forall\, x \in \{1,2, \dots N'\} \tag{53}$$

$$P_{sell,Y}^{HMG,x} = P_x(t)^* \qquad \forall\, x \in \{1,2, \dots N'\} \tag{54}$$

$$P_{buy,Y}^{HMG,y} = P_t^{net,y} \qquad \forall\, y \in \{1,2, \dots N''\} \tag{55}$$

$$f_{buy,Y}^{HMG,y} = \left(\sum_{x=1}^{N'} f_{sell,Y}^{HMG,x}\right) * \left(\frac{P_{buy,Y}^{HMG,y}}{P_{demand}}\right) \qquad \forall\, y \in \{1,2, \dots N''\} \tag{56}$$

, where $(\,.\,)^*$ denotes the value at stable Nash equilibrium.

6.5. Stability and Convergence Analysis

The stability of an equilibrium $f^*(t)$ for a non-linear dynamic system can be determined by examining the eigenvalues of the Jacobian matrix [74], given as follows:

$$\varphi\big(f^*(t)\big) = \begin{bmatrix} \frac{\partial q_1(t)}{\partial f_1(t)} & \frac{\partial q_1(t)}{\partial f_2(t)} & \frac{\partial q_1(t)}{\partial f_3(t)} & \cdots & \frac{\partial q_1(t)}{\partial f_{N'}(t)} \\ \frac{\partial q_2(t)}{\partial f_1(t)} & \frac{\partial q_2(t)}{\partial f_2(t)} & \frac{\partial q_2(t)}{\partial f_3(t)} & \cdots & \frac{\partial q_2(t)}{\partial f_{N'}(t)} \\ \frac{\partial q_3(t)}{\partial f_1(t)} & \frac{\partial q_3(t)}{\partial f_2(t)} & \frac{\partial q_3(t)}{\partial f_3(t)} & \cdots & \frac{\partial q_3(t)}{\partial f_{N'}(t)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial q_{N'}(t)}{\partial f_1(t)} & \frac{\partial q_{N'}(t)}{\partial f_2(t)} & \frac{\partial q_{N'}(t)}{\partial f_3(t)} & \cdots & \frac{\partial q_{N'}(t)}{\partial f_{N'}(t)} \end{bmatrix} \tag{57}$$

For a stable equilibrium, all eigenvalues of $\varphi(f^*(t))$ must be negative real. Thus, the equilibrium that yields negative real eigenvalues for (57) is a stable equilibrium. Even if the real part of a single eigenvalue becomes positive, then the equilibrium is rendered unstable. In general, the number of eigenvalues is equal to the number of players involved, i.e. the dimensions of the state variable.

The Nash seeking behavior of the players is modeled as the gradient search-based method (also known as the extremum seeking method), given as:

$$f_x(t) = F_x(t) + \frac{G_x}{s}\left[F_x(t)J_x(t)\right] \tag{58}$$

, where

$$F_x(t) = a_x \sin(\omega_x t + \emptyset_x) \tag{59}$$

and $x \in \{1,2,\dots,N'\}$.

Next, it is shown that when the Nash seeking behavior is modeled using the gradient-based method (58)-(59), then the system converges to one of the stable Nash equilibria depending on the initial condition. This is because, by definition, an unstable equilibrium is such that a small disturbance drives the system away from that equilibrium. In the gradient-based approach, since the players apply sinusoidal perturbations all the time (even when at stable equilibrium), hence they never converge to an unstable equilibrium. In addition to its computational simplicity, this is an additional advantage of this method.

6.5.1. Numerical Example

Consider the following example system:

$$\dot{P}_1(t) = -P_1(t) - 3f_1(t) + 6f_2(t) + 2 \tag{60}$$

$$\dot{P}_2(t) = -P_2(t) + f_1(t) + 3f_2(t) + 1 \tag{61}$$

$$J_1(t) = -4P_1(t)^2 + 8P_1(t)P_2(t) - 4P_1(t) \tag{62}$$

$$J_2(t) = -P_2(t)^3 + 5P_1(t)P_2(t) \tag{63}$$

For this system, the equilibrium values for the state variables $P_1(t)$ and $P_2(t)$ are obtained by setting $\dot{P}_1(t) = \dot{P}_1(t) = 0$ in (60)-(61) yielding

$$\begin{bmatrix} \bar{P}_1(t) \\ \bar{P}_2(t) \end{bmatrix} = \begin{bmatrix} -3f_1(t) + 6f_2(t) + 2 \\ f_1(t) + 3f_2(t) + 1 \end{bmatrix}$$

Substituting these values in (62) and (63) result in payoff functions at equilibrium which can be simplified to:

$$J_1(t) = -60f_1(t)^2 + 120f_1(t)f_2(t) + 52f_1(t) - 8$$

$$J_2(t) = -f_1(t)^3 - 27f_1(t)f_2(t)^2 - 8f_1(t) - 9f_1(t)^2f_2(t) - 33f_1(t)f_2(t) - 18f_1(t)^2$$
$$- 27f_2(t)^3 + 51f_2(t) + 63f_2(t)^2 + 9$$

Next, taking the partial derivatives with respect to the strategy set, utility functions are obtained as:

$$\frac{\partial J_1(t)}{\partial f_1(t)} = q_1(t) = -120f_1(t) + 120f_2(t) + 52$$

$$\frac{\partial J_2(t)}{\partial f_2(t)} = q_2(t) = -54f_1(t)f_2(t) - 9f_1(t)^2 - 33f_1(t) - 81f_2(t)^2 + 126f_2(t) + 51$$

Forcing $q_1(t) = 0$, $q_2(t) = 0$ and subsequently solving for $f_1(t)$ and $f_2(t)$ yields the following two solutions for equilibria

$$(f_1(t)^*, f_2(t)^*)_1 = (0.327, -0.323)$$

$$(f_1(t)^*, f_2(t)^*)_2 = (1.185, 0.752)$$

The Jacobian matrix to determine the stability of these equilibria is given as,

$$\varphi = \begin{bmatrix} \frac{\partial q_1(t)}{\partial f_1(t)} & \frac{\partial q_1(t)}{\partial f_2(t)} \\ \frac{\partial q_2(t)}{\partial f_1(t)} & \frac{\partial q_2(t)}{\partial f_2(t)} \end{bmatrix} = \begin{bmatrix} -120 & 120 \\ -54f_2(t) - 18f_1(t) - 33 & -54f_1(t) - 162f_2(t) + 126 \end{bmatrix}$$

The Jacobian matrix computed at the solution $(f_1(t)^*, f_2(t)^*)_1$ is

$$\varphi((f_1(t)^*, f_2(t)^*)_1) = \begin{bmatrix} -120 & 120 \\ -21.44 & 160.66 \end{bmatrix}$$

The eigenvalues of this matrix are 151.17 and -110.51. Since all the eigenvalues are not negative, this represents an unstable equilibrium.

At the other solution i.e. $(f_1(t)^*, f_2(t)^*)_2$, the Jacobian matrix becomes

$$\varphi((f_1(t)^*, f_2(t)^*)_2) = \begin{bmatrix} -120 & 120 \\ -94.93 & -59.81 \end{bmatrix}$$

The eigenvalues of this matrix are $-89.9 + j102.4$ and $-89.9 - j102.4$. This is a stable equilibrium since the real parts of both eigenvalues are negative. As previously stated, when the players employ the gradient search-based method, the system should converge to this stable Nash equilibrium.

This results of this numerical example are verified by simulating this system in MATLAB. Figure 20 shows the convergence to a stable Nash equilibrium i.e. $(f_1(t)^*, f_2(t)^*)_2 = (1.185, 0.752)$.

