

Minor Thesis in Genetics.

Submitted by

C. W. Holdaway, B. S.

In Application for

Master of Science Degree.

Subject: - Statistical Weighting for Age of Advanced Registry  
Cows.

Note: - A Biometrical Study of Advanced Registry data with  
a view of determining the correct age weighting  
for comparison of different animals for milk and  
fat production.

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## Statistical Weighting for Age of Advanced Registry Cows.

Any study of milk production that is made from a statistical standpoint must necessarily be complicated, for the reason that advancing age in a cow up to the time she is mature enables her to produce more milk and butter fat. A further difficulty lies in the fact that after maturity the effect of age on production has not been determined with any degree of certainty. Whether or not the increase in capacity is directly in proportion to the advance in age; at what age is the maximum of production reached; what relation is there between age and per cent of fat in milk, and at what age is a cow past the power of full productiveness, are all questions that need investigation in a broad way.

Necessarily the various breed associations must have made some comprehensive investigations to enable them to fix standards for milk and fat production, and since the only extensive authenticated records that we have are records of these associations, this study was made for the purpose of determining if their records were consistent with their standards, and if these standards could be used as a basis for weighting cows of different ages.

### Method of Collecting Data:

Seven day records only were used, these being secured from the Holstein-Friesian Blue Book, Vol. 24. For the purpose of future investigation all the animals in two direct lines of descent were tabulated, one from a female, the other from a male, both animals being noted ones in the breed. The names, herd book numbers, ages at time of record, pounds of milk, pounds of fat, and per cent of fat were all tabu-



lated. Each animal was given an arbitrary number which denoted its position in the generation, and the position of all its direct ancestors in their respective generations back to the primary ancestor of the population. All advanced registry males were tabulated also and numbered.

#### Records obtained:

From the female, Aaggie Grace, No. 2618, H.H.B., only 456 advanced registry records were obtained in 10 generations. In order to secure these records about twice as many animals were tabulated, the others consisting of the A.R.O. sires and their daughters that had not themselves made A.R.O. records but had two or more A.R.O. daughters.

The male, Paul De Kol, No. 14634, H.F.H.B., in 9 generations produced 9639 female progeny with A.R.O. records. About twice this number of animals were tabulated to secure these records.

#### Tabulation of Data:

Necessarily, before this large accumulation of data could be studied systematically, it was necessary to tabulate it in concise form, and for this purpose correlation tables were made for each population, each table involving a pair of variables. Thus age was compared to pounds of milk, age to pounds of fat, and age to percentage of fat; three tables to each population. From these tables the means of the characters in classes, class average deviations, population means, average and standard deviations and correlation coefficients were worked out. Then from this data, curves were drawn to illustrate its trend graphically.

#### Results;

For the purpose of studying the frequencies in these ta-



Table I.

Age. Years.

	1½	2	2½	3	3½	4	4½	5	5½	6	6½	7	7½	8	8½	9	9½	10	10½	11	11½	12	Fw
175	6	3	1																				10
200	1	82	8																				40
225	9	112	36	13	1																		162
250	14	209	90	34	5																		338
275	3	275	161	108	17	6	1	3	2														573
300	4	314	226	139	56	24	7	7	4	1			1	2	1	2							784
325	3	283	223	162	99	62	19	15	5	6	5	3	2	3		2	2	3					894
350	1	198	179	188	98	81	43	34	21	10	5	9	9	2	1	2		5	1				887
375	1	138	135	179	104	116	59	73	29	29	15	13	14	8	5	4	3	3	2	1			941
400	64	771	81	94	160	80	97	42	50	21	38	20	28	7	6	2	10		1	1	1		980
425	29	33	108	113	155	105	109	55	58	33	47	12	20	16	12	8	9	2	1				926
450	16	23	96	79	126	74	111	65	65	31	47	26	21	14	15	6	7	1	6	1			832
475	10	11	63	54	97	67	86	51	69	26	32	17	23	13	14	2	4	1	1	1	1		643
500	3	33	32	82	55	82	39	61	25	33	17	15	12	4	7	7	3	1	1				512
525	1	5	17	25	44	43	71	40	45	23	24	8	8	15	9	4	5	1	3	1			392
550			11	13	26	20	46	24	30	11	18	13	11	7	8	1							241
575		2	5	3	17	22	28	11	29	14	24	7	9	3	8	7	3	2					195
600			1	4	1	7	13	20	13	14	6	10	6	8	4	2	1	1					111
625			3	3	6	7	9	8	8	5	6			4	2		1						63
650			1	1	5	3	4	1	10	1	4	1	3										34
675					1	1	4		4	1	1		3	1			1						17
700					2				5	1	2	2	4										16
725									3		1												4
750						1			1	1		1											5
775																							1
800																							
825																							
850																							
875																							

Pounds Milk.



