Essays on Financial Economics

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Dissertation submitted to the faculty of the
Virginia Polytechnic Institute and State University
in partial fulfillment of the requirements for the degree of

Doctor of Philosophy
in
Economics

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March 19, 2021
Blacksburg, Virginia

Keywords: Capital Structure, Dynamic Signaling Game, Adverse Selection, Ownership and Control, Wealth Constraint, Bargaining, Shapley Value Innovation, Product Enhancement, Entry Deterrence, Creative Destruction

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ABSTRACT

This dissertation consists of three papers. In the first paper, I study firms' capital raising decisions in a two-stage signaling game. In the model, firms can issue debt or equity to finance sequentially arriving investment projects. Management is assumed to have an initial information advantage over investors. However, when a firm's decision in the first stage can change investors' beliefs and, consequently, impact the security issuance in the second stage, its optimal choice differs significantly from the strict debt-equity preference in a comparable one-stage model. In equilibrium, a dynamic pecking order arises, suggesting that the information friction can solely explain various aspects of observed corporate financing behavior.

The second paper is coauthored with Hans Haller. In this paper, we model how different wealth constraints among investors affect an entrepreneur's way of raising capital, his share of project NPV, and his ownership of the new firm. Combining cooperative and noncooperative approaches, we develop and analyze a bargaining framework and demonstrate cases in which a fair division cannot be achieved when sharing of cost and sharing of return are jointly considered. Our results cover conditions on how the entrepreneur can strategically achieve larger net wealth accumulation, and when he can obtain control of the firm. We further discuss the entrepreneur's preferences on the firm's ownership dispersion level under public financing.

The third paper argues that although innovation is costlier than imitation, the incumbent firm is endowed with an advantage of enhancing its product ahead of potential competitors. In a model that connects consumers' utility with firms' production, I show that the incumbent's product enhancement decision can foster the creation of a better product, improve consumers' utility, and deter entrance from competitors. The pace of creative activities is determined by the incumbent's potential of improving its product quality and the nature of product differentiation in the industry. Thus, creative destruction may not manifest itself as new firms replacing the incumbent, but as the incumbent constantly renovating its product.

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GENERAL AUDIENCE ABSTRACT

This dissertation consists of three papers. In the first paper I study the adverse selection problem faced by firms in a dynamic information environment, the difference between incentives provided by debt and equity securities, and how different contracts and model settings affect the equilibrium outcome, investment efficiency, and social welfare. The premise of the first paper is that dynamic elements of information asymmetry are key to better understanding how firms raise capital. This study aims to provide a more complete description and improve our understanding of the role of information in capital markets and how asymmetric information might interact with other market frictions.

In the second paper I study the origin of the firm and the bargaining problem between entrepreneurs and investors. This second paper intends to provide one possible answer for the question why firms do exist. The main point in the paper is that even when we abstract away from standard frictions like adverse selection or moral hazard, an entrepreneur still has to bargain with investors to raise the required amount of capital. The firm has to be established to enforce the bargaining outcome, which takes the form of an ownership contract, because there is a time gap between conducting the investment and when the proceed can be realized. Another purpose of this second study is to investigate fairness instead of efficiency.

Finally, in the third paper, I address the question how and when an incumbent monopolist can deter entry by means of investment in product quality enhancement. In some industries, creative destruction can be frequently observed: Incumbent firms are replaced by new firms that offer slightly different but better products. On the other hand, in a number of industries incumbent firms are at the forefront of innovation and stay ahead of potential entrants. I consider a model that allows for the latter fact combined with another frequent fact: that potential entrants more or less copy the incumbent's prior product, regardless of existence and enforcement of intellectual property rights. This third paper offers predictions on product innovation and market failure across firms and industries.

To mom and dad,

mentors and professors,

friends and colleagues,

and

the finance and economics academia

Acknowledgments

First of all, I am very grateful for the guidance and encouragement of my dissertation committee. Throughout the past few years, I had numerous discussions with Dr. Haller about how to do theoretical research and how to write models. Apart from that, we also chatted about many other issues, and I particularly enjoyed our conversation on the history and origin of the English language. I cannot thank him more for his advice during my Phd studies and I believe his insights will continue to guide me going forward. I would like to thank Dr. Bahel for his knowledge and generosity. I think his game theory course further inspired and motivated me to pursue my current research. I would also like to thank Dr. Bose for his very considerate support at the beginning of the pandemic when I was in need of a lot of help. My sincere thanks also go to Dr. Sarangi, for his suggestions to my paper and his understanding and willingness to participate in my dissertation committee.

I am also indebted to all the faculty members that have taught me one way or the other in the past years. I learned a lot of things throughout the time, whether it is a seminar, or a course, or a referee report. My apologies for not being able to list everyone's name on this page, but those names and happy memories will always be in my heart. I could never reach this stage of life without faculty members' knowledge and wisdom. Especially, I would like to dedicate my special thanks to Dr. Kats.

I also feel fortunate to have encountered many other Phd students at Virginia Tech. I can imagine that my life might have become very miserable without being able to share many thoughts with other students. I also feel lucky to have taught many lovely undergraduate students. Their smiles always light up my day.

Finally, although this is the third time that I acknowledge parents in my dissertations, and although parents never understood the contents of my dissertations, my love and appreciation for them only go deeper. As always, I couldn't have done everything without their unconditional love and support.

Contents

1 Chapter One

Corporate Financing and Investment Decisions When Equity Issuance Reveals Firms' Information to Investors

Abstract

I study firms' capital raising decisions in a two-stage signaling game. In the model, firms can issue debt or equity to finance sequentially arriving investment projects. Management is assumed to have an initial information advantage over investors. However, when a firm's decision in the first stage can change investors' beliefs and consequently impact the security issuance in the second stage, its optimal choice differs significantly from the strict debt-equity preference in a comparable one-stage model. In equilibrium, a dynamic pecking order arises, suggesting that the information friction can solely explain various aspects of observed corporate financing behavior.

1.1 Introduction

How do firms finance their investment projects in a market with frictions? Since Myers and Majluf (1984), the capital structure literature has constantly debated the validity of the pecking order theory. Despite the insightful argument about the adverse selection cost, the traditional information friction argument has been challenged by much of the empirical evidence.[1] Specifically, unlike the model prediction, equity financing has rarely been used as a last resort, and issuing activities are typically conducted during good times, e.g., high stock valuation, rather than under duress (Fama and French 2005). In this paper I propose a two-stage dynamic model to reconcile the classical pecking order theory with the seemingly contradictory empirical evidence.

I first establish a one-stage signaling model as the benchmark. In the benchmark scenario, the firm can be either a high type or a low type. The high type firm has a positive NPV

[1] At best, the literature agrees that the evidence on testing the pecking order theory is inconclusive (Leary and Roberts 2010).

project and the low type firm has a negative NPV project. The firm can raise capital from investors by issuing either debt or equity. Investors, on the other hand, do not know the firm's type but can observe the firm's choice of securities and update their beliefs. Suppose both types of firms pool at issuing debt, then if the firm turns out to be a low type firm, investors are protected by the standard debt contract feature and can capture all the available proceed. However, when both types of firms pool at issuing equity, investors only capture part of the proceed if they encounter a low type firm. Thus, the low type firm always has an incentive to issue equity if it can pool with the high type firm at a price above the investment cost. Under this logic, an equity issuance becomes a bad signal of a firm's quality. If investors update their beliefs, then the equilibrium solution in the benchmark model is that both types of firms pool at issuing debt.

I then extend the benchmark model to have two stages. In each stage the firm faces an investment project and can choose debt or equity financing. In this two-stage model, the high type firm's incentive is no longer the same as in the benchmark. Specifically, in the one-stage benchmark, the high type firm always prefers debt. However, in the two-stage model, the high type firm can first choose to issue equity at a sufficiently low price, e.g., slightly below the investment cost, at stage one to deter the low type firm from issuing equity. When investors are convinced that such behavior can only come from a high type firm, this high type firm can then issue safe debt at the second stage. This alternative equilibrium solution can only be supported in a two-stage model since it requires the high type firm to first take a small loss at stage one, or at least not to make a profit. Thus, although issuing equity is a bad signal in the benchmark model, it is possible that issuing severely underpriced equity can become a good signal in a two-stage model.

In order to see how the assumption on a firm's type affects the equilibrium outcome, I re-examine the same research question by assuming the type is distributed continuously on an interval. In the continuous-type setting, I also use a one-stage model as the benchmark for a two-stage model. The results show that if there is only one investment project, the equilibrium outcome under the continuous-type setting resembles its two-type counterpart, and all firms pool at issuing debt. However, when there are two stages, the analysis under the two-type setting is no longer applicable to the continuous-type model. Instead, there are two possibilities in equilibrium. The first possibility is that the lower spectrum of good firms (i.e., firms with positive NPV projects) issue severely underpriced equity to signal their types, whereas the higher spectrum of good firms pool with bad firms (i.e., firms with negative NPV projects) and issue debt. The second possibility is that all good firms issue severely underpriced equity to separate themselves from bad firms.

The comparison between the two-type setting and the continuous-type setting shows that both specifications have their strengths and weaknesses. For instance, the two-type setting is generally simpler and convenient to work with. This simplicity allows us to deduce major inferences quite efficiently. On the other hand, the continuous-type setting requires more computation but can provide us with finer details and predictions. For example, other than the prediction on the debt and equity preference, we can also use the equilibrium outcome under the continuous-type setting to explain the equity issuance announcement effect. Yet this detail is absent under the two-type setting.

Following the literature originated from the costly state verification model (e.g., Townsend 1979), I also study a variant of the previous signaling model by allowing investors to costly verify a firm's type before deciding whether to participate in the investment activity. I find that when firms are facing more capable investors, good firms are better off no matter which security they decide to issue. Specifically, good firms can offer a lower interest rate or a higher equity price while still providing enough incentives for investors to participate in the issuance. Since the true type of the firm can be revealed either through costly verification (when the cost is low) or through signaling (when the cost is high), the region that can support the pooling at debt equilibrium becomes much smaller compared with the previous signaling model.

Overall, my findings suggest that even if debt is preferred to equity in a one-stage model, this preference may no longer be true in a two-stage model. In most scenarios, whether a firm prefers debt or equity depends not only on its profitability type and growth opportunities, but also on the market's perception as well as the cost for investors to verify the type. It is very likely that a firm has more incentives to violate the pecking order precisely when that the adverse selection problem is most severe. Thus, the dynamic element of the asymmetric information casts serious doubt on interpretations of traditional empirical tests of the pecking order theory. Especially, evidence on firms violating the financing hierarchy perhaps only has a limited power on differentiating adverse selection from other leverage determinants. Hopefully, future research can further shed light on this issue.

This paper joins the voluminous development on the capital structure literature by studying the dynamic feature of asymmetric information. Since Akerlof (1970), many studies have investigated how information travels among economic agents, and how the uneven distribution of information has led to deviations from the Modigliani and Miller (1958) world. Earlier studies that use the multi-stage setting mainly focus on one type of security. For instance, Welch (1989) studies the IPO and SEO underpricing patterns when there exists an imitation cost for low quality firms. Chemmanur (1993) shows that IPOs are underpriced when insiders need to provide incentives for outsiders to produce information. Lucas and McDonald (1990) extend Myers and Majluf (1984) to an infinite horizon and predict the stock price behavior around the time of equity issues.[2] On the other hand, the literature that models pecking order violations typically does not involve multiple issues. For example, Nachman and Noe (1994) derive the necessary and sufficient conditions for debt to be optimal rather than equity. Fulghieri and Lukin (2001) discuss how information sensitivity of different securities can make a firm's security choice endogenous.[3] The model in this paper incorporates

[2]More recent contributions like Daley and Green (2012) model the process that firms have information that is gradually released by news, and investors/buyers can choose to wait. Bond and Zhong (2016) provide a unified framework in analyzing SEOs and repurchases.

[3]For other possibilities see, for instance, Fulghieri, Garcia and Hackbarth (2020), who investigate the scenario in which assets in place and growth options have different exposure to information asymmetry. Bolton and Dewatripont (2005) also summarize some possible scenarios for pecking order violations (e.g., p. 112-120).

both aforementioned features in describing corporate financing activities.

To model how security issuances can reveal firms' information to investors, I first focus on the signaling channel (e.g., Spence 1973, Cho and Kreps 1987), and then combine both the impact of signaling as well as costly state verification. Prior signaling games in the capital structure literature have explored the possibility that different debt levels can separate good firms from bad firms (Ross, 1977), insiders sometimes imperfectly observe a firm's cash flow (Noe 1988), and firms underprice in the IPO market to signal their types (Allen and Faulhaber 1989).[4,5] Subsequently to these early developments, the security design literature builds models that incorporate both the signaling effect and the impact of the information sensitivity of securities. For instance, DeMarzo and Duffie (1999) study how designers of securities trade off retention cost and illiquidity caused by private information. Biais and Mariotti (2005) analyze the issuance problem in the presence of both adverse selection and market power.[6] My model resorts to signaling as well, yet does not consider a possible mix of securities like the security design literature. Instead, I still use the standard debt and equity contracts and focus on how the debt-equity preference can change when firms need to interact with uninformed investors over multiple projects.[7]

Aside from the information friction, other market frictions also contribute to the determinants of real world financing patterns. Numerous alternative explanations have emerged to describe the discrepancy between Myers and Majluf (1984) and empirical stylized facts. For instance, issuing equity can alleviate the debt overhang problem (Myers 1977), reduce the bankruptcy cost (Myers 1984, Bradley, Jarrell and Kim 1984), mitigate the agency cost of debt (Jensen and Meckling 1976), enlarge a firm's investor base (Merton 1987). Baker

[4]In Allen and Faulhaber (1989) good firms have certain possibilities to become bad firms, whereas in my model the type does not change across time.

[5]Other earlier papers that include the signaling effect can be found in, for example, Leland and Pyle (1977), in which the fraction of equity retained by the entrepreneur signals project quality, and Harris and Raviv (1985), who use signaling to explain the puzzle related to convertible debt calls.

[6]For other earlier studies on the security design problem see, e.g., Boot and Thakor (1993), Nachman and Noe (1994). A model of security design with pooling and tranching is studied in DeMarzo (2005).

[7]When combining signaling with costly state verification, I currently do not model any investor heterogeneity. However, in an earlier version of this paper, I considered a static setting that allows investors to have different types as in Rock (1986) and Fulghieri and Lukin (2001).

and Wurgler (2002) propose that firms issue equity to time the market. Dittmar and Thakor (2007) develop a model by considering the belief alignment between managers and investors.[8] A dynamic pecking order can also arise when one combines information with other frictions. For example, Bolton and Freixas (2000) and Hennessy, Livdan and Miranda (2010) combine information with the liquidation cost. Faure-Grimaud and Gromb (2004) build a model in which insiders' effort can change the firm value.[9] While these explanations provide valuable insights in describing the observed financing behavior, the model in the present paper explains the pecking order violation directly based on the adverse selection problem caused by information asymmetry.

Technically, my paper provides a formal illustration of the difference between the two most commonly used model settings in games of incomplete information: two-type versus continuous-type. As shown in Tirole (2006), the two-type setting can be a powerful tool in drawing inferences, and researchers can benefit from the direct applicability of many existing solution concepts developed over the finite type space (e.g., Fudenberg and Tirole 1991). My analysis indicates that sometimes this technical detail can be irrelevant to the main conclusion, and at other times researchers will have to trade off between simplicity and generality when selecting the most appropriate model setup.

My paper contributes to the long-lasting debate of capital structure theories (for surveys on the empirical evidence see, e.g., Frank and Goyal 2008, Graham and Leary 2011), the progress on information economics, and the ongoing vivid development on leverage dynamics (e.g., DeMarzo and He 2020). As stated in Fama and French (2005), "...both the tradeoff model and the pecking order model have serious problems... Perhaps it is best to regard the two models as stable mates, with each having elements of truth that help explain some aspects of financing decisions." (p. 580-581). To resolve the inconsistency between the empirical evidence and the tradeoff theory, Hennessy and Whited (2005) develop a dynamic

[8]Another example for heterogeneous priors is Boot and Thakor (2011), who develop a dynamic pecking order model by considering the agency problem between a firm's initial owners and managers.

[9]Another example see Inderst and Mueller (2006), in which the lenders' preference on debt or equity depends on the outcome of their project screening.

tradeoff model by endogenizing several factors like investment and leverage choices.[10] Admati et al. (2018) study leverage dynamics when firms cannot commit to future funding choices. Perhaps complementary to their approach, my model builds on the traditional pecking order theory and extends the setting to incorporate dynamic debt and equity issuing activities.[11]

The rest of the paper proceeds as follows: Section 1.2 introduces a dynamic model under a two-type setting. Section 1.3 studies the same dynamic model under a continuous-type setting. Section 1.4 presents a variant model in which investors can costly verify a firm's type. Section 1.5 discusses some relations with the empirical research. Section 1.6 concludes. All proofs are embedded.

1.2 A Two-type Model

Consider a firm living in a two-stage economy, with stages occasionally referred to as today ($t = 1$) and the future ($t = 2$). At each stage the firm faces an investment project that requires external financing. These projects are identical and require I amount of investment to yield R_θ at the end of time ($t = 3$). The return of the project can be either high or low, $R_\theta \in \{R_L, R_H\}$, with $R_L < I < R_H$. Thus, the high type firm has a positive NPV project, and the low type firm has a negative NPV project. To ease notation, I use type $\theta \in \{L, H\}$ whenever there is no confusion.

The firm has no financial slack and no intermediate return is available. There are no taxes, transaction cost or time discount. I further assume that there is no managerial distortion, and throughout this paper the manager and the firm are used interchangeably. The firm can choose either equity or debt financing at $t = 1$ and $t = 2$. The timeline is illustrated in

[10] A review of related development on dynamic models and structural estimation can be found in Strebulaev and Whited (2012).

[11] To clarify a potential confusion with the terminology, the term "dynamic" in my paper borrows from the game theory literature that studies incomplete information games. Thus, the "dynamic" part of my model implies that the information is updated dynamically. On the other hand, the same term "dynamic" can be also found in the literature that relies on the continuous-time models (Brownian motion). For instance, dynamic capital structure models with changing debt levels can be found in Goldstein, Ju and Leland (2001), DeMarzo and He (2020). Dynamic real option signaling models that consider corporate decisions as signals can be found in Grenadier and Malenko (2011), Morellec and Schürhoff (2011).

Figure 1.

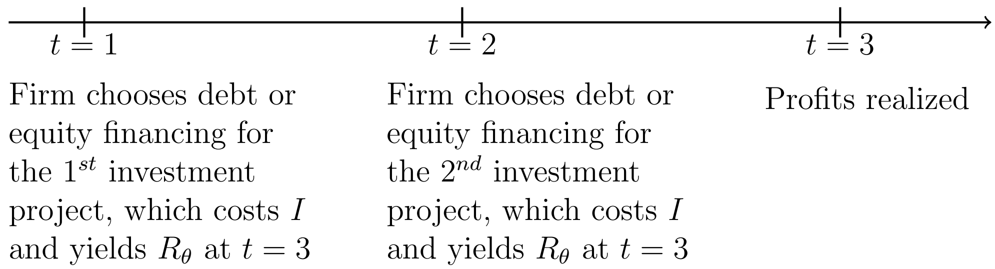


Figure 1: Timeline of events. This figure shows the timeline of events for a firm with a profitability type R_θ. There are two identical investment projects that arrive sequentially at $t = 1$ and $t = 2$. The firm can choose between equity and debt to finance each project. Profits are realized at $t = 3$.

In the following sections, I first analyze the incentive for every party in a one-stage model, then study the changes of these incentives in a two-stage setting.

1.2.1 One project

Suppose only the first project exists. To simplify the discussion, the ownership of the project is represented by one share prior to $t = 1$. The manager knows the exact realization of R_θ. If the manager chooses to issue equity, he decides the price p and the portion α of the project's share to be sold. Thus, the amount of equity issued needs to cover the investment cost $\alpha p = I$.[12] The total return for investors is $(\frac{I}{p}R_\theta - I)$, and the payoff for the firm is $\mathscr{L}^F_e(\theta) = (1 - \frac{I}{p})R_\theta$.

If the manager chooses to issue debt (or bond), he decides the interest rate r. Thus, the debt contract specifies that the firm will borrow I at $t = 1$ and return $I(1 + r)$ at $t = 3$ if $R_\theta \geq I(1 + r)$. Otherwise the firm becomes insolvent and pays all its available return R_θ to investors. The total return for investors is $\min[R_\theta - I, Ir]$. The payoff for the firm is $\mathscr{L}^F_d(\theta) = \max[0, R_\theta - I(1 + r)]$.

Denote the available action of the firm as $a_F \in \{p, r\}$. Presumably, the manager can also decide not to issue securities. However, not issuing security is weakly dominated by

[12] In this paper I only consider the case that the manager raises the required amount of capital. For a different approach, Hart and Moore (1998) study the optimal debt contract in a model that allows the entrepreneur to raise more than needed funds for the investment project.

debt issuance, as we can see from the payoff of the firm that $\mathscr{L}_d^F(\theta) \geq 0$. Thus, we can concentrate our discussion on the case of issuance.[13]

Investors have a sufficient amount of wealth to invest but are uninformed about the firm's type. Their prior belief $\mu(\theta)$ about the return distribution is that the probability of θ being a high type is μ, and being a low type is $1-\mu$, i.e., $\mu(H) = \mu$ and $\mu(L) = 1-\mu$. Denote the average of R_θ as $\bar{R}$. Assume investors' prior belief satisfies $\bar{R} = \mu R_H + (1-\mu)R_L > I$. Since all the investors are identical, they can be considered to act collectively as one party. Denote the available action for investors as $a_U \in \{P, N\}$, in which P denotes participate in investment, and N denotes not participate.

The firm moves first to determine which security to issue, and then investors can update their belief based on the firm's choice. Denote $\mu(\theta|a_F)$ as the posterior belief. Thus, we can summarize the expressions of the payoffs for the firm and investors as the following:

$$\mathscr{L}^F(\theta, a_F, a_U) = \begin{cases} \mathscr{L}_e^F(\theta) = (1 - \frac{I}{p})R_\theta & \text{if } a_F = p, a_U = P \\ \mathscr{L}_d^F(\theta) = \max[0, R_\theta - I(1+r)] & \text{if } a_F = r, a_U = P \\ 0 & \text{if } a_U = N \end{cases} \tag{1}$$

$$\mathscr{L}^U(a_F, a_U) = \begin{cases} \mathscr{L}_e^U = \sum_\theta \mu(\theta|p)(\frac{I}{p}R_\theta - I) & \text{if } a_F = p, a_U = P \\ \mathscr{L}_d^U = \sum_\theta \mu(\theta|r)\min[R_\theta - I, Ir] & \text{if } a_F = r, a_U = P \\ 0 & \text{if } a_U = N \end{cases} \tag{2}$$

Since I focus on pure-strategy equilibria, it is not necessary to specify a probability distribution over actions. Thus, I slightly abuse the notation and continue to use a_F^* and a_U^* to denote strategies for the players. Following the prior literature on signaling games (e.g.,

[13]For example, if the equilibrium is pooling, then the low type firm has no incentive to separate itself by choosing not to issue security. If the equilibrium is separating, then regardless of the issuance decision of the low type firm, investors will react optimally by not investing. Nonetheless, Noe (1988) studies a model that incorporate all three options: issuing debt, equity, and not issuing. In his model, having the option of not issuing security is necessary since the firm already has an asset that generates positive payoff. Yet this setting is absent in the current paper.

Noe 1988, Welch 1989), as well as various equilibrium definitions (e.g., Kreps and Wilson 1982, Fudenberg and Tirole 1991), I define a perfect Bayesian equilibrium (PBE) as the following:

Definition 1. *A perfect Bayesian equilibrium of the security issuance game is a strategy profile* (a_F^*, a_U^*) *and posterior beliefs* $\mu(\theta|a_F)$ *such that:*

(i) $\forall\theta$, $a_F^*(\theta) \in \arg\max_{a_F} \mathscr{L}^F(\theta, a_F, a_U^*)$,

(ii) $\forall a_F$, $a_U^*(a_F) \in \arg\max_{a_U} \mathscr{L}^U(a_F, a_U)$

(iii) Whenever a_F *is an on-the-equilibrium action, the posterior belief is given by*

$$\mu(\theta|a_F) = \begin{cases} 0 & \text{if } a_F^*(\theta) \neq a_F \\ \frac{\mu(\theta)}{\sum_{\{\theta'|a_F^*(\theta')=a_F\}} \mu(\theta')} & \text{otherwise} \end{cases}$$

and the posterior belief can be any probability distribution if a_F *is an off-the-equilibrium action.*

Conditions (i) and (ii) in the above definition are the perfection conditions, meaning that the firm maximizes its payoff given investors playing their best responses, and investors act optimally to the firm's action given their posterior beliefs. Condition (iii) means that beliefs of investors update according to Bayes' rule whenever possible.

Below Figure 2 provides an illustration of the game described above. Strictly speaking, this figure is not correct in the sense that there are infinitely many choices for each security, e.g., many possible prices and interest rates. Nevertheless, we may still find such an illustration helpful when relating the current model to some classical signaling games (e.g., the signaling game in Cho and Kreps 1987).

Since games with incomplete information often face the issue of infinitely many equilibria, and many of those equilibria may not provide meaningful implications, I first discuss two refinements on the equilibrium selection. The first refinement is about ruling out the possibility that the two types of firms can separate themselves within the same type of se-

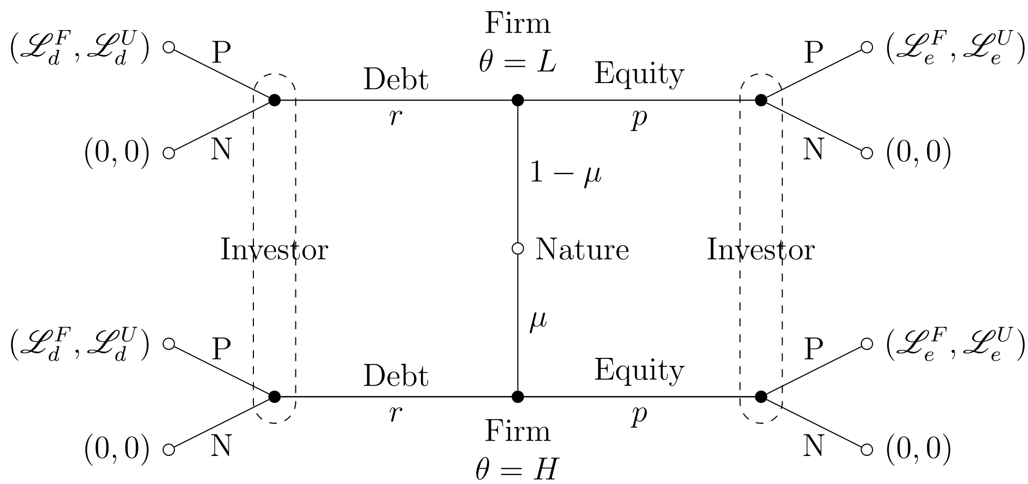


Figure 2: Decision Tree. The game is played by the firm and investors. The firm moves first and select a price p or interest rate r. Upon observing what has been offered by the firm, investors move to choose whether to participate (P) or not (N).

curity. This refinement is reasonable given that there is no private cost for selecting a price or interest rate. For instance, suppose the high type firm chooses price p_1, and the low type firm chooses price p_2. If $p_1 > p_2$, then the low type will strictly benefit from increasing the price. In other words, since investors do not invest in the low type firm whenever it separates itself from the high type firm, it is in the best interest of the low type firm to mimic the high type firm's behavior.[14] The second refinement is the Intuitive Criterion developed by Cho and Kreps (1987). I will discuss this refinement in more detail whenever it is used.

In the current model, we still encounter many equilibria even after the above two refinements. I first introduce these equilibria and then discuss a third refinement at the end of this section to further restrict our attention to only one of them. The reason for delaying the use of this third refinement is to prepare for the comparison between equilibria in the

[14]Roughly speaking, there are two types of signaling game settings. The first one is more similar to Spence (1973), in which the cost of signaling is a function of the continuously distributed signals. In this type of setting, the focus of the equilibrium solution is how to separate types within the same class of choices (e.g., education level). Models with this type of setting can be seen in, for instance, Ross (1977), DeMarzo and Duffie (1999). The second type of setting is more similar to the beer-quiche example in Cho and Kreps (1987), in which there is a fixed cost for selecting the less preferred choice. In this second type of setting, the focus of a separating equilibrium (if exists) is often a separation in between different classes (e.g., beer or quiche). A model using this type of setting is Welch (1989). In my paper, the setting is more in line with the second type of signaling games, and the focus is the firm's choice between debt and equity. Thus, the first refinement is reasonable given the focus of this paper. Especially, selecting a price or interest rate by itself does not seem to be costly.

one-stage and the two-stage models.

Equilibrium 1. ***(Pooling at debt)*** *The following strategy-belief combination constitutes an equilibrium.*

Firm's choice: $a_F^*(\theta) = r^* = \frac{1-\mu}{\mu}\frac{I-R_L}{I}$, $\forall\theta \in \{L, H\}$.

