

Chapter 2. Theoretical Principles

This chapter presents the three major concepts which underline this thesis. The first part of this chapter concerns the vibration absorber and the logical process leading to the creation of an active-passive distributed absorber. The second part introduces the variational method that has been used to model this new type of absorber. The configuration of the absorber is then optimized using a “genetic algorithm”. The last part is a brief introduction to the genetic algorithm and the way this type of algorithm was implemented.

2.1 Vibration absorbers

2.1.1 *The one degree of freedom undamped vibration absorber*

In many cases, a structure having a mechanical resonance can be approximated as a single degree of freedom system around its resonance frequency $f_r = 2\pi\sqrt{\frac{K_l}{M_l}}$. Engine vibrations and rotating machinery are often responsible for periodic excitations. If the excitation frequency matches any resonance in the structure, extreme displacements may damage the system. If the structure can not be changed, a secondary system called "vibration absorber" may be added to modify the dynamics of the main structure (cf. Figure 2.2). The absorber can be tuned so that the main structure has no motion for the excitation frequency. In the example of Figure 2.2, the main structure and the absorber have the same resonance frequency. With an absorber weighting 10% of the total mass,

the response of the main structure is zero at the former resonance frequency. For this particular frequency, the response of M_2 is finite. This property can be used to stop the vibration of a main system of mass M_1 by using an absorber of small mass M_2 .

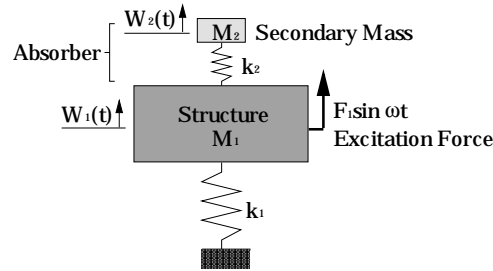


Figure 2.1: Undamped vibration absorber on a single degree of freedom structure

This basic example presents the interest in implementing absorbers. With complex systems with multiple degrees of freedom, similar properties can be observed.

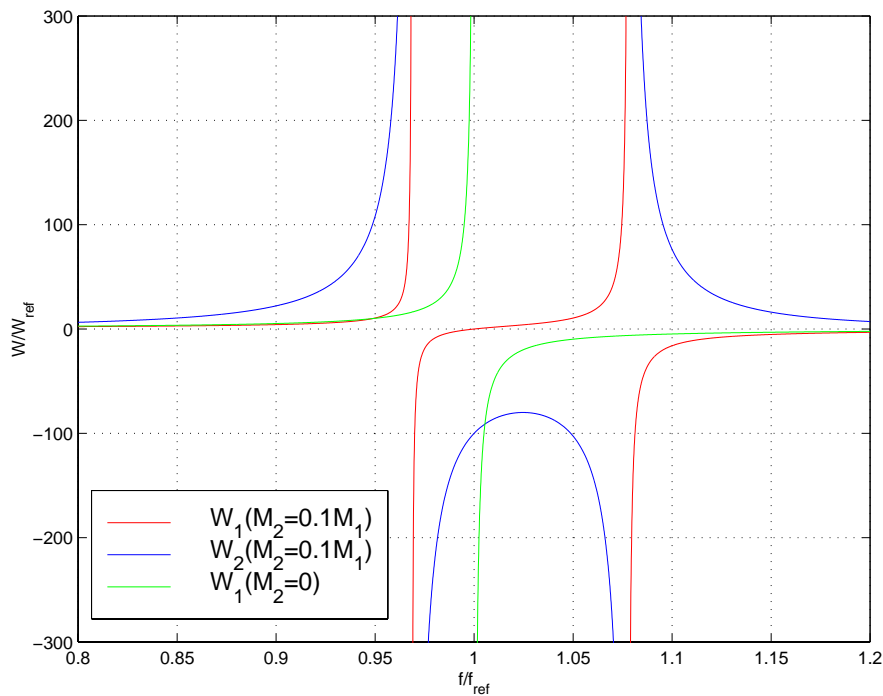


Figure 2.2: Response of system with/without tuned absorber

Figure 2.2 presents the transversal displacement of M_1 and M_2 in term of frequency. The resonance frequency of the main structure is taken as a reference. The green line presents this resonance and the 180° phase shift between the excitation and the motion of M_1 which occurs at $f/f_{\text{ref}} = 1$. No damping is included in the model so all the motions are in or out of phase with each other. The red line presents the motion of M_1 with the absorber on top of it. The weight of the absorber is 10% of the main structure. By adding a vibration absorber, the system has now two degrees of freedom. The response of M_1 presents two resonances at $f/f_{\text{ref}} = 0.93$ and $f/f_{\text{ref}} = 1.08$. At $f/f_{\text{ref}} = 1$, the mass M_1 is not moving, the absorber was properly designed to suppress the vibrations at this frequency. The motion of the secondary mass M_2 is described with the blue line. The two same resonances are observed but the motion of M_2 is never zero. In real applications, damping plays also an important role since the motion of the main structure will not be driven exactly to zero. Phase shifts between the excitation and the motion of the masses will reduce the “efficiency” of the absorber.

2.1.2 Active-passive absorber

An absorber consisting of a mass and a spring added to a main structure is a passive system. If the motion of the mass can be induced electrically as shown in Figure 2.3, the absorber has also an active component.

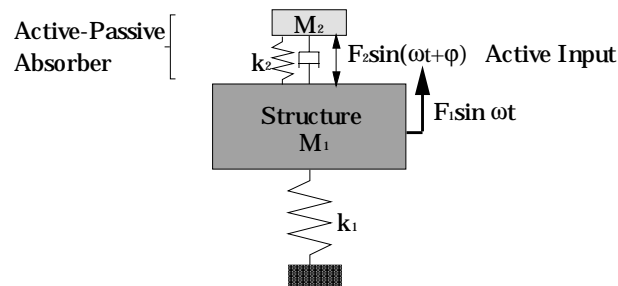


Figure 2.3: Active-passive absorber

Since such device can induce motion of the main structure, they can also be called “inertial actuators”. The denomination “active-passive absorber” emphasizes that a mechanical resonance modifies the dynamics of the main system and that the frequency of the active input is close to this resonance. The dynamic properties of the absorber is changed by the active input, therefore the whole system is affected by the active input on the absorber. *N. W. Hagood* [1] showed that electrical shunts could be used. This has been experimentally validated on the inertial actuator pictured in Figure 2.4. [2].