Table. No II.

	Age. Years.																								Fr.
	1½	2	2½	3	3½	4	4½	5	5½	6	6½	7	7½	8	8½	9	9½	10	10½	11	11½	12			
7	4	4	7																				51		
8	11	20	4	9	5																		316		
9	11	30	2	18	6	10	4	7															610		
10	5	3	2	1	5	2	6	6	1	7													770		
11	3	2	9	3	2	1	7	3	9	7	1	15											869		
12	2	2	1	0	2	0	3	1	7	2	3	5	4										1036		
13	1	1	2	1	9	7	1	0	3	2	7	6	8	7	3								994		
14	9	4	7	9	1	7	1	1	6	1	3	2	8	0	1	1	9	1	5	2	1	1	1020		
15	4	3	4	4	1	3	8	7	0	1	1	3	8	9	1	0	3	5	2	4	0	4	872		
16	3	3	2	6	9	2	7	8	1	2	6	7	4	1	3	1	5	4	6	7	1	1	846		
17	1	3	2	2	5	0	9	0	6	8	9	2	4	6	2	7	4	3	1	1	2	8	641		
18	8	8	3	1	4	5	7	0	6	2	5	8	4	8	3	3	2	1	3	8	1	6	493		
19	2	1	2	2	3	4	9	3	3	4	4	3	3	4	9	1	8	2	5	1	2	1	356		
20	3	1	1	6	1	5	4	0	2	9	4	3	2	7	3	9	1	1	9	1	0	7	281		
21	1	7	7	1	7	1	4	2	7	1	2	1	9	6	1	0	1	3	1	8	1	3	147		
22	2	4	2	0	8	2	0	1	1	5	4	1	3	1	5	4	4	5	2	3			117		
23	1	2	2	8	1	0	1	7	5	1	2	4	5	5	2								76		
24	4	4	8	7	9	2	7	7	7	7	7	7	7	7	7	1	1	1	1	1	1		54		
25	2	1	4	4	5	2	6		4	1	4	4	1	4	4	1	1	1	1				35		
26					3	2	1	2		1	2	1	2	1	2								14		
27	1	1	1	3	1	2		3	3														9		
28																							144		
29																							1		
30	3	6	1	6	5	1	3	1	1	3	1	1	2	1	1	2	1	1	2	1	1	1	8		



bles, Table No. 1 and No. 2 are given showing the distribution of the population originating in the male, Paul De Kol. From this it will be seen that the age classes are well represented at two years old and up to ten years. The one and one-half year class is small, as are also the classes over ten years of age. For this reason unbalanced and irregular results would be expected for these classes, and by referring to the curves it will be seen that the premise was justified. Special note should be made of the frequencies in the two and three year classes which are the two highest. Also of the fact that in almost all cases the half year classes are not more than 60 per cent as well represented as the full year classes.

Table No. 3 gives the average deviations, mean pounds of milk, standard deviations, correlation coefficients and regression coefficients of the population with respect to age and pounds of milk and pounds of fat. Altho the mean age is 4 years, the three and one-half year class actually reached the mean pounds of milk of the population as can be seen from Table No. 4. Correlation probably amounting to causation is shown in the tables up to six years of age, and after that age is reached the correlation is practically zero.



Table No. 3

	Correlation of Pounds of Milk to Age	Correlation of Pounds of Fat to Age
Average Deviation	69.8	2.91
Standard Deviation	92.4 $\pm$ 0.449	3.65 $\pm$ 0.018
Mean Pounds	395.5 $\pm$ 0.635	14.00 $\pm$ 0.025
Mean Age	4.0 $\pm$ 0.013	4.0 $\pm$ 0.009
Correlation Coefficient	0.604 $\pm$ 0.004	0.57 $\pm$ 0.005
Regression Weight to Age	29.84 $\pm$ 0.0006	1.11 $\pm$ 0.00003
Regression Age to Weight	0.012	0.29
Coefficient of Variability C	23.4 $\pm$ 0.001	26.0 $\pm$ 0.001



Table No. 4.

