

**STRESS INTENSITY FACTOR DISTRIBUTIONS IN
BIMATERIAL SYSTEMS - A THREE-DIMENSIONAL
PHOTOELASTIC INVESTIGATION**

by

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(ABSTRACT)

Stress-freezing photoelastic experiments are conducted using two different sets of photoelastic materials to investigate stress intensity behavior near to and coincident with bimaterial interfaces. Homogeneous, bonded homogeneous, and bonded bimaterial single edge-cracked tension specimens are utilized throughout the investigation for comparative purposes. The first series of tests involves machined cracks obliquely inclined to the direction of far field tensile loading. Mixed-mode stress intensity factors are observed and quantified using a simplified analytical algorithm which makes use of experimentally measured data. In this series of tests, the bimaterial specimens consist of a photoelastic material bonded to the same material containing a moderate quantity of aluminum powder (for elastic stiffening purposes). Moderate yet similar increases in stress intensity factors are observed in bonded homogeneous and bonded bimaterial specimens, suggesting the presence of bondline residual stresses (rather than elastic modulus mismatch) as the primary contributing factor. The second series of tests involves the bonding of mutually translucent photoelastic materials whose elastic moduli differ by a ratio of approximately four to one. Machined notches are placed both near and coincident to the bimaterial interfaces. Mode-mixity and increases in stress intensity are found only in bimaterial specimens whose cracks are placed close to the bondline. Using the materials from the first series of tests it is shown that the increases in these near-bondline experiments are due to thermally associated elastic mismatch properties (incurred during the stress freezing cycles) rather than mechanical mismatch properties.

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1.0 Introduction

An experimental investigation is undertaken which involves two series of bimaterial fracture tests. Measurements of stress intensity and mode-mixity are taken using the stress freezing technique of photoelasticity. An analytical development pertaining to the work in both test series is presented in Chapter 2. Discussions relating to the experimental procedures utilized throughout the study are likewise presented in Chapter 2. Chapter 3 deals with the first series of experiments. This test series involved the use of inclined single edge-cracked specimens in bimaterial systems for the quantification of mixed-mode stress intensity factors. Model configurations as well as material and analytical considerations are presented accordingly. Photographic and numerical results are finally presented and discussed. An investigation into single edge-cracked specimens containing cracks near to and coincident with bimaterial interfaces is considered in Chapter 4. Different crack geometries and photoelastic materials were used for this second series of experiments. An analogous format to that of Chapter 3 has been utilized for the presentation of procedures and results. Chapter 5 summarizes the material presented in this study and briefly discusses possible future work to be performed in the experimental area of bimaterial fracture.

The primary intentions of this study were to approximately model and quantify the behavior of flaws contained near and within the interface between rocket propellant and a rubber liner within solid rocket motor grains. This study was intended to be a supplement to investigations that have been and are currently being made using actual rocket motor grain materials. The photoelastic materials used in this work were selected for the resemblance of their elastic properties to those of the rocket motor grain materials. These rubber-like properties were simulated with the use of frozen stress photoelasticity. The photoelastic analyses performed in this study present quantitative results which have been used for the qualitative assessment of the significance of the bimaterial fracture scenario for the particular class of material properties found in solid rocket motor grains.

2.0 Analytical And Experimental Considerations

Material contained within this chapter primarily pertains to the mathematical developments and experimental procedures that were required to generate the results found in Chapters 3 and 4. The analytical considerations discussed in this chapter are initially presented in the general form from which several different algorithms for generating stress intensity factors from photoelastic fringe patterns have been proposed and utilized. From this general formulation, the algorithm used in this study, known as the Smith Extrapolation Method, is developed into a final form suitable for use in mixed-mode fracture analyses. Finally, a recently formulated analysis which approximates the orientation from which data is to be read in mixed-mode edge-cracked specimens is presented. This particular analysis ultimately served as an aid towards interpreting the near tip photoelastic fringe patterns found in the inclined-crack series of tests considered in Chapter 3.

For the photoelastic work considered in this study, multiple experimental procedures dealing with model construction and documentation were utilized. Since relative comparisons among models was a key feature in each series of experiments, a strict regimen of consistency in model construction was prioritized in an effort to minimize extraneous sources of error. Experimental procedures are discussed systematically beginning with the initial construction of models and ending with the methods involved in extracting data from these models.

2.1 Bimaterial Fracture

A brief review of theoretical and experimental work that has been performed in the area of interface and sub-interface bimaterial fracture mechanics is presented in the following sub-sections. The progress that has been made in this area is reviewed while simultaneously discussing the associated limitations from both theoretical and experimental standpoints. Finally, the intended application of this study to rocket motor grains is briefly discussed.

2.1.1 Interface And Sub-Interface Fracture

For decades now there has been much interest in the complex situation concerning a crack located at or near an interface between two dissimilar materials. From a theoretical standpoint, this bimaterial phenomenon presents a rather unique and complicated situation in which two infinite quantities exist at the same point: a sharp crack tip and a jump discontinuity in material properties. From a physical standpoint, analogous difficulties exist. If two materials are assumed to be joined at a common interface and are likewise assumed to be able to resist tensile and shear stresses, adherent forces of some form must be present. In practice, these adherent forces are usually derived from either a third material or from the casting and curing of one material against another. For the case in which a third material is used to fasten two materials to one another, the deviation from the theoretical bimaterial model is obvious. For the case in which one material is cast against another, residual stresses derived from varying coefficients of thermal expansion (and thus constraint against volumetric changes in the cast material) represent just one of several considerations which in practice may deviate from the theoretical model. Although variations between theory and actuality will inevitably exist in problems concerning bimaterial fracture, the quantification of these extraneous factors may in some cases be made through the combined use of judicious experiments and analytical approximations.

One of the first analytical models concerning bimaterial fracture was presented by Williams¹ in 1959. Perhaps the most common result from this work was the analytically derived presence of an oscillating stress field in the high singularity zone near the crack tip. Although no experimental evidence displaying this effect has been presented, several attempts to either include or mathematically negate this feature have been made. Several analyses were presented in the mid 1960's which dealt with the generalized stress field near the tip of a crack between dissimilar materials^{2,3,4}. These analyses however did not effectively contain traction free crack surfaces: a property which at times may not correlate to the physical reality of a fracture situation. Comninou⁵ later introduced an analysis containing a frictionless contact

zone which eliminated the oscillatory near-tip stresses found in previous work. Additional analyses^{6,7,8} have been presented more recently which deal with such factors as crack extension and direction, contact zone size and its dependence on remote loading, and the concept of a characteristic bimaterial crack length which depends upon the particular materials under consideration. A particularly comprehensive analysis and review of published works concerning bimaterial fracture phenomena is given by Hutchinson and Suo⁹. Of particular interest to the work considered in this present study are the sections pertaining to mixed-mode fracture and the fracture of sub-interface bimaterial cracks.

Experimental investigations primarily employing optical methods have more recently been applied to the field of bimaterial fracture. Liechti and Chai¹¹ have demonstrated the use of optical interferometry methods for measuring crack tip opening displacements (CTOD) and fracture toughness in an epoxy-glass bimaterial. Alternate optical techniques have also been recently used^{11,12} for the measurement of mode-mixity and complex stress intensity factors. A photoelastic study performed by Chiang et. al.¹³ is of particular interest due to their attempt at making a direct comparison between analytical and experimental results for a central crack loaded in pure remote shear. Instead of evaluating stress intensity factors, a comparison of photoelastically obtained principal shear stress magnitudes and directions was made with analytical results. The analytically predicted shear mode was indeed experimentally found by observing the presence of fringe loops in the crack tip vicinity. The comparison yielded mixed results with good correlation in some regions while in others, deviations in excess of 30% were found.

2.1.2 Applications - Rocket Motor Grains

The current study under consideration involves the use of photoelasticity to qualitatively and quantitatively measure the stress intensity conditions present at and near the bimaterial interface of similar rubber-like materials. Questions have been raised concerning the storage of inter-continental ballistic missiles (ICBM) and the role that potential gaps between

rocket propellant and its mating rubber liner may play in their structural and chemical burn-rate stability. The photoelastic study recently undertaken was intended to serve as an additional source of information for the assessment of the behavior of flaws contained within bimaterial interfaces of relatively similar elastic moduli. The state of material incompressibility ($\nu=0.5$) found in both the rocket propellant and its mating rubber liner was likewise effectively modelled by using the stress freezing technique of photoelasticity.

Depending on the lengthwise location of a flaw in a rocket motor grain, varying states of elastic plane strain and plane stress will likely exist. Single edge-cracked photoelastic specimens of moderate thickness were utilized in the photoelastic analysis to provide a measure of variation in stress intensity. Throughout the analysis, an approximate state of generalized plane strain was assumed to exist in each specimen. Moderate elevations in stress intensity over the near surface values were consistently obtained from the photoelastic analysis of thin slices extracted from the midsection of specimens. Elastic modulus ratios in the range of 2:1 to 4:1 were used to roughly simulate the magnitudes of modulus mismatch found between the actual rocket propellant and rubber liner.

The work presented in this study is ultimately intended to be of a qualitative nature although many quantitative measures have been made throughout. A qualitative standpoint has been adopted from this study primarily due to the fact that although the photoelastic materials used to model the problem are mechanically similar, they indeed are not the actual rocket motor grain materials under investigation. As is the case for all materials, unique peculiarities are bound to exist in the specific propellant material and rubber liners. As such, the measurements and conclusions presented in this work are intended to provide insight into material interactions and behaviors associated with the common elastic and mechanical properties inherent in all solid materials (e.g. stiffness) and are specifically geared to those of the rocket propellant scenario. This study is intended to be a supplement to experiments that have been and will be conducted on the actual materials.

2.2 Analytical Development

An algorithm which combines theoretical formulations with experimentally obtained information is developed using classical fracture mechanics together with the theory of photoelasticity. A general formulation which relates photoelastic parameters to the classical fracture problem is first considered. From this formulation, the development of a simplified algorithm (The Smith Extrapolation Method) is outlined and discussed. The governing equations of the extrapolation method, which were used for determining all of the experimental stress intensity factors presented in this work, are finally presented in their general form.

2.2.1 Formulation of Stress Intensity Relations

In order to make use of experimentally obtained data through the use of photoelasticity, it is necessary to express the analytical relationships derived from theoretical fracture mechanics in terms of photoelastic parameters. Consider the planar problem of a crack in a homogeneous material as given in Fig. 2.1. The generalized two dimensional equations^{14,15} relating mode I and mode II stress intensity factors to the stress field near a crack tip are given by:

$$\begin{aligned}\sigma_{xx} &= \frac{1}{\sqrt{2\pi r}} (F_1 K_I + F_2 K_{II}) \\ \sigma_{yy} &= \frac{1}{\sqrt{2\pi r}} (F_3 K_I + F_4 K_{II}) \\ \sigma_{xy} &= \frac{1}{\sqrt{2\pi r}} (F_4 K_I + F_5 K_{II})\end{aligned}\tag{2.1}$$

where K_I and K_{II} are the respective mode I and mode II stress intensity factors.

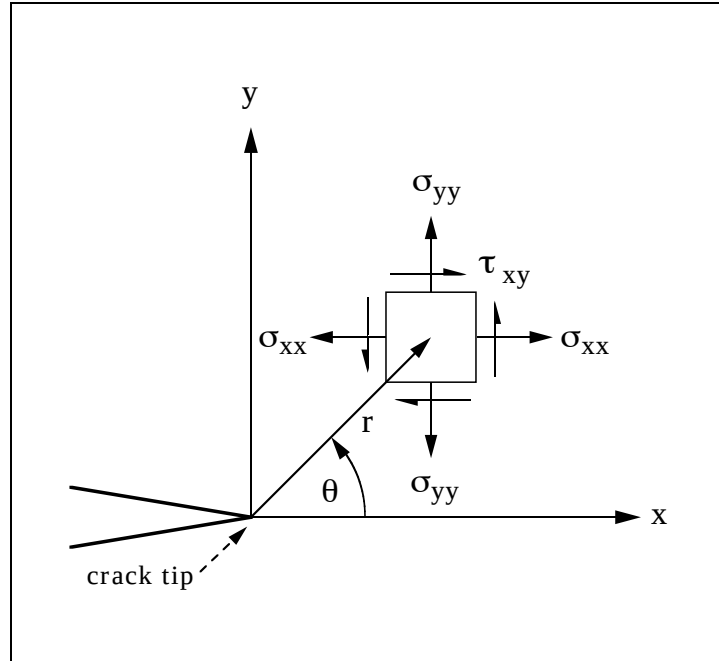


Figure 2.1: Generalized 2-D cracked-body stress field.

The trigonometric functions ($F_1 - F_5$) found in Eq. 2.1 are given by:

$$\begin{aligned}
F_1 &= \cos\left(\frac{\theta}{2}\right) \left[1 - \sin\left(\frac{\theta}{2}\right) \sin\left(\frac{3\theta}{2}\right) \right] \\
F_2 &= -\sin\left(\frac{\theta}{2}\right) \left[2 + \cos\left(\frac{\theta}{2}\right) \cos\left(\frac{3\theta}{2}\right) \right] \\
F_3 &= \cos\left(\frac{\theta}{2}\right) \left[1 + \sin\left(\frac{\theta}{2}\right) \sin\left(\frac{3\theta}{2}\right) \right] \\
F_4 &= \sin\left(\frac{\theta}{2}\right) \cos\left(\frac{\theta}{2}\right) \cos\left(\frac{3\theta}{2}\right) \\
F_5 &= \cos\left(\frac{\theta}{2}\right) \left[1 - \sin\left(\frac{\theta}{2}\right) \sin\left(\frac{3\theta}{2}\right) \right]
\end{aligned} \tag{2.2}$$

The stress field solution given by this set of equations is for the case in which the far field loading is considered to be two-dimensionally hydrostatic. To attempt to account for the possibility that the non-singular stress field away from the crack tip is not hydrostatic, a generalized non-singular stress field¹⁶ is superimposed over the singular stress field given in Eq. 2.1. This consideration yields the following modified form of the near tip stress equations:

$$\begin{aligned}
\sigma_{xx} &= \frac{1}{\sqrt{2\pi r}} (F_1 K_I + F_2 K_{II}) - \sigma_{xx}^o \\
\sigma_{yy} &= \frac{1}{\sqrt{2\pi r}} (F_3 K_I + F_4 K_{II}) - \sigma_{yy}^o \\
\sigma_{xy} &= \frac{1}{\sqrt{2\pi r}} (F_4 K_I + F_5 K_{II}) - \sigma_{xy}^o
\end{aligned} \tag{2.3}$$

where the functions F_1 through F_5 remain unchanged and the terms σ_{xx}^o , σ_{yy}^o , and σ_{xy}^o are referred to as the non-singular stress components. It is noted that for the case in which the stress field distant from the crack tip is in-plane hydrostatic, the non-singular stress components (σ_{xx}^o , σ_{yy}^o , and σ_{xy}^o) must be identically zero in order to satisfy the 2-D hydrostatic stress

field relations given by Eq. 2.1. The individual magnitudes of σ_{xx}^0 , σ_{yy}^0 , and σ_{xy}^0 will in general not be equal in magnitude to the corresponding far field loads ($\sigma_{xx}, \sigma_{yy}, \sigma_{xy}$). These terms will be carried along through the development of the algorithm for converting photoelastic data into stress intensity factors. It will be seen that the inclusion of these non-singular terms is of paramount importance over their actual magnitudes when using the Smith Extrapolation Method considered in Section 2.2.2.

The theoretical stress field solution can now be combined with the stress-optic law of photoelasticity^{17,24}. In two-dimensional form, the stress optic law can be written as:

$$\tau_{\max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{Nf}{2t} \quad (2.4)$$

where σ_1 and σ_2 are the principal in plane normal stresses, $\tau_{\max}$ is the principal in plane shear stress, and “t” is the model thickness perpendicular to the transmission direction of light. The photoelastic parameters in this relation are characterized by the fringe order “N” and the material fringe value “f”. The material fringe value is an easily determinable constant for a given photoelastic material. Fringe orders are obtained from visual inspection of the photoelastic model while in the polariscope, and usually documented photographically. Details of fringe order and material fringe value determination are given in Appendix A.

Eq. 2.4 can now be combined with the elasticity equation relating principal in-plane shear stress with the coordinate stresses σ_{xx} , σ_{yy} , and σ_{xy} to give:

$$\tau_{\max} = \frac{Nf}{2t} = \frac{\sqrt{(\sigma_{xx} - \sigma_{yy})^2 + 4\sigma_{xy}^2}}{2} \quad (2.5)$$

Combining this relation with the general stress field relations given by Eqs. 2.3 yields the following solution which relates the photoelastic variables to the cracked body stress field:

$$2\tau_{\max} = \frac{Nf}{t} = \sqrt{\frac{A}{2\pi r} + \frac{B}{\sqrt{2\pi r}}} + C \quad (2.6)$$

where:

$$\begin{aligned}
A &= [K_I(F_1 - F_3) + K_{II}(F_2 - F_4)]^2 + 4[K_I F_4 + K_{II} F_5]^2 \\
B &= -2(\sigma_{xx}^o - \sigma_{yy}^o)[K_I(F_1 - F_3) + K_{II}(F_2 - F_4)] - 8\sigma_{xy}^o[K_I F_4 + K_{II} F_5] \\
C &= (\sigma_{xx}^o - \sigma_{yy}^o)^2 + 4\sigma_{xy}^o{}^2
\end{aligned}$$

Evaluation and simplification of the functions A, B, and C yields:

$$\begin{aligned}
A &= (K_I \sin(\theta) + 2K_{II} \cos(\theta))^2 + (K_{II} \sin(\theta))^2 \\
B &= 2(\sigma_{xx}^o - \sigma_{yy}^o) \left[K_I \sin(\theta) \sin\left(\frac{3\theta}{2}\right) + K_{II} \left(2\sin\left(\frac{\theta}{2}\right) + \sin(\theta) \cos\left(\frac{3\theta}{2}\right) \right) \right] \\
&\quad - 4\sigma_{xy}^o \left[K_I \sin(\theta) \cos\left(\frac{3\theta}{2}\right) + K_{II} \left(2\cos\left(\frac{\theta}{2}\right) - \sin(\theta) \sin\left(\frac{3\theta}{2}\right) \right) \right] \\
C &= (\sigma_{xx}^o - \sigma_{yy}^o)^2 + (2\sigma_{xy}^o)^2
\end{aligned} \tag{2.7}$$

This general relationship provides the basis for several algorithms^{18,19,20,21,22} for use in the experimental determination of stress intensity factors using photoelasticity. A fairly comprehensive review and comparison of these algorithms and others is presented by Smith and Olaosebikan¹⁴. Many of the algorithms are shown to provide accurate results when compared with theoretical solutions. Analysis for use in the work presented here is based on the Smith Extrapolation Method¹⁸ for the determination of stress intensity factors using photoelasticity. The rationale for the choice of using this method lies primarily in its relative analytical simplicity coupled with its intrinsic ability to allow for a fairly simple means of acquiring the necessary data. Furthermore, closed form analytical relationships such as mode-mixity (K_{II}/K_I) are easily derived from the solution to the extrapolation method. It will be shown that the determination of the mode-mixity for a given fringe pattern can be found by a single angular measurement made near the region of a crack tip.

2.2.2 Smith Extrapolation Method

To eliminate higher order terms in Eq. 2.6, a polynomial expansion can be taken using the following general form of a binomial series:

$$(p + q)^n = p^n + np^{(n-1)}q + \frac{n(n-1)p^{(n-2)}q^2}{2!} + \frac{n(n-1)(n-2)p^{(n-3)}q^3}{3!} + \dots \quad (2.8)$$

where:

$$p = \frac{A}{2\pi r} ; q = \frac{B}{\sqrt{2\pi r}} + C ; n = \frac{1}{2}$$

Upon substitution and expansion of the first two terms in the series, the relation between stress intensity and the photoelastic quantities becomes:

$$2\tau_{\max} = \frac{Nf}{t} = \frac{\sqrt{A}}{\sqrt{2\pi r}} + \frac{B}{2\sqrt{A}} + \frac{C\sqrt{2\pi r}}{2\sqrt{A}} \quad (2.9)$$

Retaining only the first two terms on the right hand side of this relation and rearranging gives:

$$\tau_{\max} = \sqrt{\frac{A}{8\pi}} \sqrt{\frac{1}{r}} + \frac{B}{4\sqrt{A}} \quad (2.10)$$

Multiplication of Eq. 2.10 by $(\sqrt{8\pi r} / \bar{\sigma} \sqrt{\pi a})$ produces the final desired form of the Smith extrapolation equation:

$$\frac{K_{ap}}{\bar{\sigma} \sqrt{\pi a}} = \frac{K}{\bar{\sigma} \sqrt{\pi a}} + \left(\frac{\sqrt{2} B}{2\bar{\sigma} K} \right) \sqrt{\frac{r}{a}} \quad (2.11)$$

where:

$$K_{ap} = \frac{Nf}{2t} \sqrt{8\pi r}$$

$$K = \sqrt{A} = \sqrt{\left(K_I \sin(\theta) + 2K_{II} \cos(\theta) \right)^2 + \left(K_{II} \sin(\theta) \right)^2}$$

$\bar{\sigma}$ = nominal far field stress

r = distance from crack tip

a = length of crack

Eq. 2.11 represents a line whose intercept is given by “K” and whose slope is given by the quantity $(\sqrt{2} B)/(2\bar{\sigma}K)$. In order for Eq. 2.10 to represent a straight line, the slope must remain constant. Inspection of the slope term reveals dependence upon four factors: $\bar{\sigma}$, θ , K_I , and K_{II} . For a given fracture scenario, the nominal far field stress as well as the opening mode and shear mode stress intensity factors (K_I and K_{II}) will each be of constant value. Thus, the polar angular coordinate θ (as measured from the direction parallel to the crack) must remain constant. This dictates that the photoelastic data must be read along a straight line whose origin is at the crack tip. Choosing a fixed polar angle θ along which to read data yields data sets of the form: (N, r). This enables Eq. 2.11 to be plotted as a function of $\sqrt{r/a}$. A least squares curve fit to a zone of data that appears relatively linear can then be extrapolated to the origin. It is seen from Eq. 2.11 that the intercept of this linear curve with the coordinate axis $K_{ap}/\bar{\sigma}\sqrt{\pi a}$ yields a value for $K/\bar{\sigma}\sqrt{\pi a}$. With this value known, what remains to be found is a means of determining K_I and K_{II} individually.

2.2.3 Generalized Irwin Method

The generalized Irwin method¹⁶ yields the additional relationship between K_I and K_{II} as well as the reading line orientation angle θ for the computation of K_I and K_{II} individually using the Smith extrapolation method. Irwin’s criterion is given by the condition that:

$$\left. \frac{\partial \tau_m}{\partial \theta} \right|_{(r_m, \theta_m)} = 0 \quad (2.12)$$

The validity of this relationship can be rationalized with the assistance of Fig. 2.2 which represents a typical photoelastic shear stress fringe field near a crack tip. For any arbitrary fixed value of “r” within the general region of the crack tip, traversal through the angular coordinate θ from $\theta = 0$ to $\theta = \pi/2$ results in an increase in fringe order until a peak or maximum value is reached at some value of θ , referred to as θ_m . As long as $\theta_m \neq \pi/2$, the fringe order, which is proportional to τ_m , will then begin and continue to decrease in magnitude. A maximum of the shear stress τ (designated as τ_m)

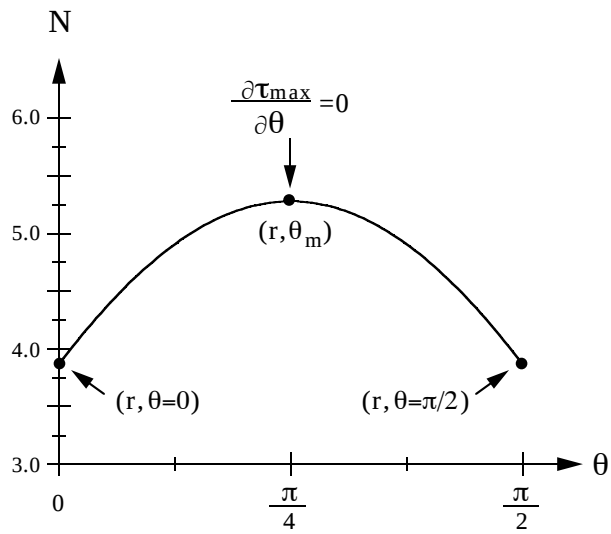
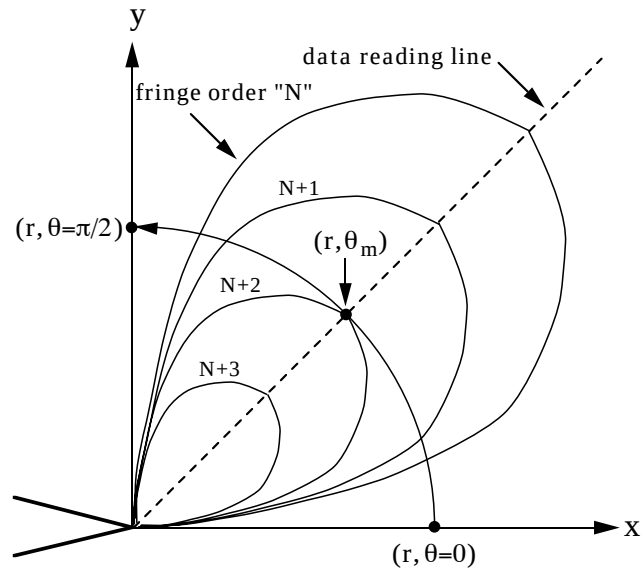


Figure 2.2: Fringe order dependence upon the polar coordinates for use in establishing Irwin's criterion.

is thus established as a function of θ for fixed values of r and can be found mathematically by evaluating the partial derivative of τ (as given by Eq. 2.5) and setting the result equal to zero. The resulting relation is given by:

$$\left(\frac{K_{II}}{K_I}\right)^2 - \frac{4}{3}\left(\frac{K_{II}}{K_I}\right)\cot(2\theta_m) - \frac{1}{3} = 0 \quad (2.13)$$

In the high singularity zone near the crack tip, the variation in θ_m from fringe loop to fringe loop tends to be small. In fact, θ_m tends to asymptotically approach a constant value as $r \rightarrow 0$. As regions further away from the singular zone are considered, the non-singular stress components (σ_{xx}^0 , σ_{yy}^0 , and σ_{xy}^0) tend to distort the fringe loops (Fig. 2.3) in such a way as to vary θ_m in relation to the distance from the crack tip. The singularity dominant zone is the implicit basis for the characterization of the fracture mechanics parameters. As such, it is desirable to experimentally determine θ_m by observing fringes in the singular zone near the crack tip in an effort to minimize error introduced by non-singular stress effects.

2.2.4 Mode I and Mode II Stress Intensity Factor Determination

Eq. 2.13 can now be coupled with Eq. 2.11 to form a system of two equations and two unknowns in K_I and K_{II} . Solving this system yields the following expressions for K_I and K_{II} :

$$K_I = \frac{K}{\sqrt{\sin^2(\theta_m)[1 + F(\theta_m)^2] + 4F(\theta_m)\cos(\theta_m)[F(\theta_m)\cos(\theta_m) + \sin(\theta_m)]}} \quad (2.14)$$

$$K_{II} = F(\theta_m)K_I \quad (2.15)$$

where:

$$F(\theta_m) = \frac{2\cot(2\theta_m) \pm \sqrt{4\cot^2(2\theta_m) + 3}}{3} \quad (2.16)$$

By obtaining photoelastic data (N, r) along θ_m , a plot may be generated using Eq. 2.10. The data to be used is selected over a range in which a straight line may be well fitted. A representative plot of data obtained from an off-bondline

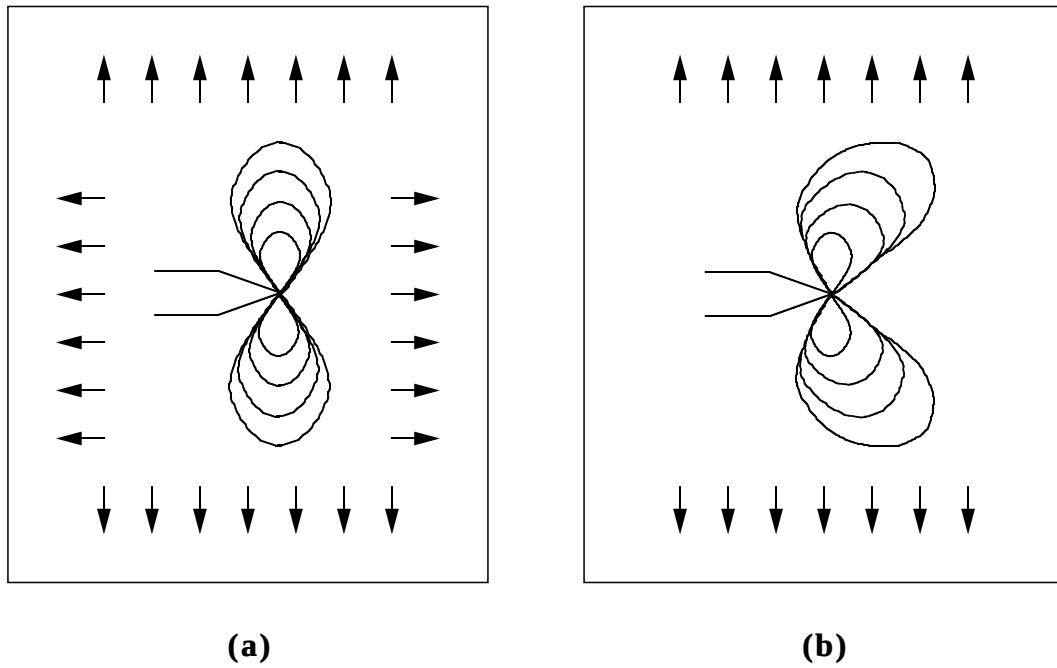


Figure 2.3: Near-tip fringe pattern dependency upon far field loading conditions.
 (a) Pure mode I singular zone found from hydrostatic far field loading.
 (b) Pure mode I singular zone found from uni-directional far field tensile loading.

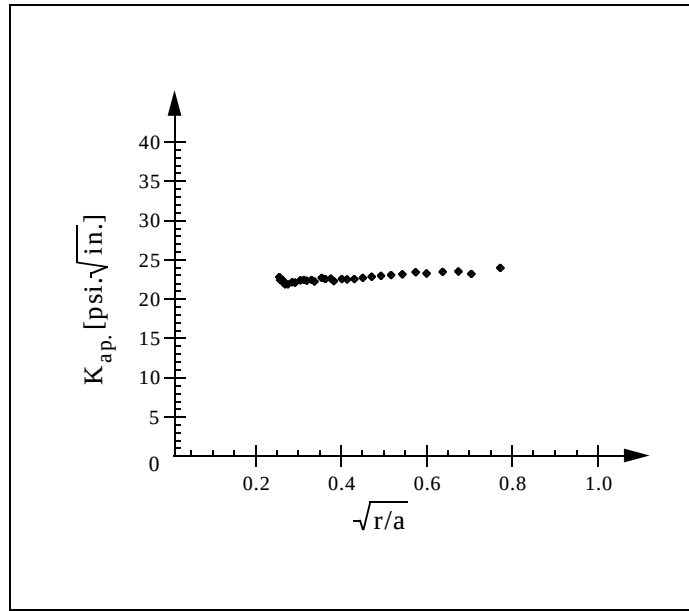
slice is given in Fig. 2.4. The extrapolation of this straight line curve fit to the $K_{ap}/\bar{\sigma}\sqrt{\pi a}$ axis yields a value for $K/\bar{\sigma}\sqrt{\pi a}$. The data reading line orientation angle θ_m as well as the value of K found from the extrapolation are then substituted into Eqs. 2.14, 2.15, and 2.16 to yield the mode I and mode II stress intensity factors. The magnitudes of the additional stress terms (σ_{xx}^0 , σ_{yy}^0 , and σ_{xy}^0) are therefore unnecessary for the determination of the stress intensity factors. It is clear though that these stress terms are embedded in the slope of the curve fit and are thus accounted for in the extrapolation method. From these equations it is noted that the mixed-mode ratio is given explicitly as a function of the reading line angle θ_m :

$$\frac{K_{II}}{K_I} = F(\theta_m) = \frac{2\cot(2\theta_m) \pm \sqrt{4\cot^2(2\theta_m) + 3}}{3} \quad (2.17)$$

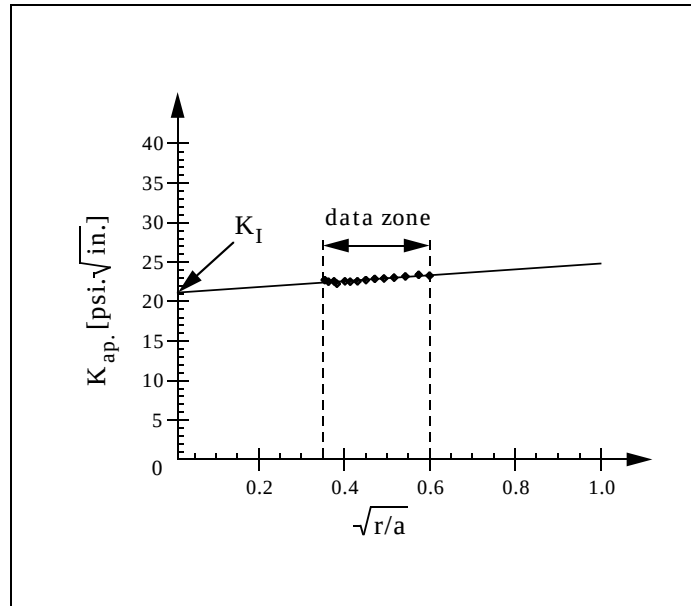
A plot of this function is given in Fig. 2.5 as are the representative fringe patterns corresponding to situations of interest. Although not shown for plot scaling purposes, it can be shown that as θ_m approaches zero, $F(\theta_m)$ asymptotically approaches infinity. Thus, for the case in which $K_{II}=0$ (pure opening mode), the reading line and thus fringe loop tips will lie along a line perpendicular to the crack direction. Likewise for the case in which $K_I=0$ (pure shear mode), the fringe loop tips will lie along a line parallel and coincident with the crack direction.

2.2.5 Data Reading Line Orientation

It has been noted that the determination of the reading line angle must be made by viewing the photoelastic fringes in the immediate vicinity of the crack tip. The classical fracture mechanics theory predicts a complete closure of each singular fringe loop at the crack tip. In actuality, such a fringe field is never quite achieved due to the finite thickness and root radius of real cracks found in materials. With this being the case, the fringes found close to the crack tip of a machined flaw in a photoelastic material tend to possess a more gradual, circular curvature (see Fig. 4.13b) rather than the more sharp and distinctive curvature that is predicted by the theoretical analysis (Fig. 2.3). To ensure that subjective interpretations of the fringe



(a)



(b)

Figure 2.4: Representative photoelastic slice data.
 (a) Photoelastic data obtained from the middle slice of a bimaterial specimen.
 (b) Least squares curve fit taken over selected range of data for stress intensity factor determination.

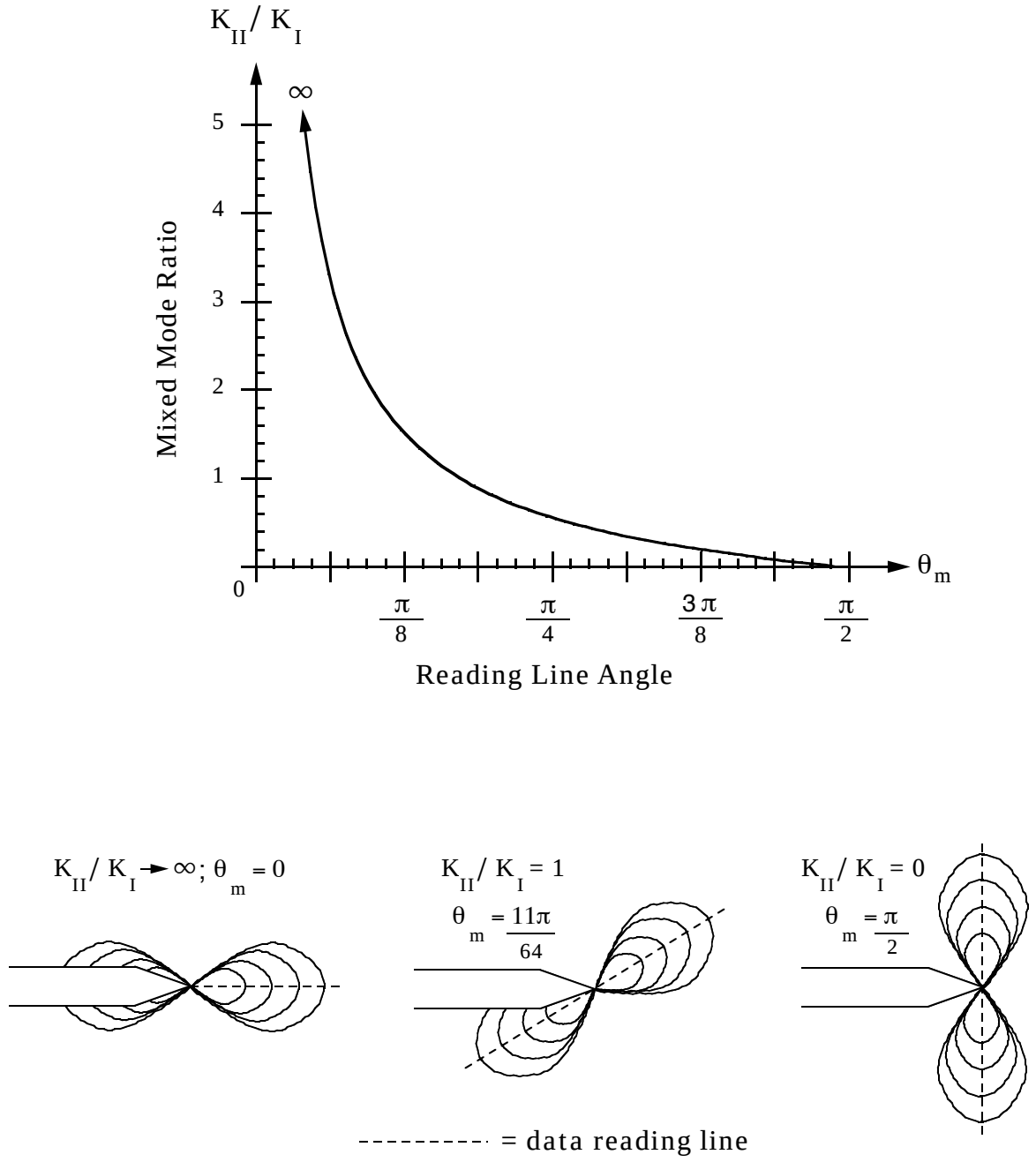


Figure 2.5: Mode-mixity dependence upon reading line orientation angle.

fields and thus the reading line angles found in this work were accurate, experimentally obtained reading line angles were compared to an analytical development which combines the Smith Extrapolation equations with the numerical solution for a semi-infinite single-edge cracked plate of arbitrary crack inclination.

