



Management Science

MANAGEMENT SCIENCE



Publication details, including instructions for authors and subscription information:
<http://pubsonline.informs.org>

Price-Matching Guarantees with Endogenous Consumer Search

Juncai Jiang, Nanda Kumar, Brian T. Ratchford

To cite this article:

Juncai Jiang, Nanda Kumar, Brian T. Ratchford (2017) Price-Matching Guarantees with Endogenous Consumer Search. Management Science 63(10):3489-3513. <https://doi.org/10.1287/mnsc.2016.2513>

Full terms and conditions of use: <http://pubsonline.informs.org/page/terms-and-conditions>

This article may be used only for the purposes of research, teaching, and/or private study. Commercial use or systematic downloading (by robots or other automatic processes) is prohibited without explicit Publisher approval, unless otherwise noted. For more information, contact permissions@informs.org.

The Publisher does not warrant or guarantee the article's accuracy, completeness, merchantability, fitness for a particular purpose, or non-infringement. Descriptions of, or references to, products or publications, or inclusion of an advertisement in this article, neither constitutes nor implies a guarantee, endorsement, or support of claims made of that product, publication, or service.

Copyright © 2016, INFORMS

Please scroll down for article—it is on subsequent pages



INFORMS is the largest professional society in the world for professionals in the fields of operations research, management science, and analytics.

For more information on INFORMS, its publications, membership, or meetings visit <http://www.informs.org>

Price-Matching Guarantees with Endogenous Consumer Search

Juncai Jiang,^a Nanda Kumar,^b Brian T. Ratchford^b

^a Virginia Tech, Blacksburg, Virginia 24061; ^b University of Texas at Dallas, Richardson, Texas 75080

Contact: jcjiang@vt.edu (JJ); nkumar@utdallas.edu (NK); btr051000@utdallas.edu (BTR)

Received: April 10, 2014

Revised: April 12, 2015; November 8, 2015

Accepted: December 29, 2015

Published Online in Articles in Advance:
August 26, 2016

<https://doi.org/10.1287/mnsc.2016.2513>

Copyright: © 2016 INFORMS

Abstract. Price-matching guarantees (PMGs) are offered in a wide array of product categories in retail markets. PMGs offer consumers the assurance that, should they find a lower price elsewhere within a specified period after purchase the retailer will match that price and refund the price difference. The goal of this study is to explain the following stylized facts: (1) many retailers that operate both online and offline implement PMG offline but not online; (2) the practices of PMG vary considerably across retail categories; and (3) some retailers launch specialized websites that automatically check competitors' prices for consumers after purchase. To this end, we build a sequential search model that endogenizes consumers' pre- and postpurchase search decisions. We find that PMG expands retail demand but intensifies price competition on two dimensions. PMG drives retailers to offer deeper promotions because it increases the overall extent of consumer search, which we call the primary competition-intensifying effect. We also find a new secondary competition-intensifying effect, which results from endogenous consumer search. As deeper promotions incentivize consumers to continue price search, retailers are forced to lower the "regular" price to deter consumers from searching. The strength of the secondary competition-intensifying effect increases with the ratio of product valuation to search cost, which explains the variation in PMG practices online versus offline and across retail categories. We show that an asymmetric equilibrium exists such that one retailer offers PMG while the other does not. In this equilibrium, the PMG retailer may benefit from launching a price check website to facilitate consumers' postpurchase search.

History: Accepted by J. Miguel Villas-Boas, marketing.

Keywords: price-matching guarantees • sequential consumer search • reservation price rule • mixed pricing strategy • price-beating guarantees

1. Introduction

Price-matching guarantees (PMGs) offer consumers the assurance that, if they find a lower price elsewhere within a grace period after purchase, the retailer will match that price. For example, Walmart promotes its Christmas Price Guarantee by advertising that "Find a lower advertised price on a gift after you've bought it? No problem. We'll give you the price difference on any identical, available product in a local competitor's store" (Russell 2011). PMGs are pervasive in a wide array of product categories ranging from durable goods to supermarket perishables and from travel agent services to financial services. In markets with widespread adoption of PMGs, consumers may possess imperfect price information even at the time of purchase. Buying with a PMG gives consumers an option to benefit from price search after purchase in the hope of finding a lower price and exercising PMG. In an effort to explore the competitive implications of this option, we develop a model in which some consumers have high search costs at the time of purchase, but can search when search costs are lower at some future date, if they take advantage of a PMG. Our model offers an explanation for the stylized facts presented below.

First, retailers like Walmart, Target, Cabela's, Home Depot, and Kohl's offer PMGs in their physical stores but not on their websites. An astute reader may argue that this could result from differences in the competitive structure of the online and offline environment. Small, low-cost online retailers may be able to afford to charge substantially lower prices for select products, which erodes the profitability of offering PMGs online. In contrast, offline retailers are more predictable and have similar cost structures to one another. However, online retailers can define the "qualified" competitors in their price-matching policies to exclude competition from these low-cost online retailers. For example, Office Depot's online store only matches the prices of "Staples.com, OfficeMax.com, BestBuy.com, Reliable.com, Quill.com, Sears.com, Target.com, KMart.com, Costco.com, or SamsClub.com."¹ Thus, it appears that differences in competitive structure cannot fully explain retailers' decisions to offer PMG offline but not online. Our study sheds light on factors that may explain this phenomenon.

Second, there is ample evidence that PMG practices vary considerably across retail categories. We collected a sample of 150 online retailers from the Top 500

Table 1. PMG Practices Across Categories

Category	No. of PMG retailers	No. of total retailers	% of PMG retailers
Office supplies	5	5	100
Hardware/Home improvement	3	6	50
Automobile/ Accessories	1	3	33
Specialty/Nonapparel	2	6	33
Health/Beauty	3	10	30
Housewares/Home furnishings	2	7	29
Mass merchant	4	19	21
Sporting goods	1	5	20
Computer/Electronics	3	17	12
Apparel/ Accessories	4	38	11
Others ^a	0	34	0
Total	28	150	19

^aOthers include books/music/videos, flowers/gifts, toys/hobbies, food/drug, and jewelry.

List[®] by *Internet Retailer* in 2010. *Internet Retailer* classifies retailers into 15 exclusive categories. In Table 1 we report the fraction of retailers in each category that offer PMGs.² In the office supplies category, all five online retailers—Staples, Office Depot, Office Max, Vistaprint, Shoplet—offer PMGs. However, none of the retailers in the jewelry category offers PMG. In the computer/electronics category, ABT, Ritz, and Crutchfield offer PMGs while others do not. Prior research argues that the variation in PMG practices across categories can be explained by the difference in the size of consumer segments with different search behaviors (Chen et al. 2001). However, this explanation seems less plausible in the online setting where consumer segments may not vary much across product categories. In this study we develop a theory that can explain the market phenomenon in Table 1 even when the sizes of different consumer segments are the same across categories.

Third, Asda, Walmart’s operation and the second largest supermarket chain in the United Kingdom, supplements its PMG with a price check website (<http://www.asdapriceguarantee.co.uk/>) along with a mobile app that facilitates consumers’ postpurchase search. This website works as follows. After grocery shopping, consumers enter the receipt details on the price check website. This website then automatically checks the prices of purchased groceries from competing retailers including Tesco, Sainsbury’s, Morrisons, and Waitrose. If Asda’s price of comparable groceries is not 10% lower relative to the competitors’ prices, consumers receive the price difference in the form of a voucher that can be used in the future shopping trips. It was reported that 800,000 consumers used this price check website in the first two months of 2011 (Asda 2011). Note that this type of price check website is useful only after

purchase and can greatly facilitate consumers’ postpurchase search. However, it is not clear why Asda would invest in such a website. Conventional wisdom might suggest that this type of website may induce consumers to search more actively after purchase, thereby intensifying price competition. As a result, Asda could suffer significant losses by hosting such website. Our paper provides a rationale for why it may be profitable for Asda to do so.

Motivated by these stylized facts, we seek to answer the following questions facing retail managers: (1) In a given product category, under what market conditions should retailers offer PMGs? For example, are we more likely to observe PMGs in markets with high search costs (e.g., offline) or low search costs (e.g., online)? (2) Why is it that all retailers in some product categories offer PMGs but few or none offer PMGs in other categories? For example, are we more likely to observe PMGs in categories with high or low product valuation (e.g., jewelry versus electronics versus office goods)? (3) Do retailers have the incentive to invest in technologies to facilitate consumers’ postpurchase search? If so, under what market conditions is it profitable for retailers to do so?

1.1. Overview of Model Setup and Major Results

To address these issues, we consider a symmetric duopoly in which retailers decide whether or not to offer PMG. Conditional on this decision, retailers simultaneously set prices. Consumers who are heterogeneous in their pre- and postpurchase search costs then search sequentially for price information. We apply the “Pandora’s rule” in Weitzman (1979) to construct consumers’ optimal search rule, which in turn determines both their equilibrium shopping decisions and retailers’ equilibrium pricing and price-matching strategies.

We examine a model in which consumers exhibit one of three search and purchase behaviors. We label these three consumer segments as *shoppers*, *nonshoppers*, and *refundees*. Shoppers are consumers who have low pre-purchase search cost. They obtain price information before purchase and directly purchase from the low-price retailer independent of the retailers’ decision to offer PMG. Nonshoppers are consumers with high pre- and postpurchase search costs. In equilibrium, these consumers do not search and therefore purchase from the retailer with the lower expected price. Refundees have high prepurchase search costs but low postpurchase search costs. As explained later, it is realistic to expect that search costs may vary over time and that consumers may have lower postpurchase search costs. For example, consumers may have a high waiting cost if an appliance breaks down unexpectedly. They may have to purchase without searching extensively. At a later time (within the grace period), they may have time to search and could benefit from PMG. The shopping behavior of refundees depends on whether or

not retailers offer PMG. In the absence of PMG, refundees act like nonshoppers. In the presence of PMG, refundees skip prepurchase search, patronize the PMG retailer(s), conduct postpurchase search, and ask for a price match if they find a lower price. We find that PMG has a demand-expansion effect: PMG attracts the refundee segment at the expense of some shoppers and nonshoppers, but the net effect of PMG on retail demand is positive. Furthermore, the demand-expansion effect increases with the size of refundees.

With competition for these three consumer segments, retailers' pricing will be in mixed strategies; that is, each retailer charges a range of prices with varying probabilities. We show that PMG intensifies price competition on two dimensions. On the one hand, PMG provides refundees with the opportunity to search after purchase and obtain a lower price if available. Consequently, both retailers are compelled to offer deeper promotions (retailers charge high prices with smaller probability and low prices with greater probability) because PMG increases the overall extent of consumer search. We call this the *primary* competition-intensifying effect. Because consumer search is endogenous in our model, the primary competition-intensifying effect alters consumers' search and purchase behaviors such that deeper promotions provide consumers greater incentives to search elsewhere for a lower price. To counter this growing incentive of consumer search, both retailers may be forced to lower their "regular" price (the common upper bound of both retailers' price distributions is lowered). We label this the *secondary* competition-intensifying effect. While the primary competition-intensifying effect increases with the size of shoppers, the strength of the secondary competition-intensifying effect increases with the ratio of product valuation to search cost.

Retailers' equilibrium price-matching strategies depend on the relative importance of the demand-expansion effect and the (primary and secondary) competition-intensifying effects. When the size of shoppers is large, PMG is not offered in equilibrium since it is unprofitable to induce even more search by offering PMG. Conversely, when the size of refundees is large, the demand-expansion effect is strong, and both retailers offering PMG is an equilibrium. Finally, when the sizes of shoppers and refundees are both small, the equilibrium outcome depends on the ratio of product valuation to search cost. When the ratio of product valuation to search cost is high, the secondary competition-intensifying effect is strong, and further intensifying competition by offering PMG is not profitable. When the ratio of product valuation to search cost is low, both retailers offering PMG is an equilibrium. When the ratio of product valuation to search cost is intermediate, there exists an asymmetric equilibrium in which one retailer offers PMG while the

other does not. In the asymmetric equilibrium, the PMG retailer always charges an average price, which is weakly greater than that of the non-PMG retailer.

Finally, we relax our assumption of price matching and expand the strategy space to allow the use of price-beating guarantees (PBGs). Through PBGs, retailers promise to beat any competitor's lower price by a certain percentage, which we call the refund depth. We find that our results are still robust in the PBG case. We show that both average retail prices and retail profits are decreasing in the refund depth. The parameter space under which PBG is profitable shrinks as the refund depth increases. In addition, PBGs are less profitable than PMGs. This finding is consistent with Hviid and Shaffer (1994) and Corts (1995) who also show that PBGs are less preferable to PMGs.

How can we relate our findings to observed market practices? First, we are able to explain why some retailers who operate both online and offline implement PMG offline but not online. As noted above, the secondary competition-intensifying effect strengthens as the ratio of product valuation to search cost increases. Since the online search cost is lower than offline search cost, for the same product the ratio of product valuation to search costs is greater in online markets than offline markets. Under these conditions our model would predict that the secondary competition-intensifying effect in the online environment is stronger than in the offline environment, thus explaining why retailers that operate both online and offline only offer PMGs offline but not online. *Note that our explanation holds even when the competitive structure in online and offline markets is the same.*

Second, our findings offer an explanation for the considerable variation in PMG practices across retail categories as is shown in Table 1. This is because product valuations differ across retail categories. For example, the product valuation of office supplies is low so the secondary competition-intensifying effect faced by retailers in this category is relatively weak, and therefore PMG is profitable. In comparison, jewelry has very high product valuation. Our model predicts that in such categories, the secondary competition-intensifying effect is strong, rendering it unprofitable to offer PMG. In the electronics category, however, product valuation is moderately high; in such markets, it is only profitable for some, but not all, retailers to offer PMGs. Thus, an asymmetric equilibrium emerges in such markets. *Note that our explanation for these differences in PMG practices holds, even if the mix of customers is the same across the product categories.*

To shed light on the seemingly counterintuitive application of the price check website by Asda, we first note that such price check website essentially transforms some nonshoppers into refundees by increasing

the proportion of consumers who have low postpurchase search cost. We find that the PMG retailer may benefit from price check technology in the asymmetric equilibrium. On the one hand, this technology increases the overall extent of consumer search and thus magnifies the (primary and secondary) competition-intensifying effect of PMG. On the other hand, the demand-expansion effect of PMG also becomes stronger as the size of refundees increases. When the size of refundees is relatively small, we show that the latter effect dominates the former and that the PMG retailer is better off with the price check technology.

1.2. Literature Review

Our paper builds on several streams of research. There is a long tradition in marketing and economics of examining the competitive implications of PMG. However, there is no consensus on whether PMG is anti- or procompetitive. Early researchers argue that PMG can facilitate tacit collusion among competing retailers and is therefore anticompetitive (see, e.g., Hay 1982, Salop 1986, Logan and Lutter 1989, Baye and Kovenock 1994, Chen 1995, Zhang 1995). When retailers are committed to match lower prices charged by competitors in the market, no one has the incentive to undercut the rivals. Hviid and Shaffer (1999) and Coughlan and Shaffer (2009) highlight the sensitivity of these findings to certain assumptions made in earlier research. Hviid and Shaffer (1999) find that an infinitesimally small hassle cost could attenuate the ability of PMG to mitigate price competition. Coughlan and Shaffer (2009) examine situations in which retailers carry multiple products and have limited shelf space. They show that when both asymmetric product substitutability and shelf-space availability are considered, retail price under PMG may even fall below the competitive level. Jain and Srivastava (2000) and Moorthy and Winter (2006) propose that PMG can serve as a credible signal of low prices in a market where consumers have incomplete price information and never conduct price search.³

By identifying a segment of refundees that takes advantage of costless postpurchase search under PMG, our study is closely related to another strand of literature that proposes PMG as a price discrimination tool. The idea is that, when consumers are heterogeneous, PMG can be used to price discriminate one consumer type over the others. Janssen and Parakhonyak (2013) and Yankelevich and Vaughan (2016) both investigate the price discrimination role of PMG within the endogenous search framework. In Janssen and Parakhonyak (2013), some consumers with high search cost do not search in equilibrium but are able to passively receive a price quote after purchase. In addition, consumers do not know whether retailers offer PMG at the time of purchase; thus, PMG does not affect consumers' search and purchase decisions. Consequently,

PMG increases consumers' option value of purchase and knowing this, all retailers charge a higher price. Yankelevich and Vaughan (2016) assume that some shoppers patronize PMG retailers only and will invoke PMG if a lower price exists. As a result, PMG increases retail prices. In contrast to their findings that PMG is anticompetitive, we show that PMG is procompetitive as it provides refundees the opportunity to search after purchase. The overall extent of consumer search increases; hence, the retail price is lowered. We also find that endogenous consumer search could lead to a secondary effect on retail competition that affects the retailers' regular price, which has been ignored in this stream of work.

Png and Hirshleifer (1987), Corts (1997), Chen et al. (2001), and Hviid and Shaffer (2012) assume that some consumers are exogenously endowed with the ability to take advantage of PMG while others cannot. Png and Hirshleifer (1987) find that PMG induces retailers to charge higher prices and is always profitable for retailers. Corts (1997) shows that PMG may lead to higher or lower prices but it is still profitable for retailers to offer PMG. Chen et al. (2001) show that both price and profit can go up or down when retailers institute PMG depending on market parameters. Hviid and Shaffer (2012) investigate the case where small local stores set prices at the local level while large chain stores set prices at the national level independent of local market conditions. They delineate the market conditions under which it is optimal for the local stores to offer PMG, price-beating guarantee (PBG), and no price guarantee.

Empirical evidence on the competitive effects of PMG is somewhat limited. Hess and Gerstner (1991) and Arbatskaya et al. (2004, 2006) provide empirical evidence that is consistent with the anticompetitive role of PMG. The latter two studies examine price quotes obtained from U.S. newspaper advertisements in late 1996. In their study of guarantees across a variety of products, Arbatskaya et al. (2004) document that about 37% of guarantees could be classified as price-beating guarantees (PBGs) rather than PMGs, although PMGs are still the mainstream. PMGs are found to be more likely than PBGs to apply to selling rather than advertised prices and to have less restrictive terms for fulfillment. Based on these findings, the authors conclude that PMGs are more likely to facilitate collusion. Using data on tire advertisements from the same period, Arbatskaya et al. (2006) find that PMGs tend to be associated with higher prices than competitors that do not offer them. They take this as evidence that is consistent with the anticompetitive role of PMG, but they also note that fostering price discrimination is an alternative explanation for this result.

While evidence in the above studies tends to favor the collusion story, Moorthy and Winter (2006) and

Moorthy and Zhang (2006) find that PMG is usually not adopted by all retailers in a market and tends to be adopted by retailers with low costs or low quality, which they take as inconsistent with PMG being anti-competitive. They argue that this and other evidence is most consistent with PMG as a signal of low prices. In a case study of British supermarket prices, Manez (2006) finds evidence that the low-price retailer initiated PMG, which is consistent with using PMG to signal low prices. In an experimental market study, Yuan and Krishna (2011) find that subjects who were designated to be sellers tended to offer PMG when they had the option to do so, and they varied prices in a way consistent with using mixed strategies for price discrimination. Buyers also searched more when PMGs were available. These results were taken as evidence in favor of the use of PMG as a price discrimination device. Thus, different studies have found evidence for and against explanations of PMG as a device for collusion, signaling, and price discrimination. However, the above empirical evidence covers only a limited selection of markets, and some of it may be obsolete because data were collected prior to the widespread use of Internet. Moreover, none of the studies discussed in this paragraph have explicitly examined the use of PMG as an option to take advantage of *post-purchase search*.

