DETERMINATION OF ACOUSTIC RAY

PATHS IN ENCLOSED SPACES

by

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1.0 INTRODUCTION

The Occupational Safety and Health Act of 1970 (OSHA¹)* requires that workers not be exposed to noise levels greater than 90 dBA over an eight hour period. Greater levels are permitted if the exposure time is reduced. The maximum level allowed is 115 dBA for exposure periods of fifteen minutes or less. Engineers with little or no background in noise abatement have found themselves in the position of having to reduce the noise levels in their plants.

Therefore, the need exists for a method to predict changes in existing sound pressure levels which might result from plant modifications. Also needed is a method for predicting sound pressure levels for factories in the planning stage.

Texts by Beranek² and others give an equation for determining total steady state sound pressure levels in "large" rooms. This equation is based on the assumption of a diffuse sound field, which can be obtained in rooms whose length, width and height do not exceed the relative proportions of 1:1.5:2. For rooms which do not conform to these ratios some error is encountered because the assumption of a diffuse sound field is violated. The magnitude of this error depends on how much the room dimensions exceed the above ratios.

Lester³, Gibbs and Jones⁴ and others have assumed that the acoustic energy from a source is propagated along rays emanating

*Numbers refer to entries in Bibliography.

from the source. Because spherical spreading is assumed, the intensity at any location is assumed to be inversely proportional to the square of the distance along the ray. This approach is known as geometric acoustics. The aforementioned authors' works have shown that the geometric acoustic approach yields more accurate predictions than the room acoustics approach and has more flexibility when irregular surface areas, such as those in a factory space, are considered. Therefore, a ray tracing technique to determine the distances along sound ray paths between sources and receivers in a factory space was developed by this author. This approach was chosen because of the accuracy and flexibility in handling rooms of many shapes.

A feature of this study was the determination of the locations where each sound ray was reflected from different surfaces. Significant noise reduction at a particular receiver position might be possible if absorption material was added to those surfaces where several rays were reflected.

Knowledge of the energy propagation paths would also indicate where barriers might be erected to minimize noise at a worker location in a plant. If barriers are not feasible the source or receiver locations might be changed.

With this program the engineer has a modeling tool to check possible modifications before physically making changes in a factory.

Before any sound pressure levels may be calculated, however, the ray paths must be determined. The emphasis in this work was determining the rays. Once a viable method for calculating ray paths was found a simplified program was devised to calculate sound pressure levels and comparisons were made with measured values in two enclosures. Reasonable agreement between the calculated and measured values indicated that it was worth proceeding with this project as a basis for further work.

2.0 SUMMARY OF PREVIOUS WORK IN RAY ACOUSTICS

The use of geometric acoustics as a tool to predict sound intensity in a factory space is relatively new. Most geometric acoustics applications have been in underwater sound propagation.

The most common use of geometric acoustics is in studies investigating sound propagation where the properties of the medium vary. However, the medium in a factory space is assumed constant enough so that the ray paths are straight lines except where reflection occurs.

The basic theory of geometric acoustics centers around the assumption of spherical spreading from point sources. Figure 1(a) shows one such point source. The wave fronts which are shown are depicted as circles with increasing radii. Huyghens principle states "every point on a wave front is to be regarded as the center of a spherical disturbance spreading outwards from it, so that the resultant disturbance at a particular instant at any point in front of the advancing waves will be produced by the summation effect of all the elementary centers of disturbance."

Figure 1b demonstrates the application of this principle to a point source. Wave front A-A' at time t is considered to consist of an infinite number of sources from which wavelets are produced. The resultant wave front at time $(t+dt)$ is the envelope of all the wavelets at $(t+dt)$. For a slowly varying or constant speed of sound, c , over surface A-A' the surface B-B' will be defined approximately by

Figure 1. Demonstration of Huyghens Principle with application to a point source.

the ends of the normals, of length $c dt$, to surface A-A'. A wave surface is the locus of points which are in the same phase of motion at a given time t . At the later time, $t+dt$, the wavelets define a new wave surface which is in the same phase of motion that the original wave surface was at time t .

2.1 The Eikonal Equation

Officer⁵ and others have shown that it is possible to denote the relationship between successive wave surface positions in equation form:

$$W(X+\alpha cdt, Y+\beta cdt, Z+\gamma cdt) - C_0(t_0+dt) = W(X, Y, Z) - C_0 t_0 \quad (1)$$

where:

$W(X, Y, Z)$: Original wave surface

C_0 : A constant

α, β, γ : Direction cosines of the normals to
original wave surface

The portion to the left of the equal sign is the new wave front and the portion to the right is the original wave front. Since $\alpha cdt, \beta cdt, \gamma cdt$ are small quantities the new wave surface can also be approximated by:

$$W(X+\alpha cdt, Y+\beta cdt, Z+\gamma cdt) = W(X, Y, Z) + \frac{\partial W}{\partial X} \alpha cdt + \frac{\partial W}{\partial Y} \beta cdt + \frac{\partial W}{\partial Z} \gamma cdt \quad (2)$$

where C = speed of sound

$\frac{\partial W}{\partial X}, \frac{\partial W}{\partial Y}, \frac{\partial W}{\partial Z}$: Direction numbers for the wave surface

If equations (1) and (2) are equated:

$$\alpha \frac{\partial W}{\partial X} + \beta \frac{\partial W}{\partial Y} + \gamma \frac{\partial W}{\partial Z} = \frac{C_0}{C} \quad (3)$$

Since the direction numbers are proportional to the direction cosines, and the direction cosines α, β, γ can be expressed in terms of derivatives along the normals, several algebraic manipulations yield:

$$\left(\frac{\partial W}{\partial X}\right)^2 + \left(\frac{\partial W}{\partial Y}\right)^2 + \left(\frac{\partial W}{\partial Z}\right)^2 = 1 \quad (4)$$

Which is known as the Eikonal equation, the fundamental equation of ray acoustics. A solution of this equation will be a good approximation to the wave equation.

2.2 Sound Pressure Levels Using Geometric Acoustics

Previous work done in sound pressure level predictions has employed a technique known as the image source method. This method consists of considering an infinite array of imaginary sources to exist, each radiating to the receiver under study. Each imaginary source yields a propagation path equivalent to one of the multiple reflection ray paths existing within the actual room.

Bolt, Doak and Westervelt⁶ used the image technique to study reverberation time and mean free path. In their study they contended that many sounds of speech and music more nearly resemble pulsed wave trains than the abruptly terminated continuous sounds commonly used in reverberation time measurement. They examined analytically the response of a room to a short pulse. The walls were replaced by an array of pulsed image sources which were then considered statistically. Experiments in a hard-walled rectangular room were used to compare with calculated values. Several discrete reflections were measured and

correlated with calculations. The pulses merged into a more or less continuous background sound pressure level after a time. This level was calculated and confirmed experimentally.

Krokstad, Strom and Sorsdal⁷ studied the space and time distribution of reflections in concert halls. Their calculation procedure used a mathematical model of a hall which was excited by a sound pulse emitted from a fixed point source. Energy was represented by rays equally distributed over the whole or over a selected part of a solid angle. The life history of each ray was calculated, assuming geometrical reflection at all surfaces, until the ray struck the audience area where it was assumed the ray was totally absorbed. The points of reflection for all emitted rays and the time delay of impingement relative to the direct sound were calculated. The direction of the rays was computed from equations derived by Stenseng⁸.

Stenseng assumed a uniform distribution of rays emanating from the center of a spherical source. With the location of each ray known its equation in polar coordinates was derived. Reflection points were calculated by combining the ray's equation and the equation for a surface. Each ray was repeatedly reflected until it was absorbed or became longer than a given maximum. In their discussion they stated that some limitations existed with this approach. The most severe of these was the limited frequency range over which results were valid. This was because in ray acoustics the wave lengths corresponding to the lowest frequency of the sound need to be small compared to the

linear dimensions of the room and its surfaces. For large rooms accuracy may be expected to be good for those frequencies above 100 hertz.

The image technique was used by Gibbs and Jones⁴ to predict variations in sound pressure levels in a rectangular room possessing various absorption configurations. Patches of acoustic absorption material were centrally located on each wall. Absorption values in the calculations were then chosen according to the location of the reflection points. Experiments were run in a rectangular room with a single patch and with six patches. The results were compared with predicted sound pressure levels using both the classical room acoustics and image techniques. Results showed that the image technique more closely approximated the experimental results than did predictions using the classical expression. The predictions from the image approach gave significantly better agreement with measured results when the absorption was distributed between all six surfaces of the room. In the conclusions it was noted that this improved agreement was gained at the expense of computing time.

An important aspect, for this investigation, of the work of Gibbs and Jones was the investigation of the effect of the angle of incidence on the absorption coefficient. They determined the sound pressure level distribution assuming a random incidence absorption coefficient, and also assuming that the absorption coefficient varied with the angle of incidence. The difference between the two distributions was said to be too slight to justify the additional computing time. But they also

noted that it was difficult to assume that the absorption is a function of the angle of incidence because of the lack of reliable information as to the nature of the angular variation of the absorption coefficient.

Lester³ used the image technique to predict sound pressure levels in rectangular and "L-shaped" rooms. Comparisons were made with the experimental works of Lowe⁹, Beranek² and with measurements made in a scale model of a large, low-ceilinged room. The results of Lester's predictions showed good agreement with the measured sound levels. Initially Lester considered using the absorption as a function of incidence angle but all of his results were computed using a random absorption coefficient.

Barron¹⁰ used the image method to derive expressions to predict the average number of reflections occurring after the emission of a signal from a source. The decay with time of the reverberant field strength was also computed as a function of the distance between source and receiver. His image model predicted a non-exponential decay of the case of a highly absorbent floor. If highly reflective surfaces existed in a room, the model predicted no more than the generally assumed exponential decay as the distance between the source and receiver increased.

3.0 DEVELOPMENT OF RAY TRACING EQUATIONS

3.1 Preliminary Investigation

Several methods were considered for calculating the reflection points of the sound rays on the surfaces within a room. The image method, along the lines of Lester's² thesis, was studied. This was an acceptable method if the walls of the room were like those shown in Figure 2a. However, the image technique does not easily handle slanted surfaces as illustrated in Figure 2b. Computer time was somewhat lengthy in this approach which placed a second limitation on its use.

Use of the direction cosines of the ray paths was considered next. Figure 3 shows a typical ray path involving a single reflection. Snell's law tells us that the angle of incidence of a sound ray equals its angle of reflection. Expressions may be written for the direction cosines of the incident and reflected rays. By manipulating these expressions the reflection point may be determined. This method proved cumbersome, and unique solutions for the path from a given source to a given receiver were indeterminate when more than one reflection from a room boundary was involved. Iteration was considered, but the computer time would have been excessive in comparison with equations which could be used to solve for the reflection points directly.

The method decided upon was based on the use of equations which could solve for the reflection points directly. Different equations were used depending upon whether the reflection locations were to be determined for one, two, or "N" number of reflections. Because no

(a) Room Acceptable to Image Technique.

(b) Room not easily handled by image technique.

Figure 2. Room Configurations and their Acceptability to Image
Source Technique.

Figure 3. Typical ray path involving a single reflection.

iteration was necessary computer time was reduced. The next section discusses the development of these ray tracing equations and subsequent application of these equations to a computer program.

3.2 Ray Tracing Equations

The final form of the ray tracing equations was determined from trigonometric and geometric considerations. Snell's law was used to determine the coordinates of all reflection points. This procedure was applied, for example, to surfaces like those shown in Figures 2a and 2b. The surfaces in Figure 2a will be discussed first.

3.2.1 Rooms with No Slanted Surfaces

Consider the three views of the room shown in Figure 3. A sound ray is shown undergoing one reflection from the back wall ($Y_2 = B$). A smooth surface is assumed.

One knows from Snell's law that the angle of incidence (ϕ_i) equals the angle of reflection (ϕ_r).

$$\phi_i = \phi_r \quad (5)$$

When a reflection occurs the incident and reflected sound rays define a plane which must be perpendicular to the wall at which the reflection occurs. With this information the coordinates of the reflection point can be determined.

For the ray shown in Figure 3 the reflection occurs on the wall which lies in the plane $Y = B$. Since the reflection point is on the wall, it is evident that the Y coordinate is also equal to B. It is only necessary to solve for the X and Z coordinates of the reflection point.

The solution proceeds as follows:

Snell's law (Equation 5) may be re-written:

$$\tan(\theta_i) = -\tan(\theta_r) \quad (5a)$$

$$\frac{Y_2 - Y_1}{X_2 - X_1} = \frac{-(Y_3 - Y_2)}{X_3 - X_2} \quad (5b)$$

There is only one unknown, X_2 , in Equation (5b). This can be solved for:

$$X_2 = \frac{X_1(Y_2 - Y_3) + X_3(Y_2 - Y_1)}{2Y_2 - Y_1 - Y_3} \quad (6)$$

The remaining unknown, Z_2 , may be determined in one of two ways. Since X_2 is known its value may be used to calculate Z_2 . The reflection, when viewed in the X-Z plane in Figure 3, is a straight line because the source, receiver and reflection point all lie in the same plane. Therefore, the slope of ray segment 1-2 is the same as the entire ray 1-3, i.e.:

$$\frac{Z_3 - Z_1}{X_3 - X_1} = \frac{Z_2 - Z_1}{X_2 - X_1} \quad (7)$$

Which, when solved for Z_2 yields:

$$Z_2 = \frac{Z_1(X_3 - X_2) + Z_3(X_2 - X_1)}{X_3 - X_1} \quad (8)$$

If the source and receiver are both the same distance from the Y-Z plane then X_3 equals X_1 and Equation (8) becomes indeterminate. To avoid this possibility Snell's law may be applied to the view of

the ray path in the Y-Z plane:

$$\frac{Z_3 - Z_2}{Y_3 - Y_2} = \frac{-(Z_2 - Z_1)}{Y_2 - Y_1} \quad (9)$$

This Equation, when rearranged may be solved for Z_2 :

$$Z_2 = \frac{Z_1(Y_3 - Y_2) + Z_3(Y_1 - Y_2)}{Y_1 + Y_3 - 2Y_2} \quad (10)$$

The denominator in this equation cannot be zero because that would mean that the source and receiver lay in the same plane as the wall from which the reflection occurs. (i.e., $Y_1 = Y_3 = Y_2 = B$).

In this manner all three coordinates of the reflection point have been determined from equations (6) and (10) and the known plane of reflection.

This procedure may be repeated for one reflection for each of the remaining walls shown in Figure 3.

Two possible reflection combinations may occur when a sound ray undergoes two reflections. Ray path 1-2-3-4 in Figure 4 is an example of reflections from parallel walls. Ray path 1-2'-3'-4 in Figure 4 is an example of reflections from perpendicular walls.

The case of reflections from parallel walls will be considered first. The known coordinates for the reflection points 2 and 3 of the ray path 1-2-3-4 are $Y_2 = B$ and $Y_3 = 0$. Snell's law may be applied to the sound ray path in the X-Y and Y-Z planes to obtain the necessary equations to derive the unique reflection points 2 and 3. Since the

Figure 4. Examples of two reflections from parallel walls (1-2-3-4) and from perpendicular walls (1-2'-3'-4).

reflections occur on parallel surfaces the view in the X-Z plane shows all points lying in the same plane. This fact is used to obtain the equations to determine points 1 and 2. These equations may be rearranged to determine the necessary values, although not as simply as in the case of one reflection. For example, in the X-Y plane we can show that:

$$\frac{Y_1 - Y_2}{X_1 - X_2} = \frac{Y_3 - Y_4}{X_3 - X_4} \quad (11)$$

In that same view we can also see that

$$\frac{Y_2 - Y_3}{X_2 - X_3} = \frac{-(Y_1 - Y_2)}{X_1 - X_2} \quad (12)$$

Equations (10) and (11) may be rearranged to eliminate one of the two unknowns, X_3 . The resulting equation when solved for X_2 is:

$$X_2 = \frac{X_1(2Y_3 - Y_4 - Y_2) + X_4(Y_1 - Y_2)}{2Y_3 - 2Y_2 + Y_1 - Y_4} \quad (13)$$

Now that X_2 has been determined X_3 may be calculated from Equation (11).

$$X_3 = \frac{X_1(Y_2 - Y_3) + X_2(Y_3 - 2Y_2 + Y_1)}{Y_1 - Y_2} \quad (14)$$

The coordinates Z_2 and Z_3 may be determined by the application of Snells in the Y-Z plane in a similar manner. These equations are:

$$Z_2 = \frac{Z_1(2Y_3 - Y_4 - Y_2) + Z_4(Y_1 - Y_2)}{2(Y_3 - Y_2) + Y_1 - Y_4} \quad (15)$$

$$Z_3 = \frac{Z_2(Y_3 - Y_4) + Z_4(Y_3 - Y_2)}{2Y_3 - Y_2 - Y_4} \quad (16)$$

To use these equations for walls parallel in the X-Y or Y-Z coordinate planes it is only necessary to interchange the coordinates. For example, suppose the two parallel walls in question were perpendicular to the X-axis. The previously derived equations (13, 14, 15, 16) may be used by interchanging each X for a Y and vice versa. This produces the same result as rotating the X-Y plane 90° so that the X and Y coordinates exchange locations.

The situation for the case of successive reflections from perpendicular walls is more complicated. A typical ray path of this kind is ray path 1-2'-3'-4 shown in Figure 4. The known coordinates (in addition to the source and receiver) are $Y_2' = B$ and $Z_3' = C$. Once again equations can be derived by applying the two aforementioned criteria - Snell's law and the fact that any three consecutive points define a plane. It is important to note that the source, the receiver, and the two intermediate reflection points do not lie in the same plane, as was the case for parallel walls. Only any three successive points lay in the same plane, e.g., points 1-2'-3' and 2'-3'-4; for the ray in question.

The general form of the ray tracing equations for two reflections involving perpendicular walls can be found by the procedure indicated above, to be:

$$X_2' = \frac{X_1(Y_2' - Y_4) + X_4(Y_2' - Y_1)}{2Y_2' - Y_1 - Y_4} \quad (17)$$

$$X_3' = \frac{X_1(Z_3' - Z_4) + X_4(Z_3' - Z_1)}{2Z_3' - Z_1 - Z_4} \quad (18)$$

$$Z_2' = \frac{Z_1(Y_4 - Y_2') + Z_3'(2Y_1 - 2Y_2') + Z_4(Y_2' - Y_1)}{Y_1 + Y_4 - 2Y_2'} \quad (19)$$

$$Y_3' = \frac{Y_1(Z_4 - Z_3') - Y_2'(Z_4 - Z_3') + Y_4(Z_2' - Z_1)}{Z_2' - Z_1} \quad (20)$$

These equations apply to successive reflections from a wall lying in the plane $Y = \text{constant}$ and a wall lying in the plane $Z = \text{constant}$. For other combinations involving perpendicular walls the equations are of the same form as equations 16-19, but, as in the case of parallel walls, the X , Y and Z values are interchanged to solve for all possible two reflection combinations.

While there are only two types of reflection combinations when two reflections occur, the situation changes when three successive reflections are considered. There are five general reflection combinations. These are shown in Figure 5. Figure 5a consists of two successive reflections from parallel walls and a third reflection from a wall perpendicular to the two parallel walls. Figure 5b is a reflection combination involving the reverse of Figure 5a, i.e., the first two successive reflections (1-2-3) are from perpendicular walls and the last two successive reflections (3-4-5) are from parallel walls. Figure 5c shows three successive reflections from parallel walls.

