

DUAL-POLARIZATION METEOROLOGICAL RADAR:

THEORY AND DESIGN

by

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Chapter I

INTRODUCTION

Dual polarization radars have become a valuable tool in the study of precipitation. Seliga and Bringi, [1976] have developed a differential reflectivity technique and McCormick and Hendry [1968] have developed a technique for measuring circular depolarization ratio. Both methods provide more data on the hydrometeors in the radar volume, than that which can be obtained with a single polarization radar.

This thesis investigates both of these techniques and present a method for measuring linear depolarization ratio to characterize more accurately the effects of raindrops in a satellite to earth propagation path. The radar can provide range gated data on the precipitation structure along the path, and a dual polarized radar can measure of the depolarizing properties of the hydrometeors. The logical conclusion of such measurements, if they can be made with sufficient accuracy, is that prediction of attenuation and depolarization on a slant path from a satellite, at any frequency, could be made from radar data alone. This would remove the need for a beacon transmission from a satellite in order to assemble slant path propagation statistics, and could considerably reduce the cost and difficulty of making

such measurements, since suitable beacon transmissions from satellites are not always available.

A more exact formulation of the differential reflectivity relationship developed by Seliga and Bringi [1976] is given, which includes the effect of elevation angle and raindrop canting angle. The cross-polarized backscatter from oblate spheroidal raindrops is analyzed in a similar fashion to derive a relationship between linear depolarization ratio (LDR) measured in the vertical-horizontal planes, and ± 45 degree planes with a dual-channel, dual linearly polarized radar and the parameters of the rain-filled medium. Also, circular polarization is considered and relationships between circular depolarization ratio (CDR) and the parameters of the rain-filled medium are investigated. These relationships are shown to be equivalent to those produced by McCormick and Hendry [1976].

The single scattering matrix for an oblate raindrop is derived first, and the result is used to calculate the back-scattered return from a collection of scatterers, in the form of a matrix radar equation. This radar equation is then applied to the problem of extracting drop size distribution and canting angle from the measurement values obtained with a dual polarization radar. The matrix radar equation developed here is general, and can be applied to

any dual polarized radar using linear, circular or elliptical polarization.

The statistical character of the backscattered return from a collection of scatterers is examined to provide insight into the design of a radar data collection system capable of logging and analyzing dual-polarization radar data. Finally, the dual-polarization radar development program underway at VPI&SU is described. Also, a detailed description of a new radar data collection system, needed to make dual-polarization radar measurements is given.

Chapter II

REVIEW OF DUAL-POLARIZATION RADAR TECHNIQUES

This chapter reviews the current techniques used in dual polarization meteorological radar studies. Although most of these studies are concerned with the characterization of hydrometeors such as rain, hail, snow, and ice crystals and their relation to storm structure, the theoretical developments put forth in these studies is readily applicable to a better understanding of the propagation effects along a satellite-earth slant path.

Early work in dual-polarization radar involved polarization measurements of precipitation, aircraft and ground clutter. Hunter [1954] conducted a circularly polarized radar experiment measuring the ratio of two circular-orthogonal components for many types of rain. Browne and Robinson [1952] investigated the cross-polarized signal return of the melting band using circular polarization. Gent et al. [1963], performed a comprehensive study of the depolarization characteristics of aircraft, precipitation and ground clutter. These studies were limited to a simplified analysis of the scattering mechanism of precipitation and its relation to measured results.

A more rigorous formulation of backscattering by rain and hail using dual-circular polarization radar techniques was developed by McCormick and Hendry [1968] and their theory will be outlined in Section 2.2.

The most recent technique employing linear-polarization was developed by Seliga and Bringi [1976]. Their technique requires the measurement of the difference in the linear co-polarized return in the horizontal and vertical planes and is termed differential reflectivity. This technique is appealing because no cross-polarized channel is needed in the radar, hence cost is reduced considerably. An outline of their technique is given in the next section.

2.1 DIFFERENTIAL REFLECTIVITY TECHNIQUE

Seliga and Bringi [1976] developed the differential reflectivity technique to determine rainfall rate. Their method involves the measurement of Z_H and Z_V , the equivalent reflectivity due to horizontally and vertically polarized transmitted waves from a radar. The differential reflectivity, defined as $10 \log(Z_H/Z_V)$, occurs because of the oblate like shape that falling raindrops take on as their size increases. Their analysis assumes that raindrops take on an oblate spheroidal shape with the axis of revolution about the drop minor axis. The minor axis points in the vertical

direction. Also raindrops fall with no canting angle in the plane of incidence (i.e., the plane perpendicular to the transmitted wave). The elevation angle to the raindrops is assumed to have little effect on this measurement technique.

A truncated, Marshal-Palmer [1948] exponential raindrop size distribution of the form:

$$N(D_{eq}) = N_0 \exp(-3.67 D_{eq}/D_0) \quad (2.1)$$

where

D_{eq} = diameter of a spherical raindrop with an equivalent volume as that volume of an oblate raindrop. ($0.0 \leq D_{eq} \leq 1.0$ cm)

N_0 = magnitude of the distribution

D_0 = median volume diameter (i.e. drops larger than D_0 contribute to half the total liquid water content per unit volume)

The raindrop distortion is measured by the raindrop axial ratio (a/b), where a is the semi-minor axis length, b is the semi-major axis length. The axial ratio is then related to

D_{eq} through the equations [Pruppacher and Beard, 1970]:

$$a/b = \begin{cases} 1.0 & 0 < D_{eq} \leq 0.028 \text{ cm} \\ [1 - (9/32)D_{eq} \rho_m v(D_{eq})^2/\mu]^{1/2} & 0.028 \leq D_{eq} \leq 0.1 \text{ cm} \quad (2.2) \\ 1.03 - 0.62 D_{eq} & 0.1 \leq D_{eq} \leq 1.0 \text{ cm} \end{cases}$$

where

$\rho_m = 1.1937 \times 10^{-3} \text{ g cm}^{-4}$ is the density of water
vapor saturated air

$\mu = 72.75 \text{ ergs cm}^{-2}$ is the surface tension of
water

$v(D_{eq})$ = terminal velocity of raindrops.

To determine the backscattered power, and the equivalent reflectivity in both polarizations, Seliga and Bringi used Rayleigh scattering theory to calculate the radar cross sections σ_H and σ_V at horizontal and vertical polarizations

respectively. The cross sections are given by:

$$\sigma_H = \left(\frac{16}{9}\right) \left(\frac{\pi^7}{\lambda^4}\right) D_{eq}^6 \left| \frac{m^2 - 1}{4\pi + (m^2 - 1)P'} \right|^2 \quad (\text{cm}^2) \quad (2.3)$$

$$\sigma_V = \left(\frac{16}{9}\right) \left(\frac{\pi^7}{\lambda^4}\right) D_{eq}^6 \left| \frac{m^2 - 1}{4\pi + (m^2 - 1)P} \right|^2 \quad (\text{cm}^2) \quad (2.4)$$

where

$$P = 4\pi - 2P' = \frac{4\pi}{e^2} \left[1 - [(a/b)/e] \sin^{-1} e \right] ;$$

e = eccentricity of the principal elliptical cross section = $1 - (a/b)^2$.

V = volume of the oblate raindrop = $4\pi D_{eq}^3/24$

m = complex index of refraction of the raindrop.

The average backscattered power from a common pulse volume filled with raindrops at a range r can then be expressed as:

$$\langle P_H(r) \rangle = (C/r^2) \left[\int_{D_{eq}} \sigma_H(D_{eq}) N(D_{eq}) d D_{eq} \right] \quad (2.5)$$

$$\langle P_V(r) \rangle = (C/r^2) \left[\int_{D_{eq}} \sigma_V(D_{eq}) N(D_{eq}) d D_{eq} \right] \quad (2.6)$$

where

C = radar constant

r = range to pulse volume

By defining C^* as:

$$C^* = C(16/9)(\pi^7/\lambda^4) \quad (2.7)$$

equations (2.5) and (2.6) become:

$$\langle P_H(r) \rangle = C^* Z_H / r^2 \quad (2.8)$$

$$\langle P_V(r) \rangle = C^* Z_V / r^2 \quad (2.9)$$

where the equivalent reflectivity factors are given by:

$$Z_H = \int_0^\infty D_{eq}^6 \left| \frac{m - 1}{4 + (m - 1)P'} \right|^2 N(D_{eq}) d D_{eq} \quad (2.10)$$

$$Z_V = \int_0^\infty D_{eq}^6 \left| \frac{m - 1}{4 + (m - 1)P} \right|^2 N(D_{eq}) d D_{eq} \quad (2.11)$$

From these two equations it can be seen that due to the factors P and P' , Z_H and Z_V will take on different values. Se-

liga and Bringi then define the shape functions

$$S_H(m, a/b) = \left| \frac{m^2 - 1}{4\pi + (m^2 - 1)P'} \right|^2 \quad (2.12)$$

and

$$S_V(m, a/b) = \left| \frac{m^2 - 1}{4\pi + (m^2 - 1)P} \right| \quad (2.13)$$

and then noted that the parameter N_0 can be brought outside the integration. Then applying the truncated limits yield:

$$\frac{Z_H}{N_0} = \int_0^{1.0\text{cm}} D_{eq}^6 S_H(m, a/b) \exp(-3.67 D_{eq}/D_0) d D_{eq} \quad (2.14)$$

$$\frac{Z_V}{N_0} = \int_0^{1.0\text{cm}} D_{eq}^6 S_V(m, a/b) \exp(-3.67 D_{eq}/D_0) d D_{eq} \quad (2.15)$$

Differential reflectivity is then defined as Z_{DR} :

$$Z_{DR} = 10 \log \left[\frac{\int_0^{1.0} D_{eq}^6 S_H(m, a/b) \exp(-3.67 D_{eq}/D_o) d D_{eq}}{\int_0^{1.0} D_{eq}^6 S_V(m, a/b) \exp(-3.67 D_{eq}/D_o) d D_{eq}} \right] \quad (2.16)$$

Note that equation 2.16 is independent of N_o and therefore the parameter D_o can be found directly by measuring Z_{DR} . Once D_o is known, N_o can be found by rewriting either equations (2.14) or (2.15) and solving for N_o . Figure 2.1 shows a plot of Z_{DR} and normalized reflectivity $10 \log(\frac{Z_H}{N_o})$ vs D_o . Seliga and Bringi [1981] have measured Z_{DR} values ranging between 0 and 3 dB.

Finally, the rainfall rate can be expressed as:

$$R = 0.6\pi N_o \int_0^{1.0} D_{eq}^3 v(D_{eq}) \exp(-3.67 D_{eq}/D_o) d D_{eq} \quad (2.17)$$

Seliga and Bringi also considered the sensitivity of N_o , and D_o to possible errors made in Z_{DR} measurements and concluded that this technique could provide better estimates of rainfall rate than single polarization radar measurements.

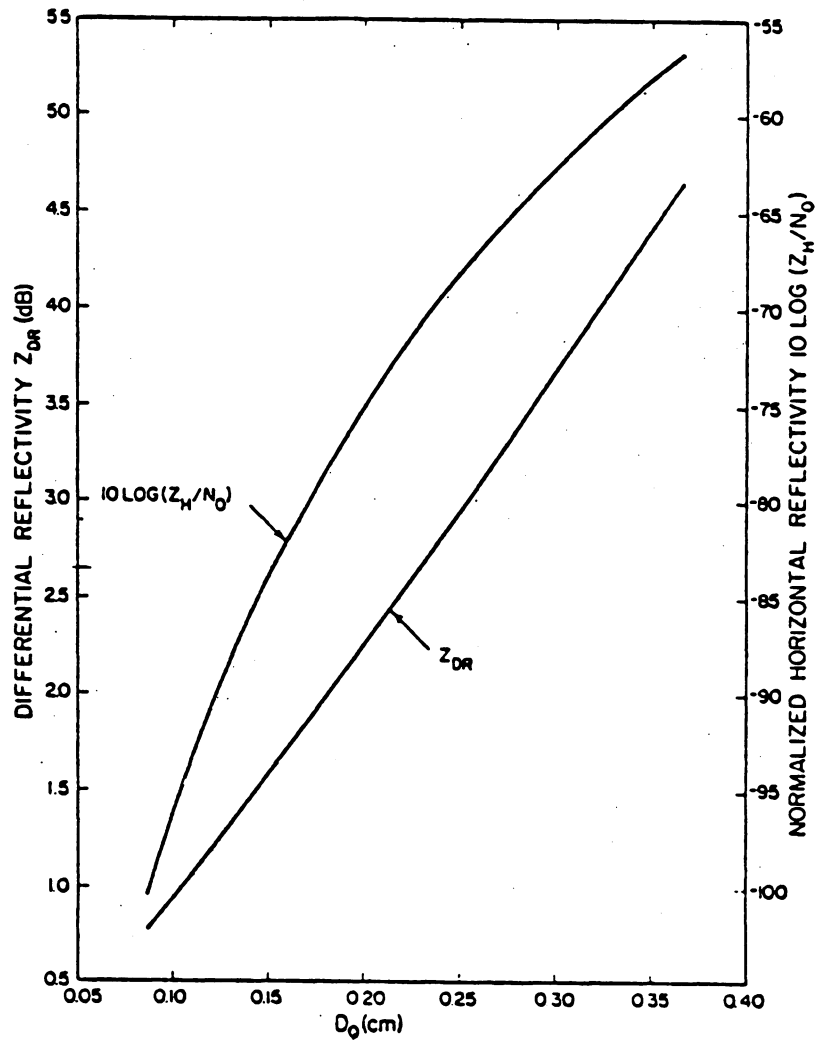


Figure 2.1. Z_{DR} (dB) and $10 \log \left(\frac{Z_H}{N_O} \right)$ vs. D_O (cm), from [Seliga and Bringi, 1976].

The development in Chapter 3 parallels this work and extends this technique to include the effects of canting angle and elevation angle on Z_{DR} measurements.

2.2 DUAL-CIRCULAR POLARIZATION TECHNIQUE

To further develop the theory of backscattering from hydrometeors using dual-circular polarization, McCormick and Hendry [1968, 1976] have developed a rigorous theoretical treatment of this phenomenon and have conducted experiments using a 16.5 GHz dual-channel radar. In their development, propagation effects induced by scatterers between the radar and those scatterers of interest are accounted for.

They first consider the backscattered field due to linear incident polarization. This field can be expressed as:

$$\begin{bmatrix} E_x^r \\ E_y^r \end{bmatrix} = \frac{-\gamma_o R}{R} \begin{bmatrix} S_{xx} & S_{xy} \\ S_{yx} & S_{yy} \end{bmatrix} \begin{bmatrix} E_x^t \\ E_y^t \end{bmatrix} \quad (2.18)$$

where the t superscript represents the transmitted field upon a scatter, the r superscript represents the received field at a distance $R = R(r, \theta, \phi)$ from the scatterer. The notation $R(r, \theta, \phi)$ will be described later. γ_o is the propagation constant $= 2\pi/\lambda$. S_{xx} , S_{xy} , S_{yx} , S_{yy} are the elements of the scattering matrix of the scatterer. The scat-

tering particle is oriented in the coordinate axes as defined in Figure 2.2. These scattering elements are the same as those developed in Chapter 3. In terms of circular polarization

$$\begin{bmatrix} E_+^r \\ E_-^r \end{bmatrix} = \frac{e^{-\gamma_0 R}}{R} \begin{bmatrix} (S_{xx} - S_{yy} - 2j S_{xy})e^{2j\alpha} & S_{xx} + S_{yy} \\ S_{xx} + S_{yy} & (S_{xx} - S_{yy} + 2j S_{xy})e^{-2j\alpha} \end{bmatrix} \begin{bmatrix} E_+^t \\ E_-^t \end{bmatrix} \quad (2.19)$$

where the + subscript represents right hand circular polarization, and the - subscript represent left hand circular polarization. In this development it is assumed that the axis of symmetry of the particle lies in the x axis. This requires the phase of the diagonal terms in equation (2.19) to be equal. Also it is assumed that the off-diagonal elements in equation (2.19) are real by adjusting the XY plane longitudinally (along the z axis). This enables equation (2.19) to be rewritten as:

$$\begin{bmatrix} E_+^r \\ E_-^r \end{bmatrix} = \frac{e^{-\gamma_0 R}}{R} \left(\frac{\sigma}{\pi} \right)^{1/2} \begin{bmatrix} v_+ e^{j(\delta + 2\alpha)} & 1 \\ 1 & v_- e^{j(\delta - 2\alpha)} \end{bmatrix} \begin{bmatrix} E_+^t \\ E_-^t \end{bmatrix} \quad (2.20)$$

where

$$\sigma = \text{backscatter cross section} = \pi(S_{xx} + S_{yy})^2$$

$$v_{\pm} e^{j\delta} = (S_{xx} - S_{yy} \mp 2j 2S_{xy}) / (S_{xx} + S_{yy})$$

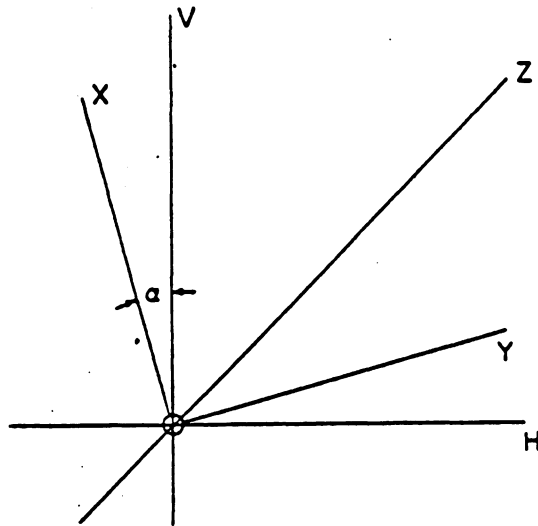


Figure 2.2. Scattering particle referred to the X,Y,Z coordinate axis, from [McCormick and Hendry, 1975].

α = canting angle (defined in the XY plane) of the particle.

If the particle is small compared to the operating wavelength then $v_+ = v_-$ and the scatterers can be considered in the Rayleigh region where Rayleigh scattering applies.

The radar employed by McCormick and Henry can transmit one and receiving any two orthogonal polarizations. This is done with a microwave circuit shown in the schematic of the radar in Figure 2.3. The scatterer is located at coordinates r, θ, ϕ denoted by $R(r, \theta, \phi)$ referenced to the x_0, y_0, z_0 coordinate axes. The microwave circuit can arbitrarily divide the input power between channels 1 and 2 and arbitrarily phase shift the two input signals to produce two output signals. This device is represented by the matrix;

$$\begin{bmatrix} m & n \\ n^* & -m^* \end{bmatrix}$$

At this point the transmitted and received field must be related to the transmitted power and received signals. The transmitted wave amplitudes are defined as:

$$\begin{bmatrix} M_1 \\ M_2 \end{bmatrix}$$

and the received wave amplitude measured at reference T in

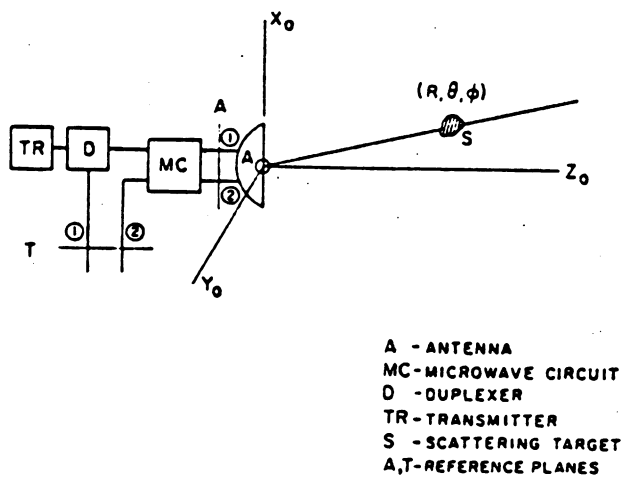


Figure 2.3. Schematic of radar and target orientation with respect to x_0 , y_0 , z_0 coordinate axis, from [McCormick and Hendry, 1975].

Figure 2.3 is:

$$\begin{bmatrix} R_1 \\ R_2 \end{bmatrix}$$

Assuming that the radar antenna has a one way power pattern $G(\theta, \phi)$ and phase pattern $\Phi(\theta, \phi)$ then the transmitted field in equation (2.20) can be expressed as:

$$\begin{bmatrix} E_+^t \\ E_-^t \end{bmatrix} = \left(\frac{G(\theta, \phi)}{2\pi \Psi_o} \right)^{1/2} \exp \left(j \Phi(\theta, \phi) \right) \frac{\exp(-\gamma_o R)}{R} \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} \Bigg|_{\text{ref plane A}} \quad (2.21)$$

where Ψ_o is the admittance of free space. The received amplitudes can be expressed as a function of the received field in equation (2.20):

$$\begin{bmatrix} R_1 \\ R_2 \end{bmatrix} \Bigg|_{\text{ref plane A}} = \frac{-j\lambda}{2} \left(\frac{G(\theta, \phi) \Psi_o}{2\pi} \right)^{1/2} \exp \left(j \Phi(\theta, \phi) \right) \begin{bmatrix} E_+^r \\ E_-^r \end{bmatrix} \Bigg|_{z=0} \quad (2.22)$$

Combining equations (2.21) and (2.22) into equation (2.20)

along with the microwave circuit transfer matrix results in:

$$\begin{bmatrix} R_1 \\ R_2 \end{bmatrix} \bigg|_{\text{ref plane T}} = \frac{j\lambda G(\theta, \phi) \exp(2j \Phi(\theta, \phi) - 2\gamma_o R)}{8\pi R^2} \left(\frac{\sigma}{\pi}\right)^{1/2} \begin{bmatrix} m & n^* \\ n & -m^* \end{bmatrix}.$$

$$\begin{bmatrix} v_+ e^{j(\delta + 2\alpha)} & 1 \\ 1 & v_- e^{j(\delta - 2\alpha)} \end{bmatrix} \begin{bmatrix} m & n \\ n^* & -m^* \end{bmatrix} \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} \bigg|_{\text{ref plane A}} \quad (2.23)$$

In general only one polarization is transmitted and R_1 , R_2 are normalized to the transmitted amplitudes. Hence either $M_1 = 1$ and $M_2 = 0$ or $M_1 = 0$ and $M_2 = 1$. Equation (2.23) then expresses the measured quantities R_1 , R_2 representing the complex amplitudes measured in channels 1 and 2 due to the characteristics of the scatterers illuminated by the radar. This equation must be averaged over the ensemble of scatterers within the radar antenna beam and pulse length of the transmitted pulse to arrive at an ensemble average of the measured quantities. To carry this out, assumptions of the statistical distribution of the scatterers must be made and McCormick and Hendry assume that the signal amplitudes, not the parameters defining the collection of scatterers, are normally distributed. From this, the resulting quanti-

ties are defined:

$$\begin{aligned}
 k_{11} &= \langle R_1 R_1^* \rangle \\
 k_{22} &= \langle R_2 R_2^* \rangle \\
 k_{12} &= \langle R_1 R_2^* \rangle \\
 \bar{\theta} &= \arg(k_{12})
 \end{aligned}
 \tag{2.24}$$

The brackets $\langle, \rangle$ represent the ensemble average of the quantity enclosed.

From equation (2.24) the final parameters can be defined:

$$\text{CAN} = 10 \log \left(\frac{k_{22}}{k_{11}} \right) = \text{circular depolarization ratio}
 \tag{2.25}$$

$$\text{ORTT} = |k_{12}| / (k_{11} k_{22})^{1/2}
 \tag{2.26}$$

$$\text{LAD} = 1/2 \arg(k_{12} - \pi/2)
 \tag{2.27}$$

where the circular depolarization ratio is a measure of the depolarization of the scatterers, ORTT is a measure of the degree of orientation or alignment of the scatterers, and ALD is a measure of the apparent canting angle of the scatterers.

Because of the short operating wavelength of the radar used by McCormick and Hendry, they must include propagation

effects. This is done by noting that for any precipitation medium there are at least two polarizations with their associated propagation constants which will pass through the medium without polarization conversion. These are known as characteristic polarizations of the medium. By transforming the received and transmitted fields to the characteristic polarizations as these fields pass through the propagation medium, then the field can be expressed as functions of the propagation constants. Therefore propagation effects can be taken into account. It is noted that in doing this, assumptions of the propagation medium must be made which could lead to misinterpretation of the received quantities. In their formulation McCormick and Hendry have included the propagation medium effects by replacing the term $(1/R) \exp(-\gamma_0 R)$ in equations (2.20) and (2.21) with the (2×2) matrix P and its transpose. Details of this matrix may be found in the literature [McCormick and Hendry, 1975].

McCormick and Hendry have used this technique to study several types of precipitation with good success in gaining further insight into the description of precipitation.

This technique is equivalent to the development presented here and theoretical results of the presented development using the definitions in equations (2.25) to (2.27) are given in Section 4.8.

Chapter III

THEORETICAL DEVELOPMENT OF A DUAL-POLARIZATION RADAR EQUATION FOR RAIN

In this chapter the single particle scattering matrix for an oblate raindrop is derived. The result is used to calculate the backscattered return from a collection of scatterers, and formulate of a matrix radar equation. In the formulation of the single particle scattering matrix a geometry was chosen to include the effects of raindrop canting angle and of elevation angle to the raindrop. The scattering matrix is developed using Rayleigh scattering approximation which holds for this scattering problem because of the relatively long wavelength (compared to raindrop size) used by the VPI&SU radar. The study scattering effects of rain at higher frequencies will require a more accurate scattering model.

3.1 SINGLE SCATTERING MATRIX FOR AN OBLATE RAINDROP

3.1.1 Backscattering from an Oblate Spheroid

In this section the backscattering coefficients for an oblate raindrop will be derived. This development is based on the Rayleigh approximation and holds fairly well for the wavelengths considered here.

The scattered field $\vec{E}$ at a distance r from the raindrop, in the far field can be expressed as [Stratton, 1941]:

$$\vec{E}_s = \frac{k_o^2 \exp(-jk_o r)}{4\pi \epsilon_o r} \{ \vec{p} - (\vec{p} \cdot \hat{k}_s) \hat{k}_s \} \quad (3.1)$$

where

$$\vec{p} = \int_v (\epsilon - \epsilon_o) \vec{E} dv \quad ; \quad \vec{E} = \text{electric field induced inside the scatterer}$$

$$k_o = \frac{2\pi}{\lambda} \quad ; \quad \lambda = \text{free space wavelength}$$

$$\epsilon_o = \text{permittivity of freespace}$$

$$\epsilon = \text{permittivity of the scatterer}$$

$$\hat{k}_s = \text{unit vector in the scattering direction}$$

Equation (3.1) holds under two assumptions. If ℓ is the maximum dimension of the scatterer than $k_o \ell$ must be much less than unity ($k_o \ell \ll 1$) and $|\frac{k_o \epsilon}{\epsilon_o} \ell| \ll 1$. These two assumptions imply that the scatterer is very small compared to the wavelength and that the induced field inside the scatterer instantaneously follows the applied field.

With these assumptions, the scattered field can be found by considering the internal field of the scatterer induced by a uniform electrostatic external field. Falling rain-drops become oblate, with an ellipsoidal shape, and can be

modeled by a dielectric ellipsoid. The electrostatic problem of a dielectric ellipsoid in a uniform electrostatic field has been considered by Stratton [1941], and Jones [1964]; the results of their development are used here.

With an incident electrostatic field $\vec{E}_0$ applied to an oblate spheroid it can be shown that [Stutzman et al., 1981]:

$$\vec{p} = \frac{4\pi ab^2}{3} \left\{ \frac{E_{on} \hat{n}}{\epsilon - \epsilon_o} + \frac{E_{ol} \hat{l} + E_{om} \hat{m}}{\epsilon - \epsilon_o \left(\frac{1 - I_1}{2} \right)} \right\} (\epsilon - \epsilon_o) \quad (3.2)$$

where

a = semiminor axis length.

b = semimajor axis length.

E_{on} , E_{ol} , E_{om} are the components of the incident field

$\epsilon = \epsilon_o \epsilon_r$ = permittivity of the dielectric ellipsoid.

$$I_1 = \frac{1}{e^2} [1 - (1 - e^2)^{1/2} e^{-1} \arcsin e]$$

$$e = [1 - (a/b)^2]^{1/2} ; \quad e = \text{eccentricity of the ellipsoid}$$

$\hat{l}$, $\hat{m}$, $\hat{n}$ is a right handed unit vector set in cartesian coordinates.

Figure 3.1 describes the geometry used in formulating equation (3.2).

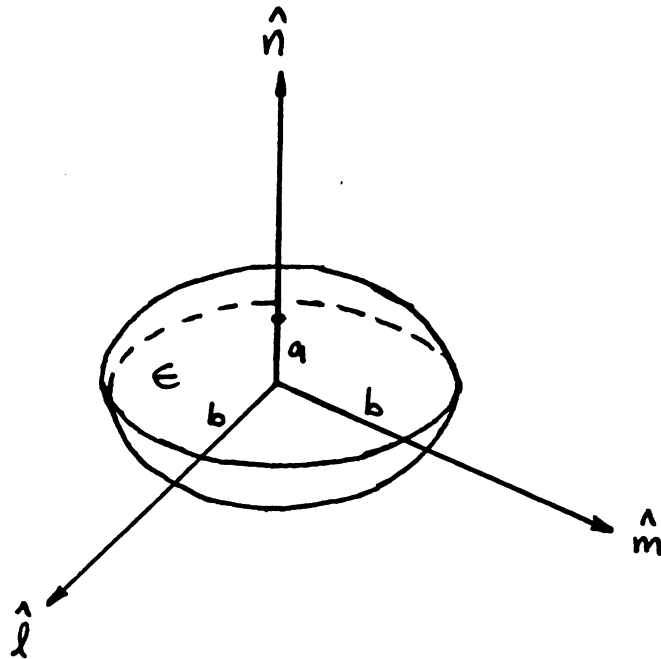


Figure 3.1. Geometry of an oblate ellipsoid with permittivity ϵ in free space. a = semi-minor axis length, b = semi-major axis length.

By examining equation (3.2) it can be seen that the incident field can be decomposed into components E_{ov} and E_{oh} which are in the direction of the spheroidal axis of revolution and the orthogonal axis of the raindrop. Therefore let the electric dipole moment be defined as:

$$\vec{p} = p_h \vec{E}_{oh} + p_v \vec{E}_{ov} \quad (3.3)$$

where

$$p_h = \frac{4\pi ab^2}{3} (\epsilon - \epsilon_o) \left\{ \frac{1}{1 + \frac{\epsilon - \epsilon_o}{\epsilon_o} \frac{1 - I_1}{2}} \right\}$$

$$p_v = \frac{4\pi ab^2}{3} (\epsilon - \epsilon_o) \left\{ \frac{1}{1 + \frac{\epsilon - \epsilon_o}{\epsilon_o} I_1} \right\}$$

The coefficients p_h and p_v can be written as a function of the complex index of refraction by making the substitutions:

$$\frac{\epsilon - \epsilon_o}{\epsilon_o} = m^2 - 1 \quad (3.4)$$

$$\epsilon - \epsilon_o = \epsilon_o (m^2 - 1) \quad (3.5)$$

This changes p_h and p_v to

$$p_h = \frac{4\pi ab^2}{3} \epsilon_o (m^2 - 1) \left\{ \frac{2}{I_1(1 - m^2) + m^2 + 1} \right\} \quad (3.6)$$

$$p_v = \frac{4\pi ab^2}{3} \epsilon_o (m^2 - 1) \left\{ \frac{1}{I_1(m^2 - 1) + 1} \right\} \quad (3.7)$$

3.1.2 Geometry for Scattering by a Single Raindrop

At this point the scattering coefficients can be calculated. The scattering problem must also include the canting angle of and elevation angle to the scatterer. Figure 3.2 shows the scattering geometry used for the oblate raindrop, following the approach of Haworth, Watson and McEwan, [1977]. The equatorial plane of the raindrop is defined by unit vectors $\hat{l}$ and $\hat{m}$, where $\hat{l} \cdot \hat{z} = 0$, and $\hat{m} = \hat{n} \cdot \hat{l}$. The vector $\hat{n}$ is in the direction of the axis of revolution of the raindrop, and is assumed vertical. Therefore $\hat{n}$, $\hat{l}$, $\hat{m}$ define the vertical and horizontal planes of the raindrop, and $\hat{x}$, $\hat{y}$, $\hat{z}$ is a set of orthogonal unit vectors associated with the incident electric field $\vec{E}_i$. The angle ψ , measured

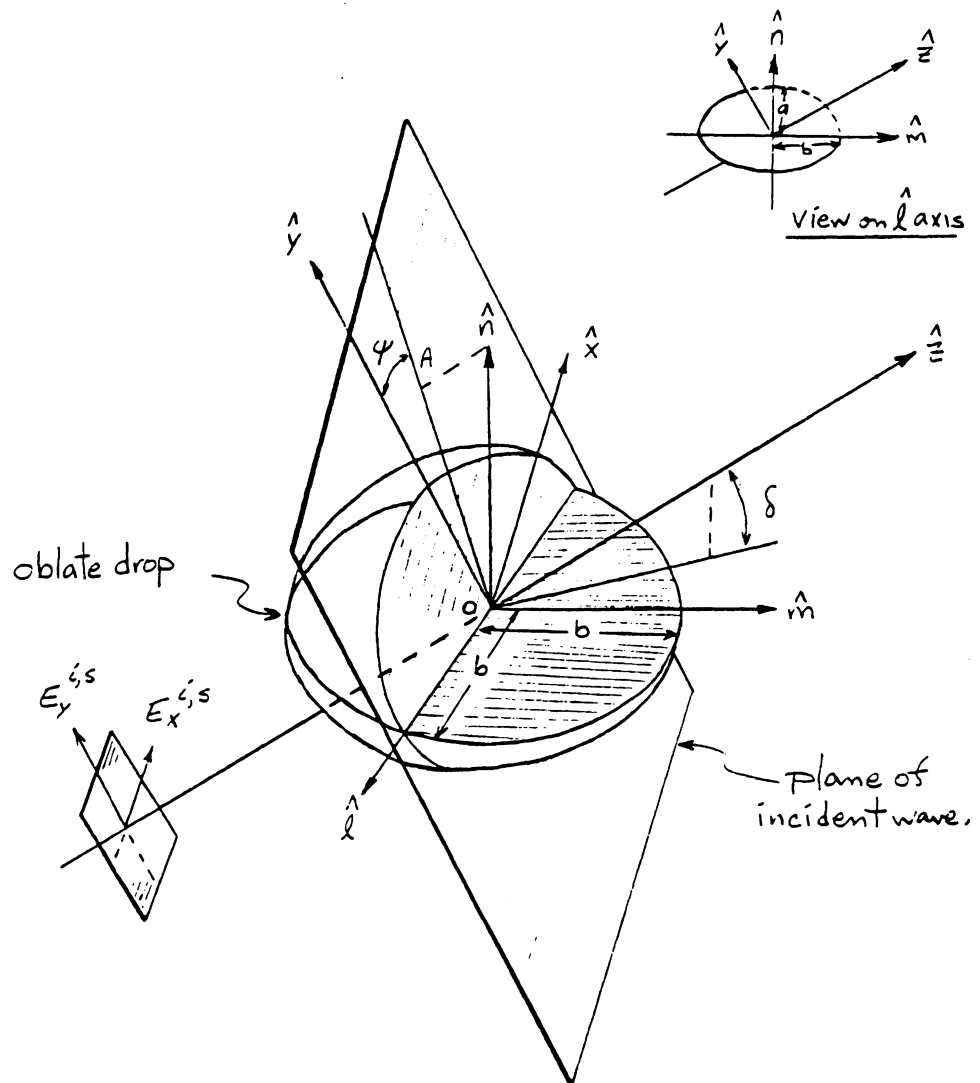


Figure 3.2. Scattering geometry of an oblate raindrop. ψ is the angle measured from the y axis to the projection of $\hat{n}$ onto the xy plane (the plane of incidence). $\hat{l}$ lies in the xy plane; δ is the elevation angle to the drop. [Figure drawn by Dr. T. Pratt, 1982].

from $\hat{y}$ axis to the projection of $\hat{n}$ onto the plane of incidence, defines the incident polarization. The angle between $\hat{n}$ and the direction of propagation ($\hat{z}$) is the complement of the elevation angle δ . This defines the orientation of the incident field with respect to the raindrop coordinate system. The incident field propagates in the $\hat{z}$ direction with the field polarized in the xy-plane. The incident field can be expressed as:

$$\vec{E}_i = E_{ox} \hat{x} + E_{oy} \hat{y} \quad (3.8)$$

The unit vector $\hat{n}$ can be written in terms of $\hat{x}$, $\hat{y}$, and $\hat{z}$ as:

$$\hat{n} = n_x \hat{x} + n_y \hat{y} + n_z \hat{z}$$

or as a function of δ and ψ :

$$\hat{n} = \cos \delta \sin \psi \hat{x} + \cos \delta \cos \psi \hat{y} + \sin \delta \hat{z} \quad (3.9)$$

From this, $\vec{E}_{oh}$ and $\vec{E}_{ov}$ can be found as follows:

$$\begin{aligned} \vec{E}_{ov} &= (\vec{E}_i \cdot \hat{n}) \hat{n} \\ &= (E_{ox} n_x + E_{oy} n_y) \\ &\quad (n_x \hat{x} + n_y \hat{y} + n_z \hat{z}) \end{aligned} \quad (3.10)$$

Since $\vec{E}_i = \vec{E}_{oh} + \vec{E}_{ov}$ we have:

$$\vec{E}_{oh} = \vec{E}_i - \vec{E}_{ov} \quad (3.11)$$

3.1.3 Calculation of the Backscattering Matrix

Using the results of equations (3.3), (3.6), (3.7), (3.10) and (3.11), equation (3.1) results in:

$$\vec{E}_s \approx \frac{k_o^2 \exp(-jk_o r)}{4\pi \epsilon_o r} \left\{ p_h \vec{E}_{oh} + p_v \vec{E}_{ov} - \left[(p_h \vec{E}_{oh} + p_v \vec{E}_{ov}) \cdot \hat{k}_s \right] \hat{k}_s \right\} \quad (3.12)$$

For the backscattering case $\hat{k}_s = -\hat{z}$ and equation (12) reduces to:

$$\vec{E}_s \approx \frac{k_o^2 \exp(-jk_o r)}{4\pi \epsilon_o r} \cdot$$

$$\left[\left\{ p_v \left[E_{ox} n_x^2 + E_{oy} n_y n_x \right] + p_h \left[E_{ox} (1 - n_x^2) - E_{oy} n_y n_x \right] \right\} \hat{x} \right. \\ \left. + \left\{ p_v \left[E_{ox} n_x n_y + E_{oy} n_y^2 \right] + p_h \left[E_{oy} (1 - n_y^2) - E_{ox} n_x n_y \right] \right\} \hat{y} \right] \quad (3.13)$$

Equation (3.13) can be written in matrix form with the sub-

stitution of the components of equation (3.9) as:

$$\begin{bmatrix} E_{s_x} \\ E_{s_y} \end{bmatrix} = \frac{k_o^2}{4\pi \epsilon_o} \begin{bmatrix} (p_v - p_h) \cos^2 \delta \sin^2 \psi + p_h & (p_v - p_h) \cos^2 \delta \frac{\sin 2\psi}{2} \\ (p_v - p_h) \cos^2 \delta \frac{\sin 2\psi}{2} & (p_v - p_h) \cos^2 \delta \cos^2 \psi + p_h \end{bmatrix} \cdot$$

$$\frac{\exp(-jk_o r)}{r} \begin{bmatrix} E_{o_x} \\ E_{o_y} \end{bmatrix} \quad (3.14)$$

This is the backscattering equation in the far field due to a plane wave incident on an oblate spheroid. By substituting equations (3.6) and (3.7) into equation (3.14) and defining:

$$M = \left\{ \frac{(m^2 - 1)(1 - 3 I_1)}{I_1(m^2 - 1)[(m^2 - 1)(1 - I_1) + 1] + m^2 + 1} \right\} \quad (3.15)$$

equation (3.14) reduces to:

$$\begin{bmatrix} E_{sx} \\ E_{sy} \end{bmatrix} = \frac{V k_o^2 (m^2 - 1)}{4\pi} \cdot$$

$$\begin{bmatrix} M \cos 2\delta \sin^2 \psi + \frac{2}{I_1 (1-m^2) + m^2 + 1} & \frac{M}{2} \sin 2\psi \cos^2 \delta \\ \frac{M}{2} \sin 2\psi \cos^2 \delta & M \cos^2 \delta \cos^2 \psi + \frac{2}{I_1 (1-m^2) + m^2 + 1} \end{bmatrix} \frac{\exp(-jk_o r)}{r} \begin{bmatrix} E_{ox} \\ E_{oy} \end{bmatrix} \quad (3.16)$$

where

m = dielectric constant

$$I_1 = \left[1 - \frac{(1 - e^2)^{1/2}}{e} \arcsin(e) \right] \left(\frac{1}{e^2} \right)$$

$$e = [(1 - (a/b)^2)]^{1/2}$$

σ, ψ = elevation and polarization angle

r = range to the observation point

$$V = \text{volume of the spheroid} = \frac{4\pi ab^2}{3}$$

By taking the limit as $a \rightarrow b$ the ellipsoid becomes a sphere. This is equivalent to letting $e \rightarrow 0$; from this it can be shown that:

$$\lim_{e \rightarrow 0} I_1 = \frac{1}{3} \quad (3.17)$$

Then the coefficient M in equation (3.16) goes to zero and

equation (3.16) reduces to the linear equation for either incident linear polarization:

$$E_{s_{x,y}} = k_o^2 \frac{(m^2 - 1)}{(m^2 + 2)} a^3 \frac{\exp(-jk_o r)}{r} E_{o_{x,y}} \quad (3.18)$$

This is a classical result in scattering from a dielectric sphere [Van de Hulst, 1957] .

Note that since the Rayleigh approximation is used the scattering matrix is symmetric and the backscattered field is equal to the forward scattered field.

3.2 BACKSCATTERING FROM A VOLUME OF SCATTERERS

In this section, a matrix formulation relating the received power from a dual-polarized radar to the particle characteristics within a cloud is developed.

