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Leptonic *CP* violation search and the ambiguity of δm^2_{31}

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We consider a search for the *CP*-violating angle δ_{CP} in long baseline neutrino oscillation experiments. We show that the subleading δ_{CP}-dependent terms in the $\nu_\mu \to \nu_e$ oscillation probability can be easily obscured by the ambiguity of the leading term which depends on $|\delta m^2_{31}|$. It is thus necessary to determine the value of δm^2_{31} with a sufficient accuracy. The ν_μ survival events, which can be accumulated simultaneously with the ν_e appearance events, can serve for this purpose owing to its large statistics. Therefore, the combined analysis of ν_e appearance and ν_μ survival events is crucial to provide a restrictive constraint on δ_{CP}. Taking a test experimental setup, we demonstrate in the δ_{CP}-δm^2_{31} plane that the analysis of ν_e appearance events leads to less restrictive constraints on the value of δ_{CP} due to the ambiguity of δm^2_{31} and that the combined analysis efficiently improves the constraints.

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I. INTRODUCTION

Neutrino oscillation has been strongly suggested by an accumulating number of experiments using a variety of neutrino sources [1–3], providing rich information on the flavor structure of the lepton sector. The mass parameters and the mixing parameters of neutrinos are nonetheless still not completely known; following the definitions of Ref. [4], the unknown parameters include the value of the mixing angle θ_{13}, the *CP*-violating angle δ_{CP}, and the sign of δm^2_{31}. The current upper bound on θ_{13} [5] is expected to be improved down to $\sin^2 2\theta_{13} \sim 0.01$ by the next generation of nuclear reactor [6–9] and accelerator experiments [9–11], while δ_{CP} and the sign of δm^2_{31} are to be investigated by future long baseline neutrino oscillation experiments [10,11].

In this paper, we consider the search for the leptonic *CP*-violation angle by long baseline neutrino oscillation experiments using a conventional ν_μ superbeam. *CP* violation can be observed only through the flavor-changing processes such as $\nu_\mu \to \nu_e$.[1] It is challenging, however, to extract the value of δ_{CP} from this oscillation since the ν_e appearance probability is suppressed by the small value of $\sin^2 2\theta_{13}$. The search for δ_{CP} is made more difficult by backgrounds such as ν_e-contamination of the incident beam, which is inseparable from the signal at the detector, and the neutral-current events that are misidentified as electron events. Further obstacles to extracting the value of δ_{CP} are the uncertainties in the other mixing parameters [12–14]. Among them, the uncertainty in the value of θ_{13}, which is bounded only from above, has been widely studied [12–14]. The value of θ_{13} enters into the leading term of the ν_e appearance oscillation probability, and the lack of knowledge on its value makes it difficult to search for the subleading effect of δ_{CP}. The ambiguity of $|\delta m^2_{31}|$ causes a similar problem: the value of $|\delta m^2_{31}|$ also enters into the leading term of the appearance probability and its current experimental uncertainty is large enough to obscure the subleading δ_{CP}-dependent effect. This uncertainty can also obscure the dependence on the sign of δm^2_{31} and make the δ_{CP} search more difficult. It is hence necessary to constrain the value of $|\delta m^2_{31}|$ in searching for the *CP*-violating angle.

The value of $|\delta m^2_{31}|$ can be precisely determined using ν_μ survival events accumulated concurrently with the ν_e events [9,13,14]. This is due to the large number of ν_μ events expected from the large ν_μ-flux available in ν_μ superbeams. A combined analysis employing ν_μ survival events as well as ν_e appearance events can thus place strong constraints on the values of δ_{CP} and δm^2_{31}. We carry out an example analysis for a test setup fixing the parameters other than δ_{CP} and δm^2_{31}. We demonstrate that the analysis of the ν_e appearance events alone does not sufficiently constrain the value of δ_{CP} in the presence of the ambiguity of $|\delta m^2_{31}|$ and that the combined analysis efficiently improves the constraint.

This paper is organized as follows. In Sec. II, we use an analytic expression of the ν_e appearance probability to show that the ambiguity in the leading term can obscure the dependence on δ_{CP} and on the sign of δm^2_{31}. We point out that precise values of $|\delta m^2_{31}|$ and $\sin^2 2\theta_{13} \sin^2\theta_{23}$ are necessary to search for δ_{CP}. We then use an analytic expression for the ν_μ survival probability to show that it can constrain the value of δm^2_{31}. In Sec. III, we consider a test setup and calculate allowed regions of parameters in the δ_{CP}-δm^2_{31} plane fixing other parameters. We show that the combined analysis of ν_e appearance and ν_μ survival

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events, in principle, gives a strong constraint on the value of δ_{CP} and on the sign of δm^2_{31}. We conclude our work and give discussions in Sec. IV.

II. RELEVANCE OF THE AMBIGUITY OF δm^2_{31} IN THE LEPTONIC *CP* VIOLATION SEARCH

Assuming that the number of neutrino generations is three, we define the mixing angles θ_{ij} ($\{i, j\} \subset \{1, 2, 3\}$), the CP-violating angle δ_{CP}, and the quadratic mass differences δm^2_{ij} as in Ref. [4]. We assume the parameter values to be $\delta m^2_{21} \simeq 8 \times 10^{-5}$ eV^2, $|\delta m^2_{31}| \simeq (2\text{–}3) \times 10^{-3}$ eV^2, $\sin^2\theta_{12} \simeq 0.3$, $\sin^2\theta_{23} \simeq 0.5$, and $\sin^2 2\theta_{13} \lesssim (0.1\text{–}0.2)$ [5,15], which accommodate all neutrino oscillation experiments except for the LSND experiment [16].

