# Appendix C

# The Higdon's absorbing boundary condition

This Appendix will derive expressions to predict field values at the absorbing boundaries from known field values inside the computation domain at previous time steps using one of the one wave equation techniques, namely the Higdon's absorbing boundary condition. The derivation will follow the exact notation in [27] .

Consider the wave equation

$$\left(\frac{\partial}{\partial x^2}+\frac{\partial}{\partial y^2}+\frac{\partial}{\partial z^2}-\frac{1}{\nu^2}\frac{\partial}{\partial t^2}\right)E=0 \tag{C.1}$$

The boundary condition proposed by Higdon for an outgoing wave in the x direction is given by

$$\prod_{i=1}^{p} B_i=\prod_{i=1}^{p}\left(\frac{\partial}{\partial x}+\frac{\cos\theta_1}{c}\frac{\partial}{\partial t}+\varepsilon_i\right)E=0 \tag{C.2}$$

where B is the boundary operator. This operator can perfectly absorb any linear combination of plane waves traveling at incidence angles $\theta_1$, $\theta_2$ ,....... $\theta_p$ . $\varepsilon_i$ is a damping factor to absorb D.C and low frequency components. Writing equation (C.2) in finite difference notation, one term in the operator can be written as

$$B_i = \frac{I - D^{-1}}{\Delta x}((1-a)I + aK^{-1}) + \frac{\cos\theta_i}{c}\left(\frac{I - K^{-1}}{\Delta t}\right)((1-b)I + bD^{-1}) + \varepsilon_i \qquad (C.3)$$

where the coefficients a and b are weighted time and space averages of the space and time differences, respectively. I, D and K are shift operators defined by

$$IE^n(i,j,k) = E^n(i,j,k), \quad DE^n(i,j,k) = E^n(i+1,j,k), \quad KE^n(i,j,k) = E^{n+1}(i,j,k) \qquad (C.4)$$

The operator in (C.3) can be written as

$$B_i = I - \alpha_i K^{-1} - \beta_i D^{-1} - \gamma_i D^{-1}K^{-1} \qquad (C.5)$$

where

$$\alpha_i = \frac{(a - g_i(1-b))}{(a - 1 - g_i(1-b) - \varepsilon_i \Delta l)}$$

$$\beta_i = \frac{(a - 1 + g_i b)}{(a - 1 - g_i(1-b) - \varepsilon_i \Delta l)}$$

$$\gamma_i = \frac{(-a - g_i b)}{(a - 1 - g_i(1-b) - \varepsilon_i \Delta l)} \qquad (C.6)$$

the parameter $g_i$ is

$$g_i = \frac{\cos\theta_i}{c}\frac{\Delta l}{\Delta t} \qquad (C.7)$$

For a first order boundary (p = 1), the field E at the mth space step and the nth time step can be expressed in terms of its neighbors along the normal to the boundary as

$$E^n(m,j,k) = \alpha_1 E^{n-1}(m,j,k) + \beta_1 E^n(m-1,j,k) + \gamma_1 E^{n-1}(m-1,j,k) \qquad (C.8)$$

For a second order boundary, the boundary operator becomes

$$B = B_1 B_2$$

$$= \begin{array}{c} I - (\alpha_1 + \alpha_2)K^{-1} + \alpha_1\alpha_2 K^{-2} - (\beta_1 + \beta 2)D^{-1} \\ +(\alpha_1\beta_2 + \alpha_2\beta_1 - \gamma_1 - \gamma_2)D^{-1}K^{-1} \\ +(\alpha_1\gamma_2 + \alpha_2\gamma_1)D^{-1}K^{-2} + \beta_1\beta_2 D^{-2} \\ +(\beta_1\gamma_2 + \beta_2\gamma_1)D^{-2}K^{-1} + \gamma_1\gamma_2 D^{-2}K^{-2} \end{array} \qquad (C.9)$$

applying the discrete operator to the electric field gives

$$
E^{n}(m,j,k) = \begin{array}{c} (\alpha_1+\alpha_2)E^{n-1}(m,j,k)+\alpha_1\alpha_2E^{n-2}(m,j,k)-(\beta_1+\beta 2)E^{n}(m-1,j,k) \\ +(\alpha_1\beta_2+\alpha_2\beta_1-\gamma_1-\gamma_2)E^{n-1}(m-1,j,k) \\ +(\alpha_1\gamma_2+\alpha_2\gamma_1)E^{n-2}(m-1,j,k)+\beta_1\beta_2E^{n}(m-2,j,k) \\ +(\beta_1\gamma_2+\beta_2\gamma_1)E^{n-1}(m-2,j,k)+\gamma_1\gamma_2E^{n-2}(m-2,j,k) \end{array} \qquad \text{(C.10)}
$$

It is important to note that in a TLM mesh, the voltage impulses incident on the absorbing boundary planes are function of both the tangential to the boundary electric and magnetic fields. Since both the tangential electric and magnetic fields satisfy the wave equation, the absorbing boundary conditions presented above can be applied to either of them or to a linear combination of them. Consequently, the absorbing boundary equations in (C.8) and (C.10) can directly be applied to the TLM voltage pulses.