Application of Load Updating to a Complex Three Dimensional Frame Structure

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ABSTRACT

This thesis presents a novel method for the correlation of FEM results to experimental test results known as the "Load updating method." Specifically, the load updating method uses the math model from the FEM and the strains measured from experimental or flight test data as inputs and then predicts the loads in the FEM which would result in strains that would correlate best to the measured strains in the least squared sense. In this research, the load updating method is applied to the analysis of a complex frame structure whose validation is challenging due to the complex nature of its structural behavior, load distributions, and error derived from residual strains. A FEM created for this structure is used to generate strain data for thirty-two different load cases. These same thirty-two load cases are replicated in an experimental setup consisting of the frame, supporting structure, and thirty actuators which are used to load the frame according to the specifications for each of the thirty-two load conditions. A force-strain matrix is created from the math model in NASTRAN using unit loads which are separately applied to each load point in order to extract strain results for each of the locations of the seventy-four strain gages. The strain data from the structural test and the force-strain matrix is then input into a Matlab code which is created to perform the load updating method. This algorithm delivers a set of coefficients which in turn gives the updated loads. These loads are applied to the FEM and the strain values extracted for correlation to the strains from test data. It is found that the load updating method applied to this structure produces strains which correlate well to the experimental strain data. Although the loads found using the load updating method do not perfectly match those which are applied during the test, this error is primarily attributed to residual strains within the structure. In summary, the load

updating method provides a way to predict loads which, when applied to the FEM, would result in strains that correlate best to the experimental strains. Ultimately, this method could prove especially useful for predicting loads in experimental and flight test structures and could aid greatly in the Federal Aviation Administration (FAA) certification process.

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GENERAL AUDIENCE ABSTRACT

The research presented in this thesis provides a new way for correlating data obtained during structural testing with results obtained from computer analysis known as the finite element method (FEM). During the process of certifying an aircraft structure with the FAA, it is important to be able to demonstrate that the results obtained for a given structure with a computer model matches the results produced by a real world experiment within a reasonable tolerance. Traditionally, differences between these two results have been accounted for by adjusting the model within the computer until its results match those from the test. However, in this research the loads which are applied on the computer model are changed instead until loads are found which produce results in the computer models that match those from testing. This method, known as the load updating method, therefore provides a way to predict loads on a structure where the loads are unknown such as a flight test article. Here, the ability of the load updating method to predict loads on a complex three dimensional frame structure is explored and the accuracy of the results studied by comparing the results to those from a structural test whose loads are known. It was found that the load updating method does indeed predict unknown loads to a reasonable accuracy and could aid future design efforts immensely.

DEDICATION

This thesis is dedicated to my mom who is the greatest teacher I have ever had and to my dad who taught me to work hard in every pursuit that I undertake.

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This thesis would not have been possible without the strong network of support that I had throughout this journey. First, I would like to thank my advisors Dr. Schetz and Dr. Kapania who went above and beyond in supporting my efforts in this research as a distance student. Their flexibility and creative teaching methods provided me with an outstanding educational opportunity and I believe expanded perceptions on the effectiveness of distance education. Also, John McCullough, Louis Tackett, and Alejandro Barbarin who mentored me in NASTRAN analysis and analytical methods of which I learned a tremendous amount about during this research. Individuals such as these make Bell Helicopter an outstanding work place and foster an environment for young engineers like myself to grow technically. My direct supervisor, Mark Upchurch, whom I am grateful to for providing me with the tools and support I needed to complete my Master's degree and this thesis. I am also appreciative of Rus Holloway and the Intellectual Property Department at Bell Helicopter which worked incredibly hard, despite their hectic schedule, to support tight deadlines in the approval process for this writing. In addition to the mentioned professionals, a few key people in my personal life also supported me on this journey who cannot go unnamed. My beautiful fiancé, Maria Tovar, who supported and encouraged me throughout this endeavor and often sacrificed time with me in order for this to be completed. My parents, who have always encouraged me in my educational pursuits and have reminded me many times of the value of this incredible opportunity. Finally, I thank God who gifted me with this opportunity and provided me with the mind and talent to make the most of it. Through him all things are possible and to him be the glory.

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LIST OF ABBREVIATIONS

Symbol	Description
ε	Strain
δ	Displacement
ϕ	Rotation
Λ	Tikhonov Regularization Parameter
c	Coefficients Vector
E	Young's Modulus
f	Force Vector
F	Force Matrix
G	Compilation Matrix for Displacement Formulation
I	Moment of Inertia
K	Stiffness Matrix
L	Length
M	Strain-Force Matrix
Ni	Shape Function *i*
N	Compilation Matrix for Strain Formulation
Pi	Loading Function *i*
P	Load
T	Reduction Matrix
u	Full Displacement Vector
u_m	Measured Displacement Vector
U	Measured Data Vector
V	Finite Element Data Vector

Acronym	Description
FAA	Federal Aviation Administration
FEM	Finite Element Method
GAG	Ground-Air-Ground
SVD	Singular Value Decomposition

CHAPTER 1

INTRODUCTION

1.1 Concept and Demand for Load Updating

When attempting to test and certify a new aircraft, one of the most daunting tasks is that of reconciling data from finite element models (FEM) to that of experimental structural testing. In an ideal world, the results of test data would mirror that of computational methods derived from theoretical calculations. In reality however, FEM often delivers results that differ from theory which leaves the analyst questioning the validity of their model. Blame for these differences typically begins with improper assumptions and includes poorly modeled structure, improperly applied boundary conditions, and discrepancies in the as built structure. However, the ability to correlate these two results gives the designer the flexibility to use further analysis from the FEM to assist in FAA certification and performance calculations. The ability for numerous loading cases to be studied in a relatively short time period greatly reduces the cost when compared to the expense of obtaining large quantities of experimental data.

Traditionally, differences between FEM and test data have been minimized with a technique known as system identification or model updating. In recent years, extensive development of model updating has taken place as engineers seek to perfect the art of model refinement. Model updating seeks to modify the FEM by changing the mathematical model which describes the structure. This improves the predictions given by the model by modifying parameters where there is uncertainty in the analytical model (Mottershead & Friswell, 1993). Although often successful, model updating can also be time consuming and computationally expensive for a large number of reasons. In addition, the results obtained from model updating are often valid only for a given load case due to the model alteration that has taken place (Chock & Kapania, Load Updating for Finite Element Models, 2003).

As a response to this technical challenge, Chock and Kapania offer the alternative of load updating. In load updating, the differences between analytical and experimental data are reduced by determining the equivalent loads on the FEM that will produce the closest values for strain or displacement that are measured on the experimental test structure. This method provides a faster more efficient alternative to the computationally expensive model updating. It also provides extended flexibility by producing results that allow additional load cases and similar structures to be quickly analyzed (Chock & Kapania, Load Updating for Finite Element Models, 2003).

In this thesis, research is discussed which builds further upon the foundation on the load updating concept. A frame structure used in the design of the Bell Helicopter 505 JetRanger X is the subject of interest. This frame provides structure which fits within the aft fuselage of the helicopter and supports the tailboom which extends directly behind it. The transmission and mast are placed on top of the forward end of this structure. This results in the frame having numerous load introduction points. A static and fatigue test setup is designed to emulate loads expected from flight test and a FEM specific to this test is created for correlation. This static test has thirty load introduction points which are connected to actuators that apply the loads. Thirty-two load cases are identified as comprising a single ground-air-ground (GAG) cycle on this aircraft. Seventy-four strain gages collect data for each load case and strain data is collected from the FEM for each of the identical seventy-four locations where gages are placed on the test article. Correlating FEM data to test data for this structure is extremely difficult which corroborates the usefulness of having alternative methods like load updating to assist in this process. The complexity of this structure makes the load updating method especially useful for this correlation. Although the FEM is designed to represent the test setup as accurately as possible, large errors are still found between the two sets of data during analysis. These errors are ultimately attributed to large residual strains within the frame structure which are introduced during the welding of the frame pieces. Therefore, even if the model perfectly represented the frame and the test structure, the model would still have to be changed in order to account for these residual strains for the data to correlate. The load updating method is therefore a prime candidate for improving the correlation of the model without changing the math model which is meticulously created to be as accurate as possible. The results from this research demonstrate the usefulness of the load updating method and also shed light on where future progress can be made.

1.2 Application of FEM Modeling in Industry

The importance of the history of finite element modeling within industry is that it shows a trajectory that (when extrapolated) can be useful in understanding where FEM will take us in the near future. As modeling techniques improve and computing power continues to climb exponentially, FEM will move closer and closer to the center stage of the certification journey for aircraft structures. This will greatly reduce the amount of time and cost associated with extensive testing and will allow for perhaps more complex analysis to be conducted that one might be unable to replicate experimentally.

The finite element method was developed by a number of practicing aerospace engineers whose goal was to advance analysis techniques by employing computers. Many were well versed, if not experts, in structural mechanics who favored the Classical Force Method which was popular at the time (Felippa, 2017). One of the early uses of FEM was the analysis of statically indeterminate structures. Although desirable for the additional strength they provide, statically indeterminate structures also are often very difficult to analyze. FEM provided an opportunity for analyzing many different load cases on these structures which reduced the quantity of testing required and thus making testing feasible. In summary, early FEM was applied to problems that could not be analyzed by hand and were too expensive to test (Rodamaker, 2002).

As FEM continued to develop, error bounding and convergence techniques were created as energy methods, the direct stiffness method, and the requirement for inter-element compatibility condition continued to mature. As the mathematical foundations for the method continued to be established, substantial progress was made in both the error estimation and the mesh adaptivity. Today, the ancestry of FEM can be traced back to matrix structural analysis, variational approximation theory, and the digital computer (Felippa, 2017).

The evolution of the FEM's role in aerospace suggests that it will continue to improve in its capabilities and gain trust as computers become more powerful and application techniques more accurate. Rodamaker (2002) notes that many FEM software developers have shifted focus from improvements to accuracy to instead improvements to user friendliness. He points out that while the long term effects of this approach are unknown, testing and FEM are complementary disciplines that should be used together at some level. The load updating method and many other novel computational techniques continue to push the boundaries of what FEM can provide, especially for certification purposes. In the future, this is likely to help reduce the number of tests necessary which will in turn dramatically reduce both the cost and the time required for certification.

1.3 FAA Certification Requirements for Airframe Structures

As discussed in the previous section, the advancements in FEM capabilities has motivated a desire to use more analytical data in the certification process. One of the challenges in adapting this new technique is conforming to the requirements established by the Federal Aviation Administration (FAA). In this section, an overview of these requirements will be presented. This will provide context and the ability to quantify improvements in the results section of this thesis.

Three statements form the structure and scope for this discussion. The first comes from 14 CFR Parts 24 and 25 §301(b) which says “Methods used to determine load intensities and distributions…must be validated...unless the methods…are shown to be reliable or conservative... ((FAA) F. A., 2017).” This means that any new method must be validated in order for it to be used as a method for certification. One requirement in showing this reliability, which will be demonstrated in this research, is that the loads used for validation are realistic ((FAA) F. A., 2017). As it turns out, this is not only a good practice for the analytical engineer, it also points to the need for the load updating method.

Second, the FAA states in 14 CFR Part 25 §305(b) “When analytical methods are used to show compliance with the ultimate load strength requirements, it must be shown that – ...The methods and assumptions used are sufficient to cover the effects of these deformations ((FAA) F. A., 2017).” The structural loads applied to the frame structure in this thesis are load conditions which emulate various flight patterns the aircraft will perform in a given Ground-Air-Ground (G.A.G.) cycle. Therefore, this requirement is not applicable to this particular research since an aircraft’s structure will not approach ultimate load strength during a typical G.A.G. cycle. However, in future analysis, where the load updating method is applied, it would be of importance to account for deformations if the data produced is to be used for certification.

Finally, 14 CFR Parts 23, 25, 27, and 29 §307(a) states that "Structural analysis may be used only if...experience has shown this method to be reliable. In other cases, substantiating load tests must be made ((FAA) F. A., 2017)." In this statement, what is considered to be "reliable" is left somewhat vague however, there are a few guidelines that could direct the engineer seeking to prove a new method. One is that strains be predicted prior to testing and then the results from the subsequent testing compared. The FAA suggests that predicted results which correlate within 10 percent of test results will be acceptable without any further evaluation. They reason that this level of correlation demonstrates the model geometry, stiffness data, internal load distribution, and boundary conditions are within an acceptable tolerance. In addition, a requirement is made for strain gages to be placed in locations of high stress and complex geometry ((FAA) F. A., 2017). In particular, this requirement of correlating test and FEM data to be within 10 percent will be key in the discussions from this research.

In short, the FAA is wanting to ensure that a given analytical method can be proven to produce repeatedly reliable results. In purpose, this differs little from the current regulations surrounding structural testing which also call for sound demonstration repeatability in experimental methods. Knowledge from engineers in the testing world can therefore continue to be useful to those engineering analysts seeking to meet these new requirements.

1.4 Overview of Prior Research

As discussed in the previous section, correlation between FEM and test data is a topic that is on the fore front of the structural analysis discipline. It seems fitting that an overview should be presented of the current efforts to solve this problem in order to critique the load updating method within the context (Rodamaker, 2002) of other methods available. Generally, a form of system identification known as model updating is used to address the discrepancies between FEM and real world data. Model updating seeks to correct these discrepancies by altering parameters in the model where uncertainties exist. Direct methods and sensitivity based methods are the two main categories of model updating techniques (Zhang, Sim, & Spencer, 2007). The direct method consists of directly updating the matrices for mass, stiffness, and damping so as to maintain symmetry and account for the test data simultaneously. The direct method is especially useful in vibration analysis where extra unwanted modes could be obtained using a sensitivity based method. Also, model reduction and expansion techniques are not required for the direct method and it preserves the original frequencies and modes throughout the updating process (Carvalho, Datta, Gupta, & Lagadapati, 2007). In a sensitivity based method, data is first acquired for both the current state of the model and the ideal state where the model would perfectly represent the real world. A sensitivity matrix is then constructed and applied to the design parameters which when correctly applied will bring the current state closer to the target state (Avitabile, 2000). Sensitivity methods are used more commonly because they are essentially employing a type of optimization whereas direct methods impose their measured values on the solution thus propagating any error from the measurement to the updated model (Zhang, Sim, & Spencer, 2007).

Load Updating was first conceptualized by Chock and Kapania (2003) as an alternative to the traditional model updating methods described above. The process is described at its most fundamental level and an example is given consisting of a beam subject to a distributed load. The math model is assumed to be accurate and the loads treated as an unknown which makes this an inverse problem. It is therefore solved using an approach opposite from traditional model updating techniques. The test data is input into the load updating algorithm along with the stiffness matrix and a guess for the force vector. The result is a solution of coefficients that can be used to determine the loads that will match the test data in a least squares sense. It was determined that singular value decomposition SVD was particularly useful in providing an initial guess when applying steepest descent method to solve the poorly conditioned system. Although regularization is not used in this particular study, it is suggested as an option for further study especially if higher order polynomials are to be used. These ideas formed the foundation for the research that is presented in this thesis.

The load updating method is extended to two dimensional problems by Chock and Kapania (2004) in their analysis of its application to plates. In addition, regularization is tested with the load updating method for the first time. Although it produced widely varying results, it is an important step in advancing the tools used in conjunction with the load updating method.

Problems containing insufficient data are discussed by Li and Kapania (2004). Although ideally one would collect as much data as possible when solving an inverse problem, this is not always feasible. Therefore, a reduced number of load coefficients (preferably less than or equal to the number of measured data points) is used to address the problem of large oscillations within the extracted load. It is found that reducing the number of load coefficients does indeed ameliorate the oscillations of the extracted loads based on the results of two separate cases studied.

The capabilities of the load updating method are expanded to non-linear problems by Li and Kapania (2007). In non-linear problems, the inverted stiffness matrix and reduced displacement vector can no longer be assumed to be constant and must be recursively solved at each step. Also introduced in this research is the ability to apply constraints on the solutions obtained from the overdetermined system. This consisted of applying what is known about the load distribution to the objective function that is to be minimized. The introduction of these additional complexities prompted the implementation of Tikhonov regularization which handled the issues of noise and also that of non-unique solutions.

The purpose of this thesis is to expand upon the load updating method and to apply it to a real world frame structure. This structure, whose correlation between FEM and test data would normally be done with model updating, will be improved by determining the equivalent loads on the FEM that will produce strains which more closely match those measured during structural testing. The success of this analysis is of key interest due to the FAA requirements described in section 1.3. It is hoped that this research will move the capabilities of the load updating method forward by providing analysts with an additional tool to use in their correlation efforts.

CHAPTER 2

METHODOLOGY

2.1 Load Updating

2.1.1 Load Updating Method

When performing traditional model updating, one assumes the loads applied in the FEM are known and alters the math model in an attempt to derive an FEM capable of being used to predict test results. Load updating however seeks to obtain the same result by approaching the problem from the opposite perspective. A given math model within a FEM is assumed to be known and the loads applied to the model are solved for so as to obtain the strain or displacement data required. Therefore, load updating is an inverse problem as it uses resulting strains or displacements in a given condition to determine the forces that caused them. While inverse problems can be difficult to initially solve, their solutions provide a number of benefits. These include preservation of the original math model, more efficient computing time for ongoing cases studied, and added confidence in the results obtained. Overcoming the challenge of the initial difficulties in obtaining the updated loads and maximizing the benefits of the results obtained is the quintessential purpose of this thesis.

To review the process of applying the load updating method as discussed by Chock and Kapania (2003), the state equation for linear FEM is expressed both in terms of a force vector and a load matrix. Where K is a stiffness matrix which relates the displacements u to the forces in vector f.

$$Ku = f = Fc \tag{1}$$

Development of the force matrix begins with a recognition of the ability to express the load distribution as a combination of an average value function and a term to allow for slight fluctuations about the mean.

$$f(x) = f_0(x) + \sum_{i=1}^{n} c_i P_i(x) \tag{2}$$

The principle of superposition allows the average function to model both the average values and the fluctuations within the same, providing the response of the structure can be approximated to be linear.

$$f(x) = \sum_{i=1}^{n} c_i P_i(x) \tag{3}$$

Finally, for a given element, the load distribution can be described as follows.

$$p^e(\bar{x}) = \sum_{i=1}^{i=m} c_i^e P_i(\bar{x}) \tag{4}$$

The elements of matrix *F* are defined as the integral along the length of a given element over the *i*th shape function and the *j*th load function.

$$F_{i,j}^e = \int_0^{le} N_i(x) P_j(x) dx \tag{5}$$

Here the size of *i* is the number of shape functions used and *j* is the number of unknown coefficients c in that particular element. The goal of this process is to minimize the square of the difference between the vector of strain or displacement data given by the test with the vector of that same data given by the FEM. This will allow the vector of coefficients *c* to be obtained from equation 1.

$$min\|U - V\|_2^2 \tag{6}$$

In order to accomplish this the gradient of equation 7 must be taken as shown. This gradient gives a relationship which describes the sensitivity for each of the displacements to the vector of coefficients c.

$$\nabla[\|U - V\|_2^2]_i^e = 2(U_i - V_i) * (\partial_h V_i)^T \tag{7}$$

This obviously requires that the size of the displacement vector from the FEM to be reduced to the size of the displacement vector of the test data.

$$u_m = Tu \tag{8}$$

$$u_m = TK^{-1}Fc \tag{9}$$

The objective is to minimize the error *e* by changing the *c* coefficients which gives.

$$\frac{\partial e}{\partial c_i} = \frac{\partial u}{\partial c_i} = G_i \tag{10}$$

Applying the minimization condition gives the symmetric square set of equations which are solved for to find the unknown coefficients c.

$$G^T G c = G^T u_m \tag{11}$$

The solution to this linear set of equations given by equation 11 is the completion of applying the load updating method. A flowchart (Chock & Kapania, 2004) is depicted below to demonstrate how the load updating method could be implemented to correlate FEM and test data for an aircraft structure.

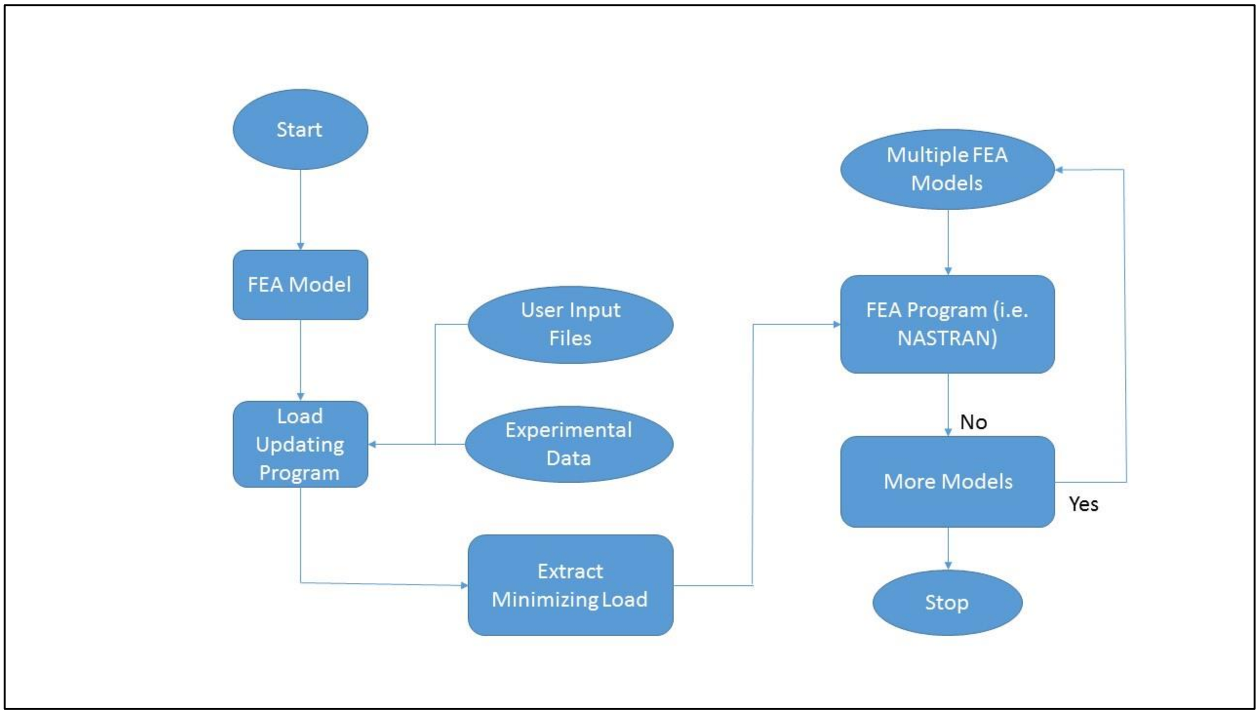


Figure 1: Flowchart for Implementing Load updating method

2.1.2 Load Updating Example

To better understand and research the load updating method, a simple experiment with a cantilever beam is first conducted. The experimental setup consists of a small rectangular aluminum beam whose strain is measured by two strain gages as shown below in figure 2 and figure 3. Mechanics of Materials equations are used to calculate the true values of these strains and then those values are compared to the measured and the FEM calculated values. The load updating method can then be applied in order to demonstrate its effectiveness in predicting the tip load at point D.

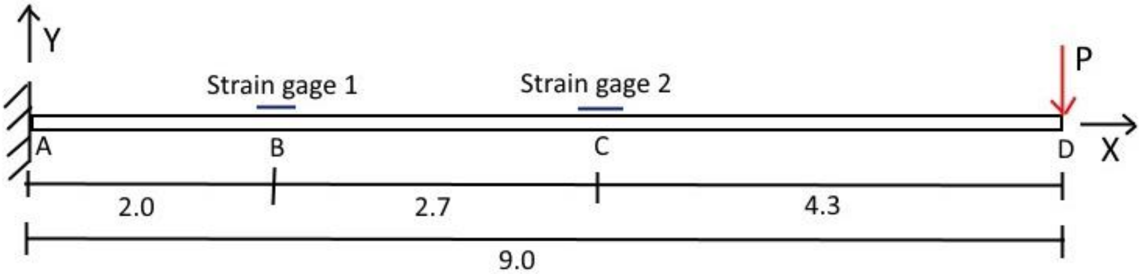


Figure 2: Cantilever Beam with Point Load

Figure 2 and Figure 3 show the setup for the cantilever beam tested. The aluminum beam is 9 inches long and has a strain gage 2.0 inches, point B, and 4.7 inches, point C, from the clamped end at point A. It is restrained at the root end by a clamp which imposes the boundary conditions of no deflection or rotation at point A.

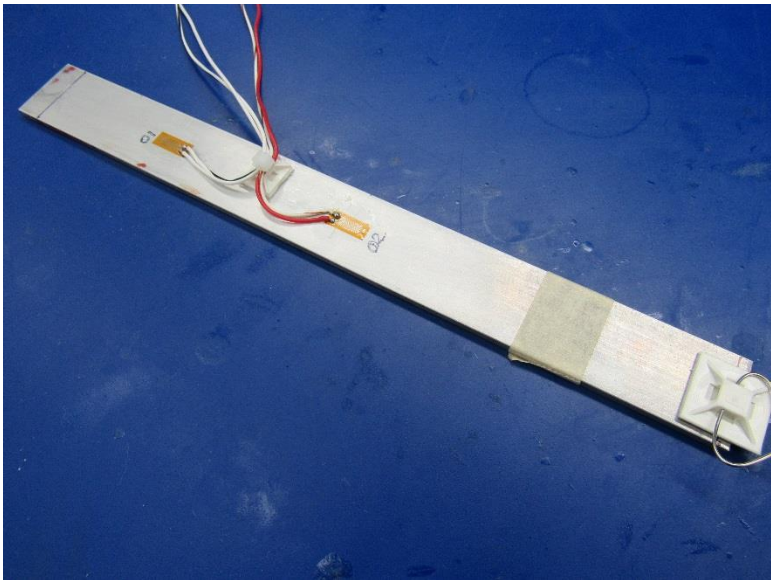


Figure 3: Cantilever Beam Setup

The properties of the beam and of the strain gages are given below in Table 1 followed by the mechanics of materials solutions for strain, rotation, and deflection. The values are provided as the standard for what is expected between the measured and the FEM calculated values.

Dimensions x, y, & z	Modulus of Elasticity, E	Moment of Inertia, I	Neutral Axis to Top Surface, y/2
9, 0.125, 1 (inches)	10,000,000 (psi)	1.6E-4 (inches^4)	6.25E-2 (inches)

Table 1: Beam Properties

$$\varepsilon = \frac{Moment\ y}{2EI} \tag{12}$$

$$\emptyset = \frac{Px(2*L-x)}{2EI} \tag{13}$$

$$\delta = \frac{Px^2(3*L-x)}{6EI} \tag{14}$$

The full stiffness matrix, load vector, and displacement vector for this three element system are as follows.

$$\begin{bmatrix} Fa \\ Ma \\ Fb \\ Mb \\ Fc \\ Mc \\ Fd \\ Md \end{bmatrix} = EI \begin{bmatrix} \frac{12}{L1^3} & \frac{6}{L1^2} & -\frac{12}{L1^3} & \frac{6}{L1^2} & 0 & 0 & 0 & 0 \\ \frac{6}{L1^2} & \frac{4}{L1} & -\frac{6}{L1^2} & \frac{2}{L1} & 0 & 0 & 0 & 0 \\ -\frac{12}{L1^3} & -\frac{6}{L1^2} & \frac{12}{L1^3}+\frac{12}{L2^3} & -\frac{6}{L1^2}+\frac{6}{L2^2} & -\frac{12}{L2^3} & \frac{6}{L2^2} & 0 & 0 \\ \frac{6}{L1^2} & \frac{2}{L1} & -\frac{6}{L1^2}+\frac{6}{L2^2} & \frac{4}{L1}+\frac{4}{L2} & -\frac{6}{L2^2} & \frac{2}{L2} & 0 & 0 \\ 0 & 0 & -\frac{12}{L2^3} & -\frac{6}{L2^2} & \frac{12}{L2^3}+\frac{12}{L3^3} & -\frac{6}{L2^2}+\frac{6}{L3^2} & -\frac{12}{L3^3} & \frac{6}{L3^2} \\ 0 & 0 & \frac{6}{L2^2} & \frac{2}{L2} & -\frac{6}{L2^2}+\frac{6}{L3^2} & \frac{4}{L2}+\frac{4}{L3} & -\frac{6}{L3^2} & \frac{2}{L3} \\ 0 & 0 & 0 & 0 & -\frac{12}{L3^3} & -\frac{6}{L3^2} & \frac{12}{L3^3} & -\frac{6}{L3^2} \\ 0 & 0 & 0 & 0 & \frac{6}{L3^2} & \frac{2}{L3} & -\frac{6}{L3^2} & \frac{4}{L3} \end{bmatrix} \begin{bmatrix} Va \\ \theta a \\ Vb \\ \theta b \\ Vc \\ \theta c \\ Vd \\ \theta d \end{bmatrix}$$

Figure 4: Cantilever Beam FEM System

The imposition of the boundary conditions at the root end of the beam eliminates rows and columns one and two from the system. This system is solved using the Matlab code in Appendix A. A 2 lbs load is applied to the tip of this beam and measurements are taken at each of the two strain gages. The values for strain, rotation, and deflection are shown below in Figure 6 through Figure 8 which compare these values to the theoretical and FEM ones for a bending moment shown in Figure 5. The theoretical, measured, and FEM values are all in close agreement as shown below in figures 5 through figure 9 and also in table 2. This verifies that the FEM is developed correctly and that the experiment to obtain the strain gage measurements is accurate. The load updating method will be applied to this system in order to demonstrate the mechanics for using it to predict the load.

