MODELING THE THERMAL PERFORMANCE OF AN INTELLIGENT MEMS PRESSURE SENSOR WITH SELF-CALIBRATION CAPABILITIES

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ABSTRACT

Recent industry trends toward more complex and interconnected systems have increased the demand for more reliable pressure sensors. One of the best methods to ensure reliability is by regularly calibrating the sensor, checking its functionality and accuracy. By integrating a micro-actuator with a pressure sensor, the sensor can self-calibrate, eliminating the complexities and costs associated with traditional sensor calibration methods. The present work is focused on furthering understanding and improving the thermal performance of a thermopneumatic actuated self-calibrating pressure sensor.

A transient numerical model was developed in ANSYS and was calibrated using experimental testing data. The model provided insights into the sensor’s performance not previously observed in experimental testing, such as the temperature gradient within the sensor and its implications. Furthermore, the model was utilized for two design studies. First, the sensor’s inefficiencies were studied, and it was found that a substrate with low thermal conductivity and high thermal diffusivity is ideal for both the sensor’s efficiency and a faster transient response time. The second design study showed that decreasing the size of the sealed reference cavity, decreases power consumption and transient response time. The study also showed that decreasing the cavity base dimension has a larger effect on decreasing power consumption and response time. Overall, the present work increases understanding of the self-calibrating pressure sensor and provides insight into potential design improvements, moving closer to true self-calibrating pressure sensors.
Pressure sensors are used in most engineering applications, and the demand is ever increasing due to emerging fields such as the Internet of things (IOT), automations, and autonomy. One drawback of current pressure sensor technology is their need to be calibrated, ensuring accuracy and function. Sensor calibration requires equipment, trained personnel, and must be done regularly, resulting in significant costs. Borrowing technology, methods, and materials from the integrated circuit industry, the costs of sensor calibration can be addressed by the development of an intelligent MEMS (micro-electromechanical system) pressure sensor with self-calibration capabilities. The self-calibrating capability is achieved by combining a micro-actuator and a micro-pressure sensor into one system.

This work focuses on complementing previously obtained experimental testing data with a thermal finite element model to provide a deeper understanding and insight. The model is implemented in the commercial software ANSYS and model uncertainties were addressed via model calibration. The model revealed a temperature gradient within the sensor, and insight into its potential effects.

The model is also used as a design tool to reduce energy inefficiencies, decrease the time it takes the sensor to respond, and to study the effects of reducing the sensor size. The studies showed that the power consumption can potentially be decreased up to 92% and the response time can be decreased up to 99% by changing the sensor’s substrate material. Furthermore, by halving the sensor reference cavity size, the cavity temperature can be increased by 45% and the time for the sensor to respond can be decrease by 59%.
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First, I want to acknowledge and thank Jesus Christ my Lord and Savior. Without His love, strength, and knowledge imparted on me, this would not have been possible. I want to thank my whole family, in particular my mother Sagé de Clerck and my father Johan de Clerck, for their unending love, support, and encouragement throughout the years.

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DEDICATION

To God my Father, for His glory.
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MODELING THE THERMAL PERFORMANCE OF AN INTELLIGENT MEMS PRESSURE SENSOR WITH SELF-CALIBRATION CAPABILITIES

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NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>Electrical resistance</td>
</tr>
<tr>
<td>π</td>
<td>Piezoresistive coefficient</td>
</tr>
<tr>
<td>σ</td>
<td>Stress</td>
</tr>
<tr>
<td>Gr</td>
<td>Grashof number</td>
</tr>
<tr>
<td>g</td>
<td>Gravitational acceleration constant</td>
</tr>
<tr>
<td>β</td>
<td>Volumetric thermal expansion coefficient</td>
</tr>
<tr>
<td>T</td>
<td>Temperature</td>
</tr>
<tr>
<td>L</td>
<td>Length</td>
</tr>
<tr>
<td>ν</td>
<td>Kinematic viscosity</td>
</tr>
<tr>
<td>ρ</td>
<td>Density</td>
</tr>
<tr>
<td>Cₚ</td>
<td>Specific heat</td>
</tr>
<tr>
<td>t</td>
<td>Time</td>
</tr>
<tr>
<td>U</td>
<td>Velocity</td>
</tr>
<tr>
<td>k</td>
<td>Thermal conductivity</td>
</tr>
<tr>
<td>q&quot;&quot;</td>
<td>Volumetric heat generation</td>
</tr>
<tr>
<td>α</td>
<td>Thermal diffusivity</td>
</tr>
<tr>
<td>h</td>
<td>Convection coefficient</td>
</tr>
<tr>
<td>θ</td>
<td>Initial temperature compensated temperature</td>
</tr>
<tr>
<td>τ</td>
<td>Thermal time constant</td>
</tr>
</tbody>
</table>

\[
\frac{ΔR}{R} = π_t σ_t + π_t σ_t
\]

Configuring the resistors in a Wheatstone bridge, the resistance change is converted to a voltage signal corresponding to the applied pressure. Even though the sensor’s operating principle is simple, much work has been done to characterize and improve piezoresistive micro-electromechanical system (MEMS) pressure sensor design [1]. Some of the work includes sensitivity and linearity optimization via structural design [2], temperature effects and compensation techniques [3], sensor packaging [4,5], and methods to optimize sensor signal-to-noise ratio [6]. However, like all pressure sensors, mechanical wear in piezoresistive sensors still results in failure, hysteresis, signal drift, and nonlinearity errors, culminating in degradation of sensor reliability and accuracy.

The best way to check the functionality of pressure sensors and eliminate the aforementioned errors is by comparing the sensor’s output to a well-defined, standard pressure. This process, known as calibration, produces a curve that quantifies the relationship between the sensor output signal and the input pressure. Traditional calibration procedures account for a significant portion of the manufacturing and maintenance costs of pressure sensors. The costs arise as sensor calibration must be done regularly over the life of the sensor to ensure accuracy and requires time, money, trained personnel, and specialized equipment.

