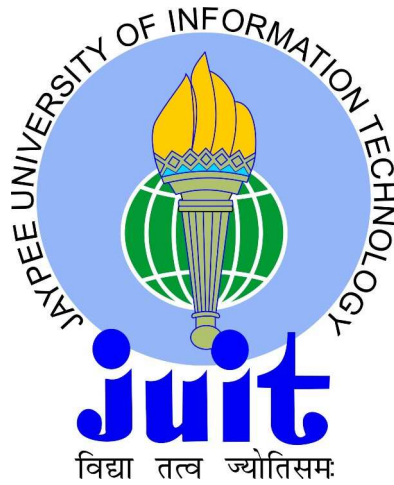# THESIS

Polynomial Based Design of Linear Phase Recursive and Non Recursive Filters

submitted by

Vinay Kumar

for the degree of
Doctor of Philosophy to the
Department of Electronics and Communication Engineering
of the Jaypee University of Information Technology,
Waknaghat, INDIA

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BY

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THESIS

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Abstract

In this dissertation, several algorithms to design linear phase Finite Impulse Response (FIR) and Infinite Impulse Response (IIR) filters have been discussed.

Contrary to various already existing standard algorithms,the proposed methods approximate magnitude and phase characteristics simultaneously. The basic mechanism used in this study is polynomial based design of digital filters. We have used several already existing polynomials; e.g., Chebyshev polynomials, Legendre polynomials, to develop linear phase digital filters and developed some two dimensional polynomials following orthogonal properties to design digital filters for image processing, their design methodology have also been discussed.

Filters of proposed type can be used for applications where exact linear phase is required. Another application of this type of filters is the design of filters with zero group delay. IIR filters are designed with absolute linear phase and zero group delay.

The algorithms proposed in the present thesis allow user to design filters with his set of constraints, which is required in practical filter design problems. Very narrow band 1D and 2D linear phase FIR filters can easily be designed by the proposed methodology. The IIR filters proposed provide the guarantee to result in a stable filter.

All the algorithms have been discussed stepwise to make sure that any one with basic programming capability can easily design them. We have not used any standard routine of any particular platform, therefore any freely available programming platform (like C, C++, Scilab, Octave, etc.) can be used to design these filters.

Acknowledgment

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1 Introduction

1.1 Digital Filters: An Introduction

The digital filter is a digital system that can be used to filter discrete-time signals. The realization of a digital filter is accomplished by burning a program on a reprogrammable device, like Field Programmable Gate Array (FPGA), or by using a software program. Therefore, if we want to change the characteristics of a digital filter, we only have to change the program which defines the circuit. Figure 1.1 shows a typical discrete time filtering system.

$$\text{Analog Signal } \frac{}{x_a(t)} \boxed{C/D} \; x[n] \; \boxed{H(e^{j\omega})} \; y[n] \; \boxed{D/C} \; \text{Analog Signal } \frac{}{y_a(t)}$$
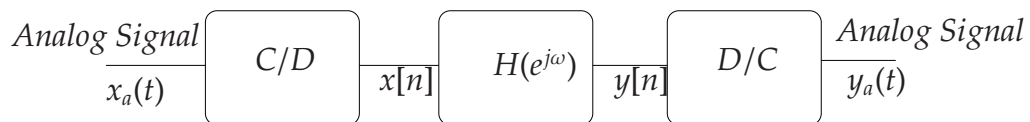
Figure 1.1: Typical system to process analog signals via a discrete time filter.

In Figure 1.1, $x_a(t)$ is the analog input signal, $y_a(t)$ is the analog output signal, $x[n]$ is the digitized $x_a(t)$, $y[n]$ is the digitized $y_a(t)$, $H(\exp(j\omega))$ is the transfer function of the filter, C/D is the continuous to digital signal converter and D/C is the digital to continuous signal converter [1].

Why are we interested in designing digital filters? As stated earlier, digital filters are used for processing digital signals. We interact with digital devices (digital TV, mobile, computers, etc.) and in turn with digital signals everyday, in fact all the time. These devices are used to interact with outer world through some form of channel; for example, space for TV and mobiles, and wire or space for computers (to connect with the internet). Channel introduces noise in the signal due to various well known reasons. These digital filters are used to remove the noise from the digital signal received at the receiver end. Filters are also used to enhance certain features of signal, like edges of an image, quality of audio (treble and bass are enhancement of high and low frequency audio signals,

respectively).

Other than everyday life applications digital filters are used for advanced scientific applications too, like for satellite and medical images. Satellite images are analyzed by applying different operations, filtering being one of the most important. Medical images (like CT scan, X-ray, etc.) and videos (like ultrasound video, etc.) are analyzed better by applying digital filters first, because they remove the noise or enhance certain features and produce better quality images and videos to be analyzed by the medical practitioner. In the present work we will show some satellite, medical and other type of images, and the output produced by applying the proposed filters.

Number of applications are increasing where digital signal processing and digital filtering are used; therefore, digital filters have to be applied for various requirements. Hence, there is a need to have designing procedures which can realize filters with required constraints. In the present thesis, we propose algorithms which can be used to design user defined type of FIR and IIR filters.

Digital filter design had been an active area of research for last several decades. One can design a digital filter having linear or non linear phase. With respect to the last statement consider the Fourier transform pair

$$X(\omega)e^{-j\alpha\omega} \Longleftrightarrow x[n - \alpha] \tag{1.1}$$

Equation (1.1) shows that applying a time delay α (in the discrete time domain) is equivalent to a multiplication of the Fourier transform by a factor of $\exp(-j\alpha\omega)$; which in turn implies the introduction of a linear phase. When we transmit a signal, a time delay is introduced. Linear phase filters delay all frequencies by the same amount, thereby preserving the waveshape maximally. Therefore, linear phase filters introduce no distortion and are preferred over non linear phase filters. All the proposed filters in the present thesis are linear phase. Primarily, there are two types of digital filters

1. Finite Impulse Response (FIR) or Non Recursive filters, and

2. Infinite Impulse Response (IIR) or Recursive filters.

FIR or non recursive filters are called by this name because to obtain the

current output sample value of the filter it uses only current and past input samples of the signal, and none of the previous output samples of the filter. Since, there is no feedback from the output there is no instability in these filters and hence they are preferred. IIR or recursive filters use past output samples of the filter also, which can cause instability due to feedback [2].

The order of a FIR filter is the number of previous inputs which have to be stored in order to generate a given output. While the order of IIR filter is the largest number of previous input or output values required to compute the current output.

The nature of the problem coupled with the desired frequency response is – generally – the primary factor to decide which type of filter, FIR or IIR, should be used. Because IIR filters use feedback they tend to be more versatile in accurately approximating the desired magnitude response, but they are not able to produce linear phase. On the other hand, it is easier to design a FIR filter having linear phase. Therefore, if our application requires a linear phase then we go for FIR filters. However, if phase is no issue then we can implement any of the two types of filters provided we make sure that the filter is stable [3].

1.1.1 Ideal Filter Characteristics

A signal is made up of one or more number of frequencies. If a filter passes certain frequencies and stops rest of the frequencies of the signal completely, it is called as an ideal filter. Frequencies which the filter allows to pass through it constitute the *pass band* of the filter and rest form the *stop band*. Figures 1.2(a), 1.3(a) and 1.4(a) show such filters.

1.1.2 Real Filter Characteristics

Real world but ideal filters are different from the ideal filters in the sense that in addition to the pass band and stop band they also have *transition band*. The transition band is the band of frequencies which necessarily has to be present because filters with sharp characteristics are unrealizable. Figures 1.2(b), 1.3(b) and 1.4(b) show these type of filters. In the present thesis these type of digital filters are designed.

(a) Ideal LP filter   (b) Realizable LP filter.
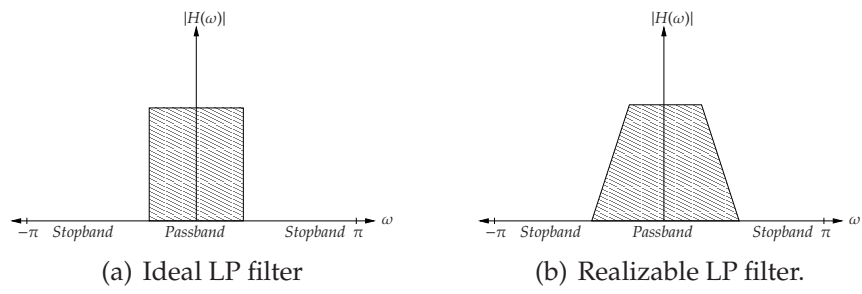
Figure 1.2: Low Pass (LP) filter.

(a) Ideal HP filter   (b) Realizable HP filter.
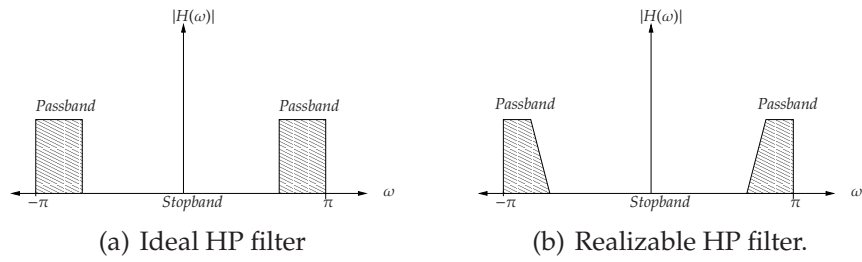
Figure 1.3: High Pass (HP) filter.

(a) Ideal BP filter   (b) Realizable BP filter.
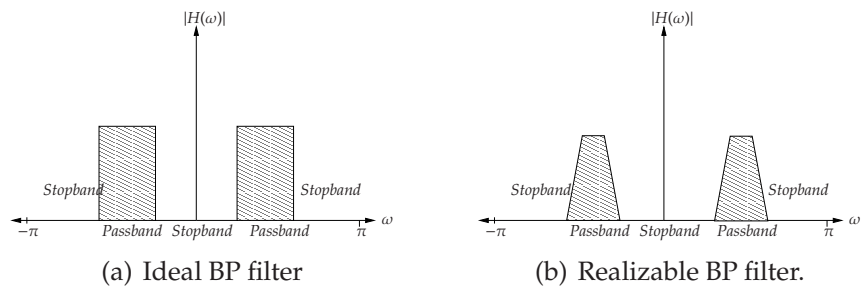
Figure 1.4: Band Pass (BP) filter.

1.2 Earlier Work in the Field of Digital Filter Design

A lot of work has been done in the field of digital filter design. Fettweis [4] gives an overview of past research in this field. We discuss some of the recent works in the present section.

In the recent past various techniques have been developed to design digital filters. Exchange algorithms [5,6], like Remez exchange algorithm, is relatively difficult to understand and apply. Although linear programming [7] produces linear phase FIR filters but the algorithm is very slow, apart from introducing a very large number of parameters to be optimized. In fact a literature survey shows that all the algorithms which have been proposed suffer from this defect.

To reduce the time requirements one can use semi-infinite programming approach [8], but this technique does not reduces the time requirement drastically. Optimization methods [9,10] work like the curve fitting technique. They are iterative methods, therefore involve a great deal of computation. To apply optimization techniques one has to give a complete description of the transfer function, which increases the complexity and computation time depending on either the number of poles and zeros, or the coefficients of the transfer function. Optimization technique further suffers from the defect that a global optimum point may not be reached.

In the Chebyshev approximation [11,12] technique, pass band loss oscillates between A_{min} and A_{max}. If one wants to put limit on stop band also, then he has to use elliptic approximation techniques [2]. A very well known technique was proposed by Parks and McCllean [5,13] and is used widely to get the optimized Chebyshev digital filters.

Weighted Chebyshev approximation [14] uses a point matching technique and generates an error function. The error function is formulated for the desired filter in terms of a linear combination of cosine functions and is then minimized using Remez exchange algorithm [15,16]. Therefore, this methods suffers from the same defects as we have outlined earlier.

The problems with these methods motivated us to propose a new method to design digital filters.

1.3   Present Work

The present thesis is focused on conceptualizing a technique which will give us a new method of designing digital filters, both recursive and non recursive. The method does not use any optimization technique, which requires recursion, therefore it does not suffer from the defects of computational time requirements. The approach proposed in this thesis is entirely new. Some of the algorithms discussed are based on the design of antenna patterns [17]. Surprisingly this cross disciplinary approach gives very good results.

We propose a polynomial based approach to design digital filters. Although "polynomial based" approaches have been discussed by other authors earlier, yet they are completely different from the present one. One prevailing method for optimized design for FIR filters is Parks-McClellan [18,19] algorithm. The Parks-McClellan algorithm is based on considering filter design problem as a problem in polynomial approximation. In this approach an approximation error function is calculated between desired frequency response and L^{th} order polynomial (in cos ω) approximation of frequency response. Various techniques have been proposed to calculate the optimal solution for this error function [20]. Another method was proposed by Hofstetter, Oppenheim, and Siegel [21] for designing maximal ripple filters. This algorithm is an iterative procedure for producing a polynomial $H^{\star}(e^{j\omega})$ that has extrema of desired values. But the number of iterations depend on the initial guess. Similarly, there are other methods [22,23] too in the litrature, but they also have no relationship with the present work. The method proposed in the present thesis neither requires any optimization technique to get the result nor any guess which has to be minimized iteratively.

The filters proposed in the present thesis are easy to implement and understand, produce excellent magnitude response and absolute linear phase without any approximation. To date there is no optimal algorithm to design absolute zero group delay IIR filters, the present dissertation proposes a technique which results in an absolute zero group delay IIR filter.

The major contribution of the present thesis is that we can design a digital filter, FIR or IIR, having user defined requirements of pass band

and stop band with linear phase. In other words, we can design a low pass, high pass, bandpass, band reject, multiband, etc. type of filters with linear phase. Because of this feature the present design procedure can be applied for a wide range of problems. The technique is extended to design filters for processing images; that is, 2 dimensional digital recursive and non recursive filters.

The outline of the procedure to design a digital filter with desired filter characteristics is as follows:

1. *First transform the filter characteristics to a function, which we call as object function, by using a special transformation.*

2. *This object function is thereafter realized by using a previously defined set of polynomials.*

3. *The realized object function is then converted back to filter characteristics using inverse of the transform used in step 1.*

Therefore, to design a filter we need to understand the transformation, which will be used to frame the object function and later its inverse to realize the filter characteristics itself. In the succeeding section we discuss the transformation.

1.3.1 Transformation

In general, transformations (like bilinear transformation) are used to convert analog filters to digital filters [24] or are used to convert one type of filter to other type of filters [25–27]; for example, low pass filter to band pass filter, low pass to high pass filter. Whereas, the transformation we are going to use in the present work has a completely different function.

The present transformation and its inverse is used to design the digital filters in the present discussion. Figure 1.5 depicts general steps required to design the desired filter characteristics. It must be clear from the Figure 1.5 that first the inverse of the transformation (Equation (1.2)) is used to find the object function, and then after approximation of the object function transformation is used to get the desired filter characteristics. This transformation is used in antenna theory [17] and we apply it to design signal processing filters.

The transformation used in the procedure is

$$x = x_0 \cos(\omega/2) \qquad\qquad (1.2)$$

where, x_0 is the maximum value of x, it will be clear how x_0 is chosen. Figure 1.6 shows the transformation under discussion. The inference which we can make from Equation (1.2) about the relationship between ω and x is as follows

- when $\omega \in [0, \pi]$ then $x \in [x_0, 0]$, and

- when $\omega \in [-\pi, 0]$ then $x \in [0, x_0]$.

From the properties of Fourier transform we know that the signal spectrum occupies a range of 2π. We assume that $\omega \in [-\pi, \pi]$ in the present work, wherever it is not explicitly mentioned.

Suppose we need to design a filter with characteristics shown in Figure 1.7. First, we apply the inverse of the transformation defined in Equation (1.2) on the filter characteristics – $H(\omega)$ – and it results in an object function – $f(x)$ – as shown in Figure 1.8. Object function thus calculated and the desired filter definition are inverse of each other; or in other words, the maximum values of filter characteristics – pass band – will be mapped to minimum values in object function and vice versa. Similarly, transition region with positive slope becomes a ramp having negative slope and vice versa in object function. This becomes clear if we look at Figures 1.7 and 1.8, which show desired filter characteristics and corresponding object function, respectively. The transformation defined, Equation (1.2), is non linear. Therefore, linearly increasing values of ω are mapped onto non linearly increasing values of x.

Digital filters are defined in terms of delay elements, or z, as they have to be realized using digital storage devices. The transformation of Equation (1.2) gives us filter characteristics directly in terms of z, as shown below
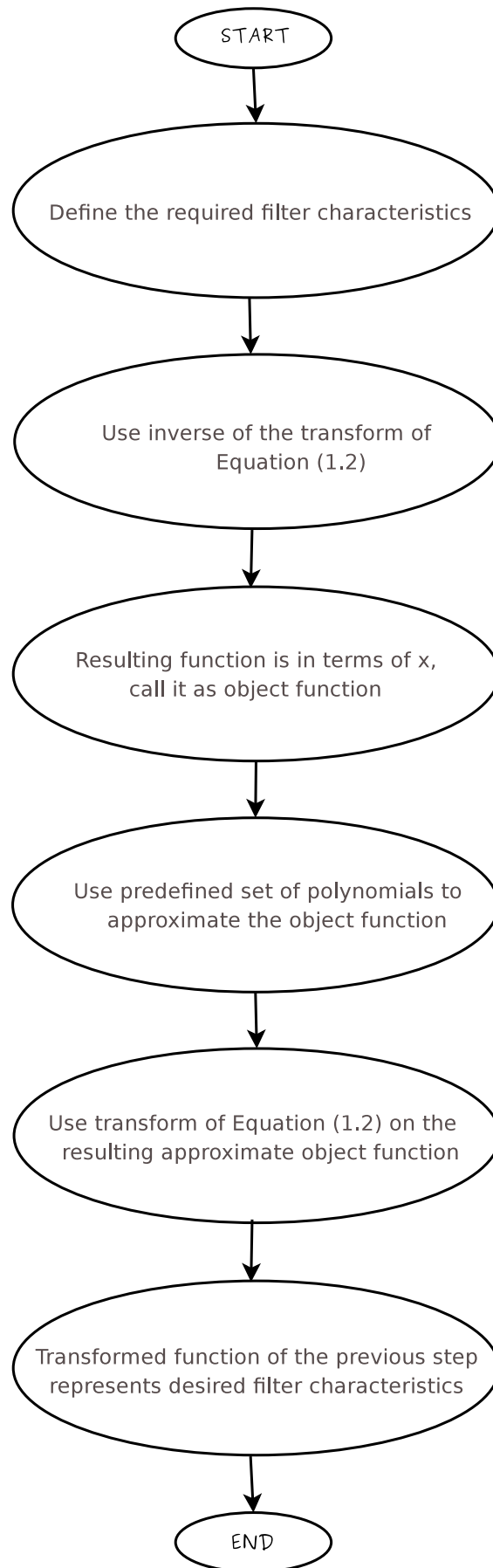
Figure 1.5: Flow chart representing steps required to design digital filters following the proposed design technique.
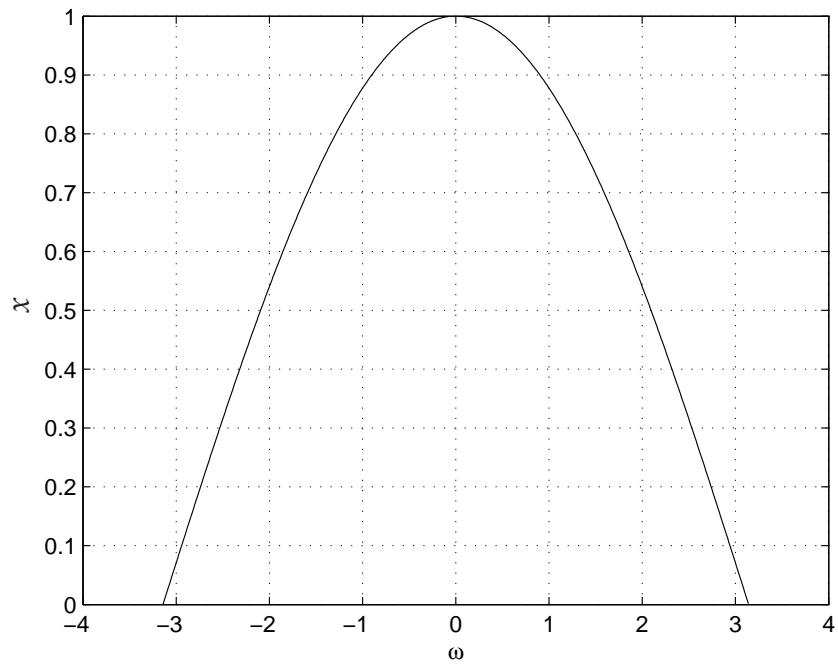
Figure 1.6: Transformation equation, $x = x_0 \cos(\omega/2)$, with value of $x_0 = 1$.
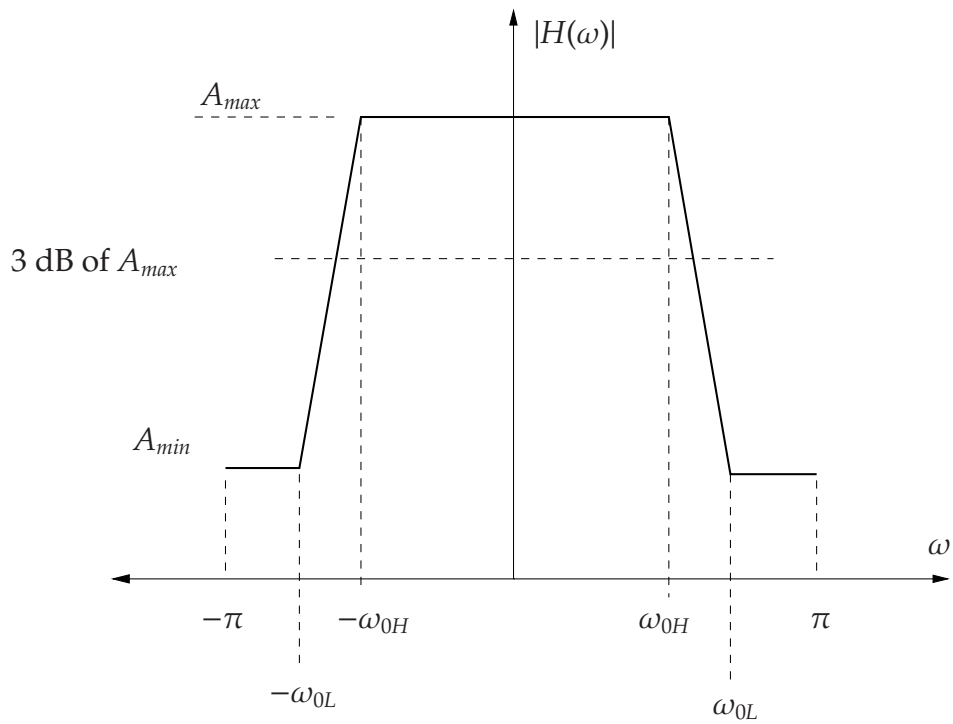
Figure 1.7: Desired filter characteristics.
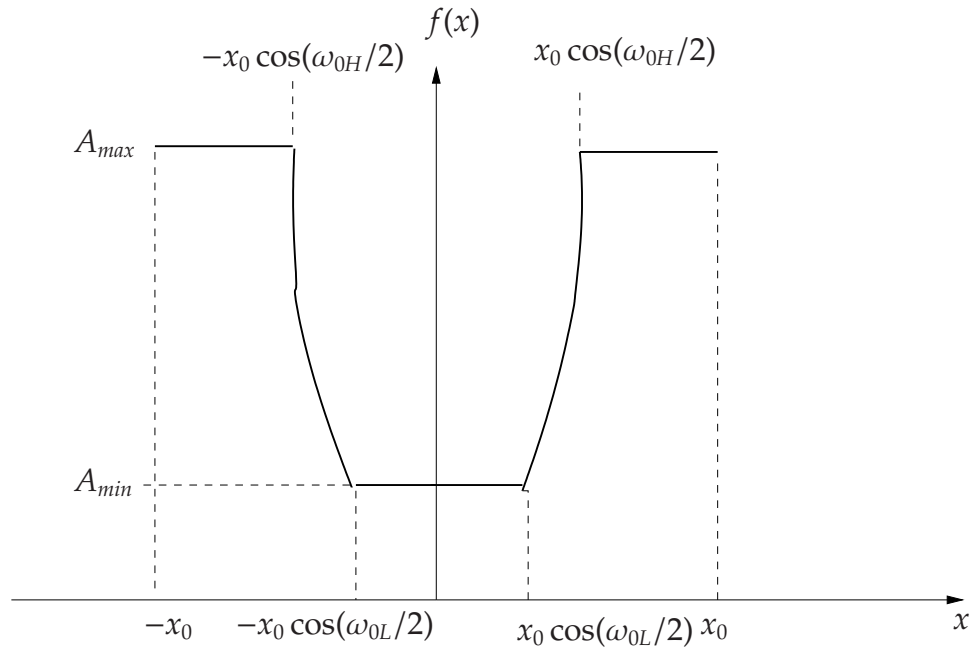
Figure 1.8: Object function for filter characteristics of Figure 1.7, showing the slight non linearity in the transition region.

$$
\begin{aligned}
x &= x_0 \cos(\omega/2) \\
\frac{x}{x_0} &= \cos(\omega/2) \\
&= \frac{e^{j\omega/2} + e^{-j\omega/2}}{2} \\
&= \frac{z^{1/2} + z^{-1/2}}{2} \quad \text{(replacing } e^{j\omega} \text{ with } z) \tag{1.3}
\end{aligned}
$$

Therefore, the object function – defined in terms of x – can directly be converted to delay elements, in terms of z. But in the present form they can not be realized (creating a delay element producing 1/2 delay is not possible). We know that filter characteristics in frequency domain always have even symmetry, therefore, while approximating the object function we use only even power terms of variable which constitutes the polynomial, like $x^0, x^2, x^4, \ldots$ (this point will be clear in the successive chapters where we design actual filters). Since we shall be using only even power terms, Equation (1.3) will always appear as

$$\left[\frac{z^{1/2} + z^{-1/2}}{2}\right]^{2n}$$

or,

$$\left[\frac{z}{4} + \frac{z^{-1}}{4} + \frac{1}{2}\right]^{n}$$

Hence, we always have our filter response in terms of z and z^{-1} and can be realized.

This transformation will be used in the successive chapters as the fundamental transformation, if there is any variation in the transform it will be duly explained in the respective chapter.

1.4   Outline

The present thesis is organized in 9 chapters, including the present one. Each of the following chapters, except Chapter 9, propose a new technique for designing digital filters. The organization of the thesis is as follows

Chapter 2 discusses a method where the object function is designed using simple or elementary polynomials. This chapter, primarily, shows the relationship between object function and filter characteristics with the help of simple polynomials.

Chapter 3 presents a design technique which uses Chebyshev polynomials to construct the object function. It also discusses the fundamental limitations of this technique.

Chapter 4 is a filter design technique where 2 dimensional (2D) filter designed by using 2D Chebyshev polynomials. .

Chapter 5 gives a design procedure by which user specific filter can be designed by the help of orthogonal polynomials.

Chapter 6 is for designing 2D filters with the help of orthogonal polynomials.

Chapter 7 gives a way to design IIR filters with zero phase, using orthogonal polynomials.

Chapter 8 address the issue of designing 2DIIR filters with zero phase.

Chapter 9 is the concluding chapter of thesis it also addresses future scope of the work.

Let us move to have a look on the design technique to generate *new type* of digital filters.

2 Design of 1 Dimensional Linear Phase FIR Filter with Elementary Polynomials

2.1 Procedure

In this chapter we present an approach to realize FIR filters using elementary polynomials. A stepwise description to design the FIR filter is discussed below

Step 1. Choose a polynomial, $f(x)$, with following properties

1. It should have zeros near the origin; that is, $0 \leq |x| \leq 1$, and

2. It should increase sharply for real values of x, which are far away from $x = 1$; that is, $x > 1$.

Such a polynomial is shown in Figure 2.1. From figure we can comprehend that as x varies from 0 to some point x_0, function $f(x)$ traces out a pattern of several side bands and one major band. The side bands are $1/b$ times down from the major band. This ratio can be selected at will by choosing the value of x_0.

Step 2. We use the transformation discussed in Chapter 1 for mapping the polynomial variable x to frequency variable ω of filter characteristics. The transformation is repeated below for convenience

$$x = x_0 \cos(\omega/2) \tag{2.1}$$

where, x_0 represents the maximum value of x. At the value when x is x_0 the polynomial has the value b, Figure 2.1.

Step 3. The zeros of the transfer function of FIR filter are calculated next. Let these zeros be represented by $z_i = \exp(j\omega_i)$, where $i = 1, 2, \ldots n$. The ω_is are calculated using the inverse of the transformation of Equation (2.1); that is,

$$\omega_i = 2 \cos^{-1}\left(\frac{x_i}{x_0}\right) \tag{2.2}$$

Note that x_is are the zeros of the original polynomial $f(x)$. The FIR filter transfer function is given by

$$H(z) = \prod_{i=1}^{n}(z - z_i) \tag{2.3}$$

or,

$$H(\omega) = \prod_{i=1}^{n}(e^{j\omega} - e^{j\omega_i}) \tag{2.4}$$
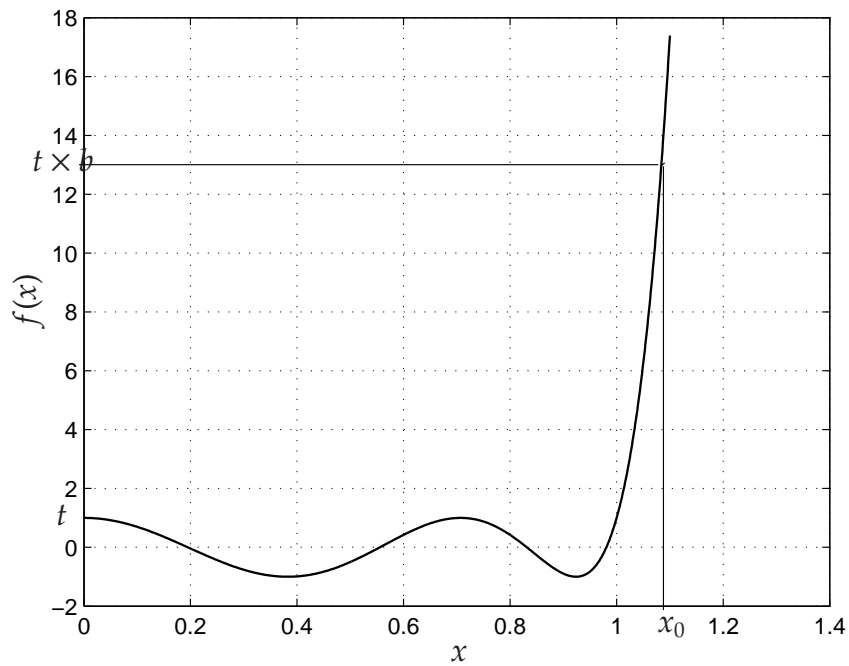
Figure 2.1: Example Polynomial.

It will be clear in the next section, where we consider some specific cases of filter design, that the values of x_0 and b are related to the bandwidth of the filter and the ripples in the stop-band, respectively.

2.2 Application

In this section, we consider different types of object functions which, in turn, will give us low pass filters having different magnitude and phase characteristics. Standard frequency transformation routines [2] can be applied on the resulting low pass filter to get the other type of filters; that is, high pass, band pass, etc. The examples below give a perspective of design from object function's side, but once one understood this he can design the filter other way round as well.

2.2.1 Design 1

Let us consider a polynomial where the values of x_i's are all equal to 0; that is, all the zeros of the object function lie on the origin. In this case the polynomial is

$$f(x) = x^n \tag{2.5}$$

This polynomial satisfies all the conditions specified in *Step 1*. Figure 2.2 shows this polynomial for $n = 6$.

The value of the function, where we want our stop band to start, can be taken at $x = 1$. This value is arbitrary, but $x = 1$ gives good results. Therefore, at the start of the stop band

$$x^n = 1 \tag{2.6}$$

Looking at the transformation $x = x_0 \cos(\omega/2)$ we observe that $x = 0$ transforms to $\omega = \pi$, $x = 1$ transforms to $\omega = 2 \cos^{-1}(1/x_0)$ which is the frequency where the stop-band starts.

2.2.1.1 Calculation of stop band frequency, ω_s, and pass band frequency, ω_p

Using above mentioned ideas, we desire that at $x = x_0$ the value of the function $f(x_0)$ should be 'b' times its value than it has at the stop band. Thus,

$$x_0^n = b \tag{2.7}$$
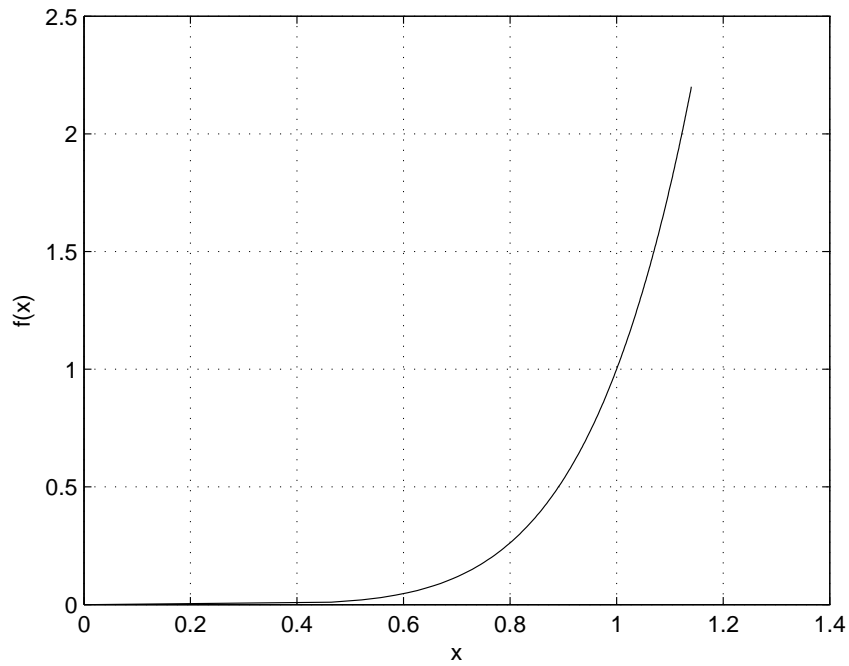
Figure 2.2: Polynomial x^6.

$$x_0 = b^{(1/n)} \tag{2.8}$$

We use the transform, Equation (2.1), to calculate stop band frequency, ω_s. The stop band starts when the value of x is 1, as discussed above, and $x_0 = b^{1/n}$. Therefore, Equation (2.1) converts to

$$1 = b^{(1/n)} \cos(\omega_s/2) \tag{2.9}$$

and,

$$\omega_s = 2\cos^{-1}(1/b^{(1/n)}) \tag{2.10}$$

The pass band frequency, ω_p, of filter characteristic occur when function is $b/\sqrt{2}$ times, or $3\,dB$ down, of its peak value than that at the stop band. Thus,

$$x_p^n = b/\sqrt{2} \tag{2.11}$$

and,

$$x_p = \frac{b^{1/n}}{2^{1/2n}} \tag{2.12}$$

where, x_p is the point on the object function which corresponds to the pass band frequency, ω_p.

Using the transform, Equation (2.1), with Equation (2.12) we can calculate the value of pass band, ω_p, as follows

$$\frac{b^{1/n}}{2^{1/2n}} = b^{1/n} \cos(\omega_p/2) \tag{2.13}$$

therefore,

$$\omega_p = 2 \cos^{-1}(1/2^{1/2n}) \tag{2.14}$$

All the zeros of $f(x)$ lie at $x = 0$, so there is no need of calculation involved in *Step 3* in this case.