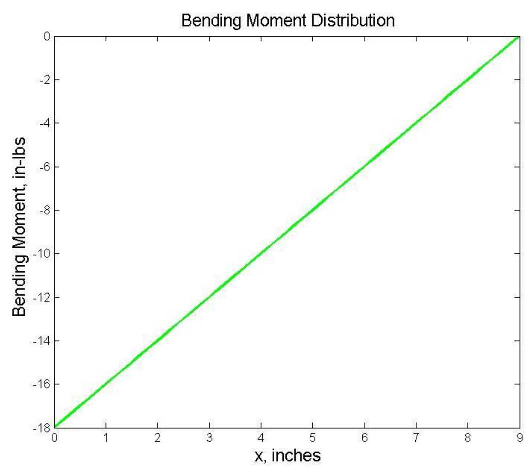


Figure 5: Cantilever Beam Bending Moment

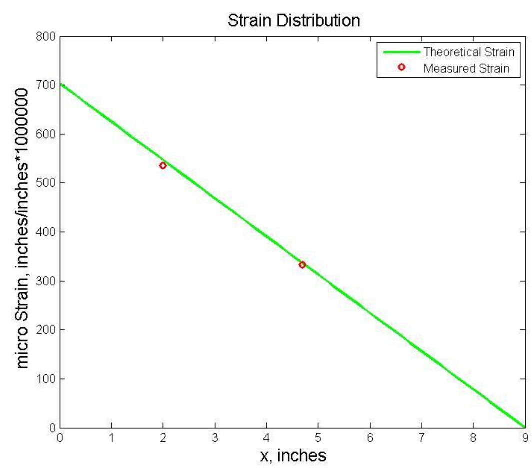


Figure 6: Cantilever Beam Strain

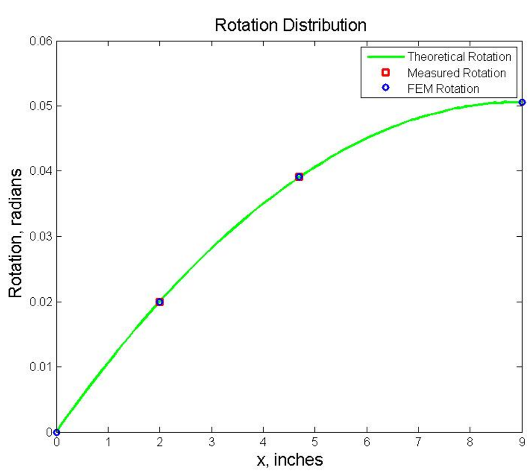


Figure 7: Cantilever Beam Rotation

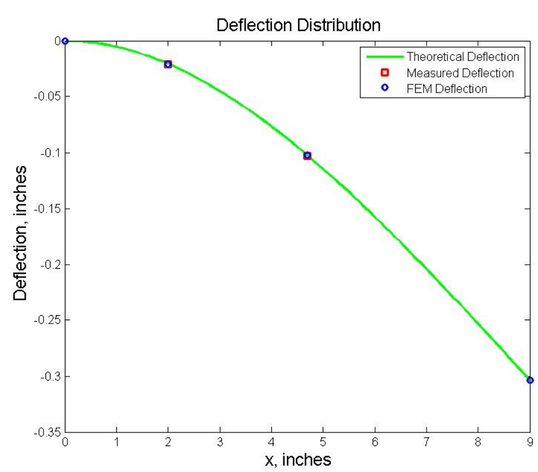


Figure 8: Cantilever Beam Deflection

	μStrain, inches/inches *1000000	Rotation, radians	Deflection, inches
Gage 1 Theoretical/Measured/Updated FEM	547/536/NA	0.0020/0.0020/0.0200	-0.02080/ -0.02080/-0.0208
Gage 2 Theoretical/Measured/Updated FEM	336/333/NA	0.0391/0.0391/0.0391	-0.1030/ -0.1030/-0.1026

Table 2: Cantilever Beam Initial Results

In order to apply the load updating method, the FEM must be reduced to the size of the system which has values for the measurements as explained by equation 8. This results in the setup described by equation 9 for this system. The load updated method is then applied to this system as follows with the updated results shown below. In this simple example, the stiffness matrix can then be simply inverted using Matlab and does not require any special algorithms. This inverse matrix can then be used in conjunction with the measured strains to solve for the load updating coefficients. These coefficients multiplied by the force matrix gives the values for the updated loads. These updated loads are then used in the FEM code to give the results in table 3.

```
>> cantileverbeam

displacement =

  -0.0208
  -0.0200
  -0.1026        Original Displacements
  -0.0391
  -0.3038
  -0.0506

>> um=[-0.0208,-0.02,-0.103,-0.0391];        Measured Strains and Force Input
>> force=[0;0;0;0;-2;0];
>> ktemp=inv(K2)

ktemp =

   0.0017  0.0013  0.0050  0.0013  0.0104  0.0013
   0.0013  0.0013  0.0046  0.0013  0.0100  0.0013
   0.0050  0.0046  0.0216  0.0069  0.0513  0.0069
   0.0013  0.0013  0.0069  0.0029  0.0195  0.0029
   0.0104  0.0100  0.0513  0.0195  0.1519  0.0253
   0.0013  0.0013  0.0069  0.0029  0.0253  0.0056

>> K3=ktemp(1:4,:)
K3 =
   0.0017  0.0013  0.0050  0.0013  0.0104  0.0013
   0.0013  0.0013  0.0046  0.0013  0.0100  0.0013     Reduced Inverted Stiffness Matrix
   0.0050  0.0046  0.0216  0.0069  0.0513  0.0069
   0.0013  0.0013  0.0069  0.0029  0.0195  0.0029

>> NN=K3*force
NN =
  -0.0208
  -0.0200
  -0.1026
  -0.0391

>> c=um'/NN        Solving for Updating Coefficeints
c =
      0      0  0.2027      0
      0      0  0.1949      0
      0      0  1.0036      0
      0      0  0.3810      0
>> updatedloads=force*c(3,:)

updatedloads =
      0      0      0      0
      0      0      0      0     Coefficients Times Force Input Gives Updated Loads
      0      0      0      0
      0      0      0      0
      0      0 -2.0073      0
      0      0      0      0
```

Figure 9: Load updating method Solution for Cantilever Beam

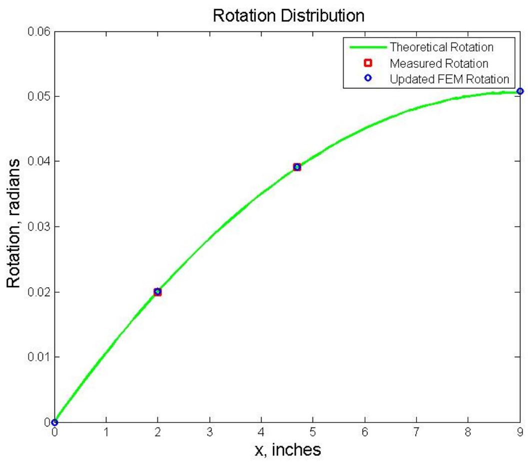


Figure 10: Updated Cantilever Beam Rotation

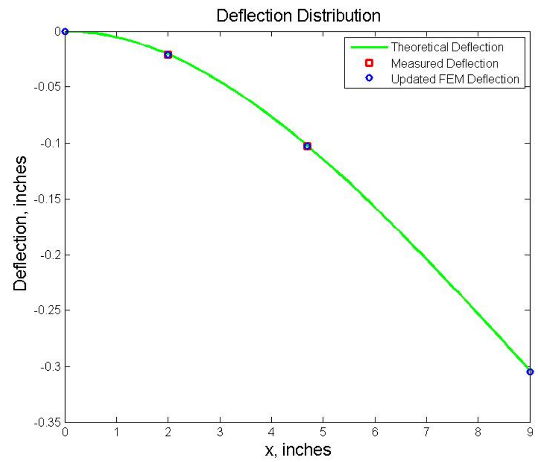


Figure 11: Updated Cantilever Beam Deflection

	µStrain, inches/inches *1000000	Rotation, radians	Deflection, inches
Gage 1 Theoretical/Measured/FEM	547/536/NA	0.0020/0.0020/0.0201	-0.02080/ -0.02080/-0.0209
Gage 2 Theoretical/Measured/FEM	336/333/NA	0.0391/0.0391/0.0392	-0.1030/ -0.1030/-0.1030

Table 3: Cantilever Beam Updated Results

The tip load is predicted to be -2.007 lbs. This result shows that the load updating method predicts the tip load very accurately using only the finite element math model and the measured strains. Although this is a rather simple example, it demonstrates the process of applying the load updating method and the potential accuracy of the results that it produces. Attention is now given to applying the load updating method to the primary problem of interest.

2.1.3 Load Updating Applied to Frame

In this experiment, the subject of interest is a frame structure which is subject to thirty load actuator inputs within a static test. Thirty-two different load conditions are measured by seventy-four strain gages scattered across the structure in an effort to better understand its response. For this analysis, the Load updating method is modified as follows to accommodate the strain data available and to meet the goal of correlating that strain data. Beginning with the state equation of the FEM the following relationship is obtained.

$$M\epsilon = f = Fc \tag{15}$$

Here, the force vector *f*, force matrix *F*, and coefficients *c* remain unchanged while the matrix is instead a relationship between these forces and the strains ϵ which the math model produces and is designated *M*. The objective is still to minimize the square of the error between the vector of strains given by the test *U* and the vector of strains given by the FEM *V*. This is done by first developing the matrix *M*. Matrix *M* is created by determining the strain for each of the seventy-four strain gage locations that is given for a unit load applied separately at each of the thirty actuator positions. In other words, a unit load is applied at a given actuator position and the data from each of the seventy-four gage locations recorded and then divided by the load applied to give a value in units of strain/pound. This results in a seventy-four by thirty matrix which although in reality is the inverse of matrix *M*, proves to be the relationship needed as will be seen shortly. It is also important to note that the matrix developed only contains values for the reduced dimension in which all the strain values are known. This relationship developed between the actuator loads and the resulting strains makes this formulation an energy method of sorts because it relates the strain within the structure to the external forces applied. This also means that the actuator forces are expressed as generalized coordinates. A more thorough discussion of this math model development is presented in the next section, 2.2. Although the formulation of the Load updating method is described assuming a distributed load, the fact that FEM reduces the loads to a finite number of nodes and that the frame consists of thirty load points means that the formulation is in essence identical to that developed by Chock and Kapania. Each of the thirty actuators contains one degree of freedom along which the actuator acts. The force matrix F is therefore reduced to a thirty by thirty matrix filled with zeroes except for the diagonal F_{ii} which is the estimated value for the load at an actuator i. Also, each force contains only one coefficient F_i which is used for the optimization problem. The system of equations to be solved is given as follows.

$$\epsilon_m = M^{-1}Fc \tag{16}$$

$$N = M^{-1}F \tag{17}$$

$$N^T\epsilon_m = N^T Nc \tag{18}$$

N is a matrix which is calculated by multiplying inverse *M* (which is directly created from the math model) by the force matrix *F*. This resulting system is highly ill-conditioned and is therefore solved using Tikhonov Regularization. In this method, a regularization term lambda, Λ is added to the system which allows for the overdetermined system to be directed towards a solution which will produce the realistic values that are desired. The regularization term Λ is a value which filters out singular components which are small relative to itself while retaining the components of larger value. Choosing Λ is a problem where a logical guess must be made and then tested in a trial and error type fashion. It is found that a value for Λ of 9740 produced results that are realistic and gave us a strain vector which correlated well to the test data.

In order to employ Tikhonov Regularization, Singular Value Decomposition (SVD) must be first applied to the system of equations. Singular Value Decomposition is a factorization method which gives a bi-orthogonal diagonalization for the force-strain matrix. The SVD method has numerous applications including least squares minimization which minimizes the L2-norm of Mc by constraining $\|c\|$=1. Once SVD is applied the resulting decomposed matrix can be passed on to a function which contains the algorithm for Tikhonov Regularization. Tikhonov Regularization will be discussed in depth in section 2.3, but in short provides a means to minimize a highly ill-conditioned overdetermined system by employing the described regularization parameters to point the solution towards the desired realistic results.

Initially, the loads recorded for each of thirty actuators at all thirty-two conditions are run in the FEM as a baseline or control point for correlating the FEM to the test data. In the ideal case, this of course would result in zero error between the test and FEM strain data. However, the motivation behind research in FEM correlation is rooted in the fact that it does not. While any initial guess might be used and in other circumstances experimentally applied loads may not be available, it is a good starting point for demonstrating both the need and the effectiveness of the load updating method because it is the best possible initial guess that could be made when choosing loads to run in the FEM and the error in the resulting data corroborates the need for applying the load updating method.

The load updating method is programed in Matlab. The code developed takes the measured vector of strain, math model, and force matrix and applies the load updating method via the process described above. Load condition 1 will be shown here as an example.

```
%Jonathan Nichols Thesis Work on Load Updating
%1-18-17

%Load Cases

%Force Matrix
Force=[.....

%Force Vectors
F1=diag(Force(1,:)');              Force Matrix Defined from Force Vector

%Force-Strain Matrix
M=[0. ...                            Force Strain Matrix Developed from Math Model

FEMstrain=[...                       Strain Data Measured During Testing

%Measured Stain Vectors
u1=transpose(FEMstrain(1,1:74));     Strain Data Measured for Load Case 1

%Compute N Matrix
N1=M*F1;                             N Matrix Computed

%Compute NN Matrix
NN1=N1'*N1;                          NN Matrix Computed

%Compute VV Vector
VV1=N1'*u1;                          VV Vector Computed

%Singular Value Decomposition
[U1,S1,V1]=csvd(NN1);                Singular Value Decomposition Applied to NN

%Tikhonov Regularization Application
c1=tikhonov(U1,S1,V1,VV1,9740);      Tikhonov Regularization Applied to System of Equations

%Compute Loads
Loads1=F1*c1;                        Updated Loads Calculated from Initial Loads and c Coefficients
```

Figure 12: Load Updating Solution for Frame Structure

2.2 Math Model Development

One of the technical challenges in applying the Load updating method is determining the values for and how to invert strain-force matrix *M*. In simple cases, the stiffness matrix can be extracted directly from the FEM or even computed by hand. However, for a large system with millions of degrees of freedom, more thought had to be given as to how to accomplish this step. Sometimes, regression analysis can be used where the loads are the dependent variables and the strains are the independent variables (predictors). Once an equation is created for each of the load inputs, the resulting matrix must be inverted. This often results in a highly ill-conditioned system and introduces a problem for structures where the strain gages haven't been strategically placed so as to predict loads coming from the actuators.

To resolve these problems an entirely different approach is pursued. First, a load case is created and run for each of the thirty actuators in the FEM. The strain data is then extracted from each of the points where the seventy-four strain gages are located. This is done by identifying the exact location of each strain gage and then identifying a few elements under the gage location which could be averaged to find the strain given by that particular strain gage. Elements that are odd or gave outlying values are dismissed from the averaged solution. The strain determined for each actuator is then divided by the load applied in order to determine a value in units of strain per pound for each gage as related to each actuator.

Actuator 1 Load		Stain Gage 1,1	Strain Gage 1,2	.	.	Strain Gage 1,74
Actuator 2 Load		Strain Gage2,1	Strain Gage 2,2	.	.	Strain Gage 2,74
.	=	.	.	.	.	.
.		.	.	.	.	.
Actuator 30 Load		Strain Gage 30,1	Strain Gage 30,2	.	.	Strain Gage 30,74

Figure 13: Strain Data Collected for Each Individual Actuator Load

		Stain Gage 1,1 / Actuator 1 Load	Strain Gage 1,2 / Actuator 1 Load	.	.	Strain Gage 1,74 / Actuator 1 Load
		Strain Gage2,1 / Actuator 2 Load	Strain Gage 2,2 / Actuator 2 Load	.	.	Strain Gage 2,74 / Actuator 2 Load
Inverse(M)	=	.	.	.	.	.
		.	.	.	.	.
		Strain Gage 30,1 / Actuator 30 Load	Strain Gage 30,2 / Actuator 30 Load	.	.	Strain Gage 30,74 /Actuator 30 Load

Figure 14: Creation of Matrix M

Once the generalized inverse of matrix M is developed it is applied in the load updating method to perform a check. In this check, the loads from each of the thirty actuators are combined into one force matrix and the resulting strains across all thirty unit loads are summed and used as the measured strain vector. This results in a multi variate system of equations given by equation 18 which is solved using the Tikhonov method (discussed in section 2.3). It is predicted and verified that all the coefficients are equal to one since the initial load guessed is in fact the load which is applied to the model. This confirms that the load updating method is applied correctly and allows for a real case to be then solved with confidence.

As briefly mentioned in the previous section, the development of this math model results in a matrix which relates strain (the dependent variable) to the load (independent variable) which is in fact the generalized inverse of the matrix M (whose dependent variable is load and independent variable is strain). This solves not only the problem of establishing this quintessential relationship, but also eliminates the problem of needing to invert the M matrix. The condition of the resulting system for condition 1 is 3.77E10. This condition is part of why singular value decomposition coupled with Tikhonov Regularization is employed in order to solve this system.

2.3 Tikhonov Regularization

The FEM state equation given by Equation 15 is developed to a square symmetric system of equations which must be solved in order to find the load updating coefficients. This as mentioned is an inverse problem which is formulated by the following state equation.

$$Ax = b \tag{19}$$

Where x and B are Hilbert spaces and A is a linear compact operator which provides a relation between the two (Applied Mathematical Sciences, 2011). Typically, an inversion algorithm for deriving loads from measured surface strains poses an innate problem of an ill-conditioned system and therefore requires stabilization. Regularization methods are often chosen as the technique to provide this stabilization and one common method of regularization is the classical Tikhonov Regularization. Tikhonov regularization imposes the minimization of the 2-norm plus a stabilizing term (Bosse & Lechleiter, 2015).

$$\|Ax - b\|^2 + \|\Lambda x\|^2 \tag{20}$$

The term Λ is the regularization parameter. In essence, it helps to point the solution towards a desirable outcome whose results are realistic. For example, in the problem it would keep the four actuators which apply the thrust load to the rotor system from having highly uneven distribution where one actuator could even apply load in the wrong direction. A solution such as this likely would not minimize the strain as is desired and also would not lend itself to the remainder of the unmonitored structure having an accurate response. Many techniques have been proposed for determining the Λ which will provide an optimum solution. Finding the optimum Λ is a trial and error problem where a guess must be made and then testing repeatedly until an acceptable solution is found.

A number of proposals are available for assistance in making this educated guess. One common suggestion is known as the L-curve. The L-curve is the shape often created by a log-log-plot of the solution norm versus the residual norm. The Λ which arrives as the point where these two converge (the corner of the L) is the guess that would be used as a starting point for Λ (Hanse, 2017). This is attempted for the system of equations created for the frame structure, but is found to give highly inaccurate unrealistic results. Therefore, it is abandoned and another idea employed.

This idea is also an optimization approach. Since the proper direction of the loads and their approximate magnitudes are known, a loop is run to find the Λ which would result in a set of loads that depart from the initial guess the least. This value is found to be 9740 which is significantly higher than what is determined using the L-curve approach. All thirty-two cases are run at this Λ and these are the results that are presented.

Another technique often used in solving ill posed systems of linear equations is the conjugate gradient method. The conjugate gradient method has some merits in that it typically outperforms Tikhonov regularization when it comes to speed as it typically reaches a given tolerance earlier. It also dodges the disadvantage of the Tikhonov regularization which is a smoothing or smearing of the results that can occur at low noise levels with discontinuous load application. However, the higher stability of the Tikhonov regularization, especially for cases with high noise and little data is present, motivated the selection of Tikhonov Regularization for use in this study. Conjugate gradient method is particularly challenging in situations where minimal strain data is available (Bosse & Lechleiter, 2015). It is found in this research that Tikhonov regularization did indeed provide better results than conjugate gradient method, steepest descent method, or any other inverse methods tested. Most all of the other methods tested failed to converge at all when solving this highly ill-conditioned system. The conjugate gradient method does converge, but to loads that are obviously unrealistic and its results are therefore discarded. Tikhonov Regularization is therefore the sole method which provided the solutions sought.

2.4 Uncertainty Analysis

An uncertainty analysis is performed to determine the effect of various sources of error on the load set extracted. This is useful for understanding the holistic accuracy of the load updating method as applied to the frame structure but also in determining where the greatest opportunity for improvement lies should additional work be done to improve accuracy. Specifically, the two sources of error which will be considered in this thesis are those from the load cells and also those from the strain gage measurements.

An uncertainty analysis begins by quantifying the max error in the input which is being studied. This error is then propagated through the system, in this case the load updating method, and the results recorded. The error can then be quantified for that particular input. Similar cases can be run for all inputs in the system allowing error bounds to be created which encompass all known errors in the system. This is visually shown in the flowchart below.

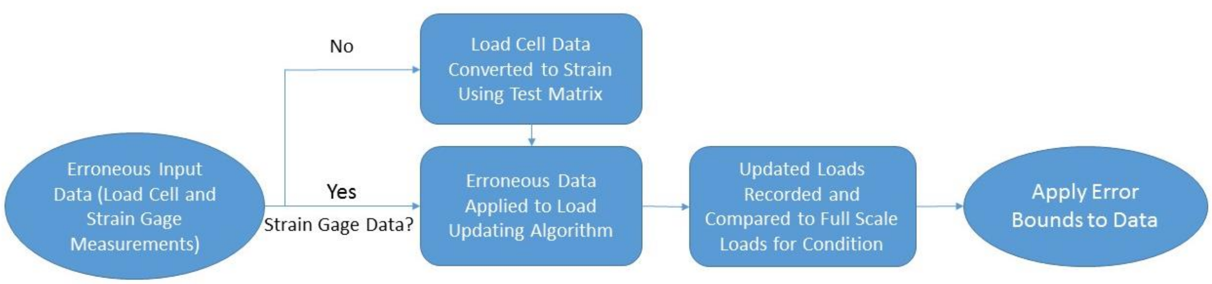


Figure 15: Uncertainty Analysis Flowchart

A load cell is used to measure the load input at each actuator location. The sizing of the load cells will be discussed in section 3.2, but the important fact for this topic at hand is that each load cell measure is rated to an accuracy of one percent. It is determined how this affects the results of the analysis as follows. For condition 1, all the load cell readings are assumed to contain an error of one percent. Therefore, each strain gage is multiplied by 0.99 and 1.01 and the resulting array input into the load updating algorithm. The initial guess for the force matrix is also reduced to one-hundredth of its normal value. This results in updated loads which give a quantified error band in pounds for the uncertainty in the load cell measurements. For a more robust uncertainty analysis, one should apply unit loads to each of the actuator locations and record the resulting strain measurements. Next, a strain-force matrix can be created for the physical test structure just like the one for the math model is in section 2.2. This requires that the resulting strains from condition one be divided by the loads applied as shown in Figure 14. This gives the sensitivity for each of the seventy-four strain gages with respect to each of the thirty actuators. Once this matrix is obtained, the resulting strain gage measurements can be obtained for these erroneous loads. These measurements must be run through the load updating algorithm, as described above, which will give values for each actuator which can be considered as the error bounds for the input for load condition one. It should be pointed out that applying the erroneous measurement to all the actuators at once may not be the maximum error possible. This is because the error in one actuator may aid the error in another by reducing erroneous strain on one or more strain gages. This possibility will not be considered here, but should be noted depending on the level of detail that one wishes to know about the error. The results of this analysis are shown in Chapter four.

A similar analysis can now be performed for the second source of error, the strain gage measurements. Each of the strain gages is itself rated to an accuracy of one micro strain. In addition, the possibility of small error due to temperature effects will motivate the conservative guess of a two percent error in each of the strain gage measurements. Once again, this error is propagated through the analysis beginning this time at the application of the load updating algorithm. The vector of strain input for condition 1 are multiplied by two percent and the load multiplier is once again used to reduce the force matrix properly. The error on each of the actuators is recorded and the results are presented in Chapter four.

While additional uncertainty analysis can be conducted, these two sources of error are believed to be the largest in the experiment and their error contributions minimal considering the scope of this research. If the load updating method is being applied to other structures in the future with unacceptable results, this approach to error analysis could prove useful in refining the experiment to the point of obtaining results with the accuracy desired.

CHAPTER 3

FEM AND EXPERIMENTAL SETUP

3.1 Finite Element Modeling

A static test FEM of the frame structure is created in NASTRAN to be used for the FEM computations. The model consisted of a structure that is representative of the frame as well as the surrounding structure used for the experimental testing. Node points are created for the actuator attach points and it is here that the loads are applied for a given test condition.

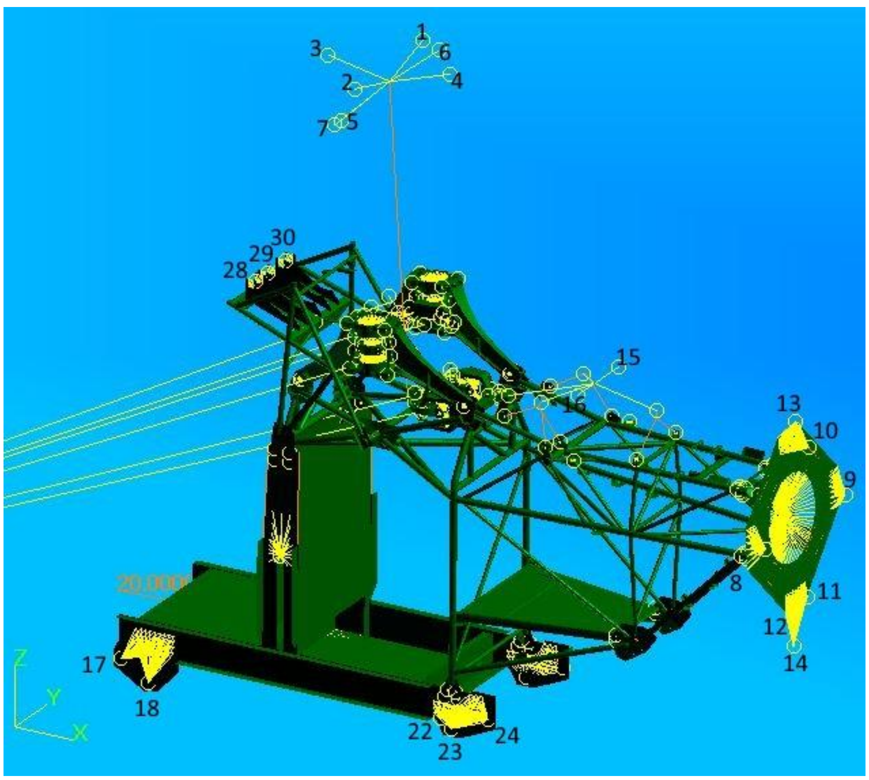


Figure 16: NASTRAN Model-Left

Figure 17: NASTRAN Model-Right

The actuators are combined and used to simulate the load inputs that this structure would see on the aircraft in flight. Table 4 shows the different load inputs expected and which actuators are combined to create these loads. This is of course applicable to both the FEM as well as the test setup (section 3.2).

Load Input:	Actuator Number
AFT KEEL RH FX=	25
AFT KEEL RH FY=	26
AFT KEEL RH FZ=	27
AFT KEEL LH FX=	22
AFT KEEL LH FY=	23
AFT KEEL LH FZ=	24
TB FX=	8+9+10+11
TB FY=	13-14
TB FZ=	-14
TB MX=	17*13+17*14

TB MY=	13.5*11-13.5*10
TB MZ=	13.5*8-13.5*9
MAST FX=	5+6
MAST FY=	-7
MAST FZ=	1+2+3+4
MAST MX=	11.25*2-11.25*1
MAST MY=	11.25*4-11.25*3
MAST MZ=	15.4375*5-15.4375*6
ENGINE FZ=	15+16
ENGINE MX=	6.75*15-6.75*16
COLLECTIVE=	29
LEFT CYCLIC=	28
RIGHT CYCLIC=	30
RH FWD KEEL FX=	19
RH FWD KEEL FY=	20
RH FWD KEEL FZ=	21
LH FWD KEEL FX=	17
LH FWD KEEL FZ=	18

Table 4: Load Inputs

When measuring strain on any structure, the strain measured can be thought of simply as an average of the strain at each infinitesimal point underneath the strain gage. It is decided that the method for determining strain at each of gage locations on the FEM should be consistent with this principle. Therefore, at each gage location several elements are identified whose shapes are not irregular and whose values are reasonable and consistent with their neighboring elements. Each time a load case is run, the values for strains at each of the elements is averaged to determine the strain value from the FEM which would be compared to the structural test measurement for the strain gage in that location. The elements used are held consistent for all gages throughout all the load cases (including unit load cases) which are run in the FEM.

As the FEM is developed and incorporated for comparison in testing, a number of iterations are made in order to improve the model of the structure. One example of this is the tailboom attachment point which is determined to lack the rigidity it needed to properly represent the real world structure. Although the load updating method is designed to ameliorate some of these issues, it is still reasonable to tweak the model in order to make it as close to representative of the structure as possible. Then, once the model is believed to represent the structure, the load updating method can be applied to correct for other issues that still remain unknown to the analysis. In other words, the load updating method should not be used to compensate for known modeling deficiencies, but rather used once these known deficiencies are resolved.

3.2 Structural Testing

Shown below are figures of the frame and thirty actuators modeled in CATIA (Figure 16 & Figure 17) and also pictures of the test figure in its entirety (Figure 18 - Figure 27).

Figure 18: CATIA Model of Frame with Actuators-Left

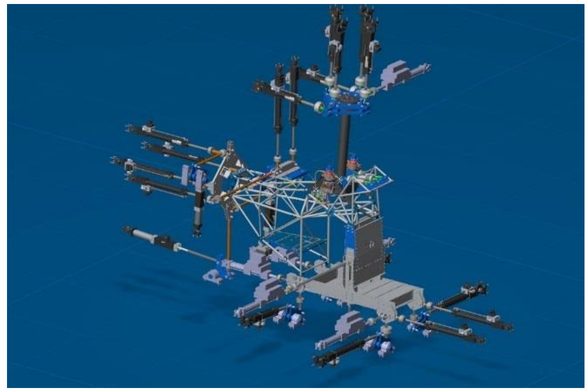


Figure 19: CATIA Model of Frame with Actuators-Right

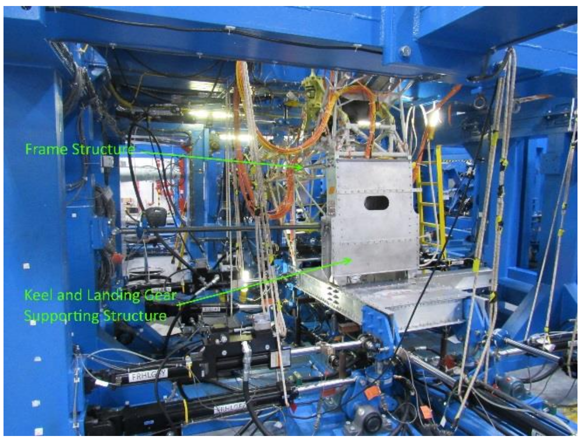


Figure 20: Test Setup-Front Right View

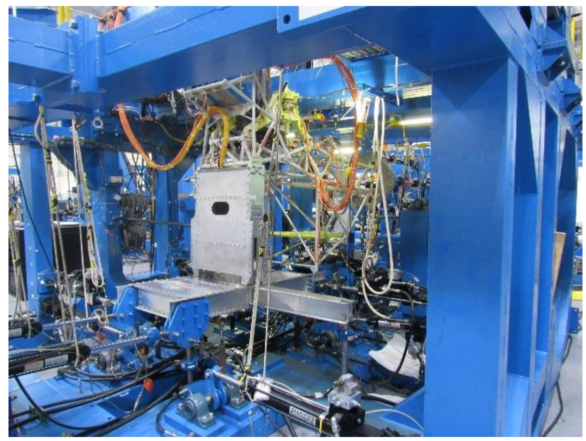


Figure 21: Test Setup-Front Left View

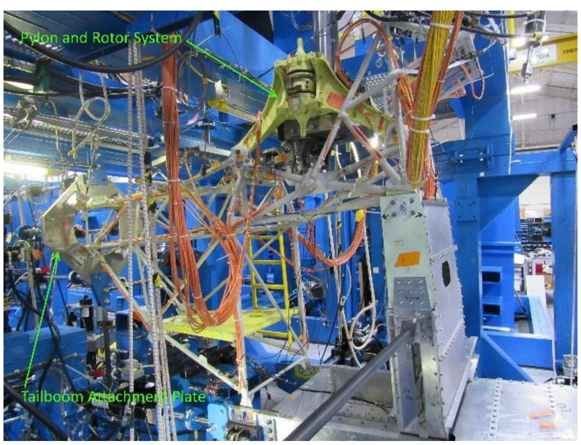


Figure 22: Test Setup-Right View

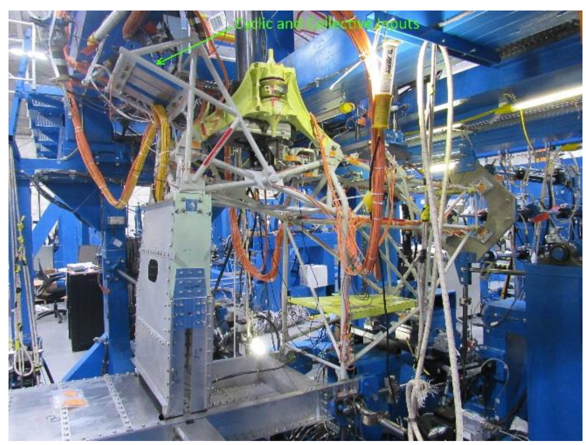


Figure 23: Test Setup-Left View

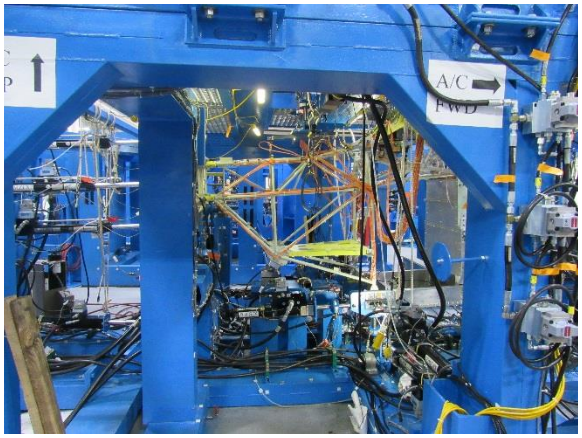


Figure 24: Test Setup-Middle Right View

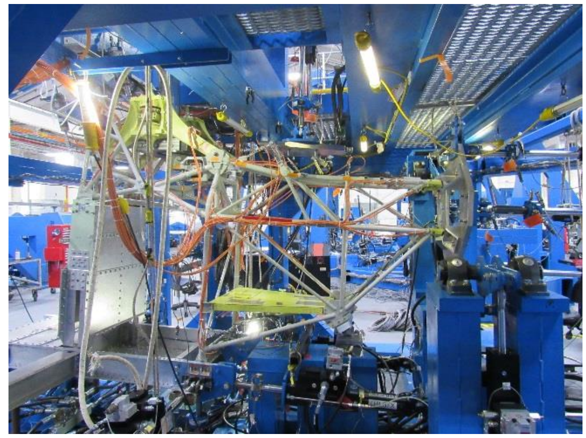


Figure 25: Test Setup-Middle Left View

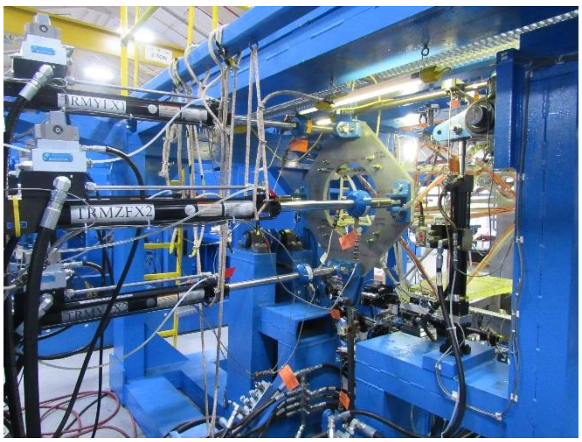


Figure 26: Test Setup-Rear Right View

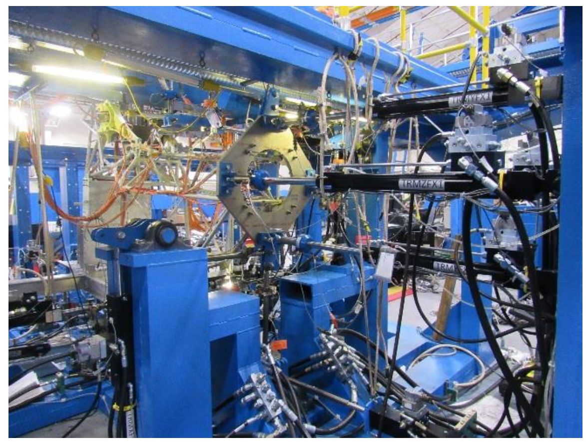


Figure 27: Test Setup-Rear Left View

Creating a structural test setup which properly replicates the load conditions experience by a particular part in flight can be a daunting task. While numerous issues must be considered for a test setup as complex as the frame structure at hand, only a few of these issues will be discussed in order to give a flavor for what is required. For starters, information must be obtained concerning all the details of the mean and oscillatory loading conditions as well as the attachment points (boundary conditions) of the structure. Characteristics of the attachment points or mating surface such as area, stiffness, and inertia must be accounted for in order to conduct a proper test. The test setup must include the desired loads for each condition as well as limits for each of those loads that will either alter the loading distribution or shut down the test depending on the severity of the divergence from the desired loading condition.