Advances in MEMS, NEMS (nano-electromechanical system), materials, and cleanroom processes have provided a means for developing self-calibrating pressure sensors, mitigating the drawback of traditional calibration procedures. By incorporating a micro-actuator and sensor into one small device, the actuator can be used to calibrate the sensor, replacing the need for an external standard as required by traditional calibration procedures. Concepts and methods for various self-calibrating sensors have been developed [7], but this new technology has yet to be fully executed and commercially produced.

Self-calibrating pressure sensors, along with appropriate software algorithms, can ensure data accuracy without the drawbacks and costs of traditional calibration procedures. The need for direct access of the sensor is eliminated and a simple calibration command initiates and performs the self-calibration sequence. Furthermore, uninstalling the sensor is not required, reducing downtime and costs. Lastly, by minimizing the time required for self-calibration, the functionality and accuracy of
the sensor can be checked often, ensuring data quality for time-sensitive and mission-critical applications, such as flight.

The many advantages of self-calibrating pressure sensors are accompanied by two challenges. First, the current self-calibrating pressure range is limited. Sensor calibration standards require that sensors be calibrated over their full sensing range, presenting a challenge as micro-actuators are limited to small pressures, thus limiting the achievable self-calibration range. The second challenge in developing a self-calibrating pressure sensor is the accuracy requirement. Since the accuracy of the sensor depends directly on the self-calibration, the micro-actuator needs to be well characterized and precisely controllable to ensure accurate and repeatable calibration curves.

Only two self-calibrating pressure sensors have been found in literature, possibly due to the challenging nature of developing a system capable of performing as previously discussed. Yameogo et al. reported a wireless self-calibrating pressure sensor for biomedical applications [8]. Used to measure intracranial pressure, the sensor is of a piezoresistive type with a pressure range of 0 - 150 mmHg (~2.9 psi). An electrostatic actuator is used to achieve self-calibration by applying a voltage between two electrodes separated by an air gap. When actuated, the pressure-sensitive membrane deflects resulting in a sensor output signal change. However, using an actuator other than pressure for a pressure sensor is not ideal as it will be unable to detect potential defects such as a leak in the reference cavity [9].

The second self-calibrating pressure sensor found in literature from previous work by the current authors [10]. A micro-heater and thermistor were integrated into the sealed reference chamber of the sensor, forming a monitorable thermopneumatic actuator, commonly used for micro-valves and pumps [11,12]. Combining a thermopneumatic actuator and a piezoresistive sensor has been done before [13,14], but the addition of the cavity thermistor allows for self-calibration. To self-calibrate, the heater is turned on, the gas in the cavity heats up, expands, and the internal pressure acts on the pressure sensor membrane producing a signal. Using the cavity thermistor, the temperature in the cavity is measured and a state equation like the Ideal Gas law is used to calculate the pressure. Knowing the pressure from the actuator and the sensor output, standard calibration procedures can be applied. The previous work presented the operating principle and conceptual design of the sensor. Furthermore, a prototype sensor was built and tested.

Predictive and consistent thermal performance is essential to the sensor’s self-calibrating capabilities. However, current thermal physics understanding is limited to simple analysis and experimental testing. For this reason, the present work uses numerical modeling complemented by experimental testing to increase understanding and investigate the transient thermal performance of self-calibrating pressure sensors. Furthermore, the model aids in assessing potential design improvements to increase thermal efficiency and decrease self-calibration response time. The model development, model calibration, and results are presented in the following sections.

2. MODEL DEVELOPMENT

This section starts with a brief overview of the self-calibrating sensor specifications and the components that make up the sensor. Next, the governing physics and model implementation in ANSYS is described. Lastly, the experimental testing, the model calibration procedure, and the results are presented.

2.1 Sensor Description

The self-calibrating, piezoresistive sensor consists of five main components as shown in Figure 1. To ensure linearity, the current pressure range is limited to 35.5 kPa (~5 psi) and the thermopneumatic actuator is limited to a heater voltage of 5V, corresponding to a heater power of 0.54 W.

![Figure 1. Picture (left) and CAD model (right) of the self-calibrating pressure sensor showing the five main components](image)

The sensing element has a square base measuring 4 mm by 4 mm with a total height of 805 μm and is the main component of interest. A close-up, cross-sectional view of the sensing element with all main components is shown in Figure 2 below.

![Figure 2. Cross-section view of the self-calibrating pressure sensor sensing element](image)

The device layer, the buried oxide (BOX) layer, and the silicon base layer are manufactured from a silicon on insulator (SOI) wafer and the gold heater and cavity thermistor are manufactured on the substrate. By bonding the substrate to the diced SOI wafer, the sealed reference cavity is created with the enclosed heater and thermistor forming the monitorable thermopneumatic
2.2 Governing Physics

To model the thermal performance, a good understanding of the governing physics and assumptions are required. Focusing on the thermal aspects only, two modes of heat transfer were considered: convection and conduction. Previous work has illustrated that radiation heat transfer is negligible at the anticipated temperature, therefore, radiation heat transfer effects were neglected for this initial study.

Two forms of convective heat transfer can occur in the air-filled reference cavity: forced convection, where a fluidic medium is forced across a surface to increase heat transfer, and buoyancy induced convection, which relies on temperature-induced density gradients to govern fluid motion. Forced convection was assumed negligible as the volume of air displaced in the cavity (due to the displacing membrane) is less than 3\% when compared to the total volume resulting in minimal bulk fluid motion.

For buoyancy induced convection, two criteria must be satisfied. First, an unstable temperature gradient must exist in the presence of a body force (gravity). For the present work, this is true as the heater is located at the bottom of the cavity resulting in higher temperatures at the bottom as compared to the top of the cavity. In addition to having an unstable temperature gradient, buoyancy forces in the fluid must be greater than the viscous forces. A non-dimensional number used for quantifying the comparison of buoyancy forces to viscous forces is the Grashof number (Equation 2).

\[
Gr = \frac{g\beta(T_1 - T_2)L^3}{\nu^2}
\]  

Large Grashof numbers (Gr\gg 1) indicate that buoyancy forces are sufficient to overcome the viscous forces in the fluid resulting in bulk fluid motion, whereas small Gr numbers (Gr\ll 1) indicate that viscous forces dominate, therefore no bulk fluid motion occurs. Due to the \( L^3 \) term in the numerator of Equation 2, buoyancy induced convection in MEMS and NEMS devices can often be neglected \([15–17]\). Similarly, the Gr number for the present work is 0.13, indicating buoyancy induced convection in the cavity is insignificant.

