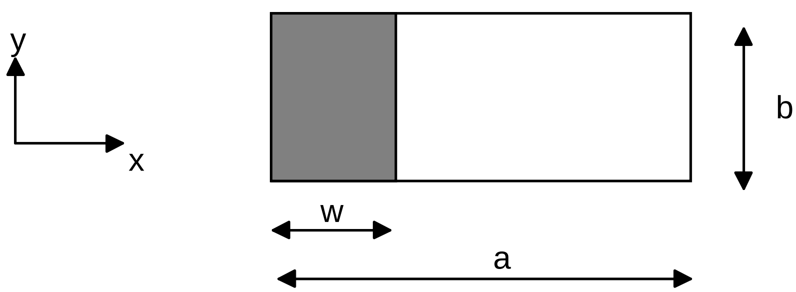
Appendix B

Formulation of the equivalent shunt inductance of a vanishingly thin inductive iris



This appendix derives an expression for the equivalent shunt inductance of a vanishingly thin inductive iris.

Assuming the waveguide is excited by a TE_{10} mode, the transverse components of the incident fields can be written as

$$E_y^i = \sin\frac{\pi x}{a} e^{-j\beta z} \quad \text{(B.1)}$$

$$H_x^i = -\frac{1}{Z_1}\sin\frac{\pi x}{a} e^{-j\beta z} \quad \text{(B.2)}$$

where

$$Z_1 = \frac{\omega\mu}{\beta_1}\ ,\ \beta_1 = \sqrt{\beta^2 - (\frac{\pi}{a})^2} \quad \text{and} \quad \beta = \omega\sqrt{\mu\varepsilon} \quad \text{(B.3)}$$

The discontinuity at the iris position will result in reflected and transmitted wave. However, because there is no y variation in the discontinuity [29], only TE_{n0} modes will be excited. The scattered fields can be written as

$$E_y^{scat} = \sum_{n=1}^{\infty} A_n \sin\frac{n\pi x}{a} e^{\pm j\beta_n z} \quad \text{(B.4)}$$

$$H_x^{scat} = \sum_{n=1}^{\infty} \pm\frac{A_n}{Z_n} \sin\frac{n\pi x}{a} e^{\pm j\beta_n z} \quad \text{(B.5)}$$

In the region $0 \le x \le w,\ 0 \le y \le b$, the boundary condition for H_x implies

$$H_x^{total}(x, y, 0^+) - H_x^{total}(x, y, 0^-) = K_y(x) \quad \text{(B.6)}$$

where $K_y(x)$ is the current flowing on the iris.

substituting from (B.5) into (B.6) yields

$$-\sum_{n=1}^{\infty} \frac{2}{Z_n} A_n \sin\frac{n\pi x}{a} = K_y(x) \quad \text{(B.7)}$$

from (B.7), the coefficient A_n can be obtained as

$$A_n = -\frac{Z_n}{a}\int_0^{\infty} k_y(x) \sin\frac{n\pi x}{a}\, dx \quad \text{(B.8)}$$

The boundary condition for the electric field implies

$$E_y^{total}(x, y, 0^-) = E_y^{total}(x, y, 0^+) \quad \text{(B.9)}$$

Substituting from (B.4) and (B.8) into (B.9) gives

$$\sin\frac{\pi x}{a} = \sum_{n=1}^{\infty} \frac{Z_n}{a} \sin\frac{n\pi x}{a} \int_0^{w} K_y(x) \sin\frac{n\pi x}{a}\, dx \quad \text{(B.10)}$$

Equation (B.10) can be solved for the current distribution $K_y(x)$ using the method of moments. Once the current distribution is obtained, the equivalent normalized shunt admittance can then be obtained from the reflection coefficient A_1 as

$$Y = \frac{-2A_1}{1 + A_1} \tag{B.11}$$

Method of moments formulation for equation (B.10)

Assume that the rectangular iris is divided into P rectangular segments and the current in each segment is assumed to be constant and equal to C_k, then the integral in (B.10) can be written as

$$\int_0^w \sin\frac{n\pi x}{a} k_y(x)\, dx = \sum_{k=1}^{p} C_k \int_{x_k}^{x_{k+1}} \sin\frac{n\pi x}{a} \tag{B.12}$$

where $x_1 = 0$, $x_{p+1} = w$ and $x_k = (k-1)w/p$. The R.H.S of equation (B.12) can be simplified as

$$\sum_{k=1}^{P} C_k \frac{a}{n\pi}\left[\cos\frac{n\pi x_k}{a} - \cos\frac{n\pi x_{k+1}}{a}\right] \tag{B.13}$$

Substituting in (B.10) yields

$$\sin\frac{\pi x}{a} = \sum_{k=1}^{P} C_k \sum_{n=1}^{\infty} \frac{Z_n}{n\pi} \sin\frac{n\pi x}{a}\left[\cos\frac{n\pi x_k}{a} - \cos\frac{n\pi x_{k+1}}{a}\right] \tag{B.14}$$

Substituting with a set of Q points exactly in the middle of the segments $x_q = (2q-1)\frac{w}{p}$

$$\sin\frac{\pi x_q}{a} = \sum_{k=1}^{P} C_k \sum_{n=1}^{\infty} \frac{Z_n}{n\pi} \sin\frac{n\pi x_q}{a}\left[\cos\frac{n\pi x_k}{a} - \cos\frac{n\pi x_{k+1}}{a}\right] \tag{B.15}$$

Equation (B.15) can be written in a matrix form as

$$[b]_{Q\,x\,1} = [A]_{Q\,x\,P}\,[C]_{P\,x\,1} \tag{B.16}$$

where

$$b_q = \sin\frac{\pi x_q}{a} \tag{B.17}$$

$$a_{k,q} = \sum_{n=1}^{\infty} \frac{Z_n}{n\pi} \sin\frac{n\pi x_q}{a}\left[\cos\frac{n\pi x_k}{a} - \cos\frac{n\pi x_{k+1}}{a}\right] \tag{B.18}$$

Equation (B.16) can be solved for the current coefficients C_k, equation (B.8) can then be used to calculate the reflection coefficient A_1. Knowing the reflection coefficient A_1, equation (B.11) can then be used to calculate the equivalent normalized shunt admittance.