

Control of sound transmission into payload fairings using distributed vibration absorbers and Helmholtz resonators.

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Abstract

A new passive treatment to reduce sound transmission into payload fairing at low frequency is investigated. This new solution is composed of optimally damped vibration absorbers (DVA) and optimally damped Helmholtz resonators (HR). A fully coupled structural-acoustic model of a composite cylinder excited by an external plane wave is developed as a first approximation of the system. A modal expansion method is used to describe the behavior of the cylindrical shell and the acoustic cavity; the noise reduction devices are modeled as surface impedances. All the elements are then fully coupled using an impedance matching method. This model is then refined using the digitized mode shapes and natural frequencies obtained from a fairing finite element model.

For both models, the noise transmission mechanisms are highlighted and the noise reduction mechanisms are explained. Procedures to design the structural and acoustic absorbers based on single degree of freedom system are modified for the multi-mode framework. The optimization of the overall treatment parameters namely location, tuning frequency, and damping of each device is also investigated using genetic algorithm. Noise reduction of up to 9dB from 50Hz to 160Hz using 4% of the cylinder mass for the DVA and 5% of the cavity volume for the HR can be achieved. The robustness of the treatment performance to changes in the excitation, system and devices characteristics is also addressed.

The model is validated by experiments done outdoors on a 10-foot long, 8-foot diameter composite cylinder. The excitation level reached 136dB at the cylinder surface comparable to real launch acoustic environment. With HRs representing 2% of the cylinder volume, the noise transmission from 50Hz to 160Hz is reduced by 3dB and the addition of DVAs representing 6.5% of the cylinder mass enhances this performance to 4.3dB. Using the fairing model, a HR+DVA treatment is designed under flight constraints and is implemented in a real Boeing fairing. The treatment is composed of 220 HRs and 60 DVAs representing 1.1% and 2.5% of the fairing volume and mass respectively. Noise reduction of 3.2dB from 30Hz to 90Hz is obtained experimentally.

As a natural extension, a new type of adaptive Helmholtz resonator is developed. A tuning law commonly used to track single frequency disturbance is newly applied to track modes driven by broadband excitation. This tuning law only requires information local to the resonator simplifying greatly its implementation in a fairing where it can adapt to shifts in acoustic natural frequencies caused by varying payload fills. A time domain model of adaptive resonators coupled to a cylinder is developed. Simulations demonstrate that multiple adaptive HRs lead to broadband noise reductions similar to the ones obtained with genetic optimization. Experiments conducted on the cylinder confirmed the ability of adaptive HRs to converge to a near optimal solution in a frequency band including multiple resonances.

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Chapter 1 Introduction

1.1 Motivation and existing solutions

The widespread use of satellite telecommunication technology in both the civil and military sectors has increased the competition between a growing number of satellite launch providers. The drive to launch heavier payloads at lower costs has led companies to develop more powerful engines and lighter launch vehicles. As a consequence the design of payload fairings, which house the satellite, has significantly evolved. For the new generation of fairings, composite materials have replaced traditional aluminum alloys and led to large weight reductions.

Due to more powerful engines, noise levels generated at launch have increased tremendously ($>140\text{dB}$). The acoustic energy transmitted through the fairing to the satellite can damage some of its components. Acoustic induced damage has to be considered during satellite design and can result in large increase in cost. The move towards lighter composite fairing exacerbates this large acoustic transmission. In a competitive market, ensuring a safe payload acoustic environment has become an important performance criterion for launch providers.

The dominant acoustic sources are the launch vehicle's supersonic jets and their interaction with the launch pad as illustrated in Figure 1.1. The pad size, geometry, and elevation of the rocket play an important role in the acoustic field characteristics. These interactions were investigated and summarized by Eldred in 1971¹. The minimization of the thrust deflection angle, the deflection of the exhaust flow through a "covered bucket" or tunnel, and the injection of water into the deflector near the nozzle were suggested as solutions for reducing the acoustic load at the fairing. The development program for the European Launcher, Ariane 5, led to research on small-scale models to characterize the

launcher's acoustic environment. This was necessary for the vibroacoustic response analysis of the fairing², and also to optimize lift-off noise reduction through means such as exhaust tunnels, and waterfalls³. Using an inverse method, a full-scale test was carried out in order to identify the acoustic sources at lift-off. Pressure measurements on the fairing as well as, on the launch pad environment⁴ were used. All these research efforts have lead to launch site configurations that reduce the sound pressure level (SPL) surrounding the fairing as much as possible. Nevertheless, external and internal SPL remains quite high and ranges generally from 120 to 140dB depending on the type of launch vehicle.

Another means to reduce the acoustic load endured by a payload is to improve fairings acoustic transmission loss. A common method is to use acoustic blankets placed on the interior walls of the payload fairing to dissipate acoustic energy as heat. The effectiveness of such blankets is directly related to the ratio of material thickness to acoustic wavelength. Because of volume and weight constraints, blankets are usually less than 5 inches thick and therefore are effective only above approximately 300 Hz. There is a need for compact lightweight solutions to improve fairings transmission loss in the low frequency range (below 300Hz).

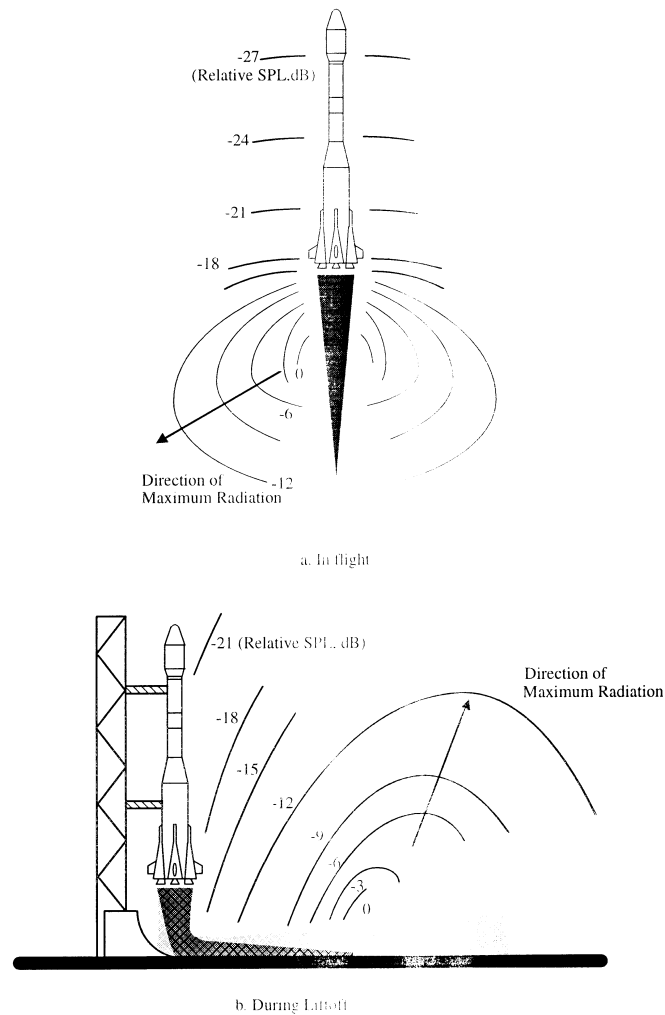


Figure 1.1 Contour of equal overall SPL (Eldred 1971)

1.2 Research development on the payload fairing noise problem

1.2.1 Active control approach

Active control has been considered a potential candidate for the low frequency payload fairing noise problem. In their preliminary review of active control technology

for the fairing acoustic problem, Niezrecki *et al.*⁵ evaluated the feasibility of different control schemes and actuators. They conclude that, piezoceramic (PZT) actuation is preferred over audio speakers, electromagnetic shakers, magnetostrictive actuators and shape memory alloys. This conclusion agrees with that of Leo *et al.*⁶ who indicated that up to 10dB broadband acoustic attenuation could be achieved with PZT actuators and local velocity feedback control of the structure. Griffin *et al.*⁷, investigated feedback control using proof mass actuators (shakers) and structural sensing, showed that the noise reduction achieved was not significant enough to justify the added complexity of the system.

Using PZT as actuators, several authors have focused on developing different control schemes for reducing noise transmission into a cavity. Clarke *et al.*⁸ modeled the fairing as a rectangular rigid cavity backed by an aluminum plate and used a two-level feedback control system. The first reduced acoustic transmission by adding damping to the structure and the second reduced acoustic reflections inside the cavity by removing structural damping at the acoustic resonance. Using the same simplified model, Wong *et al.*⁹ investigated the advantage of a dual layer feedback system that controlled a double plate wall. Vadali *et al.*¹⁰ quantified the problem of increased noise transmission into a cavity when replacing an aluminum panel by a composite one of equal thickness. Using PZT actuators, they then proposed a feedforward control approach. However, details for applying this method to the fairing problem, especially concerning the reference signal to be used, is left to future work. Johnson *et al.*¹¹ and Niezrecki *et al.*¹² have both developed and experimentally validated models of a simply supported cylinder and its acoustic cavity, and showed that PZT actuators can lead to attenuation of sound transmission. However, the conclusion is that, when scaled up to realistic fairing dimensions, the number of actuators and the power required to obtain the authority seemed impractical. More recently, Lane *et al.*¹³ investigate the use of acoustic speakers under a feedback control scheme to reduce the noise inside payload fairings. By spatially weighing the transducers, they were able to focus the control effort on specific acoustic modes of the cavity. Using 16 speakers they experimentally obtained 3dB acoustic reduction from 20Hz to 200Hz in a 5.3-meter long 1.3-meter diameter composite fairing. Other investigated approaches include active blankets capable of imposing local acoustic

impedance on the fairing wall by using collocated pressure and velocity sensors^{14,15} as well as glow-discharge plasma panels¹⁶ to control acoustic reflections.

Part of the research effort involves the development of lighter, smaller and more powerful acoustic and structural actuators¹⁷ so as to bring active control applications closer to the weight and volume constraints inherent to launch vehicles. In his master's thesis, Harris¹⁸ developed new lightweight electromagnetic actuators with high power output over a certain frequency band. In collaboration with Vibro-Acoustic Sciences, Inc. feedforward active control experiments were conducted on a large composite cylinder under high external disturbance levels (130dB). Using only eight actuators, interior noise level reduction of 5dB from 80-200Hz was achieved. However, these results were obtained using an ideal reference namely the input noise to the disturbance speakers. Similar performance was obtained at lower disturbance level (100dB) using external microphones as references. These experiments demonstrate that new powerful structural actuators may have enough authority to reduce noise transmission into payload fairing but in a feedforward active control scheme, the problem of the reference signals still needs to be solved¹⁹.

1.2.2 Passive treatment

Because of the drastic constraints, no active control system has been yet implemented in a payload fairing. Improving existing or creating new passive treatments has been the only practical means to achieve the noise reduction requirement. One example is the development of new acoustic blankets for the launch of the Cassini spacecraft²⁰ dedicated to explore the planet Saturn. In this particular case, a 3dB noise reduction at 200 and 250Hz compared to the existing treatment was necessary to avoid a costly vibration requalification of the spacecraft's radioisotope thermoelectric generators. The improved performance of the blankets was obtained by varying the thickness and density of the fiberglass batting, and the density and location of an internal barrier. Although weighing four times as much as the baseline, the new treatment was successfully implemented.

Martin *et al.*²¹ try to improve passive treatment acoustic performance by optimizing the location of impedance patches on the fairing wall, but the results were not significant. In order to reduce noise transmission of the Ariane 5 launcher of low frequencies, where acoustic blankets are ineffective, acoustic absorbers were designed and incorporated as part of the noise protection system^{3,22}. This fairing acoustic protection manufactured by Dornier, GmbH, has been patented²³. This demonstrated that even for low frequencies, passive treatment could be a viable performance/cost compromise.

The innovative idea of replacing the air inside the fairing with helium to reduce the internal SPL has also been investigated²⁴. This relatively easy to implement method, led to an experimental broadband reduction of about 10 dB in SPL. However, a major drawback was that the payload random vibration was globally increased. This technique actually reduces the damping of the payload structure due to a decrease in the gas pumping effect of the payload joints. Although a helium medium could reduce the internal SPL, it caused a more severe environment for the structure and therefore is not a suitable solution.

More recently, Gardonio *et al.*²⁵ have developed a fully coupled model to investigate the feasibility of using blocking masses to reduce noise transmission through a circular cylindrical shell excited by an acoustic plane wave. The structure is of the “sandwich-composite-with-stiff-core-construction” type developed for payload fairing. The blocking masses are used to reduce the coupling between the external acoustic field and the structure, as well as to reduce the internal coupling between the structure and the acoustic cavity. Although results show that blocking masses lead to greater noise reduction than an equivalent mass smeared over the whole fairing, the robustness of the treatment with respect to the incident angles (azimuth and elevation) of the plane wave is not assessed. Such a treatment, mainly based on modal restructuring, could be very dependent on the characteristics of the excitation. This dependence is a major downside since the launch acoustic environment remains very difficult to model.

1.3 The dynamic vibration absorber

This section presents the dynamic vibration absorber and its implementation in a variety of applications. The Distributed Vibration Absorber (DVA), which is a particular design of dynamic vibration absorber developed at Virginia Tech and used in this research, is also discussed.

1.3.1 How it works

The concept of the dynamic vibration absorber is fairly old, since Frahm²⁶ filed the first US patent in 1909 entitled “Device for damping vibrating bodies.” His absorber was a simply mass-spring system attached to a vibrating host structure, which can be modeled as in Figure 1.2. The inertia of the absorber mass reduces the net force and hence response of the host structure. The analytical model of a dynamic vibration absorber attached to a single-degree-of-freedom host structure is described by the following equations of motion

$$\begin{aligned} m_s \ddot{x}_s + (k_s + k_a)x_s - k_a x_a + (c_s + c_a)\dot{x}_s - c_a \dot{x}_a &= f(t) \\ m_a \ddot{x}_a + k_a(x_a - x_s) + c_a(\dot{x}_a - \dot{x}_s) &= 0 \end{aligned} \quad (1.3.1)$$

Assuming harmonic excitation $f(t) = fe^{-i\omega t}$, and the following standard relations between the mass (m_j), stiffness (k_j), viscous damping coefficient (c_j), natural frequency (ω_j) and damping ratio (ξ_j) of any second order system j ,

$$\omega_j = \sqrt{\frac{k_j}{m_j}} \quad \xi_j = \frac{c_j}{2m_j\omega_j}, \quad (1.3.2)$$

the impedance of the host structure without a dynamic vibration absorber is given by

$$Z_s(\omega) = \frac{m_s(\omega_s^2 - \omega^2 - 2i\omega\omega_s\xi_s)}{-i\omega}. \quad (1.3.3)$$

Thus, we define the free velocity of the host structure as

$$\dot{x}_0 = \frac{f}{Z_s}. \quad (1.3.4)$$

The impedance of the dynamic vibration absorber is defined as the reacting force through the spring and damper exerted on the primary system due to a unit velocity of its attachment point. Using Eq. (1.3.1), the absorber impedance is given by:

$$Z_a(\omega) = m_a \frac{i\omega\omega_a^2 + 2\xi_a\omega^2\omega_a}{(\omega_a^2 - \omega^2) - 2i\xi_a\omega\omega_a}. \quad (1.3.5)$$

The response of the host structure coupled to a dynamic vibration absorber is due to the primary force f as well as the reacting force of the absorber yielding

$$\dot{x}_s = \frac{f + Z_a\dot{x}_s}{Z_s} = \dot{x}_0 + \frac{Z_a\dot{x}_s}{Z_s}. \quad (1.3.6)$$

Therefore, the effect of the dynamic vibration absorber on the host structure vibration is a function of the ratio of the absorber impedance to the impedance of the structure at its attachment point since

$$\dot{x}_s = \dot{x}_0 \left[1 - \frac{Z_a}{Z_s} \right]^{-1}. \quad (1.3.7)$$

The bigger the impedance mismatch between the dynamic vibration absorber and the structure, the more the absorber affects the response of the structure.

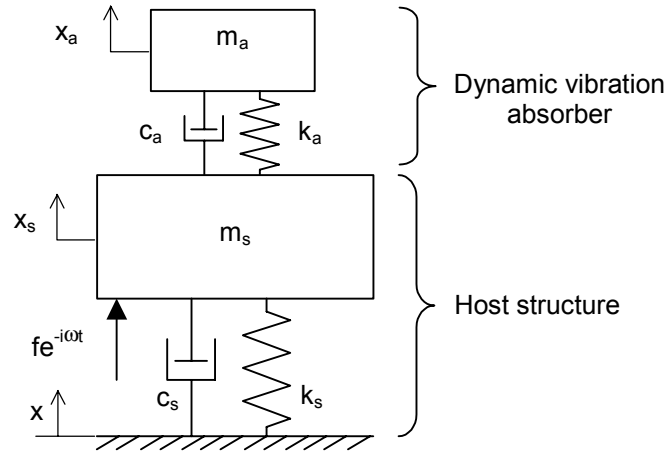


Figure 1.2 Model of a dynamic vibration absorber attached to a host structure.

Since Frahm's discovery, dynamic vibration absorbers have evolved a lot in terms of their design, but the main principle has remained the same. As von Flotow *et al.* addressed in their survey²⁷, dynamic vibration absorbers are used in two different ways with each adapted to resolve two main problems. When tuned to a specific mode of vibration of the host structure usually driven by a broadband excitation, they act as tuned dampers and are often called tuned mass dampers (TMDs). In this case, described in Figure 1.2, damping is implemented in the absorber spring to prevent a detrimental effect below and above the mode natural frequency. When they are not tuned to a mode but to a specific excitation frequency usually encountered in rotating machines dynamic vibration absorber are often called tuned vibration absorbers (TVAs). In this case TVAs are designed to present a large impedance at their attachment points in a very narrow band around the excitation frequency, and therefore the absorber damping is kept as small as possible.

Like any passive treatment, the main advantages of dynamic vibration absorbers are simplicity and lack of power consumption. Consequently, dynamic vibration absorbers are present in a myriad of applications as summarized by von Flotow *et al.*²⁷. The main drawbacks are a performance decrease with the mistuning and design constraints necessary for accurate tuning. Most of the research has been focused on optimizing

dynamic vibration absorbers to minimize these drawbacks while maintaining performance in different types of situations.

1.3.2 A well-studied system

In 1934, Den Hartog²⁸ proposed a closed form solution for the optimal frequency and damping of a tune mass damper acting on an undamped single-degree-of-freedom system subjected to harmonic excitation. His model is identical to Figure 1.2 with the viscous damping coefficient c_s being zero. The optimal parameters are a function of the mass ratio between the absorber (or secondary) mass and the structure (or primary) mass. His derivation is based on the assumption that *the most favorable curve* for the primary mass presents two equal maximum amplitudes. Because of its elegance and historical importance, Den Hartog's design procedure is the most reported in mechanical vibrations textbooks. A more complete study on the theory and applications of dynamic vibration absorbers is given in the book by Koronev and Reznikov²⁹.

Since Den Hartog, many authors have investigated different techniques to optimize TMDs. Recently, Pennestri³⁰ has derived an optimization technique that included the damping in the primary system. This technique is based on the Chebishev's min-max criterion, which ensures a unique solution but requires solving a set of non-linear equations numerically. In this study, Pennestri addresses the issue of the secondary mass displacement, usually discarded as a performance parameter but is a real concern in the design process.

To apply most of these optimization techniques to continuous structures, one has to assume that only one mode is contributing to the targeted response, and thus the structure is modeled as a single degree of freedom. Because not all problems can be simplified as such, some authors have investigated optimization methods for multi TMDs applied to multi-degree-of-freedom systems. Rice³¹ used a simplex algorithm to minimize the displacement response over a frequency band of a cantilever beam treated with two TMDs. The non-linear optimization provided optimal location, stiffness and damping for

each TMD. Using a substructure coupling technique based on either theoretical or experimental frequency response functions, Rade *et al.*³² derived a general non-linear optimization method in which different types of design constraints could be considered. This methodology could optimize the design of multiple TMDs to absorb the vibration of a structure in two disconnected frequency bands.

As an extension to these systems the idea of coupling the multiple TMDs together has generated the concept of multi-degree-of-freedom TMDs. Such a device can present up to six natural frequencies corresponding to its six degrees of freedom, and thus can dampen up to six distinct peaks in a host structure response. Using eigenvalue perturbation, Verdirame *et al.*³³ derived a methodology to design multi-degree-of-freedom TMDs. They also showed that in many cases for a given mass, a multi-degree-of-freedom TMD is more effective than multiple single degree-of-freedom TMDs. In his work, Harris¹⁸ investigated the advantages of coupling single degree-of-freedom absorbers that are spatially distributed over a vibrating structure so as to create a distributed multi-degree-of-freedom absorber. He concluded that in some cases, distributed multi-degree-of-freedom absorbers are as efficient as multiple single degree-of-freedom absorbers while having an overall reduced mass. However, the design of such devices can be fairly complex by depending on both the natural frequencies and mode shape values at the attachment points of the targeted structural resonances.

With a special interest for civil engineering applications, some authors have investigated the effect of multiple TMDs acting on a single-degree-of-freedom system subjected to a wide band input. Both Igusa *et al.*³⁴ and Joshi *et al.*³⁵ conclude that optimally designed multiple TMDs are more effective and more robust to variation in the natural frequency of the main system than an optimally designed single TMD of equal total mass.

Comparatively, a few authors have considered the use of dynamic vibration absorbers to reduce sound radiation of structure or sound transmission into a cavity. Jolly *et al.*³⁶ investigated the use of TMDs to control the sound radiation from a vibrating panel, and conclude that an overall decrease in the radiation efficiency only occurs when the TMDs are tuned to a critical mode such that its wave number at its resonant frequency equals the wave number in air. In all the other cases, the modal restructuration induced by TMDs

can lead to an increase of radiated sound power. Sun *et al.*³⁷ demonstrated the ability of dynamic vibration absorbers to improve the transmission loss of a real aircraft panel in a relatively narrow band around a structural mode. Using an optimization by neural network, Nagaya *et al.*³⁸ showed analytically and experimentally that multiple TMDs could reduce noise radiation from a plate over a broad frequency range by adding damping to structural modes.

To improve ride comfort in propeller-powered aircraft, the use of dynamic vibration absorbers acting as TVAs has been investigated. In this particular application, TVAs are tuned to the propeller blade passage frequency rather than to a particular mode of the aircraft. In his study Stubbs³⁹ addressed different practical design considerations for the implementation of TVAs to reduce the vibration induced noise for an aircraft's hydraulic and fuel pumps. After investigating the effect of tuned absorbers on the vibration of a fuselage and its coupled acoustic cavity modeled as a simply supported cylinder^{40,41}, Huang *et al.* optimize locations and parameters of the tuned absorbers⁴². When the cylindrical structure is subjected to external point force, perfectly tuned absorbers lead the greatest attenuation of the interior acoustic field. However, for uniformly distributed excitation, the best treatment is composed of slightly detuned absorbers. This result is in agreement with Fuller *et al.*⁴³ who previously showed that detuned absorbers provide more acoustic attenuation than tuned.

1.3.3 The Distibuted Vibration Absorber (DVA)

In the work presented here, a particular type of dynamic vibration absorber is used to control the sound transmission into a payload fairing. Because of the context in which it was developed at the Vibration and Acoustics Laboratories of Virginia Tech, this device is called a Distributed Vibration Absorber (DVA). A DVA is a spring mass system in which the spring is made of standard acoustic blown polyurethane foam on top of which is glued a plate made of any type of material. The block of foam acts as a distributed

spring and the top plate acts as a distributed mass layer. An example of DVA is shown in Figure 1.3.

This particular design has emerged from the investigation of a more complex device, the Distributed Active Vibration Absorber (DAVA). In his masters thesis Cambou⁴⁴ developed a DAVA made of a curved polymer piezoelectric PVDF sheet as the elastic/active layer and a sheet of thin lead as the mass layer. The sinusoidal curvature of the PVDF couples the in-plane strain and normal motion, which then excites the mass layer and simultaneously applies a normal force to the base structure. Thus it is possible to apply a control action to the base structure by sending an appropriate control voltage to the PVDF. Cambou noticed that the curved PVDF also acted as a passive distributed spring, and that different wavelengths and amplitudes of the sinusoidal curvature could change its stiffness. However, in order to control low frequencies where passive acoustic treatments are ineffective, the stiffness values of such a distributed spring were fairly high and thus a heavy mass layer was required for appropriate tuning.

To counteract this design constraint, Marcotte *et al.*⁴⁵ developed a new type of DAVA where the curved PVDF distributed spring was replaced by a sheet of PVDF embedded in a block of acoustic foam. This new device had the advantages of having a softer distributed spring, allowing the design of a light low frequency tuned absorber. In addition, this new device presents high frequency sound absorption due to the presence of the acoustic foam. Once the active PVDF sheet is removed from the block of foam, the DAVA become a simple DVA. It has been shown that for a given foam, the DVA's tuning of frequency is governed only by the foam thickness and the mass per unit area glued on top of it.

A DVA is therefore a relatively easy to built TMD, presenting large design flexibility in term of size, mass and tuning frequency. However, the damping of such a spring layer is hard to control as it is composed of both the foam structural damping as well as the air viscous losses occurring in the foam porosities. Different types of foam can provide different ranges of damping. Although it is composed of a distributed stiffness and mass, the DVA is assumed in this work to exert a uniformly distributed force to the structure it is attached to and so is modeled as a single degree of freedom system. This is a valid assumption as long as the DVA footprint is small compared to the wavelength in the

structure. Note that in this work, although the DVAs do not behave strictly as “distributed” absorbers, the acronym DVA will be used to describe this particular device. Nevertheless, the larger variable DVA footprint represents an advantage over the point TMD as it reduces stress concentration at the attachment surface and also allows a more compact device.

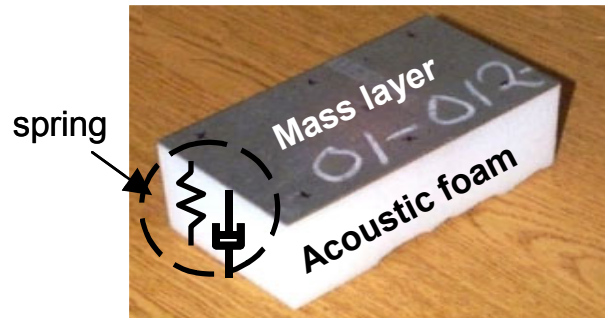


Figure 1.3 Distributed Vibration Absorber

1.4 The Helmholtz resonator

The Helmholtz resonator (HR), which is the second type of device used in this research to reduce sound transmission in payload fairings, is discussed in this section.

1.4.1 Principle and modeling

At the end of the 19th century, Helmholtz and Rayleigh⁴⁶ initiated the study of acoustic cavity resonators and described their basic physics. Since then Helmholtz resonators have been extensively analyzed. Nevertheless HRs remain a research topic even today as discrepancies between theory and experiment still exist not only in the resonant frequency but also in the sound absorption.

An acoustic resonator is composed of a rigid wall air cavity connected to the outside environment through a small opening that can be prolonged by a neck. The simplest

model of HR, as illustrated in Figure 1.4, is obtained by assuming that any characteristic dimension of the resonator is small compared to the acoustic wavelength. Under this assumption, the air trapped in the neck is modeled as a lumped mass and the air adiabatically compressed in the cavity is modeled as a spring. Helmholtz and Rayleigh⁴⁶ recognized that both interior (δ_i) and exterior (δ_e) end corrections should be added to the model so as to account for the inertia effect of the fluid in motion beyond the confines of the neck. Their length factor derivation considered a constant velocity profile over the neck circular cross-section s and was based on a radiating piston in an infinite baffle. By assuming the neck opening to be small compared to the cross-section of the HR cavity, they used the same correcting factor for the interior and exterior end given by⁴⁶

$$\delta_i = \delta_e = \frac{8r_o}{3\pi} = 0.849r_o, \quad (1.4.1)$$

where r_o is the radius of the circular neck cross-section. With c and ρ being the speed of sound and the density of the fluid containing the HR. This lumped parameter model yields the following expression for the resonant frequency of such a device:

$$f_h = \frac{c}{2\pi} \sqrt{\frac{s}{V(l + \delta_i + \delta_e)}} = \frac{c}{2\pi} \sqrt{\frac{s}{Vl_e}}. \quad (1.4.2)$$

Since this particular model depends on the volume of the cavity, the section area, and length of the neck, it does not give any information on the influence of the other geometrical characteristics such as the relative dimensions of the cavity on the acoustic behavior of the resonator.

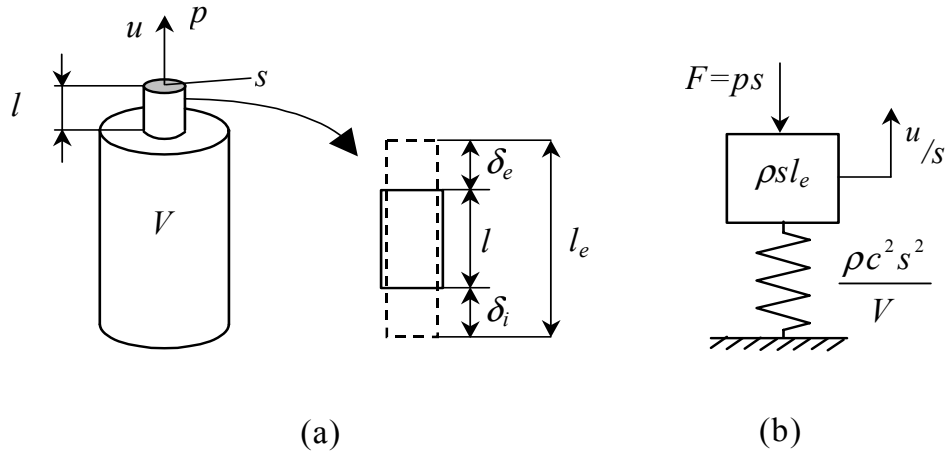


Figure 1.4 (a) Sketch of a HR exposed to an external acoustic pressure p and through whose neck flows a volume velocity u . (b) mechanical analog.

In 1953, Ingard published a complete study entitled “On the theory and design of Helmholtz resonators”⁴⁷ in which, extending the work by Rayleigh, he derives the end correction factors for various aperture geometries as well as the interaction between two distinct openings. He also presents the effect of viscosity, heat conduction and non-linearities on the scattering and dissipating properties of resonators and gives some practical guidelines for optimum resonator design. In the same year, Lambert⁴⁸ presented a study on the different experimentally significant factors influencing the damping of acoustic resonators, which is not included due to its complexity in the classic lumped model. Twenty years later, Aslter⁴⁹ developed an improved model, which includes the motion of mass particles inside the resonator and thus shows the influence of the cavity shape on the resonant frequency. However, the resonator geometry is assumed to be axisymmetric.

More recently Chanaud^{50,51}, extending the work done by Ingard, derived formulas for end correction factors and showed their improvement in the resonant frequency prediction, compared to classical equations, under extreme opening and cavity geometry as well as opening position. Selamet *et al.*⁵² investigated the influence of the cavity’s length-to-diameter ratio of a concentric cylindrical HR on its resonant frequency and its

transmission loss. Comparison between the experiment and the one-dimensional acoustic theory showed best agreement for cavity length to diameter ratio greater than 1 (when the axial propagation dominates). In a following paper Dickney and Selamet⁵³ studied resonators presenting small cavity length to diameter ratio and pointed out the effect of radial propagation on the acoustic behavior of such resonators. In a later study, Selamet *et al.*⁵⁴ derived a three dimensional analytical model to account for nonplanar wave propagation in the cavity and the neck of circular asymmetric HRs and compared these results to boundary element method numerical results. Finally, Doria⁵⁵ developed a simple model based on linear shape functions for deep cavity and long neck resonators where one of the axial lengths is not significantly smaller than the acoustic wavelength. This large amount of research signifies the use of HRs in a variety of technical application.

1.4.2 Applications

As the dynamic vibration absorbers are designed to reduce structural vibrations, HRs are designed to control acoustic vibrations. Depending on the problem, resonators can either increase or decrease room's reverberation time and can also eliminate standing waves occurring at room's resonant frequencies. As Den Hartog did for a vibration absorber on a structure, Fahy *et al.*⁵⁶ derived the interactions between an HR and a room acoustic mode. Their conclusions highlight the TVA's analogy, as a very lightly damped tuned HR greatly reduces acoustic mode pressure but increases it on each side of the tuning frequency. Thus, adding an optimal amount of damping to the resonator achieves a compromise similar to the TMD's design. The specificity of this acoustic analogy lies however in the proportional relation between the HR's performance and the HR-cavity's volume ratio.

Extending Fahy's work, Cummings⁵⁷ studied the effect of multiple HRs on several acoustic modes of an enclosure. Using an analytical derivation, he illustrated the resonators' scattering effect due to the inter-coupling of cavity modes. Thus, a single HR

can affect the response of more than one acoustic mode. In his conclusions, Cummings mentioned that the problem's complexity makes a complete parametric study impractical. Using a similar approach, Doria⁵⁸ investigated the effect of a double resonator coupling two natural frequencies to an acoustic cavity. The advantages of such a device over two independent HRs are not obvious especially when the more complicated design process is taken into consideration.

Away from room acoustic applications, HR represents an effective mean to solve different types of aeroacoustic problem. When air flows over a cavity's opening, strong acoustic oscillations can occur. This phenomenon at the base of some musical instrument's principle such as the flute can also be encountered during flight in an aircraft's weapons and landing gears bay. Hsu *et al.*⁵⁹ showed that HRs could counteract this problem. Acoustic resonators can also control flow separation over airfoils⁶⁰ and can suppress combustion-driven oscillations in industrial furnaces as well as combustion instability in rocket engines⁶¹.

When the disturbance is restricted to a particular frequency such as fan-induced noise, lightly damped tuned resonators can lead to large acoustic attenuation of the targeted tone. Thus, the use of HR in applications that encountered this type of excitation has been investigated. For instance, side branch resonators are commonly present in piping systems especially for engine silencing. The implementation of HRs in propeller aircraft has also been investigated to improve the acoustic transmission loss of panels⁶². As for the TVA, the limit of the lightly damped tuned HR is the large loss of performance with mistuning, which can be induced by variation of the disturbance frequency or error in the resonator's design. One way to counteract this drawback is to use semi active devices able to auto tune in order to maintain their efficiency.

1.4.3 Adaptive Helmholtz Resonator (AHR)

To improve the performance of passive treatments and especially to increase their robustness to uncertainty and to changes in the frequency of excitation or in the system's characteristics, adaptive-passive methods have been developed. Adaptive-passive treatments are made of passive devices such as dynamic vibration absorbers or HRs, which cannot inject energy in the system they are implemented in. Thus, they never increase the system's overall energy as opposed to active control treatments. As mentioned by Bernhard⁶³, most effective adaptive passive solutions have been developed for narrow frequency band applications where they provide a great advantage over active control means in terms of cost and power consumption and over passive treatments in term of robustness. Several authors have constructed resonators which tuning frequency is modified by varying the volume of the HR's cavity^{64,65,66} or the opening area⁶⁷. These research works involve the tuning of a single device using gradient-based search algorithm or feedback control scheme to minimize a tonal excitation at one error microphone. For the global control of structural vibration for single-frequency excitation, Dayou *et al.*⁶⁸ investigated the use of multiple tuned vibration absorbers. To avoid complex tuning algorithms, and numerous sensors to estimate global vibration, the absorbers are tuned to the excitation frequency using a simple local control strategy⁶⁹. The authors then suggested a procedure to find optimum locations and characteristics namely mass and damping of the absorbers. For any adaptive-passive system involving multiple devices, the investigation of tuning algorithms is complex as global cost functions become hyper-dimensional and the computational time involved can become impractical.

1.5 Objectives of this research

The reduction of noise transmission into payload fairings at low frequencies has become a challenging research problem due to both the increase in fairing size and the decrease in fairing mass resulting from the use of lighter composite materials. Most of the solutions proposed by researchers to reduce the low frequency noise transmission have not been implemented into today's fairings as their performance is usually far outweighed by their implementation costs. Indeed, the space and weight constraints imposed in these types of applications are extremely strenuous.

Therefore, the first and principal objective of this research is the creation of a lightweight, compact and practical low frequency noise reduction treatment using optimally damped Helmholtz resonators and DVAs. To complete this objective, the following objectives must be fulfilled:

- **Build a representative theoretical model of fairing structures.** An accurate fairing model is an effective tool to understand the noise transmission mechanisms and how noise reduction can be achieved by a treatment composed of multiple HRs and DVAs. The limitations of such a treatment and its robustness with respect to uncertainties affecting the different parts of the model need also to be addressed.
- **Development of a strategy for designing treatment.** Once the noise reduction mechanisms are understood, a methodology to design HR/DVA treatment need to be developed and applied to different structures.
- **Design, build and test near flight ready noise reduction devices.** After a particular treatment is created using model simulations, HRs and DVAs have to be built and tested under the strict flight constraints determined by Boeing.

- **Experimental validation of the model.** To be meaningful, a model of a structure representative of fairings has to be validated experimentally under realistic noise excitation.
- **Experimental verification of the treatment performance.** To validate the design of the flight-ready treatment, HRs and DVAs have to be tested experimentally on a realistic structure under acoustic disturbance near launch level (130dB).

The second objective of this research lays the groundwork for future improvements of the proposed passive noise reduction treatment. It consists of creating Helmholtz resonators that adapt to changes in the system's characteristics with time or payload size in order to maintain broadband noise reductions. To complete this objective the following tasks have to be fulfilled:

- **Development of a time domain model.** In order to investigate the real time adaptation of a multitude of HRs and the resulting noise reduction performance, a time domain model of the devices coupled to a realistic structure is necessary.
- **Investigate tuning algorithms.** Using the model, the effectiveness of different tuning laws in balance with their implementation costs can be undertaken and lead to a specific control scheme.
- **Design, build and test adaptive Helmholtz resonators.** Keeping in mind the flight constraints, the most efficient way to vary natural frequency of a resonator without altering its broadband performance needs to be found. Then, adaptive resonators and a control system have to be built and tested.
- **Experimentally verify the performance of adaptive resonators.** To validate the model simulations, the performance of multiple adaptive HRs applied to a system representative of fairings has to be demonstrated experimentally.

1.6 Contribution of the work

In fulfilling its objectives, the work carried out for this research brought several original contributions to the body of knowledge in this field:

- The development of a **fully coupled vibro-acoustic model** of a cylinder which includes:
 - the diffraction of the external acoustic field on the structure,
 - the dynamics of a composite cylinder of similar sandwich construction type used for payload fairings,
 - the dynamics of the acoustic cavity, which can be either cylindrical or annular,
 - the effect of the external fluid loading on the structure,
 - the effect of multiple DVAs and HRs.

Compared to existing models, the originality of this model lies in the simultaneous application of both structural and acoustic control devices to a fully coupled structural-acoustic system. This analytical vibro-acoustic model is also modified to work with the outputs of a fairing finite element model provided by Boeing. The resulting fairing model hence becomes a more efficient tool for the design of fairing treatments over today's finite element code.

- The **development of a methodology** to design effective and robust noise reduction treatments using HRs and DVAs. The optimization of the DVA treatment based on the studies by Johnson *et al.*⁷⁰ is extended by analogy to the HR treatment and modified in the multi-mode system. The optimization of the overall treatment parameters, namely location, tuning frequency, and damping level of each device is also investigated using a genetic algorithm.
- The **experimental validation** of the modeling work. The experiments conducted under high-level acoustic excitation (130dB) on a 10-foot long, 8-foot diameter

composite cylinder and on a full-size composite fairing represent a major contribution to this field. These experiments demonstrate the performance of a HR/DVA treatment on increasing the transmission loss of light composite structure at low frequencies. The tests conducted on the real fairing also demonstrate the effectiveness of near flight ready noise reduction treatment. In particular, the design of lightweight Helmholtz resonators is successfully validated.

- The development of **adaptive Helmholtz resonators** for broadband noise control represents a novel approach over existing tunable resonators designed for single frequency control. Because it only uses local information to each device, the proposed tuning strategy proves more practical than global strategies as each resonator becomes independent and thus can be manufactured as a generic device with integrated power supply, controller, actuator and sensor. Experiments conducted on a composite cylinder also validated the numerical simulations.

Chapter 2 Modeling

2.1 Introduction

Two models have been developed to investigate the performance of the noise reduction treatment. Both are based on coupling a modal model of the structure to a modal model of the enclosed acoustic cavity. An impedance method is used to describe the HRs and the DVAs. The matrix formulation of the system is identical for the two models. The difference arises in the mode shapes and natural frequencies of the structure and cavity and in the derivation of the external acoustic forcing. The first model illustrated on the left in Figure 2.1 is a simply supported cylinder embedded in an infinite rigid baffle excited by an acoustic plane wave. The acoustic cavity is either cylindrical or annular so this system is derived analytically. The dimensions of this system match those of a cylinder prototype provided by Boeing. The second model illustrated on the right in Figure 2.1 represents a real payload fairing. The structural and acoustic mode shapes are imported from a finite element model developed by Boeing, therefore the excitation has to be obtained using a numerical technique.

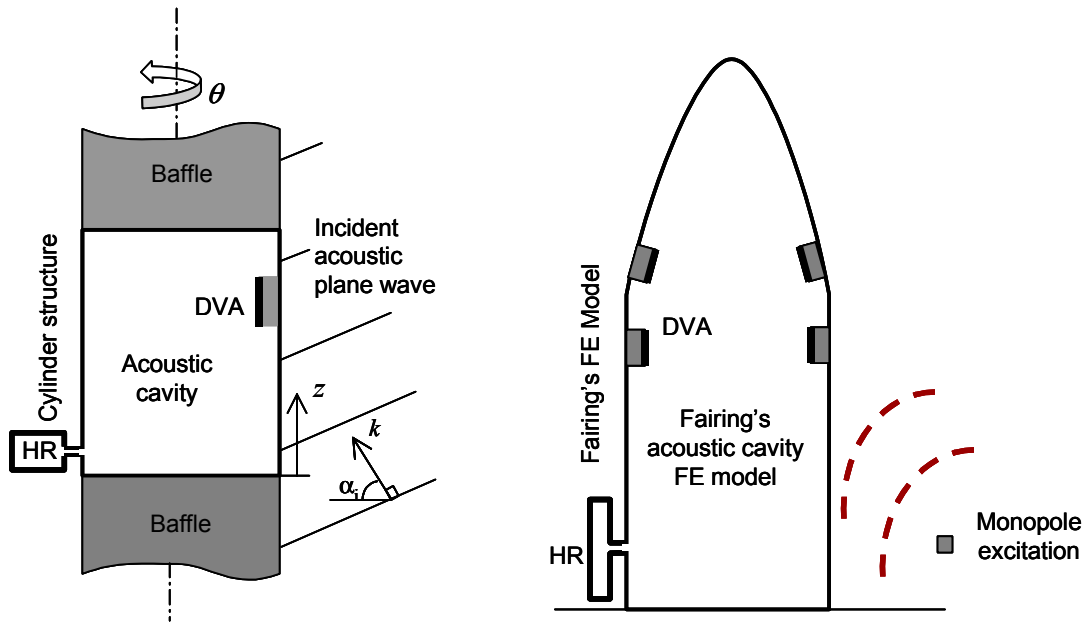


Figure 2.1 Schematic of the analytical cylinder model and the finite element based fairing

2.2 Composite cylinder vibration analysis

In this section the equation of motion of the structure is derived and solved. The effect of fluid loading is considered.

2.2.1 Equation of motion

The prototype cylinder built by Boeing is a relatively complex structure. It is made from a honeycomb core sandwiched by an inner and outer composite skin. Each skin is made of two layers of graphite epoxy with fibers oriented perpendicular to each other. This complex structure should be modeled as a five layered orthotropic cylindrical shell. However, since the thickness of the composite skins is small compare to the overall thickness of the structure, each skin can be modeled as a unique equivalent layer.

Because of the orthogonal orientation of the fibers, each skin behaves like an isotropic layer with equivalent thickness and material properties. The physical dimensions and properties of this simplified representation of the cylinder, illustrated in Figure 2.2, were provided by Boeing and are listed in Table 2.1.

Length	L	2.74 m
Internal radius	R_i	1.219 m
Thickness of the outer and inner equivalent skin	t_s	0.533 mm
Thickness of the honeycomb core	t_c	4.76 mm
Poisson ratio's in the θ_z -plane of the equivalent skin	$\nu_{\theta z}$	0.306
Young's modulus of the equivalent skin	E	44.82 GPa
Shear modulus of the equivalent skin	$G_{\theta z}$	17.448 Gpa
Axial Shear modulus of the honeycomb core	G_{zr}	96.5 MPa
Circumferential Shear modulus of the honeycomb core	$G_{\theta r}$	137.9 MPa
Density of the composite skin	ρ_s	1495.8 Kgm ⁻³
Density of the honeycomb core	ρ_c	112.2 Kgm ⁻³
Mass of the cylinder	M	75.7Kg
Mass of the top end cap	M_e	≈226Kg

Table 2.1 Physical dimensions and material characteristics of the Boeing cylinder.

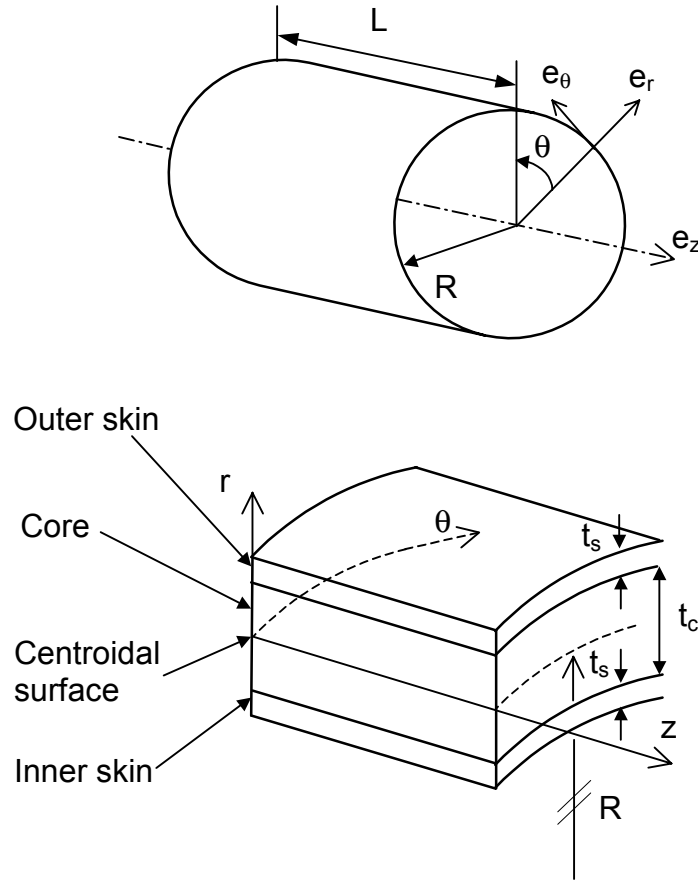


Figure 2.2 Schematic of the sandwich cylinder.

E.H. Baker and G. Herrmann⁷¹ derived the equation of motion for a cylindrical sandwich shell with orthotropic skins and core under initial stress. It was assumed in their derivation that the thickness of the skins is very small compared to the thickness of the core. This assumption ensures that all straight-lines normal to the centroidal surface remain straight after deformation, which defined the first order shear deformation theory. As a consequence, the motion of the centroidal surface of the shell can be expressed using five variables: u , v , w are the axial, tangential and radial displacements, τ_z , τ_θ are the angles of rotation in the z - r and θ - r planes. The displacements are considered small compared to the overall thickness. To simplify the expressions of the shell stresses, they assumed that the core resists only in transverse shear force and thus is free to stretch and

bend, and that the skins do not resist transverse shear force. This is a valid approximation, with respect to the material used in the design of the Boeing cylinder. Assuming very thin skin implies that the stress is uniform throughout their thickness.

As the inner and outer skin have same density and thickness, the mass and mass moment of inertia of a unit element is given by⁷¹:

$$m_\rho = t\rho_s + h\rho_c, \quad I_\rho = \rho_s I + \rho_c I_3, \quad (2.2.1)$$

where ρ_s and ρ_c are the densities of the skins and the core, respectively, and the geometrical parameters are defined by:

$$t = 2t_s, \quad h = t_s + t_c, \quad I = \frac{th^2}{4}, \quad I_3 = \frac{h^3}{12}. \quad (2.2.2)$$

The stiffness parameters for orthotropic skins and core given in ⁷¹are modified here by the isotropic property of the skins:

$$\begin{aligned} E_z = E_\theta &= \frac{Et}{1 - \nu_{\theta z}^2} & E_\nu &= \nu_{\theta z} E_z \\ D_z = D_\theta &= \frac{EI}{1 - \nu_{\theta z}^2} & D_\nu &= \nu_{\theta z} D_z \\ G_s &= tG_{z\theta} & D_{z\theta} &= IG_{z\theta} \\ G_z &= \kappa^2 G_{zr} h & G_\theta &= \kappa^2 G_{\theta r} \left(h + \frac{I}{R^2} \right) \end{aligned}, \quad (2.2.3)$$

where the shear correction factor κ equals 5/6. Using Hamilton's principle E.H. Baker and G. Herrmann⁷¹ obtained the five linearized equations of motion for the free vibration of the shell subjected to a uniform initial tension T in the axial direction. The coupled equations are expressed in the following matrix form:

$$\left(\begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} & 0 \\ A_{12} & A_{22} & A_{23} & 0 & A_{25} \\ -A_{13} & -A_{23} & -A_{33} & -A_{34} & -A_{35} \\ A_{14} & 0 & A_{34} & A_{44} & A_{45} \\ 0 & A_{25} & A_{35} & A_{45} & A_{55} \end{bmatrix} - \frac{\partial^2}{\partial t^2} \begin{bmatrix} C_{11} & 0 & 0 & C_{14} & 0 \\ 0 & C_{22} & 0 & 0 & C_{25} \\ 0 & 0 & C_{33} & 0 & 0 \\ C_{14} & 0 & 0 & C_{44} & 0 \\ 0 & C_{25} & 0 & 0 & C_{55} \end{bmatrix} \right) \begin{bmatrix} u(\theta, z, t) \\ v(\theta, z, t) \\ w(\theta, z, t) \\ \tau_z(\theta, z, t) \\ \tau_\theta(\theta, z, t) \end{bmatrix} = 0, \quad (2.2.4)$$

where each A_{ij} is a linear operator and the C_{ij} represent the mass and inertia terms:

$$\begin{aligned} C_{11} &= C_{22} = C_{33} = m_\rho \\ C_{44} &= C_{55} = I_\rho \\ C_{14} &= C_{25} = \frac{I_\rho}{R} \end{aligned} \quad (2.2.5)$$

Using the symmetry property of the sandwich cylinder described in Figure 2.2, the derived A_{ij} are:

$$\begin{aligned} A_{11} &= (E_z + T) \frac{\partial^2}{\partial z^2} + \frac{1}{R^2} \left(G_s + \frac{D_{z\theta}}{R^2} \right) \frac{\partial^2}{\partial \theta^2}, & A_{13} &= \frac{1}{R} (E_v) \frac{\partial}{\partial z}, \\ A_{12} &= \frac{1}{R} (E_v + G_s) \frac{\partial^2}{\partial z \partial \theta}, & A_{14} &= \left(\frac{D_z}{R} \right) \frac{\partial^2}{\partial z^2} - \frac{1}{R^2} \left(\frac{D_{z\theta}}{R} \right) \frac{\partial^2}{\partial \theta^2}, \\ A_{22} &= \frac{1}{R^2} \left(E_\theta + \frac{D_\theta}{R^2} \right) \frac{\partial^2}{\partial \theta^2} + (G_s + T) \frac{\partial^2}{\partial z^2} - \frac{G_\theta}{R^2}, & A_{34} &= -\frac{G_z}{R} \frac{\partial}{\partial z}, \\ A_{23} &= \frac{1}{R^2} \left(E_\theta + \frac{D_\theta}{R^2} + G_\theta \right) \frac{\partial}{\partial \theta}, & A_{35} &= -\frac{1}{R^2} \left(\frac{D_\theta}{R} + R G_\theta \right) \frac{\partial}{\partial \theta}, \\ A_{25} &= \left(\frac{D_{z\theta}}{R} \right) \frac{\partial^2}{\partial z^2} - \frac{1}{R^2} \left(\frac{D_\theta}{R} \right) \frac{\partial^2}{\partial \theta^2} + \frac{G_\theta}{R}, & A_{45} &= \frac{1}{R} (D_v + D_{z\theta}) \frac{\partial^2}{\partial z \partial \theta}, \\ A_{33} &= \frac{1}{R^2} \left(E_\theta + \frac{D_\theta}{R^2} \right) - (G_z + T) \frac{\partial^2}{\partial z^2} - \frac{1}{R^2} G_\theta \frac{\partial^2}{\partial \theta^2}, \\ A_{44} &= D_z \frac{\partial^2}{\partial z^2} + \frac{1}{R^2} D_{z\theta} \frac{\partial^2}{\partial \theta^2} - G_z, & A_{55} &= \frac{1}{R^2} D_\theta \frac{\partial^2}{\partial \theta^2} + D_{z\theta} \frac{\partial^2}{\partial z^2} - G_\theta, \end{aligned} \quad (2.2.6)$$

2.2.2 Boundary conditions, mode shapes and natural frequencies

The cylinder prototype built by Boeing is closed at both ends by two end caps made of plywood. Since the cylinder sits vertically, the weight of the top end cap adds a static compressive load to the shell. This is taken into account by the axial tension parameter T . Assuming the end cap mass is evenly spread over the circumference of the cylinder, T is given by $T \simeq -9.81M_e/2\pi R$, where M_e is the end cap mass and the negative sign indicate compression. The cylinder edges are fitted into a half-inch deep groove carved in the end cap. This design has been chosen to be as close as possible to the theoretical “simply supported by shear diaphragm” boundary conditions⁷². These boundary conditions consider the end caps stiff enough to constrain the radial displacement, w , and tangential displacement, v , of the shell. In addition, by virtue of their small thickness and the shallowness of their groove, the end caps are assumed to exert no axial stress N_{zz} in the z direction, and no bending moment $M_{\theta\theta}$ around the θ direction, on the shell. These boundary conditions are expressed as⁷¹:

$$\begin{aligned}
 v(\theta, 0) &= v(\theta, L) = 0 \\
 w(\theta, 0) &= w(\theta, L) = 0 \\
 N_{zz}(\theta, z) &= E_z \frac{\partial u}{\partial z} + \frac{D_z}{R} \frac{\partial \tau_z}{\partial z} + \frac{E_v}{R} \left(w + \frac{\partial v}{\partial \theta} \right) \bigg|_{z=0, z=L} = 0. \\
 M_{\theta\theta}(\theta, z) &= \frac{D_\theta}{R} \left(\frac{\partial \tau_\theta}{\partial \theta} - \frac{w}{R} - \frac{\partial v}{\partial \theta} \right) + D_v \frac{\partial \tau_z}{\partial z} \bigg|_{z=0, z=L} = 0
 \end{aligned} \tag{2.2.7}$$

To solve the coupled equations of motion in Eq. (2.2.4), the solution is assumed separable in θ and z and harmonic with respect to time. Because of the axisymmetry of the structure, the solution has to be periodic with respect to θ and thus must be of the $\cos(m\theta)$ or $\sin(m\theta)$ form, where m is the circumferential order ($m=0,1,2,\dots$). In order to satisfy the equation of motions as well as the boundary conditions of Eq. (2.2.7), the three displacements and two rotations must take the following form:

$$\begin{aligned}
u(\theta, z, t) &= U_{nm} \cos(m\theta) \cos\left(\frac{n\pi z}{L}\right) e^{-i\omega_{nm} t} \\
v(\theta, z, t) &= V_{nm} \sin(m\theta) \sin\left(\frac{n\pi z}{L}\right) e^{-i\omega_{nm} t} \\
w(\theta, z, t) &= W_{nm} \cos(m\theta) \sin\left(\frac{n\pi z}{L}\right) e^{-i\omega_{nm} t}, \\
\tau_z(\theta, z, t) &= T_{znm} \cos(m\theta) \cos\left(\frac{n\pi z}{L}\right) e^{-i\omega_{nm} t} \\
\tau_\theta(\theta, z, t) &= T_{\theta nm} \sin(m\theta) \sin\left(\frac{n\pi z}{L}\right) e^{-i\omega_{nm} t}.
\end{aligned} \tag{2.2.8}$$

where $n=1,2,\dots$ is the axial order, $k_n=n\pi/L$ is defined as the axial wave number and ω_{nm} is the angular frequency of oscillation. Replacing u , v , w , τ_z , and τ_θ in Eq.(2.2.4) by their expression given in Eq. (2.2.8) yields for each n,m pair the following eigenvalue problem:

$$\left(\begin{bmatrix} B_{11} & B_{12} & B_{13} & B_{14} & 0 \\ B_{12} & B_{22} & B_{23} & 0 & B_{25} \\ B_{13} & B_{23} & B_{33} & B_{34} & B_{35} \\ B_{14} & 0 & B_{34} & B_{44} & B_{45} \\ 0 & B_{25} & B_{35} & B_{45} & B_{55} \end{bmatrix} - \omega_{nm}^2 \begin{bmatrix} C_{11} & 0 & 0 & C_{14} & 0 \\ 0 & C_{22} & 0 & 0 & C_{25} \\ 0 & 0 & C_{33} & 0 & 0 \\ C_{14} & 0 & 0 & C_{44} & 0 \\ 0 & C_{25} & 0 & 0 & C_{55} \end{bmatrix} \right) \begin{bmatrix} U_{nm} \\ V_{nm} \\ W_{nm} \\ T_{znm} \\ T_{\theta nm} \end{bmatrix} = 0, \tag{2.2.9}$$

where the expression of the B_{ij} and C_{ij} are given in Eq.(2.2.10) and Eq.(2.2.5) respectively.

$$\begin{aligned}
B_{11} &= (E_z + T)k_n^2 + \left(G_s + \frac{D_{z\theta}}{R^2}\right)k_m^2, & B_{12} &= -(E_v + G_s)k_n k_m \\
B_{13} &= -\frac{1}{R}(E_v)k_n, & B_{14} &= \frac{1}{R}(D_z k_n^2 - D_{z\theta} k_m^2), & B_{34} &= \frac{G_z}{R}k_n \\
B_{22} &= (G_s + T)k_n^2 + \left(E_\theta + \frac{D_\theta}{R^2}\right)k_m^2 + \frac{G_\theta}{R^2}, & B_{23} &= \frac{1}{R}\left(E_\theta + \frac{D_\theta}{R^2} + G_\theta\right)k_m \\
B_{25} &= \frac{1}{R}(D_{z\theta} k_n^2 - D_\theta k_m^2 + G_\theta), & B_{33} &= (G_z + T)k_n^2 + G_\theta k_m^2 + \frac{1}{R^2}\left(E_\theta + \frac{D_\theta}{R^2}\right) \\
B_{35} &= -\frac{1}{R}\left(G_\theta + \frac{D_\theta}{R}\right)k_m, & B_{44} &= D_z k_n^2 + D_{z\theta} k_m^2 + G_z, \\
B_{45} &= -(D_v + D_{z\theta})k_n k_m, & B_{55} &= D_\theta k_m^2 + D_{z\theta} k_n^2 + G_\theta
\end{aligned} \tag{2.2.10}$$

Solving Eq.(2.2.9) numerically with the values given in Table 2.1 yields five positive eigenvalues ω_{nm}^α with $1 \leq \alpha \leq 5$. These eigenvalues define the five natural frequencies of vibration of the in vacuo cylinder given a spatial pattern of vibration set by the orders n and m . Each natural frequency is associated with an eigenvector, which describes the relative amplitude of the five degrees of freedom. The eigenvectors are normalized such that :

$$\left(U_{nm}^\alpha\right)^2 + \left(V_{nm}^\alpha\right)^2 + \left(W_{nm}^\alpha\right)^2 + \left(T_{znm}^\alpha\right)^2 + \left(T_{znm}^\alpha\right)^2 = 1 \quad 1 \leq \alpha \leq 5. \tag{2.2.11}$$

Depending on the relative amplitude of their component, the eigenvectors can be classified as axial ($\alpha=1$), when the amplitude U_{nm} dominates, tangential ($\alpha=2$), when the amplitude V_{nm} dominates, radial ($\alpha=3$), when the amplitude W_{nm} dominates or rotational ($\alpha=4, \alpha=5$). Therefore a simply supported cylindrical shell may vibrate with five distinct natural frequencies in the same modal pattern defined by the circumferential and axial wave number, k_m and k_n . Figure 2.3 shows the five in vacuo natural frequencies as a function of the circumferential order m for the first axial order $n=1$. Except for the $m=0$ mode, also called breathing mode, the radial eigenvector ($\alpha=3$), results in the lowest natural frequencies which are also the more dependant upon the circumferential order m . Because of the axisymmetry, the circumferential modes of cylinders orient depending on

the excitation azimuthal location. Taking this into account, cylinders are considered to have two orthogonal circumferential modes, one sine and one cosine, of the same order m_s . In order to distinguish these two modes, referred to as twin modes, an additional subscript σ is introduced. When $\sigma=0$, the mode takes the cosine form, when $\sigma=1$ it takes the sine form. The subscript σ does not apply to ω_{nm}^α as the twin modes share the same natural frequencies. Therefore, the expression for the mode shape of the sandwich cylinder are given by:

$$\psi_{nm\sigma}^\alpha(\theta, z) = \frac{1}{\sqrt{\Lambda_{nm}}} \begin{Bmatrix} U_{nm\sigma}^\alpha \cos\left(m\theta - \sigma\frac{\pi}{2}\right) \cos\left(\frac{n\pi z}{L}\right) \\ V_{nm\sigma}^\alpha \sin\left(m\theta - \sigma\frac{\pi}{2}\right) \sin\left(\frac{n\pi z}{L}\right) \\ W_{nm\sigma}^\alpha \cos\left(m\theta - \sigma\frac{\pi}{2}\right) \sin\left(\frac{n\pi z}{L}\right) \\ T_{znm\sigma}^\alpha \cos\left(m\theta - \sigma\frac{\pi}{2}\right) \cos\left(\frac{n\pi z}{L}\right) \\ T_{\theta nm\sigma}^\alpha \sin\left(m\theta - \sigma\frac{\pi}{2}\right) \sin\left(\frac{n\pi z}{L}\right) \end{Bmatrix}, \quad (2.2.12)$$

where the normalization factor Λ_{nm} is expressed using the Neumann symbol $\varepsilon_m=1$ if $m=0$ and $\varepsilon_m=2$ if $m \neq 0$ as:

$$\Lambda_{nm} = \frac{1}{2\varepsilon_m} \quad (2.2.13)$$

This normalization is introduced in order to obtain the following orthonormal properties of the mode shapes:

$$\int_S \left[\psi_{nm\sigma}^\alpha(\theta, z) \right]^T \left[\psi_{pq\eta}^\beta(\theta, z) \right] dS = S \delta_{np} \delta_{mq} \delta_{\sigma\eta} \delta_{\alpha\beta}, \quad (2.2.14)$$

where S is the mid-surface of the cylinder, the superscript T denotes the vector transpose and δ is the Kronecker's symbol.

