A Novel Thermal Method for Pipe Flow Measurements Using a Non-invasive BTU Meter

Hussain M. J. A. M. A. Alshawaf

Thesis submitted to the faculty of the Virginia Polytechnic Institute and State University in partial fulfilment of the requirements for the degree of

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in
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ABSTRACT

This work presents the development of a novel and non-invasive method that measures fluid flow rate and temperature in pipes. While current non-invasive flow meters are able to measure pipe flow rate, they cannot simultaneously measure the internal temperature of the fluid flow, which limits their widespread application. Moreover, devices that are able to determine flow temperature are primarily intrusive and require constant maintenance, which can shut down operation, resulting in downtime and economic loss. Consequently, non-invasive flow rate and temperature measurement systems are becoming increasingly attractive for a variety of operations, including for use in leak detection, energy metering, energy optimization, and oil and gas production, to name a few. In this work, a new solution method and parameter estimation scheme are developed and deployed to non-invasively determine fluid flow rate and temperature in a pipe. This new method is utilized in conjunction with a sensor-based apparatus—namely, the Combined Heat Flux and Temperature Sensor (CHFT+), which employs simultaneous heat flux and temperature measurements for non-invasive thermal interrogation (NITI). In this work, the CHFT+ sensor embodiment is referred to as the British Thermal Unit (BTU) Meter. The fluid’s flow rate and temperature are determined by estimating the fluid’s convection heat transfer coefficient and the sensor-pipe thermal contact resistance. The new solution method and parameter estimation scheme were validated using both simulated and experimental data. The experimental data was validated for accuracy using a commercially available FR1118P10 Inline Flowmeter by Sotera Systems (Fort Wayne, IN) and a ThermaGate sensor by ThermaSENSE Corp. (Roanoke, VA). This study’s experimental results displayed excellent agreement with values estimated from the aforementioned methods. Once tested in conjunction with the non-invasive BTU Meter, the proposed solution and parameter estimation scheme displayed an excellent level of validity and reliability in the results. Given the proposed BTU Meter’s non-invasive design and experimental results, the developed solution and parameter estimation scheme shows promise for use in a variety of different residential, commercial, and industrial applications.
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Hussain M. J. A. M. A. Alshawaf

GENERAL AUDIENCE ABSTRACT

This work documents the development of a novel and non-invasive method that measures fluid flow rate and temperature in pipes. While current non-invasive flow meters are able to measure pipe flow rate, they cannot simultaneously measure the internal temperature of the fluid flow, which limits their widespread application. Moreover, devices that are able to determine flow temperature are primarily intrusive and require constant maintenance, which can shut down operation, resulting in downtime and economic loss. Consequently, non-invasive flow rate and temperature measurement systems are becoming increasingly attractive for a variety of operations, including for use in leak detection, energy metering, energy optimization, and oil and gas production, to name a few. This paper presents a new method that utilizes a non-invasive British Thermal Unit (BTU) Meter based on Combined Heat Flux and Temperature Sensor (CHFT+) technology to determine fluid flow rate and temperature in pipes. The non-invasive BTU Meter uses thermal interrogation to determine different flow parameters, which are used to determine the fluid flow rate and temperature inside a pipe. The method was tested and validated for accuracy and reliability through simulations and experiments. Given the proposed BTU Meter’s non-invasive design and excellent experimental results, the developed novel sensing method shows promise for use in a variety of different residential, commercial, and industrial applications.
Acknowledgements

I would like to express my sincere gratitude to everyone that has helped me in writing my thesis. First, I would like to thank Dr. Diller for serving as my committee chair and providing me with feedback during the writing of my thesis. Next, I would like to thank Ali Roghanizad for his help in the development of the parameter estimation scheme used in this work.

I would like to thank my lab colleagues including Mohammed Alanazi and Abdulmohsen Alsaiai for their encouragement and support during my time in the lab. Without them, my life would have been much more stressful.

I would also like to thank, my good friend, Husain Almosawi for helping me create the figures presented in this work. Without him, the novelty and professionalism of my work would not have been presented as well as it has.

Furthermore, I would like to thank my beloved parents, Mohammad and Afifah Alshawaf, for their support throughout my life. Nothing I have accomplished would have been possible without their constant love and sacrifice. I love you both very much.

Last but not least, I would also like to thank my wife Noor Salman for her constant support and patience while I finished by graduate degree. Although we were 7,014 miles apart for the majority of my graduate studies, I felt her presence with all of my heart every day. Noor, I am excited to start the rest of my life with you by my side.
Preface

This Masters of Science thesis is organized in a manuscript format that includes one scientific paper that documents the focus of the thesis. This paper will be published once the intellectual property of the work is protected. Chapter 1 provides an introduction describing the main objectives of this work. The next chapter, Chapter 2, is the unpublished scientific paper and presents the focus of the thesis in developing a new solution method that incorporates a parameter estimation scheme capable of non-invasively determining fluid flow rate and temperature in a pipe by measuring convection coefficient and thermal contact resistance using a Combined Heat Flux and Temperature Sensor (CHFT+). This embodiment of the CHFT+ is called the British Thermal Unit (BTU) Meter. Finally, Chapter 3 provides a conclusion of the presented work, as well as recommendations for future work. This thesis also includes a series of Appendices that provide additional details regarding the appropriate thermal boundary conditions, the manufacture of the CHFT+ and its calibration, a detailed investigation of the assumptions discussed in Chapter 2, the data acquisition and analysis apparatus, the thermal mathematical model and solution method developed, a detailed summary of the obtained results, and a comparison between results obtained from the BTU Meter and the literature.
Attribution

Chapter 2: A Non-invasive Thermal Solution Method to Determine Fluid Flow Rate, Fluid Temperature, Convection Coefficient, and Thermal Contact Resistance of Internal Pipe Flow

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Thomas E. Diller provided advice as I was developing this novel Thermal Solution Method for the BTU Meter application of the CHFT+. 
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<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_s$</td>
<td>Surface area of heated pipe section</td>
<td>$m^2$</td>
</tr>
<tr>
<td>$A^*$</td>
<td>Ratio of the actual heater area to pipe circumferential area</td>
<td>Unit-less</td>
</tr>
<tr>
<td>$Bi$</td>
<td>Biot number</td>
<td>Unit-less</td>
</tr>
<tr>
<td>$C$</td>
<td>Specific heat capacity of pipe</td>
<td>$J/(kg - K)$</td>
</tr>
<tr>
<td>$C_{fluid}$</td>
<td>Specific heat capacity of fluid</td>
<td>$J/(kg - K)$</td>
</tr>
<tr>
<td>$C_p$</td>
<td>Specific heat capacity of pipe at constant pressure</td>
<td>$J/(kg - K)$</td>
</tr>
<tr>
<td>$c^*$</td>
<td>Specific heat capacity of semi-infinite solid</td>
<td>$J/(kg - K)$</td>
</tr>
<tr>
<td>$D_i$</td>
<td>Inner pipe diameter</td>
<td>$m$</td>
</tr>
<tr>
<td>$f_D$</td>
<td>Darcy friction factor</td>
<td>Unit-less</td>
</tr>
<tr>
<td>$h$</td>
<td>Convection heat transfer coefficient</td>
<td>$W/(m^2 - ^\circ C)$</td>
</tr>
<tr>
<td>$h_N$</td>
<td>Nominal convection coefficient</td>
<td>$W/(m^2 - ^\circ C)$</td>
</tr>
<tr>
<td>$h_{optimum}$</td>
<td>Optimal convection heat transfer coefficient</td>
<td>$W/(m^2 - ^\circ C)$</td>
</tr>
<tr>
<td>$h_P$</td>
<td>Perturbed convection coefficient</td>
<td>$W/(m^2 - ^\circ C)$</td>
</tr>
<tr>
<td>$H(t - t_j)$</td>
<td>Fundamental unit step heat flux response</td>
<td>Unit-less</td>
</tr>
<tr>
<td>$j$</td>
<td>Measurement index</td>
<td>$[0,1,2,3,...,j,M-2,M-1]$</td>
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<tr>
<td>$k$</td>
<td>Thermal conductivity</td>
<td>$W/(m - K)$</td>
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<tr>
<td>$k_{fluid}$</td>
<td>Thermal Conductivity of Fluid</td>
<td>$W/(m - K)$</td>
</tr>
<tr>
<td>$L$</td>
<td>Length of heated pipe section</td>
<td>$m$</td>
</tr>
<tr>
<td>$L_c$</td>
<td>Characteristic length of heated pipe section</td>
<td>$m$</td>
</tr>
<tr>
<td>$M$</td>
<td>Mass of heated pipe section</td>
<td>$kg$</td>
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<tr>
<td>$m$</td>
<td>Measurement index</td>
<td>$[0,1,2,3,...,m,M-2,M-1]$</td>
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<tr>
<td>$N$</td>
<td>Negative voltage output</td>
<td>$\mu V$</td>
</tr>
<tr>
<td>$Nu_D$</td>
<td>Nusselt number</td>
<td>Unit-less</td>
</tr>
<tr>
<td>$P$</td>
<td>Positive voltage output</td>
<td>$\mu V$</td>
</tr>
<tr>
<td>$Pr$</td>
<td>Prandtl number</td>
<td>Unit-less</td>
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<tr>
<td>$q_{Heater,(conduction)}$</td>
<td>Heat supplied by heater</td>
<td>$W$</td>
</tr>
<tr>
<td>$q_x$</td>
<td>Heat Transferred in the x direction</td>
<td>$W$</td>
</tr>
<tr>
<td>$q''$</td>
<td>Heat flux</td>
<td>$W/m^2$</td>
</tr>
<tr>
<td>$q''_{fluid}$</td>
<td>Heat flux going into the fluid</td>
<td>$W/m^2$</td>
</tr>
<tr>
<td>$q_o$</td>
<td>Heat Flux transferred in the centerline of sensor</td>
<td>$W/m^2$</td>
</tr>
<tr>
<td>$q''_{sensor}$</td>
<td>Heat flux read by BTU Meter</td>
<td>$W/m^2$</td>
</tr>
<tr>
<td>$q''_{Surface}$</td>
<td>Surface heat flux</td>
<td>$W/m^2$</td>
</tr>
<tr>
<td>$q''_{unit}$</td>
<td>Fundamental unit heat flux</td>
<td>$W/m$</td>
</tr>
<tr>
<td>$R$</td>
<td>Insulation thermal resistance</td>
<td>$m^2 - ^\circ C/W$</td>
</tr>
<tr>
<td>$R''$</td>
<td>Thermal contact resistance</td>
<td>$(^\circ C - m^2)/W$</td>
</tr>
<tr>
<td>$Re_D$</td>
<td>Reynolds number</td>
<td>Unit-less</td>
</tr>
<tr>
<td>$R_i$</td>
<td>Inside pipe radius</td>
<td>$m$</td>
</tr>
<tr>
<td>$RMSE$</td>
<td>Root mean squared error</td>
<td>$^\circ C$</td>
</tr>
<tr>
<td>$R''_N$</td>
<td>Nominal thermal contact resistance</td>
<td>$(^\circ C - m^2)/W$</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Unit</td>
</tr>
<tr>
<td>-----------------</td>
<td>------------------------------------------------------------------</td>
<td>----------</td>
</tr>
<tr>
<td>$R_o$</td>
<td>Outside pipe radius</td>
<td>m</td>
</tr>
<tr>
<td>$R_{optimum}$</td>
<td>Optimal thermal contact resistance</td>
<td>$(^\circ C - m^2)/W$</td>
</tr>
<tr>
<td>$R_p$</td>
<td>Perturbed thermal contact resistance</td>
<td>$(^\circ C - m^2)/W$</td>
</tr>
<tr>
<td>$S$</td>
<td>Sensitivity of Heat Flux Sensor</td>
<td>$\mu V/W - m^2$</td>
</tr>
<tr>
<td>$T_0$</td>
<td>Initial temperature</td>
<td>$^\circ C$</td>
</tr>
<tr>
<td>$T_{fluid}$</td>
<td>Temperature of fluid inside the pipe</td>
<td>$^\circ C$</td>
</tr>
<tr>
<td>$T_{pipe}$</td>
<td>Surface temperature of pipe</td>
<td>$^\circ C$</td>
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<tr>
<td>$T_{pipe(calculated)}$</td>
<td>Calculated surface temperature of pipe</td>
<td>$^\circ C$</td>
</tr>
<tr>
<td>$T_{pipe(measured)}$</td>
<td>Measured surface temperature of pipe</td>
<td>$^\circ C$</td>
</tr>
<tr>
<td>$T_{TC(calculated),m}$</td>
<td>Thermocouple calculated temperature</td>
<td>$^\circ C$</td>
</tr>
<tr>
<td>$T_{TC(measured)}$</td>
<td>Thermocouple measure temperature</td>
<td>$^\circ C$</td>
</tr>
<tr>
<td>$T_{ref}$</td>
<td>Difference between calculated and measured water temperature</td>
<td>$^\circ C$</td>
</tr>
<tr>
<td>$t$</td>
<td>Time</td>
<td>s</td>
</tr>
<tr>
<td>$t_{flow}$</td>
<td>Time it takes for the flow to pass across heater</td>
<td>s</td>
</tr>
<tr>
<td>$U$</td>
<td>Overall heat transfer coefficient</td>
<td>$(^\circ C\cdot m^2)/W$</td>
</tr>
<tr>
<td>$u_\tau$</td>
<td>Wall shear velocity</td>
<td>$m/s$</td>
</tr>
<tr>
<td>$V$</td>
<td>Volume of the heated pipe section</td>
<td>$m^3$</td>
</tr>
<tr>
<td>$v$</td>
<td>Mean flow velocity</td>
<td>$m/s$</td>
</tr>
<tr>
<td>$V_{HE}$</td>
<td>Heat flux sensor voltage output</td>
<td>$\mu V$</td>
</tr>
<tr>
<td>$\dot{V}$</td>
<td>Flow rate of fluid flow in the pipe</td>
<td>gallons/min</td>
</tr>
<tr>
<td>$x$</td>
<td>Coordinate</td>
<td>Unit-less</td>
</tr>
<tr>
<td>$x_1$</td>
<td>Half the Length of the Heat Flux Sensor</td>
<td>m</td>
</tr>
<tr>
<td>$y$</td>
<td>Effective Thickness of Laminar Sublayer</td>
<td>cm</td>
</tr>
<tr>
<td>$y^+$</td>
<td>Wall coordinate</td>
<td>Unit-less</td>
</tr>
<tr>
<td>$z$</td>
<td>Coordinate</td>
<td>Unit-less</td>
</tr>
<tr>
<td>$\Delta T_{Water}(^\circ C)$</td>
<td>Difference between calculated and measured water temperature</td>
<td>$^\circ C$</td>
</tr>
<tr>
<td>$\Delta P$</td>
<td>Pressure drop due to viscous effects</td>
<td>Pa</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Dynamic viscosity</td>
<td>$(N\cdot s)/m^2$</td>
</tr>
<tr>
<td>$\mu_s$</td>
<td>Fluid dynamic viscosity at the heat transfer boundary surface temperature</td>
<td>$(N\cdot s)/m^2$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density of the pipe</td>
<td>$kg/m^3$</td>
</tr>
<tr>
<td>$\rho_{fluid}$</td>
<td>Density of the fluid</td>
<td>$kg/m^3$</td>
</tr>
<tr>
<td>$\rho^*$</td>
<td>Density of semi-infinite solid</td>
<td>$kg/m^3$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Difference between pipe and water temperature</td>
<td>$^\circ C$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Time constant</td>
<td>s</td>
</tr>
<tr>
<td>$\tau_N$</td>
<td>Nominal time constant</td>
<td>s</td>
</tr>
<tr>
<td>$\tau_P$</td>
<td>Perturbed time constant</td>
<td>s</td>
</tr>
<tr>
<td>$\tau_w$</td>
<td>Wall shear stress</td>
<td>Pa</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Pipe thickness</td>
<td>m</td>
</tr>
<tr>
<td>$\delta_t$</td>
<td>Thermal Penetration of Heat in the Flow</td>
<td>cm</td>
</tr>
<tr>
<td>$\vartheta$</td>
<td>Kinematic viscosity</td>
<td>$m^2/s$</td>
</tr>
</tbody>
</table>
Chapter 1 – Introduction

Fluid flow rate and temperature represent critical parameters for many residential, commercial, and industrial applications such as HVAC, oil and gas production, and in assessing power plant productivity and energy efficiencies. Traditionally, the task of measuring flow rate and temperature within a pipe are recorded invasively with devices such as inline flow meters and thermocouples. Although invasive devices do feature an acceptable level of accuracy and reliability, they present numerous technical drawbacks—the majority of which are associated with fluid loss and contamination, coupled with ongoing pipe and device maintenance required with invasive systems.

Given the complications and drawbacks associated with invasive methods, non-invasive flow rate measurement methods have been described in the literature. These methods use ultrasonic, radioactive, and electric waves, as well as thermal, sonar, and vibration induced signals to determine the flow of a fluid within a pipe. While these newer non-invasive methods are able to measure the flow rate within acceptable accuracy, they are often expensive and require an extensive calibration and maintenance process. Furthermore, non-invasive meters are designed to measure the flow rate alone, rather than temperature, which further limits their widespread use in a range of residential, commercial, and industrial applications.