Age. Years.	Milk Production.				Fat Production.			
	Means	Av.Dev.	+Dev.	Curve 4	Means	Av.Dev.	+Dev.	Curve 4.
1½	268	34	302	170	8.9	1.3	10.2	4.76
2	308	44	352	220	10.7	1.74	12.44	7.00
2½	326	43	369	237	11.3	1.76	13.06	7.62
3	372	56	428	296	13.0	2.14	15.14	9.70
3½	396	55	451	319	12.8	2.37	15.17	9.73
4	428	54	482	350	15.2	2.48	17.68	12.24
4½	446	57	503	371	15.5	2.3	17.80	12.36
5	458	52	517	385	16.4	2.36	18.76	13.30
5½	461	56	517	385	16.4	2.3	18.7	13.24
6	474	62	536	404	17.0	2.55	19.55	14.09
6½	474	60	534	402	16.8	2.43	19.23	13.77
7	476	60	536	404	16.7	2.3	19.00	13.54
7½	466	64	530	398	16.8	2.69	19.49	14.03
8	475	65	540	408	17.0	2.54	19.54	14.08
8½	476	58	534	402	16.6	2.46	19.06	13.60
9	476	51	527	395	17.1	2.43	19.53	14.07
9½	475	60	535	403	17.0	2.48	19.48	14.03
10	449	76	525	393	16.6	2.7	19.30	13.85
10½	477	68	545	412	16.7	3.1	19.80	14.33
11	461	35	496	364	16.0	2.0	18.00	12.53
11½	470	36	506	374	15.7	1.43	17.13	11.66
12								

