Inlet Drag Prediction for Aircraft Conceptual Design

by

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(ABSTRACT)

A research effort aimed at enhancing ACSYNT, a computer program for aircraft conceptual design, has necessitated the development of methods for predicting inlet drag. Originally, the drag of only one inlet type, the variable-geometry conical inlet, could be calculated within ACSYNT. This prompted the present research which resulted in the creation of a modular suite of subroutines that extend the capability of ACSYNT. Using this new source code, ACSYNT can now predict the drag of subsonic and supersonic pitot inlets, fixed- and variable-geometry conical inlets, and two-dimensional supersonic inlets.

Even though the requirement of computational efficiency has necessitated that many simplifications be made in the analysis, the drag calculations have a sound physical basis. The semi-empirical methods have been extracted from a number of sources based on an extensive literature survey, and these have been enhanced to encompass the full range of inlet operating conditions. The effectiveness of the methods has been demonstrated by comparing some results of the predictions to published data.

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Nomenclature

Chapter 2

A	area
D_{add}	additive drag force
D_b	bleed and bypass drag force
D_{inlet}	inlet drag force
D_f	nacelle viscous drag force
D_p	nacelle form drag force
D_{spill}	spillage drag force
D_x	auxiliary drag force
$\dot{m}$	mass flow rate
P	static pressure
T_i	installed thrust
T_N	net standard thrust
T_o	overall thrust
V	velocity
ΔD_c	change in cowl drag at reduced mass flow rates
θ_b	bleed/bypass exit flow angle
ρ	air density
τ_w	viscous shear stress

Subscripts

b	bleed
c	plane of inlet capture face
e	exhaust plane
j	mixed-out flow station far downstream
s	over a surface
x	auxiliary (mass flow rate, drag); projected in x-direction (area)
∞	flow station far upstream

Chapter 5

A	area
A_c	area of inlet capture face
A_m	maximum cowl cross-sectional area
A_∞	area of capture streamtube far upstream
A_{lip}	lip projected area
A_s	cowl wetted area
$C_{D_{add}}$	additive drag coefficient (based on A_c)
$C_{D_{inlet}}$	inlet drag coefficient (based on A_c)
$C_{D_{lip}}$	lip wave drag coefficient (based on A_c)
$C_{D_{prof}}$	profile drag coefficient (based on A_c)
$C_{D_{spill}}$	spillage drag coefficient (based on A_c)
$C_{D_{wave}}$	wave drag coefficient (based on A_c)
C_f	skin friction coefficient (based on A_s)
C_p	pressure coefficient
C_{po}	stagnation pressure coefficient
D_{add}	additive drag force
D_{prof}	profile drag force
D_{spill}	spillage drag force
D_{wave}	wave drag force
d_c	cowl diameter at inlet capture face

d_m	maximum cowl diameter
h	height of right circular cone
K	coefficient used to evaluate profile drag; coefficient used to evaluate lip wave drag
K_{add}	spillage drag factor
K_o	coefficient used to evaluate profile drag
K_6	factor used in spillage drag calculation
K_7	factor used in spillage drag calculation
K_8	factor used in spillage drag calculation
K_{11}	factor used in subsonic wave drag calculation
L	cowl length
l	length of a right circular cone
M	Mach number
M_D	drag-rise Mach number
MFR	mass flow ratio
P	static pressure
P_o	total pressure
q_∞	freestream dynamic head
r	cowl radius; radius of right circular cone
V	velocity
x	coordinate of cowl segment
γ	ratio of specific heats
δ	local cowl slope
ρ	air density

Subscripts

c	plane of inlet capture face
$crit$	value at onset of spillage drag
D	value at the onset of drag rise
i	cowl segment index
lip	cowl lip
shk	station immediately downstream of shock
∞	flow station far upstream

Chapter 6

A	area
A_c	area of inlet capture face
d_c	maximum cowl cross-sectional area
A_∞	area of capture streamtube far upstream
A_s	cowl wetted area
a	coefficient in Willis and Randall formula
b	coefficient in Willis and Randall formula
c	coefficient in Willis and Randall formula
$C_{D_{inlet}}$	inlet drag coefficient (based on A_c)
D_{DF}	maximum disturbed flow drag force
D_{NS}	maximum normal shock drag force
$C_{D_{prof}}$	profile drag coefficient (based on A_c)
$C_{D_{spill}}$	spillage drag coefficient (based on A_c)
$C_{D_{wave}}$	wave drag coefficient (based on A_c)
$C_{D, shk2.1}$	wave drag coefficient calculated with shock expansion method for Mach 2.1
$C_{D, WR}$	wave drag coefficient calculated with Willis and Randall method
$C_{D, WR2.1}$	wave drag coefficient calculated with Willis and Randall method for Mach 2.1
$C_{D, WR1.1}$	wave drag coefficient calculated with Willis and Randall method for Mach 1.1
C_f	skin friction coefficient (based on A_s)
D_{prof}	profile drag force
D_{spill}	spillage drag force
D_{wave}	wave drag force
d	cowl diameter
d_c	cowl diameter at inlet capture face
d_m	maximum cowl diameter
f_r	fineness ratio
f_f	form factor
L	cowl length
M	Mach number
M_D	drag-rise Mach number
M_{s_i}	two-dimensional Mach number on cowl segment
n	exponent used to describe cowl profile

P	static pressure
P_{c_i}	pressure on cowl segment calculated with conical flow relations
P_{s_i}	pressure on cowl segment calculated with Prandtl-Meyer expansion
P_{seg_i}	pressure on cowl segment
q_∞	freestream dynamic head
d_c	cowl radius at inlet capture face
r_i	radius of cowl segment
d_m	maximum cowl radius
V	velocity
x	distance measured along cowl
x_i	coordinate of cowl segment
α	factor in spillage drag formulation
β	factor in spillage drag formulation; $\sqrt{M_\infty^2 - 1}$; coefficient in shock expansion method;
β_1	factor in spillage drag formulation
β_i	function used in shock expansion method
γ	ratio of specific heats
δ_i	slope of cowl segment
η	initial cowl slope
ρ	air density
ψ_i	function used in shock expansion method
Ω_i	function used in shock expansion method

Subscripts

c	plane of inlet capture face
D	value at the onset of drag rise
i	cowl segment index
shk	station immediately downstream of shock
∞	flow station far upstream

Chapter 7

A	area
A_c	area of inlet capture face
A_{lip}	lip projected area
A_m	maximum projected cowl area
A_t	inlet throat area
A_∞	area of capture streamtube far upstream
A_s	cowl wetted area; sonic flow area between cowl and bow shock sonic points
A_{os}	stream tube area bounded by inlet stagnation streamline surface and streamline passing through bow shock sonic points
$\left(\frac{A}{A^*}\right)_{os}$	contraction ratio required to decelerate the freestream flow to sonic velocity isentropically
$\left(\frac{A}{A^*}\right)_{ref}$	contraction ratio required to decelerate the reference flow to sonic velocity isentropically
abc	abscissa used to evaluate cowl suction
$C_{D_{inlet}}$	inlet drag coefficient (based on A_c)
$C_{D_{prof}}$	profile drag coefficient (based on A_c)
$C_{D_{spill}}$	spillage drag coefficient (based on A_c)
$C_{D_{wave}}$	wave drag coefficient (based on A_c)
$C_{D_{shk}}$	wave drag coefficient calculated with shock expansion
$\tilde{C}_D$	wedge transonic wave drag function
$C_{D_{wedge}}$	wave drag coefficient of a wedge (based on A_c)
$\bar{C}_{D_{wedge}}$	wave drag coefficient of a wedge (based on frontal area)
C_{suc}	cowl suction coefficient (based on A_c)
C_f	skin friction coefficient (based on A_s)
C	function used to calculate bow shock position
$coef0$	coefficient used to evaluate cowl suction
$coef1$	coefficient used to evaluate cowl suction
$coef2$	coefficient used to evaluate cowl suction
D_{prof}	profile drag force
D_{spill}	spillage drag force
D_{wave}	wave drag force
E	function used to calculate bow shock position

$exp1$	exponent used to evaluate cowl suction
$exp2$	exponent used to evaluate cowl suction
$exp3$	exponent used to evaluate cowl suction
F	function used to calculate bow shock position
F_{ramp}	force on forebody ramp
f_f	form factor
G	function used to calculate bow shock position
g_c	dimensional constant used in EES system of units
i	ratio of mass flow ratio to reference mass flow ratio
k	function used to calculate bow shock position
K	coefficient used to evaluate lip wave drag
L	displacement of bow shock along forebody ramp
L_c	cowl length
M	Mach number
MFR	mass flow ratio
MFR_{ref}	reference mass flow ratio
$\dot{m}_{max}$	maximum mass flow accepted by the inlet
n	exponent used to describe cowl profile
ord	ordinate used to evaluate cowl suction
P	static pressure
P_o	total pressure
$P_{o,ref}$	total pressure of flow after passing through inlet shock system
$\frac{P_{o,s}}{P_{o,os}}$	mean total pressure recovery through the hyperbolic portion of the bow shock between the stagnation streamline and the shock sonic point
P_{R1}	static pressure downstream of ramp oblique shock
P_{R2}	static pressure downstream of ramp normal shock
q_∞	freestream dynamic head
R	gas constant for air
r_{lip}	cowl lip radius
T	static temperature
$\frac{t}{c}$	cowl thickness-to-chord ratio
V	velocity
w	inlet width

wv	inlet throat dimension
x	distance along cowl
X_{ramp}	forebody ramp length
Y_c	height of cowl at inlet capture face
$Y_{c_{cowl}}$	height of cowl at inlet capture face plus lip thickness
Y_i	height of inlet aperture
Y_m	maximum cowl height
β	$\sqrt{M_\infty^2 - 1}$
δ_s	flow angle corresponding to ϕ_s
γ	ratio of specific heats
η	wedge half-angle
η_c	initial cowl slope
η_m	cowl slope assuming it is wedge-shaped
η_w	half-angle of equivalent wedge
θ	angle of forebody wedge
λ	flow angle at inlet throat
ρ	air density
σ	angle describing inlet geometry
σ_s	oblique shock angle
ϕ_c	mean angle of hyperbolic portion of bow shock
ϕ_s	shock angle for sonic flow
χ	function used to evaluate wedge wave drag

Subscripts

c	plane of inlet capture face
min	minimum value to support an attached shock
shk	station immediately downstream of shock; calculated using shock expansion
∞	flow station far upstream

Chapter 8

A	area
A_c	area of inlet capture face
A_{Cnx}	forebody cone projected area at inlet throat
A_s	forebody cone projected area ahead of bow shock
A_t	inlet throat area
A_∞	area of capture streamtube far upstream
$C_{D_{inlet}}$	inlet drag coefficient (based on A_c)
$C_{D_{prof}}$	profile drag coefficient (based on A_c)
$C_{D_{spill}}$	spillage drag coefficient (based on A_c)
$C_{D_{wave}}$	wave drag coefficient (based on A_c)
$C_{D_{DF}}$	maximum disturbed flow drag coefficient
$C_{D_{NS}}$	maximum normal shock drag coefficient
D_{DF}	maximum disturbed flow drag force
D_{prof}	profile drag force
D_{spill}	spillage drag force
D_{wave}	wave drag force
f	factor used in additive drag formulation
F_{cone}	force on forebody cone
g_c	dimensional constant used in EES system of units
i	ratio of mass flow ratio to reference mass flow ratio
K	function used to calculate bow shock position
L	displacement of bow shock ahead of inlet capture face
M	Mach number
M_{des}	inlet design Mach number
MFR	mass flow ratio
MFR_{ref}	reference mass flow ratio
$\dot{m}_{max}$	maximum mass flow accepted by the inlet
P	static pressure
P_{c1}	static pressure downstream of conical shock
P_{c2}	static pressure downstream of forebody normal shock
P_t	static pressure at the inlet throat
q_∞	freestream dynamic head

R	gas constant for air
R_c	cowl radius at inlet capture face
R_i	cone radius at inlet capture face
R_{Cn}	cone radius at inlet throat
T	static temperature
V	velocity
α	factor in spillage drag formulation
β	factor in spillage drag formulation
γ	ratio of specific heats
θ	forebody cone semi-angle
λ	flow angle at inlet throat
ρ	air density
σ	angle describing inlet geometry
σ_s	conical shock angle

Subscripts

c	plane of inlet capture face
t	station at inlet throat
des	value at inlet design point
∞	flow station far upstream

Chapter 9

A	area
A_c	area of inlet capture face
A_t	inlet throat area
A_∞	area of capture streamtube far upstream
$\frac{A_{aux}}{A_c}$	auxiliary mass flow ratio
$\frac{A_{bl}}{A_c}$	bleed mass flow ratio

$\frac{A_{byp}}{A_c}$	bypass mass flow ratio
$\frac{A_{vent}}{A_c}$	vent mass flow ratio
$\frac{A_{wedge}}{A_c}$	diverter wedge area ratio
$C_{D_{div}}$	diverter drag coefficient (based on A_c)
C_{D_x}	auxiliary drag coefficient (based on A_c)
D_x	auxiliary drag force
g_c	dimensional constant used in EES system of units
M	Mach number
M_{des}	inlet design Mach number
MFR_{ref}	reference mass flow ratio
$\dot{m}_c$	reference mass flow rate
$\dot{m}_{eng}$	engine mass flow rate
$\dot{m}_{eng, des}$	engine design mass flow rate
$\dot{m}_{inlet}$	inlet mass flow rate
$\dot{m}_x$	auxiliary mass flow rate
P	static pressure
P_o	total pressure
P_{ot}	total pressure at inlet throat
$P_{o\infty}$	freestream total pressure
PR_{sub}	subsonic total pressure recovery
PR_{sup}	supersonic total pressure recovery
R	gas constant for air
T	static temperature
V	velocity
γ	ratio of specific heats
ε	subsonic total pressure loss coefficient
θ	forebody cone semi-angle; forebody ramp angle
θ_p	diverter wedge half-angle
ρ	air density
σ	angle of conical/oblique shock occurring at design Mach number

Subscripts

c	plane of inlet capture face
t	station at inlet throat
des	value at inlet design point
eng	based on engine mass flow requirement only
sub	subsonic
sup	supersonic
∞	flow station far upstream

Appendix A

A	area
D_{add}	additive drag force
D_b	bleed and bypass drag force
D_f	nacelle viscous drag force
D_p	nacelle form drag force
D_x	auxiliary drag force
$\dot{m}$	mass flow rate
P	static pressure
T_i	installed thrust
T_N	uninstalled thrust
V	velocity
θ_b	bleed/bypass exit flow angle
ρ	air density
τ_w	viscous shear stress

Subscripts

b	bleed
c	plane of inlet capture face
e	exhaust plane

s	leaving control volume (mass flow rate, velocity); over a surface (area)
x	auxiliary (mass flow rate, velocity drag); projected in x-direction (area)
∞	flow station far upstream

Appendix B

A	area
$\dot{m}$	mass flow rate
P	static pressure
V	velocity
ρ	air density

Subscripts

c	plane of inlet capture face
e	exhaust plane
s	leaving control volume (mass flow rate, velocity); over a surface (area)
x	projected in x-direction
∞	flow station far upstream

Appendix C

A	area
$\frac{A}{A^*}$	contraction ratio required to decelerate or accelerate a flow to sonic velocity isentropically
C_F	compressible turbulent skin friction coefficient
C_{F_i}	incompressible turbulent skin friction coefficient
g_c	dimensional constant used in EES system of units
K_{crit}	grain diameter
L	reference length

M	Mach number
P	static pressure
R	gas constant for air
S	Sutherland's constant
T	static temperature
Re	Reynolds number
V	velocity
γ	ratio of specific heats
δ	wedge angle turned by supersonic flow
μ	air dynamic viscosity
ν	air kinematic viscosity; Prandtl-Meyer function
ρ	air density
σ	shock angle

Subscripts

1	upstream station
2	downstream station
<i>guess</i>	guessed value
<i>max</i>	maximum value
<i>n</i>	normal to shock wave
<i>ref</i>	reference value for Sutherland's Law
<i>wall</i>	value at the wall
∞	value in the freestream

1.0 Introduction

Aircraft conceptual design is the process whereby the size and general configuration of an aircraft are selected to meet a specific mission profile. This process involves the choice of a candidate aircraft layout, and an initial guess must be made as to its overall size. The aircraft is then synthesized by a series of design relations which describe important aspects of the design such as the aerodynamic performance, component weights and the propulsion system performance. This synthesis enables the aircraft weight to be calculated, and hence the initial aircraft layout can be revised accordingly. The procedure needs to be repeated until the design converges satisfactorily. This iterative design process is best performed by computer, thus enabling the design turn-around time to be minimized without foregoing the use of relatively complex design relations.

ACSYNT is one such computer program for aircraft synthesis and conceptual design. It is distinguished by the incorporation of numerical optimization techniques to allow specific characteristics of an aircraft to be optimized within certain design constraints. The major portion of ACSYNT was developed in the early 1970's at the NASA Ames Research Center, and it has since been used by several organizations involved in aircraft research and development. A typical application for which ACSYNT is used at NASA is the evaluation and comparison of alternative aircraft design concepts proposed by different organizations to fulfill a specific mission requirement.

For many years ACSYNT was run in noninteractive batch mode, with complex input file requirements and limited graphical output. However, recent progress in computer technology, manifested in the maturation of graphics workstations and device-independent

graphics software, has made the incorporation of programs such as ACSYNT into interactive, graphically-oriented environments feasible. Recently, much work has been done to develop ACSYNT's geometric modeling and pre- and post-processing capabilities (Wampler et al., 1988). The result is a completely interactive, user-friendly aircraft design system which will operate on a number of different graphics workstations.

As these new graphical capabilities have been developed, attention has also been paid to the analysis methods incorporated into the aircraft synthesis routines which form the core of ACSYNT. Specific areas within the program have been targeted for evaluation and, where necessary, reorganization or even complete revision. The prediction of inlet drag is one of the first of such areas to be investigated, and this forms the basis of the current research.

The purpose of an aircraft inlet is to capture freestream air and reduce its velocity so that the engine can process it in a stable, efficient manner. In order to minimize compressor work, the inlet diffusion should be accomplished with a minimum of total pressure loss. However, the alternative forms of inlets which work most efficiently in the required Mach number range will invariably result in an external drag penalty, especially at off-design conditions. Since this drag penalty can be severe, the selection of the inlet type, and the prediction of inlet drag, forms an important part of the aircraft design process.

The prediction of the drag of a component such as the inlet is particularly significant in a graphically-oriented conceptual design program such as ACSYNT. Here, the computer aided design (CAD) features of the program give the designer great control over the geometrical specification of the individual components of the aircraft. Consequently, changes to the geometry of these components should be adequately reflected in the synthesis of the performance of the aircraft. It was therefore the aim of this research to

investigate the methods originally embodied within ACSYNT for the purpose of inlet drag prediction, and to upgrade these methods to an appropriate level of sophistication.

The inadequacy of the original inlet drag prediction methods in ACSYNT was determined early in the research effort. As will be further discussed, only one type of inlet could be handled. This inlet type was not necessarily representative of contemporary aircraft, nor was the analysis embodied in the source code particularly sound.¹ Accordingly, the available methods for predicting inlet drag were thoroughly reviewed. The most suitable of these methods were extracted from the literature, extended where necessary, and coded into a modular suite of subroutines. These subroutines were then used to extend the capability of ACSYNT to predict the drag of a total of six inlet types, based on variations of four distinct geometrical configurations. Even though these subroutines were written with ACSYNT as the intended host program, it is believed that their well-structured and well-documented nature will make them suitable for incorporation into other aircraft design programs. A further strength of these routines is that they are based entirely on methods available in the open literature, rather than on methods proprietary to specific organizations as with many other aircraft conceptual design programs.

In this thesis, the background theory for the prediction of inlet drag is presented together with a survey of the major publications covering this topic. The methodology used for the drag prediction is explained in broad terms, and subsequent chapters describe the detailed analyses of four specific geometries; the subsonic pitot, supersonic pitot, supersonic two-dimensional and supersonic conical inlets. A further chapter outlines the calculation methods used to evaluate the additional propulsion system losses associated

¹ This issue is discussed further in the section entitled "Additive Drag Prediction Method Originally in ACSYNT" on page 110.

with the inlet. Selected results, sample applications, and recommendations for future work are also presented. Appendices A and B contain supporting analyses that were inappropriate to include in the body of the report. Appendix C contains a description of the methods and equations that are used in a library of utility routines in support of the inlet drag analysis methods. The source code listing, together with additional information for users and programmers, is contained in a separate report to the NASA Ames Research Center (Myklebust et al., 1989).

2.0 Background Theory

2.1 *Alternative Definitions of Thrust*

Figure 1 on page 6 shows the stations of relevance for the definition of the thrust of an aircraft engine. Following Newton's second law, the *overall thrust* is defined as the rate of change in momentum from the station far upstream at ∞ to the station far downstream at j , since the pressure at these stations will be equal. Thus the overall thrust is

$$T_o = \rho_j V_j^2 A_j - \rho_\infty V_\infty^2 A_\infty = \dot{m}_j V_j - \dot{m}_\infty V_\infty \quad [2.1.1]$$

This definition is, however, unsatisfactory for practical use, because the conditions at j are not easily definable due to the mixing of the jet. An alternative option would be to define the thrust in terms of the conditions at c and at e . This definition (known as *intrinsic thrust*) is also unsatisfactory because, even though the conditions at e are easily determined by engine test bed measurements, the conditions at c are dependent on the inlet configuration, and will vary according to the installation of the engine in the airframe.

The definition of thrust which is generally accepted, and which meets the needs of both the airframe and the engine manufacturers, is defined between stations ∞ and e . This is termed *net standard thrust* (or *uninstalled thrust*) and is defined as

Figure 1. Stations in the Flow for the Definition of Thrust

$$T_N = \dot{m}_e(V_e - V_\infty) + A_e(P_e - P_\infty) \quad [2.1.2]$$

2.2 Propulsion System Losses and Drag Terms

Although Equation [2.1.2] is suitable for the calculation of thrust under static conditions, the calculation of thrust under flight conditions requires the consideration of the flow field surrounding the nacelle, as well as the assignment of the drag penalties associated with the bleed, bypass and auxiliary flows.

Under these conditions, the *installed thrust* is given by

$$T_i = \dot{m}_e(V_e - V_\infty) + A_e(P_e - P_\infty) - D_{add} - D_p - D_f - D_b - D_x \quad [2.2.1]$$

This expression can be obtained by means of a control volume analysis as shown in “Appendix A. Installed Propulsion Thrust/Drag” on page 162. Using the nomenclature of Appendix A, the drag terms, which represent the difference between the net standard thrust and the installed thrust, are defined as follows:

Additive drag, $D_{add} = \int_\infty^c (P - P_\infty) dA_x$ (where dA_x is the elemental area of the bounding streamtube, projected in the x-direction);

Nacelle form drag, $D_p = \int_c^s (P - P_\infty) dA_x$ (where dA_x is the elemental area of the surface of the nacelle, projected in the x-direction);

Nacelle viscous drag, $D_f = \int_c^e \tau_w dA_s$ (where dA_s is the elemental surface area of the nacelle);

Bleed and bypass drag, $D_b = [\dot{m}_b(V_\infty - V_b) + A_b(P_b - P_\infty)] \cos \theta_b$;

Auxiliary drag, $D_x = \dot{m}_x V_\infty$.

The nacelle form drag terms can be further split into cowl pressure drag, afterbody pressure drag, and nozzle interference drag. On a podded engine the cowl is that part of the nacelle forward of the point of maximum diameter, while the afterbody is the region aft of that point.²

The subsonic form drag is traditionally combined with the viscous drag to form a quantity known as *profile drag*. At Mach numbers approaching unity, the flow accelerates around the contoured surfaces of the inlet and becomes locally supersonic. The pressure rise associated with the resulting shock formation causes a sudden increase in form drag. The Mach number at which this occurs is known as the *drag-rise Mach number*. The additional form drag due to the advent of localized or global supersonic flow is known as *wave drag*, and this becomes more severe as the Mach number increases.

² These geometrical definitions may have to be revised for integrated configurations where the cowl and afterbody are not so clearly definable.

2.3 Inlet Drag

The inlet drag is composed of the additive drag combined with the form drag and the viscous drag of the cowl.³

Figure 2 on page 11 shows that under certain conditions, for both a subsonic and a supersonic inlet, the additive drag can be zero, since there is no area change in the streamtube as the upstream air enters the inlet. This condition is known as the full-flow condition. Additive drag results when the engine accepts less than this full flow, or when the oblique forebody shocks originating from the forebody of a supersonic inlet do not coalesce at the cowl lip, causing the bounding streamlines to be curved or bent. In this case the *mass flow ratio*, defined as the ratio of the stream tube area at ∞ to the inlet capture face area, is less than unity (Figure 3 on page 12).

