

# A Novel Semi-active Suspension Design based on Decoupling Skyhook Control

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**Abstract.** A semi-active suspension design based on the traditional method of skyhook control is not capable of effectively controlling the attitude of the vehicle. However, an innovative approach called decoupling skyhook control allows the attitude of the vehicle body and its vibration characteristics to be effectively controlled. In this paper, a new decoupling skyhook controller for semi-active suspension is presented. Vehicle body motions in the three directions of vertical, pitch, and roll have been adopted to develop three skyhook controllers and directly control the vehicle body attitude. Furthermore, three orientation skyhook control forces are converted into actual damping forces of four adjustable dampers through the input decoupling transformation. The simulation results show that the developed controller is more effective than the traditional skyhook control in improving ride comfort.

**Keywords:** Skyhook control, Input decoupling, Semi-active suspension.

## 1. Introduction

Since the skyhook control scheme was introduced in the early 1970s by D. Karnopp, this method has been used in many vibration isolation applications [1]. In active and semi-active suspension control, the skyhook control scheme is an extremely effective control strategy and has been used in some commercial applications. Moreover, the skyhook scheme is often used as the reference model for other control strategies because of its simplicity and its prominent effect in improving ride comfort [2-4].

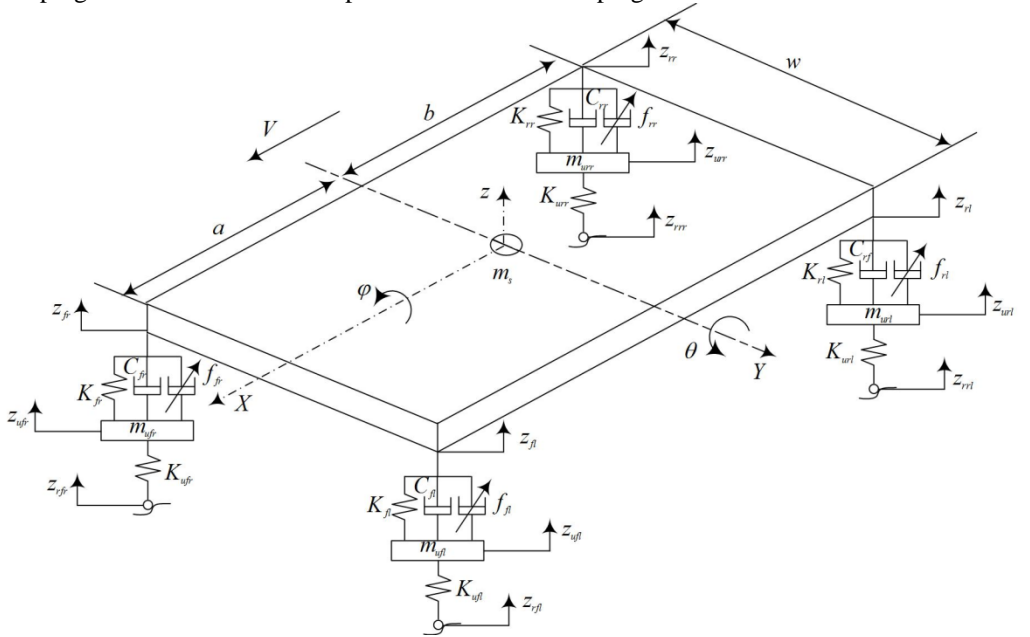
To develop the semi-active control algorithm for a full-vehicle system, two different methodologies can be used [5]. One methodology uses the quarter car model to develop the control algorithm, which is then implemented in a full vehicle model for controlling and analysis [6-8]. However, this methodology does not directly consider the pitch and roll motions of the vehicle. Furthermore, four accelerometers on the four corner points of the sprung mass are usually used to design controllers. As the aim of the suspension is to control the acceleration felt by the passengers, this acceleration should be considered at the vehicle's center of gravity and not over the wheels. Thus, the control of the body posture may be inadequate.