Considering the limitations of current non-invasive methods, a novel method that utilizes a thermal event to determine both flow rate, and the temperature of the fluid within the pipe, was developed. This method uses a non-invasive thermal interrogation (NITI) sensor referred to as the Combined Heat Flux and Temperature Sensor (CHFT+) to measure the heat flux and temperature response of a pipe when subject to a powered heater. The obtained measurements are then input to
a developed solution method and parameter estimation scheme to determine the flow rate and fluid temperature within the pipe. CHFT+ technology was developed by the Heat Transfer Measurements Laboratory at Virginia Tech in 2014. It was initially used to non-invasively measure blood perfusion in tissues but it was immediately recognized that the CHFT+ technology could be applied across a broader range of applications. However, long processing time and inaccurate results limited practical applications of the technology. Realizing this, the fundamental CHFT+ algorithms and data processing capability were redesigned by Ali R. Roghanizad to not only improve blood perfusion measurements, but to also allow for future CHFT+ applications. For the non-invasive pipe flow measurement application developed in this work, the CHFT+ is referred to as the British Thermal Unit (BTU) Meter.

The BTU Meter utilizes the heat flux and temperature measurements recorded by the sensor and thermocouple to determine values for the fluid’s convection heat transfer coefficient \( h \), fluid temperature \( T_{\text{fluid}} \), and the thermal contact resistance \( R' \) present between the thermocouple and pipe surface. These fluid and system parameters are then used to determine important pipe flow measurements. One major advantage of the BTU Meter solution and corresponding parameter estimation scheme is that it utilizes a heat flux sensor and a pipe surface thermocouple to determine the fluid’s flow rate and temperature. These measurements can determine the energy transferred with the flow. No other non-invasive thermal flow method uses measured heat flux and its corresponding temperature response to determine pipe flow measurements. Instead, they solely rely on the pipe’s measured temperature to determine different fluid flow rates.

The main body of this thesis consists of one manuscript that will remain unpublished until the intellectual property is protected. The manuscript documents the development of the new solution and parameter estimation scheme developed for the BTU Meter. The results of this work
illustrate the validity and reliability of the proposed method over the tested range, while also addressing constraints. The limitations of this work and recommended future work is presented towards the end of this thesis.
Chapter 2 – A Non-invasive Thermal Solution Method to Determine Fluid Flow Rate, Fluid Temperature, Convection Coefficient, and Thermal Contact Resistance of Internal Pipe Flow

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2.1 Abstract

This work proposes a novel solution method and parameter estimation scheme that is capable of non-invasively measuring the flow rate and temperature of fluid in a pipe. The solution method uses a Combined Heat Flux and Temperature Sensor (CHFT+), which is an apparatus that employs simultaneous heat flux and temperature measurements for non-invasive thermal interrogation (NITI). For this investigation, we refer to the non-invasive sensor as the British Thermal Unit (BTU) Meter, and employ it to directly determine values for the fluid’s convection heat transfer coefficient (h), the fluid’s temperature (Tfluid), and the thermal contact resistance (R”) between the BTU Meter and the pipe. This is done through analyzing the response of an imposed thermal event on the pipe. The measured heat flux and temperature response of the thermal event is used in conjunction with a parameter estimation scheme that compares the calculated temperature response, a function of heat flux and system parameters, to the measured temperature response. In this paper, both simulated and experimental measurements were used to validate the developed solution and parameter estimation scheme. Simulated measurements were generated with known fluid flow parameters, while the experimental measurements were recorded for water flowing through a copper pipe at different flow rates. A comparison of findings showed consistent results in terms of the water’s convection heat transfer coefficient (h), the fluid temperature (Tfluid), and the thermal contact resistance (R”). Once validation was complete, the convection heat transfer
coefficient and the corresponding flow rates were used to develop a correlation that allows the BTU Meter to directly output fluid flow rate and temperature. Results from the developed correlation were compared with two independent flow rate estimation methods, which displayed high levels of validity and agreement with a maximum difference of 5.0%.

**Keywords:** Pipe Flow Rate, Non-Invasive, Heat Flux, Fluid Temperature, Parameter Estimation, Convection Heat Transfer Coefficient, Thermal Contact Resistance, Energy Transfer, Non-Invasive Thermal Interrogation

### 2.2 Introduction

Fluid flow rate and temperature represent critical parameters for many residential, commercial, and industrial applications such as HVAC, oil and gas production, and in assessing power plant productivity and energy efficiencies. Although widely used British Thermal Unit (BTU) and flow meters can help quantify such measurements, they feature numerous drawbacks given they interfere with flow. Due to the losses and complications associated with breaking pipes and interrupted fluid flow, more non-invasive measurement techniques have been developed to quantify fluid flow that do not include these unwanted complications. Such measurement characteristics have proved to be desirable in many industrial applications as they require minimum maintenance, avoid contamination, and avoid loss of expensive or hazardous fluids [1]. While a number of non-invasive flow measurement methods have been developed, they exclusively measure flow rate alone and do not include temperature; moreover, they present numerous challenges that limit them to certain applications.

The most common non-invasive method used in industry uses ultrasonic waves to obtain the fluid flow rate in a pipe. While their accuracy can be within ±3%, ultrasonic flow meters come at a high cost and involve an extensive calibration process that requires knowledge of many
different parameters, some of which are difficult to obtain [1]. Ultrasonic flow meters also require a certain pipe geometry and fluid type that is chosen based on their color and composition [2]. These critical limitations have restricted the available applications for ultrasonic flow meters for easy and practical use.

Other types of non-invasive flow meters use sonar and vibration-based sensors to measure flow [2-4]. For example, the sonar flow meter uses an array of piezoelectric sensors that are wrapped around the pipe in order to capture the coherent vortical structures within the pipe flow. The tracked vortical structures are then used to calculate the flow speed and flow rate [2]. The sonar flow meter, however, is restricted to laminar flow and specific pipe geometry, both of which limit its scope of operation. The vibration-based meter, on the other hand, uses accelerometers to characterize vibrations induced by fully-developed turbulent flow. While such methods have been demonstrated to be promising, the uncertainties associated with different input parameters—not to mention unwanted noise from motor-related vibrations—have limited their usage [5]. Other methods in the literature include mechanical waves, electromagnetic waves, and radioactive rays to measure fluid flow non-invasively, all of which feature specific operational drawbacks that limit their usage [6-9].

During recent years, however, researchers and engineers have turned their attention to non-invasive pipe-flow measurement approaches that utilize thermal response measurements. In the literature, numerous investigations describe the use of pipe surface temperature as a means to estimate the flow flowing through it [10-12]. These methods utilize heaters and thermocouples to estimate flow rate by analyzing the temperature response of a pipe when heated. While the results from these methods may seem promising, their mathematical algorithms are based on temperature signals that tend to limit their accuracy. In their work, Roghanizad et al. (2017) confirmed that
thermal response measurements solely based on temperature signals (temperature boundary conditions) are inadequate for accurate results [13]. Specifically, the accuracy of thermal response measurement systems based only on temperature signals is limited by the time-step used in data acquisition. A more elaborate explanation and proof is shown in Appendix A.

Not only did Roghanizad et al. (2017) prove the inaccuracy of temperature boundary conditions, they also showed the advantages of using heat flux boundary conditions. In another investigation, Baker et al. (1993) reported that using a temperature solution derived from a heat flux boundary condition is smoother and less susceptible to noise when compared to a heat flux solution derived from temperature boundary condition [14].

This paper describes a novel solution method and parameter estimation scheme that determines fluid flow rate ($\dot{m}$), fluid temperature in a pipe ($T_{\text{fluid}}$), and the thermal contact resistance ($R''$) between the sensor and pipe surfaces, non-invasively and without interrupting the flow. This was accomplished by using a heat flux boundary condition and a Lumped Capacitance mathematical model for heat flux and temperature measurements obtained with a Combined Heat Flux and Temperature Sensor (CHFT+). In this work, the CHFT+ embodiment used for pipe flow measurements is referred to as the BTU Meter. The solution to the mathematical model was found with a heat flux input which makes this method more accurate than other purely temperature-based methods.

The BTU Meter records simultaneous heat flux and temperature signals that are input into a parameter estimation scheme that measures the pipe’s fluid flow convection heat transfer coefficient ($h$), fluid temperature ($T_{\text{fluid}}$) in the pipe, and the thermal contact resistance ($R''$) between the BTU Meter and pipe surface. These parameters are used to develop a correlation for the fluid flow rate in the pipe which, with a known fluid temperature, is related to the energy
transferred with the flow. The parameter estimation scheme developed for the BTU Meter is based on the fundamental approach introduced by the Roghanizad Parameter Estimation Method [13].

The results of the BTU Meter were tested with simulations and later verified with experimental testing. The experimental results were also validated with a commercially available FR1118P10 Inline Flowmeter by Sotera Systems (Fort Wayne, IN). As a final check, the experimental results were compared against measurements obtained by an independent differential sensing device developed by ThermaSENSE Corp. (Roanoke, VA) called ThermaGate.

2.3 Background of BTU Meter (CHFT+ Technology)

2.3.1 CHFT+ Design

The Combined Heat Flux and Temperature Sensor (CHFT+) is a thermal sensor that consists of a resistive heater, a heat flux sensor, and a thin-foil T-type thermocouple. Figures 1A and 1B provide a picture of a CHFT+ sensor and a schematic of the different components that make up the CHFT+ sensor, respectively.

![Figure 1. (A) Picture of a fully assembled CHFT+ sensor. (B) Schematic showing the layering of the components in a CHFT+ sensor. (Taken from Roghanizad et al. (2017))](image)

When placed on a pipe, the CHFT+ uses a heater to create a transient thermal event at the pipe surface and captures the response of both the heat flux sensor and the thin-foil thermocouple.
The response is then analyzed using a solution method and a parameter estimation scheme specifically developed for the BTU Meter. Using this approach, one is able to measure the pipe’s fluid flow convection heat transfer coefficient (h), fluid temperature ($T_{\text{fluid}}$) in the pipe, and the thermal contact resistance ($R''$) between the BTU Meter and pipe surface. Figure 2 illustrates how the BTU Meter is placed on the pipe. A detailed explanation of how the CHFT+ is produced and assembled can be found in Appendix B.

![Figure 2](image.png)

**Figure 2.** A schematic of how the BTU Meter is placed on a pipe to measure fluid flow parameters (h, $T_{\text{fluid}}$, and $R''$).

In this work, a HP 6255A Dual DC power supply is used to power the heater while a 16-channel 24-bit National Instrument DAQ (NI 9214) is used to record the CHFT+ measurements of heat flux and temperature. Details about the LABVIEW program used for the data acquisition can be found in Appendix C.

### 2.4 Development of Mathematical Model

#### 2.4.1 Determination of Appropriate Model

In order to relate the heat flux and temperature measurements obtained from the BTU Meter to pipe flow, a mathematical heat transfer model needed to be developed. Providing that the
thickness of the pipe is small relative to the heated pipe section area dimensions and the heater is larger than the heat flux sensor, one-dimensional conduction can be assumed. A more detailed investigation of the one-dimensional assumption is presented in Appendix D. Furthermore, due to the fact that pipes are typically fabricated with highly conductive materials (such as copper), the Lumped Capacitance Method (LCM) is appropriate to use. In particular, during a transient thermal event, the temperature of the heated pipe section is assumed to be spatially uniform and is only a function of time as $T(x,t) \approx T(t)$. In order to confirm this assumption, the Biot number is computed for a sample case. Equation 1 shows the Biot number equation and defines it as the ratio of conduction to convection thermal resistance,

$$Bi = \frac{hL_c}{k} = h \frac{V}{A_s} \frac{\delta}{k} = \left(\frac{\delta}{h}\right) \left(\frac{k}{h}\right)$$

where $h$ is the fluid convective heat transfer coefficient, $L_c$ is the characteristic length of the heated pipe section, $V$ is the volume of the heated pipe section, $A_s$ is the surface area of the heated pipe section, $\delta$ is pipe thickness, and $k$ is the pipe’s thermal conductivity. For the LCM to be valid, the Biot number needs to be less than $\frac{1}{10}$. For a moderate flow of water in a pipe, a typical fluid convection heat transfer coefficient can be found to be around $3000 \ \frac{W}{m^2 \cdot ^\circ C}$. Assuming a pipe’s thickness of 0.001 m and a pipe’s thermal conductivity of $200 \ \frac{W}{m \cdot ^\circ C}$, the Biot number is calculated as 0.015, which proves the validity of the LCM assumption. Appendix D further investigates the constraint imposed by the heater for the one-dimensionality criteria, and the spatial uniformity of the temperature along the thickness of the pipe for the LCM criteria.
2.4.2 Development of Lumped Capacitance Method

To apply an LCM model, a control volume analysis across the thickness of the pipe was conducted. Figures 3A and 3B show a thermal model diagram of the BTU Meter and a two-dimensional control volume of the section where the overall energy balance was conducted, respectively.

**Figure 3.** (A) Schematic illustrating the thermal model diagram of the BTU Meter setup on a pipe with fluid flowing through. (B) Schematic of the pipe thickness where the overall energy balance analysis was conducted.
By conducting an energy balance on Figure 3B, Equation 2 was developed, which states:

\[ \rho VC \frac{dT}{dt} = \frac{\partial E}{\partial t} = q_{\text{heater (conduction)}} - q_{\text{Fluid (convection)}} = q''_{\text{sensor}} A_{\text{heater}} - q''_{\text{Fluid}} A_{\text{heater}} \tag{2} \]

where \( \rho \) is the density of the pipe, \( V \) is the volume of the heated pipe section, \( C \) is the specific heat capacity of the pipe, \( A_{\text{heater}} \) is the cross sectional area of the heater, \( \frac{dT}{dt} \) is the change of temperature with respect to time; whereas \( q''_{\text{sensor}} \) and \( q''_{\text{Fluid}} \) are both the heat measured by the sensor and the heat leaving the pipe due to the fluid flow, respectively. The solution to Equation 2 for a circular pipe can be re-written as,

\[ \rho VC \frac{dT}{dt} = q''_{\text{sensor}} A_{\text{heater}} - h(T_{\text{pipe}} - T_{\text{fluid}}) A_{\text{heater}} \tag{3} \]

\[ \rho C A^* \pi (r_o^2 - r_i^2) L \frac{dT}{dt} = q''_{\text{sensor}} A^*(2\pi r_o) L - h(T_{\text{pipe}} - T_{\text{fluid}}) A^*(2\pi r_i) L \tag{4} \]

where \( L \) is the length of the heated section, \( A^* \) is the ratio of the actual area of the heater to the total circumferential area for a given \( L \), \( r_o \) is the outer radius of the pipe, \( r_i \) is the inner radius of the pipe, \( h \) is the fluid’s convection coefficient, and \( T_{\text{pipe}} \) and \( T_{\text{fluid}} \) are the pipe and fluid temperatures respectively. For a pipe with a small thickness, a ratio of the pipe’s inner and outer radii can be written as \( \frac{r_o}{r_i} \approx 1 \). Accordingly, Equation 5 was obtained, which states:

\[ \rho C \delta \frac{dT}{dt} = q''_{\text{sensor}} - h(T_{\text{pipe}} - T_{\text{fluid}}) \tag{5} \]

where \( \delta \) is the thickness of the pipe. While the heater size is included in Equation 3, the results for Equation 5 suggests that as long as the heater covers the sensor and is big enough to meet the one-dimensional conduction requirements, the heater size does not affect the derived solution.

For a spatially uniform heated pipe section, the LCM solution to Equation 5 can be written as follows,
where $T_{\text{pipe}}(t)$ is the temperature of the pipe at a given time, $T_{\text{pipe}}(0)$ is the initial pipe temperature prior to the transient thermal event, $T_{\text{pipe}}(\infty)$ is the final steady-state temperature of the pipe after the transient response, $t$ is the time it took for the transient thermal event to occur, and $\tau$ is the time constant needed for the thermal step response of the pipe to reach 63.2% of its final temperature value. For a single fundamental heat flux unit step, the pipe temperature response can be written as shown in Equation 7. Appendix E contains the derivation of the mathematical model and its solution using the fundamental unit response function.

The model developed in Equation 7 has three basic assumptions: 1) $T_{\text{fluid}}$ is constant throughout the data acquisition period, 2) $\Delta T_{across\,pipe\,thickness} \approx 0$, and 3) There is no thermal contact resistance between the BTU Meter thermocouple surface and the pipe wall surface. The first assumption can be verified by monitoring and controlling the fluid temperature during the data acquisition process, especially during short periods of testing. The second assumption can also be easily verified by choosing a pipe made with a high thermal conductive material and a thin wall thickness. The third assumption, however, is very difficult to confirm—especially with the uncertainties associated with how the BTU Meter is placed on the pipe. This factor makes the model in Equation 7 impractical as it is unrealistic to achieve, and would only work if there is a reliable way to measure the true pipe wall temperature. To account for the inherent contact resistance uncertainties associated with attaching the BTU Meter to a pipe, the model has been modified to include it, as shown in Equation 8 and Equation 9.
\[ T_{\text{pipe}}(t) = T_{TC(\text{measured})}(t) - q''(t) \times R'' \]  \[ T_{TC(\text{measured})}(t) - q''(t) \times R'' = (T_{TC(\text{measured})}(0) - q(0) \times R'') + \left( \frac{q''(0)}{R} \right) \left( 1 - e^{-\frac{t}{\tau}} \right) \]

where \( T_{TC(\text{measured})}(t) \) is the temperature recorded by the BTU Meter thermocouple, and \( R'' \) is the thermal contact resistance between the thin-foil thermocouple and the pipe. The added contact resistance value would make the temperature response measurements more accurate when analyzing fluid temperature.