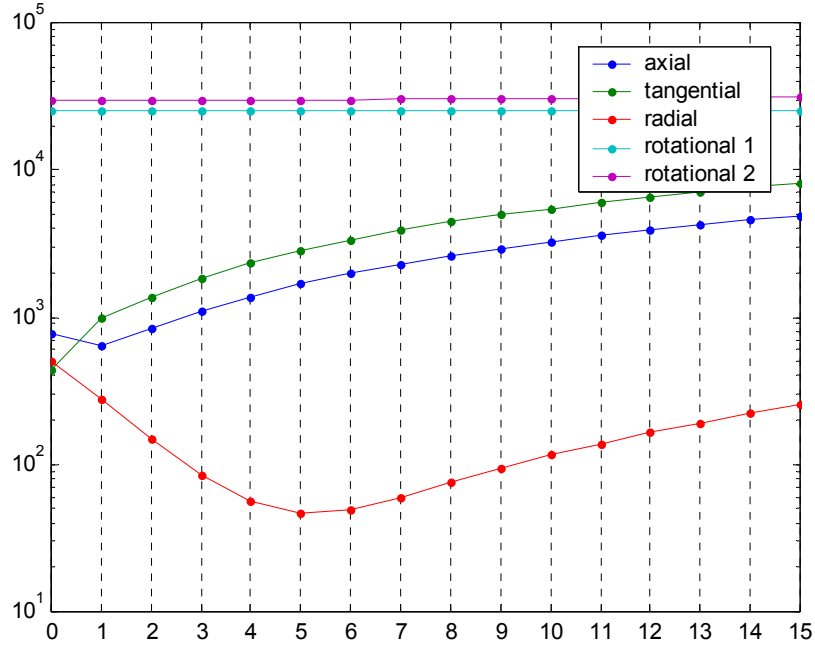


Figure 2.3 Natural frequencies of the cylinder for $n=1$.

2.2.3 Forced vibrations

For the noise transmission problem the fluid surrounding the cylinder is assumed inviscid, hence only the radial displacement of the shell is coupled to the fluid, and so only the radial component of the distributed force vector $f(z, \theta, t)$ is non zero. Assuming harmonic excitation $f(z, \theta, t) = f(z, \theta)e^{-i\omega t}$ the equation of motion for the forced vibration is given by:

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} & 0 \\ A_{12} & A_{22} & A_{23} & 0 & A_{25} \\ -A_{13} & -A_{23} & -A_{33} & -A_{34} & -A_{35} \\ A_{14} & 0 & A_{34} & A_{44} & A_{45} \\ 0 & A_{25} & A_{35} & A_{45} & A_{55} \end{bmatrix} + \omega^2 \begin{bmatrix} C_{11} & 0 & 0 & C_{14} & 0 \\ 0 & C_{22} & 0 & 0 & C_{25} \\ 0 & 0 & C_{33} & 0 & 0 \\ C_{14} & 0 & 0 & C_{44} & 0 \\ 0 & C_{25} & 0 & 0 & C_{55} \end{bmatrix} \begin{bmatrix} u(\theta, z) \\ v(\theta, z) \\ w(\theta, z) \\ \tau_z(\theta, z) \\ \tau_\theta(\theta, z) \end{bmatrix} = - \begin{bmatrix} 0 \\ 0 \\ f(\theta, z) \\ 0 \\ 0 \end{bmatrix}. \quad (2.2.15)$$

The three displacements and two rotations are expressed as summation of all the in vacuo modes:

$$\begin{bmatrix} u(\theta, z) \\ v(\theta, z) \\ w(\theta, z) \\ \tau_z(\theta, z) \\ \tau_\theta(\theta, z) \end{bmatrix} = \sum_{\alpha=1}^5 \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} \sum_{\sigma=1}^2 a_{nm\sigma}^{\alpha} \psi_{nm\sigma}^{\alpha}(\theta, z), \quad (2.2.16)$$

where $a_{nm\sigma}^{\alpha}$ is the amplitude of the mode. Applying this modal expansion in Eq.(2.2.15) and using Eq.(2.2.9), yields:

$$\begin{bmatrix} C_{11} & 0 & 0 & C_{14} & 0 \\ 0 & C_{22} & 0 & 0 & C_{25} \\ 0 & 0 & C_{33} & 0 & 0 \\ C_{14} & 0 & 0 & C_{44} & 0 \\ 0 & C_{25} & 0 & 0 & C_{55} \end{bmatrix} \sum_{\alpha=1}^5 \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} \sum_{\sigma=1}^2 \left([\omega_{nm}^{\alpha}]^2 - \omega^2 \right) a_{nm\sigma}^{\alpha} \psi_{nm\sigma}^{\alpha}(\theta, z) = \begin{bmatrix} 0 \\ 0 \\ f(\theta, z) \\ 0 \\ 0 \end{bmatrix}. \quad (2.2.17)$$

In order to simplify the equations only the radial modes, i.e $\alpha=3$, are retained in the summation. This simplification is valid for two reasons. First, the structure can be only forced in the radial direction, thus all the non-radial modes ($\alpha \neq 3$), which present a small radial contribution, are not well excited. Second, the natural frequencies of such modes are well above $\omega_{nm}^{\alpha=3}$ and so the radial response of these modes is only significant for high frequencies that are beyond the scope of the present work. As a consequence, all the

variables depending on α take their value for $\alpha=3$ and thus the superscript α is dropped. Since the modes are orthogonal, multiplying Eq.(2.2.17) by the transpose of $\psi_{nm\sigma}^\alpha$, given in Eq.(2.2.12), and integrating over the mid-surface S of the cylinder decouples the equations and yields for each mode:

$$SJ_{nm\sigma}(\omega_{nm}^2 - \omega^2)a_{nm\sigma} = f_{nm\sigma}. \quad (2.2.18)$$

The modal force $F_{nm\sigma}$ is obtained by

$$f_{nm\sigma} = \frac{W_{nm\sigma}}{\sqrt{\Lambda_{nm}}} \int_S f(\theta, z) \cos\left(m\theta - \sigma \frac{\pi}{2}\right) \sin\left(\frac{n\pi z}{L}\right) dS, \quad (2.2.19)$$

and

$$J_{nm\sigma} = m_\rho (U_{nm\sigma}^2 + V_{nm\sigma}^2 + W_{nm\sigma}^2) + I_\rho (T_{znm\sigma}^2 + T_{\theta nm\sigma}^2) + \frac{2I_\rho}{R} (U_{nm\sigma} T_{znm\sigma} + V_{nm\sigma} T_{\theta nm\sigma}). \quad (2.2.20)$$

Since the eigenvector is dominantly radial, the rotational component $T_{znm\sigma}$ and $T_{\theta nm\sigma}$ must be small compared to $W_{nm\sigma}$. For various n and m , the numerical results yield:

$$\left(\frac{T_{znm\sigma}}{W_{nm\sigma}}\right)^2 = \left(\frac{T_{\theta nm\sigma}}{W_{nm\sigma}}\right)^2 < 10^{-7}$$

$$\frac{I_\rho}{m_\rho} < 10^{-5}$$

As a consequence, one can assume $J_{nm\sigma} = m_\rho$ which give the following expression for the displacement mode amplitude with M being the total mass of the structure:

$$a_{nm\sigma} = \frac{f_{nm\sigma}}{M(\omega_{nm}^2 - \omega^2)}. \quad (2.2.21)$$

Thus the velocity mode amplitudes $v_{nm\sigma}$ are given by

$$v_{nm\sigma} = \frac{-i\omega}{M(\omega_{nm}^2 - \omega^2)} f_{nm\sigma}. \quad (2.2.22)$$

Assuming the structural damping to be viscous, the cylinder modal mobility can be defines as

$$A_{nm}^s(\omega) = \frac{-i\omega}{M(\omega_{nm}^2 - \omega^2 - 2\zeta_{nm}i\omega_{nm}\omega)}, \quad (2.2.23)$$

where ζ_{nm} is the modal damping ratio, A_{nm}^s denotes the ratio of a modal velocity to a modal force and the ‘s’ superscript signifies that the variable refers to the structure.

Because the surrounding fluid only couples to the normal displacement, the structural mode shape, $\Psi_{nm\sigma}^s$, is reduced to its radial component only:

$$\Psi_{nm\sigma}^s(z, \theta, r) = \frac{W_{nm}}{\sqrt{\Lambda_{nm}}} \sin\left(\frac{\pi n z}{L}\right) \cos\left(m\theta - \sigma \frac{\pi}{2}\right). \quad (2.2.24)$$

The radial velocity v at any point (θ_0, z_0) on the cylinder’s surface is approximated by a modal summation:

$$v(\theta_0, z_0, \omega) = \sum_{N_s} v_{nm\sigma}(\omega) \Psi_{nm\sigma}^s(\theta_0, z_0), \quad (2.2.25)$$

where N_s is the total number of structural modes considered.

For each frequency, the N_s velocity mode amplitudes are grouped into a column vector $\mathbf{v}$ and are obtained by the following matrix equation:

$$\mathbf{v} = \mathbf{A}^s \mathbf{f}, \quad (2.2.26)$$

where $\mathbf{A}^s$ is a $N_s \times N_s$ diagonal matrix regrouping the modal mobilities and $\mathbf{f}$ is a column vector of modal forces.

2.2.4 External Fluid loading effect

As the density of air is orders of magnitude smaller than the density of a structure, the fluid loading effect on the structural dynamics is usually neglected. However, the composite cylinder is a very light structure, the effect of the acoustic re-radiation into the external field may not be completely negligible. In this section the radiation impedance of a finite cylinder embedded in an infinite rigid baffle is derived. The orientation order σ , is later fixed at zero and dropped from the notation to obtain more compact formulations. Once the fluid loading is included, the equation of motion for the n, m mode becomes

$$v_{nm} = A_{nm}^s \left(f_{nm} - \int_0^{2\pi} \int_0^L p(R, \theta, z, \omega) \Psi_{nm}(z, \theta) R dz d\theta \right), \quad (2.2.27)$$

where F_{nm} is the modal force applied to the cylinder and the integral term represents the modal forcing due to the acoustic pressure on the surface of the cylinder $p(R, \theta, z, \omega)$.

Expressing Euler's equation $-\nabla p = \rho \frac{\partial v}{\partial t}$ in the radial direction at the cylinder's and baffle's surface yields

$$\begin{aligned} -\frac{\partial p}{\partial r} \Big|_{r=a} &= -i\omega\rho v(z, \theta, t) \quad 0 \leq z \leq L \\ -\frac{\partial p}{\partial r} \Big|_{r=a} &= 0 \quad z \geq L, \quad z \leq 0 \end{aligned}, \quad (2.2.28)$$

where v is radial velocity of the cylinder and ρ the air density. Using Eq. (2.2.24) and Eq. (2.2.25), the radial velocity of the cylinder is expressed as

$$v(z, \theta, t) = \sum_{N_s} \frac{W_{nm} v_{nm}}{\sqrt{\Lambda_{nm}}} \cos(m\theta) \sin(k_n z) e^{-i\omega t}. \quad (2.2.29)$$

This problem is solved using the wave number transform in the z -direction. That is

$$v(k_z, \theta, t) = \sum_{N_s} \frac{W_{nm} v_{nm}}{\sqrt{\Lambda_{nm}}} \cos(m\theta) R_n(k_z) e^{-i\omega t} \quad -\infty < k_z < +\infty, \quad (2.2.30)$$

where $R_n(k_z)$ is the wave number transform:

$$R_n(k_z) = \int_{-\infty}^{+\infty} \sin(k_n z) e^{-ik_z z} dz = \int_0^L \sin(k_n z) e^{-ik_z z} dz = \frac{k_n [(-1)^n e^{-ik_z L} - 1]}{k_z^2 - k_n^2}. \quad (2.2.31)$$

Equation (2.2.30) represents a velocity distribution of axial wave number k_z over an infinite cylinder as

$$v(z, \theta, t) = \sum_{N_s} \frac{W_{nm} v_{nm}}{\sqrt{\Lambda_{nm}}} \cos(m\theta) R_n(k_z) e^{i(k_z z - \omega t)} \quad -\infty < z < +\infty. \quad (2.2.32)$$

The linearized wave equation $\nabla^2 p = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2}$ expressed in cylindrical coordinates is

$$\frac{\partial^2 p}{\partial^2 r^2} + \frac{1}{r} \frac{\partial p}{\partial r} + \frac{1}{r^2} \frac{\partial^2 p}{\partial^2 \theta^2} + \frac{\partial^2 p}{\partial^2 z^2} + k^2 p = 0, \quad (2.2.33)$$

where $k = \omega/c$ is the wave number in the fluid in which c is the speed of sound.

For a single propagating wave in the axial direction, the acoustic pressure is separable and can be written as

$$p(r, z, \theta)e^{-i\omega t} = p_r(r)p_z(z)p_\theta(\theta)e^{-i\omega t}. \quad (2.2.34)$$

The pressure component p_r, p_z, p_θ can be expressed as a sum of modal pressure:

$$p(r, z, \theta)e^{-i\omega t} = \sum_{N_s} p_{nm}(r, z, \theta)e^{-i\omega t} = \sum_{N_s} p_{r,nm}(r)p_{z,n}(z)p_{\theta,m}(\theta)e^{-i\omega t}. \quad (2.2.35)$$

In order to satisfies the boundary condition in Eq.(2.2.28), the variation of the modal pressure along θ and z coordinate must be the same as the velocity distribution in Eq.(2.2.32),

$$-\frac{\partial p_r(r)}{\partial r} \Big|_{r=a} p_z(z)p_\theta(\theta) = -i\omega\rho \sum_{N_s} \frac{W_{nm}v_{nm}}{\sqrt{\Lambda_{nm}}} \cos(m\theta)R_n(k_z)e^{ik_z z}, \quad (2.2.36)$$

and thus using Eq.(2.2.35)

$$p_{z,n}(z)p_{\theta,m}(\theta) = \cos(m\theta)R_n(k_z)e^{ik_z z}. \quad (2.2.37)$$

Replacing Eq. (2.2.34) in Eq. (2.2.33) gives:

$$\frac{\partial^2 p_{r,nm}(r)}{\partial r^2} + \frac{1}{r} \frac{\partial p_{r,nm}(r)}{\partial r} + \left[(k^2 - k_z^2) - \left(\frac{m}{r} \right)^2 \right] p_{r,nm}(r) = 0. \quad (2.2.38)$$

The radial component of the modal pressure only depends on the circumferential m order therefore $p_{r,nm}(r) = p_{r,m}(r)$. Equation (2.2.38) is a Bessel's equation whose solution takes the form of:

$$p_{r,m}(r) = BJ_m(\sqrt{k^2 - k_z^2}r) + CN_m(\sqrt{k^2 - k_z^2}r), \quad (2.2.39)$$

where $J_m(.)$ and $N_m(.)$ are the Bessel functions of the first and second kind and $k_r = \sqrt{k^2 - k_z^2}$ is the radial wave number. The constant B and C are determined using the boundary conditions.

The first boundary condition is the radiation condition, which yields:

$$\begin{aligned} p_{r,m}(r) &\rightarrow e^{i(k_r r - \omega t)} \\ k_r r &\rightarrow \infty \end{aligned} \quad (2.2.40)$$

Considering the asymptotic expansion of the Bessel function

$$\left\{ \begin{array}{l} p_{r,m}(r) \\ k_r r \rightarrow \infty \end{array} \right\} \rightarrow \sqrt{\frac{2}{\pi k_r r}} \left\{ B \cos(k_r r - \pi/4) + C \sin(k_r r - \pi/4) \right\}, \quad (2.2.41)$$

$C=iB$ is required to satisfy the radiation condition. Moreover, the combination $J_m(.) + iN_m(.)$ is the Hankel function of the first kind, $H_m^{(1)}(.)$. Thus the modal pressure variation in the r -direction is given as:

$$p_{r,m}(r) = BH_m^{(1)} \left[\sqrt{k^2 - k_z^2} r \right]. \quad (2.2.42)$$

The modal pressure in the wave number domain is then given as:

$$p_{nm}(r, \theta, k_z, t) = BH_m^{(1)} \left[\sqrt{k^2 - k_z^2} r \right] \cos(m\theta) R_n(k_z) e^{-i\omega t}. \quad (2.2.43)$$

The second boundary condition expressed in Eq.(2.2.28) defines the amplitude B:

$$-B \frac{dH_m^{(1)} \left[\sqrt{k^2 - k_z^2} R \right]}{dr} = -i\omega\rho \frac{W_{nm} v_{nm}}{\sqrt{\Lambda_{nm}}} \quad (2.2.44)$$

or

$$k_r B H_{m,d}^{(1)} [k_r R] = i\omega\rho \frac{W_{nm} v_{nm}}{\sqrt{\Lambda_{nm}}}, \quad (2.2.45)$$

where $H_{m,d}^{(1)}$ implies the differentiation with respect to $k_r r$.

The derivative of the Hankel function of order m can be expressed as:

$$\begin{aligned} \frac{dH_0^{(1)}(\mu)}{d\mu} &= -H_1^{(1)}(\mu) \\ \frac{dH_m^{(1)}(\mu)}{d\mu} &= \frac{1}{2} \left[H_{m-1}^{(1)}(\mu) - H_{m+1}^{(1)}(\mu) \right] \end{aligned} \quad (2.2.46)$$

Solving for B in Eq.(2.2.44) and replacing into Eq.(2.2.35) yields

$$p_{nm}(r, \theta, k_z, t) = \frac{W_{nm} v_{nm}}{\sqrt{\Lambda_{nm}}} \frac{i\rho c k}{\sqrt{k^2 - k_z^2}} \frac{H_m^{(1)} \left[\sqrt{k^2 - k_z^2} r \right]}{H_{m,d}^{(1)} \left[\sqrt{k^2 - k_z^2} R \right]} \cos(m\theta) R_n(k_z) e^{-i\omega t}. \quad (2.2.47)$$

Due to the chosen $e^{-i\omega t}$ convention, the radial wave number k_r needs to satisfy the following conditions to ensure a bounded solution:

$$\begin{aligned} k_r &= \sqrt{k^2 - k_z^2} \text{ if } k \geq k_z \\ k_r &= i\sqrt{k^2 - k_z^2} \text{ if } k < k_z \end{aligned} \quad (2.2.48)$$

The radiation impedance of the cylinder for a single axial wave is given by:

$$Z_{nm}^{rad}(\omega) = \frac{p_{nm}(R, k_z, \theta, t)}{v_{nm}(k_z, \theta, t)}, \quad (2.2.49)$$

where $v_{nm}(k_z, \theta, t) = \frac{W_{nm} v_{nm}}{\sqrt{\Lambda_{nm}}} \cos(m\theta) R_n(k_z) e^{-i\omega t}$.

Using Eq.(2.2.47) yields:

$$Z_{nm}^{rad}(\omega) = \frac{i\rho c k}{\sqrt{k^2 - k_z^2}} \frac{H_m^{(1)}\left[\sqrt{k^2 - k_z^2} r\right]}{H_{m,d}^{(1)}\left[\sqrt{k^2 - k_z^2} R\right]}, \quad (2.2.50)$$

where k_r satisfies the radiation condition given in Eq.(2.2.48) This radiation impedance independent of the axial order n is equivalent to the radiation impedance of an infinite cylinder and is plotted in Figure 2.4.

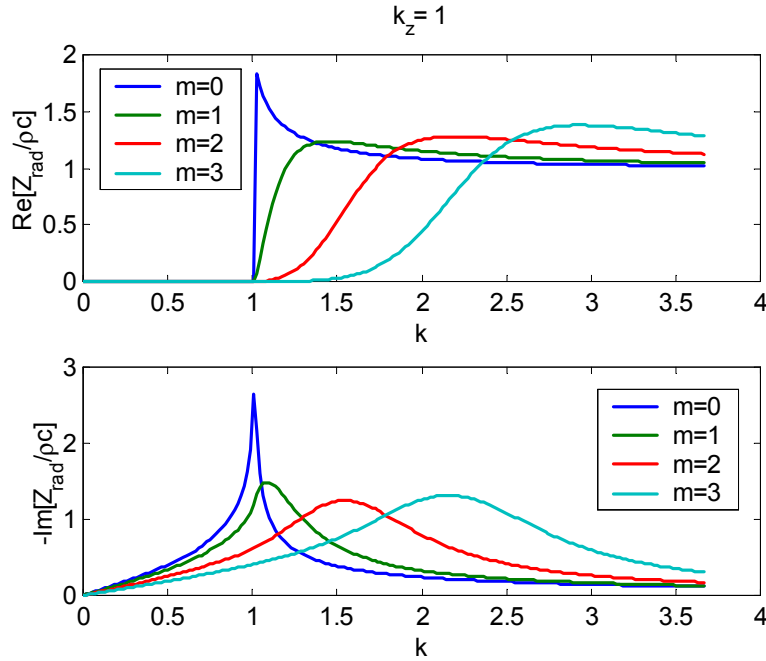


Figure 2.4 Normalized radiation impedance of an infinite cylinder for $m=0,1,2,3$ and $k_z=1$.

The surface acoustic pressure of the cylinder is obtained by taking the inverse wavenumber transform of Eq. (2.2.47) and summing over the modes taken into account:

$$p(R, \theta, z, t) = \frac{i\rho ck}{2\pi} \sum_{N_s} \frac{W_{nm} v_{nm}}{\sqrt{\Lambda_{nm}}} \cos(m\theta) \int_{-\infty}^{+\infty} \frac{R_n(k_z)}{\sqrt{k^2 - k_z^2}} \frac{H_m^{(1)}\left[\sqrt{k^2 - k_z^2} R\right]}{H_{m,d}^{(1)}\left[\sqrt{k^2 - k_z^2} R\right]} e^{ik_z z} dk_z e^{-i\omega t} \quad (2.2.51)$$

Using the expression for the normalization factor given by Eq. (2.2.13), Eq. (2.2.51) becomes:

$$p(R, \theta, z, t) = \frac{i\rho ck}{2\pi} \sum_{N_s} \sqrt{2e_m} W_{nm} v_{nm} \cos(m\theta) \int_{-\infty}^{+\infty} \frac{R_n(k_z)}{\sqrt{k^2 - k_z^2}} \frac{H_m^{(1)}\left[\sqrt{k^2 - k_z^2} R\right]}{H_{m,d}^{(1)}\left[\sqrt{k^2 - k_z^2} R\right]} e^{ik_z z} dk_z e^{-i\omega t} \quad (2.2.52)$$

Multiplying the pressure by the structure mode shape and integrating over the cylinder's surface yields the modal forcing:

$$f_{nm}^{rad}(\omega) = \frac{R}{2\pi} \int_0^{2\pi} \sum_{N_s} 2\sqrt{e_r e_m} W_{nm} W_{lr} v_{lr} \cos(r\theta) \cos(m\theta) \sin(k_n z) \int_{-\infty}^{+\infty} R_l(k_z) Z_{lr}^{rad} e^{ik_z z} dk_z dz d\theta \quad (2.2.53)$$

This expression contains the complex conjugate of the wavenumber transform:

$$\int_0^L \sin(k_n z) e^{ik_z z} dz = R_n^*(k_z), \quad (2.2.54)$$

hence

$$f_{nm}^{rad}(\omega) = \frac{R}{2\pi} \sum_{N_s} 2\sqrt{e_r e_m} W_{nm} W_{lr} v_{lr} \int_0^{2\pi} \cos(r\theta) \cos(m\theta) d\theta \int_{-\infty}^{+\infty} R_l(k_z) R_n^*(k_z) Z_{lr}^{rad}(\omega) dk_z \quad (2.2.55)$$

The circumferential mode order m and r have to be the same in order to have a non-zero modal pressure, thus

$$f_{nm}^{rad}(\omega) = i\rho ck2R \sum_{N_s} W_{nm} W_{lm} v_{lm} \int_{-\infty}^{+\infty} \frac{R_l(k_z) R_n^*(k_z)}{\sqrt{k^2 - k_z^2}} \frac{H_m^{(1)} \left[\sqrt{k^2 - k_z^2} a \right]}{H_{m,d}^{(1)} \left[\sqrt{k^2 - k_z^2} a \right]} dk_z . \quad (2.2.56)$$

The cylinder's modal radiation impedance $\mathbf{Z}^r$ is defined as the $N_s \times N_s$ symmetric matrix, which element are given by:

$$Z_{nlm}^r(\omega) = i\rho\omega 2R \int_{-\infty}^{+\infty} \frac{R_l(k_z) R_n^*(k_z) W_{nm} W_{lm}}{\sqrt{k^2 - k_z^2}} \frac{H_m^{(1)} \left[\sqrt{k^2 - k_z^2} R \right]}{H_{m,d}^{(1)} \left[\sqrt{k^2 - k_z^2} R \right]} dk_z . \quad (2.2.57)$$

The equation of motion for the n,m mode becomes:

$$\left(A_{nm}^s \right)^{-1} v_{nm} = f_{nm} - \sum_{N_s} Z_{nlm}^r v_{lm} . \quad (2.2.58)$$

Therefore the surrounding fluid couples all the structural mode of same circumferential order together. This system of N_s coupled equations is expressed in matrix form as:

$$\mathbf{v} = \left[\left[\mathbf{A}^s \right]^{-1} + \mathbf{Z}^r \right]^{-1} \mathbf{f} \quad (2.2.59)$$

The factor β_{nlm} is defined as the integral of module of $Z_{nlm}^r(\omega)$ over the frequency band of interest (in this case 0-200Hz):

$$\beta_{nlm} = \int_0^{200Hz} |Z_{nlm}^r(f)| df$$

Using the numerical value in Table 2.1, these factors are computed for $m=2$ and for different l,n combinations. Table 2.2 presents the resulting numerical values normalized by the first (highest) diagonal value β_{112} .

β_{nl2}/β_{112}	$l=1$	$l=2$	$l=3$	$l=4$	$l=5$	$l=6$
$n=1$	1	0	0.13	0	0.06	0
$n=2$	0	0.94	0	0.16	0	0.09
$n=3$	0.13	0	0.74	0	0.11	0
$n=4$	0	0.16	0	0.51	0	0.05
$n=5$	0.06	0	0.11	0	0.38	0
$n=6$	0	0.09	0	0.05	0	0.31

Table 2.2 Illustration of the diagonal dominance of the radiation impedance matrix

As expected the same circumferential order modes with axial order in odd-even combination do not coupled through the external fluid. In addition, Table 2.2 shows the diagonal dominance of the radiation impedance matrix. Retaining only the diagonal terms of $\mathbf{Z}^r$, the modes remain decoupled from one another and each modal velocity can be expressed as:

$$v_{nm}(\omega) = \frac{-i\omega f_{nm}(\omega)}{M(\omega_{nm}^2 - \omega^2 - 2i\xi_{nm}\omega_{nm}\omega) - i\omega Z'_{nnm}(\omega)}. \quad (2.2.60)$$

The real part of $\mathbf{Z}^r$ acts as a damper whereas the imaginary part acts as additional mass in the low frequencies. This additional mass and damping ratio effect are illustrated by rewriting Eq. (2.2.60), as

$$v_{nm}(\omega) = \frac{-i\omega f_{nm}(\omega)}{M\omega_{nm}^2 - \left[M - \text{Im}\left(\frac{Z'_{nnm}}{\omega}\right) \right] \omega^2 - i\omega [2M\xi_{nm}\omega_{nm} + \text{Re}(Z'_{nnm})]}. \quad (2.2.61)$$

2.3 Acoustic cavity analysis

In this section, the modal characteristics of cylindrical and annular cavities are derived and the equation governing the behavior of the enclosed fluid is presented.

2.3.1 Mode shapes and natural frequency of an acoustic cavity

Assuming the acoustic pressure p to be harmonic with time, the linearized homogenous wave equation $\nabla^2 p = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2}$ is expressed in cylindrical coordinates as

$$\frac{\partial^2 p}{\partial^2 r^2} + \frac{1}{r} \frac{\partial p}{\partial r} + \frac{1}{r^2} \frac{\partial^2 p}{\partial^2 \theta^2} + \frac{\partial^2 p}{\partial^2 z^2} + k^2 p = 0. \quad (2.3.1)$$

where $k = \omega/c$ is the wave number in the fluid in which c is the speed of sound.

Using the separation of variables, the wave equation is split into a system of three equations:

$$\begin{cases} r^2 \frac{\partial^2 P_r}{\partial r^2} + r \frac{\partial P_r}{\partial r} + [(k^2 - \mu)r^2 - \gamma] P_r = 0 \\ \frac{\partial^2 P_\theta}{\partial \theta^2} + \gamma P_\theta = 0 \\ \frac{\partial^2 P_z}{\partial z^2} + \mu P_z = 0 \end{cases} \quad (2.3.2)$$

where $p(r, \theta, z, t) = P_r(r)P_\theta(\theta)P_z(z)e^{-i\omega t}$.

Since both cylindrical and annular cavity are axisymmetric the solution to the second equation of system (2.3.2) must be periodic thus $\gamma = m^2$ with m being an integer. This leads to

$$P_\theta(\theta) = \cos\left(m\theta - \sigma \frac{\pi}{2}\right), \quad (2.3.3)$$

where, σ , the orientation order, is zero or one. Assuming hard wall boundary conditions, that is $\nabla p = 0$ at the cavity surface, the solution of third equation of the system (2.3.2) yields

$$P_z(z) = \cos\left(\frac{n\pi}{L} z\right), \quad (2.3.4)$$

where n is an integer and L is the length of the cavity. The axial modal wave number in the fluid, k_n is defined as $n\pi/L$.

Applying the change of variables $s = k_r r$, the radial wavenumber k_r is given by

$$k_r^2 = k^2 - k_n^2 \quad (2.3.5)$$

The first equation of system (2.3.2) reduces to a Bessel equation of order m :

$$s^2 \frac{\partial^2 P_r(s)}{\partial s^2} + s \frac{\partial P_r(s)}{\partial s} + [s^2 - m^2] P_r(s) = 0 \quad (2.3.6)$$

Thus, the solutions are given by a combination of Bessel functions of the first kind J_m and second kind N_m (also known as Neumann function):

$$P_r = A J_m(k_r r) + B N_m(k_r r), \quad (2.3.7)$$

where A and B are determined by applying the boundary conditions in the r -direction.

For the cylindrical acoustic cavity, since $N_m(r) \xrightarrow{r \rightarrow 0} -\infty$, the amplitude B must vanish to ensure a finite solution at $r=0$. The hard wall boundary condition reduces to

$\left. \frac{\partial}{\partial r} J_m(k_{mp}r) \right|_{r=R} = 0$ which yields the radial modal wave numbers k_{mp} . The radial variation of pressure is then given by:

$$P_r(r) = J_m(k_{mp}r) \quad (2.3.8)$$

For an annular cavity the hard wall boundary condition at the outer ($r=R$) circumference

$\left. \frac{\partial P_r}{\partial r} \right|_{r=R} = 0$ yields

$$B = -A \frac{J'_m(k_{mp}R)}{N'_m(k_{mp}R)}. \quad (2.3.9)$$

The prime superscript indicates the derivation with respect to the radial direction. The hard wall boundary condition at the inner ($r=R_i < R$) circumference yields the following transcendental equation for the modal radial wave number k_{mp} :

$$N'_m(k_{mp}R)J'_m(k_{mp}R_i) - J'_m(k_{mp}R)N'_m(k_{mp}R_i) = 0 \quad (2.3.10)$$

In this case, the radial variation of pressure are expressed as

$$P_r(r) = \Gamma_m(k_{mp}r) = \left[J_m(k_{mp}r) - \frac{J'_m(k_{mp}R)}{N'_m(k_{mp}R)} N_m(k_{mp}r) \right]. \quad (2.3.11)$$

The acoustic mode shapes for both types of cavity are obtained by recombining the circumferential, axial and radial variations. For a circular cylindrical cavity the mode shape are expressed as

$$\Psi_{nmp\sigma}^a(r, \theta, z) = \frac{1}{\sqrt{\Lambda_{nmp}}} J_m(k_{mp}r) \cos(k_n z) \cos\left(m\theta - \sigma \frac{\pi}{2}\right), \quad (2.3.12)$$

where n , m , p , σ are the axial circumferential radial and orientation mode orders respectively, and the superscript ‘ a ’ signifies that the variable refers to the acoustic cavity. The normalization factor Λ_{nmp} is chosen such that the integral of the squared of the mode shape equals the cavity’s volume V .

$$\Lambda_{nmp} = \frac{1}{V} \iiint_V [J_m(k_{mp}r) \cos\left(\frac{n\pi z}{L}\right) \cos(m_s \theta)]^2 dV = \frac{1}{\varepsilon_n \varepsilon_m} \left[1 - \left(\frac{m}{\chi_{mp} + \delta_{\chi_{mp}, 0}} \right)^2 \right] J_m^2(\chi_{mp}), \quad (2.3.13)$$

where $\frac{\partial}{\partial r}[J_m(\chi_{mp})] = 0$ and δ is the Kronecker’s symbol.

The acoustic pressure mode shape for the annular cavity is expressed as

$$\psi_{nmp\sigma}^a(r, \theta, z) = \frac{\Gamma_m(k_{mp}r)}{\sqrt{\Lambda_{nmp}}} \cos\left(\frac{n\pi z}{L}\right) \cos\left(m\theta - \sigma \frac{\pi}{2}\right), \quad (2.3.14)$$

where the modal normalization factor Λ_{nmp} is given by

$$\Lambda_{nmp} = \frac{1}{V} \iiint_V \left[\Gamma_m(k_{mp}r) \cos\left(\frac{n\pi z}{L}\right) \cos(m_s \theta) \right]^2 dV. \quad (2.3.15)$$

The complex close form solution of this integral can be found in reference⁷³.

Using Eq.(2.3.5), the resonant frequency (independent of the orientation order σ) of an nmp mode is given by:

$$\omega_{nmp} = c\sqrt{k_n^2 + k_{mp}^2} \quad (2.3.16)$$

2.3.2 Forced response of an acoustic cavity

The equation governing the behavior of an enclosed fluid⁵⁶ is given by:

$$\nabla^2 p - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = -\frac{1}{\rho} \frac{\partial q}{\partial t}, \quad (2.3.17)$$

where q is the volume velocity source strength density distribution within the volume and on the surface of the enclosure. Using a modal summation the acoustic pressure can be expressed as

$$p(r, \theta, z, \omega) = \sum_{N_a} p_{nmp\sigma}(\omega) \Psi_{nmp\sigma}^a(r, \theta, z) \quad (2.3.18)$$

where $p_{nmp\sigma}$ are the pressure mode amplitudes and N_a the number of modes considered. The mode shapes $\Psi_{nmp\sigma}^a$ derived in the previous section satisfy the homogenous linearized wave equation as

$$\nabla^2 \Psi_{nmp\sigma}^a + \frac{\omega_{nmp}^2}{c^2} \Psi_{nmp\sigma}^a = 0 \quad (2.3.19)$$

Replacing p by its modal series expression in the governing equation yields

$$\sum_{N_a} p_{nmp\sigma} \nabla^2 \Psi_{nmp\sigma}^a + \frac{\omega^2}{c^2} \sum_{N_a} p_{nmp\sigma} \Psi_{nmp\sigma}^a = \frac{i\omega}{\rho} q, \quad (2.3.20)$$

which simplifies by using Eq. (2.3.19) to

$$\sum_{N_a} [\omega_{nmp}^2 - \omega^2] p_{nmp\sigma} \Psi_{nmp\sigma}^a = \frac{-i\omega c^2}{\rho} q \quad (2.3.21)$$

The N_a equations of this system are decoupled in the usual manner

$$\int_V \Psi_{xyz\tau}^a \sum_{N_a} [\omega_{nmp}^2 - \omega^2] p_{nmp\sigma} \Psi_{nmp\sigma}^a dV = \frac{-i\omega c^2}{\rho} \int_V \Psi_{xyz\tau}^a q dV, \quad (2.3.22)$$

by using the orthogonal properties of the mode shapes and their normalization with respect to the cavity's volume. The pressure amplitude of each mode is thus obtained by:

$$V [\omega_{nmp}^2 - \omega^2] p_{nmp\sigma} = \frac{-i\omega c^2}{\rho} \int_V \Psi_{nmp\sigma}^a q dV \quad (2.3.23)$$

The integral term on the right hand side of this equation, which has the dimension of a volume velocity, represents the acoustic forcing of each mode and thus will be defined as the modal source strength $u_{nmp\sigma}$. The acoustic cavity modal impedance is also defined as

$$A_{nmp}^a(\omega) = \frac{-i\omega \rho c^2}{V(\omega_{nmp}^2 - \omega^2 - 2i\zeta_{nmp}\omega_{nmp}\omega)} \quad (2.3.24)$$

where the modal damping ratio ζ_{nmp} is incorporated to model the absorption effect of an acoustic blanket. The pressure mode amplitudes are grouped in a column vector $\mathbf{p}$ given by:

$$\mathbf{p} = \mathbf{A}^a \mathbf{u} , \quad (2.3.25)$$

where $\mathbf{A}^a$ is a $N_a \times N_a$ diagonal matrix regrouping the acoustic modal impedances of the cavity and $\mathbf{u}$ is a column vector regrouping the modal source strengths.

2.4 Structural-acoustic spatial coupling

In this section, the spatial coupling between the external acoustic field and the structure and the spatial coupling between the structure and the acoustic cavity are derived.

2.4.1 External spatial coupling

The external spatial coupling represents the cylinder's structural interaction with the external acoustic disturbance. As a first approximation, the relatively complex acoustic field emanating from a launch vehicle's engine is modeled as an incident oblique plane wave. To simplify even further, the cylinder is embedded in a rigid infinite acoustic baffle as illustrated in Figure 2.5. A traveling plane wave of elevation and azimuth incident angles (α_i, θ_i) is expressed in Cartesian coordinates as

$$P = P_0 e^{i(k_x x + k_y y + k_z z - \omega t)} \quad (2.4.1)$$

where $k_x = k \cos \alpha_i \cos \theta_i$, $k_y = k \cos \alpha_i \sin \theta_i$ and $k_z = k \sin \alpha_i$ are the projections of the wavenumber k onto the three coordinate axis and P_0 is the amplitude of the wave. Using the relation between the Cartesian and cylindrical coordinates $x = r \cos \theta, y = r \sin \theta$ yields

$$P = P_0 e^{i(k_r r \cos(\theta - \theta_i) + k_z z - \omega t)}, \quad (2.4.2)$$

where $k_r = k \cos \alpha_i$ is the projection of the wave number on the polar plane. Using the mathematical relation between the exponential the trigonometric and the Bessel function⁷⁴:

$$e^{iz \cos \theta} = \sum_{m=0}^{\infty} \varepsilon_m i^m \cos(m\theta) J_m(z), \quad (2.4.3)$$

an incident oblique plane wave P_i can be expressed as the product of an axial traveling wave and an infinite sum of cylindrical waves

$$P_i = P_0 e^{i(k_z z - \omega t)} \sum_{m=0}^{\infty} \varepsilon_m i^m \cos[m(\theta - \theta_i)] J_m(k_r r). \quad (2.4.4)$$

However, to obtain the external acoustic pressure it is necessary to account for the diffraction of the incident plane wave by the cylinder. The external pressure $P_{ext}(r, \theta, z, \omega)$ is a combination of the incoming incident wave P_i given by Eq. (2.4.4) and a diffracted outgoing wave P_d . Since the cylinder is assumed embedded in an infinite baffle there is no diffraction in the axial direction and thus the diffracted wave is of the form

$$P_d(r, \theta, z, \omega) = e^{i(k_z z - \omega t)} \sum_{m=0}^{\infty} A_m \cos[m(\theta - \theta_i)] H_m(k_r r), \quad (2.4.5)$$

where, $H_m = J_m + iN_m$, is the outgoing Hankel function of order m .

The amplitudes of each diffracted cylindrical wave, A_m , are derived by applying the boundary condition at the surface of the cylinder, $r=R$. The radial particle velocity U , is obtained using Euler's equation $-\nabla P = \rho \frac{\partial U}{\partial t}$ in the radial direction. This results in

$$U = \frac{1}{i\omega\rho} \frac{\partial}{\partial r} (P_i + P_d). \quad (2.4.6)$$

Assuming hard wall boundary condition, the radial particle velocity must vanish at $r=R$:

$$\left. \frac{\partial}{\partial r} (P_i + P_d) \right|_{r=R} = 0 \quad (2.4.7)$$

This condition leads the following expression for diffracted wave amplitudes

$$A_m = -P_0 \varepsilon_m i^m \frac{J_{m-1}(k_r R) - J_{m+1}(k_r R)}{J_{m-1}(k_r R) - J_{m+1}(k_r R) + i[N_{m-1}(k_r R) - N_{m+1}(k_r R)]}, \quad (2.4.8)$$

which can be rewritten using the notation found in Morse and Ingard⁷⁴ as

$$A_m = -\varepsilon_m P_0 i^{m+1} e^{-i\gamma_m} \sin(\gamma_m), \quad (2.4.9)$$

$$\text{where } \tan(\gamma_m) = \frac{J_{m-1}(k_r R) - J_{m+1}(k_r R)}{N_{m-1}(k_r R) - N_{m+1}(k_r R)} \text{ and } \tan(\gamma_0) = \frac{-J_1(k_r R)}{N_1(k_r R)}.$$

The pressure distribution on the exterior surface of the cylinder P_{ext} is the sum of P_i and P_d , which using Eq. (2.4.5) and (2.4.9) is expressed as

$$P_{ext} = P_0 e^{i(k_z z - \omega t)} \sum_{m=0}^{\infty} \varepsilon_m i^m \cos[m(\theta - \theta_i)] [J_m(k_r R) - i e^{-i\gamma_m} \sin(\gamma_m) H_m(k_r R)]. \quad (2.4.10)$$

The pressure can be expressed using an orientation order σ in the following form

$$P_{ext} = P_0 e^{i(k_z z - \omega t)} \sum_{\sigma=0}^1 \sum_{m=0}^{\infty} \varepsilon_m \cos\left(m\theta - \sigma \frac{\pi}{2}\right) P_{m\sigma}, \quad (2.4.11)$$

where the circumferential amplitude $P_{m\sigma}$, representing the scattering effect around the cylinder is given by:

$$P_{m\sigma} = i^m \left[J_m(k_r R) - i e^{-i\gamma_m} \sin(\gamma_m) H_m(k_r R) \right] \cos\left(m\theta_i - \sigma \frac{\pi}{2}\right). \quad (2.4.12)$$

The spatial coupling between this external field and the structural modes is obtained by the following integral:

$$f_{n_s m_s \sigma_s}^{ext}(\omega, \alpha_i, \theta_i) = \int_S P_{ext}(\omega, \alpha_i, \theta_i) \Psi_{n_s m_s \sigma_s}^s(\theta, z) dS. \quad (2.4.13)$$

$f_{n_s m_s \sigma_s}^{ext}$ represents the force applied to each mode by an oblique plane wave of elevation and azimuth incident angles (α_i, θ_i) . Using Eq. (2.4.10) and the structural modes shapes given in Eq. (2.2.24), this forcing term expands after integration as:

$$f_{n_s m_s \sigma_s}^{ext}(\omega, \alpha_i, \theta_i) = P_0 W_{n_s m_s} \frac{2R\pi e^{-i\omega t}}{\sqrt{\Lambda_{n_s m_s}}} \frac{n_s \pi / L}{k_z^2 - (n_s \pi / L)^2} [(-1)^{n_s} e^{ik_z L} - 1] P_{m_s \sigma_s}(\omega, \alpha_i, \theta_i), \quad (2.4.14)$$

where the circumferential amplitude $P_{m_s \sigma_s}$ is given by Eq. (2.4.12). These external modal forces are grouped into a column vector $\mathbf{f}^{ext}$.

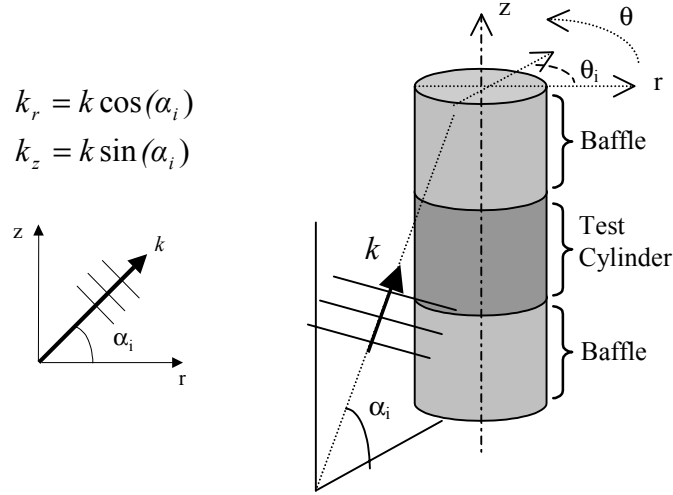


Figure 2.5 Cylinder mounted in an infinite baffle and excited by an acoustic plane wave.

2.4.2 Internal spatial coupling

Because the cylinder is embedded in a rigid baffle the sound is assumed to transmit only through the sidewall. Different spatial variations cause modes in the structure to couple to modes of the cavity with different strengths. Therefore, the coupling coefficient C , between a structural and an acoustic mode, is computed by integration of the product of their shapes over the cylinder surface at $r=R$.

$$C = \int_0^L \int_0^{2\pi} \Psi_{n_s m_s \sigma_s}^s(\theta, z) \Psi_{nmp\sigma}^a(R, \theta, z) R d\theta dz \quad (2.4.15)$$

The 's' subscript signifies that the variable refers to the structure. Since both structural and acoustic mode shapes are adimensional, the coupling coefficients have the dimension of a surface. These coefficients form an $N_a \times N_s$ coupling matrix $\mathbf{C}$, whose elements are the result of the integral in Eq. (2.4.15)

$$C([n, m, p, \sigma], [n_s, m_s, \sigma_s]) = \frac{2RLJ_m(Rk_{mp})}{\epsilon_m \sqrt{\Lambda_{n_s, m_s} \Lambda_{nmp}}} \frac{n_s \left| (-1)^n - (-1)^{n_s} \right|}{(n_s^2 - n^2)} \delta_{m, m_s} \delta_{\sigma, \sigma_s} \cdot \quad (2.4.16)$$

The Kronecker delta symbol δ_{m, m_s} is zero if $m \neq m_s$ and unity if $m = m_s$. To obtain a coupling coefficient other than zero, the circumferential orders m_s and m , as well as the orientation orders, σ_s and σ of the structural and acoustic modes must be equal. Also the axial orders n_s and n , must define an odd-even or even-odd combination. Due to the $(n_s^2 - n^2)$ term in the denominator, modes with greatly different axial mode orders will be poorly coupled. As shown by Gardonio *et al.*²⁵, the properties of the coupling coefficients C can help determining the number of modes required for the simulations. The number of acoustic modes can be reduced to those whose resonant frequencies lie inside a band slightly larger than the one of interest. However, the structural modes that are well coupled to the acoustic modes must be included even though their resonant frequencies lie well outside the band of interest.

The matrix $\mathbf{C}$ represents the link between the structural and acoustic model. It maps the $N_s \times 1$ structural modal velocity vector $\mathbf{v}$, into the $N_a \times 1$ modal acoustic source strength vector $\mathbf{u}$. Reciprocally, it converts the $N_a \times 1$ modal acoustic source pressure vector $\mathbf{p}$ into an $N_s \times 1$ internal modal force vector $\mathbf{f}^{\text{int}}$ representing the back coupling of the acoustic cavity onto the structure

$$\mathbf{u} = \mathbf{C} \mathbf{v} \quad (2.4.17)$$

$$\mathbf{f}^{\text{int}} = -\mathbf{C}^T \mathbf{p}, \quad (2.4.18)$$

where the subscript ‘T’ stands for the matrix transpose operation.

2.5 Implementation of the noise reduction devices

In this section, the modeling of DVA and HR is presented as well as their coupling to the structure and acoustic cavity.

2.5.1 Modeling

A DVA as discussed in section 1.3.3 is considered as a vibration absorber acting uniformly over an area s_d . The reaction force of the DVA induced by the velocity of the structure is a function of the DVA's impedance Z_d :

$$Z_d(\omega) = m_d \frac{i\omega\omega_d^2 + 2\zeta_d\omega^2\omega_d}{(\omega_d^2 - \omega^2) - 2i\zeta_d\omega\omega_d}, \quad (2.5.1)$$

where ω_d is the natural frequency, m_d the mass, and ζ_d the damping ratio of the DVA. The impedance of a DVA is directly proportional to its mass. As mentioned in section 1.3.1, the heavier the DVA the more effect it has on the structural dynamics. The terms $Z_d(\omega)$ are grouped into a $N_d \times N_d$ diagonal DVA impedance matrix $\mathbf{Z}_d$, where N_d is the number of DVAs.

The HR's acoustic admittance is the ratio of the volume velocity to a pressure at the HR's throat. Using the mechanical analog model detailed in section 1.4.1, the HR admittance is expressed as

$$Y_h(\omega) = \frac{s_h i\omega}{\rho l_e [(\omega_h^2 - \omega^2) - 2i\zeta_h\omega\omega_h]}, \quad (2.5.2)$$

where V_h , s_h and l_h are the volume, throat's cross section and neck's length of the resonator respectively. The HR resonant frequency is $\omega_h = c\sqrt{s_h/V_h l_e}$. Since the external

radiation loading is accounted for by the summation of the acoustic modes at the HR's throat, only the interior radiation mass effect is included and so $l_e = l_h + 0.85\sqrt{s_h/\pi}$ for a square necked resonator⁵⁰. Using the resonant frequency expression in Eq. (2.5.2), yields

$$Y_h(\omega) = V_h \frac{i\omega\omega_h^2}{\rho c^2[(\omega_h^2 - \omega^2) - 2i\zeta_h\omega\omega_h]} \quad (2.5.3)$$

This expression shows that for given resonant frequency and a damping ratio, the more voluminous the HR is the more effect it has on the cavity response. The admittance terms $Y_h(\omega)$ are grouped in a $N_h \times N_h$ diagonal HR admittance matrix $\mathbf{Y}_h$ where N_h is the number of HR.

2.5.2 Spatial coupling

The coupling between a DVA and the cylinder is obtained by integrating each structural mode shape over the DVA rectangular surface of attachment $s_d = b \times a$ at its desired location (θ_0, z_0) on the cylinder. The contact surface between the cylinder and the DVA is assumed to be flat. Normalized by s_d , the dimensionless coupling coefficients $\phi_{n_s m_s \sigma_s}^s$ are given by

$$\phi_{n_s m_s \sigma_s}^s(\theta_0, z_0, s_d) = \frac{1}{s_d} \int_{z_0-b/2}^{z_0+b/2} \int_{\theta_0-\Delta\theta}^{\theta_0+\Delta\theta} \psi_{n_s m_s \sigma_s}^s(\theta, z) R d\theta dz, \quad (2.5.4)$$

where $\Delta\theta = \sin^{-1}(a/2R)$. The structural modes in the mass layer of the DVA itself are not taken into account; it is assumed that the DVA applies a uniformly distributed normal force on the cylinder. The fully populated $N_d \times N_s$ matrix Φ^s is defined and it couples N_d DVAs to N_s structural modes.

In a similar manner, Φ^a is the fully populated $N_h \times N_a$ matrix, which couples N_h HRs to N_a acoustic modes. The elements of Φ^a are given by

$$\phi_{nmp\sigma}^a(\theta_0, z_0, s_h) = \frac{1}{s_h} \int_{z_0-a/2}^{z_0+a/2} \int_{\theta_0-\Delta\theta}^{\theta_0+\Delta\theta} \Psi_{nmp\sigma}^a(R, \theta, z) R d\theta dz, \quad (2.5.5)$$

where $s_h = a \times a$ is the HR's throat area and (r_0, θ_0, z_0) the HR's location in the cylinder. The velocity distribution is assumed uniform over the throat cross-section leading the HRs to couple like piston sources. Thus, the matrix Φ^a can also be used to couple an internal speaker to the acoustic cavity. In the simulations the resonators are assumed to lie outside of the cylinder and to couple at the circumference ($r_0=R$) as shown in Figure 2.6. This is considered to be comparable to placing HRs inside the cavity as long as the HR dimensions are small compared to the acoustic wavelength and their volume is much smaller than the cavity's volume.

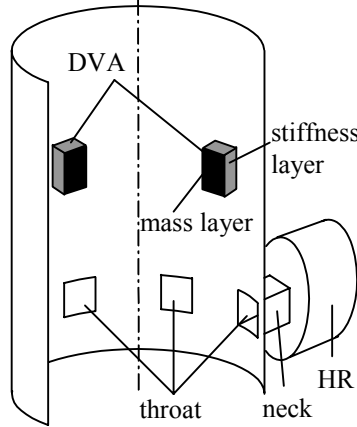


Figure 2.6 Coupling between the cylinder and the noise reduction devices: HRs and DVAs

2.6 Fully coupled system response

The coupling between all the components of the system is achieved using an impedance matching method. Therefore, the modal force $\mathbf{f}^{\text{dva}}$ exerted on the structure by the DVAs is expressed as a function of the matrix Φ^s , its transpose Φ^{sT} , the diagonal matrix $\mathbf{Z}_d$, and the structural modal vibration vector $\mathbf{v}$:

$$\mathbf{f}^{\text{dva}} = \Phi^{sT} \mathbf{Z}_d \Phi^s \mathbf{v}. \quad (2.6.1)$$

Assuming the velocity distribution in the throat of the HR to be uniform over the surface s_h , HRs act as acoustic piston sources. Therefore, the total acoustic modal source strength of the coupled system $\mathbf{u}$ is the sum of two quantities, $\mathbf{u}^h$ and $\mathbf{u}^s$. The modal source strength produced by the HR, $\mathbf{u}^h$, is a function of the acoustic modal pressure $\mathbf{p}$ and the acoustic modal source strength due to the structure $\mathbf{u}^s$ given by Eq.(2.4.17) is a function of $\mathbf{v}$,

$$\mathbf{u} = \mathbf{u}^h + \mathbf{u}^s = \Phi^{aT} \mathbf{Y}_h \Phi^a \mathbf{p} + \mathbf{C} \mathbf{v}. \quad (2.6.2)$$

Using Eq.(2.3.25), the acoustic modal pressure vector $\mathbf{p}$ due to the total acoustic source strength $\mathbf{u}$ becomes

$$\mathbf{p} = \mathbf{A}^a (\Phi^{aT} \mathbf{Y}_h \Phi^a \mathbf{p} + \mathbf{C} \mathbf{v}). \quad (2.6.3)$$

The total force $\mathbf{f}$ exciting the cylinder is the sum of the external acoustic force $\mathbf{f}^{\text{ext}}$, the internal acoustic force $\mathbf{f}^{\text{int}}$ given by Eq.(2.4.18), and the reacting force of DVAs $\mathbf{f}^{\text{dva}}$ given by Eq.(2.6.1). Expanding the vector $\mathbf{f}$ in Eq.(2.2.59) into these three components, the structural modal velocity vector $\mathbf{v}$ becomes:

$$\mathbf{v} = \left[(\mathbf{A}^s)^{-1} + \mathbf{Z}^r \right]^{-1} (\mathbf{f}^{\text{ext}} - \mathbf{C}^T \mathbf{p} + \Phi^{sT} \mathbf{Z}_d \Phi^s \mathbf{v}). \quad (2.6.4)$$

Eqs.(2.6.3) and (2.6.4) the two coupled matrix equations defining the behavior of the fully coupled system which is illustrated in block diagram form in Figure 2.7. This system is composed of four feedback loops. The first two (green and blue) acts on the structural velocity through the radiation impedance of the external fluid and the DVAs impedance. A third (red) modifies the acoustic pressure through the HRs admittance. Finally the last loop (pink) couples the structure and the cavity through the structural-acoustic coupling matrix C . Solving this system of two equations yields $\mathbf{v}$ and $\mathbf{p}$ as a function of the external acoustic modal force $\mathbf{f}^{\text{ext}}$,

$$\mathbf{v} = \left[\left[\mathbf{A}^s \right]^{-1} + \mathbf{Z}^r + \mathbf{C}^T \left[\left[\mathbf{A}^a \right]^{-1} - \Phi^{aT} \mathbf{Y}_h \Phi^a \right]^{-1} \mathbf{C} - \Phi^{sT} \mathbf{Z}_d \Phi^s \right]^{-1} \mathbf{f}^{\text{ext}} \quad (2.6.5)$$

$$\mathbf{p} = \left[\left[\mathbf{A}^a \right]^{-1} - \Phi^{aT} \mathbf{Y}_h \Phi^a \right]^{-1} \mathbf{C} \left[\left[\mathbf{A}^s \right]^{-1} + \mathbf{Z}^r + \mathbf{C}^T \left[\left[\mathbf{A}^a \right]^{-1} - \Phi^{aT} \mathbf{Y}_h \Phi^a \right]^{-1} \mathbf{C} - \Phi^{sT} \mathbf{Z}_d \Phi^s \right]^{-1} \mathbf{f}^{\text{ext}}. \quad (2.6.6)$$

The solution for the untreated system, that is without DVAs and HRs, is given by:

$$\mathbf{v} = \left[\left[\mathbf{A}^s \right]^{-1} + \mathbf{Z}^r + \mathbf{C}^T \mathbf{A}^a \mathbf{C} \right]^{-1} \mathbf{f}^{\text{ext}} \quad (2.6.7)$$

$$\mathbf{p} = \mathbf{A}^a \mathbf{C} \left[\left[\mathbf{A}^s \right]^{-1} + \mathbf{Z}^r + \mathbf{C}^T \mathbf{A}^a \mathbf{C} \right]^{-1} \mathbf{f}^{\text{ext}}. \quad (2.6.8)$$

If the system is only excited by an acoustic source placed inside the cavity ($\mathbf{f}^{\text{ext}}=0$), the equations for the untreated cylinder becomes:

$$\mathbf{p} = \left[\left[\mathbf{A}^a \right]^{-1} + \mathbf{C} \left[\left[\mathbf{A}^s \right]^{-1} + \mathbf{Z}^r \right]^{-1} \mathbf{C}^T \right]^{-1} \mathbf{u}^{\text{int}} \quad (2.6.9)$$

$$\mathbf{v} = - \left[\left[\mathbf{A}^s \right]^{-1} + \mathbf{Z}^r \right]^{-1} \mathbf{C}^T \left[\left[\mathbf{A}^a \right]^{-1} + \mathbf{C} \left[\left[\mathbf{A}^s \right]^{-1} + \mathbf{Z}^r \right]^{-1} \mathbf{C}^T \right]^{-1} \mathbf{u}^{\text{int}}, \quad (2.6.10)$$

where $\mathbf{u}^{\text{int}}$ is the modal source strength vector of the internal acoustic source.

