Positive pulse switching in coupled nonlinear Fabry-Perot cavities

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It is shown that positive pulse switching is possible in a system of two coupled nonlinear Fabry-Perot cavities. The conditions in which a signal beam can be switched on and off by positive pulses in the steady state regime are described for cavities filled with Kerr media having a long diffusion length.

1. Introduction

Nonlinear Fabry-Perot cavities, in particular the interference filters and semiconductor etalons, have been extensively studied in the recent past because of their potential as simple and versatile all optical bistable elements.^{1,2} Conventionally, a bistable Fabry-Perot cavity is operated with a constant bias. A pulse is added to the bias to switch the device from an off state of low transmittance to an on state of higher transmittance. Down switching is achieved by interrupting the bias, i.e., by adding a negative pulse to it. As this interruption is dissipative and thus, in principle, not conducive to very high speed,3 the search for systems in which both on and off switching are triggered by positive pulses is of great interest. Also, the requirement for more than two signal levels (positive, negative, and bias) is not compatible with conventional digital circuitry where gates are driven by pulses having a single polarity.

Positive pulse switching does not appear to be achievable in a conventional single input nonlinear cavity, at least not in the steady state regime. Even if the cavity is birefringent and polarization is being used as an additional design parameter, up switching in such systems is generally irreversible in the sense that the device eventually finds itself locked in the on state.⁴ It is, however, possible to down switch a cavity with a positive pulse if two different mechanisms are responsible for the up and the down switching. Down

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switching then results from a momentary shift of the hysteresis loop through the bias point.

Devices utilizing the thermal effect in semiconductors⁵ or the transient shift of the transmission peaks of an etalon⁶ have been demonstrated. The shift of the hysteresis loop can also be triggerd by the beats between the bias and a pulse of a different frequency.7 Another broad class of devices in which positive pulse switching is in principle achievable includes the coupled cavities systems. δ This possibility has been demonstrated in systems coupled by external feedback loops, 9 in coupled lasers, 10 and more recently in dual input cavities in which the bistability results from a pitchfork bifurcation^{11,12} as well as in symmetric selfelectrooptic effect devices.¹³

The aim of this paper is to show that positive pulse switching is achievable in two optically coupled nonlinear Fabry-Perot cavities in the steady state regime. The coupled cavities can be either two separate cavities or two nonoverlapping elements of the same cavity. The model used to describe the cavities and its underlying assumptions are outlined in the next section. The simplest possible model is chosen, and the analysis is limited to the steady state regime. The response of the device is described in Sec. III and demonstrated with numerical examples. Finally, some of the limitations and requirements for positive pulse switching in the steady state regime are discussed. The motivation for this analysis is twofold. Positive pulse switching is conceptually important for the reasons already mentioned. Thus, defining the types of system in which it is achievable is of definite interest. Furthermore, coupling between cavities arranged in series or between different cavity elements can occur either intentionally or accidentally in cascaded devices, which are of great practical importance.

II. Theoretical Model

Figure 1 shows schematically the geometry of the systems considered. It consists of two nonlinear

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Fig. 1. Schematic diagram of the coupled nonlinear cavities *FPj, FP2 ,* Fabry-Perot cavities filled with Kerr-type nonlinear media with long diffusion length: I_{i1} , signal input; I_{i2} , control input; *BS*, beam splitter; and *P*, polarizer transmitting I_{i1} , but not I_{i2} , I_{o1} , I_{o2} , I_{o3} , possible outputs.

Fabry-Perot cavities *FP1* and *FP2* arranged in series. Each cavity is made of two dielectric lossless mirrors of power reflectivity R_1 for FP_1 and R_2 for FP_2 . The cavities are filled with homogeneous Kerr-type media having a long diffusion length. As a result, the cavities are characterized by conventional hystereses. Polarizer P is used to isolate FP_1 from the control beam. The outputs which derive only from the signal beam are extracted by beam splitter *BS.*

The round trip phase change in each cavity depends only on the average intensity I_1 or I_2 of the radiation trapped in the cavity. The round trip phase change in cavity $j = 1,2$ can thus be written as

$$
\phi_j = 2k d_j (n_{0j} + n_{2j} I_j),\tag{1}
$$

where $k = 2\pi/\lambda$ is the wavenumber *in vacuo*, λ is the wavelength, n_{0j} is the linear index of refraction of the medium in cavity j, and n_{2i} is the value of its nonlinear index.