The relationship between the fluid temperature and the pipe temperature is modeled as a simple convection heat transfer problem. Since Equation 8 considers the pipe temperature in terms of the measured thermocouple temperature, the fluid flow temperature can be determined. Equation 10 and 11 show the fluid temperature equation,

\[ q''(0) = h(T_{\text{pipe}}(0) - T_{\text{fluid}}) \]  \[ T_{\text{fluid}} = T_{TC(\text{measured})}(0) - q''(0) \times (R'' + \frac{1}{R}) \]

where \( T_{\text{fluid}} \) is the core fluid temperature flowing through the pipe. As per the assumptions of the model, the internal fluid temperature flowing through the pipe was considered to be constant throughout the data-acquisition period.

2.4.3 Development of Superposition Model

While the proposed model can measure the temperature response for a given fundamental unit step function, it does not provide an accurate representation of how the system truly works. In reality, the combination of subsequent heat flux measurements can be modeled as a series of unit step functions, which ultimately represent the total transient heat flux response. The solution model can be adjusted to account for the series of unit steps by utilizing the Duhamel Superposition Model for measured heat flux values which can be seen in Figure 4.
Using Figure 4 and the intention of using a heat flux boundary condition, the series of measured sensor heat flux in superposition can be expressed as

\[
q_{\text{Sensor}}(t) = q_{\text{Sensor}}(0) + \sum_{j=1}^{M-1} (q_j - q_{j-1})_{\text{Sensor}} \times H(t - t_j)
\]  \[12\]

where \(H(t - t_j)\) is a unit step function, and \(M\) is the total number of measurements recorded by the BTU Meter. Inserting the derived heat flux superposition model in Equation 12 into the base unit step model in Equation 9, the total temperature solution model can be achieved. Equations 13-15 show the superposition temperature solution model used for the BTU Meter. The calculated pipe temperature with the superposition assumption expressed in Equations 13-15 was written based on indices corresponding to the measurement indexes (m) made by the BTU Meter:
\[ T_{\text{pipe(calculated)},m} = T_{\text{pipe(measured)},0} + T_{\text{transient},m} \]  
\[ \text{where:} 
T_{\text{transient},m} = \sum_{j=1}^{m} \left( \frac{q_j'' - q_{j-1}''}{h} \right) \left( 1 - e^{-\left( \frac{t_m - t_j}{\tau} \right)} \right) \]  
\[ \text{Thus:} 
T_{\text{pipe(calculated)},m} = T_{\text{pipe(measured)},0} + \sum_{j=1}^{m} \left( \frac{q_j'' - q_{j-1}''}{h} \right) \left( 1 - e^{-\left( \frac{t_m - t_j}{\tau} \right)} \right) \]

Similar to the base unit step model in Equations 8 and 9, the superposition model would need to be adjusted to account for the thermal contact resistance. The adjusted temperature refers to the thermocouple calculated temperature and is written in Equation 16. The analytically modified thermocouple temperature model is now sufficient to use on the BTU Meter since it accounts for the thermal contact resistance that accompanies the BTU Meter installation process as well as the series of time steps that are a function of the thermal event imposed on the sensor. Regardless of the change in the model due to superposition, the fluid flow temperature equation remains the same as expressed in Equation 11—the only difference being the thermocouple temperature. Equation 17, therefore, represents the indicie form of Equation 11 and is used to calculate the internal fluid flow temperature. Similar to the fundamental unit model, the superposition model assumes the fluid temperature to be constant and uniform. Appendix E contains the detailed derivation of the superposition LCM model.

\[ T_{\text{TC(calculated)},m} = [(T_{\text{TC(measured)},0} - q''_0 \times R) + \sum_{j=1}^{m} \left( \frac{q_j'' - q_{j-1}''}{h} \right) \left( 1 - e^{-\left( \frac{t_m - t_j}{\tau} \right)} \right)] + q''_m \times R'' \]  
\[ T_{\text{fluid}} = T_{\text{TC(measured)},0} - q''_0 \left( R'' + \frac{1}{h} \right) \]

While the heat flux and temperature measurements are obtained from the BTU Meter, the convective heat transfer coefficient (h), time constant (\( \tau \)), and thermal contact resistance (R'') are variable and either depend on the flow speed, or the BTU Meter setup. Both h and \( \tau \) are a function of flow rate and are inversely proportional to each other; h increases with increasing flow while \( \tau \)
decreases with increasing flow. The contact resistance, however, is solely dependent on the way the BTU Meter is setup and how much air and what type of material is found between the BTU Meter and the pipe. In order to quantify the values of the three unknown parameters, a parameter estimation scheme was developed to determine the optimum values that would yield accurate results. The values were later used to correlate the flow rate in the pipe, as well as determine the fluid temperature in the pipe non-invasively.

2.5 Development of the Parameter Estimation Scheme

2.5.1 Overview

In order to evaluate the convective heat transfer coefficient \( (h) \), fluid flow temperature \( (T_{\text{fluid}}) \), and thermal contact resistance \( (R'') \), a parameter estimation scheme was developed. This parameter estimation scheme was inspired by the work of Roghanizad et al. (2017) on the blood perfusion application of the CHFT+ [13]. Thus, it features a similar theoretical approach but was customized and optimized for the BTU Meter. Specifically, the scheme utilizes the measured heat flux and temperature results from the BTU Meter as inputs to determine the optimum solution. This process is accomplished through the reduction of the three optimization parameters \( (h, \tau, R'') \) to only one parameter, which is then used to compute the other two parameters. This reduction is performed in order to reduce both run time and avoid the constriction of the three different parameters to specific, and perhaps uncharacteristic, limits. For the BTU Meter parameter estimation scheme, the convective heat transfer coefficient \( (h) \) was used to optimize results.

The optimum value of the convection coefficient \( (h_{\text{optimum}}) \) was determined by minimizing the root mean square error (RMSE) between the measured thermocouple temperature and the analytically calculated thermocouple temperature, while at the same time accounting for contact resistance. The RMSE function can be seen in Equation 18.
\[
\text{RMSE} = \sqrt{\frac{1}{M-1} \sum_{m=1}^{M-1} \left( T_{TC(\text{measured}),m} - (T_{TC(\text{calculated}),m}) \right)^2}
\]  

[18]

Figure 5A shows a 3D plot of the RMSE value obtained when plotted against h and R”.

The global minimum of the plot signifies the most ideal point when optimizing for the two different parameters. Projecting the plot into a two-dimensional plot of the RMSE and h, a plot similar to Figure 5B would appear. Similar to Figure 5A, the minimum point on the graph when optimizing for h represents the optimum point which yields \(h_{\text{optimum}}\), and \(R^{\prime\prime}_{\text{optimum}}\).

![Figure 5A](image1)

![Figure 5B](image2)

**Figure 5.** (A) Plot of the RMSE as a function of h and R” where the global minimum indicates the optimum point that corresponds to \(h_{\text{optimum}}\) and \(R^{\prime\prime}_{\text{optimum}}\). (B) 2D plot projection of the RMSE as a function of h.

### 2.5.2 Parameter Estimation Scheme

In order to optimize RMSE with only one unknown parameter (h), the mathematical model and the RMSE function were adjusted to only have the convective heat transfer coefficient (h) in the equations. To account for the \(\tau\) in Equation 16, Equation 19 is introduced,

\[
\tau = \frac{\rho C_p \delta}{h}
\]

[19]
where \( \rho \) is pipe density, \( C_p \) is the specific heat capacity of the pipe at constant pressure, \( \delta \) is pipe thickness, and \( h \) is the convective heat transfer coefficient of the fluid. Incorporating Equation 19 into Equation 16 would yield Equation 20, which eliminates the model’s dependency on \( \tau \) and introduces \( \rho \), \( C_p \), and \( \delta \), which are known pipe parameters.

\[
T_{TC(\text{calculated}),m} = \left[ (T_{TC(\text{measured}),0} - q''_0 \times R'') + \sum_{j=1}^{m} \left( \frac{q_j - q_{j-1}}{h} \right) \left( 1 - e^{-\left( \frac{(t_m-t_j)}{\rho C_p \delta} \right)} \right) \right] + q''_m \times R'' \quad [20]
\]

To account for thermal contact resistance \( (R'') \), Equation 21 is introduced as the general definition of contact resistance. Since the contact resistance is a function of the setup and is irrespective of the fluid flow and thermal event, it is modeled as constant with respect to time of measurement. This means that the contact resistance at any moment during the measurement is equivalent to that of any other time in the measurement domain. Equation 22 represents the mathematical interpretation of the constant contact resistance.

\[
R''_j = \frac{T_{TC(\text{measured}),j} - T_{pipe(\text{calculated}),j}}{q_j} \quad [21]
\]

\[
R'' = R''_j \quad [22]
\]

where \( j = 1, 2, \ldots, M - 1 \).

Since the thermal contact resistance is uniform at any point in the domain (as shown in Equation 22), the average of the contact resistance would yield the same resistance value at any given point. Furthermore, for accuracy purposes, and to avoid any bias due to unwanted noise, the thermal contact resistance is better represented as an average of the contact resistances across the measurement time domain. Equations 23 and 24 show the mathematical representation of the average contact resistance.
\[ R'' = \frac{\sum_{j=1}^{M-1} R''_j}{M - 1} \]  

\[ R'' = \frac{\sum_{j=1}^{M-1} T_{TC(measured),j} - T_{pipe(calculated),j}}{q_j} \]  

\[ R'' = \frac{\sum_{j=1}^{M-1} T_{TC(measured),j} - T_{pipe(calculated),j}}{M - 1} \]  

Equation 24 expresses the contact resistance in terms of known variables \( T_{TC(measured)} \), \( T_{TC(calculated)} \), and \( q_j'' \). This approach reduces the unknown parameters in the RMSE to only \( h \), which is demonstrated in the \( T_{TC(calculated)} \) term. The RMSE function can now be defined as shown in Equations 25 and 26. The new expression for the RMSE function allows for the mere optimization of the convective heat transfer coefficient (\( h \)) in the parameter estimation scheme.

\[ \text{RMSE} = \sqrt{\frac{1}{M-1} \sum_{m=1}^{M-1} \left( T_{TC(measured),m} - \left( T_{pipe(calculated),m} + q''_m \times \frac{\sum_{j=1}^{M-1} T_{TC(measured),j} - T_{pipe(calculated),j}}{q_j} \right) \right)^2} \]  

where: \( T_{pipe(calculated),m} = T_{pipe(measured),h} + \sum_{j=1}^{m} \left( \frac{q_j'' - q_{j-1}''}{h} \right) (1 - e^{-\frac{(t_m - t_j) \times h}{\rho C_p \delta}}) \)  

The developed parameter estimation scheme was programmed and executed in MATLAB. The MATLAB program was coded to include a range of \( h \) values, which were then used to determine the RMSE and output results for the optimum point values. The \( h_{optimum} \) result was then employed to calculate \( \tau_{optimum} \) and provide a corresponding \( R''_{optimum} \) and \( T_{fluid} \) temperature. The graphical interpretation of the parameter estimation scheme was developed to determine an optimal value for \( h \) that would match measured temperature curve. Additionally, the appropriate contact resistance is computed during this process. This approach indicates that there could only be one combination of the three parameters that would allow for the minimization of the RMSE function. An investigation of the physical interpretation is further discussed in the Simulated Data section. Appendix C includes the MATLAB code used for parameter estimation.
In order to assess the validity and the accuracy of the parameter estimate scheme, it was first tested on simulated data, after which it verified by analyzing experimentally acquired data. The next two sections (Section 2.6 and 2.7) illustrates the employed methodology and data results for the two assessment tests.

2.6 Simulated Data

2.6.1 Validity of the Parameter Estimation Scheme

In order to test the validity of the parameter estimation scheme, simulated data (1 Hz) was generated using the fixed parameters shown in Table 1. Specifically a simulated temperature curve was created using the model for a given heat flux input curve and fixed values of h, τ, R”, and T_{fluid}. The first 20 seconds of the simulated data mimics the temperature measured prior to the transient thermal event, while the next 100 seconds mimics the response of the temperature measurements for a transient thermal event. Once the simulated temperature curve was constructed, the data was analyzed using the MATLAB program developed for the parameter estimation. Using the MATLAB program, the RMSE function was computed for a range of h values from 0 to 10,000 \( \frac{W}{m^2\cdot C} \), with increments of 1, in order to identify h_{optimum}. Figures 6 and 7 illustrate the input heat flux curve, and its corresponding temperature curve generated with the superposition model at fixed parameter values, respectively.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>( h(\frac{W}{m^2\cdot C}) )</th>
<th>( \tau (s) )</th>
<th>( T_{fluid} (^\circ C) )</th>
<th>( R'' (^\circ C\cdot \frac{m^2}{W}) )</th>
<th>( \delta (m) )</th>
<th>( \rho (\frac{kg}{m^3}) )</th>
<th>( C_p (\frac{J}{kg \cdot \circ C}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>1433</td>
<td>3.05</td>
<td>25.0</td>
<td>0.00050</td>
<td>0.00127</td>
<td>8940</td>
<td>385</td>
</tr>
</tbody>
</table>
Figure 6. Heat flux curve used in simulated data

Figure 7. Thermocouple temperature curve used in simulated data
Running the simulated data in the MATLAB program took about 0.1 seconds to execute and yielded accurate and high resolution results \((\pm 1 \ \frac{W}{m^2 \cdot ^\circ C})\) with a standard deviation of 0, as expected for simulated data. The comparison between the simulated results and the estimated results is illustrated in Table 2, where excellent agreement is shown given a RMSE error of 10\(^{-6}\).

**Table 2.** Value of the input and estimated parameters by the MATLAB parameter estimation program

<table>
<thead>
<tr>
<th>Parameter</th>
<th>(h(\frac{W}{m^2 \cdot ^\circ C}))</th>
<th>(\tau(s))</th>
<th>(R''(^\circ C \frac{m^2}{W}))</th>
<th>(RMSE(^\circ C))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input</td>
<td>1433</td>
<td>3.05</td>
<td>0.00050</td>
<td>–</td>
</tr>
<tr>
<td>Estimated</td>
<td>1433</td>
<td>3.05</td>
<td>0.00050</td>
<td>10(^{-6})</td>
</tr>
</tbody>
</table>

Figure 8 plots the simulated temperature curve, the pipe temperature curve, and the estimated temperature curve (accounting for contact resistance). As expected, the pipe temperature curve has a vertical offset from the simulated curve due to the simulated presence of thermal contact resistance. A plot of the RMSE as a function of the convection coefficient is presented in Figure 9, which matches the expected response shown in Figure 5B. These results demonstrate the validity of the parameter estimation scheme employed, as evidenced by the fact that the estimated temperature curve showed no deviation from the simulated temperature curve.
Figure 8. Plots of the simulated input, estimated, and calculated pipe temperature curves.

Figure 9. Plot of the RMSE as a function of h where the lowest point signifies the optimum h and thus the optimum values for all other parameters (functions of h).
2.6.2 Sensitivity to Noise in Temperature

In order to analyze the effect of the uncertainty in the temperature accuracy that may occur in the experimental measurements, a temperature noise sensitivity analysis was conducted. Since the accuracy of T-type thermocouples are within ± 0.5 °C, the noise introduced into the simulated data was tested within that range. Random values between ± 0.5 were generated using a random number generator, which were then added to the simulated curve. This new simulated data with noise was analyzed using the same MATLAB program used for the clean simulated data. Figure 10 provides a plot of the temperature curves, while Table 3 lists the numerical results of the parameter estimation conducted on the same temperature curves.

![Simulated Temperature Curves with and without Noise](image)

**Figure 10.** Simulated temperature curves with and without a ± 0.5°C noise variation.
Table 3. Values of estimated parameter results from the parameter estimation scheme using simulated data with and without ± 0.5°C noise.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>h</th>
<th>R”</th>
<th>RMSE(°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input (Simulated)</td>
<td>1433</td>
<td>0.00050</td>
<td>−</td>
</tr>
<tr>
<td>Estimated (No Noise)</td>
<td>1433</td>
<td>0.00050</td>
<td>10⁻⁶</td>
</tr>
<tr>
<td>Estimated (With Noise)</td>
<td>1400</td>
<td>0.00048</td>
<td>0.4140</td>
</tr>
</tbody>
</table>

The results in Table 3 show a 2% error between the h values obtained for the estimated curves with and without simulated noise. Although small, the 2% error showcases the parameter estimation scheme’s vulnerability to noise. Furthermore, the contact resistance showed an error of about 4%, and the RMSE, although minimal, showed a significant increase due to the simulated noise. These results highlight the importance of minimizing any uncertainties in temperature measurements obtained during the data acquisition period. This can be achieved by grounding and shielding the thermocouple wires to mitigate for any noise associated with electrical interference.

2.6.3 Non-dimensional Sensitivity Analysis

In order to investigate the sensitivity of $T_{Te(calculated)}$ with respect to the convective heat transfer coefficient (h), the time constant (τ), and the thermal contact resistance (R”), a non-dimensional sensitivity analysis was introduced. This approach provides a graphical feedback of how each parameter’s sensitivity changes with respect to time, thereby enabling one to identify the most appropriate section in the measurement domain to use for the solution and parameter estimation scheme. Equations 27-29 show the dimensionless sensitivity equations for h, R”, and τ respectively. Table 4 lists the values used for the parameters in the non-dimensional sensitivity analysis. To determine the non-dimensional sensitivity equation for h, Equation 27 states the following:
\[
X_R^+ = \frac{\partial T_{TC(\text{calculated})}}{\partial h/h_N} = \frac{\left(T_{TC(\text{calculated})}(h_p) - T_{TC(\text{calculated})}(h_N)\right)}{T_{Ref}}
\]

where \(h_p\) and \(h_N\) are the perturbed and nominal convection heat transfer coefficient values, respectively and \(T_{Ref} = T_{TC(\text{calculated}),M-1} - T_{TC(\text{calculated}),\text{initial}}\). Equation 28 can be used to identify the non-dimensional sensitivity equation for \(R''\), as follows:

\[
X_{R''}^+ = \frac{\partial T_{TC(\text{calculated})}}{\partial R''/R''_N} = \frac{\left(T_{TC(\text{calculated})}(R''_p) - T_{TC(\text{calculated})}(R''_N)\right)}{T_{Ref}}
\]

where \(R''_p\) and \(R''_N\) are the perturbed and nominal thermal contact resistance values, respectively.

