The Effects of Two Generative Activities on Learner Comprehension of Part-Whole Meaning of Rational Numbers Using Virtual Manipulatives

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Abstract

The study investigated the effects of two generative learning activities on students' academic achievement of the part-whole representation of rational numbers while using virtual manipulatives. Third-grade students were divided randomly in two groups to evaluate the effects of two generative learning activities: answering-questions and generating-examples while using two virtual manipulatives related to part-whole representation of rational numbers. The study employed an experimental design with pre- and post-tests. A 2x2 mixed analysis of variance (ANOVA) was used to determine any significant interaction between the two groups (answering-questions and generating-examples) and between two tests (pre-test and immediate post-test). In addition, a 2x3 mixed analysis of variance (ANOVA) and a Bonferroni post-hoc analysis were used to determine the effects of the generative strategies on fostering comprehension, and to determine any significant differences between the two groups (answering-questions and generating-examples) and among the three tests (pre-test, immediate post-test, and delayed post-test).

Results showed that an answering-questions strategy had a significantly greater effect than a generating-examples strategy on an immediate comprehension posttest. In addition, no significant interaction was found between the generative strategies on a delayed comprehension tests. However a difference score analysis between the immediate posttest scores and the delayed posttest scores revealed a significant difference between the answering-questions and the generating-examples groups suggesting that students who used generating-examples strategy tended to remember relatively more information than students who used the answering-questions strategy. The findings are discussed in the context of the related literature and directions for future research are suggested.

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Chapter 1: Introduction

Analyzing evaluations from the National Assessment of Educational Processes (NAEP), Carpenter, Coburn, Reys, and Wilson (1976) and Post (1981) found that students seem to operate with fractions without an adequate understanding of the concept of rational numbers. Research shows that learning the concept of rational numbers is a complex process, partly because it is a concept that has several meanings. In addition, it seems that students do not have enough time to play with the concept, and they jump directly to operations with fractions without having a clear understanding of these types of numbers (Kieren, 1990).

Having the ability to “see” mathematical concepts is an important skill for students to learn these concepts (Sfard, 1991). The use of tangible materials gives students opportunities to represent abstract concepts and develop mental images to understand the rational number concept (Bezuk & Cramer, 1989; Cramer & Henry, 2002). However, as discussed by different authors (e.g., Wilensky, 1991; Clements, 1999; Durmus & Karakirik, 2006), the advantages of tangible manipulatives are not due to their concreteness but to their capacity to develop students’ meaningful experiences. Alternatives such as computer-based manipulatives or virtual manipulatives are valid instructional tools to help students learn new mathematical concepts.

The use of better technologies in schools such as broadband Internet (Schofield, 2006), java-based software (Hart, Hirsch, & Keller, 2007), and computer graphics gives more options to create abstract concepts in a visible form (Sawyer, 2006). Virtual manipulatives have also become a popular new tool that gives teachers another option when working with mathematical concepts with students. About the relevance of these new technologies, the National Council of Teacher of Mathematics (2000) states, “Work with virtual manipulatives (computer simulations of physical manipulatives) or with Logo can allow young children to extend physical experience to develop an initial understanding of sophisticated ideas like the use of algorithms" (p. 26).

Research suggests that manipulatives do not have an implicit knowledge, and learners must generate mathematical knowledge by merging the use of manipulatives and their previous knowledge and experiences. One possible solution to this issue is to combine these instructional tools with instructional activities that help students to develop meaning for the tools and create relationships between the representation of the manipulatives and students' previous ideas (Freer, 2006; Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Olivier, & Human, 1997).

A relevant theory from instructional design and technology that explicitly emphasizes the importance of creating relations between elements of the new concept and between the new concept and students' previous knowledge is generative learning theory (Wittrock, 1974a, 1874b; Grabowski, 1996, 2003). Generative learning theory highlights students' activity as a primary element in the construction of knowledge. From the instructional design point of view, generative learning activities provide guidelines to allow teachers or educators to design environments where the learning and instruction principles are taken into account. A more in-depth understanding of the use of generative learning activities may lead to the development of more effective instructional environments and better opportunities for students to learn mathematical concepts.

The main goal of this study is to compare two learning activities, based on generative learning theory, that involve the use of virtual manipulatives to enhance third-grade students' comprehension of the part-whole meaning of rational numbers. This study will provide results to help teachers to decide which generative learning strategy is better to teach the part-whole concept of rational numbers. Therefore, teachers will be more able to "to create and monitor environments in which students live mathematics (Kieren, 1988). Establishing these environments in the classroom is a basic challenge for reform in teaching and learning fractions" (Steffe, Olive, Battista, & Clements, 1991, p. 24).

Chapter 2: Literature Review

To support the development of this research study, the literature review was composed of the following areas. Initially, a review of rational numbers was examined, discussing its representations, how part-whole meaning is fundamental to understanding rational numbers, and the problems that children have understanding the concept of rational numbers. Then, research behind the instructional use of manipulatives and virtual manipulatives in mathematics was analyzed, emphasizing the advantages that students have using virtual manipulatives as another representational tool. Finally, generative learning theory was discussed, emphasizing the relevance of this theory for design environments using virtual manipulatives, and research using answering-questions and generating-example activities as instructional options to help students work with virtual manipulatives.

Rational Numbers

A rational number can be defined as "a real number that can be put in the form of a common fraction *a*/*b* where *a* and *b* are integers and *b* is different from 0" (Baroody & Coslick, 1998). Rational numbers are a branch from the real number hierarchy tree, where real numbers are all the numbers in a number line consisting of rational and irrational numbers. At the same time, as part of the rational numbers set, integer numbers consist of negative integers and whole numbers, where whole numbers consist of natural numbers and the number zero (see Figure 1).

The rational number concept is one of the most important concepts that children need to learn in pre-secondary school years (Behr & Post, 1992). An understanding of rational numbers for instance, provides a relevant basis for algebraic operations in later courses (Post, Behr, & Lesh, 1982). However, at the same time, research has widely shown that children also have difficulties learning the concept of rational numbers (Cramer, Behr, Post, & Lesh, 1997; Lamon, 2007; National Research Council, 2001; Saxe, Shaughnessy, Shannon, Langer, Chinn, & Gearhart, 2007; Smith, 2002). Piaget (1973) suggested that many problems learning mathematics in the school occur because of the quick passage from qualitative or logical to the quantitative or numerical. The following sections discuss more about the problems that children have learning rational numbers and a theoretical framework to guide students' learning of rational numbers.

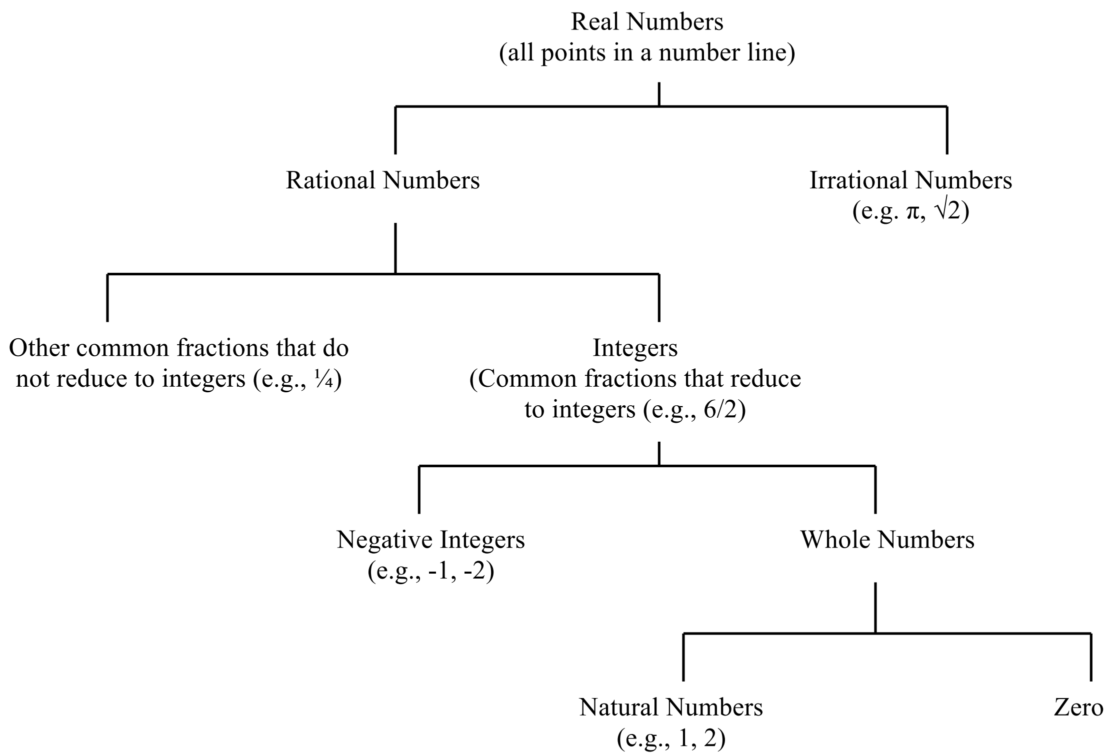

Figure 1. Tree Diagram of the Real-Number Hierarchy (Adapted from Baroody & Coslick, 1998)

Difficulties learning rational numbers

Children have difficulties learning the concept of rational number for several reasons (National Research Council, 2001). One problem is that rational numbers can represent different mathematical meanings. Another inconvenience is caused by the children's previous knowledge of whole numbers. Finally, researchers mention that children have few opportunities to use rational numbers in everyday situations in comparison with the opportunities to use whole numbers. Each of these problems will be addressed in this section.

Different meanings. Rational numbers are ambiguous because they can represent different mathematical meanings (Ohlsson, 1988). According to Kieren (1976) the different meanings or interpretations of the rational numbers include:

1. Rational numbers are fractions which can be compared, added, subtracted, etc.
2. Rational numbers are decimal fractions which form a natural extension (via our numeration system) to the whole numbers.
3. Rational numbers are equivalence classes of fraction. Thus, {1/2, 2/4, 3/6, …} and {2/3, 4/6, 6/9, …} are rational numbers.
4. Rational numbers of the form p/q, where p and q are integers and $q \neq 0$. In this form, rational numbers are “ratio” numbers.
5. Rational numbers are multiplicative operators (e.g., stretchers, shrinkers, etc.).
6. Rational numbers are elements of an infinite ordered quotient field. They are numbers of the form $x = p/q$ where x satisfies the equation $qx = p$.
7. Rational numbers are measures or points on a number line. (p. 102-103)

Several researchers (Behr, Harel, Post, & Lesh, 1993; Behr, Lesh, Post, & Silver, 1983; Freudenthal, 1983; Vergnaud, 1983; Sowder, Bezuk, & Sowder, 1993) have supported Kieren’s description of rational numbers. Although supporting Kieren’s description, Behr, Lesh, Post, and Silver (1983) renamed these meanings as part-to-whole, decimal, ratio, indicated division (quotient), operator, and measure of continuous or discrete quantities.

Previous knowledge of whole number. The second source of difficulty arises when children start learning rational number and they already have a previous knowledge of rules that relate to whole numbers. Children relate these two types of numbers and try to apply rules to rational numbers that work only with whole numbers (Behr, Wachsmuth, Post, & Lesh, 1984; Bezuk & Cramer, 1989; Mack, 1995; Hunting, 1986; Hunting & Sharply, 1991; Streefland, 1984). One example is the common confusion of adding, subtracting, multiplying, and dividing rational numbers. On a state administered standardized test, one question asks 1/5 + 3/4 = ? Forty-two percent of sixth-grade students chose 4/9 as the right answer (Marshall, 1993).

Another difficulty associated with the previous conception of rules related with whole numbers is based on counting. Behr and Post (1992) explained,

> Rational numbers are the first set of numbers children experience that are not based on a counting algorithm of some type. To this point, counting in one form or another (forward, backward, skip, combination) could be used to solve all of the problems encountered. Now with the introduction of rational numbers the counting algorithm falters (that is, there is no next rational number, fractions are added differently, and so forth). This shift in thinking causes difficulty for many students (p. 201).

Lack of practice. Children learn to work with whole numbers outside of the classroom, but the construction of a concept of rational numbers benefits from a more formal instruction. This formal instruction should provide experiences dividing elements (such as concrete models and pictures) in equal parts in meaningful contexts (National Research Council, 2001). About this situation, Clements and Del Campo (1990) stated, “Although more parents in Western cultures typically encourage their children to count (‘one, two, three, …’), very few of them encourage their children to find ‘one-half’, ‘one-quarter’, or ‘one-third’ of a whole unit” (p.181).

In summary, among the different problems encountered in learning rational numbers are the existence of several meanings, the previous knowledge of operations that apply to whole numbers, and the lack of opportunities to use rational numbers in everyday situations. This literature review will focus on the problems that students encounter when several meanings can be assigned to a rational number. The following sections will discuss how the Principles and Standards for School Mathematics (NCTM, 2000) organize the learning of rational number from prekindergarten through 12.

Rational numbers at the school

The concept of rational numbers can be taught in different ways at different school levels. For example, mathematics college students may need to handle definitions like: “Rational numbers are elements of an infinite quotient field consisting of infinite equivalent classes, and the elements of the equivalence classes are fractions” (Behr, Harel, Post, & Lesh, 1992). On the other hand, the Principles and Standards for School Mathematics (NCTM, 2000) addresses expectations about learning rational numbers for students from prekindergarten through grade 12.

According to the Principles, students from pre-k – 2 start learning specific fractions, students from 3–5 focus on learning rational numbers as part-whole meanings, and students from 6–12 emphasize using rational numbers to compare, contrast, and solve problems. More specifically, in grades pre-k-2, students are expected to “understand and represent commonly used fractions.” In grades 3-5, students should “develop understanding of fractions as parts of unit wholes, as parts of a collection, as location on number lines, and as divisions of whole numbers; use models, benchmarks, and equivalent forms to judge the size of fractions; and recognize and generate equivalent forms of commonly used fractions, decimals, and percents." In grades 6-8, students should "work flexibly with fractions, decimals, and percents to solve problems; and compare and order fractions, decimals, and percents efficiently and find their approximate locations on a number line." In grades 9-12, students should compare and contrast the properties of numbers and number systems, including the rational and real numbers, and understand complex numbers as solutions to quadratic equations that do not have real solutions" (NCTM, 2000).

Over the last two decades several researchers like Behr, Harel, Post, and Lesh (1992); Behr, Lesh, Post, and Silver (1983); Braunfeld and Wolfe (1966); Dienes (1967); Kerslake (1986); Kieren (1976); Moss (2005), and Streefland (1991) have provided information about the process of teaching rational numbers (Carraher, 1996). For example, Moss (2005), based on the work of the National Research Council (2000, 2001), summarized three general principles for teaching rational numbers: Emphasize the relevance of students' prior understandings of informal ideas of partitioning, sharing, and measuring; practice to facilitate students' translation of different forms of rational numbers; and provide opportunities for metacognitive activities.

Although these researchers' principles deal with various school levels, this literature review will focus on elementary students. Based on the Principles and Standards for School Mathematics (NCTM, 2000) and the interest of how students learn the concept of rational numbers, the following section will present models and guidelines about how to teach rational number in elementary level.

How to teach rational number in elementary school

Elementary schools teach mathematical ideas using at least two types of models: concrete and symbolic (Fennema, 1972a, 1972b). Related to this practice in classrooms, and to a more general classification of types of representation proposed by Bruner (1966b) (enactive, iconic, and symbolic), Lesh (1979a) presented a popular instructional model for teaching rational numbers (Figure 2). This translation model or multiple representations model suggested that children should be exposed to different representations of the same mathematical concepts, and at the same time, children should have the opportunity to create associations and transformations among the different representations (Cramer, 2003; Lesh, Behr, & Post, 1987; Perkins, Crismond, Simmons, & Unger, 1995; Goldenberg, 1995; Skemp, 1971). The advantage of the translation model is that it relates to the expectations of the NCTM (2000) that students in grades 3 to 5 develop understanding of the part-whole meanings of rational numbers.

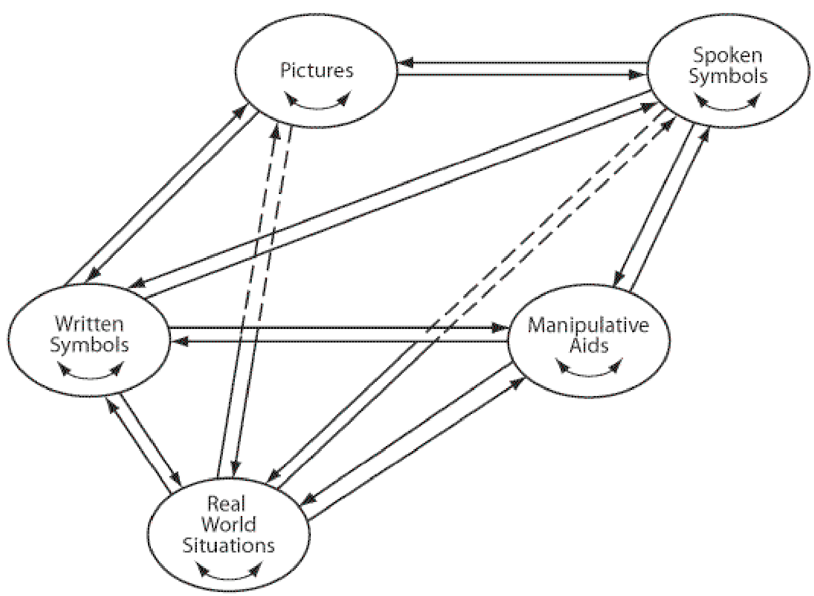

Figure 2. The Lesh Translation Model (Reprinted from Lesh, 1979a)

There is an important research group named The Rational Number Project (RNP) that explored various approaches to teaching rational numbers to elementary students. Cognitive psychological principles from Piaget (1966), Bruner (1966a), Dienes (1967), the different meanings of rational numbers defined by Kieren (1976), and the representation modes defined by Lesh (1979a) were theoretical foundations for the RNP (Behr, Post, & Lesh, 1981; Behr, Wachsmuth, Post & Lesh, 1984). This project, which has at least 86 research publications "is the longest lasting federally funded cooperative multi-university research project in the history of mathematics education" (Behr, Cramer, Harel, & Lesh, 2002, para. 2).

From the RNP, Cramer, Post, and delMas (2002) compared a commercial curriculum for learning fractions with the RNP fraction curriculum (Cramer, Behr, Post, Lesh, 1997). One thousand six hundred fourth-grade and fifth-grade students were randomly assigned to two groups with one group using the commercial curriculum and the other group using the RNP fraction curriculum. The researchers found that students who used the curriculum from RNP had statistically higher mean scores on the posttest and retention test. Based on interview data, researchers concluded that students who used the RNP lessons had a strong conceptual understanding of rational numbers, and they applied it to determine the relative size of fractions, or to estimate sums or differences of two fractions.

While emphasizing the development of rational number concepts, the RNP presented general recommendations applicable at all grade levels and explicit recommendations to teach rational numbers for primary grades (Bezuk & Cramer, 1989). Their general recommendations promoted the use of manipulatives as crucial in developing students' understanding of rational number ideas, the proper development of concepts and relationships among rational numbers, the delay of operations on fractions until rational number concepts are established, and restricting the size of denominators in computational exercises to lower than 12.

Following the general recommendations, Bezuk and Cramer (1989) presented recommendations for primary grades offering experiences to allow students to develop strong mental images of rational numbers as a basis for quantitative understanding. These recommendations included:

1. Instruction should be based on the part-whole concept using first the continuous model (circles, paper folding) and then the discrete model (counters). The discrete model should be introduced by relating it to the circles.
2. Include activities that ask students to name fractions represented by physical models and diagrams. Unit and nonunit fractions with denominators no larger than 8 should be used. Also include activities that ask students to model or draw pictures for fraction names or symbols.
3. Use words (three-fourths) initially and then introduces symbols (3/4).
4. Introduce "concept of unit" activities, that is, activities in which students name fractions when the unit is varied (p. 159)

Initial instruction of rational numbers should be based on the part-whole meaning. Based on the multiple representations of rational numbers defined by Kieren (1976), the RNP suggested that part-whole meaning along with the process of partitioning are basic to understanding the other meanings (ratio, operator, quotient, and measure) of rational numbers (Behr, Post, Silver, & Mierkiewicz, 1980). In addition, as demonstrated in Figure 3, meanings of rational numbers are also basic to promote understanding of other concepts. For example, ratio is basic to understanding the concept of equivalence, operator and measure are basic to understand multiplication and addition, and all meanings are basic to problem-solving skills.

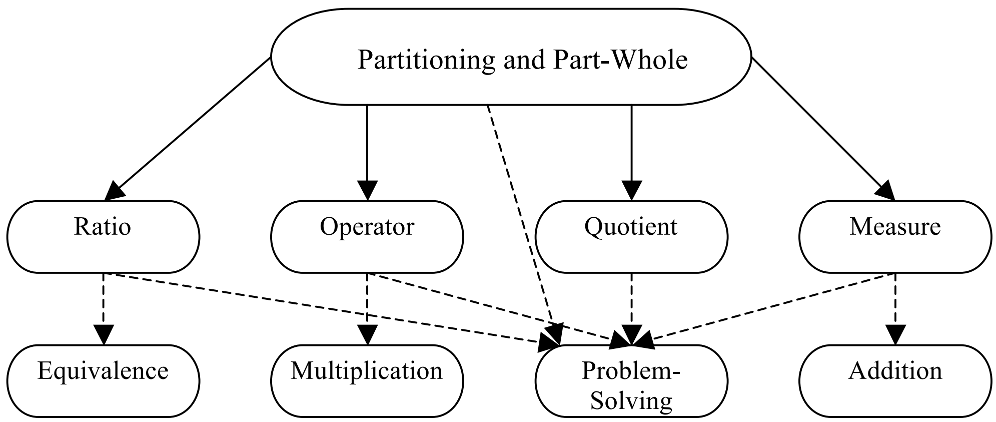

Figure 3. Conceptual Scheme for Instruction on Rational Numbers (Adapted from Behr, Post, Silver, & Mierkiewicz, 1980)

The Principles and Standards for School Mathematics (NCTM, 2000), Bezuk and Cramer (1989), Behr, Post, Silver, and Mierkiewicz (1980), as well as Kieren (1980a, 1980b) and Lovell (1971) also support the idea that the initial instruction in rational numbers should be based on the part-whole meaning. Because of its importance, the following section will discuss the instruction of part-whole meaning of rational number in elementary grades.

Instruction of part-whole meaning of rational numbers

Nowadays there is a general tendency at schools to use part-whole meaning to introduce to students the concept of rational numbers (Charalambous & Pitta-Pantazzi, 2007; Moseley, 2005). In this initial contact, children learn the ability to divide a continuous quantity or a set of discrete objects in equal parts or in subsets with equal number of elements respectively (Behr, Lesh, Post, & Silver, 1983; English & Halford, 1995; Post, Behr, & Lesh, 1982). Although part-whole meaning is the easiest for children to understand (English & Halford, 1995), as Lamon (1999) states, this first contact with rational number is not easy for students,

> Although fractions build on a child's preschool experiences with fair sharing, the more formal ideas connected with visual representations, fraction language, and symbolism, are so intellectually demanding that it takes a long time after their first formal introduction to part-whole comparisons before they can coordinate all of the essential details (p. 66).

Models to represent part-whole meaning. Part-whole situations can be represented with a continuous area like a cake, cookie, chocolate bar, etc., or a set of discrete elements like a bag of marbles, a dozen eggs, etc. Although both types of situation help students to grasp the concept of this representation, the continuous and discrete models consist of different characteristics. For example, if there is a continuous quantity (see Figure 4), children first need to identify that the parts of the whole have equal size. Second, they need to identify how many divisions comprise the whole and relate this number with the denominator of the rational number. Finally, students need to recognize how many units are shaded and relate this number with the one in the numerator of the rational number.

In a discrete model (see Figure 5), children need first to identify that the separate elements are a whole unit, where sometimes the elements do not have same size or shape. Second, they need to identify how many elements there are and relate this with the number in the denominator of the rational number. Third, they need to identify how many elements have a particular characteristic that made them different and relate this number with the number in the numerator of the rational number. The necessity of identifying discrete elements as equal parts of a whole unit make it more difficult for children to understand the concept and make it inappropriate to introduce the part-whole meaning (Hope & Owens, 1987).

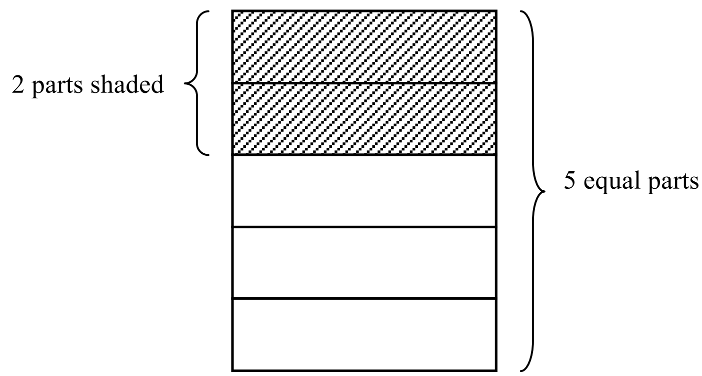

Figure 4. Representation of 2/5 using a region or continuous model

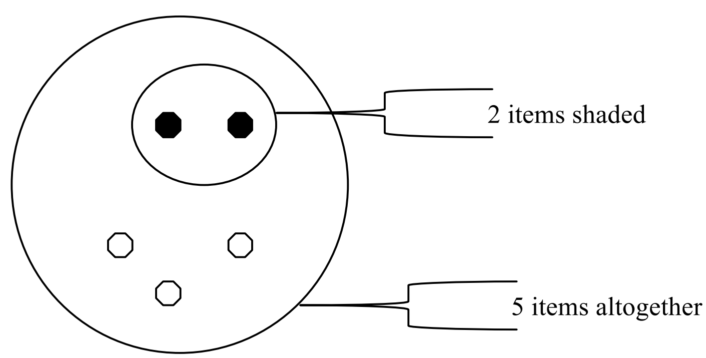

Figure 5. Representation of 2/5 using a set or discrete model

How to teach part-whole meaning. To teach part-whole meaning of rational numbers there are two elements. First, how to present representations of the part-whole, and second, what criteria are necessary to learn part-whole meaning.

Two forms of visual representation predominate in instruction involving part-whole meaning (Marshall, 1993). One is the symbol a/b, which is used as a visual pattern where different rational numbers represent different patterns. For example, 1/2, 3/5, 5/8, etc. The second visual representation is the graphic of squares, circles, or rectangles that are divided in pieces of equal size (see Figure 6).

