Chapter 3

The Transient Frequency Domain TLM Approach TFDTLM

3.1 Introduction

The TLM was initially formulated and developed in time domain. One key issue in a time domain analysis approach is the computational efficiency where a single impulsive excitation could yield information over a wide frequency range. Also, it may be more natural and realistic to model nonlinear and dispersive properties in the time domain than in the frequency domain. However, in some circumstances, frequency domain analysis may be more appealing. This might be due to the fact that the traditional teaching of electromagnetics emphasizes frequency domain concepts as complex frequency dependent impedances, reflection coefficients and frequency dispersive constitutive parameters. It might be even easier and more direct to be able to model these parameters in the frequency domain rather than trying to synthesize an equivalent time domain method. In some situations, the steady state response is required to be calculated only at distinct frequency points which makes it very inefficient to calculate all the unnecessary transients that are usually associated with an impulsive excitation in a time domain TLM (TDTLM) mesh. This was the motivation of the work done by *Jin and Vahldiek* in 1992 [11]. Their proposed approach simply uses the same TLM network as in TDTLM, the only difference is that the mesh is excited with an impulsive train of a sinusoidally modulated amplitude used to simulate a sinusoidal excitation. Consequently, the transfer characteristic of the simulated structure at the frequency of excitation is directly contained in the magnitude of the output waveform. The other FDTLM approach was introduced by *Johns* and *Christopoulos* [12-13]. The heart of the approach was the formulation of a set of complex frequency dependent simultaneous equations involving the incident voltage at each node and the source or excitation nodes. The set of equations are then solved at each frequency for the incident voltages using the Jacobi method or the conjugate gradient method.

3.2 The new FDTLM approach

In this work, a new frequency domain TLM (FDTLM) approach which combines the superior features of both the time domain and frequency domain TLM is introduced. The approach is based on a steady state analysis in the frequency domain using transient analysis techniques and hence is referred to as TFDTLM. In this approach, the link lines impedance are derived in the frequency domain as in [13] and are chosen to model the frequency dispersive material parameters. The impedance and propagation constants are allowed to be complex and consequently provide more accurate modeling for wave propagation in a frequency dispersive medium. The approach was inspired by the concept of bounce diagram in time domain and the equivalent frequency domain bounce diagram.

The time domain bounce diagram is a representation of the back and forth travel of a pulse through a structure with discontinuities causing reflections. The bounce diagram representation can be demonstrated by the following example. Consider a section of transmission line with characteristic impedance Z and length l having discontinuities at both ends as shown in Figure (3.1). An impulse incident at time t = 0 and x = 0 propagates to the end of the line in time T. When it reaches the end discontinuity, it reflects back and propagates towards the excitation point, hits the discontinuity and reflects again and so on. The steady state positive going waveform at the source position can be expressed as

$$v_o^i(t) = \delta(t) + \rho_1\rho_2\delta(t-2T) + (\rho_1\rho_2)^2\delta(t-4T) + (\rho_1\rho_2)^3\delta(t-6T) + \ldots. \qquad (3.1)$$

where ρ_1 and ρ_2 are the reflection coefficients at the two end discontinuities. It is important to note that in the expression in (3.1), the reflection coefficients are assumed to be frequency independent. Otherwise, all the multiplication operations in (3.1) should be converted to convolution operations. The bounce diagram can be also formulated in the frequency domain by expressing every delay by the propagation factor $e^{-j\beta l}$ where l is the length of the line and β is the wavenumber or propagation constant. The frequency response of the positive going waveform can then be expressed as

$$\begin{aligned} V_0^i(j\omega) &= 1 + \rho_1(j\omega)\rho_2(j\omega)e^{-2j\beta l} + (\rho_1(j\omega)\rho_2(j\omega))^2 e^{-j4\beta l} + \ldots\ldots. \\ &= \sum_{k=0}^{k=\infty} (\rho_1(j\omega)\rho_2(j\omega))^k e^{-j2k\beta l} \end{aligned} \qquad (3.2)$$

The frequency domain expression in equation (3.2) holds whether or not the reflection coefficients are frequency dependent. From the above expression, it appears that if the

