A STAND LEVEL MULTI-SPECIES GROWTH MODEL FOR APPALACHIAN HARDWOODS

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(ABSTRACT)

A stand-level growth and yield model was developed to predict future diameter distributions of thinned stands of mixed Appalachian hardwoods. The model allows prediction by species groups and diameter classes. Stand attributes (basal area per acre, trees per acre, minimum stand diameter, and arithmetic mean dbh) were projected through time for the whole stand and for individual species groups. Future diameter distributions were obtained using the three-parameter Weibull probability density function and parameter recovery method. The recovery method used employed the first two non-central moments of dbh (arithmetic mean dbh and quadratic mean dbh squared) to generate Weibull parameters. Future diameter distributions were generated for the whole stand and every species group but one; the diameter distribution of the remaining species group was obtained by subtraction from whole stand values. A system of tree volume equations which allow the user to obtain total tree volume or merchantable volume to any top height or diameter completes the model. Volumes can be calculated by species group and summed to get whole stand volume.

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CHAPTER I. INTRODUCTION

The Blue Ridge physiographic province of the Appalachian Hardwood Forest is a very important timber producing region in the eastern United States. The species composition of these forests is quite diverse, a fact that both enhances and inhibits its utilization. The more productive sites in the Blue Ridge (oak site indices of 60 and above, base age 50) produce a wide variety of forest products; however the forests have been and continue to be abused because landowners only harvest trees of highest value, causing the overall quality of the forest to decline. Due to the wide range of elevation, aspect, topography, soil types, and past harvesting practices in the region, species diversity and timber quality varies considerably within a small area. These conditions combine to create a very interesting and challenging problem for the forest stand modeler.

Growth and yield research in Appalachian hardwood forests has lagged behind research in other timber types of the South, particularly the southern pines. The economic value of southern pines is generally higher than that of Appalachian hardwoods. Thus the management of Appalachian hardwoods has been relatively extensive as compared to the intensive management of southern pines. Due to the low value of many Appalachian hardwood stands, there is a lack of mensurational data from which to develop models. Also, modeling growth of mixed species stands is inherently more difficult than modeling growth of pure species stands. However, the economic value of the Appalachian forests has increased. This increase has resulted in more intensive management, which in turn demands development of growth and yield methodology to allow forest managers to more efficiently utilize this important resource.

The objective of this research was to develop a stand level model, with values for constituent species groups and product classes within species groups, for predicting growth and yield of mixed Appalachian hardwood stands. There is a great need for this kind of model. Individual tree models provide the detailed tree and stand information desired, but these models tend to be more complex than stand level models, require more detailed (and expensive) inventory data, and require more computation time (sometimes requiring the user to have sophisticated computing facilities). Stand level models can often utilize conventional inventory information, are simpler than individual tree models, and require less computation time. Also, whole-stand models provide information in a form more useful for economic analysis than individual tree models (Munro, 1974). Stand level growth and yield information exists for unmanaged (Schnur, 1937) and managed Appalachian hardwood stands (Burk and Burkhart, 1983). However, existing stand level models ignore species composition. Species composition is very important when one is interested in the economic value of a stand, because the species present in mixed hardwood stands vary considerably in stumpage price and potential product forms. Therefore, it is appropriate to develop stand level growth and yield models because of their relatively simple input requirements and ease of application. To provide the level of resolution necessary for managers to make financial decisions, the stands must be divided into species groups and product classes.

The specific objectives of this research were to:

1. Develop a stand-level model to predict future diameter distributions of mixed Appalachian hardwood stands.

2. Develop methodology to disaggregate the predicted diameter distribution into species groups, thus allowing for computation of yield estimates by diameter classes and species groups.

CHAPTER II. LITERATURE REVIEW

Not a great deal of literature exists for growth and yield of southern hardwood stands. There are basically two reasons for this. The first is that until recently the economic value of these stands was not great enough to warrant the investment of a large amount of time and money into growth and yield research. The second is that the complexity of age and species relationships that exist in these stands greatly complicates the modeling process. Thus the opportunity and need to obtain the type of mensurational data required to develop growth and yield models has been lacking. Most of the growth and yield work existing for southern hardwoods either deals with pure stands (especially yellow-poplar) or mixed stands that ignore species composition. This literature review will examine past attempts at modeling growth and yield of southern hardwood stands, and will review techniques used to predict diameter distributions since that is the main thrust of this thesis.

Munro (1974) classified forest growth models into three categories; single tree/distance dependent, single tree/distance independent, and whole stand/distance independent models. Whole stand models assume that the stand is the primary modeling unit. Single tree/distance dependent models assume that the individual tree is the primary modeling unit and require stem charts and/or inter-tree distances as input variables. Single tree/distance independent models also assume that the individual tree is the primary modeling unit but do not require stem charts as an input requirement. For a more comprehensive review of these different model types, the reader is referred to Munro. The models discussed in the following paragraphs may be considered whole stand models except for Harrison (1984) which is a single tree/distance independent model.

SOUTHERN HARDWOOD GROWTH AND YIELD

One of the first attempts at modeling Appalachian hardwood stands was by Schnur (1937). Schnur developed yield, stand, and volume tables for even-aged upland oak forests. These tables still enjoy wide use today. Schnur used data from temporary plots in fully stocked stands located throughout central and southeastern United States. Tables were developed, using graphical techniques, which allow for the prediction of trees per acre, basal area per acre, average DBH, and volume per acre (cubic foot and board foot) given stand age and site index. Drawbacks of this yield prediction system are that they only apply to fully stocked stands, while many of the stands in the region are understocked, and, through the use of temporary rather than permanent plot data, the assumption is made that a fully stocked stand at some point in time will remain fully stocked through time, an assumption that often is questionable.

Clutter (1963) had a major impact on growth and yield research by introducing the concept of compatibility between growth and yield equations. Compatibility means that the yield at any point in time is equal to the summation of growth up to that time. This implies, in a continuous sense, that a differential-integral relationship exists between the growth and yield equations. Clutter developed his equation system for loblolly pine data; however, his equation system has been applied to other timber types because of the desirable properties it possesses.

Two growth and yield systems based on Clutter's (1963) equations were developed for yellow-poplar. The first was by Schlaegel and Kulow (1969), who developed growth and yield equations for even-aged stands of unthinned yellow-poplar in West Virginia. This model predicts total cubic foot yield inside bark. Schlaegel et al. (1969) went on to expand the model to predict merchantable yields to various top diameters. The second was by Beck and Della-Bianca (1972), who used a refined version of the basic Clutter system (Sullivan and Clutter, 1972) to predict growth and yield for thinned stands of yellow-poplar. The equation system includes equations to project basal area per acre, basal area growth, and cubic foot volume growth.

Dale (1972) developed growth and yield equations for thinned upland oak stands in Kentucky, Ohio, Missouri, and Iowa. These equations should have applicability to upland oak stands in the South, even though they were not developed from data collected in the region.

Smith et al. (1975) developed one of the first growth and yield systems for mixed species of southern bottomland hardwoods. A large number of equations are reported for four merchantability standards for nine forest site types. The four merchantability standards are board foot volume, cubic foot volume, dry weight, and green weight. The nine site classifications are muck swamp, peat swamp, wet flat, red river bottom, black river bottom, branch bottom, bottomland, coves, gulfs and lower slopes, and upland slopes and ridges. Their original equations have been updated (Gardner, et al., 1982) to include alternative merchandising options, and (Roeder and Gardner, 1984) to include growth estimates.

Burk and Burkhart (1983) applied diameter distribution methodology to predict diameter distributions of thinned mixed hardwood stands in the southern Appalachians. The Weibull and Johnson's S_b distributions were compared, as well as the parameter recovery and parameter prediction methods for obtaining parameter estimates. They concluded that the parameter recovery method was superior to the parameter prediction method, and the Weibull distribution was somewhat superior to the Johnson's S_b distribution. The data used in this thesis came from the same study as that used by Burk and Burkhart (1983).

Harrison (1984) developed a single tree/distance independent model for the same data analyzed by Burk and Burkhart (1983). He presented models to predict basal area and height growth for 12 different species and species groups. This work is very important because the model user knows how stand growth is allocated among different species, a very important consideration when economic analysis is to be performed.

DIAMETER DISTRIBUTION MODELS

Diameter distribution models provide estimates of number of trees per unit area by DBH classes. Total stand volume is calculated by obtaining a relationship between diameter and tree volume; volume per class is calculated by multiplying the volume of the tree with midpoint DBH by the number of trees per unit area in that class and total volume is calculated by summing volume per class. Three different ways of describing diameter distributions are discussed here; the use of probability density functions (particularly the Weibull), the use of Markov chains (Bruner and Moser, 1973), and a stand ratio method (Amateis et al., 1985).

Many different probability density functions (pdf's) have been used to describe diameter distributions of forest stands. The most popular ones used today are the beta, Weibull, and Johnson's S_b distributions. The beta is a flexible pdf that works well for southern pine stands; the major drawback of the beta is that no closed form exists for the cumulative distribution function (cdf), making computations difficult. The Johnson's S_b has been proposed as a diameter distribution model (Hafley and Schreuder, 1977, and Schreuder and Hafley, 1977). Burk and Burkhart (1983) applied the S_b and Weibull distributions to mixed Appalachian hardwood data and preferred the Weibull.

Bailey and Dell (1973) first used the Weibull distribution as a diameter distribution model. They showed that the Weibull has several desirable characteristics, namely flexibility of shape, parameters that are easy to relate to stand characteristics, and ease of mathematical derivation. The Weibull distribution has been used extensively in modeling growth of southern pine stands, and has recently been applied to hardwood data (Stiff, 1979, and Burk and

Burkhart, 1983) with promising results. The complete three-parameter Weibull pdf is:

$$f(y) = (c/b)((y-a)/b)^{c-1} \exp\{-[(y-a)/b]^c\}$$

where

$y \geq a$

$b,c > 0.$

The 'a' parameter is often referred to as the location parameter, 'b' is called the scale parameter, and 'c' is called the shape parameter. The location parameter defines the minimum value the function will assume. The scale parameter is approximately equal to the 63rd percentile of the distribution, and may be interpreted as the diameter where 63 percent of the trees are smaller (Bailey and Dell, 1973). The shape of the distribution depends on the value of 'c'. When 'c' < 1 the curve is an inverse j. The exponential distribution results when 'c' = 1. For 1 < 'c' < 3.6, the distribution is skewed to the right. An approximation to the normal distribution results when 'c' $\simeq$ 3.6. As 'c' increases above 3.6, the distribution becomes skewed to the left.

The Markov chain method (Bruner and Moser, 1973) has been used to project the diameter distribution of uneven-aged mixed hardwood stands by determining the probability that, over a given time span, a tree in a certain DBH class will move into a larger DBH class, stay in the same DBH class, or die. The only information one needs to predict a diameter distribution is the distribution at some point in time and the probability of a tree moving to another class or dying. Two assumptions about forest stand growth must be made to apply Markov chains. The first is that a future diameter depends only upon the present diameter, and not the past condition of the stand. The second is that the probabilities of trees moving from one diameter class to another is constant over time. Both of these assumptions are questionable. Also, it would probably prove to be very difficult to modify the appproach to yield added information about growth of individual species or species groups. Another drawback of this method is that future distributions can only be obtained for fixed multiples of the projection period length. However, one great advantage of the Markov method is that the distribution of diameters does not have to be assumed to be unimodal, as in the case of most pdf's used to model diameter distributions.

The stand ratio method (Amateis et al., 1985) uses stand level input variables (basal area and surviving trees per acre) to estimate the number of trees above any threshold diameter limit such that the basal areas by diameter class are consistent with the total stand basal area. Diameter distributions can be generated with this equation by setting the threshold limit at the lower end of successive diameter classes and obtaining the numbers of trees per class by subtraction.

CHAPTER III. DATA

Data for developing the proposed model were provided by the Southeastern Forest Experiment Station of the United States Forest Service. The data set is composed of 47 plots in the Blue Ridge physiographic province of North Carolina and Georgia (44 plots in North Carolina, 3 in Georgia). Plot locations by state and county appear in Figure 1. The plots, which range in sizes from 0.06 to 0.10 hectare (0.15 to 0.25 acres), were established in undisturbed, even-aged, mixed species stands on productive sites (oak site indices of 60 feet and above). All site indices referred to in this thesis are oak site indices, base age 50. The data set includes an initial measurement in the fall of 1974, and remeasurements in the fall of 1979 and 1984.

According to the terminology of Smith and Linnartz (1980), most of the plots are members of the yellow poplar-mixed hardwood group (site indices of 81 and above); other plots on less productive sites fall into the sugar maple-northern red oak group (site indices of 66 to 80) and the white oak-black oak group (site indices of 51 to 65). On the whole, most of the plots are in some of the most productive sites in the Blue Ridge province.

After plot establishment in 1974, 44 of the 47 plots were thinned. Thinning criteria were: (1) to leave higher-valued species; (2) to leave higher-quality stems; and (3) to improve spacing. The thinning was basically from below, although dominants and codominants were sometimes removed to satisfy the above criteria. To remove any edge

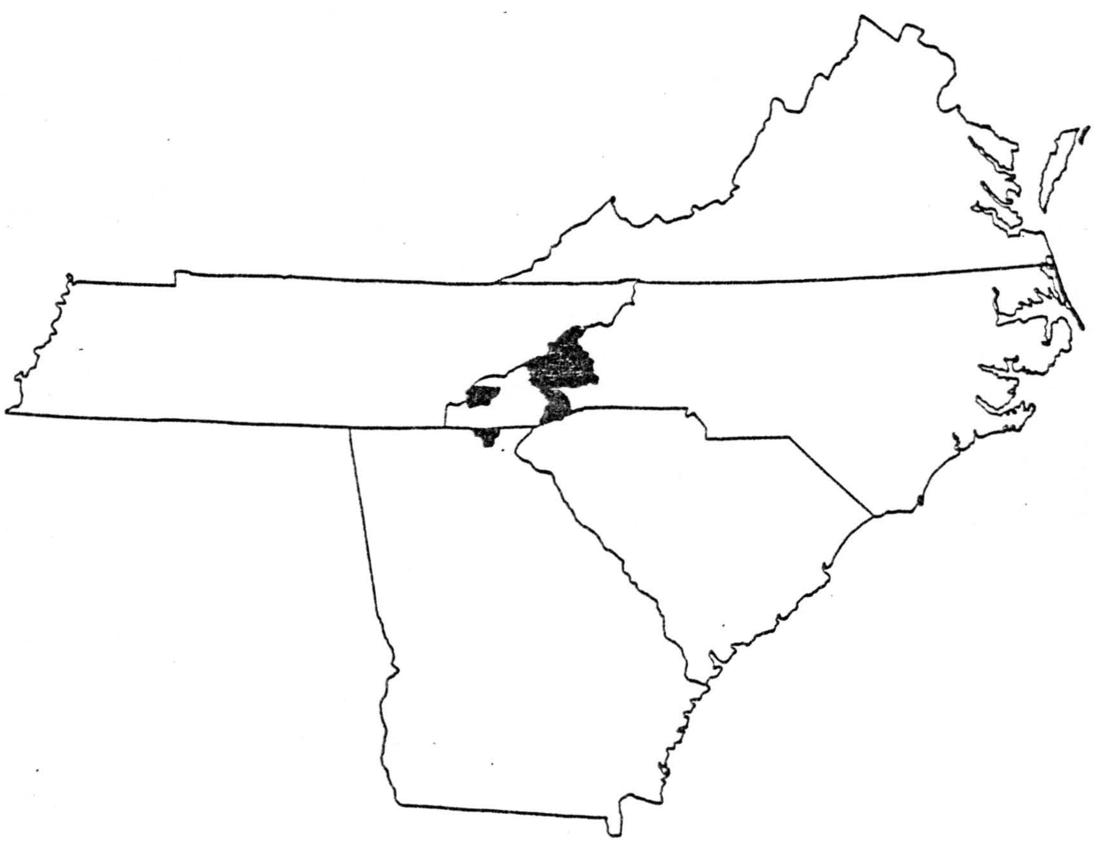

Figure 1. Locations of 47 permanent sample plots by state and county representing mixed Appalachian hardwoods in the Blue Ridge.

bias effect, a buffer strip around the plots was also thinned. Data recorded at each measurement include species, DBH (to the nearest 0.1 inch), crown class (dominant, codominant, intermediate, or suppressed), height to live crown (to the nearest foot), and total height (to the nearest foot). Tree status was also recorded at each measurement (alive, dead, wind throw, knock down, lightning damage, top dying, top broken). The thinned plots were used for model development. There were insufficient unthinned plots to develop separate models for unthinned stands; however, these plots may be useful in evaluating how well the thinned stand models perform.

Other characteristics recorded for each plot were aspect, elevation, slope position, and slope percent. Stand age for each plot was estimated from increment cores of dominant and codominant trees of major species present. Site index (base age 50) of species other than oak was determined and converted to white oak using Doolittle's (1958) species conversions. Oak site indices were computed using Olson's (1959) equation:

$$Ln(SI) = Ln(H_c) + 22.0217\ (1/A - 1/50)$$

where

SI = site index (feet), base age 50

H_c = average height of dominants and codominants (feet)

A = stand age (years)

A data summary for the 44 thinned plots appears in Tables 1 and 2.

Table 1. Data summary for 44 thinned plots used to develop a stand level growth and yield model for mixed Appalachian hardwoods in the Blue Ridge.

VARIABLE	MINIMUM	MEAN	MAXIMUM
Site index, base age 50	62	81	96
Elevation (ft.)	2040	3248	4500
Slope percent	10	38	75
Stand Age (yrs.)			
At first measurement	19	37	56
At first remeasurement	24	42	61
At second remeasurement	29	47	66
Basal Area (sq. ft./acre)			
Before thinning	83	117	193
Removed in thinning	32	59	98
At first measurement	31	57	104
At first remeasurement	38	70	127
At second remeasurement	44	82	153
Trees per acre			
Before thinning	151	470	1517
Removed in thinning	85	338	1166
At first measurement	40	133	391
At first remeasurement	40	130	391
At second remeasurement	40	129	384
Quadratic mean DBH (in.)			
Before thinning	3.4	7.3	12.6
At first measurement	4.7	9.6	14.8
At first remeasurement	5.9	10.6	15.6
At second remeasurement	6.8	11.6	16.5
Dominant/Codominant Height (ft.)			
At first measurement	44	69	93
At first remeasurement	48	78	103
At second remeasurement	54	80	98

Table 2. Distribution of 44 thinned plots by site index, age, and basal area (mixed Appalachian hardwoods in the Blue Ridge).