The angular variables used in the following development are schematically defined in Fig. 2.6. A table of values obtained from a numerical analysis which correlates mode-mixity with crack angle orientation (β) is tabulated by Sih²³ and listed in Fig. 2.7. A second order polynomial curve fit was generated using this table of values to yield an approximate analytical expression relating the mode-mixity ratio (K_{II}/K_I) to the crack orientation angle (β degrees) and is given by:

$$\frac{K_{II}}{K_I} = 10^{-3} (9.319 + 9.169\beta + 0.0481\beta^2) \quad (2.18)$$

Eq. 2.18 can now be combined with Eq. 2.17 from the Smith extrapolation method. First, rearranging Eq. 2.17 in terms of θ_m gives:

$$\cot(2\theta_m) = \frac{3F(\theta_m)^2 - 1}{4F(\theta_m)} \quad (2.19)$$

Solving Eq. 2.19 for the reading line angle θ_m yields:

$$\theta_m = \frac{90^\circ - \tan^{-1} \left(\frac{3F(\theta_m)^2 - 1}{4F(\theta_m)} \right)}{2} \quad (2.20)$$

Since $F(\theta_m) = K_{II}/K_I$, Eq. 2.18 can now be substituted into Eq. 2.20 to give the reading line angle θ_m as a function of the crack angle β :

$$\theta_m = \frac{90^\circ - \tan^{-1} \left(\frac{3(10)^{-6} [9.319 + 9.169\beta + 0.0481\beta^2]^2 - 1}{4(10)^{-3} [9.319 + 9.169\beta + 0.0481\beta^2]} \right)}{2} \quad (2.21)$$

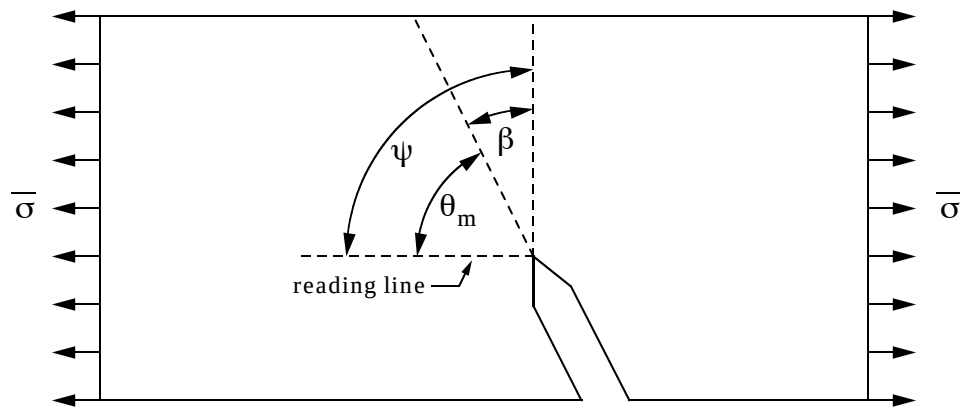


Figure 2.6: Definition of angular variables used for determining the orientation of the reading line angle.

β	F_I	F_{II}	F_{II} / F_I
80.0	0.16200	0.1740	1.0741
75.0	0.23550	0.2225	0.9448
70.0	0.30500	0.2710	0.8885
60.0	0.46200	0.3370	0.7294
52.5	0.58400	0.3620	0.6199
50.0	0.62510	0.3648	0.5836
45.0	0.70500	0.3642	0.5166
40.0	0.78180	0.3543	0.4532
30.0	0.92050	0.3059	0.3323
22.5	1.00500	0.2470	0.2458
20.0	1.02860	0.2243	0.2181
15.0	1.06900	0.1740	0.1628
10.0	1.09790	0.1188	0.1082
5.0	1.11600	0.0600	0.0538
0.0	1.12125	0.0000	0.0000

$$K_I = F_I \bar{\sigma} \sqrt{\pi a} ; K_{II} = F_{II} \bar{\sigma} \sqrt{\pi a}$$

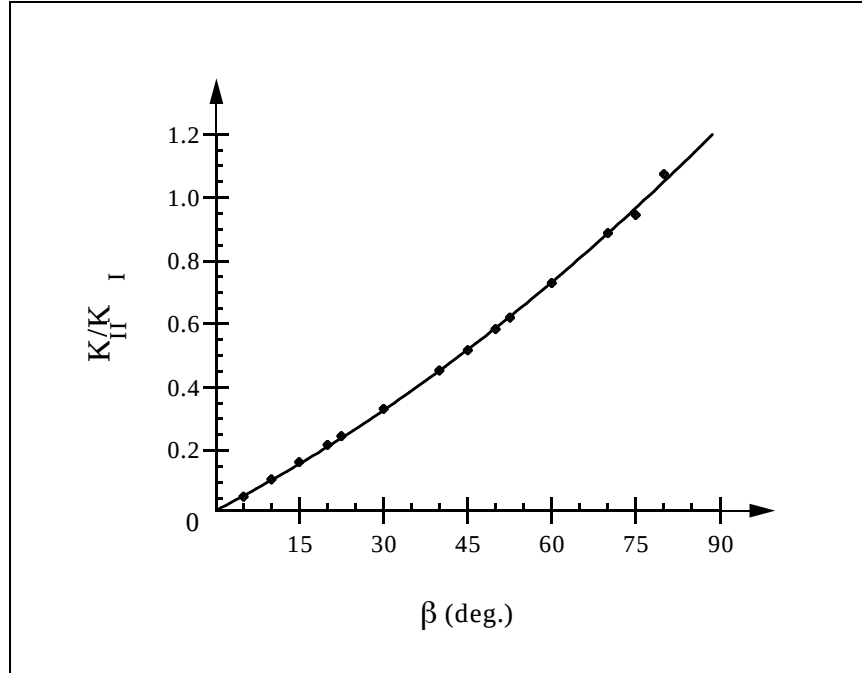


Figure 2.7: Curve fitting of numerical data from the inclined crack solution.

Finally, as seen in Fig. 2.6, the orientation of the reading line relative to a perpendicular to the free edge can be defined as:

$$\psi = \beta + \theta_m = \beta + \frac{90^\circ - \tan^{-1} \left(\frac{3(10)^{-6} [9.319 + 9.169\beta + 0.0481\beta^2]^2 - 1}{4(10)^{-3} [9.319 + 9.169\beta + 0.0481\beta^2]} \right)}{2} \quad (2.22)$$

A plot of $\psi(\beta)$ from Eq. 2.22 is shown in Fig. 2.8. Inspection of the plot reveals that for a wide range of crack orientation angles, the reading line remains virtually parallel to the direction of far field tensile loading. In fact, for all of the crack orientation angles considered in this study ($\beta=0^\circ, 15^\circ, 30^\circ$, and 45°), the appropriate reading line remains parallel to the direction of far field loading to within ± 3 degrees. Images taken near the notch tips of the inclined-angle araldite specimens seemed to confirm these results. As such, the photoelastic data for all of the araldite specimens was taken along reading lines oriented parallel to the direction of far field tensile loading.

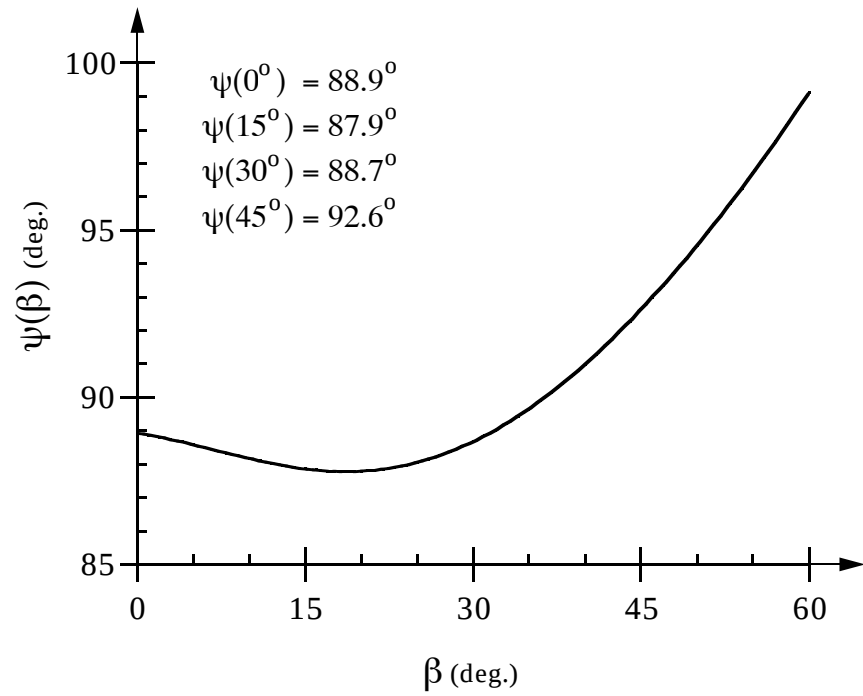


Figure 2.8: Reading line angle variation as a function of crack inclination. For the crack inclinations considered, negligible deviations between the reading line orientation and the direction of far field loading ($\beta=90^\circ$) are found.

2.3 Experimental Procedures

Three commercially available photoelastic materials were used in this study to construct models of fracture specimens. A plastic araldite material was chosen for the tests in which a mixed-mode fracture state was induced by selecting various crack orientation angles. For the off-bondline test specimens, two plastic materials available from Photolastic Inc. were used. These materials, trade named PLM-4B and PSM-9, differed from one another in their elastic moduli and critical temperatures (T_c). Above their critical temperatures, both materials possessed the elastic material property of incompressibility ($\nu=0.5$). Descriptions concerning the various methods incorporated for the preparation, construction, documentation, and analysis of the photoelastic models considered in this work are presented in the following sections.

2.3.1 Model Construction

The models used for all tests were constructed from plates of photoelastic material supplied by their respective manufacturers. A nominal plate thickness of 0.500 in. (12.7 mm) was chosen in order to generate a measurable variation in stress intensity factors through the thickness. For the homogeneous control specimens, single rectangular sections were cut out of the photoelastic plates and subsequently machined for squareness on their half inch thick outer surfaces. All bondline specimens were constructed using a method that produced uniformity and consistency in bondline thickness. Each complete bondline specimen consisted of two halves which were cut and machined from the original plates. For the bonding process, use of a simple jig was employed for maintaining proper alignment of the two pieces to be joined. In order to generate a constant thickness bondline, a feeler gauge (0.012 in. [0.3 mm]) was inserted between the two pieces to be joined. With the feeler gauge in place, a piece of cellophane tape was then placed over the feeler gauge and along the entire length of the bondline to be formed. The specimen was then able to be gently turned over while applying mild pressure to the ends of the model to ensure that the

feeler gauge remained in place between the two halves to be bonded. Removal of the feeler gauge at this point rendered a uniform gap between the two specimen halves. The tape is now seen to serve a dual purpose: it secures the two halves properly from one another and it provides a “form” or boundary for holding the adhesive while curing. The model can now be bonded by simply pouring adhesive into the thin gap between the specimen halves.

PLM-9 resin was used for the bonding of all models. When mixed with its complementary hardener and cured, the resulting bondline was compositionally similar to that of the PSM-9 material supplied by the manufacturer in sheet form. After thorough mixing and degasification, the adhesive is deposited centrally on the gap between the two halves of the specimen. To ensure that air bubbles do not become trapped at the bottom of the bondline, adhesive is placed only on the center of the top surface. This allows the adhesive to immediately flow to the middle of the lower surface of the bondline and then spread to the lower corners. The bondline is then ready to be cured once the adhesive has risen to the top and thus filled the gap initially created by the implementation of the feeler gauge.

2.3.2 Thermal Cycling

The post-curing cycle of PSM-9 as dictated by the manufacturer requires a controlled elevation in temperature followed by a dwell during which the temperature is held fixed. Finally, the post-curing process is completed by decreasing the temperature at a constant rate back down to room temperature. This time-temperature thermal cycle is depicted in Fig. 2.9. The thermal cycles used for the araldite specimens utilized the same temperature gradients found in Fig. 2.9 but used a higher maximum temperature of 250 F due to the material’s higher critical temperature. After the thermal cycle is completed, photoelastic photographs of the specimen are taken to document the residual stress field near the bondline created by the shrinkage associated with curing. Once documentation is complete, the v-notch (araldite specimens) or slot (PSM-9/PLM-4B specimens) is machined into place. The specimen is then again photographed while in the polariscope to document the effects on the stress field, if any, induced by the machining of the v-notch or slot.

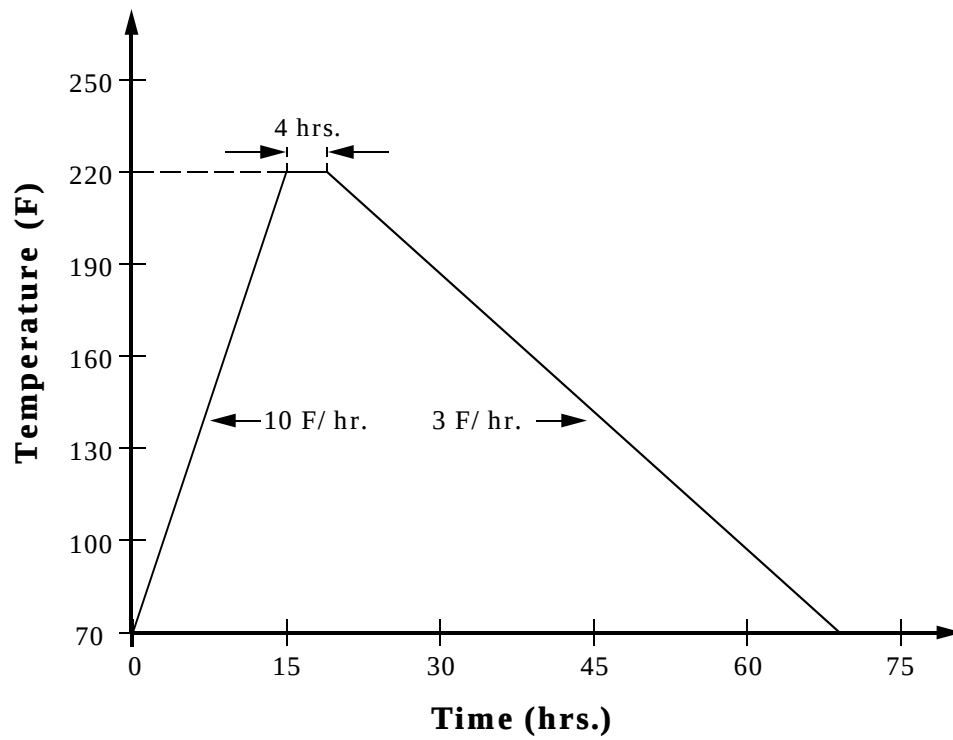


Figure 2.9: Time-temperature thermal cycle used for PSM-9/PLM-4B specimens. Araldite specimens were subjected to a peak temperature of 250 F.

2.3.3 Loading Methods

Once documentation of the residual stress field has been completed, the model is ready for loading. A fairly simplistic dead weight loading scheme is used within a large oven so that the model can be loaded while controlling its temperature (Fig. 2.10). All fracture tests considered here made use of a purely tensile far field load. The araldite series of tests utilized a three pin loading system while the PSM-9/PLM-4B series of tests utilized a single pin loading system. Due to the added kinematic freedom of the string system of Fig. 2.10, free rotation of the loaded models due to the inherent lack of symmetry of single edge-notched specimens was possible. During the commencement of the later tests conducted on the PSM-9/PLM-4B specimens, it became apparent that for the specimen geometry and loads being used, a single pin loading scheme would suffice without causing failure due to stress concentrations near the pin holes. The three pin loading scheme, although capable of handling more load, required more adjustment of the string apparatus to ensure a uniform far field tensile stress. The simplicity of the single pin scheme provided consistently uniform far field tensile stresses while requiring minimal set-up times. It is noted that multiple pin schemes become a necessity when the relative amount of load necessary to generate a sufficiently high fringe order field near a crack tip becomes large. An example of such an occurrence would be that of a specimen with a particularly low crack length vs. width ratio ($a/W \rightarrow 0$).

2.3.4 Stress Freezing - Three-Dimensional Analysis

The technique commonly referred to as the “frozen stress method”^{24,25} was employed for every photoelastic experiment conducted. Phenomenological details of photoelastic material behavior at elevated temperatures is covered in detail by Cernosek²⁶. From a physical standpoint, the frozen stress method of photoelasticity involves the loading of a photoelastic model while above its critical temperature (T_c). This temperature is dependent upon the material being used and often times the age of the material as well. When the photoelastic model reaches its T_c , it

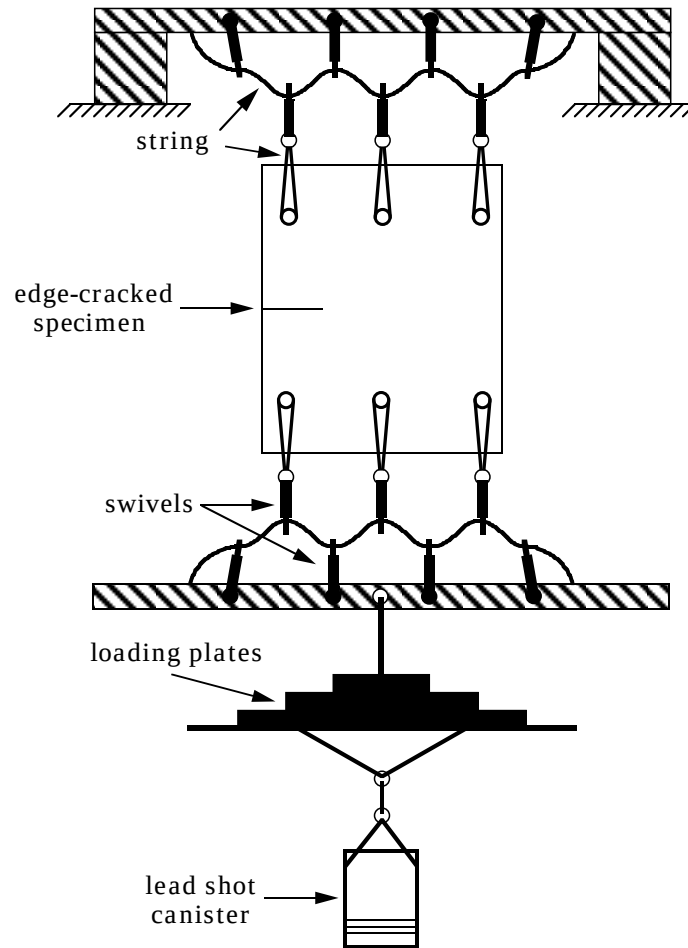


Figure 2.10: Loading apparatus used for araldite specimens. PSM-9/PLM-4B specimens were loaded similarly with the exception that a single loading pin was used on each specimen end.

becomes quite soft and rubbery. The material fringe value drops significantly thus yielding an increased sensitivity to stress. A twenty to thirty fold decrease in material fringe value is often observable with most photoelastic materials. Once the model is completely above its critical temperature regime, the desired mechanical load is placed on the model and allowed to stabilize. While maintaining the load on the specimen, a controlled cooling cycle of the model back down to room temperature will effectively “lock” or “freeze” the photoelastic stress field in place. Once room temperature has been reached, the model may be unloaded and subsequently analyzed. Since the fringe sensitivity of the material at room temperature is significantly lower than it is above its T_c , the effect of unloading the stress frozen model at room temperature is negligible. The time temperature thermal cycle used for the loading process is very similar to that of the post-cure cycle depicted in Fig. 2.10. For both the araldite and PSM-9/PLM-4B specimens, the heating and cooling rates used were the same. For the araldite specimens, a peak soaking temperature of 250 F was employed while for the PSM-9/PLM-4B specimens, a peak soaking temperature of 220 F was used. These variations are simply due to the differences in critical temperatures between the materials. The heating and cooling rates employed for all tests were the same since all models were of the same nominal thickness.

Photographs of the photoelastic shear stress field in each specimen are taken while the specimen is in the polariscope. The frozen stress method proves particularly beneficial in two ways. For one, the amount of load necessary to generate a high density fringe field is twenty to thirty times lower than would be required if the model were loaded at room temperature. Typical loads used for experiments considered in this study were in the neighborhood of 6 to 14 pounds (2.7 - 6.3 kg.). At room temperature, these loads would therefore need to be increased by a factor of twenty to thirty to generate similar fringe densities as those achieved above the critical temperature. Secondly, a stress frozen photoelastic model can inherently be photographed and analyzed in the polariscope without the use of a loading apparatus within or near the polariscope itself. This aspect of the procedure proves to be very important when considering the photographic techniques required to produce high magnification close-up images. Versatile

positioning of the stress frozen photoelastic model within the polariscope is intrinsically more simplistic than the positioning of a model undergoing a live load within a loading rig. The photographic techniques employed for producing resolute high magnification images require a close and fixed distance between the camera lens and the model itself. On the other hand, wide view images require a specific positioning of the model relative to the polariscope. These changes in position of the model relative to the camera and the polariscope would inherently require movement of either the entire loading rig or the entire polariscope if live loads were to be used at room temperature.

Perhaps the most appealing aspect of the frozen stress method is its ability to allow for the extraction of photoelastic data from within a model. Principal shear stress values at points within a photoelastic model can be determined by dissecting the model into thin slices. The cutting and removal of interior sections of a photoelastic model does not disturb the state of frozen stress previously locked into the model. By removing thin slices from planes parallel to the upper and lower surfaces of a model, one can effectively quantify the variation in principal shear stress as a function of position through the thickness. For fracture problems, these thin slices can be taken in the region of the crack and individually analyzed photoelastically to generate stress intensity factors. Each fracture specimen analyzed had three thin slices removed from its crack tip area. Fig. 2.11 illustrates their typical relative positions through the model thickness. Slice thicknesses were approximately 0.04 in. (1 mm). For each model, the first two slices near the outer surfaces (designated as “edge slices”) were taken approximately 0.04 in. (1 mm) from their respective surfaces while the third slice was taken from the middle of the specimen. All slices were extracted using a thin circular blade diamond saw that was cooled in a bath of fluid during operation.

2.3.5 Slice Analysis

Once thin slices have been extracted from a photoelastic model, data from each slice may be taken using the polariscope. Since the magnitude of the principal shear stress in a photoelastic model is inversely proportional to

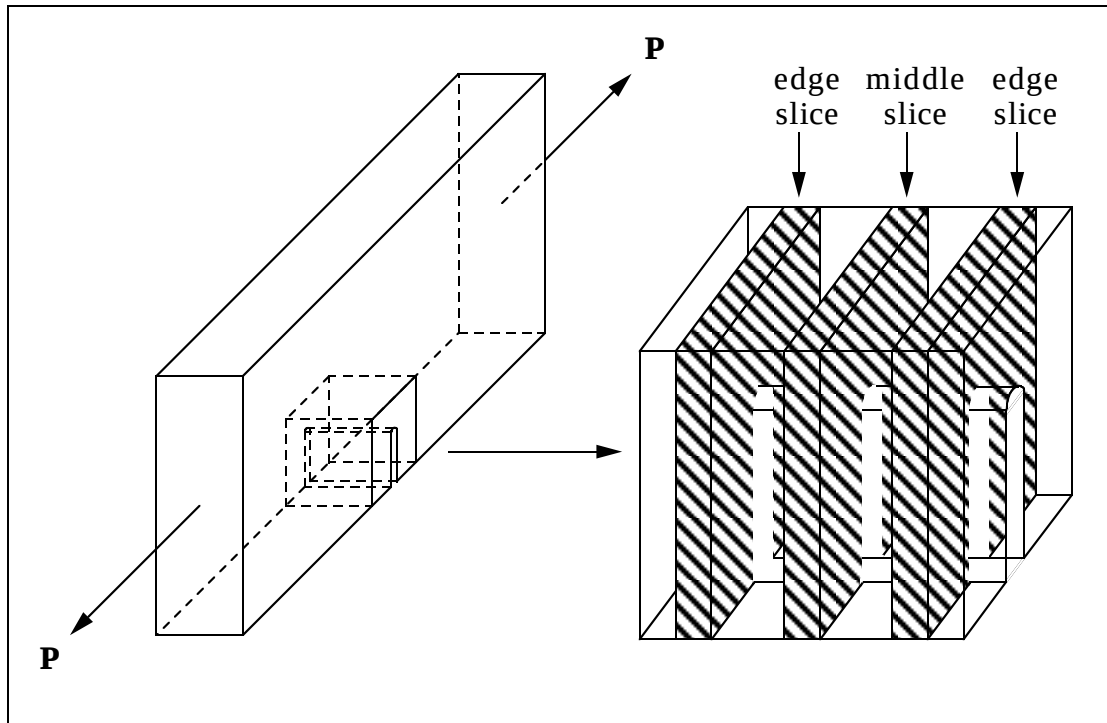


Figure 2.11: Orientation of slices taken near the crack tip region of each photoelastic specimen.

the thickness of the model, the magnitude of the principal shear stress present in an extracted slice will be proportionally lower based on the thickness of the slice as compared to the original thickness of the entire model. For the specimens considered in this study, typical thicknesses were near 0.500 in. (12.7 mm) while slice thicknesses were approximately 0.04 in. (1 mm). This thickness ratio thus yielded on average a twelve fold decrease in fringe order and thus fringe quantity when viewing slices in the polariscope. The accuracy of most if not all existing algorithms used for converting photoelastic data into stress intensity factors rely on a reasonable quantity of data to be taken near the region of the crack tip. A method of improving the quantity of fringes as seen in the individual slices was thus needed to achieve accurate stress intensity factors. The first step in achieving such an improvement is to maximize the initial stress freezing load such that the maximum stress intensity factor found at the crack tip of the model is only moderately lower than the critical stress intensity factor (K_{IC}) for the material. Other considerations such as stress concentrations at the loading pins may also dictate the maximum load that can be safely used without causing damage or failure of the model. For most photoelastic stress freezing materials, the maximum allowable damage free load for a given scenario will still require that the slices extracted for analysis be rather thick. Over a given model cross section, thicker slices will tend to more greatly discretize the distribution of information. This undesirable effect dictates that slice thicknesses remain fairly small in relation to the overall model thickness.

2.3.5.1 Tardy Method

Two methods for effectively increasing the quantity of readable data from thin slices were used in conjunction for the results presented in this study. The fringe multiplication technique²⁷ coupled with the Tardy method^{24,28} provided an adequate amount of photoelastic data from each slice without the use of large slice thicknesses.

The Tardy method involves partial rotations of the analyzer of the polariscope which in turn generate fringe fields whose fringe orders

correlate neither to the dark ($N=0, 1, 2, 3, \dots$) or light fields ($N=0.5, 1.5, 2.5, 3.5, \dots$) commonly dealt with in photoelasticity. In other words, fractional rotations of the analyzer will generate fractional fringe order fields. For example, a rotation of the analyzer through 45 degrees relative to its dark field position will produce a fringe field whose fringe order magnitudes are given by $N=0.25, 1.25, 2.25, 3.25$, etc. For full field analysis of a photoelastic model, the Tardy method can prove cumbersome due to the requirement that the isoclinics (fringes of constant principal shear stress direction) need to be “placed over” each point from which partial fringe order data is to be taken. Only the dark and light photoelastic fields are exempt from this requirement. For a thorough explanation of these phenomena, see Dally and Riley¹⁹.

Fortunately, the Smith extrapolation method for fracture analysis requires that data be taken along a straight line whose origin lies at the crack tip. This requirement coupled with the relatively small zone over which data is taken usually allows for most of the data to be read from just a single setting of the isoclinic fringe. This greatly facilitates manual data acquisition and reduces potential error associated with the otherwise repetitive but necessary adjustment of the apparatus. All data used in this study was taken using Tardy increments of 0.1. Thus, in doing so, fringe order and positional data (N, r) were found for nine discrete locations between each set of adjacent dark field fringes.

2.3.5.2 Fringe Multiplication

Unlike the Tardy method, the fringe multiplication technique developed by Post²⁷ provides a means for increasing the photoelastic data in a given slice or model without making repetitive adjustments to the elements of the polariscope. Specifics concerning the mathematics and optimization of the technique are not given here but are well documented in the literature.

The technique is based on the placement of a photoelastic model between two partial mirrors positioned at slightly inclined angles to one another. Referring to Fig. 2.12, a collimated source of monochromatic light enters from the left and passes through the first partial mirror which is

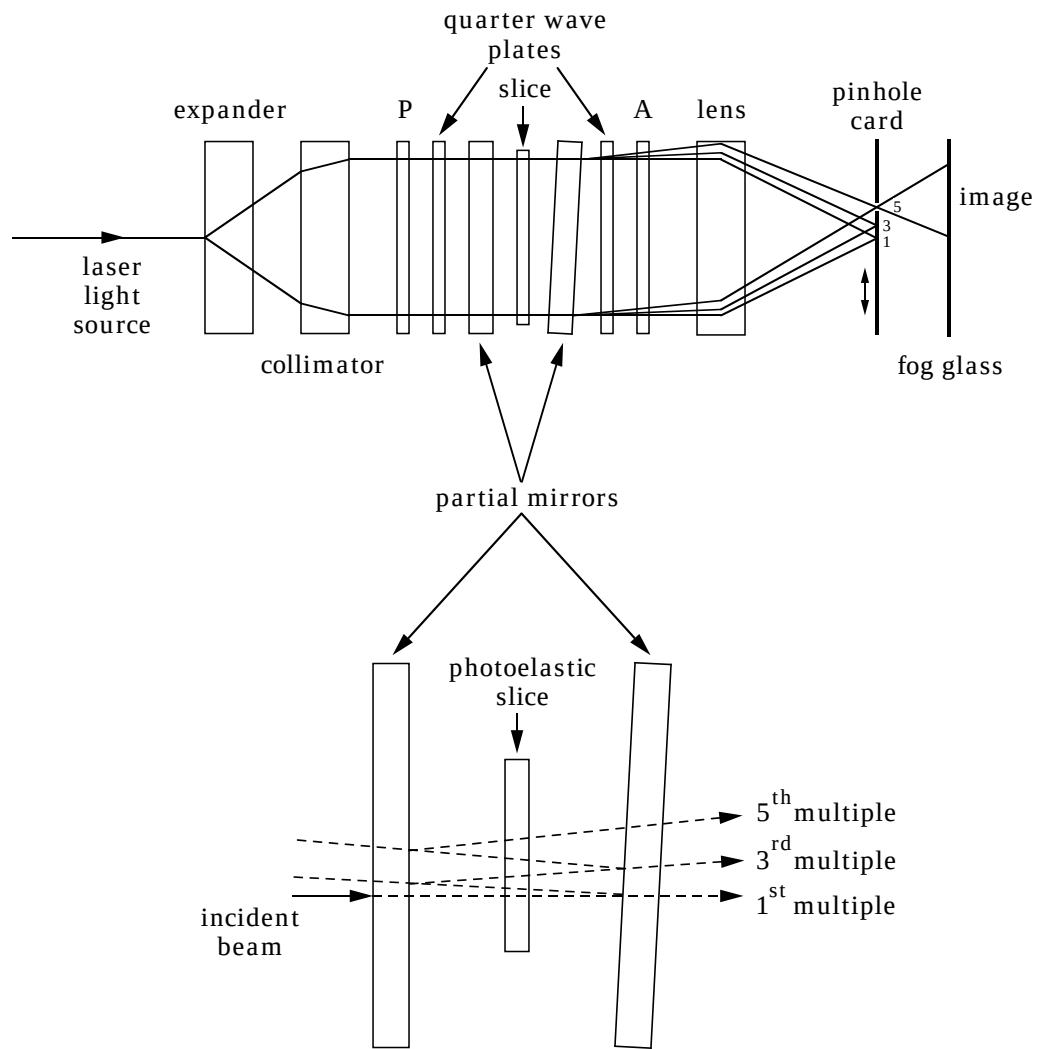


Figure 2.12: Fringe multiplication apparatus used for increasing the quantity of photoelastic data obtained from thin slices.

placed perpendicular to the direction of the incoming light source. The light next passes through the thin photoelastic slice and then approaches the inclined partial mirror. A portion of the light is transmitted through the inclined partial mirror while the rest is reflected back along an inclined path towards the slice. If the angle of inclination is small, the light will pass back through the slice at nearly the same position that it first entered. The light will now encounter the first partial mirror with similar results. A portion of the light will be transmitted while the rest will travel back through the model and once again encounter the inclined partial mirror. This sequence thus yields a series of slightly dispositioned light beams which have each passed through the photoelastic slice an increasing number of times.

By inspection it is seen that light beams that penetrate and emerge outwardly from the inclined partial mirror must have traveled through the slice an odd number of times (i.e. 1,3,5,7, ...). A light beam travelling through the slice in a small region, say, 5 times, will accordingly sum the principal shear stress in that region five times since the principal shear stress, as measured photoelastically, is inversely proportional to the model thickness. This in turn must generate a five fold increase in the number of fringes appearing in the considered region. By using a collimated monochromatic light source, the images are able to be focused and expanded as necessary, thus allowing for the multiplication effect to take place over the entire region of the slice. Furthermore, each order of multiplication can be separated from one another by focusing each emerging light beam down to a point. Due to the slight inclination of the second partial mirror, all exiting light beams will be slightly dispositioned from one another. A pinhole card can then be used to select the multiplication of choice while preventing the transmission of all other multiplied images.

2.3.6 Data Acquisition

In preparation for analysis, a slice is coated with index matching fluid and placed between the two partial mirrors. Placing the slice and partial mirrors into the rest of the apparatus (Fig. 2.12) allows for the simultaneous use of both Tardy and fringe multiplication techniques. Data correlating the

position of photoelastic fringes relative to the crack tip in a given slice is obtained manually using a translation stage. Integral with the translation stage is a micrometer that enables the user to vary the position of the slice and partial mirrors relative to the entire apparatus. Remaining fixed to the apparatus is a piece of fog glass containing thin crosshairs. Since the final image is displayed on the fog glass, distances between fringes and the crack tip can be measured by adjusting the position of the base with the micrometer.

Due to exponential decreases in output light intensity as a function of the desired fringe multiple, a fairly intense light source is generally required to generate a viewable image of highly multiplied fringe fields. With the available equipment, fringe multiplication fields of the fifth order were consistently achievable. Seventh order fields and higher were generally not visible due to substantial losses in light intensity caused primarily by the partial mirrors. Tardy increments of 18 degrees (which correspond to a 0.1 increase in the values of the fringe orders, "N") were made while using the fifth order fringe multiplication field. The combination of these techniques typically generated a total of thirty to forty data points from which ten to fifteen were eventually used for linear curve fitting in the extrapolation algorithm.

3.0 Inclined Single Edge-Notched Test Series

A series of photoelastic experiments were undertaken using moderately thick single edge-notched tension specimens. The stress freezing method of photoelasticity was employed to facilitate subsequent through-thickness analyses by extracting thin slices near the v-notch tip regions in each model. Four different angles of v-notch inclination relative to the uniform far-field tensile loading direction were studied. For each inclination angle considered, a three-specimen testing procedure was incorporated which utilized a homogeneous, bonded homogeneous, and bonded bimaterial specimen. The bonded bimaterial specimens were comprised of two elastically dissimilar isotropic materials which were bonded together at a common interface. The v-notches were machined directly into each specimen so that their tips lay within the thin, uniform bondlines which adjoined the mating halves. After stress freezing, stress intensity factors were experimentally determined for each complete specimen. Of primary interest was the measure of possible increases in stress intensity and mode-mixity due to the mechanical mismatch in elastic moduli. As such, the multiple specimen procedure was adopted in an effort to quantify and separate thermally related effects and adhesive curing stresses that might have influenced the final measurements of stress intensity and mode-mixity in the bimaterial specimens.