As in the case of two successive reflections, one coordinate location of each of the three reflection points are known. As an example, the known coordinates in Figure 5a are:

Figure 5. Five general reflection combinations for three reflections.

$$Y_2 = B \quad \text{For Point 2}$$

$$Y_3 = 0. \quad \text{For Point 3}$$

$$X_4 = 0. \quad \text{For Point 4}$$

With this information the ray tracing equations for three successive reflections can be derived. The procedure parallels that for the single and double reflection cases. The complexity of the algebra is increased since there are three reflection points to determine, but the method of determining the necessary equations is the same. The general ray tracing equations for three successive reflections are found to be:

Figure 5a: (Two X-Z planes, followed by a Y-Z plane)

$$Z_4 = \frac{Z_5(X_1 - X_4) + Z_1(X_5 - X_4)}{X_1 + X_5 - 2X_4} \quad (21)$$

$$Z_2 = \frac{Z_5(Y_2 - Y_1) + Z_1(Y_5 + Y_2 - 2Y_3)}{2Y_2 - 2Y_3 - Y_1 + Y_5} \quad (22)$$

$$Z_3 = \frac{Z_1(Y_2 - Y_3) - Z_2(2Y_2 - Y_1 - Y_3)}{Y_1 - Y_2} \quad (23)$$

$$X_2 = \frac{X_1(Y_5 - 2Y_3 + Y_2) + 2X_4(Y_2 - Y_1) + X_5(Y_1 - Y_2)}{(2Y_2 - 2Y_3 - Y_1 + Y_5)} \quad (24)$$

$$X_4 = \frac{Y_1(X_4 - X_5) + Y_2(X_5 - X_4) + Y_5(X_2 - X_1)}{(X_2 - X_1)} \quad (25)$$

$$X_3 = \frac{X_1(Y_3 - Y_2) + X_2(2Y_2 - Y_3 - Y_1)}{(Y_2 - Y_1)} \quad (26)$$

Figure 5b: (X-Z plane followed by two X-Y planes)

$$Z_2 = \frac{Z_1(X_2 - X_5) + Z_5(X_2 - X_1)}{2X_2 - X_1 - X_5} \quad (27)$$

$$Z_3 = \frac{Z_1(2Y_4 - Y_3 - Y_5) - Z_5(Y_3 - Y_1)}{2Y_4 - 2Y_3 - Y_5 + Y_1} \quad (28)$$

$$Z_4 = \frac{Z_1(Y_4 - Y_3) - Z_3(Y_4 - 2Y_3 + Y_1)}{Y_3 - Y_1} \quad (29)$$

$$X_4 = \frac{X_2(2Y_4 - 2Y_5) + X_5(Y_1 + Y_4 - 2Y_3 + X_1(Y_5 - Y_4))}{Y_1 - 2Y_3 + 2Y_4} \quad (30)$$

$$X_3 = \frac{X_4(2Y_4 - Y_3 - Y_5) + X_5(Y_3 - Y_4)}{Y_4 - Y_5} \quad (31)$$

$$Y_2 = \frac{Y_3(X_1 - X_2) - Y_4(X_1 - X_2) + Y_1(X_4 - X_3)}{X_4 - X_3} \quad (32)$$

Figure 5c: (Reflection from two parallel Y-Z planes)

$$Z_2 = \frac{Z_1(2Y_3 - 2Y_4 + Y_5 - Y_2) - Z_5(Y_2 - Y_1)}{2Y_3 - 2Y_2 - 2Y_4 + Y_5 + Y_1} \quad (33)$$

$$Z_4 = \frac{Z_5(Y_2 - Y_1) + Z_2(Y_5 - Y_4) - Z_1(Y_5 - Y_4)}{(Y_2 - Y_1)} \quad (34)$$

$$X_4 = \frac{X_5(2Y_3 - 2Y_2 - Y_4 + Y_1) + X_1(Y_5 - Y_4)}{Y_5 - 2Y_4 + 2Y_3 - 2Y_2 + Y_1} \quad (35)$$

$$X_2 = \frac{X_1(Y_5 - Y_4) + X_5(Y_1 - Y_2) + X_4(Y_2 - Y_1)}{Y_5 - Y_4} \quad (36)$$

$$X_3 = \frac{X_4(Y_1 - Y_2) + X_2(Y_4 - Y_3) + X_1(Y_3 - Y_4)}{Y_1 - Y_2} \quad (37)$$

$$Z_3 = \frac{Z_4(Y_2 - Y_1) + Z_2(Y_3 - Y_4) - Z_1(Y_3 - Y_4)}{Y_2 - Y_1} \quad (38)$$

Figure 5d: (Reflection from Y-Z, X-Z, Y-Z planes)

$$Z_3 = \frac{Z_5(Y_1 - Y_3) + Z_1(Y_5 - Y_3)}{Y_5 - 2Y_3 + Y_1} \quad (39)$$

$$Z_2 = \frac{Z_1(2X_4 - X_2 - X_5) + Z_5(X_1 - X_2)}{2X_4 - 2X_2 - X_5 + X_1} \quad (40)$$

$$Z_4 = \frac{Z_1(X_4 - X_2) - Z_2(X_4 + X_1 - 2X_2)}{X_2 - X} \quad (41)$$

$$X_4 = \frac{Y_3(2X_5 - 2X_4) + Y_1(X_4 - X_5) + Y_5(2X_2 - X_1 - X_4)}{2X_2 - 2X_4 + X_5 - X_1} \quad (42)$$

$$X_3 = \frac{X_5(Y_3 - Y_4) + X_4(2Y_4 - Y_3 - Y_5)}{Y_4 - Y_5} \quad (43)$$

$$Y_2 = \frac{Y_5(X_1 - X_2) + Y_4(X_2 - X_1) + Y_1(X_5 - X_4)}{X_5 - X_4} \quad (44)$$

Figure 5e: (Reflection from Y-Z, X-Z, X-Y planes)

$$Y_4 = \frac{Y_1(Z_5 - Z_4) + (2Z_4 - 2Z_5) + Y_5(Z_4 - Z_1)}{2Z_4 - Z_5 - Z_1} \quad (45)$$

$$Z_3 = \frac{Z_5(Y_1 - Y_3) - Z_4(Y_1 - Y_3) + Z_1(Y_4 - Y_5)}{(Y_4 - Y_5)} \quad (46)$$

$$Y_2 = \frac{Y_3(2X_1 - 2X_2) + Y_1(X_5 - X_2) + Y_5(X_2 - X_1)}{X_1 + X_5 - 2X_2} \quad (47)$$

$$X_3 = \frac{X_1(Y_3 - Y_5) - X_2(Y_3 - Y_5) + X_5(Y_1 - Y_2)}{(Y_1 - Y_2)} \quad (48)$$

$$X_4 = \frac{X_1(Y_4 - Y_5) - X_2(Y_4 - Y_5) + X_5(Y_1 - Y_2)}{(Y_1 - Y_2)} \quad (49)$$

$$Z_2 = \frac{Z_5(X_1 - X_2) - Z_4(X_1 - X_2) - Z_1(X_5 - X_4)}{(X_4 - X_5)} \quad (50)$$

The five ray tracing equation sets derived can be used to determine all of the possible three reflection combinations. For example, in Figure 5a reflections are shown occurring on the $Y_2 = B$ wall, $Y_3 = 0$. wall and $X_4 = 0$ wall. To use the derived equations, equations (21)-(26), for a reflection from the $X_2 = 0$., $X_3 = A$ and $Y_4 = 0$. walls, the Y values are substituted in the equation for the X values and vice versa. By switching the X, Y and Z's in the general equations all the possible combinations are calculated.

3.2.2 Determination of Minimum Number of Reflections

for Sound Pressure Level Calculations

The same procedure can be applied to four or more reflections. Ray tracing equations for four reflections would require equations to solve for eight unknown coordinates. The additional reflection would also add additional reflection sequence combinations; each with four unknown coordinate pairs. In view of the increased complexity of solving all of the necessary ray tracing equations for four or more reflections calculations were limited to those sound rays undergoing three or fewer reflections.

Although four or more reflections would require much additional analysis, the decision to consider only three reflections was not

made for this reason only. Once the ray paths were determined they were to be used to calculate sound pressure levels. In section 1.0 sound intensity was said to be inversely proportional to the ray path length squared:

$$I_{\text{Sound}} \propto \frac{1}{R^2} \quad (51)$$

where:

I_{Sound} = Sound Intensity

R = Distance along ray path

Unless the room in question is extremely narrow in one dimension most three reflection paths will be longer than the two reflection paths. Paths involving four reflections would generally be longer than those involving three reflections. Therefore, it can be seen from equation (51) that the acoustic energy arriving along ray paths involving more than three reflections would be appreciably lower, in general than that arriving along a shorter path.

Another factor in determining the necessary number of reflections is the absorption property of the walls. When a sound ray strikes a surface its intensity is reduced proportional to the absorptive characteristics of the surface. Most surfaces will absorb a portion of the intensity of the impinging ray so the intensity also decreases as the number of reflections increase.

Finally, it is worth noting that Lester³ used a maximum of three reflections to predict sound pressure levels. The good agreement of his results with those he measured lends credence to this procedure.

3.2.3 Ray Tracing Equations for Reflections Involving Slanted Surfaces

A rectangular room is shown in Figure 6 that contains a "slanted" surface, i.e., one that is perpendicular to only one of the coordinate planes, in this example, the X-Y plane. Slanted surfaces can occur in factory spaces as barriers and slanted walls so it was necessary to derive the necessary ray tracing equations for this application.

In the previous section the ray tracing equations were developed with the reflecting surfaces perpendicular to one of the coordinate axes. To use the same type of analysis for slanted surfaces it was necessary to use a rotated set of coordinate axes like the primed coordinates shown in Figure 6. By rotating a set of axes until the X' axis was parallel to the reflecting surface in question the necessary ray tracing equations could be developed.

The first step in this analysis was to determine a relationship between the initial coordinate axes and the rotated axes. Figure 7 shows an enlarged view of the X-Y plane and the true size and edge views of the slanted plane in Figure 6. Two axes (X' - Y') were constructed parallel and perpendicular to the slanted surface with the source as the zero location of the primed axes. The angle $\emptyset$ is the angle formed by the X and X' coordinate axes. Since point 2 must lie on the slanted plane the following expressions were derived:

Figure 6. Rectangular room containing slanted barrier.

$$X_2 = X_1 - Y_2' \sin \emptyset + X_2' \cos \emptyset \quad (52)$$

$$Y_2 = Y_1 + X_2' \sin \emptyset + Y_2' \cos \emptyset \quad (53)$$

Section A in Figure 7 shows the distances represented by each term in these equations.

Since the calculations are to be made in the primed system equations (52) and (53) are combined and rearranged to solve for X_2' and Y_2' in terms of the unprimed coordinate axes:

$$Y_2' = \sin \emptyset (X_1 - X_2) + \cos \emptyset (Y_2 - Y_1) \quad (54)$$

$$X_2' = \sin \emptyset (Y_2 - Y_1) + \cos \emptyset (X_2 - X_1) \quad (55)$$

The next problem is to determine which, if any, of the coordinates in the primed system are known. Let the end point of the slanted surface with the minimum X dimension be point (P_1, Q_1) as shown in Figure 7. The distance from point (P_1, Q_1) to point 2 in the figure is defined as "R". One may write X_2 and Y_2 as a function of R, $\emptyset$, P_1 and Q_1 ;

$$X_2 = P_1 + R \cos \emptyset \quad (56a)$$

$$Y_2 = Q_1 + R \sin \emptyset \quad (56b)$$

Upon substitution of X_2 and Y_2 into equation (54):

$$Y_2' = \sin \emptyset (X_1 - P_1 - R \cos \emptyset) + \cos \emptyset (Q_1 + R \sin \emptyset - Y_1) \quad (57)$$

Equation (57) may be reduced to:

$$Y_2' = (X_1 - P_1) \sin \emptyset + (Q_1 - Y_1) \cos \emptyset \quad (58)$$

All of the quantities to the right of the equal sign in equation (58) are known, so Y_2' may be determined.

With Y_2' known Snells law may be applied in the primed coordinate system to solve for X_2' using:

$$\frac{Y_3' - Y_2'}{X_3' - X_2'} = \frac{-(X_2' - X_1')}{Y_2' - Y_1'} \quad (59)$$

Since the coordinate axes of the primed system have their origin at (X_1, Y_1) , X_1' and Y_1' are both zero. Upon rearrangement of equation (59) to solve for X_2' :

$$X_2' = \frac{Y_2' - X_3'}{Y_3' - 2Y_2'} \quad (60)$$

Once X_2' and Y_2' are determined, X_2 and Y_2 may then be found using equations (52) and (53).

The only remaining unknown for a single reflection from a slanted surface is Z_2 . Z_2 can be found by applying Snell's law to the view in the $X' - Z$ axis:

$$Z_2 = \frac{Z_3 X_2' + Z_1 (X_3' - X_2')}{X_3} \quad (61)$$

Thus, all reflection points have been found for the case of a single reflection from a slanted surface.

The analysis becomes more involved when two reflections from slanted surfaces are considered. The possible cases involving two reflections would include reflections from:

1. A slanted surface to a "regular" surface;
2. A regular surface to a slanted surface;
3. Two successive slanted surfaces.

For cases 1 and 2 the equations are derived using the rotated axis system (primed) used in the previous section and the unrotated axes used in the "regular" room analysis. The equations for a typical "case 1" ray will be derived.

One of the possible case 1 combinations is a reflection from a slanted wall to an unslanted wall. This case is shown in Figure 8.

The known values for this case are the coordinates of the source and receiver (points 1 and 4) and Y_2' and X_3 . Y_2' can be determined in the same manner as in the single reflection case, i.e., using equation (17) which is a function of X_1 , Y_1 , P_1 , Q_1 and $\emptyset$, all known quantities. X_3 is known because it lies in the plane $X=A$.

The following equations can be derived by applying Snell's law to various views in the primed and unprimed coordinate systems. From the X-Y coordinate plane:

$$\frac{X_4 - X_3}{Y_4 - Y_3} = \frac{-(X_3 - X_2)}{Y_3 - Y_2} \quad (62)$$

From the X-Z coordinate plane:

$$\frac{Z_2 - Z_3}{X_2 - X_3} = \frac{-(Z_3 - Z_4)}{X_3 - X_4} \quad (63)$$

From the $Y' - Z$ coordinate plane:

$$\frac{Y_3' - Y_2'}{Z_3 - Z_2} = \frac{-(Y_2')}{Z_2 - Z_1} \quad (64)$$

and from the $X' - Y'$ coordinate plane:

$$\frac{X_3' - X_2'}{Y_3' - Y_2'} = \frac{-(X_2')}{Y_2'} \quad (65)$$

Figure 8. Reflection from a slanted wall to an unslanted wall.

The primed quantities are in the rotated coordinate system. Equations (20) through (23) may be used to solve for X_2 in the following manner.

Y_3 may be solved for in equation (62):

$$Y_3 = \frac{Y_4 X_3 + Y_2 X_4 - Y_2 X_3 + Y_4 X_2}{X_2 + X_4 - 2X_3} \quad (66)$$

A relationship between X and Y on the slanted wall can be derived from its edge view in Figure 8. The equation is of the form:

$$Y = M X + b \quad (67)$$

where:

Y : Y coordinate of a point on the line (plane)

X : X coordinate of a point on the line

M : Slope of line (plane)

b : Y intercept of line (plane)

This equation allows Y_2 to be expressed as a function of X_2 in (67a):

$$Y_2 = M X_2 + b$$

This equation, when substituted into equation 66, yields

$$Y_3 = \frac{a_1 X_2 + a_2}{X_2 + a_3} \quad (68)$$

where:

$$a_1 = M(X_4 - X_3) + Y_4 \quad (68a)$$

$$a_2 = b(X_4 - X_3) - Y_4 X_3 \quad (68b)$$

$$a_3 = X_4 - 2X_3 \quad (68c)$$

To eliminate Y_3 another equation for Y_3 in terms of X_2 must be

determined. From the transformation equations for the rotated axis equations (54) and (55) we know:

$$X_2' = \sin \emptyset (Y_2 - Y_1) + \cos \emptyset (X_2 - X_1) \quad (69a)$$

$$X_3' = \sin \emptyset (Y_3 - Y_1) + \cos \emptyset (X_3 - X_1) \quad (69b)$$

$$Y_3' = \sin \emptyset (X_1 - X_3) + \cos \emptyset (Y_3 - Y_1) \quad (69c)$$

These equations can be substituted into equation (65). With the additional substitution of equation (67a) an expression for Y_3 in terms of X_2 is determined.

$$Y_3 = \frac{E_1 X_2 + E_2}{E_3 X_2 + E_4} \quad (70)$$

where:

$$E_1 = 2B_1 Y_2' + B_1 C_1 X_3 - B_1 C_3 \quad (70a)$$

$$E_2 = B_2 (Y_2' - C_1 X_3 - C_3) + Y_2' (B_2 + C_2 X_3 - D_1) \quad (70b)$$

$$E_3 = C_2 B_1 \quad (70c)$$

$$E_4 = C_2 B_2 - C_1 Y_2' \quad (70d)$$

$$B_1 = -M \sin \emptyset - \cos \emptyset \quad (70e)$$

$$B_2 = \sin \emptyset (Y_1 - B) + X_1 \cos \emptyset \quad (70f)$$

$$C_1 = \sin \emptyset \quad (70g)$$

$$C_2 = -\cos \emptyset \quad (70h)$$

$$C_3 = Y_1 \cos \emptyset - X_1 \sin \emptyset \quad (70i)$$

Equations (68) and (70) can now be equated to solve for X_2 :

$$X_2 = \frac{-F_2 \pm \sqrt{F_2^2 - 4F_1 F_3}}{2F_1} \quad (71)$$

where:

$$F_1 = E_1 - E_3 A_1 \quad (71a)$$

$$F_2 = E_2 + A_3 E_1 - E_3 A_2 - E_4 A_1 \quad (71b)$$

$$F_3 = E_2 A_3 - E_4 A_2 \quad (71c)$$

Since X_3 is known quantity X_2 can be determined from equation (29).

Equations (70) and (67) can then be used to determine Y_3 and Y_2 respectively. The only remaining unknowns are Z_2 and Z_3 .

Equation (63) can be rearranged to solve for Z_2 :

$$Z_2 = \frac{G_1 Z_3 + G_2}{G_3} \quad (72)$$

where:

$$G_1 = 2X_3 - X_2 - X_4 \quad (72a)$$

$$G_2 = Z_4 (X_2 - X_3) \quad (72b)$$

$$G_3 = X_3 - X_4 \quad (72c)$$

To eliminate Z_2 from equation (72) another expression must be found for Z_2 in terms of Z_3 . Equation (65) contains the unknown quantities Z_2 and Z_3 . Y_2' is known from equation (58), and Y_3' can be determined by equation (69c). Equation (65) can be rearranged to solve for Z_2 in terms of Z_3 :

$$Z_2 = H_1 - Y_2' Z_3 \quad (73)$$

where:

$$H_1 = Z_1 (Y_3' - Y_2') \quad (73a)$$

$$H_2 = Y_3' - 2Y_2' \quad (73b)$$

Equations (72) and (73) can now be equated to solve for Z_3 :

$$Z_3 = \frac{G_3 H_1 - H_2 G_2}{G_1 H_2 - G_3 Y_2} \quad (74)$$

Either equation (72) or (73) can now be used to solve for Z_3 .

The dimensions for the other slanted reflection combinations follow the same pattern. The equations are as follows:

Case 2 - slanted wall to regular wall perpendicular to Y-axis:

$$Y_2' = (\cos \emptyset) (Y_1 - Q_1) - (\sin \emptyset) (X_1 - P_1) \quad (75)$$

$$A_1 = Y_4 - Y_3 + M X_4 \quad (76)$$

$$A_2 = X_4 (b - Y_3) \quad (77)$$

$$A_3 = b - 2Y_3 + Y_4 \quad (78)$$

$$B_1 = -M \sin \emptyset - \cos \emptyset \quad (79)$$

$$B_2 = -(\sin \emptyset) (b - Y_1) + X_1 \cos \emptyset \quad (80)$$

$$C_1 = \sin \emptyset \quad (81)$$

$$C_2 = -\cos \emptyset \quad (82)$$

$$C_3 = Y_1 \cos \emptyset - X_1 \sin \emptyset \quad (83)$$

$$D_1 = Y_1 \sin \emptyset + X_1 \cos \emptyset \quad (84)$$

$$E_1 = 2B_1 Y_2' - B_1 C_2 Y_3 - B_1 C_3 \quad (85)$$

$$E_2 = B_2 (Y_2' - C_2 Y_3 - C_3) + Y_2' (C_1 Y_3 - D_1 B_2) \quad (86)$$

$$E_3 = C_1 B_1 \quad (87)$$

$$E_4 = C_1 B_2 + C_2 Y_2' \quad (88)$$

$$F_1 = M E_1 - E_3 A_1 \quad (89)$$

$$F_2 = A_3 E_1 + M E_2 - E_3 A_2 \quad (90)$$

$$F_3 = E_2 A_3 - E_4 A_2 \quad (91)$$

$$X_2 = \frac{-F_2 \pm \sqrt{F_2^2 - 4F_1 F_3}}{2F_1} \quad (92)$$

$$Y_2 = M X_2 + b \quad (93)$$

$$X_3 = \frac{E_1 X_2 + E_2}{E_3 X_2 + E_4} \quad (94)$$

$$G_1 = X_2 - X_4 \quad (95)$$

$$G_2 = Z_4 (X_3 - X_2) \quad (96)$$

$$G_3 = X_3 - X_4 \quad (97)$$

$$Y_3' = C_1 X_3 + C_2 Y_3 + C_3 \quad (98)$$

$$H_1 = Z_1 (Y_3' - Y_2') \quad (99)$$

$$H_2 = Y_3' - 2Y_2' \quad (100)$$

$$Z_3 = \frac{G_3 H_1 - H_2 G_2}{G_1 H_2 + G_3 Y_2'} \quad (101)$$

$$Z_2 = \frac{H_1 - Y_2' Z_3}{H_2} \quad (102)$$

Case 3 - slanted plane to ceiling or floor surfaces:

$$X_4' = -(\sin \emptyset) (Y_4 - Y_1) - (\cos \emptyset) (X_4 - X_1) \quad (103)$$

$$Y_4' = -(\cos \emptyset) (Y_4 - Y_1) - (\sin \emptyset) (X_1 - X_4) \quad (104)$$

$$Y_2' = -(\cos \emptyset) (Q_1 - Y_1) - (\sin \emptyset) (X_1 - P_1) \quad (105)$$

$$X_2' = \frac{Y_2' X_4'}{2Y_2' - Y_4'} \quad (106)$$

$$X_2 = X_1 - X_2' \cos \emptyset + Y_2' \sin \emptyset \quad (107)$$

$$Y_2 = Y_1 - X_2' \sin \emptyset - Y_2' \cos \emptyset \quad (108)$$

$$A_1 = - \sin \emptyset \quad (109)$$

$$A_2 = - \cos \emptyset \quad (110)$$

$$A_3 = Y_1 \sin \emptyset + X_1 \cos \emptyset \quad (111)$$

$$B_1 = Y_4 - Y_2 \quad (112)$$

$$B_2 = X_4 Y_2 - X_2 Y_4 \quad (113)$$

$$B_3 = X_4 - X_2 \quad (114)$$

$$C_1 = A_2 Z_1 \quad (115)$$

$$C_2 = X_2' Z_3 + Z_1 (A_3 - X_2') \quad (116)$$

$$C_3 = - A_1 Z_1 \quad (117)$$

$$D_1 = B_3 C_1 - C_3 B_1 \quad (118)$$

$$D_2 = B_3 C_2 - B_2 C_3 \quad (119)$$

$$D_3 = A_1 B_1 + B_3 A_2 \quad (120)$$

$$D_4 = A_1 B_2 + B_3 A_3 \quad (121)$$

$$E_1 = 2Z_3 - Z_4 \quad (122)$$

$$E_2 = - Z_3 (X_2 + X_4 + Z_4 X_2) \quad (123)$$

$$F_1 = D_1 - E_1 D_3 \quad (124)$$

$$F_2 = D_2 - D_1 X_4 - E_1 D_4 - E_2 D_3 \quad (125)$$

$$F_3 = -D_2 X_4 - E_2 D_4 \quad (126)$$

$$X_3 = \frac{-F_2 \pm \sqrt{F_2^2 - 4F_1 F_3}}{2F_1} \quad (127)$$

$$Y_3 = \frac{B_1 X_3 + B_2}{B_3} \quad (128)$$

$$Z_2 = \frac{X_3 D_1 + D_2}{X_3 D_3 + D_4} \quad (129)$$

Case 4 - reflections from two slanted planes perpendicular to
same plane:

$$Y_2' = -\cos \theta_1 (Q_1 - Y_1) - \sin \theta_1 (X_1 - P_1) \quad (130)$$

$$Y_3'' = -\cos \theta_2 (Q_4 - Y_4) - \sin \theta_2 (X_4 - P_4) \quad (131)$$

$$A_1 = -M_2 \sin \theta_1 - \cos \theta_1 \quad (132)$$

$$A_2 = -\sin \theta_1 (M_2 - Y_1) + X_1 \cos \theta_1 \quad (133)$$

$$B_1 = -M_2 \cos \theta_1 + \sin \theta_1 \quad (134)$$

$$B_2 = -\cos \theta_1 (M_2 - Y_1 - X_1 \sin \theta_1) \quad (135)$$

$$C_1 = -M_1 \sin \theta_1 - \cos \theta_1 \quad (136)$$

$$C_2 = -\sin \theta_1 (M_1 - Y_1) + X_1 \cos \theta_1 \quad (137)$$

$$D_1 = 2C_1 Y_2' - C_1 B_2 \quad (138)$$

$$D_2 = 2C_2 Y_2' - B_2 C_2 - Y_2' A_2 \quad (139)$$

$$D_3 = B_1 C_1 \quad (140)$$

$$D_4 = B_1 C_2 + A_1 Y_2' \quad (141)$$

$$E_1 = -M_1 \sin \theta_2 - \cos \theta_2 \quad (142)$$

$$E_2 = -(\sin \theta_2)(M_1 - Y_4 + X_4) \cos \theta_2 \quad (143)$$

$$F_1 = -M_2 \sin \theta_2 - \cos \theta_2 \quad (144)$$

$$F_2 = -(\sin \theta_2)(M_2 - Y_4) + X_4 \cos \theta_2 \quad (145)$$

$$G_1 = -M_1 \cos \theta_2 + \sin \theta_2 \quad (146)$$

$$G_2 = -(\cos \theta_2)(M_1 - Y_4) - X_4 \sin \theta_2 \quad (147)$$

$$H_1 = -F_2 G_1 - Y_3'' E_1 \quad (148)$$

$$H_2 = 2F_2 Y_3'' - F_2 G_2 - Y_3'' E_2 \quad (149)$$

$$H_3 = F_1 G_1 \quad (150)$$

$$H_4 = F_1 G_2 - 2F_1 Y_3'' \quad (151)$$

$$S_1 = H_1 D_3 - H_3 D_1 \quad (152)$$

$$S_2 = H_1 D_4 + H_2 D_3 - H_3 D_2 - H_4 D_1 \quad (153)$$

$$S_3 = H_2 D_4 - H_4 D_2 \quad (154)$$

$$X_2 = \frac{-S_2 \pm \sqrt{S_2^2 - 4S_1 S_3}}{2S_1} \quad (155)$$

$$Y_2 = M_1 X_2 + b_1 \quad (156)$$

$$X_3 = \frac{H_1 X_2 + H_2}{H_3 X_2 + H_4} \quad (157)$$

$$Y_3 = M_2 X_3 + b_2 \quad (158)$$

$$X_3' = A_1 X_3 + A_2 \quad (159)$$

$$X_2' = C_1 X_2 + C_2 \quad (160)$$

$$Y_3' = B_1 X_3 + B_2 \quad (161)$$

$$T_1 = X_3' Z_1 - X_2' Z_1 \quad (162)$$

$$X_3'' = F_1 X_3 + F_2 \quad (163)$$

$$X_2'' = E_1 X_2 + E_2 \quad (164)$$

$$T_2 = X_3'' Z_4 - X_2'' Z_4 \quad (165)$$

$$Z_3 = \frac{X_3' T_2 - T_1 X_3''}{X_2' X_3'' - X_3' X_2''} \quad (166)$$

$$Z_2 = \frac{X_2'' Z_3 + T_2}{X_3''} \quad (167)$$

In the preceding equations the initial coordinate is determined by the quadratic equation (equations (71), (92), (127), (155)). Both roots were not possible solutions so in all cases one of them was extraneous. A check is made in the program to discard the extraneous root.