Investors' choice: $a_U^*(r^*) = P$.

Investors' belief along the equilibrium path: $\mu(H|r^*) = \mu$, $\mu(L|r^*) = 1-\mu$.

Investors' belief off the equilibrium path: $\forall a_F \notin \{r^*\}$, $\mu(H|a_F) \leq \mu_0$, $\mu(L|a_F) \geq 1-\mu_0$, *with* $\mu_0 = \frac{I-R_L}{R_H-R_L}$. *Investors' best response to out-of-equilibrium message is not to invest,* $a_U^*(a_F) = N$.

Payoffs for the firm and investors are: $\mathscr{L}_d^F(H) = R_H - I(1+r^*)$, $\mathscr{L}_d^F(L) = 0$, $\mathscr{L}_d^U = 0$.

Proof. The expression of r^* comes from the following:

$$\begin{aligned}\mathscr{L}_d^U &= \sum_\theta \mu(\theta|r)\min[R_\theta - I, Ir] \\ &= \min[\mu(R_H - I) + (1-\mu)(R_L - I), \mu Ir + (1-\mu)(R_L - I)]\end{aligned}$$

Investors participate whenever $\mathscr{L}_d^U \geq 0$. Since $\mu(R_H - I) + (1-\mu)(R_L - I) = \bar{R} - I > 0$, then the binding constraint $\mathscr{L}_d^U = \mu Ir^* + (1-\mu)(R_L - I) = 0$ gives the expression for r^*.[15]

The expression of μ_0 comes from letting $\mu_0 R_H + (1-\mu_0)R_L = I$. As a result, whenever $\mu(H|a_F) < \mu_0$, investors believe the average firm has a negative NPV project and choose not to invest. □

Equilibrium 2. ***(Pooling at equity)*** *The following strategy-belief combination constitutes an equilibrium.*

Firm's choice: $a_F^*(\theta) = p^* = I$, $\forall\theta \in \{L, H\}$.

[15] Note that if investors choose not to invest, the payoff is also zero. However, not to invest is not a strategy that can survive reasonable "trembles" from the firm. For instance, if the firm choose $r^* + \varepsilon$, then $\mathscr{L}_d^U > 0$ and $a_U^*(r^*) = P$. More descriptions on such trembles can be found in, for instance, Selten (1975), Kreps and Wilson (1982).

Investors' choice: $a_U^*(p^*) = P$.

Investors' belief along the equilibrium path: $\mu(H|p^*) = \mu$, $\mu(L|p^*) = 1 - \mu$.

Investors' belief off the equilibrium path: $\forall a_F \notin \{p^*\}$, $\mu(H|a_F) \leq \mu_0$, $\mu(L|a_F) \geq 1 - \mu_0$, *with* $\mu_0 = \frac{I - R_L}{R_H - R_L}$. *Investors' best response to out-of-equilibrium message is not to invest,* $a_U^*(a_F) = N$.

Payoffs for the firm and investors are: $\mathscr{L}_e^F(H) = 0$, $\mathscr{L}_e^F(L) = 0$, $\mathscr{L}_d^U = \mu(R_H - I) + (1 - \mu)(R_L - I) = \bar{R} - I$.

As shown above, the game has two pooling equilibria. Although the pooling at debt equilibrium seems more reasonable, both above equilibria can pass the Cho-Kreps Intuitive Criterion. In the current model, the Intuitive Criterion can rule out all pooling at equity equilibria if $a_F^*(\theta) = p > I$. The argument goes as follows. In case of $p > I$, the low type firm is strictly better off by choosing equity since $\mathscr{L}_e^F(L) > 0 = \mathscr{L}_d^F(L)$. Thus, it is not reasonable for the low type firm to send the message of debt. However, the high type firm can surely send the message of debt by knowing the payoff of the low type firm. Investors, on the other hand, will correctly interpret the message of debt as it can only come from the high type firm. With this logic, a pooling at equity equilibrium breaks down whenever $p > I$.

Here we can see that although the Cho-Kreps Intuitive Criterion is fairly efficient in reducing the set of equilibria, there is still one pooling at equity equilibrium that can survive this test. Specifically, if both firms choose $p^* = I$, then issuing equity is no longer a strictly better strategy for the low type firm. In this scenario, the high type firm cannot credibly send the off-equilibrium message.[16]

While the above Equilibrium 2 seems unfavorable, the game also has two separating

[16] Naturally, there are other equilibrium refinement concepts, like the idea of Divinity in Banks and Sobel (1987). For instance, if we look at Equilibrium 2, then it says that the low type firm is more likely to deviate than high type firm (since $\mu_0 < \mu$). While this tendency might be true if the off-equilibrium message is $p > I$, it might be the opposite if the off-equilibrium message is r. Thus, some of these off-equilibrium messages might not be reasonable under alternative refinement concepts. In this paper I focus on the Intuitive Criterion since it is more straight forward to apply in the current setting, and combining Intuitive Criterion with the later introduced third refinement is sufficient to restrict the set of equilibrium solutions.

equilibria that might look unreasonable.

Equilibrium 3. ***(Separating, H choose equity and L choose debt)*** *The following strategy-belief combination constitutes an equilibrium.*

Firm's choice: $a_F^*(H) = p^* = I$, $a_F^*(L) = r$.

Investors' choice: $a_U^*(p^*) = P$, $a_U^*(r) = N$.

Investors' belief along the equilibrium path: $\mu(H|p^*) = 1$, $\mu(L|r) = 1$.

Investors' belief off the equilibrium path: $\forall a_F \notin \{p^*, r\}$, $\mu(H|a_F) \leq \mu_0$, $\mu(L|a_F) \geq 1 - \mu_0$, *with* $\mu_0 = \frac{I-R_L}{R_H-R_L}$. *Investors' best response to out-of-equilibrium message is not to invest,* $a_U^*(a_F) = N$.

Payoffs for the firm and investors are: $\mathscr{L}_e^F(H) = 0$, $\mathscr{L}_d^F(L) = 0$, $\mathscr{L}_e^U = R_H - I$, $\mathscr{L}_d^U = 0$.

Proof. Observe that the high type firm cannot set any price $p > I$, for otherwise the low type can be better off by deviating. □

Equilibrium 4. ***(Separating, H choose debt and L choose equity)*** *The following strategy-belief combination constitutes an equilibrium.*

Firm's choice: $a_F^*(H) = r' = 0$, $a_F^*(L) = p$.

Investors' choice: $a_U^*(r') = P$, $a_U^*(p) = N$.

Investors' belief along the equilibrium path: $\mu(H|r') = 1$, $\mu(L|p) = 1$.

Investors' belief off the equilibrium path: $\forall a_F \notin \{p, r'\}$, $\mu(H|a_F) \leq \mu_0$, $\mu(L|a_F) \geq 1 - \mu_0$, *with* $\mu_0 = \frac{I-R_L}{R_H-R_L}$. *Investors' best response to out-of-equilibrium message is not to invest,* $a_U^*(a_F) = N$.

Payoffs for the firm and investors are: $\mathscr{L}_d^F(H) = R_H - I$, $\mathscr{L}_e^F(L) = 0$, $\mathscr{L}^U = 0$.

The reason that the above two separating equilibria may not seem reasonable is that we are not very sure why a priori the low type firm would have any incentive to pursue such option. For instance, in Equilibrium 4, the low type firm is equally better off by switching to debt financing. Yet the previous two refinements cannot filter out these two equilibria. This problem motivates the use of the third refinement, which is described as follows.

Consider the low type firm adopting the following strategy "mimic high type firm's behavior whenever possible". It can be seen that such strategy is a weakly dominating strategy. Thus, the newly introduced refinement involves both types of firms anticipating each other's behavior. Formally, I define the refinement concept as the following:[17]

Definition 2. *(Selection Criterion) The low type firm chooses the same strategy as the high type firm whenever possible, and the high type firm anticipates such behavior from the low type firm.*

We can see that the above Selection Criterion subsumes the previously undefined first refinement. Under the Selection Criterion, Equilibrium 3 and Equilibrium 4 are no longer valid. If we then revisit the two pooling equilibria, we can see that the high type firm strictly prefers Equilibrium 1.[18] Thus, a high type firm will choose to issue debt at interest rate r^* when knowing the low type firm will mimic its behavior. On the other hand, a low type firm will also choose to issue debt since it knows the high type firm's preference. Under this logic, Equilibrium 1 becomes the only equilibrium that can survive both the Intuitive Criterion and the Selection Criterion.[19] Below Proposition 1 summarizes this outcome.

Proposition 1. *(Two-type model) If there is only one project, both the high type firm and the low type firm issue debt at interest rate r^*.*

Since the combination of the Cho-Kreps Intuitive Criterion and the Selection Criterion is very effective in eliminating unfavorable equilibria, I continue to use these two refinements

[17] It is possible for other equilibrium selection concepts to overlap with the newly introduced Selection Criterion. For instance, if we look at Equilibrium 1 to Equilibrium 3, then the payoffs for firms in Equilibrium 1 Pareto dominate those in Equilibrium 2 and Equilibrium 3. However, as can be seen in the following content, the ad hoc refinement Selection Criterion works efficiently in reducing the set of equilibria.

[18] Since $R_H - I(1 + r^*) > 0$.

[19] This argument is akin to the use of iterated weak dominance (IWD). Yet IWD may generate different outcome when the order of elimination is changed. For instance, if we start with Equilibria 1 to 4, then issuing debt is a strictly dominating strategy for the high type firm. Thus after one round of elimination, Equilibria 1 and Equilibria 4 survive. In between Equilibria 1 and 4, the low type will choose Equilibria 1 to avoid revealing itself, but the high type actually prefers Equilibrium 4.

in the remaining paper when finding the equilibrium solution.

To conclude this section, we can see that in a one-stage model, one can obtain a similar pecking order theory prediction as in Myers (1984), Myers and Majluf (1984), i.e., debt is preferred to equity. However, as I will show in the next section, this preference may no longer be true in a two-stage model.

1.2.2 Two projects

Consider the two projects as shown in Figure 1. If at $t = 1$ both types of firms pool at issuing debt as in Equilibrium 1, then no information is revealed before $t = 2$ and investors possess the same prior belief when facing the second issuance. Thus, the issuance game at $t = 2$ repeats itself exactly the same way as in the one-stage model described in the previous section. Under this scenario, we know from Proposition 1 that both firms issue debt. Below Equilibrium 5 summarizes the possible outcome that firms pool at issuing debt twice.

Equilibrium 5. ***(Pooling at debt twice)*** *The following strategy-belief combination appears at both $t = 1$ and $t = 2$.*

Firm's choice: $a_F^*(\theta) = r^* = \frac{1-\mu}{\mu}\frac{I-R_L}{I}$, $\forall \theta \in \{L, H\}$.

Investors' choice: $a_U^*(r^*) = P$.

Investors' belief along the equilibrium path: $\mu(H|r^*) = \mu$, $\mu(L|r^*) = 1 - \mu$.

Investors' belief off the equilibrium path: $\forall a_F \notin \{r^*\}$, $\mu(H|a_F) \leq \mu_0$, $\mu(L|a_F) \geq 1 - \mu_0$, with $\mu_0 = \frac{I-R_L}{R_H-R_L}$. Investors' best response to out-of-equilibrium message is not to invest, $a_U^*(a_F) = N$.

Total payoffs for the firm and investors at $t = 3$ are: $\mathscr{L}_{dd}^F(H) = 2R_H - 2I(1 + r^*)$, $\mathscr{L}_{dd}^F(L) = 0$, $\mathscr{L}_{dd}^U = 0$.

On the other hand, when firms have two projects, the above Equilibrium 5 might not be the most desirable outcome for the high type firm. Imagine that if the high type firm can manage to separate itself at $t = 1$, it can raise capital from the same pool of investors at a fair price at $t = 2$. Specifically, when the type is revealed to investors, the high type firm can finance the second project by issuing safe debt and capture the entire proceed $(R_H - I)$. Since issuing debt as in Equilibrium 1 cannot effectively separate the high type from the low type, then the only possible choice for the high type firm is to issue equity at a price that is sufficiently unattractive to the low type firm. Below Equilibrium 6 summarizes this possibility.[20]

Equilibrium 6. ***(Separating, High type issue equity)***

(i) The following strategy-belief combination appears at $t = 1$.

Firm's choice: $a_F^*(H) = p^* - \varepsilon = I - \varepsilon$, $a_F^*(L) = r$.

Investors' choice: $a_U^*(p^* - \varepsilon) = P$, $a_U^*(r) = N$.

Investors' belief along the equilibrium path: $\mu(H|p^* - \varepsilon) = 1$, $\mu(L|r) = 1$.

Investors' belief off the equilibrium path: $\forall a_F \notin \{p^* - \varepsilon, r\}$, $\mu(H|a_F) \leq \mu_0$, $\mu(L|a_F) \geq 1 - \mu_0$, with $\mu_0 = \frac{I - R_L}{R_H - R_L}$. Investors' best response to out-of-equilibrium message is not to invest, $a_U^*(a_F) = N$.

(ii) The following strategy appears at $t = 2$.

The high type firm issues safe debt ($r = 0$) and investors participate.

Investors do not participate in any security issued by the low type firm.

(iii) Total payoffs for the firm and investors at $t = 3$ are: $\mathscr{L}_{ed}^F(H) = (1 - \frac{I}{I - \varepsilon})R_H + R_H - I$, $\mathscr{L}_{dd}^F(L) = 0$, $\mathscr{L}_{ed}^U = R_H \frac{I}{I - \varepsilon} - I$.

Proof. Observe that if the low type firm mimics the high type firm, it gets a negative payoff $\mathscr{L}_{ed}^F(L) = (1 - \frac{I}{I - \varepsilon})R_L + \min[0, R_L - I] < 0$. □

The above equilibrium shows that the high type firm has to issue severely underpriced equity to discourage the low type firm from mimicking. The issuing price for the first project is not just lower than the true type (R_H), but also lower than the average price $(\bar{R})$. In fact, the high type firm not only has to transfer the entire proceed of the first project to investors

[20] In this equilibrium we can also directly let $\varepsilon \to 0$, yet the current specification might look more natural at first sight.

(if issues at $p^* = I$), but also has to take a small loss (ε) so that the low type firm does not find mimicking profitable.

Naturally, a few practically concerns might arise when interpreting Equilibrium 6. For instance, negative ownership is not possible. Here the small loss of ε can be equivalently interpreted as the cost of issuing equity, e.g., fees paid to financial intermediaries.

Second, in the model the manager has to sell the entire first project to investors. This outcome may not look desirable but practically, the manager or the initial owners do not start with zero wealth. Thus, the manager can also act like investors and purchase the project's share at price $p^* - \varepsilon$. In this way, effectively only part of the project is sold to outsiders.

Finally, Equilibrium 6 can serve as a good benchmark for a comparison between some model variations. For instance, in Section 1.4, I consider investors can choose to costly verify a firm's type, and the equilibrium equity price can be well above p^*.

Comparing Equilibrium 6 with the previous Equilibrium 3 we can see that the high type firm does not have any incentive to take the loss of ε if there is only one project, as this action gives a negative total payoff (e.g., revising p^* to $p^* - \varepsilon$ in Equilibrium 3). However, when the high type firm can benefit from the second project, the previous unreasonable Equilibrium 3 might become reasonable as in Equilibrium 6. On the other hand, revisiting the previous Equilibrium 4 can show us why debt financing cannot separate the high type firm from the low type firm. Here we can see that no matter what choices a high type firm makes, it cannot force the payoff of the low type firm to become negative when deviating from the equilibrium. Thus, mimicking the high type firm is always "free" for a low type firm under debt financing, but can be "costly" under equity financing.

Since ε is very small, I will set $\varepsilon \to 0$ for the remaining discussion whenever there is no confusion. With the result of Equilibrium 5 and Equilibrium 6, we can obtain the following Proposition 2 by comparing the payoffs of the high type firm.

Proposition 2. *(Two-type model) In the presence of two projects, when $\mu \in (\mu^*, 1)$, the high* *type firm and the low type firm pool at issuing debt at* $t = 1$*. When* $\mu \in (\mu_0, \mu^*)$*, the high type firm issues equity at* $t = 1$ *to separate itself from the low type firm. Where* $\mu_0 = \frac{I - R_L}{R_H - R_L}$*, and* $\mu^* = \frac{2I - 2R_L}{R_H + I - 2R_L}$.

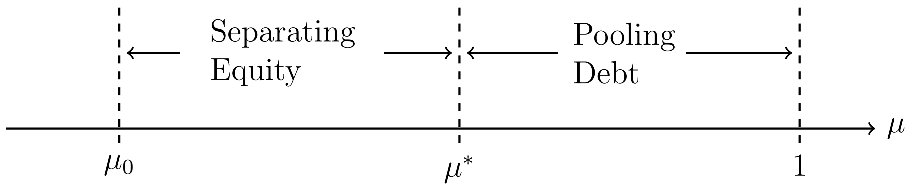


Figure 3: An illustration of Proposition 2.

Proof. (1) μ_0 is obtained from the model setting, in which $\bar{R} = \mu R_H + (1-\mu) R_L > I$. (2) μ^* is obtained by letting $\mathscr{L}^F_{ed}(H) = \mathscr{L}^F_{dd}(H) \Rightarrow (1 - \frac{I}{I - \varepsilon}) R_H + R_H - I = 2R_H - 2I(1 + r^*)$. Given $r^* = \frac{1-\mu}{\mu} \frac{I - R_L}{I}$, we have $\mu^* \to \frac{2I - 2R_L}{R_H + I - 2R_L}$ when $\varepsilon \to 0$. $\square$

Proposition 2 shows that the high type firm has more incentives to pursue the separating equilibrium when the prior belief on the probability of a firm being a high type firm is low. This result is intuitive in the sense that if investors don't have a strong prior belief, they demand a high interest rate when firms are issuing debt. Thus the potential benefit of a high type firm revealing itself is higher. On the other hand, when investors believe most firms are high type firms (μ is very large), then a high type firm prefers debt financing, since issuing equity would impose a significant amount of loss.

Comparing Proposition 1 and Proposition 2 we can see that in a two-stage model, debt financing may not be the most preferred choice, and the high type firm has incentive to violate the pecking order when issuing equity can reveal its type to investors. From the society's point of view, Equilibrium 6 is better than Equilibrium 5 because no negative NPV projects can receive funding. Thus, allowing the firm to have repeated interactions with investors can potentially improve the overall welfare and investment efficiency.

1.3 A Continuous-type Model

In corporate finance, much of the existing inference can be made by assuming that firms have two types, e.g., in various models in Tirole (2006). As shown in the previous section, we can also deduce the main prediction of this paper by employing such a setting. However, although this two-type setting can be sufficient to show the difference between the one-stage and two-stage models, such simplification on the distribution of firms might fail to capture predictions on some other dimensions. Therefore, in this section I re-examine the same research question by assuming that the type is distributed on an interval. Specifically, $R_\theta \in [R_L, R_H]$. Same as before, I use type θ and R_θ interchangeably whenever there is no confusion.

1.3.1 One project

Denote investors' prior belief about the return distribution as $f(\theta)$, which is the probability density function. To simplify the solution concept, I assume that $f(\theta)$ is continuously distributed on $[R_L, R_H]$, with $R_L \geq 0$ and $f(\theta) > 0$, $\forall R_\theta \in [R_L, R_H]$.[21] Denote the expectation of R_θ as $\bar{R}$. Similar to the previous two-type model, I assume that investors have prior belief that the average firm has a positive NPV project, i.e., $I \in (R_L, \bar{R})$.

The firm moves first with an action $a_F \in \{p, r\}$, and then investors can choose whether to participate or not, $a_U \in \{P, N\}$. Denote $f(\theta|a_F)$ the posterior belief. The expressions of payoffs for the firm and investors are summarized as follows:

$$\mathscr{L}^F(\theta, a_F, a_U) = \begin{cases} \mathscr{L}_e^F(\theta) = (1 - \frac{I}{p})R_\theta & \text{if } a_F = p, a_U = P \\ \mathscr{L}_d^F(\theta) = \max[0, R_\theta - I(1+r)] & \text{if } a_F = r, a_U = P \\ 0 & \text{if } a_U = N \end{cases} \tag{3}$$

[21]Strictly speaking, the specification of continuity is not necessary and one could allow the return distribution to have some disjoint points. Yet the continuous distribution is more convenient to work with.

$$\mathscr{L}^U(a_F, a_U) = \begin{cases} \mathscr{L}_e^U = \int_{R_L}^{R_H} f(\theta|p)(\frac{I}{p}R_\theta - I)dR_\theta & \text{if } a_F = p, a_U = P \\ \mathscr{L}_d^U = \int_{R_L}^{R_H} f(\theta|r)\min[R_\theta - I, Ir]dR_\theta & \text{if } a_F = r, a_U = P \\ 0 & \text{if } a_U = N \end{cases} \tag{4}$$

I also revise the definition of perfect Bayesian equilibrium:

Definition 3. *A perfect Bayesian equilibrium in the continuous-type model of the security issuance game is a strategy profile* (a_F^*, a_U^*) *and posterior beliefs* $f(\theta|a_F)$ *such that:*

(i) $\forall\theta$, $a_F^*(\theta) \in \arg\max_{a_F} \mathscr{L}^F(\theta, a_F, a_U^*)$,

(ii) $\forall a_F$, $a_U^*(a_F) \in \arg\max_{a_U} \mathscr{L}^U(a_F, a_U)$

(iii) Whenever a_F *is an on-the-equilibrium action, the posterior belief is given by*

$$f(\theta|a_F) = \begin{cases} 0 & \textit{if } a_F^*(\theta) \neq a_F \\ \frac{f(\theta)}{\int_{\{\theta'|a_F^*(\theta')=a_F\}} f(\theta')dR_{\theta'}} & \textit{otherwise} \end{cases}$$

and the posterior belief can be any probability distribution $g(\theta)$, *such that* $\int_{R_L}^{R_H} g(\theta)dR_\theta = 1$, *if* a_F *is an off-the-equilibrium action.*

Similar as in the previous section, conditions (i) and (ii) in the above definition are the perfection conditions, and condition (iii) means that beliefs of investors update according to Bayes' rule whenever possible.

To tackle the problem associated with infinite type sets, observe that loosely speaking, we can still classify firms into two general categories: those that have positive NPV projects (good firms, $R_\theta > I$), and those with negative NPV projects (bad firms, $R_\theta < I$). For firms with negative NPV projects, they are strictly better off if they can pool with some good firms and issue equity at a price above $p^* = I$. Thus, the previous strategy "mimicking good firms whenever possible" is still weakly dominating its alternatives. In order to focus our attention on the most meaningful equilibrium, I will continue to use the Cho-Kreps Intuitive

Criterion and the Selection Criterion as refinements.[22]

If firms only issue security once, then the below pooling at debt equilibrium is still the most attractive equilibrium to good firms. Define r^{**} to be the smallest positive value to satisfy[23]

$$\mathscr{L}_d^U\Big|_{r^{**}} = \int_{R_L}^{R_H} f(\theta)\min[R_\theta - I, Ir^{**}]dR_\theta = 0$$

then the pooling equilibrium can be summarized as the following:

Equilibrium 7. ***(Pooling at debt)*** *The following strategy-belief combination constitutes an equilibrium.*

Firm's choice: $a_F^*(\theta) = r^{**}$, $\forall R_\theta \in [R_L, R_H]$.

Investors' choice: $a_U^*(r^{**}) = P$.

Investors' belief along the equilibrium path: $f(\theta|r^{**}) = f(\theta)$.

Investors' belief off the equilibrium path: $\forall a_F \notin \{r^{**}\}$, $f(\theta|a_F)$ *can be any distribution* $g(\theta)$, *such that* $\int_{R_L}^{R_H} g(\theta)R_\theta dR_\theta \leq I$. *Investors' best response to out-of-equilibrium messages is not to invest,* $a_U^*(a_F) = N$.

Payoffs for the firm and investors are: $\mathscr{L}_d^F(\theta) = \max[0, R_\theta - I(1 + r^{**})]$, $\mathscr{L}_d^U = 0$.

Proposition 3 provides a similar summary as Proposition 1. Comparing Equilibrium 7 with Equilibrium 1, we can see that in a one-stage model, the equilibrium outcomes under the continuous-type and the two-type settings resemble each other. However, as I will show in the next section, the inference drawn from different settings can differ significantly in a two-stage model.

Proposition 3. *(Continuous-type model) If there is only one project, all firms issue debt at interest rate* r^{**}.

[22] A recent study on equilibrium solutions under infinite sets of signals and actions can be found in Myerson and Reny (2020).

[23] Since the solution of this equation may not be unique.

1.3.2 Two projects

When the firm's type is distributed on an interval, the previous analysis in the two-type setting cannot be applied directly. For instance, to discourage bad firms from issuing equity, the only choice for the good firm is to set price at $p^* - \varepsilon$. However, some very good firms, e.g., firms with $R_\theta \to R_H$, may not find such a strategy profitable. As a result, let us first look at different incentives for different types of firms.

Consider the type distribution illustrated as in Figure 4. If all firms with positive NPV projects ($R_\theta > I$) issue equity at price $p^* - \varepsilon$, then these good firms can collectively convince investors that they have good projects, since bad firms never issue equity at this price. After investors update their belief, good firms can issue safe debt at the second stage and investors will accept their offer. On the other hand, all firms can also pool at issuing debt twice and receive $\max[0, 2R_\theta - 2I(I + r^{**})]$.

If we look at firms with type $R_\theta \in [I, I(1 + r^{**})]$ as shown in the shaded area in Figure 4, these firms have positive NPV projects. However, they can not receive any positive payoff under pooling at debt equilibrium due to the existence of bad firms. If these good firms issue equity instead, they can surely get a positive payoff from the second project ($R_\theta - I$) if investors are convinced that they are good firms. Thus, firms with type $R_\theta \in [I, I(1 + r^{**})]$ strictly prefer to issue underpriced equity in the first stage. Under this logic, pooling at debt twice cannot become an equilibrium in the two-stage model. When some good firms issue equity, they drive up the equilibrium interest rate (r goes above r^{**}), and further incentivize more good firms to issue equity.

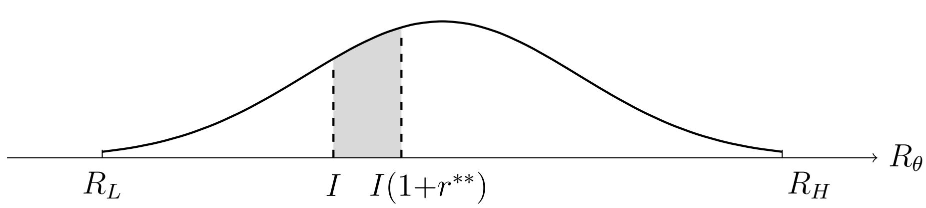


Figure 4: An illustration of the firm's type.

With the above discussion, we can see that in contrast with the two-type model, we no longer need to consider the comparison between the pooling at debt twice equilibrium and the separating equilibrium under the continuous-type setting. Instead, the question becomes whether all good firms have incentives to issue equity.

There are two possibilities. The first possibility is part of all good firms issue equity, as shown in Figure 5. In this figure R^* is denoted as the cutoff point, with firms issue debt at interest rate r^{***} when $R_\theta > R^*$ or $R_\theta < I$, and issue equity at price $p^* - \varepsilon$ when $I < R_\theta < R^*$. Before defining R^* and r^{***}, I first summarize this partial separating equilibrium as follows:

Equilibrium 8. ***(Partial Separating)***

(i) The following strategy-belief combination appears at $t = 1$.

$$\text{Firm's choice: } a_F^*(\theta) = \begin{cases} p^* - \varepsilon & \text{if } R_\theta \in [I, R^*] \\ r^{***} & \text{if } R_\theta \in [R_L, I) \cup (R^*, R_H] \end{cases}$$

Investors' choice: $a_U^*(p^* - \varepsilon) = P$, $a_U^*(r^{***}) = P$.

Investors' belief along the equilibrium path:

(a) If observe $p^* - \varepsilon$ *then the firm has a positive NPV project.*

$$f(\theta|p^* - \varepsilon) = \begin{cases} \frac{f(\theta)}{\int_I^{R^*} f(\theta')dR_{\theta'}} & \text{if } R_\theta \in [I, R^*] \\ 0 & \text{if } R_\theta \in [R_L, I) \cup (R^*, R_H] \end{cases}$$

(b) If observe r^{***} *then the firm either has a negative NPV project, or is a very good firm.*

$$f(\theta|r^{***}) = \begin{cases} 0 & \text{if } R_\theta \in [I, R^*] \\ \frac{f(\theta)}{\int_{R_L}^{I} f(\theta')dR_{\theta'} + \int_{R^*}^{R_H} f(\theta')dR_{\theta'}} & \text{if } R_\theta \in [R_L, I) \cup (R^*, R_H] \end{cases}$$

Investors' belief off the equilibrium path: $\forall a_F \notin \{p^* - \varepsilon, r^{***}\}$, $f(\theta|a_F)$ *can be any distribution* $g(\theta)$, *such that* $\int_{R_L}^{R_H} g(\theta)R_\theta dR_\theta \leq I$. *Investors' best response to out-of-equilibrium messages is not investing,* $a_U^*(a_F) = N$.