3.2.1 The Intensity Matrix

A volume filled with scatterers, as shown in Figure 3.3, is illuminated by a radar. The received energy is due to the backscattered component of all N scatterers within the volume. The volume extends from range R_1 to R_2 and within the half power beamwidth of the antenna θ_B . The parameters θ_j' , ϕ_j' , r_j define the position of the j th scatterer within the volume. Initially, it is assumed that the radar tran-

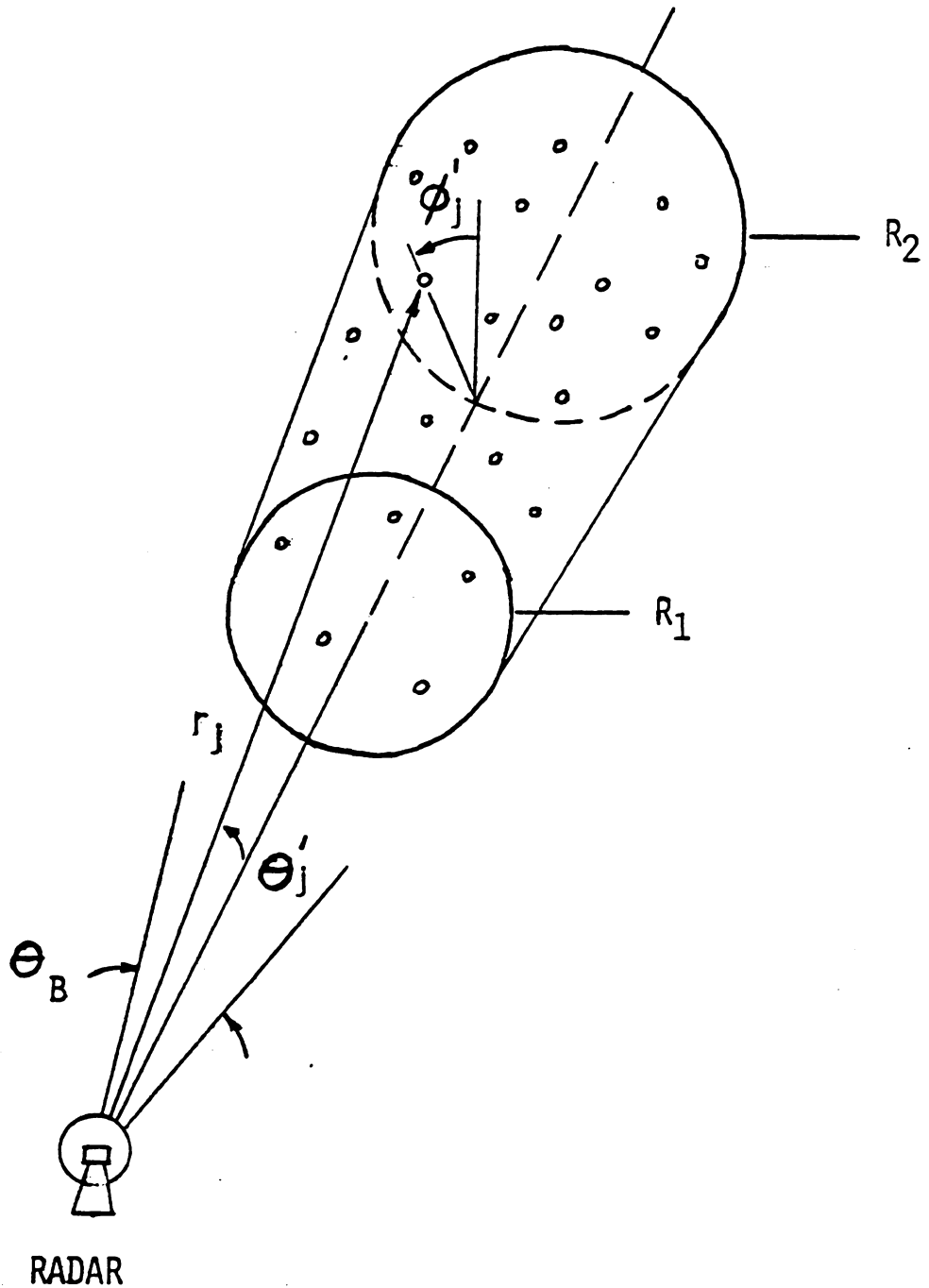


Figure 3.3. Volume filled with N scatterers illuminated by a radar. r_j , θ_j' , ϕ_j' define the coordinates of the j th scatterer within the volume.

smits a continuous wave, then with the appropriate assumptions the results will be shown to hold for the pulsed-wave case.

The scattered field from a single particle developed in the previous section can be written in matrix-form:

$$\underline{E}^s = \frac{\exp(-jk_o r)}{r} [f] \underline{E}^i \quad (3.19)$$

where $\underline{E}^i$ is the incident field, $[f]$ is the scattering matrix given in equation (3.14) and $\underline{E}^s$ is the scattered field at a great distance r from the scatterer.

In terms of scattered intensity, equation (3.19) can be written as:

$$\underline{I}^s = \underline{E}^s \underline{E}^{s^x} = \begin{bmatrix} E_1^s & E_1^{s^*} & E_1^s & E_2^{s^*} \\ E_2^s & E_1^{s^*} & E_2^s & E_2^{s^*} \end{bmatrix} =$$

$$\frac{1}{r^2} [f] \underline{E}^i \underline{E}^{i^x} [f]^x = \frac{1}{r^2} [f] \underline{I}^i [f]^x \quad (3.20)$$

The symbol $\underline{E}^x$ means take the conjugate transpose of matrix $\underline{E}$. Subscripts x and y in equation (3.14) are replaced with

subscripts 1 and 2 respectively in the general case of two orthogonal polarizations.

3.2.2 Calculation of Scattered Power Returned to a Radar

The radar parameters can be included in equation (3.2) with the assumptions that the antenna gain is the same for both polarizations and no impedance or polarization mismatch exists in the radar antenna. Then, the intensity incident upon the j th scatterer can be expressed as:

$$\underline{I}_j^i = \frac{G(\theta_j', \phi_j')}{4\pi r_j^2} \underline{P}^t \quad (3.21)$$

where

$G(\theta_j', \phi_j')$ is the gain of the antenna as a function of θ_j' and ϕ_j' ,

r_j is the distance from the j^{th} scatterer to the antenna.

$[\underline{P}]^t$ is the transmit power matrix:

$$\underline{P}^t = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$$

Note that the off-diagonal terms of the transmit power matrix can be complex, but are zero when only one polarization

is transmitted at any instant of time. The diagonal terms in this matrix correspond to the power transmitted in polarizations 1 and 2. The scatterer in turn, scatters this energy in all directions and the scattered intensity at the radar antenna is expressed as equation (3.20). Combining equations (3.20) and (3.21) results in:

$$\underline{I}_j^s = \frac{G(\theta_j', \phi_j')}{4\pi r_j^4} [f_j] \underline{P}^t [f_j]^x \quad (3.22)$$

where $[f_j]$ is the scattering matrix for the j th particle. The receiving antenna (assumed the same as the transmit antenna) will receive the scattered intensity multiplied by the antenna's effective aperture $A(\theta_j', \phi_j')$ [Stutzman and Thiele, 1981], and the power available at the receiver can be expressed as:

$$\underline{P}_j^r = \frac{G(\theta_j', \phi_j')}{4\pi r_j^4} [f_j] \underline{P}^t [f_j]^x A_e(\theta_j', \phi_j') \quad (3.23)$$

where

$$A_e(\theta_j', \phi_j') = \frac{G(\theta_j', \phi_j')}{4\pi} \lambda^2 \quad (3.24)$$

$\underline{P}_j^r$ is the received power matrix measured at the radar antenna output due to the j th particle in the volume; a discussion of this matrix follows. Therefore:

$$\underline{P}_j^r = \left(\frac{\lambda}{4\pi} \right)^2 \frac{G^2(\theta_j', \phi_j')}{r_j^4} [f_j] \underline{P}^t [f_j]^x \quad (3.25)$$

At this point the received quantities can be related to the scattering matrix and the transmit power. The transmit power matrix can be written as:

$$\underline{P}^t = \begin{bmatrix} V_1 V_1^* & V_1 V_2^* \\ V_2 V_1^* & V_2 V_2^* \end{bmatrix} \quad (3.26)$$

where

$$V_1 V_1^* = \text{power transmitted in channel 1}$$

(with polarization 1)

$$V_2 V_2^* = \text{power transmitted in channel 2}$$

(with polarization 2)

$$V_1 V_2^* , V_2 V_1^* = \text{complex power}$$

due to both channels 1 and 2.

V is the voltage across the antenna terminals referenced to an impedance of one ohm. The voltage notation is of no consequence since power is of interest here.

Since we transmit in one polarization at any instant of time, $P_{12} = 0$, $P_{21} = 0$ and either $P_{11} = P_1$ and $P_{22} = 0$ for transmitting polarization 1 or $P_{11} = 0$ and $P_{22} = P_2$ for transmitting polarization 2. P_1 and P_2 are the transmit powers (in watts) in channels 1 and 2 respectively. But in general for a single particle let:

$$[B_j] = [f_j] \underline{P}^t [f_j]^x = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \quad (3.27)$$

where

$$\begin{aligned}
 B_{11} &= f_{11}(f_{11}^* |E_1|^2 + f_{12}^* E_1 E_2^*) + f_{12}(f_{11}^* E_2 E_1^* + f_{12}^* |E_2|^2) \\
 B_{12} &= f_{11}(f_{21}^* |E_1|^2 + f_{22}^* E_1 E_2^*) + f_{12}(f_{21}^* E_2 E_1^* + f_{22}^* |E_2|^2) \\
 B_{21} &= f_{22}(f_{12}^* |E_2|^2 + f_{11}^* E_2 E_1^*) + f_{21}(f_{12}^* E_1 E_2^* + f_{11}^* |E_1|^2) \\
 B_{22} &= f_{22}(f_{22}^* |E_2|^2 + f_{21}^* E_2 E_1^*) + f_{21}(f_{22}^* E_1 E_2^* + f_{21}^* |E_1|^2)
 \end{aligned}$$

Substituting equation (3.27) into equation (3.26) yields:

$$\underline{P}_j^r = \left(\frac{\lambda}{4\pi}\right)^2 \frac{G^2(\theta_j', \phi_j')}{r_j^4} [B_j] \quad (3.28)$$

This is a generalized radar equation for any set of orthogonal polarizations with the corresponding scattering coefficients. Fortunately, equation (3.27) simplifies. First, the scattering matrix $[f_j]$ is symmetric (as shown in the last section) for particles in the Rayleigh region. There-

fore $f_{12} = f_{21}$ and equation (3.27) reduces to:

$$\begin{aligned}
 B_{11} &= |f_{11}|^2 |E_1|^2 + 2 \operatorname{Re}(f_{11} f_{12}^* E_1 E_2^*) + |f_{12}|^2 |E_2|^2 \\
 B_{12} &= f_{11} f_{12}^* |E_1|^2 + f_{11} f_{22}^* E_1 E_2^* + |f_{12}|^2 E_2 E_1^* + f_{22} f_{12}^* |E_2|^2 \\
 B_{21} &= B_{12}^* \\
 B_{22} &= |f_{22}|^2 |E_2|^2 + 2 \operatorname{Re}(f_{22} f_{12}^* E_2 E_1^*) + |f_{12}|^2 |E_1|^2
 \end{aligned}$$

Furthermore, if we transmit in polarization 1 with power P_1 (i.e. $|V_1|^2 = P_1$, $|V_2|^2 = 0$), then

$$\begin{aligned}
 B_{11} &= |f_{11}|^2 |E_1|^2 \\
 B_{12} &= f_{11} f_{12}^* |E_1|^2 = B_{21}^* \\
 B_{22} &= |f_{12}|^2 |E_1|^2
 \end{aligned} \tag{3.30}$$

Similar results hold for transmitting in polarization 2 with power P_2 (i.e. $|V_1|^2 = 0$ and $|V_2|^2 = P_2$). At this point the above results will be extended to a collection of particles within a volume.

The total scattered field from the volume shown in Figure 3.3 can be expressed as the superposition of the scattered field from all N scatterers within the volume [Van de Hulst,

1957] . Therefore, let:

$$\underline{E}^s = \sum_{j=1}^N \underline{E}_j^s \quad (3.31)$$

where

$\underline{E}_j^s$ is the scattered field from the j^{th} scatterer.

The backscattered intensity can be expressed as:

$$\underline{I}^s = \sum_{j=1}^N \sum_{i=1}^N \underline{E}_j^s \underline{E}_i^{s^*} \quad (3.32a)$$

This double summation takes into account the contribution to the total intensity from different scatterers. Equation (3.32a) can be written as:

$$\underline{I}^s = \sum_{j=1}^N \underline{E}_j^s \underline{E}_j^{s^*} + \sum_{j=1}^N \sum_{i=1}^N \underline{E}_j^s \underline{E}_i^{s^*} \quad (3.32b)$$

The total number of scatters N can be any positive integer greater than 1. The first term of equation (3.32b) is the sum of the scattered intensity from each particle. The second term is the sum of the correlation between the scattered

fields from two different particles. If N is very large and the particles within the volume are not very close to each other, as in our case, the second term will have an average value of zero. This result is due to the random phasor sums of the scattered field from particles with different distances from the antenna resulting in positive and negative correlation, hence the second term may be neglected. With the assumptions stated above, this type of scattering is known as independent scattering [Van de Hulst, 1957].

Therefore the received power due to all the scatterers within the volume is given by:

$$\underline{P}^r = \left(\frac{\lambda}{4\pi}\right)^2 \sum_{j=1}^N \frac{G^2(\theta_j', \phi_j')}{r_j^4} [B_j] \quad (3.33)$$

3.2.3 Averaging over a Volume of Scatterers

Equation (3.33) must now be averaged over an assumed particle distribution to obtain an average received intensity from the volume of scatters. The particle distribution for

raindrops can be expressed as [Ishimaru, 1977]:

$$g(r, \bar{a}, \theta, s) = \frac{n(r, \bar{a}, \theta, s)}{N} \quad (3.34)$$

where $n(r, \bar{a}, \theta, s)$ is the number of scatterers per unit volume for particles in a range r to $r + dr$, equivolumetric radius $\bar{a}$ to $\bar{a} + d\bar{a}$, canting angle θ to $\theta + d\theta$, and shape s to $s + ds$. N is the total number of particles per unit volume.

Averaging equation (33) yields (assuming N is large):

$$\langle \underline{p}^r \rangle = \left(\frac{\lambda}{4\pi} \right)^2 \left\{ \iiint \frac{G^2(\theta', \phi')}{r^4} [B(r, \bar{a}, \theta, s)] n(r, \bar{a}, \theta, s) dr d\bar{a} d\theta ds \right\} \quad (3.35)$$

This equation can be simplified by assuming that the scatterers are uniformly distributed through the volume and that the distribution in r is independent of $(\bar{a}, \theta, s)$ in equation (3.34). The integration over dr reduces to a vo-

lume integral taken over the complete volume. Therefore:

$$\begin{aligned} \langle \underline{P}^r \rangle = & \left(\frac{\lambda}{4\pi} \right)^2 \cdot \int_0^{2\pi} \int_{-\frac{\theta_B}{2}}^{\frac{\theta_B}{2}} \int_{R_1}^{R_1'} \left[\iiint \frac{G^2(\theta', \phi')}{r^4} [B(\bar{a}, \theta, s)] \right. \\ & \left. \cdot n(\bar{a}, \theta, s) d\bar{a} d\theta ds \right] r^2 \sin \theta' dr d\theta' d\phi' \end{aligned} \quad (3.36)$$

Before this integration can be carried out, the antenna gain function must be approximated by a suitable function. Probert-Jones [1962] approximated an antenna gain pattern as a Gaussian beam shape given by:

$$G(\theta', \phi') = G_0 \exp \left[-\ln 2 \left(\frac{2\theta'}{\theta_B} \right)^2 \right] \quad (3.37)$$

where

$$\frac{-\theta_B}{2} \leq \theta \leq \frac{\theta_B}{2}$$

G_0 = on axis gain of the radar antenna

Using this gain function

$$\langle \underline{P}^r \rangle = \left(\frac{\lambda}{4\pi} \right)^2 G_o^2 \left(\frac{\pi \theta_B^2}{8 \ln 2} \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) .$$

$$\left[\iiint [B(\bar{a}, \theta, s)] n(\bar{a}, \theta, s) d\bar{a} d\theta ds \right] \quad (3.38)$$

A discussion of $n(\bar{a}, \theta, s)$ is given in Section 4.1 and integration of θ and s is carried out analytically in Appendix I. The integral in the braces can be expressed as the ensemble average of $[B]$. Therefore:

$$\langle \underline{P}^r \rangle = \left(\frac{\lambda}{4\pi} \right)^2 G_o^2 \left(\frac{\pi \theta_B^2}{8 \ln 2} \right) \left(\frac{R_2 - R_1}{R_2 R_1} \right) \langle [B] \rangle \quad (3.39)$$

Equation (3.39) will now be shown to be true for the pulsed wave case. Figure 3.4 shows a pulse returned from a volume of scatterers with $|V_1|^2 = P_1$. This represents a typical return as seen on an A-scope display of a radar set, where T is the transmit pulse duration. If we assume that the sampled volume length is much smaller than the range to that volume (i.e. $R_1 \gg R_2 - R_1$), then $R_1 R_2 \approx R_1^2$. Also if

$$R_2 - R_1 = \frac{T_o C}{2} = \frac{h}{2} ; \quad h = \text{transmit pulse length in meters}$$

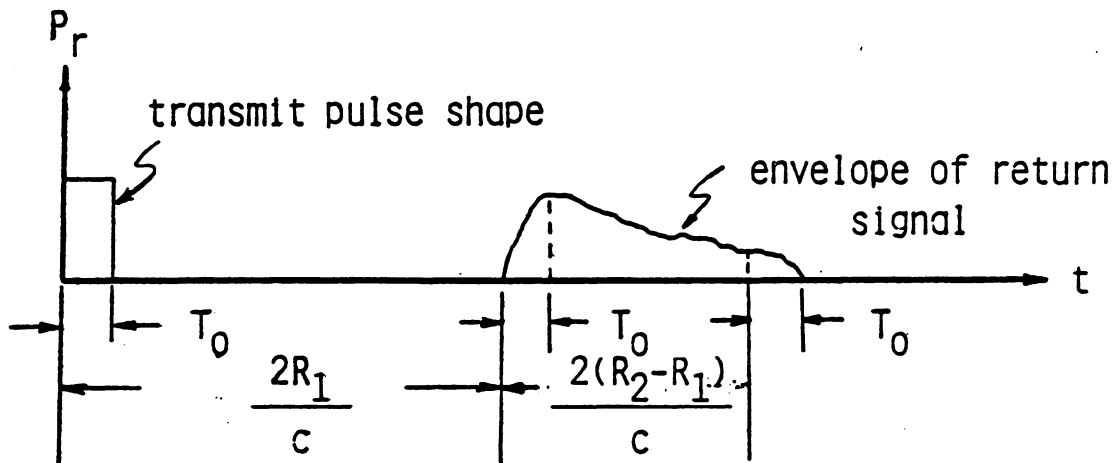


Figure 3.4. Pulse return from a volume of scatterers. Envelope of return signal vs. time.

then equation (3.38) will hold for the pulsed case [Ishimaru, 1977]. Equation (3.39) can now be written as:

$$\langle \underline{P}^r \rangle = \left(\frac{\lambda}{4\pi} \right)^2 G_o^2 \left(\frac{\pi \theta_B^2}{8 \ln 2} \right) \left(\frac{h}{2} \right) \frac{1}{r^2} \langle [B] \rangle \quad (3.40a)$$

where

$h = T_o \ c =$ pulse length in meters

$r =$ distance to the volume of scatters ($r \gg h$)
in meters.

Equation (3.40) is a matrix formulation of the familiar radar equation. Note that transmitting only in polarization 1 results in $B_{11} = |f_{11}|^2$. The radar cross section is defined as:

$$\langle \sigma_B \rangle = \langle 4\pi |f_{11}|^2 \rangle \quad (3.40b)$$

Therefore

$$\langle P_{11}^r \rangle = \frac{\lambda^2}{(4\pi)^3} G_o^2 \left(\frac{\pi \theta_B^2}{8 \ln 2} \right) P_1 \frac{1}{r^2} \langle \sigma_B \rangle \quad (3.40c)$$

This result is identical to that given by Battan [1973].

Chapter IV

RAIN CLOUD PARAMETER MEASUREMENT USING DUAL POLARIZATION TECHNIQUES

In the last chapter a radar equation was derived which relates the measured quantities in a dual polarized radar to the averaged scattering matrix of a collection of scatterers. This chapter presents a development of the techniques to extract parameters describing a collection of raindrops within a pulse volume.

Modeling of the dual polarization radar equation was carried out on an IBM-3020 mainframe computer. The computer code was written in Fortran and a source listing can be found in Appendix II. Part of this development parallels the differential reflectivity concept introduced by Seliga and Bringi [1976], but with the effects of canting angle and elevation angle included. Theoretical results of differential reflectivity measurements in the 45 degree plane are examined and it is shown that in practice real measurements of this type would be difficult to make and yield little information about the rain filled medium. This technique could possibly be used to detect non-symmetric particles in the pulse volume such as hail and grauple, with asymmetries in the ± 45 degree planes.

Linear depolarization ratio (LDR) in the horizontal plane and in the 45 degree plane ($\text{LDR}_{\pm 45}$) are considered next. It is shown that LDR measurements are also difficult to make. Measurements of $\text{LDR}_{\pm 45}$ are shown to be possible, and information about the drop size distribution can be gained with these measurements. This technique, along with the differential reflectivity technique, provides two independent means of obtaining drop size distribution parameters. The proposed dual-polarized radar at VPI&SU will be capable of making both measurements simultaneously.

The last section of this chapter considers circular depolarization ratio.

4.1 DISCUSSION OF PARAMETERS USED TO CHARACTERIZE RAINDROPS

Before the drop size shape and canting angle distributions can be determined, the particle distributions described in equation (3.34) must be formulated.

The multivariate particle distribution in equation (3.34) describes the statistical characteristics of the rain filled medium. It was shown in Section 3.2.3 that the range variable could be integrated out of the distribution assuming that r was uniformly distributed, i.e. raindrops uniformly fill the pulse volume. In order to treat the other varia-

bles, (a, s, θ) it will be assumed that they are independent; then equation (3.34) can be expressed as

$$g(\bar{a}, s, \theta) = \frac{t(\theta) n(\bar{a}) p(s)}{N} \quad (4.1)$$

where

$t(\theta)$ = canting angle density function

$n(a)$ = raindrop size density function

$p(s)$ = raindrop shape density function

The raindrop shape density function is assumed to be discrete. F_0 is the fraction of raindrops that are oblate spheroidal and the remaining fraction $(1 - F_0)$ are spherical. For spherical drops, the scattering coefficient shown in equation (3.18) is used. Note that spherical raindrops only contribute to the power returned in the co-polarized channel, and scatter no energy into the cross-polarized channel of the radar. F_0 will assume to vary from 0.6 to 1.0.

Raindrops falling at terminal velocities have been shown to have non-spherical shapes, and to vary over some distribution of size. The two parameter Marshall-Palmer drop size distribution will be used to describe the drop size distribution within the volume [Marshall and Palmer, 1948], and is

given as:

$$n(\bar{a}) = N_0 \exp(-\Lambda \bar{a}) \quad (4.2)$$

where (N_0) is the magnitude and lambda (Λ) is inversely proportional to the width of the distribution. The parameter a is the equivolumetric radius of a raindrop. The parameters to be measured are N_0 and Λ . The parameter N_0 generally takes on an average value of approximately 16000 (although it does have some dependence on rain rate), while lambda can be expressed as a rain rate [Stutzman, 1980]:

$$\Lambda = 8.2 R^{-0.21} \quad (4.3)$$

These values are based on long-term statistics of rain rate and are generally not true for instantaneous measurements; and shown by Cherry, et al. [1981] using the Z_{DR} technique. Other values of N_0 and Λ for drizzle, widespread rain and thunderstorm rain are available [Joss and Valdrogel, 1979]. Equation (4.3) will be used as an indicator of the rainrate (e.g. $\Lambda = 8.2$, rainrate ~ 1.0 mm/hr; $\Lambda = 3.6$, Rainrate ~ 50 mm/hr).

In making ratio measurements such as Z_{DR} , LDR and CDR, the ratio of two backscattered signals is formed. Because the parameter N_0 is a multiplicative factor in both back-scattered signals, once the ratio is formed, the dependence

of N_0 on this ratio drops out leaving only Λ in the drop size distribution. Once Λ is determined, N_0 can be found by rewriting one of the backscattering equations. In the following analysis, N_0 is normalized to 1.0 and the parameter Λ is varied over 1.0 to 10.0 to determine the backscattered signals and associated ratios. Seliga and Bringi [1976] have shown that equation (4.2) is a sufficient description of the drop size distribution.

This distribution must be used with caution because both N_0 and Λ are functions of rainrate [Battan, 1973]. In carrying out the ensemble average of equation (3.38) the upper limit of integration must be chosen so that the rainrate can be expressed as [Damosso, 1978]:

$$R = \int_0^{\bar{a}_{\max}} n(\bar{a}) v(\bar{a}) w(\bar{a}) d\bar{a} \text{ (mm/hr)} \quad (4.4)$$

where

$v(\bar{a})$ = terminal raindrop speed

$w(\bar{a})$ = volume of the raindrop.

Damosso [1978] has found that for $\bar{a}_{\max} = 3.5$ mm, this relationship holds fairly well for calculating differential attenuation and phase shift along forward propagation paths. Seliga et al., [1981] have used $\bar{a}_{\max} = 4$ mm and have shown

good results in predicting rainrates with the differential reflectivity technique. For this analysis, $\bar{a}_{\max}$ was chosen to be 3.5 mm.

The nonspherical shape of falling raindrops has been studied by Pruppacher and Beard [1970]. They have shown that a distorted raindrop, if large enough takes on an oblate-like shape. Although these drops become flattened at the bottom surface as the drop size increases, the scattering properties (at 10 cm wavelength) of this shape are not very different from the equivalent oblate spheroidal drop [Warner and Hizal, 1976]. The oblateness of a raindrop is described by the ratio a/b , defined in Chapter 3. Morrison and Cross [1973] give a simple relationship between a/b and $\bar{a}$ as:

$$a/b = 1 - 0.1 \bar{a} \quad (4.5)$$

where $\bar{a}$ the equivolumetric drop radius is in millimeters. This simple equation has been shown to be in good agreement with the relationship given by Pruppacher and Beard [1970].

Recently, many investigators have studied the problem of randomly canted raindrops. Measurements carried out by Semplak [1974] and a theoretical development by Brussaard [1976] have shown that the canting of raindrops is due to wind forces. Oguchi [1977] gives a canting angle distribution which takes into account the longitudinal canting angle

component and Ostberg [1976] has used a cosine-squared-distribution. A recent review, which includes the implications of canting angle distribution on forward scattering models along terrestrial paths, is given by Olsen [1982]. Although a sophisticated canting angle distribution can be incorporated into equation (3.35), a Gaussian distribution of canting angles, assumed independent of the drop size distribution, will be used here. The probability density function for canting angles is given by:

$$t(\theta) = \frac{1}{\sqrt{2\pi} \sigma} \exp \left[\frac{-(\theta - \bar{\theta})^2}{2 \sigma^2} \right] \quad (4.6)$$

where

θ = canting angle: angle between the
drop major axis and horizontal as viewed
along the transmit propagation direction

$\bar{\theta}$ = mean canting angle

σ = standard deviation of canting angle.

Standard deviation of canting angle is assumed to be in the range of 0 to 10 degrees, and the mean canting angle is assumed to vary from ± 15 degrees. Further work is needed in this area.

To study the effects of apparent canting angle on the measured quantities, the following discussion relates the canting angle θ to the polarization angle ψ . In Figure 3.2 the canting angle is defined as the angle between the horizontal axis and the semimajor axis of the raindrop in the plane of incidence. Canting angle can also be defined as the angle between the vertical axis and the semiminor axis of the raindrop in the plane of incidence since this angle is equal to θ . The second definition above is used since the projection of $\hat{n}$ onto the x, y plane is used to define the polarization angle ψ . By considering the polarization angle ψ to have a Gaussian distribution, the probability density function of θ can then be found by defining:

$$\theta = \psi - \psi' \quad (4.7)$$

where

ψ' = defined polarization angle (i.e. $\psi' = 0$ degrees or $\psi' = 90$ degrees)

ψ = a random variable with mean $\bar{\psi}$ and standard deviation σ_{ψ} .

Therefore by making the substitution $\psi = \theta + \psi'$ in equations (3.14 and 3.15) and noting that the canting angle distribu-

tion in equation (4.6) becomes [Davenport, 1970]:

$$t(\theta) = \frac{1}{\sqrt{2\pi} \sigma_{\psi}} \exp \left[\frac{-(\theta - \bar{\psi} - \psi')^2}{2\sigma_{\psi}^2} \right] \quad (4.8)$$

A direct relationship can now be seen by comparing equations (4.6) and (4.8) and noting that

$$\begin{aligned} \bar{\theta} &= \bar{\psi} - \psi' \\ \sigma &= \sigma_{\psi} \end{aligned} \quad (4.9)$$

It is now possible to infer the apparent mean canting angle of the raindrops from radar measurements.

4.2 SCATTERING MATRIX FOR DUAL LINEAR ORTHOGONAL POLARIZATION RADARS

From the development of the scattering matrix in Section 3.1 and the radar equation in Section 3.2, the following definitions will be made:

$$\langle \underline{P}^r \rangle = \frac{C}{r^2} \langle \underline{B} \rangle \quad (4.10)$$

where

$$C = \text{radar constant} = \left(\frac{\lambda}{4\pi} \right)^2 \frac{\pi \theta_B^2}{8 \ln 2} \left(\frac{h}{2} \right)$$

and

$$\begin{aligned}
 \langle P_{11}^r \rangle &= \frac{C}{r^2} \langle B_{11} \rangle \\
 \langle P_{22}^r \rangle &= \frac{C}{r^2} \langle B_{22} \rangle \\
 \langle P_{12}^r \rangle &= \frac{C}{r^2} \langle B_{12} \rangle \\
 \langle P_{21}^r \rangle &= \frac{C}{r^2} \langle B_{21} \rangle
 \end{aligned} \tag{4.11}$$

$P_{12,21}$ can be complex quantities. [B] is given in equation (3.27). A dual linearly polarized radar will transmit in one polarization with power P_1 and receive in the same polarization (co-polar) and also the orthogonal polarization (cross-polar). Then, it transmits the orthogonal polarization with power $P_2 = P_1$ and similarly receives co and cross polarized components. A theoretical development for this radar can be carried out using the simplified equations for [B] (equation 3.30). Thus, transmitting polarization 1:

$$\begin{aligned}
 \langle P_{11}^r \rangle &= \frac{C}{r^2} [\langle |f_{11}|^2 \rangle F_o + \langle |f_s|^2 \rangle (1 - F_o)] \\
 \langle P_{22}^r \rangle &= \frac{C}{r^2} [\langle |f_{12}|^2 \rangle F_o] \\
 \langle P_{12}^r \rangle &= \langle P_{21}^{r*} \rangle = \frac{C}{r^2} [\langle f_{12}^* f_{11} \rangle F_o]
 \end{aligned} \tag{4.12a}$$

where

$$f_s = k_o^2 \frac{(m^2 - 1)}{(m^2 + 2)} = \text{spherical raindrop scattering coefficient}$$

Then transmitting polarization 2:

$$\begin{aligned}
 \langle P_{11}^r \rangle &= \frac{C}{r^2} [\langle |f_{12}|^2 \rangle F_o] \\
 \langle P_{22}^r \rangle &= \frac{C}{r^2} [\langle |f_{22}|^2 \rangle F_o + \langle |f_s|^2 \rangle (1 - F_o)] \\
 \langle P_{21}^r \rangle &= \frac{C}{r^2} [\langle f_{21}^* f_{22} \rangle F_o]
 \end{aligned} \tag{4.12b}$$

where f_{11} , f_{12} , f_{22} , f_{21} are given in Section 3.1.3. The scattering coefficients can now be related to radar measurements of the co and cross signal for both polarization. From equations (4.12a, 4.12b) it is seen that for transmitting in polarization 1, the measured co and cross signals correspond to $\langle P_{11}^r \rangle$ and $\langle P_{22}^r \rangle$ respectively, while the phase information (i.e. the phase difference of the cross-polar and co-polar signal) is contained in the off-diagonal terms $\langle P_{12} \rangle$ or $\langle P_{21} \rangle$. For transmitting in polarization 2 the co and cross-polar signals correspond to $\langle P_{22} \rangle$ and $\langle P_{11} \rangle$, respectively, and the phase information is contained in the off-diagonal terms.

4.3 TRANSFORMATION TO CIRCULAR AND ELLIPTICAL POLARIZATIONS

No stipulation has been made about the defined axis of polarization thus far in this chapter. At this point the only requirement is that the transmitted polarizations be orthogonal. Thus, this development is valid for linear,

circular or elliptical polarization. The scattering coefficients were derived using linear polarization, but through the appropriate congruent transformations [Bickel, 1965], the linear polarization scattering matrix $[f]$ can be transformed to a circular polarization scattering matrix $[f]_c$ or elliptical polarization scattering matrix $[f]_e$.

To transform $[f]$ for different polarizations, let:

$$[f]_i = \underline{Q}^T [f] \underline{Q} \quad (4.13)$$

where if i = circular polarization

$$\underline{Q} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & j \\ j & 1 \end{bmatrix}$$

and if i = elliptical polarization

$$\underline{Q} = \underline{R}(\beta) \underline{H}(\alpha)$$

where

$$\underline{R}(\beta) = \begin{bmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{bmatrix}$$

β = rotational operator

$$H(\alpha) = \begin{bmatrix} \cos \alpha & j \sin \alpha \\ j \sin \alpha & \cos \alpha \end{bmatrix}$$

α = ellipticity operator

Note that if $\beta = 0$ and $\alpha = +/-(\pi/4)$, the elliptical polarization then reduces to left/right hand circular polarization respectively. These transformations will be necessary if the non-ideal nature of the radar antenna is to be taken into account. Whether we desire to use linear or circular polarizations, in practice the polarization will be elliptical. The circular representation of the scattering matrix $[f]_c$ is used by McCormick and Hendry [1975] and in Section 4.8 will show that the development here using $[f]$ instead of $[f]_c$ is equivalent to their theory. In searching for a new technique, linear polarization was investigated, and antenna effects were neglected. For any practical analysis of a dual-polarized system antenna effects should be included by using the rotational and elliptical transformations shown above.

4.4 Z_{DR} MEASUREMENTS

To determine the drop size distribution using the differential reflectivity technique including the effects of elevation angle and canting angle, we will assume the radar receives $\langle P_H \rangle$ and $\langle P_V \rangle$, due to horizontally and vertically polarized waves. For $\psi' = 0$ degrees and for transmission in polarization 1:

$$\begin{aligned} \langle P_H \rangle &= \langle P_{11}^r \rangle = \\ \frac{C}{r^2} &\left[\left\langle \left| \frac{k_o^2}{4\pi \epsilon_o} [(p_v - p_h) \cos^2 \delta \sin^2 \bar{\theta} + p_h] \right|^2 \right\rangle F_o + \right. \\ &\quad \left. \left\langle \left| k_o^2 \frac{(m^2 - 1)}{m^2 + 2} \right|^2 \right\rangle (1 - F_o) \right] \end{aligned} \quad (4.14)$$

and for transmission in polarization 2:

$$\begin{aligned} \langle P_V \rangle &= \langle P_{22}^r \rangle = \\ \frac{C}{r^2} &\left[\left\langle \left| \frac{k_o^2}{4\pi \epsilon_o} [(p_v - p_h) \cos^2 \delta \cos^2 \bar{\theta} + p_h] \right|^2 \right\rangle F_o + \right. \\ &\quad \left. \left\langle \left| k_o^2 \frac{m^2 - 1}{m^2 + 2} \right|^2 \right\rangle (1 - F_o) \right] \end{aligned} \quad (4.15)$$

Next an equivalent reflectivity for both polarizations is defined as:

$$Z_H = \left(\frac{r^2}{C} \right) \langle P_H \rangle \quad (4.16)$$

$$Z_V = \left(\frac{r^2}{C}\right) \langle P_V \rangle \quad (4.17)$$

Z_{DR} is defined by [Seliga and Bringi, 1976]:

$$Z_{DR} = 10 \log\left(\frac{Z_H}{Z_V}\right) \quad (4.18)$$

The parameter lambda can be found independently of N_0 , the radar constant, c , and range r . N_0 can be found by rewriting either equation (4.16) or (4.17) to solve for N_0 .

An investigation of the effect of elevation angle, canting angle distribution parameters, and shape distribution on Z_{DR} measurements was done with the computer model. The results are presented in the following sections.

4.4.1 Elevation Angle Effects

In making Z_{DR} measurements to estimate rain rate over a large area the elevation angle must be close to zero degrees. In this case equations (4.14) and (4.15) can be approximated (assuming that the mean canting angle is small) as:

$$\langle P_H \rangle \approx \frac{C}{r^2} \left[\left\langle \left| \frac{k_o^2}{4\pi \epsilon_o} p_h \right|^2 \right\rangle F_o + \left\langle \left| k_o^2 \frac{m^2 - 1}{m^2 + 2} \right|^2 \right\rangle (1 - F_o) \right] \quad (4.19)$$

$$\langle P_V \rangle \approx \frac{C}{r^2} \left[\left\langle \left| \frac{k_o^2}{4\pi \epsilon_o} \right|^2 \right\rangle F_o + \left\langle \left| k_o^2 \frac{m^2 - 1}{m^2 + 2} \right|^2 \right\rangle (1 - F_o) \right] \quad (4.20)$$

This approximation was used by Seliga and Bringi [1976]. However, to extend this technique to slant paths dependence of Z_{DR} measurements on elevation angle must be taken into account. Figure 4.1 shows a plot of Z_{DR} vs. λ for different elevation angles. In this plot it is assumed that the mean and standard deviation of canting angles is 0.0 degrees and the fraction of oblate raindrops is 1.0. Elevation angles of 0, 15, 30, and 45 degrees are considered. As the elevation angle increases Z_{DR} decreases; this happens because at larger elevation angles, oblate raindrops tend to look more like spherical raindrops. This plot shows that for elevation angles less than 15 degrees Z_{DR} decreases less than 0.1 dB for rain rates between 1-100 mm/hr. For elevation angles greater than 15 degrees, Z_{DR} measurements will then result in errors of greater than 0.1 dB and hence underestimate λ and rain rate.

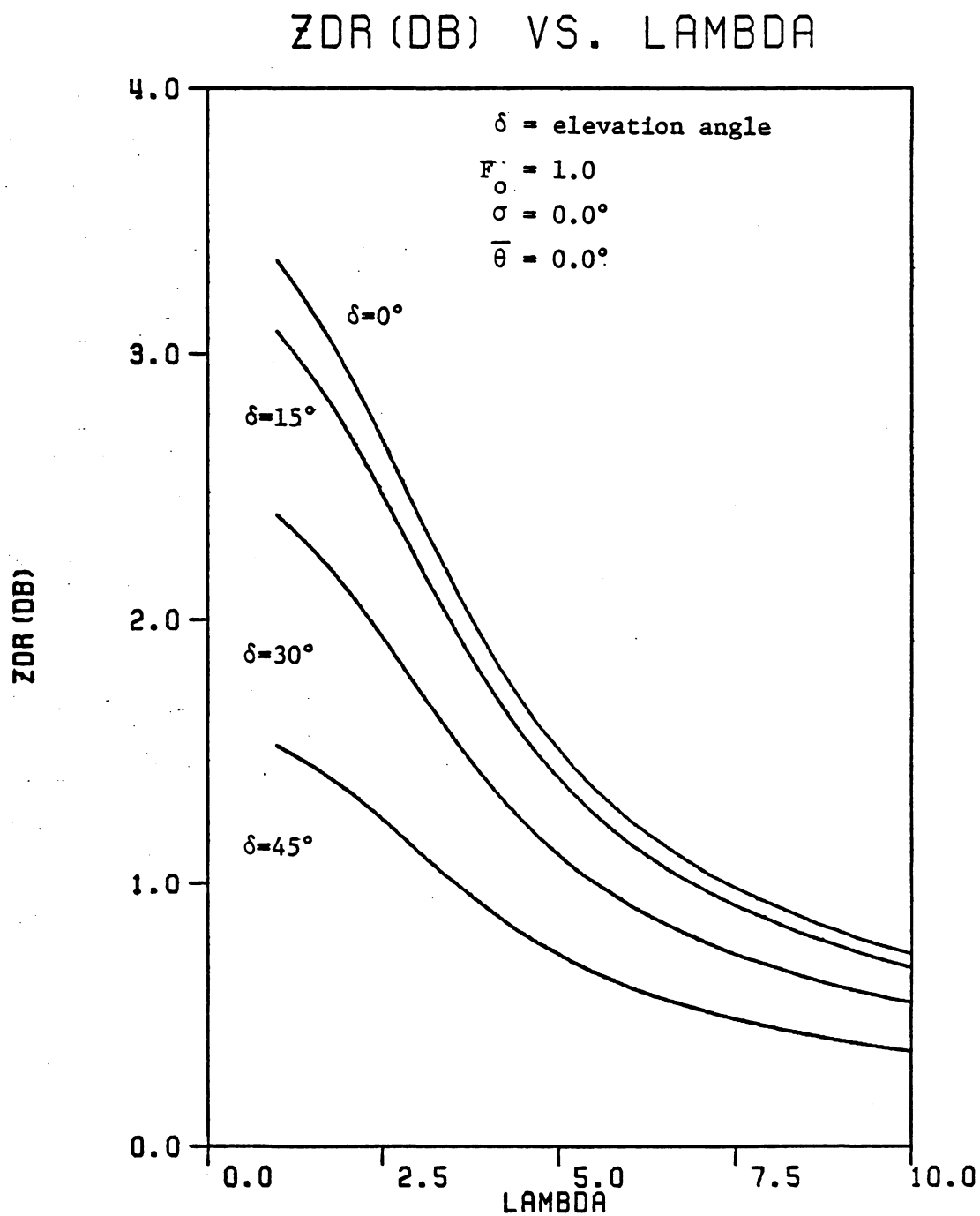


Figure 4.1. Z_{DR} vs. lambda for elevation angles of 0, 15, 30, and 45 degrees.