The previous procedures were applied in an attempt to solve for three reflections, involving at least one slanted surface. The resulting equations contained second and third order terms and multiples of them which could not be reduced to the form used for two reflections. Because of the algebraic problems involved with these equations, they were not solved. Iteration was considered but was not attempted in this work. If a solution were found the presence of third order terms indicates that additional methods of determining the

correct root would be necessary, therefore increasing computational time. Hence, ray tracing equations involving slanted surfaces were derived for only one and two reflections.

4.0 APPLICATION OF RAY TRACING EQUATIONS TO COMPUTER PROGRAM

4.1 Calculation of Rays

Since the ray tracing equations determine the coordinate locations of the reflection points, a coordinate system must be established for each room considered. To assure all coordinate points are positive the room is located in the first quadrant.* The coordinates of the source, the receiver and all of the corners of each wall are the only input necessary for the calculation of the rays.

The coordinates of each wall corner define a plane in which the wall lies. The program determines the equation of all the planes containing a wall ($X = A$, $Y = 0$., etc.) and uses these values in the ray tracing equations to determine all the possible reflections.

4.2 Validity of Calculated Rays

For mathematical reasons the walls in a room were stored as infinite planes, with the equation of the plane known. The ray tracing equations used this information to calculate reflection points. Figure 9 shows a wall and the infinite plane which contains the wall. Because the ray tracing equations calculate reflection points based on infinite planes and not walls with finite boundaries the possibility exists that a calculated reflection point may occur outside of a wall boundary.

*A complete users guide in Appendix A details the input procedure.

Figure 9. Illustration of wall within a plane concept used in computer program.

Examples of four ray paths from a point S (source) to a point R (receiver), within a room bounded by ABCDEFGHA are shown in Figure 10. These examples show some of the rays which would be calculated because reflections are determined from infinite planes. Quite obviously it shows examples of certain ray paths and reflection points that do not occur in practice. The only valid ray path is S-2-R. The others illustrate the possible violations that may occur. Thus it is necessary to include in the program a procedure to eliminate the invalid rays.

To eliminate the invalid rays as quickly as possible each ray path was divided into segments. Then, each segment was considered individually. If a segment is determined to be invalid then the entire ray path is invalid without the need of checking the remaining segments. For example, in Figure 10 ray path S-2-R is a single reflection path which consists of two line segments S-2 and S-R.

The ray segment method is used as follows. Ray segment S-2, of ray path S-2-R, does not intersect any walls except at the desired reflection point 2. In addition, the reflection point 2 occurs within the wall boundary. Thus, segment S-2 is valid. Similar arguments apply to segment 2-R. Therefore, segment 2-R is a valid segment, and, hence, ray path S-2-R is a valid ray.

Reflection path S-2'-R is an example of an invalid ray path. Point 2' was calculated as a reflection point since wall C-D was considered infinite in extent. Segment S-2' intersects no planes within wall boundaries. Since S-2' is not a direct ray path S-2' is an invalid ray.

Figure 10. Examples of valid and invalid rays.

Reflection path S-2''-R is also invalid. Ray segment S-2'' is valid by the same reasoning as for ray segment S-2. However, segment 2''-R does not qualify as a valid segment. This segment intersects two wall planes within wall boundaries and is therefore invalid.

Reflection path S-2'''-3'''-R is an example of a ray path that fails both of the previously discussed criteria. Segment 2'''-3''' intersects wall plane C-D within the wall boundary, and reflection point 3''' does not lie within the boundaries of BC or FG, although it lies on the plane common to both of them.

To eliminate invalid ray paths such as 1-2'-3 and 1-2''-3 a subprogram was developed to check each ray path after it had been calculated. The flow chart in Figure 11 indicates the steps taken to check each segment. The subprogram divided each ray path into ray segments and compared them one at a time with all of the walls in the room. The segments were checked to see if an invalid ray had been calculated.

The subprogram called on several other subprograms to determine if the ray was valid. As discussed earlier, a program calculated the equation of each wall plane. The equation of a line containing the ray segment in question was also computed by a subprogram. With these two equations the plane and line were checked to see if they were parallel. If they were parallel it was necessary to determine if the line segment lay within the plane or whether it was a finite distance from the plane. Subprograms performed each of these operations.

Figure 11. Flow chart for sub-routine ray check.

Figure 11, Cont'd.

Figure 11, Cont'd.

Figure 11, Cont'd.

Figure 11, Cont'd.

If the line and plane were not parallel then their intersection point was determined. This point was then checked to see if it:

1. Lay in the line segment.
2. Coincided with either line segment end point.

If the intersection point was not between the line segment end points the intersection was not of concern and a new wall was checked. If condition 2 were satisfied then the intersection point was checked to see if it:

1. Lay within a wall boundary.
2. Lay outside a wall boundary.

Intuitively one might think that if the intersection point between the line and the plane lay outside a particular wall boundary an invalid ray like S-2'-R as illustrated in Figure 10 had occurred. However, there might be more than one wall in a single plane. Figure 12 illustrates this point. Reflection path S-2-R is a valid ray path. Each segment is checked with all walls. Segment S-2 would be valid when checked with wall B-C. But, if wall F-G and B-C lay in the same plane segment S-2 would be invalid when checked with wall F-G. Point 2, although intersecting the plane at an end point, does not lie within the wall boundary F-G. So, instead of eliminating immediately rays which intersected a plane outside of a wall boundary, a count was made to see how many times the intersection between the wall plane and the line occurred at a ray segment end point both inside and outside of wall boundaries. In the flow chart in Figure 11 the KZ values

Figure 12. Illustration of case where intersection point between line and plane lying outside a wall boundary (wall F-G) does not imply invalid ray.

were the counting variables for this. If an intersection occurred at an end point and within a wall boundary then KZ1 was incremented by one. If the intersection occurred outside of a wall boundary at a segment endpoint, KZ2 was incremented by one. This procedure was followed for both ray segment endpoints. KZ3 and KZ4 were incremented in a similar manner for the other ray segment end point. A given ray segment was invalid if no line-plane intersections had occurred at a ray segment end point and within any wall boundary (KZ1 or KZ3 equal to zero) and if at least one time a line-plane intersection occurred at a ray segment end point outside of a plane boundary (KZ2 or KZ4 greater than zero).

The validity of ray path S-2-R in Figure 12 can be determined as follows. The intersection point (2) occurred at wall B-C within the wall boundary. Since an intersection had occurred at the first segment end point and within a boundary, KZ3 was one. Ray segment S-2 also intersected the plane containing wall FG but outside of the boundaries of FG. Therefore, KZ4, was one. Since an intersection had occurred within a boundary at an end point (KZ3=1) the ray was valid.

Those walls that were intersected by a line between segment end points and within the wall boundary were discarded immediately. Ray path S-2''-R in Figure 10 illustrates this case.

Ray paths were valid if each of its segments passed all of the aforementioned tests.

An interesting problem which arose in making these checks was determining if a ray segment end point on a given plane lay outside or inside the wall boundary. Figure 13 shows an example wall with three possible ray segment end points A, B and C.

Figure 13 is a two dimensional view because it is known that points A, B, C lay on the plane containing wall 1-2-3-4-5-6-7-1 because they were calculated assuming a reflection from that plane. The only requirement for this analysis is that the wall appear as an enclosed boundary and not as an edge view.

A reference point (Pref) is chosen outside the wall boundary and lying in the given plane. A reference line segment is constructed by joining Pref and one of the three points in question. The number of times this segment crosses the wall boundary will tell us whether the point lays inside the wall boundary. Segment Pref-C does not cross the wall boundary at all. Therefore, point C does not lie in the wall boundary. Segment Pref-A crosses the wall boundary one time while segment Pref-B crosses the wall boundary four times. By applying this technique to other points it can be seen that zero or an even number of crossings (point B) indicate that the point lies outside the wall boundary. If the number of crossings is odd (point A) then the point lies within the wall boundary.

Figure 13. Method devised to determine whether point lies inside or outside wall boundary.

5.0 SOUND PRESSURE LEVEL CALCULATIONS

As indicated in Section 1 the main emphasis in this work was to determine the sound ray paths and then check the sound pressure levels calculated from the ray paths with measured values in a few enclosures. If fairly good agreement was found then the ray tracing equations developed in this work could be used as a basis for further studies. In this section the equations used to calculate the sound pressure levels are discussed. The source was considered to be spherical and consisting of more than one discrete frequency. The latter assumption should be reasonable in most applications since most sources within a factory will not be a pure tone.

Kinsler and Frey¹¹ show that for a spherical source of radius r the average energy rate of flow is:

$$W = 4\pi r^2 I = \frac{4\pi r^2 P^2}{2\rho_o C} \quad (168)$$

where:

W = Energy flow (watt)

r = Radius of spherical surface (m)

I = Intensity of spherical wave (Watt/m²)

P = Pressure amplitude of wave (N/m²)

ρ_o = Mean density of medium of wave (kg/m³)

C = Speed of sound in medium of propagation (m/SEC)

and since the pressure amplitude of the wave is the real part of the

spherical wave equation:

$$P = \frac{A}{r} \cos(\omega t - kr) \quad (169)$$

where:

A = Amplitude constant of wave (N/M^2)

ω = Frequency of wave (hz)

t = Time (seconds)

k = Wave Number (Radians/M)

or its square may be written as:

$$P^2 = \frac{A^2}{r^2} \quad (169a)$$

Therefore equation (168) may be rewritten as:

$$W = \frac{2\pi A^2}{\rho_0 C} \quad (170)$$

If the sound power level of the source (W) is known then the amplitude constant can be determined by re-writing equation (170) as:

$$A = \sqrt{\frac{W \rho_0 C}{2\pi}} \quad (170a)$$

With the amplitude constant calculated the RMS pressure may be determined by taking the square root of equation (169a) and dividing by the square root of two or

$$P_{RMS} = \frac{A}{r} \left[\frac{1}{\sqrt{2}} \right] \quad (171)$$

The distance r is the path length of the rays and is determined from the ray segment end points using the pythagorean theorem

$$r(N) = \left[\left[X(N) - X(N-1) \right]^2 + \left[Y(N) - Y(N-1) \right]^2 + \left[Z(N) - Z(N-1) \right]^2 \right]^{\frac{1}{2}} \quad (172)$$

where:

$r(N)$: Path length (m)

$X(N)$, $Y(N)$, $Z(N)$: Coordinates of Nth point on ray path (m)

$X(N-1)$, $Y(N-1)$, $Z(N-1)$: Coordinates of point immediately preceding Nth point (m)

Equation (171) determines the RMS sound pressure along a ray path without considering intensity losses due to reflections. When sound impinges on a surface a certain amount of its energy is absorbed. The fraction of incident energy is given by (1-Absorption Coefficient) of that surface.

Thus:

$$P_{RMS} = P_{RMS} (1-\alpha)^{\frac{1}{2}} \quad (173)$$

where:

α : Absorption Coefficient of surface

This relationship is applied successively at each reflection point along each ray path. The energies arriving at a location along each ray path are then summed. The sound pressure level is then calculated by:

$$SPL = 10 \text{ LOG } \left[\frac{P_{RMS t}}{P_{REF}} \right]^2 \quad (174)$$

where:

$$P_{\text{RMS}_t} = \text{Total RMS sound pressure (N/m}^2\text{)}$$

$$P_{\text{REF}} = \text{Reference sound pressure (2 X 10}^{-5} \frac{\text{N}}{\text{m}^2}\text{)}$$

The preceding procedure may be applied to the calculation of overall sound pressure levels of octave band sound pressure levels, or of an A-weighted sound pressure level, as desired.

To calculate overall sound pressure levels the octave band acoustic power of the source must be used along with the octave band absorption characteristics of each surface. The above procedure is applied to each frequency using the appropriate power level and absorption coefficient. When the sound pressure level for each frequency has been determined the corresponding total sound pressure level is determined by adding the square of the RMS sound pressure at each frequency.

$$(P_{\text{RMS}_t})^2 = (P_{\text{RMS}_1})^2 + (P_{\text{RMS}_2})^2 + \dots + (P_{\text{RMS}_8})^2 \quad (175)$$

Equation (174) is then used to determine the total sound pressure level in dB.

The dBA calculation is performed in the same manner except that after the sound pressure levels in dB are determined at each frequency they must be weighted (adjusted) at each frequency before equations (174) and (175) are used. This procedure is illustrated in Table 1.

TABLE 1

Illustration of Conversion from dB Overall to dBA Overall

FREQUENCY HZ	SOUND PRESSURE LEVEL	A-WEIGHTING RELATIVE RESPONSE	A-WEIGHTED SOUND PRESSURE LEVEL
63	75.6	-26.2	49.4
125	74.6	-16.1	58.5
250	74.6	- 8.6	66.0
500	74.3	- 3.2	71.1
1000	74.2	0.0	74.2
2000	74.3	+ 1.2	75.5
4000	72.3	+ 1.0	73.3
8000	71.4	- 1.1	70.3
OVERALL			
SOUND PRESSURE LEVEL	83.1		80.5

The results in column 2 are the dB calculations at each frequency shown in column 1. The A-weighting relative response, e.g., the adjustment to be made of the values in column 2 at each frequency, are shown in column 3. The last column is the resulting A-weighted frequency spectra determined by adding columns 2 and 3. The overall sound pressure level for the unweighted and the A-weighted spectra is the quantity at the bottom of columns 2 and 4.

6.0 APPLICATION OF PROGRAM

6.1 Ray Tracing Plots

One of the several forms of the program output available is the plot of the calculated ray paths. These plots verify the input to the program and also allow one to see the ray paths and their reflection locations. With this knowledge it is possible to determine the best areas to add absorption material to reduce the reverberant contribution of the total sound pressure level at a given location. The reflection locations would not help in the case where the receiver is in the near field of the source and the sound pressure level is a function of the radiated sound power level of the source.

Two examples will be presented to demonstrate the use of the ray paths. The first example is a room 98.4 m. X 65.6 m. X 36 m. To illustrate more clearly the ray paths, the program was altered slightly to give three separate views of the room with the direct ray and all single reflection cases in the first view, all two reflection rays in the second view and all three reflection rays in the third view. These ray plots are shown in Figures 14, 15 and 16, respectively. A plan and elevation view is provided in each Figure showing reflections from all the surfaces. These Figures show no specific areas where absorption material would aid in reducing the reverberant part of the sound pressure level at the indicated receiver position.

Plan

Elevation

Figure 14. Direct ray and all single reflection ray paths, Randolph classroom.

Plan

Elevation

Figure 15. All valid sound rays undergoing two reflections, Randolph classroom.

Plan

Elevation

Figure 16. All valid sound rays undergoing three reflections, Randolph classroom.

The second example shown is floor plan from an existing industrial plant. Three plan views were drawn with different source and receiver locations. Instead of drawing separate elevation and plan views X's and O's were used to indicate reflections from the ceiling and floor respectively.

Figures 17, 18 and 19 show the resulting ray traces. The ray tracing plot shown in Figure 17 reveals that absorption material would be effective on areas A and B. The octave band sound pressure level predictions would indicate the necessary frequency properties of the absorption to be used. This figure shows that the addition of absorption material at any other location would be an impractical and relatively ineffective solution to a noise problem at this receiver location. In Figure 18 the source and receiver have been moved closer together. Several reflections still occur at locations A and B but additional surfaces are involved as shown in section C and D.

At this point an observer might look at Figure 18 and think that a more diffuse sound field exists there than in Figure 17. It is important to realize that the ray traces only show those rays from the source that reach the receiver in three reflections (except in the case of slanted surfaces where a maximum of two reflections are calculated). Since rays shown are a function of only the one given source at a given receiver location the ray traces do not tell us whether a diffuse sound field exists. Also, since the ray plots

1 inch = 164 M.

Ceiling Height = 65.6 M.

X = Reflection from ceiling

O = Reflection from floor

Figure 17. Floor plan from existing industrial plant, Example 1.

1 inch = 164 M.

Ceiling Height = 65.6 M.

X = Reflection from ceiling

O = Reflection from floor

Figure 18. Floor plan from existing industrial plant, Example 2.

1 inch = 164 M.

Ceiling Height = 65.6 M.

X = Reflection from
ceiling

O = Reflection from
floor

Figure 19. Floor plan from existing industrial plant, Example 3.

indicate areas where sound absorption material should be added the reader should realize that this will only reduce the noise level at that one particular receiver location. High sound pressure levels at other locations may or may not be reduced.

The program's ability to handle slanted surfaces is shown in Figure 19. This ability allows the program to handle surfaces as indicated in the figure and also factory spaces with inclined roofs. With additional development of the program this ability will allow for slanted barriers within a factory space. Another point of interest concerning Figure 19 is that the primary reflection surfaces are those in that portion of the factory space where the slanted surface is located. This can be compared with Figure 17 where no reflections occur in that same area and Figure 18 which has reflection locations in common with Figures 17 and 19.

6.2 Comparison of Predicted and Measured Sound Pressure Levels

The program's ability to determine acoustic ray paths in enclosed spaces has now been shown. In this section actual measured sound pressure levels are compared with sound pressure levels which were calculated using the ray path lengths. The results of these comparisons will help determine whether the use of this ray tracing technique to predict sound pressure levels should be investigated further.

Sound pressure level measurements were made in two different rooms and used to check the accuracy of the predicted sound pressure levels. An ILG sound source, whose octave band sound power is shown in Table 2 was used in each experiment. Measurements were taken at several locations within the room. The measured sound pressure levels were then compared with the predicted sound pressure levels determined with the computer programs.

The first room was the same room used in the initial ray tracing example (Figures 14, 15, 16) with dimensions of 98.4 m X 65.6 m X 36 m. The room was a classroom with painted concrete walls, an asphalt tiled floor and a ceiling of acoustic tile. The absorption characteristics of these surfaces are shown in Table 3. Measured and calculated sound pressure levels were determined at thirteen meter intervals throughout the room. The resulting measured sound pressure levels in dBA are shown in Figure 20. The calculated sound pressure levels in dBA are shown in Figure 21. The difference between predicted and measured levels is less than 3 dBA at all locations.

The agreement between the predicted and measured sound pressure levels can also be seen in Figure 22, which is a plot of the predicted and measured sound pressure levels at the circled positions one through seven shown in Figures 20 and 21. The results show good agreement between predicted and measured sound pressure levels in terms of the

TABLE 2

**Calibration of ILG Reference
Sound Source***

Octave Center Frequency Hz	Sound Power Level dB re 10^{-12} watt
63	82
125	81
250	81
500	81
1000	81
2000	81
4000	79
8000	78

*Manufacturer's data from ILG Industries, Inc., 2826 North Pulaski Road, Chicago, Illinois 60641.

TABLE 3

Octave Band Absorption Characteristics
of Surfaces in Randolph
Class Room*

Frequency	MATERIAL		
	Floor	Wall	Ceiling
63	.04	.01	.4
125	.04	.01	.47
250	.03	.02	.65
500	.05	.02	.75
1000	.11	.02	.84
2000	.07	.02	.83
4000	.02	.02	.81
8000	.02	.02	.8

*Taken from Acoustical Designing in Architecture by V. O. Knudsen and C. M. Harris, John Wiley and Sons, New York 1950.

☆S: Source

①, ②: Receiver Positions

Figure 20. Measured sound pressure levels at positions 1-28 in Randolph classroom and contours of constant sound pressure level.

Figure 21. Calculated sound pressure levels at positions 1-28 in Randolph classroom and constant sound pressure level contours.