When choosing the servos, actuators, and load cells for applying the loads, decisions must be made which will give the highest quality data possible. The servos and actuators must be sized in order to obtain data with a high resolution. This means that a load cell must be chosen that is rated as low as it can be without a high risk of overloading it.

When setting the loads on the test for the first time careful attention must be paid to the frequency at which the actuators are applying load. An improper frequency can result in data from the load cell that kills the desired sine wave. Also, resonance must be avoided by running at lower frequencies and by properly servicing equipment.

As the test is conducted, the instrumentation data is measured and collected in a data acquisition system. Similar to the alarms on the load cells, strain gages as well as break wire can set off alarms which notify the proper personnel and if necessary shut down the test.

It is important to understand the design tolerances of any part, to be structurally tested, in order to quantify the error due to deviations in the as-made part compared to the FEM which is absent of such errors. For this particular frame structure, each of the tubes are manufactured to extremely strict tolerances (thousandths of inches) and the material properties known to a high degree of accuracy. Therefore, the only significant manufacturing flaw whose error will be important to discuss is the weldments which hold each of the tubes together. Although specifications are given for each of the welds, it is found that these varied more than that would be desired and that the only way to truly know by how much each weld varied is to cut the truss apart once testing is completed. The process of welding also introduces the important issues of residual strains which will be discussed extensively later on in this thesis.

3.3 Instrumentation

"The strain gage has been extolled as an extremely useful tool, but in a large part its utility will depend upon what the user asks of it. The experimental stress analysis should have a broad knowledge of what can be and has been done with strain gages. Coupled with this, incidentally, he should have enough imagination to be able to adapt strain gages to his own particular uses…Basically, of course, the strain gage is used to procure accurate information about the magnitude, distribution, and directions of strains in loaded bodies (Perry & Lissner, 1962)."

The instrumentation on the frame structure includes seventy-four uniaxial strain gages strategically placed near critical points of interest. Each of the gages is placed in a location on the frame where a stress concentration is believed to exist for at least one of the load conditions. The uniaxial strain gages used have a gage factor of approximately 2. The gage factor is the value multiplied by the millivolt per volt output in order to convert the value to micro strain. A gage which has a self-temperature compensation (STC) of 06 is used in order to eliminate any error from temperature changes. The frame is made of steel and therefore has a thermal expansion coefficient of 6 ppm. Lead wires from the gage to the data acquisition system are short enough so that changes in resistance along them can be neglected.

All of the gages are placed longitudinally to the member of the frame on which they are placed. Because the frame structure is welded rigidly at all of the joints, the gages are subjected to not only strains due to axial but also bending loads. This combined with the complex geometry of the part makes it unfeasible to perform hand calculations on this structure for the strain at each of the gage locations as is done in the cantilever beam example from section 2.1.2. This also means that only the input loads at the actuator locations can be determined since it is unknown from the test data whether the load producing the strain measurement is the result of axial or bending loads.

CHAPTER 4

RESULTS AND DISCUSSION

4.1 Introduction

In a world where tests are conducted without error and where models are perfect representations of said tests, the loads initially applied would equal the loads computed with the load updating method. However, this is not the case. Therefore, this section will present and quantify these differences. The optimized solution of the resulting strain will also be compared with the initial values and the test values which are sought. Finally, the errors and causes of these differences will be discussed at length in order to understand the benefits of the load updating method and how it could be improved upon in the future.

4.2 Tikhonov Regularization Results

To determine the optimum value for Λ, an algorithm is run which compares the absolute error between the sum of the ideal loads, which are applied to the actuators in test, and the sum of the solved for updated loads. The idea is to *point* the solution to the loads which diverges as little as possible from the guessed loads which prevents unrealistic results such as poorly distributed loads where multiple load input points are used and even loads acting in the wrong direction. A value of 9740 is found to provide the best solution as shown in Figure 28.

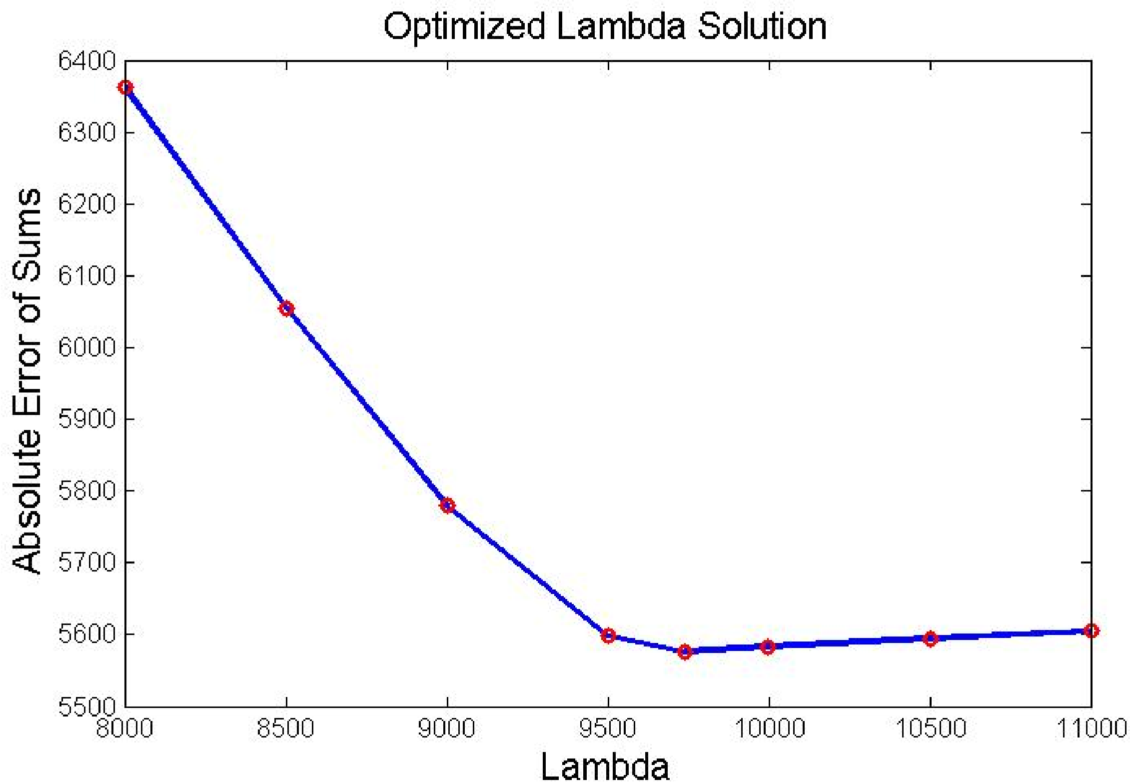


Figure 28: Optimized Lambda Solution

4.3 Updated Loads Results

The loads initially run in NASTRAN are the actual loads applied with the actuators to the test. For each of the thirty-two load cases, coefficients are computed in order to calculate the updated loads and to then re-run the NASTRAN model. This improves the strain gage correlation between the FEM and test data. The coefficients which the load updating method calculated for condition one are shown in Table 5.

Coefficient	c=
1	0.897799
2	0.855055
3	0.504298
4	0.735405
5	0.950070
6	0.949327
7	-0.01190

8	0.994780
9	0.810288
10	0.776955
11	0.999877
12	1.232323
13	0.739598
14	1.376192
15	0.954622
16	-0.01949
17	0.104251
18	0.667056
19	0.011826
20	0.299296
21	1.619773
22	-0.05545
23	0.008651
24	0.953319
25	-0.04318
26	-0.05245
27	0.812094
28	-0.08903
29	0.094692
30	0.009523

Table 5: C Coefficients for Condition 1

These coefficients are then multiplied by the initial force matrix in order to obtain the updated loads. For each of the thirty actuators, both the initial and updated scaled loads are presented for comparison in Table 6 and Figure 29. This data is plotted for all conditions in Appendix B.

Actuator:	Initial Loads/Test Loads:	Updated Loads:
1	0.294196	0.264127
2	0.387759	0.331553
3	0.221469	0.111678
4	0.245466	0.180512
5	-0.76772	-0.72939
6	0.826017	0.78416
7	0.003912	-6.5E-05
8	0.689827	0.686225
9	-0.43441	-0.352
10	0.727655	0.565351
11	-0.96807	-0.96796

12	0.095509	0.117702
13	0.172807	0.127803
14	0.201265	0.276987
15	-0.30265	-0.28892
16	0.01385	-0.00029
17	-0.02986	-0.00313
18	0.640593	0.427305
19	-0.00361	-6.1E-05
20	-0.10554	-0.0316
21	0.433631	0.702397
22	-0.01994	0.001087
23	0.001828	-2E-06
24	-0.1263	-0.12041
25	-0.01635	0.000687
26	0.049872	-0.00263
27	-0.28788	-0.23378
28	-0.11648	0.01035
29	0.037777	0.003561
30	0.009979	7.72E-05

Table 6: Initial and Updated Loads Comparison for Condition 1

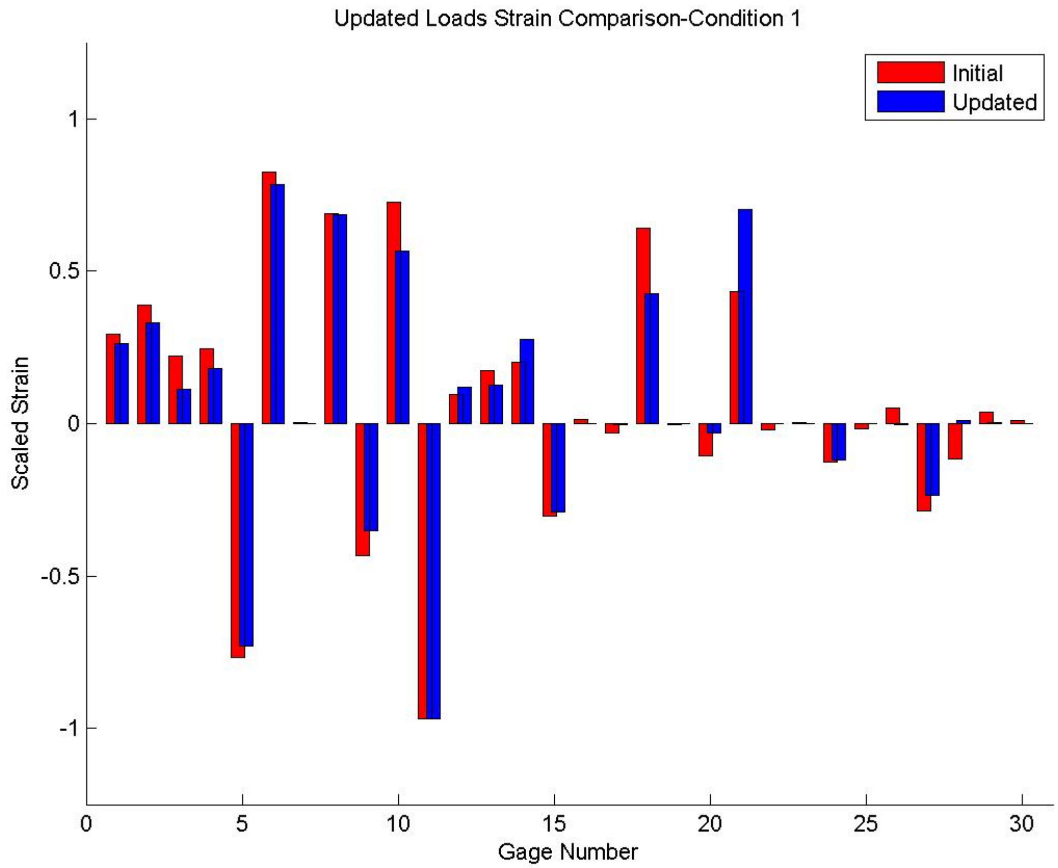


Figure 29: Initial and Updated Loads Comparison for Condition 1

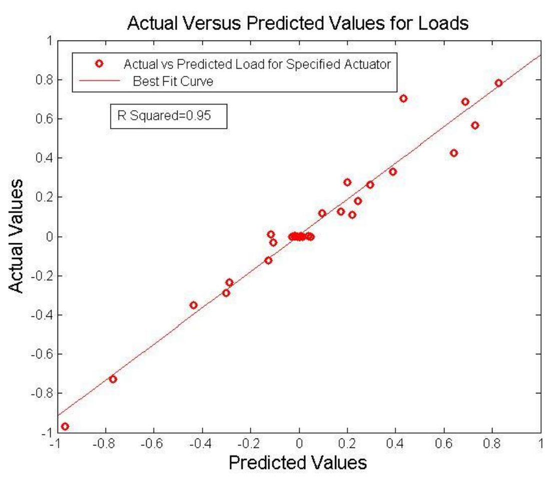


Figure 30: Actual Versus Predicted Values for Loads

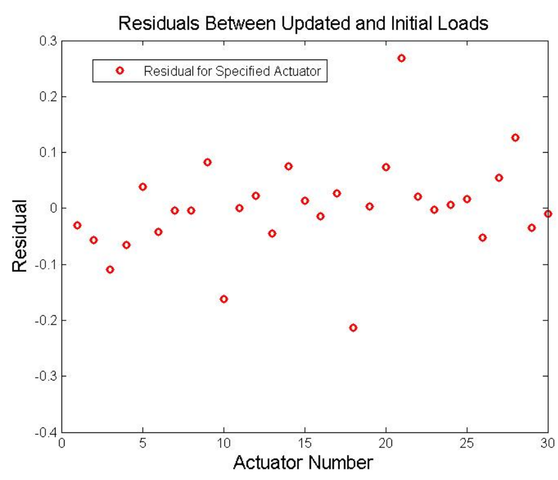


Figure 31: Residuals for Loads

It can be seen here that while the magnitudes of the loads do change, the differences do not look illogical especially within the context of the differences between the initial and updated strains.

The ability to find a realistic solution by adjusting the regularization parameter makes the Tikhonov method is especially useful in solving highly over-determined linear systems of equations. Figure 30 and Figure 31 quantify the error between the initial and the updated loads. In the ideal case, Figure 30 would have an R squared value of one and Figure 31 would consist of residuals all equal to zero.

4.4 Updated Strains Results

Now, observations are made as to how these updated loads have affected the values for strain and most importantly how these loads have reduced the error between the FEM results and the results from testing. The root mean squared error is used for comparison between the initial and the updated loads. It is given by the following equation.

$$Root\ Mean\ Squared\ Error = \sqrt{\frac{\sum_{i=1}^{74}\left(y_i' - y_i\right)^2}{74}} \tag{21}$$

Where y' is the data generated from the static test to which the correlation is made. The root mean squared error for the initial guess is compared to the root mean square error for the updated loads in Table 7 for all conditions. Normalized values for the test strain, initial guess FEM strain, and updated loads FEMs strain are compared in Table 8 and Figure 30 for condition 1 and for all conditions in Appendix B.

Root Mean Squared Error:	Initial Guess	Updated Loads
Condition 1	106.13667	69.16422
Condition 2	75.28719	36.27488
Condition 3	112.2734	69.64388
Condition 4	72.56366	48.37316
Condition 5	72.4619	35.16822

Condition 6	112.7231	74.25628
Condition 7	74.79131	54.49621
Condition 8	85.7379	52.07475
Condition 9	63.85722	31.36685
Condition 10	613.1333	58.64075
Condition 11	56.96472	27.54132
Condition 12	100.4475	79.56434
Condition 13	72.00983	28.09162
Condition 14	383.5864	53.60401
Condition 15	116.1468	67.43768
Condition 16	121.4382	60.3242
Condition 17	120.4449	65.14836
Condition 18	106.6568	56.9299
Condition 19	77.73179	45.64628
Condition 20	178.7665	66.21396
Condition 21	58.32584	32.68917
Condition 22	134.2112	69.43261
Condition 23	122.7867	52.63999
Condition 24	87.48256	52.61298
Condition 25	98.80076	61.1717
Condition 26	87.38811	43.93702
Condition 27	205.4922	64.60764
Condition 28	207.7263	64.69551
Condition 29	75.57308	30.69978
Condition 30	116.5081	32.56122
Condition 31	110.2908	30.26792
Condition 32	203.5618	59.15052

Table 7: Root Mean Squared Error

Scaled Strain:	Test	Initial	Updated
Gage 1	0.817069	1.004004✗	0.884034✓
Gage 2	0.519831	0.505002✓	0.559101✓
Gage 3	0.863993	1.056168✗	0.909037✓
Gage 4	0.843754	1.062986✗	0.869623✓
Gage 5	0.715191	0.74247✓	0.626577✗
Gage 6	0.75203	0.965601✗	0.792107✓
Gage 7	0.451098	0.386876✗	0.441985✓
Gage 8	0.287246	0.22184✗	0.247171✗
Gage 9	0.814427	0.767851✓	0.640934✗
Gage 10	0.505555	0.573574✗	0.497565✓
Gage 11	-0.18961	-0.39070✗	-0.30496✗
Gage 12	0.105093	0.01410✗	-0.00676✗
Gage 13	0.951166	0.893878✓	1.029914✓
Gage 14	-1	-0.98460✓	-1.00847✓

Gage 15	0.062772	-0.01350✗	0.074937✗
Gage 16	-0.6072	-0.35770✗	-0.40042✗
Gage 17	1	0.918189✓	0.914451✓
Gage 18	0.311678	0.301432✓	0.34087✓
Gage 19	-0.98126	-0.81750✗	-0.93862✓
Gage 20	-0.2384	0.00089✗	-0.15162✗
Gage 21	-0.35269	-0.38740✓	-0.33033✓
Gage 22	-0.79289	-0.73930✓	-0.68521✗
Gage 23	0.630967	0.482818✗	0.49971✗
Gage 24	0.176268	-0.19430✗	-0.13666✗
Gage 25	0.962889	0.952039✓	0.906226✓
Gage 26	0.622428	0.636201✓	0.596737✓
Gage 27	0.643649	0.67837✓	0.684552✓
Gage 28	0.088031	0.04709✗	0.040597✗
Gage 29	-0.02218	-0.08370✗	-0.03258✗
Gage 30	-0.33538	-0.52520✗	-0.39767✗
Gage 31	0.362929	0.341678✓	0.345494✓
Gage 32	0.232125	0.18745✗	0.187494✗
Gage 33	0.123557	0.17777✗	0.143911✓
Gage 34	0.251265	0.28803✗	0.274552✓
Gage 35	0.189708	0.12982✗	0.102139✗
Gage 36	0.237962	0.250476✓	0.230152✓
Gage 37	0.674946	0.818792✗	0.59816✗
Gage 38	-0.08622	0.706186✗	-0.05375✗
Gage 39	0.386964	0.64121✗	0.450169✗
Gage 40	0.611688	0.733273✗	0.76149✗
Gage 41	-0.43382	-0.48480✗	-0.46587✓
Gage 42	0.110079	-0.15140✗	0.116515✓
Gage 43	-0.07103	-0.13990✗	-0.07044✓
Gage 44	-0.30103	-0.30840✗	-0.4042✗
Gage 45	0.364602	0.28700✗	0.335495✓
Gage 46	-0.10436	-0.0684✗	-0.1034✓
Gage 47	0.582773	0.613442✓	0.57671✓
Gage 48	0.421671	0.53098✗	0.453744✓
Gage 49	0.325059	0.28160✗	0.353405✓
Gage 50	0.757442	0.597409✗	0.566258✗
Gage 51	0.839609	0.858689✓	0.828368✓
Gage 52	0.181001	0.13521✗	0.191562✓
Gage 53	0.704683	0.742699✓	0.660702✓
Gage 54	0.131275	0.04101✗	0.068211✗
Gage 55	-0.40345	-0.38180 ✗	-0.33102✗
Gage 56	-0.26348	-0.29130✗	-0.2301✗
Gage 57	-0.15321	-0.20870✗	-0.15094✓
Gage 58	0.614851	0.586389✓	0.601188✓
Gage 59	0.066867	0.04486✗	0.08155✗
Gage 60	0.468355	0.469562✓	0.440757✓

Gage 61	0.156018	0.00927×	0.063495×
Gage 62	0.106777	0.04566×	0.079477×
Gage 63	0.330303	0.384082×	0.346202✓
Gage 64	0.371673	0.52158×	0.426189×
Gage 65	0.520932	0.655463×	0.539211✓
Gage 66	0.687393	0.629111✓	0.577858×
Gage 67	0.822055	0.964307×	0.875711✓
Gage 68	0.967543	1.083048×	0.973002✓
Gage 69	0.048924	-0.11280×	0.080975×
Gage 70	0.417792	0.309098×	0.327669×
Gage 71	0.625478	0.63549✓	0.539447×
Gage 72	0.003423	0.02791×	0.037234×
Gage 73	0.346861	0.380663✓	0.326712✓
Gage 74	0.251501	0.416083×	0.260702✓

Table 8: Test, Initial, and Updated Loads Comparison for Condition 1

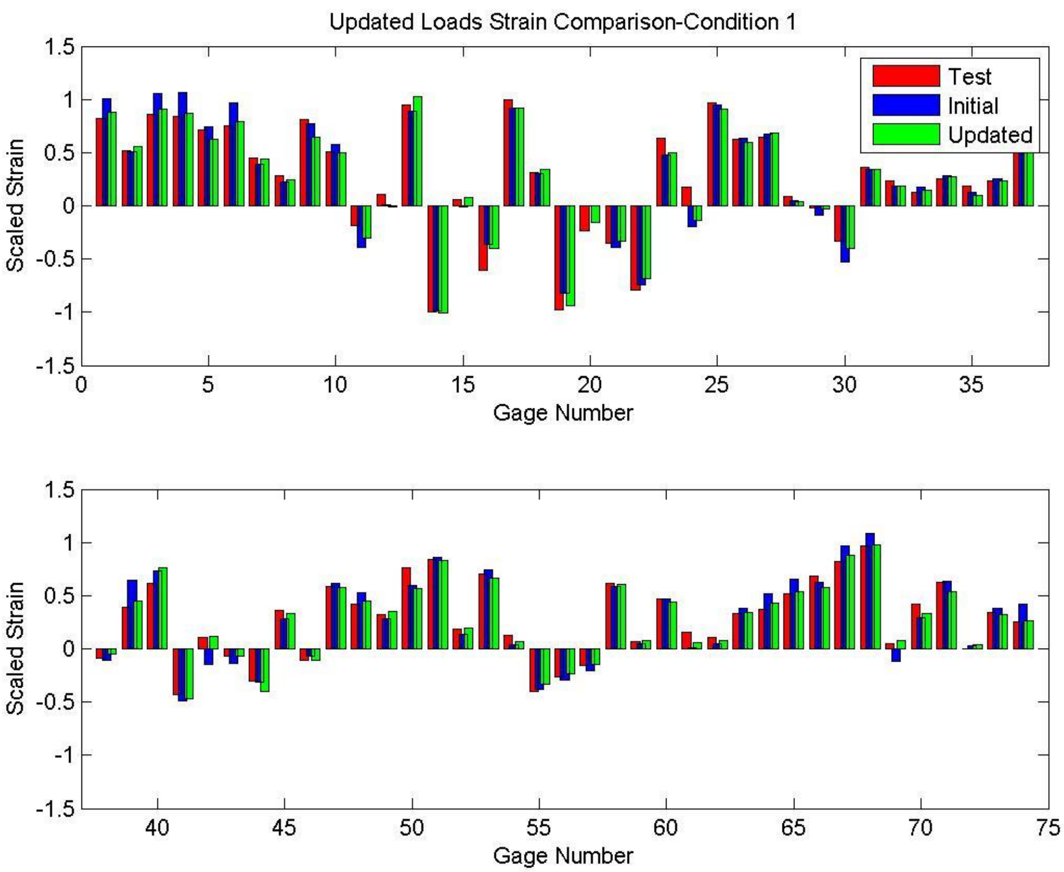


Figure 32: Test, Initial, and Updated Loads Comparison for Condition 1

In the table above, it is shown that the load updating method helped substantially to improve the correlation between the test and FEM data. This is shown visually by Figure 32 and quantitatively by Table 8. Across the thirty-two load conditions, the root mean squared error is 132 micro strain with the initial loads applied to the FEM. After applying the load updating method, the root mean squared error is reduced to 52 micro strain.

Two other sets of useful figures are presented to help quantify the error in the results obtained. The first is a plot of the actual measured values versus the values predicted. The *R* squared value from this plot is yet another way to quantify the error of this analysis. The *R* squared value using the initial loads is 0.93 which is improved by the updated loads to 0.97. This trend continues comparing the R squared values for the other thirty-one cases as well as shown in Table 9. The sources of the remaining error will be discussed in section 4.7.

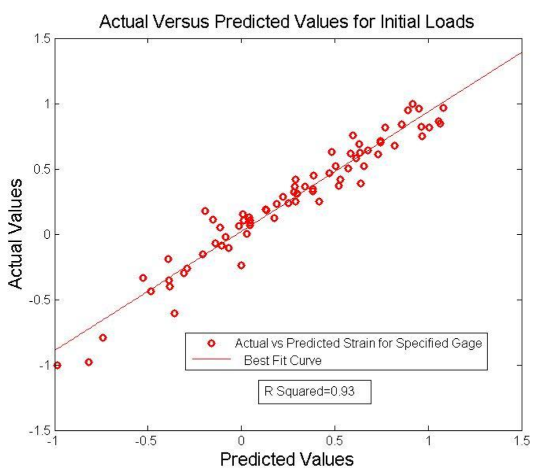


Figure 33: Initial Actual Versus Predicted Strain Values

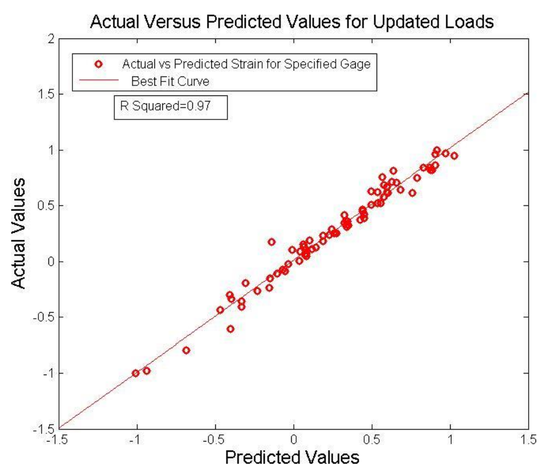


Figure 34: Updated Actual Versus Predicted Strain Values

	Initial R Squared Value	Updated R Squared Value
Condition 1	0.9398	0.9701
Condition 2	0.8066	0.9229
Condition 3	0.9367	0.9721

Condition 4	0.8630	0.9057
Condition 5	0.7285	0.8745
Condition 6	0.9364	0.9692
Condition 7	0.9459	0.9643
Condition 8	0.9255	0.9682
Condition 9	0.6468	0.8836
Condition 10	0.3382	0.9716
Condition 11	0.8824	0.9645
Condition 12	0.9173	0.9484
Condition 13	0.3545	0.8601
Condition 14	0.6303	0.9798
Condition 15	0.8644	0.9556
Condition 16	0.9021	0.9666
Condition 17	0.8158	0.9467
Condition 18	0.8159	0.9486
Condition 19	0.9218	0.9636
Condition 20	0.8421	0.9699
Condition 21	0.8713	0.9482
Condition 22	0.8483	0.9591
Condition 23	0.8634	0.9702
Condition 24	0.9220	0.9673
Condition 25	0.8883	0.9508
Condition 26	0.8899	0.9696
Condition 27	0.6026	0.9565
Condition 28	0.5755	0.9517
Condition 29	0.7355	0.9370
Condition 30	0.5425	0.9533
Condition 31	0.6415	0.9645
Condition 32	0.5339	0.9561

Table 9: R Squared Values for Thirty-Two Load Cases

The residuals for the strain given by the initial and updated loads is also shown as further evidence for improvement in the correlation. This significant improvement would be strong evidence in validating the load updating method to be used in predicting additional load cases.

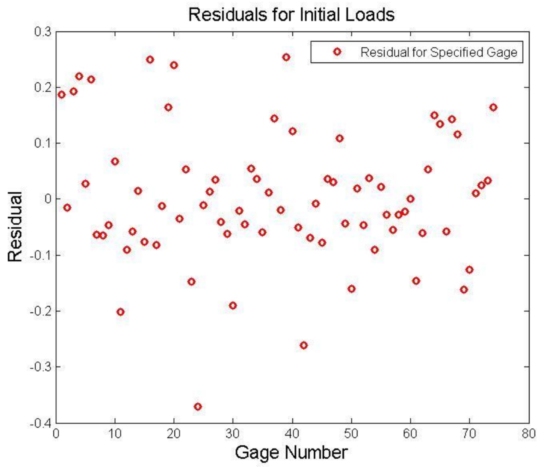


Figure 35: Initial Residuals for Strain

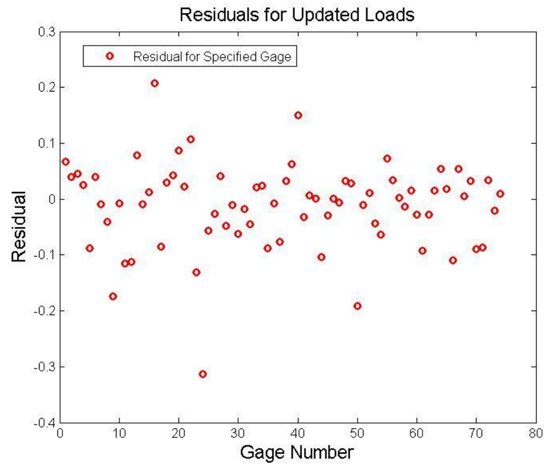


Figure 36: Updated Residuals for Strain

4.5 FAA Certification

The initial loads applied to the FEM resulted in nineteen of the seventy-four gages correlating (or thirty percent) within a ten percent tolerance to the test data. However, when the load updating method is applied forty of the seventy-four (or fifty-four percent) strain gages are measured to be within ten percent tolerance to the test data. This makes validation of this method to the FAA noticeably easier since as explained in section 1.3 meeting the ten percent tolerance results in no further evaluation necessary. It is also important to point out two other key facts regarding the results and how they relate to certification requirements. First, twenty-four of the seventy-four gages that do not meet the ten percent criteria read little strain data which is defined as 0.2 or less. The gages in this subset are obviously not in hot spots for this load condition and due to the low strain measurements taken on them would be particularly difficult to correlate to within ten percent. If these gages are thrown out of the statistic, then forty of fifty gages (or eighty percent) correlate to within ten percent of the test data. Another point to be made is that although some of the gages do not fall within the ten percent tolerance, the correlation for most of them is still improved substantially. This is strong evidence of the load updating methods effectiveness and can be presented as such to the FAA during the certification process.

The updated loads found with the Tikhonov method are also a requirement for presentation to the FAA during certification. As discussed in section 1.3, the loads applied must be realistic in order to accept the data from an FEM analysis. By prudently selecting the regularization parameter in the Tikhonov method, realistic loads are found that diverged from the initial loads as little as possible. This allows the FAA's requirement to be met. Realistic loads are also critical for another even more important reason. Since the system that is solved is highly ill-conditioned and overdetermined, an infinite number of solutions could be found. This means that there are likely a number of solutions for loads that would correlate as well or even better than the loads found with this regularization parameter. However, strains at the many other points throughout the frame structure would likely be unacceptable and in fact the structure would likely fail for many of these load cases. Unrealistic results are therefore of principle concern when applying the load updating method and discretion should be used starting with the application of realistic loads.

When evaluating the differences between the initial and updated loads, it is found that only a few fell within the ten percent target that is desired. Several sources of error are believed to contribute to these differences and they are discussed at length in section 4.7. In short, large residual strains are believe to be the largest cause of the differences and it is expected that this analysis run on a truss structure absent of these strains would correlate noticeably better. This of course would make the load updating method an even more trusted method for predicting loads on flight or test articles.

4.6 Initial Conditions and Uncertainty Analysis

In order to effectively apply the load updating method, a starting value for the force vector must be supplied. It is assumed that the analyst has some prediction of the load distribution across the structure. The question however is how accurate must this prediction be in order for the load updating method to work correctly? An attempt is made to answer this question by conducting an error analysis on the results of condition one. This consists of adding a load multiplier to the force vector which can be adjusted to change the initial loads which would be input into the load updating algorithm. The values for these load multiplier range from zero to one-hundred and twenty-eight. The updated loads are solved for each of these load multipliers and the root mean squared error calculated. This difference used for the calculation of the root mean squared error is that of the updated load minus the load that is applied to the test structure. In other words, error is quantified by how much the updated solution departed from the real world one in a manner similar to what is done when solving for the Tikhonov regularization parameter. It is found that the optimum load multiplier fell somewhere between 0.75 and 1.25. This is shown in the figure below.