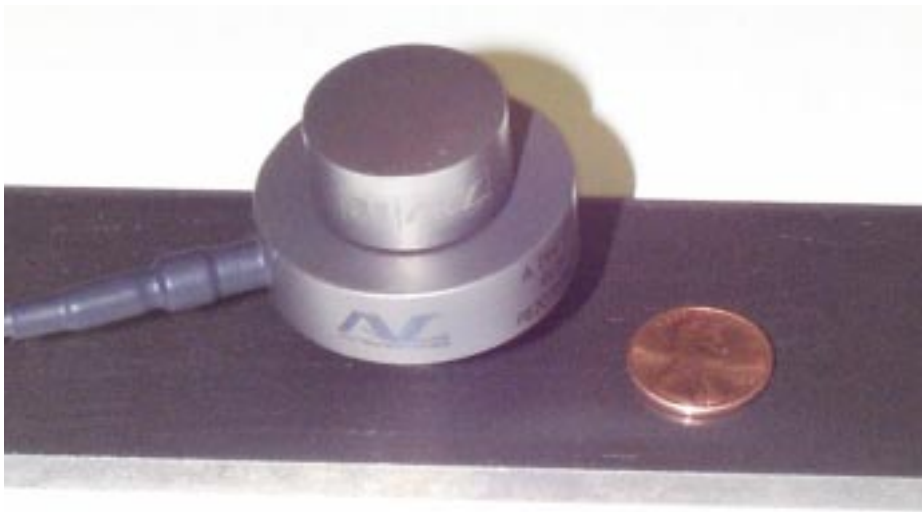


Figure 2.4: The inertial actuator from AVC can be used as an active-passive absorber

Several type of active-passive absorbers can be found on the market. The one used in the research of this thesis is the inertial actuator made by AVC (Subdivision of the PCB company) and is shown Figure 2.4. The resonance frequency depend on the added mass and is typically around 1000Hz. Details of this inertial actuator used as a point absorber are presented on figure 2.5. The piezoelectric discs act as elastic springs mounted in series. The piezoelectric excitation of these discs has the same effect as two point forces F_1 and F_2 . The electric excitation of the two piezoelectric discs can induce a vertical motion of

the steel mass. The technology and the precision with which these actuators are built makes them relatively costly.

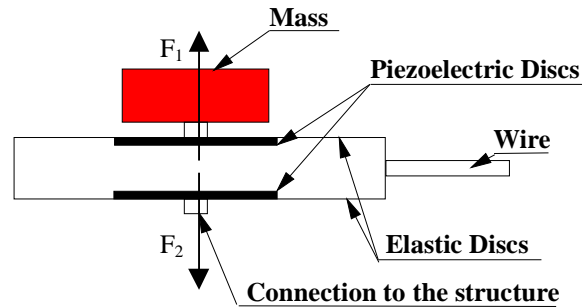


Figure 2.5: Details for the PCB “inertial actuator”

The overall size of these actuators is approximately 1" in height and 1.5" to 2" in circumference. Different mass can fit on the absorber to alter the natural resonance frequency; they typically weigh 50, 100, 150 or 200g.

2.1.3 Absorbers on continuous structures

A beam, which is one of the simplest continuous structures, has been used in this study. Absorbers are traditionally placed on the antinodes of the vibration response of the structure [3] because their modal contribution is the most important at these points. Figure 2.6 presents two possible configurations following this rule for two different modes of response.

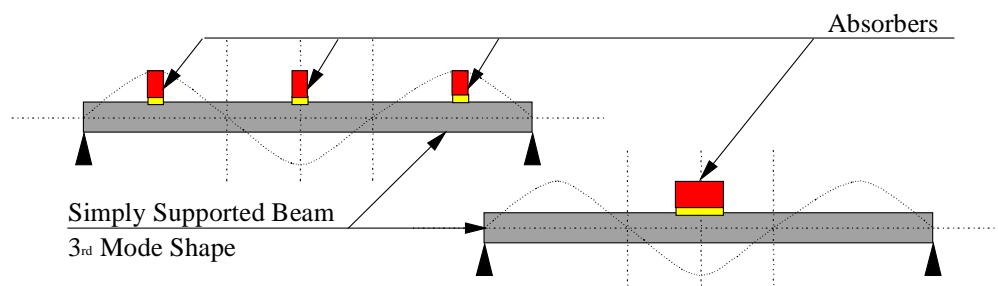


Figure 2.6: Three vs. one absorber on a simply supported beam

In order to control the third mode, three absorbers are paced on the three antinodes. the second solution is to put all the mass on the central antinode by putting the three absorbers in the middle of the beam. In this case, they act as a single absorber. The weight allowed for the absorbers is globally 10% of the beam mass.

A simulation using the model developed in Chapter 3 and the parameters of Table 2.I illustrates the efficiency of a dynamic vibration absorber and the advantage of having a distribution of absorbers. The assumption for this simulation is that the length of each absorber is negligible compared to the wavelength of the response of the beam.

Table 2.I: Properties used for simulation

Beam	Simply Supported
Length	300 mm
Width	25 mm
Thickness	3.25 mm
Material	Aluminum
Mass Absorber	10% of total
Resonance	820 Hz
Excitation	820 Hz
PZT Length	21 mm
Position	81 mm from center

In the ideal case where Q is equal to zero, the vibration at each point where an absorber is attached is driven to zero. In this case, the loss factor for the beam is 0.01. For this reason, the system with three absorbers is more efficient at reducing the vibration of the beam. Figure 2.7 presents the transversal displacement of the beam at 820Hz, which is its third resonance frequency. The blue line is the mode shape of the beam at this frequency. With one absorber in the center of the beam, the displacement of the beam is presented by the green line. The amplitude of motion decreases dramatically and corresponds to a 30dB reduction in kinetic energy. The red line shows even smaller displacements with three absorbers equally distributed (cf. Figure 2.6). A 50dB reduction in kinetic energy is achieved in this latter case.