With no radiation or convection, the sensor’s thermal performance is governed by heat conduction in both the solid and fluid domains. Derived from the energy equation and Fourier’s law the governing heat conduction equation is shown in Equation 3:

\[
\rho C_p \left( \frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T \right) = \nabla \cdot \left( k \nabla T \right) + q^\prime \prime \prime ,
\]

where \( \nabla \) is the gradient operator. When dealing with small length scales, it is important to check the validity of Equation 3 as the thermal conductivity \( k \) could vary from the bulk values. For the present work, it was determined that the length scales are sufficiently larger than the mean free path of the materials such that the bulk thermal conductivity values stand.

For this study, rather than modeling the joule heating effect for the heater, the heater is treated as an input heat flux, over the area on which the heater resides, eliminating the volumetric heating term in Equation 3. Furthermore, the velocity term in the governing equation is eliminated due to stationary material, and lastly, constant material properties were assumed. To reduce the number of parameters, thermal diffusivity, the ratio of thermal energy conducting through a material relative to the energy stored in a material, is utilized and defined in Equation 4.

\[
\alpha = \frac{k}{\rho C_p}
\]  

The resulting linear, first-order in time and second-order in space governing partial differential equation (PDE) is shown below in Equation 5.

\[
\frac{\partial T}{\partial t} = \alpha \nabla^2 T.
\]  

Along with an initial condition and two boundary conditions, the above equation for the three-dimensional sensor was solved using the finite-element method in ANSYS R19.2.

2.3 Model Setup

2.3.1 Geometry

For modeling purposes, the geometry of the sensor shown in Figure 1 and Figure 2 was simplified. The internal threads, chamfers, and wire pass-through holes of the housing were neglected. Due to the heater, cavity thermistor, surface thermistor, and device layer being on the order of 100 nm thin, their geometries were also neglected, rather boundary conditions and probe surfaces are used instead. Lastly, as the sensor’s geometry, materials, and loads are symmetric, two symmetry planes were used to quarter the sensor reducing computational resources and time. The geometry of the sensor as implemented in ANSYS is shown in Figure 3 below.
2.3.2 Material Properties

For all work presented, the thermal diffusivities of silicon, silicon dioxide, and air were taken as shown in Table 1. Due to limited information regarding the exact properties of the glass, stainless steel, and NanoSonic’s HybridSil®, the properties in Table 1 were initially assumed, knowing experimental data will be used to calibrate the model by varying these properties, as discussed later. See Table 4 and Table 5 in Appendix A for the extended material properties before and after the model was calibrated.

<table>
<thead>
<tr>
<th>Component</th>
<th>Material</th>
<th>Thermal Diffusivity $\cdot 10^4$ (m$^2$/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base layer</td>
<td>Silicon (Si)</td>
<td>89.21</td>
</tr>
<tr>
<td>Buried oxide (BOX) layer</td>
<td>Silicon dioxide (SiO$_2$)</td>
<td>0.83</td>
</tr>
<tr>
<td>Cavity</td>
<td>Air (400K)</td>
<td>38.27</td>
</tr>
<tr>
<td>Substrate</td>
<td>Glass</td>
<td>0.65</td>
</tr>
<tr>
<td>Housing</td>
<td>Stainless steel</td>
<td>4.05</td>
</tr>
<tr>
<td>Protective coating</td>
<td>HybridSil®</td>
<td>0.11</td>
</tr>
</tbody>
</table>

As shown in Figure 4, after the second mesh refinement, the percent change was less than 1% for all parts. A third refinement confined the results with less than 0.5% percent change, but due to the increased computational time, the mesh from the second refinement was selected. The selected mesh consisted of approximately 1.9 million nodes and 1.2 million elements. All the mesh statistics and data can be found in Appendix B.

2.3.4 Initial Condition and Boundary Conditions

The uniform initial temperature for the transient model and the environmental temperature were set to 22.7 °C, matching experimental conditions. Like the material properties, a convective boundary with a convection coefficient (h) of 10 W/m$^2$K was initially assumed for the top and outside surfaces of the sensor in contact with air, knowing experimental data would later be used to calibrate the model. Symmetry boundary conditions were assigned to each of the cut planes forming the quarter sensor. The heater was modeled as a uniform step input heat flux over the area where the heater is located. All other surfaces were assumed to be perfectly insulated. Refer to Figure 10 in Appendix C for an illustration of the boundary conditions implemented in ANSYS.

2.3.5 Solution Method and Verification

The model was solved using the Quasi-Linear Thermal Transient Solution method in ANSYS. Automatic time stepping was enabled to ensure sufficient time resolution over the whole simulated time while minimizing computational resources. A Distributed Sparse Direct equation solver was used to solve the system of linear equations on a 14 core, Intel Core i9-7940X 3.1GHz – 4.3GHz workstation with 128 GB of RAM.

To verify the model setup and solution method, an analytical analysis was conducted and compared to the numerical results. The long cylindrical shape of the sensor housing, with a low Biot number in the radial direction, enables the use of the transient fin equation [18] to verify the numerical solution. The numerical model and fin equation results agree well, verifying the model setup and solution method. Further details regarding model verification can be found in Appendix D.

2.4 Model Calibration

The model uncertainty from the assumed material properties and convection boundary condition was reduced using experimental testing data to calibrate the model. The testing, model calibration procedure, and results are described in the following sections.

2.4.1 Test Setup and Procedure

The prototype self-calibrating sensor was mounted on a test stand using a three-prong extension clamp. A FLIR A325sc Infrared (IR) camera was used to collect temperature data on the outer housing surface, painted with ultra-flat black paint (emissivity of 0.97 at wavelengths of 5 μm) to ensure accurate surface temperature measurements. The heater voltage, thermistors, and pressure signals were recorded at a sampling rate of 25 kHz using a PicoScope 4824 oscilloscope. The heater...
voltage was also captured on a NI USB-6210 Data Acquisition (DAQ) System and was used to temporally sync the IR data with the oscilloscope data.