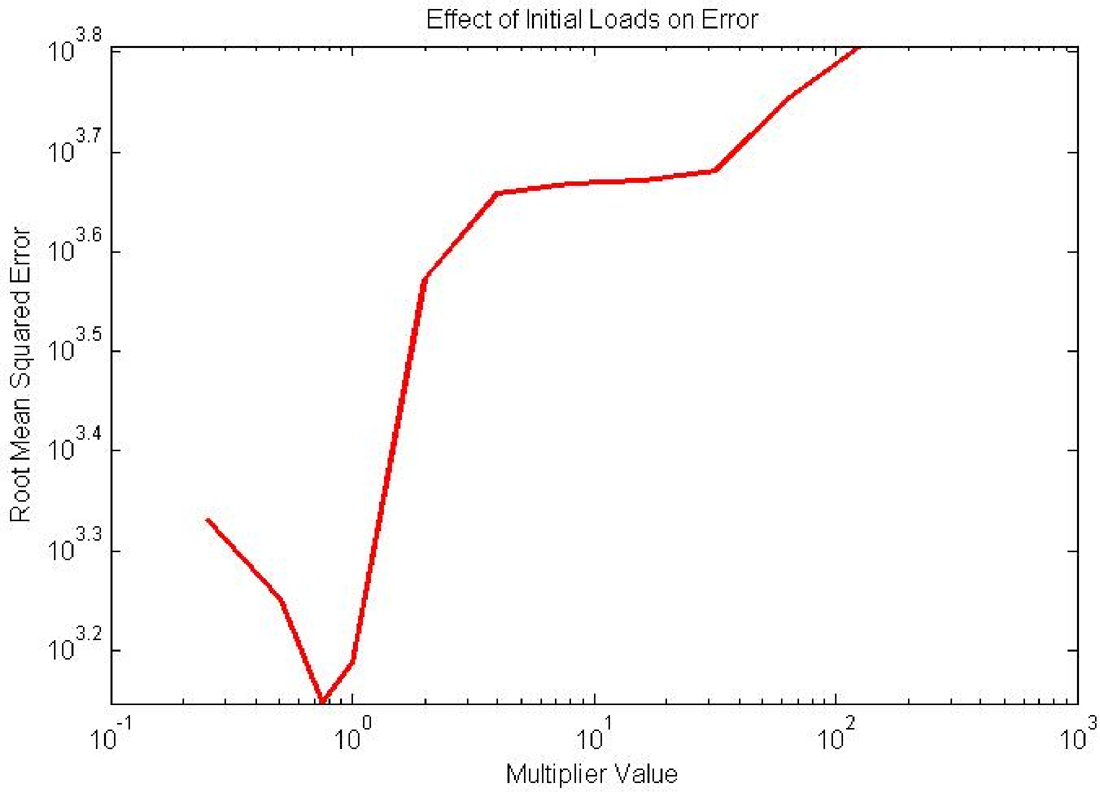


Figure 37: Error Caused by Starting Conditions for Load Updating

The uncertainty analysis performed here shows how the error in the experimental measurements are propagated through and eventually affect the data finally calculated. The two sources of error which are considered for this study are the error from the load cell measurements and also the error from the strain gage measurements. The load cells measure the input of each actuator to an accuracy of one percent and each strain gage is deemed to be accurate within two percent. The strains obtained from the load cell error and also from the strain gage error are input into the load updating algorithm with the following results shown in Table 10.

Actuator:	Load Cell Error, lbs	Strain Gage Error, lbs	Total Error, lbs
1	0.002623	0.005265	0.007906
2	0.003298	0.006613	0.009929
3	0.001099	0.002216	0.003333
4	0.001787	0.003593	0.005398

5	-0.00731	-0.01461	-0.0219
6	0.007824	0.015666	0.023507
7	-1.8E-05	-1.9E-05	-1.9E-05
8	0.006844	0.013707	0.020569
9	-0.00354	-0.00706	-0.01058
10	0.005636	0.011289	0.016943
11	-0.0097	-0.01938	-0.02906
12	0.001159	0.002336	0.003514
13	0.00126	0.002538	0.003817
14	0.002752	0.005522	0.008292
15	-0.00291	-0.0058	-0.00868
16	-2.1E-05	-2.3E-05	-2.6E-05
17	-4.9E-05	-8E-05	-0.00011
18	0.004255	0.008528	0.012802
19	-1.8E-05	-1.9E-05	-1.9E-05
20	-0.00033	-0.00065	-0.00097
21	0.007006	0.01403	0.021054
22	-7E-06	4.08E-06	1.51E-05
23	-1.8E-05	-1.8E-05	-1.8E-05
24	-0.00122	-0.00243	-0.00363
25	-1.1E-05	-3.9E-06	3.15E-06
26	-4.4E-05	-7E-05	-9.7E-05
27	-0.00236	-0.00469	-0.00703
28	8.57E-05	0.000189	0.000293
29	1.78E-05	5.36E-05	8.94E-05
30	-1.7E-05	-1.6E-05	-1.5E-05

Table 10: Uncertainty Analysis

The largest error is found to be in actuator 11 at-0.02906 lbs. Figure 29 is re-plotted with the total errors used as bounds for each of the updated data points and the load cell error used as bounds for each of the initial data points.

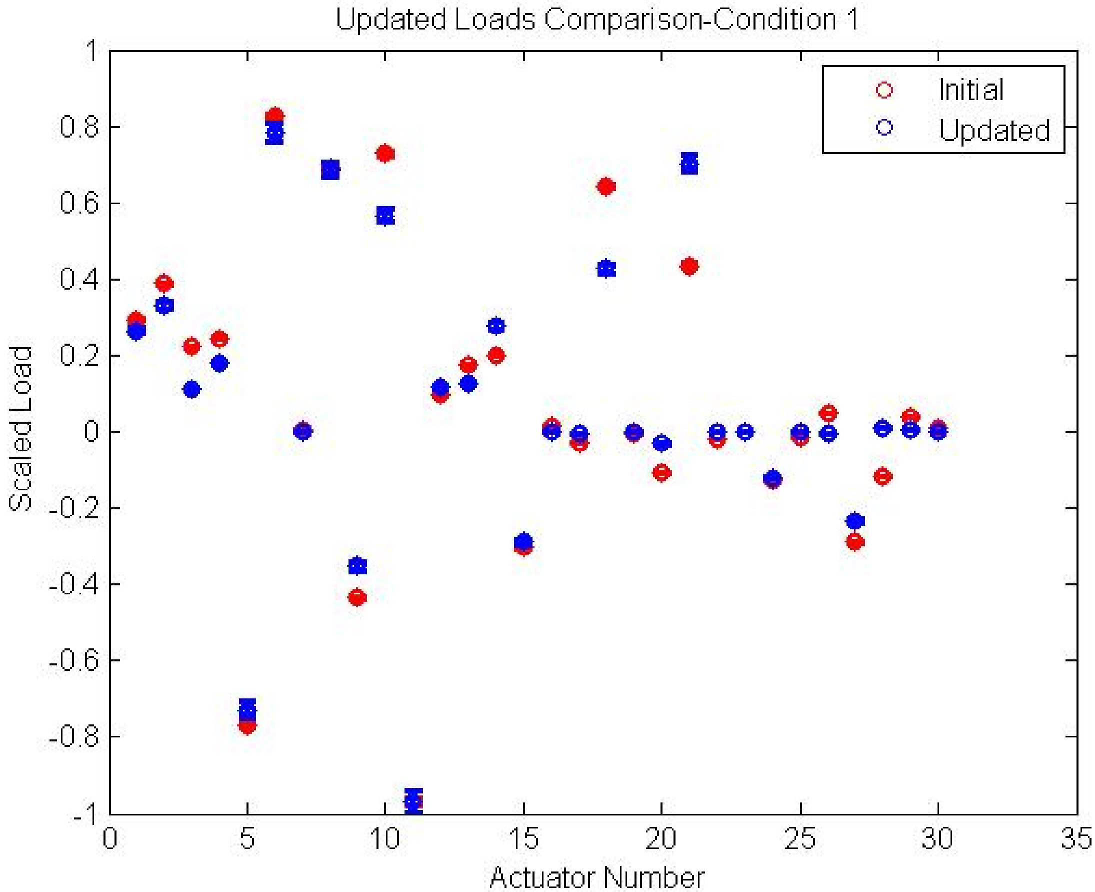


Figure 38: Initial and Updated Loads Comparison for Condition 1 With Error Bounds

Even if all the updated loads contained the max error possible, there would still be significant improvement seen after applying the load updating method for the majority of the actuators and certainly across the structure as a whole.

4.7 Experimental Error and Potential Improvements

When considering the differences between experimental and FEM data and using the load updating method to improve those differences, it is important to understand what causes are contributing to the discrepancies. The uncertainties in testing described by section 3.3 are certainly suspect for contributing to some of this error, but as shown in section 3.3 these would only be a fraction of the differences that are shown here. Another source of error is often the modeling and how well it represents the physical system being tested. While a tremendous amount of care may be given to meticulously creating the model and making proper assumptions, for complex structures a certain level of differences should always be expected. In the case of the frame test described by this thesis, yet another source of error has to be considered. This error is caused by enormous residual strains in the frame structure. During the manufacturing process, the heat generated from welding resulted in the creation and propagation of residual strains which is believed to be by far the largest contributor to the error in this test. While numerous amounts of tests were conducted in an attempt to quantify these strains, variations in the welding process resulted in too many unknowns to measure these strains or predict their effects on the structure. This is one of the primary reasons that this particular structure and test is so difficult to analyze. The load updating method is perfect for highlighting errors such as these that are known to be present, but whose effects cannot be directly obtained or resolved outside significant changes in the manufacturing process. By applying the load updating method, the loads necessary to obtain the equivalent strain in the FEM that is measured during testing can be found which can aid greatly in correlating the data and also in predicting results in future testing.

CHAPTER 5

CONCLUSIONS AND FUTURE WORK

In this study, a novel approach to correlating FEM and test data is applied to a frame structure that is subjected to thirty loads and whose response is measured by seventy-four strain gages. This approach, called the load updating method, seeks to reconcile these difference by finding what loads applied to the FEM structure will result in strains that correlate best instead of modifying the math model of the structure as is traditionally done with model updating. It is found across the thirty-two cases that the load updating method improves the mean squared error by an average of eighty micro strain as compared to the baseline cases run where the actual loads applied to the test structure are used as initial values. When considering the goal of minimizing the difference of test and FEM data to under ten percent, for the purpose of FAA certification, this method proves to be a practical method even on a highly complex aircraft structure such as this frame. The number of strain gages which correlate to within ten percent goes from thirty percent to fifty-four percent. This number is about eighty percent if the strain gages mostly insensitive to this load case are neglected. In addition, the load updating method provides an avenue for correlation despite the high residual strains present in the structure which provides strong validation to its usefulness.

While better correlation in the test and FEM data is promising, a number of important issues are discovered. These include the sensitivity of the starting loads used, selection of the Tikhonov regularization parameter, and how the uncertainty in the measurements affects the results. These issues should be understood and considered by the analysist who wishes to apply the load updating method especially if the results are to be used for certification purposes.

Some future work to be considered includes developing the strain-force matrix using regression instead of the math model from the FEM. This is impractical for this setup due to the lack of gages surrounding some actuators which made it very difficult for a good correlation that could be used for this purpose. However, many other structures such as a tailboom or helicopter blade could provide excellent correlation and would make a regression-based tool a viable goal. The advantages of attempting this would mostly be shown if flight test data is also to be analyzed. A regression with the model and the structure would allow for apples to apples comparison between the structural and FEM equations.

Improvements within the application of the load updating method to the frame structure could include accounting for displacements within the FEM and structural test, adding additional strain gages to the structure, and reducing the residual strains through heat treatment. While displacements are found to be fairly small in the FEM, they are present in the model and did therefore contribute a small amount of error. For example, the individual unit load cases used to develop the strain-force matrix create minute displacements. However, when superposition is imposed to combine the effects of all the actuators on the strain measurements simultaneously the displacements are much larger and did present some small amount of error that is assumed to be negligible. There is also a displacement error in the structure which is again assumed to be small. Therefore, removing the small displacements assumption would be one way to improve this research. Adding strain gages to the structure would increase the level of confidence in the results presented and also could allow regression to be used as an alternative for creating the strain-force matrix. As discussed already, forcing an overdetermined system to match a certain set of data does not guarantee that the strain throughout all the structure is realistic. While efforts, such as verifying the updated loads as realistic, are made to determine if reasonable results are obtained, additional strain gages would improve the confidence level of this judgement. Finally, while the residual strains present in the structure aid in demonstrating the effectiveness of the load updating method, they most certainly also hinder the accuracy at which strains for new loads can be effectively predicted. Having a structure relieved of these strains would result in an analysis where much more accurate predictions could be made. Alternatively, experiments could be run in order to try and estimate the residual strains which could be implemented into the load updating algorithm in order to more accurately predict the loads on the structure.

In addition to use with structural test predictions, FEM equivalent loads could also be obtained for flight test using the load updating method and the identical set of strain gages. While strain gages are often calibrated to be load transducers for simple component parts on aircraft the load updating method could perhaps expand these capabilities by allowing the loads on more complex structures to be predicted. If this is to be attempted, more consideration should be given to placing strain gages at locations that would predict load inputs instead of locating them at points of stress concentrations of interest. This would allow for more accuracy in the predictions and could possibly lend itself to being able to develop a regression based strain-force matrix. Also, correlation could likely be improved between flight data and FEM data, which although is outside the scope of this thesis, is also a topic of particular interest when certifying aircraft.

The purpose of this research is to implement the load updating method in order to improve the correlation between the FEM data and the structural test data. It is found that applying the load updating method does indeed reduce the differences between the FEM and test data significantly. In addition, it provides the ability to predict test loads in the frame structure which is extraordinarily difficult to do as a result of enormous residual strains within the structure. Many additional applications for the load updating method have been suggested and outlined with hopes that it can serve future certification efforts and expand the field of knowledge for correlating analytical and real world data.

Appendix A
Load Updating Algorithm

The Load updating method Applied to Cantilever Beam

```
%Cantilever Beam example for Load updating method


L=9;
L1=2;%2
L2=2.7;%2.7
L3=4.3;
I=0.00016;
E=1E7;

K=E*I*[12/power(L1,3),6/power(L1,2),-12/power(L1,3),6/power(L1,2),0,0,0,0;
   6/power(L1,2),4/L1,-6/power(L1,2),2/L1,0,0,0,0;
   -12/power(L1,3),-6/power(L1,2),12/power(L1,3)+12/power(L2,3),-6/power(L1,2)+6/power(L2,2),-
12/power(L2,3),6/power(L2,2),0,0;
   6/power(L1,2),2/L1,-6/power(L1,2)+6/power(L2,2),4/L1+4/L2,-6/power(L2,2),2/L2,0,0;
   0,0,-12/power(L2,3),-6/power(L2,2),12/power(L2,3)+12/power(L3,3),-
6/power(L2,2)+6/power(L3,2),-12/power(L3,3),6/power(L3,2);
   0,0,6/power(L2,2),2/L2,-6/power(L2,2)+6/power(L3,2),4/L2+4/L3,-6/power(L3,2),2/L3;
   0,0,0,0,-12/power(L3,3),-6/power(L3,2),12/power(L3,3),-6/power(L3,2);
   0,0,0,0,6/power(L3,2),2/L3,-6/power(L3,2),4/L3];

u=[-3.04E-1;5.06E-2;-4.33E-2;1.38E-2;-3.33E-3;2.5E-3;0;0];
f=[0;0;0;0;-2;0];
%displacement=inv(K)*f;

K2=E*I*[12/power(L1,3)+12/power(L2,3),-6/power(L1,2)+6/power(L2,2),-
12/power(L2,3),6/power(L2,2),0,0;
   -6/power(L1,2)+6/power(L2,2),4/L1+4/L2,-6/power(L2,2),2/L2,0,0;
   -12/power(L2,3),-6/power(L2,2),12/power(L2,3)+12/power(L3,3),-6/power(L2,2)+6/power(L3,2),-
12/power(L3,3),6/power(L3,2);
   6/power(L2,2),2/L2,-6/power(L2,2)+6/power(L3,2),4/L2+4/L3,-6/power(L3,2),2/L3;
   0,0,-12/power(L3,3),-6/power(L3,2),12/power(L3,3),-6/power(L3,2);
   0,0,6/power(L3,2),2/L3,-6/power(L3,2),4/L3];

displacement=inv(K2)*f
% force=K2*displacement


%Initial Plots

x0=10;
y0=10;
width=500;
height=400

x=[0;0.2…..
x2=[2,4.7];
MeasuredStrain=[536,333];
MeasuredRotation=[0.00286011, 0.014028034];
MeasuredDeflection=[-.004078899, -.044306141];
TheoreticalBending=[-18;-17….
TheoreticalStrain=[703.125000000000;695…
TheoreticalRotation=[0;6….
TheoreticalDeflection=[0;-4…

figure(1)
set(gcf,'units','points','position',[x0,y0,width,height])
plot(x,TheoreticalBending,'-g','linewidth',2);
title('\fontsize{14} Bending Moment Distribution')
ylabel('\fontsize{14} Bending Moment, in-lbs')
xlabel('\fontsize{14} x, inches')
hold on

figure(2)
set(gcf,'units','points','position',[x0,y0,width,height])
plot(x,TheoreticalStrain,'-g',x2,MeasuredStrain,'ro','linewidth',2);
title('\fontsize{14} Strain Distribution')
ylabel('\fontsize{14} micro Strain, inches/inches*1000000')
```

```
xlabel('\fontsize{14} x, inches')
legend('Theoretical Strain','Measured Strain')
hold on

figure(3)
set(gcf,'units','points','position',[x0,y0,width,height])
plot(x,TheoreticalRotation,'-g',x2,MeasuredRotation,'ro',x3,FEMRotation,'bo','linewidth',2);
title('\fontsize{14} Rotation Distribution')
ylabel('\fontsize{14} Rotation, radians')
xlabel('\fontsize{14} x, inches')
legend('Theoretical Rotation','Measured Rotation','FEM Rotation')
hold on

figure(4)
set(gcf,'units','points','position',[x0,y0,width,height])
plot(x,TheoreticalDeflection,'-
g',x2,MeasuredDeflection,'ro',x3,FEMDeflection,'bo','linewidth',2);
title('\fontsize{14} Deflection Distribution')
ylabel('\fontsize{14} Deflection, inches')
xlabel('\fontsize{14} x, inches')
legend('Theoretical Deflection','Measured Deflection','FEM Deflection')
hold on
```

The Load updating method Applied to Frame Structure

```
%Jonathan Nichols Thesis Work on Load Updating
%1-18-17

%Load Cases

%Force Matrix
Force=[….

%Force Vectors
F1=diag(Force(1,:)');
F2=diag(Force(2,:)');
F3=diag(Force(3,:)');
F4=diag(Force(4,:)');
F5=diag(Force(5,:)');
F6=diag(Force(6,:)');
F7=diag(Force(7,:)');
F8=diag(Force(8,:)');
F9=diag(Force(9,:)');
F10=diag(Force(10,:)');
F11=diag(Force(11,:)');
F12=diag(Force(12,:)');
F13=diag(Force(13,:)');
F14=diag(Force(14,:)');
F15=diag(Force(15,:)');
F16=diag(Force(16,:)');
F17=diag(Force(17,:)');
F18=diag(Force(18,:)');
F19=diag(Force(19,:)');
F20=diag(Force(20,:)');
F21=diag(Force(21,:)');
F22=diag(Force(22,:)');
F23=diag(Force(23,:)');
F24=diag(Force(24,:)');
F25=diag(Force(25,:)');
F26=diag(Force(26,:)');
F27=diag(Force(27,:)');
F28=diag(Force(28,:)');
F29=diag(Force(29,:)');
F30=diag(Force(30,:)');
F31=diag(Force(31,:)');
F32=diag(Force(32,:)');

%Force-Strain Matrix
```

```
M=[…..

FEMstrain=[…..

%Measured Stain Vectors
u1=transpose(FEMstrain(1,1:74));
u2=transpose(FEMstrain(2,1:74));
u3=transpose(FEMstrain(3,1:74));
u4=transpose(FEMstrain(4,1:74));
u5=transpose(FEMstrain(5,1:74));
u6=transpose(FEMstrain(6,1:74));
u7=transpose(FEMstrain(7,1:74));
u8=transpose(FEMstrain(8,1:74));
u9=transpose(FEMstrain(9,1:74));
u10=transpose(FEMstrain(10,1:74));
u11=transpose(FEMstrain(11,1:74));
u12=transpose(FEMstrain(12,1:74));
u13=transpose(FEMstrain(13,1:74));
u14=transpose(FEMstrain(14,1:74));
u15=transpose(FEMstrain(15,1:74));
u16=transpose(FEMstrain(16,1:74));
u17=transpose(FEMstrain(17,1:74));
u18=transpose(FEMstrain(18,1:74));
u19=transpose(FEMstrain(19,1:74));
u20=transpose(FEMstrain(20,1:74));
u21=transpose(FEMstrain(21,1:74));
u22=transpose(FEMstrain(22,1:74));
u23=transpose(FEMstrain(23,1:74));
u24=transpose(FEMstrain(24,1:74));
u25=transpose(FEMstrain(25,1:74));
u26=transpose(FEMstrain(26,1:74));
u27=transpose(FEMstrain(27,1:74));
u28=transpose(FEMstrain(28,1:74));
u29=transpose(FEMstrain(29,1:74));
u30=transpose(FEMstrain(30,1:74));
u31=transpose(FEMstrain(31,1:74));
u32=transpose(FEMstrain(32,1:74));

%Compute N Matrix
N1=M*F1;
N2=M*F2;
N3=M*F3;
N4=M*F4;
N5=M*F5;
N6=M*F6;
N7=M*F7;
N8=M*F8;
N9=M*F9;
N10=M*F10;
N11=M*F11;
N12=M*F12;
N13=M*F13;
N14=M*F14;
N15=M*F15;
N16=M*F16;
N17=M*F17;
N18=M*F18;
N19=M*F19;
N20=M*F20;
N21=M*F21;
N22=M*F22;
N23=M*F23;
N24=M*F24;
N25=M*F25;
N26=M*F26;
N27=M*F27;
N28=M*F28;
N29=M*F29;
N30=M*F30;
N31=M*F31;
N32=M*F32;
```

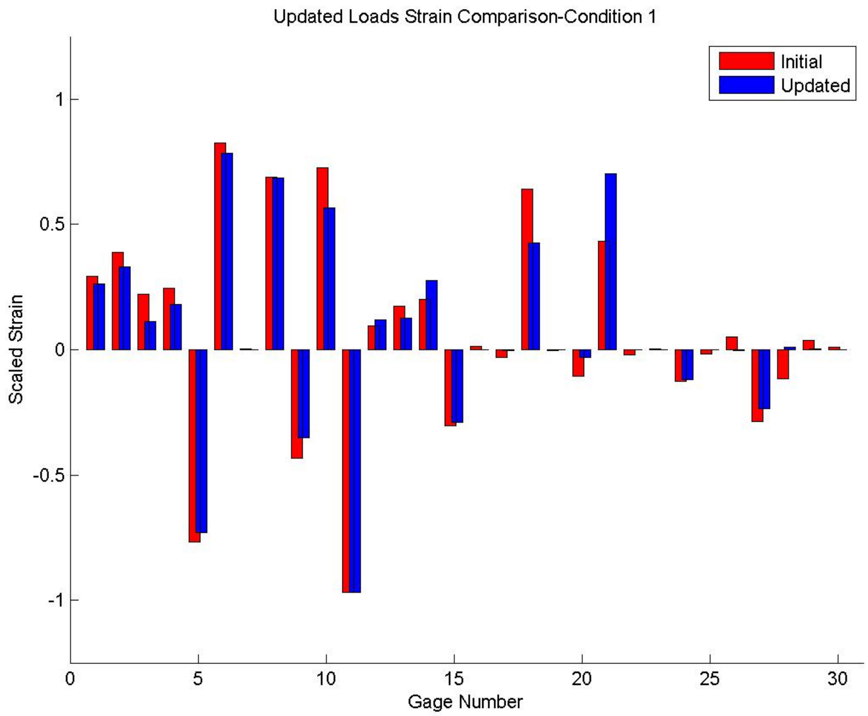
```
%Compute NN Matrix
NN1=N1'*N1;
NN2=N2'*N2;
NN3=N3'*N3;
NN4=N4'*N4;
NN5=N5'*N5;
NN6=N6'*N6;
NN7=N7'*N7;
NN8=N8'*N8;
NN9=N9'*N9;
NN10=N10'*N10;
NN11=N11'*N11;
NN12=N12'*N12;
NN13=N13'*N13;
NN14=N14'*N14;
NN15=N15'*N15;
NN16=N16'*N16;
NN17=N17'*N17;
NN18=N18'*N18;
NN19=N19'*N19;
NN20=N20'*N20;
NN21=N21'*N21;
NN22=N22'*N22;
NN23=N23'*N23;
NN24=N24'*N24;
NN25=N25'*N25;
NN26=N26'*N26;
NN27=N27'*N27;
NN28=N28'*N28;
NN29=N29'*N29;
NN30=N30'*N30;
NN31=N31'*N31;
NN32=N32'*N32;

%Compute VV Vector
VV1=N1'*u1;
VV2=N2'*u2;
VV3=N3'*u3;
VV4=N4'*u4;
VV5=N5'*u5;
VV6=N6'*u6;
VV7=N7'*u7;
VV8=N8'*u8;
VV9=N9'*u9;
VV10=N10'*u10;
VV11=N11'*u11;
VV12=N12'*u12;
VV13=N13'*u13;
VV14=N14'*u14;
VV15=N15'*u15;
VV16=N16'*u16;
VV17=N17'*u17;
VV18=N18'*u18;
VV19=N19'*u19;
VV20=N20'*u20;
VV21=N21'*u21;
VV22=N22'*u22;
VV23=N23'*u23;
VV24=N24'*u24;
VV25=N25'*u25;
VV26=N26'*u26;
VV27=N27'*u27;
VV28=N28'*u28;
VV29=N29'*u29;
VV30=N30'*u30;
VV31=N31'*u31;
VV32=N32'*u32;

%Singular Value Decomposition
```

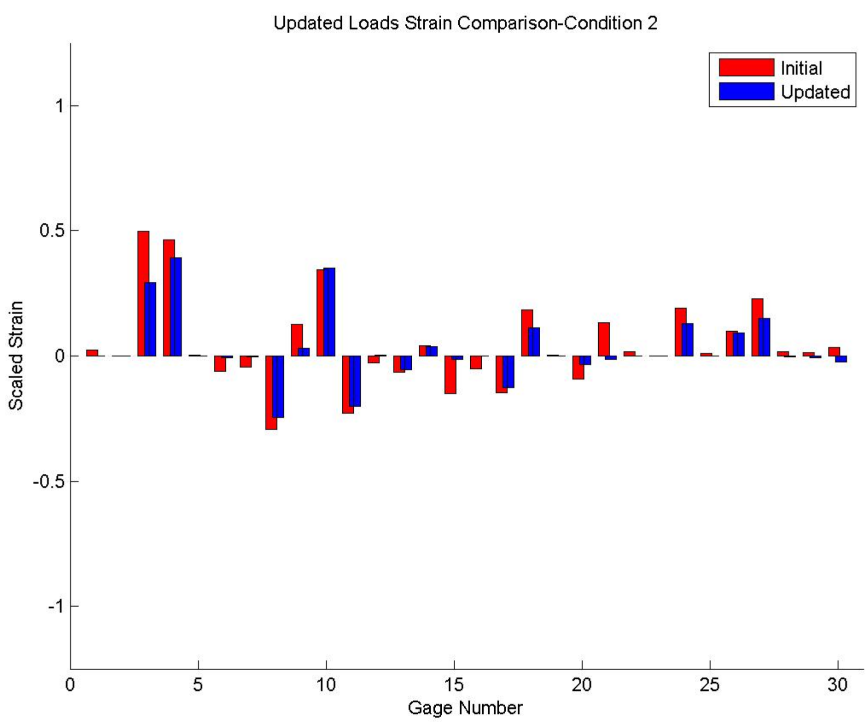
```
[U1,S1,V1]=csvd(NN1);
[U2,S2,V2]=csvd(NN2);
[U3,S3,V3]=csvd(NN3);
[U4,S4,V4]=csvd(NN4);
[U5,S5,V5]=csvd(NN5);
[U6,S6,V6]=csvd(NN6);
[U7,S7,V7]=csvd(NN7);
[U8,S8,V8]=csvd(NN8);
[U9,S9,V9]=csvd(NN9);
[U10,S10,V10]=csvd(NN10);
[U11,S11,V11]=csvd(NN11);
[U12,S12,V12]=csvd(NN12);
[U13,S13,V13]=csvd(NN13);
[U14,S14,V14]=csvd(NN14);
[U15,S15,V15]=csvd(NN15);
[U16,S16,V16]=csvd(NN16);
[U17,S17,V17]=csvd(NN17);
[U18,S18,V18]=csvd(NN18);
[U19,S19,V19]=csvd(NN19);
[U20,S20,V20]=csvd(NN20);
[U21,S21,V21]=csvd(NN21);
[U22,S22,V22]=csvd(NN22);
[U23,S23,V23]=csvd(NN23);
[U24,S24,V24]=csvd(NN24);
[U25,S25,V25]=csvd(NN25);
[U26,S26,V26]=csvd(NN26);
[U27,S27,V27]=csvd(NN27);
[U28,S28,V28]=csvd(NN28);
[U29,S29,V29]=csvd(NN29);
[U30,S30,V30]=csvd(NN30);
[U31,S31,V31]=csvd(NN31);
[U32,S32,V32]=csvd(NN32);

%Tikhonov Regularization Application
c1=tikhonov(U1,S1,V1,VV1,9740);
c2=tikhonov(U2,S2,V2,VV2,9740);
c3=tikhonov(U3,S3,V3,VV3,9740);
c4=tikhonov(U4,S4,V4,VV4,9740);
c5=tikhonov(U5,S5,V5,VV5,9740);
c6=tikhonov(U6,S6,V6,VV6,9740);
c7=tikhonov(U7,S7,V7,VV7,9740);
c8=tikhonov(U8,S8,V8,VV8,9740);
c9=tikhonov(U9,S9,V9,VV9,9740);
c10=tikhonov(U10,S10,V10,VV10,9740);
c11=tikhonov(U11,S11,V11,VV11,9740);
c12=tikhonov(U12,S12,V12,VV12,9740);
c13=tikhonov(U13,S13,V13,VV13,9740);
c14=tikhonov(U14,S14,V14,VV14,9740);
c15=tikhonov(U15,S15,V15,VV15,9740);
c16=tikhonov(U16,S16,V16,VV16,9740);
c17=tikhonov(U17,S17,V17,VV17,9740);
c18=tikhonov(U18,S18,V18,VV18,9740);
c19=tikhonov(U19,S19,V19,VV19,9740);
c20=tikhonov(U20,S20,V20,VV20,9740);
c21=tikhonov(U21,S21,V21,VV21,9740);
c22=tikhonov(U22,S22,V22,VV22,9740);
c23=tikhonov(U23,S23,V23,VV23,9740);
c24=tikhonov(U24,S24,V24,VV24,9740);
c25=tikhonov(U25,S25,V25,VV25,9740);
c26=tikhonov(U26,S26,V26,VV26,9740);
c27=tikhonov(U27,S27,V27,VV27,9740);
c28=tikhonov(U28,S28,V28,VV28,9740);
c29=tikhonov(U29,S29,V29,VV29,9740);
c30=tikhonov(U30,S30,V30,VV30,9740);
c31=tikhonov(U31,S31,V31,VV31,9740);
c32=tikhonov(U32,S32,V32,VV32,9740);

%Compute Loads
Loads1=F1*c1;
Loads2=F2*c2;
Loads3=F3*c3;
```

