Developing Active Artificial Hair Cell Sensors Inspired by the Cochlear Amplifier

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ABSTRACT

The mammalian cochlea has been the inspiration to develop contemporary cochlear implants and active dynamic sensors that operate in the sensor’s resonance region and possess favorable nonlinear characteristics. In the present work, multi-channel and self-sensing active artificial hair cells (AHCs) made of piezoelectric cantilevers and controlled by a cubic damping feedback controller are developed numerically and experimentally. These novel AHCs function near a Hopf bifurcation and amplify or compress the output by a one-third power-law relationship with the input, analogous to the mammalian cochlear amplifier. The multi-channel AHCs have extended frequency bandwidth to sense over multiple resonant frequencies, unlike conventional single-channel AHCs. Therefore, the adoption of these AHCs reduces the number of required sensors to cover the desired bandwidth of interest in an array format. Furthermore, a novel self-sensing active AHC is created in this study using quadmorph beams for future cochlear implants or sensor design applications. The self-sensing scheme allows miniaturization of the system, embedding AHCs in a limited space, and fabrication of AHC arrays by omitting external sensors from the system for practical implementation. Preliminary research on the extension of this research to MEMS AHCs and arrays of AHCs is also presented. The active AHCs can lead to transformative improvements in the dynamic range, sharpness of the response, and threshold of sound detection in cochlear implants to aid individuals with sensorineural hearing loss. Additionally, they can enhance the dynamic properties of sensors such as fluid flow sensors, microphones, and vibration sensors for various applications.
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GENERAL AUDIENCE ABSTRACT

In the mammalian auditory system, the acoustic wave that enters the ear canal is transmitted to the cochlea of the inner ear where it is decomposed into its frequency components. The cochlea then amplifies faint sounds and compresses high-level signals and as these processes stop due to damage, severe hearing loss occurs. Therefore, the present work is focused on developing artificial hair cells (AHCs) that can accurately replicate cochlea’s behavior and aid the creation of prostheses for hearing restoration. In this work, the AHC is a beam with piezoelectric layers that is integrated with a control system designed to apply the cochlea-like amplification/compression on the beam. Experimental and simulation results show that the AHC is able to amplify or compress the output based on its input level similar to the mammalian cochlea. In contrast to previous designs of AHCs where each AHC could sense a single frequency, the system developed in this work possesses multiple sensing channels to increase the frequency range of the AHC. Furthermore, the development of a novel self-sensing scheme allows the omission of the external sensor that was required for the AHC operation in previous devices. This advancement in the self-sensing AHC design paves the way for creating fully implantable AHCs to replace the damaged parts of the cochlea. These multi-channel self-sensing AHCs have the potential to be used in the creation of cochlear implants, or sensors such as accelerometers, microphones, and hydrophones with improved dynamic properties. AHCs with different lengths, i.e. different sensing frequencies, can be mounted in an array format to cover the speech frequency range for speech recognition in individuals with hearing loss.
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Chapter 1

Introduction

1.1. Introduction

This chapter introduces the mammalian cochlear amplifier and bio-inspired artificial hair cells developed in the current work. First, the motivation for this research is presented. The chapter then continues with a summary of the ear’s physiology, its signature characteristics, and the active processes of the inner ear. Next, hearing impairment and the role of commercial cochlear implants in hearing restoration and their limitations are reviewed. As a part of this chapter, a history of researchers’ efforts in developing devices to mimic cochlea’s characteristics is provided. Finally, an overview of the active artificial hair cells, significant contributions of the current work, and the dissertation outline are presented.

1.2. Motivation

The human cochlea is an astonishing and complicated component of the inner ear that plays the most significant role in hearing. Besides transducing mechanical vibrations into electrical stimuli [1], the cochlea pre-processes acoustic inputs by decomposing them into their constitutive frequency components in the basilar membrane [2-4]. Furthermore, frequency selectivity, nonlinear amplification, and compression of acoustic signals are among the signature characteristics of the mammalian cochlea [5]. It actively compresses or attenuates high sound pressure level (SPLs) signals and boosts weak SPL signals. This compressive nonlinearity allows the ear to detect a large range of sound pressure levels (0-120 dB) with its adaptive sensitivity in
a frequency range of 20 Hz to 20 kHz. These nonlinear functions stop once cochlear sensory hair cells are damaged [6]. It is known that hair cells can be damaged with age and environmental conditions and the cochlea cannot be repaired [7]. The loss of hair cells often results in sensorineural hearing loss which is a common disorder [8]. As a result, there is a need for hearing prostheses such as cochlear implants to bypass the damaged parts of the cochlea and transduce sound-induced vibrations to electrical signals. Disadvantages with commercial cochlear implants including the high cost of the implantation [9] and poor functionality in a complex acoustic situation [10] have drawn researchers’ interest in mimicking cochlear mechanisms to develop better hearing prosthesis for hearing restoration. Therefore, the development of devices such as artificial cochleae and artificial hair cells (AHCs) to create novel cochlear implants and treat hearing loss has gained traction in recent years [11-13].

Results of research on cochlear processes can span beyond medical applications such as cochlear implants to overall sensor development. Inspired by the cochlear characteristics, various vibration sensors, microphones, hydrophones, and flow sensors can be designed [14-16] for applications including but not limited to shock testing and structural health monitoring [17, 18]. Mimicking the nonlinear functions of the cochlea can lead to the development of dynamic sensors with a lower threshold of detection, sharper frequency sensitivity, and larger dynamic range operating in the resonance region than traditional sensors.

Additionally, there have been disputes in the literature as to what in the biological process of the mammalian cochlea facilitates the nonlinear amplification to take place, as discussed later in this chapter. Some of the proposed theories could be studied through mechanical surrogate systems undergoing a cochlea-like response. Since many of these questions arise from the difficulty of conducting experiments in a small living organism, the study on physical systems can provide
researchers with insights into the behavior of the biological cochlea and address the competing theories. A summary of the theories on cochlear processes is presented in the next sections.

1.3. The biological cochlea

In this section, an introduction to the auditory system’s physiology with a focus on the inner ear’s functions is presented. On this background, active processes of the cochlea of the inner ear, their features, and possible involved mechanisms in the active functions are studied.

1.3.1. Sound waves journey from the outer ear to the inner ear

This section presents a summary of the human ear’s anatomy and physiology. The outer, middle, and inner ear form the three main parts of the ear. Acoustic waves are collected by the pinna and are directed to the tympanic membrane (eardrum) located at the end of the auditory canal in the outer ear. Sound waves apply pressure on the tympanic membrane and cause it to vibrate. The tympanic membrane transmits the vibrations to the ossicles of the middle ear named malleus, incus, and stapes [2]. As the tympanic membrane vibrates, the footplate of the stapes pushes and pulls on the oval window located between the middle ear and the inner ear. As a result, acoustic waves propagate to the fluid-filled cochlea of the inner ear. The main task of the middle ear is to transmit the sound waves from air-filled outer and middle ears to the fluid-filled inner ear with minimal energy loss. Therefore, an impedance matching between the acoustic waves in the two mediums occurs in the middle ear. A significantly smaller area of the stapes compared to the tympanic membrane (about 1/17) results in an increase in the force applied to the oval window. Also, the anatomy of the middle ear is such that the ossicles provide another 1.3-fold amplification of the force due to lever action [19]. Thus, the total force is amplified by a factor of about 22 to prevent the loss of acoustic energy from the boundary of the middle ear and inner ear [2]. Other functions of the middle ear include suppressing high-intensity sounds by stapedius reflex and
balancing the pressure between the middle and outer ears by the Eustachian tube that connects the middle ear to the nasopharynx [1]. The stapes is connected to the middle ear wall by the stapedius muscle. Loud sounds cause a contraction in this muscle that prevents the stapes to transfer high-intensity acoustic waves to the inner ear. However, this reflex is relatively slow and limited to stimulus frequencies lower than about 1 kHz. Therefore, it cannot protect the inner ear from sudden and very high-intensity sounds. The stapedius reflex also occurs during vocalization. A schematic of the auditory system is shown in Fig. 1.

Fig. 1. The human ear. From “Neuroscience” by Purves D, Augustine G J, Fitzpatrick D, Hall W, LaMantia A, McNamara J and White L. [20]. Copyright © 2004 by Sinauer Associates, Inc. All rights reserved. Reproduced with permission of the Licensor through PLSclear.

The inner ear includes the cochlea and the vestibular system. The vestibular system plays an important role in balancing and providing the head’s orientation information to the brain [3], but it is not involved in hearing mechanisms. The other organ of the inner ear, the cochlea is embedded
in a spiral bony structure and contains sensory receptors. The human’s cochlea takes 2.5 turns around a bony center called modiolus to help it fit in the temporal bone [1]. The cochlea is made up of three sections, the scala vestibule and scala tympani containing perilymph, and the scala media filled with endolymph. The basilar membrane (BM) splits the scala tympani from the scala media, while the Reissner’s membrane is located between the scala vestibule and scala media. The cochlea as a rigid structure is connected to the middle ear by two windows, the oval window and the round window. Due to the cochlea’s anatomical characteristics and as the cochlear fluids are incompressible, each time the oval window is pushed inward, the round window moves outward [21]. A schematic of the uncoiled cochlea is displayed in Fig. 2.

**Fig. 2.** Schematic of the uncoiled cochlea, and tonotopic mapping along the length of the BM. From “Neuroscience” by Purves D, Augustine G J, Fitzpatrick D, Hall W, LaMantia A, McNamara J and White L [20]. Copyright © 2004 by Sinauer Associates, Inc. All rights reserved. Reproduced with permission of the Licensor through PLSclear.
As shown in Fig. 2, when the sound-induced vibrations enter the cochlea, they excite and bend the BM. The traveling wave generated in the perilymph and endolymph propagates along the length of the BM towards the Helicotrema where the scala vestibuli and scala tympani meet. The length of the BM in the human ear is about 35 mm [1]. Basilar membrane fibers are fixed at one end (near the modiolus) and free at the other end (near the outer wall of the cochlea) and their length increases from the base to the apex of the cochlea, while their diameter decreases [2]. Due to the spatially varying stiffness (about 100-fold decrease from the base to the apex) and inertia of the BM’s fibers [2], a particular site of the BM oscillates more strongly in response to a specific frequency of the traveling wave than others. Therefore, these local resonances are formed and the frequency attributed to the maximum response’s location is called the characteristic frequency (CF). Thus, a CF is defined at each site of the basilar membrane as the frequency at which the BM’s displacement at that point reaches its maximum when a traveling wave with the same frequency propagates in the BM [20, 22, 23]. As a result, a complex acoustic stimulus with multiple frequencies decomposes into its frequency content along the length of the tonotopic BM. The BM functions similar to a Fourier decomposer where high and low frequencies are detected near the base and apex of the cochlea, respectively [24]. This frequency selectivity or tonotopic mapping of the BM is one of the most important characteristics of the cochlea [22, 25] that was first observed by von Békésy [23]. On top of the BM, the cochlea's organ of Corti lies and it is composed of mechanosensory cells that transduce mechanical vibrations into electrical stimuli. Fig. 3 displays a schematic of the cochlea, its cross-section, the organ of Corti, and hair cells that are described in the next section.
Fig. 3. Schematic of the cochlea, cross-section of the cochlea, the organ of Corti, and hair cells. From “Neuroscience” by Purves D, Augustine G J, Fitzpatrick D, Hall W, LaMantia A, McNamara J and White L [20]. Copyright © 2004 by Sinauer Associates, Inc. All rights reserved. Reproduced with permission of the Licensor through PLSclear.

1.3.2. Hair cells

The organ of Corti contains one row of the flask-shaped inner hair cells (IHC) and three rows of the cylindrical outer hair cells [1, 26], as shown in Fig. 3. A human ear possesses about 15000 to 20000 hair cells [27]. The basal end of the hair cells is close to the BM and their apical end is fixed in a tissue plate called the reticular lamina. At the tip of each hair cell, a bundle of the minute hairs known as stereocilia is located. Stereocilia’s configuration in the IHCs and OHCs is different. In
the IHCs, the stereocilia are arranged in flat or U-shaped patterns, while the stereocilia of the OHCs form V-shaped or W-shaped configurations [26]. In either hair cells, the stereocilia are positioned in multiple rows that are sorted by the length with the shortest stereocilia near the center of the cochlea. Furthermore, the overall length of the stereocilia increases from the base to the apex of the cochlea. In each OHC, the tallest Stereocilia’s tip is connected to the tectorial membrane (TM) shown in Fig. 3 [26]. In contrast to OHCs, IHCs’ stereocilia are free at one end. In both types of hair cells, the tip of each stereocilium is attached to the side of the adjacent taller stereocilium in the radial direction by filaments called tip links. As the BM moves up and down, the reticular lamina moves in two directions, parallel to the BM and perpendicular to it [2, 20]. As a result, the stereocilia of the hair cells are deflected. The stereocilium deflection is translated as pivoting about its base rather than bending [28]. In OHCs, the deflection is mainly caused by shearing of the tallest stereocilium back and forth against the TM, while in IHCs it is hypothesized that the stereocilia move by the flow of the endolymph [1]. The maximum displacement of the stereocilia is about 0.3 nm [20]. The deflection of the stereocilia in IHCs results in transducing the mechanical vibrations into electrical signals as described in the following section. Moreover, function of the OHCs is presented in Section 1.3.4.

1.3.3. Mechanoelectrical transduction

The stereocilia are immersed in high potassium ion ($K^+$) concentrated endolymph with a potential of about +80 mV greater than the perilymph’s potential. Additionally, the resting potential of the IHC’s body is about -40 mV with respect to the perilymph. Therefore, the voltage between the IHC’s body and the endolymph is about -120 mV [25]. When the stereocilia bundle, also called the hair bundle, deflects towards the longest stereocilium, transduction channels at the tip of the stereocilia are opened by gating springs [1]. In early studies, researchers hypothesized that tip links
acted as gating springs and they were responsible for opening and closing the transduction channels when the tension in the tip links varied due to displacement of the hair bundle [29]. Recent studies suggest that a less stiff membrane plays the role of the gating spring [30, 31]. However, the nature of this membrane has not been determined yet [32]. The potential difference between the endolymph and the hair cell’s body causes positive ions, mostly potassium, to enter the hair cell and depolarize it. The flow of the potassium ions into the cell opens voltage-gated calcium ion ($Ca^{2+}$) channels at the basal end of the hair cell. The low concentration of the potassium ions in perilymph relative to the hair cell results in the opening of the potassium channels near the basal end of the hair cell. Therefore, the potassium ions flow out while calcium ions flow into the hair cell. As a result, repolarization occurs. The influx of calcium ions to the hair cell triggers synaptic vesicles. Therefore, the vesicles release glutamate neurotransmitters at the afferent synapse. Subsequently, action potentials are generated in the auditory nerve and are transmitted to the brain. As IHCs are responsible for sending the auditory information to the brain, approximately 95% of spiral ganglion neurons innervate the IHCs [26]. Hyperpolarization occurs as the stereocilia deflect towards the shortest stereocilium which causes the ion channels to close. The number of channels to open or close is a function of the BM vibrations level and some channels remain open at the resting state of the hair cell. A schematic of the mechanoelectrical transduction process is shown in Fig. 4.
1.3.4. **Active nonlinear processes**

OHCs in contrast to IHCs do not contribute to mechanoelectrical transduction of the sound-induced vibrations and transmitting the electrical signals to the brain. OHCs are responsible for amplifying low-level stimuli and suppressing high-level input signals in a nonlinear manner. This process is referred to as the cochlear amplifier [33]. In 1948, Gold hypothesized that an active amplification process existed in the cochlea [34]. He believed that passive amplification of the
acoustic stimuli was not effective in the cochlea due to the damping provided by the viscosity of the cochlear fluid. Gold suggested that an electromechanical transduction similar to that of piezoelectric crystals with an energy supply could overcome the effects of the damping. Experimental limitations at that time did not allow required testing on the cochlea to evaluate Gold’s hypothesis. In the 1970s, Evans and Wilson’s experiments on the cats’ cochlea showed some discrepancies between their recordings from a cochlear nerve and the BM vibrations [35]. Therefore, they suggested that a process such as a second filter should exist along with the first mechanical filter, the BM, proposed by von Békésy [23] that could be responsible for the sharp tuning of the cochlear nerve response. However, the idea of a second filter with its passive nature was rejected as research in the field was advanced [36]. In the same decade, studies such as Rhode’s measurements of the BM vibrations in squirrel monkeys [37] and distortion products Kemp recorded from the ear [38] showed nonlinearities in the cochlea’s behavior. The results obtained in Rhode’s experiments displayed a nonlinear relationship between the input SPL and the BM displacement at frequencies near the CF [37]. The nonlinearity observed in the cochlea’s behavior was not reported in von Békésy’s experiments on the cochlea of cadavers and Johnston et al.’s in vivo tests on guinea pig’s cochlea in the 1960s [39]. Since the 1980s, experimental efforts using methods such as Mössbauer technique [40] and laser interferometry [41] have confirmed highly tuned BM vibrations and the nonlinear nature of the cochlear processes [42-44]. Cochlear amplifier boosts weak sound pressure levels and sharpens the response near the CF, while it compresses the response to high level stimulus [45]. Nonlinear characteristics of the cochlea can be better explained by demonstrating the BM velocity measured by Ruggero in the basal end of the cochlea of chinchilla in response to various SPLs as shown in Fig. 5 reproduced from [46].
Fig. 5. BM velocity spectrum as a function of the input SPL in chinchilla [46]. Reproduced from Ruggero M A, Rich N C, Recio A, Narayan S S and Robles L 1997 Basilar-membrane responses to tones at the base of the chinchilla cochlea *The Journal of the Acoustical Society of America* 101 2151-63, with the permission of the Acoustical Society of America. Available online at https://doi.org/10.1121/1.418265. Fig. 5 displays that the response to low-level SPLs was sharply tuned and as the excitation level increased, the tuning became broader and the CF shifted to lower frequencies. The compressive nonlinearity that resulted in broader tuned curves in response to high-intensity input, was also investigated using input-output curves at the CF in a particular location of the BM. Input-output curves obtained from researchers’ recordings inside the cochlea of chinchilla, guinea pig, and cat are displayed in Fig. 6 (a) by square, circle, and diamond markers, respectively [6]. The magnitude of the BM-displacement/input-pressure frequency response function (FRF) for the chinchilla and guinea pig is illustrated in Fig. 6 (b). The curves on the left side of Fig. 6 (b) show a set of FRFs obtained at lower frequencies from the chinchilla’s cochlea and the ones on the right-side display another set of FRFs in a higher frequency bandwidth recorded from a guinea pig cochlea. Note that the chinchilla’s data shown in Fig. 6 (a) and (b) was from the same study, while the guinea pig data reproduced by Robles *et al.* [6] was taken from different studies.
Fig. 6. (a) BM output-input curves at the CF for chinchilla (squares), guinea pig (solid and open circles representing results of two different studies), cat (diamond). (b) BM-displacement/sound-pressure FRFs in the cochlea of chinchilla (the FRF set on the left), and guinea pig (the FRF set on the right) [6]. Reproduced from Robles L and Ruggero M A 2001 Mechanics of the mammalian cochlea *Physiological reviews* 81 1305-52, with the permission of The American Physiological Society.

The relationship between the BM displacement and input intensity in all three species was a nonlinear function. As the input level increased, the response growth with the input level had a slope of less than 1 dB/dB. Therefore, a compressive nonlinearity was observed in the cochlea’s function. Also, FRFs of Fig. 6 (b) for various input strengths were not identical and this was a good indication of the existence of nonlinearities in cochlear processes. In mammals, the basilar membrane vibrations with respect to the stimulus level are compressed by a rate of about 0.3 dB/dB near the local CF [6, 22, 46]. The nonlinear processes provide the ear with a large dynamic input range, 0 dB to 120 dB SPL, enabling it to perceive loud sounds and faint whispers [6]. This broad range of SPLs elicits BM displacements in a narrow range of about 0.1 nm to 10 nm [22].

The cochlear amplifier’s nonlinear amplification, compressive nonlinearity, and the frequency tuning of the BM are three of the key characteristics of the mammalian cochlea that are shown in Fig. 7 [22].
Fig. 7. Active processes’ characteristics, (a) amplification, (b) frequency tuning, (c) compressive nonlinearity, and (d) spontaneous otoacoustic emission [22]. Reprinted from Neuron, Vol 59, Issue 4, A.J. Hudspeth, Making an Effort to Listen: Mechanical Amplification in the Ear, Pages 530-545, Copyright (2008), with permission from Elsevier.

Nonlinear characteristics of the cochlea are only observed in response to excitations close to the CF, while the cochlea behaves linearly at other frequencies [6]. Similar features are observed in a dynamic system operating at a Hopf bifurcation [47-49]. Therefore, the mammalian cochlea similar to hearing organs of reptiles and amphibians is hypothesized to work in the vicinity of a supercritical Hopf bifurcation. A system undergoes Hopf bifurcation as varying a control parameter for the bifurcation near a critical value alters the system’s stability behavior. The system is stable on one side of the bifurcation, while it becomes unstable on the other side, as the control parameter exceeds the critical value. On the unstable side of a supercritical Hopf bifurcation, limit cycles start to form around the equilibrium of the system. The three signature characteristics of the cochlea can be observed in the cochlea operating near a bifurcation and on the stable side of it, at
frequencies close to the CF. The relationship between the output and input of the auditory system at each site of the cochlea at its CF follows a one-third power law. This enables the ear to amplify or compress input signals based on their intensity. The broad frequency range of the hearing, 20 Hz to 20 kHz, can also be explained by considering active processes in the cochlea and oscillators along the BM tuned near a Hopf bifurcation [48]. Passive BM models that attribute the frequency tuning of the cochlea to only the stiffness and inertia variations of the BM require a stiffness to inertia ratio variation of more than a million-fold to be able to account for the frequency range of the hearing [50]. However, the measured variation from the base to the apex of the BM is approximately 2 orders of magnitude lower. Therefore, the active processes are hypothesized to be responsible for the sharp tuning. Note that recent studies suggest that friction of the transduction channels are also important in frequency tuning of the hair cells in mammals, but more investigation is needed to prove this idea [51].

The fourth important characteristic of the cochlea, spontaneous otoacoustic emissions (SOAE) is another evidence for active processes inside the cochlea which occurs when the system works on the unstable side of a bifurcation [48]. SOAEs are emitted sounds from the ear in a quiet environment without the presence of an external stimulus. SOAE has been measured from the mammalian ear and auditory organs of other vertebrates such as reptiles and amphibians [52, 53]. These emitted sounds are suggested to be related to the active processes in the auditory system of these species. However, further studies need to determine the exact source of the SOAEs.

Although there are several lines of evidence for active processes inside the cochlea, the exact mechanisms of these processes are not known yet [5]. Two mechanisms have been proposed by the auditory community that may underlie cochlear processes: electromotility and hair bundle motility of the OHCs. A summary of these machines is presented in the following sections.
**1.3.4.1. Electromotility of the OHCs**

OHCs have a unique motor protein (SLC26A5) in their lateral wall called prestin [54, 55]. This motor protein is not found in the IHCs. The body of the OHCs has a resting potential of about -70 mV [25] and variation in their potential due to opening or closing of the transduction channels changes prestin’s configuration [56, 57]. The OHCs’ body is motile [57] such that it shortens at depolarization and lengthens at hyperpolarization [58]. This voltage-dependent behavior is called electromotility or somatic motility of the OHCs. This conversion of electrical charges to mechanical motion in the OHCs is similar to an inverse piezoelectric effect. A direct piezoelectric effect, conversion from mechanical forces to electrical charges, is also seen in the OHCs [59]. Therefore, it was suggested that OHCs show a piezoelectric-like behavior [36, 60-62] with a piezoelectric coefficient of about 20 fC/nN [63] and a nonlinear capacitance which is a signature of the OHCs’ electromotility [64, 65]. As a result of the electromotility, a force is applied to the BM that amplifies the initial vibrations and causes the hair bundles to deflect more, and more transduction channels to open [66]. The resultant amplified vibrations from the feedback provided by the OHCs are then transduced to electrical signals by the IHCs as discussed in the previous section.

There are several proofs showing the important role of electromotility in the active amplification process. One significant group of evidence includes results of in vivo experiments performed on mice by knocking out the prestin gene [54, 67]. Measurements of auditory sensitivity in knockout mice showed a lack of electromotility and severe hearing loss of about 40 to 60 dB. Although these experiments demonstrate the important role of electromotility in hearing, there are some uncertainties associated with interpreting the results of these experiments. Significant OHC length and stiffness change as a result of prestin knockout and the possibility of alterations in the output
of the feedback loop by modifying one of its elements are among these issues [45]. Therefore, in another experiment, knockin mice were produced from a knockin of mutated prestin and the OHC’s length and stiffness remained unchanged [45, 68]. The results of the analysis showed that the BM displacement with respect to the sound pressure level was linear and it matched the measurements after the mice’s death [69]. Thus, it was concluded that electromotility is needed for the cochlear amplifier operation [70].

There are still doubts about the source of the active processes. One of the unresolved problems regarding recognizing electromotility as the only mechanism underlying the cochlear amplification is the OHC membrane’s time constant that limits its operating frequency bandwidth. In vitro experiments demonstrated OHC’s ability to produce electromotile forces in a large bandwidth of about 20 Hz to 79 kHz [71]. However, in vivo measurements show OHCs’ electrical low-pass filtering effect with a corner frequency of more than 10-fold smaller than the upper-frequency limit of the hearing in mammals [72, 73]. Different solutions have been proposed for this problem [74-82]. Considering biological piezoelectricity that shifts the corner frequency to much higher frequencies [79] and smaller time constant of the membrane in vivo due to higher resting potential of the OHCs [76] are two of the important solutions. However, several researchers believe that a second mechanism plays an important role in active mechanisms. This mechanism is the hair bundle motility of the OHCs.

1.3.4.2. Hair bundle motility

The amplification process in the ear of amphibians [83, 84], fishes [85], reptiles [86, 87], and birds [88] is not the same as the mammalian ear. The hair bundle in these species shows active motility that is considered to be responsible for the amplification of the acoustic signals [89, 90]. Various experiments on the hair bundle of these species identified the two elements thought to be required
for the active processes: i) negative stiffness and ii) adaptation process [22, 68]. Regions of nonlinear stiffness were observed in measurements of the force applied to a hair bundle in bullfrog saccus versus the bundle’s displacement, as shown in Fig. 8 [91].

![Figure 8](image)

**Fig. 8.** Displacement of a hair bundle in bullfrog’s saccus as a function of the force measured under displacement-clamp conditions. Figure from “Negative hair-bundle stiffness betrays a mechanism for mechanical amplification by the hair cell” by Martin P, Mehta A and Hudspeth A [91]. Copyright (2000) National Academy of Sciences.

As displayed in Fig. 8, for large displacements in both directions, the hair bundle behaved linearly. At about ±10 nm from the resting position, the curve shows extremums and a negative stiffness region is observed between them. The negative stiffness corresponds to the behavior of the transduction channels. As the hair bundle deflects towards the tallest stereocilia (positive direction) in response to a force applied to it, the tension inside the tip links and the gating springs attached to the transduction channels increases. Therefore, the opening probability of the channels increases. As a result, the load distribution in the gating springs changes. Some transduction channels open and the force in their gating springs decreases. Subsequently, the tension in the gating springs of the closed channels and the probability of the rest of the channels to open increase. For a specific range of positions and open probabilities, all the channels are opened [22].
Thus, not only does not the bundle resist against the force applied to its base, but it generates a force in the same direction. A similar phenomenon happens when all of the ion-channels tend to close. The nonlinear stiffness is believed to be one of the bases for the amplification mechanism. The aforementioned force generation and mechanical work production by the hair bundle require a source of energy provided by an adaptation process. The adaptation process changes the tension of the channels’ gating springs based on their new state, shifts the displacement-force curve to the right or left, and adjusts the sensitivity range of the hair bundle [91, 92]. As a result of the adaptation, the curve shifts to the right as an excitation bends the bundle in the positive direction. Therefore, the gating springs connected to the transduction channels experience less tension and the open transduction channels are closed. Similarly, the adaptation shifts the curve to the left after the channels’ reaction to a stimulus applied in the negative direction. The adaptation process is a subject of ongoing research and researchers have proposed different models for the mechanism resulting in adaptation [93, 94]. In one of the main models called the motor model, the adaptation is attributed to the myosin molecules slipping down or climbing up an actin filament [95, 96]. This movement controls the force on the transduction channels. In the second model, the transduction channels’ reclosure depends on the intracellular calcium ions and the adaptation is hypothesized to be controlled by $Ca^{2+}$ binding to the transduction channel [97-99]. This adaptation has a low time constant, about 1 ms or less [27], and is called fast adaptation. The former adaptation’s time constant is about 10-fold greater than the later and it is called slow adaptation. A combination of these models has also been proposed [100].

Based on the difference between the strength of the external force and the force generated by the hair bundles as a result of the nonlinear stiffness and adaptation, two amplification mechanisms have been suggested [22]. If the force generated in the negative stiffness region by the hair bundle
is stronger than the stimulus, the bundle is bistable and it displays spontaneous oscillations between the two states of all open channels and all closed channels. If the forces balance each other, the nonlinear region in the displacement-force plot does not show a negative slope. Therefore, a region with approximately zero slope forms near the origin and the system is tuned near a Hopf bifurcation. At the Hopf bifurcation, the bundle amplifies weak stimuli and shows important properties discussed earlier in this chapter.

1.3.4.3. Amplification mechanism in the mammalian OHC

As mentioned in the previous section, hair bundle motility is the active amplification and suppression mechanism in non-mammal vertebrates and some insects. However, there have been debates about the mechanism of the active nonlinear processes in the mammalian OHC among the auditory community [5]. Based on the results of the experiments performed on the mammalian cochlea and discussed in section 1.3.4.1, a group of hypotheses suggests that electromotility is the main mechanism for the active processes in mammals. However, some researchers believe that electromotility of the OHC alone does not explain the frequency tuning and nonlinear nature of the amplification and compression processes. Also, active hair bundle motility has been observed in experiments on some mammals such as Mongolian gerbil and rats [101, 102]. Therefore, it was suggested that hair bundles could play an important role in nonlinear behaviors by their spontaneous movement and force generation in mammals similar to lower vertebrates and some insects [70, 102]. However, the force that an individual hair bundle can generate is not enough to contribute to the amplification and frequency tuning. A solution for solving this problem and improving the force strength and frequency tuning was to create a model of the cochlear amplifier by coupling the movement of a bundle to the other bundles via the TM [103]. An alternate solution was considering other mechanisms such as somatic motility for amplifying the generated force by
hair bundles [104]. Several researchers believe that the mechanism underlying the cochlear amplifier is a combination of both somatic motility and hair bundle motility [105-108]. Further experiments are needed to determine the role of each mechanism in the active amplification/compression and frequency tuning of the mammalian cochlea.

1.4. **Hearing impairment and cochlear implant**

Cochlear nonlinear processes stop as the OHCs are damaged [6]. Multiple factors contribute to the damage of the cochlea and OHCs including certain diseases, ototoxicity, congenital infections, radiation therapy, and noise trauma [1, 8]. Sensorineural hearing loss due to the loss of hair cells is a common hearing impairment that can impact individuals’ social and physical lives drastically. Hearing loss is one of the common disorders in the US [8] and at least 1.2 million adults in the US suffer from it [109]. In patients with hearing loss, where simple sound amplification is not effective, a cochlear implant (CI) can be used. Commercially available cochlear implant devices are composed of three main parts: i) an external microphone and radio-frequency (RF) transmitter, ii) an RF receiver part that is implanted inside the skull and connected to the electrodes inside the cochlea, and iii) electrodes that are inserted inside the cochlea [110]. The external microphone receives the acoustic signals and performs some preprocessing on the signal including filtering the complex signal into discrete frequency bandwidths, adjusting the dynamic range of the system, and applying noise reduction algorithms. The electrical signal is then encoded and sent to the internal parts as RF signals. The signals are decoded by the implanted receiver unit and are sent to the electrodes wired to this unit and distributed tonotopically inside the cochlea. As current is supplied to the electrodes near the neurons, the neurons generate action potentials and the information is sent to the brain. Major commercial cochlear implant devices are made by MED-EL Co., Advanced Bionics, Cochlear Ltd., and Nurotron Biotechnology Co. Ltd [111, 112]. The
number of electrodes and the depth of insertion of them varies between the commercially available models. The maximum number of electrode arrays belongs to Nurotron CI devices with 24 intracochlear active electrodes and 2 extracochlear reference electrodes, while MED-EL uses the minimum number of electrodes among these companies, with a maximum of 12 electrodes [113]. Although the number of electrodes differs in CIs, studies on the effects of the electrode numbers on the speech recognition in CI recipients show that the number of effective channels does not exceed 8, even in devices with 22 electrodes [114]. One of the possible explanations for this limitation is the overlapping that occurs between the electric fields. However, the reasons for this important limitation and possible solutions to it are not known yet. Another limitation with the current CIs is the depth electrodes can reach inside the cochlea that determines the lower frequency limit of the CI [115]. The diameter of electrodes, their flexibility, and the possibility of damage to the cochlear structure due to insertion make it difficult for the electrode arrays to reach apical parts of the cochlea. Amongst the CI devices, MED-EL FLEX Standard provides the maximum frequency coverage. The insertion depth of this electrode array is 31.5 mm in a 34 mm cochlea with a maximum frequency bandwidth of 70 Hz to 11.5 kHz [111]. Other limitations with cochlear implants include the high cost of the implantation [9], relatively high power consumption, and poor functionality in a complex acoustic situation [10]. Over the past decades, scientists have tried to overcome some of these challenges by characterizing different material properties, changing structures’ dimensions, and changing the number of electrodes in their proposed devices [112, 116, 117]. A summary of the development of these devices is presented in the next section.

1.5. History of physical models of the cochlea

In this section, a history of physical models of the cochlea, BM, and hair cells are presented. First, passive physical models and artificial devices inspired mainly by the frequency selectivity of the
cochlea are studied. Next, models mimicking the active processes of the cochlea are reviewed. In each section, a summary of the influential works is provided.

1.5.1. Passive physical devices

Although commercial cochlear implant devices have been developed to partially restore hearing, there are several problems associated with their functionality [111, 118]. Limited frequency bandwidth (particularly in low frequencies), cross-channel interactions, and poor functionality in noisy environments are among the limitations and disadvantages of the current prosthesis. To enhance the frequency bandwidth of hearing aids and move towards fully implantable devices, researchers have developed passive AHCs, artificial cochleas (Acochleas), and artificial basilar membranes (ABMs). These systems mimic the tonotopic mapping of the BM and electrical transduction of the acoustic-induced vibrations in IHCs. Additionally, these models and devices are useful in fabricating sensors that detect vibrations, acoustics, and fluid flow. Two approaches are popular in the modeling: i) physical models of the cochlea consisting of fluid-filled compartments and a membrane representing the BM [119-121] and ii) arrays of beams with various length working near the first natural frequencies of the system [122-126]. The next sections present a history of these passive models.

1.5.1.1. Hydromechanical models of the cochlea

George von Békésy’s pioneering measurements of the BM motion and various cochlear components’ properties, as well as creating physical models of the cochlea on a large scale [127], motivated many researchers to build physical models of the cochlea [128, 129]. Scientists believed that these models could help understand the mechanisms behind the cochlear processes and fabricate hearing aids. They examined various hydromechanical models of the cochlea by considering the configuration, geometry, and characteristics of its compartments, fluids, and
membranes in their models. Microelectromechanical System (MEMS) fabrication techniques made it possible to create life size models of the cochlea and more feasible devices for implanting inside the cochlea. Most of the physical models of the cochlea contained one to three fluid-filled ducts and a tapered structure to simulate the BM. Various membranes, beams, and combinations of these were designed to serve as ABM in physical models with an attempt to mimic the frequency selectivity and bandwidth provided by the cochlea in mammals. This section presents a summary of the significant physical models of the cochlea found in the literature.

Zhou et al. in 1993, developed the first life-size physical model of the cochlea [130]. Their model consisted of two tapered chambers as scala vestibuli and scala tympani filled with fluid, and a membrane made by MEMS fabrication techniques. The membrane represented the BM and it was fixed on the sides to a stainless steel plate with a cavity of varying width. Two holes into the entrance of the cavities with a covering rubber modeled the oval and round windows. A piezoelectric transducer was used to excite the system at the oval window and the ABM vibrations were measured by a laser vibrometer. The results of this experiment with a single tone stimulus in a bandwidth of 300 Hz to 15 kHz showed tonotopic frequency mapping of the BM similar to von Békésy’s test results on samples extracted from the BM of cadavers. However, the tuning curves obtained were not as sharp as the ones recorded from a living cochlea. In 1995, Haronian and MacDonald developed a microelectromechanics based artificial cochlea (MEMBAC) composed of fixed-fixed beams with increasing length from the base to the apex of the ABM with an exponential profile [131]. The beams were coupled together by a viscous medium. They modeled the beams as spring-mass-damper resonators coupled to the adjacent resonator by a spring and a damper. Acoustic excitation was used to drive the system and the response of the structure was measured by a laser vibrometer. A tonotopic mapping between 3 kHz to 10 kHz was observed.
Ando, Tanaka et al. (1997) created an array of microscale beams named fishbone architecture with possible applications as ABM microphone and giant impulse generator [132, 133]. They showed the direction of the energy flow and the ability of the device to decompose an input signal into its frequency components as the structure was excited from its high-frequency end. Another fishbone structure containing 43 resonators was made by the same group and displayed frequency selectivity in a narrow range of 1 kHz to 2.5 kHz [116].

Piezoelectric materials have been widely used in these artificial devices due to their rich characteristics. The piezoelectric layers in these sensors produce an electrical charge when deflected. The output voltage of these mechanoelectrical transducers can be used as a readout method without a need to use external measurement devices such as laser vibrometers. In 2005, White and Grosh mimicked the passive behavior of the mammalian cochlea via a hydromechanical model of the cochlea [120]. They fabricated a silicon oil-filled duct covered by an exponentially tapered membrane resembling the BM. The ABM was excited by acoustic stimuli and a laser vibrometer was used as the measurement device. Two different ABMs were fabricated and tested: an isotropic ABM made of silicon-nitride and an orthotropic ABM consisting of polyimide beams deposited on the membrane. Experimental results showed cochlear-like filtering behavior in a frequency bandwidth of 4 kHz to 35 kHz with slightly better sharpening seen in the orthotropic case at some tested frequencies. The effects of the viscosity of the fluid on the frequency mapping and traveling waves were also investigated. Chen et al. (2006) modeled and constructed a trapezoidal ABM made of a polyvinylidene fluoride (PVDF) film and an ACochlea using a single fluid-filled duct with a length of 20 mm [121]. A number of 32 copper (Cu) beams were deposited on the piezo-film ABM with their length ranging from 4 mm to 8 mm to provide varying stiffness along the length of the ABM. To reduce the coupling effect observed in the results, the membrane
between the beams was cut. The results of their experiments showed traveling waves propagating in the system when the window of the duct was excited by a piezo stack actuator. The Cochlear physical model developed by Wittbrodt et al. (2006) consisted of two channels filled with saline and separated by a 36 mm long trapezoidal partition as the ABM [134]. The polyimide ABM with discrete aluminum ribs showed very limited frequency selectivity when the system was excited by a coil magnet.

1.5.1.2. Artificial cochlea and artificial basilar membrane

Inspired by the tonotopic BM, sensory devices consisting of an array of AHCs [112, 116, 117, 123, 135, 136] and membranes [137, 138] have been designed to mimic the passive functions of the cochlea. These sensors offer multiple sensing channels to address the frequency bandwidth of interest. MEMS fabrication techniques miniaturize the designs to possibly embed such devices into a human ear, as well as fabricate devices such as microphones and hydrophones. A review of the influential devices in this area with a focus on their design aspects, offered number of frequency channels, and their bandwidth is presented in this section.