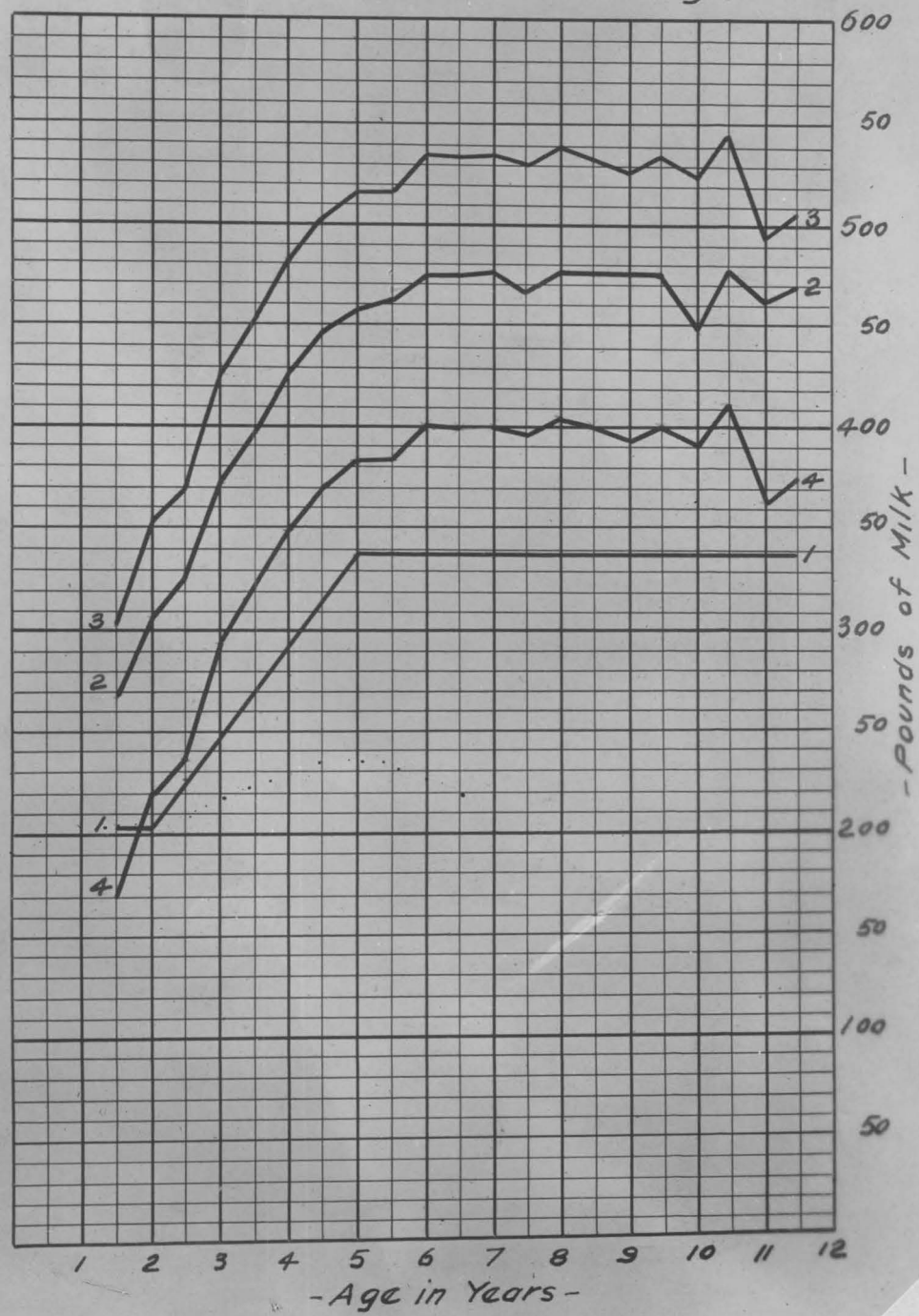


Table No. 4. This table gives the means, average deviations, and plus deviations of the different age classes for both milk and fat production. From these tables the curves for milk and fat production were plotted. They formed also the basis for calculating the curve which is used as a comparison with the Holstein-Friesian curve of fat and milk requirement. These tables also afford an interesting study from the standpoint of capacity of cows for milk production at different ages.

Considering first the curves for milk production, it will be noted that curve 1, which represents the pounds of milk required by the Holstein-Friesian Association, must be calculated from the pounds of fat required. This was done by taking the average per cent of the whole population and calculating the number of pounds of milk, having the average per cent, that would be necessary to make the required number of pounds of fat. The reason for using the average per cent of fat of the whole population as a basis for calculating the Holstein-Friesian Association requirement curve was that since the correlation coefficient between age and per cent of fat was so small in a table shown subsequently for another population, and since the popular concept is that per cent of fat is not influenced by age, we felt justified in using it. Attention is called to Table No. 5 which does not bear out this assumption entirely. For milk and fat requirement, however, there is a strong correlation to age, so the classes were considered separately, each class having its own mean and deviation. Curves 2, 3, and 4 were based on these class means and deviations. Curve No. 2 is the mean of the population. Curve No. 3 is the plus deviation from the mean. Curve No. 4



Curves of Milk Production - Fig. I





is a curve which was plotted to show what the requirements ought to be if the means, deviation and varying capacity of the different classes are taken into account. In plotting this curve it was necessary to consider the basis upon which the minimum requirements of this population ought to be placed.

The minus deviation point cannot show what ought to be required of the class as a minimum, for such point would weight individuals inversely in proportion to their capacity. A greater deviation from the mean of the class indicates here, greater capacity for production of that class, and as the capacity for production of the class increases, so should the requirements increase. Therefore, the curve of minimum requirement should be represented as following the curve of plus deviation in character and should be in a minus direction from the mean.

In order to conform to these conditions some basis must be established for calculating the minus points of the curve, or, in other words, the minimum requirements for each class. The average deviation of the whole population seems to be the logical basis upon which the minimum requirement should be based, for by its use the whole curve may be lowered an amount corresponding to the average deviation of the whole population below the mean of the population. The average deviation from the mean of the whole population is 69.8 pounds of milk. If all classes are to be given the benefit of the average deviation the calculation should start from the point at which the means are at the maximum, which is about the six year class. Hence the six year class is allowed as the minimum requirement, the 69.8 pounds below the mean of the class and the require-



ments of the other classes are worked out from this point to conform, as said before, to the maximum deviation curve.

An inspection of these curves brings out the following points

That the official requirements weight animals of an age from 18 to 21 months too heavily. The curve indicates that they are entitled to a reduction as great as for any other age. For the purpose of discouraging such early breeding, however, the requirements for this particular class should be prohibitive and they are.

That the production increases up to at least SIX YEARS of age instead of five, which the Holstein-Friesian Association requirements set as the maximum age production.

That for this reason the 5 to 6 year old animals and possibly the 7 to 8 year classes have an advantage over all other classes.