It is convenient to rewrite the inlet drag formulation as

$$\begin{aligned} D_{inlet} &= (D_p + D_f)_{c,o} + \Delta D_c + D_{add} \\ &= (D_p + D_f)_{c,o} + D_{spill} \end{aligned} \quad [2.3.1]$$

³ Although the additive drag term is treated as a drag force, it does not strictly define a force acting on any solid surface of the propulsion system. Instead it may be thought of as a correction (necessitated by the definition of installed thrust) which accounts for the fact that the airflow around the engine is disturbed. It is demonstrated in "Appendix B. Additive Drag Cancellation in Ideal Flow" on page 167 that, if it is assumed that the (ideal) downstream airflow outside of the jet uniformly has the same pressure as the upstream air, then the additive drag is exactly cancelled by the cowl drag. This occurs because the acceleration of the flow over the nacelle causes a negative drag (*cowl lip suction*) which cancels the additive drag term. In actuality, the airflow around the cowl is disturbed, and the fluid is **not** ideal. Consequently, in many cases, viscous and compressibility effects inhibit the cancellation of additive drag which must therefore be accounted for in the thrust analysis.

where the term $(D_p + D_f)_{c,o}$ defines the friction and form drag on the cowl at the full-flow condition. As mentioned in the previous section, this term consists of subsonic profile drag, D_{prof} , for Mach numbers less than the drag-rise value, and both friction and wave drag for higher Mach numbers. D_{spill} is the *spillage drag*, and it is defined as the sum of the additive drag, D_{add} , and the change in cowl drag (cowl lip suction), ΔD_c , from the cowl drag at full flow.

2.4 Inlet Types

The geometrical configuration of an inlet has a major effect on its performance, and is dictated both by the required aircraft Mach number range (i.e., the aircraft mission) and the configuration of the aircraft as a whole. Some of the most common inlet configurations are sketched in Figure 4 on page 13. Broadly speaking, inlets are classified as either axisymmetric (circular or part-circular) or two-dimensional (2-D, rectangular); and are designed for either subsonic or supersonic operation.

Inlets designed for high-supersonic Mach number operation are characterized by a forebody which is conical in the axisymmetric case and wedge-shaped for two-dimensional inlets. The forebody causes one or more oblique shocks to form ahead of the inlet. Consequently, the terminal shock at the inlet throat is weakened, and the total pressure loss of the inlet reduced.

Inlets which are **not** equipped with such forebodies are termed *pitot* inlets. Pitot inlets operating at supersonic speeds with engine mass flow rates less than the maximum have

a) Pitot Inlet – Subsonic Freestream Flow

b) Inlet with Forebody – Supersonic Freestream Flow

Figure 2. Inlets Operating at Full Flow

a) Pitot Inlet – Subsonic Freestream Flow

b) Inlet with Forebody – Supersonic Freestream Flow

Figure 3. Inlets Operating at Reduced Mass Flow Ratios

Inlet Type	Typical Positions
<p>Pitot</p>	<p>Chin</p> <p>Pod, under-wing</p>
<p>Supersonic Conical</p>	<p>Nose</p> <p>Side, semi-circular</p>
<p>Supersonic 2-D</p>	<p>Side, D-shaped</p> <p>Side, horizontal ramp</p>

Figure 4. Common Inlet Configurations: Note that the typical positions shown for the axisymmetric pitot inlet are applicable to the conical inlet, and vice versa.

a normal shock standing in front of the inlet. The high static pressure ratio and high total pressure loss associated with this shock can result in both poor inlet efficiency and high spillage drag. However, design trade studies have shown that a suitably designed pitot inlet may still perform the best in low-supersonic-cruise fighters from the point of view of inlet losses and external drag (in the most important Mach number range), weight, cost and reliability (Hawkins, 1974).⁴

The simplest form of the supersonic inlet with forebody is known as the *two-shock* inlet. The forebody only has one angle, giving rise to only one oblique shock and one terminal normal shock at the entry plane of the inlet. This is also termed an *external-compression* inlet since the diffusion nominally occurs upstream of the inlet capture face.

A more complex variation of the external-compression inlet involves the use of a forebody consisting of more than one angle, the geometry of which may be variable.⁵ This results in multiple (weaker) oblique shocks, further reducing the total pressure loss of the inlet flow.

Incorporating an infinite number of oblique shocks (Prandtl-Meyer compression) results in an isentropic compression ramp inlet, where the ramp angle varies continuously.⁶ This design results in a much higher ramp angle at the inlet plane, necessitating a higher cowl slope. The improved inlet performance must therefore be traded off against the higher cowl wave drag.

⁴ The aircraft in question is the F-16, designed for air superiority at Mach numbers up to 1.6 and a dash capability of Mach 2.0.

⁵ A modern fighter aircraft equipped with this inlet type is the F-15. Its 2-D inlet has three compression ramps of variable geometry. This is the maximum number of ramps flown on large-production-run aircraft.

⁶ The Anglo-French Concorde is one of the few aircraft to be equipped with such an inlet.

An alternative extension of the multi-shock inlet is the mixed-compression inlet where additional diffusion occurs in an oblique shock system in the duct downstream of the inlet capture plane. This results in improved performance for high Mach numbers without the cowl wave drag penalty of the multi-shock purely external-compression inlet. However, these inlets lack the inherent stability of the external-compression design, and the process known as "starting" the inlet (establishing the required shock system) requires extremely complex variable geometry, normally including large bypass doors.⁷

Axisymmetric inlets have the advantage of efficient structural shape for low duct weight and minimum wetted area per unit of flow area. The classic application for these inlets is passenger jets, but they were also used on the earliest jet fighters and also the next generation of fighters with swept wings and nose-inlets. Axisymmetric inlets equipped with conical forebodies can facilitate variable geometry in the form of a translating cone (to vary the amount of internal compression and to start the inlet), provided that the inlet configuration is suitable (as on a podded engine, for instance). On the other hand, 2-D inlets have the advantage of being able to combine variable-ramp geometry with fuselage side-mounted positioning. A further advantage of the side-mounted 2-D inlet with horizontal compression ramps (as opposed to the D-shaped configuration where the compression ramp blends with the fuselage) is the good high angle-of-attack performance. One other feature of 2-D inlets worthy of mentioning is the sideplate shape. Traditionally, these are triangular in shape, extending in a straight line from the ramp vertex to the cowl lip. However, there is evidence to suggest that cutting back the sideplate reduces the ingestion of vortices shed from the sideplate and hence improves the inlet performance during yawed flight (Whitford, 1987).

⁷ The SR-71 uses an axisymmetric variable-geometry inlet with a translating forebody. However, the mixed-compression inlet design for the B-1 was scrapped in a cost-cutting attempt. This lightened the aircraft, but limited its flight Mach number.

2.5 Accounting for Propulsion System Losses in Conceptual Design

Aircraft conceptual design codes are normally divided into several functional modules. These modules include the aerodynamics module, which generally calculates the aerodynamic lift and drag, and the propulsion module which primarily calculates the engine thrust and fuel consumption.

In the section entitled “Propulsion System Losses and Drag Terms” on page 7, many of the drag terms such as auxiliary, bleed and bypass drag are functions of engine mass flow. Since additive drag (and hence spillage drag) depends on the mass flow ratio, it is also throttle-dependent. Therefore, in most aircraft conceptual design programs, these drag terms are computed within the propulsion module so that engine mass flow rate is easily accessible. This is in contrast to the other drag terms which are normally calculated by the aerodynamics module.

3.0 Drag Prediction Methodology

As has already been stated, the aim of this research is to extend the capabilities of the aircraft conceptual design code, ACSYNT. The first step is to establish which inlet types needed to be addressed. This decision should be based both on knowledge of aircraft design practice, and the availability of suitable methods for predicting the drag of the specific inlet types. Once the candidate inlet types have been identified, the characteristics required of the drag prediction methods can be appropriately defined, and methods for calculating each of the drag contributions can be investigated.

3.1 *Selection of Candidate Inlet Types*

It is apparent from the section entitled “Inlet Types” on page 10 that, during the several decades since the inception of the jet aircraft, inlet design practice has evolved together with other facets of technology and aircraft mission requirements. For the purpose of an aircraft conceptual design program, which could be used both for evaluating existing configurations and optimizing contemporary designs, inlet types most representative of the gamut of configurations must be considered. This selection of candidate inlet types will, however, be tempered by the availability of suitable drag prediction methods.

From a thorough review of the literature, the decision was made to consider the following four inlet geometries:

- ***Subsonic Axisymmetric Pitot Inlet.*** These inlets are designed to have favorable spillage drag and good high-incidence performance, and are characterized by a rounded, blunt-lipped cowl. The blunt lip minimizes flow separation at the cowl lip, and hence maximizes cowl suction to aid in the cancellation of additive drag. This type of inlet is unsuited to high Mach number operation because the supersonic wave drag can become excessive, so it is nominally termed a "subsonic" pitot inlet. In practice, however, such inlets may be operated at low supersonic Mach numbers.

For the purpose of this work, the cowl of the subsonic pitot inlet is assumed to have a profile belonging to the NACA 1-Series family. This cowl was designed for high critical-speed operation (Baals et al., 1949), and it has become a de facto standard in the aircraft industry. Pitot inlets equipped with NACA 1-Series cowls have been studied extensively, both in the USA and in the United Kingdom, so that much experimental data and many design correlations exist, particularly for subsonic operation.

- ***Supersonic Axisymmetric Pitot Inlet.*** The main difference between this inlet and the above-mentioned inlet type is the cowl geometry. Normally, a supersonic pitot inlet will have a slender cowl profile, typically conical or parabolic, in order to minimize cowl wave drag. A small amount of lip blunting may, however, be incorporated to assist in the recovery of spillage drag.

It is acknowledged that some supersonic pitot inlets, such as the F-16's, are not truly axisymmetric. However, it is believed that the inaccuracy involved in modeling it as such should not pose a problem for conceptual design purposes. This contention is supported by the F-16 test case described in the chapter entitled "Application of the Calculation Methods" on page 146.

- ***Supersonic Two-Dimensional (2-D) Wedge Inlet.*** Only the two-shock wedge inlet was considered for the current work because the sophistication of the more advanced multi-shock inlet types results in considerably more complex drag calculations, and correlations to assist with such calculations are not widely available. The two-shock version is a fairly representative inlet type, particularly for moderate supersonic Mach numbers (say Mach 2 or less), and is considered adequate for most conceptual design applications. Furthermore, scale factors can be included in the actual routines to allow the designer to adjust the two-shock inlet drag prediction to model a more advanced inlet if necessary.⁸ Fortunately, there is a reasonable amount of data available for the two-shock inlet, and the best of the available correlations have been built into the 2-D wedge inlet drag calculations.
- ***Supersonic Axisymmetric Inlet with Conical Forebody.*** Only the two-shock axisymmetric spike inlet only was considered, because the nonuniform supersonic conical flow field is even more complex than in the 2-D case, so that drag prediction for the multi-shock variation is not practical. The original analysis within ACSYNT embodied variable inlet geometry in order to maintain a constant inlet throat Mach number. Even though this analysis is flawed (as discussed in the section entitled “Additive Drag Prediction Method Originally in ACSYNT” on page 110), a variable-geometry option (though modified) has been retained to maintain compatibility with previous test cases. An additional fixed-geometry calculation option has been provided to obtain a more realistic spillage drag prediction for less-sophisticated aircraft.

⁸ This concept is demonstrated in the chapter entitled “Application of the Calculation Methods” on page 146.

3.2 Characteristics Required of the Drag Prediction

Methods

The primary issue to be considered is the level of complexity of the calculations. However, the question as to how detailed to make the prediction of the drag of a component (such as the inlet) for conceptual design purposes is a complex one. It can be argued that only a rough estimate is required to account for the major effects which contribute to the drag, and that only a simplistic analysis need be performed. Certainly, since the propulsion module is invoked many times during the execution of the conceptual design program, methods which require excessive iteration and/or operations on large arrays cannot be used.

Surely the most economical approach is to model the inlet with about the same degree of accuracy as the rest of the aircraft. Unfortunately, this degree of accuracy is not easily quantifiable. Furthermore, if a single aspect of the inlet design is arbitrarily neglected, even though it can be relatively simply accounted for, the result may be misleading to the designer. Also worth considering is that, while a portion of the program is being upgraded, it is prudent to do the best job possible. Then, if the rest of the program is upgraded over time, the revised module does not necessarily become obsolete.

Consequently, the approach taken was to develop the best drag prediction routines possible with the available tools consisting of published correlations and methods, and one- and two-dimensional inviscid compressible flow theory. It is likely that the inlet drag analysis which results from such an approach will fall somewhere between the first-order correction required for conceptual design and the more refined prediction

needed for preliminary design. However, it should be pointed out that, in most cases, experimental data are scarce and often in a form that is difficult to interpret. Furthermore, the above-mentioned correlations are, of necessity, derived from experiments on specific geometries, and the flow analysis methods must be simplified. This means that the absolute accuracy of the drag prediction for a wide range of geometries and operating conditions cannot be guaranteed even if the calculation methods have a sound physical basis. Instead, a reasonable requirement is that the trends of the drag prediction must be realistically represented as either the geometry or the operating conditions are varied. If this is achieved, then once the designer has specified an inlet geometry and incorporated it into the aircraft using the new CAD facility of ACSYNT, the performance prediction will at least be in keeping with that geometry.

With the above issues in mind, the characteristics required of the inlet drag prediction routines can be summarized as follows:

- Computational efficiency, for the reason mentioned above.
- Quantitative realism, with the accuracy of the calculations at least as accurate as the overall accuracy of the program.
- The proper trends should be exhibited as geometry, Mach number and mass flow are varied.
- Step changes in the prediction should not be permitted, because these may interfere with the operation of the optimization routine.
- Minimal user input (in keeping with a conceptual design code) to describe the inlet.
- Well-documented source code.

3.3 General Approach to the Calculation of Inlet Drag

Broadly speaking, the approach to the calculation of inlet drag involves the evaluation of each of the terms in Equation [2.3.1]. In other words, the viscous drag, design-point form drag, and the spillage drag need to be estimated and summed together to form the total inlet drag.⁹ A flow chart which summarizes a generic approach to inlet drag prediction is shown in Figure 5 on page 23. The methods for estimating the individual drag terms will obviously vary according to the inlet type and operating conditions.

The viscous drag is generally computed from the turbulent mean skin friction coefficient which must, in turn, be corrected to compensate for both the skin surface roughness and the localized heating due to the compressibility effects at high Mach numbers (see the section entitled “Calculation of Skin Friction Coefficient” on page 170). A form factor is generally used to account for the difference between the flat-plate viscous drag and the profile drag of the inlet cowl.

Even though the spillage drag may be calculated differently for each of the inlet types, it is a function of additive drag, so that the additive drag must always be computed. The calculation of the cowl lip suction, i.e., the degree to which the additive drag is recovered, is generally a function of the cowl geometry and possibly also the inlet operating conditions.

The method of calculating wave drag will differ according to the Mach number regime and the inlet geometry. Normally, the transonic wave drag must be calculated in a dif-

⁹ This technique is known as “component build-up”, and it is used in the aerodynamics module of ACSYNT.

Figure 5. Typical Approach to Inlet Drag Prediction

ferent manner to the supersonic wave drag, and an estimate of the drag-rise Mach number is required for the detection of the onset of wave drag. It must be emphasized that, in this work, the wave drag is calculated with the assumption that the inlet is operating at full flow. This means that the effect of the bow shock, which forms at reduced mass flow ratios, is not considered in the evaluation of the wave drag coefficient. Instead, the effect of the bow shock is accounted for in the spillage drag calculation.

It is further assumed throughout the analyses that the inlet is operating at zero angle-of-attack, and that the inlet is subject to a uniform freestream flow undisturbed by upstream surfaces.

Chapters 5 through 8 describe, in detail, the approach taken to predict the drag for the individual candidate inlet types. Although not addressed in these chapters, the actual source code contains "scale factors" which allow the contributions of spillage, profile and wave drag to be individually scaled according to the requirements of the designer. Such factors are commonly employed in conceptual design programs. Applications of these scale factors are discussed in the chapter entitled "Application of the Calculation Methods" on page 146.

4.0 Literature Survey

This chapter briefly reviews the most important references covering inlet drag prediction. In the subsequent chapters which detail the methods of predicting the drag of each candidate inlet type, specific publications which have contributed to the methods used for calculating inlet drag will be further discussed.

Even though inlet drag is of considerable importance to aircraft designers, there is no single reference that covers inlet drag calculation methods thoroughly enough to be used as a basis for a conceptual design code. The book by Seddon and Goldsmith (1985) is the most useful, devoting an entire chapter to the problem. It has been referenced extensively for this work, but it is vague about many specific aspects. Furthermore, the methods described are based on the authors' experience in the British aerospace industry, and are not always compatible with the methods most commonly used in the USA. Other useful reviews by British authors are by Butler (1973) and Leyland (1988).

One of the most significant contributions from the UK is due to workers from the Royal Aircraft Establishment (RAE) and Rolls-Royce. They have successfully correlated data based upon experiments (mostly performed by NASA and Rolls-Royce) on subsonic pitot inlets equipped with NACA 1-Series cowls. Although these methods are reviewed by some of the publications mentioned above, the most comprehensive collection of the data is presented in the form of charts and equations by the Engineering Sciences Data Unit, another British organization (ESDU, 1981). This document provides sufficient information to enable a designer to calculate the subsonic drag of a pitot inlet equipped with a NACA 1-Series cowl.

The wide variation of geometrical configurations for supersonic inlets means that drag prediction methods for supersonic inlets must be derived from a number of sources. Probably the first researcher to successfully develop a method for the prediction of spillage drag for external-compression inlets is Sibulkin (1955). This work dealt with conical inlets, and it has yet to be superseded in the open literature.

An important body of work contributing to the development of spillage drag correlations for 2-D inlets has been completed by workers at North American Aviation, in cooperation with NASA. This effort was initiated by Mount (1965) and continued by Petersen and Tamplin (1966). The experimental data of Petersen and Tamplin were later used by Osmon (1967), who derived spillage drag correlations in an alternative form.

One of the few comprehensive manuals, published by a major aircraft company (Boeing), which presents detailed methods for the prediction of inlet drag, is that by Ball (1972). Even though this report derives most of its methods from the publications mentioned above, it does add some useful correlations for predicting other propulsion installation losses, such as bleed and bypass drag. Some of these methods have been used in ACSYNT. An important book by Nicolai (1975) on aircraft conceptual design also devotes a chapter to the prediction of propulsion installation losses. However, this book draws mainly from the above-mentioned references and contributes little more to the analysis methods. The most recently published book on aircraft conceptual design (Raymer, 1989) also yields little further insight into methods for inlet drag prediction.

5.0 Prediction of Drag for the Subsonic Pitot Inlet

5.1 Overview

As discussed in the section entitled “Selection of Candidate Inlet Types” on page 17, the subsonic pitot inlet is characterized by a rounded, blunt-lipped cowl assumed to be of the NACA 1-Series family of profiles. This family of cowl profiles is expressed in the form of tabulated, nondimensional coordinates, as shown in Table 1. These coordinates, combined with the selected cowl diameter at the inlet capture face, cowl maximum diameter and cowl length, define the external geometry of the inlet.

Providing that the cowl profile is suitably bluff, total cancellation of additive drag at subsonic Mach numbers can occur if the incidence of the external flow on the cowl is low enough. The parameter which is normally used to correlate the external flow incidence is the *mass flow ratio*. This is denoted MFR , and is defined as

$$MFR = \frac{A_{\infty}}{A_c} \quad [5.1.1]$$

where A_c is the inlet capture face area, and A_{∞} is the streamtube capture area. The *critical mass flow ratio*, denoted MFR_{crit} , corresponds to the onset of spillage drag during subsonic operation.

Table 1. NACA 1-Series Cowl Coordinates (from Seddon and Goldsmith (1985), Table 9.1)

$\frac{x}{L}$	$\frac{y}{Y}$
0	0
0.004	0.0663
0.008	0.0933
0.015	0.1272
0.025	0.1657
0.035	0.1994
0.050	0.2436
0.080	0.3181
0.110	0.3815
0.140	0.4366
0.170	0.4840
0.200	0.5270
0.230	0.5666
0.260	0.6035
0.300	0.6489
0.340	0.6908
0.380	0.7294
0.420	0.7648
0.460	0.7974
0.500	0.8269
0.580	0.8795
0.660	0.9220
0.740	0.9548
0.820	0.9787
0.900	0.9940
1.000	1.0000

Because the profile is rounded, the acceleration of the flow around the cowl surface can be severe, causing the onset of wave drag even while the freestream Mach number, M_∞ , is subsonic. Therefore, it is important to predict the drag-rise Mach number, M_D , and also the wave drag for subsonic Mach numbers greater than M_D .

An important issue is the prediction of **supersonic** inlet spillage, wave and profile drag for this inlet type. Even though it is termed a **subsonic** pitot inlet, it may be used for low-supersonic Mach number operation. Furthermore, for a conceptual design code

there should be no discontinuities or prematurely-occurring upper limits to the drag prediction. Even if the drag prediction is less accurate in some operational regimes than others, if the designer's selection of inlet geometry is adversely compromising his aircraft design, he should at least be made aware of this fact. Unfortunately, the body of literature referred to in the chapter entitled "Literature Survey" on page 25 only addresses the subsonic regime, because the subsonic pitot inlet is primarily intended for subsonic operation. Therefore, the existing methods have had to be extended to cover this operational regime, based on the sparse available data.

The flow chart in Figure 6 on page 30 provides an overview of the process of inlet drag prediction for this inlet type. The specific analysis methods and equations are discussed in the remainder of the chapter.

5.2 *Basic Drag Formulation*

The basic formulation for inlet drag, from the section entitled "Inlet Drag" on page 9, is again stated. However, it is now expressed in nondimensional coefficient form, which facilitates the comparison of inlet drag for various inlet dimensions and Mach numbers.

The inlet drag coefficient is expressed as

$$C_{D_{inlet}} = \frac{D_{spill} + D_{prof} + D_{wave}}{q_{\infty} A_c} = C_{D_{spill}} + C_{D_{prof}} + C_{D_{wave}} \quad [5.2.1]$$

Figure 6. Inlet Drag Prediction Process — Subsonic Pitot Inlet

where D_{spill} , D_{prof} and D_{wave} are the spillage, profile and wave drag forces, respectively. The terms $C_{D_{spill}}$, $C_{D_{prof}}$ and $C_{D_{wave}}$ are the corresponding drag coefficients referenced to the inlet capture area, A_c . The freestream dynamic head, q_∞ , is given by

$$q_\infty = \frac{1}{2} \rho_\infty V_\infty^2 = \frac{1}{2} \gamma P_\infty M_\infty^2 \quad [5.2.2]$$

Even though the spillage, profile and wave drag coefficients are summed together in the above formulation, the magnitudes of these terms are not necessarily nonzero for the entire operating range of the inlet. For instance, for operation below the drag-rise Mach number, the wave drag coefficient will be zero.

The treatment of the profile drag term for supersonic Mach numbers is discussed in the section entitled "Profile Drag" on page 39.

5.3 Determination of Critical Operating Points

This section describes the method for determining the critical mass flow ratio and the drag-rise Mach number. Both of these are based on empirical correlations derived from experimental data.

5.3.1 Critical Mass Flow Ratio for NACA 1-Series Cowls

As mentioned in the section entitled “Overview” above, the critical mass flow ratio, MFR_{crit} , is the mass flow ratio corresponding to the onset of spillage drag during subsonic operation. For $MFR \geq MFR_{crit}$, the acceleration of the flow around the cowl leading edge recovers the additive drag, and the spillage drag is cancelled. For $MFR < MFR_{crit}$, the flow separates on the cowl lip and the spillage drag is then greater than zero.

A correlation for critical mass flow ratio is presented by ESDU (1981), Figure 6, and also by Seddon and Goldsmith (1985), Figure 9.5. For the present work, this was approximated by the least-squares-fit polynomial

$$\begin{aligned} MFR_{crit} = 0.97367 - 9.22768 \times \left[\frac{\left(1 - \frac{d_c}{d_m}\right)^2}{L} \right] \\ + 22.0134 \times \left[\frac{\left(1 - \frac{d_c}{d_m}\right)^2}{L} \right]^2 \end{aligned} \quad [5.3.1]$$

This polynomial is valid in the range $0 < \left[\frac{\left(1 - \frac{d_c}{d_m}\right)^2}{L} \right] \leq 0.2$.

It should be noted that although this correlation was derived for Mach numbers less than the drag-rise Mach number, it is assumed to be valid for the entire subsonic Mach number range.