SITE INDEX (Feet)	AGE (Years)	BASAL AREA (Sq. ft. per acre) 40	60	80	100	Total
	20	-	-	-	-	-
60	40	1	-	-	-	1
	60	-	-	-	-	-
	Total	1	-	-	-	1
	20	-	-	-	-	-
70	40	-	5	1	-	6
	60	-	-	1	-	1
	Total	-	5	2	-	7
	20	4	2	1	-	7
80	40	7	2	2	1	12
	60	-	1	1	-	2
	Total	11	5	4	1	21
	20	2	2	1	-	5
90	40	2	2	2	2	8
	60	-	1	-	-	1
	Total	4	5	3	2	14
	20	-	-	-	-	-
100	40	1	-	-	-	1
	60	-	-	-	-	-
	Total	1	-	-	-	1
	TOTAL	17	15	9	3	44

CHAPTER IV. METHODS

As stated in the Introduction, the two objectives of this research were to predict future diameter distributions of mixed Appalachian hardwoods (at the stand level), and to break the predicted distribution down by diameter classes and species groups. Two different methods for predicting diameter distributions were tried: (1) the 3-parameter Weibull probability density function and the parameter recovery method, and (2) a stand-level ratio approach similar to that used by Amateis et al. (1985). The data were divided into species groups, and analyses were done both on the whole data set and individual species groups. Following is a discussion of the methods used in this thesis.

The first step in this research was to divide the data into species groups. The different species present in the data reflect a wide range of silvical characteristics and economic value. Groups were formed such that species within a group have similar economic value, growth rate, and shade tolerance. After much consideration, the trees were collected into five species groups: red oak, white oak, intolerant, tolerant, and miscellaneous. From an economic viewpoint, more species groups would have been desirable (a separate group for each species would have been ideal); from a modeling viewpoint, fewer groups would have been desirable (one group, the whole stand, would have been ideal). The groups decided upon are a compromise between the two extremes. Summary information for five species groups for which models were developed appear in Table 3. The

Table 3. Frequency of trees by species group[a] from thinned plots used to develop growth models for mixed Appalachian hardwoods in the Blue Ridge.

SPECIES GROUP	NUMBER OF TREES
Red Oak	215
White Oak	248
Intolerant	242
Tolerant	196
Miscellaneous	100
TOTAL	1001

[a] Red Oak consists of northern red (171 trees) and black oak, (44 trees).

White Oak consists of white (90 trees) and chestnut oak (158 trees).

Intolerant consists of Yellow Poplar (119 trees), Black Cherry (67 trees), Cucumbertree (5 trees), Fraser Magnolia (10 trees), and Black Locust (41 trees).

Tolerant consists of Red Maple (70 trees), Sugar Maple (7 trees), Sweet Birch (86 trees), and Yellow Birch (33 trees).

Miscellaneous consists of White Ash (3 trees), Basswood (1 tree), Beech (12 trees), Hickory (11 trees), Sassafras (4 trees), Scarlet Oak (52 trees), Mountain Silverbell (12 trees), and Sourwood (5 trees).

second step was to predict stand attributes by species group and for the whole stand to provide necessary inputs for the parameter recovery process. Basal area per acre (BA) and trees per acre (TPA) were predicted at a future time; arithmetic mean DBH (DBAR) and minimum stand DBH (DMIN) were obtained at future times with the projected BA and TPA. To provide for simplicity and consistency across species groups, equation forms were determined for the whole stand; then these equations were fitted to species groups (this was done for all equations except the basal area growth equation; the basal area growth equation chosen for the whole stand did not perform well by species groups, so the equation ultimately chosen was determined by performance by species groups, not the whole stand). Regression statistics considered in equation evaluation were R^2, mean square error (MSE), PRESS, C_p, and plots of externally studentized residuals vs. predicted values. Heavy emphasis was placed upon the PRESS statistic because it gives an impression of how well an equation will predict, while R^2 and MSE are basically fit statistics. The equations were developed largely by empirical means by the method of least squares; that is, variables were included that provided the "best" combination of regression statistics, and the variables present in any given equation may not make any apparent biological sense.

The basal area growth and mortality equations were fitted to the same data set. This data set treated the two five-year remeasurement periods as independent observations and thus had 88 observations (2 five-year remeasurements of 44 plots). The DMIN and DBAR equations were fitted to another data set, one that treated each measurement as an independent observation and thus had 132 observations (3 measurements on 44 plots). The problem of correlated errors due to repeated measurements on the same trees was ignored; resulting coefficient estimates are unbiased but have inflated variances (Sullivan and Clutter, 1972).

After projecting stand attributes, future stand diameter distributions were obtained. Two methods were evaluated: the Weibull distribution and parameter recovery method, and a stand ratio approach similar to that used by Amateis et al. (1985). The recovery method used was moment-based recovery of the Weibull 'b' (scale) and 'c' (shape) parameters (after Frazier, 1981). The 'a' (location) parameter was determined as a function of DMIN. The moment based parameter recovery method consists of obtaining expressions for non-central moments of a stand attribute (in this case DBH) which result in n equations with n unknown parameters. These equations are then solved for the unknown parameters. Specifically, the i^{th} non-central moment of x (where x is DBH) is:

$$E(x^i) = \int_0^\infty x^i f(x)dx$$

where

f(x) = Weibull probability density function

and these moments are estimated by:

$$E(x^i) = \sum_{j=0}^{N} x_j^{\,i}/N$$

where

N = number of trees on plot.

Thus, the first two noncentral moments of DBH are DBAR and quadratic mean DBH (DQ) squared. In other words,

$$DBAR = \Sigma x_j/N$$

$$DQ^2 = \Sigma x_j^2/N = BA/(0.005454\ TPA)$$

Higher moments may be estimated, but they have no familiar usage in forestry terminology.

Equations for the first two non-central moments of DBH are rewritten to obtain the following system of 2 equations in 2 unknowns which are iteratively solved to give estimates of the 'b' and 'c' parameters:

$$b = (DBAR-a)/\Gamma_1$$

$$DQ^2 - a^2 - 2a(DBAR-a) - (DBAR-a)^2 - \Gamma_2/\Gamma_1^2 = 0$$

where $\Gamma_k = (1+k/c)$ and Γ is the gamma function. Obtaining the parameters in this way insures that the projected diameter distribution has the same DBAR and BA as those predicted from the regression equations.

The 'a' parameter may be recovered along with the 'b' and 'c' parameters if the first three moments are used (generating a system with 3 equations and 3 unknown parameters). However, the 'a' parameter was estimated independently rather than obtained by recovery for several reasons. Frazier (1981) reported convergence problems when attempting to recover all 3 parameters. He ultimately used a procedure similar to that outlined above. Also, Burk and Burkhart (1983) tried both methods (recovering all 3 parameters and recovering 'b' and 'c' while obtaining 'a' independently) on a data set similar to that used in this research and concluded that recovery based on the first two moments was the superior method. Since the 'a' parameter can be interpreted as the smallest possible DBH in the stand (Bailey and Dell, 1973), DMIN is predicted and 'a' is set to some function of DMIN.

Many different methods for setting the location parameter exist in the literature. 'a' is usually defined to be somewhere between zero and DMIN. The method selected for use here was similar to that used by Cao et al. (1982):

'a' = IFIX(DMIN - 0.5) - 0.49

where

IFIX(x) = integer portion of x.

The effect of this transformation is to set 'a' at the lower end of the next smaller 1-inch diameter class than that occupied by DMIN.

The stand-level volume ratio appproach of Amateis et al. (1985) was also evaluated for these plots. Their ratio approach used the following equation to predict the number of trees in a stand above a threshold diameter (D) given the inputs of total number of trees per acre and quadratic mean DBH

$$MN=Ne^{-[(\Gamma(1+2/b)^{b}]^{1/2}(D/DQ)^{b}}$$

where

MN = trees per acre larger than D
N = total trees per acre in stand
b = parameter to be estimated
D = threshold diameter limit (inches)
DQ = quadratic mean DBH (inches).

Using equations developed above, N and DQ will be obtained at a future age; numbers of trees in each diameter class at the future age are obtained by subtraction. Amateis et al. (1985) show that this equation has the form of the 2-parameter Weibull distribution, where b' is analogous to the 'c' parameter. Thus the shape of the diameter distribution generated by this equation is constant as long as b remains the same.

Two goodness-of-fit statistics were computed to determine how well the predicted diameter distributions fit the observed data: the chi-square goodness-of-fit statistic and the Kolmogorov-Smirnov goodness-of-fit statistic (K-S statistic).

The chi-square statistic is computed as

$$X^2 = \Sigma \, (E_i - O_i)^2 / E_i$$

where

E_i = expected TPA in diameter class i
O_i = observed TPA in diameter class i

and df = n - 1 - r, where r = number of parameters estimated from the data. The statistic X^2 is used to test the hypothesis

H_0 : $f(x) = o(x)$
H_1 : $f(x) \neq o(x)$

where

f(x) = predicted diameter distribution
o(x) = observed diameter distribution

at a designated α-level.

The chi-square statistic is an effective statistic for measuring goodness-of-fit; however, it has several shortcomings when applied to these data. The statistic was designed for discrete distributions while the Weibull is a continuous distribution. To apply the test to continuous distributions the data must be categorized into arbitrary intervals (diameter classes in this case). Also, it is recommended that no more than 20 percent of the E_i should be less than 5 (Conover, 1971). If there are $E_i < 5$, classes should be collapsed so that all $E_i \geq 5$. However, for some species groups on many plots, if diameter classes were collapsed then there would not be enough degrees of freedom left to perform the test. Therefore, no collapsing was done. Chi-square values were computed for model comparisons, but they should not be regarded as true goodness-of-fit statistics.

The K-S statistic is computed as

$$K = \max \mid F_o(x) - S_n(x) \mid$$

where

$F_o(x)$ = cdf from predicted diameter distribution
$S_n(x)$ = cdf from observed diameter distribution

The statistic K tests the hypothesis

$$H_0 : F_o(x) = S_n(x) \text{ for } -\infty<x<\infty$$
$$H_1 : F_o(x) \neq S_n(x) \text{ for at least one value of } x$$

at a designated α-level. This test compares the cdf of the two distributions and determines if they differ by a significant amount. The K-S test was designed for continuous distributions. The K-S test is preferred over the chi-square test when the sample size is small (as it is on many plots in this data set)(Conover, 1971). Massey (1951) states that when the parameters have been estimated from the sample, the distribution of K is not known (Frazier, 1981). Therefore, as in the case with the chi-square test, no valid statistical tests may be performed, but the values may be used for model comparisons.

In the case of the Weibull distribution, future numbers of trees per acre are obtained from the Weibull cdf and the projected TPA for the stand. The Weibull cdf is:

$$F(x) = 1 - \exp\{-[(x-a)/b]^c\}.$$

The number of trees in the i^{th} diameter class is then

$$TPA(i) = [F(i+0.5) - F(i-0.5)]TPA.$$

The stand ratio method gives the diameter distribution by subtraction.

Future diameter distributions were predicted for the whole stand and for the five species groups. The chi-square and K-S statistics were computed to determine how well the model fit the data. These tests were used to determine if the species groups could be predicted separately and summed to get the whole stand values, or if four groups would have to be summed and the fifth obtained by subtraction from the whole stand to reduce error in whole stand prediction.

Volume estimates were then calculated by species group and diameter class from the projected diameter distribution for that species group and an equation relating tree height to diameter class midpoint and other stand variables.

CHAPTER V. RESULTS AND DISCUSSION

PREDICTION OF STAND ATTRIBUTES

The ability of the basal area growth equation to predict future stand basal area is a very important part of this model. The equations for future values of DMIN, DBAR, and DQ all include the BA term, so if basal area is poorly predicted these other variables will be poorly predicted as well. Therefore, much consideration was given to developing a good basal area growth equation.

The first basal area growth equation fitted to the data was a modified form of the equation suggested by Sullivan and Clutter (1972). The equation is:

$$\text{Ln}\ (BA_2) = A_1/A_2\ \text{Ln}\ (BA_1) + b_1(1 - A_1/A_2) \qquad (1)$$

where

A = stand age (years)
b_1 = regression coefficient
subscripts 1 and 2 on BA and A refer to times 1 and 2, respectively.

This equation is the same as the Sullivan and Clutter (1972) equation but lacks the term $b_2(1 - A_1/A_2)SI$. The term including SI was not significant for these data and its inclusion did not improve PRESS or MSE. Burk and Burkhart (1983), who used equation (1) as their basal area growth equation, also did not include the SI term; apparently the range of SI values is not wide enough in this data set to warrant its inclusion.

Equation (1) performed very well for the whole data set; however, when fitted to species groups it consistently underestimated future BA on plots with large basal area. Several modifications of the equation were examined and the form ultimately accepted was:

$$\begin{aligned} Ln(BA_{2i}) = {} & b_0 + A_1/A_2 Ln(BA_{1i}) + b_1(1-A_1/A_2) \\ & + b_2(1-BA_{1i}/BA_t) + b_3(y) \end{aligned} \tag{2}$$

where:

subscript i refers to basal area per acre of species group i
subscript t refers to total stand basal area per acre
y = years since thinning that A_1 occurs.

Inclusion of the proportional basal area term improved fit and PRESS across all species groups. When fitted to the whole stand, this term disappears ($BA_{1i}/BA_t = 1$). The years since thinning term greatly improved the PRESS for the red and white oak groups; it was nonsignificant for the other three species groups, but did not apparently affect the regressions at all (it was basically inert) so it was retained. This result implies that the basal area growth for the oaks differs from that of the other species for the first few years after thinning. The years since thinning term was significant when the equation was fitted to the whole stand; evidently the difference in growth rate of the oaks alone for the first few years after thinning is great enough to make the growth of the stand as a whole appear different. Inclusion of the intercept term greatly improved the appearance of the residual plots; without it there tended to be clumps of points. The intercept was significant for each species group, but nonsignificant for the whole stand. Model (2) lacks the desirable property that model (1) possesses, when $A_1 = A_2$, then $BA_1 = BA_2$.

However, the performance of (2) is far superior to that of (1), so model (2) was chosen as the basal area growth equation. Coefficients and regression statistics for model (2) appear in Table 4.

Two different mortality equations were evaluated. The first equation was one proposed by Clutter and Jones (1980):

$$TPA_2 = [TPA_1^{a_1} + a_2(A_2^{a_3} - A_1^{a_3})]^{1/a_1} \quad (3)$$

where:

TPA_1 and TPA_2 = TPA at ages A_1 and A_2, respectively, and

a_1, a_2, a_3 = regression coefficients.

Some desireable properties of this model are when A_1 equals A_2, TPA_1 equals TPA_2, and as A_2 approaches infinity TPA_2 approaches zero. The second equation was proposed by Piennar and Shiver (1981), which was derived from a 2-parameter Weibull density function:

$$TPA_2 = TPA_1[\exp(-a_1(t_2^{a_2} - t_1^{a_2}))] \quad (4)$$

where:

TPA_1 and TPA_2 = TPA at times t_1 and t_2, respectively, and

a_1, a_2 = regression coefficients.

Equation (4) is also conditioned so that TPA_1 equals TPA_2 when t_1 equals t_2, and as t_2 approaches infinity TPA_2 approaches zero. Both equations have the desirable property that the only inputs needed are age and initial number of trees. Lemin and Burkhart (1983) compared the performance of these two functions and two others on loblolly pine data and concluded that equations (3) and (4) were the best, with (3) being slightly better than (4).

Table 4. Coefficients and fit statistics for basal area growth equations[a] for mixed Appalachian hardwoods in the Blue Ridge.

SPECIES GROUP	COEFFICIENTS b_0	b_1	b_2	b_3
Red Oak	0.284068	4.323840	-0.476345	0.017094
White Oak	0.226187	3.693233	-0.323071	0.009908
Intolerant	0.531857	3.255270	-0.567427	-0.005942
Tolerant	0.586024	3.550618	-0.727647	0.006227
Miscellaneous	0.617423	3.119669	-0.704222	-0.004082
Whole Stand	0.021804	5.323155	-	0.004380

SPECIES GROUP	FIT STATISTICS R^2	MSE	$MSE(BA_2)$
Red Oak	.6767	0.016728	4.061
White Oak	.7998	0.005192	2.836
Intolerant	.8031	0.008472	5.685
Tolerant	.6909	0.017261	2.462
Miscellaneous	.5110	0.025597	5.264
Whole Stand	.9639	0.001675	11.234

[a] Equation is:

$$Ln(BA_{2i}) = b_o + A_1/A_2 Ln(BA_{1i}) + b_1(1-A_1/A_2) + b_2(1-BA_{1i}/BA_t) + b_3(y)$$

Where:

BA = Basal area per acre (sq.ft)

A = Stand age (yrs.)

y = Years since thinning at age A_1

Subscripts 1 and 2 refer to times 1 and 2

Subscript i refers to species group i

Subscript t refers to total stand BA

Fitting these two equations proved to be rather difficult. Convergence was not obtained for nonlinear least squares fits to either equation. This problem was due to the fact that very little mortality has occurred on the plots in the data set since they were thinned. Evidently the response surface generated was so flat that a minimum could not be found. The problem was rectified by modifying (4) so that only one parameter had to be estimated:

$$TPA_2 = TPA_1[\exp(-a_1(A_2 - A_1))] \quad (5).$$

Equation (5) easily converged and fit the data well. Coefficients and regression statistics for model (5) appear in Table 5.