Another methodology uses a full-vehicle model to directly control both the vertical motion (heave) and the angular motion (pitch and roll) of the vehicle body [9,10]. The advantage of this methodology is that a control strategy can be designed that effectively controls the attitude of the sprung mass and suppresses the vibration of the suspension. In [9], three fuzzy controllers are designed to produce three forces  $f_z$ ,  $f_\theta$  and  $f_\varphi$  to suppress the heave, pitch, and roll movements. The input variables for the controllers are velocities and accelerations in three orientations. Then, the three forces are decoupled into four forces generated by active force generators. A double-loop control scheme is presented to control the active suspension in [10]. The inner control loop adopts the active filtering of the spring and damping coefficient to design the ride controller, while the outer control loop maintains load-leveling and load distribution during vehicle maneuvers using skyhook scheme. Finally, three forces, which

control the heave, pitch, and roll movements, are also decoupled into the four active generators' forces. Reference [11] also proposes a decoupled quarter car active suspension model that introduces a new abstract variable denoted by  $\gamma$ . For semi-active suspension, a coupling design using skyhook damping is mentioned in [12]. However, no further explanation is given. According to the theory of skyhook damping forces that damp heave and angular motion proposed in [1], this study develops semi-active suspension controllers through a direct use of the skyhook damping. First, three ideal skyhook controllers are designed to command forces that damp the vehicle body's heave, pitch, and roll motions. Next, the three orientation forces are decoupled into damping forces of four adjustable dampers, which are constrained by passivity. The developed strategy is simple and effective.

## 2. Full vehicle model

The vehicle is represented as a linearized seven-degree-of-freedom system, as shown in Fig.1. This model is similar to the model used in [13]. It consists of a sprung mass (car body) and four unsprung masses. For simplicity, all pitch and roll angles are assumed to be small. The suspension is also modeled as linear spring and damper, while the tires are modeled as simple linear springs without damping. The damper is divided into two parts: the damper with a fixed damping coefficient and the damper with a variable damping coefficient.



**Fig. 1.** Vehicle model with seven degrees of freedom

Newton's second law applied to the system results in the following differential equations of motion:

$$m_s \ddot{z} = -m_s g - K_{fl}(z_{fl} - z_{ufl}) - K_{fr}(z_{fr} - z_{ufr}) - K_{rl}(z_{rl} - z_{url}) - K_{rr}(z_{rl} - z_{urr}) + \dots \\ - C_{fl}(\dot{z}_{fl} - \dot{z}_{ufl}) - C_{fr}(\dot{z}_{fr} - \dot{z}_{ufr}) - C_{rf}(\dot{z}_{rf} - \dot{z}_{urf}) - C_{rr}(\dot{z}_{rr} - \dot{z}_{urr}) + f_{fl} + f_{fr} + f_{rl} + f_{rr}$$

$$\begin{aligned}
I_y \ddot{\theta} &= K_{fl} a(z_{fl} - z_{ufl}) + K_{fr} a(z_{fr} - z_{ufr}) - K_{rl} b(z_{rl} - z_{url}) - K_{rr} b(z_{rl} - z_{urr}) + \dots \\
&\quad + C_{fl} a(\dot{z}_{fl} - \dot{z}_{ufl}) + C_{fr} a(\dot{z}_{fr} - \dot{z}_{ufr}) - C_{rl} b(\dot{z}_{rl} - \dot{z}_{url}) - C_{rr} b(\dot{z}_{rl} - \dot{z}_{urr}) - f_{fl} a - f_{fr} a + f_{rl} b + f_{rr} b \\
I_x \ddot{\phi} &= -0.5K_{fl} w(z_{fl} - z_{ufl}) + 0.5K_{fr} w(z_{fr} - z_{ufr}) - 0.5K_{rl} w(z_{rl} - z_{url}) + 0.5K_{rr} w(z_{rr} - z_{urr}) + \dots \\
&\quad - 0.5C_{fl} w(\dot{z}_{fl} - \dot{z}_{ufl}) + 0.5C_{fr} w(\dot{z}_{fr} - \dot{z}_{ufr}) - 0.5C_{rl} w(\dot{z}_{rl} - \dot{z}_{url}) + 0.5C_{rr} w(\dot{z}_{rr} - \dot{z}_{urr}) + \dots \\
&\quad 0.5wf_{fl} - 0.5wf_{fr} + 0.5wf_{rl} - 0.5wf_{rr} \\
m_{ufl} \ddot{z}_{ufl} &= -m_{ufl} g + K_{fl} (z_{fl} - z_{ufl}) + C_{fl} (\dot{z}_{fl} - \dot{z}_{ufl}) - K_{ufr} (z_{ufl} - z_{rfl}) - f_{fl} \\
m_{ufr} \ddot{z}_{ufr} &= -m_{ufr} g + K_{fr} (z_{fr} - z_{ufr}) + C_{fr} (\dot{z}_{fr} - \dot{z}_{ufr}) - K_{ufr} (z_{ufr} - z_{rfr}) - f_{fr} \\
m_{url} \ddot{z}_{url} &= -m_{url} g + K_{rl} (z_{rl} - z_{url}) + C_{rl} (\dot{z}_{rl} - \dot{z}_{url}) - K_{url} (z_{url} - z_{rrl}) - f_{rl} \\
m_{urr} \ddot{z}_{urr} &= -m_{urr} g + K_{rr} (z_{rr} - z_{urr}) + C_{rr} (\dot{z}_{rr} - \dot{z}_{urr}) - K_{urr} (z_{urr} - z_{rrr}) - f_{rr}
\end{aligned} \tag{1}$$