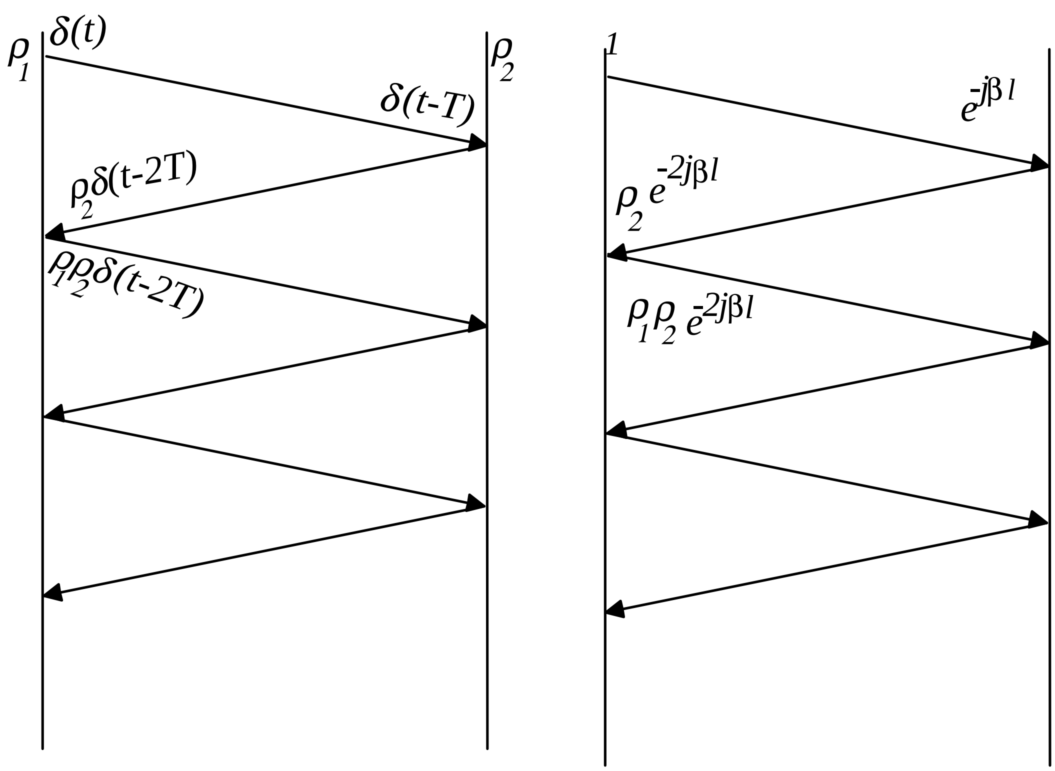


Figure 3.1 Bounce Diagram

(a) Time Domain (b) Frequency domain

reflection coefficients ρ_1 and ρ_2 are frequency independent and the scattering coefficients $(\rho_1\rho_2)^k$ at every iteration are stored, then the frequency response at any frequency point can be calculated by summing up the product of the scattering coefficient at iteration k and the propagation factor ($e^{-j\beta l}$) raised to the power 2k. Consequently, once these scattering coefficients are calculated at every iteration, there is no need to redo the simulation at every frequency point. The special feature of the TFDTLM approach introduced in this work is that it makes it possible to extract all the frequency domain information of interest over a relatively wide frequency range by performing only one simulation even if the reflection coefficients are complex and/or when dealing with inhomogeneous media with different propagation constants. This is considered the most special feature of the TFDTLM technique which makes it superior in terms of computational efficiency as compared to other frequency domain TLM approaches.

3.3 Derivation of the transient frequency domain TLM (TFDTLM)

The electrical properties of each line are indicated by three subscripts, the first subscript d indicates normalized quantities per unit length, the two following subscripts indicates line direction and polarization respectively. L, R, C, and G represent series inductance, resistance, shunt capacitance, and conductance, respectively. In terms of these quantities, the characteristic impedance of a line along the y direction carrying an x polarization is given by

$$Z_{yx} = \sqrt{\frac{R_{dyx} + j\omega L_{dyx}}{G_{dyx} + j\omega C_{dyx}}} \tag{3.3}$$

and the propagation constant along the line is given by

$$\gamma_{yx} = \sqrt{(R_{dyx} + j\omega L_{dyx})(G_{dyx} + j\omega C_{dyx})} \tag{3.4}$$

The overall capacitance and conductance of all lines responsible for an E_x polarization should satisfy the medium permitivity and conductivity as follows

$$(G_{dyx} + j\omega C_{dyx})\Delta y + (G_{dzx} + j\omega C_{dzx})\Delta z = (\sigma + j\omega\varepsilon)S_x \tag{3.5}$$

Substitution from equations (3.3) and (3.4) into (3.5) yields

$$\frac{\gamma_{yx}\Delta y}{Z_{yx}} + \frac{\gamma_{zx}\Delta z}{Z_{zx}} = (\sigma + j\omega\varepsilon)S_x \tag{3.6}$$

and setting the condition

$$\gamma_{xy}\Delta x = \gamma_{xz}\Delta x = \gamma_{yx}\Delta y = \gamma_{yz}\Delta y = \gamma_{zx}\Delta z = \gamma_{zy}\Delta z = \gamma\Delta l \tag{3.7}$$