We consider the case where the neutrino energy E and the baseline length L satisfies $(\delta m^2_{31}L)/(2E) = O(1)$ so that the oscillation signal can be observed. We calculate the $\nu_\mu \to \nu_e$ oscillation probability in matter whose density ρ, and hence the electron number density n_e, is constant. Taking up to first order in δm^2_{21} and in $a \equiv 2\sqrt{2}G_F n_e E = (7.63 \times 10^{-5}\ \text{eV}^2)(\rho/[\text{g cm}^{-3}])(E/[\text{GeV}])$, we obtain [17]

$$P(\nu_\mu \to \nu_e) = 4c^2_{13}s^2_{13}s^2_{23}\left[1 \pm \frac{2a}{|\delta m^2_{31}|}(1 - 2s^2_{13}) - \frac{\delta m^2_{21}L}{2E}\frac{c_{23}c_{12}s_{12}}{s_{13}s_{23}}\sin\delta_{CP}\right] \times \sin^2\left[\frac{|\delta m^2_{31}|L}{4E} \mp \frac{aL}{4E}(1 - 2s^2_{13}) \mp \frac{\delta m^2_{21}L}{4E}\left(s^2_{12} - \frac{c_{23}c_{12}s_{12}}{s_{13}s_{23}}\cos\delta_{CP}\right)\right], \quad (1)$$

where s_{ij} and c_{ij} denotes $\sin\theta_{ij}$ and $\cos\theta_{ij}$, respectively, and the top and the bottom of the double sign are taken when $\delta m^2_{31} > 0$ and $\delta m^2_{31} < 0$, respectively. The probability has an oscillatory dependence on the energy due to the sinusoidal factor, while the preceding factor in the first pair of brackets, depending weakly on the energy, gives the envelope of the oscillation. The leading terms of the argument and the envelope of the sinusoidal factor are given by $|\delta m^2_{31}|L/(4E)$ and $4c^2_{13}s^2_{13}s^2_{23}$, respectively. The subleading terms of both the argument and the envelope depend on δ_{CP} and the sign of δm^2_{31}. One can obtain the knowledge on the value of δ_{CP} and the sign of δm^2_{31} through subleading effects if the leading terms are known with sufficient accuracy; if not, the effects will be obscured by the ambiguity in the leading terms.

The argument of the sinusoidal factor is determined from the energy which gives the peak of the oscillation probability, or the peak energy in short. We thus evaluate the peak energy, including the corrections from energy dependence of the envelope, up to first order in δm^2_{21} and in a to obtain

$$E_{\text{peak},n} = \frac{1}{(2n+1)\pi}\frac{L}{2}\left\{|\delta m^2_{31}| \mp \left[\left(1 - \frac{4}{[(2n+1)\pi]^2}\right)\frac{|\delta m^2_{31}|}{(2n+1)\pi}a'L(1 - 2s^2_{13}) + \delta m^2_{21}s^2_{12}\right] \pm \delta m^2_{21}\frac{c_{23}c_{12}s_{12}}{s_{13}s_{23}}\left(\cos\delta_{CP} \pm \frac{2}{(2n+1)\pi}\sin\delta_{CP}\right)\right\}, \quad (2)$$

where $n = 0, 1, 2, \ldots$, $a' \equiv \sqrt{2}G_F n_e = a/(2E) = (1.93 \times 10^{-4}\ \text{km}^{-1})(\rho/[\text{g cm}^{-3}])$, and the top of the double sign is for $\delta m^2_{31} > 0$ and the bottom for $\delta m^2_{31} < 0$. Inverting the sign of δm^2_{31} changes the signs of the subleading second and third terms in the braces of Eq. (2), and the dependence on δ_{CP} appears in the third term. The ambiguity of the leading term, which we denote by $\Delta(|\delta m^2_{31}|)$, must be smaller than these subleading terms to determine δ_{CP} and the sign of δm^2_{31} from the observation of $E_{\text{peak},n}$. The conditions on $\Delta(|\delta m^2_{31}|)$ are given, for typical values of the parameters and $n = 0$, by

$$\Delta(|\delta m^2_{31}|) < \left(1 - \frac{4}{\pi^2}\right)\frac{|\delta m^2_{31}|}{\pi}a'L(1 - 2s^2_{13}) + \delta m^2_{21}s^2_{12} \simeq \left(2 \times 10^{-4}\frac{L}{[1000\ \text{km}]}\frac{\rho}{[2.6\ \text{g/cm}^3]} + 2 \times 10^{-5}\right)\text{eV}^2, \quad (3)$$

$$\Delta(|\delta m^2_{31}|) < \delta m^2_{21}\frac{c_{23}c_{12}s_{12}}{s_{13}s_{23}} \simeq \begin{cases} 2 \times 10^{-4}\ \text{eV}^2 & \text{for } \sin^2 2\theta_{13} = 0.1 \\ 8 \times 10^{-4}\ \text{eV}^2 & \text{for } \sin^2 2\theta_{13} = 0.01. \end{cases} \quad (4)$$

The current experimental uncertainty in $|\delta m^2_{31}|$ is about 6×10^{-4} eV^2 [2], which is larger or similar to the above critical values. It is therefore necessary to constrain the value of δm^2_{31} within a smaller interval.

Similar analysis of the envelope leads to another set of conditions. To determine the sign of δm^2_{31}, we must impose the following condition on the ambiguity of $4c^2_{13}s^2_{13}s^2_{23} = \sin^2 2\theta_{13}\sin^2\theta_{23}$:

$$\frac{\Delta(\sin^2 2\theta_{13}\sin^2\theta_{23})}{\sin^2 2\theta_{13}\sin^2\theta_{23}} < \frac{2a}{|\delta m^2_{31}|}(1 - 2s^2_{13}) \simeq 0.3\frac{L}{[1000\ \text{km}]}\frac{\rho}{[2.6\ \text{g/cm}^3]}, \quad (5)$$

which is evaluated at $E = E_{\text{peak},0}$ using typical values of the parameters. To constrain the value of δ_{CP}, the condition is

$$\frac{\Delta(\sin^2 2\theta_{13}\sin^2\theta_{23})}{\sin^2 2\theta_{13}\sin^2\theta_{23}} < \pi \frac{\delta m^2_{21}}{|\delta m^2_{31}|}\frac{c_{23}c_{12}s_{12}}{s_{13}s_{23}} \simeq \begin{cases} 0.3 & \text{for } \sin^2 2\theta_{13} = 0.1 \\ 0.9 & \text{for } \sin^2 2\theta_{13} = 0.01. \end{cases} \tag{6}$$