There is a behavioral literature that examines consumers' postpurchase search behavior in the presence of PMG. Kukar-Kinney et al. (2007) suggest that there exists a group of "less price-conscious" consumers who purchase from a PMG retailer to avoid costly prepurchase search because prepurchase search cost is high compared with the gain from search. Dutta and Biswas (2005) investigate the moderating role of value consciousness in the relationship between PMG and consumers' postpurchase search intentions. They find that consumers who have "high value consciousness" are prone to conduct postpurchase search, as the benefit from postpurchase search exceeds its cost. Their findings support our assumption of the existence of refundees, i.e., consumers who have high prepurchase search costs but low postpurchase search costs. Lim and Ho (2008) provide an experimental analysis in which respondents are explicitly asked to reveal the likelihood of postpurchase search. They find that consumers are generally more likely to search after purchase when faced with a PMG retailer. The central theme of these papers is to identify the type of consumers who will search after purchase in the presence of PMG. In addition, Dutta et al. (2011) evaluate the ability of PMG to reduce consumer regret when a lower price is discovered after purchase. Kukar-Kinney and Grewal (2006) study how retail factors, such as retail environment and store reputation, affect consumers' postpurchase search intentions and willingness to invoke PMG. Finally, McWilliams and Gerstner

(2006) show theoretically that PMG can be used to improve customer retention by reducing the dissatisfaction of customers who found a lower price after purchase. We build on this literature by explicitly modeling consumers' postpurchase behavior and examining the effect that postpurchase search behavior has on stores' optimal pricing and price-matching strategies.

The remainder of the paper is organized as follows. Section 2 presents the model description including key assumptions and the game structure. We present the analysis in Section 3. In Section 4 we extend the baseline model to the case of price-beating guarantees and conclude in Section 5.

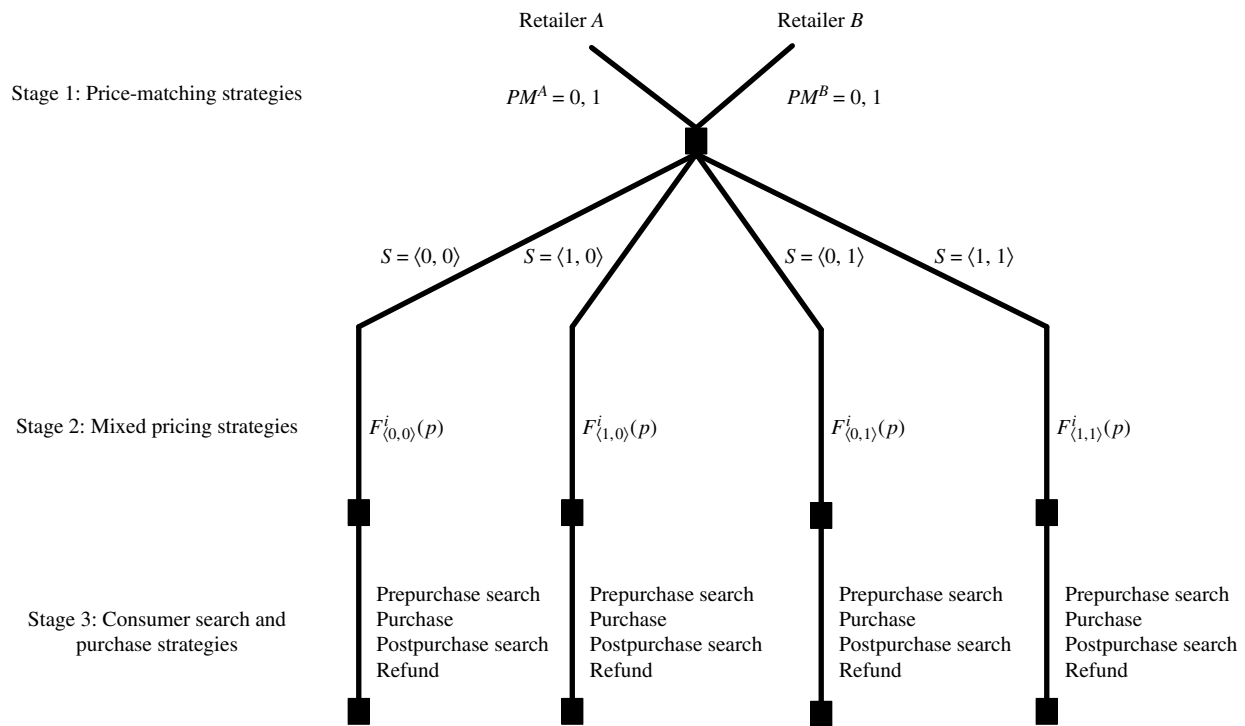
2. Model

Consider a market with two *ex ante* identical retailers indexed by i ($i = A, B$). They sell a homogeneous product to the end consumers. Consumers have a product valuation of v , which is the maximum they are willing to pay for one unit of the product. Without any loss of generality, we assume that retailers have identical marginal costs, which are normalized to zero. We assume that consumers search sequentially for price information with perfect recall. To assure full participation, we assume that consumers obtain the first price quote for free. Sequential price search, perfect recall, and the first price quote for free are common assumptions in the consumer search literature (e.g., Morgan and Manning 1985, Stahl 1989, Moorthy et al. 1997, Kuksov 2004). The measure of consumers is normalized to one without loss of generality. Consumers demand at most one unit of the product.

To allow for heterogeneity in search costs before and after purchase, we assume that some consumers have a prepurchase search cost of 0 after the initial free search, while others have a positive prepurchase search cost of $c > 0$. Consumers who have purchased from a PMG retailer may continue to search after purchase for a better deal within the grace period. We assume that the postpurchase search cost is 0 for some consumers and c for others. Discrete heterogeneity in pre- and postpurchase search costs yields four exhaustive and mutually exclusive segments of consumers: consumers who have zero pre- and postpurchase search costs; consumers who have zero prepurchase search cost but a postpurchase search cost of c ; consumer who have a prepurchase search cost of c but zero postpurchase search cost; and consumers who have pre- and postpurchase search costs of c .

A key assumption of this paper is the existence of consumers with high prepurchase search cost but low postpurchase search cost. It is not hard to envision cases in which this holds true. First, prepurchase price search is usually accompanied with product-related information search and thus can be more cognitively challenging for consumers than postpurchase price search. If the retailer offers a PMG, consumers may buy

Figure 1. Timeline of Decisions in the Game



the item and wait until time costs are lower to check other prices. Second, consumers under time pressure or making an unplanned purchase may value time before purchase more than after purchase. For example, if an appliance such as a TV, refrigerator, or air conditioner breaks down, the cost of doing without the item while a complete search is conducted may be very high. Knowing that the retailer has a PMG policy, the consumer may buy the item without searching and use it while waiting for a more opportune time to check prices at other stores. A grace period to exercise the PMG allows the consumer to wait until search costs are low. As noted in our literature review, several studies support the existence of this consumer segment.

The game unfolds in three stages, as shown in Figure 1. In the first stage, retailers simultaneously choose their price-matching strategies, which then become common knowledge to all agents in subsequent stages of the game. Let $PM^i = 1$ or 0 be an indicator of whether retailer i ($i = A, B$) offers PMG or not, respectively. We use $S = \langle PM^A, PM^B \rangle$ to index the subgame in which the price-matching strategies of retailer A and B are PM^A and PM^B , respectively. There are four possible subgames: $\langle 0, 0 \rangle$, $\langle 1, 0 \rangle$, $\langle 0, 1 \rangle$, and $\langle 1, 1 \rangle$. The fixed cost of implementing PMG is assumed to be zero without loss of generality. We further assume that PMG is legally binding and enforceable. In the second stage, both retailers simultaneously set prices conditional on their first-stage price-matching strategies. Given the discrete consumer segments, particularly the existence

of consumers with zero prepurchase search costs, we anticipate the pricing equilibrium to be in mixed strategies. We denote retailer i 's price distribution in subgame S as $F_S^i(p)$. In the third stage, consumers who have perfect knowledge of their own search cost structure make decisions on prepurchase search, purchase, postpurchase search, and refund redemption *en masse*. The equilibrium concept we employ is subgame perfect Nash equilibrium. For reference, a summary of the model notations is presented in Table A.1 of the appendix.

3. Analysis

In this section we proceed in the following manner. Since consumers' search behavior will have a significant bearing on retailers' pricing decisions, we establish consumers' optimal search rule in Section 3.1. In Section 3.2, we characterize retailers' pricing strategies while taking consumers' optimal search behavior into account and taking retailers' first-stage decisions as given. Finally, we characterize retailers' price-matching strategies in Section 3.3.

3.1. Optimal Consumer Search Rule

Consumers will engage in search as long as the expected benefit from search is greater than their search cost. Given a priori lowest price quote z at hand, consumer surplus from sampling an additional retailer that charges a price of p is

$$CS(p; z) = \max\{z - p, 0\}. \quad (1)$$

Then the marginal benefit of sampling once from retailer i 's price distribution $F^i(p)$ mixing over the interval $[l, u]$ is the expected consumer surplus of search

$$ECS(z; F^i(p)) = \int_l^u CS(p; z) dF^i(p). \quad (2)$$

For consumers with search cost c , there exists a unique z such that the marginal benefit of search is equal to search cost. We define this unique price threshold as retailer i 's reservation price r^i :

$$r^i = \arg\{ECS(z; F^i(p)) - c = 0\}. \quad (3)$$

The economic meaning of r^i is the price at which consumers with search cost c are indifferent to sampling retailer i or not. It is easy to see that r^i is an increasing function of c .

Following Weitzman (1979), we can summarize the optimal search strategy for consumers with search cost c as follows. When they base their search strategy on retailers' pricing strategies only, consumers with search cost c will sample the retailer with the lower reservation price first, followed by the retailer with the higher reservation price. They stop at the first retailer that charges a price below $\min_{i=A,B} r^i$ or else pick the retailer with the lower price after both retailers are sampled. In Lemma 6, we show that the retailer with the lower reservation price also has a lower average price. This means that consumers with search cost c first sample the retailer with the lower average price in the hope of getting a good deal and completing the costly search process as early as possible. Otherwise, if the price is not low enough, they will continue price search with the second retailer. If both retailers have the same reservation price, consumers are indifferent and will search randomly.

For consumers with zero (pre- or postpurchase) search cost, the marginal benefit of search is weakly greater than their search cost. Thus, the optimal search rule for consumers with zero search cost is to obtain price information from both retailers before stopping.⁴ Given the optimal search rule, we can proceed to characterize the equilibrium pricing strategies.

3.2. Equilibrium Price Distributions

We first characterize the equilibrium upper bound of retailers' mixed pricing strategies in each subgame in Lemma 1.

Lemma 1. *Let r_S^i be the reservation price of retailer i in subgame S . Both retailers' price distributions share a common upper bound $u_S = \min\{v, \min_{i=A,B} r_S^i\}$.*

Proof. See the appendix.

Table 2. Notation of Consumer Segments

Consumer segment	Postpurchase search cost	
	0	c
Prepurchase search cost	0	c
	Shoppers (α)	
	Refundees (β)	Nonshoppers (γ)

Lemma 1 states that the upper bound of both retailers' price distributions is the minimum of the product valuation and the minimum reservation prices of both retailers. Prices above the product valuation v are dominated since there will be zero demand. Any price above $\min_{i=A,B} r^i$ will encourage consumers to continue to search elsewhere and is therefore weakly dominated by $\min_{i=A,B} r^i$. A key implication of Lemma 1 is that, in equilibrium, consumers with pre- or postpurchase search cost c do not conduct actual search except for the first free quote based on the optimal search rule. For ease of exposition, we call a fraction of α consumers with zero prepurchase search cost "shoppers," a fraction of β consumers with positive prepurchase search cost but zero postpurchase search cost "refundees," and a fraction of γ consumers with both pre- and postpurchase search cost of c "nonshoppers." In our model, $\alpha + \beta + \gamma = 1$; see Table 2. We characterize the equilibrium search and purchase behavior of each consumer segment in Proposition 1.

Proposition 1. *Given assumptions on consumers' pre- and postpurchase search costs, the optimal consumer search and purchase behavior for each consumer segment in equilibrium is as follows:*

- (i) *Nonshoppers do not search, purchase from the retailer with the (weakly) lower reservation price, and pay the price charged by the retailer.*
- (ii) *In the absence of PMG, refundees act like nonshoppers. In the presence of PMG, refundees purchase from a PMG retailer. After purchase, they take advantage of costless postpurchase search and ask for a refund if a lower price is found.*
- (iii) *Shoppers search both retailers before purchase and purchase directly from the retailer that charges the lower price.*

Proof. See the appendix.

Since they have no price information, nonshoppers directly purchase from the retailer with the lower reservation price. If both retailers have the same reservation price, nonshoppers pick retailers randomly. The shopping behavior of refundees depends upon the presence or absence of PMG retailers. In the absence of PMG, refundees, like nonshoppers, cannot obtain any benefit from postpurchase search and will purchase from the retailer with the lower reservation price. In the presence of PMG, refundees will purchase from the PMG retailer(s) to take advantage of costless postpurchase

search and ask for a refund if a lower price is discovered after purchase. This highlights an important distinction between nonshoppers and refundees. That is, nonshoppers' search strategy hinges on retailers' pricing strategies only while refundees' shopping behavior is dependent on both pricing and price-matching strategies of retailers.

Because it can entice refundees to purchase, PMG has a direct demand effect on retailers that offer it. PMG retailers are positioned to cater to the needs of refundees who prefer to avoid costly prepurchase search and postpone search until after purchase. In the Asda example in the introduction, the 800,000 consumers who visited Asda's price check website in the first two months of 2011 to compare prices after purchase fit the definition of refundees in our model.

Shoppers collect price information from both retailers before purchase and are guaranteed to pay the lowest market price. One might wonder why shoppers do not invoke PMG at the time of purchase instead, in which case they also pay the lowest market price. The reason is as follows: Suppose that a shopper commits to purchase from a PMG retailer and decides to invoke PMG if the competing retailer charges a lower price. The shopper would now act like a "loyal" consumer, thus enabling the PMG retailer to charge a higher price. Due to the strategic complementarity of prices, the competing retailer will do the same. Although this shopper still pays the lowest market price, the lowest market price itself will be higher relative to the case when the shopper commits to directly purchase from the retailer with the lower price. Thus, shoppers are strictly worse off if they commit to purchase from the PMG retailer and invoke PMG.

Because of the need to serve all three consumer segments, retailers' pricing will be in mixed strategies; that is, each retailer charges a range of prices with varying probabilities. Because of symmetry, it is sufficient for us to consider the following three subgames in the pricing stage: $\langle 0, 0 \rangle$, $\langle 1, 1 \rangle$, and $\langle 1, 0 \rangle$.

3.2.1. Neither Retailer Offers PMG. Consider the subgame $\langle 0, 0 \rangle$ in which neither retailer offers PMG. In this subgame, like nonshoppers, refundees who cannot take advantage of costless postpurchase search randomly pick one retailer. Therefore, each retailer attracts half of the refundees and nonshoppers who pay the observed price. Shoppers visit the retailer with a lower price. When it charges a price of p^A , retailer A can attract all shoppers if retailer B charges a price greater than p^A , which occurs with probability $1 - F_{(0,0)}^B(p^A)$. Thus, the expected profit of retailer A when it charges a price of p^A is given by

$$E\Pi_{(0,0)}^A(p^A) = \left\{ \alpha \left[1 - F_{(0,0)}^B(p^A) \right] + \frac{\beta + \gamma}{2} \right\} p^A. \quad (4)$$

The equilibrium price distribution is reported in Lemma 2.

Lemma 2. When neither retailer offers PMG, the equilibrium price distribution for retailer i is given by

$$F_{(0,0)}^i(p) = 1 - \frac{\beta + \gamma}{2\alpha} \left(\frac{u_{(0,0)}}{p} - 1 \right), \quad \text{for } p \in [l_{(0,0)}, u_{(0,0)}], \quad (5)$$

where $u_{(0,0)} = \min\{r_{(0,0)}^i, v\}$ and $l_{(0,0)} = ((\beta + \gamma)/2)/(\alpha + (\beta + \gamma)/2)u_{(0,0)}$ is the lower bound of the price distribution. Here, $r_{(0,0)}^i$ is the reservation price of retailer i and is defined in the appendix. The profit of retailer i is given by

$$\Pi_{(0,0)}^i = \frac{\beta + \gamma}{2} u_{(0,0)}. \quad (6)$$

Proof. See the appendix.

3.2.2. Both Retailers Offer PMGs. Next we focus on the subgame $\langle 1, 1 \rangle$ in which both retailers implement PMGs. The effective price refundees pay is the minimum price of both retailers. Moreover, they are indifferent with purchasing from either retailer. Therefore, retailer A can attract all shoppers with probability $1 - F_{(1,1)}^B(p^A)$ when it charges a price p^A , together with half of refundees and nonshoppers. The expected profit of retailer A when it charges a price p^A is given by

$$E\Pi_{(1,1)}^A(p^A) = \left\{ \alpha \left[1 - F_{(1,1)}^B(p^A) \right] + \frac{\gamma}{2} \right\} p^A + \frac{\beta}{2} E \left[\min_{p^B \in \Sigma_{(1,1)}^B} \{p^A, p^B\} \right], \quad (7)$$

in which p^B is the price charged by retailer B chosen from the strategy set $\Sigma_{(1,1)}^B$. $E[\min_{p^B \in \Sigma_{(1,1)}^B} \{p^A, p^B\}]$ is the expected effective price refundees pay and can be further expanded as $[1 - F_{(1,1)}^B(p^A)]p^A + \int_{(1,1)}^{p^A} p^B dF_{(1,1)}^B(p^B)$. If $p^B \geq p^A$, which occurs with probability $1 - F_{(1,1)}^B(p^A)$, refundees pay p^A . Otherwise, if $p^B < p^A$ the effective price for refundees will be p^B . In this case, the effective price refundees pay on average is $\int_{(1,1)}^{p^A} p^B dF_{(1,1)}^B(p^B)$. The equilibrium pricing strategies are summarized in Lemma 3.

Lemma 3. When both retailers offer PMG, the equilibrium price distribution for retailer i is given by

$$F_{(1,1)}^i(p) = 1 - \frac{\gamma/2}{\alpha + \beta/2} \left[\left(\frac{u_{(1,1)}}{p} \right)^{(\alpha + \beta/2)/\alpha} - 1 \right], \quad \text{for } p \in [l_{(1,1)}, u_{(1,1)}], \quad (8)$$

where $u_{(1,1)} = \min\{r_{(1,1)}^i, v\}$ and $l_{(1,1)} = (\gamma/2)/(\alpha + (\beta + \gamma)/2)u_{(1,1)}$ is the lower bound of the price distribution. Here, $r_{(1,1)}^i$ is the reservation price of retailer i and is defined in the appendix. The profit of retailer i is given by

$$\Pi_{(1,1)}^i = \left(\alpha + \frac{\beta + \gamma}{2} \right) \cdot \left(\frac{\gamma/2}{\alpha + (\beta + \gamma)/2} \right)^{\alpha/(\alpha + \beta/2)} u_{(1,1)}. \quad (9)$$

Proof. See the appendix.