4.4.2 Canting Angle Effects

The differential reflectivity concept developed by Seliga and Bringi [1976, 1978] assumes that oblate drops fall with zero canting angle. This assumption will hold if the canting angle is distributed close to zero. The more exact model presented here include the effects, although small, of canting angle.

Figure 4.2 shows a plot of Z_{DR} vs. λ with different mean canting angles. The standard deviation of canting angle is zero degrees and the fraction of oblate raindrops is 1.0. This plot shows that for rainrates in the range 1 to 120 mm/hr ($8.2 \leq \lambda \leq 3.0$) an error of less than 0.1 dB is incurred if the mean canting angle is approximately ± 10 degrees. Canting angles of this magnitude are thought to be a worst-case example of canted raindrops. Figure 4.3 shows a plot of Z_{DR} vs. λ for different standard deviations. The mean canting angle is zero and the fraction of oblate raindrops is 1.0. This plot shows that for a standard deviation of 10 degrees an error of less than 0.1 dB is incurred for rainrates between 1 and 120 mm/hr.

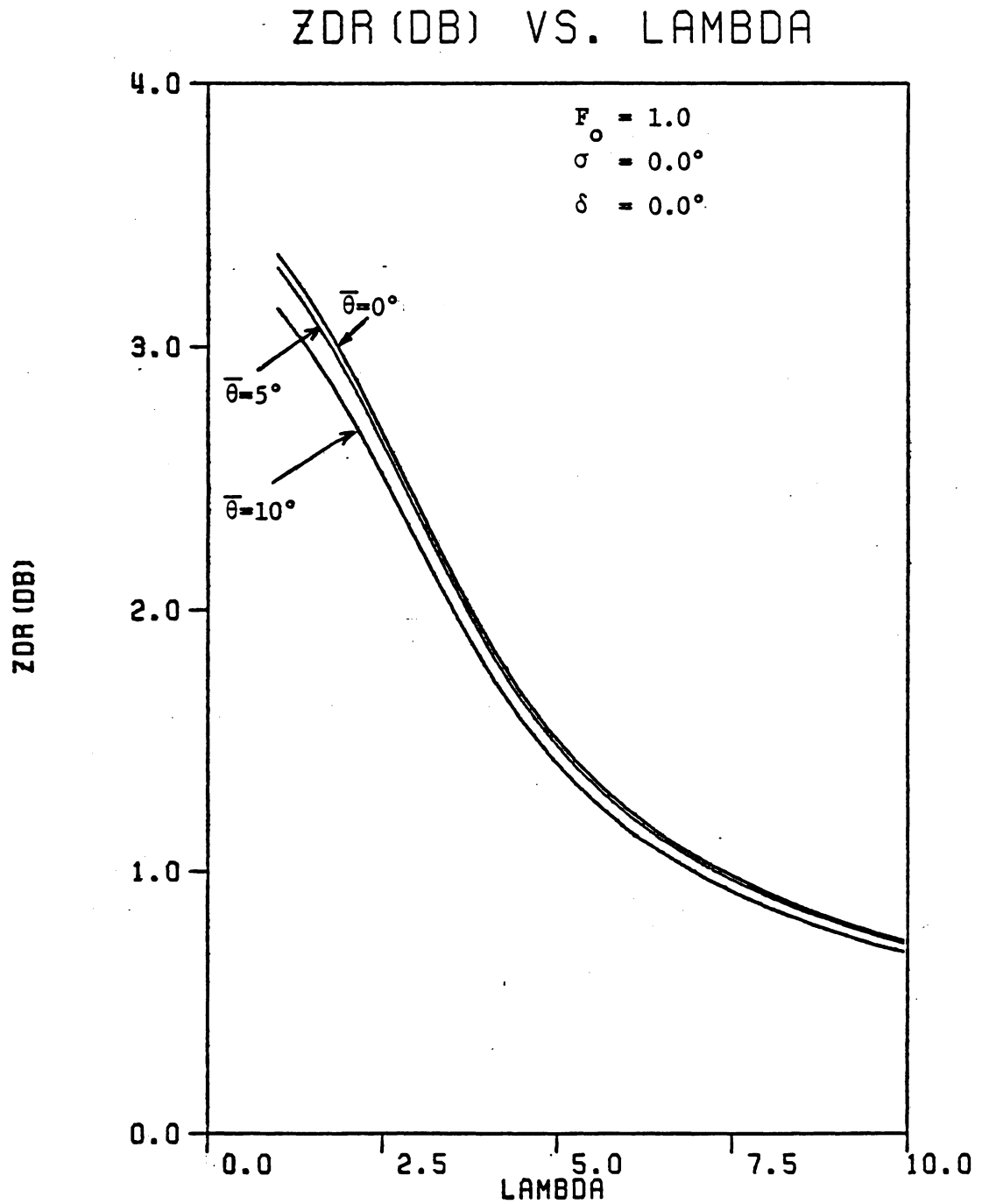


Figure 4.2. Z_{DR} vs. λ for canting angles of 0, ± 5 , and ± 10 degrees.

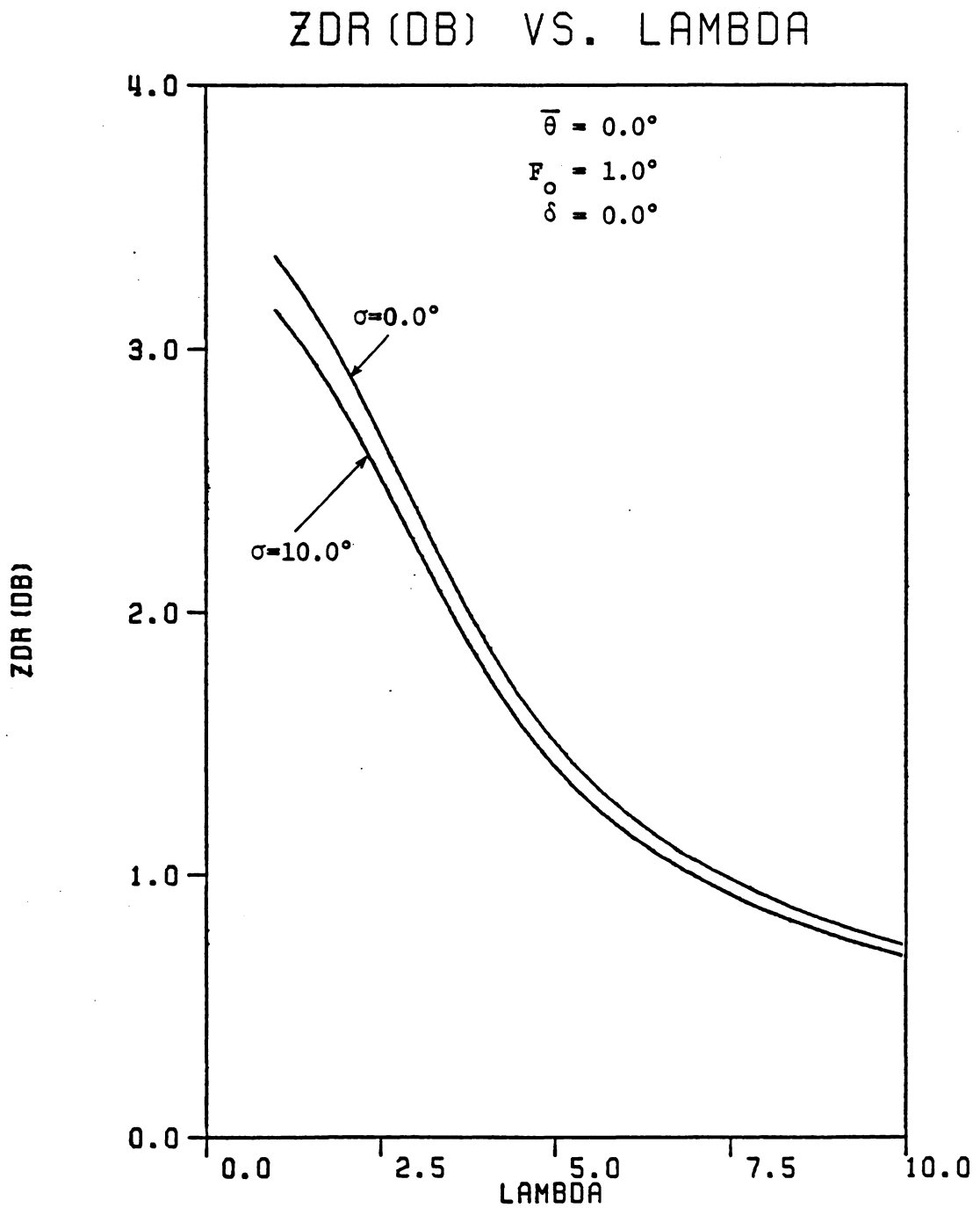


Figure 4.3. Z_{DR} vs. lambda for standard deviations in canting angle of 0 and 10 degrees.

4.4.3 Effects of the Assumed Shape Distribution

Figure 4.4 shows a plot of Z_{DR} vs. λ for different fractions of oblate raindrops. The mean canting angle and standard deviation of canting angle are assumed to be 0.0. This plot shows that significant error (≤ 0.7 dB) can be incurred for an assumed fraction of oblate raindrops between (0.6-1.0). A value of 0.6 and 1.0 represent extreme cases. These two values probably bound Z_{DR} for a given rain rate. Measurement of Z_{DR} should fall within these bounds. Further investigation in this area is needed to more accurately characterize effects of drop shape on Z_{DR} measurement.

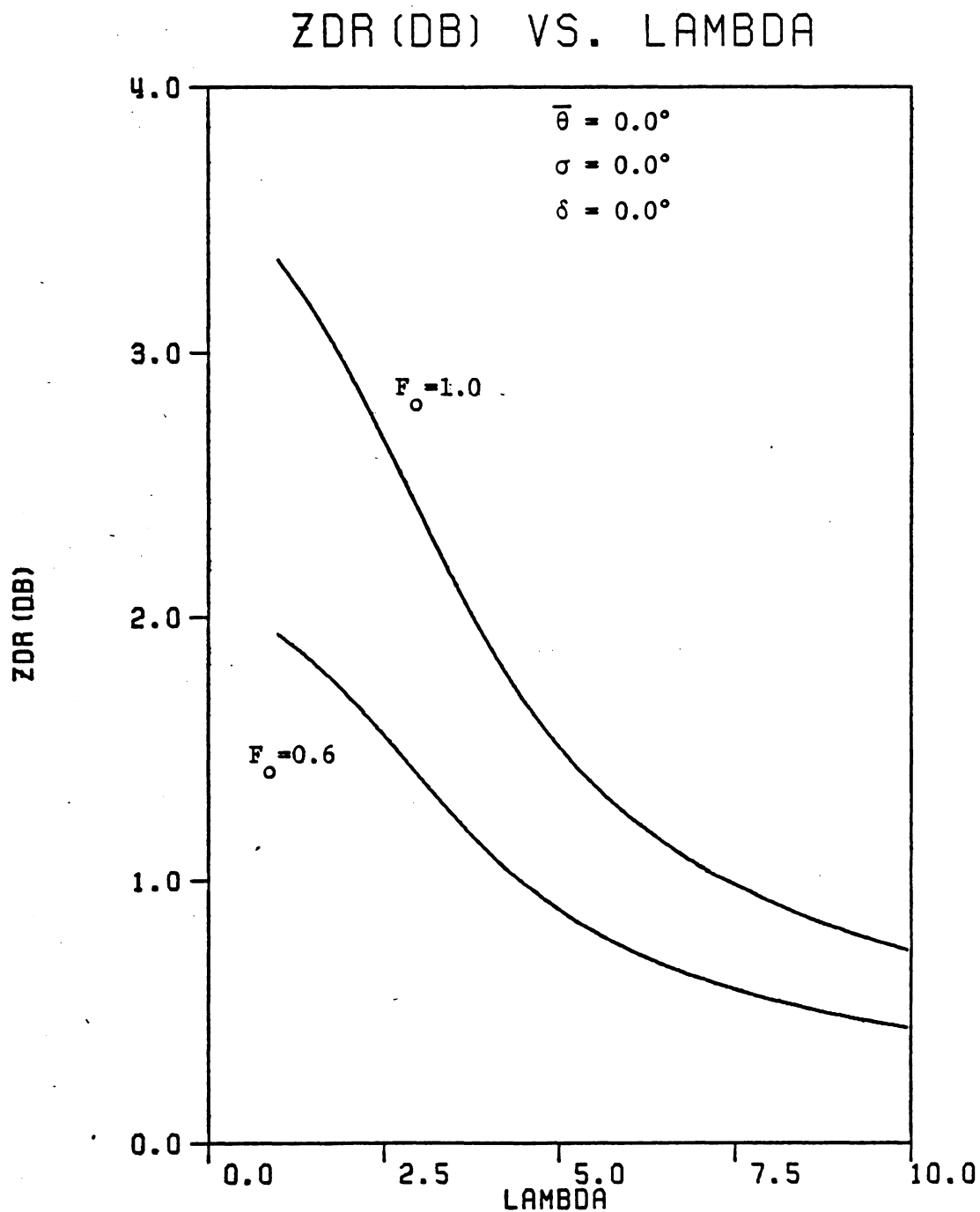


Figure 4.4. Z_{DR} vs. lambda for fractions of oblate raindrops of 0.6 and 1.0.

4.5 Z_{DR} MEASUREMENTS IN THE 45 DEGREE PLANE

By setting ψ' equal to ± 45 degrees, Z_{DR} measurements can be carried out in the 45 degree plane as follows.

$$Z_{DR_{45}} = 10 \log \left(\frac{Z_{+45}}{Z_{-45}} \right) \quad (4.21)$$

where

Z_{+45} = equivalent reflectivity received in channel 1,

Z_{-45} = equivalent reflectivity received in channel 2

The $\bar{\theta}$ term in equations (4.14) and (4.15) now become $\bar{\theta} + \frac{\pi}{4}$.

Therefore $\langle P_H \rangle$ becomes $\langle P_{+45} \rangle$ and $\langle P_V \rangle$ becomes $\langle P_{-45} \rangle$.

Theoretical results of $Z_{DR_{45}}$ measurements were examined using the computer model and are shown in Figures 4.5 and 4.6. For both plots the fraction of oblate raindrops is assumed to be 1.0. Figure 4.5 shows Z_{DR} in the 45 degree plane vs. canting angle for λ of 8.2 (rainrate ~ 1 mm/hr) and 3.0 (rainrate ~ 120 mm/hr). The standard deviation of canting angles is assumed to be 0.0 degrees. This plot shows that in order to obtain mean canting angle information near 0.0 degrees, $Z_{DR_{45}}$ must be measured very accurately but as the mean canting angle diverges from 0.0 degrees, $Z_{DR_{45}}$ becomes more dependent on Λ . If Λ is known from Z_{DR} measur-

ments, the slope of the curves in Figure 4.5 can be determined. Then, if $Z_{DR_{45}}$ can be measured, mean canting angle can be obtained. This method however is thought to be unsatisfactory because of the relative error involved with estimating Λ and measuring $Z_{DR_{45}}$. Figure 4.6 also supports this point by showing $Z_{DR_{45}}$ vs. λ for different canting angles. In this plot, the standard deviation of canting angle is zero degrees and the fraction of oblate raindrops is 1.0. The plot shows that at higher rainrates ($R > 10$ mm/hr 5.0) the range of $Z_{DR_{45}}$ values increases about 1.0 dB. If the mean canting angle is distributed about zero degrees, the $Z_{DR_{45}}$ measurement will most likely be made with a very low signal to noise ratio becoming worse as the rainrate increases. This technique could possibly be used to detect non-symmetric particles with asymmetries in the ± 45 degree planes.

4.6 LDR MEASUREMENTS

Linear depolarization ratio LDR was examined as a possible technique to determine raindrop parameters. With this technique, a cross-polarized channel is needed to measure the cross-polarized return. In this development LDR is derived as follows. For transmission in polarization 1 (with

ZDR (DB) (45 DEG)
VS.
CANTING ANGLE (DEG)

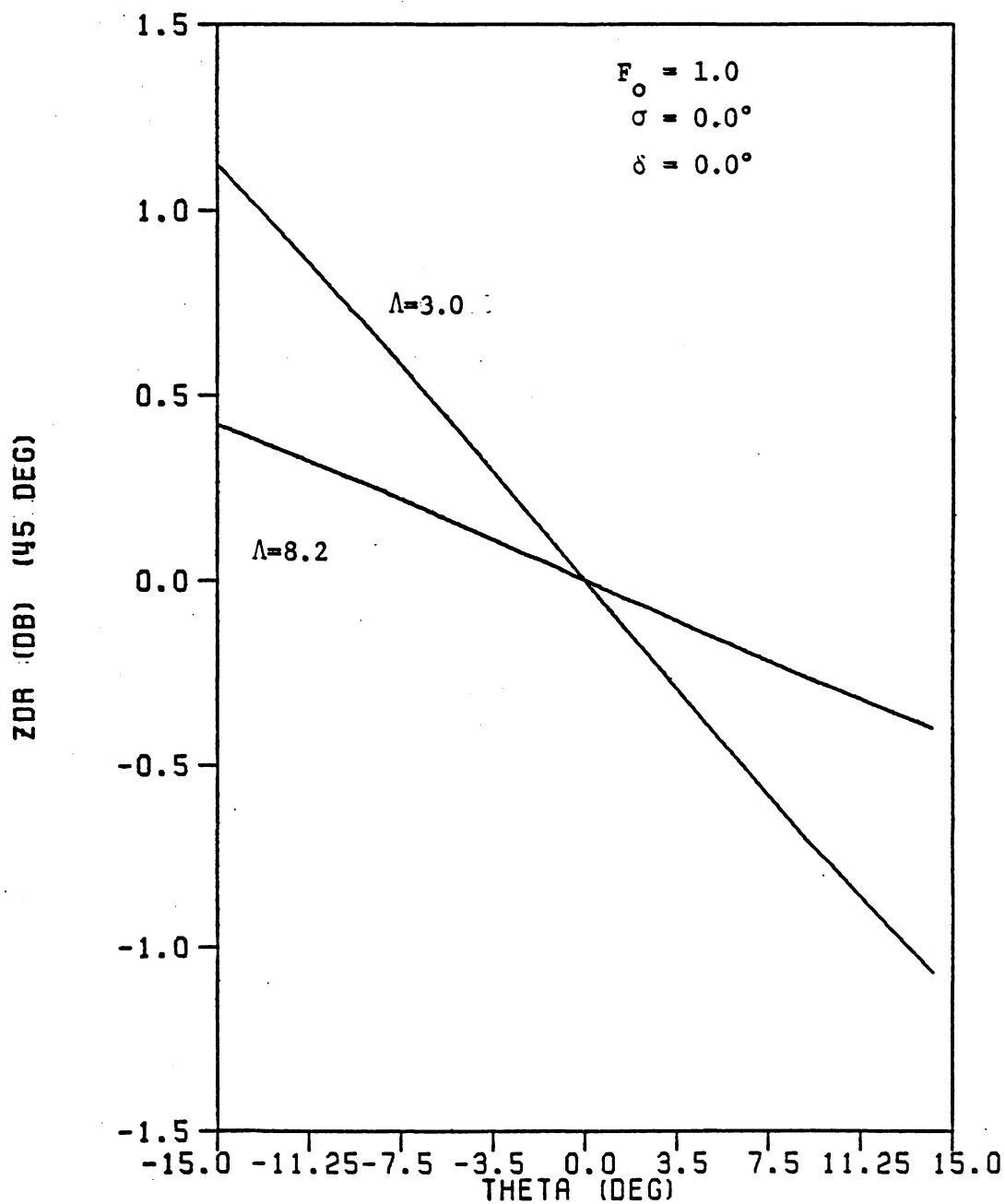


Figure 4.5. $Z_{DR_{45}}$ vs. canting angle for λ of 8.2 and 3.0 corresponding to rainrates of 1 and 120 mm/hr.

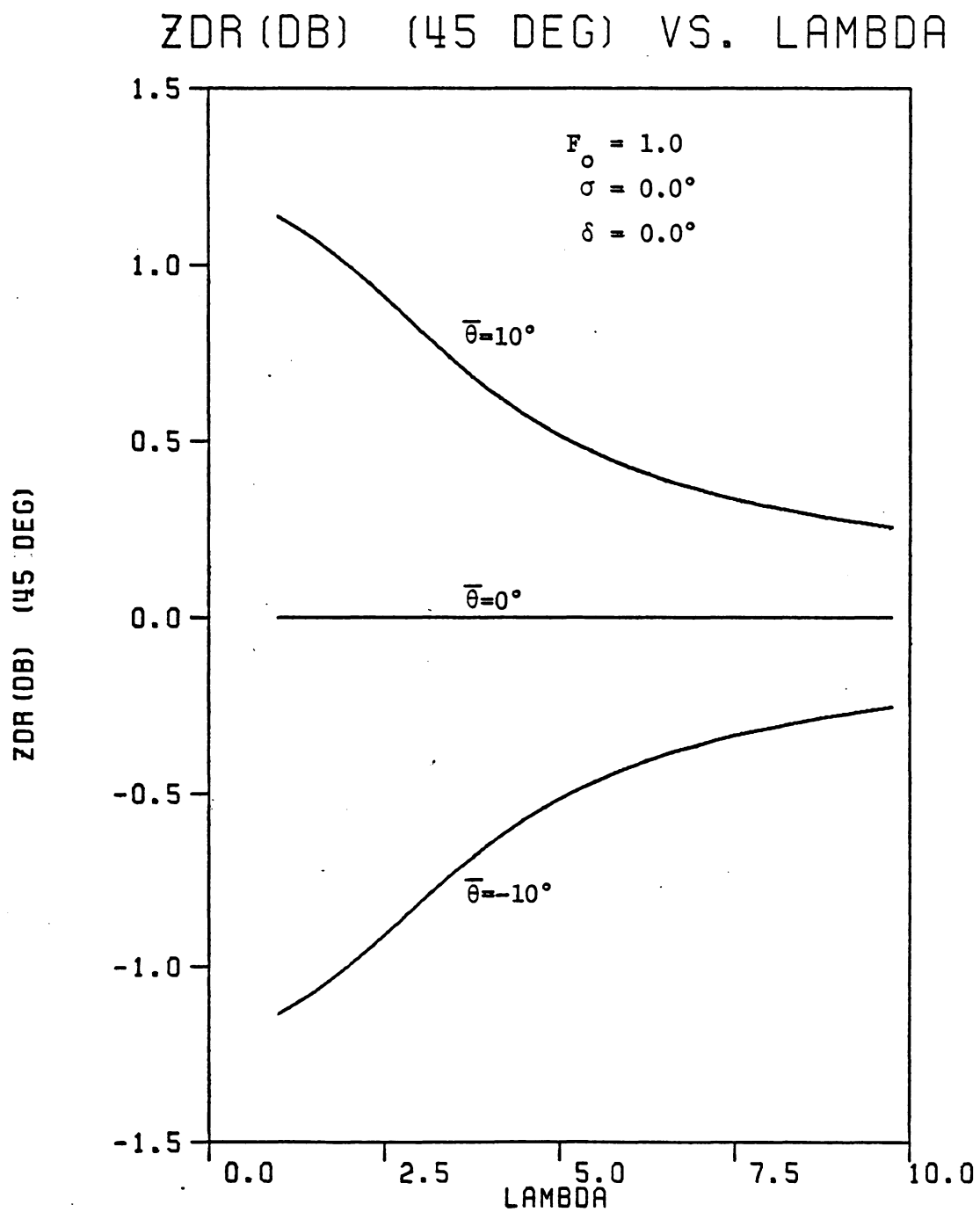


Figure 4.6. $Z_{DR_{45}}$ vs. lambda for canting angles of ± 10 , 0, and ± 10 degrees.

$\psi' = 0$ degrees)

$\langle P_{11}^r \rangle = \left(\frac{C}{r^2}\right) [\langle |f_{11}|^2 \rangle F_o + \langle |f_s|^2 \rangle (1 - F_o)] = \text{co-polarized return} =$

$$\left(\frac{C}{r^2}\right) \left[\left\langle \left| \frac{k_o^2}{4\pi \epsilon_o} [(p_v - p_h) \cos^2 \delta \sin^2 \bar{\theta} + p_h] \right|^2 \right\rangle F_o + \left\langle \left| k_o^2 \frac{m^2 - 1}{m^2 + 1} \right|^2 \right\rangle (1 - F_o) \right] \quad (4.22)$$

$\langle P_{22}^r \rangle = \frac{C}{r^2} \langle |f_{12}|^2 \rangle F_o = \text{cross-polarized return} =$

$$\left(\frac{C}{r^2}\right) \left[\left\langle \left| \frac{k_o^2}{4\pi \epsilon_o} (p_v - p_h) \cos^2 \delta \frac{\sin 2\bar{\theta}}{2} \right|^2 \right\rangle F_o \right] \quad (4.23)$$

Then LDR is defined as:

$$\text{LDR} = 10 \log \left(\frac{\langle P_{22}^r \rangle}{\langle P_{11}^r \rangle} \right) \quad (4.24)$$

Note that in LDR measurements, the range dependence drops out but this is not true for the radar constant C . In general the radar constant will be different for the co-polarized and cross-polarized channels. Care must be taken when interpreting LDR measurements (as well as circular depolari-

zation ratio measurements) and an accurate calibration of both receiver channels is needed, to obtain a reliable estimate of LDR. Since $\psi' = 0$, equation 4.22 is the LDR measured in the horizontal plane (i.e. cross-polarized component is received in the vertical plane and the co-polarized component is received in the horizontal plane). LDR measurements in the vertical plane can similarly be defined but will not be examined because the cross-polarized return for vertical polarization is smaller than for horizontal polarization.

The results of an examination of LDR for the horizontal polarization case are shown in Figures 4.7 and 4.8. These plots show LDR vs. canting angle for different values of Λ . The standard deviation of canting angle is zero degrees in Figure 4.7 and 10 degrees in Figure 4.8 and the fraction of oblate raindrops is 1.0. For these two cases, LDR can be measured (≥ -26.0 dB) only for very high rain rates and very large canting angles. The cross-polarized signal measured in this fashion is too weak to measure. This suggests that if linear polarization in the 45 degree plan is transmitted, simultaneous measurements of the return in the vertical and horizontal plane could be made. From this, Z_{DR} could be measured with negligible cross-polarization effects in these planes.

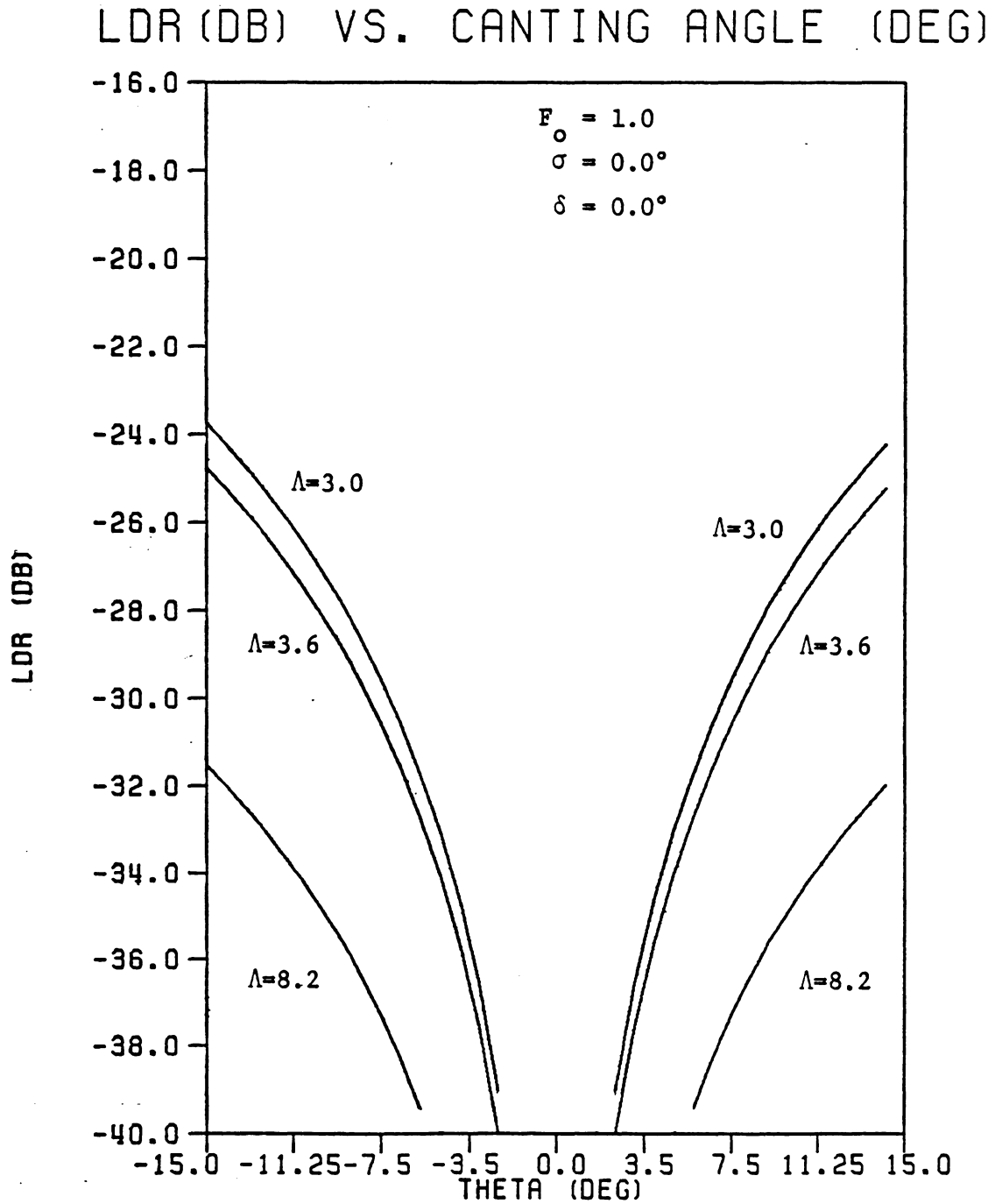


Figure 4.7. LDR vs. canting angle for Λ of 8.2, 3.6, and 3.0 corresponding to rainrates of 1, 50, and 120 mm/hr. Standard deviation of canting angle = 0 degrees.

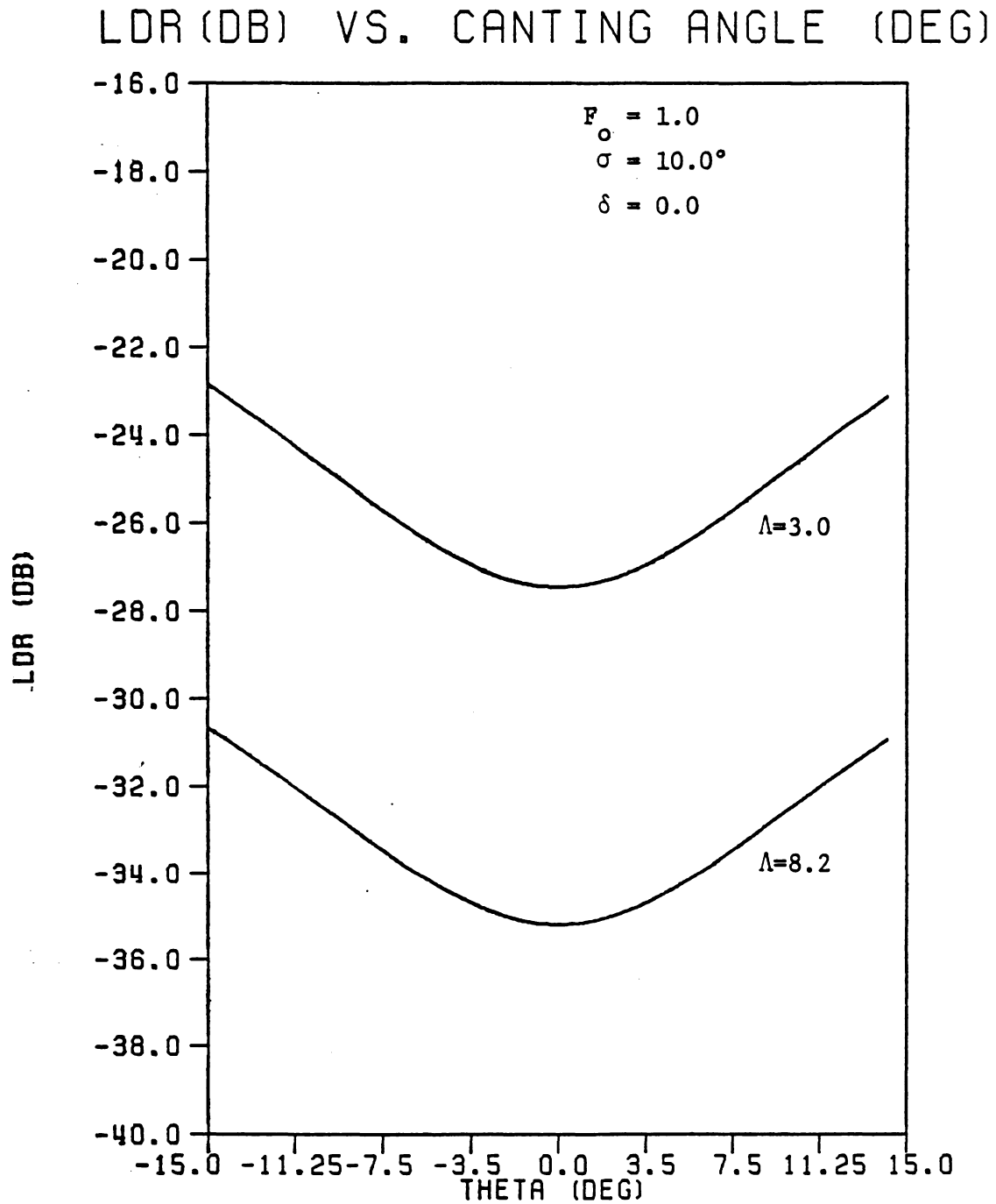


Figure 4.8. LDR vs. canting angle for lambda of 8.2 and 3.0 corresponding to rainrates of 1 and 120 mm/hr. Standard deviation of canting angle = 10 degrees.

4.7 LDR MEASUREMENTS IN THE 45 DEGREE PLANE

It was shown in the last section that LDR measurements in the horizontal plane are difficult to obtain due to the weak cross-polarized signal return. Transmitting in the ± 45 degree planes, will increase the cross-polarized signal, making LDR measurements in the ± 45 degree planes ($LDR_{\pm 45}$) an attractive technique. The theoretical results of $LDR_{\pm 45}$ can be examined by setting $\psi' = \pm \frac{\pi}{2}$ in equations (4.22) and (4.23). This was done in the computer model and the results are shown in Figures 4.9-4.12.

4.7.1 Canting Angle Effects

Figures 4.9 and 4.10 show plots of $LDR_{\pm 45}$ vs. canting angle for different values of λ . Figure 4.9 shows LDR being asymmetric about 0.0 degrees canting angle with a slowly varying peak occurring at about +5.0 degrees canting angle. Figure 4.10 show this same characteristic for LDR except with the peak occurring at about -5.0 degrees canting angle. This occurs because as the rain drop is rotates from negative canting angles to positive canting angles, the co-polarized return decreases nonlinearly in amplitude while the cross-polarized return increases nonlinearly. In forming the ratio, $LDR_{\pm 45}$, the maximum occurs at a point different from zero degrees canting angle. This holds true for LDR_{-45} .

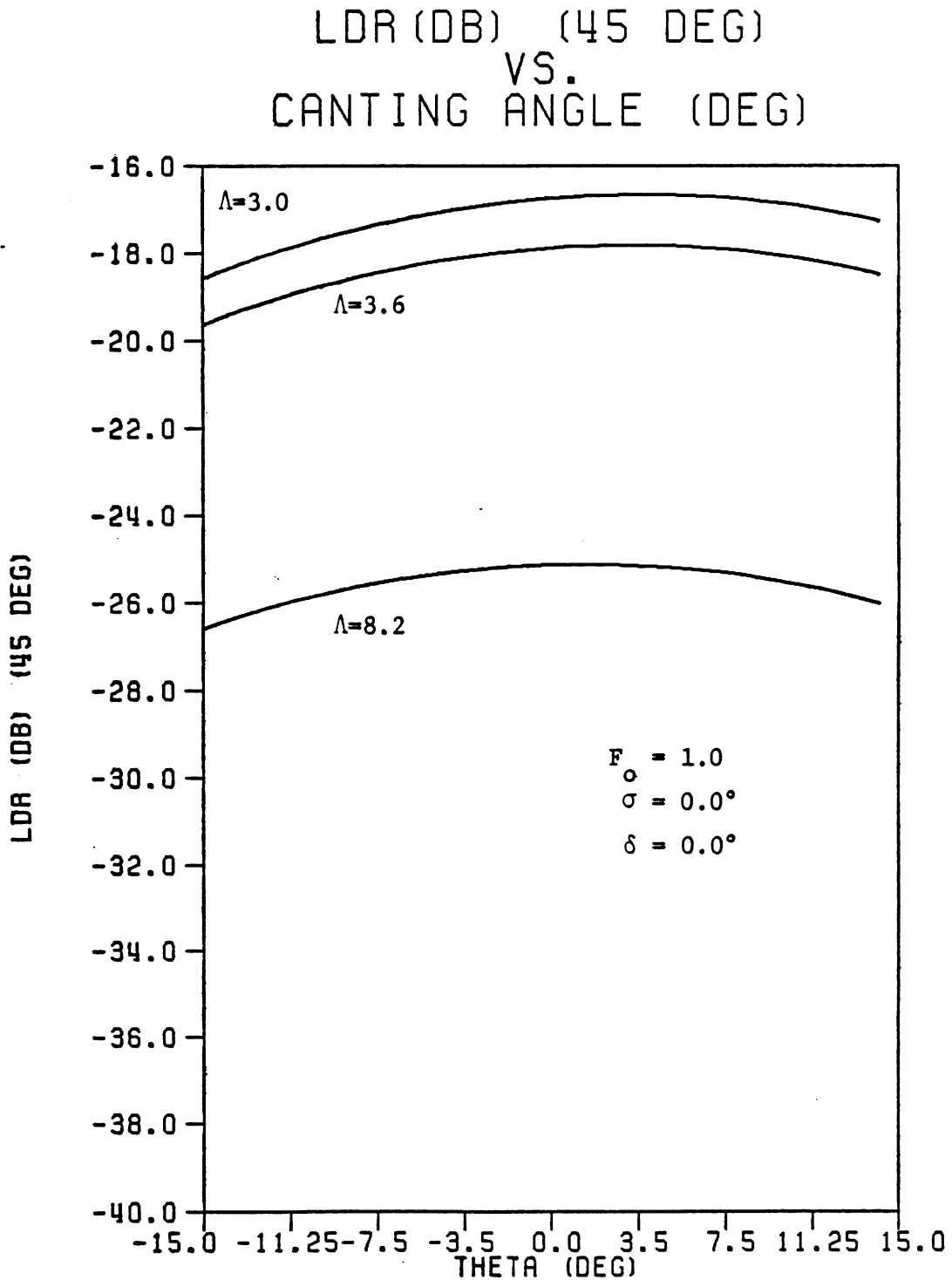


Figure 4.9. LDR_{+45} vs. canting angle for λ of 8.2, 3.6, and 3.0 corresponding to rainrates of 1, 50, and 120 mm/hr.

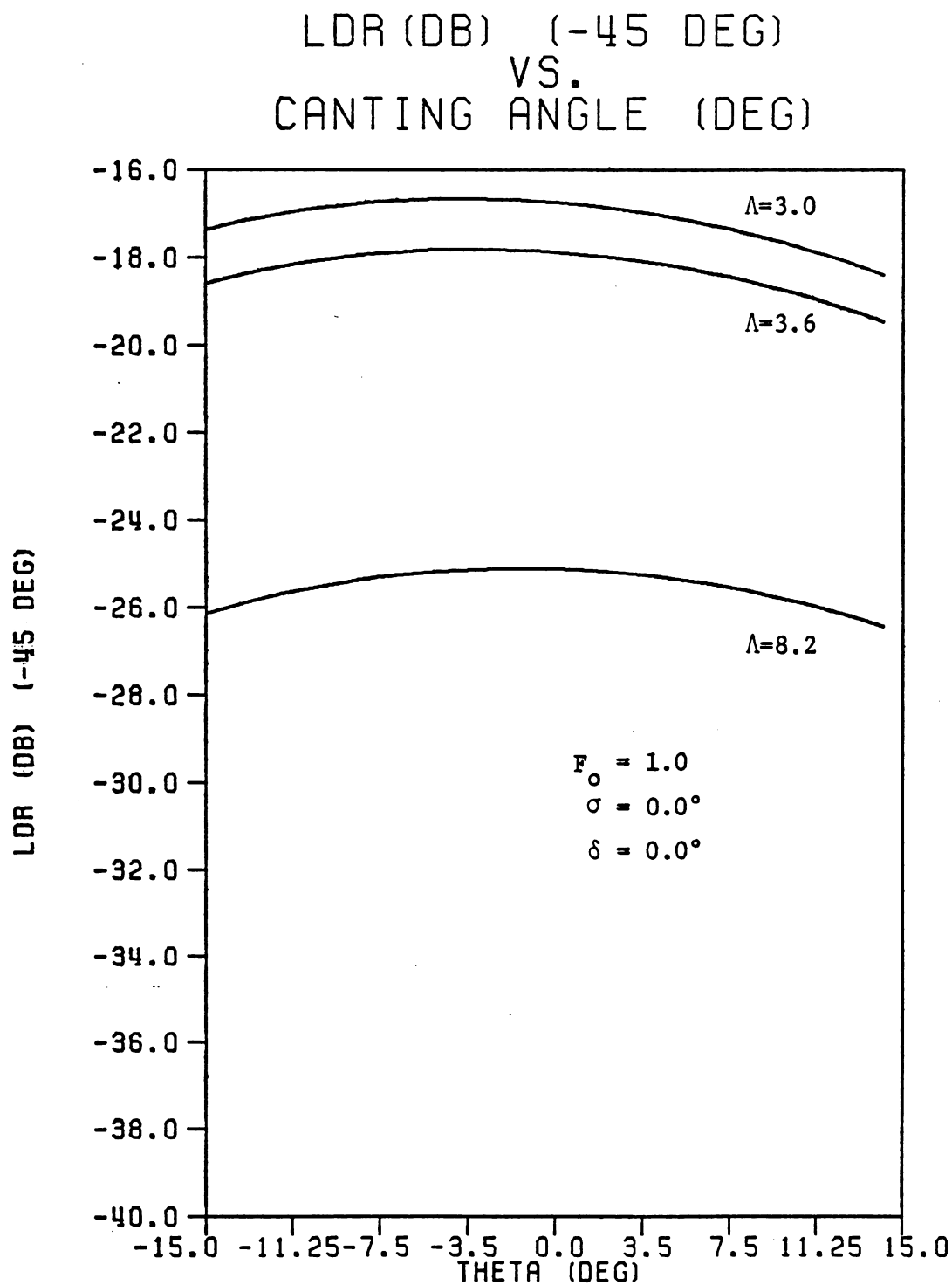


Figure 4.10. LDR₋₄₅ vs. canting angle for lambda of 8.2, 3.6, and 3.0 corresponding to rainrates of 1, 50, and 120 mm/hr.