2.2.1.2 Calculation of b

As discussed above the maximum value of the object function is b times its value than that at the stop band. Thus, if we want our stop band to be, let us say, $p\,dB$ down when compared with the maximum value of pass band, the value of b is absolute value of $p\ dB$; that is,

$$b = 10^{p/20} \tag{2.15}$$

Suppose in the polynomial discussed above, the value of n is 6; that is, our filter is of the order of 6. We want stop band to be $40\,dB$ down; that is, $b = 10^{40/20} = 100$ (Equation (2.15)). The value of ω_s is 2.17622 radians and ω_p is 0.6733 radians, calculated using Equations (2.10) and (2.14), respectively. The transfer function for this FIR filter, where $\omega_i = \pi$ and $i = 1, 2, \ldots 6$, is

$$H(\omega) = \{x_0 \cos(\omega/2)\}^6 \tag{2.16}$$

The polynomial, magnitude response in dB, and phase response for low pass FIR filter of Equation (2.16) is shown in Figures 2.2, 2.3 and 2.4, respectively. As expected the phase comes out to be zero. The magnitude characteristics is not very good but it gives us an insight into – what our transformation is going to do with the object function after application.
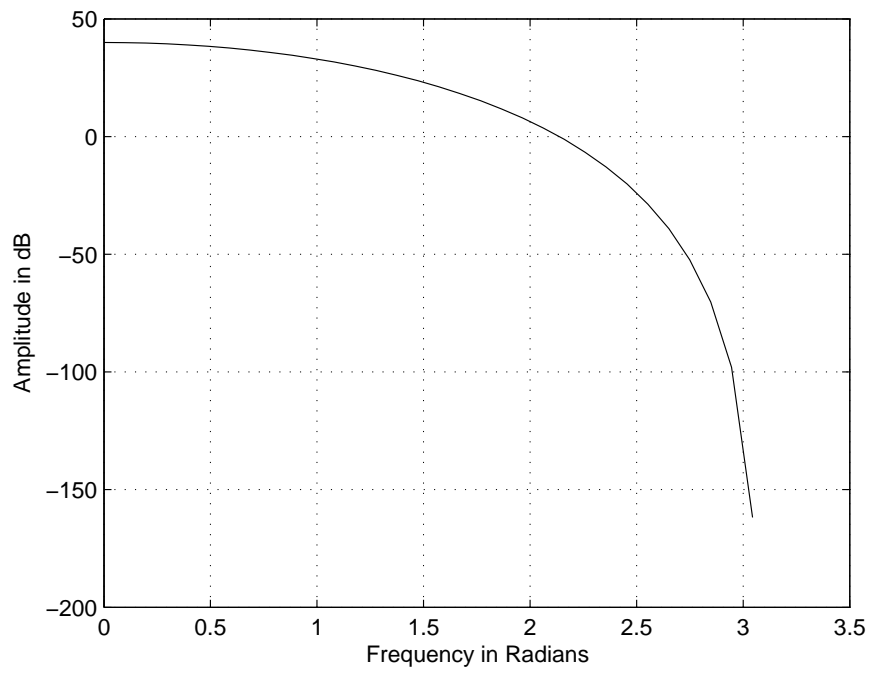
Figure 2.3: Magnitude response in *dB* for Polynomial x^6.
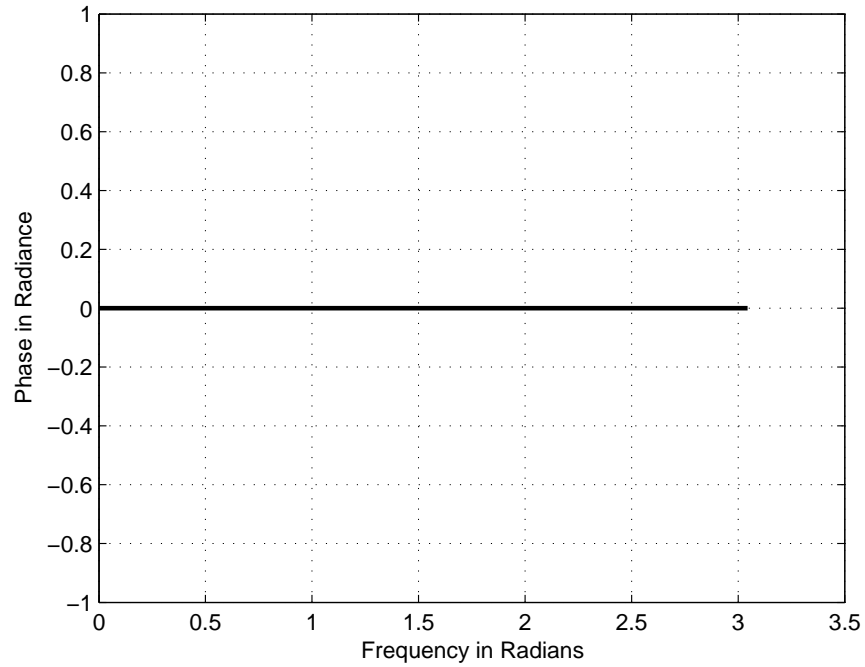
Figure 2.4: Phase response for Polynomial x^6.

2.2.2 Design 2

Let us consider another polynomial with zeros not concentrated at the origin. The polynomial is defined in Equation (2.17) and it follows the characteristics discussed in *Step 1*, as well as it has zeros at locations other than $x = 0$ except one (Figure 2.5).

$$f(x) = (x - 0)(x - 0.1)(x - 0.2)(x - 0.3)(x - 0.4)(x - 0.5) \qquad (2.17)$$

Order of the filter is 6. Suppose we want our stop band to be $40\,dB$ down; that is, $b = 100$.

We calculate the value of stop band for the above polynomial by equating, as in previous case, $f(x)$ to 1; that is,

$$(x - 0)(x - 0.1)(x - 0.2)(x - 0.3)(x - 0.4)(x - 0.5) = 1 \qquad (2.18)$$

When we solve Equation (2.18) we get six different values of x. Any of the values of x can be used, apart from the fact that every value will give rise to a different value of the frequency where the stop band ends, we choose 1.2646, this point correspond to ω_s and we call it by name x_s.

To calculate the value of x_0 we equate $f(x)$ to b (=100); that is,

$$(x - 0)(x - 0.1)(x - 0.2)(x - 0.3)(x - 0.4)(x - 0.5) = 100 \qquad (2.19)$$

The solution of Equation (2.19) gives us six different values of x, here again we consider any one positive value amongst them (see Figure 2.1), we retain $x_0 = 2.4112$.

The value of stop band, ω_s, can be calculated by using the transform discussed in *Step 2*; that is,

$$1.2646 = 2.41121 \, \cos(\omega_s/2) \qquad (2.20)$$

$$\omega_s = 2.0374 \, \text{radians} \qquad (2.21)$$

Similarly, the value of pass band, ω_p, can be calculated by equating the

polynomial to $b/\sqrt{2}$ or $100/\sqrt{2}$; that is,

$$(x-0)(x-0.1)(x-0.2)(x-0.3)(x-0.4)(x-0.5) = 100/\sqrt{2} \qquad (2.22)$$

Above equation gives us six different values, we consider any one value. In this case every value will correspond to a different value of the frequency where the stop band starts; that is, $3\,dB$ down from the maximum value of the pass band of the filter. The value of the solution is $x_p = 2.29069$. We use the same transformation as discussed in *Step 2* to get the pass band frequency; that is, ω_p

$$2.29069 = 2.41121\ \cos(\omega_p/2) \qquad (2.23)$$

$$\omega_p = 0.6350\ \text{radians} \qquad (2.24)$$

We follow *Step 2* for the transformation and *Step 3* to calculate the position of zeros for this polynomial. The zeros (in radians) for this polynomial are calculated from Equation (2.2):

$$\omega_1 = 2cos^{-1}\frac{0}{x_0} = \pi, \quad \omega_2 = 2cos^{-1}\frac{0.1}{x_0} = 3.0586,$$

$$\omega_3 = 2cos^{-1}\frac{0.2}{x_0} = 2.9755, \quad \omega_4 = 2cos^{-1}\frac{0.3}{x_0} = 2.8921,$$

$$\omega_5 = 2cos^{-1}\frac{0.4}{x_0} = 2.8083, \quad \omega_6 = 2cos^{-1}\frac{0.5}{x_0} = 2.7238. \qquad (2.25)$$

The transfer function for this FIR filter is (*Step2*)

$$H(\omega) = x_0 \cos(\omega_1/2) * x_0 \cos(\omega_2/2) * x_0 \cos(\omega_3/2) * x_0 \cos(\omega_4/2) * x_0 \cos(\omega_5/2) * x_0 \cos(\omega_6/2)$$
$$(2.26)$$

Figures 2.5 and 2.6 show the polynomial and magnitude response in dB for this low pass FIR filter. The magnitude characteristics in dB show that the transition band starts becoming steep when compared with the previous design. It is notworthy that the phase, in this case, will not be linear because zeros are not symmetrical.
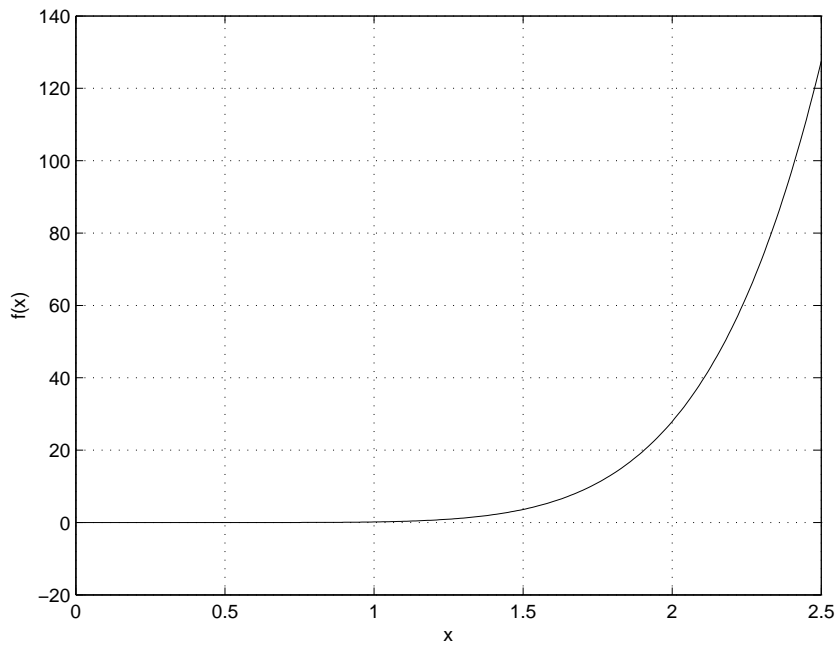
Figure 2.5: Polynomial $(x − 0)(x − 0.1)(x − 0.2)(x − 0.3)(x − 0.4)(x − 0.5)$.
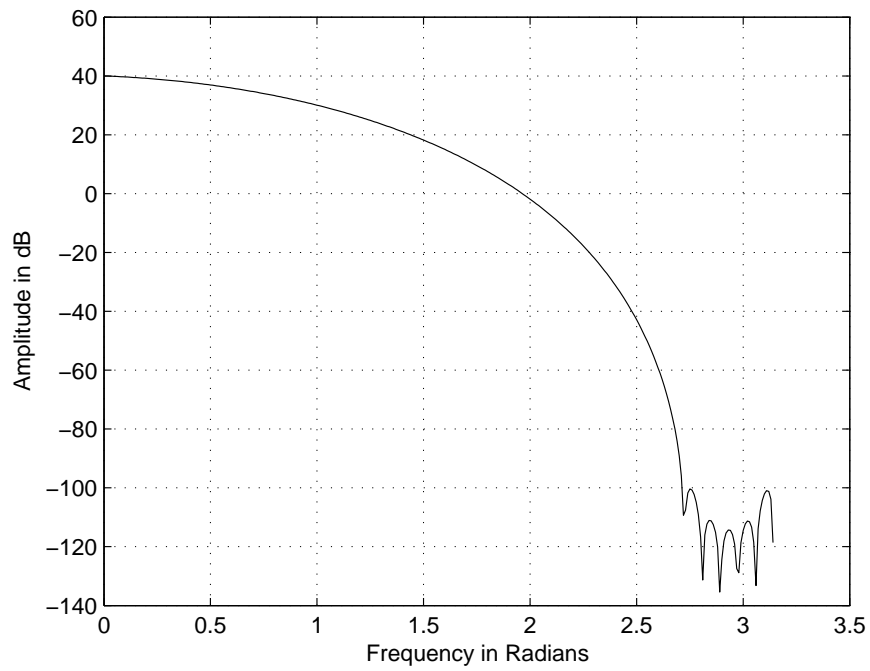
Figure 2.6: Magnitude response in dB of $H(z)$ of Equation (2.26).

2.3 Conclusion

In this chapter an insight into the connection of object function with filter, using the transformation function, is presented. With this technique we can design linear phase FIR filter with user defined values of zeros of the filter function and decide how many dB down the stop band should be with respect to the pass band; that is, the value of b.

3 Design of 1 Dimensional Linear Phase FIR Filter with Chebyshev Polynomials

3.1   Introduction

Due to large number of applications [28, 29], several approaches have been proposed for designing a Chebyshev FIR filter. To design a linear phase Chebyshev FIR filter one has to convert the filter design problem into an approximation problem [28, 29]. Imposing the linear phase restriction in the pass band it leads to a complex approximation problem. An approach discussed by Cuthbert [30] proposes a technique where real and imaginary parts of the filter frequency response are separately approximated to design a non linear phase FIR filter. This approach was further extended using Remez-exchange algorithm [31, 32] by Attikiouzel et al [33] for two real approximations. Linear programming was used by Delves et al [34] to approximate real and imaginary parts simultaneously. These papers in general discuss approximation methods, while the approach discussed in the present chapter gives exact design of FIR filter in Chebyshev sense with linear phase.

In this chapter, we are going to discuss a new approach to realize FIR filters using Chebyshev polynomials. Chebyshev polynomials play a vital role in antenna as well as in signal processing theory. The Dolph-Chebyshev distribution of currents feeding the elements of a linear array, comprising an antenna, gives a sharp main lobe and small side lobes all of which have the same power level [17], we use this concept in the present discussion.

This chapter presents a method by which we can design a filter with linear phase and,

(i) A given pass band to stop band ratio,

(ii) A given pass band to stop band transition, and to some extent

(iii) The frequency band of the pass band.

3.2 Preliminaries

A linear equispaced antenna array with n elements, labeled from left to right gives rise to a far field of [17]

$$|E| = |A_0 e^{j0} + A_1 e^{j\psi} + A_2 e^{j2\psi} + \ldots + A_{n-2} e^{j(n-2)\psi} + A_{n-1} e^{j(n-1)\psi}| \qquad (3.1)$$

where,

$$\psi = \beta d \cos\phi + \gamma \qquad (3.2)$$

$|E|$ is the magnitude of the far field, $\beta = 2\pi/\lambda$, λ is the free space wavelength, d is the spacing between elements, ϕ is the angle from the normal to the linear array, γ is the progressive phase shift from left to right, and $A_0, A_1, A_2, \ldots$ are complex amplitudes which are proportional to the current amplitudes.

If we substitute $z = e^{j\psi}$ and rewrite Equation (3.1), it leads to

$$H(z) = A_0 + A_1 z + A_2 z^2 + \ldots + A_{n-2} z^{n-2} + z^{n-1} \qquad (3.3)$$

this equation represents an FIR filter. Where, $H(z)$ represents the impulse response of the filter with $z = e^{j\omega}$, and A_0, A_1, $A_2, \ldots$ represent amplitudes at the corresponding frequencies.

The Chebyshev polynomials are given by

$$T_m(x) = \begin{cases} \cos(m \cos^{-1} x) & 0 < |x| < 1 \\ \cosh(m \cosh^{-1} x) & |x| > 1 \end{cases} \qquad (3.4)$$

3.3 Procedure

Let us assume that the order of the filter, which we intend to design, is m. The step vise procedure to design the required FIR filter is as follows:

Step 1 : As discussed earlier the pass band to stop band ratio is user dependent. Therefore, first we calculate the absolute value of attenuation,

defined by the user, in the stop band, b (refer Chapter 1),

$$b = 10^{(attenuation \ in \ dB)/20} \qquad (3.5)$$

Step 2 : We find the stop band, ω_s, and pass band, ω_p, frequencies following the steps discussed in [17]

$$\omega_s = 2cos^{-1}\left[\frac{1}{\cosh(\overline{1/m} \ \cosh^{-1} b)}\right] \qquad (3.6)$$

$$\omega_p = 2cos^{-1}\left[\frac{\cosh\left\{(1/m) \ \cosh^{-1}(b/\sqrt{2})\right\}}{\cosh(\overline{1/m} \ \cosh^{-1} b)}\right] \qquad (3.7)$$

Step 3 : The location of zeros on unit circle, ω_m, are calculated by the following equation, which is discussed in [17]

$$\omega_m = 2cos^{-1}\left\{\frac{\cos(\omega_k)}{\cosh(\overline{1/m} \ \cosh^{-1} b)}\right\} \qquad (3.8)$$

where, $\omega_k = (2k-1)\pi/2m$, and $k = 1 \dots m$.

Step 4 : Applying the relation $z_m = e^{j w_m}$ we can write frequency response, $H(z)$, in z domain using Equation (3.3) as follows

$$H(z) = (z - z_1)(z - z_2) \dots (z - z_m) \qquad (3.9)$$

where, $z_1, z_2, \dots$ are location of zeros of the transfer function $H(z)$. Replacing z by $e^{j\omega}$ and z_m's by $e^{j\omega_m}$'s in Equation (3.9) we get frequency response in frequency domain as follows

$$H(\omega) = (e^{j\omega} - e^{j\omega_1})(e^{j\omega} - e^{j\omega_2}) \dots (e^{j\omega} - e^{j\omega_m}) \qquad (3.10)$$

To get a vivid picture of the procedure discussed above, we design some actual filters in the following section.

3.4 Application

Let us design some Chebyshev low pass FIR filters with side band $40 \, dB$ down from the pass band; that is, $b = 10^{40/20} = 100$.

3.4.1 Design 1

Let us design a filter with order 6; that is, $m = 6$. From Equations (3.6) and (3.7) we calculate

$$\omega_s = 1.5732,$$

$$\omega_p = 0.5622.$$

The values of the ω_m's can be calculated by using Equation (3.8)

$$\omega_1 = 1.64,$$

$$\omega_2 = 2.0958,$$

$$\omega_3 = 2.7739,$$

$$\omega_4 = 3.5093,$$

$$\omega_5 = 4.1874,$$

$$\omega_6 = 4.6431.$$

From Equation (3.10) we can write $H(\omega)$ as

$$H(\omega) =$$
$$(e^{j\omega} - e^{j1.64})(e^{j\omega} - e^{j2.0958})(e^{j\omega} - e^{j2.7739})(e^{j\omega} - e^{j3.5093})(e^{j\omega} - e^{j4.1874})(e^{j\omega} - e^{j4.6431})$$

If we plot the filter characteristics; that is, $|H(\omega)|$ verses frequency ω, the results will become clear. The *dark continuous lines* in Figures 3.1, 3.2, and 3.3 show the magnitude response, magnitude response in *dB*, and phase response of above mentioned FIR filter, respectively. Figure 3.1 confirms that the location of ω_s and ω_p are at the points where we have calculated them. Figure 3.2 show that side bands are 40 dB down. From Figure 3.3 it is clear that the phase is linear.
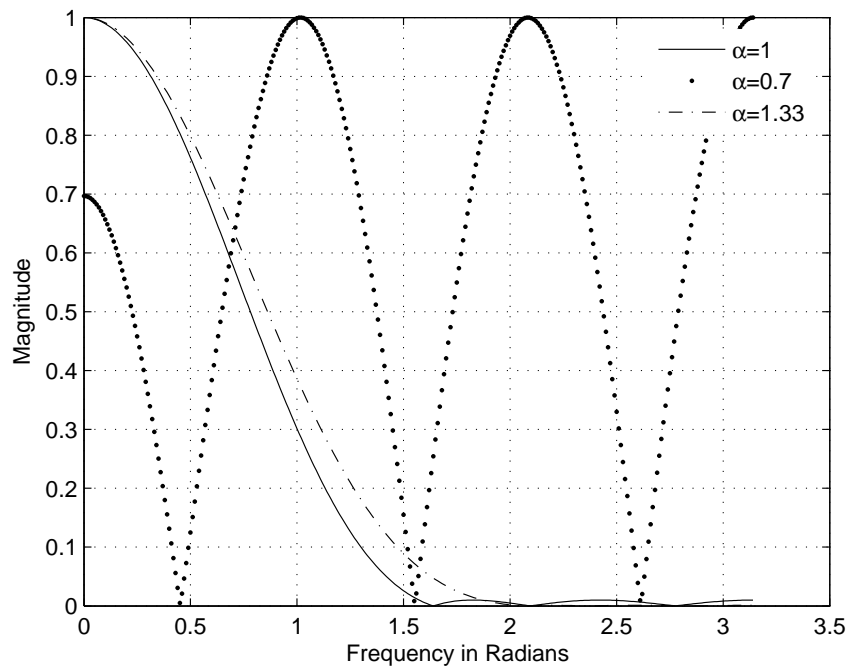
Figure 3.1: Magnitude response of 6th order Chebyshev low pass FIR filter.
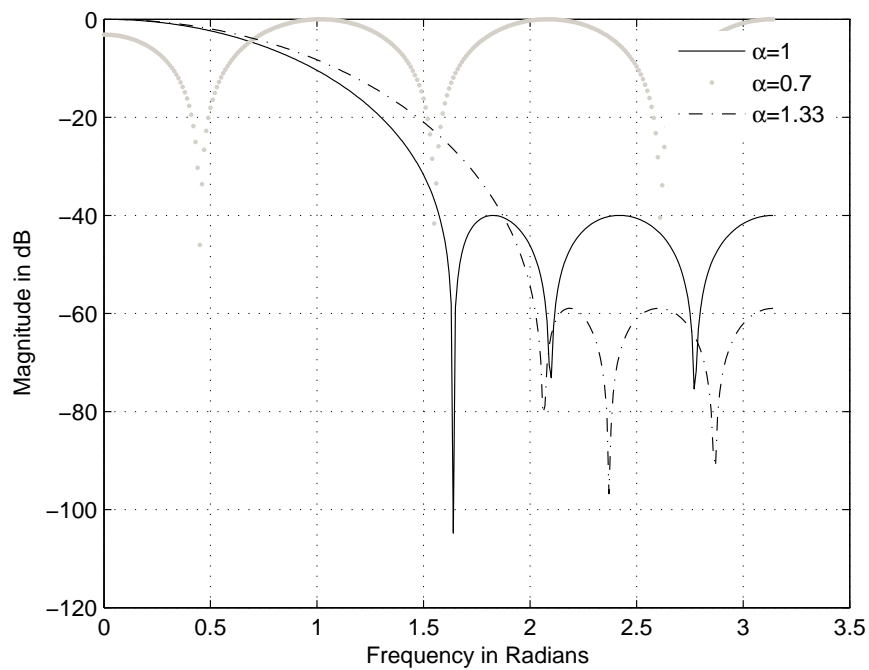
Figure 3.2: Magnitude response in dB of 6th order Chebyshev low pass FIR filter.
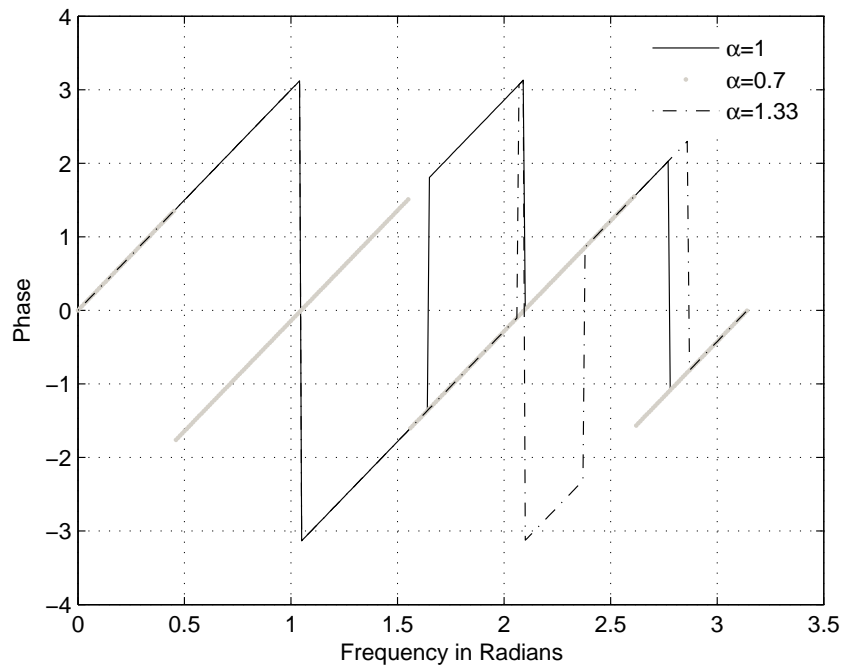
Figure 3.3: Phase response of 6th order Chebyshev low pass FIR filter.

3.4.2 Design 2

Let us calculate the frequency response of 3rd order filter by using the steps mentioned in previous section. The values of ω_s and ω_p for 3rd order filter are

$$\omega_s = 2.4641,$$

$$\omega_p = 0.9136.$$

Zeros, ω_m's, for this filter are as follows

$$\omega_1 = 2.5578,$$

$$\omega_2 = 3.1416,$$

$$\omega_3 = 3.7254.$$

While the *dark continuous lines* in Figure 3.4 represents the magnitude response of 3rd order FIR filter, Figure 3.5 shows the magnitude response in *dB*. The side bands are 40 dB down and the bandwidth of the filter is

increased when compared with Figure 3.2. The transition band is wider than the previous design, or in other words filter will pass some of the non required frequencies. Phase remains linear and can be verified easily.
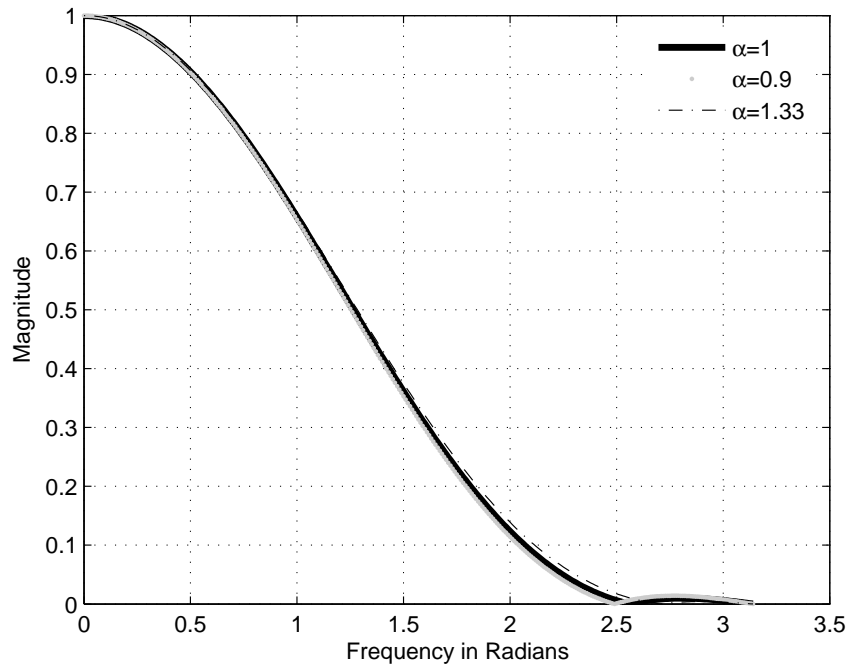
Figure 3.4: Magnitude response of 3rd order Chebyshev low pass FIR filter.

3.4.3 Design 3

Next we calculate the frequency responses of 24th order filter. Suppose we design the filter with side bands $40\,dB$ down, the values of ω_s and ω_p for the 24th order filter are as follows

$$\omega_s = 0.4380,$$

$$\omega_p = 0.1558,$$

Values of ω_m's are as follows

$$\omega_1 = 0.4568, \ \omega_2 = 0.5861;$$

$$\omega_3 = 0.7831, \ \omega_4 = 1.0088;$$
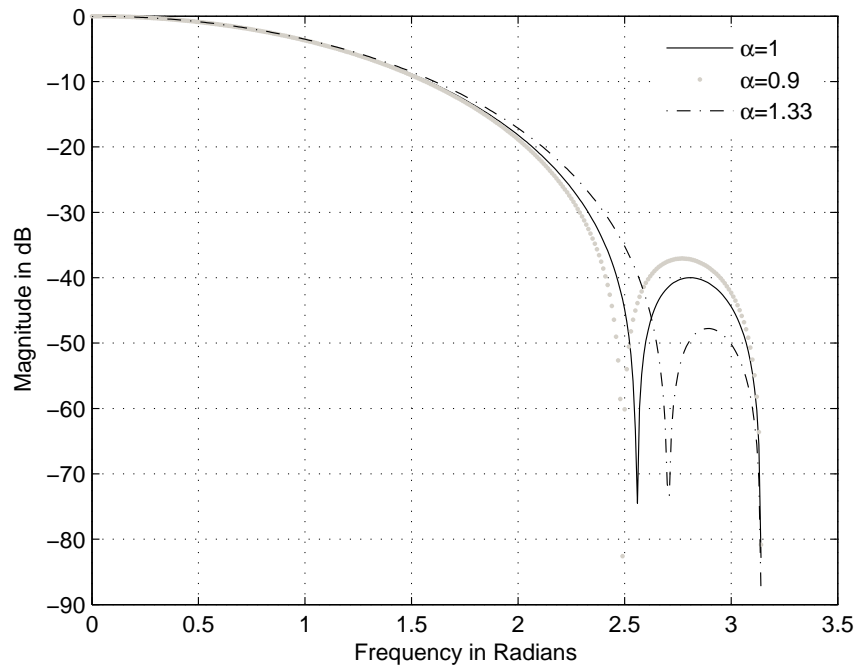
Figure 3.5: Magnitude response in dB of 3rd order Chebyshev low pass FIR filter.

$$\omega_5 = 1.2478, \ \omega_6 = 1.4935;$$

$$\omega_7 = 1.7432, \ \omega_8 = 1.9952;$$

$$\omega_9 = 2.2488, \ \omega_{10} = 2.5033;$$

$$\omega_{11} = 2.7584, \ \omega_{12} = 3.0138;$$

$$\omega_{13} = 3.2694, \ \omega_{14} = 3.5248;$$

$$\omega_{15} = 3.7799, \ \omega_{16} = 4.0344;$$

$$\omega_{17} = 4.2879, \ \omega_{18} = 4.5400;$$

$$\omega_{19} = 4.7896, \ \omega_{20} = 5.0354;$$

$$\omega_{21} = 5.2743, \ \omega_{22} = 5.5001;$$

$$\omega_{23} = 5.6970, \ \omega_{24} = 5.8264.$$

The *dark continuous lines* in Figure 3.6 represents the magnitude response of this order FIR filter and Figure 3.7 shows the magnitude response in *dB*. When Figure 3.6 is compared with Figures 3.1 and 3.4, we found that the present filter has narrow pass band and sharper transition band while side bands remains 40 dB down. Therefore, to design a narrow band filter we have to increase the order of the filter.
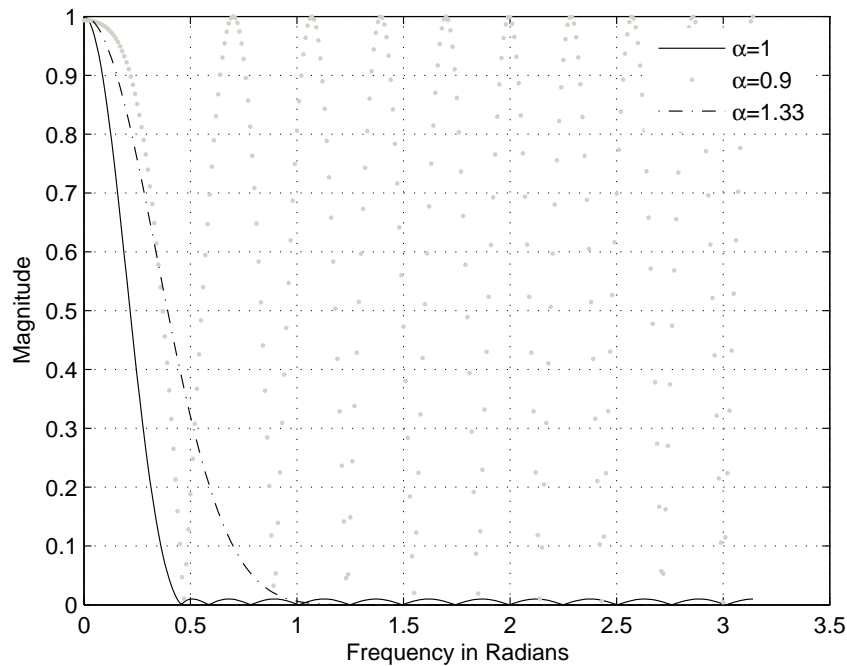
Figure 3.6: Magnitude response of 24th order Chebyshev low pass FIR filter.

3.5 Discussion

It is clear from the Figures 3.1, 3.4 and 3.6 and Figures 3.2, 3.5 and 3.7 that the width of the pass band decreases as the order of filter increases, and the transition band becomes steeper, or in other words, it follows brick-wall or ideal characteristics of a filter more closely.

Figure 3.3 shows the phase response of the 6th order FIR filter designed above, which clearly shows its linear nature. The phase responses of 3rd and 24th order filters is also linear and can be verified easily by following the same steps.
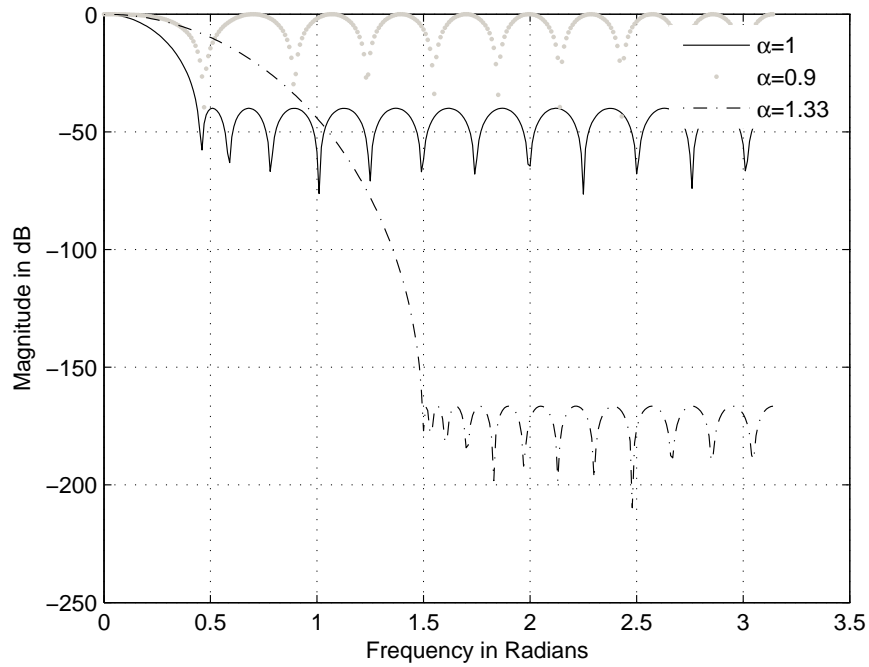
Figure 3.7: Magnitude response in dB of 24th order Chebyshev low pass FIR filter.

By using the procedure discussed, we can not control the bandwidth of the resulting filter with predefined order. In the next section we modify the design procedure to overcome this problem.

3.6 Modified Chebyshev Filter

We introduce a *new* parameter in the Chebyshev polynomial to have some control over the bandwidth, when m and b are already defined. In the original Chebyshev polynomial (Equation (3.4)) we multiply a new parameter $'\alpha'$ with parameter $'x'$ (α controls the bandwidth of the filter). Thus, Equation (3.4) becomes

$$T_m(\alpha x) = \begin{cases} \cos(m \cos^{-1} \alpha x) & 0 < |x| < 1 \\ \cosh(m \cosh^{-1} \alpha x) & |x| > 1 \end{cases} \tag{3.11}$$

When we multiply $'x'$ with $'\alpha'$, it results in a change in ω_s only, while ω_p remains the same. The reason is because $'\alpha'$ is present in both numerator

as well as in denominator terms (see Equations (3.6) and (3.7)).