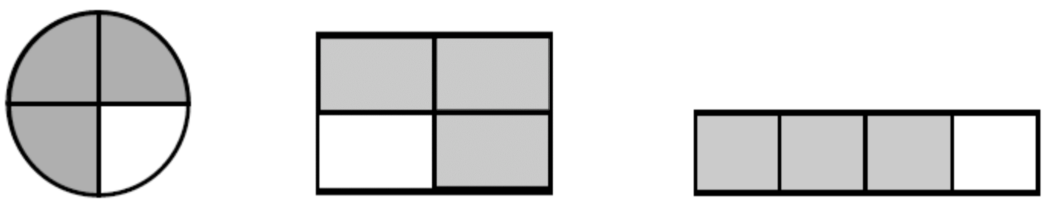

Figure 6. Visual models of three-fourths (Adapted from Cramer, Behr, Post, & Lesh, 1997)

Kieren (1984) mentioned that sometimes teachers are erroneously inclined to tell children to count the number of parts and the number of shaded parts to come up with the rational number. Instead, Kieren (1984) and Mack (1990, 1993) suggested the use of real-world problems or real-life situations to help children to develop rational number ideas. For example, "There are 2 pizzas and 3 children. How much is each child's share?" (Kieren, 1984, p. 3).

Marshall (1993) also emphasized the relevance of real-world situations to assess rational number knowledge. For example, she suggests the following type of question: "make up a story involving ¾" (p. 282). This type of question tries to connect students' own experiences and knowledge about rational numbers. In the same issue, Schoenfeld (2006) also contrasted traditional assessment that focuses on algorithms asking questions like "find (1/2)(3/5) +

(1/2)(1/5)" (p. 491), with more contemporary assessments that focus on abilities to work with different representations asking questions like "write a fraction for the shaded part of the region below" (p. 491) (Figure 7).

Figure 7. Part-whole meaning of 1/6

Two important criteria help guide instruction of the part-whole meaning. According to Piaget, Inhelder and Szeminska (1960), the part-whole meaning and the partitioning activity contain subconcepts that elementary children need to be aware of to understand part-whole meaning. The subconcepts are:

1. there can be no thought of a fraction unless there is a divisible whole, one which is composed of separable elements
2. fraction…implies a determinate number of parts
3. subdivision is exhaustive, i.e. there is no remainder
4. there is a fixed relationship between the number of parts into which a continuous whole is to be divided and the number of intersections
5. the concept of an arithmetical fraction implies over and above purely qualitative subdivision, that all of these parts are equal
6. they [fractions] are parts of the original whole and they are also wholes in their own right, and as such they too can be subdivided further

7. since fractions of an area relate to the whole from which they are drawn, that whole remains invariant; in other words, the sum of fractions equals the original whole (p. 309-311).

In other words, to relate a whole with part-whole meaning, students need to realize that this whole needs to be divided into equivalent parts or subsets, *n* divisions of the object provide *n+1* parts of the whole (1 division provides 2 parts, 2 divisions provide 3 parts, etc.), and when the equivalent parts are put back together to create the initial whole, there is nothing left over. Hiebert and Tonnessen (1978), Payne (1976), and Novillis (1976) pointed out that these subconcepts are appropriate for continuous representations like geometric figures but not for discrete representations.

Post, Behr, and Lesh (1982) proposed other criteria involving shaded parts of the whole that need to be satisfied to create correct part-whole meaning,

1. Set *A* has been divided into equivalent parts or subsets
2. Set *B* has been divided into equivalent parts of subsets
3. Each individual part or subset of *A* is equivalent to each individual part or subset of *B* (p. 62-63).

For example, Figure 8 can be interpreted as two-fourths because the shaded area (set *A*, part of the whole) consists of two equivalent parts, the whole (set *B*) consists of four equivalent parts, and each part of *A* is equivalent with each part of *B*. On the contrary, Figure 9 cannot be interpreted as two-fourths because parts of *A* are not equivalent to parts of *B* invalidating condition 3.

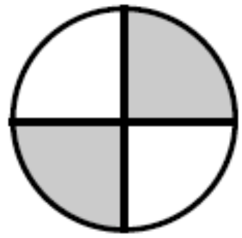

Figure 8. Correct graphical represent of two-fourths (Adapted from Cramer, Behr, Post, & Lesh, 1997)

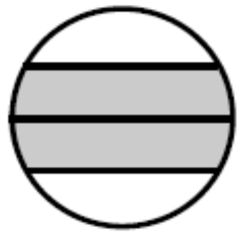

Figure 9. Incorrect graphical representation of two-fourths (Adapted from Cramer, Behr, Post, & Lesh, 1997)

In summary, the rational number is a difficult concept to learn because of different factors such as multiple meanings, previous knowledge on whole numbers, and lack of practice outside of the school. Based on the Principles and Standards for School Mathematics (NCTM, 200), the rational number concept is developed mainly in grades from 3–5, focusing on the part-whole meaning. According to different authors (Behr, Post, Silver, & Mierkiewicz, 1980; Charalambous & Pitta-Pantazzi, 2007; Kieren 1980a, 1980b; Lovell, 1971; Moseley, 2005) the instruction of rational numbers should be based on the part-whole meaning because this meaning is basic to understand the other meanings. In addition, literature review also discusses a model (Lesh, 1979a) and criteria that provide guidelines to instruct and help students to understand part-whole meaning of rational numbers (Piaget, Inhelder & Szeminska, 1960; Post, Behr, & Lesh, 1982).

Manipulatives

The graphics included in the above section of the literature review demonstrated the importance of images in understanding the rational number concept. The use of manipulatives is an important strategy to help young students develop mental images of the part-whole meaning (Bezuk & Cramer, 1989). Also, Piaget's theory of intellectual development emphasizes the necessity of concrete experiences for children at the age of 7 years to 11 years (Ginsburg & Opper, 1988). One way to create this concrete experience while promoting the development of mental images is using manipulatives, one of the representations proposed by the Lesh translations model. Therefore, this section discusses manipulatives, and their value in providing concrete experiences to help students in the process of learning rational numbers.

Manipulatives are objects or things that students are able to feel, touch, handle, move (Yeatts, 1991) and, often, stack (Clement, 2004, p. 98). The use of manipulatives, or objects that appeal to several of the senses, is not a new strategy in teaching. For example, Johann Heinrich Pestalozzi (1746-1827), a Swiss educator, opened a school for orphans and peasants where he emphasized hands-on activities using object lessons (Brosterman, 1997). Friedrich Froebel (1782-1852), who created the first kindergarten in 1837, also emphasized the use of objects to encourage expression of children's ideas (Resnick, Martin, Berg, Borovoy, Colella, Kramer, & Silverman, 1998; Hayward, 1979). Maria Montessori (1870-1952), who created a chain of schools, designed materials and activities for sensory education (Gutek, 2004). Currently, the objects used by Froebel and Montessori are popularly known as "Froebel Grifts" and "Montessori Materials" respectively (Zuckerman, Arida, & Resnick, 2005).

Manipulatives are important for multiple reasons. The senses are stimulated as the students touch the manipulative materials, move them about, rearrange them, and/or see them in various patterns and groupings. The manipulation of these materials assists students in bridging the gap from their own concrete sensory environment to the more abstract levels of mathematics. Manipulatives are, therefore, effective and motivating tools for assisting and enhancing the development of mathematical concepts (Yeatts, 1991). Children can physically manipulate these objects and, when used appropriately, they give children opportunities to compare relative sizes of objects that represent mathematical ideas such as fractions or place value. They also allow children to identify patterns and to put together representations of numbers in multiple ways (Clement, 2004).

Researchers have emphasized the advantages of manipulatives as objects to represent mathematical concepts. However, simply giving manipulatives to students is not enough to guarantee that students will learn mathematical ideas from them (National Research Council, 2001; Noss & Hoyles, 2006). Manipulatives effectively contribute to learning new mathematical ideas when used as part of instructional activities that are designed to help students to engage cognitively in creating relevant mental relationships to allow these students to develop understanding (English, 2004b; Kamii, Lewis, & Kirkland, 2001), or what Piaget (1971) calls logicomathematical knowledge.

These instructional activities are important because manipulatives do not carry any inherent knowledge (Ball, 1992), instead getting their value from “the interplay between the practical and theoretical tasks based on actions with the manipulatives” (Sierpinska, 2006, p. 123). These designed tasks need to assure that manipulatives have a high level of transparency representing just the idea that students need to interpret and to understand (Boulton-Lewis, 1998; Lesh, Behr, & Post, 1987; Meira, 1998; Uttal, Scudder, & DeLoache, 1997). In this way, students interacting with manipulatives develop knowledge relating their internal-mental representations with the external-physical representations provided by manipulatives (Goldin & Kaput, 1996; Gravemeijer, 2002). The following sections discuss the relevance of manipulative in mathematics educations, and a theoretical framework to use manipulatives based on the ideas of Jean Piaget.

Relevance of manipulatives in mathematics education

Manipulatives are important tools to learn mathematical concepts because they work as intermediaries by helping students bridge the gap between their understanding of concrete models and abstract concepts (English, 2004a; Lesh, 1979b; Kaput, 1987; Moyer & Bolyard, 2002; Resnick & Omanson, 1987). Post (1980) and Meira (2002) extended this idea by affirming that manipulatives are sense-making tools to simplify situations from the real world, and at the same time symbolize abstract concepts. Symbolization in the mathematical world allows learners to make calculations and instantiate concepts to obtain predictions that can be confirmed in the real world (Meira, 2002) (Figure 10).

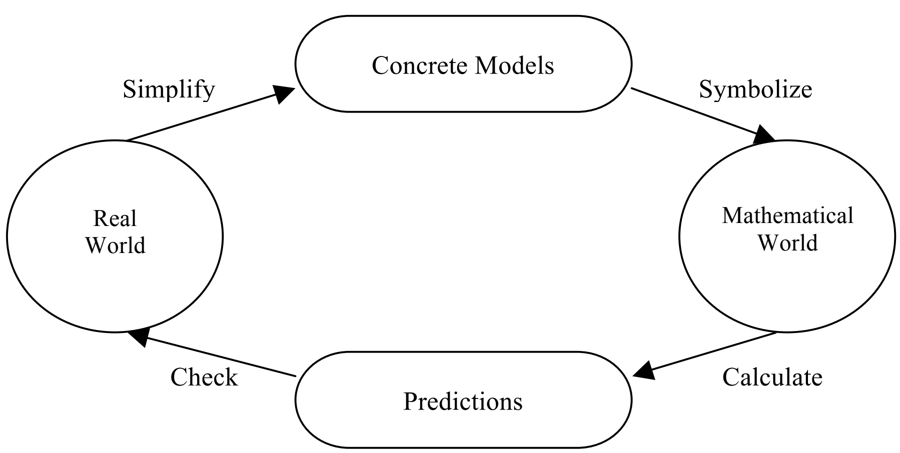

Figure 10. Relationship between the Real and Mathematical World (Adapted from Meira, 2002)

Another related model about how students interact with objects to expand mathematical understanding is proposed by Kaput (1991, 1995). He established that there is a cyclical process when students interact with physical objects. In Figure 11, the upward arrow shows that students deliberately try to interpret the meaning of the objects, or students evoke ideas just looking at the objects. When mental operations are performed students project their knowledge into the existing objects, or students elaborate their ideas by manipulating the objects (Figure 11). These mental and physical operations promote development of internal (mental images) and external representations (symbols) (Dufour-Janver, Bednarz, & Belanger, 1987). In summary, Post (1980) and Kaput's (1991, 1995) models described processes where students construct representations using manipulatives as a bridge to connect students' previous knowledge and new mathematical ideas.

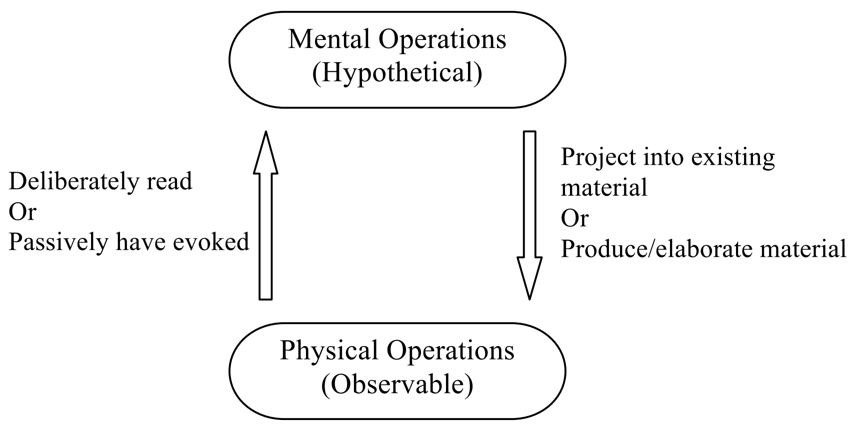

Figure 11. Relations between Physical and Mental Structures (Adapted from Kaput, 1991. Used with kind permission of Springer Science and Business Media. Springer/Kluwer Academic Publishers. *Radical Constructivism in Mathematics Education,* 1991)

Jean Piaget and manipulatives

Although Piaget did not directly study teaching or learning in the classroom, his research left two major ideas that influenced the education field (Clements, 1989). First, the child actively constructs knowledge, based on Piaget's major contention that each person constructs his/her own reality using sensory experiences (Wadsworth, 1978). This idea is related to promoting child's autonomy, one teaching principle derived from Piaget's constructivism (Kamii & Devries, 1976). A second major idea is the theory of cognitive development (Piaget, 1966), where Piaget defines that the development of the mind proceeds through stages, including assimilation and accommodation (Clements, 1989).

Assimilation and accommodation are complementary processes that encourage activity in the instruction of mathematical concepts (Lesh, 1979a). Based on the definitions developed by Siegler and Wagner (2005) and Bybee and Sund (1982), assimilation refers to how learners interpret the representations created by the manipulatives based on the learners' existing conceptions. Accommodation refers to how the existing conceptions in the learners' mind change to fit the new information coming from the interpretation of the manipulatives. From an instructional point of view, teachers should work with ideas slightly beyond the students' current conceptions to create a cognitive conflict to push students into a new phase of assimilation and accommodation (Hunt, 1961; Lovell, 1971).

According to Piaget (1966), cognitive development is divided into four periods: sensory-motor, from birth to two years; preoperational, from 2 years to 7 years; concrete-operational, from 7 years to 11 years; and formal-operational, from 11 years and above (Ginsburg & Opper, 1988; Sund, 1976). These periods put emphasis on the combination of both concrete and symbolic elements to allow children to learn, emphasizing concrete models in the first periods and symbolic models in the last periods when children have a higher level of development (Baroody, 1989; Fennema, 1972a; Sawyer, 2006).

Jean Piaget's theoretical foundations are often cited as the basis for the relevance of active children interacting with their world through concrete materials to develop understanding of mathematical concepts (Post, 1980, 1992). For example, "Major theoretical support for the use of concrete models before symbolic model in teaching mathematical ideas is provided by Piaget" (Fennema, 1972a, p. 636). "Piaget's influence in schools during the 1960s and 1970s led to the widespread use of manipulatives, blocks and colored bars to be used in math classrooms" (Sawyer, 2006, p. 12). Developmental psychologists often cite Piaget as "providing an example of a theory which places manipulative materials or concrete objects in a special prerequisite function in the development of learning structures" (Suydam & Higgins, 1977, p. 14). "A first principle drawn for Piaget's theory is the view that learning has to be an active process, because knowledge is a construction from within" (Kamii, 1973, p. 199). "Materials that children can manipulate for themselves play an absolutely fundamental role in any Piaget-inspired curriculum (Brainerd, 1978, p. 279).

Research supporting Jean Piaget's theories

Kamii and DeVries (1976) employed Piaget's principles to recommend situations to construct the concept of number in children. In addition, Piaget's (1960) cognitive psychological principles were fundamental in the Rational Number Project to develop theories and activities concerned with learning the meanings of rational numbers using manipulative materials (Behr, Wachsmuth, Post & Lesh, 1984). In other research, Olive (1999) designed computer-based activities to support construction of rational number concepts by using whole-number ideas based on the division of the unit into equal parts as described by Piaget, Inhelder, and Szeminska (1960). The research found that these activities contributed to the development of children's operations with rational numbers. Arnon, Neshier and Nirenburg (2001) also developed educational software to help elementary students learn the concept of equivalent rational numbers.

Research results clearly establish the positive impact on cognitive maturation of children manipulating concrete representations. The next section discusses in more depth the concept of concreteness and how virtual manipulatives can also be used to help students create concrete representations. Definition, types and research related with virtual manipulatives are also discussed.

Virtual Manipulatives

Recognizing the importance of allowing children to play with concrete representations to promote cognitive maturation and the desire of researchers to design activities that address important mathematical concepts, Piaget (1973) issued the following observation and warning:

> …it appears that many educators, believing themselves to be applying my psychological principles, limit themselves to showing the objects without having the children manipulate them, or, still worse, simply present audio-visual representations of objects (pictures, films, and so on) in the erroneous belief that the mere fact of perceiving the objects and their transformation will be equivalent to direct action of the learner in the experience. The latter is a grave error since actions are only instructive when they involve the concrete and spontaneous participation of the child himself with all the tentative gropings and apparent waste of time that such involvement implies. It is absolutely necessary that learners have at their disposal concrete materials experiences (and not merely pictures), and that they form their own hypothesis and verify them (or not verify them) themselves through their own active manipulations. The observed activities of

other, including those of the teacher, are not formative of new organizations in the child (p. ix).

Piaget, more than 30 years ago, discussed the relevance of concrete and spontaneous participation and concrete materials experiences for instruction, and the poor benefits of using visual representations of objects. Nowadays interactive technologies, which were not available in Piaget's time, have opened new opportunities to experiment with the use of and examine the effects of using visual representations of objects while attempting to provide concrete experiences to students. For example, Arnon, Neshier and Nirenburg's (2001) developed educational software to help students learn about rational numbers by working with equivalence-classes of fractions. Arnon et al. (2001) affirmed that the program offers concrete representations of equivalent fractions using a Cartesian system. Posterior evidence based on interviews indicated that students showed a development of the mathematical concept. In another example, Olive (1999) used a computer microworld to develop the arithmetic of rational numbers. These computer tools offer environments where students play with manipulable objects to develop their notion of rational numbers. Researchers confirm that visualization and action provided by the computer environments allow students to develop rational numbers concepts.

From concrete manipulatives to virtual manipulatives. An earlier section provided numerous examples of using concrete manipulatives for instruction. However, concreteness, as used within these examples, requires further explanation. For example, Durmus and Karakirik (2006) explained that when students have a concrete experience learning a mathematical concept, it does not refer specifically to the tangible nature of the materials used but to the meaningful relations that students create between the use of manipulatives and their previous ideas and experiences. Concreteness of tangible manipulatives is not because they are touchable, but because students are able to make sense of the ideas that manipulatives are representing. (Wilensky, 1991).

Clements (1999) classified concrete knowledge in two different types: sensory and integrated. Sensory-concrete knowledge refers to that type of knowledge that must be supported by sensory materials. For example, little kids cannot add or subtract unless they have elements to do the operations. On the other hand, integrated-concrete knowledge refers to that type of knowledge rooted in the children's mind that it is easily visualized and does not need any sensory material to support it. For example, children who have deep understanding on rational number representations can solve a problem of adding two fractions, changing those fractions to decimals.

Under the right conditions, instructional materials on the computer provide the same educational elements as that those from the tangible objects helping students to build integrated-concrete knowledge (Clements, 1999). In addition, manipulatives on the computer can have additional elements like feedback, interactivity, or flexibility. For example, Kaput (1994), Tall (1994), and Dreyfus (1994) emphasized the advantages of interactive computer environments that link different representations, helping students to recognize the same mathematical concept in different contexts. Finally, Clements (1999) defined good manipulatives as,

> Those that are meaningful to the learner, provide control and flexibility to the learner, have characteristics that mirror, or are consistent with, cognitive and mathematics structures, and assist the learner in making connections between various pieces and types of knowledge … computer manipulatives can serve that function (p. 50).

Types of manipulatives. Literature mentions three types of manipulatives that are not tangible: virtual manipulatives, computer manipulatives, and digital manipulatives. Virtual manipulatives (Moyer, Bolyard, & Spikell, 2002) and computer manipulatives (Clements and McMillen, 1996) make reference to those manipulatives that run in a computer, but as mentioned by Moyer, Bolyard, and Spikell (2001) the only difference between these two is that virtual manipulatives are freely available on the Internet. On the other hand, digital manipulatives (Resnick, Martin, Berg, Borovoy, Colella, Kramer, & Silverman, 1998) are instilled with technological capabilities. They are manipulatives that typically contains electronic devices that allow computational and communication capabilities.

The most referenced definition of virtual manipulatives is expressed by Moyer, Bolyard, and Spikell (2002). They defined a virtual manipulative "as an interactive, web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge" (p. 373). Although nowadays the virtual manipulatives are still available on the Internet, they also are sold on CD format to avoid dependency on the Internet. Currently, virtual manipulatives are also improving their functionality and they are more than just representations of a tangible manipulative (Steen, Brooks, & Lyon, 2006). In summary, virtual manipulatives could be considered as interactive computer-based tools which, when based on appropriate instructional activities, could help students represent mathematical concepts for a better understanding.

Research using manipulatives. Some research with tangible manipulatives has indicated that students who use them in their mathematics classes do better than students who do not use them (Clements & McMillen, 1996; Driscoll, 1983; Freer, 2006; Kennedy, 1986; Lee & Freiman, 2006; Suydam, 1986). Other research identified several variables to obtain positive results using manipulatives such as the expertise of the teacher (Sowell, 1989; Raphael & Wahlstrom, 1989), or the quality of thinking that the manipulatives encourage (Kamii, Lewis &

Kirkland, 2001), or teachers' beliefs about the relevance of using manipulatives in the process of teaching and learning mathematical ideas (Moyer, 2001).

Steen, Brooks, and Lyon (2006) expressed that virtual manipulatives have just started being used by teachers in the classroom and more research is needed to know the impact of these computer-based tools. However, there is an increasing use of virtual manipulatives in the classroom based on three elements: innovations in computer technology, the enhanced availability of broadband Internet at schools, and teachers' credibility in the use of concrete manipulatives in the classroom (Moyer, Niezgoda & Stanley, 2005; Moyer, Bolyard & Spikell, 2001). Other elements that increase the use of virtual manipulatives are their free use, ability to be altered, and unlimited access on the Internet (Crawford & Brown, 2003; Moyer, Bolyard & Spikell, 2001). Examples of these virtual manipulatives are found on Web pages of the National Library of Virtual Manipulatives (http://matti.usu.edu/nlvm) and the National Council of Teachers of Mathematics (http://illuminations.nctm.org). Other resources are mentioned in Lindroth (2005), Hodge (2003), and Van de Walle (2007).

In research with mathematics teachers from kindergarten to 9th grade, Crawford and Brown (2003) developed a qualitative study dealing with the rationale for using virtual manipulatives in the classroom. Teachers answered that the use of virtual manipulatives support issues such as students' motivation, instruction, teachers' productivity, and students' skill with technology. In another qualitative study, Suh, Moyer, & Heo (2005) worked with 46 fifth-grade students divided in three groups based on previous achievement (low, average, and high). They participated in three days of lessons focused on fraction equivalence and addition of fractions with unlike denominators using three different virtual manipulatives. Based on observations and students comments, researchers found that virtual manipulatives support students creating relationships between different representations, recognizing possible errors in their conceptions, developing new assumptions interacting with other students, and applying their learned concepts.

In a research study dealing specifically with rational numbers, Reimer and Moyer (2005) developed a quantitative (conceptual and procedural knowledge) and qualitative study (interviews and attitude survey) with 19 third-grade students using virtual manipulatives to learn fractions. The conceptual knowledge test showed a significant difference but the procedural knowledge assessment did not. An attitudes questionnaire showed 59% positive, 23% neutral, and 18% negative responses; and using a narrative analysis procedure, the interviews showed four consistent topics: virtual manipulatives help students learn fractions, students like the immediate feedback, virtual manipulatives are easier and faster than paper-based ones, and enjoyment.

Comparing different types of manipulatives, Moyer, Niezgoda, and Stanley (2005) reported research with a kindergarten class using wooden pattern blocks, virtual pattern blocks, and drawings. They founded that children create a greater number of patterns with the virtual pattern blocks showing more creative activities. They concluded that virtual blocks act as a bridge between the wooden blocks and the posterior use of pictorial and symbolic notations for a better understanding of mathematical ideas. In another study, Steen, Brooks, and Lyon (2006) studied the impact of virtual manipulatives on first grade students' achievement, attitudes, behaviors, and interactions. Thirty-one students were divided into an experimental group, which used the textbook and virtual manipulatives, and a control group, which used the textbook and physical manipulatives. Researchers used a pretest and posttest at both the first and second grade levels. In both cases, the experimental group outperformed the control group, though not a significant difference. In addition, Moreno and Mayer (1999) compared two types of virtual manipulatives to support addition of whole numbers. One manipulative represented problems using only a symbolic form, and the second manipulative presented problems using symbolic, visual, and verbal forms. Results showed a significant difference on high-achieving students who used manipulatives with multiple representations.

In general, research shows that virtual manipulatives are a useful tool to help students to learn mathematical concepts by externalizing their internal mental representations (Zbiek, Heid, Blume & Dick, 2007). However, these virtual manipulatives need to be accompanied by instructional activities that help students take advantages of these tools. As Crawford and Brown (2003) concluded, “Web-based mathematical manipulatives are available for integration into the learning environment. However, thoughtful consideration must be given to the instructional design of the course and the specific learning objectives for each module of instruction” (p. 179).

This section of the literature review discussed the relevance of using manipulatives and virtual manipulatives to learn mathematical concepts and how they work from a cognitive point of view, connecting mathematical ideas and students’ previous mathematical conceptions. From the instructional point of view, Lesh (1981) concluded that effective instructional strategies should promote activities with concrete materials. However, subsequent research indicates that concrete materials may be either tangible or virtual, as long as they are used as part of an effective instructional strategy.

The following section discusses a cognitive theory called generative learning theory that is based on the generation of relationships between students’ existing knowledge and new information. About the relevance of this theory in mathematics education Romberg and Carpenter (1986) stated, “Wittrock (1974[b]) was one of the first to point out the broad implications of the developing field of cognitive science for research in mathematics education”

(p. 853). The following section explains this theory and describes two instructional strategies based on this theory.

Generative Learning Theory

Based on cognitive approaches (Wittrock, 1977, 1978, 1992; Wittrock & Lumsdaine, 1977) and Luria's (1973) neural research, generative learning theory was conceived by Merlin Wittrock (1974a, 1974b, 1985, 1992). Wittrock (2000) defined and described a model of generative learning as "a functional model of learning with understanding that builds upon neural and cognitive research. The model consisted of the following processes (1) attention, (2) motivation, (3) knowledge, (4) generation, and (5) metacognition" (p. 210). Grabowski (2003) summarized a comparable model in four processes: motivation, learning, knowledge creation, and generation. These four processes are discussed in the following paragraphs.

Motivation is based on the student's perception that he/she has the control of his/her own learning process. In other words, student's success should be attributed to his/her own effort instead of external variables such as the class, the professor, or classmates (Wittrock, 1990). Specifically, motivation in the generative model is based on attributions and the delay-retention effect (Wittrock, 1974b). About attributions (Weiner, Heckenhausen, Meyer, & Cook, 1972; & Weiner, 1972), Mayer (1998) summarized motivation based on attributions as "students who attribute academic success and failure to effort are more likely to work hard on academic tasks than students who attribute academic success and failure to ability" (p. 60). About delay-retention effect, Wittrock (1974b) described a tendency to believe that reinforcements should be given immediately, discriminately, and frequently during the learning process. But other research

(Kulhavy & Anderson, 1972; Sassenrath & Younge, 1969; Surber & Anderson, 1975) has shown that the performance in retention tests is better if feedback is delayed in the immediate tests.