Equation forms to predict DMIN and DBAR are similar to those proposed by Burk and Burkhart (1983). In each case, many different combinations of independent variables were evaluated. The two equations decided upon provided the "best" combination of regression statistics. The DMIN equation is:

$$\mathrm{Ln}\,(\mathrm{DMIN}) = b_0 + b_1\mathrm{Ln}(\mathrm{DQ}) + b_2(1/\mathrm{BA}) \quad (6).$$

The DBAR equation is:

$$(\mathrm{DQ} - \mathrm{DBAR})^{1/2} = b_0 + b_1\mathrm{SI} + b_2\mathrm{BA} + b_3\mathrm{Ln}(\mathrm{SI}) \quad (7).$$

The values of DQ, DBAR, DMIN, and BA in equations (6) and (7) refer to values for the species group of interest. The reason for predicting the square root of the difference between DQ and DBAR rather than DBAR directly is to condition the equation so that DBAR cannot exceed DQ. Another way to achieve the same objective is to predict the logarithm of the difference. As would be expected, the variance inflation factors for Ln(SI) and SI were rather large; however, in order to obtain good PRESS statistics both variables had to be included in the

Table 5. Coefficients and fit statistics for mortality equations[a] for mixed Appalachian hardwoods in the Blue Ridge.

	COEFFICIENTS		
SPECIES GROUP	a_1	R^2	MSE
Red Oak	0.00221586	.997	3.896
White Oak	0.00176353	.999	3.624
Intolerant	0.00163784	.999	1.657
Tolerant	0.00324641	.998	6.282
Miscellaneous	0.00404173	.997	1.946
Whole Stand	0.00333273	.999	16.860

[a] Equation is:

$$TPA_2 = TPA_1 \, [\exp(-a_1(A_2 - A_1))]$$

Where:

TPA = Trees per acre
A = Stand Age (years)
Subscripts 1 and 2 refer to time

equation. Coefficients and regression statistics for equations (6) and (7) appear in Tables 6 and 7, respectively.

COMPARISON OF WEIBULL DISTRIBUTION AND STAND RATIO METHOD

The equations discussed previously were used to predict the stand attributes BA, TPA, DMIN, and DBAR; these values were then used to generate diameter distributions from the Weibull distribution and the stand ratio method. The distributions generated by these two methods were compared using the chi-square and K-S statistics discussed in the Methods section.

The Weibull distribution and parameter recovery method proved to be far superior to the stand ratio method for generating diameter distributions. The parameter recovery method produced a much more flexible range of curves. This result is not that surprising. The parameter recovery method requires the input of DMIN, DBAR, and DQ on each plot; DMIN helps locate the lower end of the distribution, while DBAR and DQ serve to center and describe the shape of the distribution. After fitting the ratio equation to a species group (or the whole stand), the only input that varies from plot to plot is DQ. Thus, the ratio method requires DQ to locate and center the distribution. The ratio method can be modified so that it includes a "location parameter"; perhaps this modified form would yield better results. The distributions described by the ratio method were very "flat"; that is, they were not very peaked and tended to cover a wide range of diameter classes. The parameter recovery method was far more

Table 6. Coefficients and fit statistics for minimum stand DBH equations[a] for mixed Appalachian hardwoods in the Blue Ridge.

SPECIES GROUP	COEFFICIENTS b_0	b_1	b_2
Red Oak	-0.571443	1.126911	0.464581
White Oak	-0.732170	1.171384	0.952291
Intolerant	-1.280313	1.390702	0.780550
Tolerant	-1.074034	1.347334	0.698765
Miscellaneous	-0.798974	1.249255	0.217085
Whole Stand	-1.308068	1.304337	7.688220

SPECIES GROUP	R^2	MSE	MSE(DMIN)
Red Oak	.7797	0.020919	1.538
White Oak	.8268	0.013950	0.969
Intolerant	.8506	0.023306	1.661
Tolerant	.8779	0.017776	0.868
Miscellaneous	.8659	0.021527	1.379
Whole Stand	.7413	0.032600	1.205

[a] Equation is:

$$\mathrm{Ln(DMIN)} = b_0 + b_1\ \mathrm{Ln(DQ)} + b_2(1/\mathrm{BA})$$

Where:

DMIN = Minimum stand DBH (inches)
BA = Basal area per acre
DQ = Quadratic mean DBH (inches)

Table 7. Coefficients and fit statistics for average stand DBH equations[a] for mixed Appalachian hardwoods in the Blue Ridge.

	COEFFICIENTS			
SPECIES GROUP	b_0	b_1	b_2	b_3
Red Oak	-11.503385	-0.0359087	0.00888966	3.312957
White Oak	-22.824251	-0.0876586	0.00512554	6.861026
Intolerant	30.912604	0.118744	0.00626092	-9.185541
Tolerant	- 4.984356	-0.0144908	0.00668659	1.438001
Miscellaneous	-22.728827	-0.0853015	0.0139058	6.776689
Whole Stand	-15.390317	-0.053652	0.00089795	4.590913

SPECIES GROUP	R^2	MSE
Red Oak	0.5337	0.01541
White Oak	0.2336	0.02453
Intolerant	0.3245	0.03089
Tolerant	0.2116	0.02489
Miscellaneous	0.3168	0.03151
Whole Stand	0.2210	0.00815

[a] Equation is:

$$(DQ - DBAR)^{1/2} = b_0 + b_1 SI + b_2 BA + b_3 Ln(SI)$$

Where:

DQ = Quadratic mean DBH (inches)
DBAR = Average stand DBH (inches)
BA = Basal area per acre
SI = Oak site index (feet), base age 50.

effective in describing the shape of the diameter distributions observed in these data. Therefore, the parameter recovery method was selected to produce the diameter distributions for further analysis.

EVALUATION OF PREDICTED DISTRIBUTIONS

Three projection periods were used to evaluate predictive capability of the model. Projection period (1) was a 5-year period, from the initial measurement until the first remeasurement. Period (2) was also a 5-year period, from the first remeasurement to the second remeasurement. Period (3) was a 10-year period, from the initial measurement to the second remeasurement. Distributions were predicted by species group and for the whole stand for each projection period. The predicted distributions for the five species groups were summed to get a whole stand distribution. This summed distribution was compared to the predicted whole stand distribution to determine if the groups could be projected and summed to give the whole stand distribution, or, alternatively, if all groups but one and the whole stand should be projected with the species group not projected being obtained by subtraction.

Results for the chi-square and K-S tests for periods (1), (2), and (3) appear in Tables 8, 9, and 10, respectively. It is immediately evident that the two tests give conflicting results as to the quality of fit of the predicted distributions to the observed distributions. The chi-square test rejects the null hypothesis for virtually every test performed, while the K-S test accepts the null hypothesis for

Table 8. p - values for Chi-square and Kolmogorov - Smirnov (K-S) goodness of fit statistics for projection period 1 (first measurement to first remeasurement) (comparing observed vs. predicted DBH distributions of thinned plots of mixed Appalachian hardwoods in the Blue Ridge).

CHI-SQUARE STATISTIC

SPECIES GROUP	# Plots	p<0.01	0.01<p<0.05	p>.05
Red Oak	25	25	-	-
White Oak	29	29	-	-
Intolerant	29	29	-	-
Tolerant	15	14	1	-
Miscellaneous	12	11	1	-
Groups Summed	44	43	1	-
Whole Stand	44	43	1	-

K-S STATISTIC

SPECIES GROUP	# Plots	p<0.01	0.01<p<0.05	0.05<p<0.10	p>0.10
Red Oak	39	1	-	-	38
White Oak	35	2	-	-	33
Intolerant	38	1	3	2	32
Tolerant	31	5	1	1	24
Miscellaneous	28	1	-	1	26
Groups Summed	44	-	-	-	44
Whole Stand	44	-	-	-	44

Table 9. p - values for Chi-square and Kolmogorov - Smirnov (K-S) goodness of fit statistics for projection period 2 (first remeasurement to second remeasurement) (comparing observed vs. predicted DBH distributions of thinned plots of mixed Appalachian hardwoods in the Blue Ridge).

CHI-SQUARE STATISTIC

SPECIES GROUP	# Plots	p<0.01	0.01<p<0.05	p>.05
Red Oak	28	27	1	-
White Oak	29	26	2	1
Intolerant	30	29	-	1
Tolerant	14	12	-	2
Miscellaneous	16	15	1	-
Groups Summed	44	44	-	-
Whole Stand	44	43	1	-

K-S STATISTIC

SPECIES GROUP	# Plots	p<0.01	0.01<p<0.05	0.05<p<0.10	p>0.10
Red Oak	39	-	1	1	37
White Oak	34	-	1	1	32
Intolerant	38	-	1	1	36
Tolerant	31	1	1	1	28
Miscellaneous	28	1	-	2	25
Groups Summed	44	-	-	-	44
Whole Stand	44	-	-	-	44

Table 10. p -values for Chi-square and Kolmogorov - Smirnov (K-S) goodness of fit statistics for projection period 3 (first measurement to second remeasurement) (comparing observed vs. predicted DBH distributions of thinned plots of mixed Appalachian hardwoods in the Blue Ridge).

CHI-SQUARE STATISTIC

SPECIES GROUP	# Plots	p<0.01	0.01<p<0.05	p>.05
Red Oak	26	25	-	1
White Oak	29	27	1	1
Intolerant	29	29	-	-
Tolerant	15	13	1	1
Miscellaneous	14	13	1	-
Groups Summed	44	44	-	-
Whole Stand	44	44	-	-

K-S STATISTIC

SPECIES GROUP	# Plots	p<0.01	0.01<p<0.05	0.05<p<0.10	p>0.10
Red Oak	39	2	2	2	33
White Oak	34	2	1	2	29
Intolerant	38	1	2	4	31
Tolerant	31	4	-	2	25
Miscellaneous	28	1	-	3	24
Groups Summed	44	-	-	-	44
Whole Stand	44	-	-	-	44

most tests. These conflicting results are probably due to inability to satisfy the requirements of the chi-square test (that no more than 20 percent of the E_i be < 5; if a diameter class has a small value of E_i and a large value of O_i, the X^2 value computed becomes very large, resulting in a rejection). If diameter classes were collapsed, results of the chi-square test would undoubtably improve. However, as stated earlier, if classes were collapsed there would not be many remaining plots on which the test could be performed. Also, if expected and observed number of trees per plot rather than trees per acre were used the results should differ because the power of the chi-square test is artificially increased due to the expansion factor used to convert values to a per acre basis. However, Frazier (1981) performed the chi-square test using trees per acre rather than number of trees. The K-S test, on the other hand, gives an overly optimistic impression of the fit to the observed data. The K-S test loses power when the parameters of the hypothesized distribution are estimated from the sample; the effect is to reduce the critical value against which the value of K is compared. Critical values have been derived for the 2-parameter Weibull distribution when the parameters have been estimated from the data, but not the 3-parameter Weibull distribution. Therefore, it is more useful to compare the magnitudes of the statistics than to rely on the computed p-values. Figures 2, 3, and 4 illustrate the fit of the projected distribution to the observed distribution, and the difference between the chi-square and K-S statistics for these distributions. All 3 Figures are for the red oak group and projection period 3. The following statistics apply to the

Figures: Figure 2 $X^2 = 117.5$ ($p < 0.001$) and $K = 0.318$ ($p > 0.20$); Figure 3 $X^2 = 102.9$ ($p < 0.001$) and $K = 0.494$ ($0.05 < p < 0.10$); Figure 4 $X^2 = 5.94$ ($p > 0.05$) and $K = 0.143$ ($p > 0.20$). Figure 2 is an example of a case where the chi-square test rejects H_0 and the K-S test accepts H_0. Figure 3 is an example of a case where both tests reject H_0 (if $\alpha = 0.1$ for the K-S test), whereas Figure 4 is an example of a case where both tests accept H_0.

Figures 2 and 3 show that the predicted distribution is not taking on the appropriate shape to fit the data well. The shape needed is a distribution more highly skewed to the right. The Weibull has the ability to assume this shape; if the 'c' parameter approaches 1 (from above) the distribution becomes highly skewed to the right; when 'c' $\leq$ 1 the distribution becomes an inverse j-shaped curve. The difference between DBAR and DQ exerts a great deal of influence on the value of 'c'; as the difference between DBAR and DQ increases 'c' becomes smaller (all other variables held constant). For the plots in Figures 2 and 3 there is not enough difference between DBAR and DQ to force the 'c' parameter to yield a more highly skewed distribution. An alternative recovery method using the first and fourth moments (rather than the first and second) resulted in curves that were far too highly skewed to the right. Thus it appears that even though the Weibull has the ability to take on the shape needed to fit most of the data well, there is no readily apparent way to force it to do so.

Summing the predicted distributions for the five species groups to get the whole stand distribution resulted in fits to the data almost as good as predicting the whole stand itself for periods 1 and 2.

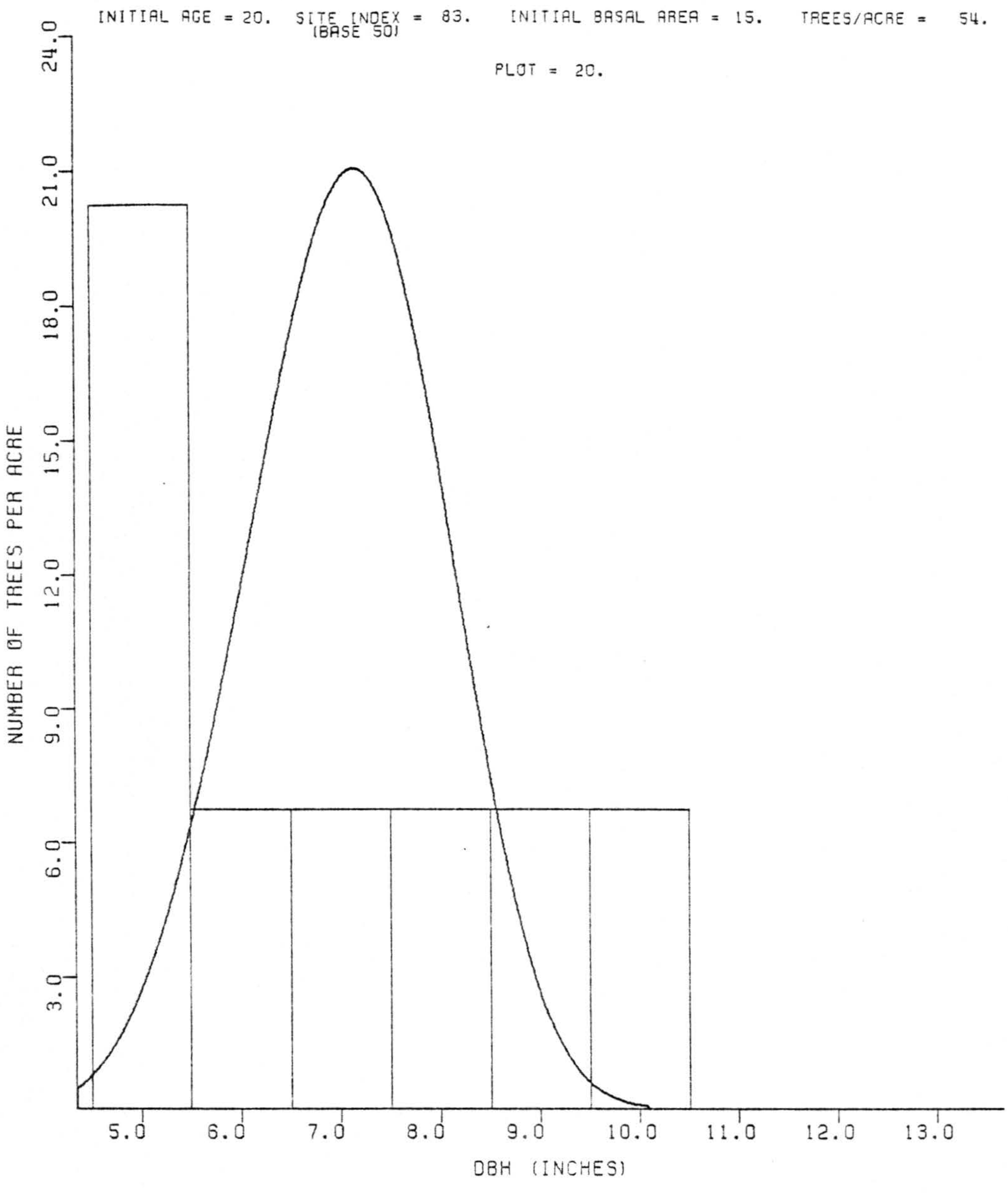

Figure 2. Observed and predicted diameter distributions for the red oak species group (plot 20), projection period 3 (first measurement to second remeasurement)(thinned plots of mixed Appalachian hardwoods in the Blue Ridge).