The kinematic equations representing the displacement at each corner are given as follows:

$$\begin{aligned}
z_{fl} &= z - a\theta + 0.5w\phi \\
z_{fr} &= z - a\theta - 0.5w\phi \\
z_{rl} &= z + b\theta + 0.5w\phi \\
z_{rr} &= z + b\theta - 0.5w\phi
\end{aligned} \tag{2}$$

where

$z$  : heave displacement of the vehicle body's mass center,

$\theta / \phi$  : pitch/roll angle,

$z_{fl}, z_{fr}, z_{rl}, z_{rr}$  : heave displacement of the vehicle body at the suspension locations,

$z_{ufl}, z_{ufr}, z_{url}, z_{urr}$  : heave displacement of the unsprung mass,

$z_{rfl}, z_{rfr}, z_{rrl}, z_{rrr}$  : excitation from the road surface,

$f_{fl}, f_{fr}, f_{rl}, f_{rr}$  : variable damping force, and

$g$ : the acceleration due to gravity

The full-vehicle mode parameters were adopted from [8] and are summarized in Table 1.

**Table 1.** Model parameters

Symbol	Description	Units
$m_s$	Sprung mass	1465kg
$m_{ufl}, m_{ufr}$ $m_{url}, m_{urr}$	Unsprung mass	40kg
$K_{fl}, K_{fr}$	Front suspension stiffness	19960N/m
$K_{rl}, K_{rr}$	Rear suspension stiffness	17500N/m
$K_{ufl}, K_{ufr}$ $K_{url}, K_{urr}$	Tire stiffness	175500N/m
$C_{fl}, C_{fr}$	Fixed damping of the front dampers	258N/m/s
$C_{rl}, C_{rr}$	Fixed damping of the rear dampers	324N/m/s
$I_y$	Pitch moment of inertia	2460 $kg \cdot m^2$
$I_x$	Roll moment of inertia	460 $kg \cdot m^2$
$a$	Longitudinal distance from vehicle c.g. to front axle	1.4m
$b$	Longitudinal distance from vehicle c.g. to rear axle	1.7m
$w$	Track width	3m

### 3. Input decoupling transformation

The actuators used to implement the semi-active suspension are continuously adjustable dampers (such as MR dampers), which are installed in all four corners of the vehicle. Each damper is independently controlled to improve the vehicle ride. The sprung mass acceleration at the body's center of gravity should be considered rather than that over the wheels, because the suspension control target is to reduce the acceleration in the location of the occupants. Thus, the damping force of four dampers should be associated with the control forces for three sprung mass accelerations. In this project, the vehicle body posture is directly controlled, considering the heave, pitch, and roll motions, and the three sprung mass accelerations ( $\ddot{z}$ ,  $\ddot{\theta}$  and  $\ddot{\varphi}$ ) at the center of gravity are also controlled.