Figure 22. Comparison of measured and calculated dBA sound pressure levels in Randolph Classroom at positions 1, 2, 3, 4, ..., 7, best agreement.

curve shape and the actual sound levels. However, the predicted sound pressure levels are always lower than the measured sound pressure levels.

The least agreement between measured and predicted dBA sound pressure levels occurred at positions twenty-one through twenty-eight, as shown in Figure 23. However, the differences between predicted and measured sound pressure levels were less than 3 dBA as mentioned previously and the curve shapes were quite similar.

To find the reason why some sound pressure level predictions in dBA were better than others, plots were made of the sound pressure levels as predicted in each octave band. This was done at points 2 and 26. The results are shown in Figures 24 and 25, respectively. The results at position 2 indicate decreased prediction accuracy at frequencies of 500 Hz or less. Since the overall levels plotted in Figure 22 were on the dBA weighted network the low frequency sound pressure levels contributed less to the resultant dBA value.

The frequency plot in Figure 25 shows fair to poor agreement between predicted and measured sound pressure levels at middle and upper frequencies as well as at the lower frequencies. For this reason the dBA results shown in Figure 23 were less accurate.

The background noise was not a contributing factor to the disagreement between predicted and measured SPL, because the difference between the background measurements and the experimental measurements were more than 10 dB at all frequencies.

Figure 23. Comparison of measured and calculated dBA sound pressure levels in Randolph Classroom 22, 23, 24, ..., 28, worst agreement.

Figure 24. Comparison of measured and calculated octave band sound pressure levels at position 2 in Randolph Classroom

Figure 25. Comparison of measured and calculated octave band sound pressure levels at position 26 in Randolph Classroom

Rather than discuss other possible reasons for errors at this point a comparison between predicted and measured sound pressure levels in a hallway will be shown and a general discussion of sources of error will follow.

A long narrow hallway 2 m X 2.4 m X 30 m was the second site used to check the predictive capabilities of the program. Figure 26 shows the room and the locations where measured and calculated sound pressure levels were determined. The walls were painted concrete, the floor was asphalt tile and the ceiling was acoustic tile. (Table 3).

The results of the measured and predicted sound pressure levels in dBA are shown in Figure 27. These results show that the predicted levels are within 4 dBA of those measured at the most remote position down the hall. The measured value at position 7 is likely an error in the meter reading. If a continuous curve is drawn through the other points this point will be excluded.

Overall sound pressure level predictions shown in Figure 28 indicate less accuracy than the dBA predictions shown in Figure 27. A look at the octave band spectra showed least accuracy at the lower frequencies. These would show up in the overall sound pressure level predictions but to a lesser extent in the dBA predictions as illustrated in the previous example. The two octave band spectra are shown in Figures 29 and 30 which were determined at points 2 and 13

Figure 26. Measurement Positions in
Derring Hallway

Figure 27. Comparison of measured and calculated dBA sound pressure levels in Derring Hallway 1, 2, 3, 4, 5, 6, ..., 14

Figure 28. Comparison of measured and calculated overall sound pressure levels in Derring Hallway at positions 1, 2, 3, ..., 14

Figure 29. Comparison of measured and calculated octave band sound pressure levels at position 2 in Derring Hallway, best agreement.

Figure 30. Comparison of measured and calculated octave band sound pressure levels at position 13 in Derring Hallway, worst agreement.

respectively. Data taken at point 2 shows excellent agreement at all but the 125 Hz frequency. Farther down the hall away from the source (point 13) the octave band spectrum reveals a 6-7 dB disagreement at most frequencies and the maximum at frequencies below 250 Hz. With these results in mind one can understand more easily the results shown earlier in Figures 27 and 28. It is evident that the further the receiver is from the source the less accurate the predictions. The dBA predictions are in better agreement further down the hall than the overall sound pressure levels.

The results in the hallway experiment and the classroom experiment indicate the program predicts sound pressure levels better at higher frequencies than lower frequencies. Possible sources of error in the low frequency spectrum may lie in the fact that the geometric acoustics theory is only valid as a predictive tool when the wavelengths of the sound are much less than the dimensions of the room. The lowest frequency considered in this study was 63 Hz. The wavelength at this frequency is about 5.5 meters. The smallest dimension in Derring Hall was 7 meters which may account for the sound pressure level inaccuracies at low frequencies. Bolt, Doak, and Westervelt⁶, as discussed in Section 2.2, indicate that for fairly large rooms, like auditoriums, results using geometric acoustics are valid for frequencies above 100 Hz.

The fact that the low frequency sound pressure levels are predicted with less accuracy is not as serious a problem as it first appears because the primary interest is dBA. This depends upon the middle to upper frequency predictive capabilities of the program.

In the case of a very long and very narrow room (hallway) the predicted sound pressure levels may be less accurate at great distances because inclusion of sound rays with no more than three reflections does not adequately describe the behavior of the sound in this duct-like environment. This problem needs further considerations. However, it is believed that a small proportion of factories will possess rooms where one dimension is so much greater than the other two. Thus, this might not prove to be a troublesome limitation.

7.0 CONCLUSIONS

The results of the theoretical and experimental investigations allow some conclusions to be drawn:

1. The program can handle irregular rooms including room boundaries with slanted surfaces.
2. The ray tracing plots can indicate areas where absorption material may be added to reduce sound levels at a given receiver location.
3. The sound pressure level predictive capabilities of the program are promising and justify further work using the methods determined in this work.

Recommendations for the improvement of the program include investigating different intensity calculations not based on a point sound source. Directivity of the source and diffraction effects in irregular rooms need to be incorporated into the program. The program should also be extended to handle barriers within a room. Investigations into the effects of air absorption should also be considered.

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APPENDIX A
USERS GUIDE AND PROGRAM DOCUMENTATION

Users Manual

The necessary steps to use the program in its current form are as follows:

The user must first assemble the following information:

1. Drawings (floor plan and elevation) of factory space;
2. Composition of all interior surfaces of factory space;
3. Location of all sound sources;
4. Octave band (if possible) or overall sound power levels of each source.

With the information from item (1) the plant should be laid out on grid paper as shown in Figure A-1. It is necessary to assure that all of the factory is positioned in the first quadrant as shown in the example. The sources from item (3) should also be located on the grid as shown (S_1 , S_2 , S_3).

The octave band absorption coefficients must then be found, either from vendor data or other sources.

The necessary data can now be put into the program on cards, or as a data set. To illustrate the procedure the factory space in Figure A-1 will serve as an example. The necessary data to be put into the program is shown in Figure A-2* and should be referred to as the procedure is explained. It is imperative that the input be placed in the indicated columns in Figure A-2.

The first data card tells the program whether a ray tracing plot is desired in the output. If a plot is desired a one is

*May be found in packet at end of thesis.

positioned in the first column. If no plot is wanted a zero goes in the first column. In all cases a plot of the room in three dimensions will be made to assure user that data was correctly entered.

The number of sources is entered in column five of the second data card.

On the third data card are entered the X, Y and Z coordinates of the first source. The X, Y and Z coordinates are entered in columns 1-10, 11-20 and 21-30 respectively. The octave band sound power level (in watts) for the source whose location was given on the third data card is then entered in columns 1-10 of the next eight data cards, one per data card beginning with 63 Hz.

The preceding procedure is repeated for each source, e.g., the coordinates of the source are entered followed by its octave band sound power level spectrum.

After the data on all sound sources has been entered the number of surfaces is entered on the next data card in column 5.

The first surface to be entered must be the floor.

The number of end points (corners) of the floor is entered on the next card in column five. An end point is defined as the point where three surfaces intersect.

The absorption coefficients followed by the corner points of each wall are entered as follows.

The octave sound absorption values of the floor are entered on the next eight cards in columns 1-10, one per data card beginning with 63 Hz.

The X, Y and Z coordinates of each corner point of the floor (surface) are entered starting at any corner and proceeding clockwise or counter-clockwise around the wall boundary until all corner points are entered. Each coordinate set is entered on a separate data card in columns 1-10, 11-20 and 21-30. This procedure describes a plane. Repeat this procedure for the ceiling and walls.

If a surface contains windows, or patches of absorption as shown in Figure A-3, it must be divided into segments so that there are no interior boundaries completely enclosed. Figure A-4 illustrates one method of segmenting the wall surface so that no interior boundaries are completely enclosed. The numbers indicate the number of walls to be entered. As explained in the preceding instructions. When one wall surface is subdivided into several smaller areas the absorption coefficients will be entered just as if it were a separate wall.

The last card contains seven entries. The desired increment of the receiver location is entered in columns 1-10. If a zero is entered then no incrementing will take place. The X, Y and Z coordinates of the initial receiver position are entered in columns 11-20, 21-30 and 31-40 respectively. The final entries on this card are the maximum X, Y and Z dimensions of the factory space.

The computer output will include the coordinates of the receiver position, the octave band sound pressure levels, and the overall sound pressure levels. If a ray tracing plot is requested the plan and elevation views of the rays will be plotted.

Surface "B" is completely enclosed within Surface "A".

FIGURE A-3

Surface "B" is not enclosed by "old" Surface "A" but is adjacent to new Surfaces "3" and "4".

FIGURE A-4

```

C*****
C THE MAIN PROGRAM READS IN THE NECESSARY DATA AND INCREMENTS
C RECEIVER POSITION
C*****
C
C      INTEGER WNTPLT
C      COMMON/ARRAY/PL,QL,PL,NPT,NPL
C      COMMON/RAY1/XR,YR,ZR,A,B,C,IL,N2,N3,N4
C      COMMON/SND/PRESTO,ALPHA
C      COMMON/SP/POWER
C      COMMON/ALL/FACT
C      COMMON/GL/WNTPLT
C      DIMENSION PL(15,15),QL(15,15),AL(15,15),A(5),R(5),C(5),P(3,15),
C      IQ(3,15),R(3,15),NPT(15),SX(50),SY(50),SZ(50),ALPHA(15,8),PWR
C      I(50,8),POWER(3),PRESTO(8)
C      DO 926 JOCT=1,8
C      PRESTO(JOCT)=0.
C      CONTINUE
C      NUNIT=8
C
C      WNTPLT= (WANT PLOT) IS TO LET THE OPERATOR CHOOSE WHETHER
C      OR NOT HE WANTS THE DATA PLOTTED BY THE CALCOMP PLOTTER.
C      WNTPLT=1 YES, PLOT THE DATA
C      WNTPLT=0 NO, DO NOT PLOT THE DATA
C
C      READ(NUNIT,200) WNTPLT
C      200 FORMAT(I1)
C      IF(WNTPLT.EQ.0) GO TO 50
C      GO TO 51
C      CONTINUE
C      50
C      READ IN NUMBER OF SOURCES (NS)
C

```

```

C      51 READ(NUNIT,201) NS
      201 FORMAT(I5)
      DO 1 IS=1,NS
C
C      READ COORDINATES OF SOURCE (X,Y,Z)
C
C      READ(NUNIT,204) SX(IS),SY(IS),SZ(IS)
      204 FORMAT(3F10.2)
      DO 255 IOCT=1,8
      255 READ(NUNIT,291) PWR(IS,IOCT)
      291 FORMAT(F10.9)
      1 CONTINUE
C
C      READ NUMBER OF WALLS (NPL)
C
C      READ(NUNIT,206) NPL
      206 FORMAT(I5)
      DO 2 I1=1,NPL
C
C      READ IN NUMBER OF CORNER POINTS OF WALL (NPT)
C
C      READ(NUNIT,208) NPT(I1)
      208 READ IN OCTAVE BAND ABSORPTION COEFFICIENTS OF SURFACE
      FORMAT(I5)
      DO 256 KOCT=1,8
      256 READ(NUNIT,290) ALPHA(I1,KOCT)
      FORMAT(F10.6)
      CONTINUE
      NPTO=NPT(I1)
      DO 3 J1=1,NPTO
C

```

```

C      BEGINNING OF SECTION OF PROGRAM TO INCREMENT RECEIVER POSITION
C      PROCEEDING SEQUENTIALLY AROUND PLANE BOUNDARY
C
C      READ IN X,Y,Z COORDINATES OF EACH CORNER POINT, PROCEEDING
C      SEQUENTIALLY AROUND PLANE BOUNDARY
C
C      READ(NUNIT,211) PL(J1,I1),QL(J1,I1),RL(J1,I1)
211  FORMAT(3F10.2)
C      3 CCNTINUE
C      2 CONTINUE
C
C      READ IN RECEIVER INCREMENT (DELTA), INITIAL
C      COORDINATES OF RECEIVER, AND MAXIMUM X,Y,Z DIMENSIONS
C      OF ROOM
C
C      READ(NUNIT,213) DELTA,XR1,YR1,ZR1,XMAX,YMAX,ZMAX
213  FORMAT(7F10.2)
C      FACT=9.5/(ZMAX+YMAX)
C      IF(WNTPLT.EQ.0) GO TO 66
C      CALL RM3DEE(PL,QL,RL,XMAX,YMAX,ZMAX,NPT,NPL)
66  XR=XR1
C      YR=YR1
C      ZR=ZR1
C      IF(WNTPLT.EQ.0) GO TO 90
C      CALL DRWPLN(PL,QL,RL,XMAX,YMAX,ZMAX,NPL,NPT)
C
C      DETERMINES RAY PATHS TO A GIVEN RECEIVER POSITION
C      (OUTER LOOP) FROM ALL SOURCE POSITIONS (INNER LOOP)
C
C      DO 100 MN=1,NS
90  WRITE(6,258) XR,YR,ZR
258  FORMAT(' ',THE X,Y,Z POSITION OF THE RECEIVER IS',3F10.3)

```

```

      DO 819 JS=1,2
      POWER(JS)=PMR(MN,JS)
      CALL RAY(PL,QL,RL,NPL,NPT,SK(IN),SY(MN),SZ(MN))
100  CONTINUE
      CALL PRSLEV
      IF(DELTA.LT.C.1) GO TO 60
      XMOVE=XMAX+2.
      7  XR=XR+DELTA
C
C      DETERMINES IF, AFTER INCREMENTING X DIMENSION, RECEIVER
C      IS WITHIN ROOM BOUNDARY
C
      CALL PTPL(PL,QL,RL,1,XR,YR,ZR,NPT,IP)
      IF(IP.EQ.0.AND.XR.GT.XMAX)GO TO 30
      IF(IP.EQ.0.AND.XR.LT.XMAX) GO TO 7
      GO TO 90
30  XR=XR1
600  YR=YR+DELTA
C
C      DETERMINES IF, AFTER INCREMENTING Y DIMENSION, RECEIVER
C      IS WITHIN ROOM BOUNDARY
C
      CALL PTPL(PL,QL,RL,1,XR,YR,ZR,NPT,IP)
      IF(IP.EQ.0.AND.YR.LT.YMAX) XR=XR+DELTA
      IF(IP.EQ.0.AND.YR.GT.YMAX) GO TO 64
      GO TO 90
64  IF(WNTPLT.EQ.0) GO TO 65
60  CALL PLOT(C.0,C.0,-4)
65  STOP
      END
      SUBROUTINE RAY(PL,QL,RL,NPL,NPT,XS,YS,ZS)
C

```



```

C*****
C      STORES INPUT FROM MAIN PROGRAM IN USABLE FORM FOR
C      RAY-TRACING CALCULATIONS
C*****
C      COMMON/RAY1/XR,YR,ZR,A,B,C,N1,M2,N3,N4
C      DIMENSION PL(15,15),QL(15,15),PL(15,15),A(5),E(5),C(5),P(3,15),
C      IQ(3,15),R(3,15),NPT(15),X(2),Y(2),Z(2),XCHK(5),YCHK(5),ZCHK(5)
C      N1=0
C      N2=0
C      N3=0
C      N4=0
C      N5=0
C      N6=0
C      N7=0

C      THIS LOOP STORES ALL WALLS (REGULAR OR SLANTED) AS PLANES
C
C      DO 16 I1=1,NPL
C      CALL PLANE(PL(I1,I1),QL(I1,I1),RL(I1,I1),PL(2,I1),QL(2,I1),RL(2,I1),
C      IL(3,I1),QL(3,I1),RL(3,I1),AP,BP,CP,DP)
C      IF(BP.EQ.0..AND.CP.EQ.0..) GO TO 10
C      IF(AP.EQ.0..AND.CP.EQ.0..) GO TO 11
C      IF(AP.EQ.0..AND.BP.EQ.0..) GO TO 12
C      IF(AP.EQ.0..) GO TO 13
C      IF(BP.EQ.0..) GO TO 14
C      IF(CP.EQ.0..) GO TO 15
C      RETURN
C
C      STORES WALLS PERPENDICULAR TO X AXIS
C
C      10 N5=N5+1

```

```

XCHK(N5)=-DP/AP
IF(N5.EQ.1) GO TO 55
DO 50 KOT=1,N1
IF(XCHK(N5).EQ.A(KOT)) GO TO 16
50 CONTINUE
55 N1=N1+1
A(N1)=XCHK(N5)
GO TO 15

C      STORES WALLS PERPENDICULAR TO Y AXIS
C
C      11  N6=N6+1
YCHK(N6)=-DP/BP
IF(N6.EQ.1) GO TO 56
DO 57 KST=1,N2
IF(YCHK(N6).EQ.B(KST)) GO TO 16
57 CONTINUE
56 N2=N2+1
B(N2)=YCHK(N6)
GO TO 16

C      STORES WALLS PERPENDICULAR TO Z AXIS
C
C      12  N7=N7+1
ZCHK(N7)=-DP/CP
IF(N7.EQ.1) GO TO 58
DO 59 KRT=1,N3
IF(ZCHK(N7).EQ.C(KRT)) GO TO 16
59 CONTINUE
58 N3=N3+1
C(N3)=ZCHK(N7)
GO TO 16

```

```

C
C
C
      STORES WALLS PERPENDICULAR TO Y-Z PLANE

13  N4=N4+1
      Q(1,N4)=AMIN1(QL(1,I1),QL(2,I1),QL(3,I1))
      IF(Q(1,N4).EQ.QL(1,I1)) GO TO 18
      IF(Q(1,N4).EQ.QL(2,I1)) GO TO 19
      GO TO 20
18  P(1,N4)=PL(1,I1)
      R(1,N4)=RL(1,I1)
      DO 27 IO=2,3
      P(IO,N4)=PL(IO,I1)
      Q(IO,N4)=QL(IO,I1)
27  R(IO,N4)=RL(IO,I1)
      GO TO 16
19  P(1,N4)=PL(2,I1)
      R(1,N4)=RL(2,I1)
29  P(2,N4)=PL(1,I1)
      Q(2,N4)=QL(1,I1)
      R(2,N4)=RL(1,I1)
      P(3,N4)=PL(3,I1)
      Q(3,N4)=QL(3,I1)
      R(3,N4)=RL(3,I1)
      GO TO 16
20  P(1,N4)=PL(3,I1)
      R(1,N4)=RL(3,I1)
30  P(2,N4)=PL(1,I1)
      Q(2,N4)=QL(1,I1)
      R(2,N4)=RL(1,I1)
      P(3,N4)=PL(2,I1)
      Q(3,N4)=QL(2,I1)
      R(3,N4)=RL(2,I1)

```

```

C
C
C
GO TO 16

      STORES PLANES PERPENDICULAR TO X-Z PLANE

14 N4=N4+1
   P(1,N4)=AMIN1(PL(1,I1),PL(2,I1),PL(3,I1))
   IF(P(1,N4).EQ.PL(1,I1)) GO TO 21
   IF(P(1,N4).EQ.PL(2,I1)) GO TO 22
   GO TO 23

21 Q(1,N4)=QL(1,I1)
   R(1,N4)=RL(1,I1)
   DO 28 I0=2,3
   P(IP,N4)=PL(IP,I1)
   Q(IP,N4)=QL(IP,I1)
28 R(IP,N4)=RL(IP,I1)
   GO TO 16

22 Q(1,N4)=QL(2,I1)
   R(1,N4)=RL(2,I1)
   GO TO 29

23 Q(1,N4)=QL(3,I1)
   R(1,N4)=RL(3,I1)
   GO TO 30

C
C
C
      STORES PLANES PERPENDICULAR TO X-Y PLANE

15 N4=N4+1
   P(1,N4)=AMIN1(PL(1,I1),PL(2,I1),PL(3,I1))
   IF(P(1,N4).EQ.PL(1,I1)) GO TO 21
   IF(P(1,N4).EQ.PL(2,I1)) GO TO 22
   GO TO 23

16 CONTINUE
31 X(1)=XS

```

```

Y(1)=YS
Z(1)=ZS
X(2)=XR
Y(2)=YR
Z(2)=ZR

      DIRECT PATH CALCULATION

      CALL RAYCHK(X,Y,Z,2,1,1)

      ONE REFLECTION - REGULAR WALLS

32 CALL AONE(XS,YS,ZS)

      TWO REFLECTIONS - REGULAR WALLS

      CALL ATWO(XS,YS,ZS)

      THREE REFLECTIONS - REGULAR WALLS

      CALL A31(XS,YS,ZS)

      IF NJ SLANTED PLANES, PROGRAM RETURNS TO MAIN

      IF (N4.EQ.0) RETURN

      ONE REFLECTION FROM SLANTED SURFACES

      CALL SLANT1(P,Q,R,XS,YS,ZS)

      TWO REFLECTIONS INVOLVING AT LEAST ONE SLANTED SURFACE

```



```

1,Q(3,J),R(3,J),1)
CALL SLANTS(X,Y,Z,P(1,J),Q(1,J),P(2,J),Q(2,J),R(2,J),P(3,J),Q(3,J),R(3,J),1)
CALL SLANTT(X,Y,Z,P(1,J),Q(1,J),P(2,J),Q(2,J),R(2,J),P(3,J),Q(3,J),R(3,J),1)
1,CONTINUE
DO 2 L=1,N4
DO 3 M=1,N4
IF(L.EQ.N) GO TO 3
CALL PLANE(P(1,L),Q(1,L),R(1,L),P(2,L),Q(2,L),R(2,L),P(3,L),Q(3,L),R(3,L),AP1,BP1,CP1,DP1)
CALL PLANE(P(1,M),Q(1,M),R(1,M),P(2,M),Q(2,M),R(2,M),P(3,M),Q(3,M),R(3,M),AP2,BP2,CP2,DP2)
IF(AP1.EQ.O..AND.AP2.EQ.O.) GO TO 5
IF(BP1.EQ.O..AND.BP2.EQ.O.) GO TO 5
IF(CP1.EQ.O..AND.CP2.EQ.O.) GO TO 5
GO TO 3
5 CONTINUE
CALL SLANTC(X,Y,Z,P(1,L),Q(1,L),P(2,L),Q(2,L),R(2,L),P(3,L),Q(3,L),R(3,L),P(1,M),Q(1,M),R(1,M),P(2,M),Q(2,M),R(2,M),P(3,M),Q(3,M),R(3,M),1)
2,CONTINUE
3,CONTINUE
2,CONTINUE
X(1)=XR
Y(1)=YR
Z(1)=ZR
X(4)=XS
Y(4)=YS
Z(4)=ZS
DO 4 J=1,N4
CALL SLANTF(X,Y,Z,P(1,J),Q(1,J),P(2,J),Q(2,J),R(2,J),P(3,J),Q(3,J),R(3,J),7)