5.3.2 Drag-Rise Mach Number for NACA 1-Series Cows

Equation 9.25 of Seddon and Goldsmith (1985), and Figure 9a of ESDU (1984) give the following expression for the drag-rise Mach number at full flow:

$$M_D' = 1 - \frac{1}{8} \frac{\sqrt{1 - \left(\frac{d_c}{d_m}\right)^2}}{\left(\frac{L}{d_m}\right)} \quad [5.3.2]$$

Section 5.1.1 of ESDU (1981) accounts for the reduction in drag-rise Mach number as the mass flow ratio is decreased as follows:

i) for $MFR \geq MFR_{crit}$

$$M_D = M_D' \quad [5.3.3]$$

ii) for $MFR < MFR_{crit}$

$$M_D = M_D' - 0.15[MFR_{crit} - MFR] \quad [5.3.4]$$

5.4 Spillage Drag

The spillage drag is a function of additive drag, representing the amount that the acceleration of the flow over the cowl surface will recover the additive drag. Instead of evaluating subsonic spillage drag using the formulation given by Equation [2.3.1], where the additive drag and the cowl lip suction are evaluated independently, the method proposed by Mount (1965) is used. Here, spillage drag is defined as

$$C_{D_{spill}} = K_{add} C_{D_{add}} \quad [5.4.1]$$

The additive drag coefficient, $C_{D_{add}}$, for a pitot inlet is found from an exact closed-form expression based on one-dimensional compressible flow theory as described in the next section. The factor K_{add} is a function of Mach number, mass flow ratio and the cowl geometry. Seddon and Goldsmith (1985) criticize this method because of the variance of K_{add} . However, for the subsonic operation of pitot inlets equipped with NACA 1-Series cowls, the large amount of experimental data has permitted K_{add} to be adequately correlated (ESDU, 1981). The method used to calculate K_{add} is described in the section entitled “Subsonic Spillage Drag” on page 37.

For supersonic operation, the correlations for K_{add} are no longer valid, and this Mach number regime must be treated differently, as described in the section entitled “Supersonic Spillage Drag” on page 39.

5.4.1 Additive Drag

Figure 7 on page 36 shows the control volume used for the analysis of additive drag. It is bounded by a station far upstream, the inlet capture face, and the bounding streamtube. A mass and momentum balance is applied across this control volume and, by evaluating the momentum fluxes and pressures on the faces of this control volume, it is no longer necessary to evaluate the integral of the pressure over the bounding streamtube. The additive drag force, D_{add} , may instead be expressed as

$$D_{add} = \int_{\infty}^c (P - P_{\infty}) dA_x = [(P_c - P_{\infty}) + \rho_c V_c^2] A_c - \rho_{\infty} V_{\infty}^2 A_{\infty} \quad [5.4.2]$$

Rewriting this equation gives the following expression for additive drag coefficient based on A_c :

$$C_{D_{add}} = \frac{[P_c \gamma M_c^2 + (P_c - P_{\infty})] A_c - P_{\infty} \gamma M_{\infty}^2 A_{\infty}}{q_{\infty} A_c} \quad [5.4.3]$$

For subsonic operation, an isentropic expansion is assumed to occur from far upstream to the inlet plane, enabling the pressure and Mach number, P_c and M_c , to be calculated.

For supersonic operation, a normal shock occurs between the upstream station and the inlet plane. In this case, the pressure and Mach number downstream of the normal shock are used in an isentropic expansion to obtain the conditions at the inlet capture face.

Figure 7. Subsonic Pitot Inlet Nomenclature and Control Volume

5.4.2 Subsonic Spillage Drag

As mentioned earlier, the spillage drag is assumed to be zero as long as the mass flow ratio is greater than the critical flow ratio. Therefore, for $MFR \geq MFR_{crit}$,

$$C_{D_{spill}} = 0 \quad [5.4.4]$$

The method for determining the spillage drag for $MFR < MFR_{crit}$ relies on the determination of the factor K_{add} . This factor is correlated against both the mass flow ratio and the inlet dimensions as follows:

$$K_{add} = K_6 K_7 K_8 \quad [5.4.5]$$

K_6 is a function of $\Delta MFR = MFR_{crit} - MFR$.

To enable the information contained in the design chart of Figure 7a of ESDU (1981) to be used in a computer program for the present work, the following polynomial was fitted to this curve:

$$K_6 = -0.0154167 + 1.654167 \Delta MFR - 0.8988095 (\Delta MFR)^2 \quad [5.4.6]$$

It is valid in the range $0.02 < \Delta MFR < 0.7$.

K_7 is a function of $\frac{d_c}{d_m}$ and, for the present work, the the following formula was obtained by polynomial regression using the data from Figure 7b of ESDU (1981):

$$K_7 = 6.51063 - 16.53089 \left(\frac{d_c}{d_m} \right) + 11.56760 \left(\frac{d_c}{d_m} \right)^2 \quad [5.4.7]$$

It is valid for $0.7 < \frac{d_c}{d_m} < 0.89$.

K_8 is a function of both M_∞ and $\frac{d_m - d_c}{L}$. The curves contained in design charts of Figure 7c of ESDU (1981) were represented as the following formulae (obtained by polynomial regression):

i) for $\frac{d_m - d_c}{L} \leq 0.1$

$$0.79 \leq M_\infty \leq 0.855, K_8 = -17.05686 + 46.83571M_\infty - 30.35714M_\infty^2$$

$$M_\infty < 0.790, K_8 = 1$$

$$M_\infty > 0.855, K_8 = 0.78 \quad [5.4.8]$$

ii) for $\frac{d_m - d_c}{L} = 0.2$

$$0.76 \leq M_\infty \leq 0.84, K_8 = 26.98857 - 61.34286M_\infty + 35.71429M_\infty^2$$

$$M_\infty < 0.76, K_8 = 1$$

$$M_\infty > 0.84, K_8 = 0.66 \quad [5.4.9]$$

iii) for $\frac{d_m - d_c}{L} \geq 0.3$

$$0.73 \leq M_\infty \leq 0.84, K_8 = -200.1889 + 777.9624M_\infty - 995.4095M_\infty^2 + 420.8754M_\infty^3$$

$$M_\infty < 0.73, K_8 = 1$$

$$M_\infty > 0.84, K_8 = 0.37 \quad [5.4.10]$$

For values of $\frac{d_m - d_c}{L}$ in the ranges $0.1 \rightarrow 0.2$ and $0.2 \rightarrow 0.3$, linear interpolation is used to obtain the appropriate value of K_8 .

5.4.3 Supersonic Spillage Drag

It is suggested by Seddon and Goldsmith (1985) that, for Mach numbers greater than about 1.8, spillage drag consists almost entirely of *normal shock drag*, (a concept discussed in the section entitled “Spillage Drag” on page 58). This implies that little or no additive drag is recovered at these Mach numbers. On the basis of this observation, it is assumed that for Mach numbers greater than 2.0, $C_{D_{spill}} = C_{D_{add}}$.

For Mach numbers between 1.0 and 2.0 it is assumed that the spillage drag coefficient varies linearly with Mach number from the spillage drag coefficient at Mach 1.0 to the additive drag coefficient at Mach 2.0.

5.5 Profile Drag

The profile drag coefficient (based on inlet capture area, A_c) of a NACA 1-Series cowl is given by Equations 3.1 and 3.7 of ESDU (1981) as

$$C_{D_{prof}} = \frac{C_f}{K_o} \frac{A_s}{A_c} \left[1 + 0.33K \frac{\left(1 - \frac{d_c}{d_m}\right)}{\frac{L}{d_m}} \right]^{1.667} \quad [5.5.1]$$

where C_f is the skin friction coefficient (see the section entitled "Calculation of Skin Friction Coefficient" on page 170);

A_s is the cowl wetted area;

and A_c is the inlet capture area.

K is a function of the cowl area ratio $\left(\frac{A_m}{A_c}\right)$ and the mass flow ratio $\left(\frac{A_\infty}{A_c}\right)$. The formula, obtained by polynomial regression using the data of Figure 1a of ESDU (1981), is

$$\text{i) for } 0 \leq \frac{\left(1 - \frac{A_\infty}{A_c}\right)}{\left(\frac{A_m}{A_c} - 1\right)} \leq 3,$$

$$K = 0.99127 + 1.00926 \frac{\left(1 - \frac{A_\infty}{A_c}\right)}{\left(\frac{A_m}{A_c} - 1\right)} - 0.21482 \left[\frac{\left(1 - \frac{A_\infty}{A_c}\right)}{\left(\frac{A_m}{A_c} - 1\right)} \right]^2 \quad [5.5.2]$$

$$\text{ii) for } \frac{\left(1 - \frac{A_\infty}{A_c}\right)}{\left(\frac{A_m}{A_c} - 1\right)} > 3,$$

$$K = 8$$

[5.5.3]

K_o is a function of diameter ratio $\left(\frac{d_c}{d_m}\right)$ and fineness ratio $\left(\frac{L}{d_m}\right)$ given in Figure 1b of ESDU (1981). An equation, obtained by polynomial regression using the data represented by this graph, gives

$$\text{i) for } 0 \leq \frac{\left(1 - \frac{d_c}{d_m}\right)}{\left(\frac{L}{d_m}\right)} \leq 1,$$

$$K_o = 0.99875 + 0.03584 \frac{\left(1 - \frac{d_c}{d_m}\right)}{\left(\frac{L}{d_m}\right)} + 0.122728 \left[\frac{\left(1 - \frac{d_c}{d_m}\right)}{\left(\frac{L}{d_m}\right)} \right]^2 \quad [5.5.4]$$

$$\text{ii) for } \frac{\left(1 - \frac{d_c}{d_m}\right)}{\left(\frac{L}{d_m}\right)} > 1,$$

$$K_o = 1.155$$

[5.5.5]

The coordinates for NACA 1-Series cowls given in Table 1 on page 28 are used in the calculation of the cowl wetted area, A_s . The cowl surface is approximated as a series of conical segments. The area of each segment is calculated and summed to get the total wetted area of the cowl.

The area of a right circular cone is $\pi r l$, where r is the radius at the base and l is the length of the side (i.e., $l = \sqrt{r^2 + h^2}$, where h is the cone height). Thus, to evaluate A_s , the following equation is used:

$$A_s = \sum_{i=1}^{25} \frac{\pi}{\sin \delta_i} (r_i^2 - r_{i-1}^2) \quad [5.5.6]$$

where r_i is the maximum radius of segment i ;

r_{i-1} is the minimum radius of segment i , which equals maximum radius of segment $i - 1$;

$$\delta_i = \tan^{-1} \left[\frac{r_i - r_{i-1}}{x_i - x_{i-1}} \right];$$

and x_i and x_{i-1} are the x coordinates of r_i and r_{i-1} , respectively.

ESDU (1981) recommends that the subsonic profile drag be evaluated at the drag-rise Mach number if the freestream Mach number exceeds this drag-rise value. This recommendation has been followed for the present work.

It is also acknowledged that the physical meaning of the contribution of the form factor term in Equation [5.5.1] to the profile drag is questionable in supersonic flow. Form factors are generally only considered valid for Mach numbers below the drag-rise value (Raymer, 1989). This is because the form factor is intended to increase the flat-plate

skin friction coefficient to proportionately account for the effect of the shape of the body on the form drag in subsonic flow. In supersonic flow, the wave drag will be the major contributor to the form drag, and the form factor is no longer strictly valid. However, since the profile drag term generally makes a small contribution compared to the wave drag, it was preferred not to neglect it for supersonic freestream flow. This approach thereby avoids a discontinuity from occurring in the drag prediction as the Mach number becomes supersonic, and the resulting overprediction is unlikely to exceed the bounds of accuracy of the calculation.

Having decided to retain the contribution of profile drag for supersonic flow, the question remains as to what Mach number it should be evaluated at. The sketch in Figure 8 on page 44 depicts a stylized representation of a subsonic pitot inlet in a supersonic flow. The subsonic bounding streamline downstream of the normal shock will stagnate on the cowl lip. The flow outside of this streamline accelerates around the nose and, at some point, the local Mach number of this flow will exceed the drag-rise value. As the sonic line is passed, the flow will become supersonic and, at some point far enough back on the cowl, the local Mach number will again equal the freestream value.

This range of local Mach numbers is at odds with the above-mentioned recommendation by ESDU (1981). However, since the variation of skin friction drag with Mach number will not have too great an effect on the overall drag prediction, the pragmatic approach was taken whereby the supersonic profile drag is always evaluated at the drag-rise Mach number.

Figure 8. Subsonic Pitot Inlet in a Supersonic Flow

5.6 Wave Drag

The drag due to compressibility effects at subsonic Mach numbers above the drag-rise Mach number is termed *subsonic wave drag*, while *supersonic wave drag* occurs at supersonic Mach numbers. Due to this difference in Mach number regimes, different analysis methods must be used in each case.

5.6.1 Subsonic Wave Drag

Subsonic wave drag can either be calculated by a fairly complex numerical scheme such as a compressible potential flow solver (which is too computationally intensive for the present work), or by correlation of experimental data. The only such correlation found in the literature is contained in ESDU (1981), Equation 5.5. It was used for the present work in the following form:

$$C_{D_{wave}} = \left[(C_{D_{add}} - C_{D_{add,D}}) \frac{P_{\infty}}{P_D} \frac{M_D^2}{M_{\infty}^2} \right] + \frac{(A_m - A_c)}{\frac{1}{2} \gamma A_c M_{\infty}^2} \left\{ \left(\frac{P_{\infty}}{P_D} - 1 \right) - K_{11}(1 - M_D) \left[1 - \left(\frac{1 - M_{\infty}}{1 - M_D} \right)^3 \frac{P_{\infty}}{P_D} \right] \right\} \quad [5.6.1]$$

where

$$\frac{P_{\infty}}{P_D} = \left[\frac{1 + \left(\frac{P-1}{2} \right) M_{\infty}^2}{1 + \left(\frac{P-1}{2} \right) M_D^2} \right]^{\frac{\gamma}{\gamma-1}}$$

$$\frac{P_{\infty}}{P_{\infty}} = \left(1 + \frac{\gamma - 1}{2} M_{\infty}^2\right)^{\frac{\gamma}{\gamma - 1}}$$

for $(M_{\infty} - M_D) \leq 0.1, K_{11} = 0.2 + M_{\infty} - M_D$
 $(M_{\infty} - M_D) > 0.1, K_{11} = 0.3.$

Note that $C_{D_{add}}$ is the additive drag coefficient calculated at M_{∞} , while $C_{D_{add,D}}$ is the additive coefficient calculated at M_D .

5.6.2 Supersonic Wave Drag

Review of Alternative Methods

Seddon and Goldsmith (1985) demonstrate that linear theory cannot predict the supersonic wave drag of a NACA 1-Series cowl with any degree of accuracy. Other than a complete flow field solution, the most accurate method of predicting the pressure distribution of such a blunt body of revolution is the second-order perturbation method of Van Dyke (1952), which is valid as long as the local cowl slope does not approach the freestream Mach angle. However, it was felt that this method is still numerically too intensive for the purposes of the present work. Since the subsonic inlet is obviously designed primarily for subsonic and transonic operation, accurate supersonic prediction should not strictly be necessary. Instead, a method which reflects an appropriate drag increase with Mach number is required, so that the higher drag value will indicate to the designer that his cowl design is incurring a drag penalty.

Unfortunately, there is a lack of experimental data quantifying the supersonic wave drag of these cowls at the full-flow condition. Seddon and Goldsmith (1985) note that the result of a force balance measurement includes additive drag, and that even with cowl surface pressure measurements, the drag at full flow must still be obtained by extrapolating the results at lower flow ratios. Consequently, the only data available for this study were derived from a plot shown by Seddon and Goldsmith (1985), which gives the drag of cowls of various $\frac{L}{d_m}$ and $\frac{d_c}{d_m}$ ratios at Mach 1.2 and Mach 1.29.

One method investigated is the modified Newtonian method. In this method, the local pressure coefficient is given by

$$C_p = C_{po} \sin^2 \delta \quad [5.6.2]$$

where δ is the local cowl slope.

C_{po} is the stagnation pressure coefficient behind a normal shock occurring at the freestream Mach number and is given by

$$C_{po} = \frac{P_{o, shk} - P_\infty}{q_\infty} \quad [5.6.3]$$

where $P_{o, shk}$ is the total pressure after the normal shock, and q_∞ is the freestream dynamic pressure. Integration of the pressure coefficient along the projected surface of the cowl will produce a drag coefficient.

Even though this theory is normally applied to hypersonic flow, Moore (1973) contends that it may be used for bluff bodies at Mach numbers as low as 1.2. However, it was found that the application of the method to NACA 1-Series cowls results in a severe underprediction of wave drag. It is presumed that the reason for Moore's success is that

he only applied the method to that region of the body where the slope was too high for perturbation methods. As the slope decreased, he blended Newtonian theory with Van Dyke's second-order method. In contrast, it was found that applying the modified Newtonian method over the entire cowl (where the slope is far less than 90° , even close to the lip), results in an underprediction due to the sine-squared term in Equation [5.6.2].

Method Used for the Present Study

Seddon and Goldsmith (1985) recommend a semi-empirical approach for estimating the drag of a blunt, circular-arc lip added to a slender cowl. This approach is very similar to the modified Newtonian method, and yields the following expression for the lip drag coefficient:

$$C_{D_{lip}} = \frac{(KP_{o, shk} - P_\infty)A_{lip}}{q_\infty A_c} \quad [5.6.4]$$

Here, K is a linear function based on the ratio of maximum cowl projected area (A_m) to the area of the inlet capture face (A_c), and is derived from experimental data. Seddon and Goldsmith (1985) give the following expression for K in their Equation 9.62 as

$$K = 0.24 \frac{A_c}{A_m} + 0.57 \quad [5.6.5]$$

where A_m is the maximum cowl cross-sectional area and A_c is the inlet capture area.

A problem which arises when applying the above expression to a NACA 1-Series cowl is that it must be approximated as a slender cowl with a circular-arc lip. For the pur-

poses of this work, the lip is approximated as that portion of the cowl where the slope is greater than 10° , and which has a diameter greater than D_c .

The above method is used for Mach numbers greater than 1.2. For Mach numbers in the range 1.0 to 1.2, the wave drag coefficient is linearly interpolated between the wave drag coefficient predicted by the subsonic wave drag method (see the section entitled "Subsonic Wave Drag" on page 45 above) at Mach 1.0 and that predicted by the supersonic method at Mach 1.2.

5.7 *Selected Results*

The source code based on the methods derived from ESDU (1981) for the prediction of subsonic pitot inlet drag at subsonic Mach numbers has been verified against the sample calculations presented in that publication. Although this verification has not been included here, some selected results, including those relating to the extension of the methods to supersonic flow, are presented in the following sections.

5.7.1 Additive Drag Prediction

Figure 9 on page 50 shows a plot of additive drag coefficients calculated for a range of Mach numbers and flow ratios. This plot is identical to the one shown by Leyland (1988), which verifies that the method used for calculating additive drag is correct.

Figure 9. Additive Drag Coefficient Calculated for Pitot Inlets

5.7.2 Transonic Spillage Drag Prediction

The assumptions mentioned in the section entitled “Supersonic Spillage Drag” on page 39 were built into the program source code so that transonic and supersonic spillage drag could be estimated. The results of these assumptions are compared to the spillage drag data (taken from Figure 9.18 of Seddon and Goldsmith (1985)) in Figure 10 on page 52. The prediction agrees very well with the data, although it should be noted that these data may not be entirely reliable. They were derived from the measurements of Sears et al. (1952), who measured the total external drag of ducted bodies in free flight, and it is not known what assumptions were made by Seddon and Goldsmith to extract the spillage drag coefficient from these data. However, most important is the fact that the overall trend exhibited by these data supports the assumption that the transonic spillage drag coefficient varies linearly with Mach number.

5.7.3 Supersonic Wave Drag Prediction

As has been mentioned, even though Seddon and Goldsmith (1985) discuss the difficulty of obtaining NACA 1-Series cowl wave drag data, they do present some experimental results. Although these data are presented in nondimensional form, it is possible to derive the $\frac{L}{r_m}$ and $\frac{d_c}{d_m}$ values, and the drag coefficients, from the data. The result is listed in Table 2, where it is compared to the drag predicted by the methods described in the

Figure 10. Transonic Spillage Drag Calculation: Data are from Sears et al. (1955), as presented by Seddon and Goldsmith (1985). The cowl specification is NACA 1-49-100.

Table 2. Wave Drag Coefficient of NACA 1-Series Cowls (from Seddon and Goldsmith (1985), Figure 9.36)

$$C_D \text{ is based on } A_m. \quad \% \text{Diff} = \frac{C_D(\text{Calc}) - C_D(\text{Exp})}{C_D(\text{Exp})} \times 100\%.$$

$\frac{d_c}{d_m}$	$\frac{L}{r_m}$	M_∞	C_D (Exp)	C_D (Calc)	%Diff
0.8	2.0	1.2	0.080	0.108	+35
0.85	1.5	1.1	0.058	0.072	+24
0.85	1.5	1.2	0.084	0.091	+8
0.85	1.5	1.29	0.084	0.092	+10
0.85	2.0	1.1	0.045	0.056	+24
0.85	2.0	1.2	0.063	0.065	+3
0.85	2.0	1.29	0.058	0.071	+22
0.85	2.5	1.2	0.048	0.054	+13
0.89	2.0	1.2	0.038	0.033	-13
0.89	2.0	1.29	0.043	0.035	-19

section entitled “Method Used for the Present Study” above. Although the maximum difference between the data and the prediction is as much as 35 percent, the average is considerably less. However, it is acknowledged that this is a very limited Mach number range, and the accuracy for higher Mach numbers and other diameter ratios is not known. Most important, though, is that the trends will at least be qualitatively predicted for most bluff cowls.

5.8 Closure

Drag analysis methods have been presented for the subsonic pitot inlet equipped with a NACA 1-Series cowl. For subsonic operation, the methods have mainly been extracted

from ESDU (1981), which presents empirical correlations for calculating spillage drag (as a function of additive drag), subsonic wave drag and profile drag.

For supersonic Mach numbers greater than 2.0, the spillage drag is assumed to be equal to the additive drag, following the observation made by Seddon and Goldsmith (1985) based on experimental data. For supersonic Mach numbers less than 2.0, a simple linear variation of spillage drag coefficient is assumed, and this gives excellent agreement with experimental data.

The prediction of supersonic wave drag relies on approximating the cowl as a slender profile equipped with a blunt, semi-circular lip. The drag of the equivalent cowl is then calculated with an empirical correlation presented by Seddon and Goldsmith (1985). The validity of this method over a comprehensive range of supersonic Mach numbers and cowl geometries has not been confirmed, due to the lack of suitable experimental data.

Even though form factors are not strictly applicable for estimating profile drag in supersonic flow, a form factor has been used over the entire Mach number range with a view to avoiding a discontinuity in the drag prediction. The viscous drag contribution is always evaluated at the drag-rise Mach number.

6.0 Prediction of Drag for the Supersonic Pitot Inlet

6.1 Overview

The essential difference between the supersonic pitot inlet and the subsonic pitot inlet (discussed in the previous chapter) is the shape of the cowl profile. The cowl is assumed to be parabolic in shape, rather than having a NACA 1-Series profile. This means that the cowl profile can be expressed by the following formula:

$$r = r_m - (r_m - r_c) \left(1 - \frac{x}{L}\right)^n \quad [6.1.1]$$

where $n = \frac{L \tan \eta}{(r_m - r_c)}$

and η is the initial cowl slope, L the length, and r_m and r_c are the cowl maximum radius and inlet capture face radius, respectively. This general equation for the cowl profile is attributed to Willis and Randall (1955). For the cowl to have a parabolic shape, the exponent n must be set to 2.

The fact that the cowl profile is relatively slender means that the assumptions made for the subsonic pitot inlet regarding critical mass flow ratio, drag-rise Mach number, sub-

sonic spillage drag and wave drag can no longer be applied. For this reason, it is treated as a totally different inlet type to the subsonic pitot inlet.

Even though it is possible to add a blunt lip to a slender axisymmetric cowl for the purpose of reducing spillage drag at the expense of wave drag, the cowl is assumed to have a sharp lip for the present work. In the absence of suitable correlations for predicting drag-rise Mach number and critical mass flow ratio, the drag-rise Mach number is assumed to be fixed at 0.9, and the critical mass flow ratio is assumed to be 1.0.

The flow chart in Figure 11 on page 57 provides an overview of the process of inlet drag prediction for this inlet type. The specific analysis methods and equations are discussed in the rest of the chapter.

6.2 Basic Drag Formulation

As in the analysis of the subsonic pitot inlet, the inlet drag coefficient is expressed as

$$C_{D_{inlet}} = \frac{D_{spill} + D_{prof} + D_{wave}}{q_{\infty} A_c} = C_{D_{spill}} + C_{D_{prof}} + C_{D_{wave}} \quad [6.2.1]$$

where D_{spill} , D_{prof} and D_{wave} are the spillage, profile and wave drag forces, respectively. The terms $C_{D_{spill}}$, $C_{D_{prof}}$ and $C_{D_{wave}}$ are the corresponding drag coefficients referenced to the inlet capture area, A_c . The freestream dynamic head, q_{∞} , is given by Equation [5.2.2].