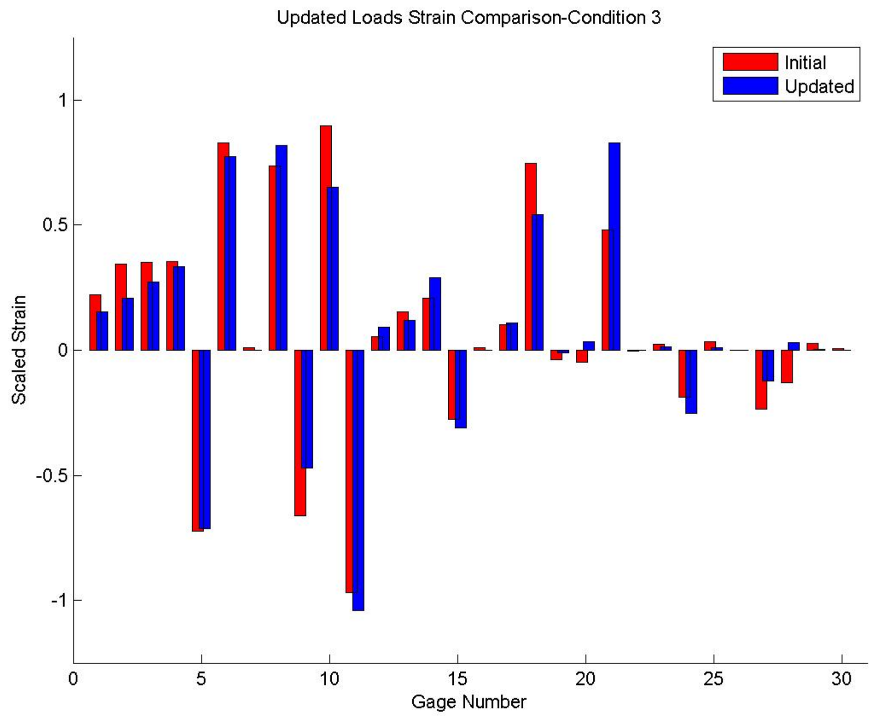
```
Loads4=F4*c4;
Loads5=F5*c5;
Loads6=F6*c6;
Loads7=F7*c7;
Loads8=F8*c8;
Loads9=F9*c9;
Loads10=F10*c10;
Loads11=F11*c11;
Loads12=F12*c12;
Loads13=F13*c13;
Loads14=F14*c14;
Loads15=F15*c15;
Loads16=F16*c16;
Loads17=F17*c17;
Loads18=F18*c18;
Loads19=F19*c19;
Loads20=F20*c20;
Loads21=F21*c21;
Loads22=F22*c22;
Loads23=F23*c23;
Loads24=F24*c24;
Loads25=F25*c25;
Loads26=F26*c26;
Loads27=F27*c27;
Loads28=F28*c28;
Loads29=F29*c29;
Loads30=F30*c30;
Loads31=F31*c31;
Loads32=F32*c32;

%Load Matrix
Loads=[Loads1,Loads2,Loads3,Loads4,Loads5,Loads6,Loads7,Loads8,Loads9,Loads10,Loads11,Loads12,Loa
ds13,Loads14,Loads15,Loads16,Loads17,Loads18,Loads19,Loads20,Loads21,Loads22,Loads23,Loads24,Load
s25,Loads26,Loads27,Loads28,Loads29,Loads30,Loads31,Loads32];
```

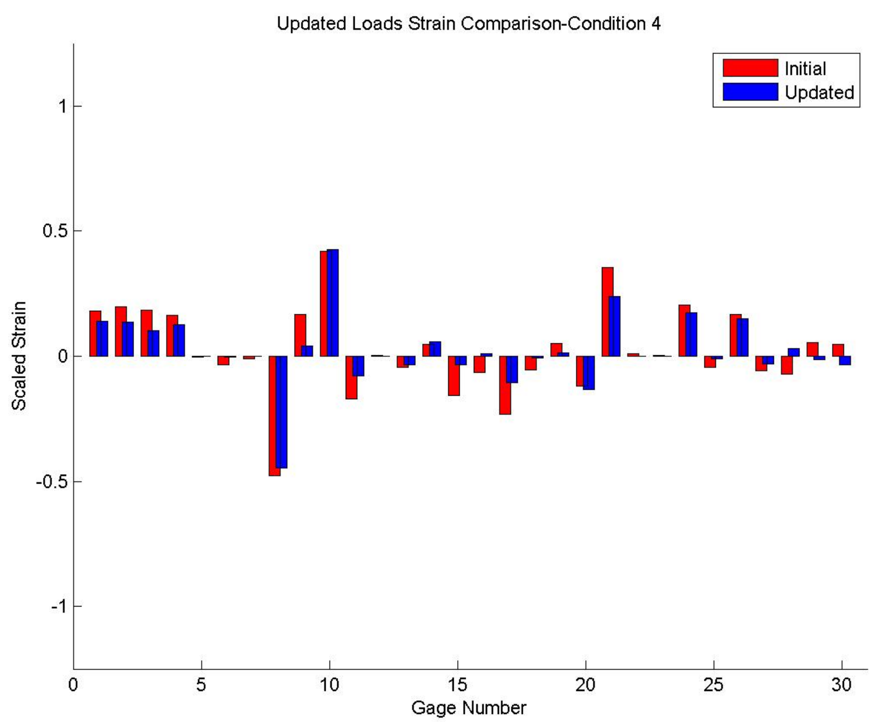
Appendix B
Load Updating Data

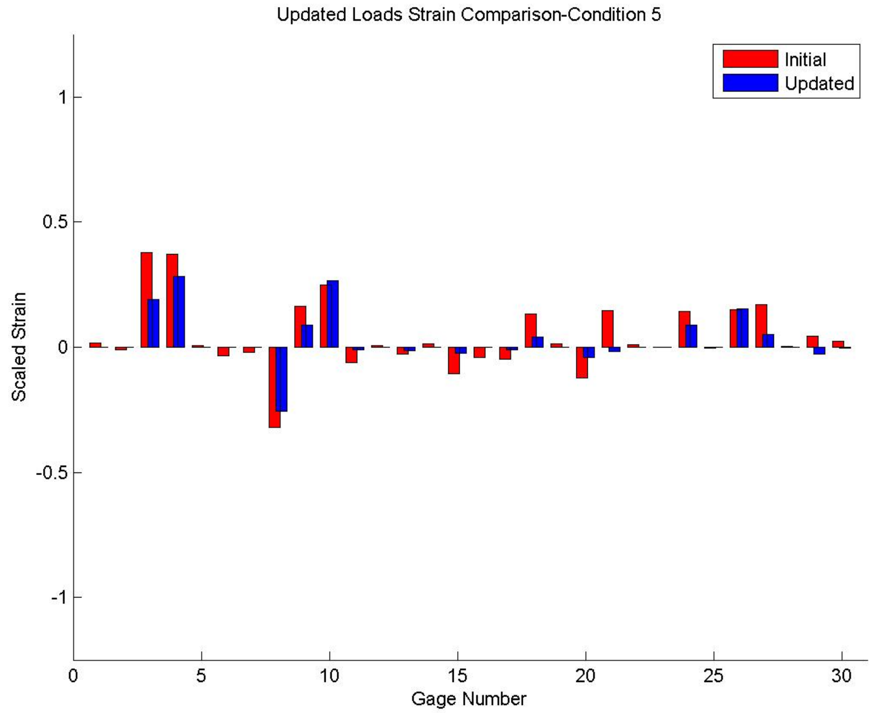
Load Updating Data

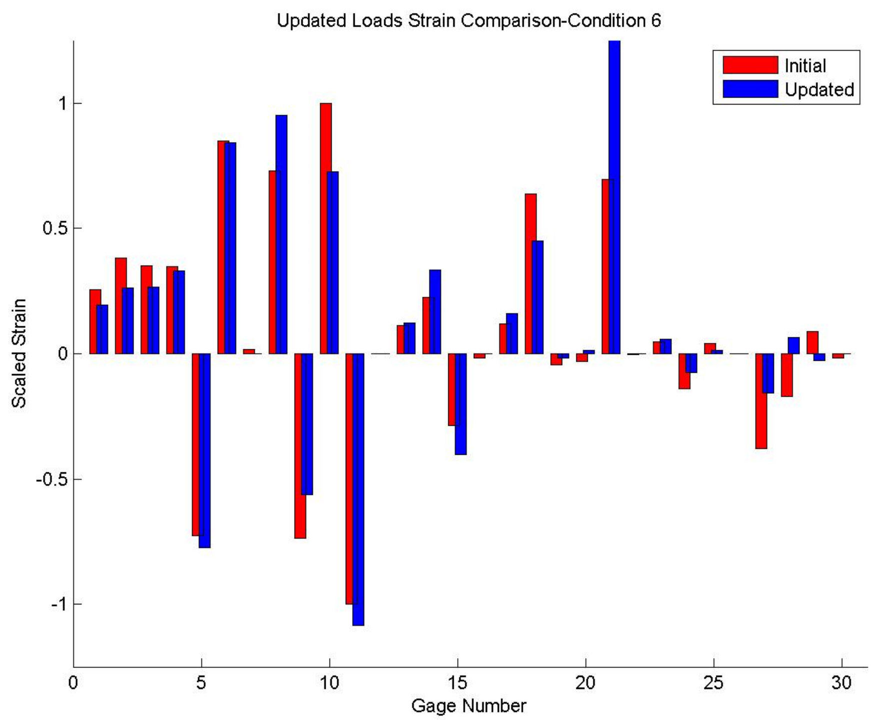





Updated Loads Strain Comparison-Condition 3
Initial
Updated
Scaled Strain
Gage Number
1
0.5
0
-0.5
-1
0
5
10
15
20
25
30


Updated Loads Strain Comparison-Condition 4
Initial
Updated
Scaled Strain
Gage Number
1
0.5
0
-0.5
-1
0
5
10
15
20
25
30


Updated Loads Strain Comparison-Condition 5
Initial
Updated
Scaled Strain
1
0.5
0
-0.5
-1
0
5
10
15
20
25
30
Gage Number


Updated Loads Strain Comparison-Condition 6
Initial
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Scaled Strain
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0.5
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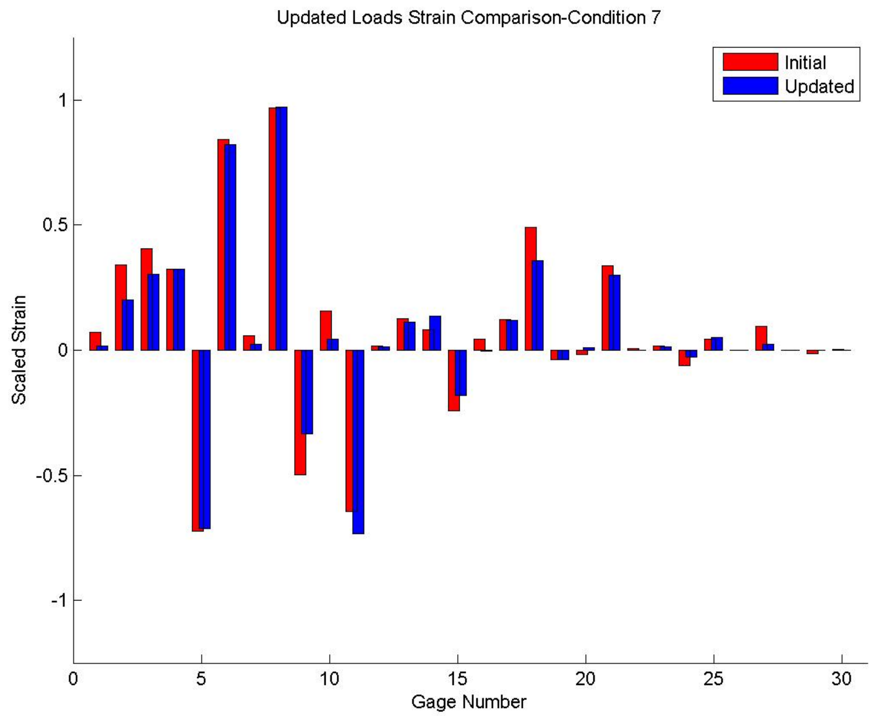
-1
0
5
10
15
20
25
30
Gage Number


Updated Loads Strain Comparison-Condition 7
Initial
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Scaled Strain
1
0.5
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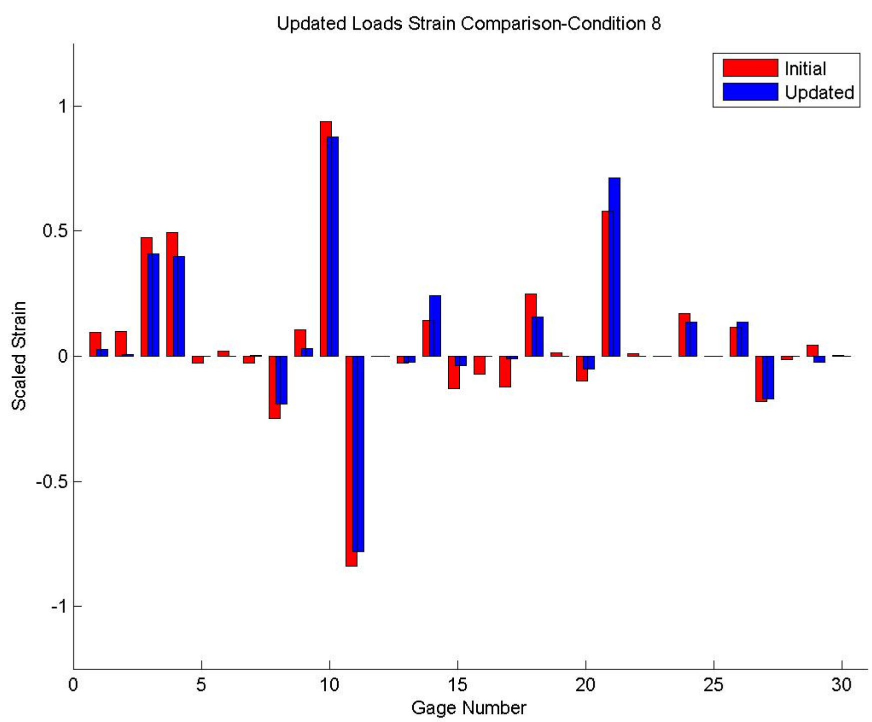
-0.5
-1
0
5
10
15
20
25
30
Gage Number


Updated Loads Strain Comparison-Condition 8
Initial
Updated
Scaled Strain
1
0.5
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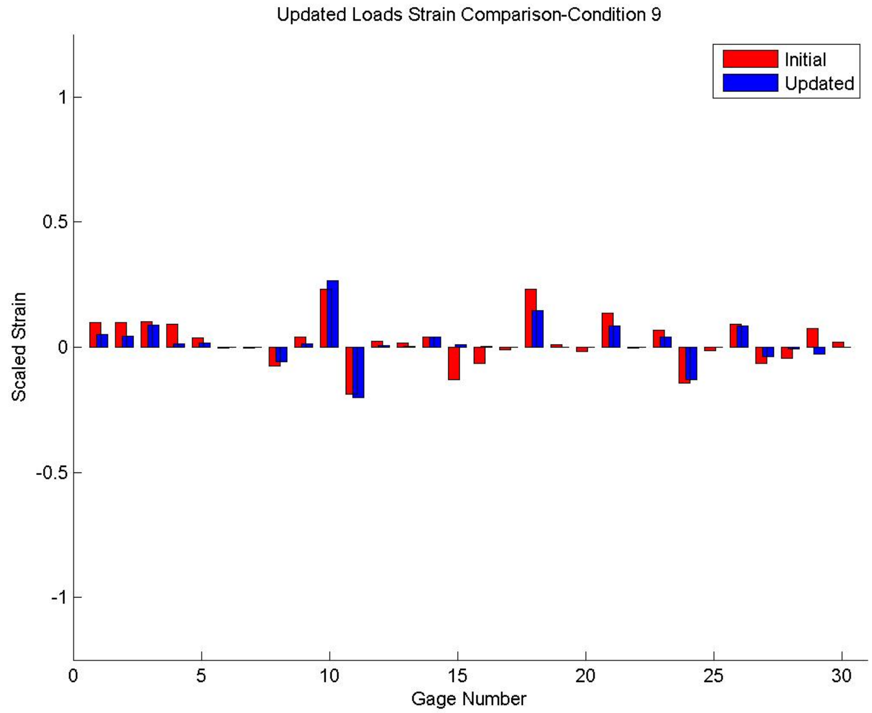
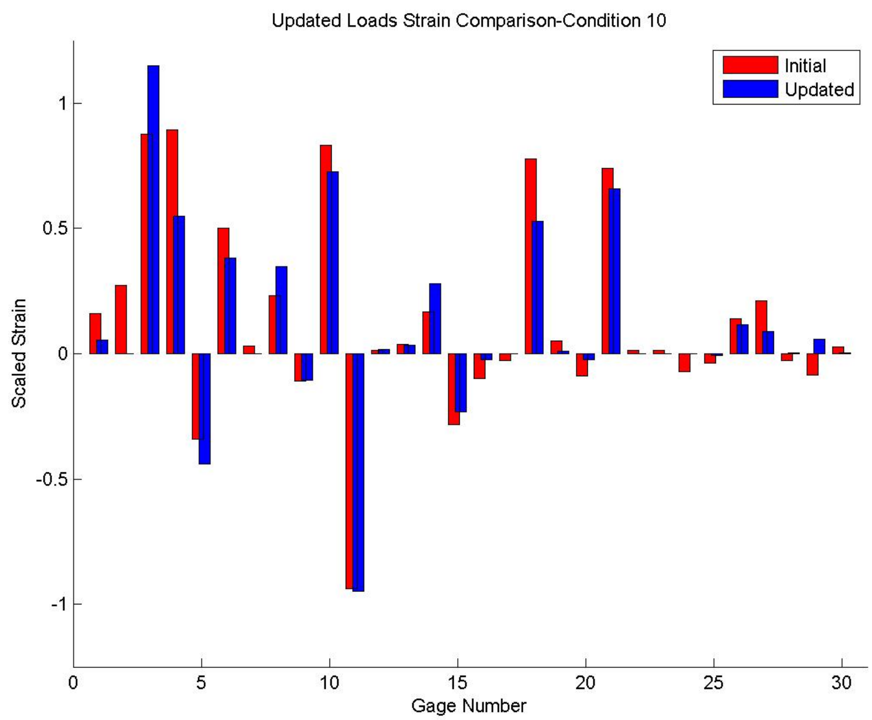
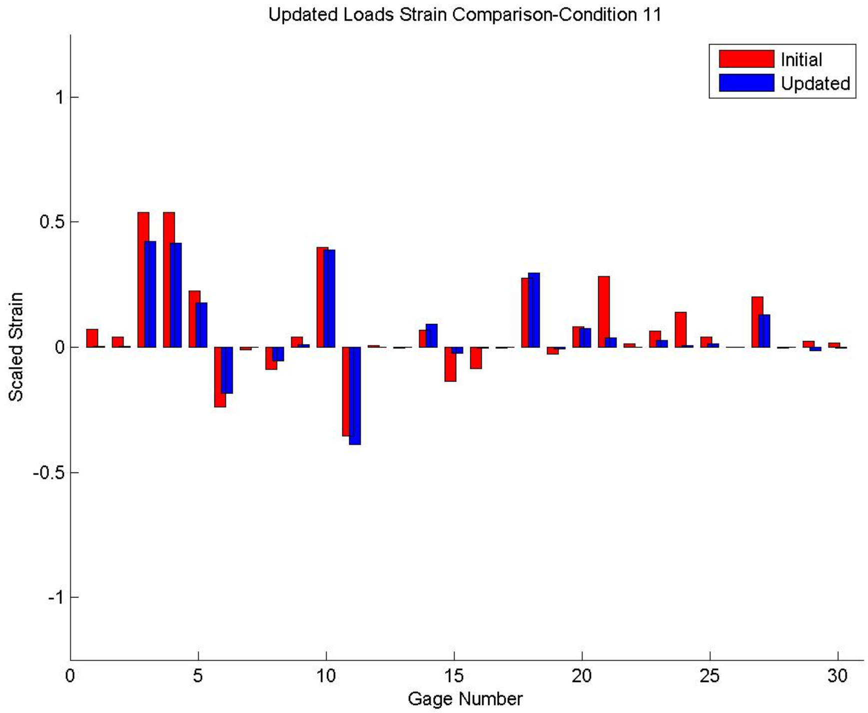
-0.5
-1
0
5
10
15
20
25
30
Gage Number


Updated Loads Strain Comparison-Condition 9
Initial
Updated
Scaled Strain
1
0.5
0
-0.5
-1
0
5
10
15
20
25
30
Gage Number


Updated Loads Strain Comparison-Condition 10
Initial
Updated
Scaled Strain
1
0.5
0
-0.5
-1
0
5
10
15
20
25
30
Gage Number


Updated Loads Strain Comparison-Condition 11
Initial
Updated
Scaled Strain
Gage Number
1
0.5
0
-0.5
-1
0
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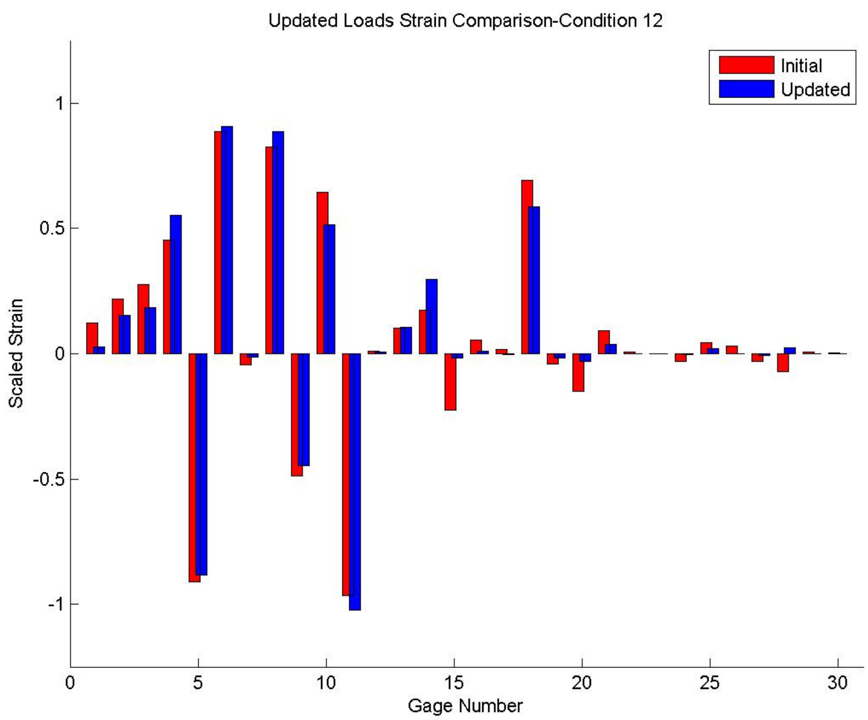
10
15
20
25
30


Updated Loads Strain Comparison-Condition 12
Initial
Updated
Scaled Strain
Gage Number
1
0.5
0
-0.5
-1
0
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10
15
20
25
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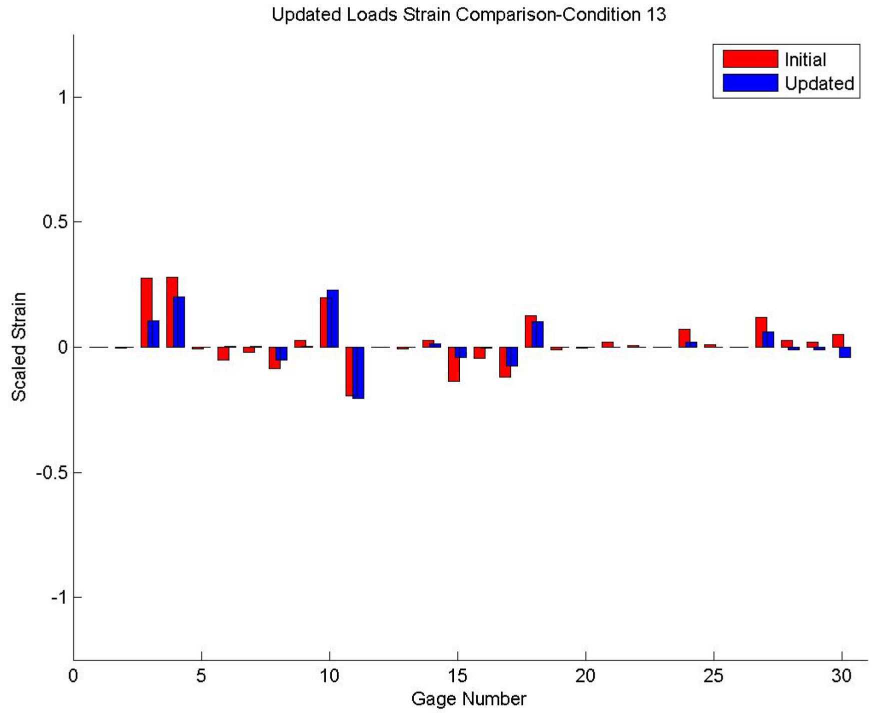
Updated Loads Strain Comparison-Condition 13
Initial
Updated
Scaled Strain
1
0.5
0
-0.5
-1
0
5
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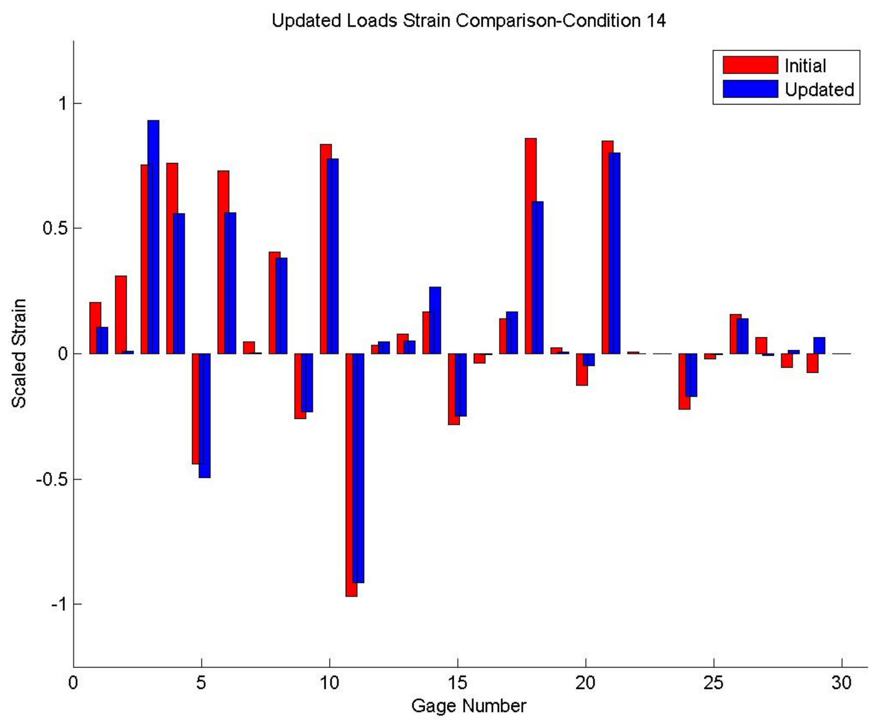
15
20
25
30
Gage Number


Updated Loads Strain Comparison-Condition 14
Initial
Updated
Scaled Strain
1
0.5
0
-0.5
-1
0
5
10
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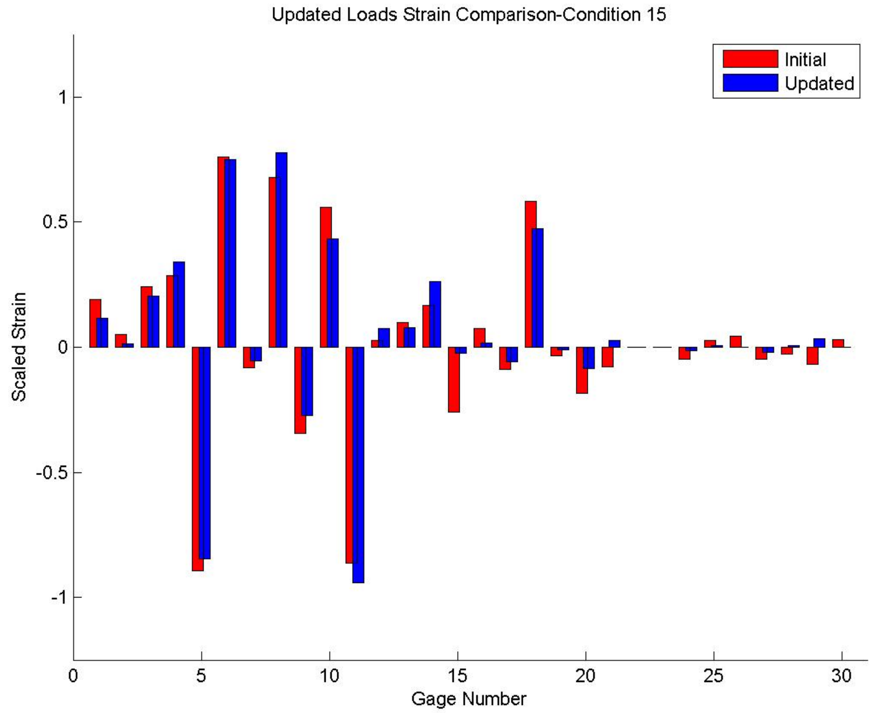
20
25
30
Gage Number


Updated Loads Strain Comparison-Condition 15
Initial
Updated
Scaled Strain
Gage Number
1
0.5
0
-0.5
-1
0
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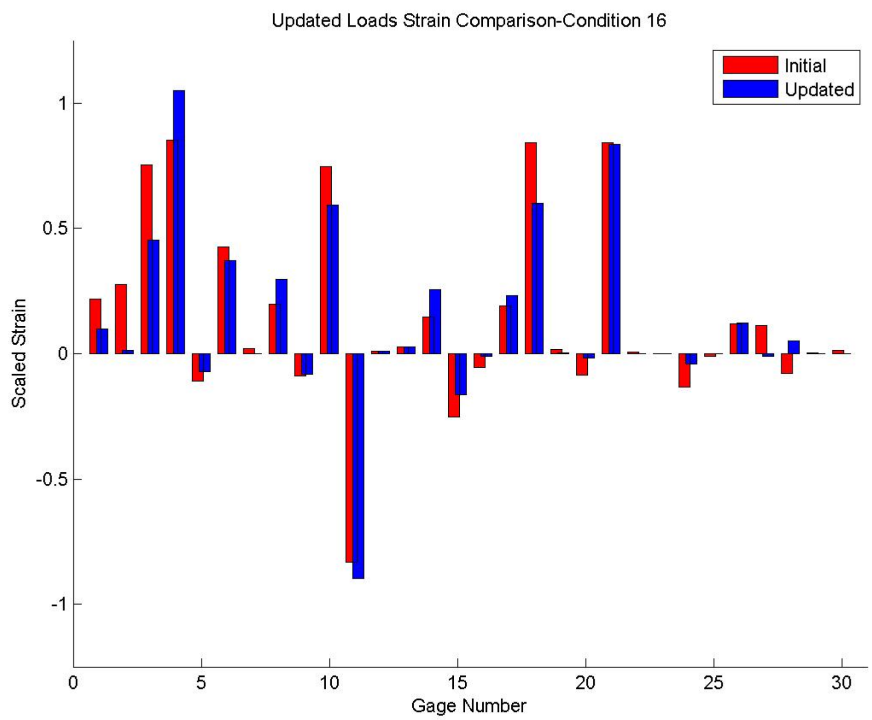
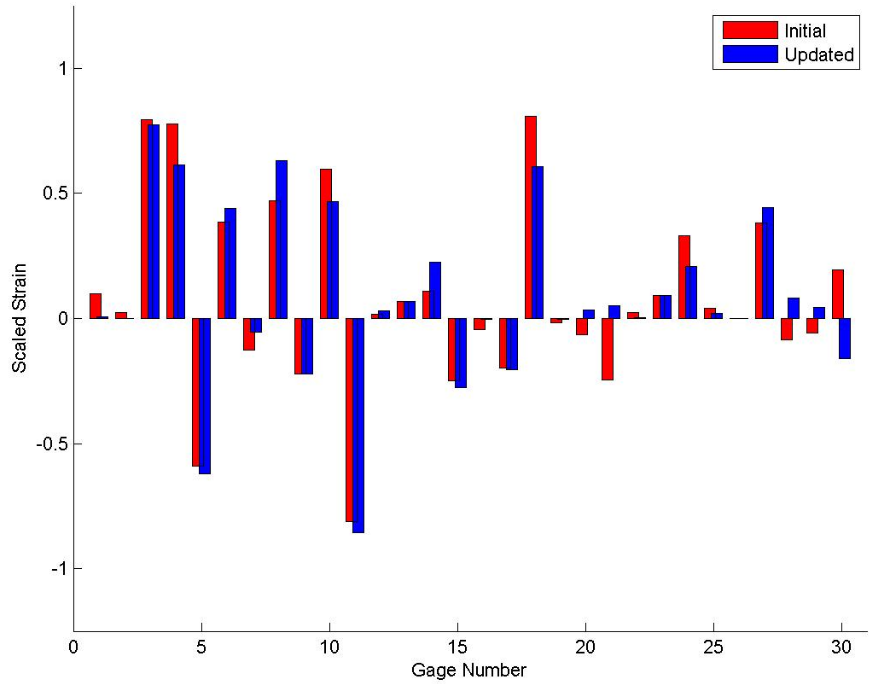
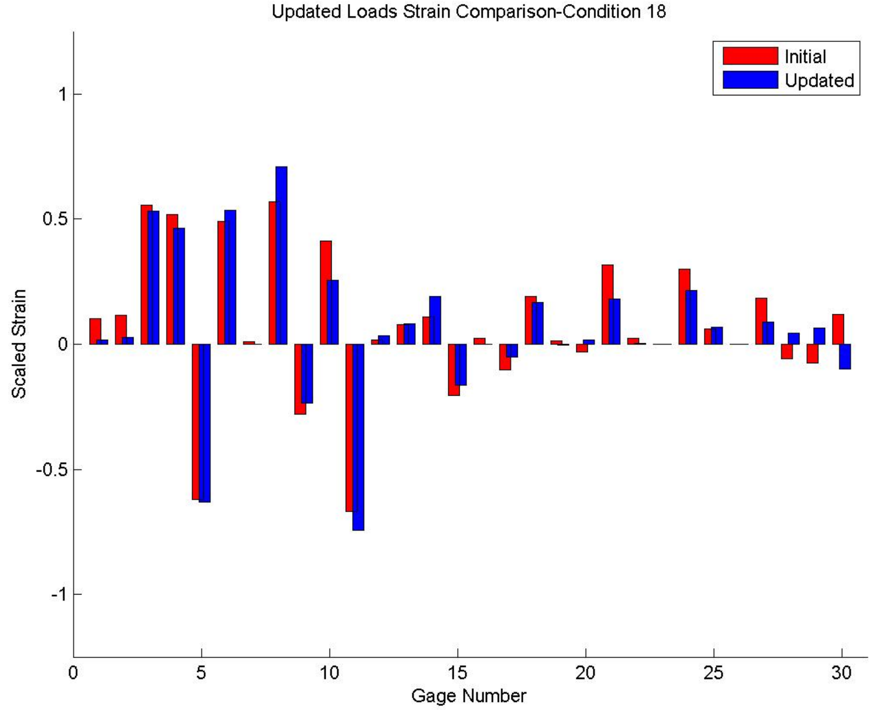
10
15
20
25
30