In order to obtain an average sound pressure level independent of the location inside the cylinder, the total time average acoustic potential energy E_p is computed as

$$E_p(\omega) = \frac{1}{4c^2\rho} \iiint_V |p(\omega, r, \theta, z)|^2 dV. \quad (2.6.11)$$

Using the orthonormal properties of the modes allows the acoustic potential energy⁷⁵ to be computed using $\mathbf{p}$ and its Hermitian transpose $\mathbf{p}^H$:

$$E_p(\omega, \alpha_i, \theta_i) = \frac{V}{4\rho c^2} \sum_{N_a} |p_{nmp\sigma}(\omega, \alpha_i, \theta_i)|^2 = \frac{V}{4\rho c^2} \mathbf{p}^H \mathbf{p}. \quad (2.6.12)$$

Similarly the total structural kinetic energy is used as an indicator of the average vibration level of the cylindrical structure. Due to the orthogonality and normalization of the structural modes, the total structural kinetic energy E_k is given by

$$E_k(\omega, \alpha_i, \theta_i) = \frac{1}{2} M \sum_{N_s} [v_{n,m,\sigma_s}(\omega, \alpha_i, \theta_i)]^2 = \frac{1}{2} M \mathbf{v}^H \mathbf{v}. \quad (2.6.13)$$

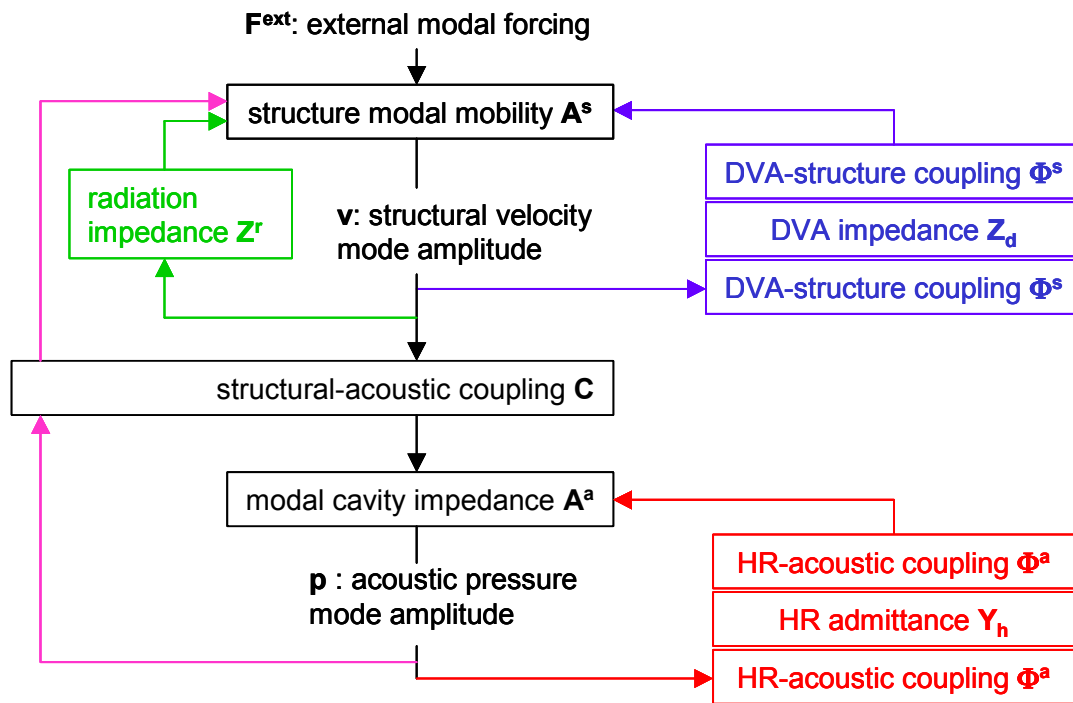


Figure 2.7 Block diagram of the fully coupled system

2.7 Model using finite element outputs

In order to evaluate the performance of distributed vibration absorbers and Helmholtz resonators in reducing the sound transmission through a real fairing, the analytical model described in the previous sections was modified to work with the outputs of a finite element model. The Boeing finite element model respectively computed the first 1109 structural modes and 293 acoustic modes with natural frequencies up to 348Hz and 219Hz. These were then exported from Nastran into Matlab.

2.7.1 Conversion of the components in discrete form

Each structural mode shape is defined by the 3-degrees-of-freedom expressed in Cartesian coordinates at the nodes of the meshed surface. Each set of 4 nodes defines an element, its area was computed using the cross product, and the modal displacement normal to the center of the element was approximated from the 3 degrees-of-freedom of the 4 nodes. The discrete structural mode shapes are represented by $\Psi_m^s(i)$ where the subscript m denotes the mode number and i is the element number. Since the structural mesh and acoustic mesh did not coincide at the surface, the modal pressures at the node the closest to the center of each structural element were retained to describe the acoustic mode shape. These mode shapes are represented by $\Psi_n^a(i)$ where the subscript n is the mode number and i the element number. The coupling coefficient between the n^{th} acoustic mode and the m^{th} structure mode is obtained by discrete integration over the 1728 elements of area s .

$$C(n, m) = \sum_{i=1}^{1728} \Psi_n^a(i) \cdot \Psi_m^s(i) s(i). \quad (2.7.1)$$

The component of the DVA and HR spatial coupling matrix Φ^s and Φ^a were defined as the values of the acoustic and structure mode shapes at the HRs and the DVAs locations respectively.

2.7.2 External excitation numerical model

The acoustic excitation is obtained by an equivalent sources method. This method based on the principle of wave superposition was introduced as an alternative to boundary element method⁷⁶ and allows computing the acoustic radiation and scattering from objects in a free-field with a reduced computational load. It was extended later, to compute the sound field inside an enclosure containing scattering objects⁷⁷. For the

acoustic scattering case, the pressure and normal particle velocity on the surface of a scattering object can be calculated by combining the external incident field with a field created by an array of sources placed inside the object. The amplitudes of the internal equivalent sources are derived to match a given boundary condition at the object surface. Assuming hard wall boundary condition, the normal particle velocity $\mathbf{v}$ at the surface of the fairing has to vanish.

$$\mathbf{v} = \mathbf{v}_p + \mathbf{v}_e = \mathbf{0}. \quad (2.7.2)$$

The vector $\mathbf{v}_p$ and $\mathbf{v}_e$ represent the normal particle velocity at each element of the fairing's meshed surface due to the primary sources and due to the internal equivalent sources respectively. Assuming the sources radiate as monopoles with source strengths $\mathbf{u}_p$ and $\mathbf{u}_e$, $\mathbf{v}_p$ and $\mathbf{v}_e$ are obtained by

$$\begin{aligned} \mathbf{v}_p &= \mathbf{T}_p \mathbf{u}_p \\ \mathbf{v}_e &= \mathbf{T}_e \mathbf{u}_e \end{aligned} \quad (2.7.3)$$

The element of the matrices $\mathbf{T}_p$ and $\mathbf{T}_e$ are obtained using the free-field Green's function G as

$$\mathbf{T}_s(n, m) = \frac{\partial G(\mathbf{x}_{0m}, \mathbf{x}_{sn})}{\partial \mathbf{n}_{0m}} = \frac{\partial}{\partial \mathbf{n}_{0m}} \left[\frac{1}{4\pi |\mathbf{x}_{0m} - \mathbf{x}_{sn}|} e^{ik|\mathbf{x}_{0m} - \mathbf{x}_{sn}|} \right], \quad (2.7.4)$$

where $\mathbf{x}_{0m}$ is the position vector of the m^{th} evaluation point, $\mathbf{n}_{0m}$ the normal to the surface at the m^{th} evaluation point and $\mathbf{x}_{sn}$ is the position vector of the n^{th} primary sources for $\mathbf{T}_p$ or of the n^{th} equivalent sources for $\mathbf{T}_e$. As mentioned by Johnson *et al.*⁷⁷, if $\mathbf{T}_e$ is a square matrix, that is, there is an equal number of equivalent sources and evaluation positions, the system is fully determined. This leads to the exact fulfillment of the boundary condition at the evaluation points but can result in large discrepancies between the evaluation positions. Therefore, an over-determined system using more evaluation

positions than equivalent sources can be used resulting in a least square approximation of the boundary condition and a more stable solution between evaluation points. The equivalent source strengths are expressed as

$$\mathbf{u}_e = -[\mathbf{T}_e^H \mathbf{T}_e]^{-1} \mathbf{T}_e^H \mathbf{v}_p, \quad (2.7.5)$$

where the associated quadratic error is

$$e = \frac{\mathbf{v}^H \mathbf{v}}{\mathbf{v}_p^H \mathbf{v}_p} \quad (2.7.6)$$

The acoustic pressure at the evaluation point is given by:

$$\mathbf{p} = \mathbf{p}_p + \mathbf{p}_e = \mathbf{Z}_p \mathbf{u}_p + \mathbf{Z}_e \mathbf{u}_e. \quad (2.7.7)$$

The elements of the impedance matrices $\mathbf{Z}_p$ and $\mathbf{Z}_e$ are obtained using the Green's function

$$\mathbf{Z}_s(n, m) = \frac{-i\omega\rho}{4\pi|\mathbf{x}_{0m} - \mathbf{x}_{sn}|} e^{ik|\mathbf{x}_{0m} - \mathbf{x}_{sn}|}, \quad (2.7.8)$$

where $\mathbf{x}_{0m}$ is the position vector of the m^{th} evaluation point, and $\mathbf{x}_{sn}$ is the position vector of the n^{th} primary sources for $\mathbf{Z}_p$ or of the n^{th} equivalent sources for $\mathbf{Z}_e$. To avoid singularities in the problem corresponding to the resonant frequency of the enclosure and of the scattering object's cavity at which the acoustic response becomes infinite, damping is introduced into the model by using a complex wave number k' in the free field Green's function:

$$k' = k(1 + i\beta), \quad (2.7.9)$$

where $0.01 \leq \beta \leq 0.03$ is the damping ratio of the room.

In this particular study, a half free-field is modeled to approximate an outdoor acoustic excitation with reflection from the ground. In this configuration, the evaluation points are only placed on the fairing's surface, the equivalent sources are positioned inside the fairing's volume and the hard wall condition on the horizontal plane (ground) is obtained by mirror imaging of the primary and equivalent sources. The number and positions of equivalent sources is function of the excitation frequency as is the evaluation point's spacing, which is always inferior or equal to a quarter of the acoustic wavelength. Figure 2.8 shows the position of the evaluation points, disturbance speaker, and the equivalent sources.

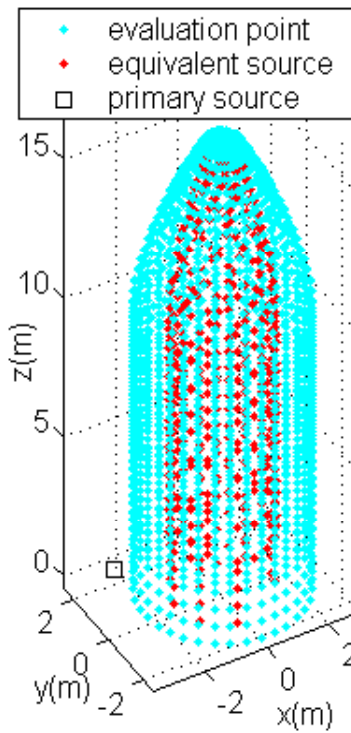


Figure 2.8 Location of evaluation points, equivalent sources and primary source used to model an outdoor acoustic excitation.

Once the external pressure $P^{ext}(\omega, i)$ at the each of fairing's surface element is computed, the external forcing of the m^{th} structural mode is then obtained by discrete integration as

$$f_m^{ext}(\omega) = \sum_{i=1}^{1728} P^{ext}(\omega, i) \Psi_m^s(i) s(i) . \quad (2.7.10)$$

This method can also be used for the cylinder model in order to get a more realistic excitation model, as the plane wave excitation is very difficult to recreate experimentally.

2.8 Conclusions

In this chapter, an analytical model of the noise transmission into an enclosed cylinder was developed. The disturbance consists in an oblique plane wave, the cylinder is embedded in an infinite baffle and the acoustic cavity is either cylindrical or annular. The effect of the external fluid loading on the structure is also taken into account. Vibration absorbers and Helmholtz resonators can be placed anywhere on the cylinder and the fully coupled system structural and acoustic response can be obtained to evaluate the performance of noise reduction treatment.

This analytical model is then modified to work with numerical mode shapes and natural frequencies obtained from a finite element model of a real fairing. The different elements of this model are derived using discrete instead of analytical integrals and the acoustic forcing from monopole sources in a half-free field is obtained using a numerical equivalent source method. All the elements are however fully coupled in the same matrix form as for the analytical cylinder model.

This technique assesses the performance of a HR and DVA treatment on a real fairing without having to model the devices in a finite element model, which represents a much more time consuming procedure. The model is therefore a very useful tool to investigate and optimize a HR and DVA noise reduction treatment.

Chapter 3 Numerical simulations

3.1 Introduction

The analytical model of sound transmission into a simply supported cylinder and into a fairing described with finite element, were developed in the previous chapter. These two models were coded using Matlab and the resulting numerical simulations are presented in this chapter. The different elements of the models were analyzed to explain the noise transmission mechanisms. HR and DVA prototypes were built and experiments conducted on the composite cylinder to validate the model.

After highlighting the noise reduction mechanisms, a design procedure for the HR and DVA treatment is presented. For both models, the treatment performance is evaluated and compared to the solution obtained with a genetic algorithm.

Finally, the robustness of the treatment to changes in the excitation, system and treatment characteristics is addressed.

3.2 Noise transmission mechanisms

In this section, the noise transmission mechanisms for the cylinder and the fairing are explained and illustrated. The validation of the numerical model with experimental results is also presented.

3.2.1. External coupling

In order to understand the cylinder's vibration response it is necessary to analyze the external coupling between an incident plane wave and the structural modes. The modal force $f_{n_s m_s}^{ext}$, expressed by Eq. (2.4.14), represents this coupling:

$$f_{n_s m_s}^{ext}(\omega, \alpha_i, \theta_i) = P_0 W_{n_s m_s} \frac{2R\pi e^{-i\omega t}}{\sqrt{\Lambda_{n_s m_s}}} \frac{n_s \pi / L}{k_z^2 - (n_s \pi / L)^2} [(-1)^{n_s} e^{ik_z L} - 1] P_{m_s \sigma_s}(\omega, \alpha_i, \theta_i).$$

The circumferential amplitude $P_{m_s \sigma_s}$ given by Eq. (2.4.12) denotes the scattering of the wave by the cylinder. It is plotted in Figure 3.1 for different m_s as a function of the normalized horizontal wave number in air, $k_r R$, obtained with an elevation angle α_i of 70° . The corresponding frequency is also plotted as a dotted line with values on the right hand y-axis. This amplitude $P_{m_s \sigma_s}$ behaves like a “high-pass filter” whose cut-on frequency increases with the circumferential mode order m_s , except for the breathing mode $m_s=0$ which has a maximum value at 0 Hz. From Figure 3.1, one can conclude that for this particular elevation angle, below 200Hz, ($k_r R < 1.5$), only the modes with circumferential order $m_s \leq 3$ are well excited. The axial component represented by the term $\frac{n_s \pi / L}{k_z^2 - (n_s \pi / L)^2} [(-1)^{n_s} e^{ik_z L} - 1]$ is the Fourier wavenumber transform of the axial mode

shape $\sin(n_s \pi / L)$. It represents the spatial coupling between the axial wavenumber in air,

$k_z = k \sin \alpha_i$, and the axial modal wave number, $k_{n_s} = n_s \pi / L$, in the cylinder. This coupling is therefore characterized by a main lobe near the coincidence frequency between k_z and k_{n_s} and side lobes of decaying amplitude. However, for $n_s=1$, the axial modal wave number k_{n_s} represents only half of a wave along the length of the cylinder and so has its main lobe at $k_z=0$. This axial component of $f_{n_s m_s}^{ext}$ is plotted in Figure 3.2 as a function of

the normalized axial wavenumber in air for $\alpha_i=70^\circ$. Therefore, below 200Hz, $(k_z L/\pi < 3)$, only modes with axial order $n_s \leq 4$ are well coupled to the incident field.

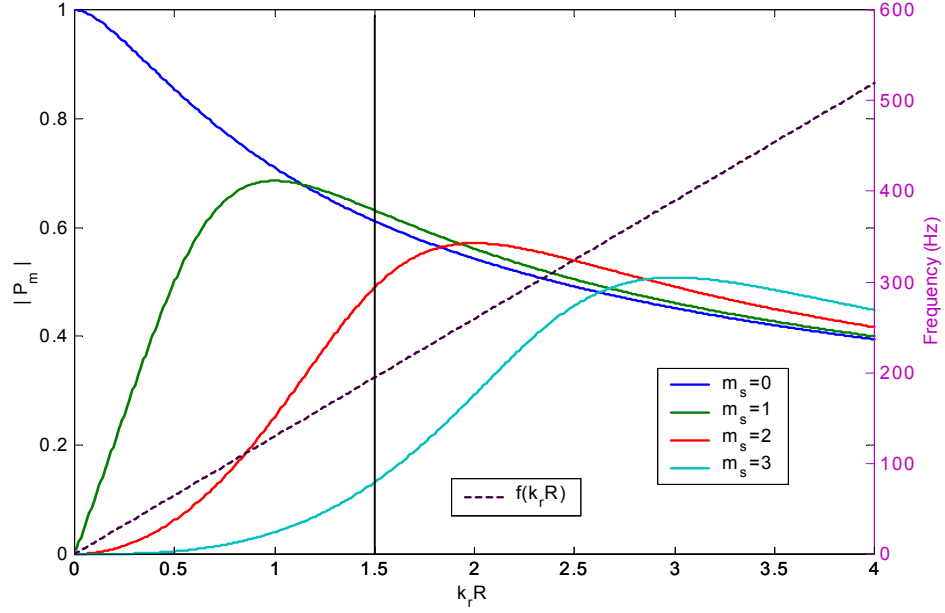


Figure 3.1 Magnitude of $P_{m_s \sigma_s}$ due to an incident plane wave of 1Pa. ($\alpha_i=70^\circ, \theta_i=0^\circ$) and frequency as a function of the normalized horizontal wave number in air $k_r R$ for $m_s=0,1,2,3$.

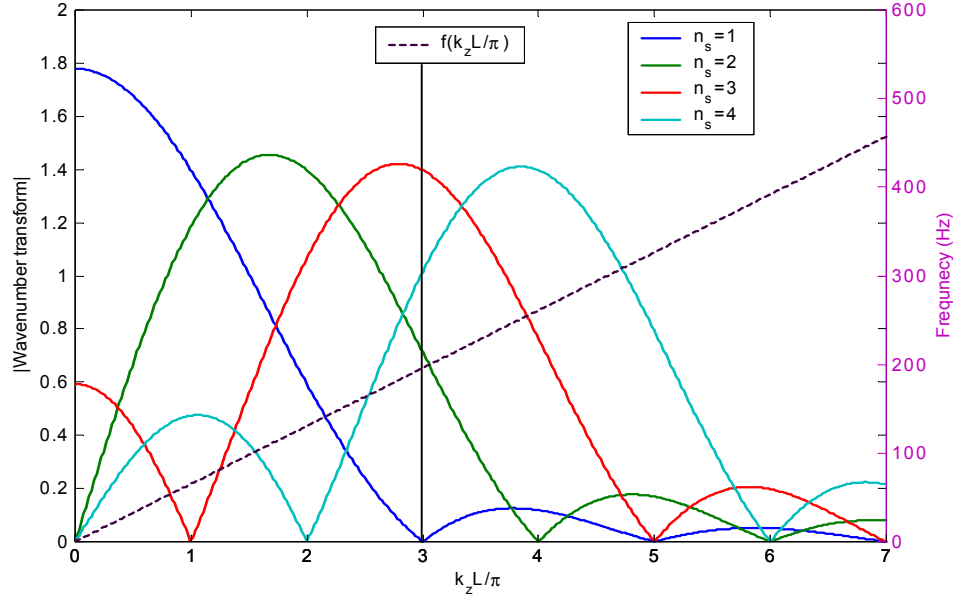


Figure 3.2 Magnitude of the axial mode shape's wavenumber transform and frequency as a function of the normalized axial wave number in air for $n_s=1,2,3,4$.

The “in vacuo” vibration response of the cylinder is obtained using Eq. (2.2.26). In the case of an excitation by an oblique plane wave, the forcing vector $\mathbf{f}$ is given by Eq. (2.4.13) in the case of a point force excitation, the vector $\mathbf{f}$ is derived using Eq. (2.2.19) where the distributed force is given by a spatial Dirac delta function at the application point (θ_0, z_0) yielding:

$$f_{n_s m_s \sigma_s} = \int_S \Psi_{n_s m_s \sigma_s}^s \delta(\theta_0, z_0) dS = \Psi_{n_s m_s \sigma_s}^s(\theta_0, z_0) \quad (3.2.1)$$

Using Eq.(2.6.13), the structural kinetic energy of the cylinder is plotted in Figure 3.3 for 3 different elevation angles of the incident wave as well as for a point force exciting at $\theta_0=0$ and $z_0=0.4$ meter. The structural damping ratio is set to 1%, which is an estimated value commonly used for such large composite structures²⁵.

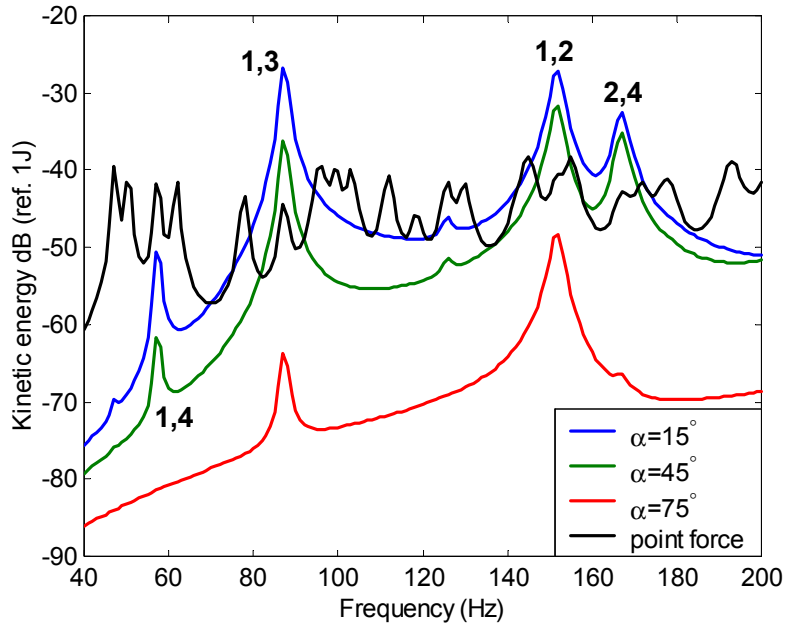


Figure 3.3 In vacuo response of the cylinder structural kinetic energy as function of frequency due a plane wave excitation of elevation angle α and due to a point force excitation.

Although 24 modes resonate below 200Hz (see Table 3.1) as the point force response illustrates, the incident oblique plane wave responses demonstrate the spatial filtering of the higher circumferential modes due to the external structure-acoustic coupling. As the elevation angle increase from 0 to 90 corresponding to a purely horizontal wavenumber to a purely vertical wavenumber, this filtering effect intensifies as shown by Figure 3.3.

Axial order n	Circumferential order m	Natural frequency (Hz)
1	5	47.3
1	6	50.4
1	4	57.4
1	7	61.7
1	8	77.7
1	3	87.3
2	7	95.5
1	9	96.8
2	8	99.5
2	6	103.2
2	9	111.9
1	10	118.5
2	5	125.7
2	10	129.8
3	8	144.5
3	9	145.4
1	2	151.7
3	7	154.9
3	10	155.2
2	4	166.9
3	6	178.3
4	9	192.1
4	10	193.5
4	8	200.7

Table 3.1 Structural resonant frequency of the Boeing cylinder below 200Hz

3.2.2. Internal coupling

The structural acoustic coupling between bending waves on a plate and an acoustic field has been extensively studied^{78,79}. Typically, at low frequencies the bending waves on a thin plate are slower than the acoustic wave speed (subsonic) and therefore do not efficiently radiate sound. Only the boundaries, which make the plate finite, allow the structure to couple effectively into the fluid in this subsonic frequency range. These boundaries permits the aliasing of the high wavenumbers under the form of side lobes into the low wavenumber region, as illustrated in Figure 3.2. This effect is also true of a cylinder where the boundaries are largely responsible for coupling the structural vibration to the acoustic cavity. Cylinders present an additional effect due to their curvature, which couples the bending and the in-plane responses. Since the structure is very stiff in the “in-

plane” direction, this coupling, effectively speeds up the low order circumferential modes that are the most coupled with the in-plane motion. This phenomenon, sometimes called the membrane effect, can cause “supersonic” behavior at lower frequencies than would be expected from a bending wave plate model. This is illustrated in Figure 3.4 where the structure wavenumbers $k_{nm} = \sqrt{\left(\frac{n\pi}{L}\right)^2 + \left(\frac{m}{R}\right)^2}$ are plotted as a function of their natural frequencies $f_{nm} = \omega_{nm}/2\pi$ and linked in n groups. The dispersion curves for the equivalent sandwich plate without core shear $k_b(\omega)$ and for the air $k(\omega)$ are also plotted. As expected, below the coincidence frequency ($k_b=k$), the bending response of the plate is sub-sonic ($k_b>k$) whereas the low circumferential order modes of the cylinder ($m<5$) present a supersonic behavior ($k_{nm}<k$) due to the membrane effect. This effect is the highest for the breathing modes $m=0$ and thus forces the first 10 breathing modes to present a very similar natural frequency. Note that the equivalent sandwich plate appears stiffer than the cylinder at high frequency due to neglecting the core shear⁷⁸.

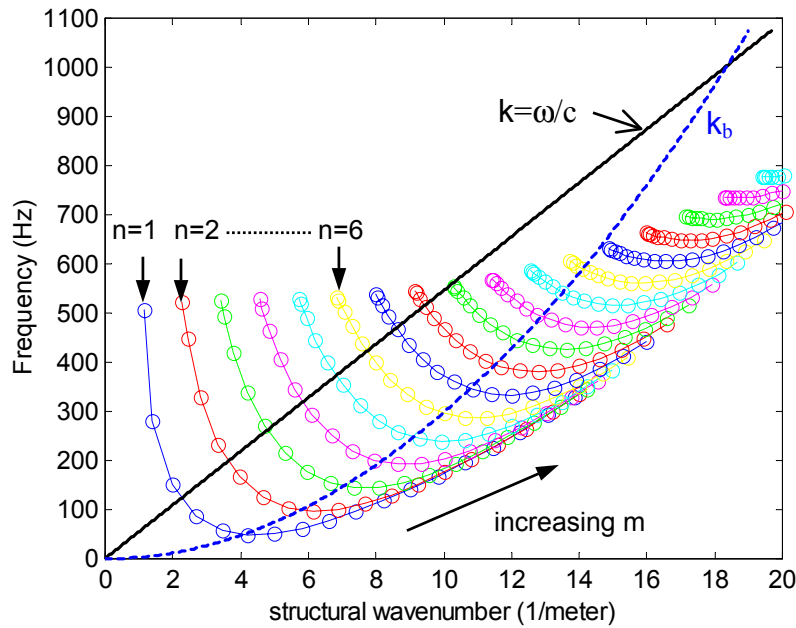


Figure 3.4 Dispersion curves for the Boeing cylinder in ‘ n ’ groups and for the equivalent infinite sandwich plate without core shear.

The level of coupling between the structural mode of the cylinder and the acoustic mode of its cavity is both spatial and frequency related. The spatial level is represented by the coefficient of the coupling matrix $\mathbf{C}$ derived in Eq.(2.4.16). In order to visualize both levels at the same time, Figure 3.5 shows two contour plots, one for the acoustic modes and one for the structural modes, superimposed on each other as a function of their axial and circumferential orders (note that only $p=0$ acoustic modes are plotted). The contours represent the resonant frequencies for each of the modes, blue for acoustic modes and red for structural modes. The modes only occur on the integer values of the plot. Acoustic modes with higher orders have higher natural frequencies but for structural modes, because of the membrane effect, this relationship between frequency and mode order is distorted. The lowest natural frequencies are for $m=5$ to $m=6$ (lower right-hand corner). As mentioned in section 2.4.2, the spatial coupling between acoustic and structural modes is such that only like circumferential mode orders (m) couple and in the axial direction (n) odd modes on the structure couple to even modes in the cylinder (and visa-versa). Therefore each intersection on the grid represents a non-zero coupling between two modes. The black line on the plots shows a line of equal natural frequency of the structural and acoustic modes. Therefore the proximity of the grid points to this line expresses the similarity in natural frequency of two spatially well-coupled modes. The red dot on the plot highlight the cases where both high spatial and frequency coupling occurs which results in large sound transmission. The problematic interactions, are between the (1,2) structural mode and the (0,2,0) acoustic mode around 140Hz, between the (2,3) and the (1,3,0) around 200Hz and between the (3,4) and the (2,4,0) around 260Hz. This plot can be repeated for acoustic modes with a radial component (i.e. $p=1,2,3,\dots$) to form new frequency coincidence lines. The main result is that the majority of coupling will come from modes that lie near to or to the left of the black line shown in Figure 3.5.

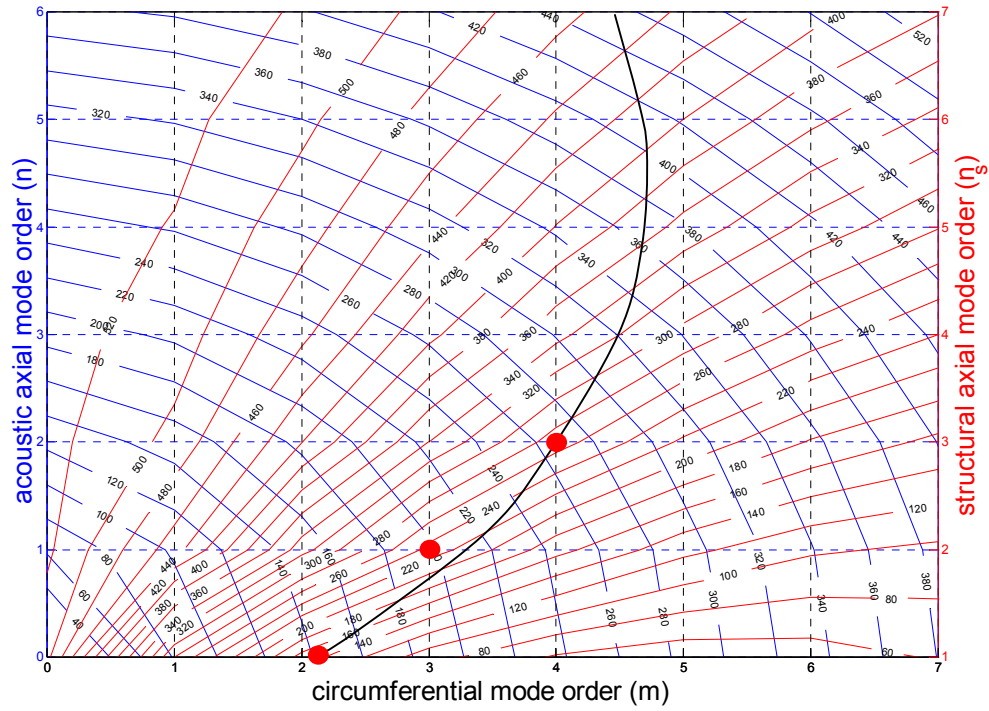


Figure 3.5 Contours of the structural and acoustic resonant frequencies as a function of their mode orders.

The vibration and acoustic responses of the fully coupled system neglecting the external fluid loading is obtained using Eq.(2.6.7) and Eq.(2.6.8) with $\mathbf{Z}^f=0$. Figure 3.6 shows the structural kinetic and acoustic potential energy of the cylinder excited by an oblique plane wave ($\alpha_i=70^\circ$). The acoustic damping is set to 1%. As pointed out by the mode orders, the internal acoustic response as well as the vibration response, are both composed of coupled acoustic resonances and structural resonances. Below 200Hz the main transmission path is due to the high structural-acoustic coupling between the (1,2) that resonates at 151Hz and (0,2,0) that resonates at 135Hz. Note that these two resonant frequencies are pushed up and down respectively as the two modes interact. Figure 3.7 shows the structural kinetic and acoustic energy of the cylinder in the case of an annular cavity of internal radius $R_i=3$ ft. As the 0,2,0 resonant frequency is lowered to 102 Hz, the coupling with the (1,2) structural mode is weaker and thus less energy is transmitted.

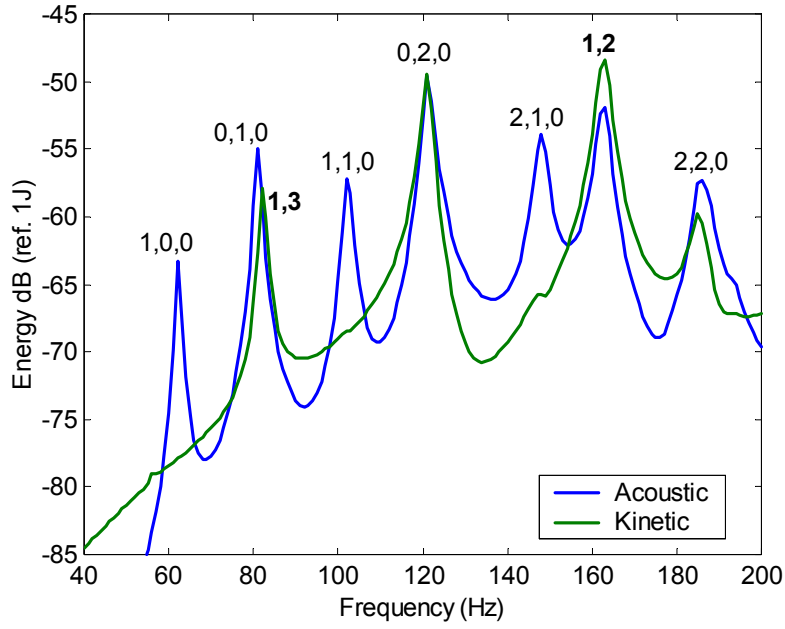


Figure 3.6 Structural kinetic energy and acoustic potential energy of an empty cylinder excited by an oblique plane wave ($\alpha_i=70^\circ$).

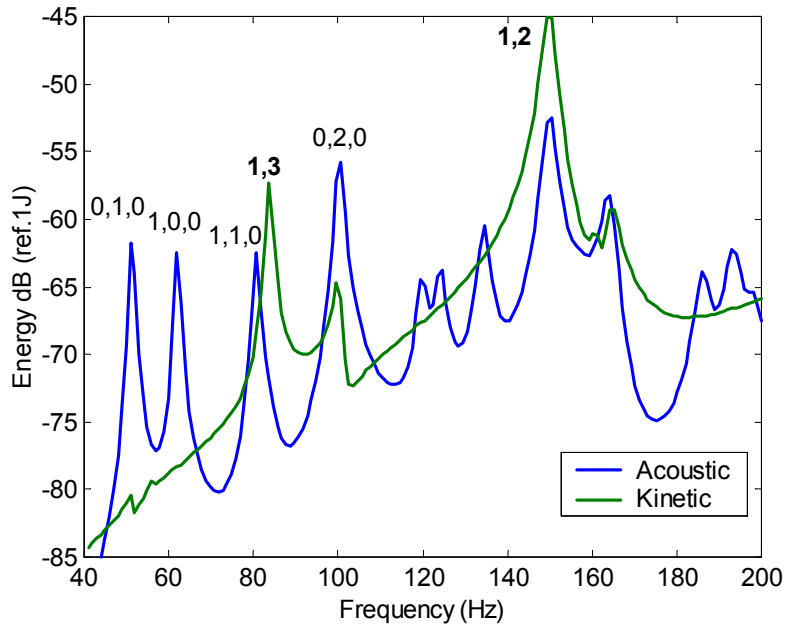


Figure 3.7 Structural kinetic energy and acoustic potential energy of an annular cylinder excited by an oblique plane wave ($\alpha_i=70^\circ$).

3.2.3. Fluid loading effect

As detailed in section 2.2.4, the surrounding fluid couples the structural mode of same circumferential order through the radiation impedance matrix $\mathbf{Z}^r$. Neglecting this cross coupling by only considering the diagonal term of $\mathbf{Z}^r$, reveals the fluid loading effect on each mode. This effect is two fold: the real part of the radiation impedance acts as a damping term as energy is lost by acoustic radiation, whereas the imaginary part is mass-like in the low frequencies. The additional damping ratio ξ^{rad} can be approximated for each mode by

$$\xi_{nm}^{rad} = \frac{\text{Re}[Z_{nnm}^r(\omega_{nm})]}{2M\omega_{nm}}. \quad (3.2.2)$$

The additional mass, m^{rad} , which lowers the mode natural frequencies, is obtained by evaluating the slope of the radiation impedance at the low frequencies as

$$m_{nm}^{rad} = \left. \frac{-d \text{Im}(Z_{nnm}^r)}{d\omega} \right|_{\omega=0}. \quad (3.2.3)$$

As the radiation impedance is linked to the coupling with the wavenumber in air it depends on the vibration pattern of the cylinder (i.e the structural mode shapes) and on the frequency. Figure 3.8 shows the real and imaginary part of the normalized radiation impedance for three low order structural modes. The scaled modal mobilities indicating the mode natural frequencies are superimposed in dotted lines. The fluid loading effect appears more important for the lower order circumferential modes. Table 3.2 lists the value of the additional radiation damping ratio and the radiation mass in percent of the mass of the cylinder for different modes.

Mode order		Approximated radiation impedance effect	
Axial (n)	Circumferential (m)	Damping ratio (%)	Additional mass (% of cylinder mass)
1	4	0.0	9.5
1	3	1.1	11.5
1	2	6.0	14.0
2	4	2.6	8.2
2	3	5.3	9.6
1	1	2.1	16.2
2	2	2.9	11.3
2	1	2.0	13.8
1	0	1.6	36.8
2	0	1.8	16.4

Table 3.2 Additional damping ratio, and mass ratio for the Boeing cylinder modes due to the fluid loading effect.

The structural kinetic energy of the cylinder is plotted for three different configurations in Figure 3.9: without the external loading (“in vacuo”) as in Figure 3.6, with the radiation impedance corresponding to Eq.(2.6.7) (“fluid loaded”), and with using only the diagonal terms of $\mathbf{Z}^r$ (“diagonally fluid loaded”). For this particular structure below 200Hz, the cross modal coupling through the radiation impedance matrix appears negligible, and so only the diagonal terms of $\mathbf{Z}^r$ will be used in the simulations. As expected the resonance of the 1,3 mode is shifted down and the 1,2 mode peak at 160Hz and its coupled acoustic resonance at 120Hz are significantly damped compare to the “in vacuo” case. Figure 3.9 also shows how these fluid loading effects modify the acoustic potential energy response. Note that in absence of notification, the reference used to plot frequency response in decibel is assumed to be 1 unit.

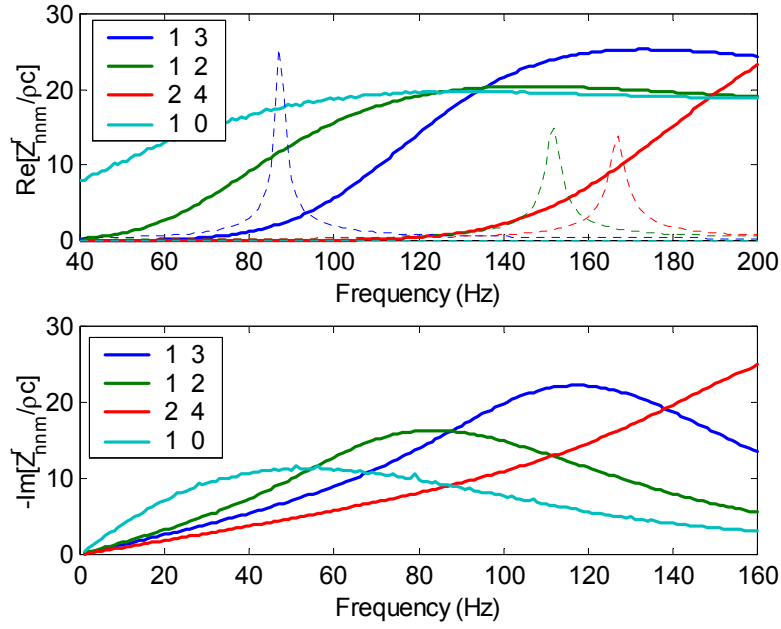


Figure 3.8 Real and imaginary part of the normalized radiation impedance as function of frequency for three different modes with scaled modal response (dotted line).

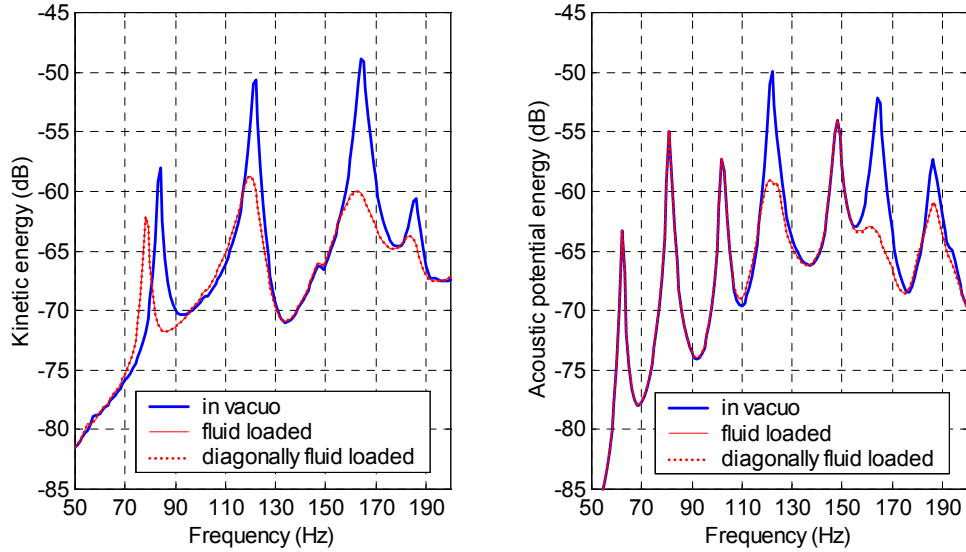


Figure 3.9 Influence of the external fluid loading on the structural kinetic energy and the acoustic potential energy of the cylinder under oblique plane wave excitation ($\alpha_i=70^\circ$).

3.2.4. Model validation

The composite cylinder prototype shown in Figure 3.10 was built by Boeing and shipped to Virginia Tech in September 2001. A dolly was built that allowed the cylinder to move in an upright position and to be lowered to the ground for testing. The bottom and top end cap are made of 5.5" thick plywood in which the edges of the cylinder are slotted. Three aluminum I-beams reinforce each end cap. Twelve threaded rods evenly spaced along the circumference secure the bottom and the top end cap together. In order to evaluate the accuracy of the model, several experiments were done.



Figure 3.10 Boeing composite cylinder on its dolly with the 5 rings of 30 accelerometers.

Validation of the acoustic cavity model

To determine the behavior of the acoustic cavity, a speaker was placed inside the cylinder and driven with 40-200Hz band passed white noise. In order to measure the

volume velocity (VV) of this source a microphone was placed into the speaker cabinet of volume V_s as shown in Figure 3.11. As long as the frequency of excitation is well below the first mode of the cabinet the pressure inside (p_{in}) is uniform and proportional to the speaker volume velocity⁸⁰ u :

$$u(\omega) = \frac{i\omega V_s}{\rho c^2} p_{in}(\omega) \quad (3.2.4)$$

Three rings of five microphones positioned at 19, 54, 92 inches from the bottom and 12 inches from the wall were used to obtain a spatial average of the acoustic pressure inside the cylinder. The transfer functions between all of the 15 microphones and the microphone placed inside the speaker cabinet were acquired. This technique allows the dynamics of the disturbance speaker to be removed from the transfer functions.

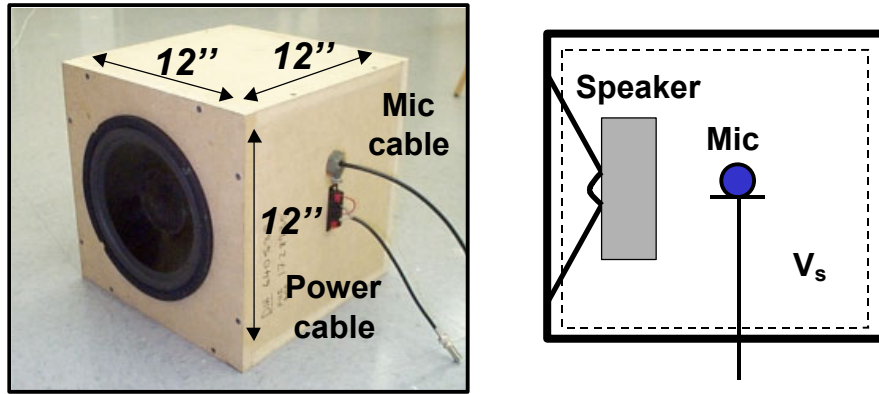


Figure 3.11 Picture and schematic of the volume velocity speaker with internal microphone.

Using the fully coupled model developed in section 2.6, the acoustic pressure is reconstructed at the 15 microphones locations. Figure 3.12 shows the experimental and predicted acoustic response at two microphones locations. The damping of the first 9 acoustic modes is evaluated from the measured transfer function and used in the model.

The typical acoustic damping ratios used to match the model range from 0.5 to 1 %. Figure 3.13 presents the measured and predicted SPL per unit volume velocity of the source obtained by averaging the 15 microphone responses. For both Figure 3.12 and Figure 3.13 the agreement between the experiment and the simulation is satisfactory.

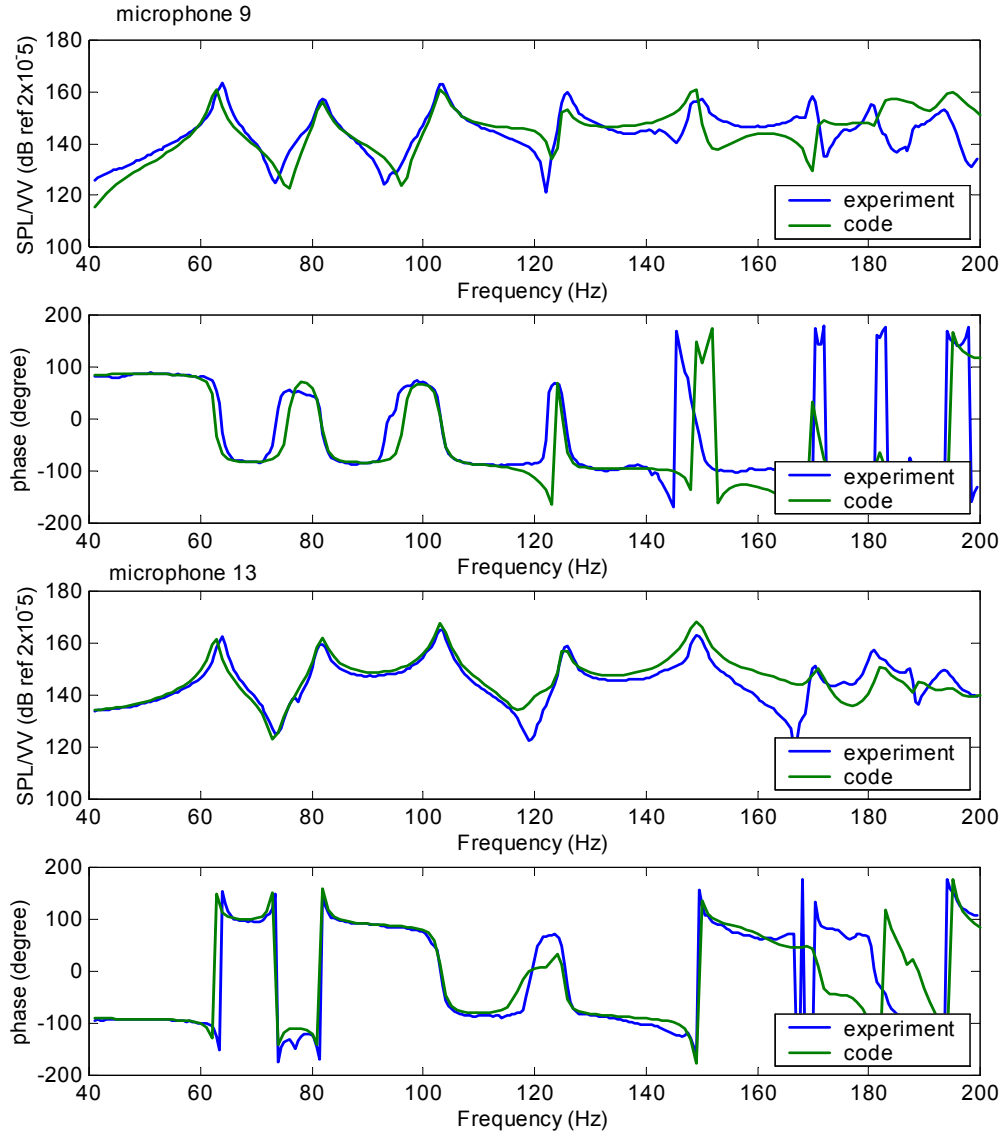


Figure 3.12 Experimental and predicted acoustic response at two microphones locations

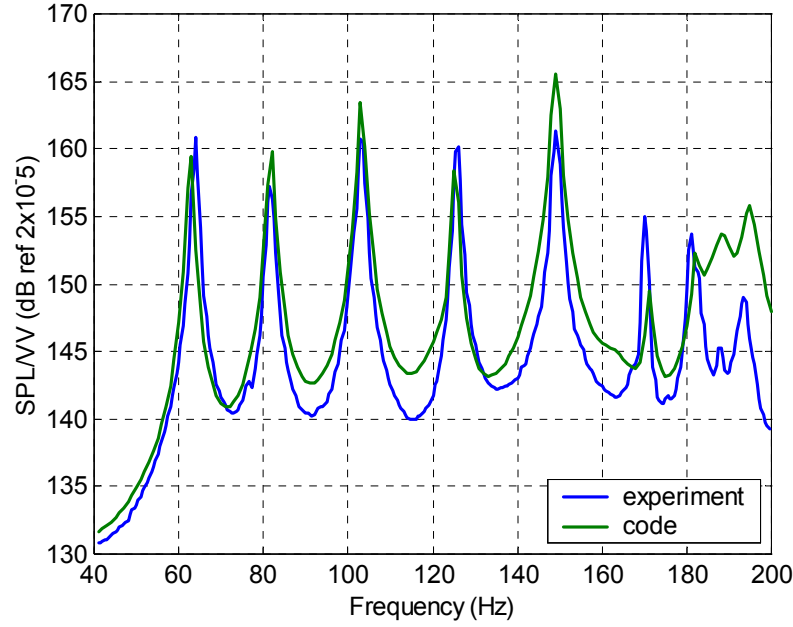


Figure 3.13 Experimental and predicted spatially averaged SPL (15 microphones) per volume velocity (VV).

Using the experimental data the amplitude of N_M modes can be extracted using an inverse technique⁸¹. At each frequency the pressure at the 15 microphones is represented in matrix form as:

$$\begin{bmatrix} \mathbf{p} \end{bmatrix} = \begin{bmatrix} \mathbf{\Phi} \end{bmatrix} \begin{bmatrix} \mathbf{a} \end{bmatrix}, \quad (3.2.5)$$

where $\mathbf{a}$ is the $N_M \times 1$ vector of complex mode amplitudes and the $15 \times N_M$ matrix $\mathbf{\Phi}$ defines the N_M mode shape values at the 15 microphone positions. The elements of $\mathbf{\Phi}$ are obtained using the theoretical mode shapes given in Eq.(2.3.12). The 15×1 vector is composed of the measure acoustic pressure response at the 15 microphones. If the number of mode N_M chosen to describe the pressure field is equal to the number of microphones, the matrix $\mathbf{\Phi}$ is square and thus the mode amplitudes are given by

$$\mathbf{a} = \Phi^{-1} \mathbf{p} \quad (3.2.6)$$

However, depending on the chosen microphone locations, two different modes can render Φ singular by having two linearly dependent columns. In this particular case, since the 15 microphones monitor only one radial position, the 1,0,0 and 1,0,1 acoustic modes are spatially identical at the microphone positions and thus cannot be simultaneously included in the modal set. Furthermore, because of errors associated with the microphone locations as well as with the theoretical mode shapes, it is usually better to use an over-determine system with more equations than unknowns, that is using more sensor measurements than modes. The extracted mode amplitudes become then a least square approximated solution expressed as:

$$\mathbf{a} = [\Phi^T \Phi]^{-1} \Phi^T \mathbf{p} \quad (3.2.7)$$

This technique applied to the 15 measured acoustic responses with 9 acoustic modes allows assigning to each peak of the acoustic response a mode order. The extracted and the predicted amplitude for three different modes are compared in Figure 3.14. Table 3.3 lists the mode order and natural frequencies of the dominant acoustic peak below 200Hz obtained experimentally, computed with Eq. (2.3.16) (hard wall condition) and obtained using the fully coupled model. Except for the 0,2,0 mode that shows three damped peaks resulting from the coupling to the 1,2 structural mode (164Hz) and the 2,2,0 acoustic mode (188Hz), the natural frequencies using Eq.(2.3.16) are fairly close to the fully coupled model predictions. The extraction of the 0,2,0 modal response from the experimental data does not permit a clear identification of the three coupled frequencies observed with the model.

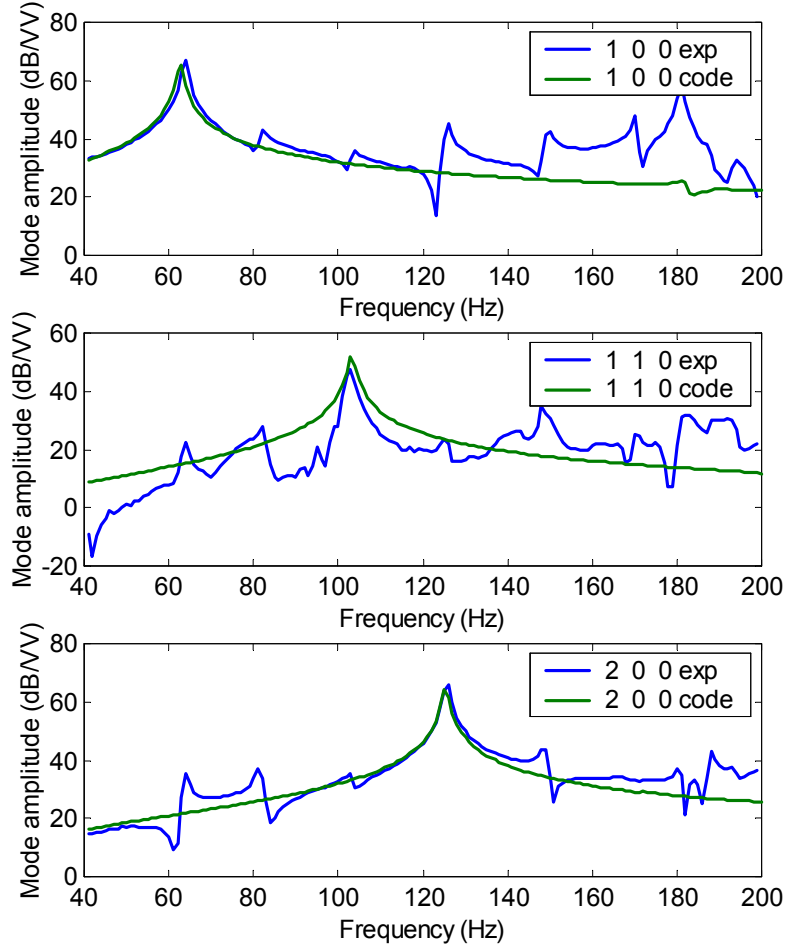


Figure 3.14 Mode amplitudes extracted from the microphone data and predicted by the numerical code.

Mode order			Natural frequency (Hz)		
n	m	p	Measured	Eq. 2.2.16	Fully coupled
1	0	0	64.0	62.8	63.0
0	1	0	82.0	82.5	82.0
1	1	0	103.2	103.6	103.0
2	0	0	125.6	125.6	125.5
0	2	0	...	136.8	121,164,188
2	1	0	148.5	150.3	149.4
1	2	0	146,149	150.5	149.0
2	2	0	183.0	185.7	188.5

Table 3.3 List of measured, theoretical and fully coupled acoustic natural frequencies.

Validation of the structural model

A shaker test was used to identify natural frequency and the mode shapes of the cylinder. A 50lb shaker positioned at $[\theta=0, z=23"]$ excited the cylinder. To avoid stress concentration, the stinger of the shaker was threaded into a 2" diameter curved aluminum disk glued on the cylinder. A total of 5 rings of 30 point each meshed the surface (see Figure 3.10). The rings were positioned every 20" starting from the bottom of the cylinder. The input force was measured with a force transducer placed between the cylinder and the shaker, which was driven with 40-500Hz band passed white noise. Transfer functions between the input force and the acceleration at the 150 locations in addition to the excitation location were acquired. As for the validation of the acoustic cavity, the fully coupled model was used to predict the velocity response at the accelerometer positions. Figure 3.15 shows the velocity response per unit force at the excitation location and at position 65 in the 3rd ring, 60° from the excitation point. The structural damping ratios are adjusted around the 1% value for a better match with the measurement. Figure 3.16 compares the spatially averaged velocities per unit force using the 151 measured transfer functions with the model predictions. Except for under predicting the first three resonant frequencies, the fully coupled model is in good agreement with the measurement. The discrepancies can be due to the free-field radiation assumption, as the data was acquired indoors. The theoretical boundary conditions, which determine the natural frequencies, are also difficult to recreate experimentally. To identify the peaks with the modes of the structure, the same inverse method used for the acoustic cavity can be applied on the accelerometer transfer functions to extract the structural mode amplitudes. Figure 3.17 shows for three different modes the experimental and predicted amplitude. To facilitate this modal identification, another modal decomposition technique, the spatial Fourier (or wavenumber) transform, can be applied to 30 experimental or predicted velocities belonging to a particular ring. The result of this transform is visualized as a contour plot of the velocity amplitudes as a function of both frequency and circumferential mode order. As for a time-frequency Fourier transform, the information is divided into a symmetric and antisymmetric content with respect to the origin. Since the excitation position is taken as the angle origin, only modes with

orientation order $\sigma_s=0$ corresponding to a cosine circumferential variation are expected to be excited. Figure 3.18 presents the symmetric components of the wavenumber transform applied to the lower ring for both the measured and predicted response. The amplitude are normalized by their maximum value and plotted with a logarithmic scale. For both the theoretical and experimental results, the modes for each axial order n lie on a “u” shaped curve characteristic of cylinders dispersion relationship as previously observed in Figure 3.4. Figure 3.18 also shows a good agreement between the model prediction and the experiment.