A discussion of model configurations and geometries is considered in Section 3.1. Material and analytical considerations are respectively discussed in Sections 3.2 and 3.3. Photographic and numerical results are then considered in the following sub-sections of Section 3.4. Finally, a discussion of the results with conclusions is undertaken in Section 3.5.

3.1 Model Configurations

A total of twelve photoelastic models were considered in this series of experiments. In order to induce a combined state of opening and shear modes, the v-notches for three of the four sets of experiments were oriented at inclined angles to the direction of far field tensile loading. The fourth set of

experiments utilized v-notches placed orthogonal to the direction of far field loading. Figures 3.1 - 3.4 illustrate the geometries and loading conditions of each specimen.

Using the general construction methods discussed in Section 2.3.1, each bondline was created using PSM-9 photoelastic adhesive and cast at a constant thickness of 0.014 in. (0.356 mm.). A v-notch was subsequently machined from one edge of each specimen into its respective bondline. As measured along the direction of the v-notch and bondline, a total notch length of 0.375 in. (9.525 mm.) was used for all models. A magnified image of the v-notch tip area and its dimensions is given in Fig. 3.5. The root radius of the notch tip saw used in these experiments was approximately 0.001 in. (0.025 mm.). Consequently, classic photoelastic fringe loop patterns correlating to high singular stress regions were observed near all v-notch tips. Specimen identifications have been assigned to each specimen for reference purposes and are tabulated in Table 3.1.

3.2 Materials

Two similar photoelastic materials were selected for this series of tests for the purpose of creating bimaterial specimens of differing elastic moduli. Araldite, a common translucent photoelastic material, was used for all homogeneous and bonded homogeneous specimens. To create a stiffer material for use in the bimaterial experiments, aluminum powder was added to araldite during the manufacturing process. As measured by weight, a 25% addition of aluminum powder to araldite generated a material with an elastic modulus twice that of pure araldite. This material, which will be referred to as “alum.-aral.”, was bonded to pure araldite to create the bimaterial specimens. A list of the properties of these materials is given in Table 3.2. Alum.-aral. was tested and chosen for use in the bimaterial specimens primarily due to the observation that the addition of aluminum powder to the araldite generated the desired stiffening effect while not altering the critical temperature of the material. The addition of aluminum powder to araldite in a sense also served to mimic the particulate nature of the propellant used in the solid rocket motor grains. By matching the critical

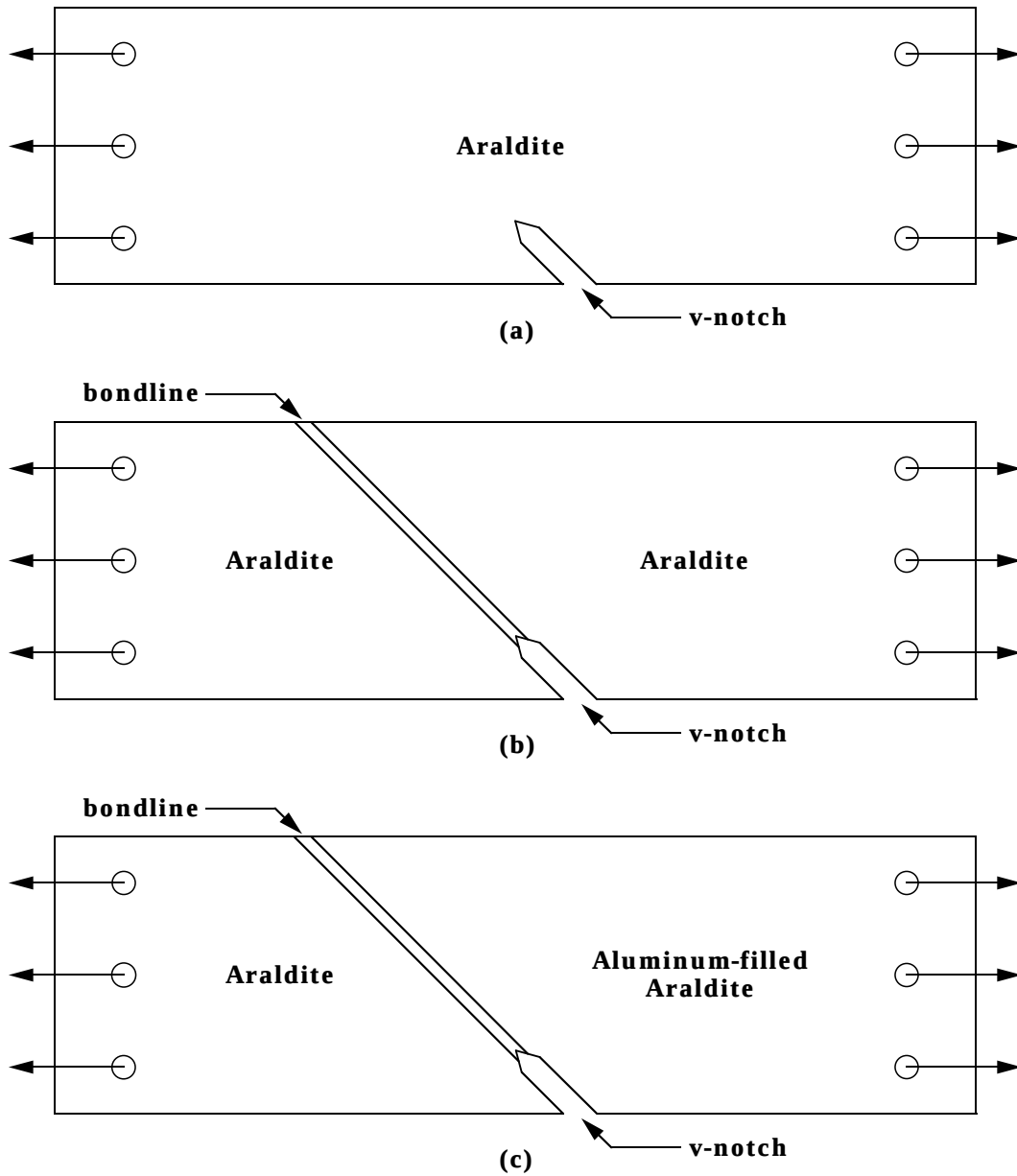


Figure 3.1: $\beta=45^\circ$ inclined-crack specimens - geometry and loading.
a) Specimen A1: Homogeneous specimen (araldite)
b) Specimen A2: Bonded homogeneous specimen (araldite)
c) Specimen A3: Bonded bimaterial specimen (araldite/alum-aral.)

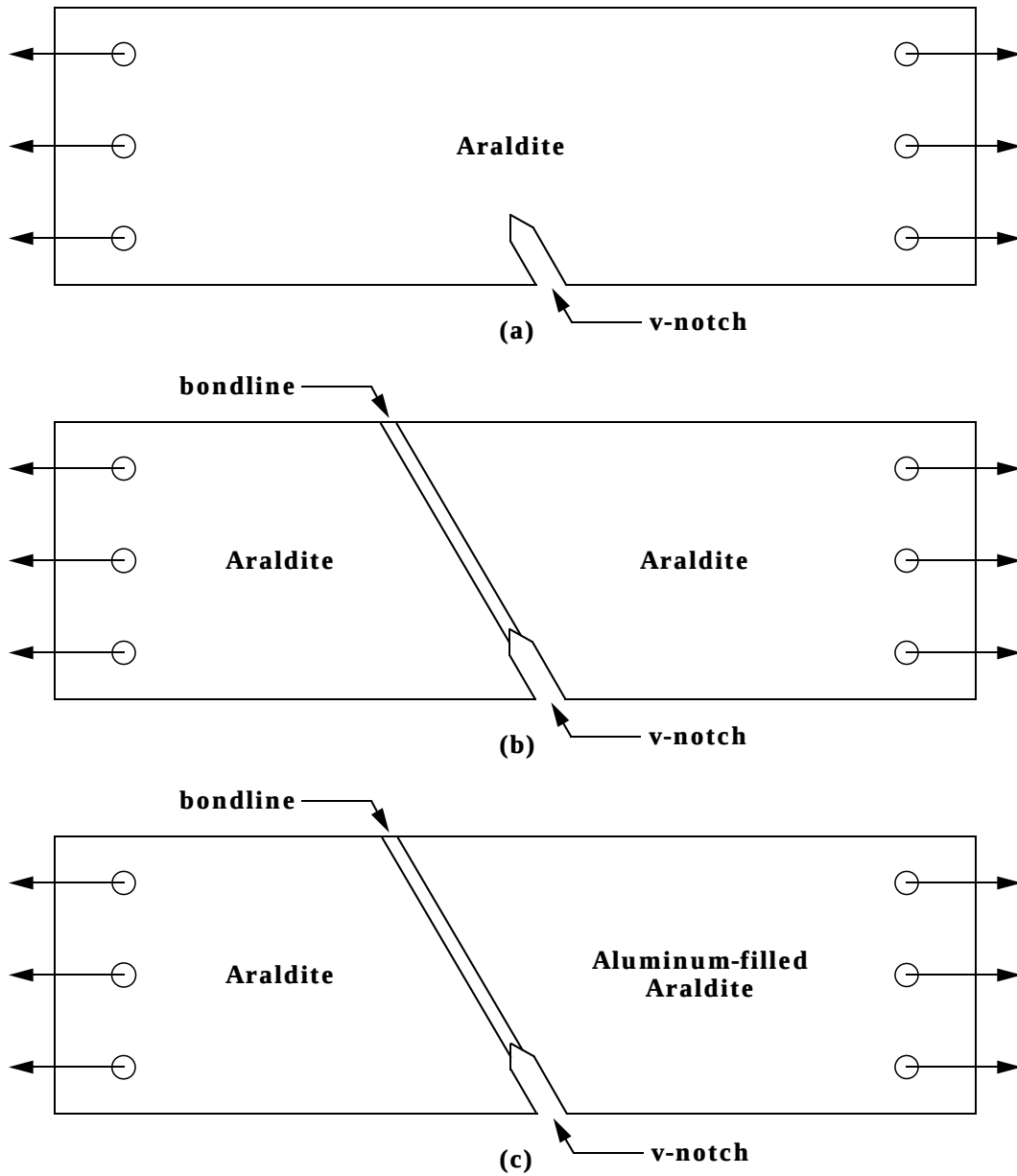


Figure 3.2: $\beta=30^\circ$ inclined-crack specimens - geometry and loading.
a) Specimen A4: Homogeneous specimen (araldite)
b) Specimen A5: Bonded homogeneous specimen (araldite)
c) Specimen A6: Bonded bimaterial specimen (araldite/alum-aral.)

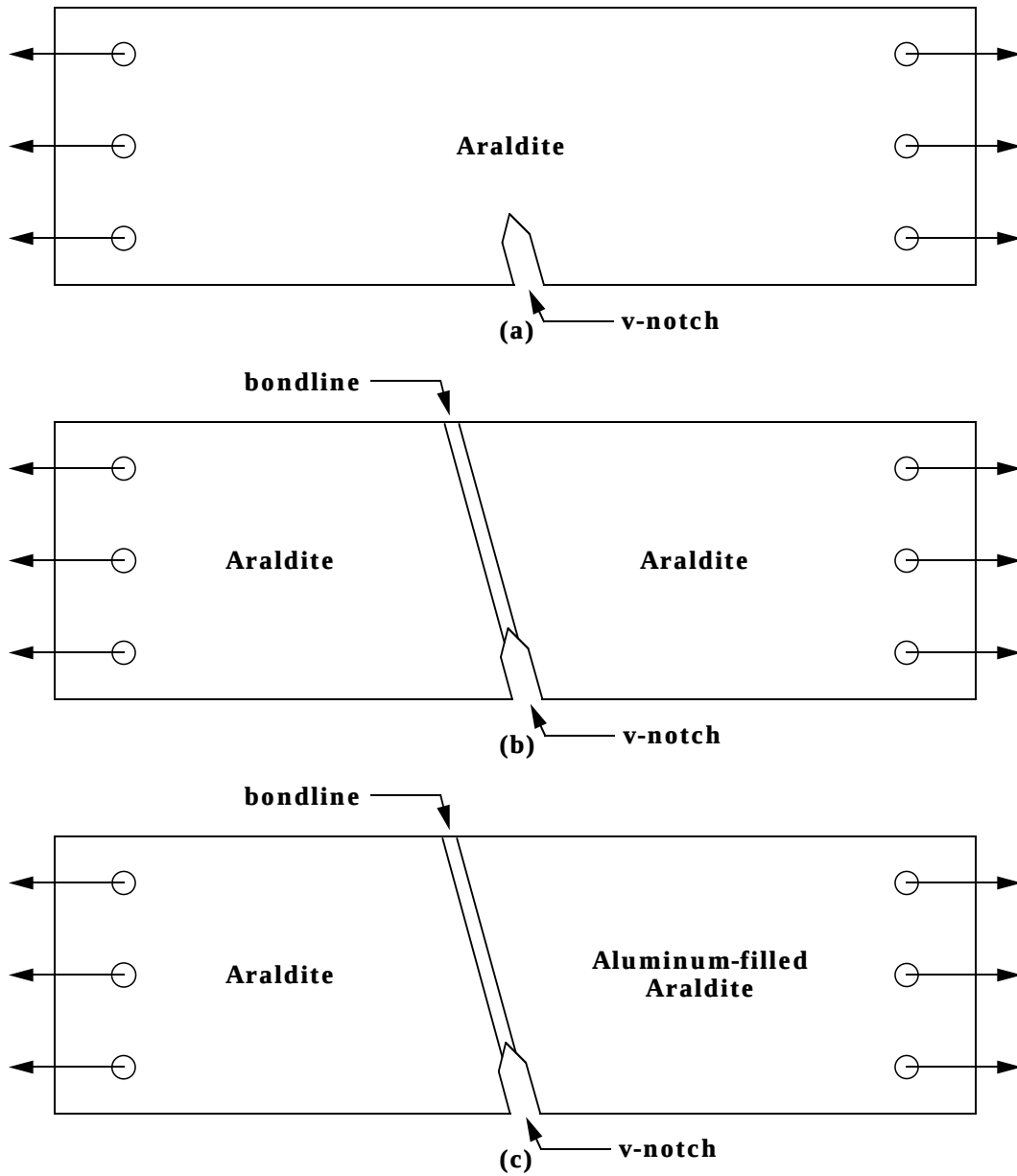


Figure 3.3: $\beta=15^\circ$ inclined-crack specimens - geometry and loading.
a) Specimen A7: Homogeneous specimen (araldite)
b) Specimen A8: Bonded homogeneous specimen (araldite)
c) Specimen A9: Bonded bimaterial specimen (araldite/alum-aral.)

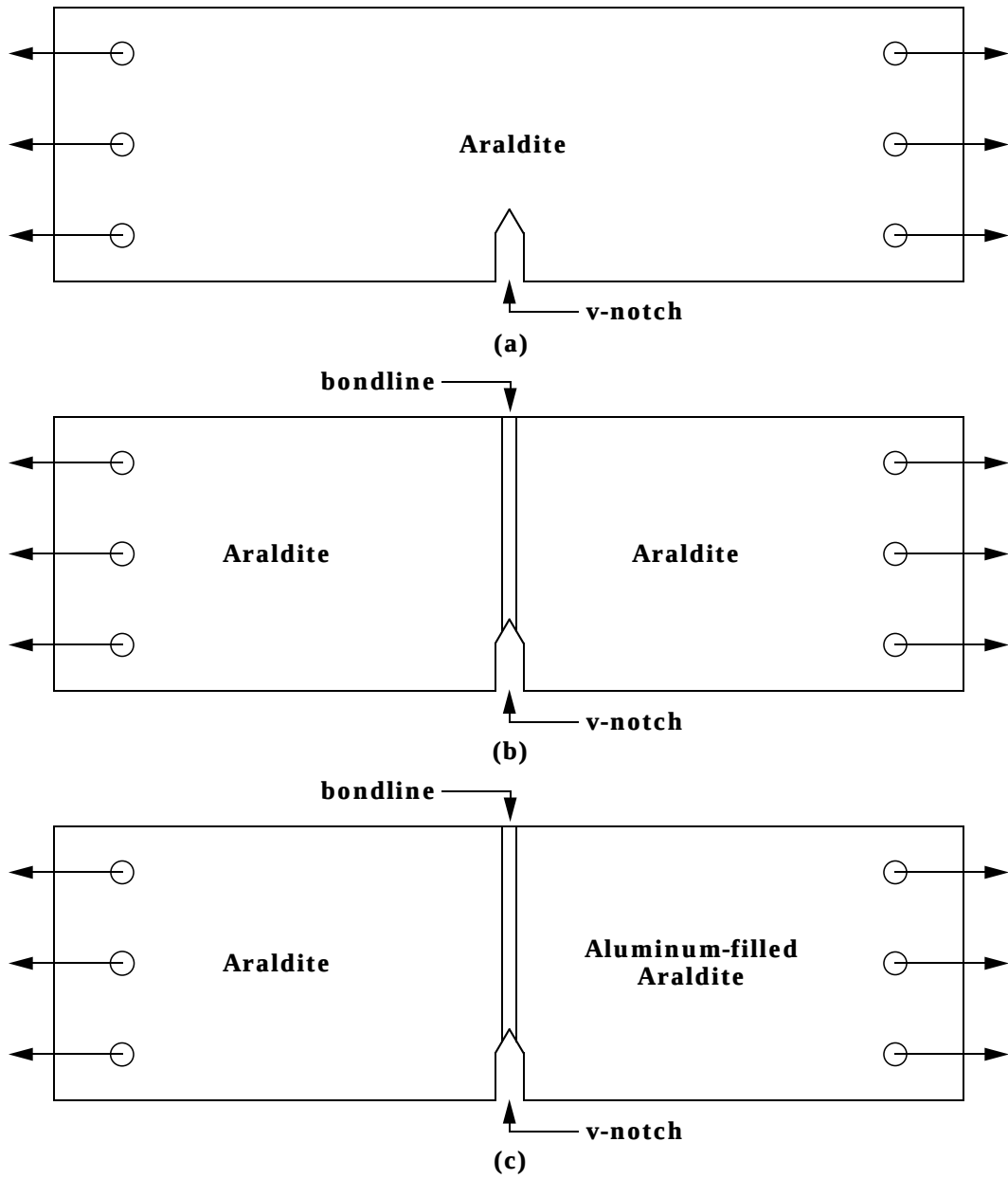


Figure 3.4: $\beta=0^\circ$ inclined-crack specimens - geometry and loading.
 a) Specimen A10: Homogeneous specimen (araldite)
 b) Specimen A11: Bonded homogeneous specimen (araldite)
 c) Specimen A12: Bonded bimaterial specimen (araldite/alum-aral.)

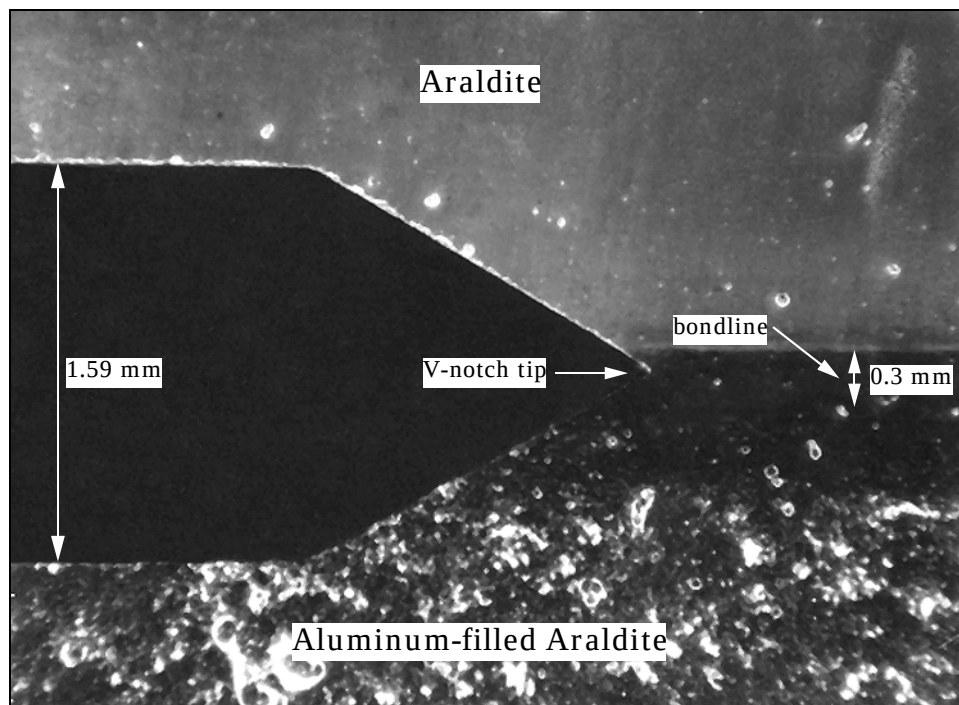


Figure 3.5: V-notch dimensions and tip placement in bimaterial specimen A12.

Table 3.1 - Test specimen identification, composition, and geometry.

Specimen Identification	Specimen Type *	Crack Angle of Inclination (β) **
A1	Homogeneous	45
A2	Bonded Homogeneous	45
A3	Bonded Bimaterial	45
A4	Homogeneous	30
A5	Bonded Homogeneous	30
A6	Bonded Bimaterial	30
A7	Homogeneous	15
A8	Bonded Homogeneous	15
A9	Bonded Bimaterial	15
A10	Homogeneous	0
A11	Bonded Homogeneous	0
A12	Bonded Bimaterial	0

* Homogeneous specimens were constructed of araldite material.
 Bonded homogeneous specimens were constructed of araldite materials bonded together with PSM-9 adhesive.
 Bonded bimaterial specimens were constructed of araldite and aluminum-powdered araldite bonded together with PSM-9 adhesive.

** Angles measured in degrees, See Figure 2.6

Table 3.2 - Araldite and aluminum-araldite material properties.

Material	Critical Temp. (T _c) Farh. (deg. C)	Modulus of Elasticity* psi. (MPa)	Material Fringe Value* (f) lbs./in. (N/cm)	Poisson Ratio* (ν)
Araldite	240 (116)	2698 (18.60)	1.64 (2.87)	~ 0.50
Alum-aral.	240 (116)	5349 (36.88)	-----	~ 0.50

*Values determined at approximately 250 F.

temperatures of the two materials, undesirable effects associated with the hardening of the two adjoined materials during the cooling stage of a stress freezing cycle would be avoided. A disadvantage associated with using aluminum-filled photoelastic materials is of course that light from a polariscope can no longer pass through the material. This obstacle was overcome by constructing the bimaterial specimens in such a way that the photoelastic data near the crack tip could be read on the side of the notch tip consisting of pure araldite. In theory, there is only one value of stress intensity associated with a given crack under a given load. As such, the same value of stress intensity should be experimentally determined regardless of which side of the notch tip the data is taken from.

Moisture absorption or “time-edge” effects were prevalent near the borders of the araldite material. As a result, residual stresses appeared in the form of photoelastic fringes when viewing a plate of araldite in the polariscope. Furthermore, the creation of new surfaces by machining typically allowed for moisture from the air to almost immediately either penetrate or vacate the specimen along the newly machined borders. Consequently, it was inevitable that residual stresses would exist along the surfaces to be later bonded in the construction of complete specimens. As will be seen in the following sections of this chapter, the homogeneous-bonded homogeneous-bonded bimaterial testing procedure outlined in Section 3.1 aided in the quantification and relative influence of these time-edge effects on the final values of stress intensity.

The thermal cycling procedure used for this set of experiments was somewhat abbreviated due to the presence of residual stresses near the bondlines and notch tips of each specimen. In a trial experiment, it was observed that by subjecting a bonded araldite specimen to one complete no-load stress freezing thermal cycle, the magnitudes of the residual stresses near the bondline were in fact exacerbated rather than alleviated. This result thus prompted the removal of the no-load stress freezing thermal cycle from all test specimen experiments. After each complete specimen bondline was post-cured, each specimen was subjected to a stress-freezing cycle under load.

3.3 Analytical Considerations

The homogeneous control specimen associated with each angle of v-notch inclination was used as a reference from which to compare stress intensity values found from the companion bonded homogeneous and bonded bimaterial specimens. The equations given in Section 2.2.4 were used to determine the mode I and mode II stress intensity factors in each specimen. However, in order to obtain this information experimentally, the analysis of Section 2.2.5 was first used to properly determine the placement and orientation of the line from which data was read for each set of v-notch inclination angles.

For simplification purposes, all stress intensity data obtained from the through-thickness slices were first normalized with respect to each specimen's notch length and far field nominal tensile stress. By normalizing the stress intensity data in this fashion, it becomes both non-dimensionalized and independent of any variations in load or far field cross-sectional area found between individual specimens. These normalized values, given as F_I and F_{II} (for modes I and II respectively), are defined as:

$$F_I = \frac{K_I}{\bar{\sigma}\sqrt{\pi a}} ; F_{II} = \frac{K_{II}}{\bar{\sigma}\sqrt{\pi a}} \quad (3.1)$$

To facilitate the determination of relative differences in stress intensity among specimens containing a specific angle of v-notch inclination, all values of F_I and F_{II} were normalized by dividing their values by the mode I through-thickness average of stress intensity found from the corresponding homogeneous control specimen. Values of F_I and F_{II} as well as their normalized values are tabulated for each specimen in Section 3.4.2. Stress intensity values for each specimen were obtained from three thin slices taken throughout the thickness as described in Sections 2.3.4 and 2.3.5.

3.4 Results

A total of twelve experiments were performed using both pure and modified forms of a common photoelastic plastic (araldite). A discussion concerning the pre-load conditions is first addressed in Section 3.4.1. In Section 3.4.2, photographic documentation and discussion concerning the post-load stress fields present in each specimen is addressed. By using fringe multiplication and Tardy compensation in conjunction, sufficient data was obtained from each photoelastic specimen for use in the Smith Extrapolation Method. Using the extrapolation method, stress intensity factors were experimentally determined by analyzing thin slices extracted from discrete positions throughout the thickness of each specimen. Numerical results pertaining to the experimentally determined values of stress intensity and mode-mixity are presented in Section 3.4.3. Finally, key features and trends found in the normalized values of stress intensity are highlighted in the context of their relative differences.

3.4.1 Pre-Load Stress Fields

Prior to testing, it was accurately assumed that special procedures would be required to perform stress intensity measurements in the direct vicinity of bondlines. Once the specimen construction details were outlined, a qualitative experimental determination of bondline residual stresses was undertaken. As was mentioned in Section 3.2, material instabilities combined with the natural shrinking processes involved with the curing of adhesives generated residual stresses near the bondlines. An evaluation of the effect of a no-load thermal cycle on the residual stress fringes was made by placing a post cured bonded specimen on a teflon coated glass plate and subjecting it to a thermal cycle above the material's critical temperature. Instead of reducing the magnitude of residual stress, this procedure seemed to moderately increase both the magnitude of stress and the affected area. As a result, after proper post-curing, specimens were subjected to loaded thermal cycles to avoid any further increases in residual stresses near the vicinity of the notch tips.

As will be seen in the results section of this chapter, the pre-load residual stress fringe fields are suspected to have contributed to elevations in stress intensity found in certain bonded specimens. Referring to Fig. 3.6, photoelastic photographs of the bondline and v-notch tip area for bonded homogeneous specimen A5 are displayed. From these photographs it is clear that a non-uniform residual stress field is present in the vicinity of the notch tip. A similar pattern was likewise found in every bonded specimen considered in this study. It is believed that the orientation of these pre-load residual stress fringes played a role in the overall distribution of stress intensity factors found between different sets of tests. Inspection of the specimen depicted in Fig. 3.6 revealed that the entire zone from which photoelastic data was ultimately taken was encompassed by the same photoelastic fringe, and thus, a relatively constant yet non-zero value of principal shear stress. The superpositioning of a constant non-zero stress field over the loaded stress field of a crack or sharp notch will not alter a stress intensity factor which was determined using the Smith Extrapolation Method. This property of the extrapolation method can be rationalized by noting that a general stress field was initially superimposed over the classical singular stress field in the initial derivation of the extrapolation method. As such, the extrapolation method can automatically account for the superpositioning of a general constant non-zero stress field over a singular stress field region. This property can be more easily demonstrated by increasing each fringe order in an extrapolation data set by a constant value. By performing a least squares linear curve fit on the adjusted data, the coordinate intersection of the extrapolated line will always be of the same value. This value is proportional to the mode I and mode II stress intensity factors. Altering the data in this manner simply changes the slope of the extrapolation line, but not its point of coordinate intersection.

Fig. 3.7 characterizes the general observations made from all four sets of inclined notch experiments. Referring to Fig. 3.7, it is seen that for the two sets of specimens containing the smaller angles of inclination ($\beta=0^\circ, 15^\circ$), the data zones were encompassed by non-uniform pre-load residual stress fields. This non-uniformity is derived from the relative positioning of the reading lines and residual principal shear stress fringes. To the contrary, the $\beta=30^\circ$

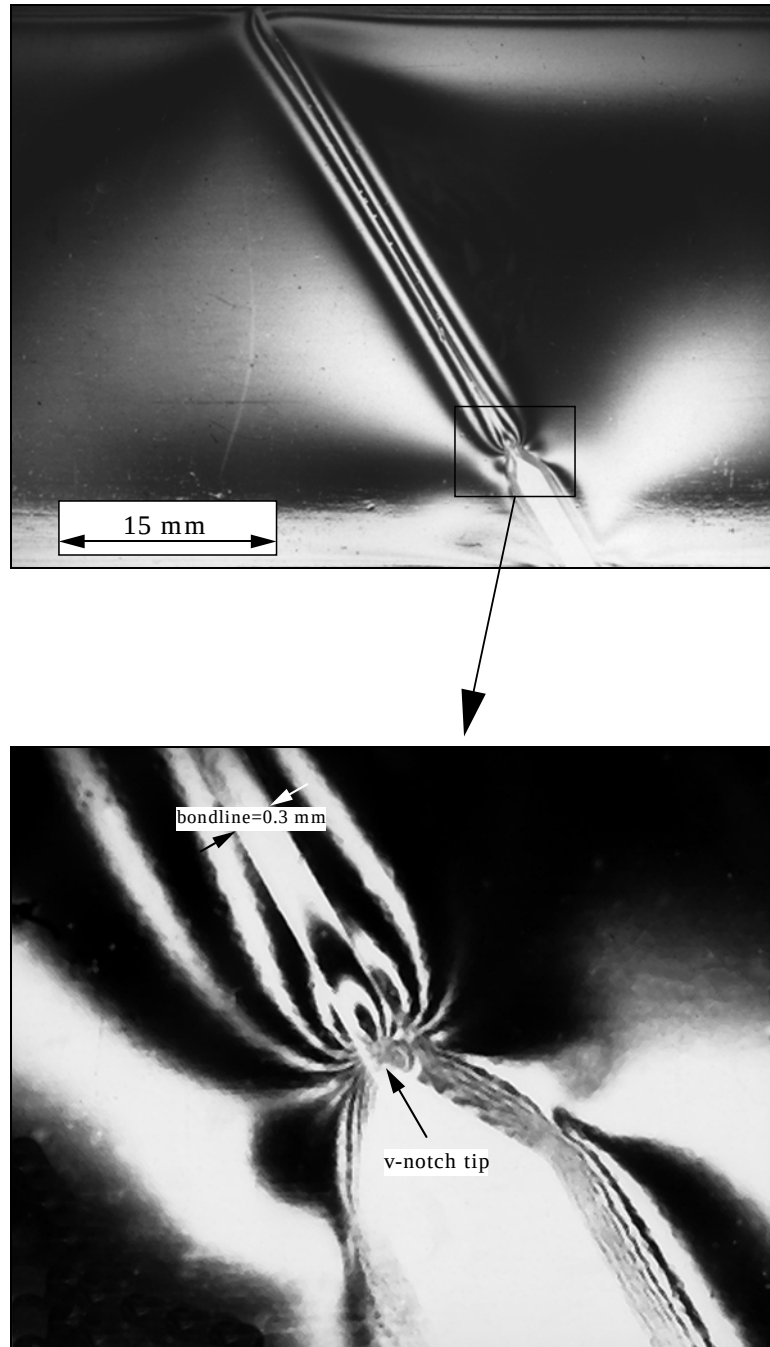


Figure 3.6: Pre-load residual stress distribution near bondline and v-notch tip areas of bonded homogeneous specimen A5. For this particular v-notch inclination angle (30°), the effects of the residual stress fringes on the measured stress intensities is minimal.

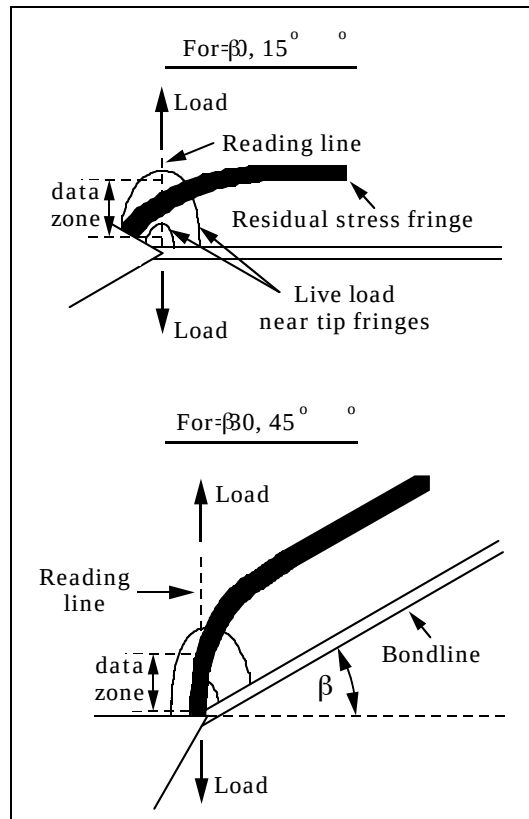


Figure 3.7: Residual stress fringe orientation as a function of crack inclination angle. For $\beta = 0^\circ$ and 15° , a stress gradient is observed over the zones from which stress intensity data is ultimately read. For $\beta = 30^\circ$ and 45° , little or no stress gradient is observed over the data zones.

and $\beta=45^\circ$ sets of experiments contained residual shear stress fringes which were parallel to and coincident with the data reading zones. The effects that these various pre-load residual stress patterns have on the final values of stress intensity will later surface when discussing the numerical results obtained from each set of photoelastic specimens. It will also be seen that by using the homogeneous-bonded homogeneous-bonded bimaterial test method, the previously mentioned residual stress effects can be accounted for in such a manner as to allow for the determination of the relative increases, if any, in stress intensities due to mechanical effects associated with the mismatch of elastic moduli in the bimaterial specimens.

3.4.2 Post-Load Stress Fields

For the stress freezing process, a thermal cycle similar to that depicted in Figure 2.9 was used for each specimen. A peak temperature of 255 F was used to insure that the araldite materials had completely transitioned from their glassy to rubbery states. Each specimen was cycled in the loading oven individually. The $\beta=15^\circ$ bonded homogeneous specimen (A8) unfortunately fractured under load and was thus unable to be used in the investigation. Only numerical results obtained for its homogeneous and bimaterial counterparts (A7 and A9) have been included.

Through-thickness photographs depicting the stress frozen photoelastic fringe fields for the $\beta=45^\circ$, 30° , and 0° specimen sets were taken after the completion of each respective stress freezing cycle but prior to the removal of slices from within each specimen. Figures 3.8 - 3.16 depict wide, medium, and close views of various specimens. The wide view images of most specimens depicted in these figures reveal a relatively uniform far field tensile stress as indicated by a single dark fringe spanning the entire width of each specimen on either side of a given v-notch. As will be seen in the experiments considered in Chapter 4, a single-pin loading arrangement was adopted rather than the three-pin arrangement used for the inclined v-notch tests. This arrangement was preferred primarily for its simplicity and consistency in generating uniform far field loads (See Section 2.3.3).

For the general determination of the proper data reading line location and orientation, Fig. 3.12 is of particular interest as it reveals in detail the nature of the fringe distribution in the highly singular region near the v-notch tip of specimen A5. By constructing a straight line connecting the vertices of the near tip fringes, it was observed that the line's orientation was parallel to the direction of far field tension loading. This property was actually found in every specimen, regardless of the values of notch tip inclinations (β) considered in this study. This visual observation was in very good agreement with the analytical derivation described in Section 2.2.5 which predicted negligible deviation in the reading line orientation relative to the direction of far field loading for all of the notch inclination angles considered (Fig. 2.8). Examination of Fig. 3.12 reveals that a proper reading

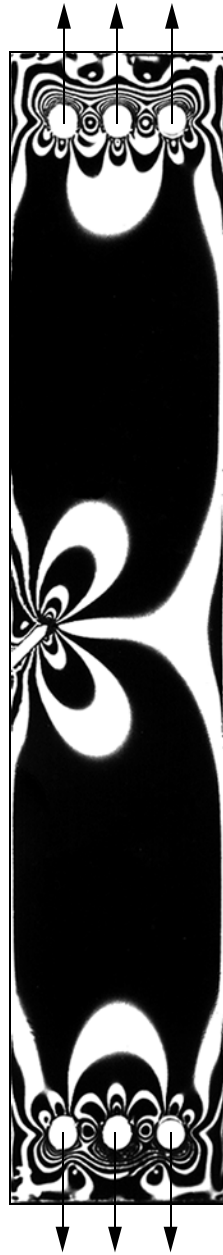


Figure 3.8: Specimen A1 post-load fringe field.



Figure 3.9: Specimen A2 post-load fringe field.