3.2.3. When Only Retailer A Offers PMG. Finally, we turn to the asymmetric subgame $\langle 1, 0 \rangle$ in which retailer A offers PMG while retailer B does not. Retailer A sells to all shoppers with probability $1 - F_{(1,0)}^B(p^A)$, and all refundees with certainty, while retailer B sells to all shoppers with probability $1 - F_{(1,0)}^A(p^B)$, and attracts no refundees. Since nonshoppers purchase from the retailer with lowest reservation price, their purchase decisions are endogenous to retailers' pricing strategies. We denote μ ($1 \geq \mu \geq 0$) as the shopping strategy of nonshoppers; that is, μ represents the probability that nonshoppers will patronize retailer A. In the aggregate, μ can be reinterpreted as the proportion of nonshoppers purchasing from retailer A and $1 - \mu$ is the fraction of nonshoppers who purchase from retailer B. Thus, the expected profit of retailer A when it charges a price of p^A is given by

$$\begin{aligned} \text{E}\Pi_{(1,0)}^A(p^A) &= \{\alpha[1 - F_{(1,0)}^B(p^A)] + \mu\gamma\}p^A \\ &\quad + \beta\text{E}\left[\min_{p^B \in \Sigma_{(1,0)}^B} \{p^A, p^B\}\right], \end{aligned} \quad (10)$$

in which p^B is the price charged by retailer B chosen from its strategy set $\Sigma_{(1,0)}^B$. The expected profit of retailer B when it charges a price p^B is given by

$$\text{E}\Pi_{(1,0)}^B(p^B) = \{\alpha[1 - F_{(1,0)}^A(p^B)] + (1 - \mu)\gamma\}p^B. \quad (11)$$

We report the equilibrium pricing strategies in the asymmetric subgame in Lemma 4.

Lemma 4. *When only retailer A offers PMG, there exists a unique μ^* such that $0.5 > \mu^* > 0$ when the size of refundees is small ($\beta < \bar{\beta}$) and $\mu^* = 0$ when the size of refundees is large ($\beta \geq \bar{\beta}$). Given μ^* , the equilibrium pricing strategies for both retailers are given by*

$$\begin{cases} F_{(1,0)}^A(p) = 1 - \frac{(1 - \mu^*)\gamma}{\alpha} \left(\frac{u_{(1,0)}}{p} - 1 \right), \\ \quad \text{for } p \in [l_{(1,0)}, u_{(1,0)}]; \\ F_{(1,0)}^B(p) = \begin{cases} 1 & \text{at } p = u_{(1,0)}, \\ \frac{\alpha + \beta + \mu^*\gamma}{\alpha + \beta} \left[1 - \left(\frac{l_{(1,0)}}{p} \right)^{(\alpha + \beta)/\alpha} \right], \\ \quad \text{for } p \in [l_{(1,0)}, u_{(1,0)}], \end{cases} \end{cases} \quad (12)$$

where $u_{(1,0)} = \min\{r_{(1,0)}^B, v\}$ and $l_{(1,0)} = ((1 - \mu^*)\gamma / (\alpha + (1 - \mu^*)\gamma))u_{(1,0)}$ is the lower bound of both retailers' price distributions. Here, $r_{(1,0)}^i$ is the reservation price of retailer i and is defined in the appendix. The profits of retailer A and B are given by

$$\begin{aligned} \Pi_{(1,0)}^A &= \frac{(\alpha + \beta + \mu^*\gamma)(1 - \mu^*)\gamma}{\alpha + (1 - \mu^*)\gamma} u_{(1,0)} \quad \text{and} \\ \Pi_{(1,0)}^B &= (1 - \mu^*)\gamma u_{(1,0)}. \end{aligned} \quad (13)$$

Proof. See the appendix.

The first sentence of Lemma 4 characterizes nonshoppers' equilibrium shopping behavior. Note that nonshoppers do not search and thus will strategically choose the retailer with the lower reservation price (and average price). Recognizing that retailer A has a greater incentive to charge a high price after obtaining all refundees by offering PMG, nonshoppers are therefore less prone to purchase from retailer A than B. This means that retailer A gets less than one half of the nonshoppers, $0.5 > \mu^* \geq 0$. We find that, when the size of refundees is relatively small ($\beta \leq \bar{\beta}$), nonshoppers adopt a mixed shopping strategy ($0.5 > \mu^* > 0$) such that the marginal nonshopper is indifferent to both retailers since their reservation prices (and average prices) are equal in equilibrium, $r_{(1,0)}^A = r_{(1,0)}^B$. When the size of refundees is relatively large ($\beta > \bar{\beta}$), nonshoppers follow a pure strategy; that is, they will purchase from retailer B with probability one ($\mu^* = 0$). Although all nonshoppers buy from retailer B in this case, retailer B's reservation price (and the average price) is still lower than that of retailer A, that is $r_{(1,0)}^A > r_{(1,0)}^B$. This result is consistent with the finding of Arbatskaya et al. (2006) that PMG retailers tend to charge prices weakly higher than non-PMG retailers.

The second part of Lemma 4 characterizes the equilibrium pricing strategies of both retailers. The equilibrium price distributions in Equation (12) imply that retailer B will have a mass point at the upper bound $u_{(1,0)}$. In line with Narasimhan (1988), the upper bound is customarily interpreted as the "regular" or "nonpromoted" price, and any price below the regular price is considered a "promotion." This result means that retailer A always runs a promotion while retailer B charges the regular price with positive probability (has a point mass at the regular price). The intuition for this finding is that, although retailer B loses all refundees to retailer A, it attracts more nonshoppers than retailer A ($\mu^* < 0.5$). Hence, retailer B has a greater incentive to charge the regular price to appropriate surplus from nonshoppers than retailer A. At the same time, after losing all refundees to retailer A, shoppers become strategically more important for retailer B. When we compare the price distributions of both retailers, we find that retailer B offers deep discounts more frequently to become more appealing to shoppers.

Next we summarize the difference in the clientele mix of retailers in the asymmetric case when only retailer A offers PMG. The customer base of retailer A comprises all shoppers with lower probability, all refundees, and a smaller proportion of nonshoppers. In contrast, retailer B serves all shoppers with higher probability and a larger fraction of nonshoppers. However, it is not a priori obvious which retailer has a larger expected demand. The following proposition compares the retail demand of both retailers in the asymmetric subgame.

Proposition 2. *When only retailer A offers PMG, retailer A has a larger expected demand than retailer B.*

Proof. See the appendix.

Even though retailer A is less attractive to shoppers and nonshoppers, it sets prices such that the store loyalty of refundees offsets the reduced demand from the other two consumer segments. This is consistent with the experimental finding that PMG encourages more in-store visits and increases purchase intentions (Srivastava and Lurie 2001). A corollary of Proposition 2 is that $\Pi_{(1,0)}^A > \Pi_{(1,0)}^B$ since retailer A has both a larger expected demand and an average price weakly greater than retailer B.

More generally, Proposition 2 implies that PMG has a demand-expansion effect. To see this, note that in subgames $\langle 0,0 \rangle$ and $\langle 1,1 \rangle$, both retailers split all three consumer segments evenly and hence have an expected demand of one half. In subgame $\langle 1,0 \rangle$ retailer A's expected demand is greater than one half while retailer B's expected demand is less than one half. Thus, retailer A's demand increases when it unilaterally offers PMG. Similarly, retailer B's demand also increases by offering PMG when retailer A is already a PMG retailer. Moreover, the demand-expansion effect of PMG increases with the size of refundees β .

3.3. Stage 1: Price-Matching Strategies

In Section 3.3, we address the issue of whether and under what conditions retailers can benefit from offering PMG in a competitive environment. That is, given the structure of three subgames developed in the preceding subsections, we derive market conditions under which neither, one, or both retailers offering PMG is an equilibrium.

Because consumer search is endogenous in our model, the reservation price plays a central role in determining the equilibrium outcome. Moreover, comparing reservation prices across subgames provides important insights into the impact of PMG on price competition. In Table 3 we report reservation prices across three subgames obtained from Lemmas 2–4. It can be easily seen from Table 3 that reservation prices

can be represented as a product of search cost c and a function of consumer segment sizes (α and β given the constraint that $\gamma = 1 - \alpha - \beta$). First we establish results about the ordering of reservation prices in three subgames in Lemma 5.

Lemma 5. *When the size of refundees is not too large ($\beta < \bar{\beta}$ where $\bar{\beta} > \underline{\beta}$ as defined in the appendix), the following relationship holds: $r_{(1,1)}^i < r_{(1,0)}^B \leq r_{(1,0)}^A < r_{(0,0)}^i$.*

Proof. See the appendix.

We assume that $\beta < \bar{\beta}$ in the remainder of the paper. This assumption simplifies our exposition of the results without changing the qualitative results. In the proof of Lemma 5 in the appendix, we discuss the market equilibrium when $\beta \geq \bar{\beta}$. The implications of Lemma 5 will be discussed after we lay out the following ancillary result that links the reservation price r_S^i to the average price \bar{p}_S^i and the regular price u_S .

Lemma 6. *Let \bar{p}_S^i be the average price of retailer i ($i = A, B$) in subgame S . Then the following relationship holds:*

$$\bar{p}_S^i = u_S \left(1 - \frac{c}{r_S^i} \right) \quad \text{or} \quad 1 - \frac{\bar{p}_S^i}{u_S} = \frac{c}{r_S^i}. \quad (14)$$

Proof. See the appendix.

Lemmas 5 and 6 allow us to make three remarks about retail prices. First, note that both retailers share the same regular price u_S in any subgame. Lemma 6 says that within a subgame the retailer with a lower reservation price also charges a lower average price. Since $r_{(1,0)}^A \geq r_{(1,0)}^B$ in the asymmetric subgame $\langle 1,0 \rangle$, Equation (14) implies that $\bar{p}_{(1,0)}^A \geq \bar{p}_{(1,0)}^B$, i.e., retailer A charges an average price weakly higher than that charged by retailer B in subgame $\langle 1,0 \rangle$. Second, given $r_{(1,1)}^i < r_{(0,0)}^i$ from Lemma 5, the average price in subgame $\langle 1,1 \rangle$ is lower than that in subgame $\langle 0,0 \rangle$, $\bar{p}_{(1,1)}^i < \bar{p}_{(0,0)}^i$. A corollary of this result is that $\Pi_{(1,1)}^i < \Pi_{(0,0)}^i$ because retailers have the same expected

Table 3. Equilibrium Reservation Prices Across Subgames

Subgame	Reservation prices
$\langle 0,0 \rangle$	$r_{(0,0)}^i = \frac{2\alpha c}{2\alpha + (\beta + \gamma) \ln((\beta + \gamma)/(2\alpha + \beta + \gamma))}$
$\langle 1,0 \rangle$	$r_{(1,0)}^A = \frac{\alpha c}{\alpha + (1 - \mu^*)\gamma \ln((1 - \mu^*)\gamma/(\alpha + (1 - \mu^*)\gamma))}$ $r_{(1,0)}^B = \frac{\beta(\alpha + \beta)[\alpha + \gamma(1 - \mu^*)]c}{\alpha(\alpha + \beta + \mu^*\gamma)\{\beta - \gamma(1 - \mu^*)[1 - (\gamma(1 - \mu^*)/(\alpha + \gamma(1 - \mu^*)))^{\beta/\alpha}]\}}$
$\langle 1,1 \rangle$	$r_{(1,1)}^i = \frac{\beta c}{(\beta + \gamma) - (2\alpha + \beta + \gamma)^{\beta/(2\alpha + \beta)}\gamma^{(2\alpha)/(2\alpha + \beta)}}$

demand of one half in subgames $\langle 0, 0 \rangle$ and $\langle 1, 1 \rangle$. Thus, if $\langle 1, 1 \rangle$ is the unique equilibrium, it is a *prisoner's dilemma*. Third, we can interpret $1 - \bar{p}_S^i / u_S$ in Lemma 6 as retailer i 's average promotion depth (in percentage terms) in subgame S . Lemma 6 implies that there is a negative relationship between a retailer's reservation price and the average promotion depth it offers. Since $r_{\langle 1, 1 \rangle}^i < r_{\langle 1, 0 \rangle}^i < r_{\langle 0, 0 \rangle}^i$, we can conclude that retailers offer deeper promotions on average when more retailers offer PMG in the market.

Next we examine the effect of PMG on retail competition. Consider the unilateral deviation by retailer A from subgame $\langle 0, 0 \rangle$ to $\langle 1, 0 \rangle$. Note that Lemma 5 implies $r_{\langle 1, 0 \rangle}^A < r_{\langle 0, 0 \rangle}^i$. Retailer A 's PMG provides refundees with the opportunity to continue search after purchase. The overall extent of consumer search increases. As discussed above, retailers facing better informed consumers offer deeper promotions in subgame $\langle 1, 0 \rangle$ than in $\langle 0, 0 \rangle$. Put differently, the average promotion depth increases as retailer A offers PMG, $1 - \bar{p}_{\langle 1, 0 \rangle}^i / u_{\langle 1, 0 \rangle} > 1 - \bar{p}_{\langle 0, 0 \rangle}^i / u_{\langle 0, 0 \rangle}$. We call this the *primary competition-intensifying effect*. Recall that $u_{\langle 0, 0 \rangle} = \min\{v, r_{\langle 0, 0 \rangle}^i\}$ and $u_{\langle 1, 0 \rangle} = \min\{v, r_{\langle 1, 0 \rangle}^B\}$. Then $r_{\langle 1, 0 \rangle}^B < r_{\langle 0, 0 \rangle}^i$ implies that $u_{\langle 1, 0 \rangle} \leq u_{\langle 0, 0 \rangle}$. Since a deeper promotion provides consumers with search cost c greater incentives to search, both retailers may be compelled to shift the "regular" price (i.e., the upper bound of their price distributions) downward ($u_{\langle 1, 0 \rangle} \leq u_{\langle 0, 0 \rangle}$) to keep these consumers from searching elsewhere. We call this the *secondary competition-intensifying effect*. This effect is a direct outcome of endogenous consumer search, where the regular price is endogenously determined. In models where search behavior is exogenously assumed, this effect does not arise because the regular price is the product valuation v across all subgames. Similarly, when the size of refundees is not too large ($\beta < \bar{\beta}$), we have $r_{\langle 1, 1 \rangle}^i < r_{\langle 1, 0 \rangle}^i$. The incentive of both retailers to charge a higher price is diminished as retailer B deviates from subgame $\langle 1, 0 \rangle$ to $\langle 1, 1 \rangle$. This means that retailer B 's PMG also has a primary and possibly a secondary competition-intensifying effect when retailer A already offers PMG. The above discussion of how PMG affects retail competition is summarized in Proposition 3.

Proposition 3. *PMG intensifies price competition on two dimensions. PMG has a primary competition-intensifying effect because it induces retailers to offer deeper promotions. There is potentially a secondary competition-intensifying effect of PMG because retailers may be forced to lower the regular price itself.*

Proof. See the appendix.

So far we have identified three effects of PMG: demand-expansion effect, primary competition-intensifying effect, and secondary competition-intensifying

Table 4. Equilibrium Demand at the Regular Price Across Subgames

Subgame	Demand at the regular price
$\langle 0, 0 \rangle$	$d_{\langle 0, 0 \rangle}^i = \frac{\beta + \gamma}{2}$
$\langle 1, 0 \rangle$	$d_{\langle 1, 0 \rangle}^A = \frac{(\alpha + \beta + \mu^* \gamma)(1 - \mu^*) \gamma}{\alpha + (1 - \mu^*) \gamma}, d_{\langle 1, 0 \rangle}^B = (1 - \mu^*) \gamma$
$\langle 1, 1 \rangle$	$d_{\langle 1, 1 \rangle}^i = \left(\alpha + \frac{\beta + \gamma}{2} \right) \left(\frac{\gamma/2}{\alpha + (\beta + \gamma)/2} \right)^{\alpha/(\alpha + \beta/2)}$

effect. Among them, the secondary competition-intensifying effect is new to the literature. To better understand how it affects retail profits, we write Π_S^i , the profit of retailer i in subgame S , as $\Pi_S^i = u_S d_S^i$. Recall that u_S is the regular price in subgame S and is given by $u_S = \min\{v, \min_{i=A, B} r_S^i\}$. d_S^i can be interpreted as the expected demand of retailer i when it charges the regular price u_S . In Table 4 we summarize d_S^i across subgames as shown in the profit Equations (6), (9), and (13). It is easy to see from Table 4 that d_S^i is a function of consumer segment sizes only.

Let $\Delta \Pi^i = \Pi_{S'}^i - \Pi_S^i$ be retailer i 's incremental profit by offering PMG, where S and S' are subgames before and after its PMG, respectively. Retailer i finds it profitable to offer PMG only if $\Delta \Pi^i > 0$. Substituting $\Pi_S^i = u_S d_S^i$ into $\Delta \Pi^i$, we obtain Equation (15) by first subtracting and then adding $u_S d_{S'}^i$:

$$\Delta \Pi^i = \underbrace{(u_{S'} - u_S) d_{S'}^i}_{\text{Secondary competition-intensifying effect}} + \overbrace{(d_{S'}^i - d_S^i) u_S}^{\text{Demand-expansion effect net of primary competition-intensifying}} \quad (15)$$

Since the regular price after offering PMG is no higher than that before offering PMG, $u_{S'} \leq u_S$, the first term in Equation (15), $(u_{S'} - u_S) d_{S'}^i$, is nonpositive (it can be zero if $u_{S'} = u_S = v$). This term represents the secondary competition-intensifying effect of PMG on retail profits. In the extant literature that does not endogenize consumer search, the regular price is usually held constant at the product valuation v . As a result, $(u_{S'} - u_S) d_{S'}^i$ is equal to zero and the secondary competition-intensifying effect does not exist. This failure to account for the secondary competition-intensifying effect could lead to the overestimation of the profitability of PMG. Given that $u_S = \min\{v, \min_{i=A, B} r_S^i\}$ and $u_{S'} = \min\{v, \min_{i=A, B} r_{S'}^i\}$, we find that $u_{S'} - u_S$ decreases (becomes more negative) as the ratio of product valuation to search cost v/c increases. This implies that the secondary competition-intensifying effect weakly increases with v/c .

The second term in Equation (15), $(d_{S'}^i - d_S^i)u_S$, represents the demand-expansion effect net of the primary competition-intensifying effect. The relative strength of demand-expansion effect and the primary competition-intensifying effect depends on $d_{S'}^i - d_S^i$. When $d_{S'}^i > d_S^i$, the demand-expansion effect is greater than the primary competition-intensifying effect. Otherwise, when $d_{S'}^i < d_S^i$, the demand-expansion effect is weaker than the primary competition-intensifying effect. Following the discussion above, we now investigate the retailers' incentive for deviations to institute PMG and the conditions under which different equilibria arise.

Proposition 4. *Adoption of PMG is beneficial if and only if (1) $d_{S'}^i > d_S^i$ and (2) $v/c < h^i(\alpha, \beta)$, where $h^i(\alpha, \beta) = r_S^B d_{S'}^i / (c d_S^i)$ is a function of consumer segment sizes only.⁵*

Proof. See the appendix.