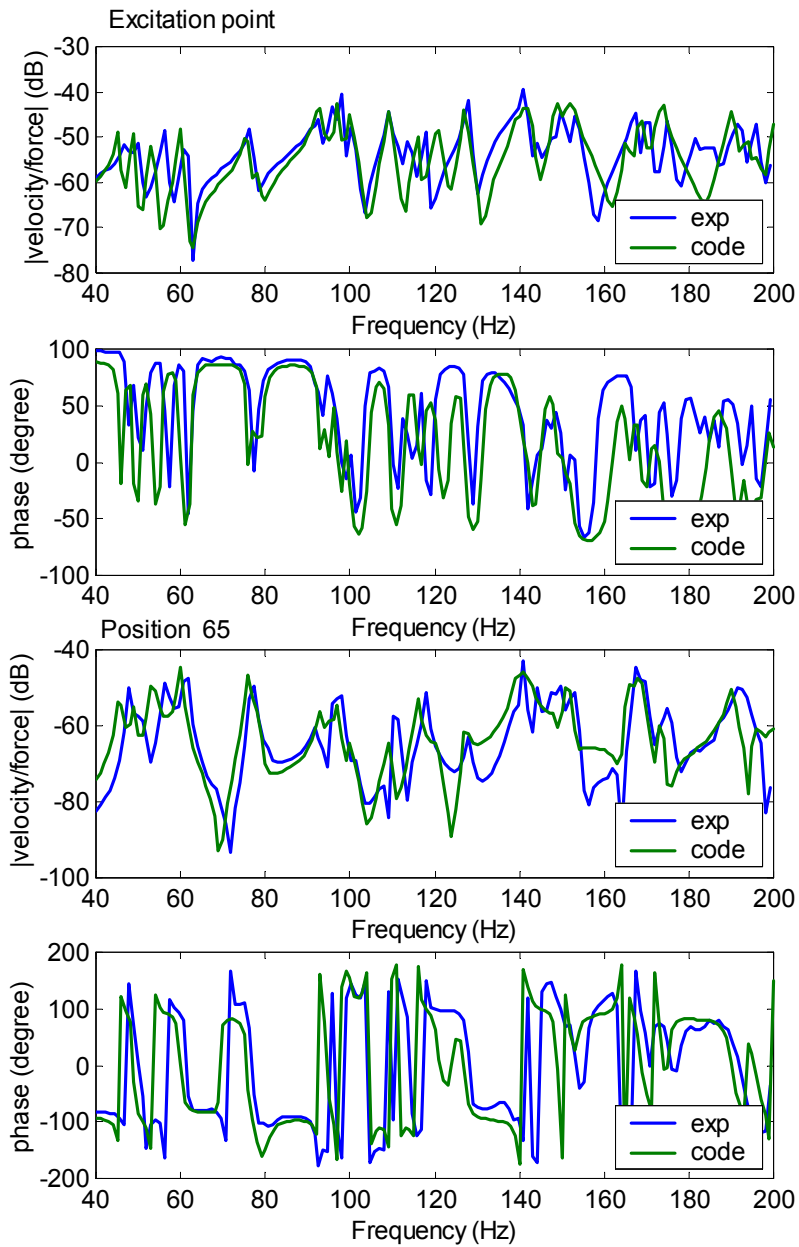


Figure 3.15 Experimental and predicted velocity response per unit force at the excitation location and at position 65 (3rd ring up).

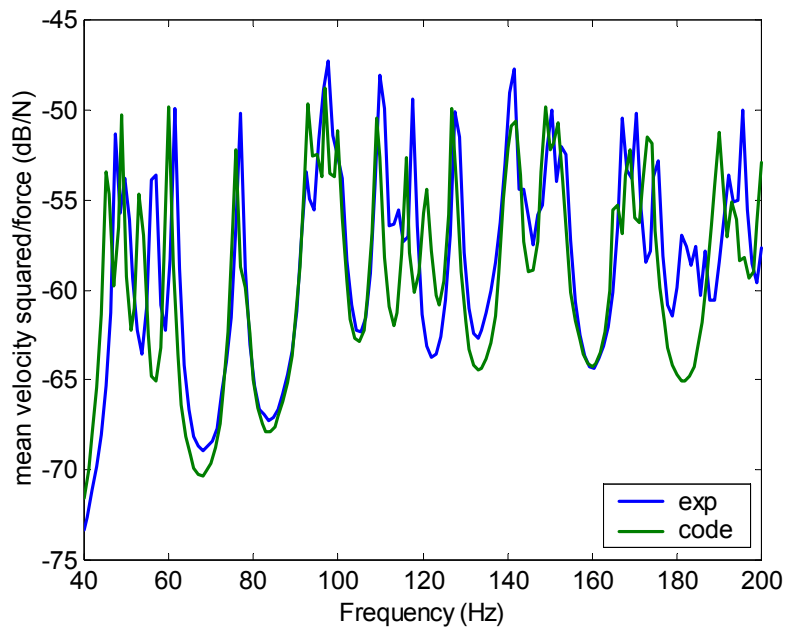


Figure 3.16 Experimental and predicted mean velocity squared per unit force (151 positions).

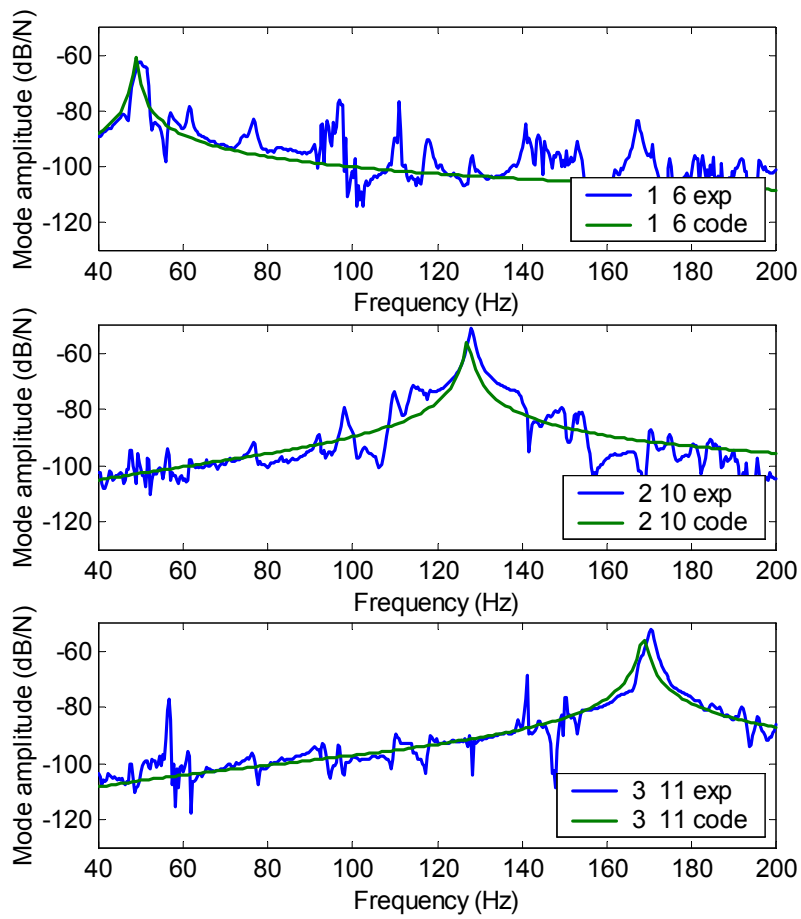


Figure 3.17 Experimental and predicted modal amplitudes.

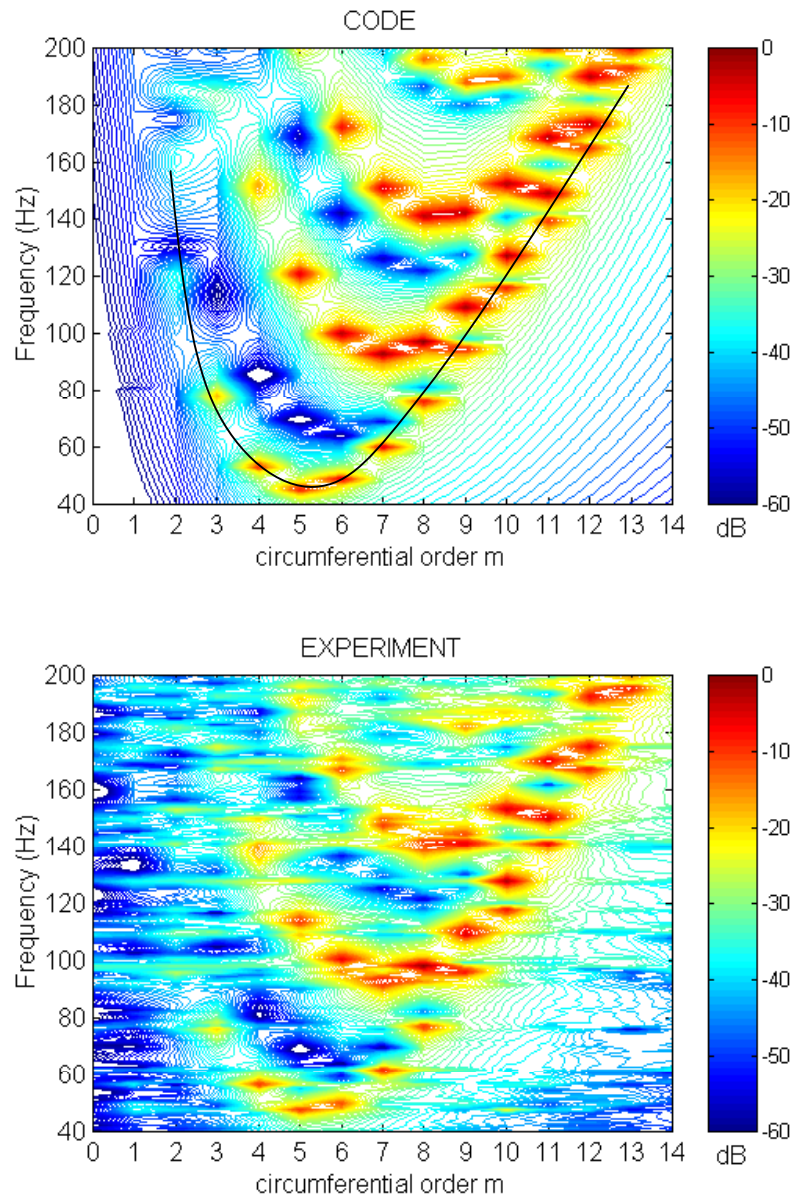


Figure 3.18 Symmetric component of the normalized velocity wavenumber transform applied on the lower ring using the predicted (top) and measured (bottom) response.

Validation of the sound transmission

The acoustic pressure inside the cylinder subjected to the shaker excitation was also measured at 96 positions covering 4 axial, 2 radial, and 12 angular different locations. The 96 transfer functions between the microphones and the input force provided an estimation of the averaged internal SPL per unit point force. Theoretically, because of the coupling matrix properties and the few low circumferential order structural modes below 200Hz, the internal SPL should be limited to acoustic modes resonating in the bandwidth. However, the comparison between the measured and predicted SPL plotted in Figure 3.19 in the top graph shows a relatively strong disagreement. The experimental spectrum presents more peaks and higher amplitudes of the 0 and 1 circumferential order acoustic modes at 60, 100 and 145Hz. the bottom graph of Figure 3.19 illustrates, these discrepancies are attenuated by using a “noisy” coupling matrix which is obtained by adding to each element of $\mathbf{C}$ a normally distributed random number of variance equals to 1% of cylinder’s surface. The acoustic transmission for this particular case seems very sensitive to the imperfections in the mode shapes and boundary conditions that couple all the modes together.

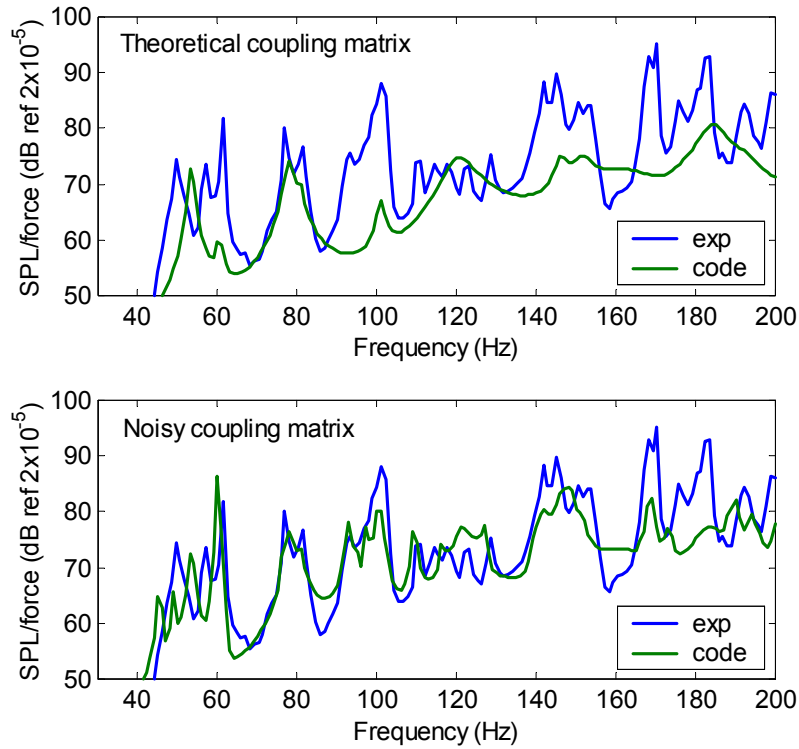


Figure 3.19 Measured and predicted internal SPL per unit input force using the theoretical coupling matrix (top) and the “noisy” coupling matrix (bottom).

3.2.5. Fairing model

In this section, the different elements of the fairing model are presented. The similarities with the cylinder are highlighted and the particularities due to the finite element base of the model are explained.

External coupling

The fairing being an axisymmetric structure, causes the orthogonal mode shapes computed by the finite element model to appear in pairs of circumferential mode order as

observed in a perfect cylinder. As shown in Figure 3.20 the connection between the cylindrical and the cone section represents a nodal line for most of the mode shapes. Consequently, the motion of in the fairing cylindrical section can be approximated using the axial and circumferential order of a simply supported cylinder. The wavenumber transform performed on velocity responses to a point force excitation ($z=0.15\text{m}$) at 36 points around the circumference confirms this similar behavior. Figure 3.21 shows that the wavenumber transform fairing presents the “u” shaped curve characteristic of cylinders dispersion relationship observed in Figure 3.18.

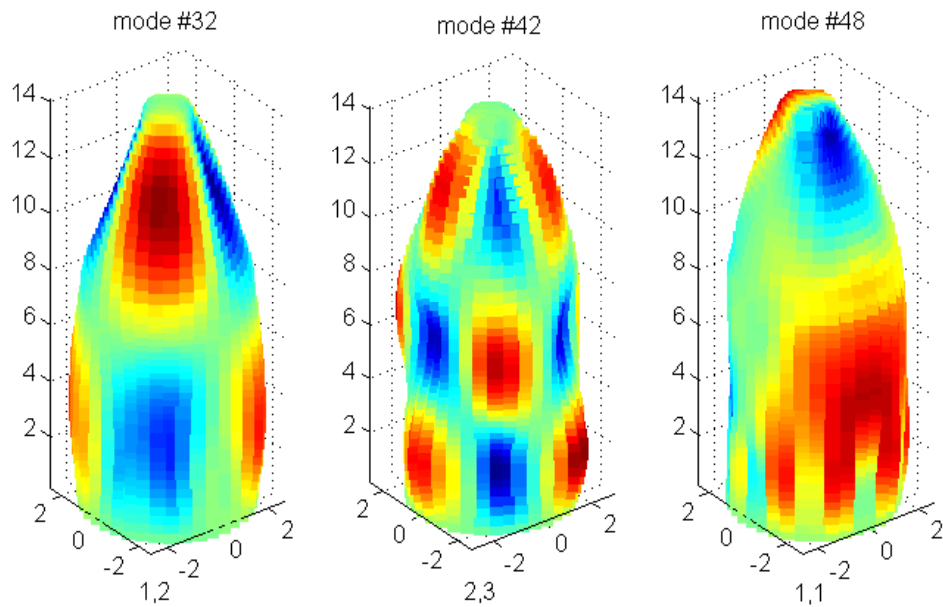


Figure 3.20 Fairing structural mode shapes with the corresponding cylindrical section mode order.

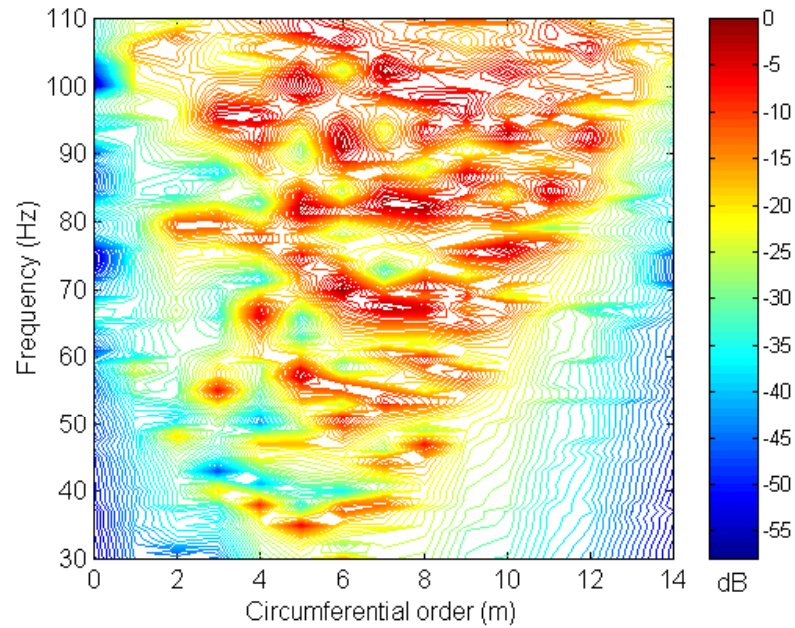


Figure 3.21 Circumferential wavenumber transform of the fairing velocity response to a point force.

In order to compute the fairing response, the external acoustic excitation was chosen to be representative of the full-scale tests detailed in section 4.4. Using the equivalent source method presented in section 2.7.2, the external acoustic field on the fairing generated by twelve speakers modeled as acoustic monopoles of constant volume acceleration was computed. Figure 3.24 shows the resulting pressure distribution on the fairing at 70Hz as well as the location of the speakers.

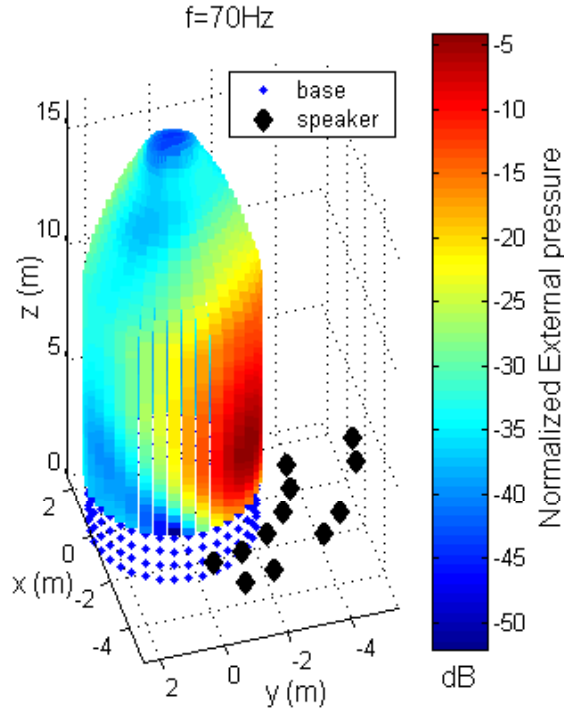


Figure 3.22 External pressure distribution on the fairing due to eight speakers at 70Hz.

Fluid loading effect approximation

The external coupling between the fairing and the external acoustic field differs from the coupling between a cylinder and a plane wave excitation that was obtained analytically. To avoid a complex and cumbersome numerical calculation of the fairing radiation impedance, only its cylindrical section is considered to approximate the fluid loading effects. Using the derivation in 3.2.3 and the mode orders describing the cylindrical section motion, Eq. (3.2.2) and (3.2.3) approximate the additional damping and mass effect of the external fluid on the fairing modes. This external damping ratio is added to the structural damping characteristic of the fairing composite construction. As an exact measurement of this quantity is complex and not crucial for the numerical predictions, the structural damping is set to 1%. Table 3.4 shows the approximated damping and mass effect of the fluid loading for the most affected modes.

FE Mode	Equivalent cylinder order		Approximated fluid loading effect	
Mode#	Axial (n)	Circumferential (m)	Damping ratio (%)	Additional mass (%)
32	1	2	8.8	17.0
33	1	2	8.8	17.0
40	2	3	3.6	10.5
42	2	3	3.6	10.5
48	1	1	8.8	28.2
49	1	1	8.8	28.2
81	2	2	6.5	13.5
84	3	3	3.6	9.0
85	3	3	3.6	9.0
86	2	2	6.5	13.5

Table 3.4 Examples of additional damping ratio, and mass ratio for fairing modes due to the fluid loading effect.

Acoustic blanket effect

The damping of the acoustic cavity occurs when energy that is transferred to the structure is lost by radiation. In addition, the fairing interior walls are covered with acoustic blankets to attenuate high frequencies noise. The experimentally measured reverberation time (T_{60}) of a payload fairing was used to evaluate the approximate damping ratio provided by such blankets. Appendix A describes the procedure followed to obtain the T_{60} in third octave bands. Theoretically, the validity of this method is arguable. The reverberation time is only meaningful at frequency where the enclosure modal density is high enough to ensure even energy distribution in all the modes. This assumption is violated for the low frequency range of the fairing. In addition, the effect of the acoustic blanket is not uniform with different mode orders. Nevertheless, in absence of a better method, the approximate acoustic damping ratio per third octave band is evaluated as⁵⁶:

$$2\xi(\omega_c) = \frac{6}{\omega_c T_{60}(\omega_c) \log_{10}(e)} \simeq \frac{13.8}{\omega_c T_{60}(\omega_c)} \quad (3.2.8)$$

The resulting third octave band damping ratios are then curved fitted in the frequency band of interest with a 4th order polynomial to extrapolate their value at the natural frequency of the acoustic modes. In the 30-120Hz, the damping ratio only varies from 0.9 to 1.5% as seen in Figure 3.23.

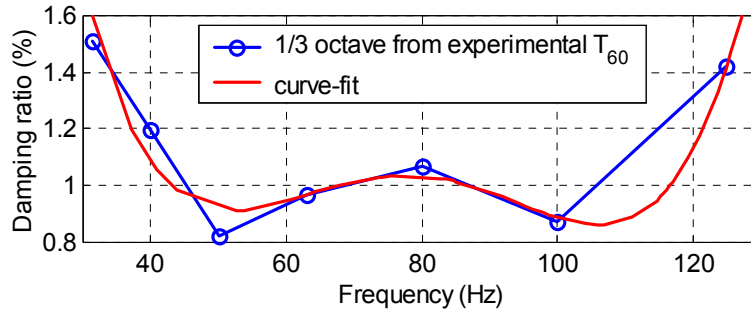


Figure 3.23 Acoustic damping ratio obtained from 1/3 octave T_{60} measurement.

Finally, as the foam treatment represents 10% of fairing mass, the structural natural frequencies of the bare fairing obtained with the finite element are multiply by $\sqrt{1/1.1}$ to drop them accordingly.

Internal coupling and mode selection

As the fairing model comprise 1109 structural modes and 293 acoustic modes, a selection of the most important modes is necessary to keep computational time low. This modal selection is a relatively complex process as it involves different levels of coupling, which do not present close-form expressions as opposed to the analytical cylinder formulation. The first selection consists in including all the acoustic and structural modes that resonate in the frequency band slightly larger than the one of interest, in this particular case up to 130Hz. This set of mode is insufficient to accurately describe the acoustic response as it discards two main transmission paths. The first and most important one comes from the forcing of an acoustic mode resonant in the bandwidth by

a structural mode resonant outside the band (as mentioned in section 2.4.2). The second path comes from an acoustic mode resonant outside the band that is forced by a structural mode resonant inside the band. To select the most important modes, the individual response of each acoustic mode, n , to each structural mode, m , is computed and integrated over the frequency band of interest.

$$\beta_{nm} = \int_{30Hz}^{130Hz} |A_n^a(\omega) \cdot C(n,m) \cdot A_m^s(\omega) \cdot F_m(\omega)| d\omega \quad (3.2.9)$$

The resulting value β_{nm} , which corresponds to the acoustic energy transmitted over the band of a particular structural-acoustic mode combination, is used to sort the modes in term of their energy contribution. Figure 3.24 shows the repartition of β_{nm} with respect to the structural and acoustic natural frequencies. In this particular case only the six most important modes are retained to create the modal subset. As illustrated by the rectangular box, the first transmission path described previously largely dominates even over the coupling between structural and acoustic modes both resonant in the band as in the case of the cylinder.

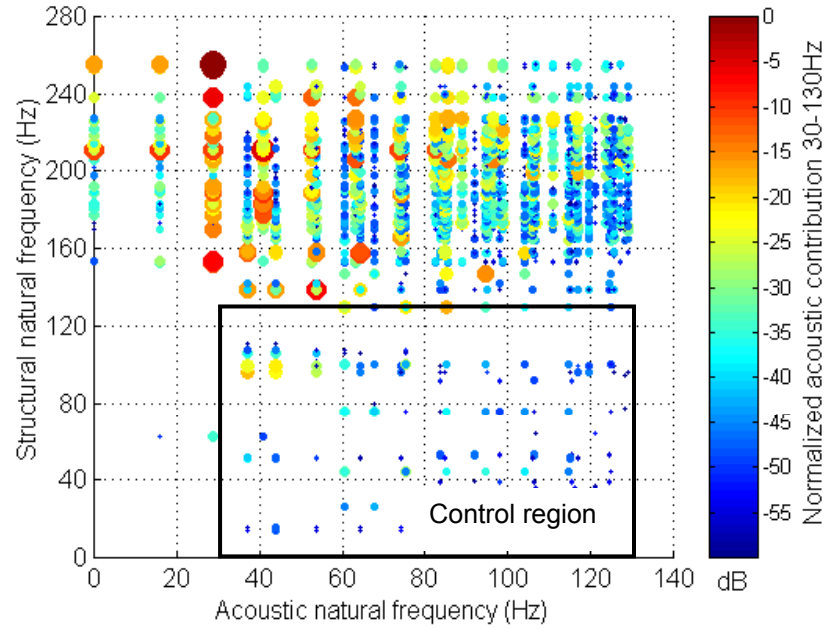


Figure 3.24 Overall coupling level between structural acoustic modes.

Figure 3.25 shows the velocity and acoustic response up to 110Hz using the complete modal set and the resulting modal subset comprising 288 structural and 79 acoustic modes with frequency up to 295 and 130Hz respectively. The differences in the velocity response between the two modal sets are negligible since all the modes below 130Hz are included and the second transmission path is marginal. As the first transmission path dominates across the band, the reduction of the structural modal set induces for particular acoustic modes slightly bigger differences in the response. These discrepancies never exceed 1.5 dB at the peak; on the other hand simulation with the modal subset are 80 times faster which is a considerable advantage when using the genetic algorithm (section 3.5.2).

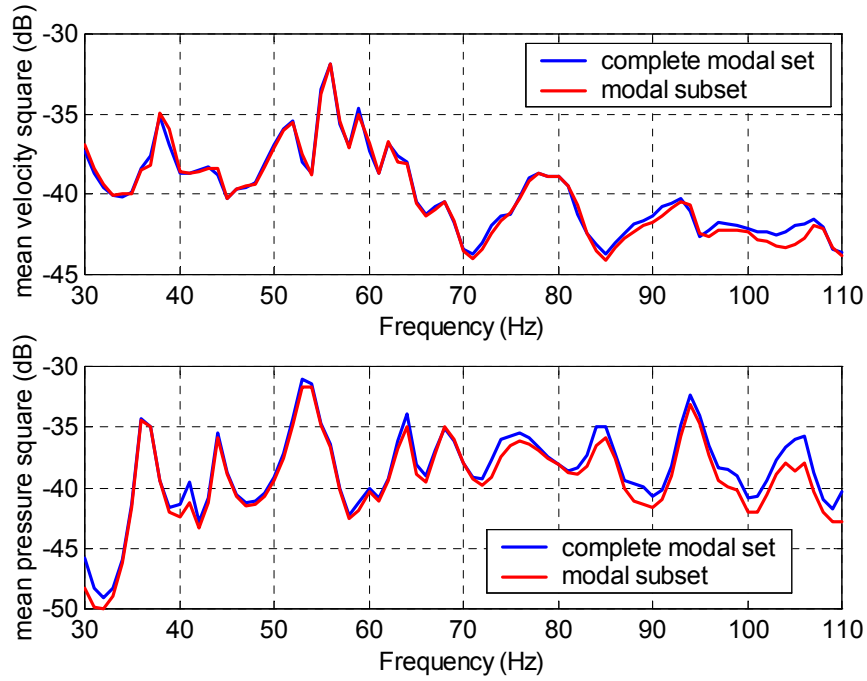


Figure 3.25 Velocity and acoustic response to speaker excitation using the complete modal set and the modal subset.

3.3 Noise reduction mechanisms

This section presents the HR and DVA effects on the system and the strategies developed to optimize the noise attenuation.

3.3.1. Optimal damping

The noise reduction mechanism of both HRs and DVAs is based on the dynamic vibration absorber system. Consider a vibration absorber of mass m_d and natural frequency ω_d attached to a mass-spring system with mass m and natural frequency ω_h as illustrated in Figure 1.2. Tuning the vibration absorber such that $\omega_d = \omega_h$ splits the

resonance of the system into two new resonances of similar amplitude either side of ω_h . The bigger the mass ratio, the further apart the two resonances of the coupled system appear. By adding damping to the absorber, (i.e. between the system and the absorber mass), both ‘new’ resonances are damped and significant broadband attenuation can be achieved. Depending on the type of excitation, several formulas for damping ratios lead to optimal vibration reduction^{28,29}. For a wide band random excitation, the optimal damping ratio ζ_d^{opt} derived by Korenev and Reznikov²⁹ to minimize the primary system’s velocity is expressed as

$$\zeta_d^{opt} = \frac{1}{2} \sqrt{\frac{\nu}{1+\nu}}, \quad (3.3.1)$$

where $\nu = m_d/m$. Once coupled to a continuous structure of mass M , the effective mass ratio ν is weighted by the normalized mode shape squared at the absorber location (θ_0, z_0)

$$\nu \approx [\Psi_{n_s m_s \sigma_s}^s(\theta_0, z_0)]^2 \frac{m_d}{M} \quad (3.3.2)$$

The coupling between a HR of volume V_h and an acoustic mode of an enclosure of volume V obeys the same mechanisms. By analogy with den Hartog’s optimized dynamic absorber²⁹, Fahy and Schofield⁵⁶ derived an optimal HR damping level ζ_h^{opt} as a solution of

$$\mu^2 \left(\frac{1}{2\zeta_h^{opt}} \right)^4 + 2\mu\xi_{nmp} \left(\frac{1}{2\zeta_h^{opt}} \right)^3 + \mu \left(\frac{1}{2\zeta_h^{opt}} \right)^2 - 1 = 0, \quad (3.3.3)$$

where ξ_{nmp} is the modal damping of the enclosure and μ the effective volume ratio given by

$$\mu \simeq [\Psi_{nmp\sigma}^a(r_0, \theta_0, z_0)]^2 \frac{V_h}{V} \quad (3.3.4)$$

Figure 3.26 plots the optimal damping ratios for the HR and the DVA as a function of the effective mass and volume ratios. The effect of the initial damping of the acoustic mode on ζ_h^{opt} is relatively small.

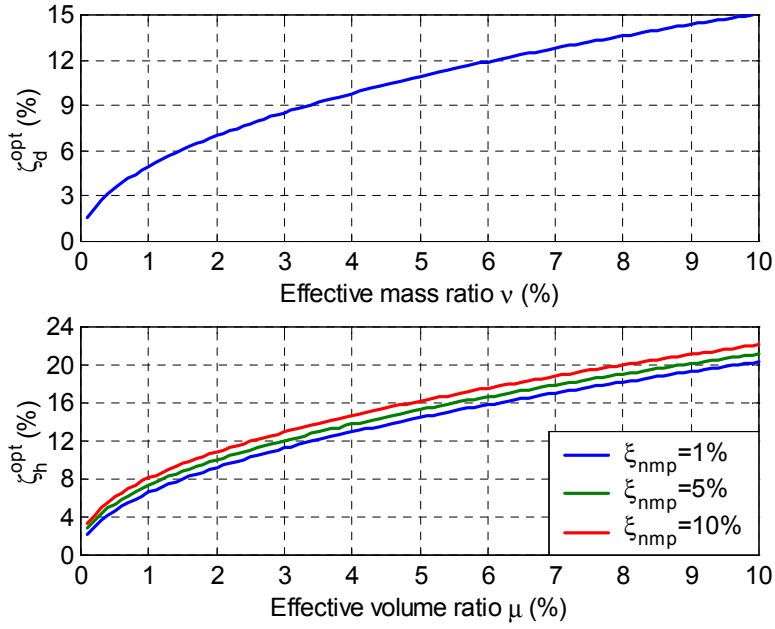


Figure 3.26 Optimal damping ratios using Eq.(3.3.1) and Eq.(3.3.3) as a function of effective mass and volume ratios.

In both the structural and acoustic cases, the amount of attenuation is a weak function of the damping ratio and so a small variation about the optimal level only marginally degrades the performance of the HRs and DVAs. Because these optimal damping ratios are derived for a single mode, they cannot be taken as true optima for the multi mode case. However, they constitute a good first solution especially for isolated modes. Figure 3.27 shows that when the damping of the devices is too low compare to the optimal value, the two ‘new’ modes are both fairly lightly damped and only small broadband

noise reduction is achieved. Alternatively, when the damping is too high, the devices become uncoupled from the mode and no longer dissipate energy.

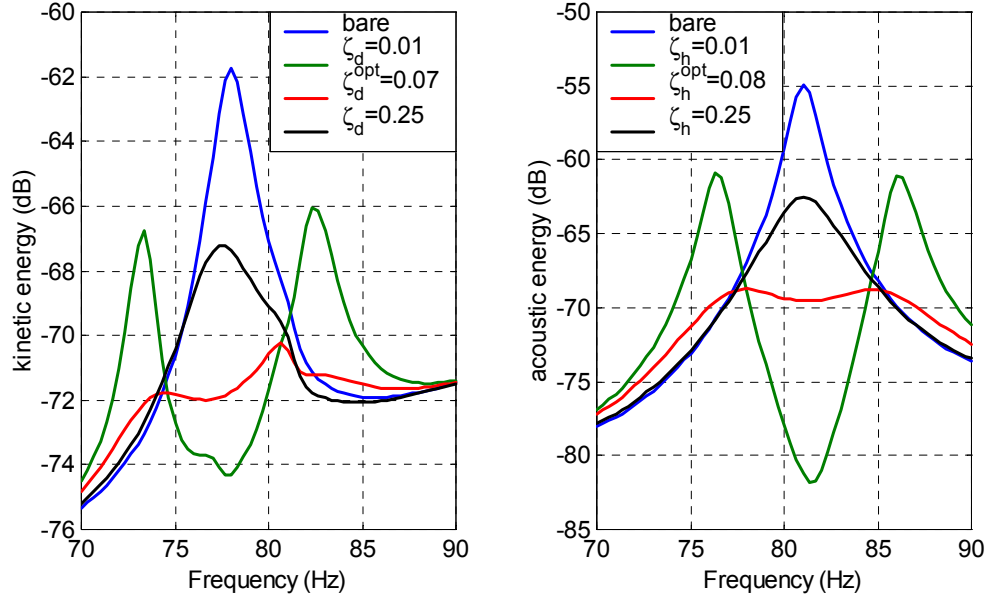


Figure 3.27 Influence of the DVA and HR damping ratios on the kinetic and acoustic energy.

In both cases the devices were split into several identical units distributed evenly around the circumference and tuned to the resonant frequency of the targeted mode. Consequently, the number of device per ring has to be greater than twice the circumferential order to avoid collocation of the units with nodes of vibration. The effective mass or volume ratio of a ring is computed using only the axial mode shape component and is weighted by $1/\varepsilon_m$ since half the mass or volume of the devices effectively acts on circumferential modes greater than zero. In Figure 3.27 the DVA treatment weighs 1% of structure's mass and is composed of 13 1.5" by 2.5" DVAs attached at $z=L/2$. The HR treatment representing 1% of the cavity's volume is split into 7 units also positioned in the middle of the cylinder.

3.3.2. Devices are used in rings.

Multiple devices are used for two reasons. First, using a symmetric ring of devices allows the treatment to be independent of the azimuth angle θ_i of the incident field, which is assumed to be unknown. Second, the devices act as discontinuities that can couple modes together by shifting energy from one circumferential mode to another. For instance, a ring of N_d DVAs targeting a mode of circumferential order $m_s=i$ redistributes the energy to all $m_s=|i \pm qN_d|$ modes where q is an integer. Thus, a DVA treatment is likely to excite structural modes that are not forced by the incident acoustic field. The closer the resonant frequency of the $m_s=|i \pm qN_d|$ mode is to the DVAs' tuning frequency, the higher the excitation. Therefore, a large number DVAs and HRs per ring ensures a weak modal coupling since only modes with greatly different circumferential order, which in most cases implies greatly different resonant frequencies, can interact. To demonstrate this, Figure 3.28 shows the amplitude of the 1,2 and 1,7 structural modes with two different treatment designed to target the 1,2 mode of the Boeing cylinder. The tuning and the total mass of the DVAs (4% of the cylinder's mass) remain the same for both treatments. Only the number of DVAs per ring differs from 13 to 5. This shows that when 5 DVAs per ring are used, the performance on the 1,2 mode is reduced compare to the 13 DVAs per ring treatment. In addition, the 5 DVAs per ring treatment increased the amplitude of 1,7 mode by 40 dB, compared to the bare or the 13 DVAs per ring treated cylinder.

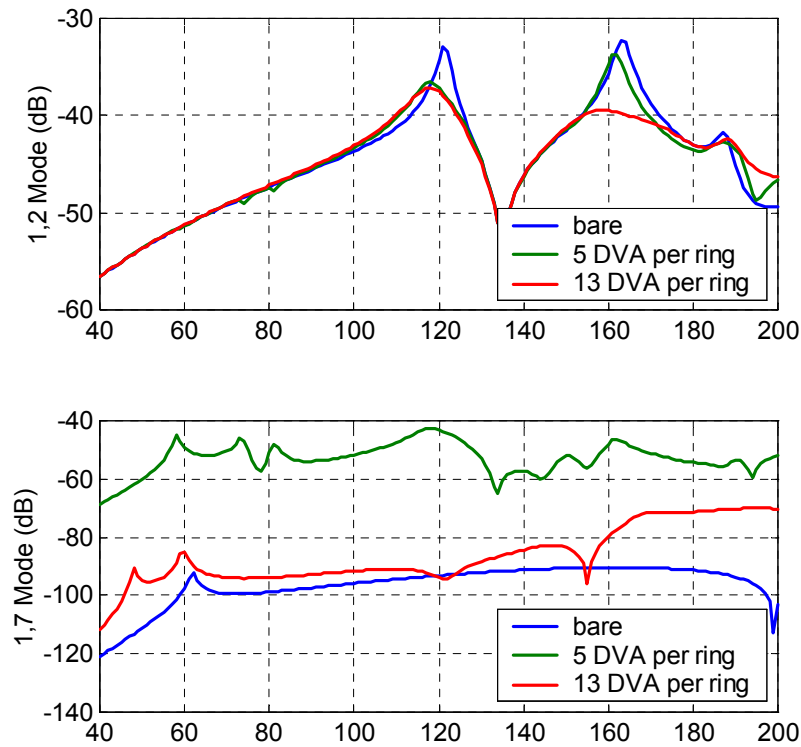


Figure 3.28 Influence of the number of DVA per ring on the amplitude of the 1,2 and 1,6 structural mode.

3.3.3. Influence of the DVA mass and HR volume.

As mentioned in section 2.5.1, the effect of a DVA on the vibration response is proportional to the DVA to structure mass ratio. Likewise, the effect of a HR on the acoustic response is proportional to the HR to cavity volume ratio. To illustrate this Figure 3.29, compare the performance of DVA and HR treatment on an isolated mode for different mass and volume ratio. In each case the devices are optimally damped with respect to their mass/volume ratios and to the shape of the mode they are targeting. This illustrates the existing trade-off between the weight and volume of a treatment and its

performance. Therefore in practice, the constraints imposed on the devices define an upper bound for the noise attenuation.

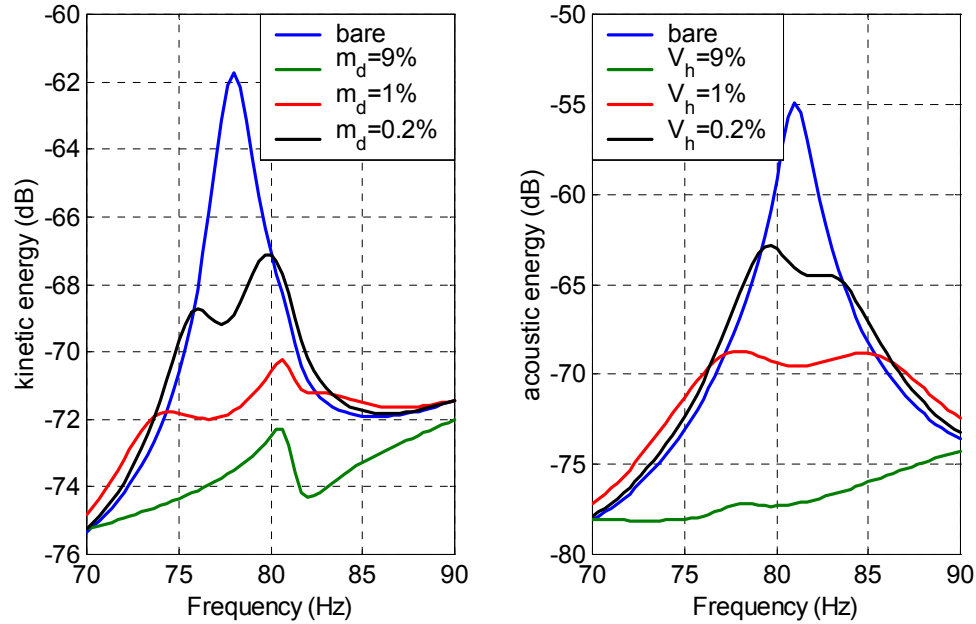


Figure 3.29 Influence of the DVA mass and HR volume in percent of the total mass and total volume of the cylinder on the kinetic and acoustic energy.

3.3.4. Experimental validation.

In order to validate the model predictions, and demonstrate the effectiveness of the DVAs and the HRs to reduce the vibration and the acoustic response, several experiments were conducted on the cylinder prototype.

DVA validation

A treatment composed of 15 DVAs was designed to target the 1,7 mode of the cylinder, which resonate at 61.5Hz. Each DVAs was built using a 2" thick 3"x5" block of polyurethane acoustic foam on top of which was glued a metallic plate as shown in Figure 3.30. Each DVA was then tuned, using the experimental set-up described in

Figure 3.31. The transfer function between two accelerometers one attached to the shaker base the other attached to the DVA mass layer was curved fitted to a single degree freedom response to extract the device's natural frequency and damping ratio. To achieve the desired frequency, the mass layer and the thickness of the foam can be varied. For a given type of foam and footprint, the thinner the foam layer is, the stiffer. The damping ratio produced by structural losses in the acoustic foam as it compresses is harder to control. Using different types of foam leads to different damping levels. The particular type used in this experiment provides a damping ratio of around 7%.

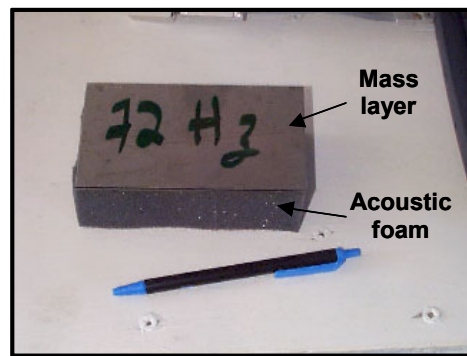


Figure 3.30 Example of a 3"x5" DVA made with 2" thick polyurethane foam.

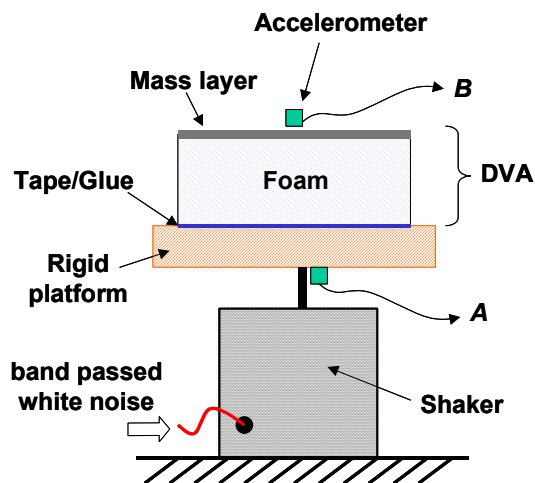


Figure 3.31 Experimental set-up used to tune DVA.

The 15 DVAs were glued on the inner wall of the bare cylinder at $z=L/2$. The cylinder was excited by a shaker in same configuration described in section 3.2.4. The transfer functions between the shaker input and the 150 accelerometers meshing the cylinder surface were used to get a spatial averaged vibration response. In addition to a good agreement between the measured and predicted response, Figure 3.32 shows that the DVAs drastically reduce the amplitude of not only the targeted 1,7 mode but all the structural modes resonating in the 50-80Hz region. The total mass of the treatment was 2.5Kg, which represents 3.3% of the cylinder's mass. This treatment leading to a vibration attenuation of 9.2 dB from 50Hz to 80Hz also reduces the acoustic level in the same band by 10dB as Figure 3.33 shows. Therefore, this experiment validates the modeling techniques used to implement the DVA in the fully coupled modal model.

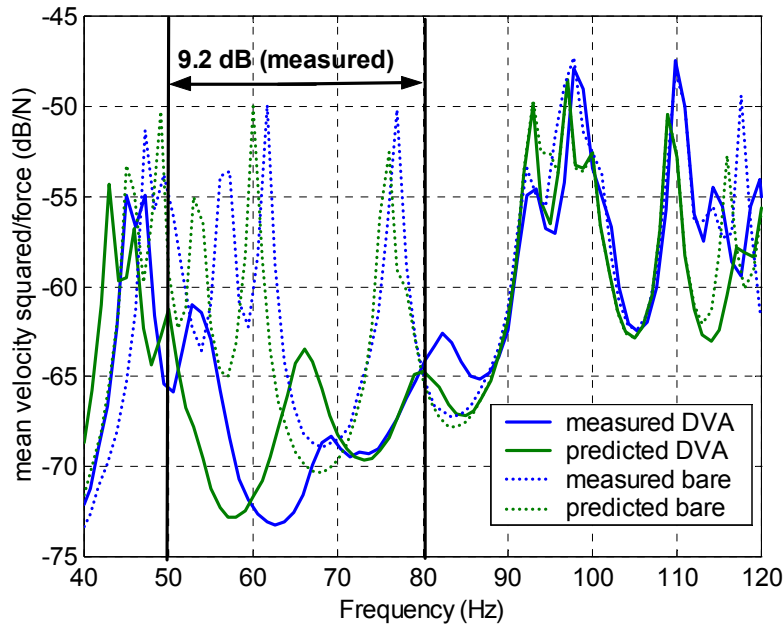


Figure 3.32 Measured and predicted vibration response of the bare (dotted line) and the DVAs treated cylinder (solid).

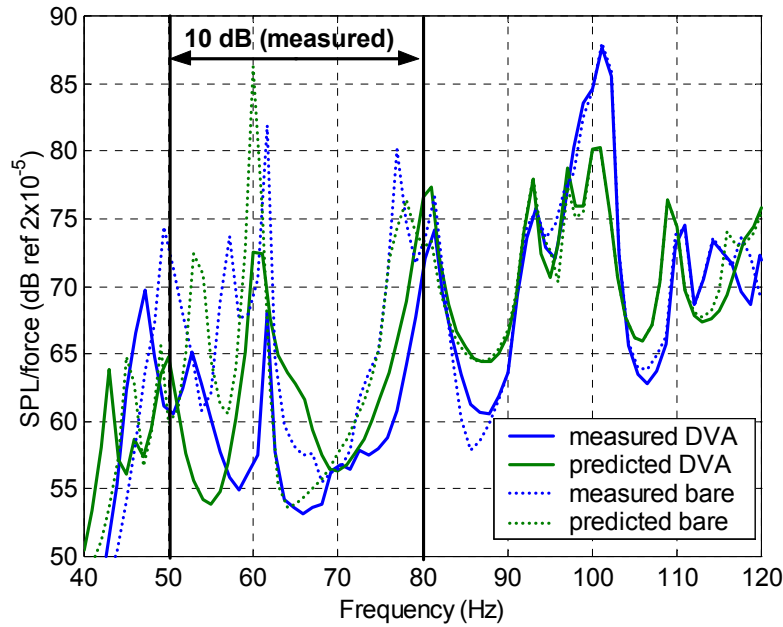


Figure 3.33 Measured and predicted acoustic response of the bare (dotted line) and the DVAs treated cylinder (solid).

HR validation

The resonators as shown in Figure 3.34 were designed using cardboard mailing tube enclosed by plastic end-cap. The throat was cut into one of the end-caps. The neck made of aluminum flexible hose was folded inside the tube to allow a more compact design. In addition to the HR natural frequency, the internal neck creates other resonances usually encountered in mufflers. However, these resonances appear at much higher frequencies and therefore do not affect the HR performance. The HRs are tuned using the experimental set-up described in Figure 3.35. The transfer function between the acoustic pressure inside the HR and in front of the throat is curve-fitted to a single degree of freedom response to obtain the device's natural frequency and damping. The tube length, the throat diameter and the neck length are the variables leading to the desired resonant

frequency. The damping of the resonator is created by viscous losses of the air moving in the neck. Therefore the desired damping is obtained by covering the throat with different types of acoustic screen (see Figure 3.34).

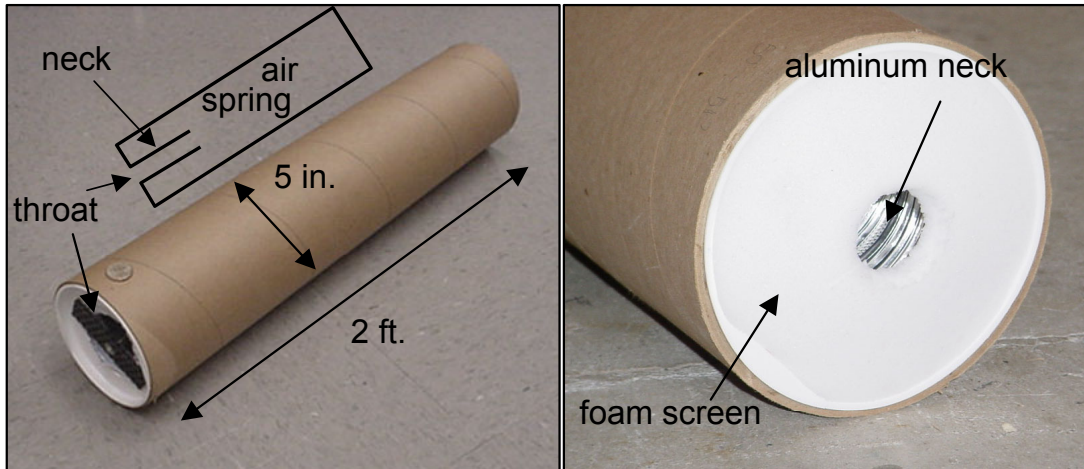


Figure 3.34 Damped Helmholtz resonator made from cardboard tube and foam screen.

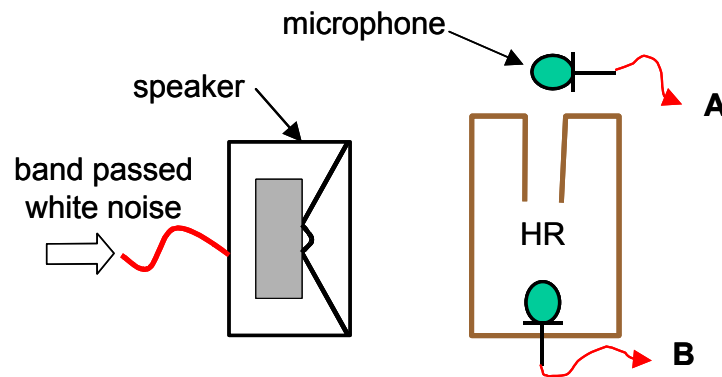


Figure 3.35 Experimental set-up used to tune HR.

Two sets of HRs were designed to target the (1,0,0) and (1,1,0) acoustic mode of the cylinder prototype at 61.5Hz and 101.5Hz. The first set was composed of 7 2' long HRs and represented 0.4% of the cavity's volume. The second composed of 5 16" long HRs represented 0.2% of the volume. Both sets were tuned to the frequency of their respective targeted mode and the damping ratios were adjusted around the optimal value to 7%. The

external acoustic excitation was provided by 600 Watts subwoofer driven with 40-250Hz band passed white noise and situated about 13' away from the cylinder. The transfer functions between the input noise and the signals of 98 microphones evenly spaced in the enclosure were measured with and without the HR treatment. These transfer functions lead to the spatially averaged acoustic response of the cylinder as shown on the left of Figure 3.36. The predicted response plotted on the right of Figure 3.36 is obtained using a plane wave excitation with a 45-degree elevation angle. This experiment demonstrates the performance of a small HR treatment as the peaks of the (1,0,0) and the (1,1,0) mode are reduced by 20 dB and 10 dB respectively. The HR modeling technique used is also validated as the HRs effect on the acoustic response is well predicted.

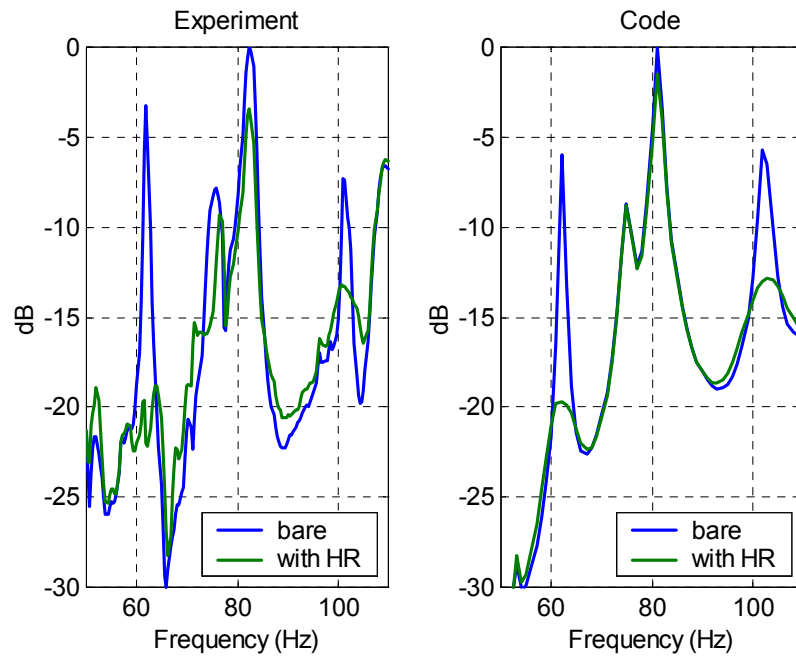


Figure 3.36 Measured (left) and predicted (right) normalized acoustic response under external acoustic excitation with and without the HR treatment.

3.4 Treatment design for the Boeing cylinder

This section presents the procedure which lead to design of a treatment using HR and DVAs. The performance and the robustness of the resulting treatment is also addressed.

3.4.1 Targeting individual modes: the “manual” treatment design

After choosing the frequency band of control, one has to set up constraints on the DVA mass and HR volume as these two variables are directly linked to the treatment performance. The treatment design involves choosing the number of target frequencies, which depends on the number of dominating resonances present in the desired band of control. The treatment parameters then become the tuning frequency, the damping ratio and the axial location for each ring of devices. As a first design, the devices are tuned to the targeted resonances, the damping ratio is set to the single-degree-of-freedom optimum given by and Eq.(3.3.1), and Eq.(3.3.3), and the rings are axially located at antinodes of the targeted modes. The treatment parameters can then be adjusted by iteration to increase the performance. This design procedure will be referred to as the “manual” design procedure.

As an example, the following treatment is designed to control sound transmission into the Boeing cylinder due to an incident plane wave ($\alpha_i=70^\circ$, $\theta_i=0^\circ$) in the 50-160Hz band. The constraints are set to 4% of the cylinder mass for the DVAs and 5% of the cavity's volume for the HRs. Figure 3.37 shows the structural kinetic and the acoustic potential energy for the bare cylinder with initial acoustic and structural damping ratios of 1%. Because the acoustic response is dominated by 5 resonances, the treatment is broken into 5 rings of 7 HR each. From the structural response, only 2 rings of 13 DVAs compose the treatment. The HR tuned to $n=0$ acoustic mode can be placed anywhere axially. However, they are deliberately placed in the middle of the cylinder corresponding to node of $n=1$ mode to avoid unfavorable modal interactions. Table 3.5 lists the parameters of the treatment and Figure 3.38 shows the narrow and third octave band acoustic potential energy attenuation. The amplitude of the first three acoustic modes are reduced by 15 dB

by the resonators, the DVAs improved the HR performance above 120Hz where the coupling between the 1,2 structural and 0,2,0 acoustic mode dominates the response. In addition, because the cylinder is so light, the amplitudes of these two particularly well-coupled modes are similar in both oscillators i.e. the structure and the cavity. As a consequence, the attenuation provided by the HRs on the acoustic cavity is also passed on the structure as shown in Figure 3.39. However, since the force is applied to the structure, the structure resonance amplitudes are more likely to be present in the acoustic response than acoustic resonance in the structure response. Hence for heavier structures, DVAs are more efficient in reducing the acoustic level than HR in reducing the vibration level. Overall the treatment reduces the transmitted noise by 6.7dB across the 50-160Hz band. This performance is also evenly distributed across the band as each third octave presents a least 6dB of attenuation.

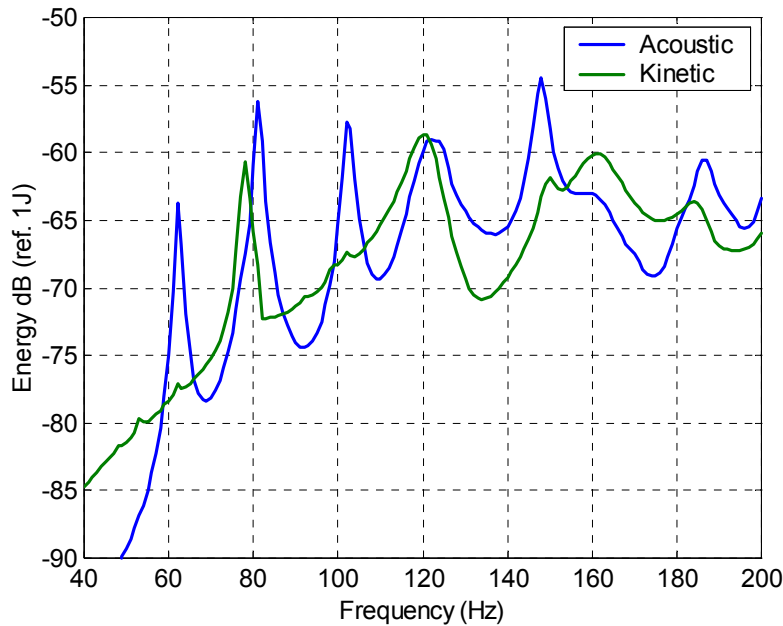


Figure 3.37 Acoustic and kinetic energy of the bare cylinder including acoustic re-radiation.