(ii) The following strategy appears at $t = 2$.

Firms that issue equity at $t = 1$ *issue safe debt* ($r = 0$) *and investors participate. Firms that issue debt at* $t = 1$ *continue to issue debt at the same interest rate, investors participate. If firms play other strategies, investors do not participate.*

(iii) Let $\varepsilon \to 0$, total payoffs for the firm and investors at $t = 3$ are:[24]

$$\mathscr{L}^F = \begin{cases} R_\theta - I & \text{if } R_\theta \in [I, R^*] \\ 2R_\theta - 2I(1 + r^{***}) & \text{if } R_\theta \in (R^*, R_H] \\ 0 & \text{if } R_\theta \in [R_L, I) \end{cases}, \ \mathscr{L}^U = \int_I^{R^*} f(\theta)(R_\theta - I)dR_\theta.$$

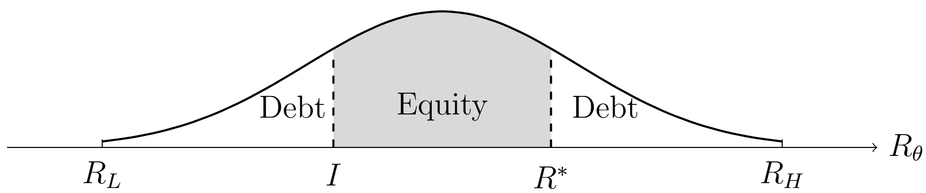


Figure 5: An illustration of Equilibrium 8.

The second possibility is that all good firms issue equity, as shown in Figure 6. In this case we obtain a separating equilibrium as summarized in the below Equilibrium 9.[25]

Equilibrium 9. ***(Separating)***

(i) The following strategy-belief combination appears at $t = 1$.

Firm's choice: $a_F^*(\theta) = \begin{cases} p^* - \varepsilon & \text{if } R_\theta \in [I, R_H] \\ r & \text{if } R_\theta \in [R_L, I) \end{cases}$

Investors' choice: $a_U^*(p^* - \varepsilon) = P$, $a_U^*(r) = N$.

Investors' belief along the equilibrium path:

(a) If observe $p^* - \varepsilon$ then the firm has a positive NPV project.

$$f(\theta|p^* - \varepsilon) = \begin{cases} \frac{f(\theta)}{\int_I^{R_H} f(\theta')dR_{\theta'}} & \text{if } R_\theta \in [I, R_H] \\ 0 & \text{if } R_\theta \in [R_L, I) \end{cases}$$

(b) If observe r then the firm has a negative NPV project.

$$f(\theta|r) = \begin{cases} 0 & \text{if } R_\theta \in [I, R_H] \\ \frac{f(\theta)}{\int_{R_L}^{I} f(\theta')dR_{\theta'}} & \text{if } R_\theta \in [R_L, I) \end{cases}$$

Investors' belief off the equilibrium path: $\forall a_F \notin \{p^* - \varepsilon, r\}$, $f(\theta|a_F)$ can be any distribution

[24] Here $\mathscr{L}^U$ is the ex ante expected payoff.

[25] Here separating only means that good firms are separated from bad firms. Yet investors still don't know the type for sure. Thus, the equilibrium is more or less a quasi-separating equilibrium.

$g(\theta)$, such that $\int_{R_L}^{R_H} g(\theta) R_\theta dR_\theta \leq I$. Investors' best response to out-of-equilibrium messages is not to invest, $a_U^*(a_F) = N$.

(ii) The following strategy appears at $t = 2$.

Firms that issue equity at $t = 1$ issue safe debt ($r = 0$) and investors participate. If firms issue debt at $t = 1$, investors do not participate.

(iii) Let $\varepsilon \to 0$, total payoffs for the firm and investors at $t = 3$ are:[26]

$$\mathscr{L}^F = \begin{cases} R_\theta - I & \text{if } R_\theta \in [I, R_H] \\ 0 & \text{if } R_\theta \in [R_L, I) \end{cases}, \text{ and } \mathscr{L}^U = \int_I^{R_H} f(\theta)(R_\theta - I) dR_\theta.$$

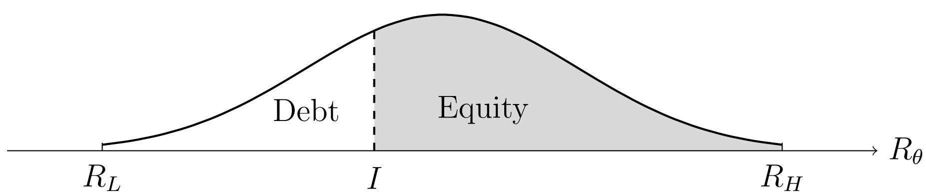


Figure 6: An illustration of Equilibrium 9.

Among the above two possibilities, only one of them can show up as the final solution. Intuitively, we can imagine that whether we obtain Equilibrium 8 or Equilibrium 9 depends on the distribution of R_θ and the amount of required investment I. For instance, if I is very large, then many firms have negative NPV projects. In this scenario, it is likely that investors require a very high interest rate. As a result, the potential benefit of issuing equity becomes larger to good firms and we are more likely to observe Equilibrium 9. Below Proposition 4 summarizes the equilibrium solution.

Proposition 4. *(Continuous-type model) In the presence of two projects, define R^* and r^{***} as the joint solution of the following two equations:*

$$R^* = I(1 + 2r^{***}) \tag{5}$$

$$I r^{***} \int_{R^*}^{R_H} f(\theta) dR_\theta = \int_{R_L}^{I} f(\theta)(I - R_\theta) dR_\theta \tag{6}$$

[26]Here $\mathscr{L}^U$ is the ex ante expected payoff.

Further, define I^* as the maximum I such that the solutions of R^* and r^{***} exist.[27] *Then we can find at least one distribution, such that there exists $I^* \in (R_L, \bar{R})$ and the following assertions hold:*

(i) If $I \in (R_L, I^*]$, then we obtain Equilibrium 8, and part of all good firms separate themselves by issuing equity.

(ii) If $I \in (I^*, \bar{R})$, then we obtain Equilibrium 9, and all firms with positive NPV projects issue equity.

Proof. (a) If we obtain Equilibrium 8, then type $R_\theta = R^*$ should be indifferent between issuing debt and equity. From payoffs of firms, we can set $R^* - I = 2R^* - 2I(1 + r^{***})$ when $\varepsilon \to 0$. This gives us Equation (5).

(b) Observe the participation constraint of investors:

$$\begin{aligned} \mathscr{L}_d^U\Big|_{r^{***}} &= \int_{R_L}^{R_H} f(\theta|r^{***}) \min[R_\theta - I, Ir^{***}] dR_\theta \\ &= \int_{R_L}^{I} f(\theta|r^{***})(R_\theta - I) dR_\theta + \int_{R^*}^{R_H} f(\theta|r^{***}) Ir^{***} dR_\theta = 0 \end{aligned}$$

Given posterior beliefs as in Equilibrium 8 we can obtain Equation (6).

(c) The example below proves the rest of this proposition. □

Example. Consider a uniform distribution $R_\theta \sim \text{Unif}[0, 2R]$, $f(\theta) = \frac{1}{2R}$. Here $\bar{R} = R$. To ease the notation I will let $\varepsilon \to 0$. Let us first consider Equilibrium 8. In this example, Equation (5) and Equation (6) can be calculated as follows:

$$R^* = I(1 + 2r^{***}) \tag{7}$$

$$r^{***}(2R - R^*) = \frac{1}{2}I \tag{8}$$

The above Equation (7) and Equation (8) have solutions whenever $I \leq I^* = \frac{2}{3}R$. Thus, if

[27] In case of multiple solutions, choose the smallest positive value of r^{***}.

we select the smallest positive value for r^{***}, we can find that

$$R^* = \frac{1}{2}[2R + I - \sqrt{(2R-3I)(2R+I)}]$$
$$r^{***} = \frac{1}{4I}[2R - I - \sqrt{(2R-3I)(2R+I)}]$$

A few detailed examples are listed below.

(a) Suppose $I = \frac{2}{3}R$, then $R^* = \frac{4}{3}R$ and $r^{***} = \frac{1}{2}$. The equilibrium looks like Figure 7.

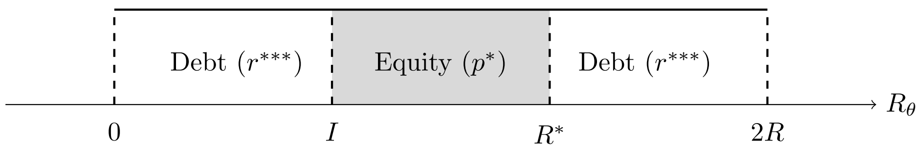


Figure 7: An illustration of Equilibrium in Example (a).

(b) Suppose $I = \frac{1}{2}R$, then $R^* = \frac{5-\sqrt{5}}{4}R \approx 0.691R$, and $r^{***} = \frac{3-\sqrt{5}}{4} \approx 0.191$. The equilibrium looks like Figure 8.

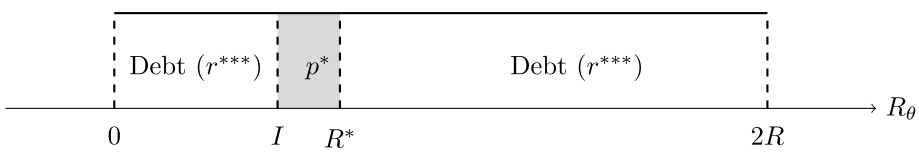


Figure 8: An illustration of Equilibrium in Example (b).

(c) Observe from Equation (7) and Equation (8) that whenever I increases, we have both r^{***} and R^* increase. Thus, whenever $I > I^*$, there does not exist a solution that can satisfy both equations.[28] Here we achieve the separating Equilibrium 9 as shown in Figure 9.

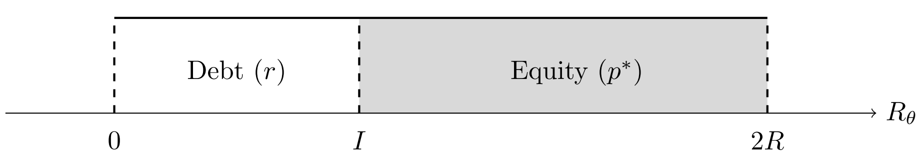


Figure 9: An illustration of Equilibrium in Example (c).

[28] To see this, suppose $I = R - \varepsilon \to R$, then in this example we can find the equilibrium interest rate in a one-stage model $r^{**} \to 1$. Thus, according to the argument at the beginning of this section, firms with $R_\theta \in (I, 2I)$ have enough incentive to switch to equity financing. Under this scenario, the remaining portion of the good firms, which is very small (e.g., $R_\theta \in [2R - 2\varepsilon, 2R]$), will be forced into choosing equity as well.

To summarize this example, we can see that $I^* = \frac{2}{3}R$, and

(i) If $I \in (0, I^*]$, we obtain Equilibrium 8. Investors' beliefs update according to: (i.1) If observe p^*, then $R_\theta \sim \text{Unif}[I, R^*]$, with $f(\theta|p^*) = \frac{1}{R^*-I}$. (i.2) If observe r^{***}, then $R_\theta \sim \text{Unif}[0, I) \cup [R^*, 2R]$, with $f(\theta|r^{***}) = \frac{1}{I+2R-R^*}$.

(ii) If $I \in (I^*, R)$, we obtain Equilibrium 9. Investors' beliefs update according to: (ii.1) If observe p^*, then $R_\theta \sim \text{Unif}[I, 2R]$, with $f(\theta|p^*) = \frac{1}{2R-I}$. (ii.2) If observe r, then $R_\theta \sim \text{Unif}[0, I)$, with $f(\theta|r) = \frac{1}{I}$. □

To conclude this section, we can see that in a two-stage model, asymmetric information between firms and investors no longer generates a strict preference of debt financing. Instead, at least part of the firms with positive NPV projects prefer to issue underpriced equity. Comparing with the two-type model in Section 1.2, we can see that allowing the firm type to have a continuous distribution can dramatically enrich the model predictions. For example, the pattern shown in the above example can also be used to explain the equity issuance announcement effect. If we consider part (b) of this example, then announcing to issue equity will revise the investors' belief to "these firms have an average type around $0.6R$", which is smaller than the prior belief that the average type is R.

1.4 When investors can verify the true type

Parallel to the development of signaling games, the literature also explores the possibility that some investors can verify a firm's type. These investors can either be informed investors that costlessly know the true type (e.g., Kyle 1985, Rock 1986), special investors that can verify the type at a cost (e.g., Townsend 1979, Chemmanur 1993, Fulghieri and Lukin 2001), or financial intermediaries like investment banks (Baron 1982, Benveniste and Spindt 1989, Chemmanur and Fulghieri 1994). Following these prior developments, in this section I examine a variant of my previous model by considering investors being able to verify the firm's type.

Formally, suppose now uninformed investors can spend a cost c to verify the true type of the firm before deciding whether to participate or not. Denote the action of costly verification as C, and not verify as D. Figure 10 shows the sequence of moves.

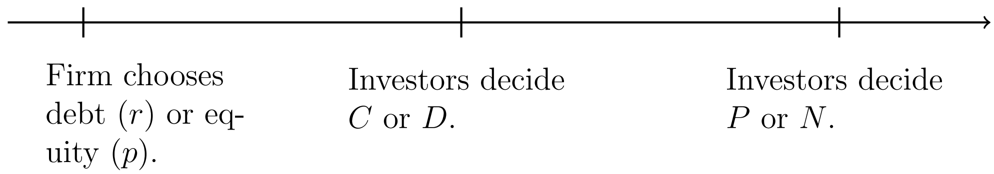


Figure 10: Sequence of moves at $t = 1$.

Thus, the augmented action for investors can be represented by $a_U \in \{CP, CN, DP, DN\}$. Clearly, investors don't have any incentives to finance firms with negative NPV projects once the type is verified, and the choice of $\{P, N\}$ depends on the outcome of verification. On the other hand, if investors decide not to verify the type (D), then the game reduces to our previous signaling game. From now on I focus on $a_U \in \{C, D\}$ whenever there is no confusion.

1.4.1 Two-type model

Let us first consider the case that a firm can be either a high type firm or a low type firm as in Section 1.2. For the low type firm, it mimics the high type firm whenever possible. For the high type firm, we can restrict its action space to $a_F \in \{r^*, \hat{r}, p^*, \hat{p}\}$, where r^* and p^* are defined in Section 1.2, and $\hat{r}$ and $\hat{p}$ are defined to be values that investors can ex ante break even if choosing $a_U = C$.[29] Specifically, $\hat{r} = \frac{c}{\mu I}$ and $\hat{p} = \frac{\mu I}{\mu I + c} R_H$.[30]

Apparently, the equilibrium also depends on the range of the cost c. Let us first focus on the range of $c < \mu(R_H - I)$. Under this scenario, the high type firm does not default under $\hat{r}$ (since $R_H - I > I\hat{r} = \frac{c}{\mu}$), and we also have $\hat{p} > p^*$.

To simplify the discussion, I first separate the choice between debt and equity. Suppose we restrict the high type firm's choice to debt financing. Then the following Lemma 1 summarizes the optimal choice for the firm and investors.

[29] Here we can also write p^* as $p^* - \varepsilon$ and $\hat{p}$ as $\hat{p} - \varepsilon$. For simplicity I let $\varepsilon \to 0$.

[30] If $a_F = \hat{r}$, and $a_U = C$, then $\mathscr{L}^U = \mu I \hat{r} - c = 0$, which gives us the expression for $\hat{r}$. If $a_F = \hat{p}$, and $a_U = C$, then $\mathscr{L}^U = \mu(R_H \frac{I}{\hat{p}} - I) - c = 0$, which gives us the expression for $\hat{p}$.

Lemma 1. *If we restrict $a_F(H) \in \{r^*, \hat{r}\}$ at $t = 1$, then*

(a) When $c \in [0, (1-\mu)(I - R_L)]$, $a_F^* = \hat{r}$, and $a_U^* = C$. Information of a firm's type is revealed at $t = 1$ by costly verification. Payoffs for firms are $\mathscr{L}_{dd}^F(H, \hat{r}, C) = 2R_H - 2I - I\hat{r}$, $\mathscr{L}_{dd}^F(L, \hat{r}, C) = 0$.

(b) When $c \in ((1-\mu)(I - R_L), \mu(R_H - I))$, $a_F^* = r^*$, and $a_U^* = D$. No information of type is revealed. Payoffs for firms are $\mathscr{L}_{dd}^F(H, r^*, D) = 2R_H - 2I(1 + r^*)$, $\mathscr{L}_{dd}^F(L, r^*, D) = 0$.

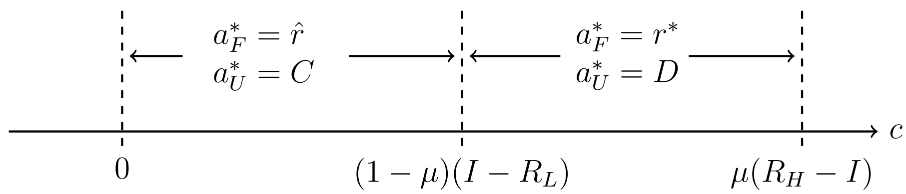


Figure 11: An illustration of Lemma 1.

Proof. Since the low type firm mimics the high type firm whenever possible, we only need to consider the incentive for the high type firm. If the high type firm chooses $a_F = \hat{r}$, then

$$\begin{aligned} \mathscr{L}^U(\hat{r}, C) \geq \mathscr{L}^U(\hat{r}, D) \quad &\Leftrightarrow \quad \mu I\hat{r} - c \geq (1-\mu)(R_L - I) + \mu I\hat{r} \\ &\Leftrightarrow \quad c \leq (1-\mu)(I - R_L) \end{aligned}$$

Given expressions of $\hat{r} = \frac{c}{\mu I}$ and $r^* = \frac{(1-\mu)(I-R_L)}{\mu I}$, we also have $\hat{r} \leq r^*$. Thus the firm can choose its preferred interest rate $\hat{r}$ and investors' best response is to verify the true type. Since the type is verified at $t = 1$, the high type firm can issue safe debt at $t = 2$. Thus the total payoff for two projects is $\mathscr{L}_{dd}^F(H, \hat{r}, C) = R_H - I(1 + \hat{r}) + R_H - I = 2R_H - 2I - I\hat{r}$.

If $c > (1-\mu)(I - R_L)$, then investors' best response to $\hat{r}$ is to choose $a_U = D$. If the firm chooses r^*, then $\mathscr{L}^U(r^*, C) = \mu I r^* - c < 0 = \mathscr{L}^U(r^*, D)$, investors' best response is to choose $a_U = D$. Since investors decide not to verify the true type, the firm will choose r^*, as we have $r^* < \hat{r}$. Here no information is revealed and the security issuance game at $t = 2$ repeats itself as in $t = 1$. $\square$

Similarly, we can also restrict the high type firm's choice to equity financing, and the following Lemma 2 summarizes the optimal choice for the firm and investors.

Lemma 2. *If we restrict* $a_F(H) \in \{p^*, \hat{p}\}$ *at* $t = 1$*, then*

(a) When $c \in [0, \mu(\frac{I}{\bar{R}}R_H - I)]$*,* $a_F^* = \hat{p}$*, and* $a_U^* = C$*. Information of type is revealed at* $t = 1$ *by costly verification. Payoffs for firms are* $\mathscr{L}_{ed}^F(H, \hat{p}, C) = 2R_H - I - \frac{I}{\hat{p}}R_H$*,* $\mathscr{L}_{ed}^F(L, \hat{p}, C) = 0$*.*

(b) When $c \in (\mu(\frac{I}{\bar{R}}R_H - I), \mu(R_H - I))$*,* $a_F^*(H) = p^*$*,* $a_F^*(L) = r$*, and* $a_U^* = D$*. Information of type is revealed through signaling. Payoffs for firms are* $\mathscr{L}_{ed}^F(H, p^*, D) = R_H - I$*,* $\mathscr{L}_{ed}^F(L, r, D) = 0$*.*

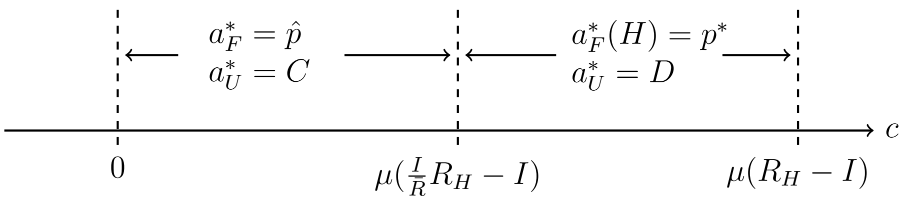


Figure 12: An illustration of Lemma 2.

Proof. Again we only need to consider the incentive for the high type firm, since the low type firm mimics the high type firm whenever possible. Note that $\hat{p} > p^*$ whenever $c < \mu(R_H - I)$. Thus the firm strictly prefers $\hat{p}$ whenever possible. If the high type firm chooses $a_F = \hat{p}$, then

$$\begin{aligned}
\mathscr{L}^U(\hat{p}, C) \geq \mathscr{L}^U(\hat{p}, D) \quad &\Leftrightarrow \quad 0 \geq \mu(R_H \frac{I}{\hat{p}} - I) + (1 - \mu)(R_L \frac{I}{\hat{p}} - I) \\
&\Leftrightarrow \quad \hat{p} \geq \bar{R} \\
&\Leftrightarrow \quad c \leq \mu(\frac{I}{\bar{R}}R_H - I)
\end{aligned}$$

Thus the firm can choose its preferred price $\hat{p}$ and investors' best response is to verify the true type.[31] Since the type is verified at $t = 1$, the high type firm can issue safe debt at $t = 2$. Thus the total payoff for two projects is $\mathscr{L}_{ed}^F(H, \hat{p}, C) = (1 - \frac{I}{\hat{p}})R_H + R_H - I$.

[31]Here the price $\hat{p}$ can pass the Cho-Kreps Intuitive Criterion because when investors' best response is to verify the type, the low type firm receives zero payoff when issuing equity. Thus, issuing equity is no longer a strictly better strategy than issuing debt given investors' best responses.

If $c > \mu(\frac{I}{R}R_H - I)$, investors' best response to $\hat{p}$ is to choose $a_U = D$. From the analysis in Section 1.2 we know that any pooling equilibria at $p > p^*$ cannot pass the Cho-Kreps Intuitive Criterion. Thus the firm can only choose p^*, and information of type is revealed at $t = 1$ through signaling. □

A few observations from the above Lemma 1 and Lemma 2 are: First, the high type firm is better off when facing more capable investors. Specifically, when investors can collect information at a smaller cost to verify the type, the high type firm can select a lower interest rate ($\hat{r}$ as in Lemma 1 part a) or a higher stock price ($\hat{p}$ as in Lemma 2 part a) to obtain a larger payoff.

Second, if μ is very high and investors have a strong belief that the firm is more likely to be a high type firm, then there is less incentive for investors to verify the type. For instance, in Lemma 1, the cutoff point $(1 - \mu)(I - R_L)$ is smaller whenever μ is higher.[32]

Third, from the expression of $\hat{p} = \frac{\mu I}{\mu I + c} R_H < R_H$ we know that the high type firm still has to issue underpriced equity whenever $c > 0$. However, since $\hat{p} > p^*$, the magnitude of this underpricing is much smaller comparing with the model in Section 1.2.

Fourth, investors stay uninformed only if the firm issues debt and the cost of verification is very high (or the prior belief sufficiently strong, i.e., μ is very high). In other words, debt financing at r^* is the only strategy that reveals no information. When the firm issues equity, investors become informed either through signaling or through costly verification. When the firm issues debt, investors can also become informed through costly verification.

Lastly, we can see that although in this modified model investors are more capable at selecting the high type firm, they are actually better off in terms of payoff if they can pretend to be unable of verifying the type. For instance, if we look at the model in Section 1.2, then investors can receive positive payoff when the high type firm chooses equity at p^* (Equilibrium 6). Yet in this section the region for a positive payoff becomes much smaller, as shown in Lemma 2. Thus, investors prefer to play the game in Section 1.2, but the

[32] Such relationship is not linear in Lemma 2.

high type firm would prefer to play the game in this section. Since the firm moves first, it naturally has a first-mover advantage. Here the advantage is that the firm can force investors to collect information and verify the type, for otherwise investors would incur a loss.

So far we have only considered the cost falls into region $c < \mu(R_H - I)$. For the region of $c \geq \mu(R_H - I)$, the choices for the firm revert back to $\{r^*, p^*\}$, as implied by Lemma 1 and Lemma 2. Thus, equilibrium results for $c \geq \mu(R_H - I)$ are similar to those in Proposition 2. For instance, we can consider the result in Proposition 2 is obtained by letting the cost $c \to +\infty$.

With Lemma 1 and Lemma 2 we can compute the high type firm's preference between all four choices $\{r^*, \hat{r}, p^*, \hat{p}\}$. Apparently, the best choice depends on the cost c and the prior belief μ. The following Proposition 5 summarizes the best choice by the high type firm.

Proposition 5. *(Two-type model) In the presence of two projects, and when investors can verify a firm's type at a cost c,*

(I) If $c \in [0, (1-\mu)(I - R_L)]$, then $a_F^*(H) \in \{\hat{r}, \hat{p}\}$.

(II) If $c \in \big((1-\mu)(I - R_L), \min[\mu(\frac{I}{\bar{R}}R_H - I), 2(1-\mu)(I - R_L)]\big]$, then $a_F^*(H) = \hat{p}$.

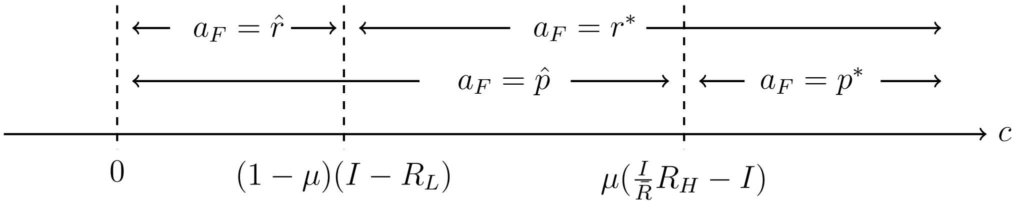
(III) If $c \in \big(\min[\mu(\frac{I}{\bar{R}}R_H - I), 2(1-\mu)(I - R_L)], +\infty\big)$ and $\mu \in (\mu_0, \mu^*)$, then $a_F^*(H) = p^*$.

(IV) If $c \in \big(\min[\mu(\frac{I}{\bar{R}}R_H - I), 2(1-\mu)(I - R_L)], +\infty\big)$ and $\mu \in (\mu^*, 1)$, then $a_F^*(H) = r^*$.

Proof. Considering Lemma 1 and Lemma 2, first we want to show the following:

$$
\begin{aligned}
& (1-\mu)(I - R_L) < \mu(\frac{I}{\bar{R}}R_H - I), \quad \forall \mu \in (\mu_0, 1) \\
\Leftrightarrow \quad & \mu(\frac{I}{\bar{R}}R_H - I) - (1-\mu)(I - R_L) > 0 \\
\Leftrightarrow \quad & \mu\frac{I}{\bar{R}}R_H - I + (1-\mu)R_L > 0 \\
\Leftrightarrow \quad & I\frac{\mu R_H - \bar{R}}{\bar{R}} + (1-\mu)R_L > 0 \\
\Leftrightarrow \quad & (1-\mu)R_L(1 - \frac{I}{\bar{R}}) > 0
\end{aligned}
$$

The last line follows naturally as $\bar{R} > I$. Thus we can remove the upper constraint on c, and combine Lemma 1 and Lemma 2 into the following figure.



From the above figure we can separate c into the three regions:

(a) If $c \in [0, (1-\mu)(I-R_L)]$, then the high type firm can choose in between $\hat{r}$ and $\hat{p}$. Since $\mathscr{L}^F(\hat{r}) = \mathscr{L}^F(\hat{p})$ can be shown as follows, the high type firm is indifferent between these two options.

$$\mathscr{L}^F(\hat{r}) = 2R_H - 2I - I\hat{r} = 2R_H - 2I - \frac{c}{\mu}$$

$$\mathscr{L}^F(\hat{p}) = 2R_H - I - \frac{I}{\hat{p}}R_H = 2R_H - I - \frac{IR_H}{\mu I R_H}(\mu I + c) = 2R_H - 2I - \frac{c}{\mu}$$

(b) If $c \in \big((1-\mu)(I-R_L), \mu(\frac{I}{R}R_H - I)\big]$, then the high type firm can choose in between r^* and $\hat{p}$. Since

$$\mathscr{L}^F(r^*) = 2R_H - 2I(1+r^*) = 2R_H - 2I - 2\frac{(1-\mu)(I-R_L)}{\mu}$$

$$\mathscr{L}^F(r^*) < \mathscr{L}^F(\hat{p}) \Leftrightarrow c < 2(1-\mu)(I-R_L)$$

Then if $\mu(\frac{I}{R}R_H - I) < 2(1-\mu)(I-R_L)$, the firm always chooses $\hat{p}$. If $\mu(\frac{I}{R}R_H - I) > 2(1-\mu)(I-R_L)$, then the firm chooses $\hat{p}$ when $c \in \big((1-\mu)(I-R_L), 2(1-\mu)(I-R_L)\big]$, chooses r^* when $c \in \big(2(1-\mu)(I-R_L), \mu(\frac{I}{R}R_H - I)\big]$

(c) If $c \in \big(\mu(\frac{I}{R}R_H - I), +\infty\big)$, then the high type firm can choose in between r^* and p^*. Here the result converges to the case as in Proposition 2. Thus the firm chooses p^* when $\mu \in (\mu_0, \mu^*)$, chooses r^* when $\mu \in (\mu^*, 1)$.