The condition in (3.7) is equivalent to the synchronization condition in the time domain TLM, γ is the propagation constant in the medium and Δl is an effective cell dimension, substituting from (3.7) into (3.6) gives

$$\frac{1}{Z_{yx}}+\frac{1}{Z_{zx}}=\frac{S_x}{Z\Delta l} \tag{3.8}$$

where Z is the intrinsic impedance of the medium given by $\sqrt{\frac{j\omega\mu}{\sigma+j\omega\varepsilon}}$, and the permeability μ can be complex if magnetic losses exist. Writing similar expressions for the characteristic impedances of all lines responsible for an E_y or an E_z polarization, it can easily be shown that

$$\frac{1}{Z_{xy}}+\frac{1}{Z_{zy}}=\frac{S_y}{Z\Delta l} \tag{3.9}$$

$$\frac{1}{Z_{xz}}+\frac{1}{Z_{yz}}=\frac{S_z}{Z\Delta l} \tag{3.10}$$

The inductance and resistance of all the link lines responsible for an I_x current must satisfy

$$(R_{dyz}+j\omega L_{dyz})\Delta y+(R_{dzy}+j\omega L_{dzy})\Delta z=j\omega\mu S_x \tag{3.11}$$

Substituting from (3.3), (3.4), and (3.7) into (3.11) yields

$$Z_{yz}+Z_{zy}=Z\frac{S_x}{\Delta l} \tag{3.12}$$

Similarly, for the link lines responsible for I_y and I_z, the following two equations can be obtained respectively

$$Z_{xz}+Z_{zx}=Z\frac{S_y}{\Delta l} \tag{3.13}$$

$$Z_{xy}+Z_{yx}=Z\frac{S_z}{\Delta l} \tag{3.14}$$

Summarizing the set of equations that need to be satisfied in order to completely model all the constitutive parameters of the medium

$$\frac{1}{\hat{Z}_{yx}}+\frac{1}{\hat{Z}_{zx}}=\frac{S_x}{\Delta l} \tag{3.15.a}$$

$$\frac{1}{\hat{Z}_{xy}}+\frac{1}{\hat{Z}_{zy}}=\frac{S_y}{\Delta l} \tag{3.15.b}$$

$$\frac{1}{\hat{Z}_{xz}}+\frac{1}{\hat{Z}_{yz}}=\frac{S_x}{\Delta l} \tag{3.15.c}$$

$$\hat{Z}_{yz}+\hat{Z}_{zy}=\frac{S_x}{\Delta l} \tag{3.15.d}$$

$$\hat{Z}_{xz} + \hat{Z}_{zx} = \frac{S_y}{\Delta l} \quad (3.15.e)$$

$$\hat{Z}_{xy} + \hat{Z}_{yx} = \frac{S_z}{\Delta l} \quad (3.15.f)$$

where $\hat{Z}$ denotes an impedance normalized by the complex intrinsic impedance of the medium. The equations above are similar to the set of equations for the link lines inductance and capacitance required to satisfy the medium permeability and permitivity, respectively. The only difference is that the link line impedances in the TFDTLM are allowed to be complex. In addition, the equations above not only satisfy the medium permeability and permitivity but the electric and magnetic losses as well.

The above set of equations can be satisfied in more than one way. One way is to choose the link line impedances to satisfy equations (3.15.a - c) thereby exactly modeling the medium permitivity and conductivity. Then the deficiency in satisfying equations (3.15.d - e) can be accounted for by adding short circuited stubs with complex characteristic impedance. This would be equivalent to Type II HSCN and might be referred to as type II FDHSCN. The second alternative is to satisfy equations (3.15.d - e) for the medium permeability and magnetic losses and the deficiency from (3.15.a - c) can then be compensated by open circuited stubs of complex characteristic impedance. The third alternative is to satisfy the six equations simultaneously in a way similar to the SSCN. It is worth mentioning that for a uniform cell, the normalized impedances of all link lines will be identical and equal to unity, the equivalent cell dimension $\Delta\ell$ will be equal = $0.5\Delta x = 0.5\Delta y = 0.5\Delta z$, the factor of 1/2 confirms that the velocity of the bulk waves on the transmission line mesh is one half the velocity of the waves on the individual transmission lines which agrees with the slow wave nature of a TDTLM mesh that was also demonstrated for *John's* FDTLM [13].