That a comparatively small number of animals made the requirement after 9 years of age, hence by selection, only the best animals were retained, thus drawing the curve down almost to a straight line. The tendency of the curve, however, is to recede, showing that the animals of these ages should not be weighted as heavily as younger animals. A study of a number of representatives of the whole breed would be necessary to determine this point.

One of the most striking points shown by these data and one which substantiates the opinion of practical breeders of Holsteins, also is brought out in the practical investigations of Eccles,<sup>1</sup> is the difference in production and capacity between 2 and 2½ year old and 3 year old cows. The difference in the means of the production between

<sup>1</sup>. Bul. No.135-Missouri Agricultural Exp. Station.



2 and  $2\frac{1}{2}$  years was 18 pounds only, while between  $2\frac{1}{2}$  and 3 years it was 46 pounds, or a total of 64 pounds between the 2 and 3 year classes. Between the 3 and 4 year classes the difference is almost as great, being 56 pounds, but the deviation of the latter class is not quite as great as the former. This seems to indicate that the 3 year animal is still at a disadvantage by reason of its immaturity in growth and body development. That the average deviation of  $2\frac{1}{2}$  year class was 43 pounds while the 3 year class deviated 56 pounds, is significant also and leads to the conclusion that at  $2\frac{1}{2}$  years of age the Holstein is still growing, and this, combined with the great strain of milk production, limits the capacity of the class.

It may be said by some that few three and four year old animals are tested for advanced registry in comparison to two year olds and aged animals, and in consequence of this, only the best of the class make the requirements. This is not borne out by the data, the number in the three year old class being second largest of all animals.

#### Curves of Fat Production:

A study of the curves based upon the actual fat production of this population brings out a number of points, many of them corroborating those brought out in the discussion of the milk production curves.

Owing to the variation of the weight classes in per cent fat, the curves of milk production and fat production agree very well when compared with the Holstein-Friesian Association requirement curve.

The requirement curve in fat production, (No. 4) crosses the Holstein-Friesian Association curve at a greater age than that worked