There are two inputs to the system. Input I_{i1} on FP_1 is called the signal beam, while input I_{i2} on FP_2 is called
the control beam. They have orthogonal polarization. Three outputs are also shown. Because the polariza-
tion isolates FP_1 from I_{i2} , the outputs derive entirely
from signal I_{i1} : I_{o1} is the signal beam transmitted by
 FP_1 , I_{o2} has experienced an additional FP_2 , and I_{o3} is the signal beam reflected by FP_1 . In the schematic diagram of Fig. 1, outputs I_{01} and I_{02} are extracted by beam splitter *BS* placed between the two cavities, but other folded geometries are also possible.

Isolating cavity FP_1 from the control beam results in the simplest possible kind of coupling between the two cavities. The control beam does not reach cavity one and thus does not affect it directly. The important point is that the control beam can change the reflectivity of *FP2,* which affects the total power incident on *FP1* and thus changes the shapes of its hysteretic response curve.

It is further assumed that the two cavities are coupled incoherently. That is, the irradiance reflected on $FP₂$ is fed back onto $FP₁$, changing the total irradiance inside that cavity, but the reflected radiation does not interfere with the incident signal beam. This situation is true when the coherence time of the radiation is shorter than the transit time between cavities or if the spacing between the cavities does not form a resonant
cavity because of misalignment or other mistuning. This contrasts with other double-cavity systems previously considered.¹⁴

For each cavity, there are four time scales in the system: The cavity round trip times $\tau_{R,i}$, the cavity lifetimes τ_c , the medium response times τ_M , and the time scale of input changes τ_{P_i} (i.e., the pulses rise and fall times). An additional time scale is the transit time τ _T between the cavities. The steady state solution is valid when τ_T , $\tau_{R_i} < \tau_{C_i} \ll \tau_{M_i} \ll \tau_{P_i}$; here $j = 1, 2$.

With these assumptions and using a plane wave approach, the average irradiance inside each cavity can be calculated by summing up the contributions after each round trip, giving the result

$$
I_1 = \frac{1 + R_1}{1 - R_1} T_1 I_{i_1} \left(1 + \frac{T^2 T_1 R_1}{1 - T^2 R_1 R_2} \right),
$$
 (2)

$$
I_2 = \frac{1 + R_2}{1 - R_2} \left(\mathcal{T}_1 I_{i_2} + \frac{T^2 \mathcal{T}_1 \mathcal{T}_2}{1 - T^2 \mathcal{R}_1 \mathcal{R}_2} I_{i_1} \right). \tag{3}
$$

In these expressions, the overall power transmittance of each cavity is given by the usual Airy function:

$$
T_j = (1 - F_j \sin^2 \phi_j / 2)^{-1}; \text{ here } j = 1, 2,
$$
 (4)

and the overall reflectance $\mathcal{R}_i = 1 - \mathcal{T}_i$ for lossless cavities. Term F_j is the finesse of cavity j , the round trip phase change ϕ_j is given by Eq. (1), an T is the transmittance of the beam splitter.
Equations (1)–(4) were solved numerically by

searching for consistent solutions for the round trip phases ϕ_j in each cavity for all pairs of input values I_{i1} and I_{i2} . The outputs were then calculated as

$$
I_{o1} = \frac{RT_1}{1 - T^2 \mathcal{R}_1 \mathcal{R}_2} I_{i1},
$$
\n(5a)

$$
I_{o2} = T\mathcal{R}_2 I_{o1},\tag{5b}
$$

where R is the reflectance of the beam splitter and T is its transmittance.