Irrespective of the method used to solve for the link line impedance, equations (3.15.a-f) show that for one homogeneous medium, the link line normalized impedances are all frequency independent and, consequently, so is the scattering matrix of the TFDTLM. Transition from one node to the next is accounted for by multiplication by the factor $e^{-\gamma\Delta l}$, which means that from one iteration to the next, the scattered pulses are modified by the factor $e^{-\gamma\Delta l}$ to become incident on the next neighboring cell. Therefore, if the scattering coefficients at the observation point are stored at each iteration, then the frequency response at any frequency of interest at the observation point can be obtained by multiplying the value stored at the first iteration by $e^{-\gamma\Delta l}$. The value at the at the second iteration is multiplied by $e^{-2\gamma\Delta l}$ and so on. The final result is then obtained by summing. An important question now arises , what if the problem under consideration has inhomogeneous regions which would result in different propagation constants in different regions as well as frequency dependent reflection coefficients, can we still apply the TFDTLM approach and perform only one simulation to extract all the frequency information of interest. This will be discussed in the next section.

3.4 TFDTLM in an inhomogeneous medium

The technique proposed to overcome the problem of inhomogeneous media, multiple propagation factors and frequency dependent reflection coefficients involves approximating all propagation factors in a TFDTLM mesh in terms of the propagation factor of some reference medium chosen to be the medium with the least propagation delay. Consider having two media 1 and 2 with propagation constants γ_1 and γ_2, respectively, and medium 1 has the least propagation delay. Then the propagation constant in medium 2 is approximated in terms of the propagation constant in medium 1 as

$$e^{-\gamma_2 l_2} = e^{-m\gamma_1 l_1} \frac{a_0 + a_1 e^{-\gamma_1 l_1} + a_2 e^{-2\gamma_1 l_1} + ... + a_n e^{-n\gamma_1 l_1}}{b_o + b_1 e^{-\gamma_1 l_1} + b_2 e^{-2\gamma_1 l_1} + ... + b_n e^{-n\gamma_1 l_1}} \quad m = 0, 1, 2...... \tag{3.16}$$

The filter coefficients are then obtained by minimizing the mean square error between the actual propagation factor of the medium and the approximated propagation factor over the frequency range of interest. The integer m is chosen to provide extra phase change (increase filter order) that can be implemented with significantly fewer computations than by increasing the integer n. Assuming m = 1, this technique is implemented in a TLM mesh similar to the implementation of a digital filter in a digital filter processing application [20]. Figure (3.2) shows two possible ways of implementing the filter.

Direct form II involves less computations than direct form I and less storage as well. For this reason, direct form II was chosen to be implemented in the TFDTLM scheme. Consider a link line in medium 2 between cells a and b. In a TDTLM mesh, the connection between two adjacent cells is implemented as follows

$$v_b^i(t + \Delta t) = v_a^r(t) \tag{3.17}$$

In a TFDTLM, the corresponding expression will be in the form

$${}^{k+1}V_b^i = e^{-\gamma_2 l_2}\ {}^{k}V_a^r \tag{3.18}$$

where k is the iteration number. Assuming the propagation constant in medium 2 is approximated in terms of that of medium 1 for m =1 and using direct form II realization, the incident voltage at node b at iteration k can be obtained as follows

$$w_0^k = \frac{1}{b_0}\left[{}^{k}V_a^r - b_1 w_1^k - b_2 w_2^k + ... - b_n w_n^k \right] \tag{3.19}$$

$${}^{k+1}V_b^i = e^{-\gamma_1 l_1}\left[a_o w_o^k + a_1 w_1^k + a_2 w_2^k ... + a_n w_n^k \right] \tag{3.20}$$

In the actual simulation, the multiplication by the factor $e^{-\gamma_1 l_1}$ is not performed at every iteration. Instead, it is only done once at the end of the simulation, where the frequency response of the voltage at the observation point is obtained as follows : the value stored at