*Curves of Fat Production - Fig. II*

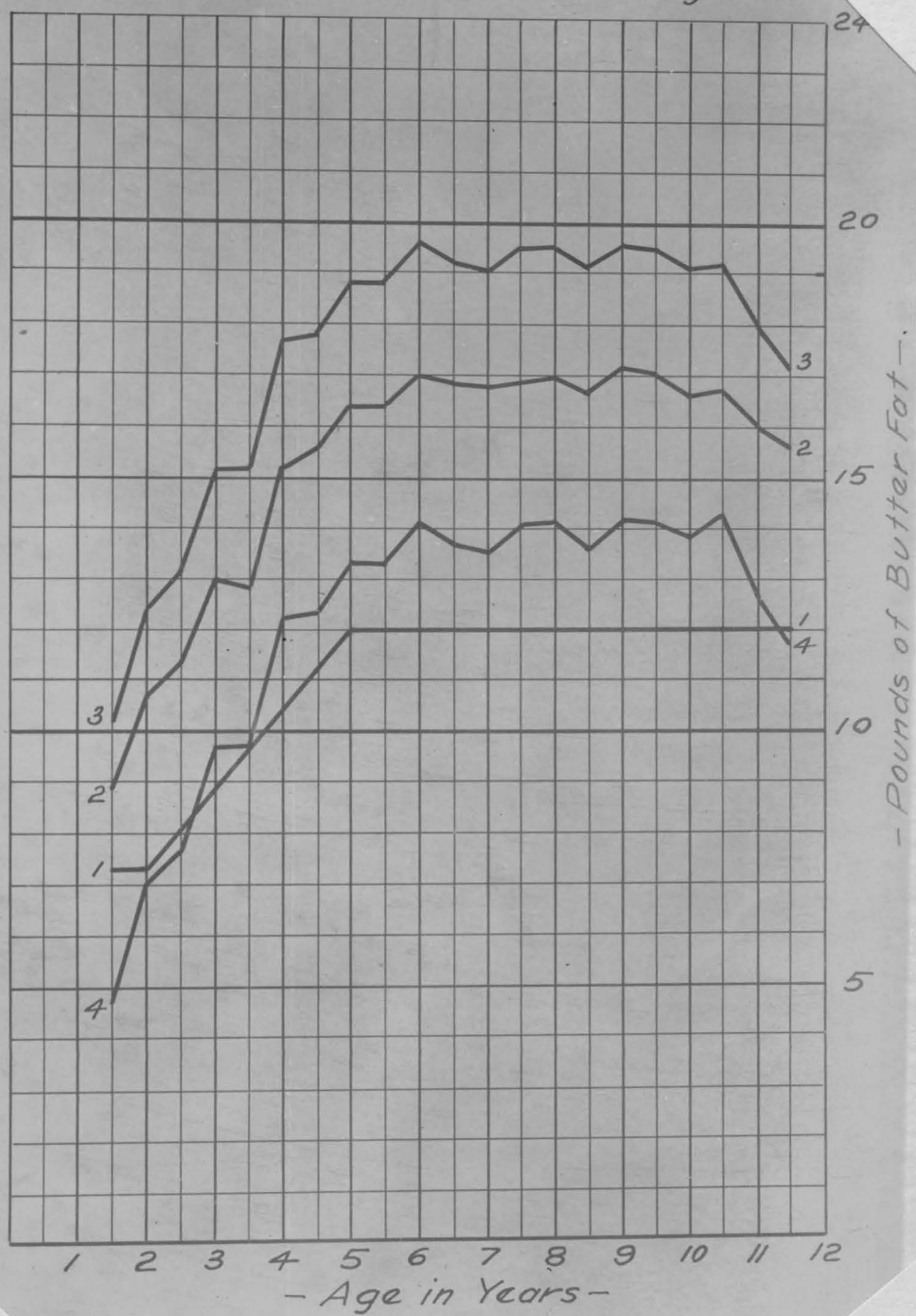




Table No. 5.

Average Per Cents Fat of the Classes.

Age. Years.	Per Cent	Age Years.	Per Cent.
$1\frac{1}{2}$	3.28	7	3.51
2	3.27	$7\frac{1}{2}$	3.61
$2\frac{1}{2}$	3.47	8	3.58
3	3.49	$8\frac{1}{2}$	3.49
$3\frac{1}{2}$	3.24	9	3.59
4	3.55	$9\frac{1}{2}$	3.58
$4\frac{1}{2}$	3.48	10	3.69
5	3.58	$10\frac{1}{2}$	3.50
$5\frac{1}{2}$	3.56	11	3.47
6	3.58	$11\frac{1}{2}$	3.34
$6\frac{1}{2}$	3.54		



out for milk production. This would indicate that the classes up to  $3\frac{1}{2}$  years produced milk containing a lower per cent fat than the mean of the whole population. This is correct, as can be found from the means of the classes. (See average per cents of class means, Table 5). A similar condition obtains with the age classes after ten years. It would appear from this that mature cows give milk slightly richer than immature cows, or than old cows past ten years of age.

A rather peculiar condition with reference to the fat production curve is shown in the mean results of the half year ages up to the  $6\frac{1}{2}$  year class. Each half year class advances but slightly, if at all, from its preceding year class, then there is a sudden drop to the next full year class. The milk production curves indicate the same condition, though to a lesser extent, and as previously noted, the frequencies in these half year classes are not more than 60 per cent of the full year classes. No good explanation is offered for this. It might be inferred that a cow freshening at  $2\frac{1}{2}$  years is not much better able to withstand the strain of milk production than a 2 year old, and that this condition continues. However, in many respects this theory does not appear sound.

Attention is called again to the points of curve 4 for fat production given in Table No. 4. This curve is plotted for the purpose of showing what the requirements ought to be according to the performance of cows that have made records. The animals involved in this curve represent 45 per cent of all the A.R.O. records that had been made up to the time of publication of Vol. 24., hence the numbers are ample.



First, the means of the classes of this population were plotted. Then their ability to deviate in a plus direction, or in other words, to produce more fat as individual classes was taken into account. The class that had the maximum production and deviation ability was allowed, as a basis for its minimum requirement, the full average deviation of the population in a minus direction from the mean, and finally the other classes that could not produce as much and had not the ability to deviate as much as this maximum class, were allowed the full minus deviation of the population plus the difference in deviation ability between their particular class and the maximum class which forms the apex of the curve.