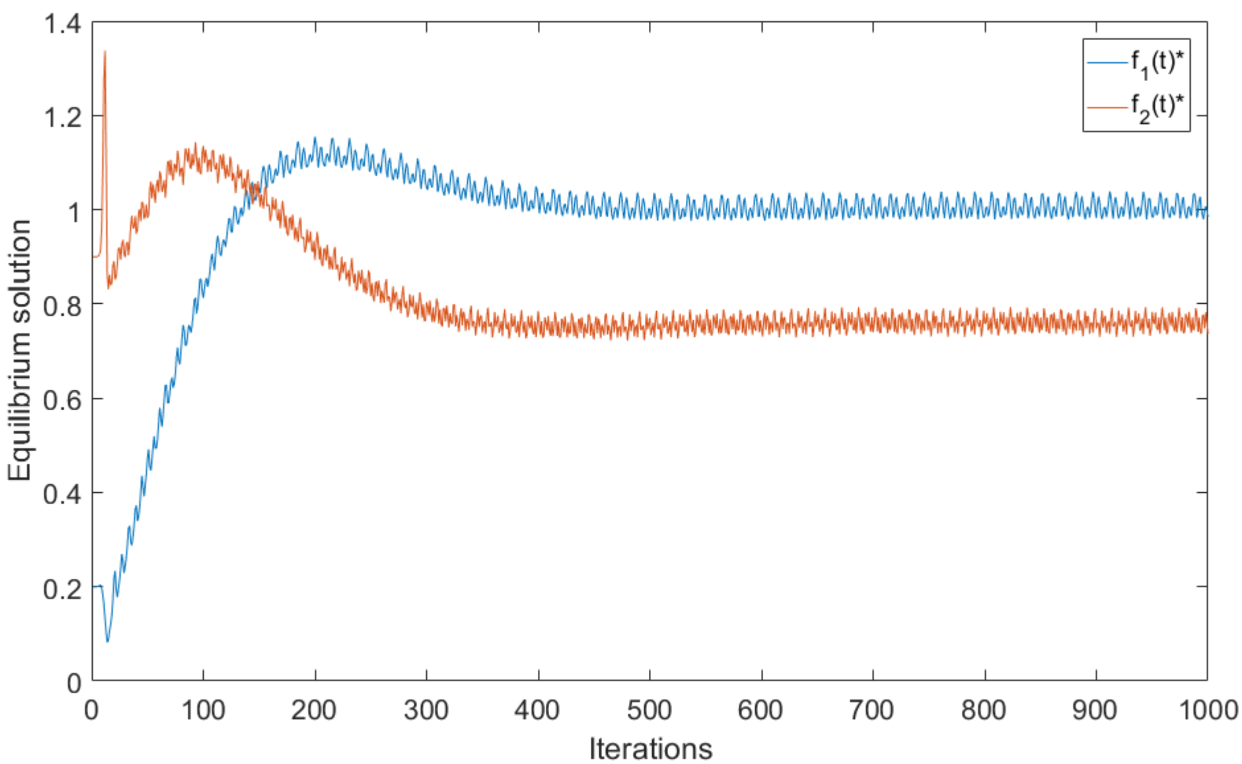

Figure 20. Verifying the result of a numerical example in MATLAB

The stability and convergence analysis in this section illustrates that when multiple Nash equilibria exist, the system converges to a stable equilibrium if gradient-based, Nash seeking approach is used. It should be noted that the computational complexities in this section do not pose any hindrance to the practical implementation of the proposed framework since the Jacobian matrix computations are not required in the actual implementation. Next, the case studies and results are discussed that provide insight into the benefits of deploying the proposed framework.

Chapter 7: Case Studies and Results

The proposed framework is implemented and case studies are performed to establish the significance of the work.

7.1.Case Study 1

Initially, in Case study 1, two cases are compared that include:

- Case-I: Without the proposed framework, 6 HMGs are connected to the UTRs to establish a base case in the IEEE 13-bus feeder shown in Figure 10.
- Case-II: The 6 HMGs and UTRs of Case-I are integrated into the proposed transactive framework.

As discussed earlier, HMGs are modeled generically, i.e. they may have all or some of the DERs installed. The case study configuration data is shown in Table 2. The cost functions for DERs are chosen such that the energy produced from DERs (subsequently HMGs) is cheaper than that offered by the UTRs. This is a valid assumption since the HMG owner would install a DER only if it produces cheaper energy than that supplied by UTRs. For simplicity, all the UTRs are combined into a single representative UTR when presenting the results.

A complete day is simulated in this case study using the case configuration profiles shown in Figure 21. These initialization profiles include hourly averaged PV output predictions, WT output predictions, and load predictions for all the HMGs. The payoffs of all DERs in this case study are assumed to be quadratic functions. To simulate realism, HMG-5 has been deliberately initiated without any PV. Similarly, HMG-3 has no WT generation. The DER generation in HMG-2 is kept lower than its net load at all times to compare HMGs with higher and lower penetrations of DERs. Once the electrical-transactive co-simulation begins, players measure their instantaneous payoffs, perturb their strategy according to gradient-based (extremum seeking method), and measure the new payoff. This process continues to iterate till convergence to Nash equilibrium occurs and no power flow constraint is violated for the 13-bus feeder. The data exchange between electrical and transactive representations is managed by HELICS.

Table 2. Configuration data for Case Study 1

Case Study Configuration data		
Game Parameters	Number of HMGs, N	6
	Number of UTRs, M	2
	Type of Game	Static
	RTOI time interval, Δt	5 minutes
	HEMS refresh interval, $\Delta t'$	5 seconds
Extremum Seeking Parameters for HMGs	a^k	0.05 $\forall$ k
	ω^k	{30, 24, 44, 36, 40, 36} rad/sec
	$\emptyset^k$	0 $\forall$ k
PV system	Maximum Power Output, $P_{max}^{PV,k}$	{3.3, 1.1, 4.2, 5.1, 0, 2} kW
WT system	Maximum Power Output, $P_{max}^{WT,k}$	{3.5, 1.0, 0, 1.6, 2.5, 2.55} kW
DG/GT system	Maximum Power Output, $P_{max}^{DG/GT,k}$	{1.5, 1.2, 1.0, 0.0, 1.0, 1.3} kW
ES system	Maximum Power Output, $P_{max}^{ES+,k}$	{1.0, 0, 1.0, 1.0, 1.0, 1.0} kW
	Maximum Power Output, $P_{max}^{ES-,k}$	{1.0, 0, 1.0, 1.0, 1.0, 1.0} kW
	ES_{cap}^{k}	{3.0, 0, 3.0, 3.0, 3.0, 3.0} kWh

	Round trip cycle efficiency, ε_{ES}	{90, 90, 90, 90, 90, 90} %
Net production cost coefficients	$\alpha^{net,k}$	{0.0003, 0.0004, 0.0004, 0.000175,0.000175, 0.000175}
	$\beta^{net,k}$	{0.015, 0.02, 0.02, 0.01, 0.01, 0.01}
	$\gamma^{net,k}$	{10, 20, 15, 12, 13, 14}
PRI initialization	$\sigma^{k}_{initial}$	{50, 40, 45, 30, 30, 40} %
PRI weighting factor	ρ^{k}	{0.5, 0.5, 0.5, 0.5, 0.5, 0.5}

Figure 22 shows the evolution of prices and power outputs of HMG-1 and HMG-6 till the convergence to equilibrium occurs. It can be seen that initially the parameters fluctuate, but once convergence is reached, the parameters stay within a narrow band. The plot for HMG payoffs looks similar to the plots in Figure 22.

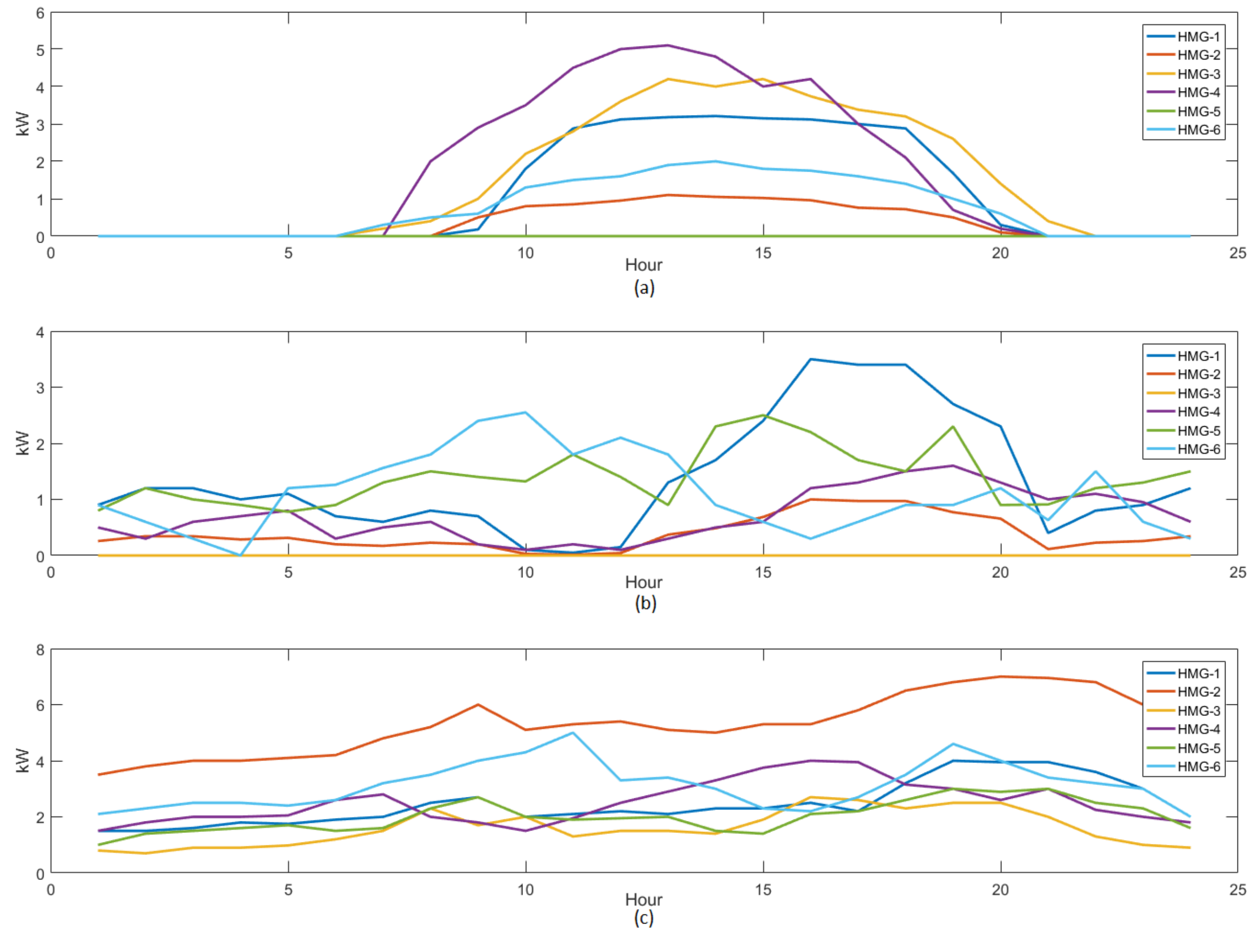

Figure 21. Predicted profiles for (a) PV, (b) WT, and (c) Loads in all HMGs

The results in this case study can be analyzed using evaluation metrics like Total Generated Energy (TGE), Change in Payoffs of the players, clearing prices, and reliability improvement. Figure 23 (a) shows the TGE for HMGs in both cases. It can be seen that the proposed framework results in an increase of TGE for all the HMGs since they are able to sell the surplus energy from DERs to their peers. This increase is proportional to the available surplus energy and PRIs of the HMGs. However, for HMG-2 the increase in TGE is zero since its generation always remains lower than the load. The percentage increase in TGE shown in Figure 23 (b) is calculated as:

$$Increase\ in\ TGE\ (\%) = \frac{TGE_{case-II} - TGE_{case-I}}{TGE_{case-I}} \times 100 \qquad (64)$$