Note that $LDR_{\pm 45}$ takes on a range of about 8 dB for λ in the range of (8.2 to 3.0) or rainrates approximately between 1 to 100 mm/hr. This observation along with the small dependence that $LDR_{\pm 45}$ has on canting angle suggests that this technique can be used to determine drop size distribution parameters. This is examined in the next section.

4.7.2 Measurement of Lambda in the Drop Size Distribution

The last section pointed out that $LDR_{\pm 45}$ can be used to determine drop size distribution parameters. This technique was modeled and the results are shown in Figures 4.11 and 4.12 by plotting $LDR_{\pm 45}$ as a function of mean canting angle.

Figure 4.11 shows LDR vs. λ for mean canting angles of -10.0, 0.0, and 10.0 degrees. The standard deviation of canting angle is zero degrees and the fraction of oblate raindrops is 1.0. This plot shows that for λ between 8.2 and 3.0 (rainrate between 1 and 120 mm/hr) $LDR_{\pm 45}$ varies by less than 1.0 dB for canting angles between ± 10 degrees. Also, $LDR_{\pm 45}$ lies in a measurable range for these rainrates. The extreme cases of ± 10 degrees mean canting angle serve as bounds in which $LDR_{\pm 45}$ should fall. Note that the $LDR_{\pm 45}$ curve for +10.0 degrees canting angle lies closer to the $LDR_{\pm 45}$ curve for 0.0 degrees canting angle than does the $LDR_{\pm 45}$ curve for -10.0 degrees canting angle. This is

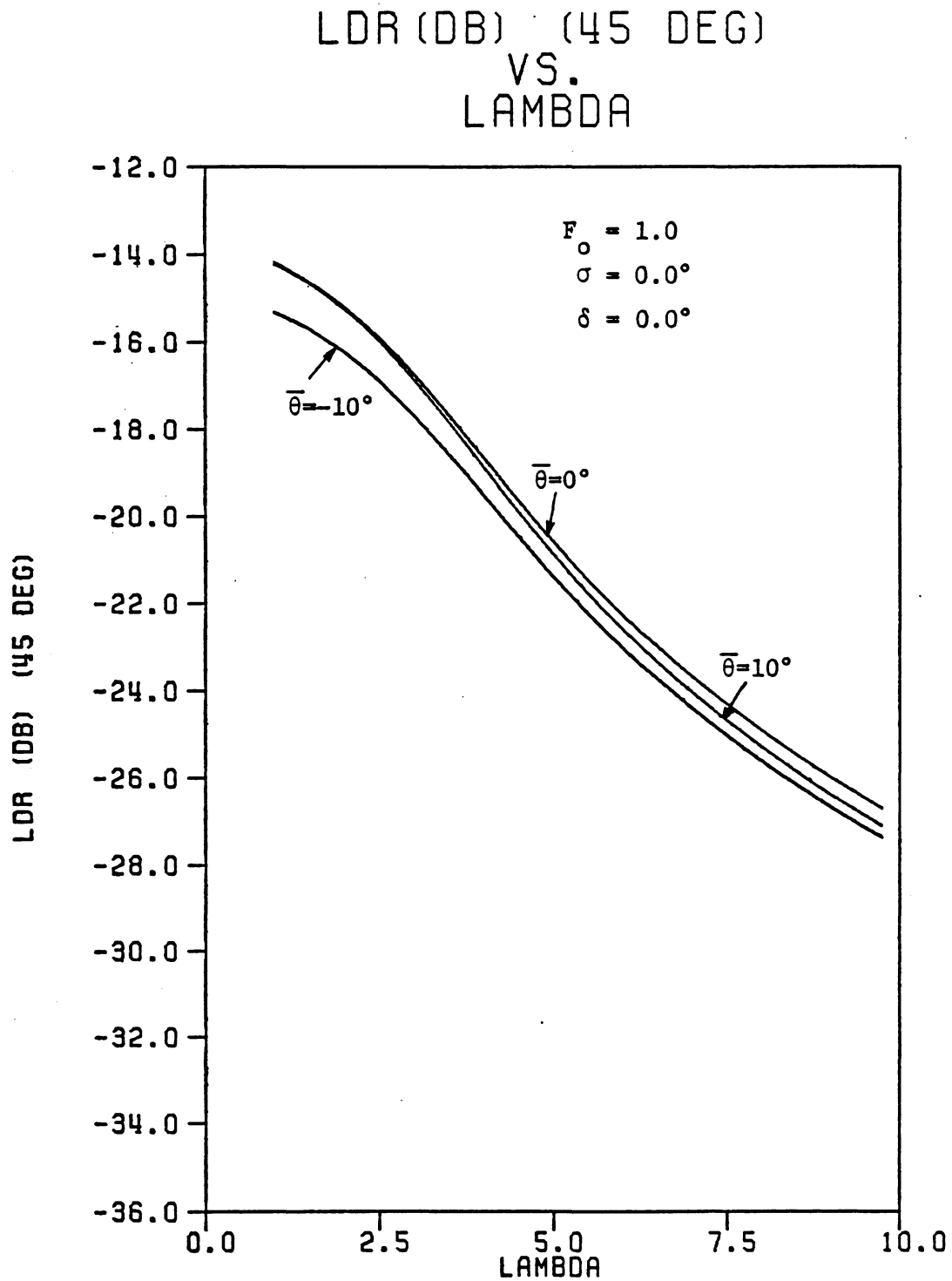


Figure 4.11. LDR_{+45} vs. lambda for canting angles of -10, 0, and +10 degrees.

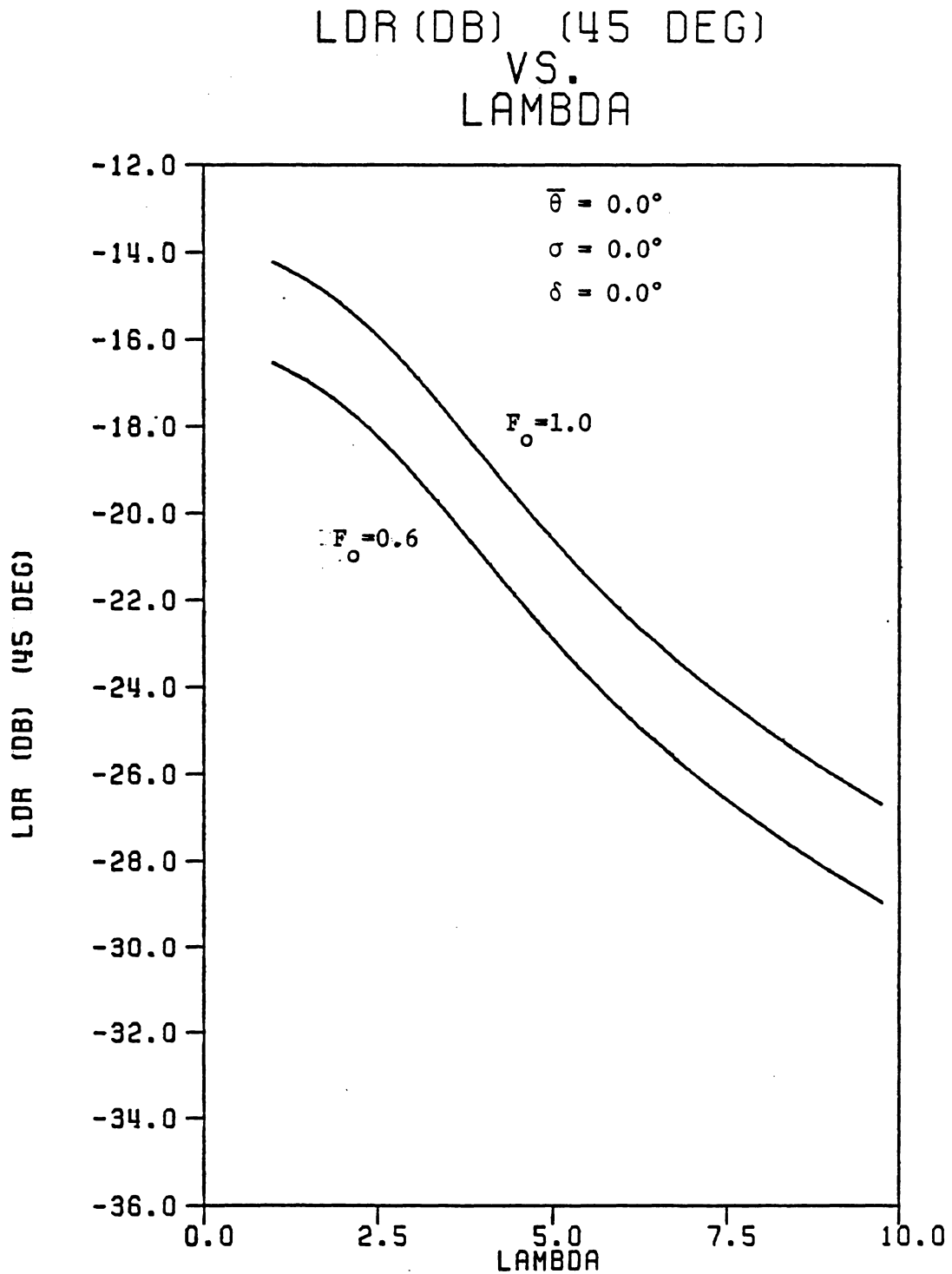


Figure 4.12. LDR_{+45} vs. lambda for fractions of oblate raindrops of 0.6 and 1.0.

due to the offset in the peak of the curves shown in Figure 4.9. A maximum LDR_{+45} curve should occur for a mean canting angle of about +5.0 degrees. Similar results hold for LDR_{-45} except the LDR_{-45} curve for -10.0 degree canting angle is closer to the LDR_{-45} curve for 0.0 degrees than is the LDR_{-45} degree curve for 10.0 degree canting angle.

Figure 4.12 shows LDR_{+45} vs. λ for the fraction of oblate drops of 0.6 and 1.0. In this plot the canting angle and the standard deviation of canting angle are zero degrees. This plot shows that there is approximately a 2.0 dB difference in the two curves for λ between 8.2 and 3.0. This serves as a bound in which LDR will fall.

4.7.3 Measurement of Apparent Degree of Orientation

It was shown in Section 2.2 that a measure of the apparent degree of orientation ORTT is possible by examining the normalized magnitude of the correlation between the two orthogonal circular returns. Similarly ORTT for the 45 degree linear polarization case can be defined as follows for transmitting polarization 1:

$$ORTT = \frac{|\langle P_{12}^r \rangle|}{(\langle P_{11}^r \rangle \langle P_{22}^r \rangle)^{1/2}} \quad (4.26)$$

where $\langle P_{11}^r \rangle$, $\langle P_{22}^r \rangle$ are defined in equations (4.22) and (4.23) with $\psi' = \pi/4$ and $\langle P_{12}^r \rangle$ is defined as

$$\langle P_{12}^r \rangle = \frac{C}{r^2} \left\{ \langle p_v^* - p_h^* \rangle \cos^2 \delta \frac{\sin 2(\bar{\theta} + \frac{\pi}{4})}{2} \cdot \right. \\ \left. [(p_v - p_h) \cos^2 \delta \sin^2(\bar{\theta} + \frac{\pi}{4}) + p_h] \right\} \quad (4.27)$$

This has been done and the results are shown in Figure 4.13. ORTT vs. the fraction of oblate raindrops is shown for lambdas of 8.2 and 3.0. It is assumed that the mean canting angle and standard deviation of canting angle is 0.0 degrees. From this plot, it can be seen that the degree of alignment can be approximated over a range of rain rates from 1 to 120 mm/hr with reasonable accuracy. The curves shown in Figure 4.13 are equal to the circular polarization case shown in Figure 4.16. This is true for the ideal case of all the raindrops being aligned with 0.0 degrees canting angle and standard deviation of canting angle equal to 0.0 degrees also. This is not true however, as the drops are allowed to be distributed about some mean canting angle. This causes ORTT for the 45 degree linear case to change, but for the circular case, ORTT is invariant to canting angle effects. This is shown in the next section.

ORTT (LINEAR POL. AT 45 DEG.) VS. F_0

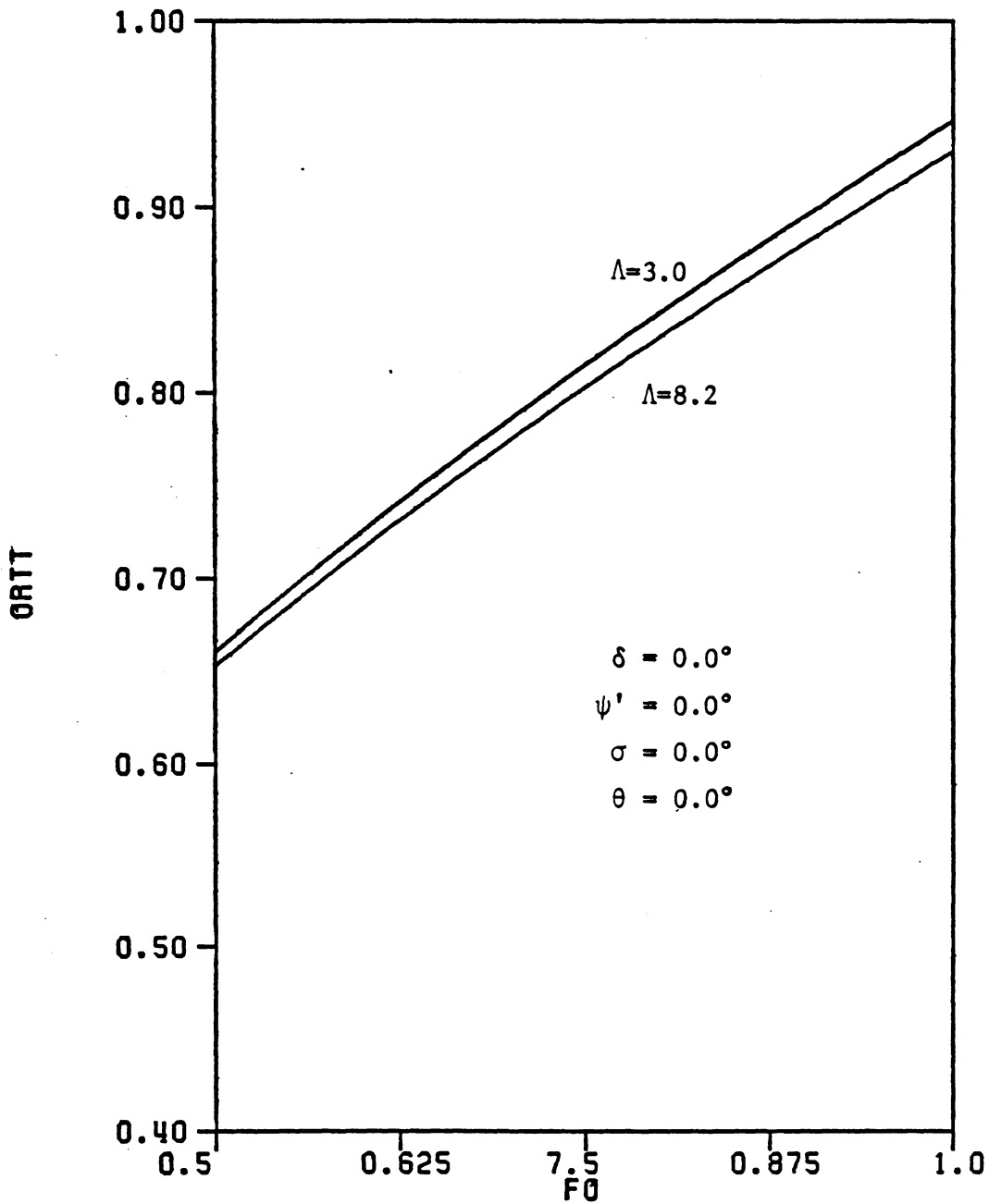


Figure 4.13. ORTT (linear polarization at 45 degrees) vs. fraction of oblate raindrops F_0 for lambda of 8.2 and 3.0 corresponding to rainrates of 1 and 120 mm/hr.

This technique should produce accurate estimates of λ , given an accurate characterization of the radar's dual-channel receiver and antenna in the radar are done. For the proposed VPI&SU radar, this technique as well as the Z_{DR} technique can be simultaneously employed to independently measure parameters of the drop size distribution.

4.8 CDR MEASUREMENTS

Circular depolarization ratio CDR, associated parameters, and phase measurement with a dual-circular polarization radar was examined. This technique is used by McCormick and Hendry [1968] and this development is equivalent to their technique. In this development CDR and the associated parameters such as apparent degree of orientation ORTT, and phase measurement between the co and cross polarized signal return is derived as follows. Using the transformations shown in Section 4.3 it can be shown that the scattering matrix of an oblate raindrop due to incident circular polari-

zation can be written as:

$$[f]_c = \left(\frac{1}{2}\right) \begin{bmatrix} f_{11} - f_{22} - 2j f_{12} & f_{11} + f_{22} \\ f_{11} + f_{22} & f_{11} - f_{22} + 2j f_{12} \end{bmatrix} \quad (4.28)$$

Therefore the scattered field can be written as:

$$\begin{bmatrix} E_{RC}^s \\ E_{LC}^s \end{bmatrix} = \frac{e^{-jk_o r}}{r} \left(\frac{1}{2}\right) \begin{bmatrix} f_{11} - f_{22} - 2j f_{12} & f_{11} + f_{22} \\ f_{11} + f_{22} & f_{11} - f_{22} + 2j f_{12} \end{bmatrix} \begin{bmatrix} E_{RC}^i \\ E_{LC}^i \end{bmatrix} \quad (4.29)$$

where

$E_{RC}^{i,s}$ = incident and scattered right hand circular polarization.

$E_{LC}^{i,s}$ = incident and scattered left hand circular polarization.

$$f_{11} = \frac{k_o^2}{4\pi \epsilon_o} [(p_v - p_h) \cos^2 \delta \sin^2 \psi + p_h]$$

$$f_{22} = \frac{k_o^2}{4\pi \epsilon_o} [(p_v - p_h) \cos^2 \delta \cos^2 \psi + p_h]$$

$$f_{12} = \frac{k_o^2}{4\pi \epsilon_o} [(p_v - p_h) \cos^2 \delta \frac{\sin 2\psi}{2}]$$

The received power due to the collection of raindrops can be achieved by applying the development of the matrix radar equation in Section 3.2. Then equation (4.10) can be written for the circular polarization case:

$$\langle \underline{P}^r \rangle = \frac{C}{r^2} \langle \underline{B}_c \rangle \quad (4.30)$$

where

$$\begin{aligned} \langle P_{11}^r \rangle &= \frac{C}{r^2} \langle B_{11_c} \rangle \\ \langle P_{22}^r \rangle &= \frac{C}{r^2} \langle B_{22_c} \rangle \\ \langle P_{12}^r \rangle &= \frac{C}{r^2} \langle B_{12_c} \rangle \\ \langle P_{21}^r \rangle &= \frac{C}{r^2} \langle B_{21_c} \rangle \end{aligned} \quad (4.31)$$

The matrix $\underline{B}_c$ can be related to the linear scattering coefficients by making the following definitions:

$$\begin{aligned} f_{11_c} &= f_{11} - f_{22} - 2j f_{12} \\ f_{22_c} &= f_{11} - f_{22} + 2j f_{12} \\ f_{12_c} &= f_{21_c} = f_{11} + f_{22} \end{aligned} \quad (4.32)$$

and noting that for transmitting right hand circular polari-

zation

$$\begin{aligned}
 B_{11_c} &= \frac{1}{4} |f_{11_c}|^2 \\
 B_{22_c} &= \frac{1}{4} |f_{12_c}|^2 \\
 B_{21_c} &= \frac{1}{4} f_{11_c}^* f_{21_c} = B_{12_c}^*
 \end{aligned} \tag{4.33}$$

It is noted that for circular polarization, the definition of co and cross polarized signal return is different than that for linear polarization. This is due to the opposite propagation directions for the scattered and incident fields. For transmitting right hand circular polarization, the co polarized return is defined as the left hand circular polarization return (orthogonal to that which was transmitted) and the crosspolarized return is defined as the right hand circular polarization return. Therefore

$$\langle P_{11}^r \rangle = \frac{C}{r^2} [\langle |f_{11_c}|^2 \rangle \frac{F_o}{4}] = \text{cross return}$$

$$\langle P_{22}^r \rangle = \frac{C}{r^2} [\langle |f_{12_c}|^2 \rangle \frac{F_o}{4} + \langle |f_s|^2 \rangle (1 - F_o)] = \text{co return}$$

$$\langle P_{12}^r \rangle = \langle P_{21}^{r*} \rangle = \frac{C}{r^2} [\langle f_{11_c}^* f_{21_c} \rangle \frac{F_o}{4}] = \langle P_{21}^{r*} \rangle \tag{4.34}$$

The phase and degree of apparent orientation parameters is contained in $\langle P_{12}^r \rangle$.

From equation (4.34) the following can be defined:

$$\text{CDR} = 10 \log \left(\frac{\langle P_{11}^r \rangle}{\langle P_{22}^r \rangle} \right) \quad (4.35)$$

$$\text{Phase} = - \arctan \left(\frac{\text{Im}(\langle P_{12}^r \rangle)}{\text{Re}(\langle P_{12}^r \rangle)} \right) \quad (4.36)$$

$$\text{ORTT} = \frac{|\langle P_{12}^r \rangle|}{(\langle P_{11}^r \rangle \langle P_{22}^r \rangle)^{1/2}} \quad (4.37)$$

All of these parameters can be measured provided an accurate calibration of the dual-channel radar is performed. Similar results hold for transmitting left hand circular polarization.

An examination of CDR, phase and ORTT was carried out for transmitting right hand circular polarization and the results are shown in Figures 4.14 - 4.16.

It is noted that in using circular polarization, the return amplitudes in the co and cross polarization field is invariant to canting angle, only the relative phase varies with canting angle parameters. The phase must be referenced to the coordinate axis of the raindrop and can only be determined through actual measurement and calibration of the radar. A discussion of the calibration procedure is given by McCormick and Hendry [1979]. In this development, the polarization angle $\psi = 0.0$ degrees and the standard deviation is set to 0.0 degrees and the elevation angle is also set to 0.0 degree.

Figure 4.14 shows a plot of CDR vs. λ for the fraction of oblate drops equal to 1.0 and 0.6. Note that these curves are the same as the case for LDR_{45} (Figure 4.12). This is true because for the ideal case (i.e. mean canting angle, and standard deviation of canting angle = 0.0 degrees) the scattering characteristics for circular and linear polarization at 45 degrees are equal. In general raindrops are distributed about some mean canting angle and therefore LDR_{45} is not equal to CDR. Because CDR does not depend on canting angle as does LDR_{45} , it is thought that CDR is a better technique for measuring λ .

The phase between the co and cross polarized return for the circular polarization case is shown in Figure 4.15. It

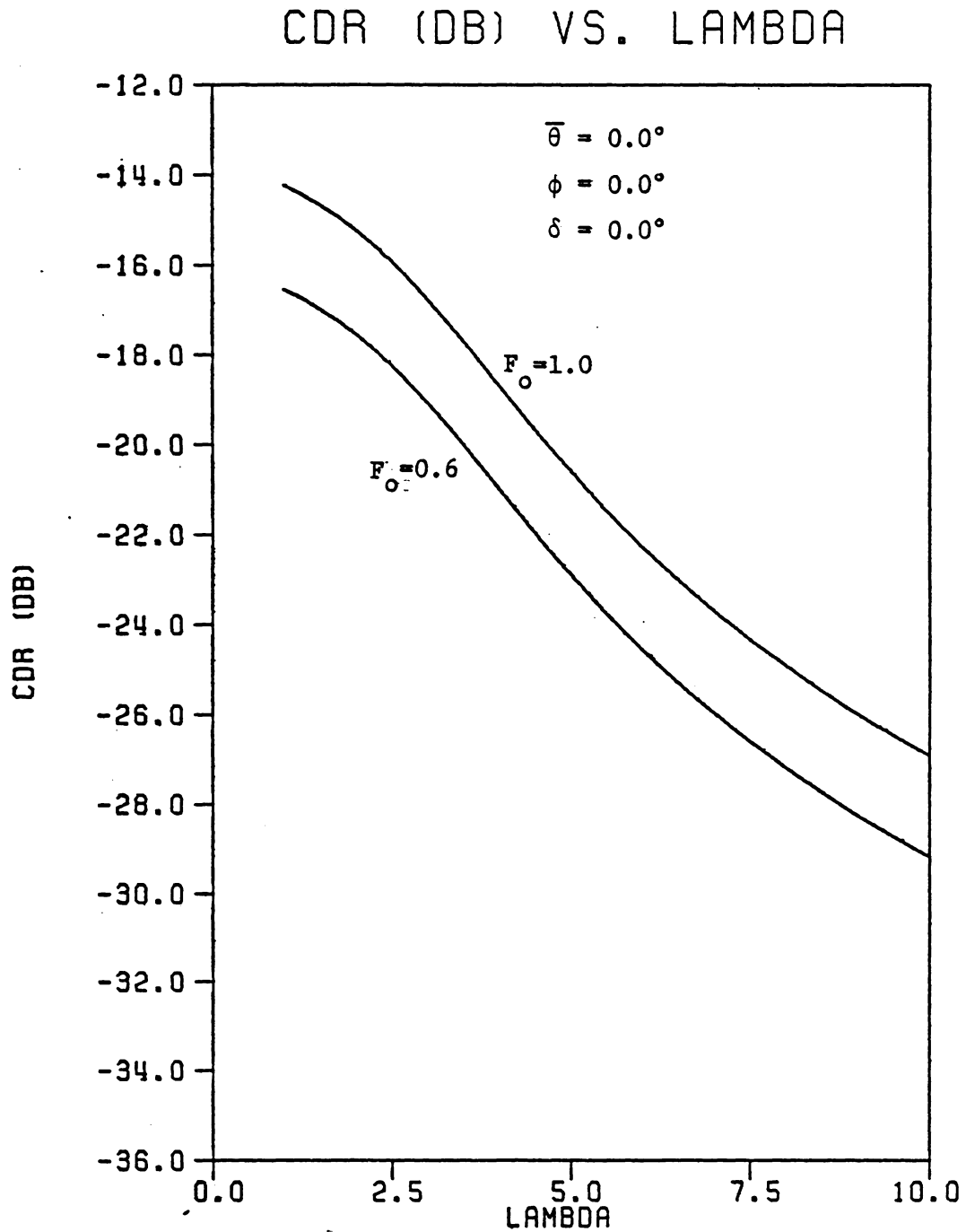


Figure 4.14. CDR vs. lambda for fractions of oblate raindrops of 0.6 and 1.0.

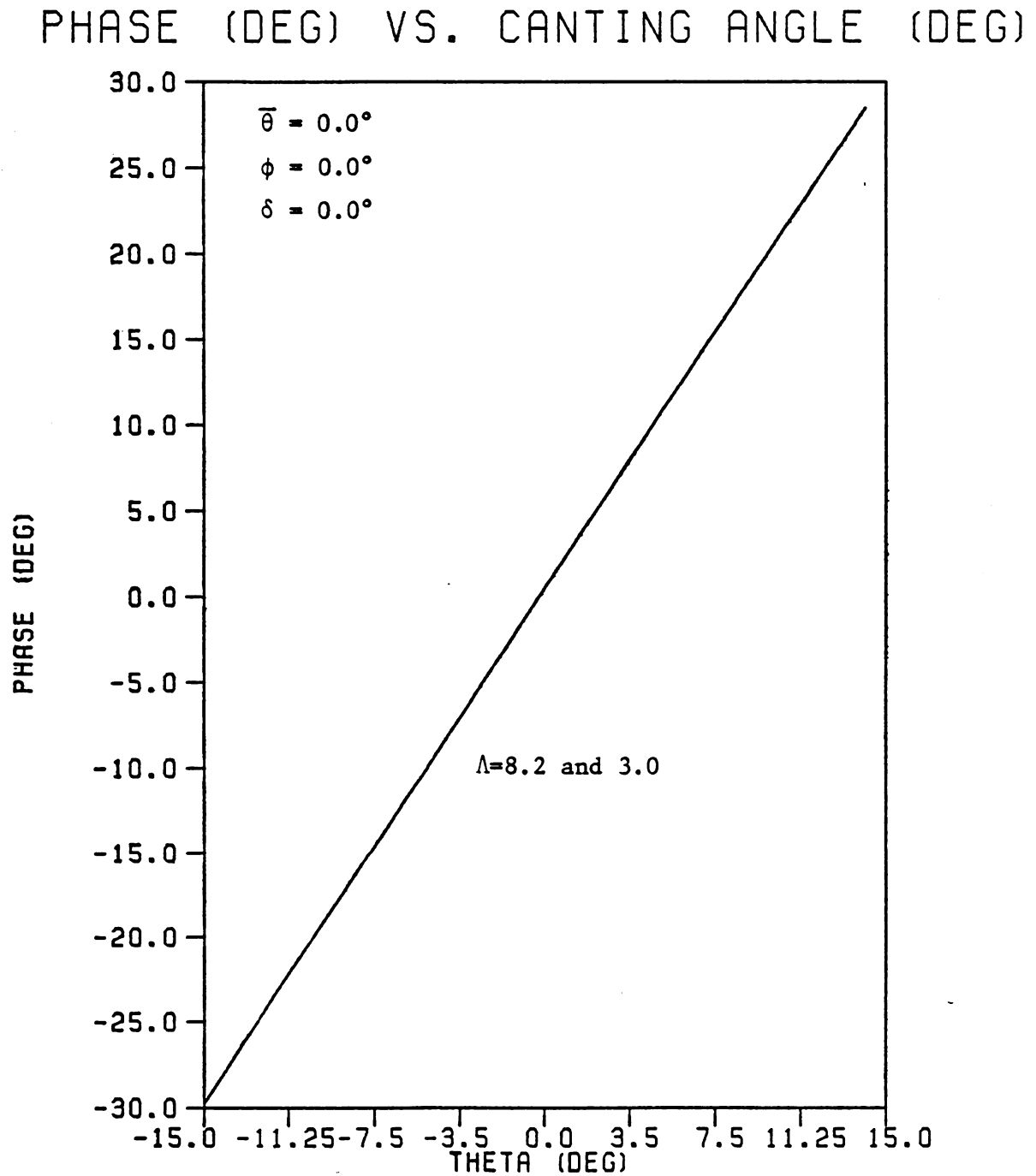


Figure 4.15. Phase vs. canting angle for lambda of 8.2 and 3.0 corresponding to rainrates of 1 and 120 mm/hr.

is noted here that the phase is invariant to the drop size distribution parameter Λ therefore, if the phase measurement can be calibrated accurately then the mean canting angle can be found with this measurement.

Figure 4.16 shows a plot of the apparent degree of orientation ORTT vs. the fraction of oblate raindrops F_0 for λ equal to 8.2 and 3.0 (or rain rates between 1 and 120 mm/hr). From this plot it is seen again ORTT for the circular polarization case is equal to ORTT for the 45 degree linear polarization case only for the ideal case of zero degrees canting angle and zero degrees standard deviation of canting angle. For the linear polarization case ORTT is a function of canting angle and will vary. Since circular polarization is invariant in amplitude to canting angle, measuring ORTT with circular polarization is thought to be superior to linear polarization.

In conclusion of this analysis, circular polarization is needed in order to perform canting angle measurements but an accurate calibration of the dual channel receiver and antenna characteristics is needed. Drop size distribution measurements can also be carried out by measuring CDR and it is thought that this technique is more accurate than LDR_{45} technique because of the independence of CDR on canting angle effects. The LDR_{45} technique does provide useful infor-

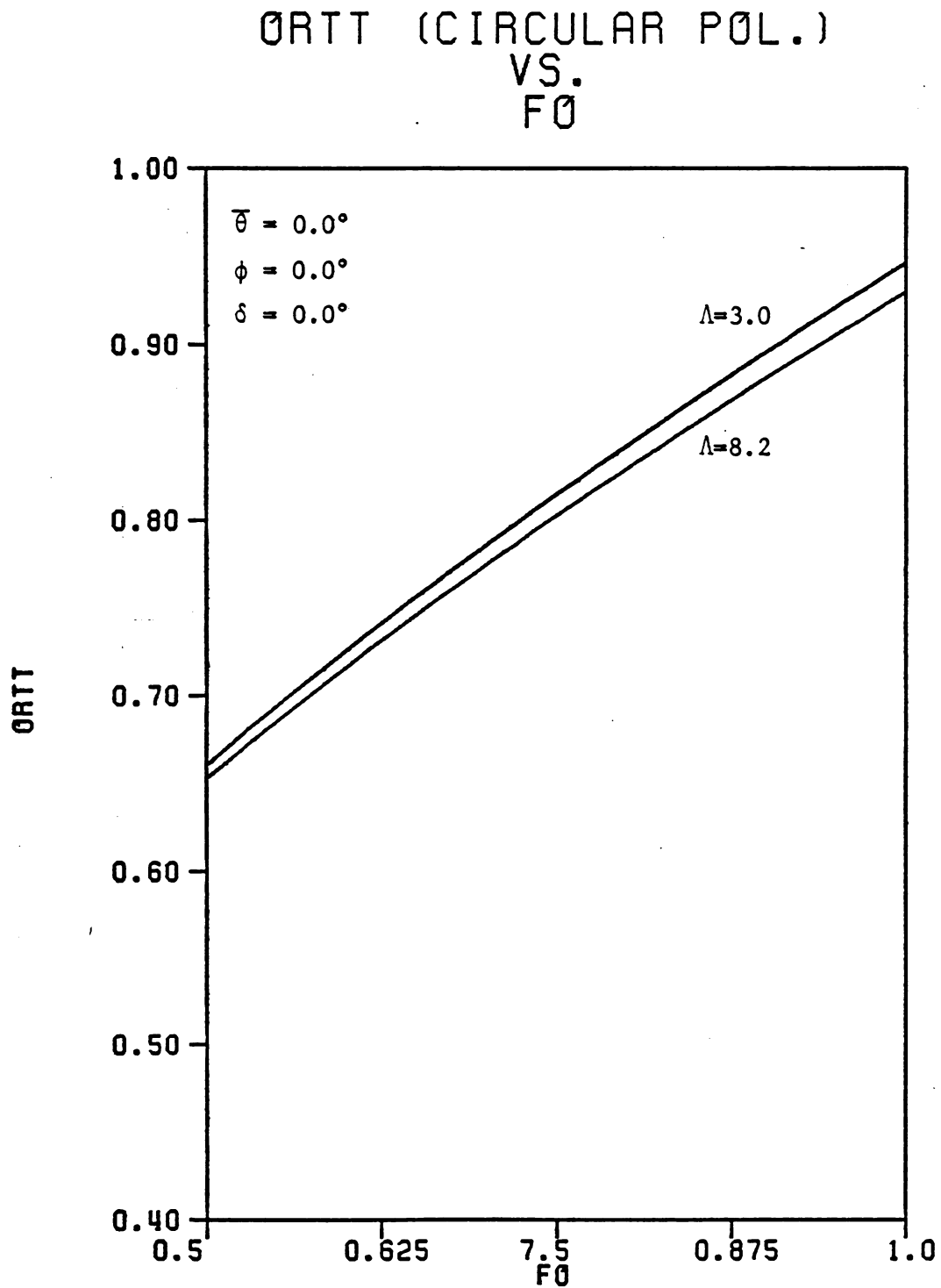


Figure 4.16. ORTT (right hand circular polarization) vs. fraction of oblate raindrops F_0 for lambda of 8.2 and 3.0 corresponding to rainrates of 1 and 120 mm/hr.

mation about precipitation parameters though, and can be used to make similar measurements as is done with the CDR technique.

Chapter V

RADAR SAMPLING CONSIDERATIONS

5.1 RADAR DATA COLLECTION

In order to measure and subsequently analyze the returned signals from a collection of scatterers, a data collection system of some form must be employed. This system is tasked with providing as much information about the returned echoes as possible while doing so efficiently and introducing negligible error. Because of the random nature of the received intensities and the required accuracy of the subsequent analysis of these intensities, a radar data collection system needs special consideration in its implementation.

This chapter addresses some of these considerations by investigating the statistical characteristics of a random fluctuating echo and the differential reflectivity radar signal. In light of these considerations, Chapter 6 offers a detailed design of a data acquisition system which will replace/upgrade the existing data acquisition system, along with a description of the unique antenna feed system developed at VPI&SU.

5.2 STATISTICAL CHARACTERISTICS OF RADAR SIGNALS AND SIGNAL SAMPLING CONSIDERATIONS

5.2.1 Uncorrelated Samples Case

Chapter 3 showed that the backscattered field from a distribution of scatterers within a pulse volume is incoherent and that the average intensity is the sum of the intensities resulting from each scatterer. In the next two sections, the backscattered signals are assumed to be due to a scatterer-filled pulse volume bounded by the half power beamwidths of the radar antenna and one half the transmitted pulse length. The path is divided into N range gates and that for each transmitted pulse an instantaneous sample for each range gate is taken and represents the amplitude (A) of the returned echo due to the pulse volume centered around any one of N range gates. Expressions for the average amplitude and average intensity of the returned signal due to a transmitted pulse have been developed by Marshall and Hitschfeld [1953]. In their work, the statistical characteristics of the amplitude of the returned pulse from a distribution of scatterers is modeled as a narrow-band Gaussian process. This is reasonable because its spectral width is small compared to the transmit frequency. From this model probability density functions of (A), the amplitude, (A^2) the intensity and $\log(A^2)$ were developed. It was then shown

that little information can be gained from a single sample of these quantities due to the large widths of the corresponding density functions. The corresponding variance of the intensity samples is comparable with their mean.

In order to reduce the sample variance, a number of independent samples must be averaged. Marshall and Hitschfeld [1953] derived the density function $p(J_K)$ where J_K is the average of a sum of intensity samples given by

$$J_K = \frac{1}{K} \sum_{i=1}^K A_i^2 \quad (5.1)$$

and

$$p(J_K) = \frac{K^K}{(\bar{A})^K (K-1)!} J_K^{K-1} \exp\left(-\frac{K J_K}{\bar{A}^2}\right) \quad (5.2)$$

and

$$\bar{A}^2 = \text{mean squared value of the received amplitude}$$

A plot of $p(J_K)$ is given in Figure 5.1. As K increases, the width of $p(J_K)$ and hence the variance decreases. The case for $K=1$ results in an exponential density function. Mars-

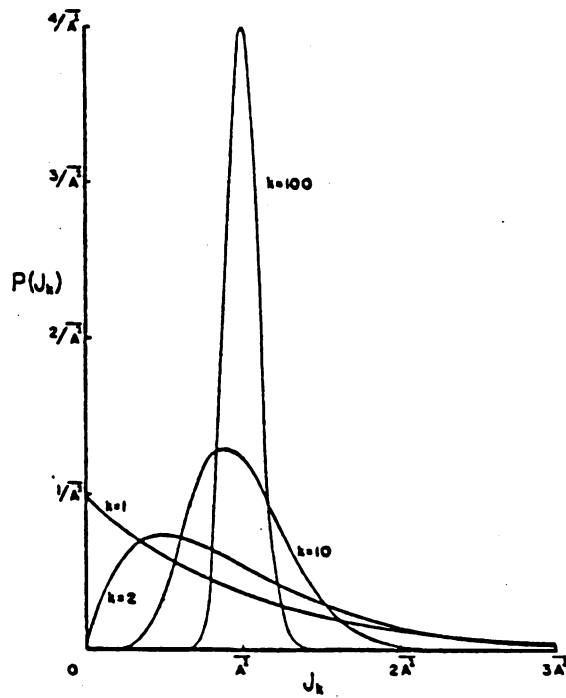


Figure 5.1. The probability density function of J_K , the average of K independent values of signal intensity, when the true mean intensity is $\bar{A}^2$ from [Marshall and Hitschfeld, 1953].

hall and Hitschfeld determined $p(J_{K_{\log}})$ numerically for

$$J_{K_{\log}} = \frac{1}{K} \sum_{i=1}^K \log (A_i)^2 \quad (5.3)$$

and the result was very close to $p(J_K)$. For large values of K the curves are identical.

5.2.2 Correlated Samples Case

The results shown above are based on independent samples. Then the variance ($\hat{\sigma}_A^2$) of the sample mean amplitude can be shown to be reduced by a factor of $1/K$ [Davenport and Root, 1958], K being the number of samples taken. This is termed variance reduction. But when the samples are highly correlated, the variance of the samples is approximately the variance of the amplitude regardless of the number of samples taken.