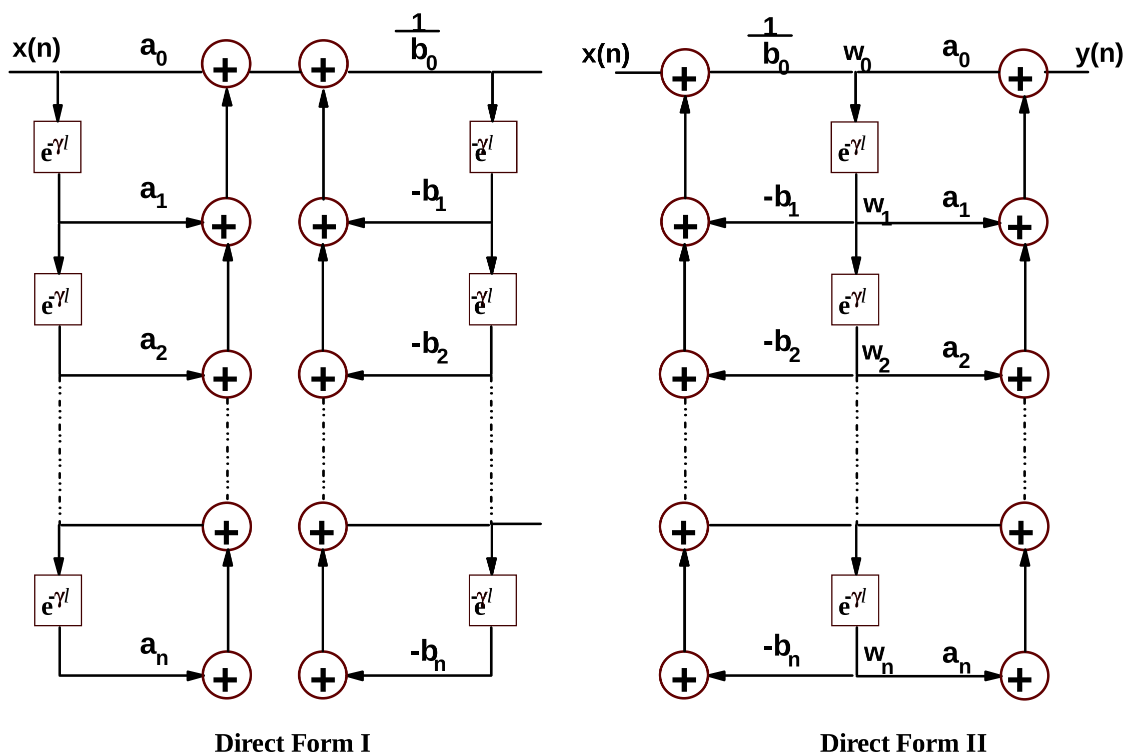


Figure 3.2 Implementation of the approximation filter in a FDTLM mesh

iteration 1 is multiplied by $e^{-\gamma_1 l_1}$ and that at iteration 2 by $e^{-2\gamma_1 l_1}$, etc. These terms are then summed . The values of the intermediate variables w_n are updated at each iteration by pushing them one step downwards to simulate the multiplication by the factor $e^{-\gamma_1 l_1}$ as follows

$$w_n^{k+1} = w_{n-1}^{k},\ w_{n-1}^{k+1} = w_{n-2}^{k},\ \ldots w_1^{k+1} = w_0^{k} \tag{3.21}$$

By the same token, complex frequency reflection coefficients at the interface between two different media can also be approximated by a similar filter and implemented the same way.

3.5 Derivation of the approximation filter coefficients

As mentioned before, the basic idea of the TFDTLM is to be able to extract all the frequency domain information in the entire frequency range of interest by performing only one simulation. This is achieved by expressing the propagation delay in all media in terms of the propagation delay of some reference medium, the medium with the least propagation delay.

Let's assume that medium 2 is any medium with frequency dispersive constitutive parameters, and medium 1 is the reference medium, the goal is to be able to express the propagation delay in medium 2 as follows

$$e^{-\gamma_2 l_2} = e^{-m\gamma_1 l_1}\frac{a_0 + a_1 e^{-\gamma_1 l_1} + a_2 e^{-2\gamma_1 l_1} + \ldots. + a_n e^{-n\gamma_1 l_1}}{b_o + b_1 e^{-\gamma_1 l_1} + b_2 e^{-2\gamma_1 l_1} + \ldots. + b_n e^{-n\gamma_1 l_1}} \quad m = 0, 1, 2\ldots\ldots \tag{3.22}$$

The approximation filter coefficients are then obtained by minimizing the root mean square error between the actual propagation delay and the approximated one over the frequency range of interest. The mean square error is given by

$$\sum_{\kappa=1}^{K}\left| e^{-y_k} - e^{-mx_k}\frac{a_0 + a_1 e^{-x_k} + a_2 e^{-2x_k} + \ldots. + a_n e^{-nx_k}}{b_o + b_1 e^{-x_k} + b_2 e^{-2x_k} + \ldots. + b_n e^{-nx_k}}\right|^2 \tag{3.23}$$

where $x = \gamma_1 l_1$, $y = \gamma_2 l_2$ and k is the number of frequency points

For simplicity, only the numerator of the above function will be minimized. This will consequently minimize the whole function.