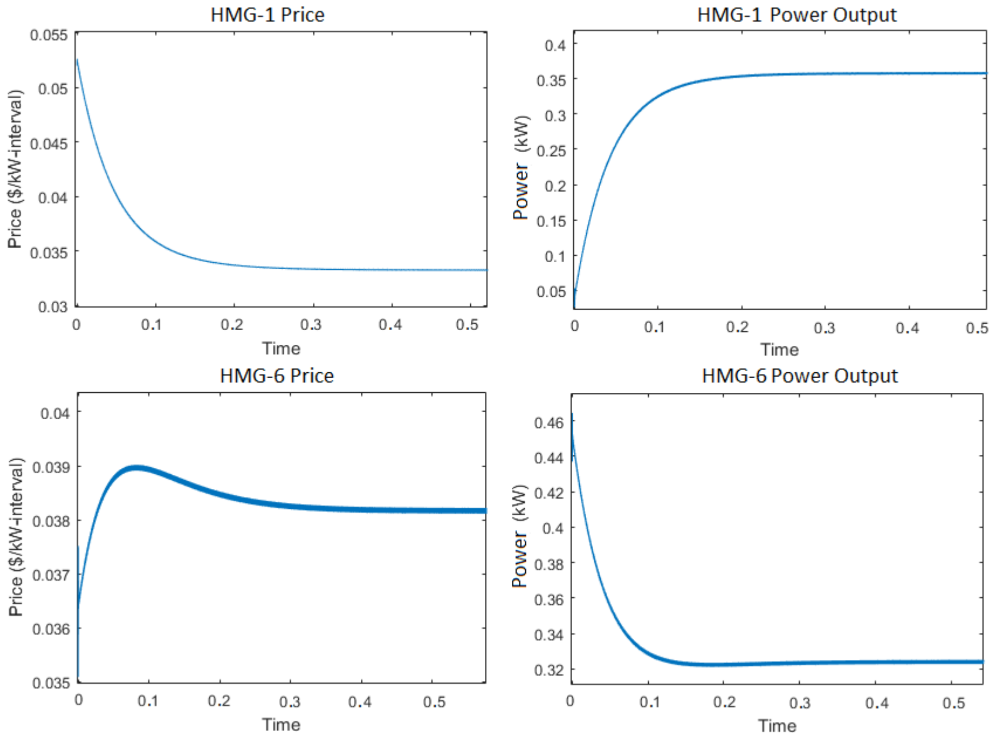

Figure 22. Convergence of power outputs and selling prices for HMG-1 and HMG-6

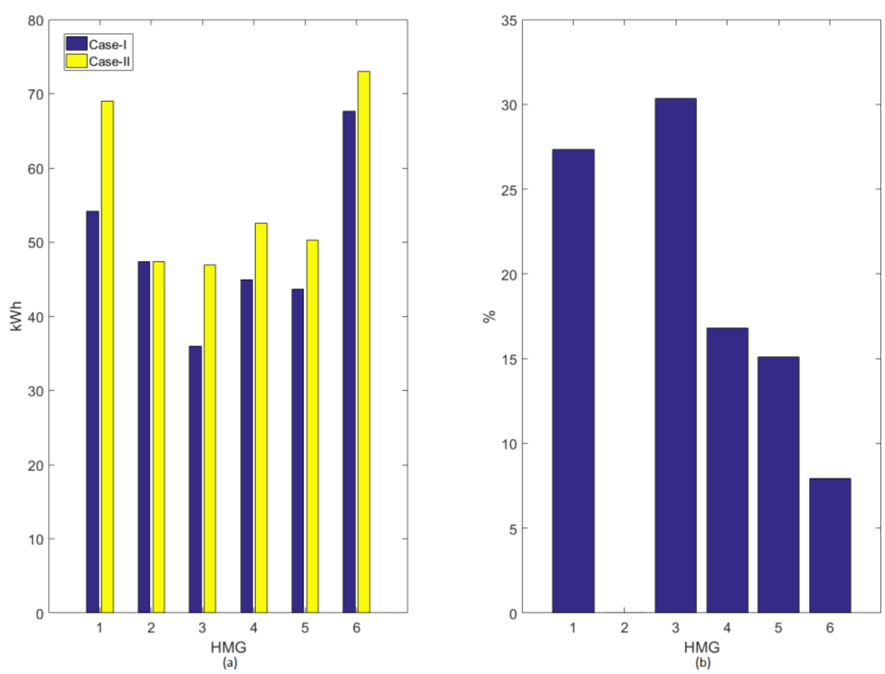

Figure 23. (a) TGE in both cases for all the HMGs (b) Percentage increase in TGE in case-II for all the HMGs

Examining the payoffs of the participating players, it can be seen that the cumulative daily payoffs for all the HMGs increase as shown in Figure 24 (a). This is because HMGs can sell surplus energy to their peers and they can also consume cheaper energy as opposed to buying from UTRs all the time. Though HMG-2 has a low DER penetration level and its total generation always remains lower than its load, still, it sees an increase in net payoff due to the consumption of cheaper energy from other HMGs. This increase in Figure 24 (b) is lower compared to the increase seen by other HMGs due to the low DER penetration of HMG-2. Thus, if the owner of HMG-2 would like to benefit more from the proposed framework, then HMG-2 should invest in installing more generation. On the other hand, the cumulative payoffs for UTRs decrease as shown in Figure 25. This is expected since HMGs are less reliant on UTRs in case-II as compared to case-I. The UTRs could recover some of the lost profit with subscription fees charged to the HMGs.

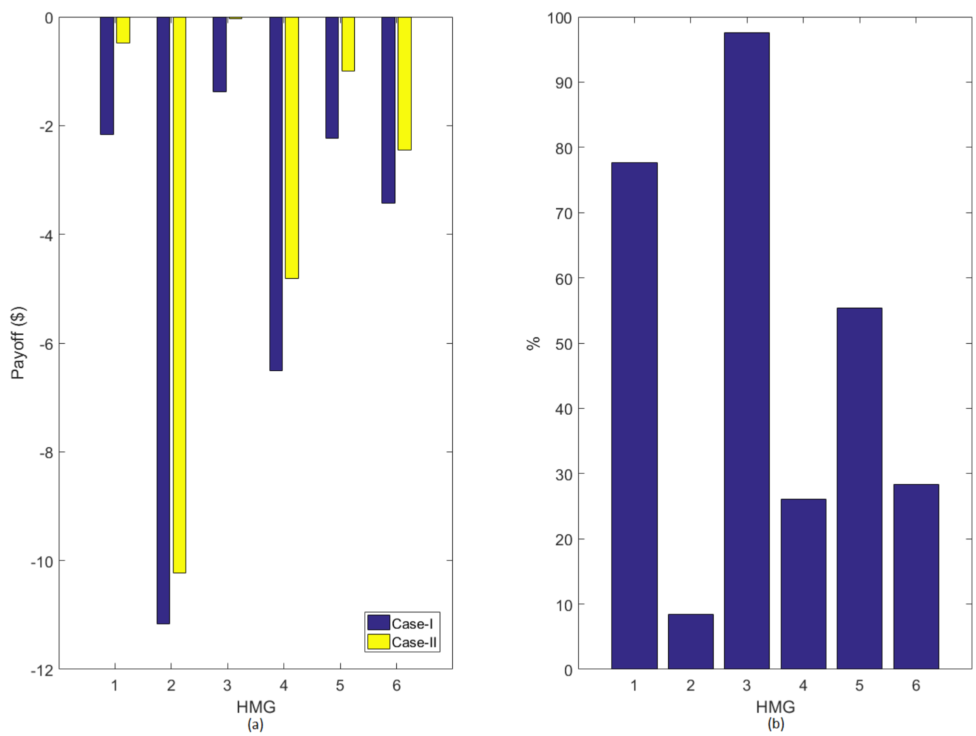

Figure 24. (a) Cumulative daily payoffs for all the HMGs in both cases (b) Percentage increase in Payoffs due to the proposed framework