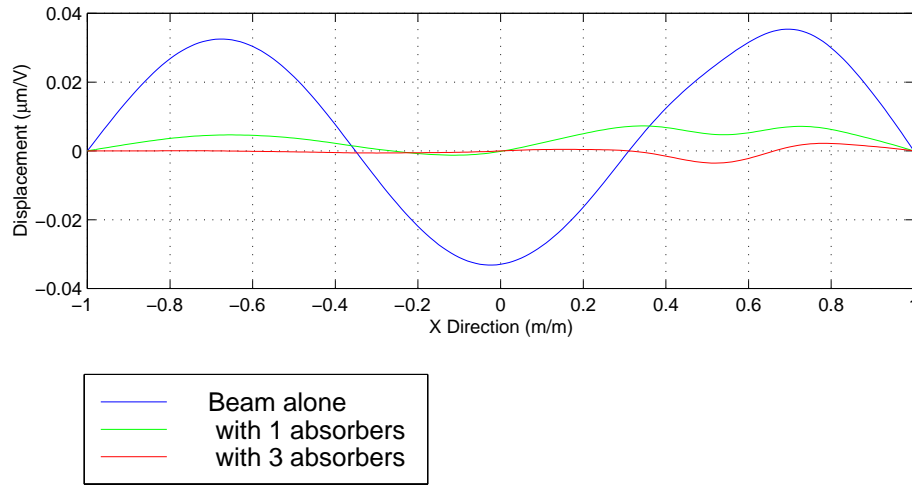


Figure 2.7: Displacement of the beam presented figure 2.6

In the article of *C.R. Fuller and J.P. Maillard* [34], a discussion on the attenuation induced by an absorber is provided. The velocity v_{base} of the base structure at the attachment point of the absorber is considered. Without the absorber, this velocity is noted v_{free} . The mechanical impedance of the base is noted Z_b . The mechanical impedance of the absorber is noted Z_a . The force exerted at the attachment point of the absorber is the combination of the force causing v_{free} and the force exerted by the absorber to the base. This relationship is presented equation (2.1).

$$Z_b v_{base} = Z_b v_{free} - Z_a v_{base} \quad (2.1)$$

From equation (2.1), the ratio of the base velocity with the absorber to that without it can be derived. This ratio is presented in equation (2.2).

$$\frac{v_{base}}{v_{free}} = \frac{Z_b}{Z_a + Z_b} \quad (2.2)$$

In order to minimize this ratio the sum $Z_a + Z_b$ has to be maximized. Since Z_a (and so is Z_b)

is a complex number, its amplitude and phase can be changed. Increasing the amplitude of Z_a will minimize the ratio $\frac{v_{base}}{v_{free}}$. Nevertheless, the efficiency of this approach is determined by the phase difference between Z_a and Z_b . The phase of Z_a has to be 0 for optimum reduction. The mechanical impedance Z_a is a function of frequency and is expressed in equation (2.3).

$$Z_a = M_a j\omega \left[\frac{1 + j\alpha_a / Q_a}{1 - \alpha_a^2 + j\alpha_a / Q_a} \right] \quad (2.3)$$

M_a is the mass of the absorber and K_a its stiffness. $\omega_r = \sqrt{\frac{K_a}{M_a}}$ is the resonance frequency

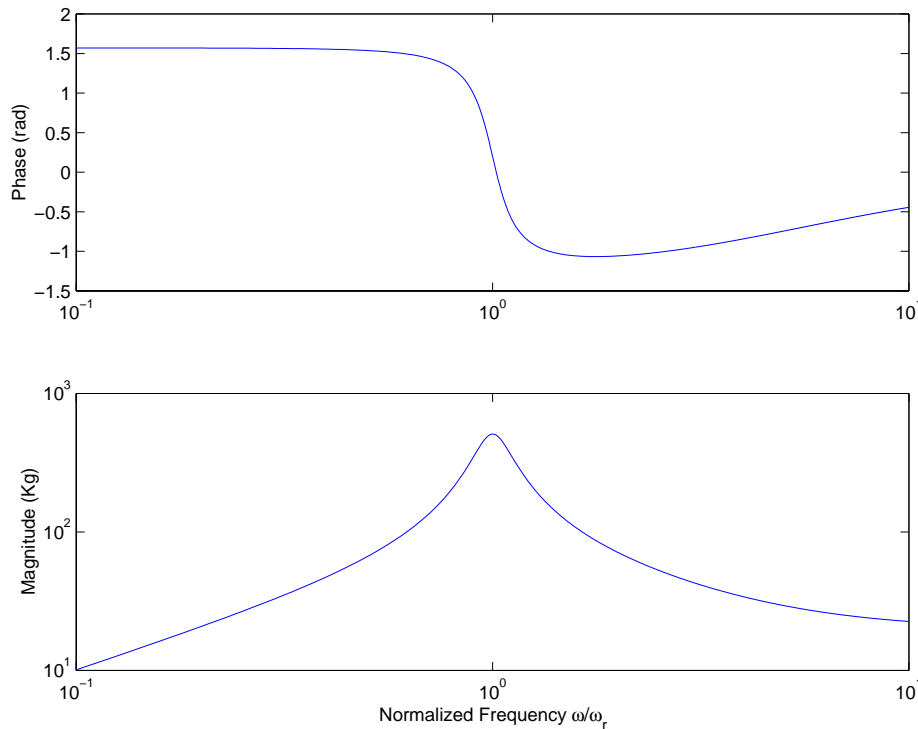


Figure 2.8: Absorber impedance ($M_a=1$ and $Q_a=5$)