The current experimental bound on the value of θ_{23} is $\sin^2 2\theta_{23} > 0.9$ [2,15]. The value of θ_{13}, on the other hand, is bound only from above to date. Ignorance of the value of θ_{13} is shown to be a major obstacle in determining the value of δ_{CP} through long baseline experiments [12–14]. A possible approach to this difficulty is to combine results from reactor neutrino experiments [6–9]. Future experiments searching for the disappearance of $\bar{\nu}_e$ from reactors are expected to constrain the value of θ_{13} independently of the values of other parameters such as δ_{CP} and matter density. Reference [8] suggests that future experiments can constrain the value of $\sin^2 2\theta_{13}$ with $\lesssim$ 10% accuracy if the value of $\sin^2 2\theta_{13}$ is as large as 0.1. Such reactor experiments can be performed in advance or concurrently with accelerator-based δ_{CP} searches. In this prospect, we assume that the value of $\sin^2 2\theta_{13}\sin^2\theta_{23}$ will be known with a reasonable accuracy by the time of δ_{CP} searches, and we keep the value of $\sin^2 2\theta_{13}\sin^2\theta_{23}$ fixed in this paper to explore the best-case scenario for the long baseline experiments.

The ν_μ survival events, which can be accumulated simultaneously with the ν_e appearance events, can be used to constrain the value of δm^2_{31} owing to the large statistics available. The energy dependence of the ν_μ survival probability is calculated, up to first order in δm^2_{21} and in a, as [17]

$$P(\nu_\mu \to \nu_\mu) = 1 - 4c^2_{13}s^2_{23}(1 - c^2_{13}s^2_{23})\left[1 \mp 2\frac{a}{|\delta m^2_{31}|}\frac{s^2_{13}(1 - 2c^2_{13}s^2_{23})}{1 - c^2_{13}s^2_{23}}\right] \times \sin^2\left[\frac{|\delta m^2_{31}|L}{4E} \pm \frac{aL}{4E}\frac{s^2_{13}(1 - 2c^2_{13}s^2_{23})}{1 - c^2_{13}s^2_{23}} \mp \frac{\delta m^2_{21}L}{4E}\frac{s^2_{13}s^2_{23}s^2_{12} + c^2_{23}c^2_{12} - 2c_{23}s_{23}c_{12}s_{12}s_{13}\cos\delta_{CP}}{1 - c^2_{13}s^2_{23}}\right], \tag{7}$$

where the top and the bottom of the double sign in Eq. (7) are taken when $\delta m^2_{31} > 0$ and $\delta m^2_{31} < 0$, respectively. Since the value of $as^2_{13}(1 - 2c^2_{13}s^2_{23})/(1 - c^2_{13}s^2_{23})$ is negligibly small compared to the leading terms under the current experimental limits, the observation of the energy dependence of this mode gives the value of

$$\left| \delta m^2_{31} - \delta m^2_{21}\frac{s^2_{13}s^2_{23}s^2_{12} + c^2_{23}c^2_{12} - 2c_{23}s_{23}c_{12}s_{12}s_{13}\cos\delta_{CP}}{1 - c^2_{13}s^2_{23}} \right|. \tag{8}$$

We then obtain two possible values of δm^2_{31}, one being positive and the other being negative; their absolute values differ from each other by twice the δm^2_{21} factor in Eq. (8). This constraint on δm^2_{31} would contribute to determining the value of δ_{CP} and the sign of δm^2_{31}. Consequently, a combined analysis of the ν_e appearance and ν_μ survival events is crucial for measuring δ_{CP}. It gives the simultaneous constraint on δ_{CP} and δm^2_{31}, and thus the result shall be presented in the δ_{CP}-δm^2_{31} plane.

III. NUMERICAL ANALYSIS OF AN EXAMPLE SETUP

In this section, we consider a test experiment setup and obtain constraints on δ_{CP} and δm^2_{31} from the oscillation event spectra, which are numerically calculated without employing any approximation formulas of the oscillation probabilities. We fix other parameters including θ_{13} and θ_{23} as mentioned in Sec. II. We show that the values of δ_{CP} and δm^2_{31} are significantly constrained by performing the combined analysis of ν_e appearance and ν_μ survival events.

A. Example setup and the method of analysis

We consider the following example setup. A wide band beam of neutrinos is produced at the upgraded Alternating Gradient Synchrotron (AGS) at Brookhaven National Laboratory (BNL). We assume the flux of neutrinos given in Fig. 1, which is obtained by fitting the flux presented in Fig. 3 of Ref. [18]. Neutrinos are detected by a water Čerenkov detector which has 500 kt of fiducial mass and is placed 770 km away from BNL in the Kimballton mine, Virginia [19]. We exclusively consider quasielastic events $\nu_l + \mathrm{n} \to l^- + \mathrm{p}$ as signals. We assume that its detection efficiency is unity, the data acquisition time is 5×10^7 sec, and the matter density is 2.6 g/cm^3.

The expected number of ν_l events, where $l \in \{\mathrm{e}, \mu\}$, within an energy bin $E_i < E < E_{i+1}$ is evaluated as

$$\langle N_i(\nu_\mu \to \nu_l)\rangle \equiv T\mathcal{N}\int_{E_i}^{E_{i+1}} dE\varepsilon(E)\frac{f^{(\nu_\mu)}(E)}{L^2}P(\nu_\mu \to \nu_l) \times \frac{d\sigma^{(\nu_l)}}{dE}, \tag{9}$$

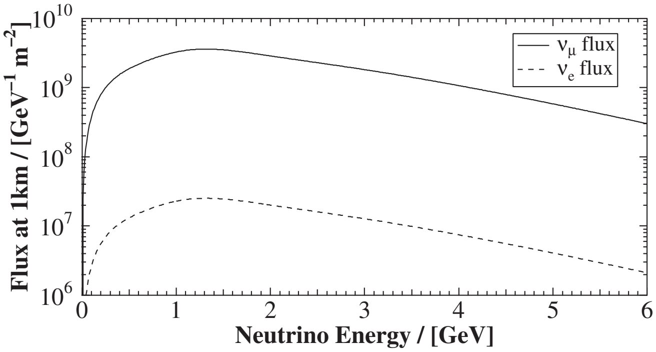

FIG. 1. The neutrino fluxes used in the example analysis. The flux of muon neutrinos (solid line) is obtained by fitting the flux presented in Fig. 3 of Ref. [18]. The flux of electron neutrinos (dotted line) is assumed to be 0.7% of that of muon neutrinos with the same energy dependence.