Rings of 13 DVAs 4% of Mass=75Kg				
Targeted mode	Tuning frequency (Hz)	Mass ratio (%)	Damping ratio (%)	Axial position (m)
(1,2)	120	2	8.7	1.45
(1,2)	151	2	8.7	1.45
Rings of 7 HRs 5% of Volume=13m ³				
Targeted mode	Tuning frequency (Hz)	Volume ratio (%)	Damping ratio (%)	Axial position (m)
(1,0,0)	62	1	9.2	2.74
(0,1,0)	81	1	7.8	1.37
(1,1,0)	102	1	10.9	0.015
(0,2,0)	120	1	8.6	1.37
(1,2,0)	147	1	12.0	2.71

Table 3.5 Parameters of the “manual” treatment design.

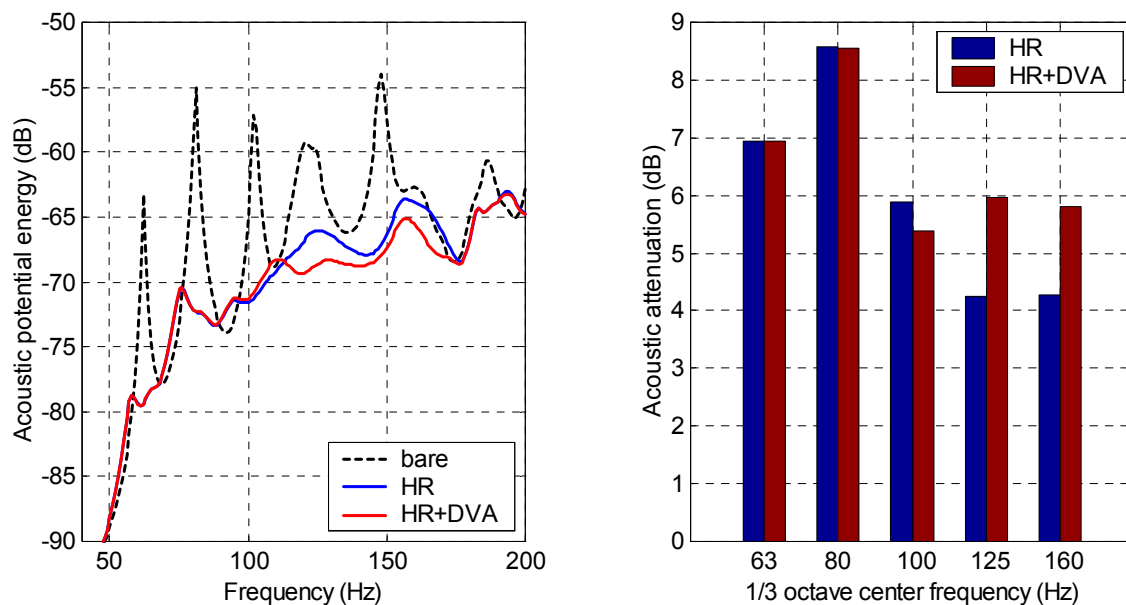


Figure 3.38 Narrow and third octave acoustic energy reduction for the “manual” treatment design.

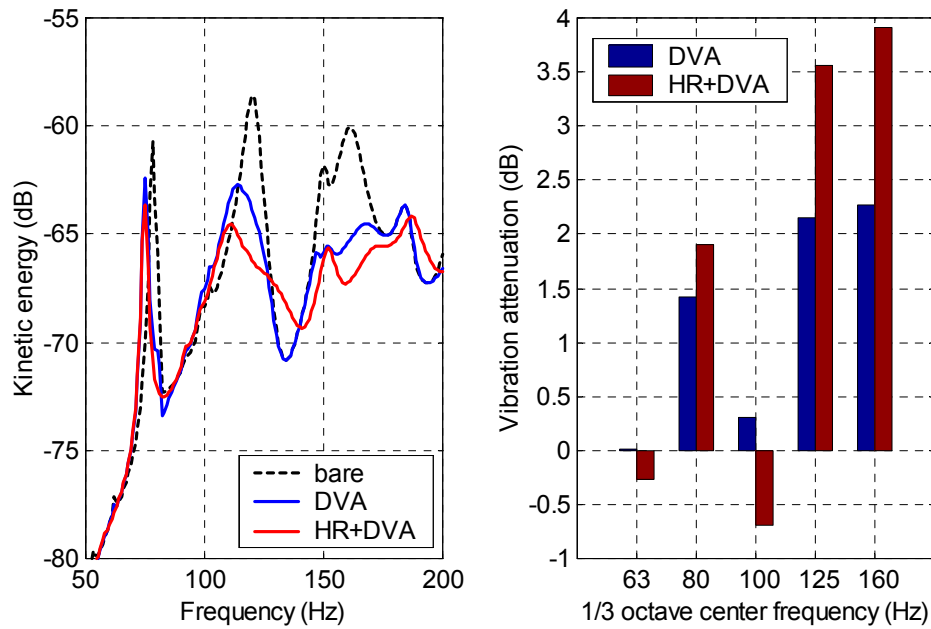


Figure 3.39 Narrow and third octave band kinetic energy reduction for the “manual” treatment design.

Influence of the initial structural and acoustic damping.

As explained previously, the DVA and HR noise reduction mechanism is based on adding damping to sharp structural and acoustic resonances responsible for the majority of the interior noise. Therefore, such treatment can be adapted to different type of excitation by targeting in each different case the unfavorable resonances. However, the performance of these devices is directly related to the amount of damping initially present in the structure and in the acoustic cavity. The higher the damping ratios of the system, the less reduction the treatment can provide. This is illustrated by Figure 3.40, which displays the noise attenuation in the 50-160 Hz band provided by the “manual” treatment described in Table 3.5 for different structural and acoustic initial damping ratios ranging from 0.1 to 30%. The green line outlines the linear decaying trend of the acoustic attenuation in decibel with the logarithm of the initial damping ratios of the system.

When the structure and the acoustic cavity are lightly damped, damping ratios below 3%, the DVAs and HRs can provide significant noise attenuation, above 6dB.

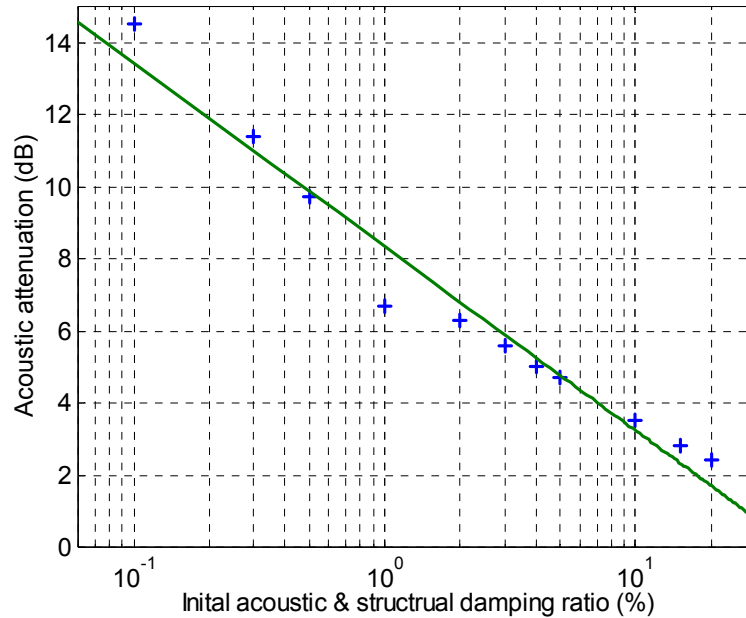


Figure 3.40 Acoustic attenuation as a function of the initial acoustic & structural damping ratio for the treatment in Table 3.5 with adjusted optimal damping ratios.

Treatment trade-off

Once the damping, the frequency, and location of the devices are optimized, the performance of a treatment can only be improved by increasing both the total mass of DVAs and total volume of HRs as explained in section 3.3.3. Using the treatment parameter listed in Table 3.5, the mass of the DVA and volume of HR was varied and the optimal damping ratios adjusted accordingly. The acoustic attenuation in the 50-160Hz band obtained with the different treatments are listed in Table 3.6.

50-160Hz Attenuation (dB)	Mass of DVA (% of cylinder's mass)		
Volume of HRs (% of cavity's volume)	2%	4%	8%
2.5%	5.0	5.3	5.8
5%	6.3	6.7	7.2
10%	7.9	8.5	9.1

Table 3.6 Acoustic attenuation in the 50-160 Hz band obtained with the treatment in Table 3.5 for different total mass of DVAs and total volume of HRs with optimal damping ratios computed accordingly.

3.4.2 Treatment design using a genetic algorithm

Due to the complexity of the system with respect to the different treatment parameters the cost function surface is hyper dimensional and presents a multitude of local minima. Indeed, it was shown in reference 82 that even for the simple case of two resonators coupled to a multi-mode system, the cost function presents two localized minima. This particularity of the system makes the optimal solution difficult to find. Gradient-based search algorithms, which move from point to point in the steepest descent direction, are thus ineffective. In addition, the fairly large number of parameter renders the exhaustive search of all possible solution impractical. In order to find the optimal treatment parameters a genetic algorithm was used. Genetic algorithms, which are based on the natural selection and genetics principle, have demonstrated their efficiency in the optimization of multi-modal search problem⁸³. All the design parameters to be optimized form a solution or individual. The starting point is the creation of a set (generation) of different individual whose parameters are chosen randomly within given limits. The value of the cost function obtained for each solution defines the individual fitness. Depending on their fitness, the best individuals are probabilistically selected to reproduce in the next generation. These particular individuals (parents) undergo different genetic reproduction functions such as crossover (exchange of parameters between two

individuals), mutations (random variation among one individual) and generate a second generation with an improved averaged fitness. The process repeats itself with the new generation until a termination criterion is met usually a specified maximum number of generations. In general, the genetic algorithm forces the majority of a generation to converge to a single solution.

A detailed analysis of genetic algorithms being beyond the scope of this work, an off-the-shelf genetic optimization Matlab toolbox obtained from reference 84 was used. The algorithm's cost function is defined as the attenuation across the desired frequency band of control. This cost function is only used in the algorithm to rank the different individual. The probability of each individual to be selected is derived using a normalized geometric ranking⁸⁴. The optimization is run twice with identical inputs to verify the validity of the solution. For both runs, the parameters of the treatment listed in Table 3.5 namely the tuning frequency damping ratio and axial location of the devices are optimized over 300 generations of 80 individual each. This represents a search over 24000 different sets of parameters. The convergence of the solution is illustrated in Figure 3.41, which plot for each generation the best and the averaged attenuation. Both runs converge at similar rate to the same optimal attenuation. Figure 3.42 shows the narrow and third octave band acoustic reduction obtained with the run#1 genetically optimized treatment. The attenuation from 50-160Hz of 9.1dB represents a 2.4dB improvement from the "manual" treatment design. However, the performance of the genetic solution is not as equally shared across the third octave frequency bands. This is due to the definition of the cost function, which tends to flatten the response rather than targeting individual peaks. Another difference is the focus of all the DVAs on only the 125Hz frequency band as shown in Figure 3.43, which results in a 4dB enhancement of the HR performance in this band. Using different mass of DVAs and volume of HRs, the noise attenuation obtained with the genetic algorithm is compared in Table 3.8 to the performance of the "manual" treatment design. It is interesting to notice the performance enhancement of the genetic solution over the manual one increases with the DVA mass and HR volume.

Rings of 13 DVAs 4% of Mass=75Kg				
Ring#	Tuning frequency (Hz)	Mass ratio (%)	Damping ratio (%)	Axial position (m)
1	113/113.5	2	4.3/5.3	1.57/1.58
2	122/124	2	4.2/3.3	1.58/1.58
Rings of 7 HRs 5% of Volume=13m ³				
Ring#	Tuning frequency (Hz)	Volume ratio (%)	Damping ratio (%)	Axial position (m)
1	84/86	1	10/7	2.74/2.75
2	115.6/117.6	1	15/10	2.73/2.72
3	142.6/149.6	1	8.6/7	1.54/1.63
4	144/154.6	1	10/18	0.03/0.0
5	146.7/156.6	1	7.7/9.3	2.71/2.75

Table 3.7 Parameters of the genetically optimized treatment for run#1 (in blue) and run#2 in red.

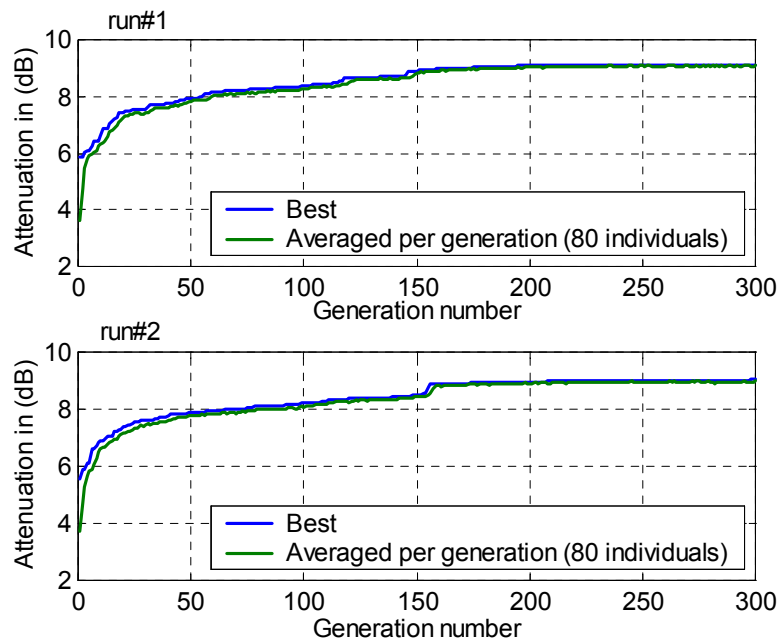


Figure 3.41 Convergence of the genetic algorithm for run#1 and 2.

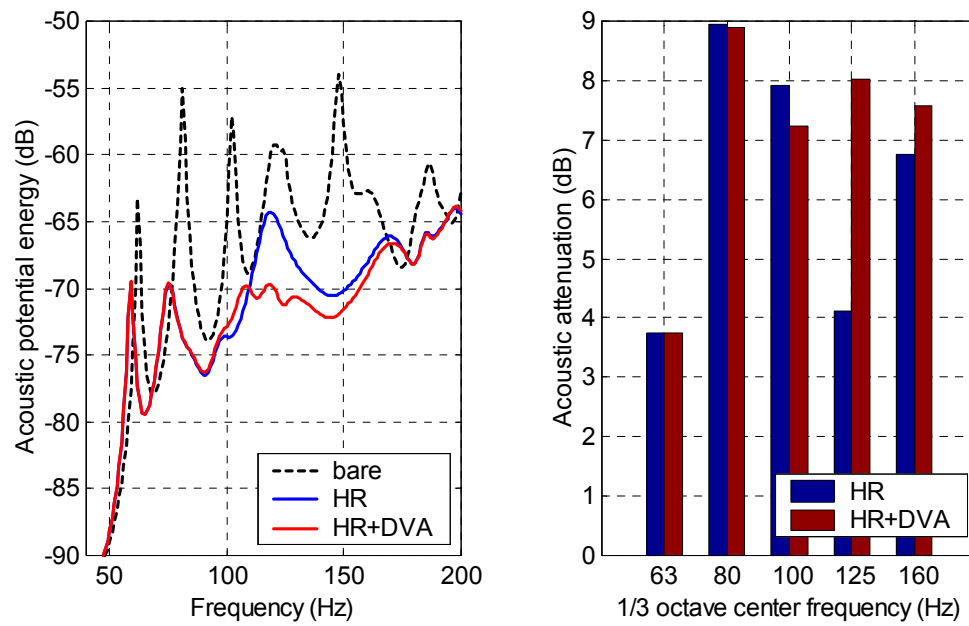


Figure 3.42 Narrow and third octave acoustic energy reduction for a genetically optimized treatment.

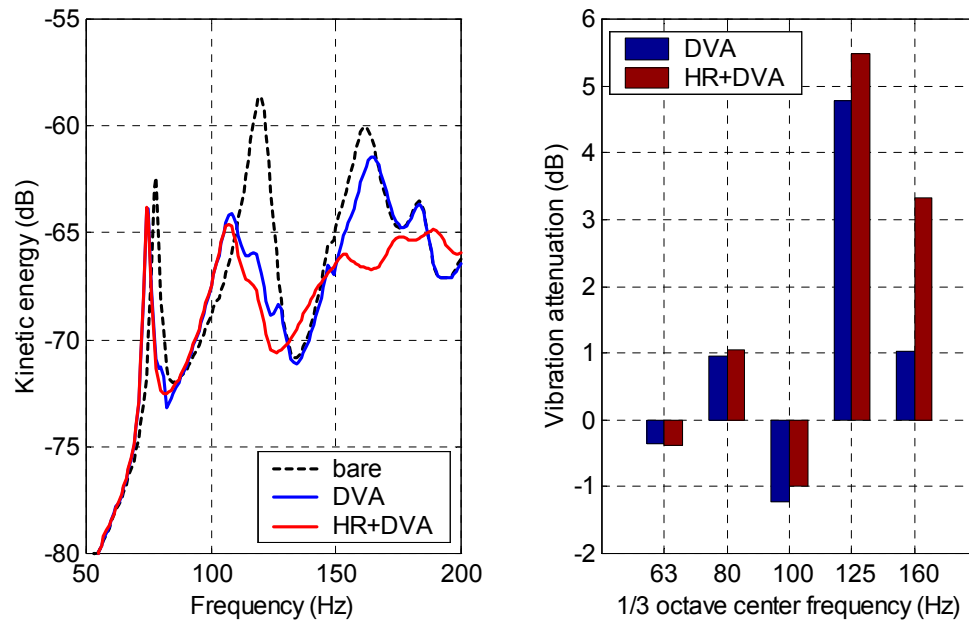


Figure 3.43 Narrow and third octave band vibration attenuation for the genetically optimized treatment.

Volume of HRs (% of cavity's volume)	Mass of DVA (% of cylinder's mass)	50-160Hz Attenuation (dB)	
		"Manual"	Genetic
2.5	2	5.0	6.8
5	4	6.7	9.1
10	8	9.1	12.2

Table 3.8 Performance comparison between the "manual" treatment in Table 3.5 and the genetic algorithm solution for different mass of DVAs and volume of HRs.

To verify that the number of different tuning frequencies allowed in the treatment does not limit the optimal performance, another optimization was done. In this case, the treatment is composed of twice as many rings as in run#1 but with the same overall DVA mass and HR volume. The convergence of the solution to 9.0 dB is illustrated in Figure 3.44 and the treatment parameters are listed in Table 3.9. Although a couple of HR rings

seems to be the duplicate of each other as rings 2 and 3 and rings 9 and 10, overall, the tuning of the devices spread across the band. However, the axial position of the rings corresponds consistently in every run to antinodes of the modes. It is interesting to notice that the increase in possible tuning frequencies did not increase the performance of the treatment for this cylinder within this frequency range.

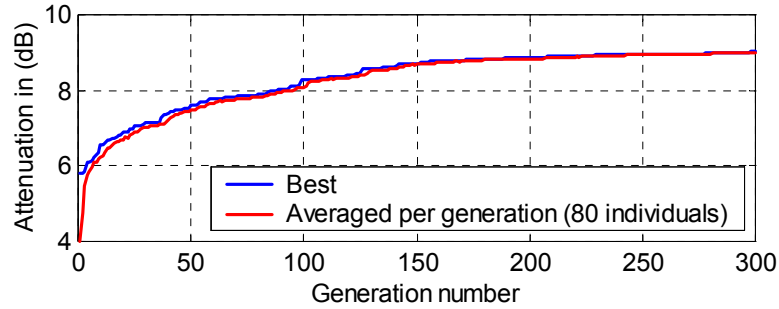


Figure 3.44 Convergence of the genetic algorithm for run#3.

Rings of 13 DVAs 4% of Mass=75Kg				
Ring#	Tuning frequency (Hz)	Mass ratio (%)	Damping ratio (%)	Axial position (m)
1	99.5	1	24.1	1.59
2	111.5	1	6.1	1.44
3	125.6	1	2.9	1.60
4	140.8	1	5.6	1.70
Rings of 7 HRs 5% of Volume=13m³				
Ring#	Tuning frequency (Hz)	Volume ratio (%)	Damping ratio (%)	Axial position (m)
1	86.0	0.5	9.0	2.74
2	102.3	0.5	17.1	2.74
3	109.0	0.5	18.4	2.74
4	116.3	0.5	17.4	2.74
5	118.6	0.5	5.0	1.45
6	138.8	0.5	4.4	1.58
7	145.0	0.5	12.7	2.73
8	152.8	0.5	8.4	0.02
9	156.8	0.5	8.9	2.68
10	159.7	0.5	6.5	2.71

Table 3.9 Parameters of the genetically optimized treatment for run#3.

To investigate this matter even further, a final genetic optimization was done. This case, called run#4, uses the same number of devices, 36 DVAs and 35 HRs as for run#1 and #2. However, the imposed ring positioning is no longer required and thus each device becomes completely independent. Choosing the tuning frequency, damping ratio, axial and angular locations for each device represent an optimization of 244 parameters. Even though 600 generations of 80 individuals are used to search over 48000 different sets of parameter, the solution has not quite converged as shown in Figure 3.45. With 10.7dB, the final solution slightly out performs the treatments of both run#1 and run#2 that use rings of device. The main reason is the chosen angular location of the devices as illustrated in Figure 3.46. The DVAs orient mostly with the azimuth angle of the incident wave representing a maximum value of all the mode shapes. In addition, DVAs and HRs tuned to the 110-150Hz-frequency band dominated by the coupling of circumferential mode of order 2 are positioned at the according antinodes. Figure 3.47 illustrates further this dependence of the solution with respect to the wave azimuth angle by plotting the treatment attenuation as a function this angle. Therefore, to obtain a robust treatment, the use of device in rings has to be maintained.

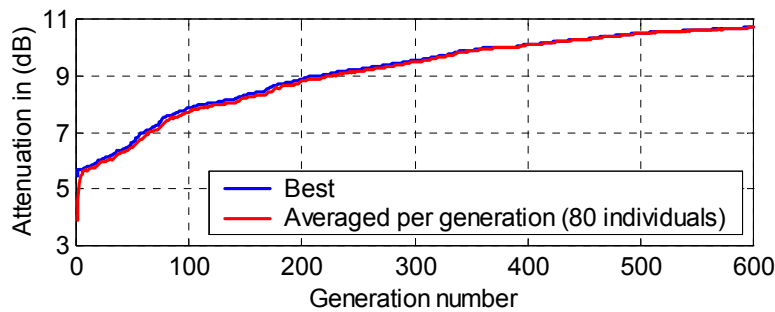


Figure 3.45 Convergence of the genetic algorithm for run#3.

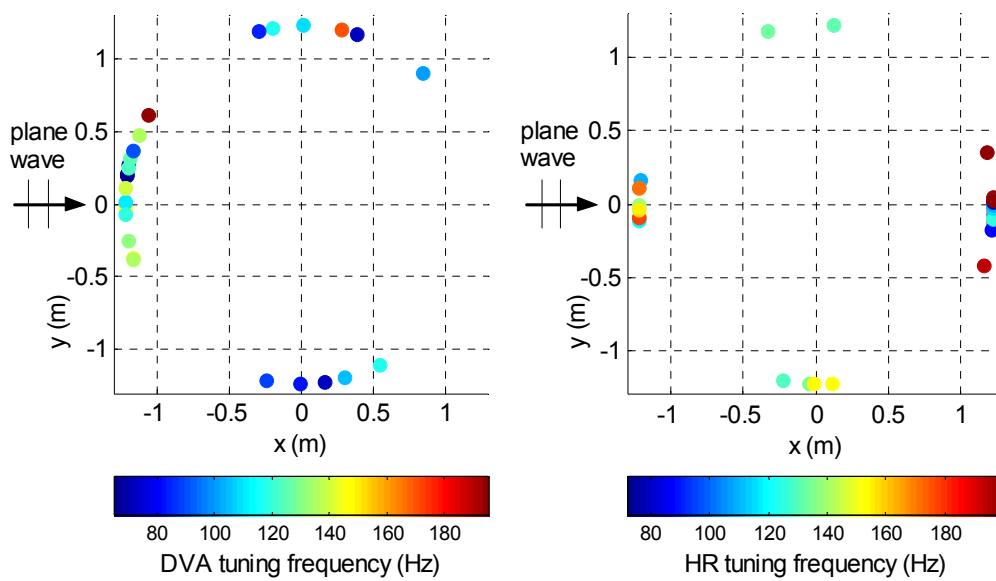


Figure 3.46 Position of the DVA and HR tuning frequencies around the cylinder circumference for the genetic solution of run#4.

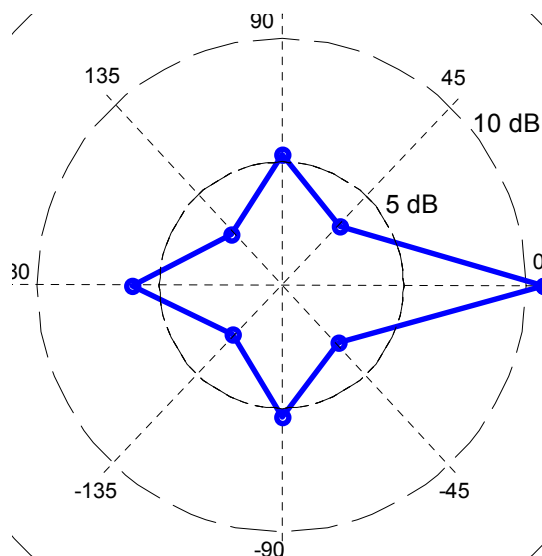


Figure 3.47 Noise attenuation of the run#4 treatment as a function of the azimuth angle θ_i of the incident wave.

3.4.3 Performance robustness to uncertainty in the excitation, system and treatment

As illustrated with the genetic results presented previously, the solution to the optimal noise reduction is not unique. Using rings of devices ensures the treatment will perform identically regardless of the azimuth angle of the incident plane wave. To check the treatment robustness to the elevation angle, the performance of the run#1 solution genetically optimized for an angle of 70° is evaluated for other angles. The resulting attenuation is then compared to the optimal performance achievable at each angle. Figure 3.48 illustrates these results and demonstrates the performance robustness of a combined HR/DVA treatment to the elevation angle of the incident wave. For all angles ranging from 25° to 85° , the treatment optimized form 70° remains within 1dB of the optimal solution. This result is to be expected since the main mechanism of control is damping and not modal restructuring which is more sensitive to changes in the primary excitation.

It was demonstrated in 3.4.1 that the effectiveness of a HR/DVA treatment depends on the amount of damping initially present in the system. To evaluate the performance sensitivity to another system variation, the treatment obtained in run#1 is evaluated with the structural and acoustic natural frequencies of the cylinder perturbed. This perturbation is obtained using a uniform random distribution constrained within a certain percentage of the modes initial natural frequency. Averaging over 20 different systems with permutations up to 20%, the treatment averaged attenuation decrease from 9.2dB to 8.0dB with a variance of 1.3dB. When reduced to perturbations of 10%, the performance is almost unaffected with an averaged attenuation of 8.8dB of variance 0.2dB. These results show the fairly good robustness of an HR/DVA treatment to random variations in the system natural frequencies.

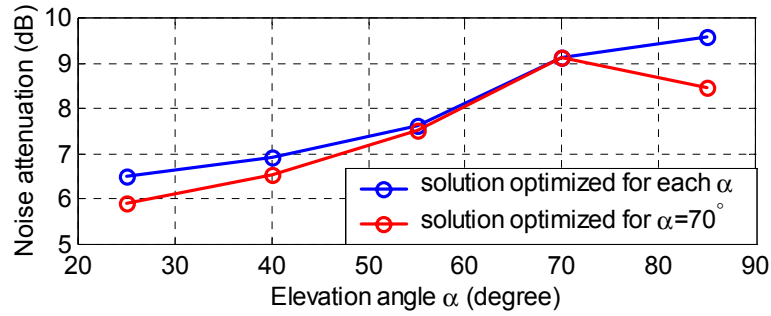


Figure 3.48 Robustness of the treatment with respect to the elevation angle of the incident plane wave.

To assess the robustness to variation in the treatment parameters, the solution obtained in run#1 is randomly perturbed and the performance averaged over 20 cases. The tuning frequencies and damping ratios of the devices are perturbed using uniform random variations around their nominal value. For variation up to 40% of the initial value of their tuning frequencies the average attenuation only drops by 2dB to 7.2dB and remains within 0.2dB of the optimal value for perturbation within 10%. The treatment robustness to the damping of the devices is even stronger as the averaged attenuation is almost unaffected (within 0.1dB) with variations of up to 40%. The performance is slightly more sensitive to the HR and DVA axial locations as the averaged attenuation obtained by randomly placing the devices along the cylinder length is 6.6dB.

3.4.4 Performance of random treatments

The performance of treatments where the tuning frequency, damping ratio and axial position of the devices are arbitrarily set is given by the first generation obtained with each genetic optimization. The first generation is indeed constituted of 80 solutions with parameters randomly chosen within the given limits. For all the runs performed, the tuning range of the device was set from 40Hz to 200Hz, the damping ratios ranged from 0.1% to 100% and the axial position could vary along the total length of the cylinder. As

seen in Figure 3.41 and Figure 3.44, the averaged attenuation over these 80 random solutions is around 3.5dB. Out of these 80 random treatments, the best one achieved an attenuation of 5.5dB. This shows that with enough devices that at least equal the number of the lightly damped resonances in the frequency band of control, a HR/DVA treatment designed randomly might provide some attenuation. However, as Figure 3.41 and Figure 3.44 illustrate there is room for optimization. Treatments designed using some knowledge of the system can indeed perform better than random ones while remaining quite robust to changes in the excitation, the system or in treatment parameters.

3.5 Treatment design for the payload fairing

In this section the design process developed previously is applied to the much larger real size fairing. Manually and genetically design treatment are discussed and the performance and robustness are addressed.

3.5.1 The “manual” solution

Using a similar approach to the cylinder treatment design, the parameters are chosen by examining the fairing response to the acoustic excitation described in section 3.2.5. In the defined 30-90Hz frequency band of control, the number of dominating acoustic peaks appearing in Figure 3.25 sets the number of HR rings to 8. The number of DVAs ring is set to 4 each representing 1% of the fairing mass. The total volume of the HR treatment is restricted to 2% of the fairing volume. The number of device per ring is 36 in order match with the finite element mesh of the structure and acoustic cavity. To position each ring at the modal antinodes, each dominating mode is visualized and identified using and equivalent cylinder mode order. To avoid a concentration of the devices on one antinode common to several modes, compromises are made to ensure a different axial position for each rings. Using the approximate mode orders, the optimal damping ratios of the devices

are obtained using the derivation presented in section 3.3.1. Table 3.10 lists the parameters of the resulting treatment.

Rings of 36 DVAs 4% of fairing mass				
Targeted mode	Tuning frequency (Hz)	Mass ratio (%)	Damping ratio (%)	Axial position (m)
(3,3)	52	1	7.0	2.0
(1,5)	56	1	7.0	3.9
(3,2)	78	1	7.0	7.0
(6,4)	93	1	7.0	10.0
Rings of 36 HRs 2% of fairing volume				
Targeted mode	Tuning frequency (Hz)	Volume ratio (%)	Damping ratio (%)	Axial position (m)
(0,1,0)	37	0.25	7	0.15
(1,1,0)	44	0.25	9	7.50
(2,1,0)	53	0.25	9	4.44
(0,2,0)	60	0.25	8	1.37
(3,1,0)	65	0.25	9	11.2
(1,2,0)	70	0.25	12	6.48
(2,2,0)	75	0.25	12	9.20
(3,2,0)	85	0.25	12	10.0

Table 3.10 Parameters of the “manual” treatment design.

Figure 3.49 shows the internal SPL in narrow and third octave band for the HR and combined HR+DVA treatment. In the four third octave bands comprised within 30 and 90Hz, the acoustic attenuation exceeds 3dB. The HR treatment achieved reductions up to 10dB at the acoustic peaks, and the 100Hz third octave band although outside the band of control presents some attenuation. The overall noise reduction in the 30-90Hz band obtained with the HR treatment is 3.5dB. The DVA treatment improves this performance to 3.9dB, which is equally spread across the band. The vibration attenuation illustrated in Figure 3.50, shows the lower coupling level between the acoustic cavity and the structure than for the cylinder, as the HRs do not significantly affect the vibration levels. This lower level of coupling between the cavity and the structure is in part due to the surface density of the fairing, which is more than twice the surface density of the cylinder.

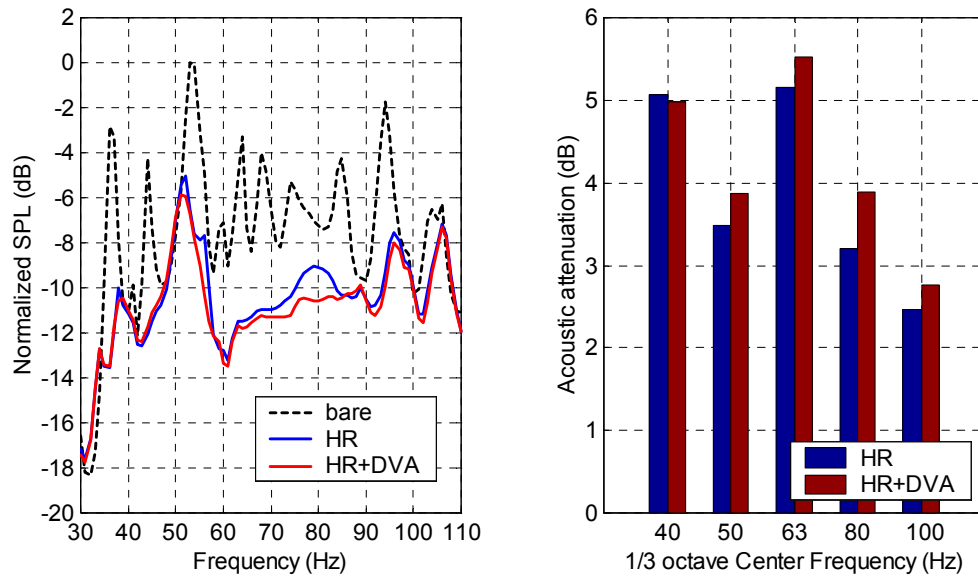


Figure 3.49 Narrow and third octave fairing internal SPL reduction for the “manual” treatment design.

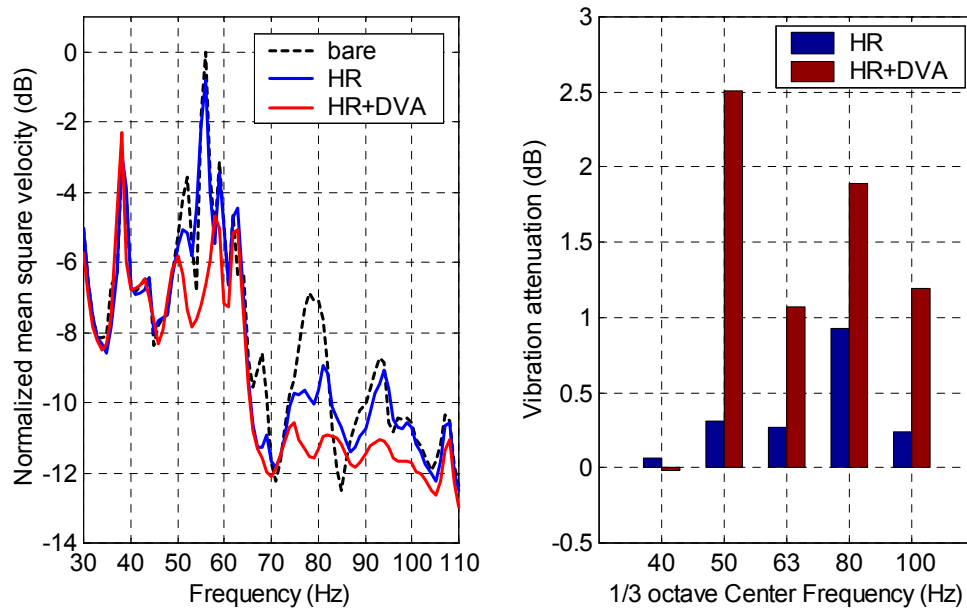


Figure 3.50 Narrow and third octave fairing vibration reduction for the “manual” treatment design.

3.5.2 Genetic algorithm solution

In order to compare the performance with the optimal solution, the tuning frequency, damping ratio and axial position of the manual treatment were optimized using the genetic algorithm. The optimization was carried over 300 generations of 80 individual each representing a comparison over 24000 different solutions and the cost function was the acoustic attenuation in decibel over the 30-90Hz band. Table 3.11 lists the parameter of the final solution and Figure 3.51 shows the convergence of the search. Attenuation of 5.7dB for the combined HR+DVA treatment and 4.8dB for the HRs only were achieved. Figure 3.52 and Figure 3.53 show the acoustic and vibration narrow band responses as well as the attenuation in third octave bands. Across the band of control the third octave band the acoustic attenuation exceeds 4dB and reached 6.5 dB for the 50Hz and 63Hz band. The tuning frequency of the devices corresponding approximately to the “manual” treatment, this increase in performance may be due to slightly smaller damping ratios but more likely to the axial positioning of the rings. As opposed to the “manual” treatment, no constraint was implemented in the genetic algorithm in terms of placing the rings at the same axial location. Consequently the HR sets#1 and #5 were both placed at the bottom of the faring and set#2 and #3 in the cone section resulting in better attenuation in the 50Hz and 63Hz third octave bands. Through lower damping ratios and different axial positions, the genetic optimization also finds a better use of the DVAs as they enhance the HR performance by 18% compared to 9% for the manual treatment.

Rings of 36 DVAs 4% of fairing mass				
Set#	Tuning frequency (Hz)	Mass ratio (%)	Damping ratio (%)	Axial position (m)
1	51.1	1	2.0	5.71
2	55.5	1	2.0	6.23
3	73.4	1	2.3	2.29
4	81.9	1	5.3	6.73
Rings of 36 HRs 2% of fairing volume				
Set#	Tuning frequency (Hz)	Volume ratio (%)	Damping ratio (%)	Axial position (m)
1	37.7	0.25	8.1	0.15
2	48.9	0.25	5.9	9.56
3	53.6	0.25	5.6	9.56
4	60.9	0.25	5.6	5.70
5	63.7	0.25	12.3	0.15
6	67.8	0.25	5.5	7.00
7	79.0	0.25	5.9	8.00
8	87.0	0.25	5.0	6.73

Table 3.11 Parameters of the genetically optimized treatment.

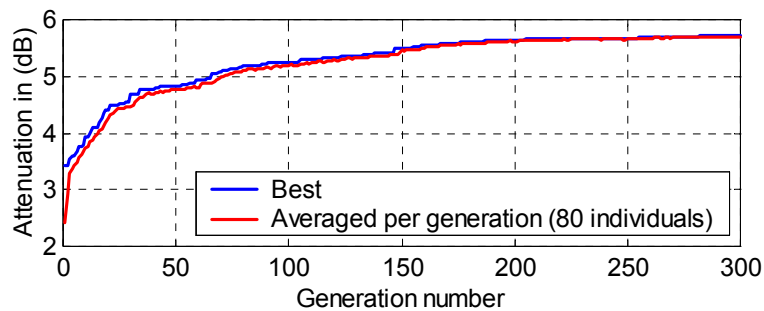


Figure 3.51 Convergence of the genetic algorithm.

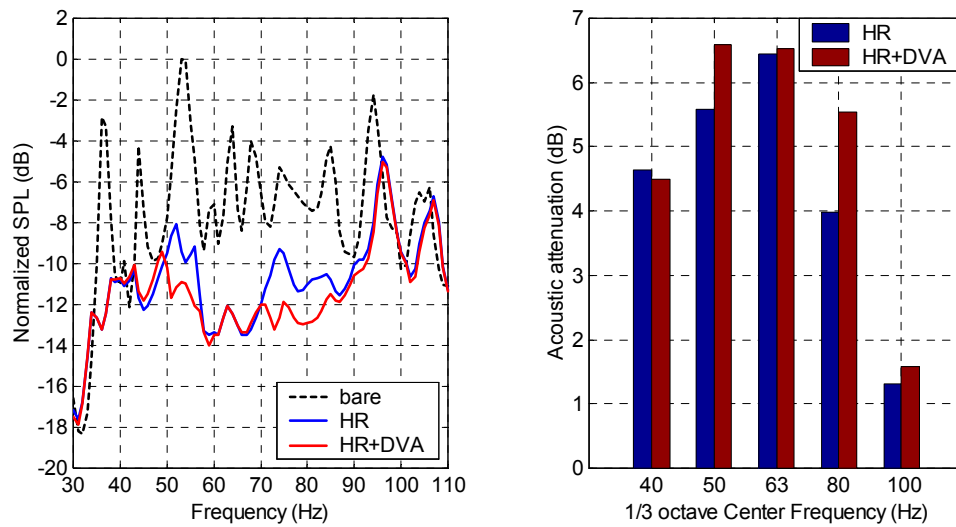


Figure 3.52 Narrow and third octave fairing internal SPL reduction for the genetically optimized treatment.

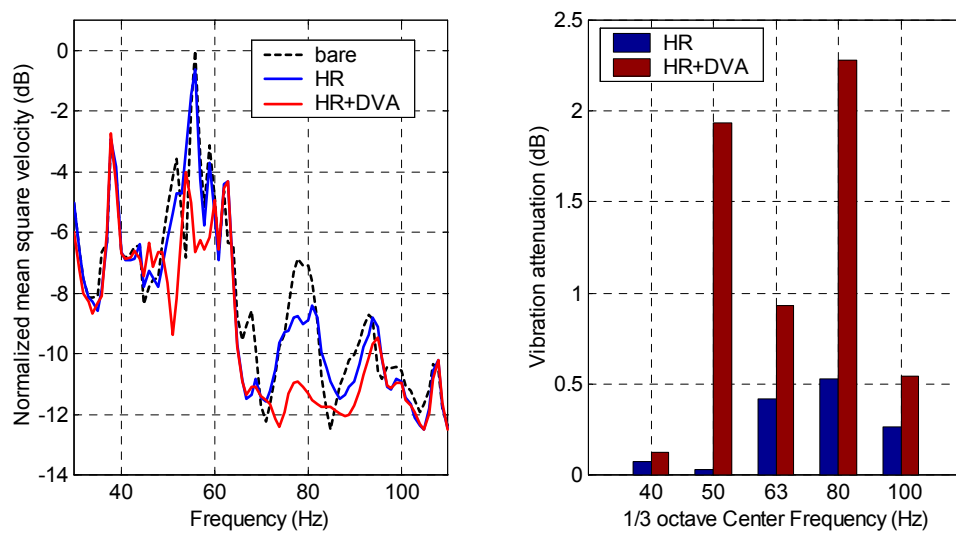


Figure 3.53 Narrow and third octave fairing vibration reduction for the genetically optimized treatment.

3.5.3 Performance robustness to variation in the excitation, system and treatment.

To check the robustness of the genetic solution to changes in the disturbance speaker locations, the performance of the treatment is evaluated for four different external fields. Figure 3.54 shows the location of the speakers for the different cases and the resulting attenuation provided by the treatment. Setup#1 corresponds to the initial configuration used in the genetic optimization, which uses 12 monopoles. This particular configuration was chosen to approximate the experimental setup detailed in section 4.4. Setup#3 and #4 represents slight variations around the nominal position, whereas setup#2 and setup#5 use only 3 and 1 monopole respectively. The corresponding attenuation listed in Figure 3.54 shows the robustness of the treatment to changes in the acoustic external field.

To assess the sensitivity of the solution to variations in the system, the same method used for the cylinder is applied. It consists in perturbing the structural and acoustic natural frequencies of the fairing within a certain percentage of their initial value with a uniform random distribution. To evaluate the robustness of the performance to treatment variation, the tuning frequency and damping are also perturbed using this method. Averaging over 20 instances. The reference unperturbed treatment is the genetically obtained design which achieved acoustic attenuation of 5.7dB in the 30-90Hz band and whose parameters are listed in Table 3.11. The acoustic attenuation resulting from different level of variations for each case are detailed in Table 3.12. A first remark is that the performance of the treatment is more sensitive to changes in the resonant frequencies of the system than in the variation in devices' tuning frequency. This is due to the difference in the number of devices (288 HRs and 144 DVAs) and the number of important acoustic and structural resonances (less than 20). When a resonance is shifted a whole set of HRs or DVAs becomes detuned whereas the probability of having all the devices targeting a specific mode detuned at the same time is very small. A second remark is that as for the cylinder case, the performance sensibility to variation in the devices' damping ratio is very small. In addition to the aforementioned probabilistic effect due to the very large number of devices, the amount of attenuation is a fairly weak function of the device's damping ratio as it was mentioned in section 3.3.1. Therefore,

variations of 40% around the optimal value do not translate to dramatic loss of performance. It is also interesting to note that the performance sensitivity to axial positions of the devices is similar for both the cylinder and fairing case. Indeed, the averaged attenuation obtained by randomly placing the devices along the fairing length drops by 2.7dB from the optimal attenuation, representing in both case a reduction of the performance by approximately half. Finally, these results demonstrate, as it was the case with the cylinder, the robustness of the treatment to changes in the system and device characteristics.

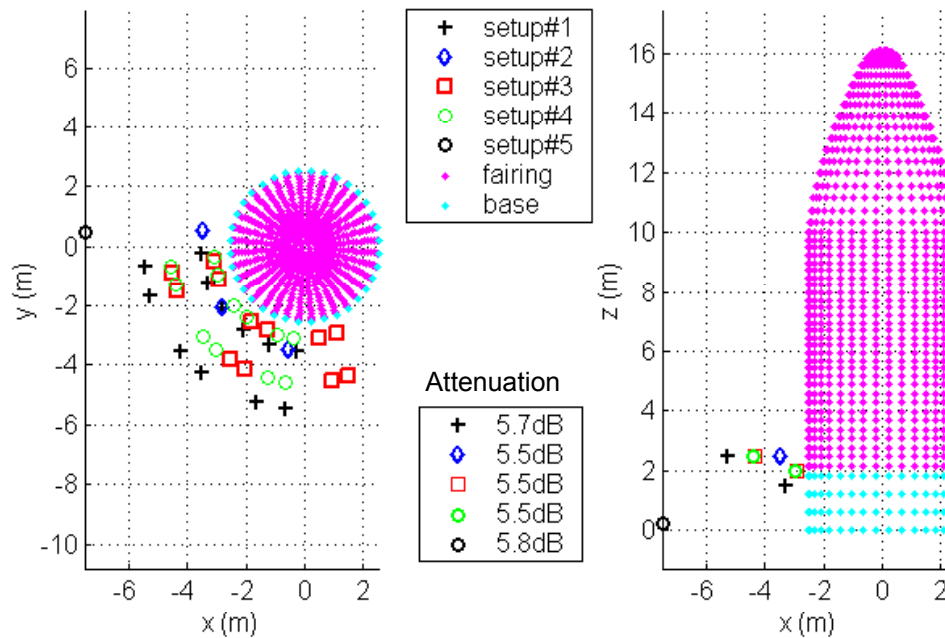


Figure 3.54 Fairing top and side view for different disturbance speaker configurations and the resulting attenuation obtained with genetic solution.

Attenuation (dB) (Unperturbed: 5.7dB)		Variable parameter		
		System natural frequencies	Device tuning frequency	Device damping ratio
Variation	$\pm 10\%$	4.9	5.5	5.6
	$\pm 20\%$	4.4	5.1	5.6
	$\pm 40\%$	3.6	4.4	5.6

Table 3.12 Performance robustness to changes in the system and treatment characteristics.

3.5.4 Performance of random treatments

As for the cylinder, the results for the first generation of the genetic optimization are used to illustrate how arbitrarily designed treatment perform. Figure 3.51 shows that 80 different treatments with random tuning, damping and axial positions yield an averaged attenuation of 2.4dB. The best out of these 80 different solutions achieves an attenuation of 3.5dB. The smaller difference in the performance between the random and the optimized treatment obtained with the fairing in comparison with the cylinder can be due to a higher number of devices with respect to the number of dominant resonances in the control band. However, as illustrated by Figure 3.51, some knowledge of the system can lead to treatment that greatly outperform random ones. In addition, as demonstrated in the previous paragraph, variation around the optimal treatment parameters to not reduced drastically the performance.

3.6 Conclusions

In this chapter, the analytical model of the sound transmission into an enclosed cylinder developed in Chapter 2 is validated experimentally. Using numerical simulations the noise transmission mechanisms were analyzed for the cylinder and for the fairing case. After the description of the noise control mechanisms of the DVA and HR, a

strategy for designing acoustic treatments with these structural and acoustic devices was developed. The results obtained on both the cylinder and the fairing model, were compared to the performance of treatments optimized with a genetic algorithm. Finally, the robustness of the solutions with respect to variation in the excitation, system characteristics, and in the devices parameters was addressed.

Using the cylinder model, the characteristics of the sound transmission through this particular structure were highlighted. The behavior of this system, composed of two resonant sub-systems namely the structure and the acoustic cavity, is dictated first by the coupling of the disturbance acoustic field with the structure (external coupling) and second by the coupling between the structure and the acoustic cavity (internal coupling). The external coupling presents a filtering effect of the higher circumferential order structural modes due to the mismatch of the wavenumber in the structure and in air. This effect is stronger for axisymmetric structures as spill over of the higher wave numbers only occurs in their axial direction, as opposed to plates or curved panels, which present this spill over in both directions. This filtering effect is important as it permits a reduction in the number of modes necessary to describe the sound transmission.

The internal coupling is both spatial and frequency related. Spatially, it represents the matching level between the structural and the acoustic modes of the cavity over the surface of the cylinder, and is also particular to axisymmetric system. Indeed, only modes with equal circumferential orders and with axial orders in an even-odd (or odd-even) combination will couple resulting in a sparse coupling matrix between the two modal systems. However, experimentally, the acoustic transmission proved to be sensitive to imperfections in the mode shapes and boundary conditions that couple all the modes together. Consequently, a better sound transmission prediction was obtained when the zeros of the sparse coupling matrix were replaced by small random numbers to simulate this cross modal coupling. The level of internal coupling is also a function of the distance between the natural frequencies of spatially coupled modes. Acoustic modes with higher orders have higher natural frequency but for structural modes, because of the membrane effect, this relationship between natural frequency and mode order is distorted. The first acoustic modes are predominantly excited by structural modes with much higher natural

frequency resulting in a low frequency coupling. This situation was also observed in the fairing. However, for the smaller and lighter cylinder, the noise transmission is dominated by two spatially well-coupled modes with very close resonant frequencies. All these coupling mechanisms are important for the selection of a modal subset that leads to a good approximation of the system response with the minimum computation.

Finally, the damping of the structure and the acoustic cavity is an important parameter to evaluate the treatment performance. As one source of structural damping comes from energy loss by acoustic radiation, the effect of the fluid loading was implemented in both models. It was found that because the cylinder is such a light structure, the surrounding fluid add a lot of damping especially to the low circumferential order modes (up to 5%) and can also lower their natural frequency by acting as an additional mass. Including this commonly neglected effect helped to validate the model and allowed the performance of the acoustic treatment to be accurately predicted.

The noise reduction mechanisms of the control devices were then presented. Based on the principle of tuned-mass dampers, the damping of the HR and DVA can be optimized for broadband attenuation. By applying a multitude of these small devices onto a large system, sharp acoustic and structural resonance present in the frequency band of control can be damped thus reducing sound transmission. DVAs and HRs were hence built and applied to the cylinder in order to validate experimentally the model. For both the structural and acoustic response, good agreement with the model was observed.

After validating the model, a procedure to designing an acoustic treatment composed of both HRs and DVAs was then proposed and applied to both the cylinder and the fairing. The strategy was to target each of the dominant resonances present in the acoustic spectrum with the control devices. Each device was split into several units evenly distributed around the circumference to be independent of the azimuth angle of the disturbances and to avoid unfavorable modal interactions. The number of devices in each ring depended on the circumferential order of the targeted mode. Starting with the optimal solution derived in the literature for broadband control of a single mode, the optimal damping ratios were then adapted to the ring positioning of the devices. The axial location of the devices was chosen to correspond to an antinode of the targeted mode.

Fixing the mass of the DVAs and the volume of the HRs as these variable are directly linked to the device effectiveness, the treatment performance (defined as the noise attenuation in the control band) was then evaluated and compared to the optimal solution obtained with a genetic algorithm. For the cylinder, the genetic solution presented a 2.4dB improvement over the 6.7dB attenuation obtained by the “manual” design across the 50-160Hz band. However, the attenuation provided by the genetic solution was not equally shared across the third octave bands due to the cost function that tend to flatten the response. For the fairing, the genetic achieved a 2.2dB improvement over the “manual” design 3.5dB attenuation from 30-90Hz. In this case, because no particular band dominated the response, the manual and genetic performance were uniformly spread across the third octave bands. As the noise reduction mechanisms are based on adding damping to sharp resonances, the amount of damping initially present in the system greatly affects the performance of a treatment. Simulations showed the performance variation of the “manual” cylinder treatment as a function of the initial structural and acoustic damping ratios. It showed that for ratios below 3% the HR and DVA can provided significant noise attenuation, above 6dB.

Finally the robustness of the treatment to changes in the excitation characteristics, in the system natural frequencies, in the tuning frequency, damping ratio and axial location of the devices was addressed for both the cylinder and the fairing. For both systems, the performance proved to be robust to any combination of changes but particularly to the damping ratio of the devices. This robustness is inherent of the noise reduction mechanism, which is based on adding damping to lightly damped resonances of the system. Therefore, as long as the initial system is lightly damped the treatment will perform well. Due to the large number of devices applied to the system, the performance of random treatments i.e. where the axial locations, tuning frequencies and damping ratios of the devices are set arbitrarily within limits was investigated. It was showed that even if on average a random treatment can provide up to 3.5dB of attenuation, the proposed design procedure based on some knowledge of the system, can greatly out perform these random treatments. In addition, this optimal solution does not lie in a steep narrow valley of the cost function surface, and therefore was proved to be robust.

Chapter 4 Outdoor high level tests

4.1 Introduction

This chapter describes the two series of tests conducted at Virginia Tech on the Boeing cylinder and the final test conducted at the Boeing facilities in California on a launch vehicle fairing.

The objective of the cylinder tests was to demonstrate the performance of HR/DVA treatment under realistic launch conditions. As a consequence the experiments took place outdoors at the Virginia Tech airport as it provided the space and isolation necessary to conduct such loud and large-scale experiments. The disturbance noise level reached 130dB.

The first test in 2001, involved the empty cylinder, whereas in the second one in 2002, the cylinder was partially filled with a mock payload. The treatment, designed using the developed numerical code, was adjusted using preliminary experiments.

The objective of the fairing test organized in collaboration with the Boeing team, was to assess the performance of a pre-flight treatment. Prototypes of the noise reduction devices were designed at Virginia Tech under strict flight constraints, and then mass-produced and incorporated in the launch vehicle fairing by Boeing designers.

4.2 Empty cylinder test November 14th-30th, 2001

This section covers the outdoor test done on the empty cylinder under high-level acoustic excitation.

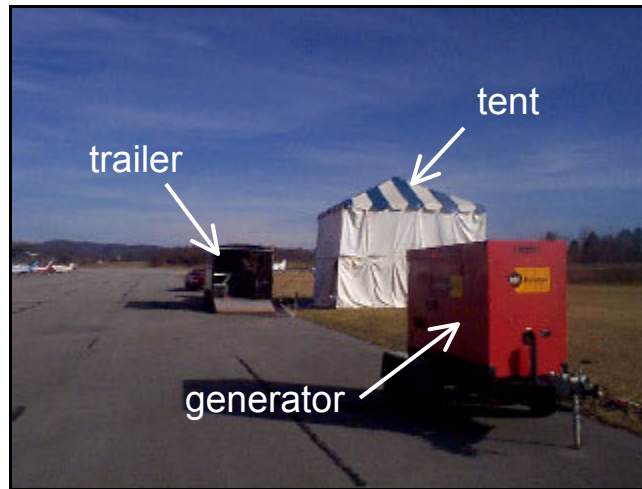
4.2.1. Test setup

The cylinder transported to the outdoor test site was craned onto a 2.5 ft wood base and rolled under a large tent as shown in Picture 4.1.



Picture 4.1 Cylinder being craned onto the base and then placed under the large tent at the outdoor test site.

A large speaker system was hired and set up on one side of the cylinder. The speaker system consisted of eight enclosures each with four 18 inch 400 watt drivers totaling a 12.8 Kilowatts capability. The speaker system was designed to work in the 40-250Hz range. The power amplification and measurement equipment was housed in a trailer placed next to the tent and the electrical power supplied by a generator (see Picture 4.2).



Picture 4.2 Test site showing the tent, equipment trailer and the power generator

As seen in Picture 4.3, the cylinder was instrumented with a ring of 30 accelerometers evenly spaced 40 inches up the cylinder and a line of 5 accelerometers running the length of the cylinder and facing the speaker array. Outside the cylinder 6 B&K microphones monitored the acoustic field exciting the cylinder. A set of 16 PCB microphones were placed on a tree inside the cylinder and rotated to measure a total of 32 interior positions. The top end cap was also instrumented with 4 accelerometers to measure the vibration and to assess the potential flanking (i.e unwanted sound transmission through the end cap). Figure 4.1 is a schematic showing all of the instrumentation positions and speaker positions used for this test.



Picture 4.3 Instrumented cylinder showing the accelerometers and one of the external monitoring microphones

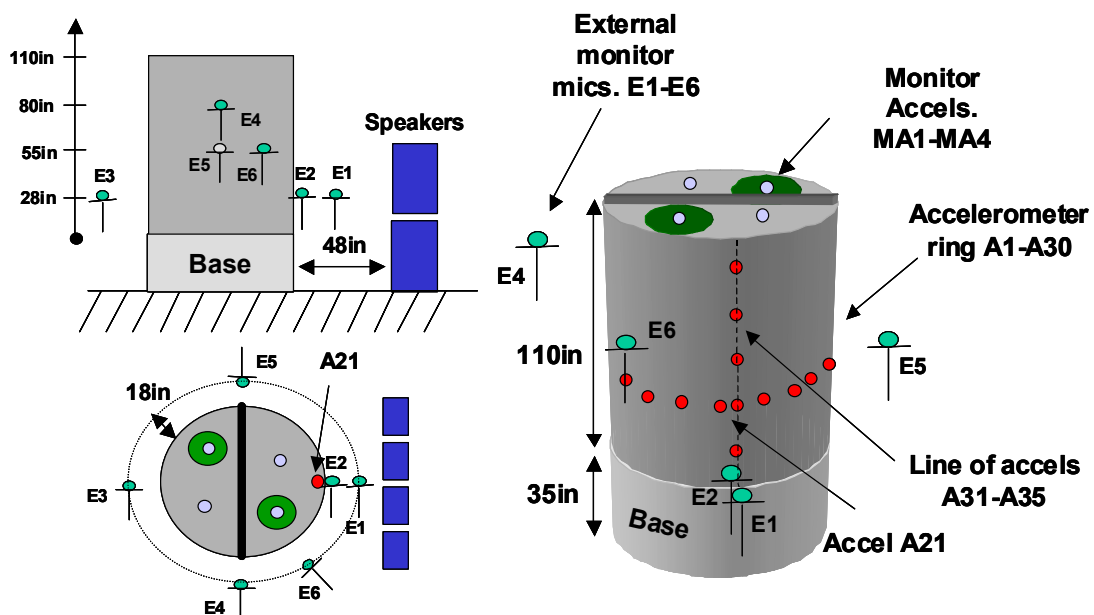


Figure 4.1 Schematic showing the positions of all of the measurement equipment.