Learning process is based on the students' attention during instruction (Wittrock, 1992). How and what students learn is based on the voluntary attention that students put in the process (Picton, Stuss, & Marshall, 1986). Although there are different types of attention such as sustained attention, short-term attention, voluntary and involuntary attention, and selective attention, voluntary sustained attention is particularly important in academic tasks (Wittrock, 1991). Research indicates there is a higher correlation between attention and achievement than time-to-learn or time-on-task and achievement (Wittrock, 1986a; Simon, 1986).

Knowledge creation, the third factor, is based on elements of cognitive theory such as metacognition, previous knowledge, or beliefs that influence the construction of new relations in memory. Wittrock (1986a) claimed that students learn and remember information by associating ideas to one another, and learning and memory increase when students relate information to their knowledge and experience. Specifically, as Bonn and Grabowski (2001) mentioned, this knowledge creation process on learning mathematics is based on the use of previous mathematical concepts to help the students generate new mathematical understanding.

Finally, the generation process distinguishes generative learning theory from other cognitive theories (Grabowski, 2003). Wittrock (1974b) explains the generative process as activities that involve the creation of meaningful relationships between different parts of the instructional material, and the creation of meaningful relationships between different parts of the instructional material and students' previous knowledge and experiences. Grabowski (2003) calls these processes coding and integrating respectively.

Based on this generation process, other research has applied these processes to other specific environments. For example, the model of generative reading comprehension (Wittrock, Marks, & Doctorow 1975; Wittrock, 1990) involves building relations among the parts of the text, and between the text and readers' previous beliefs as fundamental constructions in the process of reading comprehension; and the generative teaching model (Wittrock, 1991; Kourilsky & Wittrock, 1992) that teach students to generate relationships between the ideas they are learning and between the new ideas and their previous knowledge using learning strategies. Concluding, Wittrock and Alesandrini (1990) stated,

> The model of generative learning from teaching and instruction (Wittrock, 1974a, 1974b, 1981, 1983a, 1983b, 1990) states that people comprehend and remember information when they generate relations (a) among the propositions and sentences of the text, and (b) between the text and their knowledge base (semantic memory) and their experience (episodic memory). (p. 490)

Confirming the importance of establishing these relationships inside of the cognitive theory, Weinstein and Mayer (1986) emphasized that these kinds of relations made by the student to his/her previous established schemas are relevant to developing new knowledge. However, the above discussion of the generation process does not establish how to help students to generate these relationships. The following section discusses learning strategies that help students in the process of generation.

Generative learning strategies.

Generative learning strategies are those strategies that promote generative learning. Generative learning strategies are learning activities that guide students to work with static information and transform this information into a relevant, dynamic and usable knowledge (Grabinger, 1996). Grabowski (1997) narrowed down the definition of generative learning activities and maintained that activities can be considered generative learning activities only if they guide students to generate meaning from the instructional environment.

Wittrock (1990) defined several instructional activities that stimulate the generation of relationships between elements of the concepts, and between the concept and the student's prior knowledge. As shown in Table 1, Wittrock (1990) also affirmed that the generation of relationships could be fostered by the students' generating suggested instructional activities, or by the teachers giving instructional elements to stimulate the generation of relationships.

Table 1

Ways to Stimulate Generation (Adapted from Wittrock, 1990)

Teacher Given	**Learner Constructed**
Among Concepts Presented in Instruction	
Titles	Compose titles
Headings	Compose headings
Questions	Write questions
Objectives	State objectives
Summaries	Write summaries
Graphs	Draw graphs
Tables	Prepare tables
Main ideas	Construct main ideas
Between Instruction and Prior Knowledge	
Demonstrations	Student demonstrations
Metaphors	Compose metaphors
Analogues	Propose analogues
Examples	Give examples
Pictures	Draw pictures
Applications	Solve problems
Interpretations	Develop explanations
Paraphrases	Put into own words
Inferences	Draw inferences

Although generative learning theory proposes two types of relationship building (between elements of the concepts and between the concept and the student's prior knowledge), Grabowski (1997, 2006) identified five different types of cognitive levels where these relationships may be created. These levels are based on the type of mental effort required for each generative activity. She defined coding, organization, and conceptualization levels when the student is developing relationships among parts of the information; and integration and translation when the student is developing relationships between parts of the information and his/her previous knowledge (Table 2).

In a recent classification, Mayer and Wittrock (2006) classified generative learning activities in four groups: elaboration, summary note-taking, self-explaining, and questioning. Elaboration strategies explicitly ask students how the new material is related with their previous knowledge. Examples of elaboration are the construction of images, summaries, analogies, or metaphors after an instructional event. The note-taking method asks students to take notes from the instructional source such as textbooks, lecture or videotape. In self-explanation, students are asked to express in their own words what they get from the instruction. Finally, questioning methods encourage students to generate their own questions from an instructional event. Research has shown that these generative methods help students to increase comprehension (Linden & Wittrock, 1981), metacognition (Thiede & Anderson, 2003), and problem solving performance (Chi, 2000).

Table 2

Match of Activities with Level of Processing (Adapted from Grabowski, 2003)

Activities	**Levels of Cognitive Processing**
Underlining Highlighting Tracing Answering questions Creating titles and headings Creating mnemonics	Coding
Outlining Summarizing Diagramming	Organization
Paraphrasing Explaining/Clarifying Creating semantic networks Creating concept maps Identifying important information Creating images or creative interpretations	Conceptualization
Creating relevant examples Relating to prior knowledge	Integration
Creating analogies Creating metaphors Synthesizing	Integration & Translation
Evaluating Questioning Analyzing Predicting Inferring	Translation

Previous studies comparing generative learning strategies

As shown in Table 1 and Table 2, generative learning activities can be differentiated as teacher given and student generated, or as the level of cognitive processing that they can promote. The following selected research studies explored comparisons using generative learning strategies in different settings. Table 3 summarizes these research studies.

Table 3

Summary of Selected Generative Research Studies

Researchers	**Comparing**		**Processing Levels**	**Students**
	Instructor-provided	**Learner-generated**		
Ritchie and Volkl (2002)		Object manipulation and concept mapping	Coding	Sixth grade
Wittrock and Alesandrini (1990)		Summaries and analogies	Integration	College students
King (1992)		Questions and summaries	Coding and integration	College students
Ray (2005)	Answering questions and underlining	Analogies and summarizing	Coding vs. Integration	Ninth grade
Boujaoude and Tamin (1998)	Answering questions	Analogies and summarizing	Coding vs. Coding and integration	Seventh grade
Johnsey (1990)	Embedded and detached elaboration strategies	Embedded and detached elaboration strategies		Adult learners
Higginbotham-Wheat (1991)	Visual and verbal elaborations	Visual and verbal elaborations		College students

Ritchie and Volkl (2002) compared laboratory experiments involving object manipulation and students-generated concept maps with 80 sixth-grade students. Both strategies are considered coding strategies that help students to create relationships between different elements of the instruction (Grabowski, 2003). In an experimental study with pretest, posttest, and retention test, students using concept maps showed a significant difference on achievement in the retention test. As Ritchie and Volkl (2002) concluded, although some generative learning strategies have been shown to improve students' understanding, more research should be done with different generative learning strategies. They suggested that research should provide evidence of the appropriate generative strategies for specific settings such as content, students' grade level, type of strategy, etc.

In another study, Ray (2005) compared four generative strategies: learner-generated analogies, learner-generated summaries, experimenter-provided adjunct questions, and experimenter-provided underlined material. On one hand, experimenter-provided adjunct questions and underlined material help students to create relationships among the ideas from the reading material (see Table 2). On the other hand, as Wittrock and Alesandrini (1990) stated that generation of analogies and summaries help students to create relationships among the ideas in the text and between the text and student's previous knowledge promoting deeper processing. 135 ninth-grade students were randomly assigned to one of the four experimental groups or the control group. Results showed that the group who generated summaries performed significantly better than the control group and better than the group that worked with underlined material in both immediate and delayed posttests.

However, there are other studies that did not show significant differences among the strategies. For example, Wittrock and Alesandrini (1990) compared generating summaries and generating analogies on 57 college students. The authors emphasized that both generative strategies help students to create relationships among the sentences of the text and between the text and the students' previous knowledge. Although researchers found no significant difference between the use of summaries and analogies, they did find that both experimental groups outperformed the control group that just read the text.

Boujaoude and Tamin (1998) also compared students-generated summaries, students-generated analogies and answering questions strategies on seventh-grade students. Results indicated that there were no significant differences among students' scores in the groups of analogy generation, summaries generation and answering questions. Researchers also allow students to work with the three different generative strategies and run a perceptions questionnaire. Results showed that in first place students preferred answering questions because they were easy and it needed little time; in second place, they preferred generate analogies because they were interesting and help them to compare things, and in third place students preferred generate summaries because they stressed the main idea and the important concepts.

King (1992) compared student-generated questioning and student-generated summaries strategies on 56 college students. Author emphasized that students generating their own questions and answering them, and generating summaries facilitate comprehension and retention organizing the new material and integrating the new information with students' previous knowledge. Results showed no significant difference between the strategies in any of the posttests.

Generative learning strategies have also been studied in computer-based instruction. For example, Allen and Merrill (1985) defined a theoretical framework for designing computer-based instructional environments using generative learning strategies (such as paraphrasing, asking questions, showing diagrams, or providing examples) based on the learner's information processing skills. Based on Rigneys' (1978) distinction of embedded strategies (proposed by the instruction) and detached strategies (proposed by the learner), the researchers defined three types of instructional intervention in a computer-based instructional environment: embedded strategies into the computer-based instruction, system-assigned detached strategies, and student-assigned detached strategies.

For example, Johnsey (1990) studied the generative learning activities in computer-based instruction. She designed her study with five groups: control group, experimenter-generated detached and embedded groups, and learner-generated detached and embedded groups. Learner-generated subjects participated in a one-hour class on how to work with generative learning activities. Results showed that learner-generated groups outperformed the control group on the recall and total scores. However, there were no significant differences between the experimenter-provided groups and the control group. The author concluded that students learn best when instructional environments combine instructing students in generative learning activities with embedded strategies.

In a related study, Higginbotham-Wheat (1991) examined the learning efficacy of generative learning strategies in a computer-based instruction by studying the effects of learner-generated and experimenter-generated visual and verbal elaborations. Results showed that the group that interacted with learner-generated activities with verbal conditions outperformed the control group on the recall, recognition, inference, and total scores. Additionally, in the inference posttest there was a significant difference between the learner-generated visual group and the experimenter-provided visual group. The authors concluded that a greater level of processing could be reached through the use of learner-generated strategies with verbal elaborations.

Although research shows that some generative learning strategies are relevant instructional tools that improve students' comprehension and retention in different types of learning environments, there is a need to investigate which generative learning strategies match better with specific contents. The need is especially great with elementary-school children and math education. Grabowski (2003) summarized 39 research studies using generative learning strategies. From these studies 6 are related to elementary-school students and just 1 is related to mathematics education. Also, it is not clear that all generative learning strategies improve comprehension or retention. For example, Wittrock (1990) identified several generative strategies that need further research to establish their relevance in the generative process of comprehension, including summaries, generating examples and applications, titles, and asking questions.

Answering questions and generating examples strategies

Despite the fact that Wittrock identified questioning and exemplifying as needing more study, these strategies have been used successfully for decades in mathematics (Halmos, 1994). The following sections examine the use of questioning and exemplifying generative strategies in instruction and specifically in the teaching of mathematics.

Answering questions. Questions in mathematics education can be defined as “pedagogic instruments both for engaging students in and assessing students’ grasp of, ideas and techniques” (Mason, 2000, p. 97). The value of this popular strategy to teach math is to bring consciously student’s prior and relevant knowledge to the problem at hand (Hyde, 2006). Specifically with elementary students, research shows that those who are asked questions that require higher order thinking perform better than those students who do not answer such questions (Perry, Scott, VanderStoep, & Shirley, 1993). In addition, new investigations adopting reading strategies to teach mathematics to students from kindergarten to sixth grade, promote the use of asking questions because the approach has been identified as a highly effective cognitive strategy for students in reading comprehension (Hyde, 2006).

The classifications of learning activities from Table 2 shows that teacher given questions or student answering questions is an activity that fosters coding processes helping to relate concepts presented in the instruction. This type of activity is also known in the literature as adjunct questions. Providing questions to students induce the generative process by generating relationships between the ideas from the instructional materials (Grabowski, 2003).

The effects of adjunct questions have been widely studied in different areas and age levels (Anderson & Biddle, 1975; Rickards, 1979). Grabowski (2003) summarized results from the use of this instructional activity when students use written materials,

> The more frequent the questions, the better; feedback increased learning, but so did inserted questions without feedback; whereas most of the research focused on fact-level questions, there was also a positive effect for higher-level questions; free recall was generally better than multiple choice; a need for overt responding was dependent on how the questions did motivate learner in some cases. They also found that these effects held across age level, content, length of the text, and medium used. (p. 733)

Using adjunct questions with computer-based materials also has been investigated. For example, Holliday and McGuire (1992) explored the use of adjunct questions in a sequence of computer visuals in an earth science class with junior high school students. Results showed that the group with more adjunct questions outperformed the group with less adjunct questions.

Adjunct questions have also been studied with two types of knowledge: intentional and incidental. The first one is the knowledge that students learn from the adjunct questions. The second one is the knowledge that was not stated in the adjunct questions. For example, Duchastel and Nungester (1984) studied the impact of adjunct questions, at the end of the text and inserted in the text, on the retention of intentional and incidental knowledge. Results indicated that both types of adjunct questions improved the retention of intentional knowledge, but they did not improve the retention of incidental knowledge.

Other important characteristics of questioning are the purpose and the type of question. Although students perceive questions as testing, Mason (2000) described three principal purposes for asking questions to students: focusing attention, testing, and enquiry. On the other hand, Hsu and Dwyer (2004) pointed out the adjunct questions can be divided in factual questions, those that focus on rote learning and ask students to answer exactly as it was in the instruction; and comprehension or higher-order questions, those that focus more on understanding and ask students to make inferences and use their new knowledge in different settings. Evaluating understanding, Hsu and Dwyer (2004) found that students who use higher-order adjunct questions outperformed the students who use factual questions. In addition, students who use factual adjunct questions outperformed those who received no questions.

Generating examples. Examples are fundamental educational tool in the formation of mathematical concepts. One principal of the learning mathematics proposed by Skemp (1971) stated, "concepts of a higher order than those which a person already has cannot be communicated to him by definition, but only by arranging for him to encounter a suitable collection of examples" (p. 32). The relevance of using collection of examples to teach mathematical concepts is based on the advantage of constructing generalizations from these examples to give details to students about a concept, a technique or a principal that they want to learn (Watson & Mason, 2005).

Showing examples to students has been an instructional strategy widely used for teachers in mathematics classrooms to illustrate and clarify mathematical concepts. But these instructor-provided examples could restrict students' thinking. Instead, activities involving student-generated examples "could offer opportunities to experience structure of mathematical examples, to discern what is invariant and what can be varied and more importantly, and so to reveal their awareness of mathematical concept" (Rahman, 2006).

Watson and Mason (2005) stated that generating examples

> … describes and elaborates on an important and effective pedagogical strategy whose potential is rarely exploited yet which promotes active engagement in mathematics. It arises from the perspective that mathematics is a constructive activity and is most richly learned when learners are active constructing objects, relations, questions, problems, and meanings (p. ix)

They also discussed that examples inside of the instructional strategy also could involve illustrations, placeholders, worked examples, exercises, or specific contextual situations. Table 1 and Table 2 show that the students-generated examples activity is another generative activity that fosters the integration of elements between instruction and prior knowledge.

There are different types of research using generative learning strategies. For example, DiVesta and Peverly (1984) assessed learner-organized and instructed-organized examples on comprehension and retention. Results indicated that the group that organized examples did significantly better in both tests than the group with preorganized examples. In another study, Gorrell (1991) worked with 26 fifth-grade students. During three weeks students generated their own examples for some science concepts. In a retention test, students recalled better those concepts for which they generated their own examples than concepts for which they do not.

There are studies that show the advantages of learner-generated examples (Dahlberg & Housman, 1997; Hazzan & Zazkis, 1997). For example, using multiple cases from primary, secondary and college levels, Watson and Mason (2005) showed the relevance of learner-generated examples for extending and developing knowledge structures. Watson and Mason (2005) emphasized some characteristics of examples:

1. Examples from the learners depend on their knowledge, experience, and predisposition.
2. Requests to exemplify can bring students a specific example or a class of examples.
3. Based on experiences, students develop an "example space" to help them to create knowledge. This knowledge facilitates students to develop more examples with related or unrelated characteristics.
4. Generated examples could be divided between those found easily or those that need to be constructed based on knowledge, experience, and predisposition.

Watson and Mason (2005) summarized,

> Our theory is that asking learners to exemplify aspects of what they have studied encourages them to search through the structure from varying points of view, using a new dimension, and hence see, perhaps for the first time, what might be there by discerning features and aspects. Thus, learners might find that being asked to exemplify gives them an opportunity to search in unfamiliar ways through what is familiar to get a more complex sense of the range of possibilities in the topics studied (p. 31).

Generative learning theory emphasizes the design of instruction that focuses on the generation of relationships among the elements in the learning material and between these elements from the new material and students' previous knowledge. In addition, research has established that answering questions and generating examples are valuable instructional strategies to use in the learning of mathematical concepts. The following section presents a summary of this literature review and conclusions based on the discussions of rational numbers, manipulatives, and generative learning theory.

Summary and Conclusions

Rational numbers represent an important concept in the mathematics field. Unfortunately, research has shown that this is a difficult concept for students to learn for different reasons. First, students have few opportunities to face this type of numbers in their daily life. Second, students do not work long enough to familiarize themselves with the concept of rational numbers. Third, previous knowledge on natural numbers can interfere with the concept of rational numbers. And fourth, there are different meanings associated with rational numbers.

The literature review shows different alternatives to help students understand this concept. First, work with the different meanings of the rational numbers. Specifically, mathematics education researchers have shown that the part-whole meaning of rational numbers is the most basic and important one to start learning this concept. Second, work with different representations of the same rational number, allowing children to explore and create connections among these different representations. The Rational Number Project, an extensive research program on rational numbers, shows that this translation model improves the students' achievement and retention on the rational number concept. And third, use manipulatives to help students to represent abstract ideas in tangible materials.

Manipulatives have been used in the teaching and learning of mathematical concepts for several decades. In addition, based on the work of different authors like Jean Piaget, manipulatives have been confirmed as an important instructional tool. A number of studies have shown that the use of manipulatives help students in the learning process by promoting a more active participation, and at the same time, allow them to represent abstract concepts in tangible forms. Researchers have confirmed that the advantages of concrete manipulatives also can be reached using computer-based manipulatives. Studies using these new versions of manipulatives, usually called virtual manipulatives, have also shown the effectiveness in the students' learning process of mathematical concepts, representing abstract ideas through concrete models. However, virtual manipulatives are not magical tools and they need to be accompanied with instructional activities that allow students to develop connections between the representations of the manipulative and their previous knowledge. Learning activities involving virtual manipulatives and based on generative learning theory should allow students to develop these connections between their existing ideas and new information.

Generative learning theory emphasizes the construction of relations among the parts of the new material and between the new material and students' previous knowledge to enhance comprehension. In addition, there are different generative learning activities to help students to construct these relations. These activities are divided according to the level of the cognitive processing that students need to do such as coding, organization, conceptualization, integration, and translation. Among these generative learning strategies, there are two popular activities in the mathematics classroom: questioning and exemplifying.

The review of relevant literature found no studies that involved both questioning and exemplifying generative strategies to help young students comprehend and recall the part-whole meaning of rational numbers while using either tangible or virtual manipulatives. Additional research is needed to determine if questioning and exemplifying generative strategies help elementary-level students comprehend the part-whole meaning of rational numbers. The hypothesis underlying this research is that students who use these generative learning activities will improve comprehension and retention of part-whole meaning of rational numbers. The answering questions strategy within this research would determine if student's comprehension were improved by promoting connections among ideas within the instructional activity. The generating examples strategy within this research would determine if students' comprehension were improved by promoting connections among ideas within the instructional activity and between this new concept of rational numbers and previous knowledge and experiences.

Research is also needed to compare possible differences in the effects of questioning and exemplifying generative strategies to help elementary-level students comprehend the part-whole meaning of rational numbers. The hypothesis underlying this research is based on the likelihood that different cognitive processing requirements will lead to different effects and predicts that students using the deeper cognitive level of the generating-examples strategy will achieve higher scores on immediate and delayed tests measuring comprehension than students using answering-questions strategy.

Chapter 3: Method

The purpose of this study was to examine the impact of using two generative learning activities, answering questions and generating examples, on third-grade students' comprehension using virtual manipulatives. This chapter describes the research questions, the participants, the design of the study, the procedure of the study, and the analysis of the data.

Research Questions

Based on the literature review, this study sought to answer the following research questions:

1. Are there significant differences between the scores that students obtain in the pretest and the scores that students obtained in the immediate and delayed posttests?
2. Are there significant differences on *an immediate comprehension test* between the students who use virtual manipulatives with an answering questions generative learning activity and the students who use virtual manipulatives with a generating examples generative learning activity during the learning of part-whole representations of rational numbers?
3. Are there significant differences in *a delayed comprehension test* between the students who use virtual manipulatives with an answering questions generative learning activity and the students who use virtual manipulatives with a generating examples generative learning activity during the learning of part-whole representations of rational numbers?

Participants

Sixty native Spanish-speaking students from four third-grade classrooms in San Juan, Puerto Rico, participated in the research study. The study occurred during the second half of the third-graders' academic year. Five dollars (USD) was provided to each student as an incentive to participate in the study.

Research Design

Type of design

This study is a two-way mixed ANOVA (see Table 4). The study used two different generative learning strategies as the treatments. The answering-questions group used a treatment based on answering questions while using virtual manipulatives, and the generating-examples group used a treatment based on generating examples while using virtual manipulatives. The participants from four third-grade classrooms were randomly assigned to either one of the answering-questions or generating-examples groups to provide internal validity of the study (Howell, 2002).

Variables

The independent variable was the generative learning strategy that students employed while using the virtual manipulatives, either answering questions or generating examples. The dependent variable was the students' comprehension. This variable was measured by a pretest, a posttest one day after completing the virtual manipulative activities, and a posttest two weeks after completing the virtual manipulative activities.

Table 4

Design of the Experiment

	Pretest	**Immediate Posttest**	**Delayed Posttest**
Answering-question strategy			
Generating-examples strategy			

Treatment

The experiment consisted of two different generative learning treatments using a common set of virtual manipulatives. These manipulatives were developed by the National Library of Virtual Manipulatives (NVLM) to help students understand the concept of part-whole representation of rational numbers. The answering-questions group used activities that required answering questions using two virtual manipulatives. On the other hand, the generating-examples group used activities that required creating examples using the same two virtual manipulatives. Although each activity was designed to be finished in approximately one hour, participants had additional time to finish all the activities. This ensured that all students went through the planned activities.

Groups worked with two virtual manipulatives from the National Library of Virtual Manipulatives (NLVM) web site. Students from the answering-questions group answered twelve multiple-choice questions divided in two different sections. The first section contained six multiple-choice questions that were answered using the “Fraction Pieces” virtual manipulative (see Figure 12 and Figure 14). The second section also contained six multiple-choice questions that were answered using the “Parts of a Whole” virtual manipulative (see Figure 13 and Figure 15).

Students from the generating-examples group had equivalent activities to the answering-questions group. The only difference was that students from the generating-examples group interacted with the “Fraction Pieces” virtual manipulative by generating six examples, and another six examples when they interacted with the “Part of a Whole” virtual manipulative.