Proposition 4 delineates the necessary and sufficient conditions for PMG to be profitable. The first necessary condition $d_{S'}^i > d_S^i$ indicates that the demand-expansion effect outweighs the primary competition-intensifying effect. Otherwise, retailers cannot benefit from adopting PMG regardless of the secondary competition-intensifying effect. The second necessary condition says that the ratio of product valuation to search cost must be sufficiently low (i.e., $v/c < h^i$). Here, h^i is the critical value of v/c at which $\Delta\Pi^i = 0$. In addition, we have $\Delta\Pi^i > 0$ when $v/c < h^i$ and $\Delta\Pi^i < 0$ when $v/c > h^i$. Recall that the secondary competition-intensifying effect increases with the ratio of product valuation to search cost v/c . Hence, we can take h^i as a measure of the demand-expansion effect net of the primary competition-intensifying effect. The second necessary condition $v/c < h^i$ means that the secondary competition-intensifying effect is weaker than the demand-expansion effect net of the primary competition-intensifying effect. These two conditions together are sufficient to guarantee a profitable deviation by retailers to offer PMG.

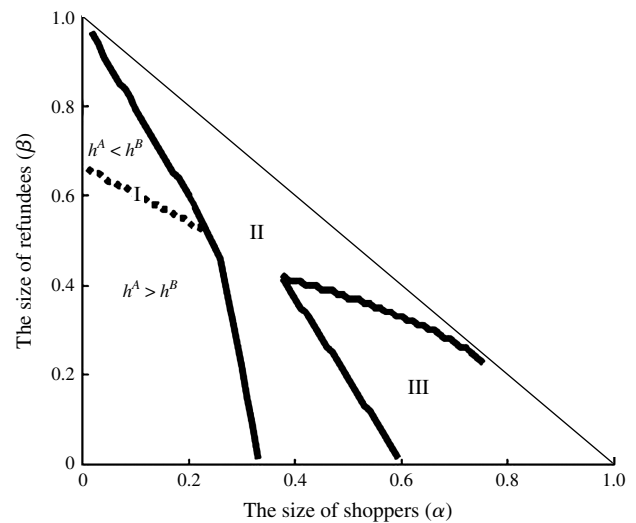
Proposition 5. *The equilibrium price-matching strategies are given by*

- (i) When $d_{(1,0)}^A > d_{(0,0)}^i$ and $d_{(1,1)}^i > d_{(1,0)}^B$, $\langle 1, 1 \rangle$ is the equilibrium if $v/c < \min_i h^i$; $\langle 1, 0 \rangle$ is the equilibrium if $h^A > v/c > h^B$; both $\langle 1, 1 \rangle$ and $\langle 0, 0 \rangle$ are equilibria if $h^B > v/c > h^A$; $\langle 0, 0 \rangle$ is the equilibrium if $v/c > \max_i h^i$.
- (ii) When $d_{(1,0)}^A < d_{(0,0)}^i$ and $d_{(1,1)}^i > d_{(1,0)}^B$, both $\langle 0, 0 \rangle$ and $\langle 1, 1 \rangle$ are equilibria if $v/c < h^B$; $\langle 0, 0 \rangle$ is the equilibrium if $v/c > h^B$.
- (iii) When $d_{(1,0)}^A < d_{(0,0)}^i$ and $d_{(1,1)}^i < d_{(1,0)}^B$, $\langle 0, 0 \rangle$ is the equilibrium.

Proof. See the appendix.

This proposition summarizes the retailers' equilibrium price-matching strategies while highlighting the

Figure 2. Three Regions of Consumer Segment Sizes



role that the ratio of product valuation to search cost v/c plays in driving the results. As v/c grows larger, we see fewer retailers adopting PMG in equilibrium. This could explain why many retailers that operate both online and offline implement PMG offline but not online. Since Internet lowers consumer search cost (Ratchford 2009), online retailers have higher v/c than offline retailers. As a result, the secondary competition-intensifying effect in the online setting is stronger than that in the offline setting. Therefore, holding all else equal, a retailer is less likely to offer PMG online than offline. In Proposition 5 we also identify three regions that differ in how v/c affects the equilibrium outcome: (I) $d_{(1,0)}^A > d_{(0,0)}^i$ and $d_{(1,1)}^i > d_{(1,0)}^B$; (II) $d_{(1,0)}^A < d_{(0,0)}^i$ and $d_{(1,1)}^i > d_{(1,0)}^B$; and (III) $d_{(1,0)}^A < d_{(0,0)}^i$ and $d_{(1,1)}^i < d_{(1,0)}^B$.⁶ These three regions are illustrated in Figure 2, which is plotted on a two-dimensional plane with α and β as axes.

Consider region (I) in which the size of shoppers is small ($d_{(1,0)}^A > d_{(0,0)}^i$ and $d_{(1,1)}^i > d_{(1,0)}^B$). When there are few shoppers in the market, the primary competition-intensifying effect of PMG is weak and is dominated by the demand-expansion effect. Thus, retailers' equilibrium price-matching strategies in this region depend on the strength of the secondary competition-intensifying effect, which is determined by the ratio of product valuation to search cost v/c . In particular, when v/c is small enough ($v/c < \min_i h^i$), the secondary competition-intensifying effect is so weak that it is dominated by the demand-expansion effect net of the primary competition-intensifying effect. In this case, both retailers in equilibrium pursue the PMG strategy. Conversely, when the secondary competition-intensifying effect is sufficiently strong (i.e., v/c is large enough or equivalently, $v/c > \max_i h^i$), it is dominant for both retailers to give up the PMG strategy. For intermediate v/c , there are two possibilities depending on the

relative magnitude of h^A and h^B . In Figure 2, the dotted line in region (I) denotes the line in the $\alpha - \beta$ space where $h^A = h^B$. We have $h^A > h^B$ ($h^A < h^B$) below (above) the dotted line.

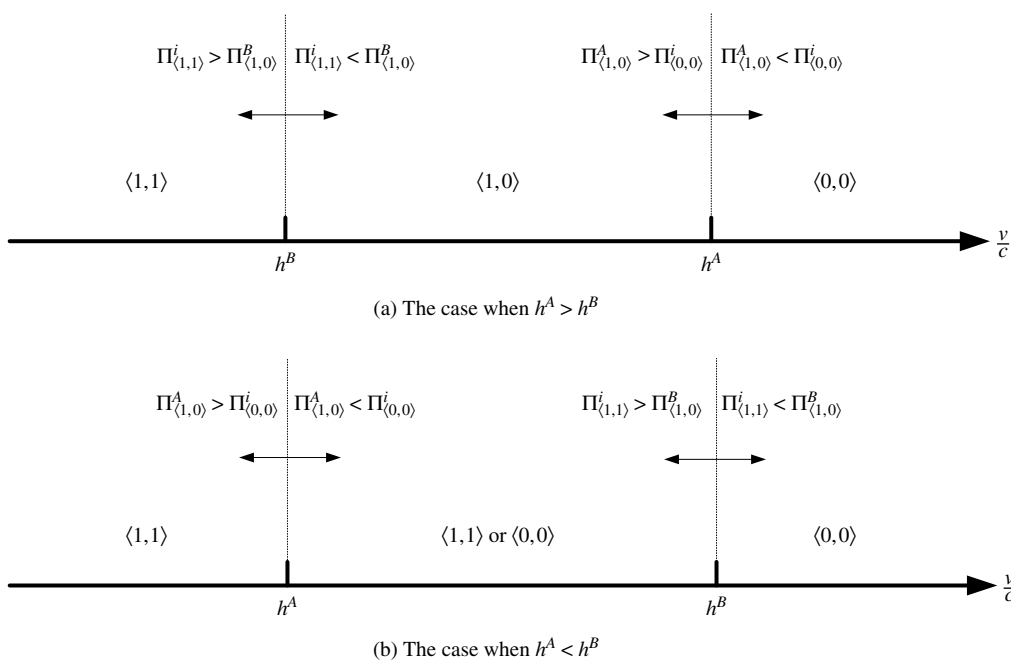
Interestingly, when $h^A > v/c > h^B$ (with an implied condition of $h^A > h^B$), there exists an asymmetric equilibrium in which two *ex ante* symmetric retailers endogenously differentiate in their choices of PMG; that is, one retailer adopts PMG while the other does not. The asymmetric equilibrium reflects a trade-off between the secondary competition-intensifying effect and the demand-expansion effect net of the primary competition-intensifying effect. Because the secondary competition-intensifying effect for the first PMG in the market is still weak, a retailer benefits from adopting PMG only if the competitor does not. However, if a retailer offers PMG when its competitor is already a PMG retailer, the secondary competition-intensifying effect it faces will be inevitably too strong. As a result, symmetric retailers adopt asymmetric price-matching strategies in equilibrium. This is illustrated in the middle section of Figure 3(a). Finally, when $h^B > v/c > h^A$, the secondary competition-intensifying effect is such that no retailers can benefit from unilaterally deviating to offer PMG but will do so if the competitor already pursued the PMG strategy. Thus, as shown in the middle section of Figure 3(b), both $\langle 1, 1 \rangle$ and $\langle 0, 0 \rangle$ are possible equilibria in this situation.

To sum up, region (I) highlights the value of endogenizing consumer search and considering the secondary competition-intensifying effect. If consumer search were not modeled endogenously, the upper bound in all subgames would be product valuation v ,

and the secondary competition-intensifying effect is essentially assumed away. The profitability of offering PMG is overestimated, and the equilibrium prediction would be $\langle 1, 1 \rangle$ regardless of the ratio of product valuation to search cost. We demonstrate that, as the ratio of product valuation to search cost increases, the equilibrium price-matching strategies shift from $\langle 1, 1 \rangle$ to $\langle 1, 0 \rangle$ and to $\langle 0, 0 \rangle$ in markets where the size of the shoppers and refundees are both small (i.e., $d_{(1,0)}^A > d_{(0,0)}^i$, $d_{(1,1)}^i > d_{(1,0)}^B$ and $h^A > h^B$). This offers an explanation of the variation in PMG practices across retail categories in Table 1; that is, we see all, some, and no retailers offering PMG in some categories. The higher the product valuation in a retail category, the greater the secondary competition-intensifying effect retailers in the category face, and the less likely they are to offer PMG.

Part (ii) of Proposition 5 deals with region (II) in which $d_{(1,0)}^A < d_{(0,0)}^i$ and $d_{(1,1)}^i > d_{(1,0)}^B$. This occurs when the size of shoppers is intermediate or when the sizes of shoppers and refundees are both large. In this region, the demand-expansion effect is dominated by the primary competition-intensifying effect for the first PMG retailer in the market (i.e., $d_{(1,0)}^A < d_{(0,0)}^i$). Thus, no retailer has an incentive to switch to the PMG strategy when its competitor is a non-PMG retailer. However, since $d_{(1,1)}^i > d_{(1,0)}^B$, the demand-expansion effect a retailer faces by offering PMG is stronger than the primary competition-intensifying effect when the competitor is already a PMG retailer. As a result, a retailer is better (worse) off by offering PMG when its competitor already does so when $v/c < h^B$ ($v/c > h^B$). Thus, both $\langle 0, 0 \rangle$ and $\langle 1, 1 \rangle$ are possible equilibria when the ratio of product valuation to search cost is low (i.e., $v/c < h^B$)

Figure 3. The Equilibrium Price-Matching Strategies in Region (I)



and $\langle 0, 0 \rangle$ is the unique equilibrium when the ratio of product valuation to search cost is high (i.e., $v/c > h^B$).

Lastly, region (III) arises when the size of shoppers is large while the size of refundees is small ($d_{(1,0)}^A < d_{(0,0)}^i$ and $d_{(1,1)}^i < d_{(1,0)}^B$). When the size of shoppers is large, the primary competition-intensifying effect is strong. Since the size of refundees is small, the demand-expansion effect is weak. In this region, the primary competition-intensifying effect dominates the demand-expansion effect and neither retailer has the incentive to offer PMG in equilibrium. The secondary competition-intensifying effect has no impact on the equilibrium outcome in this region.

As a robustness check, we also characterize the equilibrium outcome in extreme cases when each of the three consumer segments is set to zero. When the size of shoppers is zero, retailers have no incentive to compete on prices and thus charge the product valuation v in equilibrium. Since refundees still weakly prefer to purchase from a PMG retailer if there is any, both retailers offering PMG is a weakly dominant Nash equilibrium. When the size of refundees is zero, our model reduces to Stahl (1989). It is easy to see that both retailers are indifferent to offering PMG or not in a market without refundees. Finally, we consider the case when the size of nonshoppers is zero. In subgame $\langle 0, 0 \rangle$, both retailers follow a mixed pricing strategy and obtain positive profit because refundees behave like nonshoppers. In subgames $\langle 1, 0 \rangle$ and $\langle 1, 1 \rangle$, fierce competition for shoppers will drive the retail price and retail profit down to zero. Thus, it is a weakly dominant strategy for retailers not to offer PMG. The above discussion is summarized in Corollary 1.⁷

Corollary 1. *When the size of shoppers is zero, both retailers find offering PMG a weakly dominant strategy; when the size of refundees is zero, retailers are indifferent to offering PMG or not; and when the size of nonshoppers is zero, not offering PMG is a weakly dominant strategy for both retailers.*

One question left unaddressed is whether retailers can benefit from launching price check websites that facilitate consumers' postpurchase search. We posit that such price check websites essentially transform some nonshoppers into refundees by increasing the proportion of consumers who have low postpurchase search cost. In Result 1, we lay out the conditions under which retailers have the incentive to do so.

Result 1. In the asymmetric equilibrium, the profit of the PMG retailer first increases and then decreases with the size of refundees while holding the size of shoppers constant.

Our numerical analysis indicates that there exists an inverted-U relationship between the PMG retailer's profit and the size of refundees while holding the size

of shoppers constant. Thus, when the size of refundees is still small, retailers may benefit from providing price check websites to facilitate consumers' postpurchase search in the asymmetric equilibrium. On the one hand, the price check website increases the overall extent of consumer search and thus magnifies the primary and secondary competition-intensifying effects of PMG. On the other hand, the demand-expansion effect of PMG also becomes stronger as the size of refundees increases. We show that, when the size of refundees is small, the latter dominates the former and the PMG retailer is better off with the price check technology in the asymmetric equilibrium. At the same time, technology that facilitates consumers' postpurchase search works to the detriment of the non-PMG retailer in the asymmetric equilibrium because it not only intensifies price competition but also lowers the non-PMG retailer's demand. In addition, retailers in equilibrium $\langle 1, 1 \rangle$ do not find it profitable to offer the price check technology because it intensifies price competition without any contribution to the demand (both retailers still evenly split the market).

4. Price-Beating Guarantees

In this section we discuss how our model can be extended to the case of price-beating guarantees (PBGs). Following the previous literature (e.g., Hviid and Shaffer 2012, Corts 1997), we consider PBGs through which retailers beat competitors' prices by a percentage ρ ($\rho > 0$) of the price difference. Here ρ can be interpreted as the refund depth. In the special case when $\rho = 0$, our PBG model reduces to the PMG case.

In solving this game, we use backward induction as in Section 3. In stage 3, consumers' equilibrium shopping behavior characterized in Proposition 1 still holds in the case of PBG.⁸ In stage 2, while subgame $\langle 0, 0 \rangle$ remains unaffected, PBG influences the equilibrium pricing strategies in subgames $\langle 1, 1 \rangle$ and $\langle 1, 0 \rangle$ by shifting the effective price refundees pay. Given that retailer A is a PBG retailer, let us consider the profit of retailer A when it charges a price of p^A . The effective price refundees pay when they purchase from retailer A is $p^A - \max_{p^B \in \Sigma_S^B} \{(1 + \rho)(p^A - p^B), 0\}$ where p^B is the price charged by retailer B chosen from the strategy set Σ_S^B . If retailer A charges a lower price than B , then refundees still pay p^A . Otherwise, if retailer A is undercut by retailer B , refundees pay p^A less the refund, which is $(1 + \rho)(p^A - p^B)$. Then we can write down both retailers' profit functions in subgames $\langle 1, 1 \rangle$ and $\langle 1, 0 \rangle$ and solve for the equilibrium outcomes accordingly (see the proof of Lemma 7 in the appendix for details). This part of the result is reported in Lemma 7.

Lemma 7. In subgame $\langle 1, 1 \rangle$, when $\rho < \gamma/\beta$, the equilibrium pricing strategies of retailer i are given by

$$F_{\langle 1,1 \rangle}^i(p) = 1 - \frac{\gamma/2 - \rho(\beta/2)}{\alpha + (1 + \rho)(\beta/2)} \cdot \left[\left(\frac{u_{\langle 1,1 \rangle}}{p} \right)^{(\alpha + (1 + \rho)(\beta/2))/\alpha} - 1 \right],$$

for $p \in [l_{\langle 1,1 \rangle}, u_{\langle 1,1 \rangle}]$, (16)

where

$$u_{\langle 1,1 \rangle} = \min\{v, r_{\langle 1,1 \rangle}^i\} \quad \text{and}$$

$$l_{\langle 1,1 \rangle} = \left(\frac{\gamma/2 - \rho(\beta/2)}{\alpha + \beta/2 + \gamma/2} \right)^{\alpha/(\alpha + (1 + \rho)(\beta/2))} u_{\langle 1,1 \rangle}.$$

Here, $r_{\langle 1,1 \rangle}^i$ is the reservation price of retailer i . The profit of retailer i is given by

$$\Pi_{\langle 1,1 \rangle}^A = \left(\alpha + \frac{\beta}{2} + \frac{\gamma}{2} \right) \cdot \left(\frac{\gamma/2 - \rho(\beta/2)}{\alpha + \beta/2 + \gamma/2} \right)^{\alpha/(\alpha + (1 + \rho)(\beta/2))} u_{\langle 1,1 \rangle}. \quad (17)$$

Otherwise, if $\rho \geq \gamma/\beta$, retailer i charges a price of zero and obtains a profit of zero.

In subgame $\langle 1, 0 \rangle$ where only retailer A offers PMG, there exists a unique shopping strategy $\mu^*(\rho)$ such that $1 \geq \mu^*(\rho) \geq 0$ and $\partial \mu^*/\partial \rho \geq 0$. When $1 > \mu^*(\rho) \geq 0$, the equilibrium pricing strategies for both retailers are given by

$$\left\{ \begin{array}{l} F_{\langle 1,0 \rangle}^A(p) = 1 - \frac{(1 - \mu^*(\rho))\gamma}{\alpha} \left(\frac{u_{\langle 1,0 \rangle}}{p} - 1 \right) \\ \quad \text{for } p \in [l_{\langle 1,0 \rangle}, u_{\langle 1,0 \rangle}]; \\ F_{\langle 1,0 \rangle}^B(p) = \begin{cases} 1 & \text{at } p = u_{\langle 1,0 \rangle}, \\ \frac{\alpha + \beta + \mu^*(\rho)\gamma}{\alpha + (1 + \rho)\beta} \left[1 - \left(\frac{l_{\langle 1,0 \rangle}}{p} \right)^{(\alpha + (1 + \rho)\beta)/\alpha} \right] & \text{for } p \in [l_{\langle 1,0 \rangle}, u_{\langle 1,0 \rangle}), \end{cases} \end{array} \right. \quad (18)$$

where

$$u_{\langle 1,0 \rangle} = \min\{r_{\langle 1,0 \rangle}^B, v\} \quad \text{and}$$

$$l_{\langle 1,0 \rangle} = \frac{(1 - \mu^*(\rho))\gamma}{\alpha + (1 - \mu^*(\rho))\gamma} u_{\langle 1,0 \rangle}.$$

Here, $r_{\langle 1,0 \rangle}^B$ is the reservation price of retailer B and is defined in the appendix. The profits of retailer A and B are given by

$$\Pi_{\langle 1,0 \rangle}^A = \frac{(\alpha + \beta + \mu^*(\rho)\gamma)(1 - \mu^*(\rho))\gamma}{\alpha + (1 - \mu^*(\rho))\gamma} u_{\langle 1,0 \rangle} \quad \text{and} \quad (19)$$

$$\Pi_{\langle 1,0 \rangle}^B = (1 - \mu^*(\rho))\gamma u_{\langle 1,0 \rangle}.$$

Otherwise, if $\mu^*(\rho) = 1$, both retailers charge a price of zero and obtain a profit of zero.