The testing procedure started by setting a trigger on the oscilloscope and starting the DAQ and IR camera recordings. When the heater voltage was turned on, the oscilloscope triggered recording. After approximately 15 minutes, the heater was turned off and the sensor was left to cool down before all recording was stopped. Starting at a heater voltage of zero, as a baseline, the voltage was increased by 1V increments up to 5V, and then back down to 1V to ensure repeatability. Later, the data was post-processed in MATLAB. Previously obtained, NIST traceable calibration curves were used to convert voltages into pressure, temperature, and heater power signals [10]. Following the voltage to signal conversion, a moving mean filter was employed to eliminate high-frequency EMI, as well as other potential white noise sources.

2.4.2 Calibration Procedure and Results

The Response Surface Optimization feature in ANSYS, in conjunction with the collected testing data, was utilized to calibrate the model. The model was first parameterized with the inputs being the thermal conductivities of three materials with uncertain properties, stainless steel, glass, and HybridSil®, as well as the convection boundary condition. These input parameters were assigned a lower and upper bound based on reasonable expected values, as shown in Table 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{\text{glass}}$ (W/ m·°C)</td>
<td>0.8</td>
<td>1.5</td>
</tr>
<tr>
<td>$k_{\text{stainless}}$ (W/ m·°C)</td>
<td>12.5</td>
<td>17.5</td>
</tr>
<tr>
<td>$k_{\text{HybridSil}}$ (W/ m·°C)</td>
<td>0.1</td>
<td>0.5</td>
</tr>
<tr>
<td>$h$ (W/ m²·°C)</td>
<td>1</td>
<td>200</td>
</tr>
</tbody>
</table>

For the outputs, various discrete locations and times were selected such that a direct comparison with the test data were possible. Using a Central Composite Design of Experiment, the parameterized model was solved 25 times, effectively covering the design space for each of the four input variables. Next, the relationship between the input variables and the output responses were quantified using full second-order polynomial response surfaces. The lowest coefficient of determination is 0.9997, indicative of a good fit of the response surfaces to the 25 generated design points.

![Figure 5](image)
Knowing the input-response relationships from the model, the Multi-Objective Generic Algorithm (MOGA) was used to solve for the three unknown thermal conductivities and the convection boundary condition such that the response from the model matched the testing data, within reasonable uncertainty.

The model was calibrated with the testing data for the highest heater power (5V) and a detailed comparison was done to ensure the model is representative of the actual sensor. Figure 5 shows the comparison between the model and the test data, where \( \theta \) is defined as the temporally local temperature minus the initial temperature.

The housing temperature contour plots in Figure 5a show that the model slightly underpredicts the temperature after 60 seconds, however, the model prediction is well within the uncertainty range of the IR camera. (\( \pm 2^\circ \text{C} \)). Furthermore, the main interest is in the sensing element so the slight difference in the housing temperatures is of little consequence.

The calibrated model’s prediction of the surface and cavity thermistor temperatures (Figure 5b) agrees well with the data in both trend and magnitude with the largest difference of approximately 3°C for the cavity thermistor. Lastly, the model and data were compared over the range of heater powers (Figure 5c) to ensure the model is valid for all heater powers, not just the 5V case used for calibration. Comparing the slopes of the best fit lines, there is only an 8.6 °C/W and 10.4 °C/W difference for the cavity and surface thermistors respectively, which is acceptable for the current purpose.

The differences between the model and the data are possibly due to experimental uncertainty, imprecise selection of material properties, and the convection coefficient, during the calibration procedure, or the lack of a precise model for the thermistors. The calibrated model was taken to be representative of the actual sensor’s thermal performance and is now used for further development and investigation.

3. RESULTS AND DISCUSSION

After the FE model was developed and calibrated via experimental testing data, the model was utilized to provide insight and understanding regarding the sensor’s temperatures, temperature distribution, potential sources of error during self-calibration, response time, and the efficiency of the thermopneumatic actuator. Two design studies were then conducted seeking to increase the efficiency of the sensor and to decrease the thermal response time of the actuator. The results and findings are described in the following section.

3.1 Baseline Sensor

The calibrated model was used to analyze and understand the thermal performance of the sensor, not observable during experimental testing. In particular, the temperature distribution in the sensing element and its effects were investigated. The temperature contour plot at the maximum heater voltage (5V) is shown in Figure 6. Interested in the fastest possible response time (< 30 seconds), the analysis hereafter was limited to the first 30 seconds.

Figure 6. Temperature (°C) contour plot for the maximum heater voltage of 5 V

A large temperature gradient is observed in the reference cavity with a maximum temperature of 199.2 °C at the bottom of the cavity where the heater is located, and a minimum temperature of 29.4 °C. The temperature gradient is physical, as no convection (forced or buoyancy induced) occurs in the cavity as discussed previously. However, this temperature gradient is significant as the current self-calibration procedure assumes a uniform cavity temperature as measured by the cavity thermistor located at the bottom of the cavity, resulting in potential self-calibration error.

By measuring the temperature at the bottom of the cavity in the presence of a temperature gradient, the thermistor will have an error associated with its measurement. To estimate this error, it was assumed that the actual reference cavity temperature is the volume average temperature in the cavity. The percent error between the cavity thermistor and the volume average temperature was calculated as a function time as shown in Figure 7.

Figure 7. Estimated error between the average cavity temperature and the thermistor measured temperature

6
After about 10 seconds, the estimated error between the average cavity temperature and the thermistor measurement is approximately constant with a magnitude of 10%. This estimated error for the model is consistent with the approximate 6% error from previous experimental data [10]. The cavity thermistor over predicts the cavity temperature and is attributed to the thermistor’s proximity to the heater. To get a more precise cavity temperature measurement, the location of the cavity thermistor should be moved away from the heater, and ideally surrounded by the cavity gas only (suspended). Future work will address this, but given the error is approximately constant after 10 seconds, it can be accounted for in the self-calibration algorithms.