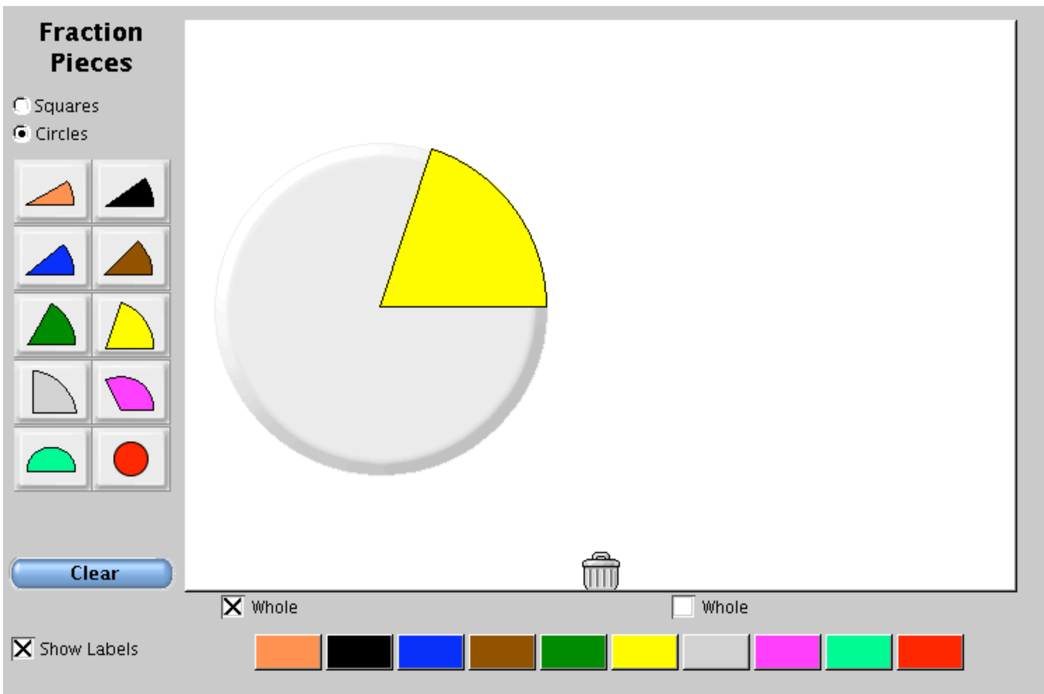

Figure 12. Virtual manipulative in English where students fill a circle using fraction pieces (Reprinted from National Library of Virtual Manipulatives http://nlvm.usu.edu)

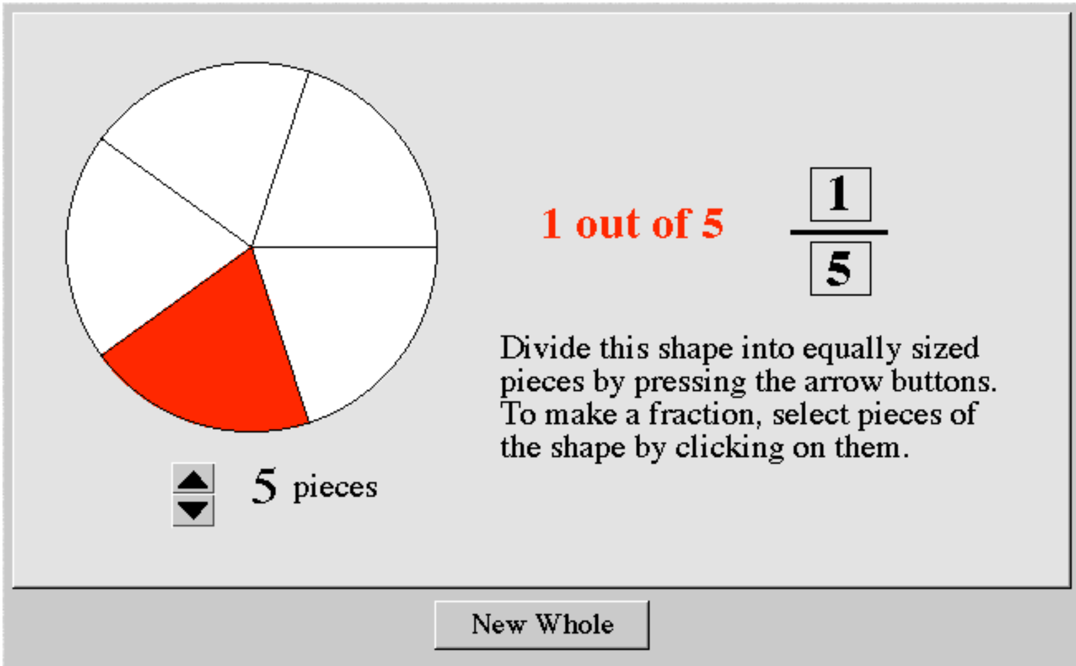

Figure 13. Virtual Manipulative in English where students observe a fraction in three different representations (Reprinted from National Library of Virtual Manipulatives http://nlvm.usu.edu)

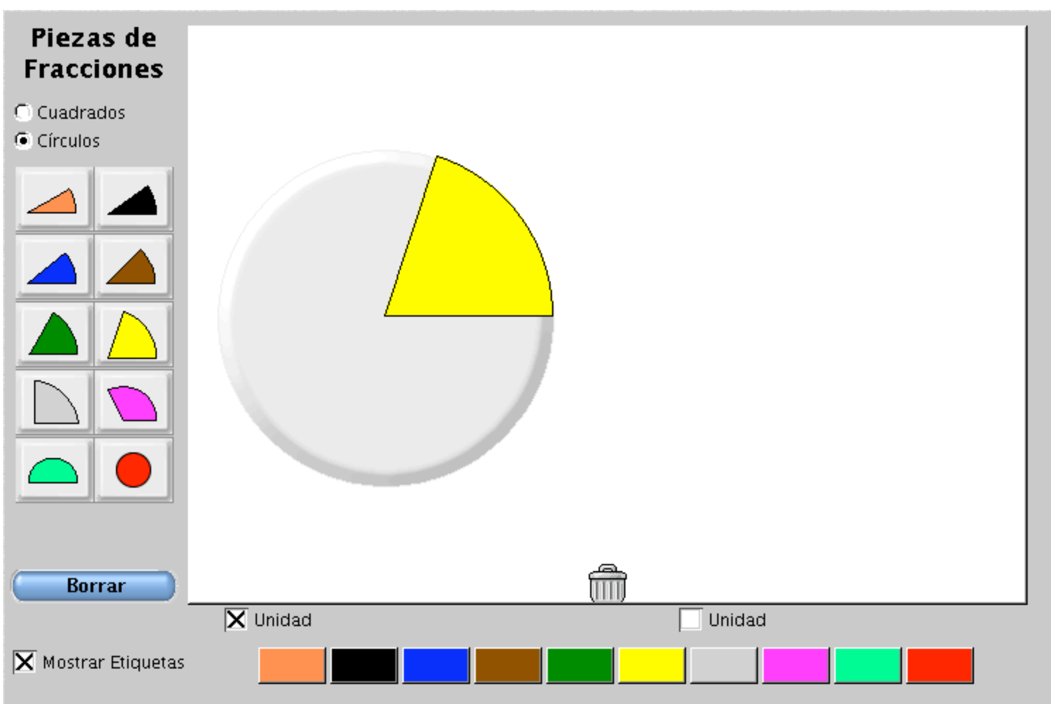

Figure 14. Virtual manipulative in Spanish where students fill a circle using fraction pieces (Reprinted from National Library of Virtual Manipulatives http://nlvm.usu.edu)

Figure 15. Virtual Manipulative in Spanish where students observe a fraction in three different representations (Reprinted from National Library of Virtual Manipulatives http://nlvm.usu.edu)

Materials

Virtual manipulatives

The study used two virtual manipulatives from the NLVM website. Based on the classification of cognitive technologies for mathematics education developed by Pea (1987), the first manipulative, called "Fraction Pieces" (see Figure 12 and Figure 14), is a "tool for developing conceptual fluency" (p. 106). This virtual manipulative is designed to help students both to play with the part and whole concepts using pieces from different colors and sizes, and to become fluent in the use of these types of virtual manipulatives. The second manipulative, "Parts of a Whole" (see Figure 13 and Figure 15), is a "tool for integrating different mathematics representations" (p. 109). This virtual manipulative is designed to help students to relate three different representations, such as graphic, symbolic, and verbal, of the same fraction number.

Written activities

Readability of instructions. Because of the age of the students, the written instructions were short and with an appropriate vocabulary to help the students better understand the activities. The readability level of the English version of the instruction was analyzed using the readability statistics from Microsoft Word®. The Flesch-Kincaid grade level for the answering-questions activity was 3.5, with a Flesch Reading Ease of 84.6. For the generating-examples activity, a Flesch-Kincaid grade level of 3.7 was obtained, with a Flesch Reading Ease of 81.9.

Flesch Reading Ease has a 100-point scale from 0 to 100, where a higher score indicates a lower readability level (Klare, Rowe, John, & Stolurow, 1969; Klare, 1974). Flesch-Kincaid grade level converts the Flesch Reading Ease score to a grade-school level in the United States (Wilson, Rosenberg, & Hyatt, 1997). A score of 4.2 for instance, indicated that an average student from fourth grade could read the material. The Flesch-Kincaid scores of 3.5 and 3.7 obtained in the activities indicated that an average student in the second half of the third grade could read these activities. In addition, a professor specializing in elementary reading and three professors and two doctoral students from the mathematics department at Virginia Tech reviewed the activities and judged them as appropriate instruments.

The researcher, a native Spanish speaker, translated the English version of the instructions into Spanish. A Spanish professor from the Foreign Language Department at Virginia Tech reviewed the translated instruments for accuracy. The Fernandez-Huerta (1959) readability index for Spanish texts was used to verify the readability level of the instructions. To use Fernandez-Huerta formula, software from the Computer Science department from the University of Texas at Austin was used. The Fernandez-Huerta readability index for the generating-examples activity was 91.06, and for the answering-questions activity was 90.34. According to Fernandez-Huerta scale, these readability values are classified as "Very Easy".

Activity sections. Each activity was divided in two sections. The goal of the first section was for students to answer multiple-choice questions or generate examples to help them to develop the concept of parts and whole. This section also was intended to help students to familiarize themselves with virtual manipulatives. The goal of the second section was for students to answer multiple-choice questions or generate examples to help them to understand different representations of the same rational number. Students were expected to manipulate the graphic representation of a rational number and to observe how they also can be represented verbally (expression) and symbolically (fraction). The English versions of answering-questions and generating-examples activities are included in Appendix C and Appendix D, respectively.

Scripts

To guarantee that students received the same explanations for the assessments and the activities, scripts describing the assessments (see Appendix A) and the work with the activities using the virtual manipulatives (see Appendix B) were developed. The researcher read these scripts to the students before each assessment and activity with the virtual manipulatives.

Assessments

The items on the assessments used in this study to evaluate part-whole representation knowledge had two main sources. The first source was from the Standards of Learning (SOL) from the state of Virginia. These standards of learning for Virginia public schools describe the expectations that students are expected to achieve in grades K-12 in several areas such as

English, mathematics, or science (Virginia Department of Education, 2008). The second source was from the curriculum of the Rational Number Project (RNP) (Cramer, Post, & delMas, 2002). Cramer et al. developed a test to evaluate the rational number concepts, including part-whole representation, between the curricula from RNP and a commercial curricula. Although they did not publish the test, they point out that the items were taken from previously published RNP teaching experiments.

Three comparable versions (Colman, 2001; Feuer, Holland, Green, Bertenthal, & Hemphill, 1999) of the assessment were used during the evaluation sessions to avoid decreasing the internal validity of the study because of the testing effect (Slavin, 2007). To create these three comparable versions, three similar items were developed for each question and randomly assigned to each assessment. These three comparable assessments were given randomly to students during the pretest, immediate posttest, and delayed posttest. Although there was a chance that students would be assigned the same exam version more than once, randomization provided that students from different treatments had the same chance to receive any of the three comparable assessments (Wiersma, 2000).

Each test contained three different sections. The first section contained five multiple-choice questions with different representations of a rational number (i.e., graphic, symbolic, and verbal) related to each other. The second section contained five multiple-choice questions with different representations of a rational number related to each other using real life events. The last section contained five open questions where students needed to write a real life example related to a given representation of a rational number (see Appendices E, F, and G).

The same procedures were used to analyze the readability of the tests as were used to analyze the activity instructions. Because of the age of the students, the assessments had a simple vocabulary to help the students to better understand the questions. The readability level for the English versions of the tests was analyzed using the readability statistics from Microsoft Word®. The Flesch-Kincaid grade level for the tests was 3.3. The Flesh Reading Ease indexes for the tests were 85.6, 86.1, and 85.7 for versions 1, 2 and 3, respectively. As written activities, a specialist in elementary reading and three professors and two doctoral students from the mathematics department at Virginia Tech reviewed the tests and judged them as appropriate instruments providing content validity to tests (Thorkildsen, 2005).

The researcher translated the English version of these assessments into Spanish. A Spanish professor from the Foreign Language Department at Virginia Tech reviewed these translations for accuracy. In addition, the Fernandez-Huerta (1959) readability indexes for the three tests versions were 92.33, 92.20, and 92.20. The readability of all three of these versions was evaluated as “Very Easy” on the Fernandez-Huerta scale.

The assessment instruments designed for this study were based on the goals stated by the Principles and Standards for School Mathematics (National Council of Teachers of Mathematics, 2000). Specifically, the Standards for Number and Operation for grades 3-5 state,

> Recognize equivalent representations for the same number and generate them by decomposing and composing numbers; develop understanding of fractions as parts of unit wholes, as parts of a collection, as locations on number lines, and as divisions of whole numbers (p. 148).

Puerto Rico, as an associate state of the United States, follows the same educational standards as the other 50 states. In addition, schools from Puerto Rico follow two more standards. One is the Mathematics Curricular Framework (INDEC, 2003) that proposes guidelines for the processes and content of mathematics based on levels. Level I, from kindergarten to third grade for instance, describes a major emphasis on numbers and operations to give a basis in following concepts. Another standard is the General Expectation by Grade for Learning Mathematics (Departamento de Educación de Puerto Rico, 2007). The Number and Operations section for third grade states that students need to understand, interpret, and represent fractions as parts of unit wholes, as part of a collection, and as points on number line.

Scoring

The first and second parts of the assessments used in this study, which involved multiple-choice questions, were graded with 2 points for a correct answer and 0 points for an incorrect answer. The third part of the assessments, which involved open questions, used scoring rubrics or scoring guides (Haladyna, 1999) (see Tables 6, 7 and 8).

Table 5

Rubric to Assess Answers to Question 11 and 12

2 Points	1 Point	0 Points
The story made an appropriate and clear reference to the number of pieces and the shaded number of pieces from the graphic and there was an explicit and correct symbolic representation of the fraction on the story.	Either the story made an appropriate and clear reference to the number of pieces and the shaded number of pieces from the graphic, but there was not an explicit and correct symbolic representation of the fraction on the story, or student wrote a correct symbolic representation but did not include a clear reference to the number of pieces and the shaded pieces.	Nothing was written, or the story and the symbolic representation were not related to the picture.

Table 6

Rubric to Assess Answers to Question 13

2 Points	1 Point	0 Points
Student expressed that he/she could not write an example related with the part-whole concept and he/she explained why not.	Student expressed that he/she could not write an example related with the part-whole concept but he/she did not express why.	Nothing was written, or the student wrote a story based on the graphic.

Table 7

Rubric to Assess Answers to Question 14 and 15

2 Points	1 Point	0 Points
The story made an appropriate and clear reference to the number of pieces and the whole from the rational number and there was a correct graphic representation of the rational number.	Either the story made an appropriate and clear reference to the number of pieces and the whole from the rational number but there was not a correct graphic representation of the rational number, or student drew a correct graphic representation of the rational number but the story did not include an appropriate and clear reference to the number of pieces and the whole.	Nothing was written, or the story and the symbolic representation were not related to the picture.

Pilot Study

A pilot study was performed to test procedures, activities, and assessments. Fourteen third-grade students (7 boys and 7 girls) participated in the pilot study during the last week of January 2007. The students were attending one school in Montgomery County in Virginia, USA. The principal and the teacher agreed to collaborate in the research study. Approval from the Institutional Review Board (IRB) at Virginia Tech (Appendix H), from the Montgomery County Public School (Appendix I), and from the students' parents was obtained (Appendix J). Students were told that by completing the activity using the computer and three assessments, they would be eligible to win five dollars cash.

Students who participated in the pilot study were asked to complete three evaluations using a pilot assessment (Appendix K) that was the initial version of the ones used in the study, and one pilot activity from either answering-questions or generating-examples activities (see Appendices L and M) that were also the initial versions of the ones used in the study.

The pilot assessment was composed of twelve questions divided in two sections. The first section contained six multiple-choice questions, and the second section contained six fill-in-the-blank questions. The generating-examples pilot activity consisted of 14 open questions. The answering-questions pilot activity consisted of 14 multiple-choice questions. Students completed these activities using two virtual manipulatives from NLVM.

The first day of the pilot study, a pretest was administered to the students. The pretest attempt to determine the prior knowledge of the students on part-whole meaning of rational numbers. Three days later, students were divided randomly into two groups and the computer-based pilot activities were administered to the students. One group was assigned to the answering-questions activity and the second group was assigned to the generating-examples activity using virtual manipulatives. The next day, the pilot assessment was administered as an immediate posttest to students to evaluate the comprehension of the students. Finally, two weeks later, the pilot assessment was administered again as a delayed posttest.

Because of the absences of some students, scores from only 11 students were included in the data analysis, five students belonging to the generating-examples group and six students belonging to the answering-questions group. Scores from the tests (see Table 8) were changed to percentages and analyzed using SPSS version 11 for Windows XP. A Cronbach's alpha reliability test was run on the immediate posttest scores. An alpha reliability of 0.72 was obtained from the posttest. No further analysis was run because of the low amount of data.

Table 8

Results from the Pilot Study

	Pretest	Immediate Posttest	Delayed Posttest
Generating-examples group	67.22	75.02	74.02
Answering-questions group	67.6	73.13	72.23

Observations made from this pilot study led to several conclusions and suggested some changes for the research study. First, during the second and third evaluation sessions, several students identified and claimed that these assessments were the same as the one used in the first evaluation session. To avoid this situation, the gap from the first evaluation session to the second evaluation session was increased from four to six days and three different but comparable tests were designed and randomly distributed during the assessment activities. Second, some students using the generating-examples pilot activity exceeded the time that was provided to complete this activity. The number of the questions in both activities was reduced from 14 questions to 12 questions. Third, in a closer analysis of the activities, real life situations were used on both of them. This design did not establish a clear difference between the activities. To create a more clear difference between the designs of the activities, real life situations were eliminated from the answering-questions activity. Finally, because of the freedom to create examples from real-life situations in the generating-examples activity, it was difficult to observe equivalence between the items in the activities from the two groups. Therefore, the items of the generating-examples activities were changed and the same representation (verbal or graphical) used to ask questions in the answering-questions activity, was also used to ask student to create real-life examples based on the same representations. For example, the question seven from the answering-questions activity showed a graphical representation of a fraction and the student needed to find its verbal representation. The question seven from the generating-examples activity showed the same graphical representation of a fraction and the student needed to generate an example based on that graphical representation. This change confirmed that the only difference between the activities is the type of treatment.

Procedure

The revised procedures resulting from the pilot study were followed for the main research study. After reviewing the revised materials, an amendment approval from the Institutional Review Board (IRB) at Virginia Tech was obtained (Appendix N). The participants of the study were third grade Spanish-speaking students from a public elementary school in San Juan, Puerto Rico. The school was identified during the academic year 2006-2007 as one needing improvement. To this group of schools that need improvement belong those where students have not shown an adequate yearly progress on annual tests. Approvals from Puerto Rico Department of Education (Appendix O), and from the public elementary school were also obtained.

Spanish versions of the parents' approval were sent previously to students' parents (Appendix P). In March, the names of the 79 students who agreed to participate in the study were put in a list and assigned randomly to either the answering-questions or generating-examples group. The number of students in the groups was approximately the same. Of the 79 students, 60 students who completed the treatment and the three tests assigned for the study were included in the analysis. Initially, students were informed that they were free to leave the study anytime without penalty, and their names and the results of their tests were kept confidential.

In the first day of the study, Spanish versions of the three assessments (see Appendices Q, R, and S) were administered to both groups. Five days later, students from each of the four courses were divided randomly into two groups to use one of the Spanish versions of the treatments (see Appendices T and U). Activities using virtual manipulatives were administered to one course at a time in the computer laboratory of the elementary school. The next day after the activities, an immediate posttest was administered to evaluate the comprehension of the students. Finally, two weeks later, a delayed comprehension posttest was administered. A schedule of the research study is shown in Table 9. Spanish versions of the script were read before of the tests (Appendix V) and before the computer-based activities (Appendix W).

Table 9

Schedule of the Research Study

Activity	**Month**
IRB and Montgomery County Public Schools approvals	December
School, parents, and students approvals	January
Pilot test	End of January
Research Study	
- Presentation and pretest activity	March 7
- Treatments for control and experimental groups	March 12
- Immediate posttest activity	March 13
- Delayed posttest activity	March 26

Data Analysis

A 2x3 mixed analysis of variance (ANOVA) was used to answer research questions one and three. A 2x2 mixed ANOVA was used to answer research questions two. The design contains two independent variables. One independent variable (strategy) has independent measures (between-subjects). On the other independent variable (time of testing) has repeated measures (within-subject). Bonferroni post-hoc was used to determine any significant differences between the pretest and the two posttests (Cardinal & Aitken, 2006; Roberts & Russo, 1999). The instructional strategy is the between variable and time is the within variable. This repeated-measures design in the mixed ANOVA allowed comparing the groups using the two different generative learning strategies, and the scores from the pretest, immediate and delayed posttest for each group.

Chapter 4: Results

The purpose of this study was to investigate the effects of two generative learning treatments on the comprehension of the part-whole concept of rational numbers. The first treatment consisted of an answering-questions generative strategy combined with the use of two virtual manipulatives. The second treatment consisted of a generating-examples generative strategy combined with the use of the same two virtual manipulatives. Specifically, the study was conducted to answer the following three research questions:

1. Are there significant differences between the scores that students obtain in the pretest and the scores that students obtained in the immediate and delayed posttests?
2. Are there significant differences on *an immediate comprehension test* between the students who use virtual manipulatives with an answering questions strategy and the students who use virtual manipulatives with a generating examples strategy during the learning of part-whole representation of rational numbers?
3. Are there significant differences on *a delayed comprehension test* between the students who use virtual manipulatives with an answering questions strategy and the students who use virtual manipulatives with a generating examples strategy during the learning of part-whole representation of rational numbers?

Analysis of Instruments

Statistical evaluation of the assessment instruments

Three separate, but comparable, instruments were used to collect data for this study. Before the data could be analyzed to answer the research questions, it was important to examine the equivalence of the instruments by analyzing the data produced by the three instruments. Two separate statistical tests were performed to determine the equivalence of the instruments. The first test, a one-way analysis of variance (ANOVA), showed that the mean scores on the three separate instruments used in the pretest were not significantly different, F (1, 58) = 0.486, p = 0.618 (Appendix X). The second test, a Levene's test, showed that they also had the same variance (Appendix X).

In addition to these assumptions, it was important to examine the reliability of the various versions of the assessment instruments. Table 10 describes the reliability of the three comparable assessments. Coefficients above 0.7 are considered good indexes for research purposes (Ary, Jacobs, & Razavieh, 2002).

Table 10

Reliability Coefficients of Three Equivalent Tests

	Test Version 1	**Test Version 2**	**Test Version 3**
Cronbach's alpha	0.78	0.81	0.80

Data Analysis

Descriptive statistics and assumptions

Table 11 shows the descriptive statistics of the study. The Mean column represents the mean scores that each group obtained on the tests in the assessment sessions. Test scores were based on the scoring rubrics found in the Methods chapter. Based on these rubrics, the maximum value for any test was 30 points.

Table 11

Table of Means and Standard Deviations

Type of Test	**Experimental Groups**	**N**	**Mean**	**Std. Deviation**
Pretest	Answering-Questions	31	9.77	5.33
	Generating-Examples	29	9.79	4.97
	Total	60	9.78	5.12
Immediate Posttest	Answering-Questions	31	17.29	6.68
	Generating-Examples	29	14.72	6.35
	Total	60	16.05	6.59
Delayed Posttest	Answering-Questions	31	14.74	6.58
	Generating-Examples	29	14.83	7.35
	Total	60	14.78	6.90

The study design employed a 2x3 mixed analysis of variance (ANOVA) to analyze the effects of the generative learning strategies (answering-questions and generating-examples) on the delayed comprehension (pretest, immediate posttest, and delayed posttest). For this design, one independent variable is a between-subjects factor (generative strategy) and the other independent variable is a repeated or within-subjects factor (time of testing). In addition, this design analyzes the differences between the pretest and the immediate posttest, and between the pretest and the delayed posttest.

A separate 2x2 mixed ANOVA was conducted to examine the effects of the generative learning strategies on the immediate comprehension (pretest and immediate posttest). With this type of mixed analysis of variance, specific assumptions are made with respect to the responses generated by the assessment instruments. These assumptions include homogeneity of variance and sphericity.

Homogeneity of variance assumption. For each test, ANOVA assumes that each of the experimental groups have similar variance. To verify this assumption a Levene's test was used (Cardinal & Aitken, 2006). Levene's analysis showed no significant results (see Table 12), demonstrating that compared groups have similar variance, satisfying this major ANOVA assumption.

Table 12

Levene's Test of Equality of Error Variances

Type of Test	**F**	**df1**	**df2**	**Sig.**
Pretest	1.39	1	58	.24
Immediate Posttest	.39	1	58	.53
Delayed Posttest	.06	1	58	.81

Sphericity assumption. Another specific assumption for within-subjects design that needed to be evaluated was sphericity. As Levene's test evaluates the similar variance for between-subject design, Mauchly's test of sphericity assesses the equality of variance across the levels of the repeated measure (Meyers, Gamst, & Guarino, 2006). If this assumption is violated, results showing significant difference might not be true. Therefore a correction factor should be used if the assumption is violated. Mauchly's test of sphericity showed that the sphericity assumption was violated for the 2x3 mixed ANOVA (see Table 13). Because of this violation, the Huynh-Feldt correction factor was used (Cardinal & Aitken, 2006). It is important to notice that a more conservative correction factor such as the Greenhouse-Geisser also shows a similar pattern (see Table 14). For the 2x2 mixed ANOVA, the sphericity assumption cannot be violated for a within-subjects factor that has only two levels (pretest, immediate posttest) (Cardinal & Aitken, 2006). For the following analyses, an alpha level of 0.05 was used to determine statistical significance in all assessments.

Table 13

Mauchly's Test of Sphericity

Within Subjects Effect	**Mauchly's W**	**df**	**Sig.**	**Epsilon**
				Huynh-Feldt
Test	0.72^{*}	2	.01	.81

Note. $^{*}p < .05$

Comparing pretest and posttests

Research Question 1 asked if there were significant differences between the scores that students obtained in the pretest and the scores that students obtained in the immediate and delayed posttests. This question was answered using a 2x3 mixed ANOVA.

The test of within-subjects effects (Table 14) showed that a significant difference existed for assessments, $F\ (1.6, 93.9) = 49.328$, $\hat{\varepsilon} = 0.8$, $p < 0.001$, partial $\eta^2 = .46$. Because there was a significant difference, a post hoc test was needed to identify where the differences occurred. The Bonferroni *post hoc* test showed that there were significant differences between the pretest and the immediate posttest and between the pretest and the delayed posttest (see Table 15).

Additionally, Table 15 also shows that there was a significant difference between the immediate posttest and delayed posttest.

Table 14

Tests of Within-Subjects Effects for the 2x3 Mixed ANOVA

Source		**Df**	**F**	**Sig.**
Tests	Sphericity Assumed	2	49.33^{*}	.01
	Huynh-Feldt	1.62	49.33^{*}	.01
	Greenhouse-Geisser	1.56	49.33^{*}	.01
Tests*Strategy	Sphericity Assumed	2	2.96^{*}	.06
	Huynh-Feldt	1.62	2.96^{*}	.07

Note. $^{*}p < .05$

Table 15

Bonferroni post hoc Test

Test	**Tests**	**Mean Difference**	**Std. Error**	**Sig.**
Pretest	Immediate Posttest	-6.07^{*}	.69	.01
	Delayed Posttest	-4.76^{*}	.75	.01
Immediate Posttest	Pretest	6.07^{*}	.69	.01
	Delayed Posttest	1.31^{*}	.44	.01
Delayed Posttest	Pretest	4.76^{*}	.75	.01
	Immediate Posttest	-1.31^{*}	.44	.01

Note. $p < .05$

Comparing answering-questions and generating-examples groups on comprehension

Research Question 2 asked if there were significant differences on *an immediate comprehension test* between the students who use virtual manipulatives and answer questions and the students who use virtual manipulatives and generate examples during the learning of part-whole representation of rational numbers?

Research Questions 2 was answered using a 2x2 mixed ANOVA. The test of within-subjects effects (Table 16) indicated that there is a significant differences for the interactions between strategies and time of testing (pretest, immediate posttest), F (1, 58) = 4.310, p = 0.042. Therefore, based on the descriptive statistics (Table 11), the answering-questions group outperformed the generating-examples group.

Table 16

Tests of Within-Subjects Effects for the 2x2 Mixed ANOVA

Source	**df**	**F**	**Sig.**
Tests*Strategy	1	4.31^{*}	.04

Note. $^{*}p < .05$

Research Question 3 asked if there were significant differences on *a delayed comprehension test* between the students who use virtual manipulatives answering questions and the students who use virtual manipulatives generating examples during the learning of part-whole representation of rational numbers?

Research Questions 3 was answered using a 2x3 mixed ANOVA. Using the Huynh-Feldt correction factor, the test of within-subjects effects (Table 14) indicated that no significant differences existed for the interactions between strategies and any of the three tests, *F* (1.619, 93.909) = 2.963, *p* = 0.067. That is, when analyzing all possible combinations of tests and strategies in this experiment, no significant differences on comprehension scores were detected between the interactions of the pre, immediate, or delayed tests with either the answering-questions group or the generating-examples group. Therefore, when testing for delayed comprehension, there was no significant difference between students using answering questions strategy and students using the generating examples strategy.