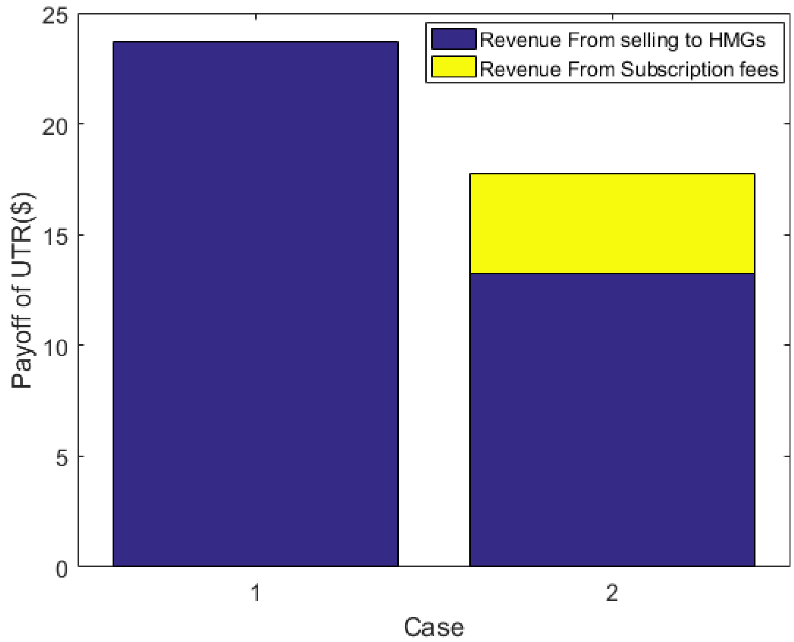

Figure 25. Payoffs of UTRs decrease in case-II compared to case-I

Figure 26 relates the payoffs of HMGs with their PRI values. It shows that as the PRIs of the HMGs improve, they receive a higher payoff. The payoff for all the HMGs is maximized at the maximum value of PRI, i.e. 100 %. At lower PRI values, the payoffs decrease as HMGs get penalized by the CA. At the same value of PRI, the payoffs of different HMGs can differ as shown in Figure 26. This difference is due to the different penetration levels of installed DERs, different cost functions of installed DERs, and load profile diversity in the HMGs. For example, HMG-1 has a higher installed generation as shown previously in Figure 23. Thus, compared to HMG-3 and HMG-5, its payoff is higher at the same value of the PRI. The optimum purchase prices in case-II for all HMGs are shown in Figure 27. For some hours of the day, i.e. hours 21 and 22, there are no seller HMGs available, thus all HMGs meet their demand by buying energy from UTRs. That forces purchase prices for all HMGs equal to the UTR selling optimum price. For other hours of the day, HMGs can partially or completely meet their demand by consuming cheaper energy from their peers. This decreases their optimum price compared to the prices offered by UTRs. When

HMGs act as sellers (i.e. have no demand), the purchase optimum prices become zero. However, for HMG-2 this price is always non-zero since it always buys energy due to lower installed DER generation.

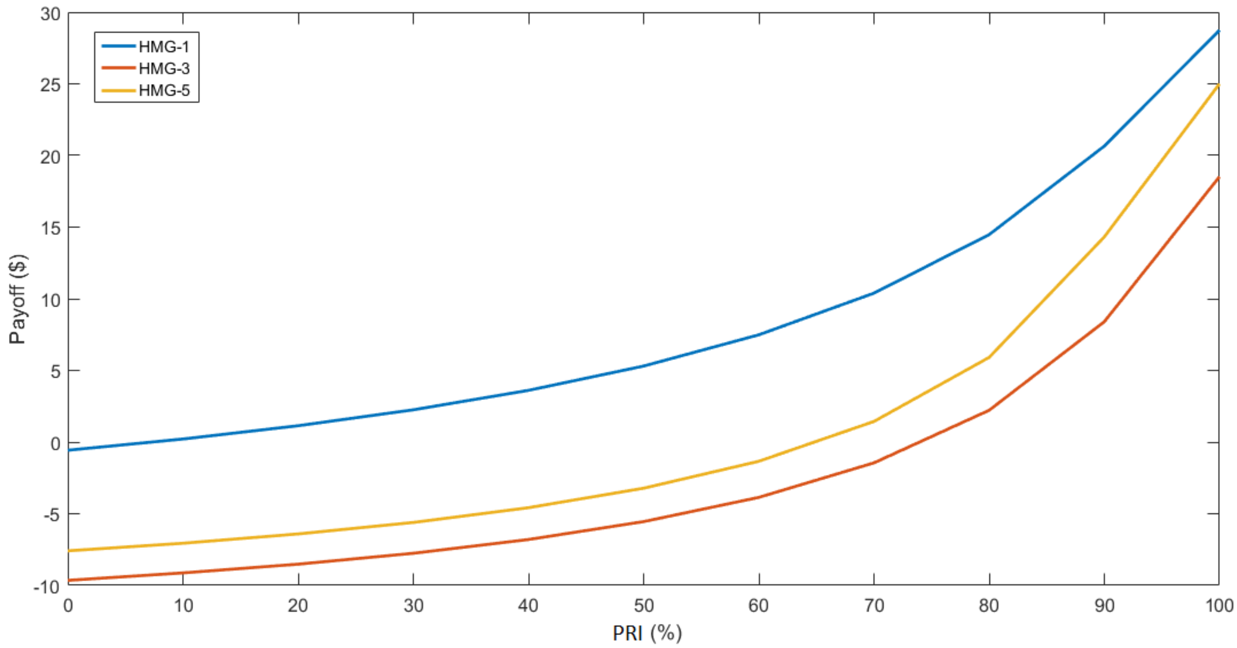

Figure 26. Relationship between the PRIs and Payoffs of the HMGs

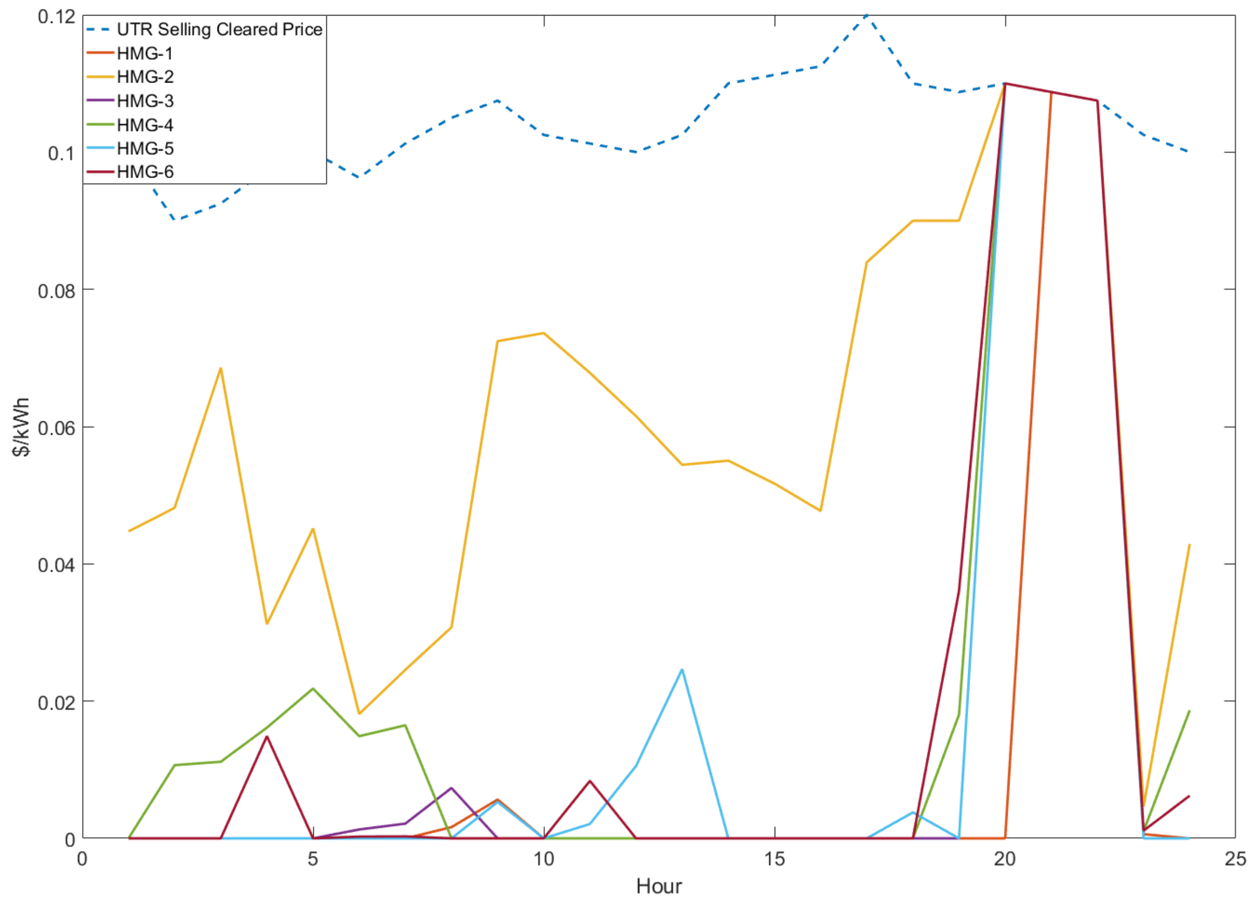

Figure 27. Optimum purchase prices for all HMGs

For this case study, an error distribution according to a Gaussian distribution is assumed between the optimum (committed) and the actual power output of the HMGs. The variance of this distribution corresponds to the accuracy of the forecasting equipment installed in each HMG. Figure 28 shows the evolution of PRIs throughout the simulated day calculated using (34). The HMGs that stick to their commitments and do not deviate often, i.e. HMG-1 and HMG-3, accumulate a net gain in their PRI over the day. Whereas other HMGs, such as HMG-4, show a decreased PRI due to a deviation from the committed power output. The rate of gains and losses in PRI values are controlled by the parameter ρ^k defined earlier in (34). Since HMG-2 does not sell any power, thus its PRI remains constant.