To find the non-dimensional sensitivity equation for \(\tau\), Equation 29 states the following:

\[
X_{\tau}^+ = \frac{\partial T_{TC(\text{calculated})}}{\partial \tau/\tau_N} = \frac{\left(T_{TC(\text{calculated})}(\tau_p) - T_{TC(\text{calculated})}(\tau_N)\right)}{T_{Ref}}
\]

where \(\tau_p\) and \(\tau_N\) are the perturbed and nominal time constant values, respectively.

**Table 4.** Value of parameters used in non-dimensional sensitivity analysis.

<table>
<thead>
<tr>
<th>Convection Coefficient ((h))</th>
<th>Thermal Contact Resistance ((R''))</th>
<th>Time Constant ((\tau))</th>
<th>(T_{ref} = T_{Sensor,M-1} - T_{Sensor,0})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(h_p\left(\frac{W}{m^2 - C}\right))</td>
<td>(h_N\left(\frac{W}{m^2 - C}\right))</td>
<td>(R''_p(\text{C} m^2 W))</td>
<td>(R''_N(\text{C} m^2 W))</td>
</tr>
<tr>
<td>Value 1434</td>
<td>1433</td>
<td>0.00051</td>
<td>0.00050</td>
</tr>
</tbody>
</table>

The results of the non-dimensional sensitivity analysis are illustrated in Figure 11. Note that the first 20 seconds show that all the sensitivity values are equal to zero; this is due to the fact that the parameters were modeled for the transient event—and thus only show their sensitivity
after the heater was powered. With regard to the thermal contact resistance and the convection coefficient curves, both parameters displayed a small sensitivity in the transient state and increased with time as the temperature stabilized to steady-state. As the solution achieved the final steady-state temperature, the convection coefficient (h) and the thermal contact resistance (R") sensitivities remained relatively stable. The thermal contact resistance and convection heat transfer coefficient have opposite sign sensitivities due to their opposing effects on the curve. As h increased, the height of the steady-state temperature decreased; conversely, as R” increased, the height of the temperature steady-state increased.

**Figure 11.** Non-dimensional sensitivity curves for convection heat transfer coefficient (h), thermal contact resistance (R”), and time constant (τ).
Furthermore, the results show that the sensitivity of the time constant ($\tau$) was greatest in the transient domain (since it is a function of the transient response), and converged to zero as time increased and steady-state temperature conditions were achieved. Thus, the curves illustrate three conclusions: 1) The most appropriate domain for obtaining the contact resistance ($R''$) is within the final steady-state temperature region; 2) The time constant ($\tau$) can only be estimated during the transient domain; and 3) While the convection coefficient ($h$) shows high sensitivity within the final steady-state temperature domain, it cannot be distinguished from contact resistance ($R''$). Thus, the convection coefficient must be estimated as a function of $\tau$, and thus determined during the transient region.

### 2.6.4 Graphical Interpretation of $h$, $\tau$ and $R''$

To further investigate the effects of $h$, $\tau$, and $R''$, simulated temperature curves were generated using different values for each parameter. When simulating a parameter (e.g. $h$), all other parameters (e.g., $R''$ and $\tau$) kept constant in order to understand the real and total effect of the simulated parameter.

#### 2.6.4.1 Graphical Interpretation of Convection Heat Transfer Coefficient ($h$)

Three different values of the convection heat transfer coefficient ($h$) were used to generate new simulated data. The temperature response curves for each data value are plotted in Figure 12 and analyzed. The results show that for a given fixed value of $R''$ and $\tau$, an increase in $h$ would decrease the steady-state final temperature of the curve. Since the magnitude of the convection coefficient is a function of flow rate, higher values of $h$—and consequently lower steady-state final temperatures (at the same heat flux)—are associated with higher flow rates.
2.6.4.2 Graphical Interpretation of Time Constant (τ)

Three sets of simulated data were generated with a different time constant (τ) for each. The results for the three different data sets are illustrated in Figure 13. Specifically the figure confirms that for the three different τ values, the temperature curves matched before and after the transient domain, which was expected as τ is a parameter of the transient region only. In the transient domain, however, the τ was susceptible to value difference. From the figure, it can be concluded that as τ decreases, the curve reaches the steady-state temperature faster. Since τ is inversely proportional to h (as previously expressed in Equation 19), a decrease in τ signifies an increase in h, which in turn indicates an increase in the fluid flow rate.

Figure 12. Simulated temperature response curves for three different convection coefficient.
Figure 13. Simulated temperature response curves for three different time constants.

2.6.4.3 Graphical Interpretation of Thermal Contact Resistance (R")

Similarly, different values for the thermal contact resistance (R") were used to generate new simulated data. Figure 14 shows the plotted curves for the new simulated data. The results illustrate that an increase in R" results in an increase in the steady-state final temperature. Since R" is a function of how well the thermocouple reads the temperature values, an increase or decrease in the thermal contact resistance depends on how well the thermocouple is placed on the system. Contrary to h and τ, the thermal contact resistance of the measurements does not give an indication of the fluid flow in the pipe.
2.6.4.4 Distinction between Convection Coefficient (h) and Thermal Contact Resistance (h)

As illustrated in Sections 2.6.4.1 and 2.6.4.3, both h and R” affected the height of the final steady-state temperature value. Since the parameter estimation scheme is designed to estimate both values simultaneously, a more detailed analysis is needed to distinguish between the independent effects of the two values.

While the non-dimensional sensitivity analysis presented in Section 2.6.3 shows the highest sensitivity of h in the final steady-state domain, it constrains the \( \tau \) in the model and makes h only a function of the steady-state temperature height. In reality, however, the estimated h value

**Figure 14.** Simulated temperature response curves for three different thermal contact resistance values.
is dependent on the \( \tau \) value that matches in the transient region. Therefore, \( h \) is most suitably estimated during the transient domain through matching \( \tau \) to the transient rise of the curve.

To demonstrate the differences between \( h \) and \( R'' \) in a graph, the overall heat transfer coefficient is detailed in Equation 30, which states the following:

\[
U = \frac{1}{h} + R'' \tag{30}
\]

where \( U \) is the overall heat transfer coefficient, \( h \) is the convection heat transfer coefficient, and \( R'' \) is the thermal contact resistance. For a fixed value of \( U \), the final steady-state temperature is the same, regardless of the \( h \) and \( R'' \) values. Therefore, for a given fixed \( U \), the value of \( h \) must be determined based on behavior in the transient region. This difference allows one to differentiate the determination of \( h \) and \( R'' \) in the parameter estimation scheme. Thus it can be concluded that \( h \) is best estimated for in the transient region and \( R'' \) is best estimated for in the steady-state region. Although, \( h \) is optimized in the entirety of the thermal response domain, it is differentiated from \( R'' \) in the transient region. Figure 15 illustrates the regions where \( h \) and \( R'' \) are determined, and three of their possible combinations that provide the same overall \( U \) value.
Figure 15. Simulated temperature response curves with different $h$ and $R''$ combinations for the same overall $U$ value.

2.7 Experimental Data

2.7.1 Experimental Methodology and Setup

An experimental setup was constructed to analyze the results obtained from the BTU Meter at different fluid flow rates. The parameter estimation scheme developed in Section 2.5 was employed to obtain the optimum results for the pipe’s fluid flow convection heat transfer coefficient ($h$), fluid temperature ($T_{\text{fluid}}$) in the pipe, and the thermal contact resistance ($R''$) between the BTU Meter and pipe surface. The optimal corresponding values were then utilized to create a correlation that relates the convection heat transfer coefficient to the flow rate in the pipe, and the flow’s Nusselt number to its Reynolds number.
The experimental setup included a submersible pump with a check valve to control the fluid flow rate. The pump was connected to a ¾” (0.01905m) inner diameter L type copper pipe with a 0.05” (0.00127m) wall thickness. This water was delivered through the pipe from a 25-gallon water reservoir, where the discharge was recirculated back into the same water reservoir. The BTU Meter was installed on the copper pipe with double-sided Kapton (polyimide) tape, and wrapped with self-fusing silicon compression seal tape. Table 5 provides the copper pipe’s physical parameters that were used in the parameter estimation scheme. Figures 16 and 17 provide images of the schematic and the actual experimental setup, respectively. A HP 6255A Dual DC power supply, supplied power to the resistive heater whereas the BTU Meter was connected to a 16-channel, 24-bit NI-DAQ for data acquisition. The DAQ was grounded to the pipe and the thermocouple wires were shielded to minimize noise from electrical interference.

Table 5. Values of the copper pipe’s physical parameters used for the data analysis.

<table>
<thead>
<tr>
<th>$\rho (\frac{kg}{m^3})$</th>
<th>$c_p (\frac{J}{KgK})$</th>
<th>$\delta (m)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>8940</td>
<td>385</td>
</tr>
</tbody>
</table>
Figure 16. Schematic of the experimental setup where recirculated water runs through the copper pipe with the BTU Meter attached to it.

Figure 17. The actual BTU Meter experimental setup used for data acquisition.
The BTU Meter was used to take measurements at flow rates of \( \frac{1 \text{ gal}}{\text{min}} \), \( \frac{4 \text{ gal}}{\text{min}} \), \( \frac{7 \text{ gal}}{\text{min}} \), \( \frac{10 \text{ gal}}{\text{min}} \), \( \frac{13 \text{ gal}}{\text{min}} \), and \( \frac{16 \text{ gal}}{\text{min}} \). The flow rates were determined using a 5-gallon bucket and a stop watch. The time it took the water to fill the 5 gallon bucket was recorded to compute the water flow rate. At each flow rate, the BTU Meter recorded heat flux and temperature measurements for 130 seconds total. The first 20 seconds of the data acquisition captured the initial steady-state values prior to the introduction of the thermal event, which was then followed by 110 seconds under heating supply. The temperature of the water was also monitored and recorded via a thermocouple in the reservoir. Monitoring the water temperature allowed for the verification of the constant water temperature assumption. The recorded water temperature was then compared with the calculated \( T_{\text{fluid}} \) obtained through the parameter estimation. It should be noted that the water temperature would rise during long periods of data acquisition due to the heat and friction released from the pump. This rise, however, was accounted for in the data analysis.

Once measurements for the convection heat transfer coefficient (h) were obtained and the calculated \( T_{\text{fluid}} \) value was verified to match recorded water temperature, a convection coefficient-flow rate correlation was developed. This enabled the BTU Meter to determine the flow rate of water in the pipe as a function of \( h_{\text{optimum}} \) as determined when analyzing the recorded measurements. The flow rates obtained from the correlation were compared to results acquired from a FR1118P10 Inline Flowmeter by Sotera Systems (Fort Wayne, IN), and an independent differential thermal sensing device by ThermaSENSE Corp. (Roanoke, VA) called ThermaGate.

2.7.2 Experimental Results

The experimental data was collected over a period of three days, which involved conducting two runs for a given flow rate for each data set. Additionally, the measurements for
each flow rate were repeated at different days and at different water temperature conditions to ensure the repeatability of the results. The collected data showed high repeatability and consistently reliable results. For example, across all flow rates, the highest standard deviation in the convection coefficient (h) was $11 \frac{W}{m^2\cdot{C}}$. Table 6 lists the summary analysis results from the parameter estimation scheme for the collected data sets at the six desired flow rates. The same program used to simulate the data (see Section 2.6) was also employed for experimental data processing. To reiterate, the parameter estimation program has a resolution of $(1 \frac{W}{m^2\cdot{C}})$ and executes in about 0.1 seconds. A complete table of all the data results is presented in Appendix F.

Figures 18 and 19 depict a sample heat flux and temperature measurements recorded and analyzed with the MATLAB program at two different flow rates. The figures show how for a given heat flux, temperature response measurements change as a function of h and flow rate.

| Flow Rate (gal.min$^{-1}$) | Number of Trials | $h_{mean}$ (W/m$^2\cdot{C}$) | $h_{std}$ (W/m$^2\cdot{C}$) | $\tau_{mean}$ (s) | $R''_{mean}$ (m$^2$/{°C}.W) | RMSE$_{mean}$ (°C) | $|\Delta T_{Water,mean}^{(°C)}|=|T_{measured} - T_{estimated}|$ |
|--------------------------|-----------------|--------------------------|--------------------------|-----------------|-------------------------|----------------|-------------------------------------------------
| 1                        | 6               | 828                      | 5                        | 5.3             | 0.000331                | 0.119         | 0.11                                           |
| 4                        | 6               | 1383                     | 11                       | 3.2             | 0.000329                | 0.039         | 0.12                                           |
| 7                        | 6               | 1590                     | 5                        | 2.8             | 0.000329                | 0.023         | 0.05                                           |
| 10                       | 6               | 1688                     | 4                        | 2.6             | 0.000332                | 0.022         | 0.06                                           |
| 13                       | 6               | 1796                     | 7                        | 2.4             | 0.000330                | 0.019         | 0.13                                           |
| 16                       | 6               | 1867                     | 4                        | 2.4             | 0.000330                | 0.021         | 0.14                                           |

Table 6. Summary of results for the mathematical mean for data collected at the six desired flow rates.
Figure 18. Sample of heat flux measurement recorded and analyzed in the parameter estimation MATLAB code at different flow rates.

Figure 19. Sample of temperature measurement recorded and analyzed in the parameter estimation MATLAB code at different flow rates.
The results displayed in Table 6 confirm the expected correlation between the fluid’s heat transfer convection coefficient (h) and flow rate. As described in Section 2.6.4 (and shown in Table 6), the convection transfer coefficient (h) increased as the flow rate of the fluid increased. In addition, the obtained standard deviation confirmed the consistency of results and showed the parameter estimation’s ability to distinguish h from R” in each of the flow rate as h values were confined within the small standard deviation values. Furthermore, the time constant (τ) was observed to decrease as the flow rate increased; this outcome was also expected as described in Section 2.6.4.

The thermal contact resistance (R”) was observed to be relatively constant with a small standard deviation (±8.35E-07), which is likely attributable to the noise in the results. This fairly constant thermal contact resistance value validates the consistency of the parameter estimation scheme, as it was able to provide matching and accurate values for the resistance at different flow rates. Figure 20 represents the relationship of the thermal contact resistance as a function of flow rate.
Figure 20. A scattered plot of the thermal contact resistance as a function of flow rate, where resistance is observed to be constant at all flow rates.

Table 6 also provides RMSE values obtained by matching the $T_{TC(measured)}$ curve to $T_{TC(calculated)}$. For flow rates of $4 \frac{gal}{min}$ and higher, the RMSE between the two curves was small, which indicates a strong agreement between the measured and calculated thermocouple temperatures obtained with the experimental results. However, with $1 \frac{gal}{min}$ the RMSE value was higher, which meant that there is a higher discrepancy between the measured and calculated temperature values. The error between the two temperature curves was only 3% of the total temperature rise and thus was still considered minimal. Similar to the simulated data, the minimum RMSE value is plotted in Figure 21 as a function of the convection heat transfer coefficient (h) to show the minimum RMSE yielding optimized results. The global minimum in the experimental results is not expected to be as distinct as shown in simulated results. This is attributed to the drop in quality of the parameter estimation scheme as result of measurement noise. However, since the
BTU Meter’s measurement noise was minimized by grounding, the global minimum in experimental results was still distinct in an excellent manner.

**Figure 21.** Plot of the RMSE as a function of h for a sample of experimental data at h iterations of one.

Furthermore, Table 6 shows the mean of the absolute difference between the calculated and measured temperatures $|\Delta T_{\text{Water}}|$. The results showed a maximum absolute difference of 0.14°C between the two values. This relative accuracy is excellent given T-type thermocouple based measurements are only accurate within half a degree centigrade. The minimal difference in measured and calculated temperatures suggests strong agreement between the two values. Therefore, it can be concluded that the fluid temperature results further verify the accuracy of the results obtained from the model and parameter estimation scheme. Figure 22 shows the
relationship between the measured and estimated temperature difference as a function of flow rate, which confirms the independency of the fluid calculated temperature results from flow rate.

**Figure 22.** Relationship between the measured and estimated absolute temperature difference as a function of flow rate.

The MATLAB program not only delivered optimized values for the desired parameters, but it also provided visual representations of the measured thermocouple temperature \((T_{TC(measured)})\), the calculated thermocouple temperature \((T_{TC(calculated)})\), and its corresponding true pipe temperature \((T_{pipe})\). All three curves are presented in Figure 23, which shows a sample of the plotted curve output from the MATLAB program. This figure documents that the measured and calculated thermocouple temperatures match, as well as shows their relationship with the pipe temperature. As discussed in Section 2.6.4, the difference between the temperature curves signifies the presence of thermal contact resistance \((R'')\) in the thermocouple temperature measurements.
2.7.3 Experimental Correlations

2.7.3.1 Correlations found in Literature

There are a number of correlations described in the literature that help relate the convective heat transfer coefficient (h) to pipe flow rate. Typically, these relationships are devised through the use of non-dimensional parameters such as the Nusselt and the Reynolds numbers. In a pipe, the Nusselt number is directly proportional to h and can be defined as follows:

$$ Nu_D = \frac{hD_i}{k} $$

where $Nu_D$ is the Nusselt number, $h$ is the convective heat transfer coefficient, $D_i$ is the inner diameter of the pipe, and $k$ is the thermal conductivity of the fluid. Furthermore, the Reynolds number in a pipe is directly proportional to its flow and is defined as in Equation 32:
\[ Re_D = \frac{4\dot{m}}{\pi D_i \mu} \]  

where \( Re_D \) is the Reynolds number, \( \dot{m} \) is the fluid flow rate, \( D_i \) is the inner diameter of the pipe and \( \mu \) is the fluid’s dynamic viscosity at the bulk fluid temperature. Therefore, for a correlation that combines the Nusselt and Reynolds numbers, the flow rate of a fluid in a pipe with known dimensions and fluid temperature can be determined by the convective heat transfer coefficient of the fluid.