of the absorber. The tuning ratio α_a is defined as ω/ω_r . Q_a is the quality factor of the absorber and is defined as $\frac{M_a \omega_r}{C_a}$, with C_a being the damping of the absorber. Figure (2.8) presents the magnitude and phase of Z_a . The magnitude is maximized at resonance, and the phase has a 180° phase shift. Trying to match the phase has little effect since the magnitude drops quickly when the excitation is not at resonance. According to *C.R. Fuller and J. P. Maillard* [34] the optimum solution is to tune the absorber to the disturbance and to maximize the quality factor. In order to improve the performances of these absorbers, a very fine-tuning is ideally needed, and for this reason tunable vibration absorbers (TVA) have been implemented. These TVAs are enhanced absorbers with the ability of having their resonance frequency varied. This modification can be done through mechanical means [35] or electronic means [38]. The TVAs can be tuned exactly to the excitation and minimize the vibration of a structure. [34]

These same TVAs can be used to minimize the sound radiated from a structure. Minimizing the vibration minimizes the radiated sound only to a certain point. The tuning approach thus is not perfect in minimizing the sound radiated from a structure.

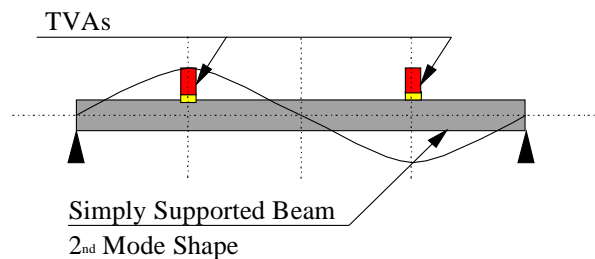


Figure 2.9: Two tunable vibration absorbers on a simply supported beam

Figure 2.9 presents a simulation case where two TVAs are positioned in order to minimize the vibration of a beam. The beam is excited at its second resonance frequency. The properties for the beam and the absorber are similar to Table 2.I, except for the excitation frequency, which is now 342Hz. The tuning ratios $\alpha=\omega/\omega_r$ are varied from 0.5 to 1.5 for the two absorbers. The radiated power of the beam is calculated from the vibration data

using the method presented in Appendix A. The surface graph presented Figure 2.10 shows all the possible tuning configurations within this range.

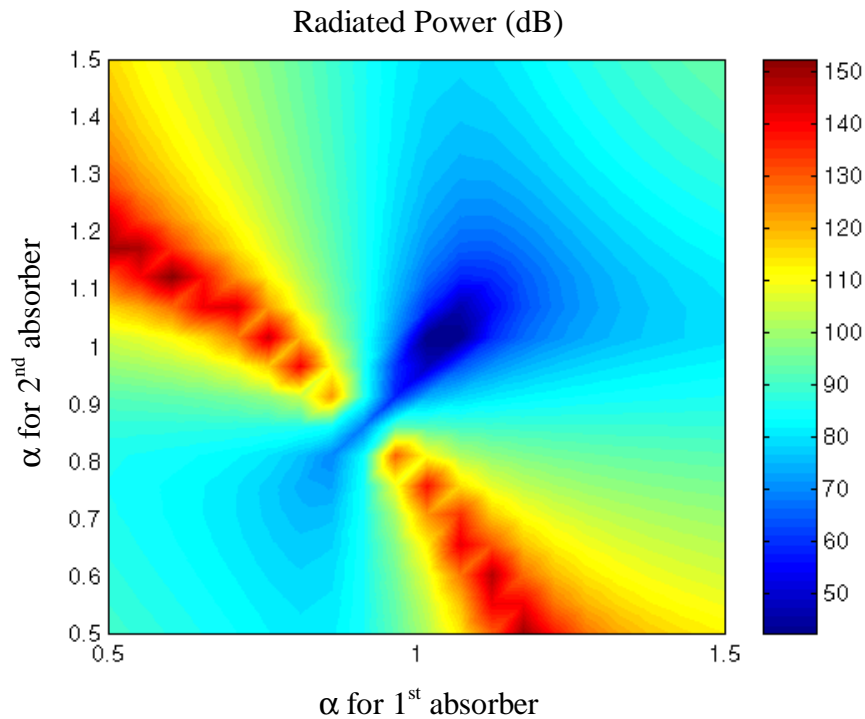


Figure 2.10: Tuning optimum for sound radiation control

The optimum tuning for the two absorbers is seen from Figure 2.10 to be not (1,1). Damping in the structure and in the absorber ($Q=100$) is only part of the reason. The different efficiency of the modes to radiate noise has to be taken into account. Trying to stop the vibration of the second mode, which is a mode that does not radiate well, is not the optimum solution. The first and third modes are non-volumetric and radiate sound more efficiently. The optimum solution in this case is to let the second mode radiate slightly so it destructively interferes with the radiation of the other modes. The situation is very similar to a comparison of active vibration control and active structural acoustic control [6]. The relation between the vibration and the sound radiation is nonlinear. For this reason, the minimization rules applied on the vibration do not apply for the sound

radiation. By changing the tuning of the absorber, the optimal mode-shape response is seen from Figure 2.10 to be a tuning factor of (1.07,1.07).

Figure 2.10 also shows the difficulty of the optimization process. A simple gradient search might not be sufficient to find the optimum. In this example, a local minimum exists for the tuning ratio (0.84,0.84). For larger structure with more absorbers, many local minimums can be observed. In these cases, the optimization process requires powerful optimization techniques such as the genetic algorithm or the simulated annealing technique in order to find a global minimum.