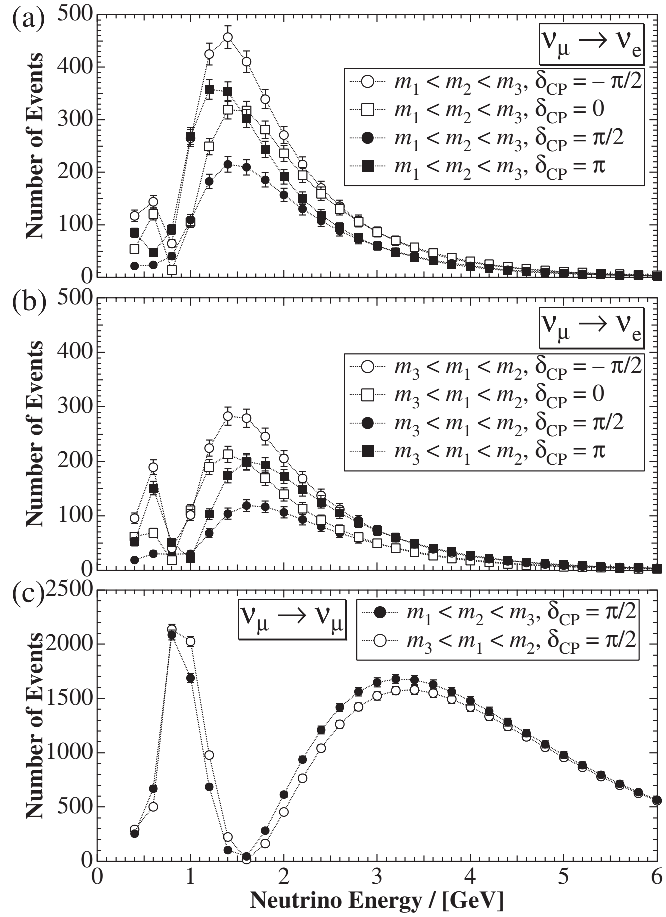

FIG. 2. Calculated energy spectra of ν_e appearance events [(a), (b)] and ν_μ survival events (c) for the setup described in text with $\delta m^2_{31} = \pm 2.5 \times 10^{-3}$ eV^2 [+ in (a), − in (b), and both in (c)]. Other parameters are taken as $\delta m^2_{21} = 8.2 \times 10^{-5}$ eV^2, $\sin^2 2\theta_{12} = 0.84$, $\sin^2 2\theta_{23} = 1.00$, and $\sin^2 2\theta_{13} = 0.06$. The ν_μ survival event spectra in (c) are presented only for $\delta_{CP} = \pi/2$ since they depend little on δ_{CP}. The error bars include only statistical errors.

where T is the data acquisition time, $\mathcal{N}$ is the number of target nucleons in the fiducial volume of the detector, $\varepsilon(E)$ is the detection efficiency, $f^{(\nu_\mu)}(E)$ is the incident ν_μ flux, and $d\sigma^{(\nu_l)}/dE$ is the cross section of the detection reaction [20]. We present in Fig. 2 the calculated event number spectra of ν_e appearance events and those of ν_μ survival events. The parameters are taken as $|\delta m^2_{31}| = 2.5 \times 10^{-3}$ eV^2, $\delta m^2_{21} = 8.2 \times 10^{-5}$ eV^2, $\sin^2 2\theta_{12} = 0.84$, $\sin^2 2\theta_{23} = 1.00$, and $\sin^2 2\theta_{13} = 0.06$. These figures show clear signals of ν_e appearance and ν_μ disappearance between 1 and 2 GeV.

It is necessary in the analysis to take backgrounds into consideration. Significant sources of backgrounds in the ν_e appearance signal are ν_e-contamination in the incident ν_μ beam, and misidentification of neutral pions produced through neutral-current interaction. In the following, we take into account only the ν_e-contamination which is not separable from signal at the detector. The number of the neutral-pion background events is difficult to estimate theoretically since it depends on the model of the pion-production process, details of the detector design, and methods of the event selection; we leave the consideration of the neutral-pion background to a future work. The expected number of ν_l events in an energy bin $E_i < E < E_{i+1}$ is then given by $\langle N_i^{(\nu_l)} \rangle \equiv \langle N_i(\nu_\mu \to \nu_l) \rangle + \langle N_i(\nu_e \to \nu_l) \rangle$, where $\langle N_i(\nu_e \to \nu_l) \rangle$ is defined as in Eq. (9) with the initial ν_μ replaced by ν_e. We assume in the following that the flux of ν_e-contamination in the incident beam is 0.7% of the ν_μ flux with the same energy dependence, as shown in Fig. 1 by a dotted line.

We obtain the constraint on δ_{CP} and δm^2_{31} by the following procedure. We generate the event number spectrum $N_i^{(\nu_l)}(\delta_{CP}, \delta m^2_{31})$ for various values of δ_{CP} and δm^2_{31} while keeping the other parameters fixed. The number of events in each bin $N_i^{(\nu_l)}(\delta_{CP}, \delta m^2_{31})$ includes statistical errors and is distributed around $\langle N_i^{(\nu_l)}(\delta_{CP}, \delta m^2_{31}) \rangle$. We carry out hypothesis testing with the null hypothesis: *the parameter values are* $\delta_{CP}^{(\mathrm{true})}$ *and* $\delta m^{2(\mathrm{true})}_{31}$. We reject this null hypothesis if the deviation of $N_i^{(\nu_l)}(\delta_{CP}, \delta m^2_{31})$ from $\langle N_i^{(\nu_l)}(\delta_{CP}^{(\mathrm{true})}, \delta m^{2(\mathrm{true})}_{31}) \rangle$ is of statistical significance, which we examine by the χ^2 test using

$$\chi^{2(\nu_l)}(\delta_{CP}, \delta m^2_{31}) = \sum_{i=1}^{n_{\mathrm{bin}}} \frac{|N_i^{(\nu_l)}(\delta_{CP}, \delta m^2_{31}) - \langle N_i^{(\nu_l)}(\delta_{CP}^{(\mathrm{true})}, \delta m^{2(\mathrm{true})}_{31}) \rangle|^2}{N_i^{(\nu_l)}(\delta_{CP}, \delta m^2_{31})}, \tag{10}$$

where $n_{\rm bin}$ is the number of the energy bins. The allowed region is obtained as a region in which the null hypothesis cannot be rejected for a certain confidence level.