From Eq. (1), the equivalent forces for heave, pitch, and roll can be defined by

$$\begin{aligned} f_z(t) &= f_{fl}(t) + f_{fr}(t) + f_{rl}(t) + f_{rr}(t) \\ f_\theta(t) &= -af_{fl}(t) - af_{fr}(t) + bf_{rl}(t) + bf_{rr}(t) \\ f_\varphi(t) &= 0.5w(f_{fl}(t) - f_{fr}(t) + f_{rl}(t) - f_{rr}(t)) \end{aligned} \quad (3)$$

and expressed in a matrix form:

$$\begin{bmatrix} f_z \\ f_\theta \\ f_\varphi \end{bmatrix} = W \begin{bmatrix} f_{fl} \\ f_{fr} \\ f_{rl} \\ f_{rr} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -a & -a & b & b \\ 0.5w & -0.5w & 0.5w & -0.5w \end{bmatrix} \begin{bmatrix} f_{fl} \\ f_{fr} \\ f_{rl} \\ f_{rr} \end{bmatrix} \quad (4)$$

Eq. (4) takes the form ' $b = Ax$ ', where  $b$  is the  $m \times 1$  vector,  $x$  is the  $n \times 1$  vector, and  $A$  is the  $n \times m$  matrix. The pseudo right inverse can be obtained using  $W = A^T(AA^+)^{-1}$ ,  $WA=I$ . Thus, Eq. (4) can be expressed as

$$\begin{bmatrix} f_{fl} \\ f_{fr} \\ f_{rl} \\ f_{rr} \end{bmatrix} = \begin{bmatrix} \frac{b}{2(a+b)} & -\frac{1}{2(a+b)} & \frac{1}{2w} \\ \frac{b}{2(a+b)} & -\frac{1}{2(a+b)} & -\frac{1}{2w} \\ \frac{a}{2(a+b)} & \frac{1}{2(a+b)} & \frac{1}{2w} \\ \frac{a}{2(a+b)} & \frac{1}{2(a+b)} & -\frac{1}{2w} \end{bmatrix} \begin{bmatrix} f_z \\ f_\theta \\ f_\varphi \end{bmatrix} \quad (5)$$

By entering the decoupling transformation expressed by (5), the control for heave, pitch, and roll of the car body is converted into the control for the four dampers.

### 4. Skyhook control design

In a full-vehicle suspension system, the rigid body of the vehicle (i.e., the sprung mass) has three DOFs of heave, pitch, and roll, as HPR in the rest of this paper.

According to the theory of skyhook damping presented in [1], the ideal skyhook control forces in three orientations (i.e., heave, pitch, and roll) for the suspension system of a full car model can be expressed as

$$F_j = C_j \dot{Z}_j \quad j = z, \theta \text{ and } \varphi \quad (6)$$

where  $C_j$  is the skyhook control gain,  $\dot{Z}_j$  consists of the HPR velocities of the vehicle body, and  $z$ ,  $\theta$  and  $\varphi$  represent the heave, pitch, and roll orientations, respectively.

We obtain an over-actuated system with three degrees of freedom controlled by four actuators. Thus, the three ideal skyhook control forces must be decoupled. Meanwhile, the actual adjustable dampers cannot produce active forces, so the semi-active suspension controller can only be conducted in an on-off control. Thus, the actual skyhook controller needs the

velocities and relative suspension velocities of the sprung mass. Given the problem of measuring relative velocity in a practical application, velocities of sprung mass and relative velocities at each suspension corner are adopted to build a skyhook controller to achieve a direct input decoupling. In this decoupling skyhook control, seven sensors are needed: three sprung mass accelerometers (measuring  $\ddot{z}$ ,  $\ddot{\theta}$  and  $\ddot{\phi}$ ) and four displacement sensors (measuring suspension relative displacement).

The semi-active skyhook control forces that are constrained by passivity can be expressed as

$$f_j = \begin{cases} C_j \cdot \dot{Z}_j & \dot{Z}_j \dot{Z}_i \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

where  $i = fl, fr, rl, rr$ , and  $j = z, \theta$  and  $\phi$ .