Proof. See the appendix.

In subgame $\langle 1, 1 \rangle$, retailers can charge positive prices and obtain positive profits only if the refund depth is small (i.e., $\rho < \gamma/\beta$). This threshold value increases with the size of nonshoppers γ and decreases with the size of refundees β . This is intuitive because retailers' profits are positively related to the size of nonshoppers while more refundees implies a larger payment of refunds. Moreover, deeper refunds lower average retail prices and retail profits because retailers are penalized for charging a high price because refundees can claim larger refunds.

In subgame $\langle 1, 0 \rangle$, the refund depth has an impact on nonshoppers' shopping strategy. As the refund depth increases, retailer A (the PBG retailer) has less incentive to charge a high price and thus becomes more preferable to nonshoppers relative to retailer B (the non-PBG retailer). Thus, more nonshoppers will purchase from retailer A when the refund depth is larger, or equivalently, $\partial \mu^*/\partial \rho \geq 0$. As a result, retailer A 's average demand increases with the depth of refund while retailer B 's average demand decreases with the refund depth. This implies that PBG has a demand-expansion effect and this effect strengthens as the refund depth increases. This result is consistent with the experimental finding of Kukar-Kinney and Walters (2003) that a deeper refund can increase consumers' store patronage intentions. Similar to the above subgame, we find that the average retail prices and retail profits in subgame $\langle 1, 0 \rangle$ both decrease with the refund depth. When the refund depth is very large (i.e., $\mu^*(\rho) = 1$), the average retail prices and retail profits both drop to zero. In addition, in subgame $\langle 1, 0 \rangle$, the average price of retailer A is still weakly greater than that of retailer B .

As in the PMG case, PBG has a primary and secondary competition-intensifying effect on retail profits. Both dimensions of competition-intensifying effects are enhanced by the refund depth. Retailers decide whether to offer PBGs by balancing the demand-expansion effect and the primary and secondary competition-intensifying effect. Since retail profits are decreasing in the refund depth, the parameter space in which it is profitable to offer PBG shrinks as the refund depth grows larger. An immediate corollary is that PBGs are less profitable than PMGs. This result is in line with Hviid and Shaffer (1994) and Corts (1995), who also show that PBGs are less preferable to PMGs. When characterizing the equilibrium price-beating strategies, we find the nature of the equilibrium outcome is similar to the equilibrium price-matching strategies in the baseline model. Overall, our qualitative results still hold when the model is extended to the PBG case.

5. Concluding Remarks

It is a common practice for retailers to offer PMG accompanied with a grace period for consumers to search

after purchase. In this study we develop a theory to explain three stylized facts pertinent to PMG in the retail markets. First, many retailers that operate both online and offline offer PMG in their brick-and-mortar stores but not in their online stores. Second, PMG practices vary considerably across retail categories; that is, a majority of retailers offer PMG in some categories while few or none of the retailers offer PMG in other categories. Finally, in addition to offering PMG, some retailers have launched price check websites that allow consumers to easily check competitors' prices after purchase. This practice is counterintuitive because such websites reduce consumers' postpurchase search cost and increase the overall extent of consumer search.

To this end, we develop a game-theoretic model in which retailers decide whether or not to offer PMG and, depending on this decision, simultaneously set prices. Given retailers' price-matching and pricing strategies, consumers who are heterogeneous in pre- and postpurchase search costs make search and purchase decisions. By endogenizing consumer's pre- and postpurchase search, we find that there are three consumer segments, each formulating a different search and purchase strategy. Most importantly, we examine a segment of refundees, with high prepurchase search cost but low postpurchase search cost, who purchase from a PMG retailer (if one exists), and search actively postpurchase to take advantage of PMG.

We find that PMG expands retail demand but intensifies price competition on two dimensions. PMG has a primary competition-intensifying effect because it increases the overall extent of consumer search and thus drives retailers to offer deeper promotions. By modeling consumer search decisions endogenously, we also find a secondary competition-intensifying effect that limits the profitability of PMG. As deeper promotions incentivize consumers to continue search, retailers are forced to lower the "regular" price to deter consumer search. One key prediction drawn from this result is that both the promotion depth (in percentage term) and the regular price are lowered after the institution of PMG, compared with those before PMG. The ideal way to empirically test this insight would be a natural experiment in which a retailer plans to offer PMG. Then our prediction can be tested by comparing prices of a number of product categories over a period of time that spans both before and after the focal retailer offered PMG.

The strength of the secondary competition-intensifying effect increases with the ratio of product valuation to search cost. This key prediction is consistent with several retail practices. Given lower online search costs, online retailers face a stronger secondary competition-intensifying effect than their offline counterparts. This explains why many retailers offer PMG offline but not online, which is one of the puzzles we posed at

the beginning of this paper. We also asked why PMG varies across product categories in the introduction of this paper. Differences across categories in the ratio of product valuation to search costs provide an explanation for the variation in PMG practices across retail categories. Thus, we should only observe PMG in categories with a relatively low product valuation relative to search costs.

Retailers balance the demand-expansion effect and the primary and secondary competition-intensifying effects when choosing whether or not to implement PMG. There may exist an asymmetric equilibrium in which one retailer offers PMG and cultivates the refundee segment, while the other retailer does not. This helps to explain why not all retailers in a market offer PMG. The asymmetric equilibrium arises when the sizes of shoppers and refundees are both small and the ratio of product valuation to search cost is in the intermediate range.

Finally, we show that in the asymmetric equilibrium, the PMG retailer may benefit from launching a price check website to facilitate consumers' postpurchase search. Such price check websites transform some nonshoppers into refundees by lowering their search cost. Therefore, they increase both the demand-expansion effect and competition-intensifying effect of PMG. When the size of refundees is sufficiently small, retailers can benefit from such price check websites.

To sum up, we study retailers' decisions to offer PMGs in the context of endogenous consumer search. This allows us to identify a secondary competition-intensifying effect, in which competition leads to a lowering of the regular or nonpromoted price (the common upper bound of the price distributions of retailers in a market). Our results indicate that ignoring this effect will overestimate the profitability of offering PMG. Our results also provide an explanation for many stylized facts in retail markets where PMG is a viable option.

Acknowledgments

The authors thank the department editor, the associate editor, and two anonymous reviewers for their comments. They also thank Dmitri Kuksov, Frank May, Ram C. Rao, Rebecca Rabino, Upender Subramanian, and Daniel Villanova, as well as seminar participants at Cornell University, the University of Texas at Dallas, and Washington University in St. Louis for their comments. Summer research support by Naveen Jindal School of Management is gratefully acknowledged.

Appendix

Lemma 1. Let r_S^i be the reservation price of retailer i in subgame S . Both retailers' price distributions share a common upper bound $u_S = \min\{v, \min_{i=A,B} r_S^i\}$.

Proof of Lemma 1. First, it is easy to see that there is no hole within the price distribution. We note that retailers that

Table A.1. A Summary of Model Notations

Notation	Definition
i	Index of retailers, $i = A, B$
PM^i	Price-matching strategy of retailer i
S	Index of subgames that differ in retailers' price-matching strategies
c	Positive pre- (post-)purchase search cost for some consumers
v	Product valuation
p	An exogenous price quote of a retailer
z	Lowest price offer at hand at a certain search stage
$CS(p)$	Consumer surplus when paying an effective price of p
$ECS(z; F^i(p))$	Expected consumer surplus of searching from a price distribution $F^i(p)$ under z
α	Size of shoppers who have zero prepurchase search cost
β	Size of refundees who have positive prepurchase search cost and zero postpurchase search cost
γ	Size of nonshoppers who have positive pre- and postpurchase search cost; $\alpha + \beta + \gamma = 1$
p^i	Price charged by retailer i
$F_S^i(p)$	Mixed pricing strategy of retailer i in subgame S
u_S	Common equilibrium upper bound of both retailers' price distributions in subgame S
l_S	Common equilibrium lower bound of both retailers' price distributions in subgame S
Π_S^i	Profit of retailers of retailer i in subgame S
Σ_S^i	Pricing strategy set of retailer i in subgame S
μ	Probability that each nonshopper search and purchase from the PMG retailer
$1 - \mu$	Probability that each nonshopper search and purchase from the non-PMG retailer
r_S^i	Reservation price of $F_S^i(p)$ as the solution of $ECS(r_S^i; F_S^i(p)) = 0$
\bar{p}_S^i	Average price of retailer i in subgame S
d_S^i	Expected demand of retailer i when it charges a price at the upper bound u_S
h^i	Threshold value of the ratio of product valuation to search cost
ρ	Depth of refund for price-beating guarantee

charge a price at the upper bound only expect consumers with a prepurchase search cost of c to buy since consumers with zero prepurchase search cost can obtain price information from both retailers and can find a lower price from the competitor with probability 1.

(1) Suppose that the upper bound u_S is below $\min\{v, \min_{i=A, B} r_S^i\}$ or equivalently, $u_S < \min\{v, \min_{i=A, B} r_S^i\}$. Retailers can increase the price without reducing demand. Therefore, we can always find a price p' lying between u_S and $\min\{v, \min_{i=A, B} r_S^i\}$ ($u_S < p' < \min\{v, \min_{i=A, B} r_S^i\}$) that retailers can charge without sacrificing demand. It means that p' dominates p . Therefore, $u_S \geq \min\{v, \min_{i=A, B} r_S^i\}$.

(2) Suppose $\min_{i=A, B} r_S^i < v$ and $v > u_S > \min_{i=A, B} r_S^i$. When retailers charge a price at the upper bound u_S , those consumers who have pre- and postpurchase search costs of c will continue to sample the competitor before purchase. Note that consumers who sampled both retailers will directly purchase from the retailer with lower price. They will not invoke PMG even though they have the opportunity to do so; the reason for this will be explained when we discuss the pur-

chase behavior of shoppers. For consumers with prepurchase search cost of c and postpurchase of 0, there are two possibilities.

Case I. Consumers with prepurchase search cost of c and postpurchase of 0 sample both retailers before purchase and directly purchase from the retailer with lower price, no matter the focal retailer offers PMG or not.

In this case it is easy to see that it is never profitable for the retailer to charge a price at u_S because there is at least some probability that consumers with a prepurchase search cost of c find a lower price at the competing retailer and will not return, the demand drops suddenly as the upper bound goes slightly above $\min_{i=A, B} r_S^i$.

Case II. When the focal retailer offers PMG, consumers with prepurchase search cost of c and postpurchase of 0 stop searching before purchase, purchase from the focal retailer, and search the other retailer and ask for refund after purchase.

Suppose that the focal retailer charges a price at u_S . In this case, we can also find a price p' ($\min_{i=A, B} r_S^i < p' < u_S$). If the competitor charges a price p' , the focal retailer serves consumers with prepurchase search cost of c and postpurchase of 0 who pay the competitor's price p' and consumers with pre- and postpurchase search costs of c purchase from the competitor. However, if the focal retailer also charges a price at p' , consumers with prepurchase search cost of c and postpurchase of 0 still pay p' . Moreover, the focal retailer can also serve half of consumers with pre- and postpurchase search costs of c . In other words, p' dominates u_S .

In both possibilities, there is a drop in profits if retailers charge a price above $\min_{i=A, B} r_S^i$. Therefore, $u_S \leq \min_{i=A, B} r_S^i$.

(3) Suppose $v < \min_{i=A, B} r_S^i$ and $\min_{i=A, B} r_S^i > u_S > v$. When a retailer charges a price at u_S , no consumers can afford to buy since the price is set above the product valuation. Therefore, any price above v is dominated by a price at v . Therefore, $u_S \leq v$.

Hence, there exists a unique upper bound for $F_S^i(p)$ in subgame S ($i = A, B$), $u_S = \min\{v, \min_{i=A, B} r_S^i\}$. Q.E.D.

Proposition 1. Given assumptions on consumers' (pre- and post-purchase) search cost, the optimal consumer search and purchase behavior for each consumer segment is as follows.

(i) Nonshoppers search exactly once, choose the retailer with the (weakly) lower reservation price, and purchase at the price charged by the retailer.

(ii) In the absence of PMG, refundees act like nonshoppers. In the presence of PMG, refundees purchase from a PMG retailer. After purchase, they take advantage of costless postpurchase search and ask for a refund if a lower price is found.

(iii) Shoppers search both retailers before purchase and purchase directly from the one that charges the lower price.

Proof of Proposition 1. Given that the pricing strategy is mixed, we link consumers' optimal search rule to their search and purchase decisions.

(1) Nonshoppers (consumers who have positive pre- and postpurchase search cost) follow the reservation price rule to search for one retailer for free. Since it is not optimal to continue to search before purchase as is shown in Lemma 1, they directly purchase from the sampled retailer. After purchase, nonshoppers in equilibrium do not search either.

(2) Refundees (consumers who have positive prepurchase search cost and zero postpurchase search cost) have different strategies in the absence and presence of PMG. In subgame $\langle 0, 0 \rangle$, refundees randomly purchase from a retailer and pay the price charged by the retailer. In subgame $\langle 1, 1 \rangle$, refundees also randomly purchase from a retailer and pay the price charged by the retailer. After purchase, refundees have zero search cost and will continue to search the other retailer. If a lower price is found, refundees will invoke the PMG and redeem the price difference. In subgame $\langle 1, 0 \rangle$, refundees need to choose which retailer to purchase from, the PMG retailer or the non-PMG retailer. Anticipating that they can utilize costless postpurchase search by patronizing the PMG retailer, refundees purchase from the PMG retailer. After purchase, refundees continue to search the other retailer and invoke the PMG if a lower price is found.

(3) Shoppers (consumers who have zero pre- and post-purchase search cost) search both retailers before purchase. In subgame $\langle 0, 0 \rangle$, our model reduces to Stahl (1989); hence, shoppers will directly purchase from the retailer with lower price. Therefore, we focus next on their purchase decisions in the presence of PMG (in subgame $\langle 1, 0 \rangle$ and $\langle 1, 1 \rangle$). In the presence of PMG, shoppers can manage to pay the lowest market price in two ways: (1) purchase from the low-price retailer and (2) purchase from a PMG retailer and behave like refundees. However, these two types of purchase behavior will yield qualitatively different market outcomes. The second option is strictly dominated since the PMG retailer will increase the price knowing that shoppers will purchase no matter what price it charges. Because of strategic complementarity effect, the other retailer will also inflate the price. Consider the extreme case when all shoppers behave like refundees. In this case, it is a Nash equilibrium for both retailers to charge a price at the product valuation v . Shoppers are strictly worse off by behaving like refundees. This implies that the only reason retailers lower the price below the product valuation is to compete for the shoppers. Q.E.D.

Lemma 2. When neither retailer offers PMG, the equilibrium price distribution for retailer i is given by

$$F_{(0,0)}^i(p) = 1 - \frac{\beta + \gamma}{2\alpha} \left(\frac{u_{(0,0)}}{p} - 1 \right), \quad \text{for } p \in [l_{(0,0)}, u_{(0,0)}], \quad (20)$$

where $u_{(0,0)} = \min\{r_{(0,0)}^i, v\}$ and $l_{(0,0)} = ((\beta + \gamma)/2)/((\alpha + (\beta + \gamma)/2))u_{(0,0)}$. $r_{(0,0)}^i$ is the reservation price of retailer i and is defined below. The profit of retailer i is given by

$$\Pi_{(0,0)}^i = \frac{(\beta + \gamma)u_{(0,0)}}{2}. \quad (21)$$

Proof of Lemma 2. Suppose that there already exists an exogenous upper bound $u_{(0,0)}$. Given $u_{(0,0)}$, $F_{(0,0)}^i(p; u_{(0,0)})$ can be derived for each retailer from the regularity condition $F_{(0,0)}^i(u_{(0,0)}) = 1$. Hence, we have $F_{(0,0)}^i(p; u_{(0,0)}) = 1 - ((\beta + \gamma)/(2\alpha))(u_{(0,0)}/p - 1)$. From the other regularity condition $F_{(0,0)}^i(l_{(0,0)}) = 0$, we can establish the relationship between the lower bound $l_{(0,0)}$ and the upper bound $u_{(0,0)}$ as $l_{(0,0)} = (((\beta + \gamma)/2)/(\alpha + (\beta + \gamma)/2))u_{(0,0)}$. Given $F_{(0,0)}^i(p; u_{(0,0)})$, we can derive the equilibrium reservation price of retailer i in subgame S , $r_{(0,0)}^i$. Then, following Lemma 1, $u_{(0,0)} = \min\{v, r_{(0,0)}^i\}$.

Next we discuss how to compute $r_{(0,0)}^i$ for retailer i . We consider two possible cases.

Case I. $r_{(0,0)}^i < v$.

When $r_{(0,0)}^i < v$, the upper bound is $u_{(0,0)} = r_{(0,0)}^i$. Recall that $r_{(0,0)}^i = \arg_z \{ECS(z; F_{(0,0)}^i(p)) - c = 0\}$. Thus, the following equation must hold that

$$\bar{r}_{(0,0)}^i = \arg_z \left\{ \int_{((\beta + \gamma)/2)/(\alpha + (\beta + \gamma)/2)\bar{r}_{(0,0)}^i}^{\bar{r}_{(0,0)}^i} F_{(0,0)}^i(p) dp - c = 0 \right\}.$$

There exists a unique $\bar{r}_{(0,0)}^i$ that satisfies the above equation: $\bar{r}_{(0,0)}^i = 2\alpha c / (2\alpha + (\beta + \gamma) \ln((\beta + \gamma)/(2\alpha + \beta + \gamma)))$.

Thus, when $\bar{r}_{(0,0)}^i < v$, $r_{(0,0)}^i = \bar{r}_{(0,0)}^i$.

Case II. $r_{(0,0)}^i > v$.

When $r_{(0,0)}^i > v$, the upper bound is $u_{(0,0)} = v$. In other words, the reservation price does not exist within the convex interval of $[(\beta + \gamma)/2/(\alpha + (\beta + \gamma)/2)v, v]$. In other words, consumers with search cost c do not need to follow a reservation price rule because they will never find a price above $r_{(0,0)}^i$. To facilitate our analysis that follows, we impute $r_{(0,0)}^i = \bar{r}_{(0,0)}^i$ when $r_{(0,0)}^i > v$. Note here that this will not affect our comparison of reservation prices across subgames in Lemma 5 and relating $r_{(0,0)}^i$ to the average price $\bar{p}_{(0,0)}^i$ and the upper bound in Lemma 6. This further gives rise to the equilibrium upper and lower bound as $u_{(0,0)} = \min\{r_{(0,0)}^i, v\}$ and $l_{(0,0)} = (((\beta + \gamma)/2)/(\alpha + (\beta + \gamma)/2))u_{(0,0)}$ where $r_{(0,0)}^i = 2\alpha c / (2\alpha + (\beta + \gamma) \ln((\beta + \gamma)/(2\alpha + \beta + \gamma)))$. The profit of retailer i is $\Pi_{(0,0)}^i = (\beta + \gamma)u_{(0,0)}/2$. Q.E.D.