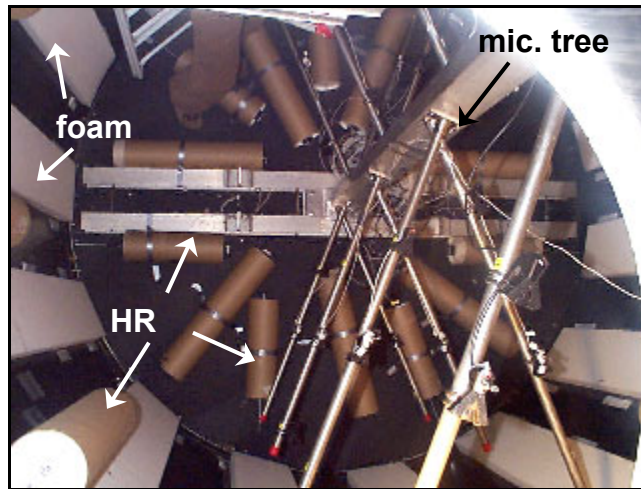
4.2.2. Noise reduction treatment

Helmholtz resonators

Five sets of Helmholtz resonators designed with cardboard tube as in section 3.3.4 were tuned using the experimental set-up described by Figure 3.35 to the first 5 acoustic modes of the cavity. Table 4.1 summarized the HR treatment characteristics, which were chosen using numerical simulations, and adjusted using preliminary experiments. The second set of HR was deliberately not attached to the sidewall of the cylinder but hung from the ceiling so that they did not affect the cylinder vibration directly. These HRs occupied a very small fraction of the total volume of the cylinder (about 2%).

HR set#	Targeted mode	Tuning Frequency (Hz)	Quantity	Volume ratio (% of cavity)	Axial location
1	(1,0,0)	61	7	0.4	floor
2	(0,1,0)	81	12	0.6	middle
3	(1,1,0)	105	7	0.3	floor
4	(0,2,0)	120	7	0.3	ceiling
5	(2,1,0)	146	12	0.3	ceiling

Table 4.1 Description of the Helmholtz resonator treatment.



Picture 4.4 View looking down in the cylinder with the HR treatment.

Distributed vibration absorbers

The DVA treatment consisted in three rings of 15 devices. Each DVA has a 3inch by 5inch footprint and were tuned to 72Hz, 95Hz and 145Hz corresponding to the (1,3), (2,3) and (1,2) structural modes. Representing 3, 2.5, and 1% of the cylinder's mass respectively, the 3 rings were placed around the midpoint as shown in Figure 4.2.

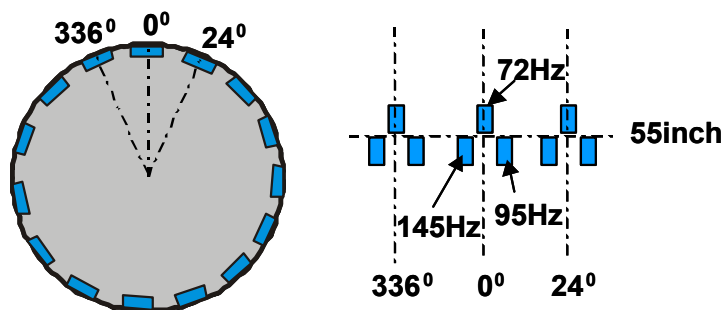


Figure 4.2 DVAs spaced every 24° and around the midpoint up the cylinder

4.2.3. Results

Acoustic forcing characterization

The external coupling between an incident plane and the structural modes of the cylinder was discussed in section 3.2.1. In order to characterize this external coupling in the experiment configuration, the transfer function between the speakers signal, 40-250Hz band-passed white noise, and microphones situated at the surface of the cylinder were measured. Seven rings of 12 microphones evenly space along the length of the cylinder meshed the pressure distribution. Assuming the structural mode shapes, a spatial Fourier transform can determine the structural modal component of the incident acoustic field. Due to the limited number of measurement in the axial (7) and circumferential (12) direction, the spatial Fourier transform is only valid that is with minimum aliasing, up to $n_s=3$ and $m_s=6$. Figure 4.3 shows a contour plot of these modal components for four different frequencies. Although, the speaker acoustic field differs from a theoretical plane wave, due in particular to the reflection from the ground, the main characteristic of the external coupling remains similar. Only the low circumferential order ($m_s < 4$) are well excited up to 180Hz. The variation with the axial order is harder to analyze partly due to the under sampling. In this direction, the acoustic field differs more from a plane wave excitation, since the pressure decays along the cylinder axis as Figure 4.4 illustrates. However, this particularity of the field might be closer to a real launch acoustic environment.

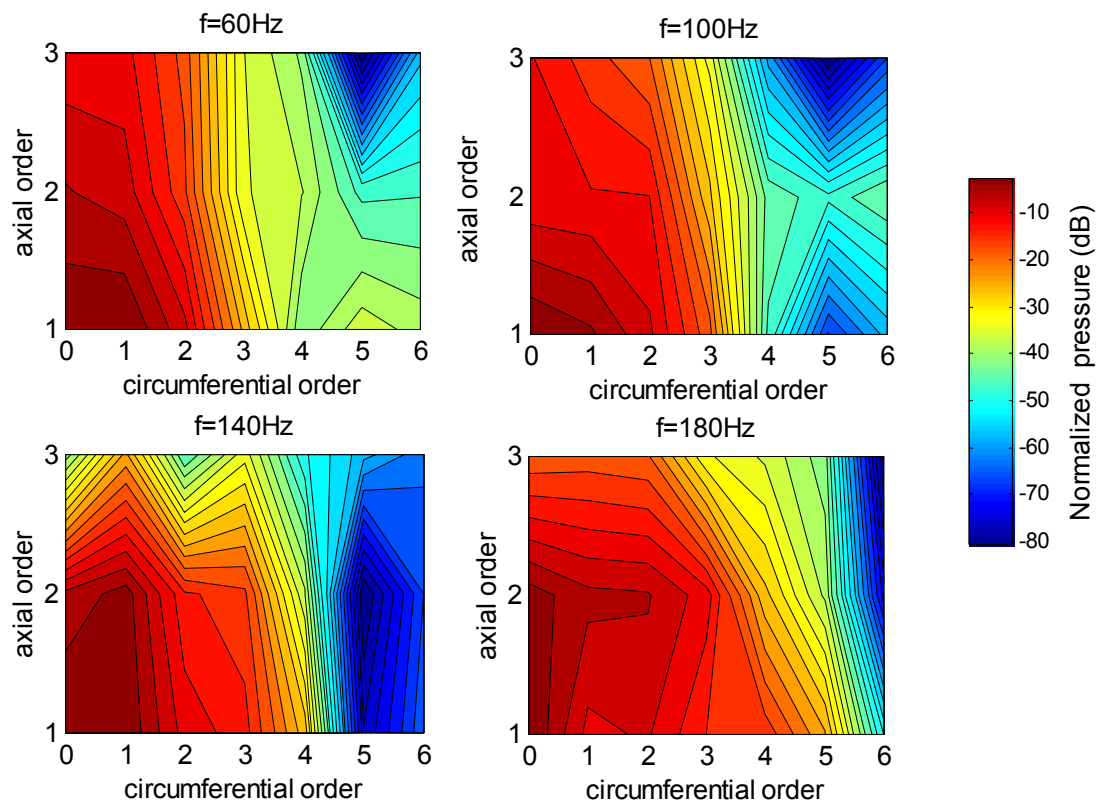


Figure 4.3 Structural modal components of the external acoustic field at 4 different frequencies.

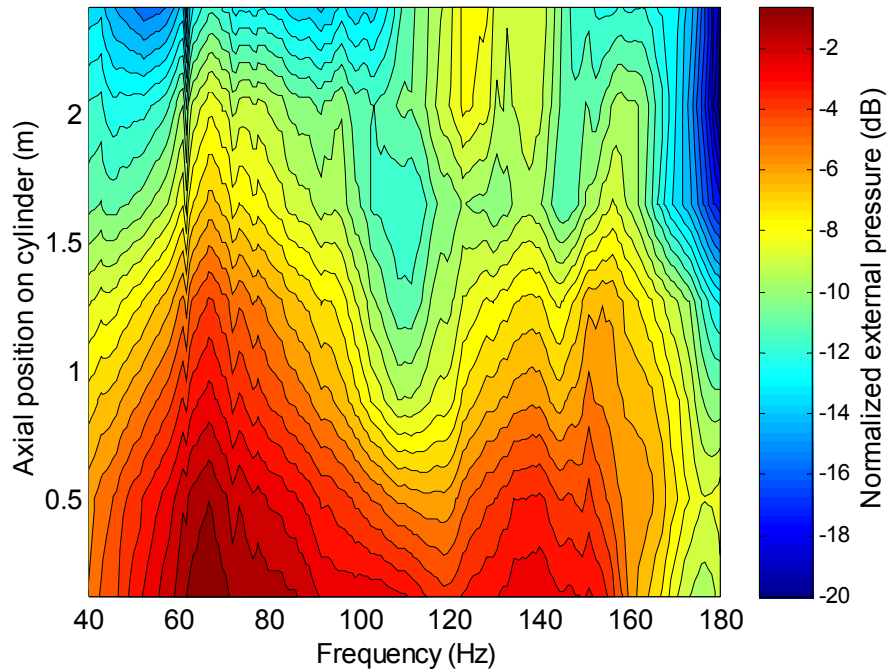


Figure 4.4 Variation of the external pressure distribution along the length of the cylinder as a function of frequency.

Bare baseline

The performance of the different treatments is compared in this paragraph to the non-treated cylinder that is with “bare” walls. Since the measurement were taken over a number of days, the external microphones ensure that changes in interior acoustic levels were not due to changes in the primary field. Figure 4.17 shows the consistency (within 0.5dB) of the external acoustic level measured at microphone E#2 (see Figure 4.1) for the three different configurations.

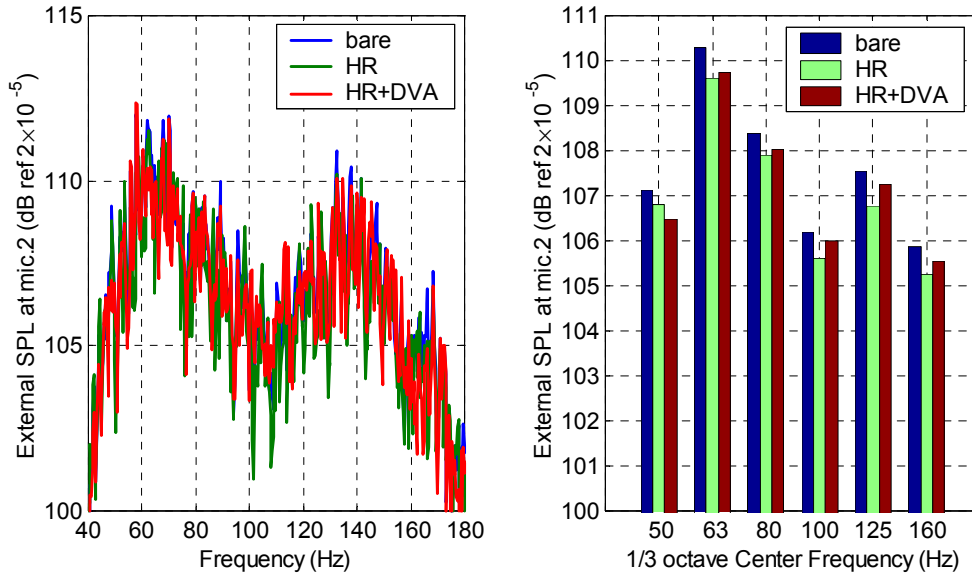


Figure 4.5 Narrow and third octave band external SPL for the three different configurations.

Using the ring of accelerometers, a spatial Fourier transform is performed to identify the modal content of the cylinder's vibration. As shown in Figure 4.6, the velocity of the structure is dominated across the band by higher order circumferential modes ($m=7,8,9,10$). This is contradictory with the circumferential filtering characteristic of the external coupling illustrated by Figure 4.3. The imperfection in the boundary conditions at the end-caps and the circumferential discontinuities induced by the seven sheets composing the cylinder skin can be responsible for the inter-structural modal coupling pointed out by the arrows in Figure 4.6. As a consequence the vibration response of the cylinder becomes hard to predict, as the orthogonal trigonometric assumption of the mode shapes seems violated experimentally.

The performance of the DVA and HR+DVA treatment on the velocity response averaged over the 35 accelerometers is illustrated in Figure 4.7. In addition to their targeted mode at 75, 95 and 145Hz, the DVAs attenuate greatly all the lightly damped peaks from 50 to 120Hz. Since the (1,2) structural mode is barely excited by the external acoustic field, the measure velocity response is relatively different than the response

predicted by the model. Indeed, the predicted response was only composed of coupled of well-excited modes and was dominated especially by the interaction between the 1,2 structural mode and 0,2,0 acoustic mode in the 120Hz range. As a consequence, the HRs do not enhance the vibration reduction provided by the DVAs as it was observed in section 3.4.1. Furthermore, the (0,2,0) acoustic mode at 121 Hz predominantly coupled to the (1,2) structural mode, was therefore unexcited and hence did not appear in the interior acoustic spectrum. Consequently, the fourth set of resonator was removed from the treatment.

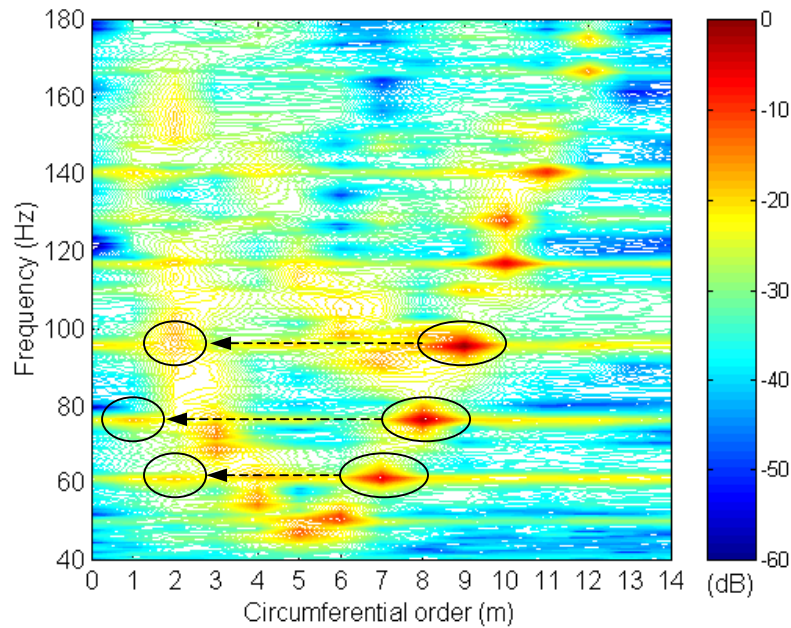


Figure 4.6 Normalized velocity wavenumber transform applied on the ring of accelerometers.

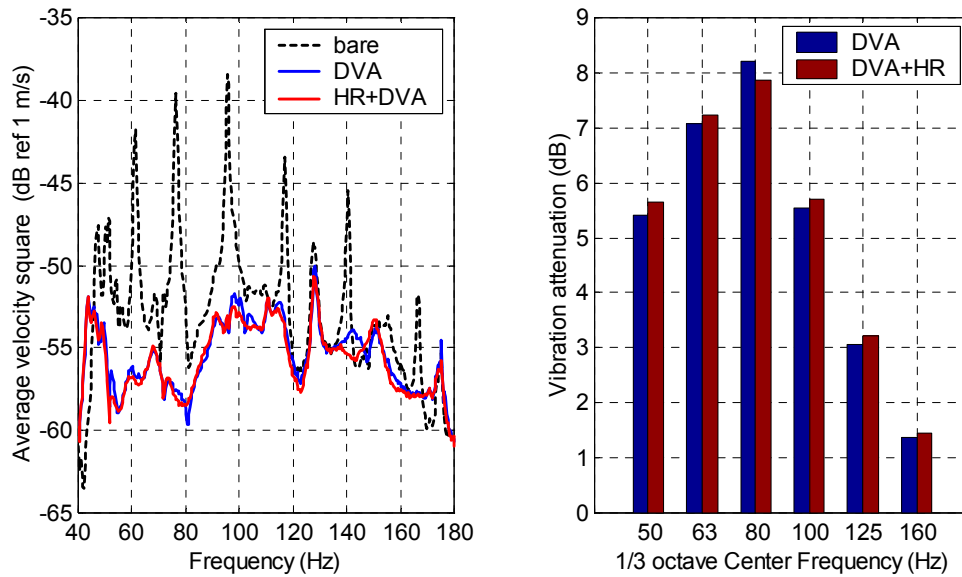


Figure 4.7 Narrow and third octave band vibration performance of the DVA and DVA+HR treatment compare to the bare baseline.

Averaging over 28 microphone locations, the response of the acoustic cavity is plotted in Figure 4.8 for the bare, HR and HR+DVA treatment. The 4 sets of HR greatly reduce the lightly damped resonances at 60, 80, 100 and 145Hz. A single resonance at 75Hz remains unaffected by the HR as it corresponds to the (1,3) structural mode. The DVAs tackle this lightly damped peak and improve the HR performance across the band. The HRs provides a 3.1dB noise reduction from 50-160Hz adding the DVAs to the treatment enhances this performance to 4.3dB.

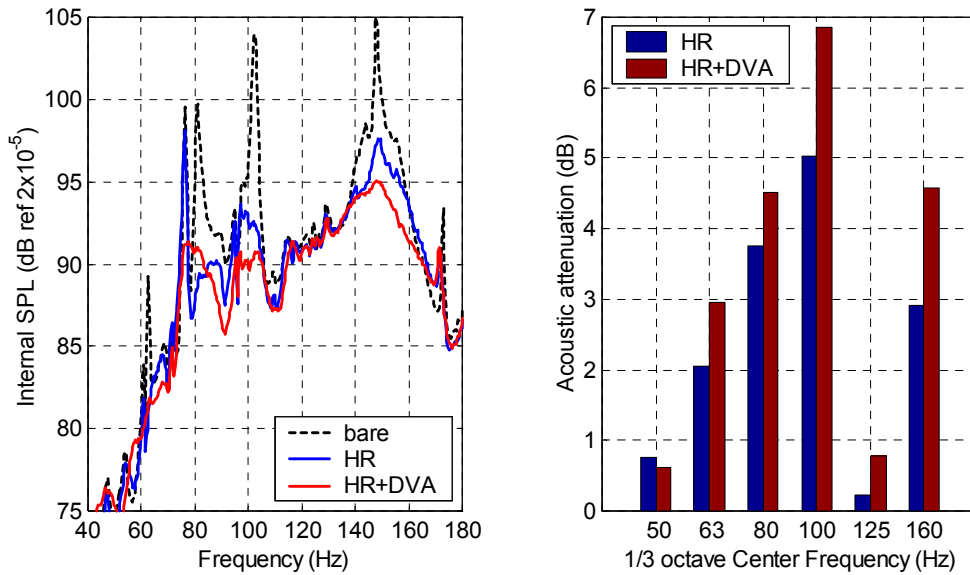


Figure 4.8 Narrow and third octave band acoustic performance of the HR and HR+DVA treatments compare to the bare baseline.

Half foam baseline

In an effort to make the cylinder behave as close as a real fairing as possible, a second baseline was used. It consisted in covering part of the cylinder's interior wall with some acoustic foam such that the level of acoustic damping approaches the one observed in fairings. This was achieved by matching reverberation time measurements and is detailed in Appendix A. The resulting baseline, called half-foam is composed of 2inch-thick melamine foam covering approximately half of the cylinder's wall surface.

Figure 4.9 presents, using the same scale as in Figure 4.7, the vibration response for the HR and combined HR+DVA treatment compared to the half foam baseline. The vibration level achieved by the combined HR+DVA treatment remains similar than when used with bare walls. Because the foam provides some damping to the initially lightly damped structural modes especially at the higher frequencies, the treatment performance in third octave bands is reduced compared to the bare foam baseline. Nevertheless, from 50 to 120Hz, the vibration attenuation exceeds 3dB in each band.

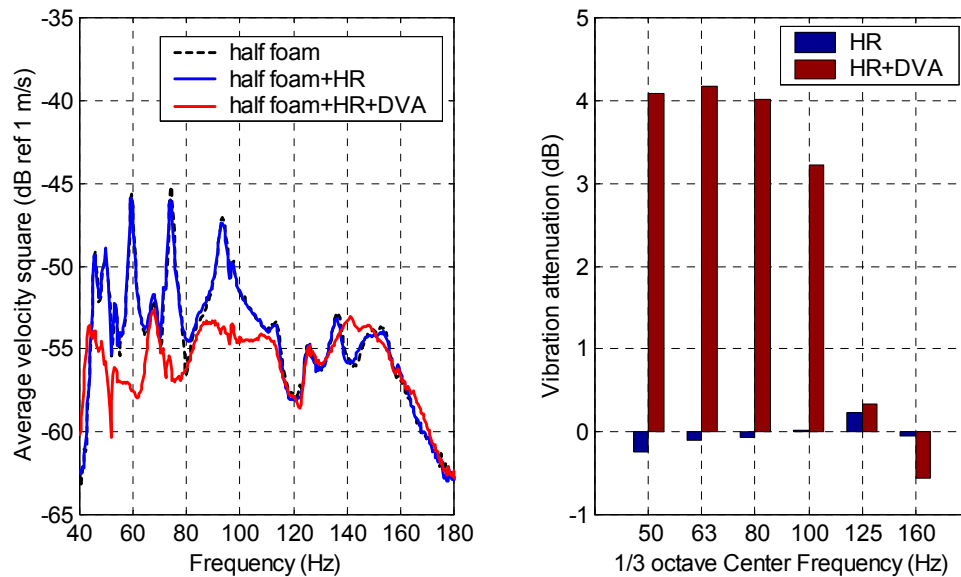


Figure 4.9 Narrow and third octave band vibration performance of the HR and HR+DVA treatments compare to the half foam baseline.

The acoustic response of the cavity presented in Figure 4.10 (also using same scale as Figure 4.8) illustrates the damping effect of the foam in the higher frequencies. The 145Hz resonance is for instance reduced by 15dB. The acoustic levels obtained with the combined HR+DVA treatment remain similar to one obtained with the bare baseline. The performance of the treatments reduces especially in the 160Hz third octave band. From 50Hz to 110Hz, the HR provides 2.2dB noise attenuation, which is enhanced to 3.3dB by the DVAs.

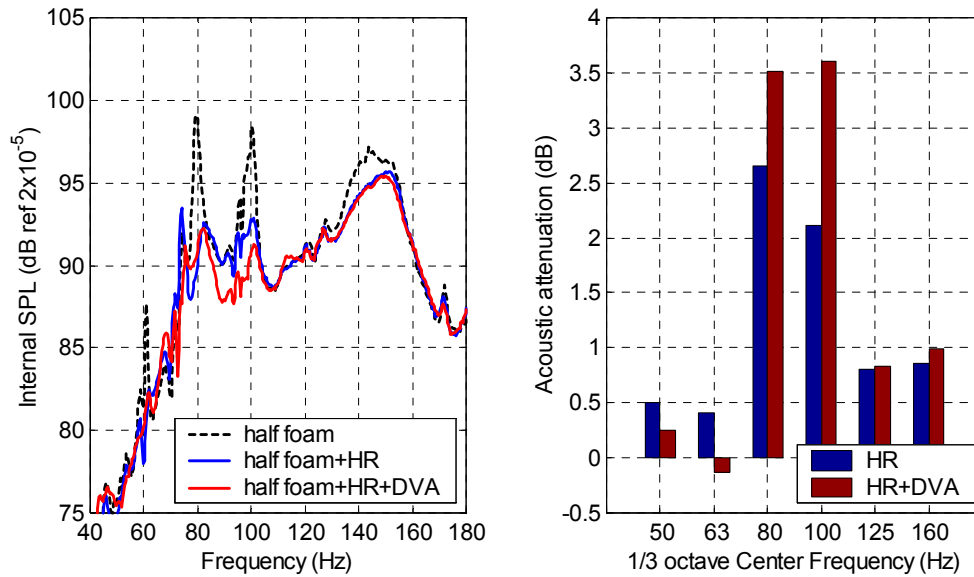


Figure 4.10 Narrow and third octave band acoustic performance of the HR and HR+DVA treatments compare to the half foam baseline.

Conclusion

These tests demonstrated that a small HRs and lightweight DVAs could reduce the noise transmission into a large composite cylinder. As shown previously with numerical simulations, the level of attenuation is function of the initial damping of the structure and of the acoustic cavity. The results obtained with the bare baseline clearly highlight this dependence with respect to damping and not frequency. Consequently, HRs and DVAs are complementary to existing acoustic blankets as they work in the low frequencies range where the foam treatment does not perform well.

4.3 Partially filled cylinder test October 2002

This section describes the outdoor tests done on the partially filled cylinder under high-level acoustic excitation.

4.3.1 Test setup

As for the empty cylinder test of October 2001, the cylinder was housed under a 30'x30'x16' tent at the Virginia Tech airport. The data acquisition equipment was also secured in a trailer. A 60KW Watt generator was used to power all the equipment used during testing and positioned 100' away from the test setup. The cylinder was excited by a total of 16 bass speakers equipped with horn loaded 15" driver and this time placed on each side of cylinder as shown in Figure 4.11.

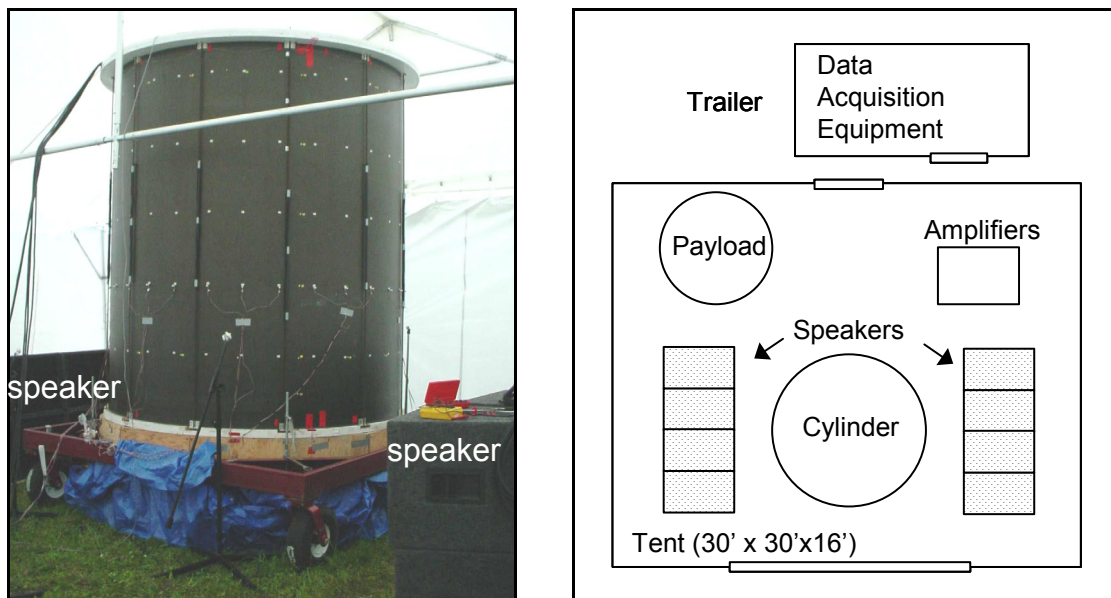
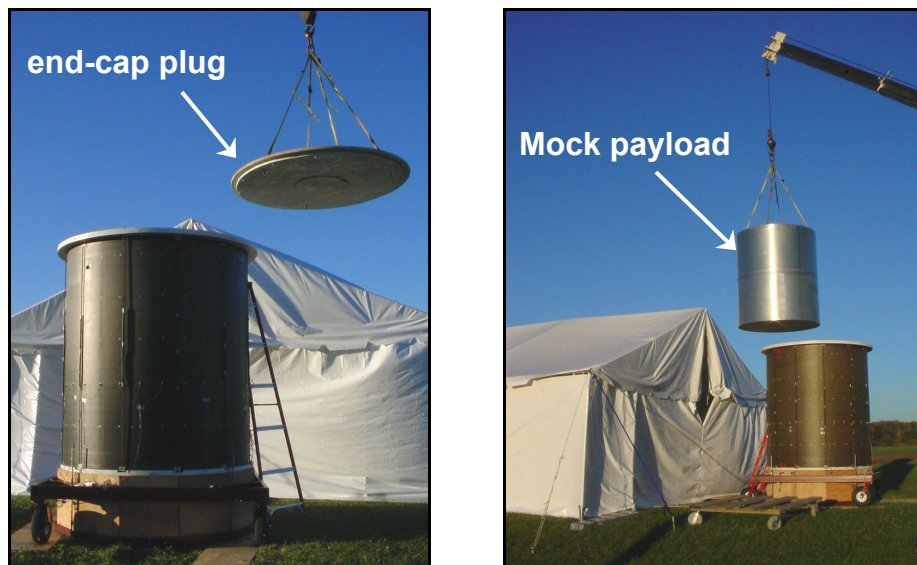


Figure 4.11: Cylinder inside the tent and experiment layout diagram.

One of the focuses of this test was to characterize the effectiveness of the treatment when the cylinder's cavity is partially filled with a mock payload. The mock payload provided by Boeing is a 6' diameter, 7' tall cylinder with a wooden frame, wooden end-caps, and an aluminum-sheet shell. The interior is lined with 3" melamine foam to damp any acoustic and structural resonances. A crane truck was rented for the period of the test, as it was necessary to remove the payload from the cylinder in order to work on the different acoustic treatment as shown in Picture 4.5.



Picture 4.5 Craning operation of the mock payload from the cylinder

For these tests, 15 internal microphones were flush mounted to the payload, a ring of 30 and a line of 7 accelerometers monitored the cylinder's vibration, and 4 external microphones monitored the external SPL. The two walls of speakers were driven with two independent white noise band passed between 40 and 400Hz. The time histories of the two inputs as well as all the sensors were recorded and post-processed using Matlab.

4.3.2 Noise reduction treatment

Partially filled acoustic cavity natural frequencies

In order to design a Helmholtz resonator treatment, the natural frequencies of the partially filled cylinder cavity needed to be identified. The mock payload being shorter than the cylinder, the cavity resonant frequencies could not be accurately predicted using the theoretical annular cavity equation derived in section 2.3.1. Therefore, Boeing provided the natural frequencies and mode shapes of the partially filled cavity obtained with a finite element (FE) model as shown in Figure 4.12. In order to check these results, and quantify the effect of the foam, a low-level acoustic transmission test as described in Figure 4.13, was conducted. The acoustic disturbance was created by a 18" Yamaha subwoofer, the transfer function between a line of 8 microphones positioned at the payload surface and the white noise input to the speaker was acquired for 8 different angles θ . The average of the square of the 64 transfer functions is plotted in Figure 4.14.

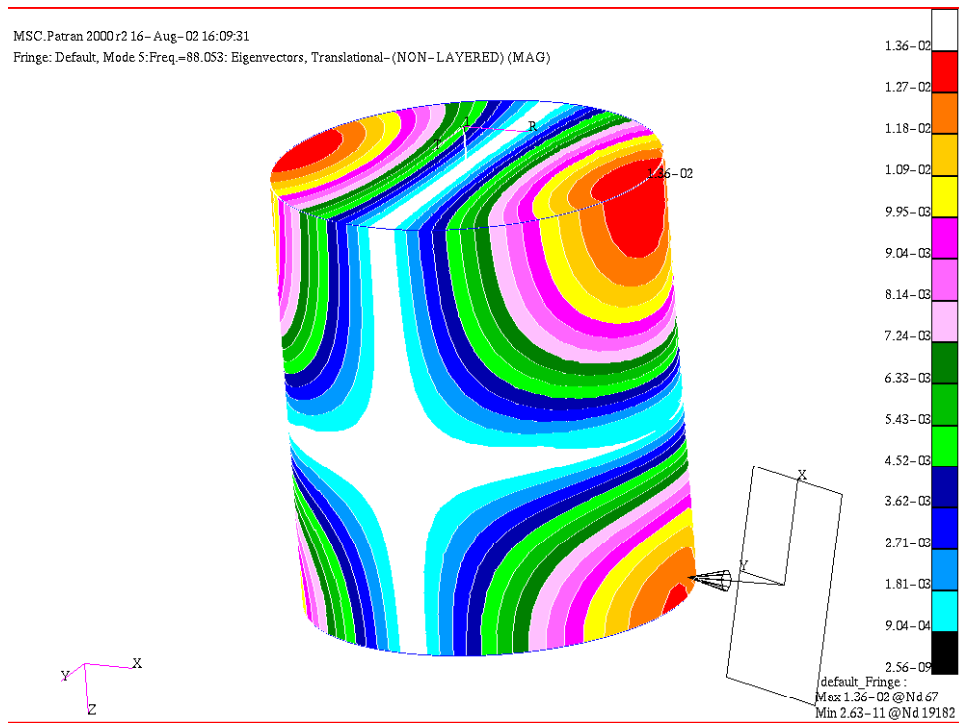


Figure 4.12 Acoustic mode shape of the partially filled cavity obtained from Boeing finite element model.

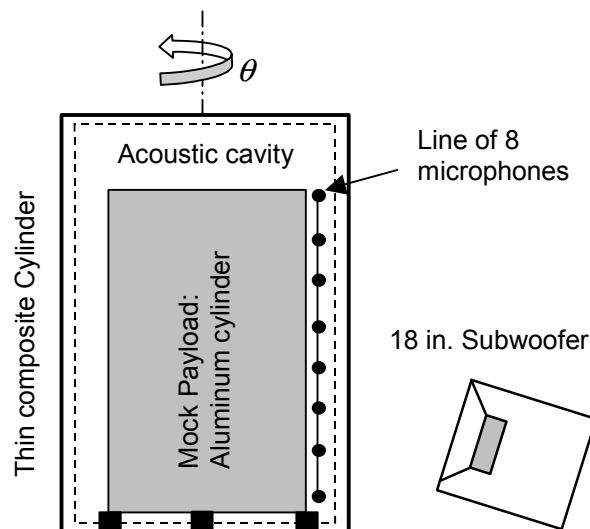


Figure 4.13 Low-level acoustic transmission with payload set up.

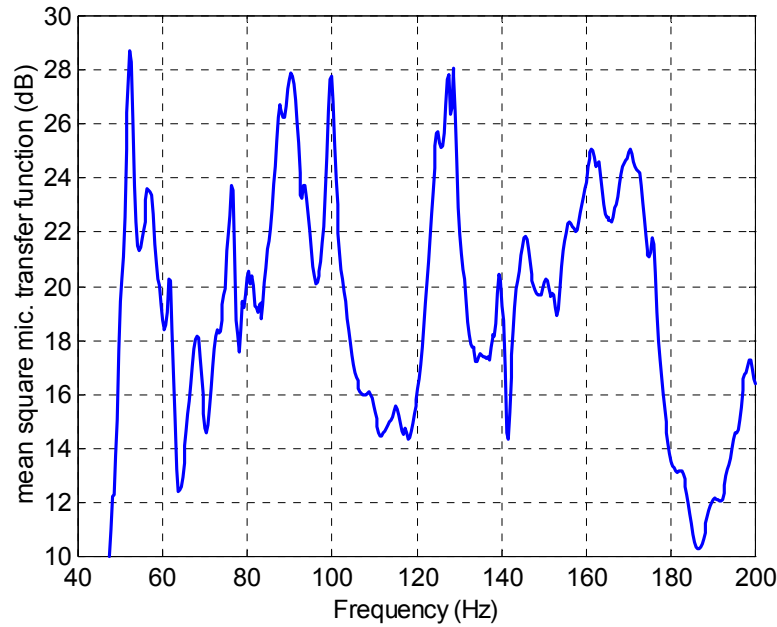


Figure 4.14: mean square pressure transfer functions (64 microphones)

Because of the asymmetry of the acoustic cavity the mode shape extraction as explained in section 3.2.4 is not as accurate as with the empty symmetric cavity. However, the dominant acoustic modes could be identified and their natural frequencies for the bare and half foam covered cavity are compared with the FE model in Table 4.2. As the cavity remains axisymmetric, the mode shape can be described using a circumferential order. Although the axial nodal lines are slightly shifted compared to a perfectly annular cavity, the axial order is retained to describe the mode shape. The discrepancies between the FE and the experimental results may be due to the small difference in the position of the payload, 4" from the cylinder bottom in the FE and 7.5" in the experiment.

Mode order (n,m,p)	Resonant frequency (Hz)		
	FE	Bare experimental	Half foam experimental
(1,0,0)	48.9	52.5	57.5
(0,1,0)	60.2	62	60.5
(1,1,0)	88.0	87.5	85
(2,0,0)	102.6	99.5	98
(2,1,0)	129.2	128.5	127
(1,2,0)	132.6	127	125
(2,2,0)	163.9	162	145

Table 4.2 Resonant frequencies of the cavity filled with the mock payload

Helmholtz resonators

Using the same design strategy as for the empty cylinder tests, 6 sets of Helmholtz resonators were designed and tuned to the frequencies of the cavity with half foam wall coverage. The characteristics of this treatment are listed in Table 4.3 and Picture 4.6 shows the first 5 sets of HRs as well as the half foam treatment attached to the cylinder.

HR set#	Targeted mode	Tuning Frequency (Hz)	Quantity	Volume ratio (% of cavity)	Axial location
1	(1,0,0)	57.5	4	1.1	floor
2	(0,1,0)	60.5	4	1.0	floor
3	(1,1,0)	85	12	1.5	ceiling
4	(2,0,0)	98	12	1.1	payload top
5	(1,2,0)	125	18 (6+12)	1.2	floor+ceiling
6	(2,2,0)	144	15	0.7	middle

Table 4.3 Description of the Helmholtz resonator treatment.



Picture 4.6 Five different HR (left). Half foam treatment (right).

The 6th set of resonators was designed as a hybrid HR/DVA. The idea is that the HR mass could be used as the mass layer in a distributed vibration absorber. By varying the foam thickness underneath the HR, the natural frequency of the resulting vibration absorber could be varied. Figure 4.15 shows the HR/DVA used in this test with its bouncing mode response measured using the experimental set up described in Figure 3.31. It is mainly this bouncing mode that couples into the structural vibration. Therefore, this device targets the acoustic mode at 144Hz while simultaneously adds damping to the (1,3) structural mode around 75Hz. Consequently, this set of HR could not be separated from the DVA treatment. As for the previous tests, the first 5 sets of HRs were deliberately not attached to the cylinder's wall to prevent any mass loading effect on the structural vibration. The HR volume ratios quoted in Table 4.3 are with respect to the partially filled cavity, which with 5.7m^3 represents 44% of the empty cylinder volume. Therefore, the HRs only occupy 2.8% of the empty cylinder volume.

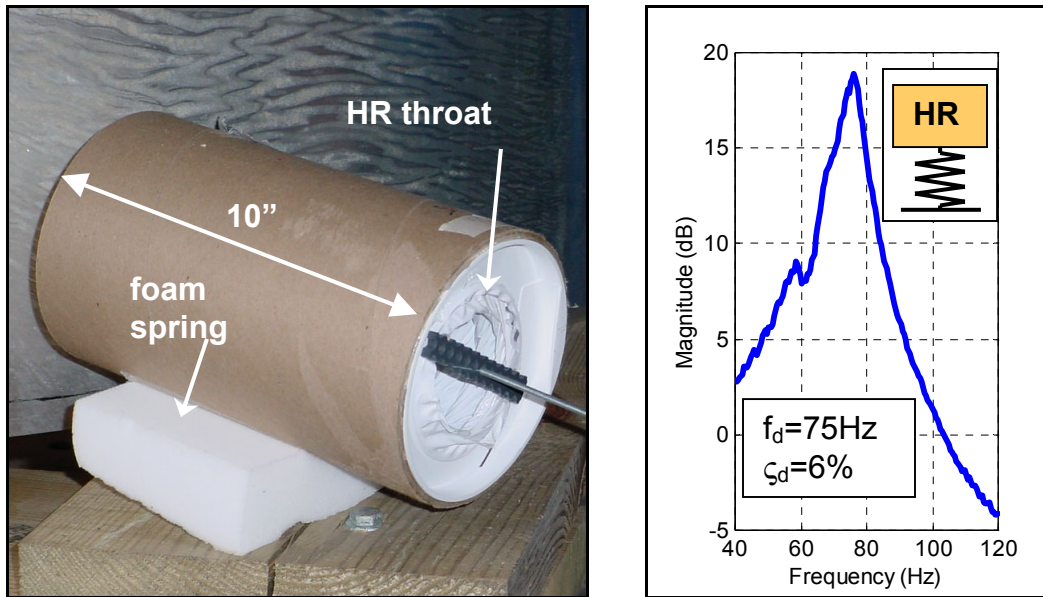


Figure 4.15 HR/DVA and its bouncing natural frequency curve.

Distributed vibration absorbers

The DVA treatment consists of 2 rings of 15 DVAs. The first ring tuned to the (1,3) structural mode at 74Hz is constituted of the HR/DVAs each weighing 400g. The second ring tuned to 145Hz is constituted of 3" by 5" footprint DVA made of melamine foam and a 60g aluminum plate. Picture 4.7 shows the two DVA rings position around the midpoint of the cylinder. The total mass of the treatment was 7 Kg representing approximately 9% of the cylinder's mass.



Picture 4.7 HR/DVAs and DVAs spaced every 24° around the mid point up the cylinder

4.3.3 Results

Acoustic forcing characterization

The two walls of speakers being symmetrically position around the cylinder and driven with two independent white noises, the incident acoustic field is only characterized for one side. As for the previous test the transfer function between the speakers signal and microphones at the cylinder surface are measured. This time, the pressure distribution is meshed with 7 axial and 30 angular positions. Figure 4.16 highlights the main characteristics of the external acoustic field. With a higher spatial sampling than with the previous test, the wavenumber transform shows that, below 180Hz, the external acoustic field excites only the lower circumferential order modes. The wavenumber transform in the axial direction does not bring any interesting information. However the pressure axial variation illustrates the amplitude decays from the bottom to the cylinder top by approximately 15dB.

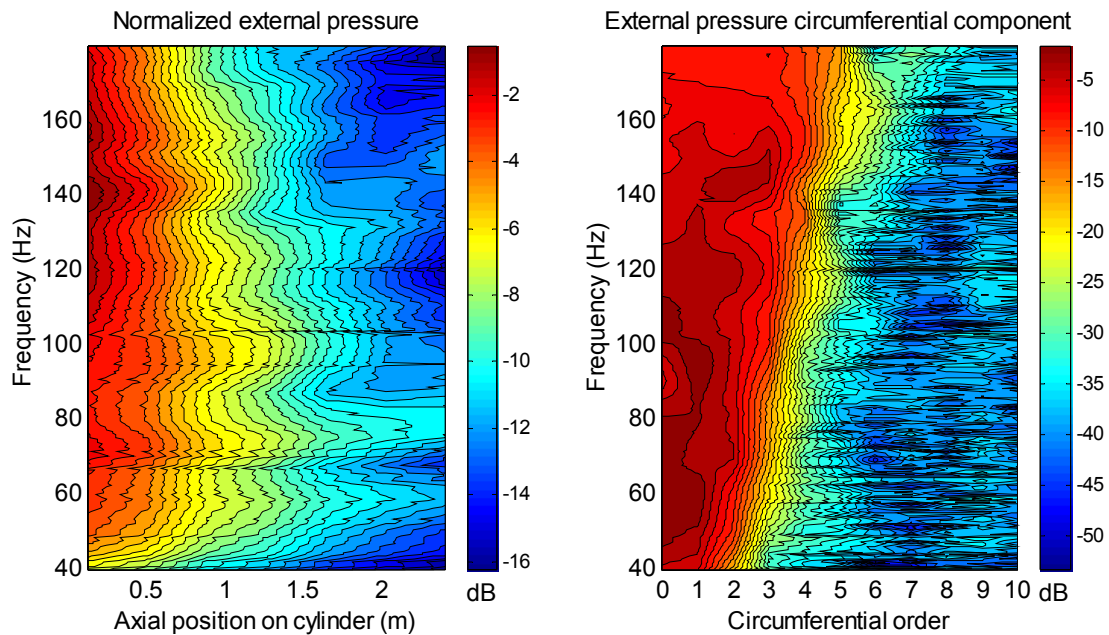


Figure 4.16 Characteristics of the external acoustic field created on the cylinder by one side of speaker.

Treatment performance

As the different tests took place over several days, two microphones in front of each speaker wall (mic#1 and #3), were consistently position 5" away from the cylinder surface to monitor the external disturbance levels. The sound system was different than with the previous tests, and the control of the level was not as easy. Therefore, all the tests had to be scaled to one particular run using the two monitoring microphones. Figure 4.17 shows the narrow and third octave band of averaged over mic#1 and #3 for the three different runs. An overall SPL of 132.5dB constituting a satisfactory approximation of real launch conditions was achieved and the level difference between the runs in each third octave band always remains within 1dB.

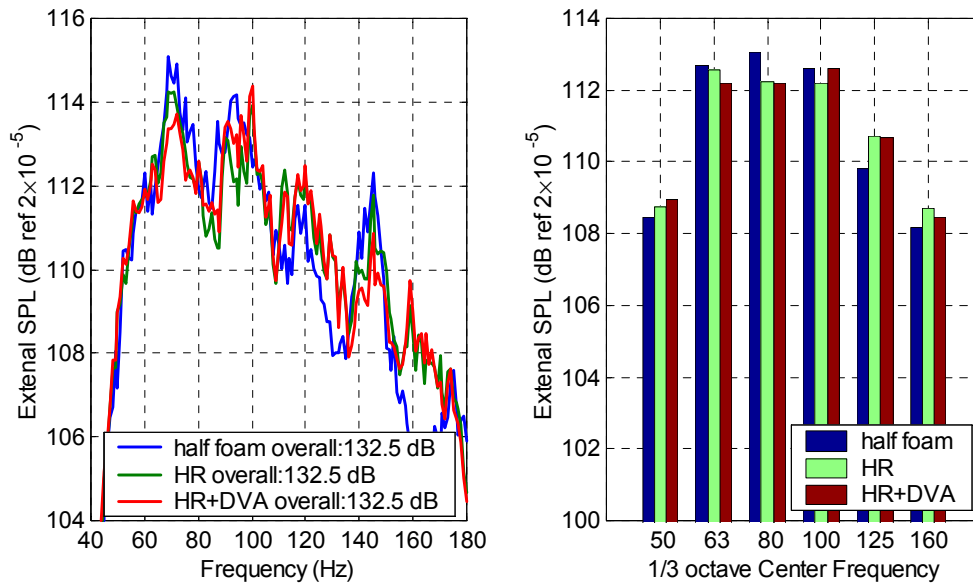


Figure 4.17 Narrow and third octave external SPL averaged between mic#1 and mic#3.

For this particular test, the performance of the HR and combined HR+DVA treatment were compared only to a half-foam baseline, which consist of a 50% coverage of the cylinder walls with 2inch thick melamine acoustic foam as shown in Picture 4.7. Although the mock payload does no add any acoustic damping directly, by reducing the volume of the cavity, it increase the effectiveness of the half foam treatment. As this effect is also likely to occurs in the real application the 50% coverage is maintained as the baseline to which the HR/DVA treatment is compared. As seen in Figure 4.18, the DVA treatment leads to large vibration attenuation below 100Hz where lightly damped modes resonate. This fairly high attenuation is to be expected as the DVAs represent about 9% of the cylinder mass. The 145Hz-targeted resonance being already quite damped by the foam blankets, the corresponding DVAs could be removed from the treatment without affecting the performance and would reduce the treatment mass by 15%.

The acoustic performance is illustrated in Figure 4.19. Due to the rearrangement of the acoustic mode due to the presence of the mock payload, the lightly damped resonances dominating the acoustic spectrum appears at 60 and 80Hz as opposed to 80 and 100Hz for the empty cavity case (see Figure 4.10). These lightly damped peaks

below 100Hz are well tackled by the HRs. In the first three lower third octave band, the DVAs enhance the HR attenuations by reducing the structural resonances well coupled to the acoustic cavity such as the 75Hz 1,3 mode. These attenuation improvements are up to 2.5dB. As the acoustic modes above 100Hz are already damped by the foam treatment the HRs performance in those frequency bands is undermined.

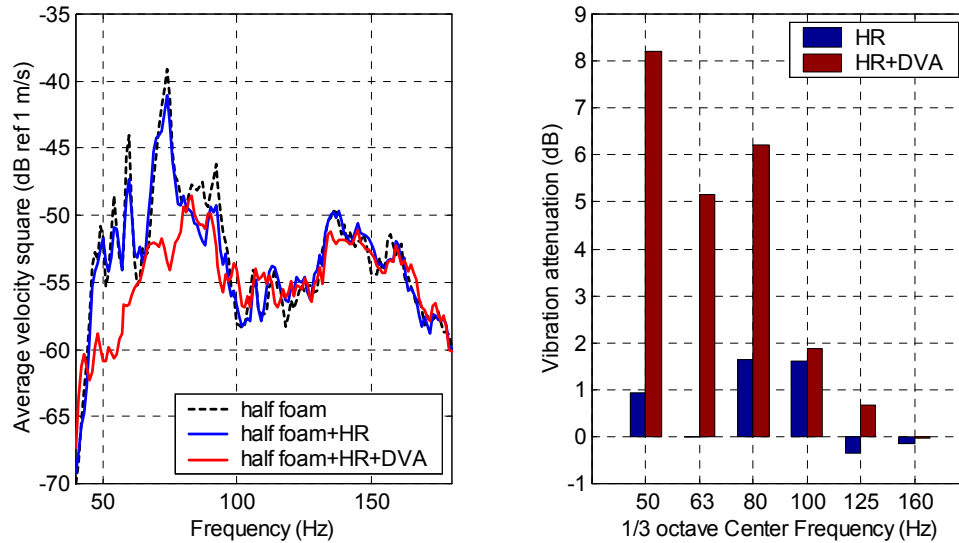


Figure 4.18 Narrow and third octave band vibration performance of the HR and combined HR+DVA treatment compared to the half foam baseline.

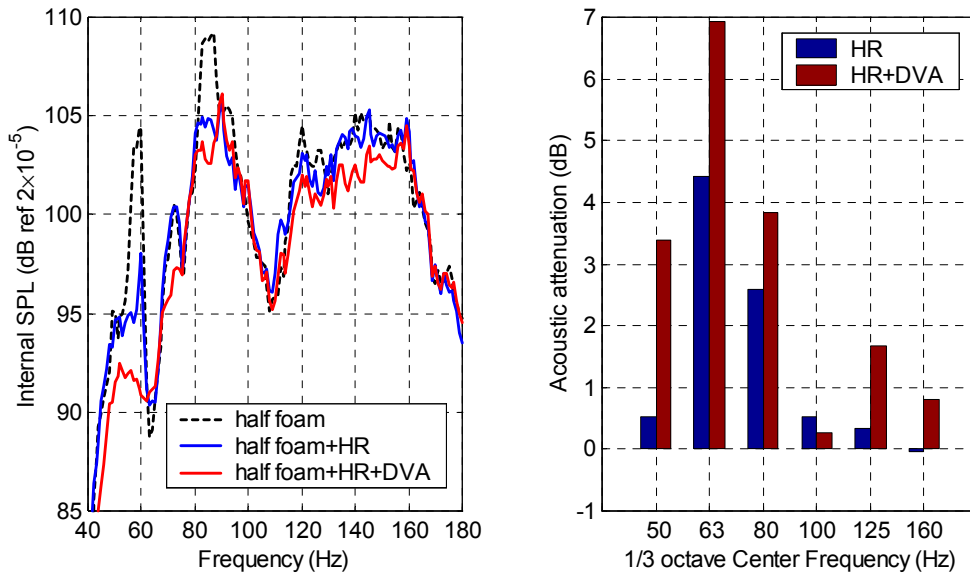


Figure 4.19 Narrow and third octave band acoustic performance of the HR and HR+DVA treatments compared to the half foam baseline.

Conclusion

These tests confirmed the conclusions drawn from the results obtained for the empty cavity. The rearrangement of the acoustic modes induced by the mock payload led to more structural acoustic coupling in the low frequency range. This allowed a clearer demonstration of the DVAs role in noise reduction. This test also showed the feasibility of hybrid HR/DVA that makes effective use of the HR mass to reduced noise transmission. This issue is important in the long run as the overall mass of the treatment will include the mass of the HR casing. The hybrid HR/DVA can be one solution to reduce this overall mass.

4.4 Fairing test November-December 2003

This section describes the test conducted on a full-scale fairing at the Boeing facilities in Huntington Beach CA. The design processes that lead to a pre-flight HR/DVA noise reduction treatment is detailed. Then, the experimental set up and the results are presented.

4.4.1 Lightweight Helmholtz resonator design

The design of a HR treatment for a full-size faring differs from the approach used with the scaled cylinder prototype. The volume of the fairing being 16 times larger than the cylinder prototype, a treatment representing a couple of percent of this volume translates automatically into a very large number of resonators. In addition, Boeing designers' will to integrate the HRs to the pre-existing acoustic blanket treatment gives the treatment a very high weight penalty. As a consequence, the feasibility of lightweight HR was investigated.

The effect of wall elasticity on the effectiveness of a Helmholtz resonator motivated by underwater applications has been subject to a couple of studies^{85,86}. Both studies conclude the wall compliance decreases the HR effective stiffness reducing its frequency in comparison with an identically shaped rigid cavity by $\gamma^{1/2}$. This factor is a function of the shell impedance Z_s and the impedance of the fluid Z_f and is given by

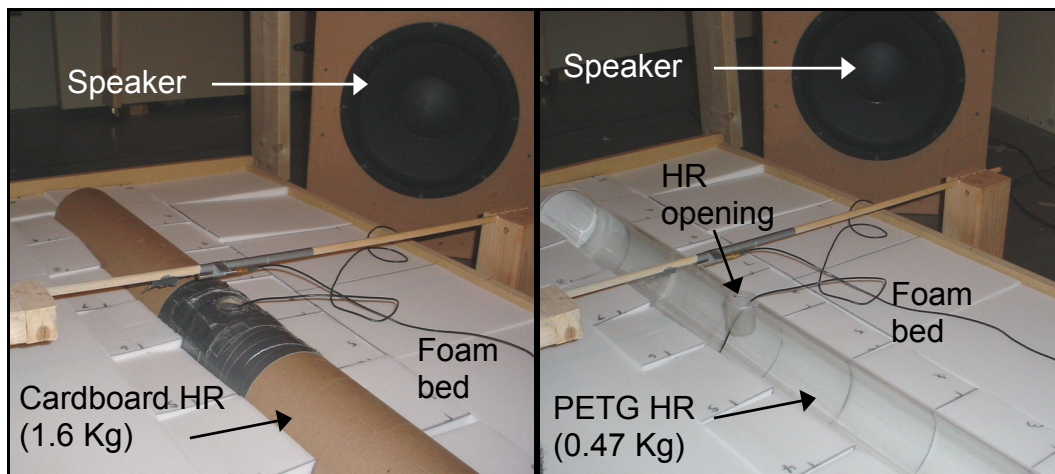
$$\gamma = \frac{Z_s}{Z_s + Z_f}, \quad (4.4.1)$$

Photiadis⁸⁵ shows that when the internal dissipation produced by viscosity and heat conduction exceeds the radiation loss in air, which is mostly the case^{47,48}, the damping ratio of the elastic HR is increased. In addition, he also demonstrates that the resonator net volume outflow is reduced by γ^2 . As a consequence, the design of lightweight efficient HR is based on the trade-off between the mass and the stiffness of the resonator

shell. After a spherical HR, not suited for this application, a cylindrical HR with hemispherical end-cap presents the second stiffest type of cavity for a given material. In collaboration with Boeing designers, a compromise between cost and effectiveness was reached in using off the shelf plastic tube enclosed with regular slightly curved end-cap.

As the HR were to be embedded in the foam blankets the opening had to be placed on the side of the tube rather than on the end-cap. To conserve symmetry and avoid quarter wavelength tube behavior, the opening was placed in the middle of the tube section. The type of plastic used is called PETG (for polyethyleneterephthalateglycolate) and presents a density of 1270Kg/m^3 , the inner diameter of the tube is 5" as for the cardboard HR, and its thickness is 0.025". The 5" diameter was the maximum dimension intruding in the payload bay, which was allowed by Boeing. The plastic end-caps are identical than the one used for the cardboard HR, and the necks were made of acrylic tubes of different diameter and length.

In order to verify the effectiveness of a PETG HR in the configuration of the test, a foam bed consisting in a rectangular wood box filled with 3.5" thick foam was built. As shown in Picture 4.8, a cardboard and a PETG HR of identical dimensions, 48" long with a 1.5" diameter and 2" long neck, were tested in the foam bed using the experimental setup described in Figure 3.35.



Picture 4.8 Cardboard and PETG HR in the test foam bed.

The transfer function between the internal and external microphone is shown in Figure 4.20 for both HRs. In agreement with the theory, the PETG HR presents a lower natural frequency at 48Hz than the cardboard HR, assumed infinitely stiff, at 51Hz. The amplitude at resonance, proportional to the HR net outflow, reduces from 9 to 7. It is interesting to note that quantitatively these results agreed with Photiadis⁸⁵:

$$\gamma = \left(\frac{48}{51} \right)^2 \approx \sqrt{\frac{7}{9}} \approx 0.88.$$

Note that this agreement is to take with caution as small error on the evaluated resonant frequencies can lead to quite large variations in γ . The damping ratio is also increased for the elastic HR. This experiment illustrates the previously mentioned trade-off. The PETG HR, approximately 20% less effective, represent a 70% weight reduction compared cardboard HR. This compromise was retained for the fairing test.

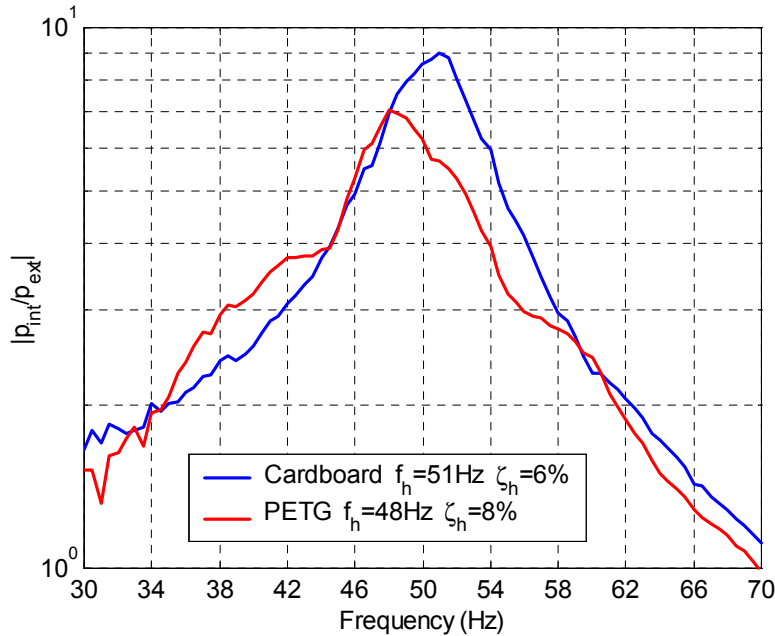
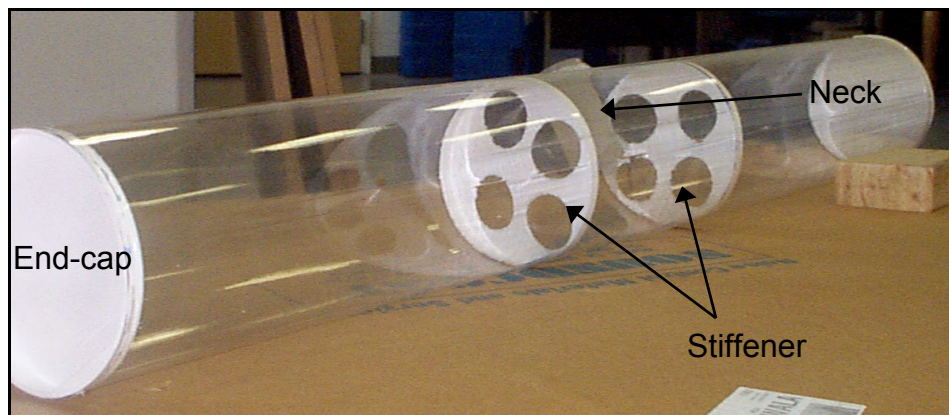


Figure 4.20 Transfer function between internal and external pressure obtained with the cardboard and PETG HR.