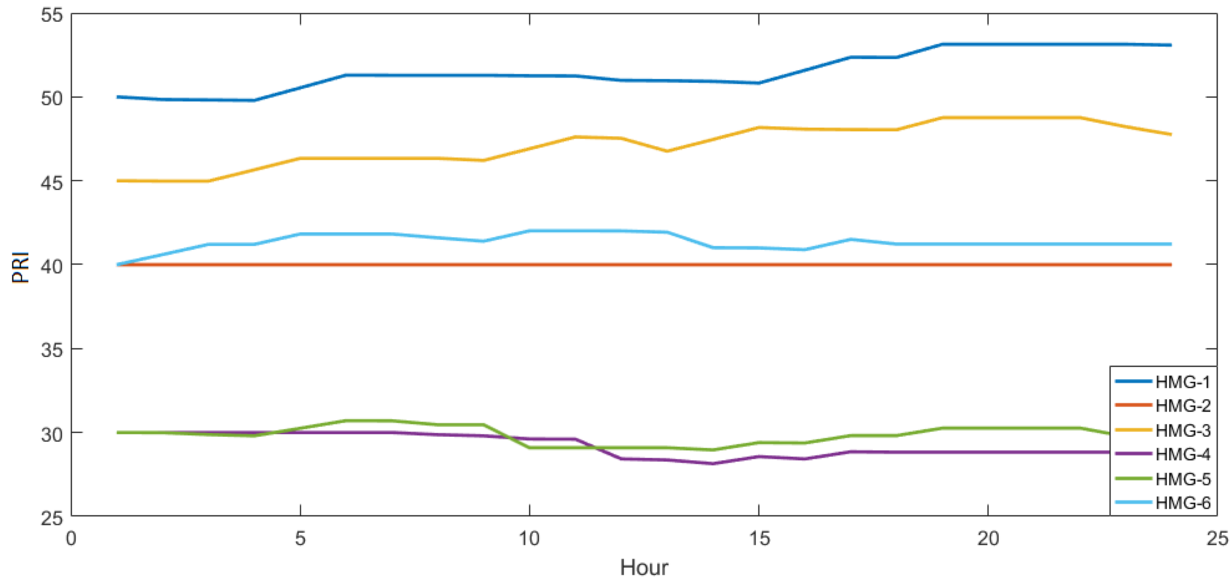

Figure 28. PRIs of all the HMGs throughout the simulated day

7.2.Case Study 2: PRIs and Reliability

To demonstrate the role of PRIs in ensuring reliability and fairness, this case study compares the proposed framework with two alternate frameworks in which:

- The role of PRIs is dampened to 50 % compared to the base case
- PRIs are not considered at all

Figure 29 compares the percentage increase in payoffs for all HMGs due to the proposed framework (previously shown in Figure 24) with that in the two alternate frameworks. For this case study, HMG-1, HMG-3, and HMG-6 are seeded as the good players. It is seen that although the payoffs increase in all the cases, the gains in payoff decrease when the role of PRIs is dampened to half as compared to the case with PRIs. Moreover, if the role of PRIs is completely removed, then the payoff gains drop further. This trend is the opposite for not so good players, i.e. HMG-2, HMG-4, and HMG-5. As PRIs are neglected, the bad players with lower PRIs achieve a higher increase in payoffs. This validates the fact that the concept of PRIs is important to ensuring that the players are rewarded according to their reputation that is built upon their historical performance.

In addition to enforcing fairness, PRIs also improve the reliability. This is shown in Figure 30, which shows the cumulative energy output from all HMGs over the course of the day. It is seen that if the role of PRIs is ignored, then more energy is bought from unreliable players, i.e. HMG-4 and HMG-5. The contribution from HMG-2 remains zero since it is not able to sell. Similarly, the energy output from good players such as HMG-1, HMG-3, and HMG-6, reduces when PRIs are completely ignored. Since PRIs ensure that a higher percentage of demand is met by players with healthy PRIs (higher reliability), hence the concept of PRIs is helpful in enhancing the reliability of the proposed framework.

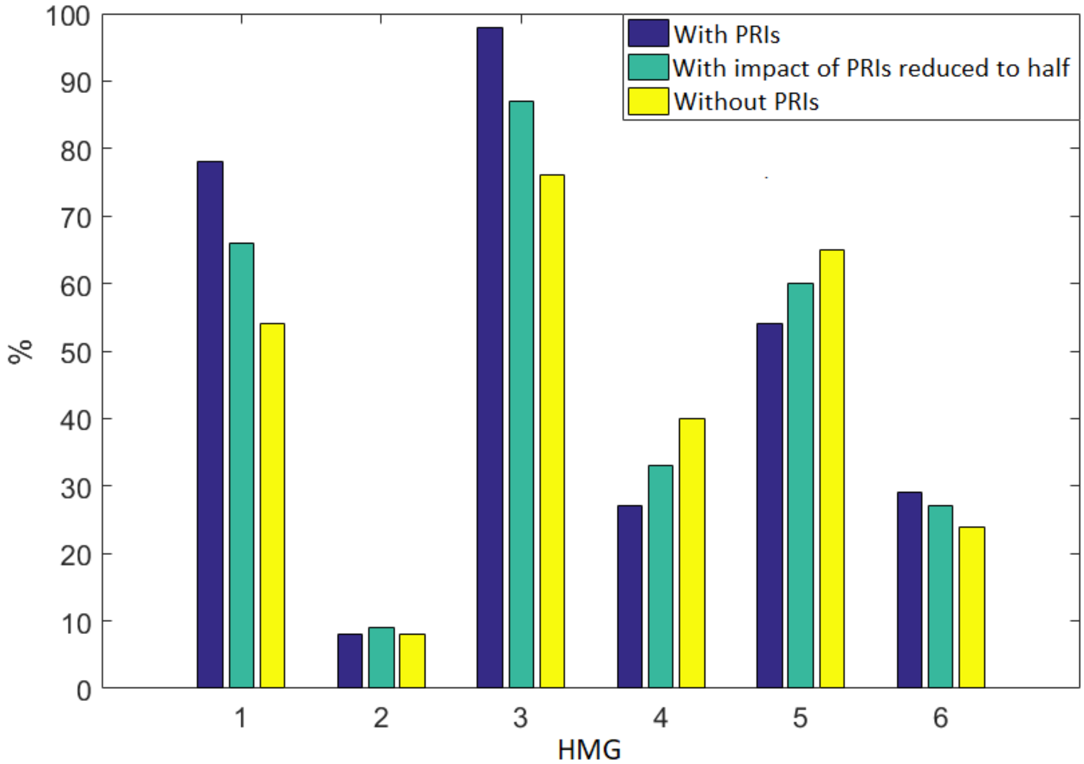

Figure 29. Percentage increase in the payoffs for all HMGs in the proposed and the alternate frameworks

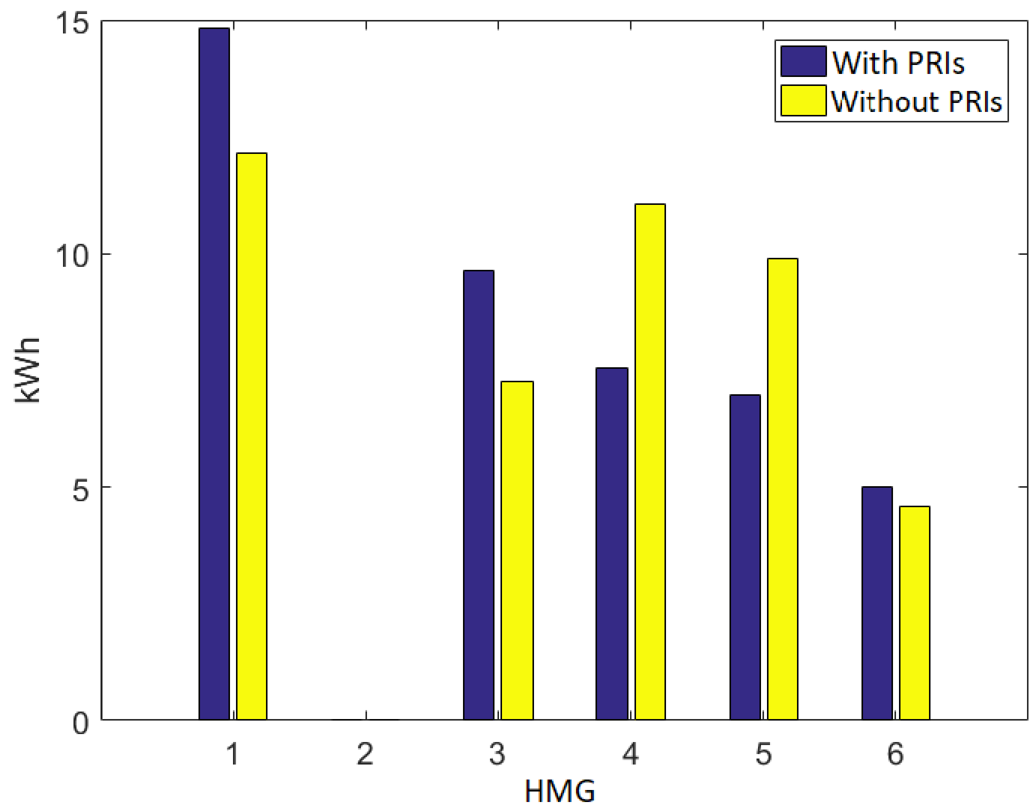

Figure 30. Cumulative energy output from all HMGs with and without PRIs

7.3.Case Study 3: Scalability

The simulation times for the proposed framework are shown in Figure 31 as a function of the number of players. These times are recorded on an Intel core-i7 CPU when all the computations are performed on a single processor. Even without distributed computation, it is seen that the simulation times in the order of few seconds are not significant compared to the real-time operating interval, i.e. 5 minutes. Moreover, the simulation times seem to scale linearly with the size of the system.