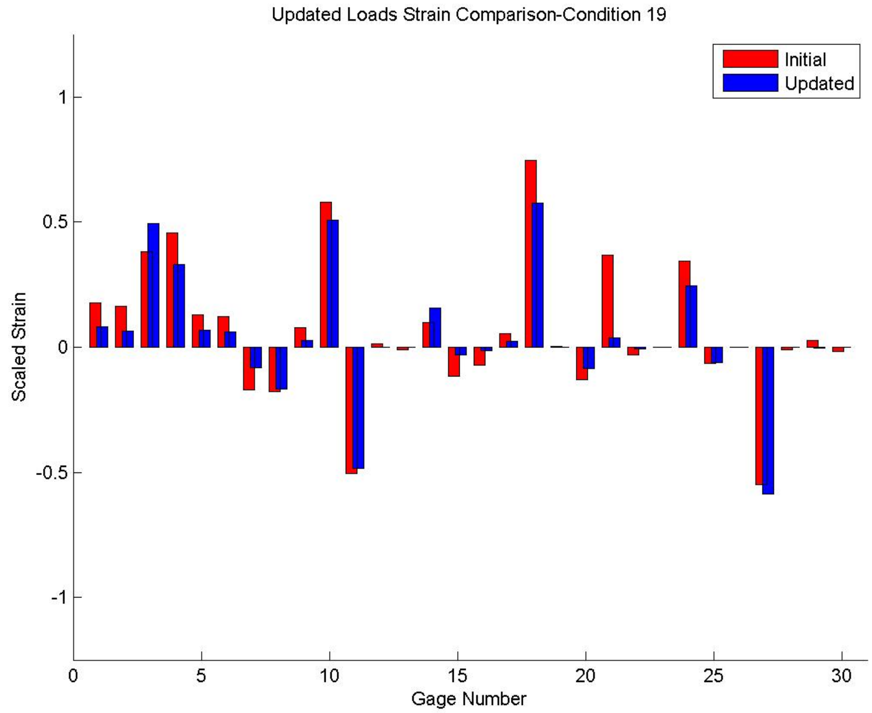

Updated Loads Strain Comparison-Condition 16
Initial
Updated
Scaled Strain
Gage Number
1
0.5
0
-0.5
-1
0
5
10
15
20
25
30

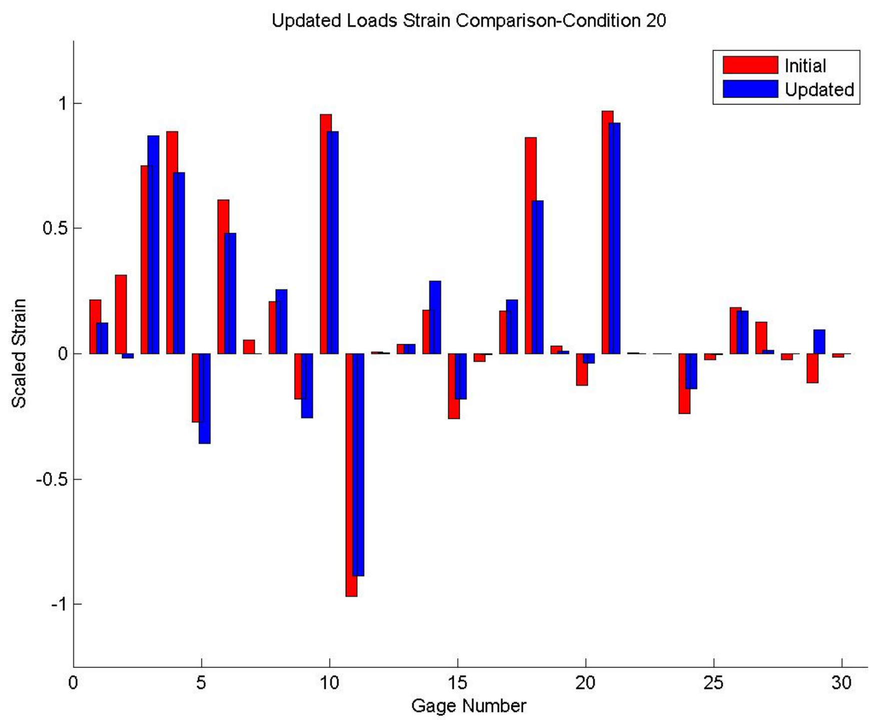
Updated Loads Strain Comparison-Condition 17



Updated Loads Strain Comparison-Condition 18




Updated Loads Strain Comparison-Condition 19
Initial
Updated
Scaled Strain
1
0.5
0
-0.5
-1
0
5
10
15
20
25
30
Gage Number


Updated Loads Strain Comparison-Condition 20
Initial
Updated
Scaled Strain
1
0.5
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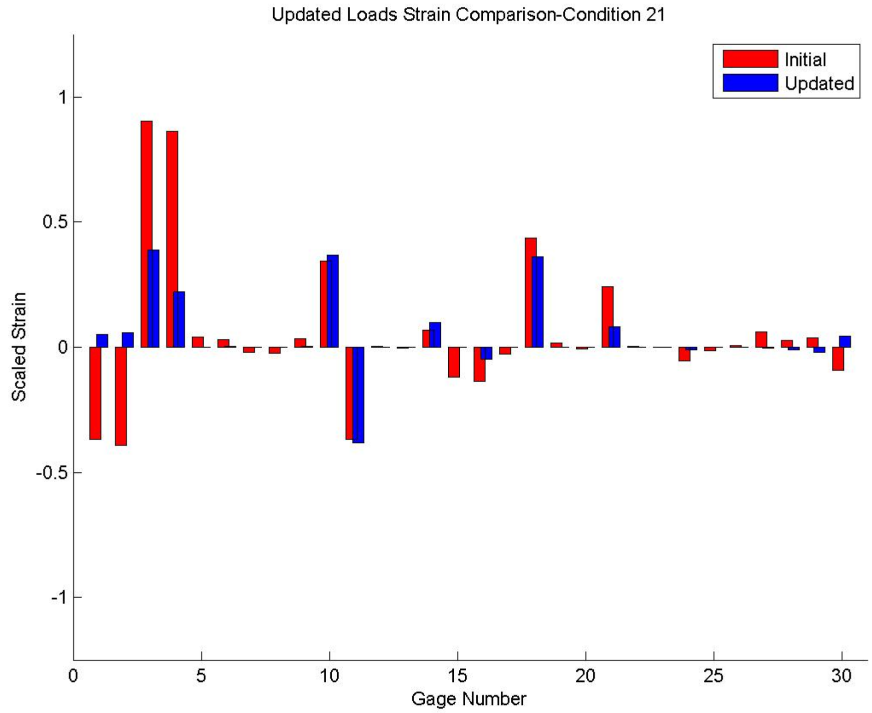
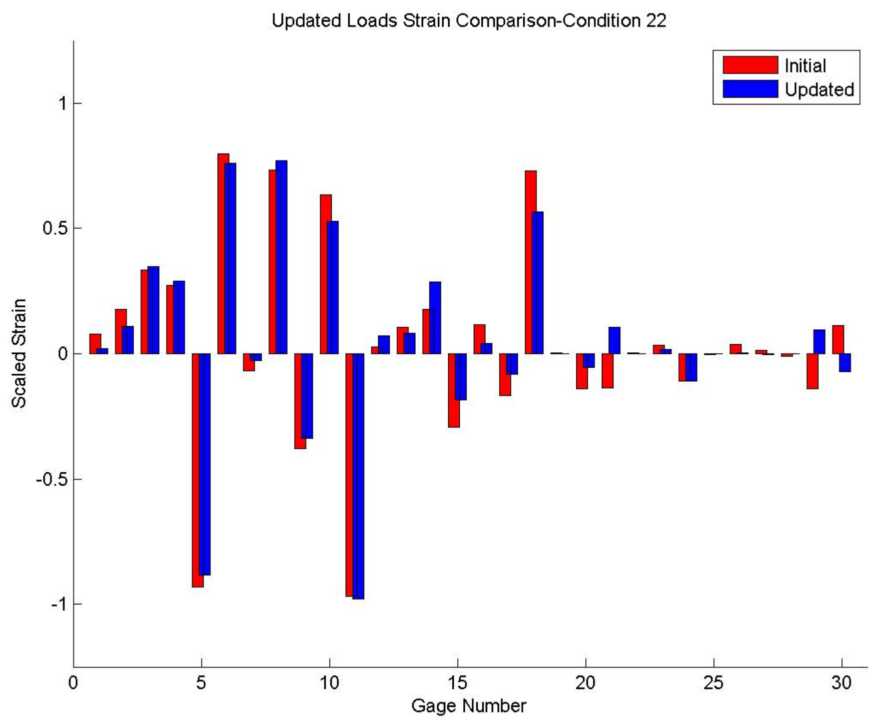
-0.5
-1
0
5
10
15
20
25
30
Gage Number


Updated Loads Strain Comparison-Condition 21
Initial
Updated
Scaled Strain
1
0.5
0
-0.5
-1
0
5
10
15
20
25
30
Gage Number


Updated Loads Strain Comparison-Condition 22
Initial
Updated
Scaled Strain
1
0.5
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-0.5
-1
0
5
10
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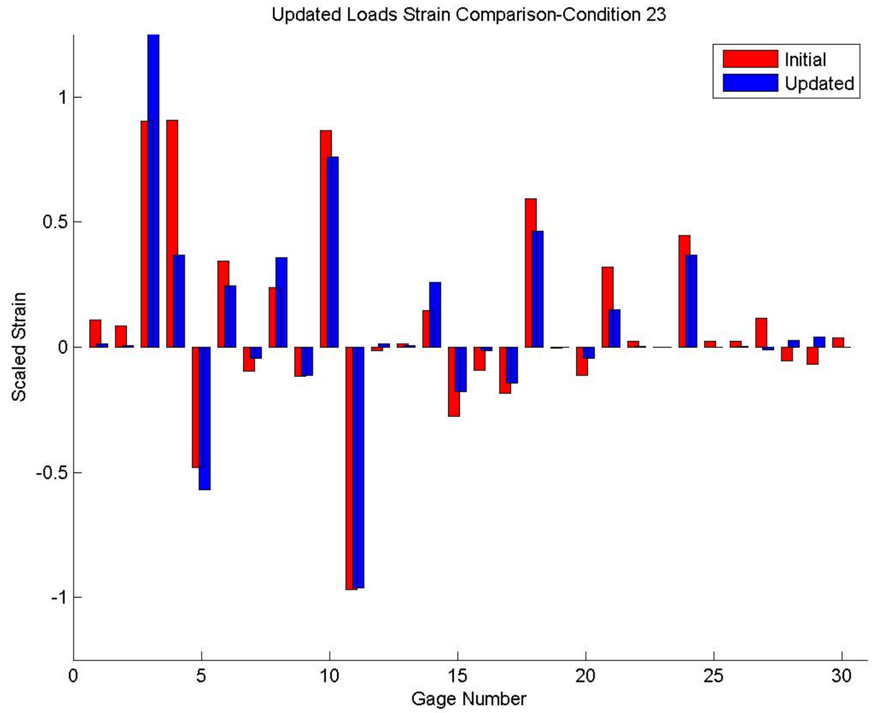
20
25
30
Gage Number


Updated Loads Strain Comparison-Condition 23
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Updated
Scaled Strain
1
0.5
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0
5
10
15
20
25
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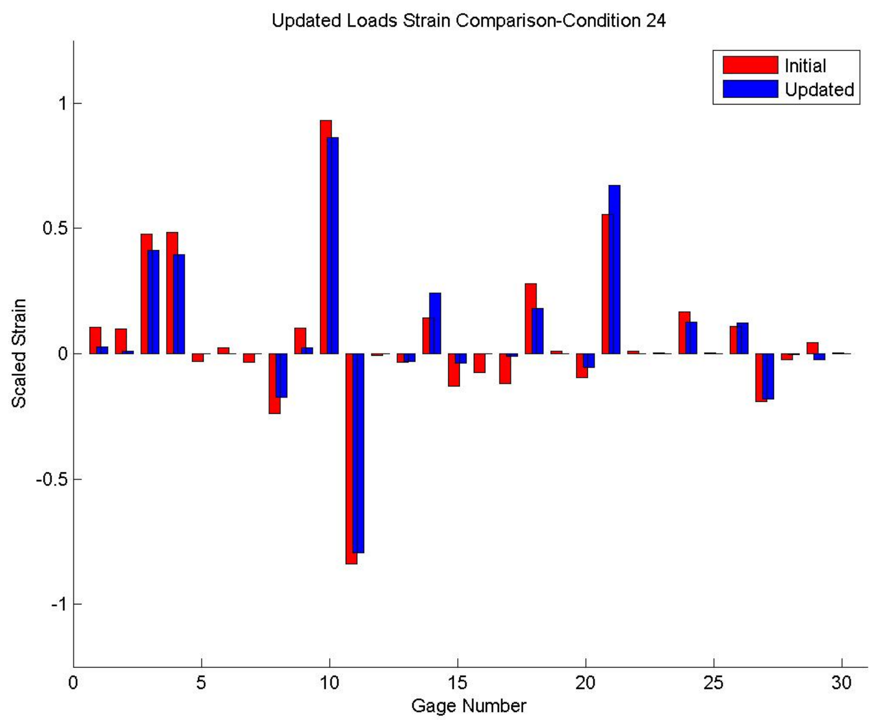
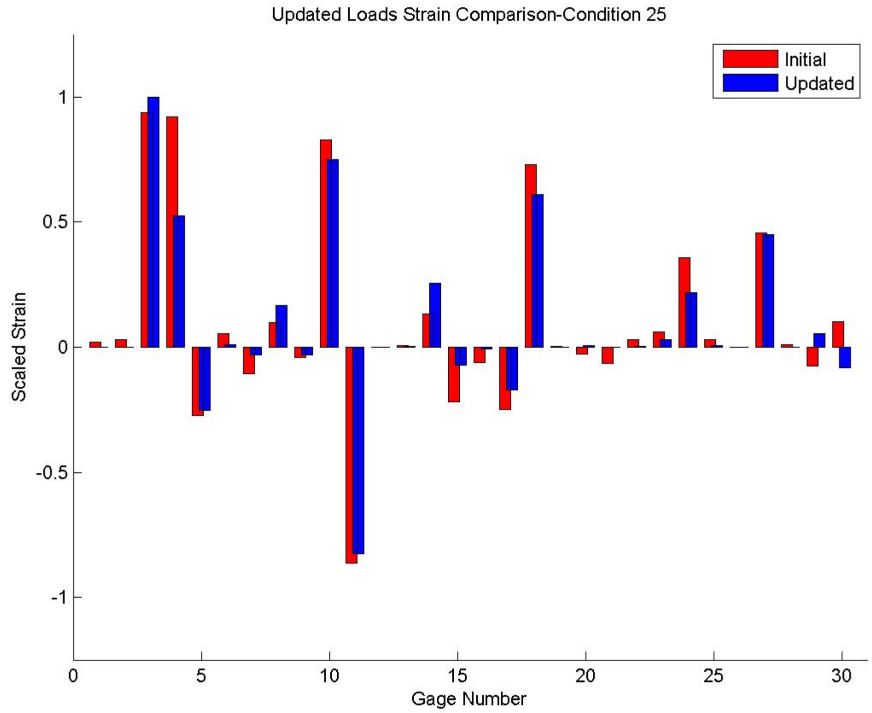
Gage Number


Updated Loads Strain Comparison-Condition 24
Initial
Updated
Scaled Strain
1
0.5
0
-0.5
-1
0
5
10
15
20
25
30
Gage Number


Updated Loads Strain Comparison-Condition 25
Initial
Updated
Scaled Strain
1
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0
-0.5
-1
0
5
10
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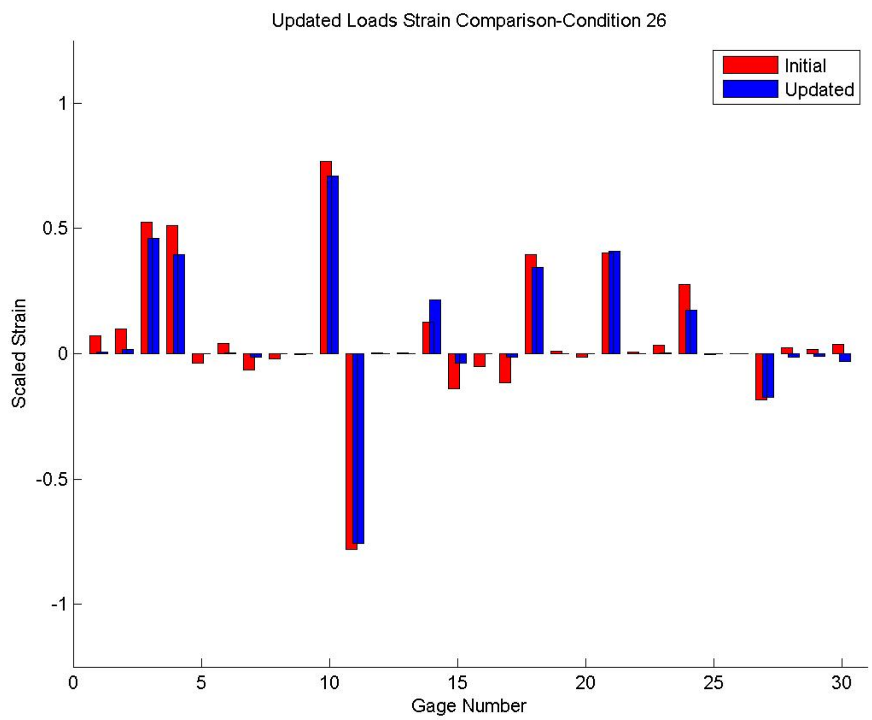
20
25
30
Gage Number


Updated Loads Strain Comparison-Condition 26
Initial
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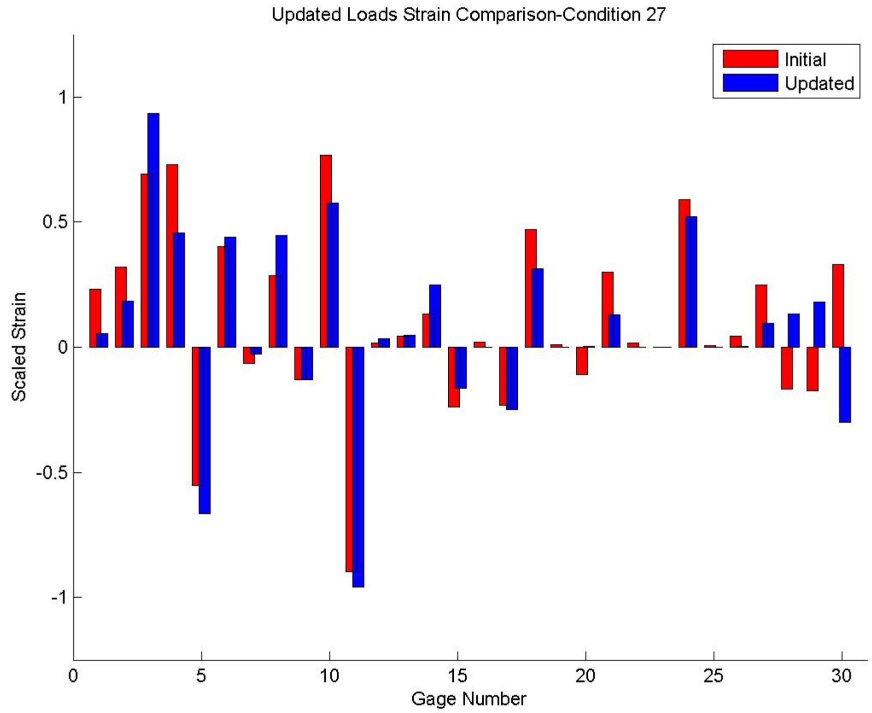
Scaled Strain
1
0.5
0
-0.5
-1
0
5
10
15
20
25
30
Gage Number


Updated Loads Strain Comparison-Condition 27
Initial
Updated
Scaled Strain
Gage Number
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0.5
0
-0.5
-1
0
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10
15
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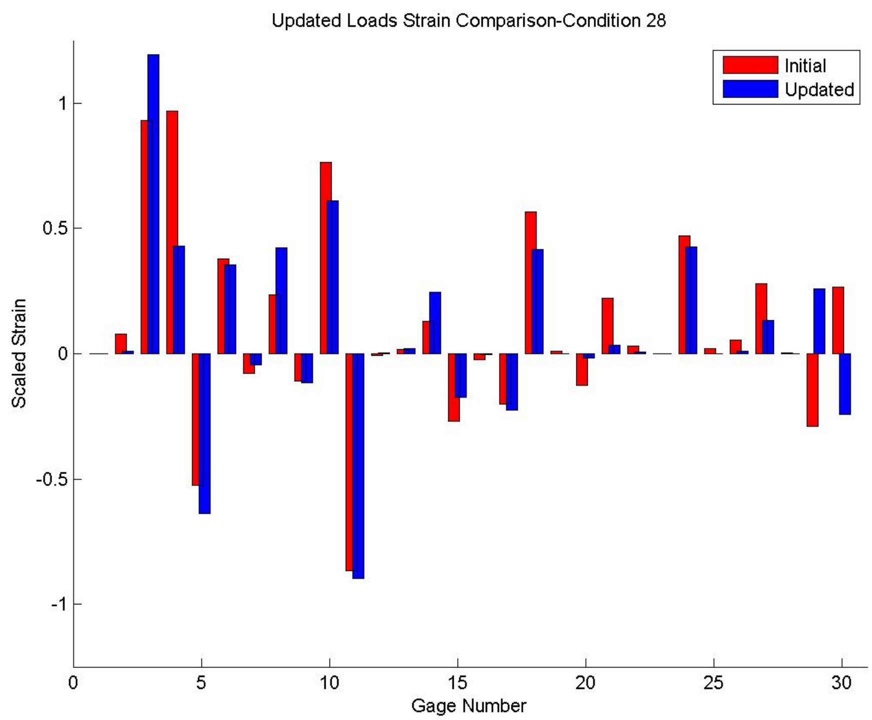
25
30


Updated Loads Strain Comparison-Condition 28
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Scaled Strain
Gage Number
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0
-0.5
-1
0
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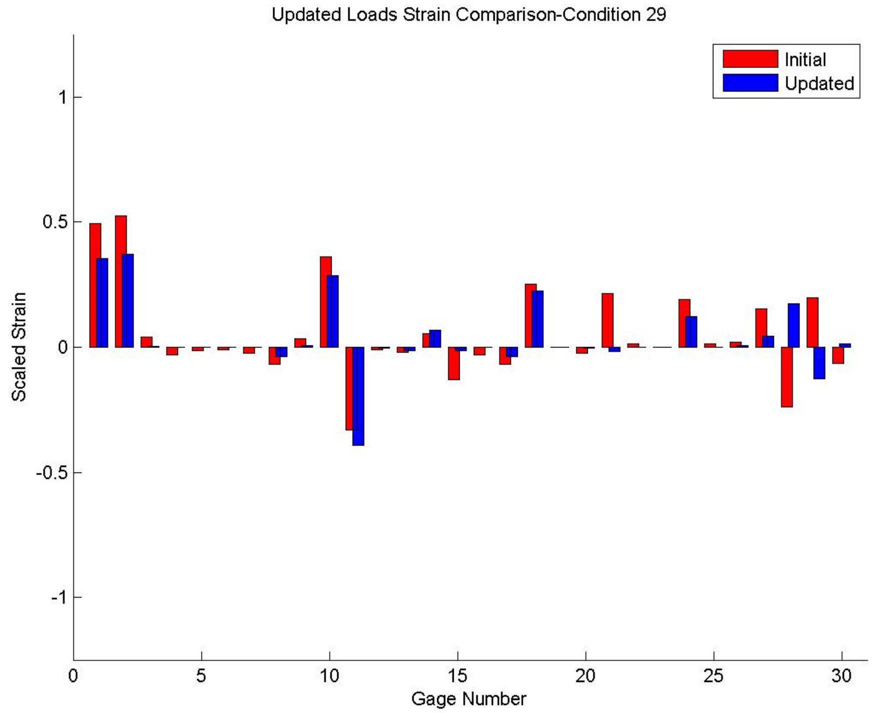
10
15
20
25
30


Updated Loads Strain Comparison-Condition 29
Initial
Updated
Scaled Strain
1
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-0.5
-1
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5
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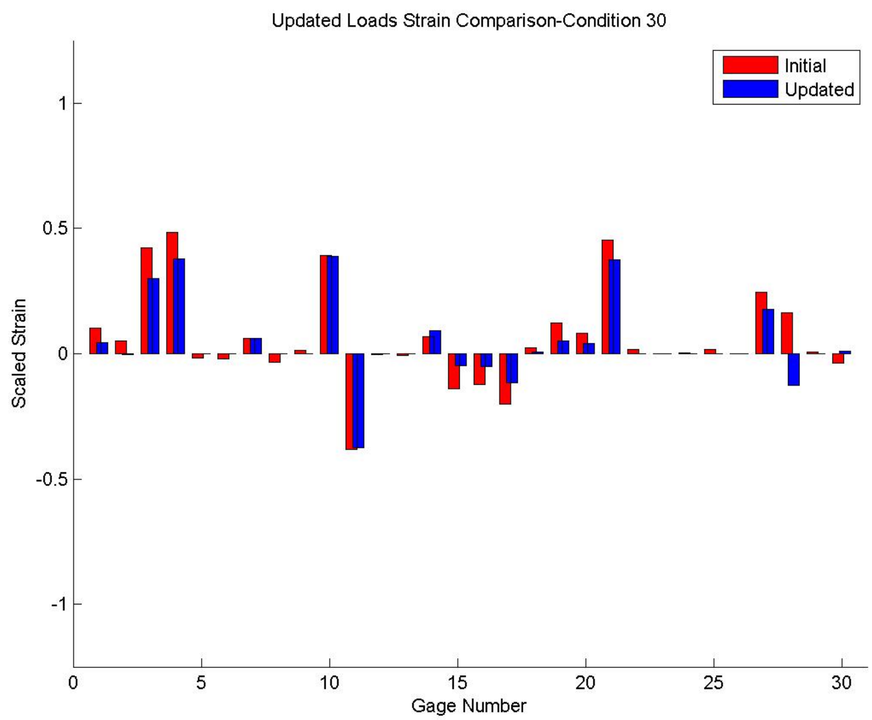
25
30
Gage Number


Updated Loads Strain Comparison-Condition 30
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-1
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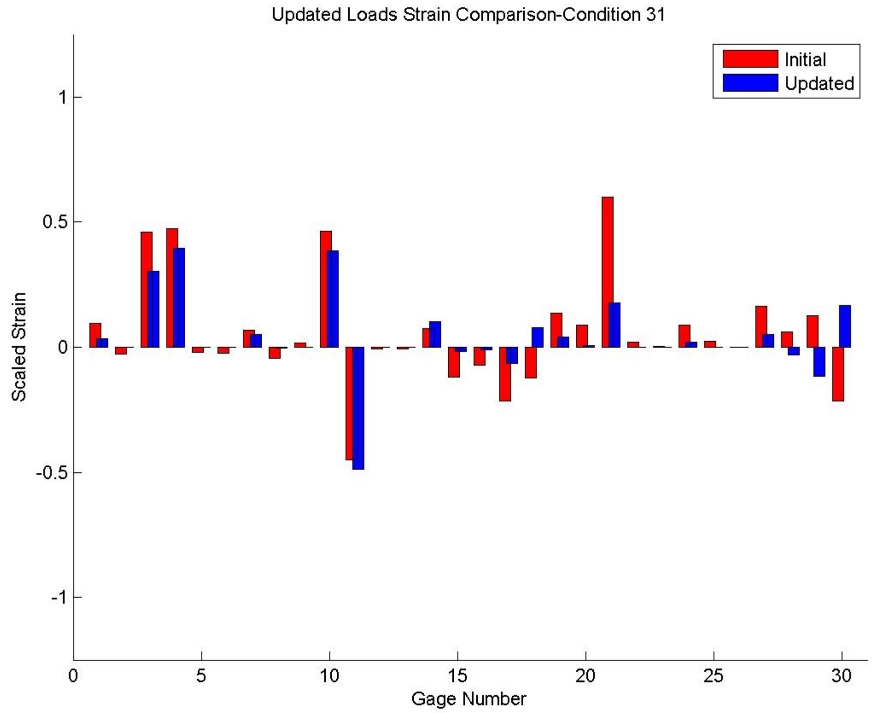
20
25
30
Gage Number


Updated Loads Strain Comparison-Condition 31
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-0.5
-1
0
5
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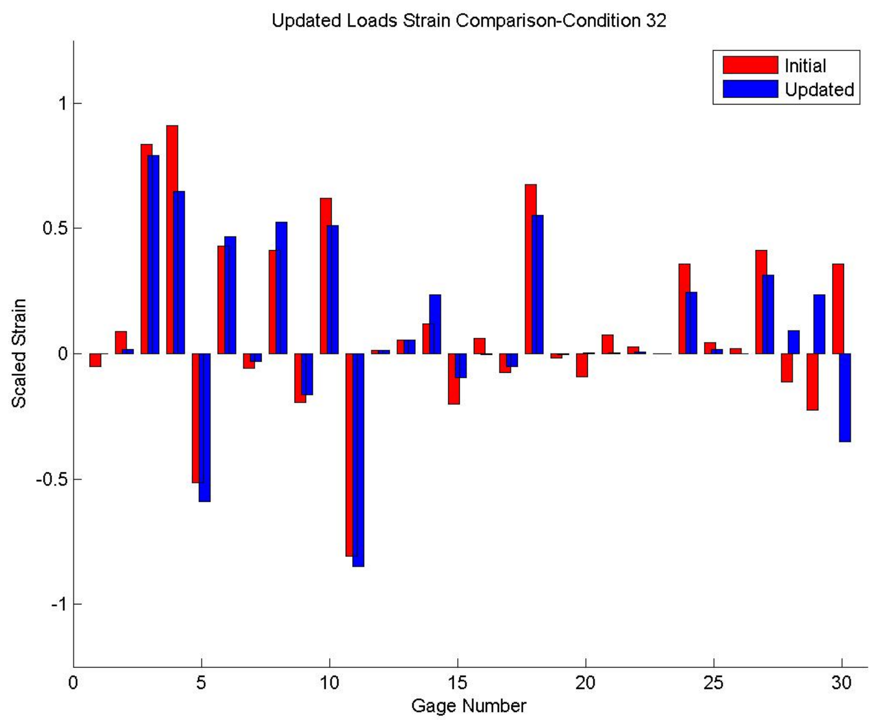
15
20
25
30
Gage Number