$\dot{Z}_j$  consists of the HPR velocities of the vehicle body, and  $\dot{Z}_i$  of the relative velocities at each suspension corner.

From Eq. (2), the HPR velocities of the four corner points of the vehicle body can be obtained:

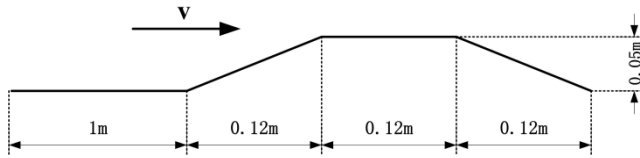
$$\begin{aligned} \dot{z}_{fl} &= \dot{z} - a\dot{\theta} + 0.5w\dot{\phi} \\ \dot{z}_{fr} &= \dot{z} - a\dot{\theta} - 0.5w\dot{\phi} \\ \dot{z}_{rl} &= \dot{z} + b\dot{\theta} + 0.5w\dot{\phi} \\ \dot{z}_{rr} &= \dot{z} + b\dot{\theta} - 0.5w\dot{\phi} \end{aligned} \quad (8)$$

Finally, the actual control force demanded from each damper can be calculated by Eqs. (5) and (7).

## 5. Simulation

The simulation focuses on the vehicle ride comfort and is representative of the vehicle body accelerations. To validate the decoupling skyhook control (DSKY) scheme proposed in this study, a passive suspension is also simulated with fixed damping forces acting on the four dampers. The passive suspension is treated as the baseline for DSKY control. Furthermore, a traditional skyhook control (TSKY) for a full-vehicle system is simulated too. It is controlled from its four corners according to the skyhook control scheme for the quarter-car model. This controller uses eight sensors (i.e., four acceleration and four deflection sensors). For simplicity, the MRDs are considered as ideal linear adjustable dampers, although a polynomial model has been proposed in our previous research [14].

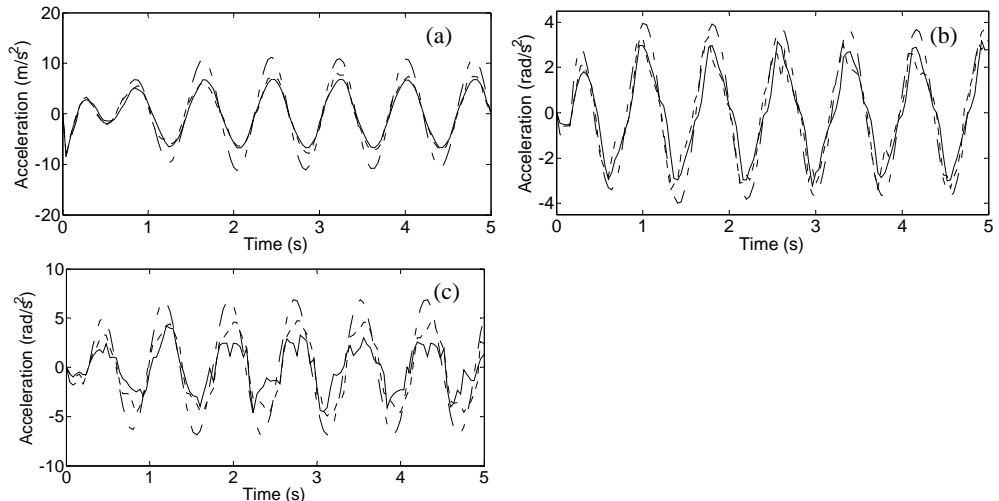
The suspension system in this study has been simulated for two cases, in which a delay is introduced between the front and rear axles. In the first case, the periodic road inputs have three different values of frequency, that is,  $\omega = 8\text{rad/s}$ ,  $\omega = 69\text{rad/s}$ , and  $\omega = 140\text{rad/s}$ . The input terrain disturbances have been defined as sinusoids given by  $z_{rl}(t) = A_{rl} \sin(\omega t)$ ,  $z_{fr}(t) = A_{fr} \sin(\omega t)$ ,  $z_{rl}(t) = A_{rl} \sin(\omega(t + \tau))$ ,  $z_{rr}(t) = A_{rr} \sin(\omega(t + \tau))$ , where  $\omega$  is the terrain disturbance frequency and  $A_{fl}$ ,  $A_{fr}$ ,  $A_{rl}$  and  $A_{rr}$  are the terrain disturbance amplitudes at the front-left, front-right, rear-left, and rear-right wheels, respectively. The time delay is given by  $\tau = (a + b) / v$  and the speed at which the vehicle travels is given by  $v$ . The vehicle is driving along a straight longitudinal trajectory at a constant speed  $v = 24\text{m/s}$ . Hence, longitudinal and lateral dynamics do not interfere with the vertical dynamics. Other simulation parameter values are selected as  $A_{fl} = A_{rl} = 0.05\text{m}$ ,  $A_{fr} = A_{rr} = 0.15\text{m}$ . In the second case, the road disturbance is the ‘‘speed bump’’ used in [15], the profile and dimension of which are shown in Fig.2. Only the left side of the vehicle meets a bump, so that both pitch and roll motion are produced at the same time. Moreover, the skyhook gains for heave, pitch, and roll are  $C_z = 2000\text{N/m/s}$ ,  $C_\theta = 3000\text{N/m/s}$ , and  $C_\phi = 3000\text{N/m/s}$ , respectively.