Combining the cases from (a) to (c) we obtain the four regions as shown in this Proposition. $\square$

I use Figure 13 and Figure 14 as two examples to illustrate Proposition 5. In Figure 13, we have the specification $I = 4$, $R_L = 2$, and $R_H = 10$. In Figure 14, we have the specification $I = 4$, $R_L = 3.2$, and $R_H = 10$. Both these two figures can show the four regions summarized by Proposition 5.

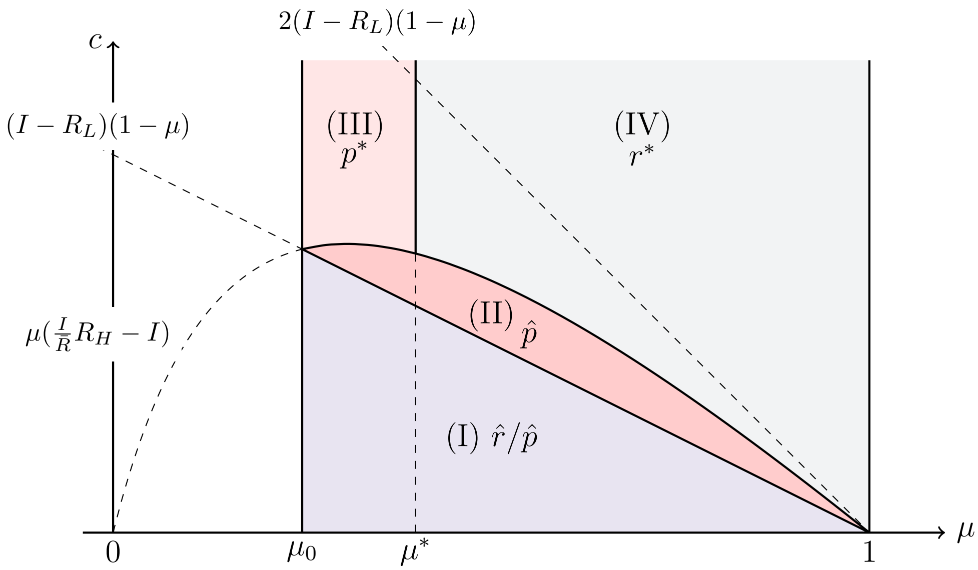


Figure 13: An illustration of Proposition 5.

To conclude this section, we can see that when investors can verify the firm's type, the region for a pooling equilibrium becomes much smaller. As shown in Proposition 5, among all four regions, only region (IV) supports the pooling at debt equilibrium. Thus, unless the cost of verification is very high and the prior belief is sufficiently strong, the high type firm can always reveal its type to investors. From Figure 13 and Figure 14 we can also see that in this model, there does not exist a strict preference for debt financing. In fact, equity financing can sometimes be the only best option for the high type firm, as shown in regions (II) and (III) in Proposition 5.

1.4.2 Continuous-type model

With results from the above two-type model, let us consider the continuous-type specification. As shown in Section 1.3, the belief updating process is rather complicated. Thus,

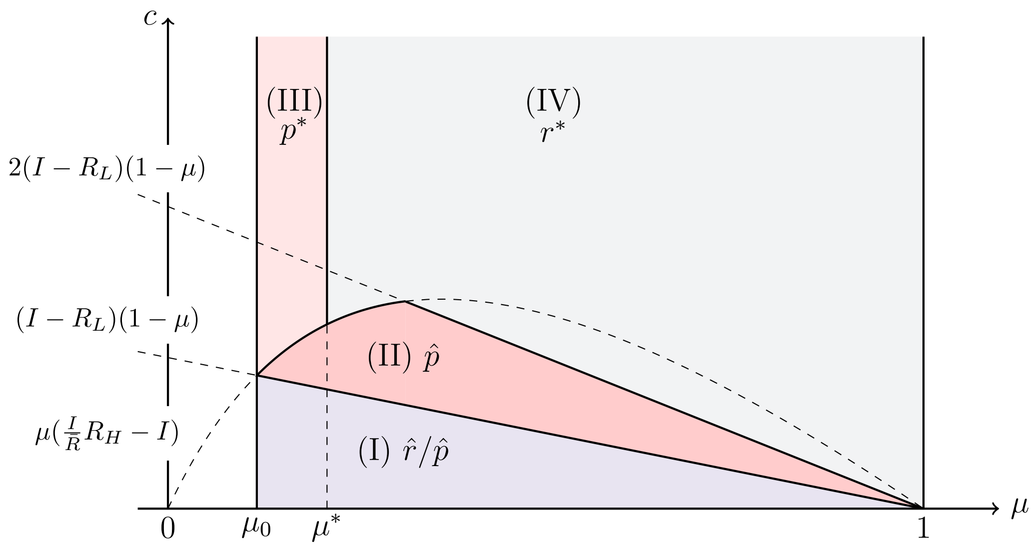


Figure 14: Another illustration of Proposition 5.

in this section I focus on debt and equity issuances at $t = 1$ separately. Presumably, the comparison between debt and equity would show a similar pattern as in the two-type model. In addition, since a very high cost c implies that the equilibrium converges to the case in Proposition 4, here I only discuss the case in which $c < \int_{R_L}^{I} f(\theta)(I - R_\theta)dR_\theta$.

First, consider all types of firms issue debt at interest rate $r^\dagger$, which is defined as the following:

$$\mathscr{L}^U(r^\dagger, C) = \int_I^{R_H} f(\theta)\min[R_\theta - I, Ir^\dagger]dR_\theta \; - c = 0$$

Then the equilibrium can be summarized as in Equilibrium 10.

Equilibrium 10. ***(Debt only)*** *When $c < \int_{R_L}^{I} f(\theta)(I - R_\theta)dR_\theta$, the following strategy-belief combination constitutes an equilibrium.*

Firm's choice: (a) $a^*_{F,t=1}(\theta) = r^\dagger$, $\forall R_\theta \in [R_L, R_H]$. (b) Good firms issue safe debt at $t = 2$.

Investors choice: $a^*_{U,t=1}(r^\dagger) = C$, and participate only in issuances by good firms.

Investors' belief along the equilibrium path: $f(\theta|r^\dagger) = f(\theta)$.

Payoffs for the firm:

$$\mathscr{L}^F(\theta, r^\dagger, C) = \begin{cases} \max[0, R_\theta - I(1+r^\dagger)] + R_\theta - I, & \text{if } R_\theta \in [I, R_H] \\ 0, & \text{if } R_\theta \in [R_L, I] \end{cases}.$$

Proof. With the definition of $r^\dagger$, we need the following for investors to choose to verify the type:

$$\begin{aligned} &\mathscr{L}^U(r^\dagger, D) < \mathscr{L}^U(r^\dagger, C) \\ &\Leftrightarrow \int_{R_L}^{I} f(\theta)(R_\theta - I)dR_\theta + \int_{I}^{R_H} f(\theta)\min[R_\theta - I, Ir^\dagger]dR_\theta < 0 \\ &\Leftrightarrow c < \int_{R_L}^{I} f(\theta)(I - R_\theta)dR_\theta \end{aligned}$$

If a firm deviates to any $r < r^\dagger$, then investors' best response is to choose $a_U = DN$.[33] Comparing with Figure 4 we can see that, since $r^\dagger < r^{**}$, firms with types $R_\theta \in [I, I(1+r^{**})]$ no longer have incentives to issue underpriced equity at price $p^* - \varepsilon$ to reveal its type. $\square$

The above Equilibrium 10 shows that firms can select an interest rate to incentivize investors to collect information when the cost of doing so is not very high. This prediction is in the same direction as in Lemma 1 in the two-type model.

Second, consider all types of firms issue some kinds of equity. Here we can imagine that since investors can verify the true type, some good firms should be able to issue equity at a price that is larger than p^* but smaller than their actually type R_θ. On the other hand, bad firms will also want to issue at a higher price to take advantage of the asymmetric information. Thus, the equilibrium has to require investors to respond with $a_U = C$ for any price $p > p^*$ in order to pass the Intuitive Criterion. I present one reasonable equilibrium as in Equilibrium 11. However, it is worth mentioning that such equilibrium is not unique. I will briefly explain other possible solutions after this presentation.

Equilibrium 11. ***(Equity only)*** *When $c < \int_{R_L}^{I} f(\theta)(I - R_\theta)dR_\theta$, the following strategy-belief combination constitutes an equilibrium.*

[33] Here it is not necessary to specify off-equilibrium beliefs.

(i) Firm's choice:

$$\text{(a) } a^*_{F,t=1}(\theta) = \begin{cases} p^\dagger = \frac{\eta I}{\eta I + c} R_\theta, & \textit{if } R_\theta \in [I + \frac{c}{\eta}, R_H] \\ p^* - \varepsilon, & \textit{if } R_\theta \in [I, I + \frac{c}{\eta}] \\ \textit{random } p \in [I, \frac{\eta I}{\eta I + c} R_H], & \textit{if } R_\theta \in [R_L, I) \end{cases} .$$

(b) Good firms issue safe debt at $t = 2$.

(ii) Investors choice:

(a) $a^*_{U,t=1}(p^* - \varepsilon) = DP$, $a^*_{U,t=1}(p^\dagger/p) = C$ and participate only in issuances by good firms.

(b) $a^*_{U,t=2}(p^* - \varepsilon) = P$, and

$$a^*_{U,t=2}(p^\dagger/p) = \begin{cases} P, & \textit{if } R_\theta \in [I + \frac{c}{\eta}, R_H] \textit{ and } \mathscr{L}^U_{t=1}(p^\dagger/p, C) \geq 0 \\ N, & \textit{if } R_\theta \in [R_L, I] \textit{ or } \mathscr{L}^U_{t=1}(p^\dagger/p, C) < 0 \end{cases}$$

(iii) Investors' belief along the equilibrium path:

$$\text{(a) } f(\theta | p^* - \varepsilon) = \begin{cases} \frac{f(\theta)}{\int_I^{I + \frac{c}{\eta}} f(\theta') dR_{\theta'}} & \textit{if } R_\theta \in [I, I + \frac{c}{\eta}) \\ 0 & \textit{if } R_\theta \in [R_L, I) \cup [I + \frac{c}{\eta}, R_H] \end{cases}$$

(b) If observe $p^\dagger/p$ (since investors cannot distinguish them), then investors believe the firm is of type $R_\theta = \frac{\eta I + c}{\eta I} p^\dagger$ with probability $\eta = \frac{\int_{I + \frac{c}{\eta}}^{R_H} f(\theta) dR_\theta}{1 - \int_I^{I + \frac{c}{\eta}} f(\theta) dR_\theta}$, of type $R_\theta \in [R_L, I)$ with $f(\theta | p^\dagger/p) = \frac{f(\theta)}{1 - \int_I^{I + \frac{c}{\eta}} f(\theta) dR_\theta}$.

(iv) Payoffs for the firm when $\varepsilon \to 0$:

$$\mathscr{L}^F = \begin{cases} 2R_\theta - 2I - \frac{c}{\eta}, & \textit{if } R_\theta \in [I + \frac{c}{\eta}, R_H] \\ R_\theta - I & \textit{if } R_\theta \in [I, I + \frac{c}{\eta}) \\ 0, & \textit{if } R_\theta \in [R_L, I) \end{cases} .$$

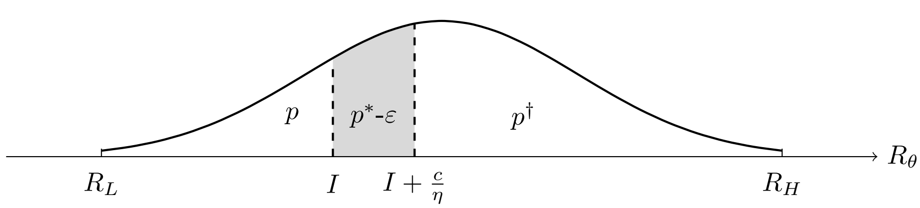


Figure 15: An illustration of Equilibrium 11.

Proof. Payoffs for investors at $t = 1$ are

$$\begin{aligned}
\mathscr{L}^U_{t=1}(p^* - \varepsilon) &= \int_I^{I+\frac{c}{\eta}} f(\theta|p^* - \varepsilon)(\frac{I}{I-\varepsilon}R_\theta - I)dR_\theta > 0 \\
\mathscr{L}^U_{t=1}(p^\dagger/p, C) &= \eta(\frac{I}{p^\dagger}R_\theta - I) - c = 0 \; \Leftrightarrow \; p^\dagger = \frac{\eta I}{\eta I + c}R_\theta \\
\mathscr{L}^U_{t=1}(p^\dagger/p, D) &= \eta(\frac{I}{p^\dagger}R_\theta - I) + \int_{R_L}^{I} f(\theta|p^\dagger/p)(\frac{I}{p}R_\theta - I)dR_\theta < 0 \\
&\Leftrightarrow c < \int_{R_L}^{I} f(\theta|p^\dagger/p)(I - \frac{I}{p}R_\theta)dR_\theta
\end{aligned}$$

Since $f(\theta|p^\dagger/p) > f(\theta)$, $(I - \frac{I}{p}R_\theta) > (I - R_\theta)$ when $p > I$, then the above last line holds whenever $c < \int_{R_L}^{I} f(\theta)(I - R_\theta)dR_\theta$.[34] The rest of the proof is described in the main context.

□

From Equilibrium 11 we can see that some good firms issue underpriced equity at $p^\dagger$. However, these good firms might have incentives to deviate to $p' = R_\theta$, since investors cannot tell the true type without verification and it is still rational for investors to invest after the verification (although investors would lose money ex ante, any price $p \leq R_\theta$ is acceptable ex post). This is why we need investors' strategy at $t = 2$ to be conditional on payoffs at $t = 1$. As shown in the above Equilibrium 11, investors can threaten firms by not participating at $t = 2$ if they lose money at $t = 1$.[35] Such strategy can prevent firms from deviating and is credible since investors earn zero payoffs at $t = 2$ whether they participate or not. Here we can see that the existence of the second stage is crucial for establishing Equilibrium 11, as a one-stage model cannot support $p^\dagger$.

One observation here is that allowing investors' strategy at $t = 1$ to be conditional on the payoffs at $t = 1$ can also be the potential source for alternative equilibrium solutions. For instance, investors can demand firms to further lower the price at $t = 1$, and threaten

[34] Here we should also specify that $I + \frac{c}{\eta} < R_H \Leftrightarrow c < \eta(R_H - I)$. Yet this restriction is most likely to be weaker than $c < \int_{R_L}^{I} f(\theta)(I - R_\theta)dR_\theta$. For instance, if we let $R_H \to +\infty$, then $c < \eta(R_H - I)$ is most likely true. I abbreviate this restriction to simplify the discussion.

[35] $\mathscr{L}^F(p^\dagger) = (1 - \frac{I}{p^\dagger})R_\theta + (R_\theta - I) > (R_\theta - I) = \mathscr{L}^F(p')$

firms by not participating at $t = 2$ if firms refuse to do so. Firms, on the other hand, might be able to ignore these threats but will have to come up with a different offer at $t = 2$ to provide incentives for investors to participate. This multiplicity of equilibrium problem is most severe in the continuous-type model, since a continuum of type distribution provides more "space" for firms to deviate from underpricing their equity and for investors to imagine the true type. Comparing with the two-type model, we can see that although the two-type model may not be completely immune to the multiple equilibria problem, it can be largely exempt from it.

To conclude this section, we can see that firms are better off when investors are able to verify the type before deciding whether to participate in any investment. From the firm's perspective, whether to use debt or equity financing depends on investors' priors as well as the cost of verification. Typically, there does not exist a strict pecking order of financing. These predictions hold regardless of whether we employ a two-type setting or a continuous-type setting.

1.5 Relations with Empirical Research

First, this paper can be related to the IPO and SEO literature. Empirical evidence of underpricing in IPOs and SEOs can be traced back to the early 1970s (e.g., Ibbotson 1975, Smith 1977, and Ritter 1984). Since then, the motivation for issuing underpriced equity has been a puzzle to academics. The results from previous sections provide additional support to the argument that the IPO underpricing and the subsequent SEO announcement effect are due to the asymmetric information problem between firms and investors.[36,37]

Although the IPO and SEO literature can be developed separately without the debt market, many empirical findings suggest that there exists cross impact between debt and

[36]For instance, Rock (1986) considers an information asymmetry model and proposes that issuing equity will suffer from the underpricing problem due to the additional compensation required by uninformed investors. A comprehensive review of IPO motives can be find in Ritter and Welch (2002).

[37]Historically, the SEO announcement return has been used as a proxy for the adverse selection cost (Choe, Masulis and Nanda 1993). Alternative determinants for the SEO announcement effect can be seen in, e.g., Kim and Purnanandam (2013).

equity financing. For instance, Pagano, Panetta and Zingales (1998) examine a panel of Italian firms and find that IPOs are followed by lower costs of credit. Kisgen (2006) shows that firms near a credit rating change are more likely to issue equity than debt. At the aggregate level, macro evidence (e.g., Erel et al. 2012) also shows that leverage changes according to the fluctuation of the cost of equity and cost of debt. Thus, my paper is also related to the vast majority of empirical capital structure papers that investigate the leverage determinants (e.g., Rajan and Zingales 1995).

For example, to test the importance of adverse selection, a significant amount of empirical studies have been focusing on how financial deficit predicts leverage dynamics. Starting from Shyam-Sunder and Myers (1999), who argue that the pecking order is a good descriptor, the literature has arrived at different conclusions. For instance, the sample size and time period can lead to a disagreement (Frank and Goyal 2003), ignoring debt capacity can reduce the testing power (Lemmon and Zender 2010, Leary and Roberts 2010), shocks and adjustment costs can make firms deviating from their desired capital structure (Leary and Roberts 2005), and simulations from structural models (Strebulaev 2007) or random assignments (Chang and Dasgupta 2009) can produce testing results largely comparable with real data.

As mentioned by Graham and Leary (2011), an important and still debatable issue in the capital structure literature is the relative importance of different market frictions, e.g., tax, bankruptcy cost, information asymmetry, and agency conflict. While the past two decades have extensively studied different dynamic versions of the tradeoff theory, tests of a dynamic version of the pecking order theory have encountered various difficulties (e.g., Frank and Goyal 2008). Although this paper does not aim to empirically differentiate the adverse selection cost versus other frictions, my model suggests that testing the financing hierarchy may not be a reliable method, and perhaps it is better for researchers to find more direct tests of asymmetric information. To the minimum, any tests on the debt-equity preference should consider (or be conditional on) certain intertemporal features.

1.6 Conclusion

In this paper I develop a two-stage signaling model to describe corporate financing and investment decisions. In each stage the firm faces an investment project that requires outside financing. Equity issuances are accompanied by higher adverse selection costs when there exists information asymmetry between the firm and the security market. Thus, if one stage is viewed separately, the classical pecking order theory holds. However, security issuances are not independent events in a firm's history that has no impact on its future cost of capital. I show that when allowing a firm's behavior to change investors' beliefs, a two-stage model no longer yields a strict preference for debt financing in equilibrium. The firm has incentive to issue underpriced equity if the benefit from future project could outweigh the current adverse selection cost. Technically, a formal comparison between different settings, i.e., two-type versus continuous-type, highlights their pros and cons. Lastly, I briefly discuss the model's implications to empirical research.

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2 Chapter Two

A Model of Ownership Formation

Abstract

We model how different wealth constraints among investors affect an entrepreneur's way of raising capital, his share of project NPV, and his ownership of the new firm. Combining cooperative and noncooperative approaches, we develop and analyze a bargaining framework and demonstrate cases in which a fair division cannot be achieved when sharing of cost and sharing of return are jointly considered. Our results cover conditions on how the entrepreneur can strategically achieve larger net wealth accumulation, and when he can obtain control of the firm. We further discuss the entrepreneur's preferences on the firm's ownership dispersion level under public financing.

2.1 Introduction

Two problems simultaneously arise when a wealth constrained entrepreneur aims to raise capital for his investment project: How to share the cost, and how to share the return. The classical theories of the firm tend to start with one or multiple existing firms and analyze the efficiency of their transactions, conditional on a frictionless or frictional contracting environment. The question of why firms emerge in the first place has yet to receive a satisfying explanation. Consequently, the origin of the firm, i.e., the initial establishment of a partnership or small enterprise, has been developed separately following the principle agent literature and hence integrated less into the existing theories of the firm. Our paper argues that when the investment cost expenditure is due prior to profit realization, the emergence of a firm can become necessary to enforce repayments agreed upon. We provide a framework to describe a firm's ownership formation, and study the detailed bargaining process when an entrepreneur raises capital from different types of investors.

We shall use the following example to illustrate our formal model. Suppose there is an entrepreneur, who successfully developed a prototype and is granted a patent. The prototype requires \$100 to be put into production and can generate \$200 at a later date. There are two types of investors to whom the entrepreneur (he) can reach out for financing: A strategic and wealthy investor, and small (or atomic) investors. The strategic investor has sufficient wealth, can bargain efficiently, and does not require any wealth from the entrepreneur. On the other hand, small investors have less bargaining skills and thus would not invest unless the entrepreneur has at least \$50 in his pocket. The entrepreneur establishes a firm if he can reach an agreement with any type of investors on how to share the cost and return. The firm is private when there is no atomic investor as shareholder, and is public vice versa. While the cost is expended today, each owner of the firm receives a claim to their future share of profit. Given these settings, we investigate the entrepreneur's responses when facing different investors, his share of project NPV (\$100), and the ownership structure of the firm.

We first look at how the entrepreneur would interact with the strategic investor, Hermione. Suppose Hermione is a venture capitalist. If the entrepreneur has \$0 and Hermione has \$100. Then it has to be the case that Hermione first spends all her money on the investment project and the two of them share the project return. Both the entrepreneur and Hermione have reasons to bargain for part of the return because the entrepreneur owns the prototype and Hermione provides funding. Our model shows that in this case, Hermione can obtain \$150 from the project return, which translates to a 75% ownership of the firm. Suppose in the second case that the entrepreneur has \$90 and Hermione has \$30. Then the two of them can share both the cost and the return. It is natural to expect that the entrepreneur should be able to bargain for a larger slice of the \$200 when he has more wealth to start with. At the same time, Hermione might have incentive to provide less money and still demand quite a high fraction of the return. Indeed, our analysis shows that here the entrepreneur contributes all his money in exchange for a 70% ownership of the firm, whereas Hermione contributes \$10 and obtains a 30% ownership.

More formally, the bargaining power of each agent is limited by his or her wealth. On the one hand, when the entrepreneur contracts with one strategic investor, he cannot control the firm if his wealth level is below his financier Hermione. Under this scenario, the entrepreneur can sometimes transfer some ownership rights to his wealthy counterpart in exchange for a larger share of the project NPV. On the other hand, if the entrepreneur starts with a higher wealth level, he often has to contribute more to the initial investment in order to attract Hermione's funding. As shown in the previous example, different wealth levels together with the different timing between investment cost expense and profit realization generate possibilities for an uneven distribution of project contribution and firm ownership among agents. Thus, an ex ante fair sharing rule may not lead to ex post fair outcome. In the cases above, the cost and return are never shared proportionally.

When the entrepreneur has at least \$50, we can consider how he would raise funds from small investors. Suppose there are two small investors, Harry and Ron, each with \$50. Then the entrepreneur in effect only needs to reach out to one investor. He can establish the firm with Harry alone, or establish the firm with both of them, as shown in Figure 16. The fact that Harry and Ron can substitute each other should render them unable to bargain for a very large slice. Thus, it becomes the entrepreneur's question how to select the ownership structure of the firm. Continuing from the previous example when the investment cost is \$100, and suppose the entrepreneur has \$80, then our results indicate that when the project returns \$200, the entrepreneur prefers to establish the firm with both Harry and Ron. More specifically, the entrepreneur obtains a NPV of \$57.5 when he contracts with Harry, and obtains a NPV of \$83.3 when he contracts with both Harry and Ron. Alternatively, suppose the project only generates \$110 instead of \$200. Then the entrepreneur's selection would be to contract with just Harry.

Figure 16: Concentrated and Dispersed Ownership Structure

When contracting with small investors becomes feasible, the entrepreneur strictly prefers these smaller partners. Specifically, he can appropriate a larger share of the profit and can easily become the controlling shareholder of the firm. We further extend our analysis of small investors to atomic investors. When the entrepreneur establishes his firm with atomic investors, the firm represents a publicly traded corporation. For example, suppose the entrepreneur has $60 and there are 100 atomic investors each with $1. Would he prefer to have all those investors as shareholders (very dispersed ownership structure) or only 40 of them (relatively concentrated ownership structure)? Our results show that issuing new shares, which increases the ownership dispersion level, is optimal when the profitability of the investment project becomes higher. Meanwhile, the entrepreneur might prefer to repurchase shares when he accumulates more wealth.

Our paper extends the current theory of the firm by providing a coherent model that can both describe a firm's initial formation process and its later public financing stage. For established corporations, the separation of ownership and control induces incentive misalignment between managers, shareholders, and debtholders (e.g., Berle and Means 1932, Jensen and Meckling 1976).[38] As suggested by the earlier literature (Coase 1937, Klein et al. 1978, Williamson 1979), opportunistic behaviors might be prevalent when situations are not predictable or contractable. Thus, the property rights view of the firm has been largely focused on resolving the principal-agent problem (Grossman and Hart 1983) when there are residual rights associated with assets (Grossman and Hart 1986), and studying the optimal integration decisions between firms (Hart and Moore 1990).[39] Our paper does not deal with uncertainty related to the future. Instead, we explicitly model the difference between ownership and control, and study its implications.[40]

Our approach combines both cooperative (e.g., Nash 1950, Nash 1953, Shapley 1953) and noncooperative (e.g., Rubinstein 1982) bargaining features in describing the entrepreneur's

[38] Reviews of the theory of the firm can be found in, for instance, Holmstrom and Tirole (1989).

[39] The literature proposes different mechanisms to overcome this moral hazard problem inside corporations. For instance, Shleifer and Vishny (1986) model how one large shareholder can raise firm value through mitigating the free-rider problem caused by diffused ownership. Burkart et al. (1998) study how higher takeover premia can translate into higher ownership concentration and be beneficial to minority shareholders.

[40] In related work for instance, Aghion and Tirole (1997) discuss the difference between formal and real authority. Burkart et al. (1997) consider the potential costs associated with tight shareholder monitoring.

interactions with investors.[41] The related literature has applied bargaining theories to describe the shareholder constraint (Leech 1987), differences between transactions within the firm and through the market (Hart and Moore 1990), the access to critical corporate resources (Rajan and Zingales 1998), and the bargaining for control among large block-holders (Bennedsen and Wolfenzon 2000). Our analysis resorts to bargaining as well, but does not take existence of the firm as fait accompli. Rather, we study how the firm is established when the entrepreneur and the market investors can strategically interact with each other.[42]

Furthermore, our paper joins the burgeoning development on entrepreneurship (e.g., Hellmann 1998, Hall and Lerner 2010, Da Rin et al. 2013), firms' going public decisions (e.g, Chemmanur and Fulghieri 1999, Boot et al. 2006), as well as the effort on unifying the theory of the firm with those of consumers and markets (e.g., Spulber 2009, Hart 2011). Significant progress has been made in examining the role of venture capitalists (or investment banks) in mitigating adverse selection and moral hazard problems. For instance, Admati and Pfleiderer (1994) focus on finding the optimal contract that can incentivize the entrepreneur to make the optimal continuation and investment decisions. Cornelli and Yosha (2003) show that an appropriately designed convertible security can discourage the entrepreneur from window dressing activities.[43,44] Although the literature often assumes the agent to be selfish and only pursuing private benefit, real entrepreneurs may not always be the culprit that causes welfare losses. Our paper proposes an approach that rebalances the different roles

[41] Theoretical literature that focuses on establishing the strategic foundations for cooperative bargaining solutions can be found in, for instance, Binmore et al. (1986), Gul (1989), Stole and Zwiebel (1996), Brügemann et al. (2019).

[42] Although we focus on bargaining under symmetric information, a vast amount of studies have modeled bargaining with incomplete information, e.g., Fudenberg and Tirole (1983), Myerson (1984).

[43] The adverse selection literature focuses primarily on issues like the lemon's problem (Akerlof 1970). Both the debt (e.g., Bester 1985) and equity (e.g., Myers and Majluf 1984) market could face serious adverse selection problems. More recent papers incorporate dynamic features of the market (e.g., Daley and Green 2012) and consider the impact of information acquisition (e.g., Chakraborty and Yilmaz 2011, Yang and Zeng 2018).