```

```

      CALL SLANTS(X,Y,Z,P(1,J),Q(1,J),R(1,J),P(2,J),Q(2,J),R(2,J),P(3,J),
1,Q(3,J),R(3,J),7)
      CALL SLANTT(X,Y,Z,P(1,J),Q(1,J),R(1,J),P(2,J),Q(2,J),R(2,J),P(3,J),
1,Q(3,J),R(3,J),7)
4 CONTINUE
      RETURN
      END
      SUBROUTINE SLANTF(X,Y,Z,P1,Q1,R1,P2,Q2,R2,P3,Q3,R3,ISR)
C *****
C *****
C ***** DETERMINES TWO REFLECTIONS - SLANTED SURFACE TO REGULAR
C ***** WALL PERPENDICULAR TO Y AXIS *****
C *****
C *****
      COMMON/RAVL/XR,YR,ZR,A,B,C,N1,N2,N3,N4
      DIMENSION X(4),Y(4),Z(4),A(5),B(5),C(5),P(3),Q(3),R(3)
      P(1)=P1
      Q(1)=Q1
      P(2)=P2
      Q(2)=Q2
      R(2)=R2
      P(3)=P3
      Q(3)=Q3
      R(3)=R3
      IF(P1.EQ.P2.AND.Q1.EQ.Q2) GO TO 10
      IF(P1.EQ.P2.AND.P1.EQ.R2) GO TO 20
      IF(Q1.EQ.Q2.AND.Q1.EQ.R2) GO TO 30
      GO TO 40
10 P(2)=P3
   Q(2)=Q3
   GO TO 40
20 P(2)=P3

```



```

R(2)=R3
GO TO 40
30 Q(2)=Q3
R(2)=R3
40 CALL PLANE(P1,Q1,P1,P2,Q2,R2,P3,Q3,P3,AP,PP,CP,D)
IF(AP.EQ.0.) GO TO 1
IF(BP.EQ.0.) GO TO 2
IF(CP.EQ.0.) GO TO 3
RETURN
1 HYP=SQRT(ABS(Q(2)-Q(1))**2.+ABS(R(2)-R(1))**2.)
CTHETA=(Q(2)-Q(1))/HYP
STHETA=(R(2)-R(1))/HYP
SLOPE=-BP/CP
AXIS=-D/CP
DO 4 I=1,N3
Z(3)=C(I)
CALL FSLANT(Y,Z,X,Q(1),R(1),SLOPE,AXIS,CTHETA,STHETA)
CALL RAYCHK(X,Y,Z,4,ISR,1)
4 IF(ISR.EQ.7) CALL RAYCHK(X,Y,Z,4,7,0)
RETURN
2 HYP=SQRT(ABS(P(2)-P(1))**2.+ABS(R(2)-R(1))**2.)
CTHETA=(P(2)-P(1))/HYP
STHETA=(R(2)-R(1))/HYP
SLOPE=-AP/CP
AXIS=-D/CP
DO 5 J=1,N3
Z(3)=C(J)
CALL FSLANT(X,Z,Y,P(1),R(1),SLOPE,AXIS,CTHETA,STHETA)
CALL RAYCHK(X,Y,Z,4,ISR,1)
5 IF(ISR.EQ.7) CALL RAYCHK(X,Y,Z,4,7,0)
RETURN
3 HYP=SQRT(ABS(Q(2)-Q(1))**2.+ABS(P(2)-P(1))**2.)

```



```

E3=C1*B1
E4=C1*B2+C2*Y2P
F1=E1*SLOPE-E3*A1
F2=A3*E1+SLOPE*E2-E3*A2-E4*A1
F3=E2*A3-E4*A2
ROOT=F2**2-4.*F1*F3
IF(ROOT.LT.0.) GO TO 300
I=1
X(2)=(-F2+SQRT(ROOT))/(2.*F1)
GO TO 7
6  X(2)=(-F2-SQRT(ROOT))/(2.*F1)
7  Y(2)=SLOPE*X(2)+AXIS
X(3)=(E1*X(2)+E2)/(E3*X(2)+E4)
110 G1=X(2)-X(4)
G2=Z(4)*(X(3)-X(2))
G3=X(3)-X(4)
Y3P=C1*X(3)+C2*Y(3)+C3
H1=Z(1)*(Y3P-Y2P)
H2=Y3P-2.*Y2P
CHK1=ABS(G1*H2+G3*Y2P)
IF(CHK1.LT.1.E-5) GO TO 300
Z(3)=(G3*H1-H2*G2)/(G1*H2+G3*Y2P)
Z(2)=(H1-Y2P*Z(3))/H2
I=I+1
IF (I.EQ.3) GO TO 8
99 DIFF1=ABS(X(3)-X(2))
DIFF2=ABS(Y(3)-Y(2))
IF(DIFF1.LT. .001 .AND. DIFF2.LT. .001) GO TO 6
222 DIFF=ABS((Z(3)-Z(2))/(Y3P-Y2P)+(Z(2)-Z(1))/Y2P)
IF(DIFF-1.) 11,6,6
8 DIFF1=ABS(X(3)-X(2))
DIFF2=ABS(Y(3)-Y(2))

```

```

71      IF(DIFF1.LT..001 .AND.DIFF2.LT..001)GO TO 300
      DIFF=ABS((7(3)-Z(2))/(Y3P-Y2P)+(Z(2)-Z(1))/Y2P)
      IF (DIFF-1.) 11,300,300
11      Y1P=0.
      IF(Y2P.GT.Y1P.AND.Y2P.LT.Y3P.GT.Y2P.LT.Y1P.AND.Y2P.GT.Y3P)GO TO
1300
      IF(Y(3).GT.Y(2).AND.Y(3).LT.Y(4).OR.Y(3).LT.Y(2).AND.Y(3).GT.Y(4)
1GO TO 300
      RETURN
300  Y(2)=-1.5
      X(3)=-1.8
      Z(2)=-3.3
      Y(3)=-2.5
      Z(3)=-4.
      X(2)=-7.
      RETURN
9
      END
      SUBROUTINE SLANTS(X,Y,Z,P1,Q1,R1,P2,Q2,R2,P3,Q3,R3,ISR)
C*****
C*****
C***** DETERMINES TWO REFLECTIONS - SLANTED PLANE TO REGULAR
C***** WALL PERPENDICULAR TO X AXIS
C*****
C*****
C*****
      DIMENSION X(4),Y(4),Z(4),A(5),B(5),C(5),P(3),Q(3),R(3)
      COMMON/RAY1/XR,YR,ZR,A,B,C,N1,N2,N3,N4
      P(1)=P1
      Q(1)=Q1
      R(1)=R1
      P(2)=P2
      Q(2)=Q2
      R(2)=R2

```

```

P(3)=P3
Q(3)=Q3
R(3)=R3
IF(P1.EQ.P2.AND.Q1.EQ.Q2) GO TO 10
IF(P1.EQ.P2.AND.P1.EQ.R2) GO TO 20
IF(Q1.EQ.Q2.AND.P1.EQ.R2) GO TO 30
GO TO 40
10 P(2)=P3
   Q(2)=Q3
   GO TO 40
20 P(2)=P3
   R(2)=R3
   GO TO 40
30 Q(2)=Q3
   R(2)=R3
   CALL PLANE(P1,Q1,P1,P2,Q2,R2,P3,Q3,R3,AP,RP,CP,U)
   IF(AP.EQ.O.) GO TO 1
   IF(BP.EQ.O.) GO TO 2
   IF(CP.EQ.O.) GO TO 3
   RETURN
1  HYP=SQRT(ABS(Q(2)-Q(1))**2.+ABS(R(2)-P(1))**2.)
   CTHETA=(Q(2)-Q(1))/HYP
   STHETA=(P(2)-P(1))/HYP
   SLOPE=-BP/CP
   AXIS=-D/CP
   DO 4 I=1,N2
   Y(3)=B(I)
   CALL SSLANT(Y,Z,X,O(1),R(1),SLOPE,AXIS,CTHETA,STHETA)
   CALL RAYCHK(X,Y,7,4,ISR,1)
   IF(ISR.EQ.7) CALL RAYCHK(X,Y,Z,4,7,C)
4  RETURN
2  HYP=SQRT(ABS(P(2)-P(1))**2.+ABS(P(2)-P(1))**2.)

```



```

A1=SLOPE*X(4)-SLOPE*X(3)+Y(4)
A2=-Y(4)*X(3)+AXIS*(X(4)-X(3))
A3=X(4)-2.*X(3)
B1=-SIGN*SLOPE*STHETA-SIGN*CTHETA
B2=-SIGN*STHETA*(AXIS-Y(1))+SIGN*X(1)*CTHETA
C1=SIGN*STHETA
C2=-SIGN*CTHETA
C3=SIGN*Y(1)*CTHETA-SIGN*X(1)*STHETA
D1=SIGN*Y(1)*STHETA+SIGN*X(1)*CTHETA
E1=2.*B1*Y2P-E1*C1*X(3)-B1*C3
E2=B2*(Y2P-C3-C1*X(3))+Y2P*(B2-C2*X(3)-D1)
E3=B1*C2
E4=C2*B2-C1*Y2P
F1=E1-E3*A1
F2=E2+A3*E1-E3*A2-F4*A1
F3=E2*A3-E4*A2
ROOT=F2**2-4.*F1*F3
IF(ROOT.LT.0.) GO TO 300
I=1
X(2)=(-F2-SQRT(F2**2-4.*F1*F3))/(2.*F1)
GO TO 7
X(2)=(-F2+SQRT(ROOT))/(2.*F1)
Y(2)=SLOPE*X(2)+AXIS
G1=2.*X(3)-X(2)-X(4)
G2=Z(4)*(X(2)-X(3))
G3=X(3)-X(4)
Y(3)=(E1*X(2)+E2)/(F3*X(2)+E4)
Y3P=C1*X(3)+C2*Y(3)+C3
H1=Z(1)*(Y3P-Y2P)
H2=Y3P-2.*Y2P
CHK1=ABS(G1*H2+G3*Y2P)
IF(CHK1.LT.1.E-5) GO TO 300

```

6
7


```

C      OR FLOOR SURFACES
C*****
C
      DIMENSION X(4),Y(4),Z(4),A(5),B(5),C(5),P(3),Q(3),R(3)
COMMON/RAY1/XR,YR,ZR,A,B,C,N1,N2,N3,N4
P(1)=P1
Q(1)=Q1
R(1)=R1
P(2)=P2
Q(2)=Q2
R(2)=R2
P(3)=P3
Q(3)=Q3
R(3)=R3
IF(P1.EQ.P2.AND.Q1.EQ.Q2) GO TO 10
IF(P1.EQ.P2.AND.P1.EQ.R2) GO TO 20
IF(Q1.EQ.Q2.AND.R1.EQ.R2) GO TO 30
GO TO 40
10  P(2)=P3
    Q(2)=Q3
    GO TO 40
20  P(2)=P3
    Q(2)=Q3
    GO TO 40
30  Q(2)=Q3
    R(2)=R3
40  CALL PLANE(P1,Q1,R1,P2,Q2,R2,P3,Q3,R3,AP,BP,CP,D)
    IF(AP.EQ.D.) GO TO 1
    IF(BP.EQ.D.) GO TO 2
    IF(CP.EQ.D.) GO TO 3
    RETURN
1  HYP=SQRT(ABS(Q(2)-Q(1))**2.+ABS(R(2)-R(1))**2.)

```

```

CTHETA=(Q(2)-G(1))/HYP
STHETA=(P(2)-E(1))/HYP
SLOPE=-BP/CP
AXIS=-D/CP
DO 4 I=1,N1
  X(3)=A(I)
  CALL TSLANT(Y,Z,X,Q(1),R(1),SLOPE,AXIS,CTHETA,STHETA)
  CALL RAYCHK(X,Y,Z,4,ISR,1)
  IF(ISR.EQ.7) CALL RAYCHK(X,Y,Z,4,7,0)
4 RETURN
2 HYP=SQRT(ABS(P(2)-P(1))**2.+ABS(R(2)-P(1))**2.)
CTHETA=(P(2)-P(1))/HYP
STHETA=(R(2)-P(1))/HYP
SLOPE=-AP/CP
AXIS=-D/CP
DO 5 J=1,N2
  Y(3)=B(J)
  CALL TSLANT(X,Z,Y,P(1),R(1),SLOPE,AXIS,CTHETA,STHETA)
  CALL RAYCHK(X,Y,Z,4,ISR,1)
  IF(ISR.EQ.7) CALL RAYCHK(X,Y,Z,4,ISR,0)
5 RETURN
3 HYP=SQRT(ABS(Q(2)-Q(1))**2.+ABS(P(2)-P(1))**2.)
CTHETA=(P(2)-P(1))/HYP
STHETA=(Q(2)-Q(1))/HYP
SLOPE=-AP/EP
AXIS=-D/EP
DO 6 K=1,N3
  Z(3)=C(K)
  CALL TSLANT(X,Y,7,P(1),Q(1),SLOPE,AXIS,CTHETA,STHETA)
  CALL RAYCHK(X,Y,7,4,ISR,1)
  IF(ISR.EQ.7) CALL RAYCHK(X,Y,Z,4,7,0)
6 RETURN

```



```

F2=D2-D1*X(4)-E1*D4-E2*D3
F3=-D2*X(4)-E2*D4
ROOT=F2**2-4.*F1*F3
IF (ROOT.LT.0.) GO TO 300
X(3)=(-F2+(SQRT(F2**2-4.*F1*F3)))/(2.*F1)
Y(3)=(R1*X(3)+F2)/B3
Z(2)=(X(3)*D1+D2)/(X(3)*D3+D4)
X3P=-SIGN*STHETA*(Y(3)-Y(1))-SIGN*CTHETA*(X(3)-X(1))
Y3P=-SIGN*STHETA*(Y(3)-Y(1))-SIGN*CTHETA*(X(3)-X(1))**2)
UX1=(X(3)-X(2))/(SQRT((X(3)-X(2))**2+(Y(3)-Y(2))**2))
UY1=(Y(3)-Y(2))/(SQRT((X(3)-X(2))**2+(Y(3)-Y(2))**2))
UX2=(Y(4)-Y(3))/(SQRT((X(4)-X(3))**2+(Y(4)-Y(3))**2))
UY2=(X(4)-X(3))/(SQRT((X(4)-X(3))**2+(Y(4)-Y(3))**2))
PROT1=UX1*UX2
PROT2=UY1*UY2
IF (PROT1.LT.-.00001.OR.PROT2.LT.-.00001) GO TO 100
DIFF=ABS((Z(3)-Z(2))/(Y3P-Y2P)+(Z(2)-Z(1))/Y2P)
IF (DIFF.GE..1) GO TO 100
GO TO 110
100 X(3)=(-F2-(SQRT(F2**2-4.*F1*F3)))/(2.*F1)
Z(2)=(X(3)*D1+D2)/(X(3)*D3+D4)
Y(3)=(B1*X(3)+F2)/B3
X3P=-SIGN*STHETA*(Y(3)-Y(1))-SIGN*CTHETA*(X(3)-X(1))
Y3P=-SIGN*STHETA*(Y(3)-Y(1))-SIGN*CTHETA*(X(3)-X(1))**2)
UX1=(X(3)-X(2))/(SQRT((X(3)-X(2))**2+(Y(3)-Y(2))**2))
UY1=(Y(3)-Y(2))/(SQRT((X(3)-X(2))**2+(Y(3)-Y(2))**2))
UX2=(X(4)-X(3))/(SQRT((X(4)-X(3))**2+(Y(4)-Y(3))**2))
UY2=(Y(4)-Y(3))/(SQRT((X(4)-X(3))**2+(Y(4)-Y(3))**2))
PROU1=UX1*UX2
PROU2=UY1*UY2
IF (PROU1.LT.-.00001.OR.PROU2.LT.-.00001) GO TO 300
DIFF=ABS((X2P-X3P)/(Y2P-Y3P)-(X2F-X4P)/(Y2P-Y4P))

```

```

IF (DIFF.LT..1) GO TO 300
Y1P=0.
IF(Y2P.GT.Y1P.AND.Y2P.LT.Y3P.OR.Y2P.LT.Y1P.AND.Y2P.GT.Y3P)
  1 GO TO 300
IF(Z(3).LT.Z(2).AND.Z(3).GT.Z(4).OR.Z(3).GT.Z(2).AND.Z(3).LT.Z(4))
  1 GO TO 300
300 Y(2)=-2.
X(3)=-3.
Z(2)=-5.
Y(3)=-6.
Z(3)=-25.
X(2)=-2.2
110 RETURN
END
SUBROUTINE SLANTQ(X,Y,Z,P1,Q1,P2,Q2,P3,Q3,P4,Q4,P5,Q5
C *****
C DETERMINES TWO REFLECTIONS - FROM TWO SLANTED PLANES
C PERPENDICULAR TO THE SAME PLANE
C *****
C
  1R5,P6,Q6,R6,ISR)
  DIMENSION X(4),Y(4),Z(4),P(6),Q(6),R(6)
  P(1)=P1
  Q(1)=Q1
  R(1)=R1
  P(2)=P2
  Q(2)=Q2
  R(2)=R2
  P(3)=P3
  Q(3)=Q3
  R(3)=R3

```

```

P(4)=P4
Q(4)=Q4
R(4)=R4
P(5)=P5
Q(5)=Q5
R(5)=R5
P(6)=P6
Q(6)=Q6
R(6)=R6
IF(P1.EQ.P2.AND.O1.EQ.Q2) GO TO 10
IF(P1.EQ.P2.AND.P1.EQ.R2) GO TO 20
IF(Q1.EQ.Q2.AND.R1.EQ.R2) GO TO 30
GO TO 40
10 P(2)=P3
   Q(2)=Q3
   GO TO 40
20 P(2)=P3
   R(2)=R3
   GO TO 40
30 Q(2)=Q3
   R(2)=R3
   CALL PLANE(P1,Q1,R1,P2,Q2,R2,P3,Q3,R3,A,E,C,D)
   IF(P4.EQ.P5.AND.Q4.EQ.Q5) GO TO 50
   IF(P4.EQ.P5.AND.R4.EQ.R5) GO TO 60
   IF(Q4.EQ.Q5.AND.R4.EQ.R5) GO TO 70
   GO TO 80
50 P(5)=P6
   Q(5)=Q6
   GO TO 80
60 P(5)=P6
   R(5)=R6
   GO TO 80

```

```

70 Q(5)=Q6
   R(5)=R6
80 CALL PLANE(P4,Q4,X4,P5,Q5,R5,P6,Q6,R6,AP,SP,CP,DP)
   IF (A.EQ.0.) GO TO 1
   IF (B.EQ.0.) GO TO 2
   IF (C.EQ.0.) GO TO 3
   RETURN
1 HYP1=SQRT(ABS(Q(2)- Q(1))**2.+/RS(R(2)-P(1))**2.)
  CTHETA=(Q(2)-Q(1))/HYP1
  STHETA=(R(2)-R(1))/HYP1
  SLOPE1=-B/C
  AXIS1=-D/C
  HYP2=SQRT(ABS(Q(5)-Q(4))**2.+ABS(P(5)-P(4))**2.)
  CHETA=(Q(5)-Q(4))/HYP2
  SHETA=(R(5)-R(4))/HYP2
  SLOPE2=-BP/CP
  AXIS2=-DP/CP
  CALL OSLANT(Y,Z,X,Q(1),Q(4),R(1),R(4),SLOPE1,SLOPE2,AXIS1,AXIS2,C
1 HETA,CHETA,STHETA,SHETA)
  CALL RAYCHK(X,Y,7,4,1,1)
  RETURN
2 HYP1=SQRT(ABS(P(2)- P(1))**2.+ABS(P(2)-R(1))**2.)
  CTHETA=(P(2)-P(1))/HYP1
  STHETA=(R(2)-R(1))/HYP1
  SLOPE1=-A/C
  AXIS1=-D/C
  HYP2=SQRT(ABS(P(5)-P(4))**2.+ABS(P(5)-P(4))**2.)
  CHETA=(Q(5)-Q(4))/HYP2
  SHETA=(R(5)-R(4))/HYP2
  SLOPE2=-AP/CP
  AXIS2=-DP/CP
  CALL OSLANT(X,Z,Y,P(1),P(4),R(1),R(4),SLOPE1,SLOPE2,AXIS1,AXIS2,C

```

```

1 THETA,CHETA,STHETA,SHETA)
CALL RAYCHK(X,Y,Z,4,1,1)
RETURN
3 HYP1=SQRT((ABS(Q(2))-Q(1))**2.+ABS(P(2)-P(1))**2.)
CTHETA=(P(2)-P(1))/HYP1
STHETA=(Q(2)-Q(1))/HYP1
SLOPE1=-A/R
AXIS1=-D/B
HYP2=SQRT(ABS(Q(5))-Q(4))**2.+ABS(P(5)-P(4))**2.)
CHETA=(P(5)-P(4))/HYP2
SHETA=(Q(5)-Q(4))/HYP2
SLOPE2=-AP/BP
AXIS2=-DP/BP
CALL OSLANT(X,Y,Z,P(1),Q(1),P(4),Q(4),SLOPE1,SLOPE2,AXIS1,AXIS2,C
1 THETA,CHETA,STHETA,SHETA)
CALL RAYCHK(X,Y,Z,4,1,1)
RETURN
END
SUBROUTINE OSLANT(X,Y,Z,P 1 ,Q 1 ,P 4 ,Q 4 ,SLOPE1,SLOPE2,AXIS1,A
C ***
C *** PART OF SLANTQ ***
C ***
C ***
1 IS2,CTHETA,CHETA,STHETA,SHETA)
DIMENSION X(4),Y(4),Z(4),P(5),Q(6),R(5)
IF(STHETA.GT.0.) SIGN1=1.
IF(STHETA.LT.0.) SIGN1=-1.
IF(SHETA.GT.0.) SIGN2=1.
IF(SHETA.LT.0.) SIGN2=-1.
Y2P=-SIGN1*CTHETA*(Q 1 -Y(1))-SIGN1*STHETA*(X(1)-P 1 )
Y3PP=-SIGN2*CHETA*(Q 4 -Y(4))-SIGN2*SHETA*(X(4)-P 4 )