Figure 11. Inlet Drag Prediction Process – Supersonic Pitot Inlet

Even though the spillage, profile and wave drag coefficients are summed together in the above formulation, the magnitudes of these terms are not necessarily nonzero for the entire operating range of the inlet. For instance, for operation below the drag-rise Mach number, the wave drag coefficient will be zero. The treatment of the profile drag term for supersonic Mach numbers is discussed in the section entitled "Profile Drag" on page 62.

6.3 *Spillage Drag*

The most common approaches for evaluating spillage drag have been mentioned in previous chapters. These involve either reducing the additive drag by a calculated cowl suction coefficient (as in Equation [2.3.1]), or by multiplying the additive drag by a factor less than or equal to unity (as in Equation [5.4.1]). However, for this analysis, a slightly different approach has been followed.

Seddon and Goldsmith (1985) show that the additive drag of a pitot inlet can be divided into two components, the *disturbed flow drag* and the *normal shock drag* (see Figure 12 on page 60). The disturbed flow drag represents the loss of cowl suction associated with the separation of the flow at the cowl lip. In subsonic flow, the maximum disturbed flow will be identically equal to the additive drag. In supersonic flow, the maximum normal shock drag is the difference between the additive drag and the disturbed flow drag, and is therefore related to the pressure rise through the normal shock wave which forms ahead of the inlet capture face. The treatment of these two cases is as follows:

i) The maximum disturbed flow drag force for $0 \leq M_\infty \leq 1.0$ is

$$D_{DF} = [P_c \gamma M_c^2 + (P_c - P_\infty)] A_c - P_\infty \gamma M_\infty^2 A_\infty \quad [6.3.1]$$

and for $M_\infty > 1.0$

$$D_{DF} = [P_c \gamma M_c^2 + (P_c - P_{shk})] A_c - P_{shk} \gamma M_{shk}^2 A_\infty \quad [6.3.2]$$

where P_{shk} and M_{shk} are the pressure and Mach number downstream of a normal shock occurring at M_∞ .

ii) The normal shock drag is given by

$$D_{NS} = (P_{shk} - P_\infty)(A_c - A_\infty) \quad [6.3.3]$$

For $M_\infty < 1$, a normal shock will not form ahead of the inlet capture face, and therefore, by definition, $D_{NS} = 0$ in subsonic flow.

These drag components are combined to give the spillage drag as follows:

$$D_{spill} = \alpha D_{DF} + \beta D_{NS} \quad [6.3.4]$$

The factors α and β are functions of the cowl initial slope. They represent the amount that additive drag is recovered due to the cowl slope. In this case, since the cowl is assumed to have a sharp lip, there is no finite cowl lip radius to reduce separation and hence promote additive drag recovery. If α and β are set to 1.0, then the spillage drag will be identically equal to the additive drag as expressed in Equation [5.4.2].

a) Subsonic Freestream Flow

b) Supersonic Freestream Flow

Figure 12. Spillage Drag of Slender, Sharp-Lipped Pitot Inlets

Seddon and Goldsmith (1985) correlated α and β according to experimental data. From their Figure 9.23, the following simple relation was obtained for α as a function of the cowl initial slope, η :

$$\eta \leq 5.0^\circ, \alpha = 1$$

$$5^\circ < \eta < 12.5^\circ, \alpha = \frac{(12.5 - \eta)}{7.5}$$

$$\eta \geq 12.5^\circ, \alpha = 0 \quad [6.3.5]$$

The value of β is obtained as follows:

$$\beta_1 = \frac{\alpha + 1}{2} \quad [6.3.6]$$

$$M_\infty < 1.0, \beta = 0.0$$

$$1.0 \leq M_\infty \leq 1.8, \beta = \beta_1 + \frac{(1 - \beta_1)}{0.8} (M_\infty - 1)$$

$$M_\infty > 1.8, \beta = 1.0 \quad [6.3.7]$$

These relationships imply that β is assumed to increase linearly from a value of β_1 at $M_\infty = 1$ to a value of 1.0 at $M_\infty = 1.8$.

This formulation is used for the entire Mach number range, even though Seddon and Goldsmith (1985) neglect D_{DF} for Mach numbers greater than 1.8, based on experimental evidence that, for high supersonic Mach numbers, spillage drag consists almost entirely of normal shock drag. Indeed, in the present formulation, D_{DF} only contributes about 10% to the total additive drag at $M_\infty = 1.8$ and decreases asymptotically to zero as the

freestream Mach number further increases. Therefore, the approach used does not actually contradict the trend observed in the experimental data, and also has the advantage of preventing a discontinuity from occurring in the drag prediction at $M_\infty = 1.8$.

The spillage drag coefficient, based on A_c , is then

$$C_{D_{spill}} = \frac{D_{spill}}{q_\infty A_c} \quad [6.3.8]$$

6.4 Profile Drag

The profile drag is estimated using the following:

$$\text{fineness ratio, } f_r = \frac{5}{3} \frac{L}{D_m} + 1$$

$$\text{form factor, } f_f = 1 + 3f_r^{-\frac{3}{2}} + 7f_r^{-3}$$

This form factor was taken from the original ACSYNT code for a complete podded nacelle of parabolic profile. Its source is unknown and, although it produces reasonable results, the appropriateness of its use here requires confirmation.

The profile drag coefficient is then

$$C_{D_{prof}} = C_f \frac{A_s}{A_c} f_f \quad [6.4.1]$$

where C_f is the skin friction coefficient (see the section entitled “Calculation of Skin Friction Coefficient” on page 170);

A_s is the cowl wetted area;

and A_c is the inlet capture area.

A_s is calculated in much the same way as for the subsonic pitot inlet, except that Equation [6.1.1] is used to describe the cowl shape, rather than the set of NACA 1-Series cowl coordinates.

It should be noted that this profile drag coefficient is also evaluated for supersonic Mach numbers. As discussed in the section entitled “Profile Drag” on page 39, which deals with the subsonic pitot inlet, the form factor is not strictly valid for supersonic flow. However, viscous drag must still be taken into account, and its value is likely to be greater than just the equivalent flat-plate value. Furthermore, the reasoning applied to the subsonic pitot inlet, with regard to the undesirability of a discontinuous drag prediction, again applies.

The main difference between the present method and the one used for the subsonic pitot inlet is that, in this case, the actual freestream Mach number is used to evaluate the skin friction coefficient, rather than the drag-rise value. This fine point does not, however, make a big difference to the overall drag prediction.

6.5 *Cowl Wave Drag*

6.5.1 Review of Alternative Methods

Aside from complex numerical methods, there is no single simplified theory which adequately predicts the wave drag of axisymmetric bodies, even for inviscid flow. For supersonic flow, the method of characteristics has long been regarded as the most accurate, but this was deemed too numerically intensive for the present work (for the reasons discussed in the section entitled “Characteristics Required of the Drag Prediction Methods” on page 20). Alternative simplified methods are listed by Seddon and Goldsmith (1985). Of these, shock expansion methods are the most promising for high Mach numbers, and perturbation methods are more suited to lower Mach numbers. Presley and Mossman (1958) compare the results of these methods to the family of cowl profiles which are of interest.

Shock expansion methods for axisymmetric bodies were first reported by Eggers and Savin (1951). When applied to cowls, their method equates to the assumption that a two-dimensional oblique shock is formed at the cowl lip, with a Prandtl-Meyer expansion occurring at successive incremental stations along the cowl. This is effectively two-dimensional shock expansion and neglects the three-dimensional effects associated with bodies of revolution. The result is an analysis which severely overpredicts the drag at low Mach numbers. Another obvious restriction of shock expansion methods is that for any specified initial cowl slope, the analysis is only valid for the Mach number regime in which an oblique shock will attach to the cowl lip.

Figure 13. Illustration of the Shock Expansion Method

The basic method was extended by Syvertson and Dennis (1957), who developed the second-order shock expansion method. In this method, the pressure after the shock falls exponentially towards the pressure that would be experienced by a cone (for the same Mach number and local slope) on each segment. After the first segment, Prandtl-Meyer expansion is used to evaluate the change in Mach number and pressure as the cowl slope changes from one cowl segment to another (see Figure 13 on page 65). According to Presley and Mossman (1958), the best results are obtained when the product of initial cowl slope and Mach number is greater than about 0.3. Although this method lends itself to incorporation into a computer program, such as the projectile analysis code of Jackson et al. (1969), a simpler approach was preferable. A promising adaptation of the second-order method is presented by Faro (1961), based on original work by Fenter (1957). In Fenter's method, the exponential decay of the pressure on each cowl segment from the Prandtl-Meyer value, P_s , to the cone pressure, P_c , is governed by a function of the segment length, s , the body length, L , and the hypersonic similarity parameter, $K = \frac{Md}{L}$, such that

$$P_n = P_c + (P_s - P_c)e^{\frac{-s}{KL}} \quad [6.5.2]$$

This method was investigated in the present study, but found unacceptable since the resulting pressure distribution (and thus the drag) appears to be dependent on the number of incremental segments into which the cowl surface is divided. A similar approach is proposed by Brook (1958), but his method is limited to conical cowls.

In their original paper, Syvertson and Dennis (1958) propose a simpler *two-step* variation of their second-order method. In this method, the pressure distribution on any cowl segment is approximated by

$$P_n = P_c + (P_s - P_c)e^{-\beta\psi} \quad [6.5.3]$$

where β is a function of radius, distance along the cowl segment, and local cowl angle relative to initial cowl slope; and ψ is a function of local cowl angle, initial cowl angle, the pressure difference $(P_s - P_c)$, the Mach number downstream of the oblique shock, and the Mach number calculated by the Prandtl-Meyer expansion from the initial cowl angle to the local cowl angle. This method gives excellent agreement with the full second-order theory as long as the cowl is curved. If the cowl is conical, however, the parameter ψ is zero, and the method reverts to first order. This was not considered a serious problem for the present work, since the cowl shapes were generally assumed parabolic. The method was therefore adopted for the present work because it was seen to provide the best compromise between simplicity and accuracy.

The use of the simplified second-order shock expansion theory requires that another method be found to predict wave drag for Mach numbers between 1.0 and 2.0. This is the Mach number range in which, for cowls of moderately high slope, an oblique shock may not attach to the cowl lip. Historically, perturbation methods have been used in this range, and their range of validity is for local cowl slopes less than the freestream Mach angle. Unlike the shock expansion methods, the accuracy of perturbation methods declines as Mach number and initial cowl slope increase. The perturbation method with the best accuracy is Van Dyke's second-order method (Van Dyke, 1952), but as has been mentioned, the complexity of this approach made it undesirable. A compromise was therefore made by using the method of Willis and Randall (1955), who obtained a closed-form slender body solution for the wave drag coefficient of the family of cowl profiles of interest.

Slender body theory also becomes inaccurate in the transonic range. Therefore, for Mach numbers near to one, some other approximation needs to be made. This approximation is described further in the section entitled “Transonic Wave Drag Approximation” on page 71.

6.5.2 Willis and Randall Method

Seddon and Goldsmith (1985) quote the results of Willis and Randall who applied slender body theory to obtain a closed-form expression for the wave drag coefficient based on A_c for the family of cowl profiles described by

$$d = d_m - (d_m - d_c) \left(1 - \frac{x}{L}\right)^n \quad [6.5.4]$$

For $n > 1.25$, this expression is

$$C_{D_{wave}} = \frac{1}{2} \left(\frac{nd_m}{L} \right)^2 \left(1 - \frac{d_c}{d_m}\right)^2 \times \left\{ a + b \frac{d_c}{d_m} + \left[c + \log\left(\frac{4L}{\beta d_c} \right) \right] \frac{A_c}{A_m} \right\} \frac{A_m}{A_c} \quad [6.5.5]$$

where $\beta = \sqrt{M_\infty^2 - 1}$ and the coefficients a , b , and c are functions of the exponent n , and are presented graphically. For the purposes of this work, these curves were manually digitized (see Table 3) and linear interpolation was used to get the appropriate value for a specific n .

Table 3. Digitized Values of Willis and Randall Coefficients (from Seddon and Goldsmith (1985), Figure 9.29)

n	a	b	c
1.25	1.5	0.72	-1.0
1.5	1	0.65	-1.3
2	0.565	0.565	-1.7
3	0.41	0.54	-2.2
4	0.35	0.52	-2.54
5	0.32	0.505	-2.76
6	0.30	0.49	-2.96
7	0.28	0.478	-3.13
8	0.26	0.474	-3.28
9	0.25	0.457	-3.41
11	0.235	0.434	-3.61
13	0.22	0.42	-3.78
15	0.22	0.41	-3.93
17	0.22	0.41	-4.07

Conical profiles may also be handled by the Willis and Randall method, although this feature is not actually used for the present the inlet drag calculations. In the case of $n < 1.25$, the cowl may be assumed conical, and the drag coefficient is given by

$$C_{D_{wave}} = \frac{1}{2} \left(\frac{d_m}{L} \right)^2 \left(1 - \frac{d_c}{d_m} \right) \times \left\{ \frac{d_c}{d_m} - \left(1 + \frac{A_c}{A_m} \right) \left[\frac{1}{2} - \log \left(\frac{4L}{\beta d_m} \right) \right] - \frac{A_c}{A_m} \log \left(\frac{d_c}{d_m} \right) \right\} \frac{A_m}{A_c} \quad [6.5.6]$$

6.5.3 Modified Second-Order Shock Expansion Method

As discussed earlier, the implementation of this method relies on the division of the cowl into a finite number of segments, each forming an angle δ_i with the freestream flow direction. The pressure on the leading edge of each segment is approximated by the pressure that would be predicted by assuming that the flow is two-dimensional. The cowl surface pressure is then assumed to fall exponentially along the surface from the leading edge towards the pressure that would exist on the surface of a cone of the same angle in supersonic flow.

For the first cowl segment (with index $i = 1$), the pressure over the entire segment is assumed to be uniformly two-dimensional. This is calculated as the pressure behind an oblique shock formed on a wedge of angle δ_1 in a flow with a freestream Mach number of M_∞ .

For successive segments, the pressure on segment i is

$$P_{seg_i} = P_{c_i} + (P_{c_i} - P_{s_i})e^{-\beta_i\psi_i} \quad [6.5.7]$$

where P_{c_i} is the pressure on a cone of an equivalent angle to segment i ;
 P_{s_i} is the pressure, given by a two-dimensional Prandtl-Meyer expansion of the flow from the conditions on segment 1, through an angle $\delta_i - \delta_1$; and

$$\beta_i = \frac{x_i \sin \delta_1 - r_i \cos \delta_1}{r_i \cos \delta_i - x_i \sin \delta_i} \quad [6.5.8]$$

$$\psi_i = \frac{B_i}{(P_{c_i} - P_{s_i})} \left(\frac{\Omega_1}{\Omega_i} - \frac{\sin \delta_i}{\sin \delta_1} \right) \quad [6.5.9]$$

$$B_i = \frac{\gamma P_{s_i} M_{s_i}^2}{2(M_{s_i}^2 - 1)} \quad [6.5.10]$$

$$\Omega_i = \frac{1}{M_{s_i}} \left[\frac{1 + \left(\frac{\gamma - 1}{2} \right) M_{s_i}^2}{\frac{\gamma + 1}{2}} \right]^{\frac{\gamma + 1}{2(\gamma - 1)}} \quad [6.5.11]$$

Finally, M_{s_i} is the two-dimensional Mach number on the segment, corresponding to the pressure P_{s_i} .

The pressure, thus calculated at the mid-point of each segment (with coordinates x_i, r_i), is assumed to act over the entire projected area of the segment. Summing up the product of the mean pressure on the segment and the segment projected area results in the total cowl drag force.

6.5.4 Transonic Wave Drag Approximation

In the absence of suitable theory or correlations in the transonic range, the slender body solution was used for Mach numbers close to unity. Subsonic drag rise is approximated by assuming a fixed drag-rise Mach number, $M_d = 0.9$, and allowing the predicted drag to increase from zero at the drag-rise Mach number to a peak at Mach 1.05. The result is a transonic behavior which is at least qualitatively close to that observed for slender bodies.

Hence, for $M_d \leq M_\infty \leq 1.05$, the wave drag is given by

$$C_{D_{wave}} = \left[\frac{M_{\infty} - M_D}{1.05 - M_D} \right]^{\frac{5}{3}} C_{D, WR1.1} \quad [6.5.12]$$

where $C_{D, WR1.1}$ is the wave drag calculated by the Willis and Randall method for Mach 1.1.

For $1.05 \leq M_{\infty} \leq 1.1$, the wave drag is assumed nominally equal to the value calculated for Mach 1.1. However, the "fairing" process described in the next section may cause the drag coefficient to peak at Mach 1.05 and then decline towards Mach 1.1, rather than maintaining a plateau in this region.

6.5.5 Combining the Wave Drag Calculation Methods

The shock expansion method is applied for Mach numbers of 2.1 or greater, since its validity is dependent only on the attachment of a shock at the cowl lip. For this reason, the cowl initial slope is limited to 23° , which guarantees an attached oblique shock at this Mach number.

The applicability of slender body theory becomes questionable as the freestream Mach angle approaches the maximum slope of the cowl. At Mach 2.1, the limiting cowl slope for the Willis and Randall method is 28° . This means that placing a limit of 23° on the cowl slope definitely ensures an overlapping region of validity for both the Willis and Randall and the shock expansion methods.

To ensure a smooth transition between the two methods, the Willis and Randall prediction of wave drag was "faired" into the shock expansion prediction by using a linear function to ensure a smooth transition with no discontinuities. Therefore, for $1.1 < M_\infty < 2.1$,

$$C_{D_{wave}} = C_{D, WR} + (M_\infty - 1.1)(C_{D, shk2.1} - C_{D, WR2.1}) \quad [6.5.13]$$

where $C_{D_{wave}}$ is the Willis and Randall wave drag coefficient calculated at M_∞ ;

$C_{D, shk2.1}$ is the shock expansion wave drag calculation for Mach 2.1;

and $C_{D, WR2.1}$ is the Willis and Randall wave drag coefficient calculated at Mach 2.1.

6.6 *Selected Results*

6.6.1 **Modified Shock Expansion Method**

The modified shock expansion method was tested on the Willis and Randall family of cowls. In Figure 14 on page 74, the wave drag of a cowl of fixed fineness ratio has been computed for varying cowl initial slopes at Mach 2.0. The result is extremely close to the value computed by Presley and Mossman (1958) using the method of characteristics.

Similar calculations were performed for a cowl of fixed initial slope (21.16°) at Mach 2.5 for a range of fineness ratios (Figure 15 on page 75). The agreement with the method of characteristics solution is still good, although not as good as in the previous comparison. The reason for the larger discrepancy is not known.

Figure 14. Shock Expansion Method Compared to Method of Characteristics: The method of characteristics solution is by Presley and Mossman (1958) for varying cowl slope. Fineness ratio is 5, freestream Mach number is 2.0, and diameter ratio is 0.742.

Figure 15. Shock Expansion Method Compared to Method of Characteristics: The method of characteristics solution for varying fineness ratio is by Presley and Mossman (1958). Cowl initial slope is 21.16° , freestream Mach number is 2.5, and diameter ratio is 0.742.

6.7 *Closure*

The supersonic pitot inlet considered in this study is assumed to have a parabolic cowl profile, and a sharp cowl lip. The inlet drag is comprised of spillage, wave and profile drag contributions.

The spillage drag coefficient is estimated using a novel formulation presented by Seddon and Goldsmith (1985). Coefficients derived from experimental data are used to reduce the additive drag to a value corresponding to the spillage drag. Using this formulation, the spillage drag becomes equal to the additive drag at high Mach numbers, as has been observed experimentally.

The profile drag coefficient is calculated from a form factor obtained from the original version of ACSYNT. As with the subsonic pitot inlet, it is applied over the entire Mach number range. However, in this case, the viscous drag contribution is evaluated at the freestream Mach number rather than at the drag-rise Mach number.

The wave drag is estimated by a combination of methods. For high Mach numbers, a simplified form of the second-order shock expansion method, developed by Syvertson and Dennis (1958), is used. For lower supersonic Mach numbers, the slender body solution of Willis and Randall (1955) is used. These methods are combined with a simple approximation in the transonic Mach number regime, and blended together to obtain a continuous drag prediction as the freestream Mach number is varied.

7.0 Prediction of Drag for the Supersonic 2-D Inlet

7.1 Overview

The external geometry of the two-shock supersonic 2-D inlet comprises a wedge-shaped ramp of angle θ , and two cowls. The so-called *upper cowl* is the parabolic cowl which, together with the ramp, captures the inlet flow. The wedge-shaped *lower cowl* may or may not be part of the inlet geometry, since it is only used with the "side-mounted" inlets discussed in the section entitled "Inlet Types" on page 10. The parabolic upper cowl may also be equipped with a blunt lip, to aid in the reduction of spillage drag. This means that it is necessary to estimate both the lip suction and the wave drag due to this blunt lip.

It should be noted that, for a purely external-compression inlet, such as the one considered in this analysis, the inlet throat is actually coincident with the inlet capture face.

An important parameter which must be calculated is the maximum mass flow rate which can be handled by a given supersonic inlet at any Mach number. This parameter can then be used to determine whether or not the mass flow rate required by the engine is feasible, and can thus be used to prevent unrealistic drag estimates.

The flow chart in Figure 16 on page 79 provides an overview of the process of inlet drag prediction for this inlet type. The specific analysis methods and equations are discussed in the rest of the chapter.

7.2 Basic Drag Formulation

The inlet drag coefficient is expressed as

$$C_{D_{inlet}} = \frac{D_{spill} + D_{prof} + D_{wave}}{q_{\infty} A_c} = C_{D_{spill}} + C_{D_{prof}} + C_{D_{wave}} \quad [7.2.1]$$

where D_{spill} , D_{prof} and D_{wave} are the spillage, profile and wave drag forces, respectively. The terms $C_{D_{spill}}$, $C_{D_{prof}}$ and $C_{D_{wave}}$ are the corresponding drag coefficients referenced to the inlet capture area, A_c . The freestream dynamic head, q_{∞} , is again given by Equation [5.2.2].

The spillage drag is the algebraic sum of the additive drag and cowl suction coefficients. In this analysis, the absolute value of the cowl suction coefficient, termed C_{suc} , is used so that

$$C_{D_{spill}} = C_{D_{add}} - C_{suc} \quad [7.2.2]$$

The wave drag is comprised of contributions from the upper (curved) cowl, the lower cowl, and the drag of the blunt lip so that

$$C_{D_{wave}} = C_{D_{wave, uppercowl}} + C_{D_{wave, lowercowl}} + C_{D_{wave, lip}} \quad [7.2.3]$$

Figure 16. Inlet Drag Prediction Process — Supersonic 2-D Inlet

Similarly, both the upper and lower cowl contribute to the profile drag, so

$$C_{D_{prof}} = C_{D_{prof, uppercowl}} + C_{D_{prof, lowercowl}} \quad [7.2.4]$$

Even though the spillage, profile and wave drag coefficients are summed together in the above formulation, the magnitudes of these terms are not necessarily nonzero for the entire operating range of the inlet. For instance, for operation below the drag-rise Mach number, the wave drag coefficient will be zero. The treatment of the profile drag term for supersonic Mach numbers is discussed in the section entitled “Upper Cowl Profile Drag” on page 94.

7.3 *Spillage Drag*

Since the spillage drag is a function of the additive drag, the method of calculating additive drag is an important consideration. One method of calculating the additive drag is to perform the pressure integration along the bounding streamline as in the definition contained in the section entitled “Propulsion System Losses and Drag Terms” on page 7. This method is suggested by Seddon and Goldsmith (1985).

The pressure is nominally constant downstream of the oblique shock which forms from the forebody vertex, and this enables the pressure integral along part of the bounding streamline to be calculated very easily. However, as the inlet mass flow rate is reduced from full flow, a bow shock forms as depicted in Figure 3 on page 12. While it is possible to estimate the position of this bow shock, the intersection of the bounding streamline

Figure 17. 2-D Inlet Nomenclature and Control Volume

with the oblique and normal shock structure is also of importance. The several geometrical possibilities for this intersection, even with a simple two-shock, 2-D inlet, make the problem of evaluating the streamline pressure fairly complex.

The estimation of the pressure integral in the region downstream of the normal shock is also a problem with this approach. Since the flow is subsonic here, the pressure will not be constant along this portion of the streamline. Seddon and Goldsmith (1985) avoid the problem by assuming this pressure constant. Then, acknowledging that this will cause an underprediction of additive drag, they recommend that the cowl suction term, ΔD_c , be neglected in the evaluation of spillage drag.