Additionally, various equations in the literature document the correlation between the Nusselt number and the Reynolds number for both internal and external flow. For a turbulent forced-convection flow in a pipe, the Dittus-Boelter and the Sieder-Tate are commonly used. Both correlations are only valid for fully developed flow, where \( \frac{L}{D_i} \geq 10 \), and a fully turbulent flow of \( Re_D \geq 10,000 \). Equations 33 and 34 present the Dittus-Boelter and the Sieder-Tate correlations for heating, respectively. The equations state,

\[ Nu_D = 0.023 \, Re_D^{0.8} \, Pr^{0.4} \]  \[ \text{[33]} \]

\[ Nu_D = 0.027 \, Re_D^{0.8} \, Pr^{0.3} \left( \frac{\mu}{\mu_s} \right)^{0.14} \]  \[ \text{[34]} \]

where \( Pr \) is the Prandtl number, and \( \mu_s \) is the fluid viscosity at the heat transfer boundary surface temperature. By combining Equations 31-32 with 33-34 and solving for flow rate, one would be able to determine a fluid’s flow rate using the other known parameters, which include the fluid’s convection heat transfer coefficient (\( h \)), and fluid temperature (\( T_{\text{fluid}} \)) with known properties and a pipe with known dimensions.
2.7.3.2 Development of the BTU Meter Correlation

As previously noted, the BTU Meter system records the heat flux and temperature response signals and, using a parameter estimation scheme, determines (a) the optimal values for the convection heat transfer coefficient \( h \), (b) the internal fluid temperature \( T_{\text{fluid}} \), and (c) the thermal contact resistance \( R'' \) of the system. One underlying assumption associated with BTU Meter measurements obtained at turbulent flows is that the rapid flow does not allow the heat to dissipate into the center of the fluid. Thus, the rapid removal of heat from the water column in turbulent flow does not permit the BTU Meter to determine the total convection heat transfer coefficient of the flow. This is due to the limited thermal penetration of the BTU Meter resulting in measurements that are more descriptive of the laminar sublayer present in the turbulent flow. Therefore, the BTU Meter only determines the convection coefficient dominant in the sublayer. Given this assumption, the use of the convection coefficient \( h \) obtained from the BTU Meter would not be suitable to use in the correlations described in Section 2.7.3.1, since the aforementioned correlations were developed using the true convection coefficient of the fluid flow. However, a custom correlation between the sublayer convection coefficient and overall flow rate can be found.

A correlation for the Nusselt number as a function of the Reynolds number was developed and compared to the correlations found in literature. The BTU Meter correlation was constructed using the optimized results obtained from the experimental setup at different flow rates (see Table 6 and Appendix F). Only the results obtained for flow rates of \( \frac{4 \text{ gal}}{\text{min}} \) and higher were used for the correlation as they alone were able to satisfy the fully turbulent condition criteria in the literature; the \( \frac{1 \text{ gal}}{\text{min}} \) flow rate is considered to be in the transitional region between laminar and fully turbulent
flow and was thus not used in this paper. However, correlations that include the $\frac{1\ \text{gal}}{\text{min}}$ flow rate are presented in Appendix G. Figure 24 displays a plot of the obtained BTU Meter correlation for forced internal convection of water through a pipe.

![Nu vs Re for BTU Meter](image)

**Figure 24.** The BTU Meter Nusselt vs Reynolds correlation for forced internal convection of water flow through a pipe.

Figure 24 provides a plot for the Nusselt and Reynolds numbers for the different flow rates with the best-fit curve that matches the results. The corresponding best-fit line was determined to be the Nusselt-Reynolds correlation for the BTU Meter, which is shown in Equation 35.

$$Nu = 5.4607 \ Re^{0.2137} \quad [35]$$

By comparing the BTU Meter correlation in Equation 35 to the Dittus-Boelter and the Sieder-Tate correlations in Equations 33 and 34, respectively, it is clear that there is a demonstrable difference in correlation assumptions. Specifically, Equations 33 and 34 show the Reynolds having
a high power number closer to one while the BTU Meter correlation shows a much smaller Reynolds power. In fact, the power of the Reynolds in the BTU correlation is very close to the $\frac{1}{3}$ power that is characterized by the laminar flow described in literature investigations. The power of the Reynolds number in the BTU Meter correlation strongly suggests that the results obtained in the setup behave more like a laminar flow than a turbulent flow. Taking into account that the tested Reynolds numbers actually surpassed the minimum turbulent flow criteria, one can corroborate that the assumption of convection coefficient ($h$) being predominantly measured in the laminar sublayer is valid. Results of the investigation of the thickness of thermal penetration in the laminar sublayer is noted in Appendix D. This finding indicates that the turbulent correlations presented in prior literature reports are not applicable for the BTU Meter, as it does not correctly describe the obtained results. Comparisons between the convection coefficient values obtained from the BTU Meter measurements and the Dittus-Boelter equation are presented in Appendix H.

In an attempt to revise this correlation in terms of values directly optimized by the parameter estimation scheme—as well as to non-invasively determine the flow of the water irrespective of the pipe diameter—a correlation between the convection coefficient and flow rate was created. Since the Nusselt and the Reynolds are non-dimensional indicators of $h$ and flow rate, respectively, the solution for the new correlation should have the same power value as the $Nu - Re$ correlation. Similar to the method used to create the correlation in Equation 35, the $h - \dot{V}$ correlation in Equation 36 is plotted in Figure 25. For a more practical correlation in determining the flow rate from the BTU Meter output, Equation 37 was developed and is plotted in Figure 26.

$$h = 1035.2 \left[ \frac{W}{m^2 - ^\circ \text{C}} \left( \frac{\text{min}}{\text{gal}} \right)^{0.2137} \right] \dot{V}^{0.2137}$$

[36]
\[
\dot{V} = 1 \times 10^{-14} \left[ \frac{gal}{min} \left( \frac{m^2}{W} \right)^{4.6475} \right] h^{4.6475}
\]

\[h = 1035.2 \sqrt{v^{0.2137}}\]
\[R^2 = 0.9933\]

**Figure 25.** The BTU Meter convection coefficient vs flow rate correlation for forced internal convection of water flowing through a pipe.

**Figure 26.** The BTU Meter flow rate vs convection coefficient correlation for forced internal convection of water flowing through a pipe.
2.7.4 Flow Rate Comparisons with other Methods

Based on the developed correlations, the BTU Meter was tested against two independent flow rate and fluid temperature measurement devices. The first device utilizes a commercially available FR1118P10 Inline Flowmeter by Sotera Systems (Fort Wayne, IN) combined with a fluid thermocouple. The second device used was ThermaGate by ThermaSENSE Corp. (Roanoke, VA). ThermaGate technology relies on a differential sensing method to determine fluid flow rate and temperature in a pipe.

The Sotera Inline Flowmeter was used to quantitatively verify flow rate results obtained with the BTU Meter. Furthermore, in order to verify the accuracy of the BTU Meter’s calculated water values, they were compared against water temperature measurements taken with a T-type thermocouple. The ThermaGate sensor was used as a final check to make sure all results agreed. A summary of the results obtained from a sample run for all three methods is presented in Table 7. Note that the flow rate values for the BTU Meter in Table 7 were obtained using Equation 37.

Table 7. Comparison of results between the BTU Meter System, an inline flow meter, and the ThermaGate technology.

<table>
<thead>
<tr>
<th>Inline Flowmeter</th>
<th>BTU Meter Correlations</th>
<th>ThermaGate Technology</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow Rate (GPM)</td>
<td>T_{Water} (measured) (°C)</td>
<td>Flow Rate (GPM)</td>
</tr>
<tr>
<td>4.0</td>
<td>23.84</td>
<td>3.8</td>
</tr>
<tr>
<td>7.0</td>
<td>24.55</td>
<td>7.3</td>
</tr>
<tr>
<td>10.0</td>
<td>29.69</td>
<td>9.8</td>
</tr>
<tr>
<td>13.0</td>
<td>26.40</td>
<td>13.1</td>
</tr>
<tr>
<td>16.0</td>
<td>26.42</td>
<td>15.9</td>
</tr>
</tbody>
</table>
The highest percentage difference recorded for the flow rate between the BTU Meter correlations and the Sotera Inline Flowmeter was 5.0%. The mean of the percent errors between the Sotera Flow meter and the BTU Meter flow rates was 2.5% with standard deviation of ±2.0%. The BTU Meter accuracy presented (±1.25%) shows comparable accuracy to the inline flowmeter of ±1%. The highest absolute temperature difference between the measured and calculated water temperature was 0.18°C, which is fairly minimal given the inherent thermocouple measurement uncertainties. Furthermore, the mean absolute difference between the measured and calculated water temperatures was 0.152 ± 0.06 °C. Results obtained from the ThermaGate sensor showed a high level of agreement with the results obtained from both the BTU Meter and Sotera Inline Flowmeter. This confirms the accuracy of the compared methods and further validates the BTU Meter. Thus, based on the presented consistency and accuracy of the results presented in Tables 6 and 7, it can be concluded that the solution method and parameter estimation scheme in conjunction with the BTU Meter are both valid and reliable in determining the fluid flow and temperature in a pipe, non-invasively.

2.8 Conclusions

This work detailed the development of a novel solution method and parameter estimation scheme combined with CHFT+ technology for non-invasively determining the flow rate and flow temperature of fluid in a pipe. Experimental testing showed consistent convection coefficient values with a maximum standard deviation of ± 11 W/m²·°C and a high agreement between expected and determined flow rates and flow temperatures values. The largest discrepancy in flow rates between obtained values from the proposed BTU Meter method and other flow rate estimation methods values was 5.0%. The largest absolute temperature difference between the measured and BTU Meter methods was 0.18°C. Thus, with the BTU Meter, energy transferred
with flow can be calculated with high accuracy and repeatability. The most noteworthy limitation for the BTU Meter solution and parameter estimation pertains to the thermocouple temperature measurement, since any significant noise in the temperature signal will offset the heat flux results and yield inaccurate convection coefficient values. Overall, however, the results detailed herein confirm that the solution method and parameter estimation scheme in conjunction with the BTU Meter was able to capture consistent and accurate values over the tested flow rate ranges.

Up to this point, there has been no method that non-invasively and simultaneously determines the flow rate and temperature of a fluid in a pipe. Thus, the newly developed solution and parameter estimation scheme involving the BTU Meter offers major enhancements in non-invasive internal fluid flow measurements. Utilizing the BTU Meter solution and parameter estimation approach described herein, important parameters such as the fluid’s flow rate, temperature, and convection coefficient, as well as the system’s thermal contact resistance can be determined non-invasively, making the solution and technology desirable to use in a wide range of residential, commercial, and industrial systems.

2.9 References


Chapter 3 – Conclusions, Limitations, and Suggestions for Future Work

3.1 – Conclusions

This investigation presents a new solution method and parameter estimation scheme for determining pipe fluid flow rate and temperature utilizing a non-invasive thermal interrogation (NITI) sensor called the BTU Meter. The new and novel method is based on CHFT+ technology and is used to determine values for the fluid’s convection heat transfer coefficient (h), fluid temperature (T_{fluid}), and the thermal contact resistance (R’”) between the BTU Meter and the pipe surface. These determined parameters are then used to obtain values for the flow rate of the fluid within the pipe using a developed correlation. When tested over the specified range, the results of the flow rate and temperature of the fluid in the pipe have excellent accuracy and reliability; which in turn provide accurate and reliable measures of energy transferred with fluid flow.

The newly developed solution and parameter estimation scheme involving the BTU Meter suggests major ramifications for the development of non-invasive internal fluid flow measurement devices and strategies. Utilizing the BTU Meter solution method described herein would allow for rapid, reliable, and accurate internal fluid flow measurements without the need of interrupting the fluid flow. Moreover, eliminating the need to break pipes and introduce devices and probes within the flow reduces required maintenance and shut-down periods that negatively compromise residential, commercial and industrial systems. Such measurement method criteria are desirable for a range of applications that include, but are not limited to, HVAC systems, oil and gas production, and building energy management.
Although initial testing methods do display reasonable results, they feature limitations that underscore the feasibility of the BTU Meter for the aforementioned applications. Therefore, more work needs to be done to verify the functionality of the system under different conditions and operating assumptions. Some of this potential work is listed in the next section.

3.2 – Limitations and Future Work

Limitations:

Based on our theoretical and experimental findings, the proposed method showed a high level of consistency and accuracy. It must be noted, however, that it was only tested and verified over a specific range of flow rates (4GPM to 16 GPM). While these results are nonetheless quite promising, the lack of experimental results for other flow rates limit the verified functionality of the system to the specified ranges. Additionally, the solution method assumed a steady flow rate in the pipe during the time of data acquisition, which might not always be the case across a range of operating conditions. Furthermore, the solution approach described herein determined the fluid temperature using a constant temperature assumption, limiting its applicability to systems that satisfy this particular criterion.

Additional assumptions that limit the functionality of the solution include (a) the time of measurement, (b) the thickness and material of the pipe, (c) the fluid type, and (d) the accuracy of temperature measurements. First, for this study, the time of measurement of the data acquisition system was fixed at 130 seconds, which represents the duration of all acquired data. Second, pipe thickness and material are critical factors, since the developed solution only works for pipe embodiments that satisfy a $\text{Bi} < 0.1$ condition. With materials and thicknesses that do not satisfy this condition, new thermal models would have to be developed. Third, the correlations developed were for water measurements only; they are not applicable for other types of fluids featuring
different fluid properties. Lastly, the method’s susceptibility to noise has the potential to obfuscate the accuracy of the measured temperature, yielding potentially inaccurate results.

**Future Work:**

Despite the promising results described in this investigation, further work is recommended to address the limitations discussed in the prior section. Firstly, more testing needs to be conducted in order to obtain reliable results for laminar and turbulent flows outside the tested flow rate region. Secondly, a more detailed analysis should be conducted to determine the minimum time required for data acquisition that would yield accurate and consistent results. Moreover, pipes of differing thicknesses should be tested for the purpose of quantifying any deviations in results. Lastly, further work should be devoted to developing an algorithm that would automatically update the flow rate correlations based on the different fluid properties. This can be done with a specialized parameter estimation scheme.
Appendices

Appendix A – Comparison between the Semi-Infinite Analytical Solutions using Temperature and Heat Flux Boundary Conditions

The contents of this Appendix is taken from Roghanizad et. al (2017) thesis appendices with his permission.

The purpose of this Appendix is to document and highlight the advantage of using heat flux boundary conditions instead of temperature boundary conditions when deriving and using analytical solutions with experimental data. There is a common misconception that both boundary conditions provide exact and accurate solutions of the corresponding mathematical model. Although this is true for purely analytical applications, this is not true when using the derived analytical solutions with experimental data. This is because experimental data applications are subject to a frequency of data acquisition (time-step) whereas purely analytical applications are not. It is shown in this work that the only analytical solutions effected by the size of the time-step are the analytical solutions based on temperature boundary conditions. The analytical solutions derived using heat flux boundary conditions are not affected by the time-step and output the same results for a given input with different time-steps.

In order to demonstrate the effect of the time step on analytical solutions, simulations were performed on the analytical solutions derived using a heat flux and temperature boundary condition for a semi-infinite solid with the properties in Table 8.

<table>
<thead>
<tr>
<th>$p^* (kg/m^3)$</th>
<th>$c^*(J/Kg - K)$</th>
<th>$k(W/mK)$</th>
<th>$T_0(°C)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>900</td>
<td>4168</td>
<td>0.68</td>
<td>20</td>
</tr>
</tbody>
</table>
A series of heat flux step functions were modeled as being imposed on the surface of a semi-infinite solid. This heat flux profile was initially modeled using a time step of 0.01 seconds. Figure 27 documents the heat flux profile modeled at the surface of the semi-infinite solid and Table 9 summarizes the mathematical models of the semi-infinite solid subject to a heat flux at its surface with varying boundary conditions.