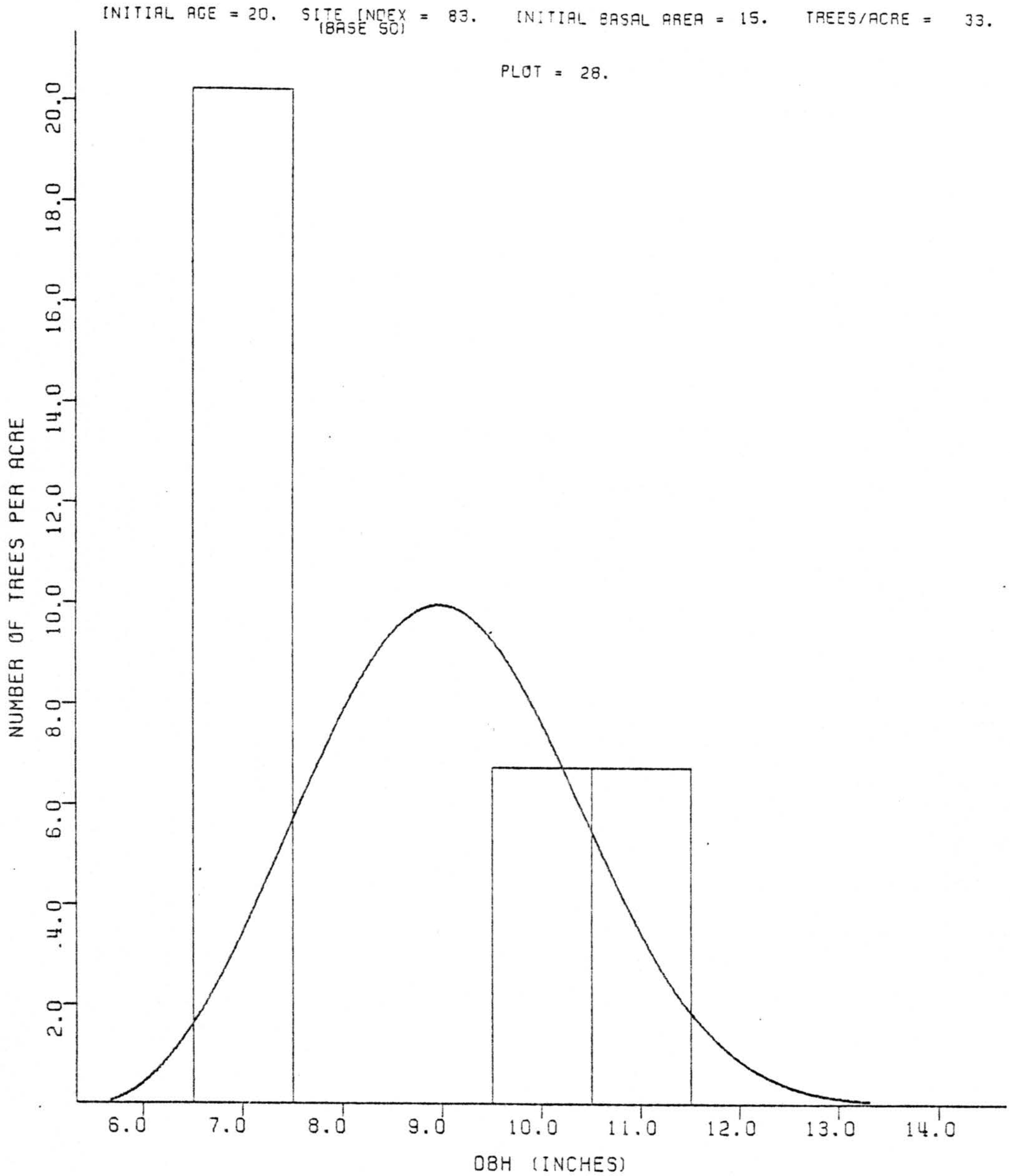

Figure 3. Observed and predicted diameter distributions for the red oak species group (plot 28), projection period 3 (first measurement to second remeasurement) (thinned plots of mixed Appalachian hardwoods in the Blue Ridge).

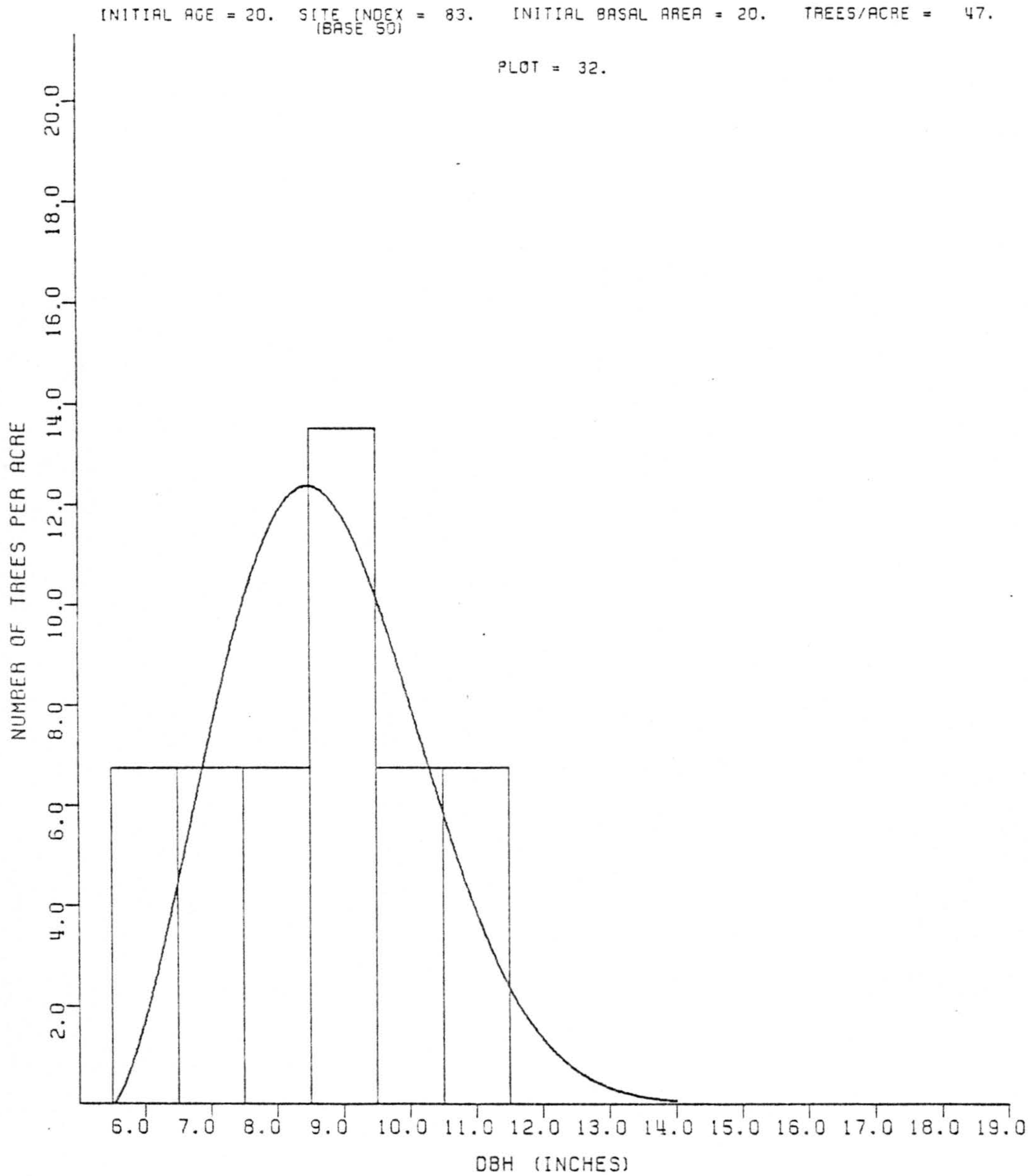

Figure 4. Observed and predicted diameter distributions for the red oak species group (plot 32), projection period 3 (first measurement to second remeasurement) (thinned plots of mixed Appalachian hardwoods in the Blue Ridge).

Magnitudes of the X^2 and K values computed for each method were comparable. However, for period 3 projecting the whole stand was considerably better than summing the groups. The magnitude of the X^2 values for summed groups was much greater than those for the whole stand (even though $p < 0.01$ for either case). Since it is desireable to have a projection system that performs well for long projection periods, the stand as a whole was predicted to provide whole stand values; four species groups were predicted as before and summed and the distribution of the fifth group was obtained by subtraction from the whole stand.

None of the distributions of different species groups obtained by subtraction fit the observed data well. The miscellaneous group was selected to be obtained by subtraction because none of the species in the group are of primary economic importance. Thus errors in prediction are not as important as for the other groups.

HEIGHT PREDICTION

In order to obtain volume estimates, an equation relating tree height to diameter class midpoint was needed. The equation selected was one developed by Harrison (1984):

$$H = 4.5 + H_c[1 + a_1 e^{(a_2 H_c)}][1 - e^{(BD/H_c)}] \quad (8)$$

where

H = total height (feet)
H_c = height of dominant and codominant oaks (feet)
D = diameter class midpoint (inches)
a_1, a_2, B = parameters to be estimated.

The value of H_c was obtained by rearranging Olson's site index equation:

$$\ln H_c = \ln(SI) - 22.0217(1/A - 1/50).$$

For each plot, a value of H_c was obtained given the values of SI and A; then H_c and various values of D are used to obtain estimates of tree height by diameter class. The height equation was fitted to species groups separately. Parameter estimates and fit statistics for the height equation are presented in Table 11.

VOLUME PREDICTION

The equations selected to estimate predicted volume are a compatible system of volume ratio equations developed by Burk et al. (1985). The equations were developed from upper stem diameter measurements using a Barr and Stroud dendrometer on some of the same plot trees in the data set examined in this thesis. The system has three equations; a total volume equation, a merchantable height ratio equation, and a taper equation. Coefficients for the system were estimated simultaneously by species group so that the same volume estimate will be calculated whether a merchantable height or diameter is specified. The total volume equation is:

$$V = \frac{.005454(4.5)^2}{-b_{0i}b_{1i}} \frac{(DBH)^2}{H} \left(\frac{H-4.5}{4.5}\right)^{b_{1i}+1} e^{-b_{0i}\left(\frac{4.5}{H-4.5}\right)^{b_{1i}}} \qquad (9)$$

where

V = total cubic foot volume outside bark
H = total tree height (feet)
b_{0i}, b_{1i} = coefficients for species group i.

Table 11. Coefficients and fit statistics for height prediction equations[a] for mixed stands of Appalachian hardwoods in the Blue Ridge.

SPECIES GROUP	COEFFICIENTS a_1	a_2	B
Red Oak	1.07929254	-0.01546219	-7.29097774
White Oak	1.06190129	-0.02253277	-10.33367444
Intolerant	1.17739446	-0.01381045	-8.16534630
Tolerant	1.36028975	-0.02091233	-9.05520533
Miscellaneous	1.19610263	-0.01870700	-9.01152482

SPECIES GROUP	R^2	FIT STATISTICS MSE	RMSE*
Red Oak	.985	81.39	9.02
White Oak	.987	66.71	8.17
Intolerant	.983	106.32	10.31
Tolerant	.984	66.00	8.12
Miscellaneous	.982	91.39	9.56

* RMSE = MSE $^{1/2}$

[a]Equation is:

$$H = 4.5 + H_c[1 + a_1 e^{(a_2 H_c)}][1 - e^{(BD/H_c)}]$$

Where:

H = Total tree height (feet)
D = DBH class midpoint (inches)
H_c = Height of dominant and codominant oaks (feet)

Merchantable cubic foot volume to any height h is:

$$V_m = V\ (1 - \exp(b_{0i}(h/(H - h))^{b_{1i}})) \qquad (10)$$

where

V_m = merchantable cubic foot volume to any height h
h = height on bole above ground (feet).

The taper equation is:

$$\frac{d^2}{D^2} = \left(\frac{h}{4.5}\right)^{b_{1i}-1} \left(\frac{H-4.5}{H-h}\right)^{b_{1i}+1} e^{b_{0i}\left[\left(\frac{h}{H-h}\right)^{b_{1i}} - \left(\frac{4.5}{H-4.5}\right)^{b_{1i}}\right]} \qquad (11)$$

where

d = diameter outside bark at height h (inches).

The coefficients by species groups for this equation system are presented in Table 12.

MODEL EVALUATION

A typical volume calculation would proceed as follows. Inputs needed for the model are stand age, site index, values of BA and TPA for the whole stand, and the proportion of BA and TPA in each species group. Estimates of future stand BA by species group and the whole stand are calculated from equation (2), and future values of TPA by species group and the whole stand are calculated from equation (5). The miscellaneous group does not need to be projected because it will be obtained by subtraction. Values for DBAR and DMIN are calculated using equations (12) and (6) based on the BA computed from equation (2). The Weibull parameters are then recovered and the future diameter distributions for each species group generated from the Weibull cdf and the TPA computed from equation (5). Tree heights by

Table 12. Coefficients for total tree volume equation system[a] for mixed Appalachian hardwoods in the Blue Ridge.

SPECIES GROUP	COEFFICIENTS b_0	b_1
Red Oak	-1.7320	0.7989
White Oak	-1.6334	0.7959
Intolerant	-1.7427	0.8540
Tolerant	-1.8285	0.8416
Miscellaneous	-1.7854	0.8304

[a]System is:

$$V = \frac{.005454(4.5)^2}{-b_{0i}b_{1i}} \frac{(DBH)^2}{H} \left(\frac{H-4.5}{4.5}\right)^{b_{1i}+1} e^{-b_{0i}\left(\frac{4.5}{H-4.5}\right)^{b_{1i}}} \qquad (9)$$

$$V_m = V\,(1 - \exp(b_{0i}(h/(H-h))^{b_{1i}})) \qquad (10)$$

$$\frac{d^2}{D^2} = \left(\frac{h}{4.5}\right)^{b_{1i}-1} \left(\frac{H-4.5}{H-h}\right)^{b_{1i}+1} e^{b_{0i}\left(\left(\frac{h}{H-h}\right)^{b_{1i}} - \left(\frac{4.5}{H-4.5}\right)^{b_{1i}}\right)} \qquad (11)$$

Where:

V = Total tree volume (cubic feet, outside bark)
D = DBH class midpoint (inches)
H = Total tree height (feet)
V_m = Volume to height h (cubic feet, outside bark)
h = Height on bole above ground (feet)
d = Diameter outside bark on bole at height h (inches)
b_{0i}, b_{1i} = Coefficients for species group i

diameter class are calculated for each species group from equation (8). Volume estimates desired are then computed from equations (9), (10), and (11).

In order to examine properties of the model, total volume estimates were calculated under two different scenarios. First, all input variables were held constant except SI to evaluate the effect of site index on model behavior. For these calculations, initial age was set at 40 years and future age at 50 years. Three different SI values were used; 60, 80, and 100. Second, all input variables were held constant except for the projection period length to evaluate model behavior through time. Four different projection periods were examined; 10 years (ages 40 to 50), 20 years (ages 40 to 60), 30 years (ages 40 to 70), and 40 years (ages 40 to 80). SI was set at 80 for these calculations. In both cases, the BA and TPA values used as input were mean values from the data and years since thinning was set to zero.

The predicted diameter distributions behaved inconsistently with respect to SI. This inconsistency was due to problems with the DBAR equation. The equations that predict future values of BA, TPA, and DMIN are independent of SI; DBAR is the only variable in the diameter distribution projection system dependent on SI. The problem with the DBAR equation is that it has two terms that include SI, and the collinearity between these terms results in unstable coefficient estimates that yield inconsistent results across the values of SI tested. The trends for the whole stand, red oak, white oak, and tolerant groups are similar; the intolerant group is opposite that of the others. The reason for the discrepancy is that the signs on the SI and Ln(SI) coefficients in the DBAR equation are opposite for the intolerant group of what they are for the other groups. For the intolerant group, as SI increases from 60 to 80, DBAR increases; as SI increases from 80 to 100, DBAR decreases. The other groups behave in an opposite manner; as SI increases from 60 to 80, DBAR decreases; as SI increases from 80 to 100, DBAR increases.

In order to alleviate this problem, an equation with only one SI term was fitted to the data. This equation is:

$$(DQ - DBAR)^{1/2} = b_0 + b_1 BA + b_2 Ln(SI). \qquad (12)$$

The regression statistics for this equation are almost as good as those for the previous equation (equation (7)), but equation (12) exhibits consistent behavior both across species groups and SI. The decision was made to replace equation (7) with equation (12), and to incorporate equation (12) into the final model. Coefficient estimates and regression statistics for equation (12) appear in Table 13. No effort was made to go back and recalculate goodness of fit statistics calculated earlier. Tables 14-21 were all generated from the model incorporating equation (12).

Overall trends in volume estimates with increasing SI are as expected; as SI increases, volume increases. This volume increase is due both to increases in predicted height due to raising the SI, and to changes in the predicted diameter distributions. As SI increases, DBAR decreases, causing the variance of the predicted distributions to increase (DQ and DMIN do not change as SI changes). As the variance increases, the value of the shape parameter decreases, yielding

Table 13. Coefficients and fit statistics for average stand DBH equations[a] with one SI term for mixed Appalachian hardwoods in the Blue Ridge.

	COEFFICIENTS		
SPECIES GROUP	b_0	b_1	b_2
Whole Stand	-1.225427	0.000978128	0.373726
Red Oak	-2.052667	0.00886028	0.498249
White Oak	-0.0293827	0.00509698	0.052979
Intolerant	-1.254560	0.00616552	0.331129
Tolerant	-1.014511	0.00671482	0.266649
Miscellaneous	1.113313	0.0136102	-0.224281

SPECIES GROUP	R^2	MSE
Whole Stand	0.1766	0.0085
Red Oak	0.5267	0.0155
White Oak	0.2016	0.0253
Intolerant	0.2982	0.0318
Tolerant	0.2111	0.0246
Miscellaneous	0.3069	0.0316

[a] Equation is:

$$(DQ - DBAR)^{1/2} = b_0 + b_1 BA + b_2 Ln(SI)$$

Where:

DQ = Quadratic mean DBH (inches)
DBAR = Average stand DBH (inches)
BA = Basal area per acre
SI = Oak site index (feet), base age 50.

distributions more highly skewed to the right. Thus, as SI increases from 60 to 100, the diameter distributions become less peaked and skewed further to the right, depositing trees in larger diameter classes (the total TPA does not change as SI changes, just the distribution). These trends are illustrated in Tables 14, 15, and 16. Table 20 illustrates the difference in volume estimates with increasing SI for the red oak species group; the other groups are similar.

Overall trends in volume estimates with increasing projection period length are also as expected; as projection period length increases, volume increases. This increase is due to changes both in the diameter distribution and predicted tree height. As the stand moves through time, projected BA increases, causing DBAR and DMIN to increase, while TPA decreases. These are the expected trends. Volume estimates increase due to the movement of trees into larger diameter classes. The effect of increasing projection period length on projected diameter distributions is illustrated in Tables 15, 17, 18, and 19. These projected distributions seem to indicate that the red oak group grows faster than the other groups; the white oak and intolerant groups grow at about the same rate, and the tolerant group grows at a slower rate than the rest. Table 21 illustrates volume growth through time for the red oak group; other groups are similar.

Table 14. Effect of site index on projected diameter distributions (site index = 60, initial age = 40, future age = 50) (frequencies are trees/acre).

			GROUP FREQUENCIES			
DBH Class	Whole Stand	Red Oak	White Oak	Intol-erant	Tolerant	Misc.
6	1.6	-	-	-	0.2	1.3
7	5.8	-	-	-	4.2	1.5
8	13.2	-	3.5	0.6	16.3	-
9	21.9	0.5	7.8	3.1	4.9	-
10	27.8	4.3	9.7	8.3	-	3.9
11	26.7	13.1	7.2	11.3	-	-
12	18.7	8.8	3.0	6.7	-	-
13	9.1	0.5	0.7	1.2	-	1.8
14	2.9	-	-	-	-	2.9
15	0.6	-	-	-	-	0.6

Table 15. Effect of site index and projection period length on projected diameter distributions (site index = 80, initial age = 40, future age = 50) (frequencies are trees/acre).