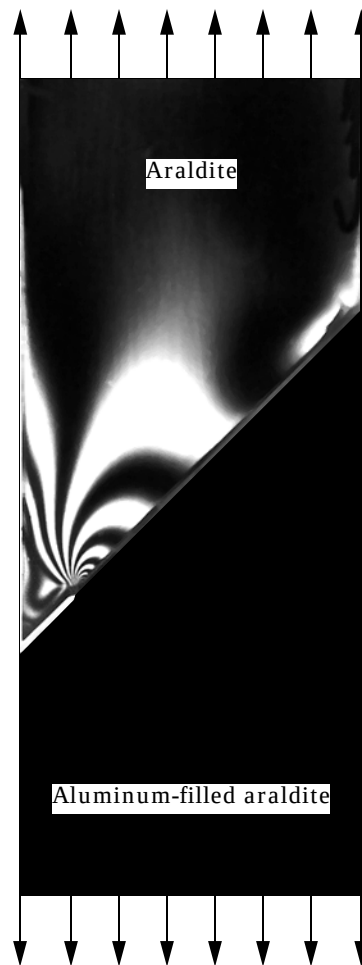


Figure 3.10: Specimen A3 post-load fringe field.

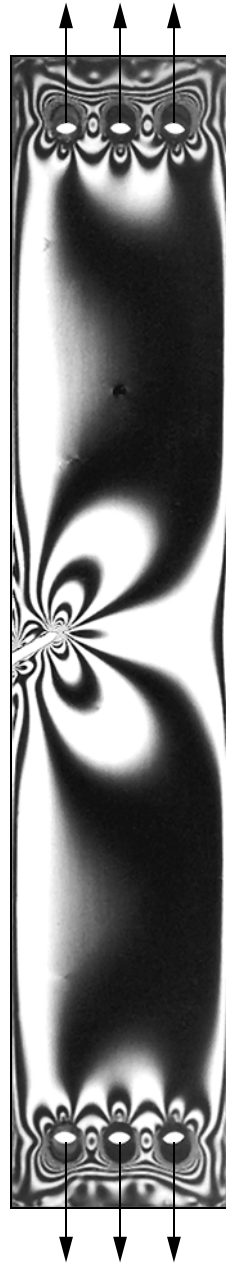


Figure 3.11: Specimen A4 post-load fringe field.



Figure 3.12: Specimen A5 post-load fringe field near v-notch tip.

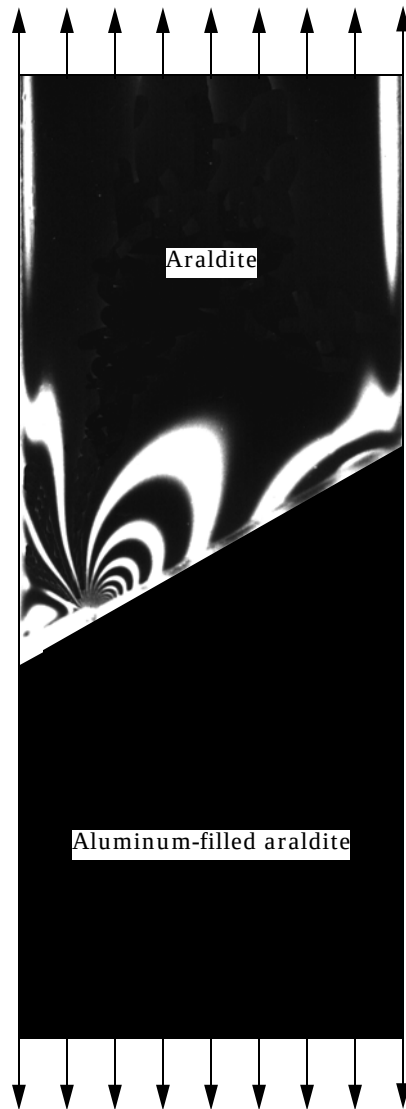


Figure 3.13: Specimen A6 post-load fringe field.

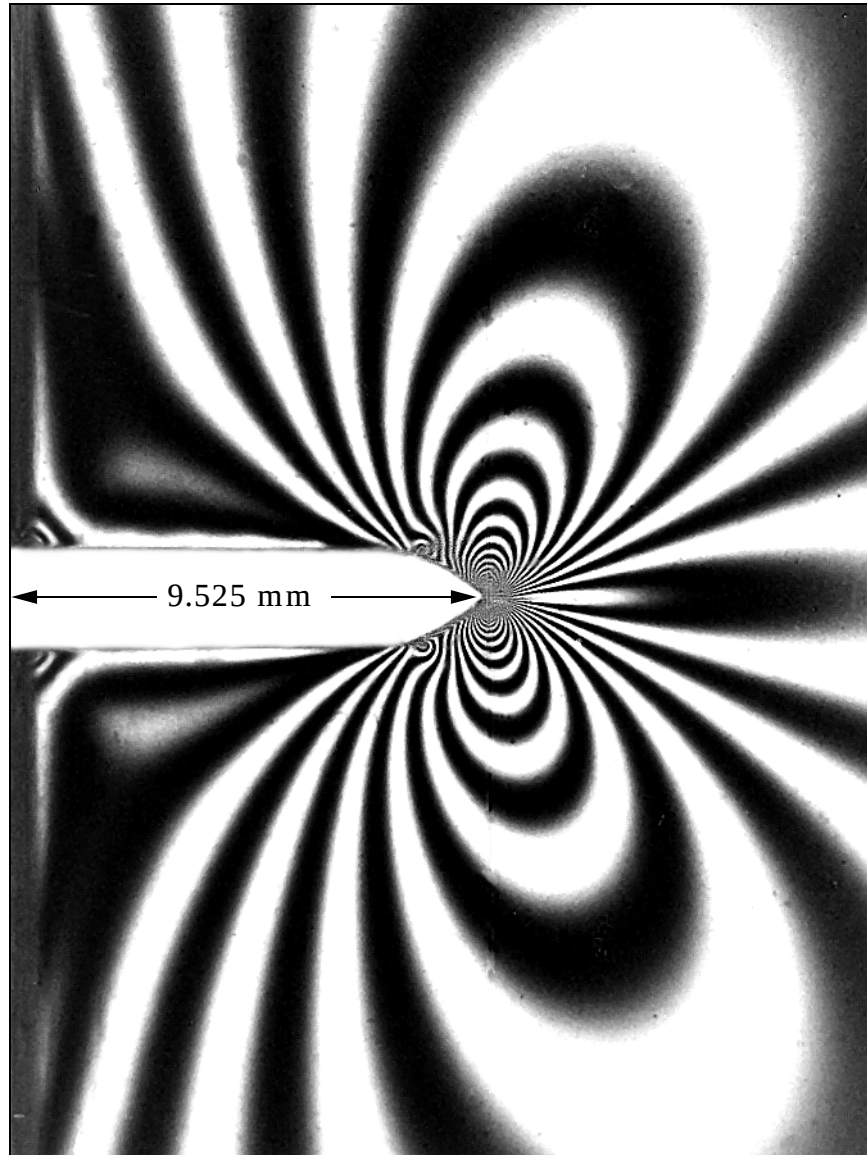


Figure 3.14: Specimen A10 post-load fringe field.

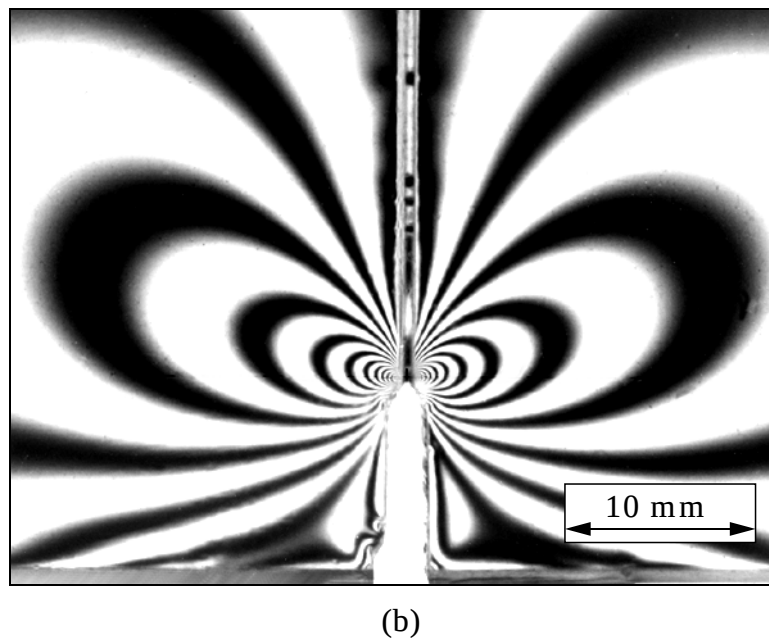
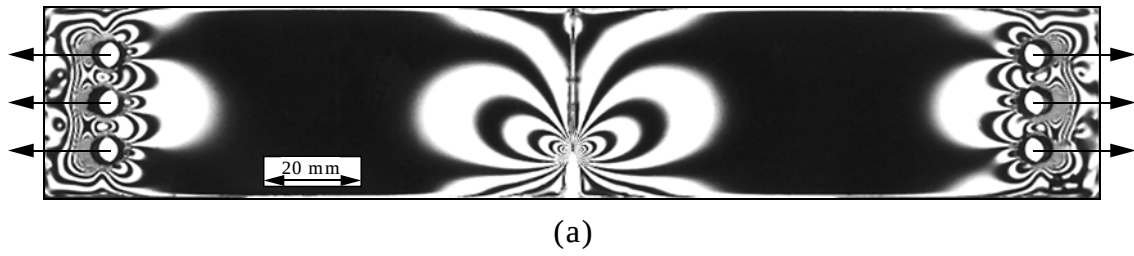


Figure 3.15: Specimen A11 post-load fringe field.
a) Wide view of complete specimen.
b) General crack-tip region.

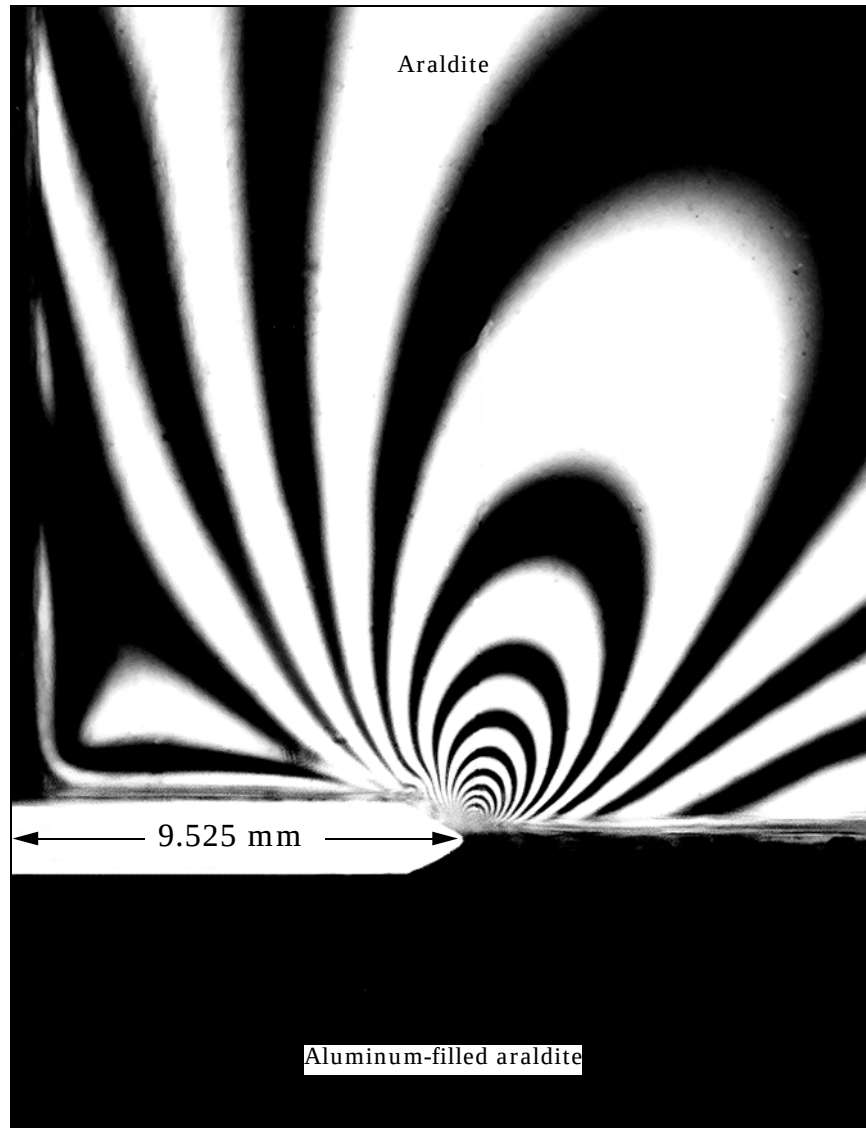


Figure 3.16: Specimen A12 post-load fringe field.

line orientation will be typically based upon fringes within 1 millimeter of the flaw tip in question. Beyond this region, non-singular far field effects tend to reorient the fringe extrema. Using these fringes as a guide for determining a reading line orientation would thus yield varying, inaccurate angular values which would be incorrectly based on non-singular rather than singular stress effects. It is thus important to be able to acquire high magnification images such as that in Fig. 3.12 so as to insure that an accurate reading line is selected for subsequent data acquisition and stress intensity factor determination.

Using the fringe multiplication apparatus outlined in Section 2.3.5.2, a typical photoelastic image of a slice is shown in Fig. 3.17. The pixelation found in this image is primarily due to the monochromatic nature of the laser light used and required for fringe multiplication. The near tip fringe extrema to the upper side of the v-notch tip in this image again demonstrate the parallel nature of a properly selected reading line and the corresponding direction of remote tensile loading.

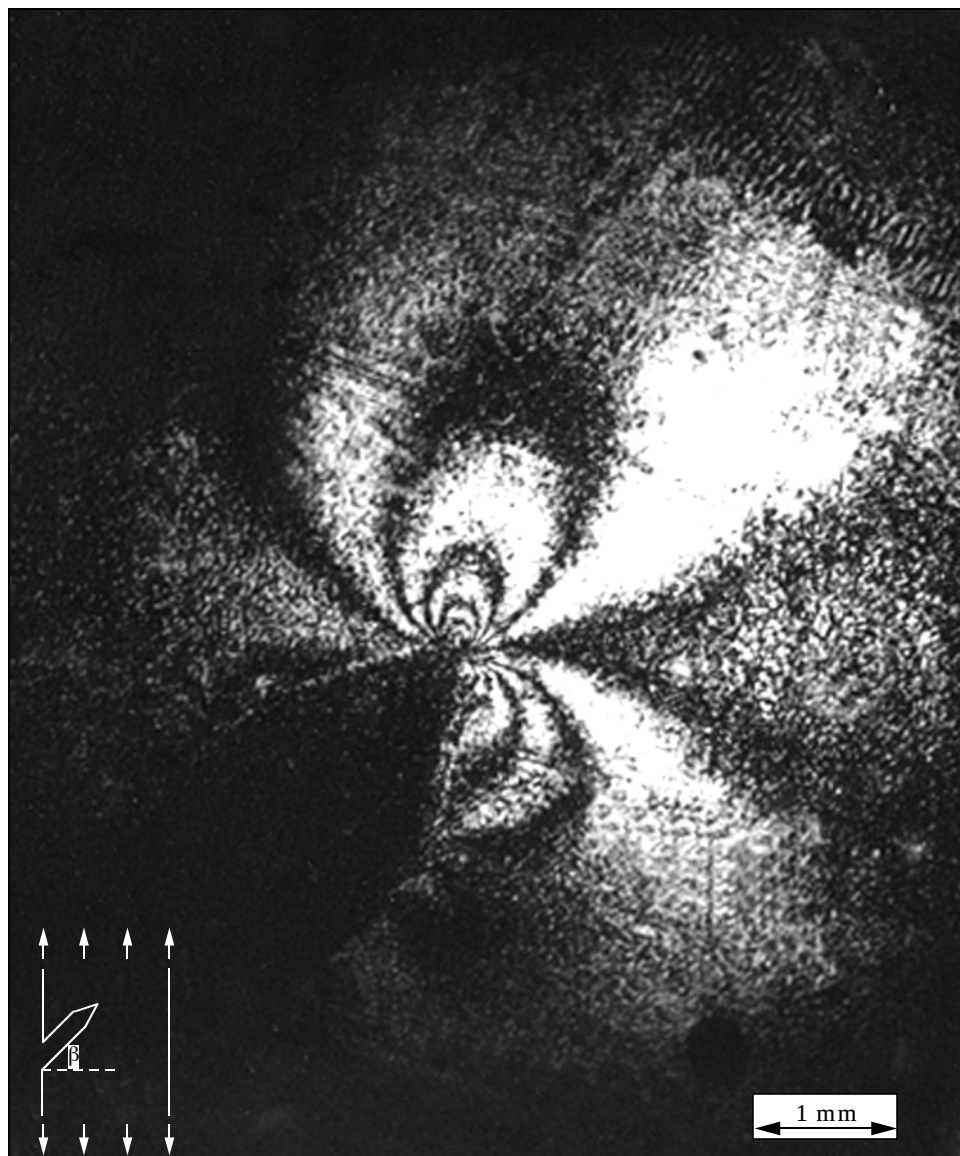


Figure 3.17: Specimen A1 middle slice image.
Fifth order fringe multiplication, $\beta=45^\circ$

3.4.3 Normalized Stress Intensity Factors

Stress intensity factors were obtained from thin slices taken throughout the thickness of the v-notch tip regions of each specimen. Three slices in total were taken from each cross section. One slice was removed from each specimen edge with the third slice being extracted from the middle of the cross section (See Fig. 2.11). Edge slices have been designated by “L” and “R” for left and right, while middle slices have been designated by “M”. Thus, combining the previously adopted specimen designation with the slice designation completely defines the source of data. For example, data designated by “A9-LR” is representative of the average values of stress intensity obtained from the left and right side edge slices of the $\beta=15^\circ$ bonded bimaterial specimen.

As discussed in Section 3.3, the SIF's (stress intensity factors) obtained from slices were non-dimensionalized in such a manner as to facilitate direct comparison between all specimen results regardless of any possible small variations in cross sectional areas, notch lengths, and applied loads. These non-dimensionalized values have been designated as “ F_I ” and “ F_{II} ” for correlation with their respective mode I and mode II stress intensity factors. For a given set of specimens possessing the same notch angle of inclination β , values of F_I and F_{II} have been normalized with respect to the LMR three slice average of the homogeneous control specimen. These values are of particular interest as they simplify the direct comparison of relative differences found within a three-specimen set of tests.

Tables 3.3a - 3.3d contain M, LR, and LMR slice data obtained from each specimen in both non-dimensionalized form (F_I and F_{II}) and normalized form ($F_{I \text{ norm.}}$, $F_{II \text{ norm.}}$). By examining the normalized values of stress intensity ($F_{I \text{ norm.}}$, $F_{II \text{ norm.}}$), several important trends were found, namely:

- With the exception of only one test, all middle slice data (M) displayed moderately elevated values over those of the corresponding LR edge slice averages. Relative elevations ranged between approximately 5 and 20 percent.

Table 3.3a - Normalized stress intensity values for the
 $\beta=45$ degree test series.

Specimen ^{*1}	F_I ^{*2}	$(F_I)_{\text{norm.}}$ ^{*3}	F_{II} ^{*4}	$(F_{II})_{\text{norm.}}$ ^{*5}
A1-M	0.92	1.05	0.53	0.61
A1-LR	0.86	0.98	0.49	0.56
A1-LMR	0.88	1.00	0.51	0.58
A2-M	0.87	0.99	0.50	0.57
A2-LR	0.84	0.96	0.49	0.55
A2-LMR	0.85	0.97	0.49	0.56
A3-M	0.86	0.98	0.49	0.56
A3-LR	0.89	1.02	0.52	0.59
A3-LMR	0.88	1.00	0.51	0.58

- *1 A1 = Homogeneous specimen, $\beta=45$ deg. (See Figure 2.6)
A2 = Bonded homogeneous specimen, $\beta=45$ deg. (See Figure 2.6)
A3 = Bonded bimaterial specimen, $\beta=45$ deg. (See Figure 2.6)
M = Middle slice
LR = Average value of left and right side slices
LMR = Average value of left, middle, and right side slices

*2 $F_I = K_I / \bar{\sigma} \sqrt{\pi a}$

*3 $(F_I)_{\text{norm.}} = F_I / [(F_I)_{\text{LMR}}]_{\text{Cntrl. Spec. "A1"}} = F_I / 0.88$

*4 $F_{II} = K_{II} / \bar{\sigma} \sqrt{\pi a}$

*5 $(F_{II})_{\text{norm.}} = F_{II} / [(F_I)_{\text{LMR}}]_{\text{Cntrl. Spec. "A1"}} = F_{II} / 0.88$

Table 3.3b - Normalized stress intensity values for the
 $\beta=30$ degree test series.

Specimen ^{*1}	F_I ^{*2}	$(F_I)_{\text{norm.}}$ ^{*3}	F_{II} ^{*4}	$(F_{II})_{\text{norm.}}$ ^{*5}
A4-M	1.14	1.07	0.35	0.33
A4-LR	1.03	0.96	0.32	0.30
A4-LMR	1.07	1.00	0.33	0.31
A5-M	1.16	1.08	0.36	0.33
A5-LR	1.05	0.98	0.32	0.30
A5-LMR	1.08	1.01	0.33	0.31
A6-M	1.16	1.08	0.36	0.33
A6-LR	1.02	0.95	0.32	0.30
A6-LMR	1.07	1.00	0.33	0.31

- *1 A4 = Homogeneous specimen, $\beta=30$ deg. (See Figure 2.6)
A5 = Bonded homogeneous specimen, $\beta=30$ deg. (See Figure 2.6)
A6 = Bonded bimaterial specimen, $\beta=30$ deg. (See Figure 2.6)
M = Middle slice
LR = Average value of left and right side slices
LMR = Average value of left, middle, and right side slices

*2 $F_I = K_I / \bar{\sigma} \sqrt{\pi a}$

*3 $(F_I)_{\text{norm.}} = F_I / [(F_I)_{\text{LMR}}]_{\text{Cntrl. Spec. "A1"}} = F_I / 1.07$

*4 $F_{II} = K_{II} / \bar{\sigma} \sqrt{\pi a}$

*5 $(F_{II})_{\text{norm.}} = F_{II} / [(F_I)_{\text{LMR}}]_{\text{Cntrl. Spec. "A1"}} = F_{II} / 1.07$

Table 3.3c - Normalized stress intensity values for the
 $\beta=15$ degree test series.

Specimen ^{*1}	F_I ^{*2}	$(F_I)_{\text{norm.}}$ ^{*3}	F_{II} ^{*4}	$(F_{II})_{\text{norm.}}$ ^{*5}
A7-M	1.62	1.14	0.22	0.16
A7-LR	1.31	0.92	0.18	0.13
A7-LMR	1.42	1.00	0.19	0.14
A9-M	1.79	1.26	0.24	0.17
A9-LR	1.52	1.07	0.21	0.15
A9-LMR	1.61	1.13	0.23	0.16

- *1 A7 = Homogeneous specimen, $\beta=30$ deg. (See Figure 2.6)
A8 = Bonded homogeneous specimen, $\beta=30$ deg. (See Figure 2.6)
(A8 data missing due to specimen failure - See text)
A9 = Bonded bimaterial specimen, $\beta=30$ deg. (See Figure 2.6)
M = Middle slice
LR = Average value of left and right side slices
LMR = Average value of left, middle, and right side slices

*2 $F_I = K_I / \bar{\sigma} \sqrt{\pi a}$

*3 $(F_I)_{\text{norm.}} = F_I / [(F_I)_{\text{LMR}}]_{\text{Cntrl. Spec. "A1"}} = F_I / 1.42$

*4 $F_{II} = K_{II} / \bar{\sigma} \sqrt{\pi a}$

*5 $(F_{II})_{\text{norm.}} = F_{II} / [(F_I)_{\text{LMR}}]_{\text{Cntrl. Spec. "A1"}} = F_{II} / 1.42$

Table 3.3d - Normalized stress intensity values for the $\beta=0$ degree test series.

Specimen ^{*1}	F_I ^{*2}	$(F_I)_{\text{norm.}}$ ^{*3}
A10-M	1.67	1.14
A10-LR	1.37	0.93
A10-LMR	1.47	1.00
A11-M	1.95	1.33
A11-LR	1.73	1.18
A11-LMR	1.80	1.22
A12-M	1.82	1.24
A12-LR	1.72	1.17
A12-LMR	1.75	1.19

- *1 A10 = Homogeneous specimen, $\beta=0$ deg. (See Figure 2.6)
A11 = Bonded homogeneous specimen, $\beta=0$ deg. (See Figure 2.6)
A12 = Bonded bimaterial specimen, $\beta=0$ deg. (See Figure 2.6)
M = Middle slice
LR = Average value of left and right side slices
LMR = Average value of left, middle, and right side slices

*2 $F_I = K_I / \bar{\sigma} \sqrt{\pi a}$

*3 $(F_I)_{\text{norm.}} = F_I / [(F_I)_{\text{LMR}}]_{\text{Ctrl. Spec. "A1"}} = F_I / 1.47$

- As was anticipated, for larger v-notch angles of inclination, greater percentages of shear mode (mode II) were encountered. A pure mode I state was found for the entire $\beta=0^\circ$ set of tests while a maximum mode-mixity ratio of $K_{II}/K_I = 0.58$ was found for each of the $\beta=45^\circ$ specimens.
- For reasons discussed in Section 3.4.1, relative differences in the normalized LMR stress intensity values for the $\beta=45^\circ$ and $\beta=30^\circ$ specimens were negligible. That is to say, within each of these two sets of tests, the differences in the normalized LMR values of the homogeneous, bonded homogeneous, and bonded bimaterial specimens were only on the order of experimental scatter ($\pm 5\%$).
- Again, as discussed in Section 3.4.1, normalized LMR results for the $\beta=0^\circ$ set of specimens displayed relative elevations. Compared to the homogeneous control specimen, the elevations found in the bonded homogeneous and bonded bimaterial specimens were both approximately 20%. Although only assumptions may be made concerning the $\beta=15^\circ$ set of experiments (due to the failure and loss of the bonded homogeneous specimen), it is probable that the 13% elevation found in the $\beta=15^\circ$ bimaterial specimen over its homogeneous counterpart would have similarly been found in the bonded homogeneous specimen.

A graphical comparison of the normalized mode I LMR slice averages for each set of specimens is given in Fig. 3.18. Comments and conclusions concerning the above observations are discussed in the following summarizing section.

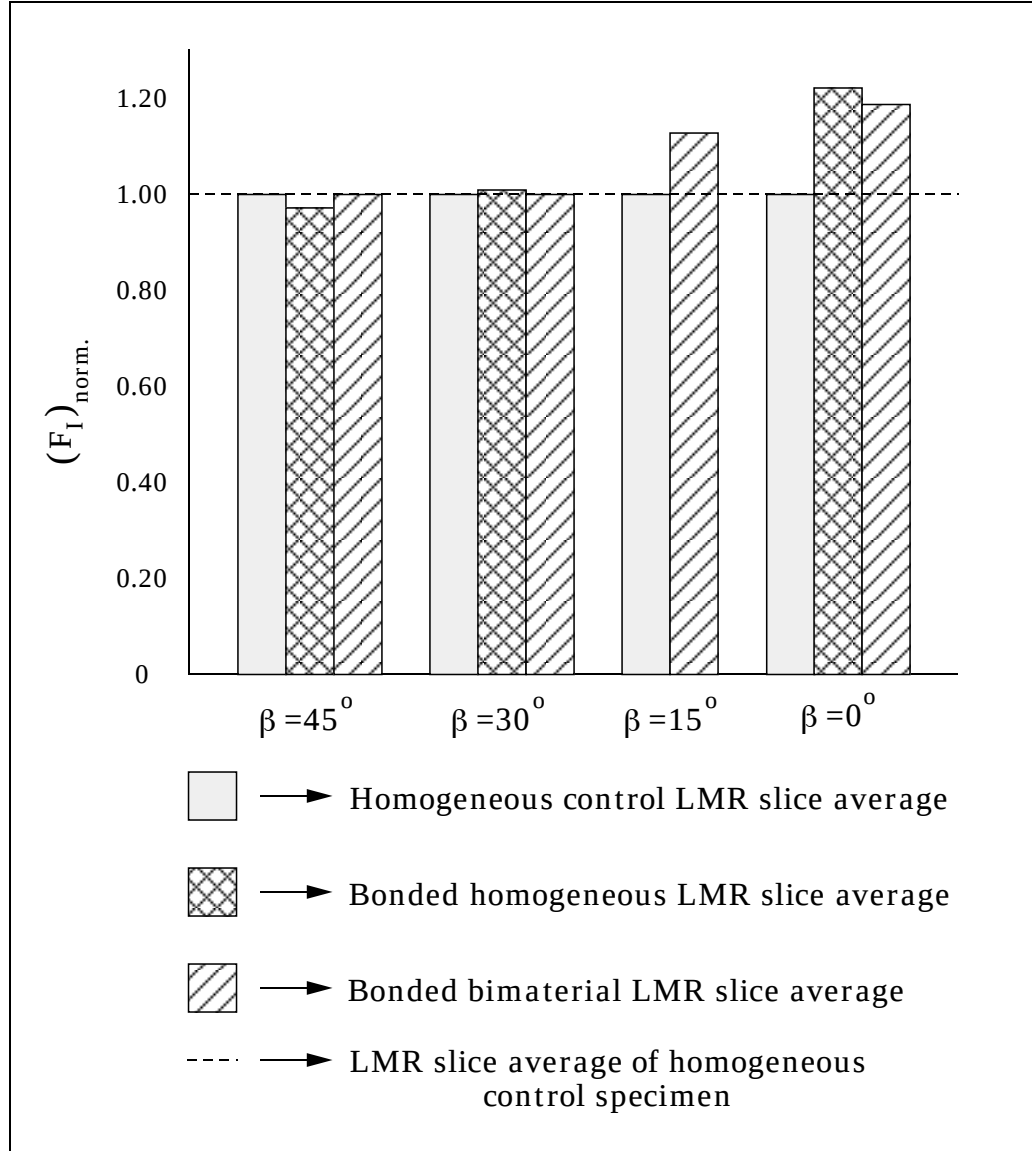


Figure 3.18: LMR values of normalized mode I stress intensity factors for various v-notch inclination angles.

3.5 Discussion

The primary goal of the three-specimen test method was to establish the magnitude and source of elevations in stress intensity. This particular method of separating and subtracting sources of SIF elevation which were not related to the mechanical mismatch in elastic moduli was achieved with reasonable success. The constraint of moderate cross-sectional thickness imposed upon each specimen and the consequential elevations in stress intensity were likewise measured with success. The anticipated elevation in stress intensity as measured at the mid-thickness of each specimen was consistent with the assumed state of generalized plane strain.

The absence of any general elevations in stress intensity in specimens A1 through A6 ($\beta=45^\circ$, 30°) strongly suggests that negligible influences on the singular stress state were imposed by the presence of the mismatch in elastic moduli. For the non-inclined notch experiments ($\beta=0^\circ$, specimens A10 - A12), an equal elevation in stress intensity factors found in the bonded homogeneous and bonded bimaterial specimens over their homogeneous control specimen implies similar conclusions to be drawn. The fact that the average elevations in these two specimens were nearly identical suggests that bondline residual stress effects were the dominant contributor to the elevations in stress intensity. Furthermore, the amounts of mode-mixity found in each specimen were quite consistent with estimates made using the analytical algorithm for obtaining stress intensity factors.

Neither qualitative nor quantitative effects associated with the mechanical mismatch in elastic moduli were observed. The fact that the absolute values of the mode I stress intensity factors found from specimens A11 and A12 were elevated over the values obtained from the homogeneous control specimen is perhaps worth greater attention than the mechanical mismatch in modulus itself. For the modest ratios of elastic modulus mismatch considered in this study, it is apparent that residual stresses associated with the bonding process vastly overshadow any influences that may be presumed from elasticity or fracture theory. As such, it is likely that problems of this nature are more accurately handled on an individual basis rather than in a strictly generalized, theoretical sense. Models which

attempt to generalize the bimaterial scenario while ignoring the inevitable and potentially influential presence of residual stresses imposed during a casting or curing process will be potentially inadequate in their attempt to accurately estimate stress intensity factors. To the contrary, bimaterial systems involving large elastic modulus ratios and minimal residual stresses may exhibit behavior which is dominated by purely elastic phenomena rather than residual stress effects. With such a wide variety of potential sources of influence, it is anticipated that each particular bimaterial system demands an evaluation of such factors as the materials, the casting or curing processes, and the thermal conditions under which that particular bimaterial system is to be subjected. Taking all of these factors into consideration for a given fracture scenario will likely result in a more accurate estimation of stress intensity and mode-mixity.

4.0 Parallel Crack Test Series

A series of photoelastic experiments were conducted on specimens containing edge cracks near and coincident with bimaterial interfaces. Of primary concern was the determination of mode-mixity along with the corresponding magnitudes of stress intensity factors. A single edge-cracked specimen was employed for all tests by using a machined crack aligned perpendicular to the direction of tensile loading. A systematic multiple specimen testing procedure was employed in an attempt to separate various sources of influence over the resulting stress intensity factors.

The test methodology and model geometries considered in this set of experiments are discussed in Section 4.1. Details concerning the photoelastic materials used in this work are then followed by a brief description concerning the particular analytical methods used for determining the mode-mixity and stress intensity found in this series of tests. Experimental results covering both pre-load and post-load principal shear stress fields are then given in pictorial form and discussed. Finally, numerical results obtained from the photoelastic experiments are then tabulated in Section 4.4.3 and discussed in Section 4.5.

4.1 Model Configurations

The primary goal of the experiments at hand was to determine the level of mode-mixity and elevation in stress intensity, if any, that would be generated by the presence of a bimaterial interface near the vicinity of a crack tip. As is seen in figures. 4.1 and 4.2, specimens containing edge cracks aligned perpendicular to the direction of tensile loading were employed for all tests. Furthermore, three different distances between the bonded interfaces and the machined cracks were considered. Models containing cracks machined directly into their respective bondlines were also considered for completeness.