For the above function to be minimized, the set of derivatives with respect to the set of coefficients a_0, a_1, ------- a_n, b_0, b_1, ---------- b_n must be zero i.e.

$$\frac{\partial}{\partial a_i} = 0 \quad i = 0, 1, 2 --m \qquad \frac{\partial}{\partial b_j} = 0 \quad j = 0, 1, 2 --m \tag{3.24}$$

$$\frac{\partial}{\partial a_i} = \sum_{k=1}^{K} \{e^{-y_k}(b_0 + b_1 e^{-x_k} + \text{——} b_n e^{-nx_k}) - e^{-n x_k}(a_0 + a_1 e^{-x_k} + \text{——} a_n e^{-nx_k})\}.(-2e^{-(n+i)x_k}) \tag{3.25}$$

$$\frac{\partial}{\partial b_j} = \sum_{k=1}^{K} \{e^{-y_k}(b_0 + b_1 e^{-x_k} + \text{——} b_n e^{-nx_k}) - e^{-n x_k}(a_0 + a_1 e^{-x_k} + \text{——} a_n e^{-nx_k})\}.(-2e^{-(y_k+jx_k})) \tag{3.26}$$

From the above set of derivatives, a set of 2n+2 in 2n+2 unknowns, the filter coefficients, can be obtained. The set of equations can be written in a matrix form as

$$A_{2n+2\ *\ 2n+2}\ C_{2n+2\ *\ 1} = 0 \tag{3.27}$$

where C is a column vector having the filter coefficients in the following order

$$C = \begin{bmatrix} a_0 \\ a_1 \\ . \\ . \\ a_n \\ b_0 \\ . \\ . \\ b_n \end{bmatrix}$$

A is an 2n+2 by 2n+2 matrix whose coefficients have the form

$$A_{p,q} = \sum_{k=1}^{K} 2e^{-(2n+p+q)x_k} \qquad 0 \le p \le n \quad 0 \le q \le n \tag{3.28a}$$

$$A_{p,q} = \sum_{k=1}^{K} 2e^{-\{(n+p+q-(m+1))x_k+y_k\}} \qquad 0 \le p \le n \quad n \le q \le 2n+1 \tag{3.28b}$$

$$A_{p,q} = \sum_{k=1}^{K} 2e^{-\{(n+p+q-(m+1))x_k+y_k\}} \qquad n \le p \le 2n+1 \quad 0 \le q \le n \tag{3.28c}$$

$$A_{p,q} = \sum_{k=1}^{K} 2e^{-\{(n+p+q-(m+1))x_k+y_k\}} \qquad n \le p \le 2n+1 \quad n \le q \le 2n+1 \tag{3.28d}$$

Equation (3.27) has a nontrivial optimum solution only if the determinant of the matrix A is identically zero. However, this is not necessarily the case. In order to get a near optimum solution, the eigenvalue of matrix A that has minimum absolute value (closest to zero) is calculated. The corresponding eign vector would consequently make the L.H.S of equation (3.27) closest to zero and therefore is considered a near optimum solution for the set of filter coefficients.

3.5 On the accuracy of the approximation filter

This section will investigate the accuracy of the filter coefficients in approximating the propagation constant of lossless inhomgeneous media as well as lossy inhomogeneous media. The approximation filter coefficients are optimized in a frequency range where the maximum cell dimension is less than 0.125 the corresponding wavelength. For a satisfactory order of accuracy, the TLM node is usually operated in a frequency range where the maximum cell dimension is less than 0.1 times the corresponding wavelength.

Figure (3.3) shows the error obtained from a first order filter in approximating the propagation constant of a lossless inhomgeneous medium with different relative dielectric constant. The filter used has the form F_1 given by

$$F_1 = e^{-\gamma_1 l_1} \frac{a_0 + a_1 e^{-\gamma_1 l_1}}{b_o + b_1 e^{-\gamma_1 l_1}} \tag{3.29}$$

Figure (3.3) shows the error obtained from a first order approximation filter for $\varepsilon_r = 5$ is always less than 1e-3 % which is a very small amount of error. The error reaches a maximum of 5e-3 % for $\varepsilon_r = 10$ and is still less than 1e-2 for $\varepsilon_r = 20$. These extremely small amount of errors emphasize the superior performance of a first order filter in approximating the propagation constant of inhomgeneous media even with a relatively high dielectric constant. The fact that only a first order approximation filter can provide a satisfactory order of accuracy will help enhance the computational efficiency of the TFDTLM.

Figure (3.4a) and (3.4b) show the error obtained from a first and second order filter (with m =1), respectively in approximating a lossless medium with $\varepsilon_r = 80$. Figure (3.4a) shows that even for such a high relative dielectric constant, a first order approximation filter can provide an error less than 0.02 % which is still a low level of error. Figure (3.4b) shows that a second order filter can drop the level of the maximum error to less than 2.5e-4 %. It is worth mentioning that the choice between a first and second order filter in an actual simulation would be a matter of compromise, for although a second order filter can provide less amount of error, it would degrade the computational efficiency as compared to a first order filter. Also the amount of error obtained from a first order filter is still very satisfactory.