**Figure 27.** Modeled Surface Heat Flux (0.01 Second Time-Step)

**Table 9.** Mathematical Models of Simulated Semi-Infinite Solid

<table>
<thead>
<tr>
<th></th>
<th>Heat Flux Boundary Condition</th>
<th>Temperature Boundary Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>PDE</strong></td>
<td>( \frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} )</td>
<td>( \frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} )</td>
</tr>
<tr>
<td><strong>BCs</strong></td>
<td>(-k \frac{\partial T}{\partial x} = q_{\text{Sensor}}(t) , \ x = 0)  (T \rightarrow T_0 , \ x \rightarrow \infty)</td>
<td>(T = T_{\text{Sensor}}(t) , \ x = 0)  (T \rightarrow T_0 , \ x \rightarrow \infty)</td>
</tr>
<tr>
<td><strong>IC</strong></td>
<td>(T = T_0 , \ t = 0)</td>
<td>(T = T_0 , \ t = 0)</td>
</tr>
</tbody>
</table>
The analytical solutions were derived from the mathematical models summarized in Table 9 using the Duhamel Method of Superposition. Equation A1 and Equation A2 express the analytical solutions derived for temperature and heat flux at the surface using the heat flux and temperature boundary condition, respectively.

\[ T_{\text{Surface},n} = T_{\text{Surface},0} + \sum_{j=1}^{n-1} (q_{\text{Surface},j}^* - q_{\text{Surface},j-1}^*) \times \sqrt{t_n - t_j} \times \frac{2}{\sqrt{\pi} \sqrt{kp^*c^*}} \quad [A1] \]

\[ q_{\text{Surface},n}^* = \sum_{j=1}^{n} \frac{(T_{\text{Surface},j} - T_{\text{Surface},j-1})}{\sqrt{t_n - t_{j-1}}} \times \frac{\sqrt{kp^*c^*}}{\sqrt{\pi}} \quad [A2] \]

where \( q_{\text{Surface},j}^* \) and \( T_{\text{Surface},j} \) correspond to the surface heat flux and temperature at the time step of \( j \), respectively. Figure 28 and Figure 29 illustrate the analytical solutions for temperature (Equation A1) and heat flux (Equation A2) when subject to a step input, respectively.

![Figure 28. Analytical Temperature Response to Step Heat Flux Input (Equation E1)](image)
Using the modeled heat flux profile with a time-step of 0.01 seconds in Figure 27 as the input of Equation E1, the corresponding temperature profile at the surface of the semi-infinite solid was output. Figure 30 displays the input heat flux profile and the temperature profile output.
As expected, the output temperature profile also has a time step of 0.01 seconds corresponding to the input heat flux profile. Using the output temperature profile in Figure 30 as the input of Equation A2, a heat flux profile is output. In theory, this output heat flux profile should exactly match the modeled surface heat flux as displayed in Figure 27 and input into Equation A1. Figure 31 displays both heat flux profiles in addition to the output temperature curve in Figure 30.

![Modeled Surface Heat Flux and Output Surface Heat Flux Profile (0.01 Second Time-Step)](image)

**Figure 31.** Modeled Input Surface Heat Flux, Output Surface Temperature Profile, and Corresponding Output Surface Heat Flux Profile (0.01 Second Time-Step)

In terms of the analytical solutions where the time step is an infinitely small value (dt), the heat flux profiles in Figure 31 would match exactly. In this case, where a small time-step of 0.01 seconds is used, there is good but not exact agreement between the two heat flux profiles. As time progresses, the agreement is less. Specifically, the heat flux profile output from Equation A2 has a ramp up pattern where it ramps up to the modeled heat flux profile in the first few time steps.
Clearly, the smaller the time-step the faster this ramp up occurs and consequently more accurate the output heat flux profile is. This indicates that Equation A2’s accuracy, which was derived using the temperature boundary condition, is dependent on size of the time-step.

To further expose the dependence of Equation A2’s accuracy on the time-step, the simulation routine described above was repeated using the same heat flux profile modeled at the surface but with a larger time step of 1 second. Figure 32 documents this modeled heat flux profile with the 1 second time-step as well as the original 0.01 second time-step profile.

![Figure 32. Modeled Input Surface Heat Flux and Output Heat Flux Profile (1.0 Second Time-Step)](image)

Using Equation A1, the modeled heat flux curve with the time step of 1 second was used to develop a corresponding surface temperature profile with the time step of 1 second. Figure 33
displays this output temperature profile as well as the temperature profile output using the modeled heat flux with a time step of 0.01 seconds.

**Figure 33.** Modeled Input Surface Heat Flux and Output Surface Temperature Profile (0.01 and 1.0 Second Time-Step)

As can be seen, the two temperature profiles correspond to one another which indicates that the accuracy of Equation A1, which was derived using a heat flux boundary condition, is not compromised by the time-step used. The only time-step related effect seen is in the resolution of the heat flux profile where the 0.01 second curve is better defined in terms of shape.

Using the output temperature profile with a time step of 1 second as the input of Equation A2, a corresponding heat flux profile is output. Figure 34 consists of the initially modeled heat flux curve at the surface of the semi-infinite solid and the heat flux curves output from Equation A2 using the temperature profiles with the 0.01 and 1 second time-steps as inputs.
Figure 34. Modeled Input Surface Heat Flux, Output Temperature Profile, and Corresponding Output Surface Heat Flux Profile (0.01 and 1.0 Second Time-Step)

From Figure 34, it is clear that the heat flux curve output when using the temperature curve with a time step of 1 second as the input is much less accurate than the heat flux profile with a time step of 0.01 seconds. Consequently, it is concluded that the greater the time-step of the input data, the less accurate Equation A2 which is based on a mathematical model with a temperature boundary condition.

The reason for this time-step dependent accuracy can be seen when referring to Equation A2. The \( \sqrt{t} \) component of Equation A2 causes it to have a shape similar to the \( \frac{1}{x} \) function. Given that \( \lim_{x \to 0} \frac{1}{x} = \infty \), it can be concluded that the smaller the time-step, the greater the output of the initial heat flux. This has a direct role in the ramp up pattern of the heat flux output from Equation
A2 and consequently in Equation A2’s accuracy. Figure 35 provides an illustration of the heat flux output (0.01 time-step) from Equation A2 given the temperature input (0.01 time-step) also illustrated in Figure 35. Figure 35 also illustrates the temperature output (0.01 time-step) of Equation A1 when using the output heat flux curve as the input (0.01 time-step).

![Figure 35](Image)

**Figure 35.** Modeled Input Surface Temperature, Output Heat Flux, and Corresponding Output Surface Temperature (0.01 Second Time-Step)

Expectedly, when using an infinitely small time-step (dt), the output of Equation A1 would be a temperature step function similar to the temperature step function modeled at the surface of the semi-infinite solid. This is because when using an infinitely small time-step, the output heat flux from Equation A2 would have an initial value of infinity, hence corresponding to a step temperature output when input into Equation A1.
Appendix B – CHFT+ Production and Assembly Manual

The contents of this Appendix is taken from Roghanizad et al. (2017) thesis appendices with his permission.

The purpose of this Appendix is to serve as a reference for the independent development of the CHFT+. This document was purposely laid out and written in a fashion so that it is easy to understand and follow. In case of any questions or concerns, contact Ali Roghanizad (aroegani@vt.edu). It should be noted that at the time this manual was originally written (2014), the CHFT+ was made using an RdF corporation heat flux sensor.

The CHFT+ is the combination of a heat flux sensor, thermocouple, and heater. It is capable of thermal response measurement for a variety of applications and is able to measure the value of different parameters within thermal models. Blood perfusion, flow rate, thermal diffusivity, and convection coefficient are just a few examples of parameters the CHFT+ can determine when used in a variety of applications. When making the CHFT+, it is important to follow each step in order as outlined below. Any deviation from the predetermined steps may result in a misaligned or otherwise defective CHFT+. Figure 36 shows the completed device after assembly and curing.

Figure 36. A fully assembled and cured CHFT+
- **Step 1: Determine the heat flux sensor, thermocouple, and heater to be used**

*Heat Flux Sensor:*

The heat flux sensor must have one side that is free of wire leads and wire connections. This warrants that the side of the heat flux sensor placed on the measurement surface is smooth and without deformities. The heat flux sensor must also be as flexible as possible without undermining its structural integrity. This allows for better contact between the heat flux sensor and measurement surface. Figure 37 showcase an example of heat flux sensors from RdF Corporation that can be used for a CHFT+.

![Figure 37. Example of Heat Flux Sensors used in the CHFT+ manufactured and sold by RdF Corporation](image)

A simple resistivity test of the heat flux sensor using a multi-meter will determine full functionality of the heat flux sensor. Heat flux sensors are susceptible to a multitude of problems after increased use and time. A constant resistivity value indicates proper function of the heat flux sensor before use.
It addition to the resistivity test, it is important to determine and verify the heat flux sensor’s sensitivity and accuracy before assembly. This will ensure that the heat flux sensor is operating normally before it is used in the CHFT+. Calibration can be done using an RMATIC heat flow meter or any other NIST traceable calibration method. The value obtained from the calibration should be compared to the manufacture’s specification. If the manufacture’s specification is not available, conduct the calibration at a series of different temperature differences in order make sure of a relatively constant and accurate sensitivity. Step 7 in this manual details the calibration procedure in greater depth.

**Thermocouple:**

Since the thermocouple will be situated between the measurement surface and heat flux sensor, a thin-film thermocouple must be used in order to facilitate full contact. It is important that full contact occur between the two respective surfaces. Similar to the heat flux sensor, the thermocouple must be tested in order to determine proper function and accuracy before it is considered for CHFT+ development. Figure 38 below shows a thin-film thermocouple made from constantan and copper. The two materials have been spot welded at their apex and have been designed so that the wire leads are close to each other upon exit from the CHFT+. It should be noted that almost any thermocouple could be used as long as its thickness does not alter the tissue surface.

![Figure 38. Thin-Film Thermocouple](image)
**Heater:**

In order to facilitate and guarantee uniform heat flux, it is important that the heater be slightly larger than the heat flux sensor. This provides for one-dimensional heat transfer through the heat flux sensor which is required for the thermal event designed to take place. The heater’s resistance should be measured in order to test for proper function. Additionally, it may be of interest to calculate the specific watt density provided by the heater given an input voltage and heater resistance. The specific watt density can be used to estimate different design values given related thermal conductivity and heat capacity values of the thermal system. Figure 39 shows a resistive heater that would be suitable for use. Like the heat flux sensor, it is important that one side of the heater be free of wire leads in order to experience full contact with the heat flux sensor.

![Resistive Heater](image)

**Figure 39. Example of a Resistive Heater**

Each of the three elements above should be checked for nicks, cuts, scrapes, or any other deformities that may cause an electrical short and otherwise compromise the CHFT+’s long term functionality.

- **Step 2: Collect and prepare the needed materials and tools:**
Ahead of assembly, it is vital that all needed materials and tools are collected and prepared. The materials and tools needed are:

- Acetone

![Acetone](image)

**Figure 40.** Acetone

- 4 pieces of Mylar (0.0508 mm thickness)

![Mylar](image)

**Figure 41.** Mylar

The size of each Mylar sheet should approximately be 4 times the size of the heater.

- Two – part Epoxy (CC-1095 A/B - Produced by John C. Dolph Company)
Figure 42. Two-Part Epoxy

- Epoxy Brush

Figure 43. Epoxy Brush

Wash and rinse with hot water/acetone and then dry before use.

- Teflon® (2 sets of 4 pieces that increase in size)

Figure 44. Two consecutive Teflon® pieces

Each set should consist of 4 pieces. Each piece should be slightly larger than its predecessor. The smallest piece should be roughly the size of the CHFT+.

- Kapton® Tape
- **Hot Press**

[Figure 45. Roll of Kapton Tape]

- **Step 3: Clean hot press foundation and top surface**

  Make sure no residual Kapton® tape or Mylar is present on hot press clamping surfaces.

- **Step 4: Clean heat flux sensor, thin-film thermocouple, and heater**

  Using acetone and a non-residue leaving cloth, clean the surfaces of each component. This step removes any greases, oils or other substances that may have accumulated on their respective surfaces.

- **Step 5: Understand CHFT+ design**

  The following schematic shown in Figure 47 represents the layered assembly of the
CHFT+. Make sure you understand the assembly before attempting to create it.

![Figure 47. Schematic of CHFT+ Assembly](image)

- **Step 6: Assemble and Produce CHFT+:**

  Using Figure 47 as a guide, start assembling the CHFT+.

  1. Place the 4 bottom pieces of Teflon® on the bottom face of the hot press, followed by a sheet of Mylar. Press the Mylar, eliminating as many air bubbles as possible.
  2. Using the epoxy brush, dab the epoxy onto the Mylar. Make sure you dab the epoxy in the area that has Teflon® directly under it. Epoxy use should be medium, not too much and not too little.
  3. Place the heater on the epoxy dabbed Mylar. MAKE SURE heater side without wire leads is upward. If desired, you may lightly dab the wired side with epoxy before placing it on the Mylar.
4) After placing heater, apply epoxy to the remaining heater surface. This is the surface without wire leads. Again, dabbing works best when applying the epoxy.

5) Place and press a sheet of Mylar on epoxy dabbed heater surface.

6) Repeat steps 2-5 for the heat flux sensor. AGAIN MAKE SURE heat flux sensor face with wire leads is the bottom surface. When placing heat flux sensor, ensure that it is placed exactly in the center of the heater. Meaning, the heat flux sensor should be aligned and centered on the heater surface. When sensor is placed on the epoxy surface, you can move it around slightly in order to determine ideal configuration.

7) Using the epoxy brush, dab a small amount of epoxy onto the Mylar. Make sure you dab the glue in the area that is above the center of the heat flux sensor.

8) Place the thermocouple above the desired area and determine its centrality with regard to the heat flux sensor. When aligned, slowly lower and press into Mylar.

9) Dab epoxy on top of thermocouple.

10) Place last Mylar sheet.
11) Apply Kapton® tape to all edges of exposed Mylar.

12) Place remaining pieces of Teflon® smallest first on top of one another.

![Image of clamped hot press](image)

**Figure 51.** Clamped Hot Press

14) Open Hot Press and remove top Teflon pieces.

15) Confirm assembly is still aligned. If not, realign and re-do steps 12-14.

16) If aligned, replace Teflon pieces as done in step 12.

17) Clamp hot press shut.

18) Plug hot press into power outlet.

19) Set temperature to 160 °C and turn off after 45 minutes.
20) Let press cool down, in closed configuration, for 1-2 hours.

21) Slowly un-clamp and open press. You may feel some resistance.

22) Remove top Teflon® pieces.

23) Remove Kapton® tape and separate assembly from hot press.
24) Trim excess Mylar.

25) Test heat flux sensor, thermocouple and heater functionality.
- **Step 7: Sensor Calibration**

The equipment used to determine the sensitivity of the heat flux sensor component of the CHFT+ are conduction based calibration systems. An example is the R-MATIC guarded Heat Flow Meter Thermal Conductivity Instrument from Dynatech R/D Company. Which is based on ASTM C177. However, any conduction calibration system that is traceable to NIST will suffice.

The R-MATIC allows samples to be held between two temperature-controlled plates and measures the amount of heat flow through the central portion of the sample. The lower plate can be mechanically lowered and raised as needed to fit the sample snuggly between the two plates. The R-MATIC is equipped with a Freon cooling system that allows for the cold plate (upper plate) of the R-MATIC to be maintained at temperatures well below ambient conditions. The guarded hot plate (lower plate) of the R-MATIC is heated by means of a multitude of heating elements present within the plate. Both the upper and lower plate temperatures are measured using platinum resistance temperature sensors and are adjusted with 10-turn potentiometers that are situated on the front side of the instrument. The R-MATIC’s chamber size is 24”x24”. The heat flow through the samples is measured with a specially constructed heat flow meter that completely covers a 10”x10” area in the center of the chamber. Based on the specifications of the R-MATIC instrument, at least a 10"x10" surface size for the samples is required to effectively measure the thermal resistance.

The sensors are calibrated using insulation material with tabulated specifications provided by the National Institute of Standards and Technology (NIST). The specifications include mean temperature and thermal resistance. The heat flux sensors with unknown sensitivity are placed directly under the sample insulation and centered on the guarded hot plate (bottom plate). The voltage output of the heat flux sensor is compared with the known heat flux due to the insulation
material and temperatures. The relationship between the voltage output and actual heat flux values define the sensitivity of the heat flux sensors as seen in Equation B1.

\[ S = \frac{V}{q''} \]  

Where \( S \) is the sensitivity of the heat flux sensor \( \left( \frac{\mu V}{m^2/W} \right) \), \( V_{HF} \) is the output voltage of the heat flux sensor \( (\mu V) \), and \( q'' \) is the experienced heat flux \( \left( \frac{W}{m^2} \right) \). \( V_{HF} \) can be calculated with Equation B2 and \( q'' \) can be calculated with Equation B3.

\[ V_{HF} = \frac{P - N}{2} \]  

Where \( P \) (\( \mu V \)) is the positive voltage reading and \( N \) (\( \mu V \)) is the negative voltage reading as provided by the heat flux sensor when measured across the output terminals in both directions.

\[ q'' = -\frac{\Delta T}{R} \]  

Where \( \Delta T \) is the temperature difference \( (\degree C) \) across the NIST insulation and \( R \) is the provided insulation resistance value \( (\degree C \cdot m^2/W) \).
Appendix C – Data Acquisition and Analysis Apparatus

The purpose of the Appendix is to document the BTU Meter data acquisition and data analysis apparatus used to make measurements of the convection heat transfer coefficient (h), and the temperature of the fluid flowing through the pipe (T_{\text{fluid}}), as well as the system’s thermal contact resistance (R”).

This Appendix is composed of two sections:

**Data Acquisition System**

A 24 bit, 16 channel National Instrument Data Acquisition System (NI 9214) was used as the means to record measurements of heat flux and temperature. A picture of the DAQ is presented in Figure 56.

![National Instruments DAQ (NI 9214)](Taken from National Instruments)

LABVIEW was used as the apparatus to display and record the heat flux and temperature measurements recorded by the NI DAQ. Figure 57 shows the front panel of the LABVIEW apparatus developed for the BTU Meter. The program includes an adjusted sensitivity that
accounts for the change in heat flux sensor sensitivity as a function of temperature. Results from the LABVIEW program are saved in an excel sheet which is then analyzed in MATLAB.