A major group of ABMs and Acochleas has been made of piezoelectric membranes with a number of electrodes deposited on the membrane to measure the electrical outputs of the system at different locations. Bio-compatibility of the piezoelectric material is a very important design consideration in devices aimed to be implanted into the auditory organs. Therefore, PVDF and aluminum nitride (AlN) have been drawn researchers’ attention in the literature. A piezoelectric Acochlea made of a 30 mm long trapezoidal PVDF membrane that was attached to a frame, was built by Shintaku et al. in 2010 [16]. The membrane had 24 aluminum electrodes on it that were distributed along the length of the ABM. The output voltages of the electrodes in response to an acoustic excitation were later used in their analysis. Frequency analysis in air and silicon oil was conducted.
Frequency selectivity in the frequency range of 6.6 kHz to 19.8 kHz in air and 1.4 kHz to 4.9 kHz in the fluid was reported. This device was tested by Inaoka, Shintaku, et al. in deafened guinea pig cochlea [139]. One of the mid-electrodes of the array was connected to platinum-iridium ball electrodes implanted into the scala tympani. The response of the system to a 104.4 dB SPL input after amplification by a gain of 1000 could stimulate auditory brain-stem responses (ABRs). The same group fabricated and tested another device that was made specifically for implant inside the basal turn of the guinea pig cochlea. This new curved PVDF membrane fixed to a silicon frame was implanted inside a healthy cochlea such that the membrane was very close to the BM. The optical measurement from the BM and the membrane showed similar frequency selectivity. In another group of tests on the ex vivo temporal bone of the guinea pig, a pair of electrodes was wired to the device to measure the membrane’s output voltage in response to acoustic excitation. The recorded electrical outputs in response to a 100 dB SPL input in 5 kHz, 10 kHz, and 20 kHz were very low, with a maximum of 29.3 μV at 20 kHz. The authors emphasized a need for higher output (about $10^5$ times greater) to be able to stimulate auditory primary neurons. Jung et al. (2013), modeled and fabricated a multi-channel piezoelectric acoustic sensor (McPAS) to mimic the frequency separation function of the BM [135]. Their ABM was a trapezoidal shape, thin film PVDF membrane with a length of 28 mm and varying width of 1 mm to 8 mm. From 23 thin metal electrodes that were fabricated, 12 of them were used for measurement in each experiment. McPAS’s frequency bandwidth was 2.5 kHz to 13.5 kHz. In response to a 94 dB SPL acoustic stimulus, the maximum output read from an electrode was 6.33 mVpp. In 2015, they developed a piezoelectric artificial cochlea (PAC) with the ability to separate input frequencies in the range of about 300 Hz to 6 kHz into 13 bands [140]. The authors reported the detection of 19 peaks in their frequency analysis and by further analysis, they could recognize 13 frequency bands. A similar
membrane as their previous design was fabricated in this device. However, the number of electrodes was modified to 13 electrodes and a packaged part containing a fluid-filled cavity was added. In another work by this group, the membrane between the electrodes in McPAS was partially etched (reduced from 25 μm to 5 μm) to improve the frequency selectivity function of the device [141]. In 2013, a group of researchers from South Korea designed ABMs composed of 16 Aluminum Nitride (AlN) beams with narrow supports [142]. Varying the length of the beams from 305 μm to 3200 μm provided a frequency bandwidth of 2 kHz to 20 kHz. ABM-displacement/input-voltage and ABM-displacement/sound-pressure FRFs of the system showed the frequency selectivity of the ABM with better results obtained from the electrical excitation. Due to the large size of the device (5.81 cm × 6.27 cm) and its lower frequency limit, it needed significant modifications. In the same year, the group designed a new ABM array consisting of 10 fixed-fixed AlN beams on a silicon-on-insulator wafer [143]. This design eliminated rigid body modes of the previous device observed due to the boundary conditions that the narrow supports provided. Therefore, the device showed improved output strength. A signal processor was used to generate pulse width modulation (PWM) signals from the output of the piezoelectric beams in response to acoustic stimuli for future stimulation of brain nerves. A similar design with 16 beams of varying lengths from 450 μm to 3300 μm was fabricated and named RAIB which stands for resonator array of isolated beams [125]. This device was later compared to a resonator array of coupled beams (RACB). In RACB each beam was coupled to the adjacent beams by a 200 nm thick SiO₂ membrane. Beam-displacement/sound-pressure FRFs and frequency selectivity analysis between 4 kHz to 20 kHz showed poor frequency separation of the RACB, while the RAIB showed a desired frequency selectivity behavior. In 2020, Saadatzi et al. proposed an ABM made up of a piezoelectric trapezoidal membrane fixed from the sides and the basal end to a ducted
base [144]. The spatially distributed electrodes measured the tonotopic response of the system to acoustic stimuli. The system was studied analytically and numerically. Additionally, a prototype of the device created by MEMS fabrication techniques with a 45 μm PVDF membrane, Polydimethylsiloxane (PDMS) base, and 30 Gold electrodes was tested in a frequency range of 1-10 kHz. The system showed frequency selectivity between 3 kHz and 8 kHz for inputs above ~58 dB SPL. The authors concluded that replacing a thinner membrane can improve the minimum excitation level and signal to noise ratio required for the operation of the system.

Hair-like piezoelectric cantilever beams have been widely used in ABM and Acochlea devices to harness their rich electro-mechanical coupling. These devices offer many advantages over the membrane-based devices mentioned in this section. The coupling between the channels’ stiffness due to the presence of a membrane was a drawback in ABM membranes that could negatively affect the frequency selectivity of the device. This issue was not dominant in beam-based devices. Another significant feature of these devices is the ability to adjust the natural frequency of each channel (beam) without complications. Therefore, in a device consisting of similar cantilever beams with the same width and thickness, the length of the beam can be adjusted to tune its fundamental frequency to a desired value. Some of the significant ABM and Acochleas are described as follows.

Bachman et al. in 2006 built a four-channel array of cantilever beams made of epoxy with different lengths (2-7 mm) to mechanically filter the acoustic stimuli into its frequency sub-bands [145]. Two measurement techniques were used and compared to each other: optical readout and capacitive readout. The advantage of the latter method over the former was its ease of use with other electronics and independence from an external measurement device. Individual amplifier channels were utilized for amplification of each beam’s output. Therefore, the device was capable
of gain tuning in addition to frequency tuning. The authors envisioned the application of this device as a speech processor part of a future CI. Hur et al. created an array of four piezoelectric cantilevers made of lead magnesium niobate-lead titanate (PMN-PT) material [146]. They deposited a pair of interdigitated electrodes near the basal part of each beam to enhance the sensitivity of the device. In their analysis, for each beam length, a pair of identical cantilevers was wired together to investigate the accuracy of the fabrication. Discrepancies in resonant frequency and slightly in the amplitude of the response were observed for each pair of beams that were attributed to the MEMS fabrication method. The beam was first characterized and the displacement- piezoelectric charge relation was determined. The array was stimulated by a speaker with a sound pressure of 1 Pa from 1 kHz to 6 kHz and the charge on the electrodes was measured. The array displayed high sensitivity. Lee et al. in 2014, used PZT material to fabricate flexible acoustic nanosensors to mimic the function of hair cells in the human cochlea [147]. The inorganic-based piezoelectric acoustic nanosensors (iPANS) fabricated by this group were bonded to a flexible trapezoidal substrate at 3 locations. The system was excited by acoustic stimuli and the response of the structure at 23 points was measured by a laser vibrometer. Piezoelectric outputs were also measured at 3 locations. The tonotopic response of the membrane was shown by optical measurements. The iPANS output was in good agreement with optical results measured at corresponding locations. The sensor showed high sensitivity to 40 SPL inputs. However, to mimic the BM frequency selectivity, more sensors should be used and the device needed to be miniaturized. Although these sensors showed high sensitivity due to their high piezoelectric constant, they were not usually considered for biomedical applications as they contained lead. A completely implantable organ of Corti (CIAO) was developed and tested in vitro and in vivo in guinea pig cochlea by Katherine Knisely et al. at the University of Michigan [112, 148, 149]. A
number of 5 AlN bimorph micro-cantilevers were fixed to a silicon substrate. The length of the cantilevers was adjusted between 150 μm to 210 μm such that they covered a frequency range of about 20 kHz to 40 kHz (frequency range in guinea pig) in water. When CIAO was implanted in the straight part of the basal turn of the guinea pig cochlea, the electrodes that were in contact with the scala tympani fluid could stimulate the spiral ganglion. The authors stated another application of this device in measuring perilymph’s pressure. In another work by Jang et al., an array of 8 AlN microcantilevers with varying length (600 μm to 1350 μm) was created such that each pair of the beams with different lengths were located in front of each other with a gap between their tips to reduce the size of the device (2.5 mm × 2.5 mm × 0.6 mm) [150]. In this study, cantilevers were used to reduce the frequency range of the system (2.92 kHz to 12.6 kHz) and increase its sensitivity. A signal processor unit was integrated with the array to amplify and convert the electrical output of the system to stimulation signals. The device was tested on a deafened guinea pig and an intra-cochlear electrode array connected to the output of the device was inserted into the scala tympani. Electrically evoked auditory brainstem response (EABR) was generated as the system was excited with an acoustic stimulus in the range of 75 dB SPL to 95 dB SPL with a frequency equal to the characteristic frequency of the channels. EABRs were not recorded when the system was excited at other frequencies. Therefore, frequency selectivity was reported. In addition, this device was 3 times more sensitive than their 2013 design. However, the signal processor unit needed miniaturization for implantation in the cochlea. In 2017, they improved the design and fabricated the first cantilever-based flexible artificial basilar membrane (FABM) [151, 152]. They increased the number of channels to 10 and fabricated a flexible SU-8 substrate instead of the silicon used in their earlier design. The device showed filter-like behavior in a frequency range of 659.4 Hz to 2375 Hz and covered most of the frequencies in the human communication
range (300 Hz to 3 kHz). Results were obtained by measuring the displacement using a laser vibrometer in response to an acoustic or electrical stimulus. In a most recent study by Zhao, Knisely, et al. at the University of Michigan, a 4 channel MEMS transducer was built for a completely implantable device [13, 15]. Both sensing and actuation tests were performed on the device in air and water and the functionality of this xylophone-like transducer in frequency decomposition was shown. The device was also used to measure the intracochlear pressure in the cochlea of guinea pigs. The group enhanced the device by fabricating a Parylene ribbon cable and bonding it to the xylophone-like probe [124]. This electrical connection could help easier measurements and actuation of the device inside the cochlea. The authors stated various possible applications for this device as sensors and actuators.

The passive devices reviewed in this section offered different degrees of tonotopy, sensitivity, and frequency bandwidth. Among these, bio-compatible devices with a frequency bandwidth covering the speech range in humans, can potentially substitute the microphone and front end of cochlear implants and act as an integrated acoustic sensor and mechanical filter-bank. Although these devices mimic the frequency selectivity of the cochlea, two of its significant characteristics, nonlinear amplification, and compressive nonlinearity, cannot be mimicked using passive devices. Therefore, mimicking the cochlear amplifier is crucial in creating cochlear implants with the amplification/compression capabilities analogous to the biological cochlea.

1.5.2. Active physical devices

Developing models of the active hair cells with nonlinear characteristics inspired by the cochlea and constructing active AHCs and artificial cochlea are subjects of ongoing research. Incorporating the unique aspects of the cochlear amplifier to develop novel cochlear implants or sensors with a
sharper frequency sensitivity, broader dynamic range, and lower detection threshold than traditional transducers has been very challenging for researchers over the past few decades.

In 1993, Lechner created an active hydromechanical model of the cochlea [153]. He used a 465 mm long natural rubber membrane as the tapered BM and a fluid-filled duct as the scala vestibule. An electromechanical shaker was used to generate traveling waves in the system by pushing on the window of the fluid-filled duct. Two sets of PVF$_2$ piezoelectric bending bimorphs were attached to the membrane in order to, i) measure the response of the membrane at different locations and ii) apply the feedback control voltage to the membrane. The piecewise linear feedback control algorithm used in his work was introduced by Zwicker [154]. The results of Lechner’s experiments showed nonlinear amplification of the weak input and sharpened tuning near the CF. Some nonlinear compression of the high-level stimulus was also seen but it was not effective.

In 2009, Jian-si modeled a hair bundle as an array of 10 cantilever beams with a PVDF strain sensor and a PZT bimorph actuator mounted on two different sides of the basal part of each beam [155]. A feedback constant gain control method was used to control the amplitude and phase of the voltage sent to the PZT actuator in response to a base or acoustic excitation. The controller parameters were tuned to achieve maximum sensitivity, but the system did not show nonlinear amplification and compression as a result of the control law used in this work. Note that, although a piezoelectric sensor was embedded in their array, the tip response measured by a laser vibrometer was used in the experiments.

Inspired by the hair bundle motility and spontaneous oscillations of the stereocilia, Lim et al. (2009) fabricated an array of inverted pendulums, each connected at the tip to the neighbor pendulum by bistable buckling springs [156]. The model and experimental results showed the
nonlinear amplification of faint inputs as a result of stiffness softening when the pendulum was deflected. This mechanism was not effective for larger input levels, as it needed a resetting mechanism. Three years later, Lee and Park made a physical model of the stereocilia consisting of an inverted pendulum with a magnet at its tip, a pair of pivotal springs, a T-bar with another magnet, and a stepper motor to move the magnet in the T-bar groove horizontally [157]. The interaction between the restoring force of the springs and the repulsive force of the magnets simulated the negative stiffness of hair bundles and varying the magnetic force by moving the T-bar magnet worked similar to the adaptation mechanism. The displacement-force curves measured in displacement clamping experiments were similar to the ones measured in biological hair bundles.

In 2010, Tapson et al. simulated the cochlear amplifier as an electromechanical acoustic sensor [158]. Displacement of a latex band representing the BM and excited by an acoustic stimulus was measured using an electro-optical displacement sensor. It was then rectified and used in the control signal. The control signal sent to the electromechanical shaker that was connected to the band to vary the tension and produce the desired response. They showed that the controlled system could amplify the output level by one-third power of the input. However, due to some stability issues, they stated that a negative feedback control loop is needed to further control the shaker’s amplifier power.

Kim, Song, et al., developed a force sensor capable of detecting forces as low as $10^{-10}$ N (pressures as low as $10^{-6}$ Pa) in water with a sharp frequency selectivity [159]. A 23 mm long cantilever beam with a copper wire loop embedded between the two layers of it represented a hair bundle. The displacement of the beam in response to a force stimulus was measured using a quadrant photodetector that detected the reflection of laser light from a silicon mirror attached to the beam.
The displacement data was then used in a feedback control law that actively applied a force to the beam by controlling the wire loop’s current in a magnetic field. The control law design was inspired by the hair bundle motility and mimicked the negative stiffness region of the displacement-stiffness curve and adaptation process [160]. The experimental results showed an inverse relationship between the output and the sensitivity defined as output/input. In another work, the artificial hair cell was integrated with an artificial neuron [161].

Joyce et al. examined several models of the active AHC inspired by the somatic motility and hair bundle motility of the AHC to simulate nonlinear characteristics of the cochlea [162, 163]. They experimentally developed a single-channel, active artificial hair cell by applying a phenomenological cubic damping control law and to a bimorph piezoelectric beam subject to a base excitation [11, 164-169]. Their numerical and experimental results showed compressive nonlinearity and active amplification of the input signal. The input of the AHC had a cubic power-law relationship with its output similar to the relationship observed in the mammalian cochlea.

Similar to active processes seen in mammalian cochlea, research on some other species’ hearing physiology, such as insects demonstrated compressive nonlinearity and amplification of the acoustic stimuli. Inspired by the mosquito’s auditory system, Guerreiro et al. controlled the response of a MEMS microphone by implementing a feedback computation algorithm through an embedded system called spike generator [14, 170]. Generated phase-lock pulses were added to the initial vibrations of the microphone’s diaphragm to achieve adaptive quality factors at the resonance and mimic the nonlinear functions of the mosquito ear.

Tsuji et al. in 2018, developed an artificial cochlea epithelium constructed of a trapezoidal PVDF membrane [171]. Electrodes were patterned such that each adjacent pair of them served as a sensor for detecting the deflection of the membrane and actuator for locally applying the feedback control
signal. Six pairs of electrodes were used to mimic the frequency selectivity of the cochlea. The feedback control of the system showed one-third amplification behavior.

1.6. **Active artificial hair cells**

The previous section reviewed researchers’ attempts in developing various devices and control algorithms to achieve a cochlea-like performance. However, the great majority of research into AHCs and Acochlea faces major challenges in matching the performance of the complex nonlinear cochlea. Among these devices, passive models attribute the frequency tuning of the cochlea to the stiffness and inertia variations of the BM and eliminate the role of the active processes. Furthermore, not only are there a limited number of studies on active AHCs or artificial cochleae in the literature, but most of them are unable to mimic the compressive nonlinearity of the cochlear amplifier, their design is not feasible, or their operation depends on the feedback signal measured by an external sensor that needs to be embedded permanently near the AHC.

Therefore, the primary objective of the present study is to mimic the compressive nonlinearity of the cochlea by developing piezoelectric based active artificial hair cells. Inspired by the stereocilia geometry of the hair cells, AHCs can be represented as piezoelectric cantilever beams. Such adaptation is widely used in mechanical sensors such as microphones, flow meters, and hydrophones. The AHC design consists of a substrate material and two or four layers of piezoceramics. Similar to the OHCs, piezoelectric materials possess an electromechanical coupling that enables augmented composite beams to transduce electrical signals into mechanical vibrations and mimic the amplification functions of the OHCs. As a voltage is supplied to the piezoelectric layers, they experience a strain proportional to the applied voltage [172]. This strain produces a bending moment and causes the beam to deflect. Furthermore, the piezoelectric layers
in the sensor will produce an electrical charge when deflected and they can be used to convert mechanical vibrations into electrical signals analogous to mammalian IHCs.

The response of an AHC to a stimulus is heavily dependent on its frequency content, and the AHC is most sensitive to the frequencies close to its natural frequencies. In contrast to typical dynamic sensors that operate linearly in the sensor’s quasi-static response region, the AHCs are designed to operate in the resonance region, where weak inputs are significantly amplified. As the resonance frequencies of the structure are a function of its geometry, the length of the beam can be tailored to tune its fundamental frequency to a targeted value and mimic the frequency selectivity of the cochlea.

In order to mimic the cochlear amplifier and create an active AHC, a nonlinear feedback control law is applied to the piezoelectric layers to create a cochlea-like response near the resonance. As a result, these active sensors can detect smaller inputs than passive sensors and compression of high-intensity stimulus allows the same sensor to capture a larger range of signal levels. Moreover, the AHC sharpens the resonance peak and results in a higher frequency sensitivity.

In contrast to the previous studies where AHCs operate near the fundamental frequency, in the present work a novel multi-channel active AHC is developed that is capable of applying its nonlinear compressibility and amplification behavior to the sensor’s output near its first and second natural frequencies. It should be noted that although two-channel AHCs are created in the current work, the number of channels can be increased to include more frequency bandwidths. Previous studies have shown that systems tuned to a Hopf bifurcation can be used as the basis to mimic the characteristics of the mammalian cochlea [173-176]. Therefore, after examining models of the biological cochlea, a nonlinear feedback controller is designed based on a cubic damping control
law [11, 165, 166] which applies the appropriate forcing conditions to the beam to amplify vibrations initially induced by an external stimulus. A schematic of the AHC is shown in Fig. 9.

![Schematic of the AHC](image)

**Fig. 9.** Schematic of the AHC

This nonlinear feedback control law uses the tip-velocity of the AHC as the feedback signal and eliminates the linear damping of the system and introduces cubic damping into the system. Consequently, it drives the response of the AHC to a Hopf bifurcation near the first and second natural frequencies and replicates the cochlear amplifier by maintaining a one-third power relationship between the output and input of the AHC.

In the next step, a novel self-sensing active AHC scheme is developed in this research that tunes the AHC near a Hopf bifurcation. It is the first time that a self-sensing active AHC is modeled as a quadmorph piezoelectric cantilever controlled by a phenomenological cubic damping controller. The self-sensing AHC is designed such that it has four piezoelectric layers and the control signal is supplied to a pair of its piezoelectric layers and the output of the AHC is the sensed voltage of the second piezoelectric pair. This is in contrast to the present studies where the tip velocity of the AHC was measured with external sensors. The requirement of a permanent external sensor in the AHC system, to acquire velocity, was a strict limitation that is eliminated in the current work for the practical implementation of the AHCs. Finally, the self-sensing scheme is combined with the
multi-channel control law to create self-sensing multi-channel active AHCs to increase the number of sensing channels in the self-sensing AHC.

1.7. Contributions

This section summarizes the significant contributions of this work as follows.

- In this work, active piezoelectric-based AHCs are developed. The inclusion of a cochlea-like amplification scheme will allow the active AHCs to achieve sensitivity and performance comparable to the biological hair cells and thus form a better replacement for the damaged hair cells. The nonlinear amplification in active AHCs will lower the threshold of sound detection, improve the frequency sensitivity, and increase the range of sound pressures the sensor can detect.

- In the present work, multi-channel active AHCs with nonlinear characteristics analogous to the biological cochlea are created for the first time. The multiple degrees of freedom approach considered in this study expands the ability of each AHC to capture more than one frequency. The multi-channel AHCs can be embedded in an array to mimic the tonotopic function of the cochlea along with its nonlinear properties over a large frequency bandwidth. In the present literature, AHCs work close to their fundamental frequency and the total number of sensing channels in an array is equal to the number of the AHCs used. Therefore, multi-channel AHCs broaden the frequency bandwidth in an array with the same number of sensory cantilevers.

- In this research, self-sensing active AHCs are developed for the first time. The realization and validation of a self-sensing active AHC will have significant impacts on sensor development. This work provides a solution to the challenging problem of omitting permanent external sensors from the feedback loop of the AHC systems while mimicking the cochlear amplifier. Self-sensing AHCs offer several advantages over previous designs including the ability to
embed them in a limited space and to combine several AHCs in an array without the need for the same number of external measurement devices. Furthermore, in contrast to the systems with velocity feedback [164, 166, 177], the output of the self-sensing AHC is the piezo-electric voltage which can directly be transferred from the electrodes to the auditory nerves to stimulate them. Moreover, miniaturization of the active AHCs and fabrication of a MEMS AHC for cochlear implants or sensor applications is only possible if external sensors such as laser vibrometers are omitted from the system.

- To the best of the researcher’s knowledge, the current work is the first instance of the adoption of a quadmorph as a self-sensing self-actuating element in a feedback control system. Therefore, the successful implementation of the system opens the possibility of using quadmorphs in various self-sensing systems with different control schemes.

- This study advances research to mimic the nonlinear cochlear amplifier. It opens the possibility to further explore hearing restoration in the mammalian cochlea via multi-channel self-sensing AHCs capable of mimicking the biological counterpart. The multiple transduction channels and improved frequency resolution in these AHCs will offer significant improvements over the limitations of current cochlear implants. As a result, the development of self-sensing multi-channel AHCs could lead to a better understanding of human hearing as well as aiding people with hearing disorders, increasing their social activities, levels of hope, and happiness in life. Furthermore, the realization of a multi-channel self-sensing active AHC will have a significant impact on sensor development. These AHCs can be used as active sensors in microphones, hydrophones, and fluid flow sensors due to their broader input range and frequency selectivity.
• The preliminary study on passive MEMS AHCs provides the basis required for designing active MEMS artificial hair cells to mimic the nonlinear amplification of human cochlea. It also builds a framework that guides the testing of the MEMS AHCs for future work.

• The numerical study on the active AHC array can be used as a starting point for implementing the control laws on an actual array and tuning the AHC parameters. It provides valuable insight into array performance in response to complex stimuli.

1.8. Chapter summary and dissertation outline

The present work is divided into eight main chapters. In this chapter, first, an introduction to this research including a summary of the physiology of the ear, characteristics of the cochlear amplifier, and its possible mechanisms were presented. Next, sensorineural hearing loss and a need to develop novel cochlear implant devices were stated. Physical models of the cochlea and basilar membrane attempting to mimic various characteristics of the hearing organs were then presented. Most of the passive devices showed acceptable band-pass filter-like behavior and therefore, their application in replacing the front-end part of the cochlear implants was envisioned. It was shown that only a few numbers of the physical models aimed to replicate active nonlinear features of the cochlear processes. Therefore, an active artificial hair cell developed in this research was introduced at the end of the chapter.

Chapter 2 describes the model development of the active AHC. The nonlinear feedback control law is also discussed in that chapter. Furthermore, conditions required to tune the system to a Hopf bifurcation, stability of the feedback control system, and its steady-state response are investigated. In Chapter 3, a Simulink model of the system is created and numerical simulation of the single-channel and multi-channel AHC is presented. Chapter 4 is dedicated to the real-time implementation of the multi-channel AHC system. The response of the AHC to various stimulus
levels is studied and an experimental approach is taken to calculate the controller gains. At the end of the chapter, the experimental results are compared with the numerical results obtained in Chapter 3 for model validation. Chapter 5 builds the theoretical and numerical framework for developing active self-sensing AHCs. The simulation results of the self-sensing AHC are presented in that chapter. Subsequently, Chapter 6 validates the self-sensing AHC model constructed in Chapter 5. Furthermore, the work in that chapter is extended to create multi-channel AHCs. In Chapter 7, preliminary extensions of the current research including an introduction to MEMS AHCs and passive AHC arrays, and active AHC arrays are discussed. Finally, a conclusion of the work done in this research is presented in Chapter 8.
Chapter 2

Model development of active artificial hair cell

2.1. Introduction

In this chapter, a finite element (FE) model of a piezoelectric cantilever beam representing an AHC will be created. The FE model will be used to create a single degree of freedom (SDOF) and two degrees of freedom (2DOF) models of the system. Then, a nonlinear feedback control law will be presented that can tune the SDOF system to a Hopf bifurcation in an attempt to mimic the nonlinear functions of the outer hair cells discussed in the previous chapter. Next, the stability of the feedback control system will be investigated and then the steady-state response of the system to a harmonic input will be studied. Finally, an extended control law will be introduced to be applied to the 2DOF representation of the system to increase the number of frequencies a single AHC can detect.

2.2. Artificial hair cell model

An active AHC is modeled as a cantilevered bimorph beam augmented with piezoceramics poled in opposite directions. The bimorph composite is shown schematically in Fig. 10.
The brass substrate is length $L$, width $w_b$ and thickness $2t_b$ and is sandwiched between two piezoelectric layers, each $t_p$ thick. The poling direction of the piezoceramics, $p$, is also displayed in Fig. 10. The parameters of the composite bimorph structure with brass shim and PSI-5A4E piezoceramics are tabulated in Table 1. Details of the piezoelectric material properties can be found in [178].

**Table 1.** The model parameters of the beam and piezoelectric actuators

<table>
<thead>
<tr>
<th>Properties</th>
<th>Beam</th>
<th>Piezoelectric Actuator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young Modulus ($GPa$)</td>
<td>$C_{b11}^E$</td>
<td>$Y_p$</td>
</tr>
<tr>
<td>Density ($Kg/m^3$)</td>
<td>$\rho_b$</td>
<td>$\rho_p$</td>
</tr>
<tr>
<td>Length (m)</td>
<td>$L_p$</td>
<td>$L_p$</td>
</tr>
<tr>
<td>Width (m)</td>
<td>$w_b$</td>
<td>$w_p$</td>
</tr>
<tr>
<td>Thickness (mm)</td>
<td>$2t_b$</td>
<td>$t_p$</td>
</tr>
<tr>
<td>Piezoelectric strain constant ($C/N$)</td>
<td>$d_{31}$</td>
<td>$-190 \times 10^{-12}$</td>
</tr>
<tr>
<td>Capacitance (series operation) ($nF$)</td>
<td>$C_p$</td>
<td>2.5</td>
</tr>
</tbody>
</table>

As the structure is excited through a base excitation ($\ddot{z}(t)$) in the vertical (z (3)) direction, the piezoelectric actuators apply a control voltage ($V(t)$) to the system and introduce nonlinear dynamics to the AHC. The control algorithm applied to the system is presented in section 2.5. The governing electromechanical equations of the Euler-Bernoulli beam augmented with piezoceramics connected in series is given by [164, 172, 179].

\[
\rho A \frac{\partial^2 u(x,t)}{\partial t^2} = - \frac{\partial^2}{\partial x^2} C_{b11}^E I \frac{\partial^2 u(x,t)}{\partial x^2} + C \frac{\partial u(x,t)}{\partial t} + \frac{\partial^2}{\partial x^2} \left( \vartheta \chi_{[a,b]} V(t) \right) - \rho A \dddot{z}(t),
\]

\[
i_p = -\vartheta \left( \frac{\partial^2 u(x,t)}{\partial x \partial t} \bigg|_{a}^{b} \right) - C_p \frac{dV(t)}{dt},
\]

where, $u(x,t)$ is the deflection of the beam in z (3) direction, $\rho$ is the material density, $A$ is the cross-sectional area of the composite, $C_{b11}^E$ is Young’s modulus of the composite, $I$ is the moment of inertia of the composite, $C$ is the distributed damping operator, $\vartheta$ is the electromechanical coupling term, $V(t)$ is the control voltage supplied to the piezoelectric actuator, $i_p$ is the current...
flowing through the piezoelectric layers, \( C_p \) is the equivalent capacitance of the piezoceramics in series operation, and \( \chi_{[a,b]} \) is a characteristic function defined as,

\[
\chi_{[a,b]} = \begin{cases} 
1 & (x_1, x_3) \in \left( [a, b] \times [-t_b - t_p, -t_b] \cup [a, b] \times [t_b, t_b + t_p] \right) \\
0, & \text{otherwise}
\end{cases}
\]  

(2)

where, \( a \) is the starting location of the piezoceramic on \( x \) (1) axis and \( b \) is the location on the same axis where the piezoceramic ends. In the bimorph AHC shown in Fig. 10, \( a = 0 \), and \( b = L_b \). The electromechanical coupling factor for series connection of piezoelectric layers is given by,

\[
\vartheta = \frac{\kappa e_{31}}{2 t_p},
\]  

(3)

where, \( \kappa \) is the cross-sectional first moment of inertia, and \( e_{31} \) is the piezoelectric stress constant related to the piezoelectric strain constant \( (d_{31}) \) by \( e_{31} = C_{11}^E d_{31} \). At the clamped end of the beam, boundary conditions become,

\[
u(0,t) = 0, \frac{\partial u}{\partial x}(0,t) = 0,
\]  

(4)

and at the free end, boundary conditions are stated as,

\[
C_{11}^E I \frac{\partial^2 u}{\partial x^2}(L_b, t) = 0, \text{ and } C_{11}^E I \frac{\partial^3 u}{\partial x^3}(L_b, t) = 0.
\]  

(5)

Details of the distributed parameter model derivation for a piezoelectric cantilever beam using Newton’s method can be found in [164, 179].

2.3. Finite element model of the system

The finite element model of the bimorph beam is developed by approximating the displacement of the composite \( u(x, t) \) as nodal displacements \( (\mathbf{x}(t)) \) using the Galerkin approach [164] with finite element shape functions built from cubic order Hermite splines. The equation of motion for the approximated system is in the form of,

\[
M\ddot{\mathbf{x}} + C\dot{\mathbf{x}} + K\mathbf{x} = B V(t) - P_2 \ddot{z}(t),
\]  

(6)
where, $\mathbf{x} = [u_1 \ \theta_1 \ u_2 \ \theta_2 \ ... \ u_n \ \theta_n]'$ is the vector of nodal translational and rotational displacements ($u_i$ and $\theta_i$) in an $n$ element FE model, $M$ is the mass matrix, $C$ is the damping matrix, $K$ is the stiffness matrix, $B$ is the control influence matrix, $P_z$ is the base acceleration influence matrix, $\ddot{z}(t)$ is the base acceleration, and $V(t)$ is the control voltage shown in Fig. 10 and defined in Section 2.5. These global matrices are built by assembling the corresponding elemental matrices. The elemental mass, stiffness, control influence and base acceleration influence matrices are given by,

$$m_{el} = \frac{\rho Ah}{420} \begin{bmatrix} 156 & 22h & 54 & -13h \\ 22h & 4h^2 & 13h & -3h^2 \\ 54 & 13h & 156 & -22h \\ -13h & -3h^2 & -22h & 4h^2 \end{bmatrix}, \quad k_{el} = \frac{c_{11}^E h^3}{h^3} \begin{bmatrix} 12 & 6h & -12 & 6h \\ 6h & 4h^2 & -6h & 2h^2 \\ -12 & -6h & 12 & -6h \\ 6h & 2h^2 & -6h & 4h^2 \end{bmatrix}$$

$$b_{el} = \frac{\kappa d_{31} c_{11}^E}{2t_p} \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \quad p_{el} = \rho Ah \begin{bmatrix} 1/2 \\ -\frac{1}{12} h \\ -\frac{1}{12} h \\ -\frac{1}{12} h \end{bmatrix},$$

where, $h$ is the length of each element ($h = L_b/n$). The following relations hold in calculating the elemental matrices of Equation (7) of the composite,

$$\rho A = \rho_b (2w_b t_b) + \rho_p (2w_p t_p),$$

$$c_{11}^E I = c_{b11}^E I_b + Y_p I_p, \quad I_b = \frac{1}{12} w_b (2t_b)^3, \quad I_p = \frac{1}{12} w_p (2t_p)^3,$$

and,

$$\kappa = w_b \left( (t_b + t_p)^2 - t_b^2 \right).$$

The parameter values of the composite bimorph structure are tabulated in Table 1. In the FE model, a modal damping ratio of $\zeta_1 = 3\%$ for the first mode and $\zeta_k = 1.24\%$ for all the other modes are considered. This model represents the dynamics of the AHC from a numerical perspective. These parameter values are set equal to damping estimations from experimental data obtained by a
system identification method in Chapter 4. Subsequently, the damping coefficient matrix can be determined by substituting,

\[ x = \psi \eta, \]  

into the free vibration equation of,

\[ M\ddot{x} + C\dot{x} + Kx = 0, \]  

where, \( \psi = [\psi_1 \psi_2 \ldots \psi_n] \) is the eigenvector matrix obtained by solving the eigenvalue problem of,

\[ M\omega_i^2 \psi_i = K\psi_i. \]  

Substituting (11) into (12) and pre-multiplying the obtained equation by \( \psi^T \) yields,

\[ \psi^T M\psi \ddot{\eta} + \psi^T C\psi \dot{\eta} + \psi^T K\psi \eta = 0. \]  

Now Equation (14) is rewritten as,

\[ M_r \ddot{\eta} + C_r \dot{\eta} + K_r \eta = 0, \]  

where, \( M_r, C_r, \) and \( K_r \) are diagonal and,

\[ C_r = 2 \xi diag [\omega_i] M_r. \]  

Comparing Equation (14) to Equation (15), the damping coefficient matrix is calculated as,

\[ C = \psi^{-T} C_r \psi^{-1}. \]  

Details of the Galerkin FE model created from the distributed parameter model of the AHC can be found in [164].

2.4. Two degrees of freedom model of the artificial hair cells

The equations of motion developed in the previous sections are transformed into the modal domain to decouple the system of equations. The damping matrix is not considered in initial derivations, but it is added to the decoupled equations at the final steps. The simplified undamped equation of motion is given by,
\[ M\ddot{x} + Kx = B V(t) - P_z \ddot{z}(t). \]  

(18)

The decoupling procedure discussed in chapter 4 by Inman [180] is adopted in the present work. The Cholesky decomposition \((L)\) of the symmetric and positive definite mass matrix \((M = LL^T)\) results in a transformation \(x(t) = (L^T)^{-1} q(t)\) that simplifies Equation (18) to,

\[ M(L^T)^{-1} \ddot{q} + K(L^T)^{-1} q = B V(t) - P_z \ddot{z}(t). \]  

(19)

Pre-multiplying Equation (19) by \(L^{-1}\) yields,

\[ I \ddot{q} + \tilde{K} q = L^{-1} B V(t) - L^{-1} P_z \ddot{z}(t), \]  

(20)

where, \(\tilde{K} = L^{-1} K(L^T)^{-1}\) is the symmetric mass-normalized stiffness matrix. As a result, the physical coordinate system is transformed into a coordinate system in which the left side matrices of Equation (19) are represented by a single symmetric matrix in Equation (20). Assuming a solution for Equation (20) in the form of \(q = v e^{j\omega t}\) and substituting it into \(I \ddot{q} + \tilde{K} q = 0\), the eigenvalue problem for \(\tilde{K}\) is calculated as,

\[ -\omega^2 v + \tilde{K} v = 0. \]  

(21)

Eigenvalues \((\lambda = \omega^2)\) and orthonormal eigenvectors \(v \neq 0\) are determined from the characteristic equation (21). The eigenvectors matrix \(P = [v_1 \ v_2 \ldots v_{2n}]\) can decouple Equation (20) using the transformation,

\[ q(t) = Pr(t). \]  

(22)

Substituting (22) into Equation (20) and pre-multiplying it by \(P^T\) yields,

\[ P^T I P \ddot{r} + P^T \tilde{K} P r = P^T L^{-1} B V(t) - P^T L^{-1} P_z \ddot{z}(t). \]  

(23)

Matrix \(P\) is an orthogonal matrix where, \(P^T P = I\) and \(P^T \tilde{K} P = diag[\lambda_i]\). As a result, Equation (23) can be rewritten as,

\[ I \ddot{r} + \Lambda r = P^T L^{-1} B V(t) - P^T L^{-1} P_z \ddot{z}(t), \]  

(24)
where, $\Lambda = \text{diag} [\lambda_i] = \text{diag} [\omega_i^2]$ and $\omega_i$ is the natural frequency of mode $i$. The modal damping ($\zeta_i$) is now introduced to the decoupled system as,

$$I \ddot{r} + \text{diag} [2\zeta_i \omega_i] \dot{r} + \Lambda r = B_y V(t) - B_x \ddot{z}(t).$$  \hspace{1cm} (25)

In this equation, $B_y = P^T L^{-1} B = [\beta_{1v} \beta_{2v} \ldots \beta_{(2n)v}]'$ and $B_x = P^T L^{-1} P_z = [\beta_{1z} \beta_{2z} \ldots \beta_{(2n)z}]'$.

The decoupled equations of motion can be written as,

$$\ddot{r}_i + 2\zeta_i \omega_i \dot{r}_i + \omega_i^2 r_i = \beta_{iv} V(t) - \beta_{iz} \ddot{z}(t), \hspace{1cm} i = 1, 2, \ldots, 2n$$  \hspace{1cm} (26)

Next, the model is reduced to only the first two modes. The final decoupled set of the equations of motion are,

$$\ddot{r}_1 + 2\zeta_1 \omega_1 \dot{r}_1 + \omega_1^2 r_1 = \beta_{1v} V(t) - \beta_{1z} \ddot{z}(t),$$  \hspace{1cm} (27)

$$\ddot{r}_2 + 2\zeta_2 \omega_2 \dot{r}_2 + \omega_2^2 r_2 = \beta_{2v} V(t) - \beta_{2z} \ddot{z}(t).$$  \hspace{1cm} (28)

The values of the coefficients used in this work are tabulated in Table 2. As mentioned earlier, the modal damping ratios are estimated from the experimental data and all the other coefficients are calculated using the FE model with properties listed in Table 1.