A lot of the PETG tube presented permanent oval deformation yielding large reduction of their stiffness and therefore HR performance. To overcome this problem, stiffeners made out of plastic end-caps were inserted on each side of the neck to maintain the circular cross section of the tube as shown in Picture 4.9. Due to the four large holes, the stiffeners are acoustically invisible below 100Hz and thus do not alter the HR behavior. The gain in performance is illustrated in Figure 4.21, which shows the transfer function between the internal and external pressure for a PETG HR with and without stiffeners. In the first case, it can be clearly seen that the shell dynamics impair the HR resonance, whereas with the stiffen HR these dynamics occur outside the band, resulting in a sharper acoustic resonance.



Picture 4.9 PETG HR with stiffeners.

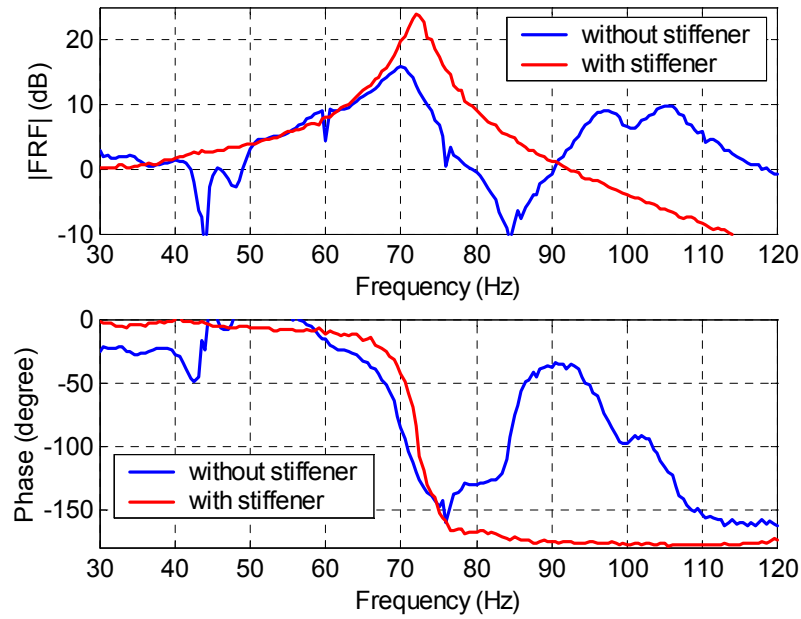
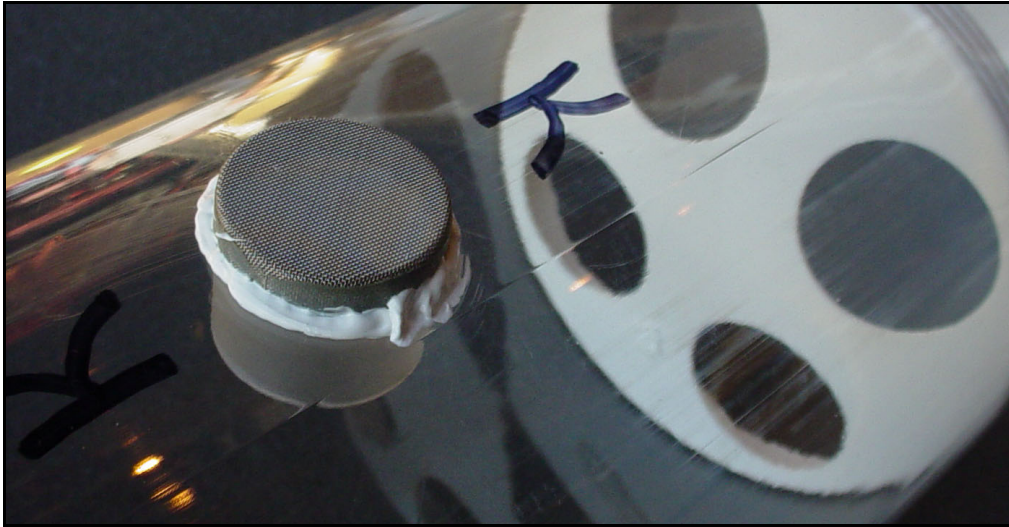


Figure 4.21 Transfer function of a PETG HR with and without stiffener.

Placing acoustic foam in the HR opening to adjust the damping to the desired value was no longer possible in the fairing application as the payload environment had be cleared of any dust and particles. Consequently, the damping was created by covering the HR opening with metal wire mesh of different percentage open area. Picture 4.10 shows and example of 30% open wire mesh covering the HR neck.



Picture 4.10 Resonator neck covered with 30% open wire mesh.

4.4.2 Noise reduction treatment design

In order to design a HR+DVA treatment for the fairing the excitation configuration had to be defined. Figure 4.22 describes the proposed test configuration that was adopted by Boeing. As for the partially filled cylinder test, two independent white noises were to be used to drive the speakers on each side of the fairing. Using the symmetry, one side of the particular configuration was used in section 3.2.5 for the excitation model.

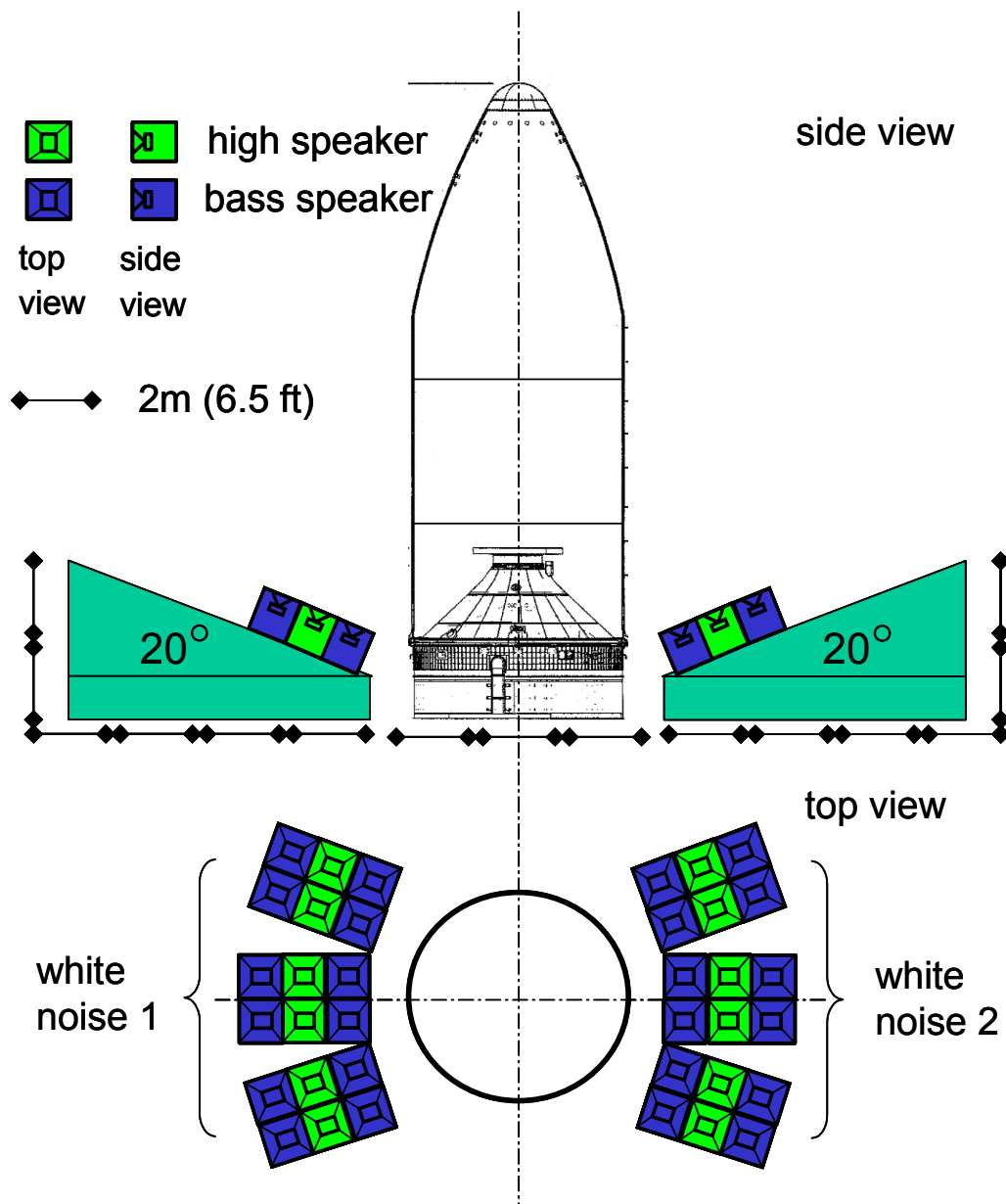


Figure 4.22 Speaker layout for the payload fairing test.

The existing cell division of the fairing blanket, dictated the positioning of the HR and DVA. In the cylindrical section, the 30"by 80" rectangular cells could only host 2 HRs and one DVA. To maximize their volume, two 40" long HR were placed in all the cylinder cells. The DVA positioning was also constrained to the top or bottom of a cell.

In the cone section, only the first three rows of blanket cells could be modified and 22” long HRs were therefore placed in all of these to maximize again the volume of the treatment. The mass of each DVA was limited to 1.2lb in the cylindrical section and 0.9lb in the cone and the overall thickness of the devices had to be 3.5” for compatibility with the attachment procedure. Without further investigation on other types of foam, these constraints lower bounded the DVA tuning frequency range to 50Hz.

Based on the design procedures and the numerical simulation insights described in section 3.5, a HR+DVA treatment fulfilling all the requirements was designed and is presented in Figure 4.23. The device characteristics listed in Table 4.4 and Table 4.5 were determined experimentally and then mass-produced by Boeing designers. The mentioned damping ratios were approximated from experimental measurements and are situated around the desired optimal value.

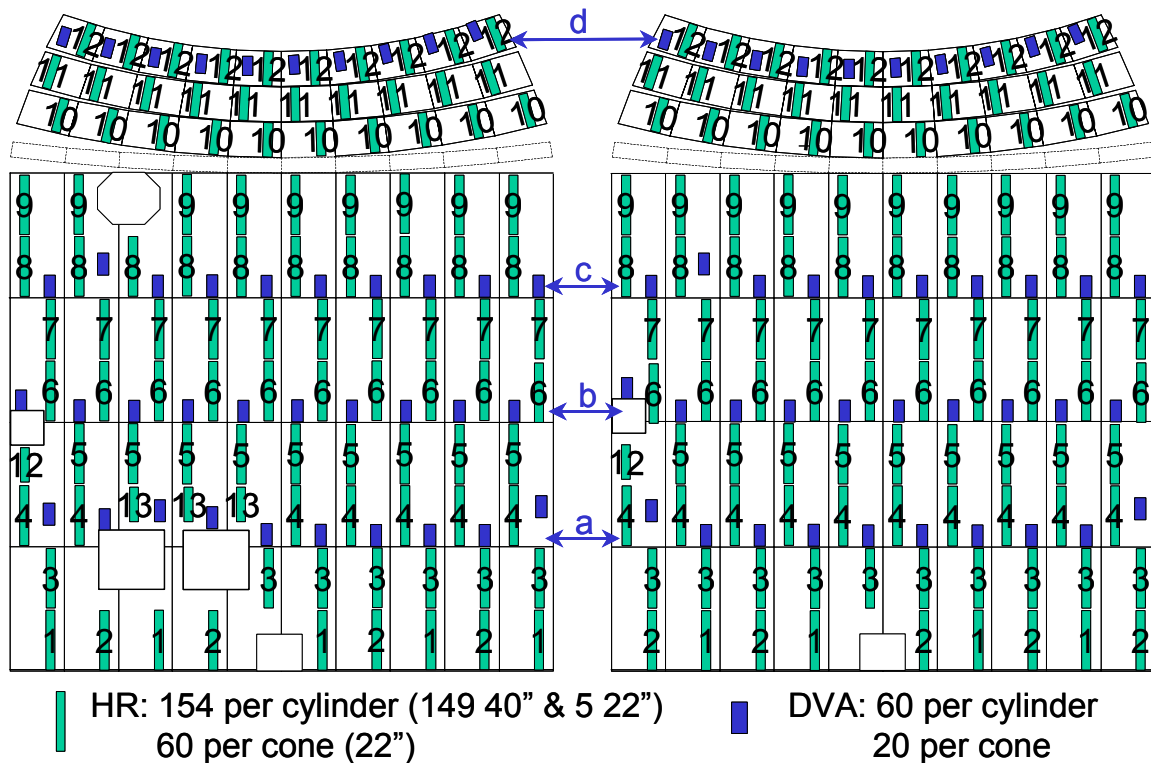


Figure 4.23 HR and DVA layout in the fairing blanket.

40 inch long HR, 440g (149 units)						
Set#	Frequency (Hz)	Damping ratio (%)	Opening ID (in.)	Neck (in)	Quantity	% of fairing volume
1	37	9.5	1	2-3/8	9	0.05
2	61	11	1-1/2	1-1/2	9	0.05
3	43	12	1	1-1/2	17	0.1
4	52	7	1-1/2	2-1/2	17	0.1
5	85	8	2	7/8	18	0.11
6	54	9	1-1/2	2-1/8	20	0.12
7	63	13	1-1/2	1-3/8	20	0.12
8	65	7.4	1-1/2	1-1/4	20	0.12
9	70	13	2	2	19	0.11
22 inch long HR, 260g (63 units)						
10	71	12	1	7/8	20	0.07
11	75	12	1-1/2	2-3/8	20	0.07
12	85	7	1-1/2	2-3/8	20	0.07
13	52	12	1	2-1/4	3	0.01

Table 4.4 HR parameters of the launch vehicle treatment

1.2lb (545g) DVA (20 units per set)						
Set#	Frequency (Hz)	Damping ratio (%)	Foam thickness (in)	Aluminum plate thickness (in)	Footprint (in ²)	% of fairing mass
a	56.3	2	3-1/8	.375	32	0.7
b	49.7	2	3	.50	24	0.7
c	74.5	2	3-1/2	.188	63-3/4	0.7
0.9lb (409g) DVA (20 units per set)						
d	91.4	3	3-1/2	.125	72	0.5

Table 4.5 DVA parameters of the launch vehicle treatment

The 41Kg overall mass of the 80 DVAs, represents 2.2% of the mass of the fairing with its blanket. With 82Kg the 212 HRs adds 4.5% mass and represents 1.1% of the fairing volume. Figure 4.24 shows the narrow and third octave band performance of this particular treatment predicted using the model. As it will be considered in the experiments, the 31.5Hz third octave band is included. However in the simulation, this band only contain the tail of the first acoustic mode as it ranges from 28Hz to 35Hz. The expected overall acoustic attenuation from 30-90Hz is of 3.2dB out of which 3.0dB is obtained with the HRs only.

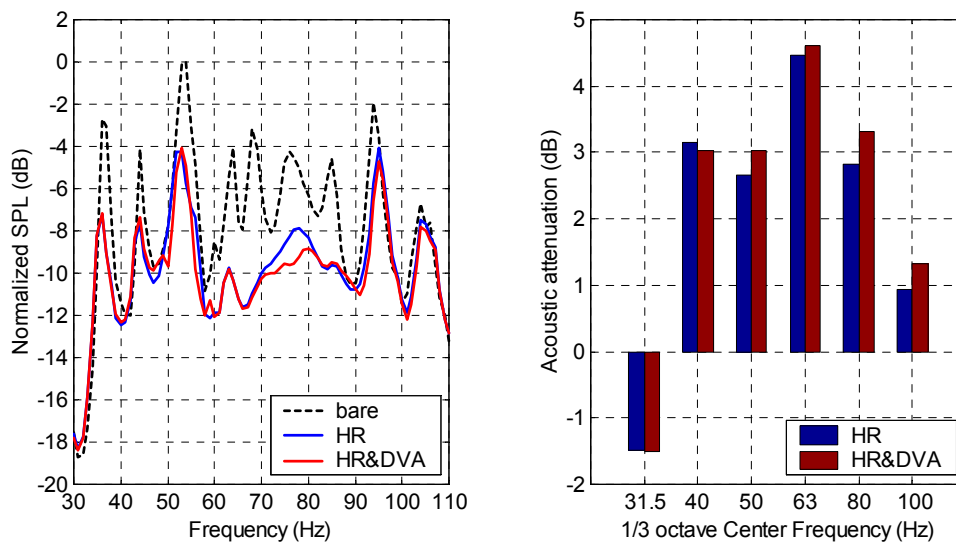


Figure 4.24 Prediction of the acoustic performance of the treatment tested in the fairing.

4.4.3 Experimental setup

The tests took place inside a large building at the Boeing facilities in Huntington Beach California. The fairing, and the speakers on their stands are shown in Picture 4.11. In addition to the HR and DVA treatment, these tests were the opportunity for Boeing to evaluate the performance of a new type of blanket. Unfortunately for cost reason, only two noise treatments could be evaluated. The first corresponds to the existing acoustic blanket made of 4.5-inch thick melamine foam and was conducted in October 2003 and is called “bare”. The second corresponds to the new blanket composed of 3-inch thick melamine foam and 0.5-inch thick polyamide foam, with the HRs and DVAs. As a consequence, the contribution of each device to the acoustic reduction could not be determined experimentally. The new acoustic blanket is however supposed to differ from the existing blanket above 110Hz. The performance of the combined HR+DVA treatment designed to work in the low frequencies would therefore be considered independent from

the new blanket effect. This second test, occurred two month later in December, as it takes at least 6 weeks to disassemble and reassemble the fairing with a new treatment.

For both tests, an axial line of 8 and 2 rings of 36 accelerometers were used to measure the fairing vibration level. A tree holding 30 microphones was used to record the internal acoustic level while 12 other microphones placed around the fairing monitored the external level. All the sensor positions are illustrated in Figure 4.25. To get a better evaluation of the internal SPL, three measurements corresponding to three different angles of the microphone tree were acquired for each test. This resulted in a total of 90 internal microphone positions.



Picture 4.11 Fairing inside the building with speakers.

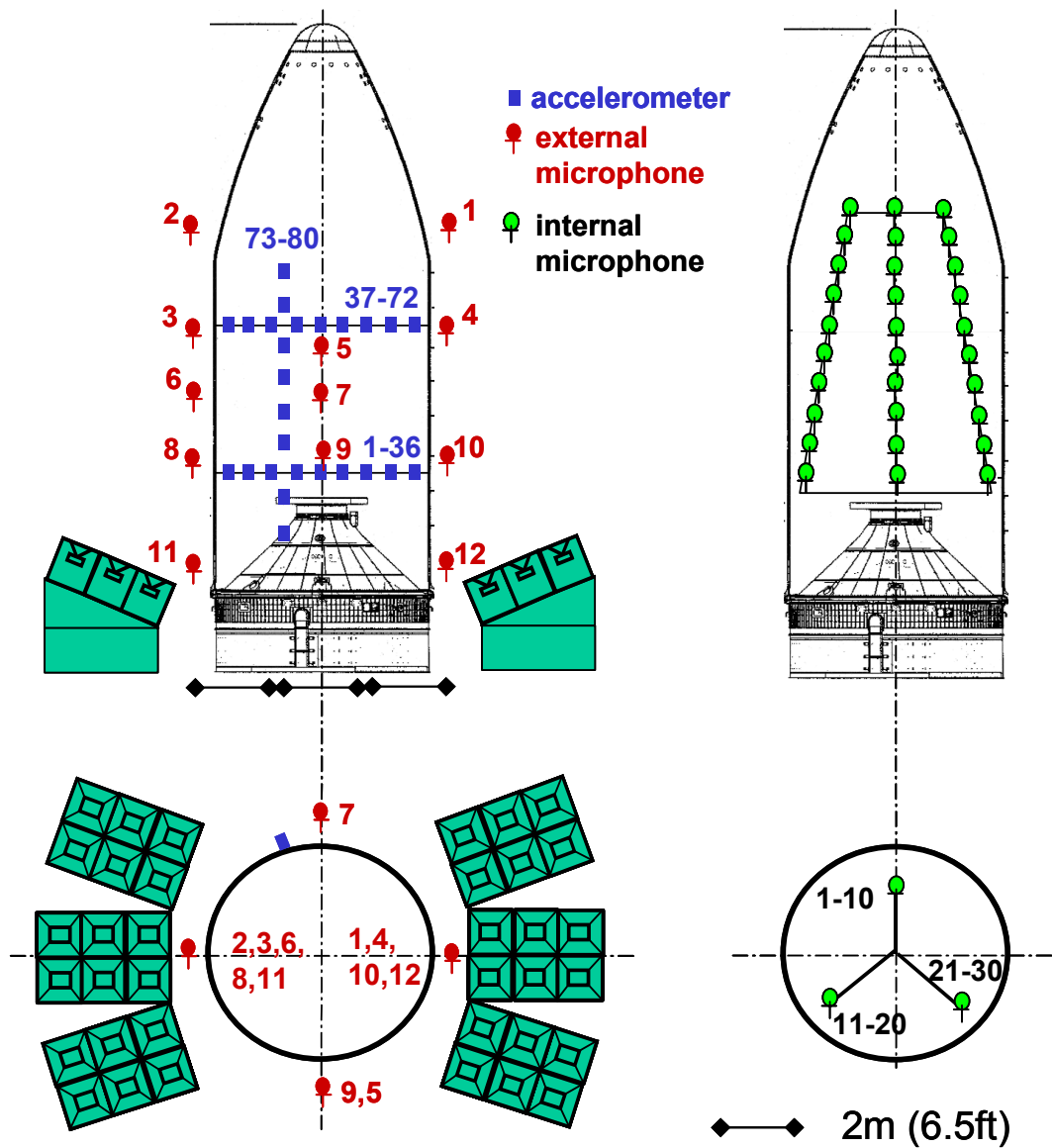


Figure 4.25 Location of the accelerometers and microphones for the fairing test.

4.4.4 Results.

The two tests being spaced by two months, the external SPL averaged over the 12 microphones was slightly higher for the second (HR+DVA) test as shown in Figure 4.26. Therefore, for each frequency, the microphone and accelerometer signals of the second

test were scaled down by the ratio of the averaged external SPL of the bare test with the averaged external SPL of HR+DVA test.

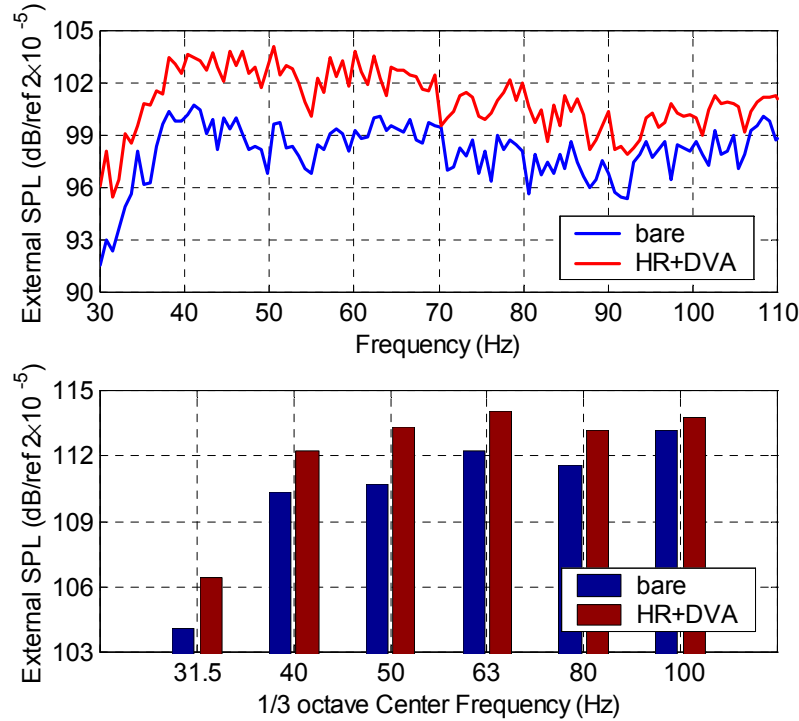


Figure 4.26 Narrow and third octave band level of the average external SPL for both tests.

Using the upper ring of accelerometers, the circumferential wavenumber transform of the velocity response is performed for both tests and plotted in Figure 4.27. From these plots, the DVA tuned to 50 and 55Hz (set a and b) seems effective in reducing the vibration in these bands. However, the attenuation is only significant on the higher circumferential order modes ($m=6,7$, and 8) and minimum for the dominant lower order modes ($m=1$ and 3). This could be due to the axial position of the DVAs with respect to the axial antinodes of those modes. The DVA tuned to 75Hz seems to reduce the amplitude of a couple peaks in the 65-75Hz band but the vibration levels are already low in this range. These remarks are illustrated also in Figure 4.28, which plots the narrow band vibration response and the third octave frequency band attenuation obtained with

the treatment. The difference of performance compared to the results obtained on the cylinder illustrated in Figure 4.18, are mainly due to the difference in the mass ratio of the DVAs. Indeed in this experiment, the DVA treatment represents only 2.2% of the fairing mass whereas in the partially filled cylinder test, the DVAs represented 9% of the cylinder mass. Furthermore, the effect of the last set of DVA (set d) tuned to 90Hz are imperceptible as no resonance seemed to occurs in this band and the devices only represent 0.5% of the fairing mass. The 3dB vibration increase in the 31.5Hz third octave band (28-35Hz) is to take with caution as the speaker system can lack consistency in this low range. In addition, the consistency in assembling the two fairing halves can affect the structural dynamics in this low frequency range.

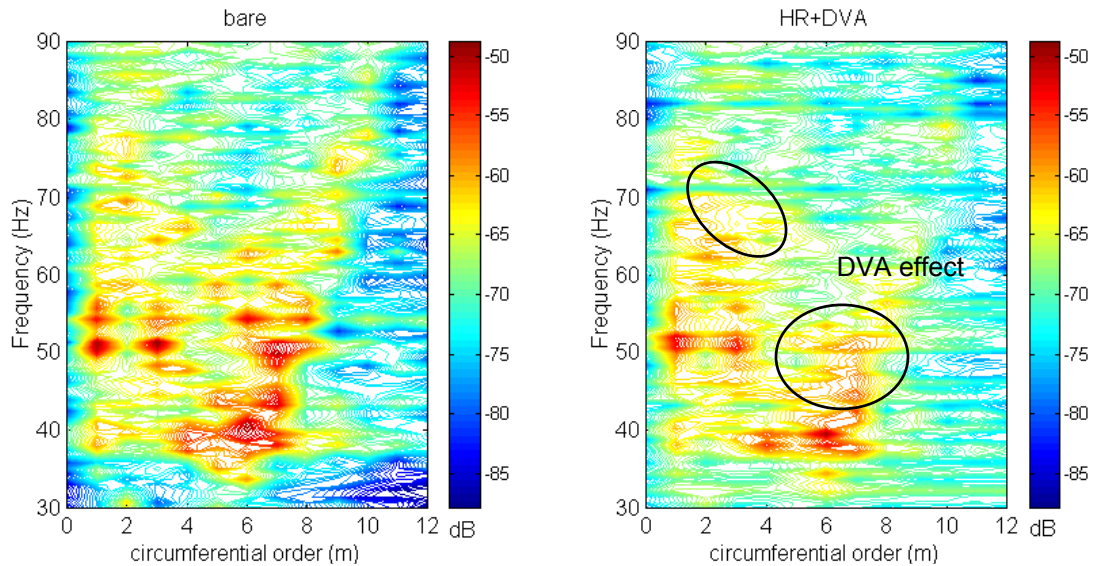


Figure 4.27 Circumferential wavenumber transform of the velocity response of the upper ring for both treatments.

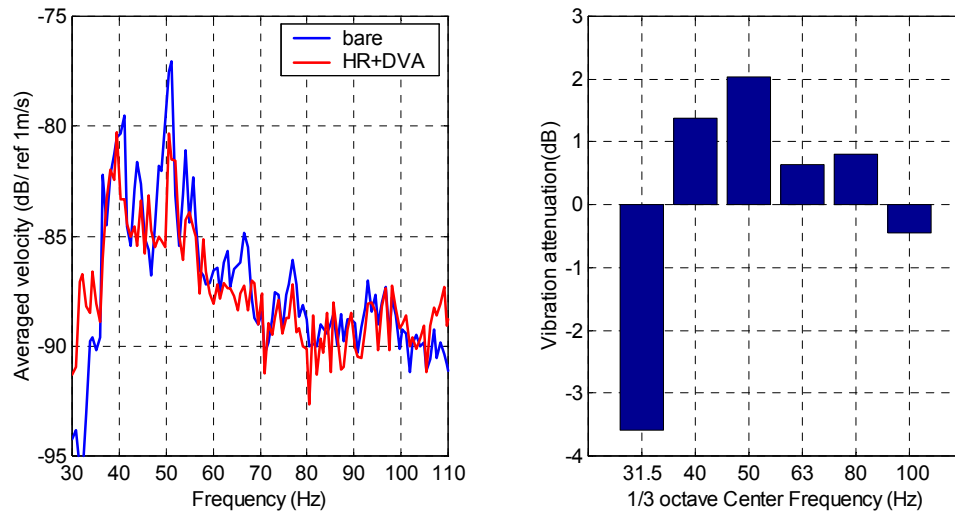


Figure 4.28 Narrow and third octave vibration performance of the fairing treatment.

For proprietary reasons the acoustic experimentally measured narrowband performance of the treatment cannot be shown in this dissertation. As expected, the resonators damp the sharp resonances in the band of control, reducing two peaks by 5dB and another by 10dB. The overall attenuation from 30-90Hz is 3.2dB. A maximum of 5.2dB is obtained in 31.5Hz third octave band. Attenuation of 1.1dB, 3.6dB, 4.2dB and 2dB were obtained in the 40Hz, 50Hz, 63Hz, and 80Hz third octave band respectively. By lowering the acoustic natural frequency in the model by 5% a good prediction of the acoustic response was obtained and is shown in Figure 4.29. Also observed for the cylinder, this 5% drop in the natural frequency is attributed to the foam blanket. The poor performance in the 40Hz band comes from the almost undamped 41Hz resonance. This peak can be associated with a structural resonance since it also occurs in the vibration response as Figure 4.28 shows. The 40Hz band performance may be improved by DVAs tuned to this particular mode. The 100Hz band even though outside the frequency band of control, presents a minor attenuation of 1dB and is exempted from any treatment spillover. Given the complexity of the system the third octave performance are relatively well predicted by the model.

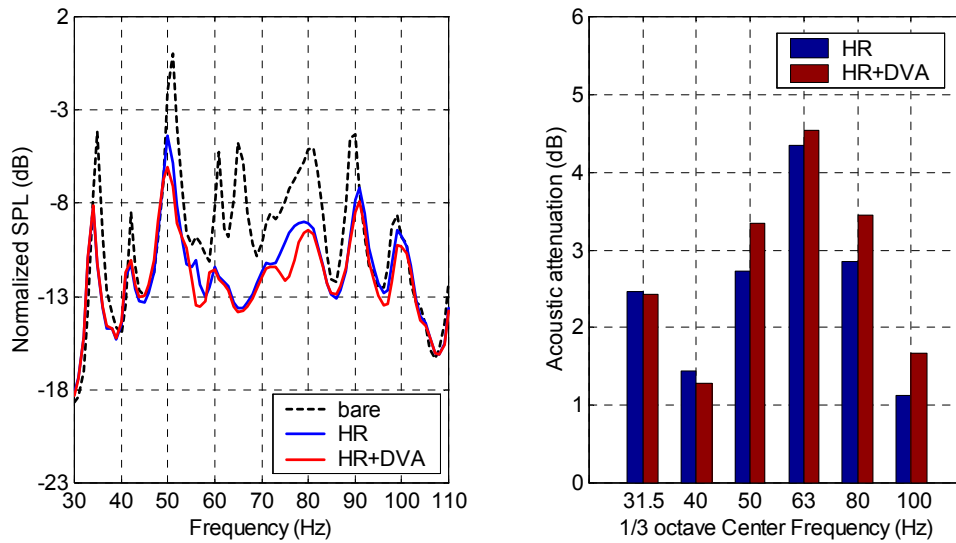


Figure 4.29 Acoustic performance prediction with lowered acoustic natural frequencies.

4.5 Conclusions

These tests have demonstrated that lightweight compact HR and DVA can increase the transmission loss of a real fairing below 100Hz by absorbing half of the acoustic energy in that band. Using the model, the effect of the DVA is evaluated. Due to small mass ratio (2.5%) of the DVA and a fairly damped vibration response most of the acoustic attenuation is provided by the HRs. It was also shown that using simple and cheap solution relatively light and effective resonators could be built. A more involved design process maximizing the HR stiffness to mass ratio could yield large weight reduction of the treatment. These experiments have also permitted the fairing model to be adjusted and then verified. The model then becomes a useful and fast tool for the design of a wide variety of noise reduction treatments. In particular, the design procedure presented in Chapter 3 using multiple Helmholtz resonators and vibration absorbers proved its effectiveness experimentally on three different systems. Indeed, the results obtained on the empty and partially filled cylinder demonstrated that by adding damping

to sharp acoustic and structural resonances can lead to efficient noise attenuation in the frequency band of control without adding significant weight and volume to the system. The last tests conducted on the fairing, have shown how this design procedure remains valuable when scaled to a much larger structure.

Chapter 5 Adaptive Helmholtz resonator

5.1 Introduction

This chapter represents a natural extension of the work presented previously in anticipation of the final payload fairing application. It was demonstrated that a noise treatment composed of distributed vibration absorbers and Helmholtz resonators can be optimized based on a fairing finite element model. Although the treatment proved robust to changes in the system, a treatment that could adapt might provide performance in a wider range of situations. For instance every payload being different in size and shape, shifts cavity natural frequencies; these shifts can be different for each launch.

In this chapter, the use of adaptive Helmholtz resonators that automatically tune to resonances in an enclosure is investigated. The time domain version of the cylinder model presented in Chapter 2 is developed to allow real time simulation of the HRs adaptation. Keeping in mind the drastic constraints of payload fairing applications, a local tuning law was adopted, which renders each HR an independent generic device with sensor, controller and actuator integrated.

An adaptive HR prototype was built and a control system to control up to eight devices was developed. Experiments were done on the Boeing cylinder, to validate the model simulations.

5.2 Time domain model

The modal description presented in Chapter 2 is extended here to the time domain. The time domain set of equation is solved using a 4th order Runge-Kutta method.

5.2.1 Governing equations

The acoustic cavity equation

In order to simulate the real time tuning of the resonators, the equations of the system need to be expressed in the time domain. The equation governing the behavior of an enclosed fluid was introduced in section 2.3.2 (Eq. (2.3.17)):

$$\nabla^2 p - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = -\frac{1}{\rho} \frac{\partial q}{\partial t},$$

where q is the acoustic source strength density distribution within the volume and on the surface of the enclosure. For the time domain derivation, the acoustic velocity potential φ is introduced and is related to the acoustic pressure p by:

$$p(\mathbf{r}, t) = -\rho \dot{\varphi}(\mathbf{r}, t). \quad (5.2.1)$$

The dot indicates the differentiation with respect to time and the $\mathbf{r}$ is the spatial vector regrouping the coordinates. The governing equation then becomes:

$$\nabla^2 \varphi - \frac{1}{c^2} \ddot{\varphi} = q(\mathbf{r}, t) \quad (5.2.2)$$

The acoustic velocity potential can be expanded in terms of the pressure modes of the cavity acoustic Ψ^a derived in section 2.3.1:

$$\varphi(\mathbf{r}, t) = \sum_N a_N(t) \Psi_N^a(\mathbf{r}), \quad (5.2.3)$$

where $a_N(t)$ is the amplitude and Ψ_N^a is the shape of the N^{th} mode which satisfies:

$$(c^2 \nabla^2 \Psi_N^a - \tilde{\omega}_N^2 \Psi_N^a) a_N = 0 \quad (5.2.4)$$

$\tilde{\omega}_N$ being the complex natural frequency of mode N . Using hard wall boundary conditions, the acoustic modes are assumed to be othorgonal and therefore multiplying Eq.(5.2.2) by Ψ_N^a and integrating it over the volume isolates each mode N :

$$V \Lambda_N (\ddot{a}_N - \tilde{\omega}_N^2 a_N) = -c^2 \int_V \Psi_N^a(\mathbf{r}) q(\mathbf{r}, t) dV \quad (5.2.5)$$

where Λ_N is the normalization factor given by

$$\Lambda_N = 1/V \int_V [\Psi_N^a]^2(\mathbf{r}) dV$$

The acoustic forcing on the right-hand side of Eq.(5.2.5) is composed of the forcing due to the vibration of the cylinder wall Q_N , and the forcing due to the HRs Q_N^r as

$$\int_V \Psi_N^a(\mathbf{r}) q(\mathbf{r}, t) dV = Q_N + Q_N^r$$

The primary forcing Q_N is expressed as

$$Q_N = \int_S \Psi_N^a(\mathbf{r}) \sum_{N_s} \Psi_{N_s}^s(\mathbf{r}) v_{N_s} dS = \sum_{N_s} C_{N, N_s} v_{N_s} \quad (5.2.6)$$

where v_{N_s} is the velocity amplitude of the N_s structural mode $\Psi_{N_s}^s$ and C_{N, N_s} is the N^{th} , N_s^{th} element of the coupling matrix $\mathbf{C}$ in Eq.(2.4.16). The term Q_N is the time domain version of an element of the modal acoustic source strength vector $\mathbf{u}^s$ introduced in section 2.6. The excitation of the system can also be provided by an acoustic source

placed inside the cavity. The modal acoustic forcing created by a piston source of velocity v_s at $\mathbf{r}_s$ inside the cavity is then given by

$$Q_N^s = \left(\int_{s_s} \Psi_N^a(\mathbf{r}_s) ds_s \right) v_s, \quad (5.2.7)$$

where s_s is the piston area. In order to model the possible damping of the acoustic cavity, the complex frequency $\tilde{\omega}_N$ is replaced by an equivalent real frequency ω_N and an equivalent arbitrary damping coefficient c_N , where $\tilde{\omega}_N = i\omega_N - c_N/2$. The modal damping ratio ζ_N is related to c_N by $c_N = 2\zeta_N\omega_N$. By rearranging Eq.(5.2.5), each acoustic mode amplitude are obtained using

$$\ddot{a}_N + c_N \dot{a}_N + \omega_N^2 a_N = -\frac{c^2}{V\Lambda_N} (Q_N + Q_N^r). \quad (5.2.8)$$

The Helmholtz resonator equation

The HRs are modeled as piston sources, therefore, the modal forcing term Q_N^r generated by N_r resonators is expressed as a summation:

$$Q_N^r = \sum_{h=1}^{N_r} \left(\int_{s_h} \Psi_N^a(\mathbf{r}_h) ds_h \right) \dot{\xi}_h = \sum_{h=1}^{N_r} \phi_{h,N}^a \dot{\xi}_h \quad (5.2.9)$$

where $\dot{\xi}_h$ is the particle velocity at the opening of the h^{th} HR, and s_h is its area. The term $\phi_{h,N}^a$, is one element of the acoustic-HR coupling matrix Φ^a given in Eq.(2.5.5). Using the resonator's mechanical analog model of section 1.4.1, the HR behavior is described by

$$\rho l_h s_h \ddot{\xi}_h + s_h R_h \dot{\xi}_h + (\rho c^2 s_h^2 / V_h) \xi_h = - \int_{s_h} p(\mathbf{r}_h) ds_h \quad (5.2.10)$$

where l_h is the effective length of the neck accounting for the internal radiation loading, R_h is the resistance of the HR responsible for its damping level, V_h is the HR volume, and $p(\mathbf{r}_h)$ is the acoustic pressure at the HR opening. Using Eq.(5.2.1) and (5.2.3) in Eq.(5.2.10) yields:

$$\ddot{\xi}_h + c_h \dot{\xi}_h + \omega_h^2 \xi_h = \frac{1}{s_h l_h} \sum_N \dot{a}_N(t) \phi_{h,N}^a(\mathbf{r}_h) \quad (5.2.11)$$

where $\omega_h = c\sqrt{s_h/V_h l_h}$ is the resonant frequency of the HR and c_h its damping coefficient related to the damping ratio ζ_h by $c_h = 2\zeta_h \omega_h$.

5.2.2 Matrix formulation of the system

The one-way coupled system

To simplify the formulation and obtain reasonable computational time, the acoustic back coupling onto the structure represented by the internal forcing $\mathbf{f}^{\text{int}}$ in section 2.6 is neglected. The external fluid loading effect is also excluded from this model. The system becomes then one-way coupled and the structure is independent of the acoustic cavity dynamics. As a consequence, the vibration response is simplified to structural mode only whereas the acoustic response of the cavity remains a mix of acoustic and structural resonances. The greater the density of the structure compared to air, the smaller the discrepancies in the cavity response introduced by this simplification. Using Eq.(5.2.8) and (5.2.11) the acoustic cavity-HRs coupled system is put in matrix form as

$$\begin{bmatrix} \mathbf{I}_N & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_R \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{a}} \\ \ddot{\xi} \end{Bmatrix} + \begin{bmatrix} \mathbf{C}_N & \mathbf{C}_1 \\ \mathbf{C}_2 & \mathbf{C}_R \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{a}} \\ \dot{\xi} \end{Bmatrix} + \begin{bmatrix} \omega_N^2 & 0 \\ 0 & \omega_R^2 \end{bmatrix} \begin{Bmatrix} \mathbf{a} \\ \xi \end{Bmatrix} = \begin{Bmatrix} \mathbf{F}_N \\ 0 \end{Bmatrix} \quad (5.2.12)$$

$\mathbf{I}_N$ and $\mathbf{I}_R$ are the identity matrix of dimension N , the total number of acoustic modes, and N_r total number of HRs respectively. The vectors $\mathbf{a}$ and $\boldsymbol{\xi}$ group the amplitudes of the velocity potential and the particle velocities at the HR throats. $\mathbf{C}_N$ and $\mathbf{C}_R$ are the diagonal matrices of damping coefficient of the N acoustic modes and of the N_r HRs respectively. Similarly $\boldsymbol{\omega}_N^2$ and $\boldsymbol{\omega}_R^2$ are the diagonal matrices of mode natural frequencies squared and the resonant frequency squared of the HRs.

The off diagonal matrices $\mathbf{C}_1$ and $\mathbf{C}_2$ represent the coupling between the HRs and the cavity. This coupling involves the velocity at the opening of the HRs and the time derivative of the velocity potential mode amplitudes, which, from Eq.(5.2.1), are proportional to the acoustic pressure. Using the HR-acoustic coupling matrix $\boldsymbol{\Phi}^a$ yields

$$\begin{aligned}\mathbf{C}_1 &= \boldsymbol{\Lambda}_N^{-1} \boldsymbol{\Phi}^{a^T} c^2 / V . \\ \mathbf{C}_2 &= -\mathbf{L}_R^{-1} \mathbf{S}_R^{-1} \boldsymbol{\Phi}^a\end{aligned}\tag{5.2.13}$$

$\mathbf{L}_R$, $\mathbf{S}_R$ and $\boldsymbol{\Lambda}_N$ are the diagonal matrices grouping the effective neck length, the throat area of the HR and the normalization factor of the acoustic mode respectively.

The forcing vector $\mathbf{F}_N$ in Eq.(5.2.12) is given by

$$\mathbf{F}_N = -\boldsymbol{\Lambda}_N^{-1} \mathbf{Q}_N c^2 / V .\tag{5.2.14}$$

If the acoustic cavity is excited from the inside by an acoustic piston source, the elements of $\mathbf{Q}_N$ are obtained using Eq.(5.2.7). When the cavity is excited through the structure, these elements are obtained using Eq.(5.2.6). For this particular case, the time domain structural mode amplitudes v_{N_s} are required. These amplitudes are obtained from the frequency domain model. Using the frequency response of the structural mode amplitude $\mathbf{v}$, impulse response filters are created by inverse Fourier transform. The convolution of these filters with a uniformly random signal leads to the time domain structural mode amplitudes. The cylinder scattering of the incident plane wave, the structural coupling

with the external pressure field, and the dynamics of the structure are thus included in the forcing vector $\mathbf{F}_N$. Moreover the effect of a DVA treatment can be taking into account by using the frequency response of the structural mode with DVAs.

State space formulation

The real time adaptation of the HR natural frequencies uses a fourth order Runge-Kutta technique⁸⁷. For this purpose Eq.(5.2.12) is rearranged in state space:

$$\dot{\mathbf{y}}_i = \mathbf{A}_i \mathbf{y}_i + \mathbf{f}_i, \quad (5.2.15)$$

where

$$\mathbf{y}_i = \begin{bmatrix} \mathbf{a} \\ \xi \\ \dot{\mathbf{a}} \\ \dot{\xi} \end{bmatrix}, \mathbf{A}_i = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{I}_N & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}_R \\ -\omega_N^2 & \mathbf{0} & -\mathbf{C}_N & -\mathbf{C}_1 \\ \mathbf{0} & -\omega_R^2 & -\mathbf{C}_2 & -\mathbf{C}_R \end{bmatrix}, \mathbf{f}_i = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{F}_N \\ \mathbf{0} \end{bmatrix} \quad (5.2.16)$$

The state of the system at the following time step $i+1$, which is Δt into the future, is calculated by estimating the gradient of the state vector $\mathbf{y}$ and considering the input force,

$$\mathbf{y}_{i+1} = \mathbf{y}_i + \Delta t \left(\frac{\mathbf{g}_1}{6} + \frac{\mathbf{g}_2}{3} + \frac{\mathbf{g}_3}{3} + \frac{\mathbf{g}_4}{6} \right). \quad (5.2.17)$$

The four estimates of the gradient $\mathbf{g}_1$ - $\mathbf{g}_4$ are given by,

$$\begin{aligned} \mathbf{g}_1 &= \mathbf{A}_i \mathbf{y}_i + \mathbf{f}_i, \\ \mathbf{g}_2 &= \mathbf{A}_i (\mathbf{y}_i + 0.5 \Delta t \mathbf{g}_1) + \mathbf{f}_{i+1/2}, \\ \mathbf{g}_3 &= \mathbf{A}_i (\mathbf{y}_i + 0.5 \Delta t \mathbf{g}_2) + \mathbf{f}_{i+1/2}, \\ \mathbf{g}_4 &= \mathbf{A}_i (\mathbf{y}_i + \Delta t \mathbf{g}_3) + \mathbf{f}_{i+1}. \end{aligned} \quad (5.2.18)$$

At each time step, the matrix A_i can be modified as the natural frequency of the HRs are adjusted to minimized some cost function.

5.3 Tuning algorithm

The following section investigates different strategies to adapt the HR and presents the particular tuning law used.

5.3.1 Local strategy for global control

As a preliminary study for the new adaptive resonator concept presented in this chapter, a comparison between local and global strategies for the control of a set of modes was undertaken. This paragraph present the main conclusion of this investigation detailed in reference 82. The idea was to investigate the interaction of multiple single degree-of-freedom systems with a continuous system described by a set of modes. The shape of different local cost functions due to tuning of these single degree-of-freedom systems was studied and compared to a global cost function. In reference 82 the concrete example of a vibrating structure on which multiple vibration absorbers are attached is adopted to describe the different cost functions. The acoustic analogue corresponds to an acoustic cavity coupled to multiple Helmholtz resonators. The pressure mode amplitude of the cavity are then equivalent to the velocity amplitude of the structure, the pressure at the opening of a HR is equivalent the base velocity of a vibration absorber, and finally the pressure inside a HR is equivalent to the velocity of the absorber mass. To remain consistent with reference 82, the cost functions are here presented using the structural example and are illustrated in Figure 5.1.

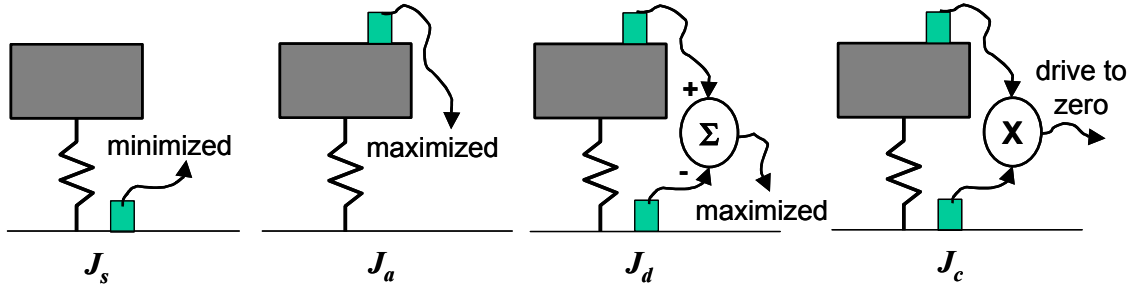


Figure 5.1 Schematic of the different local cost functions for a tunable vibration absorber.

The global cost function proportional to the kinetic energy of the structure in the frequency band of control was intended to be minimized and evaluated using the structural mode amplitudes vector $\mathbf{v}$ by

$$J_g = \sum_{\omega_1}^{\omega_2} \mathbf{v}^H \mathbf{v}. \quad (5.3.1)$$

Four different local cost functions were considered. The structure velocity at the absorber location J_s , monitoring the discrete vibration of the host structure was intended to be minimized and given by

$$J_s = \sum_{\omega_1}^{\omega_2} \Phi^s \mathbf{v}. \quad (5.3.2)$$

The absorber velocity J_a , and the difference in velocity from absorber to structure J_d , both linked to the power dissipated by the absorber were intended to be maximized and obtained by

$$\begin{aligned} J_a &= \sum_{\omega_1}^{\omega_2} \frac{Z_d}{-i\omega m_d} \Phi^s \mathbf{v} \\ J_d &= \sum_{\omega_1}^{\omega_2} \left[\frac{Z_d}{-i\omega m_d} - 1 \right] \Phi^s \mathbf{v}. \end{aligned} \quad (5.3.3)$$

Finally the product between the velocity of the absorber and the structure, was intended to be driven to zero and expressed as

$$J_c = \sum_{\omega_i}^{\omega_b} \left[\frac{Z_d}{-i\omega m_d} \Phi^s \mathbf{v} \right] \times \Phi^s \mathbf{v}. \quad (5.3.4)$$

As shown in Figure 5.2, this latest cost function called the dot-product is the only local cost functions of the set that converge very close to the global solution for a multi-mode single absorber case. This remained valid for the multi-mode multi-absorber case. The dot-product method was therefore retained as a local tuning law and its mechanisms are analyzed in detailed in the next section.

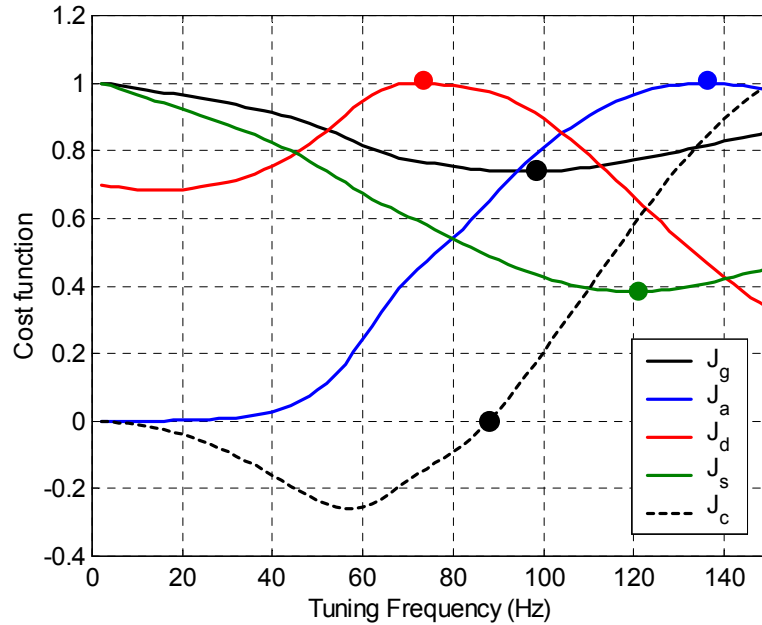


Figure 5.2 Cost functions as a function of absorber tuning frequency using 13 modes.

5.3.2 The dot-product method

Single-frequency excitation

For a single degree of freedom vibration absorber, the tuning is achieved when the motion of the structure and the absorber mass are in quadrature. The dot-product method is a tuning law used for adaptive vibration absorbers⁸⁸ in single frequency control applications. It works by evaluating the integral over a period of the excitation frequency of the product between the velocity amplitudes of the absorber mass $\dot{x}_a(t)$ and the host structure $\dot{x}_s(t)$. Taking the dot-product of the two discrete time signals approximates this integral. Expressing for single-frequency excitation these time signals as $\dot{x}_s(t) = \dot{X}_s \cos(\omega t)$ and $\dot{x}_a(t) = \dot{X}_a \cos(\omega t - \phi)$, Long⁶⁹ showed that the dot-product is proportional to a cosine function of the phase angle between the host structure and the absorber mass as

$$\frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \dot{x}_s(t) \dot{x}_a(t) dt = \frac{|\dot{X}_s| |\dot{X}_a|}{2} \cos(\phi). \quad (5.3.5)$$

When the two signals are in quadrature, the absorber is tuned and, assuming a lightly damped absorber, the dot-product is approximately zero. As a consequence, the sign of the product can be used to tune the absorber to the excitation frequency. When the absorber is over tuned, the two signals are almost in phase and the dot-product is positive. When the absorber is under tuned, the two signals are mostly out of phase and the dot-product becomes negative. The dot product also gives information on the magnitude of the tuning error. Using the acoustic analogue, this tuning algorithm is applied to a tunable HR by evaluating the dot product between the pressure amplitudes inside the HR, p_{int} , and at its opening, p_{ext} .

Broadband excitation

Under broadband excitation, the product of the two signals is integrated over an arbitrary period T . This integral provides an estimate at $\tau=0$ of the cross correlation function $X_{p_{int}, p_{ext}}(\tau)$ between p_{int} and p_{ext} ; the longer the period, the more accurate the estimation. The cross correlation can be expressed as the inverse Fourier transform of the cross spectrum $G(\omega)$ of the two signals:

$$X_{p_{int}, p_{ext}}(\tau) = \int_{-\infty}^{+\infty} G_{p_{int}, p_{ext}}(\omega) e^{j\omega\tau} d\omega. \quad (5.3.6)$$

Using the even and odd properties of the real and imaginary part of the cross spectrum, the dot product reduces to

$$X_{p_{int}, p_{ext}}(\tau = 0) = \int_0^{+\infty} \text{Re}(G_{p_{int}, p_{ext}}(\omega)) d\omega. \quad (5.3.7)$$

As explained in section 3.2.4, for frequencies well below the first mode of the HR cavity, Eq.(3.2.4) links the internal pressure p_{int} and the volume velocity of the HR opening u . Rearranging this equation yields

$$p_{int}(\omega) = \frac{\rho c^2}{i\omega V_h} u(\omega). \quad (5.3.8)$$

The HR admittance Y_h derived in section 2.5.1, relates the HR volume velocity to the impinging pressure at its throat, p_{ext} as

$$u(\omega) = Y_h(\omega) p_{ext}(\omega) = \frac{i\omega\omega_h^2 V_h}{\rho c^2 [(\omega_h^2 - \omega^2) - 2i\zeta_h \omega\omega_h]} p_{ext}(\omega) \quad (5.3.9)$$

Substituting Eq.(5.3.9) in Eq.(5.3.8) the cross spectrum of the two pressure signals reduces to

$$G_{p_{int}p_{ext}} = p_{int}(\omega)p_{ext}^*(\omega) = \frac{\omega_h^2}{[(\omega_h^2 - \omega^2) - 2i\zeta_h\omega\omega_h]} |p_{ext}(\omega)|^2. \quad (5.3.10)$$

This shows that the real part of the cross spectrum is positive below the HR tuning frequency and negative above. Driving the dot-product in Eq.(5.3.7) to zero, tunes the HR such that an equal amount of the real part of the cross-spectrum lies above and below its natural frequency. As a consequence, HRs are attracted to peaks in the spectrum of p_{ext} . Placed in a resonant environment, the HRs tune to the lightly damped modes of the cavity where they are the most effective in adding damping and thus have a global effect on the noise level as illustrated in Figure 5.3.

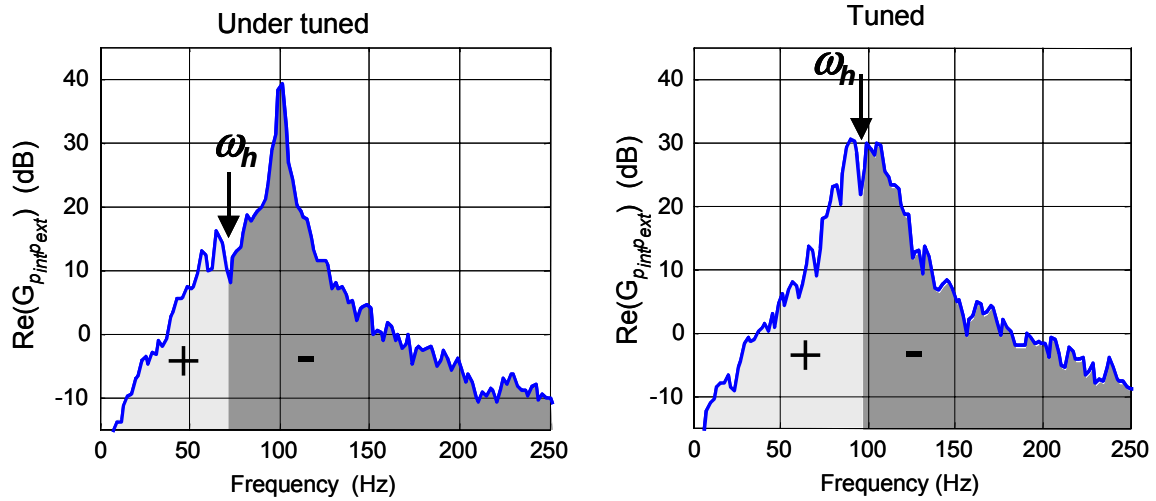


Figure 5.3 Cross-spectrum of and under tuned HR (left) and tuned HR right with respect to a single resonance at 100Hz.

Since the dot-product evaluation is local, the HR will not tune to uncontrollable modes that are modes with nodes at the HR position. Another property of the dot-product method is the high frequency filtering of the broadband excitation due to the natural

dynamics of the HR. Indeed the displacement-force admittance of the HR provides a -6dB per decade filter above the tuning frequency. Excited with a constant volume velocity source, the acoustic resonances of the cavity present a velocity-force admittance behavior with symmetric slopes of 3dB and -3dB per decade before and after the natural frequency. Combining these two type of responses with Eq.(5.3.10) leads to band-passed filter properties for the cross-spectrum. However if the cavity is excited by a constant volume acceleration source, the acoustic resonances present the same displacement-force admittance behavior as the resonator. As a consequence, the cross-spectrum behave as a 6 order low-pass filter. This may cause an under tuning of the HR by biasing the dot product to the low frequencies. Therefore, when the cavity is excited with a constant acceleration source such as a speaker, the dot-product can be evaluated using the derivative of the pressure signals. The resulting cross-spectrum presents band-passed filter properties, which ensure a more accurate tuning.

In the model, the pressures inside the HR and at its throat are expressed as

$$\begin{aligned} p_{int} &= \rho c^2 / V_h \xi_h(i=1, \dots, n) \\ p_{out} &= -\rho \sum_N \phi_{h,N}^a \dot{a}_N(i=1, \dots, n) \end{aligned} \quad (5.3.11)$$

where n is the number of time steps defining the period T . In order to limit its absolute value by unity, the dot product, x_p , is also normalized

$$x_p = \frac{p_{int} \cdot p_{out}}{\sqrt{(p_{int} \cdot p_{int})(p_{out} \cdot p_{out})}}. \quad (5.3.12)$$

5.4 Numerical simulations

The following numerical simulations correspond to the Boeing composite cylinder. The natural frequencies and mode shapes for the acoustic cavity are obtained from

section 2.3. The structural modal characteristics differ slightly from those presented in section 2.2 as the simpler Donnell-Musthari shell theory⁷² was used. The length of the HR neck was chosen to modify their tuning frequency.