Summary

A 2x3 mixed ANOVA and a 2x2 mixed ANOVA were used to investigate the effects of two generative learning strategies on the comprehension of third-grade students about the part-whole concept of rational numbers. Two treatments were administered, one involving answering questions combined with the use of two virtual manipulatives and the other involving generating examples combined with the use of the same virtual manipulatives, A Bonferroni *post hoc* test was also conducted, when appropriate, to investigate the significant differences among the repeated measures. The following results were inferred from the data:

1. The analysis of within-subjects effects test did reveal a significant difference effect between the pretest and the immediate posttest, and between the pretest and the delayed posttests. Results also showed a significant difference between the immediate posttest and the delayed posttest.

2. The 2x2 mixed ANOVA reveal significant difference on the interaction between the generative learning strategies and the time of the tests (pretest, immediate posttest). Based on the descriptive statistics (Table 11), the group that used an answering-questions generative strategy

performed significantly better on the immediate posttest for comprehension than the group that used a generating-examples generative strategy.

3. The 2x3 mixed ANOVA did not reveal a significant difference on the interaction between the generative learning strategies and the delayed posttest.

Chapter 5: Discussion

The purpose of this study was to investigate the effects of two generative learning strategies on the comprehension of the part-whole concept of rational numbers while using virtual manipulatives. Third-grade Spanish-speaking students participated in this study that encompassed four interactions. Initially, a pretest was administered to students to establish the previous knowledge on part-whole concept. Five days later, students were randomly divided in two groups where each group received a treatment consisting of one generative strategy (either answering-questions or generating-examples) while interacting with two identical virtual manipulatives. The next day after the activity, students were assessed to establish how much they comprehended from the activity sessions. Finally, two weeks later, students' comprehension was assessed again. Differences in comprehension between the experimental groups were investigated, as well as the instructional utility of using these two generative learning strategies to learn part-whole meaning of rational numbers while using virtual manipulatives.

Effectiveness of the generative learning activities

The first research question of whether there are significant differences between the scores that students obtain in the pretest and the scores that students obtained in the immediate and delayed posttests was answered by performing a Bonferroni *post-hoc* analysis from a 2x3 mixed analysis of variance (ANOVA). The analysis showed significant differences between the pretest and immediate posttest, and between the pretest and delayed posttest.

From these results it can be concluded that both treatments used in this study are effective in helping third-grade students to comprehend the part-whole concept of rational numbers. This finding confirmed that the generation of relationships provided by generative learning activities fosters new learning (Wittrock, 1990, 1991, 1992; Grabowski, 2003). Moreover, this result also showed that virtual manipulatives are valid computer-based instructional tools to help students learn mathematical concepts, confirming the results of previous studies (Moyer, Niezgoda, & Stanley, 2005; Reimer & Moyer, 2005; Suh, Moyer, & Heo, 2005).

Effects on comprehension

The second research question of whether there are significant differences on an *immediate comprehension posttest* between the students who use virtual manipulatives answering questions and the students who use virtual manipulatives generating examples during the learning of part-whole representation of rational numbers was answered by performing a 2x2 mixed ANOVA. The analysis showed a significant difference between the two experimental groups. From these results it can be concluded that in terms of the immediate comprehension, the group that used answering-questions strategy did significantly better than the group who used generating-examples strategy.

Particularly for this study, it appears that students who focus on the content, deal with the instructions to answer questions, and then answer questions, initially comprehend better on an immediate test than students who focus on the content, deal with the instruction and examples to generate examples, then relate the new content with their previous knowledge, and finally generate their own examples. In other words, the work of students to generate relationships using virtual manipulatives while answering instructor-given questions is a better strategy for an immediate comprehension than having students generate relationships while using virtual manipulatives and creating their own examples. These results are consistent with the findings made in other studies (Peper & Mayer, 1978, 1986; Stull & Mayer, 2007). For example, Peper and Mayer (1986) found that note takers performed better than non-note takers on far-transfer tasks but worse on near-transfer tasks. Note taking is considered a generative activity where students need to paraphrase, organize and make sense of the material. These findings are also consistent with Stull and Mayer's (2007) conclusion that excessive cognitive activity can disrupt the generative processing.

The third research question of whether there are significant differences on *delayed comprehension posttests* between the students who use virtual manipulatives answering questions and the students who use virtual manipulatives generating examples during the learning of part-whole representation of rational numbers was answered by performing a 2x3 mixed ANOVA. The analysis showed no significant difference between the two experimental groups. From these results it can be concluded that in terms of delayed comprehension both of the two generative learning strategies provide equivalent levels of longer-term comprehension.

Although the 2x3 mixed ANOVA did not reveal significant differences on a delayed comprehension test between the two groups using the generative learning activities, the profile analysis from the repeated measures showed that the students in the generating-examples group tended to remember relatively more of what they have comprehended than the students in the answering-questions group (see Figure 16). A difference scores analysis (Howell, 2002) between the immediate posttest scores and the delayed posttest scores, and a two-sample *t* test revealed a significant difference between the answering-questions and the generating-examples groups (Table 17). From this result it can be concluded that the students who used the generating-examples strategy tended to remember relatively more of the information that they had previously comprehended than students who used the answering-questions strategy. As discussed in the literature review, Wittrock (1990) indicated that activities like answering questions focus on the generation of organizational relationships between the elements of the new information; and activities like generating examples focus on the generation of integrated relationships between new information and previous knowledge. Jonassen, Mayes, and McAleese (1993), and Grabowski (1997, 2006) added that these generations of relationships have different levels of processing (Craik & Lockhart, 1972; Cermak & Craik, 1979), where integrated relationships require deeper level of processing than organizational relationships. Results from the difference score analysis and from the within-subjects design, support the idea that concepts that have been comprehended using integrated relationships can be recalled longer because more meaning is related to it (Jonassen, 1998).

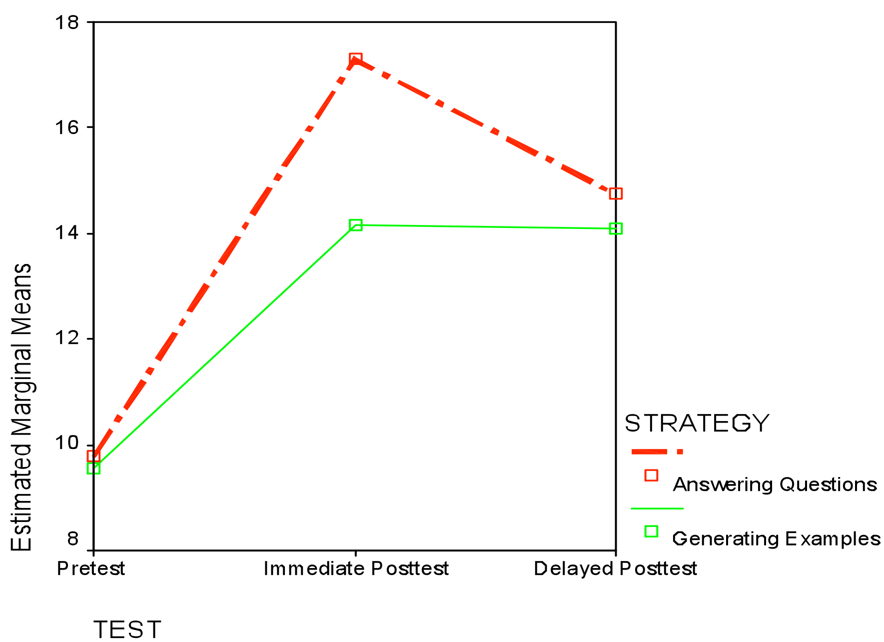

Figure 16. Means from the experimental groups in each evaluation session

Table 17

Difference Score Analysis Two-Sample t Test

	t	df	Sig.	Mean Difference
Difference	-2.79*	58	.01	-2.48

*p < .05

Future Research

In the present study, students who used an answering-questions generative activity showed a significantly greater effect on an immediate posttest of comprehension than students who used a generating-examples generative activity. However, a two-weeks delayed comprehension posttest did not show a significant effect between the generative learning activities (see Figure 16). Additional research may be needed using either a longer gap between the immediate and delayed assessment or another repeated measures longer than three weeks later after the immediate posttest. This new study would help to verify if these two generative learning activities have an effect on comprehension over a longer time.

A difference scores analysis between the immediate posttest scores and the delayed posttest scores revealed a significant difference between the answering-questions and the generating-examples groups. Additional analysis will be needed to generalize the tendency that students using generating-examples activities retain longer the part-whole meaning of rational numbers using virtual manipulatives than students using answering-questions activities.

Summary

The answering-questions and generating-examples strategies designed for this study fostered comprehension of the part-whole meaning of rational numbers using virtual manipulatives. The research study found that, on an immediate posttest, answering questions had a greater effect on comprehension than generation of examples. However, the study also revealed that students using a generating-examples strategy appear to retain longer what they have comprehended than students using an answering-questions strategy.

The study presented in this report added evidence of the value of using generative learning strategies. However, generalizability of the findings may be limited to specific settings with similar language, environment, grade of students, and the concept learned. It is hoped that the findings of this study help future researchers to provide instructional options that help elementary students to learn mathematical concepts.

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Appendix A

Pretest, immediate posttest, and delayed posttest Script

Good morning everyone.

Today, we are going to work on some paper-pencil activities. These activities will help me to understand how much you…

- …know about the concept of fractions (pretest)
- …learned from the activities on the computer about the concept of fractions (posttest)
- …remember from the activities on the computer about the concept of fractions (retention test)

The test has 12 questions. It has multiple-choice questions and open questions. In the multiple-choice questions, read carefully each question and options, and then fill the circle close to the option that you think is the best answer. In the open questions, you need to create a short story based on a picture or a fraction. The test will have one example to show you how to write the story.

Is this clear? Do you have any questions?

Please, try to answer all the questions and work individually. Let me know if you have any questions at any time.

Thank you.

Appendix B

Activity Script

Good morning everyone.

Today, we are going to work with two computer programs.

The first program is called "*Fraction Pieces*" and it will help you to answer the first section of the activity (I'll show the virtual manipulative to them). You can use this tool in the following way (I'll show an example on each step):

- You can click once on the piece you want, and automatically it shows up on the working area
- You can select a piece on the working area by clicking on the piece. Once in the working area you also can move pieces by clicking and holding the mouse button on the piece you want to move, and then moving the mouse to the place you want to locate the piece.
- You can also rotate the figure by placing the mouse in one of the borders of the figure. When a small black circle shows up, hold the mouse button on the circle and move the mouse to rotate the piece.
- You can click the "clear" button if you want to clear the working area (I'll show how).
- Is this clear? Do you have any questions?

The second program is called "*Parts of a Whole*", and it will help you to answer the second section of the activity (I'll show them the virtual manipulative). You can use this tool in the following way (I'll show an example on each step):

- You can change the figure (circle, rectangle) by clicking on the "*New Whole*" button.
- You can change the number of divisions, clicking on the arrows buttons below the figure. Clicking on the up-arrow button, increase the divisions by 1. Clicking on the down-arrow button, decrease the divisions by 1.
- You can select (shade) the parts of the whole by clicking on them and immediately other expressions of the graphics will show up
- Is this clear? Do you have any questions?

As I said before, you are going to use these two computer programs to complete the activities in your sheets. Please, let me know if you have any questions at any time.

Appendix C

Answering-Questions Activity

SECTION I

Read carefully and answer each question. Fill in the circle next to the best answer. Use the "Fraction Pieces" activity on the computer.

1. How many blue pieces do you need to cover the circle?

- ❍ 2
- ❍ 5
- ❍ 7
- ❍ 9
- ❍ None of these answers

---

2. How many yellow pieces do you need to cover the circle?

- ❍ 2
- ❍ 1
- ❍ 5
- ❍ 3
- ❍ None of these answers

---

3. How many brown pieces do you need to cover the circle?

- ❍ 2
- ❍ 8
- ❍ 4
- ❍ 3
- ❍ None of these answers

---

4. How many blue pieces do you need to cover one pink piece?

- ❍ 3
- ❍ 2
- ❍ 6
- ❍ 0
- ❍ None of these answers

---

5. How many pink pieces cover one red piece?

- ❍ 2
- ❍ 1
- ❍ 0
- ❍ 3
- ❍ None of these answers

---

6. How many black pieces do you need to cover one yellow piece?

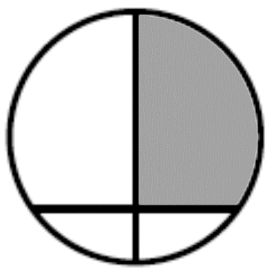
- ❍ 1
- ❍ 3
- ❍ 2
- ❍ 4
- ❍ None of these answers

SECTION II

Use the “Parts of a Whole” activity on the computer. Read carefully and answer each question. Fill in the circle next to the best answer.

---

7. Look at the figure.



What option shows the shaded part of the figure?

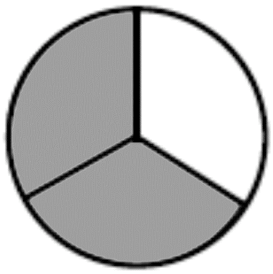
❍ 1/3

❍ 1/4

❍ 3/4

❍ None of these answers

8. Look at the figure.



What option shows the shaded part of the figure?

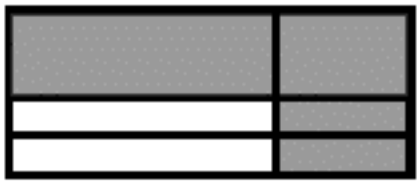
❍ 1 out of 2

❍ 2 out of 3

❍ 1 out of 3

❍ None of these answers

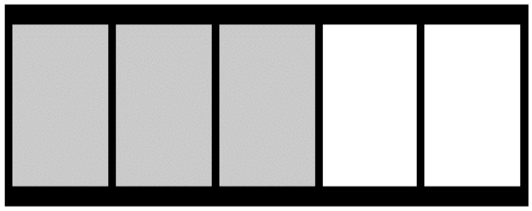
9. Look at the figure.



What option shows the shaded part of the figure?

- ❍ 2/4
- ❍ 2/6
- ❍ 4/6
- ❍ None of these answers

10. Look at the figure.

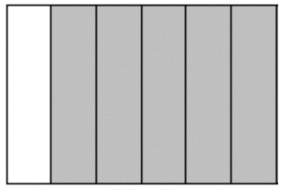


What option shows the shaded part of the figure?

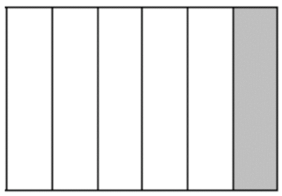
- ❍ 3 out of 2
- ❍ 2 out of 3
- ❍ 3 out of 5
- ❍ None of these answers

11. Which figure has 1 out of 6 of the picture shaded?

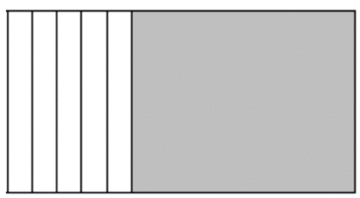
❍



❍



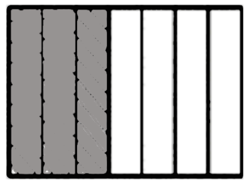
❍



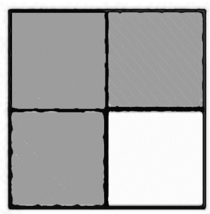
❍ None of these answers

12. Which figure has 3/4 of the picture shaded?

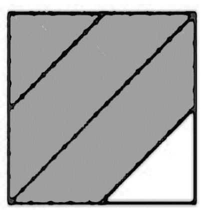
❍



❍



❍



❍ All of these answers are correct

This is the end of the activity.
Thank you so much for working hard and thinking!

Appendix D

Generating-Examples Activity

SECTION I
Use the "Fraction Pieces" computer activity. Write three different sentences about how the pieces relate to the circle. One example might be: Five yellow pieces cover the entire circle.

1.

2.

3.

Use the same activity on the computer. Write three different sentences about how the pieces relate to each other. One example might be: Two black pieces cover one entire yellow piece.

4.

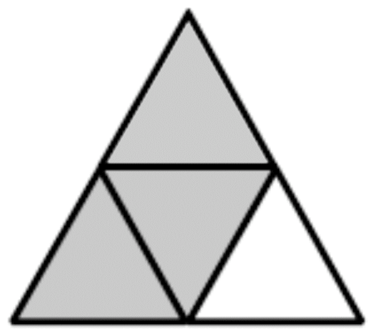
5.

6.

SECTION II

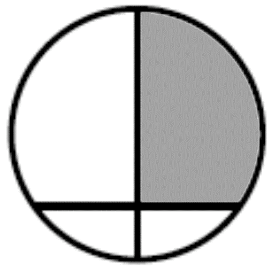
In the following questions you will need to create different stories from real life. Use a fraction number in your example.
Use the "Parts of a Whole" computer activity to verify your answers.

For example, look at the graphic:

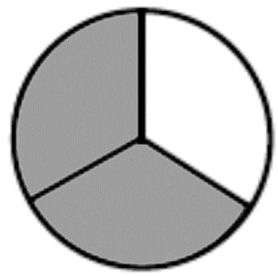


One example from real life could be: My mom made a cake as a triangle. She cut the cake in 4 pieces of the same size. My friends Jay, Peter and Michael ate one piece each. They ate **3/4** or **3 out of 4 pieces** of the cake.

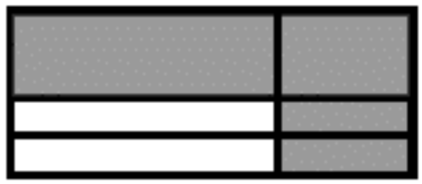
7. Look at the graphic. Write an example from real life. Use a fraction number in your example. If it is not possible, please explain why.



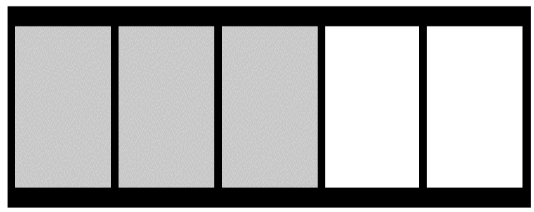
8. Look at the graphic. Write an example from real life. Use a fraction number in your example. If it is not possible, please explain why.



9. Look at the graphic. Write an example from real life. Use a fraction number in your example. If it is not possible, please explain why.



10. Look at the graphic. Write an example from real life. Use a fraction number in your example. If it is not possible, please explain why.

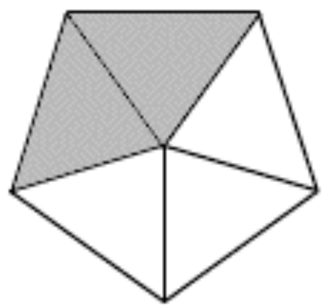


In the following questions you will need to create stories from the real life. Draw a graphic that describes the fraction.

For example, look at the fraction:

2 de 5 o **2/5**

One example from real life could be: My father constructed a kite made of 5 equal pieces . He painted two of those pieces. A graphic of the kite could be:



11. Look at the fraction. Write an example from real life. Draw a graphic that describes the fraction. If it is not possible, please explain why.

1 out of 6 or **1/6**

12. Look at the fraction. Write an example from real life. Draw a graphic that describes the fraction. If it is not possible, please explain why.

3/4 or **3 out of 4**

____

____

____

____

____

____

____

____

____

____

This is the end of the activity.
Thank you so much for working hard and thinking!

Appendix E

Assessment 1

Name ____________________________________ **Date** _____________

SECTION I

Read carefully and answer each question. Fill in the circle next to the best answer.

---

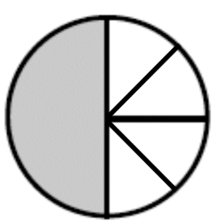
1. Look at this picture:

Which option shows the shaded part of the picture?

- ❍ 2 out of 5
- ❍ 3 out of 2
- ❍ 3 out of 5
- ❍ None of these answers

---

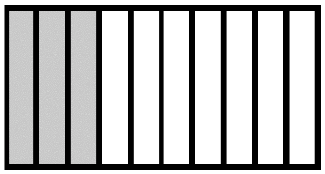
2. Look at this picture:



Which option shows the shaded part of the picture?

- ❍ 1/5
- ❍ 4/5
- ❍ 1/4
- ❍ None of these answers

3. Look at this picture.



Which fraction shows the shaded part of the figure?

- ❍ 3/7
- ❍ 3/10
- ❍ 7/10
- ❍ None of these answers

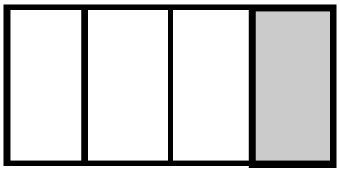
---

4. Which fraction means 2 parts out of 5?

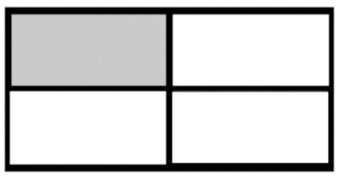
- ❍ 5/2
- ❍ 2/7
- ❍ 2/5
- ❍ None of these answers

5. Which option does **not** represent 1/4?

❍



❍



❍

❍ None of these answers

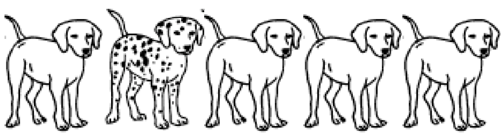
SECTION II

Read carefully and answer each question. Fill in the circle next to the best answer.

---

6. Which figure shows 1/4 of the dogs with spots?

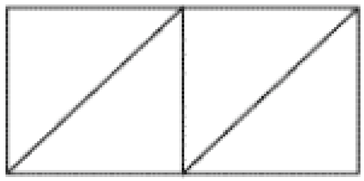
❍

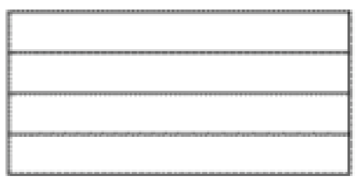

❍


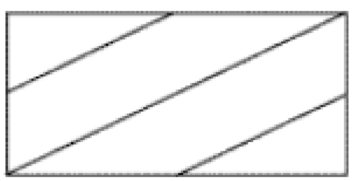
❍


❍ None of these answers

7. Judith is dividing a chocolate bar to share fairly. Which division should **not** Judith use?

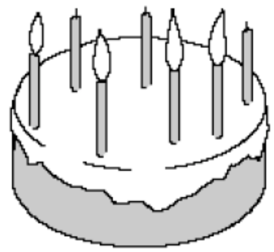
   ❍ 

   ❍ 

   ❍ 

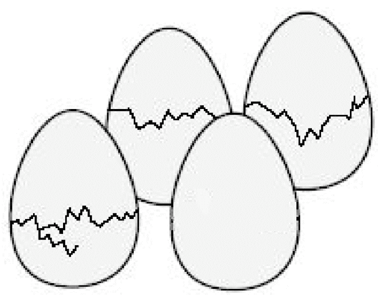
   ❍ None of these answers

8. What fraction of the candles on the cake is lit?



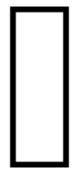
- ❍ 3 out of 7
- ❍ 4 out of 3
- ❍ 4 out of 7
- ❍ None of the answers

9. What fraction of the group of eggs is cracked?

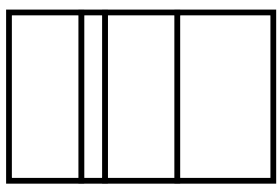


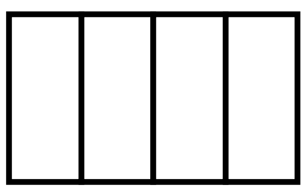
- ❍ 1/4
- ❍ 3/1
- ❍ 3/4
- ❍ None of these answers

10. The picture shows 1 out of 4 parts of a candy bar that John left for his brother.



What graphic represents the whole candy bar?

❍ 

❍ 

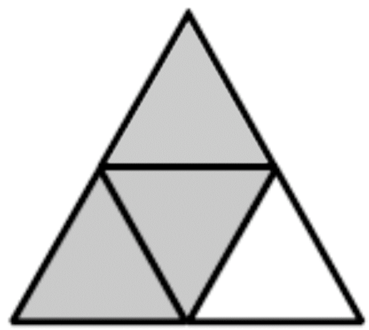
❍ 

❍ None of the answers is correct

SECTION III

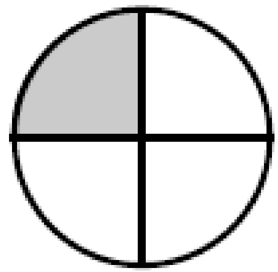
In the following questions you will need to create different short stories from real life. Use a fraction number in your example.

For example, look at the following graphic:

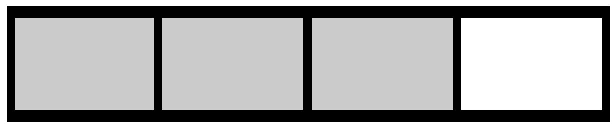


One example from real life could be: My mom made a cake as a triangle. She cut the cake in 4 pieces of the same size. My friends Jay, Peter and Michael ate one piece each. They ate **3/4** or **3 out of 4 pieces** of the cake.

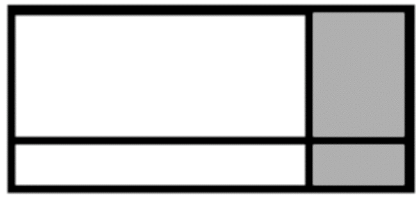
11. Look at the following graphic. Write an example from real life. Use a fraction number in your example. If it is not possible, please explain why.



12. Look at the graphic. Write an example from real life. Use a fraction number in your example. If it is not possible, please explain why.



13. Look at the graphic. Write an example from real life. Use a fraction number in your example. If it is not possible, please explain why.

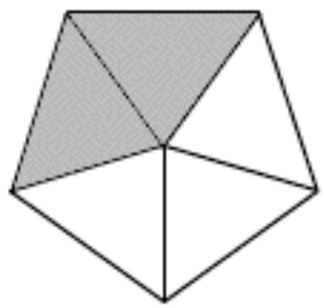


In the following questions you will need to create short stories from the real life. Draw a graphic that describes the fraction.

For example, look at the following fraction:

2 de 5 o **2/5**

One example from real life could be: My father constructed a kite made of 5 equal pieces . He painted two of those pieces. A graphic of the kite could be:



14. Look at the fraction. Write an example from real life. Draw also a graphic that describes the fraction. If it is not possible, please explain why.

1 out 4 or **1/4**

15. Look at the fraction. Write an example from real life. Draw also a graphic that describes the fraction. If it is not possible, please explain why.

2 out of 8 or **2/8**

This is the end of the test.
Thank you so much for working hard and thinking carefully!

Appendix F

Assessment 2

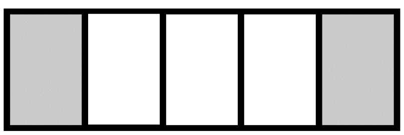
Name ______________________________ Date ____________

SECTION I

Read carefully and answer each question. Fill in the circle next to the best answer.

---

1. Look at this picture:

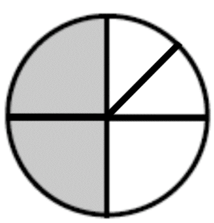


Which option shows the shaded part of the picture?