B. Constraints on the values of δ_{CP} and δm^2_{31}

We carry out the analysis illustrated in the previous subsection for three cases: using $\nu_{\rm e}$ appearance events only, using ν_μ survival events only, and using both $\nu_{\rm e}$ appearance and ν_μ survival events. In the following, the values of the parameters except δm^2_{31} and δ_{CP} are fixed to those given in Fig. 2, and we take $|\delta m^{2({\rm true})}_{31}| = 2.5 \times 10^{-3}$ eV^2 and $\delta^{({\rm true})}_{CP} = -\pi/2$, 0, $\pi/2$, and π.

First, we show the analysis of the $\nu_{\rm e}$ appearance spectrum only using $\chi^{2(\nu_{\rm e})}(\delta_{CP}, \delta m^2_{31})$. We present in Figs. 3 and 4 allowed regions for 68.3% and 95% confidence levels obtained from the $\nu_{\rm e}$ appearance spectrum. We see that $\nu_{\rm e}$ appearance events cannot always determine the sign of δm^2_{31} and the value of δ_{CP}. While there are indeed cases in which the wrong sign of δm^2_{31} does not give any allowed region in the presented figures [e.g. Fig. 3(a)], there are cases where the wrong sign of δm^2_{31} also gives allowed regions [e.g. Fig. 4(b)]. The allowed region in such cases, however, is not a single extended region but a group of small isolated spots, which indicates that they are due to statistical errors and background. We also note that the allowed interval of δ_{CP} is enlarged by the ambiguity of δm^2_{31} in some cases such as Fig. 3(a); in this case, the allowed intervals of the parameters are $\delta_{CP} \simeq (-0.5 \pm 0.25)\pi$ and $\delta m^2_{31} \simeq (2.5 \pm 0.2) \times 10^{-3}$ eV^2 for the 95% confidence level.

The allowed regions in Figs. 3 and 4 can be understood in terms of analytic expressions of Eqs. (1) and (2) as follows. We expect that the two oscillation spectra resemble each other and give a small value of $\chi^{2(\nu_{\rm e})}$ when their peak energies and peak oscillation probabilities are equal, i.e.

$$E_{\rm peak} = E^{({\rm true})}_{\rm peak}, \tag{11}$$

and

$$P(\nu_\mu \to \nu_{\rm e}; \delta_{CP}, \delta m^2_{31}; E_{\rm peak}) = P(\nu_\mu \to \nu_{\rm e}; \delta^{({\rm true})}_{CP}, \delta m^{2({\rm true})}_{31}; E^{({\rm true})}_{\rm peak}), \tag{12}$$

where $E_{\rm peak}$ and $E^{({\rm true})}_{\rm peak}$ are given by Eq. (2) with $n = 0$. Solutions to these equations are shown in Figs. 3 and 4 by a dotted line for Eq. (11) and a broken line for Eq. (12). The allowed values of δ_{CP} and δm^2_{31} are well constrained around the region where the dotted and the broken lines intersect or get close to each other, even for cases with the wrong sign of δm^2_{31}. It shows that it is difficult to distinguish two spectra satisfying the peak-matching conditions, Eqs. (11) and (12). The difference of the two spectra away from the peak energy is not significant under the limited statistics and background.