In total, four sets of tests were conducted in an effort to isolate the effects associated with bondline curing stresses from those associated with the mechanical effects due to the presence of a bimaterial interface. Each set

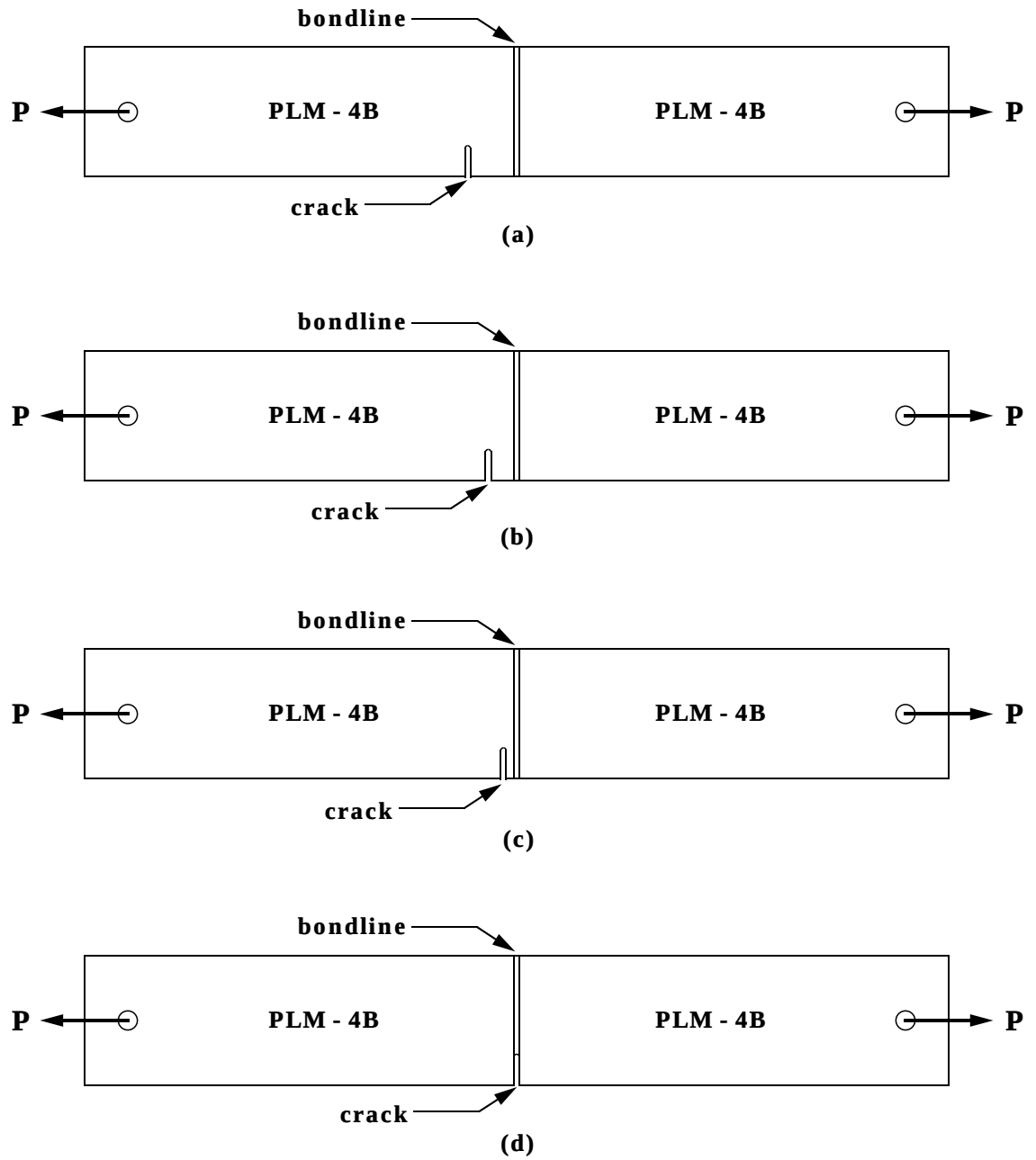


Figure 4.1: Bonded homogeneous specimens - geometry and loading.
a) Specimen B1: Crack/bondline separation distance = 0.500 in. (12.7 mm)
b) Specimen B2: Crack/bondline separation distance = 0.250 in. (6.35 mm)
c) Specimen B3: Crack/bondline separation distance = 0.125 in. (3.18 mm)
d) Specimen B4: Crack/bondline separation distance = 0

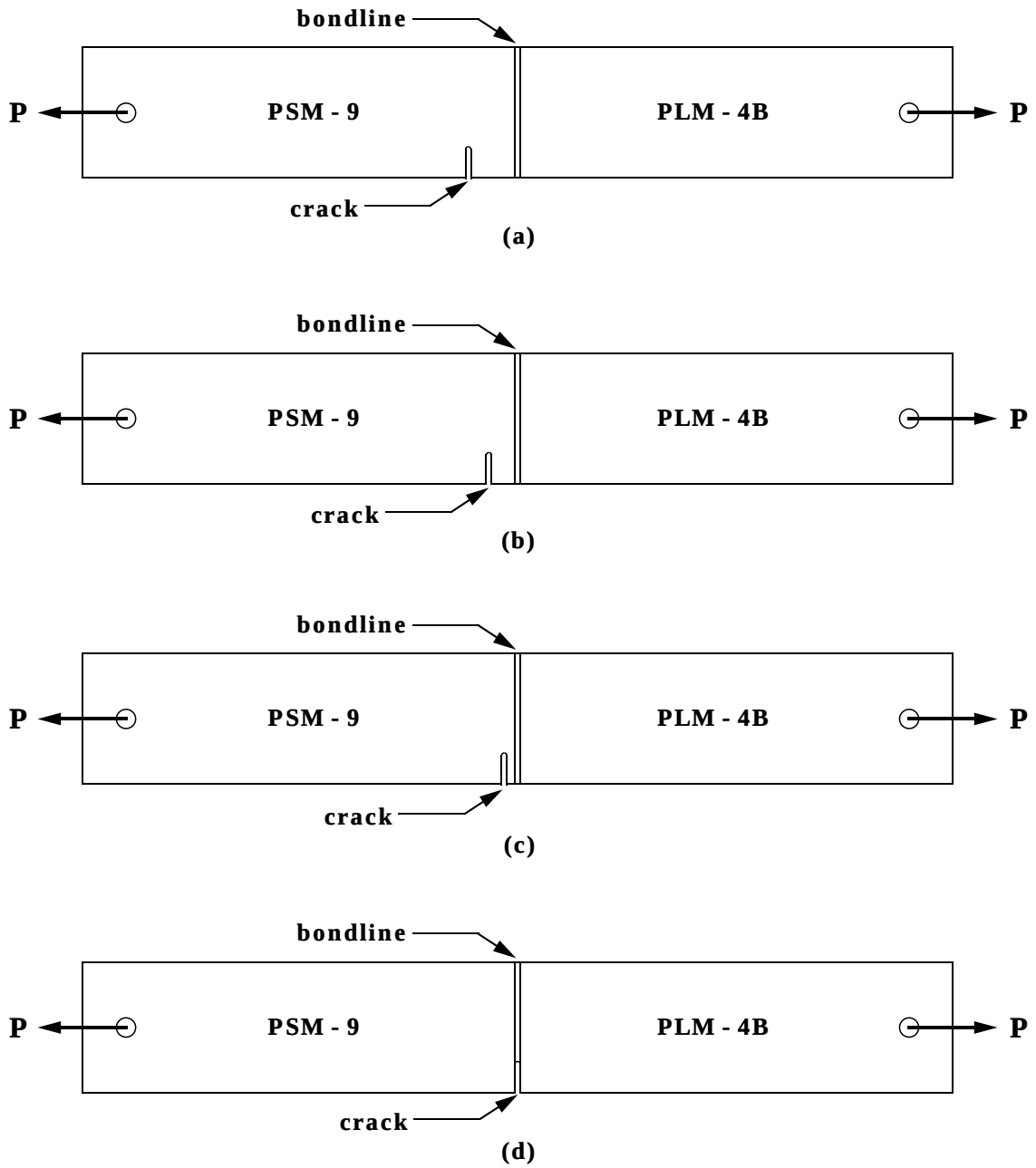


Figure 4.2: Bonded bimaterial specimens - geometry and loading.
a) Specimen B5: Crack/bondline separation distance = 0.500 in. (12.7 mm)
b) Specimen B6: Crack/bondline separation distance = 0.250 in. (6.35 mm)
c) Specimen B7: Crack/bondline separation distance = 0.125 in. (3.18 mm)
d) Specimen B8: Crack/bondline separation distance = 0

of tests consisted of two complete specimens, both containing equal length cracks positioned identically within models of consistent overall geometry. The only difference between each of the two geometrically identical specimens was that of the materials used on each side of the centered bondlines. In addition to the four sets of tests described thus far, an additional “control specimen” was constructed and analyzed for the purpose of obtaining reference values for stress intensity factors. The control specimen was homogeneous and of the same overall geometry as the rest of the specimens considered. All cracks were of the same length and oriented perpendicular to the direction of tensile loading. Additionally, all specimens were subjected to identical far field tensile loads. For organizational purposes, identifications and specifications for the test specimens under consideration are comprehensively tabulated in Table 4.1.

In Table 4.1 it is seen that for the off-bondline experiments, the distances used between cracks and bondlines were 0.500 in. (12.7 mm), 0.250 in. (6.35 mm), and 0.125 in. (3.18 mm). These distances were strategically selected in an effort to characterize the dependence of the potential presence of mode-mixity as a function of crack and bondline positioning. A rather large distance of 0.500 in. (12.7 mm) for specimens B1 and B5 was selected with the anticipation that results from these models would contain only an opening mode stress intensity factor (K_I) due to their relative isolation from their respective bonded interfaces. Specimens B2 and B6 were of intermediate crack and bondline separation distance while specimens B3 and B7 utilized cracks in fairly close proximity to their respective bondlines. Specimens B4 and B8 had their edge cracks machined directly into their respective bondlines. All bondline widths were kept as similar as possible to that of the cutting blade and thus the pre-load crack width of 0.012 in. (0.305 mm).

4.2 Materials

Two commercially available photoelastic materials with the trade names PLM-4B and PSM-9 were used for the off-bondline fracture experiments. The homogeneous control specimens as well as all bonded

Table 4.1 - Test specimen identification, composition, and geometry.

Specimen Identification	Specimen Type	Bondline/Crack Separation Distance, Inches (mm.)
B	Homogeneous Control	-----
B1	Bonded Homogeneous	0.500 (12.7)
B2	Bonded Homogeneous	0.250 (6.35)
B3	Bonded Homogeneous	0.125 (3.18)
B4	Bonded Homogeneous	0
B5	Bonded Bimaterial	0.500 (12.7)
B6	Bonded Bimaterial	0.250 (6.35)
B7	Bonded Bimaterial	0.125 (3.18)
B8	Bonded Bimaterial	0

homogeneous specimens were constructed using PLM-4B. All bimaterial specimens naturally consisted of a half-block of PSM-9 bonded to a geometrically equivalent half-block of PLM-4B. For all bimaterial specimens, the machined crack was placed on the PSM-9 side as is depicted in Fig. 4.2. All specimens with the exception of the control specimen (B) were bonded using a commercially available photoelastic adhesive designated by the manufacturer as PLM-9. The mechanical properties of this adhesive in its hardened form proved to be nearly identical to those of PSM-9. A tabulation of the properties of PLM-4B and PSM-9 above their respective critical temperatures is given in Table 4.2.

Upon referring to Table 4.2 it is seen that the critical temperatures (T_c) of the two materials under consideration differed by 25 F. Thus, during the initial cooling stage of a thermal cycle for a bimaterial specimen, there will be a period of time during which the PSM-9 will have transitioned from the rubbery to solid state while the PLM-4B remains rubbery. When the thermal cycle then drops below 180 F, the PLM-4B will transition from the rubbery to solid state while mated at the bonded interface to the hardened PSM-9. Ideally it would be desirable to use materials possessing identical critical temperatures to avoid any excess shear stress introduced from the material transitioning routine just described. PLM-4B and PSM-9 were ultimately chosen despite their differing T_c 's due to their superior resistance to time-edge effects as described in Section 3.2. The technique of preparing and testing the bonded homogeneous counterparts to the bimaterial specimens provided a partial basis from which to evaluate the relative influence of the critical temperature mismatch. An additional measure taken in an attempt to separate thermal and mechanical effects was accomplished by performing "no-load" thermal cycles on all specimens. As discussed in more detail in Section 4.4, the no-load thermal cycles were performed on each specimen prior to their loading cycles to document and evaluate the residual stress fields present in each specimen due to bondline shrinkage and thermal mismatch effects.

It was naturally desirable to maintain a certain level of material property similarity between the photoelastic experiments and the actual scenario of rocket motor propellant bordering a rubber liner. Most, if not all

Table 4.2 - PLM-4B and PSM-9 material properties.

Material	Critical Temp. (T _c) Farh. (deg. C)	Modulus of Elasticity* psi. (MPa)	Material Fringe Value* (f) lbs./in. (N/cm)	Poisson Ratio* (ν)
PLM-4B	180 (82)	1881 (12.97)	2.40 (4.20)	~ 0.50
PSM-9	205 (96)	6867 (47.35)	2.97 (5.20)	~ 0.50

*Values determined ~ 15 F above each material's respective T_c.

common photoelastic materials exhibit rubber-like properties above their T_c 's. In particular, Poisson ratios near 0.5 are typically found in most rubber materials as well as in photoelastic materials while above their critical temperatures. The deformation field corresponding to the rubbery state above the T_c is the field that is "locked" in place when slowly cooled to room temperature. As such, the photoelastic data obtained at room temperature corresponds not to the room temperature material properties, but to the properties of the material above its critical temperature. Data obtained at room temperature from the model slices thus corresponded to a state of material incompressibility. Elastic moduli ratios of approximately 4 to 1 were observed for all of the bimaterial experiments that were conducted using PLM-4B and PSM-9.

4.3 Analytical Considerations

In order to obtain relative comparisons among the stress intensity factors obtained from each specimen, the control specimen "B" was first tested. Specimen "B" was geometrically identical to the rest of the specimens considered in this work with the exception that it did not possess a bondline. As such, the experimental stress intensity values obtained from this specimen were able to be directly compared to numerical solutions^{23,29,30}. Upon doing so, the average through-thickness stress intensity value obtained was approximately 10% higher than the values predicted by several analytical and numerical solutions. For the geometry and loading of the photoelastic experiment, theoretical non-dimensionalized values (F_I) of the mode I stress intensity factor ranged from between 1.494 and 1.505. From a through-thickness averaging of slices taken from specimen "B", F_I was found to equal 1.64. This moderate rise (10%) over the predicted value was likely due to a combination of finite thickness effects in conjunction with the relative bluntness of the machined crack tip. However, by using the normalized value of F_I obtained from specimen "B" as the control value against which to compare all other results, the relative differences in F_I between the experimental and theoretical values are accounted for. Thus, by using a control specimen in this manner, any effects associated with crack tip

bluntness or variation in the through thickness stress state are effectively removed from the analysis.

To account for the presence of a shear mode (K_{II}) in addition to the expected opening mode (K_I), the mixed mode equations derived in Section 2.3.4 were employed. For cases in which a shear mode appeared in addition to the opening mode, a reading line angle was estimated from the photographs taken near each crack tip and then substituted into the governing analytical expressions along with fringe orders and positional data. Mode I and mode II stress intensity factors were then obtained and subsequently normalized into non-dimensional form in order to facilitate a direct comparison of results between all nine specimens.

4.4 Results

In total, nine experiments using PLM-4B and PSM-9 were performed. All experiments were conducted using nearly identical construction and testing procedures in an effort to minimize experimental error. As described in Section 2.4, a no-load thermal cycle was employed in each test to relieve residual bondline curing stresses and establish a reference “pre-load” stress field. After documenting the no-load stress fields, the specimens were stress frozen at a tensile load of 8 lbs. (35.6 N). Photographs documenting the post-load stress fields were taken for all specimens after each stress freezing cycle had been completed. Finally, three slices were taken from each specimen near their crack tip regions as described in Section 2.4.5. For each specimen, the stress intensity factors obtained from the three slices were subsequently normalized to a quantity containing the original specimen thickness. This was done in such a way as to remove physical units from the results. The thickness was also considered in the normalization in order to account for small variations in the actual thicknesses found between specimens. Details pertaining to the use of the stress intensity normalization factor as well as a comprehensive tabulation of results obtained from the slices are given in Section 4.4.3. Through-thickness stress intensity factor distributions were successfully measured using the stress freezing method. Moderate elevations in stress intensity were consistently found from the middle slices

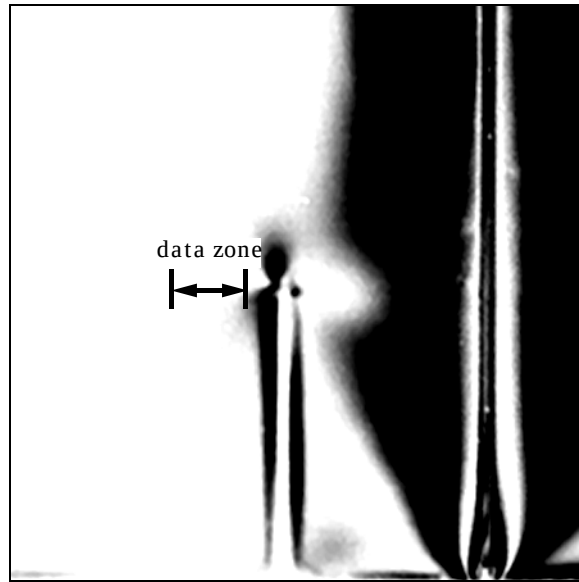
of specimens. Sections 4.4.1 and 4.4.2 document and discuss key features found in the photoelastic images obtained from each specimen.

4.4.1 Pre-Load Stress Fields

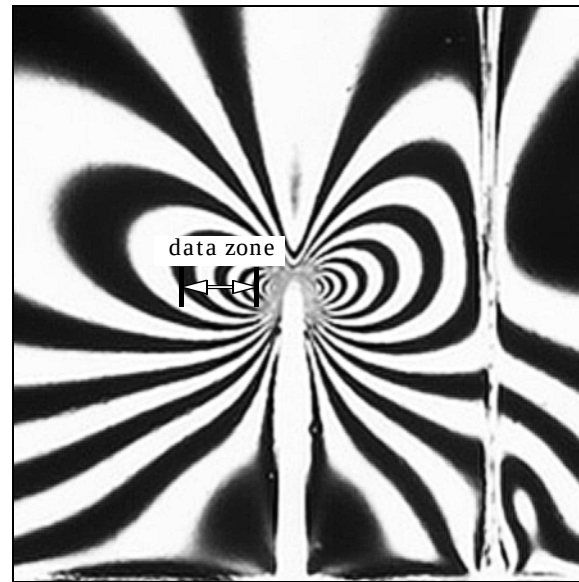
After the model halves of each specimen were bonded together, a thermal cycle with a maximum temperature of 250 F was used to fully cure the bondline. To relieve bondline curing stresses, each complete specimen was then subjected to a thermal cycle with a maximum temperature of 220 F. Typical thermal cycles for frozen stress photoelasticity utilize a peak temperature of 10 F - 15 F above the T_c of the material, which is consistent with the T_c of PSM-9 (205 F). Although the bonded homogeneous specimens were constructed primarily of PLM-4B (the bondline was made using PLM-9), an identical thermal cycle to that of the bimaterial specimens was used to maintain consistency in procedure and to also ensure that the bondline (PLM-9) was elevated above its critical temperature.

Photographs of the specimens after having experienced no-load thermal cycles provided information about the potential effects of bondline residual stresses on the final stress intensity factors. Fig. 4.3a illustrates the crack tip region of specimen B2 after a no-load thermal cycle while Fig. 4.3b displays the same specimen and region after having been stress frozen under loading. Comparison of the two images over the zone from which data were ultimately read reveals that little or no variation in the magnitude of stress was present directly prior to the loading cycle. The data zones depicted in Fig. 4.3 were identically used over a range of $0.35 \leq \sqrt{r/a} \leq 0.60$ for all specimens. Inspection of the no-load images of all nine specimens reveals a maximum fringe order variation of 0.5 across the length of the data zones.

The quantification of this variation is useful for approximating the influence of the pre-load residual stress field on the stress intensity factors found from each specimen after loading. As was shown in Section 2.3.1, Irwin¹⁶ suggested that an attempt to account for non-singular stress effects can be made by superimposing a generalized yet constant stress state ($\sigma_{xx}^0, \sigma_{yy}^0, \sigma_{xy}^0$) over the singular stress field equations. Typically, the only non-singular stress term that is retained is that which is oriented parallel to the



(a)

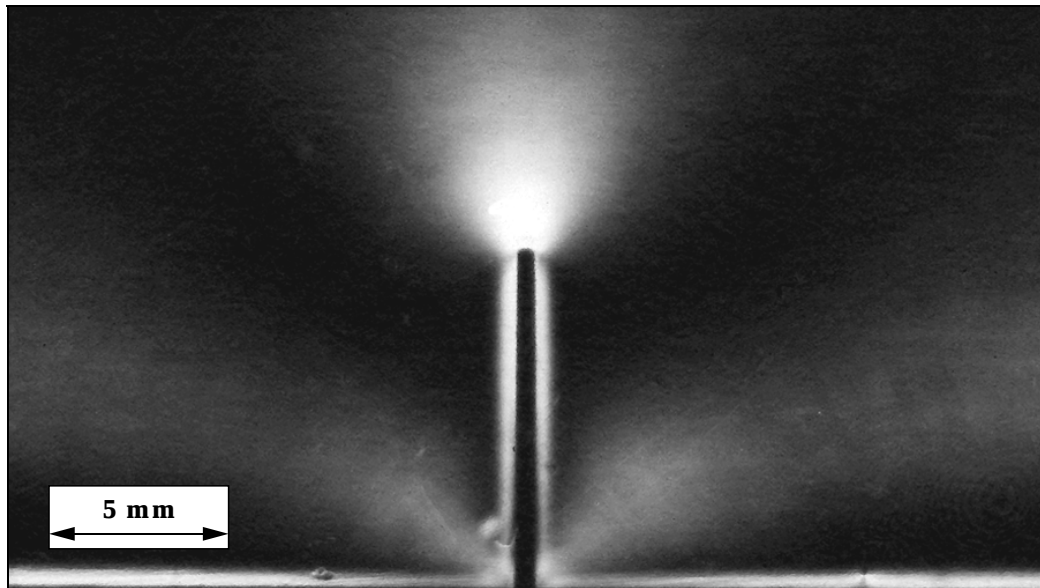


(b)

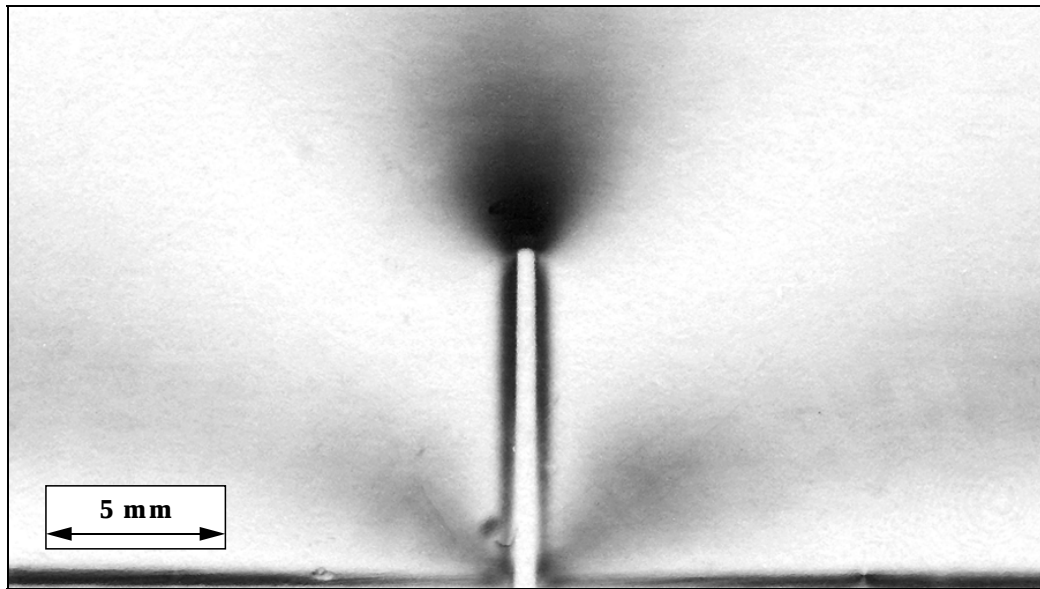
Figure 4.3: Location and length of data zone used for SIF determination.
a) Specimen B2 after a no-load thermal cycle.
b) Specimen B2 after a loaded stress freezing cycle.

lengthwise direction of the crack (given by σ_{xx}^0 for the coordinate system considered in this work). Most if not all current analytical algorithms which incorporate photoelastic data make use of the assumption that the non-singular stress term (σ_{xx}^0) is constant. Extensive use of such algorithms have produced stress intensity factor results which closely concur with known solutions derived from both analytical and numerical methods, thus lending credence to the use of a constant non-singular stress term in classical fracture analyses. However, these algorithms (including the Smith Extrapolation Method) cannot account for the possibility of a non-singular stress field which varies as a function of position throughout a cracked body. This occurrence will effectively arise if the residual stress field present in the data zone of a specimen is not constant. All nine specimens under consideration displayed minimal variation in the residual stresses found over the region from which data was ultimately taken. As such, the algorithm used for analysis is expected to accurately account for the residual stresses present prior to loading. Figures 4.4 through 4.12 document the no-load stress field present in each of the nine specimens prior to undergoing their loaded stress freezing cycles.

The use of bonded homogeneous specimens in addition to bonded bimaterial specimens was intended to provide additional information as to the source of any potential variations in stress intensity among the bimaterial specimens. Naturally, the goal at hand was to separate and quantify any possible variations in mode-mixity and stress intensity due entirely to mechanical effects; that is, due to the jump discontinuity in the elastic modulus at the bondline. By comparing results obtained from bonded homogeneous specimens to their bimaterial counterparts, one can quantitatively establish the combined effect of bondline residual stresses and the critical temperature mismatch on the final stress intensity factors. In order to achieve the goal of completely separating these undesirable effects from purely mechanical effects, additional information is required. Qualitative as well as quantitative arguments and results which deal with this separation are discussed in Section 4.5.



(a)



(b)

Figure 4.4: Specimen "B" pre-load fringe field.
a) Dark field image of crack area.
b) Light field image of crack area.

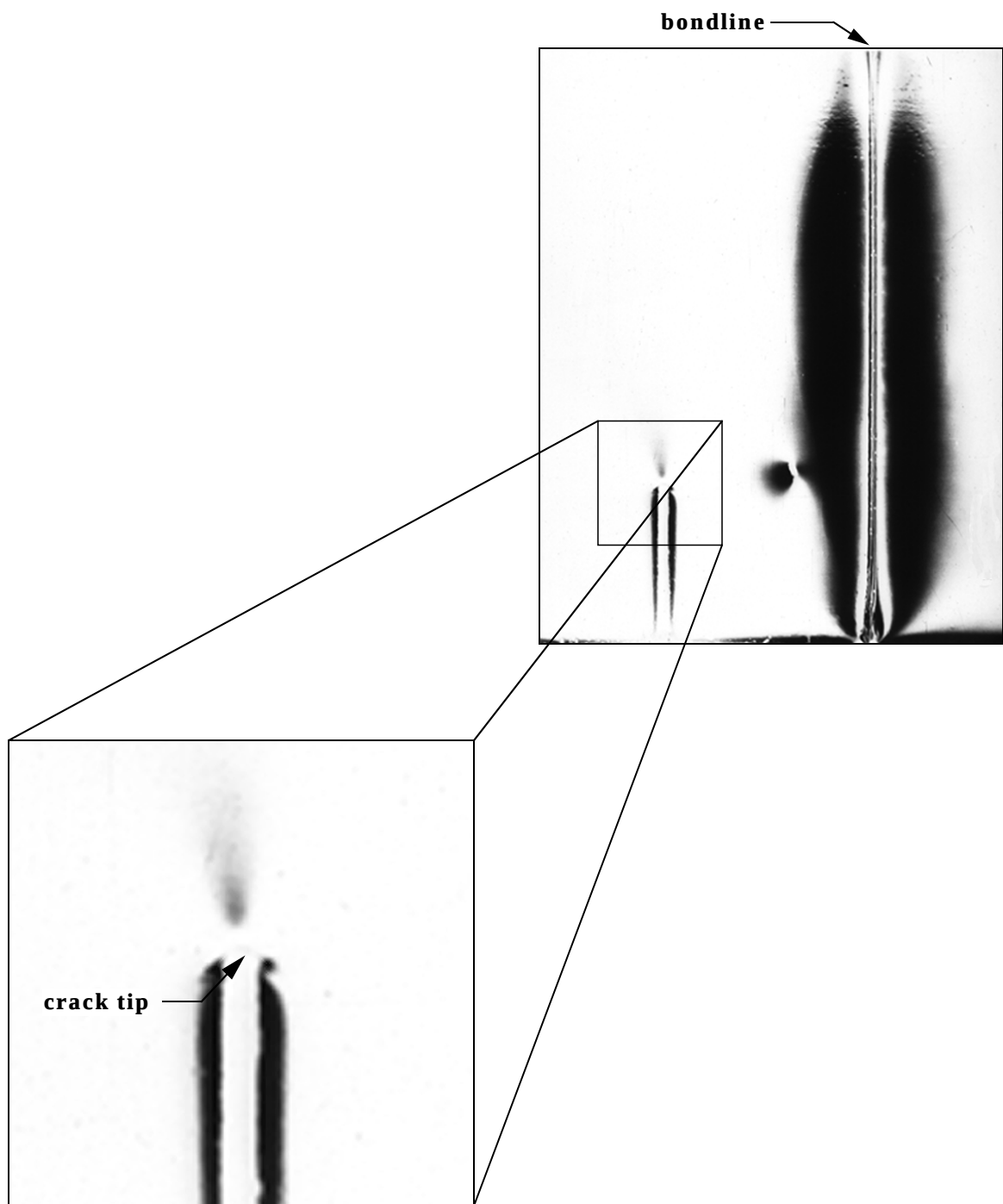


Figure 4.5: Specimen B1 pre-load fringe field.

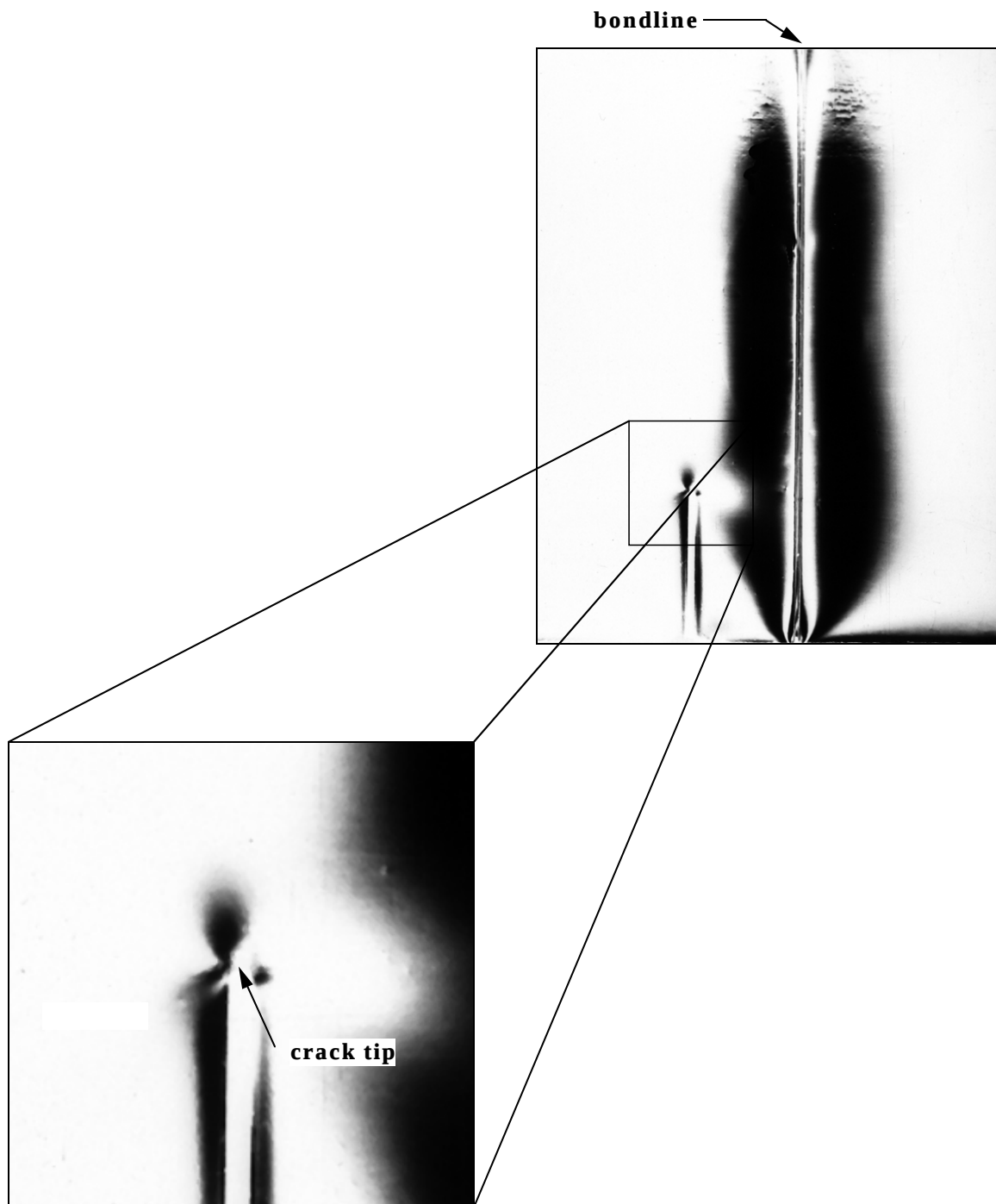


Figure 4.6: Specimen B2 pre-load fringe field.

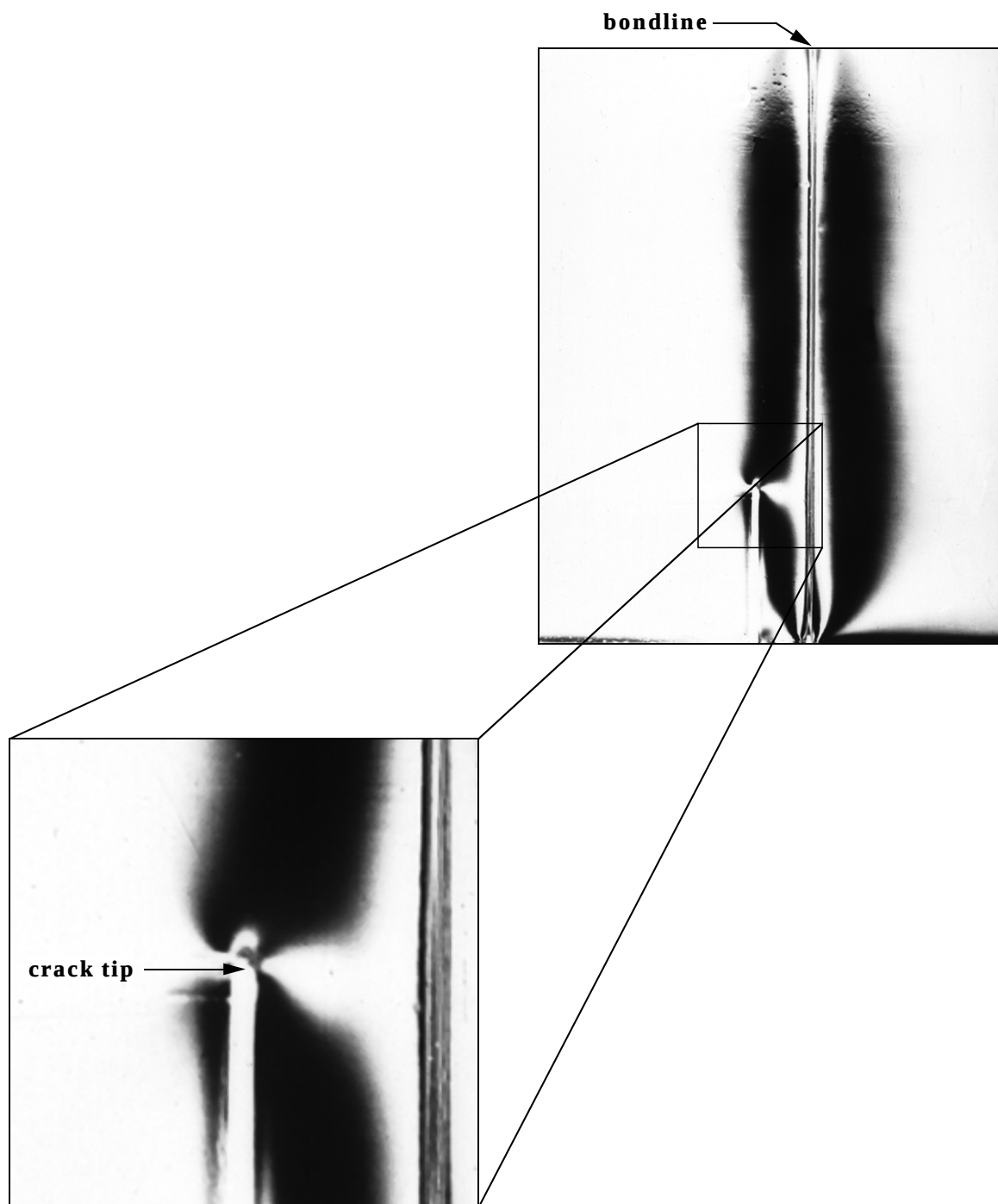


Figure 4.7: Specimen B3 pre-load fringe field.

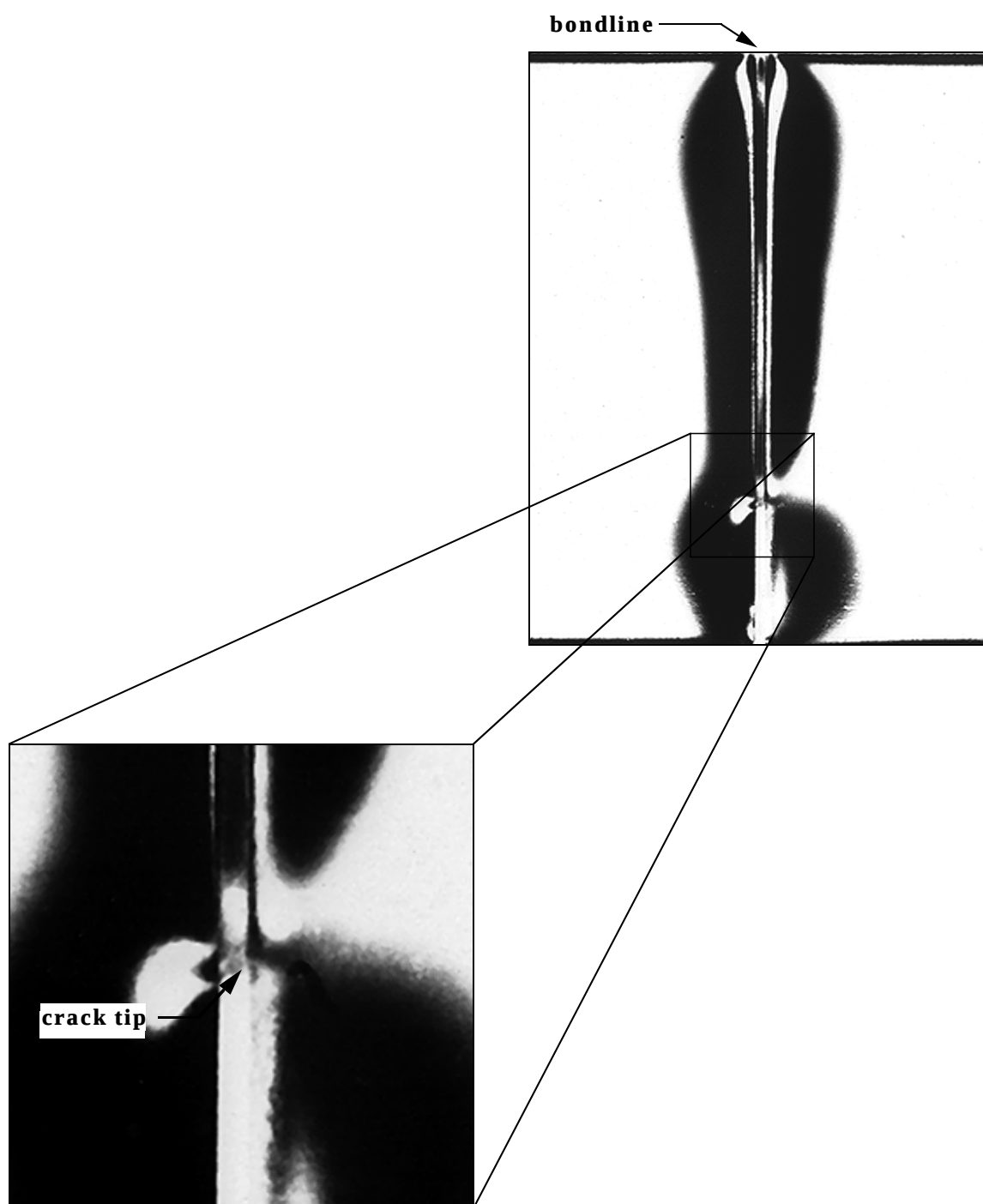


Figure 4.8: Specimen B4 pre-load fringe field.