In regards to real-world deployment, it should be noted that:

- Real-world implementation is meant to be distributed since it involves multiples processors, such as HEMSs, intelligent transactive agents, and CA. Thus, simulation times for larger system sizes simulated on a single processor do not provide an accurate representation of real-world implementation, which being distributed is expected to be faster. However, it should be noted that the real-world implementation is dependent on speed and reliability of communication infrastructure used by the smart grid.

- For very large system sizes, a hierarchical structure can be created with multiple sub-CAs to substantially decrease the implementation time. Each sub-CA can operate within its jurisdiction while closely coordinating the operation with other sub-CAs and the root CA.
- CAs can effectively pipeline the bidding process by estimating the time to clearing. For example, if the CA knows that it takes 3-5 seconds to converge for 100 players, then it can initiate the process 5 or 10 seconds earlier than the time it is supposed to communicate the schedule to the players.

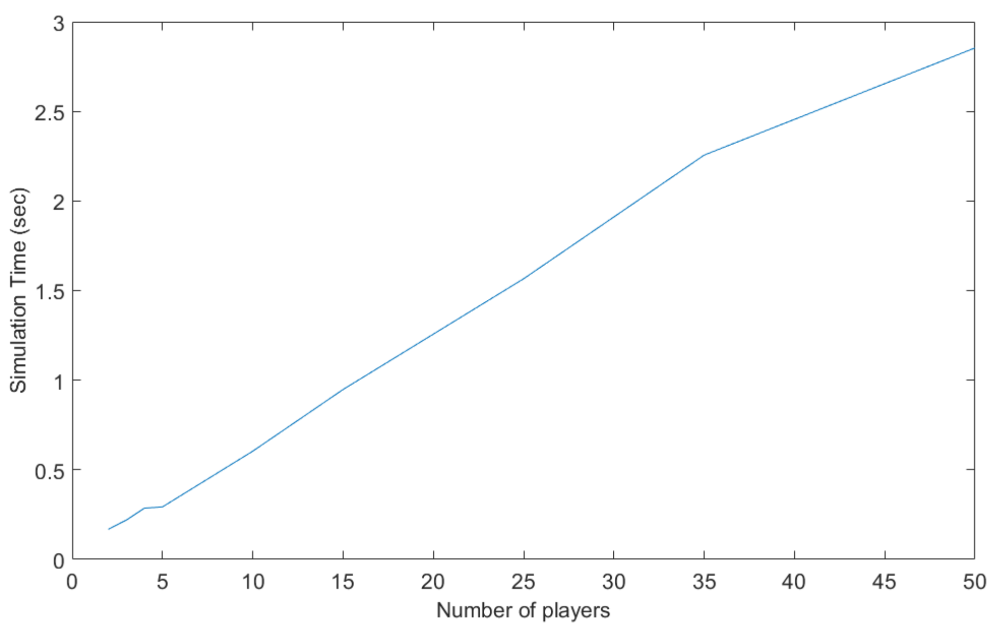

Figure 31. Simulation time in seconds as a function of the number of players

Chapter 8: Conclusions and Future Work

A novel electrical-transactive co-simulation framework has been proposed that enables DERs and their aggregators to supply the load in the distribution systems. The load supply is optimal in the electrical and transactive sense while considering constraints at various levels. To this end, a novel game model is proposed with an integrated notion of the reputation of each player. Maintaining the reputation of each player enforces fairness and transparency. Thus, the non-cooperative game model rewards the players according to their historical performance. The two types of players include the prosumers, i.e. HMGs/DER aggregators, and the suppliers, i.e. UTRs.

An iterative solution algorithm is proposed to solve the framework. The optimal solution satisfies all the constraints, i.e. power flow and the transactive constraints. This is ensured by the co-simulation of the electrical and transactive network representations. In addition to discussing the DERs with quadratic cost functions, the theory is also extended to a generic class of DERs with non-quadratic cost functions. For both the scenarios it is demonstrated that when a gradient-based, Nash seeking method is employed by the players, the game converges to a solution resulting in optimum power outputs and payoffs for all the players. Moreover, for the case with non-quadratic payoffs, multiple equilibria are demonstrated, and it is shown that the gradient-based method converges to one of the stable solutions, where the stable solution obtained is dependent upon the initial condition. In addition to rewarding players according to their historical behaviors, the proposed index, i.e. PRI, improves reliability by ensuring that more loads are supplied from reliable players with higher PRIs as compared to the unreliable players. Being inherently embedded in the game model, PRI ensures that reliable (good) players can inject more energy into the distribution system compared to unreliable players, and thus obtain a higher payoff.

The framework is implemented using existing software packages, i.e. HELICS, MATLAB, and GridLAB-D. Several case studies are performed to establish the results and usefulness of the proposed work. It is shown that the proposed work results in higher consumption of local generation, i.e. the total generated energy from DERs, lower buying prices, and higher payoffs for the players. Moreover, it results in a demand decrease for UTRs that relieves the burden on the transmission system, which can result in transmission upgrade deferrals. The role of PRIs in increasing reliability is also discussed. The practical considerations in the implementation of the proposed work and scalability are also considered. Though the complete framework is simulated on a single processor, the practical implementation is meant to be distributed across multiple processors, such as HEMS, transactive agents, and CAs. Thus, it is the smart grid communication system and its latency that can potentially impact the solution speed, where device-level computations are not complex.

This dissertation made the following eight contributions to the state of art:

- Proposed a novel electrical-transactive co-simulation architecture to enable distribution system load supply from DERs and their aggregators.
- Introduced a novel game model to realize the load supply process. The model is based on non-cooperative game theory and is rewarding to players who behave in a beneficial way towards the system.
- A new idea of ranking players according to their historical performance is introduced. It is embedded in the game model and thus impacts the behavior of the players and the overall reliability of the process.
- A low-complexity, gradient-based scheme is demonstrated to solve for the solution of the proposed framework.

- DERs with non-quadratic payoff functions are considered i.e. games inclusive of players with non-quadratic DER cost functions.
- A generic, dynamic mapping of the strategy set to payoff functions is discussed. The game model is revised accordingly to accommodate dynamic mapping.
- For generic, non-quadratic games, the existence of multiple solutions is discussed. Convergence properties of stable solutions are studied.
- It is demonstrated that when the gradient-based method is used to solve the system, the system converges to one of the stable solution points.

The following future research directions exist for the work presented in this dissertation:

- Modeling the communication system, i.e. adding a communication system dimension to the electrical-transactive co-simulation
- In addition to energy, an extension of the concept to other parameters, such as frequency regulation services from DERs
- Extending co-simulation to incorporate transmission and other infrastructures, such as transportation
- Distributed computations to closely match the real-world implementation.

References

[1] "Annual Energy Outlook 2020." https://www.eia.gov/outlooks/aeo/ (accessed Oct. 25, 2020).

[2] "Electricity – World Energy Outlook 2019 – Analysis," *IEA*. https://www.iea.org/reports/world-energy-outlook-2019/electricity (accessed Oct. 25, 2020).

[3] "Global electricity consumption continues to rise faster than population - Today in Energy - U.S. Energy Information Administration (EIA)." https://www.eia.gov/todayinenergy/detail.php?id=44095 (accessed Oct. 25, 2020).

[4] "International Energy Outlook 2019." https://www.eia.gov/outlooks/archive/ieo19/ (accessed Oct. 25, 2020).

[5] G. Brinkman *et al.*, "Grid Modeling for the SunShot Vision Study," National Renewable Energy Lab. (NREL), Golden, CO (United States), NREL/TP-6A20-53310, Feb. 2012. doi: https://dx.doi.org/10.2172/1036369.

[6] CAISO, "Integration of renewable resources: Operational requirements and generation fleet capability at 20% RPS," 2010. [Online]. Available: https://www.caiso.com/Documents/Integration-RenewableResources-OperationalRequirementsandGenerationFleetCapabilityAt20PercRPS.pdf.

[7] Y. V. Makarov, C. Loutan, J. Ma, and P. de Mello, "Operational Impacts of Wind Generation on California Power Systems," *IEEE Trans. Power Syst.*, vol. 24, no. 2, pp. 1039–1050, May 2009, doi: 10.1109/TPWRS.2009.2016364.

[8] K. Kok and S. Widergren, "A Society of Devices: Integrating Intelligent Distributed Resources with Transactive Energy," *IEEE Power Energy Mag.*, vol. 14, no. 3, pp. 34–45, May 2016, doi: 10.1109/MPE.2016.2524962.