<table>
<thead>
<tr>
<th>$\omega_1 (\text{rad/s})$</th>
<th>$\zeta_1$</th>
<th>$\beta_{1v} (\text{kg}^{1/2}/\text{s}^{1/2})$</th>
<th>$\beta_{1z} (\text{kg})$</th>
<th>$\omega_2 (\text{rad/s})$</th>
<th>$\zeta_2$</th>
<th>$\beta_{2v} (\text{kg}^{1/2}/\text{s}^{1/2})$</th>
<th>$\beta_{2z} (\text{kg})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2783.72</td>
<td>0.03</td>
<td>0.0378</td>
<td>0.0163</td>
<td>17446.35</td>
<td>0.0124</td>
<td>-0.1313</td>
<td>0.0091</td>
</tr>
</tbody>
</table>

By only considering the contribution of the first two modes, the tip velocity of the beam is approximated by,

$$\dot{u}_t = s_{x_{t_1}} \dot{r}_1 + s_{x_{t_2}} \dot{r}_2,$$  \hspace{1cm} (29)

where, $s_{x_{t_1}}$ and $s_{x_{t_2}}$ are the mode participation factors of the first and second modes on the tip velocity. These coefficients are calculated using the inverse of the transformation from the physical to the modal domain as,

$$x = [u_1 \theta_1 \ u_2 \ \theta_2 \ldots u_n \ \theta_n]' = (L^T)^{-1} Pr = Sr.$$  \hspace{1cm} (30)
Therefore, the tip displacement can be obtained as,

\[ u_t = u_n = s_{2n-1,1} r_1 + s_{2n-1,2} r_2 = s_{x_{t_1}} r_1 + s_{x_{t_2}} r_2. \] (31)

Elements \( s_{2n-1,1} \) and \( s_{2n-1,2} \) of \( S \) matrix are named as \( s_{x_{t_1}} \) and \( s_{x_{t_2}} \) for simplification and are calculated to be equal to \( 85.82 \text{ kg}^{-1/2} \) and \( -61.80 \text{ kg}^{-1/2} \), respectively.

### 2.5. Nonlinear feedback control law

Previous studies have shown that a system tuned to a Hopf bifurcation can be used as the basis to mimic the characteristics of the mammalian cochlea [173-176]. This can be further explained through a single degree of freedom AHC system in the modal domain as,

\[ \ddot{r}_1(t) + 2 \zeta_1 \omega_1 \dot{r}_1(t) + \omega_1^2 r_1(t) = \beta_1 v V(t) - \beta_2 \ddot{z}(t). \] (32)

Pre-multiplying equation (32) by \( s_{x_{t_1}} \) and substituting \( u_t = s_{x_{t_1}} r_1 \) in it, the equation of motion for the SDOF AHC in the physical domain becomes,

\[ \ddot{u}_t = 2 \zeta_1 \omega_1 \dot{u}_t + \omega_1^2 u_t = \beta_1 v V(t) - \beta_2 \ddot{z}(t), \] (33)

where, \( u_t \) is the tip displacement of the SDOF approximation, \( \omega_1 \) is its fundamental frequency in \( \text{rad/s} \), and \( \zeta_1 \) is the corresponding modal damping, \( V(t) \) is the voltage supplied to the piezoelectric actuators, \( \ddot{z}(t) \) is the base acceleration, \( \beta_1 v = \beta_1 s_{x_{t_1}} \) is the control voltage influence term of the SDOF approximation, and \( \beta_2 \) is the acceleration influence term. In the following equations, \( u(t) \) notation is used instead of \( u_t \) for simplification. A phenomenological feedback control law that tunes the system to a Hopf bifurcation is [11, 166],

\[ V(t) = \frac{1}{\beta_v} (a_1 \dot{u}(t) - a_3 \dot{u}^3(t)). \] (34)

In Equation (34), the controller gain \( \frac{a_1}{\beta_v} \) is called the linear damping gain of the SDOF system and \( \frac{a_3}{\beta_v} \) is the cubic damping gain. The control law can be rewritten in the form of,
\[ V(t) = \alpha_1 \dot{u}(t) - \alpha_3 \dot{u}^3(t), \]  
(35)

where, \( \alpha_1 = a_1/\beta \nu \) and \( \alpha_3 = a_3/\beta \nu \). This control law will be extended in Section 2.6 to develop novel two-channel active AHCs.

### 2.5.1. Tuning the AHC to a Hopf bifurcation

To investigate the behavior of the controlled system for different values of the controller’s gains, a state-space model of the system is created in this section. For the controlled system given by,

\[ \ddot{u}(t) + (2\zeta_1 \omega_1 - a_1)\dot{u}(t) + a_3 \dot{u}^3(t) + \omega_1^2 u(t) = -\beta_z \ddot{z}(t), \]  
(36)

variables \( x_1 = u(t) \) and \( x_2 = \dot{u}(t) \) are taken as state variables. The state-space equations become,

\[ \dot{x}_1 = x_2, \]  
\[ \dot{x}_2 = -(2\zeta_1 \omega_1 - a_1)x_2 - a_3 x_2^3 - \omega_1^2 x_1 - \beta_z \ddot{z}(t). \]  
(37)

By setting \( \ddot{z}(t) = 0 \) and defining \( \mu = a_1 - 2\zeta_1 \omega_1 \), the state equations for the autonomous system \( \dot{x} = f(x, \mu) \) are obtained as,

\[ \dot{x}_1 = x_2 = f_1(x), \]  
\[ \dot{x}_2 = \mu x_2 - a_3 x_2^3 - \omega_1^2 x_1 = f_2(x, \mu). \]  
(38)

Equilibrium points for the above system are obtained by setting \( f_1 = f_2 = 0 \) and solving Equation (38) for \( x \). As a result, the origin, \( x_0 = (x_{1_0}, x_{2_0}) = (0,0) \) is the unique equilibrium point of the system. To study the behavior of the system in the neighborhood of the equilibrium point, it is assumed that the behavior of the nonlinear system is similar to its linearized system near that point. However, the linearization fails for some borderline cases and further investigations are needed to determine the behavior and stability of the system near the equilibrium. To linearize the system of differential equations (38) near the origin, the Jacobian matrix is given by,

\[ A(\mu) = \frac{\partial f}{\partial x} \bigg|_{x_0} = \begin{bmatrix} 0 & 1 \\ -\omega_1^2 & \mu - 3a_3 x_2^2 \end{bmatrix} \bigg|_{x_0} = \begin{bmatrix} 0 & 1 \\ -\omega_1^2 & \mu \end{bmatrix}. \]  
(39)
Therefore, the linearized state equations, \( \dot{x} = A(\mu) x \) in the neighborhood of the equilibrium is a function of \( \mu \). Eigenvalues of \( A(\mu) \) at the origin determine stability properties of the linearized system and are calculated to be,

\[
\lambda_{1,2} = \frac{\mu \pm j \sqrt{4\omega_1^2 - \mu^2}}{2}.
\] (40)

The type of equilibrium in the linearized system and its stability for different ranges of \( \mu \), are tabulated in Table 3.

Table 3. Behavior of the linearized system near the equilibrium for different values of \( \mu \).

<table>
<thead>
<tr>
<th>Parameter range</th>
<th>Eigenvalues</th>
<th>Type of the equilibrium for the linearized system</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu &lt; -2\omega_1 )</td>
<td>distinct negative real</td>
<td>Stable node</td>
</tr>
<tr>
<td>( \mu = -2\omega_1 )</td>
<td>Repeated, real, negative. One linearly independent eigenvector.</td>
<td>Stable Improper node</td>
</tr>
<tr>
<td>(-2\omega_1 &lt; \mu &lt; 0 )</td>
<td>Complex conjugate, negative real parts</td>
<td>Stable focus</td>
</tr>
<tr>
<td>( \mu = 0 )</td>
<td>Complex conjugate, zero real parts</td>
<td>Stable center</td>
</tr>
<tr>
<td>( 0 &lt; \mu &lt; 2\omega_1 )</td>
<td>Complex conjugate, positive real parts</td>
<td>Unstable focus</td>
</tr>
<tr>
<td>( \mu = 2\omega_1 )</td>
<td>Repeated, real, positive. One linearly independent eigenvector.</td>
<td>Unstable Improper node</td>
</tr>
<tr>
<td>(-2\omega_1 &lt; \mu )</td>
<td>distinct positive real</td>
<td>Unstable node</td>
</tr>
</tbody>
</table>

As shown in Table 3, the behavior of the system changes from stable to unstable at \( \mu = 0 \). Therefore, linearization fails and determining the stability of the nonlinear system for \( \mu = 0 \) needs further studies as presented in the next section. The nonlinear system’s phase portraits in the neighborhood of the origin are demonstrated in Fig. 11 to Fig. 13 by numerically solving the state equations (38) for various initial conditions and different values of \( \mu \). The natural frequency and the modal damping ratios are assumed to be 443 Hz and 3%, respectively. The cubic damping gain is chosen to be \( a_3 = 1000 \, s/m^2 \).
Fig. 11. Phase portraits of the nonlinear system for different values of $\mu$, (a) $\mu = -3\omega_1$, (b) $\mu = -2\omega_1$, (c) $\mu = -0.035\omega_1$.

Fig. 11 shows that for the first three ranges of $\mu$ listed in Table 3, the origin is stable in the nonlinear system. Examples of the last three cases of Table 3 are displayed in Fig. 12.

Fig. 12. Phase portraits of the nonlinear system for different values of $\mu$, (a) $\mu = 0.035\omega_1$, (b) $\mu = 2\omega_1$, (c) $\mu = 3\omega_1$.

As shown in Fig. 12, the origin of the nonlinear system is unstable for the $\mu > 0$ but a limit cycle exists for each value of $\mu$ that attracts all the trajectories. Fig. 13 illustrates the phase portrait for $\mu = 0$. The origin of the nonlinear system for $\mu = 0$ and $a_3 = 1000 \text{ s/m}^2$ resembles a stable
However, as mentioned earlier, the stability of the system at this borderline point needs further studies and it is the subject of the next section.

As shown in Table 3, eigenvalues of the linearized system pass through the imaginary axis with \( \lambda_{1,2} = \pm j \omega_1 \) for \( \mu = 0 \). Therefore, the origin that was a stable focus for \(-2\omega_1 < \mu < 0\) (Fig. 11), loses its stability and becomes an unstable focus for \( 0 < \mu \), and a limit cycle forms around the origin as shown in Fig. 12. As a result, the system may undergo a Hopf bifurcation. According to the Hopf bifurcation theorem [181], the system has a Hopf bifurcation if the following conditions are satisfied for a certain value of \( \mu = \mu_0 \):

1. For \( \lambda_{1,2}(\mu_0) = \alpha_h(\mu_0) \pm j \beta_h(\mu_0) \) at the equilibrium \((x_0)\), \( \alpha_h(\mu_0) = 0 \) and \( \beta_h(\mu_0) = \omega \neq 0 \)
2. \( \frac{d\alpha_h(\mu)}{d\mu} \bigg|_{\mu_0} = d_h \neq 0 \),
3. \( a_h \neq 0 \), where,

\[
a_h = \left( \frac{1}{16} \frac{\partial^3 f_1}{\partial x_1^3} + \frac{\partial^3 f_1}{\partial x_1 \partial x_2^2} + \frac{\partial^3 f_2}{\partial x_1^2 \partial x_2} + \frac{\partial^3 f_2}{\partial x_1 \partial x_2^2} \right) + \frac{1}{16} \omega \left( \frac{\partial^2 f_1}{\partial x_1 \partial x_2} \left( \frac{\partial^2 f_1}{\partial x_1^2} + \frac{\partial^2 f_1}{\partial x_2^2} \right) - \frac{\partial^2 f_2}{\partial x_1 \partial x_2} \left( \frac{\partial^2 f_2}{\partial x_1^2} + \frac{\partial^2 f_2}{\partial x_2^2} \right) \right) \bigg|_{\mu_0, x_0}.
\]

If \( a_h d_h < 0 \), the limit cycle exists and it is stable for \( \alpha_h > 0 \). For the nonlinear system of (36), \( \mu_0 = 0 \), \( \lambda_{1,2}(0) = \pm j \omega_1 \), \( d_h = 0.5 \), and \( a_h = -6/16 a_3 \neq 0 \) for \( a_3 > 0 \). Therefore, the system

![Fig. 13. Phase portraits of the nonlinear system for \( \mu = 0 \).](image)
undergoes a Hopf bifurcation at $\mu = 0$ and a stable limit cycle exists for $\mu > 0$. This type of the Hopf bifurcation is called supercritical as the stable limit cycle orbits around the unstable equilibrium point and its size grows by increasing the bifurcation parameter $\mu$. The system of equations (38) is solved with the initial condition of $(x_1, x_2) = (10^{-6}, 0)$ for different values of $\mu$. The supercritical Hopf bifurcation is displayed in Fig. 14 as $\mu$ varies from $-200 \text{ 1/s}$ to $167 \text{ 1/s}$. The envelope of the trajectories is shown in red in Fig. 14.

![Fig. 14. The supercritical Hopf bifurcation: (a) 3D view, (b) side view.](image)

2.5.2. **Stability of the system tuned to the Hopf bifurcation**

In this section, the stability of equilibrium in the autonomous system tuned to the Hopf bifurcation and the boundness of the system’s states in the nonautonomous system excited by a bounded input are studied. First, the base excitation input in Equation (38) is set to zero and then the system is driven to the Hopf bifurcation by setting $\mu = 0$ to mimic the behavior of the cochlea. As a result, the state equations of the autonomous system are given by,

$$\begin{align*}
\dot{x}_1 &= x_2 = f_1(x), \\
\dot{x}_2 &= -a_3 x_2^3 - \omega_1^2 x_1 = f_2(x),
\end{align*}$$

(41)
where, \(a_3 > 0\) to add cubic damping to the system. Based on Barbashin and Krasovskii theorem, if \(x = 0\) is an equilibrium point for system \(\dot{x} = f(x)\), and Lyapunov function \(V: R^n \rightarrow R\) is a continuously differentiable, radially unbounded, positive definite function such that \(\dot{V}(x) \leq 0, \forall x \in R^n\), and suppose that no solution other than the trivial solution, \(x(t) \equiv 0\), can stay identically in \(S = \{x \in R^n | \dot{V}(x) = 0\}\), then the origin is globally asymptotically stable [182]. As mentioned previously in Section 2.5.1, the origin is the unique equilibrium point for the autonomous system. To prove the stability of the origin, a positive definite Lyapunov function in the form of,

\[
V(x) = \frac{1}{2} \omega_1^2 x_1^2 + \frac{1}{2} x_2^2,
\]

is defined. This function is continuously differentiable and radially unbounded; i.e. \(V(x) \rightarrow \infty\) as \(|x| \rightarrow \infty\). The derivative of the Lyapunov function is given by,

\[
\dot{V}(x) = \omega_1^2 x_1 \dot{x}_1 + x_2 \dot{x}_2 = -a_3 x_2^4,
\]

and it is negative semi-definite. Furthermore,

\[
S = \{x \in R^n | \dot{V}(x) = 0\} = \{x \in R^2 | x_2 = 0\}.
\]

Substituting \(x_2 = 0\) and subsequently, \(\dot{x}_2 = 0\) in the state equations (41) results in \(x_1 = 0\). As a result, \(x(t) = (x_1, x_2) = 0\) is the only solution that can stay identically in \(S\) and all of the conditions of the Barbashin and Krasovskii theorem are met. Thus, the origin is globally asymptotically stable.

To study if the states of the system remain bounded when an external bounded input excites the system, the input-state stability (ISS) of the nonautonomous system should be assessed. This can be done by using a Lyapunov-like theorem discussed in reference [182]. The theorem states that a nonautonomous system in the form of \(\dot{x} = f(t, x, u)\) is ISS if \(V: [0, \infty) \times R^n \rightarrow R\) is a continuously differentiable function and for \(\forall (t, x, u) \in [0, \infty) \times R^n \times R^m\) the following hold,
\[ \alpha_{s1}(\|x\|) \leq V(t, x) \leq \alpha_{s2}(\|x\|), \]  

\[
\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(t, x, u) \leq -W_3, \ \forall \|x\| \geq \rho(\|u\|) > 0,
\]  

where, \( \alpha_{s1} \) and \( \alpha_{s2} \) are class \( \mathcal{K}_\infty \) functions, \( \rho \) is a class \( \mathcal{K} \) function, and \( W_3(x) \) is a continuous positive definite function on \( \mathbb{R}^n \). This theorem is used to prove the ISS of,

\[ \dot{x}_1 = x_2, \]

\[ \dot{x}_2 = -a_3 x_2^3 - a_1 x_1 + u(t), \]  

where, \( u(t) = -\beta z \dot{z} \) is the bounded input. The Lyapunov function is chosen as before (Equation (42)). Therefore,

\[ \dot{V} = -a_3 x_2^4 + x_2 u \]

\[ \leq -a_3 (1 - \theta) x_2^4 - a_3 \theta x_2^4 - a_3 (1 - \theta) x_1^4 + a_3 (1 - \theta) x_1^4 + x_2 u \]

\[ \leq -a_3 (1 - \theta) x_1^4 + x_2^4 + a_3 (1 - \theta) x_1^4 - a_3 \theta x_2^4 + x_2 u \]

\[ \leq -a_3 (1 - \theta) (x_1^4 + x_2^4) + a_3 \|x\|_2^4 - a_3 \theta (x_1^4 + x_2^4) + \|x\|_2 \|u\| \]

\[ \leq -W_3 + a_3 \|x\|_2^4 - a_3 \theta (x_1^4 + x_2^4) + \|x\|_2 \|u\|, \]

where, \( 0 < \theta < 1 \) and \( W_3(x) = a_3 (1 - \theta) (x_1^4 + x_2^4) \) is a positive definite function. In order for \( \dot{V} \leq -W_3 \) to hold, the following condition must be met,

\[ a_3 \|x\|_2^4 - a_3 \theta (x_1^4 + x_2^4) + \|x\|_2 \|u\| \leq 0 \]

\[ \rightarrow 0 < \|u\| \leq \frac{a_3 \theta (x_1^4 + x_2^4) - a_3 \|x\|_2^4}{\|x\|_2} \leq \frac{2a_3 \theta \|x\|_2^4 - a_3 \|x\|_2^4}{\|x\|_2} \]

\[ \rightarrow 0 < \|u\| \leq a_3 (2\theta - 1) \|x\|_2^3 \]  

\[ \rightarrow 0 < \frac{\|u\|}{a_3 (2\theta - 1)} \leq \|x\|_2^3, \quad \frac{1}{2} < \theta < 1 \]  

\[ \rightarrow \|x\|_2^3 \geq \left( \frac{\|u\|}{a_3 (2\theta - 1)} \right)^\frac{1}{3} > 0, \]

\[ 57 \]
where, \( \|x\|_2 = \sqrt{x_1^2 + x_2^2} \) is the norm of \( x = (x_1, x_2) \). Comparing inequality (49) to (46), 
\[
\rho(\|u\|) = \left( \frac{\|u\|}{(2\theta - 1) a_3} \right)^{\frac{1}{\theta}},
\]
where \( \frac{1}{2} < \theta < 1 \) and \( a_3 > 0 \) is a class \( \mathcal{K} \) function; i.e. it is strictly increasing and \( \rho(\|u\|) \big|_{u = 0} = 0 \). Therefore, condition (46) of the theorem holds and condition (45) needs to be satisfied as,
\[
V(x) = \frac{1}{2} \omega_1^2 x_1^2 + \frac{1}{2} x_2^2 \leq \frac{1}{2} \omega_1^2 \|x\|^2, \quad (50)
\]
and,
\[
\frac{1}{2} \omega_1^2 x_1^2 + \frac{1}{2} \omega_2^2 x_2^2 \leq \frac{1}{2} \omega_1^2 x_1^2 + \frac{1}{2} x_2^2, \quad \text{if } \omega_1 < 1 \\
\frac{1}{2} x_1^2 + \frac{1}{2} x_2^2 \leq \frac{1}{2} \omega_1^2 x_1^2 + \frac{1}{2} x_2^2, \quad \text{if } \omega_1 \geq 1
\]
(51)
As a result,
\[
\min\left\{ \left( \frac{1}{2} \omega_1^2 \|x\|^2 \right), \left( \frac{1}{2} \|x\|^2 \right) \right\} \leq V(x) = \frac{1}{2} \omega_1^2 x_1^2 + \frac{1}{2} x_2^2 \leq \frac{1}{2} \omega_1^2 \|x\|^2 + \frac{1}{2} \|x\|^2. \quad (52)
\]
Comparing Equation (52) to Equation (45), \( \alpha_{s1}(\|x\|) = \min\left\{ \left( \frac{1}{2} \omega_1^2 \|x\|^2 \right), \left( \frac{1}{2} \|x\|^2 \right) \right\} \), and \( \alpha_{s2}(\|x\|) = \frac{1}{2} \omega_1^2 \|x\|^2 + \frac{1}{2} \|x\|^2 \). Both \( \alpha_{s1} \) and \( \alpha_{s2} \) are class \( \mathcal{K}_\infty \) functions as they are defined in the range of \([0, \infty)\), are strictly increasing, \( \lim_{\|x\| \to \infty} \alpha_{s1}(\|x\|) = \infty \), and \( \lim_{\|x\| \to \infty} \alpha_{s2}(\|x\|) = \infty \).

Finally, all of the conditions of the Lyapunov-like theorem are satisfied and the system is input-to-state stable.

2.5.3. Steady-state response of the cubic damping system

In this section, the harmonic balance method [183, 184] is used to study the steady-state response of the AHC to a harmonic base acceleration input. The equation of the motion for a SDOF AHC with base excitation is given by Equation (36) and rewritten as,
\[
\ddot{u}(t) + 2 \xi \omega_1 \dot{u}(t) + a_3 \dot{u}^3(t) + \omega_1^2 u(t) = -\beta_2 \ddot{z}(t). \quad (53)
\]
where, the bifurcation parameter is defined as,
\[ 2\xi_1\omega_1 = 2\zeta_1\omega_1 - a_1. \] (54)

Note that the bifurcation parameter \( \mu \) defined in the previous sections is equal to \(-2\xi_1\omega_1\). Exciting the base with a harmonic acceleration with driving frequency \( \Omega \) as,
\[ \ddot{z}(t) = Z \sin(\Omega t), \] (55)
results in,
\[ \dddot{u}(t) + 2\xi_1\dot{u}(t) + a_3 \dddot{u}(t) + \omega_1^2 u(t) + \beta_2 Z \sin(\Omega t) = 0. \] (56)

In order to approximate the steady-state response of the system given in Equation (56) using the harmonic balance method, a periodic solution is assumed. Fourier series expansion for the approximated solution is given by,
\[ u(t) = \frac{c_0}{2} + \sum_{n=1}^{N} (c_n \cos(n\Omega t) + d_n \sin(n\Omega t)), \] (57)
and its first and second derivatives are,
\[ \dot{u}(t) = \sum_{n=1}^{N} n\Omega (-c_n \sin(n\Omega t) + d_n \cos(n\Omega t)), \] (58)
\[ \ddot{u}(t) = \sum_{n=1}^{N} -n^2\Omega^2 (c_n \cos(n\Omega t) + d_n \sin(n\Omega t)). \] (59)

The Fourier series expansion of the cubic term in Equation (40) can be written in the form of,
\[ \dddot{u}(t) = \sum_{n=1}^{N} (\tilde{c}_n \cos(n\Omega t) + \tilde{d}_n \sin(n\Omega t)). \] (60)

Substituting Equation (57) to (60) into Equation (56) yields,
\[ \sum_{n=1}^{N} [-n^2\Omega^2 (c_n \cos(n\Omega t) + d_n \sin(n\Omega t)) + 2\xi_1 n\Omega (-c_n \sin(n\Omega t) + d_n \cos(n\Omega t)) + a_3 \left(\tilde{c}_n \cos(n\Omega t) + \tilde{d}_n \sin(n\Omega t)\right) + \omega_1^2 \frac{c_0}{2} + \omega_1^2 (c_n \cos(n\Omega t) + d_n \sin(n\Omega t)) + \beta_2 Z \sin(\Omega t) = 0. \] (61)

Therefore,
\[ c_0 = 0, \] (62)
\[-\Omega^2 d_1 - 2\xi_1 \omega_1 c_1 + a_3 \ddot{d}_1 + \omega_1^2 d_1 + \beta Z = 0,\]
\[-n^2 \Omega^2 c_n + 2\xi_1 n\Omega d_n + a_3 \ddot{c}_n + \omega_1^2 c_n = 0, \quad n = 1, 2, ..., N\]
\[-n^2 \Omega^2 d_n - 2\xi_1 n\Omega c_n + a_3 \ddot{d}_n + \omega_1^2 d_n = 0. \quad n = 2, 3, ..., N\]

For \(N = 1\), equations become,
\[c_0 = 0,\]
\[-\Omega^2 d_1 - 2\xi_1 \omega_1 c_1 + a_3 \ddot{d}_1 + \omega_1^2 d_1 + \beta Z = 0,\]  \((63)\)
\[-\Omega^2 c_1 + 2\xi_1 \omega_1 d_1 + a_3 \ddot{c}_1 + \omega_1^2 c_1 = 0.\]

To write \(\ddot{c}_1\) and \(\ddot{d}_1\) in terms of \(c_1\) and \(d_1\), \(\ddot{u}^3(t)\) can also be obtained from Equation (58) as,
\[\ddot{u}^3(t) = (\sum_{n=1}^{N} n\Omega (-c_n \sin(n\Omega t) + d_n \cos(n\Omega t)))^3, \quad N = 1.\]  \((64)\)

By substituting \(\ddot{u}^3(t)\) from Equation (64) into Equation (60), the Fourier coefficients \(\ddot{c}_1\) and \(\ddot{d}_1\) are determined as,
\[\ddot{c}_1 = \frac{2n}{2\pi} \int_0^{2\pi} 3 (\Omega (-c_1 \sin(\Omega t) + d_1 \cos(\Omega t)))^3 \cos(\Omega t) dt,\]  \((65)\)
\[\ddot{d}_n = \frac{2n}{2\pi} \int_0^{2\pi} 3 (\Omega (-c_1 \sin(\Omega t) + d_1 \cos(\Omega t)))^3 \sin(\Omega t) dt.\]

Using orthogonality relations,
\[\ddot{c}_1 = \frac{\Omega^4}{\pi} \int_0^{2\pi} (d_1^3 \cos^4 (\Omega t) + 3c_1^2 d_1 \sin^2(\Omega t) \cos^2 (\Omega t)) dt.\]  \((66)\)

After further simplification,
\[\ddot{c}_1 = \frac{\Omega^4}{\pi} 3c_1^2 d_1 \int_0^{2\pi} \left( \cos^2(\Omega t) - \cos^4(\Omega t) \right) dt = \frac{3\Omega^4}{4} \left( d_1^3 + c_1^2 d_1 \right).\]  \((67)\)

Also, the Fourier coefficient \(\ddot{d}_1\) is obtained as,
\[\ddot{d}_1 = -\frac{\Omega^4}{\pi} 3c_1 d_1 \int_0^{2\pi} \left( \cos^2 (\Omega t) - \cos^4 (\Omega t) \right) dt = \frac{3\Omega^4}{4} \left( -c_1^3 - c_1 d_1^2 \right).\]  \((68)\)

Substituting Fourier coefficients into Equation (63) yields,
\[ c_0 = 0, \quad (69) \]
\[ (\omega_1^2 - \Omega^2) d_1 - 2\xi \omega_1 \Omega c_1 - a_3 \frac{3\Omega^3}{4} (c_1^3 + c_1 d_1^2) + \beta_z Z = 0, \quad (70) \]
\[ (\omega_1^2 - \Omega^2) c_1 + 2\xi \omega_1 \Omega d_1 + a_3 \frac{3\Omega^3}{4} (d_1^3 + c_1^2 d_1) = 0. \quad (71) \]

Multiplying Equation (70) by \( c_1 \) and Equation (71) by \(-d_1\) and summing them results in,
\[ -2\xi \omega_1 \Omega (c_1^2 + d_1^2) - a_3 \frac{3\Omega^3}{4} (c_1^2 + d_1^2)^2 + c_1 \beta_z Z = 0. \quad (72) \]

As \( N = 1 \) and \( c_0 = 0 \), Equation (57) can be written as,
\[ u(t) = c_1 \cos(\Omega t) + d_1 \sin(\Omega t) = U \sin(\Omega t + \phi), \quad (73) \]
where, \( U \) is the amplitude of the response and \( \phi \) is the phase angle given by,
\[ U = \sqrt{(c_1^2 + d_1^2)}, \quad \phi = \tan^{-1}\left(\frac{c_1}{d_1}\right). \quad (74) \]

Substituting \((c_1^2 + d_1^2) = U^2\) from Equation (74) into (72), coefficient \( c_1 \) is determined to be,
\[ c_1 = \frac{a_3 \frac{3\Omega^3}{4} - 2\xi \omega_1 \Omega U^2}{\beta_z Z}. \quad (75) \]

Multiplying Equation (70) by \( d_1 \) and (71) by \( c_1 \) and summing them yields,
\[ (\omega_1^2 - \Omega^2)(c_1^2 + d_1^2) + d_1 \beta_z Z = 0, \quad (76) \]
and coefficient \( d_1 \) becomes,
\[ d_1 = \frac{-(\omega_1^2 - \Omega^2)U^2}{\beta_z Z}. \quad (77) \]

Substituting the coefficients into Equation (70) yields,
\[ \frac{9}{16} \Omega^6 a_3^2 (U^2)^3 + \frac{3\Omega^4}{2} 2\xi \omega_1 a_3 (U^2)^2 + ((\omega_1^2 - \Omega^2)^2 + (2\xi \omega_1 \Omega)^2)(U^2) = (\beta_z Z)^2. \quad (78) \]

At the Hopf bifurcation \( \xi = 0 \) and therefore, \( a_1 = 2\xi \omega_1 \). Therefore, Equation (78) becomes,
\[ \frac{9}{16} \Omega^6 a_3^2 (U^2)^3 + (\omega_1^2 - \Omega^2)^2(U^2) = (\beta_z Z)^2. \quad (79) \]

At the resonance, \( \Omega = \omega_1 \) and Equation (79) simplifies to,
\[ U = \left( \frac{4\beta_x}{3\omega_1^2 \alpha_3} Z \right)^{\frac{1}{3}}. \]  

Equation (78) can be modified for the magnitude of the velocity response \( (V = \Omega U) \) as,

\[
\frac{9}{16} a_3^2 (V^2)^3 + \frac{3\Omega^4}{2} 2\xi \omega_1 a_3 (V)^2 + \left( \frac{(\omega_1^2 - \Omega^2)^2 + (2\xi\omega_1\Omega)^2}{\Omega^2} \right) (V^2) = (\beta_x Z)^2. \tag{81}
\]

Subsequently, at the resonance and Hopf bifurcation, the following relation holds,

\[ V = \omega_1 U = \left( \frac{4\beta_x}{3\alpha_3} Z \right)^{\frac{1}{3}}. \tag{82} \]

Equations (80) and (82) show that the system undergoing Hopf bifurcation scales the response by one-third power of the input acceleration at the resonance and mimics the compressive nonlinearity characteristic of the cochlea [6]. The uncontrolled AHC’s magnitude velocity/acceleration FRF is obtained by substituting \( \xi = \xi_1 \) and \( \alpha_3 = 0 \) in Equation (81) as,

\[ \frac{V}{Z} = \frac{\beta_x \Omega}{\sqrt{(\omega_1^2 - \Omega^2)^2 + (2\xi_1\omega_1\Omega)^2}}. \tag{83} \]

As an instance, the velocity/acceleration FRF for an AHC with properties listed in Table 1 is determined by solving Equation (81) with \( \xi = 0 \) and \( \alpha_3 = 1.8 \times 10^5 \, \text{s/m}^2 \). FRFs of the system for various base acceleration inputs are shown in Fig. 15.

![Fig. 15. Magnitude tip-velocity/base-acceleration FRF of the SDOF AHC for various input levels.](image-url)
As shown in Fig. 15, the system’s behavior varies for different input levels. The response is compressed near the fundamental frequency for \( Z = 4.95 \, \text{m/s}^2 \), while it is amplified for other cases. Also, the input strength has an inverse relation with the damping or sharpness of the response at the natural frequency. Comparing the velocity response of the active AHC to the passive or uncontrolled AHC displayed in Fig. 16, amplification of output to low-level inputs and compression of response to inputs higher than a certain level can be observed.

Thus, the nonlinear feedback control system examined in this section shows a nonlinear amplification/compression behavior and provides a cubic relation between the output and input of the AHC as shown by the harmonic balance method. Therefore, the phenomenological cubic control law is used as the basis for designing the active AHCs discussed in the present work.

### 2.6. Two-channel active AHC

In section 2.5, the AHC was approximated as a SDOF system and the control law applied to the system was able to tune the system to a Hopf bifurcation. It was shown that the controlled system can mimic the cubic amplification and compression of the output similar to the function of the
cochlea. In this section, the control law presented in Equation (34) is extended to develop a two-channel active AHC in which the response of the system is controlled near the first and second natural frequencies [185]. To achieve this goal, the 2DOF model of the AHC developed in section 2.4 is used. The equations of motion are rewritten here as,

\[ \ddot{r}_1 + 2 \zeta_1 \omega_1 \dot{r}_1 + \omega_1^2 r_1 = \beta_{11} V(t) - \beta_{12} \ddot{z}(t), \]
\[ \ddot{r}_2 + 2 \zeta_2 \omega_2 \dot{r}_2 + \omega_2^2 r_2 = \beta_{21} V(t) - \beta_{22} \ddot{z}(t). \]

In order to be able to control the response of the system near the first two natural frequencies independently, the control law is determined to be,

\[ V(t) = \begin{cases} V_1(t) & \omega < \omega_m \\ V_2(t) & \omega > \omega_m \end{cases}, \]

where, \( \omega_m \) is a frequency between the first and second natural frequencies. Switching from \( V_1(t) \) to \( V_2(t) \) in the control law can be conducted using filters. As the frequency content of the input signal is not known before passing through the AHC in the actual implementation of the controlled system, filters should be used to ensure that the control laws are applied in the desired frequency bandwidth. A schematic of the feedback control system is presented in Fig. 17. A low-pass (LP) filter with a cut-off frequency of \( \omega_m \) is used to filter frequencies above \( \omega_m \), and a band-pass (BP) filter is utilized to pass the frequencies between \( \omega_m \) and an edge frequency between the second and third natural frequencies.

Fig. 17. Schematic of the two-channel active AHC system
As a base excitation is applied to the system, the tip velocity of the two-channel AHC, \( \dot{u}_{t_1} \) in Equation (25), is passed through a low-pass (LP) Butterworth filter with a cut-off frequency of \( \omega_m \) to filter frequencies above \( \omega_m \). The LP filtered velocity (\( \dot{u}_{t_1} \)) is then used in the first control voltage (\( V_1 \)). The velocity is also filtered in a band-pass (BP) Butterworth filter, as shown in Fig. 17 by \( \dot{u}_{t_2} \), before the second control law (\( V_2 \)) is applied. The BP filter is utilized to pass the frequencies between \( \omega_m \) and an edge frequency between the second and third natural frequencies. The summation of \( V_1(t) \) and \( V_2(t) \) results in the feedback control signal (\( V(t) \)) applied to the system. The control voltages \( V_1(t) \) and \( V_2(t) \) are defined as,

\[
V_1(t) = \frac{1}{\beta_{1v}} \left( \frac{a_{11}}{x_{t_1}} \dot{u}_{t_1} - \frac{a_{31}}{x_{t_1}^3} \dot{u}_{t_1}^3 \right),
\]
\[
V_2(t) = \frac{1}{\beta_{2v}} \left( \frac{a_{12}}{x_{t_2}} \dot{u}_{t_2} - \frac{a_{32}}{x_{t_2}^3} \dot{u}_{t_2}^3 \right).
\]

The control law can be stated in the following form,

\[
V_1(t) = \alpha_{11} \dot{u}_{t_1} - \alpha_{31} \dot{u}_{t_1}^3,
\]
\[
V_2(t) = \alpha_{12} \dot{u}_{t_2} - \alpha_{32} \dot{u}_{t_2}^3,
\]

where, controller gains \( \alpha_{11} = \frac{1}{\beta_{1v} s_1} a_{11} \) and \( \alpha_{31} = \frac{1}{\beta_{1v} s_1^3} a_{31} \) are the linear damping and cubic damping gains acting on the first mode, and \( \alpha_{12} = \frac{1}{\beta_{2v} s_2} a_{12} \) and \( \alpha_{32} = \frac{1}{\beta_{2v} s_2^3} a_{32} \) are the gains corresponding to the second mode.

Near the first natural frequency, the contribution of the first mode on the tip velocity is maximum and the contribution of the other modes is negligible, therefore,

\[
\dot{u}_{t_1} \approx s_{x t_1} \hat{r}_1.
\]

Similarly, near the second natural frequency, \( \dot{u}_{t_2} \) is given by,
\[
\dot{u}_{t_2} \approx s_{x_{t_2}} \dot{r}_2. 
\] 

(89)

Therefore, to eliminate the linear damping of the system near each natural frequency and tune the AHC system near the Hopf bifurcation, the linear damping gains are determined to be,

\[
\alpha_{11} = \frac{2\zeta_1 \omega_1}{\beta_{1v} s_{x_{t_1}}},
\]

\[
\alpha_{12} = \frac{2\zeta_2 \omega_2}{\beta_{2v} s_{x_{t_2}}},
\]

(90)

The choice of cubic damping gains and filter parameters affect the behavior of the system that is discussed in the next chapters.

2.7. Conclusion

In this chapter, an artificial hair cell was modeled as a piezoelectric cantilever beam and a finite element model of the beam was created. The model was then reduced to only the first two modes of the system and used to first develop a single-channel AHC and then a two-channel AHC. To mimic the nonlinear amplification and compressive nonlinearity of the cochlear hair cells, a nonlinear feedback control law for the single-channel AHC was introduced. The control law consisted of two parts: a linear damping part to eliminate the linear damping of the system and a cubic damping term to introduce nonlinear cubic damping to the AHC near the first natural frequency. The behavior of the controlled system for different linear damping gains was examined and conditions required for tuning the system to a Hopf bifurcation were studied. As a part of this chapter, the stability of the unforced controlled system (autonomous system) was shown and the input-state stability of the nonautonomous system was proved. The harmonic balance method was then used to study the steady-state response of the system. The analysis showed that the output and input of the system hold a cubic relationship at the Hopf bifurcation at resonance. Finally, the control law was extended to control the two degrees of freedom model of the AHC and develop
two-channel active AHCs capable of nonlinear amplification and compression near the first two natural frequencies of the AHC.
Chapter 3

Numerical simulation of the artificial hair cells

3.1. Introduction

This chapter presents numerical simulations of active AHCs. The phenomenological control law presented in the previous chapter will be applied to the AHC model and the response of the system excited by a base acceleration input will be simulated. First, a single-channel AHC modeled as a SDOF system controlled by a cubic damping controller with or without a filter will be implemented in Simulink. Next, a new method for determining the linear damping gain of the controller for the filtered case will be presented. Cubic damping will then be added to the system and simulation results will be displayed. In the next step, a numerical model of a two-channel AHC will be created. Using the same gain tuning method as the single-channel AHC with filter, linear damping gains of the two-channel active system will be obtained. After introducing the cubic damping to the AHC system, the cubic relation between the input and output of the AHC will be investigated. Finally, the controllers’ effect on each other and their effect on frequencies other than the natural frequencies will be discussed. Some contexts of this chapter are adapted from Sheyda Davaria, Vijaya VN Sriram Malladi, Seyedmostafa Motaharibidgoli, and Pablo A. Tarazaga, “Cochlear amplifier inspired two-channel active artificial hair cells.”, Mechanical Systems and Signal Processing 129 (2019): 568-589 [177] and Sheyda Davaria, VVN Sriram Malladi, and Pablo A. Tarazaga, “Bio-inspired Nonlinear Control of Artificial Hair Cells.”, In Structural Health Monitoring, Photogrammetry & DIC, Volume 6, pp. 179-184. Springer, Cham, 2019 [185].
3.2. Simulink model of the system

To numerically study the behavior of the active AHC, the control algorithms discussed in the previous chapter are implemented on Simulink models developed based on the equations of motion of the single-channel and two-channel AHCs. The Simulink model of a single-channel active AHC with the parameters listed in Table 1 and Table 2 is shown in Fig. 18.

