# Appendix A. Radiated Power Computation

The radiated power is a numeric quantity that reflects the energy of the sound radiated by a structure. In this appendix, the structure to be considered is a beam located in an infinite rigid baffle.

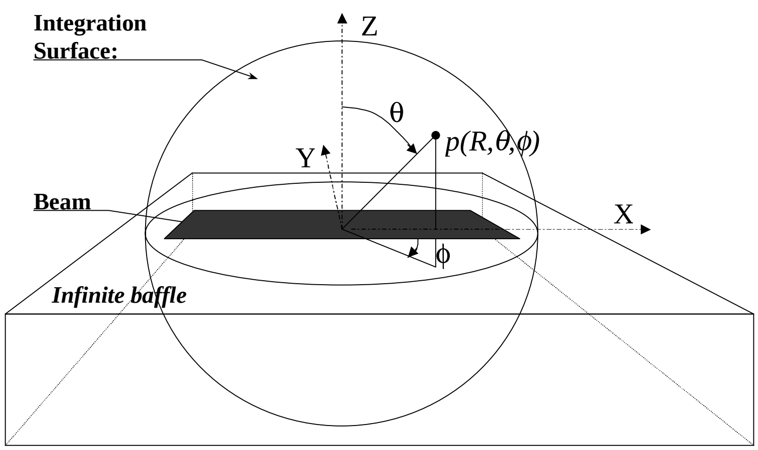


Figure A.1: Geometry for radiated power computation

Figure A.1 presents the geometry associated with the beam. The position of a point p is defined in term of the distance $R$ from the center of the beam and the two angular values $\theta$ and $\phi$. The radiated power in the half-space Z>0 is defined as the integral of the acoustic intensity over a half-sphere [11]:

$$\Pi = \frac{R^2}{2\rho_o c_o} \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \left| p(R,\theta,\phi) \right|^2 sin(\theta) d\theta d\phi \qquad \text{(A.1)}$$

The pressure at any point in the half-space is expressed in equation (A.2)

$$p(R,\theta,\phi) = \frac{\omega^2 \rho_0 e^{jk_0 R}}{2\pi R} \overline{w}(k_x, k_y) \tag{A.2}$$

In this equation, $k_x$ and $k_y$ are the projections of $k$ in respect to the $x$ and $y$ direction. $\overline{w}$ is the spatial Fourier's transform of $w$.

$$\overline{w}(k_x, k_y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} w(x,y) e^{jk_x x} e^{jk_y y} dxdy \tag{A.3}$$

$$\overline{w}(k_x, k_y) = \frac{2\sin\left(k_y \frac{L_y}{2}\right)}{k_y} \int_{-\frac{L_x}{2}}^{\frac{L_x}{2}} w(x,y) e^{jk_x x} dx \tag{A.4}$$

The displacement $w$ is already known in term of the Psin functions as it is presented in equation (A.5).

$$w(x,y) = \sum_{n=1}^{N} A_n * \frac{1}{2}\left[\cos\left(\alpha_n \frac{2x}{L_x} + \beta_n\right) - \cos\left(\gamma_n \frac{2x}{L_x} + \delta_n\right)\right] = \sum_{n=1}^{N} A_n * P\sin_n\left(\frac{2x}{L_x}\right) \tag{A.5}$$

Since every variable is known, the radiated power can be computed for the beam. The main analytical work is to compute the different integrals. The first integration is the spatial Fourier’s transform of the Psin functions.

$$F(P\sin_n)(k_x) = \frac{L_x}{4}\left[\operatorname{sinc}\left(\alpha_n + k_x \frac{L_x}{2}\right) e^{i\beta_n} + \operatorname{sinc}\left(\alpha_n - k_x \frac{L_x}{2}\right) e^{-i\beta_n} - \operatorname{sinc}\left(\gamma_n + k_x \frac{L_x}{2}\right) e^{i\delta_n} - \operatorname{sinc}\left(\gamma_n - k_x \frac{L_x}{2}\right) e^{-i\delta_n}\right] \tag{A.6}$$

This integration is simple because the Psin functions are very similar to the trigonometric functions. This is not the case when polynomials are used as trial functions (a recursive method is then required). The integral over the half-sphere can not be found analytically.

A numerical integration method has been used. The algorithm used comes from the mathematics library NAG (under the name d01fcft.f).