Ill. Numerical Results

There are two possible modes of operation for the device of Fig. 1. In the first mode, the device is switched on and off by two control pulses of different magnitudes, while in the second mode it is switched on by a signal pulse and switched off by a control pulse.
In both cases, cavity FP_1 is biased within its bistable

region with a constant input signal beam and with the reflection of that beam on cavity FP_2 . The cavity FP_2 need not be biased and acts as a mirror of variable reflectance. The control pulses are used to change the reflectance of *FP2* and consequently switch the device.

3A. On and Off Switching with Control Pulses

In the first mode of operation, cavity *FP2* must also be bistable. Control pulses of different magnitude are used to change the reflectance of FP_2 . This changes the total bias of FP_1 and thus shifts the operating point on the hysteretic transfer function of the device, re-

Fig. 2. Transmittance of FP_1 (uncoupled) with the following cavity parameters: $R_1 = 0.4$; initial detuning $\phi_{01} = 2kd_1n_{01} = 1.50rd$ mod 2π ; and nonlinear index $n_{21} = 10^{-3}$ r.u.

Fig. 3. Steady state output I_{o1} vs signal input I_{i1} for different values of the control beam intensity. Signal bias, *I*: (1) output for $I_{i2} = 0$; (2) output for $I_{i2} = 3.4$; (3) output for $I_{i2} = 11$ corresponding to the upper branch of the hysteresis loop of \mathcal{R}_2 ; and (4) output for $I_{i2} = 4.2$ corresponding to the lower branch of the hysteresis loop of \mathcal{R}_2 . The output is switched on $(A \text{ to } B)$ by control beam intensity $I_{i2} \approx 11$. It relaxes to *C* as the control pulse ends. The output is switched off by control beam intensity $I_{i2} > 11$. When the pulse ends, the output follows the lower branch of the hysteresis loop of \mathcal{R}_2 (C to D). Switch down occurs near point 4 (inset). The output shifts from *D* to E as \mathcal{R}_2 switches back up and finally relaxes back to A . For the output not to switch back on, FP, must be biased between the switch-up points of curve 2 and 3.

Inset. Reflectance of *FP2* (uncoupled) with the following cavity parameters: $R_1 = R_2 = 0.4$, initial detuning $\phi_{02} = 3.94$ rd mod 2π , and nonlinear index $n_{22} = 5 \times 10^{-4}$ r.u.

Fig. 4. Steady state output I_{o1} vs control signal input I_{i2} . The output is switched on by a small control pulse $(A \rightarrow B \rightarrow C)$ and switched off by a larger control pulse $(C \rightarrow D \rightarrow E \rightarrow A)$.

Fig. 5. Steady state outputs illustrating a switching sequence in which up and down switching are both triggered by control pulses of different magnitudes. The cavity parameters are the same as those for Figs. 2 and 3.

sulting in the device switching either on or off depending on the magnitude of the control beam.

The result of a numerical simulation is illustrated in Figs. 2-5. Transmittance T_1 of FP_1 and the reflectance \mathcal{R}_2 of FP_2 are shown in Fig. 2 and in the inset of Fig. 3, respectively, as functions of the incident intensity on the respective uncoupled cavity. The parameters of each cavity are chosen so that both *FP1* and *FP2* are bistable. Namely, $R_1 = R_2 = 0.4$, the initial detunings ($\phi_{0i} = 2k d_i n_{0i}$) are $\phi_{01} = 1.50rd$, $\phi_{02} = 3.94rd$ mod

 2π , and the nonlinear indices are $n_{21} = 10^{-3}$, $n_{22} = 5 \times$ 10^{-4} . With the normalized intensity units used, $n_2 \approx$ 10^{-3} corresponds approximately to the nonlinear coefficient due to thermal effects in a typical nematic liquid crystal (e.g., MBBA). The output coupling beam splitter has a transmittance of 80%.

As is evident from Eqs. (2) and (3), the reflectance of *FP2 ,* which is a function of the total irradiance inside that cavity, can be controlled by varying the intensity of the control beam. Since FP_1 is biased by two beams, the incident bias signal beam and the signal beam reflected by FP_2 , changing the reflectance of FP_2 alters
the shape of FP_1 's hysteretic response. In particular, it changes the values of the critical intensities at which the outputs, I_{01} , I_{02} , or I_{03} switch on or off. Figure 3 shows four hysteretic response curves for the output I_{o1} as a function of input signal intensity I_{i1} . Each curve corresponds to a different value of control beam intensity I_{i2} and thus to different values of the reflectance of $F\overline{P}_2$, as indicated by the corresponding number in the inset of Fig. 3.