In general, samples from a distribution of scatterers are not independent. Their correlation is described by the autocorrelation function $R_A(\tau)$ where τ is the time difference between samples. Some properties of the autocorrelation function are: a) that a maximum value of $R_A(\tau)$ occurs at $\tau = 0$ and $R(0) = \sigma_A^2$; the variance, and b) $R_A(\tau)$ is an even function; c) $R_A(\tau)$ is the inverse Fourier transform of the

power spectral density $S_A(f)$ of the amplitude A . The parameter can be put in terms of multiples of the pulse repetition frequency (PRF) by letting

$$\tau = i t_o = \frac{i}{\text{PRF}} \quad (5.4)$$

where t_o = period of the PRF.

The power spectral density $S_A(f)$ is directly proportional to the Doppler spectrum of the received echo. This is a function of the random reorientation of the scatterers within the pulse volume due to raindrop velocity, turbulent air motion, wind shear, and parameters associated with the radar, i.e., antenna pattern, elevation angle to the scatterers, transmit frequency, receiver characteristics, etc.

Although the sample variance is not reduced by $1/K$ for partially correlated samples it can be written as [Davenport and Root, 1958]:

$$\hat{\sigma}_A^2 = \frac{A}{K} + \frac{2}{K} \sum_{i=1}^{K-1} \left(1 - \frac{i}{K}\right) [R_A(i t_o) - \bar{A}^2] \quad (5.5)$$

where

σ_A^2 = true variance of A

$\bar{A}^2$ = mean squared value of A .

This expression shows that the second term contributes to the sample variance significantly if the samples over a large number of pulses are correlated, i.e. $R_A(i t_o) \approx 1$ for large i . If $R_A(i t_o) \approx 0$ for $i \geq 2$ then the sample variance approaches $\frac{\sigma_A^2}{K}$ and variance reduction is realized.

A measure of the spread of the autocorrelation function $R_A(\tau)$ is termed the decorrelation time. It is generally defined as τ_d , the time required to make $R_A(\tau)$ drop to $1/e$ of $R_A(0)$.

Evaluation of τ_d requires many parameters and is too complex to treat here. The main point here is that little information is gained in averaging over a minimum number samples with a small sampling period, and that if the sampling period is large the time required to collect a large number of independent samples may be longer than the time in which the statistics of the signal change. A more detailed analysis of this phenomena can be found in Nathanson [1969] and Lhermitte [1963]. In the analysis of raindrop scatterers, τ_d is in the range of 5 to 30 milliseconds for the transmit frequency of 2.8 GHz.

In summary a radar data acquisition system should be capable of employing a pre-selected number of samples in determining the average received intensity with a sample spacing within an acceptable range. The proposed design will meet these requirements.

5.3 STATISTICAL CHARACTERISTICS OF THE DIFFERENTIAL REFLECTIVITY RADAR SIGNAL

Accuracy is very critical in differential reflectivity measurements because of the small range of values Z_{DR} can take on, typically 0-3 dB. In order to determine the sample statistics of Z_{DR} a model was constructed by Bringi et al. [1980]. In their model both the horizontal and vertical return amplitudes (A_H , A_V) are narrow band Gaussian noise processes with different mean squared values (σ_H^2 , σ_V^2). The normalized ratio is formed:

$$u = \frac{A_H/\sigma_H}{A_V/\sigma_V} \quad (5.6)$$

and the density function $p(u)$ is examined. $p(u)$ takes on the form of a bivariate-chi distribution with two degrees of freedom, and a cross-correlation coefficient ρ . An estimate of Z_{DR} ($\hat{Z}_{DR}$ in their notation) is determined by assuming A_{H1} , A_{Vi} (where $i = 1, M$) form a paired set of independent samples of horizontal and vertical polarization returns. A

square-law estimator

$$\hat{Z}_{DR} = \frac{\frac{1}{M} \sum_{i=1}^M A_{H_i}^2}{\frac{1}{M} \sum_{i=1}^M A_{V_i}^2} \quad (5.7)$$

and a log-ratio estimator

$$\hat{Z}_{DR}(\text{dB}) = \frac{1}{M} \sum_{i=1}^M 10 \log \left(\frac{A_{H_i}}{A_{V_i}} \right)^2 \quad (5.8)$$

are examined. The standard deviation of Z_{DR} in dB was then considered and plotted against the number of samples M for different cross correlation coefficients ρ for both the square-law estimator (Figure 5.2a) and the log-ratio estimator (Figure 5.2b). This figure shows that for high correlation between each paired set of horizontal and vertical returns, the estimate variance or sample variance decreases. The square law estimator has a smaller sample variance than the log ratio estimator for a given M and ρ . Although the square-law estimator, resembling a square law detector in the receiving chain of a radar, out performs the log-ratio estimator, the log-ratio estimator is of interest because it

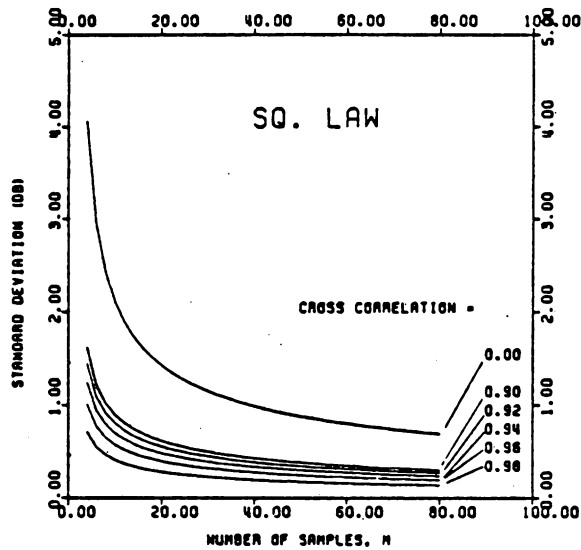


Figure 5.2a.

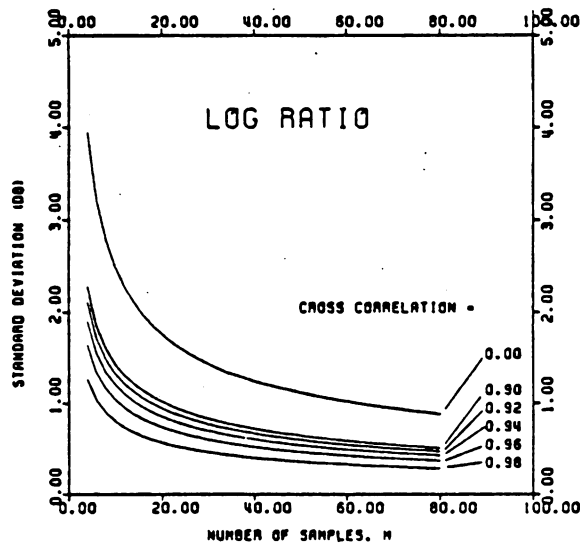


Figure 5.2b.

Figure 5.2. The standard deviation of Z_{DR} plotted against the number of samples M measured with a square law receiver (a) and a logarithmic receiver (b) for different cross-correlation coefficients, from [Seliga et al., 1980].

has the characteristic of a conventional log-receiver which, in general has a greater dynamic range than a square-law receiver.

The theoretical results were supported by measurements of Z_{DR} for various storm conditions with the Chilbolton radar located in the U.K. and with the CHILL radar located in Oklahoma. Details of both radars can be found in Bringi et al. [1978] and Hall [1981]. For the Chilbolton radar, horizontal and vertical returns were separated by 1.6 milliseconds resulting in an estimated cross correlation coefficient of about 0.97. The time between successive horizontal or vertical returns was 3.2 milliseconds. Their results show that a statistical model of this type is adequate in investigating Z_{DR} statistics and that mean values of Z_{DR} (using a log-receiver) can be obtained within a standard error range of 0.1 to 0.3 dB using approximately 60 independent samples.

The data used in the above analysis may not have achieved independence between succeeding returns (in this case $\tau = 3.2$ milliseconds) hence the number of samples may be larger than 60 for 0.1 to 0.3 dB accuracy in Z_{DR} measurements.

In order to carry out Z_{DR} measurements of this type using a log-receiver, horizontal and vertical returns which are correlated must be obtained by a radar that switches polari-

zations rapidly (on a pulse by pulse basis), stores individual returns in a range gate buffer and later forms samples of Z_{DR} from M paired sets of independent returns. With this in mind, the proposed radar/data acquisition system design will allow partial correlation between horizontal and vertical returns. It will also permit Z_{DR} estimates from horizontal and vertical returns, with arbitrary time separation.

An investigation of the autocorrelation function between successive samples and the cross correlation function between successive vertical and horizontal returns resulting in a cross-correlation coefficient less than 0.9 should be carried out. This will determine how fast the transmit polarization must be switched and how many samples must be obtained to achieve a desired accuracy on Z_{DR} measurements.

Chapter VI

DUAL-POLARIZATION RADAR DEVELOPMENT PROGRAM

To utilize the theoretical developments described in previous chapters, a program to upgrade the original VPI&SU radar design has been implemented. The main goal of the program is to develop a dual-polarization radar capable of measuring Z_{DR} , LDR and relative phase using a combination of linear polarizations. The program is being carried out in three phases:

- I. Slow Polarization-Switching Radar capable of measuring Z_{DR} from horizontal and vertical planes.
- II. Fast Dual-Polarization-Switching Radar capable of measuring Z_{DR} in H-V plane and the 45° - 135° planes.
- III. Fast Dual-Polarization-Switching Radar capable of measuring the parameters in II and measuring the crosspolarized components of amplitude and relative phase in four orthogonal planes.

Phase I is currently in operation and phase II is scheduled to be completed by August 1982. The next three sections will briefly describe the radar hardware used in the program.

This chapter will consider the feed system implementation and also give a detailed design of the new radar data acqui-

sition system. Considerations of the RF-front end, down converting circuitry, log-amplifier and phase extraction circuitry at IF will not be covered in this thesis.

6.1 ORIGINAL VPI&SU RADAR SYSTEM

The original VPI&SU radar was designed to collect single polarization backscatter data along the SIRIO path (11.6° elevation) and the COMSTAR path (46° elevation). The data acquisition system uses a hardwired TTL digital integrator. An eight-bit analog-to-digital converter and fast sample-and-hold circuit quantizes the received intensity, after log-amplification. A pulse repetition frequency of 375 Hz and a transmit pulse of 0.8 microseconds are used. The data acquisition system divides the slant path into 128 range gates, starting one kilometer from the radar and numbered consecutively up the slant path. Each range gate has a selectable length of 500 meters or 125 meters, used for the SIRIO or COMSTAR satellite link slant paths respectively. A sampling scheme accumulating 64 samples per range gate with 13.3 milliseconds between each sample is used. Details of the VPI&SU radar are discussed by Marshall et al. [1981] and a detailed description of the original data acquisition system is given by Alliss et al. [1980].

6.2 SLOW-POLARIZATION SWITCHING RADAR SYSTEM

In order to carry out differential reflectivity measurements using the existing radar a slow polarization-switching scheme was developed to operate with a slightly modified data acquisition system. A total of 64 samples per range gate is accumulated for one polarization; this takes 0.86 seconds. The polarization is then switched to the orthogonal state; switching takes about two seconds to complete. Then data collection resumes for another 0.86 seconds. From this set of samples, one Z_{DR} value is calculated. This process takes three seconds. Although paired sets of correlated returns of orthogonal polarization are not measured, it is thought that Z_{DR} measurements of some value can be made. In an earlier study using this polarization switching technique, Seliga, et al. [1981] have shown that more accurate estimates of rainfall rate can be made by measuring Z_{DR} in this fashion than estimating rainfall rates from the reflectivity Z .

The unique feature of this polarization switching scheme is in the antenna feed system. The radar antenna was originally designed to have a conical scan capability by inserting an offset of about 0.7 degrees from boresight in the splash-plate feed assembly. This assembly was then rotated about the boresight to produce a conically-scanned antenna

pattern. Another of the original features of this feed system is that linear or circuit polarization could be selected. This is achieved by launching linear polarization into a circular waveguide containing a three-stub polarizer. By changing the angle of incident linear polarization relative to the plane of symmetry of the polarizer, the desired polarization is developed and radiated toward the target. With linear polarization (set by a linear polarization launcher in the plane of the polarizer stubs), the polarizer has no effect and vertical or horizontal polarization results. When the incident linear polarization is set to ± 45 degrees relative to the plane of the polarizer stubs circular polarization results.

The feed system was adapted to give switched linear polarization by removing the offset angle from the feed and inserting a second polarizer in the rotating section of the feed. Figure 6.1 shows the modified feed system. The original, fixed polarizer-adjustable launching stub hardware is set to generate circular polarization which is incident upon the second polarizer in the rotating section. The second polarizer converts the circular polarization back to linear polarization. The resulting linear polarization angle is a function of the angle of rotation of the rotatable section of the feed.

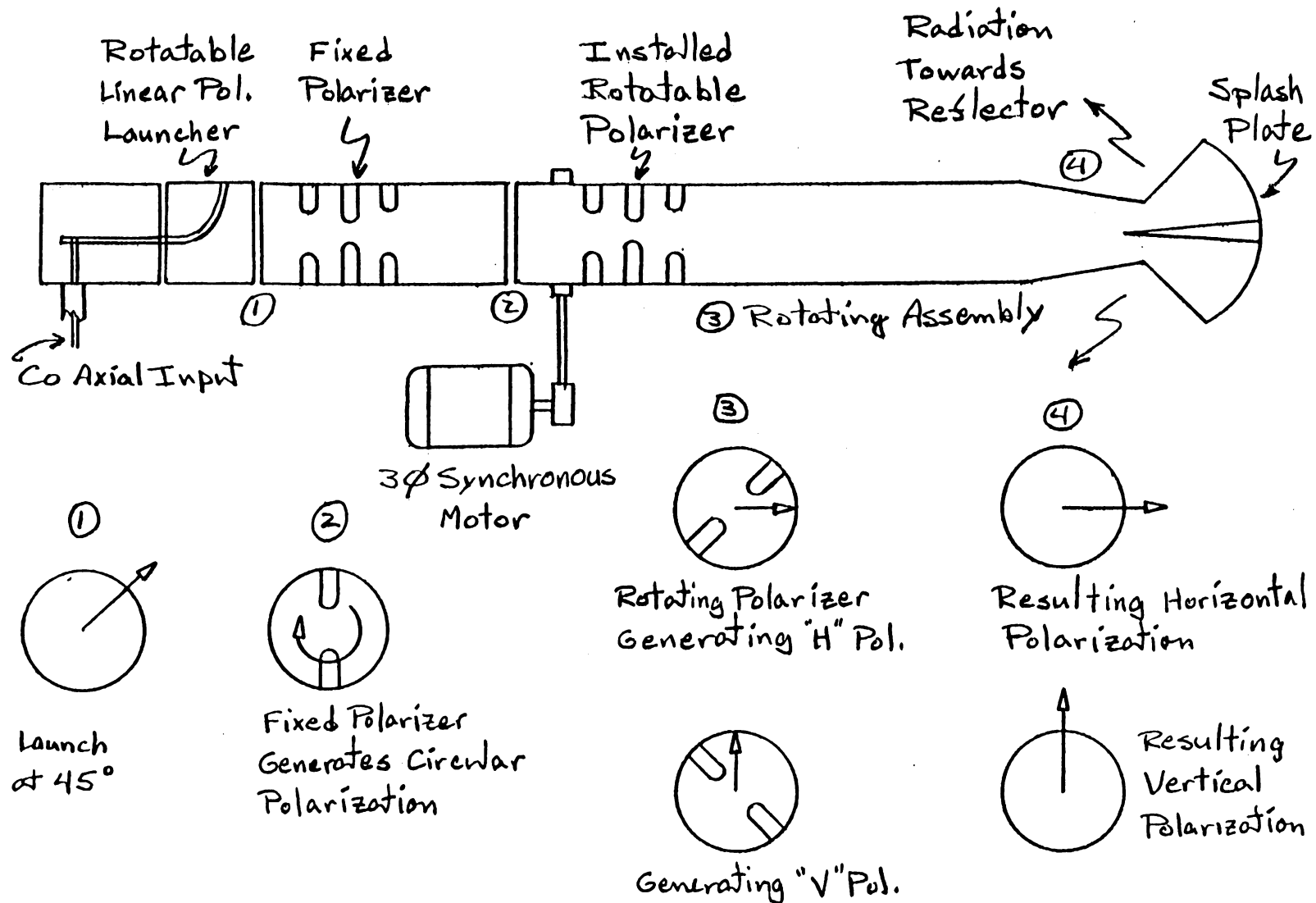


Figure 6.1. Diagram of the feed system developed at VPI&SU. Operation in the vertical and horizontal polarization positions shown.

The scanner motor rotates the splash plate-second polarizer assembly by operating at a reduced current level. This allows the motor to turn at a slow rate from an initial starting position. Two rubber-backed stops on the stationary assembly allow the splash-plate assembly to rotate through 90 degrees; with correct adjustment of the second polarizer, horizontal and vertical polarization results. Interchanging two phases of the three-phase power input to the scanner motor reverses the rotation direction. An optical sensor on the scanner motor shaft indicates feed position.

The original data acquisition system was modified to collect data only when the feed is in position. Polarization state (vertical or horizontal) is determined by monitoring a reversing relay driving the scanner motor.

This switching scheme is currently in operation and has shown considerable promise in making differential reflectivity measurements.

6.3 PROPOSED FAST-POLARIZATION SWITCHING SYSTEM

Several different schemes for rapidly switching between two orthogonal states have been devised. Hall [1981] uses a rotary waveguide switch employing a high speed shutter feeding a turnstile to obtain a switching time of 1.6 milliseconds. Although this scheme performs well, a complex switching system is needed. A fast electronic switch using ferrite devices has been used and provides very fast polarization switching times (5 microseconds) [Mueller, 1981]. This switch also performs very well but is limited in its power handling capability and is very expensive.

The unique fast-polarization switching scheme developed at VPI&SU does not perform as well as the switches described above with regard to switching times but is relatively inexpensive and easy to implement. The design is identical to that of the slow-polarization switching scheme except that the splash-plate is free to rotate and the scanner motor operates at 1800 RPM (30 Hz). The feed position indicator will produce a pulse every time the feed is in position to emit vertical polarization. A phase lock loop circuit, locked to this 30 Hz signal, will generate the transmitter trigger and polarization information used in the data acquisition system. A description of this circuit follows in Section 6.4.3.

With a rotation rate of 30 revolutions per second, the time between horizontal and vertical returns would be 8.33 milliseconds. This should allow partial cross-correlation between horizontal and vertical returns. As we noted in Section 5.3, an analysis of the cross-correlation coefficient for horizontal and vertical polarization samples spaced 8.33 milliseconds apart should be carried out in order to determine the expected error in Z_{DR} measurements. If the rotation rate is doubled (3600 RPM) the time between horizontal and vertical returns is halved. Rotation of the splash plate assembly at this speed could be difficult and therefore it is suggested that if only the second polarizer is rotated, a rotation rate of this speed (or even faster) could be achieved. This requires the development of a new feed system which could be expensive.

In addition to transmitting and receiving both vertical and horizontal polarization, the feed system will also transmit and receive in the 45 degree, 135 degree planes. This feature is included in the feed system to investigate differential reflectivity and cross-polarization measurements in these two planes and to verify the theory for this form of measurement presented in Chapter 4. This feature is implemented in the phase lock loop circuit described in Section 6.4.3.

The fast dual polarized switching radar can be implemented by replacing the fixed polarizer-adjustable-launcher with a turnstile. If the turnstile is configured to transmit circular polarization then this is equivalent to the Phase II system except that the orthogonal channel (the cross-polarized component) can be extracted easily. It is difficult to measure the cross-polarization return when transmitting in the vertical or horizontal plane but information in this channel for the 45 degree or 135 degree polarization case could be of value.

6.4 RADAR DATA COLLECTION SYSTEM CONFIGURATION

This section briefly describes the radar data acquisition system on a system level and how it will interface with the processing equipment used at the VPI&SU Satellite Tracking Station. Also, a description of the phase lock loop circuit used to synchronize the feed system is given. Details of the radar system operation will not be covered in this thesis.

A block diagram of the radar data collection system is shown in Figure 6.2. The S-100 microcomputer, STD microcomputer and the phase lock loop circuit will be covered in the following subsections while the data acquisition system will be covered in Section 6.5. A description of the PDP-11 min-

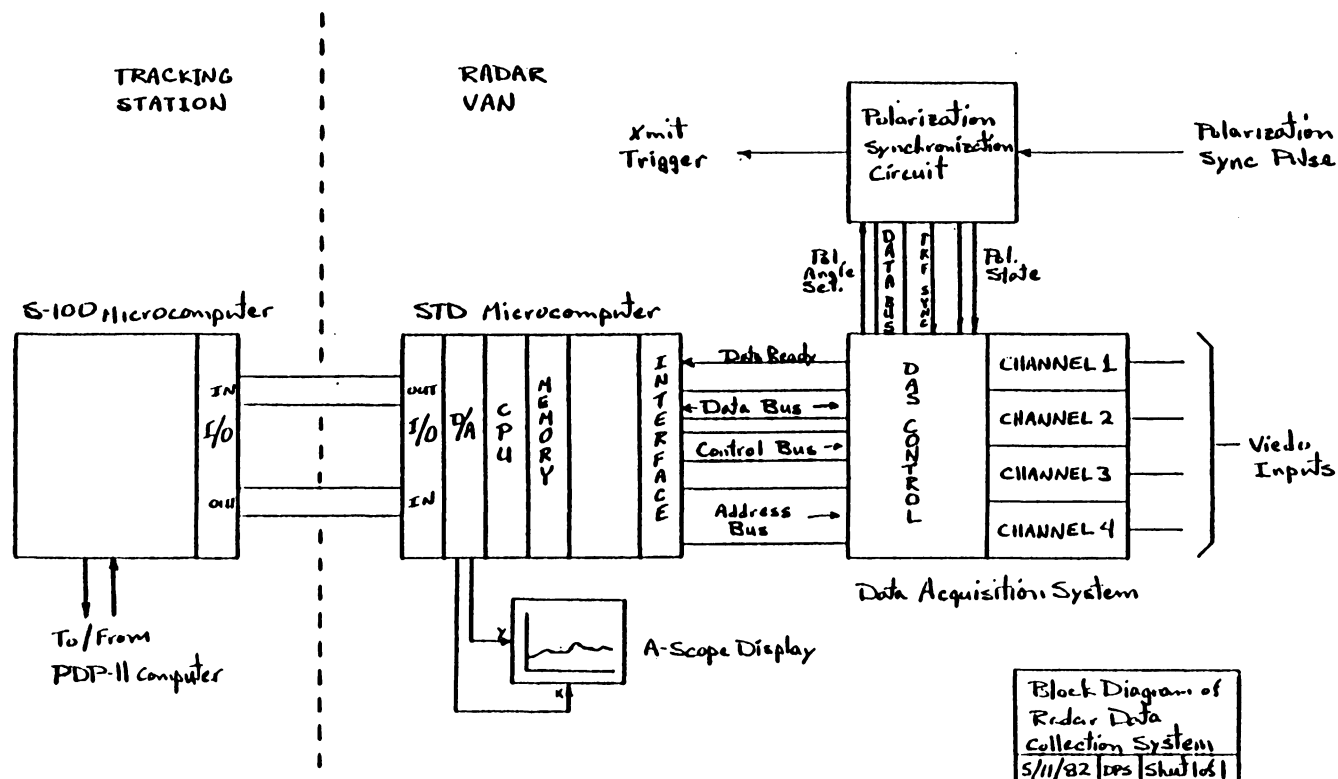


Figure 6.2. Block diagram of the radar data collection system.

icomputer and the details of hardware associated with the propagation experiments can be found in [Bostian, et al., 1980].

6.4.1 S-100 Microcomputer

The S-100 microcomputer system will function in the capacity as a front-end processor to the PDP-11 minicomputer which is used to control, monitor and collect data from all the ongoing propagation experiments. The S-100 will sample signal data, rain gauge data, and weather data, along with accepting radar data from the data acquisition system, reformat these values and pass them to the PDP-11 computer for storage. In addition to real time processing of propagation data, the S-100 system will interface with a digitizer which will record previously collected propagation data from strip charts. Table 6.1 lists the main features of the S-100 microcomputer. Additional information can be found in [Inter Systems, 1981]. The S-100 system will interface with the STD microcomputer through two I/O ports. The S-100 microcomputer will also be used to develop software for the STD microprocessor as well as to store and possibly process radar data. Programs developed on the S-100 microcomputer used to operate the radar will be down-line loaded into the STD microcomputer for execution. This enables a variety of

Table 6.1.

S-100 Microcomputer Features

Type:	Ithica 840H
CPU:	One Z80A Processor (6 MHz operating speed)
Memory:	a) 40 M byte Unformatted Hard Disk b) 12 M byte Cassette Cartridge c) 8 inch single density single side floppy disk d) 512 K byte Random Access Memory
Input/Output:	2 Serial I/O Ports Various Parallel I/O Ports Printer
Software:	CPM Operating System Pascal Compiler Assembler

experiments to be developed in software and provides the flexibility of conducting radar experiments under computer control.

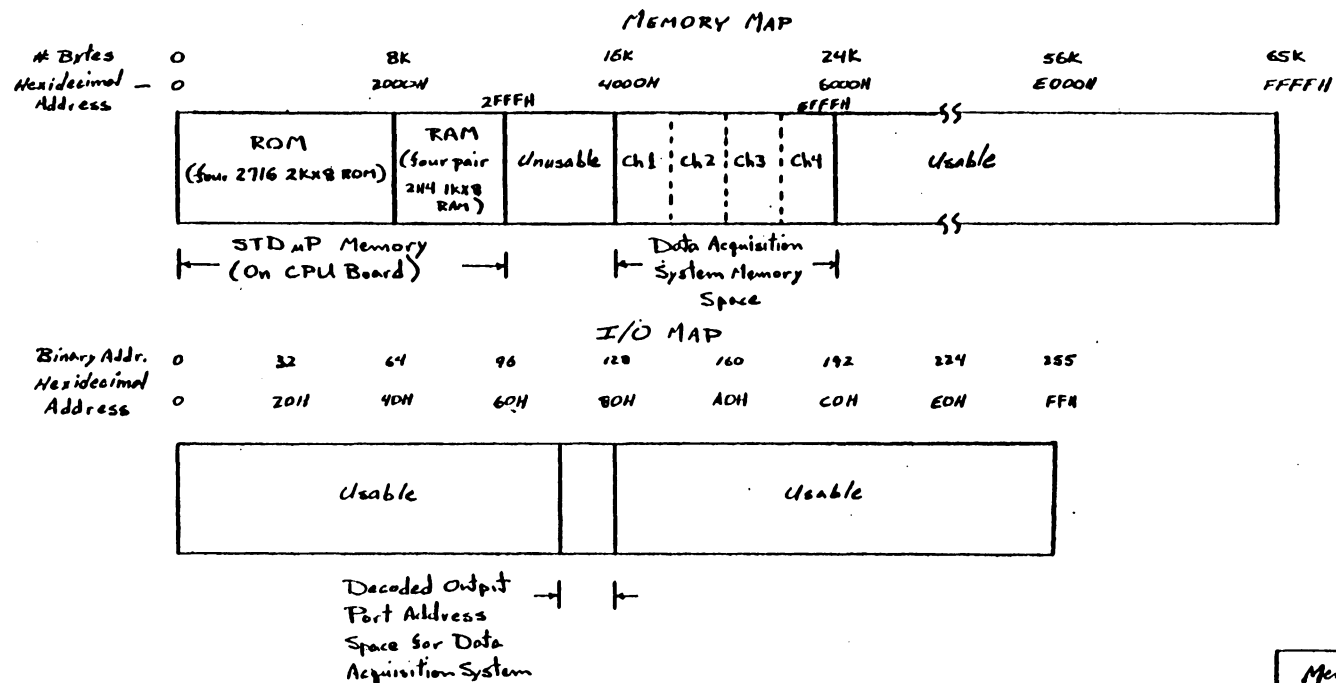
6.4.2 STD Microcomputer

The STD microcomputer extracts data from the data acquisition system and may perform some processing in the form of range normalization, accumulation, real time display, etc. Once data is in the appropriate form it is passed to the S-100 for storage.

The STD microcomputer bus system architecture was chosen because many STD microcomputer products are available from a variety of manufacturers, and this system enables the hardware to be reconfigured at a relatively low cost. The minimum system configuration consists of a Z80A (4 MHz) CPU board with 8K bytes of on board ROM and 4 K bytes of on board RAM; a universal parallel I/O board, used for the S-100 system interface; and a special interface board used for the data acquisition system interface. A description of these boards can be obtained from Pro-Log Corporation [1981]. Information concerning the Z80 A cpu can be found in [Zilog, 1978]. Additional functions can be implemented by including the appropriate I/O (A/D-D/A converters, serial, parallel I/O) and developing software on the S-100 com-

puter to drive the I/O. A real time display of reflectivity or differential reflectivity be implemented in the STD microcomputer using two D/A converters. One D/A converter may drive the x axis with range gate position and the other may drive the y axis with measured signal level data for each range gate.

A memory and I/O map of the STD microcomputer is shown in Figure 6.3. Output ports 70H to 7FH are decoded by the interface board and are used to set parameters in the data acquisition system and the phase lock loop circuit. Also, some output ports are reserved for handling the reset sequence in the STD microcomputer. The reset sequence allows the STD microcomputer to start program execution (upon receiving a reset) at up to seven selectable memory locations. The operator can then load up to seven programs into the STD microcomputer and select which program to execute. Note that some programs can be permanently located in ROM and some dynamically located in RAM through down-line loading from the S-100 system. In order to do this, the eight restart locations (0, 8, 16, 24, 32, 40, 48), termed restart locations 0-7, must be programmed in ROM as follows in Table 6.2. The restart location will be set by a thumbwheel switch located on the front panel next to a reset push button. See Figures 6.15, 6.16 for the detailed schematic of



Memory / I/O
Map

4/27/82 DRS sheet 1 of 1

Figure 6.3. Memory and I/O map of the STD microcomputer.

Table 6.2.

Assembly language code required for restart select option.

<u>Restart Location</u>	<u>Address</u>	<u>Instruction</u>	<u>Comments</u>
0	0000,0001H	OUT 7B,A	; Check if restart vector ; flag is set, if so ; execute a restart x.
	0002,0003,0004H	JP BOOTSTRAP	; If not set jump to boot- ; strap program.
8	0008,0009H	OUT 7C,A	; Restart vector flag was ; set and the restart ; vector was set to 1. ; Clear restart flag.
	000A,000B,000CH	JP PROGRAM1	; Jump to program one.
16	0010H,0011H	OUT 7C,A	; Restart vector flagset ; and restart vector was ; set to 2. Clear restart ; flag.
	0012,0013,0014H	JP PROGRAM2	; Jump to program two.
			⋮
48	0030,0031H	OUT 7C,A	; Restart vector flag was ; set and restart vector ; was set to 7. Clear ; restart flag.
	0032,0033,0034H	JP PROGRAM7	; Jump to program seven.

the interface board. Assuming that all seven programs are located in memory, the restart location is selected and the reset push button is depressed causing the Z80A CPU to execute the instruction located at 00H. This output instruction causes (A16), a one-shot, to pulse if the restart enable switch (also located on the front panel) is on and causes the RST X instruction to be executed; X is the restart location value (1-7). Program execution then jumps to the appropriate restart location and executes another OUT instruction. This operation disables the restart location selection until the rest push button is depressed again. Finally, the JP PROGRAMX is executed completing the restart sequence and passing control of the processor to the code located at Program X in memory (ROM or RAM).

The RAM used to store the accumulated values of data are accessed by the STD microcomputer between memory locations 4000H - 5FFFH, for all four channels. Each channel contains four polarization states with up to 256 range gates per polarization. Each range gate takes up two bytes in memory to realize up to 16 bits of accuracy. Details of how the processor handles the range gated data will be given in Section 6.5.

The RAM located on the CPU board resides in memory locations 2000H to 2FFFH. ROM is located on the CPU board and

resides in memory locations from 0H to 1FFFH. Locations 0H to 34H must be programmed as shown before in Table 6.2. Additional memory may be placed in the STD microcomputer system between locations 6000H to FFFFH.

6.4.3 Polarization Synchronization/PRF Generation

The polarization synchronization circuit consists of a phase lock loop circuit which provides a constant clock signal to a bank of counters. The counters provide an address input to a ROM which will provide the polarization state information, the PRF sync pulse which drives the data acquisition system, and a transmit trigger which fires the transmitter at the PRF of 480 Hz. A schematic of the polarization synchronizer is shown in Figure 6.4. The synchronizing pulse on the input to the circuit is generated in the feed position sensing circuit located in the feed assembly. A sync-pulse is input to the polarization synchronization circuit every time the feed rotates through vertical polarization resulting in a 30 Hz signal for a feed rotation rate of 1800 RPM. This signal is divided by two with flip-flop (A1) to produce a 15 Hz square wave. This signal is then input to the PLL circuit (A2).

The other input to the PLL is the signal generated by the divide by 720 circuit. The PLL output (VCO output) is a

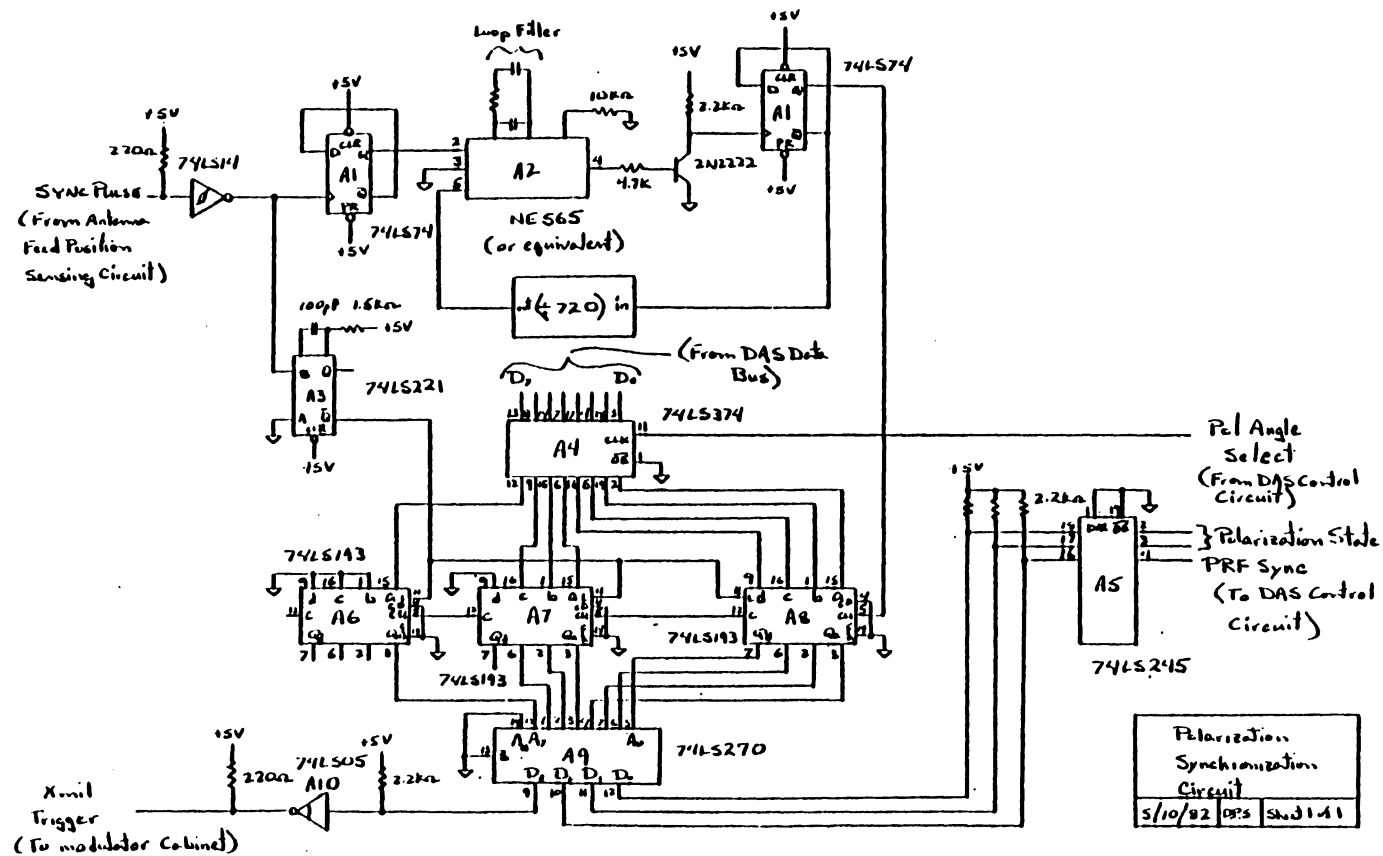


Figure 6.4. Schematic diagram of the polarization synchronizer/PRF generator circuit.

21.6 KHz square wave. Flip-flop (A3) divides this signal by 2 to generate a 10.8 KHz square wave. This signal drives the divide by 720 circuit to close the loop and is also used to drive a second bank of counters. Note that the 10.8 KHz signal provides one transition for every degree that the feed rotates. The second bank of counters divides this signal by 360 to obtain an address of the counter output which is used to access the ROM (A15). The ROM outputs are used to drive the data acquisition system and the transmitter. A truth table showing the contents of the ROM is shown in Table 6.3. Note that the transmit trigger fires the transmitter about every 22.5 degrees of feed rotation resulting in 16 transmit pulses per revolution (or 480 Hz PRF). A polarization synchronization timing diagram is shown in Figure 6.5. The PRF sync drives the data acquisition system so that data is collected on every other transmit pulse resulting in the collection of data at 0, 45, 90, 135, 180, 225, 270, and 315 degrees rotation from local vertical. The polarization state indicates the feed polarization in for the most recent transmit trigger and PRF sync pulses. With this setup, two returns per polarization are received for one revolution of the feed; therefore 60 samples per polarization can be accumulated each second.

Table 6.3.

Polarization synchronizer/PRF generator ROM truth table.

Address Input	ROM Output			Polarization	Address Input	ROM Output			Polarization
	Transmit Trigger	PRF Sync.	Polarization State			Transmit Trigger	PRF Sync.	Polarization State	
0	0	0	00	V	•	•	•	•	135°
1	1	1	00		•	•	•	•	
2	1	1	00		179	1	1	11	
•	•	•	•		180	0	0	00	
•	•	•	•		181	1	1	00	
•	•	•	•		•	•	•	•	
21	1	1	00		•	•	•	•	
22	0	1	00		201	1	1	00	
23	0	1	00		202	0	1	00	
•	•	•	•		203	1	1	00	
44	1	1	00	45°	•	•	•	•	45°
45	0	0	01		•	•	•	•	
46	1	1	01		224	1	1	00	
•	•	•	•		225	0	0	01	
•	•	•	•		226	1	1	01	
66	1	1	01		•	•	•	•	
67	0	1	01		•	•	•	•	
68	1	1	01		246	1	1	01	
•	•	•	•		247	0	1	01	
•	•	•	•		248	1	1	01	
89	1	1	01	90°	•	•	•	•	H
90	0	0	10		•	•	•	•	
91	1	1	10		269	1	1	01	
•	•	•	•		270	0	0	10	
•	•	•	•		271	1	1	10	
•	•	•	•		•	•	•	•	
111	1	1	10		•	•	•	•	
112	0	1	10		•	•	•	•	
113	1	1	10		291	1	1	10	
•	•	•	•		292	0	1	10	
•	•	•	•	135°	293	1	1	10	
134	1	1	10		•	•	•	•	
135	0	0	11		•	•	•	•	
136	1	1	11		•	•	•	•	
•	•	•	•		•	•	•	•	
•	•	•	•		314	1	1	10	
156	1	1	11		315	0	0	11	
157	0	1	11		316	1	1	11	
158	1	1	11		•	•	•	•	
•	•	•	•		•	•	•	•	
•	•	•	•		358	1	1	11	135°
•	•	•	•		359	1	1	11	

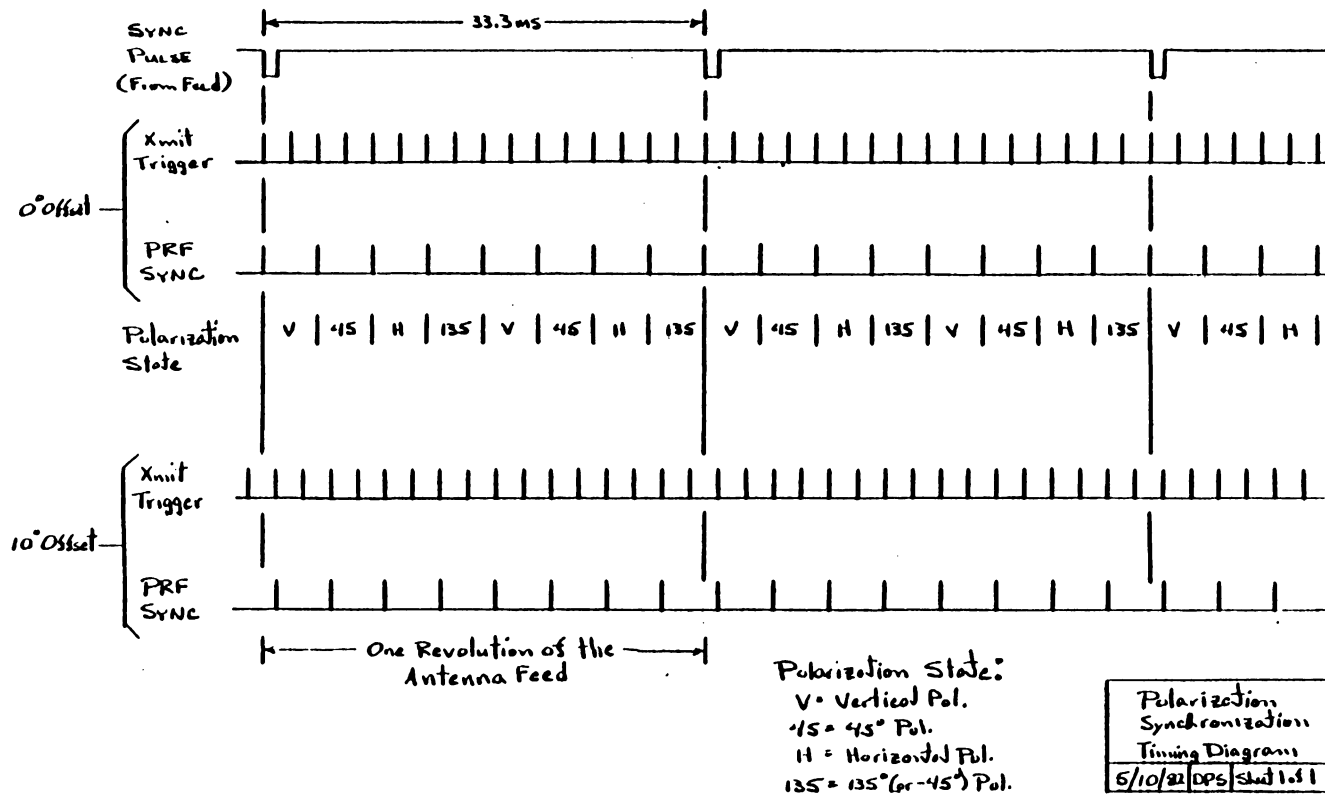


Figure 6.5. Polarization synchronizer timing diagram.