![Simulink model of the single-channel active AHC system](image)

**Fig. 18.** Simulink model of the single-channel active AHC system

The Simulink model applies the control law presented in Equation (35) to the SDOF model of the system formulated in the state-space as,

\[
\dot{x} = \begin{bmatrix} 0 & 1 \\ -\omega_1^2 & -2\zeta_1\omega_1 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ \beta_1v & -\beta_1z \end{bmatrix} u_{\text{in}},
\]

\[
y = [0 \ 1] x,
\]

where, \( x = \begin{bmatrix} r_1 \\ \dot{r}_1 \end{bmatrix} \) and \( u_{\text{in}} = \begin{bmatrix} V(t) \\ \dot{z}(t) \end{bmatrix} \). To apply the control law to the AHC in a particular frequency bandwidth, a LP filter is used before the controller with a cut-off frequency between the first and second natural frequencies of the AHC. Although filtering is not mandatory in simulating the single-channel active AHC, it is necessary for the actual implementation of the active AHC. In the real-time implementation of the system, a cantilever beam (continuous system) will be used as an
AHC. Therefore, using a LP filter with a cut-off frequency below the second natural frequency ensures that the controller affects the frequencies only in the desired bandwidth and the control spillover to higher modes of the AHC will be reduced [164].

Similarly, the state-space form of the equations of motion for a 2DOF AHC given by Equation (84) is obtained as,

\[ \dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\omega_1^2 & -2\zeta_1\omega_1 & 0 & 1 \\ 0 & 0 & -\omega_2^2 & -2\zeta_2\omega_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ \beta_{1v} & -\beta_{1z} \\ 0 & 0 \\ \beta_{2v} & -\beta_{2z} \end{bmatrix} u_{in}, \]

\[ y = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x, \]

where, \( x = \begin{bmatrix} r_1 \\ \dot{r}_1 \\ r_2 \\ \dot{r}_2 \end{bmatrix} \) and \( u_{in} = \begin{bmatrix} V(t) \\ \ddot{z}(t) \end{bmatrix} \). Fig. 19 presents the Simulink model of the two-channel AHC system displayed previously in Fig. 17.

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**Fig. 19.** Simulink model of the two-channel active AHC system
The filters used in the model are designed such that the bandwidth of the filtered velocities lies in the frequency bandwidth of the first two natural frequencies of the AHC. The low-pass filter is a 6th order Butterworth filter with a corner frequency of 650 Hz (4084.07 \text{ rad/s}) and the band-pass filter is a 6th order Butterworth filter with pass-band edge frequencies of 2000 Hz (12566.37 \text{ rad/s}) and 3500 Hz (21991.15 \text{ rad/s}). In the two-channel AHC, the control voltage applied to the piezoelectric layers is a summation of control signals \( V_1 \) and \( V_2 \). As discussed earlier in Chapter 2, the control signal \( V_1 \) is in the form of \( V_1 = \alpha_{11} \dot{u}_{t1} - \alpha_{13} \dot{u}_{t1}^3 \), where \( \dot{u}_{t1} \) is the LP filtered velocity and \( V_2 \) is given by \( V_2 = \alpha_{12} \dot{u}_{t2} - \alpha_{32} \dot{u}_{t2}^3 \), where \( \dot{u}_{t2} \) is the tip velocity filtered by the PB filter. The controller gains \( \alpha_{11} \) and \( \alpha_{12} \) are chosen such that they remove the linear dampings of the first and second modes, respectively. The cubic damping gains \( \alpha_{31} \) and \( \alpha_{32} \) are selected such that the dynamics of the active AHC mimic the cochlear amplifier. Gain tuning will be discussed later in this chapter. The next sections present the simulation.

### 3.3. Single-channel active AHC

This section numerically studies the behavior of the single-channel active AHC. The Simulink model in Fig. 18 is used with and without the LP filter and simulation results are discussed.

#### 3.3.1. Single-channel active AHC without any filters

In this section, the LP filter shown in Fig. 18 is eliminated and the linear damping gain of the controller is set to its theoretical value, \( \alpha_{11} = \frac{2 \zeta_1 \omega_1}{\beta_{1V} s_1} \) and the cubic damping gain, \( \alpha_{31} \) is set to three different values. The system is excited by stepped sine inputs near its first natural frequency between 410 Hz and 480 Hz and the tip-velocity/base-acceleration FRFs of the AHC are obtained. Fig. 20 shows the changes in the magnitude of the tip-velocity/base-acceleration FRFs of the AHC.
as the amplitude of the base excitation level \((Z)\) increases for \(\alpha_{31} = 1 \times 10^4 \, V/m^3/s^3\), \(\alpha_{31} = 5 \times 10^4 \, V/m^3/s^3\), and \(\alpha_{31} = 2 \times 10^5 \, V/m^3/s^3\).

**Fig. 20.** Magnitude tip-velocity/base-acceleration FRF of the single-channel AHC without the filter due to changing the input amplitude near the first natural frequency for, (a) \(\alpha_{31} = 1 \times 10^4 \, V/m^3/s^3\), (b) \(\alpha_{31} = 5 \times 10^4 \, V/m^3/s^3\), and (c) \(\alpha_{31} = 2 \times 10^5 \, V/m^3/s^3\).

The effects of the nonlinear dynamics introduced by the control law around the first natural frequency of the structure are evident in Fig. 20. The response of the system is amplified near the fundamental frequency for the input signals corresponding to the FRFs above the uncontrolled system’s FRF and it is compressed otherwise. The FRFs of the controlled system with \(\alpha_{31} = 5 \times 10^4 \, V/m^3/s^3\) displayed in Fig. 20 (b) is very similar to the FRFs obtained by the harmonic
balance method for the same base excitation and cubic damping gain shown in Fig. 15. This can verify the accuracy of the simulation results. Fig. 21 compares the response of the active AHC against the passive or uncontrolled AHC. For $\alpha_{31} = 5 \times 10^4 \text{ V/m}^3 / \text{s}^3$, the controlled system amplifies the response for RMS base acceleration below 1.55 m/s$^2$ and compresses it for base accelerations above 1.55 m/s$^2$. The AHC with $\alpha_{31} = 1 \times 10^4 \text{ V/m}^3 / \text{s}^3$ amplifies the response for all the simulated input levels. Furthermore, the switching point from amplification to compression for the case with $\alpha_{31} = 2 \times 10^5 \text{ V/m}^3 / \text{s}^3$ is 3.11 m/s$^2$ RMS base acceleration.

![Input to output curves at the first natural frequency for the simulated single-channel AHC](image)

Fig. 21. Input to output curves at the first natural frequency for the simulated single-channel AHC

To examine the relationship between the output and input of the simulated model, the least-square power-law fits to the input-output curves are obtained in Fig. 21. As a result, the output is scaled by about one-third power of the input for multiple values of the cubic damping gain. This result is similar to the analytical cube root relationship between the input and output of the AHC shown previously in Equation (82) using the harmonic balance method.

3.3.2. **Single-channel active AHC with an LP filter**

In this section, a 6th order LP filter with a 650 Hz cut-off frequency is added to the system before the controller, as shown in the Simulink model of Fig. 18. To obtain comparable results with the
system without a filter, the linear damping gain, cubic damping gains, and base excitation levels are kept the same as the previous section. The simulated FRFs for cubic damping gains of $\alpha_{31} = 1 \times 10^4 \, V/m^3/s^3$, $\alpha_{31} = 3 \times 10^4 \, V/m^3/s^3$, and $\alpha_{31} = 5 \times 10^4 \, V/m^3/s^3$ are demonstrated in Fig. 22.

As shown in Fig. 22, in contrast to the results displayed in Fig. 20 for the unfiltered case, the response in the system with filter is not compressed as the input strengthens. According to Fig. 22, the controlled system’s natural frequency is lower than the uncontrolled system and as the input

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**Fig. 22.** Magnitude tip-velocity/base-acceleration FRF of the single-channel AHC due to changing the input amplitude near the first natural frequency in the system with an LP filter for, (a) $\alpha_{31} = 1 \times 10^4 \, V/m^3/s^3$, (b) $\alpha_{31} = 3 \times 10^4 \, V/m^3/s^3$, and (c) $\alpha_{31} = 5 \times 10^4 \, V/m^3/s^3$. As shown in Fig. 22, in contrast to the results displayed in Fig. 20 for the unfiltered case, the response in the system with filter is not compressed as the input strengthens. According to Fig. 22, the controlled system’s natural frequency is lower than the uncontrolled system and as the input...
level increases, the natural frequency shifts more towards higher frequencies. The shift in the natural frequency and the hardening behavior, i.e. increase in the natural frequency as the input level increases, in the controlled system with the filter were not observed in the system without the filter. Additionally, for a fixed base acceleration, the response grows as the cubic damping gain increases. This can also be concluded from the input-output curves plotted in Fig. 23.

![Graph showing input to output curve at the first natural frequency of the single-channel AHC with LP filter](image)

**Fig. 23.** Input to output curve at the first natural frequency of the single-channel AHC with LP filter

Therefore, the simulated AHC with the filter does not show the desired nonlinear amplification/compression behavior. The main reason for this unwanted behavior is the phase delay introduced by the filter to the system. This phase delay depends on the filter’s order and cut-off frequency and can change the amplification/compression trend of the response and create softening, i.e. decrease in the natural frequency when the input increases, or hardening effects. In the present work, to compensate for the filtering effect on the amplification/compression behavior of the system, a linear damping gain tuning method is developed in the next section.

### 3.3.3. Single-channel active AHC with an LP filter and tuned linear damping gain

As mentioned in the previous chapter, the equation of motion for an active single-channel AHC is given by,
\[ \ddot{u}_{ts}(t) + 2\zeta_1 \omega_1 \dot{u}_{ts}(t) + \omega_1^2 u_{ts}(t) = \beta_v (\alpha_{11} \dot{u}_{ts}(t) - \alpha_{31} \ddot{u}_{ts}(t)) - \beta_z \ddot{z}(t), \]  

(93)

where, \( \beta_v = \beta_{1v} s_{x_{t1}} \) and \( \beta_z = \beta_{1z} s_{x_{t1}} \) are control influence and acceleration influence terms, respectively. To eliminate the linear damping of the system and tune it to a Hopf bifurcation, \( \alpha_{11} \) is set to \( 2\zeta_1 \omega_1 / \beta_v \), as discussed previously. Although this gain value can be used in controlling the system without any filters, as shown in Section 3.3.1, adopting the theoretical value is not effective for the system with the filter, as discussed in Section 3.3.2. Therefore, a new linear damping gain tuning method is developed based on the SDOF approximation of the filtered system. The equation of motion of the controlled system with the filter is approximated by,

\[ \ddot{u}_{ts}(t) + \left( 2\zeta_1 \omega_1 - \beta_{veq} \alpha_{11} \right) \dot{u}_{ts}(t) + \omega_1^2 u_{ts}(t) = - \beta_{veq} \alpha_{31} \ddot{u}_{ts}(t) - \beta_z \ddot{z}(t), \]

(94)

where, \( \beta_{veq} \) is the equivalent control influence term of the approximated system that implies the filtering effect on the system. To determine \( \beta_{veq} \) and the linear damping gain that tunes the system near a Hopf bifurcation, the cubic damping gain, \( \alpha_{31} \) is set to zero and Equation (94) is rewritten as,

\[ \ddot{u}_{ts}(t) + \left( 2\zeta_{1eq} \omega_{1eq} \right) \dot{u}_{ts}(t) + \omega_{1eq}^2 u_{ts}(t) = - \beta_z \ddot{z}(t) \]

(95)

where,

\[ 2\zeta_{1eq} \omega_{1eq} = - \beta_{veq} \alpha_{11} + 2\zeta_1 \omega_1. \]

(96)

In this equation, \( \omega_{1eq} \) is the equivalent natural frequency and \( \zeta_{1eq} \) is the corresponding equivalent modal damping. Method of determining \( \omega_{1eq} \) and \( \zeta_{1eq} \) is discussed later in this section. Hereinafter, \( \alpha_{11} \) notation is used only for the tuned linear damping gain and other values of the linear damping gains are noted as \( \alpha'_{11} \). Therefore, Equation (96) becomes,

\[ 2\zeta_{1eq} \omega_{1eq} = - \beta_{veq} \alpha'_{11} + 2\zeta_1 \omega_1. \]

(97)
As a result, plotting \((2\zeta_{1eq}\omega_{1eq})\) with respect to \(\alpha'_{11}\) is a straight line with a slope of \(-\beta_{veq}\) and y-intercept of \((2\zeta_1\omega_1)\). The linear damping gain \(\alpha_{11}\) that drives the system to the Hopf bifurcation is the x-intercept of the line as,

\[
\alpha_{11} = \frac{(2\zeta_1\omega_1) - (2\zeta_{1eq}\omega_{1eq})}{\beta_{veq}}.
\]  

(98)

The aforementioned procedure is used to determine the linear damping gain of the controller for the single-channel AHC of Section 3.3.2. Therefore, as the AHC is excited, \(\alpha'_{11}\) is varied and the FRF of the system is plotted for multiple values of \(\alpha'_{11}\) as shown in Fig. 24.

![Fig. 24. Magnitude tip-velocity/base-acceleration FRF of the AHC for various values of \(\alpha'_{11}\).](image)

Fig. 24 displays that as the linear damping gain increases, the damping of the controlled system decreases. To find the \(\alpha_1\) of interest, circle-fit system identification method [186, 187] is used to approximate the reduced damping system as an equivalent SDOF system for each linear damping gain (\(\alpha'_{11}\)) cases. The system identification method used in this work will be discussed in detail in Section 4.4.1 of the next chapter. Natural frequency \((\omega_{1eq})\) and damping ratio \((\zeta_{1eq})\) of the
approximated system are calculated and \((2\zeta_{1_{eq}}\omega_{1_{eq}})\) versus \(\alpha'_{1}\) is plotted. For the system shown in Fig. 24, \((2\zeta_{1_{eq}}\omega_{1_{eq}})\) versus \(\alpha'_{11}\) and the least square fit to the data are plotted in Fig. 25.

![Graph](image)

**Fig. 25.** Finding linear damping gain \((\alpha_{11})\) that removes the damping near the first natural frequency using the simulated data obtained from the tip-velocity/base-acceleration FRF.

As displayed in Fig. 25, \(\alpha_{11}\) is equal to \(-52.51\ V/m/s\), and \(\beta_{v_{eq}} = -3.156 \ m/s^2/V\), while the theoretical linear damping gain value used in the previous section was \(\alpha_{11} = 51.46 \ V/m/s\) and \(\beta_{v}\) of the system without the filter was calculated from Table 2 to be \(3.246 \ m/s^2/V\). Although the absolute values of the parameters calculated for the system with the filter are close to the system’s model, they have opposite signs. As a result, the linear damping of the controlled system illustrated in Fig. 22 of the previous section increased after applying the control voltage and the system did not show desired amplification/compression behavior after applying the cubic damping. The nonlinear behavior of the controlled system with \(\alpha_{11} = -52.51 \ V/m/s\) and \(\alpha_{31} = -5 \times 4\) is shown in Fig. 26.
Fig. 26. Magnitude tip-velocity/base-acceleration FRF of the AHC with LP filter due to changing the input level near the fundamental frequency for $\alpha_{11} = -52.51 \, V/m/s$ and $\alpha_{31} = -5 \times 10^4 \, V/m^3/s^3$.

As illustrated in Fig. 26, the controlled system demonstrates the nonlinear behavior expected from an active AHC. As the input amplitude increases, the damping of the system increases and the system compresses the response for $4.95 \, m/s^2$ base acceleration, while it amplifies the output for the other cases. The compressive rate of the output in the controlled system is shown in Fig. 27.

Fig. 27. Input-output curve at the fundamental frequency of the single-channel AHC with the LP filter
Comparing the magnitude FRF of the system with the filter in Fig. 26 to the unfiltered case displayed in Fig. 20, the magnitude FRFs for the same excitation level at the resonance are very close. However, a softening behavior is observed in the filtered case due to the phase delay the filter introduced to the system. The same method of determining the linear damping gain will be used in the next section for a simulated two-channel active AHC.

3.4. Two-channel active AHC

In this section, first, the results of the numerical simulations for the two-channel active AHC near the two natural frequencies are presented and discussed. Next, the response of the controlled system is analyzed in a broader bandwidth.

3.4.1. Two-channel active AHC with filters

The Simulink model shown in Fig. 19 is used in this section. The procedure for determining the linear damping gains for the controllers, \( V_1 \) and \( V_2 \), is similar to the one described in the previous section. Subsequently, the cubic damping gains, \( \alpha_{13} \) and \( \alpha_{32} \) are set to zero, the linear damping gains, \( \alpha'_{11} \) and \( \alpha'_{12} \) are varied, and equivalent system’s parameters are identified. Varying \( \alpha'_{11} \) and plotting the resultant FRFs of the system near the first natural frequency yields in the same FRFs as Fig. 24 for the single-channel active AHC with the filter. FRFs of the system near the second natural frequency for various \( \alpha'_{12} \) values are illustrated in Fig. 28.
A hardening behavior is observed in the FRFs of the system near the second natural frequency as the linear damping gain increases. The \((2ζ_{2\text{eq}} \omega_{2\text{eq}})\) estimates derived from the FRFs of Fig. 28 are displayed in Fig. 29. The linear damping gain corresponding to the second natural frequency is calculated from Fig. 29 to be \(α_{12} = 61.14 \, \text{V/m/s}\). The obtained linear damping gains are used in the numerical simulations for the rest of this section.
The cubic damping is then introduced to the system near both natural frequencies. The nonlinearity of the system is easily observed when the amplitude of the base excitation is varied. Fig. 30 (a), (b), and (c) show the results near the first natural frequency. In Fig. 30 (a), the system amplifies the output near the first natural frequency for all of the simulated input levels. The response is compressed nonlinearly for the $4.95 \text{ m/s}^2$ input acceleration for $\alpha_{31} = -5 \times 10^4 \text{ V/m}^3/\text{s}^3$ shown in Fig. 30 (b), while it is amplified for the lower input levels. Fig. 30 (c) shows that the response is amplified for $0.85 \text{ m/s}^2$ and $1.7 \text{ m/s}^2$ and it is compressed for other cases. FRFs of the controlled system near the second natural frequency are displayed in Fig. 30 (d), (e), and (f). For $\alpha_{32} = 5 \times 10^6 \text{ V/m}^3/\text{s}^3$, the response is amplified for $Z = 0.71 \text{ m/s}^2$ to $2.26 \text{ m/s}^2$. The system compresses the response for $2.26 \text{ m/s}^2$ base acceleration and amplifies it for lower inputs. Amplification occurs near the second natural frequency for $0.71 \text{ m/s}^2$ and $1.13 \text{ m/s}^2$ base accelerations shown in Fig. 30 (f).
Fig. 30. Magnitude tip-velocity/base-acceleration FRFs of the AHC due to changing the input amplitude near the first natural frequency for, (a) $\alpha_{31} = -1 \times 10^4 \text{ V/m}^3/\text{s}^3$, (b) $\alpha_{31} = -3 \times 10^4 \text{ V/m}^3/\text{s}^3$, (c) $\alpha_{31} = -5 \times 10^5 \text{ V/m}^3/\text{s}^3$, and near the second natural frequency for, (d) $\alpha_{32} = 5 \times 10^6 \text{ V/m}^3/\text{s}^3$, (e) $\alpha_{32} = 1 \times 10^7 \text{ V/m}^3/\text{s}^3$, and (f) $\alpha_{32} = 3 \times 10^7 \text{ V/m}^3/\text{s}^3$.

Fig. 30 displays that as the input level increases, the first natural frequency of the controlled system shifts towards lower frequencies, while the second natural frequency shifts towards higher frequencies. The first natural frequency ($f_1$) of the controlled system for various cubic damping
gains and base excitation levels is tabulated in Table 4. The shift in the natural frequency of the system relative to the natural frequency of the uncontrolled system is also shown.

**Table 4.** Controlled system’s first natural frequency \( (f_1) \) and its shift with respect to the uncontrolled system’s natural frequency for various cubic damping gains and input base excitation levels.

<table>
<thead>
<tr>
<th>Base Acc. ((m/s^2))</th>
<th>( \alpha_{31} = -1 \times 10^4(V/m^3/s^3) )</th>
<th>( f_1 ) (Hz)</th>
<th>( f_1 ) shift (%)</th>
<th>( \alpha_{31} = -5 \times 10^4(V/m^3/s^3) )</th>
<th>( f_1 ) (Hz)</th>
<th>( f_1 ) shift (%)</th>
<th>( \alpha_{31} = -2 \times 10^5(V/m^3/s^3) )</th>
<th>( f_1 ) (Hz)</th>
<th>( f_1 ) shift (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.85</td>
<td>446.91</td>
<td>0.848</td>
<td>445.62</td>
<td>0.557</td>
<td>445.62</td>
<td>0.557</td>
<td>445.63</td>
<td>0.560</td>
<td></td>
</tr>
<tr>
<td>1.7</td>
<td>445.63</td>
<td>0.560</td>
<td>444.40</td>
<td>0.282</td>
<td>444.39</td>
<td>0.280</td>
<td>444.39</td>
<td>0.280</td>
<td></td>
</tr>
<tr>
<td>2.55</td>
<td>445.63</td>
<td>0.560</td>
<td>444.40</td>
<td>0.282</td>
<td>444.39</td>
<td>0.280</td>
<td>444.39</td>
<td>0.280</td>
<td></td>
</tr>
<tr>
<td>3.25</td>
<td>445.62</td>
<td>0.557</td>
<td>444.39</td>
<td>0.280</td>
<td>444.39</td>
<td>0.280</td>
<td>444.39</td>
<td>0.280</td>
<td></td>
</tr>
<tr>
<td>4.1</td>
<td>444.40</td>
<td>0.282</td>
<td>444.39</td>
<td>0.280</td>
<td>443.17</td>
<td>0.000</td>
<td>443.17</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>4.95</td>
<td>444.39</td>
<td>0.280</td>
<td>443.17</td>
<td>0.000</td>
<td>443.17</td>
<td>0.000</td>
<td>443.17</td>
<td>0.000</td>
<td></td>
</tr>
</tbody>
</table>

The existence of softening behavior can also be concluded from the decreasing trend of the natural frequency corresponding to each cubic damping gain listed in Table 4. Natural frequency shift with respect to the uncontrolled system’s natural frequency shows that the maximum shift of the first natural frequency for the base excitation between 0.85 \( m/s^2 \) and 4.95 \( m/s^2 \), and \( \alpha_{31} \) between \(-2 \times 10^5(V/m^3/s^3)\) and \(-1 \times 10^4(V/m^3/s^3)\) is 0.848\%. Similar analysis is conducted for the second frequency of the controlled system and the results are listed in Table 5.

**Table 5.** Controlled system’s second natural frequency \( (f_2) \) and its shift with respect to the uncontrolled system’s natural frequency for various cubic damping gains and input base excitation levels.

<table>
<thead>
<tr>
<th>Base Acc. ((m/s^2))</th>
<th>( \alpha_{32} = 5 \times 10^6(V/m^3/s^3) )</th>
<th>( f_2 ) (Hz)</th>
<th>( f_2 ) shift (%)</th>
<th>( \alpha_{32} = 1 \times 10^7(V/m^3/s^3) )</th>
<th>( f_2 ) (Hz)</th>
<th>( f_2 ) shift (%)</th>
<th>( \alpha_{32} = 3 \times 10^7(V/m^3/s^3) )</th>
<th>( f_2 ) (Hz)</th>
<th>( f_2 ) shift (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.71</td>
<td>2761.8</td>
<td>-0.551</td>
<td>2763.3</td>
<td>-0.497</td>
<td>2767.9</td>
<td>-0.331</td>
<td>2774.0</td>
<td>-0.112</td>
<td></td>
</tr>
<tr>
<td>1.13</td>
<td>2764.9</td>
<td>-0.439</td>
<td>2767.9</td>
<td>-0.331</td>
<td>2774.0</td>
<td>-0.112</td>
<td>2781.7</td>
<td>0.166</td>
<td></td>
</tr>
<tr>
<td>1.55</td>
<td>2766.4</td>
<td>-0.385</td>
<td>2771.0</td>
<td>-0.220</td>
<td>2781.7</td>
<td>0.166</td>
<td>2786.2</td>
<td>0.328</td>
<td></td>
</tr>
<tr>
<td>1.84</td>
<td>2769.5</td>
<td>-0.274</td>
<td>2774.0</td>
<td>-0.112</td>
<td>2786.2</td>
<td>0.328</td>
<td>2793.9</td>
<td>0.605</td>
<td></td>
</tr>
<tr>
<td>2.26</td>
<td>2771.0</td>
<td>-0.220</td>
<td>2777.1</td>
<td>-0.000</td>
<td>2793.9</td>
<td>0.605</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Based on Table 5, the system shows a hardening behavior as the base excitation level increases. It can be observed that for a fixed base excitation level, increasing the cubic damping gain results in shifting the natural frequency towards the higher frequencies. However, the maximum shift of the
second natural frequency is 0.605%. The effect of changing the cubic damping gain on the response of the system can be observed in Fig. 31. As the absolute value of the cubic damping gains increases, the system’s overall behavior tends to compress.

**Fig. 31.** Magnitude of the RMS tip-velocity of the active AHC due to varying the excitation level near the first natural frequency for, (a) $\alpha_{31} = -1 \times 10^4 \frac{V}{m^3/s^3}$, (b) $\alpha_{31} = -3 \times 10^4 \frac{V}{m^3/s^3}$, (c) $\alpha_{31} = -5 \times 10^5 \frac{V}{m^3/s^3}$, and near the second natural frequency for, (d) $\alpha_{32} = 5 \times 10^6 \frac{V}{m^3/s^3}$, (e) $\alpha_{32} = 1 \times 10^7 \frac{V}{m^3/s^3}$, (f) $\alpha_{32} = 3 \times 10^7 \frac{V}{m^3/s^3}$. 
Nonlinear compression and amplification for the active and passive AHC at the first and second natural frequencies are shown in Fig. 32. For each input level and cubic damping gain, RMS velocity is calculated at the shifted peak frequency. The input and output of the passive system have a linear relationship, as shown in Fig. 32 by a black dashed line. This line divides the graph into two regions: (i) amplification region above the black line corresponding to the passive system, (ii) compression region below the line. For each curve corresponding to a particular cubic damping gain, the behavior of the system switches from amplification to compression where the input to output curve intersects with the passive system’s line. Therefore, the switching RMS base acceleration levels corresponding to \( \alpha_{31} = -1 \times 10^4 \, V/m^3/s^3 \), \( \alpha_{31} = -3 \times 10^4 \, V/m^3/s^3 \), and \( \alpha_{31} = -5 \times 10^5 \, V/m^3/s^3 \) are calculated from Fig. 32 (a) to be 7.41 m/s\(^2\), 3.35 m/s\(^2\), and 1.62 m/s\(^2\), respectively. At the second natural frequency, the cross-over RMS base acceleration levels for \( \alpha_{32} = 5 \times 10^6 \, V/m^3/s^3 \), \( \alpha_{32} = 1 \times 10^7 \, V/m^3/s^3 \), and \( \alpha_{32} = 3 \times 10^7 \, V/m^3/s^3 \) shown in Fig. 32 (b) are obtained as 2.22 m/s\(^2\), 1.5 m/s\(^2\), and 0.89 m/s\(^2\), respectively.

**Fig. 32.** Input to output curves for the simulated two-channel AHC, (a) at the first natural frequency, (b) at the second natural frequency.
Results shown in Fig. 32 demonstrate a one-third compression rate at the first and second natural frequencies. Therefore, the simulations show the ability of the two-channel active AHC to mimic the nonlinear functions of the cochlea near its natural frequencies.

3.4.2. Effects of the controllers on a broader bandwidth

To investigate the effect of the controller on frequencies other than the ones near the natural frequencies, the response of the system is simulated in a broad bandwidth. The base excitation of the beam is simulated as a stepped sine signal in a frequency range of 0 to 3500 Hz with an amplitude of 1.13 m/s². The passive system is also excited by the same input. FRFs of the uncontrolled and controlled systems with \( \alpha_{31} = -5 \times 10^4 \) V/m³/s³ and \( \alpha_{32} = 1 \times 10^7 \) V/m³/s³ are presented in Fig. 33.

![Fig. 33. Magnitude of the tip-velocity/base-acceleration FRF of the two-channel active AHC for 1.13 m/s² input with \( \alpha_{31} = -5 \times 10^4 \) V/m³/s³ and \( \alpha_{32} = 1 \times 10^7 \) V/m³/s³.](image)

In this figure, the controller amplifies the response of the system only near the first and second natural frequencies and the FRFs of the active and passive systems match for the frequencies between the natural frequencies. Additionally, the control law can be applied such that the AHC shows nonlinear behavior only near one of the natural frequencies. To further illustrate this, Fig. 34 (a) shows the FRF of the system with only the first controller (\( V_1 \)) and Fig. 34 (b) shows the
results by only having the second controller ($V_2$). Therefore, each of the controllers only affects the response close to the corresponding natural frequency, as expected.

![Diagram of controller $V_1$ and $V_2$](image)

**Fig. 34.** Magnitude of the tip-velocity/base-acceleration FRF of the two-channel active AHC for 1.13 m/s² input for, (a) $\alpha_{31} = -5 \times 10^4$ V/m³/s³ and $\alpha_{32} = 0$, (b) $\alpha_{31} = 0$ and $\alpha_{32} = 1 \times 10^7$ V/m³/s³.

### 3.5. Conclusion

In this chapter, models of the active single-channel and two-channel AHCs were created in Simulink. The response of the single-channel AHC was simulated with and without an LP filter embedded before the controller. For the system with the filter, a new method of determining the linear damping gains of the controller was presented to compensate for the filtering effect. In the
two-channel AHC model, the tip velocity response of the system to a base excitation was passed through two filters separately, a low-pass and a band-pass filter. The filtered velocities were then used in calculating the control signals of the two controllers used in the system. The controllers were responsible for removing the linear damping of the first and second modes and adding nonlinear damping to the system near the corresponding natural frequencies of the AHC. Based on the literature, the cochlea works similar to a dynamic system tuned to a Hopf bifurcation. Therefore, a control law that eliminates the linear damping of the system and introduces cubic damping into the system of equations can drive the response of the system to a Hopf bifurcation and produces the cochlea-like response. In this work, the controller was able to introduce cubic damping at multiple frequency bandwidths as an attempt to move towards MDOF hair cell mimicry. Simulation results of the linear and nonlinear systems were compared and it was shown that the amplitude of the response near each of the natural frequencies can be controlled without affecting the response near the other natural frequencies. Also, exciting the system with different levels of the input showed the nonlinear amplification and compression functions of the system.
Chapter 4

Real-time implementation of the multi-channel active AHC

4.1. Introduction

The nonlinear feedback control law creating a cubic relation between the output and input of the AHC at resonance was discussed in chapter 2 and the simulated case studies implementing the control law were presented in chapter 3. Subsequently, this chapter is dedicated to the real-time implementation of the controlled system. First, a stepped sine analysis will be performed on the uncontrolled AHC and the tip velocity with respect to the base acceleration FRF of the uncontrolled AHC will be obtained near the first two natural frequencies. Next, an experimental approach will be taken to calculate the linear damping gains of the controller for various input levels. After adding the cubing damping to the AHC, the performance of the system will be evaluated. Finally, the experimental results will be compared to the numerical simulation results to validate the model of the AHC developed in the previous chapter. The context of this chapter is adapted from Sheyda Davaria, Vijaya VN Sriram Malladi, Seyedmostafa Motaharibidgoli, and Pablo A. Tarazaga, “Cochlear amplifier inspired two-channel active artificial hair cells.”, Mechanical Systems and Signal Processing 129 (2019): 568-589 [177].

4.2. Test setup

The experimental setup for the real-time implementation of the control algorithm is discussed in this section. The following system is developed to apply the control algorithm to an actual system.
A piezoelectric bimorph beam, 31.8 mm long, 3.2 mm width, and 0.66 mm thick, from Piezo Systems, Inc. is set as the hair cell structure. The beam consists of a 0.04 mm brass shim sandwiched between two PSI-5A4E piezoceramic layers poled in opposite directions. The beam is fixed at an end to a test fixture such that AHC has a suspended length of 26 mm. A 2007E mini shaker from The Modal Shop, Inc. provides the base excitation to the AHC. As a power amplifier (2100E21) drives the shaker, a PCB shear accelerometer mounted on the base measures the input acceleration. A Polytec PDV-100 laser Doppler vibrometer measures the tip velocity of the beam and provides feedback to the controller. Fig. 35 (a) shows the test setup and Fig. 35 (b) shows the schematics of the test.

**Fig. 35.** Experimental setup: (a) test setup, (b) schematic of the test setup

A NI-PXI 8109 embedded controller and a NI-PXI 6259 M series multifunction DAQ implements the real-time control algorithm. A laptop with LabVIEW real-time module serves as a host
computer and communicates with the PXI modulus (real-time target) through a standard Ethernet connection. The velocity of the tip of the AHC, the acceleration of the base, and the input voltage of the shaker amplifier are collected at the rate of 20 kS/s. The NI system also supplies the excitation signal to the shaker amplifier and the control signal to the amplifier connected to the piezoceramic layers of the AHC.

4.3. Passive AHC

The preliminary focus of the experiment is to passively (without any controllers) test AHC near the first and the second natural frequencies. Stepped frequency analysis with 1 Hz resolution, determined the frequency response of the passive AHC in these ranges: [420 to 465 Hz] and [2670 to 2715 Hz]. As the strength of the voltage signal supplied to the shaker is varied from 0.25 V to 1.5 V, the tip velocity and the base acceleration of the AHC are measured. Fig. 36 shows the frequency response functions between the tip response (velocity) and base-excitation (acceleration) of the AHC near the first and the second natural frequencies.

![Magnitude of the experimental tip-velocity/base-acceleration FRF of the uncontrolled AHC for different input voltages to the shaker: (a) near the first natural frequency, (b) near the second natural frequency.](image)

**Fig. 36.** Magnitude of the experimental tip-velocity/base-acceleration FRF of the uncontrolled AHC for different input voltages to the shaker: (a) near the first natural frequency, (b) near the second natural frequency.
The peak location and the corresponding magnitude of the FRF in Fig. 36 (a) reduces with increasing strength of the input voltage. The passive AHC displays such a nonlinear softening behavior in this frequency bandwidth. This can be a result of the inherent material damping, air damping, or bonded boundary condition. Piezoelectric layers of the beam, shaker, and amplifier may also contribute to the nonlinear behavior. However, a similar nonlinear characteristic is not prominent near the second natural frequency as shown in Fig. 36 (b).

As a result of this nonlinearity in the passive system, it is not straightforward to estimate a constant linear damping gain that drives the AHC to a Hopf bifurcation for all voltage levels. Due to the inherent nonlinearity, the value of this gain varies with the voltage levels and have to be estimated individually. Therefore, the first step is to reduce this passive damping from the AHC and drive the system towards Hopf bifurcation. The second step is to introduce a bio-inspired cubic damping to the active AHC.

Due to the nonlinear nature of the passive system, a method is needed to remove the damping of the AHC close to each of the natural frequencies. Therefore, an experimental approach is taken in which the nonlinear system is approximated as an SDOF linear system close to each of the natural frequencies for different input voltage levels to the shaker. The linear damping gains of the controllers near the two natural frequencies ($\alpha_{11}$ and $\alpha_{12}$) are then calculated for each voltage level based on the approach of Section 4.4.

### 4.4. Determining the linear damping gains of the controller ($\alpha_{11}$ and $\alpha_{12}$)

One of the first steps in the controller development is determining the gains ($\alpha_{11}$ and $\alpha_{12}$) that remove the linear damping near the first two natural frequencies, while the values of the cubic damping gains ($\alpha_{31}$ and $\alpha_{32}$) are set to zero. The schematic of the resulting partial controller that drives the stable system into a Hopf bifurcation is shown in Fig. 37.
Fig. 37. Schematic picture of the system used to find the linear damping gains of the controllers that eliminate the linear damping near the first and second natural frequencies.

For different input voltage levels, as the linear damping gains ($\alpha'_{11}$ and $\alpha'_{12}$) are varied, the corresponding FRFs of the AHC with reduced damping are obtained. Increasing the values of $\alpha'_{11}$ and $\alpha'_{12}$ reduces the linear damping and subsequently, there will be a point ($\alpha_{11}$ and $\alpha_{12}$) where the dynamics of the AHC will be close to an undamped system and will undergo a Hopf bifurcation.

Additionally, LP and BP filters are designed according to the frequency bandwidth of the two natural frequencies of the AHC. Therefore, the low-pass filter is a 6th order Butterworth filter with a corner frequency of 650 Hz and the band-pass filter is a 6th order Butterworth filter with edge frequencies of 2 kHz and 3.5 kHz. The cut-off frequencies are chosen by analyzing the transfer function of the filters numerically and experimentally to ensure there is no interference between their functions.

The next sections describe the circle-fit system identification method used in the gain tuning algorithm and the gain tuning approach, respectively.

4.4.1. Circle-fit system identification method

In this section, the circle-fit method used to approximate the AHC’s modal parameters is presented. This method can be used to identify system parameters such as damping, and natural frequency and estimate the fit to the FRF near its natural frequencies. Near the resonance, the Nyquist plot
of the FRF data is close to a circle. Therefore, a circle is fitted to the complex data near the resonance and its properties are used to estimate the SDOF fit in the circle-fit method. For a $N$ DOF system, the mobility of the system is given by [186, 187],

$$Y_{pq}(\omega) = \sum_{i=1}^{N} \frac{j i C_{pq} \omega_i^2}{\omega_i^2 - \omega^2 + j 2 \zeta_i \omega_i^2},$$  

(99)

where, $\omega_i$ is the natural frequency of the $i^{th}$ mode in $rad/s$, $\zeta_i$ is the corresponding modal damping, $\omega$ is the frequency in $rad/s$, and the modal constant $i C_{pq}$ is defined as,

$$i C_{pq} = (C_i e^{i \phi_i})_{pq},$$  

(100)

where, $C_i$ is a real constant that scales the circle and $e^{i \phi_i}$ corresponds to a rotation about the origin. To estimate the SDOF near the $i^{th}$ natural frequency, the FRF in equation (99) can be written as,

$$Y_{pq}(\omega) = \frac{j i C_{pq} \omega_i^2}{\omega_i^2 - \omega^2 + j 2 \zeta_i \omega_i^2} + i D_{pq},$$  

(101)

where, $i D_{pq}$ is a complex number associated with the contribution of the other modes on the FRF and corresponds to the translation of the origin as shown in Fig. 38 for $i = 1$. Using a least square method, the circle shown in Fig. 38 is fitted to the tip-velocity/base-acceleration FRF of the passive AHC excited by a 0.25 $V$ input voltage near the AHC’s first natural frequency. The data points, 20 here, are chosen on both sides of the peak frequency. In Fig. 38, the data points are shown in black and the peak frequency is shown in red.
Fig. 38. Circle-fit to the tip-velocity/base-acceleration FRF of the passive AHC excited by a 0.25 V input voltage near the first natural frequency.