Therefore, new stop band frequency, ω_{s-new}, is

$$\omega_{s-new} = 2 \cos^{-1}\left[1/\alpha\left\{\cosh(\overline{1/m}\cosh^{-1}b)\right\}\right] \tag{3.12}$$

and,

$$\omega_{p-new} = \omega_p \tag{3.13}$$

We can calculate the location of zeros for modified Chebyshev polynomials by

$$\omega_{m-new} = 2 \cos^{-1}\left[\cos(\omega_k)/\left\{\alpha(\cosh(\overline{1/m}\cosh^{-1}b))\right\}\right] \tag{3.14}$$

where, $\omega_k=(2k-1)\pi/2m$, and $k=0\ldots m$. We can write $H(\omega)$ as

$$H(\omega) = \prod_{m-new=0}^{n} (e^{j\omega} - e^{j\omega_{m-new}}) \tag{3.15}$$

3.7 Application

Plotting the magnitude response of Design 1 – with the new parameter α taken into consideration – we get the magnitude response and magnitude response in dB as shown in Figures 3.1 and 3.2, respectively ("**dash followed by dot**" for values $\alpha > 1$ and "**dots**" for values $\alpha < 1$, respectively). It is evident from figures that the bandwidth of our filter is increased in case of $\alpha > 1$ and the stop band is further down.

Figure 3.3 shows that our modified Chebyshev FIR filter has linear phase characteristics, further it confirms that the filter retains its linear phase even after introducing the new parameter α. Similarly Figures 3.4 and 3.5, Figures 3.6 and 3.7 show the magnitude response and magnitude response in *dB* for 3rd and 24th order FIR filter having side bands $40\,dB$ down, respectively. For lower order filter α makes very small difference, this can be verified by looking at Figures 3.4 and 3.5.

It is noteworthy that when α has value less than 1, see Figures 3.1 and 3.6, the magnitude of the stop band is approximately equal to the pass band. Thus we *should not* use filters designed using value of $\alpha < 1$.

3.8 Discussion

Figure 3.8 shows the magnitude response in *dB* for various values of α for 32nd order low pass FIR filter. It is clear from the figure that as we increase the value of our new parameter α, the bandwidth of our filter increases and the level of sidebands goes further down. One should note that the value of α less than 1 will give us a poor design, where pass band and stop band are approximately at the same level, and thus should not be used. The value of α is obtained by trial to obtain the desired bandwidth. A filter was designed and synthesized based on this technique to control burst noise, and it produced good results.
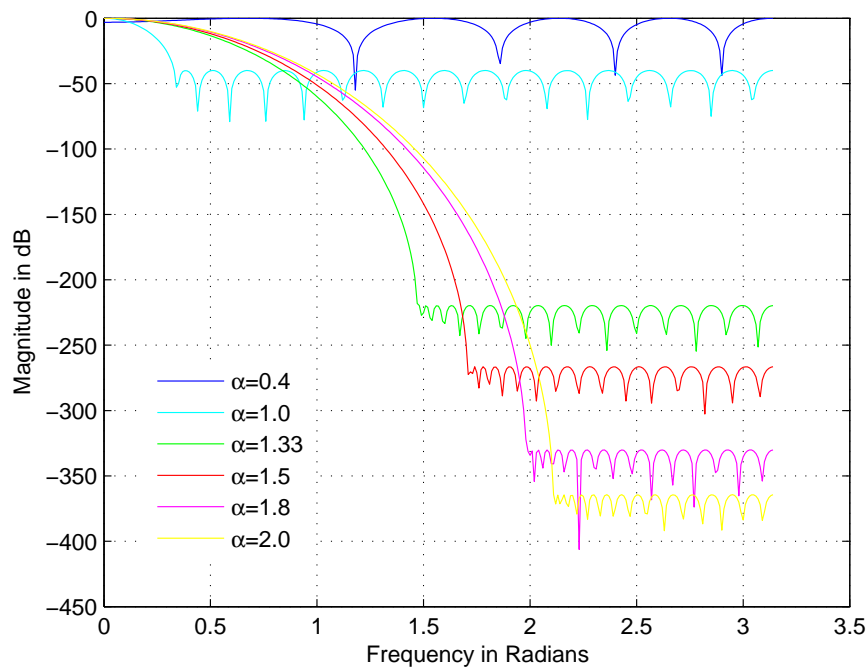
Figure 3.8: Magnitude responses of 32nd order Chebyshev low pass FIR filter for various values of α.

3.9 Conclusion

We can say that the FIR filter design technique discussed in the present chapter can easily be implemented for given sideband specifications and bandwidth. The new parameter α further pulls the side band levels down

and increases the bandwidth, while maintaining the phase linear. By this technique we can design band pass, band reject, multiband and other filters using the standard frequency transformation techniques.

We extend this concept to two dimensions, thus making use of such filters for image processing applications as well, in the next chapter.

4 Design of 2 Dimensional Linear Phase FIR Filter with Chebyshev Polynomials

4.1 Introduction

The design of 2 dimensional (2D) linear phase Chebyshev FIR filters has been discussed previously by some authors. An application of linear programming to design such filters was introduced by Hu and Rabiner [35]. But due to its very high computational requirements Fiasconaro [36] developed a better algorithm. Though this algorithm was an improvement but it was still slow. Lu [37] and, Lu and Hinamoto [38] introduced methods based on semi definite programming (SDP) and sequential quadratic programming (SQP). These methods work quite well except for the fact that the design complexity becomes rather high even for filters of moderate order. The approach we discuss in this chapter presents the Chebyshev filter design with a new purview, it results in a linear phase Chebyshev 2DFIR filter. The computation time is less, as the parameters involved to design the filter are few.

4.2 Procedure

1 dimensional (1D) Chebyshev polynomials are given by

$$T_m(x) = \begin{cases} \cos(m \, \cos^{-1} x) & 0 < |x| < 1 \\ \cosh(m \, \cosh^{-1} x) & |x| > 1 \end{cases} \tag{4.1}$$

We desire to design a 2D linear phase FIR filter and for that we require 2D polynomials. To convert 1D Chebyshev polynomial, defined in Equation (4.1), to 2D Chebyshev polynomial we change the variable x with a new variable ρ, which will represent the Chebyshev polynomial in cylin-
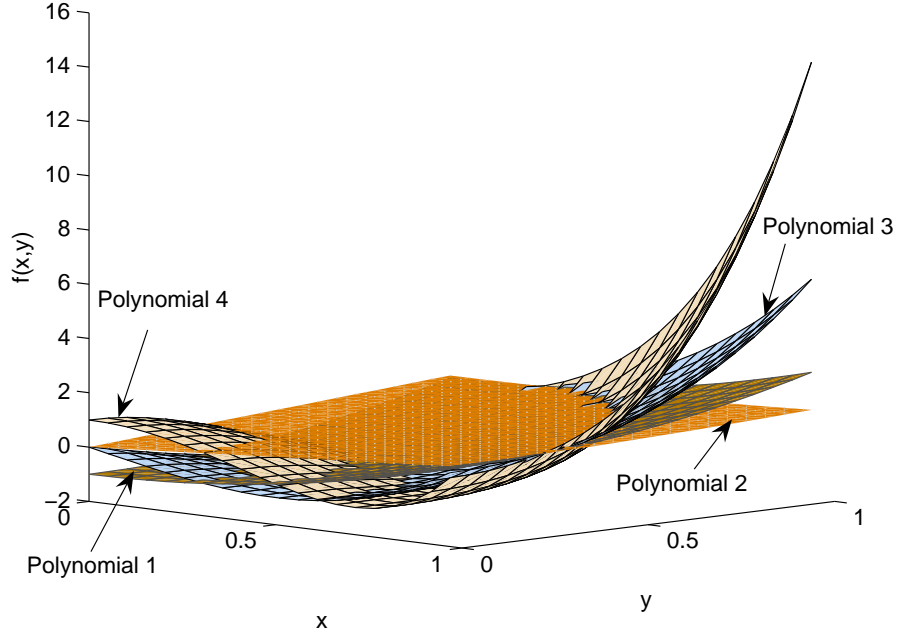
Figure 4.1: First few Chebyshev polynomials in 2 dimension.

drical coordinate system. ρ is a mapping function, in other words, the values of ρ are mapped onto variables x and y (Cartesian coordinates) by the relationship $\rho = \sqrt{x^2 + y^2}$. Thus, Equation (4.1) with new variable ρ is represented as

$$T_m(\rho) = \begin{cases} \cos(m \ \cos^{-1} \rho) & 0 < |\rho| < 1 \\ \cosh(m \ \cosh^{-1} \rho) & |\rho| > 1 \end{cases} \tag{4.2}$$

Some of these polynomials are shown in Figure 4.1.

We call these polynomials as object functions and use them to design our filter. The design is performed by transforming the object function to filter characteristics using the transformation function in the cylindrical coordinate system, in terms of ρ. The transformation is defined as follows

$$\rho = \rho_0 \cos\left(\frac{\omega}{2}\right) \quad -\pi \leq \omega \leq \pi \tag{4.3}$$

where ρ_0 is the maximum value of ρ.

It is apparent from the discussion of Chapter 1 that this transformation converts the object function, Equation (4.2), to low pass filter; that is, lower

values of the object function will be converted to higher values in filter characteristics and vice versa.

After transformation is applied, object function (Equation (4.2)) transforms to filter characteristics and is given by

$$H_m(\omega) = \begin{cases} \cos\left[m \, \cos^{-1}\left\{\rho_0 \, \cos(\omega/2)\right\}\right] & -\pi < |\omega| < \pi \\ \cosh\left[m \, \cosh^{-1}\left\{\rho_0 \, \cosh(\omega/2)\right\}\right] & |\omega| > \pi \end{cases} \qquad (4.4)$$

ω (Equation (4.4)) is mapped onto a 2D frequency domain – defined by u and v – by the relationship $\omega = \sqrt{u^2 + v^2}$. After mapping Equation (4.4) converts to

$$H_m(u, v) = \begin{cases} \cos\left[m \, \cos^{-1}\left\{\rho_0 \, \cos\left(\frac{\sqrt{u^2+v^2}}{2}\right)\right\}\right] & -\pi < |u, v| < \pi \\ \cosh\left[m \, \cosh^{-1}\left\{\rho_0 \, \cosh\left(\frac{\sqrt{u^2+v^2}}{2}\right)\right\}\right] & |u, v| > \pi \end{cases} \qquad (4.5)$$

To realize the filter defined in Equation (4.5) we first have to find out the value of ρ_0. Following the procedure given in [17] we can state that the value of ρ_0 is given by

$$\rho_0 = \cosh\{\cosh^{-1}(b/m)\}$$

where, m is the order of the filter, and b is the absolute value of the attenuation and is given by

$$b = 10^{(attenuation \; in \; dB/20)} \qquad (4.6)$$

We use this approach of design to model a filter whose stop band attenuation is user defined. Following the procedure outlined in [17] we can find out the values of stop band, ω_s, and pass band, ω_p, frequencies

$$\omega_s = 2\cos^{-1}\left[\frac{1}{\cosh(\overline{1/m} \cosh^{-1} b)}\right] \qquad (4.7)$$

$$\omega_p = 2\cos^{-1}\left[\frac{\cosh\left\{(1/m) \cosh^{-1}(b/\sqrt{2})\right\}}{\cosh(\overline{1/m} \cosh^{-1} b)}\right] \qquad (4.8)$$

Therefore, to design such filters we need to work out following steps

Step 1 : Calculate the absolute value of attenuation in the stop band, b,

Equation (4.6).

Step 2 : Calculate the stop band, ω_s, and pass band, ω_p, frequencies, Equations (4.7) and (4.8).

Step 3 : The location of zeros, ω_m, on unit circle can be calculated by the following equation [17]

$$\omega_m = 2cos^{-1}\left\{\frac{\cos(\omega_k)}{\cosh(\overline{1/m}\,\cosh^{-1}b)}\right\} \qquad (4.9)$$

where, $\omega_k = (2k-1)\pi/2m$, and $k = 1\ldots m$.

Step 4 : Calculate the value of $H_m(u,v)$ using Equation (4.5).

To make the approach and procedure clear we design some such linear phase FIR filters in the next section.

4.3   Application

Suppose we want to design low pass FIR filters with stop band $40\,dB$ down; that is, $b = 10^{40/20} = 100$.

4.3.1   Design 1

Let us design a filter of order 6; that is, $m = 6$. The values of ω_s and ω_p are calculated by using Equations (4.7) and (4.8) and result in 1.5732 radian and 0.5622 radian, respectively. When we realize the filter characteristics of Equation (4.5) with these values of m and b, and plot the magnitude characteristics they result as shown in Figure 4.2, the side bands are 40 dB down. The phase characteristics are shown in Figure 4.3, it is clear from the figure that phase is linear in pass band and stop band.

If we replace $\rho_0 \cos(\sqrt{(u^2 + v^2)}/2)$ in Equation (4.5) by $\rho_0 sin(\sqrt{(u^2 + v^2)}/2)$ we get high pass filter, since *sinusoidal* function transforms low values in object function to low values of filter magnitude characteristics and vice versa. This high pass filter is shown in Figure 4.4.

4.3.2   Design 2

We extend our design to a filter whose order is 20; that is, $m = 20$. Again we follow the steps of previous section and calculate the filter characteristics.
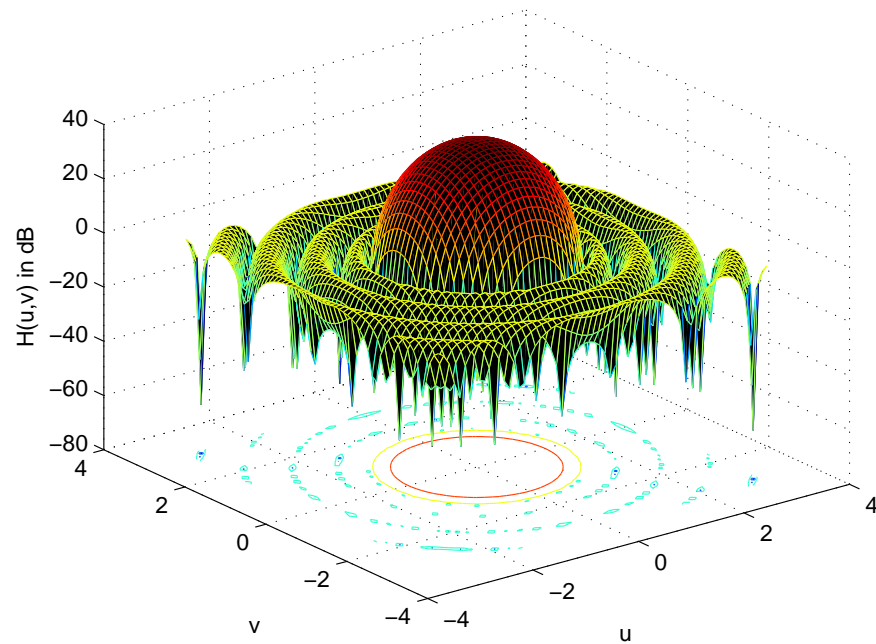
Figure 4.2: Magnitude response in dB of 6th order low pass FIR filter.
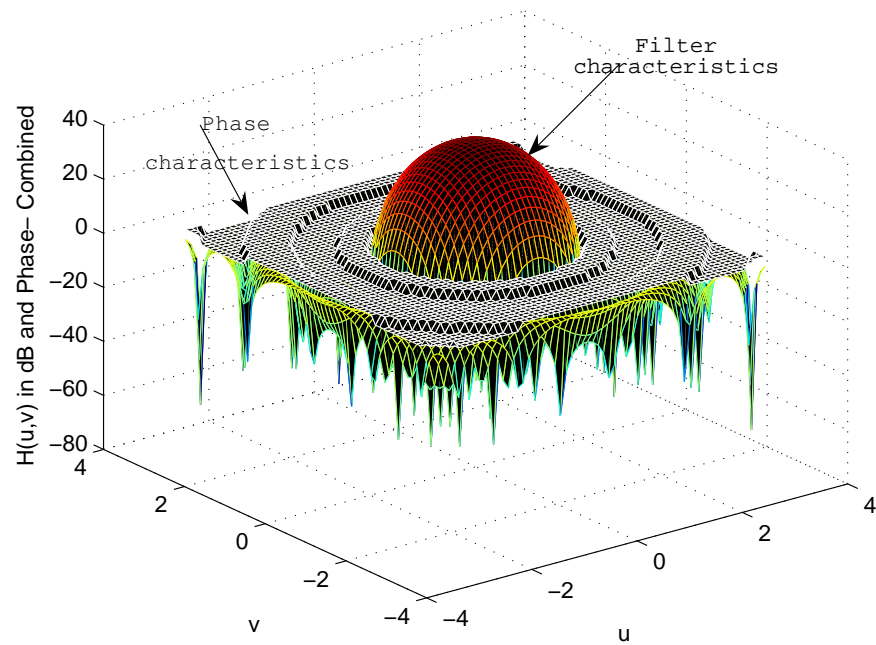
Figure 4.3: Phase response of 6th order low pass FIR filter.
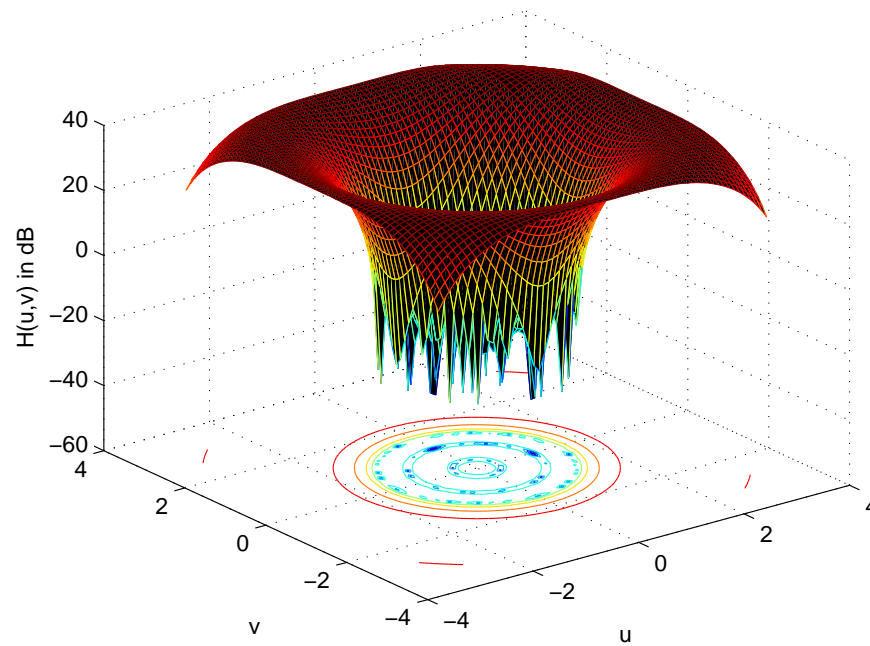
Figure 4.4: Magnitude response in dB of 6th order Chebyshev high pass FIR filter.

The low pass and high pass filter magnitude characteristics of this order are shown in Figures 4.5 and 4.6, respectively. The resulted filter has side bands which are 40 dB down and filter pass band becomes narrower when compared with the previous design. Transition bands are also sharper than the previous design.

4.3.3 Design 3

We design a filter of order 40, following the steps outlined in the the previous section. Figures 4.7 and 4.8 show the magnitude response of low and high pass filters of 40th order, respectively. Note the very narrow pass band of resulting filters.

4.4 Discussion

If we look at Figures 4.2, 4.5 and 4.7, and Figures 4.4, 4.6 and 4.8 we can easily state that the pass band of the filters successively become narrower
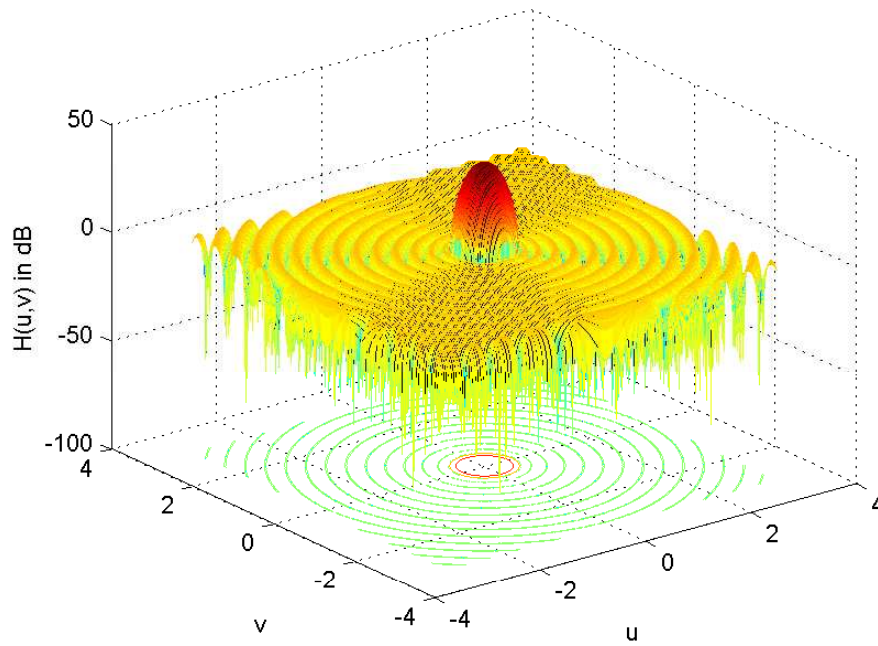
Figure 4.5: Magnitude response in dB of 20th order Chebyshev low pass FIR filter.
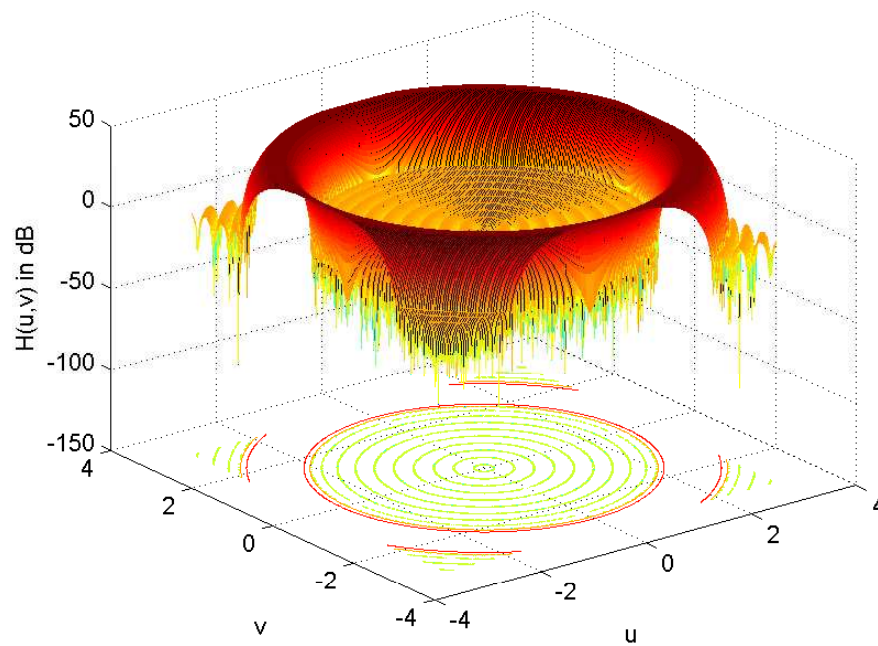
Figure 4.6: Magnitude response in dB of 20th order Chebyshev high pass FIR filter.
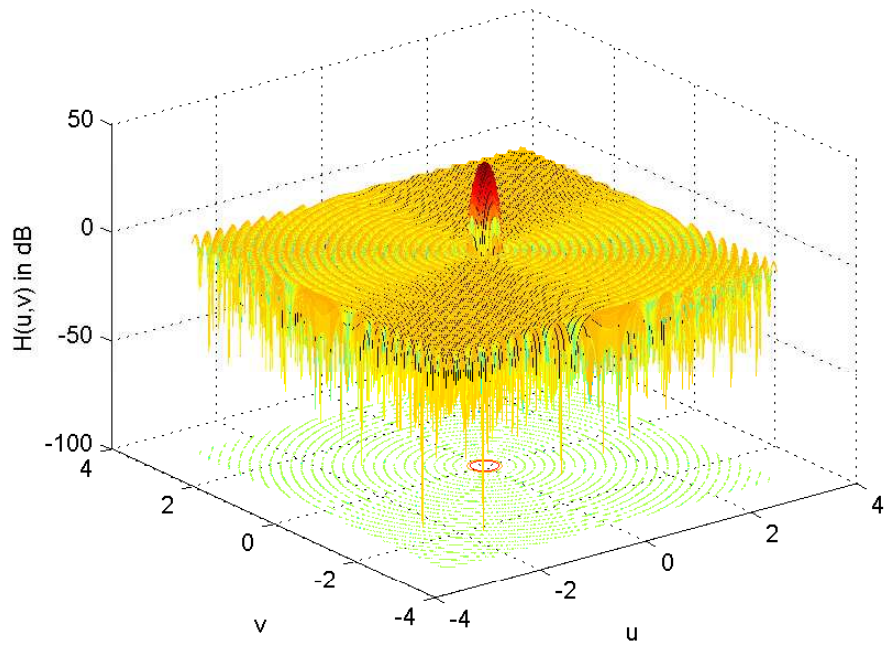
Figure 4.7: Magnitude response in dB of 40th order Chebyshev low pass FIR filter.
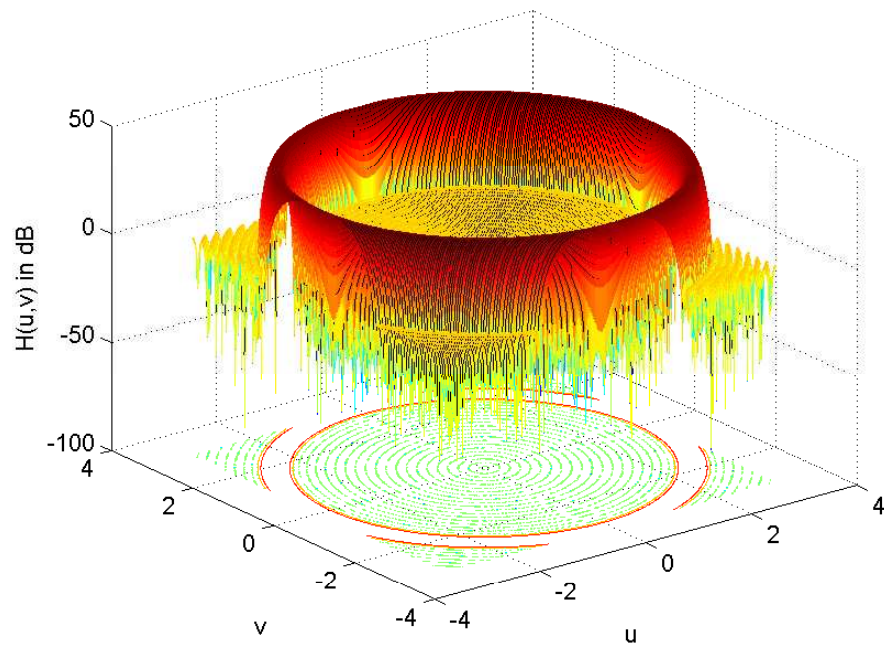
Figure 4.8: Magnitude response in dB of 40th order Chebyshev high pass FIR filter.

and narrower as we increase the order of the Chebyshev polynomials, which are used to design the filter.

To show the performance of the filters designed by using the proposed methodology in the present chapter, we pass an image through some of these filters. Figure 4.9 shows an image, which we pass through our 6th order filter, Figures 4.10 and 4.11 are low pass and high pass filtered results, respectively. In an image large blurred details are low frequencies, and small, sharp details represent high frequencies. In image of Figure 4.10 only large overview of the image is available for view, while Figure 4.11 shows the sharp details of the original image.

Figure 4.9: Image-1.
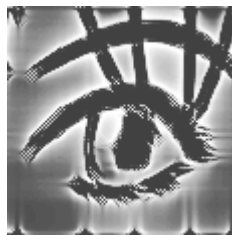
Figure 4.10: Image-1 passed through Chebyshev low pass filter of 6th order.

Application of high pass filter becomes more clear if we apply it to an image having a lot of high frequency components. A satellite image is such an image, it consists of a large number of sharp details. Figures

Figure 4.11: Image-1 passed through Chebyshev high pass filter of 6th order.

Figure 4.12: Image-2.

Figure 4.13: Image-2 passed through Chebyshev high pass filter of 6th order.

4.12 and 4.13 show such an image and its high pass filtered outcome when it is passed through a 6th order high pass FIR filter, respectively. The filtered outcome shows, in general, the edges present in the original image. Therefore, we can use this filter to enhance the edges of a satellite image. This sharper image can be used for further analysis, like finding a street or building on a city map. Further, an edge detection algorithm together with the filter may generate excellent results.

4.5   Conclusion

An alternate approach to design 2D linear phase FIR filters, using Chebyshev polynomials, with the help of the transformation has been examined. By this approach filters can directly be designed from defined 2D Chebyshev polynomials. The filters designed are equiripple and as we increase the order of the filter the sideband goes further down, thus improving the filter characteristic by the rejection of parts which are not required. One can design very narrow band filter using the present technique, thus passing only those frequencies which are required (useful for satellite, medical, astronomical, etc. type of images). Any other type of filter can be realized by shifting the pass band of filter using any of the standard frequency transformation techniques.

5 Design of 1 Dimensional Linear Phase FIR Filter with Orthogonal Polynomials

5.1   Introduction

To design linear phase FIR filters, various techniques have been discussed by several authors. Some of them, like Remez exchange algorithm [18] [6], are generally used for efficient implementation of linear phase FIR filters, but as the number of conditions grow the computational time increases exponentially.

Linear programming (LP), semi-infinite programming (SIP) and iterative re-weighted least square (IRLS) algorithms are generally used to design filters with constraints. Liang *et al.* [7] used LP technique to design some filters with constraints, like optimal Nyquist filters, partial response filters, etc. but this method consumes a great deal of CPU time. Potchinkov [8] applied SIP technique for constrained filters but the time requirements were still high. IRLS algorithms by Burrus et al [39] and many others have good computational efficiency. But they are bad when convergence is considered, because it is not guaranteed that they will converge.

Some techniques to design FIR filter are discussed by various authors, where either the filter has good magnitude response but poor phase characteristics or if they possess exact linear phase then magnitude characteristics are poor or they can not be controlled completely. Chan and Tsui [40] discusses an approach where the group delay is not constant and consequently the phase is not linear. Hanna [41] discusses a method where as bandwidth of the filter increases, the level of sidebands increases and also when the cutoff frequency goes far from the origin pass band characteristic becomes less flat. These requirements motivate us to discuss a new technique to design an arbitrary magnitude response and exact linear phase FIR filter using a very simple technique.

5.2 Preliminaries

A sequence of orthogonal polynomials with weight W(x) satisfies the relation

$$\left\langle f_i(x), f_j(x) \right\rangle = \int_{x_2}^{x_1} f_i(x)f_j(x)W(x)dx = 0 \qquad i \neq j \tag{5.1}$$

where, $f_i(x)$ and $f_j(x)$ are any two members of the orthogonal set, and $\langle \bullet, \bullet \rangle$ represents the inner product.

In the present discussion, we consider only Legendre polynomials as an example of orthogonal polynomials. Since they are the simplest orthogonal polynomials among all – generally used – yet the procedure is general in nature and one can use any of the other orthogonal polynomials and consider their interval of orthogonality.

For Legendre polynomials the interval of orthogonality is $[-1, 1]$ and weight function is 1. We can generate the Legendre polynomials by using Rodrigues' Formula [42] given by

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n \tag{5.2}$$

where, $n=1, 2, 3, \dots$. The first few polynomials are defined below:

$$P_0(x) = 1 \tag{5.3}$$

$$P_1(x) = x \tag{5.4}$$

$$P_2(x) = \frac{3x^2 - 1}{2} \tag{5.5}$$

$$P_4(x) = \frac{5x^3 - 3x}{2} \tag{5.6}$$

$$\vdots$$

These all are orthogonal to each other over $[-1, 1]$; that is

$$\int_{-1}^{1} P_m(x)P_n(x)dx = 0 \qquad whenever \ m \neq n \tag{5.7}$$

The plot of the first few Legendre polynomials is shown in Figure 5.1.
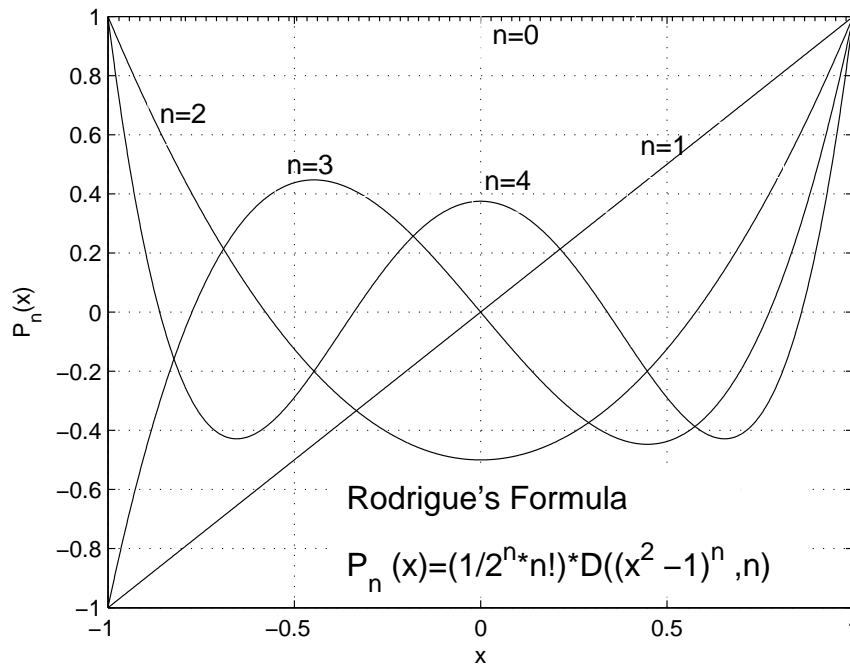
Figure 5.1: Plot of first few Legendre polynomials.

5.3 Procedure

Suppose user needs to design a low pass filter (LPF), whose magnitude response, $H(\omega)$, is shown in Figure 5.2[1] where,

A_{max} represents the amplitude of the pass band of the filter,

A_{min} represents the amplitude of the stop band of the filter,

ω_{OH} is the end of pass band and the start of the transition band, and

ω_{OL} is the end of transition band and start of the stop band.

$H(\omega)$ (Figure 5.2) is first converted to an object function (independent variable x), shown in Figure 5.3. The mapping from Figure 5.2 to 5.3 is done by previously defined transformation (repeated below)

$$x = x_0 cos(\omega/2) \qquad -\pi < \omega < \pi \qquad (5.8)$$

The object function is then approximated using a linear combination of several Legendre polynomials[2]. Note that we approximate the object

---

[1]Though here an LPF is considered, any other ideal magnitude characteristic may be used. The procedure is general in nature.

[2]The proof that any object function can be approximated by a linear combination of

function with even Legendre polynomial terms only; that is, $P_0, P_2, P_4, \ldots$, since the characteristics of Figure 5.3 are symmetric.