In addition to the synchronization of a set of four polarizations, this circuit also can rotate this set ± 45 degrees from vertical. This allows the operator to rotate the linear polarization angle in real-time to investigate rain-drop oblateness and symmetry. This is achieved by loading an offset into the second bank of counters every time the sync pulse is present. This causes an offset in the actual polarization to take place. For example: If the counter is loaded with zero during a sync pulse then V, 45, H, 135 degree polarization results; if the counter is loaded with an offset of -10 then -10, 35, 80, 125 degree polarization results. The input to the counters is set by writing the offset value to output port 7DH with the STD microcomputer. This causes latch (A4) to latch the offset word present on the DAS data bus. The offset word consists of the MSB (D_7) as the sign bit; 0 = negative, 1 = positive, and 7 least significant bits ($D_0 - D_6$) as the magnitude of the offset word.

This circuit along with the unique feed design provides a simple way to control the linear polarization angle of the transmit pulse at a far less expense than the switching technique used by others.

6.5 DATA ACQUISITION SYSTEM

This section completes the chapter with a description of the data acquisition system. This system, under control of the STD microcomputer, accepts up to four video channels on input, converts these signals to a digital format, and accumulates these values in memory. Each channel is independent of the other channels and all channels operate in parallel during data collection. The video signals input to this system will be a co-polarized return, cross-polarized return, and two quadrature components of the correlation of the co- and cross-polarized returns from which phase information is extracted. Note that only one channel is needed to make differential reflectivity measurements though.

A control circuit drives a common bus structure shared by all channels and provides the timing signals to the channels as well as interfacing with the STD microcomputer and the polarization synchronization circuit.

The data acquisition system was designed with the previous considerations discussed in Sections 5.2 and 5.3 along with some practical considerations which include:

- I. Flexibility - capable of easily reconfiguring the hardware and software to accomodate a variety of experiments.
- II. Implementable - designed with simple architecture and standard components.
- III. Inexpensive - total cost of the system as small

as possible without sacrificing performance.

6.5.1 Modes of Operation

The data acquisition system (DAS) has two modes of operation. They are: a) the data collect mode and b) the data transfer mode. During the data transfer mode, the STD microcomputer has control of the data acquisition system. The microcomputer's address, data, and control bus are connected to the data acquisition system address, data, and control bus enabling the STD microcomputer to access the DAS memory (which accumulates the range gated data) with read/write operations. Also, in this mode the STD microcomputer can modify the parameters of the DAS by writing the parameter to the appropriate output port. During the data collection mode the DAS control circuit has control of the DAS bus, and the STD microcomputer is then free to process previous data. The DAS control circuit uses the PRF sync signal to start the data collection process for all range gates for each return while in the data collection mode.

6.5.2 Control Circuit and Timing Sequence Generation

During the data collection mode period, the control circuit assumes control of the DAS bus and directs the appropriate control signals to all channels. Also it keeps track of which range gated data is being collected and counts the number of samples being collected.

Data collection begins when the PRF sync signal is logical true, setting flip-flop (A11) in Figure 6.7. This causes the Q output to go high and the scan reset to become inactive. This enables the sequencer to generate the correct timing sequence. Figure 6.6 shows a schematic diagram of the sequencer, which is active during the data collection mode. The clock (A1) generates a 10 MHz square wave which is input to the AND gate (A10) and two counters (A3 and A4). These counters form the blanking range length measured from the radar to the first range gate, and are loaded with a blanking range (BR) parameter (0-255) from the STD microcomputer through latch (A7). This is done by writing this parameter to output port 70H during the previous data transfer period.

Once the scan reset goes inactive (low) the counters (A3, A4) begin to count down to zero (from the range blanking parameter value). After $BR+1$ cycles of the clock, a borrow in (A4) occurs and this clears flip-flop (A11) on Figure 6.6

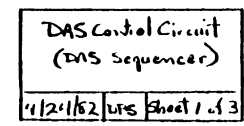


Figure 6.6. DAS control circuit (Sequencer).

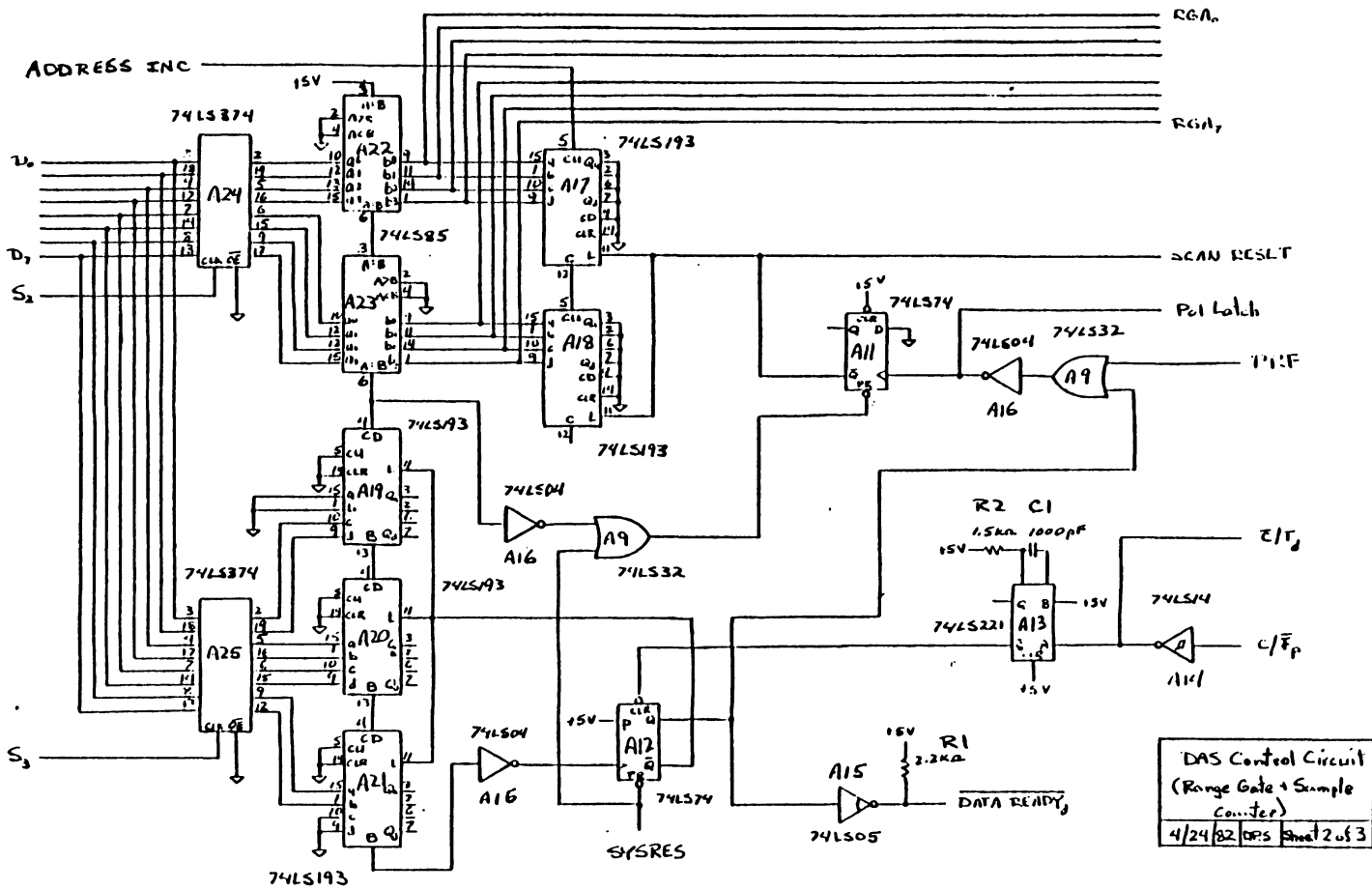


Figure 6.7. DAS control circuit (Range Gate and Sample Counter).

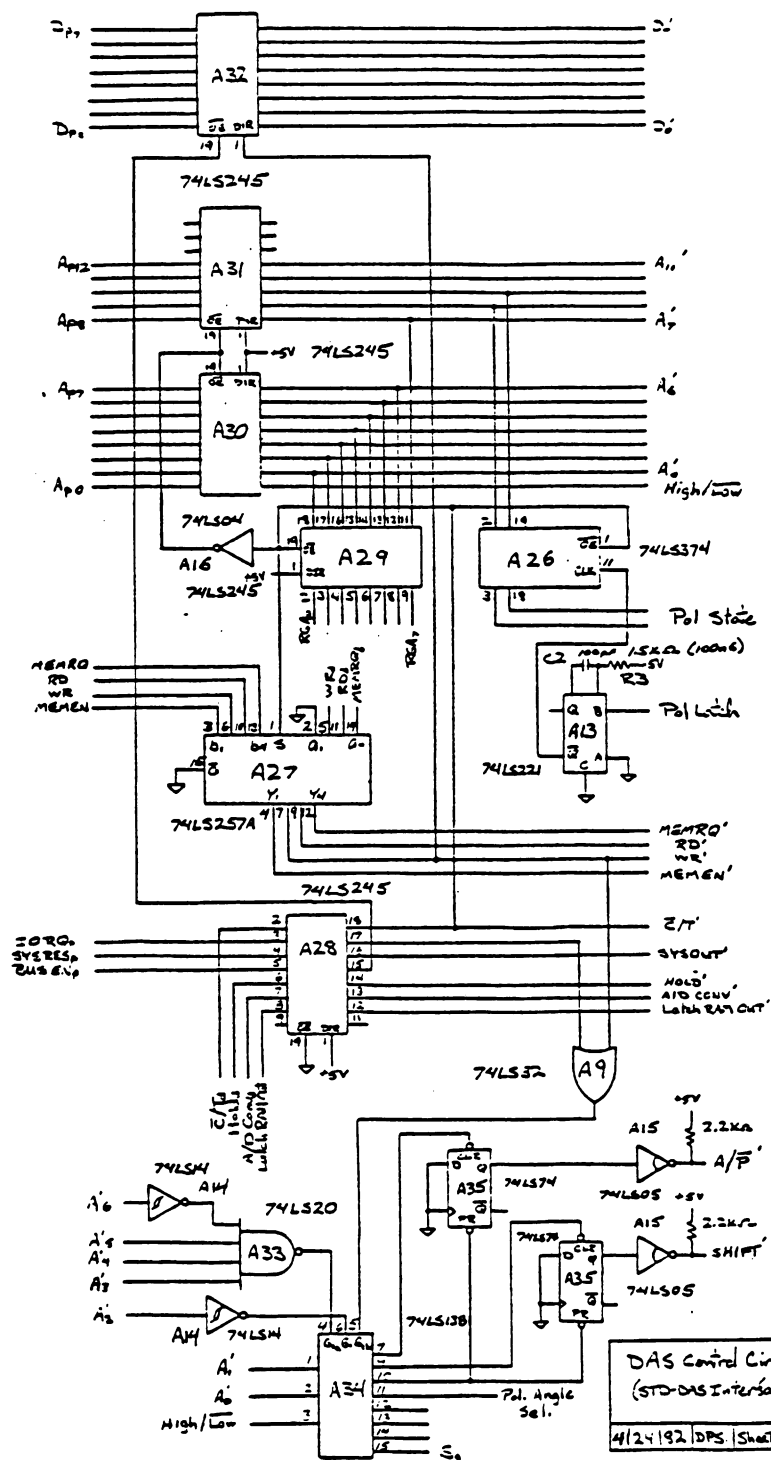


Figure 6.8. DAS control circuit (STD-DAS Interface).

thereby enabling the AND gate A10 to pass the clock signal. The output of (A10) is complementary and is used to: a) count through the sequence ROM address and b) latch the output of the sequence ROM (A2) at the correct time. This method "pipelines" the sequence ROM addressing and allows the address input to change while holding the data output constant. The sequence sequence ROM (A2) has a bank of output latches built in. The address is formed by counters (A5) and (A6) which count from zero up to 127. The ROM (A2) has 512 words which are eight bits wide. The two most significant bits of the input address to the ROM (A2) are latched by (A8) which is controlled by the STD microcomputer writing a range gate length (RGL) parameter (0-3) into output port 71H.

The output sequence of the ROM (A2) is shown in Figure 6.9. The sequence ROM address, generated by counters (A5 and A6), sequences the ROM to produce the active low timing signals: HOLD, A/D CONV, MEMRQ, RD, WR, LATCH RAM OUT, ADDRESS INC and Range Gate RESET signals. The HOLD and A/D CONV signals cause the sample and hold and the analog-to-digital in each channel to sample the video and convert this to a 12 bit digital word which is input to one input of the digital accumulator. See Figures 6.11-6.14. This takes approximately 2 microseconds. While this is going on (shown

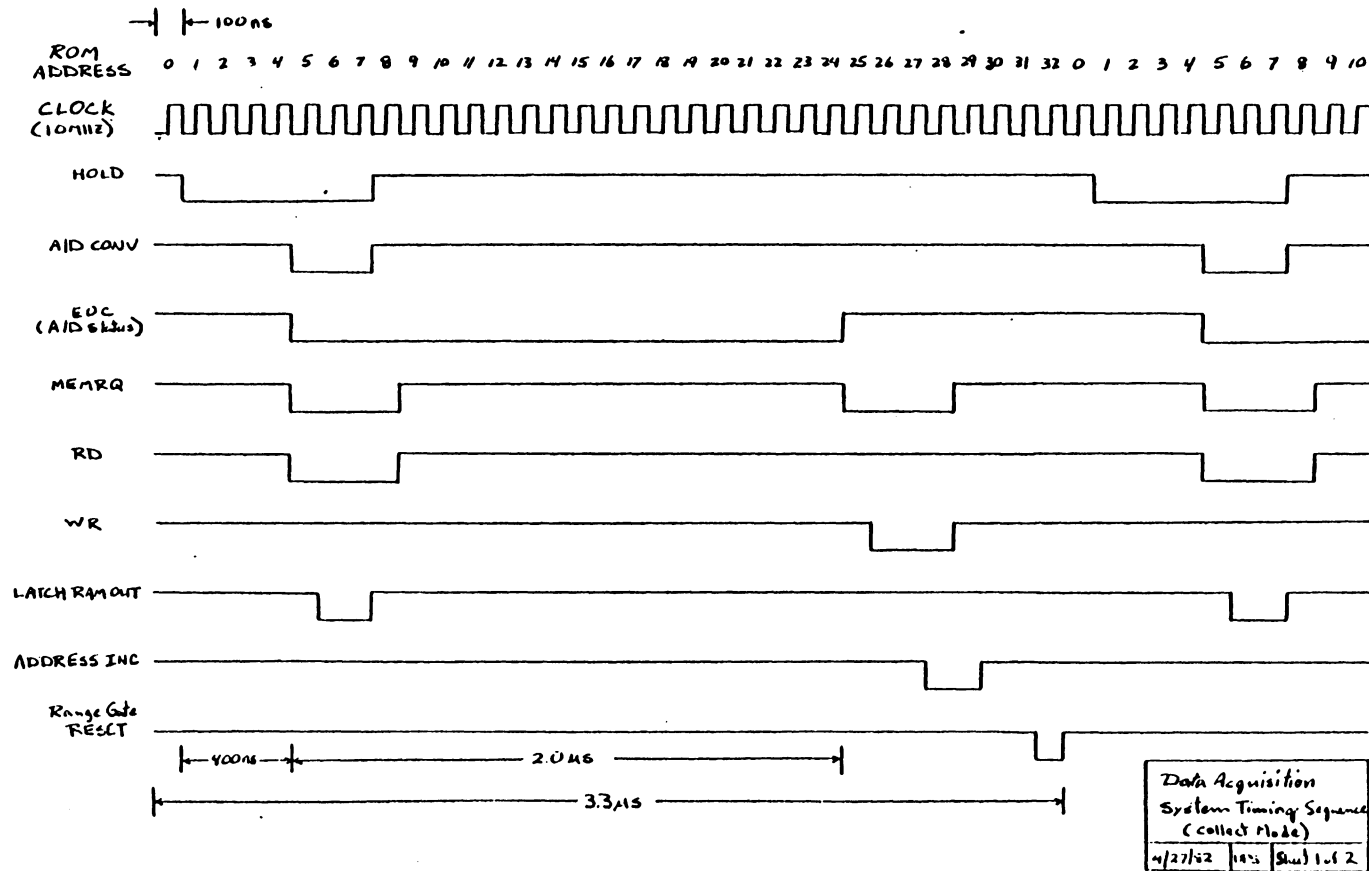


Figure 6.9. DAS timing sequence for one range gate during data collection mode.

by the A/D status signal shown in Figure 6.9), the previously accumulated value is retrieved from the DAS memory and latched with the LATCH RAM OUT signal to the other input of the digital accumulator. The new sample for one range gate is then added to the previously accumulated data almost immediately and then this result is returned to the same memory location. Once this sequence is finished the memory address, generated by counters (A17) and (A18) in Figure 6.7, is incremented by one to point to the next range gate. Then a Range Gate RESET pulse causes counters (A5) and (A6) to reset to zero and thereby reinitiate the sequence for the next range gate.

Figure 6.9 shows the case for a range gate length of 500 meters, which takes 33 clock cycles to complete. For range gate lengths greater than 500 meters, the Range Gate RESET is not asserted until more than 32 clock pulses have occurred. Therefore, extending the range gate length is equivalent to inserting "no-operation" (ROM output-FFH) in address locations between the ADDR INC operation and the Range Gate RESET operation. This is programmed into different blocks of the ROM, with each block being 128 locations, therefore the range gate length (RGL) parameter (RGL=0-3) is used to select which block of memory in the sequence ROM to execute. It is suggested that the first block (0-127), ac-

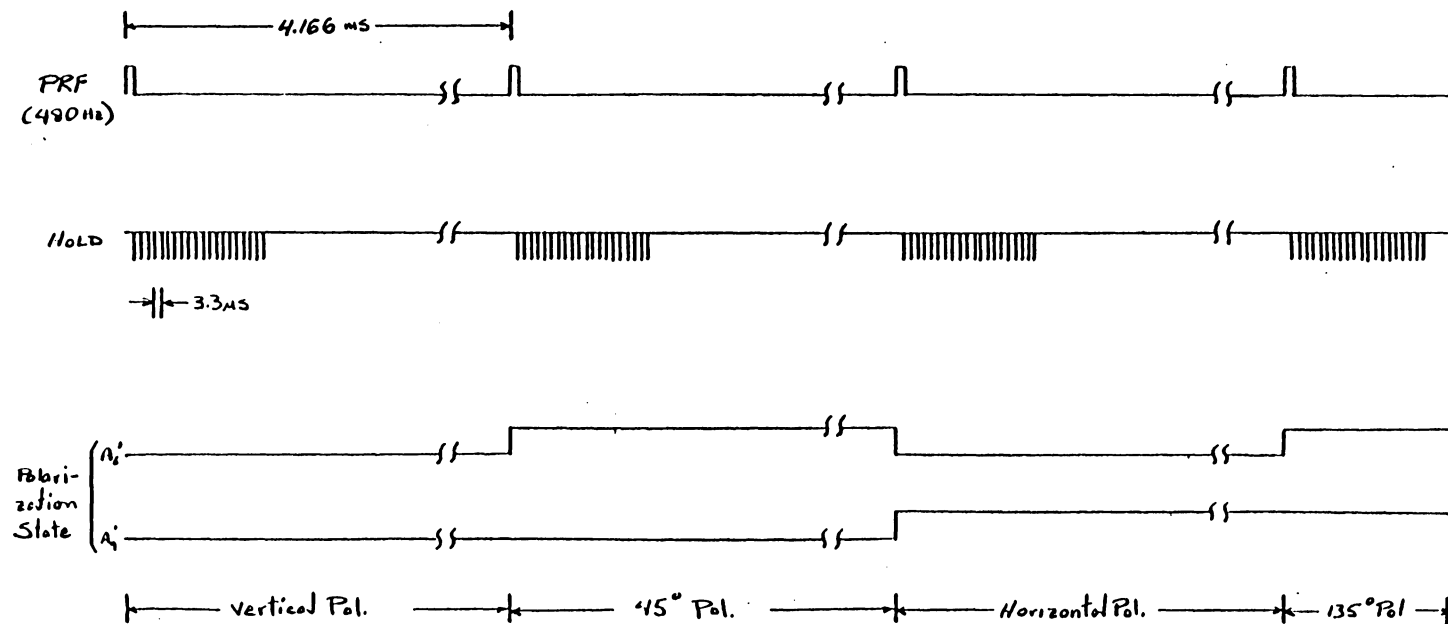
cessed by setting RGL=0, be used for 500 meter range gate length and programmed according to the sequence shown in Figure 6.9. The second block accessed by setting RGL=1 could be programmed for 750 meter range gates by inserting "no-operations" between locations 160 and 177 and then programming the Range Gate Reset operation at location 178. The same sequence is programmed into locations 128-159 as programmed into locations 0-30. A similar procedure is done for locations 256-383 and 384-512 to realize four selectable range gate lengths. If less than 500 meter range gates are needed for more resolution along the slant path, an A/D converter with a fast conversion time (≤ 1.0 microsecond) must be used. To achieve faster conversion times, a 10 or 8-bit A/D converter must be used, at the expense of accuracy. One segment of the ROM could be designated to operate with a smaller range gate length (say with only 20 operations programmed into the sequence ROM) and selected with the RGL parameters when the faster A/D circuits are being used.

Magnitude comparators (A22) and (A23) in Figure 6.7 compare the memory address with a range gate number (RGN) parameter latched by (A24). The RGN parameter is used to select the number of range gates used for all polarization states and is set by the STD microcomputer writing the RGN parameter (1-255) into output port 73H.

The sequence described above continues until the memory address equals RGN parameter and generates an output pulse on pin 6 of (A23). This causes the bank of counters (A19, A20, A21) to count down one, and presets flip-flop (A11) in Figure 6.7 causing the scan reset to go low, stopping the range gate counting sequence. Since the flip-flop (A11) has been preset, the sequence will not continue until the next PRF sync pulse. Figure 6.10 shows this for the case when data for 20 range gates (RGN=20) are collected.

Counters (A19, A20, A21) decrement one for each sequence through the set number of range gates, and serve as sample counters. This bank of counters then counts four counts before one complete sample for each range gate in four polarizations is complete. The sample number parameter loaded into this counter bank is latched by latch (A25). The SN parameter is used to select the number of samples used for all polarization states and is set by the STD microcomputer writing the sample number (SN) parameter (1-255) into output port number 74H.

The SN parameter is counted down until the desired number of samples are accumulated and a borrow occurs at the output of counter (A21). This clocks flip-flop (A12) which inhibits any more PRF sync pulses from entering the system, reloads the SN parameter into counters (A19, A20, A21) and



Data Acquisition		
System Timing Sequence		
(One Sample Complete)		
4/27/62	1015	Sheet 2 of 2

Figure 6.10. DAS timing sequence for one complete sample of four polarizations.
Note: 20 range gates per polarization shown.

signals the STD microcomputer that data is ready. At this point the STD microcomputer must put the system in the data transfer mode and extract the accumulated data from the DAS memory.

Figure 6.8 shows the interface circuit between the DAS bus and the STD microcomputer bus. Inverters (A14), NAND gate (A33), decoder (A34), and OR gate (A9), decode the STD microcomputer output port addresses 70H to 77H. Buffers (A32, A31, A30) buffer the data and address bus to/from the STD microcomputer and are active during the data transfer mode. Buffers (A29, A26) buffer the address bus to the DAS memory during the data collection mode. Note that the polarization state (generated by the polarization synchronization circuit) form the two most significant bits of the DAS address bus during the data collection mode. The state is latched with one-shot (A13) just after the PRF sync pulse is true. Data selector (A27) selects part of the STD microcomputer control bus during the data transfer mode or part of the sequencer output during the data collection mode. Buffer (A28) buffers additional control functions generated by the STD microcomputer (IORQp, SYSRESp, BUSENp) or the sequencer output (HOLD, A/D CONV, LATCH RAM OUT) and the data collect-transfer signal ($\overline{C}/T'$).

If fewer than 16 samples are being accumulated to complete one scan an optional four-bit shift of the analog-to-digital converter output is possible to realize full accuracy of the data that is transferred to the STD microcomputer. Also if only one sample is to be collected, then no accumulation is needed. Only the most recent sample is needed to be stored in the DAS memory. These functions are implemented by executing an output 75H instruction in the STD microcomputer during the data transfer mode. This causes flip-flops (A35) to preset forcing the SHIFT signal to become true (low) and change the $A/\bar{P}$ signal to the $\bar{P}$ state (low). This shifts the A/D converter output four bits and therefore disables the divide by 16 function built into the digital integrator, see Figure 6.13. Also since the P function is selected, new data is directly passed to the memory and not added with previous values stored in the DAS memory. Executing an output 76H instruction in the STD microcomputer causes the A/D converter to be stored or added to the 12 least significant bits of the DAS memory and executing an output 77H instruction causes the digital integrator to add new samples to the previous samples being collected, therefore placing the DAS into normal operation. A detailed discussion of the digital integrator is found in Section 6.5.3.

6.5.3 Digital Integrator and A/D Converter

The digital integrator performs the accumulation and storage of range gated data for all four polarizations. The analog-to-digital (A/D) converter converts the sampled video signal into a 12 bit digital word. Figures 6.11-6.13 show a schematic diagram of the digital integrator and Figure 6.14 shows a schematic diagram of the A/D converter circuit.

Figure 6.11 shows the DAS memory which holds the range gated data for four polarizations (one channel). Memories (A1-A5) are 1024 x 4 bit RAM storage devices which must be set to zero during the initialization process. The two most significant DAS address bits of the RAM (A_8' , A_9') form the polarization state. The 8 least significant bits of the DAS address form the range gate number (0-255). Buffers (A6) and (A7) are used to select the 16 most significant bits (MSB) of the DAS memory. The output of buffers (A6, A7) are tied to the 8 bit DAS data bus. The High/Low signal is tied to the STD microcomputer address bit A_0 . When A_0 is 0 then buffer (A7) is active and the low-order byte of the 16 most significant bits are selected. If A_0 is 1 then buffer (A6) is active and the high-order byte of the 16 most significant bits are selected. Therefore all even address location in the STD microcomputer memory space 4000H to 5FFFH access the low order byte of the 16 MSB of the DAS memory and all odd

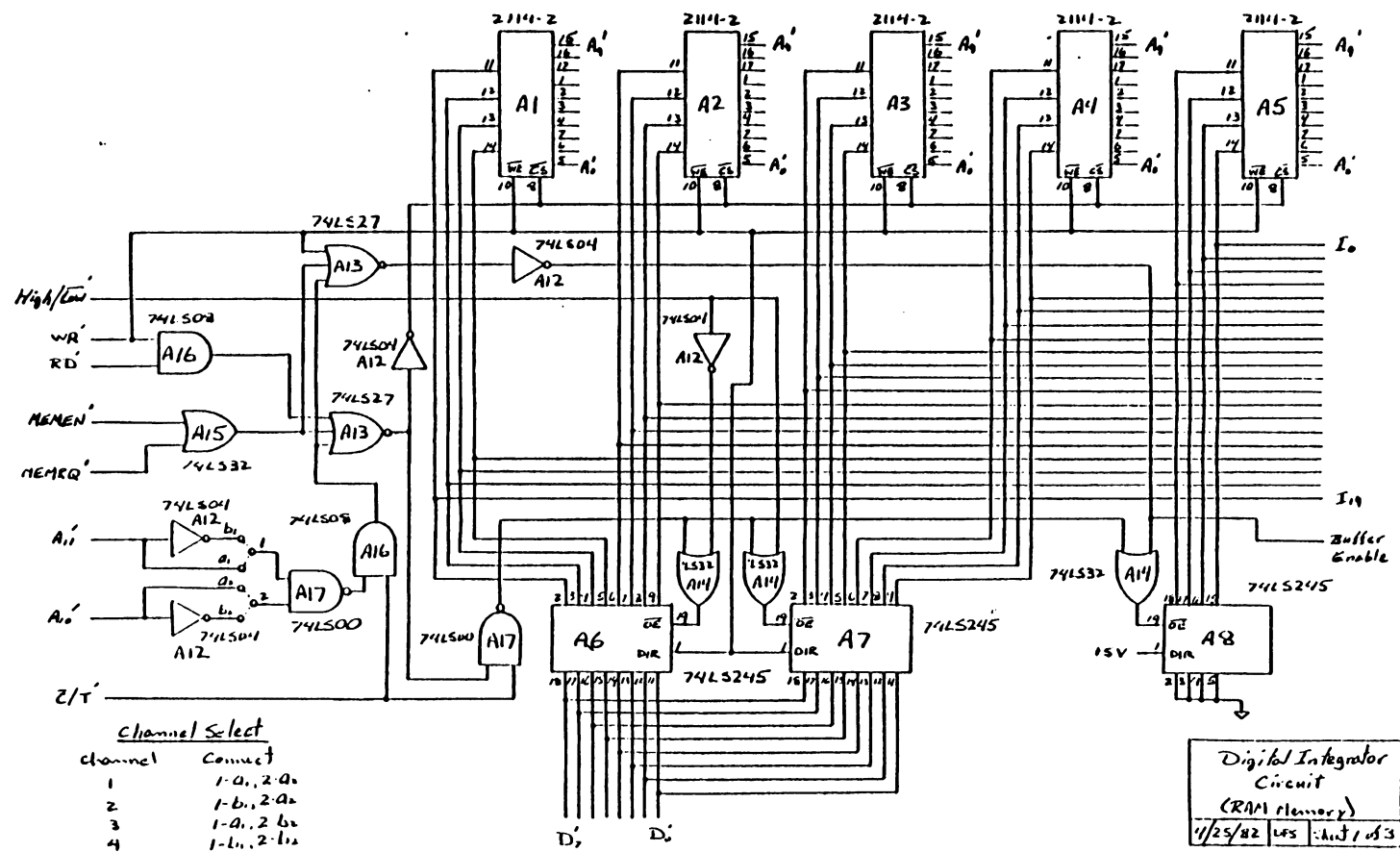


Figure 6.11. Digital integrator (Memory).

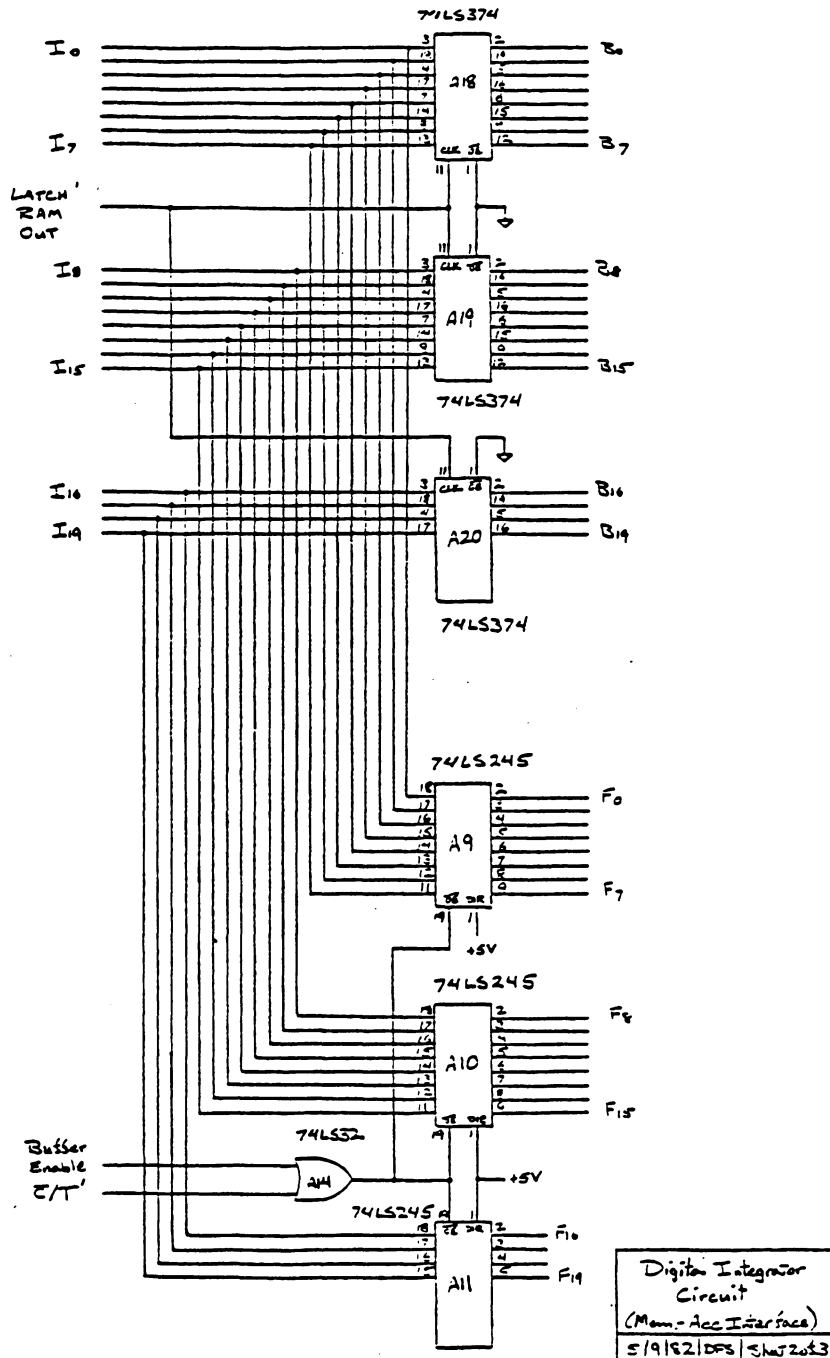


Figure 6.12. Digital integrator (Memory-Accumulator Interface).

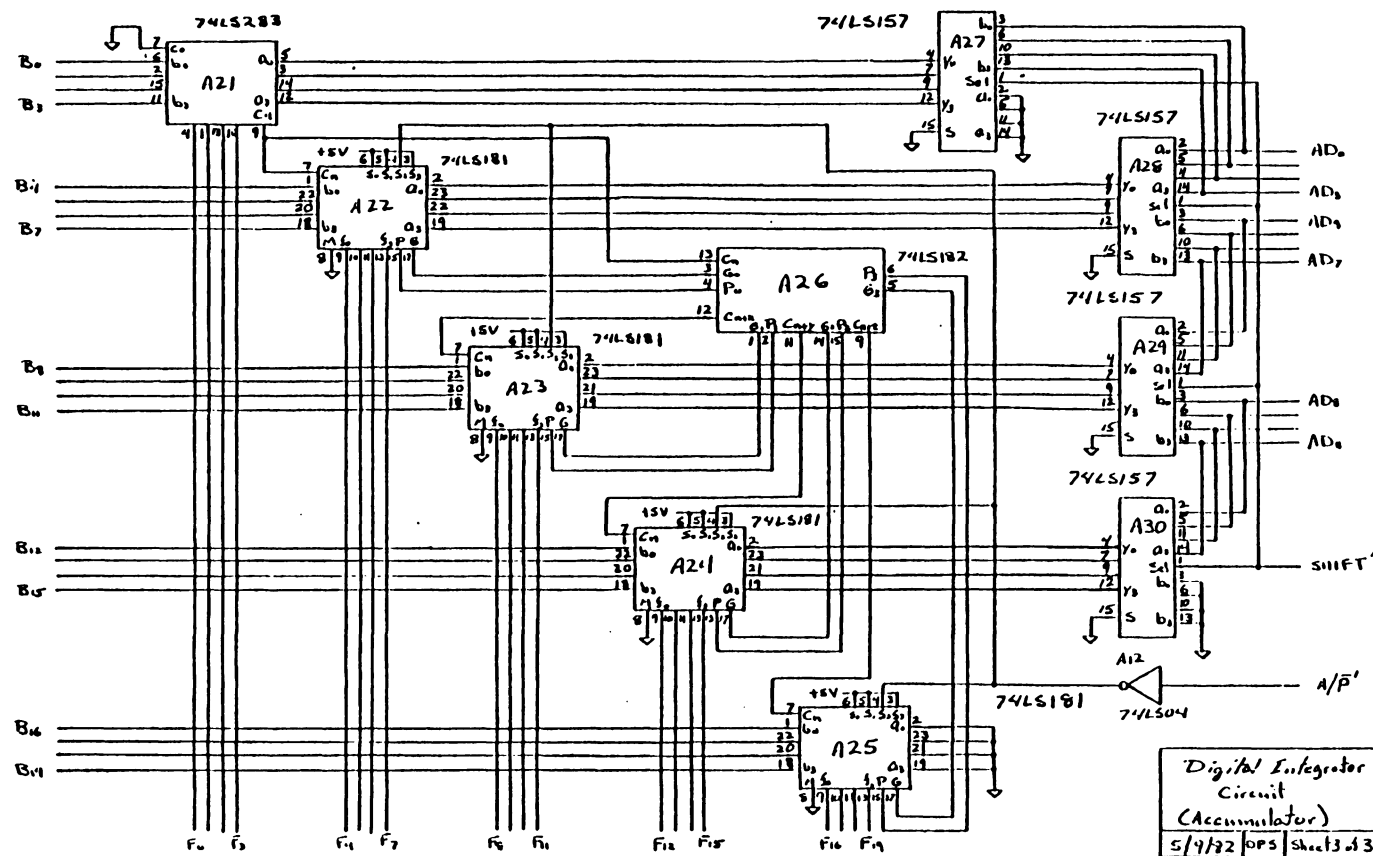


Figure 6.13. Digital integrator (Accumulator).

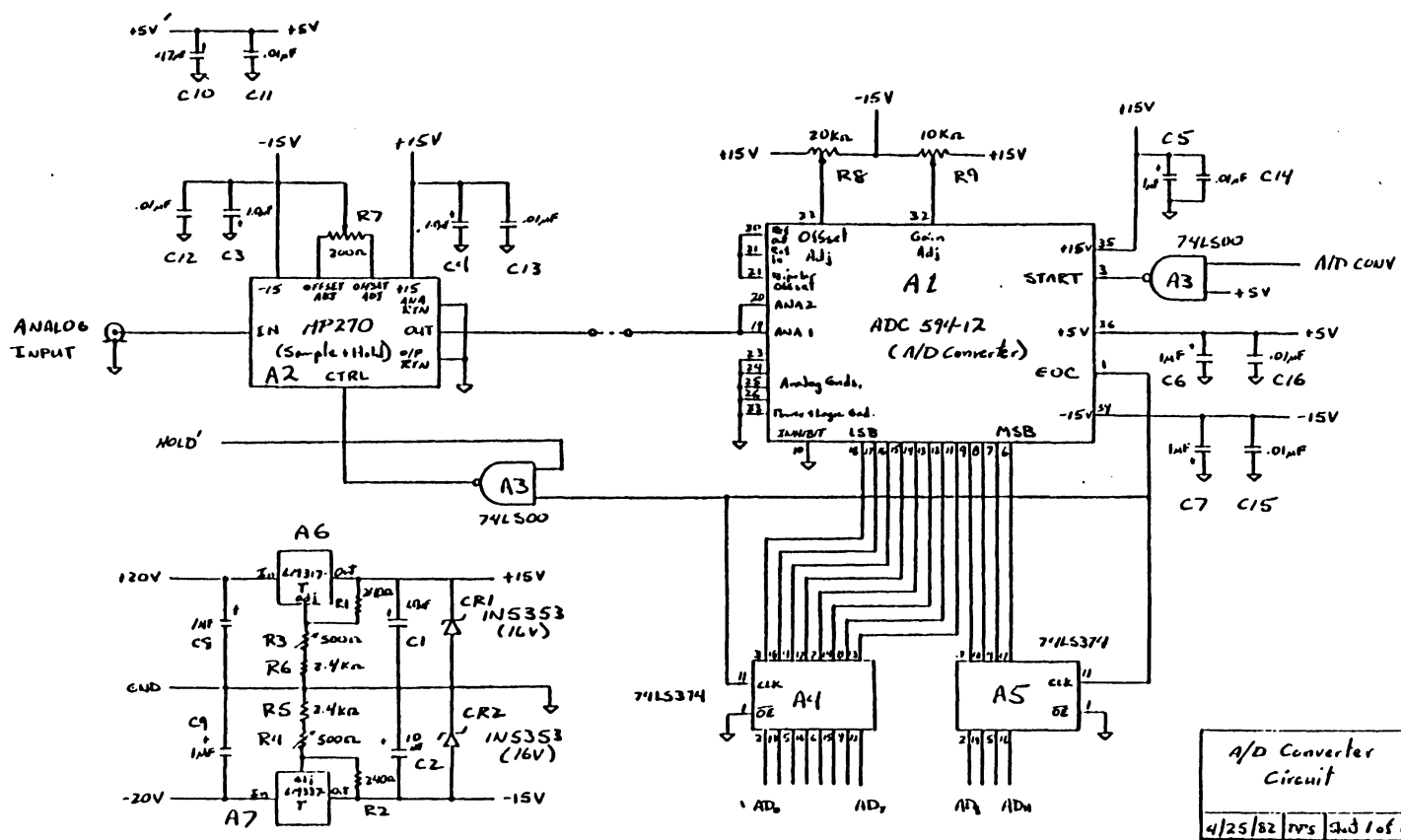


Figure 6.14. A/D converter.

address locations in the STD microcomputer memory space 4000H to 5FFFH access the high-order byte of the 16 MSB of the DAS memory. Buffer (A8) will cause zeros to be written into RAM (A5) whenever the STD microcomputer performs a memory write operation to the low-order byte in the DAS memory. DAS address bits A_{10}' and A_{11}' form the channel select portion of the address and are active during the data transfer mode. They are used by the STD microcomputer to access the DAS memory in series as shown in the memory map in Figure 6.3. Two straps are provided on the input of NAND gate (A17) to program the hardware for channels 1-4. During data collection A_{10}' and A_{11}' are inactive and all four channels operate in parallel.