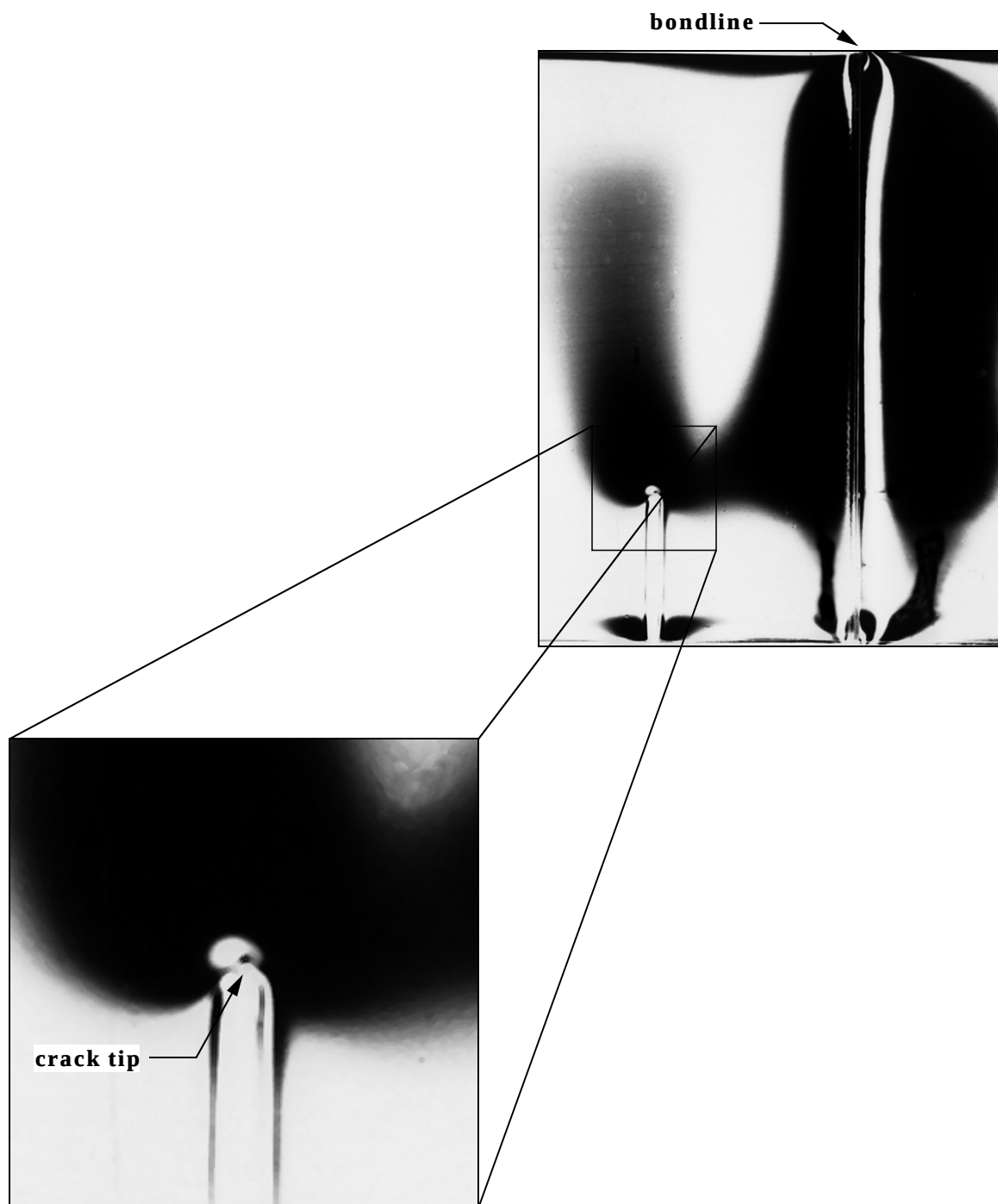


Figure 4.9: Specimen B5 pre-load fringe field.

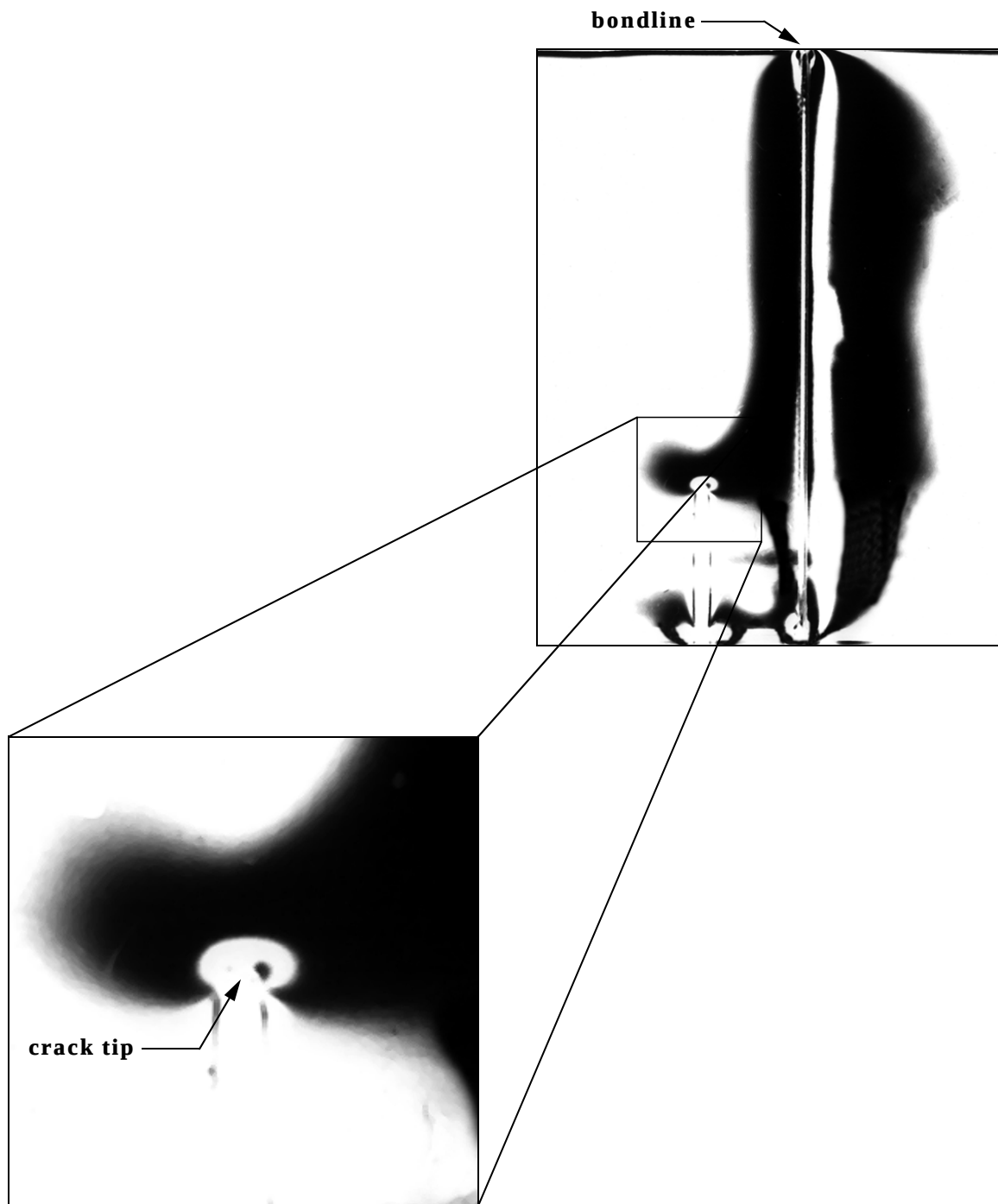


Figure 4.10: Specimen B6 pre-load fringe field.

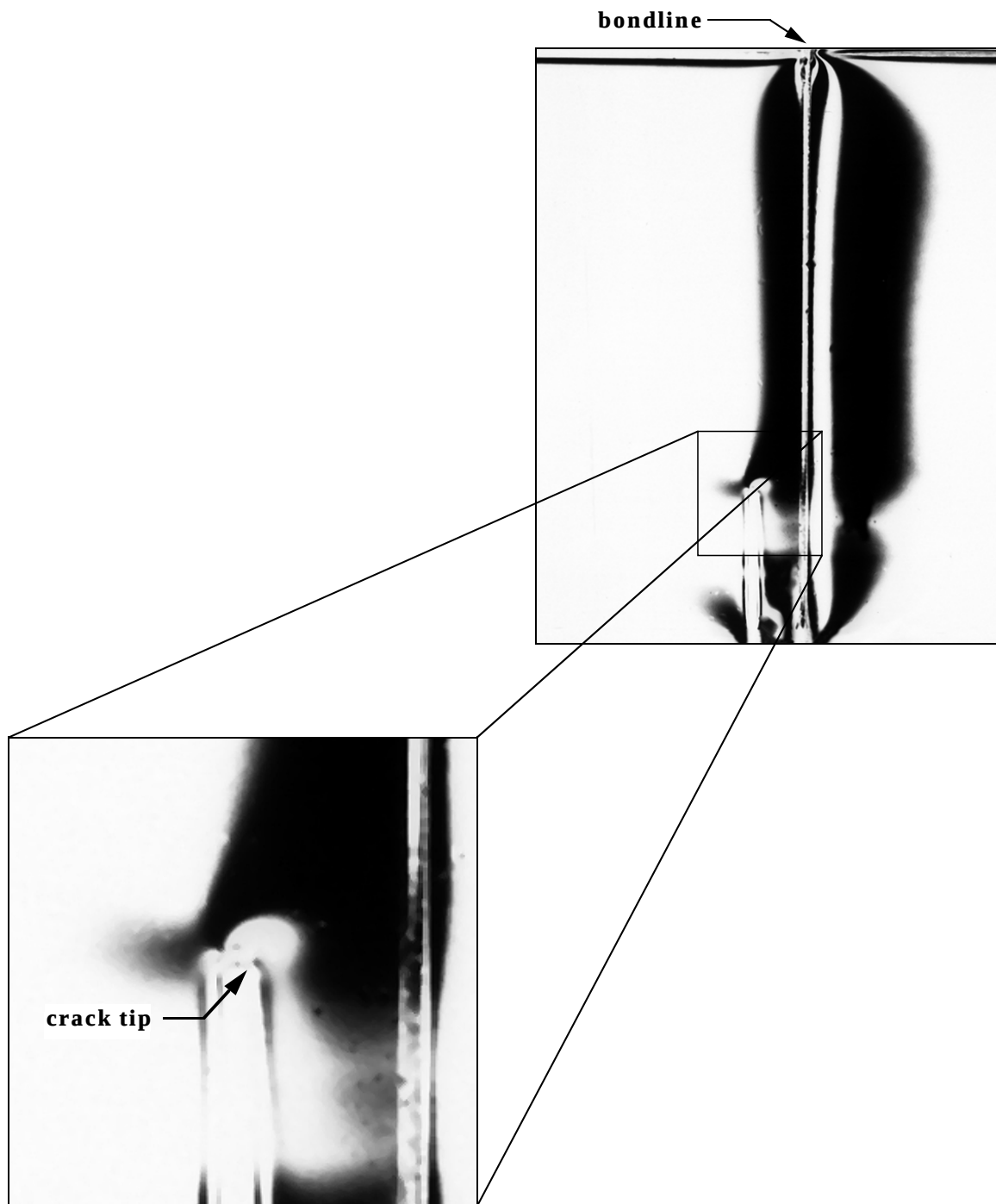


Figure 4.11: Specimen B7 pre-load fringe field.

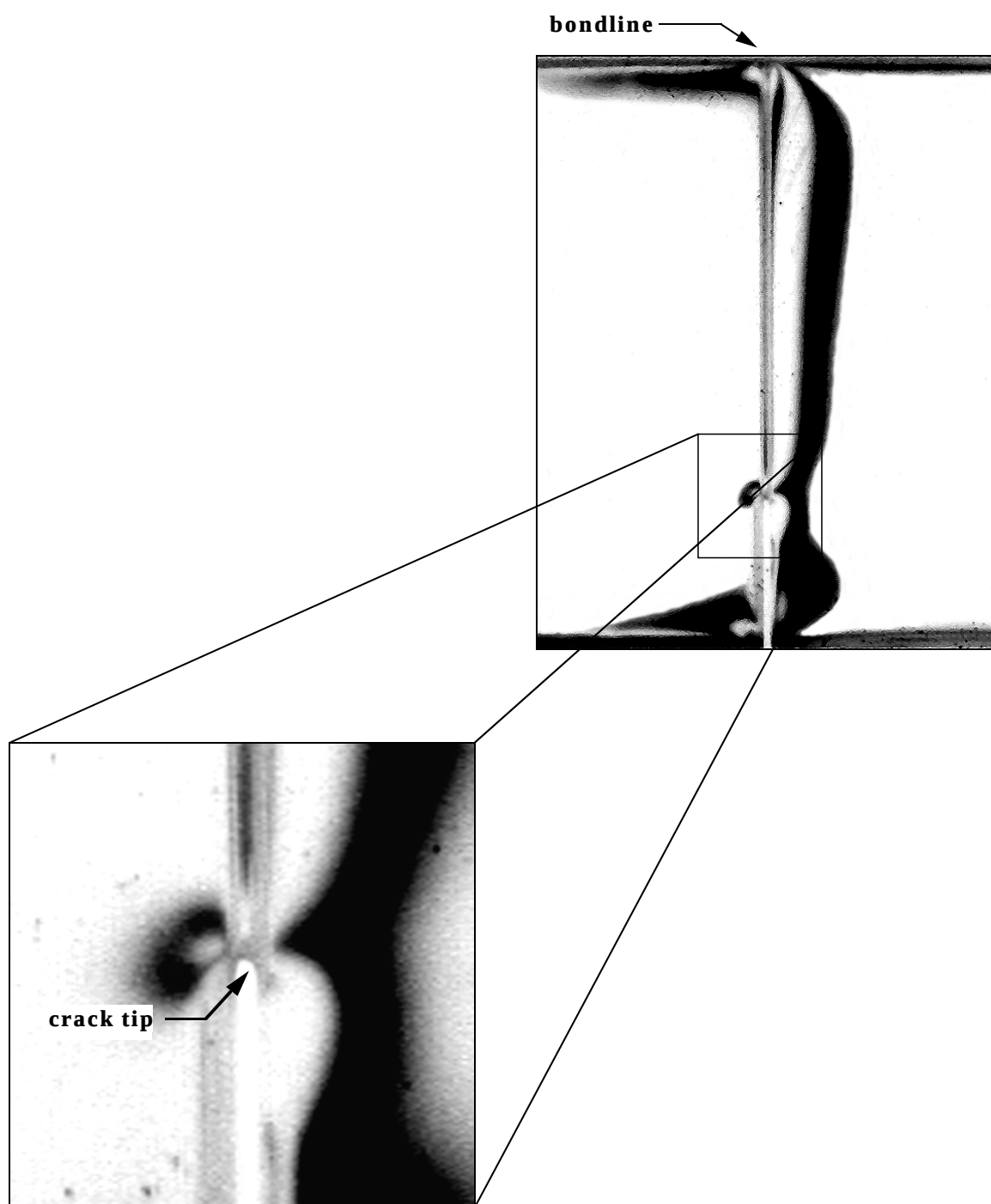


Figure 4.12: Specimen B8 pre-load fringe field.

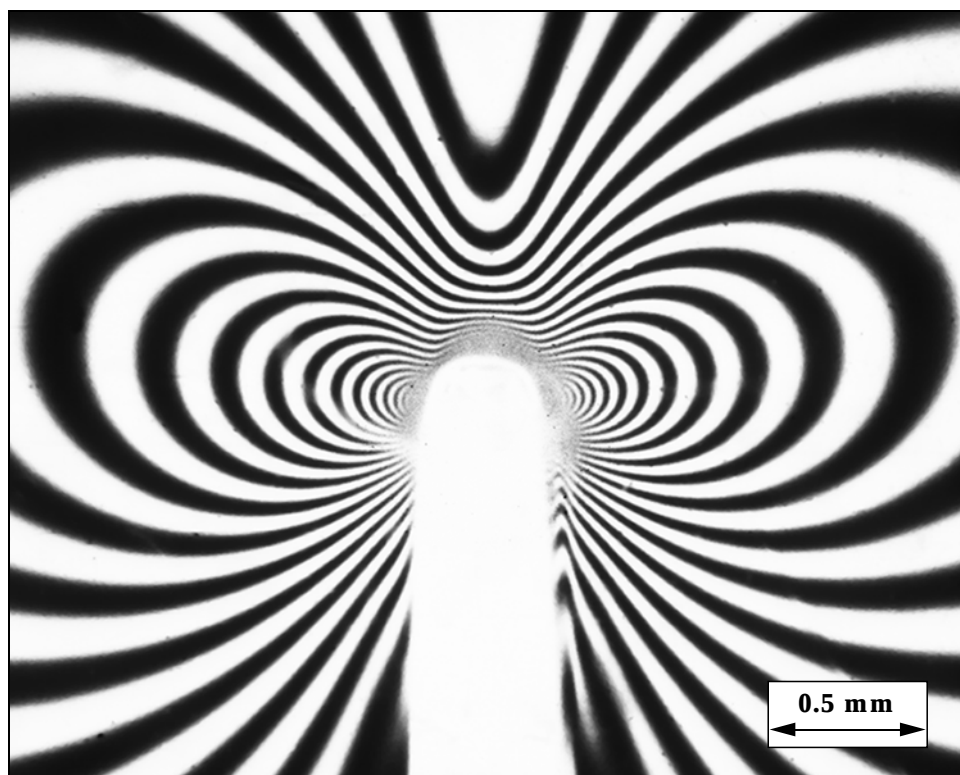
4.4.2 Post-Load Stress Fields

By making use of the experimental and photographic techniques described in Appendix A, high magnification images of the photoelastic fringe fields were obtained for all nine specimens near their respective crack tips. These images as well as full field photoelastic photographs are depicted in Figures 4.13 through 4.21. For clarity purposes, the light field was used for all photoelastic images. As such, the dark fringes in these images correspond to the half fringe orders given by: ($N = 0.5, 1.5, 2.5, \dots$).

Inspection of the homogeneous control specimen (B) as well as all four bonded homogeneous specimens (B1 through B4) near their crack tips reveals consistent, symmetric fringe patterns. As discussed in Section 2.3, the data reading line from a crack tip fringe pattern is determined by joining the points at the “top” of the near tip fringes. A straight line is thus constructed and extended out beyond the singular region near the crack tip. From visual inspection of the bonded homogeneous near tip fringe patterns it is apparent that the appropriate reading lines were all approximately perpendicular to the direction of the crack length and likewise parallel to the direction of far field tensile loading. A data reading line oriented perpendicular to the length of any crack is indicative of a pure mode I state of stress intensity. As such, specimens B1 through B4 demonstrated a complete absence of a shear mode (K_{II}) as well as a complete lack of dependency of mode-mixity upon the spacing between the crack and the bondline. Furthermore, data analysis of the slices extracted from B1 through B4 indicated virtually no variation in stress intensity between specimens. Details concerning the stress intensity results for all nine specimens are discussed in Section 4.4.3. The results obtained from specimens B1 through B4 do not provide a complete means of separating the effects of the critical temperature mismatch found in the bimaterial specimens from the possible effects due to the mismatch in elastic modulus values. The results do however reveal that the bondline residual stresses found prior to loading have a negligible effect upon both the mode-mixity and stress intensity factors found after loading.



(a)



(b)

Figure 4.13: Homogeneous control specimen "B" post-load fringe field.
 a) Wide view displaying presence of uniform far-field tensile stress.
 b) Close view of principal shear stress field near the crack tip.

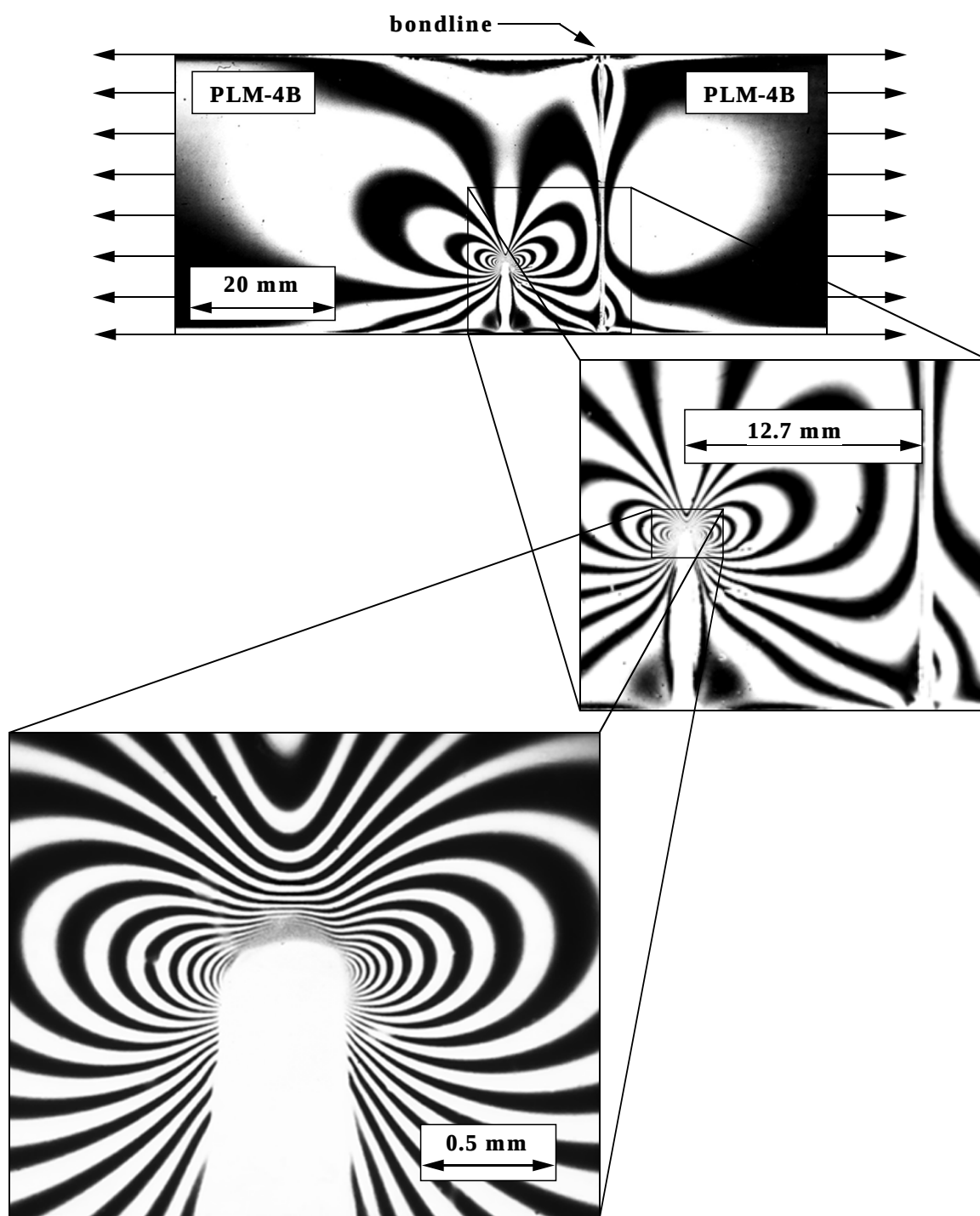


Figure 4.14: Specimen B1 post-load fringe field.

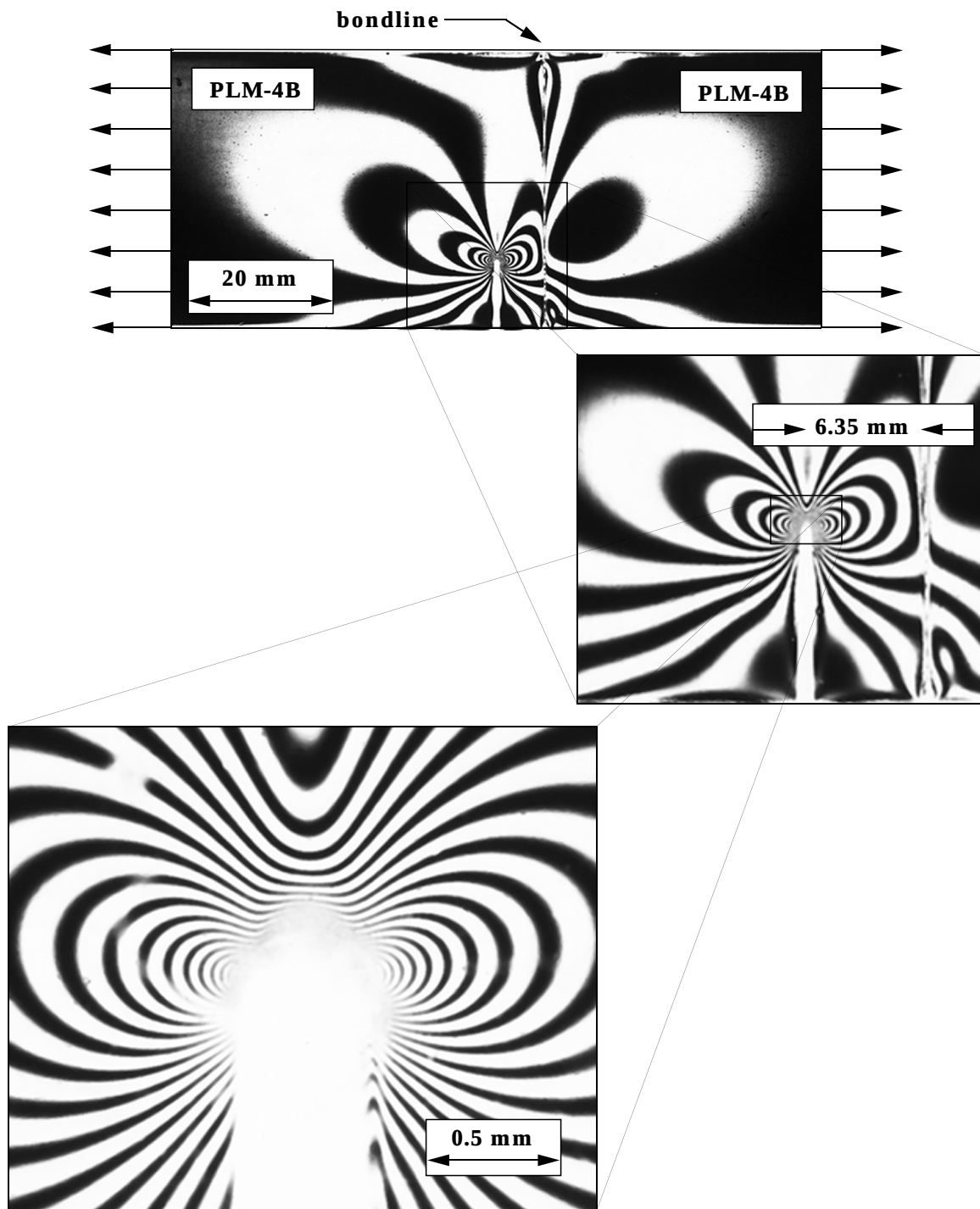


Figure 4.15: Specimen B2 post-load fringe field.

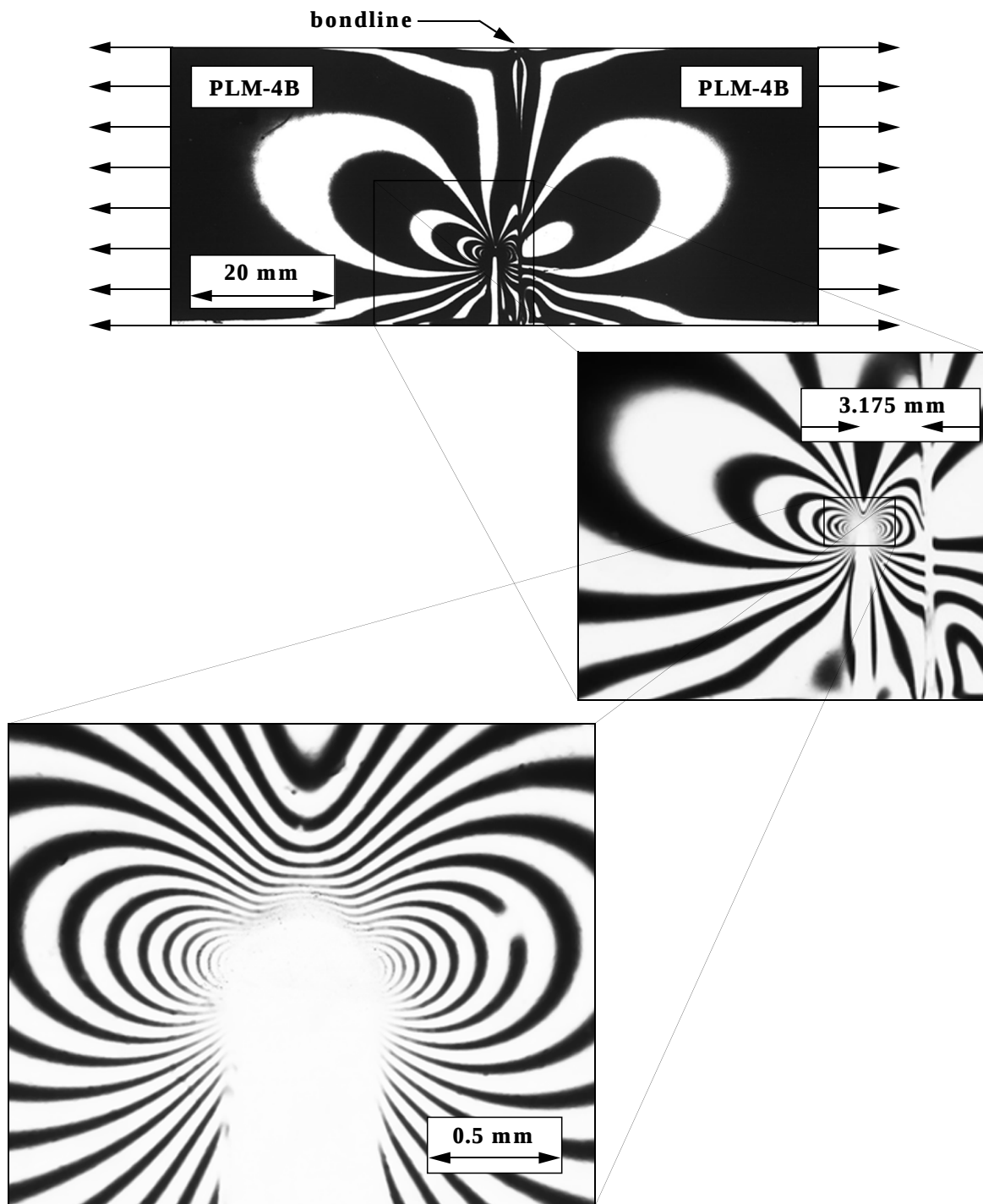


Figure 4.16: Specimen B3 post-load fringe field.

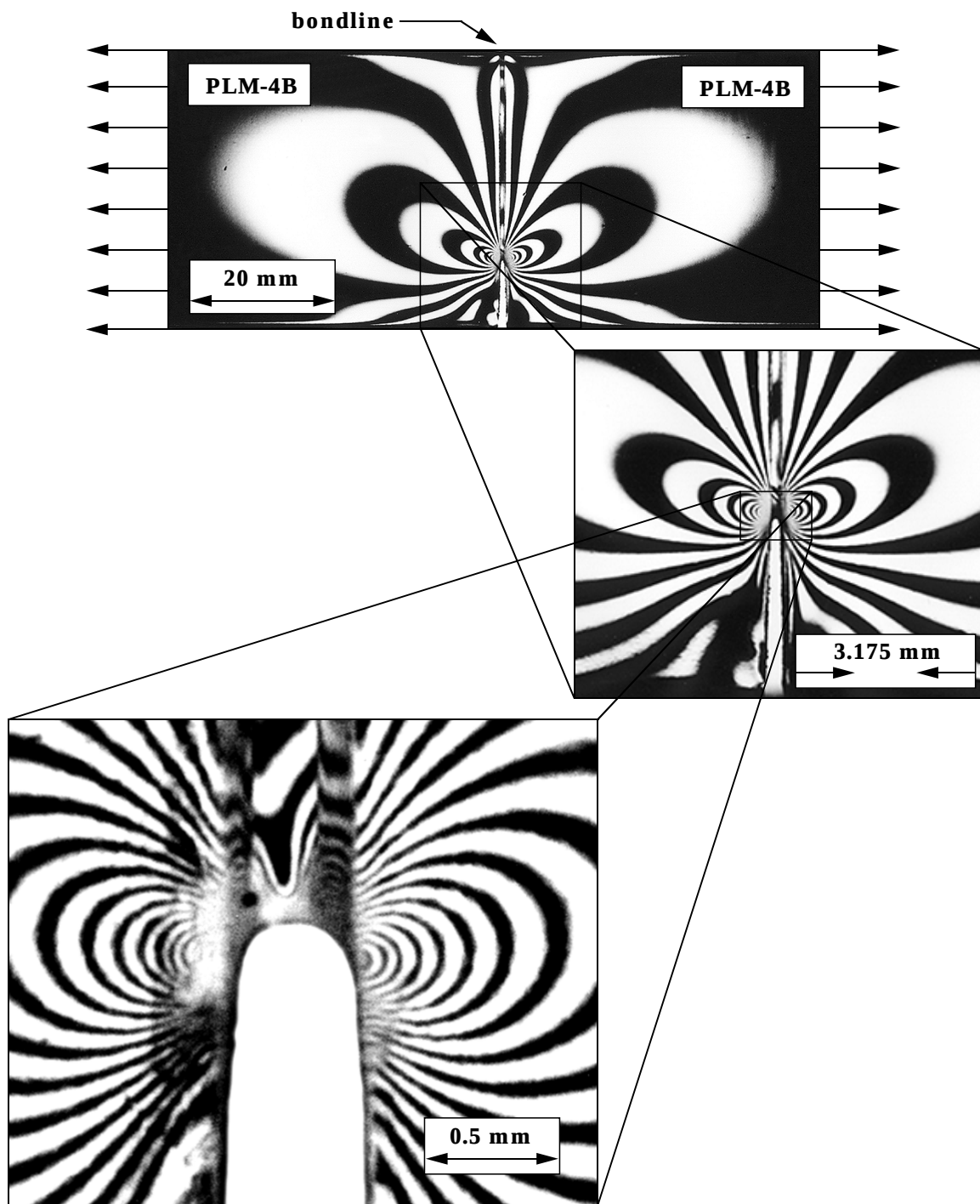


Figure 4.17: Specimen B4 post-load fringe field.