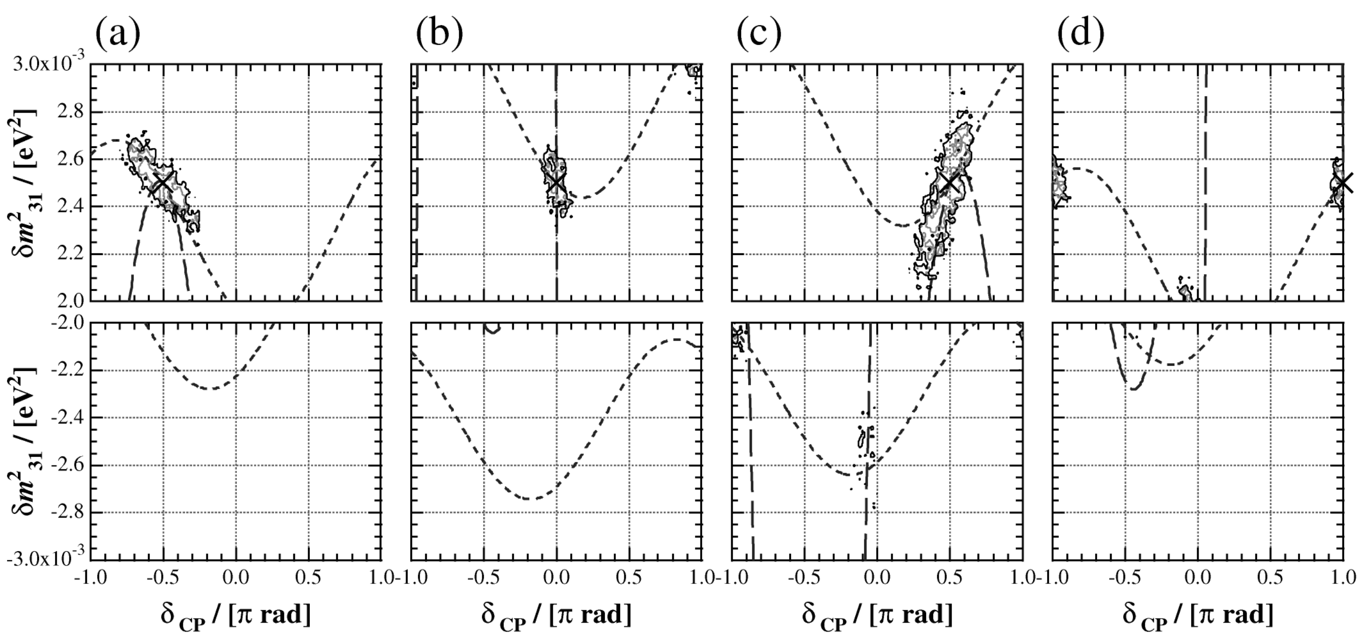

FIG. 3. Allowed regions for 68.3% (gray solid line) and 95% (black solid line) confidence levels obtained from $\nu_{\rm e}$ appearance events between 0.7 and 3.1 GeV. The width of energy bins is taken as 0.2 GeV. Incident $\nu_{\rm e}$-contamination and statistical errors are taken into account. The true parameters (cross) are taken to be $\delta m^{2({\rm true})}_{31} = 2.5 \times 10^{-3}$ eV^2 and $\delta^{({\rm true})}_{CP} = -\pi/2$, 0, $\pi/2$, and π in (a), (b), (c), and (d), respectively, while δm^2_{21} and $\sin^2 2\theta_{ij}$'s are fixed to those given in Fig. 2. The dotted line and the broken line show the solutions to Eqs. (11) and (12) in text, respectively.

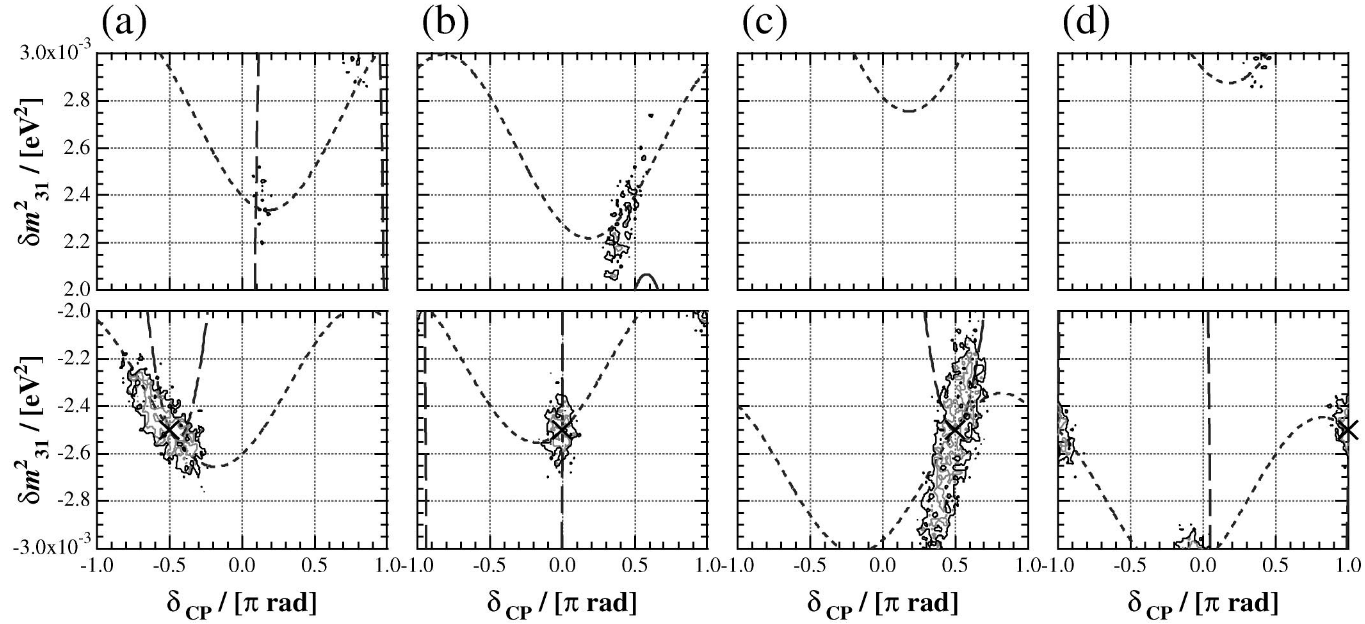

FIG. 4. The same as Fig. 3, but $\delta m_{31}^{2(\text{true})} = -2.5 \times 10^{-3}$ eV^2.

Second, we present allowed regions obtained from ν_μ survival spectra in Fig. 5. We show the result only for $\delta_{CP}^{(\text{true})} = \pi/2$ since the results depend little on $\delta_{CP}^{(\text{true})}$. The allowed region separates into two horizontal bands, one with $\delta m_{31}^2 > 0$ and the other with $\delta m_{31}^2 < 0$. The error of the two values of δm_{31}^2 shrinks down to about $\pm 3 \times 10^{-5}$ eV^2 at the 95% confidence level. This small error is due to the large statistics available. In terms of Eq. (7), the two bands correspond to the two possible values of δm_{31}^2 that give the same value of Eq. (8) and, consequently, the same energy dependence of the ν_μ survival probability given in Eq. (7). These two values are indicated in Fig. 5 by dotted lines which lie inside the allowed regions.

Finally, we present the combined analysis by evaluating $\chi^2 = \chi^{2(\nu_e)} + \chi^{2(\nu_\mu)}$. The allowed regions from the combined analysis are presented in Figs. 6 and 7. It is shown