A pressure sensor that can diagnose and correct measurement errors in real-time is valuable for time-sensitive applications such as flight. For this reason, the transient response of the self-calibration sensor is analyzed to minimize the time required to self-calibrate. The thermal time constant (τ), defined as the time required to change 63.2% from the initial to the final temperature, is used to determine how fast the actuator can respond thermally, and thus how close to real-time self-calibration can be achieved.

The temperature associated with 63.2% of change from the initial temperature to the temperature at 30 seconds was calculated and linear interpolation was used to estimate τ between the discrete data points. The thermal time constant for the average cavity temperature and the cavity thermistor is 0.209 seconds and 0.264 seconds, respectively. The slight delay in the cavity thermistor response is attributed to the influence of the glass substrate with a much lower thermal diffusivity, as compared to that of air. The calculated thermal time constant for the baseline sensor is later used to evaluate various design options to minimize the thermal response time for the sensor.

Lastly, previous testing data indicated that more than 90% of heat is lost, rather than used to heat the cavity. This heat loss reduces the energy efficiency of the sensor, critical for remote sensing applications. The developed model confirmed this inefficiency with the vertical direction heat flux contour plot shown in Figure 8.

Using point heat flux values from the contour plot, the model shows that 95.4% of heat is lost through the substrate. The heat loss is attributed to the low unit area thermal resistance of the substrate (4.48 x 10^4 m² °C/W), compared to that of air in the cavity (8.58 x 10^3 m² °C/W). The substrate is 94.3% less resistant to heat flow. This heat loss is significant and can be addressed by increasing the unit area thermal resistance of the substrate as presented in the next section.

### 3.2 Substrate Material Selection

The observed heat loss was addressed by utilizing the model to determine potential design configurations resulting in higher efficiency. As previously mentioned, to increase the efficiency the substrate’s thermal conductivity must decrease, increasing the unit area thermal resistance. Three alternative substrate materials were selected focusing on materials with decreasing thermal conductivity: Polyimide, a close-cell foam, and air.

Polyimide was selected for its good thermal and mechanical stability at high temperatures, its established use in electronics, and its proven manufacturing processes and techniques. Close-cell foam was selected for its high thermally insulating properties while providing a structure on which the heater can be manufactured. Lastly, the properties of air were used even though it is not possible to have an air substrate. This analysis was done as a first approximation of a suspended heater, where air surrounds the micro-heater as done in MEMS gas sensors [19,20]. The suspended heater should provide higher efficiencies as most of the heat will be transferred into the reference cavity gas, rather than being conducted away through the surrounding structures. The selected material and their properties are listed in Table 3.

To analyze the effect of the substrate material on the actuator’s efficiency, the average cavity temperature results were plotted over the full range of heater powers and the data were fitted with linear trend lines. Comparing the slopes (a) of the trend lines, it is observed that for decreasing thermal conductivity, higher cavity temperatures are achieved per watt of heater power. By changing from the current glass substrate to a polyimide, foam, or air (suspended heater) substrate, the power consumption can be decreased by 81.5%, 85.2%, or 92% respectively. The authors attribute this to the increasing unit area thermal resistance of the substrate, redirecting heat up into the cavity. The results are summarized in Table 3.

Interested in the fastest possible self-calibration time, the effect of changing the substrate material on the transient response was also considered. By calculating the thermal time constant (τ) from the temporal temperature data, the transient response is evaluated. The results show that a lower thermal conductivity, required for less heat loss, does not always result in a faster response. Rather, a substrate material with high thermal diffusivity (α) exhibits a faster response time. Thus, according to Equation 4, for a material with low thermal conductivity to have a high thermal diffusivity, the density and specific heat needs to be small.
The results from the substrate material study are summarized in Table 3 and show that optimal efficiency and fast transient response can be achieved by utilizing a substrate with low thermal conductivity and high thermal diffusivity.

### 3.3 Cavity Size

A smaller cavity is of interest, as the amount of air to be heated is decreased, potentially increasing efficiency, and reducing the actuator’s response time. Secondly, the size reduction is also beneficial for applications like aerodynamic measurements, where large sensors can affect the flow field. Reducing the cavity size, can also increase the sensor’s fundamental frequency, increasing the sensor’s frequency response. Using the previously calibrated numerical model, a parametric design study was conducted to predict the effects of altering cavity dimensions on self-calibration.

The heater power was held constant at 0.5W while varying both the square cavity base dimension (L) and the cavity height (H). Starting from the current sensor configuration, L and H were incrementally reduced independently and simultaneously. Like the substrate material study, the effects of the cavity size on the efficiency and response time were evaluated. The cavity base and height dimensions were normalized using the dimensions of the baseline sensor. The resulting average cavity temperature (Figure 9a) and the thermal time constant (Figure 9b) results were plotted against a ratio of the normalized height to the normalized base dimensions. The colored lines in the vertical direction indicate a constant cavity height (H) and the black lines in the horizontal direction indicate a constant cavity base dimension (L).

The results generally indicate that cavities with smaller volumes (smallest volume investigated marked by a dashed circle in Figure 9) achieve higher cavity temperatures (higher heating efficiencies) and faster response times as compared to larger cavities (largest cavity volume investigated marked by a dashed square in Figure 9). The trends in Figure 9 also show the sensitivity for changing the cavity base versus changing the cavity height. Starting from the current sensor’s cavity size, indicated by the dashed squares, the average cavity temperature and the thermal time constant are substantially more sensitive to a decrease in the base dimension as compared to the height, as evidenced by the substantial increase in cavity temperature and decrease in thermal time constant following the magenta line and lack thereof following the black horizontal line. In fact, a 50% decrease in the base dimension is predicted to result in a 45% increase in temperature and a 59% decrease in the thermal time constant whereas an 83% decrease in cavity height leads to a marginal predicted temperature increase of only 8% and a 10% decrease of the thermal time constant. The higher sensitivity on the cavity base dimension is attributed to the L² term when calculating the volume average temperature, on which the thermal time constant is also dependent. This parabolic nature of the constant-height (altering base dimension) lines indicates an optimal point at which a maximum temperature and a minimal thermal time constant will be achieved for a given cavity height and heater power. Decreasing the base cavity dimension any further will have minimal to negative effects on the heating efficiency and response times. With progression toward smaller cavity base dimensions (L), increasing temperature effect and decreasing thermal time constants is attributed to decreasing the cavity height, illustrated by the progressively larger magnitude slopes of the horizontal black lines. This can be attributed to the cavity height becoming a larger factor as the base dimension moves toward zero.