![Figure 57](image.png)

**Figure 57.** Front panel of the BTU Meter LABVIEW program.

**Data Analysis Program (Parameter Estimation Scheme Code)**

Presented below is the code used in the BTU Meter Parameter Estimation Scheme. It should be noted that the program has a resolution of $\pm 1 \frac{W}{m^2 \cdot ^\circ C}$ and the parameter estimation takes about 0.1 seconds to execute on a Microsoft Surface Pro 3 (4GB RAM- 1.9 GHz).

```matlab
%This program analyzes the temperature transient response of a BTU Meter
%sensor attachment on a copper pipe with water running through it. The code
%imports the sensor's excel data and conducts a parameter estimation that
%works on minimizing the RMSE between the recorded and
%calculated thermocouple temperature in order to find optimal values of h,
%Tau, and R". The solution assumes presence thermal contact resistance
%between the thin foil thermocouple and the true pipe wall temperature,
```
% assumes no temperature drop across the pipe wall, and assumes no change
% in temperature of the water flowing through the pipe.
% This code was written by Hussain Alshawaf on July 5, 2017.

% Use of this code and its incorporated methods require my explicit
% permission. – HMA August 2017

clear all;
clc;

Importing data

filename = input('Enter name of excel data file: ', 's');
A = xlsread(filename,1);
Time= A(:,1);
Heat_Flux = A(:,2);
Ts = A(:,3);
Table = [Time, Heat_Flux, Ts];

Constants

Den=8940;
thic=0.00127;
Cp=385;

Detecting when heater turns on

index_modified=1;
for count= 3:length(Table(:,1))
    Delta_Heat_Flux = (Heat_Flux(count, :) - Heat_Flux(count-1, :));
    if Delta_Heat_Flux > 50
        Delta_index(index_modified,1)= count-1;
        index_modified=index_modified+1;
    end
end

Parameter Estimation

% For loop that iteratively finds the optimal value for h

index=1;
x=0;
y=100;
z=10000;
%Starts a stopwatch timer
tic
for s = 1:1:3;
for h = x:y:z;
    %For loop that iteratively finds the optimal value for Tau (range have
    %to be changed manually)
    [Solution_Cell, Tsensor_calculated, Rth_Avg, RMSE_Sum] = BTU_Meter_T_Model(h, Time, Heat_Flux, Ts, Table, Delta_index, Den, thic, Cp);
    Iteration_RMSE(index) = RMSE_Sum;
    Iteration_Rth(index) = Rth_Avg;
    Iteration_Solution_Cell(:,index) = Solution_Cell;
    Iteration_h(index) = h;
    Iteration_Tsensor_calculated(:,index) = Tsensor_calculated;
    index=index+1;
end
%Finds the minimum value in the RMSE vector
[Minimum, ind] = min(Iteration_RMSE);
%Saves optimal values
Rth_optimum = Iteration_Rth(ind);
h_optimized = Iteration_h(ind);
Solution_Cell = Iteration_Solution_Cell(:,ind);
Tsensor_calculated = Iteration_Tsensor_calculated(:,ind);
x=h_optimized-y;
z=h_optimized+y;
y=y/10;
end
%Computes Tau and Fluid temperature
Tau_optimum= Den*thic*Cp/h_optimized;
Tfluid=Solution_Cell(1,1)-Heat_Flux(1,1)/h_optimized;
%outputs solution values
fprintf('
The optimal values that reduce the RMSE are: \n\nH= %d W/m^2-C, Tau= %d s and Rth= %d
', h_optimized, Tau_optimum, Rth_optimum);
%reads the elapsed time from the stopwatch timer
toc

Plotting

plot(Time ,Ts)
hold on
plot(Time, Tsensor_calculated)
hold on
plot(Time, Solution_Cell)
hold off
BTU_Meter_Model Function

```matlab
function [ Solution_Cell, Tsensor_calculated, Rth_Avg, RMSE_Sum ] = BTU_Meter_T_Model( h, Time, Heat_Flux, Ts, Table, Delta_index, Den, thic, Cp)

% This function imports the time, heat flux, temperature, h, and Tau from the
% superposition code to calculate the wall temperature and RMSE error
% between the calculated and recorded thermocouple temperature values.
% This code was written by Hussain Alshawaf on July 5, 2017.
%
% Use of this code and its incorporated methods require my explicit
% permission. - HMA August 2017
%
%writing the time constant equation
Tau=Den*thic*Cp/h;

Computing and storing the first two cells in the solution column

First_Cell = Ts(Delta_index(1,1),:);
Solution_Cell (1,1)= First_Cell;
Second_Cell = First_Cell +((Heat_Flux(2,1) - Heat_Flux(1,1))/h)*(1-exp(-((Time(2,1) - Time(1,1))/Tau)));
Solution_Cell (2,1)= Second_Cell;

Computing values from cell 3 until the heater is turned on

for count_steady= 3:Delta_index(1,1)

    % First loop counter
    N = count_steady - 1;
    S = 1;
    Bn = 0;

    %while loop that computes the second part of the temperature response
    %solution

    while S < N
        Z = S+1;
        Bn = ((Heat_Flux(Z, :) - Heat_Flux(S, :))/h) * (1 - exp(-(Time(count_steady, :) - Time(S, :))/Tau))\n
            S = S+1;
        end
```
%The first part of the temperature response solution
An = Ts(Delta_index(1,1,:), :) + ((Heat_Flux(count_steady, :) - Heat_Flux(N, :))/h) * (1 - exp(-((Time(count_steady, :) - Time(N, :))/ Tau));

%Combining both parts of the temperature response solutions
Ttotal= An + Bn;

%Storing the temperature solutions in a vector
Solution_Cell(count_steady,1) = Ttotal;
end

Computing values for when the heater is turned on

for count= Delta_index(1,1)+1:length(Table(:,1))
    % First loop counter
    N = count - 1;
    S = 1;
    Bn = 0;

    %While loop that computes the second part of the temperature response solution
    while S < N
        Z = S+1;
        Bn = ((Heat_Flux(Z, :) - Heat_Flux(S, :))/h) * (1 - exp(-((Time(count, :) - Time(S, :))/Tau)))+ Bn;
        S = S+1;
    end

    %The first part of the temperature response solution
    An = Ts(Delta_index(1,1,:), :) + ((Heat_Flux(count, :) - Heat_Flux(N, :))/h) * (1 - exp(-((Time(count, :) - Time(N, :))/ Tau));

    %Combining both parts of the temperature response solutions
    Ttotal= An + Bn;

    %Storing the temperature solutions in a vector
    Solution_Cell(count,1) = Ttotal;

    %Computing the thermal contact resistance
    Rth(count,1) = (Ts(count,1) - Solution_Cell(count,1))/Heat_Flux(count,1);
end

Computing R^" and RMSE

% Averaging the thermal contact resistance in the last 40 seconds
Rth_Avg = mean(Rth(end-40:end));

%Calculated thermocouple temperature
\[ \text{Ts} = T_{\text{Solution\_Cell}}(:,1) + R_{\text{th\_Avg}} \cdot \text{Heat\_Flux}(:,1) \]

\%Calculating RMSE after the heater turns on

\[
W = \text{Delta\_index}(1,1) + 5;
\]

\[
\text{for } K = W : \text{length(Table(:,1))}
\]

\[
\text{RMSE}(K,1) = (\text{Ts}(K,1) - T_{\text{s}}(K,1))^2;
\]

\text{end}

\%Summing RMSE

\[
\text{RMSE\_Sum} = \sqrt{\frac{\text{sum(RMSE)}}{\text{length(Table(:,1))} - W}};
\]

\text{end}
Appendix D – Investigation of Temperature Spatial Uniformity, One-dimensionality of Heat Transfer, and Laminar Sublayer Thickness Assumptions

The purpose of this Appendix is to provide further elaborations on key assumptions made for the BTU Meter model and solution.

**Temperature Spatial Uniformity:**

The spatial uniformity of temperature in a solid is directly related to the Biot number. Figure 58 shows how different Biot numbers affect the temperature distribution on a solid with surface convection. As discussed in Chapter 2.4, the Biot number is the ratio of the conduction to convection resistance. This means that for a small Biot number (Bi << 1), the resistance of conduction is small relative to the convection resistance and therefore the temperature gradient across the solid thickness is minimal. Furthermore, this allows for the temperature of the solid to only be a function of time, as the major temperature gradient observed is between the surface and the fluid; hence \( T(x,t) \approx T(t) \). In Chapter 2.4, the Biot number for moderate flow inside a copper pipe was found to be less than \( \frac{1}{10} \), it can be concluded that the temperature is spatially uniform across the thickness of the pipe. To further verify the spatial uniformity assumption, the temperature drop across the thickness of the heated pipe section was calculated using Equation D1, which states

\[
q''_{conduction} = \frac{k\Delta T}{\delta}
\]  

[D1]

where \( q''_{conduction} \) is heat flux due to conduction, \( k \) is thermal conductivity of the pipe, \( \Delta T \) is the temperature gradient across the thickness of the pipe, and \( \delta \) the thickness of the pipe. For a
measured heat flux of $3,000 \frac{W}{m^2 \cdot ^\circ C}$, pipe thickness of 0.00127 m and pipe thermal conductivity of $200 \frac{W}{m \cdot ^\circ C}$, the temperature drop across the thickness of the pipe is found to be 0.02°C which is considered to be negligible and beyond thermocouple accuracy and measurement capabilities.

![Diagram](image)

**Figure 58.** Effect of Biot number on steady-state temperature distribution on a solid with conduction and convection.

**One-dimensionality of Heat Transfer:**

One of the assumptions used in Chapter 2 for the BTU Meter solution was the one-dimensionality of the system. This was assumed in order to simplify the solution to only account for the heat going radially and disregard axial effects. In order to verify the assumption, a more detailed analysis was conducted on the temperature distribution along the length of the heated pipe section. Figure 59 illustrate the thermal model used and the energy balance conducted to determine the temperature distribution along the length of the heater.
Figure 59. (A) Schematic illustrating the thermal model diagram used to analyze the one-dimensionality of heat transfer in the system. (B) Schematic of the pipe cross section used for the energy balance conducted to determine the temperature distribution along the length of the heater.

Conducting the energy balance on the pipe cross section and simplifying the equation using Fourier’s Law and convection equations and \( \theta = T_{\text{pipe}} - T_{\text{fluid}} \), Equation D2 is written.

\[
\frac{d\theta^2}{dx^2} - \frac{h}{k\delta}(\theta) + \frac{q^*}{k\delta} = 0 \quad [D2]
\]
Applying Boundary conditions in Equations D3 – D5, the solution can be simplified to Equation D6.

\[ \frac{d\theta}{dx}|_{x=0} = 0 \]  \hspace{1cm} [D3]

\[ \theta(z) = \theta_1 e^{-\frac{h}{\sqrt{k\delta}} z} \]  \hspace{1cm} [D4]

\[ \theta|_{x=L} = \theta_1 = \theta|_{z=0} \]  \hspace{1cm} [D5]

\[ \theta(x) = \left( \frac{q_o}{h} \tanh \left( \frac{\sqrt{\frac{h}{k\delta}} L}{h} \right) \right) - \left( \frac{q_o}{h} \tanh \left( \frac{\sqrt{\frac{h}{k\delta}} x}{h} \right) \right) \cosh \left( \frac{\sqrt{\frac{h}{k\delta}} L}{h} \right) + \frac{q_o}{h} \]  \hspace{1cm} [D6]

Since the water temperature in the solution is modeled constant, changes in \( \theta \) represent changes in the pipe temperature. Plotting \( \theta \) along the length of the heater illustrates the temperature distribution present in the heated section. Since the heat applied is constant at the surface, the change in pipe temperature across the heated section allows for quantification of the axial conduction present. Figure 60 plots the temperature distribution along the length of the heated pipe section using a 3 inch x 3 inch heater and a sensor of 1 inch x 1 inch for the highest and lowest flow rate. As illustrated in Figure 60, discrepancies in the temperature distribution within the sensor region were small for both the minimum and maximum tested flow rates. In fact, the temperature distribution in the sensor region only varied a maximum of 3.5% from the sensor centerline across all tested flow rates. Since the change in temperature distribution was minimal along the length of the sensor, it can be concluded that one-dimensionality effects dominate the model’s behavior. Therefore, the model can be simplified to a one-dimensionality heat transfer assumption.
Figure 60. Temperature distribution along the heated pipe section for the highest and lowest flow rates.

**Boundary Layer Thickness:**

In Chapter 2.7, there was a discrepancy between the results of the literature and BTU Meter correlations. This difference was attributed to the measurements of the BTU Meter being predominantly measured in the laminar sublayer, as the heat transferred is dominated in the laminar sublayer present in the fully turbulent flow. In order to verify the assumption, a law of the wall and thermal penetration analyses were conducted. The law of the wall determines the thickness of the laminar sublayer present in a flow (Equations D7 – D10), while the thermal penetration
analyzes the thickness at which heat penetrates and measures the convection coefficients (Equation D11). Equations D7 states:

$$\Delta P \frac{\pi D_i^2}{4} = \tau_w \pi D_i L$$

[D7]

where $\Delta P$ is the pressure drop due to viscous effects and $\tau_w$ is the wall shear stress. Equation D8 rewrites Equation D7 in terms of the shear stress. Equation D7 states:

$$\tau_w = \frac{\Delta P D_i}{L \frac{4}{4}}$$

[D8]

In order to determine the pressure drop across the length Equation D9 is introduced. Equation D9 states,

$$\frac{\Delta P}{L} = f_D \frac{\rho_{fluid} v^2}{2 D_i}$$

[D9]

where $f_D$ is Darcy friction factor, $\rho_{fluid}$ is the density of the fluid, and $v$ is the mean flow velocity. Once the wall shear stress was found, the effective laminar sublayer can be calculated using Equation D10. Equation D10 states,

$$y = \frac{\theta y^+}{u_r} = \frac{\theta y^+}{\sqrt{\frac{\tau_w}{\rho_{fluid}}}}$$

[D10]

where $y$ is the effective laminar sublayer thickness, $\theta$ is the fluid’s kinematic viscosity, $y^+$ is the wall coordinate, and $u_r$ is shear velocity. Once the effective laminar sublayer is determined, it can be compared to the thermal penetration of the heat in the flow. Equation D11 computes the thermal penetration in the flow and states,
\[ \delta_t = \frac{k_{fluid}}{\sqrt{\rho_{fluid} C_{fluid}}} t_{flow} \]  

\[ [D11] \]

where \( \delta_t \) is the thermal penetration of the heat in the flow, \( C_{fluid} \) is the heat capacity of the fluid, and \( t_{flow} \) is the time it takes for the flow to pass through the length of the heater. Figure 61 illustrates the velocity profile of turbulent flow as well as the heat penetration into the flow. Table 10 summarizes the laminar sublayer analysis on the data obtained from the BTU Meter for 4 \( \frac{gal}{min} \) and 16 \( \frac{gal}{min} \) assuming a smooth pipe, a water temperature of 20 °C, and a laminar \( y^* \) of 5.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{schematic.png}
\caption{Schematic of the velocity profile present in turbulent flow in a pipe with heat penetration.}
\end{figure}

\begin{table}[h]
\centering
\caption{Summary of laminar sublayer and thermal penetration analysis on BTU Meter results.}
\begin{tabular}{|c|c|c|c|}
\hline
Flow Rate (GPM) & \( y \) (cm) & \( \delta_t \) (cm) & Buffer Layer (cm) at \( y^* = 9 \) \\
\hline
4 & 0.109 & 0.113 & -- \\
16 & 0.032 & 0.057 & 0.057 \\
\hline
\end{tabular}
\end{table}

The results in Table 10 show that for the 4 \( \frac{gal}{min} \) the thermal penetration thickness was almost entirely present in the effective laminar sublayer thickness. On the other hand, results for
the 16 $\frac{gal}{min}$ show that the thermal penetration thickness surpasses the laminar sublayer. This is partially due to the decrease in thickness of the effective laminar sublayer as a function of increasing flow. In fact, the heat penetrated in the 16 $\frac{gal}{min}$ does not surpass the buffer layer present in the flow. The buffer layer is defined as the layer in between the laminar and the turbulent layers at $5 < y^+ < 30$, where turbulent effects start to overcome laminar ones. As seen in Table 10, the $y^+$ that would match the sublayer thickness to the thermal penetration was 9, which is in the early stages of the buffer layer and close to the laminar sublayer. Nonetheless, the majority of the thermal penetration occurred in the laminar sublayer. Therefore, the results concur that the convection heat transfer coefficient measured in the BTU Meter across the tested flow rates is predominantly dominated in the laminar sublayer and in areas very close to the sublayer. This agrees with the assumption that the BTU Meter measurements are not indicative of the true convection coefficient assumed in the literature correlations.
Appendix E – Derivation and Overview of the Mathematical Model

The purpose of this Appendix is to provide a detailed derivation and overview of the mathematical solution model and parameter estimation scheme used for the BTU Meter.

Physical system:

![Diagram of BTU Meter setup with fluid flow through a pipe](image)

**Figure 62.** (A) Illustration of the thermal model diagram of the BTU Meter setup on a pipe with fluid flowing through. (B) Schematic of the pipe thickness where the overall energy balance analysis was applied.