2.1.3 Distributed Absorber

The control ability of a localized absorber positioned on a distributed structure is very good when only one mode is of concern. As has been previously discussed, the resonance frequency of the absorber has to be tuned to the excitation. If several modes have to be controlled, the use of different absorbers with distinct tuning frequencies has to be considered. Figure 2.7 showed that several absorbers with the same resonance frequency could better reduce the vibration of a beam. Generally and more particularly for modally dense structures such as plates, several absorbers will achieve better control. An absorber can be positioned at each antinode of the structure and then in-between anti-nodes in order to control the vibration entirely. Ideally, the absorbers would be positioned continuously all over the structure. Adding more absorbers would result in the limit, to the creation of a distributed absorber. Another remark is that the resonance frequency of all the absorbers is not necessary the excitation frequency especially when sound radiation has to be minimized. The resonance frequency of the absorbers might differ considering their position on the structure. This is also true if several modes have to be controlled. The modal contribution of each absorber is related to his position and its resonance frequency. Therefore, an optimal distributed absorber has a varying resonance frequency in space. The concept of distributed absorber comes from the need of multiple absorbers as it is

presented Figure 2.11. The resonance frequency of a distributed absorber is then a function of space. The numerical values of the mass and stiffness characterizing the single degree of freedom absorber are also a function of space for the distributed absorber. Mass or stiffness of the distributed absorber can be varied along the structure. The absorber presented in this thesis has a varying mass distribution and a constant stiffness. This choice was made for practical reason. It appeared easier to vary the continuous mass rather than the continuous stiffness of a distributed layer.

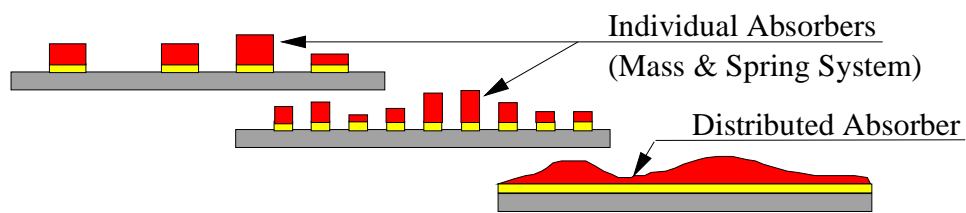


Figure 2.11: Distributed absorber concept

The continuous mass variation induces a continuous spatial variation of resonance frequency.

Another reason in varying the mass distribution is the limited amount of mass that can be used to damp the structure. Typically, the absorber should not weight more than 10% of the overall mass. Therefore, the mass has to be “wisely” distributed. For the area with the most motion and therefore with potentially large modal contributions, the mass is expected to be bigger compared to area with small motions. The efficiency of the absorber is bigger if the local resonance of the distributed absorber in this area is close to the excitation frequency. For the other areas, the resonance frequency might be higher than this excitation. The non-linearity between the vibration and the radiated sound makes the optimization of the mass distribution even more uncertain. Tuning such an absorber is particularly difficult since the number of parameters to be varied is infinite. The technique used to optimize the mass re-distribution is therefore heuristic. The genetic algorithm has been chosen for this purpose and is discussed in Section 2.4.

2.3 Variational approach

2.2.1 Hamilton's principle

As a first step to model the behavior of complex structures, this work was focused on a one-dimensional problem. The vibration of a beam and its associated radiated sound was of concern. The beam equation (2.1) for transversal vibration can be solved for simply supported boundary conditions.

$$m(x) \frac{\partial^2 w(x,t)}{\partial x^2} - c_{11} \frac{bh^3}{12} \frac{\partial^4 w(x,t)}{\partial x^4} = F(x,t) \tag{2.4}$$

This derivation can be found in any good textbook on elementary vibration problems [4]. An example is detailed in Appendix C.

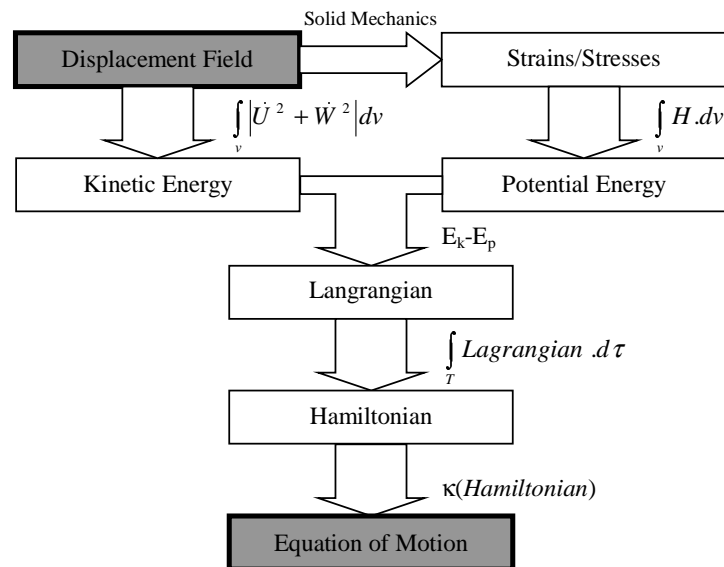


Figure 2.12: Diagram for derivation process using a variational method

Nevertheless, the equation (2.1) is not solvable for more complicated boundary conditions (springs for example) and when the transversal motion is coupled to the longitudinal motion. Approximate methods permit to solve this kind of problem. The Hamilton's Principle provide the link between the physics and its mathematical formulation [4]:

The motion of any mechanical system occurs such that the definite integral of the Lagrangian becomes stationary for all admissible configuration of this system.

Assuming the displacement field to be known, Hamilton's principle leads to the derivation of the equation of motion. Figure 2.12 presents the derivation process used in the models presented in details in Chapter 3. The starting point is the displacement field of the beam and the different added elements such as the absorbers, piezoelectric patch, or constrained layer damping. From the displacement field, the motion of any point in the system is known and the kinetic energy can be integrated over the volume of the system. Using the mechanical properties of the different materials, the stresses can be determined at any point. The mechanical enthalpy for each point of the system is the product of strain and stress. The potential energy of the system is obtained by integrating the enthalpy over the volume. From the kinetic and potential energies, the Lagrangian is computed. The Lagrangian is simply the difference between the kinetic and potential energy $E_k - E_p$. The Lagrangian is then integrated in time in order to obtain the Hamiltonian. The variation of the Hamiltonian is equal to zero. Therefore, an equation of motion is obtained. This equation of motion is a functional that can not be solved analytically. The Rayleigh-Ritz method is a method that can solve this type of problem. The downside of this method is that the solution will only be an approximation.