By far the most common and practical method for additive drag calculation parallels the approach followed for pitot inlets. A momentum analysis across the control volume shown in Figure 17 (with the bounding streamline as one of its boundaries) is used to rewrite the additive drag expression in terms of the momentum fluxes across the control volume, the mean ramp pressure, and the pressure forces on the upstream and downstream flow faces. This is the approach used in the present study and, using the nomenclature of Figure 17, it yields the following expression for additive drag:

$$\begin{aligned}
 D_{add} &= \int_{\infty}^c (P - P_{\infty}) dA_x \\
 &= F_{Ramp} + (P_c - P_{\infty} + \gamma M_c^2 P_c) w Y_i \cos^2 \theta - \gamma M_{\infty}^2 A_{\infty} P_{\infty}
 \end{aligned}
 \tag{7.3.1}$$

where F_{Ramp} is the force on the forebody ramp, and w is the inlet width. In this analysis, the flow angle at the throat is assumed equal to the ramp angle, θ .

In the region upstream of the bow shock, but downstream of the forebody oblique shock, the ramp pressure is constant, and is easily calculated from oblique shock relations. However, for the evaluation of the total ramp force, it is important to calculate the bow shock position in supersonic flow, and to make a suitable approximation for the mean ramp pressure in the region downstream of the bow shock. In this region, the flow will be subsonic and the pressure nonuniform. It can be argued that, because of the necessity of making such an approximation, this method offers little improvement over the method of Seddon and Goldsmith (1985) discussed above. However, the formulation used appears to de-emphasize the pressure in the subsonic region and thus minimizes the error incurred by the approximation.

Further approximations must also be made to estimate the additive drag in a fully-subsonic flow, and also in a transonic flow where the supersonic Mach number is not high enough for an oblique shock to attach to the vertex of the forebody ramp. The following sections discuss the approximations made, together with methods for calculating the position of the bow shock. Also discussed is the empirical correlation used to evaluate the cowl suction coefficient which, in turn, is used to reduce the additive drag to finally estimate the spillage drag.

7.3.1 Additive Drag

The formulation for additive drag is given in Equation [7.3.1]. The unknown parameters in this equation are the ramp force, F_{ramp} , and the pressure at the inlet capture face, P_c . These need to be calculated for subsonic, transonic and supersonic flows.

For *subsonic* operation, the pressure at the inlet capture face, P_c , is calculated by assuming an isentropic expansion from the freestream conditions P_∞ , M_∞ , A_∞ , to P_c , M_c , A_c . Following the approach suggested by Petersen and Tamplin (1966), the pressure on the ramp is assumed to be equal to the arithmetic average of the freestream pressure, P_∞ , and the pressure at the inlet face, P_c . The ramp force is thus

$$F_{Ramp} = [\frac{1}{2}(P_\infty + P_c) - P_\infty](Y_c - Y_i \cos^2 \theta)w \quad [7.3.2]$$

For *transonic* operation, i.e., when $M_\infty > 1$ but the Mach number is too low for an oblique shock to be attached to the ramp vertex, a normal shock is assumed to occur in front of the ramp. The pressure at the inlet capture face, P_c , is calculated from an isentropic expansion from the conditions downstream of the normal shock to the inlet entry plane. Again, following Petersen and Tamplin (1966), the ramp pressure is then taken to be the average of the pressure downstream of the detached normal shock and the pressure at the inlet capture face. Thus

$$F_{Ramp} = [\frac{1}{2}(P_c + P_{R2}) - P_\infty](Y_c - Y_i \cos^2 \theta)w \quad [7.3.3]$$

where P_{R2} is the pressure behind a normal shock occurring at a Mach number of M_∞ .

For *supersonic* operation, the oblique shock is attached to the forebody ramp vertex. At mass flow ratios reduced from the maximum, a bow shock forms in front of the inlet face. The ramp force then becomes a function of the bow shock position, L .

$$F_{Ramp} = \{[\frac{1}{2}(P_c + P_{R2}) - P_\infty]L \sin \theta + (P_{R1} - P_\infty)(Y_c - Y_i \cos^2 \theta - L \sin \theta)\}w \quad [7.3.4]$$

where P_{R1} is the pressure downstream of the oblique shock;

and P_{R2} is the pressure downstream of the bow normal shock.

Note that the pressure in the region between the bow shock and the inlet face is again assumed to be the average of the pressures at these two stations. The conditions at the inlet face (M_c, P_c) are calculated by taking the pressure and Mach number changes through the shock system into consideration, and by application of the continuity equation in the subsonic region downstream of the bow shock, assuming isentropic flow in this subsonic region.

7.3.2 Prediction of Bow Shock Position

As has been mentioned, the prediction of bow shock position is fundamental to the evaluation of mean ramp pressure, and hence the calculation of additive drag, in supersonic flow. The classic theory for predicting the shock detachment ahead of a blunt body is attributed to Moeckel (1949). In this analysis, applied both to axisymmetric and plane bodies, an assumption is made with regard to the actual shape of the shock wave and the sonic line. Moeckel extended this analysis to 2-D and axisymmetric inlets by expressing the bow shock position as a linear function of mass flow ratio. At full flow the bow shock detachment distance is obviously zero, while at zero flow the distance is equivalent to that of a blunt body of the same cross-section as the inlet face.

Strictly speaking, the linear theory developed by Moeckel is applicable only to a perfectly two-dimensional inlet, i.e., an inlet with sideplates of infinite extent and infinite spacing. Osmon (1967) adapted Moeckel's theory to account for the effect of the lateral spillage due to the sideplate configuration. He used the data of Petersen and Tamplin (1966) to develop an empirical correlation which relates the extent of the side spillage as a function of Mach number. It should be noted that this correlation may only be valid

for the inlet sideplate configurations (triangular, "full" sideplates) and the Mach number range investigated by Petersen and Tamplin.

Both of the above methods were incorporated into the source code for the present study. These methods are detailed in the following sections.

Linear Theory (Moeckel's Theory)

As presented by Osmon (1967), Moeckel's theory gives the following expression for bow shock detachment distance:

$$\frac{L}{wv} = (1 - i) \frac{C + E}{1 - F} \quad [7.3.5]$$

where L is the distance from the bow shock to inlet plane;

wv is the inlet plane height, $wv = Y_i \cos \theta$;

and i is the ratio of mass flow ratio to the reference (maximum) mass flow ratio, i.e.,

$$i = \frac{MFR}{MFR_{ref}} = \frac{\frac{A_\infty}{A_c}}{\left(\frac{A_\infty}{A_c} \right)_{ref}} \quad [7.3.6]$$

Also,

$$C = \beta \left[\beta \tan \phi_s - \sqrt{\beta^2 \tan^2 \phi_s - 1} \right] \quad [7.3.7]$$

$$\beta = \sqrt{M_\infty^2 - 1} \quad [7.3.8]$$

The quantity ϕ_s is the shock angle for sonic flow, given by the equation

$$\sin^2(\phi_s) = \frac{1}{4\gamma M_\infty^2} \left\{ (\gamma + 1)M_\infty^2 - (3 - \gamma) + \sqrt{(\gamma + 1)[(\gamma + 1)M_\infty^4 - 2(3 - \gamma)M_\infty^2 + (\gamma + 9)]} \right\} \quad [7.3.9]$$

$$E = \frac{A_s}{A_{os}} \sin \delta_s \quad [7.3.10]$$

$$F = \frac{A_s}{A_{os}} \cos \delta_s \quad [7.3.11]$$

The quantity δ_s is the flow angle which corresponds to ϕ_s , so

$$\delta_s = \tan^{-1} \left[\frac{M_\infty^2 \sin 2\phi_s - 2 \cot \phi_s}{2 + M_\infty^2 [\gamma + \cos(2\phi_s)]} \right] \quad [7.3.12]$$

$$\frac{A_s}{A_{os}} = \frac{1}{\left(\frac{A}{A^*} \right)_{os} \left(\frac{P_{o,s}}{P_{o,os}} \right)} \quad [7.3.13]$$

$$\left(\frac{A}{A^*} \right)_{os} = \frac{1}{M_\infty} \left[\frac{1 + \frac{\gamma - 1}{2} M_\infty^2}{\frac{\gamma + 1}{2}} \right]^{\frac{\gamma + 1}{2(\gamma - 1)}} \quad [7.3.14]$$

Finally, $\frac{P_{o,s}}{P_{o,os}}$ is the mean total pressure recovery through the hyperbolic portion of the bow shock between the stagnation streamline and the shock sonic point.

Applying Moeckel's theory, the angle of the shock at this point is

$$\phi_c = \tan^{-1} \sqrt{\frac{(4\beta^2 - 3 \cot^2 \phi_s)}{\beta \cot \phi_s}} \quad [7.3.15]$$

Therefore, $\frac{P_{o,s}}{P_{o,os}}$ is obtained by calculating the total pressure recovery through a normal shock occurring at a Mach number of $M_\infty \sin \phi_c$.

Osmon's Theory

Applying Osmon's modifications to Moeckel's linear equation, the equation for the bow shock position becomes

$$\frac{L}{wv} = (1 - i) \frac{C + \left[k + \frac{1-i}{\eta} \right] E}{1 - \left[k - \frac{1-i}{\eta} \right] F} \quad [7.3.16]$$

where i, C, E and F are defined as before. The value of η is given by

$$\eta = \frac{E + \sqrt{\frac{E(E + CF)}{1 - kF}}}{kE + C}$$

For $M_\infty < 2.0$, $k = \frac{G}{F}$

where G comes from the graph in Figure 8 of Osmon's paper. For this work, a polynomial was fitted to this curve to give

$$G = 1.012 - 6.905 \times 10^{-3} M_\infty^{3.5} - 4.405 \times 10^{-3} M_\infty^7 \quad [7.3.17]$$

For $M_\infty \geq 2.0$, $k = 1.0$.

7.3.3 Cowl Suction

The cowl suction coefficient is determined using the correlations of Osmon (1967). He presented these correlations in graphical form for freestream Mach numbers of 0.692, 0.845, 1.093, 1.294, 1.393 and 1.687.

The abscissa of these graphs is

$$abc = \frac{MFR \ M_c \ \frac{A_c}{A_m}}{\left(\frac{A}{A^*} \right)_{ref}} \quad [7.3.18]$$

where $MFR = \frac{A_\infty}{A_c}$, the mass flow ratio;

M_c is the Mach number at the inlet capture plane;

A_c is the projected inlet capture area;

A_m is the maximum projected upper cowl area; and

$$\left(\frac{A}{A^*} \right)_{ref} = \frac{1}{M_{ref}} \left[\frac{1 + \left(\frac{\gamma - 1}{2} \right) M_{ref}^2}{\frac{\gamma + 1}{2}} \right]^{\frac{\gamma + 1}{2(\gamma - 1)}} \quad [7.3.19]$$

For $M_\infty \leq 1$, $M_{ref} = M_\infty$

$$M_\infty > 1, M_{ref} = \left[\frac{2}{\gamma - 1} \left(\frac{P_{o, ref}}{P_\infty} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right]^{\frac{1}{2}} \quad [7.3.20]$$

where P_∞ is the freestream static pressure;
and $P_{o,ref}$ is the total pressure resulting from the freestream flow passing
through the inlet shock system.

The ordinate of each graph is

$$ord = \frac{C_{suc} (\tan \theta)^{exp1} \left(\frac{A_c}{A_m} \right)^{exp3}}{\left(\frac{t}{c} \right)^{exp2}} \quad [7.3.21]$$

where C_{suc} is the magnitude of the cowl suction coefficient;
 θ is the forebody ramp angle;
 $\frac{t}{c}$ is the cowl thickness-to-chord ratio, defined by $\frac{t}{c} = \frac{Y_m - Y_c}{L_c}$;
and $exp1, exp2, exp3$ are tabulated in Table 4.

Each of the curves presented in the paper by Osmon (1967) was manually digitized, and a least-squares technique was used to fit parabolas to the data, resulting in the formula

$$ord = coef0 + coef1(abc) + coef2(abc)^2 \quad [7.3.22]$$

The coefficients of these parabolas, together with the limits of the validity of the fitted curves, are given in Table 4. The cowl suction coefficients are calculated from the ordinates of the curves. To compensate for the fact that the correlations are given for discrete Mach numbers, linear interpolation is used between cowl suction coefficients calculated at similar mass flow ratios for the closest of these discrete Mach numbers.

For Mach numbers less than 0.692, a linear interpolation between zero cowl suction at $M_\infty = 0$ and the value at $M_\infty = 0.692$ is used. For Mach numbers above 1.687, a linear

Table 4. Parameters for Cowl Suction Correlation (from Osmon (1967), Figures 11 to 16)

Mach No	0.692	0.845	1.903	1.294	1.393	1.687
exp1	0.466	0.531	0.619	0.679	0.705	0.774
exp2	0.471	0.805	1.474	2.127	2.385	3.690
exp3	0.238	0.340	0.525	0.682	0.760	0.987
coef0	0.17175	0.1566	0.09767	0.05893	0.02875	0.0969
coef1	-0.38689	-0.34576	-0.19361	-0.089845	-0.01125	-0.2410
coef2	0.21606	0.18908	0.088719	0.019588	-0.03125	0.150
Lo Limit	0.1	0.1	0.1	0.1	0.2	0.5
Hi Limit	0.8	0.8	0.8	0.8	0.8	0.8

interpolation is used from $M_\infty = 1.687$ to zero at $M_\infty = 2.0$. This again embodies the assumption that no additive drag is recovered for $M_\infty \geq 2.0$.

The cowl suction is not permitted to be greater than the additive drag. Complete cancellation of the additive drag is assumed instead of allowing this to occur.

7.4 Calculation of Maximum Mass Flow Rate

In calculating the maximum mass flow rate permitted to enter an inlet, it is convenient to define a reference mass flow ratio corresponding to the maximum mass flow rate. This reference mass flow ratio is defined as

$$MFR_{ref} = \left(\frac{A_\infty}{A_c} \right)_{ref} \quad [7.4.1]$$

In the above equation, the value of A_c is fixed according to the geometry of the inlet, so that MFR_{ref} is directly proportional to the value of the streamtube capture area, A_c , at the reference condition. The method of evaluating MFR_{ref} is discussed in the following sections.

The equation which relates the maximum mass flow rate to the reference mass flow ratio is

$$\dot{m}_{\max} = MFR_{ref} A_c P_{\infty} M_{\infty} \sqrt{\frac{\gamma g_c}{RT_{\infty}}} \quad [7.4.2]$$

where $M_{\infty}, P_{\infty}, T_{\infty}$ are the freestream Mach number, pressure and temperature, respectively;

R is the gas constant for air, and has the value 287 Nm/(kgK) in SI units and 53.3 ft-lbf/(lbm °R) in the EES;

and g_c is the dimensional constant, and has the dimensionless value 1 when using SI units, and 32.2 lbm ft/(s²lbf) in the EES.

7.4.1 Subsonic/Transonic Reference Mass Flow Ratio

For *subsonic* and *transonic* conditions, the limit on the mass flow rate is imposed by choking at the throat of the inlet. However, it is worth considering the scenario prior to choking. As the mass flow ratio approaches 1.0, the flow will start *accelerating* into the inlet, because the throat area will be less than the capture area. In this case, the pressure on the bounding streamline will be less than P_{∞} , so that the additive drag term may be negative and become *additive suction*. As mass flow rate further increases toward the

choking limit, the capture streamtube will expand, and the definition of additive drag may no longer be entirely meaningful.

For the sake of simplicity, the limiting subsonic mass flow rate is assumed to occur when the area of the capture streamtube equals the inlet throat area. In other words, at this point, there is neither deceleration or acceleration of the flow as it enters the inlet. So the reference mass flow ratio is then

$$MFR_{ref} = \left(\frac{A_{\infty}}{A_c} \right)_{ref} = \frac{A_t}{A_c} = \frac{Y_i \cos \theta}{Y_c} \quad [7.4.3]$$

7.4.2 Supersonic Reference Mass Flow Ratio

From geometric considerations, the maximum mass flow ratio which a 2-D inlet can handle at a given supersonic Mach number is given by Seddon and Goldsmith (1985) as

$$MFR_{ref} = \left(\frac{A_{\infty}}{A_c} \right)_{ref} = \frac{\cot \theta - \cot \sigma}{\cot \theta - \cot \sigma_s} \quad [7.4.4]$$

where θ is the forebody ramp wedge angle;

σ_s is the shock angle corresponding to a flow at the freestream Mach number M_{∞} turning an angle θ ;

and σ is the angle formed by a line from the ramp vertex to the cowl lip, and the horizontal.

$$\text{So,} \quad \sigma = \tan^{-1} \left(\frac{X_{Ramp}}{Y_c} \right)$$

where $X_{Ramp} = \frac{(Y_c - Y_i)}{\tan \theta}.$

7.5 *Upper Cowl Profile Drag*

The profile drag of the upper cowl is determined from an adaptation of a form factor commonly used for wings,

$$f_f = 1 + \frac{t}{c} + 100\left(\frac{t}{c}\right)^4 \quad [7.5.1]$$

where $\frac{t}{c}$ is the cowl thickness-to-chord ratio calculated as $\frac{t}{c} = \frac{Y_m - Y_c}{L_c}.$

The profile drag coefficient is given by

$$C_{D_{prof}} = C_f \frac{A_s}{A_c} f_f \quad [7.5.2]$$

where C_f is the skin friction coefficient (see the section entitled “Calculation of Skin Friction Coefficient” on page 170);

and A_s is the wetted area, calculated by the summation of segment lengths obtained by approximating the cowl profile as a series of straight lines.

The approach taken here regarding the evaluation and application of the profile drag coefficient at supersonic Mach numbers is similar to that used for the supersonic pitot inlet (see “Profile Drag” on page 62). The only difference lies in the evaluation of the form factor, as described above.

7.6 Upper Cowl Wave Drag

As with the axisymmetric inlets, there are three Mach number ranges of importance to consider in the calculation of wave drag. These are subsonic Mach numbers greater than the drag-rise value, supersonic Mach numbers where the Mach number is not high enough to support an attached oblique shock, and supersonic Mach numbers when a shock will be attached to the cowl lip. For the present analysis, the first two are combined and termed "transonic wave drag", while the third is termed "supersonic wave drag".

7.6.1 Supersonic Wave Drag

For supersonic Mach numbers at which an oblique shock is attached to the cowl lip (assuming that the upstream flow is undisturbed), the cowl wave drag is calculated using the traditional 2-D shock expansion method. The method is completely analogous to that used for the supersonic pitot inlet, without the complication of the three-dimensional flow effects.

The cowl is divided into a series of segments, and the pressure after the oblique shock is assumed to be uniform over the first segment. For successive segments, the pressure on each segment is calculated by assuming a Prandtl-Meyer expansion of the flow as it

turns from one segment angle to the next. The drag force is calculated by summing the pressures acting over the projected area of each segment.

In this routine, the following equation, similar to the one used for the supersonic pitot inlet, is used to specify the cowl profile:

$$Y = Y_m - (Y_m - Y_c) \left(1 - \frac{x}{L_c} \right)^n \quad [7.6.1]$$

To simplify the input requirements, a parabolic profile is normally assumed, so that $n = 2$.

7.6.2 Transonic Wave Drag

For the calculation of wave drag for Mach numbers above the drag-rise Mach number, yet less than the Mach number at which an oblique shock will attach to the cowl lip, transonic similarity laws were used.

In the absence of suitable data for a general family of cowl profiles, data for the transonic drag of a wedge were used. Liepmann and Roshko (1963) show a combination of exact numerical solutions plotted in similarity coordinates. The drag coefficient of a wedge, based on the projected frontal area, is given as

$$\bar{C}_{D_{wedge}} = \frac{\tilde{C}_D (\tan \eta)^{\frac{2}{3}}}{[(\gamma + 1)M_\infty^2]^{\frac{1}{3}}} \quad [7.6.2]$$

where M_∞ is the freestream Mach number and η is the wedge half-angle.

$\tilde{C}_D$ is a function of χ , where

$$\chi = \frac{M_\infty^2 - 1}{[(\gamma + 1)M_\infty^2 \tan \eta]^{\frac{2}{3}}} \quad [7.6.3]$$

Values of $\tilde{C}_D$ as a function of χ were manually digitized, and these data are presented in Table 5. Interpolation is used to get the appropriate value of $\tilde{C}_D$ for arbitrary χ values, enabling the transonic wave drag to be calculated for any wedge angle and any freestream Mach number.

An advantage of this approach is that it is not necessary to explicitly evaluate the drag-rise Mach number, M_D . If $M_\infty < M_D$, then the resulting value of χ will be less than the minimum value tabulated. Should this occur, the wave drag coefficient is set to zero.

The wave drag coefficient based on the inlet capture area is given by

$$C_{D_{wedge}} = \bar{C}_{D_{wedge}} \frac{(Y_m - Y_c)}{Y_c} \quad [7.6.4]$$

For Mach numbers greater than those in the transonic range, the wedge wave drag coefficient is obtained from linear theory, so for $\chi > 2.5$

$$\bar{C}_{D_{wedge}} = \frac{2 \tan \eta}{\sqrt{M_\infty^2 - 1}} \quad [7.6.5]$$

Table 5. Digitized Data for Transonic Drag of a 2-D Wedge (from Liepmann and Roshko (1963), Figure 10.2b)

χ	$\tilde{C}_D$
-1.75	0.0
-1.0	0.17
-0.75	0.37
-0.5	0.78
0.0	1.78
0.25	2.25
0.5	2.73
0.75	3.1
1.0	3.2
1.25	2.56
1.5	2.0
2.0	1.57
2.5	1.27

7.6.3 Combining the Wave Drag Calculation Methods

A suitable wedge angle must be chosen so that the transonic drag, estimated by treating the cowl as a wedge, is representative of curved profiles. The following weighted average is used:

$$\eta_w = \frac{(\eta_c + 2\eta_m)}{3} \quad [7.6.6]$$

where η_w is the equivalent cowl wedge angle;

η_c is the initial slope of the cowl;

η_m is the cowl slope assuming that the cowl is wedge shaped, therefore

$$\eta_m = \tan^{-1} \left(\frac{Y_m - Y_c}{L_c} \right).$$

The above expression for η_w was obtained by simple numerical experiments which tested cowls of various profiles using both the shock expansion and the wedge approximation methods. The shock expansion method was used as a datum, and the effect of varying the weighting factors on the wedge wave drag prediction in the region of overlapping validity of the two methods was monitored. The resulting expression produces good agreement between the methods for parabolic cowl profiles, and any error in the approximation will be compensated for by the fairing process described in the next paragraph.

Defining $M_{\min}$ as the minimum Mach number which will support an attached shock on the cowl lip, the transonic and supersonic solutions are faired together to ensure a minimum of discontinuity. Thus, for $M < M_{\min}$, the following function is used to calculate the cowl wave drag coefficient:

$$C_{D_{cowl}} = C_{D_{wedge}} + \frac{(M_{\infty} - 0.6)}{(M_{\min} - 0.6)} (C_{D_{shk, \min}} - C_{D_{wedge, \min}}) \quad [7.6.7]$$

where $C_{D_{cowl}}$ is the resulting cowl wave drag at $M = M_{\infty}$;

$C_{D_{wedge}}$ is the drag of the equivalent wedge at $M = M_{\infty}$ calculated using the transonic similarity law;

$C_{D_{shk, \min}}$ is the wave drag calculated using shock expansion at $M = M_{\min}$;

and $C_{D_{wedge, \min}}$ is the drag of the equivalent wedge calculated at $M = M_{\min}$.

For $M \geq M_{\min}$ the drag is calculated using shock expansion methods.

7.6.4 Supersonic Blunt Lip Drag

The Seddon and Goldsmith (1985) analysis for the calculation of supersonic wave drag increment (due to a blunt lip of circular arc added to an axisymmetric cowl of low angle)¹⁰ was adapted to 2-D cowls as follows:

$$C_{D_{lip}} = \frac{(KP_{o, shk} - P_{\infty})A_{lip}}{q_{\infty}A_c} \quad [7.7.1]$$

where $K = 0.24 \frac{Y_c}{Y_{c_{cowl}}} + 0.57;$

$P_{o, shk}$ is the total pressure behind a normal shock occurring at freestream Mach number M_{∞} ;

$Y_{c_{cowl}} = Y_c + 2r_{lip}$ where r_{lip} is the lip radius;

and $A_{lip} = 2r_{lip}w.$

7.7 *Profile and Wave Drag of the Lower Cowl*

Since this cowl is assumed to be wedge shaped, the method used for transonic wedge drag is used to calculate the wave drag (see the section entitled “Transonic Wave Drag” on page 96). This method automatically uses linear theory for the supersonic regime, so that it can account for the entire Mach number range.

¹⁰ See the section entitled “Method Used for the Present Study” on page 48.

The profile drag for this cowl is calculated in a manner identical to that for the upper cowl, described in the section entitled “Upper Cowl Profile Drag” above.