- ❍ 2 out of 5
- ❍ 3 out of 5
- ❍ 2 out of 3
- ❍ None of these answers

---

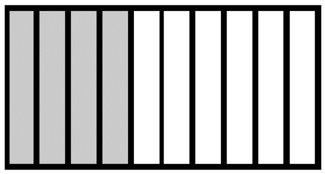
2. Look at this picture:



Which option shows the shaded part of the picture?

- ❍ 2/5
- ❍ 3/5
- ❍ 2/3
- ❍ None of these answers

3. Look at this picture.



Which option shows the shaded part of the picture?

- ❍ 4/6
- ❍ 6/10
- ❍ 4/10
- ❍ None of these answers

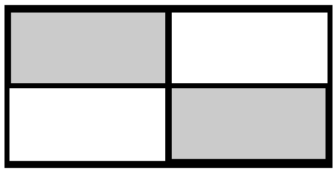
---

4. Which options shows 2 parts out of 3?

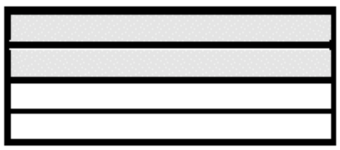
- ❍ 3/2
- ❍ 2/3
- ❍ 2/5
- ❍ None of these answers

5. Which option does **not** show 2/4?

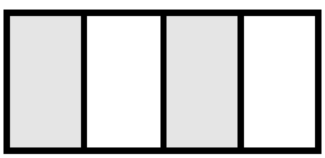
❍



❍



❍



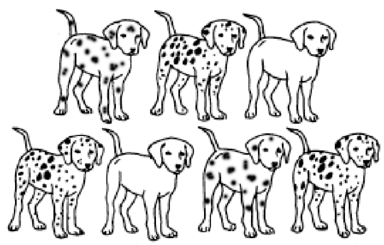
❍ None of these answers

SECTION II

Read carefully and answer each question. Fill in the circle next to the best answer.

---

6. Which picture shows 2/5 of the dogs with spots?

❍


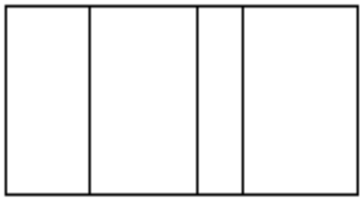
❍


❍

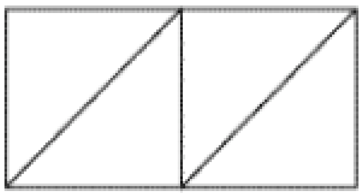

❍ None of these answers

7. Judith is dividing a chocolate bar to share fairly. Which division should **not** Judith use?

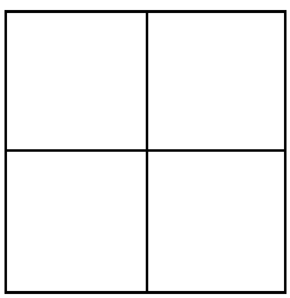
❍



❍

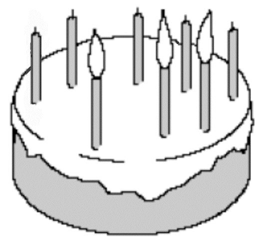


❍



❍ None of these answers

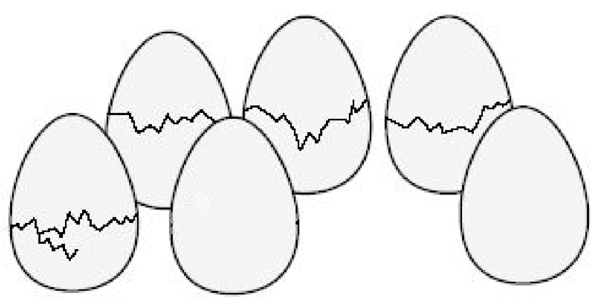
8. What fraction of the candles on the cake is lit?



- ❍ 3 out of 8
- ❍ 5 out of 8
- ❍ 3 out of 5
- ❍ None of the answers is correct

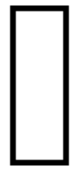
---

9. What fraction of the group of eggs is cracked?

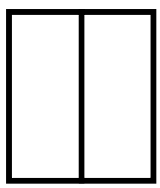
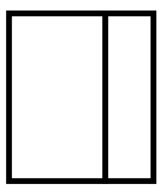


- ❍ 4/6
- ❍ 2/6
- ❍ 2/4
- ❍ None of these answers

10. The picture shows 1 out of 2 parts of a candy bar that John left for his brother.



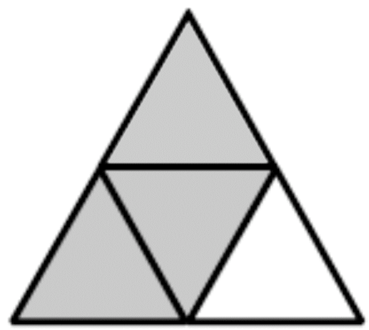
What graphic represents the whole candy bar?

- ❍ 
- ❍ 
- ❍ 
- ❍ None of the answers is correct

SECTION III

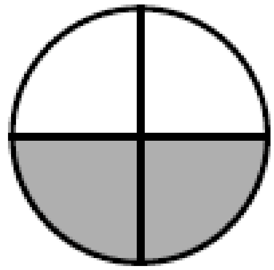
In the following questions you will need to create different short stories from the real life. Use a fraction number in your example.

For example, look at the following graphic:



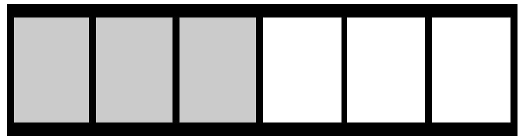
One example from real life could be: My mom made a cake as a triangle. She cut the cake in 4 pieces of the same size. My friends Jay, Peter and Michael ate one piece each. They ate **3/4** or **3 out of 4 pieces** of the cake.

11. Look at the following graphic. Write an example from real life. Use a fraction number in your example. If it is not possible, please explain why.



____

12. Look at the graphic. Write an example from real life. Use a fraction number in your example. If it is not possible, please explain why.



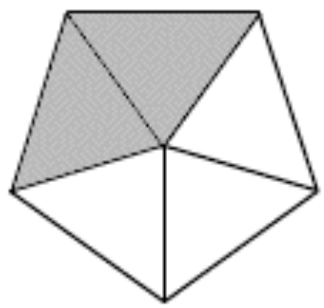
13. Look at the graphic. Write an example from real life. Use a fraction number in your example. If it is not possible, please explain why.

In the following questions you will need to create short stories from the real life. Draw a graphic that describes the fraction.

For example, look at the following fraction:

2 de 5 o **2/5**

One example from real life could be: My father constructed a kite made of 5 equal pieces . He painted two of those pieces. A graphic of the kite could be:



14. Look at the fraction. Write an example from real life. Draw also a graphic that describes the fraction. If it is not possible, please explain why.

2 out 4 or **2/4**

15. Look at the fraction. Write an example from real life. Draw also a graphic that describes the fraction. If it is not possible, please explain why.

4 out of 8 or **4/8**

This is the end of the test.
Thank you so much for working hard and thinking carefully!

Appendix G

Assessment 3

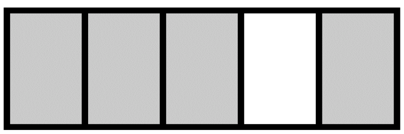
Name ________________________________ **Date** ____________

SECTION I

Read carefully and answer each question. Fill in the circle next to the best answer.

---

1. Look at this picture:

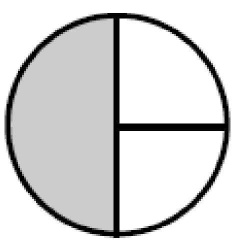


Which option shows the shaded part of the picture?

- ❍ 1 out of 5
- ❍ 4 out of 5
- ❍ 1 out of 4
- ❍ None of these answers

---

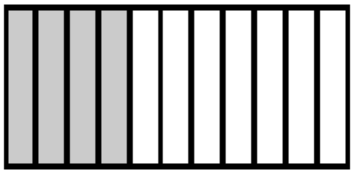
2. Look at this picture:



Which option shows the shaded part of the picture?

- ❍ 1/3
- ❍ 2/3
- ❍ 3/2
- ❍ None of these answers

3. Look at this picture.



Which option shows the shaded part of the figure?

- ❍ 4/11
- ❍ 7/11
- ❍ 4/7
- ❍ None of these answers

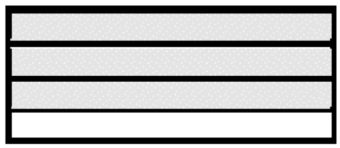
---

4. Which option shows 3 parts out of 7?

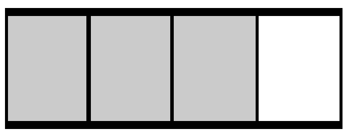
- ❍ 3/7
- ❍ 4/7
- ❍ 7/3
- ❍ None of these answers

5. Which figure does **not** represent 3/4?

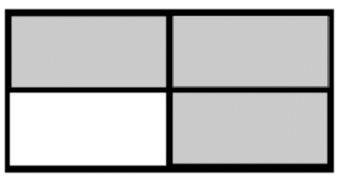
❍



❍



❍



❍ None of these answers

SECTION II

Read carefully and answer each question. Fill in the circle next to the best answer.

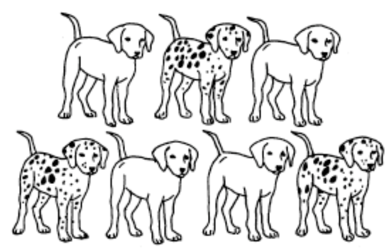
---

6. Which figure shows 3/4 of the dogs with spots?

❍

❍

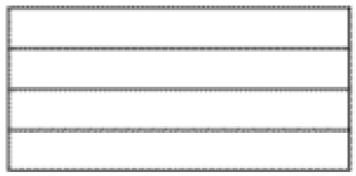
❍



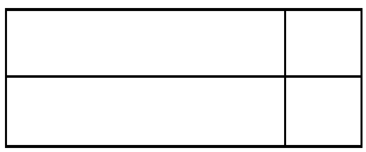
❍ None of these answers

7. Judith is dividing a chocolate bar to share fairly. Which division should **not** Judith use?

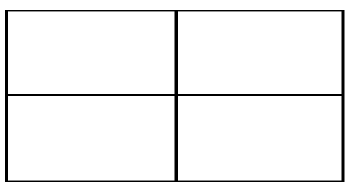
❍



❍

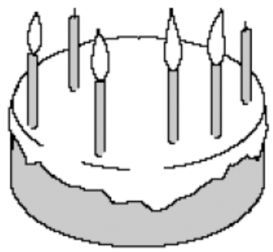


❍



❍ None of these answers

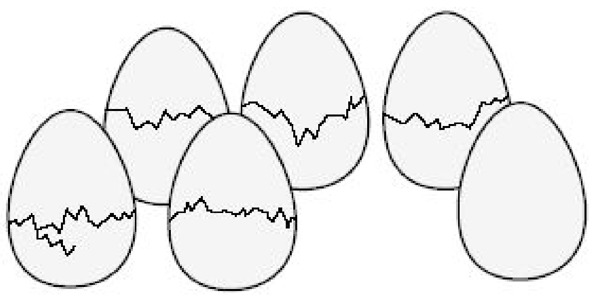
8. W hat fraction of the candles on the cake is lit?



- ❍ 2 out of 6
- ❍ 4 out of 6
- ❍ 2 out of 4
- ❍ None of the answers is correct

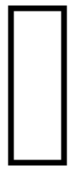
---

9. What fraction of the group of eggs is cracked?

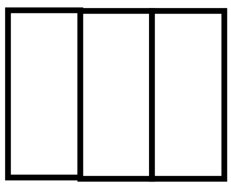


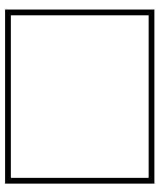
- ❍ 1/6
- ❍ 5/6
- ❍ 1/5
- ❍ None of these answers

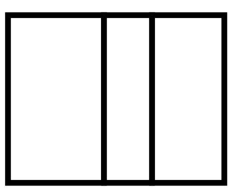
10. The picture shows 1 out of 3 parts of a candy bar that John left for his brother.



What graphic represents the whole candy bar?

❍ 

❍ 

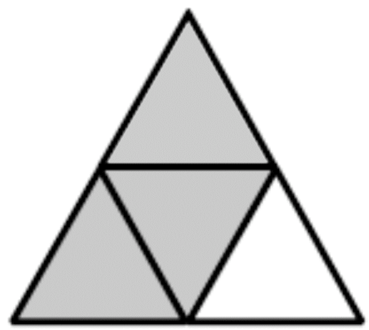
❍ 

❍ None of the answers is correct

SECTION III

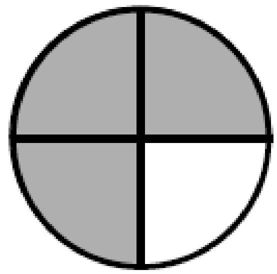
In the following questions you will need to create different short stories from the real life. Use a fraction number in your example.

For example, look at the following graphic:

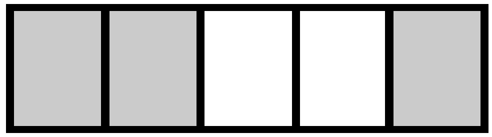


One example from real life could be: My mom made a cake as a triangle. She cut the cake in 4 pieces of the same size. My friends Jay, Peter and Michael ate one piece each. They ate **3/4** or **3 out of 4 pieces** of the cake.

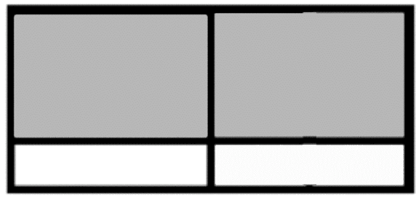
11. Look at the following graphic. Write an example from real life. Use a fraction number in your example. If it is not possible, please explain why.



12. Look at the graphic. Write an example from real life. Use a fraction number in your example. If it is not possible, please explain why.



13. Look at the graphic. Write an example from real life. Use a fraction number in your example. If it is not possible, please explain why.

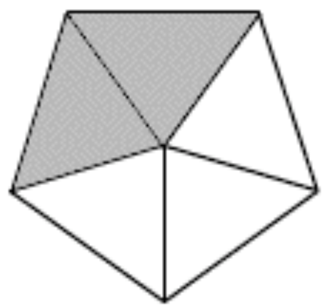


In the following questions you will need to create short stories from the real life. Draw a graphic that describes the fraction.

For example, look at the following fraction:

2 de 5 o **2/5**

One example from real life could be: My father constructed a kite made of 5 equal pieces . He painted two of those pieces. A graphic of the kite could be:



14. Look at the fraction. Write an example from real life. Draw also a graphic that describes the fraction. If it is not possible, please explain why.

3 out 4 or **3/4**

15. Look at the fraction. Write an example from real life. Draw also a graphic that describes the fraction. If it is not possible, please explain why.

3 out of 8 or **3/8**

This is the end of the test.
Thank you so much for working hard and thinking carefully!

Appendix H

Approval from the IRB at Virginia Tech

VirginiaTech

Office of Research Compliance
Institutional Review Board
1880 Pratt Drive (0497)
Blacksburg, Virginia 24061
540/231-4991 Fax: 540/231-0959
E-mail: moored@vt.edu
www.irb.vt.edu
FWA00000572(expires 7/20/07)
IRB # is IRB00000667.

DATE: December 20, 2006

MEMORANDUM

TO: Kenneth R. Potter
Jesus Trespalacios

Approval date: 12/19/2006
Continuing Review Due Date:12/4/2007
Expiration Date: 12/18/2007

FROM: David M. Moore

SUBJECT: **IRB Expedited Approval:** "The Effects of Two Generative Activities on Learner Achievement and Retention of Part-Whole Representation of Rational Numbers Using Virtual Manipulatives" , IRB # 06-743

This memo is regarding the above-mentioned protocol. The proposed research is eligible for expedited review according to the specifications authorized by 45 CFR 46.110 and 21 CFR 56.110. As Chair of the Virginia Tech Institutional Review Board, I have granted approval to the study for a period of 12 months, effective December 19, 2006.

As an investigator of human subjects, your responsibilities include the following:

1. Report promptly proposed changes in previously approved human subject research activities to the IRB, including changes to your study forms, procedures and investigators, regardless of how minor. The proposed changes must not be initiated without IRB review and approval, except where necessary to eliminate apparent immediate hazards to the subjects.
2. Report promptly to the IRB any injuries or other unanticipated or adverse events involving risks or harms to human research subjects or others.
3. Report promptly to the IRB of the study's closing (i.e., data collecting and data analysis complete at Virginia Tech). If the study is to continue past the expiration date (listed above), investigators must submit a request for continuing review prior to the continuing review due date (listed above). It is the researcher's responsibility to obtained re-approval from the IRB before the study's expiration date.
4. If re-approval is not obtained (unless the study has been reported to the IRB as closed) prior to the expiration date, all activities involving human subjects and data analysis must cease immediately, except where necessary to eliminate apparent immediate hazards to the subjects.

Important:
If you are conducting **federally funded non-exempt research**, this approval letter must state that the IRB has compared the OSP grant application and IRB application and found the documents to be consistent. Otherwise, this approval letter is invalid for OSP to release funds. Visit our website at http://www.irb.vt.edu/pages/newstudy.htm#OSP for further information.

cc: File

Invent the Future

VIRGINIA POLYTECHNIC INSTITUTE AND STATE UNIVERSITY

An equal opportunity, affirmative action institution

Appendix I

Montgomery County Public Schools Approval

Laura Williams, Grant Writer
Montgomery County Public Schools
208 College Street
Christiansburg, VA 24073
(540) 381-6158

November 21, 2006

Jesus Trespalacios, Doctoral Student
Instructional Design and Technology
Virginia Tech
Blacksburg, VA 24061

Dear Jesus,

I have reviewed the proposal you submitted to MCPS to conduct doctoral research regarding the effect of virtual manipulatives on student achievement and retention of part-whole representation of rational numbers. Your proposed study activities will take place with approximately 50 MCPS third graders on four separate occasions, taking approximately one hour each. I completed this review in consultation with Betti Kreye, MCPS Supervisor of Math. This letter serves as notification that MCPS has approved your request at the district level, pending receipt of your approval letter from the Virginia Tech IRB.

As I understand it, you have already had preliminary conversations about the study with one third grade teacher from Kipps Elementary and one from Price's Fork Elementary. This letter allows you to approach Brian Kitts, KES Principal, and Dollie Cottrill, PFE Principal, to request their approval of your project. The final decision as to whether to participate in the project will rest with the principals of each of these schools. If Mr. Kitts and Ms. Cottrill approve the project, it is the district's understanding that 1) parent and teacher written consent and student verbal or written assent will be obtained for all study participants, and 2) that no personally identifying results or information will be shared. In addition, one of the documents you submitted indicates that you plan to reward students for participation in the study. If you receive principal approval for your research, I suggest you discuss with each principal whether such a reward is appropriate, and, if so, what form it should take.

If you have questions or need any further assistance, please don't hesitate to contact me.

Sincerely,

Laura Williams
Cc: Jeanette Warwick, Betti Kreye, Brian Kitts, Dollie Cottrill.

Appendix J

Parents' approval

Letter to Parents Attached to Parental Permission Forms

Dear parents:

I am Jesus Trespalacios, a doctoral student in the Instructional Design and Technology program at Virginia Polytechnic Institute and State University (Virginia Tech). For my dissertation, I am planning to collect data from third-grade students in the elementary school in which your son/daughter is enrolled. The elementary school and the teacher have approved the study.

Participation in the study is voluntary. Your son/daughter will also receive a verbal explanation of the study and assent from him/her should be solicited in the presence of the teacher. Please read the attached consent form that explains details of the study. If you wish to give permission for his/her participation in this study, please sign the form and give it to him/her. Your son/daughter will take the signed forms to his/her teacher.

If you have questions regarding the study, please feel free to contact me via email jtrespal@vt.edu or by phone 5402302394. Thank you so much for your time and consideration.

Sincerely,

____

Jesus Trespalacios
Ph.D. Student
Instructional Design and Technology
Virginia Tech
Blacksburg, VA 24060
jtrespal@vt.edu

VIRGINIA POLYTECHNIC INSTITUTE AND STATE UNIVESITY

Parental Permission in Research Projects Involving Human Subjects

Title of the Project
The Effects of Two Generative Activities on Learners Achievement and Retention of Part-Whole Representation of Rational Numbers Using Virtual Manipulatives.

Investigator(s):
Ken Potter, Ph.D., Jesus H Trespalacios

I. Purpose of this Research/Project

The purpose of the study is to examine the success of two instructional strategies using computers. The participants of the study will be third-grade students in an elementary school. Study will include just participants who agree to participate in the study and obtain consent from his/her parents. The expected number of participants is around 60.

II. Procedures

Students are divided in two groups. They will be asked to complete a list of activities based on answering questions and generating examples. The study will be divided in four parts.

1. In the last week of January, a pretest will be administered.

2. The next day the activities will be administered. The first group will receive fourteen multiple-choice questions. The second group will receive fourteen open questions. Students will answer these questions using two virtual manipulatives on the computer. The activity requires around forty-five minutes. The activity will be conducted in the elementary school's classroom.

3. The next day after the activities sessions, a posttest will be administered.

4. Two weeks later, another posttest will be administered to find out how much the students remember of the material they learned while doing the activities of the previous two weeks. Test and activities will take approximately 45 minutes each.

III. Risks

There are no anticipated risks for the participants other than those associated with regular school or class activities in this study.

IV. Benefits

Studies show that these types of activities enhance students' academic achievement. Although there is no guarantee that the proposed activities have significant effects, researchers believe that the proposed activities using manipulatives provide academic benefits to students. If the child does not receive permission to participate, the child will participate in the activities but data from the child will not be used.

V. Anonymity/Confidentiality

For this study, the names of the participants will be confidential. Data collected will be kept confidential. Only the researchers associated with the project will have access to the data. These tests will be accessible only by the investigators for the purpose of analysis. Information gathered from the study may be used in reports, presentations, or journal articles. However, data will be published in aggregate form only and it will never identify a participant.

VI. Compensation

There is a five dollars compensation for participating in this study.

VII. Freedom to Withdraw

Participants are free to leave from the study at any time without penalty in their grades.

VIII. Subject Responsibilities

The participants who voluntarily agree to participate in this study will have the following responsibilities:

- Answer completely the pretest, posttest, and retentions tests
- Answer completely the questions from the activities using the computer.

IX. Parent's Permission

I have read and I understand this form and the conditions of this project. I have had all my questions answered. I hereby acknowledge the above and give my consent. This project has been explained to my child, in language he/she can understand. He/she has been encouraged to ask questions both now, and in the future, about the research study.

____

Signature of Parent Date

____

Child's name (Name of Child) School

Should I have any pertinent questions about this research or the manner it which it is conducted, about the rights of the research subjects, or about whom to contact in the event of a problem, I may contact:

Jesus H Trespalacios____ (540) 230-2394 / jtrespal@vt.edu
Investigator(s) Telephone / e-mail

Ken Potter, Ph.D.____ (540) 230-2394 / jtrespal@vt.edu
Faculty Advisor Telephone / e-mail

David M. Moore (540) 231-4991 / moored@vt.edu
Chair, Virginia Tech Institutional Telephone/e-mail
Board for the Protection of Human Subjects
Office of Research Compliance
1880 Pratt Drive, Suite 2006 (0497)
Blacksburg, VA 24061

Appendix K

Pilot Test

I. MULTIPLE CHOICE QUESTIONS

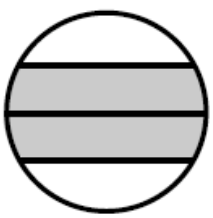
Read carefully and answer the questions. Fill in the circle next to the best answer.

1. Look at this picture:

Which fraction shows the shaded part of the picture?

- ❍ 2/5
- ❍ 3/5
- ❍ 4/5
- ❍ None of these answers

2. Look at this picture:



The shaded part of the circle shows:

- ❍ 2 out of 2
- ❍ 2 out of 4
- ❍ 4 out of 2
- ❍ None of these answers

3. Look at this picture:

Which fraction shows the shaded part of the picture?

- ❍ 1/2
- ❍ 1/3
- ❍ 2/1
- ❍ None of these answers

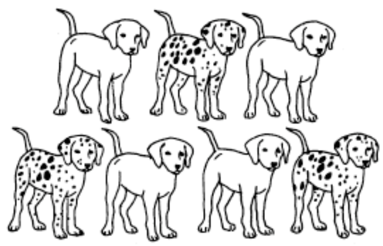
4. Which fraction means 2 parts out of 3?

- ❍ 3/2
- ❍ 2/3
- ❍ 2/5
- ❍ None of these answers

5. Which picture shows 3/4 of the dogs with spots?

❍

❍

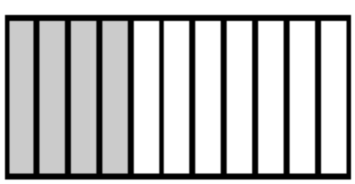


❍

❍ None of these answers

---

6. Look at this picture:



Which fraction shows the shaded part of the figure:

❍ 4/7
❍ 7/11
❍ 4/11
❍ None of these answers

II. OPEN QUESTIONS

Read carefully and solve each question. Then write in the blank space(s) the best answer(s)

---

7. 1 part out of 2 parts of a cake was left over after a party. That portion looked like this:

Draw a picture of the whole cake:

---

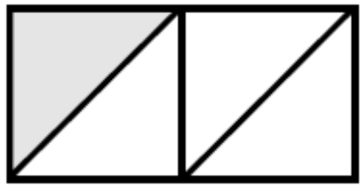
8. This is a picture of a candy bar that John wants to cut into pieces to share fairly with 3 friends:

Each piece is __________ of the whole candy bar
(write a fraction in the blank)

Draw lines in the above picture that represent how John cuts the candy bar.

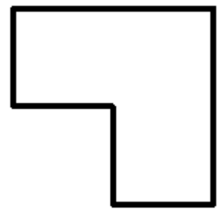
---

9. Look at the picture:



Write a short story about the parts in this picture using fractions (You could use the name of your friends, family and objects like cookies, chocolate bars, etc.).

10. A cookie comes in the shape of a square. After Janis ate one piece of the cookie it looked like this:

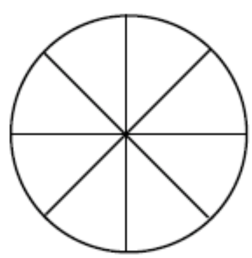


The piece that Janis ate is _________ of the whole cookie
(write a fraction in the blank)

Draw lines on the cookie to show the pieces left. They should be the same size of the piece that Janis ate.

---

11. In this picture, shade any number of pieces.



When you finish shading, fill the following blanks:

The shaded part represents ______ out of ______

The shaded part is the same as this fraction: ___ / ___

---

12. Draw lines on this rectangle to make any number of same size pieces. Shade as many pieces as you want.

Now, fill the following blanks:

Each piece is _____ out of _____

The shaded part is the same as this fraction: ___ / ___

This is the end of the test.
Thank you so much for working hard and thinking carefully!