		GROUP FREQUENCIES				
DBH Class	Whole Stand	Red Oak	White Oak	Intolerant	Tolerant	Misc.
6	4.3	-	-	-	1.2	3.1
7	9.9	-	-	-	6.0	3.9
8	15.9	-	3.8	1.8	11.1	-
9	20.5	3.0	7.8	4.7	6.6	-
10	22.0	6.0	9.2	7.3	0.8	-
11	20.0	7.3	6.8	7.8	-	-
12	15.4	5.9	3.2	5.7	-	-
13	10.1	3.1	0.9	2.8	-	-
14	5.5	1.1	0.1	0.9	-	1.9
15	2.5	0.2	-	0.2	-	2.1
16	0.9	-	-	-	-	0.9
17	0.3	-	-	-	-	0.3

Table 16. Effect of site index on projected diameter distributions (site index = 100, initial age = 40, future age = 50) (frequencies are trees/acre).

		GROUP FREQUENCIES				
DBH Class	Whole Stand	Red Oak	White Oak	Intolerant	Tolerant	Misc.
6	7.0	-	-	-	2.3	4.8
7	12.5	-	-	-	6.4	6.1
8	16.8	-	4.1	3.0	8.7	1.0
9	18.9	4.6	7.8	5.2	6.0	-
10	18.7	5.6	8.8	6.5	1.9	-
11	16.5	5.3	6.6	6.2	0.2	-
12	13.1	4.1	3.2	4.7	-	-
13	9.4	2.7	1.0	2.9	-	-
14	6.1	1.6	0.2	1.4	-	-
15	3.6	0.8	-	0.6	-	-
16	1.9	0.4	-	0.2	-	-
17	0.9	0.1	-	-	-	0.8
18	0.4	-	-	-	-	0.4
19	0.2	-	-	-	-	0.2

Table 17. Effect of projection period length on projected diameter distributions (initial age = 40, future age = 60, site index = 80)(frequencies are trees/acre).

			GROUP FREQUENCIES			
DBH Class	Whole Stand	Red Oak	White Oak	Intol-erant	Tolerant	Misc.
7	4.7	-	-	-	1.5	3.3
8	10.0	-	1.7	1.2	5.6	1.5
9	15.1	-	4.6	3.3	9.4	-
10	18.7	2.5	7.6	6.0	6.6	-
11	19.7	4.7	8.3	7.4	1.5	-
12	18.1	5.9	6.0	6.6	-	-
13	14.5	5.5	2.8	4.0	-	-
14	10.2	3.9	0.8	1.6	-	-
15	6.2	2.2	0.1	0.4	-	0.7
16	3.3	0.9	-	-	-	2.4
17	1.5	0.3	-	-	-	1.3
18	0.6	-	-	-	-	0.6
19	0.2	-	-	-	-	0.2

Table 18. Effect of projection period length on projected diameter distributions (initial age = 40, future age = 70, site index = 80) (frequencies are trees/acre).

		GROUP FREQUENCIES				
DBH Class	Whole Stand	Red Oak	White Oak	Intol-erant	Tolerant	Misc.
8	5.8	-	-	-	2.2	3.7
9	10.9	-	3.0	2.7	6.1	-
10	15.2	-	6.0	5.4	8.3	-
11	17.7	2.7	7.7	7.0	5.5	-
12	17.9	4.4	6.9	6.6	1.6	-
13	16.1	5.1	4.4	4.6	0.2	-
14	12.9	4.8	2.0	2.4	-	-
15	9.3	3.7	0.6	0.9	-	-
16	6.0	2.4	0.1	0.2	-	0.7
17	3.5	1.3	-	-	-	2.2
18	1.8	0.6	-	-	-	1.2
19	0.8	0.2	-	-	-	0.6
20	0.4	-	-	-	-	0.4
21	0.1	-	-	-	-	0.1

Table 19. Effect of projection period length on projected diameter distributions (initial age = 40, future age = 80, site index = 80) (frequencies are trees/acre).

		GROUP FREQUENCIES				
DBH Class	Whole Stand	Red Oak	White Oak	Intol-erant	Tolerant	Misc.
8	3.3	-	-	-	-	3.3
9	7.0	-	1.8	2.0	3.3	-
10	11.0	-	4.3	4.5	6.7	-
11	14.4	-	6.6	6.4	7.1	-
12	16.3	3.1	7.2	6.7	4.2	-
13	16.4	4.3	5.7	5.2	1.3	-
14	14.7	4.6	3.2	3.0	0.2	-
15	11.9	4.1	1.3	1.3	-	-
16	8.6	3.2	0.3	0.4	-	-
17	5.6	2.2	-	-	-	1.7
18	3.3	1.4	-	-	-	1.9
19	1.7	0.8	-	-	-	0.9
20	0.8	0.4	-	-	-	0.4
21	0.3	0.2	-	-	-	0.1
22	0.1	-	-	-	-	0.1

Table 20. Comparison of projected total cubic foot volume/acre (o.b.) estimates at 3 site index levels for the red oak species group (site indices of 60, 80,and 100, initial age = 40, future age = 50).

	SITE INDEX		
DBH CLASS	60	80	100
9	5.6	38.4	60.2
10	65.4	98.3	95.6
11	242.5	148.1	112.3
12	198.2	144.3	106.9
13	13.2	91.8	85.6
14	-	37.4	58.9
15	-	9.4	35.2
16	-	-	18.5
17	-	-	8.5
TOTAL	524.9	567.7	581.7

Table 21. Comparison of projected total cubic foot volume/acre (o.b.) estimates for 4 projection period lengths, red oak species group (initial age = 40, future ages = 50, 60, 70, and 80; site index = 80).

	PROJECTION PERIOD LENGTH			
DBH Class	10 yrs.	20 yrs.	30 yrs.	40 yrs.
9	38.4	-	-	-
10	98.3	42.0	-	-
11	148.1	97.6	55.7	-
12	144.3	148.8	110.7	79.0
13	91.8	165.8	155.9	132.2
14	37.4	139.4	172.4	167.2
15	9.4	89.1	155.5	175.3
16	-	43.1	116.3	158.1
17	-	15.6	72.7	124.8
18	-	-	38.1	87.3
19	-	-	16.7	54.4
20	-	-	-	30.4
21	-	-	-	15.3
TOTAL	567.7	741.4	894.0	1,024.0

CHAPTER VI. CONCLUSIONS AND RECOMMENDATIONS

Diameter distributions of thinned stands of mixed Appalachian hardwoods can be modeled reasonably well using the Weibull distribution and parameter recovery method. Distributions of individual species groups cannot be modeled as well as the whole stand, because the distributions of trees within species groups assume shapes that the Weibull distribution cannot approximate. However, if the distribution of trees within species groups is constrained so that the sum of species group distributions equals the whole stand values, the system should yield accurate estimates of stand growth. This thesis explored the feasibility of predicting all species groups but one and obtaining the distribution of the group not predicted by subtraction from the projected whole stand distribution. This technique produces a system logically constrained so that the sum of the parts equals the whole; however, the distribution of the group obtained by subtraction was very inaccurate. In this case, the group obtained by subtraction (the miscellaneous group) was of little economic significance so errors in prediction were not of great importance.

One recommendation that should improve the performance of the above model is to increase the size of plots measured to provide input data. Hopefully, larger plots with more trees per species group per plot would yield smoother distributions that are more easily modeled using a probability density function. Another recommendation for future research is to attempt to employ various mathematical techniques that exist for taking an entity and breaking it down into its consituent parts constrained so that the parts sum to the whole. For example, Amateis et al. (1984) used a simultaneous system of equations to break the total contents of loads of weight-scaled timber down into product contents so that the sum of the contents equalled the whole load. Similar methods could probably be used to break total volumes by species group down by size or product class.

Stand-level models, which have relatively simple input and computational requirements, need to be developed for mixed stands of Appalachian hardwoods. This research is an initial attempt at addressing this problem. As more remeasurement data becomes available from the sample plots, a data base should exist that will facilitate improvement of these kinds of models.

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APPENDIX I. USE OF STAND PROJECTION MODEL

The model requires two or three cards per stand as input, depending on whether the user wishes to input the basal area and trees/acre for each species group or use default values (percentages of the whole stand values input) defined in the program. The first card consists of stand level variables (SI, initial age) and variables defining the projection process (merchantable volumes, years since thinning at the initial age, and the projection period length). The second card consists of whole stand characteristics (BA and TPA), while the third card (optional) consists of stand characteristics (BA and TPA) by species group. A table displaying required format for input data is in Appendix II.

If use of default values for BA and TPA by species group is desired, the percentages of the whole stand BA and TPA (input on card 2) used are: red oak (25% BA, 21% TPA), white oak (26% BA, 25% TPA), intolerant (28% BA, 24% TPA), and tolerant group (12% BA, 20% TPA).

After defining all initial values of BA and TPA, stand characteristics (BA, TPA, DMIN, DBAR) are projected for the time period specified on the first card. These future values are then used to recover the 'b' and 'c' Weibull parameter estimates of the predicted distribution (subroutine WEIB) and generate diameter distributions using the Weibull cdf from these parameters (subroutine FREQ). Subroutine DIST sums the diameter distributions of the four species groups projected and subtracts the summed distribution from the whole stand distribution to obtain the distribution of the miscellaneous group. DIST defines the endpoints (minimum and maximum DBH classes) of the miscellaneous group distribution to be those of the whole stand; the routine works through the distribution from left to right (from small to large DBH classes) collecting and distributing excess numbers of trees per class as it goes; when it gets to the end (the largest DBH class in the whole stand), it goes back through from right to left doing the same thing until it reaches the smallest DBH class in the whole stand. Excess numbers of trees per class result when the summed distribution has more trees in a class than the whole stand does. If an excess number of trees remains after passing through the distribution twice (the summed distribution has more trees than the whole stand), the TPA in one or more species groups is adjusted and the DBH distribution recalculated so that no excess will remain after repeating subroutine DIST. The program first seeks to adjust the TPA of the tolerant group. If the tolerant group has no trees or lacks a sufficient number to correct the excess, the program next goes to the intolerant group. If the problem remains uncorrected, the program goes to the white oak group and finally to the red oak group.

Finally, total and merchantable (if requested) volumes are computed and the volume table is generated. If more than one stand is to be projected, the 3-card sequence is repeated for each stand.

The program generates error messages which should be checked after each run. The error messages are written to an output file. These messages come from either the WEIB subrountine or the GMMMA function. The WEIB subroutine generates three possible error messages. Error number 1 results when the secant method fails to converge in 70 iterations; a solution is reported, and the variable X2P is the value of DQ^2 corresponding to the solution reported. X2P should be checked to ensure that it is close enough to DQ^2 to result in a reasonable solution. Error number 2 results when a solution is found after perturbating DBAR. DBAR is perturbated in increments of Ø.Ø1 until a solution is obtained. X1P is the perturbated value of DBAR corresponding to the solution. Error number 3 results when a solution in the allowable range could not be found even after perturbating DBAR. The allowable range defined in the WEIB subroutine is $0.1 \leq$ 'c' $\leq$ 1Ø.Ø. The user should modify this range if he feels that reasonable values of 'c' might fall outside the range, or if he wishes to further restrict the range.

The GMMMA function generates two possible error messages. The first results when the argument of the gamma function is close to being a negative integer. The error message is reported and the gamma function is not solved. The second error message results when the argument is too large for the routine to solve. Again, the error is reported and the gamma function is not solved. If the user has access to IMSL subroutines or some other method of evaluating the gamma function, he may wish to substitute that method for this function.

APPENDIX II. INPUT VARIABLE FORMATS AND DESCRIPTION

Data Card	Column	Format	Variable	Description
1	1-2	I2	ISTAND	Stand I.D. number being analyzed.
	3-5	F3.0	AGE1	Stand age at start of projection period.
	6-8	F3.0	AGE2	Stand age at end of projection period.
	9-11	F3.0	SI	Site index (oak, in feet, base age 50 years).
	12-13	F2.0	Y	Years since stand was thinned at AGE1.
	14-17	F4.1	DTOP*	Merchantable top diameter (inches).
	18-21	F4.1	HTOP*	Merchantable height on bole (feet).
	22	I1	KFLAG	0 = Species group BA and TPA input. 1 = Default values desired.
2	1-7	F7.2	TOBA1	Total stand basal area/acre at AGE1.
	8-14	F7.2	TOTPA.1	Total stand trees/acre at AGE1.
3	1-7	F7.2	RBA1	Red oak BA at AGE1.
	8-14	F7.2	RTPA1	Red oak TPA at AGE1.
	15-21	F7.2	WBA1	White oak BA at AGE1.
	22-28	F7.2	WTPA1	White oak TPA at AGE1.
	29-35	F7.2	IBA1	Intolerant BA at AGE1.

* Either DTOP or HTOP may be specified, not both.

APPENDIX II. INPUT VARIABLE FORMATS AND DESCRIPTION (continued)

Data Card	Column	Format	Variable	Description
3	36-42	F7.2	ITPA1	Intolerant TPA at AGE1.
3	43-49	F7.2	TBA1	Tolerant BA at AGE1.
	50-56	F7.2	TTPA1	Tolerant TPA at AGE1.

