Appendix C. Point absorbers on SS beam

The system modeled in this appendix is a simply supported beam with P absorbers and a piezoelectric patch (PZT) for the excitation. The model for the piezoelectric patch was obtained by *Gibbs and Fuller* [49].

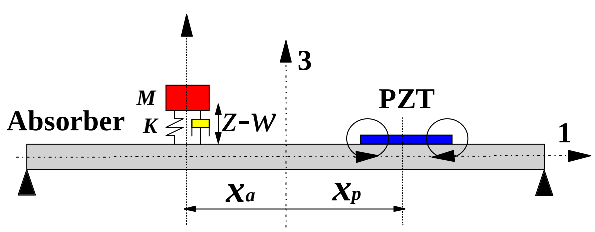

Figure C.1: Modeled system

Figure C.1 present the model investigated in the appendix. The devices added to the simply supported beam are referred to the center of the beam. The differential equation for the system presented by figure C.1 is expressed in equation (C.1).

$$\frac{\partial^4 w}{\partial x^4}+\frac{\rho_b b h_b}{Y_b I_b}\frac{\partial^2 w}{\partial \tau^2}=\frac{K}{Y_b I_b}(z-w)\delta(x-x_a)-D_{fl}\,\frac{d_{31}\Delta\varphi^z}{h_z}\left\{\delta'\left[x-\left(x_z-\frac{L_z}{2}\right)\right]-\delta'\left[x-\left(x_z+\frac{L_z}{2}\right)\right]\right\}$$

$$with \quad D_{fl}=\frac{6h_zY_bY_zh_z(h_b+h_z)}{h_b^4Y_b^2+4h_b^3Y_bY_zh_z+6h_b^2Y_bY_zh_z^2+4h_bY_bY_zh_z^3+h_z^4Y_b^2} \tag{C.1}$$

This equation is reduced using several assumptions and change of variables:

$$w = we^{-i\omega\tau} \tag{C.2}$$

$$z-w=D(\omega)w \quad with \quad D(\omega)=\frac{\alpha^2}{1-\alpha^2+j\dfrac{\alpha}{Q}}, \quad \alpha=\frac{\omega}{\omega_r}, and \quad Q=\frac{M\omega_r}{C} \tag{C.3}$$

$$\beta^4 = \omega^2 \frac{\rho_b b h_b}{Y_b I_b} \tag{C.4}$$

$$\frac{\partial^4 w}{\partial x^4} + \beta^4 w = \frac{K}{Y_b I_b} D(\omega) w \delta(x - x_a) - D_{fl} \frac{d_{31} \Delta\varphi^z}{h_z} \left\{ \delta' \left[x - \left(x_z - \frac{L_z}{2} \right) \right] - \delta' \left[x - \left(x_z + \frac{L_z}{2} \right) \right] \right\} \tag{C.5}$$

The general solution for this equation is:

$$w(x) = A_1 \sin\beta\left(x + \frac{L_b}{2} \right) + A_2 \cos\beta\left(x + \frac{L_b}{2} \right) + A_3 \sinh\beta\left(x + \frac{L_b}{2} \right) + A_4 \cosh\left(x + \frac{L_b}{2} \right) \tag{C.6}$$

Applying the boundary conditions:

$$\begin{cases} w\left(-\frac{L_b}{2} \right) = 0 \\ w\left(\frac{L_b}{2} \right) = 0 \end{cases} \Rightarrow \begin{cases} A_2 = A_3 = A_4 = 0 \\ \beta_n = \frac{n\pi}{L_b} \end{cases} \tag{C.7}$$

Therefore, the solution for w has the following form:

$$w(x) = \sum_{n=1}^{+\infty} A_n \phi_n(x) \quad with \quad \phi_n(x) = \sin\left(\frac{n\pi}{L_b} \left(x - \frac{L_b}{2} \right) \right) \tag{C.8}$$

Using the orthogonality of the mode shapes equation (C.5) is multiplied by ϕ_m and integrated over the length of the beam:

$$\begin{aligned} \frac{\rho_b b h_b}{Y_b I_b} \left[\omega_m^2 - \omega^2 \right] \sum_{n=1}^{+\infty} A_n \int_{-\frac{L_b}{2}}^{\frac{L_b}{2}} \phi_n(x) \phi_m(x) dx = \frac{K D(\omega)}{Y_b I_b} \sum_{n=1}^{+\infty} A_n \int_{-\frac{L_b}{2}}^{\frac{L_b}{2}} \phi_n(x) \phi_m(x) \delta(x - x_a) dx \\ - D_{fl} \frac{d_{31} \Delta\varphi^z}{h_z} \int_{-\frac{L_b}{2}}^{\frac{L_b}{2}} \phi_m(x) \left\{ \delta' \left[x - \left(x_z - \frac{L_z}{2} \right) \right] - \delta' \left[x - \left(x_z + \frac{L_z}{2} \right) \right] \right\} dx \end{aligned} \tag{C.9}$$

$$\frac{\rho_b b h_b}{Y_b I_b}\left[\omega_m^2-\omega^2\right]A_m\frac{L_b}{2}=\frac{KD(\omega)}{Y_b I_b}\sum_{n=1}^{+\infty}A_n\phi_n(x_a)\phi_m(x_a)$$
$$-D_{fl}\frac{d_{31}\Delta\varphi^z}{h_z}\frac{n\pi}{L_b}\left[\cos\frac{n\pi}{L_b}\left(x_z-\frac{L_z}{2}+\frac{L_b}{2}\right)-\cos\frac{n\pi}{L_b}\left(x_z+\frac{L_z}{2}+\frac{L_b}{2}\right)\right]$$
(C.10)

The order of the system is truncated to N. A set of N linear equations is then obtained. This system can be presented in matrix form. This time P different absorbers are taken into account.

$$\{A\}_N=\left[[I]_{N,N}-[H]_{N,N}[\phi]_{N,P}[Z]_{P,P}[\phi_{i,n}]_{P,N}\right]^{-1}\{M\}_N \quad \text{(C.11)}$$

$[I]_{N,N}$ is the identity matrix

$[H]_{N,N}$ is a diagonal matrix $H_{i,i}=\frac{2}{\rho_b b h_b L_b}\frac{1}{\omega_i^2-\omega^2}$

$[\phi]_{N,P}$ is a NxP matrix $\phi_{i,j}=\sin\frac{i\pi}{L_b}\left(x_j+\frac{L_b}{2}\right)$

$[Z]_{P,P}$ is diagonal matrix $Z_{i,i}=K_i D_i(\omega)$

$\{M\}_N$ is a vector $M_i=\frac{\pi Y_b h_b^2 D_{fl} d_{31}\Delta\varphi^z}{6\rho_b L_b^2 h_z}\frac{i\left[\cos\frac{i\pi}{L_b}\left(x_z-\frac{L_z}{2}+\frac{L_b}{2}\right)-\cos\frac{i\pi}{L_b}\left(x_z+\frac{L_z}{2}+\frac{L_b}{2}\right)\right]}{\omega_i^2-\omega^2}$

With the A_n coefficients known, the transversal displacement of the beam w is known (cf. equation (C.8)).