Figure 6.12 shows the memory-accumulator interface. When the DAS memory is read during the data collection mode, it is latched with latches (A18-A20) to provide the accumulator with the previously sampled data. Once the A/D converter provides the accumulator with new data and the addition has taken place buffers (A9-A11) are enabled to write the accumulated value back into the DAS memory.

Figure 6.13 shows the accumulator. Adder (A21) and arithmetic units (AU) (A22-A25), along with the look-ahead carry circuit (A26), are configured to add 20 bits of previous data (from the DAS memory) and 12 bits of new data (from

the A/D converter). The $A/\bar{P}'$ signal, under STD microcomputer control causes the (AU) to add previous data to new data ($A/\bar{P}'=1$) or to pass the new data ($A/\bar{P}'=0$) to the memory. Data selectors (A27-A30) perform the A/D output 4 bit shift through the SHIFT signal under STD microcomputer control.

Figure 6.14 shows the analog-to-digital converter circuit. The sample and hold circuit (A2) samples the video input and provides a constant analog voltage to the A/D converter input. The A/D converter (A1) quantizes this value in a conversion time of 2 microseconds. Once this is complete, the A/D converter output is latched by latches (A4) and (A5). Voltage regulators (A6) and (A7) provide regulated power ($\pm 15V$) to (A1) and (A2). R7 provides an offset adjustment for (A2) and R8 and R9 provide an offset and gain adjustment for (A1).

6.5.4 STD Microcomputer Interface

The STD microcomputer interface is intended to fit on a 4.5" x 6.5" standard STD microcomputer board and take up one card slot in the STD microcomputer. This card's function is to process the restart sequence (described in Section 5.5.2), to allow the STD microcomputer read/write access to the data acquisition system memory, and to control the DAS mode of operation.

Figure 6.15, 6.16 show a schematic diagram of the interface circuit. Buffers (A1, A2, A3) buffer the data and address bus to/from the DAS and are active during the data transfer mode. Inverter (A10), NAND gate (A11), decoder (A9) and OR gate (A13) decode output ports 78-7FH. When an output 78H instruction is executed, this causes flip-flop (A7) to preset, placing the DAS in the data collect mode and executing an output 79H instruction causes flip-flop (A7) to clear, placing the DAS in the data transfer mode. This function is provided to allow manual access and control of the DAS. In the data collect mode, the address decoding circuitry (A10, A12, A13, A14) and the data and address buffers are inactive. During data transfer mode the buffers are active and the address decoding circuitry generates the BUSEN and MEMEN signals to the DAS every time the processor accesses memory address space 4000-5FFFH. Inverter (A18) on Figure 6.15 drives a Data Collect Mode indicator on the front panel. Buffer (A4) buffers the STD microcomputer's control bus to the DAS and is active during the data transfer mode.

Executing an output 7AH instruction causes flip-flop (A8) in Figure 6.16 to clock a high on its Q output enabling a pulse from the one-shot (A17) to travel through the NOR gate (A13). This enables the data acquisition system to collect

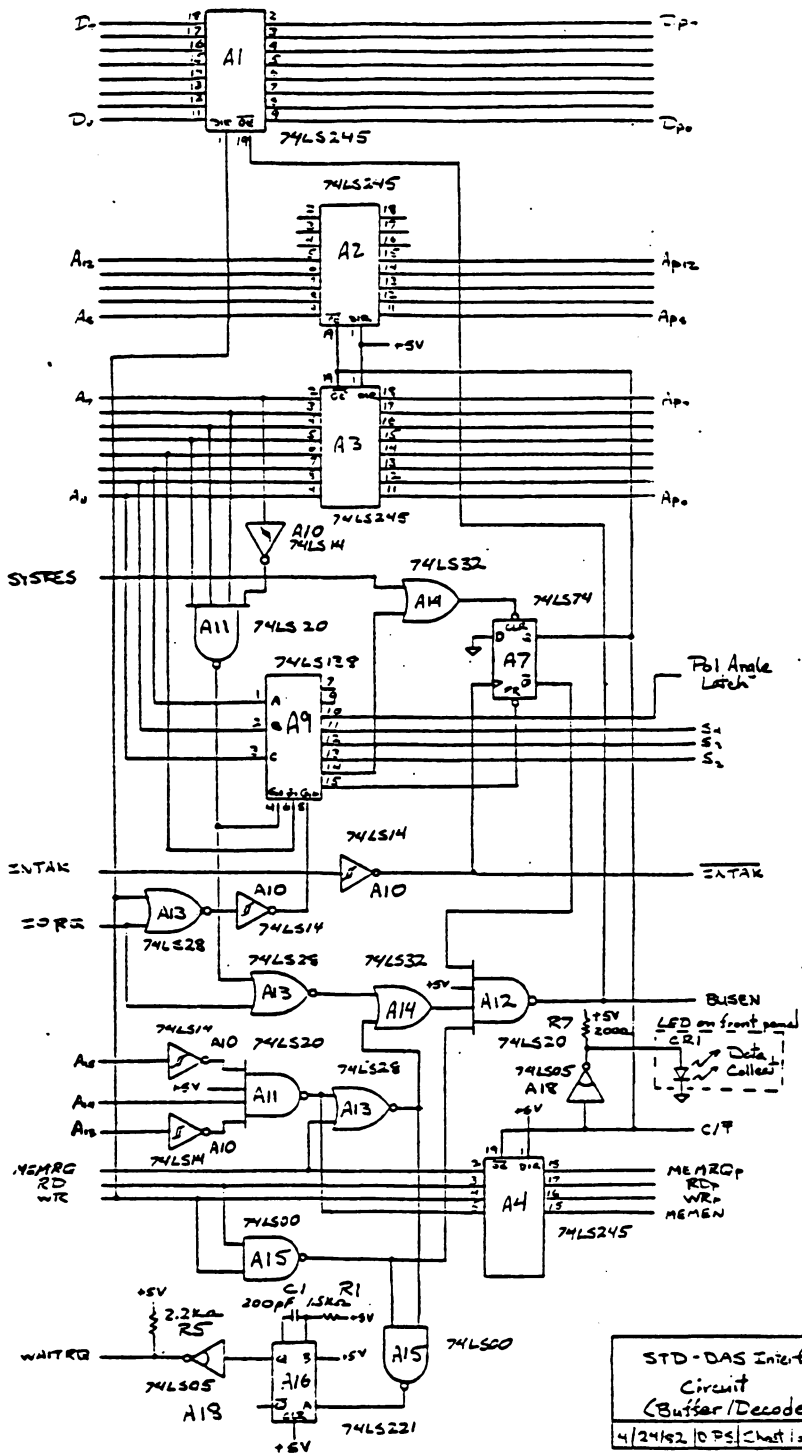


Figure 6.15. STD-DAS interface (Buffer/Decoder).

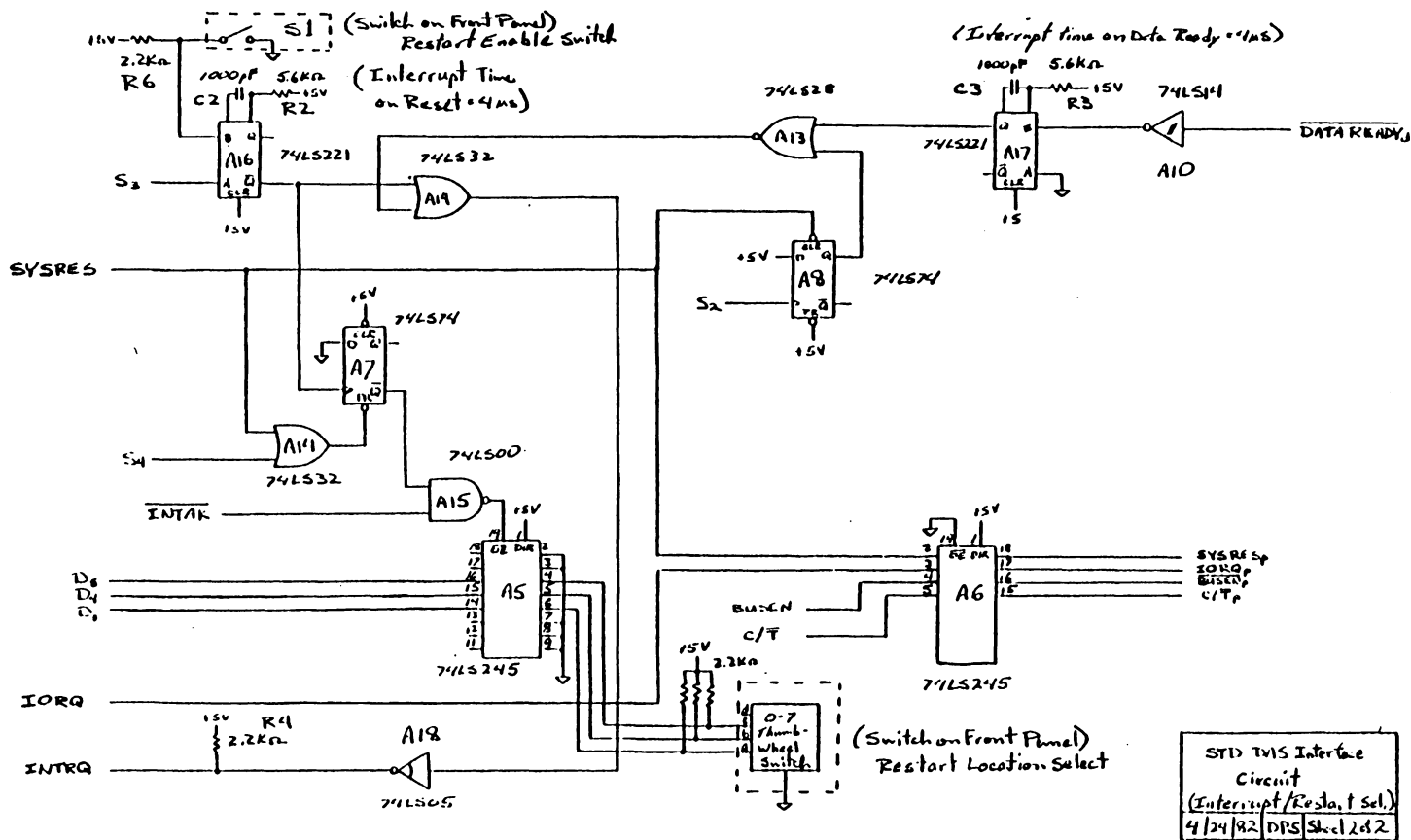


Figure 6.16. STD-DAS interface (Interrupt/Restart Select).

data. When the DATA READY signal is true, meaning that the data acquisition system has acquired a sampled set of data, and flip-flop (A8) has been clocked, an interrupt request will be generated. The STD microcomputer will be programmed to acknowledge this as a mode I interrupt and vector to location 66H and execute the interrupt service routine. During this time the processor will acknowledge with an INTAK signal which causes flip-flop (A7) in Figure 6.15 to place the DAS in the data transfer mode. Then the STD microcomputer has control of the DAS memory and will move the contents of locations 4000H to 5000H (for one channel: locations 4000H to 43FFH) to another section in memory (for one channel: locations 2000H-23FFH). After this is done the STD microcomputer must zero the DAS memory, or preset a range normalizing term in each location, before commanding the DAS to collect more data by executing an output 78H instruction, returning the DAS to the data collect mode.

NAND gate (A15), one-shot (A16) in Figure 6.15 and Inverter (A18) serve to insert wait states into the processor state sequence in case that the DAS memory cannot respond fast enough to the memory read/write cycle time of the processor. This is provided but most likely not needed; if it is not needed, then the "B" input on one-shot (A16) in Figure 6.15 should be tied low instead of tied high.

The interface circuit automatically places the DAS into the data transfer upon receiving a system rest (SYSRES) signal from the processor.

6.5.5 Design Implementation

Special consideration must be given to the construction of the data acquisition system. First a suitable enclosure which provides good RF shielding must be used to package the STD microcomputer, polarization synchronization circuit and the data acquisition system. This is needed because of the large ambient EM radiation present when the radar transmitter is on. The half-rack enclosing the existing DAS should be modified for this use.

The STD microcomputer will fit in a 8-slot card rack mounted close to the DAS hardware. Connections to the front panel or a CRT connector to the rear panel are made with flat cable connected to an I/O board. Two parallel I/O ports are connected through the rear panel to the S-100 system in the same fashion. Power to the STD microcomputer is furnished by a common power supply located at the bottom of the half rack, through a suitable power harness.

A 56 connection flat cable less than approximately 2-3 feet in length will be used to interface the STD microcomputer and the DAS hardware. Also a 14 connection flat cable

is needed to interface the polarization synchronization circuit to the DAS hardware.

The DAS should be constructed in its own card rack and have enough card slots with 100-pin sockets to accomodate up to four channels of video input. These card slots should be wired together to form a mother board bus structure enabling all four channels to operate in parallel. The polarization synchronization circuit should fit on a single board and also mounted in this rack taking up one card slot with the feed positioning signal input provided with coaxial cable. The control circuit should take up approximately two card slots and provide the bus signals through the mother board to the channels.

Each channel should take up approximately two card slots, one slot being the digital section and the other being the analog section. The A/D output from the analog section is passed to the digital section through a 14 connection flat cable. Analog signals are connected to each channel through coaxial cable.

This totals to at least eleven card slots connected to a common motherboard. Power will be supplied to the DAS from the common power supply through a power harness.

Each board in the DAS will be constructed using wire wrap techniques with the exception of parts of the polarization

synchronization circuit and the analog section in each channel. Due to the presence of analog signals in these sections, printed circuit board techniques should be used. For all boards, plenty of by-pass capacitors and possibly some inductors should be used to provide power supply filtering. This is critical in the analog section in each channel.

Chapter VII

CONCLUSIONS

An improved formulation for radar backscatter from a volume of oblate spheroidal raindrops has been developed, taking account of the beam elevation angle and the raindrop canting angle. The formulation is in terms of scattering coefficients in a matrix, and can be applied to systems using linear, circular, or elliptical polarization. It has been shown that the general form reduces to results previously published in the literature for linear polarization differential reflectivity (Z_{DR}) and for circular polarization depolarization ratio (CDR).

Analysis of the linear polarization case for a dual-polarization radar which can transmit and receive orthogonal linear polarizations in the ± 45 degree planes shows that the drop size distribution can be obtained from measurements of LDR with the radar.

This technique can be used to verify differential reflectivity (Z_{DR}) measurements. Analysis of the circular polarization case for a dual polarization radar which can transmit and receive circular polarizations show that both the drop size distribution and the drop mean canting angle can be obtained from measurements of CDR and the relative phase bet-

ween the co and cross-polarized return. The calculation of attenuation and depolarization of a radio signal from a satellite along a co-located path can be improved considerably by the knowledge of these parameters of the rain in the path. Similar measurements of ice cloud parameters should also be possible.

Finally, the dual-polarized radar development program at VPI&SU was discussed and a detailed design of a new radar data collection system was given.

With the theory of dual-polarization radar measurement and the practical implementation of such a radar described within this thesis, a more accurate characterization of the propagation effects along a satellite to earth path can be carried out.

REFERENCES

- Alliss, D. N., E. A. Manus, P. H. Wiley, C. W. Bostian [1980], "An S-Band Radar for Investigating of Rain and Ice Effects on Millimeter Wave Propagation," Interim Report 1981-1, under Contract NAS5-22577, EE Dept., VPI&SU, November.
- Battan, L. J. [1973], Radar Observation of the Atmosphere, The University of Chicago press, Chicago, 324 pp.
- Bickel, S. H. [1965], Some invariant properties of the polarization scattering matrix, Proc. IEEE, 53 (8), 1070-1072.
- Bringi, V. N., S. M. Cherry, M. P. M. Hall, T. A. Seliga [1978], A new accuracy in determining rainfall rates and attenuation due to rain by means of dual-polarization radar measurements, IEE International Conference on Antennas and Propagation, November 28-30, pp. 120-124.
- Bringi, V. N., T. A. Seliga, M. G. SriRam [1980], Statistical characteristics of the differential reflectivity radar signal, 19th Conference on Radar Meteorology, April 15-18, pp. 692-696.
- Browne, I. C., N. P. Robinson [1952], Cross-polarization of the radar melting band, Nature, Vol. 170, November, p. 1078-1079.
- Brussaard, G. [1976], A meteorological model for rain-induced cross polarization, IEEE Trans. on Antennas and Propagation, AP24(1), 5-11.
- Cherry, S. M., J. W. Goddard, M. P. M. Hall [1981], Use of dual polarization radar data for evaluation of attenuation on a satellitelink, Ann. Telecommunic., 36(1-2), 33-39.
- Damosso, E., [1978], Dependence of specific rain-attenuation and phase shift on electrical, meteorological and geometrical parameters, CSELT Rapporti tecnici, Volume VI-N.3, September, p. 249-255.
- Davenport, W. B. [1970], Probability and Random Processes, McGraw-Hill, New York, 542 pp.

- Davenport, W. B., W. L. Root [1958], "An Introduction to the Theory of Random Signals and Noise," McGraw-Hill Book Co., New York, pp. 393.
- Gent, H., I. M. Hunter, N. P. Robinson [1963], Polarization of radar echoes, including aircraft precipitation and terrain, Proc. of IEE, Vol. 110, No. 12, pp. 2139-2148.
- Hall, M. P. M. [1981], Dual-Polarization radar helps gauge rainfall rates, Microwaves, September, p. 93-101.
- Haworth, D. P., P. A. Watson, N. J. McEwan [1977], Model for the effects of electric fields on satellite-earth microwave radio propagation, Electronics Letters, Vol. 13, 562-564.
- Hunter, I. M. [1954], Polarization of Radar Echoes from Meteorological Precipitation, Nature, Vol. 172, January, p. 167-166.
- Intersystems [1981], Ithica 840H User's Manual, Ithica, NY.
- Ishimaru, A. [1977], Theory and application of wave propagation and scattering in random media, Proc. IEEE, 65 (7), 1030-1061.
- Jones, D. S. [1964], The Theory of Electromagnetics The MacMillan Company, New York.
- Joss, J., A. Valdvogel, [1969], Raindrop size distribution and sampling size errors, Journal of the Atmospheric Sciences, Vol. 26, May, p. 566-569.
- Lhermitte, R. M. [1963], "Motions of Scatterers and the Variance of the Mean Intensity of Weather Radar Signals," Sperry Rand Research Center Program 38310, SRRC-RR-63-57, Atmospheric Physics Dept., Sudbury Mass., November.
- Marshall, J. S., W. Hitschfeld [1953], Interpretation of the fluctuating echo from randomly distributed scatterers. Part I, Canadian Journal of Physics, Volume 31, p. 962-994.
- Marshall, J. S., W. M. K. Palmer [1948], The distribution of raindrops with size, J. Meteorology, Vol. 5, 165-166.

- Marshall, R. E., E. A. Manus, P. H. Wiley, C. W. Bostian [1981], An S-band radar for studying the effects of weather on the SIRIO and COMSTAR satellite downlink signals, 20th Conference on Radar Meteorology, November 30-December 3, p. 498-501.
- McCromick, G. C., A. Hendry [1968], Polarization properties of precipitation scattering, National Research Council, Vol. 18, No. 4, 25-37.
- McCormick, G. C., A. Hendry [1975], Principles for the radar determination of the polarization properties of precipitation, Radio Science, 10(4), 421-434.
- McCormick, G. C., A. Hendry [1979], Techniques for the determination of the polarization properties of precipitation, Radio Science, Vol. 14, No.6, November, p. 1027-1040.
- Morrison, J. A., M. J. Cross [1974], Scattering of a plane electromagnetic wave by axisymmetric raindrops, B.S.J.T., Vol. 53, July-August, 955-1019.
- Mueller, E. A. [1981], Implementation of differential reflectivity on the CHILL radar, 20th Conference on Radar Meteorology, November 30-December 3, p. 666-667.
- Nathanson, F. E. [1969], "Radar Design Principles," McGraw-Hill Book Co., New York, pp. 626.
- Oguchi, T. [1977], Scattering properties of Pruppacher-and-Pitter form raindrops and crosspolarization due to rain: calculations at 11, 13, 19.3 and 34.8 GHz, Radio science, 12(1), 41-51.
- Olsen, R. L. [1982], A review of theories of coherent radiowave propagation through precipitation media of randomly oriented scatterers, and the role of multiple scattering. To be published in Radio Science.
- Ostberg, K. [1976], Effects of drop canting-angle distribution on depolarization of microwaves in rain, FOA Rep. (Sweden), 10, 1-8.
- Probert-Jones, J. R. [1962], The radar equation in meteorology, Quart. J. Roy. Meteorological Soc., 88, 485-495.

- Pro-Log Corporation [1981], Series 7000 STD Bus Technical Manual and Product Catalog, Pro-Log Corporation, Monterey, CA, pp. 131.
- Pruppacher, H. R., K. V. Beard [1970], A wind tunnel investigation of the internal circulation and shape of water drops falling at terminal velocities in air, Quart. J. Roy. Meteorological soc., 96, 247-256.
- Reid, J. D. [1970], Decorrelation time of weather radar signals, 14th Conference on Radar Meteorology, p. 419-420.
- Seliga, T. A., V. N. Bringi [1976], Potential use of radar differential reflectivity measurements at orthogonal polarizations for measuring precipitation, Journal of Applied Meteorology, January, 69-76.
- Seliga, T. A., V. N. Bringi [1978], Differential reflectivity and differential phase shift: applications in radar meteorology, Radio Science, 13(2), 271-275.
- Seliga, T. A., V. N. Bringi [1980], Statistical characteristics of the differential reflectivity radar signal, 19th Proc. of Radar Meteorology Conf., 692-696.
- Seliga, T. A., V. N. Bringi, H. H. Al-Khatib [1981], A preliminary study of comparative measurements of rainfall rate using the differential reflectivity radar technique and a raingauge network, Journal of Applied Meteorology, November, p. 1362-1368.
- Semplak, R. A. [1974], Measurement of rain-induced polarization rotation at 30.9 GHz, Radio Science, 9(4), 425-429.
- Stratton, J. A. [1974], Measurement of rain-induced polarization rotation at 30.9 GHz, Radio science, 9(4), 425-429.
- Stratton, J. A. [1941], Electromagnetic Theory, McGraw-Hill, New York, 615 pp.
- Stutzman, W. L., G. A. Thiele [1981], Antenna Theory and Design, John Wiley, New York, 597 pp.

- Stutzman, W. L., W. P. Overstreet, C. W. Bostian, A. Tsolakis, E. A. Manus [1981], Ice Depolarization on Satellite Radio Paths, Final Report for Intelsat Contract, INTEL-123, VPI&SU, EE Dept., April, 151 pp.
- Van de Hulst, H. C. [1957], Light Scattering by Small Particles, John Wiley, New York, 470 pp.
- Warner, C., A. Hizal [1976], Scattering and depolarization of microwaves by spheroidal raindrops, Radio Science, 11(11), 921-930.
- Zilog [1978], Z80-CPU, Z80 A-CPU Technical Manual, Zilog, Inc., Capertino, CA, pp. 75.

APPENDIX I

AVERAGING OVER RAINDROP PARAMETERS

1.1 Linear Polarization Case

The backscattered power due to a volume of falling raindrops is

$$\langle \underline{P}^r \rangle = \frac{C}{r^2} \langle \underline{B} \rangle \quad (\text{A.1})$$

where

$$C = \text{radar constant} = \frac{\lambda^2}{4\pi} \frac{G_o^2}{8 \ln 2} \frac{\pi \theta_B}{2} \left(\frac{h_\ell}{2} \right)$$

$$h_\ell = \text{pulse length of transmit pulse (m)}$$

$$\theta_B = \text{half-power beamwidth (rad)}$$

$$G_o = \text{on axis antenna gain}$$

$\langle \underline{B} \rangle$ is calculated, given the elevation angle , drop size and canting angle distributions.

Transmitting in polarization 1 with $P_{11}^t = 1$, $P_{22}^t = 0$ results in:

$$B_{11} = f_{11} f_{11}^* = |f_{11}|^2 \quad (\text{A.2})$$

$$B_{22} = f_{12}^* f_{21} \quad (\text{A.3})$$

$$B_{21} = f_{11}^* f_{21} = B_{12}^* \quad (\text{A.4})$$

where

$$f_{11} = \frac{k_o^2}{4\pi \epsilon_o} \left[(p_v - p_h) \cos^2 \delta \sin^2 \psi + p_h \right] \quad (\text{A.5})$$

$$f_{22} = \frac{k_o^2}{4\pi \epsilon_o} \left[(p_v - p_h) \cos^2 \delta \cos^2 \psi + p_h \right] \quad (A.6)$$

$$f_{12} = f_{21} = \frac{k_o^2}{4\pi \epsilon_o} \left[(p_v - p_h) \cos^2 \delta \frac{\sin 2\psi}{2} \right] \quad (A.7)$$

$k_o = 2\pi/\lambda$; λ = free space wavelength (m)

ϵ_o = permittivity of free space

δ = elevation angle (rad)

ψ = polarization angle (related to the canting angle)

$$p_h = \frac{4\pi \bar{a}^3}{3} \epsilon_o (m^2 - 1) \left[\frac{2}{I_1 (1 - m^2) + m^2 + 1} \right] \quad (A.8)$$

$$p_v = \frac{4\pi \bar{a}^3}{3} \epsilon_o (m^2 - 1) \left[\frac{1}{I_1 (m^2 - 1) + 1} \right] \quad (A.9)$$

m = complex index of refraction

$\bar{a}$ = equivolumetric radius

$$I_1 = \frac{1}{e^2} [1 - (1 - e^2)^{1/2} e^{-1} \sin^{-1} e] \quad (A.10)$$

$$e = [1 - (1 - 0.1 \bar{a})^2]^{1/2} \quad (A.11)$$

B_{11} can be expressed as:

$$B_{11} = \left[\cos^4 \delta \sin^4 \psi (|p_v|^2 - 2 \operatorname{Re}(p_v p_h^*) + |p_h|^2) \right. \\ \left. + \cos^2 \delta \sin^2 \psi (2 \operatorname{Re}(p_v p_h^*) - |p_h|^2) + |p_h|^2 \right] \quad (A.12)$$

and B_{22} (the cross-polar component) is:

$$B_{22} = \frac{k_o^4}{(4\pi \epsilon_o)^2} \cos^4 \delta \frac{\sin^2 2\psi}{2} \left[|p_v|^2 - 2 \operatorname{Re}(p_v p_h^*) + |p_h|^2 \right] \quad (\text{A.13})$$

and the off-diagonal term containing the phase is:

$$B_{21} = \frac{k_o^4}{(4\pi \epsilon_o)^2} \left[(|p_v|^2 - 2 \operatorname{Re}(p_v p_h^*) + |p_h|^2) \cos^4 \delta + \right. \\ \left. 1/4(\sin 2\psi - \frac{1}{2} \sin 4\psi) + (p_h^* p_v - |p_h|^2) \cos^2 \delta \frac{\sin 2\psi}{2} \right] \quad (\text{A.14})$$

B_{11} , B_{22} , B_{21} must be averaged. This is done by computing

$$\int_{\text{shape}} \int_0^\infty \int_{\langle \theta \rangle - \alpha}^{\langle \theta \rangle + \alpha} B_{11,22,21} \bar{n}(\bar{a}, \theta, s) d\theta d\bar{a} ds \quad (\text{A.15})$$

The integration over θ is carried out analytically but the integration over $\bar{a}$ will be done numerically. The integration over a shape distribution is done by assuming a discrete distribution which has $N F_o$ oblate shaped raindrops and $N(1 - F_o)$ spherical raindrops. Spherical raindrops only contribute to the co-polarized return, and is independent of canting angle parameters. All the distributions are assumed to be independent:

$$\bar{n}(\bar{a}, \theta, s) = t(\theta) \bar{n}(\bar{a}) P(s) \quad (\text{A.16})$$

The integration over canting angle is carried out first. The canting

angle (θ) is related to the polarization angle ψ by:

$$\theta = \psi - \psi' \quad \text{or} \quad \psi = \theta + \psi' \quad (\text{A.17})$$

where ψ' defines the reference from which the polarization state is determined (i.e. $\psi = 0^\circ$ horizontal polarization, $\psi = 90^\circ$ vertical polarization) ψ is a random variable as is θ .

Therefore assuming that θ is Gaussian distributed the integration over θ can be written as:

$$\int_{\langle \theta \rangle - \alpha}^{\langle \theta \rangle + \alpha} B_{11,22,21} \frac{1}{\sigma\sqrt{2\pi}} \exp \left[\frac{-(\theta - \bar{\theta})^2}{2\sigma^2} \right] d\theta \quad (\text{A.18})$$

To carry out this integral a substitution is made; let:

$$u = \theta - \bar{\theta} \quad \text{or} \quad \theta = u + \bar{\theta} \quad (\text{A.19})$$

Therefore

$$\psi = u + (\bar{\theta} + \psi') = u + \psi'' \quad ; \quad \psi'' = \bar{\theta} + \psi' \quad (\text{A.20})$$

Using equations (A.12-A.14, A.18 and A.19) and defining

$$a = \cos^4 \delta (|p_v|^2 - 2 \operatorname{Re}(p_v p_h^*) + |p_h|^2) \quad (\text{A.21})$$

$$b = 2 \cos^2 \delta (\operatorname{Re}(p_v p_h^*) - |p_h|^2) \quad (\text{A.22})$$

$$c = |p_h|^2 \quad (\text{A.23})$$

$$d = \frac{k_o^4}{(4\pi \epsilon_o)^2} \quad (\text{A.24})$$

$$e' = [p_h p_v^* - |p_h|^2] \cos^2 \delta \quad (\text{A.25})$$

$$s = k_o^4 \left| \frac{m^2 - 1}{m^2 + 2} \right| \frac{1}{a^6} \quad ; \quad s = \text{scattering coefficient for spherical raindrops}$$

then $B_{11,22,21}$ become:

$$B_{11} = d[a \sin^4(u + \psi'') + b \sin^2(u + \psi'') + c]F_o + s(1 - F_o) \quad (\text{A.26})$$

$$B_{22} = d \left[a \frac{\sin 2(u + \psi'')}{4} \right] F_o$$

$$B_{21} = d \left[a \left\{ \frac{1}{4} \left(\sin 2(u + \psi'') - \frac{1}{2} \sin 4(u + \psi'') \right) \right\} + \frac{e'}{2} \sin 2(u + \psi'') \right] F_o \quad (\text{A.27})$$

Using the appropriate trigonometric identities and recognizing that any term containing a $\sin(nu)$ factor will contribute zero to the integral taken over symmetric limits, which is the case here since we assume that the standard deviation σ is small enabling the limits of integration to be taken from $\pm\infty$. $B_{11,22,21}$ can be written as (dropping all $\sin(nu)$ terms):

$$B_{11} = d \left[a \left\{ \frac{1}{8} (3 - 4 \cos 2u \cos 2\psi'' + \cos 4u \cos 4\psi'') \right\} + b \left\{ \frac{1}{2} (1 - \cos 2u \cos 2\psi'') \right\} + c \right] F_o + s(1 - F_o) \quad (\text{A.28})$$

$$B_{22} = d \left[\frac{a}{4} \left\{ \frac{1}{2} (1 - \cos 4u \cos 4\psi'') \right\} \right] F_o \quad (\text{A.29})$$

$$B_{21} = d \left[a \left\{ \frac{1}{4} (\cos 2u \sin 2\psi'' + \frac{1}{2} \cos 4u \sin 4\psi'') \right\} + \frac{e'}{2} \cos 2u \sin 2\psi'' \right] F_o \quad (A.30)$$

Collecting terms:

$$B_{11} = d \left[-\frac{1}{2} (a + b) \cos 2\psi'' \cos 2u + \frac{a}{8} \cos 4\psi'' \cos 4u + \frac{3}{8} a + \frac{1}{2} b + c \right] F_o + s(1 - F_o) \quad (A.31)$$

$$B_{22} = d \left[\frac{a}{8} (1 - \cos 4\psi'' \cos 4u) \right] F_o \quad (A.32)$$

$$B_{21} = d \left[\left(\frac{a}{4} + \frac{e'}{2} \right) \sin 2\psi'' \cos 2u - \frac{a}{8} \sin 4\psi'' \cos 4u \right] F_o \quad (A.33)$$

At this point the integral in equation (A.18) can be carried out by assuming σ is small and extending the limits of integration to $\pm\infty$ and using the integral forms:

$$\int_{-\infty}^{\infty} \xi \exp(-\eta^2 x^2) \cos gx \, dx = \xi \frac{\sqrt{\pi}}{\eta} \exp\left(\frac{-g^2}{4\eta^2}\right) \quad (A.34)$$

$$\int_{-\infty}^{\infty} \xi \exp(-\eta^2 x^2) \, dx = \frac{\xi}{\eta} \sqrt{\pi} \quad (A.35)$$

Combining equations (A.34) and (A.36) in carrying out the integration over θ in equations (A.31) - (A.33) gives:

$$\begin{aligned} \langle B_{11} \rangle_{\theta} = d \left[-\frac{1}{2} (a + b) \cos 2\psi'' \exp(-2\sigma^2) + \frac{a}{8} \cos 4\psi'' \exp(-8\sigma^2) + \frac{3}{8} a + \frac{1}{2} b + c \right] F_o + s(1 - F_o) \quad (A.36) \end{aligned}$$

$$\langle B_{22} \rangle_{\theta} = d \left[\frac{a}{8} - \frac{a}{8} \cos 4\psi'' \exp(-8\sigma^2) \right] F_o \quad (A.37)$$

$$\begin{aligned} \langle B_{21} \rangle_{\theta} = d & \left[\left(\frac{a}{4} + \frac{e'}{2} \right) \sin 2\psi'' \exp(-2\sigma^2) \right. \\ & \left. - \frac{a}{8} \sin 4\psi'' \exp(-8\sigma^2) \right] F_o \end{aligned} \quad (A.38)$$

Transmitting in polarization 2 results in:

$$B_{11} = f_{21}^* f_{12} \quad (A.39)$$

$$B_{22} = f_{22} f_{22}^* \quad (A.40)$$

$$B_{12} = f_{22}^* f_{12} \quad (A.41)$$

Since $f_{12} = f_{21}$ then B_{11} for polarization 2 is equal to B_{22} for polarization 1. Therefore:

$$\langle B_{11} \rangle_{\theta} = d \left\{ \frac{a}{8} - \frac{a}{8} \cos 4\psi'' \exp(-8\sigma^2) \right\} F_o \quad (A.42)$$

Following the same steps previously given for polarization, the components of $\underline{B}$ can be written as:

$$\begin{aligned} \langle B_{22} \rangle_{\theta} = d & \left[\frac{1}{2} (a + b) \cos 2\psi'' \exp(-2\sigma^2) + \frac{a}{8} \cos 4\psi'' \exp(-8\sigma^2) \right. \\ & \left. + \frac{3}{8} a + \frac{1}{2} b + c \right] F_o + s(1 - F_o) \end{aligned} \quad (A.43)$$

and

$$\langle B_{12} \rangle_{\theta} = d \left[\left(\frac{a}{4} + \frac{e'}{2} \right) \sin 2\psi'' \exp(-2\sigma^2) + \frac{a}{8} \sin 4\psi'' \exp(-8\sigma^2) \right] F_o \quad (A.44)$$

Note: for $\psi' = 0$; $\psi'' = \bar{\theta}$ and transmitting in polarization 1 results in horizontal polarization and transmitting in polarization 2 results in vertical polarization.

Integration over the drop size distribution is done numerically. This is done in a computer program, shown in Appendix II. The functions a, b, c, e' are integrated separately and then combined in equations (A.36) - (A.38) to give the coefficients $B_{11,22,21}$. By letting $\psi' = 90^\circ$ results in transmitting in polarization 2. In addition to the calculation of $B_{11,22,21}$; the radar constant and range are input to this program to determine the backscattered power in dBm.

1.2 Circular Polarization Case

The scattering matrix equation for linear polarization is

$$\begin{bmatrix} E_x^s \\ E_y^s \end{bmatrix} = \frac{e^{-jk_o r}}{r} \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} \begin{bmatrix} E_x^i \\ E_y^i \end{bmatrix} \quad (\text{A.45})$$

To represent circular polarization as a function of linear polarization:

$$\begin{bmatrix} E_{RC}^{BS} \\ E_{LC}^{BS} \end{bmatrix} = \frac{e^{-jk_o r}}{r} \left(\frac{1}{2}\right) \begin{bmatrix} 1 & -j \\ 1 & +j \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -j & j \end{bmatrix} \begin{bmatrix} E_{RC}^i \\ E_{RC}^i \end{bmatrix} \quad (\text{A.46})$$

where the subscript RC represents right hand circular polarization and LC represents left hand circular polarization. BS represents the back-scattered field.

Or

$$\begin{bmatrix} E_{RC}^{BS} \\ E_{LC}^{BS} \end{bmatrix} = \frac{e^{-jk_o r}}{r} \left(\frac{1}{2}\right) \begin{bmatrix} f_{11} - f_{22} - j(f_{12} + f_{21}) & f_{11} + f_{22} + j(f_{12} - f_{21}) \\ f_{11} + f_{22} + j(f_{21} - f_{12}) & f_{11} - f_{22} + j(f_{12} + f_{21}) \end{bmatrix} \begin{bmatrix} E_{RC}^i \\ E_{LC}^i \end{bmatrix} \quad (A.47)$$

Since only Rayleigh scattering is considered $f_{12} = f_{21}$

$$\begin{bmatrix} E_{RC}^{BS} \\ E_{LC}^{BS} \end{bmatrix} = \frac{e^{-jk_o r}}{r} \left(\frac{1}{2}\right) \begin{bmatrix} f_{11} - f_{22} - 2jf_{12} & f_{11} + f_{22} \\ f_{11} + f_{22} & f_{11} - f_{22} + 2jf_{12} \end{bmatrix} \begin{bmatrix} E_{RC}^i \\ E_{LC}^i \end{bmatrix} \quad (A.48)$$

This gives the backscattered circularly polarized field in terms of the incident circularly polarized field and the linear polarization scattering coefficients.