[44] The moral hazard literature often analyzes the inefficiencies caused by the different objectives of principals and agents (e.g., Holmstrom 1982, Aghion and Bolton 1992). Examples for later progress include the optimal contracting problem when an entrepreneur finances an infinite horizon investment project (Biais et al. 2007), and the continuous-time agency problem in which the manager balances his information advantage on his private action and the costs of early termination by investors (DeMarzo and Sannikov 2017).

traditionally played by principals and agents, and outlines the beginning of the evolutionary process of small enterprises towards corporations.

Finally, notice that under the assumption of certainty, symmetric information and enforceable contracts, the irrelevance proposition of Modigliani and Miller (1958) holds trivially. Practically, debt financing is not always accessible to entrepreneurs because the investment phase prior to cash flow realization often can last very long time. Thus, we focus our discussions on the equity market and reserve the classical distinction between debt and equity financing for future extensions. We further elaborate on this point in the Conclusion.

The rest of the paper proceeds as follows: Section 2.2 details the entrepreneur's raising capital problem with one wealthy investor. Section 2.4 studies the entrepreneur's choice of small investors. Section 2.5 extends the framework to atomic investors. Section 2.6 deals with some ramifications. Section 2.7 concludes. All proofs can be found in Appendix A. Appendix B provides additional graphical examples.

2.2 One Wealthy Investor

2.2.1 Model setup

The model has two dates: t and $t+1$. At time t, an entrepreneur (E) is endowed with a wealth of w_E and a positive NPV investment project that costs I at t and yields the return R at $t+1$, with $R > I > 0$. The entrepreneur is wealth constrained, $w_E \in [0, I)$, and needs to raise capital from a market investor (M) to finance this investment project. In this section, we consider how the entrepreneur interacts with one wealthy investor (namely, a venture capitalist).[45] The venture capitalist is endowed with wealth w_M. We assume $w_M \geq I - w_E$ so that the total wealth level is sufficient to finance the project.

We further assume that both the entrepreneur and the venture capitalist are risk neutral. The sharing of investment cost and return is determined through a bargaining process. Since

[45] Although in this paper we consider the venture capitalist as the wealthy investor, our framework is applicable to other scenarios. For instance, as in Rajan (1992), we can also consider the bank to be the wealthy counterpart having bargaining power with respect to the firm's profit distribution.

$w_E + w_M \geq I$, both the entrepreneur and the venture capitalist can decide how much wealth to consume and how much wealth to use in the bargaining game at time t. Let B_i denote the amount of wealth that agent $i \in \{E, M\}$ decides to use as a stake to bargain, and $w_i - B_i$ represent the amount to consume (or invest elsewhere) at time t.

We first represent a cooperative game by means of a value function (characteristic function) and explain the specific sharing rule.[46] Denote by $N = \{E, M\}$ the set of players and consider the value function $v : 2^N \to \mathbb{R}$ given by

$$v(\{E, M\}) = R,\ v(\{E\}) = B_E,\ v(\{M\}) = B_M,\ v(\varnothing) = 0. \tag{9}$$

This value function can be interpreted as follows: If the entrepreneur and the venture capitalist agree to invest, they jointly share a wealth of R at $t+1$. If they don't cooperate, the entrepreneur still has B_E and the venture capitalist still has B_M at $t+1$. If at time t both parties put up large enough stakes ($B_E + B_M \geq I$), a firm is established, in which the entrepreneur becomes the manager-shareholder and venture capitalist becomes the shareholder. The project return is shared according to the Shapley value defined as [47]

$$\phi_i^{Sh} = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|!\,(|N| - |S| - 1)!}{|N|!} \big[v(S \cup \{i\}) - v(S)\big] \tag{10}$$

where S is a possible coalition in N, that is $S \in 2^N$.

Alternatively, one might consider a different value function specified as $v'(\{E, M\}) = R - I + B_E + B_M$. Specifically, the utility function (14) below suggests that the total value for consumption at $t+1$ can be expressed as the project NPV ($R - I$) plus the reservation utilities B_E and B_M. In this specification, Nash bargaining would simply divide the NPV

[46] Hart and Moore (1990) provide a detailed framework to derive the particular value function. Here we adopt their general structure and modify several assumptions to better suit our purposes.

[47] In two-person games with transferable utility, the Shapley value reduces to the Nash bargaining solution. (See formula (8.5) in Myerson (1991), for instance.) More generally, Harsanyi (1977, Th.11.2) has shown that the Shapley value can be viewed as "multilateral bargaining equilibrium". Here we use the Shapley value so that the later discussions with multiple agents are coherent.

equally between the two parties. For example, suppose that $I = 1$, $R = 2$, $w_E = 0$, $w_M \geq 1$. For the project to be realized, the investor would have to provide $I_M = 1$, and then both parties would receive $\frac{1}{2}(R - I) = \frac{1}{2}$ in NPV. However, once we alter the input parameters, e.g., assuming $w_E = 0.5$, this seemingly more straightforward solution can no longer provide any details regarding how the entrepreneur and the investor share the project return and cost. In contrast, our approach, i.e., $v(\{E, M\}) = R$, does not suffer from this problem. Moreover, following the specification in (9), we later obtain Proposition 6 which implies that $B_i^* = I_i \; \forall i \in \{E, M\}$ in the majority of equilibria, so that $v'(\{E, M\}) = v(\{E, M\})$ and the second term in (14) becomes $U_{t+1}^i(\phi_i^{Sh})$. That is, both sides care about revenue shares in equilibrium.

We define ownership and control of this new firm as follows:

Definition 4. *Ownership of agent $i \in N$ is defined by $s_i = \phi_i^{Sh}/R$, with the property that $\sum_i s_i = 1$.*

Definition 5. *The control right of the firm belongs to the agent with the largest ownership, i.e., the firm is controlled by agent i if $s_i > s_j, \forall j \in N \setminus \{i\}$. In the event of a tie between the entrepreneur and a market investor, the control right belongs to the entrepreneur.*

Both the ownership and the control definition are natural consequences of the bargaining game. However, the control right could also belong to a certain coalition instead of one agent.[48] This definition of control especially affects our later discussion on small investors. Yet we differ from the branch of literature in which agents are nearly identical with respect to bargaining power and can form a coalition at no cost (e.g., in Bennedsen and Wolfenzon 2000).[49]

[48] The firm may be controlled by a group of agents holding a minority of shares, if preferred and common stock are issued.

[49] In general, control allows to influence the future direction of the firm and, consequently, yields private benefits. In the current model, no further decisions remain to be made, once the funding and revenue distribution of the firm are determined. Insofar, control rights are purely nominal. But the current model can serve as a building block in a more elaborate model where future decisions are made. In a variant of the model, one could assume a reduced form where a continuation utility of control rights is included. For

We assume monotonicity of v which requires that $B_i \leq R$ for both agents — which only imposes an additional restriction on the investor, since $B_E \leq w_E < I < R$. Then both the entrepreneur and the market investor obtain at least a non-negative share of returns under all circumstances. Thus, the investment project is conducted if the following three conditions are satisfied:

$$B_E \in [0, w_E],\ B_M \in [0, \min(w_M, R)],\ B_M + B_E \geq I. \tag{11}$$

In other words, both parties use B_i at time t as a stake in exchange for receiving ϕ_i^{Sh} at time $t+1$.

With the above determination of sharing the return, we next turn to the cost sharing problem. Prior literature typically assumes a cost function that is convex and twice differentiable with respect to an agent's action. For instance, in Hart and Moore (1990) an action is the agent's investment in human capital. In this paper we consider a different cost function. Specifically, we assume that the investment cost is shared proportional to the Shapley value unless one party hits his or her bargaining wealth constraint.[50] Formally, agent i has to contribute I_i defined as follows:

$$I_E = \max\big[\min\big(\frac{I}{R}\phi_E^{Sh}, B_E\big), I - B_M\big] \tag{12}$$

$$I_M = \min\big[\max\big(\frac{I}{R}\phi_M^{Sh}, I - B_E\big), B_M\big] \tag{13}$$

The above functional forms guarantee that $I_E + I_M = I, \forall B_i$ under conditions (11). If neither B_i is very small, the cost functions are simply reduced to $I_i = \frac{I}{R}\phi_i^{Sh}$, which is proportional to agent i's share of the project return. In this scenario, both agents' investments have the

the sake of analytic simplicity and transparency, we refrain from doing so in the main context. Still, we are able to find out how certain funding opportunities give rise to specific ownership and control rights. In Section 2.6, we discuss possible model extensions that allow ownership to yield additional benefits.

[50]This cost sharing rule is the fairest assumption among all alternatives. We focus on this assumption in the spirit of the Nash bargaining solution. Specifically, if there is an impartial arbitrator, how would he determine the cost sharing function? It might be assumed that one party will first deplete his or her stake before the other party contributes, but this would go against the spirit of most cooperative bargaining solutions.

same profitability as the entire project: $\frac{\phi_i^{Sh}}{I_i} = \frac{R}{I}$.

On the other hand, if one of the B_i is very small (smaller than the proportional cost), then this party only contributes B_i and the other party bears the remaining cost $I - B_i$. For example, if the entrepreneur starts with zero wealth, then all the investment cost has to be contributed by the venture capitalist. In this case, $I_E = B_E = 0$, and $I_M = I$. Thus, the two tails of B_i create a wedge in between an absolutely fair division and a practically feasible division. Conceivably, both parties may have incentives to strategically choose a small B_i and force the other party to expense more wealth.

If the investment project is undertaken, we can represent both the entrepreneur's and the market investor's utility as follows:

$$U^i = U_t^i(w_i - B_i) + U_{t+1}^i(\phi_i^{Sh} + (B_i - I_i)) \tag{14}$$

The utility function of either party has two components: (i) time t consumption U_t^i, which is a function of $w_i - B_i$, and (ii) time $t+1$ consumption U_{t+1}^i, which is a function of the share of project return ϕ_i^{Sh} plus any amount of bargaining wealth in excess of the investment cost, $B_i - I_i$. In this paper we consider the simplest scenario that $U^i(x) = x$. Thus, we can reduce the entrepreneur's and the venture capitalist's utility to the following:

$$U^i = w_i - B_i + \phi_i^{Sh} + B_i - I_i = w_i + \phi_i^{Sh} - I_i.$$

The latter expression indicates that the utility of each shareholder is increased by his or her share of the project NPV, $\phi_i^{Sh} - I_i$.

Given the above setting, there are two possible objectives for the entrepreneur and the venture capitalist. The first possibility is that both the entrepreneur and the venture capitalist choose B_E^* and B_M^* to maximize their utilities. Formally, we can summarize the problem for each $i \in \{E, M\}$ as:

$$\begin{array}{lll} \text{Find} & B_i^* = \arg\max U^i & \\ \text{s.t.} & U^i = \begin{cases} w_i + \phi_i^{Sh} - I_i & \text{if (11) is satisfied,} \\ w_i & \text{otherwise.} \end{cases} \end{array} \tag{15}$$

The second possibility is that the agent may be interested in maximizing ownership instead of maximizing utility. For instance, if both the entrepreneur and the venture capitalist maximize their ownership, the problem can be summarized as follows, for $i = E, M$:

$$\text{Find} \quad \widehat{B}_i = \arg\max s_i \quad \text{s.t. (11) is satisfied.} \tag{16}$$

Notice that as long as (11) is satisfied and no B_i is very small, $U^i = w_i + \phi_i^{Sh} - I_i = w_i + \phi_i^{Sh} - \frac{I}{R}\phi_i^{Sh} = w_i + (1 - \frac{I}{R})\phi_i^{Sh} = w_i + (1 - \frac{I}{R})s_i R = w_i + (R - I)s_i$. This means that the two objectives, utility maximization and ownership maximization coincide provided that the firm is formed and both parties assume a sufficiently large stake. The reasons are the assumed linearity of the U^i and the absence of benefits from exercising future control. In general, however, utility maximization and ownership maximization need not coincide as a comparison of Lemma 4 and Proposition 6 shows.

2.2.2 Equilibrium

We first demonstrate that making an investment in the positive NPV project is weakly preferred by both the entrepreneur and the market.

Lemma 3. *If the pair (B_E, B_M) satisfies condition (11), then $U^i \geq w_i$ for $i = E, M$.*

Lemma 3 shows that the sharing rules for cost and return are efficient. There is no deadweight loss. Any investment project will be undertaken as long as it is a positive NPV project, i.e. $R > I$. Therefore, our discussion shall be centered on the bargaining and sharing problem conditional on efficiency. In this regard, we deviate from much of the contracting literature, in which the entrepreneur's private benefit can create inefficiencies (e.g., Aghion and Bolton 1992, Hart 2001).

We next summarize the solution for problem (16) in the following Lemma 4. This solution is intuitive because the Shapley value is a linear combination of an agent's marginal contributions. If any party wants to bargain for a larger share of ownership, he or she has to enter the game with the maximal allowed stake:

Lemma 4. *If an agent wants to maximize his or her ownership, the best he or she can do is to choose* $\widehat{B}_E = w_E$ *or* $\widehat{B}_M = \min(w_M, R)$*, respectively.*

We now turn to the solution of problem (15). Because of Lemma 3, no party will have any incentives to deviate to regions outside the set delineated by (11). Thus, we can confine our discussion to values of B_E and B_M such that condition (11) is satisfied. Accordingly, B_E and B_M also denote generic strategies in the two-player strategic game with player set $\{E, M\}$, respective strategy sets $[0, w_E]$ and $[0, \min(w_M, R)]$, and payoff functions $U^i = w_i + \phi_i^{Sh} - I_i$, $i = E, M$.

The following proposition presents the Nash equilibria (B_E^*, B_M^*) of that game under various parameter values:

Proposition 6. *The Nash Equilibria (NE) are:*

(a) *If* $w_M > R - \frac{R-I}{I} w_E$*, then* $B_E^* = w_E$ *and* $B_M^* = \min(w_M, R)$*;*

(b) *If* $w_M < R - \frac{R-I}{I} w_E$*, then:*

   (b.1) *If* $w_E < w_M$*, then* $B_E^* = \max(I - w_M, 0)$ *and* $B_M^* = w_M$*;*

   (b.2) *If* $w_E > w_M$*, then* $B_E^* = w_E$ *and* $B_M^* = I - w_E$*;*

   (b.3) *If* $w_E = w_M$*, then there are three NE:*

      (i) $B_E^* = w_E$, $B_M^* = I - w_E$,

      (ii) $B_E^* = w_E$, $B_M^* = w_E$,

(iii) $B_E^* = I - w_E$, $B_M^* = w_E$.

(c) *If* $w_M = R - \frac{R-I}{I} w_E$, *then there are two NE:*

(i) $B_E^* = w_E$ *and* $B_M^* = w_M$,

(ii) $B_E^* = 0$ *and* $B_M^* = w_M$.

We draw some diagrams to illustrate the Nash equilibria in Proposition 6. Figure 17 depicts an example in which $w_E < w_M$ and $I < w_M < R - \frac{R-I}{I} w_E$, an instance of case (b.1) — like in Numerical Example 2 below. The shaded area indicates the feasible set under condition (11). The blue line is the best response from the entrepreneur and the red horizontal line is the best response from the market investor. The Nash Equilibrium is $(B_E^*, B_M^*) = (0, w_M)$. Figure 18 shows another example with $w_M < w_E < I$, an instance of case (b.2) as in Numerical Example 4 below. Here the Nash Equilibrium is $(B_E^*, B_M^*) = (w_E, I - w_E)$. 3.B provides additional graphical examples and Section 2.3 provides further numerical examples.

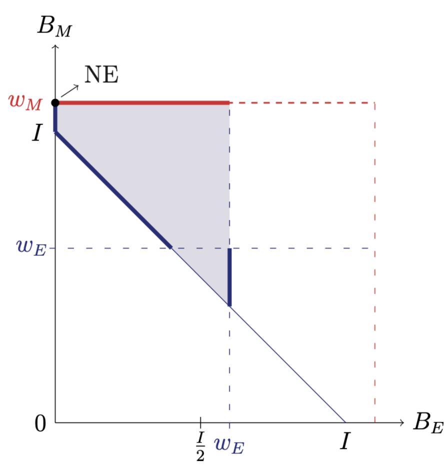


Figure 17: NE Example 1

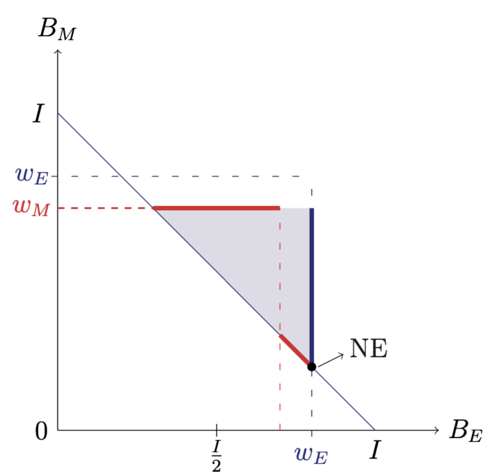


Figure 18: NE Example 2

A comparison between Proposition 6 and Lemma 4 shows the difference between maximizing utility and maximizing ownership. For instance, if we look at the responses of the entrepreneur, maximizing utility can sometimes be equivalent to maximizing ownership when the entrepreneur has wealth exceeding the market investor's, like in case (b.2). In the knife-edge case (b.3), there exists a Nash equilibrium where both sides maximize ownership as well. If the entrepreneur has very limited wealth whereas the venture capitalist has considerable wealth as in case (a), the entrepreneur is better off by putting all his wealth on the bargaining table; that is, not conceding any ownership right to the venture capitalist yields higher utility for the entrepreneur. On the other hand, if the entrepreneur has very limited wealth and the venture capitalist is not too wealthy, it would be in the entrepreneur's interest to forego part of the feasible ownership: Take, for instance, $w_E = 0.2\,I$ and $w_M = 0.9\,I$ in case (b.1), resulting in $(\widehat{B}_E, \widehat{B}_M) = (0.2\,I, 0.9\,I)$ and $(B_E^*, B_M^*) = (0.1\,I, 0.9\,I)$.

2.2.3 Entrepreneur's Choice of VC and Control

In this section we augment the previous setting by allowing the entrepreneur to choose the wealth level of the market investor. Specifically, given the equilibrium result, we ask what is his optimal choice if the entrepreneur can choose whom to deal with. Proposition 7 answers this question:

Proposition 7. *If the entrepreneur can choose w_M before entering a bargaining process with the market investor, his optimal choice is $w_M^* \in [I - w_E, I]$ regardless of the level of his initial endowment w_E. The entrepreneur's maximum utility then is $U_{max}^E = U^E(w_M^*) = w_E + \frac{1}{2}(R - I)$.*

Proposition 7 indicates that the entrepreneur would choose an investor with a wealth level sufficient to finance the investment project, but that does not exceed the cost I. In addition, the best this entrepreneur can achieve is to extract half of the total surplus, that is half of the project NPV$=R - I$.[51] As reflected in Propositions 6 and 7, the bargaining strength of a market investor stems from wealth. Empirically, Hsu (2004) finds that high-reputation VCs can acquire start-up equity with a significant 10-14% discount. Thus, it

[51] Proposition 7 also implies that when the entrepreneur can choose his preferred counterpart, our bargaining solution coincides with the solution of maximizing the classical Nash product, $(U^E - w_E)(U^M - w_M)$, with (w_E, w_M) being the disagreement point (or *status quo*).

is reasonable to imagine that given a fixed investment project (R does not change) and no asymmetric information, the entrepreneur has incentive to bargain with a less powerful venture capitalist.[52]

In general, maximizing ownership need not yield control of the firm in the sense of Definition 5. Maximum ownership may fall short of control of the firm. With a large endowment, maximum ownership may exceed the threshold for control of the firm. The next proposition distinguishes three different wealth scenarios:

Proposition 8. (a) *If $w_E < \frac{1}{2}I$, then the entrepreneur never obtains control of the firm.*

(b) *If $w_E = \frac{1}{2}I$, then the entrepreneur's optimal choice is $w_M^* = \frac{1}{2}I$ with respect to both utility and ownership. The two agents equally split the ownership and the entrepreneur controls the firm.*

(c) *If $w_E > \frac{1}{2}I$, then the entrepreneur can achieve maximum utility and obtain control by choosing an investor who has less wealth than himself, that is $w_M^* \in [I - w_E, w_E)$.*

Proposition 8 (a) conforms with the fact that venture capitalists often have strong control and veto power vis-à-vis early start-up entrepreneurs. When the entrepreneur's wealth level is very limited, financing his investment project is also at risk of losing his idea. This possible downside outcome may occassionaly deter innovations and perhaps impose an emotional burden on entrepreneurs. For instance, Cumming (2008) empirically examines the entrepreneur's and VC's preferences with different exit options. On the other hand, as suggested by Proposition 8 (c), when the entrepreneur's project becomes more successful — and the entrepreneur is wealthy enough — venture capitalists often gradually shift control back to the entrepreneur. These predicted patterns are largely consistent with empirical evidence (e.g., Kaplan and Strömberg 2003).[53]

[52] On the other hand, in the presence of asymmetric information, the entrepreneur may prefer a more powerful venture capitalist to signal project quality.

[53] Notice, however, that even if the entrepreneur's endowment is rather large, he may be unable to gain control in case his only possible choice is a much wealthier venture capitalist.

2.3 Some Numerical Examples

Numerical Example 1. Suppose the entrepreneur starts with wealth $w_E = 0$ and the market investor starts with wealth $w_M = 6$. The investment project costs $I = 6$ at time t and has return $R = 12$ at time $t+1$. Then at equilibrium, the market investor allocates all her wealth to bargain for return, i.e., $B_M^* = 6$ and $B_E^* = 0$. Investment costs are solely borne by the market investor, i.e., $I_M = 6$ and $I_E = 0$. Finally, the return is shared according to $\phi_M^{Sh} = 9$ and $\phi_E^{Sh} = 3$. Although this example is trivial in the sense that the investment project cannot be conducted unless the market investor invests all her wealth, we can see that by cooperation, both parties enjoy higher utilities, with $U^E = 3$ and $U^M = 9$.

Numerical Example 2. Suppose the entrepreneur starts with wealth $w_E = 1$ and the market investor starts with wealth $w_M = 8$. The investment project has cost $I = 5$ at time t and return $R = 10$ at time $t+1$. Here we can see that $w_M < R - \frac{R-I}{I} w_E = 9$. Then at equilibrium, the market investor allocates all her wealth to bargain for return, $B_M^* = 8$. On the other hand, the entrepreneur would consume all his current wealth and put zero on the bargaining table, $B_E^* = 0$. The investment costs are solely borne by the market, i.e., $I_M = 5$ and $I_E = 0$. Finally, the return is shared according to $\phi_M^{Sh} = 9$ and $\phi_E^{Sh} = 1$. The equilibrium utility level of the entrepreneur is $U^E = w_E + \phi_E^{Sh} - I_E = 1 + 1 - 0 = 2$. Here it would be worthwhile to examine whether the entrepreneur can have a profitable deviation — which should not occur in equilibrium. For instance, suppose the entrepreneur put $B_E' = 1$ on the bargaining table instead. Then both parties will share the return according to $(\phi_E^{Sh}, \phi_M^{Sh}) = (1.5, 8.5)$. Costs will be shared proportionally, $(I_E, I_M) = (0.75, 4.25)$. Thus, the entrepreneur's new utility level is $U^E = 1 + 1.5 - 0.75 = 1.75$, which is smaller than before, as to be expected.

Numerical Example 3. We continue with the parameters of the previous example with the exception of w_E. (i) Suppose now $w_E = 3$. Then we have $w_M > R - \frac{R-I}{I} w_E = 7$.

Hence at equilibrium, both agents allocate all their wealth to bargain for return, $B_E^* = 3$ and $B_M^* = 8$. The return is shared according to $(\phi_E^{Sh}, \phi_M^{Sh}) = (2.5, 7.5)$. The investment cost is shared proportionally as $(I_E, I_M) = (1.25, 3.75)$. (ii) Suppose next that $w_E = 2$. Then we have $w_M = R - \frac{R-I}{I} w_E = 8$. In that case, the entrepreneur is indifferent between $B_E^* = 0$ or $B_E^* = 2$. Specifically, when $B_E^* = 0$, we can find that $(\phi_E^{Sh}, \phi_M^{Sh}) = (1, 9)$, $(I_E, I_M) = (0, 5)$, and $U^E = 3$. When $B_E^* = 2$,, we can find that $(\phi_E^{Sh}, \phi_M^{Sh}) = (2, 8)$, $(I_E, I_M) = (1, 4)$, and $U^E = 3$.

Numerical Example 4. Suppose that the entrepreneur has $w_E = 4$ and the market investor starts with $w_M = 3$. The investment project costs $I = 5$ at time t and return $R = 10$ at time $t + 1$. Then the equilibrium condition indicates that the entrepreneur will put all his wealth on the bargaining table, $B_E^* = 4$. The market investor, instead, will only put $B_M^* = I - B_E^* = 5 - 4 = 1$ and consume the rest. The project return is shared by $(\phi_E^{Sh}, \phi_M^{Sh}) = (6.5, 3.5)$. To find the sharing of investment cost, since $\frac{I}{R} = \frac{1}{2}$ we have $I_E = \max[\min(\frac{1}{2} \times 6.5, 4), 5 - 1] = \max[3.25, 4] = 4$. Similarly, $I_M = \min[\max(\frac{1}{2} \times 3.5, 5 - 4), 1] = \min[1.75, 1] = 1$. Thus the cost shares are given by $(I_E, I_M) = (4, 1)$. Here we can see that the entrepreneur is paying a cost more than proportional to his share of return. On the other hand, the market investor is paying less than her proportional cost share. The sharing of surplus is then $(\phi_E^{Sh} - I_E, \phi_M^{Sh} - I_M) = (6.5 - 4, 3.5 - 1) = (2.5, 2.5)$, which shows that the market investor can profit from strategically choosing a smaller B_M. However, the entrepreneur does obtain a larger ownership of this firm. Specifically, the ownership structure of the firm is $(s_E, s_M) = (\frac{6.5}{10}, \frac{3.5}{10}) = (0.65, 0.35)$, which means that the entrepreneur owns 65% of this firm.

2.4 Small Investors

2.4.1 Setup

In this section, we study how the entrepreneur raises capital from small investors. Denote a small investor as M_j. Assume that there are a total number of J identical small investors, each with wealth level $w_j = \frac{I}{J}$. Recall that in the previous section, the large investor could choose the level of wealth (B_M) to be put on the table before entering the bargaining process. To model the difference between large and small investors, we assume that the small investors cannot bargain efficiently. Instead of maximizing utility, all small investors choose $B_{M_j} = w_j = \frac{I}{J}$ to maximize their ownership (Lemma 4). This assumption closely resembles reality. For instance, large investors like Warren Buffett can typically negotiate the terms better than smaller investors. Alternatively, one could consider that there is a private cost of changing B_{M_j} and only the large investor can afford that cost.

It is further assumed that the entrepreneur needs to have a certain amount of wealth before he can raise capital from small investors. Specifically, we assume $w_E \in [\frac{I}{2}, I)$. This restriction also guarantees that the entrepreneur can obtain the control of the firm under all circumstances. In addition, we assume that small investors cannot actually form a coalition to take over the firm. Practically, venture capitalists and angels sometimes syndicate their investment (see, e.g., Gompers and Lerner 2001). However, control by a coalition would be better represented by our model in the previous section. Here we only consider agents to interact with each other in a noncooperative way. Nonetheless, including alternative control structures and potential collusions properties (e.g., Haller 1994, Segal 2003) might generate interesting future extensions. Lastly, although we focus on the entrepreneur's decision of raising capital, our analysis can be applied to a wide range of circumstances like establishing a partnership or merger activities among firms.

2.4.2 Equilibrium

We first restrict attention to $J = 2$ to explore the implications of moving from one to several investors. Section 2.5 considers the scenario in which J can be very large. Here the entrepreneur needs to choose (1) how much wealth to use in bargaining, $B_E \in [0, w_E]$, and (2) how many small investors to become the shareholders of the firm. For instance, when $J = 2$, there are two identical small investors in the market, and the entrepreneur can choose to reach out to either one of them or both. Denote by n the entrepreneur's choice of the number of small investors. Since the entrepreneur is wealth constrained, we have $n \in \{1, 2\}$.

We first look at the minimum bargaining wealth $B_E^{\min}$ for the entrepreneur to maintain control of the firm.

Lemma 5. *If the entrepreneur wants to obtain control of the firm, his minimum choice of B_E is*

(i) $B_E^{\min}\big|_{n=1} = \frac{1}{2}I$ *when establishing the firm with one of the small investors;*

(ii) $B_E^{\min}\big|_{n=2} = \max(\frac{3}{2}I - R, 0)$ *when establishing the firm with two small investors.*

In addition, we have the property that $B_E^{\min}\big|_{n=1} > B_E^{\min}\big|_{n=2}$, since $R > I$.