The natural frequency is determined using the fact that \( \frac{d(\omega^2)}{d\theta_l} \) is minimum at the resonance \( (\omega_i) \), where, \( \theta_l \) is the phase angle associated with mode \( i \). Choosing two immediate points at each side of the peak frequency and using Newton’s divided difference formula yields,

\[
\theta = \theta_0 + (\omega^2 - \omega_0^2)(\theta_0, \theta_1) + (\omega^2 - \omega_0^2)(\omega^2 - \omega_1^2)(\theta_0, \theta_1, \theta_2) + (\omega^2 - \omega_0^2)(\omega^2 - \omega_1^2)(\omega^2 - \omega_2^2)(\theta_0, \theta_1, \theta_2, \theta_3),
\]

where, points 0 to 3 are indexed counter-clockwise starting from the left of the peak frequency and,

\[
(\theta_0, \theta_1) = \frac{\theta_0 - \theta_1}{\omega_0^2 - \omega_1^2},
\]

\[
(\theta_0, \theta_1, \theta_2) = \frac{(\theta_0, \theta_1) - (\theta_1, \theta_2)}{\omega_0^2 - \omega_2^2},
\]
\[ (\theta_0, \theta_1, \theta_2, \theta_3) = \frac{(\theta_0, \theta_1, \theta_2, \theta_3) - (\theta_1, \theta_2, \theta_3)}{\omega_0^2 - \omega_3^2}. \]  

(105)

The \( i^{th} \) natural frequency is calculated to be,

\[ \omega_i^2 = \frac{1}{3} \left( \omega_0^2 + \omega_1^2 + \omega_2^2 - \frac{(\theta_0, \theta_1, \theta_2)}{(\theta_1, \theta_2, \theta_3)} \right). \]

(106)

To determine the viscous damping of the system, \( \zeta_i \) is calculated as,

\[ \zeta_i = \frac{1}{n} \sum_{m=1}^{n} \frac{(\omega_a)^2_m - (\omega_b)^2_m}{2 \omega_i \left( (\omega_a)^m \tan \left( \frac{\theta_a}{2} \right) + (\omega_b)^m \tan \left( \frac{\theta_b}{2} \right) \right)}, \]

(107)

where, \( \omega_a \) and \( \theta_a \) are the frequency and phase angle of a point above the natural frequency and \( \omega_b \) and \( \theta_b \) correspond to a point below the natural frequency. Parameter \( n \) is the number of data pairs used for determining the damping. The real part of the modal constant in equation (101) is given by,

\[ C_i = 2 d_{circle} \omega_i^2 \zeta_i. \]

(108)

where, \( d_{circle} \) is the diameter of the fitted circle. The rotation angle is defined as,

\[ \phi_i = \tan^{-1} \left( \frac{y_c - y_D}{x_c - x_D} \right), \]

(109)

where, \((x_c, y_c)\) is the coordinate of the circle’s center and \((x_D, y_D)\) is the coordinate of the translated origin displayed in Fig. 38. The estimated mobility for the system shown in Fig. 38 is calculated to be,

\[ Y_{pq}(\omega) = \frac{4620.84 \times 10^{-4} j}{2789.73^2 - \omega^2 + j \frac{402050.24}{4620.84}} + (-8.04 + 4.82 J) \times 10^{-4}. \]

(110)

The circle-fit method is used in the following section to identify system parameters.

### 4.4.2. Gain tuning

Fig. 39 (a) and (b) show the FRFs of the system for various values of \( \alpha'_{11} \) near the first natural frequency. These figures are generated when the shaker is supplied with a 1 V input signal. A similar analysis is performed for the rest of the cases.
Fig. 39. Magnitude of the experimental FRFs of the AHC: (a) tip-velocity/voltage for different values of $\alpha'_{11}$ near the first natural frequency for 1V input voltage to the shaker amplifier, (b) tip-velocity/base-acceleration for different values of $\alpha'_{11}$ near the first natural frequency for 1V input voltage to the shaker amplifier, (c) tip-velocity/voltage due to changing $\alpha'_{12}$ near the second natural frequency for 1.5V input voltage to the shaker, (d) tip-velocity/base-acceleration due to changing $\alpha'_{12}$ near the second natural frequency for 1.5V input voltage to the shaker.

As expected with an increase in $\alpha'_{11}$, the damping of the controlled system decreases. Along with this, an unexpected softening behavior is also observed in Fig. 39 (a) and (b). In literature, previous studies have shown that the nonlinear response of the piezoelectric structures displays both softening and hardening behavior [188-190]. Wang et al. attributed such behavior to the material properties of the substrate and the piezoceramics plate [191]. Abdelkefi et al. has shown that the
The net behavior of the system is determined by the dominant type of nonlinearity [192]. Also, in the current work filters contribute to the distortions observed in the FRFs, as shown previously in Chapter 3. The damping estimates of each of these FRFs are determined using the circle-fit identification approach presented in Section 4.4.1. The SDOF fit on the tip-velocity/base-acceleration FRF for the uncontrolled system for $\alpha_1' = 20 \, V/m/s$, both excited by 1 V input voltage signal are shown in Fig. 40.

![Fig. 40. Experimental tip-velocity/base-acceleration FRF of the AHC near the first natural frequency for a 1 V input voltage to the shaker and the SDOF fit obtained by the circle-fit method: (a) uncontrolled system ($\alpha_1' = 0$), (b) $\alpha_1' = 20 \, V/m/s$. The natural frequency and the damping ratio are estimated from two FRFs: (i) the FRF between the tip velocity and the base acceleration, and (ii) the FRF between the tip velocity and the voltage input. Fig. 41 presents the linear trend of $2\zeta_{1eq}\omega_{1eq}$ estimates, as the value of the gain $\alpha_{11}'$ is increased for the 1V base excitation case, where $\zeta_{1eq}$ and $\omega_{1eq}$ stand for the equivalent modal parameters.](image)

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damping and natural frequency of the first mode, respectively. Linear least-squares fits corresponding to these estimates are also presented alongside the data in this plot. The linear damping gain $\alpha_{11}$ that drives the system to a Hopf bifurcation corresponds to the zero-crossing value (x-intercept) of the linear fit. It is not practical to experimentally drive the undamped AHC near its resonant frequency as that could permanently damage the structure. Therefore, it is only possible to estimate the linear damping gain by extrapolating the fitted curve to determine the x-intercept.

**Fig. 41.** Finding linear damping gain: (a) $\alpha_{11}$ that removes the damping near the first natural frequency using data obtained from experimental tip-velocity/voltage FRF and experimental tip-velocity/base-acceleration FRF, (b) $\alpha_{12}$ that removes the damping near the second natural frequency.

For the 1V input case, the gain that drives the first mode to Hopf bifurcation is estimated to be $(\alpha_{11})_v = 52.03 \, V/m/s$ from the tip-velocity/voltage FRF, and $(\alpha_{11})_a = 51.47 \, V/m/s$ from the tip-velocity/acceleration FRF. The value of $(\alpha_{11})_a$ is later used as the linear damping gain $(\alpha_{11})$ in the complete controller. Similar analysis at other input voltage signal levels determines the corresponding values of the linear damping gain $\alpha_{11}$. These estimated gains are tabulated in Table 6. Furthermore, the estimates $(\alpha_{11})_v$ and $(\alpha_{11})_a$ are very close to each other; this shows that the
contribution of the clamping mechanism, shaker, and power amplifier on the response of the AHC near the first natural frequency is negligible. Additionally, it can be seen that the estimated linear damping gain ($\alpha_{11}$) values increase with the input voltage level. Such a trend is expected due to the inherent softening nonlinearity seen in Fig. 36.

A similar approach is adapted to determine the linear damping gain ($\alpha_{12}$) that drives the second natural frequency to a Hopf bifurcation. FRFs of the system for a 1.5 V input voltage to the shaker for various values of $\alpha'_1$ near the second natural frequency are shown in Fig. 39 (c) and (d). Compared to the previous case (Fig. 39 (a) and (b)) there is no significant nonlinear softening behavior seen when the damping of the system is actively reduced from the AHC (Fig. 36 (b)).

Fig. 41 (b) shows the $2\zeta_{2,eq}\omega_{2,eq}$ estimates derived from the FRFs of Fig. 39 (c) and (d). The linear damping gain corresponding to the second natural frequency is calculated using the circle-fit method. The linear damping gain ($\alpha_{12}$) estimates for various input voltage levels are presented in Table 6. Note that the positive or negative sign of the linear damping gain ($\alpha_{11}$ or $\alpha_{12}$) depends on the sign of the control influence term ($\beta_v$) of the SDOF fit. The control influence terms ($\beta_v$) and ($\beta_v$) are the slope of the linear fit on $2\zeta_{1,eq}\omega_{1,eq}$ versus $\alpha'_{11}$ (Fig. 41 (a)) and $2\zeta_{2,eq}\omega_{2,eq}$ versus $\alpha'_{12}$ (Fig. 41 (b)), respectively.

As shown in Table 6, increasing the voltage from 0.5 V to 1.5 V increases the linear damping gain $\alpha_{12}$ by a maximum of 1.00%, while the maximum increase in $\alpha_{11}$ is 11.55% due to the inherent nonlinearity seen in the system near the first natural frequency.
Table 6. Linear damping gain required to remove the damping near the two natural frequencies

<table>
<thead>
<tr>
<th>Voltage (V)</th>
<th>$(\alpha_{11})_v$ (V/m/s)</th>
<th>$(\alpha_{11})_a$ (V/m/s)</th>
<th>$(\alpha_{12})_v$ (V/m/s)</th>
<th>$(\alpha_{12})_a$ (V/m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>47.33</td>
<td>47.63</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>0.5</td>
<td>49.49</td>
<td>49.00</td>
<td>-55.70</td>
<td>-61.86</td>
</tr>
<tr>
<td>0.75</td>
<td>50.34</td>
<td>49.89</td>
<td>-56.32</td>
<td>-61.91</td>
</tr>
<tr>
<td>1</td>
<td>52.03</td>
<td>51.47</td>
<td>-57.12</td>
<td>-61.70</td>
</tr>
<tr>
<td>1.25</td>
<td>52.47</td>
<td>52.50</td>
<td>-57.63</td>
<td>-61.26</td>
</tr>
<tr>
<td>1.5</td>
<td>53.02</td>
<td>53.13</td>
<td>-58.32</td>
<td>-61.24</td>
</tr>
</tbody>
</table>

4.5. Results of implementing the cubic damping controllers

The next step, once the linear damping gains of the controllers are estimated, is to introduce cubic damping to the active AHC (Fig. 39). Similar to the previous test procedure, a stepped frequency analysis in the range of 420 Hz to 465 Hz and 2670 Hz to 2715 Hz is performed on the AHC. The voltage supplied to the shaker is varied from 0.25 V to 1.5 V and tip-velocity/base-acceleration FRF is obtained.

4.5.1. Controlling the AHC near the first natural frequency

The magnitude response of the tip-velocity/acceleration FRFs (near the first natural frequency) for different cubic damping gains $\alpha_{31} = 1 \times 10^4 \ V/m^3/s^3$, $\alpha_{31} = 5 \times 10^4 \ V/m^3/s^3$, and $\alpha_{31} = 2 \times 10^5 \ V/m^3/s^3$ are shown in Fig. 42. These gain values are chosen by taking experimental constraints such as shaker limits into consideration.
Fig. 42. Magnitude of the experimental tip-velocity/base-acceleration FRF of the active AHC due to changing the input voltage level to the shaker amplifier for, (a) $\alpha_{31} = 1 \times 10^4 \, V/m^3/s^3$, (b) $\alpha_{31} = 5 \times 10^4 \, V/m^3/s^3$, and (c) $\alpha_{31} = 2 \times 10^5 \, V/m^3/s^3$. Linear damping gains ((\alpha_{11})_a) are listed in Table 6.

As the input voltage increases, the damping of the controlled system increases and the natural frequency shifts towards the lower frequencies as a softening behavior is observed. Varying the cubic damping gain changes the nonlinear amplification and compression behavior of the system for the same input level. For $\alpha_{31} = 1 \times 10^4 \, V/m^3/s^3$, AHC amplifies the response for all the given input voltages in the range of 0.25 V to 1.5 V, while for $\alpha_{31} = 5 \times 10^4 \, V/m^3/s^3$ and $\alpha_{31} = 2 \times 10^5 \, V/m^3/s^3$ the response is amplified for voltages equal to and below 1 V and 0.5 V, respectively. Input to output curves of the controlled system at resonance for different cubic gains are illustrated in Fig. 43. Also, Fig. 42 and Fig. 43 are color-coordinated. Relationships between the input (acceleration) values and outputs (velocities) are estimated as power functions.
The curve fitting shows that the acceleration is compressed by a factor of 0.34, 0.36, and 0.37 for $\alpha_{31} = 1 \times 10^4 \ V/m^3/\text{s}^3$, $\alpha_{31} = 5 \times 10^4 \ V/m^3/\text{s}^3$ and $\alpha_{31} = 2 \times 10^5 \ V/m^3/\text{s}^3$, respectively. These values are close to 0.33 as expected from a dynamic system tuned on a Hopf bifurcation with cubic damping. The amplitudes of the fitted curves represent the sensitivity of the active AHC. For instance, when $\alpha_{31} = 1 \times 10^4 \ V/m^3/\text{s}^3$, the cubic relationship is $vel = 0.031 a^{0.37} \text{ m/s}$, and the sensitivity is $0.031 \left( \frac{m}{s^2} \right)^{0.37}$. The sensitivity guides the value of the acceleration where the dynamic system switches from compression to amplification of the base excitation. The sensitivity depends on the cubic damping gain of the controller as shown in Fig. 43. The effect of cubic damping gain on the response of the system can be observed in Fig. 44. As the value of the gain ($\alpha_{31}$) increases, the sensitivity of the dynamic system reduces, which in-turn increases the damping of the tip-velocity (output). As a result, the sensitivity of the AHC can be tuned by varying cubic damping gains of the controller around each of the natural frequencies.

**Fig. 43.** Input to output curves of the controlled AHC at the first natural frequency.
Fig. 44. Magnitude of the experimental tip-velocity of the active AHC around the first natural frequency due to varying the input voltage level for, (a) $\alpha_{31} = 1 \times 10^4 \, V/m^3/s^3$, (b) $\alpha_{31} = 5 \times 10^4 \, V/m^3/s^3$, and (c) $\alpha_{31} = 2 \times 10^5 \, V/m^3/s^3$. Linear damping gains ($\alpha_{11}$) are listed in Table 6.

4.5.2. Controlling the AHC near the second natural frequency

To control the system near the second natural frequency and find the desired controller gain, the cubic damping gain is set to three different values that are pre-selected using numerical simulations and the corresponding FRFs of the active system are presented in Fig. 45.

Based on a similar approach, the input-output curves for different cubic damping gains are shown in Fig. 46. As expected, increasing the value of the cubic damping coefficient increases the damping of the system, and setting the controller gain on different values can change the location
of switching from amplification to compression of the system. This allows for tuning of these AHCs.

**Fig. 45.** Magnitude of the experimental tip-velocity/base-acceleration FRF of the active AHC due to changing the shaker input voltage level for, (a) \( \alpha_{32} = -5 \times 10^6 \, V/m^3/s^3 \), (b) \( \alpha_{32} = -1 \times 10^7 \, V/m^3/s^3 \), and (c) \( \alpha_{32} = -3 \times 10^7 \, V/m^3/s^3 \). Linear damping gains are listed in Table 6. The fitted power function equations are also provided alongside the data points in Fig. 46. As seen in the figure, the compressive rate (the power of the equation) for three cubic gains, \( \alpha_{32} = -5 \times 10^6 \, V/m^3/s^3 \), \( \alpha_{32} = -1 \times 10^7 \, V/m^3/s^3 \) and \( \alpha_{32} = -3 \times 10^7 \, V/m^3/s^3 \), are 0.37, 0.36 and 0.33, respectively.
4.5.3. Filtering effect

In the approach adopted in this study, the choice of filters affects the dynamics of the AHC. A series of experiments are conducted on a single-channel AHC, the two-channel AHC with $\alpha_{12} = 0$ and $\alpha_{32} = 0$, near the first natural frequency in the presence and the absence of the filters. Linear damping gains of the controller are calculated using the circle-fit approach and are tabulated in Table 7.

Table 7. Linear damping gain required to remove the damping near the first natural frequency in the absence of the filters.

<table>
<thead>
<tr>
<th>Voltage (V)</th>
<th>$(\alpha_{11})_v$ (V/m/s)</th>
<th>$(\alpha_{11})_a$ (V/m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>-51.19</td>
<td>-51.5</td>
</tr>
<tr>
<td>0.5</td>
<td>-53.09</td>
<td>-53.33</td>
</tr>
<tr>
<td>0.75</td>
<td>-55.29</td>
<td>-55.09</td>
</tr>
<tr>
<td>1</td>
<td>-56.07</td>
<td>-56.07</td>
</tr>
<tr>
<td>1.25</td>
<td>-56.92</td>
<td>-56.62</td>
</tr>
<tr>
<td>1.5</td>
<td>-58.87</td>
<td>-58.44</td>
</tr>
</tbody>
</table>
Comparing the \((\alpha_{11})_a\) gains listed in Table 7 to the ones shown in Table 6, the absolute values of the estimated linear damping gains around the first natural frequency in the absence of the filters are approximately 8\% to 10\% higher than the values obtained for the active AHC with LP and BP filters. In addition, the gains’ sign difference in the two cases is due to the phase shift added to the AHC system by the LP filter. Next, the cubic damping gains are added to the system \((\alpha_{31} = -5 \times 10^4 \ V/m^3/s^3)\) and the tip-velocity/base-acceleration FRF of the system is compared to the filter-adopted system with \(\alpha_{31} = 5 \times 10^4 \ V/m^3/s^3\). Fig. 47 shows that the nonlinear behavior of the system is very similar in both cases.

![Graph](image)

**Fig. 47.** Magnitude of the experimental tip-velocity/base-acceleration FRF of the active AHC due to changing the input voltage level to the shaker amplifier for, (a) the system without filters \((\alpha_{31} = -5 \times 10^4 \ V/m^3/s^3\) and linear damping gains \((\alpha_{11})_a\) listed in Table 7), (b) the system with LP and BP filters \((\alpha_{31} = 5 \times 10^4 \ V/m^3/s^3\) and linear damping gains \((\alpha_{11})_a\) listed in Table 6).
Fig. 48 illustrates the similar cubic power relationship between the tip-velocity and the base-acceleration of the AHCs with and without filters. The sensitivity of the AHC remains unchanged in the absence of the filters. As a result, a well-tuned filter can reduce the unwanted effects of the filters on the dynamics of the system. It should be noted that filters are necessary to apply the control law efficiently. In developing a multi-channel AHC, filters are required to ensure that each controller affects the response in a desired frequency bandwidth.

![Graph showing the relationship between RMS Velocity and RMS Acceleration]

**Fig. 48.** Input to output curves of the controlled AHC with and without filter at the first natural frequency.

### 4.5.4. Effect of the controller on the complete bandwidth

To investigate the effect of the controllers over the complete frequency bandwidth of interest, experimental FRFs of the controlled and the uncontrolled systems are presented in Fig. 49. As shown in Fig. 49, the controller amplifies the response near the natural frequencies but does not affect the dynamics of the AHC at other frequencies.
Fig. 49. Magnitude of the experimental tip-velocity/base-acceleration FRF of the active AHC for 1 V input voltage with $\alpha_{11} = 51.47 \text{ V/m/s}$, $\alpha_{31} = 1 \times 10^4 \text{ V/m}^3/\text{s}^3$, $\alpha_{12} = -61.70 \text{ V/m/s}$, and $\alpha_{32} = -5 \times 10^6 \text{ V/m}^3/\text{s}^3$.

To show the capabilities of the two controllers to work independently, the response of the AHC with the application of just the second part ($\alpha_{31} = 0$ and $\alpha_{32} = -1 \times 10^7 \text{ V/m}^3/\text{s}^3$) in the frequency bandwidth near the first natural frequency is shown in Fig. 50 (a). Similarly, Fig. 50 (b) plots the FRF of the system in the presence of the first part ($\alpha_{31} = 5 \times 10^4 \text{ V/m}^3/\text{s}^3$ and $\alpha_{32} = 0$) near the second natural frequency. The response of the controlled system near the first natural frequency is matched with the uncontrolled system and is not influenced by the controller around the second natural frequency. Similarly, the first part of the controller has no effect on the response near the second natural frequency. This displays the potential of the control procedure to target specific frequency bandwidth in a multi-degree of freedom system. Such a feature is especially beneficial in an arrayed structure with multiple frequencies relatively close to each other.
Fig. 50. Magnitude of the experimental tip-velocity/base-acceleration FRF of the active AHC for 1 V input with: (a) $\alpha_{31} = 0$ and $\alpha_{32} = -1 \times 10^7 \, \text{V/m}^3/\text{s}^3$, (b) $\alpha_{31} = 5 \times 10^4 \, \text{V/m}^3/\text{s}^3$ and $\alpha_{32} = 0$.

4.6. **Comparison with numerical simulation results**

In this section, the model of the two-channel active AHC is experimentally validated by adopting the linear damping gain tuning method discussed earlier in this chapter. Although numerical simulation results were shown in Section 3.4, some results are exhibited in this section again for ease of comparison. First, modal parameters of the modeled passive AHC are compared with the experimental values and results are listed in Table 8.

<table>
<thead>
<tr>
<th></th>
<th>$f_1$ (Hz)</th>
<th>$\zeta_1$ (%)</th>
<th>$f_2$ (Hz)</th>
<th>$\zeta_2$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Experiment</strong></td>
<td>442</td>
<td>3.00</td>
<td>2693</td>
<td>1.24</td>
</tr>
<tr>
<td><strong>Simulation</strong></td>
<td>443</td>
<td>*3.00</td>
<td>2777</td>
<td>*1.24</td>
</tr>
<tr>
<td><strong>Error (%)</strong></td>
<td>0.23</td>
<td>N/A</td>
<td>3.12</td>
<td>N/A</td>
</tr>
</tbody>
</table>

*Added as modal damping from experimental results.

The percent error values corresponding to the natural frequencies are less than 3.2%. As a result, the model parameters demonstrate a close match with the experimentally obtained parameters. Next, the control influence terms and linear damping gains computed numerically and
experimentally are compared against each other in Table 9. As mentioned earlier, due to the existence of inherent nonlinearities in the AHC, the value of the linear damping gain in experiments depends on the input level. Therefore, the mean value of the linear damping gains listed in Table 6 is used here for comparison purposes.

Table 9. Comparison of the absolute values of the linear damping gains and control influence terms of the AHC obtained numerically and experimentally.

<table>
<thead>
<tr>
<th></th>
<th>$(\alpha_{11})_a (V/m/s)$</th>
<th>$(\beta_v)_1 (m/s^2/V)$</th>
<th>$(\alpha_{12})_a (V/m/s)$</th>
<th>$(\beta_v)_2 (m/s^2/V)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment</td>
<td>50.60</td>
<td>2.91</td>
<td>-61.59</td>
<td>-6.62</td>
</tr>
<tr>
<td>Simulation</td>
<td>-52.51</td>
<td>-3.16</td>
<td>61.14</td>
<td>6.93</td>
</tr>
<tr>
<td>Error (%)</td>
<td>3.77</td>
<td>8.59</td>
<td>0.73</td>
<td>4.68</td>
</tr>
</tbody>
</table>

The percent error values listed in Table 9 show that the maximum error between the linear damping gains is less than 3.8%, and the maximum error between the experimental and numerical control influence term is 8.59%. To compare the behavior of the active AHC implemented experimentally with the simulated AHC’s behavior near the first natural frequency, FRFs of the AHCs for $|\alpha_{31}| = 5 \times 10^4 \ V/m^3/s^3$ are plotted in Fig. 51. Note that the absolute values of the cubic damping gains used in simulations and experiments are equal, but their signs are opposite as shown in Table 9. Similarly, the signs of the experimental and numerical linear damping gains and control influence terms are different. These sign differences correspond to wiring and electrical connections of the AHC in the experiments. Therefore, the absolute values of the parameters are used for comparisons.
Fig. 51. Magnitude of the tip-velocity/base-acceleration FRF of the active AHC near the first natural frequency due to changing the input level for $|\alpha_{31}| = 5 \times 10^4 \text{ V/m}^3 /\text{s}^3$ obtained, (a) experimentally, with linear damping gains $(\alpha_{11})_a$ listed in Table 6, (b) numerically, with $\alpha_{11} = -52.51 \text{ (V/m/s)}$.

In Fig. 51 (b), the simulated base accelerations are taken from the accelerations measured at the natural frequency for each experiment. As shown in Fig. 51, there is a very close correlation between the simulated and experimental FRFs. A numerical comparison between the FRFs is presented in Table 10.

Table 10. Comparison of the natural frequency and amplitude of the FRFs near the first natural frequency obtained numerically and experimentally.

<table>
<thead>
<tr>
<th>Base Acc. ($\text{m/s}^2$)</th>
<th>$f_1$ (Hz)</th>
<th>Amplitude (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Experiment</td>
<td>Simulation</td>
</tr>
<tr>
<td>Passive</td>
<td>442.00</td>
<td>443.00</td>
</tr>
<tr>
<td>0.85</td>
<td>442.00</td>
<td>446.91</td>
</tr>
<tr>
<td>1.7</td>
<td>442.00</td>
<td>445.63</td>
</tr>
<tr>
<td>2.55</td>
<td>442.00</td>
<td>445.63</td>
</tr>
<tr>
<td>3.25</td>
<td>441.00</td>
<td>445.62</td>
</tr>
<tr>
<td>4.1</td>
<td>441.00</td>
<td>444.40</td>
</tr>
<tr>
<td>4.95</td>
<td>441.00</td>
<td>444.39</td>
</tr>
</tbody>
</table>
According to Table 10, an error of less than 3.9% is observed in the natural frequency values and the amplitude of the simulated and experimental FRFs at the natural frequencies. This shows the model’s accuracy in estimating the response of the AHC. A similar analysis is conducted for the response of the AHCs near the second natural frequency. Fig. 52 demonstrates the results obtained numerically and experimentally near the second natural frequency.

**Fig. 52.** Magnitude of the tip-velocity/base-acceleration FRF of the active AHC near the second natural frequency due to changing the input level for $|\alpha_{32}| = 1 \times 10^7 \text{ V/m}^3/\text{s}^3$ obtained, (a) experimentally, with linear damping gains $(\alpha_{12})$ listed in Table 6, (b) numerically, with $\alpha_{12} = 61.14 \text{ (V/m/s)}$.

Fig. 52 shows that although the corresponding numerical and experimental FRFs have approximately the same amplitude, the natural frequency values are higher in the simulated FRFs. The FRFs obtained from simulation are distorted towards higher frequencies, while the ones obtained from experiments are distorted towards lower frequencies. These discrepancies can be due to the existence of inherent nonlinearities in the tested AHC that is not considered in modeling, the contribution of the clamping, shaker, and power amplifier. Also, the asymmetry and distortion of the simulated FRFs in Fig. 51 and Fig. 52 are due to the effects of the LP and BP filters on the response of the AHC. To further show the filter’s contribution on the simulated FRF of the
modeled AHC, two SDOF models are developed and tuned to the first and the second natural frequencies. As these models are SDOF systems, filters are not used in simulating the FRFs shown in Fig. 53. As a result, the effect of filters is evident when Fig. 51 (b) and Fig. 52 (b) are compared against Fig. 53 (a) and Fig. 53 (b), respectively. Therefore, the observed shift in peak values is a result of the filter application.

**Fig. 53.** Magnitude of the simulated tip-velocity/base-acceleration FRF due to changing the input level for: (a) the SDOF model of the AHC near the first natural frequency without LP filter for $|\alpha_{31}| = 5 \times 10^4$ V/m³/s³, (b) the SDOF model of the AHC near the second natural frequency without BP filter for $|\alpha_{32}| = 1 \times 10^7$ V/m³/s³.

The experimental natural frequency values and FRF amplitudes at the resonance are tabulated in Table 11 along with the corresponding numerical values. As shown in Table 11, discrepancies between the natural frequencies and the amplitude of the FRFs at the natural frequency are less than 3.4% and 1.8%, respectively. As a result, there is a good match between the simulation results and the experimental results.
Table 11. Comparison of the natural frequency and amplitude of the FRFs near the second natural frequency obtained numerically and experimentally.

<table>
<thead>
<tr>
<th>Base Acc. (m/s²)</th>
<th>f₂ (Hz)</th>
<th>Magnitude (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Experiment</td>
<td>Simulation</td>
</tr>
<tr>
<td>Passive</td>
<td>2693.00</td>
<td>2777.00</td>
</tr>
<tr>
<td>0.71</td>
<td>2684.03</td>
<td>2763.30</td>
</tr>
<tr>
<td>1.13</td>
<td>2684.04</td>
<td>2767.90</td>
</tr>
<tr>
<td>1.55</td>
<td>2685.05</td>
<td>2771.00</td>
</tr>
<tr>
<td>1.84</td>
<td>2684.99</td>
<td>2774.00</td>
</tr>
<tr>
<td>2.26</td>
<td>2688.01</td>
<td>2777.10</td>
</tr>
</tbody>
</table>

Furthermore, a comparison of the AHC’s sensitivity and compressive rate obtained numerically and experimentally is conducted. Therefore, the output versus input curves for three cubic damping gain values at the first natural frequency are plotted in Fig. 54.

**Fig. 54.** Comparison of the input to output curves of the AHC at the first natural frequency obtained experimentally and numerically.

Based on the results shown in Fig. 54, the sensitivity percent errors are calculated to be 3.23% for |α₃₁| = 1 × 10⁴ V/m³/s³, 5.26% for |α₃₁| = 5 × 10⁴ V/m³/s³, and 8.33% for |α₃₁| = 2 × 10⁵ V/m³/s³. A similar study is performed at the second natural frequency and the input-output curves are illustrated in Fig. 55. The sensitivity percent errors corresponding to |α₃₂| = 5 ×
$10^6 \ V/m^3 /s^3$ and $|\alpha_{32}| = 3 \times 10^7 \ V/m^3 /s^3$ are approximately zero and the one corresponding to $|\alpha_{32}| = 1 \times 10^7 \ V/m^3 /s^3$ is 5.56%.

**Fig. 55.** Comparison of the input to output curves of the AHC at the second natural frequency obtained experimentally and numerically.

The errors between the numerical and experimental data are attributed mainly to the inherent nonlinearities of the tested AHC, and dynamics of the fixture and parts of the experimental setup. However, as shown in this section, the numerical results match the experimental results closely and the simulated system can predict the behavior of the AHC accurately.

### 4.7. Conclusion

In this chapter, the control law discussed in the previous chapters was implemented on an AHC represented by a piezoelectric bimorph cantilever beam. First, the dynamics of the passive system was characterized. The obtained FRFs near the first two natural frequencies showed the existence of inherent nonlinearities. Next, to create a two-channel active AHC the controller’s linear damping gains had to be determined such that the system could be tuned near a Hopf bifurcation. To accomplish this, an approach was used similar to the one introduced in the previous chapter to estimate the linear damping gains of the AHC system. In that method, the linear damping gain of
the AHC corresponding to each mode was varied and modal parameters of the system were estimated using SDOF fit to the system near the natural frequency. Then the linear damping gain was calculated as the gain that eliminated the damping of the system. This method was adopted for each input level and near each natural frequency. Next, the cubic damping was introduced to the system near each of the natural frequencies. The stepped sine analysis showed the amplification/compression behavior of the active AHC for various cubic damping gains. Furthermore, the compressive rate and sensitivity of the active AHC near each natural frequency were shown by plotting the velocity of the tip versus the base acceleration for various cubic damping gains. The obtained compressive rates were close to the desired one-third rate. To evaluate the effects of the controller on the response near other frequencies than the natural frequencies, the FRF of the active AHC was compared to the passive AHC in a broader bandwidth. It was concluded that the active AHC only shows nonlinear amplification and compression near the natural frequencies. Therefore, the two-channel active AHC could mimic the nonlinear function of the cochlea near the natural frequencies. Finally, a series of comparisons were conducted between the numerical results obtained in the previous chapter and the experimental results obtained in this chapter. It was shown that the simulation results are in good agreement with the experimental data.
Chapter 5

Theoretical and numerical realization of self-sensing active artificial hair cells

5.1. Introduction
The multi-channel AHC with an optical readout presented in the previous chapters showed promising results in mimicking the cochlear amplifier. However, implanting an additional sensor such as a laser vibrometer in the cochlea or embedding it in a final AHC product for sensor applications is not feasible. Furthermore, creating an array of AHCs to mimic the cochlear amplifier in a broad bandwidth, requires multiple external sensors, i.e. the same number as AHCs. Therefore, the research presented in this chapter is focused on developing self-sensing active AHCs as a fundamental step towards creating self-contained AHCs.

Various strategies have been examined by researchers to develop beam-based self-sensing actuators for different applications. These methods include embedding strain gauges at the base of the beams [193], adopting a capacitive method [145, 194], using piezoelectric beams for both sensing and actuation along with a method to compensate for direct feedthrough terms contributed to the response due to the asymmetric actuation [195], and electrode patterning of the piezoelectric beams [172]. In this paper, a 4-layer piezoelectric beam is used as a self-sensing active AHC to provide symmetric piezoelectric actuation and sensing. To the best of the author’s knowledge, this is the first time a quadmorph self-sensing active AHC is developed capable of mimicking the cochlear amplifier in a self-sensing scheme. This novel self-sensing AHC could be a candidate to
embed in a new generation of cochlear implants. Furthermore, these AHCs can be used as active sensors in microphones, hydrophones, and fluid flow sensors due to their broader input range and frequency selectivity. The study presented in this chapter builds a theoretical framework for self-sensing active AHCs and examines it analytically and numerically. The current chapter is divided into seven sections. Following the brief introduction provided in this section, Section 5.2 presents the quadmorph piezoelectric beam configuration adopted in the current work and shows its advantages over a split-quadmorph through preliminary experiments. Section 5.3 then discusses the modeling of the self-sensing quadmorph AHC. First, a distributed parameter model of the system is illustrated. Then, a SDOF approximation of the active AHC with a cubic damping controller is derived. Subsequently, the steady-state response of the system is investigated in Section 5.4. Section 5.5 presents numerical simulation results of the system and successful replication of the cochlear amplifier through the self-sensing active AHC. Finally, Section 5.6 concludes the current chapter.

5.2. Quadmorph self-sensing artificial hair cell

This section presents the self-sensing artificial hair cell structure used in this study. In a previous attempt to create self-sensing active AHCs, Joyce et al. theoretically examined the potential use of a split-bimorph piezoelectric cantilever as a self-sensing system, where one layer served as a sensor and the other layer was the actuator to apply the control signal, as presented in [164]. According to their study, in a split-bimorph shown schematically in Fig. 56 (a), the actuator layer located on one side of the beam generates a longitudinal (in-plane) force and therefore, contributes to a longitudinal motion in addition to the bending displacement. As a result, a direct feedthrough term appears in the sensed voltage in the form of $D_{sb}V_a$, where $D_{sb}$ is a constant and $V_a$ is the piezoelectric actuator voltage. To mimic the cochlear amplifier, this direct feedthrough term is
required to be compensated by the controller, as shown in the detailed derivations in [164]. However, in a non-self-sensing bimorph displayed in Fig. 56 (b) where both piezoelectric layers act as actuators, the in-plane forces generated by each layer cancel each other due to the symmetric actuation about the neutral axis and the beam undergoes a pure transverse (out-of-plane) motion. Therefore, the control law developed initially for a non-self-sensing bimorph AHC with an optical readout is inapplicable to the split-bimorph case due to the presence of the feedthrough term and the system will not be able to replicate the behavior of the cochlear amplifier.

\[ V_s(t) = A_{sb} \frac{\partial u(x, t)}{\partial x} \big|_0 + D_{sb} V_a(t) \]

\( A_{sb}, D_{sb} \): constants, \( L \): length, \( u(x, t) \): displacement, \( V_a \): actuator voltage, \( V_s \): sensor voltage, \( z \): base acceleration

**Fig. 56.** Schematic of: (a) a split-bimorph, (b) a bimorph.

The inefficiency and impracticality of split-bimorphs for vibration control applications have been investigated in various studies and some feasible sensor-actuator configurations have been proposed and studied including the adoption of 4-layer piezoelectric beams [196-198]. To address the asymmetric actuation issue and create an AHC independent of external measurement, a 4-layer piezoelectric beam is used as a self-sensing, self-actuating artificial hair cell in the present study. This system provides symmetric actuation and sensing about the neutral axis and the sensed
voltage does not contain any feedthrough terms, as shown later by Equation (115) in Section 5.3.1. The two inner piezoelectric layers of the quadmorph are used as the actuator to apply the control voltage on the AHC, while the two outer piezoelectric layers operate as the sensor to convert the mechanical motion of the beam to an electrical signal similar to mammalian hair cells. A schematic of the 4-layer AHC with a brass shim and PSI-5A4E piezoceramics [178] is shown in Fig. 57. Further details are tabulated in Table 12.

The 0.029 m long quadmorph is clamped at one end and free at the other end and is extended along the x (1) axis, as displayed in Fig. 57. The base acceleration $\ddot{z}(t)$ excites the system in z (3) direction, and $u(x, t)$ denotes the composite’s displacement response in the same direction. The generated voltage (sensed piezoelectric voltage) in the outer piezoelectric layers in response to the

---

**Table 12. Self-sensing AHC properties.**

<table>
<thead>
<tr>
<th>Properties</th>
<th>Beam</th>
<th>Piezoelectric Actuator</th>
<th>Piezoelectric Sensor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young Modulus (GPa)</td>
<td>$Y_b$</td>
<td>$Y_a$</td>
<td>$Y_s$</td>
</tr>
<tr>
<td>Density (Kg/m$^3$)</td>
<td>$\rho_b$</td>
<td>$\rho_a$</td>
<td>$\rho_s$</td>
</tr>
<tr>
<td>Length (m)</td>
<td>$L_b$</td>
<td>$L_a$</td>
<td>$L_s$</td>
</tr>
<tr>
<td>Width (m)</td>
<td>$w_b$</td>
<td>$w_{pa}$</td>
<td>$w_s$</td>
</tr>
<tr>
<td>Thickness (mm)</td>
<td>$2t_b$</td>
<td>$t_a$</td>
<td>$t_s$</td>
</tr>
<tr>
<td>Piezoelectric starting location (m)</td>
<td>$a_a$</td>
<td>0</td>
<td>$a_s$</td>
</tr>
<tr>
<td>Piezoelectric ending location (m)</td>
<td>$b_a$</td>
<td>0.029</td>
<td>$b_s$</td>
</tr>
<tr>
<td>Piezoelectric strain constant (nC/N)</td>
<td>$d_{31a}$</td>
<td>$-0.19$</td>
<td>$d_{31s}$</td>
</tr>
<tr>
<td>Capacitance (parallel operation) (nF)</td>
<td>$C_a$</td>
<td>38.5</td>
<td>$C_s$</td>
</tr>
</tbody>
</table>
base excitation is shown by $V_s(t)$, where the $s$ subscript stands for the sensor. Similarly, $V_a(t)$ is the actuator voltage or the control voltage supplied to the inner piezoelectric layers, and $I_a$ is the current in the actuator layers. Each pair of the piezoelectric layers are poled in the same direction for parallel operation with $p_s$ and $p_a$ indicating the polarization direction for the sensor and actuator layers, respectively. The method used for the wiring of the electrodes is demonstrated in Fig. 58. As shown in the side view of the composite in Fig. 58, the electrodes located between the sensor and actuator layers are used as common electrodes between the piezoelectric sensor and actuator.