With the control beam intensity equal to zero, I_{i2} = 0, the device's response is described by curve 1 in Fig. 3. Input I_{i1} biases FP_1 within its bistable region, while the reflectivity *FP2* is at 1 in the inset.

A switching cycle may start with the device in a low state (e.g., at point A in Fig. 3). A control pulse of relatively small amplitude increases reflectance \mathcal{R}_2 up to region 3 in the inset. This pulse must not be large enough to carry \mathcal{R}_2 beyond its bistable region. With an increased feedback from the reflection on *FP2 ,* the critical switch-up intensity of *FP1* drops. This results in a shift of the output hysteresis loop toward lower values of I_{i1} , as shown by curve 3. Since FP_1 is still biased at I, the output switches to the upper branch of its hysteresis loop (point B) as soon as the critical intensity for up switching drops below the bias. When the control pulse ends, the output response curve returns to curve 1, leaving the output in a high state (point *C).*

Down switching is triggered by a more intense control pulse which brings \mathcal{R}_2 beyond its bistable region. Reflectance \mathcal{R}_2 then switches to its lower branch. As the control pulse ends, reflectance \mathcal{R}_2 drops to low values along its lower branch. This reduces the bias of $FP₁$ and increases its critical switch-down intensity (curve 4). When this critical switch-down intensity exceeds bias I , the output cannot be maintained in the upper branch and it switches back to a low state (point D). Eventually, as \mathcal{R}_2 returns to its original states, cavity *P2* will switch back up and its reflectance will increase momentarily. This increased bias on *FP1* must not be sufficient to switch the output back on. For this, it is necessary that $\mathcal{R}_2(2) < \mathcal{R}_2(3)$, and cavity FP_1 must be biased between the crtical up-switching intensity of curve 2 and that of curve 3.

Another way to visualize the switching sequence is to plot the output vs the control beam intensity. This is shown in Fig. 4 for output I_{01} and the same cavity system. This figure is self-explanatory: a small control pulse $I_{\text{up}} < I_{i2} < I_{\text{down}} (I_{\text{up}} \sim 5.4, I_{\text{down}} \sim 11.9)$

Fig. 6. Transmittance of FP_1 with the following cavity parameters: $R_1 = 0.4$, initial detuning $\phi_{01} = 5.22$ rd mod 2π , and nonlinear index $n_{21} = 10^{-3}$ r.u.

switches the output up from A to B and to C when the control pulse ends. A large control pulse $I_{i2} > I_{\text{down}}$ switches the output down along the path C to D then to E and eventually back to A when the pulse ends.

Note that this figure also indicates that a constant bias $(I_{bias} < I' \sim 4)$ could be used to maintain the control cavity closer to its switching point and thus to allow
switching with smaller pulses.

The second output I_{o2} experiences an additional reflection on *FP2 .* Consequently, the effect of the control pulses is not only to shift the hysteresis of the response curve but also to alter considerably the out-
put intensity while the control pulses are present. Figure 5 illustrates the steady state switching sequence
of the device just described.

B. On and Off Switching With a Signal and a Control Pulse

In this second mode of operation, the device is switched on by a signal pulse added incoherently to the bias signal in a manner similar to the switch-on of a single nonlinear cavity. It is then switched off by a control pulse in the same way as that described in the

previous section.
The transmittance of FP_1 and the reflectance of FP_2 . uncoupled, are shown in Fig. 6 and the inset of Fig. 7, respectively. The cavity parameters for this example are $R_1 = R_2 = 0.4$, $\phi_{01} = \phi_{02} = 5.22$ rd mod 2π , $n_{21} = 10^{-3}$, $n_{22} = 5 \times 10^{-4}$. Note that with this initial detuning, \mathcal{R}_2 remains fairly constant on the upper branch of its hystersis. It would thus not be possible to switch on this device with a control pulse incident on FP_2 .