2.2.2 Rayleigh-Ritz method

The Rayleigh-Ritz method is a very popular method for obtaining an approximate solution to a differential equation. In the model presented in this thesis, the displacement field is described using up to three functions. The final equation of motion is a functional of these functions. The Rayleigh-Ritz method uses series of trial function to approximate the functions describing the displacement field. Those series are approximations because they are truncated to their N first terms. This truncation is necessary to limit computation time but also numerical errors.

$$f = \sum_{i=1}^{\infty} A_i f_i \quad \rightarrow \quad f \approx \sum_{i=1}^N A_i f_i \quad (2.2)$$

where

N is the order of approximation for the model

f_i are the trial functions

A_i are the unknown coefficients to be solved for

The Hamilton's principle relates to the physics of the problem. Thanks to this principle, the equation of motion is obtained. The Rayleigh-Ritz method is the mathematical tool that permits the solution of this equation of motion. This method only gives an approximate solution in the form of a series of trial function. The accuracy of model relies on the choice of the trial functions. Figure 2.13 presents the different steps in the derivation and the importance of the trial functions. Let us call H_1 the Hamiltonian of the system. Let us assume that it is in term of the function f , which describes the displacement field. The variation (noted κ) of the Hamiltonian is equal to zero. The first step is to project f on the trial function basis. This projection is truncated to the N first terms as it is presented in equation (2.4). The Hamiltonian is now noted H_2 and is in term of a vector $\{A_n\}$ of rank N . The variation of the Hamiltonian is then a simple derivation in respect to the A_n coefficients. The matrix equation that is obtained can then be solved using a computer. From the A_n coefficients, the function f can be evaluated. The displacements are then entirely known and any vibration or acoustic value can be computed. For each of these

steps in obtaining a solution, the choice of trial function is critical. Some sets might be better than another and only the use of it can determine if a basis is practical or not. The only mathematical requirement is for the basis to be complete. A physical requirement is that it should be able to describe the mode shapes of a beam with few coefficients. This last requirement cannot be expressed in mathematical term and only the use of the trial functions validates their choice.

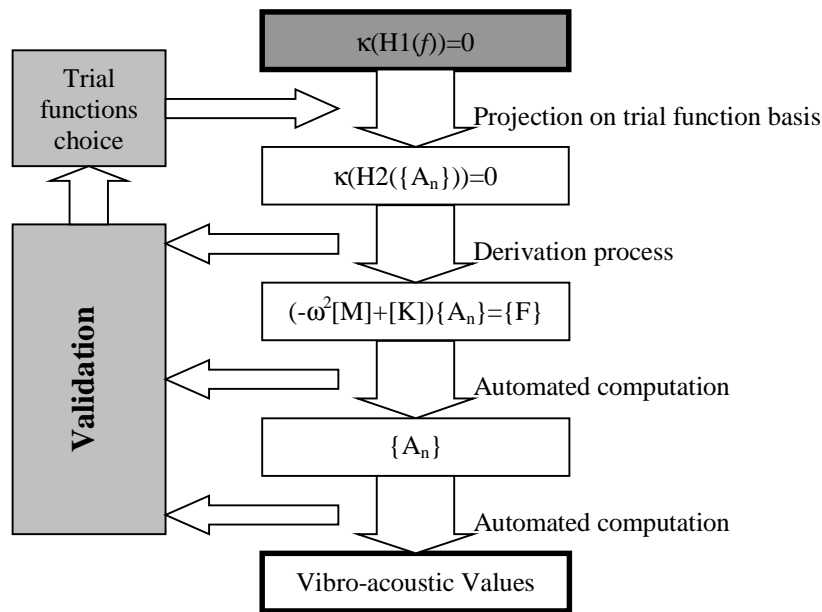


Figure 2.13: Obtaining a solution and choosing a set of trial functions

2.2.3 Trial functions

In previous work [5] the trial functions were simple polynomial functions.

$$f_n(x) = P_n(x) = x^n \quad (2.3)$$

This basis of functions offers the main advantage of being simple but has also some drawbacks. The first one is that it is relatively unstable for high order system (for $N \approx 20$

the matrices become singular due to the dynamics of its coefficients). The second drawback is the loss of precision for every derivation. The coefficient A_{N-i} is lost at the i^{th} derivation.

A new set of functions that are called "Psin" in this thesis has been created and used. The formulation in equation (2.3) has been first used.[43] This formulation did not help the derivation of a variational approach for the problem of this thesis so the formulation of equation (2.4) has been used all throughout this thesis. The Psin functions as they are presented in equations (2.4) and (2.5), are defined between -1 and $+1$.

$$f_n(x) = P \sin_n(x) = \sin(a_n x + b_n) * \sin(c_n x + d_n) \tag{2.4}$$

$$= \frac{1}{2} [\cos(\alpha_n x + \beta_n) - \cos(\gamma_n x + \delta_n)] \tag{2.5}$$

Table 2.II: Coefficients for (2.3).

n	a_n	b_n	c_n	d_n
1	$\frac{\pi}{4}$	$\frac{3\pi}{4}$	$\frac{\pi}{4}$	$\frac{3\pi}{4}$
2	$\frac{\pi}{4}$	$\frac{3\pi}{4}$	$-\frac{\pi}{2}$	$-\frac{3\pi}{2}$
3	$\frac{\pi}{4}$	$-\frac{3\pi}{4}$	$\frac{\pi}{4}$	$-\frac{3\pi}{4}$
4	$\frac{\pi}{4}$	$-\frac{3\pi}{4}$	$\frac{\pi}{2}$	$-\frac{3\pi}{2}$
≥ 5	$\frac{\pi}{2}(n-4)$	$\frac{\pi}{2}(n-4)$	$\frac{\pi}{2}$	$\frac{\pi}{2}$