Figure (3.5) shows the amount of error obtained from a second order filter approximation for $\varepsilon_r = 10$ and three different conductivities $\sigma = 0.025$ s/m, $\sigma = 1$ s/m and $\sigma = 2$ s/m. It

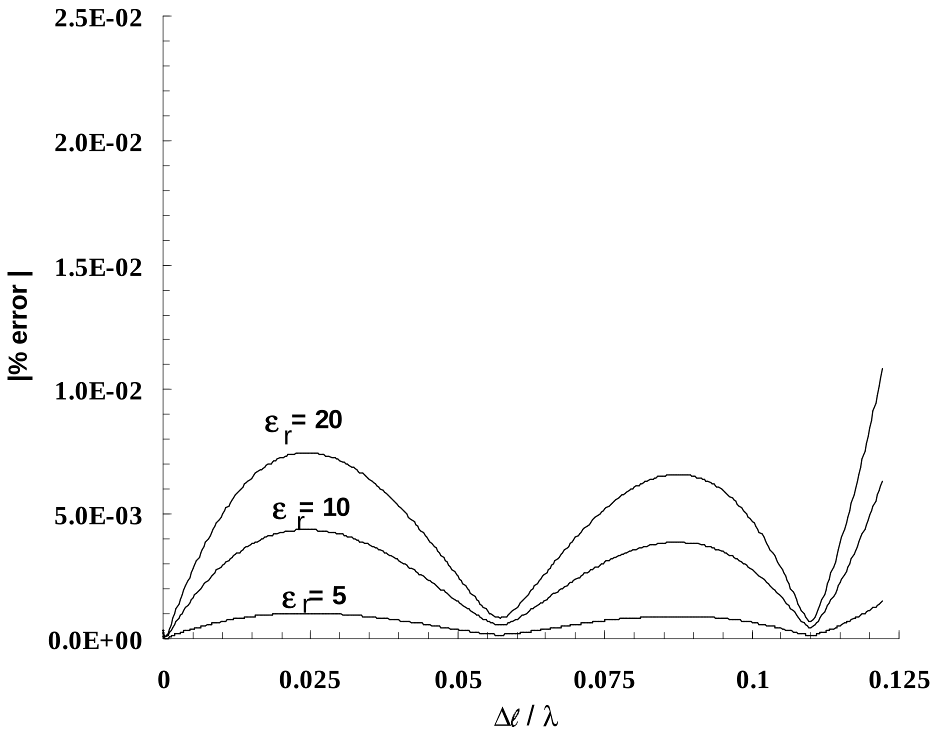


Fig 3.3 Error obtained from a first order approximation filter
for different relative dielectric constant

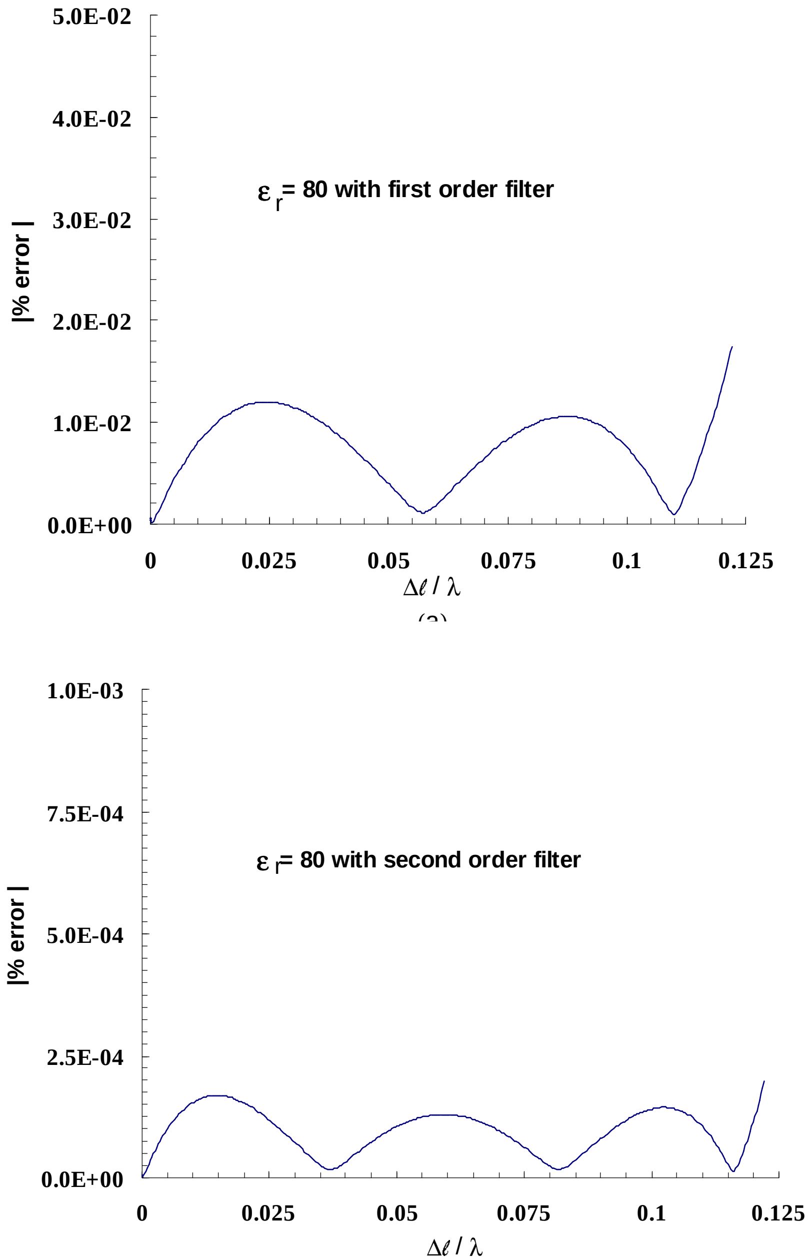
appears from the figure that a second order filter approximation can provide a relatively low level of error even for a relatively high loss tangent. The error increases for higher conductivities although it remains within an acceptable level. The error starts with a relatively higher level near zero frequency and then drops significantly after a frequency where the maximum cell dimension is larger than 0.025 times the corresponding wavelength.