Let us call the object function of Figure 5.3 as $f(x)$, and it is given by

$$f(x) = \sum_{n=0}^{\infty} a_{2n} P_{2n}(x) \tag{5.9}$$

where, $a_0, a_2, a_4, \ldots$ are coefficients which have to be multiplied with the corresponding Legendre polynomials to get the required characteristics, shown in Figure 5.3.
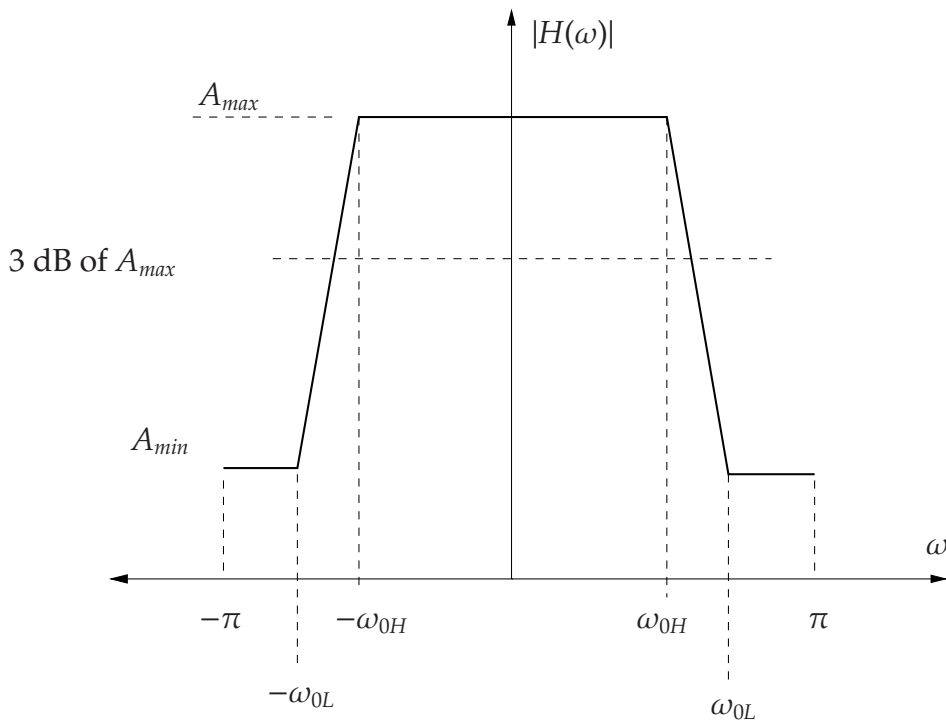
Figure 5.2: Desired characteristics of low pass filter.

Let us discuss the procedure to design the filter step by step.

Step 1 : To calculate the coefficients $(a_0, a_1, a_2, \ldots)$ we obtain a set of equations by multiplying Equation (5.9) by $P_0(x)$, $P_2(x), \ldots$ one by one and integrating over the interval $[-1, 1]$ [3]this leads to following array of equations

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even powered Legendre polynomial terms is given in Appendix A.

[3]See Appendix A
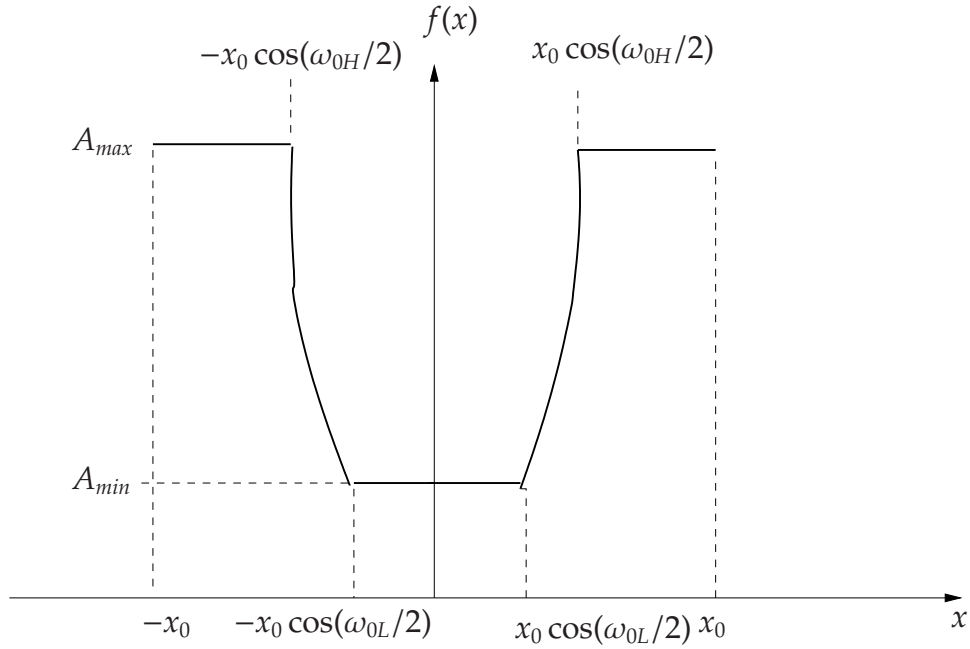
Figure 5.3: Object function, for filter characteristics of Figure 5.2, to be approximated using Legendre polynomials.

$$2 \int_0^1 f(x)P_0(x)dx = 2 \int_0^1 \left(\sum_{n=0}^{\infty} a_{2n} P_{2n}(x) \right) P_0(x)dx \qquad (5.10)$$

$$2 \int_0^1 f(x)P_2(x)dx = 2 \int_0^1 \left(\sum_{n=0}^{\infty} a_{2n} P_{2n}(x) \right) P_2(x)dx \qquad (5.11)$$

$$2 \int_0^1 f(x)P_4(x)dx = 2 \int_0^1 \left(\sum_{n=0}^{\infty} a_{2n} P_{2n}(x) \right) P_4(x)dx \qquad (5.12)$$

$$\vdots$$

The number of equations which we have to solve depends on how many coefficients we desire, or in other words how many Legendre polynomials we are going to use for the approximation of our object function, this point will be clear soon.

Using the orthogonality property Equations (5.10), (5.11), ... reduce to

$$\int_0^1 f(x)P_0(x)dx = \int_0^1 a_0(P_0(x)P_0(x))^2 dx \qquad (5.13)$$

$$\int_0^1 f(x)P_2(x)dx = \int_0^1 a_2(P_2(x)P_2(x))^2 dx \qquad (5.14)$$

$$\vdots$$

From the above equations, it is clear that the coefficients $a_0, a_2, \ldots$ are calculated directly by using

$$a_i = \frac{\int_0^1 f(x)P_i(x)dx}{\int_0^1 P_i(x)P_i(x)dx} \qquad (5.15)$$

where, $i = 0, 2, 4, \ldots$.

Step 2 : The coefficients are replaced in Equation (5.9), and the resulting approximate polynomial $f_a(x)$ is calculated. Note that $f_a(x)$ has a limited number of terms, up to N, when compared with $f(x)$ Equation (5.9); that is,

$$f_a(x) = \sum_{n=0}^{N} a_{2n}P_{2n}(x) \qquad (5.16)$$

Step 3 : To transform from polynomial (or $'x'$) domain to frequency (or $'\omega'$) domain we use the transformation, Equation (5.8). We choose x_0 to be 1, albeit one can choose any value.

Step 4 : The zeros of $f_a(x)$ are calculated using any standard routine and the zeros of $f_a(\omega)$ are obtained by using

$$x_i \xrightarrow{\mathcal{T}} \omega_i$$

here x_is are the zeros of $f_a(x)$ and $\mathcal{T}$ is the transformation. The zeros of $H(z)$ are $z_i = exp(j\omega_i)$, where $i = 1, 2, 3 \ldots$.

Step 5 : Using these zeros we calculate the transfer function in z domain; that is,

$$H(z) = (z - z_1)(z - z_2)(z - z_3) \ldots \qquad (5.17)$$

and replace $z = e^{j\omega}$ to find $H(\omega)$.

5.4 Application and Discussion

Let us design a filter with following values (refer Figure 5.2)

$$A_{max} = 1000,$$

$$A_{min} = 0,$$

$$\omega_{OL} = 2.3186,$$

$$\omega_{OH} = 2.0007,$$

$$\omega_{3dB} = 2.0944.$$

Then, the "object function" characteristics are calculated to be

$$f(x) = \begin{cases} 0 & \text{if } 0 \le x < 0.4 \\ 7142.86(x - 0.4) & \text{if } 0.4 \le x < 0.54 \\ 1000 & \text{if } 0.54 \le x < 1.0 \end{cases} \qquad (5.18)$$

Let us consider some specific cases for above mentioned values.

5.4.1 Design 1

As we keep increasing the number of terms to approximate the object function we get better and better results. It is up to the designer, where he would like to stop. For example, when ten even Legendre polynomial terms are used to approximate the object function then

$$f_a(x) = a_0 P_0 + a_2 P_2 + a_4 P_4 + a_6 P_6 + \ldots + a_{18} P_{18} \qquad (5.19)$$

where, $f_a(x)$ is the approximation to $f(x)$.

Using Equation (5.15) we calculate the coefficients a_0 to a_{18}, and

$$f_a(x) = 530 + 909.685 P_2 - 586.385 P_4 - 4.6429 P_6 + 401.108 P_8 + \ldots \qquad (5.20)$$

The approximated, together with ideal, object function is shown in Figure 5.4. It is clear from the figure that approximated object function follows

the user defined characteristics, but not very closely (in Design 2 we try to improve this).

Using the transformation described in Step 3, we transform $f_a(x)$ to frequency domain. Step 4 will give the position of the eighteen zeros in the frequency domain.

Following Step 5 we can write $H(z)$ as

$$H(z) = \prod_{i=1}^{18}(z - z_i) \tag{5.21}$$

where, z_i's are $e^{j\omega_i}$; $i = 1, 2, \ldots 18$. This $H(z)$ gives the magnitude response shown in Figure 5.5 and is following the required filter characteristics, also shown in the figure. Magnitude response in dB is shown in Figure 5.6. If we look at it we found that the pass band is very linear and stop bands are approximately 25dB down from the pass band.
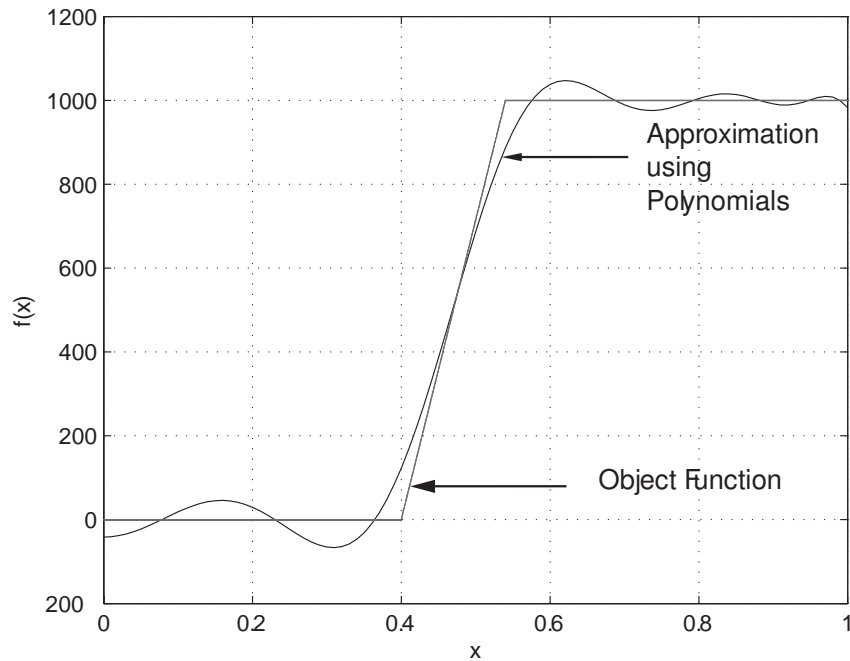
Figure 5.4: Approximation of object function using first 10 Legendre polynomials terms, P_0 to P_{18} (the object function here is not showing the non linearities for simplicity).
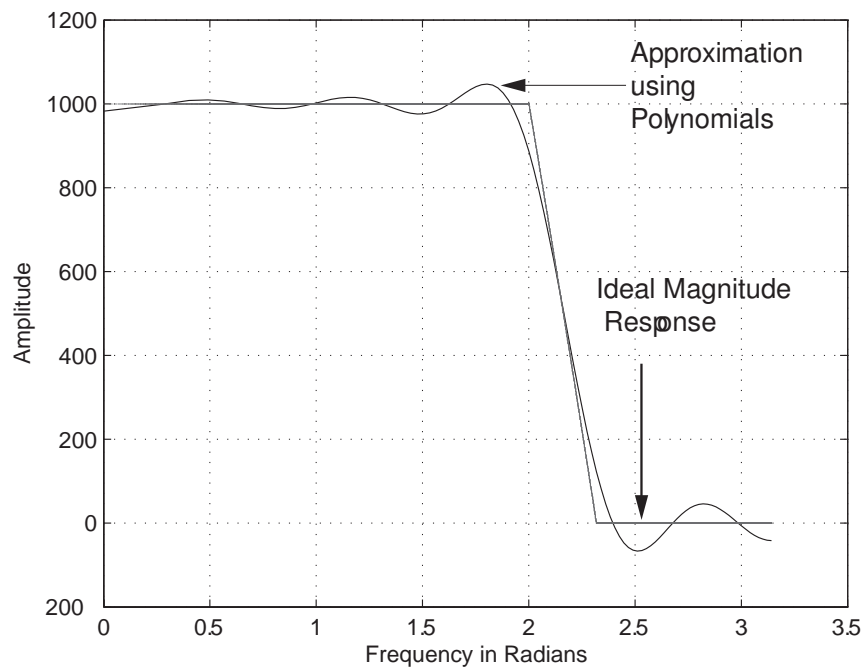
Figure 5.5: Magnitude response of the low pass FIR filter corresponding to the object function shown in Figure 5.4.
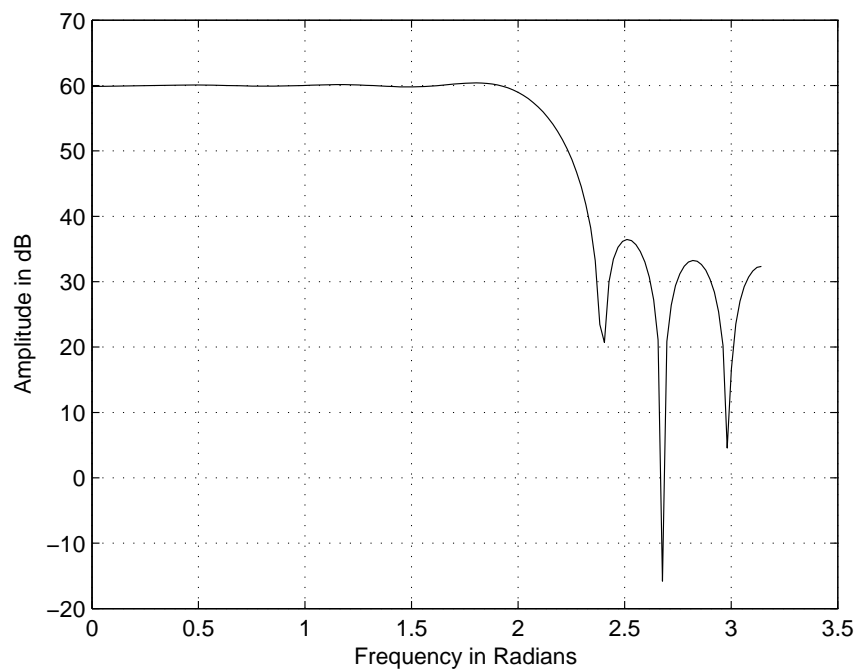
Figure 5.6: Magnitude response in dB of the low pass FIR filter corresponding to the object function shown in Figure 5.4.

5.4.2   Design 2

Let us approximate our object function of Equation (5.18) using 17 even Legendre polynomial terms. We get the magnitude response and the magnitude response in dB as shown in Figures 5.7 and 5.8, respectively. The object function and thus magnitude response are following the required characteristics more closely than in Design 1 (Figure 5.5), which shows that as we increase the number of terms to approximate our object function we get better approximation of magnitude response. The dB magnitude response (Figure 5.8) shows that the first side band is approximately 35dB down; that is, there is an improvement of 10dB over previous design. Figure 5.9 shows the phase response for the present design, the phase is exactly linear.
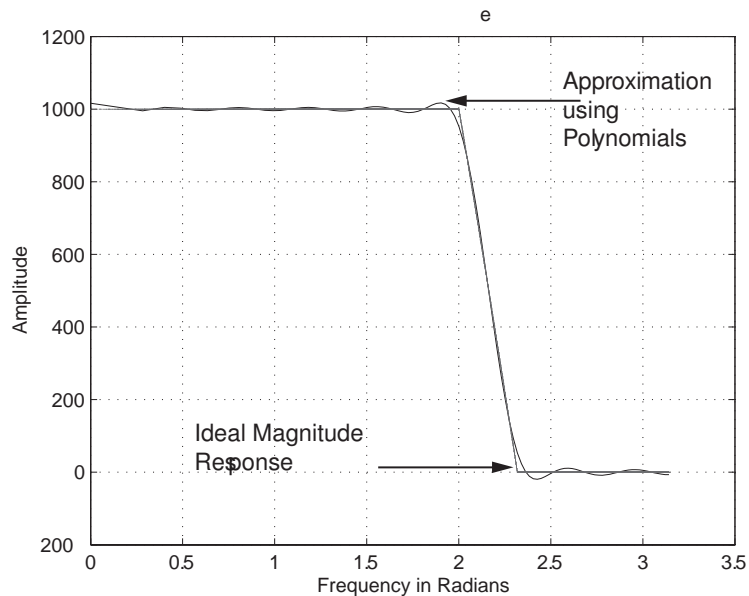
Figure 5.7: Magnitude response of low pass FIR filter when object function is approximated using 17 Legendre polynomial terms, P_0 to P_{32}.

5.4.3   Design 3

Following the above procedure we can easily design a high pass filter also. Let us consider the following values for the filter to be designed

$$A_{max} = 1000,$$
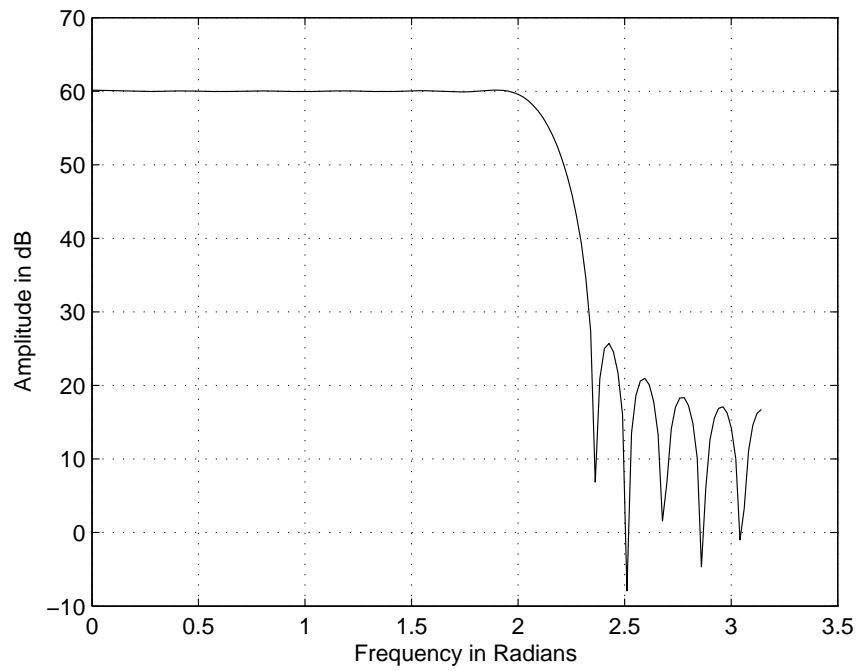
Figure 5.8: Magnitude response in dB of low pass FIR filter when object function is approximated using 17 Legendre polynomial terms, P_0 to P_{32}.
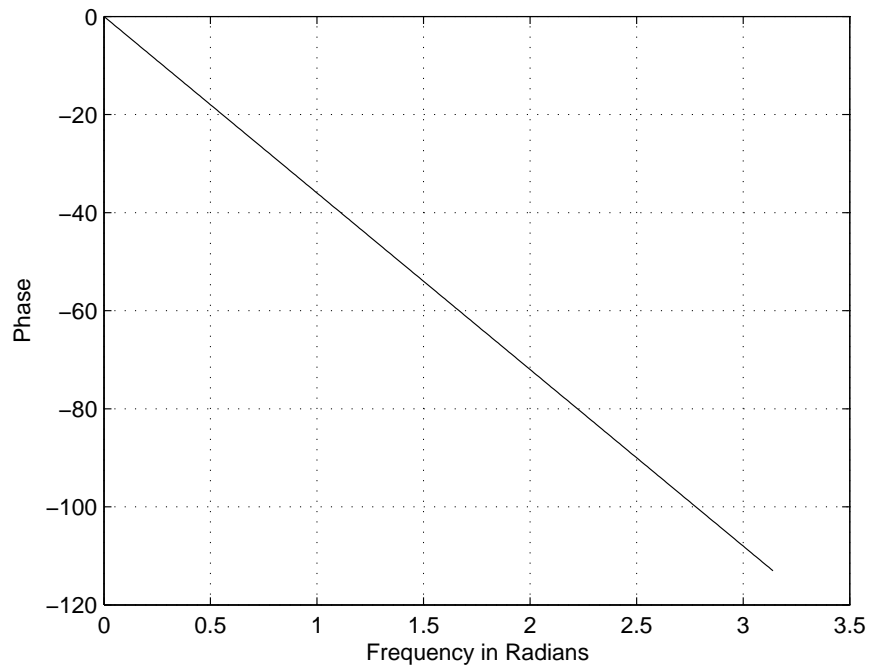
Figure 5.9: Phase response of low pass FIR filter when object function is approximated using 17 Legendre polynomial terms, P_0 to P_{32}.

$$A_{min} = 0,$$

$$\omega_{OH} = 2.3186,$$

$$\omega_{OL} = 2.0007,$$

$$\omega_{3dB} = 2.0944.$$

Figures 5.10 and 5.11 show the magnitude response, and magnitude response in dB, respectively, of the high pass filter. The high pass filter is derived from the object function which is approximated using 21 Legendre polynomials.
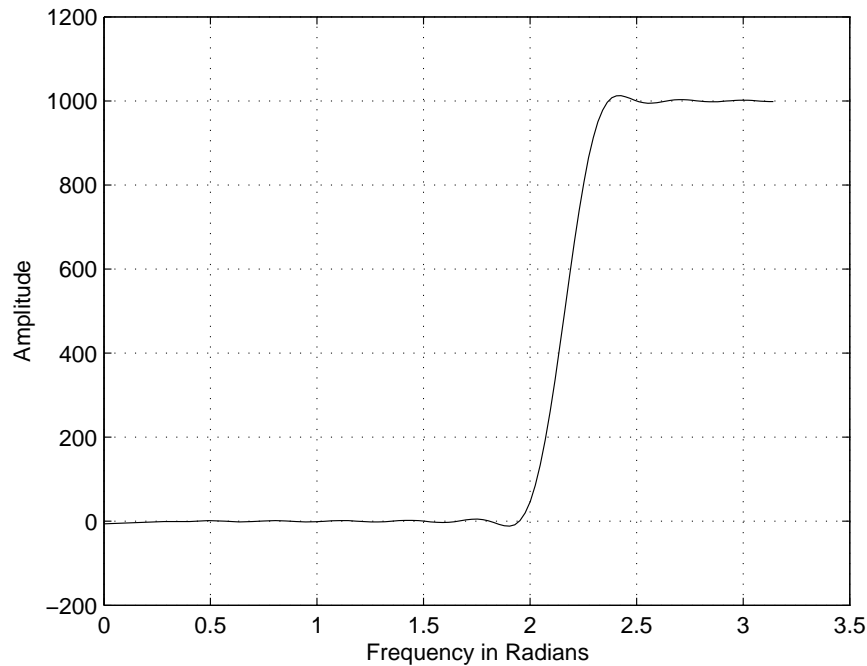
Figure 5.10: Magnitude response of high pass FIR filter when object function is approximated using 21 Legendre polynomial terms, P_0 to P_{40}.

It is clear from the Figures 5.10 and 5.11 that the pass band of the resulting high pass filter is very flat, phase of this filter is absolutely linear, and the realized filter closely follows the restrictions; that is, cutoff frequency and pass band frequency.
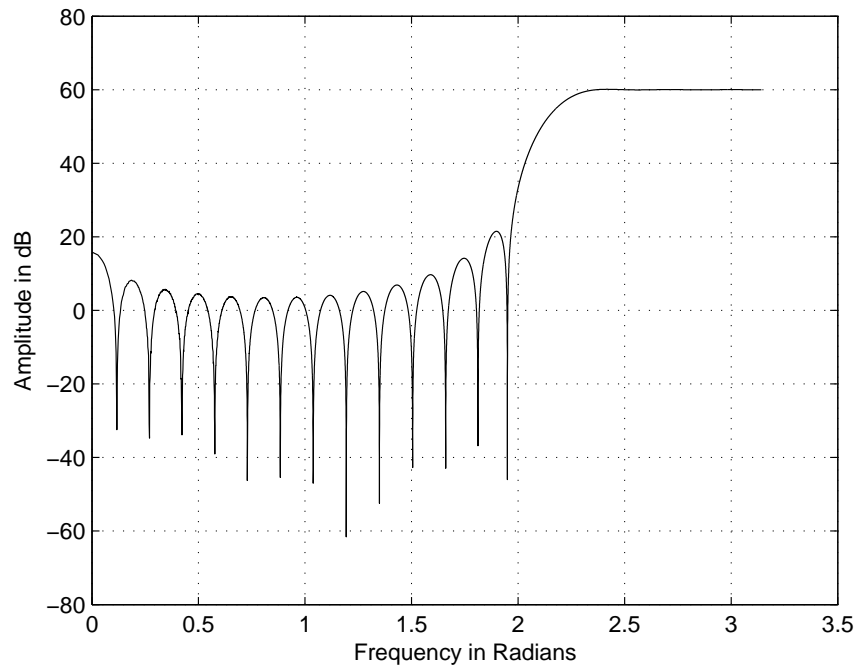
Figure 5.11: Magnitude response in dB of high pass FIR filter when object function is approximated using 21 Legendre polynomial terms, P_0 to P_{40}.

5.4.4 Design 4

Next we design a band pass filter with the transition band, stop band, and pass band characteristics as follows

$$A_{max} = 1000,$$

$$A_{min} = 0,$$

$$\omega_{OL1} = 1.5908,$$

$$\omega_{OH1} = 2.0944,$$

$$\omega_{OL2} = 2.7389,$$

$$\omega_{OH2} = 2.5322.$$

Note that the filter has unequal transition bands.

The magnitude response, and magnitude response in dB are calculated using 21 Legendre polynomial terms to approximate the corresponding
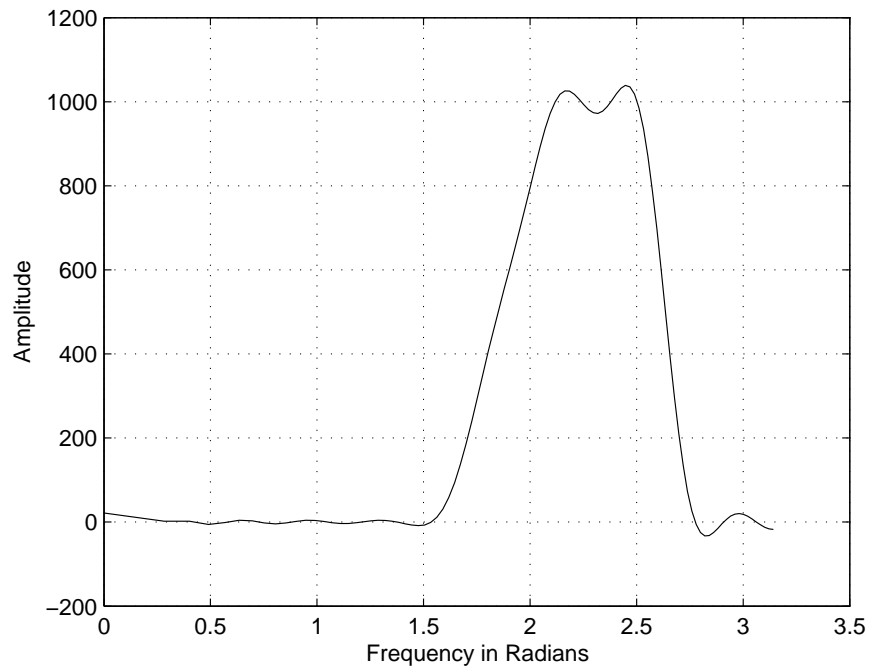
Figure 5.12: Magnitude response of band pass FIR filter when object function is approximated using 21 Legendre polynomial terms, P_0 to P_{40}.
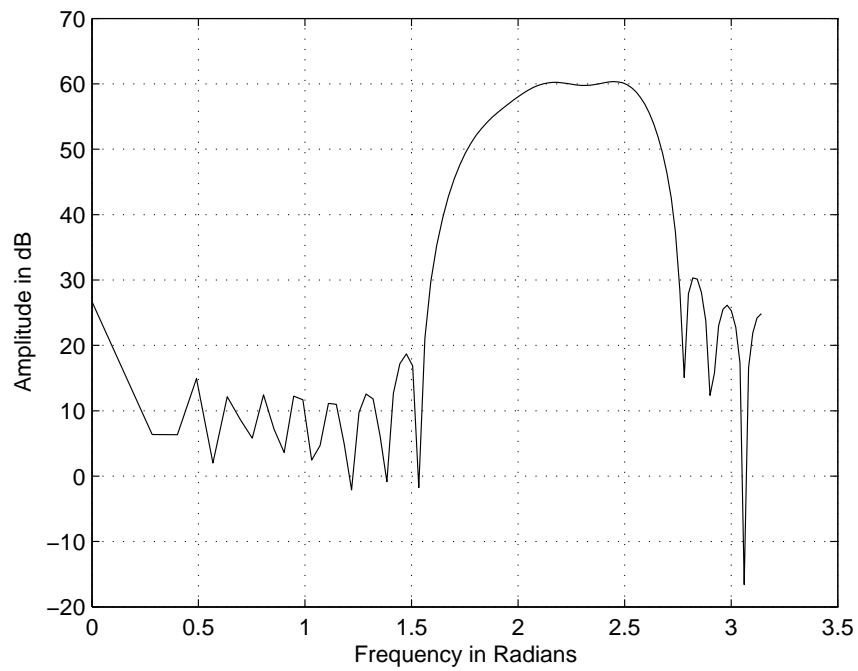
Figure 5.13: Magnitude response in dB of band pass FIR filter when object function is approximated using 21 Legendre polynomial terms, P_0 to P_{40}.

object function and are shown in Figures 5.12 and 5.13, respectively. Un-
equal side bands are distinguishable in Figure 5.12. The pass band is not as
flat as we need but if we increase the number of terms, used to approximate
object function, it will become better. The phase response remains linear
and can be verified easily.

5.5 Results

We can conclude that as we increase the number of terms in our polynomial,
the approximation of the object function becomes more and more like the
desired object function (see Figures 5.5 and 5.7).
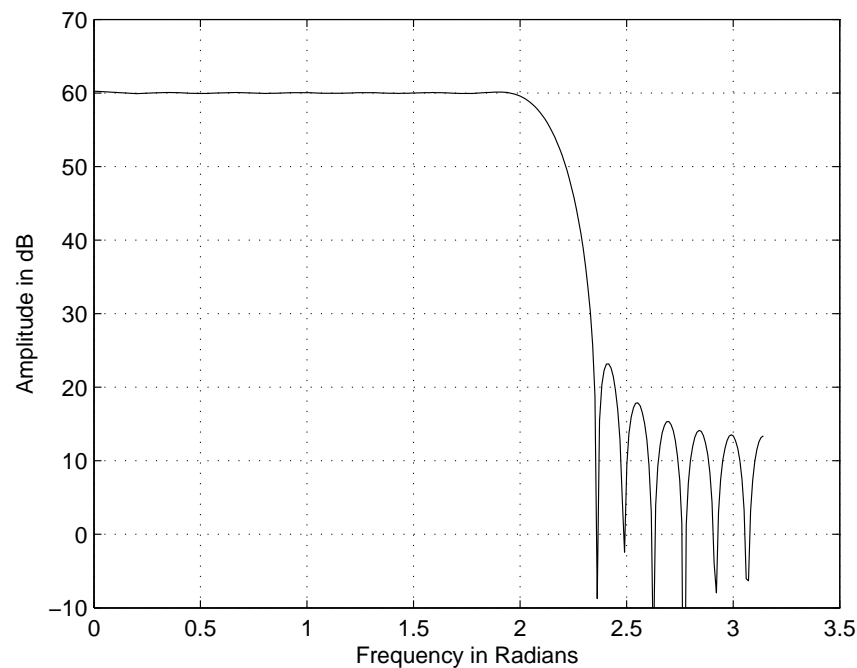
Figure 5.14: Magnitude response in dB of low pass FIR filter when object
function is approximated using 21 Legendre polynomial terms, P_0 to P_{40}.

Figures 5.6, 5.8, and 5.14 show a steady improvement in all round
performance in the magnitude responses. It is evident from the figures
that the sharpness of the transition band, side band level and ripple in the
pass band are all improved. We can also easily deduce from the figures
that the filter characteristics are in accordance with the defined values of
ω_{OL}, ω_{OH} and ω_{3dB}.

Figures 5.11 and 5.13 make it clear that the method proposed can easily be implemented to design FIR filters of another kind; that is, high pass, bandpass, as well as band reject, multiband, and any other type, all we have to do is write down the corresponding object function characteristics and follow the procedure discussed.

Important Note: If we approximate the "ideal brick wall characteristics"; that is, with an abrupt change at a particular point in place of a gradual change, the results show serious limitations because of Gibbs type of phenomenon. In the case of an abrupt change the approximation gives us a flat pass band but it turns out that the stop band does not reduce below a certain value irrespective of how ever many terms we use.

5.6   Conclusion

An alternative simple approach has been presented to design the linear phase FIR filters, whose characteristics are modeled using orthogonal polynomials. The orthogonal polynomials give us good frequency domain characteristics in terms of sharp cutoff, low stop band, low ripple in the pass band and linear phase. These may be improved to any desired level by increasing the number of terms used to approximate the object function. Further, there is no restriction on the type of filter to be designed.

6 Design of 2 Dimensional Linear Phase FIR Filter with Orthogonal Polynomials

6.1   Introduction

Due to rapid development in the field of image processing, sonar signal processing and other related fields, the design of 2 dimensional (2D) digital filters has become very important. 2D finite impulse response (FIR) filters are preferred over infinite impulse response (IIR) filters because of their inherent stability. These 2DFIR filters may have linear phase[1].

Various techniques have been discussed in the literature to design 2DFIR digital filters. If we take into consideration least square (LS) error criterion, we observe that an overshoot of the frequency response at the pass band and the stop band edges occur due to Gibb's phenomenon [43]. Whereas, a minmax design approach results in an equiripple solution [44, 45].

Efficient 2DFIR filters can also be designed by using transformations. J. H. McClellen [46, 47] has proposed a powerful technique, where 1DFIR filters are used with a transformation to design 2DFIR filters. This technique is very efficient and fast. This technique was implemented by [48]. But in this technique the transformation steps are very complicated and difficult to implement, and can be used to design 2D square filters only.

What we are presenting here is a new method to design 2DFIR filter with linear phase through the transformation and orthogonal polynomials. The whole procedure is simple and produce excellent results. Any type of filter can be designed by this approach.

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[1]It is intuitively clear that non linear phase will introduce a distortion.

6.2 Procedure

Because we are going to design an orthogonal polynomial based 2D FIR filter, therefore first we have to calculate 2D orthogonal polynomials.

We know that a sequence of orthogonal polynomials, $P_n(\rho)$ in cylindrical coordinates, must satisfy the relation

$$\langle P_i(\rho), P_j(\rho) \rangle = \int_0^{2\pi} \int_0^1 P_i(\rho)P_j(\rho)\rho d\rho d\phi = 0 \qquad i \neq j \qquad (6.1)$$

where,

$P_i(\rho)$ and $P_j(\rho)$ are any two members of the orthogonal set,

$\langle \bullet, \bullet \rangle$ represents the inner product, and

$\rho d\rho d\phi$ is the element of area.

The polynomials can be obtained by applying the well known *Graham-Schmidt* procedure [49] over the interval $[-1, 1]$ for ρ. Equation (6.2) represents some of the even polynomials.

$$P_0(\rho) = 1,$$
$$P_2(\rho) = 1 + 2\rho^2,$$
$$P_4(\rho) = 1 - 6\rho^2 + 6\rho^4,$$
$$P_6(\rho) = 1 - \tfrac{192}{31}\rho^2 + \tfrac{190}{31}\rho^4 + \tfrac{20}{31}\rho^6$$
$$\vdots \qquad\qquad\qquad (6.2)$$

Figure 6.1 shows some of these polynomials.