7.8 *Selected Results*

7.8.1 Bow Shock Prediction Using Osmon’s Theory

The results of the calculation of bow shock position ($\frac{L}{wv}$), using the theory of Osmon(1967) for a range of operating conditions, are shown in Figure 18 on page 102. In this graph, the calculations (based on Osmon’s methods) are compared to the data presented by Osmon (1967). The trends are well represented and this graph compares well to a similar one shown by Osmon to demonstrate the validity of his methods. However, for an unknown reason, Osmon’s prediction tracks the experimental data somewhat better than the present calculation for $M_\infty = 1.29$. This discrepancy is most likely due to a slightly different value of reference mass flow ratio being calculated, which was indicated by Osmon’s additive drag prediction curves.

Figure 18. Calculated Bow Shock Position Compared to Experimental Data: The data are from Osmon (1967).

7.8.2 Additive Drag Prediction

Figure 19 on page 104 shows the additive drag prediction for the data presented by Osmon (1967). The trends are well represented. For this calculation, Osmon's method for predicting bow shock position was used.

7.9 Closure

Drag analysis methods have been presented for a two-shock external-compression 2-D inlet, featuring a parabolic upper cowl and a wedge-shaped lower cowl. Provision has been made for the inclusion of a blunt lip on the upper cowl.

The additive drag coefficient is calculated and then decremented by the value of the cowl suction coefficient to obtain the spillage drag coefficient. The calculation of additive drag in a supersonic flow is strongly dependent on the prediction of the bow shock position, and two alternative methods have been presented for this. The first expresses the bow shock detachment distance as a linear function of mass flow ratio (Moeckel, 1949). The second takes spillage over the triangular sideplates into account using an empirically-based nonlinear function (Osmon, 1967). The cowl suction coefficient is also obtained from empirical correlations (Osmon, 1967).

M = 1.29	M = 1.39	M = 1.69	M = 1.29	M = 1.39	M = 1.69
Exp	Exp	Exp	Calc	Calc	Calc
□ □ □	○ ○ ○	△ △ △		-----	————

Figure 19. Calculated Additive Drag Compared to Experimental Data: The data are from Osmon (1967).

The cowl profile drag is estimated using a form factor derived for wing sections, and is applied in a manner identical to that used for the supersonic pitot inlet, where the viscous drag contribution is calculated at the freestream Mach number.

The cowl wave drag is calculated with two different methods. For the lower (wedge-shaped) cowl, a similarity law is used. This covers the transonic Mach number range for calculating the drag of wedges, and merges with linear theory at high supersonic Mach numbers. This method is also used for the calculating the transonic wave drag of the upper (parabolic) cowl (which is approximated as a wedge). For high supersonic Mach numbers, a simple 2-D shock expansion method is used. The two methods are blended together to avoid the occurrence of a discontinuity in the prediction of the wave drag of the upper cowl.

The wave drag due to the blunt cowl lip is estimated using a method proposed by Seddon and Goldsmith (1985) for axisymmetric cowls, adapted for 2-D cowls.

8.0 Prediction of Drag for the Supersonic Conical Inlet

8.1 Overview

The supersonic conical inlet has characteristics common to both the supersonic pitot inlet and the supersonic 2-D inlet. Therefore, the approach used for predicting the drag draws heavily on the methods used for these two other inlet types. The assumption is made that the conical inlet is equipped with a slender, sharp-lipped parabolic cowl. This means that the cowl profile and wave drag methods used for the supersonic axisymmetric inlet can be employed for the conical inlet.

Even though the conical and 2-D inlets have similar forebody configurations, and the same basic additive drag formulation is applicable to both, the principle difference is the supersonic flow field. For the 2-D inlet, simple oblique shock relations can be used to determine the pressure and Mach number downstream of these shocks, and the pressure is constant in this supersonic region. However, for the axisymmetric case, a conical flow field results. The governing equations need to be solved numerically, and the pressure in the region downstream of conical shocks is not constant. Since the solution of the conical flow relations is a complex task, the relevant conical flow data have been stored digitally within the program, enabling the conditions immediately behind the conical

shock, and also on the cone surface, to be determined.¹¹ Further approximations must be made regarding the flow enclosed by these boundaries.

The flow chart in Figure 20 on page 108 provides an overview of the process of inlet drag prediction for this inlet type. The specific analysis methods and equations are discussed in the remainder of the chapter.

8.2 Basic Drag Formulation

The inlet drag coefficient is expressed as

$$C_{D_{inlet}} = \frac{D_{spill} + D_{prof} + D_{wave}}{q_{\infty} A_c} = C_{D_{spill}} + C_{D_{prof}} + C_{D_{wave}} \quad [8.2.1]$$

where D_{spill} , D_{prof} and D_{wave} are the spillage, profile and wave drag forces, respectively. The terms $C_{D_{spill}}$, $C_{D_{prof}}$ and $C_{D_{wave}}$ are the corresponding drag coefficients referenced to the inlet capture area, A_c . The freestream dynamic head, q_{∞} , is again given by Equation [5.2.2].

Even though the spillage, profile and wave drag coefficients are summed together in the above formulation, the magnitudes of these terms are not necessarily nonzero for the entire operating range of the inlet. The approach taken for the evaluation of the wave and profile drag terms, and their applicability to the various Mach number regimes, is identical to that used for the supersonic pitot inlet.

¹¹ See the section entitled "Conical Flow Functions" on page 175.

Figure 20. Inlet Drag Prediction Process – Supersonic Conical Inlet

8.3 *Spillage Drag*

As with the other inlet types, the spillage drag is considered to be a function of the additive drag. In the original version of ACSYNT, only the supersonic conical inlet was considered, and additive drag was the only drag contribution calculated. This analysis was examined thoroughly for the present study, and is discussed in the next section. An important facet of this original analysis is that it embodied the use of a translating forebody. Even though this is a fairly uncommon inlet type, and a new fixed-geometry calculation option has been added, a variable-geometry option has also been retained to provide compatibility with previous test data.

As with the 2-D inlet, the assumption is made that the compression is purely external for this inlet. Hence, the throat is coincident with the inlet capture face. This throat location is easily accommodated with a fixed-geometry conical inlet. However, it is extremely difficult to implement in practice if the geometry of the inlet is variable, since the cone dimensions would have to change as the cone is translated. While it is unlikely that such an inlet would be constructed, it can be regarded as an approximation to a multi-shock, mixed-compression, translating-cone inlet.

The additive drag formulation for conical inlets is identical to the one used for 2-D inlets (see the section entitled “Spillage Drag” on page 80). When applied to the inlet geometry shown in Figure 23 on page 116, the following equation results:

$$\begin{aligned}
D_{add} &= \int_{\infty}^c (P - P_{\infty}) dA_x \\
&= F_{cone} + (P_t - P_{\infty} + \gamma M_t^2 P_t) A_t \cos \lambda - \gamma M_{\infty}^2 A_{\infty} P_{\infty}
\end{aligned}
\tag{8.3.1}$$

where F_{cone} is the force on the forebody cone. This formulation is used for both the fixed- and the variable-geometry inlet analyses, the major difference being in the evaluation of F_{cone} .

8.3.1 Additive Drag Prediction Method Originally in ACSYNT

Pioneering work on the additive drag of conical inlets was done by Sibulkin (1954). One of his contributions was to compute the additive drag coefficients for conical inlets operating with conical flow at the inlet capture face. (For this to occur, the inlet must operate without a bow-wave and the conical forebody must therefore translate to appropriately adjust the inlet throat area.) He performed this calculation by numerically integrating supersonic conical flow data, and used a formulation similar to that of Equation [8.3.1].

The result of this calculation is a theoretical minimum additive drag for a conical inlet. Plotting these additive drag coefficients for inlets of varying design Mach number, it can be seen that the additive drag coefficient *decreases* with increasing Mach number.¹²

¹² This seemingly anomalous result is perhaps best understood in the light of the fact that the drag coefficient is obtained by nondimensionalizing with the square of the Mach number. A similar case is the pressure coefficient of a right circular cone at zero angle of attack in a supersonic flow, which also decreases with Mach number (Figure 22 on page 114).

For the case of a fixed-geometry inlet, a bow shock forms as the mass flow ratio is reduced from the maximum. The increased pressure, due to the shock, along the bounding streamline increases the additive drag considerably, and the trend of decreasing additive drag coefficient with increasing Mach number no longer occurs.

In the original ACSYNT calculation, the assumption made by Morris et al. (1977) is that the inlet throat Mach number (i.e., the inlet capture plane Mach number for an external-compression inlet) is kept constant at a value specified by the user. A variable inlet geometry, incorporating a translating forebody cone, is required to keep the throat Mach number constant and, depending on the inlet throat Mach number, the flow generally remains supersonic up to the entry plane of the inlet. However, in the event that a bow shock **does** form, provision is made to calculate the effect of the bow shock on the additive drag.

In Figure 21 on page 112, the predictions of ACSYNT for an inlet throat Mach number of 0.85 are compared to the theoretical minimum, as well as to experimental data for a fixed-geometry configuration at a freestream Mach number of 1.8. The ACSYNT prediction is closer to the minimum than it is to the fixed-geometry data, and the reason it is slightly higher than the minimum is due to the conical flow approximation made in ACSYNT, rather than the presence of a bow shock (which does not even form in this test case).

The expression for the forebody cone pressure coefficient contained in ACSYNT is derived from a slender body solution quoted by Lighthill (1960). Figure 22 on page 114 compares this approximation, for the 20° cone semi-angle used in ACSYNT, to data from the supersonic conical flow tables of Sims (1964) using least-squares regression. This plot shows that the cone pressure coefficient is overpredicted by slender body the-

Figure 21. Additive Drag Coefficient Predicted by ACSYNT: The calculation is for a throat Mach number of 0.85, and is compared to the calculation of Sibulkin (1954) for the case of minimum additive drag, and also to his experimental data for a fixed-geometry inlet operating at its design Mach number of 1.8.

ory, especially at high Mach numbers. Therefore, the over-estimation of the cone pressure tends to drive the additive drag estimation higher than would otherwise be predicted.

It can be seen, therefore, that the original method for calculating additive drag in ACSYNT has two major faults. Firstly, the assumption that the throat Mach number is kept constant (by varying the inlet geometry) means that a bow shock will not form in most cases, and the resulting additive drag prediction approaches the theoretical minimum for low supersonic freestream Mach numbers. Secondly, the relation used for cone pressure coefficient is inappropriate for 20° cones, resulting in a prediction of additive drag which is higher than the theoretical minimum, especially at high supersonic Mach numbers. Consequently, the analysis produces results which fall somewhere between the theoretical minimum and the fixed-geometry additive drag, depending on the freestream Mach number.

For the above reasons, the decision was made to not use the existing routines, but to create new additive drag calculation routines for conical inlets. These new routines are also more consistent with the methodology and coding standards of the other new modules. The methods used for these new routines are discussed in the following sections.

Figure 22. Comparison of Cone Pressure Coefficients: The formulation used in ACSYNT (Lighthill, 1960), is compared to the exact solution (Sims, 1964).

8.3.2 Additive Drag for the Fixed-Geometry Inlet

This additive drag formulation given by Equation [8.3.1] is rewritten as follows:

$$D_{add} = F_{cone} + (P_t - P_\infty + \gamma M_t^2 P_t) A_t f - \gamma M_\infty^2 A_\infty P_\infty \quad [8.3.2]$$

In this equation, the cosine of the flow angle at the throat ($\cos \lambda$) has been replaced by a factor f . The reason for this is that if the value of $\cos \lambda$ is based purely on geometric considerations (say $\cos \lambda = 1$, or $\lambda = \theta$) then the additive drag at the design point (i.e., the shock-on-cowl case operating at maximum mass flow) is not guaranteed to be zero. This is because the assumptions made, both in the calculation of the mean cone pressure and the conditions at the throat, may not be entirely accurate. The calculation of f and its physical meaning is discussed in the section entitled "Calculation of the Factor f " on page 120. At this stage, it is sufficient to state that the use of the factor f is an artifice which can be used to guarantee zero additive drag at the design point.

For the calculation of throat area and cone projected area at the throat, the plane of the throat is assumed to be perpendicular to the cone surface ($\lambda = \theta$). Therefore, the throat area is

$$A_t = (A_c - \pi R_t^2) \cos \theta \quad [8.3.3]$$

and the projected cone area at this station is

$$A_{Cnx} = A_c - A_t \cos \theta \quad [8.3.4]$$

For *subsonic* operation, the pressure on the cone is assumed to be equal to the arithmetic average of the freestream pressure, P_∞ , and the pressure at the inlet face, P_t , the latter

Figure 23. Supersonic Conical Inlet Nomenclature and Control Volume

being calculated by assuming an isentropic expansion for conditions A_∞ , P_∞ , M_∞ to A_t , P_t , M_t . The cone force is thus

$$F_{cone} = [\frac{1}{2}(P_\infty + P_t) - P_\infty]A_{Cnx} \quad [8.3.5]$$

For *transonic* operation, i.e., when $M_\infty > 1$ but the Mach number is too low for a conical shock to be attached to the vertex of the cone, a normal shock is assumed to occur in front of the cone. The cone pressure is calculated as the average of the pressure immediately downstream of the shock, P_{C2} , and the pressure at the inlet capture face. Thus

$$F_{cone} = [\frac{1}{2}(P_\infty + P_{C2}) - P_\infty]A_{Cnx} \quad [8.3.6]$$

The relationship used to express the minimum Mach number for an attached shock as a function of cone angle is a polynomial, fitted to the data of Sims (1964) using least-squares regression. This is

$$M_{min} = 1.0 + 0.35955\theta + 1.89161\theta^2 - 0.24534\theta^3 \quad [8.3.7]$$

where θ is the cone angle in radians.

For *supersonic* operation, the conical shock is attached to the forebody cone vertex. At mass flow ratios reduced from the maximum, a bow shock forms in front of the inlet face, and the cone force then becomes a function of the bow shock position, L . Following Sibulkin (1954), the cone static pressure in the subsonic flow downstream of the normal shock is calculated as the average of the throat static pressure and the static pressure just downstream of the bow shock:

$$F_{cone} = [\frac{1}{2}(P_{C2} + P_t) - P_\infty](A_{Cnx} - A_s) + (P_{C1} - P_\infty)A_s \quad [8.3.8]$$

In this expression, A_s is the projected cone area ahead of the normal shock, so that

$$A_s = \pi(R_{Cn} - L \tan \theta)^2 \quad [8.3.9]$$

where R_{Cn} is the cone radius corresponding to A_{Cnx} .

The bow shock position, L , is obtained from the empirically-based expression given by Sibulkin (1954):

$$\frac{L}{R_c} = K(1.0 - i) \quad [8.3.10]$$

where $i = \frac{MFR}{MFR_{ref}}$

and for $M_\infty \leq 1.6$, $K = 1.13$

$$1.6 < M_\infty < 3.0, K = 0.2505M_\infty^2 - 1.492625M_\infty + 2.8921$$

$$3.0 \leq M_\infty, K = 0.65.$$

The above polynomial curve fit for K is given by Morris et al. (1977) and is based on the data of Sibulkin (1954).

In order to calculate the conditions at the throat (M_t , P_t), the recommendations of Sibulkin (1954) are followed. The total pressure recovery downstream of the conical shock is first calculated by treating it as an oblique shock of the same angle as the conical shock. The bow shock is assumed to be a normal shock, and is assumed to occur at a Mach number equivalent to the average of the cone surface Mach number and the Mach number directly downstream of the conical shock. This enables the total pressure recovery at the inlet face to be calculated. The throat pressure and Mach number can

then be calculated from one-dimensional flow relations. (The cone shock angle, cone surface Mach number, and Mach number downstream of the conical shock are obtained from the conical flow data of Sims (1964).)

Note that the assumption implicit in this analysis is that **all** of the flow entering the inlet passes first through the conical shock, and then through the bow shock. The validity of this assumption is discussed in the section entitled “Shock System Recovery” on page 133.

8.3.3 Additive Drag for the Variable-Geometry Inlet

For the subsonic and transonic case, the inlet geometry is assumed to be fixed, using dimensions determined from the design point of the inlet. This means that the additive drag calculations are identical to those in the previous section for these Mach number regimes.

For supersonic flow, the cone position is calculated by assuming that the inlet always operates at its reference mass flow ratio for that freestream Mach number. Seddon and Goldsmith (1985) present an analytical solution for the inlet reference mass flow ratio, derived from the assumption that the streamlines downstream of the conical shock are portions of hyperbolas. In the nomenclature of Figure 24 on page 121,

$$MFR_{ref} = \left(\frac{A_{\infty}}{A_c} \right)_{ref} = \frac{\cot^2 \theta - \cot^2 \sigma}{\cot^2 \theta - \cot^2 \sigma_s} \quad [8.3.11]$$

where θ is the forebody cone semi-angle;

σ_s is the conical shock angle resulting from a flow at the freestream Mach number, M_∞ , impinging on a cone of semi-angle θ ;
and σ is the angle formed by a line from the cone vertex to the cowl lip, and the horizontal.

The length of the forebody cone is thus

$$X_{cone} = R_c \sqrt{(1 - MFR_{ref}) \cot^2 \theta + MFR_{ref} \cot^2 \sigma_s} \quad [8.3.12]$$

This enables the cone projected area, A_{cnx} , and the throat area, A_t , to be determined so that the additive drag can be calculated from Equation [8.3.2]. The pressure and Mach number at the inlet face (P_t , M_t) are calculated in the same way as the fixed-geometry case, and the normal shock is assumed to occur right at the inlet face, i.e., at the inlet throat.

Since there will be no bow shock, the cone pressure is simply the cone surface pressure in the conical flow downstream of the conical shock, P_{c1} , and the force on the cone is

$$F_{cone} = (P_{c1} - P_\infty) A_{cnx} \quad [8.3.13]$$

8.3.4 Calculation of the Factor f

As stated in the section entitled “Additive Drag for the Fixed-Geometry Inlet” on page 115, the purpose of the factor f is to guarantee that the additive drag of the inlet is zero at design point. It is calculated by rearranging Equation [8.3.2], with the additive drag set to zero, and (since the inlet is operating at design point) $M_\infty = M_{des}$ and $A_\infty = A_c$, so

Figure 24. Calculation of Maximum Flow Conditions for a Conical Inlet

$$f = \frac{\gamma M_{des}^2 A_c P_\infty - F_{cone, des}}{(P_{t, des} - P_\infty + \gamma M_{t, des}^2 P_{t, des}) A_t} \quad [8.3.14]$$

Note that the force on the cone, $F_{cone, des}$, is calculated by making the assumption that there is no bow shock displaced ahead of the inlet entry plane.

It should be noted that this analysis contains a minor inconsistency. The throat area, A_t , is calculated using Equation [8.3.3], which makes the assumption that the flow angle at the throat equals the cone semi-angle ($\lambda = \theta$). Physically, f should also be equal to the cosine of this angle. Thus, for this calculation to be strictly correct, the above assumption that $\lambda = \theta$ should not be made, and Equation [8.3.2] should rather be solved for $\cos \lambda$, with D_{add} set to zero. However, since A_t is also a function of $\cos \lambda$, and is required to solve for the conditions at the throat, this complicates the calculation excessively, and requires an iterative solution. The simplicity and efficiency of the present approach was preferred, especially since it is unlikely to introduce major errors in the analysis.

8.3.5 Spillage Drag

In the absence of suitable correlations for the cowl suction of conical inlets, a spillage drag model was developed based on an extension of the model used for sharp-lipped pitot inlets as detailed in the section entitled “Spillage Drag” on page 58. This spillage drag model is applied to the fixed-geometry conical inlet, and also to the subsonic and transonic Mach number regimes of the variable-geometry configuration.

a) Subsonic

b) Transonic

c) Supersonic

Figure 25. Idealized Representation of Conical Inlets at Various Mach Numbers

Figure 25 on page 123 shows a sketch of a conical inlet operating in subsonic, transonic and supersonic conditions. It is proposed that, when the inlet is in supersonic operation, the bounding streamline will be turned by the forebody cone and it will enter the inlet roughly parallel to the cone surface. Assuming that the difference between the cone semi-angle (θ) and the cowl initial slope (η) is small (say less than 5°), and the cowl lip is sharp, then it is likely that under supersonic conditions the flow will nearly always separate. At any rate, the capability of an inlet to recover additive drag through cowl suction is known to diminish as the freestream Mach number increases. Therefore, it is assumed that for *supersonic* flow,

$$C_{D_{spill}} = C_{D_{add}} \quad [8.3.15]$$

For subsonic operation, the effect of the forebody in deflecting the bounding streamline is assumed to be negligible. Following the analysis in the section entitled “Spillage Drag” on page 58, it can be seen that the maximum disturbed flow drag for a pitot inlet, D_{DF} , is analogous to the additive drag, D_{add} , when in subsonic operation. Therefore it is assumed that

$$C_{D_{spill}} = \alpha C_{D_{add}} \quad [8.3.16]$$

The factor α is calculated as a function of cowl initial slope, just as for the pitot inlet (Equation [6.3.4]).

For *transonic* operation, the analogy between the conical and pitot inlets is maintained even though in actuality the detached shock is not really a normal shock, and the bounding streamline will be deflected. The following formula (analogous to Equation [6.2.1] for pitot inlets) is used to calculate the maximum disturbed flow drag:

$$D_{DF} = (P_t - P_{shk} + \gamma P_t M_t^2) A_f + F_{cone} - \gamma P_{shk} M_{shk}^2 A_\infty \quad [8.3.17]$$

Combining this with Equation [8.3.2], and nondimensionalizing, gives

$$C_{D_{DF}} = C_{D_{add}} - \frac{(P_{shk} - P_\infty) A_f + (P_\infty M_\infty^2 - P_{shk} M_{shk}^2) \gamma A_\infty}{q_\infty A_c} \quad [8.3.18]$$

From Equation [6.2.1], and assuming $\beta = 1$, the spillage drag coefficient is then

$$\begin{aligned} C_{D_{spill}} &= \alpha C_{D_{DF}} + C_{D_{NS}} \\ &= \alpha C_{D_{DF}} + (C_{D_{add}} - C_{D_{DF}}) \\ &= (\alpha - 1) C_{D_{DF}} + C_{D_{add}} \end{aligned} \quad [8.3.19]$$

The assumption that $\beta = 1$ represents the most conservative approach. In this case, there is neither a physical basis nor experimental data to support an assertion that $\beta < 1$.

8.4 Calculation of Maximum Mass Flow Rate

The calculation of the maximum mass flow ratio proceeds exactly as with the 2-D inlet described in the section entitled "Calculation of Maximum Mass Flow Rate" on page 91. The only difference is the way in which the reference mass flow ratio for supersonic flow is calculated, since Equation [8.3.11] must be used for the conical inlet.

8.5 Cowl Wave and Profile Drag

The profile and wave drag contributions are calculated just as for the supersonic pitot inlet.

8.6 Selected Results

8.6.1 Additive Drag for the Fixed-Geometry Inlet

Figure 26 on page 127 shows the calculation of additive drag using the present method compared to the data of Sibulkin (1954) for conical inlets at three different design Mach numbers. It should be noted that the experimental data set used in this figure is for a different freestream Mach number than the data shown in Figure 21 on page 112. This data set was chosen to illustrate the effectiveness of the calculation at an off-design Mach number, but the results are as good for the design-point case.

Figure 26. Calculated Additive Drag Coefficient Compared to Experimental Data: The data are from Sibulkin (1954). The fixed-geometry inlets have design Mach numbers of 1.8, 2.8 and 4.5, and cone semi-angles of 20° . The freestream Mach number is 1.6.

8.6.2 Additive Drag for the Variable-Geometry Inlet

The result of the additive drag calculation for a variable-geometry inlet at one specific Mach number is shown in Figure 27. It is compared to the theoretical minimum predicted by Sibulkin (1954). The slight overprediction could be a result of the assumption that the flow at the inlet face is uniform. However, the trend of decreasing additive drag coefficient with increasing freestream Mach number is correctly predicted.

The Mach 1.8 case corresponds to the one shown in Figure 21 on page 112, and comparison of these figures demonstrates that the present method agrees much more closely with the theoretical minimum additive drag than does the original calculation within ACSYNT.

8.7 Closure

Since the supersonic conical inlet is equipped with a parabolic sharp-lipped cowl, the cowl wave drag and profile drag coefficients are calculated just as for the supersonic pitot inlet.

Two geometrical configurations are considered for the supersonic conical inlet. The first assumes that the geometry is fixed, and the additive drag is calculated using the bow shock prediction method of Sibulkin (1954). The second configuration allows the conical forebody to translate in supersonic flow, minimizing the additive drag by ensuring that

Figure 27. Calculated Additive Drag Coefficient for Variable-Geometry Inlets: The calculation is compared to the "exact" numerical solution of Sibulkin (1954).

the flow is conical at the inlet capture face. For supersonic operation, the spillage drag coefficient is assumed to be equal to the additive drag coefficient. For subsonic and transonic operation, a new spillage drag model has been developed, based on the methods used for supersonic pitot inlets.