```



```

A1=-SIGN1*SLOPE2*STHETA-SIGN1*CTHETA
A2=-SIGN1*STHETA*(AXIS2-Y(1))+SIGN1*X(1)*CTHETA
B1=-SIGN1*SLOPE2*CTHETA+SIGN1*STHETA
B2=-SIGN1*CTHETA*(AXIS2-Y(1))-SIGN1*STHETA*X(1)
C1=-SIGN1*SLOPE1*STHETA-SIGN1*CTHETA
C2=-SIGN1*STHETA*(AXIS1-Y(1))+SIGN1*X(1)*CTHETA
D1=2.*C1*Y2P-C1*B2
D2=2.*C2*Y2P-B2*C2
D3=B1*C1
D4=B1*C2+A1*Y2P
E1=-SIGN2*SLOPE1*STHETA-SIGN2*CHETA
E2=-SIGN2*STHETA*(AXIS1-Y(4))+SIGN2*X(4)*CHETA
F1=-SIGN2*SLOPE2*STHETA-SIGN2*CHETA
F2=-SIGN2*STHETA*(AXIS2-Y(4))+SIGN2*X(4)*CHETA
G1=-SIGN2*SLOPE1*CHETA+SIGN2*SHETA
G2=-SIGN2*CHETA*(AXIS1-Y(4))-SIGN2*X(4)*SHETA
H1=-F2*G1-Y3PP*E1
H2=2.*F2*Y3PP-F2*G2-Y3PP*E2
H3=F1*G1
H4=F1*G2-2.*F1*Y3PP
S1=H1*D3-H3*D1
S2=H1*D4+H2*D3-H3*D2-H4*D1
S3=H2*D4-H4*D2
ROOT=S2**2-4.*S1*S3
IF(ROOT.LT.0.) GO TO 300
I=1
X(2)=(-S2-SQRT(S2**2-4.*S1*S3))/(2.*S1)
GO TO 7
6
X(2)=(-S2+SQRT(ROOT))/(2.*S1)
7
Y(2)=SLOPE1*X(2)+AXIS1
X(3)=(H1*X(2)+H2)/(H3*X(2)+H4)
Y(3)=SLOPE2*X(3)+AXIS2

```

```

X3P=A1*X(3)+A2
X2P=C1*X(2)+C2
Y3P=B1*X(3)+B2
T1=X3P*Z(1)-X2P*Z(1)
X3PP=F1*X(3)+F2
X2PP=E1*X(2)+E2
T2=X3PP*Z(4)-X2PP*Z(4)
Z(3)=(X3P*T2-T1*X3PP)/(X2P*X3PP-X3P*X2PP)
Z(2)=(X2PP*Z(3)+T2)/X3PP
I=I+1
  DIFF1=ABS(X(3)-X(2))
  DIFF2=ABS(Y(3)-Y(2))
  IF(DIFF1.LT..001.AND.DIFF2.LT..001.AND.I.LT.3) GO TO 6
  IF(DIFF1.LT..001.AND.DIFF2.LT..001.AND.I.GE.3) GO TO 300
  IF(DIFF1.LT..001.AND.DIFF2.LT..001.AND.Y3PP.OR.Y3PP.LT.Y2PP.AND.Y3PP.GT.Y4PP
Y1P=0.
Y4PP=0.
Y2PP=G1*X(2)+G2
  IF(Y2P.GT.Y1P.AND.Y2P.LT.Y3P.OR.Y2P.LT.Y1P.AND.Y2P.GT.Y3P)
    1 GO TO 300
  IF(Y3PP.GT.Y2PP.AND.Y3PP.LT.Y4PP.OR.Y3PP.LT.Y2PP.AND.Y3PP.GT.Y4PP
    1 GO TO 300
  RETURN
300 Y(2)=-2.
    X(3)=-3.
    Z(2)=-5.
    Y(3)=-7.
    Z(3)=-9.
    X(2)=-2.2
  RETURN
END
SUBROUTINE RAYCHK(X,Y,Z,N,ISR,ISS)

```

```

C*****
C      DETERMINES VALIDITY OF ALL CALCULATED RAY PATHS -
C      ELIMINATES RAYS PASSING THROUGH WALLS
C*****
C
C      INTEGER WNTPLT
C      COMMON/ARF/DMAX
C      COMMON/ARAY/PL,QL,RL,NPT,NPL
C      COMMON/SND/PRESTO,ALPHA
C      COMMON/GL/WNTPLT
C      DIMENSION PL(15,15),QL(15,15),RL(15,15),NPT(15),X(N),Y(N),Z(N)
C      1,XR(5),YR(5),ZR(5),ALFA(15,8),ALPHA(15,8),PRESTO(8)
C
C      COMPUTED GO TO RE-ORIENTS ALL COORDINATES TO ACTUAL
C      LOCATION IN X,Y,Z COORDINATE SYSTEM
C
C      1  GO TO (1,2,3,4,5,6,7),ISR
C      2  DO 8 I=1,N
C          XR(I)=X(I)
C      8  YR(I)=Y(I)
C          DO 9 J=1,N
C              Y(J)=XR(J)
C      9  X(J)=YR(J)
C          GO TO 10
C      3  DO 11 I=1,N
C          ZR(I)=Z(I)
C      11 YR(I)=Y(I)
C          DO 12 J=1,N
C              Z(J)=YR(J)
C      12 Y(J)=ZR(J)
C          GO TO 10
C      4  DO 13 I=1,N

```

```

      ZR(I2)=Z(I2)
13  XR(I2)=X(I2)
      DO 14 J2=1,N
      X(J2)=ZR(J2)
14  Z(J2)=XR(J2)
      GO TO 10
      DO 15 I3=1,N
      ZR(I3)=Z(I3)
      XR(I3)=X(I3)
15  YR(I3)=Y(I3)
      DO 16 J3=1,N
      X(J3)=ZR(J3)
      Y(J3)=XR(J3)
16  Z(J3)=YR(J3)
      GO TO 10
      DO 17 I4=1,N
      YR(I4)=Y(I4)
      ZR(I4)=Z(I4)
17  XR(I4)=X(I4)
      DO 18 J4=1,N
      X(J4)=YR(J4)
      Y(J4)=ZR(J4)
18  Z(J4)=XR(J4)
      GO TO 10
7    XR1=X(1)
      YR1=Y(1)
      ZR1=Z(1)
      XR2=X(2)
      YR2=Y(2)
      ZR2=Z(2)
      X(1)=X(4)
      Y(1)=Y(4)

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```

Z(1)=Z(4)
X(2)=X(3)
Y(2)=Y(3)
Z(2)=Z(3)
X(3)=XR2
Y(3)=YR2
Z(3)=ZR2
X(4)=XR1
Y(4)=YR1
Z(4)=ZR1
      CONTINUE
10  IF(ISS.EQ.0) RETURN
      NA=1
      DO 888 JOCT=1,8
888  ALFA(NA,JOCT)=0.
C
C      DETERMINES RAY VALIDITY BY DIVIDING RAY PATH INTO N-1
C      SEGMENTS (N=TOTAL NUMBER OF SOURCE, RECEIVER, AND
C      REFLECTION POINTS ALONG A RAY PATH) AND CHECKING WITH EACH
C      PLANE IN ROOM BOUNDARY
C
      DO 19 LI=2,N
C
C      EACH LINE SEGMENT CHECKED AGAINST EACH PLANE IN
C      ROOM BOUNDARY
C
      KZ1=0
      KZ2=0
      KZ3=0
      KZ4=0
      DO 20 I1=1,NPL
      L2=L1-1

```

```

C
C      DETERMINES EQUATION OF LINE FROM SEGMENT END POINTS
C
C      CALL ALINE(X(L2),Y(L2),Z(L2),X(L1),Y(L1),Z(L1),ALL,BLL,CLL,DL1,AL
1,BL2,CL2,DL2)
C
C      DETERMINES EQUATION OF PLANE CONTAINING WALL BEING CHECKED
C
C      CALL PLANE(PL(1,11),QL(1,11),RL(1,11),PL(2,11),QL(2,11),RL(2,11),
1PL(3,11),QL(3,11),RL(3,11),AP1,BP1,CP1,DPI)
C
C      DETERMINES IF LINE AND PLANE:
C      1. PARALLEL (FINITE DISTANCE APART OR LYING IN PLANE)
C      2. NOT PARALLEL
C
C      CALL PARLEL(AP1,BP1,CP1,DPI,X(L2),Y(L2),Z(L2),X(L1),Y(L1),Z(L1),L
1,LQ)
C
C      IF PLANE AND LINE PARALLEL AT FINITE DISTANCE,
C      CHECK NEXT WALL
C
C      IF(LP.EQ.1.AND.LQ.EQ.0) GO TO 20
C
C      IF LINE IS IN PLANE DETERMINE IF SEGMENT LIES ANYWHERE
C      WITHIN WALL BOUNDARY
C
C      IF(LP.EQ.1.AND.LQ.EQ.1) GO TO 40
C
C      DETERMINES INTERSECTION OF LINE AND PLANE
C
C      144 CALL POINT(X(L2),Y(L2),Z(L2),X(L1),Y(L1),Z(L1),7(L1),ALL,BLL,CLL,AL
1,BL2,CL2,DL2,AP1,BP1,CP1,DPI,XP,YP,ZP)

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```

C      DETERMINES IF INTERSECTION OCCURS ON LINE SEGMENT-
C      IF NOT, CHECK NEXT WALL
C
C      33  CALL PLINE(XP,YP,ZP,X(L1),Y(L1),Z(L1),X(L2),Y(L2),Z(L2),IES)
C          IF(IES.EQ.0) GO TO 20
C
C      DETERMINES IF INTERSECTION POINT OCCURS WITHIN
C      WALL BOUNDARY
C
C      CALL PTPL(PL,QL,FL,I1,XP,YP,ZP,NPT,IP)
C
C      IF INTERSECTION OCCURS OUTSIDE WALL BOUNDARY AND RAY
C      PATH IS DIRECT (N=2) CHECK NEXT WALL
C
C      IF(IP.EQ.0.AND.N.EQ.2) GO TO 20
C
C      IF INTERSECTION OCCURS WITHIN WALL BOUNDARY AND RAY
C      PATH IS DIRECT - INVALID RAY - WENT THROUGH A WALL
C
C      IF(IP.EQ.1.AND.N.EQ.2) RETURN
C      CK1=ABS(XP-X(L2))
C      CK2=ABS(YP-Y(L2))
C      CK3=ABS(ZP-Z(L2))
C
C      COUNTS NUMBER OF TIMES INTERSECTION OCCURRED WITHIN WALL
C      BOUNDARY AT FIRST END POINT OF LINE SEGMENT
C
C      IF(IP.EQ.1.AND.CK1.LT..01.AND.CK2.LT..01.AND.CK3.LT..01)
C          1KZ1=KZ1+1
C
C      COUNTS NUMBER OF TIMES INTERSECTION OCCURRED OUTSIDE WALL

```

```

C      BOUNDARY AT FIRST END POINT OF LINE SEGMENT
C      IF(IP.EQ.0.AND.CK1.LT..01.AND.CK2.LT..01.AND.CK3.LT..01)
        IKZ2=KZ2+1
        CK4=ABS(XP-X(L1))
        CK5=ABS(YP-Y(L1))
        CK6=ABS(ZP-Z(L1))
C      COUNTS NUMBER OF TIMES INTERSECTION OCCURRED WITHIN WALL
C      BOUNDARY AT SECOND END POINT OF LINE SEGMENT
C      IF(IP.EQ.1.AND.CK4.LT..01.AND.CK5.LT..01.AND.CK6.LT..01)
        IKZ3=KZ3+1
        IF(IP.EQ.1.AND.CK4.LT..01.AND.CK5.LT..01.AND.CK6.LT..01)
          1 NA=NA+1
        IF(IP.EQ.1.AND.CK4.LT..01.AND.CK5.LT..01.AND.CK6.LT..01)
          IGO TO 55
          GO TO 66
          55 DO 75 JT=1,8
              ALFA(NA,JT)=ALPHA(I1,JT)
              75 CONTINUE
C      COUNTS NUMBER OF TIMES INTERSECTION OCCURRED OUTSIDE WALL
C      BOUNDARY AT SECOND END POINT OF LINE SEGMENT
C      IF(IP.EQ.0.AND.CK4.LT..01.AND.CK5.LT..01.AND.CK6.LT..01)
        IKZ4=KZ4+1
C      IF LINE AND PLANE INTERSECT WITHIN WALL BOUNDARY AND NOT AT
C      EITHER LINE SEGMENT END POINT - RAY IS INVALID
C      IF(IP.EQ.1.AND.(CK1.GT..01.OR.CK2.GT..01.OR.CK3.GT..01).

```



```

1AND.(CK4.GT..01.OR.CK5.GT..01.OR.CK6.GT..01)) RETURN
GO TO 20
40 CALL PTPL(PL,QL,RL,I1,X(L2),Y(L2),Z(L2),NPT,IP)
   IF(IP.EQ.1) RETURN
CALL PTPL(PL,QL,RL,I1,X(L1),Y(L1),Z(L1),NPT,IP)
   IF(IP.EQ.1) RETURN
NPR=NPT(I1)
PMAX=PL(1,I1)
PMIN=PL(1,I1)
QMAX=QL(1,I1)
QMIN=QL(1,I1)
RMAX=RL(1,I1)
RMIN=RL(1,I1)
DO 101 I=2,NPR
   IF(PMAX.LT.PL(I,I1)) PMAX=PL(I,I1)
   IF(QMAX.LT.QL(I,I1)) QMAX=QL(I,I1)
   IF(RMAX.LT.RL(I,I1)) RMAX=RL(I,I1)
   IF(PMIN.GT.PL(I,I1)) PMIN=PL(I,I1)
   IF(QMIN.GT.QL(I,I1)) QMIN=QL(I,I1)
   IF(RMIN.GT.RL(I,I1)) RMIN=RL(I,I1)
101 CALL APLAN(PMAX,PMIN,QMAX,QMIN,RMAX,RMIN,AP1,SP1,CPL,X(L1),Y(L1),
   1(L1),X(L2),Y(L2),Z(L2),NO)
   IF(NO.EQ.1) RETURN
20 CONTINUE
C
C   INVALIDATES RAYS WHERE REFLECTION POINT OCCURS ON
C   PLANE OF WALL BUT NOT WITHIN WALL BOUNDARIES
C
   IF(KZ1.EQ.0.AND.KZ2.GT.0.AND.L2.NE.1) RETURN
   IF(KZ3.EQ.0.AND.KZ4.GT.0.AND.L1.NE.N) RETURN
19 CONTINUE
   IF(WNTPLT.EQ.0) GO TO 65

```

```

39      CALL RAPL(X,Y,N)
      CALL PLOT(0.,-DMAX,-3)
      CALL RAPL(X,Z,N)
      CALL PLOT(0.,DMAX,-3)
      CONTINUE
65      CONTINUE
322     CALL SNPPRS(ALFA,X,Y,Z,N)
      RETURN
      END
      SUBROUTINE XLINE(X1,Y1,X2,Y2,SLOPE,AXIS,CCN,KT)
C *****
C ***** DETERMINES EQUATION OF LINE IN TWO DIMENSIONS *****
C *****
C *****
      IF(X2.EQ.X1) GO TO 1
      KT=C
      SLOPE=(Y2-Y1)/(X2-X1)
      AXIS=Y2-SLOPE*X2
      CCN=J.
      RETURN
1      AXIS=0.
      SLOPE=0.
      CCN=X1
      KT=1
      RETURN
      END
      SUBROUTINE ALINPT(X1,Y1,X2,Y2,X3,Y3,X4,Y4,XI,YI)
C *****
C ***** DETERMINES INTERSECTION OF TWO LINES IN TWO DIMENSIONS *****
C *****
C *****

```



```

2      X=- (8*Y+C*Z+D)/A
      GO TO 3
4      Z=- (A*X+B*Y+D)/C
3      AL1=((A*(X1-X)+C*(Y1-Y)+C*(Z1-Z))/(SORT(A**2+B**2+C**2)))
      AL2=((A*(X2-X)+B*(Y2-Y)+C*(Z2-Z))/(SORT(A**2+B**2+C**2)))
      IF (AL1.EQ.(-AL2)) RETURN
      AL1=ABS(AL1)
      AL2=ABS(AL2)
      CHK=ABS(AL2-AL1)
      IF (CHK.LT..0001.AND.AL1.GT..0001) GO TO 5
      IF (CHK.LT..0001.AND.AL1.LT..0001) GO TO 6
      RETURN
5      LP=1
      LQ=0
      RETURN
6      LP=1
      LQ=1
      RETURN
      END
      SUBROUTINE PLINE(XP,YP,ZP,X1,Y1,Z1,X2,Y2,Z2,IES)
C**
C**
C** DETERMINES IF A GIVEN POINT LIES ON LINE SEGMENT
C**
C**
      IES=0
      CHK1=ABS(X1-X2)
      CHK2=ABS(Y1-Y2)
      CHK3=ABS(Z1-Z2)
      CHK4=(X1-XP)
      CHK5=(X2-XP)
      CHK6=(Z1-ZP)

```



```

S4=A2*B3-A3*B2
S5=B3*D2-B2*D3
S6=B2*C3-B3*C2
CHK1=S1*S6-S4*S3
IF(CHK1.NE.0.) GO TO 1
S4=B1*A3-B3*A1
S5=B1*D3-B3*D1
S6=B3*C1-B1*C2
CHK1=S1*S6-S4*S3
IF(CHK1.NE.0.) GO TO 1
S1=A2*B3-A3*B2
S2=B3*D2-B2*D3
S3=B2*C3-B3*C2
CHK1=S1*S6-S4*S3
IF(CHK1.NE.0.) GO TO 1
4 T1=B2*C1-B1*C2
T2=C1*D2-C2*D1
T3=A1*C2-A2*C1
T4=B3*C2-B2*C3
T5=C2*D3-C3*D2
T6=A2*C3-A3*C2
CHK2=T1*T6-T4*T3
IF(CHK2.NE.0.) GO TO 2
T4=B3*C1-B1*C3
T5=C1*D3-C3*D1
T6=A1*C3-A3*C1
CHK2=T1*T6-T4*T3
IF(CHK2.NE.0.) GO TO 2
T1=B3*C2-B2*C3
T2=C2*D3-C3*D2
T3=A2*C3-A3*C2
CHK2=T1*T6-T4*T3

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```

IF(CHK2.NE.O.) GO TO 2
5 G1=A2*C1-A1*C2
  G2=A2*D1-A1*D2
  G3=A1*B2-A2*B1
  G4=A2*C3-A3*C2
  G5=A2*D3-A3*D2
  G6=A3*B2-A2*B3
  CHK3=C1*G6-G4*G3
  IF(CHK3.NE.O.) GO TO 3
  G4=A1*C3-A3*C1
  G5=A1*D3-A3*D1
  G6=A3*B1-A1*B3
  CHK3=G1*G6-G4*G3
  IF(CHK3.NE.O.) GO TO 3
  G1=A2*C3-A3*C2
  G2=A2*D3-A3*D2
  G3=A3*B2-A2*B3
  CHK3=G1*G6-G4*G3
  IF(CHK3.NE.O.) GO TO 3
  GO TO 6
1  XP=(S5*S3-S2*S6)/CHK1
  GO TO 4
2  YP=(T5*T3-T2*T6)/CHK2
  GO TO 5
3  ZP=(G5*G3-G2*G6)/CHK3
6  IF(CHK1.NE.O..AND.CHK2.NE.O..AND.CHK3.NE.O.) RETURN
  IF(CHK1.EQ.O..AND.CHK2.NE.O..AND.CHK3.NE.O.) GO TO 7
  IF(CHK2.EQ.O..AND.CHK1.NE.O..AND.CHK3.NE.O.) GO TO 8
  IF(CHK3.EQ.O..AND.CHK1.NE.O..AND.CHK2.NE.O.) GO TO 9
  GO TO 10
7  IF(A1.NE.O.) XP=-((B1*YP+C1*ZP+D1)/A1
  IF(A2.NE.O.) XP=-((B2*YP+C2*ZP+D2)/A2

```

```

      IF(A3.NE.C.) XP=-(B3*YP+C3*ZP+D3)/A3
      RETURN
8  IF(B1.NE.O.) YP=-(A1*XP+C1*ZP+D1)/B1
   IF(B2.NE.O.) YP=-(A2*XP+C2*ZP+D2)/B2
   IF(B3.NE.O.) YP=-(A3*XP+C3*ZP+D3)/B3
      RETURN
9  IF(C1.NE.O.) ZP=-(A1*XP+B1*YP+D1)/C1
   IF(C2.NE.O.) ZP=-(A2*XP+B2*YP+D2)/C2
   IF(C3.NE.O.) ZP=-(A3*XP+B3*YP+D3)/C3
      RETURN
10 IF(CHK1.EQ.O..AND.CHK2.EQ.O..AND.CHK3.NE.O.) GO TO 11
   IF(CHK1.EQ.O..AND.CHK3.EQ.O..AND.CHK2.NE.O.) GO TO 12
   IF(CHK2.EQ.O..AND.CHK3.EQ.O..AND.CHK1.NE.O.) GO TO 13
11 IF(X1.EQ.X2.AND.Y1.EQ.Y2) GO TO 14
12 IF(X1.EQ.X2.AND.Z1.EQ.Z2) GO TO 15
13 IF(Y1.EQ.Y2.AND.Z1.EQ.Z2) GO TO 16
   GO TO 17
14 YP=Y1
   XP=X1
   GO TO 9
15 XP=X1
   ZP=Z1
   GO TO 8
16 YP=Y1
   ZP=Z1
   GO TO 7
17 WRITE(6,20) A1,B1,C1,D1
20 FORMAT(' ',THE LINE AND PLANE COEFFICIENTS ARE',4F20.10)
   WRITE(6,20) A2,B2,C2,D2
   WRITE(6,20) A3,B3,C3,D3
      RETURN
      END