The filters which we propose to design are circularly symmetric 2DFIR filters, half of the symmetric magnitude response of such a filter is shown in Figure 6.2, other half being mirror image. Since the design is circularly symmetric, we use the cylindrical co-ordinate system $-(\rho, \phi)$ – to represent the object function in 2D. ρ is a mapping function and is related to Cartesian coordinates (x, y) by

$$\rho^2 = x^2 + y^2 \quad -1 \leq \rho \leq 1 \qquad (6.3)$$

Since the filter is circularly symmetric, therefore, if we take the cross section of the 2D FIR filter perpendicular to $u - v$ plane passing through
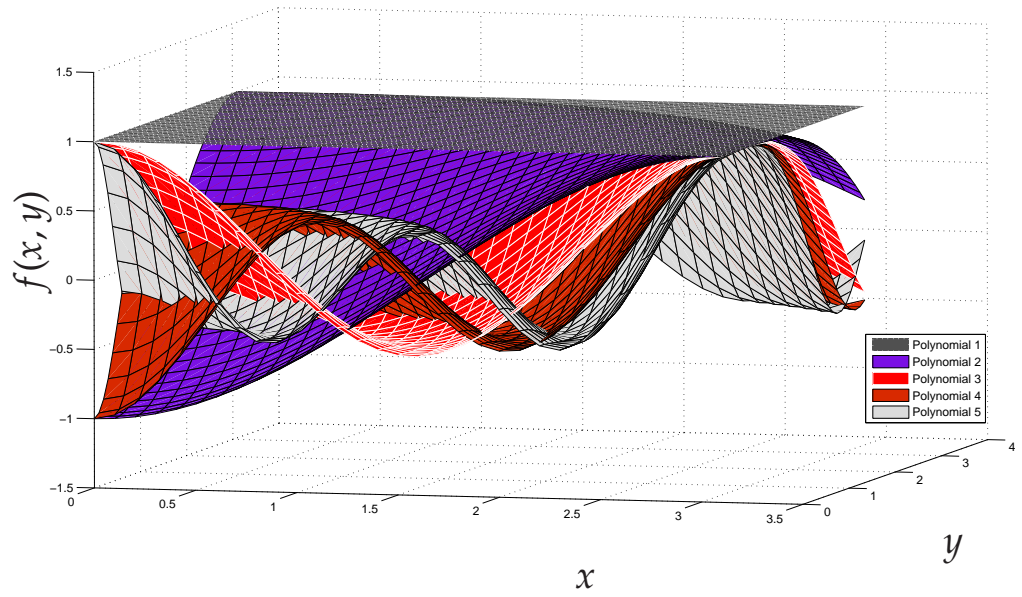
Figure 6.1: First few 2D orthogonal polynomials calculated using Equation (6.1).

the origin, it will look like a 1D filter. Figure 6.3 shows the 1D equivalent magnitude response for the filter shown in Figure 6.2. In Figure 6.3

A_{max} represents the amplitude of the pass band of the filter,

A_{min} represents the amplitude of the stop band of the filter,

ω_{OH} is the end of pass band and start of the transition band, and

ω_{OL} is the end of transition band and start of the stop band.

The relationship between the 1D frequency axis, ω-axis, of Figure 6.3 and 2D frequency axis, u, v-axis, of Figure 6.2 is given by

$$\omega^2 = u^2 + v^2 \quad -\pi \leq \rho \leq \pi \quad (6.4)$$

The transformation used for conversion from object function $-\rho$ domain $-$ to frequency characteristics $-\omega$ domain $-$ is carried out by

$$\rho = \rho_0 \cos\left(\frac{\omega}{2}\right) \quad -\pi \leq \omega \leq \pi \quad (6.5)$$

Because the filter is circularly symmetric, therefore, the object function will also be circularly symmetric around the z-axis. Figure 6.4 shows this object function in 3D and its corresponding 1D representation in Figure 6.5.
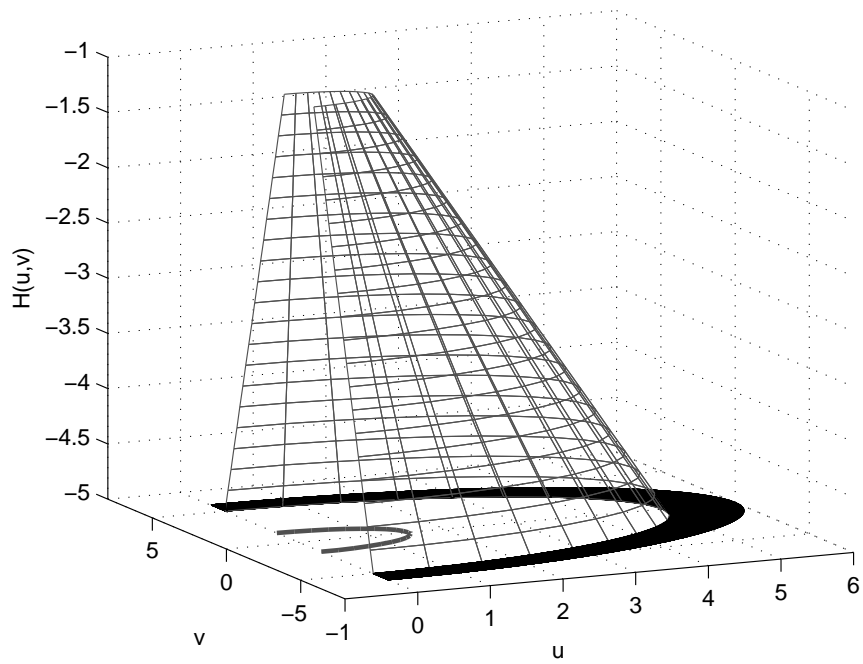
Figure 6.2: 3D desired filter response.
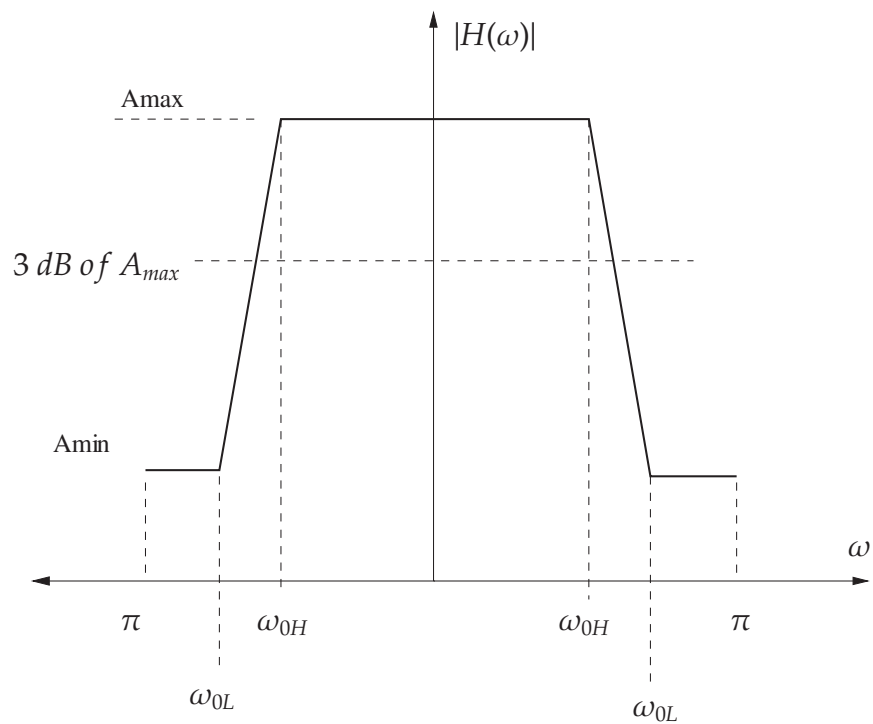
Figure 6.3: 1D representation of a low pass 2D filter characteristics.
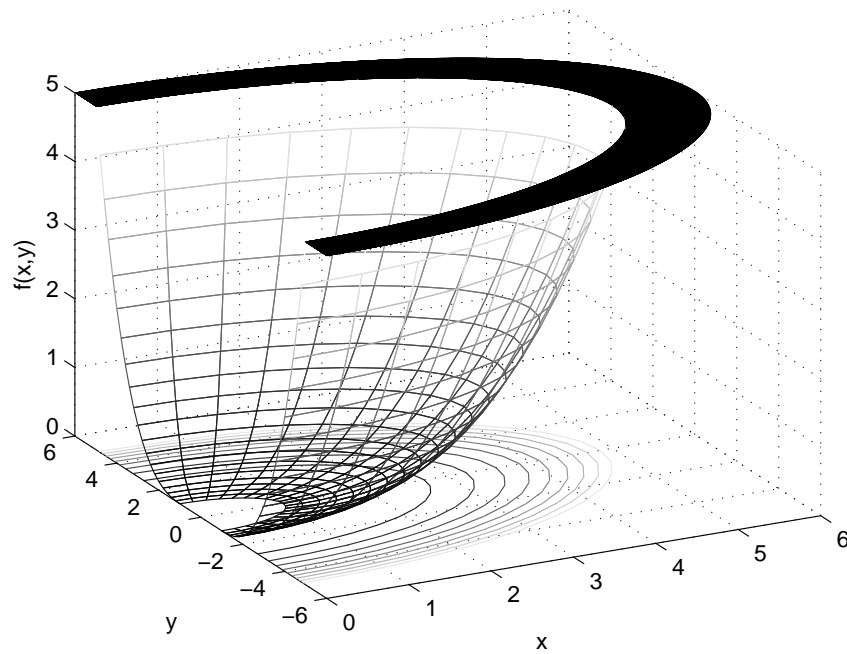
Figure 6.4: 3D object function for filter characteristics of Figure 6.2, showing slight non linearity in the transition region.
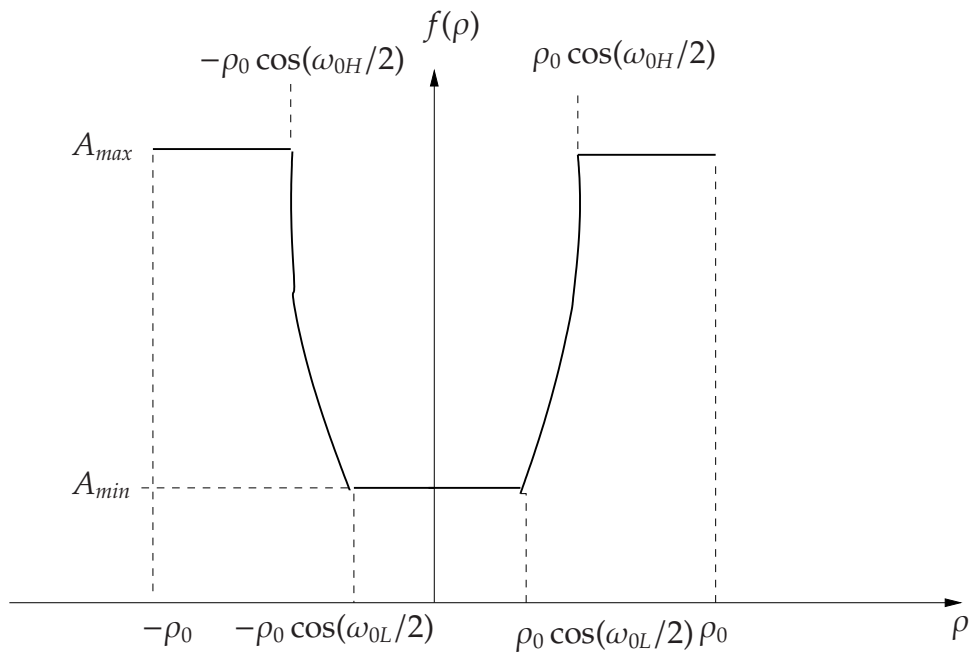
Figure 6.5: 1D object function, to be approximated using Legendre polynomials.

To transform the 2D filter function, in terms of $u - v$, to 2D object function, in terms of $x - y$, we use

$$u = 2cos^{-1}(x/\rho_0) \quad \pi \le |u|, \rho_0 < |x|$$
$$v = 2cos^{-1}(y/\rho_0) \quad \pi \le |v|, \rho_0 < |y| \tag{6.6}$$

$$f(x, y) \xrightarrow{\mathcal{T}} H(u, v)$$

where, ρ_0 is the maximum value of ρ and $\mathcal{T}$ is the transformation of Equation (6.6), it is also the maximum value of x and y, Equation (6.3).

We approximate the object function using a linear combination of several orthogonal polynomials [2], calculated above. Note that approximation of the object function will be done using even polynomials only; that is, $P_0(\rho), P_2(\rho), \dots$, because the object function and frequency response both are symmetric in nature, and polynomials with even power will give us the required symmetrical characteristics.

Let us denote the object function of Figure 6.4 by $f(\rho)$ (recall Equation (6.3)), which can be written as

$$f(\rho) = \sum_{n=0}^{\infty} a_{2n} P_{2n}(\rho) \tag{6.7}$$

where, $a_0, a_2, \dots$ are coefficients to be multiplied with orthogonal polynomials to get the required characteristics, shown in Figure 6.4. To calculate these coefficients, $a_0, a_2, \dots$, we use [3]

$$a_i = \frac{\int_0^1 f(\rho) P_i(\rho) \rho d\rho}{\int_0^1 P_i(\rho) P_i(\rho) \rho d\rho} \tag{6.8}$$

where, $i = 0, 2, 4, \dots$.

We calculate the approximate object function $f_a(\rho)$ by

$$f_a(\rho) = \sum_{i=0}^{N} a_i P_i(\rho) \tag{6.9}$$

---

[2]Any function can be represented as a linear combination of several orthogonal polynomials, refer Appendix A.

[3]Refer Appendix A.

Note that $f_a(\rho)$ tends to $f(\rho)$ when we use infinite number of terms to approximate the object function. Therefore, as we increase number of terms in $f_a(\rho)$ we get better approximation of $f(\rho)$.

This approximate object function is then transformed from (x, y) or ρ domain to frequency domain, or (u, v) or ω domain, by using Equation (6.5).

The zeros of $f_a(\rho)$ can be calculated by using any standard routine, and zeros of $f_a(\omega)$ are obtained by using

$$\rho_i \xrightarrow{\mathcal{T}} \omega_i$$

here ρ_is are the zeros of $f_a(\rho)$, and ρ is transformation of Equation (6.5).

6.3 Application and Discussion

Suppose we want to design a 2DFIR filter with transfer function defined as

$$|H(\omega)| = \begin{cases} 1000 & 0 \le \omega < 1.5908 \\ \text{transition band} & 1.5908 \le \omega < 2.0944 \\ 10 & 2.0944 \le \omega < \pi \end{cases} \tag{6.10}$$

Comparing these characteristics with Figure 6.2, we note that

$$A_{max} = 1000,$$

$$A_{min} = 10.$$

First we find the corresponding object function by using the transformation given in Equation (6.5). The object function comes out to be

$$f(\rho) = \begin{cases} 10 & 0 \le \rho < 0.5 \\ \text{transition band} & 0.5 \le \rho < 0.7 \\ 1000 & 0.7 \le \rho < 1.0 \end{cases} \tag{6.11}$$

Let us design this filter.

6.3.1 Design 1

To calculate the value of $f_a(\rho)$ (Equation (6.9)), we first calculate the values of a_i's using Equation (6.8). As mentioned earlier, when we increase the number of terms to approximate our object function the approximation becomes better and better. Therefore, it is up to the designer where he wants to stop.

Let us approximate the object function using 8 orthogonal polynomials. We calculate the coefficients using Equation (6.8). The approximate object function is

$$
\begin{aligned}
f_a(\rho) \;=\; & 750P_0 - 562.5P_2 - 468.75P_4 + 59.04P_6 + 263.6712P_8 \\
& +306.152P_{10} + 66.65048P_{12} - 198.4407P_{14} \qquad\qquad (6.12)
\end{aligned}
$$

We apply Equations (6.3) and (6.6) on the approximated object function and get the filter in frequency domain. The magnitude response of the filter is shown in Figure 6.6, its cross section in Figure 6.7 and the magnitude response in dB is shown in Figure 6.8. When we look at Figures 6.6 and 6.7, we find that the pass band of the filter has some ripples. Figure 6.8 shows that first side band is approximately 35dB down.

In the next classification we consider the same problem while using more number of orthogonal polynomial terms to approximate the object function.

6.3.2 Design 2

In the present case we use 15 orthogonal polynomial terms to approximate the object function. This object function is then transformed using the transformation of Equation (6.5). The resulting 2DFIR is shown in Figure 6.9, the cross section in Figure 6.10 and the magnitude response in dB is shown in Figure 6.11, the first side band is approximately 45dB down.

Comparing Figures 6.7 and 6.10, it is clear that as we increase the number of terms to calculate object function, which in turn is used to construct the required filter, the sharpness of the transition band of the filter increases and ripples in the pass band decreases. The filter shown in Figure 6.11 has lower side bands than the filter represented in Figure 6.8.

Figure 6.6: Magnitude response of low pass FIR filter when object function is approximated using 8 orthogonal polynomial terms.

6.3.3 Design 3

We can easily design high pass, bandpass, band reject, and other type of filters using the same approach. We only have to calculate the coefficients required to design the filter.

Let us design a high pass filter with characteristics as below

$$|H(\omega)| = \begin{cases} 10 & 0 \leq \omega < 2.0944 \\ \text{transition band} & 2.0944 \leq \omega < 2.5322 \\ 1000 & 2.5322 \leq \omega < \pi \end{cases} \qquad (6.13)$$

The corresponding object function is

$$f(\rho) = \begin{cases} 1000 & 0 \leq \rho < 0.3 \\ \text{transition band} & 0.3 \leq \rho < 0.5 \\ 10 & 0.5 \leq \rho < 1.0 \end{cases} \qquad (6.14)$$

Suppose this object function is approximated using 15 orthogonal polynomial terms. The resulting high pass filter is shown in Figure 6.12, and

Figure 6.7: Cross section of the magnitude response of low pass FIR filter when object function is approximated using 8 orthogonal polynomial terms.
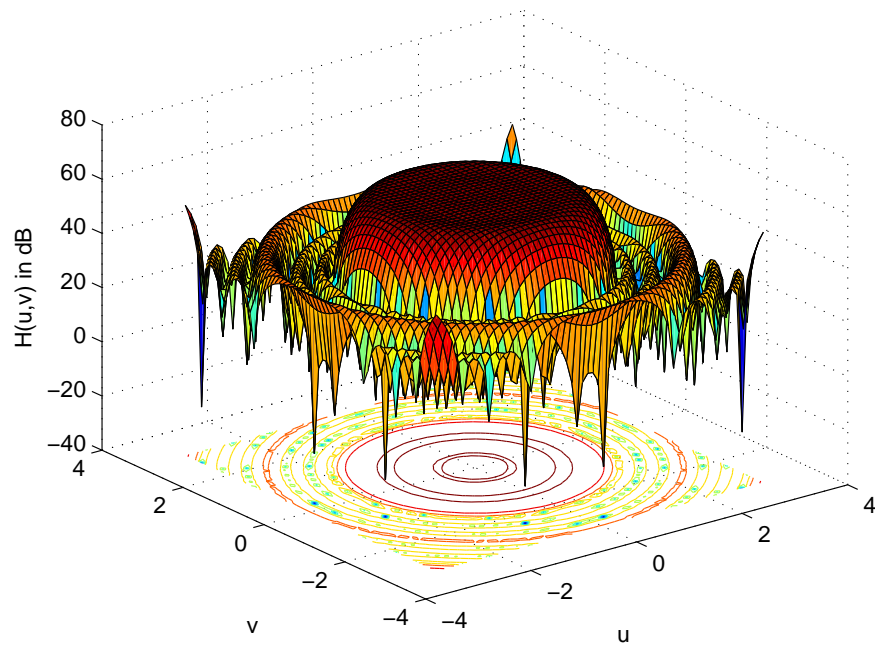
Figure 6.8: Magnitude response in dB of low pass FIR filter when object function is approximated using 8 orthogonal polynomial terms.
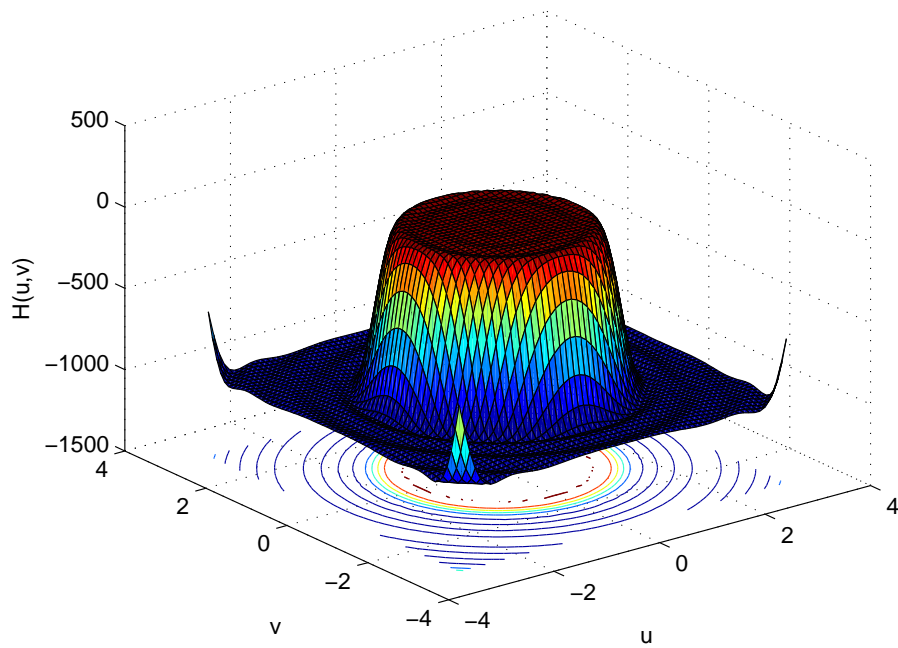
Figure 6.9: Magnitude response of low pass FIR filter when object function is approximated using 15 orthogonal polynomial terms.
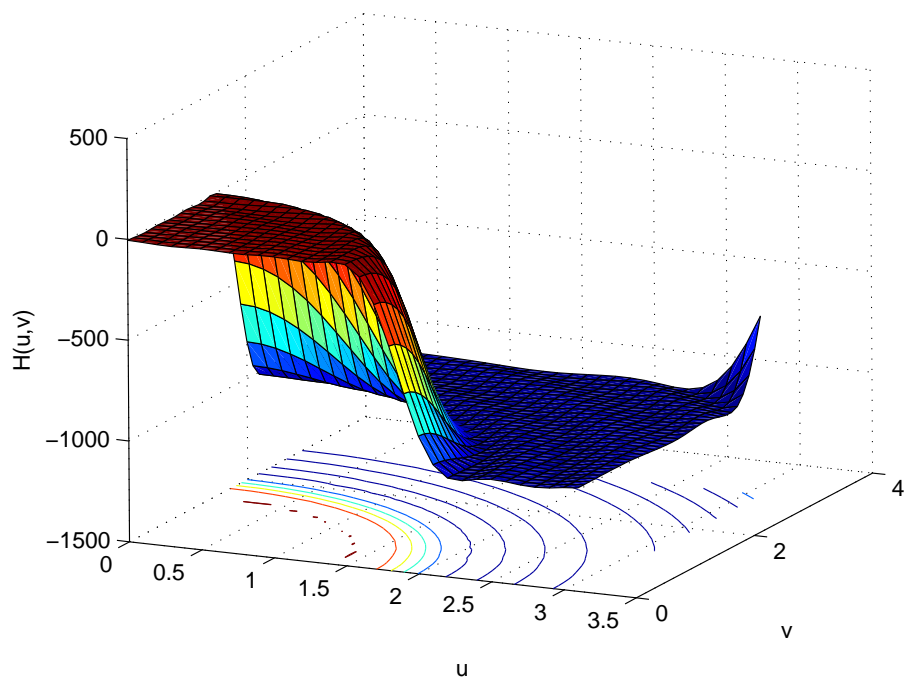
Figure 6.10: Cross section of the magnitude response of low pass FIR filter when object function is approximated using 15 orthogonal polynomial terms.
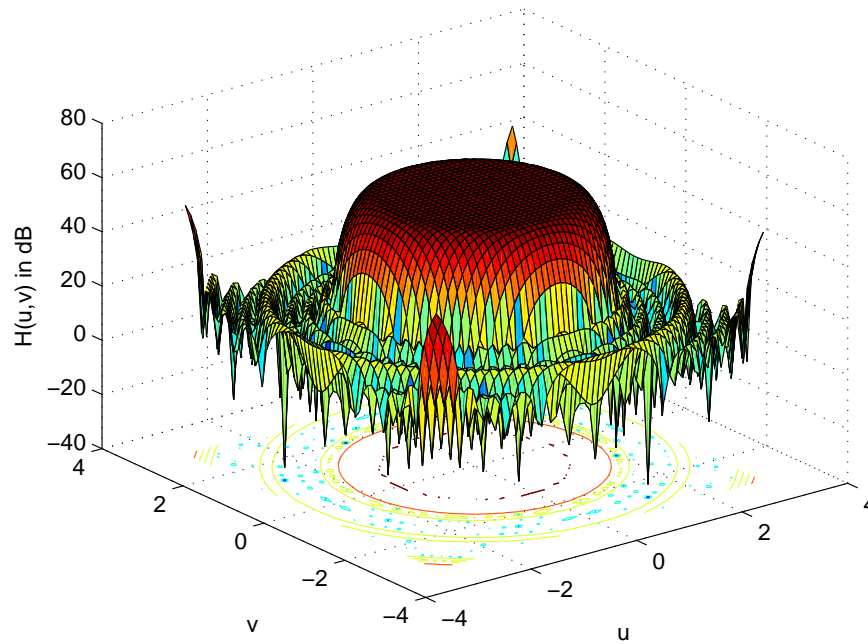
Figure 6.11: Magnitude response in dB of low pass FIR filter when object function is approximated using 15 orthogonal polynomial terms.

the cross section of this filter is represented in Figure 6.13. From the figures it is clear that the filter has flat pass band and if we pass an image through it the filter will be able to cut off non required frequencies (we will pass some images through this filter in the next section). Phase remains linear and can easily be verified.

6.3.4 Design 4

Let us design a band pass filter with frequency domain characteristics

$$|H(\omega)| = \begin{cases} 1000 & 1.4455 \le \omega < 2.6362 \\ \text{transition bands} & 1.1096 \le \omega < 1.4455 \text{ and } 2.6362 \le \omega < 2.8405 \\ 10 & 2.8405 \le \omega < \pi \text{ and } 0 \le \omega < 1.1096 \end{cases}$$

(6.15)

The corresponding object function is
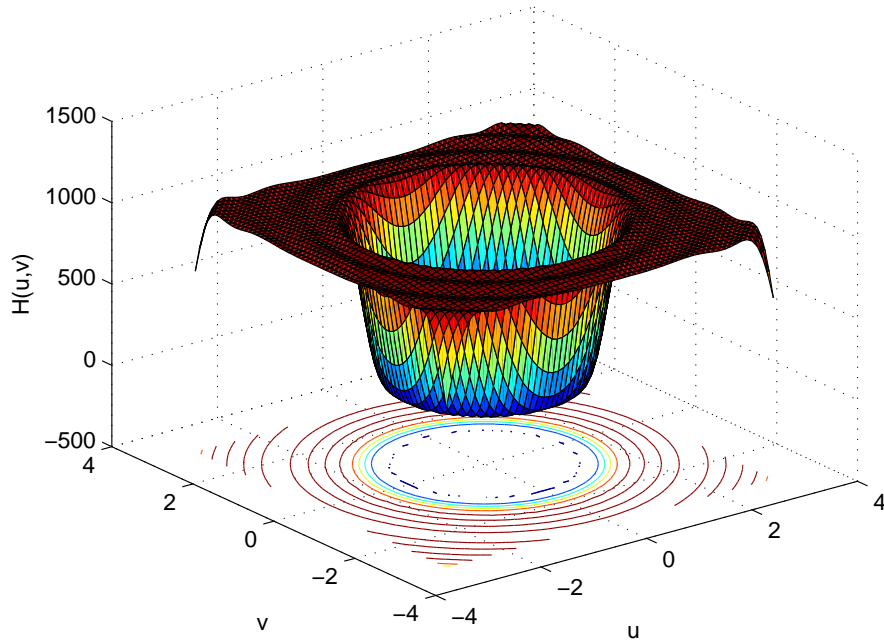
Figure 6.12: Magnitude response of high pass FIR filter when object function is approximated using 15 orthogonal polynomial terms.

$$f(\rho) = \begin{cases} 1000 & 0.25 \leq \rho < 0.75 \\ \text{transition bands} & 0.15 \leq \rho < 0.25 \text{ and } 0.75 \leq \rho < 0.85 \\ 10 & 0.0 \leq \rho < 0.15 \text{ and } 0.85 \leq \rho < 1.0 \end{cases} \quad (6.16)$$

Using the procedure discussed and approximating the object function of Equation (6.16) with 15 polynomial terms we get the required filter characteristics, which are shown in Figure 6.14. The magnitude response in dB is shown in Figure 6.15, the pass band of filter is very flat and side bands are approximately 25 dB down.

6.4   Results

The results produced by the algorithm, proposed in this chapter, give promising results. By increasing the number of terms to approximate the object function we can design better filters; that is, filters with sharp cutoff, lower side band and less ripples in the pass band. This technique can
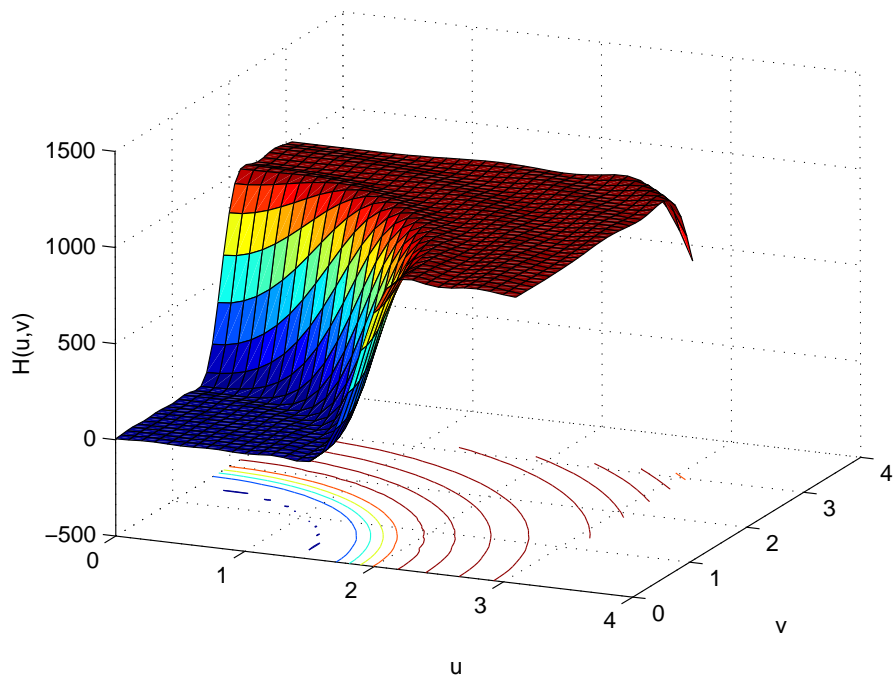
Figure 6.13: Cross section of magnitude response of high pass FIR filter when object function is approximated using 15 orthogonal polynomial terms.

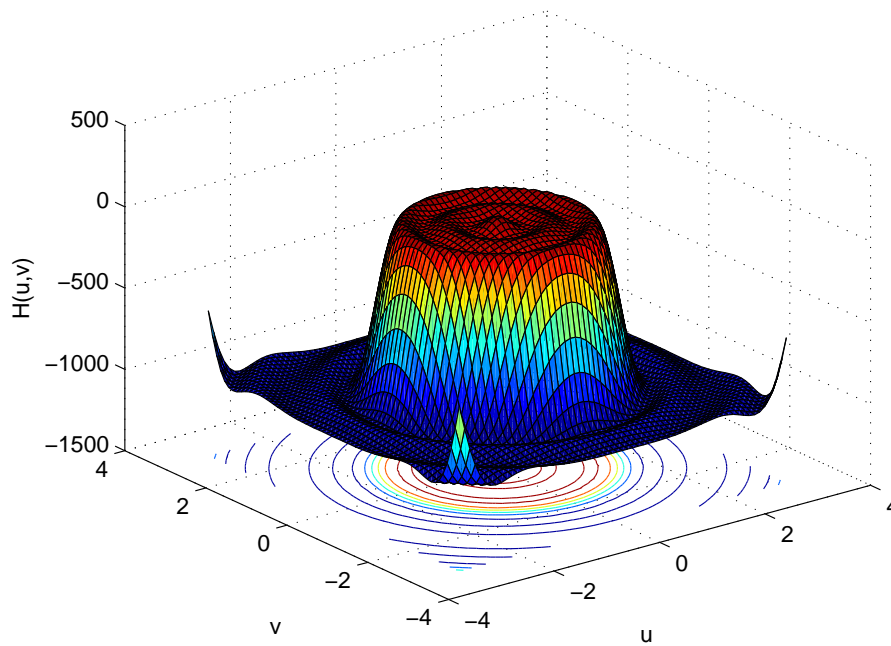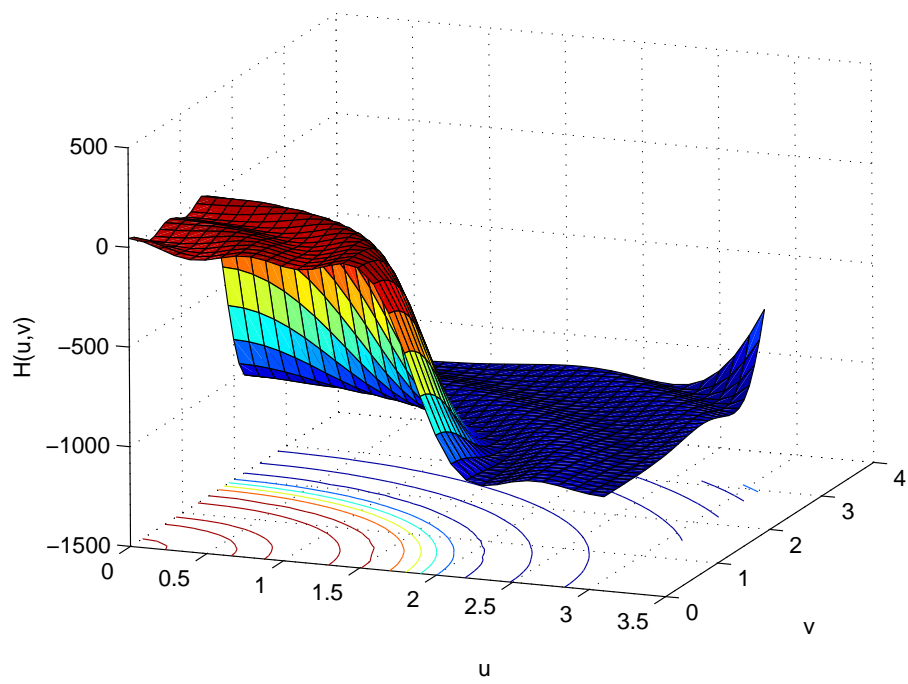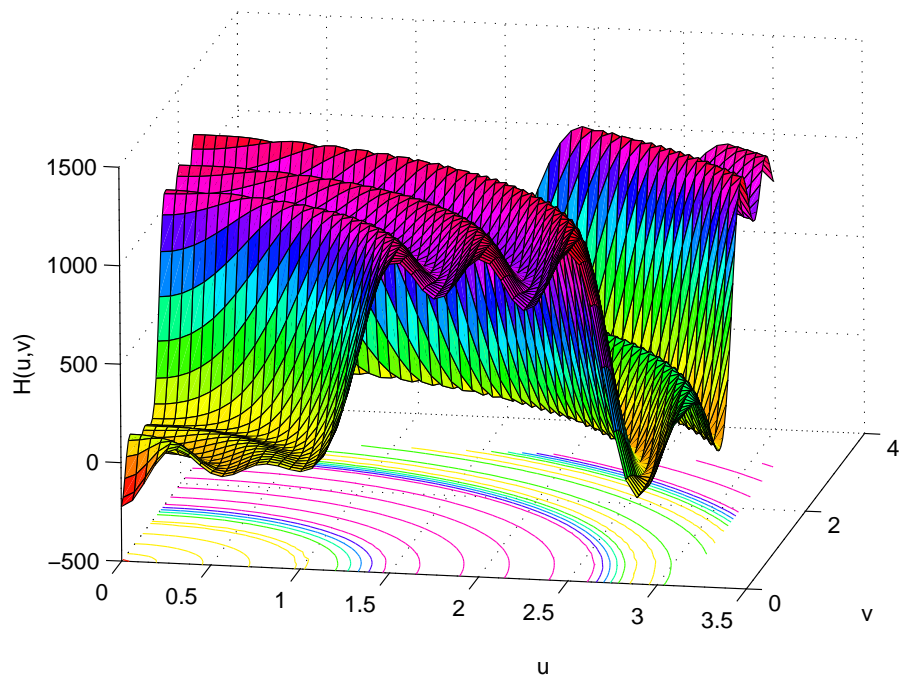Figure 6.14: Cross section of magnitude response of band pass FIR filter when object function is approximated using 15 orthogonal polynomial terms.
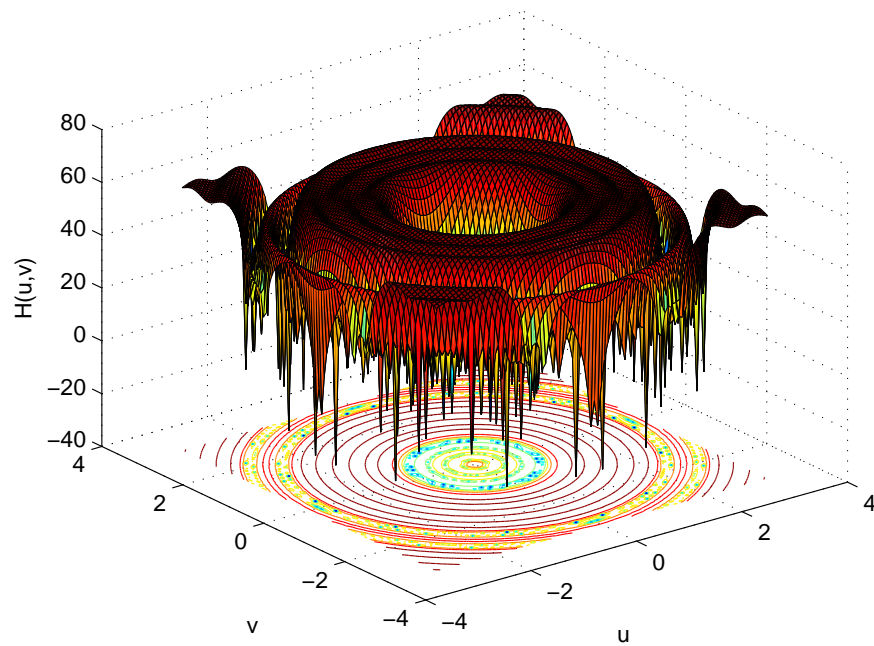
Figure 6.15: Full view of magnitude response in dB of band pass FIR filter when object function is approximated using 15 orthogonal polynomial terms.

easily be used to design low pass, high pass, band pass, band reject, and other type of filters.