APPENDIX III. PROGRAM LISTING

```
C**************************************************************
C
C     PROGRAM
C
C      DESCRIPTION:
C         THIS PROGRAM IS A WHOLE STAND GROWTH AND YIELD MODEL
C         FOR THINNED STANDS OF MIXED APPALACHIAN HARDWOODS
C         ON GOOD SITES.  THE MODEL PREDICTS GROWTH AND YIELD
C         FOR 5 SPECIES GROUPS; THE GROUPS MAY BE SUMMED TO
C         OBTAIN ESTIMATES OF STAND LEVEL YIELD.  THE FIVE
C         SPECIES GROUPS ARE:
C                1) RED OAK (NORTHERN RED AND BLACK OAK)
C                2) WHITE OAK (WHITE AND CHESTNUT OAK)
C                3) INTOLERANT (BLACK CHERRY, YELLOW POPLAR,
C                     CUCUMBERTREE, BLACK LOCUST, AND FRASER MAGNOLIA)
C                4) TOLERANT (RED MAPLE, SUGAR MAPLE, YELLOW BIRCH,
C                        AND SWEET (BLACK) BIRCH)
C                5) MISCELLANEOUS (OTHER SPECIES, MOSTLY SCARLET OAK,
C                        HICKORY, AND BEECH).
C
C         THE PROGRAM FIRST PROJECTS STAND VARIABLES FOR THE WHOLE
C         STAND AND SPECIES GROUPS (EXCEPT MISCELLANEOUS); PREDICTED
C         DIAMETER DISTRIBUTIONS ARE OBTAINED USING THE WEIBULL
C         PROBABILITY DENSITY FUNCTION.  WEIBULL PARAMETERS ARE
C         CALCULATED USING THE PARAMETER RECOVERY METHOD (MOMENT
C         BASED RECOVERY OF THE FIRST AND SECOND NONCENTRAL MOMENTS
C         OF DBH).  PREDICTED SPECIES GROUPS ARE SUMMED AND
C         SUBTRACTED FROM THE PREDICTED WHOLE STAND DISTRIBUTION
C         TO OBTAIN THE MISCELLANEOUS DISTRIBUTION.  FINALLY,
C         TOTAL AND MERCHANTABLE CUBIC FOOT VOLUME (IF DESIRED)
C         IS CALCULATED BY SPECIES GROUPS.
C
C        INPUT/OUTPUT UNITS:
C              1 = READ
C              2 = OUTPUT (STAND AND STOCK TABLES)
C              3 = OUTPUT (ERROR MESSAGES)
C
C        WRITTEN BY ERNEST BOWLING.
C        DATE COMPLETED:  NOVEMBER 4, 1985.
C
C******************************************************************
```

```
      REAL*8 DQ(6), DBAR(6), TPA2(6),TOTVOL(6,30),BA2(6)
      REAL*8 SHAPE(6),SCALE(6),XN(6,30),MERVOL(6,30),IBA1,ITPA1
      REAL DMIN(6),LOCA(6),HDOM(6,30)
      CHARACTER*13 TITLE(6)
      DATA TITLE(2) /'RED OAK'/,TITLE(3)/'WHITE OAK'/,
     &TITLE(4)/'INTOLERANT'/,TITLE(5)/'TOLERANT'/TITLE(6)/'
     &MISCELLANEOUS'/,TITLE(1)/'WHOLE STAND'/
C
C     READ IN STAND CHARACTERISTICS; DEFINE PROJECTION
C     PERIOD LENGTH, MERCHANTABLE HEIGHTS OR DIAMETERS;
C
    5 READ(1,10,END=999)ISTAND,AGE1,AGE2,SI,Y,DTOP,HTOP,KFLAG
   10 FORMAT(I2,3F3.0,F2.0,2F4.1,I2)
      IF(HTOP.GT.0.0.AND.DTOP.GT.0.0) THEN
        WRITE(2,7)
    7   FORMAT(1X,'  BOTH DTOP AND HTOP ARE SPECIFIED. INPUT ONE OR THE
     &OTHER, NOT BOTH')
        GO TO 999
      END IF
      PERIOD=AGE2-AGE1
C
C   INITIALIZE ALL ARRAYS TO ZERO
C
      DO 15 I=1,6
      BA2(I)=0.DO
      DQ(I)=0.DO
      DBAR(I)=0.DO
      TPA2(I)=0.DO
      DMIN(I)=0.0
      SHAPE(I)=0.DO
      SCALE(I)=0.DO
      DO 18 J=1,30
      XN(I,J)=0.DO
      TOTVOL(I,J)=0.DO
      MERVOL(I,J)=0.DO
      HDOM(I,J)=0.DO
   18 CONTINUE
   15 CONTINUE
C
C     READ IN WHOLE STAND VALUES FOR NUMBER OF TREES AND BASAL AREA;
C     COMPUTE FUTURE VALUES FOR BA, TPA, DMIN, DBAR FOR WHOLE STAND
C
      READ(1,20,END=999)TOBA1,TOTPA1
   20 FORMAT(2F7.2)
      BA2(1)=EXP(0.021804+(AGE1/AGE2)*LOG(TOBA1)+5.323155*(1-AGE1/AGE2)
     &+0.004380*Y)
      TPA2(1)=TOTPA1*(EXP(-0.00333273*(AGE2-AGE1)))
      DQ(1)=SQRT(BA2(1)/TPA2(1)/0.005454)
      DMIN(1)=EXP(-1.308068+1.304337*LOG(DQ(1))+7.688220*(1/BA2(1)))
      DBAR(1)=DQ(1)-(-1.225427+0.373726*LOG(SI)+0.000978128*BA2(1))**2
```

```
C
C    READ IN BASAL AREA/ACRE AND TREES/ACRE FOR
C       SPECIES GROUPS (IF INPUT)
C
      IF(KFLAG.EQ.0)READ(1,30,END=999)RBA1,RTPA1,
     &WBA1,WTPA1,IBA1,ITPA1,TBA1,TTPA1
   30 FORMAT(8F7.2)
C
C   COMPUTE DEFAULT VALUES FOR BASAL AREA/ACRE AND TREES/ACRE
C       (IF REQUESTED)
C
      IF(KFLAG.EQ.1) THEN
         RBA1=0.25*TOBA1
         RTPA1=0.21*TOTPA1
         WBA1=0.26*TOBA1
         WTPA1=0.25*TOTPA1
         IBA1=0.28*TOBA1
         ITPA1=0.24*TOTPA1
         TBA1=0.12*TOBA1
         TTPA1=0.20*TOTPA1
      END IF
C
C    COMPUTE FUTURE VALUES OF BA, TPA, DMIN, DBAR FOR RED OAK GROUP
C
      IF(RBA1.EQ.0.0) GO TO 22
      BA2(2)=EXP(0.284068+(AGE1/AGE2)*LOG(RBA1)+4.323840*(1-AGE1/AGE2)
     &-0.476345*(1-RBA1/TOBA1)+0.017094*Y)
      TPA2(2)=RTPA1*(EXP(-0.00221586*(AGE2-AGE1)))
      DQ(2)=SQRT(BA2(2)/TPA2(2)/0.005454)
      DMIN(2)=EXP(-0.571443+1.126911*LOG(DQ(2))+0.464581*(1/BA2(2)))
      DBAR(2)=DQ(2)-(-2.052667+0.498249*LOG(SI)+0.00886028*BA2(2))**2
      GO TO 29
   22 TPA2(2)=0.D0
      BA2(2)=0.D0
      DQ(2)=0.D0
      DMIN(2)=0.0
      DBAR(2)=0.D0
C
C    COMPUTE FUTURE VALUES OF BA, TPA, DMIN, DBAR FOR WHITE OAK GROUP
C
   29 IF(WBA1.EQ.0.0) GO TO 34
      BA2(3)=EXP(0.226187+(AGE1/AGE2)*LOG(WBA1)+3.693233*(1-AGE1/AGE2)
     &-0.323071*(1-WBA1/TOBA1)+0.009908*Y)
      TPA2(3)=WTPA1*(EXP(-0.00176353*(AGE2-AGE1)))
      DQ(3)=SQRT(BA2(3)/TPA2(3)/0.005454)
      DMIN(3)=EXP(-0.732170+1.171384*LOG(DQ(3))+0.952291*(1/BA2(3)))
      DBAR(3)=DQ(3)-(-0.0293827+0.052979*LOG(SI)+0.00509698*BA2(3))**2
      GO TO 40
   34 TPA2(3)=0.D0
      BA2(3)=0.D0
```

```
      DQ(3)=0.D0
      DMIN(3)=0.0
      DBAR(3)=0.D0
C
C    COMPUTE FUTURE VALUES OF BA, TPA, DMIN, DBAR FOR INTOLERANT GROUP
C
   40 IF(IBA1.EQ.0.0) GO TO 42
      BA2(4)=EXP(0.531857+(AGE1/AGE2)*LOG(IBA1)+3.255270*(1-AGE1/AGE2)
     &-0.567427*(1-IBA1/TOBA1)-0.005942*Y)
      TPA2(4)=ITPA1*(EXP(-0.00163784*(AGE2-AGE1)))
      DQ(4)=SQRT(BA2(4)/TPA2(4)/0.005454)
      DMIN(4)=EXP(-1.280313+1.390702*LOG(DQ(4))+0.780550*(1/BA2(4)))
      DBAR(4)=DQ(4)-(-1.25456+0.331129*LOG(SI)+0.00616552*BA2(4))**2
      GO TO 50
   42 TPA2(4)=0.D0
      BA2(4)=0.0
      DQ(4)=0.D0
      DMIN(4)=0.0
      DBAR(4)=0.D0
C
C    COMPUTE FUTURE VALUES OF BA, TPA, DMIN, DBAR FOR TOLERANT GROUP
C
   50 IF(TTPA1.EQ.0.0)GO TO 52
      BA2(5)=EXP(0.586024+(AGE1/AGE2)*LOG(TBA1)+3.550618*(1-AGE1/AGE2)
     &-0.727647*(1-TBA1/TOBA1)+0.006227*Y)
      TPA2(5)=TTPA1*(EXP(-0.00324641*(AGE2-AGE1)))
      DQ(5)=SQRT(BA2(5)/TPA2(5)/0.005454)
      DMIN(5)=EXP(-1.074034 + 1.347334*LOG(DQ(5))+0.698765*(1/BA2(5)))
      DBAR(5)=DQ(5)-(-1.014511+0.266649*LOG(SI)+0.00671482*BA2(5))**2
      GO TO 60
   52 TPA2(5)=0.D0
      BA2(5)=0.D0
      DQ(5)=0.D0
      DMIN(5)=0.0
      DBAR(5)=0.D0
C
C   OBTAIN PARAMETERS FOR PREDICTED FUTURE DIAMETER DISTRIBUTIONS
C   FOR WHOLE STAND AND SPECIES GROUPS (EXCEPT MISCELLANEOUS
C   GROUP).
C    VARIABLES ARE:
C      X1 = FIRST NON-CENTRAL MOMENT OF DBH (DBAR COMPUTED ABOVE)
C      X2 = SECOND NON-CENTRAL MOMENT OF DBH (QUADRATIC MEAN DBH
C               COMPUTED ABOVE SQUARED)
C      LOCA(I) = WEIBULL LOCATION PARAMETER
C      SCALE(I) = WEIBULL SCALE PARAMETER
C      SHAPE(I) = WEIBULL SHAPE PARAMETER
C
   60 DO 100 I=1,5
      X1=DBAR(I)
      X2=DQ(I)**2
```

```
      LOCA(I) = IFIX(DMIN(I) - 0.5) - 0.49
      IF(LOCA(I).LT.0.0) LOCA(I)=0.0
C
C    SET PARAMETERS EQUAL ZERO IF SPECIES GROUP HAS NO TREES
C
      IF(TPA2(I).EQ.0.0)THEN
        LOCA(I)=0.0
        SCALE(I)=0.0
        SHAPE(I)=0.0
        GO TO 100
      END IF
C
C    OBTAIN WEIBULL PARAMETERS OF PREDICTED DISTRIBUTIONS
C
      CALL WEIB(X1,X2,LOCA,SCALE,SHAPE,IER,I,ISTAND)
C
C    IF ERRORS RESULT, WRITE THEM OUT
C
C    IER = 1, CONVERGENCE NOT OBTAINED.  X2P IS THE VALUE OF X2
C             FOR THE SOLUTION FOUND.
C    IER = 2, SOLUTION FOUND AFTER PERTURBATING X1.  X1 IS PERTURBATED
C             BY INCREMENTS OF .01 UNTIL A SOLUTION CAN BE FOUND.
C    IER = 3, NO SOLUTION COULD BE FOUND, EVEN AFTER PERTURBATION.
C
      IF(IER.EQ.1)WRITE(3,97)IER,ISTAND,I
   97 FORMAT(' SOLUTION DID NOT CONVERGE , ERROR = ',I2,' FOR PLOT ',
     &' SPECIES ',I1)
      IF(IER.EQ.3)WRITE(3,99)IER,ISTAND,I
   99 FORMAT(' ERROR = ',I2,' NO SOLUTION FOR PLOT ',I2,' SPECIES ',I1)
      IF(IER.EQ.2)WRITE(3,98)IER, X1, X1P, ISTAND,I
   98 FORMAT(' ERROR= ',I2,' DBAR CHANGED FROM ',F5.2,' TO ',F5.2,
     &' PLOT ',I2,' SPECIES ',I1)
  100 CONTINUE
C
C     COMPUTE PREDICTED DIAMETER DISTRIBUTIONS FROM WEIBULL CDF
C     USING PARAMETERS AND FUTURE TREES/ACRE COMPUTED ABOVE
C
   70 DO 200 I=1,5
      CALL FREQ(I,LOCA,SCALE,SHAPE,TPA2,XN)
  200 CONTINUE
C
C     SUM PREDICTED DIAMETER DISTRIBUTIONS AND SUBTRACT FROM WHOLE
C     STAND VALUES TO OBTAIN DISTRIBUTION OF MISCELLANEOUS GROUP
C
      CALL DIST(XN,JFLAG,EXCESU,TOTAL)
C
C    IF AN EXCESS NUMBER OF TREES REMAINS AFTER ALLOCATING TREES
C    TO THE MISCELLANEOUS GROUP, ADJUST THE NUMBER OF TREES IN
C    SPECIES GROUPS SO THAT NO EXCESS REMAINS AND REPEAT
C    SUBROUTINES FREQ AND DIST.  FIRST REMOVE TREES FROM THE
```

```
C     TOLERANT GROUP; THEN GO TO INTOLERANT; THEN TO WHITE OAK;
C     FINALLY TO RED OAK.
C
       IF(JFLAG.EQ.1) THEN
          IF(TPA2(5).GT.0.D0) THEN
             TPA2(5)=TPA2(5)-EXCESU
             IF(TPA2(5).LE.0.D0) THEN
                EXCESU=ABS(TPA2(5))
                TPA2(5)=0.D0
             END IF
          END IF
          IF(TPA2(5).EQ.0.D0) THEN
            TPA2(4)=TPA2(4)-EXCESU
            IF(TPA2(4).LE.0.D0) THEN
              EXCESU=ABS(TPA2(4))
              TPA2(4)=0.D0
            END IF
          END IF
          IF(TPA2(5).EQ.0.D0.AND.TPA2(4).EQ.0.D0) THEN
            TPA2(3)=TPA2(3)-EXCESU
            IF(TPA2(3).LE.0.D0) THEN
              EXCESU=ABS(TPA2(3))
              TPA2(3)=0.D0
            END IF
          END IF
          IF(TPA2(5).EQ.0.D0.AND.TPA2(4).EQ.0.D0.AND.TPA2(3).EQ.0.D0)THEN
                TPA2(2)=TPA2(2)-EXCESU
                IF(TPA2(2).LE.0.D0) THEN
                   EXCESU=ABS(TPA2(2))
                   TPA2(2)=0.D0
                END IF
          END IF
       GO TO 70
       END IF
C
C      CALCULATE TOTAL AND MERCHANTABLE VOLUMES BY SPECIES GROUPS
C
       CALL TVOL(XN,SI,AGE2,TOTVOL,MERVOL,DTOP,HTOP,HDOM,HC)
C
C      WRITE THE VOLUME ESTIMATES BY SPECIES GROUP AND STAND
C
  205 FORMAT('1',65('_'))
          WRITE(2,710)TITLE(1),ISTAND
  710     FORMAT(//,'  PREDICTED DIAMETER DISTRIBUTION FOR THE ',A11,'
     &, STAND NUMBER ',I2)
          WRITE(2,214)PERIOD
  214     FORMAT(/,'  PROJECTION PERIOD LENGTH IS = ',F4.0,' YEARS.')
          WRITE(2,215)Y
  215     FORMAT(/,'  YEARS AFTER THINNING PROJECTION PERIOD STARTS = ',F
     &4.0)
```

```
      WRITE(2,220)AGE2
      WRITE(2,230)
      WRITE(2,740)SI,BA2(1),AGE1,TPA2(1),AGE2,DBAR(1),TOBA1,DMIN(1)
   &,TOTPA1,HC,AGE2
740  FORMAT(/,6X,'SITE INDEX = ',F4.0,6X,'BASAL AREA/ACRE = ',F5.
   &     1/5X,'INITIAL AGE = ',F4.0,11X,'TREES/ACRE = ',F5.1/2X,'
   &     FUTURE AGE = ',F4.0,10X,'AVERAGE DBH = ',F6.2/'
   &BASAL AREA/ACRE = ',F5.1,9X,'MINIMUM DBH = ',F5.1/2X,'
   &     TREES/ACRE = ',F5.1,4X,'DOM/CODOM OAK HT = ',F5.1//5X,
   &'STAND TABLE, AGE ',F4.0/5X,21('_')//3X,'
   &     DBH',6X,'NUMBER'/
   &     6X,'CLASS',4X,'OF TREES'/
   &     6X,'(IN.)',4X,'PER ACRE'
   &/6X,'_____',4X,'________'/)
     DO 799 J=1,30
     IF(XN(1,J).GT.0.0)THEN
     WRITE(2,750)J,XN(1,J)
750 FORMAT(/7X,I2,6X,F5.1)
     END IF
799 CONTINUE
     WRITE(2,265)
     WRITE(2,205)
     IF(TPA2(2).GT.0.0) THEN
        WRITE(2,210)TITLE(2),ISTAND
210     FORMAT(//,'  CUBIC FOOT (O.B.) VOLUME TABLE FOR ',A7,'
   &SPECIES GROUP, STAND NUMBER ',I2)
        WRITE(2,214)PERIOD
        WRITE(2,215)Y
        IF(KFLAG.EQ.0)WRITE(2,218)
218 FORMAT(/,'  NO MERCHANTABLE VOLUMES REQUESTED')
        IF(KFLAG.EQ.1)WRITE(2,216)DTOP
216     FORMAT(/,'  MERCHANTABLE VOLUMES ARE TO A ',F4.1,' INCH TOP DIA
   &METER')
        IF(KFLAG.EQ.2)WRITE(2,217)HTOP
217     FORMAT(/'  MERCHANTABLE VOLUMES ARE TO ',F4.1,' FEET ON BOLE
   & ABOVE GROUND')
        WRITE(2,220)AGE2
220     FORMAT(//,'    INPUT VARIABLES          PREDICTED VARIABLES,
   & AGE ',F4.0)
      WRITE(2,230)
230     FORMAT(1X,'   _______________          ',29('_'))
        WRITE(2,240)SI,BA2(2),AGE1,TPA2(2),AGE2,DBAR(2),RBA1,DMIN(2),
   &RTPA1,HC,AGE2
240  FORMAT(/,6X,'SITE INDEX = ',F4.0,6X,'BASAL AREA/ACRE = ',F5.
   &     1/5X,'INITIAL AGE = ',F4.0,11X,'TREES/ACRE = ',F5.1/2X,'
   &     FUTURE AGE = ',F4.0,10X,'AVERAGE DBH = ',F6.2/'
   &BASAL AREA/ACRE = ',F5.1,9X,'MINIMUM DBH = ',F5.1/2X,'
   &     TREES/ACRE = ',F5.1,4X,'DOM/CODOM OAK HT = ',F5.1
   &//18X,'STAND AND STOCK TABLE, AGE ',F4.0/18X,31('_')//3X,'
   &     DBH',6X,'NUMBER',6X,'AVERAGE',6X,'TOTAL',6X,'MERCHANTABLE'/
```

```
  &    6X,'CLASS',4X,'OF TREES',5X,'HEIGHT',7X,'VOLUME',8X,'VOLUME'/
  &    6X,'(IN.)',4X,'PER ACRE',5X,'(FEET)',7X,'(O.B.)',8X,'(O.B.)'
  &/6X,'_____',4X,'________',5X,'_______',6X,'______',5X,12('_')/)
      DO 299 J=1,30
      IF(XN(2,J).GT.0.0)THEN
      WRITE(2,250)J,XN(2,J),HDOM(2,J),TOTVOL(2,J),MERVOL(2,J)
250 FORMAT(/7X,I2,6X,F5.1,9X,F5.1,7X,F5.1,9X,F5.1)
      SUMVOL=SUMVOL+TOTVOL(2,J)
      SUMMER=SUMMER+MERVOL(2,J)
      END IF
299   CONTINUE
    WRITE(2,260)SUMVOL,SUMMER
260 FORMAT(/40X,6('_'),8X,6('_')/
  & 6X,'TOTAL',29X,F6.1,8X,F6.1)
    WRITE(2,265)
265 FORMAT(/,65('_'))
    END IF
    IF(TPA2(2).EQ.0.0)WRITE(2,270)TITLE(2),ISTAND
270 FORMAT('1','  NO TREES OF THE ',A13,' SPECIES GROUP PRESENT ON
  & STAND ',I2)
    IF(TPA2(3).GT.0.0)THEN
    TREES=0.0
    SUMVOL=0.0
    SUMMER=0.0
    WRITE(2,305)
305 FORMAT('1',65('_'))
      WRITE(2,310)TITLE(3),ISTAND
310   FORMAT(//,'  CUBIC FOOT (O.B.) VOLUME TABLE FOR ',A9,'
  &SPECIES GROUP, STAND NUMBER ',I2)
      WRITE(2,214)PERIOD
      WRITE(2,215)Y
      IF(KFLAG.EQ.0)WRITE(2,218)
      IF(KFLAG.EQ.1)WRITE(2,216)DTOP
      IF(KFLAG.EQ.2)WRITE(2,217)HTOP
      WRITE(2,220)AGE2
      WRITE(2,230)
      WRITE(2,240)SI,BA2(3),AGE1,TPA2(3),AGE2,DBAR(3),WBA1,DMIN(3),
  &WTPA1,HC,AGE2
      DO 399 J=1,30
      IF(XN(3,J).GT.0.0)THEN
      WRITE(2,250)J,XN(3,J),HDOM(3,J),TOTVOL(3,J),MERVOL(3,J)
      SUMVOL=SUMVOL+TOTVOL(3,J)
      SUMMER=SUMMER+MERVOL(3,J)
      TREES=TREES+XN(3,J)
      END IF
399   CONTINUE
    WRITE(2,260)SUMVOL,SUMMER
    WRITE(2,265)
    END IF
    IF(TPA2(3).EQ.0.0)WRITE(2,270)TITLE(3),ISTAND
```

```
      IF(TPA2(4).GT.0.0)THEN
      TREES=0.0
      SUMVOL=0.0
      SUMMER=0.0
      WRITE(2,305)
         WRITE(2,410)TITLE(4),ISTAND
410      FORMAT(//,'  CUBIC FOOT (O.B.) VOLUME TABLE FOR ',A10,'
     &SPECIES GROUP, STAND NUMBER ',I2)
         WRITE(2,214)PERIOD
         WRITE(2,215)Y
         IF(KFLAG.EQ.0)WRITE(2,218)
         IF(KFLAG.EQ.1)WRITE(2,216)DTOP
         IF(KFLAG.EQ.2)WRITE(2,217)HTOP
         WRITE(2,220)AGE2
         WRITE(2,230)
         WRITE(2,240)SI,BA2(4),AGE1,TPA2(4),AGE2,DBAR(4),WBA1,DMIN(4),
     &ITPA1,HC,AGE2
         DO 499 J=1,30
         IF(XN(4,J).GT.0.0)THEN
         WRITE(2,250)J,XN(4,J),HDOM(4,J),TOTVOL(4,J),MERVOL(4,J)
         SUMVOL=SUMVOL+TOTVOL(4,J)
         SUMMER=SUMMER+MERVOL(4,J)
         TREES=TREES+XN(4,J)
         END IF
499      CONTINUE
      WRITE(2,260)SUMVOL,SUMMER
      WRITE(2,265)
      END IF
      IF(TPA2(4).EQ.0.0)WRITE(2,270)TITLE(4),ISTAND
      IF(TPA2(5).GT.0.0)THEN
      TREES=0.0
      SUMVOL=0.0
      SUMMER=0.0
      WRITE(2,305)
         WRITE(2,510)TITLE(5),ISTAND
510      FORMAT(//,'  CUBIC FOOT (O.B.) VOLUME TABLE FOR ',A8,'
     &SPECIES GROUP, STAND NUMBER ',I2)
         WRITE(2,214)PERIOD
         WRITE(2,215)Y
         IF(KFLAG.EQ.0)WRITE(2,218)
         IF(KFLAG.EQ.1)WRITE(2,216)DTOP
         IF(KFLAG.EQ.2)WRITE(2,217)HTOP
         WRITE(2,220)AGE2
         WRITE(2,230)
         WRITE(2,240)SI,BA2(5),AGE1,TPA2(5),AGE2,DBAR(5),TBA1,DMIN(5),
     &TTPA1,HC,AGE2
         DO 599 J=1,30
         IF(XN(5,J).GT.0.0)THEN
         WRITE(2,250)J,XN(5,J),HDOM(5,J),TOTVOL(5,J),MERVOL(5,J)
         SUMVOL=SUMVOL+TOTVOL(5,J)
```

```
          SUMMER=SUMMER+MERVOL(5,J)
          TREES=TREES+XN(5,J)
          END IF
  599     CONTINUE
        WRITE(2,260)SUMVOL,SUMMER
        WRITE(2,265)
        END IF
        IF(TPA2(5).EQ.0.0)WRITE(2,270)TITLE(5),ISTAND
        IF(TOTAL.GT.0.0)THEN
        TREES=0.0
        SUMVOL=0.0
        SUMMER=0.0
        WRITE(2,305)
          WRITE(2,610)TITLE(6),ISTAND
  610     FORMAT(//,'  CUBIC FOOT (O.B.) VOLUME TABLE FOR ',A13,'
       &SPECIES GROUP, STAND NUMBER ',I2)
          WRITE(2,615)
  615 FORMAT(/,'  DIAMETER DISTRIBUTION OBTAINED BY SUBTRACTION
       & FROM WHOLE STAND VALUES')
          WRITE(2,214)PERIOD
          WRITE(2,215)Y
          IF(KFLAG.EQ.0)WRITE(2,218)
          IF(KFLAG.EQ.1)WRITE(2,216)DTOP
          IF(KFLAG.EQ.2)WRITE(2,217)HTOP
          WRITE(2,220)AGE2
          WRITE(2,230)
          WRITE(2,640)SI,TOTAL,AGE1,HC,AGE2,AGE2
  640     FORMAT(/,6X,'SITE INDEX = ',F4.0,11X,'TREES/ACRE = ',F5.1
       &/5X,'INITIAL AGE = ',F4.0,5X,'DOM/CODOM OAK HT = ',F5.1/6X,
       &'FUTURE AGE = ',F4.0//18X,'
       &STAND AND STOCK TABLE, AGE ',F4.0/18X,31('_')//3X,'
       &    DBH',6X,'NUMBER',6X,'AVERAGE',6X,'TOTAL',6X,'MERCHANTABLE'/
       &     6X,'CLASS',4X,'OF TREES',5X,'HEIGHT',7X,'VOLUME',8X,'VOLUME'/
       &     6X,'(IN.)',4X,'PER ACRE',5X,'(FEET)',7X,'(O.B.)',8X,'(O.B.)'
       &/6X,'_____',4X,'________',5X,'_______',6X,'______',5X,12('_')/)
          DO 699 J=1,30
          IF(XN(6,J).GT.0.0)THEN
          WRITE(2,250)J,XN(6,J),HDOM(6,J),TOTVOL(6,J),MERVOL(6,J)
          SUMVOL=SUMVOL+TOTVOL(6,J)