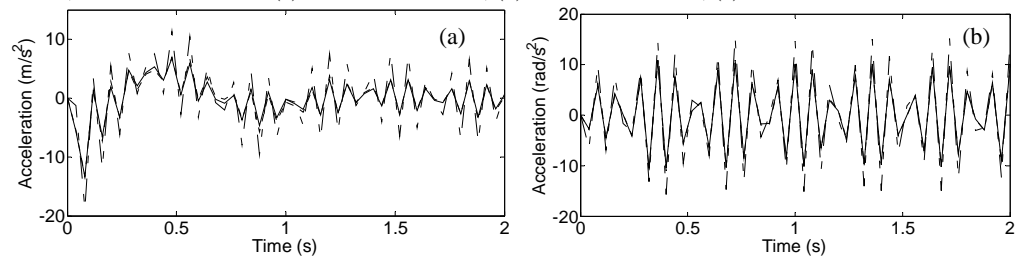
**Fig. 2.** 'speed bump' Profile

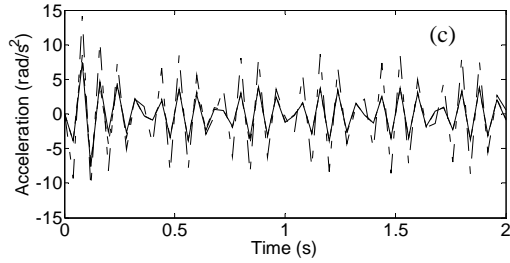
Figs.3 to 5 show the heave, pitch, and roll accelerations at low and high frequencies when the vehicle is given periodic road inputs. As can be seen, responses from low to high frequencies are considerably better for the new configuration compared with those from the traditional skyhook and passive cases.

Figure 6 shows the vehicle acceleration response for a speed bump. The vertical acceleration is shown in Fig.6(a). The DSKY configuration clearly yields the best results. The reduction of peak acceleration is approximately 20% more than that in the passive case and 10% more than that in the traditional skyhook case. Both the DSKY and TSKY configurations attenuate the vertical motions faster than the passive configuration does. The pitch response is shown in Fig.6 (b). As can be seen, the DSKY configuration yields the smallest peak and the fastest time in eliminating the pitch motions. Fig.6(c) shows the roll response. As can be seen, the DSKY configuration also yields the smallest peak. However, the passive configuration converges faster than do the DSKY and TSKY cases.