Appendix L

Answering-questions Pilot Activity

ACTIVITY ONE

Color the little circle next to the answer for every question. Use the “Fraction Pieces” activity on the computer.

1. How many blue pieces do you need to cover the circle?

- ❍ 2
- ❍ 5
- ❍ 7
- ❍ 9
- ❍ None of the above is correct

2. How many black pieces do you need to cover one yellow piece?

- ❍ 1
- ❍ 3
- ❍ 2
- ❍ 4
- ❍ None of the above is correct

3. How many brown pieces do you need to cover the circle?

- ❍ 2
- ❍ 8
- ❍ 4
- ❍ 3
- ❍ None of the above is correct

4. How many blue pieces do you need to cover one pink piece?

- ❍ 3
- ❍ 2
- ❍ 6
- ❍ 0
- ❍ None of the above is correct

5. How many pink pieces cover one red piece?

- ❍ 2
- ❍ 1
- ❍ 0
- ❍ 3
- ❍ None of the above is correct

6. How many yellow pieces do you need to cover the circle?

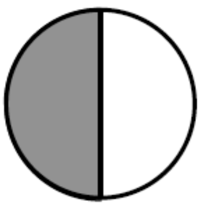
- ❍ 2
- ❍ 1
- ❍ 5
- ❍ 3
- ❍ None of the above is correct

ACTIVITY TWO

Please answer these questions. Use the "Parts of a Whole" activity on the computer.

> **Ms. Hickman made an apple pie. Her daughter, Jane, ate part of the pie. The circle in the picture shows the apple pie. The shaded part of the circle is the part that Jane ate.**

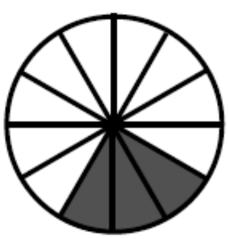
7. In this picture, what answer shows the part that Jane ate?



- ❍ 1 out of 3
- ❍ 2 out of 1
- ❍ 1 out of 2
- ❍ None of the above is correct

---

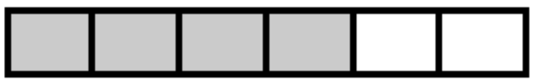
8. The next day, Ms Hickman made another apple pie. In this new picture, what answer shows the part that Jane ate?



- ❍ 3 out of 6
- ❍ 3 out of 12
- ❍ 9 out of 3
- ❍ None of the above is correct

Matthew and Kathy shared a candy bar. Matthew ate a piece. The rectangle in the picture shows the candy bar. The shaded part of the rectangle shows the part that Matthew ate.

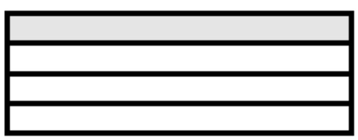
9. Look at this picture of the candy bar. In this picture, what answer shows the part that Mathew ate?



- ❍ 4 out of 2
- ❍ 2 out of 4
- ❍ 4 out of 6
- ❍ None of the above is correct

---

10. The next day, Matthew and Kathy shared another candy bar. In this new picture, what answer shows the part that Mathew ate?



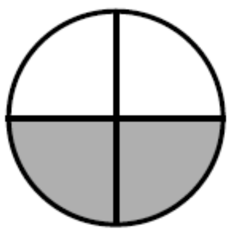
- ❍ 1 out of 5
- ❍ 3 out of 4
- ❍ 1 out of 3
- ❍ None of the above is correct

ACTIVITY THREE

Please answer these questions. Use the "Parts of a Whole" activity on the computer.

Jessica and Kim shared a large pizza. Jessica ate part of the pizza. The circle shows the pizza. The shaded part of the circle shows the part that Jessica ate.

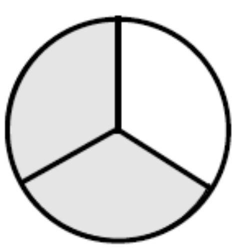
11. For this picture, what fraction shows the part of the pizza that Jessica ate?



- ❍ 2/4
- ❍ 1/4
- ❍ 2/2
- ❍ None of the above is correct

---

12. The next day, Jessica and Kim shared another pizza. In this new picture, what answer shows the part that Jessica ate?



- ❍ 3/2
- ❍ 1/2
- ❍ 2/3
- ❍ None of the above is correct

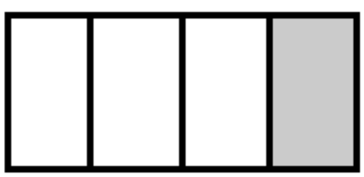
Mr. Smith bought a candy bar. His daughter Lilly ate a piece of the candy. The rectangle in the picture shows the candy. The shaded part of the rectangle shows the part of the candy that Lilly ate.

13. In this picture, what fraction shows the part of the candy that Lilly ate?

- ❍ 2/2
- ❍ 2/4
- ❍ 1/4
- ❍ None of the above is correct

---

14. The next day, Mr. Smith bought another candy bar. In this new picture, which fraction shows the part of the candy that Lilly ate?



- ❍ 3/1
- ❍ 1/3
- ❍ 3/4
- ❍ None of the above is correct

This is the end of the activity.
Thank you so much for working hard and thinking!

Appendix M

Generating-examples Pilot activity

ACTIVITY ONE

Use the "Fraction Pieces" computer activity. Write three different sentences about how the pieces relate to the circle. One example might be: Five yellow pieces cover the entire circle.

1.

2.

3.

Use the same activity on the computer. Write three different sentences about how the pieces relate to each other. One example might be: Two black pieces cover one entire yellow piece.

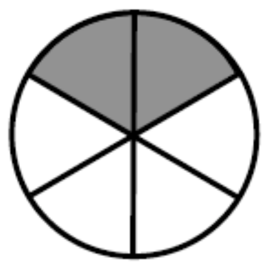
1.

2.

3.

ACTIVITY TWO
Use the "Parts of a Whole" computer activity. Write four different examples about parts of a whole in real life. **Draw a picture** for your example.

One example could be: I cut a pizza in 6 pieces of the same size. My friends Jay and Michael each ate one piece of pizza. They ate **2 out of the 6** pieces of the pizza. It is like this figure:



1.

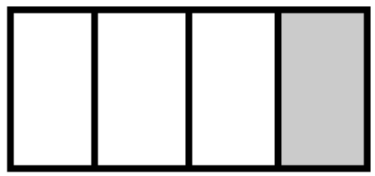
2.

3.

4.

ACTIVITY THREE
Use again the "Parts of a Whole" activity on the computer. Write four different examples from real life situations. **Draw a picture** and **write the fraction number** that goes with your example.

One example would be: I have a candy bar. The candy bar is cut in 4 pieces of the same size. My friend Janis ate **1 out of 4** pieces of the candy. It is like this figure:



The fraction is: **1/4**

1.

2.

3.

4.

This is the end of the activity.
Thank you so much for working hard and thinking!

Appendix N

Amendment Approval from the IRB at Virginia Tech

VirginiaTech

Office of Research Compliance
Institutional Review Board
1880 Pratt Drive (0497)
Blacksburg, Virginia 24061
540/231-4991 Fax: 540/231-0959
E-mail: moored@vt.edu
www.irb.vt.edu
FWA00000572(expires 7/20/07)
IRB # is IRB00000667.

DATE: March 5, 2007

MEMORANDUM

TO: Kenneth R. Potter
Jesus Trespalacios

Approval date: 12/19/2006
Continuing Review Due Date:12/4/2007
Expiration Date: 12/18/2007

FROM: David M. Moore

SUBJECT: **IRB Amendment 1 Approval:** "The Effects of Two Generative Activities on Learner Achievement and Retention of Part-Whole Representation of Rational Numbers Using Virtual Manipulatives" , IRB # 06-743

This memo is regarding the above referenced protocol which was previously granted approval by the IRB on December 19, 2006. You subsequently requested permission to amend your IRB application. Since the requested amendment is nonsubstantive in nature, I, as Chair of the Virginia Tech Institutional Review Board, have granted approval for requested protocol amendment, effective as of March 2, 2007. The anniversary date will remain the same as the original approval date.

As an investigator of human subjects, your responsibilities include the following:

1. Report promptly proposed changes in previously approved human subject research activities to the IRB, including changes to your study forms, procedures and investigators, regardless of how minor. The proposed changes must not be initiated without IRB review and approval, except where necessary to eliminate apparent immediate hazards to the subjects.
2. Report promptly to the IRB any injuries or other unanticipated or adverse events involving risks or harms to human research subjects or others.
3. Report promptly to the IRB of the study's closing (i.e., data collecting and data analysis complete at Virginia Tech). If the study is to continue past the expiration date (listed above), investigators must submit a request for continuing review prior to the continuing review due date (listed above). It is the researcher's responsibility to obtained re-approval from the IRB before the study's expiration date.
4. If re-approval is not obtained (unless the study has been reported to the IRB as closed) prior to the expiration date, all activities involving human subjects and data analysis must cease immediately, except where necessary to eliminate apparent immediate hazards to the subjects.

cc: File

Invent the Future

VIRGINIA POLYTECHNIC INSTITUTE UNIVERSITY AND STATE UNIVERSITY
An equal opportunity, affirmative action institution

Appendix O

Approval from Puerto Rico Department of Education

Estado Libre Asociado de Puerto Rico
DEPARTAMENTO DE EDUCACIÓN

SUBSECRETARÍA PARA ASUNTOS ACADÉMICOS

16 de febrero de 2007

Prof. Judith Andújar
Directora
Esc. Francisco Matías Lugo
Distrito Escolar de Carolina II
C/ Almendro
Valle Arriba Heights
Carolina, Puerto Rico 00983-0000

Estimada señora Andújar:

Autorizamos al Sr. Jesús Trespalacios, estudiante de la Universidad Virginia Tech Blaskburg. V.A. 24061 a realizar su trabajo de investigación relacionado a ***Los efectos de dos actividades generativas en el rendimiento y memorización de la representación de parte de la Unidad en los números racionales usando manipulativos vituales***, en la Escuela Francisco Matías Lugo en un periodo de cuatro (4) horas durante los meses de febrero a marzo de 2007.

Esperamos pueda ofrecerle al Sr. Trespalacios todo el apoyo necesario para que pueda realizar su trabajo de investigación.

Atentamente,

Waldo A. Torres Vázquez
Subsecretario

TV/cng

P.O. BOX 190759, SAN JUAN, PUERTO RICO 00919-0759 • TEL.: (787) 759-2000 EXTS.: 2749, 4749 • FAX: (787) 753-1804

El Departamento de Educación no discrimina por razón de raza, color, sexo, nacimiento, origen nacional, condición social, ideas políticas o religiosas, edad o impedimento en sus actividades, servicios educativos y oportunidades de empleo.

Appendix P

Parents' approval in Spanish

Cartas a los Padres de Familia Adjunta al Permiso a los Padres de Familia

Estimados padres de familia:

Mi nombre es Jesús Trespalacios, soy un estudiante doctoral en el programa de Tecnología Educativa en la Universidad Politécnica de Virginia (Virginia Tech). Para mi disertación estoy planeando hacer un estudio con los estudiantes de tercer grado en la escuela Francisco Matías Lugo donde su hijo esta matriculado. El estudio ha sido aprobado por la directora de la escuela y por las maestras. Este estudio permitirá a su hijo(a) trabajar con un programa educativo en la computadora para mejorar su aprendizaje del concepto de números racionales.

La participación en el estudio es voluntaria. Su hijo(a) recibirá una explicación verbal del estudio y su consentimiento también será solicitado en la presencia de su maestro(a). Por favor lea la forma adjunta a esta carta donde se explica detalladamente el estudio. Si desea(n) que su hijo(a) participe en el estudio, por favor firme la forma adjunta y devuélvasela a su hijo(a). Su hijo(a) le devolverá esta forma firmada al maestro.

Si tiene preguntas sobre el estudio, por favor comuníquese conmigo al teléfono 5402302394 o a mi correo electrónico jtrespal@vt.edu. Muchas gracias por su tiempo y consideración.

Respetuosamente,

____

Jesus Trespalacios

VIRGINIA POLYTECHNIC INSTITUTE AND STATE UNIVESITY

Permiso de los padres de familia para investigaciones que usan seres humanos

Titulo del estudio
Los efectos de dos actividades generativas en el rendimiento y memorización de la representación de partes de la unidad en los números racionales usando manipulativos virtuales.

Investigadores
Dr. Ken Potter, Jesús H Trespalacios

I. Propósito del Estudio

El propósito del estudio es analizar las ventajas de dos estrategias educativas usando la computadora. Los participantes en el estudio serán estudiantes de tercer grado de una escuela elemental. En el estudio sólo participarán los estudiantes que quieran participar y cuyos padres hayan firmado este permiso. Se espera una participación aproximada de 60 estudiantes.

II. Procedimientos

Los estudiantes serán divididos en dos grupos. Completarán una serie de actividades que consisten en responder preguntas y crear ejemplos. Todo el estudio será dividido en cuatro partes:

1. Al comienzo del estudio, a los estudiantes se les hará una prueba preliminar.

2. Después de esta evaluación preliminar, al día siguiente, el primer grupo responderá 14 preguntas tipo opción múltiple. El segundo grupo recibirá 14 preguntas tipo respuesta abierta. Los estudiantes responderán estas preguntas usando dos manipulativos virtuales en la computadora. Esta actividad requiere aproximadamente 60 minutos. La actividad se llevará acabo en el salón de clases de la escuela.

3. Al siguiente día a los estudiantes se les hará otra prueba para medir su aprovechamiento.

4. Dos semanas después, a los estudiantes se les hará una última prueba para medir cuánto recuerdan. Todas las pruebas y actividades durarán aproximadamente 60 minutos cada una.

III. Riesgos

Se cree que no habrá ningún riesgo para los participantes de este estudio diferentes de los relacionados con las actividades normales de la escuela.

IV. Beneficios

Las investigaciones muestran que el uso de estas actividades mejoran el rendimiento académico de los estudiantes. Aunque no hay garantías de que este estudio tenga efectos significativos, los investigadores creen que las actividades diseñadas ofrecerán beneficios académicos a los estudiantes.

V. Anonimato/Confidencialidad

Para este estudio los nombres de los estudiantes se considerará información confidencial. Los datos recogidos se mantendrán en secreto. Solo los investigadores asociados con el estudio tendrán acceso a los datos. La información obtenida podría usarse en informes, presentaciones o publicaciones científicas. Sin embargo, sólo las conclusiones se publicarían y nunca se identificaría por nombre a ningún estudiante.

VI. Compensación

Los participantes recibirán cinco dólares por participar en el estudio.

VII. Opción para retirarse del estudio

Los estudiantes tendrán el derecho de retirarse del estudio en cualquier momento sin recibir ninguna penalidad en sus calificaciones.

VIII. Responsabilidades del estudiante

Los estudiantes que participen voluntariamente en el estudio tendrán las siguientes responsabilidades:

- Completar las tres evaluaciones
- Completar las actividades usando la computadora.

IX. Aprobación de los padres de familia

He leído y entendido este permiso y las condiciones del estudio. Todas mis dudas se han aclarado. Entiendo todo lo que aparece arriba y doy mi consentimiento. Este estudio ha sido explicado a mi hijo(a) en un lenguaje que el/ella puede entender. Se le ha explicado que puede hacer preguntas tanto al comienzo del estudio como al finalizar el estudio.

____

Firma del padre de familia   Fecha

____

Nombre del niño(a)   Nombre de la Escuela

Se releva al Departamento de Educación de Puerto Rico de toda responsabilidad por cualquier reclamación que surja debido a este estudio. Si tengo alguna pregunta sobre el estudio o su procedimiento, puedo comunicarme con:

Jesus H Trespalacios____ — (540) 230-2394 / jtrespal@vt.edu
Investigador — Teléfono / e-mail

Ken Potter, Ph.D.________ — (540) 230-2394 / jtrespal@vt.edu
Profesor — Teléfono / e-mail

David M. Moore_______ — (540) 231-4991 / moored@vt.edu
Director, Virginia Tech Institutional — Télefono / e-mail
Board for the Protection of Human Subjects
Office of Research Compliance
1880 Pratt Drive, Suite 2006 (0497)
Blacksburg, VA 24061

Appendix Q

Assessment 1 in Spanish

Nombre ______________________________ **Fecha** __________

SECCION I

Lee cuidadosamente y responde a las preguntas. Marca el círculo que está en frente de la mejor respuesta.

---

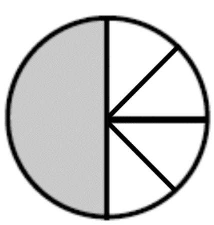
1. Mira la gráfica

¿Qué opción muestra la parte sombreada de la gráfica?

- ❍ 2 de 5
- ❍ 3 de 2
- ❍ 3 de 5
- ❍ Ninguna de las anteriores

---

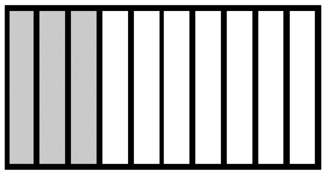
2. Mira la gráfica:



¿Qué opción muestra la parte sombreada de la gráfica?

- ❍ 1/5
- ❍ 4/5
- ❍ 1/4
- ❍ Ninguna de las anteriores

3. Mira la gráfica



¿Qué opción muestra la parte sombreada de la gráfica?

- ❍ 3/7
- ❍ 3/10
- ❍ 7/10
- ❍ Ninguna de las anteriores

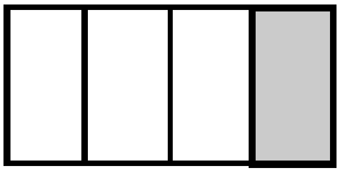
---

4. ¿Cuál de las opciones muestra 2 de 5?

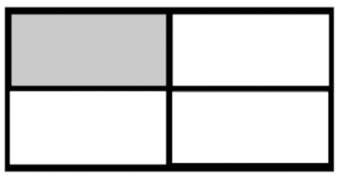
- ❍ 5/2
- ❍ 2/7
- ❍ 2/5
- ❍ Ninguna de las anteriores

5. ¿Cual opción **no** representa 1/4?

❍



❍



❍

❍ Ninguna de las anteriores

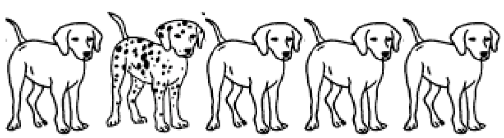
SECCION II

Lee cuidadosamente y responde a las preguntas. Marca el círculo que está en frente de la mejor respuesta.

---

6. ¿En cuál figura 1/4 de los perros tienen manchas?

❍

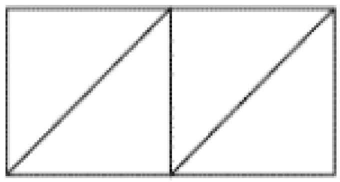

❍


❍

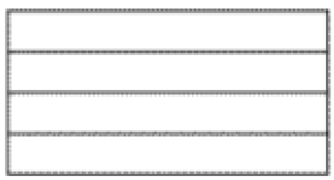

❍ Ninguna de las anteriores

7. Juan está partiendo una barra de chocolate en pedazos iguales. ¿Cual de estas divisiones Juan **no** debería usar?

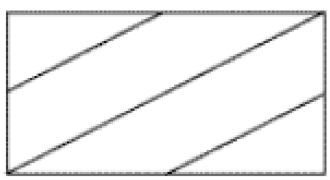
❍



❍

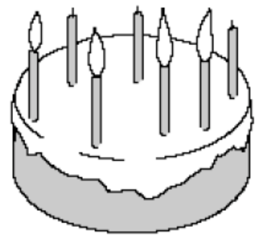


❍



❍ Ninguna de las anteriores

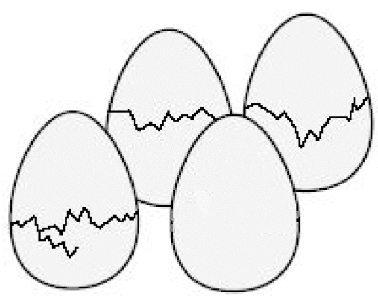
8. ¿Qué fracción de las velas en el pastel están prendidas?



- ❍ 3 de 7
- ❍ 4 de 3
- ❍ 4 de 7
- ❍ Ninguna de las anteriores

---

9. ¿Qué fracción de los huevos están partidos?

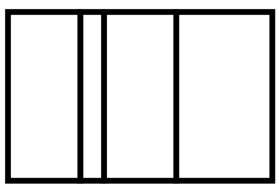
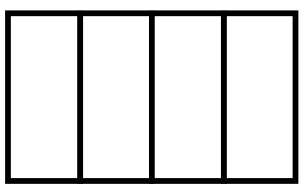


- ❍ 1/4
- ❍ 3/1
- ❍ 3/4
- ❍ Ninguna de las anteriores

---

10. La siguiente figura muestra 1 de 4 partes de una barra de chocolate que Juan le dejó a su hermano.

¿Cuál gráfica representa la barra completa de chocolate?

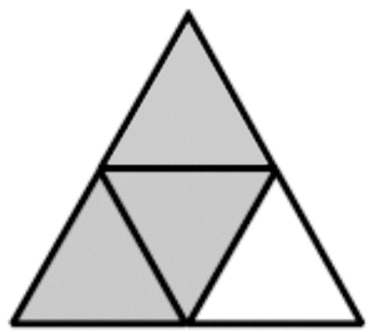
- ❍ 
- ❍ 
- ❍ 
- ❍ Ninguna de las anteriores

SECCION III

EJEMPLO

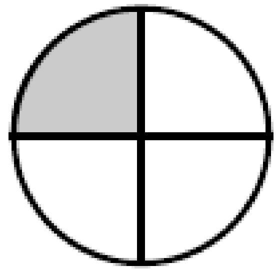
En las siguientes preguntas tu necesitarás crear ejemplos de tu vida diaria. Usa un fraccionario en tu ejemplo.

Por ejemplo, mira la siguiente gráfica:

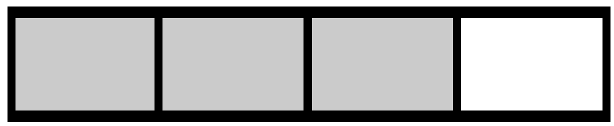


Un ejemplo de la vida diaria podría ser: Mi mamá hizo un bizcocho en forma de triangulo. Ella lo cortó 4 pedazos iguales. Mis amigos Iván, Darío y Jorge se comieron un pedazo cada uno. Ellos se comieron **3/4** o **3 de 4** pedazos del bizcocho.

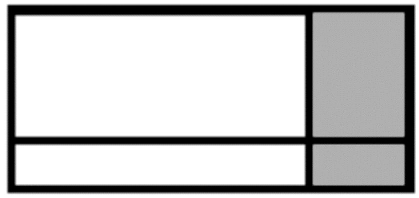
11. Mira la siguiente gráfica. Escribe una ejemplo de tu vida diaria. Usa un fraccionario en tu ejemplo. Si no es posible, por favor explica por qué.



12. Mira la siguiente gráfica. Escribe un ejemplo de tu vida diaria. Usa un fraccionario en tu ejemplo. Si no es posible, por favor explica por qué.



13. Mira la siguiente gráfica. Escribe un ejemplo de tu vida diaria. Usa un fraccionario en tu ejemplo. Si no es posible, por favor explica por qué.



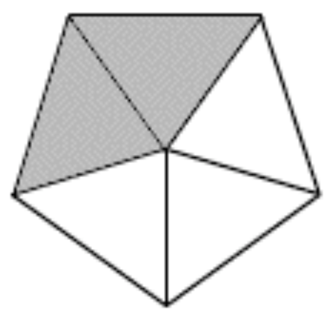
EJEMPLO

En las siguientes preguntas necesitarás crear diferentes ejemplos de tu vida diaria. Dibuja una gráfica que muestre este fraccionario.

Por ejemplo, mira el siguiente fraccionario:

2 de 5 o **2/5**

Un ejemplo de la vida diaria podría ser: Mi papá hizo una chiringa compuesta de 5 pedazos iguales. Mi papá le pintó 2 pedazos. Una gráfica de la chiringa podría ser:



14. Mira el siguiente fraccionario. Escribe un ejemplo de tu vida diaria. Dibuja también una gráfica que muestre este fraccionario. Si no es posible, por favor explica por qué.

1 de 4 o **1/4**

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15. Mira el siguiente fraccionario. Escribe una ejemplo de la vida real. Dibuja también una grafica que muestre tu fraccionario. Si no es posible, por favor explica por qué.

2 de 8 o **2/8**

Fin de la prueba.
¡Muchas gracias por trabajar duro y con cuidado!

Appendix R

Assessment 2 in Spanish

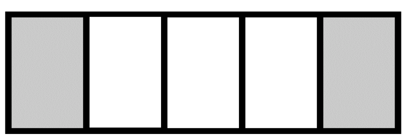
Nombre _______________________________________ Fecha ____________

SECCION I

Lee cuidadosamente y responde a las preguntas. Marca el círculo que está en frente de la mejor respuesta.

---

1. Mira la gráfica

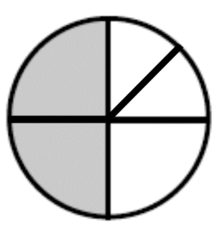


¿Qué opción muestra la parte sombreada de la gráfica?

- ❍ 2 de 5
- ❍ 3 de 5
- ❍ 2 de 3
- ❍ Ninguna de las anteriores

---

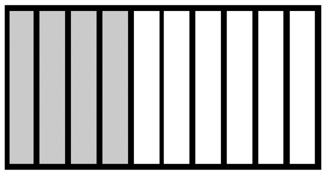
2. Mira la gráfica:



¿Qué opción muestra la parte sombreada de la gráfica?

- ❍ 2/5
- ❍ 3/5
- ❍ 2/3
- ❍ Ninguna de las anteriores

3. Mira la gráfica



¿Qué opción muestra la parte sombreada de la gráfica?

- ❍ 4/6
- ❍ 6/10
- ❍ 4/10
- ❍ Ninguna de las anteriores

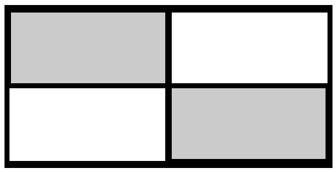
---

4. ¿Cuál de las opciones muestra 2 de 3?

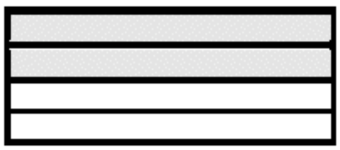
- ❍ 3/2
- ❍ 2/3
- ❍ 2/5
- ❍ Ninguna de las anteriores

5. ¿Cual opción **no** representa 2/4?

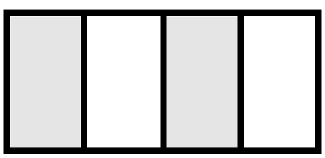
❍



❍



❍



❍ Ninguna de las anteriores

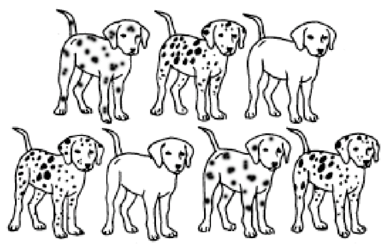
SECCION II

Lee cuidadosamente y responde a las preguntas. Marca el círculo que está en frente de la mejor respuesta.