          SUMMER=SUMMER+MERVOL(6,J)
          TREES=TREES+XN(6,J)
          END IF
  699     CONTINUE
        WRITE(2,260)SUMVOL,SUMMER
        WRITE(2,265)
        END IF
        IF(TOTAL.EQ.0.0)WRITE(2,270)TITLE(6),ISTAND
C         GO TO 5
  999 STOP
        END
```

```
C
C   SUBROUTINE WEIB CALCULATES ESTIMATES OF WEIBULL PARAMETERS
C   FOR FUTURE DBH DISTRIBUTIONS.
C
      SUBROUTINE WEIB(X1,X2,LOCA,SCALE,SHAPE,IER,I,ISTAND)
      REAL*8 A,B,C,D1,D2,XN,FXN,XN1,FXN1,TEMP,FTEMP
      REAL*8 SHAPE(5),SCALE(5)
      REAL LOCA(5)
      COMMON/BLKM22/A,B,C,D1,D2
C
C    INITIALIZE VARIABLES
C
      SHAPEL = 0.1
      SHAPEU = 10.0
      IER=0
      A=DBLE(LOCA(I))
      SCALE(I)=0.0
      SHAPE(I)=0.0
      D2=DBLE(X2)
      X1P=X1
      X2P=X2
      IFLAG=0
C
C   PERTURBATE X1 IF NECESSARY; IF NO SOLUTION CAN
C   BE FOUND, RETURN TO MAIN PROGRAM
C
   10 D1=DBLE(X1P)
      XN=DBLE(SHAPEL)
      FXN=FCV(XN)
      IF(FXN.LT.0.D0)GO TO 30
      IER=2
      IF(IFLAG.EQ.0)GO TO 20
      IER=3
      RETURN
   20 X1P=X1P+0.01
      GO TO 10
   30 XN1=DBLE(SHAPEU)
      FXN1=FCV(XN1)
      IF(FXN1.GT.0.D0)GO TO 40
      IER=2
      IFLAG=1
      X1P=X1P-0.01
      GO TO 10
C
C     5 BISECTION ITERATIONS TO GET STARTED
C
   40 DO 60 J=1,5
      TEMP=(XN+XN1)/2.D0
      FTEMP=FCV(TEMP)
      IF(FTEMP*FXN.LE.D0)GO TO 50
```

```
      XN=TEMP
      FXN=FTEMP
      GO TO 60
   50 XN1=TEMP
      FXN1=FTEMP
   60 CONTINUE
C
C     SECANT ITERATIONS TO FIND THE FINAL SOLUTION
C
      DO 70 J=1,100
      TEMP=XN-FXN*(XN-XN1)/(FXN-FXN1)
      XN1=XN
      FXN1=FXN
      XN=TEMP
      FXN=FCV(XN)
      IF(DABS(FXN).LE.0.00001)GO TO 80
   70 CONTINUE
      IER=1
      X2P=SNGL(D2-FXN)
   80 SHAPE(I)=C
      SCALE(I)=B
      RETURN
      END
C
C    FUNCTION FCV COMPUTES THE VALUE OF THE FUNCTION AT THE CURRENT
C        VALUE OF THE SHAPE PARAMETER; IF THIS IS CLOSE ENOUGH TO
C        ZERO, THE SOLUTION HAS BEEN FOUND.  IF NOT, DO ANOTHER
C        SECANT ITERATION.
C
      DOUBLE PRECISION FUNCTION FCV(ZX)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/BLKM22/A,B,C,D1,D2
      C=ZX
      GAM=1.0+1.0/C
      G1=GMMMA(GAM)
      GAM=1.0+2.0/C
      G2=GMMMA(GAM)
      B=(D1-A)/G1
      FCV=D2-A*A-2.0*A*B*G1-B*B*G2
      RETURN
      END
C
C     FUNCTION GMMMA EVALUATES THE GAMMA FUNCTION FOR A GIVEN ARGUMENT.
C       GAM = ARGUMENT OF THE GAMMA FUNCTION.
C     ERROR CODES ARE:
C         IIER=0 NO ERROR.
C         IIER=1 GAM IS CLOSE TO BEING A NEGATIVE INTEGER.
C         IIER=2 OVERFLOW.
C
      DOUBLE PRECISION FUNCTION GMMMA(GAM)
```

```
      IMPLICIT REAL*8 (A-H,O-Z)
      IF(GAM-57.)6,6,4
    4 IIER=2
      GMMMA=1.D35
      WRITE(3,99)ISTAND,I
   99 FORMAT(' OVERFLOW ERROR FOR STAND ',I2,'SPECIES
     & GROUP ',I2)
      RETURN
    6 X=GAM
      ERR=1.0D-6
      IIER=0
      GMMMA=1.0
      IF(X-2.0)50,50,15
   10 IF(X-2.0)110,110,15
   15 X=X-1.0
      GMMMA=GMMMA*X
      GO TO 10
   50 IF(X-1.0)60,120,110
C
C     SEE IF X IS NEAR NEGATIVE INTEGER OR ZERO
C
   60 IF(X-ERR)62,62,80
   62 Y=FLOAT(INT(X))-X
      IF(ABS(Y)-ERR)130,130,64
   64 IF(1.0-Y-ERR)130,130,70
C
C     X NOT NEAR NEGATIVE INTEGER OR ZERO, COMPUTE THE GAMMA FUNCTION.
C
   70 IF(X-1.0)80,80,110
   80 GMMMA=GMMMA/X
      X=X+1.0
      GO TO 70
  110 Y=X-1.0
      GY=1.0+Y*(-0.5771017+Y*(0.9858540+Y*(-0.8764218+Y*(0
     &.8328212+Y*(-0.5684729+Y*(0.2548205+Y*(-0.05149930))
     &)))))
      GMMMA=GMMMA*GY
  120 RETURN
  130 IIER=1
      IF(IIER.EQ.1) WRITE(3,98)ISTAND,I
   98 FORMAT(' X NEAR NEGATIVE FOR STAND ',I2,' SPECIES GROUP ',I2)
      RETURN
      END
C
C     SUBROUTINE FREQ GENERATES FUTURE DIAMETER DISTRIBUTIONS USING THE
C     WEIBULL CDF AND PARAMETERS COMPUTED IN SUBROUTINE WEIB
C
C          XN(I,J) = NUMBER OF TREES/ACRE OF SPECIES GROUP I IN DIAM
C                    CLASS J
C
```

```
C          SPECIES GROUPS ARE:
C             1 = WHOLE STAND
C             2 = RED OAK
C             3 = WHITE OAK
C             4 = INTOLERANT
C             5 = TOLERANT
C             6 = MISCELLANEOUS
C
      SUBROUTINE FREQ(I,LOCA,SCALE,SHAPE,TPA2,XN)
      REAL*8 TPA2(6),XN(6,30),SCALE(5),SHAPE(5)
      REAL LOCA(5)
      IF(SCALE(I).EQ.0.0) GO TO 200
   20 DO 100 J=1,30
      X=J-0.5
      IF(X.LE.LOCA(I)) THEN
            XN(I,J)=0.D0
            GO TO 100
      END IF
      Z=J+0.5
      FX=1-EXP(-((X-LOCA(I))/SCALE(I))**SHAPE(I))
      FZ=1-EXP(-((Z-LOCA(I))/SCALE(I))**SHAPE(I))
      XN(I,J)=TPA2(I)*(FZ-FX)
      IF(XN(I,J).LT.0.1)THEN
          XN(I,J)=0.D0
      END IF
  100 CONTINUE
      RETURN
  200 DO 300 K=1,30
      XN(I,K)=0.D0
  300 CONTINUE
      RETURN
      END
C
C    SUBROUTINE DIST SUMS THE DISTRIBUTIONS OF THE FOUR SPECIES GROUPS,
C    THEN SUBTRACTS THIS SUMMED GROUP FROM THE WHOLE STAND TO GIVE THE
C    DISTRIBUTION OF THE MISCELLANEOUS GROUP.  THE ROUTINE GOES THROUGH
C    THE DISTRIBUTION FROM LEFT TO RIGHT ACCUMULATING AND DISTRIBUTING
C    NUMBERS OF TREES AS IT GOES THROUGH.  IT THEN GOES BACK THROUGH
C    FROM RIGHT TO LEFT AND REPEATS THE PROCEDURE.  ANY "GAPS" LEFT
C    BETWEEN THE WHOLE STAND AND SUMMED DISTRIBUTIONS ARE ALLOCATED
C    TO THE MISCELLANEOUS GROUP.
C
      SUBROUTINE DIST(XN,JFLAG,EXCESU,TOTAL)
      DIMENSION D(30),DT(30),SUME(31)
      REAL MN,MNT,NTI(30),NT(30)
      REAL*8 XN(6,30)
      JFLAG=0
C
C  ZERO OUT ARRAYS
C
```

```
    5 DO 10 I=1,30
        D(I)=0.0
        DT(I)=0.0
        NTI(I)=0.0
        NT(I)=0.0
        SUME(I)=0.0
        SUME(0)=0.0
   10 CONTINUE
C
C    DEFINE VARIABLES
C
        IDMIN=30
        IMAX=0
        IDMAX=0
        L=-2
        EXCESL=0.0
C
C    COMPUTE SUMMED DISTRIBUTION AS D(I);
C    DT(I) IS WHOLE STAND DISTRIBUTION.  TIE DOWN ENDPOINTS OF WHOLE
C    STAND DISTRIBUTION (IDMIN AND IDMAX).
C
   20 DO 100 I=1,30
        D(I)=XN(2,I) + XN(3,I) + XN(4,I) + XN(5,I)
        IF(D(I).GT.0.0)IMAX=I
        DT(I)=XN(1,I)
        IF(DT(I).GT.0.0) THEN
            IF(IDMIN.GT.I)IDMIN=I
            IF(IDMAX.LT.I)IDMAX=I
        END IF
  100 CONTINUE
        IRANGE=IDMAX-IDMIN
C
C    IF THERE ARE TREES IN DIAMETER CLASSES OF SUMMED DIST. SMALLER
C    THAN IDMIN, COLLECT EXCESS NUMBER OF TREES FROM THESE CLASSES
C    (EXCESL) TO DUMP INTO DIAM CLASS OF SUMMED GROUP = IDMIN.
C
        DO 200 J=1,30
        IF(J.GE.IDMIN)GO TO 250
        EXCESL=EXCESL+D(J)
  200 CONTINUE
C
C    BEGIN MOVING THROUGH FROM LEFT TO RIGHT
C
  250 DO 300 I=IDMIN,IDMAX
       NTI(I)=D(I)
           IF(NTI(I).LT.DT(I))THEN
               NTI(I)=NTI(I)+EXCESL
               IF(NTI(I).GT.DT(I))THEN
                   EXCESL=NTI(I)-DT(I)
                   NTI(I)=DT(I)
```

```
            END IF
            IF(NTI(I).LT.DT(I)) EXCESL=0.0
         END IF
      IF(NTI(I).GT.DT(I)) THEN
         EXCESL=EXCESL+NTI(I)-DT(I)
         NTI(I)=DT(I)
      END IF
  300 CONTINUE
C
C  WE MADE IT THROUGH ONCE.  SET EXCESU EQUAL TO ANY EXCESS LEFT OVER
C  AT END.
C
      EXCESU=EXCESL
C
C  COLLECT EXCESS NUMBER OF TREES FOR SUMMED DIST. IN DIAMETER CLASSES
C  LARGER THAN IDMAX TO DUMP INTO DIAM CLASS OF SUMMED GROUP = IDMAX.
C
      DO 400 J=1,30
      JJ=J-1
      IZ=IMAX-JJ
      IF(IZ.LE.IDMAX) GO TO 450
      EXCESU=EXCESU+D(IZ)
  400 CONTINUE
  450 M=IDMAX+IRANGE
C
C  NOW GO BACK THROUGH FROM RIGHT TO LEFT
C
      DO 500 J=IDMAX,M
      L=L+2
      N=J-L
      NT(N)=NTI(N)
      IF(NT(N).LT.DT(N)) THEN
         NT(N)=NT(N) + EXCESU
         IF(NT(N).GT.DT(N)) THEN
            EXCESU=NT(N)-DT(N)
            NT(N)=DT(N)
         END IF
         IF(NT(N).LT.DT(N)) EXCESU=0.0
      END IF
      IF(NT(N).GT.DT(N)) THEN
         EXCESU=EXCESU+NT(N)-DT(N)
         NT(N)=DT(N)
      END IF
  500 CONTINUE
C
C  WE MADE IT BACK.  NOW COMPUTE DIAM DISTRIBUTION FOR MISCELLANEOUS
C  GROUP AS DIFFERENCE BETWEEN WHOLE STAND (DT(I)) AND SUMMED DIST.
C  (NT(I)).
C
      DO 525 I=1,30
```

```
      XN(6,I)=DT(I)-NT(I)
      IF(XN(6,I).LT.0.1) XN(6,I)=0.0
      SUME(I)=SUME(I-1)+XN(6,I)
      TOTAL=SUME(30)
  525 CONTINUE
      IF(EXCESU.GT.0.0)JFLAG=1
      RETURN
      END
C
C   SUBROUTINE TVOL TAKES SPECIES GROUP DIAMETER DISTRIBUTIONS,
C   CALCULATES VOLUME FOR THE TREE OF MEAN DBH IN EACH DBH
C   CLASS, AND THEN COMPUTES VOLUME PER CLASS BY SPECIES GROUP.
C   MERCHANTABLE VOLUMES ARE COMPUTED IF REQUESTED.  IF A TOP
C   DIAMETER IS SPECIFIED, A BISECTION ROUTINE IS USED ON THE
C   TAPER EQUATION TO DETERMINE HEIGHT ON THE STEM AT THAT DIAMETER.
C   THIS HEIGHT IS USED IN THE RATIO EQUATION TO GIVE MERCHANTABLE
C   VOLUME.  IF MERCHANTABLE HEIGHT IS SPECIFIED, MERCHANTABLE
C   VOLUMES ARE CALCULATED DIRECTLY.
C
C   VARIABLE DEFINITIONS ARE:
C      HDOM(I,J) = TREE HEIGHT OF I SPECIES GROUP IN J DIAM CLASS.
C      VROAK(J) = VOLUME OF TREE OF MEAN DBH IN J DIAM CLASS, RED OAK
C                   GROUP.
C      VWOAK(J), VINT(J), VTOL(J), VMIS(J), SAME AS VROAK(J) BUT FOR
C         WHITE OAK, INTOLERANT, TOLERANT, AND MISCELLANEOUS GROUPS,
C         RESPECTIVELY.
C      HC = HEIGHT OF DOMINANT AND CODOMINANT OAKS FROM OLSON'S EQN.
C      A, B, HTOP = POINTS ON TREE STEM USED IN BISECTION ROUTINE.
C            A IS INITIALLY SET AT TOP OF TREE, B AT STUMP, AND
C            HTOP AT MIDPOINT.
C      FA, FB, FH = FUNCTION EVALUATED AT A, B, HTOP.
C      TOTVOL(I,J) = TOTAL VOLUME OF I SPECIES IN J DIAMETER CLASS.
C      MERVOL(I,J) = MERCH VOLUME OF I SPECIES IN J DIAMETER CLASS.
C
      SUBROUTINE TVOL(XN,SI,AGE2,TOTVOL,MERVOL,DTOP,HTOP,HDOM,HC)
      REAL*8 XN(6,30),TOTVOL(6,30),VROAK(30),VWOAK(30),VINT(30),
     &VTOL(30),VMIS(30),MERVOL(6,30)
      REAL*4 HTOP, A, B, HDOM(6,30)
      HC = EXP (LOG(SI) - 22.0217*(1/AGE2 - 1/50))
C
C  FIRST THE RED OAK GROUP.
C
      DO 100 J=1,30
      IF (XN(2,J).GT.0.D0) THEN
          HDOM(2,J)=4.5 + HC*(1+1.07929254*EXP(-0.01546219*HC))*(1 -
     &          EXP(-7.92097774*J/HC))
          VROAK(J)=((0.005454*4.5**2)/(1.7320*0.7989))*(J**2/HDOM(2,J))
     &          *((HDOM(2,J)-4.5)/4.5)**(0.7989+1.0)
     &          *EXP(1.7320*(4.5/(HDOM(2,J)-4.5))**0.7989)
          TOTVOL(2,J)=XN(2,J)*VROAK(J)
```

```
C
C  BEGIN BISECTION ITERATIONS
C
          IF (DTOP.GT.0.0)THEN
            IF(DTOP.GT.J) GO TO 10
            A=HDOM(2,J)-0.1
            B=0.1
   40       HTOP=(A + B)/2
            FA = (J**2)*((A/4.5)**-0.2011)*(((HDOM(2,J)-4.5)/(HDOM(2,
     &           J)-A))**1.7989)*EXP(-1.7320*((A/(HDOM(2,J)-A))**0.7989-
     &            (4.5/(HDOM(2,J)-4.5))**0.7989))-DTOP**2
            FH = (J**2)*((HTOP/4.5)**-0.2011)*(((HDOM(2,J)-4.5)/(HDOM(2,
     &            J)-HTOP))**1.7989)*EXP(-1.7320*((HTOP/(HDOM(2,J)-HTOP))
     &            **0.7989 - (4.5/(HDOM(2,J)-4.5))**0.7989))-DTOP**2
            FB = (J**2)*((B/4.5)**-0.2011)*(((HDOM(2,J)-4.5)/(HDOM(2,J)
     &            -B))**1.7989)*EXP(-1.7320*((B/(HDOM(2,J)-B))**0.7989 -
     &            (4.5/(HDOM(2,J)-4.5))**0.7989))-DTOP**2
            IF(ABS(FH-FA).LT.0.001.OR.ABS(FB-FH).LT.0.001)GO TO 30
            FZ=FA*FH
            IF(FZ.LT.0.0)THEN
               B=HTOP
               GO TO 40
            END IF
            IF(FZ.GT.0.0)THEN
               A=HTOP
               GO TO 40
            END IF
        END IF
C
C    CALCULATE RED OAK MERCHANTABLE VOLUMES
C
   30   IF(HTOP.GE.HDOM(2,J))GO TO 10
        IF(HTOP.GT.0.0)
     &MERVOL(2,J)=TOTVOL(2,J)*(1 - EXP(-1.7320*(HTOP/(HDOM(2,J)-HTOP))
     &                **0.7989))
        END IF
   31   IF(XN(2,J).EQ.0.D0) TOTVOL(2,J)=0.D0
        IF(TOTVOL(2,J).EQ.0.D0) MERVOL(2,J)=0.D0
        GO TO 20
   10   MERVOL(2,J)=0.D0
        IF(XN(2,J).EQ.0.D0) TOTVOL(2,J)=0.D0
C
C  NEXT THE WHITE OAK GROUP
C
   20   IF(XN(3,J).GT.0.D0) THEN
            HDOM(3,J)=4.5 + HC*(1+1.06190129*EXP(-0.02253277*HC))*(1 -
     &             EXP(-10.33367444*J/HC))
            VWOAK(J)=((0.005454*4.5**2)/(1.6334*0.7959))*(J**2/HDOM(3,J))
     &             *((HDOM(3,J)-4.5)/4.5)**(0.7959+1.0)
     &             *EXP(1.6334*(4.5/(HDOM(3,J)-4.5))**0.7959)
```

```
          TOTVOL(3,J)=XN(3,J)*VWOAK(J)
C
C   BISECTION ITERATIONS
C
        IF (DTOP.GT.0.0)THEN
          IF(DTOP.GT.J) GO TO 70
          A=HDOM(3,J)-0.1
          B=0.1
 50       HTOP=(A + B)/2.0
          TEMP=-1.6334*((A/(HDOM(3,J)-A))**0.7959 - (4.5/(HDOM(3,J)-4.5)
     &          )**0.7959)
          TTEMP=EXP(TEMP)
          Z = (J**2)*((A/4.5)**-0.2041)*(((HDOM(3,J)-4.5)/(HDOM(3,J
     &          )-A))**1.7959)*TTEMP
          FA = Z - DTOP**2
          FH = (J**2)*((HTOP/4.5)**-0.2041)*(((HDOM(3,J)-4.5)/(HDOM(3,
     &          J)-HTOP))**1.7959)*EXP(-1.6334*((HTOP/(HDOM(3,J)-HTOP))
     &          **0.7959 - (4.5/(HDOM(3,J)-4.5))**0.7959))-DTOP**2
          FB = (J**2)*((B/4.5)**-0.2041)*(((HDOM(3,J)-4.5)/(HDOM(3,J)
     &          -B))**1.7959)*EXP(-1.6334*((B/(HDOM(3,J)-B))**0.7959 -
     &          (4.5/(HDOM(3,J)-4.5))**0.7959))-DTOP**2
          IF(ABS(FH-FA).LT.0.001.OR.ABS(FB-FH).LT.0.001)GO TO 60
          FZ=FA*FH
          IF(FZ.LT.0.0)THEN
             B=HTOP
             GO TO 50
          END IF
          IF(FZ.GT.0.0)THEN
             A=HTOP
             GO TO 50
          END IF
      END IF
C
C   COMPUTE WHITE OAK MERCH VOLUME
C
 60   IF(HTOP.GE.HDOM(3,J)) GO TO 70
      IF(HTOP.GT.0.0)
     &MERVOL(3,J)=TOTVOL(3,J)*(1 - EXP(-1.6334*(HTOP/(HDOM(3,J)-HTOP))
     &               **0.7959))
      END IF
 61   IF (XN(3,J).EQ.0.D0)TOTVOL(3,J)=0.D0
      IF(TOTVOL(3,J).EQ.0.D0) MERVOL(3,J)=0.D0
      GO TO 90
 70   MERVOL(3,J)=0.D0
      IF (XN(3,J).EQ.0.D0)TOTVOL(3,J)=0.D0
C
C  NOW THE INTOLERANT GROUP
C
 90   IF(XN(4,J).GT.0.D0) THEN
```

```
           HDOM(4,J)=4.5 + HC*(1+1.17739446*EXP(-0.01381045*HC))*(1 -
     &           EXP(-8.16534630*J/HC))
           VINT(J)=((0.005454*4.5**2)/(1.7427*0.8540))*(J**2/HDOM(4,J))
     &          *((HDOM(4,J)-4.5)/4.5)**(0.8540+1.0)
     &          *EXP(1.7427*(4.5/(HDOM(4,J)-4.5))**0.8540)
           TOTVOL(4,J)=XN(4,J)*VINT(J)
C
C  BISECTION ITERATIONS
C
         IF (DTOP.GT.0.0)THEN
           IF(DTOP.GT.J) GO TO 110
           A=HDOM(4,J)-0.1
           B=0.1
 140       HTOP=(A + B)/2
           FA = (J**2)*((A/4.5)**-0.1460)*(((HDOM(4,J)-4.5)/(HDOM(4,
     &          J)-A))**1.8540)*EXP(-1.7427*((A/(HDOM(4,J)-A))**0.8540-
     &          (4.5/(HDOM(4,J)-4.5))**0.8540))-DTOP**2
           FH = (J**2)*((HTOP/4.5)**-0.1460)*(((HDOM(4,J)-4.5)/(HDOM(4,
     &          J)-HTOP))**1.8540)*EXP(-1.7427*((HTOP/(HDOM(4,J)-HTOP))
     &          **0.8540 - (4.5/(HDOM(4,J)-4.5))**0.8540))-DTOP**2
           FB = (J**2)*((B/4.5)**-0.1460)*(((HDOM(4,J)-4.5)/(HDOM(4,J)-
     &          -B))**1.8540)*EXP(-1.7427*((B/(HDOM(4,J)-B))**0.8540 -
     &          (4.5/(HDOM(4,J)-4.5))**0.8540))-DTOP**2
           IF(ABS(FH-FA).LT.0.001.OR.ABS(FB-FH).LT.0.001)GO TO 130
           FZ=FA*FH
           IF(FZ.LT.0.0)THEN
              B=HTOP
              GO TO 140
           END IF
           IF(FZ.GT.0.0)THEN
              A=HTOP
              GO TO 140
           END IF
        END IF
C
C    INTOLERANT MERCH VOLUME
C
 130    IF(HTOP.GE.HDOM(4,J)) GO TO 110
        IF(HTOP.GT.0.0)
     &MERVOL(4,J)=TOTVOL(4,J)*(1 - EXP(-1.7427*(HTOP/(HDOM(4,J)-HTOP))
     &               **0.8540))
        END IF
 131    IF(XN(4,J).EQ.0.D0) TOTVOL(4,J)=0.D0
        IF(TOTVOL(4,J).EQ.0.D0) MERVOL(4,J)=0.D0
        GO TO 120
 110    MERVOL(4,J)=0.D0
        IF (XN(4,J).EQ.0.D0)TOTVOL(4,J)=0.D0
C
C  NOW THE TOLERANT GROUP
C
```

```
 120  IF(XN(5,J).GT.0.D0) THEN
          HDOM(5,J)=4.5 + HC*(1+1.36028975*EXP(-0.02091233*HC))*(1 -
     &           EXP(-9.05520533*J/HC))
          VTOL(J)=((0.005454*4.5**2)/(1.8285*0.8416))*(J**2/HDOM(5,J))
     &          *((HDOM(5,J)-4.5)/4.5)**(0.8416+1.0)
     &          *EXP(1.8285*(4.5/(HDOM(5,J)-4.5))**0.8416)
          TOTVOL(5,J)=XN(5,J)*VTOL(J)
C
C   BISECTION ITERATIONS
C
        IF (DTOP.GT.0.0)THEN
          IF(DTOP.GT.J) GO TO 210
          KOUNT=0
          A=HDOM(5,J)-0.1
          B=0.1
  240     HTOP=(A + B)/2
          FA = (J**2)*((A/4.5)**-0.1584)*(((HDOM(5,J)-4.5)/(HDOM(5,
     &         J)-A))**1.8416)*EXP(-1.8285*((A/(HDOM(5,J)-A))**0.8416 -
     &          (4.5/(HDOM(5,J)-4.5))**0.8416))-DTOP**2
          FH = (J**2)*((HTOP/4.5)**-0.1584)*(((HDOM(5,J)-4.5)/(HDOM(5,
     &          J)-HTOP))**1.8416)*EXP(-1.8285*((HTOP/(HDOM(5,J)-HTOP))
     &          **0.8416 - (4.5/(HDOM(5,J)-4.5))**0.8416))-DTOP**2
          FB = (J**2)*((B/4.5)**-0.1584)*(((HDOM(5,J)-4.5)/(HDOM(5,J)
     &          -B))**1.8416)*EXP(-1.8285*((B/(HDOM(5,J)-B))**0.8416 -
     &          (4.5/(HDOM(5,J)-4.5))**0.8416))-DTOP**2
          IF(ABS(FH-FA).LT.0.001.OR.ABS(FB-FH).LT.0.001)GO TO 230
          FZ=FA*FH
          IF(FZ.LT.0.0)THEN
             B=HTOP
             GO TO 240
          END IF
          IF(FZ.GT.0.0)THEN
             A=HTOP
             GO TO 240
          END IF
       END IF
C
C   TOLERANT GROUP MERCH VOLUME
C
 230  IF(HTOP.GE.HDOM(5,J)) GO TO 210
      IF(HTOP.GT.0.0)
     &MERVOL(5,J)=TOTVOL(5,J)*(1 - EXP(-1.8285*(HTOP/(HDOM(5,J)-HTOP))
     &              **0.8416))
       END IF
 231  IF(XN(5,J).EQ.0.D0) TOTVOL(5,J)=0.D0
      IF(TOTVOL(5,J).EQ.0.D0) MERVOL(5,J)=0.D0
      GO TO 220
  210 MERVOL(5,J)=0.D0
      IF (XN(5,J).EQ.0.D0)TOTVOL(5,J)=0.D0
C
```