Lemma 3. When both retailers offer PMG, the equilibrium price distribution for retailer i is given by

$$F_{(1,1)}(p) = 1 - \frac{\gamma/2}{\alpha + \beta/2} \left[\left(\frac{u_{(1,1)}}{p} \right)^{(\alpha + \beta/2)/\alpha} - 1 \right], \quad \text{for } p \in [l_{(1,1)}, u_{(1,1)}], \quad (22)$$

where $u_{(1,1)} = \min\{r_{(1,1)}^i, v\}$ and

$$l_{(1,1)} = \left(\frac{\gamma/2}{\alpha + (\beta + \gamma)/2} \right)^{\alpha/(\alpha + \beta/2)} u_{(1,1)}.$$

Here, $r_{(1,1)}^i$ is the reservation price of retailer i and is defined below. The profit of retailer i is given by

$$\Pi_{(1,1)}^i = \left(\alpha + \frac{\beta + \gamma}{2} \right) \left(\frac{\gamma/2}{\alpha + (\beta + \gamma)/2} \right)^{\alpha/(\alpha + \beta/2)} u_{(1,1)}. \quad (23)$$

Proof of Lemma 3. From the equiprofit condition, $\Pi_{(1,1)}^A(p^A) = \Pi_{(1,1)}^B(u_{(1,1)})$ so we have

$$\begin{aligned} & (\alpha + \beta/2)p^A [1 - F_{(1,1)}^B(p^A)] \\ &= (\gamma/2)(u_{(1,1)} - p^A) + (\beta/2) \int_{p^A}^{u_{(1,1)}} p^B dF_{(1,1)}^B(p^B), \end{aligned}$$

in which

$$\int_{p^A}^{u_{(1,1)}} p^B dF_{(1,1)}^B(p^B) = u_{(1,1)} - p^A F_{(1,1)}^B(p^A) - \int_{p^A}^{u_{(1,1)}} F_{(1,1)}^B(p^B) dp^B$$

by using the chain rule. Then the above equation can be rewritten as

$$\begin{aligned} & (\alpha + \beta/2 + \gamma/2)p^A - \alpha p^A F_{(1,1)}^B(p^A) \\ &= (\beta/2 + \gamma/2)u_{(1,1)} - (\beta/2) \int_{p^A}^{u_{(1,1)}} F_{(1,1)}^B(p^B) dp^B. \end{aligned}$$

Solving the above differential equation, we have

$$F_{(1,1)}^i(p) = 1 - \frac{\gamma/2}{\alpha + \beta/2} \left[(u_{(1,1)}/p)^{(\alpha+\beta/2)/\alpha} - 1 \right].$$

From $F_{(1,1)}^i(l_{(1,1)}) = 0$, we can establish the relationship between $l_{(1,1)}$ and $u_{(1,1)}$ as

$$l_{(1,1)} = \left(\frac{\gamma/2}{\alpha + \beta/2 + \gamma/2} \right)^{\alpha/(\alpha+\beta/2)} u_{(1,1)}.$$

Similar to the proof of Lemma 2, $r_{(1,1)}^i$ is the unique solution of

$$\int_{((\gamma/2)/(\alpha+\beta/2+\gamma/2))^{\alpha/(\alpha+\beta/2)} r_{(1,1)}^i}^{r_{(1,1)}^i} F_{(1,1)}^i(p) dp - c = 0.$$

Hence, $r_{(1,1)}^i = \beta c / ((\beta + \gamma) - (2\alpha + \beta + \gamma)^{\beta/(2\alpha+\beta)} \gamma^{(2\alpha)/(2\alpha+\beta)})$. Therefore, the equilibrium upper and lower bound is $u_{(1,1)} = \min\{r_{(1,1)}^i, v\}$ and

$$l_{(1,1)} = \left(\frac{\gamma/2}{\alpha + (\beta + \gamma)/2} \right)^{\alpha/(\alpha+\beta/2)} u_{(1,1)}.$$

The profit of retailer i is

$$\Pi_{(1,1)}^i = \left(\alpha + \frac{\beta + \gamma}{2} \right) \left(\frac{\gamma/2}{\alpha + (\beta + \gamma)/2} \right)^{\alpha/(\alpha+\beta/2)} u_{(1,1)}. \quad \text{Q.E.D.}$$

Lemma 4. When only retailer A offers PMG, there exists a unique μ^* such that $0.5 > \mu^* > 0$ when the size of refundees is small ($\beta < \bar{\beta}$) and $\mu^* = 0$ when the size of refundees is large ($\beta \geq \bar{\beta}$). Given μ^* , the equilibrium pricing strategies for both retailers are given by

$$\begin{cases} F_{(1,0)}^A(p) = 1 - \frac{(1 - \mu^*)\gamma}{\alpha} \left(\frac{u_{(1,0)}}{p} - 1 \right), \\ \quad \text{for } p \in [l_{(1,0)}, u_{(1,0)}]; \\ F_{(1,0)}^B(p) = \begin{cases} 1 & \text{at } p = u_{(1,0)}, \\ \frac{\alpha + \beta + \mu^*\gamma}{\alpha + \beta} \left[1 - \left(\frac{l_{(1,0)}}{p} \right)^{(\alpha+\beta)/\alpha} \right], \\ \quad \text{for } p \in [l_{(1,0)}, u_{(1,0)}], \end{cases} \end{cases} \quad (24)$$

where $u_{(1,0)} = \min\{r_{(1,0)}^B, v\}$ and $l_{(1,0)} = ((1 - \mu^*)\gamma / (\alpha + (1 - \mu^*)\gamma)) u_{(1,0)}$. Here, $r_{(1,0)}^B$ is the reservation price of retailer i and is defined below. The profits of retailer A and B are given by

$$\begin{aligned} \Pi_{(1,0)}^A &= \frac{(1 - \mu^*)\gamma(\alpha + \beta + \mu^*\gamma)}{\alpha + (1 - \mu^*)\gamma} u_{(1,0)} \quad \text{and} \\ \Pi_{(1,0)}^B &= (1 - \mu^*)\gamma u_{(1,0)}. \end{aligned} \quad (25)$$

Proof of Lemma 4. Suppose there already exists a common upper bound $u_{(1,0)}$. There is at most one retailer who can charge the upper bound with a mass point. Given an exogenous upper bound $u_{(1,0)}$ and nonshoppers' search strategy μ , we first calculate the lowest acceptable prices of retailer A and B as $\underline{p}_{(1,0)}^A = (\mu\gamma / (\alpha + \beta + \mu\gamma))^{\alpha/(\alpha+\beta)} u_{(1,0)}$ and $\underline{p}_{(1,0)}^B = ((1 - \mu)\gamma / (\alpha + (1 - \mu)\gamma)) u_{(1,0)}$, respectively. An endogenously determined μ will, in turn, have an influence on the equilibrium price distribution. Next we discuss the following two cases: (1) $\underline{p}_{(1,0)}^A \geq \underline{p}_{(1,0)}^B$ and (2) $\underline{p}_{(1,0)}^A < \underline{p}_{(1,0)}^B$.

Case I. $\underline{p}_{(1,0)}^A \geq \underline{p}_{(1,0)}^B$.

Here, $\underline{p}_{(1,0)}^A \geq \underline{p}_{(1,0)}^B$ suggests that the following three regularity conditions must hold in equilibrium: $F_{(1,0)}^B(u_{(1,0)}) = 1$, $F_{(1,0)}^B(l_{(1,0)}) = 0$, and $F_{(1,0)}^A(l_{(1,0)}) = 0$. Then we have

$$F_{(1,0)}^A(p; \mu) = \begin{cases} 1 & \text{at } p = u_{(1,0)}, \\ \frac{\alpha + (1 - \mu)\gamma}{\alpha} \left[1 - \left(\frac{\mu\gamma}{\alpha + \beta + \mu\gamma} \right)^{\alpha/(\alpha+\beta)} \frac{u_{(1,0)}}{p} \right], \\ \quad \text{for } p \in [l_{(1,0)}, u_{(1,0)}], \end{cases}$$

and

$$F_{(1,0)}^B(p; \mu) = 1 - \frac{\mu\gamma}{\alpha + \beta} \left[\left(\frac{u_{(1,0)}}{p} \right)^{(\alpha+\beta)/\alpha} - 1 \right].$$

The lower bound is

$$l_{(1,0)}(\mu) = (\mu\gamma / (\alpha + \beta + \mu\gamma))^{\alpha/(\alpha+\beta)} u_{(1,0)}(\mu).$$

Then we can construct a consistent reservation price $r_{(1,0)}^i(\mu)$ for retailer i 's price distribution $F_{(1,0)}^i(p; \mu)$. Given that $r_{(1,0)}^A(\mu)$ is a strictly increasing function in μ and $r_{(1,0)}^B(\mu)$ is a strictly decreasing function of μ , there exists a unique $\mu^\#$ that satisfies $r_{(1,0)}^A(\mu^\#) = r_{(1,0)}^B(\mu^\#)$. Numerically solving this equation shows that $\underline{p}_{(1,0)}^A < \underline{p}_{(1,0)}^B$ for $\mu^\#$, which contradicts our supposition.

Case II. $\underline{p}_{(1,0)}^A < \underline{p}_{(1,0)}^B$.

Here, $\underline{p}_{(1,0)}^A < \underline{p}_{(1,0)}^B$ means that the following three regularity conditions hold in equilibrium: $F_{(1,0)}^A(u_{(1,0)}) = 1$, $F_{(1,0)}^A(l_{(1,0)}) = 0$, and $F_{(1,0)}^B(l_{(1,0)}) = 0$. Then we solve for retailers' pricing strategies given by $F_{(1,0)}^A(p | \mu) = 1 - ((1 - \mu)\gamma / \alpha)(u_{(1,0)} / p - 1)$ and

$$F_{(1,0)}^B(p | \mu) = \begin{cases} 1 & \text{at } p = u_{(1,0)}, \\ \frac{\alpha + \beta + \mu\gamma}{\alpha + \beta} \left[1 - \left(\frac{l_{(1,0)}}{p} \right)^{(\alpha+\beta)/\alpha} \right], \\ \quad \text{for } p \in [l_{(1,0)}, u_{(1,0)}]. \end{cases}$$

Thus, the equilibrium lower bound is given by $l_{(1,0)} = ((1 - \mu)\gamma / (\alpha + (1 - \mu)\gamma)) u_{(1,0)}$. Then we compute the reservation prices of both retailers as a function of μ , and we have $r_{(1,0)}^A(\mu) = \alpha c / (\alpha + (1 - \mu)\gamma \ln((1 - \mu)\gamma / (\alpha + (1 - \mu)\gamma)))$ and $r_{(1,0)}^B(\mu) = (\beta(\alpha + \beta)[\alpha + \gamma(1 - \mu)]c) / (\alpha(\alpha + \beta + \mu\gamma)\{\beta - \gamma(1 - \mu)[1 - (\gamma(1 - \mu) / (\alpha + \gamma(1 - \mu)))^{\beta/\alpha}]\})$. Moreover, it is easy to show that $r_{(1,0)}^A(\mu)$ and $r_{(1,0)}^B(\mu)$ are both strictly decreasing in μ . Together with $r_{(1,0)}^A(1) = c > r_{(1,0)}^B(1) = c(\alpha + \beta)$, it is sufficient to show that there exists a unique $\mu^\nabla(\alpha, \beta)$ that satisfies $r_{(1,0)}^A(\mu^\nabla) = r_{(1,0)}^B(\mu^\nabla)$, i.e., $\mu^\nabla(\alpha, \beta) = \arg\{r_{(1,0)}^A(\mu) = r_{(1,0)}^B(\mu)\}$. Given μ^∇ , we find that there exists a $\bar{\beta} = \arg\{\mu^\nabla(\beta) = 0\}$ such that $0.5 > \mu^\nabla \geq 0$ when $\beta \leq \bar{\beta}$ and $\mu^\nabla < 0$ when $\beta > \bar{\beta}$. Since nonshoppers' search strategy μ is bounded between zero and one, the equilibrium search strategy of nonshoppers is given by $\mu^* = \max\{0, \mu^\nabla\}$.

Given μ^* , the equilibrium pricing strategies of both retailers are given by $F_{(1,0)}^A(p) = 1 - ((1 - \mu^*)\gamma/\alpha)(u_{(1,0)}/p - 1)$ and

$$F_{(1,0)}^B(p) = \begin{cases} 1 & \text{at } p = u_{(1,0)}, \\ \frac{\alpha + \beta + \mu^*\gamma}{\alpha + \beta} \left[1 - \left(\frac{l_{(1,0)}}{p} \right)^{(\alpha+\beta)/\alpha} \right], & \text{for } p \in [l_{(1,0)}, u_{(1,0)}], \end{cases}$$

where the upper bound is $u_{(1,0)} = \min\{r_{(1,0)}^B, v\}$ and the lower bound is $l_{(1,0)} = ((1 - \mu^*)\gamma/(\alpha + (1 - \mu^*)\gamma))u_{(1,0)}$. Take μ^* into $r_{(1,0)}^A(\mu)$ and $r_{(1,0)}^B(\mu)$, we have

$$r_{(1,0)}^A(\mu^*) = \frac{\alpha c}{\alpha + (1 - \mu^*)\gamma \ln((1 - \mu^*)\gamma/(\alpha + (1 - \mu^*)\gamma))}$$

and

$$r_{(1,0)}^B(\mu^*) = \frac{\beta(\alpha + \beta)[\alpha + \gamma(1 - \mu^*)c]}{\alpha(\alpha + \beta + \mu^*\gamma)\{\beta - \gamma(1 - \mu^*)[1 - (\gamma(1 - \mu^*)/(\alpha + \gamma(1 - \mu^*)))^{\beta/\alpha}]\}}.$$

The equilibrium profits of both retailers are given by $\Pi_{(1,0)}^A = (((1 - \mu^*)\gamma(\alpha + \beta + \mu^*\gamma))/(\alpha + (1 - \mu^*)\gamma))u_{(1,0)}$ and $\Pi_{(1,0)}^B = (1 - \mu^*)\gamma u_{(1,0)}$, respectively. Q.E.D.

Proposition 2. When only retailer A offers PMG, retailer A has a larger demand than retailer B.

Proof of Proposition 2. The total expected demand of retailer A is given by

$$D_{(1,0)}^A = \alpha \int_{l_{(1,0)}}^{u_{(1,0)}} [1 - F_{(1,0)}^B(p)] dF_{(1,0)}^A(p) + \beta + \mu^*\gamma.$$

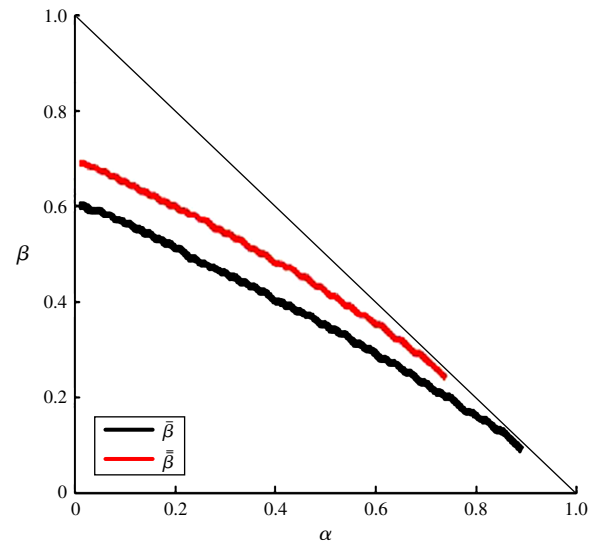
The first component is the demand from shoppers. Retailer A serves all the refundees and can obtain all the shoppers with probability $\int_{l_{(1,0)}}^{u_{(1,0)}} [1 - F_{(1,0)}^B(p)] dF_{(1,0)}^A(p)$ and a proportion of μ^* the nonshoppers. Similarly, the total expected demand of retailer B is given by $D_{(1,0)}^B = \alpha \int_{l_{(1,0)}}^{u_{(1,0)}} [1 - F_{(1,0)}^A(p)] dF_{(1,0)}^B(p) + (1 - \mu^*)\gamma$. The difference in the expected demand of both retailers is given by $D_{(1,0)}^A - D_{(1,0)}^B = (\int_{l_{(1,0)}}^{u_{(1,0)}} [1 - F_{(1,0)}^A(p)] dF_{(1,0)}^B(p) - \int_{l_{(1,0)}}^{u_{(1,0)}} [1 - F_{(1,0)}^B(p)] dF_{(1,0)}^A(p))\alpha + \beta + (2\mu^* - 1)\gamma > 0$. Thus, we have $D_{(1,0)}^A > D_{(1,0)}^B$. Q.E.D.

Lemma 5. When the size of refundees is not too large ($\beta < \bar{\beta}$ where $\bar{\beta} > \bar{\beta}$ is defined in the appendix), the following relationship holds that $r_{(1,1)}^i < r_{(1,0)}^B \leq r_{(1,0)}^A < r_{(0,0)}^i$.

Proof of Lemma 5. Reservation prices across retailers and across subgames are reported in Table 3. We first show $r_{(1,0)}^A < r_{(0,0)}^i$. Let $G(x) = \alpha c/(\alpha + x \ln(x/(\alpha + x)))$ be a function of x . Furthermore, it can be shown that $G(x)$ is increasing in x , $dG/dx > 0$. It is easy to see that $r_{(0,0)}^i = G((\beta + \gamma)/2) > r_{(1,0)}^A = G((1 - \mu^*)\gamma)$ because $(\beta + \gamma)/2 > (1 - \mu^*)\gamma$. The relationship that $r_{(1,0)}^B \leq r_{(1,0)}^A$ has been established in the proof of Lemma 4. In the end we compare $r_{(1,1)}^i$ and $r_{(1,0)}^B$ and find that there exists a $\bar{\beta}$ ($1 > \bar{\beta} > \beta$) such that $r_{(1,0)}^B > r_{(1,1)}^i$ when $\beta < \bar{\beta}$ and $r_{(1,0)}^B < r_{(1,1)}^i$ when $\beta > \bar{\beta}$. We plot $\bar{\beta}$ and $\bar{\beta}$ on a two-dimensional plane with α and β as axes.

Next we discuss the market equilibrium when $\beta > \bar{\beta}$, which represents the region above the red curve in Figure A.1. In this region we have $r_{(1,0)}^B < r_{(1,1)}^i < r_{(1,0)}^A < r_{(0,0)}^i$. It can be easily

Figure A.1. (Color online) A Plot of $\bar{\beta}$ and $\bar{\beta}$



seen that the only difference from our main model is that the order between $r_{(1,0)}^B$ and $r_{(1,1)}^i$ is reversed.

Given $r_{(1,0)}^B < r_{(1,0)}^A < r_{(0,0)}^i$, it is easy to see that the average promotion depth has increased as retailer A deviates to offer PMG, $1 - \bar{p}_{(1,0)}^i/u_{(1,0)} > 1 - \bar{p}_{(0,0)}^i/u_{(0,0)}$. This implies that retailer A's PMG has a primary competition-intensifying effect on both retailers' profits (when retail B is a non-PMG retailer). Moreover, since $r_{(1,0)}^B < r_{(0,0)}^i$, it holds that $u_{(0,0)} = \min\{v, r_{(0,0)}^i\} \geq u_{(1,0)} = \min\{v, r_{(1,0)}^B\}$. This means that there is potentially a secondary competition-intensifying effect of retailer A's PMG by shifting the regular price downward.