If we pass some images through these filters, we can estimate the quality of our filter. Passing an image through a high pass filter means that the details of the image are kept, while the large scale gradients are removed, while in the case of low pass filter opposite of this happens.

Figure 6.16 shows an image which when passed through a high pass filter, designed using 15 orthogonal polynomial terms, becomes as shown in Figure 6.17, which depicts that only high frequency components (changes in the different gray levels of the image) are retained. When this image is passed through a low pass filter the resulting image is shown in Figure 6.18, which removes high frequency components (small details of image) of the image and only blurred details of the image are retained.

Similarly, an image with more details than the previous one is shown in Figure 6.19, its high pass filtered output is shown in Figure 6.20 and low pass in Figure 6.21, respectively.

A daily life image with a fair combination of changes is shown in Figure 6.22, its high pass filtered result is shown in Figure 6.23 and low pass filtered output is the Figure 6.24.

One more image is shown in Figure 6.25 its high pass and low pass filtered outputs are shown in Figures 6.26 and 6.27, respectively. We have shown various type of images to show that the filter designed can be used for any type of image. The filters used above are designed using 15 orthogonal polynomial terms to approximate the corresponding object function.

Figure 6.16: Image-1.

Figure 6.17: Image-1 passed through high pass filter designed using 15 orthogonal polynomial terms.

Figure 6.18: Image-1 passed through low pass filter designed using 15 orthogonal polynomial terms.

Figure 6.19: Image-2.

6.5   Conclusion

A simple new approach has been discussed here for designing 2D linear phase FIR filters. The cut off characteristics and ripples in the pass band can be controlled by using the method proposed in this chapter. As we increase the number of terms, the resulting filter becomes more flat in pass band and the transition region approximates the defined value. We can design any type of filter; that is, band pass, band reject, etc., using the present technique.

Figure 6.20: Image-2 passed through high pass filter designed using 15 orthogonal polynomial terms.

Figure 6.21: Image-2 passed through low pass filter designed using 15 orthogonal polynomial terms.

Figure 6.22: Image-3.

Figure 6.23: Image-3 passed through high pass filter designed using 15 orthogonal polynomial terms.

Figure 6.24: Image-3 passed through low pass filter designed using 15 orthogonal polynomial terms.

Figure 6.25: Image-4.

Figure 6.26: Image-4 passed through high pass filter designed using 15 orthogonal polynomial terms.

Figure 6.27: Image-4 passed through low pass filter designed using 15 orthogonal polynomial terms.

7 Design of 1 Dimensional Zero or Linear Phase IIR Filter with Orthogonal Polynomials

7.1   Introduction

In the past two to two and half decades a great deal of work has been carried out in the field of design of linear phase IIR filters. In general, design of exact linear phase IIR filter is not possible, schemes have been proposed to approximate pass band linearity. Conventionally, first the magnitude specifications of an IIR filter are met, and then all pass equalizers are applied to linearize the phase response [50,51]. Mostly IIR filters are designed with equiripple or maximally flat group delay [52]. But their magnitude characteristics are poor. Optimization techniques are used to simultaneously approximate magnitude and phase response characteristics [53,54]. To meet with the magnitude and phase characteristics at the same time, generally, linear programming is used [55]. Lu *et al* [56] gives an iterative procedure to directly design a linear phase IIR filter, it is based on a weighted least-squares algorithm. Xiao *et al* [57] discusses a method to design a linear phase IIR filter with frequency weighted least square error optimization using Broyden-Flether-Goldfarb-Shanno (BFGS) [58] method.

Model reduction approach has also been proposed by various authors [59,60]. A procedure to design linear phase IIR filter from linear phase FIR filter has been discussed by Holford *et al* [61] using frequency weighting model reduction, for highly elective filters. He [61] gives good compromise for order of the filter, pass band maximum ripple and stop band minimum attenuation. None of the techniques discussed above give perfect zero group delay IIR filter. The use of zero group delay filters makes it possible to obtain much higher data rates than with ordinary filters, since the rise time is extremely fast and there is less accumulated loss due to rise time [62,

63], various problems arising due to non zero group delay are disscussed in [64]. In the present chapter, we discuss a new technique to design the IIR filters having perfectly linear or zero phase response which consequently will lead to zero group delay. Group delay is defined as:

$$\tau_g = -\left|\frac{d\phi(\omega)}{d\omega}\right| \tag{7.1}$$

where, $\phi(\omega)$ is the phase as a function of frequency and ω is frequency in rad/s.

7.2 Preliminaries

Proposed zero group delay IIR filter design uses linear phase high pass and low pass FIR filters. To design a linear phase low pass IIR filter ($H_{iirlowpass}(\omega)$), we divide linear phase low pass FIR filter characteristics ($H_{LP(FIR)\mathbf{N}}(\omega)$, Figure 7.1) [1] with linear phase high pass FIR filter characteristics, ($H_{HP(FIR)\mathbf{D}}(\omega)$, Figure 7.2) [2]. The required FIR filters can be designed by using the procedure discussed in Chapter 5. The magnitude characteristics of two FIR filters are such that the pass band of low pass FIR filter is the stop band of high pass FIR filter and vice-versa; the transition bands overlap, when one transition band goes from lower value to higher value other goes from higher to lower. Therefore, the low pass IIR filter function is given by

$$
\begin{aligned}
H_{iirlowpass}(\omega) &= \frac{H_{LP(FIR)N}(\omega)}{H_{HP(FIR)D}(\omega)} \\
&= \frac{\left|H_{LP(FIR)N}\right| \angle H_{LP(FIR)N}}{\left|H_{HP(FIR)D}\right| \angle H_{HP(FIR)D}} \\
&= \frac{\left|H_{LP(FIR)N}\right|}{\left|H_{HP(FIR)D}\right|} \angle \left(H_{LP(FIR)N} - H_{HP(FIR)D}\right)
\end{aligned} \tag{7.2}
$$

It is clear from Equation (7.2) that at every point the amplitudes of the low pass and high pass filters will be divided and phase subtracted, to get

---

[1] The subscript indicates Low Pass FIR filter to be used as Numerator.
[2] The subscript indicates High Pass FIR filter to be used as Denominator.
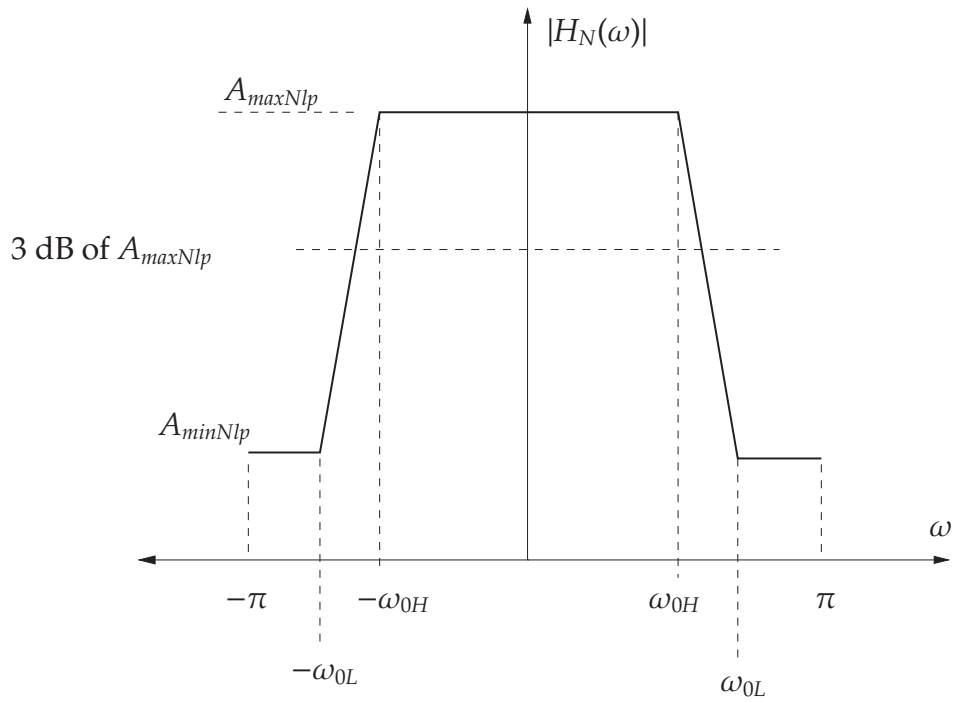
Figure 7.1: Desired filter characteristics for LPF to be used as numerator.
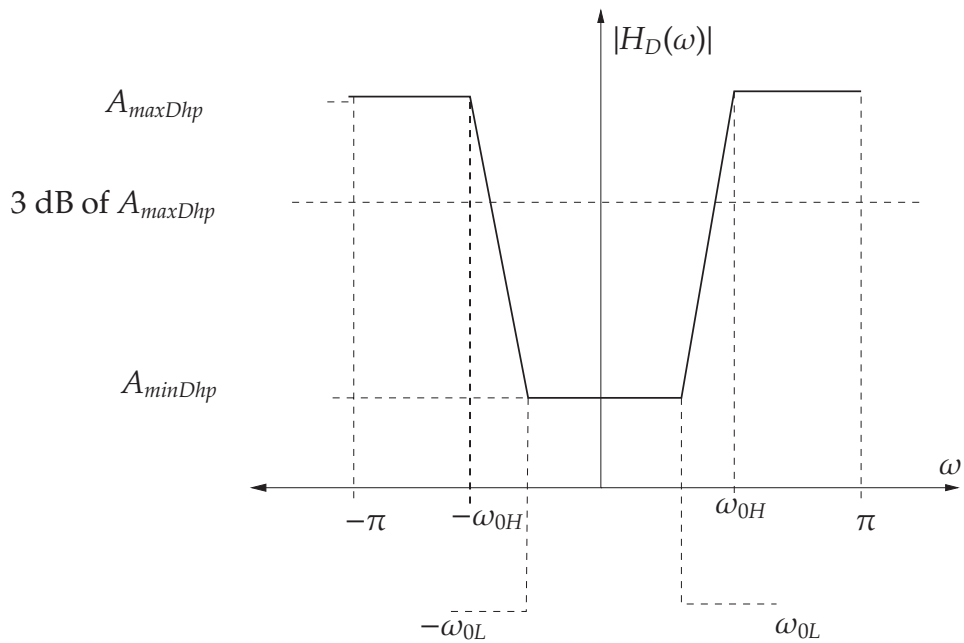
Figure 7.2: Desired filter characteristics for HPF to be used as denominator.
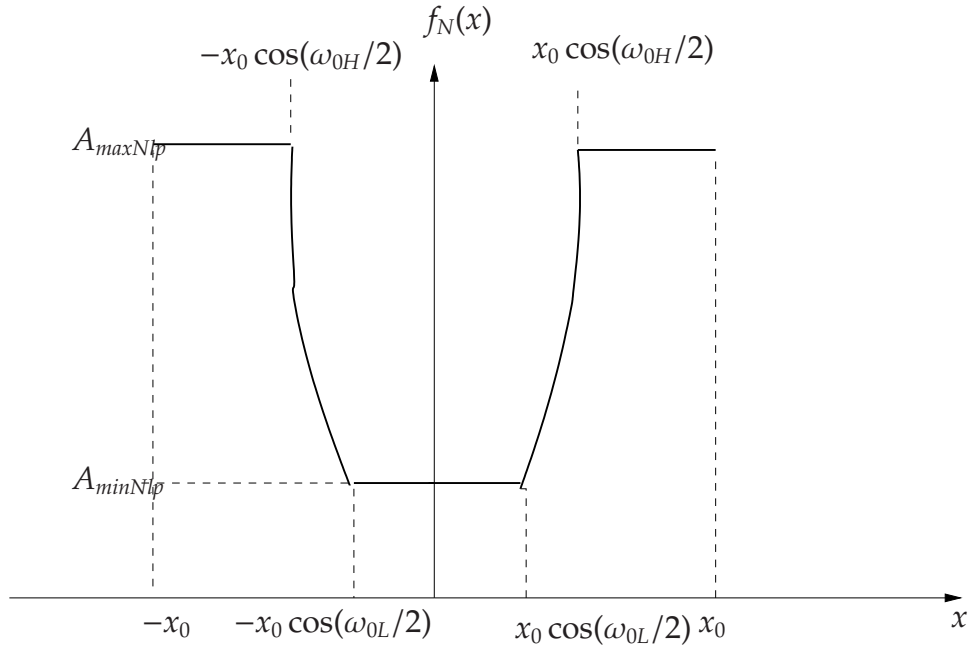
Figure 7.3: Object function for LPF of Figure 7.1, showing slight non linearity in the transition region.

the amplitude and phase characteristics of the low pass IIR filter at these points, respectively. To calculate the pass band values of the IIR filter we divide pass band values of $|H_{LP(FIR)\mathbf{N}}|$ with stop band values of $|H_{HP(FIR)\mathbf{D}}|$ and vice versa to get the stop band. Because both, high pass and low pass, filters have linear phase, therefore, resulting IIR filter will have a zero phase (if the phase response is same for both of the numerator and denominator) or linear phase, Equation (7.2).

From Equation (7.2) we can deduce that while designing the FIR high pass filter; that is, $H_{HP(FIR)\mathbf{D}}(\omega)$, we have to ensure that the filter has no zeros in $0 \le \omega \le \pi$ region, otherwise these zeros will make the resulting IIR filter unstable. If $H_{HP(FIR)\mathbf{D}}(\omega)$ has zero crossings, then before designing the IIR filter we add a constant term (a term with zero frequency) to make sure that $H_{HP(FIR)\mathbf{D}}(\omega)$ is above ω-axis; that is, there is no zero in $0 \le \omega \le \pi$. Also, because the phase of both of the FIR filters is linear the division will result in a linear phase IIR filter.

If our application requires a zero/constant phase IIR filter, then we can design such filter as well. It is clear from Equation (7.2) that to get zero phase we have to make sure that the phase characteristics of low pass and high pass FIR filters are identical; furthermore the linear phase

low pass FIR filter must not cross the ω-axis, because every such crossing results in a phase change of π. We can control magnitude as well as phase characteristics of resulting IIR filter, it will be discussed shortly. Since we need to have FIR filters to get the IIR filter; we have to design them first.

To design a low pass FIR filter ($H_{LP(FIR)\mathbf{N}}(\omega)$), shown in Figure 7.1, we first have to construct a corresponding object function ($f_l(\omega)$), shown in Figure 7.3, as discussed and designed in Chapter 5. Similarly, we can design the high pass FIR filter, Figure 7.2, and its corresponding object function shown in Figure 7.4. Once we have the required FIR filters we move to design the IIR filter.
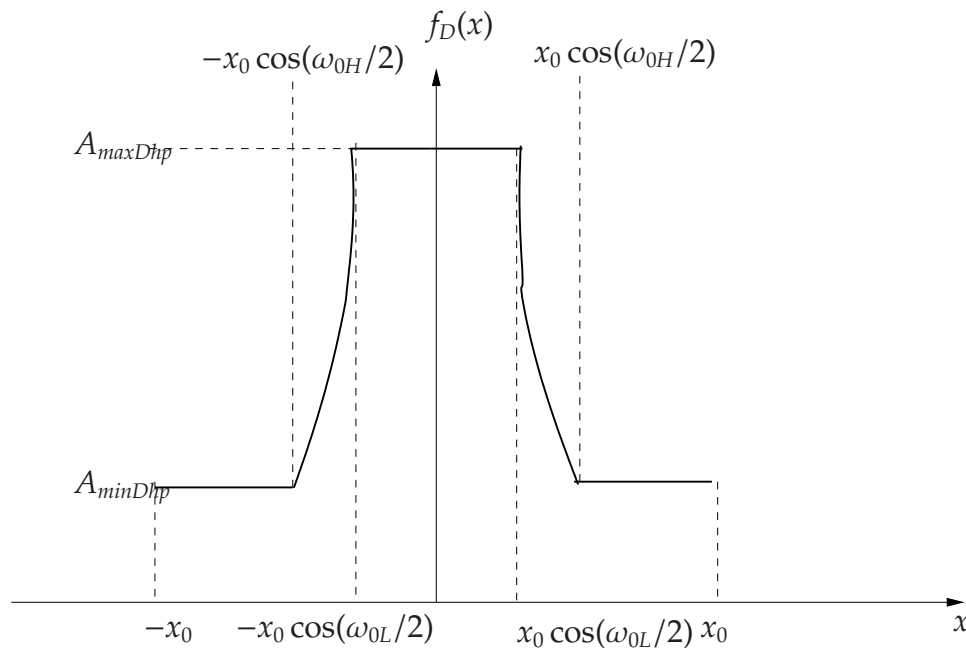
Figure 7.4: Object function for HPF of Figure 7.2, showing slight non linearity in the transition region.

Based on the method discussed above, we discuss the procedure to design the IIR filters in detail in the next section.

7.3 Procedure

Let us design a low pass, zero group delay IIR filter. To have a zero phase and consequently zero group delay, we have to make sure that pass/stop band of numerator and corresponding stop/pass band of denominator,

and transition bands of both numerator and denominator must match. From here onwards we represent $H_{LP(FIR)\mathbf{N}}(\omega)$ by $H_N(\omega)$ and $H_{HP(FIR)\mathbf{N}}(\omega)$ by $H_D(\omega)$ for simplicity. To design such a filter we divide low pass FIR filter characteristics ($H_N(\omega)$), shown in Figure 7.1, by high pass FIR filter characteristics ($H_D(\omega)$), shown in Figure 7.2.

As discussed in the last section these FIR filter characteristics will be realized by their corresponding object functions – $f_N(x)$ for $H_N(\omega)$ and $f_D(x)$ for $H_D(\omega)$ – as shown in Figures 7.3 and 7.4, respectively. The object functions are given by

$$f_N(x) \;=\; \sum_{n=0}^{\infty} a_{2n} P_{2n} \tag{7.3}$$

$$f_D(x) \;=\; \sum_{n=0}^{\infty} b_{2n} P_{2n} \tag{7.4}$$

where, $P_0, P_2, P_4 \ldots$ are the Legendre polynomials of the order of $0, 2, 4, \ldots$, and a_{2n} and b_{2n} are coefficients which are multiplied with the Legendre polynomials to approximate the required object function characteristics shown in Figures 7.3 and 7.4, respectively. Only even terms of Legendre polynomials are used, considering the fact that filter characteristics in frequency domain are symmetrical in nature.

Following steps outline the procedure to design zero group delay IIR filters.

Step 1 : Because polynomials are orthogonal to each other we can calculate the coefficients $a_0, a_2, a_4, \ldots$ and $b_0, b_2, b_4, \ldots$ by using the following formulae[3]

$$a_i = \frac{\int_0^1 f_N(x) P_i dx}{\int_0^1 P_i P_i dx} \tag{7.5}$$

$$b_i = \frac{\int_0^1 f_D(x) P_i dx}{\int_0^1 P_i P_i dx} \tag{7.6}$$

where, $i = 0, 2, 4, \ldots$

Step 2 : We use finite number of a_i's and b_i's in Equations (7.3) and

---

[3]see Appendix A.

(7.4), therefore, we get the approximate object functions $f_{Na}(x)$ and $f_{Da}(x)$ in place of $f_N(x)$ and $f_D(x)$, respectively; that is,

$$f_{Na}(x) = \sum_{n=0}^{N} a_{2n} P_{2n} \tag{7.7}$$

$$f_{Da}(x) = \sum_{n=0}^{M} b_{2n} P_{2n} \tag{7.8}$$

Step 3 : The polynomials $f_{Na}(x)$ and $f_{Da}(x)$ are converted to $H_{Na}(\omega)$ and $H_{Da}(\omega)$, respectively, using the transformation discussed in Chapter 1 and repeated below for convenience

$$x = x_0 cos(\omega/2) \tag{7.9}$$

We consider x_0 to be equal to 1, one can assume other values also.

The discussion in the previous section makes it clear that $H_{Na}(\omega)$ and $H_{Da}(\omega)$ represent the approximate LPF and HPF characteristics corresponding to $f_{Na}(x)$ and $f_{Da}(x)$, respectively.

Step 4 : Calculate the rational function in $cos(\omega/2)$

$$H_{iirlowpass}(\omega) = \frac{H_{Na}(\omega)}{H_{Da}(\omega)} \tag{7.10}$$

Notice that the denominator must not have a zero in $0 \leq \omega \leq \pi$ and $0 \leq x \leq \pi$, since this will lead to instability.

Step 5 : We find the zeros of $H_{Na}(\omega)$ and $H_{Da}(\omega)$ by solving $H_{Na}(\omega) = 0$ and $H_{Da}(\omega) = 0$, respectively. They are represented by $z_{iN} = exp(j\omega_{iN})$ and $z_{iD} = exp(j\omega_{iD})$; where, $i = 1, 2, 3, \ldots$, and ω_{iN} and ω_{iD}'s are the zeros calculated.

Step 6 : Transfer function of the resulting IIR filter is

$$H_{iirlowpass}(z) = \frac{(z - z_{1N})(z - z_{2N})(z - z_{3N}) \ldots}{(z - z_{1D})(z - z_{2D})(z - z_{3D}) \ldots} \tag{7.11}$$

The transfer function given in the above equation is used to find out the magnitude response and phase response of our IIR filter.

To design high pass filter we have to divide high pass FIR filter characteristics by low pass FIR filter characteristics.

For clear understanding of the above procedure we design some IIR filters using the procedure discussed above.

7.4   Application and Discussion

Suppose we intend to design an IIR filter with following characteristics

$$
\begin{aligned}
A_{max} &= 500 \text{ (pass band amplitude)} \\
A_{min} &= 0 \text{ (stop band amplitude)} \\
\omega_{OL} &= 2.3186 \text{ (start of transition band)} \\
\omega_{OH} &= 2.0007 \text{ (end of transition band)}
\end{aligned}
$$

Therefore, as per the discussion of the previous section we have to design the low pass and high pass FIR filters first. The, assumed, filter characteristics are as follows

Low pass FIR (Figure 7.1) filter

$$
\begin{aligned}
A_{maxN} &= 1000 \\
A_{minN} &= 0 \\
\omega_{OL} &= 2.3186 \\
\omega_{OH} &= 2.0007
\end{aligned}
$$

High pass FIR (Figure 7.2) filter

$$
\begin{aligned}
A_{maxD} &= 1 \\
A_{minD} &= 2 \\
\omega_{OL} &= 2.0007 \\
\omega_{OH} &= 2.3186
\end{aligned}
$$

from the previous section it must be clear to the reader that above values will lead to zero phase IIR filter.

Note that as of now we show only positive half of the graph the negative half being a mirror image. Now we consider some specific cases in the next

section.

7.4.1 Design 1

Suppose we take 10 Legendre polynomial terms to approximate the object functions (both for LPF and HPF characteristics). The object functions are approximated in the following manner

$$f_{Na}(x) = a_0 P_0 + a_2 P_2 + \ldots + a_{18} P_{18} \tag{7.12}$$

$$f_{Da}(x) = B + b_0 P_0 + b_2 P_2 + \ldots + b_{18} P_{18} \tag{7.13}$$

where, $f_{Na}(x)$ and $f_{Da}(x)$ are discussed in the previous section, B is added to the object function to make sure that the condition stated in Step 4 is met. According to Equations (7.3) and (7.4) we calculate the coefficients $a_0, a_1, a_2, \ldots$ and $b_0, b_1, b_2, \ldots$. Then,

$$f_{Na}(x) = 530 P_0 + 909.685 P_2 - 586.385 P_4$$
$$-4.64292 P_6 + 401.108 P_8 - 379.026 P_{10} + \ldots \tag{7.14}$$

$$f_{Da}(x) = 100 + 470 P_0 - 909.685 P_2 + 586.385 P_4$$
$$+4.64292 P_6 - 401.108 P_8 + 379.028 P_{10} + \ldots \tag{7.15}$$

Here we have assumed the value of B as 100, one can assume any value of B provided the condition stated in Step 4 is satisfied.

Using the transformation (Equation (7.9)) we transform our object functions to approximate LPF and HPF characteristics; that is, $H_{Na}(\omega)$ and $H_{Da}(\omega)$, respectively. The zeros (in frequency domain) ω_{iN} and ω_{iD}'s corresponding to $H_{Na}(\omega)$ and $H_{Da}(\omega)$, as given in Step 5 and Step 6, are calculated next which in turn gives the transfer function $H_{iirlowpass}(z)$ as follows

$$H_{iirlowpass}(z) = \frac{\prod_{i=1}^{18}(z - z_{iN})}{\prod_{i=1}^{18}(z - z_{iD})} \tag{7.16}$$

where, z_i's are $e^{j\omega_i}$; $i = 1, 2 \ldots 18$.

This $H_{iirlowpass}(z)$ gives the magnitude response and magnitude response in dB, which are shown in Figures 7.5 and 7.6, respectively. The pass band

of resulting filter has many ripples, although side bands are very low, but even then the design is not good. Let us approximate the object functions with more number of terms in the following section to have a better filter.
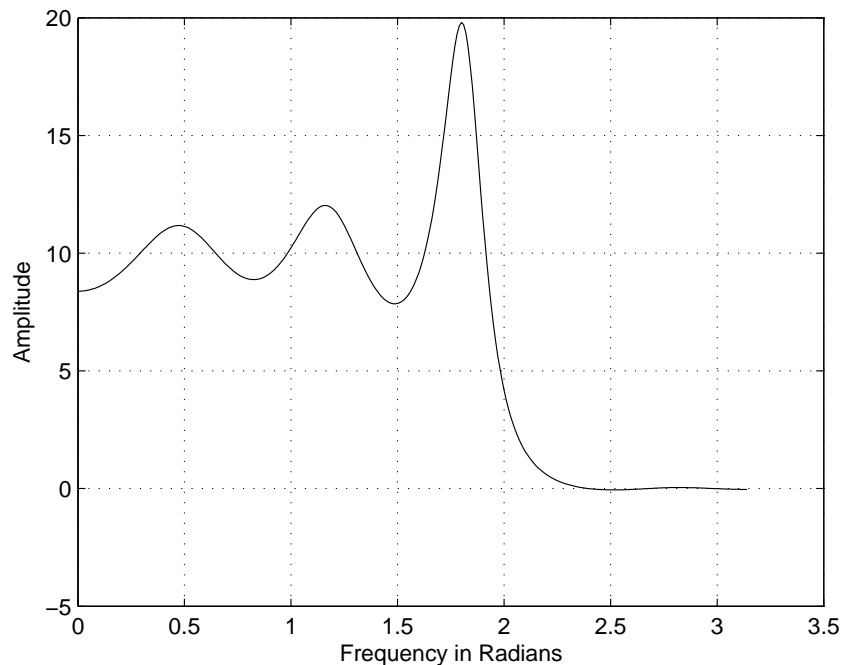
Figure 7.5: Magnitude response of low pass IIR filter designed using 10 orthogonal polynomial terms to approximate object function.

7.4.2 Design 2

Twenty Legendre polynomial terms are used to approximate the object functions (both for LPF and HPF characteristics) in the present example. After following the steps outlined in Section 2, we get the approximate object functions $f_{Na}(x)$ and $f_{Da}(x)$ and eventually $H_{iirlowpass}(\omega)$. The magnitude response, magnitude response in dB and phase response are shown in Figures 7.7, 7.8 and 7.9, respectively.

It becomes evident from the comparison of Figures 7.6 and 7.8 that the magnitude response becomes better in terms of flatness of pass band, and the second side band levels (they go almost 20dB further down when compared with the previous design). The transition band becomes sharper in the present design when compared with the previous design. In the same
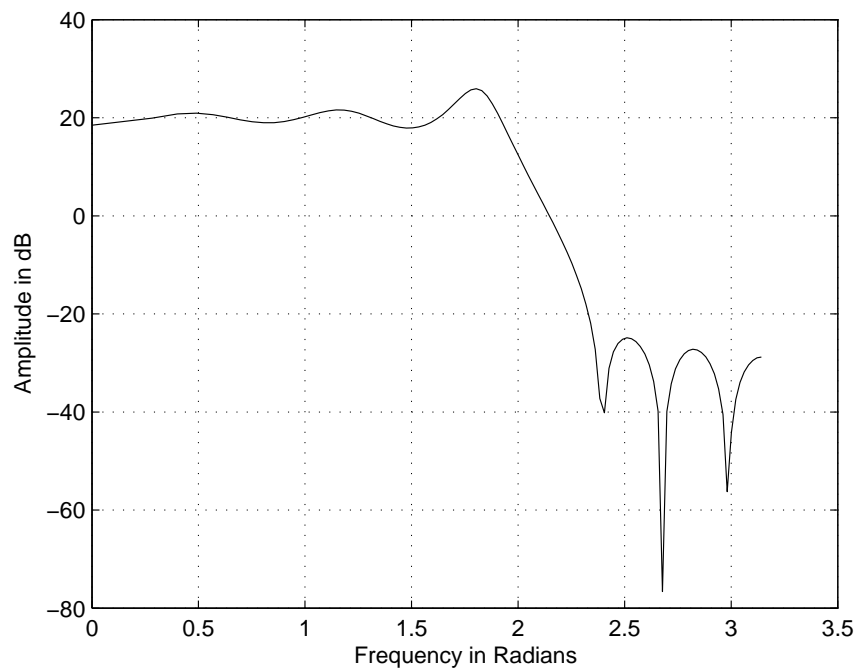
Figure 7.6: Magnitude response in dB of low pass IIR filter corresponding to the object function approximated using 10 orthogonal polynomial terms.
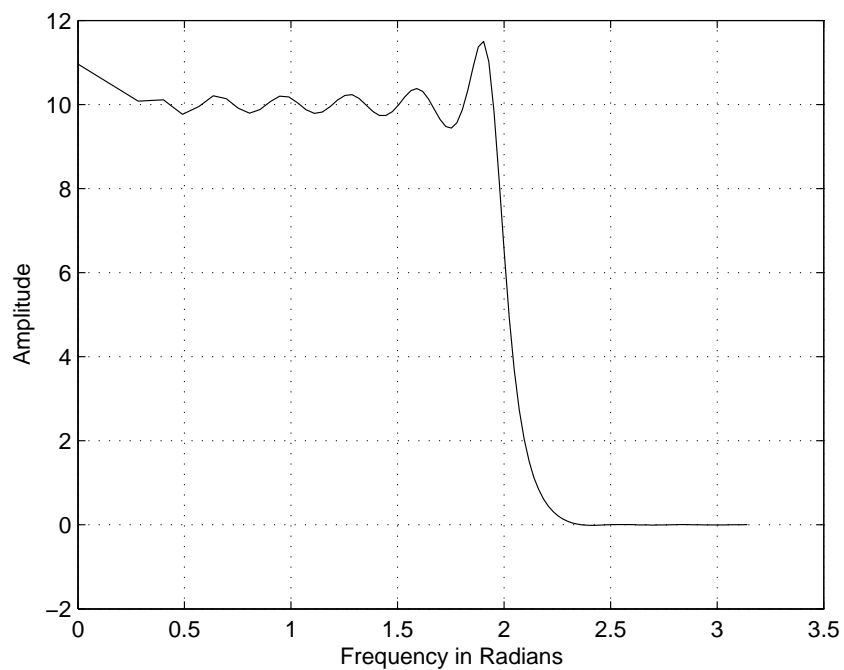
Figure 7.7: Magnitude response of low pass IIR filter corresponding to the object function approximated using 20 orthogonal polynomial terms.
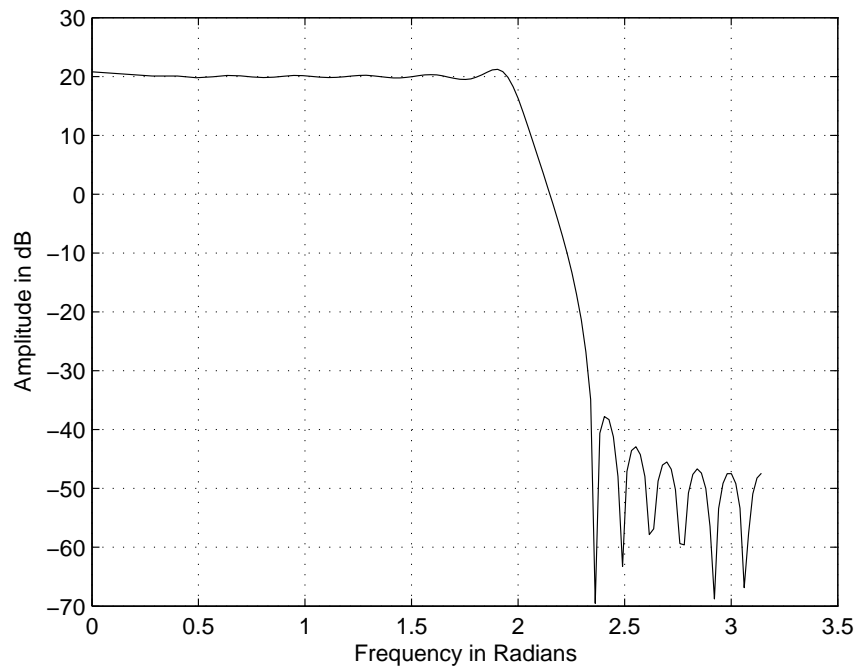
Figure 7.8: Magnitude response in dB of low pass IIR filter corresponding to the object function approximated using 20 orthogonal polynomial terms.
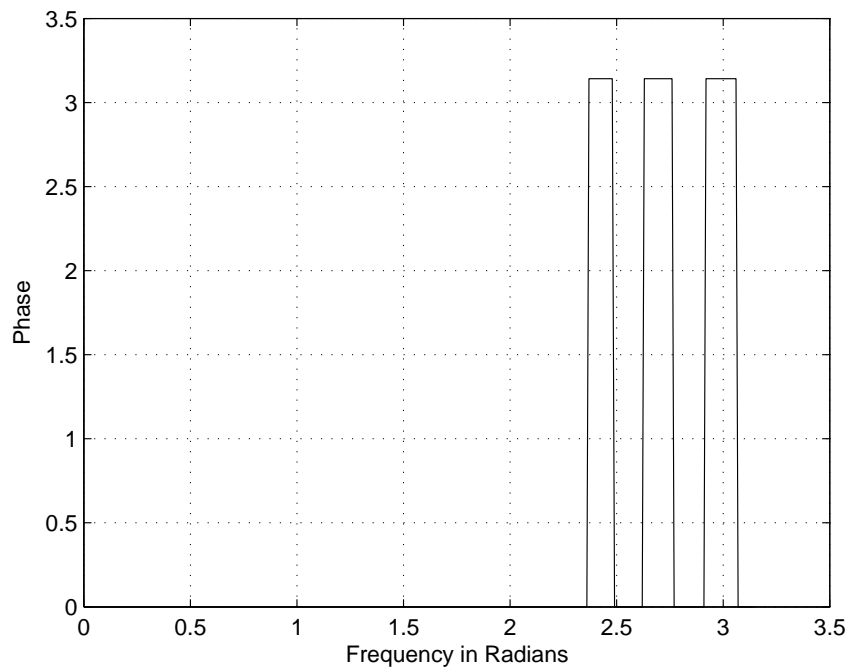
Figure 7.9: Phase response of low pass IIR filter corresponding to the object function approximated using 20 orthogonal polynomial terms.

fashion, if we increase the number of terms to approximate object functions further we get better and better results. The phase is zero and is shown in Figure 7.9.