Updated Loads Strain Comparison-Condition 32
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-0.5
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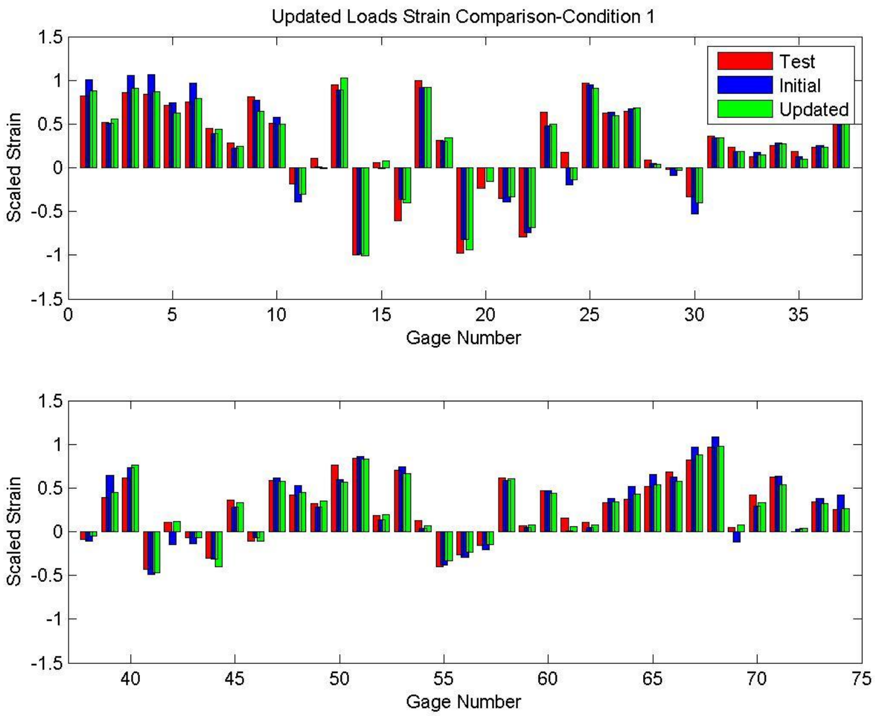
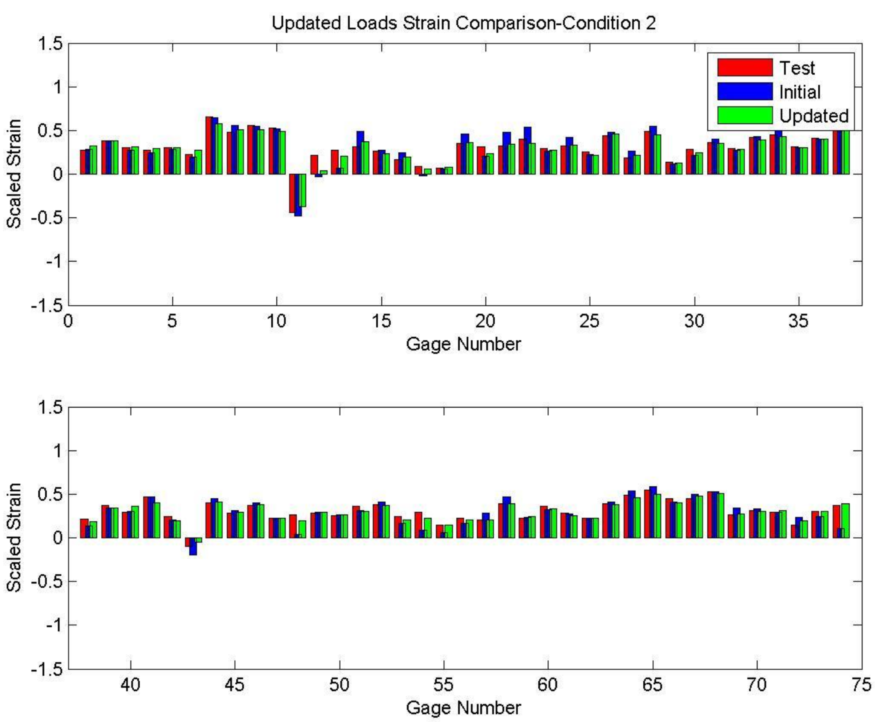
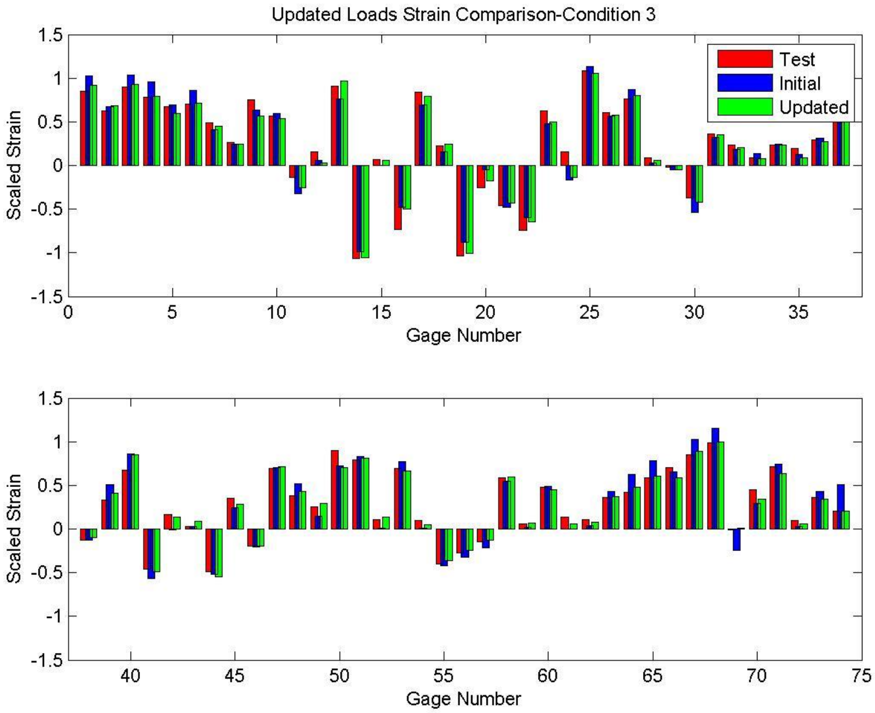
10
15
20
25
30
Gage Number

Strain Correlation Data






Updated Loads Strain Comparison-Condition 3
Test
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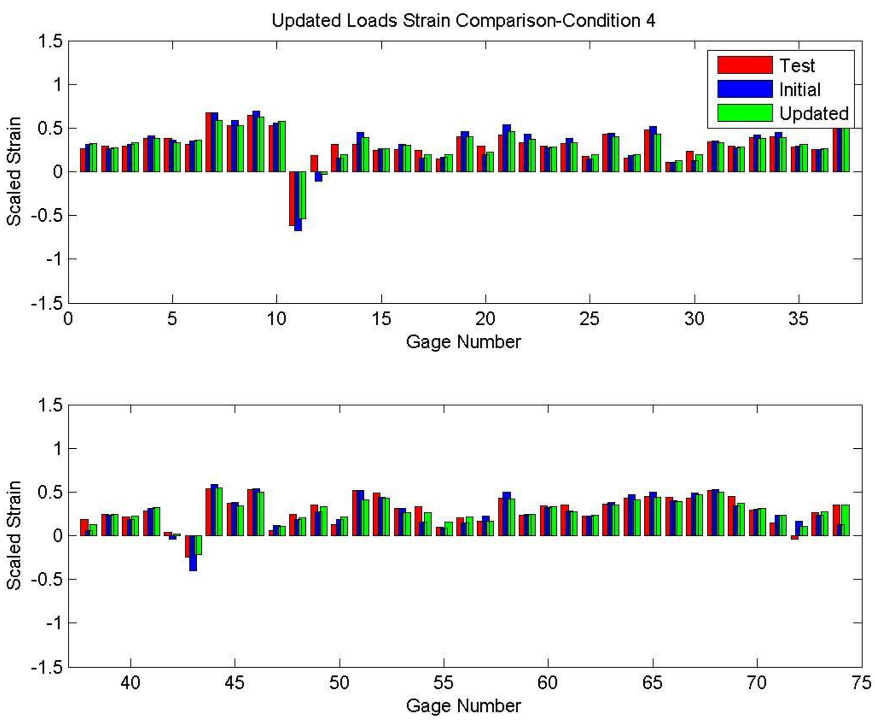
Scaled Strain
Gage Number
Scaled Strain
Gage Number


Updated Loads Strain Comparison-Condition 4
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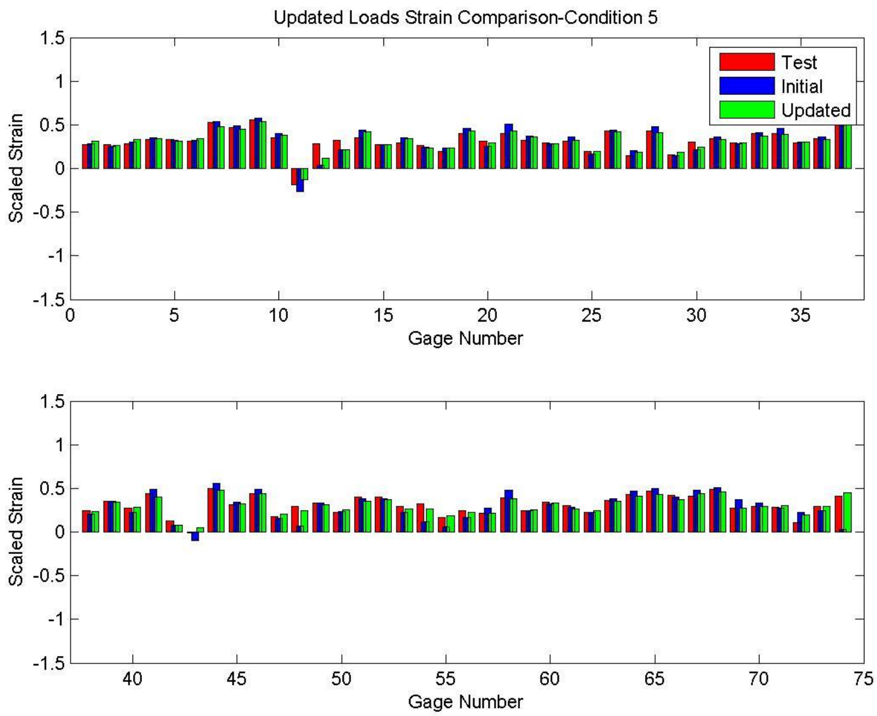
Scaled Strain
Gage Number
Scaled Strain
Gage Number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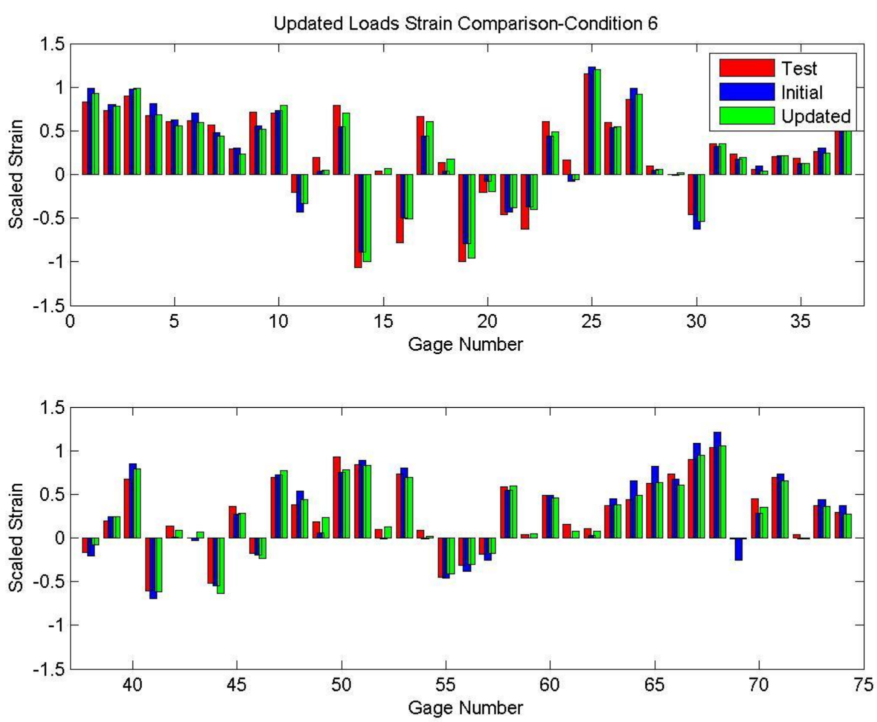
Updated
Scaled Strain
Gage Number


Updated Loads Strain Comparison-Condition 6
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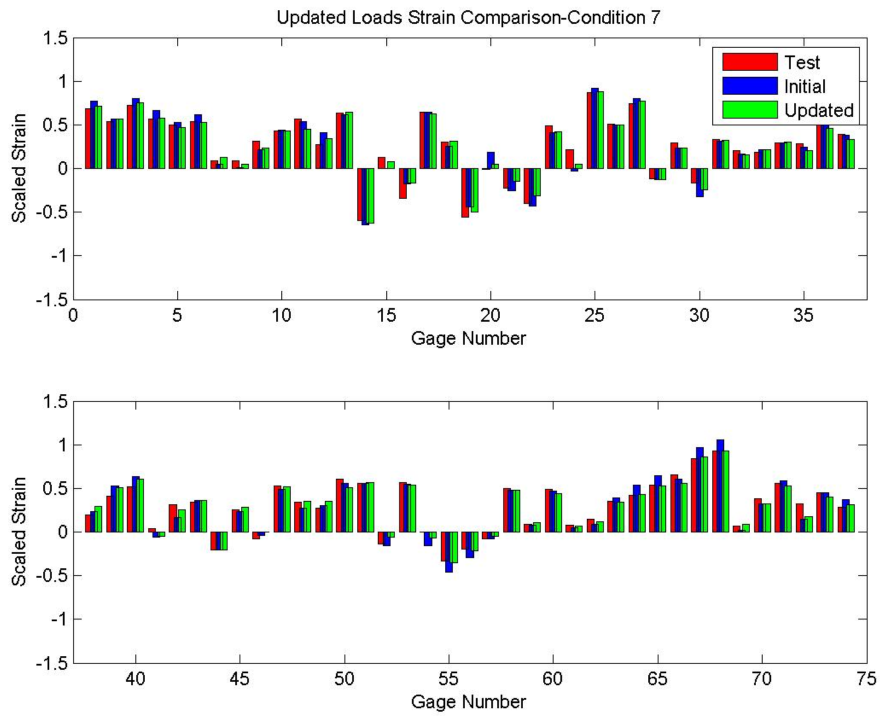
Updated
Scaled Strain
Gage Number


Updated Loads Strain Comparison-Condition 7
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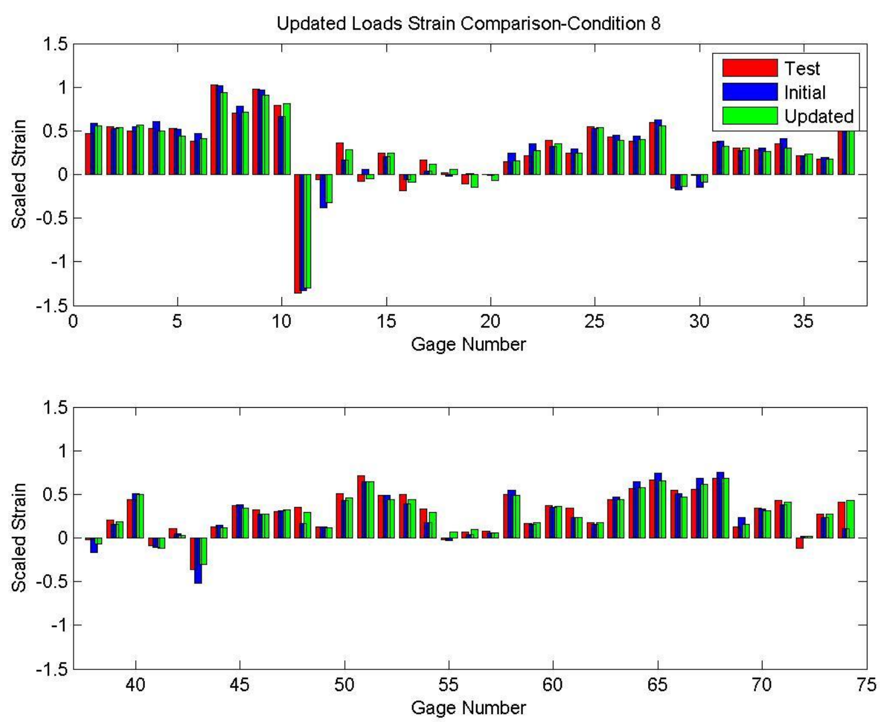
Scaled Strain
Gage Number
Scaled Strain
Gage Number


Updated Loads Strain Comparison-Condition 8
Test
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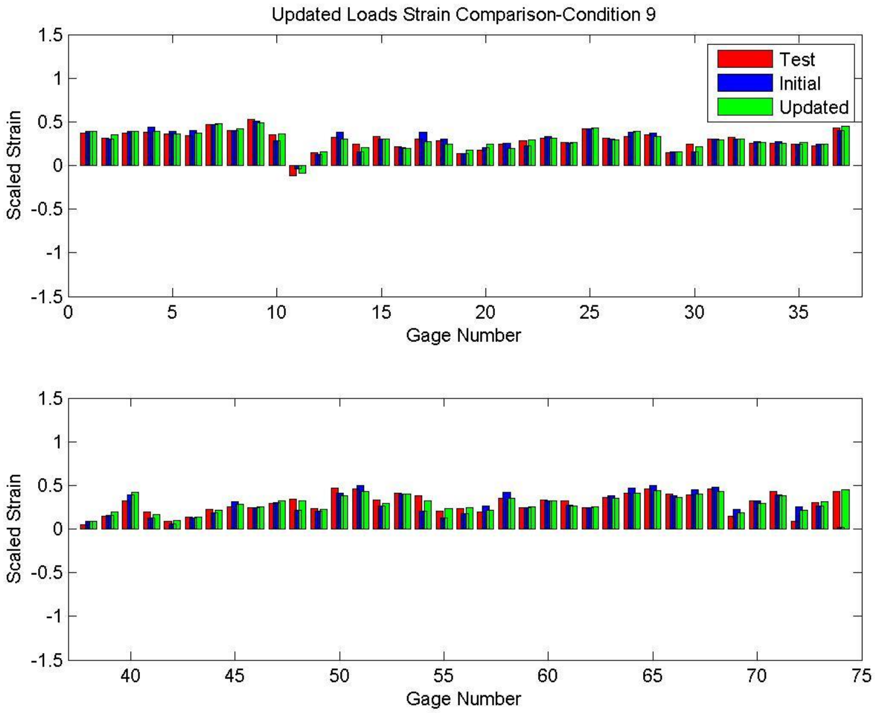
Scaled Strain
Gage Number
Scaled Strain
Gage Number


Updated Loads Strain Comparison-Condition 9
Test
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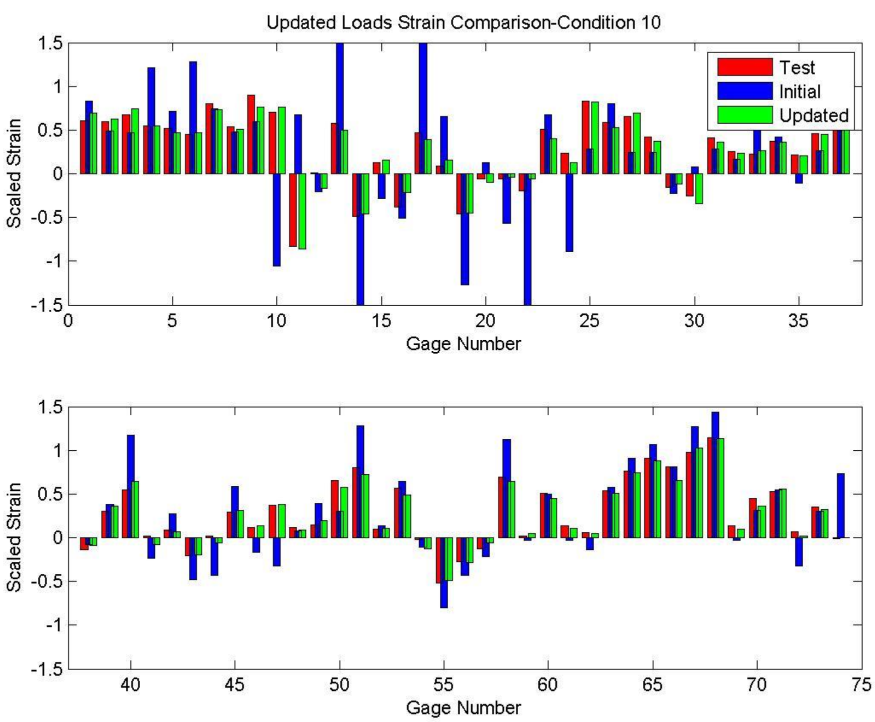
Updated
Scaled Strain
Gage Number
Scaled Strain
Gage Number


Updated Loads Strain Comparison-Condition 10
Test
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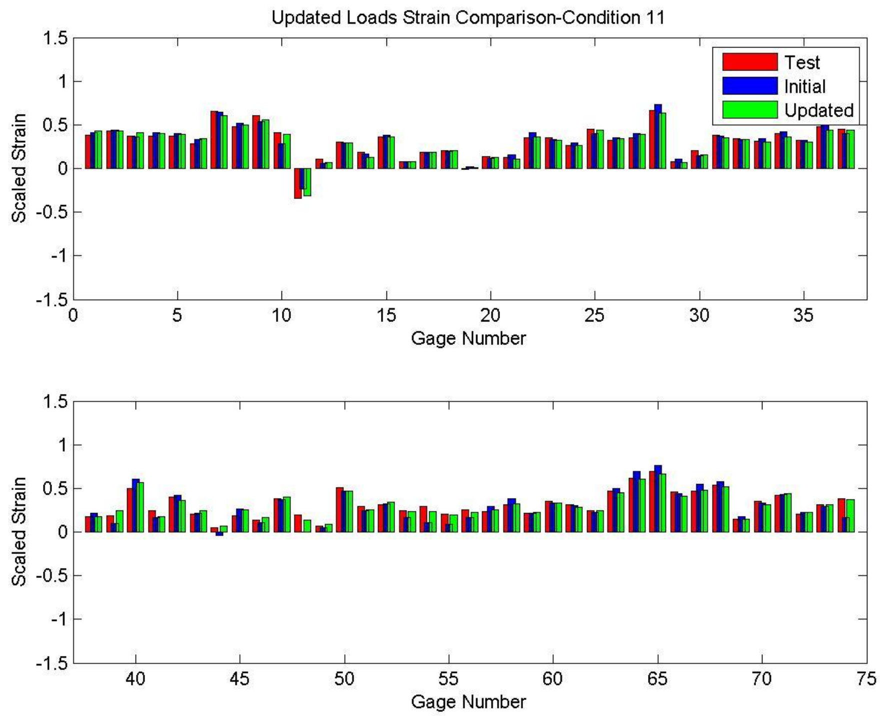
Gage Number
Scaled Strain
Gage Number


Updated Loads Strain Comparison-Condition 11
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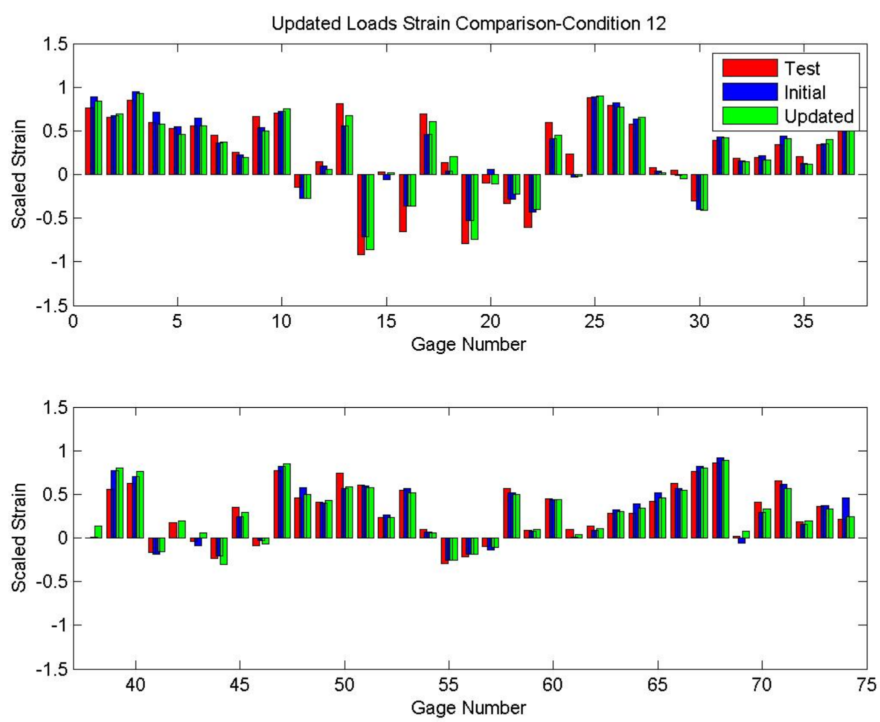
Scaled Strain
Gage Number
Scaled Strain
Gage Number


Updated Loads Strain Comparison-Condition 12
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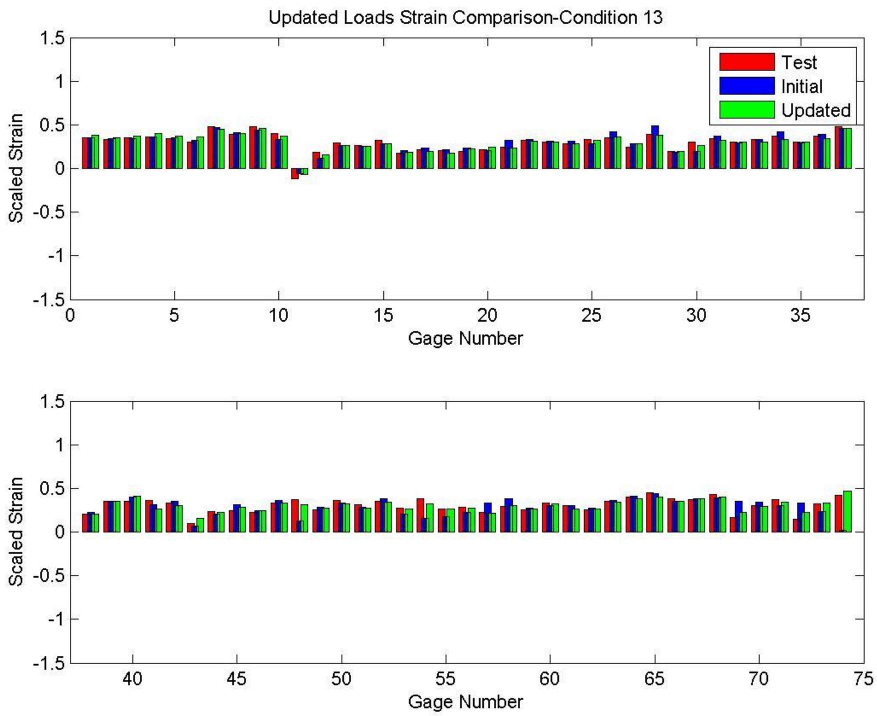
Scaled Strain
Gage Number
Scaled Strain
Gage Number


Updated Loads Strain Comparison-Condition 13
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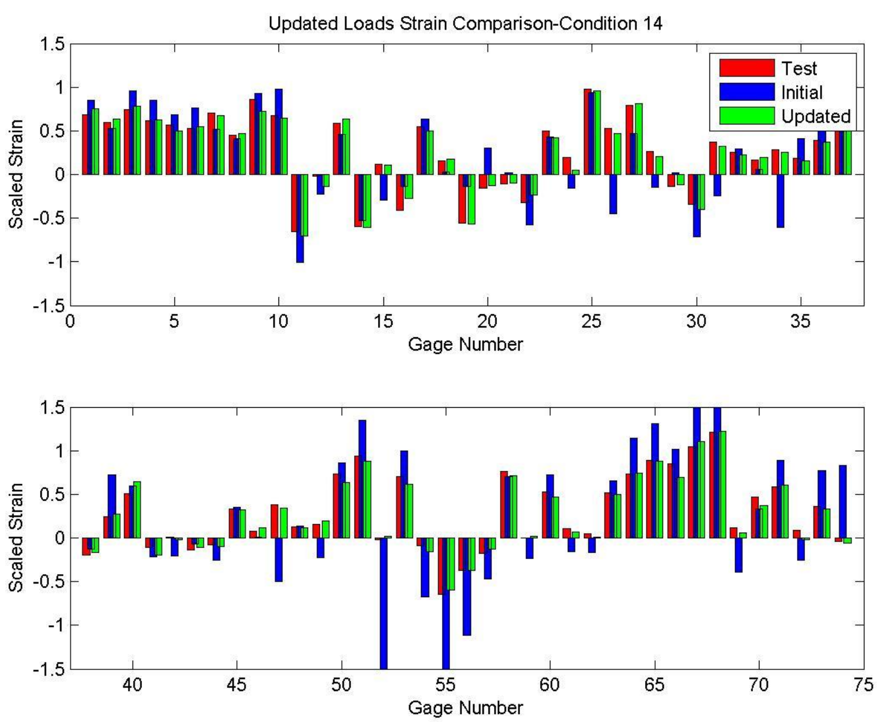
Scaled Strain
Gage Number
Scaled Strain
Gage Number


Updated Loads Strain Comparison-Condition 14
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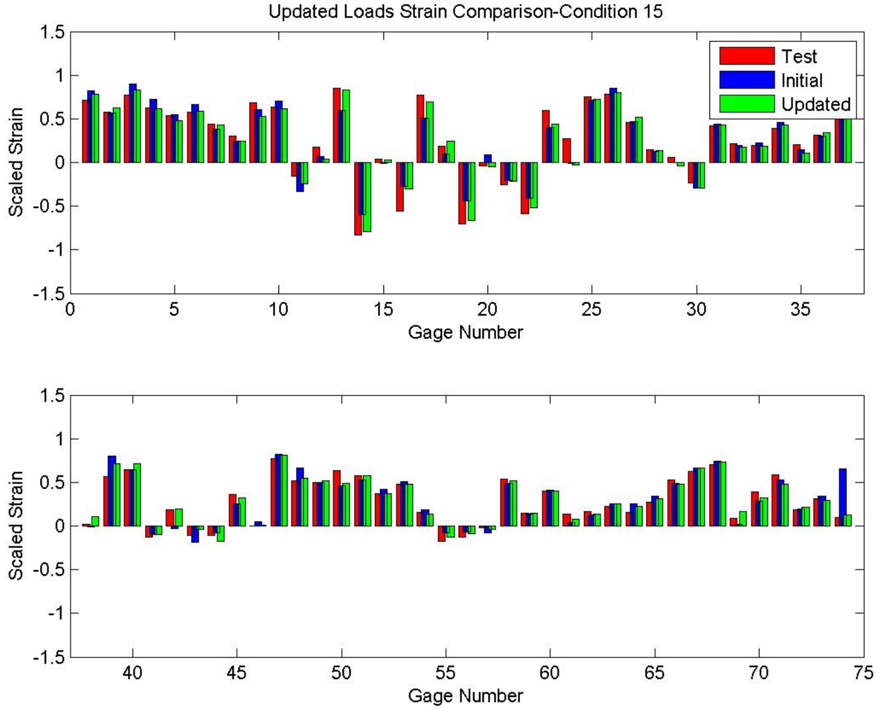
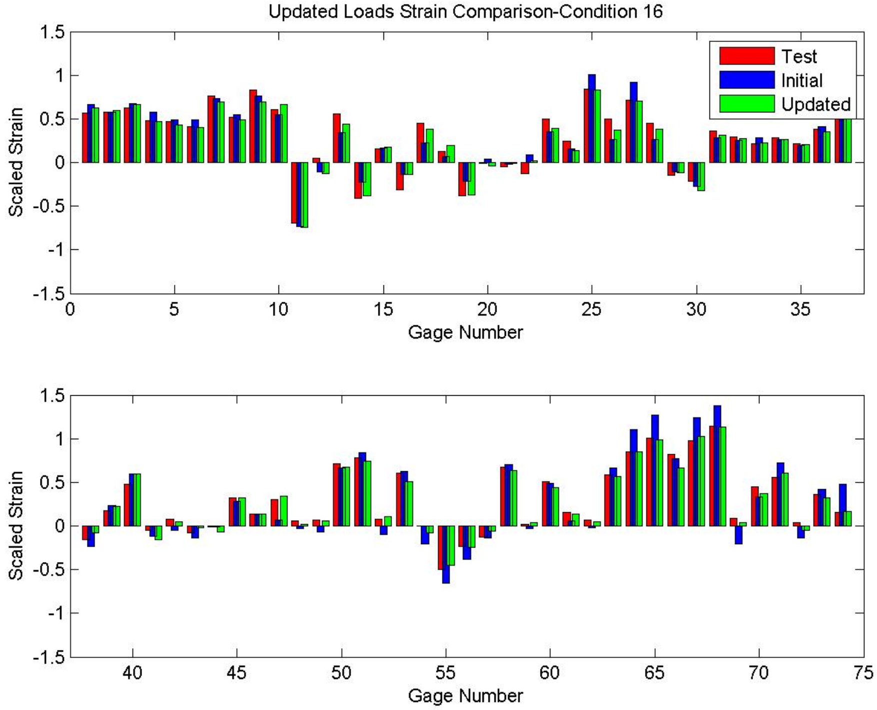
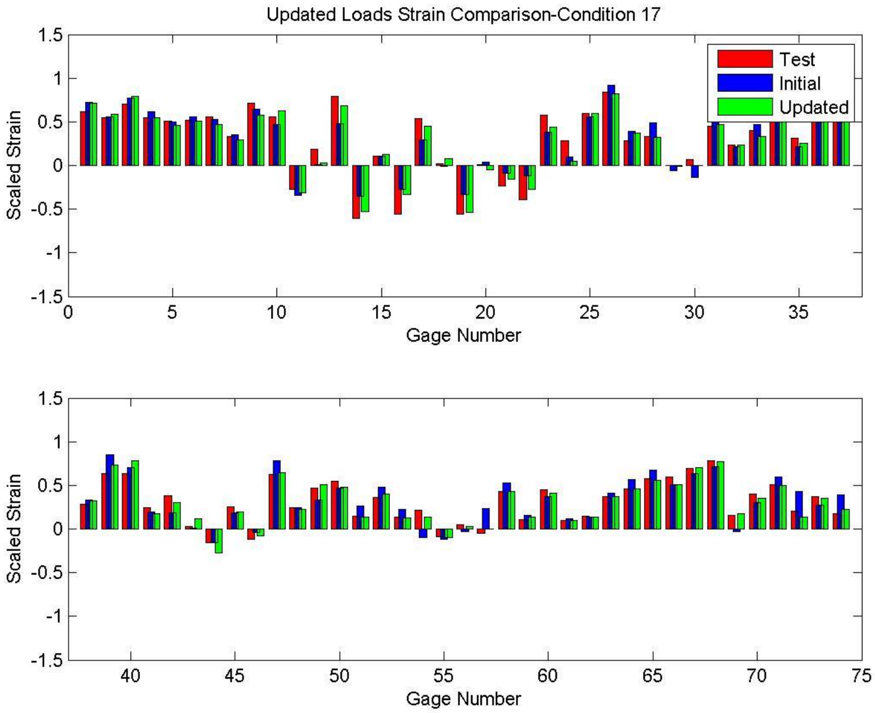
Scaled Strain
Gage Number
Scaled Strain
Gage Number