![Schematic of the electrode wiring.](image)

Fig. 58. Schematic of the electrode wiring.

To examine the symmetric actuation in the aforementioned quadmorph, a brief experimental study is conducted by exciting the beam with a chirp signal in a frequency range of 200 Hz to 40 kHz supplied to the inner piezoelectric layers. The sensed piezoelectric voltage of the outer piezoelectric layers and velocity response of the beam at various locations are measured and frequency response functions of the quadmorph are plotted. The experimental setup is shown in Fig. 59.
Piezoelectric-sensor-voltage/piezo-actuator-voltage FRF of the quadmorph and average velocity/piezo-actuator-voltage FRF are shown in Fig. 60 (a) and (b), respectively. Furthermore, operational deflection shapes (ODS) of the quadmorph are obtained by scanning the quadmorph using a Polytec PSV-400 scanning laser doppler vibrometer and measuring the velocity at various locations along the beam. The ODSs at natural frequencies of the quadmorph are displayed in Fig. 60 (b).
**Fig. 60.** (a) Magnitude of the piezo-sensor-voltage/piezo-actuator-voltage FRF of the quadmorph self-sensing AHC, (b) Magnitude of the average velocity/piezo-actuator-voltage FRF of the quadmorph and operational deflection shapes at the natural frequencies.

The ODSs in Fig. 60 (b) show that the six peaks observed in both FRFs of Fig. 60 (a) and (b) correspond to the first six bending modes of the quadmorph. Therefore, the symmetric actuation of the piezoelectric layers about the neutral axis results in a pure transverse or bending motion, as mentioned earlier in this section.
In the next experiment, the wiring of the beam is altered to form a split-quadmorph. In the split-quadmorph configuration, instead of using both outer piezoelectric layers for sensing and inner layers for actuation, one of the inner layers is used as the actuator and the outer layer on the opposite side serves as the sensor. While only one piezoelectric layer on each side is adopted as the sensor or actuator in the split-quadmorph, it functions similar to a split-bimorph discussed previously. The schematic of the setup and the wiring configuration is shown in Fig. 59 (d) and piezo-sensor-voltage/piezo-actuator-voltage FRFs are illustrated in Fig. 61.

![Fig. 61. Magnitude of the piezo-sensor-voltage/piezo-actuator-voltage FRFs of the split-quadmorph AHC.](image)

A comparison between the FRFs of Fig. 61 and Fig. 60 (a) shows that there are more peaks in the FRFs of the split-quadmorph. This observation is expected according to the theoretical studies on the existence of an in-plane motion in the beam when excited asymmetrically. These additional peaks can be attributed to other modes of the beam including in-plane modes. However, further investigation is required to determine the exact type of modes and it is beyond the scope of the current study. Moreover, the FRFs obtained from opposite sensor-actuator configurations are in
very good agreement. This can also show that piezoelectric actuator layers on either side of the beam excite the structure in a similar manner. Based on the preliminary experimental results and theoretical derivations in previous studies, the quadmorph configuration is chosen as the self-sensing AHC due to its pure transverse motion when excited. The next section presents model development of the quadmorph self-sensing active AHC.

5.3. Self-sensing artificial hair cell model

To study the behavior of the self-sensing active AHC proposed in this paper, an analytical framework is created on a model of a piezoelectric beam. This section explains the self-sensing AHC’s distributed parameter model and its single degree of freedom approximation.

5.3.1. Distributed parameter model of the system

This section uses the Euler-Bernoulli beam assumption to write the governing equations of motion of the piezoelectric quadmorph cantilever as [164, 172, 177, 179],

$$\rho A \frac{\partial^2 u(x,t)}{\partial t^2} = -\frac{\partial^2}{\partial x^2} C_{11}^E I \frac{\partial^2 u(x,t)}{\partial x^2} + C \frac{\partial u(x,t)}{\partial t} + \frac{\partial^2}{\partial x^2} \left( \vartheta_a \chi_{[a_a,b_a]} V_a(t) \right) - \rho A \ddot{z}(t), \quad (111)$$

with the boundary conditions given by,

$$\rho(0,t) = 0, \quad \frac{\partial u}{\partial x}(0,t) = 0, \quad C_{11}^E I \frac{\partial^2 u}{\partial x^2}(L_b, t) = 0, \quad \text{and} \quad C_{11}^E I \frac{\partial^3 u}{\partial x^3}(L_b, t) = 0. \quad (112)$$

In Equation (111), the electromechanical coupling factor for the actuator, \( \vartheta_a \) and the characteristic function are defined as,

$$\vartheta_a = \frac{\kappa_a Y_a d_{31a}}{t_a} \quad \text{and} \quad \chi_{[a_a,b_a]} = \begin{cases} 1 & (x,z) \in \{(a_a, b_a) \times [-t_b - t_a, -t_b] \cup [a_a, b_a] \times [t_b, t_b + t_a] \} \\ 0 & \text{otherwise} \end{cases} \quad (113)$$

In these equations, \( \rho \) is the density of the quadmorph, \( A \) is the cross-sectional area, \( C_{11}^E \) is the Young’s modulus of the composite, \( I \) is the second moment of inertia, \( C \) is the distributed damping
operator, \( \kappa_a \) is the cross-sectional first moment of inertia of the actuator layers, \( d_{31,a} \) is the piezoelectric strain constant of the actuator, \( Y_a \) is the Young’s modulus of the actuator, \( a_a \) and \( b_a \) are the piezoelectric actuator starting and ending locations, \( t_b \) is the substrate thickness, and \( t_a \) is the actuator layer thickness. Parameters used in the equations of this section and their values are listed in Table 12.

The electrical domain equation in terms of the current passing through the actuator layers can be written as,

\[
i_a = -\vartheta_a \left( \frac{\partial^2 u(x,t)}{\partial x \partial t} \bigg|_{a_a} \right) - C_a \frac{dV_a(t)}{dt},
\]

where, \( C_a \) is the capacitance of the actuator layers in parallel operation. The voltage across the sensor layers is given by,

\[
V_s = -\vartheta_s \frac{\partial u(x,t)}{\partial x} \bigg|_{a_s} - \vartheta_s \left( \frac{\partial u(x,t)}{\partial x} \bigg|_{b_s} - \frac{\partial u(x,t)}{\partial x} \bigg|_{a_s} \right),
\]

where the electromechanical coupling factor for the sensor layers is \( \vartheta_s = \frac{\kappa_s Y_s d_{31,s}}{t_s} \). The definitions of parameters for the sensor layers, with “s” subscript, are similar to those for the actuator layer.

In the next section, the AHC’s dynamic behavior is approximated using a single degree of freedom model based on a finite element model of the system.

### 5.3.2. Single degree-of-freedom approximation of the active AHC

In this section, a Galerkin finite element (FE) method is used to make a weak form of the equations of motion of the AHC. Let \( \mathbf{x} = [u_1 \theta_1 u_2 \theta_2 \ldots u_n \theta_n]^\top \) be the vector of nodal point translational and rotational displacements, and \( n \) be the number of elements with the length of \( h = L_b/n \), the equations of motion is approximated by,

\[
M \ddot{\mathbf{x}} + C \dot{\mathbf{x}} + K \mathbf{x} = B_a V_a(t) - P \ddot{z}(t),
\]

and the voltage across the sensor piezoelectric layers is denoted by,
\[ V_s(t) = \frac{1}{c_s} B_s^T \mathbf{x}. \]  

(117)

In Equation (116) and Equation (117), \( M, C, K, B_a, P_z \), and \( B_s \) are the global mass, damping, stiffness, control influence, base acceleration influence, and sensor influence matrices of the system constructed from the following elemental matrices,

\[
\begin{bmatrix}
156 & 22h & 54 & -13h \\
22h & 4h^2 & 13h & -3h^2 \\
54 & 13h & 156 & -22h \\
-13h & -3h^2 & -22h & 4h^2 \\
\end{bmatrix}, \\
\begin{bmatrix}
12 & 6h & -12 & 6h \\
6h & 4h^2 & -6h & 2h^2 \\
-12 & -6h & 12 & -6h \\
6h & 2h^2 & -6h & 4h^2 \\
\end{bmatrix}, \\
\begin{bmatrix}
0 \\
-1 \\
0 \\
1 \\
\end{bmatrix}, \\
\begin{bmatrix}
1/2 \\
1/12 \\
1/2 \\
-1/12 \\
\end{bmatrix}, \begin{bmatrix}
0 \\
1 \\
0 \\
-1 \\
\end{bmatrix}.
\]

(118)

Values of the parameters used in the model are tabulated in Table 12. For a quadmorph cantilever, \( B_s \) is a vector assembled from elemental vectors \( b_{sel} \) in Equation (118) and the only nonzero element in \( B_s \) is the last element (index \( 2n \)). This simplifies Equation (117) to,

\[ V_s(t) = -\frac{\theta_s}{c_s} \theta_t = \frac{1}{c_s} \frac{-\kappa_s d_{31s} Y_s}{t_s} \theta_t, \]

(119)

where, \( \theta_t = \theta_n \) is the last element in the nodal point displacements vector \( \mathbf{x} \) that corresponds to the rotational displacement of the quadmorph’s tip. The damping coefficient matrix in Equation (116) is based on the experimental modal damping ratios.

The next step in creating a SDOF model of the AHC is to convert the coupled equations of motion (116) stated in the physical domain into decoupled equations in the modal domain as presented earlier in Section 2.4. The decoupled equation corresponding to the first natural frequency is given by,

\[ \ddot{r}_1 + 2 \zeta_1 \omega_1 \dot{r}_1 + \omega_1^2 r_1 = \beta_{1v} V_a(t) - \beta_{1x} \dot{z}(t), \]

(120)

and the sensed voltage, \( V_s \) presented in Equation (119), is approximated by \( V_{s_s} \) as,
\[ V_{s_1} = -\frac{\theta_s}{C_s} s_{\theta_1} r_1 = p_s r_1, \quad (121) \]

where, \( s_{\theta_1} \) is the modal contribution of the first mode on the tip rotational displacement obtained from the transformation \( \mathbf{x} = (L^T)^{-1} \mathbf{P} = \mathbf{S} \mathbf{r} \). Equation (120) is re-written in terms of the sensed voltage approximation as,

\[ \ddot{V}_{s_1}(t) + 2 \zeta_1 \omega_1 \dot{V}_{s_1}(t) + \omega_1^2 V_{s_1}(t) = \beta_{1v} p_s V_a - \beta_{1z} p_s \ddot{z}(t). \quad (122) \]

Equation (122) represents the SDOF approximation of the equation of motion for the AHC sensed voltage. Using the relationship between the approximated tip displacement, \( u_{t_s} \) and the approximated sensed voltage as,

\[ u_{t_s}(t) = s_{xt_1} r_1 = \frac{s_{xt_1}}{p_s} V_{s_1}, \quad (123) \]

the SDOF approximation of the equation of motion for the AHC tip displacement is formulated as,

\[ \ddot{u}_{t_s}(t) + 2 \zeta_1 \omega_1 \dot{u}_{t_s}(t) + \omega_1^2 u_{t_s}(t) = \beta_{1v} s_{xt_1} V_a - \beta_{1z} s_{xt_1} \ddot{z}(t), \quad (124) \]

where, \( s_{xt_1} \) is the modal participation factor of the first mode on the tip displacement.

The nonlinear feedback control law presented in the previous chapters uses AHC’s tip velocity feedback to mimic the cochlear nonlinear processes. To adapt the control law for the self-sensing system, the relationship between the sensed voltage and the tip displacement in Equation (123) is used. Consequently, the derivative of the sensed voltage is adopted as feedback to the system.

Therefore, the control law for the self-sensing system takes the form,

\[ V_a(t) = \alpha_1 \dot{V}_{s_1} - \alpha_3 \dot{V}_{s_1}^3, \quad (125) \]

where, \( \alpha_1 \) and \( \alpha_3 \) are the linear damping and cubic damping gains of the active self-sensing AHC.

The linear damping gain required to tune the system to the Hopf bifurcation is obtained as,
\[ \alpha_1 = \frac{2\zeta_1 \omega_1}{\beta_{1v} p_s}. \] (126)

Substituting Equation (125) into Equation (122) yields,
\[ \ddot{V}_{s_1}(t) + (2\xi_1 \omega_1 - \beta_{1v} p_s \alpha_1) \dot{V}_{s_1}(t) + \beta_{1v} p_s \alpha_3 \dot{V}_{s_1}^3 + \omega_1^2 V_{s_1}(t) = -\beta_{1x} p_s \ddot{z}(t). \] (127)

The next section studies the steady-state response of the self-sensing AHC undergoing a Hopf bifurcation.

5.4. Steady-state response of the active self-sensing AHC

In this section, the harmonic balance method [183, 184] is used to study the steady-state response of the self-sensing AHC to a harmonic base acceleration input. The equation of the motion for the sensed voltage of a SDOF AHC with base excitation is given by Equation (127) and rewritten as,
\[ \ddot{V}_{s_1}(t) + 2\xi \omega_1 \dot{V}_{s_1}(t) + \beta_{1v} p_s \alpha_3 \dot{V}_{s_1}^3 + \omega_1^2 V_{s_1}(t) = -\beta_{1x} p_s \ddot{z}(t), \] (128)

where, the bifurcation parameter is defined as \(2\xi \omega_1 = 2\xi_1 \omega_1 - \beta_{1v} p_s \alpha_1\). Based on the harmonic balance method, exciting the base of the AHC with a harmonic acceleration with driving frequency \(\Omega\) as, \(\ddot{z}(t) = Z \sin(\Omega t)\), results in a periodic response in the form of,
\[ V_{s_1}(t) = V \sin(\Omega t + \phi), \] (129)

where, \(V\) is the amplitude of the sensed voltage and \(\phi\) is the phase angle. Adopting the harmonic balance method on Equation (128) results in,
\[ \frac{9}{16} \Omega^6 (\beta_{1v} p_s \alpha_3)^2 (V^2)^3 + \frac{3\Omega^4}{2} 2\xi \omega_1 (\beta_{1v} p_s \alpha_3)(V^2)^2 + ((\omega_1^2 - \Omega^2)^2 + (2\xi \omega_1 \Omega)^2)(V^2) = (\beta_{1x} p_s Z)^2. \] (130)

At the Hopf bifurcation, \(\xi = 0\) and therefore, \(\alpha_1 = \frac{2\zeta_1 \omega_1}{\beta_{1v} p_s}\). At the resonance, \(\Omega = \omega_1\), and Equation (130) simplifies to,
\[ V = \left( \frac{4\beta_{1x}}{3 \omega_1^2 \beta_{1v} \alpha_3} Z \right)^{\frac{1}{3}}. \] (131)
Equation (131) shows that the system undergoing Hopf bifurcation scales the sensed voltage by one-third power of the input acceleration at resonance and mimics the compressive nonlinearity characteristic of the cochlea [6]. For an active AHC with properties listed in Table 12, the response spectrum and sensed voltage/base acceleration FRF can be built point-by-point by solving Equation (130) with $\xi = 0$ for different base acceleration amplitudes and driving frequencies in a bandwidth in the vicinity of the first natural frequency. The uncontrolled AHC’s magnitude FRF is obtained by substituting $\xi = \zeta_1$ and $\alpha_3 = 0$ in Equation (130) as,

$$\frac{V}{Z} = \frac{\beta_1 z_p s}{\sqrt{\left(\omega_1^2 - \Omega^2\right)^2 + \left(2\zeta_1 \omega_1 \Omega\right)^2}}.$$  

(132)

The next section studies the response of the active system, numerically.

### 5.5. Numerical realization of a self-sensing AHC

To evaluate the response of the modeled active self-sensing AHC numerically, the controlled system represented by Equation (127) is simulated using Simulink. The implemented system is shown in Fig. 62. Note that the filter block highlighted in Fig. 62 will be added to the system and discussed in Section 5.5.2 and it is not present in the closed-loop system for the numerical simulations of Section 5.5.1.

![Fig. 62. Active AHC implemented in Simulink.](image-url)
The state-space form of the equation of motion is used in the Simulink model. Base acceleration and control voltage are the inputs to the system and sensed voltage is the output. Voltage derivative, the second state of the system, is used in the control law, as mentioned previously. The linear damping gain of the controller is obtained as $2.45 \times 10^{-4}$ s using Equation (126) and three different cubic damping gains are examined. For each case, the response of the AHC is simulated for various excitation levels to examine the amplification/compression behavior of the system in a frequency range of $410 - 440$ Hz, near the first natural frequencies. The next section presents the simulation results of the active AHC without any filters.

5.5.1. **Numerical simulation of the active AHC**

This section discusses the artificial hair cell’s ability to mimic the cochlear amplifier. The 3D visualization of the response to various base acceleration levels is as shown in Fig. 63 (a), (b), and (c) for three cubic damping gains, $\alpha_3 = 6 \times 10^{-11}$ s$^3$/V$^2$, $\alpha_3 = 1 \times 10^{-10}$ s$^3$/V$^2$, and $\alpha_3 = 1.8 \times 10^{-10}$ s$^3$/V$^2$. Solid lines show the controlled system’s response, while dashed lines correspond to the uncontrolled system. The peak of each curve is projected onto a plane perpendicular to the frequency and sensed piezo voltage axes and color-coordinated with the response curve. The least-square power-law fit to the controlled system’s projected data is also displayed on the input-output plane. Furthermore, the least-square linear fit to the uncontrolled data is shown on the same plane. These curves represent input-output plots of the AHC for different cubic damping gains. Input-output curves for the aforementioned cubic damping gains are illustrated in Fig. 63 (d) for ease of comparison.
Fig. 63. Magnitude of the sensed piezo-voltage of the active self-sensing AHC due to changing the input level and corresponding input-output plots for: (a) $\alpha_3 = 6 \times 10^{-11} \, s^3/V^2$, (b) $\alpha_3 = 1 \times 10^{-10} \, s^3/V^2$, (c) $\alpha_3 = 1.8 \times 10^{-10} \, s^3/V^2$. (d) Input-output plots for the three cubic damping cases.

Amplification or compression behavior of the AHC for each cubic damping gain and input level can be determined by comparing the controlled and uncontrolled response of the AHC near the natural frequency. For instance, the AHC amplifies the response for the lowest base acceleration applied to the system for all the three cases displayed in Fig. 63 (a) to (c). Moreover, amplification and compression zones are marked on the input-output plot in Fig. 63 (d). Comparing the trend of the sensed piezoelectric voltage of the controlled AHC to the uncontrolled system for each cubic damping case of Fig. 63, a compression in the response of the active AHC is observed as the base
acceleration increases. The rate of this compressive nonlinearity at the natural frequency is one-third power of the input, as shown on the input-output plane. The obtained rate matches the mammalian cochlear amplifier compressive rate. Furthermore, as the cubic damping gain increases from $6 \times 10^{-11} \, s^3/V^2$ in Fig. 63 (a) to $1.8 \times 10^{-10} \, s^3/V^2$ in Fig. 63 (c), the response experiences more compression. This is also evident from the input-output plot in Fig. 63 (d). The FRFs of these cases are shown in Fig. 64. According to Fig. 64, the amplification/compression function of the AHC is observed near the natural frequency and the nature of this function depends on both the cubic damping gain and the input level at which the AHC is excited. At frequencies farther from the natural frequency, the controlled system FRF follows the uncontrolled AHC. The next section discusses the addition of a filter to the AHC and its necessity for the practical implementation of the active AHC.

**Fig. 64.** Magnitude of the Piezo-voltage/base-acceleration FRF of the active self-sensing AHC for various input base accelerations and cubic damping gains.

### 5.5.2. Addition of a filter to the active AHC system

Numerical simulation results showed the desired behavior of the self-sensing active AHC in the previous section. As mentioned previously, a single AHC operates near a particular frequency, i.e.
its first natural frequency. In practice, to decompose a signal into its frequency content and apply the compressive nonlinearity of the AHC to the response, a series of active AHCs with various lengths is required. Therefore, to limit each AHC’s processes to the frequencies near its first natural frequency, a filter is needed in the control loop. Also, the application of filters is required in a multi-channel AHC working near multiple natural frequencies which was discussed in the previous chapters. Therefore, in this section, a 6th order low-pass Butterworth filter with a cut-off frequency of 650 Hz is added to the system and response of the system is studied. Fig. 62 shows the controlled AHC implemented in Simulink. The next sections present a linear damping gain tuning method for the AHC with a filter and simulation results, respectively.

5.5.2.1. Linear damping gain tuning

To tune the AHC near a Hopf bifurcation, first, the linear damping of the AHC needs to be removed. As mentioned in Section 5.3.2, this is performed by setting the linear damping gain of the controller to a theoretical value. However, using a constant linear damping gain estimated by this method is only effective on a linear system without any filters, as shown in Section 3.3.2. Therefore, to partially compensate for the filter’s effect on the system and being able to tune the AHC near a Hopf bifurcation, an approach similar to the gain tuning method developed in the previous chapters is adopted in this study, with the difference that vector-fitting [199-201] is used as the system identification method instead of circle-fitting. It is important to note that vector-fitting is an accurate and very robust algorithm for curve-fitting [200]. However, no significant difference was observed in the obtained results in this study with both vector-fitting and circle-fitting algorithms. To use this method, a set of simulations are conducted by setting the controller’s cubic damping gain to zero and varying the linear damping gain, $\alpha'_1$. For each set of simulated sensed-voltage/base-acceleration FRF, the system with a filter is approximated as an equivalent
system by fitting a SDOF curve to the FRF data using vector-fitting. In the next step, natural frequency \((\omega_{1eq})\) and damping ratio \((\zeta_{1eq})\) of the estimated or equivalent system are calculated. Next, \(2\zeta_{1eq}\omega_{1eq}\) is plotted as a function of \(\alpha_1'\) and a linear least-square fit to the data points is obtained. Finally, the x-intercept of the line is identified as the linear damping gain, \(\alpha_1\) that tunes the system near a Hopf bifurcation. The reduced damping FRFs, as well as the linear fit to the estimated \(2\zeta_{1eq}\omega_{1eq}\) for the AHC with a 6th order Butterworth filter, are displayed in Fig. 65 (a) and (b), respectively. Note that the fitted line’s equation has a form of \(2\zeta_{1eq}\omega_{1eq} = -\beta_{1veq}p_s \alpha_1' + 2\zeta_1\omega_1\), where \(\beta_{1veq}p_s\) is the control influence term in Equation (127) and \(2\zeta_1\omega_1\) corresponds to the uncontrolled system. This equation can be obtained by setting the cubic damping gain to zero in Equation (127). Therefore, the slope of \(2\zeta_{1eq}\omega_{1eq}\) versus \(\alpha_1'\) represents \(-\beta_{1veq}p_s\).

**Fig. 65.** (a) Magnitude of the sensed piezo voltage/base acceleration FRF of the AHC for different values of \(\alpha_1'\) for 1 V input voltage to the shaker amplifier, (b) \(2\zeta_{1eq}\omega_{1eq}\) versus \(\alpha_1'\) plot used for finding the linear damping gain \(\alpha_1\) that eliminates the linear damping near the first natural frequency.
The linear damping gain obtained for the system with filter is $-2.9 \times 10^{-4}$ s. As shown in Fig. 65 (b), the approximated control influence term for the reduced damping AHC with filter is negative. Subsequently, the estimated linear damping gain is negative. Therefore, the term $\beta_{1v} p_s \alpha_1$ in Eq. (18), $\ddot{V}_{ss}(t) + (2\zeta_1 \omega_1 - \beta_{1v} p_s \alpha_1) \dot{V}_{ss}(t) + \beta_{1v} p_s \alpha_3 \dot{V}_{ss}^3 + \omega_1^2 V_{ss}(t) = -\beta_{1z} p_s \ddot{z}(t)$, is always positive to reduce the linear damping of the system.

5.5.2.2. Simulation results

This section studies the behavior of the AHC with the filter. The linear damping gain obtained from the previous section is used in the control loop of Fig. 62 in Simulink and FRFs of the system excited with various stimulus levels for $\alpha_3 = -6 \times 10^{-11} \text{s}^3/V^2$, $\alpha_3 = -1 \times 10^{-10} \text{s}^3/V^2$, and $\alpha_3 = -1.8 \times 10^{-10} \text{s}^3/V^2$ are shown in Fig. 66 (a) by dotted lines, solid lines, and dashed lines, respectively. The input-output curves corresponding to these gains are displayed in Fig. 66 (b).

![Fig. 66. Magnitude of the Piezo-voltage/base-acceleration FRF of the active self-sensing AHC with the LP filter for various input base accelerations and cubic damping gains.](image)
Fig. 66 (a) and (b) illustrate the amplification/compression behavior of the AHC for various input levels and cubic damping gains. The desired one-third power-law relationship between the input and output holds for the AHC with the filter and the system can replicate the cochlear amplification. Note that the negative sign in cubic damping gains corresponds to the negative control influence term and cubic damping coefficient, $\beta_{1v} p_3 \alpha_3$ is always positive to add cubic damping to the system. For each cubic damping gain of Fig. 66 (a), the active system shows softening behavior as the input strengthens. This behavior can be explained by the phase shift the filter adds to the system. Therefore, phase plots of the FRFs of the AHC for $|\alpha_3| = 1 \times 10^{-10} \text{s}^3/\text{V}^2$ with and without the filter are shown in Fig. 67.

![Phase of FRFs](image)

**Fig. 67.** Phase of the Piezo-voltage/base-acceleration FRF of the active self-sensing AHC with $|\alpha_3| = 1 \times 10^{-10} \text{s}^3/\text{V}^2$ without and with the LP filter for various input levels.

For the system without filter, the phase at the natural frequency of the uncontrolled system, 424.6 Hz, remains the same for all the input levels given to the system and no shift in the natural
frequency of the active AHC is observed in Fig. 64. However, the filtered system’s phase varies at 424.6 Hz for different stimulus levels, as displayed in Fig. 67 (b) and its close-up near the uncontrolled system’s natural frequency in Fig. 67 (c). The phase of the active AHC for 0.33 m/s² and 0.74 m/s² base acceleration is higher than the phase of the uncontrolled system. As a result, the system hardens with respect to the uncontrolled system for these base accelerations. In a similar manner, the decreased phase for the four other input levels causes a softening behavior. Overall, the AHC with filter shows softening behavior as the input level increases. Furthermore, to show the effect of the filter on the system’s time response, AHC’s sensed piezoelectric voltage for 1.64 m/s² excitation level and $|\alpha_3| = 1 \times 10^{-10} \text{s}^3/\text{V}^2$ at 424.6 Hz, the natural frequency of the uncontrolled and unfiltered active AHC, is presented in Fig. 68.

![Fig. 68. Sensed piezoelectric voltage of the AHC with and without the filter at 424.6 Hz for 1.64 m/s² base acceleration level and $|\alpha_3| = 1 \times 10^{-10} \text{s}^3/\text{V}^2$.](image)

As shown in Fig. 68, the time-delay between the sensed piezoelectric voltage for the active system with and without the filter is about $5 \times 10^{-5}$ s. As the time-delay and consequently, the phase shift are small, the system can still be tuned near the Hopf bifurcation, as shown in Fig. 66 (b). The time-delay in the system with the filter causes a shift in the natural frequency with respect to the system without the filter. This shift for the cases shown in Fig. 66 (a) lays between $-2 \text{Hz}$ to
0.6 Hz, i.e. less than 0.48% shift. Therefore, the quadmorph self-sensing active AHC with filter is a good candidate for mimicking the cochlear amplifier.

5.6. Conclusion

The work presented in this chapter enhances the previous piezo-based AHC designs to take into account self-sensing for future cochlear implants or sensor design applications. This chapter focused on developing a novel self-sensing active AHC scheme that implemented the nonlinear amplification/compression function of the cochlear amplifier. The AHC was modeled as a quadmorph piezoelectric beam for the first time in the literature to ensure symmetric excitation of the AHC in a self-sensing system. The controller introduced in the previous chapters was modified to use the sensed voltage of the AHC instead of the tip-velocity. The system was studied analytically and numerically and its FRFs and input-output curves in response to various stimuli were shown. The AHC was able to function near a Hopf bifurcation and amplify or compress the output piezoelectric voltage by a one-third power-law relationship with the input, similar to the mammalian cochlear amplifier. The effect of using the filter on the system’s magnitude and phase FRFs were also discussed.
Chapter 6

Experimental validation of the self-sensing active artificial hair cell

6.1. Introduction

The self-sensing AHC simulations discussed in the previous chapter showed promising results. Subsequently, experimental validation of the self-sensing AHC is the main objective of this chapter. Furthermore, an extension of the self-sensing scheme for developing multi-channel self-sensing AHCs is examined in the current chapter that provides additional sensing channels and a larger frequency range for the AHC.

This chapter is presented in six sections. The next section discusses the experimental validation of the self-sensing quadmorph. The AHC’s behavior is examined under different base excitation levels with and without the control voltage, and the ability of the system in mimicking the cochlear one-third compressive rate is investigated. Then, the experimental results are compared to the numerical results in Section 6.4 to validate the model created in the previous chapter. Section 6.5 briefly discusses multi-channel self-sensing AHCs in an attempt to increase the bandwidth of the AHC. The last section provides a conclusion to the present chapter.

6.2. Experimental validation of an active self-sensing AHC

In this section, the performance of a quadmorph AHC is examined experimentally following the theory of Chapter 5. First, the real-time implementation of the AHC is explained in detail. Next,
the AHC is tested without any control signal and its dynamic behavior is examined. Lastly, the cubic damping feedback control law is applied to the system and the active AHC’s nonlinear characteristics are investigated. Results obtained from the uncontrolled and controlled models of the AHC discussed in Chapter 5 are shown next to the experimental results in this section. Equations derived by adopting the harmonic balance method are used to calculate the response.

6.3. Experimental setup and procedure

In order to characterize the AHC’s dynamic behavior with and without the feedback controller, the experimental setup illustrated schematically in Fig. 69 is employed. The setup consists of an AHC clamped to a Modal Shop Inc. mini shaker, an amplifier powering the shaker, a ceramic flexural ICP® accelerometer placed on the clamp and connected to a PCB signal conditioner, an amplifier connected to the AHC’s piezoelectric actuator layers, a Polytec PDV-100 laser Doppler vibrometer, and a NI- PXI data acquisition system.

![Fig. 69. Schematic and photograph of the experimental setup.](image)

The AHC is a 4-layer piezoelectric beam cantilevered by the clamp at 15.5 mm from its base. The composite is fabricated of PSI-5A4E piezoceramics and a brass center shim by Piezo Systems, Inc.
with the unmounted dimensions of \(44.5 \text{ mm} \times 5.1 \text{ mm} \times 0.86 \text{ mm}\) [178]. Properties of the AHC can also be found in Table 12. The 4-layer beam was initially designed by the company to be used as an actuator capable of providing higher deflection and force compared to a bimorph. Therefore, to be able to use this composite as a self-sensing self-actuating AHC considering the polarization of the layers, the wiring method for the electrodes is altered from its pre-set configuration available in [178] to the one depicted in Fig. 69 (b). The polarization of the outer piezoelectric layers shown in green in Fig. 69 is in the same direction for parallel function. This pair of piezoelectric layers are used as the sensor layers to measure the generated voltage due to the bending strain they experience while the system is excited. The inner piezoceramics are also poled in the same direction, but opposite the sensor layers. These layers are utilized as an actuator that provides the control voltage to the AHC and they are connected in parallel. Electrodes sandwiched between the inner and outer layers are connected to the black wires shown in Fig. 69 (b) and are common electrodes between those layers. Therefore, these electrodes with the center electrode and the outer electrodes are connected to the piezo amplifier and the DAQ system, respectively.

A set of experimental tests are performed by driving the shaker and measuring the base acceleration input \((\ddot{z}(t))\), piezoelectric sensor layers’ induced voltage \((V_s(t))\), and tip velocity of the AHC \((\dot{u}_t(t))\) in real-time at a sampling rate of 20 kHz. The piezoelectric sensed voltage is filtered by a 6th order, low-pass Butterworth filter with a 650 Hz corner frequency. As mentioned in the previous chapter, a filter is used in this study to ensure that the control law is applied to the AHC only at frequencies below its second natural frequency. The filter parameters are the same as the previous chapter. Next, the derivative of the filtered voltage, \(\dot{V}_{sf}(t)\) is fed to the nonlinear controller implemented in LabVIEW on a host computer with the real-time module installed, as shown schematically in Fig. 70.
The program is executed on the NI-PXI 8109 real-time target connected to the host computer by an Ethernet cable. The control voltage, \( V_a(t) = \alpha_1 \dot{V}_{sf}(t) - \alpha_3 \dot{V}_{sf}^2(t) \) is amplified by the piezo amplifier and supplied to the AHC’s actuator layers. The control law was discussed in detail in Chapter 5. It should be emphasized that although the tip velocity is acquired by the laser vibrometer, it is not adopted in the feedback control law and its measurement is not required. This data is only used to analyze the AHC’s performance in terms of the velocity output in the following sections.

### 6.3.1 Experimental characterization of the uncontrolled self-sensing AHC

This section examines the dynamic behavior of the AHC under various excitation levels and frequencies to characterize the uncontrolled self-sensing AHC, and explore the existence of inherent nonlinearities in the system. For each input level, a stepped sine signal with a frequency bandwidth of 410 Hz to 440 Hz is supplied to the shaker’s amplifier and the sensed voltage of the AHC is measured. The frequency resolution was chosen to be 0.2 Hz near the first natural frequency and 1 Hz elsewhere. The experiment is repeated for six stimulus levels ranging from 0.25 V to 1.5 V and the frequency response function of the system in terms of the piezoelectric sensed voltage over the base acceleration is obtained. The experimental results along with the uncontrolled SDOF AHC model’s FRF are shown in Fig. 71 and later compared in Section 6.4. The modeled FRF is obtained using Equation (132).
**Fig. 71.** Magnitude FRF of the sensed piezo voltage/base acceleration for the uncontrolled AHC excited with various stimulus levels.

Fig. 71 shows an inherent nonlinear softening behavior in the AHC’s FRFs. Increasing the input amplitude from 0.25 $V$ to 1.5 $V$ shifts the first natural frequency from 424.6 $Hz$ to 421.4 $Hz$ and decreases the magnitude of the piezo-voltage/base-acceleration by 28.13 %. Similar nonlinear behavior was seen in the study on developing multi-channel AHCs and in various studies on piezoelectric structures [202-204]. Similarly, this inherent nonlinearity can be attributed to the dynamics of the clamp, nonlinear piezoelectricity of the piezoceramics, geometrical nonlinearities, and possible nonlinearities in the shaker’s dynamics. Since the contribution of multiple sources in these nonlinearities must be considered, identifying and modeling the inherent nonlinearities requires extensive analysis of the passive AHCs that are not within the scope of the current work. Therefore, the AHC is approximated as a linear system in the present study. The inherent nonlinearities affect the gain tuning of the controller and add complexity to the process of eliminating the linear damping of the system which is discussed in the next session.
6.3.2. **Experimental characterization of the active self-sensing AHC**

After characterizing the uncontrolled AHC, the dynamic behavior of the active self-sensing AHC is studied. First, the linear damping gain of the AHC is determined. Then, the experimental results of the active system with the cubic damping controller are provided and the behavior of the AHC is investigated.

6.3.2.1. **Linear damping gain tuning**

In this section, a summary of the gain tuning method described in Section 5.5.2.1 to compute the linear damping gain of the controller is presented. This method is used to account for the inherent nonlinearity of AHC seen in the previous section, compensate for the filter effects, and considerably decrease the linear damping of the system. This method is visualized in the flowchart of Fig. 72.

![Flowchart of Linear Damping Gain Determination](image)

**Fig. 72.** Linear damping gain determination flowchart.

The step marked with (I) in Fig. 72 is shown in Fig. 65 (a) for the case of 1 V input to the amplifier. For the same system, \((2\zeta_{1 ecl} \omega_{1 ecl})\) versus \(\alpha'_1\) is displayed in Fig. 65 (b) which corresponds to steps
(II) to (IV) in Fig. 72. Note that $\zeta_{1\text{eq}}$ and $\omega_{1\text{eq}}$ are the modal damping and natural frequency of the equivalent system. Results of the final step marked by (V) are tabulated in Table 13.

![Graph](image)

**Fig. 73.** (a) Magnitude FRF of the experimental sensed piezo voltage/base acceleration for multiple values of $\alpha_1'$ for 1 V input to the amplifier connected to the shaker, (b) $(2\zeta_{1\text{eq}}\omega_{1\text{eq}})$ vs $\alpha_1'$ used for finding the gain $\alpha_1$ that eliminates the linear damping near the first natural frequency. Term $\beta_{1\text{eq}}p_s$ is the equivalent system’s control influence term.

Table 13 presents the linear damping gains obtained for various stimulus levels. According to this table, the linear damping gain’s absolute value increases by 37.5% as the input level strengthens from 0.25 V to 1.5 V. For ease of reference, the value of $\alpha_1$ in Fig. 65 (b) is highlighted in blue in Table 13.

**Table 13.** Linear damping gains of the controller.

<table>
<thead>
<tr>
<th>Input Voltage (V)</th>
<th>0.25</th>
<th>0.5</th>
<th>0.75</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1(s)$</td>
<td>$-2.32 \times 10^{-4}$</td>
<td>$-2.55 \times 10^{-4}$</td>
<td>$-2.81 \times 10^{-4}$</td>
</tr>
<tr>
<td>Input Voltage (V)</td>
<td>1</td>
<td>1.25</td>
<td>1.5</td>
</tr>
<tr>
<td>$\alpha_1(s)$</td>
<td>$-2.88 \times 10^{-4}$</td>
<td>$-2.99 \times 10^{-4}$</td>
<td>$-3.19 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

The gains displayed in this table are considered as final linear damping gains for the controller. It should be pointed out that the controller gain’s negative or positive sign matches the control
influence term and a discussion on this matter can be found in Section 5.5.2.1. Section 6.4 compares $\alpha_1$ gains to its theoretical value.

6.3.2.2. Experimental results with cubic damping control

In the present section, the linear damping gains corresponding to the various stimulus levels determined in the previous section are adopted to reduce the AHC’s linear damping. Then, different values are assigned to the controller’s cubic damping gain to study the nonlinear function of the active system. Cubic damping gains are chosen with the aid of the AHC model. Nine different cases are implemented in real-time with cubic damping gains ranging from $-4 \times 10^{-11} s^3/V^2$ to $-2 \times 10^{-10} s^3/V^2$. FRFs of three cases are plotted in Fig. 64. For $\alpha_3 = -4 \times 10^{-11} s^3/V^2$, $-1 \times 10^{-10} s^3/V^2$, and $-2 \times 10^{-10} s^3/V^2$, magnitude FRFs of the AHC’s Piezo-voltage/Base-acceleration are shown in Fig. 64 (a) to (c). These FRFs are obtained experimentally by changing the input voltage to the shaker amplifier. In addition to the experimental FRFs, corresponding model FRFs computed adopting the harmonic balance method are illustrated in Fig. 64 (d) to (f) and discussed in the next section. The cubic damping gain for the modeled AHC without inherent nonlinearity and filter is positive, as the theoretically calculated control influence term is positive. It is important to note that the numerical results displayed in this section are obtained using the acceleration inputs measured in the experiments. Therefore, for each cubic damping gain, Equation (130) is solved for the sensed voltage $V$ by substituting the driving frequency, $\Omega$, and the corresponding measured acceleration level, $Z$, at that frequency in the equation. Subsequently, the magnitude Piezo-voltage/Base-acceleration ($V/Z$) FRF is built point-by-point, similar to the stepped sine analysis.
Fig. 74. Magnitude FRF of the Piezo-voltage/base-acceleration of the active self-sensing system for various input levels to the shaker amplifier obtained: (a) experimentally for $\alpha_3 = -4 \times 10^{-11} \text{s}^3/\text{V}^2$, (b) experimentally for $\alpha_3 = -1 \times 10^{-10} \text{s}^3/\text{V}^2$, (c) experimentally for $\alpha_3 = -2 \times 10^{-10} \text{s}^3/\text{V}^2$, (d) numerically for $\alpha_3 = 4 \times 10^{-11} \text{s}^3/\text{V}^2$, (e) numerically for $\alpha_3 = 1 \times 10^{-10} \text{s}^3/\text{V}^2$, and (f) numerically for $\alpha_3 = 2 \times 10^{-10} \text{s}^3/\text{V}^2$.