```



```

C *****
C SUBROUTINE ALINE(X1,Y1,Z1,X2,Y2,Z2,A1,B1,C1,D1,A2,B2,C2,D2)
C *****
C DETERMINES EQUATION OF LINE IN THREE DIMENSIONS
C *****
C
      CALL PLANE(C,-20.,-30.,X1,Y1,Z1,X2,Y2,Z2,A1,B1,C1,D1)
      CALL PLANE(30.,30.,50.,X1,Y1,Z1,X2,Y2,Z2,A2,B2,C2,D2)
      IF(A1.NE.0.) RATIO=A2/A1
      IF(B1.NE.0.) RATIO=B2/B1
      IF(C1.NE.0.) RATIO=C2/C1
      IF(D1.NE.0.) RATIO=D2/D1
      CK1=RATIO*A1
      CK2=RATIO*B1
      CK3=RATIO*C1
      CK4=RATIO*D1
      DIFF1=ABS(A2-CK1)
      DIFF2=ABS(B2-CK2)
      DIFF3=ABS(C2-CK3)
      DIFF4=ABS(D2-CK4)
      IF(DIFF1.LE.1.E-3.AND.DIFF2.LE.1.E-3.AND.DIFF3.LE.
11.E-3.AND.DIFF4.LE.1.E-3) GO TO 1
      RETURN
1 CALL PLANE(8.5,7.2,3.,X1,Y1,Z1,X2,Y2,Z2,A1,B1,C1,D1)
      RETURN
      END
C SUBROUTINE PTPL(PL,QL,RL,I1,X1,Y1,Z1,IPT,IP)
C *****
C DETERMINES IF REFLECTION POINT LIES WITHIN WALL BOUNDARY
C *****
C

```



```

CALL XLINF(PRX,PRY,X1,Y1,SLOPE1,AXIS1,C1,KS)
PL(NP,II)=PL(1,II)
QL(NP,II)=QL(1,II)
CALL XLINF(PL(J,II),QL(J,II),PL(L,II),QL(L,II),SLOPE2,AXIS2,C2,KS)
SLOPE1=ABS(SLOPE1)
SLOPE2=ABS(SLOPE2)
DIFF=ABS(SLOPE1-SLOPE2)
IF(DIFF.LT..001) GO TO 1
CALL ALINPT(PRX,PRY,X1,Y1,PL(J,II),QL(J,II),PL(L,II),QL(L,II),XP,
1P)
IF(DIFF.EQ.0..AND.SLOPE1.EQ.SLOPE2..AND.XP.GE.PL(J,II)..AND.XP.LE.P
1(L,II)) GO TO 10
IF(DIFF.EQ.0..AND.SLOPE1.EQ.SLOPE2..AND.XP.GE.PL(L,II)..AND.XP.LE.P
1(J,II)) GO TO 10
IF(XP.LT.(-.0001)..OR.YP.LT.(-.0001)..OR.XP.GT.X1) GO TO 1
CX=ABS(XP-X1)
CY=ABS(YP-Y1)
IF(XP.GT.PL(J,II)..AND.XP.LE.PL(L,II)..AND.CX.LT..0001..AND.CY.LT..0
1001) RETURN
IF(XP.GT.PL(J,II)..AND.XP.LE.PL(L,II)) NC=NC+1
IF(XP.GE.PL(L,II)..AND.XP.LT.PL(J,II)..AND.CX.LT..0001..AND.CY.LT..0
101) RETURN
IF(XP.GE.PL(L,II)..AND.XP.LT.PL(J,II)) NC=NC+1
IF(PL(J,II).EQ.PL(L,II)..AND.YP.GE.QL(L,II)..AND.YP.LT.QL(J,II)..
1AND.CX.LT..0001..AND.CY.LT..0001) RETURN
IF(PL(J,II).EQ.PL(L,II)..AND.YP.GE.QL(L,II)..AND.YP.LT.QL(J,II)) NC
1NC+1
IF(PL(J,II).EQ.PL(L,II)..AND.YP.GT.QL(J,II)..AND.YP.LE.QL(L,II)..
1AND.CX.LT..0001..AND.CY.LT..0001) RETURN
IF(PL(J,II).EQ.PL(L,II)..AND.YP.GT.QL(J,II)..AND.YP.LE.QL(L,II)) NC
1NC+1
1 CONTINUE
1

```



```

      RETURN
    END
  SUBROUTINE APLA(P1,P2,Q1,Q2,F1,F2,A,B,C,X1,Y1,Z1,X2,Y2,Z2,M)
C ***
C ***
C *** DETERMINES IF LINE SEGMENT IS PARTIALLY CONTAINED
C *** WITHIN BOUNDED PLANE ***
C ***
C ***
      NJ=0
      IF(A.EQ.0..AND.B.EQ.0..) GO TO 2
      IF(A.EQ.0..AND.C.EQ.0..) GO TO 3
      IF(B.EQ.0..AND.C.EQ.0..) GO TO 1
      IF(B.EQ.0..) GO TO 2
      IF(A.EQ.0..) GO TO 3
      IF(C.EQ.0..) GO TO 1
      1 IF(X1.LT.P2.AND.X2.GT.P1) NJ=1
        IF(X1.GT.P1.AND.X2.LT.P2) NJ=1
      RETURN
      2 IF(Z1.LT.P2.AND.Z2.GT.P1) NJ=1
        IF(Z1.GT.P1.AND.Z2.LT.P2) NJ=1
      RETURN
      3 IF(Y1.LT.Q2.AND.Y2.GT.Q1) NJ=1
        IF(Y1.GT.Q1.AND.Y2.LT.Q2) NJ=1
      RETURN
    END
  SUBROUTINE ACME(XS,YS,ZS)
C ***
C ***
C *** DETERMINES ALL SINGLE REFLECTION PAYS IN REGULAR ROOM ***
C ***
C ***

```

```

COMMON/RAY1/XP,YF,ZR,A,B,C,N1,N2,N3,N4
DIMENSION X(3),Y(3),Z(3),A(5),B(5),C(5)
X(3)=XR
Y(3)=YR
Z(3)=ZR
X(1)=XS
Y(1)=YS
Z(1)=ZS
DO 1 I=1,N1
  X(2)=A(I)
  IF(XS.EQ.A(I)).OR.XR.EQ.A(I)) GO TO 1
  IF(X(2).LT.X(3).AND.X(2).GT.X(1)).OR.X(2).LT.X(1).AND.X(2).GT.X(3)
    1GO TO 1
    CALL REFL1(Z,X,Y)
    CALL RAYCHK(X,Y,Z,3,1,1)
  1 CONTINUE
  DO 2 J=1,N2
    Y(2)=B(J)
    IF(YS.EQ.B(J)).OR.YP.EQ.B(J)) GO TO 2
    IF(Y(2).LT.Y(3).AND.Y(2).GT.Y(1)).OR.Y(2).LT.Y(1).AND.Y(2).GT.Y(3)
      1GO TO 2
      CALL REFL1(X,Y,Z)
      CALL RAYCHK(X,Y,Z,3,1,1)
    2 CONTINUE
    DO 3 K=1,N3
      Z(2)=C(K)
      IF(ZS.EQ.C(K)).OR.ZR.EQ.C(K)) GO TO 3
      IF(Z(2).LT.Z(3).AND.Z(2).GT.Z(1)).OR.Z(2).LT.Z(1).AND.Z(2).GT.Z(3)
        1GO TO 3
        CALL REFL1(X,Z,Y)
        CALL RAYCHK(X,Y,Z,3,1,1)
      3 CONTINUE
    8

```

```

      RETURN
    END
    SUBROUTINE REFL1(X,Y,Z)
C*****
C    RAY TRACING EQUATIONS FOR ONE REFLECTION BETWEEN SOURCE
C    AND RECEIVER
C*****
C
C    DIMENSION X(3),Y(3),Z(3)
C    A1=X(1)
C    A3=X(3)
C    B1=Y(1)
C    C1=Z(1)
C    B3=Y(3)
C    C3=Z(3)
C    B2=Y(2)
C    A2=(A1*(B2-B3)+A3*(B2-B1))/(2.*B2-B1-B3)
C    X(2)=A2
C    C2=(C1*(B2-B3)+C3*(B2-B1))/(2.*B2-B1-B3)
C    Z(2)=C2
C    RETURN
C    END
    SUBROUTINE ATWO(XS,YS,ZS)
C*****
C    RAY PATHS FOR TWO REFLECTIONS FROM WALLS PARALLEL TO
C    COORDINATE PLANES
C*****
C
C    COMMON/RAY1/XR,YR,ZR,A,B,C,N1,N2,N3,N4
C    DIMENSION X(4),Y(4),Z(4),A(5),B(5),C(5)

```

```

X(4)=XR
Y(4)=YR
Z(4)=ZR
X(1)=XS
Y(1)=YS
Z(1)=ZS
CALL ATWQA(Z,Y,X,N2,B,1)
CALL ATWQA(Y,Z,X,N3,C,3)
CALL ATWQA(Z,X,Y,N1,A,2)
CALL ATWOB(X,Y,Z,N2,N3,B,C,1)
CALL ATWOB(Z,Y,X,N2,N1,B,A,4)
CALL ATWOB(Y,Z,X,N3,N1,C,A,5)
CALL ATWOB(Y,X,Z,N1,N3,A,C,2)
CALL ATWOB(Z,X,Y,N1,N2,A,B,6)
CALL ATWOB(X,Z,Y,N3,N2,C,B,3)
RETURN
END
SUBROUTINE ATWQA(Z,Y,X,N2,B,ISR)
C *****
C ***** DETERMINES RAY PATHS BETWEEN PARALLEL WALLS FOR TWO REFLECTIONS *****
C *****
C
  DIMENSION X(4),Y(4),Z(4),B(5)
  DO 1 I=1,N2
  DO 2 N=1,N2
    Y(2)=B(I)
    Y(3)=B(N)
    IF(Y(1).GE.Y(2).AND.Y(1).GE.Y(3).AND.Y(4).GE.Y(2).AND.Y(4).GE.Y(3)
1) GO TO 2
    IF(Y(1).LE.Y(2).AND.Y(1).LE.Y(3).AND.Y(4).LE.Y(2).AND.Y(4).LE.Y(3)
1) GO TO 2

```



```

IF(Y(1).EQ.Y(2)) GO TO 1
IF(Y(3).EQ.Y(4)) GO TO 2
IF(Y(2).EQ.Y(3)) GO TO 2
CHK1=2.*Y(2)-2.*Y(3)+Y(4)-Y(1)
CHK2=2.*Y(3)-Y(2)-Y(4)
IF(CHK1.EQ.0.) GO TO 1
IF(CHK2.EQ.0.) GO TO 1
CALL REFL21(Z,Y,X)
IF(Y(2).LT.Y(3).AND.Y(2).GT.Y(1).OR.Y(2).GT.Y(3).AND.Y(2).LT.Y(1)
1 GO TO 2
IF(Y(3).LT.Y(4).AND.Y(3).GT.Y(2).OR.Y(3).GT.Y(4).AND.Y(3).LT.Y(2)
1 GO TO 2
CALL RAYCHK(X,Y,Z,4,ISR,1)
CALL RAYCHK(X,Y,Z,4,ISR,0)
2 CONTINUE
1 CONTINUE
RETURN
END
SUBROUTINE ATWOB(X,Y,Z,N2,N3,B,C,ISR)
C*****
C DETERMINES RAY PATHS BETWEEN PERPENDICULAR WALL FOR TWO
C REFLECTIONS
C*****
C
C DIMENSION X(4),Y(4),Z(4),B(5),C(5)
DO 1 I=1,N2
DO 2 N=1,N3
Y(2)=B(I)
Z(3)=C(N)
IF(Y(1).EQ.Y(2)) GO TO 1
IF(Z(3).EQ.Z(4)) GO TO 2

```



```

D1=Z(1)
D4=Z(4)
E4=Y(4)
E1=Y(1)
F1=X(1)
F4=X(4)
E2=Y(2)
E3=Y(3)
D2=(D4*(E2-E1)+D1*(E4+E2-2.*E3))/(2.*E2-2.*E3+E4-E1)
Z(2)=D2
D3=(D2*(E3-E4)+D4*(E3-E2))/(2.*E3-E2-E4)
Z(3)=D3
F2=(F4*(E2-E1)+F1*(E4+E2-2.*E3))/(2.*E2-2.*E3+E4-E1)
X(2)=F2
F3=(F1*(E2-E3)+F2*(E3-2.*E2+E1))/(E1-F2)
X(3)=F3
RETURN
END
SUBROUTINE REFL22(X,Y,Z)
C ***
C *** RAY TRACING EQUATIONS FOR TWO REFLECTIONS -
C *** PERPENDICULAR WALLS ***
C ***
C
DIMENSION X(4),Y(4),Z(4)
D1=X(1)
D4=X(4)
E1=Y(1)
E4=Y(4)
F1=Z(1)
F4=Z(4)

```

```

E2=Y(2)
F3=Z(3)
D2=(D1*(E2-E4)+D4*(E2-E1))/(2.*E2-E1-E4)
X(2)=D2
D3=(D1*(F3-F4)+D4*(F3-F1))/(2.*F3-F1-F4)
X(3)=D3
F2=(F1*(E4-E2)+F3*(2.*E1-2.*E2)+F4*(E2-E1))/(E1+E4-2.*E2)
Z(2)=F2
E3=(E1*(F4-F3)-E2*(F4-F3)+E4*(F2-F1))/(F2-F1)
Y(3)=E3
RETURN
END
SUBROUTINE A31(XS,YS,ZS)
C *****
C *****
C ***** DETERMINES RAY PATHS FOR THREE REFLECTIONS - WALLS
C ***** PARALLEL TO COORDINATE PLANES
C *****
C *****
COMMON/RAY1/XR,YR,ZR,A,B,C,N1,N2,N3,N4
DIMENSION X(5),Y(5),Z(5),A(5),B(5),C(5)
X(5)=XR
Y(5)=YR
Z(5)=ZR
X(1)=XS
Y(1)=YS
Z(1)=ZS
CALL A31A(X,Y,Z,N1,N2,N3,A,B,C,1,1,2,3)
CALL A31A(Z,X,Y,N3,N1,N2,C,A,B,1,1,2,3)
CALL A31A(Y,X,Z,N2,N1,N3,B,A,C,2,1,2,3)
CALL A31A(Y,Z,X,N2,N3,N1,B,C,A,5,1,2,3)
CALL A31A(X,Z,Y,N1,N3,N2,A,C,B,3,1,2,3)

```

```

CALL A31A(Z,Y,X,N3,N2,N1,C,3,A,4,1,2,3)
CALL A31A(X,Y,Z,N2,N2,N1,B,3,A,1,1,1,2)
CALL A31A(Y,Z,X,N3,N3,N2,C,C,8,5,1,1,2)
CALL A31A(Y,X,Z,N1,N1,N2,A,A,3,2,1,1,2)
CALL A31A(7,Y,X,N2,N2,N3,B,C,4,1,1,2)
CALL A31A(Z,X,Y,N1,N1,N3,A,A,C,6,1,1,2)
CALL A31A(X,Z,Y,N3,N3,N1,C,C,A,3,1,1,2)
CALL A31A(X,Y,Z,N2,N2,N2,B,3,B,1,1,1,1)
CALL A31A(Y,X,Z,N1,N1,N1,A,A,A,2,1,1,1)
CALL A31A(X,Z,Y,N3,N3,N3,C,C,C,3,1,1,1)
CALL A31A(X,Y,Z,N1,N2,N1,A,B,A,1,1,2,1)
CALL A31A(Y,Z,X,N2,N3,N2,B,C,B,5,1,2,1)
CALL A31A(Z,Y,X,N3,N2,N3,C,B,C,4,1,2,1)
CALL A31A(X,Z,Y,N1,N3,N1,A,C,A,3,1,2,1)
CALL A31A(Y,X,7,N2,N1,N2,B,4,B,2,1,2,1)
CALL A31A(Z,X,Y,N3,N1,N3,C,A,C,6,1,2,1)
CALL A31A(X,Y,Z,N1,N2,N2,A,B,B,1,1,2,2)
CALL A31A(Z,Y,X,N3,N2,N2,C,B,8,4,1,2,2)
CALL A31A(Y,Z,X,N2,N3,N3,B,C,C,5,1,2,2)
CALL A31A(Y,X,Z,N2,N1,N1,B,A,A,2,1,2,2)
CALL A31A(Z,X,Y,N3,N1,N1,C,A,A,6,1,2,2)
CALL A31A(X,Z,Y,N1,N3,N3,A,C,C,3,1,2,2)

```

```

RETURN

```

```

END

```

```

SUBROUTINE A31A(X,Y,Z,N1,N2,N3,A,B,C,ISK,I1,I2,I3)

```

```

C

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```

C *****

```

```

C CALLS RAY TRACING EQUATIONS FOR ALL THREE REFLECTION

```

```

C COMBINATIONS

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C *****

```

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C

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```

      DIMENSION A(5),B(5),C(5),X(5),Y(5),Z(5)

```

```

1  IF(I1.NE.I2.AND.I1.NE.I3.AND.I2.NE.I3) GO TO 1
   IF(I1.EQ.I2.AND.I1.EQ.I3) GO TO 2
   IF(I1.EQ.I2) GO TO 3
   IF(I1.EQ.I3) GO TO 4
   IF(I2.EQ.I3) GO TO 5
   DO 10 I=1,N1
   DO 20 J=1,N2
   DO 30 K=1,N3
   X(2)=A(I)
   Y(3)=B(J)
   Z(4)=C(K)
   IF(X(1).EQ.X(2)) GO TO 10
   IF(Z(4).EQ.Z(5)) GO TO 30
   CHK1=2.*Z(4)-Z(5)-Z(1)
   CHK2=X(1)+X(5)-2.*X(2)
   IF(CHK1.EQ.0.) GO TO 30
   IF(CHK2.EQ.0.) GO TO 10
   CALL REFL35(X,Y,Z)
   IF(X(2).GT.X(1).AND.X(2).LT.X(3).OR.X(2).GT.X(3).AND.X(2).LT.X(1)
1  GO TO 30
   IF(Y(3).GT.Y(2).AND.Y(3).LT.Y(4).OR.Y(3).LT.Y(2).AND.Y(3).GT.Y(4)
1  GO TO 30
   IF(Z(4).GT.Z(3).AND.Z(4).LT.Z(5).OR.Z(4).LT.Z(3).AND.Z(4).GT.Z(5)
1  GO TO 30
   CALL RAYCHK(X,Y,7,5,ISR,1)
   IF(ISR.EQ.5) GO TO 21
   IF(ISR.EQ.6) GO TO 22
   CALL RAYCHK(X,Y,Z,5,ISR,0)
   GO TO 30
21  CALL RAYCHK(X,Y,Z,5,6,0)
   GO TO 30
22  CALL RAYCHK(X,Y,Z,5,5,0)

```

```

30 CONTINUE
20 CONTINUE
10 CONTINUE
  RETURN
2  DO 70 I=1,N1
   DO 80 J=1,N2
    DO 90 K=1,N3
     Y(2)=A(I)
     Y(3)=B(J)
     Y(4)=C(K)
     IF(Y(1).EQ.Y(2)) GO TO 70
     IF(Y(4).EQ.Y(5)) GO TO 90
     IF(Y(2).EQ.Y(3)) GO TO 80
     IF(Y(3).EQ.Y(4)) GO TO 90
     CK1=2.*(Y(3)-Y(2)-Y(4))+Y(5)+Y(1)
     IF(CK1.EQ.C.) GO TO 90
     CALL REFL33(X,Y,Z)
     IF(Y(2).GT.Y(1).AND.Y(2).LT.Y(3).OR.Y(2).GT.Y(3).AND.Y(2).LT.Y(1)
1    GO TO 90
     IF(Y(3).GT.Y(2).AND.Y(3).LT.Y(4).OR.Y(3).GT.Y(4).AND.Y(3).LT.Y(2)
1    GO TO 90
     IF(Y(4).GT.Y(3).AND.Y(4).LT.Y(5).OR.Y(4).GT.Y(5).AND.Y(4).LT.Y(3)
1    GO TO 90
     CALL RAYCHK(X,Y,7,5,ISR,1)
     CALL RAYCHK(X,Y,Z,5,ISR,0)
90 CONTINUE
80 CONTINUE
70 CONTINUE
  RETURN
3  DO 40 I=1,N1
   DO 50 J=1,N2
    DO 60 K=1,N3

```

```

Y(2)=A(I)
Y(3)=B(J)
X(4)=C(K)
CHK3=ABS(Y(1)-Y(2))
IF(CHK3.LE..0001) GO TO 40
IF(X(4).EQ.X(5)) GO TO 60
IF(Y(2).EQ.Y(3)) GO TO 50
CHK1=ABS(X(1)+X(5)-2.*X(4))
CHK2=ABS(2.*Y(2)-2.*Y(3)-Y(1)+Y(5))
IF(CHK1.LE..0001) GO TO 60
IF(CHK2.LE..0001) GO TO 50
CALL REFL31(X,Y,Z)
IF(Y(2).GT.Y(1).AND.Y(2).LT.Y(3).OR.Y(2).GT.Y(3).AND.Y(2).LT.Y(1)
1 GO TO 60
IF(Y(3).GT.Y(2).AND.Y(3).LT.Y(4).OR.Y(3).GT.Y(4).AND.Y(3).LT.Y(2)
1 GO TO 60
IF(X(4).GT.X(3).AND.X(4).LT.X(5).OR.X(4).GT.X(5).AND.X(4).LT.X(3)
1 GO TO 60
CALL RAYCHK(X,Y,Z,5,ISR,1)
IF(ISR.EQ.5) GO TO 61
IF(ISR.EQ.6) GO TO 62
CALL RAYCHK(X,Y,Z,5,ISR,0)
GO TO 60
61 CALL RAYCHK(X,Y,Z,5,6,0)
GO TO 60
62 CALL RAYCHK(X,Y,Z,5,5,0)
60 CONTINUE
50 CONTINUE
40 CONTINUE
RETURN
4 DO 100 I=1,N1
DO 110 J=1,N2

```



```

DO 120 K=1,N3
X(2)=A(I)
Y(3)=B(J)
X(4)=C(K)
IF(X(1).EQ.X(2)) GO TO 100
IF(X(4).EQ.X(5)) GO TO 120
CHK1=Y(5)-2.*Y(3)+Y(1)
CHK2=2.*X(4)-2.*X(2)-X(5)+X(1)
IF(CHK1.EQ.0.) GO TO 110
IF(CHK2.EQ.0.) GO TO 120
CALL REFL34(X,Y,Z)
IF(X(2).GT.X(1).AND.X(2).LT.X(3).OR.X(2).GT.X(3).AND.X(2).LT.X(1)
1 GO TO 120
IF(Y(3).GT.Y(2).AND.Y(3).LT.Y(4).OR.Y(3).GT.Y(4).AND.Y(3).LT.Y(2)
1 GO TO 120
IF(X(4).GT.X(3).AND.X(4).LT.X(5).OR.X(4).GT.X(5).AND.X(4).LT.X(3)
1 GO TO 120
CALL RAYCHK(X,Y,Z,5,ISR,1)
IF(ISR.EQ.5) GO TO 101
IF(ISR.EQ.6) GO TO 102
CALL RAYCHK(X,Y,Z,5,ISR,0)
GO TO 100
101 CALL RAYCHK(X,Y,Z,5,6,C)
GO TO 100
102 CALL RAYCHK(X,Y,Z,5,5,0)
120 CONTINUE
110 CONTINUE
100 CONTINUE
RETURN
5 DO 130 I=1,N1
DO 140 J=1,N2
DO 150 K=1,N3

```