Nonlinear cavity FP_1 is again biased within its bistable region. A trigger pulse added incoherently to the bias signal beam switches on the output from A to B in Fig. 7. This trigger pulse also increases the total input on FP_2 and thus may reduce the reflectance of that cavity. This reduction, which acts as a negative

(3) and (3), output for $I_{12} = 3.3$ corresponding to the upper and the different amplitude levels to turn the device on and of $\frac{1}{2}$ by a signal pulse $(A \rightarrow B)$ and switched off by a control pulse $(B \rightarrow C$ which is not compatible with binary logics. Another Fig. 7. Steady state output I_{01} vs signal input I_{i1} for different values of the control beam intensity. Signal bias, I ; (1) output for $I_{i2} = 0$;

Inset. Reflectance of FP_2 with the following parameters: $R_1 = R_2$

Fig. 8. Steady state outputs illustrating a switching sequence in which up switching is triggered by a signal pulse and down switching is triggered by a control pulse.

feedback on *FP1,* can be maintained small, however, by a proper choice of cavity parameters. As in the previous example, the output is switched back to a low state along the path *BCDA* by a control pulse bringing \mathcal{R}_2 beyond its bistable region. The resulting steady state switching sequence of the device is shown in Fig. 8.

IV. Discussion and Conclusion

0O V3 **⁵**/ \ We have demonstrated with numerical examples that positive pulse switching in the steady state regime is realizable in a system of two optically coupled nonlinear Fabry-Perot cavities filled with Kerr media having a long diffusion length.

Two modes of operation were described. In the first, a signal beam transmitted by the first cavity and extracted as an output is switched on and off by control pulses of different magnitudes incident on the second cavity. For this to be possible, both cavities must be bistable. Cavity one should be biased within its bi-0) **0.2 OA 0.6 0.8** ¹stable region, and cavity two can be biased before its bistable region or can be left unbiased. Cavity one INPUT I_i , must also be biased in such a way that when R_2 switch-
 I_2 and I_3 are the abight reflectance the output will remain es back up to a high reflectance, the output will remain in the lower branch of the hysteresis curve. Besides critical biasing requirements, this design needs two
different amplitude levels to turn the device on and off. (α) and (β), output in $I_{12} = 3.5$ corresponding to the dependent of different amplitude levels to turn the device on and off,
lower branch of the hysteresis loop of \Re . The output is switched on which is not, co by a signal pulse $(A \rightarrow B)$ and switched off by a control pulse $(B \rightarrow C)$ type of coupled cavity, which can be switched with $\rightarrow D \rightarrow A$). pulses of equal magnitude, has been demonstrated experimentally.⁶ This device, which utilizes the finite There. Reflectance of F1 2 with the following parameters. $\frac{1}{2}$ $\frac{1}{2}$ experimentally.⁶ This device, which utilizes the finite = 0.4, $\phi_{02} = 5.22$ rd mod 2 π , and nonlinear index $n_{22} = 5 \times 10^{-4}$ r.u. recovery time of the cavity, requires two different wavelengths.

> A second mode of operation of the system, in which the output is switched on by a signal pulse incident on one cavity and switched off by a control pulse incident on the other cavity was also described. The cavity parameter requirements are less critical for this device, $\overline{0.5}$ $\overline{0.5}$ since the control pulse must only shift the hysteresis loop in one direction by an amount just sufficient for α ^t the critical switch-down intensity to exceed the bias. The control cavity FP_2 , for example, need not be bistable. An additional advantage of this design is that 5¹ if FP_2 has a narrow bistable region, it can be biased close to its switching point. Consequently, only a σ^{\dagger} small intensity control pulse is needed to switch the device of Similarly if FP_1 is biased close to its device off. Similarly, if *FP1* is biased close to its switching point, it can be switched on with a small intensity signal pulse. It is also possible, in principle, to adjust the parameters of the system to switch the **_-** device on and off with pulses of equal magnitude.

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