7.4.3 Design 3

Hitherto we have considered that the order of object functions, corresponding to HPF and LPF, are same. Now we consider the case when the object function $f_{Na}(x)$ is synthesized using less number of Legendre polynomials than $f_{Da}(x)$. Suppose the order of $f_{Na}(x)$ is 10 and that of $f_{Da}(x)$ is 20, Figure 7.10 shows the magnitude response in dB.

Comparing Figures 7.8 and 7.10, we infer that if we decrease the number of terms to approximate numerator, side bands are not as lower as they were when both numerator and denominator were approximated using equal number terms. The transition band is also not that sharp in the present case, the reason becomes clear if we look at Equation (7.2). The phase, in this case, is linear.
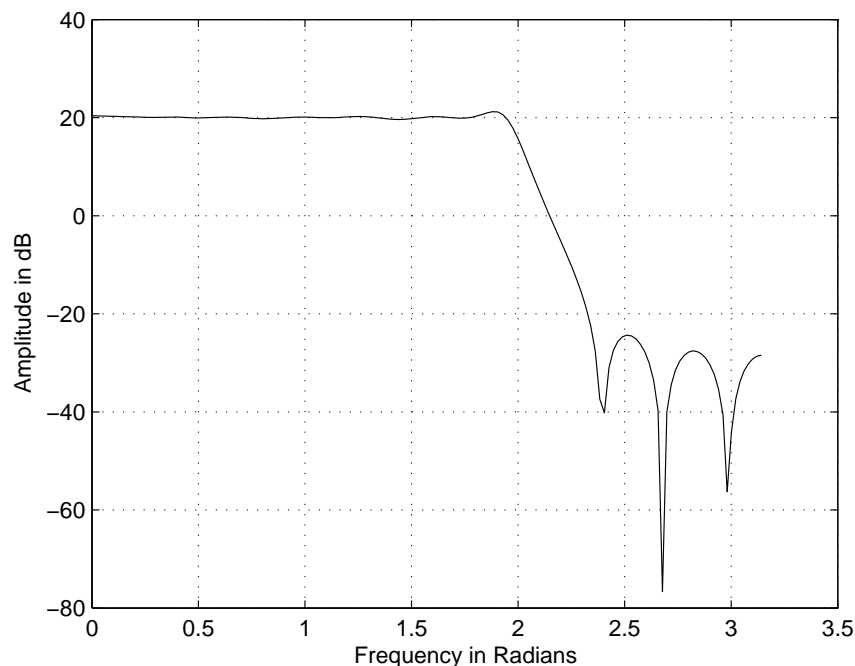
Figure 7.10: Magnitude response in dB when numerator and denominator object functions are approximated using 10 and 20 orthogonal polynomial terms, respectively.

7.4.4 Design 4

In this design we consider that the object function $f_{Da}(x)$ is synthesized using 10 and $f_{Na}(x)$ using 20 orthogonal polynomial terms, respectively. The magnitude response in dB for this kind of IIR filter is shown in Figure 7.11.
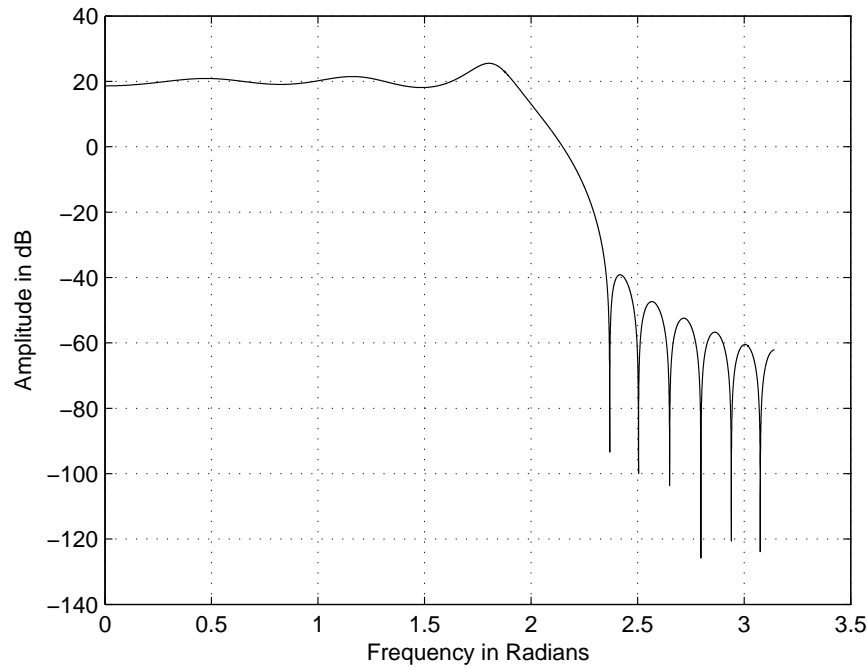
Figure 7.11: Magnitude response in dB when numerator and denominator object functions are approximated using 20 and 10 orthogonal polynomial terms, respectively.

Figures 7.10 and 7.11 make it clear that as we increase the number of terms in the denominator, or in other words as we increase number of zeros in the denominator of our IIR filter, it has pass band which is more flat than of Design 4. The side band level is better if we design the IIR filter with less number of terms to approximate denominator object function (corresponding to high pass FIR filter). User has to decide, depending on application, which amongst them he would like to use. If we increase the number of terms used to approximate object function then definitely both type of filters will become better.

7.5   Conclusion

Above discussion makes it clear that an IIR filter with linear or zero phase and consequently zero group delay can easily be designed by using the orthogonal polynomials. The proposed IIR filter gives good cutoff characteristics. By increasing the number of polynomial terms, we can approximate our object function very closely, which in turn will produce good frequency characteristics both in the pass band and transition band. The ripples in the pass band becomes negligible as we increase the number of terms to approximate our object function. Second side band amplitudes decreases as we increase the number of terms in our object functions. In all, we may state that the alternate approach discussed in this chapter gives a much better design of IIR filter, with absolute zero group delay, when compared with the currently available methods [55, 57, 60, 65, 66], none of them have zero group delay.

8 Design of 2 Dimensional Zero or Linear Phase IIR Filter with Orthogonal Polynomials

8.1 Introduction

Medical imaging [67], face recognition [68], and image processing [69] are some of the potential applications areas where 2 dimensional (2D) filters are used. Some applications; for example, multirate signal processing, and biomedical applications (ECG, EEG etc.) [70, 71] need zero group delay characteristics coupled with good pass band characteristics. Huang *et al* [72] pointed out that for restoration of images, linear phase plays an important role. In the last few years various approaches have been discussed by various researchers to design a linear phase 2D IIR filter [73–77]. The procedures discussed in these papers either first design a 2D filter with desired amplitude response which is followed by 2D all pass transfer function to linearize the phase, or minimize the error of desired magnitude and phase response. Computer aided optimization techniques [78–80] are also used, but they are computationally very extensive.

Present method discusses a new approach which results in absolute linear phase and zero group delay 2D IIR filters without using any optimization.

8.2 Procedure

We extend the proposed idea of previous chapter to design 2D IIR filters with zero phase. To design such filters, first we need to calculate the 2D orthogonal polynomials (in cylindrical or ρ domain). These polynomials, as previously, will be used for realizing object functions. The procedure and the resulting polynomials are given in Chapter 6.

To design the low pass IIR filter, we need to divide the low pass FIR filter characteristics with the high pass FIR filter characteristics, as discussed in the previous chapter and repeated below

$$
\begin{aligned}
H_{iirlowpass}(\omega) &= \frac{H_{LP(FIR)\mathbf{N}}(\omega)}{H_{HP(FIR)\mathbf{D}}(\omega)} \\
&= \frac{\left|H_{LP(FIR)\mathbf{N}}\right| \angle H_{LP(FIR)\mathbf{N}}}{\left|H_{HP(FIR)\mathbf{D}}\right| \angle H_{HP(FIR)\mathbf{D}}} \\
&= \frac{\left|H_{LP(FIR)\mathbf{N}}\right|}{\left|H_{HP(FIR)\mathbf{D}}\right|} \angle \left(H_{LP(FIR)\mathbf{N}} - H_{HP(FIR)\mathbf{D}}\right) \qquad (8.1)
\end{aligned}
$$

We proceed to design the object functions, $f_N(\rho)$ and $f_D(\rho)$, corresponding to numerator and denominator object functions respectively, as discussed in the previous chapter. ρ is in cylindrical co ordinate and is related with Cartisan co-ordinates (x,y) by $\rho^2 = x^2 + y^2$. For low pass and high pass filters (shown in Figures 8.1 and 8.2, respectively) example object functions are shown in Figures 8.3 and 8.4, respectively. Figures show half of the filter characteristics and object function, rest being mirror image of it.

From the previous chapter, we know that the object functions, $f_N(\rho)$ and $f_D(\rho)$, are weighted linear combinations of polynomials, and are represented as follows

$$
\begin{aligned}
f_N(\rho) &= \sum_{m=0}^{\infty} a_{2m} P_{2m}(\rho) \\
f_D(\rho) &= \sum_{m=0}^{\infty} b_{2m} P_{2m}(\rho) \qquad (8.2)
\end{aligned}
$$

where, P_{2m} are the orthogonal polynomials calculated in Chapter 6, and a_{2m} are the coefficients.

Because we use finite number of polynomials to design object functions, therefore, $f_N(\rho)$ and $f_D(\rho)$ become $f_{Na}(\rho)$ and $f_{Da}(\rho)$, respectively, and are given by
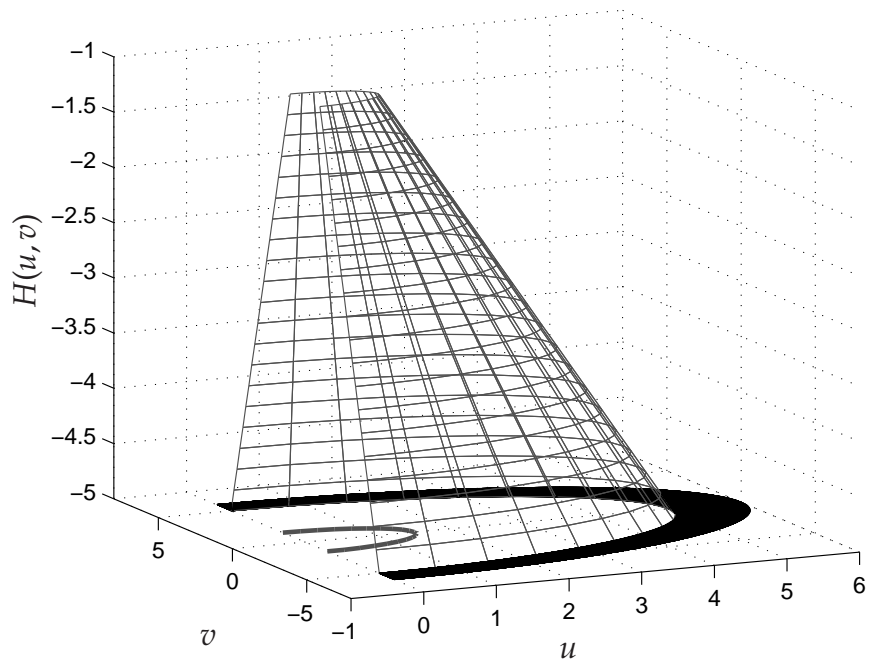
Figure 8.1: 3D desired low pass filter response.
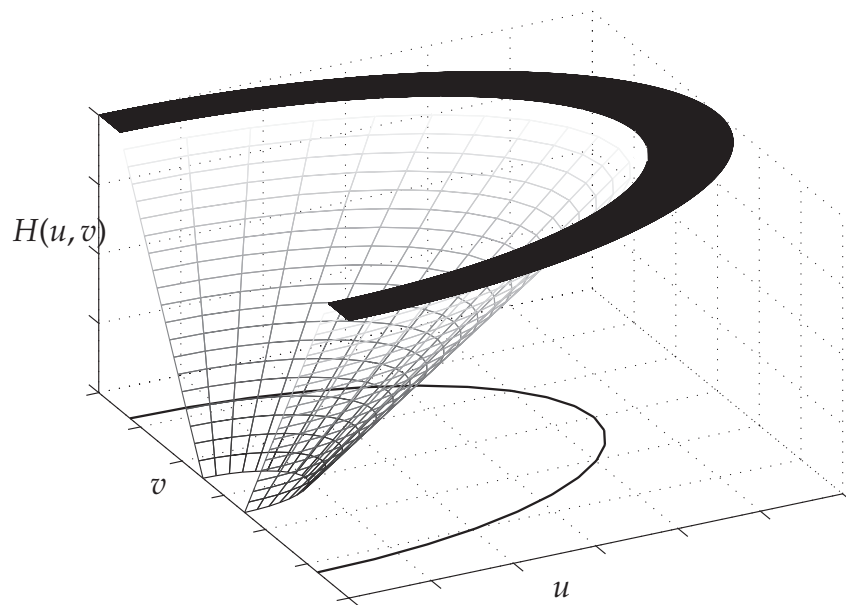
Figure 8.2: 3D desired high pass filter response.
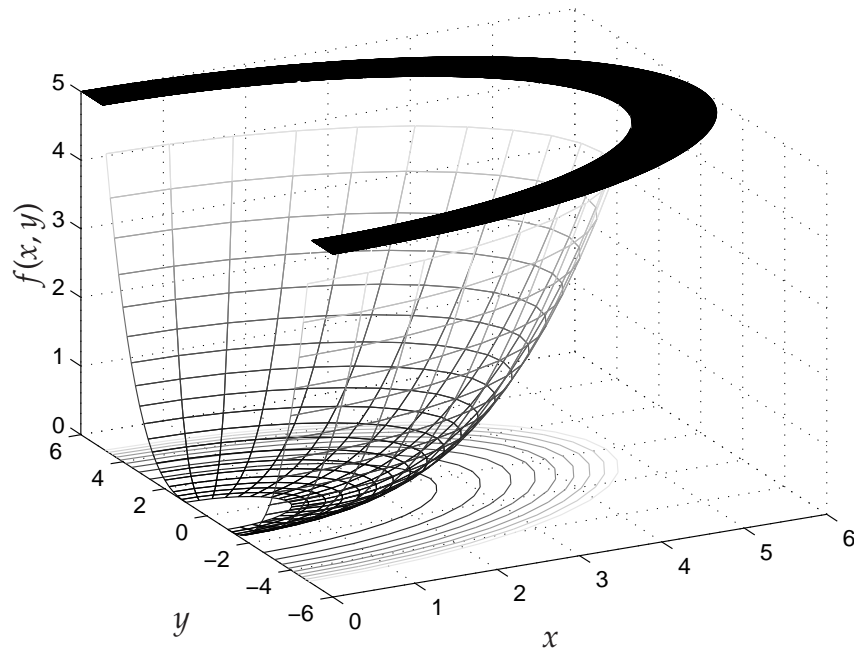
Figure 8.3: 3D object function for low pass filter of Figure 8.1, showing the slight non linearity in the transition region.
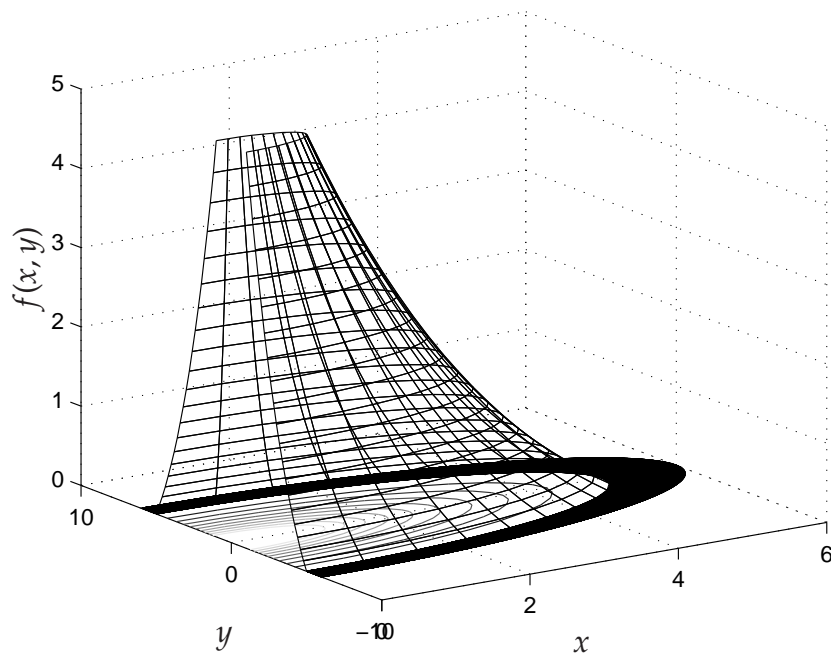
Figure 8.4: 3D object function for high pass filter of Figure 8.2, showing the slight non linearity in the transition region.

$$f_{Na}(\rho) \;=\; \sum_{m=0}^{N} a_{2m}P_{2m}(\rho) \tag{8.3}$$

$$f_{Da}(\rho) \;=\; \sum_{m=0}^{N} b_{2m}P_{2m}(\rho) \tag{8.4}$$

The values of a_m and b_m are calculated by[1]

$$a_m = \frac{\int_0^1 f_N(\rho)P_m(\rho)d\rho}{\int_0^1 P_m(\rho)P_m(\rho)d\rho} \tag{8.5}$$

$$b_m = \frac{\int_0^1 f_D(\rho)P_m(\rho)d\rho}{\int_0^1 P_m(\rho)P_m(\rho)d\rho} \tag{8.6}$$

The object functions – $f_{Na}(x, y)$ and $f_{Da}(x, y)$ or $f_{Na}(\rho)$ and $f_{Da}(\rho)$ – are transformed to filter characteristics – $H_{Na}(u, v)$ and $H_{Da}(u, v)$ or $H_N(\omega)$ and $H_D(\omega)$ – by using the transformation discussed previously and repeated below

$$\rho = \rho_0 cos(\omega/2) \tag{8.7}$$

where, $\omega^2 = u^2 + v^2$ and we assume ρ_0 equal to 1.

The zeros of $f_{Na}(\rho)$ and $f_{Da}(\rho)$ can be calculated by using any standard routine, and zeros of $H_N(\omega)$ and $H_D(\omega)$ are obtained by using the transformation

$$\rho_j \xrightarrow{\;\mathcal{T}\;} \omega_j$$

where ρ_js are the zeros of $f_a(\rho)$ and $\mathcal{T}$ is transformation defined in Equation (8.7).

Next, we calculate the rational function by

$$H_{iirlowpass}(u, v) = \frac{H_{Na}(u, v)}{B + H_{Da}(u, v)} \tag{8.8}$$

B in Equation (8.8) is added to make sure that the division does not yield an infinite value and consequently an unstable filter, whenever $|H_{Da}(u, v)|$ crosses zero.

---

[1]see Appendix A.

From the previous discussion we know that where $|H_{Da}(u,v)|$ has small values (in its stop band), $|H_{iirlowpass}(u,v)|$ will have large values (in its pass band). Very large oscillations in the pass band will correspond to a pass band which is not smooth. Thus to have a smooth pass band we need a sufficiently large value of B.

We can design a high pass filter by dividing high pass FIR filter characteristics by low pass FIR filter characteristics. If we are going to use same object functions then for high pass filter Equation (8.8) converts to

$$H_{iirhighpass}(u,v) = \frac{B + H_{Da}(u,v)}{H_{Na}(u,v)} \qquad (8.9)$$

As discussed earlier to avoid zero crossing to occur in the denominator term, which is $|H_{Na}(u,v)|$, we have to add a constant term, say A. Thus, Equation (8.9) becomes

$$H_{iirhighpass}(u,v) = \frac{B + H_{Da}(u,v)}{A + H_{Na}(u,v)} \qquad (8.10)$$

and we write Equation (8.8) as, assuming that same object functions are going to be used to realize both high pass and low pass filter,

$$H_{iirlowpass}(u,v) = \frac{A + H_{Na}(u,v)}{B + H_{Da}(u,v)} \qquad (8.11)$$

From the previous discussion it is clear that A should be sufficiently large so as to make sure that there are no oscillations in the pass band. How different values of A and B will change the characteristics of resulting filter is shown in Table 8.1.

Type of Filter	A	B	Figure Number
Low Pass	0	1100	8.5
Low Pass	0	2000	8.6
Low Pass	2000	2000	8.7
High Pass	1100	0	8.8
High Pass	2000	0	8.9
High Pass	2000	2000	8.10

Table 8.1: Values of A and B vs. figures.

The stepwise procedure to design the low pass IIR filter is as follows

Step 1 : Calculate a_{2m} and b_{2m}.
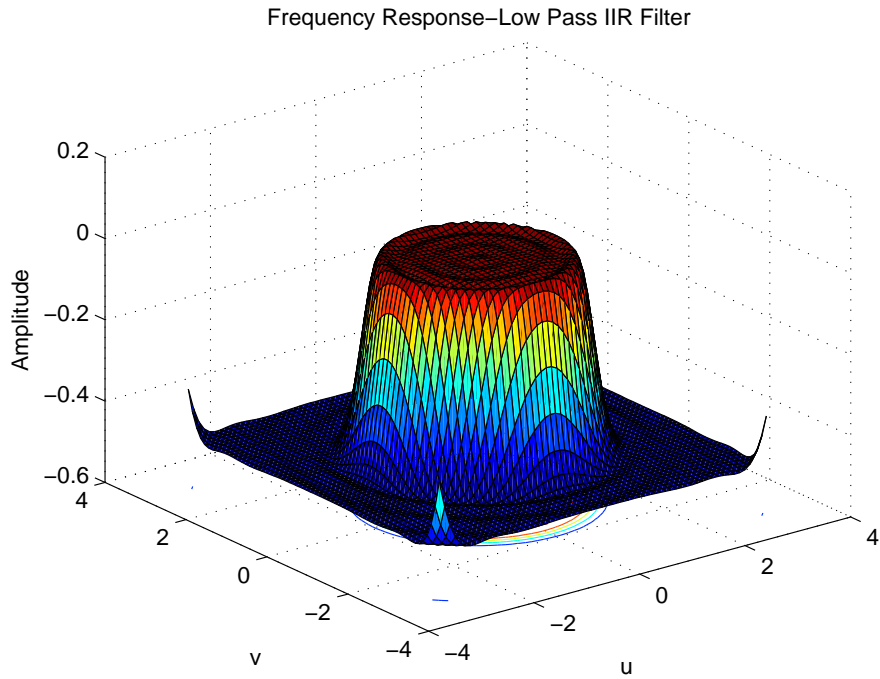
Figure 8.5: Example low pass filter with A=0 and B=1100.
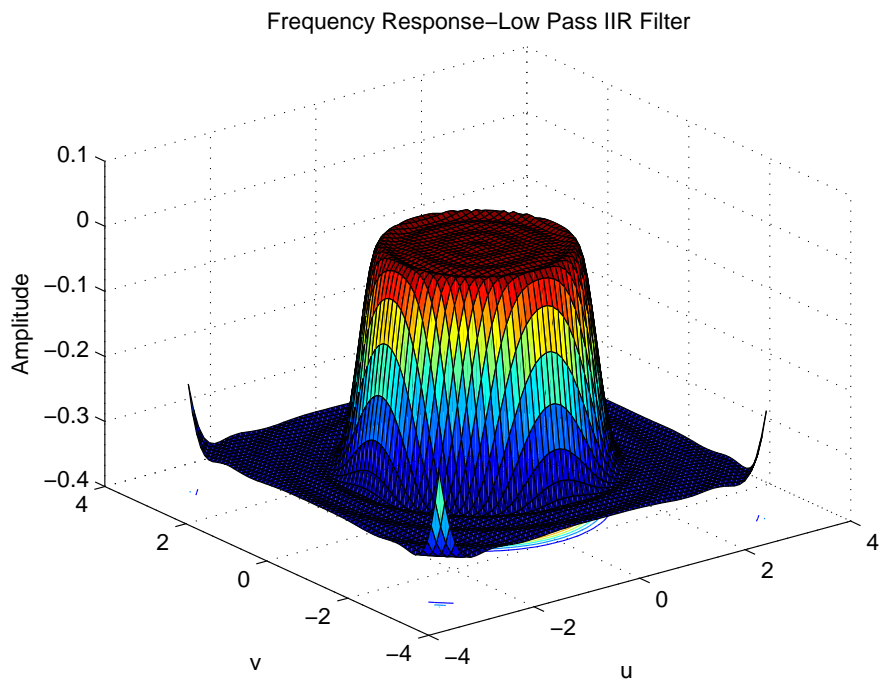
Figure 8.6: Example low pass filter with A=0 and B=2000.
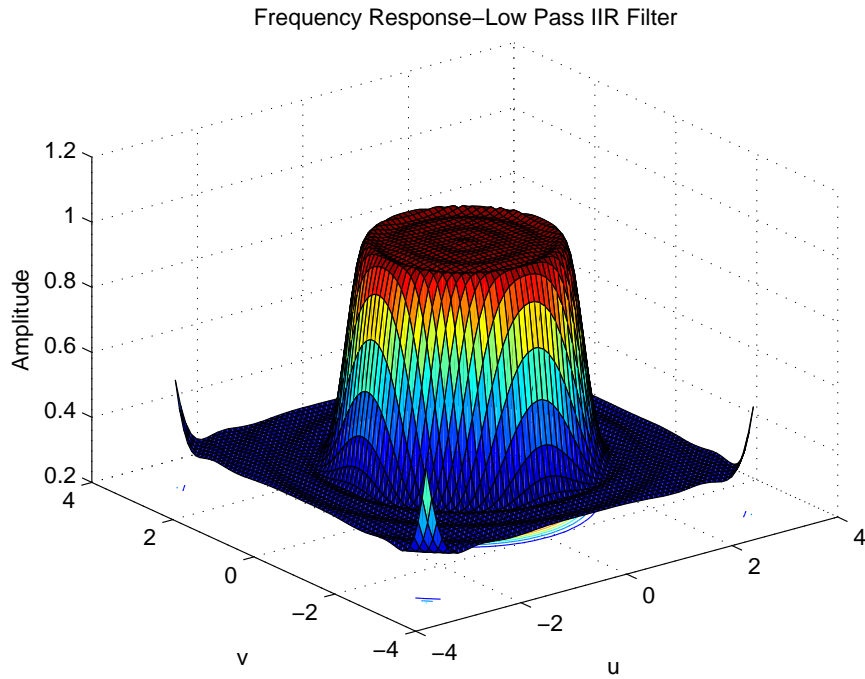
Figure 8.7: Example low pass filter with A=2000 and B=2000.
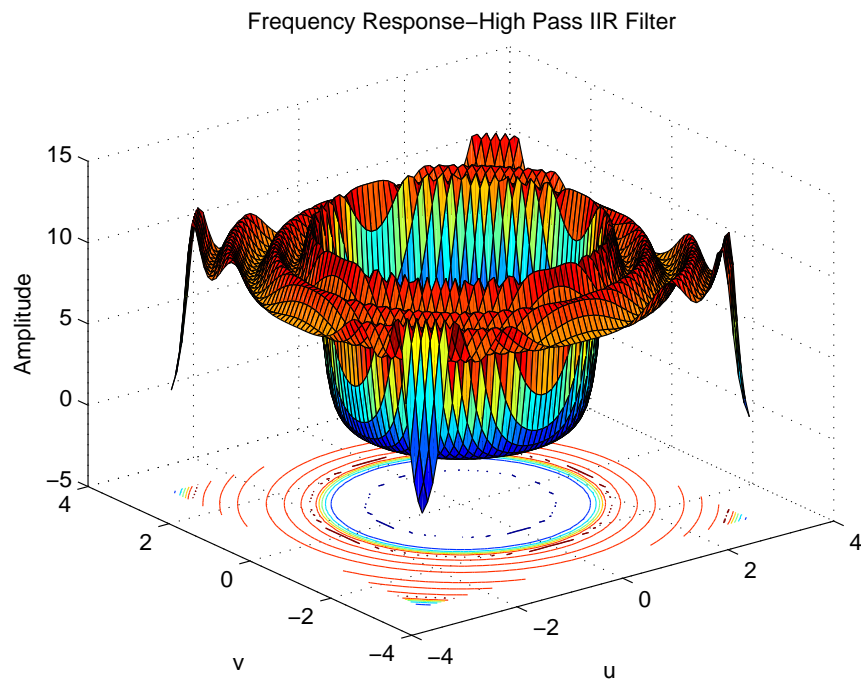
Figure 8.8: Example high pass filter with A=1100 and B=0.
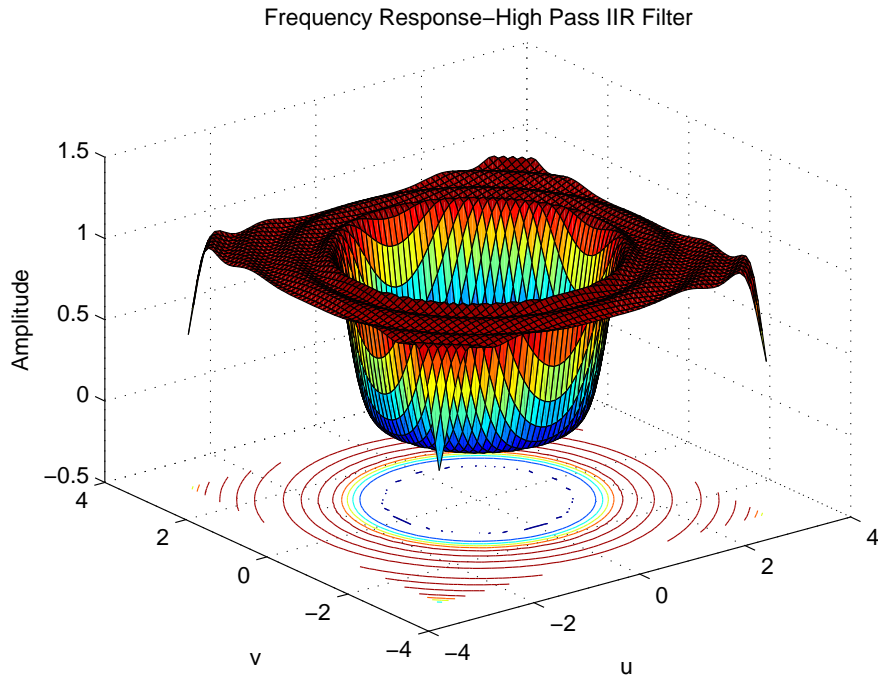
Figure 8.9: Example high pass filter with A=2000 and B=0.
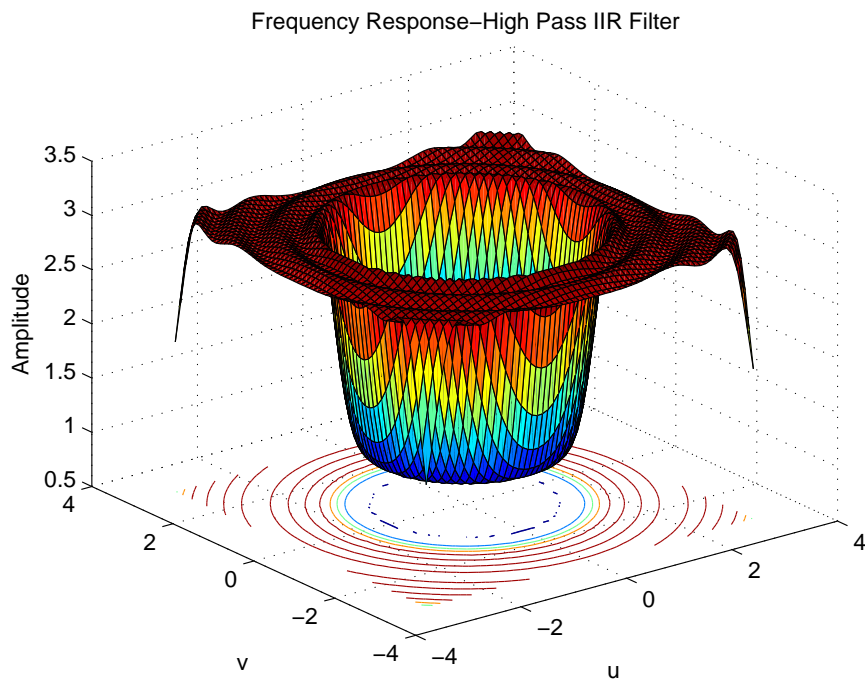
Figure 8.10: Example high pass filter with A=2000 and B=2000.

Step 2 : Calculate the approximate object functions; that is, $f_{Na}(\rho)$ and $f_{Da}(\rho)$.

Step 3 : Calculate

$$H_{iirlowpass}(u, v) = \frac{A + H_{Na}(u, v)}{B + H_{Da}(u, v)}$$

Step 4 : Calculate the transfer function in z domain; that is, $H_{iirlowpass}(z)$.

Step 5 : Find the zeros of $H_{Na}(\omega)$ and $H_{Da}(\omega)$ represented by $z_{iN} = exp(j\omega_{iN})$ and $z_{iD} = exp(j\omega_{iD})$; where, $i = 1, 2, 3, \ldots$, and ω_{iN} and ω_{iD}'s are the calculated zeros.

Step 6 : The transfer function of the resulting low pass filter is given as

$$H_{iirlowpass}(z) = \frac{A + (z - z_{1N})(z - z_{2N})(z - z_{3N}) \ldots}{B + (z - z_{1D})(z - z_{2D})(z - z_{3D}) \ldots} \tag{8.12}$$

in the similar fastion we can design a high pass filter.