The results from this cavity size study confirm that a sensor with a smaller cavity will be more efficient and actuate faster. Moreover, the study shows when decreasing the cavity size, efforts should be focused on decreasing the cavity base dimension first.

### 4. CONCLUSION AND FUTURE WORK

The potential uses and benefits of a thermopneumatic actuated self-calibrating pressure sensor are significant. However, predictive and consistent thermal performance is required for accurate self-calibration. The present work used a calibrated finite element model to increase understanding of the thermal performance of the sensor. The model shows that a temperature gradient in the reference cavity resulted in an error estimated to be 10%, as a point measurement is taken as the bulk cavity temperature. The model also confirmed that majority of the heat is lost, as approximately 95.4% of the heat transfers through the substrate due to its low thermal resistance when compared to air. Using the model as a design tool, two potential design improvements, to increase efficiency and minimize the thermal response time, were thoroughly analyzed. The substrate material design study showed that optimal efficiency, high temperature, and a fast response can be achieved with a substrate with low thermal conductivity and high thermal diffusivity. Lastly, it was found that the cavity size should be decreased to increase cavity temperature and decrease response time, by first focusing on

### Table 3. Material properties and results from the substrate material design study

<table>
<thead>
<tr>
<th>Material</th>
<th>k (W/m·°C)</th>
<th>a·10⁶ (m/s)</th>
<th>a (°C/W)</th>
<th>Power Consumption Decrease (%)</th>
<th>τ (sec.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glass (baseline)</td>
<td>1.034</td>
<td>0.559</td>
<td>63.38</td>
<td>N/A</td>
<td>0.209</td>
</tr>
<tr>
<td>Polyimide</td>
<td>0.12</td>
<td>0.078</td>
<td>341.93</td>
<td>81.46</td>
<td>0.706</td>
</tr>
<tr>
<td>Foam</td>
<td>0.085</td>
<td>1.504</td>
<td>428.89</td>
<td>85.22</td>
<td>0.034</td>
</tr>
<tr>
<td>Air</td>
<td>0.034</td>
<td>38.265</td>
<td>792.32</td>
<td>92.00</td>
<td>0.002</td>
</tr>
</tbody>
</table>
decreasing the cavity base dimension. A third design study was conducted to study the effects of the reference cavity gas on the sensor’s responses, however the study revealed insignificant changes. The reference cavity gas study can be found in Appendix E.

The presented work increased understanding and provided insight into potential design improvements for the sensor by Kang et al., moving one step closer to a commercially available self-calibrating pressure sensor. Future work on the presented model can include material property refinement and increasing the model’s complexity by modeling the joule heater, thermistors, and the effects of non-perfect thermal contact between parts. Lastly, a multi-physics (thermal, fluid, and structural) model can be developed to accurately predict cavity pressures and account for effects such as thermal induced stress and the fluid and structural damping effects on the transient response of the sensor.

ACKNOWLEDGEMENTS

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REFERENCES


ATTRIBUTIONS

Several individuals contributed to the presented work. Ridge Sibold contributed to all aspects of this project, especially in the experimental testing, data processing, and editing aspects. Dr. Kang and Dr. Ruan from NanoSonic Inc. were responsible for the self-calibrating sensor design and manufacturing and providing funding for the project. Scott Mouring assisted with experimental testing and with data processing. Dr. Ng was involved in all aspects of the project and provided guidance and insights, making this research project possible.
# APPENDIX A: ADDITIONAL MATERIAL PROPERTY DATA

## Table 4. Initially assumed material properties

<table>
<thead>
<tr>
<th>Material</th>
<th>ρ (kg/m³)</th>
<th>k (W/m·°C)</th>
<th>C_p (J/kg·°C)</th>
<th>α·10⁶ (m²/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silicon (Si)</td>
<td>2330</td>
<td>148</td>
<td>712</td>
<td>89.213</td>
</tr>
<tr>
<td>Silicon dioxide (SiO₂)</td>
<td>2220</td>
<td>1.38</td>
<td>745</td>
<td>0.834</td>
</tr>
<tr>
<td>Air (400K)</td>
<td>0.8711</td>
<td>0.0338</td>
<td>1014</td>
<td>38.266</td>
</tr>
<tr>
<td>Glass</td>
<td>2230</td>
<td>1.2</td>
<td>830</td>
<td>0.648</td>
</tr>
<tr>
<td>Stainless steel</td>
<td>8000</td>
<td>16.2</td>
<td>500</td>
<td>4.050</td>
</tr>
<tr>
<td>HybridSil®</td>
<td>970</td>
<td>0.15</td>
<td>1460</td>
<td>0.106</td>
</tr>
</tbody>
</table>

## Table 5. Material properties used after model calibration

<table>
<thead>
<tr>
<th>Material</th>
<th>ρ (kg/m³)</th>
<th>k (W/m·°C)</th>
<th>C_p (J/kg·°C)</th>
<th>α·10⁶ (m²/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silicon (Si)</td>
<td>2330</td>
<td>148</td>
<td>712</td>
<td>89.213</td>
</tr>
<tr>
<td>Silicon dioxide (SiO₂)</td>
<td>2220</td>
<td>1.38</td>
<td>745</td>
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</tr>
<tr>
<td>Air (400K)</td>
<td>0.8711</td>
<td>0.0338</td>
<td>1014</td>
<td>38.266</td>
</tr>
<tr>
<td>Glass</td>
<td>2230</td>
<td>1.2</td>
<td>830</td>
<td>0.648</td>
</tr>
<tr>
<td>Stainless steel</td>
<td>8000</td>
<td>12.539</td>
<td>500</td>
<td>3.135</td>
</tr>
<tr>
<td>HybridSil®</td>
<td>970</td>
<td>0.30255</td>
<td>1460</td>
<td>0.214</td>
</tr>
</tbody>
</table>
### APPENDIX B: GRID INDEPENDENCE STUDY DATA