Lemma 5 shows that the entrepreneur needs less bargaining wealth when facing more investors. This result is intuitive because the entrepreneur effectively only needs one small investor to meet the investment requirement. Thus when the entrepreneur is establishing the firm with two small investors, each of the small investors' marginal contribution drops, which makes it easier for the entrepreneur to obtain control. Lemma 5 also shows that when there are more shareholders, the entrepreneur can control the firm with contributing less than half of the initial cost. This finding is consistent with prior theory (e.g., Almeida and Wolfenzon 2006) and many of the empirically observed pyramidal ownership structures among corporations, in which a single individual or a family can use their wealth to control a disproportionately large number of firms (e.g., La Porta et al. 1999).

Second we pose the question of the optimal ownership dispersion level for the entrepreneur. From the entrepreneur's perspective, he can establish the firm with a more dispersed ownership structure by reaching out to both of the two small investors. But would he want to do so? The following proposition summarizes the entrepreneur's choice:

Proposition 9. *Denote by $\rho = \frac{R}{I} \in (1, +\infty)$ the profitability of the investment project. Conditional on maintaining control of the firm, let n^* denote the entrepreneur's optimal choice. There exist ρ^* and ρ^{**} such that $\frac{8}{7} \leq \rho^* < \rho^{**} \leq \frac{3}{2}$ and the following assertions hold.*

(i) *When $\rho < \rho^*$, $n^* = 1$: the entrepreneur prefers a more concentrated ownership structure.*

(ii) *When $\rho > \rho^{**}$, $n^* = 2$: the entrepreneur prefers a more dispersed ownership structure.*

(iii) *When $\rho \in [\rho^*, \rho^{**}]$, $\exists\, w_E^*(\rho)$, s.t. $n^* = 1$ if $w_E > w_E^*(\rho)$, $n^* = 2$ if $w_E < w_E^*(\rho)$, and the entrepreneur is indifferent between $n^* = 1$ and $n^* = 2$ if $w_E = w_E^*(\rho)$.*

Proposition 9 shows that a more dispersed ownership structure ($n^* = 2$) is preferred by the entrepreneur only if the project is sufficiently profitable. Otherwise, he prefers a more concentrated ownership structure ($n^* = 1$). The intuition behind this proposition is that when the entrepreneur is facing more investors, he can bargain for a larger share of the return but at the same time also has to shoulder a larger share of the cost. When profitability rises, the entrepreneur's net profits (NPVs) under concentrated and dispersed ownership structure are not increasing at the same speed. Thus the entrepreneur has more incentive to reduce the marginal contribution of the small investors (by reaching out to both of them) when the profitability is higher. As a result, there exists a cutoff profit ratio to separate the entrepreneur's optimal ownership choice. Proposition 9 also shows that when profitability is not sufficiently high and the entrepreneur has more bargaining power (reflected by a higher wealth level w_E), he is more likely to be better off with a concentrated ownership structure.

Finally we consider the entrepreneur's choice between one large investor (venture capitalist) versus small investors. As stated in Corollary 1, when the entrepreneur has a wealth level above half of the project cost, he strictly prefers to raise capital from small investors.

Corollary 1. *When $w_E \in (\frac{I}{2}, I)$, the entrepreneur is strictly better off raising capital from small investors.*

Corollary 1 is a natural result when we consider small investors to be less efficient in their bargaining skills and unable to form a coalition to control the firm. In conclusion, the findings in this section show that going public (more dispersed ownership structure) is a desired outcome as the firm becomes more profitable. On the other hand, when profitability is restricted, the entrepreneur may prefer to reduce outsiders' ownership when he gradually accumulates a sufficient amount of wealth. Section 2.5 further illustrates this dynamic.

2.5 Atomic Investors

2.5.1 Setup

In this section, we consider the case where the number of investors (J) is very large. We call these investors atomic when $J \to \infty$. Recall from the previous section that n represents the entrepreneur's choice of the number of small investors when establishing the firm. We further denote by $\gamma = \frac{n}{J}$ the degree of ownership dispersion. For instance, $\gamma \approx 1$ means that the firm has a very dispersed ownership structure. Like in the case for small investors, atomic investors cannot bargain efficiently and the entrepreneur must have a certain amount of wealth $w_E \in [\frac{I}{2}, I)$ before he can raise capital from the public. In practice, raising capital from the public is a costly process and is not likely to be accessible to very small startups. For example, during an IPO process, atomic investors like households can only submit their order but have no control over the share allocation.[54]

[54]While we focus on the ownership formation process under symmetric information, a significant amount of literature studies the problems generated by asymmetric information during the IPO process. For instance, Rock (1986) explains why IPOs are underpriced by modeling how the informed investors can take advantage of the uninformed ones.

Second, we consider here that once the entrepreneur decides to raise capital from the atomic investors, he cannot bargain efficiently as well. For instance, issuing equity requires detailed disclosure by the SEC (Securities and Exchange Commission) and bargaining with atomic investors can be subject to legal constraints. Alternatively, the entrepreneur could also face a reputation cost in addition to the investment cost. Hence we impose the restriction $B_E = w_E$, which guarantees a maximum ownership $s_E = \phi_E^{Sh}/R$ for the entrepreneur.[55] Our interest in this section is to study how the entrepreneur chooses the ownership dispersion level γ.[56]

2.5.2 Equilibrium

Since maximizing ownership s_E is equivalent to maximizing ϕ_E^{Sh}, we first derive the discrete expression of ϕ_E^{Sh} under n small investors and then take $J \to \infty$ to derive its continuous limit. Observe that the investment project requires $w_E + nw_j \geq I$, with $w_j = \frac{I}{J}$. We first denote the lower bound of the feasible set of n as $n^c = \lceil J(1 - \frac{w_E}{I}) \rceil$, which is the ceiling of $J(1 - \frac{w_E}{I})$. Thus, $n \in \{n^c, n^c + 1, ..., J\}$. Extending the notation introduced in Section 2.2, we now have the total set of players $N = \{E, M_1, M_2, ..., M_n\}$ and for any coalition with k small investors we can find the expression for the value function as:

$$v(\{E, M_1, ..., M_k\}) = \begin{cases} R & \text{if } n^c \leq k \leq n, \\ w_E + \frac{k}{J}I & \text{if } 0 \leq k < n^c; \end{cases}$$

$$v(\{M_1, ..., M_k\}) = \frac{k}{J}I.$$

[55]From Section 2.2.2 we know that maximizing ownership can also be equivalent to maximizing utility under certain conditions.

[56]In related work, Müller and Wärneryd (2001) also discuss the trade-off between inside and outside ownership. In their model, outside ownership might be beneficial if it can incentivize insiders to reduce the amount of wasted resources. Our model insofar does not include a standard moral hazard problem. However, in Secton 2.6, we reconcile it with the prior literature by discussing potential extensions that include the effort level of agents, as well as allowing the entrepreneur to be able to divert part of the project cash flow.

From the above results and equation (10), we can find both the discrete value and the continuous limit of ϕ_E^{Sh} as the following (derivations are provided in the proof of Proposition 10):

$$\phi_E^{Sh}(n) = \frac{n-n^c+1}{n+1}R - \frac{(n+n^c)(n-n^c+1)}{2(n+1)}\frac{I}{J} + \frac{n^c}{n+1}w_E, \tag{17}$$

$$\phi_E^{Sh}(\gamma) = \lim_{J\to\infty}\phi_E^{Sh}(n) = R - \frac{I}{2}\gamma - (1-\frac{w_E}{I})(R-\frac{I}{2}-\frac{w_E}{2})\frac{1}{\gamma}. \tag{18}$$

Equation (18) shows how the entrepreneur can maximize ϕ_E^{Sh} by choosing the degree of ownership dispersion γ. Formally, we obtain

Proposition 10. *The optimal ownership dispersion level (γ^*) can be expressed as*

$$\gamma^* = \min\left[\,1\,,\ \frac{1}{I}\sqrt{(I-w_E)(2R-I-w_E)}\,\right]. \tag{19}$$

Specifically, $\gamma^* \to 1$ represents very dispersed ownership structure, and $\gamma^* \to (1-\frac{w_E}{I})$ represents relatively concentrated ownership structure. The optimal ownership dispersion level (γ^*) weakly increases when (1) the profitability ($\rho = \frac{R}{I}$) increases and (2) the entrepreneur's wealth (w_E) decreases.

Proposition 10 shows that a firm (the entrepreneur) is more likely to increase its ownership dispersion level when its profitability increases. On the other hand, a firm (the entrepreneur) with a higher amount of wealth may prefer to reduce the ownership dispersion level. These results allow to describe many observed financing activities for large corporations. For instance, one may observe a firm issuing equity during a high growth period and repurchasing shares once the cash reserves have become large.

Related to our analysis on dispersed shareholders, prior literature also discusses the cost and benefit related to dispersed debtholders, e.g., Dewatripont and Maskin (1995). For example, when a firm chooses debt financing, returns are risky, and there is a chance of default or violation of a debt covenant, a further consideration may become relevant: It could be more difficult to renegotiate with dispersed debtholders whose cost of coordination is high. This consideration, however, does not play a role in our model where the entrepreneur may prefer to bargain with dispersed shareholders (or debtholders) because he can benefit from the very fact that atomic shareholders cannot coordinate effectively.

2.6 Discussion

Our formal description of the investment project simply consists in the parameters I and R. In this section, we elaborate on some more detailed specifications of the model and their potential implications.

1. Continuation payoff. As pointed out in footnote 12, control rights are nominal in the main context. However, we can imagine how future pecuniary benefits, or non-pecuniary utility, would influence the equilibrium outcome. Essentially, maximizing ownership differs from maximizing utility only when the entrepreneur cannot gain enough profit from the bargaining process, i.e., entering the bargaining game with maximum wealth (w_E) is not the best move. Thus, if we allow ownership to yield an additional benefit, then it will increase the entrepreneur's incentive to bargain with his entire wealth.

Consider the following specification. Suppose that ceteris paribus the project yields a non-pecuniary benefit (e.g., esteem of owning a company) of P, and this additional benefit is proportional to an agent's ownership. Then there exists a cutoff point P^* such that the entrepreneur will always choose $B_E^* = w_E$ whenever $P > P^*$, and maximizing utility will be equivalent to maximizing ownership. To illustrate this trade-off, let us assume that the non-pecuniary benefit does not enter the bargaining process. Numerical Example 5 below shows that under this ad hoc specification, a large enough additional benefit might change the bargaining solution.

Numerical Example 5. Suppose that the entrepreneur starts with wealth $w_E = 3$ and the market investor starts with wealth $w_M = 4$. The investment project costs $I = 6$ at time t and has return $R = 12$ at time $t+1$. Without an additional benefit, we find that $(B_E^*, B_M^*) = (2,4)$, $(\phi_E^{Sh}, \phi_M^{Sh}) = (5,7)$, $(I_E, I_M) = (2,4)$, and $(U^E, U^M) = (6,7)$. Alternatively, if the entrepreneur deviates from his optimal strategy and chooses $B'_E = 3$, then we obtain $(\phi_E^{Sh'}, \phi_M^{Sh'}) = (5.5, 6.5)$, $(I'_E, I'_M) = (2.75, 3.25)$, and $(U^{E'}, U^{M'}) = (5.75, 7.25)$.

With additional benefit P, each agent obtains another amount of utility $P_i = s_i P = \frac{\phi_i^{Sh}}{R} P$. Thus the entrepreneur will prefer to deviate if $U^E + P_E < U^{E'} + P'_E$, which means that $6 + \frac{5}{12}P < 5.75 + \frac{5.5}{12}P$ in this example. Hence the entrepreneur prefers $B'_E = 3$ whenever $P > P^* = 6$.

2. Effort. A significant amount of literature has investigated the potential outcome when allowing the effort level of agents to affect the return, e.g., Hart and Moore (1990), Rajan and Zingales (1998). Conceivably, the entrepreneur makes a substantial managerial, advisory or supervisory contribution to the completion of the project. For instance, this might be the case if the entrepreneur has obtained a patent after some research and needs funding for the development stage where his specific input is indispensable. The value of R may already implicitly capture the impact of that contribution. Though it would also be interesting to allow the effort level to explicitly affect the return. However, it is a priori unclear why the entrepreneur should be the only potential owner that would like to maximize the return. Suppose the investor becomes the owner. Then this investor also has an incentive to spend effort to run the firm and generate a larger profit. In practice, venture capitalists often provide business advice to the start-ups they are funding.

More formally, suppose the return is determined by the effort level e of the entrepreneur: $R = R(e)$. Then we know from the proof of Proposition 1 that the utility of the entrepreneur is:

$$U^E \quad = \quad w_E + \min \big[\max \big(\underbrace{\frac{R-I}{R}(\frac{R}{2} - \frac{B_M}{2} + \frac{B_E}{2})}_{①}, \underbrace{\frac{R}{2} - \frac{B_M}{2} - \frac{B_E}{2}}_{②} \big),$$

$$\underbrace{\frac{1}{2}R - I + \frac{B_M}{2} + \frac{B_E}{2}}_{③}\ ]$$

Here we can see that all the components, ①, ②, and ③, are increasing functions of $R(e)$, meaning that the utility of the entrepreneur U^E is also an increasing function of $R(e)$. Thus, disregarding a private cost of spending the effort, the entrepreneur always has an incentive to maximize the return. The same logic goes for the investor.

3. Principal-agent issues. The traditional literature often models the entrepreneur as the "bad guy", who diverts cash flow from the principals, a kind of principal-agent problem — though moral hazard or adverse selection are absent in our context. To shed some light on this problem, we consider another specification that can contribute to the discussion of the role of entrepreneurship in addition to the above points 1 and 2.

Suppose the entrepreneur can divert a cash flow d from the total return to enjoy a private benefit of $\frac{d}{2}$, while the other $\frac{d}{2}$ becomes a deadweight loss. Similar as in point 1, we assume that this diversion technology does not enter the bargaining process. The following Numerical Example 6 shows that the investor might have an incentive to let the entrepreneur control the firm.

Numerical Example 6. Suppose that the entrepreneur starts with wealth $w_E = 3$ and the market investor starts with wealth $w_M = 4$. The investment project costs $I = 6$ at time t and has return $R = 12$ at time $t + 1$. From Numerical Example 5 we know that, without the diversion technology, $(B_E^*, B_M^*) = (2, 4)$, $(s_E, s_M) = (\frac{5}{12}, \frac{7}{12})$, and $(U^E, U^M) = (6, 7)$. If the entrepreneur can divert d from R, then $U^E = 3 + \frac{5}{12}(12 - d) - 2 + \frac{d}{2} = 6 + \frac{d}{12}$, and $U^M = 4 + \frac{7}{12}(12 - d) - 4 = 7 - \frac{7}{12}d$. Thus, we can write payoffs as $(U^E, U^M) = (6 + \frac{d}{12}, 7 - \frac{7}{12}d)$. Here we can see that the entrepreneur strictly prefers to divert the cash flow.

Alternatively, consider the possibility that the market investor decides to yield the control right and choose $B'_M = 3$. Then the entrepreneur can control the firm by choosing $B'_E = 3$. Under $(B'_E, B'_M) = (3, 3)$, we can find that $(s_E, s_M) = (\frac{1}{2}, \frac{1}{2})$, and $(I_E, I_M) = (3, 3)$. If the entrepreneur does not divert any cash flow, then $(U^E, U^M) = (6, 7)$. If the entrepreneur diverts the cash flow, we can similarly find $(U^E, U^M) = (6, 7 - \frac{d}{2})$. Comparing with the outcome before (namely, investor control the firm), we can see that in this example, the entrepreneur no longer has any incentive to divert the cash flow if he can control the firm. The investor, on the other hand, strictly prefers to let the entrepreneur control the firm to obtain a higher payoff.

2.7 Conclusion

The foregoing analysis is focused on the funding and ownership structure of a nascent firm. We model an entrepreneur's problem of raising capital when the entrepreneur and potential investors have conflicting interests. Specifically, when the investment cost is required and expended today and the return is realized tomorrow, both the entrepreneur and a venture capitalist have an incentive to contribute less cost but bargain for more return. Our findings show that when allowing for strategic interactions, it is not always feasible to achieve a fair allocation among agents. Under these scenarios, the entrepreneur often faces a trade-off between maximizing ownership and maximizing net wealth accumulation. When the entrepreneur has a sufficient amount of wealth, he prefers to raise capital from smaller investors or the public capital market as these shareholders often have limited bargaining capability.

As mentioned earlier, one issue that remains unexplored is the choice between different financial instruments. On the one hand, under the assumption of certainty, symmetric information and enforceable contracts, the traditional distinction between debt and equity financing of firms gets blurred and the irrelevance proposition of Modigliani and Miller (1958) holds trivially. Thus, the financial arrangement we have described can be interpreted as a debt contract as well. Namely, suppose the firm is formed with $s_M > 0$ where a venture capitalist provides external funds, with s_M representing the venture capitalist's share of stock. It can also be argued that the venture capitalist provides the amount $s_M I$ of funding at time t when shares in costs and returns are proportional. At time $t+1$, she is entitled to the amount $s_M R = s_M I + s_M(R - I)$, composed of the repayment of the principal $s_M I$ and the interest payment $s_M(R - I)$. On the other hand, our model does not encompass mature firms' financing patterns involving a variety of securities as different securities often yield different ownership and control structures. We leave the more detailed exploration of this possibility to future research.

Last but not least, our model may not be applicable to certain industries. For instance, firms are not always founded to implement a product or process innovation. Funding may not be an issue. According to Coase (1937), one rationale for the existence of a firm is a reduction in contracting costs. A law firm may benefit from the complementary skills of its associates and avoid contracting with outside specialists. Productive cooperatives are formed to realize economies of scale and to improve pricing of inputs and outputs. Alternative ways of firm formation and funding raise novel issues, e.g., hold-up problems,[57] which lie beyond the scope of the current paper.

[57]See for example, Ewens et al. (2016), Gersbach and Haller (2020).

3.A Proofs

Proof of Lemma 3. The full expressions of agents' utility functions become

$$
\begin{aligned}
U^E &= w_E + \phi_E^{Sh} - I_E \\
&= w_E + \phi_E^{Sh} - \max\big[\min\big(\frac{I}{R}\phi_E^{Sh}, B_E\big), I - B_M\big] \\
&= w_E + \min\big[\phi_E^{Sh} - \min\big(\frac{I}{R}\phi_E^{Sh}, B_E\big), \phi_E^{Sh} - I + B_M\big] \\
&= w_E + \min\big[\max\big(\phi_E^{Sh} - \frac{I}{R}\phi_E^{Sh}, \phi_E^{Sh} - B_E\big), \phi_E^{Sh} - I + B_M\big] \\
&= w_E + \min\big[\max\big(\frac{R-I}{R}\phi_E^{Sh}, \phi_E^{Sh} - B_E\big), \phi_E^{Sh} - I + B_M\big]
\end{aligned}
$$

and similarly,

$$
\begin{aligned}
U^M &= w_M + \phi_M^{Sh} - I_M \\
&= w_M + \phi_M^{Sh} - \min\big[\max\big(\frac{I}{R}\phi_M^{Sh}, I - B_E\big), B_M\big] \\
&= w_M + \max\big[\min\big(\frac{R-I}{R}\phi_M^{Sh}, \phi_M^{Sh} - I + B_E\big), \phi_M^{Sh} - B_M\big].
\end{aligned}
$$

In the case of one large investor, the Shapley values reduce to the Nash bargaining solutions, and are calculated as:

$$
\begin{aligned}
\phi_E^{Sh} &= \frac{1!0!}{2!}\big[v(\{E, M\}) - v(\{M\})\big] + \frac{0!1!}{2!}\big[v(\{E\}) - v(\varnothing)\big] \\
&= \frac{1}{2}[R - B_M] + \frac{1}{2}[B_E - 0] \\
&= \frac{1}{2}R - \frac{1}{2}B_M + \frac{1}{2}B_E \\
\phi_M^{Sh} &= \frac{1}{2}R - \frac{1}{2}B_E + \frac{1}{2}B_M
\end{aligned}
$$

To prove this Lemma, we start with the entrepreneur's utility function:

$$
U^E = w_E + \min\big[\max\big(\frac{R-I}{R}\phi_E^{Sh}, \phi_E^{Sh} - B_E\big), \phi_E^{Sh} - I + B_M\big]
$$

(i) Since $\phi_E^{Sh} \geq 0$ and $R > I$, we have $\max\left(\frac{R-I}{R}\phi_E^{Sh}, \phi_E^{Sh} - B_E\right) \geq 0$.

(ii) The term $\phi_E^{Sh} - I + B_M$ can be written as:

$$\begin{aligned}\phi_E^{Sh} - I + B_M &= \frac{1}{2}R - \frac{1}{2}B_M + \frac{1}{2}B_E - I + B_M \\ &= \frac{1}{2}(R-I) + \frac{1}{2}(B_M + B_E - I)\end{aligned}$$

Since $R > I$ and $B_M + B_E \geq I$, we have $\phi_E^{Sh} - I + B_M > 0$.

(iii) $U^E \geq w_E$ follows from (i) and (ii).

Similarly, we can prove the market's utility function satisfies $U^M \geq w_M$. We have

$$U^M = w_M + \max\left[\min\left(\frac{R-I}{R}\phi_M^{Sh}, \phi_M^{Sh} - I + B_E\right), \phi_M^{Sh} - B_M\right].$$

(iv) The term $\phi_M^{Sh} - I + B_E$ can be written as

$$\begin{aligned}\phi_M^{Sh} - I + B_E &= \frac{1}{2}R - \frac{1}{2}B_E + \frac{1}{2}B_M - I + B_E \\ &= \frac{1}{2}(R-I) + \frac{1}{2}(B_M + B_E - I).\end{aligned}$$

Thus, $R > I$, $B_M + B_E \geq I$ and $\phi_M^{Sh} \geq 0$ yield $\min\left(\frac{R-I}{R}\phi_M^{Sh}, \phi_M^{Sh} - I + B_E\right) \geq 0$.

(v) $\min\left(\frac{R-I}{R}\phi_M^{Sh}, \phi_M^{Sh} - I + B_E\right) \geq 0$ implies $U^M \geq w_M$. □

Proof of Lemma 4. From Lemma 3, we know that

$$s_E = \frac{\phi_E^{Sh}}{R} = \frac{1}{R}\left(\frac{1}{2}R - \frac{1}{2}B_M + \frac{1}{2}B_E\right) = \frac{1}{2R}(R - B_M + B_E).$$

Thus we have $\widehat{B}_E = \arg\max s_E = w_E$. Similarly we can get $\widehat{B}_M = \arg\max s_M = \min(w_M, R)$.

□

Proof of Proposition 6. To find the Nash equilibria, we first look at the best responses of each party.

(i) For the entrepreneur, the utility function can be decomposed as

$$\begin{aligned} U^E &= w_E + \min\left[\max\left(\frac{R-I}{R}\phi_E^{Sh}, \phi_E^{Sh} - B_E\right), \phi_E^{Sh} - I + B_M\right] \\ &= w_E + \min\Big[\max\Big(\underbrace{\frac{R-I}{R}\Big(\frac{R}{2} - \frac{B_M}{2} + \frac{B_E}{2}\Big)}_{\textcircled{1}}, \underbrace{\frac{R}{2} - \frac{B_M}{2} - \frac{B_E}{2}}_{\textcircled{2}}\Big), \\ &\qquad \underbrace{\frac{1}{2}R - I + \frac{B_M}{2} + \frac{B_E}{2}}_{\textcircled{3}}\Big] \end{aligned}$$

Thus, we can solve for the intersection points of the above three lines:

$$\begin{aligned} \textcircled{1} = \textcircled{2} \quad &\Rightarrow \quad B_E = \frac{R}{2R-I}I - \frac{I}{2R-I}B_M \\ \textcircled{3} = \textcircled{2} \quad &\Rightarrow \quad B_E = I - B_M \end{aligned}$$

There are two possibilities to draw these three lines. The first case is when $I - B_M < \frac{R}{2R-I}I - \frac{I}{2R-I}B_M$. Observe that the slope of line $\textcircled{1}$ is always smaller than that of line $\textcircled{3}$. The following Figure A1 depicts this first case. Since the constraint on B_E requires that $B_E \geq I - B_M$, the thick line in Figure A1 shows the utility level of the entrepreneur.

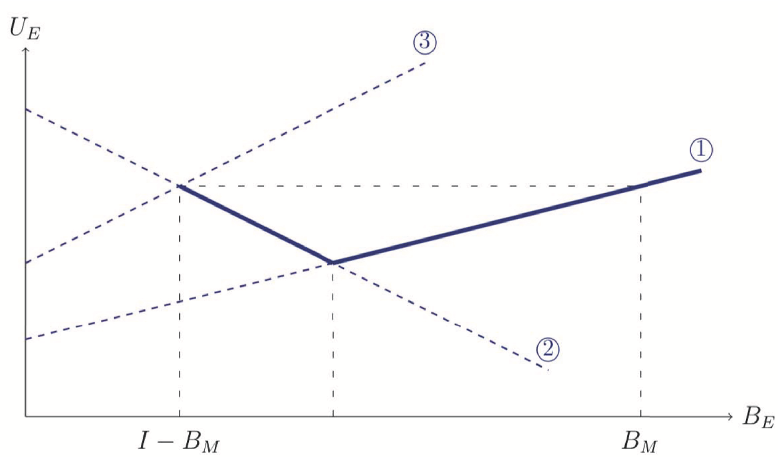


Figure A1: Entrepreneur's Utility: Case 1

Since $I - B_M < \frac{R}{2R-I}I - \frac{I}{2R-I}B_M$ leads to $B_M > \frac{I}{2}$, from Figure A1 we can see that the entrepreneur has two options to maximize utility: (i) $\max(I-B_M, 0)$, (ii) w_E. Thus the best response narrows down to the comparison between these two options. From line ② and line ① in Figure A1 we can find the entrepreneur's utility as follows:

$$\begin{aligned}
U^E\big|_{B_E=\max(I-B_M,0)} &= w_E + \frac{R}{2} - \frac{B_M}{2} - \frac{1}{2}\max(I-B_M,0) \\
&= w_E + \frac{1}{2}[R - \max(I, B_M)] \\
U^E\big|_{B_E=w_E} &= w_E + \frac{R-I}{R}(\frac{R}{2} - \frac{B_M}{2} + \frac{w_E}{2})
\end{aligned}$$

(a) If $B_M \leq I$, then

$$\begin{aligned}
U^E\big|_{B_E=\max(I-B_M,0)} - U^E\big|_{B_E=w_E} &= \frac{R-I}{2} - \frac{R-I}{R}(\frac{R}{2} - \frac{B_M}{2} + \frac{w_E}{2}) \\
&= \frac{R-I}{2R}(B_M - w_E) \geq 0 \quad \Leftrightarrow \quad B_M \geq w_E
\end{aligned}$$

Thus we have the best response:

$$B_E^* = \begin{cases} \max(I-B_M, 0) & \text{if } w_E \leq B_M \leq I, \\ w_E & \text{if } B_M \leq w_E \end{cases}$$

(b) If $B_M > I$, then

$$\begin{aligned}
U^E\big|_{B_E=\max(I-B_M,0)} \quad - \quad U^E\big|_{B_E=w_E} &= \frac{R}{2} - \frac{B_M}{2} - \frac{R-I}{R}(\frac{R}{2} - \frac{B_M}{2} + \frac{w_E}{2}) \\
&= \frac{1}{2}(I - \frac{I}{R}B_M - \frac{R-I}{R}w_E) \geq 0 \quad \Leftrightarrow \quad B_M \leq R - \frac{R-I}{I}w_E
\end{aligned}$$

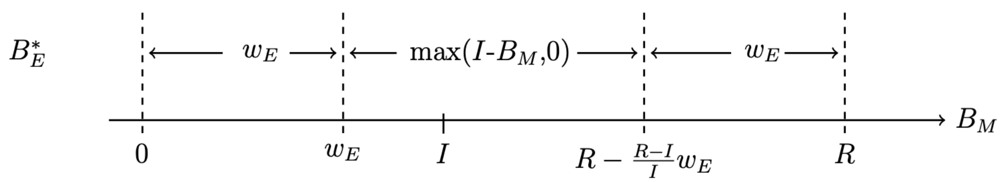
Since $w_E < I \Rightarrow \frac{R-I}{I}w_E < R - I \Rightarrow R - \frac{R-I}{I}w_E > I$, the best response is:

$$B_E^* = \begin{cases} \max(I-B_M, 0) & \text{if } I < B_M \leq R - \frac{R-I}{I}w_E, \\ w_E & \text{if } B_M \geq R - \frac{R-I}{I}w_E \end{cases}$$

Combining (a) and (b), the entrepreneur's best response in case 1 can be summarized as follows:

$$\text{If } B_M > \frac{I}{2}, \qquad B_E^* = \begin{cases} \max(I - B_M, 0) & \text{if } w_E \leq B_M \leq R - \frac{R-I}{I} w_E, \\ w_E & \text{if } B_M \leq w_E, \text{ or } B_M \geq R - \frac{R-I}{I} w_E \end{cases}$$

Graphically, an illustration of the above best response is shown below.