3.6 Summary

In this chapter, the transient frequency domain TLM (TFDTLM) was introduced. A set of equations relating the link line impedances of the TFDTLM to the equivalent cell dimension was derived. The technique used to overcome the problem of inhomogeneous media, multiple propagation factors and frequency dependent reflection coefficients was discussed. This involved approximating all propagation factors in a TFDTLM mesh in terms of the propagation factor of some reference medium chosen to be the medium with the least propagation delay. The approximation was done with the aid of a digital filter.

From the results discussed above, it can be concluded that the filter coefficients can be optimized in such a way to minimize the error between the actual propagation factor and the approximated one. A first order filter with an optimized coefficients can perfectly approximate a lossless inhomogeneous medium. A second order filter can provide a very satisfactory level of error in approximating the propagation factor of a lossy inhomogeneous medium.

It is also worth mentioning that one important advantage of the approximation filter is that it can approximate a general frequency dispersive constitutive parameter, a feature that can not provided by a traditional time domain TLM scheme with open circuited, short circuited and lossy stubs.



(b)

Fig 3.4 Error obtained from a first and second order approximation filter for $\varepsilon_r = 80$

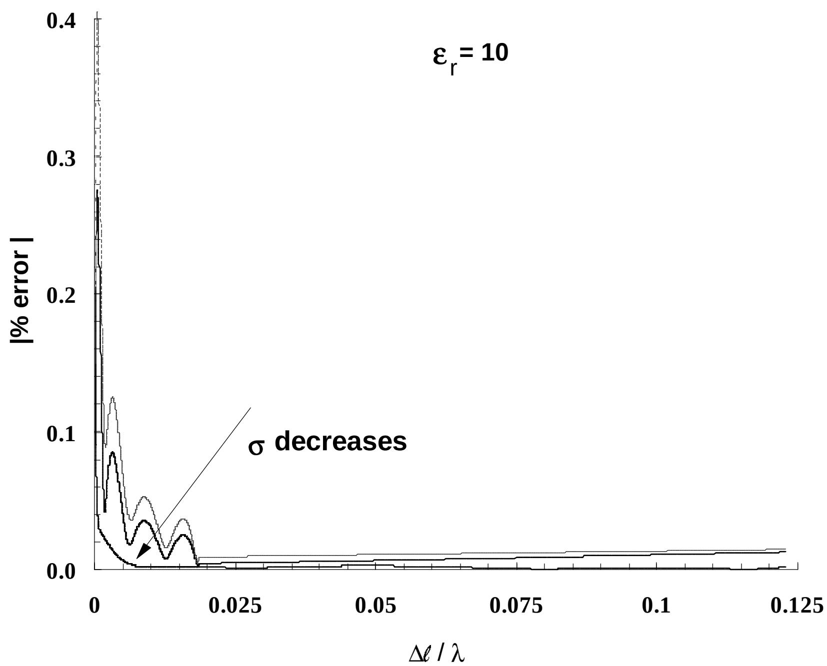


Fig 3.5 Error obtained from a second order approximation filter

for ε_r = 10 and different conductivities