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C   FINALLY THE MISCELLANEOUS GROUP
C
  220 IF(XN(6,J).GT.0.D0) THEN
          HDOM(6,J)=4.5 + HC*(1+1.19610263*EXP(-0.01870700*HC))*(1 -
     &           EXP(-9.01152482*J/HC))
          VMIS(J)=((0.005454*4.5**2)/(1.7854*0.8304))*(J**2/HDOM(6,J))
     &           *((HDOM(6,J)-4.5)/4.5)**(0.8304+1.0)
     &           *EXP(1.7854*(4.5/(HDOM(6,J)-4.5))**0.8304)
          TOTVOL(6,J)=XN(6,J)*VMIS(J)
C
C   BISECTION ITERATIONS
C
        IF (DTOP.GT.0.0)THEN
          IF(DTOP.GT.J) GO TO 310
          A=HDOM(6,J)-0.1
          B=0.1
 340      HTOP=(A + B)/2
          FA = (J**2)*((A/4.5)**-0.1696)*(((HDOM(6,J)-4.5)/(HDOM(6,
     &          J)-A))**1.8304)*EXP(-1.7854*((A/(HDOM(6,J)-A))**0.8304-
     &           (4.5/(HDOM(6,J)-4.5))**0.8304))-DTOP**2
          FH = (J**2)*((HTOP/4.5)**-0.1696)*(((HDOM(6,J)-4.5)/(HDOM(6,
     &           J)-HTOP))**1.8304)*EXP(-1.7854*((HTOP/(HDOM(6,J)-HTOP))
     &           **0.8304 - (4.5/(HDOM(6,J)-4.5))**0.8304))-DTOP**2
          FB = (J**2)*((B/4.5)**-0.1696)*(((HDOM(6,J)-4.5)/(HDOM(6,J)
     &           -B))**1.8304)*EXP(-1.7854*((B/(HDOM(6,J)-B))**0.8304 -
     &           (4.5/(HDOM(6,J)-4.5))**0.8304))-DTOP**2
          IF(ABS(FH-FA).LT.0.001.OR.ABS(FB-FH).LT.0.001)GO TO 330
          FZ=FA*FH
          IF(FZ.LT.0.0)THEN
             B=HTOP
             GO TO 340
          END IF
          IF(FZ.GT.0.0)THEN
             A=HTOP
             GO TO 340
          END IF
      END IF
C
C  MISCELLANEOUS GROUP MERCH VOLUME
C
 330  IF(HTOP.GE.HDOM(6,J)) GO TO 310
      IF(HTOP.GT.0.0)
     &MERVOL(6,J)=TOTVOL(6,J)*(1 - EXP(-1.7854*(HTOP/(HDOM(6,J)-HTOP))
     &                **0.8304))
      END IF
 331  IF(XN(6,J).EQ.0.D0) TOTVOL(6,J)=0.D0
      IF(TOTVOL(6,J).EQ.0.D0) MERVOL(6,J)=0.D0
      GO TO 100
 310  MERVOL(6,J)=0.D0
      IF (XN(6,J).EQ.0.D0)TOTVOL(6,J)=0.D0
```

```
100 CONTINUE
    RETURN
    END
```

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