**Table 6. Maximum allowable element size**

<table>
<thead>
<tr>
<th>Mesh</th>
<th>Housing (mm)</th>
<th>HybridSil (mm)</th>
<th>Substrate (mm)</th>
<th>Silicon Base (mm)</th>
<th>Membrane (mm)</th>
<th>Cavity (mm)</th>
<th>Silicon Dioxide (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.000</td>
<td>2.000</td>
<td>0.500</td>
<td>0.200</td>
<td>0.020</td>
<td>2.100</td>
<td>0.040</td>
</tr>
<tr>
<td>2</td>
<td>1.000</td>
<td>1.000</td>
<td>0.250</td>
<td>0.100</td>
<td>0.010</td>
<td>0.100</td>
<td>0.020</td>
</tr>
<tr>
<td>3</td>
<td>0.500</td>
<td>0.500</td>
<td>0.125</td>
<td>0.050</td>
<td>0.005</td>
<td>0.050</td>
<td>0.010</td>
</tr>
<tr>
<td>4</td>
<td>0.250</td>
<td>0.250</td>
<td>0.063</td>
<td>0.025</td>
<td>0.003</td>
<td>0.025</td>
<td>0.005</td>
</tr>
</tbody>
</table>

**Table 7. Statistics for each of the four meshes**

<table>
<thead>
<tr>
<th>Mesh</th>
<th>Total Nodes (#)</th>
<th>Total Elements (#)</th>
<th>Solution Time (sec.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>70874</td>
<td>35091</td>
<td>30</td>
</tr>
<tr>
<td>2</td>
<td>336094</td>
<td>181604</td>
<td>108</td>
</tr>
<tr>
<td>3</td>
<td>1906475</td>
<td>1154428</td>
<td>604</td>
</tr>
<tr>
<td>4</td>
<td>12647921</td>
<td>8367678</td>
<td>12356</td>
</tr>
</tbody>
</table>

**Table 8. Grid independent study results A**

<table>
<thead>
<tr>
<th>Mesh</th>
<th>Housing</th>
<th>Substrate</th>
<th>Silicon Base</th>
<th>Cavity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Maximum (°C)</td>
<td>Percent Change (%)</td>
<td>Maximum (°C)</td>
<td>Percent Change (%)</td>
</tr>
<tr>
<td>1</td>
<td>35.042</td>
<td>N/A</td>
<td>248.790</td>
<td>N/A</td>
</tr>
<tr>
<td>3</td>
<td>38.386</td>
<td>0.094</td>
<td>212.149</td>
<td>0.126</td>
</tr>
<tr>
<td>4</td>
<td>38.385</td>
<td>0.003</td>
<td>212.348</td>
<td>0.097</td>
</tr>
</tbody>
</table>

**Table 9. Grid independence study results B**

<table>
<thead>
<tr>
<th>Mesh</th>
<th>Membrane</th>
<th>Silicon Dioxide</th>
<th>HybridSil</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Maximum (°C)</td>
<td>Percent Change (%)</td>
<td>Maximum (°C)</td>
</tr>
<tr>
<td>1</td>
<td>34.648</td>
<td>N/A</td>
<td>34.706</td>
</tr>
<tr>
<td>2</td>
<td>33.291</td>
<td>3.919</td>
<td>33.452</td>
</tr>
<tr>
<td>3</td>
<td>33.274</td>
<td>0.050</td>
<td>33.290</td>
</tr>
<tr>
<td>4</td>
<td>33.350</td>
<td>0.228</td>
<td>33.349</td>
</tr>
</tbody>
</table>
APPENDIX C: BOUNDARY CONDITIONS FIGURE

Figure 10. Boundary conditions as implemented in ANSYS

- Yellow: Convection coefficient ($h$) (Assumed 10 W/m$^2$ °C)
- Red: Symmetry
- Blue: Step input heat flux
- Light blue: All other boundary surfaces were taken as insulated
APPENDIX D: MODEL VERIFICATION

To ensure the numerical model setup and solution method works as expected, a semi-analytical analysis was done and the results were compared to that of the numerical model. Given the long, cylindrical shape of the self-calibrating pressure sensor, a transient fin analysis seemed most appropriate for estimating the numerical solution. However, to do this analysis two simplifying assumptions were made.

First, it was assumed that the geometry of the sensor is a solid, constant radius cylinder with constant and uniform material properties. Figure 11a illustrates this assumption. Secondly, to perform the fin analysis, the boundary conditions had to be simplified. It was assumed that a uniform, constant heat flux is applied over the base of the fin with a total power of 0.5 W. For the numerical model, however, the total heater power (0.5W) is imparted to the sensor only over the square micro-heater’s location in the center of the sensor measuring 1 mm by 1 mm. The fin model used for the analysis is shown in Figure 11b with the boundary conditions.

To ensure the validity of the fin analysis, the Biot number (a nondimensional ratio of conductive heat transfer in a body to the convective heat transfer out of a body) in the radial direction was calculated to be 0.0045. The calculated Biot number is much less than unity, indicating the radial heat transfer occurs much faster than the convective heat transfer at the surface, making the fin analysis appropriate. The exact nondimensional governing equation and variables as presented by Cole et al. is used as shown below:

\[
\theta(\xi, \tau) = \left[1 - e^{-M^2\tau}\right] + 2 \sum_{n=1}^{\infty} \frac{1 - \exp \left[-(M^2 + n^2\pi^2)\tau\right]}{M^2 + n^2\pi^2} \cos(n\pi\xi) \\
\theta = \frac{T - T_e}{q_o L/k}; \quad \xi = \frac{x}{L}; \quad \tau = \frac{\alpha t}{L^2} \\
M = \sqrt{Bi} \left(\frac{L}{V/A_h}\right); \quad Bi = \frac{h(V/A_h)}{k}
\]