---

6. ¿En cuál figura 2/5 de los perros tienen manchas?

❍



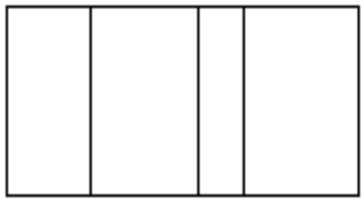
❍

❍

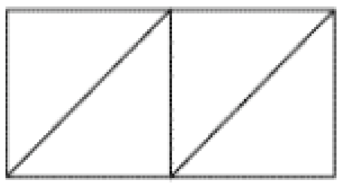
❍ Ninguna de las anteriores

7. Juan está partiendo una barra de chocolate en pedazos iguales. ¿Cuál de estas divisiones Juan **no** debería usar?

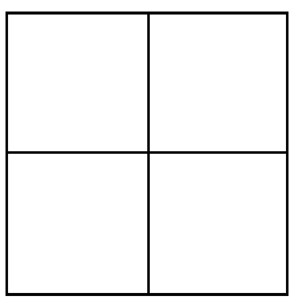
❍



❍

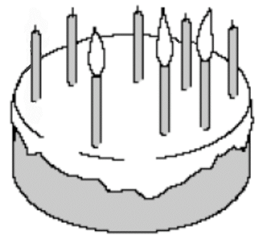


❍



❍ Ninguna de las anteriores

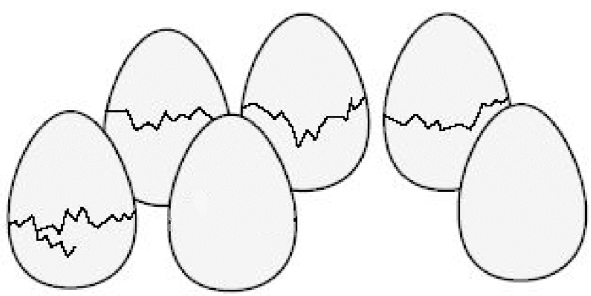
8. ¿Qué fracción de las velas en el pastel están prendidas?



- 3 de 8
- 5 de 8
- 3 de 5
- Ninguna de las anteriores

---

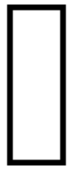
9. ¿Qué fracción de los huevos están partidos?



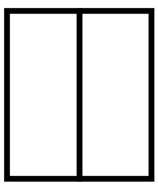
- 4/6
- 2/6
- 2/4
- Ninguna de las anteriores

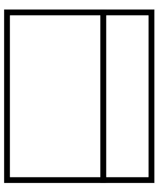
---

10. La siguiente figura muestra 1 de 2 partes de una barra de chocolate que Juan le dejó a su hermano.



¿Cuál gráfica representa la barra completa de chocolate?

❍ 

❍ 

❍ 

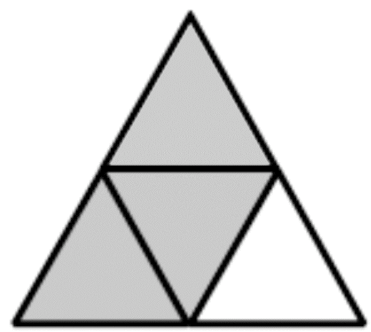
❍ Ninguna de las anteriores

SECCION III

EJEMPLO

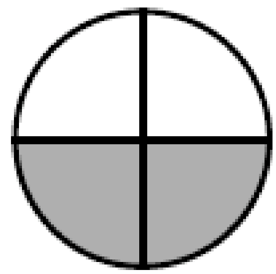
En las siguientes preguntas tu necesitarás crear ejemplos de tu vida diaria. Usa un fraccionario en tu ejemplo.

Por ejemplo, mira la siguiente gráfica:



Un ejemplo de la vida diaria podría ser: Mi mamá hizo una bizcocho en forma de triangulo. Ella lo cortó 4 pedazos iguales. Mis amigos Iván, Darío y Jorge se comieron un pedazo cada uno. Ellos se comieron **3/4** o **3 de 4** pedazos de la torta.

11. Mira la siguiente gráfica. Escribe una ejemplo de tu vida diaria. Usa un fraccionario en tu ejemplo. Si no es posible, por favor explica por qué.



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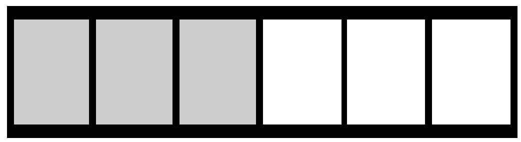
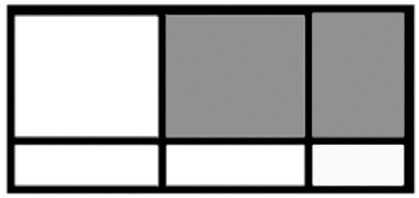
12. Mira la siguiente gráfica. Escribe un ejemplo de tu vida diaria. Usa un fraccionario en tu ejemplo. Si no es posible, por favor explica por qué.



13. Mira la siguiente gráfica. Escribe un ejemplo de tu vida diaria. Usa un fraccionario en tu ejemplo. Si no es posible, por favor explica por qué.



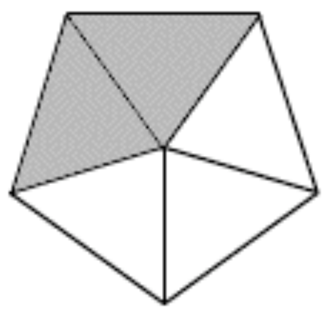
EJEMPLO

En las siguientes preguntas necesitarás crear diferentes ejemplos de tu vida diaria. Dibuja una gráfica que muestre este fraccionario.

Por ejemplo, mira el siguiente fraccionario:

2 de 5 o **2/5**

Un ejemplo de la vida diaria podría ser: Mi papá hizo una chiringa compuesta de 5 pedazos iguales. Mi papá le pintó 2 pedazos. Una gráfica de la chiringa podría ser:



14. Mira el siguiente fraccionario. Escribe un ejemplo de tu vida diaria. Dibuja también una gráfica que muestre este fraccionario. Si no es posible, por favor explica por qué.

2 de 4 o **2/4**

15. Mira el siguiente fraccionario. Escribe una ejemplo de la vida real. Dibuja también una grafica que muestre tu fraccionario. Si no es posible, por favor explica por qué.

4 de 8 o **4/8**

Fin de la prueba.
¡Muchas gracias por trabajar duro y con cuidado!

Appendix S

Assessment 3 in Spanish

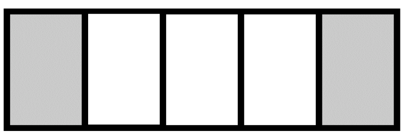
Nombre ______________________________________ **Fecha** ____________

SECCION I

Read carefully and answer each question. Fill in the circle next to the best answer.

---

1. Look at this picture:

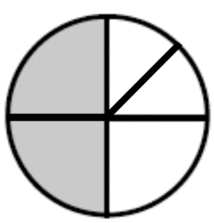


Which option shows the shaded part of the picture?

- ❍ 2 out of 5
- ❍ 3 out of 5
- ❍ 2 out of 3
- ❍ None of these answers

---

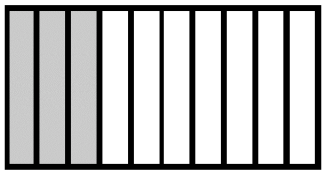
2. Look at this picture:



Which option shows the shaded part of the picture?

- ❍ 2/5
- ❍ 3/5
- ❍ 2/3
- ❍ None of these answers

3. Look at this picture.



Which fraction shows the shaded part of the figure?

- ❍ 3/7
- ❍ 3/10
- ❍ 7/10
- ❍ None of these answers

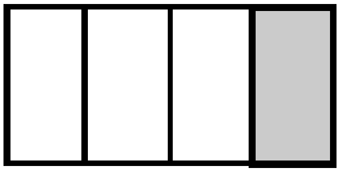
---

4. Which fraction means 2 parts out of 5?

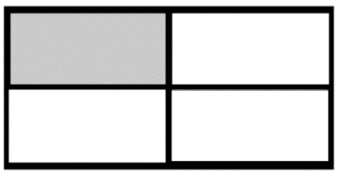
- ❍ 5/2
- ❍ 2/7
- ❍ 2/5
- ❍ None of these answers

5. Which option does **not** represent 1/4?

❍



❍



❍

❍ None of these answers

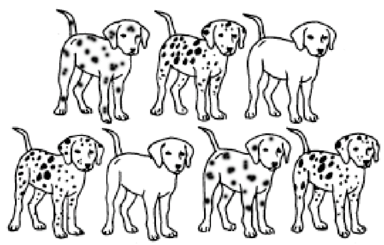
SECTION II

Read carefully and answer each question. Fill in the circle next to the best answer.

---

6. Which picture shows 2/5 of the dogs with spots?

❍

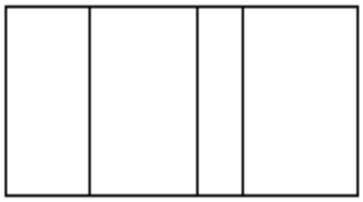


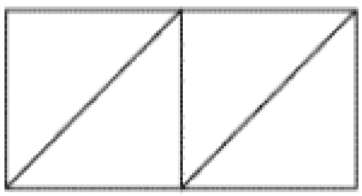
❍

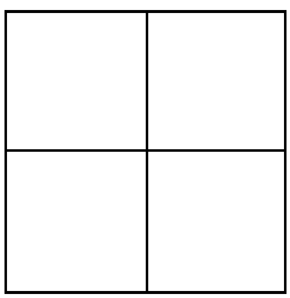
❍

❍ None of these answers

7. Judith is dividing a chocolate bar to share fairly. Which division should **not** Judith use?

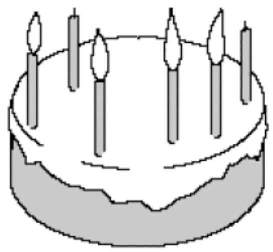
   ❍ 

   ❍ 

   ❍ 

   ❍ None of these answers

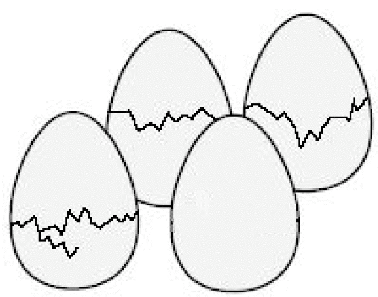
8. W hat fraction of the candles on the cake is lit?



- ❍ 2 out of 6
- ❍ 4 out of 6
- ❍ 2 out of 4
- ❍ None of the answers is correct

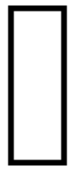
---

9. What fraction of the group of eggs is cracked?

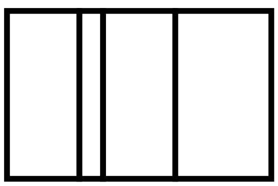


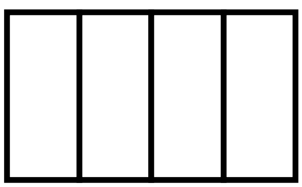
- ❍ 1/4
- ❍ 3/1
- ❍ 3/4
- ❍ None of these answers

10. The picture shows 1 out of 4 parts of a candy bar that John left for his brother.



What graphic represents the whole candy bar?

❍ 

❍ 

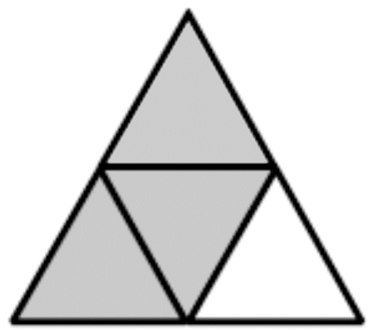
❍ 

❍ None of the answers is correct

SECTION III

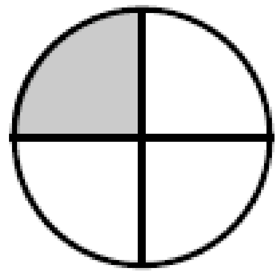
In the following questions you will need to create different short stories from the real life. Use a fraction number in your example.

For example, look at the following graphic:

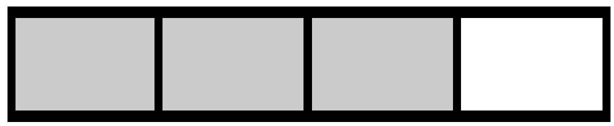


One example from real life could be: My mom made a cake as a triangle. She cut the cake in 4 pieces of the same size. My friends Jay, Peter and Michael ate one piece each. They ate **3/4** or **3 out of 4 pieces** of the cake.

11. Look at the following graphic. Write an example from real life. Use a fraction number in your example. If it is not possible, please explain why.



12. Look at the graphic. Write an example from real life. Use a fraction number in your example. If it is not possible, please explain why.



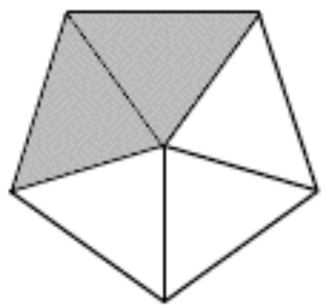
13. Look at the graphic. Write an example from real life. Use a fraction number in your example. If it is not possible, please explain why.

In the following questions you will need to create short stories from the real life. Draw a graphic that describes the fraction.

For example, look at the following fraction:

2 de 5 o **2/5**

One example from real life could be: My father constructed a kite made of 5 equal pieces . He painted two of those pieces. A graphic of the kite could be:



14. Look at the fraction. Write an example from real life. Draw also a graphic that describes the fraction. If it is not possible, please explain why.

1 out 4 or **1/4**

15. Look at the fraction. Write an example from real life. Draw also a graphic that describes the fraction. If it is not possible, please explain why.

4 out of 8 or **4/8**

This is the end of the test.
Thank you so much for working hard and thinking carefully!

Appendix T

Generating-examples activity in Spanish

ACTIVIDAD

SECCION I
Usa la actividad "Piezas de Fracciones" en el computador. Escribe tres oraciones diferentes que muestren relaciones entre el círculo y los pedazos de colores. Un ejemplo podría ser: Cinco pedazos amarillos cubren todo el círculo.

1.

2.

3.

Usa la misma actividad en el computador. Escribe tres oraciones diferentes que muestren relaciones entre los pedazos de círculo de diferentes colores. Un ejemplo podría ser: Dos pedazos negros cubren todo un pedazo amarillo.

4.

5.

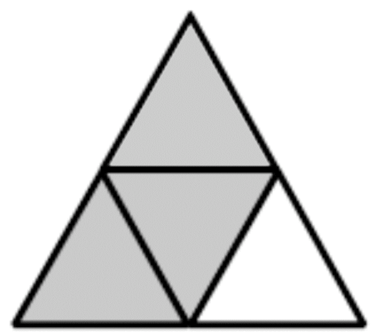
6.

SECCION II

EJEMPLO

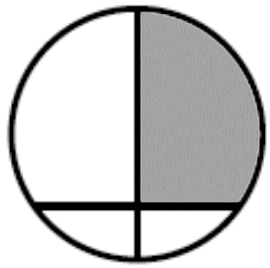
En las siguientes preguntas tu necesitarás crear ejemplos de tu vida diaria. Usa una fraccion en tu ejemplo.
Usa la actividad "Fracciones – Partes de la Unidad" en el computador para verificar tus respuestas.

Por ejemplo, mira la siguiente gráfica:

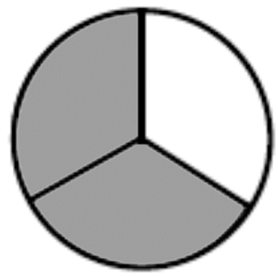


Un ejemplo de la vida diaria podría ser: Mi mamá hizo un bizcocho en forma de triangulo. Ella lo cortó 4 pedazos iguales. Mis amigos Iván, Darío y Jorge se comieron un pedazo cada uno. Ellos se comieron **3/4** o **3 de 4** pedazos del bizcocho.

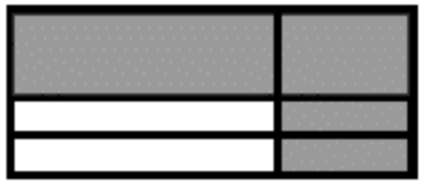
7. Mira la gráfica. Escribe una ejemplo de la vida real. Usa una fracción en tu ejemplo. Si no es posible, por favor explica por qué.



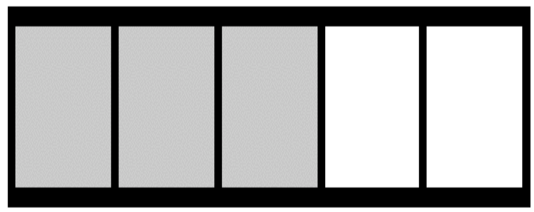
8. Mira la gráfica. Escribe una ejemplo de la vida real. Usa una fracción en tu ejemplo. Si no es posible, por favor explica por qué.



9. Mira la gráfica. Escribe una ejemplo de la vida real. Usa una fracción en tu ejemplo. Si no es posible, por favor explica por qué.



10. Mira la gráfica. Escribe una ejemplo de la vida real. Usa una fracción en tu ejemplo. Si no es posible, por favor explica por qué.



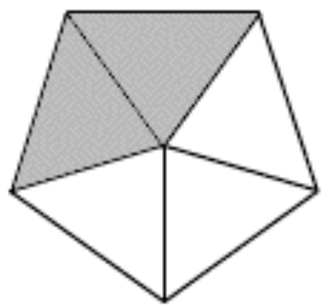
EJEMPLO

En las siguientes preguntas necesitarás crear diferentes ejemplos de tu vida diaria. Dibuja una gráfica que muestre esta fracción.

Por ejemplo, mira la siguiente fracción:

2 de 5 o **2/5**

Un ejemplo de la vida diaria podría ser: Mi papá hizo una chiringa compuesta de 5 pedazos iguales. Mi papá le pintó 2 pedazos. Una gráfica de la chiringa podría ser:



11. Mira la fracción. Escribe una ejemplo de la vida real. Dibuja también una grafica que muestre esta fracción. Si no es posible, por favor explica por qué.

1 de 6 o **1/6**

12. Mira la fracción. Escribe una ejemplo de la vida real. Dibuja una grafica que muestre esta fracción. Si no es posible, por favor explica por qué.

3/4 o **3 de 4**

Este es el final de la actividad.
¡Muchas gracias por trabajar duro y con cuidado!

Appendix U

Answering-questions activity in Spanish

ACTIVIDAD

SECCION I

Lee cuidadosamente y responde cada pregunta. Colorea el círculo que está al frente de la mejor respuesta. Usa la actividad "Piezas de Fracciones" en el computador.

1. ¿Cuántos pedazos azules necesitas para cubrir completamente el circulo?

- ❍ 2
- ❍ 5
- ❍ 7
- ❍ 9
- ❍ Ninguna de las anteriores

---

2. ¿Cuántos pedazos amarillos necesitas para cubrir completamente el círculo?

- ❍ 2
- ❍ 1
- ❍ 5
- ❍ 3
- ❍ Ninguna de las anteriores

---

3. ¿Cuántos pedazos marrones necesitas para cubrir completamente el círculo?

- ❍ 2
- ❍ 8
- ❍ 4
- ❍ 3
- ❍ Ninguna de las anteriores

---

4. ¿Cuántos pedazos azules necesitas para cubrir completamente un pedazo rosado?

- ❍ 3
- ❍ 2
- ❍ 6
- ❍ 0
- ❍ Ninguna de las anteriores

---

5. ¿Cuántos pedazos rosados necesitas para cubrir completamente un pedazo rojo?

- ❍ 2
- ❍ 1
- ❍ 0
- ❍ 3
- ❍ Ninguna de las anteriores

---

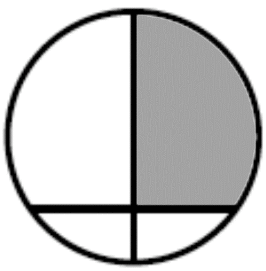
6. ¿Cuántos pedazos negros necesitas para cubrir totalmente un pedazo amarillo?

- ❍ 1
- ❍ 3
- ❍ 2
- ❍ 4
- ❍ Ninguna de las anteriores

SECCION II

Lee cuidadosamente y responde cada pregunta. Colorea el círculo que está al frente de la mejor respuesta. Usa la actividad llamada "Fracciones -Partes de la Unidad" en la computadora.

7. Mira la figura.



¿Qué opción muestra la parte sombreada de la figura?

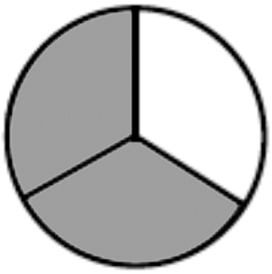
❍ 1/3

❍ 1/4

❍ 3/4

❍ Ninguna de las anteriores

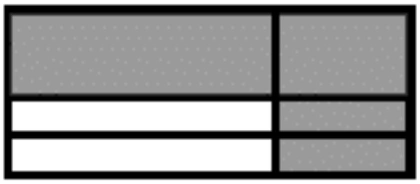
8. Mira la figura.



¿Qué opción muestra la parte sombreada de la figura?

- ❍ 1 de 2
- ❍ 2 de 3
- ❍ 1 de 3
- ❍ Ninguna de las anteriores

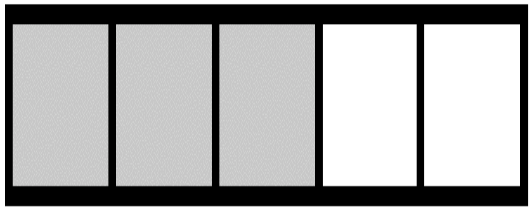
9. Mira la figura.



¿Qué opción muestra la parte sombreada de la figura?

- 2/4
- 2/6
- 4/6
- Ninguna de las anteriores

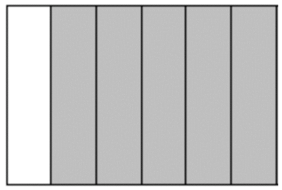
10. Mira la figura.

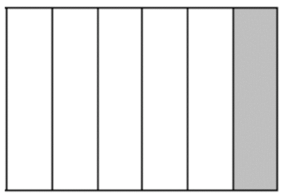


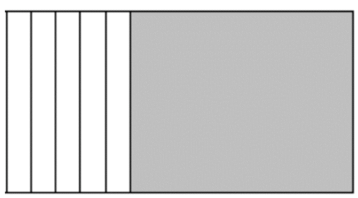
¿Qué opción muestra la parte sombreada de la figura?

- ❍ 3 de 2
- ❍ 2 de 3
- ❍ 3 de 5
- ❍ Ninguna de las anteriores

11. Que figura sombreada muestra la fracción 1 de 6?

❍


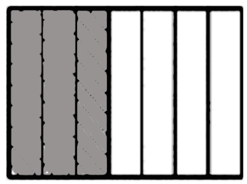
❍


❍


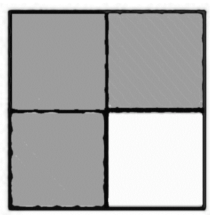
❍ Ninguna de las anteriores

12. Que figura sombreada muestra la fracción 3/4?

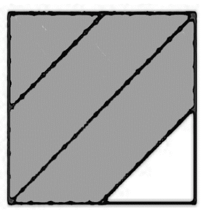
❍



❍



❍



❍ Todas las anteriores

Este es el final de la actividad.
Muchas gracias por trabajar duro y con cuidado!

Appendix V

Pretest, immediate posttest, and delayed posttest script in Spanish

Buenas tardes para todos.

Hoy vamos a trabajar contestando una actividad usando papel y lápiz. Estas actividades me ayudarán a entender cuanto …

- … saben sobre el concepto de fracciones (pre-prueba)
- … aprendieron de las actividades en la computadora relacionadas con fracciones (post-prueba inmediata)
- … recuerdan de las actividades en la computadora relacionadas con fracciones.

La prueba tiene 12 preguntas, algunas son de selección múltiple y otras son preguntas abiertas. En las preguntas de selección múltiple, lee cuidadosamente cada pregunta y llena el circulo que esta al frente de la opción que tu creas es la mejor respuesta. En las preguntas abiertas, necesitaras escribir ejemplos de tu vida diaria basado en una figura o en una fracción. En el cuestionario tu tendrás dos ejemplos que te ayudaran a escribir tus propias situaciones.

Esta todo claro? Tienen preguntas?

Por ultimo, por favor, intenten responder todas las preguntas en forma individual y en silencio. Déjenme saber levantando la mano desde tu silla si tienen alguna duda o sino entienden alguna pregunta del cuestionario.

Gracias.

Appendix W

Activity Script in Spanish

Buenos días.

Hoy vamos a trabajar con programas en la computadora.

El primer programa se llama "*Piezas de Fracciones*" y te ayudará a contestar la primera sección de la actividad (les mostraré el manipulativo virtual). Tu puedes usar el manipulativo de la siguiente manera (les mostraré un ejemplo de cada paso):

- Tu puedes darle un clic sobre la figura y automáticamente esta aparece en tu área de trabajo.
- Tu puedes seleccionar una figura del área de trabajo dando un clic sobre la figura. Una vez en el área de trabajo tu puedes mover la pieza tomándolas con el ratón y desplazarla al lugar que quieras.
- Tu también puedes rotar la figura poniendo el ratón en uno de los bordes de la figura. Cuando un pequeño circulo negro aparece, das un clic sosteniendo el botón del ratón y moviendo el ratón la pieza empieza a girar.
- Tu puedes limpiar tu área de trabajo dando un clic sobre el botón de "borrar" (les mostraré un ejemplo).
- Esta claro? Tienen alguna pregunta?

El segundo programa se llama "Fracciones Parte de la Unidad" y te ayudará a contestar la segunda sección de la actividad (les mostraré el manipulativo virtual). Tu puedes usar el manipulativo de la siguiente manera (les mostraré un ejemplo de cada paso):

- Tu puedes cambiar la figura (circulo o rectángulo) dando un clic sobre el botón "*Unidad Nueva".*
- Tu puedes cambiar el número de divisiones, dando un click en las flechas debajo de la figura. Dando un clic sobre la flecha que mira hacia arriba, las divisiones se incrementan en 1. Dando un clic sobre la flecha que mira hacia abajo, las divisiones se reducen en 1.
- Tu puedes sombrear las divisiones dando un clic sobre ellas y automáticamente las otras expresiones cambiarán.
- Esta claro? Tienen alguna pregunta?

Como dije antes, vas a usa los dos programas de computadora para completar las actividades que están en papel. Por favor, déjenme saber si tienen alguna pregunta en cualquier momento.

Appendix X

Statistics to determine the equivalence of the tests

Group Statistics

	STRATEGY	N	Mean	Std. Deviation	Std. Error Mean
DIFERENC	.00	31	-2.5484	3.24385	.58261
	1.00	29	-.0690	3.62463	.67308

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
									95% Confidence Interval of the Difference	
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
DIFERENC	Equal variances assumed	.120	.730	-2.796	58	.007	-2.4794	.88688	-4.25470	-.70414
	Equal variances not assumed			-2.785	56.220	.007	-2.4794	.89021	-4.26257	-.69627