9.0 Other Installation Losses and Inlet Sizing

9.1 Overview

The previous four chapters dealt only with the calculation of inlet drag. However, in an aircraft conceptual design program such as ACSYNT, other losses associated with the inlet design also need to be considered, together with the calculation of the inlet dimensions. Understandably, the inlet size has a large effect on the prediction of overall inlet drag. However, the inlet size is a function of the propulsion system requirements, and must be calculated rather than specified by the conceptual designer.

The methods for calculating the installation losses as well as the methods for sizing the inlet are elucidated within this chapter.

9.2 Inlet Supersonic Total Pressure Recovery

This section deals with the calculation of the supersonic total pressure recovery, defined as the ratio of total pressure at the inlet throat to the freestream total pressure, i.e.,

$$PR_{\text{sup}} = \frac{P_{ot}}{P_{o\infty}} \quad [9.2.1]$$

As in the original version of ACSYNT, four options are provided for the supersonic total pressure recovery. The first two are standard pressure recovery schedules expressed as functions of freestream Mach number and are independent of the actual inlet type specified. The third option is a modification of the "normal shock" recovery option of the original version of the code. The fourth option, as before, performs linear interpolation on a user-supplied table consisting of values of PR_{sup} versus M_{∞} .

It should be stated that the choice of inlet supersonic pressure recovery option does not affect the inlet drag calculations, because the drag calculations implicitly always use the third option. The inlet sizing and engine performance prediction are, however, affected.

9.2.1 AIA Standard Ram Recovery

Originally obtained from Hesse and Mumford (1964), this total pressure recovery schedule is as follows:

For $M_{\infty} \leq 1.0$, $PR_{\text{sup}} = 1.0$

$$M_{\infty} > 1.0, PR_{\text{sup}} = 1.0 - 0.1(M_{\infty} - 1.0)^{1.5} \quad [9.2.2]$$

9.2.2 Military Specification 5008B

The source of this schedule is again Hesse and Mumford (1964).

For $M_\infty \leq 1.0$, $PR_{\text{sup}} = 1.0$

$$M_\infty > 1.0, PR_{\text{sup}} = 1.0 - 0.075(M_\infty - 1.0)^{1.35} \quad [9.2.3]$$

9.2.3 Shock System Recovery

For the pitot-type inlets, the total pressure recovery is calculated as the total pressure ratio across a normal shock occurring at a Mach number of M_∞ . This relation is given in the section entitled “Normal Shock Relations” on page 176.

For the supersonic 2-D or conical inlet types, if the inlet is operating under transonic conditions (i.e., $M_\infty > 1$, but the Mach number is too low for a shock to attach to the forebody), a normal shock recovery is assumed. In this case, the total pressure recovery will be identical to that of a pitot inlet. If the freestream Mach number is high enough for a forebody shock to be attached, then the total pressure recovery is the product of the total pressure ratio across an oblique shock (of appropriate angle) and the total pressure ratio across a normal shock. For the conical inlet, this calculation is performed just as in the additive drag calculation (see the section entitled “Additive Drag for the Fixed-Geometry Inlet” on page 115).

Here, the terminal (normal) shock is assumed to occur at a Mach number that is the arithmetic average of the cone surface Mach number and the Mach number just downstream of the conical shock.

For both the 2-D and the conical inlets, all of the flow is assumed to first pass through the forebody shock, and then through the bow or terminal shock as depicted schematically in Figure 17 on page 81 and Figure 23 on page 116. This may not actually happen in practice since, in subcritical operation, the forebody and bow shocks may intersect inside the streamtube which bounds the entering flow. A portion of the air entering the inlet may instead pass through the single shock formed by the intersection of the forebody and the bow shocks. In this case, the mass-averaged total pressure of the entering flow will be lower than in the full flow case. This effect was ignored because the complexity that would result from including it in the calculation is not warranted, especially in view of the many other approximations made in the analysis.

9.3 *Subsonic Total Pressure Recovery*

The empirical method of Ball (1972), used in the original version of ACSYNT, is retained. For $\gamma = 1.4$,

$$PR_{\text{sub}} = 1.0 - \varepsilon \left\{ 1.0 - \frac{1.0}{[1.0 + 0.2M_t^2]^{3.5}} \right\} \quad [9.3.1]$$

where PR_{sub} is defined as the ratio of the total pressure at the inlet throat to the total pressure at the engine face;

M_t is the inlet throat Mach number (calculated as described in the following section, entitled “Inlet Throat Mach Number”;
and ε is a loss coefficient and is a function of M_t .

Morris et al. (1977) fitted the following polynomial to the data of Ball (1972):

$$\varepsilon = 0.37148M_t^2 - 0.231428M_t + 0.06 \quad [9.3.2]$$

It is noted that ε depends also on the subsonic diffuser geometry, which will affect both flow separation and the friction losses. Nicolai (1975) provides curves of ε versus diffuser length-to-height ratio for various throat Mach numbers. However, this level of detail is inappropriate for conceptual design, and therefore the above relation for ε is considered to be satisfactory.

9.4 Inlet Throat Mach Number

The inlet throat Mach number, M_t , is a function of the inlet geometry, the mass flow ratio, and the supersonic total pressure recovery. This Mach number is required for the calculation of the subsonic diffuser total pressure recovery described in the previous section. Subsequently, the total pressure at the engine face, and hence the engine mass flow rate, can be estimated. Because the mass flow ratio cannot be determined without this mass flow rate, M_t cannot be calculated without iteration.

To avoid iterating, M_t is assumed to be constant at the value corresponding to the reference mass flow ratio, MFR_{ref} , even if the mass flow ratio is less than its reference value.

This represents the most conservative approach, since, in supersonic operation, reduction of the mass flow ratio permits a subsonic expansion downstream of the bow shock, thereby reducing M_t . According to Equation [9.3.1], the reduction in M_t will, in turn, improve the subsonic total pressure recovery.

It should be noted that the value of M_t , calculated in this way, is only used for determining the subsonic total pressure recovery. It is independently calculated in the inlet drag routines, once the inlet has been sized.

For *subsonic* operation, the ratio of the inlet throat area to the streamtube capture area, $\frac{A_t}{A_\infty}$, is assumed to be unity. Since there is zero total pressure loss, this assumption implies that the inlet throat Mach number is taken to be the same as the freestream Mach number.

For *transonic* operation, $\frac{A_t}{A_\infty}$ is again assumed to be unity. The total pressure recovery at the inlet throat is calculated by the methods described earlier in this chapter, and the inlet throat Mach number is obtained from mass continuity.

For *supersonic* operation, $\frac{A_t}{A_\infty}$ varies according to the inlet type, and the calculation of inlet throat Mach number must be handled differently for each inlet type. These calculations are discussed in the following sections.

9.4.1 Pitot Inlet in Supersonic Flow

For the pitot inlet, the reference mass flow ratio, MFR_{ref} , is assumed to be unity (Figure 28 on page 138). This means that the ratio of the inlet throat area to the

streamtube capture area, $\frac{A_t}{A_\infty}$, is also unity. The throat Mach number is calculated from mass continuity, based on the required supersonic total pressure recovery.

9.4.2 Supersonic 2-D Inlet in Supersonic Flow

For the supersonic 2-D inlet in a supersonic flow, the reference mass flow ratio, MFR_{ref} , is given by Equation [7.4.4]. The value of σ in this equation is the oblique shock angle corresponding to a flow at the inlet design Mach number turning an angle θ .

As before, the flow angle at the throat is assumed to be equal to the ramp angle, θ . By consideration of the inlet geometry (Figure 29 on page 139), the ratio of the throat area to the streamtube capture area is given by

$$\frac{A_t}{A_\infty} = \left(\frac{1}{MFR_{ref}} - \frac{\tan \theta}{\tan \sigma} \right) \cos \theta \quad [9.4.1]$$

The total pressure ratio is calculated as described earlier in this chapter, and the throat Mach number is subsequently obtained from mass continuity.

9.4.3 Supersonic Conical Inlet in Supersonic Flow

The calculation for the conical inlet proceeds just as for the 2-D inlet. In this case, however, the mass flow ratio is given by Equation [8.3.11], and σ is obtained from con-

Figure 28. Calculation of Throat Mach Number for a Pitot Inlet

Figure 29. Calculation of Throat Mach Number for a Supersonic Inlet with Forebody

ical shock tables. The ratio of the throat area to the streamtube capture area (see Figure 29 on page 139) is given by

$$\frac{A_t}{A_\infty} = \left(\frac{1}{MFR_{ref}} - \frac{\tan^2 \theta}{\tan^2 \sigma} \right) \cos \theta \quad [9.4.2]$$

9.5 Inlet Sizing and Mass Flow Calculation

The basic information made available by the engine performance prediction module includes the information required to calculate the engine design mass flow rate $\dot{m}_{eng, des}$, and the mass flow rate for the current conditions, $\dot{m}_{des}$. To size the inlet (i.e., calculate the inlet capture face area, A_c) and calculate the total inlet mass flow rate, the additional mass flows handled by the inlet must be taken into account. These additional requirements include the bleed, bypass, auxiliary and vent air flows, and are generally treated as they were in the original version of ACSYNT.

The engine vent mass flow ratio is set internally as

$$\frac{A_{vent}}{A_c} = 0.03 \quad [9.4.1]$$

The auxiliary mass flow ratio, $\frac{A_{aux}}{A_c}$, is a user input, and should be a small number in the range 0.005 to 0.01. It is incumbent on the conceptual designer to select an appropriate value for this variable.

The bleed mass flow ratio is set as follows. For an inlet design Mach number in the range $1.0 < M_{des} < 2.8$,

$$\frac{A_{bl}}{A_c} = 0.1 \left(\frac{M_{des}}{3.0} \right)^3 \left(\frac{M_\infty - 1.0}{M_{des} - 1.0} \right) \quad [9.4.2]$$

For $M_{des} \geq 2.8$,

$$\frac{A_{bl}}{A_c} = 0.1(0.344M_{des} - 0.0113) \quad [9.4.3]$$

For subsonic inlets, the bleed mass flow ratio is set to zero.

Equation [9.4.2] comes from a straight-line curve fit to the data of Ball (1972), while the source of Equation [9.4.3] is not known. However, the agreement exhibited by these two relations in the overlapping Mach number region seems to support the validity of the latter.

The subsonic bypass mass flow ratio is set to zero.

For supersonic operation, the bypass mass flow ratio is calculated from

$$\frac{A_{byp}}{A_c} = \frac{1}{2} \left[1.0 - \left(\frac{A_\infty}{A_c} \right)_{eng} \right] \quad [9.4.4]$$

where $\left(\frac{A_\infty}{A_c} \right)_{eng}$ is the mass flow ratio based only on the engine mass flow requirements.

Before $\left(\frac{A_\infty}{A_c} \right)_{eng}$ can be calculated, the area of the inlet capture face, A_c , must be found.

It is calculated using the following method:

$$A_c = \frac{\dot{m}_{eng, des}}{(\rho_\infty V_\infty)_{des} \left(1 - \frac{A_{bl}}{A_c} - \frac{A_{aux}}{A_c} - \frac{A_{vent}}{A_c} \right)_{des}} \quad [9.4.5]$$

where $\dot{m}_{eng, des}$ is the design-point engine mass flow rate;

and $(\rho_\infty V_\infty)_{des}$ is the product of the freestream density and velocity at the design Mach number. It is given by

$$(\rho_\infty V_\infty)_{des} = M_{des} P_\infty \sqrt{\frac{\gamma g_c}{R T_\infty}}$$

where $M_{des}, P_\infty, T_\infty$ are the inlet design Mach number, and the freestream pressure and temperature, respectively;

R is the gas constant for air, and has the value 287 Nm/(kgK) in SI units and 53.3 ft-lbf/(lbm °R) in the EES;

and g_c is the dimensional constant, and has the dimensionless value 1 in SI units, and 32.2 lbm ft/(s²lbf) in the EES.

This enables $\left(\frac{A_\infty}{A_c} \right)_{eng}$ to be calculated as follows:

$$\left(\frac{A_\infty}{A_c} \right)_{eng} = \frac{\dot{m}_{eng}}{\rho_\infty V_\infty A_c} \quad [9.4.6]$$

where $\dot{m}_{eng}$ is the engine mass flow rate;

$\rho_\infty V_\infty$ is calculated as above, but in this case the current freestream Mach number, M_∞ , is used rather than the design point value, M_{des} .

The inlet mass flow rate is calculated as follows:

$$\dot{m}_{inlet} = \dot{m}_{eng} + \dot{m}_c \left(\frac{A_{bl}}{A_c} + \frac{A_{byp}}{A_c} + \frac{A_{vent}}{A_c} + \frac{A_{aux}}{A_c} \right) \quad [9.4.7]$$

where $\dot{m}_c$ is a reference mass flow rate, such that $\dot{m}_c = \rho_\infty V_\infty A_c$.

9.6 *Bleed and Bypass Drags*

The method and source code used to calculate the bleed and bypass drags has been left totally unchanged from the original version of ACSYNT. The methods are consistent with the definition of the drag terms in “Propulsion System Losses and Drag Terms” on page 7, and are fully documented by Morris et al. (1972).

9.7 *Auxiliary Systems Drag*

As suggested by Morris et al. (1972), the basis of the auxiliary systems drag calculation is the assumption that the total momentum to these systems is lost (see “Appendix A. Installed Propulsion Thrust/Drag” on page 162). In this case,

$$C_{D_x} = \frac{D_x}{q_\infty A_c} = \frac{\dot{m}_x V_\infty}{\frac{1}{2} \rho_\infty V_\infty^2 A_c} = 2 \frac{A_{aux}}{A_c} \quad [9.6.1]$$

9.8 Boundary Layer Diverter Drag

On many aircraft, the inlet will be positioned adjacent to a larger component of the aircraft, such as the fuselage or wing. In this case, the inlet will be subject to the ingestion of low-momentum boundary layer fluid forming on the surface of the large component upstream of the inlet, and this may result in a nonuniform inlet flow which can degrade of the performance of the propulsion system. To avoid this undesirable effect, a channel or step (termed a *diverter*) may be added to lift the inlet away from the surface that generates the boundary layer. The geometry of this boundary layer diverter causes the flow to turn and adds drag to the aircraft.

The method for predicting boundary layer diverter drag is also unchanged from the original code. Morris et al. (1972) combined data from two different sources to obtain the following relationships for the diverter drag coefficient, $C_{D_{div}}$, based on A_c :

$$M_\infty < 0.8, C_{D_{div}} = 0.0 \quad [9.7.1]$$

$$0.8 < M_\infty < 0.95, C_{D_{div}} = 0.499 \frac{(M_\infty - 0.8)}{(0.95 - 0.80)} \frac{\theta_p}{20} \frac{A_{wedge}}{A_c} \quad [9.7.2]$$

$$0.95 < M_\infty < 1.55, C_{D_{div}} = 0.499 \frac{\theta_p}{20} \frac{A_{wedge}}{A_c} \quad [9.7.3]$$

$$1.55 \leq M_\infty, C_{D_{div}} = \frac{1.2}{M_\infty^2} \frac{\theta_p}{20} \frac{A_{wedge}}{A_c} \quad [9.7.4]$$

where θ_D is the diverter wedge half-angle, set to 20° within the program;
and $\frac{A_{wedge}}{A_c}$ is the ratio of the wedge frontal area to the inlet capture area, and is a
user input.

It is implied in this calculation that the ratio of diverter height to boundary layer height is 0.5.

10.0 Application of the Calculation Methods

10.1 Application to Alternative Geometries

During the course of the code development, the results of each of the calculation methods were, wherever possible, compared to the experimental data or the sample calculations presented in the source of that specific method. Examples of such comparisons are shown in Chapters 5 through 8. The question arises, however, as to how well the calculation methods will perform in the prediction of the drag of inlets other than those used as a basis for the development of the correlations.

The acquisition of suitable sample data sets is, in itself, problematic. Due to the expense of large-scale, supersonic wind-tunnel tests, complete drag measurements of inlets are rarely performed and, because of their proprietary nature, seldom documented in the open literature. In spite of this difficulty, two test cases for the supersonic 2-D and conical inlets, respectively, were investigated. The fact that these geometries differ from the simple two-shock configuration for which the code has been developed means that it is perhaps unreasonable to expect close agreement, especially in view of the empirical nature of the calculation methods. Nevertheless, it is still informative to see how well the calculation methods perform when the test geometries are approximated as two-shock inlets.

10.1.1 2-D Three-Shock External-Compression Inlet

An inlet model with varying ramp configurations was tested by Hawkins et al. (1976). Although nominally a four-shock inlet, in its simplest configuration it performed as a three-shock inlet with ramp angles of 7° and 11° . This inlet was also equipped with a wedge-shaped lower cowl with a 4° wedge angle. For the present work, the inlet was modeled by treating it as a two-shock inlet with a forebody ramp angle of 7° . This also gave a different inlet throat area to the test configuration.

The results of this calculation are shown in Figure 30 on page 148 for freestream Mach numbers of 0.85, 1.2 and 1.39. The results for $M_\infty = 0.85$ show very good agreement with the data, although the predicted gradient of the variation of drag with mass flow ratio is higher than that of the test data. This is to be expected in all cases due to the difference in the throat area between the test and the calculated configurations. The level of the drag prediction is also satisfactory. The two components of the inlet drag which would contribute to this level, the profile and the subsonic wave drag, are therefore in reasonable agreement with the experimental data.

The transonic case ($M_\infty = 1.2$) also exhibits good agreement, in spite of the simplifying assumptions made in the transonic additive drag analysis. As mentioned, the slope of the drag prediction is understandably higher than the data shown.

The supersonic case ($M_\infty = 1.39$) shows the poorest agreement with the experimental data. Unexpectedly, the slope of the drag prediction matches the data very well, but the

Figure 30. Inlet Drag Coefficient for a Three-Shock Inlet: The data are from Hawkins et al. (1976). The parameters used in the calculations are: $M_{des} = 2.2$, $\theta = 7^\circ$, $\psi = 4^\circ$, $AR = 0.98$, $n = 1.3$, $\frac{Y_c}{Y_m} = 0.692$, $\frac{L}{Y_m} = 2.73$, $\frac{2r_{lip}}{Y_m - Y_c} = 0.025$.

absolute level reflects as much as a 40 per cent overprediction. It was first thought that this was indicative of an error in the profile or wave drag estimates. However, the profile drag coefficient agrees well with the calculation of Hawkins et al. (1976), and the wave drag prediction also appears to be reasonable. It is therefore possible that the discrepancy arises due to the favorable effect of the multiple forebody shocks on the spillage drag. These weaker, multiple shocks will cause a reduction in the pressure integral along the bounding streamline, so reducing the additive drag from the value calculated using the two-shock assumption.

10.1.2 Conical Mixed-Compression Inlet

Inlet drag data for tests on an axisymmetric, mixed-compression inlet with a design Mach number of 2.7 and a forebody cone angle of 20° are presented by McVey et al. (1969). These tests were run at freestream Mach numbers of 0.7, 0.9, 1.05 and 2.00.

Figure 31 on page 150 shows that, at subsonic Mach numbers, the drag is slightly overpredicted. Here, the calculation registers a complete recovery of additive wave drag due to the high cowl slope, and zero wave drag due to the low Mach number. Therefore, the only drag component is that due to the viscous effects. A negative drag for $M_\infty = 0.7$ is indicated by the experimental data. The reason for this is not completely understood, but it is possibly due to an error in the calculation of cowl suction from the cowl surface pressure taps. In spite of this, the prediction is still reasonably close to the data.

Figure 31. Inlet Drag Coefficient for a Conical-Mixed Compression Inlet: The data are from McVey et al. (1969). The parameters used in the calculations are: $M_{des} = 2.7$, $\theta = 20^\circ$, $\Delta\eta = 20^\circ$, $n = 2$, $\frac{R_c}{R_m} = 0.833$, $\frac{L}{R_m} = 0.833$.

For both the transonic and supersonic cases, the total inlet drag is greatly overpredicted. For supersonic flow, this overprediction can be explained in terms of the effect of the compression being mixed, which will reduce the displacement of the bow shock ahead of the inlet capture face. The discrepancy in the transonic case is most likely due to the throat area which could not be faithfully modeled.

As an exercise, the total inlet drag coefficient was multiplied by a factor of 0.6, using the scale factors provided in the program. The result, shown in Figure 32 on page 152, is definitely acceptable, and thus demonstrates that the external-compression inlet represented by the calculations can be simply extended to effectively model a mixed-compression inlet.

10.2 Test Case — F-16 Inlet Spillage Drag

The individual subroutines containing the methods described in this thesis have been successfully merged with ACSYNT, replacing the inlet drag routines contained in the original version of the cycle calculation module.

As an example with which to test these new routines, the F-16 was synthesized using both the original and the new versions of ACSYNT. In actuality, the chin-mounted F-16 pitot inlet is not truly axisymmetric, and has a slightly blunted lip. For this test case, however, it had to be modeled with the most appropriate inlet type available. In the original version of ACSYNT, the inlet was modeled as a variable-geometry conical inlet. Using the new routines, the inlet was modeled as a semi-circular supersonic pitot inlet

Figure 32. Inlet Drag Coefficient for a Conical-Mixed Compression Inlet: The inlet drag coefficient is scaled by a factor of 0.6.

(with a sharp cowl lip). The resulting spillage drag distributions are shown in Figure 33 compared to the data of Hunter and Cawthon (1984).

The inlet drag predicted by the original version of ACSYNT is actually additive drag. However, since the requirement of constant inlet throat Mach number serves to minimize the additive drag, it is not unreasonable to treat it as a spillage drag term. It does not, however, agree well with the data.

The new inlet drag prediction methods are remarkably successful in modeling the F-16 inlet, even though the exact inlet geometry is not replicated within the analysis. The major discrepancy occurs in the transonic region, and it is likely that this is due to a weakness in the spillage drag model for this difficult-to-handle Mach number range.

Figure 33. F-16 Inlet Spillage Drag Comparison: Data are from Hunter and Cawthon (1984). Inputs to original ACSYNT: XMDES = 2.0, XMT = 0.85. Inputs to new version: MDES = 2.0, YCOYM = 0.85, SFSPIL = 0.5. Combat phase, altitude 30,000ft. Reference area $S_{ref} = 300ft^2$.

11.0 Recommendations for Future Work

11.1 Additional Test Cases

It is believed that the inlet drag calculation routines are, in general, more than adequate for the purposes of conceptual design. However, should further experimental data become available, benefit would be derived from modeling the inlets using the new routines and comparing the results of the calculations to this data. These comparisons would assist in determining the limits of the routines and guide in selecting the best combination of scale factors to model variations of the inlet geometry.

11.2 Source Code Improvements

It is believed that the present drag prediction analyses embody the best of the methods reported in the literature. However, since the empirical methods are based on experimental data, it is likely that these will be superseded as more data become available. In this event, the appropriate individual subroutines should be updated. The source code is written in a modular fashion so that such improvements will be easy to incorporate.

11.3 Further Integration of the Inlet Drag Prediction

Routines

Even though the initial phase of the integration of the inlet drag calculations into ACSYNT has been completed, it is proposed that further attention should be given to the following areas:

1. A standard for the thrust and drag accounting within ACSYNT should be established and documented. This standard would address such matters as which drag components should be charged to the propulsion system, and how the cowl and afterbody are to be defined for integrated propulsion systems. Without such a standard, there is a potential for accounting for the same drag term more than once. For instance, because cowl wave drag is now calculated in the propulsion module, the airframe wave drag analysis should not include the inlet. Unless care is exercised, this error could be made both with the present component-buildup method, and also with the newer wave drag method proposed by Grieshaber et al. (1989).
2. Presently, there are three distinct engine routines within ACSYNT. One of these uses a cycle calculation approach, and the other two perform lookups on tabulated engine data. Presently, only the cycle routine allows propulsion installation effects to be computed. It is therefore proposed that the propulsion module be reorganized

and upgraded so that the installation effects are also properly accounted for in the table-lookup routines.

3. The inlet drag routines should be linked more closely to the geometric modeling routines. CAD could be used to specify the required proportions of an unsized inlet. The output of ACSYNT could then be used to merge the sized inlet with the airframe.

It is further suggested that the number of inlets be specified independently of the number of engines, and that "fractions" of an inlet be permitted so that, for instance, semi-annular or quarter-annular configurations can be specified. The scale factors could then be appropriately used to account for these configurations in the analysis. Provision should also be made for the geometric representation of inlets such as the F-16's, which is neither axisymmetric nor two-dimensional. In this case the inlet could be treated as the most appropriate inlet type within the analysis.