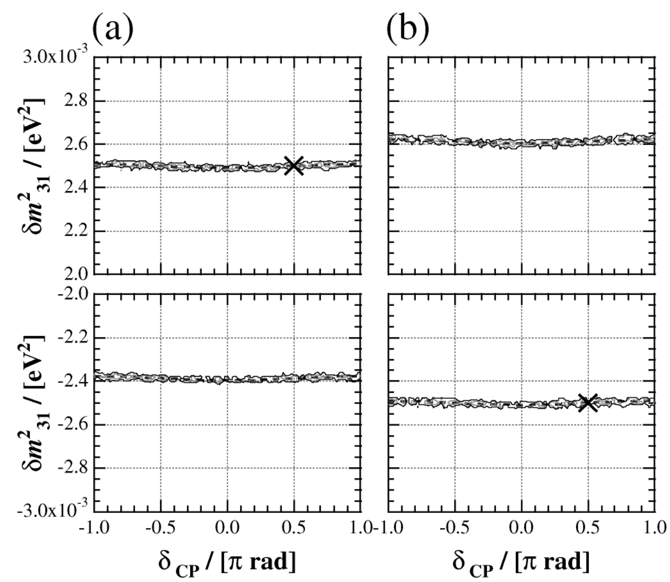

FIG. 5. Allowed regions for 68.3% (gray solid line) and 95% (black solid line) confidence level obtained from ν_μ survival events between 0.7 and 4.1 GeV. Incident ν_e-contamination as well as statistical errors are taken into account. The true parameters (cross) are $\delta_{CP}^{(\text{true})} = \pi/2$ and (a) $\delta m_{31}^{2(\text{true})} = 2.5 \times 10^{-3}$ eV^2, (b) $\delta m_{31}^{2(\text{true})} = -2.5 \times 10^{-3}$ eV^2, while δm_{21}^2 and $\sin^2 2\theta_{ij}$'s are fixed to those given in Fig. 2. The dotted line is explained in the text.

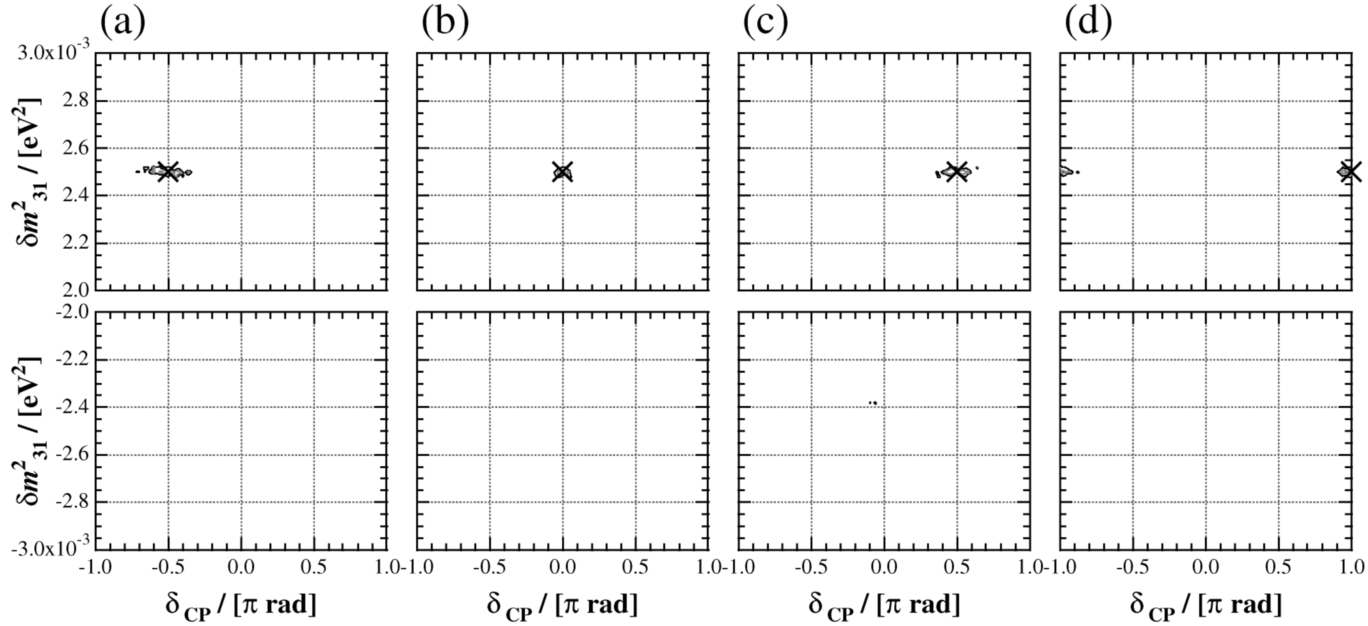

FIG. 6. Allowed regions for 68.3% (gray solid line) and 95% (black solid line) confidence level. The values of the parameters are the same as in Fig. 3.

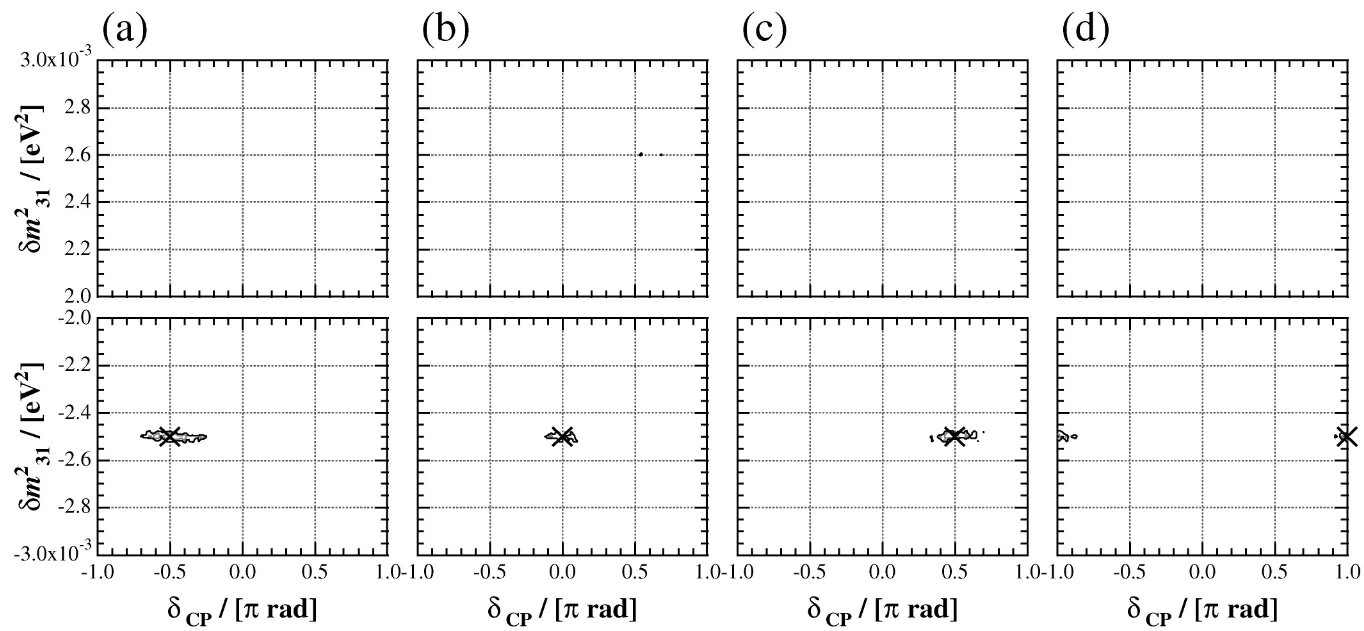

FIG. 7. The same as Fig. 6, but $\delta m_{31}^{2(\mathrm{true})} = -2.5 \times 10^{-3}$ eV^2.