When $\beta > \bar{\beta}$, the effect of retailer B's PMG on the price competition has changed. Because $r_{(1,0)}^B < r_{(1,1)}^i < r_{(1,0)}^A$, retailer A's average promotion depth increased but retailer B's average promotion depth decreased as retailer B also offers PMG, $1 - \bar{p}_{(1,1)}^i/u_{(1,1)} > 1 - \bar{p}_{(1,0)}^i/u_{(1,0)}$ and $1 - \bar{p}_{(1,1)}^i/u_{(1,1)} < 1 - \bar{p}_{(1,0)}^i/u_{(1,0)}$, respectively. This means that retailer B's PMG has a primary competition-intensifying effect on retailer A but has a primary competition-dampening effect on retailer B (when retailer A is already a PMG retailer). Given that $r_{(1,0)}^B < r_{(1,1)}^i$, $u_{(1,0)} = \min\{v, r_{(1,0)}^B\} \leq u_{(1,1)} = \min\{v, r_{(1,1)}^i\}$. This means that retailer B's PMG has a secondary competition-dampening effect on both retailers by increasing the regular price.

Based on the above discussion, it can be shown that when $\beta > \bar{\beta}$, it is a dominant strategy for retailer B to offer PMG when retailer A is already a PMG retailer. This is because, by offering PMG, retailer B charges a higher price and obtains a large expected demand than before. Since $r_{(1,1)}^i < r_{(0,0)}^i$, the average price in subgame $\langle 1, 1 \rangle$ is still lower than in $\langle 0, 0 \rangle$. Given that retailers have the same expected demand in both subgames, we conclude that retailers in subgame $\langle 1, 1 \rangle$ obtain a lower profit than in $\langle 0, 0 \rangle$, $\Pi_{(1,1)}^i < \Pi_{(0,0)}^i$. A corollary of this finding is that if $\langle 1, 1 \rangle$ is the equilibrium, then it is a prisoner's dilemma.

The equilibrium price-matching strategies we characterize later in Proposition 4 take into account the case of $\beta > \bar{\beta}$. To simplify our exposition, we mainly focus on the case when $\beta \leq \bar{\beta}$ in the baseline model. Q.E.D.

Lemma 6. Let \bar{p}_S^i be the average price of retailer i ($i = A, B$) in subgame S . Then the following relationship holds: $c/r_S^i = 1 - \bar{p}_S^i/u_S$.

Proof of Lemma 6. The average price \bar{p}_S^i can be rewritten as $\bar{p}_S^i = \int_{l_{(0,0)}^i}^{u_{(0,0)}^i} p f_S^i(p) dp = u_S - \int_{l_{(0,0)}^i}^{u_{(0,0)}^i} F_S^i(p) dp$. To prove Lemma 6, we only need to show that $\int_{l_{(0,0)}^i}^{u_{(0,0)}^i} F_S^i(p) dp = cu_S/r_S^i$. To facilitate our analysis, let us take, for example, subgame $\langle 0, 0 \rangle$ where $F_{(0,0)}^i(p) = 1 - ((\beta + \gamma)/2\alpha)(u_{(0,0)}^i/p - 1)$ and $l_{(0,0)}^i = (((\beta + \gamma)/2)/(\alpha + (\beta + \gamma)/2))u_{(0,0)}^i$.

Recall that $r_{(0,0)}^i$ satisfies

$$\int_{(((\beta+\gamma)/2)/(\alpha+(\beta+\gamma)/2))r_{(0,0)}^i}^{r_{(0,0)}^i} [1 - ((\beta + \gamma)/2\alpha)(r_{(0,0)}^i/p - 1)] dp = c.$$

The LHS can be further rewritten as

$$r_{(0,0)}^i \int_{((\beta+\gamma)/2)/(\alpha+(\beta+\gamma)/2)}^1 [1 - ((\beta + \gamma)/(2\alpha))(x - 1)] dx$$

where $x = r_{(0,0)}^i/p$. This means that we have

$$\int_{((\beta+\gamma)/2)/(\alpha+(\beta+\gamma)/2)}^1 [1 - ((\beta + \gamma)/(2\alpha))(x - 1)] dx = c/r_{(0,0)}^i.$$

Thus, we have

$$\begin{aligned} & \int_{(((\beta+\gamma)/2)/(\alpha+(\beta+\gamma)/2))u_{(0,0)}^i}^{u_{(0,0)}^i} F_{(0,0)}^i(p) dp \\ &= u_{(0,0)}^i \int_{((\beta+\gamma)/2)/(\alpha+(\beta+\gamma)/2)}^1 [1 - (\beta + \gamma)/(2\alpha)(x - 1)] dx \\ &= cu_{(0,0)}^i/r_{(0,0)}^i. \end{aligned}$$

The result of subgame $\langle 1, 0 \rangle$ and $\langle 1, 1 \rangle$ can be shown in the similar way. Q.E.D.

Proposition 3. PMG intensifies price competition on two dimensions. On one hand, PMG has a primary competition-intensifying effect that it induces retailers to offer deeper promotions than before. On the other hand, there is a secondary competition-intensifying effect of PMG that the regular price is lowered.

Proof of Proposition 3. The proof of Proposition 3 follows directly from Lemma 6. Q.E.D.

Proposition 4. An adoption of PMG is beneficial if and only if (1) $d_{S'}^i > d_S^i$ and (2) $v/c < h^i(\alpha, \beta)$, where $h^i(\alpha, \beta) = r_{S'}^i d_{S'}^i / (c d_S^i)$ is a function of consumer segment sizes.

Proof of Proposition 4. Retailer i will deviate to offer PMG if and only if $\Delta\Pi^i = \Pi_{S'}^i - \Pi_S^i = u_{S'} d_{S'}^i - u_S d_S^i > 0$ where S and S' are subgames before and after retailer i offers PMG, respectively. From Proposition 3, we have $u_{S'} \leq u_S$. Thus, if $d_{S'}^i < d_S^i$ then $\Delta\Pi^i < 0$ so retailer i will not deviate to offer

PMG. Thus, $d_{S'}^i > d_S^i$ is necessary for retailer i 's deviation to be profitable. Substituting $u_S = \min\{v, \min_{i=A,B} r_S^i\}$ and $u_{S'} = \min\{v, \min_{i=A,B} r_{S'}^i\}$ into $\Delta\Pi^i$, we find that $\Delta\Pi^i > 0$ when $v/c < h^i = r_{S'}^i d_{S'}^i / (c d_S^i)$ and $\Delta\Pi^i < 0$ when $v/c > h^i$. Q.E.D.

Proposition 5. The equilibrium price-matching strategies are given by the following:

(i) When $d_{(1,0)}^A > d_{(0,0)}^i$ and $d_{(1,1)}^i > d_{(1,0)}^B$, $\langle 1, 1 \rangle$ is the equilibrium if $v/c < \min_i h^i$; $\langle 1, 0 \rangle$ is the equilibrium if $h^A > v/c > h^B$; both $\langle 1, 1 \rangle$ and $\langle 0, 0 \rangle$ are equilibria if $h^B > v/c > h^A$; $\langle 0, 0 \rangle$ is the equilibrium if $v/c > \max_i h^i$.

(ii) When $d_{(1,0)}^A < d_{(0,0)}^i$ and $d_{(1,1)}^i > d_{(1,0)}^B$, both $\langle 0, 0 \rangle$ and $\langle 1, 1 \rangle$ are equilibria if $v/c < h^B$; $\langle 0, 0 \rangle$ is the equilibrium if $v/c > h^B$.

(iii) When $d_{(1,0)}^A < d_{(0,0)}^i$ and $d_{(1,1)}^i < d_{(1,0)}^B$, $\langle 0, 0 \rangle$ is the equilibrium.

Proof of Proposition 5. To prove this proposition, we will evaluate whether conditions (1) and (2) in Proposition 4 are satisfied when retailer A deviates to offer PMG from $\langle 0, 0 \rangle$ to $\langle 1, 0 \rangle$ and retailer B deviates to offer PMG from $\langle 1, 0 \rangle$ and $\langle 1, 1 \rangle$.

Region (I): $d_{(1,0)}^A > d_{(0,0)}^i$ and $d_{(1,1)}^i > d_{(1,0)}^B$.

In this region, condition (1) in Proposition 4 is satisfied for both retailers' deviation. Thus, both retailers' decisions to deviate depend on condition (2). When $v/c < \min_i h^i$, condition (2) is satisfied for both retailers so $\langle 1, 1 \rangle$ is the unique equilibrium. When $v/c > \max_i h^i$, condition (2) is satisfied for neither retailer so $\langle 0, 0 \rangle$ is the unique equilibrium. When $h^A > v/c > h^B$, condition (2) is satisfied for retailer A but not retailer B so $\langle 1, 0 \rangle$ is the unique equilibrium. When $h^B > v/c > h^A$, condition (2) is satisfied for retailer B but not retailer A so both $\langle 1, 1 \rangle$ and $\langle 0, 0 \rangle$ are possible equilibria. The above discussion is summarized in Table A.2.

Region (II): $d_{(1,0)}^A < d_{(0,0)}^i$ and $d_{(1,1)}^i > d_{(1,0)}^B$.

In this region, condition (1) is satisfied for retailer B but not for retailer A . Thus, retailer A desires not to innovatively offer PMG while retailer B 's deviation depends on condition (2). When $v/c < h^B$, condition (2) is satisfied for retailer B so both $\langle 1, 1 \rangle$ and $\langle 0, 0 \rangle$ are possible equilibria. When $v/c > h^B$, condition (2) is not satisfied for retailer B so $\langle 0, 0 \rangle$ is the unique equilibrium. The above discussion is summarized in Table A.3.

Region (III): $d_{(1,0)}^A < d_{(0,0)}^i$ and $d_{(1,1)}^i < d_{(1,0)}^B$.

Since condition (1) is not satisfied for both retailers, $\langle 0, 0 \rangle$ is the unique equilibrium. Q.E.D.

Result 1. In the asymmetric equilibrium, the profit of the PMG retailer first increases and then decreases as more non-shoppers are transformed into refundees.

Table A.2. Equilibrium When $d_{(1,0)}^A > d_{(0,0)}^i$ and $d_{(1,1)}^i > d_{(1,0)}^B$

	When $h^A > h^B$			When $h^A < h^B$		
	$v/c < h^B$	$h^A > v/c > h^B$	$v/c > h^A$	$v/c < h^A$	$h^B > v/c > h^A$	$v/c > h^B$
$\Pi_{(1,0)}^A - \Pi_{(0,0)}^i$	+	+	-	+	-	-
$\Pi_{(1,1)}^i - \Pi_{(1,0)}^B$	+	-	-	+	+	-
Equilibrium	$\langle 1, 1 \rangle$	$\langle 1, 0 \rangle$	$\langle 0, 0 \rangle$	$\langle 1, 1 \rangle$	$\langle 1, 1 \rangle, \langle 0, 0 \rangle$	$\langle 0, 0 \rangle$

Note. "+" means "greater than zero"; "-" means "lower than zero."

Table A.3. Equilibrium When $d_{(1,0)}^A < d_{(0,0)}^i$ and $d_{(1,1)}^i > d_{(1,0)}^B$

	$v/c < h^B$	$v/c > h^B$
$\Pi_{(1,0)}^A - \Pi_{(0,0)}^i$	–	–
$\Pi_{(1,1)}^i - \Pi_{(1,0)}^B$	+	–
Equilibrium	$\langle 1, 1 \rangle, \langle 0, 0 \rangle$	$\langle 0, 0 \rangle$

Note. “+” means “greater than zero”; “–” means “lower than zero.”

Proof of Result 1. Ideally, we would demonstrate Result 1 in the following way. First, we fix α and v/c , to derive the parameter space of β in which the asymmetric equilibrium arises, $\beta \in (\beta_{\min}, \beta_{\max})$. Then we evaluate how $\Pi_{(1,0)}^A$ changes when β shifts within the parameter space $\beta \in (\beta_{\min}, \beta_{\max})$. However, as can be seen in Proposition 5, the asymmetric equilibrium arises when $h^A > v/c > h^B$, where h^i in itself is a function of β . This makes analytically evaluating $\partial \Pi_{(1,0)}^A / \partial \beta$ impossible. Therefore, we resort to numerical method to prove Result 1. Note that the parameter space of consumer segment sizes is a compact set: $\alpha > 0, \beta > 0$, and $\alpha + \beta < 1$. This means that we can find numerical values to span the whole compact set. We first draw α from 0 to 1 with a step size of 0.01, and then for each α we draw β from 0 to $1 - \alpha$ with a step size of 0.01. Also note that v does not affect the existence of asymmetric equilibrium for any value above $r_{(0,0)}^i$ or below $r_{(1,1)}^i$, which is a function of α and β only. This means that the ratio of product valuation to search cost v/c is also drawn from an interval set. Thus, for each α and β , we draw v from $r_{(1,1)}^i$ to $r_{(0,0)}^i$ at a step size of 0.1. For each set of α, β , and v/c , we can determine the parameter space in which the asymmetric equilibrium arises (at which two conditions must hold are $\Pi_{(1,0)}^A > \Pi_{(0,0)}^i$ and $\Pi_{(1,1)}^i > \Pi_{(1,0)}^B$). Within the parameter space, we then evaluate $\Pi_{(1,0)}^A$ as β increases. We find that when the asymmetric equilibrium arises, $\Pi_{(1,0)}^A$ is first increasing and then decreasing in β while holding α constant. Thus, the PMG retailer has the incentive to transform nonshoppers into refundees when β is small (within the parameter space in which the asymmetric equilibrium arises).

Next we provide several numerical examples of the parameter values in the following table. For each pair of α and v/c in columns 1 and 2, we report the range of β in which the asymmetric equilibrium arises $[\beta_{\min}, \beta_{\max}]$ in column 3, the size of refundees at which $\Pi_{(1,0)}^A$ is maximized β^* in column 4, and $\Pi_{(1,0)}^A$ evaluated at β_{\min}, β^* , and β_{\max} in columns 5–7. Although this result holds for all parameter values, in Table A.4 we only present the six sets of parameter values: (a) $\alpha = 0.2$ and $v/c = 4$; (b) $\alpha = 0.1$ and $v/c = 8$; (c) $\alpha = 0.08$ and $v/c = 12$; (d) $\alpha = 0.05$ and $v/c = 16$; (e) $\alpha = 0.04$ and $v/c = 20$; and (f) $\alpha = 0.03$ and $v/c = 30$. These numerical

examples are also illustrated in Figure A.2 in which $\Pi_{(1,0)}^A$ is plotted against β . Q.E.D.

Lemma 7. In subgame $\langle 1, 1 \rangle$, when $\rho < \gamma/\beta$, the equilibrium pricing strategies of retailer i is given by

$$F_{(1,1)}^i(p) = 1 - \frac{\gamma/2 - \rho(\beta/2)}{\alpha + (1 + \rho)(\beta/2)} \left[\left(\frac{u_{(1,1)}}{p} \right)^{(\alpha + (1 + \rho)(\beta/2))/\alpha} - 1 \right],$$

for $p \in [l_{(1,1)}, u_{(1,1)}]$, (26)

where $u_{(1,1)} = \min\{v, r_{(1,1)}^i\}$ and $l_{(1,1)} = ((\gamma/2 - \rho(\beta/2))/(\alpha + \beta/2 + \gamma/2))^{\alpha/(\alpha + (1 + \rho)(\beta/2))} u_{(1,1)}$. $r_{(1,1)}^i$ is the reservation price of retailer i . The profit of retailer i is given by

$$\Pi_{(1,1)}^A = \left(\alpha + \frac{\beta}{2} + \frac{\gamma}{2} \right) \cdot \left(\frac{\gamma/2 - \rho(\beta/2)}{\alpha + \beta/2 + \gamma/2} \right)^{\alpha/(\alpha + (1 + \rho)(\beta/2))} u_{(1,1)}. \quad (27)$$

Otherwise, if $\rho \geq \gamma/\beta$, retailer i charges a price of zero and obtains a profit of zero.

In subgame $\langle 1, 0 \rangle$ where only retailer A offers PMG, there exists a unique shopping strategy $\mu^*(\rho)$ such that $1 \geq \mu^*(\rho) \geq 0$ and $\partial \mu^* / \partial \rho \geq 0$. When $1 > \mu^*(\rho) \geq 0$, the equilibrium pricing strategies for both retailers are given by

$$\begin{cases} F_{(1,0)}^A(p) = 1 - \frac{(1 - \mu^*(\rho))\gamma}{\alpha} \left(\frac{u_{(1,0)}}{p} - 1 \right), \\ \text{for } p \in [l_{(1,0)}, u_{(1,0)}]; \\ F_{(1,0)}^B(p) = \begin{cases} 1 & \text{at } p = u_{(1,0)}, \\ \frac{\alpha + \beta + \mu^*(\rho)\gamma}{\alpha + (1 + \rho)\beta} \left[1 - \left(\frac{l_{(1,0)}}{p} \right)^{(\alpha + (1 + \rho)\beta)/\alpha} \right], & \text{for } p \in [l_{(1,0)}, u_{(1,0)}], \end{cases} \end{cases} \quad (28)$$

where $u_{(1,0)} = \min\{r_{(1,0)}^B, v\}$ and $l_{(1,0)} = ((1 - \mu^*(\rho))\gamma)/(\alpha + (1 - \mu^*(\rho))\gamma) u_{(1,0)}$. Here, $r_{(1,0)}^B$ is the reservation price of retailer B and is defined in the appendix. The profits of retailer A and B are given by

$$\Pi_{(1,0)}^A = \frac{(\alpha + \beta + \mu^*(\rho)\gamma)(1 - \mu^*(\rho))\gamma}{\alpha + (1 - \mu^*(\rho))\gamma} u_{(1,0)} \quad \text{and} \quad (29)$$

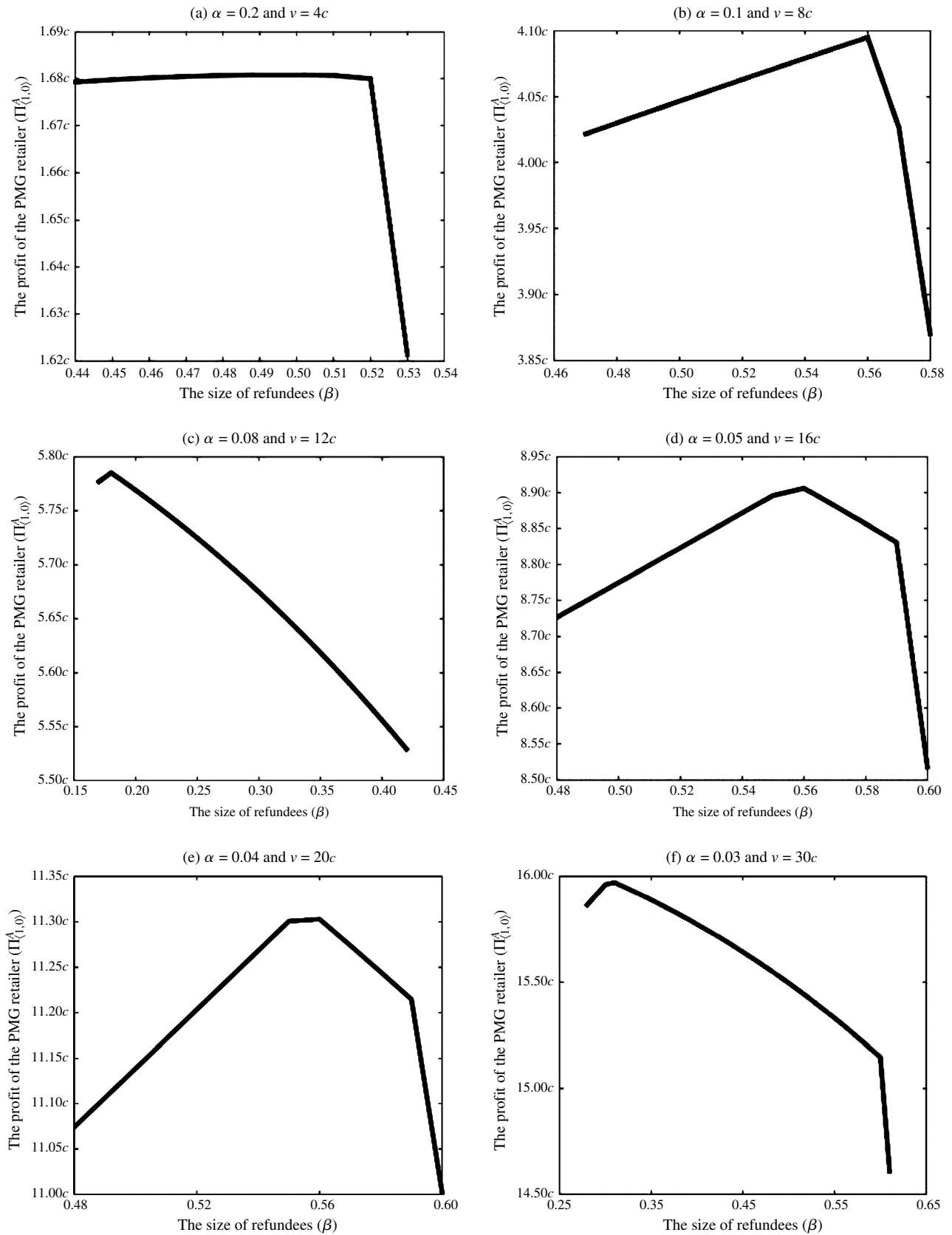
$$\Pi_{(1,0)}^B = (1 - \mu^*(\rho))\gamma u_{(1,0)}.$$

Otherwise, if $\mu^*(\rho) = 1$, both retailers charge a price of zero and obtain a profit of zero.