Table 2.III: Coefficients for (2.4).

n	α_n	β_n	γ_n	δ_n
1	0	0	$\frac{\pi}{2}$	$-\frac{\pi}{2}$
2	$\frac{3\pi}{4}$	$\frac{\pi}{4}$	$-\frac{\pi}{4}$	$-\frac{3\pi}{4}$
3	0	0	$\frac{\pi}{2}$	$\frac{\pi}{2}$
4	$-\frac{\pi}{4}$	$\frac{3\pi}{4}$	$\frac{3\pi}{4}$	$-\frac{\pi}{4}$
≥ 5	$\frac{\pi}{2}(n-5)$	$\frac{\pi}{2}(n-5)$	$\frac{\pi}{2}(n-3)$	$\frac{\pi}{2}(n-3)$

The Psin functions shown in Figure 2.14 are very well suited to describe the mode shapes of a beam since the boundary conditions can be totally determined knowing the 4 first coefficients. This set of functions eliminates the previous drawbacks. It introduces more stability since the model can be solved for order higher than 100. No coefficient is lost in the derivatives. The spatial Fourier's transform (useful to compute the radiated power) is very easily computed. The drawback for this set of functions is the complexity it

introduces in the derivation of the model. This complexity vanishes when some integrals of those Psin functions are known (cf. Appendix D).

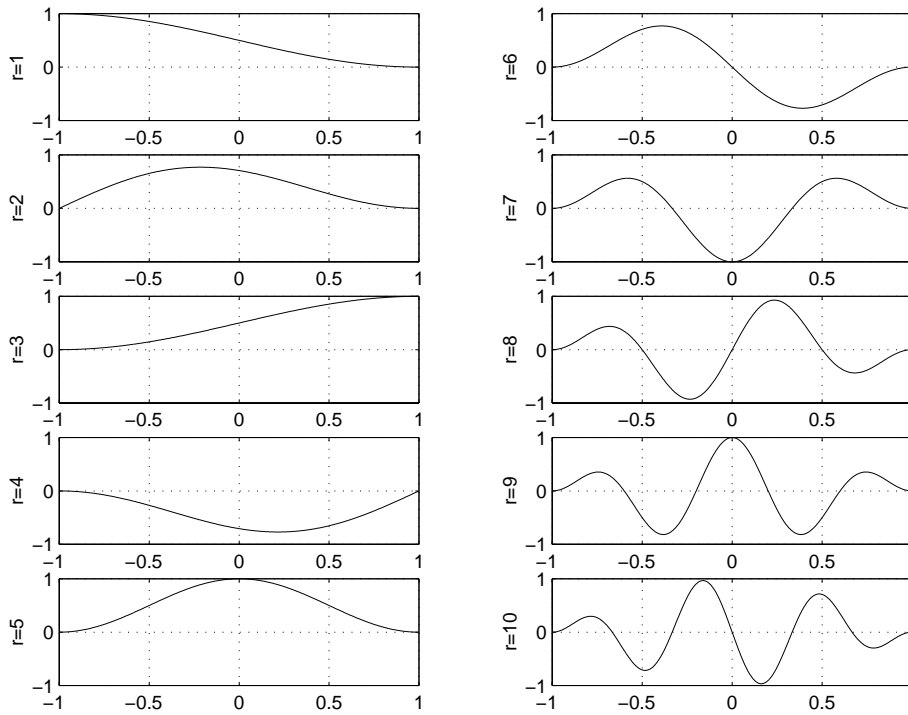


Figure 2.14: Presentation of the 10 first Psin_r functions

2.3 Genetic algorithm

The genetic algorithm was first presented by *H.J. Holland* [7] in 1975. This algorithm led to large advances in the artificial intelligence field. Attempts to create artificial life have been undertaken with limited success. The science fiction dream of self-replicating machines has not come true yet. One of the limitations is the amount of information that can be handled. Even the simplest life form contains billions of elementary genetic information. Most of the artificial life form that have been created simply degenerate after a certain time. However, the genetic algorithm (GA) has found very useful applications in engineering. The basic principles of the genetic algorithm showed ability to solve optimization problems. Spacecraft control has been one of the major accomplishments of genetic algorithm in its early days. Nowadays genetic algorithms are part of the engineer optimization toolbox. To explain it simply, the genetic algorithm mimics the mating process of chromosomes to improve a set of parameters. The following section presents more in detail of this fascinating algorithm applied on the optimization problem faced in developing a distributed absorber.

2.3.1 Chromosomes

In the classical genetic algorithm, the parameters information is coded using binary strings. An individual corresponds to one particular configuration of this string. In this research, the object to be optimized is a function. As it has been discussed in the first part of this chapter, the distributed absorber has a varying mass distribution. The function to be optimized is the mass distribution. The mass can be considered as negative if the excitation is out of phase with the disturbance. For this reason, any continuous function is acceptable. Figure 2.15 presents the coding process used to create an individual chromosome which is understandable by the genetic algorithm. In order to reduce the number of parameter from infinity to a finite number, the function was projected in a

certain basis. Logically the basis used for the variational problem (cf. 2.2.3) was used for this purpose.