Fig. 64 (a) to (c) shows that changing the cubic damping gain alters the behavior of the active AHC in response to different stimuli. To determine whether the AHC amplifies or compresses the response near the natural frequency for a given input, these FRFs need to be compared to the corresponding uncontrolled system’s FRFs shown in Fig. 71. Comparison between the experimental FRFs in Fig. 71 and Fig. 64 displays the amplification function of the AHC with
\[ \alpha_3 = -4 \times 10^{-11} \text{s}^3/\text{V}^2, \quad \alpha_3 = -1 \times 10^{-10} \text{s}^3/\text{V}^2, \quad \text{and} \quad \alpha_3 = -2 \times 10^{-10} \text{s}^3/\text{V}^2 \]

for input strengths equal to or less than 1 \text{V}, 0.5 \text{V}, and 0.25 \text{V}, respectively. The results show that increasing the cubic damping gain’s absolute value, shifts the system’s overall behavior towards compression for a wider range of stimulus voltages. In addition, it is observed that adding more cubic damping to the system shifts the natural frequency of the active AHC, and distorts the FRFs. This shift in the natural frequency and the FRF distortion was observed in two-channel AHC results. Simulated FRFs of an active AHC with and without filter adoption are available in the previous chapter. The results show that for the active system with a filter, the natural frequency shifts farther from the uncontrolled system’s natural frequency as the cubic damping gain increases. However, the natural frequency for the AHC without filter remains constant. The other factor that may influence the behavior of the controlled system is the inherent nonlinearities of the AHC.

The magnitude spectrum of the sensed piezo voltage of the active system for the aforementioned cubic damping gains along with the uncontrolled system’s response are plotted in Fig. 75. Furthermore, backbone curves defined as the peak response data versus the corresponding frequency for both uncontrolled and active AHCs are shown in the same figure. As displayed in Fig. 75, the backbone curve for the active system deviates from the uncontrolled system’s curve and bends towards lower frequencies as the cubic damping gain value increases. Additionally, intensification of the active AHC’s softening behavior due to an increase of the nonlinear damping is evident in this figure. Moreover, Fig. 75 shows that for a fixed cubic damping gain, increasing the input by a constant factor does not increase the output of the active AHC with the same factor near the natural frequency due to the compressive nonlinearity characteristic of the AHC. Consequently, compression in the response is seen near the first natural frequency by comparing the output at frequencies in the vicinity of the natural frequency to farther frequencies. Fig. 75 also
demonstrates the effects of changing the cubic damping gain on the sensitivity of the AHC. As the magnitude of the cubic damping gain increases, the peak area broadens and the sensitivity of the AHC decreases near the natural frequency.

![Fig. 75. Magnitude of the RMS sensed piezo voltage of the uncontrolled and active self-sensing AHC](image)

Near the first natural frequency obtained experimentally in response to various input voltage levels for, (a) \( \alpha_3 = -4 \times 10^{-11} \, \text{s}^3/\text{V}^2 \), (b) \( \alpha_3 = -1 \times 10^{-10} \, \text{s}^3/\text{V}^2 \), and (c) \( \alpha_3 = -2 \times 10^{-10} \, \text{s}^3/\text{V}^2 \).

To evaluate the performance of the self-sensing active AHC, input-output curves at the natural frequency are generated from experimental data for the active AHC with nine different values of cubic damping gains and the uncontrolled system, as shown in Fig. 76 (a). The input-output curves obtained for the uncontrolled and active AHC models at the natural frequency are plotted in Fig.
For each cubic damping gain value, RMS values of the piezo-voltage and base acceleration are plotted at the natural frequency for various input voltages to the shaker. Note that the data points are extracted from the FRF data for the nine cubic damping gains. However, from the nine experimental and modeled FRFs, only three of them are shown in Fig. 64. Data points corresponding to 0.25 V, 0.5 V, 0.75 V, 1 V, 1.25 V, and 1.5 V input to the shaker’s amplifier are displayed in cyan, red, orange, magenta, blue, and green, respectively. A power function of the form \( volt = S a^c \) with fitting parameters, \( S \) and \( c \) is fitted to the data points via least-squares for each cubic damping gain. The obtained equation is exhibited next to each curve. In the power-law relationship, \( volt \) represents the RMS piezo-voltage, \( a \) is the RMS base acceleration, \( S \) stands for the sensitivity of the AHC, and \( c \) is the compressive rate. As shown in Fig. 76 (a), the compressive rate of the self-sensing active AHC ranges from 0.36 to 0.39 for various cubic damping gains. This rate is very close to the one-third power-law relation between the response and input of the cochlea at the local CF [6, 22, 46].

In addition to the input-output curves for the active AHC, data points for the uncontrolled AHC along with the fitted curve are shown in Fig. 76 (a) with triangle markers and a black solid line, respectively. Curve-fitting to the uncontrolled data shows a power-law relationship in the form of \( volt = 0.706 a^{0.78} \) between the output and input of the system. This finding confirms the existence of an inherent nonlinearity in the system, as expected from the FRFs obtained in Section 6.3.1. The fitted curve to the uncontrolled AHC’s data can be used to determine the amplification/compression performance of the active system. Therefore, as the black solid curve represents the neutral border, the area below it, is the compression region, while the area above it, is the amplification area. The equations on the right-hand side of Fig. 76 marked with an asterisk correspond to the FRFs of Fig. 64.
Fig. 76. Input base-acceleration to output piezo-voltage curves of the self-sensing AHC at the first natural frequency obtained: (a) experimentally, (b) numerically.

The behavior of the system can be studied further by plotting the sensitivity values, i.e. the coefficients of $volt = S\alpha^c$ equations shown in Fig. 76 (a) and Fig. 76 (b), with respect to the cubic damping gains and fitting a least-square power curve to the data, as displayed in Fig. 77. As a result, the input-output relation for any cubic damping gain that lies between $\alpha_3 = -4 \times$
$10^{-11} \text{s}^3/\text{V}^2$ and $\alpha_3 = -2 \times 10^{-10} \text{s}^3/\text{V}^2$, and is not examined in this study, can be approximated in two steps: i) finding the compressive rate by interpolation from Fig. 76, and ii) calculating the sensitivity using the power-law relationship between the cubic damping gain and the sensitivity obtained in Fig. 77. The intersection of the uncontrolled AHC’s input-output curve of Fig. 76 and the estimated curve determines the switching point between the amplification and compression modes. Fig. 76 shows that for a specific stimulus level, the AHC can be adapted to work in amplification or compression mode by tuning the controller’s cubic damping gain. The aforementioned procedure with reversed order can be used to guide the gain tuning for a desired mode switching point. For instance, if the AHC’s desired switching point from amplification to compression is set to $0.8 \text{ m/s}^2$ RMS base excitation level, the cubic damping gain can be obtained by first finding the sensitivity from Fig. 76 (a) and then the cubic damping gain from Fig. 77. The switching point is the intersection of the vertical blue line drawn at $0.8 \text{ m/s}^2$ on Fig. 76 (a) and the uncontrolled system’s curve and is displayed by a plus marker. This point lies between the curves for $a_3 = -8 \times 10^{-11} \text{s}^3/\text{V}^2$ and $a_3 = -6 \times 10^{-11} \text{s}^3/\text{V}^2$, and its equation can be written as $\text{volt} = S a^{0.38}$. Therefore, sensitivity denoted by $S$, can be determined by solving system of equations \begin{align*} \text{volt} &= S a^{0.38} \\ \text{volt} &= 0.706 a^{0.78} \end{align*} at $a = 0.8$. The calculated sensitivity, $S = 0.65 \left( \frac{V}{(\frac{m}{s^2})^{0.38}} \right)$, is then substituted in equation $S_{\exp} = 0.000412 |a_3|^{-0.314}$ shown in Fig. 77 to find the cubic damping gain as $a_3 = -0.65 \times 10^{-10} \text{s}^3/\text{V}^2$. The sensitivity and its corresponding cubic damping gain are shown graphically in Fig. 77 by the thin blue and orange lines.
Fig. 77. Sensitivity of the AHC versus cubic damping gain at the first natural frequency.

Furthermore, Fig. 77 indicates a nearly inverse cubic relation, in the form of $S_{\text{exp}} = 0.000412 |\alpha_3|^{-0.314}$ between the sensitivity and the absolute cubic damping gain obtained experimentally. The above inverse cubic relation closely follows the behavior of an oscillator at a Hopf bifurcation with cubic damping as shown implicitly in Equation (131). The numerical results shown in this figure and their comparison to the experimental results are presented in Section 6.4.

6.3.2.3. Comparison of the AHC’s sensed voltage to the velocity measurements

According to the results presented in this section, the cubic damping controller with the sensed piezoelectric voltage feedback demonstrates a cochlea-like amplification/compression behavior. Next, the relationship between the tip-velocity of the AHC that the laser vibrometer measures and the base acceleration input in the same self-sensing system is investigated. Fig. 78 (a) shows the tip-velocity/base-acceleration FRF of the active self-sensing AHC. This figure is plotted by varying the voltage to the amplifier powering the shaker for $\alpha_3 = -1 \times 10^{-10} \, s^3/V^2$. The Piezo-voltage/Base-acceleration FRF of the system is also displayed in Fig. 78 (b).
Comparison between the magnitude Tip-velocity/Base-acceleration and Piezo-voltage/Base-acceleration FRFs shows a similar trend in the magnitude FRFs. To evaluate the performance of the self-sensing mechanism, the relationship between the AHC’s measured tip velocity and the sensed voltage rate is investigated and compared to the time derivative of Equation (123). The tip velocity of the system shown in Fig. 78 is plotted against the sensed piezo voltage rate for various input levels, as illustrated in Fig. 79 (a) and color-coordinated with Fig. 78. Furthermore, this study is extended to examine this relationship between the measurements with various cubic damping gains. For instance, the tip velocity versus the sensed voltage rate for 1 V input voltage to the shaker’s amplifier for various cubic damping gains in the frequency range of 410 Hz to 440 Hz is displayed in Fig. 79 (b).
Fig. 79. RMS Tip-velocity versus RMS Piezo voltage rate between 410 Hz and 440 Hz for (a) \( \alpha_3 = -1 \times 10^{-10} \text{s}^3/V^2 \) for various input voltage levels supplied to the shaker’s amplifier corresponding to Fig. 78, (b) 1 V input level to the amplifier for various cubic damping gains.

The least-square fit to the experimental data in Fig. 79 (a) and (b), shows a linear relationship between the tip velocity, \( vel \), and the sensed piezo voltage rate, \( \dot{volt} \), in the form of \( vel = 1.79 \times 10^{-5} \dot{volt} \). The theoretical relationship calculated using Equation (123) is \( \dot{u}_{ts} = 1.87 \times 10^{-5} \dot{V}_{ss} \) where \( \dot{u}_{ts} \) is the tip velocity of the SDOF approximation of the AHC and \( \dot{V}_{ss} \) is the sensed piezo voltage rate. Comparison between the conversion factors from sensed piezo voltage rate to tip velocity obtained experimentally and theoretically shows a good agreement between the results with a 4.5% error. The input-output plots at the first natural frequency in terms of the AHC’s tip velocity are displayed in Fig. 80.
Fig. 80. Experimental input base-acceleration to output tip-velocity curves of the self-sensing AHC at the first natural frequency.

As shown in Fig. 80, the tip velocity response is compressed at a rate of about one-third power of the base acceleration at the first natural frequency. In addition, curve fitting to the uncontrolled system data results in a similar relationship between the tip velocity and base acceleration as the sensed piezo voltage versus acceleration, i.e. a power function with the same power, but different sensitivity. This shows a linear relationship between the tip velocity and the sensed voltage at the first natural frequency, as expected from Fig. 79.

The results of this section demonstrate that the self-sensing AHC’s performance is comparable to the AHC with velocity read-out and verifies the effectiveness of the self-sensing control law.

6.4. Model validation of the self-sensing active AHC

This section quantitatively compares the experimental results discussed in Section 6.2 to the ones obtained from the SDOF model of the AHC created in Chapter 5 and displayed in Section 6.2. To validate the model of the self-sensing AHC, the modeled FRFs and input-output curves are plotted
by substituting the exact measured base acceleration profiles into the equations derived by the harmonic balance method in Section 5.4, as mentioned previously. First, the uncontrolled systems are compared, then the model validation for the active AHC is presented.

6.4.1. Comparison of the uncontrolled AHC results

To validate the initial AHC models without the controller, the uncontrolled system’s experimental and numerical FRFs are displayed in Fig. 71 and compared. As discussed previously in Section 6.3.1, the uncontrolled system’s experimental FRFs for different stimulus levels show a softening behavior. Although the system is inherently nonlinear, it acts nearly linear at 0.25 \( V \) as the FRF is approximately symmetric in the vicinity of the first natural frequency. At this voltage, the experimental FRF closely matches the SDOF model’s FRF as displayed in Fig. 71. A comparison between the experimental and numerical FRFs is presented in Table 14.

Table 14. Comparison of the experimental FRF for 0.25 V and the numerical FRF for the uncontrolled AHC.

<table>
<thead>
<tr>
<th></th>
<th>( f_1 ) (Hz)</th>
<th>( \zeta_1 ) (%)</th>
<th>Amplitude (V/m/s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment (0.25 V)</td>
<td>424.60</td>
<td>0.53</td>
<td>0.854</td>
</tr>
<tr>
<td>Model</td>
<td>424.52</td>
<td>0.53</td>
<td>0.860</td>
</tr>
<tr>
<td>Error (%)</td>
<td>0.02</td>
<td>N/A</td>
<td>-0.702</td>
</tr>
</tbody>
</table>

Table 14 shows a good agreement between the experimental data and the model in the linear region. It should be noted that the model’s damping ratio is set equal to the damping ratio extracted from the experimental data at 0.25 \( V \) using a vector-fitting system identification method.

6.4.2. Comparison of the active AHC results

This section compares the experimental results for the active self-sensing AHC to model predictions. It was shown in the previous section that the linear SDOF model of the AHC can represent its behavior for low-level inputs. Consequently, the model’s linear damping gain is
calculated to be $\alpha_{1,\text{theo}} = 2.45 \times 10^{-4}$, which lies between the $\alpha_1$ values obtained for the two lowest stimulus levels in the experiment. The errors between the absolute values of the gain calculated analytically and experimentally for the low driving voltages are below 6%. The linear damping gains for higher voltage levels exhibited in Table 13 deviate more from the model’s gain. However, due to the adoption of the gain tuning method described in Section 5.5.2.1, the experimental linear damping gains are expected to partially compensate for the effects of the inherent nonlinearity and the filter. Furthermore, instead of assuming a constant base acceleration as the input to the modeled AHC, the base acceleration profiles obtained experimentally are used in the numerical analysis, as mentioned previously. Therefore, the experimental and numerical results for the active AHCs with cubic damping are comparable and presented as follows.

For comparison, Fig. 64 demonstrates the experimental FRFs of the system with three different cubic damping gains against the FRFs of the active AHC model. The FRFs show a close match between the results. However, the softening effect in the experimental FRFs is greater than in modeled FRFs, due to the combination of filtering effect and the inherent nonlinearity of the AHC. A quantitative comparison of the FRFs for $|\alpha_3| = 1 \times 10^{-10} s^3/V^2$ displayed in Fig. 64 (b) and Fig. 64 (e) is illustrated in Table 15.

**Table 15.** Comparison of the experimental and the numerical FRFs for the self-sensing active AHC for $|\alpha_3| = 1 \times 10^{-10} s^3/V^2$ shown in Fig. 64 (b) and Fig. 64 (e), respectively.

<table>
<thead>
<tr>
<th>Voltage ($V$)</th>
<th>$f_1$ (Hz)</th>
<th>Amplitude ($V/m/s^2$)</th>
<th>Error (%)</th>
<th>Experiment</th>
<th>Model</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>423.8</td>
<td>423.8</td>
<td>0</td>
<td>1.425</td>
<td>1.451</td>
<td>-1.825</td>
</tr>
<tr>
<td>0.5</td>
<td>422.8</td>
<td>423.2</td>
<td>-0.09</td>
<td>0.853</td>
<td>0.814</td>
<td>4.572</td>
</tr>
<tr>
<td>0.75</td>
<td>422.2</td>
<td>423.0</td>
<td>-0.19</td>
<td>0.642</td>
<td>0.605</td>
<td>5.763</td>
</tr>
<tr>
<td>1</td>
<td>421.8</td>
<td>423.0</td>
<td>-0.28</td>
<td>0.520</td>
<td>0.496</td>
<td>4.615</td>
</tr>
<tr>
<td>1.25</td>
<td>421.4</td>
<td>423.0</td>
<td>-0.38</td>
<td>0.445</td>
<td>0.427</td>
<td>4.045</td>
</tr>
<tr>
<td>1.5</td>
<td>421.0</td>
<td>423.0</td>
<td>-0.48</td>
<td>0.394</td>
<td>0.378</td>
<td>4.060</td>
</tr>
</tbody>
</table>
Based on Table 15, a maximum error of $-0.48\%$ and $5.763\%$ between the natural frequencies and magnitude of the FRFs of the model and experiment are observed, respectively. Note that although the error between the natural frequency of the model and experiment is negligible, it increases as the input strengthens. As the errors remain at $\sim 5\%$, the model accurately represents the implemented self-sensing AHC.

Next, the power-law relationships between the sensed piezoelectric voltage and the base acceleration experimental data are compared to the corresponding numerical results for various cubic damping gains. A comparison of the compressive rates and the sensitivities of the input-output curves displayed in Fig. 76 (a) and (b), is shown in Table 16.

**Table 16.** Comparison of the input-output relations obtained experimentally and numerically for various gains, shown in Fig. 76 (a) and (b).

| $|\alpha_3|$ ($s^3/V^2$) | Experiment | Model | Error (%) | Experiment | Model | Error (%) |
|-----------------------------|------------|-------|-----------|------------|-------|-----------|
| $4 \times 10^{-11}$         | 0.752      | 0.753 | -0.133    | 0.39       | 0.39  | 0         |
| $6 \times 10^{-11}$         | 0.67       | 0.665 | 0.746     | 0.38       | 0.37  | 2.63      |
| $8 \times 10^{-11}$         | 0.611      | 0.607 | 0.655     | 0.38       | 0.37  | 2.63      |
| $1 \times 10^{-10}$         | 0.569      | 0.566 | 0.527     | 0.36       | 0.36  | 0         |
| $1.2 \times 10^{-10}$       | 0.537      | 0.538 | -0.186    | 0.37       | 0.35  | 5.41      |
| $1.4 \times 10^{-10}$       | 0.512      | 0.509 | 0.586     | 0.37       | 0.36  | 2.70      |
| $1.6 \times 10^{-10}$       | 0.485      | 0.488 | -0.619    | 0.38       | 0.36  | 5.26      |
| $1.8 \times 10^{-10}$       | 0.471      | 0.471 | 0         | 0.36       | 0.35  | 2.78      |
| $2 \times 10^{-10}$         | 0.455      | 0.454 | 0.220     | 0.36       | 0.36  | 0         |

As shown in Table 16, the maximum error between the sensitivities is $0.746\%$, while the maximum error in compressive rates is $5.41\%$. Based on the results illustrated in Fig. 76 and Table 16 for the active AHC, the fitted curve on the model’s data closely follows the power-law fit to the experimental results. However, as the inherent nonlinearity of the uncontrolled system is not considered in creating the model, the model can only predict the dynamic behavior of the uncontrolled system when excited by low-level signals.
Additionally, sensitivity versus cubic damping gain for the modeled system and implemented AHC at the first natural frequency are shown together in Fig. 77. Comparing the power-law relationship obtained from the curve fitting to the data, the error between the coefficients is $-2.184\%$, while the power is $-0.31$ for both experimental and numerical results. The theoretical relation between the sensitivity and the cubic damping gain can be derived from Equation (131) as $S = \gamma a_3^{-\frac{1}{3}}$, where $\gamma = \left(\frac{4\beta_{1z}}{3\omega_1^3\beta_{1v}}\right)^{\frac{1}{3}}$ is a constant coefficient. The power-law functions obtained experimentally and numerically, closely follow the inverse cubic relation between the sensitivity and the cubic damping gain. As a result, the analysis of this section exhibits that the experimental results validate the model of the active AHC built in Section 5.3.

6.5. Multi-channel self-sensing AHC

The self-sensing AHC developed in this work and presented in previous sections has shown promising results in mimicking the nonlinear processes of the cochlea. In order to increase the number of sensing channels of the AHC, the self-sensing scheme developed in this study is combined with the work on multi-channel AHCs and develop multi-channel self-sensing AHCs. These AHCs offer advantages over previous generations of AHCs with their extended bandwidth and self-sensing properties. To create multi-channel self-sensing AHCs, the feedback control system shown in Fig. 70 is modified to amplify or compress the output close to the second natural frequency. The extended active system is illustrated schematically in Fig. 81. The first controller and the LP filter are the same as the previous sections. In order to control the self-sensing system near its second natural frequency, the sensed voltage rate is filtered by a Butterworth band-pass filter. The filter’s order is 6 and cut-off frequencies are $2\ kHz$ and $3.5\ kHz$ to restrict the application of the second control signal to frequencies near the second natural frequency. The
filtered voltage rate is then passed through Controller 2, where $\alpha_{12}$ and $\alpha_{32}$ are the linear and cubic damping gains of the AHC related to the second natural frequency, respectively. In the last step, the sum of the control voltages from Controller 1 and 2 is fed back to the actuator layers of the AHC.

**Fig. 81.** Schematic of the extended control loop implemented in LabVIEW.

The system is first characterized near the second natural frequency using a stepped sine signal for various input levels. The method used in section 5.5.2.1 for gain tuning is then used in the vicinity of the second natural frequency to approximate the linear damping gains needed to reduce the damping of the system near the natural frequency. The gains are tabulated in Table 17.

**Table 17.** Linear damping gains of the controller for linear damping reduction near the second natural frequency.

<table>
<thead>
<tr>
<th>Input Voltage (V)</th>
<th>0.75</th>
<th>1</th>
<th>1.25</th>
<th>1.5</th>
<th>1.75</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{12}(s)$</td>
<td>$1.57 \times 10^{-4}$</td>
<td>$1.61 \times 10^{-4}$</td>
<td>$1.66 \times 10^{-4}$</td>
<td>$1.70 \times 10^{-4}$</td>
<td>$1.78 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

The linear damping gains are used in the feedback control law and cubic damping is also added to the system. Magnitude FRFs of the Piezo-voltage/base-acceleration of the controlled AHC for three different cubic damping gains, $\alpha_{32} = 8 \times 10^{-11} \text{s}^3/\text{V}^2$, $\alpha_{32} = 2 \times 10^{-10} \text{s}^3/\text{V}^2$, and $\alpha_{32} = 4 \times 10^{-10} \text{s}^3/\text{V}^2$ are shown in Fig. 82. The uncontrolled system’s FRF for 1 V voltage to the shaker’s amplifier is also displayed in Fig. 82. For $\alpha_{32} = 8 \times 10^{-11} \text{s}^3/\text{V}^2$ the system amplifies the response for the given stimulus levels, while for $\alpha_{32} = 2 \times 10^{-10} \text{s}^3/\text{V}^2$ and $\alpha_{32} = 4 \times 10^{-10} \text{s}^3/\text{V}^2$
$10^{-10} \text{s}^3/\text{V}^2$ the AHC compresses the response for the input levels corresponding to the FRFs below the uncontrolled AHC’s FRF and amplifies the response for the FRFs above it.

![Magnitude FRF of the Piezo-voltage/base-acceleration of the self-sensing system for various input voltage level to the shaker amplifier obtained experimentally near the second natural frequency for.](image)

(a) $\alpha_{32} = 8 \times 10^{-11} \text{s}^3/\text{V}^2$, (b) $\alpha_{32} = 2 \times 10^{-10} \text{s}^3/\text{V}^2$, (c) $\alpha_{32} = 4 \times 10^{-10} \text{s}^3/\text{V}^2$.

The hardening behavior observed in Fig. 82 FRFs can be attributed to the BP filter in the control loop. To investigate the compressive rate of the active AHC near the second natural frequency, the input-output graphs at this frequency for the three cubic damping gains are exhibited in Fig. 83. The fitted power-law curves to the data and the obtained equations are also displayed.
As shown in Fig. 83, the AHCs compressive rate is 0.38 or 0.39 for various cubic damping gains. This compressive rate is very close to the one-third compressive rate observed in the mammalian cochlea. Furthermore, the FRF of the multi-channel self-sensing AHC to 1 V input voltage to the shaker amplifier for $\alpha_{31} = -1 \times 10^{-10} \, \text{s}^3/\text{V}^2$ and $\alpha_{32} = 8 \times 10^{-11} \, \text{s}^3/\text{V}^2$ in a frequency range of 100 Hz to 2720 Hz is displayed in Fig. 84.
Fig. 84. Experimental magnitude FRF of the piezo-voltage/base-acceleration of the self-sensing system for 1 V input voltage to the shaker amplifier for $\alpha_{31} = -1 \times 10^{-10} \, s^3/V^2$ and $\alpha_{32} = 8 \times 10^{-11} \, s^3/V^2$. Fig. 84 shows that for the 1 V input voltage to the shaker amplifier, the AHC compresses the response near the first natural frequency while it amplifies the output near the second natural frequency. Additionally, the FRF of the controlled system closely matches the uncontrolled system’s FRF for other frequencies far from the natural frequencies. It is important to note that the peak observed at about 1700 Hz corresponds to the clamping mechanism and it was seen in the author’s previous experiments using the same clamp [28]. Therefore, the self-sensing scheme is effective in creating multi-channel AHCs with a broader operational range and mimicking the cochlear amplifier.

6.6. Conclusion

The work presented in this chapter was a major contribution to developing self-sensing AHCs to replicate the mammalian cochlea. The research validated the phenomenological nonlinear feedback control algorithm established in Chapter 5 to achieve a cochlea-like response from the quadmoph piezoelectric active AHCs with self-sensing characteristics. As the beam was excited at its base, the output voltage of the piezoelectric layers was measured and used to generate a feedback control voltage. Consequently, the phenomenological control law applied cubic damping to the AHC, while reducing linear damping in the vicinity of its first natural frequency to replicate the biological cochlea’s function. Moreover, the experimental results were used to validate the model built in Chapter 5 and the work was extended to implement a multi-channel AHC with more active sensing channels. The self-sensing AHC system works independent of external sensors, such as laser vibrometers, and therefore offers significant advantages over the previous generations of active AHCs. Some of these advantages include the ability to embed AHCs in a limited space.
and to combine several AHCs in an array without the need for the same number of external measurement devices.
Chapter 7

Preliminary extension of the current research

7.1. Introduction

This chapter presents the preliminary research performed on developing MEMS AHCs and arrays of AHCs. Although in the early stage, the work described in this chapter forms a basis for future fabrication of MEMS AHCs and array design. Contexts of Section 7.2 are adapted from Sheyda Davaria, VVN Sriram Malladi, Lukas Avilovas, Phillip Dobson, Andrea Cammarano, and Pablo A. Tarazaga, “Study on Developing Micro-Scale Artificial Hair Cells.”, in Special Topics in Structural Dynamics & Experimental Techniques, Volume 5, pp. 95-99. Springer, Cham, 2020 [12]. Furthermore, Section 7.3 is adapted from a previously published paper entitled “MEMS scale artificial hair cell sensors inspired by the cochlear amplifier effect” by Sheyda Davaria and Pablo A. Tarazaga in Bioinspiration, Biomimetics, and Bioreplication 2017, Volume 10162, p. 101620G, International Society for Optics and Photonics [123].

7.2. Towards developing MEMS artificial hair cells

The work presented in Section 7.2 is part of an ongoing effort in the development of MEMS scale artificial hair cells. In this work, the use of Scanning Thermal Microscopy (SThM) probes as potential AHCs is investigated. These devices offer a ubiquitous, modestly priced MEMS cantilever structure with a wealth of background literature and these make them ideal as a starting point for subsequent development and modification to achieve micro-scale AHCs. In the present work, to move towards implementing an active AHC at the micro-scale, modal characteristics and
linearity of a MEMS cantilever developed by Dobson et al. [205] are studied by exciting the structure’s base and measuring its response. In the next section, a brief description of the MEMS cantilever and its fabrication process is presented. Then, the test setup and the procedure for high-frequency modal testing are described. The first four natural frequencies of the cantilever structure and its corresponding mode shapes are extracted. Finally, the linearity of the structure is studied by a stepped sine analysis around its fundamental frequency. The results of this study can be used in the future for developing MEMS active AHCs.

7.2.1. Micro-cantilever structure

The cantilever structures consist of 400 nm thick Si$_3$N$_4$ beams of dimensions shown on Fig. 85, tethered at one end to a silicon substrate.

![Micro-cantilever structure](image)

**Fig. 85.** Micro-cantilever, (a) schematic with dimensions in µm, (b) structure under scanning electron microscope (SEM), and (c) Polytec MSA-500 built-in microscope with a 20x objective.

The structures’ original design was intended for use as SThM probes, so they also incorporate a sharp tip, with integrated Palladium (Pd) resistive heater at their free end. Electrical access to the tip heater is achieved through two thin-film Gold (Au) pads and wires that run from the probe body, along the cantilever, to the tip. These gold wires also double as a mirror, providing a strong
reflected signal when interrogating the cantilever optically. The complete fabrication process is available elsewhere [205] but in brief, a thin film of Low Pressure Chemical Vapor Deposition (LPCVD) Si$_3$N$_4$ on top of a micromachined Si wafer is patterned using ebeam- and photolithography followed by anisotropic dry-etching to form the cantilever shape. Wires, pads, and resistive heaters are patterned on top of the cantilever using ebeam-lithography followed by lift-off of evaporated thin film metals. Finally, the entire cantilever structure is released from the underlying substrate using a Tetramethylammonium hydroxide (TMAH) wet etch to selectively remove the silicon.

7.2.2. Experimentation and analysis

To investigate the possible use of MEMS cantilevers in AHC design, the dynamics of the cantilever are characterized in a high-frequency modal test. Therefore, the probe body is mounted on a piezoelectric membrane to provide a base excitation, as shown in Fig. 86. Another piezoceramic is attached to the membrane as a sensor to measure input actual excitation of the MEMS structure. The base is excited by a chirp signal with a 0 to 400 kHz bandwidth and the response of the system is obtained by a Polytec MSA-500 scanning laser vibrometer at various points on the structure.

![Test setup including the MEMS structure](image)

Fig. 86. Test setup including the MEMS structure, (a) under the Polytec MSA-500 laser vibrometer with a 20x objective, (b) shown schematically.
The frequency response functions of the microstructure around the first four natural frequencies of the cantilever along with its corresponding mode shapes are illustrated in Fig. 87.

**Fig. 87.** FRFs and mode shapes of the cantilever near the first four natural frequencies

The mode shapes shown in Fig. 87 are determined using Siemens LMS Test.Lab PolyMAX modal parameter estimation [206]. The extracted modes are the first bending mode, torsional mode, second bending, and third bending mode of the structure, respectively. To demonstrate the quality of the experimental results, the Modal Assurance Criterion (MAC) [207] between the modes is calculated and the MAC chart is displayed in Fig. 88.
The MAC chart shows that the correlation between each mode with itself is very high, while its cross-correlations with other modes are very low. As a result, the modes are distinct and easily identifiable. To further characterize the dynamics of the structure near the natural frequencies, the modal damping of the structure corresponding to each mode is tabulated in Table 18. PolyMAX’s stabilization diagrams constructed from the experimental data are used to extract natural frequencies and damping ratios.

**Table 18.** Natural frequencies and modal damping of the cantilever structure

<table>
<thead>
<tr>
<th>Mode #</th>
<th>Natural frequency (kHz)</th>
<th>Damping ratio (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25.048</td>
<td>0.79</td>
</tr>
<tr>
<td>2</td>
<td>112.211</td>
<td>0.39</td>
</tr>
<tr>
<td>3</td>
<td>147.312</td>
<td>0.02</td>
</tr>
<tr>
<td>4</td>
<td>377.533</td>
<td>0.07</td>
</tr>
</tbody>
</table>

To investigate the existence of nonlinearities of the system at voltage inputs of interest, a stepped sine analysis is conducted around the fundamental frequency of the cantilever. Various input voltages, ranging between 1V and 9V are supplied to the piezoelectric membrane. The resultant FRFs are shown in Fig. 89.
Fig. 89. Magnitude tip-velocity/base-voltage experimental FRF for different input levels.

Fig. 89 shows that as the input level increases, the natural frequency shifts to higher frequencies and the magnitude of the FRF increases. The maximum natural frequency shift and the maximum dB amplitude shift in this input level range are 0.65% and 1.45%, respectively. This is a welcomed type of response as strong nonlinearities will be hard to mitigate if an approach to implement a nonlinear feedback controller as in [185] is used to mimic the hair cell behavior.

### 7.3. Towards developing arrays of MEMS AHCs

The main goal of this research is to model the AHCs as MEMS piezoelectric cantilever beams and simulate their behavior in an array when they are excited by a stimulus. This work is the preliminary step of extensive research in which a nonlinear feedback control law will be used with a design to mimic the cochlear amplifier in a piezoelectric MEMS array of artificial hair cells. In this work, MEMS AHCs are modeled in the finite element software, Abaqus, and the fundamental frequency of each AHC is adjusted to a particular frequency in the human hearing range. These beams are then used to form arrays of AHCs to mimic the function of the basilar membrane in the ear. To obtain natural frequencies of the arrays, a frequency analysis is performed in Abaqus. In
the next step, basic tests are conducted on a macro-scale single AHC and AHC arrays to examine the concept of the research as a precursor to the MEMS built device. Results of the tests on a single and array of AHCs are shown and discussed.

7.3.1. Development of a Fourier transformer using Piezoelectric based AHCs

In this research, artificial hair cells are modeled as piezoelectric cantilevered beams with different lengths. As the resonance frequencies of any engineering structure are a function of its geometry, the length of the beam can be tailored to tune its fundamental frequency to a targeted value.

7.3.1.1. MEMS Artificial Hair Cell Design

As a part of this research, a multi-layer piezoelectric cantilever beam is designed by considering two layers of Lead Zirconate Titanate (PZT), so that it can be used as a bimorph for future nonlinear control purposes. A common electrode layer sandwiched between the two PZT layers acts as the ground to the composite. Two additional layers, one on the top and another at the bottom are connected with a wire to form a parallel electrode configuration. The beam layers are shown in Fig. 90. It is important to note that two more piezoelectric layers and electrodes need to be added to this design for self-sensing purposes.

![Fig. 90. Layers of the piezo-MEMS beam](image)

The composite is modeled in the finite element package Abaqus to determine the dynamic frequency range of the design. The final model for a single AHC with its electrode pads is
illustrated in Fig. 91. Electrode pads will be deposited on a Silicon (Si) base and the suspended beam works as an AHC.

Dimensions of the electrode pads are 500×500 µm, and the spacing between them is 250 µm. The larger dimension of the electrode pads compared to the beam enables the soldering of the electrodes. To mimic the tonotopic basilar membrane of the cochlea, an array of artificial hair cells are designed. Two models are proposed for the array of AHCs as displayed in Fig. 92. The target frequencies for the cantilever beams are set to 2, 3, 4, 5, 8, 10, 15, and 20 kHz. These frequencies are chosen because the human ear is most sensitive in the range from 2 to 5 kHz [24]. For a cantilever beam, the $i^{th}$ natural frequency of the beam can be obtained by the following formula,

$$ f_i = \frac{\lambda_i^2}{2\pi L^2} \sqrt{\frac{EI}{m'}} $$  \hspace{1cm} (133)

where, $f_i$ is the $i^{th}$ natural frequency of the cantilevered beam, $\lambda_i$ is a constant for the $i^{th}$ mode, $L$ is the length of the beam, $E$ is Young’s modulus, $I$ is the second moment of inertia, and $m$ is the mass per unit length of the beam. Equation (133) along with the results for the 500 µm length beam is used to adjust the length of the beams in the array. As a starting point, the fundamental frequency of the single beam with 500 µm length and 40 µm width (Fig. 91) is determined by frequency analysis in Abaqus and is calculated to be 21.175 kHz. The lengths of individual hair cells in each
array are adjusted such that the first natural frequency of each beam satisfies one of the eight target frequencies mentioned previously.

![Diagram](image)

**Fig. 92.** Array of artificial hair cells. a) comb-like device, b) fishbone device

### 7.3.1.2. MEMS AHC array simulations

The comb-like and fishbone structures in Fig. 92 are modeled in Abaqus and a frequency analysis is performed on them. In order to model the arrays, different layers of the AHCs (Fig. 90) and the base are created in Abaqus. Characteristics of the materials used in designing the device (Fig. 92) are defined in the software and assigned to the parts. material model/constants, elements, loads and BCs used in the models. After assembling the parts and defining the constraints between them,
parts are partitioned and a structured mesh is applied to each array. Due to the interest in investigating the behavior of the AHCs in an array and also their small size relative to the base, a finer mesh is used on the beams while a coarser mesh is applied to the array base. Fig. 93 shows the mesh on the comb-like array.

![Mesh on Comb-like Array](image)

**Fig. 93.** Fixed boundary condition and the elements used for the analysis of the comb-like AHC array.

Finally, fixed boundary conditions, shown by markers in Fig. 93, are defined at the edges of the base, and linear perturbation frequency analysis is used to extract the natural frequencies. Results of the frequency analysis are summarized in Table 19. Interested readers are encouraged to contact the researcher for more information about the material properties and details of the array models.

**Table 19. Simulation Results for MEMS AHC Arrays**

<table>
<thead>
<tr>
<th>AHC Number</th>
<th>Length (µm)</th>
<th>Target Frequency (Hz)</th>
<th>Comb-like Array Frequency (Hz)</th>
<th>Fishbone Array Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1600</td>
<td>2000</td>
<td>2009.8</td>
<td>2010.7</td>
</tr>
<tr>
<td>2</td>
<td>1310</td>
<td>3000</td>
<td>3001.8</td>
<td>3001.8</td>
</tr>
<tr>
<td>3</td>
<td>1130</td>
<td>4000</td>
<td>4037.7</td>
<td>4036.7</td>
</tr>
<tr>
<td>4</td>
<td>1010</td>
<td>5000</td>
<td>5042</td>
<td>5041.9</td>
</tr>
<tr>
<td>5</td>
<td>800</td>
<td>8000</td>
<td>8040.6</td>
<td>8040.4</td>
</tr>
<tr>
<td>6</td>
<td>720</td>
<td>10000</td>
<td>9951.8</td>
<td>9953.4</td>
</tr>
<tr>
<td>7</td>
<td>580</td>
<td>15000</td>
<td>15294</td>
<td>15295</td>
</tr>
<tr>
<td>8</td>
<td>510</td>
<td>20000</td>
<td>19769</td>
<td>19770</td>
</tr>
</tbody>
</table>
Fig. 94 displays the variation of the fundamental natural frequency as a function of the length of the cantilever beam. The trend between the frequency and the length of the beam closely follows the analytical relationship shown in Equation (133).