[9] Jianming Chen, F. N. Lee, A. M. Breipohl, and R. Adapa, "Scheduling direct load control to minimize system operation cost," *IEEE Trans. Power Syst.*, vol. 10, no. 4, pp. 1994–2001, Nov. 1995, doi: 10.1109/59.476068.

[10] Wen-Chen Chu, Bin-Kwie Chen, and Chun-Kuei Fu, "Scheduling of direct load control to minimize load reduction for a utility suffering from generation shortage," *IEEE Trans. Power Syst.*, vol. 8, no. 4, pp. 1525–1530, Nov. 1993, doi: 10.1109/59.260955.

[11] C. N. Kurucz, D. Brandt, and S. Sim, "A linear programming model for reducing system peak through customer load control programs," *IEEE Trans. Power Syst.*, vol. 11, no. 4, pp. 1817–1824, Nov. 1996, doi: 10.1109/59.544648.

[12] K. Kalsi, M. Elizondo, J. Fuller, S. Lu, and D. Chassin, "Development and Validation of Aggregated Models for Thermostatic Controlled Loads with Demand Response," in *2012 45th Hawaii International Conference on System Sciences*, Jan. 2012, pp. 1959–1966, doi: 10.1109/HICSS.2012.212.

[13] D. S. Callaway, "Tapping the energy storage potential in electric loads to deliver load following and regulation, with application to wind energy," *Energy Convers. Manag.*, vol. 50, no. 5, pp. 1389–1400, May 2009, doi: 10.1016/j.enconman.2008.12.012.

[14] W. Zhang, J. Lian, C. Chang, and K. Kalsi, "Aggregated Modeling and Control of Air Conditioning Loads for Demand Response," *IEEE Trans. Power Syst.*, vol. 28, no. 4, pp. 4655–4664, Nov. 2013, doi: 10.1109/TPWRS.2013.2266121.

[15] S. Bashash and H. K. Fathy, "Modeling and Control of Aggregate Air Conditioning Loads for Robust Renewable Power Management," *IEEE Trans. Control Syst. Technol.*, vol. 21, no. 4, pp. 1318–1327, Jul. 2013, doi: 10.1109/TCST.2012.2204261.

[16] J. Liu, S. Li, W. Zhang, J. L. Mathieu, and G. Rizzoni, "Planning and control of Electric Vehicles using dynamic energy capacity models," in *52nd IEEE Conference on Decision and Control*, Dec. 2013, pp. 379–384, doi: 10.1109/CDC.2013.6759911.

[17] S. Vandael, B. Claessens, M. Hommelberg, T. Holvoet, and G. Deconinck, “A Scalable Three-Step Approach for Demand Side Management of Plug-in Hybrid Vehicles,” *IEEE Trans. Smart Grid*, vol. 4, no. 2, pp. 720–728, Jun. 2013, doi: 10.1109/TSG.2012.2213847.

[18] H. Chao, “Price-Responsive Demand Management for a Smart Grid World,” *Electr. J.*, vol. 23, no. 1, pp. 7–20, Jan. 2010, doi: 10.1016/j.tej.2009.12.007.

[19] W. W. Hogan, “Demand Response Compensation, Net Benefits and Cost Allocation: Comments,” *Electr. J.*, vol. 23, no. 9, pp. 19–24, Nov. 2010, doi: 10.1016/j.tej.2010.10.002.

[20] S. Borenstein, M. Jaske, and A. Rosenfeld, “Dynamic Pricing, Advanced Metering, and Demand Response in Electricity Markets,” Oct. 2002, Accessed: Oct. 26, 2020. [Online]. Available: https://escholarship.org/uc/item/11w8d6m4.

[21] H. Allcott, “Rethinking real-time electricity pricing,” *Resour. Energy Econ.*, vol. 33, no. 4, pp. 820–842, Nov. 2011, doi: 10.1016/j.reseneeco.2011.06.003.

[22] The GridWise Architecture Council, “GridWise transactive energy framework Version 1.0,” Pacific Northwest National Laboratory, Technical Report PNNL-22946, 2015. [Online]. Available: https://www.gridwiseac.org/pdfs/te_framework_report_pnnl-22946.pdf.

[23] D. J. Hammerstrom *et al.*, “Pacific Northwest GridWise™ Testbed Demonstration Projects; Part I. Olympic Peninsula Project,” Pacific Northwest National Lab. (PNNL), Richland, WA (United States), PNNL-17167, Jan. 2008. doi: 10.2172/926113.

[24] J. C. Fuller, K. P. Schneider, and D. Chassin, “Analysis of Residential Demand Response and double-auction markets,” in *2011 IEEE Power and Energy Society General Meeting*, Jul. 2011, pp. 1–7, doi: 10.1109/PES.2011.6039827.

[25] S. E. Widergren, M. C. Marinovici, J. C. Fuller, K. Subbarao, D. P. Chassin, and A. Somani, “Customer Engagement in AEP gridSMART Residential Transactive System,” GridWise Architecture Council,

Richland, WA, United States(US)., PNNL-SA-107064, Dec. 2014. Accessed: Oct. 26, 2020. [Online]. Available: https://www.osti.gov/biblio/1179531.

[26] S. Widergren, J. Fuller, C. Marinovici, and A. Somani, “Residential transactive control demonstration,” in *ISGT 2014*, Feb. 2014, pp. 1–5, doi: 10.1109/ISGT.2014.6816405.

[27] R. Melton, “Pacific Northwest Smart Grid Demonstration Project Technology Performance Report Volume 1: Technology Performance,” Pacific Northwest National Lab. (PNNL), Richland, WA (United States), PNW-SGDP-TPR-Vol.1-Rev.1.0; PNWD-4438, Volume 1, Jun. 2015. doi: 10.2172/1367568.

[28] P. Huang *et al.*, “Analytics and Transactive Control Design for the Pacific Northwest Smart Grid Demonstration Project,” in *2010 First IEEE International Conference on Smart Grid Communications*, Oct. 2010, pp. 449–454, doi: 10.1109/SMARTGRID.2010.5622083.

[29] K. Kok *et al.*, “Dynamic pricing by scalable energy management systems — Field experiences and simulation results using PowerMatcher,” in *2012 IEEE Power and Energy Society General Meeting*, Jul. 2012, pp. 1–8, doi: 10.1109/PESGM.2012.6345058.

[30] W. Su and A. Q. Huang, “A game theoretic framework for a next-generation retail electricity market with high penetration of distributed residential electricity suppliers,” *Appl. Energy*, vol. 119, pp. 341–350, Apr. 2014, doi: 10.1016/j.apenergy.2014.01.003.

[31] N. Zhang, Y. Yan, and W. Su, “A game-theoretic economic operation of residential distribution system with high participation of distributed electricity prosumers,” *Appl. Energy*, vol. 154, pp. 471–479, Sep. 2015, doi: 10.1016/j.apenergy.2015.05.011.

[32] Tao Chen, H. Pourbabak, and W. Su, “A game theoretic approach to analyze the dynamic interactions of multiple residential prosumers considering power flow constraints,” in *2016 IEEE Power and Energy Society General Meeting (PESGM)*, Jul. 2016, pp. 1–5, doi: 10.1109/PESGM.2016.7741082.

[33] Ni Zhang, Yu Yan, Shengyao Xu, and Wencong Su, “Game-theory-based electricity market clearing mechanisms for an open and transactive distribution grid,” in *2015 IEEE Power Energy Society General Meeting*, Jul. 2015, pp. 1–5, doi: 10.1109/PESGM.2015.7285598.

[34] M. Marzband, M. Javadi, S. A. Pourmousavi, and G. Lightbody, “An advanced retail electricity market for active distribution systems and home microgrid interoperability based on game theory,” *Electr. Power Syst. Res.*, vol. 157, pp. 187–199, Apr. 2018, doi: 10.1016/j.epsr.2017.12.024.

[35] H. Yang, R. Zhou, and J. Xin, “Dynamic cournot game behavior of electric power providers in retail electricity market,” 2005, vol. 1, pp. 578–583.

[36] N. Yaagoubi and H. T. Mouftah, “A distributed game theoretic approach to energy trading in the smart grid,” in *2015 IEEE Electrical Power and Energy Conference (EPEC)*, Oct. 2015, pp. 203–208, doi: 10.1109/EPEC.2015.7379950.

[37] W. Lin *et al.*, “Game-theory based trading analysis between distribution network operator and multi-microgrids,” *Energy Procedia*, vol. 158, pp. 3387–3392, Feb. 2019, doi: 10.1016/j.egypro.2019.01.945.

[38] M. Motalleb, A. Annaswamy, and R. Ghorbani, “A real-time demand response market through a repeated incomplete-information game,” *Energy*, vol. 143, pp. 424–438, Jan. 2018, doi: 10.1016/j.energy.2017.10.129.

[39] F. Wei, Z. X. Jing, P. Z. Wu, and Q. H. Wu, “A Stackelberg game approach for multiple energies trading in integrated energy systems,” *Appl. Energy*, vol. 200, pp. 315–329, Aug. 2017, doi: 10.1016/j.apenergy.2017.05.001.

[40] M. Yu and S. H. Hong, “Incentive-based demand response considering hierarchical electricity market: A Stackelberg game approach,” *Appl. Energy*, vol. 203, pp. 267–279, Oct. 2017, doi: 10.1016/j.apenergy.2017.06.010.