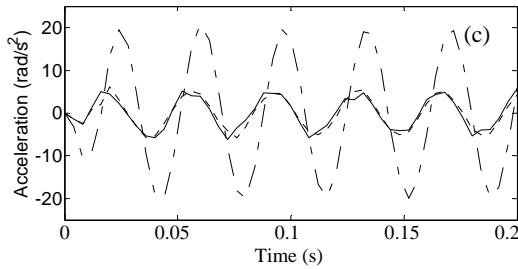
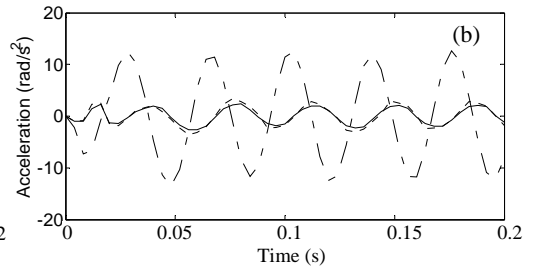
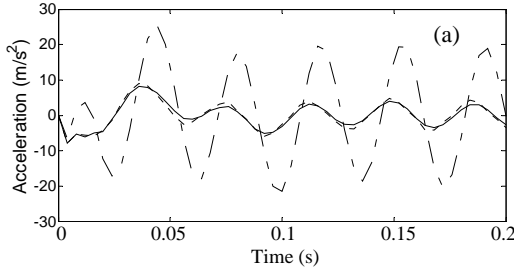


**Fig.3.** Vehicle Acceleration Responses for input frequency  $\omega = 8\text{rad/s}$ . -----: DSKY; - . - . : TSKY; — — — : Passive. (a)Heave acceleration; (b) Pitch acceleration; (c) Roll acceleration

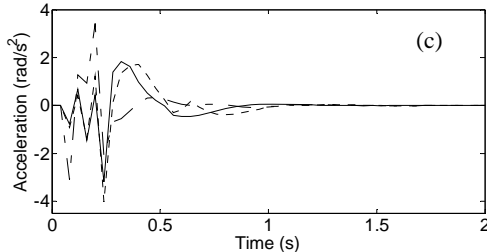
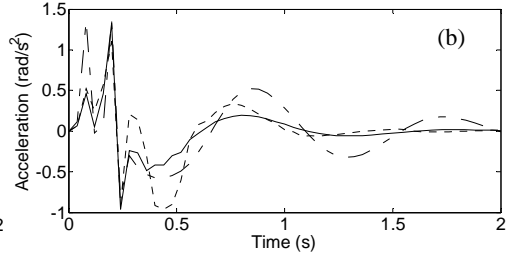
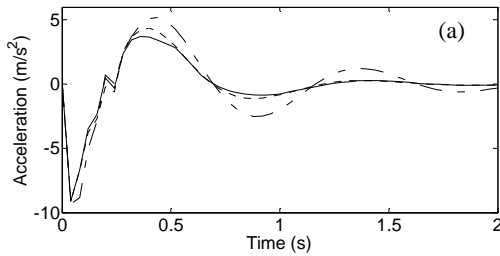




**Fig.4.** Vehicle Acceleration Responses for input frequency  $\omega = 69\text{rad/s}$ . -----: DSKY; - - - - : TSKY; - · - · : Passive. (a)Heave acceleration; (b) Pitch acceleration; (c) Roll acceleration



**Fig.5.** Vehicle Acceleration Responses for input frequency  $\omega = 140\text{rad/s}$ . -----: DSKY; - - - - : TSKY; - · - · : Passive. (a)Heave acceleration; (b) Pitch acceleration; (c) Roll acceleration



**Fig.6.** Vehicle Acceleration Responses for 'speed bump' road exciting. -----: DSKY; - - - - : TSKY; - · - · : Passive. (a)Heave acceleration; (b) Pitch acceleration; (c) Roll acceleration

## 6. Conclusion

A semi-active suspension using a novel methodology called DSKY was developed to control the attitude of the vehicle body and to suppress the vibration of the suspension. In

accordance with the vehicle body motion (vertical, pitch, and roll), three skyhook controllers were designed. An input decoupling transformation converted the three orientation skyhook control forces into actual damping forces of four adjustable dampers. The simulations were conducted under two kinds of road inputs: periodic road inputs with three different frequencies and a “speed bump” input. The results showed that the proposed semi-active system provides better ride control in the heave, pitch, and roll motions of the vehicle body compare with the traditional semi-active and passive systems.

### **Acknowledgement**

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