References

- Baals, D. D., Smith, N. F., and Wright, J. B.
"The Development and Application of High-Critical-Speed Nose Inlets"
NACA Technical Report 920, 1949
- Ball, W. H.
"Propulsion System Installation Corrections", vols 1 - 4
Boeing Company, Seattle Washington Research and Engineering Division AFFL-TR-72-147, December 1972
- Bertram, M. H.
"Calculations of Compressible Average Turbulent Skin Friction"
NASA Technical Report R-123, 1961
- Brook, J. W.
"An Approximate Method for Determining the Wave Drag of Axisymmetric Conical Cowls"
J. Aero. Sci., No. 5, Vol. 25, May 1958
- Butler, S. F. J.
"Aircraft Drag Prediction for Project Appraisal and Performance Evaluation"
AGARD CP-124, 1973
- Eggers, A. J. and Savin, R. C.
"Approximate Methods for Calculating the Flow Around Nonlifting Bodies of Revolution at High Supersonic Speeds"
NACA Technical Note 2579, 1951
- ESDU
"Drag of Axisymmetric Cowls at Zero Incidence for Subsonic Mach Numbers"
Engineering Sciences Data Unit, Item 81024, 1981
- Faro, I. D. V.
"Handbook of Supersonic Aerodynamics, Section 8, Bodies of Revolution"
Navweps Report 1488, Volume 3, October 1961
- Fenter, F. W.
"An Approximate Method for the Calculation of the Aerodynamic Characteristics of Ogive-Cylinders Near Zero Lift"
Report 390, Defense Research Laboratory, University of Texas, (JHU/APL Bumblebee CF-2547), January 1957
- Grieshaber, M. M., Jayaram, S., Jayaram, U., Myklebust, A., and Mahan, J. R.
"Interactive Aircraft Section Calculations for Drag"
Presented at the *University Programs in Computer-Aided Engineering, Design, and Manufacturing ASCE Seventh Annual Conference*, Laramie, Wyoming, 1989

- Hawkins, J. E.
 "YF-16 Inlet Design and Performance"
AIAA Paper 74-1062, 1974
- Hawkins, J. E., Kirkland, F. P., and Turner, R. L.
 "Inlet Spillage Drag Tests and Numerical Flow-Field Analysis at Subsonic and Transonic Speeds of a 1/8-scale, Two-Dimensional, External-Compression, Variable Geometry, Supersonic Inlet Configuration"
NASA CR-2680, 1976
- Hesse, W. J., and Mumford, N. V. S.
 "Jet Propulsion for Aerospace Applications"
Pitman Publishing Corp., 1964
- Hoerner, S. F.
 "Fluid-Dynamic Drag"
Published by the author, 1965
- Hunter, L. G., and Cawthon, J. A.
 "Improved Supersonic Performance Design for the F-16 Inlet Modified for the J-79 Engine"
AIAA Paper 84-1272, 1984
- Jackson, C. M. Jr., Sawyer, W. C., and Smith, R. S.
 "A Method for Determining Surface Pressures on Blunt Bodies of Revolution at Small Angles of Attack in Supersonic Flow"
NASA-TN-D-4865, 1969
- Leyland, D. C.
 "Intake Drag"
Von Karman Institute Lecture Series LS 1988-04, 1988
- Liepmann, H. W., and Roshko, A.
 "Elements of Gasdynamics"
John Wiley and Sons, 1957
- Lighthill, M. J.
 "Higher Approximations in Aerodynamic Theory"
Princeton University Press, 1960
- McVey, F. D., Rejeske, J. V., and Phillips, E. J.
 "Experimental Evaluation of Inlet Drag Characteristics in the Transonic Mach Number Regime"
AFAPL-TR-68-119, Supplement 1, 1969
- Moeckel, W. E.
 "Approximate Method for Predicting Form and Location of Detached Shock Waves Ahead of Plane or Axially Symmetric Bodies"
NACA Technical Note 1921, 1949

- Moore, F. G.
 "Aerodynamic Drag and Lift of General Body Shapes at Subsonic, Transonic and Supersonic Mach Numbers"
AGARD CP-124, 1973
- Morris, D. N.
 "A Summary of the Supersonic Pressure Drag of Bodies of Revolution"
J. Aero. Sci., July 1961
- Morris, S. J. Jr., Nelms, W. P. Jr, and Bailey, R. O.
 "A Simplified Analysis of Propulsion System Installation Losses for Computerized Aircraft Design"
NASA TM X-73136, 1977
- Mount, J. S.
 "Effect of Inlet Drag on Aircraft Performance"
J. Aircraft, vol 2, no 3, Sept-Oct 1965, pp 374-378
- Myklebust, A., Mahan, J. R., Brown, E. F., Jayaram, S., Taylor, A. K., Kolady, K., Malan, P., Jayaram, U., Gloudemans, J. R., Grieshaber, M. M.,
 "A CAD System for Automated Conceptual Aircraft Design"
Report No. 313161-1E, CAD/CAM Laboratory, Virginia Polytechnic Institute and State University, July 1989.
- Nicolai, L. M.
 "Fundamentals of Aircraft Design"
Published by the author, 1975
- Osmon, R. V.
 "Improved Methods of Spillage Drag Prediction for Two-Dimensional Supersonic Inlets"
AIAA Paper 67-449, 1967
- Petersen, M. W., and Tamplin, G. C.
 "Experimental Review of Transonic Spillage Drag of Rectangular Inlets"
North American Aviation Inc, Los Angeles, CA., AFAPL-TR-66-30, May 1966
- Presley, L. L., and Mossman, E. A.
 "A Study of Several Theoretical Methods for Computing the Zero-Lift Wave Drag of a Family of Open-Nosed Bodies of Revolution in the Mach Number Range of 2.0 to 4.0"
NACA Technical Note 4368, 1958
- Raymer, D. P.
Aircraft Design: A Conceptual Approach
 AIAA, 1989
- Sears, R. I., Merlet, C. F., and Putland, L. W.
 "Flight Determination of Drag of Normal-Shock Nose Inlets with Various Cowling Profiles at Mach Numbers from 0.9 to 1.5"
NACA Report 1281, 1955

- Seddon, J., and Goldsmith, E. L.
Intake Aerodynamics
 AIAA and Collins, 1985
- Sibulkin, M.
 "Theoretical and Experimental Investigation of Additive Drag"
NACA Report 1187, 1954
- Sims, J. L.
 "Tables for Supersonic Flow Around Right Circular Cones at Zero Angle of Attack"
NACA SP-3004, 1964
- Sorenson, S. C., and Short, B. J.
 "Free-Flight Measurements of Turbulent-Boundary-Layer Skin Friction in the Presence of Severe Aerodynamic Heating at Mach Numbers from 2.8 to 7.0"
NACA Technical Note 3391, 1954
- Syvertson, S. A., and Dennis, D. H.
 "A Second-Order Shock-Expansion Method Applicable to Bodies of Revolution Near Zero Lift"
NACA Report 1328, 1957
- Van Dyke, M. D.
 "Practical Calculation of Second-Order Supersonic Flow Past Non-Lifting Bodies of Revolution"
NACA Technical Note 2744, 1952
- Wampler, S. G., Myklebust, A., Jayaram, S., and Gelhausen, P.
 "Improving Aircraft Conceptual Design - A PHIGS Interactive Graphics Interface for ACSYNT"
 Presented at the *AIAA/AHS/ASEE Aircraft Design, Systems and Operations Meeting*, Atlanta, Georgia, 1988
- Whitford, R.
Design for Air Combat
 Jane's, 1987
- Willis, J. H., and Randall, D. G.
 "The Theoretical Wave Drag of Open Nose Axisymmetrical Forebodies with Varying Fineness Ratio, Area Ratio and Nose Angle"
RAE TN Aero 2360, 1955

Appendix A. Installed Propulsion Thrust/Drag

The purpose of this appendix is to derive the installed thrust equation, and to define the drag terms associated with the propulsion system. Figure 34 on page 163 shows a schematic of a propulsion system and the control volumes used for this analysis. The outer control volume is assumed cylindrical in shape to simplify the notation used, but it could have a completely arbitrary cross-section. Since the bleed and bypass drags are treated identically, they are lumped together to simplify the analysis.

The flow at face ed may be disturbed by the propulsion system, and therefore no assumptions can be made about the velocity and pressure distribution along this face. A mass flow $\dot{m}_s$ leaves the control volume with a mean velocity V_s . Far upstream, the flow at face $a\infty$ is undisturbed, and the area of this face is A . A_∞ is the cross-sectional area of the streamtube bounding the flow which ultimately enters the inlet.

Applying the principle of conservation of momentum to the control volume $a\infty ced$,

$$\begin{aligned}
 & \int_e^d \rho V^2 dA_x - \rho_\infty (A - A_\infty) V_\infty^2 + \dot{m}_s V_s - \dot{m}_b V_b \cos \theta_b \\
 &= P_\infty (A - A_\infty) - \int_c^e P dA_x - \int_\infty^c P dA_x - \int_e^d P dA_x \\
 & \quad - \int_c^e \tau_w dA_s + P_b A_b \cos \theta_b
 \end{aligned} \tag{A.1}$$

Figure 34. Control Volume for Installed Propulsion Analysis

Note that the integrals from c to e include only the *surfaces* of the nacelle, pylon and other protuberances.

Similarly, for the control volume $aadd$,

$$\begin{aligned} \int_e^d \rho V^2 dA_x - \rho_\infty V_\infty^2 A + \rho_e V_e^2 A_e + \dot{m}_s V_s + \dot{m}_x V_x \\ = P_\infty A - \int_e^d P dA_x - P_e A_e + T_i \end{aligned} \quad [A.2]$$

Subtracting Equation [A.2] from [A.1] gives

$$\begin{aligned} T_i = \rho_e V_e^2 A_e - \rho_\infty V_\infty^2 A_\infty + \rho_e A_e - P_\infty A_\infty - \int_\infty^c P dA_x - \int_c^e P dA_x \\ - \int_c^e \tau_w dA_s + \dot{m}_x V_x + (\dot{m}_b V_b + P_b A_b) \cos \theta_b \end{aligned} \quad [A.3]$$

Applying the principle of mass continuity to the control volume $a\infty ced$,

$$\dot{m}_s - \rho_\infty V_\infty (A - A_\infty) - \dot{m}_b + \int_e^d \rho V dA_x = 0 \quad [A.4]$$

And for the control volume $aadd$,

$$\dot{m}_s + \dot{m}_x + \rho_e V_e A_e - \rho_\infty V_\infty A + \int_e^d \rho V dA_x = 0 \quad [A.5]$$

Subtracting Equation [A.4] from [A.5] gives

$$\rho_e V_e A_e + \dot{m}_x + \dot{m}_b = \rho_\infty V_\infty A_\infty \quad [A.6]$$

or,

$$\dot{m}_e + \dot{m}_x + \dot{m}_b = \dot{m}_\infty \quad [A.7]$$

Assuming that all the momentum to the auxiliary system is lost, $V_x = 0$.

Equation [A.3] then becomes

$$\begin{aligned} T_i = \dot{m}_e V_e - V_\infty (\dot{m}_e + \dot{m}_x + \dot{m}_b) + P_e A_e - P_a A_\infty - \int_\infty^c P dA_x - \int_c^e P dA_x \\ - \int_c^e \tau_w dA_s + (\dot{m}_b V_b + P_b A_b) \cos \theta_b \end{aligned} \quad [A.8]$$

Noting that

$$\begin{aligned} \int_\infty^c P dA_x - \int_c^e P dA_x \\ = \int_\infty^c (P - P_\infty) dA_x + \int_c^e (P - P_\infty) dA_x + P_\infty (A_e - A_\infty + A_b \cos \theta) \end{aligned} \quad [A.9]$$

the thrust equation becomes

$$\begin{aligned}
T_i = & \dot{m}_e(V_e - V_\infty) + A_e(P_e - P_\infty) - \int_\infty^c (P - P_\infty) dA_x - \dot{m}_x V_\infty \\
& - \int_c^e \tau_w dA_s - [\dot{m}_b(V_\infty - V_b) + A_b(P_b - P_{p\infty})] \cos \theta_b
\end{aligned} \tag{A.10}$$

i.e., $T_i = T_N - D_{add} - D_p - D_f - D_b - D_x$

where the individual terms are:

Uninstalled thrust, $T_N = \dot{m}_e(V_e - V_\infty) + A_e(P_e - P_\infty)$

Additive drag, $D_{add} = \int_\infty^c (P - P_\infty) dA_x$

Nacelle form (or pressure) drag, $D_p = \int_c^e (P - P_\infty) dA_x$

Nacelle viscous drag, $D_f = \int_c^e \tau_w dA_s$

Auxiliary drag, $D_x = \dot{m}_x V_\infty$

Bleed and bypass drag, $D_b = [\dot{m}_b(V_\infty - V_b) + A_b(P_b - P_{p\infty})] \cos \theta_b$.

Appendix B. Additive Drag Cancellation in Ideal Flow

The purpose of this appendix is to demonstrate the principle of additive drag recovery. Figure 35 on page 168 shows a propulsion system with its associated control volume. Assuming that the flow is ideal, and that the flow through the control volume *aadd* is totally undisturbed by the presence of the engine, the pressure and velocities on the faces *aa* and *dd* of the control volume will be uniform and equal. Applying conservation of momentum to the control volume *a∞ced*,

$$\begin{aligned} & \rho_{\infty}(A - A_e)V_{\infty}^2 - \rho_{\infty}(A - A_{\infty})V_{\infty}^2 + \dot{m}_s V_{\infty} \\ &= P_{\infty}(A - A_e) - P_{\infty}(A - A_{\infty}) - \int_{\infty}^c P dA_x - \int_c^e P dA_x \end{aligned} \quad [B.1]$$

Doing a mass continuity balance on the same control volume,

$$\dot{m}_s = \rho_{\infty} V_{\infty} (A_e - A_{\infty}) \quad [B.2]$$

Substituting [B.2] in [B.1] gives

$$P_{\infty}(A_e - A_{\infty}) = \int_{\infty}^c P dA_x + \int_c^e P dA_x \quad [B.3]$$

Noting that

Figure 35. Control Volume for Undisturbed Flow Analysis

$$P_{\infty}(A_e - A_{\infty}) = P_{\infty}(A_e - A_c) + P_{\infty}(A_c - A_{\infty}) = P_{\infty} \int_{\infty}^c dA_x + P_{\infty} \int_c^e dA_x \quad [B.4]$$

then combining Equations [B.3] and [B.4],

$$\int_{\infty}^c (P - P_{\infty}) dA_x + \int_c^e (P - P_{\infty}) dA_x = 0 \quad [B.5]$$

Equation [B.5] indicates that, based on the assumption that the external flow is undisturbed, the cowl suction will cancel the additive drag.

Appendix C. Methods and Equations for Supporting Routines

C.1 Overview

In Chapters 5 through 9, aspects of the drag analysis such as the evaluation of skin friction coefficient and compressible flow relations were not discussed in detail. These form the core of a set of utility subroutines which are accessed by the specific drag calculation routines. The methods and equations used in these subroutines are discussed in this appendix.

C.2 Calculation of Skin Friction Coefficient

This module is based entirely upon the one originally contained in the ACSYNT aerodynamics module. It has been restructured and only accounts for turbulent skin friction.

The T' method of Sommer and Short (1985) is used to correct for compressibility, and the relation for the turbulent skin friction coefficient comes from Bertram (1962). The

skin friction is further limited to a minimum imposed by the grain size criterion recommended by Hoerner (1965).

From Bertram (1962), the incompressible turbulent skin friction coefficient is

$$C_{F_i} = \frac{0.088}{(\log_{10} Re - 1.5)^2} \quad [C.7.1]$$

The reference temperature, T' , is evaluated from the empirical correlation of Sommer and Short (1955) as follows:

$$\frac{T'}{T_\infty} = 1 + 0.035M_\infty^2 + 0.45\left(\frac{T_{wall}}{T_\infty} - 1\right) \quad [C.2.2]$$

Assuming a wall-recovery factor of 0.89 gives

$$\frac{T_{wall}}{T_\infty} = 1 + \frac{(\gamma - 1)}{2} (0.89) M_\infty^2 \quad [C.2.3]$$

So that with $\gamma = 1.4$,

$$\frac{T'}{T_\infty} = 1 + 0.115M_\infty^2 \quad [C.2.4]$$

The reference Reynolds number, Re' is evaluated from

$$\frac{Re'}{Re} = \frac{\rho'}{\rho_\infty} \frac{\mu_\infty}{\mu'} = \frac{1}{\left(\frac{T'}{T_\infty}\right) \left(\frac{\mu'}{\mu_\infty}\right)} \quad [C.2.5]$$

Using the Sutherland viscosity law,

$$\frac{\mu'}{\mu_\infty} = \left(\frac{T'}{T_\infty} \right)^{\frac{3}{2}} \left(\frac{T_\infty + S}{T' + S} \right) \quad [C.2.6]$$

where S = Sutherland's constant. It has the value 110.4 K in SI units and 198.6 °R in the EES.

Then,

$$\frac{Re'}{Re} = \frac{1}{\left(\frac{T'}{T_\infty} \right)^{\frac{5}{2}} \left(\frac{T_\infty + S}{T' + S} \right)} \quad [C.2.7]$$

So that

$$Re' = \frac{Re [T_\infty(1 + 0.115M_\infty^2) + S]}{(1 - 0.115M_\infty^2)(T_\infty + S)} \quad [C.2.8]$$

The value of the Reynolds number, Re , is obtained from

$$Re = \frac{\rho_\infty V_\infty L}{\mu_\infty} \quad [C.2.9]$$

where $\rho_\infty = \frac{P_\infty}{RT_\infty}$

$$V_\infty = M_\infty \sqrt{\gamma R g_c T_\infty}$$

$$\mu = \mu_{ref} \frac{(T_{ref} + S)}{(T_\infty + S)} \left(\frac{T_\infty}{T_{ref}} \right)^{1.5}$$

R is the gas constant for air, and has the value 287 Nm/(kgK) in SI units and 53.3 ft-lbf/(lbm °R) in the EES;

g_c is the dimensional constant, and has the nondimensional value 1 in SI units, and 32.2 lbf ft/(s²lbf) in the EES;

L is the reference length;

and μ_{ref} is a known viscosity at a specific temperature T_{ref}

Also, Re' is limited to a value of Re'_{crit} due to skin roughness. To obtain Re'_{crit} , a grain diameter of $K_{crit} = 0.001$ inches (based on typical aircraft paint finish) is assumed. From Hoerner (1965)

$$\left(\frac{V}{\nu} \right)_{crit} = \frac{100}{K_{crit}} \quad [C.2.10]$$

Therefore,

$$Re'_{crit} = \frac{100 \times 12}{0.001} L = 1,200,000L \quad [C.2.11]$$

where L is expressed in feet. (For L in meters, $Re'_{crit} = 3,937,008L$.)

According to Bertram (1962), the compressible skin friction coefficient, C_F , is given by the equation

$$C_F = C_{F_i}' \frac{T_\infty}{T'} \quad [C.2.12]$$

where C_{F_i}' is the incompressible skin friction coefficient evaluated at Re' and T' . Hence

$$C_F = \frac{0.088}{(\log_{10} Re' - 1.5)^2 (1 + 0.0115 M_\infty^2)} \quad [C.2.13]$$

C.3 Compressible Flow Continuity Equation

It is often necessary to solve for a downstream Mach number (M_2), given an upstream Mach number (M_1), an area ratio $\left(\frac{A_2}{A_1}\right)$, and a total pressure recovery $\left(\frac{P_{o2}}{P_{o1}}\right)$.

This is traditionally accomplished with the * reference concept where

$$\frac{A}{A^*} = \frac{1}{M} \left[\frac{1 + \left(\frac{\gamma - 1}{2}\right) M^2}{\frac{\gamma + 1}{2}} \right]^{\frac{\gamma + 1}{2(\gamma - 1)}} \quad [C.3.1]$$

and since $P_o A^* = \text{constant}$,

$$\left(\frac{A}{A^*}\right)_2 = \frac{A_2}{A_1} \left(\frac{A}{A^*}\right)_1 \left(\frac{P_{o2}}{P_{o1}}\right) \quad [C.3.2]$$

Equation [C.3.1] is then solved to obtain a value of M_2 . This solution is performed numerically using Newton-Raphson iteration. Noting that either a subsonic or a supersonic solution is obtained depending on the initial guess, this option must be specified. The initial guess for each option is based on a rough curve fit of the solution to Equation [C.3.1].

For a subsonic solution,

$$M_{2, guess} = 0.765 \left(\frac{A}{A^*}\right)_2^{-1.083} \quad [C.3.3]$$

and for a supersonic solution,

$$M_{2,guess} = 1.225 \ln \left[2.3 \left(\frac{A}{A^*} \right)_2 \right] \quad [C.3.4]$$

C.4 Conical Flow Functions

Data consisting of cone surface Mach number, the Mach number immediately behind a conical shock, the conical shock angle, and the static pressure ratio across the shock on the cone surface, are tabulated for a range of freestream Mach numbers and cone semi-angles. The range of cone semi-angles is from 2.5° to 30° . The Mach number range is from the minimum for an attached shock, to a Mach number of 5.0. The data comes from Sims (1964).

The interpolation routine which does linear interpolation on the conical flow data is also used by the other subroutines in the program. It assumes that the arrays are pre-sorted on the abscissa values, and simply does a sequential search. If the abscissa value being searched for is out of range, the ordinate corresponding to the appropriate extreme abscissa is returned.

C.5 Prandtl-Meyer Expansion

The Prandtl-Meyer function is given by

$$v = \left[\frac{\gamma + 1}{\gamma - 1} \right]^{\frac{1}{2}} \tan^{-1} \left[\frac{\gamma - 1}{\gamma + 1} (M^2 - 1) \right]^{\frac{1}{2}} - \tan^{-1} (M^2 - 1)^{\frac{1}{2}} \quad [C.5.1]$$

This equation is used to obtain the reference angle of the entering flow, v . The value of v is then decremented by the flow turning angle, and Equation [C.5.1] is solved by the Newton-Raphson method for the resulting Mach number.

The pressure ratio for this isentropic expansion is given by

$$\frac{P_2}{P_1} = \left[\frac{1 + \frac{\gamma - 1}{2} M_1^2}{1 + \frac{\gamma - 1}{2} M_2^2} \right]^{\frac{\gamma}{\gamma - 1}} \quad [C.5.2]$$

C.6 Normal Shock Relations

Given an upstream Mach number M_1 , the Mach number downstream of a normal shock is

$$M_2 = \frac{2 + (\gamma - 1)M_1^2}{2\gamma M_1^2 - (\gamma - 1)} \quad [C.6.1]$$

The temperature, static pressure and total pressure ratios are then

$$\frac{T_2}{T_1} = \frac{[2 + (\gamma - 1)M_1^2][2\gamma M_1^2 - (\gamma - 1)]}{(\gamma + 1)^2 M_1^2} \quad [C.6.2]$$

$$\frac{P_2}{P_1} = \frac{[2\gamma M_1^2 - (\gamma - 1)]}{(\gamma + 1)} \quad [C.6.3]$$

$$\frac{P_{o2}}{P_{o1}} = \left[\frac{(\gamma + 1)M_1^2}{2 + (\gamma - 1)M_1^2} \right]^{\frac{\gamma}{\gamma - 1}} \left[\frac{\gamma + 1}{2\gamma M_1^2 - (\gamma - 1)} \right]^{\frac{1}{\gamma - 1}} \quad [C.6.4]$$

C.7 Oblique Shock Relations

The maximum shock angle for a given freestream Mach number, M_∞ , is

$$\sigma_{\max} = \sin^{-1} \left\{ \frac{(\gamma + 1)}{4\gamma} \left[1 - \frac{4}{(\gamma + 1)M_\infty^2} \right] + \sqrt{1 + \frac{8(\gamma - 1)}{(\gamma + 1)M_\infty^2} + \frac{16}{(\gamma + 1)M_\infty^4}} \right\}^{\frac{1}{2}} \quad [C.7.1]$$

The corresponding maximum wedge angle is

$$\delta_{\max} = \tan^{-1} \left\{ \frac{(M_\infty^2 \sin^2 \sigma_{\max} - 1) \cot \sigma_{\max}}{M_\infty^2 [\gamma + \cos(2\sigma_{\max}) + 2]} \right\} \quad [C.7.2]$$

To find the shock angle σ for any wedge angle δ , Equation [C.7.2] is solved using Newton-Raphson iteration. To ensure a weak solution, the following initial guess is made:

$$\sigma_{guess} = 0.8 \tan^{-1} \left[\frac{1}{\sqrt{M_{\infty}^2 - 1}} \right] + \frac{180^\circ}{10} \quad [C.7.3]$$

The component of the upstream Mach number (M_1) normal to the shock is then $M_{1n} = M_1 \sin \sigma$.

The properties downstream of the shock, including the Mach number, M_{2n} , are calculated with the normal shock relations described above.

The Mach number after the oblique shock is obtained from

$$M_2 = \frac{M_{2n}}{\sin(\sigma - \delta)} \quad [C.7.4]$$

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