Updated Loads Strain Comparison-Condition 15



Updated Loads Strain Comparison-Condition 16




Updated Loads Strain Comparison-Condition 17
Test
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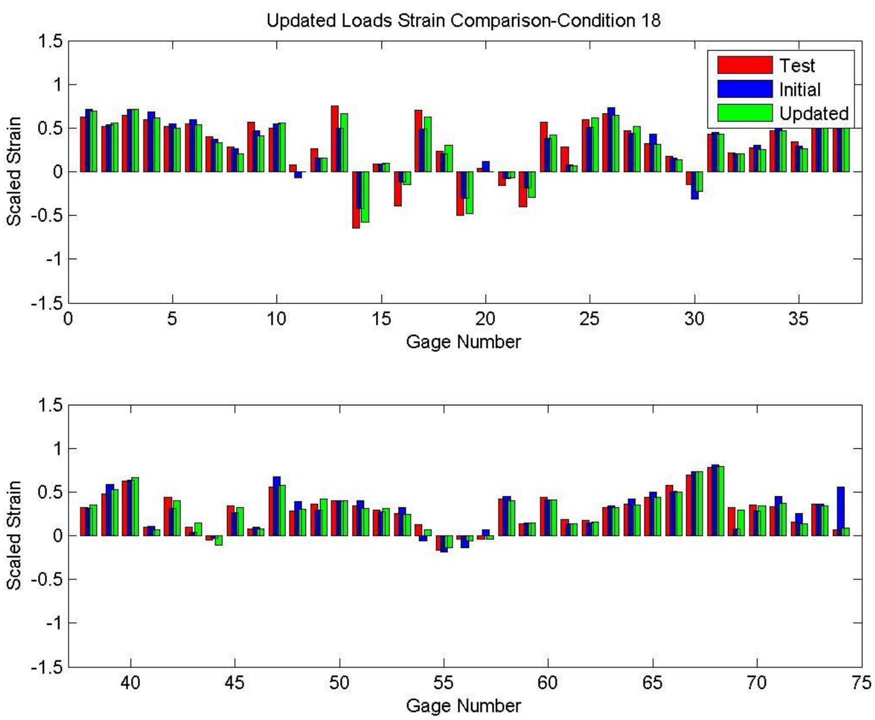
Scaled Strain
Gage Number
Scaled Strain
Gage Number


Updated Loads Strain Comparison-Condition 18
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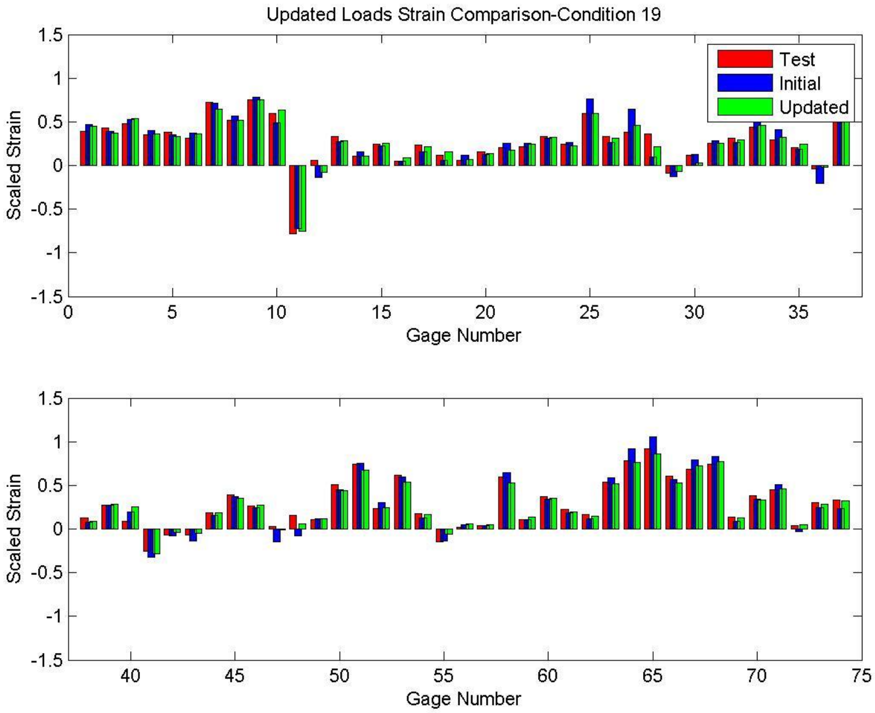
Scaled Strain
Gage Number
Scaled Strain
Gage Number


Updated Loads Strain Comparison-Condition 19
Test
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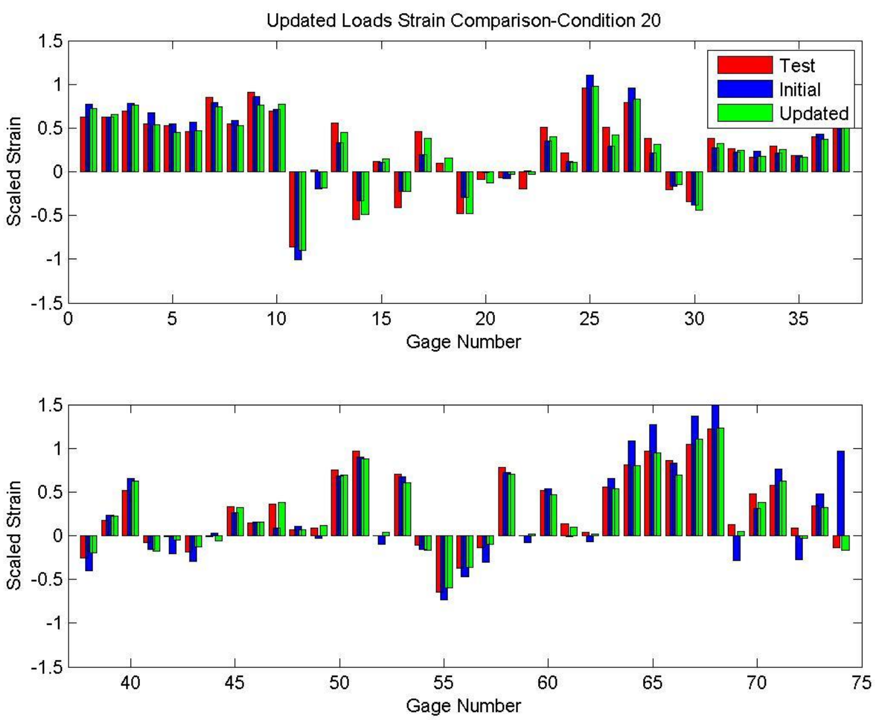
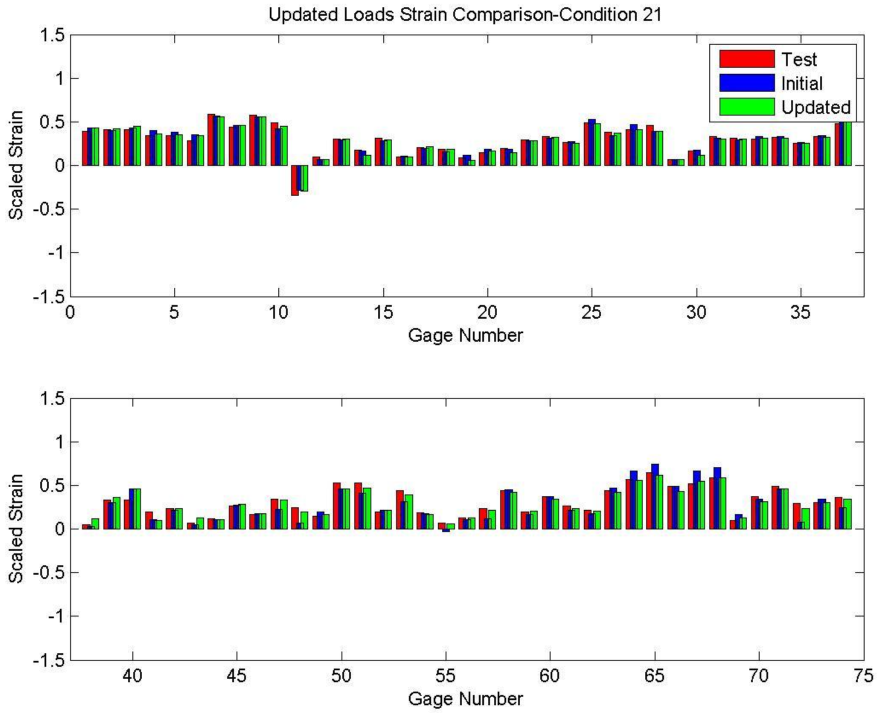
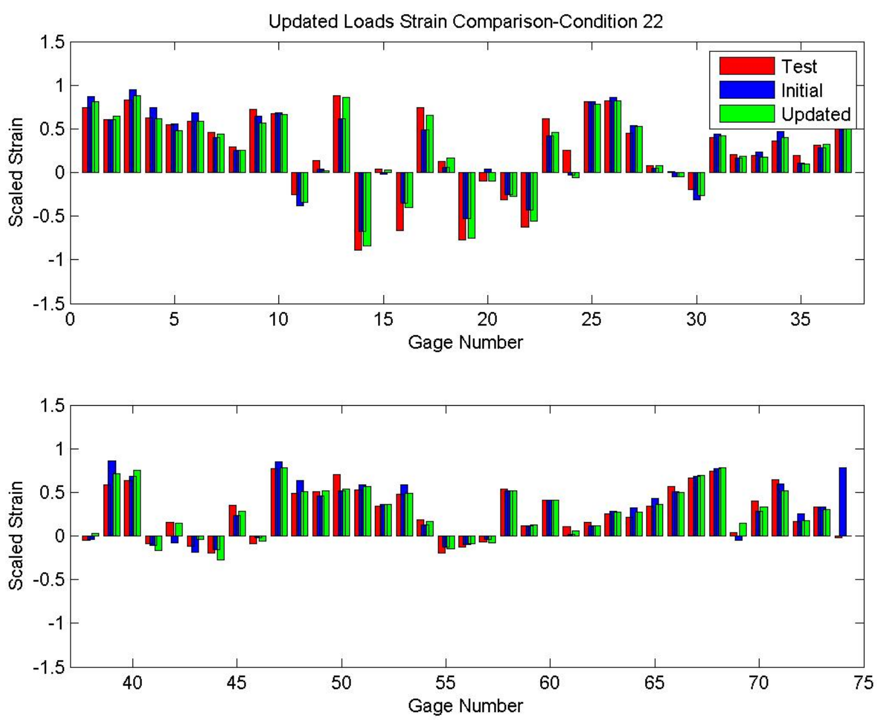
Updated
Scaled Strain
Gage Number
Scaled Strain
Gage Number


Updated Loads Strain Comparison-Condition 20
Test
Initial
Updated
Scaled Strain
Gage Number
Scaled Strain
Gage Number


Updated Loads Strain Comparison-Condition 21
Test
Initial
Updated
Scaled Strain
Gage Number


Updated Loads Strain Comparison-Condition 22
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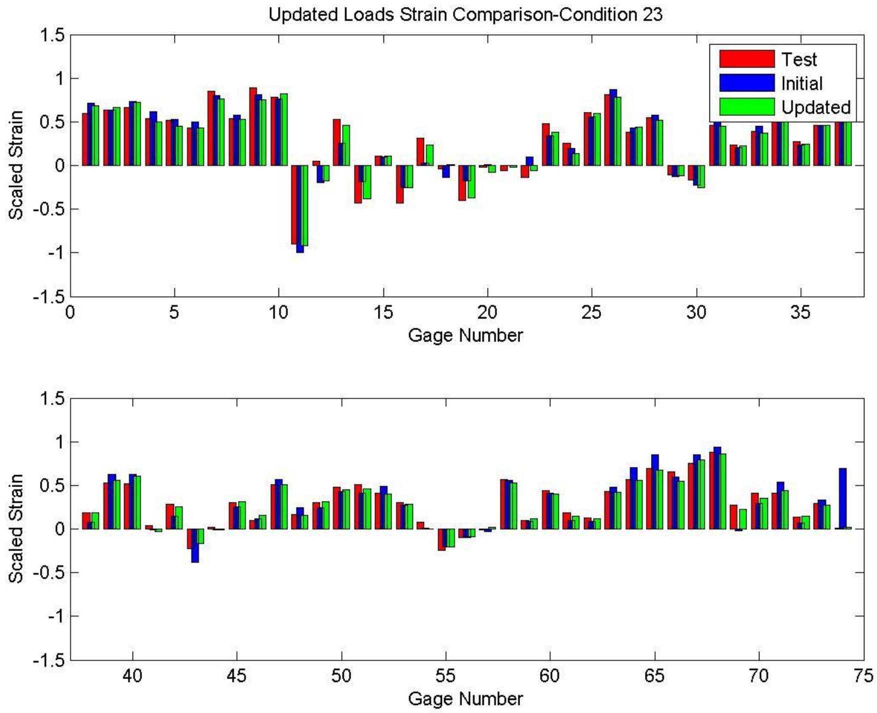
Scaled Strain
Gage Number


Updated Loads Strain Comparison-Condition 23
Test
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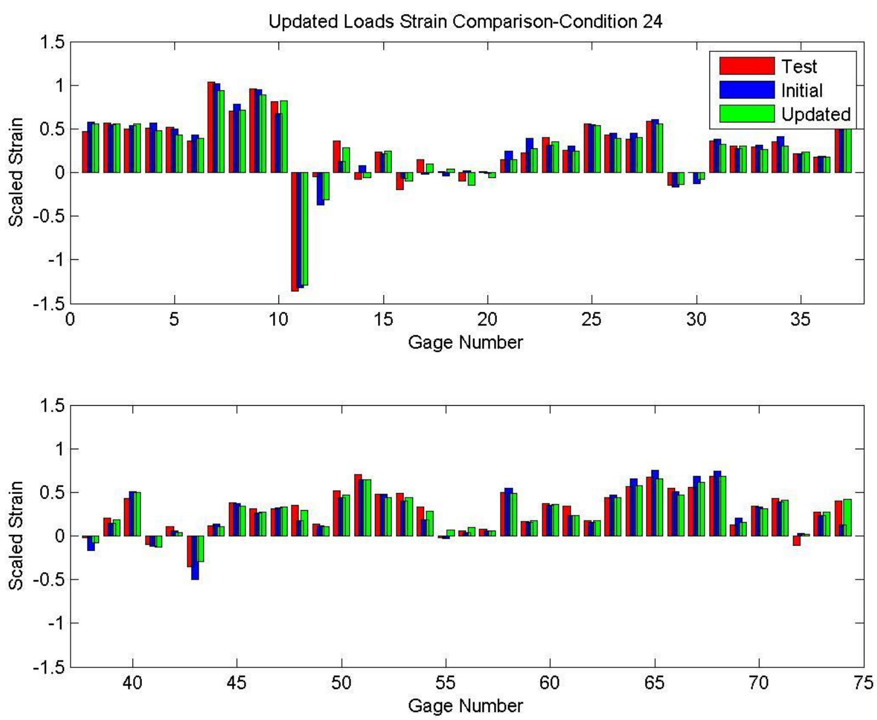
Scaled Strain
Gage Number
Scaled Strain
Gage Number


Updated Loads Strain Comparison-Condition 24
Test
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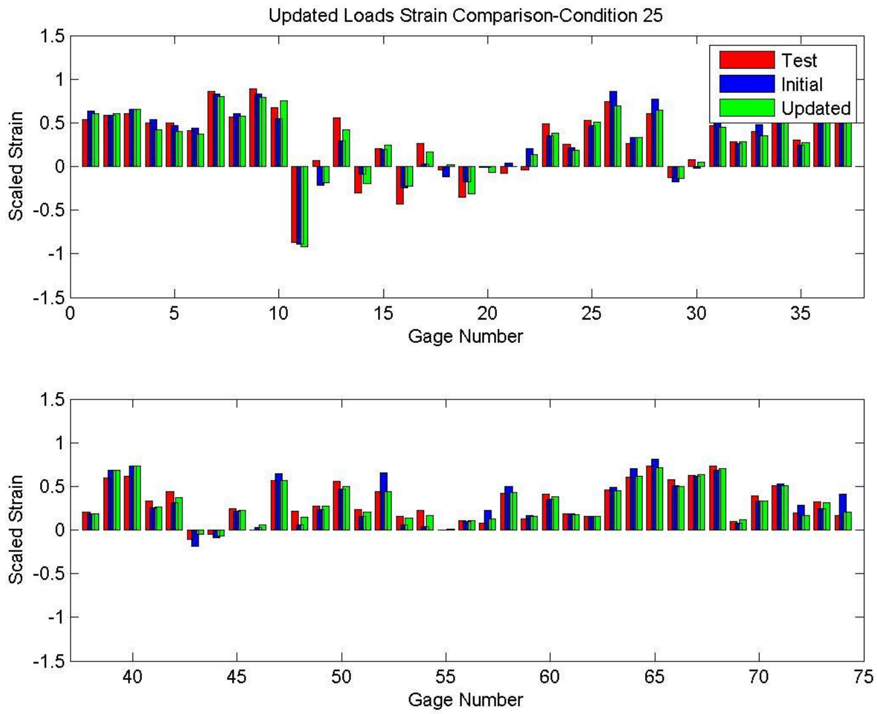
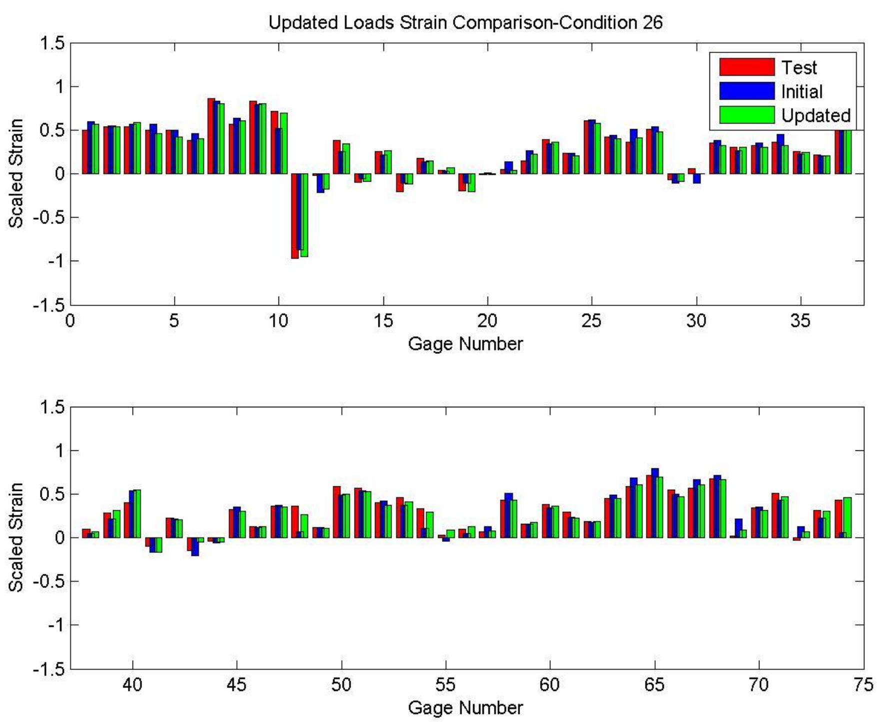
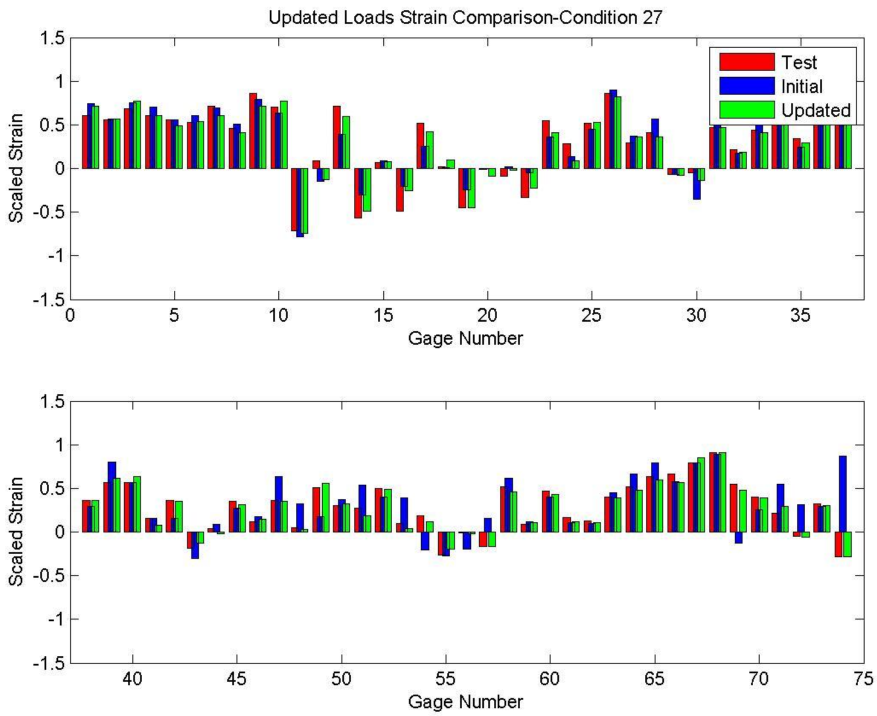
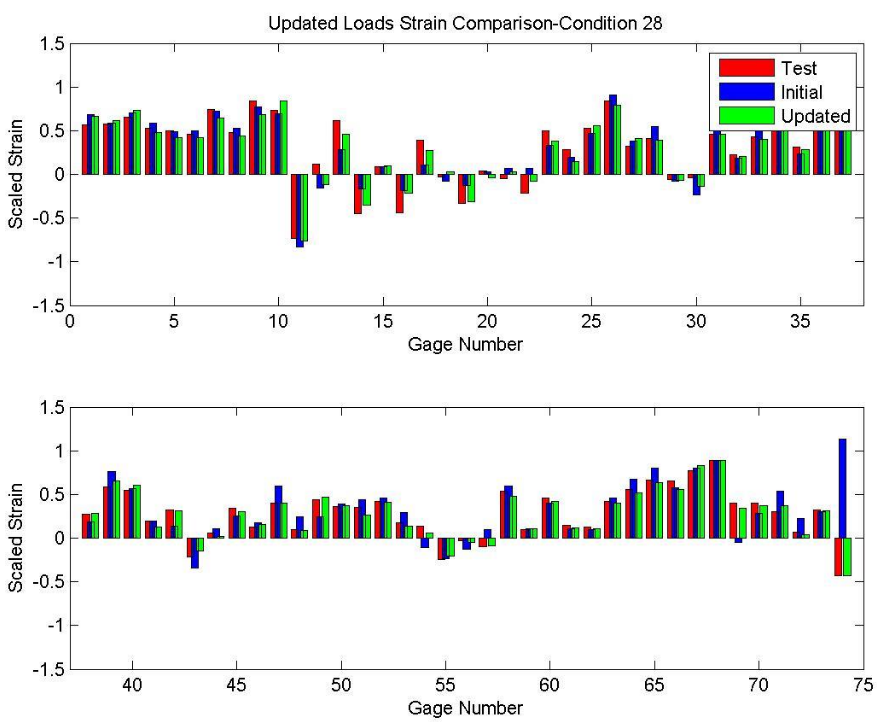
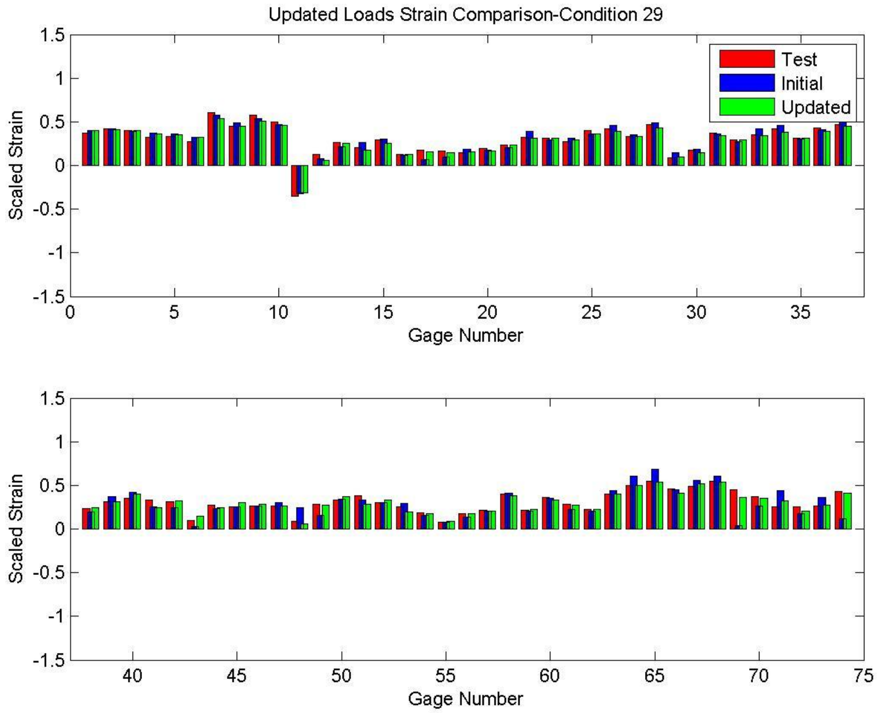
Scaled Strain
Gage Number
Scaled Strain
Gage Number

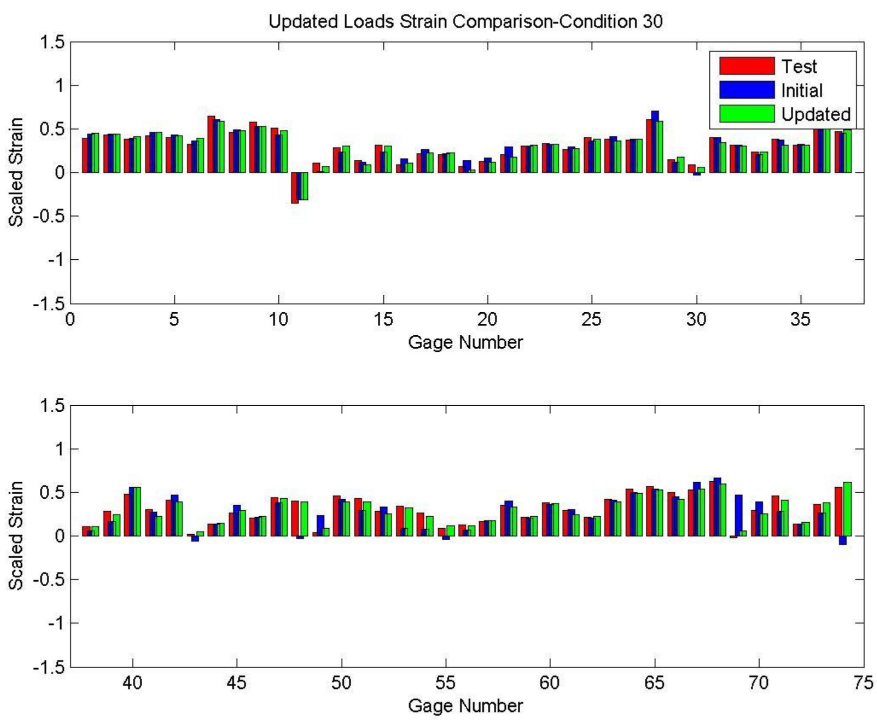

Updated Loads Strain Comparison-Condition 25
Test
Initial
Updated
Scaled Strain
Gage Number
Scaled Strain
Gage Number

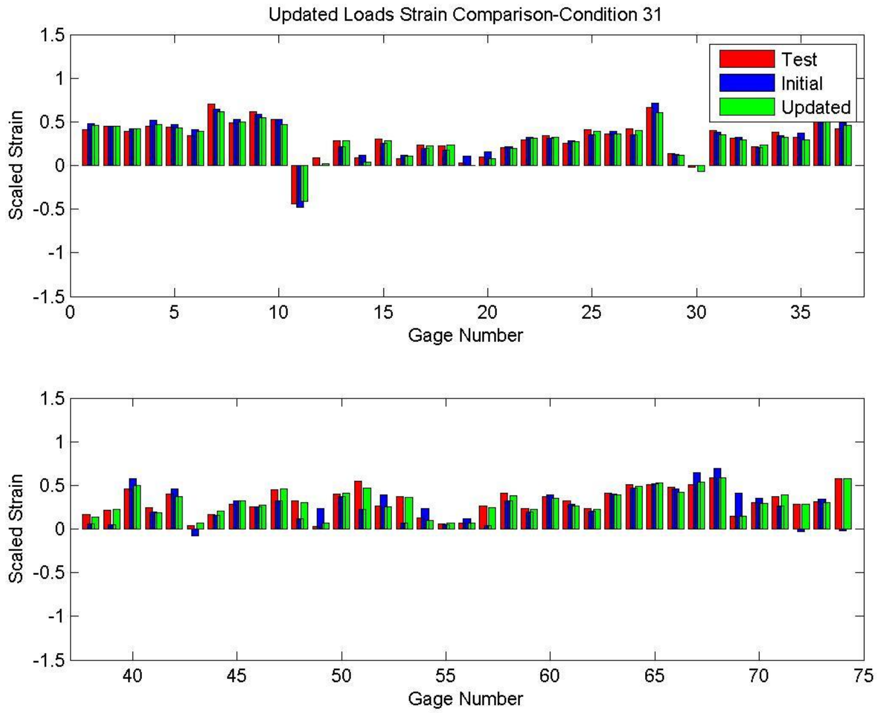

Updated Loads Strain Comparison-Condition 26
Test
Initial
Updated
Scaled Strain
Gage Number
Scaled Strain
Gage Number

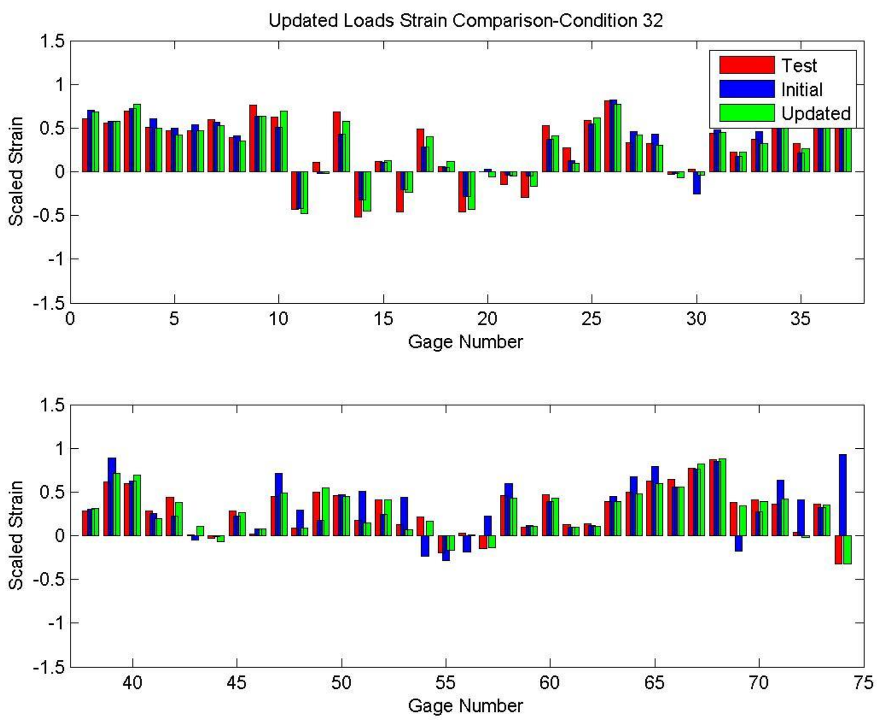

Updated Loads Strain Comparison-Condition 27
Test
Initial
Updated
Scaled Strain
Gage Number


Updated Loads Strain Comparison-Condition 28
Test
Initial
Updated
Scaled Strain
Gage Number


Updated Loads Strain Comparison-Condition 29
Test
Initial
Updated
Scaled Strain
Gage Number
Scaled Strain
Gage Number


Updated Loads Strain Comparison-Condition 30
Test
Initial
Updated
Scaled Strain
Gage Number
Scaled Strain
Gage Number


Updated Loads Strain Comparison-Condition 31
Test
Initial
Updated
Scaled Strain
Gage Number


Updated Loads Strain Comparison-Condition 32
Test
Initial
Updated
Scaled Strain
Gage Number

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