The second case is $B_M \leq \frac{I}{2}$, which is shown in Figure A2. Here the entrepreneur's utility is monotonically increasing with respect to B_E. Thus his best response is to set $B_E^* = w_E$.

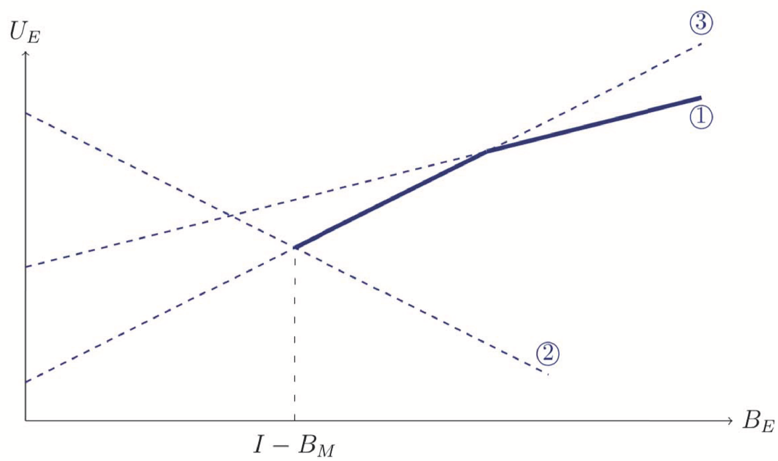


Figure A2: Entrepreneur's Utility: Case 2

To summarize, the entrepreneur's best response given B_M is the following:

$$\text{In case } B_M > \frac{I}{2}, \qquad B_E^* = \begin{cases} \max(I - B_M, 0) & \text{if } w_E \leq B_M \leq R - \frac{R-I}{I} w_E, \\ w_E & \text{if } B_M \leq w_E, \text{ or } B_M \geq R - \frac{R-I}{I} w_E. \end{cases}$$

$$\text{In case } B_M \leq \frac{I}{2}, \qquad B_E^* = w_E.$$

(ii) For the market, we can similarly write the utility function as:

$$
\begin{aligned}
U^M &= w_M + \max\big[\min\big(\frac{R-I}{R}\phi_M^{Sh}, \phi_M^{Sh} - I + B_E\big), \phi_M^{Sh} - B_M\big] \\
&= w_M + \max\big[\min\big(\underbrace{\frac{R-I}{R}(\frac{R}{2} - \frac{B_E}{2} + \frac{B_M}{2})}_{①}, \underbrace{\frac{R}{2} - I + \frac{B_E}{2} + \frac{B_M}{2}}_{③}\big), \\
&\qquad \underbrace{\frac{R}{2} - \frac{B_E}{2} - \frac{B_M}{2}}_{②}\big]
\end{aligned}
$$

Thus, the above three lines are the exact symmetric analogues of the earlier ones. Note that the only difference between the market's problem and the entrepreneur's problem is the wealth constraint. For instance, since $w_E < I$, we only need to consider the case of $B_E < I$ in computing the best response. Accordingly, we can write the best response of the market as follows:

$$
\begin{aligned}
&\text{In case } B_E > \frac{I}{2}, \qquad B_M^* = \begin{cases} I - B_E & \text{if } \min(w_M, R) \le B_E, \\ \min(w_M, R) & \text{if } \min(w_M, R) \ge B_E. \end{cases} \\
&\text{In case } B_E \le \frac{I}{2}, \qquad B_M^* = \min(w_M, R).
\end{aligned}
$$

(iii) Given these best responses, the Nash Equilibria are as stated in the proposition.

□

Proof of Proposition 7. We distinguish the three cases specified in Proposition 6.

Case (a): $w_M > R - \frac{R-I}{I}w_E$. From Proposition 6, $B_E^* = w_E$ and $B_M^* = \min(w_M, R)$.

Since here $B_M^* > \frac{I}{2}$, the utility of the entrepreneur looks like Figure A1. Following line ①, we have

$$
U^E = w_E + \frac{R-I}{R}\Big(\frac{R}{2} - \frac{\min(w_M, R)}{2} + \frac{w_E}{2}\Big)
$$

To maximize U^E, the entrepreneur will choose w_M as small as possible. Consider $w_M =$

$R - \frac{R-I}{I} w_E + \varepsilon$, then

$$\begin{aligned} U^E &= w_E + \frac{R-I}{2R}(R - w_M + w_E) = w_E + \frac{R-I}{2R}(\frac{R-I}{I} w_E - \varepsilon + w_E) \\ &= w_E + \frac{R-I}{2R}(\frac{R}{I} w_E - \varepsilon) = w_E + \frac{R-I}{2}\frac{w_E}{I} - \frac{R-I}{2R}\varepsilon \end{aligned}$$

Since $w_E < I \Rightarrow \frac{w_E}{I} < 1$, we have $U^E < w_E + \frac{R-I}{2}$.

Case (c): $w_M = R - \frac{R-I}{I} w_E$. From Proposition 6, $B_E^* = w_E$ or $B_E^* = 0$ and $B_M^* = w_M$.

Since the entrepreneur is indifferent between choosing $B_E^* = 0$ or $B_E^* = w_E$, the utility can be expressed as (from line ②)

$$\begin{aligned} U^E &= w_E + \frac{R}{2} - \frac{w_M}{2} - \frac{0}{2} = w_E + \frac{1}{2}(R - w_M) \\ &= w_E + \frac{1}{2}\frac{R-I}{I} w_E = w_E + \frac{R-I}{2}\frac{w_E}{I} < w_E + \frac{R-I}{2} \end{aligned}$$

Combining Case (a) and (c), we can see that if $w_M \geq R - \frac{R-I}{I} w_E$, the best choice for the entrepreneur is to choose $w_M = R - \frac{R-I}{I} w_E$, and the utility will be $U^E = w_E + \frac{R-I}{2}\frac{w_E}{I}$. However, the entrepreneur can do better (achieving a higher utility level) by choosing an even smaller w_M, as shown in the following Case (b).

Case (b): $w_M < R - \frac{R-I}{I} w_E$. From Proposition 6 we can further divide this case into the following three sub-cases:

(b.1) If $w_E < w_M$, then $B_E^* = \max(I - w_M, 0)$, $B_M^* = w_M$.

Since $w_E < w_M$ and $w_E + w_M \geq I$ imply that $w_M > \frac{I}{2}$, the utility of the entrepreneur follows Figure A1. From line ② we have

$$U^E = w_E + \frac{R}{2} - \frac{w_M}{2} - \frac{\max(I - w_M, 0)}{2} = w_E + \frac{1}{2}[R - \max(I, w_M)]$$

If the entrepreneur wants to maximize utility he can choose $w_M \leq I$, and then $U_{max}^E = w_E + \frac{1}{2}(R - I)$. Thus, as long as $w_M \leq I$, the entrepreneur is indifferent between any feasible w_M.

(b.2) If $w_E > w_M$, then $B_E^* = w_E$ and $B_M^* = I - w_E$.

Since $w_E > w_M$ and $w_E + w_M \geq I$ imply that $w_E > \frac{I}{2}$, $\Rightarrow B_M^* = I - w_E < \frac{I}{2}$, the utility of the entrepreneur follows Figure A2. Thus,

$$\begin{aligned}
U^E &= w_E + \min[①, ③] \\
&= w_E + \min\left[\frac{R-I}{2R}(R - B_M^* + B_E^*), \frac{1}{2}R - I + \frac{B_M^*}{2} + \frac{B_E^*}{2}\right] \\
&= w_E + \min\left[\frac{R-I}{2R}(R - I + w_E + w_E), \frac{1}{2}R - I + \frac{I - B_E^*}{2} + \frac{B_E^*}{2}\right] \\
&= w_E + \min\left[\frac{R-I}{2R}(R - I + 2w_E), \frac{1}{2}(R - I)\right] \\
&= w_E + \frac{1}{2}(R - I).
\end{aligned}$$

The last line follows from $w_E \in (\frac{I}{2}, I)$. We conclude that the entrepreneur is indifferent between any w_M that is feasible, i.e., $w_M^* \in [I - w_E, w_E)$, and the maximum utility is $U_{max}^E = w_E + \frac{1}{2}(R - I)$.

(b.3) If $w_E = w_M$, Proposition 6 asserts that there are three NEs. Since both the entrepreneur and the market investor are indifferent between choosing w_E or $I - w_E$, and $U^E + U^M = w_E + w_M + (R - I)$ always holds, from the symmetry of the two agents we can conclude that $U_{max}^E = w_E + \frac{1}{2}(R - I)$.

Comparing (b.1), (b.2) and (b.3) we can see that in Case (b), the entrepreneur's optimal choice is $w_M^* \in [I - w_E, I]$, and the maximum utility is $U_{max}^E = w_E + \frac{1}{2}(R - I)$.

Finally we can combine Cases (a) to (c) and find the entrepreneur's optimal choices to be as in Proposition 7. □

Proof of Proposition 8. From Definitions 4 and 5 we know that the ownership of the entrepreneur is

$$s_E = \frac{\phi_E^{Sh}}{R} = \frac{1}{R}\left[\frac{1}{2}R - \frac{1}{2}B_M^* + \frac{1}{2}B_E^*\right] = \frac{1}{2} + \frac{B_E^* - B_M^*}{2R}.$$

Thus if the entrepreneur wants to obtain control, he needs to have $B_E^* \geq B_M^*$.

From Proposition 7 we know that the entrepreneur would choose $w_m^* \in [I - w_E, I]$ to maximize his utility. Thus we can divide the analysis into the following cases:

Case 1: $w_E \in [0, \frac{I}{2}]$. Since now we have $B_M^* = w_M$, and $B_E^* = I - w_M$, $B_E^* \geq B_M^*$ is equivalent to $I - w_M \geq w_M$ or $w_M \leq \frac{I}{2}$. This combined with $w_M^* \in [I - w_E, I]$ implies:

(1) If $w_E < \frac{I}{2}$, then the entrepreneur never obtains control when maximizing utility. In addition, if the entrepreneur maximizes ownership, the best he can do according to Lemma 4 is to set $B_E = w_E$. Since $w_E < \frac{I}{2}$, it has to be the case that $w_M > \frac{I}{2}$, which means $B_E < B_M$. Again the entrepreneur cannot obtain control.

(2) If $w_E = \frac{I}{2}$, then by the tie-breaking rule of Definition 5, the entrepreneur's only option to obtain the control right is to choose $w_M^* = \frac{I}{2}$.

Case 2: $w_E \in (\frac{I}{2}, I)$. Here we further consider three sub-cases:

(i) When $w_E < w_M$, we have $B_M^* = w_M$, and $B_E^* = I - w_M$ by Propositions 6 and 7 . Thus when $w_M > w_E > \frac{I}{2}$, it is impossible for $B_E^* \geq B_M^*$ to hold. As a result the entrepreneur will not obtain control if he picks $w_M > w_E$.

(ii) When $w_E > w_M$, we have $B_M^* = I - w_E$, and $B_E^* = w_E$ by Propositions 6 and 7. Thus $B_E^* \geq B_M^*$ is equivalent to $w_E \geq I - w_E$ or $w_E \geq \frac{I}{2}$, which is satisfied for $w_E \in (\frac{I}{2}, I)$.

(iii) When $w_E = w_M$, we know from proposition 6 that there are three possible NE. One of the possible NE is $B_M^* = w_E$ and $B_E^* = I - w_E$. Similar to (i), the entrepreneur never has control in this NE. Here we can see that if the entrepreneur choose $w_M = w_E$, there is a positive probability that he may lose the control right. As a result, although the entrepreneur can still obtain control under the other two NE, we consider that he would be better off not choosing this option.

Combining (i), (ii), and (iii) we conclude in Case 2 that if $w_E \in (\frac{I}{2}, I)$, the entrepreneur can achieve maximum utility and obtain control by choosing an investor whose wealth is less than his own, i.e., $w_M^* \in [I - w_E, w_E)$. □

Proof of Lemma 5. Since $J=2$, $w_j=\frac{I}{2}$, there are two cases:

Case 1: The entrepreneur establishes the firm with one small investors, i.e., $n=1$.

Similar to the analysis in the case for one large investor, we can represent the value function as:

$$v(\{E,M_j\})=R,\ v(\{E\})=B_E,\ v(\{M_j\})=\frac{I}{2},\ v(\varnothing)=0.$$

Thus equation (10) yields:

$$\phi_E^{Sh}=\frac{1}{2}R-\frac{1}{2}\frac{I}{2}+\frac{1}{2}B_E=\frac{1}{2}(R-\frac{I}{2}+B_E).$$

From Definitions 4 and 5 we know that the condition for the entrepreneur to maintain control is:

$$s_E=\frac{\phi_E^{Sh}}{R}\geq\frac{1}{2}\quad\Rightarrow\quad\phi_E^{Sh}\geq\frac{R}{2}\quad\Rightarrow\quad\frac{I}{2}+B_E\geq 0\quad\Rightarrow\quad B_E\geq\frac{I}{2},$$

which means that $B_E^{\min}\big|_{n=1}=\frac{I}{2}$.

Case 2: The entrepreneur establishes the firm with two small investors, i.e., $n=2$.

Here we can represent the value function as:

$$\begin{aligned}&v(\{E,M_1,M_2\})=R,\ v(\{E,M_j\})=R,\ v(\{M_1,M_2\})=I,\\&v(\{E\})=B_E,\ v(\{M_j\})=\frac{I}{2},\ v(\varnothing)=0.\end{aligned}$$

In other words, we have $N=\{E,M_1,M_2\}$, and $S\subseteq N\setminus\{E\}$ means S can be $\{M_1,M_2\}$, $\{M_1\}$, $\{M_2\}$ or $\varnothing$. Thus equation (10) implies

$$\begin{aligned}\phi_E^{Sh}&=\frac{2!}{3!}(R-I)+\frac{1!1!}{3!}(R-\frac{I}{2})\times 2+\frac{2!}{3!}(B_E-0)\\&=\frac{1}{3}(R-I)+\frac{1}{3}(R-\frac{I}{2})+\frac{1}{3}B_E\\&=\frac{2}{3}R-\frac{1}{2}I+\frac{1}{3}B_E,\\\phi_{M_j}^{Sh}&=\frac{1}{2}(R-\phi_E^{Sh})=\frac{1}{2}(\frac{1}{3}R+\frac{1}{2}I-\frac{1}{3}B_E).\end{aligned}$$

From Definitions 4 and 5 we know that the condition for the entrepreneur to maintain control is:

$$\begin{aligned}
& s_E \geq s_{M_j} \quad \Rightarrow \quad \frac{\phi_E^{Sh}}{R} \geq \frac{\phi_{M_j}^{Sh}}{R} \quad \Rightarrow \quad \phi_E^{Sh} \geq \phi_{M_j}^{Sh} \\
& \Rightarrow \quad \frac{2}{3}R - \frac{1}{2}I + \frac{1}{3}B_E \geq \frac{1}{2}(\frac{1}{3}R + \frac{1}{2}I - \frac{1}{3}B_E) \quad \Rightarrow \quad B_E \geq \frac{3}{2}I - R,
\end{aligned}$$

which means that $B_E^{\min}\big|_{n=2} = \max(\frac{3}{2}I - R, 0)$. Since $\frac{3}{2}I - R = \frac{I}{2} + (I - R) < \frac{I}{2}$, we obtain $B_E^{\min}\big|_{n=2} < B_E^{\min}\big|_{n=1}$. □

Proof of Proposition 9. To find the optimal n, we first have to determine the entrepreneur's utility under both $n = 1$ and $n = 2$ and compare these two utilities. Like in Lemma 5 we can again break up the analysis into two cases:

Case 1: The entrepreneur establishes the firm with one small investor, i.e., $n = 1$. From the proof of Lemma 5 we know that the entrepreneur's share of return can be written as

$$\phi_E^{Sh} = \frac{1}{2}(R - \frac{I}{2} + B_E).$$

Conditional on maintaining control, the entrepreneur can choose $B_E \in [\frac{I}{2}, w_E]$. Thus the entrepreneur's share of cost can be derived from equation (12) and simplified as follows:

$$\begin{aligned}
I_E &= \max\big[\min\big(\frac{I}{R}\phi_E^{Sh}, B_E\big), \frac{I}{2}\,\big] \\
&= \min\big(\frac{I}{R}\phi_E^{Sh}, B_E\big) \;=\; \frac{I}{R}\phi_E^{Sh}.
\end{aligned}$$

The second equality follows because when $B_E \geq \frac{I}{2}$, we have $\phi_E^{Sh} \geq \frac{R}{2}$ and thus $\frac{I}{R}\phi_E^{Sh} \geq \frac{I}{2}$. The third equality can be proven as follows:

$$\begin{aligned}
\frac{I}{R}\phi_E^{Sh} \leq B_E \;&\Leftrightarrow\; \frac{I}{R}(R - \frac{I}{2} + B_E) \leq 2B_E \;\Leftrightarrow\; \frac{I}{R}\big[\frac{1}{2}(2R - I) + B_E - \frac{2R}{I}B_E\big] \leq 0 \\
&\Leftrightarrow\; \frac{1}{2}(2R - I) + \frac{B_E}{I}(I - 2R) \leq 0 \;\Leftrightarrow\; \frac{1}{2} \leq \frac{B_E}{I} \;\Leftrightarrow\; B_E \geq \frac{I}{2}.
\end{aligned}$$

Thus the entrepreneur's utility can be written as:

$$U^E\big|_{n=1} = w_E + \phi_E^{Sh} - I_E = w_E + \phi_E^{Sh} - \frac{I}{R}\phi_E^{Sh} = w_E + \frac{R-I}{2R}(R - \frac{I}{2} + B_E).$$

Maximizing utility results in $B_E^* = w_E$ and the following:

$$U_{\max}^E\big|_{n=1} = w_E + \frac{R-I}{2R}(R - \frac{I}{2} + w_E) = w_E + \frac{R-I}{R}(\frac{R}{2} - \frac{I}{4} + \frac{1}{2}w_E). \quad \text{(A1)}$$

Case 2: The entrepreneur establishes the firm with two small investors, i.e., $n = 2$. From the proof of Lemma 5 we know that the entrepreneur's share of return can be written as

$$\phi_E^{Sh} = \frac{2}{3}R - \frac{1}{2}I + \frac{1}{3}B_E.$$

Conditional on maintaining control, the entrepreneur can choose $B_E \in [\max(\frac{3}{2}I - R, 0), w_E]$. Thus the entrepreneur's share of cost can be derived from a variation of equation (12) and simplified as follows:

$$\begin{aligned} I_E &= \max\big[\min\big(\frac{I}{R}\phi_E^{Sh}, B_E\big), I - B_{M_1} - B_{M_2}\big] \\ &= \max\big[\min\big(\frac{I}{R}\phi_E^{Sh}, B_E\big), I - \frac{I}{2} - \frac{I}{2}\big] \\ &= \min\big(\frac{I}{R}\phi_E^{Sh}, B_E\big) \end{aligned}$$

The third line follows immediately because the combined wealth of two small investors equals the investment cost.

Thus the entrepreneur's utility can be written as

$$\begin{aligned} U^E\big|_{n=2} &= w_E + \phi_E^{Sh} - I_E = w_E + \max(\frac{R-I}{R}\phi_E^{Sh}, \phi_E^{Sh} - B_E) \\ &= w_E + \max\big[\underbrace{\frac{R-I}{R}(\frac{2}{3}R - \frac{1}{2}I + \frac{1}{3}B_E)}_{①}, \underbrace{\frac{2}{3}R - \frac{1}{2}I - \frac{2}{3}B_E}_{②}\big]. \end{aligned}$$

Observe that if the utility is given by line ① then the optimal choice is to set $B_E^* = w_E$, if the utility is given by line ② then the optimal choice is to set $B_E^* = \max(\frac{3}{2}I - R, 0)$. Thus the maximum utility can be expressed as:

$$\begin{aligned} U^E_{\max}\Big|_{n=2} &= w_E + \max\big[\frac{R-I}{R}(\frac{2}{3}R - \frac{1}{2}I + \frac{1}{3}w_E)\,,\,\frac{2}{3}R - \frac{1}{2}I - \frac{2}{3}\max(\frac{3}{2}I - R, 0)\,\big] \\ &= w_E + \max\big[\frac{R-I}{R}(\frac{2}{3}R - \frac{1}{2}I + \frac{1}{3}w_E), \min\big(\frac{4}{3}R - \frac{3}{2}I, \frac{2}{3}R - \frac{1}{2}I\big)\big]. \end{aligned} \tag{A2}$$

(1) To find the cutoff points, we observe first that if $R \geq \frac{3}{2}I$, then $B_E^* = 0$ and the maximum utility can be simplified as:

$$\begin{aligned} U^E_{\max}\Big|_{n=2} &= w_E + \max\big[\frac{R-I}{R}(\frac{2}{3}R - \frac{1}{2}I + \frac{1}{3}w_E)\,,\,\frac{2}{3}R - \frac{1}{2}I\,\big] \\ &= w_E + \frac{2}{3}R - \frac{1}{2}I. \end{aligned}$$

The above second line can be derived as follows: Since $R \geq \frac{3}{2}I$ and $I > w_E$,

$$\begin{aligned} & R > \frac{1}{2}I \quad \Rightarrow \quad R - I < 2R - \frac{3}{2}I \\ \Rightarrow \quad & \frac{R-I}{R}(\frac{2}{3}R - \frac{1}{2}I + \frac{1}{3}w_E) \\ = \quad & \frac{R-I}{3R}w_E + \frac{R-I}{R}(\frac{2}{3}R - \frac{1}{2}I) \\ < \quad & \frac{R-I}{3R}I + \frac{R-I}{R}(\frac{2}{3}R - \frac{1}{2}I) \\ < \quad & \frac{I}{R}(\frac{2}{3}R - \frac{1}{2}I) + \frac{R-I}{R}(\frac{2}{3}R - \frac{1}{2}I) \\ = \quad & \frac{2}{3}R - \frac{1}{2}I. \end{aligned}$$

Combining Case 1 and Case 2 we know that when $R \geq \frac{3}{2}I$ holds, the maximum utility can be expressed as:

$$U^E_{\max} = \max\big(\,U^E_{\max}\Big|_{n=1}\,,\,U^E_{\max}\Big|_{n=2}\,\big) = U^E_{\max}\Big|_{n=2}$$

The second equality can be demonstrated as follows when $R \geq \frac{3}{2}I$ and $I > w_E$:

$$
\begin{aligned}
& 2R^2 - 3RI + 3I^2 \; > \; 2(R^2 - 2RI + I^2) \; = \; 2(R-I)^2 \; > \; 0 \\
\Rightarrow \;\; & \frac{1}{2}(2R^2 - RI - I^2) < \frac{1}{3}(4R^2 - 3RI) \\
\Rightarrow \;\; & (R-I)\frac{2R+I}{4} < R(\frac{4R-3I}{6}) \\
\Rightarrow \;\; & \frac{R-I}{R}(\frac{R}{2} - \frac{I}{4} + \frac{1}{2}I) < \frac{2}{3}R - \frac{1}{2}I \\
\Rightarrow \;\; & \frac{R-I}{R}(\frac{R}{2} - \frac{I}{4} + \frac{1}{2}w_E) < \frac{2}{3}R - \frac{1}{2}I \\
\Rightarrow \;\; & U^E_{\max}\big|_{n=1} < U^E_{\max}\big|_{n=2}.
\end{aligned}
$$

Thus we can define profitability as $\rho = \frac{R}{I}$ and conclude that there exists $\rho^{**} \leq \frac{3}{2}$ such that when $\rho > \rho^{**}$, the entrepreneur prefers a more dispersed ownership structure, $n^* = 2$.

(2) When $R < \frac{3}{2}I$, we have

$$
U^E_{\max}\big|_{n=2} \;\; = \;\; w_E + \max\big[\frac{R-I}{R}(\frac{2}{3}R - \frac{1}{2}I + \frac{1}{3}w_E)\, , \; \frac{4}{3}R - \frac{3}{2}I\,\big].
$$

First we prove that when $R \leq \frac{8}{7}I$, the above expression can be further simplified into:

$$
U^E_{\max}\big|_{n=2} \;\; = \;\; w_E + \frac{R-I}{R}(\frac{2}{3}R - \frac{1}{2}I + \frac{1}{3}w_E).
$$

The proof goes as follows:

$$
\begin{aligned}
& R \leq \frac{8}{7}I \\
\Rightarrow \;\; & 2R^2 - \frac{3}{2}RI - I^2 \; = \; 2R^2 - [\frac{3}{2}RI + I^2] \\
& \leq \; 2R^2 - \frac{3}{2}R \cdot \frac{7}{8}R - \frac{49}{64}R^2 \; = \; R^2[2 - \frac{21}{16} - \frac{49}{64}] \; = \; R^2[2 - \frac{133}{64}] < 0 \\
\Rightarrow \;\; & (R-I)(2R-I) \geq R(4R - \frac{9}{2}I) \\
\Rightarrow \;\; & \frac{R-I}{R}(\frac{2}{3}R - \frac{1}{2}I + \frac{1}{3}\frac{I}{2}) \geq \frac{4}{3}R - \frac{3}{2}I \\
\Rightarrow \;\; & \frac{R-I}{R}(\frac{2}{3}R - \frac{1}{2}I + \frac{1}{3}w_E) \geq \frac{4}{3}R - \frac{3}{2}I.
\end{aligned}
$$

Combining Case 1 and Case 2 we find that when $R \leq \frac{8}{7}I$, the maximum utility can be expressed as:

$$U^E_{\max} = \max\left(U^E_{\max}\big|_{n=1}, U^E_{\max}\big|_{n=2} \right) = U^E_{\max}\big|_{n=1}$$

The second equality can be shown as follows:

$$\begin{aligned}
& R \leq \frac{8}{7}I \Rightarrow R < \frac{3}{2}I \Rightarrow w_E \geq 0 > R - \frac{3}{2}I \\
& \Rightarrow \frac{1}{6}w_E > \frac{1}{6}R - \frac{1}{4}I \Rightarrow \frac{R}{2} - \frac{I}{4} + \frac{1}{2}w_E > \frac{2}{3}R - \frac{1}{2}I + \frac{1}{3}w_E \\
& \Rightarrow \frac{R-I}{R}(\frac{R}{2} - \frac{I}{4} + \frac{1}{2}w_E) > \frac{R-I}{R}(\frac{2}{3}R - \frac{1}{2}I + \frac{1}{3}w_E) \Leftrightarrow U^E_{\max}\big|_{n=1} > U^E_{\max}\big|_{n=2}.
\end{aligned}$$

Thus we can conclude that there exists $\rho^* \geq \frac{8}{7}$, such that when $\rho < \rho^*$, the entrepreneur prefers a more concentrated ownership structure, $n^* = 1$.

(3) Finally, we prove the third statement in Proposition 9. From the previous proof we know that $n^* = 1$ when $\rho < \rho^*$ and $n^* = 2$ when $\rho > \rho^{**}$. If we fix the profitability, then equations (A1) and (A2) show that both utilities are continuous in w_E. Thus there exists $w^*_E(\rho)$ such that the entrepreneur is indifferent between $n = 1$ and $n = 2$. In addition, when w_E increases, the maximum utility under $n = 1$ is increasing faster than the maximum utility under $n = 2$. Thus we conclude that when $w_E > w^*_E(\rho)$ the optimal is $n^* = 1$, and $w_E < w^*_E(\rho)$ the optimal is $n^* = 2$. □

Proof of Corollary 1. From Proposition 7 we know that the maximum utility for the entrepreneur is $U^E_{\max} = w_E + \frac{1}{2}(R - I)$ when establishing the firm with one wealthy investor. From the proof of Proposition 9 we know that when raising capital from small investors, the maximum utility is $U^E_{\max} = \max\left(U^E_{\max}\big|_{n=1}, U^E_{\max}\big|_{n=2} \right)$. By equation (A1), if $w_E \in (\frac{I}{2}, I)$, then $U^E_{\max}\big|_{n=1} > w_E + \frac{R-I}{R}(\frac{R}{2} - \frac{I}{4} + \frac{1}{2}\frac{I}{2}) = w_E + \frac{1}{2}(R - I)$. Therefore, the entrepreneur is strictly better off raising capital from small investors when $w_E \in (\frac{I}{2}, I)$. □

Proof of Proposition 10. We first derive Equations (17) and (18). Since $N = \{E, M_1, ..., M_n\}$, $S \subseteq N \setminus \{E\}$ means S can be (i) $\{M_1, ..., M_n\}$, (ii) any $n-1$ investors like $\{M_1, ..., M_{n-1}\}$, (iii) any $n-2$ investors like $\{M_1, ..., M_{n-2}\}$, ... , or finally $\varnothing$. Since investors are identical, we can write ϕ_E^{Sh} by using the binomial coefficient C_n^k as follows:[58] There are C_n^k sets of k investors so that

$$\begin{aligned}
\phi_E^{Sh} \quad = \quad & \frac{n!0!}{(n+1)!}[R - \frac{n}{J}I] + \frac{(n-1)!1!}{(n+1)!}[R - \frac{n-1}{J}I] \times C_n^1 \\
& +... + \frac{n^c!(n-n^c)!}{(n+1)!}[R - \frac{n^c}{J}I] \times C_n^{n^c} \\
& + \frac{(n^c-1)!(n-n^c+1)!}{(n+1)!}[w_E + \frac{n^c-1}{J}I - \frac{n^c-1}{J}I] \times C_n^{n^c-1} \\
& +... + \frac{1!(n-1)!}{(n+1)!}[w_E + \frac{1}{J}I - \frac{1}{J}I] \times C_n^1 + \frac{0!n!}{(n+1)!}[w_E - 0].
\end{aligned}$$

The coefficient on w_E then is

$$\sum_{k=0}^{n^c-1} \frac{k!(n-k)!}{(n+1)!} C_n^k = \sum_{k=0}^{n^c-1} \frac{k!(n-k)!}{(n+1)!} \frac{n!}{k!(n-k)!} = \sum_{k=0}^{n^c-1} \frac{1}{n+1} = \frac{n^c}{n+1}.$$

The coefficient on R is

$$\sum_{k=n^c}^{n} \frac{k!(n-k)!}{(n+1)!} C_n^k = \sum_{k=n^c}^{n} \frac{k!(n-k)!}{(n+1)!} \frac{n!}{k!(n-k)!} = \sum_{k=n^c}^{n} \frac{1}{n+1} = \frac{n-n^c+1}{n+1}.$$

The coefficient on $\frac{I}{J}$ is

$$\begin{aligned}
-\sum_{k=n^c}^{n} \frac{k!(n-k)!}{(n+1)!} k C_n^k &= -\sum_{k=n^c}^{n} \frac{k!(n-k)!}{(n+1)!} \frac{n!}{k!(n-k)!} k \\
&= -\sum_{k=n^c}^{n} \frac{k}{n+1} = -\frac{(n^c+n)(n-n^c+1)}{2(n+1)}.
\end{aligned}$$

[58] Another common notation is $\binom{n}{k}$.