that the isolated allowed regions are eliminated in these figures. The sign of δm_{31}^2 is hence well determined, and its absolute value is restricted to $(2.5 \pm 0.03) \times 10^{-3}$ eV^2, whose small error also reduces the error of δ_{CP} down to about $\pm 0.15\pi$ or less around the true value. The synergy of ν_e appearance events and ν_μ survival events excludes the fake allowed regions with the wrong sign of δm_{31}^2 and improves the precision of the parameters. The observation of both event types and the combined analysis are thus crucial.

IV. CONCLUSION AND DISCUSSIONS

We considered a search for the *CP*-violating angle δ_{CP} in long baseline neutrino oscillation experiments. We pointed out that it is necessary to take the ambiguities in

$|\delta m_{31}^2|$, θ_{13}, and θ_{23} into consideration in CP-violation searches. Their ambiguities can obscure the dependence of the $\nu_\mu \to \nu_e$ oscillation probability on δ_{CP} and on the sign of δm_{31}^2. We then showed that the ν_μ survival events can be employed to precisely determine the value of δm_{31}^2, and be combined with ν_e appearance events to improve constraints on the value of δ_{CP} and to determine the sign of δm_{31}^2.

We numerically verified the significance of the combined analysis for an example setup. We assumed that the neutrino beam generated by the upgraded AGS beam at BNL is observed for 5×10^7 sec by a 500 kt water Čerenkov detector, placed 770 km away at Kimballton mine. We took into account the ambiguity of $|\delta m_{31}^2|$ and fixed parameters other than δm_{31}^2 and δ_{CP}. We considered only the ν_e-contamination in the incident ν_μ beam as a source of background. We obtained allowed regions in the δ_{CP}-δm_{31}^2 plane through a χ^2 analysis. The analysis of ν_e appearance events alone led to less restrictive constraints on the value of δ_{CP} and the sign of δm_{31}^2 due to the ambiguity of $|\delta m_{31}^2|$. In contrast, the combined analysis of ν_e appearance and ν_μ survival events was capable of constraining the values of δ_{CP} and $|\delta m_{31}^2|$ with the errors of $\pm 0.15\pi$ or less and $\pm 3 \times 10^{-5}$ eV^2, respectively, and also of determining the sign of δm_{31}^2.

The values of θ_{13} and θ_{23}, which we have kept fixed in the present analysis, may not be known with sufficient precision by the time of δ_{CP} searches. The ambiguity in the value of $\sin^2 2\theta_{13} \sin^2\theta_{23}$ can actually be a hurdle in searching for the value of δ_{CP} and the sign of δm_{31}^2, as discussed in Sec. II and in other studies [12–14]. This, however, does not change our major conclusions; i.e., the ambiguity in the value of $|\delta m_{31}^2|$ is also a significant obstacle to the δ_{CP} search, and that ambiguity can be diminished by employing the ν_μ survival events. First, note that the ambiguities of $\sin^2 2\theta_{13}\sin^2\theta_{23}$ and of $|\delta m_{31}^2|$ affect two separate features of the $\nu_\mu \to \nu_e$ oscillation probability, namely, the amplitude and the phase, as can be seen in Eq. (1). Since they have independent impacts on the δ_{CP}-search in our approximation, the ambiguity of $|\delta m_{31}^2|$ will still have significant effects independently of θ_{13} and θ_{23}. Second, the phase of the ν_μ survival probability, Eq. (8), depends little on θ_{13} and θ_{23} for small θ_{13} below the current upper bound. Hence the value of $|\delta m_{31}^2|$ will be tightly constrained from the ν_μ disappearance probability regardless of the ambiguity in θ_{13} and θ_{23}. Our considerations are also supported by Ref. [14], although its authors analyzed a counting experiment using low-energy neutrinos and did not consider an experiment measuring the energy of neutrinos as we did.

Since the ambiguity of $|\delta m_{31}^2|$ has been shown to be controllable, the ambiguities of θ_{23} and θ_{13} are the remaining obstacles to δ_{CP} searches. Of the two, the value of θ_{23} can be determined from the amplitude of the ν_μ disappearance probability [cf. Eq. (7)] [9,13,14,21]. This hardly interferes with the determination of $|\delta m_{31}^2|$ since the value of $|\delta m_{31}^2|$ is determined from the phase of the oscillation. The improvement of the uncertainty in θ_{13} must await future experiments as discussed in Sec. II.

The combined analysis presented in this paper can be applied to more realistic case studies. For that purpose, it is necessary to take account of backgrounds other than ν_e-contamination, especially that from single pion-production events. A neutral pion becomes a source of background when the two photons from its decay are not separately detected due to the limited resolution of the detector and thereby misidentified as a ν_e appearance signal. The estimation of this background requires careful treatments. It is difficult to theoretically estimate the number of single-π^0-production events owing to its dependence on how strong-interaction processes are modeled. The number of misidentified events among them further depends on the details of the setup and method of experiments such as the design of the detector and the criteria of event selection. Nevertheless, there is a case study claiming that the number of background events from the neutral pions can be suppressed to be comparable to that from the incident ν_e-contamination [10]. We expect that the analysis presented in this paper shall remain effective when such excellent suppression of background is possible. We leave a realistic analysis with the consideration of π^0-background for a future work.

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