Where \(\theta\) is the nondimensional temperature, \(\xi\) and \(\tau\) are the nondimensional length and time, \(M\) is the fin parameter and \(Bi\) is the Biot number. All the variables used for the fin analysis were extracted from the numerical model in ANSYS and are listed in Table 10.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Thermal conductivity (W/m°C)</th>
<th>(\alpha) (10^6) (m²/s)</th>
<th>Length (m)</th>
<th>Radius (m)</th>
<th>Cross-section area (m²)</th>
<th>Convective surface area (m²)</th>
<th>Total volume (m³)</th>
<th>Total heater power (W)</th>
<th>Heat flux (W/m²)</th>
<th>Environmental Temperature (°C)</th>
<th>Convection coefficient (W/m²°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbol</td>
<td>(k)</td>
<td>(\alpha)</td>
<td>(L)</td>
<td>(r)</td>
<td>(A_c)</td>
<td>(A_h)</td>
<td>(V)</td>
<td>(q)</td>
<td>(q_o)</td>
<td>(T_e)</td>
<td>(h)</td>
</tr>
<tr>
<td>Value</td>
<td>16.2</td>
<td>3.135</td>
<td>4.45E-02</td>
<td>7.62E-03</td>
<td>1.82E-04</td>
<td>6.46E-04</td>
<td>1.52E-06</td>
<td>0.5</td>
<td>2741.007</td>
<td>22.765</td>
<td>10</td>
</tr>
</tbody>
</table>

Along with the initial condition and two boundary conditions, the governing equation was solved in MATLAB for a 120 second period with a summation index (n) of 10 million.

![Figure 11. Geometry simplification (a) and fin model with boundary conditions (b)](image-url)
The fin and numerical results were compared for four cases as shown in Figure 12. First, two locations on the sensor housing were selected to compare as a function of time. The first location corresponds to the location where the heat flux is applied (Figure 12a) and the second location is ~0.5 inches for the heat source in the axial direction (Figure 12b). For both cases, the magnitude and trends match well. However, Figure 12a shows an approximate 1°C difference between the two temperature solutions at the heat source. This difference is attributed to the assumptions (simplified geometry, uniform material properties, and location of heat flux boundary condition) that were made in developing the fin model. The assumptions are less accurate, the closer one is to the heat source. As we move away from the heat source, the assumptions are more valid, resulting in better agreement between the numerical model and the fin analysis as shown in Figure 12b.

The second set of cases is the temperature in the axial direction of the sensor at two times: 30 seconds (Figure 12c) and 120 seconds (Figure 12d). Like the previous two cases, the trend and magnitude match well, but the assumptions result in a larger difference between the two solutions near the heat source (x=0).

![Figure 12. Comparison of the numerical and fin solutions for (a) the temperature at the heat source over a 120 seconds period, (b) the temperature ~0.5 inches from the top of the sensor housing over the 120 seconds period, (c) the temperature along the axial direction after 30 second, and (d) the temperature along the axial direction after 120 seconds.](image)

The comparison between the numerical results from ANSYS and the fin analysis shows good agreement. The differences between the results near the heat source (x = 0) are attributed to the simplifying assumptions that had to be made to do the fin analysis. This analysis verified the numerical model setup and solution method, providing confidence to proceed to model calibration.
APPENDIX E: REFERENCE CAVITY GAS STUDY

A third design study was conducted, evaluating the effects of the reference cavity gas on the thermal performance of the self-calibrating sensor. A parametric study was used in which all variables were held constant except for the reference cavity gas properties. The selected gases are shown in Table 11, along with their material properties. The gases were limited to inert gases to avoid any reactions in the reference cavity when the heater is switched on.

The average cavity temperatures are plotted as a function of thermal diffusivity as shown in Figure 13 below.

![Figure 13. Average cavity temperature for different reference cavity gas](image)

Noting the y-axis scale, only a ~2°C range is observed for all the seven selected gases. Furthermore, looking to increase the temperature in the reference cavity, the largest increase in temperature is only 0.66%, when switching from the current sensor gas (air) to Xenon. The thermal response time also is insensitive to the changing gas in the cavity, with the largest decrease only equating to 0.23%. All the results are listed in Table 11.

<table>
<thead>
<tr>
<th>Gas</th>
<th>ρ (kg/m³)</th>
<th>k (W/m·°C)</th>
<th>C_p (J/kg·°C)</th>
<th>μ·10^6 (m²/s)</th>
<th>Cavity Temperature (°C)</th>
<th>Percent Change (%)</th>
<th>τ (sec.)</th>
<th>Percent Change (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>0.865</td>
<td>0.034</td>
<td>1016</td>
<td>38.687</td>
<td>56.58</td>
<td>N/A</td>
<td>0.19949</td>
<td>N/A</td>
</tr>
<tr>
<td>Helium</td>
<td>0.119</td>
<td>0.192</td>
<td>5193</td>
<td>310.459</td>
<td>54.90</td>
<td>-2.96</td>
<td>0.20547</td>
<td>3.00</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>0.829</td>
<td>0.033</td>
<td>1040</td>
<td>38.670</td>
<td>56.59</td>
<td>0.02</td>
<td>0.19946</td>
<td>-0.02</td>
</tr>
<tr>
<td>Argon</td>
<td>1.177</td>
<td>0.023</td>
<td>520</td>
<td>37.579</td>
<td>56.73</td>
<td>0.27</td>
<td>0.19903</td>
<td>-0.23</td>
</tr>
<tr>
<td>Neon</td>
<td>0.603</td>
<td>0.061</td>
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<td>97.409</td>
<td>56.24</td>
<td>-0.61</td>
<td>0.20041</td>
<td>0.46</td>
</tr>
<tr>
<td>Carbon Dioxide</td>
<td>1.300</td>
<td>0.025</td>
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<td>0.21</td>
<td>0.19930</td>
<td>-0.10</td>
</tr>
<tr>
<td>Xenon</td>
<td>3.920</td>
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<td>159</td>
<td>12.033</td>
<td>56.96</td>
<td>0.67</td>
<td>0.19928</td>
<td>-0.11</td>
</tr>
</tbody>
</table>

Changing the reference cavity gas from air to one of the selected inert gases has minimal improvements regarding both increasing temperature and decreasing the thermal response time. The insignificant benefits are outweighed by the increased complexity and costs that would be required during sensor manufacturing.