By defining:

$$f_{11_c} = f_{11} - f_{22} - 2j f_{12}$$

$$f_{12_c} = f_{11} + f_{22} = f_{21_c} \quad (A.49)$$

$$f_{22_c} = f_{11} - f_{22} + 2j f_{12}$$

Equation (A.48) becomes:

$$\begin{bmatrix} E_{RC}^{BS} \\ E_{LC}^{BS} \end{bmatrix} = \frac{e^{-jk_o r}}{r} \left(\frac{1}{2}\right) \begin{bmatrix} f_{11_c} & f_{12_c} \\ f_{21_c} & f_{22_c} \end{bmatrix} \begin{bmatrix} E_{RC}^i \\ E_{LC}^i \end{bmatrix} \quad (A.50)$$

Applying equation (A.1) to equation (A.50) and transmitting in right hand circular polarization (considering all scatterers oblate):

$$\begin{aligned}
 B_{11_c} &= f_{11_c} f_{11_c}^* = \frac{1}{4} |f_{11_c}|^2 & (\text{Cross}) \\
 B_{22_c} &= \frac{1}{4} f_{12_c}^* f_{21_c} & (\text{Co}) \\
 B_{21_c} &= \frac{1}{4} f_{11_c}^* f_{21_c} = B_{12}^* &
 \end{aligned} \tag{A.51}$$

In terms of the linear polarization scattering coefficients:

$$\begin{aligned}
 |f_{11_c}|^2 &= |f_{11}|^2 + |f_{22}|^2 - (f_{11} f_{22}^* + f_{22} f_{11}^*) + 2j(f_{11} f_{12}^* - f_{11}^* f_{12}) \\
 &\quad - 2j(f_{22} f_{12}^* - f_{22}^* f_{12}) + 4|f_{12}|^2 & (\text{A.52})
 \end{aligned}$$

$$\text{Note that } f_{11} f_{22}^* + f_{22} f_{11}^* = 2 \operatorname{Re}(f_{11} f_{22}^*)$$

$$f_{11} f_{12}^* - f_{11}^* f_{12} = 2j \operatorname{Im}(f_{11} f_{12}^*)$$

$$f_{22} f_{12}^* - f_{22}^* f_{12} = 2j \operatorname{Im}(f_{22} f_{12}^*)$$

$\therefore$

$$\begin{aligned}
 |f_{11_c}|^2 &= |f_{11}|^2 + |f_{22}|^2 - 2 \operatorname{Re}(f_{11} f_{22}^*) - 4 \operatorname{Im}(f_{11} f_{12}^*) \\
 &\quad + 4 \operatorname{Im}(f_{22} f_{12}^*) + 4|f_{12}|^2 & (\text{A.53})
 \end{aligned}$$

$\therefore$

$$\begin{aligned}
 B_{11_c} &= \frac{1}{4}(|f_{11}|^2 + |f_{22}|^2) - \frac{1}{2} \operatorname{Re}(f_{11} f_{22}^*) - \operatorname{Im}(f_{11} f_{12}^*) \\
 &\quad + \operatorname{Im}(f_{22} f_{12}^*) + |f_{12}|^2 & (\text{A.54})
 \end{aligned}$$

$$f_{12_c}^* f_{21_c} = |f_{11}|^2 + |f_{22}|^2 + 2 \operatorname{Re}(f_{11} f_{22}^*) \quad (\text{A.55})$$

∴

$$B_{22_c} = \frac{1}{4}(|f_{11}|^2 + |f_{22}|^2) + \frac{1}{2} \operatorname{Re}(f_{11} f_{22}^*) \quad (\text{A.56})$$

$$f_{11_c}^* f_{21_c}^* = |f_{11}|^2 - |f_{22}|^2 - 2j \operatorname{Im}(f_{11} f_{22}^*) + 2j f_{12}^* (f_{11} + f_{22}) \quad (\text{A.57})$$

∴

$$B_{21_c} = \frac{1}{4}(|f_{11}|^2 - |f_{22}|^2) - \frac{j}{2}(\operatorname{Im}(f_{11} f_{22}^*) - f_{12}^* f_{11} - f_{12}^* f_{22}) \quad (\text{A.58})$$

Similarly transmitting left hand circular polarization:

$$B_{11_c} = \frac{1}{4} f_{21_c}^* f_{12_c} \quad (\text{Co}) \quad (\text{A.59})$$

$$B_{22_c} = \frac{1}{4} f_{22_c} f_{22_c}^* = \frac{1}{4} |f_{22_c}|^2 \quad (\text{Cross}) \quad (\text{A.60})$$

$$B_{12_c} = \frac{1}{4} f_{22_c}^* f_{12_c} = B_{21}^* \quad (\text{A.61})$$

where

$$B_{11_c} = \frac{1}{4}(|f_{11}|^2 + |f_{22}|^2) + \frac{1}{2} \operatorname{Re}(f_{11} f_{22}^*) \quad (\text{A.62})$$

∴

$$B_{22_c} = \frac{1}{4}(|f_{11}|^2 + |f_{22}|^2) - \frac{1}{2} \operatorname{Re}(f_{11}f_{22}^*) - \operatorname{Im}(f_{22}f_{12}^*) \\ + \operatorname{Im}(f_{11}f_{12}^*) + |f_{12}|^2 \quad (\text{A.63})$$

$$B_{12_c} = \frac{1}{4}(|f_{11}|^2 - |f_{22}|^2) - \frac{j}{2}(\operatorname{Im}(f_{11}f_{22}^*) + f_{12}^*f_{11} + f_{12}^*f_{22}) \quad (\text{A.64})$$

$|f_{11}|^2$, $|f_{22}|^2$, $f_{11}f_{12}^*$, and $f_{22}f_{12}^*$ are defined for the linear polarization case and

$$f_{11}f_{22}^* = d[\frac{1}{4}(|p_v|^2 - 2 \operatorname{Re}(p_v p_h^*) + |p_h|^2) \cos^4 \delta (1 - \cos^2 2\psi) \\ + \cos^2 \delta (\operatorname{Re}(p_h p_v^*) + j \cos 2\psi \operatorname{Im}(p_h p_v^*)) \\ + |p_h|^2 (1 - \cos^2 \delta)] \quad (\text{A.65})$$

In terms of a , g , h , k , and ψ :

$$f_{11}f_{22}^* = d[\frac{a}{4}(1 - \cos^2 2\psi) + g + j \cos 2\psi h + k] \quad (\text{A.66})$$

Let $\psi = u + \psi''$, $\psi'' = \bar{\theta} + \psi'$

∴

$$f_{11}f_{22}^* = d[\frac{a}{4}(1 - \frac{1}{2}(1 + \cos 4(u + \psi'')) \\ + j \cos 2(u + \psi'')h + g + k] \quad (\text{A.67})$$

Integrating over u as was done for the linear polarization case

$$f_{11}f_{22}^* = d[\frac{a}{8} - \frac{a}{8} \cos 4\psi'' \exp(-8\sigma^2) + jh \cos 2\psi'' \exp(-2\sigma^2) + g + k] \quad (\text{A.68})$$

The parameters g , h , and K are:

$$g = \operatorname{Re}(p_h p_v^*) \cos^2 \delta$$

$$h = \operatorname{Im}(p_h p_v^*) \cos^2 \delta \quad (\text{A.69})$$

$$k = |p_h|^2 (1 - \cos^2 \delta)$$

Note that:

$$|f_{11}|^2 + |f_{22}|^2 = 2d \left[\frac{a}{8} \cos 4\psi'' \exp(-8\sigma^2) + \frac{3}{8} a + \frac{1}{2} b + c \right] \quad (\text{A.70})$$

$$|f_{11}|^2 - |f_{22}|^2 = -d[(a + b) \cos 2\psi'' \exp(-2\sigma^2)] \quad (\text{A.71})$$

∴ Transmitting RHC polarization:

$$\begin{aligned} B_{11_c} &= \frac{1}{2} d \left[\frac{a}{8} \cos 4\psi'' \exp(-8\sigma^2) + \frac{3}{8} a + \frac{1}{2} b + c \right] \\ &\quad - \frac{1}{2} \operatorname{Re} \left\{ d \left[\frac{a}{8} - \frac{a}{8} \cos 4\psi'' \exp(-8\sigma^2) + jh \cos 2\psi'' \exp(-2\sigma^2) + g + k \right] \right\} \\ &\quad + d \left[\frac{a}{8} - \frac{a}{8} \cos 4\psi'' \exp(-8\sigma^2) \right] \quad (\text{Cross}) \quad (\text{A.72}) \end{aligned}$$

$$\begin{aligned} B_{22_c} &= \frac{1}{2} d \left[\frac{a}{8} \cos 4\psi'' \exp(-8\sigma^2) + \frac{3}{8} a + \frac{1}{2} b + c \right] \\ &\quad + \frac{1}{2} \operatorname{Re} \left\{ d \left[\frac{a}{8} - \frac{a}{8} \cos 4\psi'' \exp(-8\sigma^2) + jh \cos 2\psi'' \exp(-2\sigma^2) + g + k \right] \right\} \\ &\quad (\text{Co}) \quad (\text{A.73}) \end{aligned}$$

$$B_{21_c} = -\frac{1}{4} d[(a + b) \cos 2\psi'' \exp(-2\sigma^2)] \quad (\text{A.73})$$

$$\begin{aligned} & -\frac{j}{2} \text{Im} \left\{ d \left[\frac{a}{8} - \frac{a}{8} \cos 4\psi'' \exp(-8\sigma^2) + jh \cos 2\psi'' \exp(-2\sigma^2) + g + k \right] \right\} \\ & + jd \left[\left(\frac{a}{4} + \frac{e_1'^*}{2} \right) \sin 2\psi'' \exp(-2\sigma^2) \right] \end{aligned} \quad (\text{A.74})$$

and transmitting LHC polarization:

$$\begin{aligned} B_{11_c} &= \frac{1}{2} d \left[\frac{a}{8} \cos 4\psi'' \exp(-8\sigma^2) + \frac{3}{8} a + \frac{1}{2} b + c \right] \\ &+ \frac{1}{2} \text{Re} \left\{ d \left[\frac{a}{8} - \frac{a}{8} \cos 4\psi'' \exp(-8\sigma^2) + jh \cos 2\psi'' \exp(-2\sigma^2) + g + k \right] \right\} \end{aligned} \quad (\text{Co}) \quad (\text{A.75})$$

$$\begin{aligned} B_{22_c} &= \frac{1}{2} d \left[\frac{a}{8} \cos 4\psi'' \exp(-8\sigma^2) + \frac{3}{8} a + \frac{1}{2} b + c \right] \\ &- \frac{1}{2} \text{Re} \left\{ d \left[\frac{a}{8} - \frac{a}{8} \cos 4\psi'' \exp(-8\sigma^2) + jh \cos 2\psi'' \exp(-2\sigma^2) + g + k \right] \right\} \\ &+ d \left[\frac{a}{8} - \frac{a}{8} \cos 4\psi'' \exp(-8\sigma^2) \right] \end{aligned} \quad (\text{Cross}) \quad (\text{A.76})$$

$$\begin{aligned} B_{12_c} &= -\frac{1}{4} d[(a + b) \cos 2\psi'' \exp(-2\sigma^2)] \\ &- \frac{j}{2} \text{Im} \left\{ d \left[\frac{a}{8} - \frac{a}{8} \cos 4\psi'' \exp(-8\sigma^2) + jh \cos 2\psi'' \exp(-2\sigma^2) + g + k \right] \right\} \\ &- jd \left[\left(\frac{a}{4} + \frac{e_1'^*}{2} \right) \sin 2\psi'' \exp(-2\sigma^2) \right] \end{aligned} \quad (\text{A.77})$$

These equations can be reduced further by noting:

$$\text{Re}\{f_{11}f_{22}^*\} = d \left[\frac{a}{8} - \frac{a}{8} \cos 4\psi'' \exp(-8\sigma^2) + g + k \right]$$

$$\text{Im}\{f_{11}f_{22}^*\} = d[h \cos 2\psi'' \exp(-2\sigma^2)]$$

Therefore including the shape distribution and transmitting RHC polarization:

$$B_{11_c} = d[\frac{a}{4} + \frac{b}{4} + \frac{c}{2} - \frac{q}{2} - \frac{k}{2}]F_o \quad (\text{Cross}) \quad (\text{A.78})$$

$$B_{22_c} = d[\frac{a}{4} + \frac{b}{4} + \frac{c}{2} + \frac{q}{2} + \frac{k}{2}]F_o + s(1 - F_o) \quad (\text{Co}) \quad (\text{A.79})$$

$$B_{21_c} = -d[(j \frac{h}{2} + \frac{a}{4} + \frac{b}{4}) \cos 2\psi'' \exp(-2\sigma^2) + j(\frac{a}{4} + \frac{e'^*}{2}) \sin 2\psi'' \exp(-2\sigma^2)]F_o \quad (\text{A.80})$$

and transmitting LHC polarization:

$$B_{11_c} = d[\frac{a}{4} + \frac{b}{4} + \frac{c}{2} + \frac{q}{2} + \frac{k}{2}]F_o + s(1 - F_o) \quad (\text{Co}) \quad (\text{A.81})$$

$$B_{22_c} = d[\frac{a}{4} + \frac{b}{4} + \frac{c}{2} - \frac{q}{2} - \frac{k}{2}]F_o \quad (\text{Cross}) \quad (\text{A.82})$$

$$B_{12_c} = -d[(j \frac{h}{2} + \frac{a}{4} + \frac{b}{4}) \cos 2\psi'' \exp(-2\sigma^2) - j(\frac{a}{4} + \frac{e'^*}{2}) \sin 2\psi'' \exp(-2\sigma^2)]F_o \quad (\text{A.83})$$

APPENDIX II

FORTRAN COMPUTER TO CALCULATE THE BACKSCATTERED POWER FROM A
COLLECTION OF RAINDROPS RECEIVED BY A DUAL-POLARIZATION RADAR

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C PROGRAMMER: D. P. STAPOR
C
C FEBRUARY 12, 1982
C REVISED TO INCLUDE CIRCULAR POLARIZATION MAY 16, 1982.
C
C VERSION 1.3
C
C THIS PROGRAM CALCULATES THE BACK-SCATTERED POWER RECEIVED BY A
C DUAL-POLARIZATION RADAR USING LINEAR OR CIRCULAR POLARIZATION. THE
C AVERAGE OVER THE CANTING ANGLE DISTRIBUTION HAS BEEN CARRIED OUT
C ANALYTICALLY AND THE AVERAGE OVER THE DROP SIZE DISTRIBUTION HAS
C CARRIED OUT NUMERICALLY. A DISCRETE SHAPE DISTRIBUTION IS ALSO
C SPECIFIED. THE BACKSCATTERED POWER IS DUE TO A PULSE VOLUME
C FILLED WITH RAINDROPS, LOCATED A DISTANCE R FROM THE RADAR.
C
C IMPLICIT REAL*8 (A-H,O-Z)
C EXTERNAL PHA,PVA,RPHV,ALPHV,PSPHA
C
C REAL*8 CO(70),CISO(70),PHASE(70),CROS(70),RE(70),AI(70)
C REAL*8 PR1(70),PR2(70),ZDR(70),THDEG(70)
C REAL*8 RECOR(70),AIMCOR(70),ORTT(70)
C REAL*8 K,NO,I1,LAMBDA,KK
C COMPLEX*16 M,MR,PH,PV,EE,RPX,RPXC,CORR
C REAL X(52),Y(52)
C REAL XNAME(6,7),YNAME(6,9),XNAM(6),YNAM(6)
C COMPLEX SJ
C
C DATA YNAME/' ', ' ', CO, ' ', '(DB)', ' ', LIN, ' ', 'EAR', ' ', ' ',
1 ' ', C, ' ', ROSS, ' ', '(DB)', ' ', LL, ' ', 'NEAR', ' ', ' ',
1 ' ', CO, ' ', '(DB)', ' ', CIRC, ' ', 'ULAR', ' ', ' ',
1 ' ', CR, ' ', OSS, ' ', '(DB)', ' ', CIR, ' ', 'CULA', ' ', R, ' ',
1 ' ', ' ', ' ', LDR, ' ', '(DB)', ' ', ' ', ' ', ' ',
1 ' ', ' ', ' ', ZDR, ' ', '(DB)', ' ', ' ', ' ', ' ',
1 ' ', ' ', ' ', CDR, ' ', '(DB)', ' ', ' ', ' ', ' ',
1 ' ', P, ' ', HASE, ' ', '(DE', ' ', G) L, ' ', 'INEA', ' ', R, ' ',
1 ' ', PH, ' ', ASE, ' ', '(DEG', ' ', ) CI, ' ', 'RCUL', ' ', AR, ' '/
C
C DATA XNAME/' ', EL, ' ', 'EVAT', ' ', 'ION', ' ', 'ANGL', ' ', 'E (D', ' ', EG) ',
1 'POLA', ' ', RIZA, ' ', 'TION', ' ', 'ANG', ' ', 'LE (', ' ', DEG)',
1 ' ', ' ', RAIN, ' ', RATE, ' ', (MM, ' ', /HR)', ' ', ' ',
1 ' ', ' ', LAM, ' ', BDA, ' ', ' ', ' ', ' ',
1 'CA', ' ', 'NTIN', ' ', 'G AN', ' ', 'GLE', ' ', '(DEG', ' ', )',
1 ' ', ' ', S, ' ', 'IGMA', ' ', '(DE', ' ', G)',
1 'FRAC', ' ', 'TION', ' ', OF, ' ', 'OBLA', ' ', 'TE D', ' ', 'ROPS'/'
C
C COMMON/ABC/ RR,LAMBDA,IRAIN
C COMMON/DEF/ MR
C
C CONSTANCE:
C
C PI = 3.141592653589793
C DEG = PI/180.
C SJ = (0.0,1.0)
C

```

C RADAR PARAMETERS

```

C
    IFLG = 0
    IPLT = 0
    READ(5,990) NAME1,NAME2
990  FORMAT(2A4)
    READ(5,996) ISET,IOUTPT
996  FORMAT(12,8X,11)
997  CONTINUE
    READ(5,10,END=998) G,THB,TAU,POW,FREQ,ELEV,PSI
    10  FORMAT(7F10.4)
        IF(IOUTPT.EQ.2)GO TO 160
        IF(IFLG.EQ.0) CALL PLOTS(0,0,0)
        IF(IFLG.EQ.0) CALL PLOT(2.0,3.0,-3)
160  CONTINUE
        IFLG = 1
        IPLT = IPLT + 1
C
    WRITE(6,11)
11  FORMAT(//, '*****')
    WRITE(6,12)
12  FORMAT(/, 'MATRIX RADAR EQUATION CALCULATION PROGRAM',//,
1  ' RADAR PARAMETERS:',/)
    WRITE(6,13) G,THB,TAU,POW,FREQ,ELEV,PSI
13  FORMAT(' ANTENNA GAIN = ',F8.2,' DB',/,
1  ' ANTENNA BEAMWIDTH = ',F8.2,' DEGREES',/,
1  ' TRANSMIT PULSE-LENGTH = ',F8.2,' MICROSECONDS',/,
1  ' TRANSMIT POWER = ',F8.2,' KILOWATTS',/,
1  ' FREQUENCY = ',F8.2,' GHZ',/,
1  ' ELEVATION ANGLE = ',F8.2,' DEGREES',/,
1  ' POLARIZATION ANGLE = ',F8.2,' DEGREES')

```

C

C CONVERT RADAR PARAMETERS TO USEABLE QUANTITIES

C

```

    G = 10.0**(G/10.0)
    THB = THB*DEG
    H = TAU*3.0E02
    POW = POW*1000.
    WAV = 0.3/FREQ
    K = 2.0*PI/WAV

```

C

C ASSUME COMPLEX INDEX OF REFRACTION AT 5 DEG C AND FREQ = 2.8 GHZ.

C

```

    READ(5,144) M
144  FORMAT(F10.4,F10.4)
    MR = M**2
    READ(5,145) IRAIN
145  FORMAT(11)
    IF(IRAIN.EQ.1) READ(5,51) RR
    IF(IRAIN.EQ.2) READ(5,52) NO,LAMBDA
51  FORMAT(F10.4)
52  FORMAT(2F10.4)
    READ(5,51) FO
    READ(5,52) THTA,SIG
    READ(5,51) RANGE

```

```

C
C PLOT TYPE:
C
C X AXIS-
C   IVAR = 1  ELEVATION
C   IVAR = 2  PSI
C   IVAR = 3  IRAIN = 1 RAINRATE
C   IVAR = 3  IRAIN = 2 LAMBDA
C   IVAR = 4  THETA
C   IVAR = 5  SIGMA
C   IVAR = 6  FO
C
C Y AXIS-
C   IOUT = 1  CO (LINEAR)
C   IOUT = 2  CROSS (LINEAR)
C   IOUT = 3  CO (CIRCULAR)
C   IOUT = 4  CROSS (CIRCULAR)
C   IOUT = 5  LDR, RE(CORR), IM(CORR), MAG(CORR)
C   IOUT = 6  ZDR
C   IOUT = 7  CDR, RE(CORR), IM(CORR), MAG(CORR)
C   IOUT = 8  PHASE (LINEAR)
C   IOUT = 9  PHASE (CIRCULAR)
C
C XSTR = STARTING VALUE OF X AXIS
C XSTP = STOPPING VALUE OF X AXIS
C
C   READ(5,53) IVAR,IOUT
53  FORMAT(11,9X,11)
C   READ(5,52) XSTR,XSTP
C
C N = NUMBER OF DATA POINTS
C
C   N = 35
C
C   XLNTH = 4.0
C   YLNTH = 5.0
C
C   XINC = (XSTP - XSTR)/FLOAT(N - 1)
C   FVALX = XSTR
C   DVX = (XSTP - XSTR)/XLNTH
C   X(N+1) = FVALX
C   X(N+2) = DVX
C   X(1) = SNGL(XSTR)
C
C   WRITE(6,14) IVAR,IOUT
C   14  FORMAT(/,' IVAR = ',11,' IOUT = ',11,/)
C
C OUTPUT RAIN CLOUD PARAMETERS
C
C   WRITE(6,17)
17  FORMAT(/,' RAIN CLOUD PARAMETERS:',/)
C   WRITE(6,23) M
23  FORMAT(' COMPLEX INDEX OF REFRACTION',/,
1   ' M = (' ,F8.2,' ,',F8.2,' )')
C   WRITE(6,20) FO

```

```

20  FORMAT(' FRACTION OF OBLATE RAINDROPS = ',F8.2)
    WRITE(6,21) RANGE
21  FORMAT(' RANGE TO CLOUD = ',F8.2,' KILOMETERS')
    RANGE = RANGE*1000.
    WRITE(6,29) THTA,SIG
29  FORMAT(' CANTING ANGLE = ',F8.2,' DEGREES',/,
1    ' STANDARD DEVIATION = ',F8.2,' DEGREES')
    IF(IRAIN.EQ.1) WRITE(6,22) RR
    IF(IRAIN.EQ.2) WRITE(6,24) LAMBDA,NO
24  FORMAT(' LAMBDA = ',F9.3,/, ' NO = ',F9.3,/)
22  FORMAT(' RAINRATE = ',F8.2,' (MM/HR)',/)

C
C  CALCULATE THE RADAR CONSTANT
C
    PSI = PSI*DEG
    THTA = THTA*DEG
    SIG = SIG*DEG
    ELEV = ELEV*DEG

C
    RC = POW*(WAV/(4.*PI))**2*(G*THB)**2*H*PI/(16.*ALOG(2.0))
    IF(IRAIN.EQ.1) RC = RC*16000.0
    IF(IRAIN.EQ.2) RC = RC*NO
C    WRITE(6,60) RC
C 60  FORMAT(' RC = ',E10.4,/)
C
C  CALCULATE THE INTEGRATION OVER THE DROP SIZE DISTRIBUTION
C
C  |PH|**2,|PV|**2,RE(PH*CONJG(PV)),IM(PH*CONJG(PV)) ARE COMPUTED
C
    AP = 1.0E-06
    BP = 3.5
    DEL = 1.0E-06
    IMAX = 20
    IZDR = 1
31  CONTINUE
    IF(IVAR.EQ.2.OR.IVAR.EQ.4.OR.IVAR.EQ.5.OR.IVAR.EQ.6) GO TO 211
    IF(IVAR.EQ.1) ELEV = XSTR*DEG
    IF(IVAR.EQ.3) GO TO 210
    GO TO 211
210  IF(IRAIN.EQ.1) RR = XSTR
    IF(IRAIN.EQ.2) LAMBDA = XSTR
211  I = 1
32  CONTINUE

C
    CALL SMPSN(PHA,AP,BP,DEL,IMAX,S11,PHE,NN,IERROR)
    CALL SMPSN(PVA,AP,BP,DEL,IMAX,S11,PVE,NN,IERROR)
    CALL SMPSN(RPHV,AP,BP,DEL,IMAX,S11,RPHVE,NN,IERROR)
    CALL SMPSN(ALPHV,AP,BP,DEL,IMAX,S11,ALPHVE,NN,IERROR)
    CALL SMPSN(PSPHA,AP,BP,DEL,IMAX,S11,PSPHE,NN,IERROR)

C
C    PHE = DCADRE(PHA,AP,BP,0.0,1.0E-06,ERROR,IERROR)
C    PVE = DCADRE(PVA,AP,BP,0.0,1.0E-06,ERROR,IERROR)
C    RPHVE = DCADRE(RPHV,AP,BP,0.0,1.0E-06,ERROR,IERROR)
C    ALPHVE = DCADRE(ALPHV,AP,BP,0.0,1.0E-06,ERROR,IERROR)
C    PSPHE = DCADRE(PSPHA,AP,BP,0.0,1.0E-06,ERROR,IERROR)

```

```

C      IF(IERROR.EQ.0) GO TO 200
      IF(IERROR.NE.0) WRITE(6,201)
201  FORMAT(' AN ERROR OCCURRED IN THE INTEGRATION ROUTINE')
      STOP
200  CONTINUE

C
C  CALCULATE THE COEFFICIENTS AA,BB,CC,DD,EE,GG,HH, KK
C
      AA = DCOS(ELEV)**4*(PVE - 2.0*RPHVE + PHE)
      BB = 2.0*DCOS(ELEV)**2*(RPHVE - PHE)
      CC = PHE
      DD = K**4
      EE = DCOS(ELEV)**2*(DCMPLX(RPHVE,ALPHVE) - PHE)
      GG = DCOS(ELEV)**2*RPHVE
      HH = DCOS(ELEV)**2*ALPHVE
      KK = PHE*(1.0 - DCOS(ELEV)**2)

C
C  TESTING
C
C      WRITE(6,800) AA,BB,CC,DD,EE,GG,HH, KK
C 800  FORMAT(/,1X,'AA = ',E12.4,/, 'BB = ',E12.4,/, 'CC = ',E12.4,/,
C      1 ' DD = ',E12.4,/, 'EE = ',E12.4,/, 'GG = ',E12.4,/,
C      1 ' HH = ',E12.4,/, 'KK = ',E12.4,/)
C
C 70  CONTINUE
      IF(IVAR.EQ.1.OR.IVAR.EQ.3) GO TO 212
      IF(IVAR.EQ.2) PSI = XSTR*DEG
      IF(IVAR.EQ.4) THTA = XSTR*DEG
      IF(IVAR.EQ.5) SIG = XSTR*DEG
      IF(IVAR.EQ.6) FO = XSTR
212  CONTINUE

C
C  CALCULATE THE RETURN POWER
C
C  PCO = CO SIGNAL AFOR LINEAR POLARIZATION
C  PCROS = CROSS SIGNAL FOR LINEAR POLARIZATION
C  RPX = OFF DIAGONAL TERM FOR LINEAR POLARIZATION
C
      PCO = (-1.*(AA+BB)/2.*DCOS(2.*(THTA+PSI))*DEXP(-2.*SIG**2)
1 + AA/8.*DCOS(4.*(THTA+PSI))*DEXP(-8.*SIG**2)+AA*3./8.+BB/2.
1 + CC)*FO + PSPHE*(1.- FO)
C
      PCROS = FO*AA/8.*(1.-DCOS(4.*(THTA+PSI))*DEXP(-8.*SIG**2))
C
      RPX = FO*((AA/4.+EE/2.)*DSIN(2.*(THTA+PSI))*DEXP(-2.*SIG**2)
1 -AA/8.*DSIN(4.*(THTA+PSI))*DEXP(-8.*SIG**2))
C
C  PCOC = CO SIGNAL FOR CIRCULAR POLARIZATION
C  PCROSC = CROSS SIGNAL FOR CIRCULAR POLARIZATION
C  RPXC = OFF DIAGONAL TERM FOR CIRCULAR POLARIZATION
C
      PCOC = (AA/4. + BB/4. + CC/2. + GG/2. + KK/2.)*FO +
1 PSPHE*(1.0 - FO)
C

```

```

      PCROSC = (AA/4. + BB/4. + CC/2. - GG/2. - KK/2.)*FO
C
      RPXC = (-1.*(SJ*HH/2. + AA/4. + BB/4.)*DCOS(2.*(THTA + PSI))*
1 DEXP(-2.*SIG**2) + SJ*(AA/4. + DCONJG(EE)/2.))*
1 DSIN(2.*(THTA + PSI))*DEXP(-2.*SIG**2))*FO
C
C CORR = COMPLEX CORRELATION FACTOR
C RECOR(1) = REAL PART OF THE CORRELATION COEFFICIENT
C AIMCOR(1) = IMAGINARY PART OF THE CORRELATION COEFFICIENT
C ORTT(1) = APPARENT DEGREE OF ORIENTATION
C
C CALCULATE THE CORRELATION BETWEEN THE CO AND CROSS SIGNALS
C
      IF(IOUT.EQ.7) GO TO 85
      CORR = RPX/DSQRT(PCO*PCROS)
      RECOR(1) = DREAL(CORR)
      AIMCOR(1) = DIMAG(CORR)
      ORTT(1) = CDABS(CORR)
      GO TO 86
C
      85 CORR = RPXC/DSQRT(PCOC*PCROSC)
      RECOR(1) = DREAL(CORR)
      AIMCOR(1) = DIMAG(CORR)
      ORTT(1) = CDABS(CORR)
      86 CONTINUE
C
C SAVE REAL AND IMAGINARY PARTS TO CALCULATE THE PHASE
C
      IF(IOUT.EQ.9) GO TO 207
      REPX = DREAL(RPX)
      AIEPX = DIMAG(RPX)
      GO TO 208
      207 REPX = DREAL(RPXC)
      AIEPX = DIMAG(RPXC)
      208 CONTINUE
C
C STORE OUTPUT IN VARIABLES CO(1), CROS(1), RE(1), AI(1)
C
C NOTE THAT THE ADDED FACTORS IN ( PR added ) ARE TO CONVERT MM-6 TO M-6
C (THE 180.0) AND TO CONVERT DBW TO DBM (THE 30.0).
C
      PR added = 10.0*DLOG10(RC*DD/RANGE**2) - 180.0 + 30.0
      IF(IOUT.EQ.1.OR.IOUT.EQ.5.OR.IOUT.EQ.6.OR.IOUT.EQ.8)
1 CO(1) = PR added + 10.0*DLOG10(PCO)
      IF(IOUT.EQ.3.OR.IOUT.EQ.7.OR.IOUT.EQ.9)
1 CO(1) = PR added + 10.0*DLOG10(PCOC)
      IF(IOUT.EQ.4.OR.IOUT.EQ.7.OR.IOUT.EQ.9)
1 GO TO 205
      IF(PCROS.LE.1.0E-30) GO TO 25
      CROS(1) = PR added + 10.0*DLOG10(PCROS)
      GO TO 26
      205 CONTINUE
      IF(PCROSC.LE.1.E-30) GO TO 25
      CROS(1) = PR added + 10.0*DLOG10(PCROSC)
      GO TO 26

```

```

25  CROS(I) = -300.0
26  CONTINUE
    CISO(I) = CROS(I) - CO(I)
    IF(CISO(I).LT.-40.0) CISO(I) = -40.0
C
C  CALCULATE THE PHASE
C
    IF(REPX.EQ.0.0) GO TO 80
    PHASE(I) = -1.*DATAN2(AIEPX,REPX)/DEG
    GO TO 81
80  PHASE(I) = 90.0
81  CONTINUE
    RE(I) = (RC*DD/RANGE**2)*(REPX)*1.0E-15
    AI(I) = (RC*DD/RANGE**2)*(AIEPX)*1.0E-15
    X(I+1) = SNGL(X(I) + XINC)
    IF(IZDR.EQ.1) PR1(I) = CO(I)
    IF(IZDR.EQ.2) PR2(I) = CO(I)
    PSIDEG = PSI/DEG
    THTDEG = THTA/DEG
    ELVDEG = ELEV/DEG
C
C  TESTING
C
C    WRITE(6,800) I,PSIDEG,THTDEG,SIG,ELVDEG,CO(I),AA,BB,CC,PSPHE,FO
800  FORMAT(' I=',I2,' PSI=',F7.2,' THETA=',F8.2,' SIG=',F6.1,
C    1 ' ELEV=',F6.1,' CO=',F7.2,' AA=',E8.2,' BB=',E8.2,' CC=',E8.2,
C    1 ' /, ' PSPHE=',E8.2,' FO=',F6.1)
C
    IF(IVAR.EQ.1.OR.IVAR.EQ.3) GO TO 213
    IF(IVAR.EQ.2) PSI = PSI + XINC*DEG
    IF(IVAR.EQ.4) THTA = THTA + XINC*DEG
    IF(IVAR.EQ.5) SIG = SIG + XINC*DEG
    IF(IVAR.EQ.6) FO = FO + XINC
    I = I + 1
    IF(I.GT.N) GO TO 74
    GO TO 212
213  CONTINUE
C
    IF(IVAR.EQ.1) ELEV = ELEV + XINC*DEG
    IF(IVAR.EQ.3) GO TO 215
    GO TO 214
215  IF(IRAIN.EQ.1) RR = RR + XINC
    IF(IRAIN.EQ.2) LAMBDA = LAMBDA + XINC
214  CONTINUE
    I = I + 1
    IF(I.GT.N) GO TO 74
    IF(IVAR.EQ.1) GO TO 200
    IF(IVAR.EQ.3) GO TO 32
74  IF(IOUT.EQ.1.OR.IOUT.EQ.2.OR.IOUT.EQ.3.OR.IOUT.EQ.4.OR
1  .IOUT.EQ.5.OR.IOUT.EQ.7.OR.IOUT.EQ.8.OR.IOUT.EQ.9) GO TO 71
    IF(IZDR.EQ.2) GO TO 71
    IF(IZDR.EQ.1) PSI = PSI + 90.0*DEG
    I = 1
    IZDR = 2
    IF(IVAR.EQ.1) ELEV = XSTR*DEG

```

```

      IF(IVAR.EQ.1) GO TO 200
      IF(IVAR.EQ.3) GO TO 31
      GO TO 70
71  CONTINUE
      DO 73 J=1,N
73  ZDR(J) = PR1(J) - PR2(J)
C
      DO 232 I=1,6
      IF(IVAR.EQ.1) XNAM(I) = XNAME(I,1)
      IF(IVAR.EQ.2) XNAM(I) = XNAME(I,2)
      IF(IVAR.EQ.3) GO TO 231
      IF(IVAR.EQ.4) XNAM(I) = XNAME(I,5)
      IF(IVAR.EQ.5) XNAM(I) = XNAME(I,6)
      IF(IVAR.EQ.6) XNAM(I) = XNAME(I,7)
      GO TO 232
231  IF(IRAIN.EQ.2) XNAM(I) = XNAME(I,4)
      IF(IRAIN.EQ.1) XNAM(I) = XNAME(I,3)
232  CONTINUE
C
      DO 240 I=1,N
      IF(IOUT.EQ.1.OR.IOUT.EQ.3) Y(I) = SNGL(CO(I))
      IF(IOUT.EQ.2.OR.IOUT.EQ.4) Y(I) = SNGL(CROS(I))
      IF(IOUT.EQ.5.OR.IOUT.EQ.7) Y(I) = SNGL(CISO(I))
      IF(IOUT.EQ.6) Y(I) = SNGL(ZDR(I))
      IF(IOUT.EQ.8.OR.IOUT.EQ.9) Y(I) = SNGL(PHASE(I))
240  CONTINUE
C
      DO 241 I=1,6
      IF(IOUT.EQ.1) YNAM(I) = YNAME(I,1)
      IF(IOUT.EQ.2) YNAM(I) = YNAME(I,2)
      IF(IOUT.EQ.3) YNAM(I) = YNAME(I,3)
      IF(IOUT.EQ.4) YNAM(I) = YNAME(I,4)
      IF(IOUT.EQ.5) YNAM(I) = YNAME(I,5)
      IF(IOUT.EQ.6) YNAM(I) = YNAME(I,6)
      IF(IOUT.EQ.7) YNAM(I) = YNAME(I,7)
      IF(IOUT.EQ.8) YNAM(I) = YNAME(I,8)
      IF(IOUT.EQ.9) YNAM(I) = YNAME(I,9)
241  CONTINUE
C
      WRITE(6,991) NAME1,NAME2
991  FORMAT(/,1X,'OUTPUT DATA FILE NAME = ',2A4,' OUTPUT',/)
      WRITE(6,401) XNAM,YNAM
401  FORMAT(/,' X AXIS: ',6A4,' Y AXIS: ',6A4,/)
      WRITE(6,402) IPLT,ISET
402  FORMAT(/,' PLOT ',12,' SET ',12,/)
      IF(IOUT.EQ.5.OR.IOUT.EQ.7) WRITE(6,165) (I,X(I),Y(I),RECOR(I),
1  AIMCOR(I),ORTT(I),I=1,N)
165  FORMAT(' X(',12,') = ',F10.4,' Y(I) = ',F10.4,' RE(COR) = ',
1  F10.4,' IM(COR) = ',F10.4,' ORTT = ',F10.4)
      IF(IOUT.NE.5.AND.IOUT.NE.7) WRITE(6,400) (I,X(I),I,Y(I),I=1,N)
400  FORMAT(' X(',13,') = ',F10.4,' Y(',13,') = ',F10.4)
      WRITE(10,156) NAME1,NAME2
156  FORMAT(' :READ ',2A4,' OUTPUT A1')
      WRITE(10,157) ISET
157  FORMAT('//GO.FT',12,'F001 DD *')

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      IF(IOUT.NE.5.AND.IOUT.NE.7) WRITE(10,162) (X(I),Y(I),I=1,N)
      IF(IOUT.EQ.5.OR.IOUT.EQ.7) WRITE(10,163) (X(I),Y(I),RECOR(I),
1  AIMCOR(I),ORTT(I),I=1,N)
163  FORMAT(5(F10.4,5X))
162  FORMAT(F10.4,5X,F10.4)
      WRITE(10,158)
158  FORMAT('/*')
      GO TO 997
161  CONTINUE
      FCTOR = 1.0
      CALL FACTOR(FCTOR)
      CALL PLOT(0.0,0.0,-3)
      CALL SYMBOL(1.7,-1.5,.12,4HPLT,0.,4)
      CALL SYMBOL(1.8,-1.85,.12,3HSET,0.,3)
      PSET = FLOAT(ISET)
      PIPLT = FLOAT(IPLT)
      CALL NUMBER(2.3,-1.5,.12,PIPLT,0.,-1)
      CALL NUMBER(2.3,-1.85,.12,PSET,0.,-1)
      CALL SCALE(Y, YLNTH,N,1)
      CALL SCALE(X, XLNTH,N,1)
      CALL AXIS(0.0,0.0,XNAM,-24,XLNTH,0.0,X(N+1),X(N+2))
      CALL AXIS(0.0,0.0,YNAM,24,YLNTH,90.0,Y(N+1),Y(N+2))
      CALL LINE(X,Y,N,1,0,0)
      CALL PLOT(8.5,0.0,-3)
      GO TO 997
998  IF(IOUTPT.EQ.2) STOP
      CALL PLOT(9.0,0.0,999)
      STOP
      END

C
C
C FUNCTION SUBPROGRAMS
C
      REAL FUNCTION RDIST(A)
      REAL*8 RR,NO,LAMBDA,RDIST,A
      COMMON/ABC/ RR,LAMBDA,IRAIN
      IF(IRAIN.EQ.2) GO TO 10
      RDIST = DEXP(-8.2*RR**(-0.21)*A)
      RETURN
10  RDIST = DEXP(-LAMBDA*A)
      RETURN
      END

C
      REAL FUNCTION PHA(A)
      REAL*8 A,PHA,RDIST
      COMPLEX*16 PH
      PHA = CDABS(PH(A))**2*RDIST(A)
      RETURN
      END

C
      REAL FUNCTION PVA(A)
      REAL*8 A,PVA,RDIST
      COMPLEX*16 PV
      PVA = CDABS(PV(A))**2*RDIST(A)
      RETURN

```

```

      END
C
      REAL FUNCTION RPHV(A)
      REAL*8 A,RPHV,RDIST
      COMPLEX*16 PH,PV
      RPHV = DREAL(PH(A)*DCONJG(PV(A)))*RDIST(A)
      RETURN
      END
C
      REAL FUNCTION AIPHV(A)
      REAL*8 A,RDIST,AIPHV
      COMPLEX*16 PH,PV
      AIPHV = DIMAG(PH(A)*DCONJG(PV(A)))*RDIST(A)
      RETURN
      END
C
      REAL FUNCTION PSPHA(A)
      REAL*8 A,RDIST,PSPHA
      COMPLEX*16 PSPH
      PSPHA = CDABS(PSPH(A))**2*RDIST(A)
      RETURN
      END
C
      COMPLEX FUNCTION PSPH(A)
      REAL*8 A
      COMPLEX*16 PSPH,MR
      COMMON/DEF/ MR
      PSPH = (MR-1.)/(MR+2.)*A**3.
      RETURN
      END
C
      COMPLEX FUNCTION PV(A)
      REAL*8 I1,V,A,E
      COMPLEX*16 MR,PV
      COMMON/DEF/ MR
      IF(A.EQ.0.0) GO TO 10
      V = A**3/3.
      E = DSQRT(1. - (1. - 0.1*A)**2)
      I1 = (1. - DARSIN(E)*(DSQRT(1. - E*E)/E))/(E*E)
      PV = V*(MR - 1.)*(1./((I1*(MR - 1.) + 1.)))
      RETURN
10  PV = 0.0
      RETURN
      END
C
      COMPLEX FUNCTION PH(A)
      REAL*8 I1,A,V,E
      COMPLEX*16 MR,PH
      COMMON/DEF/ MR
      IF(A.EQ.0.0) GO TO 10
      V = A**3/3.
      E = DSQRT(1. - (1. - 0.1*A)**2)
      I1 = (1. - DARSIN(E)*(DSQRT(1. - E*E)/E))/(E*E)
      PH = V*(MR - 1.)*(2./((I1*(1. - MR) + MR + 1.)))
      RETURN

```

```

10  PH = 0.0
    RETURN
    END

```

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    INTEGRATION BY SIMPSON'S RULE

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    SUBROUTINE SMPSN(F,A,B,DEL,IMAX,S11,S,N,IER)
    IMPLICIT REAL*8 (A-H,O-Z)
    EXTERNAL F
    S11=0.
    S=0.
    N=0
    BA=B-A
    IF(BA) 20,19,20
19  IER=1
    RETURN
20  IF(DEL) 22,22,23
22  IER=2
    RETURN
23  IF(IMAX-1) 24,24,25
24  IER=3
    RETURN
25  X=BA/2.+A
    NHALF=1
    SUMK=F(X)*BA*2./3.
    S=SUMK+(F(A)+F(B))*BA/6.
    DO 28 I=1,IMAX
    S11=S
    S=(S-SUMK/2.)/2.
    NHALF=NHALF*2.
    ANHLF=NHALF
    FRSTX=A+(BA/ANHLF)/2.
    SUMK=F(FRSTX)
    XK=FRSTX
    KLAST=NHALF-1
    FINC=BA/ANHLF
    DO 26 K=1,KLAST
    XK=XK+FINC
26  SUMK=SUMK+F(XK)
    SUMK=SUMK*2.*BA/(3.*ANHLF)
    S=S+SUMK
27  IF(DABS(S-S11)-DABS(DEL*S)) 29,29,28
28  CONTINUE
    IER=4
    GO TO 30
29  IER=0
30  N=2*NHALF
    RETURN
    END

```

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DUAL-POLARIZATION METEOROLOGICAL RADAR:

THEORY AND DESIGN

by

Daniel P. Stapor

(ABSTRACT)

An improved formulation for radar backscatter from a volume of oblate spheroidal raindrops is developed, taking account of the beam elevation angle and the raindrop canting angle. The formulation is in terms of scattering coefficients in a matrix, and can be applied to systems using linear, circular, or elliptical polarization. The general formulation reduces to results previously published in the literature for linear polarization differential reflectivity (Z_{DR}) and for circular polarization depolarization ratio (CDR).

Analysis of the linear polarization case for a dual-polarization radar which can transmit and receive orthogonal linear polarizations in the horizontal-vertical (HV) planes, and the (± 45) degree planes are considered. Results are presented showing the effect raindrop parameters have on Z , and linear polarization ratio (LDR) measured in (HV) and ($\pm 45^\circ$) planes. Analysis of the circular polarization case for a dual polarization radar which can transmit and receive circular polarizations is also considered. Results show that LDR measured in the ($\pm 45^\circ$) planes is similar to CDR.

The dual-polarized radar development program at VPI&SU was discussed and a detailed design of a new radar data collection system is given.

With the theory of dual-polarization radar measurement and the practical implementation of such a radar described within this thesis, a more accurate characterization of the propagation effects along a satellite to earth path can be carried out.