```

X(2)=A(I)
Y(3)=B(J)
Y(4)=C(K)
IF(X(1).EQ.X(2)) GO TO 130
IF(Y(4).EQ.Y(5)) GO TO 150
IF(Y(3).EQ.Y(4)) GO TO 150
CHK1=2.*X(2)-X(1)-X(5)
CHK2=2.*Y(4)-2.*Y(3)-Y(5)+Y(1)
CHK3=Y(3)-Y(1)
IF(CHK1.EQ.0.) GO TO 130
IF(CHK2.EQ.0.) GO TO 150
IF(CHK3.EQ.0.) GO TO 140
CALL REFL32(X,Y,Z)
IF(X(2).GT.X(1).AND.X(2).LT.X(3).OR.X(2).GT.X(3).AND.X(2).LT.X(1)
1 GO TO 150
IF(Y(3).GT.Y(2).AND.Y(3).LT.Y(4).OR.Y(3).GT.Y(4).AND.Y(3).LT.Y(2)
1 GO TO 150
IF(Y(4).GT.Y(3).AND.Y(4).LT.Y(5).OR.Y(4).GT.Y(5).AND.Y(4).LT.Y(3)
1 GO TO 150
CALL RAYCHK(X,Y,Z,5,ISR,1)
IF(ISR.EQ.5) GO TO 151
IF(ISR.EQ.6) GO TO 152
CALL RAYCHK(X,Y,Z,5,ISR,0)
GO TO 150
151 CALL RAYCHK(X,Y,Z,5,6,0)
GO TO 150
152 CALL RAYCHK(X,Y,Z,5,5,C)
150 CONTINUE
140 CONTINUE
130 CONTINUE
RETURN
END

```

```

C
C      SUBROUTINE REFL31(X,Y,Z)
C      *****
C      DETERMINES RAY PATHS FOR THREE REFLECTIONS FROM TWO
C      PARALLEL WALLS AND ONE WALL PERPENDICULAR TO THE PARALLEL
C      WALLS
C      *****
C      DIMENSION X(5),Y(5),Z(5)
C      D1=X(1)
C      D5=X(5)
C      E1=Y(1)
C      E5=Y(5)
C      F1=Z(1)
C      F5=Z(5)
C      D4=X(4)
C      E2=Y(2)
C      E3=Y(3)
C      F4=(F5*(D1-D4)+F1*(D5-D4))/(D1+D5-2.*D4)
C      F2=(F5*(E2-E1)+F1*(F5+E2-2.*E3))/(2.*E2-2.*E3-E1+E5)
C      F3=(F1*(E2-E3)-F2*(2.*E2-E1-E3))/(E1-E2)
C      D2=(D1*(E5-2.*E3+E2)+2.*D4*(E2-F1)+D5*(E1-E2))/(2.*E2-2.*E3-F1+E5)
C      CHK1=ABS(D2-D1)
C      IF(CHK1.LE..0001) GO TO 1
C      E4=(E1*(D4-D5)+E2*(D5-D4)+E5*(D2-D1))/(D2-D1)
C      D3=(D1*(E3-E2)+D2*(2.*E2-E3-E1))/(E2-E1)
C      Z(4)=F4
C      Z(2)=F2
C      Z(3)=F3
C      X(2)=D2
C      Y(4)=E4
C      X(3)=D3

```



```

Z(3)=F3
Z(4)=F4
X(4)=D4
X(3)=D3
Y(2)=E2
GO TO 2
1 Y(2)=-1.5
2 RETURN
END
SUBROUTINE REFL33(X,Y,Z)
C ***
C *** RAY PATHS FOR THREE REFLECTIONS FROM PARALLEL WALLS ***
C ***
C ***
C
DIMENSION X(5),Y(5),Z(5)
D1=X(1)
D5=X(5)
E1=Y(1)
E5=Y(5)
F1=Z(1)
F5=Z(5)
E2=Y(2)
E3=Y(3)
E4=Y(4)
F2=(F1*(2.*E3-2.*E4+E5-E2)-F5*(E2-E1))/(2.*E3-2.*E2-2.*E4+E5+E1)
F4=(F5*(E2-E1)+F2*(E5-E4)-F1*(E5-E4))/(E2-E1)
D4=(D5*(2.*E3-2.*E2-E4+E1)+D1*(E5-E4))/(E5-2.*E4+2.*E3-2.*E2+E1)
D2=(D1*(E5-E4)+D5*(E1-E2)+D4*(E2-E1))/(E5-E4)
D3=(D4*(E1-E2)+D2*(E4-E3)+D1*(E3-E4))/(E1-E2)
F3=(F4*(E2-E1)+F2*(E3-E4)-F1*(E3-E4))/(E2-E1)
Z(2)=F2

```



```

F4=(F1*(D4-D2)-F2*(D4+D1-2.*D2))/(D2-D1)
E4=(E3*(2.*D5-2.*D4)+E1*(D4-D5)+E5*(2.*D2-D1-D4))/(2.*D2-2.*D4+D5
1D1)
CHK1=E4-E5
IF(CHK1.EQ.0.) GO TO 1
D3=(D5*(E3-E4)+D4*(2.*E4-E3-E5))/(E4-E5)
E2=(E5*(D1-D2)+E4*(D2-D1)+E1*(D5-D4))/(D5-D4)
Z(3)=F3
Z(2)=F2
Z(4)=F4
Y(4)=E4
X(3)=D3
Y(2)=E2
GO TO 2
1 X(3)=-1.2
Y(2)=-2.5
2 RETURN
END
SUBROUTINE REFL35(X,Y,Z)
C ***
C *** RAY PATHS FOR REFLECTION FROM THREE MUTUALLY PERPENDICULAR
C *** WALLS ***
C ***
C ***
C
DIMENSION X(5),Y(5),Z(5)
D1=X(1)
D5=X(5)
E1=Y(1)
E5=Y(5)
F1=Z(1)
F5=Z(5)

```

```

D2=X(2)
E3=Y(3)
F4=Z(4)
E4=(E1*(F5-F4)+E3*(2.*F4-2.*F5)+F5*(F4-F1))/(2.*F4-F5-F1)
CHK1=E4-E5
IF(CHK1.EQ.0.) GO TO 1
F3=(F5*(E1-E3)-F4*(E1-E3)+F1*(E4-E5))/(E4-E5)
E2=(E3*(2.*D1-2.*D2)+E1*(D5-D2)+E5*(D2-D1))/(D1+D5-2.*D2)
CHK2=E1-E2
IF(CHK2.EQ.0.) GO TO 1
D3=(D1*(E3-E5)-D2*(E3-E5)+D5*(E1-E2))/(E1-E2)
D4=(D1*(E4-E5)-D2*(E4-E5)+D5*(E1-E2))/(E1-E2)
CHK3=D4-D5
IF(CHK3.EQ.0.) GO TO 1
F2=(F5*(D1-D2)-F4*(D1-D2)-F1*(D5-D4))/(D4-D5)
Y(4)=E4
Z(3)=F3
Y(2)=E2
X(3)=D3
X(4)=D4
Z(2)=F2
GO TO 2
1  Z(2)=-1.2
   Y(4)=-3.8
   Z(3)=-2.2
   Y(2)=-1.8
   X(3)=-4.3
   X(4)=-2.9
2  RETURN
   END
SUBROUTINE SLANTI(P,Q,R,XS,YS,ZS)

```



```

C ***
C *** RAY PATH CALCULATION FOR ONE REFLECTION FROM SLANTED
C *** SURFACE
C ***
C ***
C ***
COMMON/RAY1/XR,YR,ZR,A,B,C,N1,N2,N3,N4
DIMENSION X(3),Y(3),Z(3),A(5),B(5),C(5),P(3,15),R(3,15)
X(1)=XS
Y(1)=YS
Z(1)=ZS
X(3)=XR
Y(3)=YR
Z(3)=ZR
DO 1 J=1,N4
  CALL ASLANT(X,Y,Z,P(1,J),Q(1,J),R(1,J),P(2,J),R(2,J),P(3,J)
  1,Q(3,J),R(3,J))
  CALL RAYCHK(X,Y,Z,3,1,1)
1 CONTINUE
  RETURN
END
SUBROUTINE ASLANT(X,Y,Z,P1,Q1,R1,P2,Q2,R2,P3,Q3,R3)
C ***
C ***
C *** RAY PATH FOR ONE REFLECTION FROM SLANTED SURFACE
C ***
C ***
C ***
DIMENSION X(3),Y(3),Z(3),P(3),Q(3),R(3)
P(1)=P1
Q(1)=Q1
R(1)=R1
P(2)=P2
Q(2)=Q2

```

```

R(2)=R2
P(3)=P3
Q(3)=Q3
R(3)=R3
  IF (P1.EQ.P2.AND.Q1.EQ.Q2) GO TO 6
  IF (P1.EQ.P2.AND.R1.EQ.R2) GO TO 7
  IF (Q1.EQ.Q2.AND.R1.EQ.R2) GO TO 8
  GO TO 9
6  P(2)=P3
   Q(2)=Q3
   GO TO 9
7  P(2)=P3
   R(2)=R3
   GO TO 9
8  Q(2)=Q3
   R(2)=R3
   CALL PLANE(P1,Q1,R1,P2,Q2,R2,P3,Q3,Q3,A,B,C,E)
   IF (A.EQ.O.) GO TO 1
   IF (B.EQ.O.) GO TO 2
   IF (C.EQ.O.) GO TO 3
   RETURN
1  HYP=SQRT(ABS(Q(2)-Q(1))*2.+ABS(R(2)-R(1))*2.)
   CTHETA=(Q(2)-Q(1))/HYP
   STHETA=(R(2)-R(1))/HYP
   CALL SLANTA(STHETA,CTHETA,Y,Z,X,Q1,R1)
   RETURN
2  HYP=SQRT(ABS(P(2)-P(1))*2.+ABS(R(2)-R(1))*2.)
   CTHETA=(P(2)-P(1))/HYP
   STHETA=(R(2)-R(1))/HYP
   CALL SLANTA(STHETA,CTHETA,X,Z,Y,P1,R1)
   RETURN
3  HYP=SQRT(ABS(Q(2)-Q(1))*2.+ABS(P(2)-P(1))*2.)

```

```

CTHETA=(P(2)-P(1))/HYP
STHETA=(Q(2)-Q(1))/HYP
CALL SLANTA(STHETA,CTHETA,X,Y,Z,P1,Q1)
RETURN
END
SUBROUTINE SLANTA(STHETA,CTHETA,X,Y,Z,P1,Q1)
C ***
C *** RAY TRACING EQUATIONS FOR ONE REFLECTION FROM SLANTED SURFACE ***
C ***
C
  DIMENSION X(3),Y(3),Z(3)
  IF(STHETA.GT.0.) SIGN=1.
  IF(STHETA.LT.0.) SIGN=-1.
  Y3P=-SIGN*CTHETA*(Y(3)-Y(1))-SIGN*STHETA*(X(1)-X(3))
  X3P=-SIGN*STHETA*(Y(3)-Y(1))-SIGN*CTHETA*(X(3)-X(1))
  Y2P=-SIGN*CTHETA*(Q1-Y(1))-SIGN*STHETA*(X(1)-P1)
  X2P=(-X3P*Y2P)/(Y3P-2.*Y2P)
  X(2)=X(1)+X2P*CTHETA*(-SIGN)+Y2P*SIGN*STHETA
  Y(2)=Y(1)+X2P*STHETA*(-SIGN)-Y2P*CTHETA*SIGN
  Z(2)=(Z(3)+X2P+Z(1))*(X3P-X2P)/X3P
  RETURN
  END
SUBROUTINE SNPPES(ALPH,X,Y,Z,N)
C ***
C *** THIS SUBROUTINE CALCULATES THE TOTAL SOUND PRESSURE ***
C *** AT EACH RECEIVER POSITION BY CALCULATING THE PATH ***
C *** LENGTHS AND MULTIPLYING BY THE APPROPRIATE ALPHAS ***
C *** FOR REFLECTIONS OFF OF WALLS ETC. ***
C ***
C

```

```

COMMON/SP/POWER
COMMON/SND/PRESTO,ALPHA
DIMENSION ALPHA(15,8),X(N),Y(N),Z(N),ALPHA(15,8)
1,POWER(8),PRESTO(8)
PATH1=SQRT((X(2)-X(1))**2+(Y(2)-Y(1))**2+Z(2)-Z(1))**2)
PATH2=C.
PATH3=C.
PATH4=C.
70 IF (N-2) 74,77,70
PATH2=SQRT((X(2)-X(3))**2+(Y(2)-Y(3))**2+Z(2)-Z(3))**2)
71 IF (N-3) 74,77,71
PATH3=SQRT((X(4)-X(3))**2+(Y(4)-Y(3))**2+Z(4)-Z(3))**2)
72 IF (N-4) 74,77,72
PATH4=SQRT((X(4)-X(5))**2+(Y(4)-Y(5))**2+Z(4)-Z(5))**2)
74 IF (N-5) 74,77,74
WRITE(6,73)
73 FORMAT(5X,'SOMETHING IS WRONG IN SNOOPS')
77 CONTINUE
PATH=PATH1+PATH2+PATH3+PATH4
DC 25 JOCT=1,8
AMP=8.037672*SQRT(POWER(JOCT))
PRMS=(AMP/(PATH*1.414214))*3.28084
PREF=0.00002
SPL=20.*ALOG10(PRMS/PREF)
IF (N.EQ.2) GO TO 3
LLLL=N-1
DO 1 J=2,LLLL
PRMS=PRMS*SQRT(1.-ALPHA(J,JOCT))
1 CONTINUE
3 CONTINUE
PRESTO(JOCT)=SQRT(PRESTO(JOCT)**2+PRMS**2)
25 CONTINUE

```

```

      RETURN
    END
    SUBROUTINE PRSLEV
C *****
C **      THE TOTAL SOUND PRESSURE LEVEL (PL) IS CALCULATED
C **      HERE FOR EACH RECEIVER POSITION.
C *****
C
      COMMON/SND/PRESTO,ALPHA
      DIMENSION ALPHA(15,8),PRESTO(8)
      TOTP=0.
      PREF=0.00002
      DO 10 JOCT=1,8
        TOT SPL=20.*ALOG10(PRESTO(JOCT)/PREF)
        TOTP=TOTP+(PRESTO(JOCT))**2
        WRITE(6,333)
333   FORMAT(' *****
1     WRITE(6,1) JOCT,TOT SPL
1     FORMAT(' ',THE OCTAVE BAND AND CORRESPONDING SOUND PRESSURE
1     LEVEL IS',2X,12,' ',F5.1,' ',',',DB')
10    CONTINUE
      TOTAL=10.*ALOG10(TOTP/(PREF**2))
      WRITE(6,2) TOTAL
2     FORMAT(' ',THE TOTAL OVERALL SOUND PRESSURE LEVEL IS',2X,F5.1,'
1     ',DB')
      WRITE(6,334)
334   FORMAT(' *****
20    DO 20 JOCT=1,8
20    PRESTO(JOCT)=0.
      RETURN
    END

```

```

SUBROUTINE RM3DDEE(XX,YY,ZZ,XMAX,YMAX,ZMAX,NPT,NPL)
C*****
C** THIS SUBROUTINE PLOTS THE ROOM BOUNDARYS IN A THREE
C** DIMENSIONAL, ISOMETRIC DRAWING. ITS PURPOSE IS TO
C** GIVE A VISUAL CHECK AS TO WHETHER THE BOUNDARY
C** LOCATIONS WERE FED INTO THE PROGRAM CORRECTLY.
C*****
C
COMMON/DUM/FACTOR
DIMENSION XX(15,15), YY(15,15), ZZ(15,15), A(15), B(15), NPT(15)
DIMENSION X(15,15), Y(15,15), Z(15,15)
I=1
DO 30 J=1,NPL
  I=I+1
  K=NPT(I-1)
  DO 31 L=1,K
    X(L,J)=XX(L,J)
    Y(L,J)=YY(L,J)
    Z(L,J)=ZZ(L,J)
  CONTINUE
CONTINUE
  UP=(ZMAX+YMAX*(.5))
  FACTOR=10./UP
  I=1
  DO 6 J=1,NPL
    I=I+1
    K=NPT(I-1)
    DO 7 L=1,K
      X(L,J)=(X(L,J)+Y(L,J)*.866)*FACTOR
      Z(L,J)=(Z(L,J)+Y(L,J)*.500)*FACTOR
    CONTINUE
  31
  30
  7

```

```

6 CONTINUE
C
C ESTABLISHES ORIGIN & PLOTS BOUNDARYS
C
      I=1
      CALL PLOT(.75,.75,-3)
      DO 8 J=1,NPL
        I=I+1
        K=NPT(I-1)
        DO 9 L=1,K
          A(L)=X(L,J)
          B(L)=Z(L,J)
          A(K+1)=X(1,J)
          B(K+1)=Z(1,J)
        9 CONTINUE
        CALL LINE(A,B,K+1,1)
      8 CONTINUE
      CALL AXIS(0.,-.5,SHSCALE,5,5,0.,0.,1./FACTOR)
      RETURN
      END
      SUBROUTINE DRWPLA(XA,YA,ZA,XMAX,YMAX,ZMAX,NPL,NPT)
C
C *****
C ** IN THIS SUBROUTINE, THE PLAN VIEW AND THE FRONT
C ** ELEVATION ARE DRAWN. SUBROUTINE 'RAYCHK' DRAWS
C ** THE RAY PATHS ON THESE TWO VIEWS.
C *****
C
      COMMON/ARF/CMAX
      COMMON/ALL/FACT
      COMMON/DUM/FACTOR
      DIMENSION X(15,15), Y(15,15), Z(15,15), A(15), P(15), NPT(15)

```

```

DIMENSION XA(15,15),YA(15,15),ZA(15,15)
I=1
DO 30 J=1,NPL
I=I+1
K=NPT(I-1)
DO 31 L=1,K
X(L,J)=XA(L,J)
Y(L,J)=YA(L,J)
Z(L,J)=ZA(L,J)
CONTINUE
31 CCNTINUE
30 I=1
DO 21 J=1,NPL
I=I+1
K=NPT(I-1)
DO 22 L=1,K
X(L,J)=X(L,J)*FACT
Y(L,J)=Y(L,J)*FACT
Z(L,J)=Z(L,J)*FACT
CONTINUE
22 CCNTINUE
21 DMAX=(ZMAX)*FACT+.5
I=1
YMOVE=(YMAX*.866+XMAX)*FACTOR+2.
CALL PLOT(YMOVE,0.,-3)
DO 23 J=1,NPL
I=I+1
K=NPT(I-1)
DO 24 L=1,K
A(L)=X(L,J)
B(L)=Z(L,J)
A(K+1)=X(L,J)

```



```

      B(K+1)=Z(1,J)
24  CONTINUE
      CALL LINE(A,B,K+1,1)
23  CONTINUE
      CALL AXIS(C,-.5,5HSCALE,5,5,.0,.0,.0,1./FACT)
      I=1
      CALL PLOT(C,DMAX,-3)
      DO 25 J=1,NPL
      I=I+1
      K=NPT(I-1)
      DO 26 L=1,K
      A(L)=X(L,J)
      B(L)=Y(L,J)
      A(K+1)=X(1,J)
      B(K+1)=Y(1,J)
26  CONTINUE
      CALL LINE(A,B,K+1,1)
25  CONTINUE
      RETURN
      END

```

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the scanned document**

DETERMINATION OF ACOUSTIC RAY
PATHS IN ENCLOSED SPACES

by

Bruce Wayne Mitchell

(Abstract)

Equations were derived to calculate the reflection points of acoustic rays emanating from a source to a receiver. Reflection points for up to three reflections could be determined if the location of the source, receiver and all reflecting surfaces were known. Assuming a point source, the distances along the calculated ray paths were used along with the power level of the source and the absorptive characteristics of all the reflecting surfaces to determine the sound pressure levels at specified receiver locations. A computer program was developed to perform the necessary calculations for reflection points and sound pressure levels. The output was in the form of ray tracing plots which showed the unique reflection paths for up to three reflections and the sound pressure levels at each receiver location. Comparisons were made between predicted and calculated sound pressure levels in a rectangular parallelepiped shaped room and a long narrow hallway.

The results of this investigation showed a promising potential in the area of sound pressure level predictions and the use of ray tracing plots to provide a means of reducing the sound pressure levels. A particularly interesting point of the program was its ability to handle very irregularly shaped rooms which include slanted surfaces.

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PROGRAMMER		DATE	PAGE 1 OF 7
COMMENT STATEMENT NUMBER	CONTINUATION	FORTRAN STATEMENT	
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*NOTE: Cards with "C" in the 1st column should not be used in data set.

PROGRAMMER

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PAGE 2 OF 7

COMMENT | STATEMENT
NUMBER

CONTINUATION

FORTRAN STATEMENT

IDENTIFICATION

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ENTER NUMBER OF SURFACES FORMAT 15

ENTER NUMBER OF CORNER POINTS FORMAT 15 (THE FIRST SURFACE MUST BE FLOOR)

ENTER OCTAVE BAND ABSORPTION VALUES OF SURFACE FORMAT F10.6

ENTER SEQUENTIAL Y COORDINATES OF SURFACE

PROGRAMMER

DATE

PAGE 3 OF 7

COMMENT STATEMENT
NUMBER CONTINUATION

FORTRAN STATEMENT

IDENTIFICATION

72173 80

12 367 0. 0. 0.

17. 0. 0.

17. 7. 0.

9. 7. 0.

6. 10. 0.

0. 10. 0.

REPEAT FOR ALL SURFACES

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0. 0. 6.

17. 0. 6.

17. 7. 6.

9. 7. 6.

6. 10. 6.

0. 10. 6.

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PROGRAMMER

DATE

PAGE 4 OF 7

COMMENT STATEMENT CONTINUATION
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FORTRAN STATEMENT

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IDENTIFICATION

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INCREMENT OF RECEIVER POSITION, COORDINATES OF RECEIVER, MAX. X, Y AND Z. DIM OF SPACE