If these fundamental allowances are fair, impartial and accurate, the curve is accurate, and the only question that remains is whether or not it should alter the requirements of the Holstein-Friesian Association. If curve 4 touches the Holstein-Friesian Association curve at any point and does not coincide with it throughout, then the latter should be changed. It does touch it at both beginning and end, showing that all classes after the  $2\frac{1}{2}$  years and up to  $11\frac{1}{2}$  years have an advantage over the others. This advantage is greatest for the classes between  $5\frac{1}{2}$  and 11 years of age.

The next consideration in connection with curve 4 is its application, and when dealing with this, two things should be kept in mind; first, the practical, and secondly, the more concise and mathematical application. The practical application finds its expression in the endeavor of the Holstein-Friesian Association to make a uniform advance per day in the fat requirement for the seven day test up to the age at which it was considered the maximum production was reached. Table 6 compares the in-



crease in the amount of fat required each year over that required in the previous year from two up to six years, with the increase in amount of fat that the year classes are able to produce as calculated from Curve 4.

Table No. 6.

Age years	H.F.A. Requirements.		Curve 4 Requirements	
	Fat increase. yearly.	Fat increase daily.	Fat increase yearly.	Fat increase daily.
2 to 3	1.6 lbs.	0.00438	2.70 lbs.	0.00740
3 to 4	1.6 "	0.00438	2.54 "	0.00696
4 to 5	1.6 "	0.00438	1.06 "	0.00290
5 to 6	0.0 "	0.0	0.79 "	0.00216

The table shows plainly that the daily increased requirement from 2 to 3 years should be 0.00696 instead of 0.00438, or  $1\frac{1}{2}$  times as much. From 4 to 5, 0.0029 instead of 0.00438, or nearly  $\frac{1}{2}$ , and from 5 to 6 years, 0.00216 instead of no increase.



## Population No. 2

The second population tabulated is that which began with Aaggie Grace, No. 2618, H. H. B., as the primary ancestress, and consisted of only 456 animals. Tables 7, 8, 9, and 10 are given showing all the data necessary for comparison with the previous population. Of course, it must be borne in mind that the comparison cannot be too exacting, for this population is altogether too few in numbers to secure smooth results, especially when comparing classes. In fact, the class means and deviations, Table 9, included only the classes up to 9 years because of the low frequencies after that age. If Tables 3 and 4 are compared with 9 and 10, a remarkable agreement is noticed throughout, especially in the essential points which have been discussed.

The correlation table for age to per cent of fat is not shown, but the coefficients of this table may be seen in Table 10. The correlation coefficient is so small that it may seem negligible, but Table 5 shows that even with a low correlation, important points might be brought out if the data are sufficient.

No endeavor will be made in this paper to enlarge on the exact mathematical application of these data. This will be taken up later in connection with a further study of the two populations.



Table VII

Frequency Distribution, Population 2.  
Age, Years.

[illegible]



### Table. VIII

[illegible]



Table No. 9.

## Class Means and Deviations of Population 2.

Age to pounds milk			Age to pounds fat.	
Age Years.	Mean Pounds milk	Average Deviation	Mean Pounds fat.	Average Deviation
2	286	47.3	10.35	1.65
2½	310	48.6	11.35	1.63
3	392	52.5	13.16	1.87
3½	403	53.5	14.22	1.97
4	413	60.0	14.30	2.20
4½	470	57.8	16.5	2.28
5	469	56.6	16.42	2.30
5½	484	41.4	16.27	2.15
6	504	66.4	17.23	2.26
6½	406	93.9	17.75	3.37
7	480	59.2	16.45	1.56
7½	471	47.0	17.22	2.14
8	468	60.7	16.27	2.48
8½	500	40.0	18.20	3.92
9	447	47.6	15.00	2.29

Table No. 10.

Population Coefficients : Population 2.

	Age to Pounds Milk	Age to Pounds Fat	Age to Per Cent Fat.
Means.	405.4 $\pm 2.8$	13.89 $\pm 0.113$	3.455 $\pm 0.014$
Standard Deviations	90.39 $\pm 2.02$	3.593 $\pm 0.08$	0.436 $\pm 0.009$
Correlation Coefficients N	0.592 $\pm 0.02$	0.581 $\pm 0.02$	0.08 $\pm 0.031$
Coefficient of Variability C	0.223 $\pm 0.005$	0.258 $\pm 0.006$	0.126 $\pm 0.003$
Regression Weights to Age	26.51 $\pm 0.042$	1.057 $\pm 0.002$	
Regression Age to Weight.	0.022	0.548	