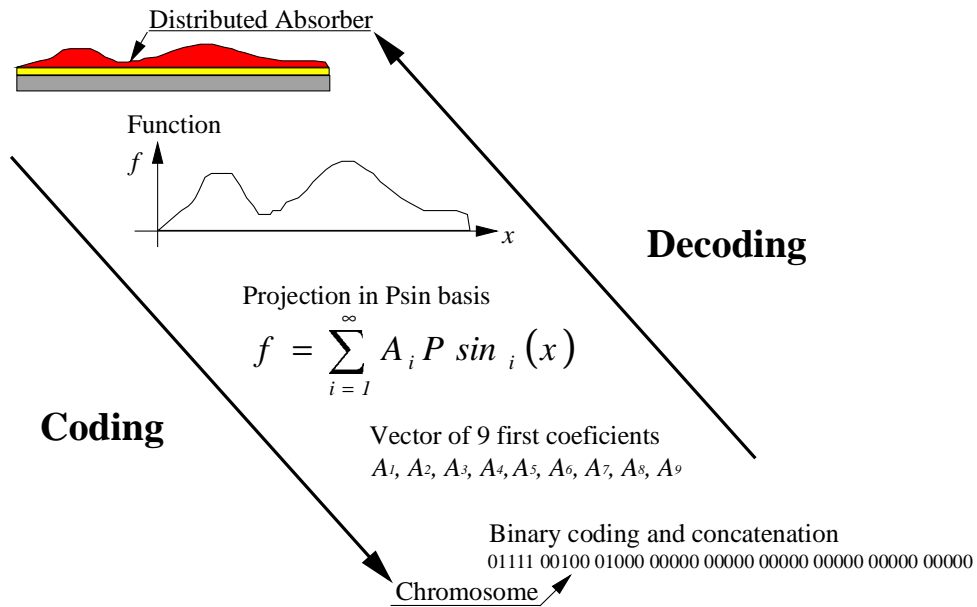


Figure 2.15: Coding and decoding a Chromosome

Each coefficient is discretized using one of 31 possible values between -1 and $+1$. Each of these values corresponds to a binary number coded on five bits ($2^5=32$). In this example, 9 binary numbers describe a specific mass distribution. By chaining all the binary numbers together (concatenation) a large binary number containing $5 \times 9 = 45$ bits represent the mass distribution. This large binary number is called the chromosome of the distributed absorber. It is interesting to notice that this type of coding can lead to 35 trillions different individuals ($2E45$). A classical optimization process such as a systematic search is in this case not realistic.

2.3.2 Fitness

The genetic algorithm is based on natural selection; only “the fittest survives”. The fitness is a particular number attached to each individual that quantifies its suitability. The probability that an individual will transmit its genes will depend on the fitness. An active-passive distributed absorber is optimized so that it minimizes a cost function. This cost function for the objectives of this thesis can be any combination of the parameters presented in the following list.

- Radiated power over a certain frequency range with active control off
- Radiated power over a certain frequency range with active control on
- Velocity squared over a certain frequency range with active control off
- Velocity squared over a certain frequency range with active control on
- Electric power required by the active part of the system

This list is not intended to be exhaustive. The fitness used in this research is the inverse of the cost function.

2.3.3 Algorithm

The genetic algorithm does not need any information concerning the system itself. All it needs is a starting generation and the fitness information. A generation is a group of individuals that will be mated in order to create a new generation. The mating process uses numerical analogy to biochemical processes known as reproduction, crossover and mutation. The reproduction is done at random but the chance for an individual to reproduce itself is proportional to the importance of its fitness compared to the rest of the generation. Figure 2.16 presents three individuals A, B, and C with different fitness. The probability of reproduction for these individuals can be visualized on the wheel chart. This

probability is 0.6 for A, 0.3 for B and 0.1 for C. This probability is proportional to the fitness of each individual. The random process in choosing the individuals that will mate is nevertheless random. It can be visualized as a spin of the wheel chart of Figure 2.16. The odds for the chart to stop on A, B, or C are directly proportional to their fitness.

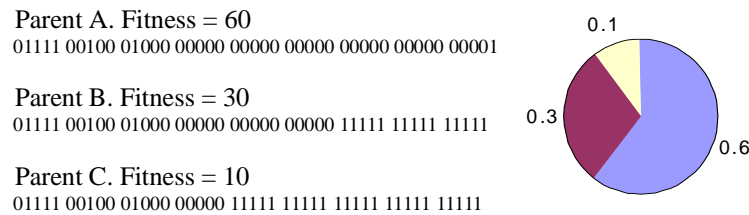


Figure 2.16: Reproduction probability for three individuals

The crossover between two individual of the new generation is also a random process. The two individuals picked at random will exchange a random part of their binary string with the other. The last modification on the chromosomes of the new generation is the Mutation of a bit. A randomly picked bit will switch from 0 to 1 if the bit is a 0, and from 1 to 0 if it is a 1.

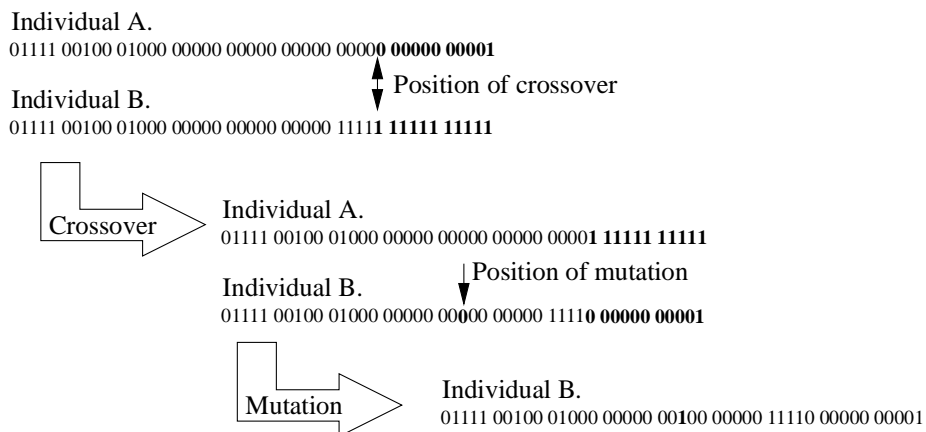


Figure 2.17: Example of crossover and mutation

Figure 2.17 presents an example of mating between the chromosome A and B. The position for the crossover is between the 11th and the 12th bit starting from the right of the chromosome. The right part of each chromosome therefore delimited is exchanged with the other chromosome. The Chromosomes A and B have exchanged part of their genetic content. The mutation is another genetic alteration which is presented in this example. The individual B has a mutation of the 23th bit. from a zero, it is transformed into a 1. The probability that a mutation occurs is usually very low (typically 0.03). This is implemented as a precaution that the system will not converge toward a local minimum.