Table A.4. Numerical Examples of Parameter Values

α	v/c	$\beta \in [\beta_{\min}, \beta_{\max}]$	β^*	$\Pi_{(1,0)}^A(\beta = \beta_{\min})$	$\Pi_{(1,0)}^A(\beta = \beta^*)$	$\Pi_{(1,0)}^A(\beta = \beta_{\max})$
0.20	4	[0.44, 0.53]	0.49	1.6793c	1.6808c	1.6214c
0.10	8	[0.33, 0.56]	0.39	3.8442c	3.8894c	3.6655c
0.08	12	[0.17, 0.42]	0.18	5.7771c	5.7853c	5.5291c
0.05	16	[0.48, 0.60]	0.56	8.7262c	8.9063c	8.5180c
0.04	20	[0.48, 0.60]	0.56	11.0744c	11.3029c	11.0021c
0.03	30	[0.28, 0.61]	0.31	15.8647c	15.9705c	14.6076c

Figure A.2. $\Pi_{(1,0)}^A$ as a Function of β



Downloaded from informs.org by [73.152.126.154] on 21 December 2017, at 18:46. For personal use only, all rights reserved.

Proof of Lemma 7. First, consider subgame $\langle 1, 1 \rangle$ in which both retailers offer PBGs. The expected profit of retailer A when it charges a price p^A is

$$\text{E}\Pi_{(1,1)}^A(p^A) = \{\alpha[1 - F_{(1,1)}^B(p^A)] + \gamma/2\}p^A + (\beta/2) \cdot \text{E}[p^A - \max_{p^B \in \Sigma_{(1,1)}^B} \{(1 + \rho)(p^A - p^B), 0\}].$$

Three regularity conditions that hold in equilibrium are $F_{(1,0)}^A(u_{(1,0)}) = 1$, $F_{(1,0)}^A(l_{(1,0)}) = 0$, and $F_{(1,0)}^B(l_{(1,0)}) = 0$. As in the proof of Lemma 3, we work out the model solution by solving the differential equation given the above three regularity conditions. We find that when $\rho < \gamma/\beta$, the equilibrium mixed pricing strategies are given by

$$F_{(1,1)}^i(p) = 1 - \frac{\gamma/2 - \rho(\beta/2)}{\alpha + (1 + \rho)(\beta/2)} \left[\left(\frac{u_{(1,1)}}{p} \right)^{(\alpha + (1 + \rho)(\beta/2))/\alpha} - 1 \right].$$

The upper and lower bound is $u_{(1,1)} = \min\{v, r_{(1,1)}^i\}$ and $l_{(1,1)} = ((\gamma/2 - \rho(\beta/2))/(\alpha + \beta/2 + \gamma/2))^{\alpha/(\alpha + (1 + \rho)(\beta/2))} u_{(1,1)}$ where $r_{(1,1)}^i$ is the reservation price. The profit of retailer i is

$$\Pi_{(1,1)}^A = \left(\alpha + \frac{\beta}{2} + \frac{\gamma}{2} \right) \left(\frac{\gamma/2 - \rho(\beta/2)}{\alpha + \beta/2 + \gamma/2} \right)^{\alpha/(\alpha + (1 + \rho)(\beta/2))} u_{(1,1)}.$$

When the depth of refund is sufficiently high (i.e., $\rho \geq \gamma/\beta$), both retailers charge a price of zero and have zero profit.

Next we examine subgame $\langle 1, 0 \rangle$ in which retailer A offers PBG while retailer B does not. As in the baseline model, we use μ to denote nonshoppers' shopping strategy; that is, a proportion of μ nonshoppers purchase from retailer A and the remaining ones patronize retailer B . The expected profit of retailer A is

$$\text{E}\Pi_{(1,0)}^A(p^A) = \{\alpha[1 - F_{(1,0)}^B(p^A)] + \mu\gamma\}p^A + \beta \text{E}[p^A - \max_{p^B \in \Sigma_{(1,0)}^B} \{(1 + \rho)(p^A - p^B), 0\}]$$

in which p^B is the price charged by retailer B chosen from the strategy set $\Sigma_{(1,0)}^B$. The expected profit of retailer B when it charges a price p^B is given by $\text{E}\Pi_{(1,0)}^B(p^B) = \{\alpha[1 - F_{(1,0)}^A(p^B)] + (1 - \mu)\gamma\}p^B$. Given nonshoppers' shopping strategy μ , we first calculate the lowest acceptable prices of both retailers as $\underline{p}_{(1,0)}^A = ((\mu\gamma - \rho\beta)/(\alpha + \beta + \mu\gamma))^{\alpha/(\alpha + (1 + \rho)\beta)} u_{(1,0)}$ and $\underline{p}_{(1,0)}^B = ((1 - \mu)\gamma/(\alpha + (1 - \mu)\gamma)) u_{(1,0)}$, respectively. An endogenously determined μ will, in turn, have an influence on the equilibrium price distribution. Recall that in the baseline model, we show that in equilibrium $\underline{p}_{(1,0)}^B > \underline{p}_{(1,0)}^A$. This relationship still holds in the case of PBGs because deeper refund lowers $\underline{p}_{(1,0)}^A$ but has no impact on $\underline{p}_{(1,0)}^B$ (i.e., $\partial \underline{p}_{(1,0)}^A / \partial \rho < 0$ and $\partial \underline{p}_{(1,0)}^B / \partial \rho = 0$). Thus, the following three regularity conditions hold in equilibrium: $F_{(1,0)}^A(u_{(1,0)}) = 1$, $F_{(1,0)}^A(l_{(1,0)}) = 0$, and $F_{(1,0)}^B(l_{(1,0)}) = 0$. Then we have $F_{(1,0)}^A(p|\mu) = 1 - ((1 - \mu)\gamma/\alpha)(u_{(1,0)}/p - 1)$ and

$$F_{(1,0)}^B(p|\mu) = \begin{cases} 1 & \text{at } p = u_{(1,0)}, \\ \frac{\alpha + \beta + \mu\gamma}{\alpha + (1 + \rho)\beta} \left[1 - \left(\frac{l_{(1,0)}}{p} \right)^{(\alpha + (1 + \rho)\beta)/\alpha} \right] & \\ \text{for } p \in [l_{(1,0)}, u_{(1,0)}). \end{cases}$$

Thus, the equilibrium lower bound is given by $l_{(1,0)} = ((1 - \mu)\gamma/(\alpha + (1 - \mu)\gamma)) u_{(1,0)}$. Given μ , we can construct a consistent reservation price $r_{(1,0)}^i(\mu)$ for retailer i 's price distribution $F_{(1,0)}^i(p)$. Moreover, both $r_{(1,0)}^A(\mu)$ and $r_{(1,0)}^B(\mu)$ are strictly decreasing in μ and $r_{(1,0)}^B(\mu)$ is a strictly increasing function of μ . Together with $r_{(1,0)}^A(1) < r_{(1,0)}^B(1)$, it is sufficient to show that there exists at most a unique μ^∇ that satisfies $r_{(1,0)}^A(\mu^\nabla) = r_{(1,0)}^B(\mu^\nabla)$. Since it is bounded between 0 and 1, the optimal search strategy of nonshoppers is given by $\mu^* = \min\{1, \max\{0, \mu^\nabla\}\}$; μ^* is a function of consumer segment sizes (α and β) and the depth of refund ρ . We further find that $\partial \mu^* / \partial \rho > 0$, which means that more nonshoppers purchase from retailer A when the depth of refund increases. This result is straightforward, given that retailer A will incur a large amount of refund if it charges a high price. To highlight the effect of the depth of refund on μ^* , we write $\mu^*(\rho)$ as a function of ρ only without causing any confusion. Given $\mu^*(\rho)$, we can derive the equilibrium reservation price $r_{(1,0)}^i$ for retailer i . The equilibrium upper bound is $u_{(1,0)} = \min\{r_{(1,0)}^B, v\}$. Hence, we have $l_{(1,0)} = ([1 - \mu^*(\rho)]\gamma/(\alpha + [1 - \mu^*(\rho)]\gamma)) u_{(1,0)}$. The profit of retailer A is $\Pi_{(1,0)}^A = ([1 - \mu^*(\rho)]\gamma(\alpha + \beta + \mu^*(\rho)\gamma))/(\alpha + [1 - \mu^*(\rho)]\gamma) u_{(1,0)}$ and the profit of retailer B is $\Pi_{(1,0)}^B = [1 - \mu^*(\rho)]\gamma u_{(1,0)}$. Q.E.D.

Endnotes

¹Source: <http://www.officedepot.com/renderStaticPage.do?file=/customerservice/lowPrice.jsp> (accessed January 2014).

²Internet Retailer takes into account manufacturers' online direct channel such as www.apple.com, www.dell.com, etc. Our data suggests that PMGs with a grace period after purchase are the mainstream form. In this data only 28 online retailers explicitly offer PMGs with a grace period.

³Our model yields two key empirical predictions that are different from those of the signaling theory of PMG. First, the signaling theory of PMG suggests that, in equilibrium, consumers do not need to collect price information in the presence of PMG after purchase and PMG is never invoked. This is counter to the UK supermarket industry example above. Anecdotal evidence also suggests that consumers do search for price information after purchase and invoke PMG if they can. Second, the signaling theory of PMG implies that the PMG retailer charges a lower price than the non-PMG retailer. However, Arbatskaya et al. (2006) find that PMG retailers usually charge a price at least as high as non-PMG retailers. This implies that there is a need for a theory to complement the signaling theory of PMG. In our paper, the refundee segment is actively engaged in postpurchase search and invokes the PMG if they find a lower price. The PMG retailer who can attract all refundees charges a price weakly greater than the non-PMG retailer in the asymmetric equilibrium. Our theory, therefore, serves to complement the signaling theory of PMG.

⁴See Stahl (1989, p. 702) for a discussion of this point. As we will show later, because consumers with zero search cost will sample both retailers, retailers' pricing will be in mixed strategies.

⁵This can be seen as follows. The reservation price r_s^i can be written as the product of search cost c and a function of consumer segment sizes as is in Table 3 so the ratio of reservation prices to search cost r_s^i/c is a function of consumer segment sizes only. Since d_s^i is a function of consumer segment sizes, h^i is also a function of consumer segment sizes.

⁶There does not exist a region in which both $d_{(1,0)}^A > d_{(0,0)}^i$ and $d_{(1,1)}^i < d_{(1,0)}^B$ hold.

⁷Corollary 1 shows that all three consumer segments played an important role in retailers' decisions to offer PMG or not. We thank an anonymous reviewer for suggesting this analysis.

⁸In the PMG case, shoppers get the minimum price regardless of whether they purchase from the PMG retailer or the lower-priced retailer. Therefore, there is no incentive to deviate from their equilibrium search strategy. However, in the PBG case the shoppers have an incentive to purchase from the PBG retailer because in the event that the non-PBG retailer's price is lower they can get a price strictly lower than the minimum price they would obtain if they purchased from the lower priced retailer. Consequently, it may seem that the equilibrium search strategy is not deviation proof. However, if shoppers were to commit to purchasing from the PBG retailer, they essentially act like loyal customers and so neither retailer has an incentive to compete for them. In equilibrium, both retailers will charge a price at the product valuation v and shoppers are strictly worse off. Forward-looking shoppers can anticipate this outcome and have an incentive to commit ex-ante to purchase from the lower priced retailer. In this section we assume that shoppers are able to credibly precommit to directly purchasing from the retailer with the lower price. In other words, their shopping behavior does not depend on whether or not the retailer offers PBG.

References

Arbatskaya M, Hviid M, Shaffer G (2004) On the incidence and variety of low-price guarantees. *J. Law Econom.* 47(1):307–332.

Arbatskaya M, Hviid M, Shaffer G (2006) On the use of low-price guarantees to discourage price cutting. *Internat. J. Indust. Organ.* 24(6):1139–1156.

Asda (2011) Asda returns to like-for-like market out-performance. (February 22), <http://your.asda.com/press-centre/asda-returns-to-like-for-like-market-out-performance>.

Baye MR, Kovenock D (1994) How to sell a pickup truck: "Beat-or-pay" advertisements as facilitating devices. *Internat. J. Indust. Organ.* 12(1):21–33.

Chen Z (1995) How low is a guaranteed-lowest-price? *Canadian J. Econom.* 28(3):683–701.

Chen Y, Narasimhan C, Zhang ZJ (2001) Research note: Consumer heterogeneity and competitive price-matching guarantees. *Marketing Sci.* 20(3):300–314.

Corts KS (1995) On the robustness of the argument that price-matching is anti-competitive. *Econom. Lett.* 47(3–4):417–421.

Corts KS (1997) On the competitive effects of price-matching policies. *Internat. J. Indust. Organ.* 15(3):283–299.

Coughlan AT, Shaffer G (2009) Price-matching guarantees, retail competition, and product-line assortment. *Marketing Sci.* 28(3):580–588.

Dutta S, Biswas A (2005) Effects of low price guarantees on consumer post-purchase search intention: The moderating roles of value consciousness and penalty level. *J. Retailing* 81(4):283–291.

Dutta S, Biswas A, Grewal D (2011) Regret from postpurchase discovery of lower market prices: Do price refunds help? *J. Marketing* 75(6):124–138.

Hay GA (1982) Oligopoly, shared monopoly, and antitrust law. *Cornell Law Rev.* 67(3):439–481.

Hess JD, Gerstner E (1991) Price-matching policies: An empirical case. *Managerial Decision Econom.* 12(4):305–315.

Hviid M, Shaffer G (1994) Do low-price guarantees facilitate collusion? Working paper, University of Michigan, Ann Arbor.

Hviid M, Shaffer G (1999) Hassle costs: The Achilles' heel of price-matching guarantees. *J. Econom. Management Strategy* 8(4):489–521.

Hviid M, Shaffer G (2012) Optimal low-price guarantees with anchoring. *Quant. Marketing Econom.* 10(4):393–417.

Jain S, Srivastava J (2000) An experimental and theoretical analysis of price-matching refund policies. *J. Marketing Res.* 37(3):351–362.

Janssen MCW, Parakhonyak A (2013) Price matching guarantees and consumer search. *Internat. J. Indust. Organ.* 31(1):1–11.

Kukar-Kinney M, Walters RG (2003) Consumer perceptions of refund depth and competitive scope in price-matching guarantees: Effects on store patronage. *J. Retailing* 79(3):153–160.

Kukar-Kinney M, Grewal D (2006) Consumer willingness to claim a price-matching refund: A look into the process. *J. Bus. Res.* 59(1):11–18.

Kukar-Kinney M, Walters RG, MacKenzie SB (2007) Consumer responses to characteristics of price-matching guarantees: The moderating role of price consciousness. *J. Retailing* 83(2):211–221.

Kuksov D (2004) Buyer search costs and endogenous product design. *Marketing Sci.* 23(4):490–499.

Lim N, Ho T-H (2008) A theory of lowest-price guarantees. Working paper, University of Houston, Houston.

Logan JW, Lutter RW (1989) Guaranteed lowest prices: Do they facilitate collusion? *Econom. Lett.* 31(2):189–192.

Manez JA (2006) Unbeatable value low-price guarantee: Collusive mechanism or advertising strategy? *J. Econom. Management Strategy* 15(1):143–166.

McWilliams B, Gerstner E (2006) Offering low price guarantees to improve customer retention. *J. Retailing* 82(2):105–113.

Moorthy S, Winter RA (2006) Price-matching guarantees. *RAND J. Econom.* 37(2):449–465.

Moorthy S, Zhang X (2006) Price matching by vertically differentiated retailers: Theory and evidence. *J. Marketing Res.* 43(2):156–167.

Moorthy S, Ratchford BT, Talukdar D (1997) Consumer information search revisited: Theory and empirical analysis. *J. Consumer Res.* 23(4):263–277.

Morgan P, Manning R (1985) Optimal search. *Econometrica* 53(4):923–944.

Narasimhan C (1988) Competitive promotional strategies. *J. Bus.* 61(4):427–449.

Png IPL, Hirshleifer D (1987) Price discrimination through offers to match price. *J. Bus.* 60(3):365–383.

Ratchford BT (2009) Consumer search behavior and its effect on markets. *Foundations Trends in Marketing* 3(1):1–74.

Russell B (2011) Is Walmart's new Christmas price guarantee good for tech-lovers? *Techno Buffalo* (October 30), <http://www.technobuffalo.com/tag/black-friday-2011/page/2/>.

Salop SC (1986) Practices that (credibly) facilitate oligopoly coordination. Stiglitz J, Mathewson GF, eds. *New Development in the Analysis of Market Structure* (MIT Press, Cambridge, MA), 265–294.

Srivastava J, Lurie N (2001) A consumer perspective on price-matching refund policies: Effect on price perceptions and search behavior. *J. Consumer Res.* 28(2):296–307.

Stahl II, DO (1989) Oligopolistic pricing with sequential consumer search. *Amer. Econom. Rev.* 79(4):700–712.

Yankelevich A, Vaughan B (2016) Price-matching announcements in a consumer search duopoly. *Southern Econom. J.* 82(4):1186–1211.

Yuan H, Krishna A (2011) Price-matching guarantees with endogenous search: A market experiment approach. *J. Retailing* 87(2):182–193.

Weitzman ML (1979) Optimal search for the best alternative. *Econometrica* 47(3):641–654.

Zhang ZJ (1995) Price-matching policy and the principle of minimum differentiation. *J. Indust. Econom.* 43(3):287–299.