[41] M. H. Cintuglu, H. Martin, and O. A. Mohammed, “Real-Time Implementation of Multiagent-Based Game Theory Reverse Auction Model for Microgrid Market Operation,” *IEEE Trans. Smart Grid*, vol. 6, no. 2, pp. 1064–1072, Mar. 2015, doi: 10.1109/TSG.2014.2387215.

[42] H. Wang, T. Huang, X. Liao, H. Abu-Rub, and G. Chen, “Reinforcement Learning for Constrained Energy Trading Games With Incomplete Information,” *IEEE Trans. Cybern.*, vol. 47, no. 10, pp. 3404–3416, Oct. 2017, doi: 10.1109/TCYB.2016.2539300.

[43] C. Li, Y. Xu, X. Yu, C. Ryan, and T. Huang, “Risk-Averse Energy Trading in Multienergy Microgrids: A Two-Stage Stochastic Game Approach,” *IEEE Trans. Ind. Inform.*, vol. 13, no. 5, pp. 2620–2630, 2017, doi: 10.1109/TII.2017.2739339.

[44] C. P. Mediwaththe, E. R. Stephens, D. B. Smith, and A. Mahanti, “Competitive Energy Trading Framework for Demand-Side Management in Neighborhood Area Networks,” *IEEE Trans. Smart Grid*, vol. 9, no. 5, pp. 4313–4322, Sep. 2018, doi: 10.1109/TSG.2017.2654517.

[45] N. Liu, X. Yu, C. Wang, and J. Wang, “Energy Sharing Management for Microgrids with PV Prosumers: A Stackelberg Game Approach,” *IEEE Trans. Ind. Inform.*, vol. 13, no. 3, pp. 1088–1098, 2017, doi: 10.1109/TII.2017.2654302.

[46] W. Tushar *et al.*, “A motivational game-theoretic approach for peer-to-peer energy trading in the smart grid,” *Appl. Energy*, vol. 243, pp. 10–20, Jun. 2019, doi: 10.1016/j.apenergy.2019.03.111.

[47] W. Tushar, C. Yuen, D. B. Smith, and H. V. Poor, “Price Discrimination for Energy Trading in Smart Grid: A Game Theoretic Approach,” *IEEE Trans. Smart Grid*, vol. 8, no. 4, pp. 1790–1801, Jul. 2017, doi: 10.1109/TSG.2015.2508443.

[48] W. Tushar *et al.*, “Energy Storage Sharing in Smart Grid: A Modified Auction-Based Approach,” *IEEE Trans. Smart Grid*, vol. 7, no. 3, pp. 1462–1475, 2016, doi: 10.1109/TSG.2015.2512267.

[49] Q. Wu, H. Ren, W. Gao, and J. Ren, "Benefit allocation for distributed energy network participants applying game theory based solutions," *Energy*, vol. 119, pp. 384–391, Jan. 2017, doi: 10.1016/j.energy.2016.12.088.

[50] F. Pourahmadi, M. Fotuhi-Firuzabad, and P. Dehghanian, "Application of Game Theory in Reliability-Centered Maintenance of Electric Power Systems," *IEEE Trans. Ind. Appl.*, vol. 53, no. 2, pp. 936–946, Mar. 2017, doi: 10.1109/TIA.2016.2639454.

[51] S. R. Dabbagh and M. K. Sheikh-El-Eslami, "Risk-based profit allocation to DERs integrated with a virtual power plant using cooperative Game theory," *Electr. Power Syst. Res.*, vol. 121, pp. 368–378, Apr. 2015, doi: 10.1016/j.epsr.2014.11.025.

[52] J. Mei, C. Chen, J. Wang, and J. L. Kirtley, "Coalitional game theory based local power exchange algorithm for networked microgrids," *Appl. Energy*, vol. 239, pp. 133–141, Apr. 2019, doi: 10.1016/j.apenergy.2019.01.208.

[53] C. Long, Y. Zhou, and J. Wu, "A game theoretic approach for peer to peer energy trading," *Energy Procedia*, vol. 159, pp. 454–459, Feb. 2019, doi: 10.1016/j.egypro.2018.12.075.

[54] J. Lee, J. Guo, J. K. Choi, and M. Zukerman, "Distributed Energy Trading in Microgrids: A Game-Theoretic Model and Its Equilibrium Analysis," *IEEE Trans. Ind. Electron.*, vol. 62, no. 6, pp. 3524–3533, Jun. 2015, doi: 10.1109/TIE.2014.2387340.

[55] S. Park, J. Lee, G. Hwang, and J. K. Choi, "Event-Driven Energy Trading System in Microgrids: Aperiodic Market Model Analysis with a Game Theoretic Approach," *IEEE Access*, vol. 5, pp. 26291–26302, 2017, doi: 10.1109/ACCESS.2017.2766233.

[56] C. Zhang, J. Wu, Y. Zhou, M. Cheng, and C. Long, "Peer-to-Peer energy trading in a Microgrid," *Appl. Energy*, vol. 220, pp. 1–12, Jun. 2018, doi: 10.1016/j.apenergy.2018.03.010.

[57] A. W. Tucker and R. D. Luce, *Contributions to the Theory of Games*. Princeton University Press, 1959.

[58] J. von Neumann and O. Morgenstern, *Theory of Games and Economic Behavior (Commemorative Edition)*. Princeton University Press, 2007.

[59] C. Antonopoulos, “Modelling and Analysis of Complex Systems,” Sep. 14, 2016, doi: 10.13140/RG.2.2.13668.37769.

[60] B. A. Bhatti, S. Hanif, J. Alam, T. McDermott, and P. Balducci, “A Combined Day-ahead and Real-time Scheduling Approach for Real and Reactive Power Dispatch of Battery Energy Storage,” in *IEEE Power & Energy Society General Meeting (PESGM) 2020*, Montreal, Aug. 2020, pp. 1–5.

[61] R. G. Lutes *et al.*, “VOLTTRON: User Guide,” Pacific Northwest National Lab. (PNNL), Richland, WA (United States), PNNL-23182, Apr. 2014. doi: 10.2172/1130247.

[62] P. Top *et al.*, “Hiearchical Engine for Large Scale Infrastructure Simulation,” Lawrence Livermore National Lab.(LLNL), Livermore, CA (United States), 2017.

[63] B. A. Bhatti and R. Broadwater, “Energy trading in the distribution system using a non-model based game theoretic approach,” *Appl. Energy*, vol. 253, p. 113532, Nov. 2019, doi: 10.1016/j.apenergy.2019.113532.

[64] W. Shi, N. Li, C. Chu, and R. Gadh, “Real-Time Energy Management in Microgrids,” *IEEE Trans. Smart Grid*, vol. 8, no. 1, pp. 228–238, Jan. 2017, doi: 10.1109/TSG.2015.2462294.

[65] C. A. Macana, S. M. Mohiuddin, H. R. Pota, and M. A. Mahmud, “Online energy management strategy for islanded microgrids with feedback linearizing inner controllers,” in *2017 IEEE Innovative Smart Grid Technologies - Asia (ISGT-Asia)*, Dec. 2017, pp. 1–6, doi: 10.1109/ISGT-Asia.2017.8378319.

[66] R. de Azevedo, M. H. Cintuglu, T. Ma, and O. A. Mohammed, “Multiagent-Based Optimal Microgrid Control Using Fully Distributed Diffusion Strategy,” *IEEE Trans. Smart Grid*, vol. 8, no. 4, pp. 1997–2008, Jul. 2017, doi: 10.1109/TSG.2016.2587741.

[67] *Wind Energy Handbook*, 1st ed. John Wiley & Sons, Ltd.

[68] F. D. Santillán-Lemus, H. Minor-Popocatl, O. Aguilar-Mejía, and R. Tapia-Olvera, “Optimal Economic Dispatch in Microgrids with Renewable Energy Sources,” *Energies*, vol. 12, no. 1, Art. no. 1, Jan. 2019, doi: 10.3390/en12010181.

[69] P. Frihauf, M. Krstic, and T. Basar, “Nash equilibrium seeking in noncooperative games,” *IEEE Trans. Autom. Control*, vol. 57, no. 5, pp. 1192–1207, 2012, doi: 10.1109/TAC.2011.2173412.

[70] A. K. Naimzada and L. Sbragia, “Oligopoly games with nonlinear demand and cost functions: Two boundedly rational adjustment processes,” *Chaos Solitons Fractals*, vol. 29, no. 3, pp. 707–722, Aug. 2006, doi: 10.1016/j.chaos.2005.08.103.

[71] T. Offerman, J. Potters, and J. Sonnemans, “Imitation and Belief Learning in an Oligopoly Experiment,” *Rev. Econ. Stud.*, vol. 69, no. 4, pp. 973–997, Oct. 2002, doi: 10.1111/1467-937X.00233.

[72] B. A. Bhatti and R. Broadwater, “Distributed Nash Equilibrium Seeking for a Dynamic Micro-grid Energy Trading Game with Non-quadratic Payoffs,” *Energy*, vol. 202, p. 117709, Jul. 2020, doi: 10.1016/j.energy.2020.117709.

[73] E. Kohlberg and J.-F. Mertens, “On the Strategic Stability of Equilibria,” *Econometrica*, vol. 54, no. 5, pp. 1003–1037, 1986, doi: 10.2307/1912320.

[74] Roussel, M. R., “Stability analysis for ODEs. Nonlinear Dynamics,” University Hall, Canada, 2005. Accessed: Nov. 13, 2020. [Online]. Available: https://fac.ksu.edu.sa/sites/default/files/stability.pdf.