![Graph showing the variation of natural frequency with length](image)

\[ f = 5.055 \times 10^3 L^{-1.997} \]

**Fig. 94.** Natural frequency versus length of the cantilever beams in the comb-like array

The steady-state linear dynamic analysis is then conducted on the comb-like array to obtain the FRF of the structure due to base excitation. A concentrated force is applied on the base and the response is measured at the tip of the eight AHCs (Fig. 93) and also at the location where the point force is applied. The resultant FRFs are illustrated in Fig. 95.
Fig. 95. Frequency response function of the comb-like AHC array

Fig. 95 shows that the array decomposes the input signal into its frequency content and works similar to a Fourier analyzer for the base excitation. As the nature of the array is to work as a Fourier analyzer, it is expected that the array shows a similar behavior when it is excited by acoustic signals.

7.3.2. Proposed fabrication process flow

In the previous section, an AHC and two different hair cell arrays were proposed. The piezoelectric beams have lengths ranging from 560 μm to 1600 μm. These beams are made of a seven-layer stack of material that will be deposited via sputtering. The electrode deposition will be completed by lift-off of the subsequent layers. The detailed fabrication process flow proposed for the AHCs is shown in the tables below.

Table 20. The detailed proposed fabrication process flow of the AHC

<p>| a. | silicon | a. RCA cleaning |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
</table>
| **b.** | b. LPCVD silicon oxide (SiO$_2$)  
1.5 µm  |
|   |   |
| **c.** | c. Photolithography: spin, expose,  
and develop 5µm of photoresist on  
the front and back sides  
(AZ9260 – AZ400K)  |
|   |   |
| **d.** | d.1. RIE of 1.5 µm of silicon oxide  
d.2. Strip photoresist using acetone  
d.3. RCA cleaning  |
|   |   |
| **e.** | e.1. Photolithography: spin, expose,  
and develop 5µm of photoresist  
(AZ9260 – AZ400K)  
e.2. LPCVD polysilicon 1.5µm  
e.3. Strip resist with acetone  
e.4. RCA cleaning  |
|   |   |
| **f.** | c.1. Sputtering 20nm of titanium  
c.2. Sputtering 200nm of platinum  
c.3. Sputtering 5µm of PZT  
c.4. Annealing at 650C  
c.5. Sputtering 200nm of platinum  
c.6. Sputtering 5µm of PZT  |
Table 21. The detailed proposed fabrication process flow of the electrodes

<table>
<thead>
<tr>
<th>a.</th>
<th>b.</th>
</tr>
</thead>
<tbody>
<tr>
<td>c.7. Annealing at 650C</td>
<td>a.1. Photolithography: spin, expose, and develop 4μm of photoresist (SPR220 – MF26A)</td>
</tr>
<tr>
<td>c.8. Sputtering 200nm of titanium</td>
<td>a.2. Ion beam milling gold/titanium 30nm/200nm ~ 10.9 mins.</td>
</tr>
<tr>
<td>c.9. Sputtering 30nm of gold</td>
<td>a.3. Strip photoresist with acetone</td>
</tr>
<tr>
<td>a.1. Photolithography: spin, expose, and develop 4μm of photoresist (SPR220 – MF26A)</td>
<td>b.1. RCA cleaning</td>
</tr>
<tr>
<td>a.2. Ion beam milling gold/titanium 30nm/200nm ~ 10.9 mins.</td>
<td>b.2. Photolithography: spin, expose, and develop 4μm of photoresist (SPR220 – MF26A)</td>
</tr>
<tr>
<td>a.3. Strip photoresist with acetone</td>
<td>b.3. Ion beam milling PZT 5μm ~ 55mins. *If PZT was not fully etched use a dilute solution of hydrofluoric acid to etch (Platinum will act as etch stop)</td>
</tr>
<tr>
<td>b.4. Strip photoresist with acetone</td>
<td>b.4. Strip photoresist with acetone</td>
</tr>
</tbody>
</table>
### c.
- c.1. RCA cleaning
- c.2. Photolithography: spin, expose, and develop 4μm of photoresist (SPR220 – MF26A)
- c.3. Ion beam milling platinum 200nm ~ 6.5mins.
- c.4. Strip photoresist with acetone

### d.
- d.1. RCA cleaning
- d.2. Photolithography: spin, expose, and develop 4μm of photoresist (SPR220 – MF26A)
- d.3. Ion beam milling PZT 5μm ~ 55mins. *If PZT was not fully etched use a dilute solution of hydrofluoric acid to etch (Platinum will act as etch stop)
- d.4. Strip photoresist with acetone

### e.
- e.1. RCA cleaning
- e.2. Photolithography: spin, expose, and develop 4μm of photoresist (SPR220 – MF26A)
- e.3. Ion beam milling of multi-layer stack 10.65μm ~ 8hrs.
Table 22. The detailed proposed fabrication process flow for the release of the micro cantilevers

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>a. LPCVP of silicon oxide (SiO$_2$) 1.5μm</td>
</tr>
<tr>
<td>b.</td>
<td>b. KOH etching on the backside of the wafer. 45% KOH solution at 90°C ~ 3.3hrs.</td>
</tr>
<tr>
<td>c.</td>
<td>c. BHF etching of silicon dioxide (SiO$_2$) on both backside and front side (1.5μm). 5:1 solution at room temperature ~15mins.</td>
</tr>
</tbody>
</table>
7.3.3. Experimental Setup and procedure

As the microfabrication of the MEMS devices is an extensive project, the concept of this research is first tested on a macro-scale AHC array. Preliminary tests are performed on a single macro-scale AHC and an array of four AHCs.

7.3.3.1. Test setup

Four bimorph piezoceramic beams from Piezo Systems, Inc. consisted of a 0.04 mm brass shim sandwiched between two PSI-5A4E piezoceramic layers, poled in opposite directions (for series operation) are used as the hair cells. The width and the total thickness of the beams are equal for all of the beams (3.2 mm and 0.66 mm, respectively). The lengths of the beams used are 31.75, 25.4, 19.05, and 12.7 mm [25]. Each beam is first clamped individually as a single AHC (Fig. 96 (a)) at the length of 25, 20, 15, and 10 mm, respectively. In the second test; all of the beams are clamped at the lengths mentioned previously to form an array of 4 AHCs (Fig. 96 (b)).

Fig. 96. AHCs, (a) Single AHC, (b) Array of 4 AHCs.
Fig. 97 shows the setup for the preliminary tests on the AHCs. The clamp is mounted on a 2007E mini shaker from The Modal Shop, Inc. to provide a base excitation for the AHCs and a 2100E21 amplifier from the same company is used to drive the shaker. A shear accelerometer is used to measure the base acceleration while a Polytec PDV-100 laser Doppler vibrometer is used to measure the tip velocity of the beams.

In order to measure the actual output voltage of the amplifier, a BK PR-60 active differential probe from B&K Precision Co. is used to attenuate the input voltage to the data acquisition system. A LMS SCADAS Mobile data acquisition system with 16 input channels is utilized to measure the acceleration of the base, velocity of the tip of the beams, input and output voltage of the amplifier, and sensed voltages from the piezoceramics. It is also used to apply the excitation signal to the amplifier connected to the shaker.

**7.3.3.2. Procedure**

In the first experiment, each beam is clamped at the length determined in the previous section, and a chirp signal in the range of 100 to 4096 Hz with a voltage level of 0.5 V is sent to the amplifier.
Measured acceleration at the base of the beam and the velocity of the tip are used to plot the FRF of the beam. In addition, the piezoceramic voltage and base acceleration are used as another means of capturing the FRF. In order to investigate the effects of the clamp, the FRF of the base is calculated using the acceleration of the base and the input voltage to the shaker.

In the second experiment, the beams are clamped at different lengths as mentioned previously and the spacing between them is 5.8 mm. A stepped sine analysis in the range of 150 to 3800 Hz is performed on the array, and the FRFs are obtained. To measure the velocity of the tip of each AHC in the array, the experiment is repeated four times by changing the location of the laser vibrometer to the tip locations at each different AHC.

7.3.4. **Experimental results and discussion**

The FRFs of the array of AHCs due to a base excitation is shown in Fig. 98. The clamping effect on the results can be better investigated from the FRF of the base shown in Fig. 98 (c). This effect is evident in the FRFs between 1500 Hz and 1700 Hz. The FRF of the base at 484, 746, 1213, 2569, and 3050 Hz does not show dominant peaks as seen in the FRFs of the beams. These frequencies correspond to the natural frequencies of the beams and are not affecting the clamp dynamics, showing very little coupling between the subsystems as designed.
Fig. 98. FRF of the AHC array obtained by measuring, (a) velocity of the tip of AHCs and base acceleration, (b) piezoceramic AHC voltages and base acceleration, and (c) input voltage to the shaker and base acceleration.

FRFs for the 25 mm AHC are tested as an individual beam, and the same beam is later tested in the AHC array of different lengths. The results for the beam are shown in Fig. 99. The response of AHC in both configurations show similar results for the fundamental frequency. The discrepancies around 1500 Hz are most likely due to the clamping mechanism.
Towards developing an active array of AHCs

The work presented in this section is an extension of the studies on creating active self-sensing AHCs and examines the behavior of an array of active AHCs subjected to complex stimuli rather than a stepped sine input. AHCs in the array are modeled as quadmorph self-sensing piezoelectric beams. Therefore, the piezoelectric layers are used for sensing the output of the AHC and supplying the control voltage to the system. This modeling approach is beneficial in future implementation of the array, as the control voltage for each beam is a function of its voltage output. This section consists of four sub-sections. The next section presents the AHC array modeled in this work. The third section discusses the response of a single AHC to a complex input before applying a multi-tone signal to the array. Subsequently, the fourth section displays the simulation results for the active AHC array.

7.4.1. Array of self-sensing artificial hair cells

In this work, an eight-channel active AHC array is designed and its behavior is studied numerically. Each AHC in the array is modeled as a self-sensing single-channel AHC and its length
is determined such that the fundamental frequency of the beam matches a particular frequency. Array frequencies are selected in the human speech range and are tuned to 200 Hz, 500 Hz, 800 Hz, 1 kHz, 3 kHz, 5 kHz, 8 kHz, and 10 kHz by adjusting the length of the AHC cantilevers. The active array is shown schematically in Fig. 100.

![Fig. 100. Schematic of the AHC array with its fundamental frequencies.](image)

In order to create an active array, a cubic damping controller is used for each AHC and its gains are tuned based on the dynamics of the AHC. The controllers adopted for self-sensing AHCs were introduced in Chapter 5 and are in the form of, \( V_c = \alpha_1 \dot{V}_{sensed} - \alpha_3 \ddot{V}_{sensed}^3 \). The linear damping gains used for the AHCs in this work are tabulated in Table 23.

<table>
<thead>
<tr>
<th>No.</th>
<th>Length (mm)</th>
<th>Fundamental Freq. (kHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>42.37</td>
<td>0.2</td>
</tr>
<tr>
<td>2</td>
<td>26.80</td>
<td>0.5</td>
</tr>
<tr>
<td>3</td>
<td>21.19</td>
<td>0.8</td>
</tr>
<tr>
<td>4</td>
<td>18.95</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>10.94</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>8.47</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>6.70</td>
<td>8</td>
</tr>
<tr>
<td>8</td>
<td>5.99</td>
<td>10</td>
</tr>
</tbody>
</table>

**Table 23. Schematic of the AHC array with its fundamental frequencies.**

<table>
<thead>
<tr>
<th></th>
<th>AHC1</th>
<th>AHC2</th>
<th>AHC3</th>
<th>AHC4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_1 (s) )</td>
<td>(9.27 \times 10^{-5} )</td>
<td>(7.18 \times 10^{-5} )</td>
<td>(5.68 \times 10^{-5} )</td>
<td>(5.08 \times 10^{-5} )</td>
</tr>
<tr>
<td>( \alpha_3 (s^3/V^2) )</td>
<td>(6 \times 10^{-11} )</td>
<td>(5 \times 10^{-11} )</td>
<td>(1 \times 10^{-9} )</td>
<td>(5 \times 10^{-9} )</td>
</tr>
</tbody>
</table>
The next section investigates the response of a single AHC to complex stimuli before embedding the AHC in the array.

### 7.4.2. Single AHC simulation results

This section evaluates the response of a single AHC to complex inputs as a starting point prior to studying the behavior of the array. The AHC used in this study is the second beam of the array shown in Fig. 100. Therefore, it is expected that this AHC affects the input signals with a frequency component near 500 Hz. Two types of signals are used in this study: a chirp signal and a periodic input. The chirp signal is applied to the AHC between 250 Hz and 2184.5 Hz and its amplitude is changed from 0.025 m/s² to 7 m/s². The time response of the AHC in response to the highest examined input level is shown in Fig. 101(a).

![Fig. 101](attachment:image.png)

**Fig. 101.** (a) Time-domain response of the active and passive AHC for a 7 m/s² chirp input acceleration, (b) Input-output plot.

As displayed in Fig. 101 (a), the maximum response of the uncontrolled AHC is compressed in the active AHC’s output. This maximum response corresponds to the input frequencies near the fundamental frequency of the AHC. The relationship between the input and output of the AHC is
studied using the plot illustrated in Fig. 101 (b). The input-output plot shows a nearly one-third power-law relationship between the base acceleration and the voltage. This shows that the AHC can mimic the mammalian cochlea’s behavior when excited by a complex stimulus. It is important to note that as the response spectra of the AHC is time-variant, the values for the sensed piezoelectric voltage are calculated using the short-time Fourier transform (STFT).

In the second step, a multi-tone sine signal is used as the input and the response of the AHC was computed. The results for this part of the study are shown in Fig. 102 (b), Fig. 103 (b), and Fig. 104 (b) in the next section. As the single AHC showed desired behavior in response to complex stimuli, a similar study is performed on an array of AHCs in the next section with the periodic input.

7.4.3. **AHC array simulation results**

In this section, the behavior of the AHC array for various input levels is studied. A signal in the form of \( \ddot{z}(t) = A(\sin(2\pi \times 200t) + \sin(2\pi \times 500t) + \sin(2\pi \times 1000t) + \sin(2\pi \times 3000t) + \sin(2\pi \times 5000t) + \sin(2\pi \times 8000t) + \sin(2\pi \times 10000t) \), where \( \ddot{z}(t) \) is the base acceleration and \( A \) is the excitation amplitude is applied to the array and the voltage response of each AHC is obtained. First, the signal’s amplitude is set to 0.025 m/s\(^2\) and the response of the AHCs to the stimulus is shown in Fig. 102.
Fig. 102. Magnitude sensed voltage of the AHC array for 0.025 m/s² input level.

Fig. 102 illustrates the output of the AHCs in two modes, i.e. passive and active or uncontrolled and controlled, against each other. As shown in this figure, the responses of each AHC is amplified at its fundamental frequency. For each AHC, amplification occurs in two steps: (1) amplification by the passive system due to the resonance-based nature of the AHC, (2) nonlinear amplification by the control law with respect to the passive system. As the AHC works in resonance region, the response is amplified when the frequency of the input matches the natural frequency of the beam. This provides an initial amplification for the signal and replicates the frequency selectivity of the cochlea. Subsequently, the controller injects nonlinearity to the AHC near the natural frequency such that the response is amplified or compressed with respect to the passive system. The amplification or compression provided by the controller depends on the input level and the cubic damping gain of the controller. To show the compression function of the active AHC array, the input level is increased to 5.5 m/s² and the results are displayed in Fig. 103.
Fig. 103. Magnitude sensed voltage of the AHC array for 5.5 m/s² input level.

As shown in Fig. 103, the response of each AHC is compressed relative to the passive system for 5.5 m/s² input level. It is evident from Fig. 103 that even though the active system compresses the output for each AHC at its fundamental frequency, the response level at the natural frequency remains significantly higher than any other frequencies. Therefore, the active system can detect the output at fundamental frequencies of the AHCs. To investigate the compressive rate of the array, the input-output plot for each AHC at the fundamental frequency is plotted in Fig. 104. In these plots, the input level is varied from 0.025 m/s² to 5.5 m/s².
According to Fig. 104, the response of the AHCs is amplified or compressed by a one-third power function of the excitation level and the AHCs can mimic the biological cochlea’s compressive nonlinearity. As shown in Fig. 104, each AHC’s mode changes from amplification to compression at a particular input strength that depends on the dynamic of the AHC and the cubic damping gain used in the controller corresponding to that AHC. The input-output plots show the good performance of the array when the system is excited by a complex stimulus.

7.5. Conclusion

The work discussed in this chapter investigated the potential of transforming MEMS scale cantilevers, initially designed for use as scanning thermal microscopy probes, into micro-scale artificial hair cells. These cantilever structures were fabricated by employing electron beam- and photo-lithography, together with Low Pressure Chemical Vapor Deposition, metal evaporation, dry- and wet-etching on n-type silicon wafer substrates. In this work, the dynamic characterization
of these micro-structures was the focus. A series of dynamic tests were conducted on the MEMS micro-structure as it was subjected to base excitation and its first four modes were identified. Furthermore, the dynamic characteristics of the MEMS system were investigated by varying the excitation levels and evaluating the limits of the structure's linearity. Based on the experimental findings, although the fundamental frequency of the tested cantilever was not in the hearing frequency range, the MEMS cantilevers can be used as active artificial hair cells with some modifications in the future. The proposed research on the MEMS AHCs is available in Chapter 8. In the second part of this chapter, developing piezoelectric, MEMS AHC arrays capable of mimicking the cochlea’s behavior was the main focus of the work. The AHC design consisted of a substrate and two layers of PZT deposition that represented AHCs. Subsequently, a fabrication process flow was proposed for the MEMS AHC. Abaqus finite element software was used to model the AHC arrays. Fundamental frequencies of the AHCs and FRFs due to a base excitation were obtained by linear perturbation frequency analysis and steady-state linear dynamic analysis, respectively. As a proof of concept, a series of dynamic tests were conducted on larger scale single AHCs and an array of four AHCs to measure the response of the sensors to an external stimulus. This preliminary study can be used in future design and fabrication of AHC arrays as described in Chapter 8. Finally, in the last study presented in this chapter, an array of eight self-sensing AHCs was modeled. The AHCs’ lengths were chosen based on the target frequencies considered for the array. A single AHC’s response to complex stimuli was presented and the AHC showed nonlinear behavior as it was excited by a chirp signal and a multi-tone sine input. Next, the array was excited by a multi-tone signal with different amplitudes. The input-output curves showed a compressive rate of about one-third and the output of the system amplified or compressed based on the input
level. The results also displayed that the AHC could mimic the frequency selectivity of the cochlea, as well as its compressive nonlinearity. The results obtained in the study showed the compatibility of the control laws developed in previous chapters with AHC arrays excited by complex stimuli. Therefore, this numerical study provided a framework for future experimental analysis of AHC arrays.
Chapter 8

Conclusion

A brief conclusion of the work presented in this dissertation and proposed future work are presented in this chapter.

8.1. Conclusions

Work herein has focused on leveraging the understanding of the complex mammalian cochlea in order to create systems that can accurately replicate its unique compressive nonlinear behavior. This work has been successful in advancing this front and creating, via electromechanically coupled systems, the first of its kind self-sensing multi-channel artificial hair cells that can mimic the cochlea’s compressive nonlinearity. The fundamental research carried out in this project has laid out the initial steps towards revolutionizing hearing restoration and has consequently leveraged the cochlea’s characteristics to create sensors with highly advanced dynamic range capabilities for a myriad of applications. This section summarizes the research presented in the previous chapters.

- Model development of active AHCs

A piezoelectric-based sensor inspired by the nonlinear behavior of the hair cells of the mammalian cochlea was designed. Inspired by the motion of the cochlear hair cells, an AHC was modeled as a beam excited by a base acceleration. Then, a nonlinear feedback control law was presented that could tune the SDOF system to a Hopf bifurcation in an attempt to mimic the nonlinear functions
of the outer hair cell. As part of this study, the stability of the autonomous controlled system was shown and the input-state stability of the nonautonomous system was proved. Then the steady-state response of the system to a harmonic input was studied. Finally, an extended control law was introduced that was applied to the 2DOF system with the goal of increasing the number of frequencies a single AHC can detect. To the best of the researcher’s knowledge, this is the first time a multi-channel single AHC is developed.

- **Numerical simulation of the AHCs**

Models of active single-channel and two-channel AHCs were created in Simulink. The response of the single-channel AHC was simulated with and without using a filter before the controller. For the system with filter, a new method of determining the linear damping gains of the controller was presented to account for the filtering effect. Comparison between the simulation results of the linear and nonlinear systems showed that the amplitude of the response close to each of the natural frequencies can be controlled without affecting the response close to the other natural frequencies. Also, it was concluded that exciting the system with different input levels showed the nonlinear amplification function of the system.

- **Real-time implementation of the multi-channel active AHC**

A novel two-channel active artificial hair cell consisting of a 26 mm piezoelectric bimorph cantilever was developed. To apply the phenomenological nonlinear feedback control law to the AHC, the controller was implemented in real-time. Stepped sine base excitation analysis in the frequency range of 320 Hz to 3.1 kHz was conducted on the AHC, and the feedback control voltage as a function of the linear and cubic velocity of the tip was applied to the piezoelectric actuators. Passive AHC analysis (without the control voltage) showed an inherent nonlinearity near the first natural frequency, while the system behaved almost linear near the second natural frequency. Due
to the nonlinearities observed in the FRFs of the preliminary tests, it was challenging to estimate the linear damping gains of the controller using analytical relations. Therefore, for each voltage level, a set of experiments were performed by gradually reducing the damping of the system in the absence of the cubic damping terms of the controller. Then, the circle-fit system identification method was used to accurately predict the system parameters, and the gain values required to reduce the linear damping of the system. As a result, the experimental approach adopted in the present work, including the calculation of the appropriate linear damping gains of the controller for various input levels, helps to better compensate for the filters’ and inherent nonlinearity’s effect on the behavior of the controlled system. In future, some of the parametric identification methods focussed on nonlinear systems will be explored for this application [208, 209]. The cubic damping was then added to the system tuned near the Hopf bifurcation at the first two natural frequencies. Exciting the system with different levels of input showed that the AHC’s sensitivity is a nonlinear function. The response magnitude was scaled by a power of about 0.34 of the input amplitude at the resonance. Below an excitation level (base acceleration) threshold, the AHC compressed the response close to the natural frequency, and above that threshold, the output was amplified, and the resonance peak was sharpened. As a result, the controlled system works very similar to the biological cochlea and is capable of mimicking the active nonlinear processes inside the cochlea. The stimulus level at which the behavior of the system switches from amplification to compression and vice versa could be adjusted by tuning the cubic damping gain of the controller.

It was seen that in the experimental FRFs of the controlled system, particularly the one close to the second natural frequency, the peak shifted towards the lower frequencies. This shift occurred due to the phase shift introduced into the system by the LP and BP filters. However, the phase shift introduced by the low-pass filter was about 180 degrees, and therefore, the maximum frequency
shift with respect to the passive system at the first natural frequency was about 0.23%, while for the second natural frequency it was about 0.33%.

Removing each of the controllers and comparing the FRF of the system with that of the passive case showed that the interference of the filters and controllers with each other was negligible. Thus, gains for each of the controllers can be tuned in the presence of the other controller without complications. Also, stepped sine analysis of the full controlled system demonstrated that, as desired, the controllers affected the response only near the natural frequencies of interest, while remaining intact at all the other frequencies.

Finally, to validate the model of the two-channel active AHC, values of the parameters used in simulations were set on the ones used in the experiment. The same approach, similar to the one adopted in the experiments, was taken to calculate the linear damping gains in the simulation results. The comparisons conducted between the experimental and numerical results show that the simulations match closely with the experimental data.

In conclusion, the novel two-channel active AHC shows advantages over a single-channel active AHC. Therefore, embedding a number of them in an array instead of a series of single-channel AHCs will offer larger frequency bandwidth in a more compact setting. These two-channel AHCs have the potential to be used in the creation of novel cochlear implants, accelerometers, microphones, hydrophones, and other sensors.

- **Theoretical and numerical realization of self-sensing active AHCs**

In this work, a self-sensing active artificial hair cell capable of amplifying and compressing the response based on the stimulus signal level was developed. The active AHC consisted of a quadmorph piezoelectric cantilever controlled by a cubic damping closed-loop controller. The derivative of the sensed voltage from the outer piezoelectric layers was used in the
phenomenological feedback control law to tune the AHC near a Hopf bifurcation and mimic the compressive nonlinearity of the cochlear amplifier. The control voltage was then supplied to the inner piezoelectric layers and provided symmetric actuation of the AHC.

To examine the adoption of a quadmorph for symmetric actuation, first, a series of preliminary tests were conducted on the quadmorph with a symmetric configuration of the sensing and actuation layers. Next, the asymmetric configurations, split-bimorph, were tested, where one layer on each side of the beam was used as the sensor or actuator. Experimental results of the quadmorph showed that only bending modes were excited, while that was not the case for the split-quadmorph. The FRFs of the split-quadmorph showed some peaks that did not correspond to bending modes. Therefore, due to the asymmetric actuation, the response of the split-quadmorph contained longitudinal components that would be very challenging and difficult to compensate by the controller. Therefore, the quadmorph was identified as an appropriate candidate for self-sensing AHC.

Next, a finite element model of the self-sensing quadmorph AHC was developed. Subsequently, it was used to derive a SDOF model of the controlled system. The steady-state response of the active AHC was obtained via the harmonic balance method and showed that the input and output of the AHC hold a one-third power-law relationship at the Hopf bifurcation at the natural frequency, analogous to its biological counterpart. The equations derived in that chapter were used in Chapter 6 to compare the experimental results with the AHC model for model validation.

The active AHC was then implemented in Simulink and the response of the system, its input-output plots, and FRFs were displayed for different stimulus levels and cubic damping gain. To make the design of the AHC system more feasible for the ultimate goal of this research, i.e. developing sensors with broader dynamic range and contemporary cochlear implants, a filter was
added to the control system to define an operating frequency bandwidth for the AHC. Effects of
the filter on the active system’s magnitude and phase FRFs were discussed and it was shown that
the system softens as the input level increases. However, the percent change in the natural
frequency was very small and as the AHC operates over a frequency range, it is expected that this
shift in the natural frequency will not hinder the behavior of the AHC, especially in cochlear
implant applications were a few hertz changes in the frequency will not drastically affect one’s
speech perception [210, 211]. The active AHC with filter mimicked the amplification/compression
behavior of the biological cochlear amplifier with similar power-law relationships.

Finally, Chapter 5 has laid the foundational work to replicate the mammalian cochlea’s
compressive nonlinear behavior via self-sensing artificial hair cells. Furthermore, this study has
shown that quadmorph piezoelectric beams are appropriate candidates for use in control
applications, especially in creating active AHCs. The self-sensing AHC does not require additional
space for embedding external sensing modules that were required for the feedback loop in non-
self-sensing systems. Furthermore, this important feature allows future miniaturization of the
AHCs for embedding them inside the human cochlea. Promising results obtained from the
numerical analysis of the self-sensing AHCs paved the way for real-time implementation of the
AHCs and its experimental validation that were examined in Chapter 6.

- **Experimental validation of the self-sensing active AHC**

Chapter 6 was dedicated to the experimental implementation and model validation of the self-
sensing artificial hair cell made of a piezoelectric quadmorph cantilever driven to a Hopf
bifurcation with a nonlinear feedback controller. The stepped sine analysis near the first natural
frequency of the AHC showed the amplification/compression behavior of the self-sensing system
for various cubic damping gains. Furthermore, the compressive rates for the nine examined cubic
damping gains were between 0.36 and 0.39, a very close approximation to the desired one-third rate seen in the biological cochlea. Examining the sensitivity of the AHC as a function of the cubic damping gain showed approximately an inverse one-third power-law relationship between these parameters. Therefore, the obtained least square fit on the sensitivity data, in conjunction with the compressive rates estimated from the output-input curves could be used to approximate the behavior of the AHC at the natural frequency for any given cubic damping gains. The intersection of the output-input curves for the uncontrolled AHC and the controlled system could then determine the input level at which the AHC’s mode switched from amplification to compression. To evaluate the performance of the self-sensing algorithm, the sensed voltage rate and the tip velocity measurements were compared against each other. The linear relationship between the piezoelectric sensed voltage rate and the tip velocity for various input levels and cubic damping gains showed excellent performance of the self-sensing system.

To validate the models created in Chapter 5, a series of comparisons were conducted between the numerical results and the experimental results. The results for the uncontrolled systems showed a close match between the experimental and numerical results for the lowest examined input level. It is important to point out that in this chapter the base accelerations measured during the active AHC experiments were used as the input for the model to partially account for the dynamics of the clamp and shaker. The model could predict the behavior of the self-sensing AHC at the natural frequency for the nine different gains with less than 1% error in the sensitivity and less than 5.5% error in the compressive rate. Overall, the SDOF model of the AHC was able to closely predict the dynamics of the active AHC.

Lastly, the number of channels in a self-sensing AHC was increased and multi-channel self-sensing AHCs were developed. The control law was extended to remove the linear damping of the
system near the second natural frequency, as well as the first natural frequency. The input-output plots demonstrated a compressive rate of about one-third close to the second natural frequency. As the multi-channel self-sensing AHCs provides additional sensing channels for each AHCs, a series of these AHCs with different lengths can be mounted in an array format to increase the sensing bandwidth for the device. Using the sensed piezoelectric voltage from each of the self-sensing AHCs will make it possible for the device to simultaneously amplify or compress the output near each of the characteristic frequencies.

In summary, Chapter 6 validated the approach of using a piezoelectric quadmorph cantilever controlled by a cubic damping feedback controller to create self-sensing active AHCs through experiments. To the best of the researcher’s knowledge, such a self-sensing active AHC was created for the first time in the literature. Advantages of the self-sensing AHC including independence from an external sensor, the voltage output of the system that could be transmitted directly from the AHCs electrode, and the feasibility of adoption of these AHCs in an array format as mentioned earlier makes this design superior to the prior designs of the bimorph AHCs with external velocity feedback.

- Preliminary extension of the AHC research

In Chapter 7, a preliminary study on developing MEMS AHCs was conducted. The dynamics of a MEMS cantilever structure initially developed for scanning thermal microscopy was characterized by modal analysis. The first four natural frequencies, mode shapes, and modal damping of the structure were obtained by exciting the base of the structure and measuring its velocity response at different locations. Results of the analysis showed that the first natural frequency of the cantilever was about 25 kHz which was not in the bandwidth of the human auditory system. Although this specimen was not in the auditory range, the MEMS cantilever seems to be an ideal
candidate for AHC studies especially given its linear behavior. In future studies, the MEMS beams will be designed in the human auditory range and the cubic damping feedback control law will be adopted to create active AHCs, as mentioned later in this chapter. Therefore, the current work was a preliminary study to develop MEMS AHCs.

In another preliminary study presented in Chapter 7, a unique piezoelectric-based MEMS sensor inspired by the hair cell’s function of the mammalian cochlea was designed and simulated in Abaqus. Inspired by the geometry of the mammalian hair cells, a set of artificial hair cells was designed as micro cantilevered beams. Each beam consisted of a multi-layer stack of a substrate, piezoelectric material, and electrodes. Furthermore, a detailed fabrication process flow for MEMS AHC fabrication was proposed as a part of Chapter 7. For a fixed cross-sectional area, the length of the beam determined the sensitivity of the AHC to a particular frequency. For sensing a broader range of frequency, similar to the function of the tonotopic BM in the cochlea, two arrays of AHCs were designed consisting of different length cantilever beams. Frequency analysis was then conducted on both the comb-like and fishbone AHC arrays. FRFs obtained by applying a constant force to the base of the comb-like array and measuring the responses at the AHC tips showed the filtering function of the array on the stimuli. Moreover, as a baseline for testing arrays of AHCs, some experiments were performed on the macro scale AHCs with a maximum length of 25 mm. Experimental results showed that the length of the beams could be adjusted individually to obtain a particular fundamental frequency and then these beams can be used in an array of AHCs to function as a discrete mechanical filter with a bandwidth defined by the range of the natural frequencies of the beams.

Finally, to investigate the possibility of adopting the control laws developed in this work in an AHC array, an array was modeled and studied numerically. The array consisted of eight
quadmorph cantilevers with fundamental frequencies between 200 Hz and 10000 Hz in the speech range. The AHCs’ lengths were chosen based on the target frequencies considered for the array. A cubic damping controller was used for each AHC and the system was tuned to the Hopf bifurcation. As an AHC array is intended to mimic the tonotopic BM and the cochlear amplifier, a complex input was used in Section 7.4. First, the response of a single AHC to a chirp input and a periodic multi-tone sine signal was obtained. Results showed that the AHC was able to detect the signal’s component that matched the AHC’s fundamental frequency. Furthermore, it could amplify or compress the detected signal component at a compressive rate of about one-third. In the next step, the periodic signal was applied to the array and its amplitude was varied. The input-output curves showed that the targeted compressive rate (about one-third) was achieved and the output of the system was amplified or compressed depending on the input level. Therefore, the array’s AHCs showed desired behavior near the fundamental frequencies of the beams. The results also displayed that the AHC could mimic the frequency selectivity of the cochlea. Furthermore, it was observed that even for the highest input amplitude studied in this work where all the AHCs in the system compressed the response, the output at each fundamental frequency was detectable. In summary, this study showed the compatibility of the control law developed for single AHCs with AHC arrays that were excited by complex stimuli. Therefore, the active AHCs can be used as suitable candidates in an array format for sensor development or cochlear implants, as they can mimic the frequency selectivity and compressive nonlinearity of the cochlea. Lastly, the numerical study presented in this study provided a framework for future experimental analysis of AHC arrays.
8.2. Future work

This section presents some of the possible directions for future work that can lead to the fabrication of a fully implantable cochlear prosthesis, as well as sensor development. These research areas are described as follows.

- **Designing and fabricating a micro-scale active artificial hair cell**

The present work showed that the macro active AHC with a phenomenological nonlinear feedback control law could mimic the cochlear active functions. As practical implementation of the AHCs needs miniaturization of these sensors, the development of MEMS active AHCs is proposed as a part of future research. In a collaboration with the University of Glasgow, the potential of transforming MEMS scale cantilevers, initially designed for use as scanning thermal microscopy probes, into micro-scale artificial hair cells was investigated in Section 7.2. A series of dynamic tests were conducted on a MEMS micro-structure developed by Dobson et al. [205] as it was subjected to a base excitation. A MSA-500 Polytec laser vibrometer was used to measure the response of the cantilevers. The dynamic characteristics of the MEMS cantilever were studied by varying the excitation levels and evaluating the limits of the structure's linearity. This was a starting point for subsequent development and modification to achieve micro-scale AHCs. In future studies, the MEMS structures will be designed in the human auditory range (20 Hz to 20 kHz) and a phenomenological control law inspired by the mammalian cochlea will be applied on the cantilevers via heat excitation using feedback actuation. The possibility of using heat excitation as the control input will be investigated and the control law will be modified accordingly. Moreover, if successful, the possibility of depositing piezoelectric material on these probes will be explored in an attempt to fabricate self-sensing AHCs. Furthermore, the proposed fabrication process for creating piezoelectric MEMS AHCs shown in Section 7.3.2, will be used as a starting point for
fabricating piezoelectric MEMS AHCs. These AHCs will be tested using the MSA-500 laser vibrometer as they are excited by a base excitation or an acoustic stimulus.

- **Designing, modeling, and fabricating an active artificial hair cell array**

To replicate the mammalian cochlear amplifier in a broad bandwidth and create novel cochlear implants or dynamic sensors, an array of active AHCs is required. To move towards creating an array of active AHCs, it is proposed that a sample array be constructed using a few sensors to transduce a small set of frequencies. In the first step, an array of macro AHCs needs to be tested with the cubic damping control law applied to the response of each AHC in the array. System identification techniques developed in the present research can be used to determine the model parameters and tune the controller gains. The response of the array to various signals including single tone and multi-tone signals should be evaluated. In the next step, MEMS AHC arrays will be constructed and tested. The preliminary work on AHC arrays described in Section 7.3 proposed two possible designs that should be evaluated experimentally as a part of future work. The response of the beams will be monitored using the sensors’ self-sensing mechanism and verified using a laser vibrometer. Besides driving the system by a base excitation, an acoustic test will be performed on the array. Furthermore, at micro scales, the role of surrounding air can be significant. Therefore, physics-based models along with system identification methods for characterizing the role of air will be examined. In addition, some experiments will be conducted in a partial vacuum to study the role of air on the sensors’ performance.

- **Advancing multi-physics understanding of fluid-embedded active AHCs to mimic the cochlear amplifier**

Initial work on active artificial hair cells in air showed great promise in mimicking the cochlear amplifier, but applying these sensors in a fluidic environment similar to biological hair cells
requires the understanding of the effects of fluid-structure interaction. Therefore, understanding of the fluid-structure interaction and control laws in an environment similar to the operational environment of the cochlear hair cells is a direction for future work. Understanding the multi-physics of AHCs in the biological cochlea’s inherent environment can lead to the fabrication of cochlear implants with nonlinear characteristics and highly improved sensors such as hydrophones and flow sensors. Furthermore, it is hypothesized that the fluid immersion will have a positive effect on the frequency selectivity by increasing overall system damping and allowing the nonlinear controller to increase response at the natural frequency and therefore improving selective frequency and sensitivity of the AHCs. This hypothesis will also be evaluated in the future.

In order to perform this study, the AHC will be coated with a waterproof layer and embedded in a fluid with similar characteristics as the cochlear perilymph and its dynamics will be characterized. Subsequently, the effects of mass loading and damping introduced by the fluid on the system’s dynamics will be considered and added to the prior AHC model in an increased difficulty first by taking into account the effects of mass loading and subsequently by a full fluid-structure interaction model. This will provide a fundamental understanding of both the coupled fluid-structure interaction and the nonlinear control law.

- Advancing understanding of inherently nonlinear piezoelectric AHCs

Studies on the AHCs presented in previous chapters showed that the inherent nonlinearity observed in the AHC, combined with the effect of filters used in the system, shift the sensed frequency. This can hinder the ability of an AHC array to accurately sense a complex signal in a narrow bandwidth. Therefore other areas for future work are to investigate the sources and types of the inherent nonlinearities associated with AHCs, to understand their effects on the active system’s response and their sensitivity to various input and boundary conditions. Prior work on the active AHCs has
shown that the piezoceramics are viable options for active AHCs that can reproduce very closely the one-third compressive nonlinearity of the cochlea. However, the inherent nonlinearities associated with the AHC system leave much work to be done in coping with these nonlinearities to increase the sensed frequency’s accuracy and improve the compressive rate of the AHC, whether these nonlinearities are geometrical, boundary, or inherent to the material. Therefore, it is proposed that the dynamics of the passive AHC under different loading conditions be characterized and the effect of various parameters and conditions on the inherent nonlinearities, such as boundary conditions, piezoelectric material, and geometry of the AHC be investigated. For instance, the effects of boundary conditions at the base of the AHC on its inherent nonlinearities should be studied by characterizing the system with different boundary conditions such as a clamped or bonded. These studies will lead to identifying the sources of the nonlinearities and their dominance in the system. Furthermore, to model the AHC, Timoshenko beam theory should be considered in future work to create a more realistic model of the beam.

Accordingly, the results of this study can be combined with the previous proposed work and create a full model of the active fluid-embedded artificial hair cell. This can be accomplished by exploring nonlinear models or adopting data-driven modeling techniques to model the AHC in the fluid and compensate for the effect of nonlinearities in the control law, theoretically and experimentally.
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