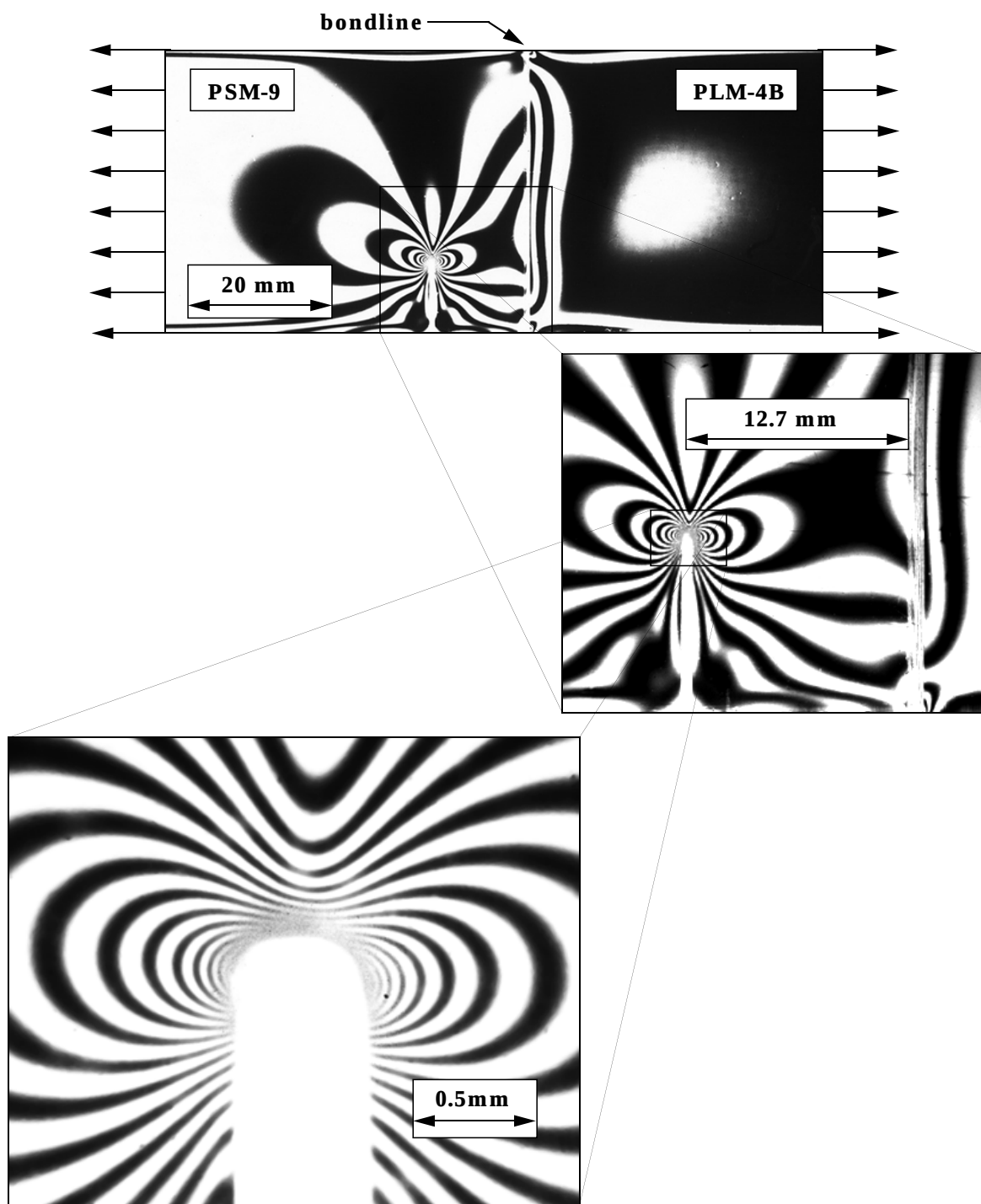


Figure 4.18: Specimen B5 post-load fringe field.

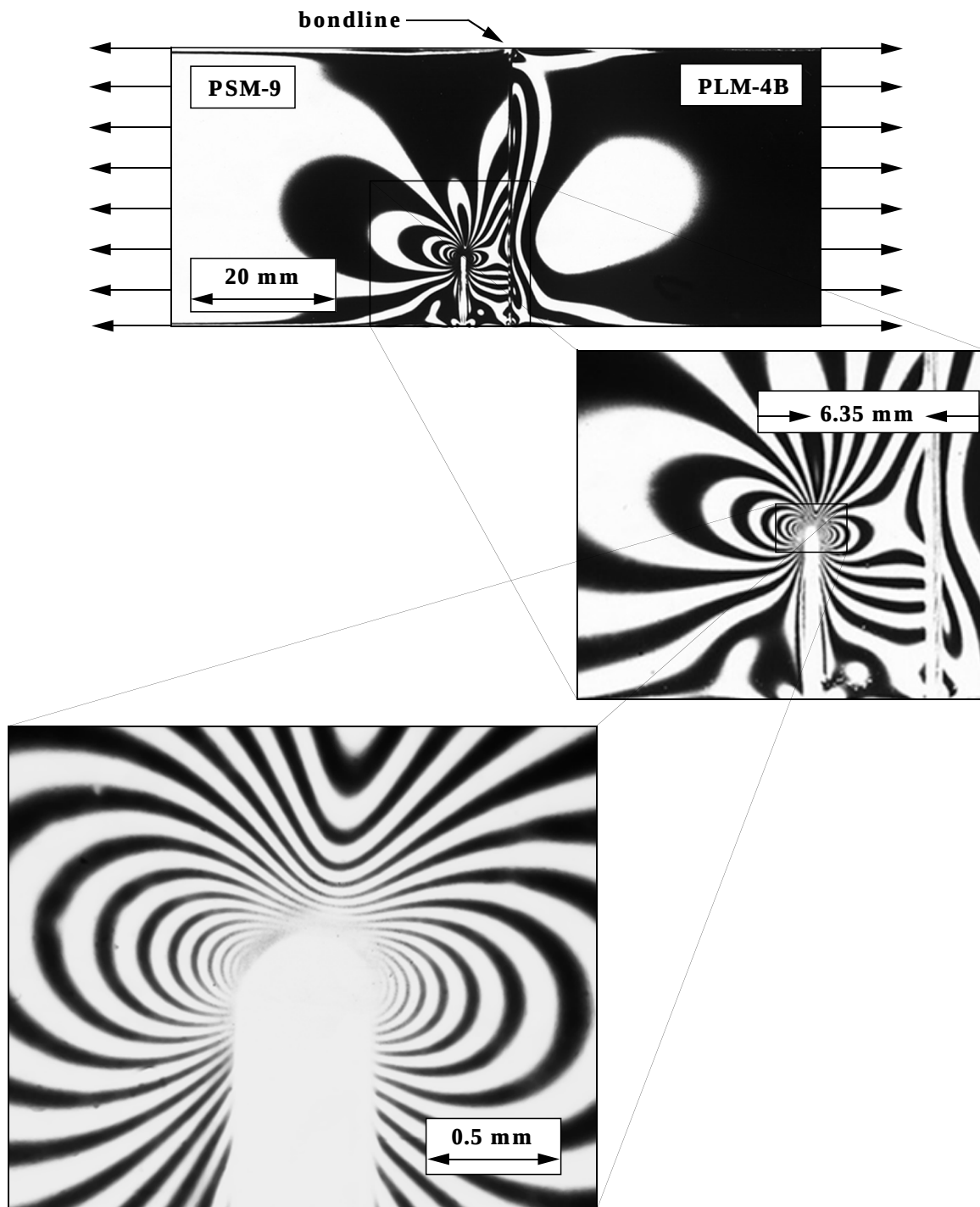


Figure 4.19: Specimen B6 post-load fringe field.

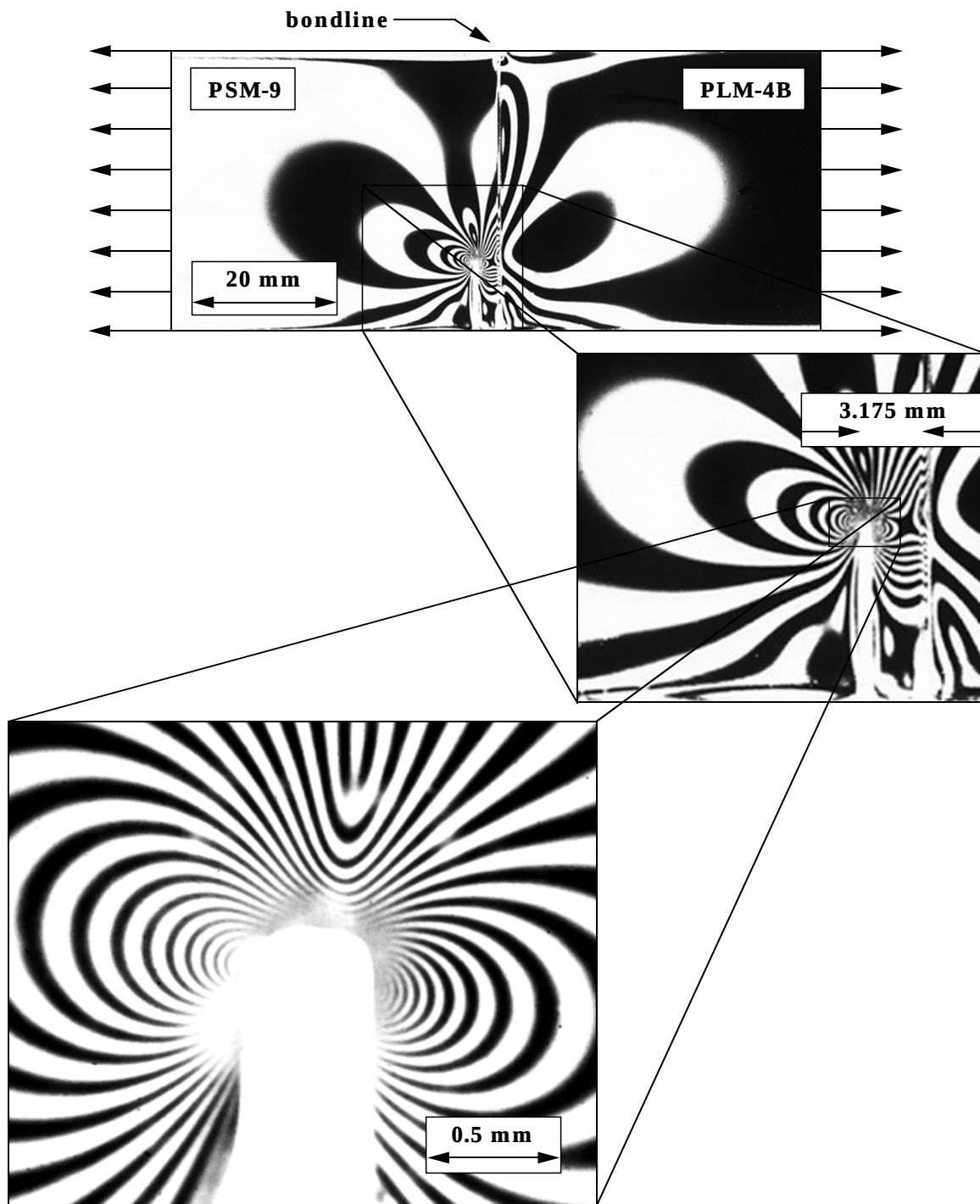


Figure 4.20: Specimen B7 post-load fringe field.

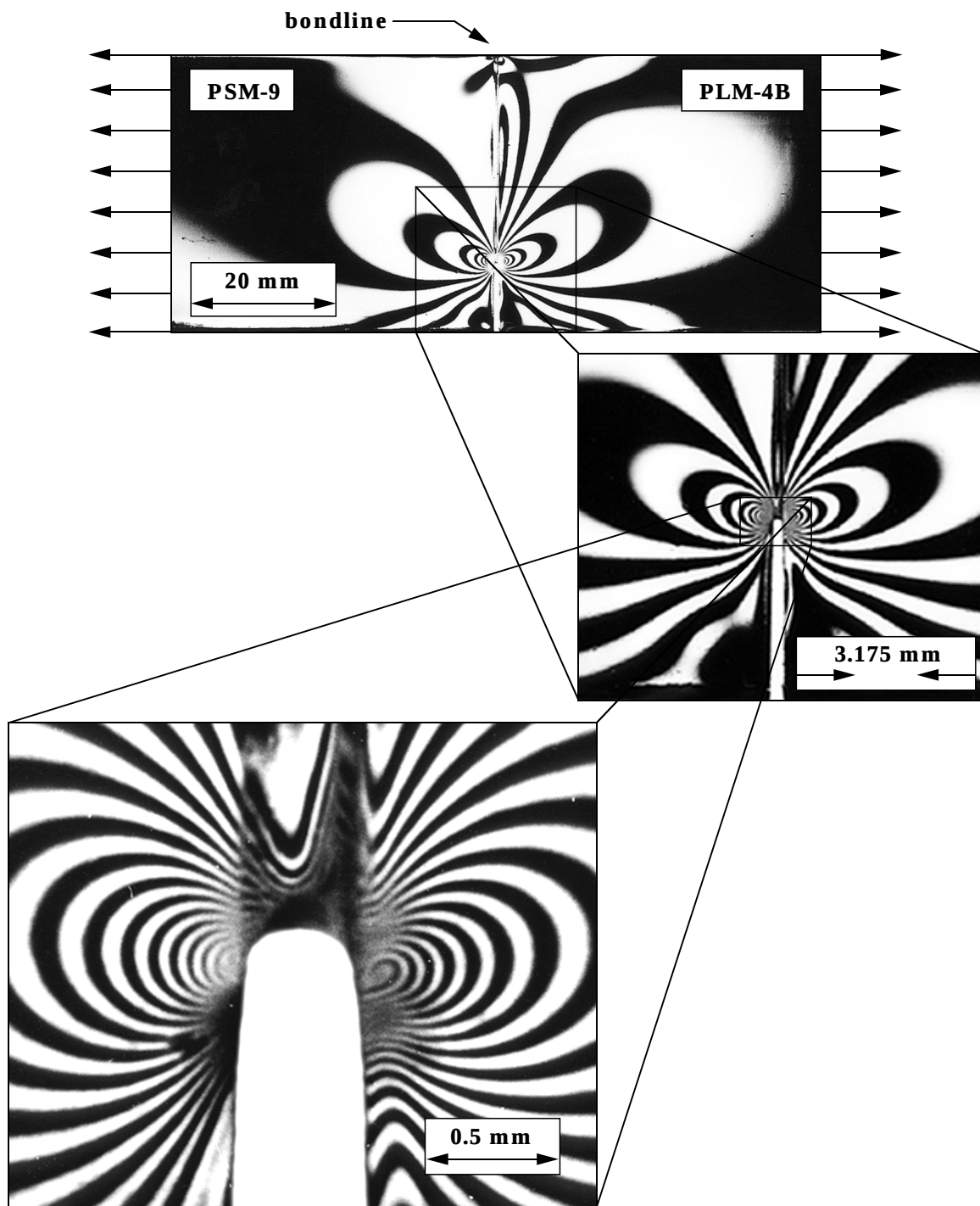


Figure 4.21: Specimen B8 post-load fringe field.

The bonded bimaterial specimens shown in Figures 4.18 through 4.20 reveal noticeable variations in their fringe patterns when compared to both one another and to their bonded homogeneous counterparts. By first observing specimen B5 (Fig. 4.18) which contains the crack positioned farthest from the bondline, it is seen that the near-tip fringe loops as a whole are slightly rotated with respect to the symmetry line parallel to and coincident with the crack. When moving the crack closer to the bondline, as was done with specimens B6 and B7, the rotation of the fringe pattern found in B5 becomes greater as the separation distance between the crack and the bondline decreases. The presence of these fringe field rotations (and thus, data reading line rotations) indicates a measurable presence of shear mode in addition to the expected opening mode. The increase in fringe rotation found from positioning the crack closer and closer to the bondline indicates a corresponding increase in mode mixity (K_{II}/K_I). The actual amounts of mode-mixity were quantified by visually estimating the angles of fringe field rotation for each specimen and then substituting those values into Eq. 2.16. The angles of rotation observed through the thickness for specimens B5, B6, and B7 as well as the corresponding mode-mixities are given in Table 4.3. The mixed-mode near-tip stress fields and corresponding angles of fringe loop rotation remained in the slices. The shear modes encountered in these specimens were therefore not a result of live loads, but rather of permanently frozen loads locked into the specimens during their high temperature loading cycles.

The final specimen tested (B8) was of particular interest due to its comparability with the araldite specimens discussed in Chapter 3. The photoelastic fringe field for B8 is given in Fig. 4.21. The crack of B8 was machined directly into the bondline in an attempt to once again produce a model which simulates the interface fracture problem. Inspection of the photoelastic fringe field near the crack tip reveals a return to the pure mode I symmetry found in the bonded homogeneous specimens. Stress intensity results are given in Section 4.4.3. By observing all four bimaterial specimens together, a mode-mixity pattern is established. As the crack is placed closer to the bondline, a greater amount of shear mode is encountered. However, when the crack is actually placed coincident with the bondline, the shear

Table 4.3 - Mixed-mode properties measured from off-bondline bimaterial specimens.

Specimen	Reading Line Rotation Angle (Degrees)*	Mode-Mixity Ratio (K_{II}/K_I)
B5	8	0.071
B6	13	0.117
B7	17	0.156

*As measured from a line perpendicular to the crack length direction

mode disappears. A peak or optimum distance for generating a shear mode thus likely exists somewhere between the closest off-bondline crack considered (0.125 in. [3.18 mm]) and the bondline itself. Interpretations and conclusions concerning these phenomena are discussed in Section 4.5.

4.4.3 Normalized Stress Intensity Factors

Stress intensity factors (SIF) were obtained from three slices taken from each of the nine specimens tested. For simplicity, the slices taken from each edge of a specimen have been designated as the “L” and “R” slices (left and right) while the middle slice from each specimen is designated as the “M” slice. Thus, data obtained from a given slice has been classified by its specimen and slice identity; for example, data obtained from the right edge slice of the bondline crack bimaterial specimen is designated as the B8-R data. An average of the data from the edge slices of this specimen is given by the B8-LR designation and so on.

Once stress intensity factors were obtained from each slice, they were normalized by the applied far field load, crack length, and overall specimen width and thickness according to the standard definition of stress intensity for a rectangular cross section specimen subjected to a far field tensile load:

$$K_I = F_I \bar{\sigma} \sqrt{\pi a} = F_I \frac{P}{Wt} \sqrt{\pi a} \quad (4.1)$$

All specimens were subjected to equivalent far field loads ($P=8.0$ lbs. [35.6 N]) and contained equivalent crack lengths ($a=0.375$ in. [9.525 mm]) and cross sectional widths ($W=1.50$ in. [38.1 mm]). Through thickness values (t) varied slightly between specimens. Substituting these values into Eq. 4.1 and rearranging in terms of the dimensionless factor F_I gives:

$$\begin{aligned} F_I &= 0.1727 K_I t \quad (K_I [\text{psi} \cdot \sqrt{\text{in.}}], t [\text{in.}]) \\ F_I &= 0.1959 K_I t \quad (K_I [\text{MPa} \cdot \sqrt{\text{m m}}], t [\text{m m}]) \end{aligned} \quad (4.2)$$

Similarly, mode II stress intensity factors can be non-dimensionalized:

$$\begin{aligned} F_{II} &= 0.1727 K_{II} t \left(K_{II} [\text{psi} \cdot \sqrt{\text{in.}}], t [\text{in.}] \right) \\ F_{II} &= 0.1959 K_{II} t \left(K_{II} [\text{MPa} \cdot \sqrt{\text{m m}}], t [\text{m m}] \right) \end{aligned} \quad (4.3)$$

By using these non-dimensionalized terms which account for the variation in specimen through thickness values, direct comparisons of results between all specimens can be readily made. Tables 4.4a-4.4c list the slice thicknesses and normalized stress intensity factors found from all nine experiments. Table 4.4a gives the values found from the homogeneous control specimen (B) test. Of particular importance is the LMR value found for F_I from this test. This value ($F_I=1.64$) is representative of a through-thickness average of the normalized mode I stress intensity factor for a homogeneous specimen. As such, it was subsequently used as a normalizing value against which all other data was compared. Data normalized in this manner is given by:

$$\begin{aligned} (F_I)_{\text{norm.}} &= \frac{F_I}{(F_I)_{\text{LMR}}} = \frac{F_I}{1.64} \\ (F_{II})_{\text{norm.}} &= \frac{F_{II}}{(F_I)_{\text{LMR}}} = \frac{F_{II}}{1.64} \end{aligned} \quad (4.4)$$

Tables 4.4b and 4.4c contain the bonded homogeneous and bonded bimaterial test results respectively. From the results given in Tables 4.4a - 4.4c and Fig. 4-22, several trends were found, namely:

- As anticipated, middle slice SIF values were consistently higher than their edge slice counterparts for a given specimen. When comparing the average of edge slices (LR) to that of the middle slice (M) for each specimen, the increases in the middle slice stress intensity factors ranged between 1% and 22% with the majority of specimens displaying elevations in the range of 15% - 22%.
- All four bonded homogeneous specimens displayed a state of pure opening mode (mode I). No shear mode (mode II) was present.

- The LMR values for all three off-bondline homogeneous specimens were within a range of $\pm 5\%$ of the control specimen value. A range of values within $\pm(3\% - 5\%)$ is usually considered to be within the scope of experimental error for the equipment and techniques used for obtaining this photoelastic data. The LMR value associated with the bondline crack homogeneous specimen (B4) displayed a modest increase of 11% over the homogeneous control specimen.
- The off-bondline bimaterial specimens each displayed the presence of both shear and opening modes. The mode-mixity ratio (K_{II}/K_I) increased as the bondline/crack separation distance decreased (Table 4.3). A return to pure mode I was found for the bondline crack bimaterial specimen (B8).
- The LMR values of the normalized mode I stress intensity factors for specimens B5, B6, and B8 were within 6% of the control specimen value. The one significant increase in mode I stress intensity was found in the bimaterial specimen containing the off-bondline crack closest to the bondline (B7). In this case, a 30% increase in the normalized mode I SIF was found over the control specimen value.

Of most notable importance from this series of tests is that with the exception of one case (B7), all average normalized stress intensity factors were relatively similar to one another. The bimaterial specimens containing cracks of large and intermediate distance to their respective bondlines displayed no significant elevation in mode I stress intensity over that of the control specimen and bonded homogeneous specimens. The actual source of the mode-mixity found in the off-bondline bimaterial specimens deserves particular attention and is discussed in Section 4.5. One final yet significant photoelastic experiment that was performed in an effort to definitively determine the source of mode-mixity in the off-bondline bimaterial specimens is likewise discussed.

Table 4.4a - Normalized stress intensity values for the homogeneous control specimen.

Specimen ^{*1}	Slice Thickness, Inches (mm) ^{*2}	F_I ^{*3}	$(F_I)_{\text{norm.}}$ ^{*4}
B-L	0.037 (0.940)	1.41	0.86
B-M	0.0385 (0.978)	1.70	1.04
B-R	0.036 (0.914)	1.80	1.10
B-LR	0.0365 (0.927)	1.61	0.98
B-LMR	0.0372 (0.945)	1.64	1.00

* Specimen Through-Thickness (TT) = 0.483 in. (12.27 mm)

*1 B = Homogeneous single edge-crack control specimen (no bondline)

L = Left side slice (edge slice) extracted from crack-tip area

M = Middle slice

R = Right side slice (edge slice)

LR = Average value of left and right side slices

LMR = Average value of left, middle, and right side slices

*2 Slices extracted using a thin, fluid cooled, rotating blade

*3 $F_I = K_I / \bar{\sigma} \sqrt{\pi a}$

*4 $(F_I)_{\text{norm.}} = F_I / (F_I)_{\text{LMR}} = F_I / 1.64$

Table 4.4b - Normalized stress intensity values for the bonded homogeneous specimens.

Specimen*	Slice Thickness, Inches (mm)	F_I	$(F_I)_{\text{norm.}}$
B1-L	0.0388 (0.986)	1.71	1.05
B1-M	0.0375 (0.953)	1.93	1.18
B1-R	0.0370 (0.940)	1.35	0.82
B1-LR	0.0379 (0.963)	1.53	0.93
B1-LMR	0.0378 (0.960)	1.66	1.01
B2-L	0.0410 (1.041)	1.52	0.93
B2-M	0.0385 (0.978)	1.81	1.10
B2-R	0.0380 (0.965)	1.43	0.87
B2-LR	0.0395 (1.003)	1.47	0.90
B2-LMR	0.0392 (0.996)	1.58	0.97
B3-L	0.0375 (0.953)	1.57	0.96
B3-M	0.0380 (0.965)	1.85	1.13
B3-R	0.0365 (0.927)	1.74	1.06
B3-LR	0.0370 (0.940)	1.65	1.01
B3-LMR	0.0373 (0.947)	1.72	1.05
B4-L	0.0410 (1.041)	1.67	1.02
B4-M	0.0410 (1.041)	2.01	1.23
B4-R	0.0420 (1.067)	1.78	1.09
B4-LR	0.0415 (1.054)	1.73	1.05
B4-LMR	0.0413 (1.049)	1.82	1.11

* See Table 4.4a and Figure 4-1 for variable definitions and specimen nomenclature.

B1 = Bonded homog. spec., $h = 0.500$ in. (12.70 mm), $TT = 0.500$ in. (12.70 mm)

B2 = Bonded homog. spec., $h = 0.250$ in. (6.35 mm), $TT = 0.504$ in. (12.80 mm)

B3 = Bonded homog. spec., $h = 0.125$ in. (3.18 mm), $TT = 0.507$ in. (12.88 mm)

B4 = Bonded homogeneous specimen, $h = 0$, $TT = 0.466$ in. (11.84 mm)

$$F_I = K_I / \bar{\sigma} \sqrt{\pi a}$$

$$(F_I)_{\text{norm.}} = F_I / [(F_I)_{\text{LMR}}]_{\text{Cntrl. Spec. "B"}} = F_I / 1.64$$

Table 4.4c - Normalized stress intensity values for the bonded bimaterial specimens.

Specimen*	Slice Thickness, Inches (mm)	F_I	$(F_I)_{\text{norm.}}$	F_{II}	$(F_{II})_{\text{norm.}}$
B5-L	0.0350 (0.889)	1.53	0.93	0.11	0.07
B5-M	0.0390 (0.991)	1.96	1.20	0.14	0.08
B5-R	0.0350 (0.889)	1.52	0.93	0.11	0.07
B5-LR	0.0350 (0.889)	1.53	0.93	0.11	0.07
B5-LMR	0.0363 (0.922)	1.67	1.02	0.12	0.07
B6-L	0.0360 (0.914)	1.64	1.00	0.19	0.12
B6-M	0.0380 (0.965)	1.81	1.10	0.21	0.13
B6-R	0.0345 (0.876)	1.76	1.07	0.21	0.13
B6-LR	0.0353 (0.897)	1.70	1.04	0.20	0.12
B6-LMR	0.0362 (0.919)	1.74	1.06	0.20	0.12
B7-L	0.0338 (0.859)	1.95	1.19	0.30	0.19
B7-M	0.0355 (0.902)	2.35	1.44	0.37	0.22
B7-R	0.0365 (0.927)	2.07	1.26	0.32	0.20
B7-LR	0.0351 (0.892)	2.01	1.23	0.31	0.19
B7-LMR	0.0353 (0.897)	2.12	1.30	0.33	0.20
B8-L	0.0410 (1.041)	1.70	1.04	0	0
B8-M	0.0440 (1.118)	1.74	1.06	0	0
B8-R	0.0390 (0.991)	1.75	1.07	0	0
B8-LR	0.0400 (1.016)	1.73	1.05	0	0
B8-LMR	0.0413 (1.049)	1.73	1.06	0	0

* See Table 4.4a and Figure 4-2 for variable definitions and specimen nomenclature.

B5 = Bonded bimaterl. spec., h = 0.500 in. (12.70 mm), TT = 0.494 in. (12.55 mm)

B6 = Bonded bimaterl. spec., h = 0.250 in. (6.35 mm), TT = 0.485 in. (12.32 mm)

B7 = Bonded bimaterl. spec., h = 0.125 in. (3.18 mm), TT = 0.485 in. (12.32 mm)

B8 = Bonded bimaterial specimen, h = 0, TT = 0.465 in. (11.81 mm)

$$F_I = K_I / \bar{\sigma} \sqrt{\pi a} ; F_{II} = K_{II} / \bar{\sigma} \sqrt{\pi a}$$

$$(F_I)_{\text{norm.}} = F_I / [(F_I)_{\text{LMR}}]_{\text{Cntrl. Spec. "B"}} = F_I / 1.64$$

$$(F_{II})_{\text{norm.}} = F_{II} / [(F_I)_{\text{LMR}}]_{\text{Cntrl. Spec. "B"}} = F_{II} / 1.64$$

4.5 Discussion

The impetus behind the use of bonded homogeneous specimens in addition to the bimaterial specimens was to provide a means of separating purely mechanical phenomena from thermomechanical factors as well as the contraction effects associated with the curing of adhesives. Although the bonded homogeneous specimens allowed for some conclusions to be drawn about the influence of various factors on stress intensity, additional key information was obtained from an off-bondline test performed with the same araldite materials used for the inclined crack tests described in Chapter 3.

The pre-load thermal cycling performed on each specimen proved effective in alleviating much of the residual stress that was initially present after the final post-cure of the bondline adhesive. Not only did the pre-load thermal cycles reduce the magnitude of residual stresses found near the bondlines and crack tips of each specimen, but more importantly they reduced the stress gradients found in the areas from which data was ultimately taken. As discussed previously, the presence of only minimal stress gradients prior to loading will have negligible effects on the SIF values obtained from the corresponding post-load fringe patterns. As such, documentation and assessment of the pre-load, post-thermal cycle images effectively eliminated bondline residual stress effects as a possible factor in affecting the values obtained for stress intensity.

The bonded homogeneous specimens displayed particular importance in this study both qualitatively and quantitatively. From a quantitative standpoint, these specimens (B1 - B4) provided direct confirmation to the prior postulation that the nature of the bondline residual stresses present in each specimen would have negligible effect on the post-load stress intensity factors. Indeed, as seen in Fig. 4.22, these specimens displayed SIFs of negligible difference when compared with the SIF found in the homogeneous control specimen. The bonded homogeneous specimens perhaps proved most vital in their role as a qualitative measure for determining the source of mode-mixity found in the off-bondline bimaterial specimens. With bondline residual stresses ruled out as a possible factor for the presence of mode-mixity and elevated mode I SIF's, only two sources seemed likely to have

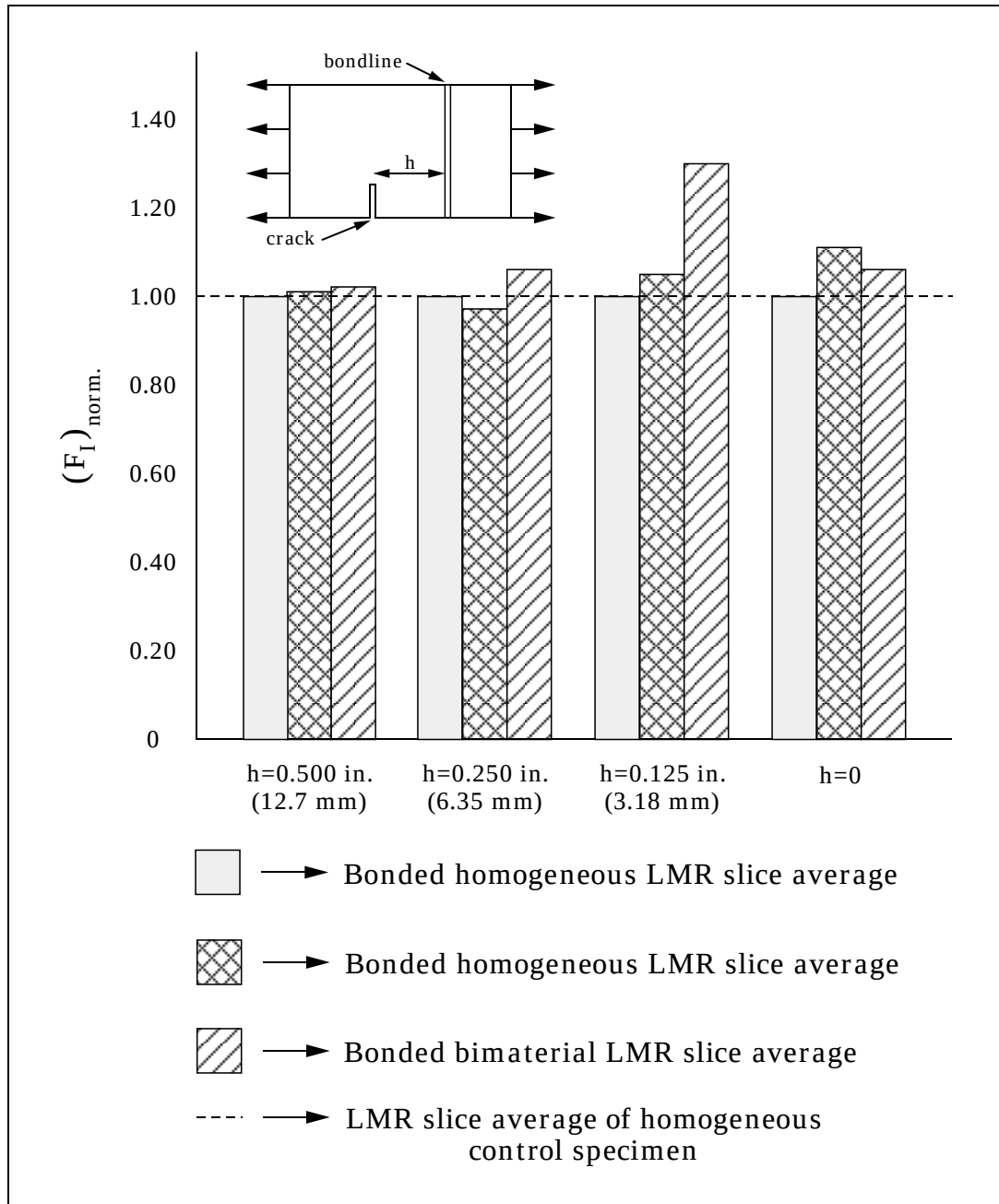


Figure 4.22: LMR values of normalized mode I stress intensity factors. Significant variation from the control specimen value of 1 is found only in the B7 specimen (1.30).

caused these phenomena. One possible source was that which was sought to be measured: the purely mechanical effects associated with a jump discontinuity in elastic moduli. The other source was that of the critical temperature mismatch between the two materials comprising the bimaterial specimens. More specifically, it is conjectured that a potentially moderate amount of shear stress may have been introduced into the bimaterial specimens during the cooling stage of the final loaded thermal cycles. During this cooling stage of a thermal cycle, the PSM-9 (which contained each off-bondline crack) will be the first of the two materials to transition from the rubbery to solid state due to its critical temperature of 205 F (the T_c of the PLM-4B is approximately 180 F). While making this transition, the PSM-9 will be somewhat constrained at the bonded interface by the rubbery PLM-4B. Since this constraint is present both before, and during the transition, whatever stresses are induced are likely to permanently remain in the PSM-9 material. When the thermal cycle finally falls through and below 180 F, the PLM-4B transitions from the rubbery to solid state, further inducing shear stress into the solid state PSM-9. It is likely that the induced shear stress from this transition however is at least partially released upon slicing the specimen apart near the bondline and crack for through thickness analysis. Even if the induced shear stress from the PLM-4B transition is completely relieved after slicing, the shear stress induced during the PSM-9 transition will remain embedded in the slices and thus, in the photoelastic data used for determining stress intensity.

In an effort to qualitatively determine the source of the mode-mixity present in the PSM-9/PLM-4B bimaterial specimens, a bimaterial specimen was made using the same materials from the inclined crack tests discussed in Chapter 3 (araldite and aluminum filled araldite). Once the specimen was constructed, a crack of roughly similar length was machined approximately 0.125 in. (3.18 mm) from the bondline on the clear araldite side. The specimen was constructed using sections of the same 0.5 inch (12.7 mm) thick rectangular blocks that were used for a previous inclined crack test. Before joining the two halves, they were thermal cycled above their T_c to remove the frozen stresses that were present from their initial inclined crack loading. The advantage in using the araldite materials was that they both possessed

the same critical temperatures (240 F) and would thus likely answer the question as to whether or not the mode-mixity found in the PSM-9/PLM-4B specimens was due to either mechanical or critical temperature mismatch effects. The initial decision to use PSM-9 and PLM-4B for the off-bondline series of crack tests (as opposed to the araldite materials) was made primarily due to excessive residual stresses found in the inclined-crack araldite tests near the bondline. The araldite blocks used for this more current experiment however had been subjected to several thermal cycles prior to bonding. An accompanying alteration in the coloring of the araldite which had been subjected to multiple thermal cycles suggested that the material had not been fully post-cured upon reception. After bonding the now well cured araldite halves together, it was observed that a substantial reduction in bondline residual stresses had occurred when compared to the initial araldite tests performed with inclined cracks. A proper post-curing of the original araldite material would likely have inspired its use in the off-bondline experiments.