**Fundamental Unit Step Function Response:**

\[
\rho VC \frac{\delta T}{\delta t} = \frac{\delta E}{\delta t} = q_{\text{heater\,(conduction)}} - q_{\text{fluid\,(convection)}} = q_{'' \text{sensor \,A}_{\text{heater}}} - q_{'' \text{fluid \,A}_{\text{heater}}}
\]
\[ \rho V C \frac{dT}{dt} = q''_{sensor} A_{heater} - h(T_{pipe} - T_{fluid}) A_{heater} \]

\[ \rho CA\pi (r_o^2 - r_i^2) L \frac{dT}{dt} = q''_{sensor} A^* (2\pi r_o) L - h(T_{pipe} - T_{fluid}) A^* (2\pi r_i) L \]

\[ \rho C\delta \frac{dT}{dt} = q''_{sensor} - h(T_{pipe} - T_{fluid}) \]

\[ \frac{\rho C\delta}{h} \frac{dT}{dt} = \frac{q''_{sensor}}{h} - (T_{pipe} - T_{fluid}) \]

At \( t = \infty \), temperature response is in steady state

\[ T_{fluid} = T_{pipe}(\infty) - \frac{q''_{sensor}}{h} \]

\[ \frac{\rho C\delta}{h} \frac{dT}{dt} = \frac{q''_{sensor}}{h} - (T_{pipe} - T_{pipe}(\infty)) - \frac{q''_{sensor}}{h} \]

\[ \frac{\rho C\delta}{h} \frac{dT}{dt} = -(T_{pipe} - T_{pipe}(\infty)) \]

Since \( \theta = T_{pipe} - T_{pipe}(\infty) \)

\[ \frac{\rho C\delta}{h} \frac{d\theta}{dt} = -\theta \]

\[ \int_{\theta_i}^{\theta} \frac{d\theta}{\theta} = - \int_0^t \frac{dt}{\rho C\delta} \frac{1}{h} \]

\[ \ln \theta - \ln \theta_i = - \frac{t}{\frac{\rho C\delta}{h}} \]

\[ \ln \frac{\theta}{\theta_i} = - \frac{t}{\frac{\rho C\delta}{h}} \]
\[ \frac{\theta}{\theta_i} = -e^{\frac{t}{\rho C \delta}} \]

\[ \tau = \frac{\rho C \delta}{h} \]

\[ \frac{\theta(t)}{\theta(0)} = \frac{T_{\text{pipe}}(t) - T_{\text{pipe}}(\infty)}{T_{\text{pipe}}(0) - T_{\text{pipe}}(\infty)} = e^{-\frac{t}{\tau}} \]

\[ T_{\text{pipe}}(t) = T_{\text{pipe}}(\infty) + (T_{\text{pipe}}(0) - T_{\text{pipe}}(\infty))e^{-\left(\frac{t}{\tau}\right)} \]

\[ T_{\text{pipe}}(t) = T_{\text{fluid}} + \frac{q''_{\text{sensor}}}{h} + (T_{\text{pipe}}(0) - (T_{\text{fluid}} + \frac{q''_{\text{sensor}}}{h}))e^{-\left(\frac{t}{\tau}\right)} \]

\[ T_{\text{pipe}}(t) = \frac{q''_{\text{sensor}}}{h} \left(1 - e^{-\left(\frac{t}{\tau}\right)}\right) + T_{\text{fluid}}(1 - e^{-\left(\frac{t}{\tau}\right)}) + T_{\text{pipe}}(0)e^{-\left(\frac{t}{\tau}\right)} \]

Since \( T_{\text{fluid}} = T_{\text{pipe}}(0) - \frac{q''(0)}{h} \)

\[ T_{\text{pipe}}(t) = T_{\text{pipe}}(0) + \frac{q''_{\text{sensor}} - q''(0)}{h}(1 - e^{-\left(\frac{t}{\tau}\right)}) \]

For unit step

\[ T_{\text{pipe}}(t) = T_{\text{pipe}}(0) + \left(\frac{q''_{\text{unit}}}{h}\right)(1 - e^{-\left(\frac{t}{\tau}\right)}) \]

Accounting for contact resistance,

\[ T_{\text{pipe}}(t) = T_{\text{TC(measured)}}(t) - q''(t) \times R'' \]

\[ T_{\text{TC(measured)}}(t) - q''(t) \times R'' = (T_{\text{TC(measured)}}(0) - q(0) \times R) + \left(\frac{q''_{\text{unit}}}{h}\right)(1 - e^{-\left(\frac{t}{\tau}\right)}) \]

A similar solution was found using steady state condition at \( t = 0 \).
Superposition:

For first step:
\[ T_{\text{pipe}}(t) = T_{\text{pipe}}(0) + \left( \frac{q''_{1} - q''_{0}}{h} \right) \left( 1 - e^{-\frac{(t-0)}{\tau}} \right) \]

For second step:
\[ T_{\text{pipe}}(t) = T_{\text{pipe}}(0) + \left( \frac{q''_{1} - q''_{0}}{h} \right) \left( 1 - e^{-\frac{(t-0)}{\tau}} \right) + \left( \frac{q''_{2} - q''_{1}}{h} \right) \left( 1 - e^{-\frac{(t-1)}{\tau}} \right) \]

For third step:
\[ T_{\text{pipe}}(t) = T_{\text{pipe}}(0) + \left( \frac{q''_{1} - q''_{0}}{h} \right) \left( 1 - e^{-\frac{(t-0)}{\tau}} \right) + \left( \frac{q''_{2} - q''_{1}}{h} \right) \left( 1 - e^{-\frac{(t-1)}{\tau}} \right) + \left( \frac{q''_{3} - q''_{2}}{h} \right) \left( 1 - e^{-\frac{(t-2)}{\tau}} \right) \]

Thus it can be concluded that,
\[ T_{\text{pipe(calculate)},m} = T_{\text{pipe(measured)},0} + T_{\text{transient},m} \]

Therefore:
\[ T_{\text{pipe,0}} = T_{\text{pipe}}(0) \]

Using Duhamel superposition:
\[ q_{\text{Sensor}}(t) = q_{\text{Sensor}}(0) + \sum_{j=1}^{M-1} (q''_{j} - q''_{j-1})_{\text{sensor}} \times H(t - t_j) \]

Therefore:
\[ T_{\text{transient},m} = \sum_{j=1}^{m} \left( \frac{q''_{j} - q''_{j-1}}{h} \right) (1 - e^{-\frac{(t-m-1)}{\tau}}) \]

Thus:
\[ T_{\text{pipe(calculate)},m} = T_{\text{pipe(measured)},0} + \sum_{j=1}^{m} \left( \frac{q''_{j} - q''_{j-1}}{h} \right) (1 - e^{-\frac{(t-m-1)}{\tau}}) \]

Accounting for thermal contact resistance:

\[ T_{\text{TC(calculate)},m} = [(T_{\text{pipe(calculate)},m}] + q''_{m} \times R'' \]

\[ T_{\text{TC(calculate)},m} = [(T_{\text{pipe(measured)},0}) + \sum_{j=1}^{m} \left( \frac{q''_{j} - q''_{j-1}}{h} \right) (1 - e^{-\frac{(t-m-1)}{\tau}})] + q''_{m} \times R'' \]

\[ T_{\text{TC(calculate)},m} = [(T_{\text{TC(measured),0}} - q''_{0} \times R'') + \sum_{j=1}^{m} \left( \frac{q''_{j} - q''_{j-1}}{h} \right) (1 - e^{-\frac{(t-m-1)}{\tau}})] + q''_{m} \times R'' \]
Calculating Thermal Contact Resistance ($R''$):

From fundamental convection:

$$q''_j = \frac{T_{TC,j} - T_{pipe,j}}{R''_j}$$

$$R''_j = \frac{T_{TC(measured),j} - T_{pipe(calculated),j}}{q_j}$$

For constant contact resistance: 

$$R'' = R''_j \quad \text{where} \quad j = 1, 2, \ldots, M - 1.$$ 

Mathematically, this is also true:

$$R'' = \sum_{j=1}^{M-1} R''_j = \frac{\sum_{j=1}^{M-1} \frac{T_{TC(measured),j} - T_{pipe(calculated),j}}{q_j}}{M - 1}$$

Parameter Estimation Scheme:

Objective:
Estimate:

- $h$ = convection heat transfer coefficient ($\frac{W}{m^2 \cdot ^\circ C}$)
- $\tau$ = Time constant (s)
- $R''$ = thermal contact resistance between thermocouple and pipe ($^\circ C \cdot \frac{m^2}{W}$)
- $T_{fluid}$ = fluid flow temperature ($^\circ C$)

Procedure:

1. Use $q''_{sensor}$ response and initial surface temperature as known inputs in the mathematical model.
2. Assume a convection coefficient value ($h$) and calculate contact resistance ($R''$) to obtain $T_{TC(calculated)}$.
3. Compare ($T_{TC(calculated)}$) and ($T_{TC(measured)}$) and find RMSE.
4. Find the $h_{optimum}$ that would best fit ($T_{TC(calculated)}$) to ($T_{TC(measured)}$) [Minimum RMSE].
5. $R''$, $\tau$, and $T_{fluid}$ are directly calculated using $h_{optimum}$.
Equations:

\[ \text{RMSE} = \sqrt{\frac{1}{M-1} \sum_{m=1}^{M-1} \left( T_{TC(\text{measured}),m} - (T_{TC(\text{calculated}),m}) \right)^2} \]

\[ \text{RMSE} = \sqrt{\frac{1}{M-1} \sum_{m=1}^{M-1} \left( T_{TC(\text{measured}),m} - T_{\text{pipe}(\text{calculated}),m} + q''_m \times \frac{\sum_{j=1}^{M-1} T_{TC(\text{measured}),j} - T_{\text{pipe}(\text{calculated}),j}}{M-1} \right)^2} \]

\[ \text{RMSE} = \sqrt{\frac{1}{M-1} \sum_{m=1}^{M-1} \left( -q''_m \times R''_c + \sum_{j=1}^{M-1} \frac{q'_j - q'_{j-1}}{h_1} \left( 1 - e^{-\left( \frac{L_{m-1}}{R''_c} \right)} \right) + q''_m \times \frac{\sum_{j=1}^{M-1} T_{TC(\text{measured}),j} - (T_{TC(\text{measured}),0} - q''_0 \times R''_c) + q''_0 \times \left( 1 - e^{-\left( \frac{L_{m-1}}{R''_c} \right)} \right)}{M-1} \right)^2} \]

As discussed in Chapter 2.6 the thermal contact resistance is best estimated in the steady state region, while h is estimated in the transient region. Since the RMSE is a function of both h and R”, the solution was constrained to only account for h in the first iteration (i.e. R\_j=1=0). In the following iterations, the contact resistance is estimated using the aforementioned contact resistance equations. This allows the minimization of the RMSE to only include h as the estimated parameter, as the contact resistance values are computed theoretically through calculated and measured temperatures. This is also the case for the \( \tau \) parameter. It should be noted that all three parameters (h, \( \tau \), R”) can be independently estimated if desired.

It is worth mentioning that this parameter estimation scheme could be further enhanced and altered by the following ways: 1) Weighting different parts of the curve differently to maximize optimization of desired parameters; 2-)Neglecting parts of the curves for more consistent and accurate results, for example this could include the first few data points recorded by the BTU Meter; and 3) Making use of minimization functions in MATLAB and other programs.
(0.1 seconds). However, in this work it was found that these augmentations were not necessary due to the already fast, consistent and accurate results. Furthermore, while minimization functions may improve parameter estimation time, they were not employed in this work due to their unreliability in consistently determining the optimum values.

**Calculated Fluid Temperature (T\textsubscript{fluid})**

Convection Equation: \[ q''(0) = h(T_{\text{pipe}}(0) - T_{\text{fluid}}) \]

\[ T_{\text{fluid}} = T_{TC(measured)}(0) - q''(0)\times(R'' + \frac{1}{h}) \]
Appendix F – Detailed Summary of Experimental Results

The purpose of this Appendix is to provide a detailed summary of the experimental results. The results include the convection coefficient (h), time constant (τ), thermal contact resistance (R”), Root Mean Square Error (RMSE), T\text{water(estimated)} ,T\text{water(measured)}, the difference between the measured and calculated temperatures, Reynolds number and Nusselt number obtained for each data set analyzed. The summary of the results obtained from the MATLAB parameter estimation scheme is presented in Table 11. Furthermore, a correlation estimated flow rate column is added to present the flow rate determined using the h in the BTU Meter correlations. It should be noted that the correlation used to determine the values for the 1 \frac{gal}{min} is presented in Appendix G.

Table 11. Detailed summary of the experimental results obtained from the MATLAB program and BTU Meter correlation.

<table>
<thead>
<tr>
<th>Flow rate (GPM)</th>
<th>h (W/(m²·°C))</th>
<th>τ(s)</th>
<th>R” (°C m²/W)</th>
<th>RMSE (°C)</th>
<th>T\text{fluid estimated} (°C)</th>
<th>T\text{water measured} (°C)</th>
<th></th>
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<th>Re</th>
<th>Nu</th>
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Table 11 cont’d. Detailed summary of the experimental results obtained from the MATLAB program and BTU Meter correlation.

| Flow rate (GPM) | h (W/(m²·°C)) | τ(s) | R" (°C·m²/W) | RMSE (°C) | $T_{\text{fluid estimated}}$ (°C) | $T_{\text{water measured}}$ (°C) | $|ΔT_{\text{water}}|$ (°C) | Re | Nu | Correlation Estimated Flow Rate (GPM) |
|-----------------|----------------|------|--------------|-----------|----------------------------------|----------------------------------|-------------------------------|----|----|--------------------------------------|
| 10              |                |      |              |           |                                  |                                  |                               |     |    |                                      |
| 1687            | 1695           | 1690 | 1688         | 1683      | 1682                             | 1786                             | 1796                          | 1808| 1794| 1800                                 | 1869| 1864| 1863| 1857                              | 1857 |
| 2.59            | 2.58           | 2.59 | 2.59         | 2.60      | 2.60                             | 2.45                             | 2.43                          | 2.35| 2.35| 2.35                                 | 2.34| 2.35| 2.35| 2.35                              | 2.35 |
| 0.000332        | 0.000333       | 0.000331 | 0.000333 | 0.000331 | 0.000331 | 0.000332                        | 0.000333                    | 0.000333 | 0.000333 | 0.000333                          | 0.000332 | 0.000333 | 0.000333 | 0.000333 | 0.000333 |
| 0.020           | 0.021          | 0.021 | 0.023        | 0.022     | 0.025                             | 0.019                           | 0.019                        | 0.020| 0.020| 0.020                               | 0.019 | 0.020| 0.020| 0.020                             | 0.020 |
| 0.11            | 0.03           | 0.05  | 0.03         | 0.02      | 0.02                              | 0.15                            | 0.12                         | 0.13| 0.09| 0.15                               | 0.18 | 0.11 | 0.09| 0.11                             | 0.17 |
| 46453           | 46453          | 46453 | 46453        | 46453     | 46453                             | 60388                           | 60388                        | 60388| 60388| 60388                              | 74324 | 74324 | 74324 | 74324                             | 74324 |
| 54.09           | 54.35          | 54.19 | 54.12        | 53.96     | 53.93                             | 57.26                           | 57.58                        | 57.97| 57.52| 57.52                              | 59.92 | 59.76 | 59.73 | 59.54                             | 59.54 |
| 10.0            | 10.2           | 10.0  | 10.0         | 9.8       | 9.8                               | 13.0                            | 13.3                         | 13.7| 13.2| 13.5                               | 16.0  | 15.8 | 15.8 | 15.6                              | 15.6  |
Appendix G – BTU Meter Correlations with 1 GPM

The purpose of this Appendix is to present the BTU Meter correlations that include the 1 $\frac{gal}{min}$ results. As discussed in Chapter 2, the 1 $\frac{gal}{min}$ results were not included in the correlations since they did not satisfy the fully turbulent criteria constricting the Dittus-Boelter correlation. For completion purposes, the $Nu – Re$ and $h – \dot{V}$ correlations are presented in Figures 63 and 64.

![Nusselt vs Reynolds with 1 GPM Results](image)

Figure 63. The BTU Meter Nusselt-Reynolds correlation for forced internal convection of water flow through a pipe with 1GPM results.

where: $Nu = 2.3555 Re^{0.292}$
Figure 64. The BTU Meter convection coefficient vs flow rate correlation for forced internal convection of water flow through a pipe with 1GPM results.

where:

\[
h = 864.47 \left[ \frac{W}{m^2 \cdot ^\circ C} \left( \frac{min}{gal} \right)^{0.292} \right]^{0.292}
\]
Appendix H – Comparison between the Convection Coefficient Measured by the BTU and the Convection Coefficient Calculated with Dittus-Boelter

The purpose of this Appendix is to show the differences between the h obtained from the BTU Meter, and the h obtained from the Dittus-Boelter correlation for the same flow rate. As mentioned in Chapter 2, the difference between the two values is due to the BTU Meter not measuring the true convection coefficient of the flow, rather the results obtained from the BTU Meter measure the convection coefficient in the laminar sublayer. This accounts for the significant difference between the convection coefficient values. Table 12 shows the average h value obtained from the BTU Meter and the Dittus-Boelter correlation.

Table 12. A comparison between the convection coefficients obtained from the BTU Meter and the Dittus-Boelter correlation.

| Flow Rate (GPM) | h (BTU Meter) [W/(m²-°C)] | h (Dittus-Boelter) [W/(m²-°C)] | |Δh| [W/(m²-°C)] |
|-------------|-----------------|-----------------|-----|-----------------|
| 1           | 828.33          | 1252.79         | 424.46 |
| 4           | 1382.50         | 3797.74         | 2415.24 |
| 7           | 1589.67         | 5942.32         | 4352.65 |
| 10          | 1687.50         | 7904.55         | 6217.05 |
| 13          | 1796.33         | 9750.61         | 7954.28 |
| 16          | 1862.17         | 11512.60        | 9650.43 |