5.4.1 Single mode forced by an internal source

In this section the acoustic forcing is provided by a piston source inside the cavity; therefore, because the model is only one-way coupled, the dynamics of the structure do not intervene in the computation. The source modeled as a random signal is positioned at the bottom of the cylinder (axial position $z=0$, radial position $r=1.23$, and circumferential position $\theta=\pi$). Only the first mode of the cavity with a natural frequency of 61.7 Hz and a damping ratio of 1% is taken into account. Three identical HRs are positioned at the boundary (i.e. $r=1.23$), at the bottom of the cylinder ($z=0$) and are evenly distributed around the circumference ($\theta=0, 2\pi/3, 4\pi/3$). The total volume of the HRs represents 0.6% of the cavity volume. The optimal damping ratio is set to 5.2% according to Eq.(3.3.3). The initial length of the necks are 12cm, 20 cm and 15cm, which provide initial tuning frequencies of 66.7Hz, 55.1Hz and 62.0Hz respectively. This configuration represents the over-tuned, under-tuned and tuned cases for the HRs. Using the Runge-Kutta model with a time step of 1ms, the necks of HRs are sequentially adapted by 2.5mm increments according to the value of the dot product evaluated over a period of 0.5s. The HRs stop tuning when the absolute value of the normalized dot product drops below 0.01. Figure 5.4 shows the evolution of the dot product and the tuning frequency for the 3 HRs as a function of time. As expected the first HR drops its natural frequency down to 62.1Hz, the second raises it to 59.6Hz and the third one maintains it at 62.0Hz. The adaptation time for the three HRs is less than 25 s (during the first 5 seconds, HRs are inactive).

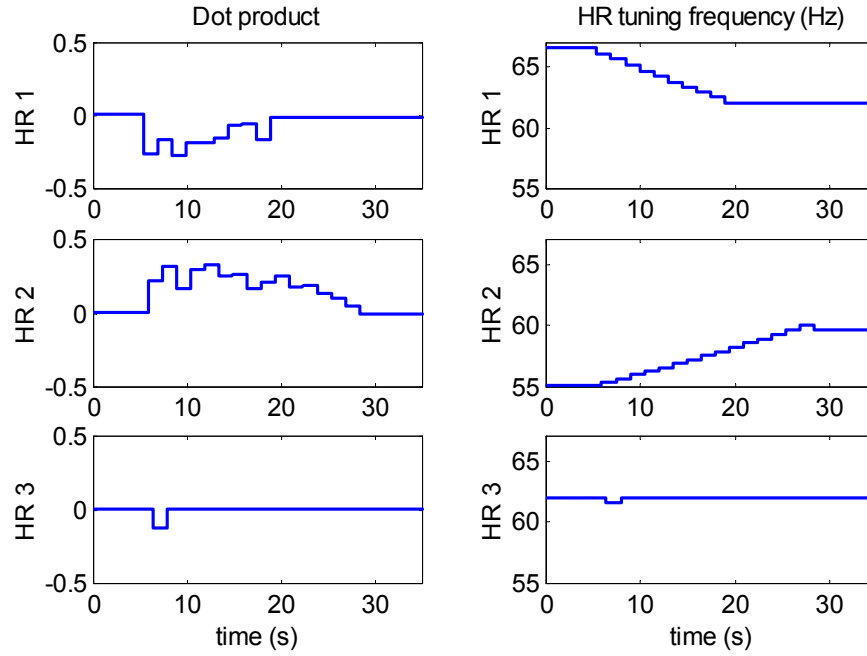


Figure 5.4 Change in the dot-product and tuning frequency of the 3 HRs as a function of time.

5.4.2 Multi-mode forced by an external plane wave

In this section the acoustic cavity is forced through the structure by an incident plane wave ($\alpha_i=70^\circ$). Figure 5.5 shows the kinetic and acoustic energy obtained with the one-way coupled frequency domain model with and without the DVA treatment. The simulation includes 36 structural modes and 17 acoustic modes with natural frequencies below 200Hz. The DVA treatment, representing 2% of the cylinder mass, consists of a ring of 13 absorbers distributed evenly around the circumference and placed halfway up the cylinder wall. The damping ratio of the structural mode is set to 1% whereas the damping ratio of the DVAs is set to its optimal value of 10% computed using Eq.(3.3.1). The DVAs target the most excited 1,2 structural mode at 112.5Hz, which also couples best to the acoustic cavity. Once this structural resonance is damped by the DVAs, the

remaining peaks in the acoustic energy spectrum correspond to the acoustic modes of the cavity and thus can be damped by HRs. Without DVAs, the HR could tune to any structural peaks in the acoustic spectrum even if they cannot damp them efficiently.

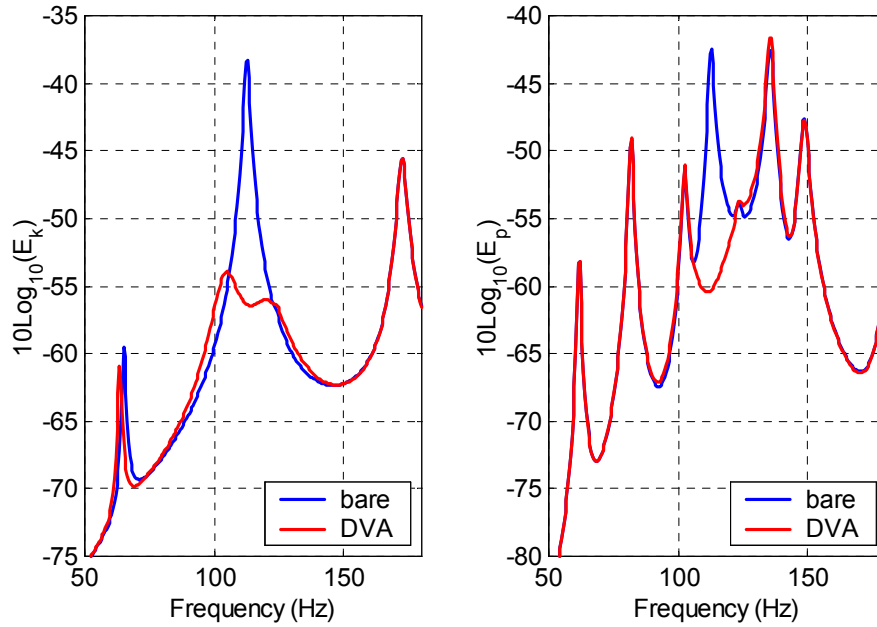


Figure 5.5 Structural kinetic (E_k) and acoustic potential (E_p) energy with and without DVA treatment (one-way coupled model).

The HR treatment consists of 2 rings of 5 resonators placed at the bottom and halfway up the cylinder walls. As for the DVA, the 5 HRs are evenly distributed around the circumference. The total volume of the HR treatment represents 4% of the cavity volume, and their damping ratio is set to 9%. The time domain velocity amplitudes of the structural modes with the DVA treatment are substituted in the Runge-Kutta model. The time step is set to 0.5ms. The adaptation is continued until all the HRs are considered tuned, when the absolute value of their normalized dot product is less than 0.01. In this particular case, the adaptation lasts around 2 minutes, (around 12s per HR). Figure 5.6 shows the acoustic potential energy for the bare cylinder and the cylinder treated with HRs and DVAs before and after the adaptation. The initial tuning frequency of the

bottom and middle ring illustrated by the blue arrows in Figure 5.6 are 50 and 170Hz respectively. The final tuning frequencies (red arrows) are for the bottom ring [107.2, 89.6, 109.5, 101.1, 76.5] Hz and for the middle ring, [148.1, 146.3 136.9, 136.1, 130.1] Hz.

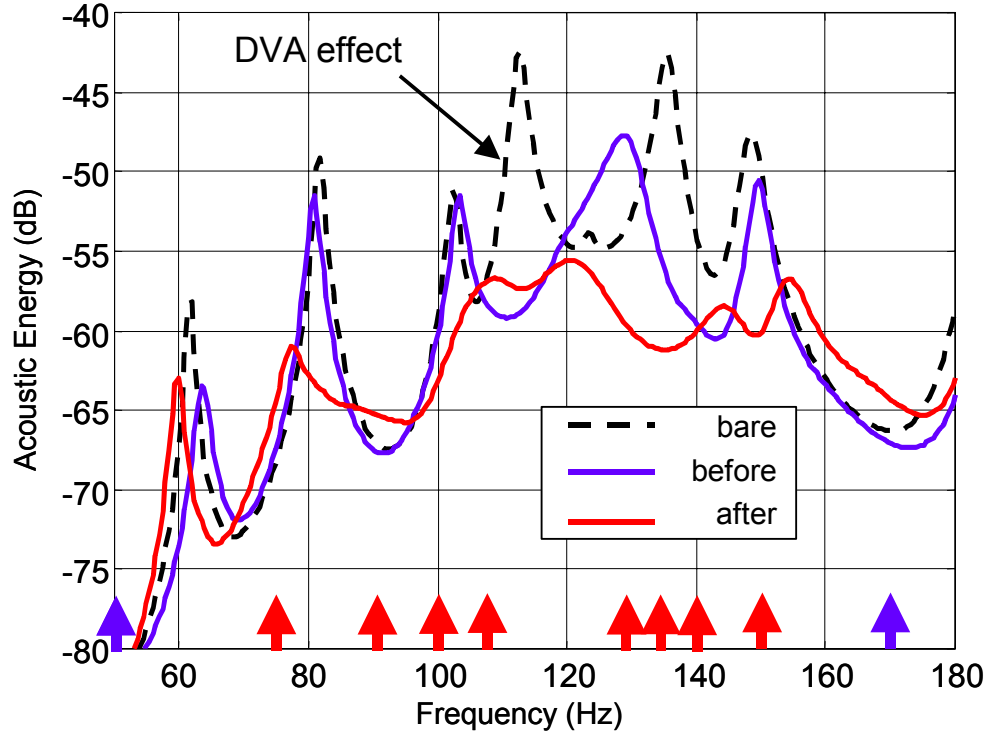


Figure 5.6 Acoustic potential energy before and after adaptation compared to the bare case.

Initially, with the HR rings tuned to 50Hz and 170Hz, the acoustic attenuation from 40Hz to 160Hz is 3.4dB, where 1dB is due to the DVAs. Even though they are not tuned, the HRs can still absorb some energy of the neighboring peaks because of their high damping level, which broadens their frequency range of action. However, once the adaptation is over the acoustic attenuation from 40Hz to 160Hz increases up to 7.5dB. This performance is similar to 7.7dB obtained in reference 89 where 5 rings of 5 HRs each, representing 6% of the cavity volume, were tuned to the five dominant acoustic resonances between 40Hz-160Hz. These simulations demonstrate that, as long as the structure modes, which strongly couple to the acoustic cavity, are damped with a DVA treatment, the dot-product method appears to converge to a near optimal solution.

5.5 Experimental results

This section describes the experiment conducted on the composite cylinder. The adaptive resonator prototype and the control system are also presented.

5.5.1 Adaptive Helmholtz resonator prototype

According to Eq.(1.4.2), changing the resonant frequency of a Helmholtz resonator can be achieved by modifying the volume, the opening area or the neck length. De Bedout *et al.* built a tunable resonator with a variable volume⁶⁵. This solution probably allows the widest tuning range and ensures the natural frequency to vary as the square root of the volume. However, the HR efficiency is proportional to its volume (section 2.5.1), and therefore this device has reduced efficiency when tuned above its lowest frequency. In addition, the weight of the machinery required to change a sealed volume renders such a device unsuited for payload fairing applications.

Several designs of tunable HR with fixed volume were investigated. The best compromise between tuning range and compactness was obtained for a variable opening HR as shown in Figure 5.7. Based on the previous design, the resonator was made of a cardboard tube with plastic end-caps. On the top end-cap, an iris diaphragm provides the variable opening whose diameter ranges from 9 to 58mm. The length of the tube (22”) was chosen to obtain a tuning range including the first three acoustic mode of the cylinder at 63, 82, and 103Hz. Using a pulley mechanism, a lightweight (28g) small step motor is used to rotate the iris control arm as shown in Figure 5.8. This mechanism represented an easy, off the shelf and cheap solution. The adaptive HR is equipped with two microphones; one placed in front of the opening, and the other flush mounted to the back end cap to monitor the internal pressure. These two microphones provide the two signals needed to evaluate the dot product.

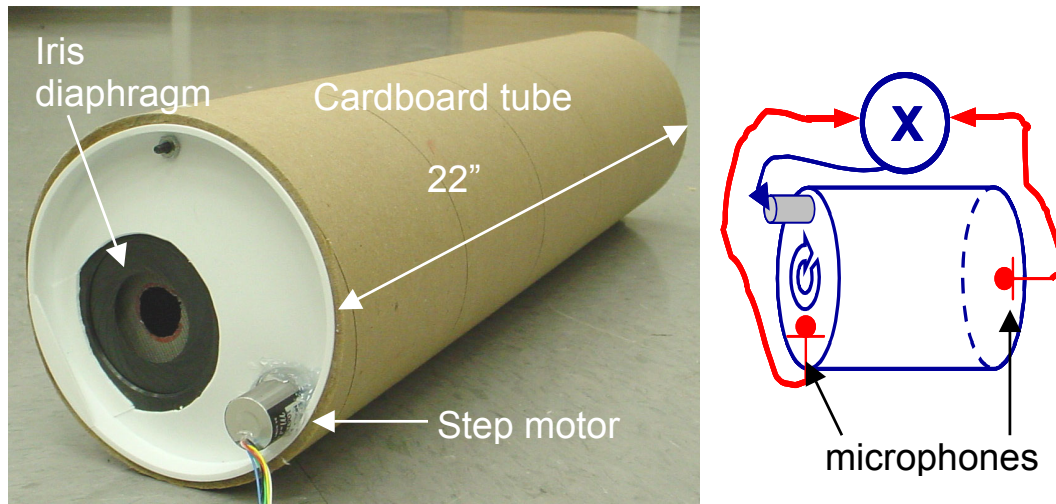


Figure 5.7 Adaptive Helmholtz resonator prototype.

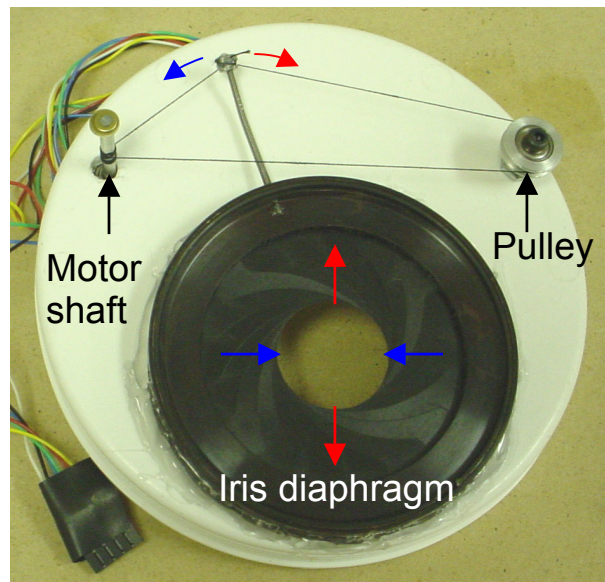


Figure 5.8 Motorized iris diaphragm mechanism.

Using the same experimental set-up as described in Figure 3.35, the adaptive HR natural frequency and damping ratio were measured for different opening diameters. Because of the 90° minimum step angle of the motor, only 30 different diameters can be obtained. The resulting frequency resolution of approximately 2Hz is sufficient for this

particular application, which requires damped HRs. As shown in Figure 5.9, the good agreement between the measured and the predicted natural frequency using Eq.(1.4.2) is lost for opening diameters greater than 30mm. This is due to the increase of the aperture thickness as the iris opens. Indeed, in Eq.(1.4.2), the physical neck is fixed to 1mm, which corresponds to the thickness of the iris for small diameter. However, as the leaves of the iris fold to open, the physical neck increase to a maximum that includes the frame and end-cap thickness. Using a neck length corresponding to each opening diameter in Eq.(1.4.2) can correct this discrepancy. As illustrated in Figure 5.9, the damping of the resonator also varies significantly with the tuning frequency. Created by viscous losses, the damping increase non-linearly as the HR opening is reduced. A fairly constant level of damping through out the frequency range is preferable to ensure the optimal performance of the HR. As a solution, a wire mesh screen is placed over the opening to increase the damping levels, for diameters greater than 20mm. To avoid a further increase of damping, already too high for smaller diameters, a 20mm diameter disk is cut out of the center of the screen as shown in Figure 5.10. The measured natural frequency and damping ratio with and without screen is plotted in Figure 5.11 as well as the theoretical frequency obtained with adjusted neck length. The resonance of the HR is not affected by the presence of the screen and can be well predicted by Eq. (1.4.2). In addition, the screen ensures a more constant and favorable damping level of 5% throughout the tuning range except for the lowest three frequencies.

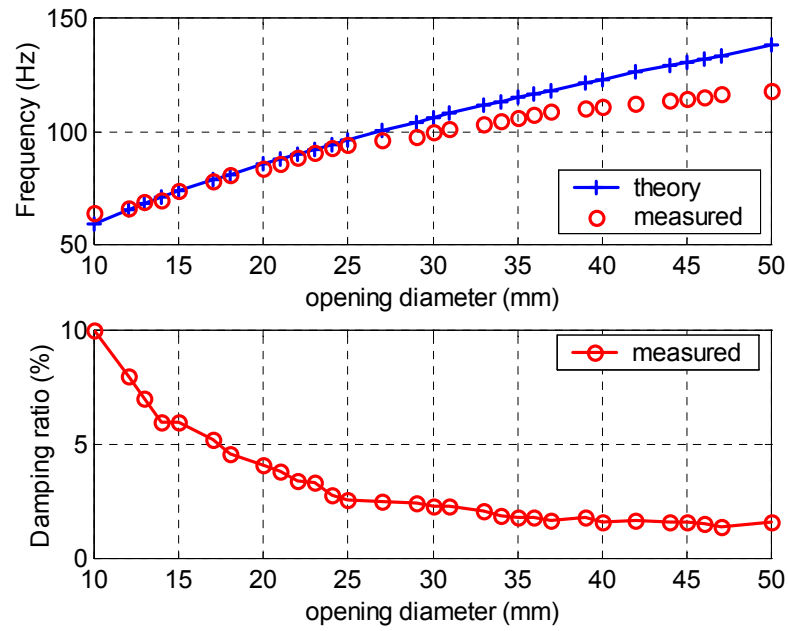


Figure 5.9 Measured natural frequency and damping ratio without screen for different opening diameters.

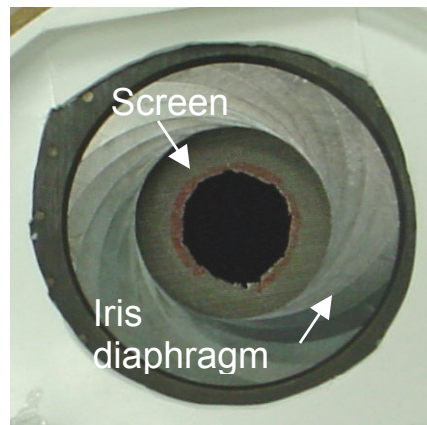


Figure 5.10 Iris diaphragm with wire mesh screen for constant damping.

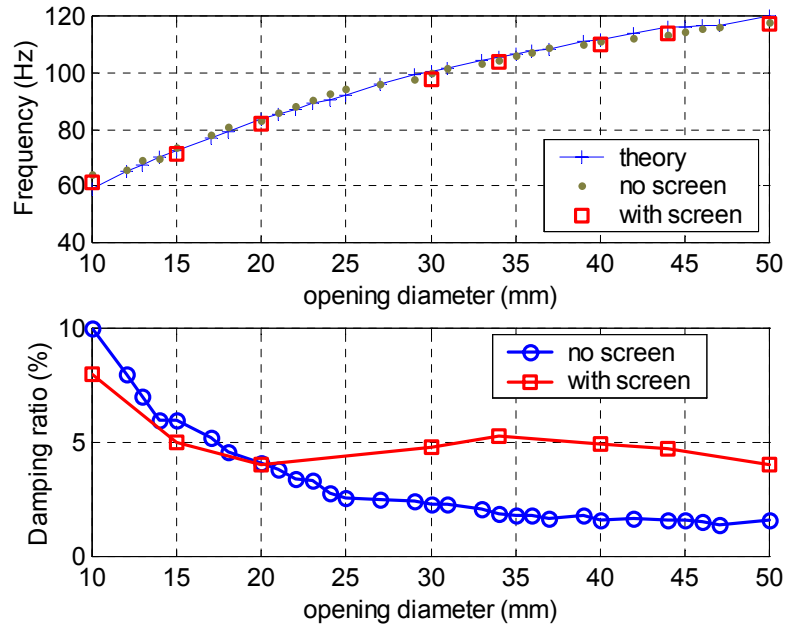


Figure 5.11 Measured natural frequency and damping ratio with screen for different opening diameters.

5.5.2 Control system

The main advantage of this method is its possible implementation using analogue circuitry, which reduces the controller complexity, cost, and weight. Indeed, the analogue signals of the two microphones can be summed and then low pass filtered yielding a positive or negative continuous voltage that can then control the rotation of a standard DC motor. To avoid tuning the HR outside of its functional range band pass filtering of the two microphone signals around this range would be sufficient. This simple scheme permits the integration of sensor, actuator and controller into one generic device.

However, in this work, a digital centralized controller was developed to mimic the behavior of multiple independent analogue controllers. This centralized controller was more suitable to observe the tuning mechanisms of several HRs simultaneously. This

control system uses a Labview ® interface and is composed of two parts. The first part is the National Instrument data acquisition used to acquire the time signals of the microphone. The second is the stepper motor driver, which uses the parallel port of the computer and an electronic circuit composed of decoders and transistor arrays. A 5 volt and 28 volt power supply is required for the electronics and the stepper motor respectively. This system allows the control of up to eight motors sequentially. Given the initial diameter of the opening, the system tracks the iris variations and stops the motor whenever the physical limits of the opening are reached.

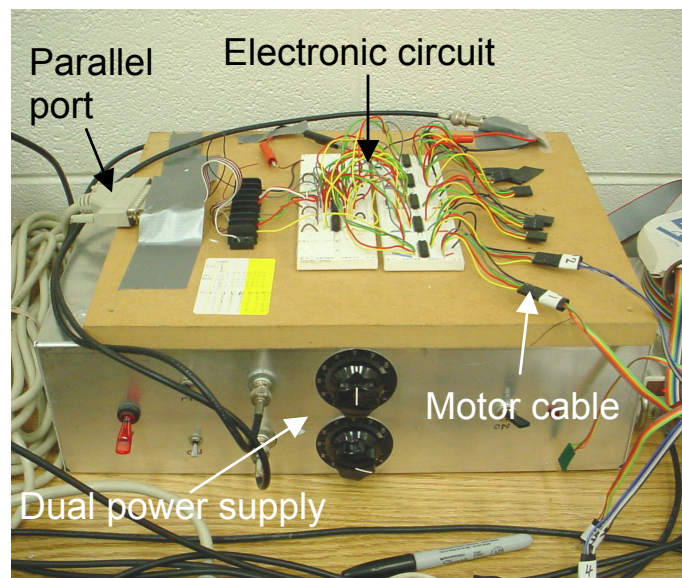


Figure 5.12 Control system for 8 resonators.

5.5.3 Experimental set-up

Eight adaptive HRs identical to the one in Figure 5.7 each equipped with two microphones were built. A speaker was placed at the bottom of the cylinder as seen in Figure 5.13 and driven with 40-200Hz band passed white noise to excite the acoustic cavity. Three rings of five microphones positioned at 19, 54, 92 inches from the bottom and 12 inches from the wall were used to obtain a spatial average of the acoustic pressure

inside the cylinder. A last microphone was placed inside the disturbance speaker cabinet in order to estimate the volume velocity (VV) of the source as explained in section 3.2.4. The auto-spectra of all the microphones and their cross-spectrum with respect to the microphone placed inside the speaker cabinet can be acquired simultaneously. The total volume of the resonator represents only 0.4% of the cylinder's cavity. As a consequence, to obtain the best efficiency, the 8 HRs are positioned as illustrated in Figure 5.7 with their opening towards the cylinder wall. In this configuration, the HRs are placed at antinodes of the first three acoustic modes $(1,0,0)$, $(0,1,0)$ and $(1,1,0)$, which orient with respect to the disturbance speaker.

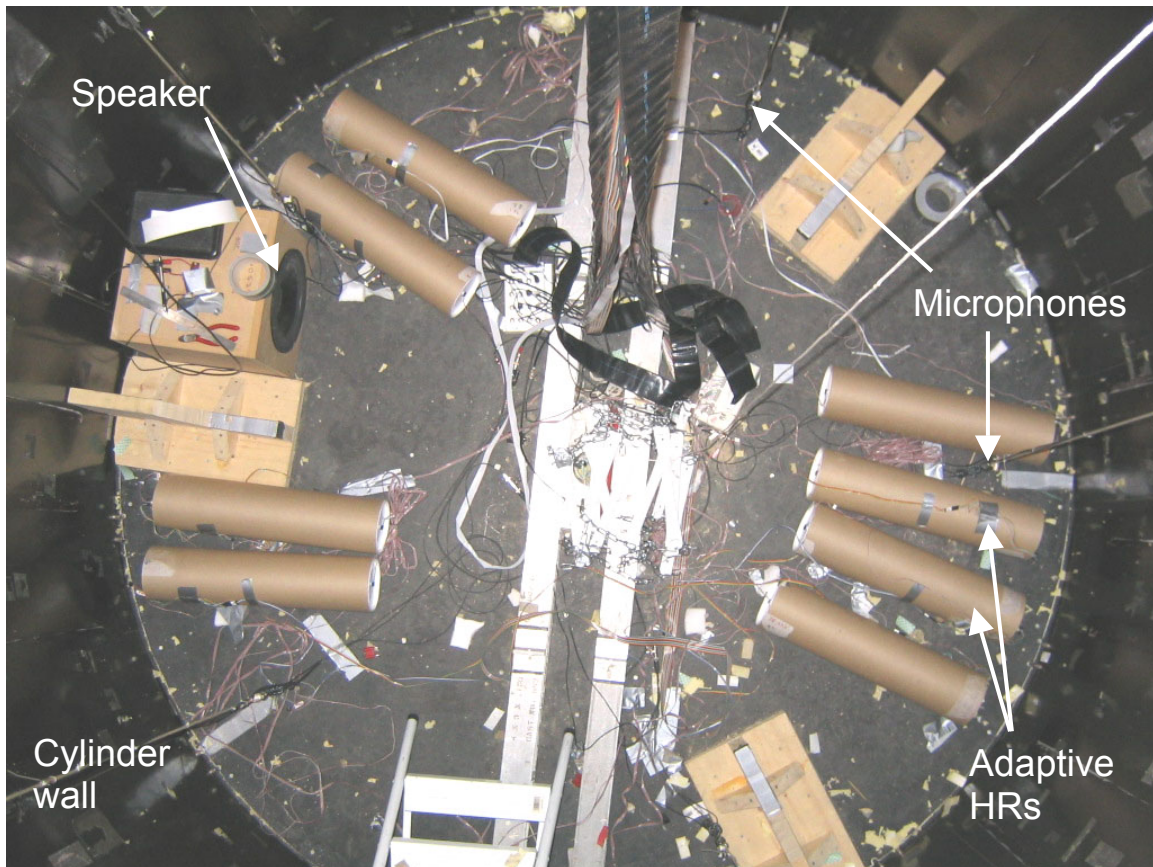


Figure 5.13 Top view of the inside of the cylinder

5.5.4 Results

The microphone signals are first acquired when the HRs are inside the cylinder with tape over their opening which renders them inactive. This configuration is called “bare” since it is equivalent to the empty cylinder apart from the small volume taken by the HR. The tape is then removed from the HR throats and the opening diameters are set to some initial value. The data is acquired in this configuration, which is called “before adaptation”. The control system is turned on, and using the time signal of the HR microphones the adaptation begins. When the opening diameter of each HR has converged, the control is turned off and all the sensor signals are acquired to evaluate this “after adaptation” configuration.

Single mode tuning

The objective of this experiment was to verify the tuning algorithm on a single mode. The HR microphones signals were therefore digitally band-pass filtered around the third mode of the acoustic cavity at 103Hz. The cut-on frequencies of the 6th order Butterworth filters used were 95 and 115Hz. As explained previously the time derivative of the microphone signals are used to compute the dot-product. The opening diameters of the HR were set initially to 12 mm and the 8 HRs were adapted sequentially. With a sampling frequency of 1024Hz, the dot-product was evaluated over one second using 1024 points and its threshold below which the HRs were considered tuned was set to 0.05. The adaptation process was carried on for about 5 minutes, each iteration lasting about one second. Figure 5.14 plots the opening diameter of the 8 HRs for every iteration. and Table 5.1 lists the corresponding initial and final tuning frequencies obtained from the transfer function between the two microphones at each HR. As an example, the transfer function of two HR before and after adaptation is plotted in Figure 5.15 showing the shift in their natural frequency. Note that these transfer functions are obtained as a ratio of the cross spectrum of the two microphone with respect to the microphone placed in the speaker cabinet, and the spike around 90Hz comes the ratio of a zero in both cross

spectra. To further illustrate the convergence of the HRs toward the 104Hz-targeted resonance, Figure 5.16 plot for two HRs the real part of the normalized cross-spectrum after band-passed filtering. Initially under tuned, the arithmetic area under the blue curve corresponding to the dot-product value x_p is negative. Once tuned, the positive and negative area under the red curve balances out yielding a dot-product value close to zero.

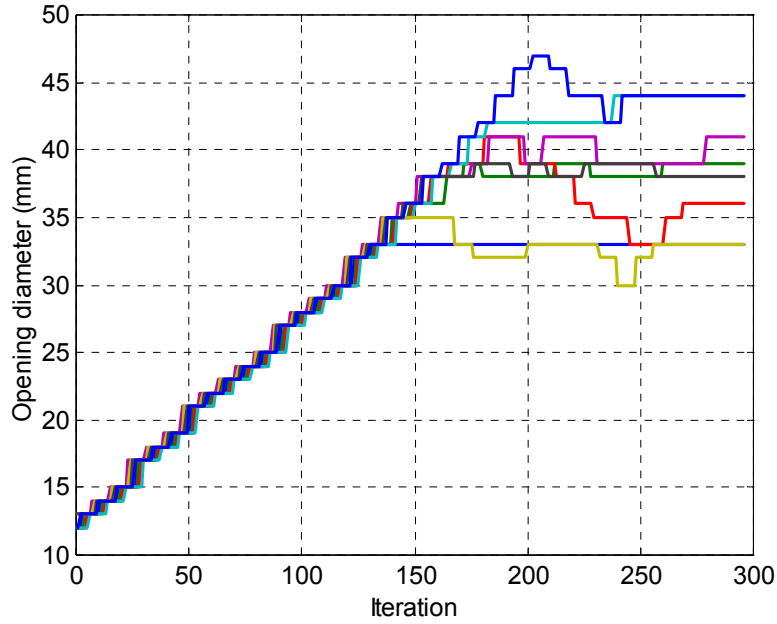


Figure 5.14 Evolution of the opening diameter for the 8 HRs.

HR #	Tuning frequency (Hz)	
	Before adaptation	After adaptation
1	72	105.5
2	69	105
3	66	104.5
4	67	105
5	69	108
6	70	101.5
7	67	101
8	67	106

Table 5.1 Tuning frequency of the 8 HR before and after the single mode adaptation.

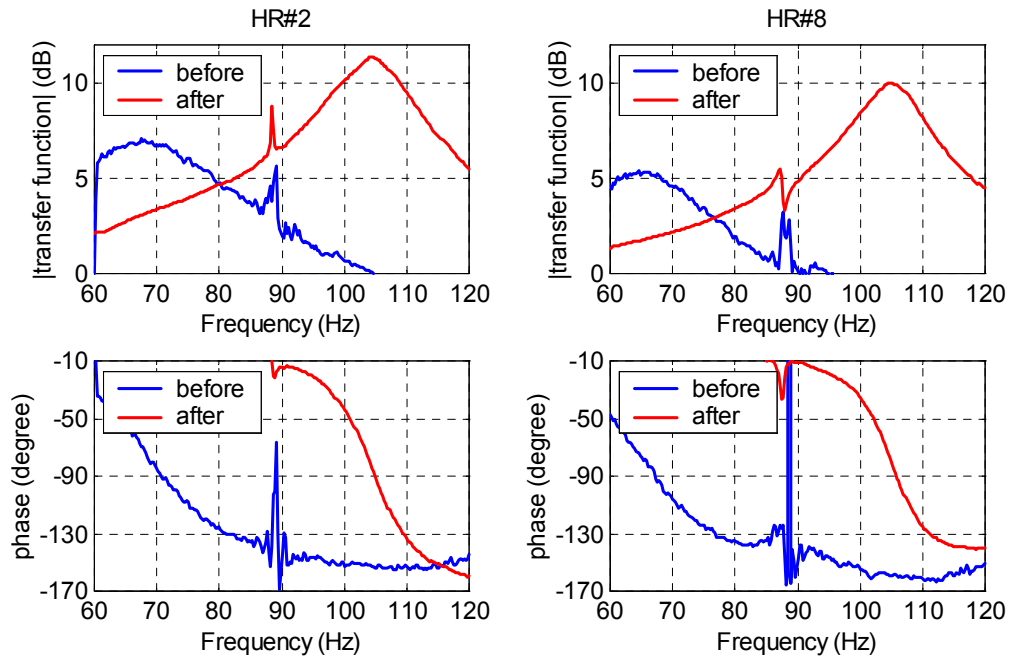


Figure 5.15 Magnitude and phase of the transfer function of HRs #2 and #8.

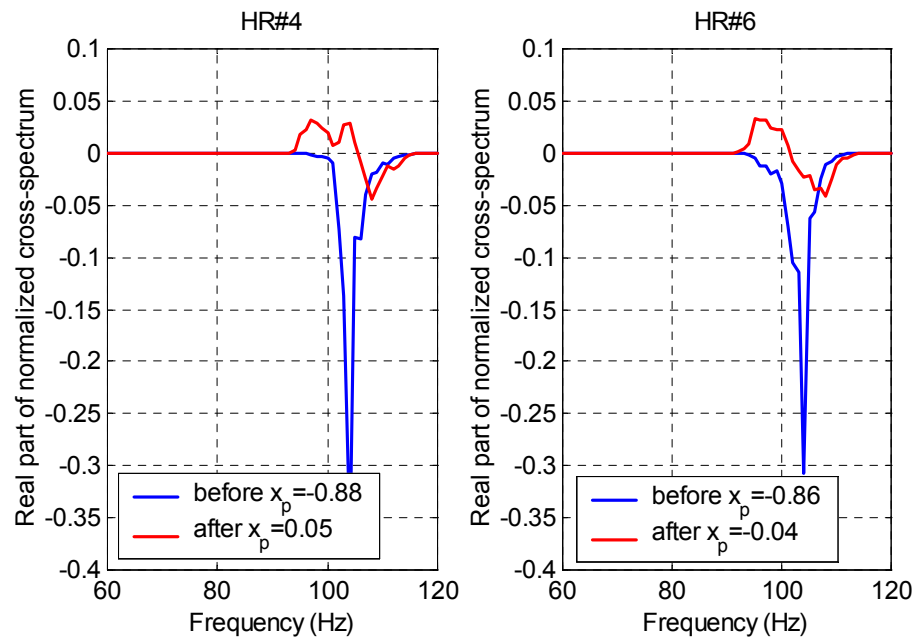


Figure 5.16 Normalized cross-spectrum of HR#4 and HR#6.

The transfer functions between the 14 monitoring microphones (one microphone had a bad signal) and the microphone placed inside the speaker cabinet are used to get the average sound pressure level inside the cylinder. Figure 5.17 compares the acoustic response of the cavity for the bare, before and after adaptation cases. Even though the HRs are initially detuned around 70Hz, they add some damping to the three acoustic peaks compare to the bare case. Once tuned, the HRs provide a 6.5dB attenuation in the 95-115Hz bandwidth of control.

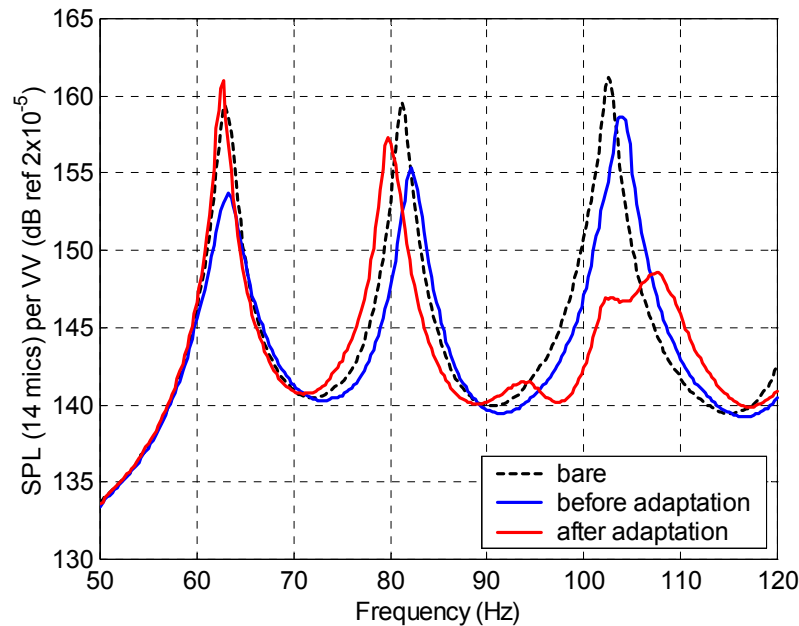


Figure 5.17 Acoustic response of the cavity before and after the adaptation

Multi-mode tuning

The objective of this experiment is to verify that the tuning algorithm would still performs as expected in a multi mode case. Although the HRs could tune to any of the first three acoustic resonances of the cylinder, the bandwidth of control was set around

the last two. The unavoidable high damping level of the HR near 64Hz and the sharing of the already small 0.4% volume ratio on three different modes would not lead to significant attenuation of the 3 peaks. The digital filter cut-on frequencies for the HR signals were set to 75 and 115Hz, and the adaptation was carried on using the same procedure as with the single mode experiment. Figure 5.18 illustrates the evolution of the opening diameter and Table 5.2 lists the corresponding initial and final HR tuning frequencies. As observed in the simulation, the HRs spread their natural frequencies across the bandwidth of control. Figure 5.19 shows the real part of the normalized cross-spectrum for HR#2 tuned to the second mode and HR#4 tuned to the third. In both cases the dot-product initially negative converges towards zero. The acoustic response of the cavity plotted in Figure 5.20 demonstrates the global effect of the tuning algorithm as both targeted peaks are reduced by 6 and 8dB compared to before the adaptation. The resulting overall attenuation in the 75-115Hz bandwidth of control is 4.2dB.

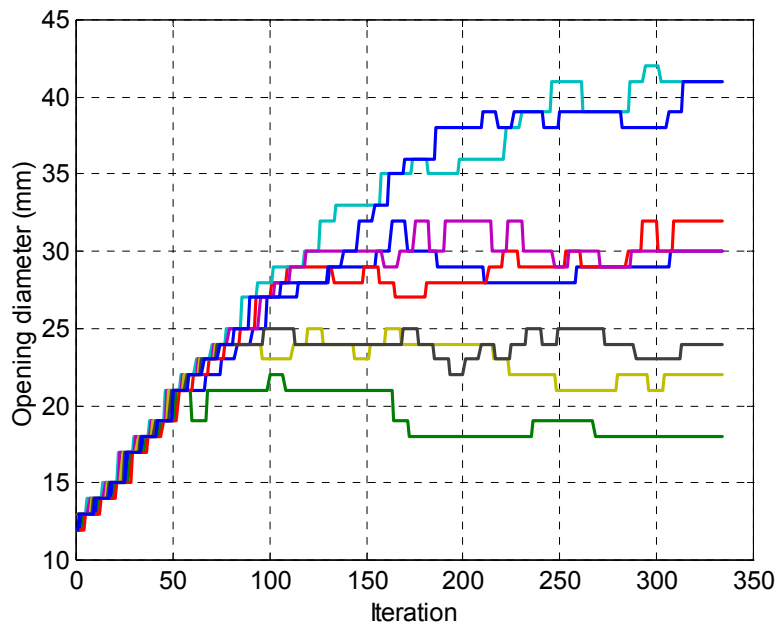


Figure 5.18 Evolution of the opening diameter for the 8 HRs.

HR #	Tuning frequency (Hz)	
	Before adaptation	After adaptation
1	72	102.5
2	69	81
3	66	90
4	67	102
5	69	97
6	70	88
7	67	86
8	67	103

Table 5.2 Tuning frequency of the 8 HRs before and after the multi mode adaptation.

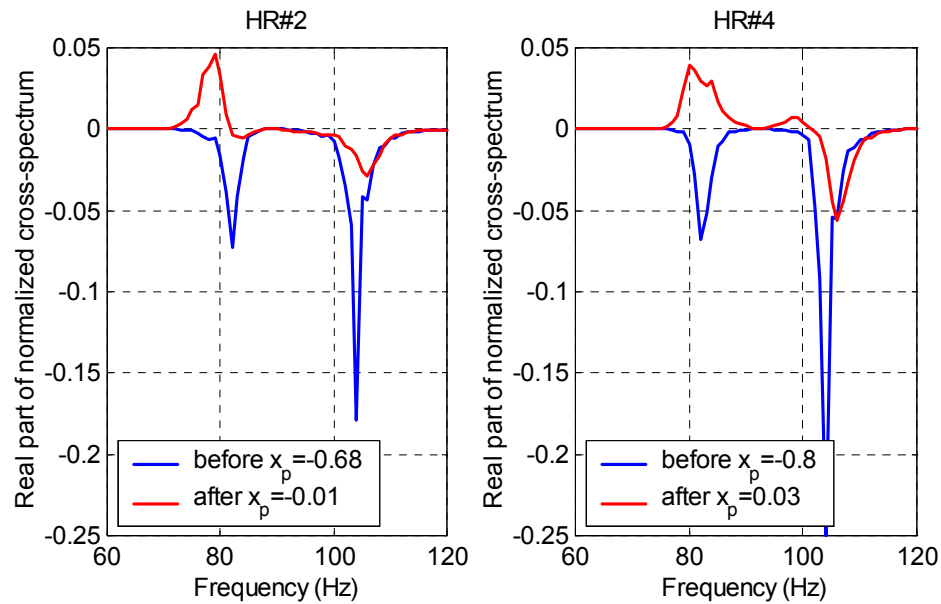


Figure 5.19 Normalized cross-spectrum of HR#2 and #4

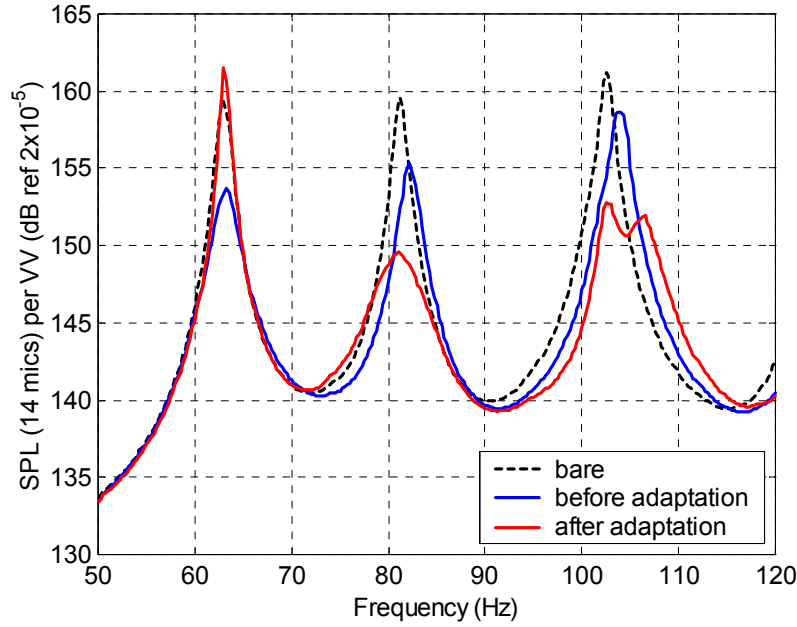


Figure 5.20 Acoustic response before and after adaptation.

5.6 Conclusions

A time domain version of the cylinder model presented in Chapter 2 was developed. To simplify the formulation and obtain reasonable computational time, this time domain model was only one way coupled. The real time simulation was obtained by a fourth order Runge-Kutta technique after rearranging the system in state-space domain.

The dot-product method commonly used to tune absorber to single frequency excitation was applied in a new fashion to Helmholtz resonators for broadband frequency excitation. Using only information local to each resonator, this tuning algorithm was more practical than global strategies for payload fairing applications. Simulations demonstrated that multiple resonators independently controlled by the dot-product, can lead to global broadband noise attenuation in a lightly damped enclosure. The level of attenuation obtained was close to near optimal solution. In the case of an externally

forced cavity, the structural modes which couple well to the internal acoustic spectrum must be damped by DVAs to ensure a proper tuning of the HR to acoustic peaks.

A prototype of adaptive Helmholtz resonator was built and characterized. A centralized controller mimicking the behavior of multiple independent local controllers was developed in order to track and observe the tuning mechanisms of up to 8 HRs simultaneously. The experiments carried on the Boeing cylinder with 8 adaptive resonators validated the simulations and demonstrated the ability of the dot-product method to tune resonators to near optimal solution over a frequency band including multiple resonances.

Chapter 6 Conclusions and future work

6.1 Conclusions

This work has demonstrated that a lightweight and compact noise reduction treatment can significantly increase low frequency transmission loss of a composite cylinder representative of a payload fairing. This is possible because in such structures the lower part of the internal acoustic spectrum is composed of sharp structural and acoustic resonances, which cannot be effectively damped by traditional acoustic blankets. Therefore, damped vibration absorbers and Helmholtz resonators represent an efficient way to add damping to these resonances without adding a significant amount of weight or volume.

An analytical model of a composite cylinder excited by an oblique plane wave was developed as a first approximation to the problem. The vibration absorbers and Helmholtz resonators were implemented in the analytical model using an impedance matching method. The effect of these devices on the vibration and acoustic response were investigated using a fully coupled system. Several characteristics of such a treatment were illustrated.

First, the noise reductions are linked to the DVA-cylinder mass ratio and to the HR-cavity volume ratio. Trade-offs between performance and added mass and volume were dictated by the strict weight and volume constraints associated with payload fairing applications.

Second, the performance of the treatment is directly related to the amount of damping initially present in the structure and in the acoustic cavity. The higher the damping ratios of the system, the less reduction the treatment can provide. As a consequence HRs and DVAs represent a complimentary treatment to the existing acoustic blankets, as they are designed to work at low frequencies where these blankets are ineffective.

Third, as the noise reduction devices are targeting structural and acoustic modes, their spatial locations are of importance. Because of the axisymmetry of cylinders, the devices are split in a certain number of units that are distributed evenly around the circumference to avoid unfavorable modal interactions. The general rule of thumb is to position absorbers axially at antinodes of the targeted modes. However, due to the interaction and the modal scattering of the devices, alternative optimal positions can be found.

Finally, the presence of damping in the vibration absorbers as well as in the Helmholtz resonators is a determining factor for broadband frequency performance. Formulas for the optimal damping ratio of both types of devices can be found in the literature but are restricted to the control of a single mode. Nevertheless, these formulas provide a good estimate for the optimal damping in multi-modal cases.

Finding the best treatment, given mass and volume constraints, requires a simultaneous optimization of the tuning frequencies, locations, and damping levels of the devices, and therefore a genetic algorithm was used. The algorithm was set to maximize the acoustic attenuation over a specified frequency band. Due to the complexity of the cost function, different genetically optimized treatments led to similar attenuation levels. The genetic solutions represented a 2.4dB improvement over an iterative optimization based on design experience. The performance robustness to changes in excitation characteristics, the system natural frequencies, tuning frequency, damping ratio and axial location of the devices was also investigated. In all the different cases, the performance was shown to be fairly robust to any particular changes. However, it was demonstrated that random treatments, where the axial positions, frequency and damping of the HRs and DVAs were set arbitrarily, did not perform as well on average. Therefore, some knowledge of the system, primarily natural frequency and mode shapes, permit treatment to be found that lie in relatively shallow bowls of the cost function surface with improved performance and good robustness. This robustness of the treatment mainly comes from the noise reduction mechanism that is based on adding damping to lightly damped resonances of the system.

As the interior acoustic spectrum is dominated by acoustic resonances, the Helmholtz resonators provide more acoustic reduction than the DVAs. However, vibration absorbers enhance the performance by damping the remaining structural resonances present in the

acoustic spectrum. Thus, the performance of the DVAs is closely linked to the level of structural-acoustic coupling of the system.

The conclusions drawn from the analytical model as well as the experimental results obtained on the Boeing cylinder prototype were used to design a HR/DVA treatment for a full size fairing. The analytical model was modified to use mode shapes and natural frequencies derived using a finite element model of a fairing, and the numerical equivalent source technique was applied in order to model the external acoustic excitation. Although the treatment design procedure was similarly based on both iterative and genetic optimizations, more implementation constraints such as size and location of the devices were taken into consideration. In this regard, an effort was made to minimize the weight of the HR treatment. Indeed, using light PETG tubes, the 212 HRs only represent 5% of the bare fairing weight when the former foam treatment represents 10%. The new fairing treatment composed of HRs embedded in the new, lighter foam blanket is only 5% heavier than the former blanket and 25% by addition of the DVAs. This final design was tested in a payload fairing and lead to 3.2dB attenuation from 30Hz to 90Hz in agreement with the model predictions.

The other part of this work was the investigation of a new type of adaptive Helmholtz resonator. It was shown that the dot product method usually used to track single frequency disturbances could be useful in tracking modes driven by broadband excitation. This could be of particular interest as the acoustic natural frequencies of a fairing's cavity are shifted with varying payload fills. In addition, this tuning law only requires information local to the resonator, greatly simplifying its implementation. A time domain model of adaptive Helmholtz resonators coupled to a cylinder was developed. The simulations demonstrated that multiple adaptive HRs could lead to broadband noise reductions similar to the ones obtained by genetic optimization. Experiments using adaptive HR prototypes confirmed their ability to converge to a near optimal solution in a frequency band including multiple modes.

6.2 Future work

As a first improvement of the acoustic treatments using HRs and DVAs, the sensitivity to arbitrary payload fills could be investigated. Although the robustness of a treatment with respect to changes in the cavity mode shapes was not strictly investigated in this work, it can be related to the treatment robustness with respect to the axial positions of the devices. Indeed, the treatment performance was showed to be most sensitive to this particular parameter. Therefore, to evaluate the robustness of a treatment designed with an empty fairing model to changes in the mode shapes and resonant frequencies due to a payload, a finite element model could be used to model the cavity with a variety of fills representative of typical payloads and the modes incorporated into the analytical model developed in this work. Depending on the loss in performance, it could be determined whether or not the treatments need to be tailored for fairings filled with a particular payload.

For further improvement of treatments, a better knowledge of the external disturbance acoustic field would be of interest. In this work, the devices are used in rings designed to be independent of the azimuth angle of excitation, which is assumed to be unknown. An accurate description of the acoustic disturbance, characteristic of a particular launch site and launch vehicle, could lead to a better use of the noise reduction devices. Developing an analytical model of this type of external disturbance is relatively complex and therefore the measured acoustic field based on scaled model tests has been used previously². The measured disturbance can then be approximated in an analytical model with multiple plane waves.

To increase the transmission loss of existing composite payload fairing, the use of active/passive hybrid treatment could also be investigated. The goal would be to increase the performance of the passive devices by using structural and acoustic actuators under a feedforward/feedback active control scheme. The location of the reference and error sensors is of primary importance for the effectiveness of such a treatment. To implement this system into a real application, the development of lightweight powerful actuators is necessary and has already been subject to investigations.

Another means by which to assess the problem would involve a new design of the fairing structure itself, taking into account the acoustic transmission loss while meeting the structural requirements. This concept is being evaluated through the use of a Chamber Core⁹⁰, where the structure is fabricated from multi-layered composite face sheets separated by channel-shaped chambers. In addition to providing a strong and lightweight fairing, the chambers could act as acoustic resonators able to attenuate low frequency acoustic resonances. Another way would be to replace the honeycomb core present in actual composite fairings with a more irregular grid. The shape and characteristics of this grid would modify the fairing stiffness locally and could be optimized to improve the transmission loss at low frequencies.

Furthermore, the use of thick (couple of feet) acoustic blankets placed at the bottom and in the nose of the fairing, could damp the first axial acoustic modes and hence increase the acoustic absorption in the low frequencies.

Finally, the concept of an adaptive Helmholtz resonator under local control for broadband noise reduction could be extended to adaptive vibration absorbers to control broadband vibrations of the structure. The investigation of different control architectures (namely, hierarchical or decentralized architectures) has already commenced⁹¹. The impact on the acoustic radiation and transmission would be of a special interest for payload fairing applications.

Appendix A Acoustic damping scaling

The objective of this experiment is to determine the amount of 2-inch thick foam needed in the cylinder prototype in order to match the reverberation time measured in the real payload fairing.

Reverberation time of a payload fairing

The reverberation time in the payload fairing was evaluated by measuring the response to the shut off of an acoustic source at 32 microphones. The data provided by Boeing initially sampled at 48000 Hz was down-sampled to 3000 Hz using Matlab. Figure A.1 shows the spatial mean squared using 31 microphone signals (one microphone being noisy).

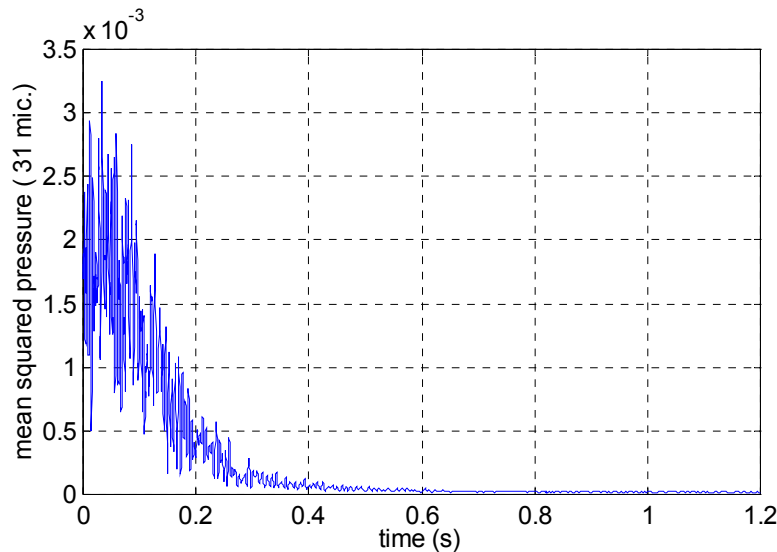


Figure A.1 Mean square pressure response to the source shut-off for the payload fairing.

The microphone signals are then digitally filtered in each $1/3^{\text{rd}}$ octave band. The 31 filtered signals are squared and summed. The resulting signal proportional to the acoustic

potential energy is then convolved with a rectangular window. The slope of the linear portion of this decaying signal approximates the reverberation time (T60), which is defined as the time necessary for the sound to decay by 60 dB. Figure A.2 shows an example of the decaying signal.

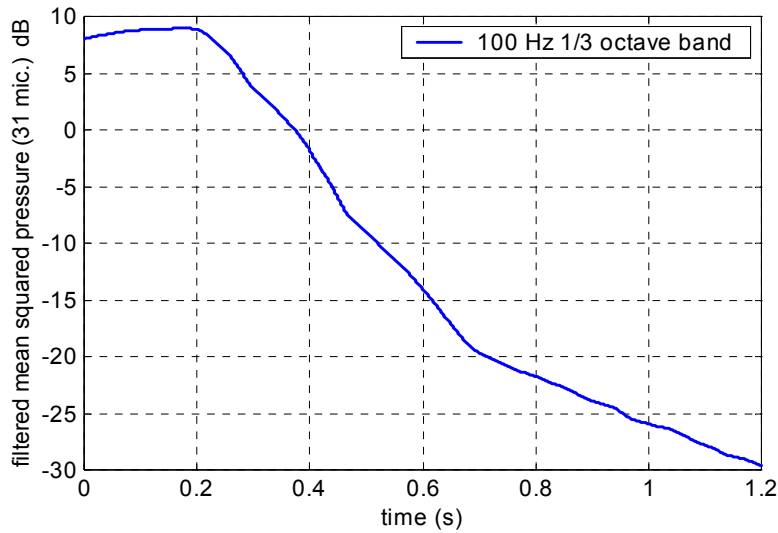


Figure A.2 100 Hz third octave band decaying signal for the payload fairing.

Reverberation time in the cylinder prototype:

The reverberation time in the cylinder prototype was evaluated by measuring the response of 16 microphones randomly spaced in the cavity to the shut off of an acoustic source. A speaker placed inside the cylinder with an input band passed from 31.5 to 300Hz white noise signal provided the source. The sampling frequency was 3000 Hz, Figure A.3 shows the spatial mean squared signal.

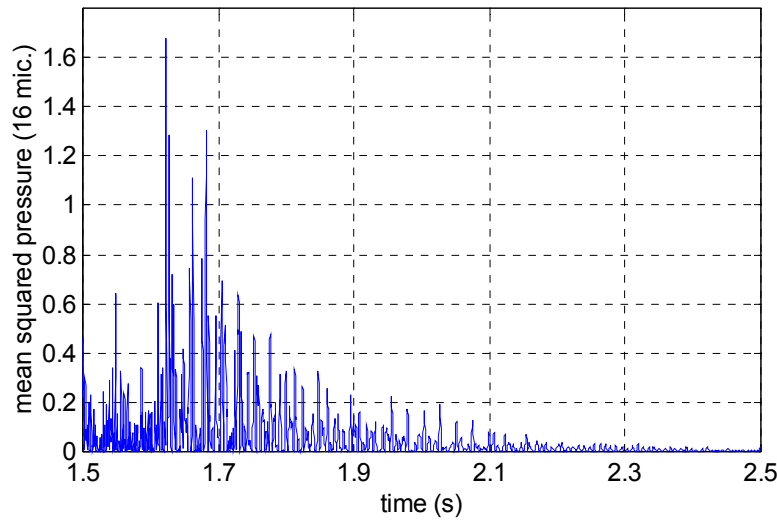


Figure A.3 Mean square pressure response to the source shut-off for the bare cylinder.

The same processing as for the payload fairing data was applied to extract the third octave band reverberation time. Because the volume of the cylinder prototype is 16 times smaller than the payload fairing, the lowest significant $1/3^{\text{rd}}$ octave band for the cylinder is the 40 Hz band. To determine the amount of foam necessary for matching the fairing reverberation time, a iterative percentage of the cylinder's wall were covered with 2-inch thick acoustic foam panels and the T60 was evaluated in each case. Table A.1 lists the T60 for the fairing, the bare cylinder, and the cylinder covered with 20% 40% a 100% of foam. From these results, an approximate half coverage of the cylinder wall was chosen to be the most representative of the damping provided by the acoustic blankets in the fairing.

Third octave band center frequency (Hz)	Reverberation time (second)				
	Payload fairing	Bare Cylinder	20% foam Cylinder	40%foam Cylinder	100%foam Cylinder
25	3.1	-	-	-	-
31.5	2.3	-	--	-	-
40	2.7	3.5	3.2	2.7	1.2
50	2.7	3.6	2.7	2.2	1
63	1.8	3.5	3.1	2.1	0.9
80	1.0	1.8	1.1	1.1	1.4
100	1.3	2.1	1.6	1.2	1.4
125	0.6	1.8	1.8	1.1	2.0
160	0.4	1.6	1.6	0.8	2.1
200	0.3	4.2	1.5	0.3	1.5

Table A.1 Comparison of the third octave band reverberation time.

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