The results from this off-bondline araldite test were indeed compelling. Unlike the corresponding PSM-9/PLM-4B specimen with a crack placed 0.125 in. (3.18 mm.) from the bondline, the comparable araldite/aluminum-filled araldite specimen displayed a state of pure mode I stress intensity at its crack tip. This total lack of a shear mode was exemplified by the symmetry of the fringe field as well as the complete absence of fringe loop rotation. A photoelastic fringe field comparison of the geometrically identical araldite/aluminum-araldite and PSM-9/PLM-4B specimens near their crack tips is shown in Fig. 4.23. A measurement of the mode I stress intensity value from the araldite specimen revealed a negligible increase of less than 1% over that of the values predicted by analytical solutions^{23,29,30}. Since the araldite materials possess identical T_c 's, these results strongly suggest that the presence of mode-mixity as well as the increase in mode I stress intensity (30%) found in the corresponding PSM-9/PLM-4B specimen was entirely due to the mismatch in the critical temperatures of the PSM-9 and PLM-4B materials. Typically, only modest relative differences in thermal expansion coefficients are found in most epoxy materials over their relatively narrow temperature range of glass transition. As such, the large relative change in

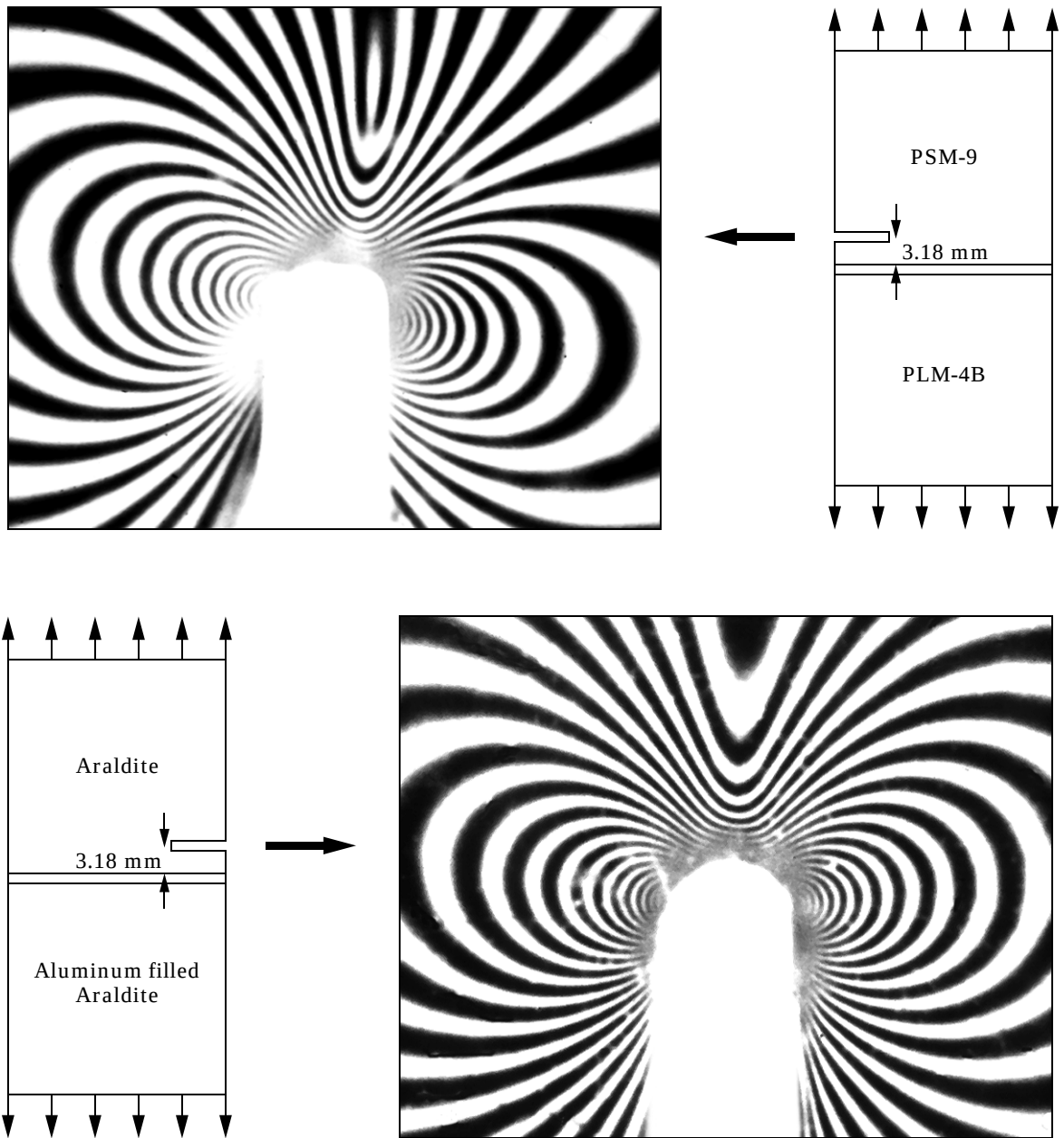


Figure 4.23: Comparison of araldite/aluminum-filled araldite near tip fringe field with corresponding PSM-9/PLM-4B specimen. Fringe loop rotation (and thus a partial shear mode) is found only in the 9/4B specimen suggesting a mismatch in critical temperatures in the 9 and 4B as the source of the shear mode.

the elastic modulus ratio between the two materials in question during the PSM-9 transition state is conjectured to have been the primary source of the shear state found at the crack tips of the bimaterial specimens.

It is interesting to note that a return to the pure mode I stress intensity state was found when placing the crack within the bondline (B8) instead of slightly offset as in the case of the B7 specimen. The most notable geometric difference between these two specimens was that the entire length of the crack of the B7 specimen was contained within the PSM-9 material but situated rather closely to the bimaterial interface. The crack in the B8 bondline specimen however had one face along its crack length consisting of PLM-4B while the opposite face consisted of PSM-9. It is perhaps possible that less constraint and thus more freedom of motion to conform to a classical mode I condition was permitted by the effective removal of the bimaterial interface along the length of the bondline crack.

In summary, the results obtained from the PSM-9/PLM-4B series of experiments yielded desirable information concerning the behavior of cracks and flaws near bimaterial interfaces of relatively similar elastic moduli. When considering the off-bondline araldite bimaterial test in conjunction with the results obtained from the PSM-9/PLM-4B tests, it is evident that the source of mode-mixity found in the off-bondline bimaterial specimens can be attributed to the mismatch in critical temperatures between the two materials comprising each complete specimen. Specifically, this state of mode-mixity is suspected to have resulted from the large relative mismatch in elastic modulus which inherently exists during the glass transitioning state between two materials with differing critical temperatures. In order to specifically measure modulus-mismatch related effects on stress intensity, materials possessing identical material properties with the exception of elastic stiffness would be most preferred. Based on the evidence presented by the off-bondline araldite bimaterial specimen, indeed, it would likely have been more desirable to have used materials with equivalent critical temperatures such as the araldite materials. Although unverified, it is also likely that if the T_c of a common photoelastic material such as araldite is unaltered by the addition of aluminum powder (for stiffening purposes), other photoelastic materials may demonstrate similar attributes if mixed

with powdered metals. It is concluded that for bimaterial investigations such as those considered in this work, obtaining comparable critical temperatures between materials is perhaps most crucial. Furthermore, the relative effects associated with the inevitable presence of bondline residual stresses due to adhesive curing may be accurately accounted for in most circumstances by constructing comparable bonded homogeneous specimens.

In addition to clarifying the origin of the mode-mixity found in the PSM-9/PLM-4B specimens, the araldite/araldite aluminum powder off-bondline specimen importantly displayed a negligible increase in the mode I stress intensity factor found at the crack tip. The mode I, or “opening mode” stress intensity factor generally governs the magnitude of far field load required to commence crack propagation. Furthermore, as a crack propagates, it will travel in such a manner as to maintain a state of pure opening mode at its instantaneous crack tip. Therefore, from a design perspective, an increase in the mode I stress intensity factor for an interfacial fracture scenario generally deserves greater attention than the mere appearance of a shear mode prior to propagation. Although further testing would be desirable, the negligible increase in mode I stress intensity found in the off-bondline bimaterial araldite specimen suggests that for the ratio of elastic moduli used, a simple analytical or numerical solution to a homogeneous fracture problem could be used to accurately determine failure loads in a comparable bimaterial scenario.

Finally, it is interesting to note that the mode-mixity and increase in mode I stress intensity found in the B7 specimen due to the T_c mismatch perhaps presents evidence that large differences in elastic moduli and thermal expansion coefficients of bimaterial systems may indeed serve to notably intensify near-crack tip stress fields. During the stress freezing process, a series of fairly complex constraint conditions surely existed near the bimaterial interfaces of every specimen. Temporary but large elastic moduli ratios are the probable source of the mode-mixity and increase in mode I stress intensity found in the B7 specimen. As such, it is quite possible that much larger modulus ratios may produce elevations in stress intensity over that of a comparable homogeneous specimen.

5.0 Summary and Future Work

Stress intensity factors (SIF's) were experimentally determined from bimaterial photoelastic specimens of moderate thickness. The stress freezing technique of photoelasticity successfully allowed for the determination of SIF's throughout the thickness of single edge cracked and notched specimens. Flaws were machined into specimens both near and coincident with the bimaterial interfaces. A shear mode was geometrically induced in several bondline-notched specimens by orienting notches at various angles of inclination to the direction of far field tensile loading. Moderate increases in stress intensity factors were consistently found at the mid-thickness of specimens. Mode-mixity and elevated values of mode I stress intensity were found in several specimens. Through the use of a multiple test-specimen procedure, the sources of mode-mixity and elevated stress intensity were determined to be completely due to T_c induced elastic modulus mismatch effects and bondline residual stress effects. It was therefore concluded that for the elastic modulus ratios considered (2:1 and 4:1), any influences incurred by the mechanical mismatch in elastic moduli were of negligible magnitude as compared to T_c induced elastic modulus mismatch effects and bondline shrinkage phenomena. As a result of these factors, relative increases in mode I stress intensity values were found to be as high as 30% when compared to homogeneous control specimens. The dominance of these factors over the otherwise negligible effects of the mechanical mismatch in moduli suggests that for bimaterial scenarios of such low mismatch ratios, the particular casting, curing, and thermal properties of the materials in question are of primary concern.

Future bimaterial photoelastic experiments will be conducted on part-through crack specimens containing surface flaws of varying depth. Further work will be performed using bonded homogeneous specimens containing edge cracks. In an effort to simulate potential end conditions in rocket motor grains, experiments incorporating uniform displacement far-field boundary conditions will be performed using polyurethane materials specifically designed for photoelastic analysis. Through-thickness stress intensity values obtained from the photoelastic models will be compared to numerical

solutions for various crack lengths and locations. Strength and fracture testing of the actual materials will likewise be performed using similar boundary conditions and specimen geometries. The results obtained from these various experiments and analyses is intended to provide information to be used for estimating the mechanical strength and stability of the rocket motor grain structures under consideration.

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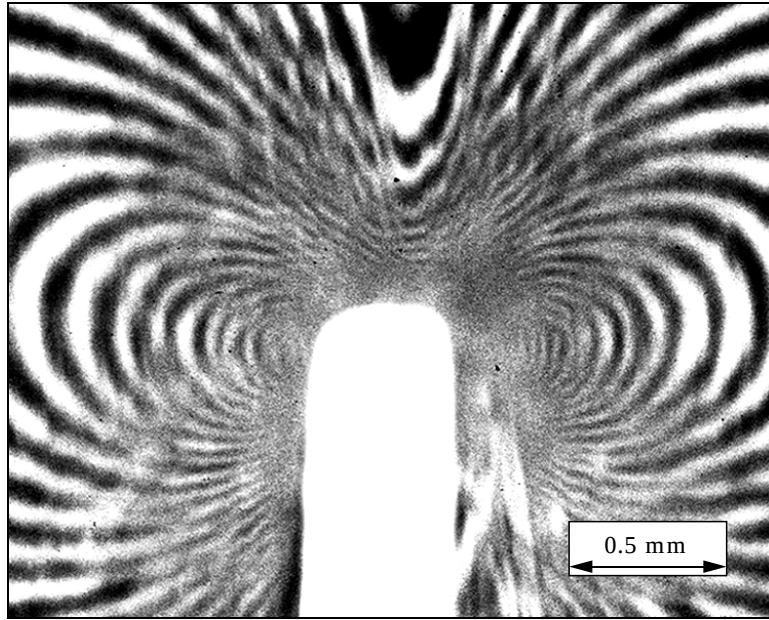
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Appendix A: Photographic Techniques

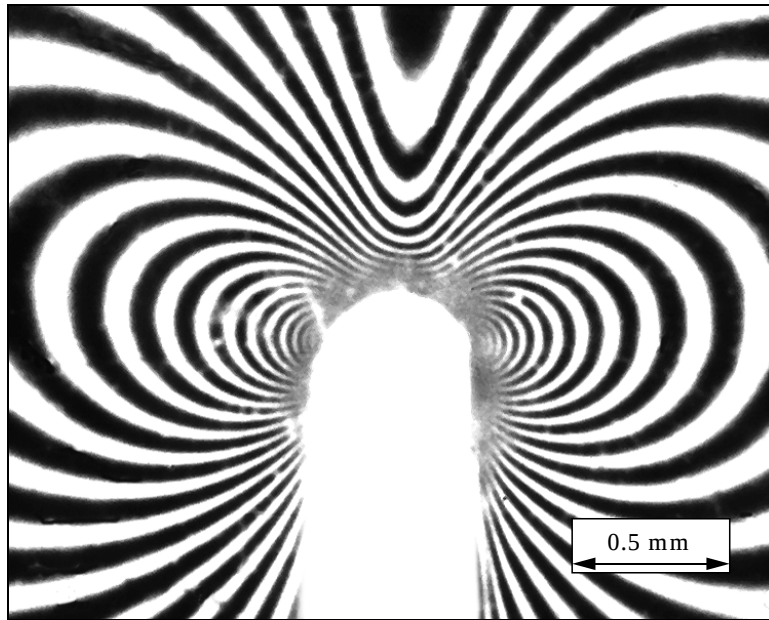
As discussed in the section concerning the analytical development of the Smith Extrapolation Method, fringe patterns near the tip of a crack reveal important information that can be used to determine mode-mixity and the associated stress intensity factors. As already noted, all models used in this study were of one half inch nominal thickness. Such large relative thicknesses create the need for specific photographic techniques to be utilized to ensure sharp, high contrast images when high magnification photos are desired.

For clarity purposes, it is desirable to create an image for which all features are sharp and focused. The plane of focus for photoelastic fringes is typically near the mid-thickness of the model. For a high magnification photograph of a crack tip, it is desirable to keep not only the fringes focused in the final image, but also the crack edges at the front and rear of the specimen. Maintaining focused front and rear surfaces is critical for the determination of the crack boundaries and the position of the effective crack tip. This focusing criteria necessitates a minimum focused depth of field of one half inch. Fig. A.1 portrays a typical high magnification crack tip photograph with and without full depth of field focus.

The problem of achieving an adequate depth of field for thick specimens is compounded by the inherent decrease in the depth of the focusing field associated with locating the film plane of the camera near the specimen to be photographed. Although this problem can be overcome to a certain degree through the use of large focal length lenses, it was deemed practical to utilize more conventional photographic equipment. The camera configuration used for the close crack tip photographs is depicted in Fig. A.2. A series of matched multipliers and extension tubes were used in conjunction with a standard, variable aperture 50 mm focal length lens. Before extracting slices for analysis, all specimens were photographed in a polariscope consisting of 12.0 in. (30.5 cm) diameter quarter-wave and polarizer plates. This polariscope provided light through the use of a white fluorescent bulb while the apparatus for slice analysis made use of a monochromatic laser light source. Due to the white light source, a



(a)



(b)

Figure A.1: High magnification crack-tip fringe fields.
a) Aperature f-stop =3.5 , exposure time ~ 20 sec.
b) Aperature f-stop =22 , exposure time ~ 1 hour.

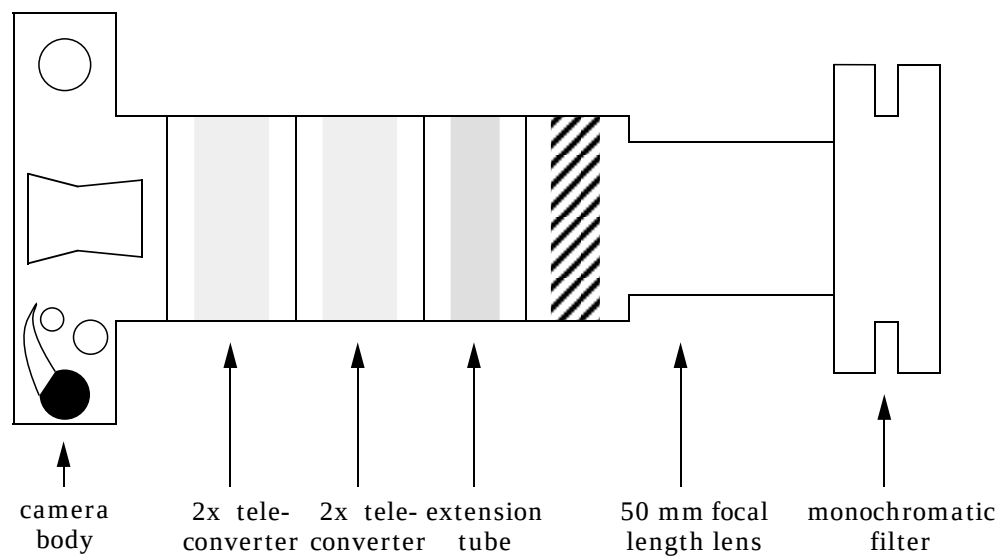


Figure A.2: Optical arrangement used for close-up photography of crack tip regions.

monochromatic filter (monochromator) was used on the front of the camera to enhance the contrast of black and white negatives. Additionally, Kodak Technical Pan Film was used to further maximize black and white contrast while also maximizing fine grain resolution.

In order to achieve a depth of field value of at least one model thickness (0.500 in.), high magnification crack tip photographs were taken using a relatively small aperture setting. This simple yet effective technique is commonly used in portrait photography in order to assure that objects beyond the primary subject remain sharply focused. Likewise, a blurring background effect can be achieved by using a relatively small aperture setting (large aperture diameter). The minimum available setting for the lens aperture diameter, designated “f 22”, was used for all high magnification photographs such as those found in the figures of Sections 4.4.1 and 4.4.2. The aperture setting, or “f stop” value, is simply given by:

$$\text{aperture setting} = \frac{\text{lens focal length}}{\text{aperture diameter}} \quad (\text{A.1})$$

For an aperture setting of 22 and a lens focal length of 50 mm (1.97 in.), Eq. A.1 yields an aperture diameter of only 2.27 mm (0.089 in.). Fig. A.1 illustrates the same image taken using large and small aperture diameters (f3.5 and f22 respectively).

There are several factors which influence the exposure durations required to properly produce the photographic negatives. The use of teleconverters and a monochromatic filtering device tend to reduce the net light intensity at the film plane. Likewise, using small aperture diameters to achieve larger focus depths translates into an exponential decrease in light intensity. Nonlinearities in film sensitivity as a function of time of exposure also tend to increase exposure durations. The net result is a vast increase in exposure times when using small aperture diameters for high magnification photographs. The exposure time for the photograph of Fig. A.1 using an aperture setting of f3.5 was of the order of twenty seconds while that of the photograph taken at an aperture setting of f22 was approximately one hour.

Index Matching Fluid and the Plate Glass Technique

When creating the bonded models for this study, it was necessary to deposit excess adhesive along the length of the upper bondline surface. This was performed to ensure that a partial void would not form along the top surface due to adhesive shrinkage during the curing process. After having cured, wet sanding was performed to remove the excess adhesive. Plates of photoelastic material received from the manufacturers possess naturally smooth and clear outer surfaces. The sanding process naturally tended to dull the original smooth and clear finish of the each specimen near its bondline and crack tip. To regain image clarity for photographic purposes, an index matching fluid was used in conjunction with thin plates of glass. Index matching fluid is simply a fluid whose index of refraction is closely matched to that of the solid photoelastic material itself. When applied to a photoelastic model, this fluid tends to effectively smooth the surface by filling the microscopic valleys induced by sanding. The result is a crisp and clear image that rivals the clarity originally found in the material prior to sanding.

For low magnification photographs of either the entire model or large sections of the model, index matching fluid can be applied to the specimen simply by using a cotton tipped swab. For high magnification photographs of very localized regions, index matching fluid applied with a cotton tipped swab will create quite noticeable fringe distortions due to microscopic streaking and bubbling. A method of fluid entrapment using plates of standard glass was utilized to overcome this obstacle. (Fig. A.3). Using a dropper, index matching fluid is deposited on the surface of the specimen near the area of interest. A clear plate of glass is then lowered at a partially inclined angle onto the fluid and surface. The fluid is thus forced to spread out, forming a thin layer upon which the glass rests. At this point, the entire model may be carefully turned over so that the identical procedure can be performed on the opposite face. The glass plates tend to hold the fluid in place for long periods of time, even if placed in an upright position. This proves particularly useful for creating the necessary high magnification photographs of lengthy time exposure. Fig. A.4 compares photographs taken both with and without the use of the glass plates.

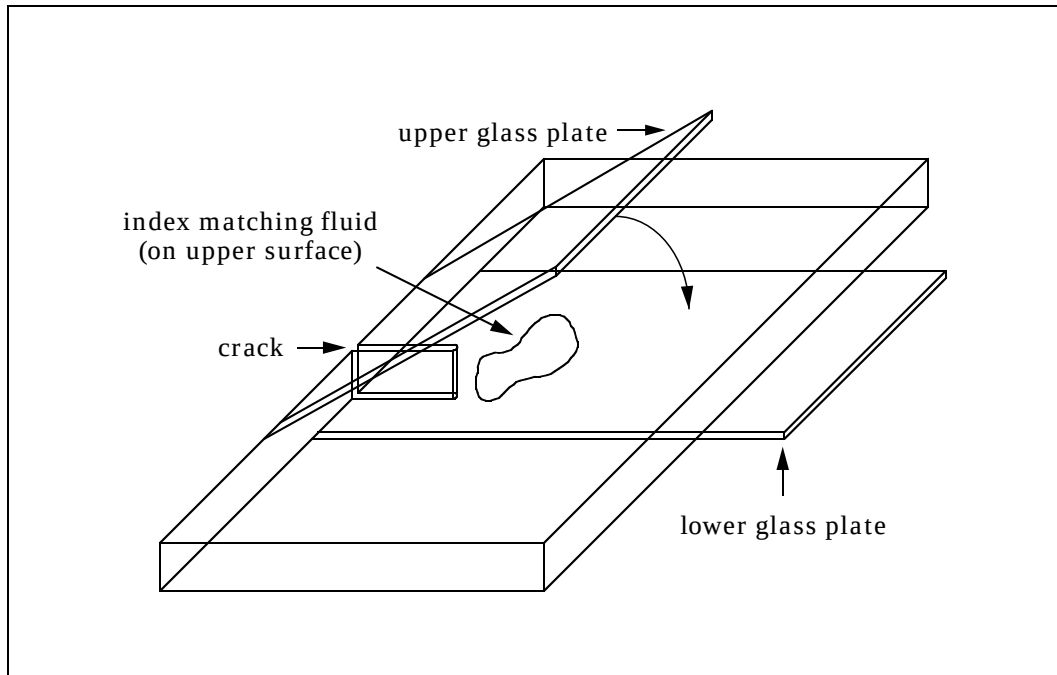
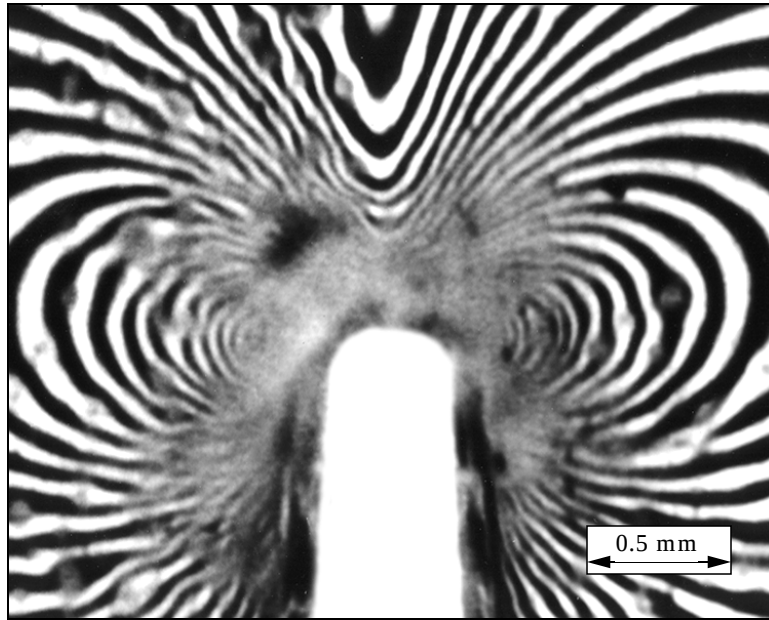
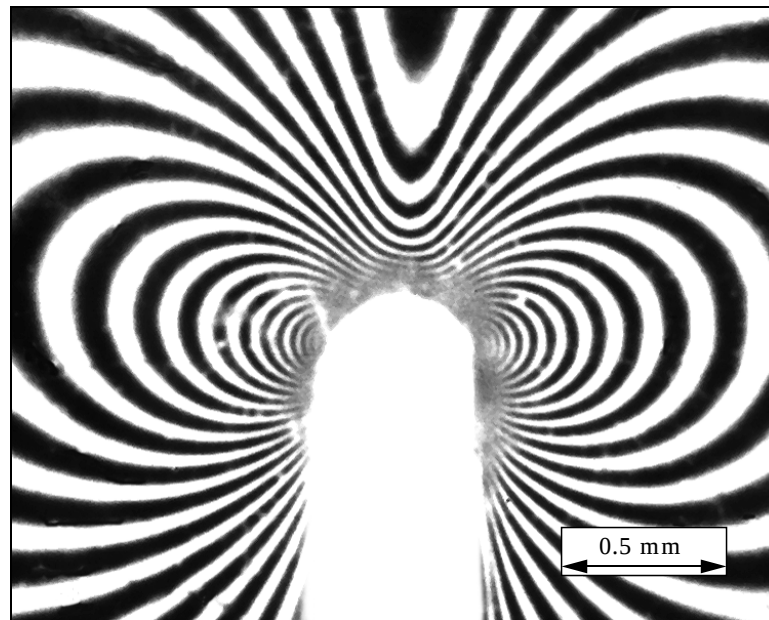


Figure A.3: Plate glass technique used for creating a uniform coating of index matching fluid.



(a)



(b)

Figure A.4: High magnification crack-tip fringe fields.
 a) Image taken without use of the plate glass technique.
 b) Image taken using plate glass technique.

Appendix B: Material Fringe Value Determination

Whenever a photoelastic study is performed, the optical property of the material chosen for use is required in order to quantify stress magnitudes. This optical property, referred to as the material fringe value, is dependant upon internal and environmental factors, as well as the wavelength of light used in the analysis. The manufacturers of photoelastic materials are often able to alter the chemical composition of the materials to bring about changes in material fringe value and material stiffness. Environmental factors such as the age of the material as well as the moisture content of the air surrounding the material will often have a measurable effect on the material fringe value in particular. The combination of these factors necessitates the use of a method for accurately determining the properties of the material. Typically, a fairly simple control specimen is machined out of the same plate of material from which the primary test specimens are to be taken. A control specimen geometry for which a known closed-form analytical solution exists is usually chosen so that the desired material properties may be solved for as the only unknowns in the governing analytical stress field equations. Tension specimens, diametrical disk-compression specimens, and four point bend specimens are the most commonly employed geometries for determining material fringe values and elastic moduli.

For the work considered in this study, four point bend specimens were chosen for determining both the material fringe values and elastic moduli of all of the photoelastic materials used in the study. The bend specimen possesses unique qualities that allows the accurate determination of both of these properties in a very timely fashion. The tension specimen generally yields only one data point from which to determine the material fringe value. The disk-compression specimen is somewhat more flexible in that multiple points may be used for obtaining data. Such an exercise has its drawbacks in this instance though as an accurate method for determining coordinate positions within the body would need to be incorporated. The bend specimen however easily yields multiple data points which can themselves be averaged and then substituted into a single governing equation for the determination of the material fringe value. As will be seen in the following analytical

development, fringe orders and their corresponding distances from the neutral axis can be taken along a line perpendicular to the neutral axis and located at the mid-span of the specimen. Furthermore, the bend test is unique among the three configurations considered due to the moderate displacements that result from loading. By measuring the mid-span deflection, the elastic modulus of the material can be determined simply by knowing the geometry and loading of the bend specimen.

One final and rather unique attribute of the bend specimen worth noting is that fringe counting may be performed by starting within the specimen rather than at a corner (where the fringe order must be zero in all bodies). The fringes found at the corners of photoelastic specimens are often somewhat difficult to decipher. Ideally, when viewed in the dark field, the corners should be distinctly dark. Unfortunately this often is not the case¹⁷ and what is found at the corners is instead a small zone of varying fringe intensity. These disturbances are likely caused by the presence of small time-edge (moisture related) residual stresses whose magnitudes are similar to those of the zero and near-zero order mechanical stress field. Although these rather minimal residual stresses will have a negligible impact on the rest of the body, they can result in a miscount of fringe orders that will usually be offset by a value of 1 from the proper order. The solution to the bend specimen specifies that the zero order fringe will be present at the neutral axis. As such, one needs merely to find the midpoint of a bend specimen to find the zero order fringe assuming that a doubly symmetric cross section (e.g. rectangular) is utilized. From there, a constant increase in fringe order will be found on the tension side of the neutral axis while a constant decrease (negative) will be found on the compression side.

Four-Point Bend Solution

Four point bend specimens were machined from each photoelastic plate and tested using a simple loading scheme. A rectangular cross section uniformly spanning the length of each beam was used for simplicity. A single dead weight was suspended from the center of a symmetric string system which in turn provided two equal forces acting at equal and opposite

distances from the mid-span of the beam. Fig. B.1a depicts the geometry, loading, and definitions of the variables used for the following development of the equations governing the elastic modulus and material fringe value.

Positioning the x-z coordinate system coincident with the neutral axis at the left hand side of the beam in Fig. B.1a with the z coordinate running along the length of the beam, the following internal moment equation can be written for the mid-span cross section:

$$M_{m.s.} = \left[Pb + \frac{w}{2} (Lz - La - z^2) \right] \quad (B.1)$$

where:

$$w = W/L = \text{weight of beam / unit length}$$

It will be seen that for the relative magnitude of the loads “P” used, the weight of the beam itself has a measurable effect on the final results and is thus considered throughout the development. Equating the simple bend stress field to that of the general photoelastic parameters gives:

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{Nf}{2t} = \frac{\sigma_{zz}}{2} = \frac{M_{m.s.} x}{2I} \quad (B.2)$$

Substituting Eq. (B.1) as well as the second area moment of inertia ($I = th^3/12$) for a rectangular cross section into Eq. (B.2) and rearranging in terms of the material fringe value “f” yields the desired relationship:

$$f = \left[\frac{12}{h^3} \left(\frac{x}{N} \right)_{avg.} \right] \left[Pb + \frac{WL}{2} \left(\frac{L}{4} - a \right) \right] \quad (B.3)$$

The quantity $(x/N)_{avg.}$ found in Eq. B.3 is experimentally determined at the mid-span of the beam. Measuring along a line perpendicular to the neutral axis, the distance (x) to each successive fringe is divided by its respective fringe order (N). Working in the dark field and starting from a value of zero at the neutral axis, the fringe orders increase by a factor of 1 as one moves farther away from the neutral axis in either direction. Since Euler beam theory stipulates that the distribution of stress as a function of x is linear with a value of zero at the neutral axis³¹, it is expected that the x/N ratios be identical. Indeed, as measured experimentally, the values of these ratios taken along the mid-span were consistently found to be quite similar to one

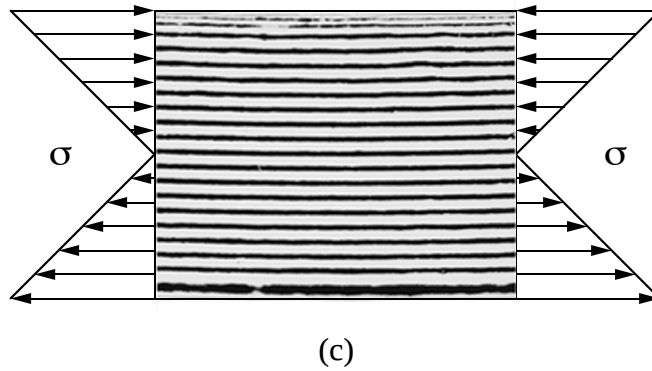
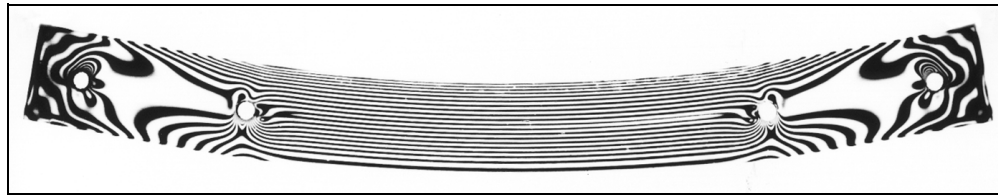
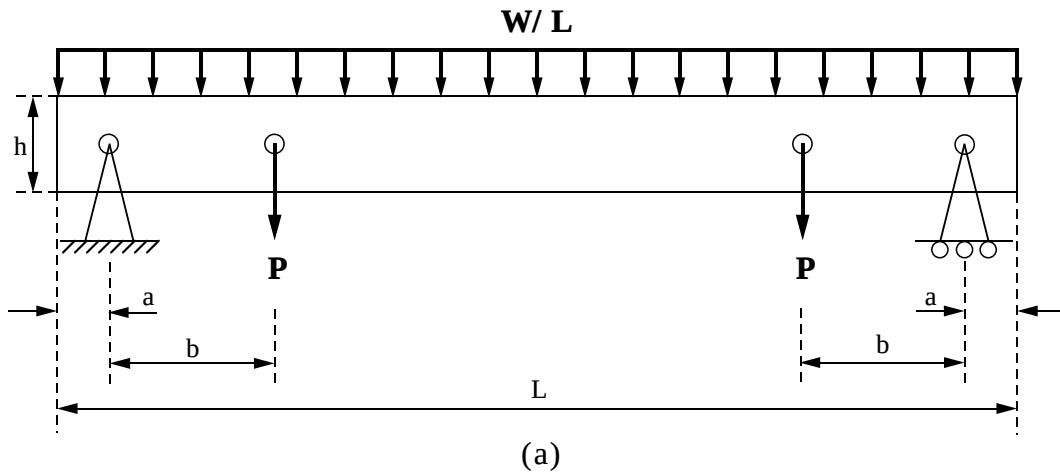


Figure B.1: a) 4-Point bend specimen geometry and loading.
 b) Frozen-stress photoelastic image of a bend specimen after loading.
 c) Close-up of mid-span region illustrating the linear stress distribution predicted by beam theory.

another. As such, the average of all values was taken and substituted into Eq. B.3 for the final determination of the material fringe values for each bend test. By doing so, the material fringe value was effectively determined by several data points as opposed to a single data point obtained from a tension specimen. A typical fringe pattern for an entire beam is shown in Fig. B.1b while a magnified section of the mid-span region is shown in Fig. B.1c. It is seen from the mid-span image that approximately twenty fringes are present and thus twenty data points are present for use in determining the material fringe value.

The same bend specimen used to determine the material fringe value of a given material may also be used for finding its elastic modulus. Starting with the governing differential equation for an Euler beam, we have:

$$\frac{d^2x}{dz^2} = \frac{M(z)}{EI} \quad (B.4)$$

Substituting Eq. B.1 into Eq. B.4 and integrating gives:

$$EI \frac{dx}{dz} = Pbz + \frac{wLz^2}{4} - \frac{wLaz}{2} - \frac{wz^3}{6} + C_1 \quad (B.5)$$

Integrating Eq. B.5 yields:

$$EIx = \frac{Pbz^2}{2} + \frac{wLz^3}{12} - \frac{waLz^2}{4} - \frac{wz^4}{24} + C_1z + C_2 \quad (B.6)$$

Substituting the natural boundary conditions ($[@z=a, x=0]$; $[@z=L/2, dx/dz=0]$) into Eqs. B.5 and B.6 yields the constants C_1 and C_2 . After some rearrangement and simplification, substituting these constants back into Eq. B.6 and evaluating at the mid-span ($z=L/2, x=\delta$) gives the desired equation relating the elastic modulus to the specimen geometry, loading, and mid-span deflection (δ):

$$E = \frac{6 \left[P \left(abL - a^2b - \frac{bL^2}{4} \right) + W \left(\frac{a^3}{3} + \frac{a^4}{12L} - \frac{aL^2}{24} - \frac{5L^3}{192} - \frac{a^2L}{2} \right) \right]}{th^3 \delta} \quad (B.7)$$

Eqs. B.3 and B.7 were used to determine the material fringe values and elastic moduli for all of the materials used in this study. Ideally, a bend specimen should be subjected to the same type and same number of thermal cycles as the actual models being tested. The PLM-4B exhibited excellent environmental and thermal stability thus yielding negligible material property dependence upon thermal cycling. The araldite and PSM-9 materials however exhibited variations primarily in their material fringe values when subjected to multiple thermal cycles. Although the elastic moduli seemed to be consistent, there appeared to be some variation in the material fringe value among different plates of PSM-9. The material fringe value listed for PSM-9 in Table 4.1 is representative of the value found from most plates. It is finally noted that for each single-edge cracked specimen, the material fringe value was further verified by establishing the fringe order associated with the uniform far field stresses in the actual single edge-cracked specimens. By placing the isoclinics over the uniform tensile stress zones, the correct fractional fringe order for each specimen was obtained by rotating the analyzer (Tardy compensation) until a dark fringe was obtained. Using the known load and cross sectional area in combination with the governing photoelastic relation allowed for the determination of the material fringe value. Values determined using this method generally were found to be within 3% of the values found from the beam specimens. As mentioned previously though, the bend specimens prove instrumental in definitively establishing the correct fringe orders since the zero fringe order need not be determined from a corner of the specimen. Without knowing the material fringe value from a bend specimen, it may in some cases be difficult to determine the proper fringe order magnitude from a tension specimen alone. Only after knowing the correct value from a bend test can a confirmation with a tension specimen be made.

Vita

Born on March 14, 1973 in Troy, New York, Eric Finlayson was raised and educated in the same region. Upon receiving the Rensselaer Medal for excellence in science and mathematics during his senior year of high school, the author began his collegiate studies during the fall of 1991 at Rensselaer Polytechnic Institute, located in Troy, New York. After receiving his Bachelor's of Science degree in Mechanical Engineering in 1995, the author began graduate studies in the Department of Engineering Mechanics at Virginia Polytechnic Institute. The author intends to pursue a career in industrial machine design and analysis.