8.3   Application and Discussion

Let us design IIR filter with the following values:

Low pass FIR filter (to be used as numerator to design the IIR filters), Figure 8.11

$$
\begin{aligned}
A_{maxN} &= 0dB \text{ (pass band magnitude)} \\
A_{minN} &= -60dB \text{ (stop band magnitude)} \\
\omega_{OL} &= 2.0944 \\
\omega_{OH} &= 1.5908
\end{aligned}
$$

High pass FIR filter (to be used as denominator to design the IIR filters), Figure 8.12

$$
\begin{aligned}
A_{maxD} &= 60dB \text{ (pass band magnitude)} \\
A_{minD} &= 0dB \text{ (stop band magnitude)} \\
\omega_{OL} &= 1.5908 \\
\omega_{OH} &= 2.0944
\end{aligned}
$$

and

$$A = 2000,$$

$$B = 2000.$$
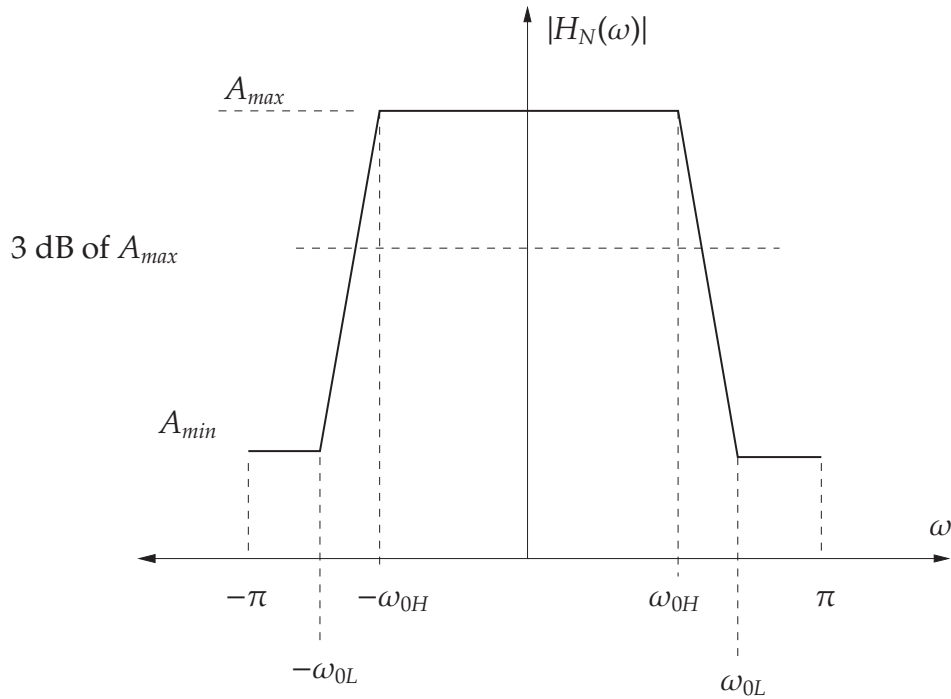
Figure 8.11: Desired filter characteristics for LPF.

8.3.1 Design 1

Let us design an IIR filter with 9 Legendre polynomial terms used to approximate both the object functions; that is, to calculate $f_{Na}(\rho)$ and $f_{Da}(\rho)$. Following the procedure discussed in the previous section we design the resulting filter.

Figures 8.13 and 8.14 represent the designed IIR low pass and high pass filters, respectively. Figures 8.15 and 8.16 show the low and high pass filter response in dB. It is clear from the Figures 8.13 and 8.15 that the pass band of the resulting IIR filter has oscillations in the pass band and stop band. Similarly, if we look at Figures 8.14 and 8.16 we find the same.

When an image (Figure 8.17) is passed through the low pass IIR filter, we have designed using 9 Legendre polynomial terms, the output is shown
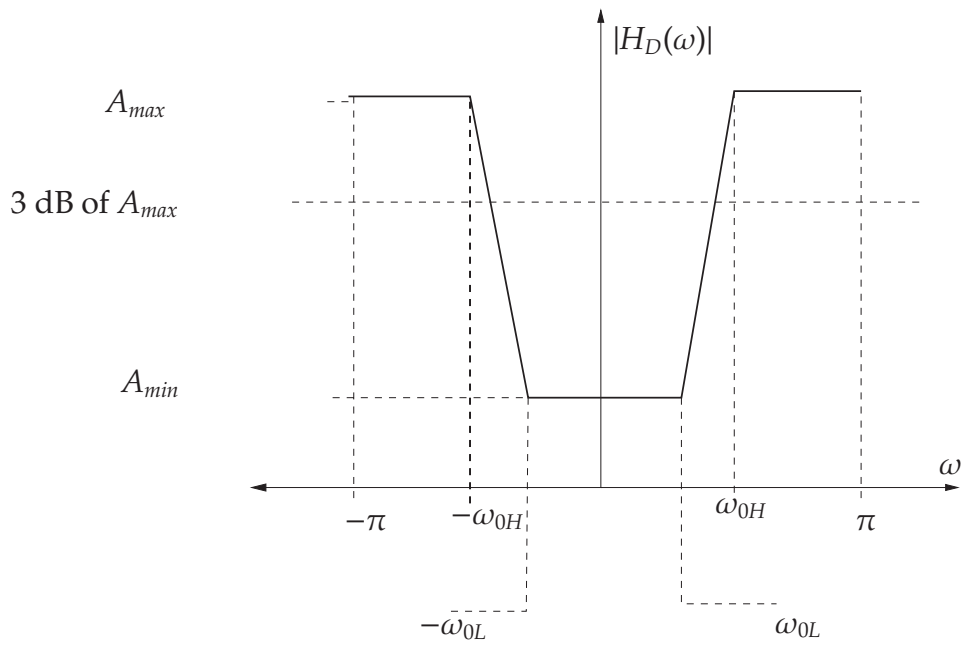
Figure 8.12: Desired filter characteristics for HPF.

Figure 8.13: Magnitude response of low pass IIR filter when object functions are approximated using 9 orthogonal polynomial terms.

Figure 8.14: Magnitude response of high pass IIR filter when object functions are approximated using 9 orthogonal polynomial terms.

Figure 8.15: Magnitude response in dB of low pass IIR filter when object functions are approximated using 9 orthogonal polynomial terms.

Figure 8.16: Magnitude response in dB of high pass IIR filter when object functions are approximated using 9 orthogonal polynomial terms.

in Figure 8.18 and if it is passed through the high pass filter then the output is represented in Figure 8.19. In Figure 8.18 we can clearly see that the high frequency components; that is, the changes or edges of the original image, are lost and only the slow changes or low frequency components are intact. Figure 8.19 shows that only the high frequency components (small, sharp details) of the image are present and low frequency components are lost.

8.3.2 Design 2

Let us calculate approximate object functions using 15 polynomial terms and then design the filter. The resulting IIR low pass filter is shown in Figure 8.20. Similarly, if we design a high pass filter using 15 polynomial terms then the filter is shown in Figure 8.21.

From Figures 8.20 and 8.21 it becomes evident that there is an improvement in the pass band and stop band characteristics of the filter. If we draw the frequency characteristics in dB, Figures 8.22 and 8.23, then the improvement can be seen clearly. The pass band is very flat and has less

Figure 8.17: Image-1.

oscillations, and also the stop band is lower than in the previous case.

Passing the previous image, Figure 8.17, through this filter we get Figures 8.24 and 8.25 resulting from high pass and low pass filtering, respectively. The images make it clear that increment in the number of terms to approximate the object function improves the filter performance. The difference between the images of Figures 8.18 and 8.25 is very small and in general we can not make out the difference in one look. We have shown one such difference by encircling it in Figures 8.18 and 8.25. The human eye behaves as a low pass filter, therefore it is very difficult to see any difference between images of Figures 8.19 and 8.24, the high passed results, through naked eyes.

If we filter an image with greater details (like an image having text) through our 15th order high pass filter, the resultant images can show the quality of our filter in a better way. Figure 8.26 is such an image while Figure 8.27 shows the high pass filtered output. It is evident from the high pass filtered image, Figure 8.27, that it is clearer than the original image; that is, we can view the text better. Figures 8.28 and 8.29 shows the closer
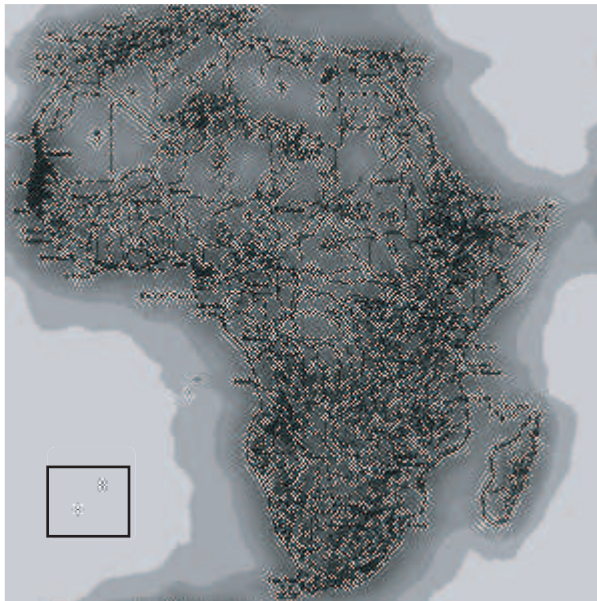
Figure 8.18: Image-1 passed through low pass filter designed using 9 orthogonal polynomial terms.

Figure 8.19: Image-1 passed through high pass filter designed using 9 orthogonal polynomial terms.
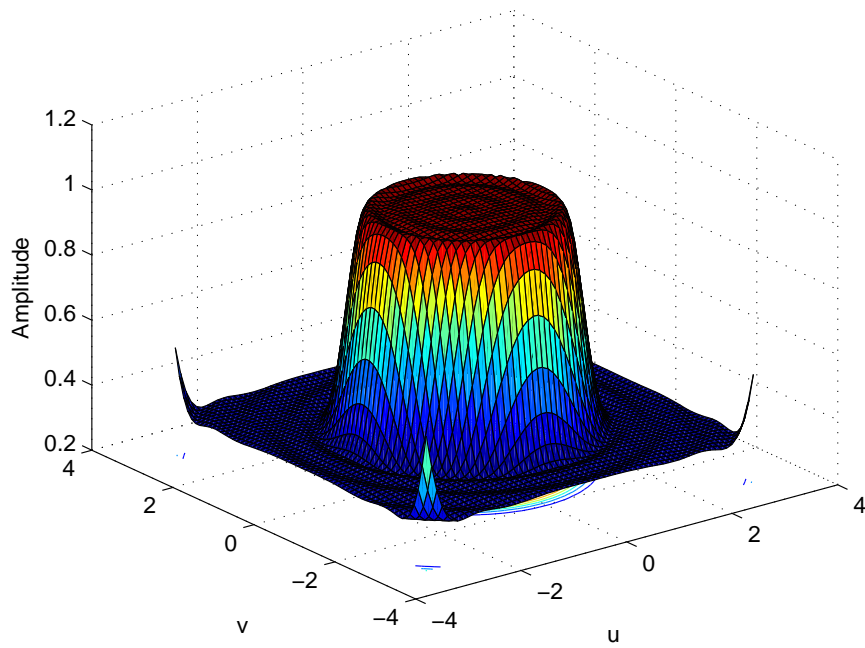
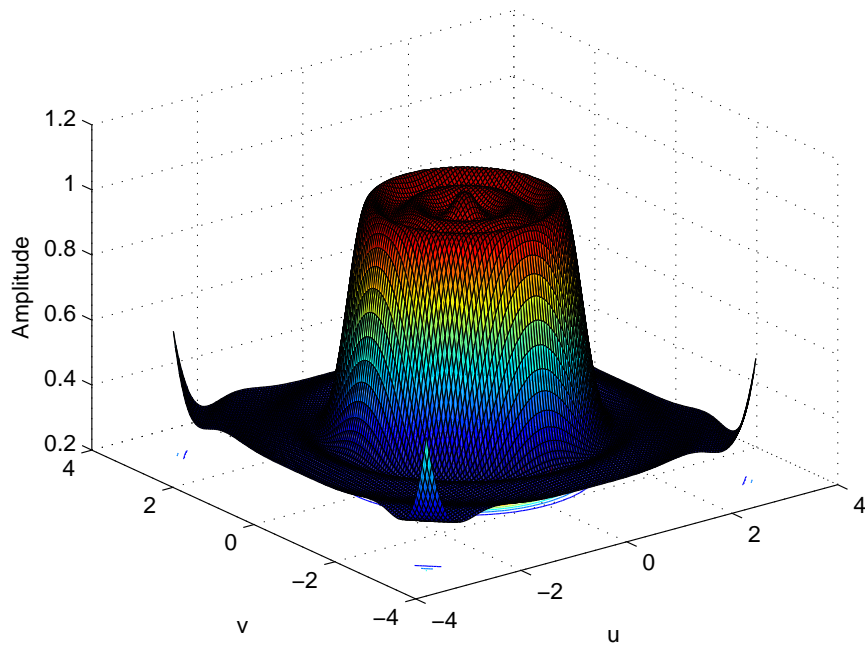Figure 8.20: Magnitude response of low pass IIR filter when object functions are approximated using 15 orthogonal polynomial terms.
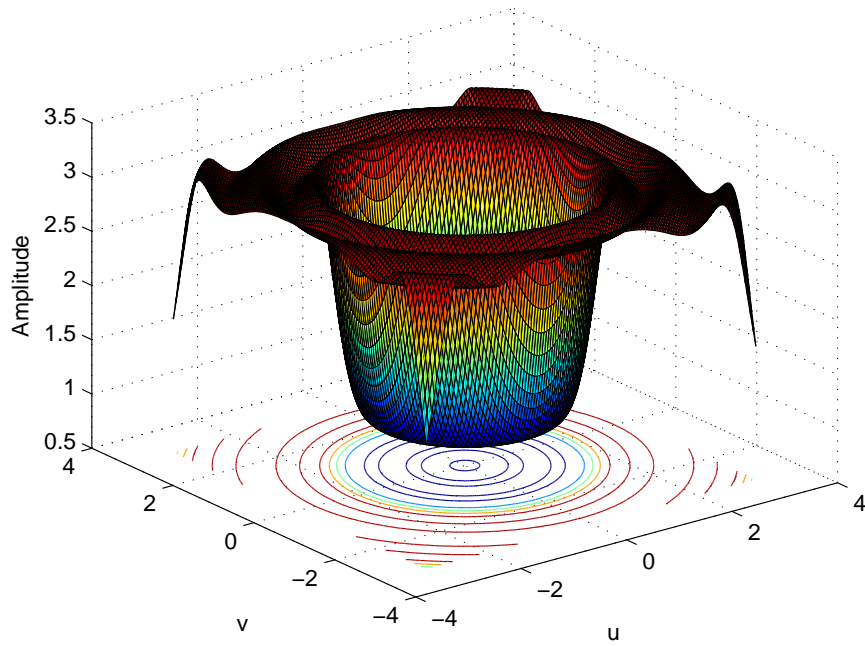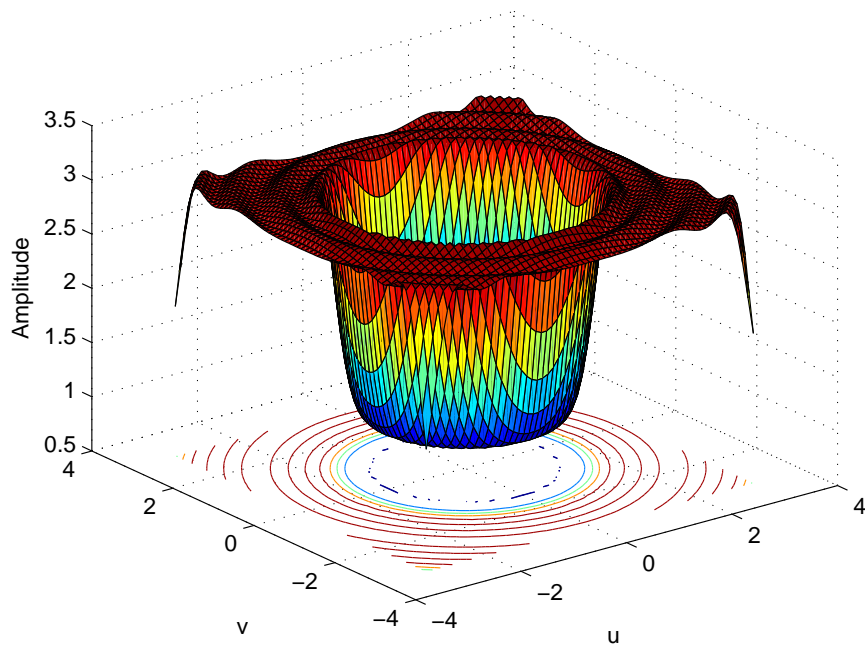

Figure 8.21: Magnitude response of high pass IIR filter when object functions are approximated using 15 orthogonal polynomial terms.
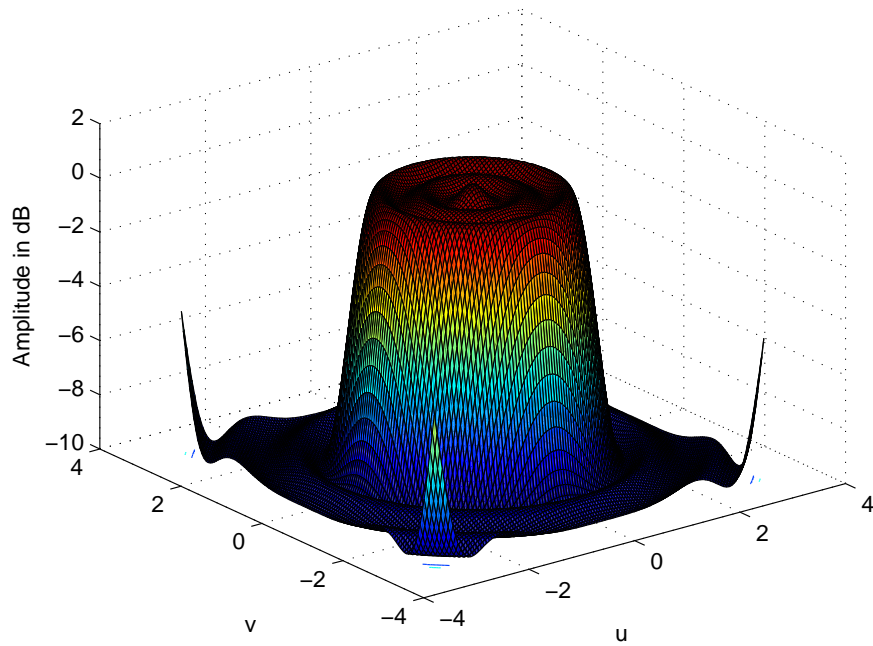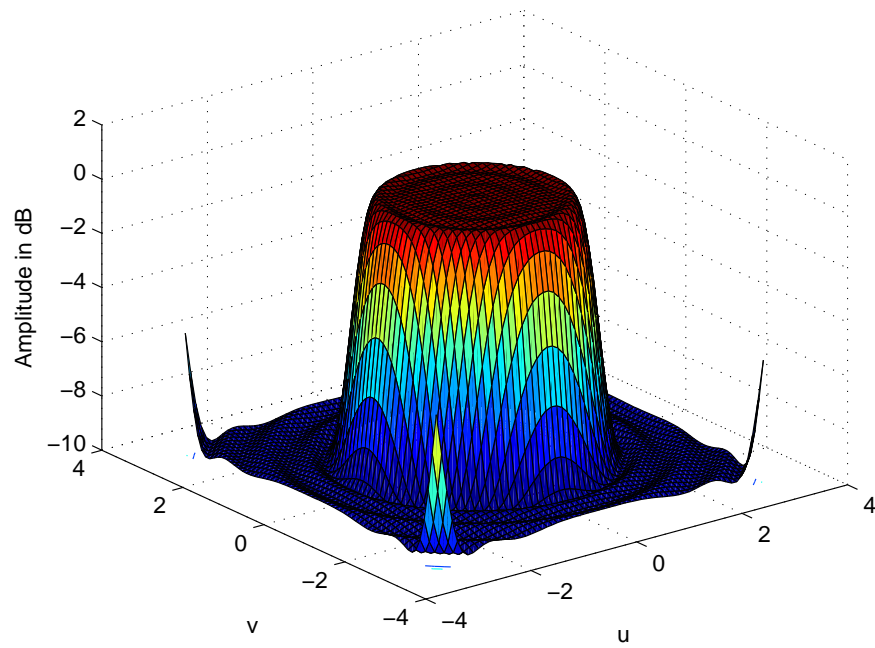
Figure 8.22: Magnitude response in dB of low pass IIR filter when object functions are approximated using 15 orthogonal polynomial terms.
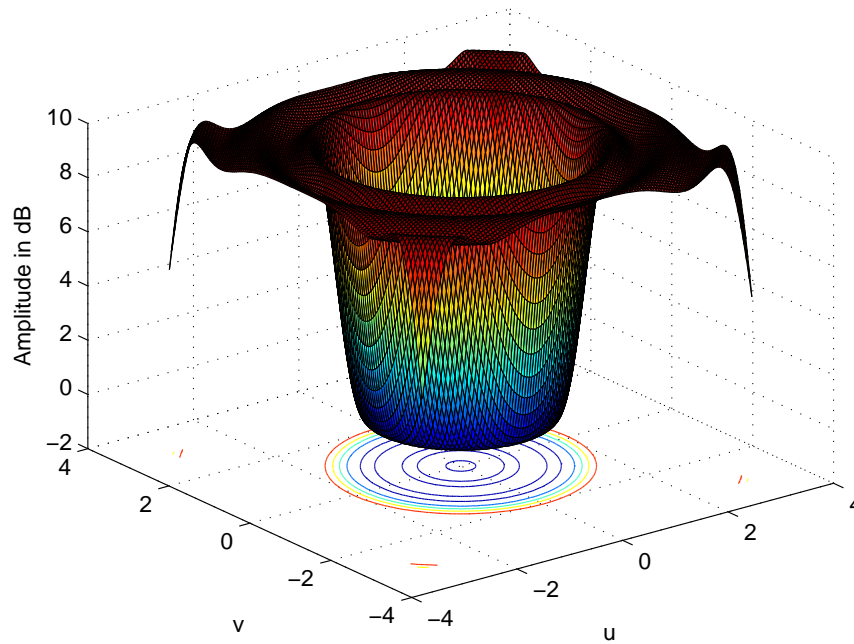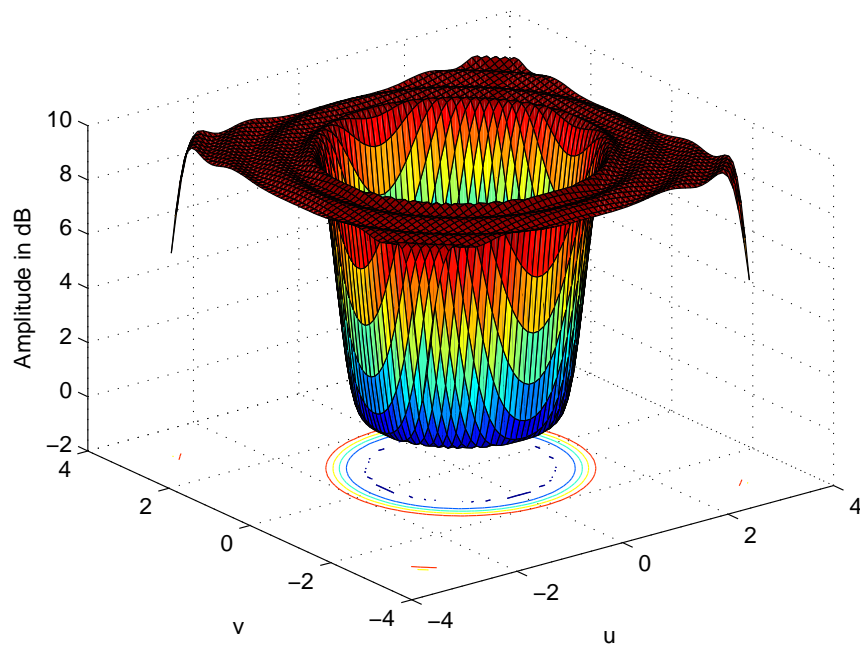

Figure 8.23: Magnitude response in dB of high pass IIR filter when object functions are approximated using 15 orthogonal polynomial terms.

Figure 8.24: Image-1 passed through high pass filter designed using 15 orthogonal polynomial terms.
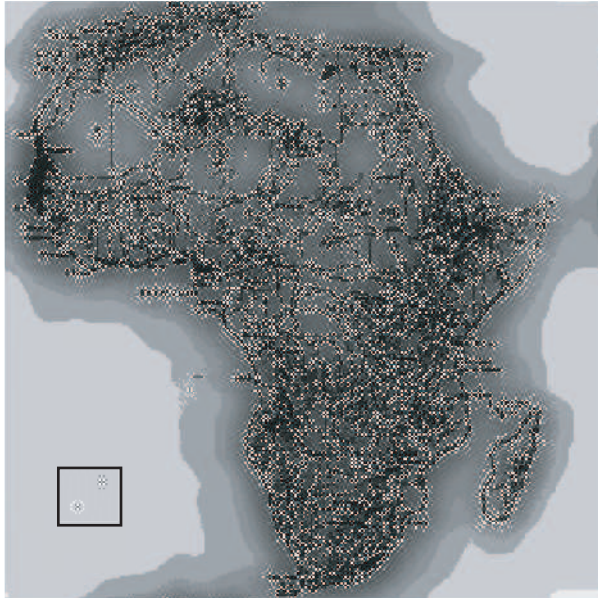
Figure 8.25: Image-1 passed through low pass filter designed using 15 orthogonal polynomial terms.

view of a part of these images shown in Figures 8.26 and 8.27, respectively.
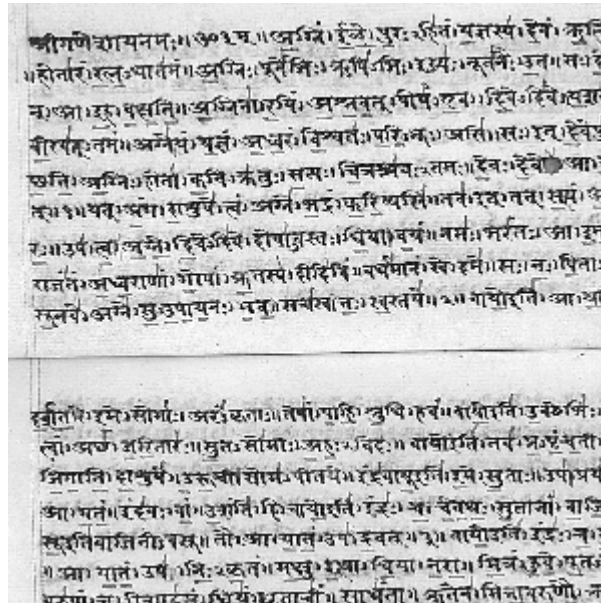
Figure 8.26: Image-2.

If we apply our high pass filter on an ultrasound image of a fetus, Figure 8.30, the result is shown in Figure 8.31. Another image which is of human bone structure and its high pass filtered output is shown in Figures 8.32 and 8.33, respectively. The filtered output show that the edges present in the original image are enhanced and therefore it is easier to find different parts of fetus in Figure 8.30 and defects, if there is any, from the bone structure.

Next we pass a satellite image through our IIR filter. The satellite images have a lot of high frequency details (like outlines of the street of a city, river, buildings, etc.). Figure 8.34 shows the original satellite image and when it is passed through 2DIIR filter the outcome is shown in Figure 8.35. The edges of the original image are retained in the resulting image.

Figure 8.27: Image-2 passed through high pass filter designed using 15 orthogonal polynomial terms.
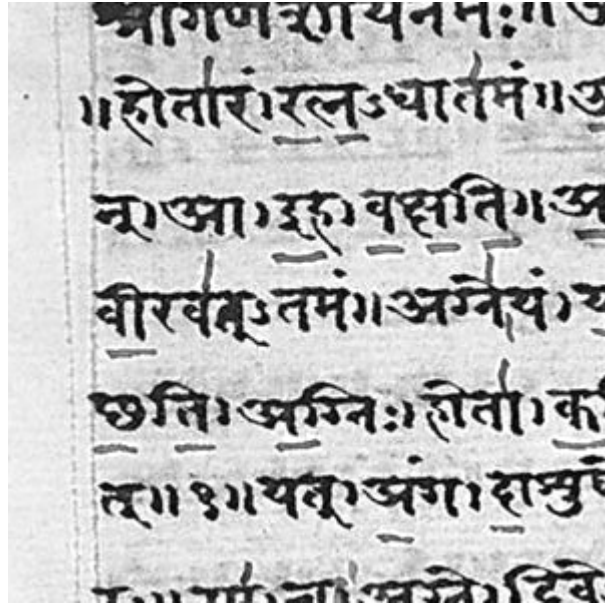
Figure 8.28: Image-3.

8.4 Conclusion

Above discussion makes it clear that an IIR filter with zero phase and consequently zero group delay can easily be designed by using the orthogonal polynomials. The IIR filter designed above gives good cutoff characteristics. By increasing the number of polynomial terms, we can approximate our object function very closely, which in turn will produce good frequency characteristics both in the pass band and the transition region. The ripples in the pass band becomes negligible as we increase the number of terms to approximate our object function. Stop band amplitude decreases as we increase the number of terms in our object functions. One can use these filters for specific purposes like, satellite images, medical image enhancement also. In all, we may state that the alternate approach discussed in this chapter gives a much better design of IIR filter, with absolute zero group delay, when compared with the currently available methods.

Figure 8.29: Image-3 passed through high pass filter designed using 15 orthogonal polynomial terms.
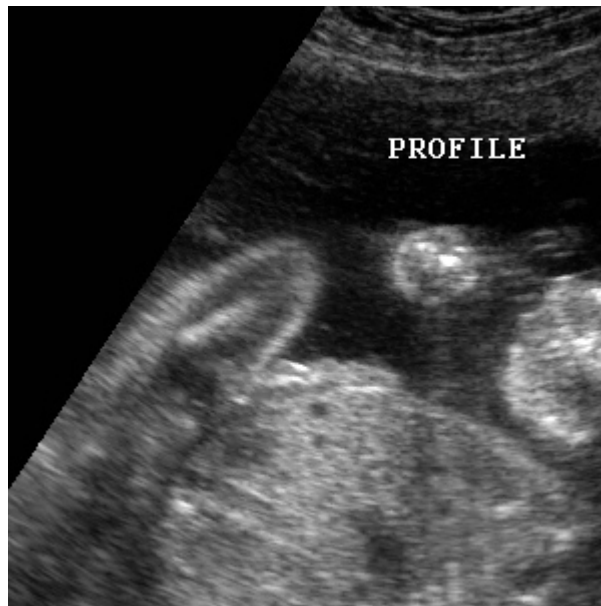
Figure 8.30: Image-4.

Figure 8.31: Image-4 passed through high pass filter designed using 15 orthogonal polynomial terms.

Figure 8.32: Image-5.

Figure 8.33: Image-5 passed through high pass filter designed using 15 orthogonal polynomial terms.

Figure 8.34: Image-6

Figure 8.35: Image-6 passed through high pass filter designed using 15 orthogonal polynomial terms.

9 Conclusion and Future Work

In this dissertation, we have proposed design methodologies to design the linear phase, 1D and 2D, FIR and IIR digital filters. Due to the simplicity of the procedure and excellent magnitude and phase characteristics, filters designed in the present work have a broad range of applications [2]. Most standard methods deal only either on the magnitude or phase characteristics, whereas, we have proposed algorithms which produce digital filters with user defined magnitude characteristics with the linear phase. The methods discussed give us unique design in terms of magnitude characteristics. Method to design zero group delay IIR filters is also discussed.

The proposed 1D and 2D Chebyshev FIR filter design methodology can be used to realize filters having equripple side bands, and for designing narrow to very narrow band filters with side band level defined by the user. Narrow band filters are very useful if we want to remove a particular frequency (it may be due to noise or in case of image representing a particular color or gray level) from the signal spectrum. The bandwidth of the filter can be controlled by using the modified approach to design the Chebyshev filters. When a satellite image is passed through this type of high pass filter edges (high frequency components of an image) were well retained.

We have proposed 1D and 2D FIR and IIR filter design based on orthogonal polynomials. It can be used for designing digital filters with linear phase and used specific magnitude characteristics. This type of design technique can be used to design multiband, notch, low pass, high pass, band pass and band reject digital filters. Depending on the application requirements we can design required type of filters.

We have passed different types of images through proposed 2D filters and shown the simulated results. Medical images, satellite images, text as images, everyday life images, and other type of images are passed through these digital filters. The filtered images show the quality of our filters. When an ultrasound image is passed through our 2DIIR digital

filter, it is enhanced by fairly good amount. If one realizes application specific digital filter, the results will be promising. Satellite images were passed and a better and enhanced outcome was received. The filter design parameters are less in number, therefore, computational requirements are less.

There are various fields where present approach can be extended. Adaptive filters have various applications [81], one can extend the concept presented in the present thesis to design such filters. Development of video processing filters is another area where present technique can be extended. One can design a wavelet filter bank, designing wavelet using polynomials, which can be used to generate very narrow band to wide band filters. The discussed 2D filters can be used in conjunction with edge detecting algorithm to enhance edges of an image.

A   Appendix

The sum of Legendre polynomials multiplied by suitable coefficients approximates the ideal polynomial in the least mean square error sense.

Proof :

Suppose a polynomial $f(x)$ is approximated by $f_a(x)$ and its representation is

$$f_a(x) = \sum_{n=0}^{N} a_{2n} P_{2n}(x)$$

The mean squared error (MSE) polynomial between the original polynomial and approximated polynomial is given by

$$E(x) = \{f(x) - f_a(x)\}^2$$

or,

$$E(x) = \left[f(x) - \sum_{n=0}^{N} a_{2n} P_{2n}(x) \right]^2 \tag{A.1}$$

Note that $E(x)$ is either positive or zero hence the minimum value of its integral is zero. We define

$$\epsilon(a_0, a_2, \ldots) = \int_{-1}^{1} E(x)dx = \int_{-1}^{1} \left[f(x) - \sum_{n=0}^{N} a_{2n} P_{2n}(x) \right]^2 dx \tag{A.2}$$

We need to find the coefficients $a_0, a_2, \ldots$ from the above equation so that this integral is minimized. The general way of solving this problem is well known, and is described below

$$\frac{\partial}{\partial a_i} \int_{-1}^{1} \left[f(x) - \sum_{n=0}^{N} a_{2n} P_{2n}(x) \right]^2 dx = 0 \quad i = 1 \ldots N \tag{A.3}$$

or

$$\frac{\partial}{\partial a_i} \int_{-1}^{1} \left[\{f(x)\}^2 + \left\{ \sum_{n=0}^{N} a_{2n} P_{2n}(x) \right\}^2 \right.$$

$$\left. -2f(x) \left\{ \sum_{n=0}^{N} a_{2n} P_{2n}(x) \right\} \right] dx = 0 \, i = 1 \dots N \qquad (A.4)$$

which is

$$\int_{-1}^{1} \left[0 + \frac{\partial}{\partial a_i} \left\{ \sum_{n=0}^{N} a_{2n} P_{2n}(x) \right\}^2 \right.$$

$$\left. -2 \frac{\partial}{\partial a_i} f(x) \left\{ \sum_{n=0}^{N} a_{2n} P_{2n}(x) \right\} \right] dx = 0 \, i = 1 \dots N \qquad (A.5)$$

spelt out this is

$$\int_{-1}^{1} \left[\frac{\partial}{\partial a_i} \left\{ a_0{}^2 P_0{}^2 + 2a_0 a_2 P_0 P_2 + \dots + a_i{}^2 P_i{}^2 \right. \right.$$

$$+ 2a_i a_{i+2} P_i P_{i+2} + 2a_i a_{i+4} P_i P_{i+4} + \dots \}\, dx$$

$$\left. -2 \int_{-1}^{1} f(x) a_i P_i \right] dx = 0 \qquad (A.6)$$

The orthogonality property of the Legendre polynomials reduces this integral to

$$2 \int_{-1}^{1} a_i P_i{}^2 dx - 2 \int_{-1}^{1} f(x) P_i dx = 0 \quad i = 1 \dots N \qquad (A.7)$$

$$a_i = \frac{\int_{-1}^{1} f(x) P_i dx}{\int_{-1}^{1} P_i{}^2 dx} \quad i = 1 \dots N \qquad (A.8)$$

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