Thus the expression in equation (17) becomes

$$\phi_E^{Sh}(n) = \frac{n - n^c + 1}{n+1}R - \frac{(n+n^c)(n-n^c+1)}{2(n+1)}\frac{I}{J} + \frac{n^c}{n+1}w_E.$$

To find the limit of equation (17), observe that when $J \to \infty$, we have $\frac{n+1}{J} \to \frac{n}{J} = \gamma$ and $\frac{n^c}{J} \to 1 - \frac{w_E}{I}$. Strictly speaking, when trying to fix γ, one encounters an integer problem that disappears in the limit. For instance, one can obtain precisely $\gamma = 3/4$ only if J is a multiple of 4. As a result:

$$\begin{aligned}
\phi_E^{Sh}(\gamma) &= \lim_{J\to\infty} \phi_E^{Sh}(n) = \lim_{J\to\infty}\left[\frac{n-n^c+1}{n+1}R - \frac{(n+n^c)(n-n^c+1)}{2(n+1)}\frac{I}{J} + \frac{n^c}{n+1}w_E\right] \\
&= \lim_{J\to\infty}\left[\frac{\frac{n+1}{J} - \frac{n^c}{J}}{\frac{n+1}{J}}R - \frac{(\frac{n}{J} + \frac{n^c}{J})(\frac{n+1}{J} - \frac{n^c}{J})}{2\frac{n+1}{J}}I + \frac{\frac{n^c}{J}}{\frac{n+1}{J}}w_E\right] \\
&= \frac{\gamma - 1 + \frac{w_E}{I}}{\gamma}R - \frac{(\gamma + 1 - \frac{w_E}{I})(\gamma - 1 + \frac{w_E}{I})}{2\gamma}I + \frac{1 - \frac{w_E}{I}}{\gamma}w_E \\
&= R - (1 - \frac{w_E}{I})R\frac{1}{\gamma} - \left[\gamma^2 - (1 - \frac{w_E}{I})^2\right]\frac{I}{2}\frac{1}{\gamma} + (1 - \frac{w_E}{I})w_E\frac{1}{\gamma} \\
&= R - \frac{I}{2}\gamma - (1 - \frac{w_E}{I})(R - \frac{I}{2} - \frac{w_E}{2})\frac{1}{\gamma}.
\end{aligned}$$

To find the FOC, we differentiate equation (18) with respect to γ as follows:

$$\begin{aligned}
\frac{\partial \phi_E^{Sh}(\gamma)}{\partial \gamma} &= -\frac{I}{2} - (1 - \frac{w_E}{I})(R - \frac{I}{2} - \frac{w_E}{2})(-\frac{1}{\gamma^2}), \\
\frac{\partial^2 \phi_E^{Sh}(\gamma)}{\partial \gamma^2} &= -(1 - \frac{w_E}{I})(R - \frac{I}{2} - \frac{w_E}{2})\frac{2}{\gamma^3} < 0.
\end{aligned}$$

Thus, we can find γ^* as:

$$\begin{aligned}
\frac{\partial \phi_E^{Sh}(\gamma)}{\partial \gamma} = 0 \quad &\Leftrightarrow \quad (1 - \frac{w_E}{I})(R - \frac{I}{2} - \frac{w_E}{2}) = \frac{I}{2}\gamma^2 \\
&\Leftrightarrow \quad \gamma^2 = \frac{1}{I^2}(I - w_E)(2R - I - w_E) \\
&\Leftrightarrow \quad \gamma = \frac{1}{I}\sqrt{(I - w_E)(2R - I - w_E)} \\
&\Rightarrow \quad \gamma^* = \min\left[1, \frac{1}{I}\sqrt{(I - w_E)(2R - I - w_E)}\right].
\end{aligned}$$

The last line follows because by definition $\gamma \in [1-\frac{w_E}{I}, 1]$. Thus when we have $\frac{1}{I}\sqrt{(I-w_E)(2R-I-w_E)} > 1$, the maximum point is at the corner.

With $\rho = \frac{R}{I}$, γ^* can also be expressed as

$$\gamma^* = \min\left[1, \sqrt{(1-\frac{w_E}{I})(2\frac{R}{I}-1-\frac{w_E}{I})}\right] = \min\left[1, \sqrt{(1-\frac{w_E}{I})(2\rho-1-\frac{w_E}{I})}\right]$$

as asserted in Proposition 10. □

3.B Additional Examples for the Nash Equilibria in Proposition 1

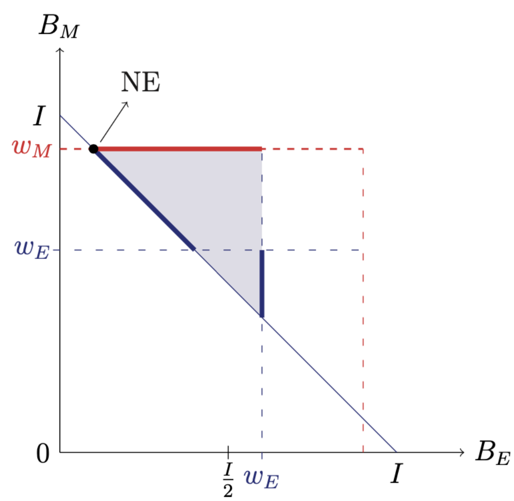


Figure A3: Nash Equilibrium Example 3. In this example, $\frac{I}{2} < w_E < w_M < I$. Here the Nash Equilibrium is $(B_E^*, B_M^*) = (I - w_M, w_M)$.

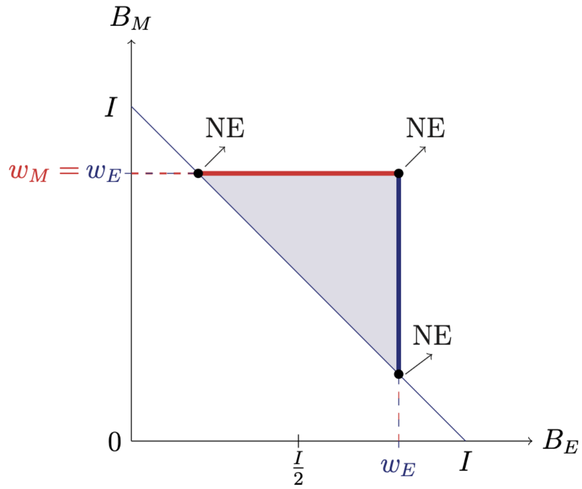


Figure A4: Nash Equilibrium Example 4. In this example, $w_M = w_E > \frac{I}{2}$. Here the Nash Equilibria are (i) $(B_E^*, B_M^*) = (w_E, I - w_E)$, (ii) $(B_E^*, B_M^*) = (w_E, w_M)$, and (iii) $(B_E^*, B_M^*) = (I - w_E, w_M)$.

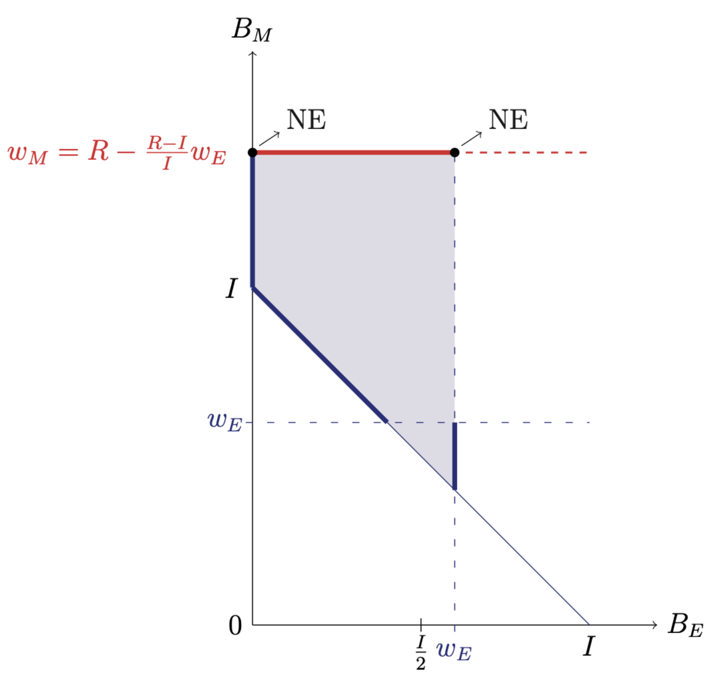


Figure A5: Nash Equilibrium Example 5. In this example, $w_E > \frac{I}{2}$ and $w_M = R - \frac{R-I}{I} w_E$. Here the Nash Equilibria are (i) $(B_E^*, B_M^*) = (w_E, w_M)$, and (ii) $(B_E^*, B_M^*) = (0, w_M)$.

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3 Chapter Three

The Role of Product Enhancement in Innovation and Entry Deterrence

Abstract

Although innovation is costlier than imitation, the incumbent firm is endowed with an advantage of enhancing its product ahead of potential competitors. In a model that connects consumers' utility with firms' production, I show that the incumbent's product enhancement decision can foster the creation of better products, improve consumers' utility, and deter entrance from competitors. The pace of creative activities is determined by the incumbent's potential of improving its product quality and the nature of product differentiation in the industry. Thus, creative destruction may not manifest as new firms replacing the incumbent, but as the incumbent constantly renovating itself.

3.1 Introduction

Many innovative ideas that keep the capitalist engine in motion cannot seek the protection from existing devices such as patents or long-term contracts. A few questions then naturally arise. For instance, given the fact that many production processes involve several phases of investment, what motivates firms to innovate despite the constant learning by competitors? Why do we observe innovations come in waves? What keeps a firm alive in the perennial gale of creative destruction? Why do some industries or firms develop faster than others? What are the fundamental impulses that sustain the growth and prosperity of an economy? These questions have surrounded economists for at least decades. Other than our existing knowledge of industrial organization and market structures, the main point I focus on in this paper is that the initial innovator is endowed with an advantage of improving its product ahead of competitors.

Since Schumpeter (1942), the evolution of the corporate landscape has been described as the manifestation of creative destruction. Yet as observed by studies such as Stein (1997), the real world industrial sectors are much less fluid than those pictured by the creative destruction models, e.g., Aghion and Howitt (1992). Thus, there have to be other key factors that can prevent the destruction process and allow an innovation to grow to a scale that can provide stable output. My paper focuses on the possibility that the incumbent's product enhancement activity can, to a certain extent, deter potential entrance and in turn ensure its profit and prospect.

More precisely, my model involves two periods. In the first period an incumbent firm generates an idea and starts a costly innovation process. This firm can sell a beta version of the product at the end of the first period. In the second period the incumbent firm can enhance its product, which improves the utility of a representative consumer. On the other hand, at the beginning of the second period the beta version of the product is observed by a competing firm, and this second firm can mimic the same product and enter the product market to compete with the incumbent at the end of the second period. Since the incumbent is using an improved version of the product to compete with the potential entrant, the consumer might exclusively prefer the better product and thus drive the competitor's profit to zero. In this way, entry is effectively deterred even if there is no patent protection and innovation is costlier than imitation.

On the contrary, if the incumbent cannot substantially improve the product quality and deter entrance, my model shows that competition between the two products shrinks the incumbent's profit and thus reduces the incumbent's incentive to innovate. However, the price and output of the incumbent's second period product need not fall. Deadweight loss and market failure can occasionally arise when the shrinked profit cannot recoup the cost of innovation. Let me provide an example for this scenario.

The concept of bike sharing is an unpatentable idea. The bike sharing market in China first started as a way of resolving the traffic problem in the main campus of Peking University, in which a group of four students established a company named Ofo sharing bicycles in 2014. The initial success of Ofo inside the university attracted a large amount of funding from investors, and Ofo expanded its business rapidly outside the campus. In 2017, Ofo bikes can be found in almost all the large cities in China. However, since the sharing bike concept can be easily adopted by another company, Ofo naturally faces the entrance of many competitors. One of the largest competitors is a company named Mobike, which started its operation in 2016. The competition between Ofo and Mobike quickly pushed the two companies to produce a massive amount of poor quality bikes in the hope of winning the market. Yet consumers' dissatisfaction with poor bikes as well as the squeeze of public spaces by overly supplied bikes eventually eroded both companies' potential profit. In 2019 Ofo was no longer capable of running its business and Mobike was acquired by Meituan, which is a shopping platform company.

The phenomena of competitions and potential market failures related with some types of ideas are not unique to the bike sharing market. Smart phones, E-book readers, and many new business concepts are all examples of ideas that require many phases of product enhancement activities. As stated in Arrow (1962), indivisibilities, inappropriability, and uncertainty are three classical reasons for the possible allocation failure in a perfectly competitive market. Economic theories have discussed for decades whether competition or monopoly rent is the key driver for inventive activities (e.g., Schumpeter 1942, Dasgupta and Stiglitz 1980), yet many studies on innovations have overlooked one fact that was stressed by Schumpeter (1942, p. 92):

> "... the great majority of new consumers' goods... are at first introduced in an experimental and unsatisfactory form in which they could never conquer their potential markets. Improvement in the quality of products is hence a practically universal feature of the development of individual concerns and of industries."

Thus, this paper focuses on the problem related with inappropriability and illustrates how potential competition affects the pace of innovation, which is reflected by the incumbent's product enhancement decisions.

The model's results show that competition can be considered as a double-edged sword, and the pace of innovation and product enhancement differs across firms and industries. I summarize these predictions in the following Figure 19. For instance, when the incumbent firm has limited potential to enhance its existing product (low potential), or when the industry is a sector that has limited ability to permit similar products to compete with each other (high differentiation), competition and entrance by a new firm typically reduces the incentive for the incumbent to innovate and improve its product quality (slow pace). The bike sharing market can be considered as an example of an industry that has limited potential of improving its product quality. As a result, the pace of innovation is at most mild, and market failure is more likely to happen. Industries that are already mature are also likely to fall into the south-east corner of Figure 19, and we can expect to observe a slow pace of any form of innovation.

	Low differentiation	High differentiation
High Potential	Fast	Mild
Low potential	Mild	Slow

Figure 19: **The Pace of Innovation and Product Enhancement.**

On the other hand, when the incumbent firm has a high potential to improve its product and the industry has a low product differentiation among different firms, competition most likely can motivate the incumbent to devote many of its resources to product enhancement (fast pace). For instance, any new business concept in its earliest form is likely to have a high growth potential. When such an industry just starts to develop, the product differentiation between different firms is very low. Then we can expect to observe a very fast pace of R&D and innovation since these firms fall into the north-west corner of Figure 19. These predictions are consistent with the fact that creative activities often come in waves. Overall, my findings can help to explain why certain industries develop earlier and are more mature than other industries, and why some firms innovate more often and grow faster than other firms.

This paper can be first related with many classical and more recent studies on competition and entry deterrence strategies. For instance, Dixit (1979) studies a duopoly model and analyzes how the fixed cost of entry affects the incumbent's optimal production level. Spence (1977) introduces the concept of capacity, and argues that existing firms can scale up their productions to the full industry capacity to make new entrants unprofitable.[59] Gilbert and Newbery (1982) model preemptive patenting as a method of entry deterrence strategy.[60] Rasmusen (1988) considers the scenario when the incumbent can buy out the entrant, the entrant might be able to blackmail the incumbent despite a negative entering profit.[61,62] Fudenberg and Tirole (2000) study the incumbent's pricing incentives when there are network externalities. My model considers product enhancement as another way of entry deterrence strategy. Similar to the spirit addressed by Rajan (2012) that an entrepreneur has to differentiate her enterprise from the ordinary in order to generate value, the incumbent firm in my model innovates and improves its product quality in order to outperform possible imitators.

Second, this paper can also be related with the literature on innovation, creative destruction and growth. For example, Aghion and Howitt (1992) introduce the factor of obsolescence into the endogenous growth theory. Segal and Whinston (2007) study the endogenous impact of antitrust policy on innovations.[63] Grossman and Helpman (1991) model the interaction

[59] Dixit (1980) further studies how the incumbent's pre-entry decision on the capacity affects the prospective entrant's decisions on entrance.

[60] Tirole (1988) provides a summary of related literature on patent races.

[61] Literature that relates product market competition with financial securities can be found in, for instance, Mathews (2006), who studies how strategic alliances between an entrepreneurial firm and an established firm can motivate interfirm equity sales.

[62] More recently, Fulghieri and Sevilir (2011) study the impact of mergers on employee's incentives to innovate. Cunningham, Ederer, and Ma (2020) propose that an acquirer might purchase another firm only to terminate its project to reduce potential competition. Other papers that study the influence of takeover on innovation can be found in, for instance, Phillips and Zhdanov (2013), in which small firms innovate more in order to sell themselves to large firms, and Rajan, Kamepalli and Zingales (2020), who argue that once a winner product (e.g., a digital platform) is identified, investors reduce investment on similar products and result in a kill zone.

[63] Segal and Whinston (2007) show that an antitrust policy, which increases the profit of the entrant, makes it a lot more attractive for the entrant to enter, but since the entrant becomes the incumbent in the future, it also lowers the entrant's incentive to become the incumbent.

between innovation and imitation in a two-country model.[64] In my paper I do not take new firms replacing the incumbent as a given fact.[65] Instead, I consider that the incumbent can improve and replace its own product. Thus, creative destructions happen within the firm. In this regard, my setting is also related to Aghion et al. (2001), in which the authors analyze the relationship between the intensity of product market competition and growth.[66]

At least dating back to Arrow (1962), the literature on industrial structures often models innovative activities as a means of reducing the production cost (e.g., Dasgupta and Stiglitz 1980), and the cost-reduction model can occasionally be interpreted as improving the product quality (e.g., Vives 2008).[67] Prior studies have also tried to explicitly incorporate product quality into the innovation framework. For instance, in Sutton (1996), firms can decide the quality of the product after entry, and all firms compete in a Cournot equilibrium.[68] Grossman and Helpman (1991) and Aghion et al. (2001) model the quality improvement as firms climbing the ladders.[69,70] Davis, Murphy and Topel (2004) analyze how product

[64] Models such as Segal and Whinston (2007) and Grossman and Helpman (1991) focus on the Bertrand type of competition and the market has the winner-take-all feature, whereas in my model, the two firms compete in the Cournot style and share the market.

[65] Another earlier paper that considers entrants replacing the incumbent can be found in Reinganum (1985). Perhaps in contrast with these prior papers that focus more on drastic innovations that allow the entrant to overthrow the incumbent, my model does not differentiate whether a quality improvement belongs to dramatic or incremental innovations.

[66] My model differs from Aghion et al. (2001) mostly in the way that I focus more on the incumbent's ability of improving its product, whereas Aghion et al. (2001) focus more on the ease of imitation from competitors. They find that when this ease of imitation goes to infinity the growth rate falls to zero. In my paper, the competitor always has a perfect ability of imitating the past product, but this imitation may or may not affect the growth rate of the incumbent.

[67] Vives (2008) summarizes many model settings and discusses the possibility of interpreting the cost-reduction model (e.g., Spence 1984) as investment in quality. My paper also argues that the cost reduction model may not be accurate enough to capture the impact of product enhancement on innovation and firms' behaviors. For instance, a consumer is more likely to be better off with purchasing one high quality cellphone than buying three low quality cellphones, even if the former is tagged with a higher price.

[68] Sutton's (1996) model doesn't have different stages of investment and only consider the product quality as one choice variable. Moreover, studies such as Sutton (1996) feature a free entry market and the equilibrium is developed given all firms earning zero profit. My model uses a more exogenous market structure and only considers the possible entrance of one competitor.

[69] Grossman and Helpman (1991) mainly study how the two regions learn from each other to take over market shares. They do not emphasize the possibility of entry deterrence.

[70] More recent papers that use the quality ladder setting can be found in, e.g., Acemoglu and Cao (2015), in which incumbents improve their products whereas entrants engage in radical innovations to replace incumbents. Acemoglu et al. (2018) further introduce firm heterogeneity into the growth model. These papers focus more on the macroeconomic patterns.

design and improvements change the incumbent's and the entrant's exclusionary strategies and consumer welfare.[71] The difference between my model, which is more similar to the approach by Singh and Vives (1984), and the previous papers on quality improvement is that I focus more on the incumbent's potential to improve the product quality as well as the influence of product differentiation on the equilibrium outcome.

Finally, the literature also proposes a few possible reasons to explain why innovations often come in waves. For instance, Shleifer (1986) suggests that firms can synchronize their implementations of innovations to take advantage of the consumer demand spillovers. Stein (1997) develops a model that considers firms having a distribution cost that diminishes over time and cannot spill over to new firms or new innovations. In his model, innovations often come in waves since it is easier to gain access to the market when the incumbent hasn't established its customer base. My model's result also includes the possibility of innovation waves. Yet the cause of these waves is that the incumbent does not have a high enough potential to improve its product quality and thus cannot successfully deter competitors' entrance.[72]

The rest of the paper proceeds as follows: Section 3.2 presents the main model, and an example is provided at the end. Section 3.3 concludes.

3.2 Model

There are three dates in the model as shown in Figure 20. At $t = 0$ the incumbent firm, firm 1, has an innovative idea. The fixed cost of innovation is c_0. It takes one period before the initial product (beta version) can be sold to consumers. Period 1, which is from $t = 0$ to $t = 1$, is called the innovation period. Starting from $t = 1$, firm 1 can invest in product

[71] Davis, Murphy and Topel (2004) consider heterogeneous consumers and focus on the price competition among firms. Although their model includes product design changes, the cost of initiating those changes are not explicitly discussed.

[72] Although my paper does not investigate the financing problem of innovations, there are also strands of literature that studies this problem. For instance, Bergemann and Hege (2005) study the funding and stopping problem in both the relationship financing and the arm's-length financing modes. Manso (2011) shows that contracts that tolerate early failures may provide better incentives for managers to innovate.

enhancement and sell an improved version of the product at $t = 2$. Period 2, which is from $t = 1$ to $t = 2$, is called the enhancement period.

At $t = 1$, a potential competitor, firm 2, can observe the product sold by firm 1. Since this paper focuses on the problem generated by inappropriability, I consider firm 2 can directly start mimicking firm 1's product. To model the lagged timing of production, let us assume it takes another period for firm 2 to be able to sell its product. For simplicity, I model firm 2's product at $t = 2$ as the same beta version product of firm 1. Thus, if firm 1 does not engage in product enhancement, both firms compete at $t = 2$ with exactly the same product. In addition, I assume firm 2 does not have to spend the fixed cost of innovation to produce its product. This setting reflects the fact that the initial innovator typically has to spend more money on innovation.[73]

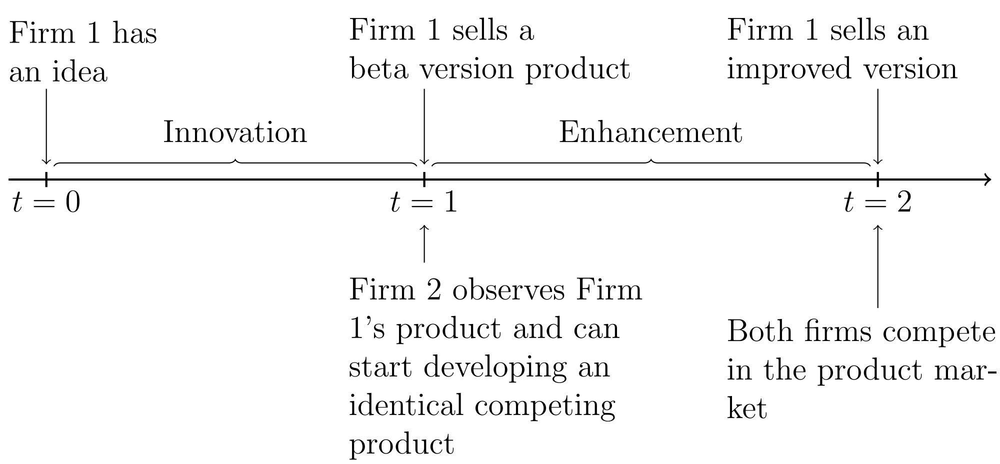

Figure 20: **Timeline of events.**

With the above timing, I focus my analysis on a few questions. The first question is how does firm 2's presence change firm 1's incentive in its product enhancement decisions. Second, under what condition can the incumbent firm successfully deter the entrance from firm 2. Third, when is competition severe enough to stifle innovation. To study these questions, I

[73]Firm 2 in my model is akin to those copycats in Stein (1997). However the copycats in Stein's model serve the purpose of limiting the price of the incumbent and do not capture any market share in equilibrium. In my setting, the imitator plays a significant role in either motivating the incumbent to innovate, or competing with the incumbent and sharing the market.

separate the analysis into three steps. In Step I, I develop a simple benchmark model for firm 1's innovation process in period 1. In Step II, I consider a monopoly case of firm 1's investment on product enhancement. In Step III, I add firm 2's possible entrance into the model in Step II. An example is provided in Section 3.2.4.

3.2.1 Step I

In the first step, I consider the production problem of firm 1 at $t = 1$. Other than the fixed cost of c_0, consider all firms having constant marginal costs of production. Thus, we can equivalently focus on prices net of marginal costs as in Singh and Vives (1984).[74] Suppose the product is a non-durable good and there is a representative consumer with a simple quadratic utility function as follows:

$$U_1(q) = \alpha_1 q - \frac{1}{2}q^2$$

where α_1 is the consumer's preference on the product, and q is the quantity being consumed. The consumer's objective is to maximize the consumer surplus:

$$\max_q [U_1(q) - pq]$$

where p denotes the price. A standard first order condition gives us the linear inverse demand function:

$$p = \alpha_1 - q$$

Firm 1's objective is to maximize the net profit given the inverse demand:

$$\max_q \pi(q) = \max_q [pq - c_0] = \max_q [(\alpha_1 - q)q - c_0]$$

[74]Literature sometimes simplify the constant marginal cost to zero in order to focus on other factors. For instance, the production is assumed to be costless in Fudenberg and Tirole (2000).

Since the first order condition is straight forward, I summarize the solutions as the following

$$q^N = p^N = \frac{1}{2}\alpha_1 \qquad \pi^N = (q^N)^2 - c_0 = \frac{1}{4}\alpha_1^2 - c_0$$

Here I temporarily forgo the discussion of the fixed cost c_0, since a long-run maximizer also considers the profit generated in the second period. This discussion is deferred to Section 3.2.3. The consumer's utility and the consumer surplus have the expressions of the following:

$$U_1(q^N) = \frac{3}{2}(q^N)^2 \qquad [U_1(q^N) - p^N q^N] = \frac{1}{2}(q^N)^2$$

3.2.2 Step II

Suppose firm 1 does not face any possible entrance from firm 2. Denote the representative consumer's utility function as follows:[75]

$$U_2(q_1) = \alpha_2(I)q_1 - \frac{1}{2}q_1^2$$

where I denotes firm 1's effective investment on increasing $\alpha_2(I)$, which is the consumer's preference on firm 1's product at $t = 2$. Thus, I represents firm 1's product enhancement decision. Here I assume the preference $\alpha_2(I)$ is either a concave or a linear function. Assumption 1 below summarizes the properties of $\alpha_2(I)$:

Assumption 1. $\alpha_2(I)$ *is twice differentiable and increasing with respect to* I*.* $\alpha_2(0) = \alpha_1$*,* $\alpha_2'(I) > 0$*,* $\alpha_2''(I) \leq 0$*.*

The above assumption shows that if firm 1 does not enhance its product, the effective investment $I = 0$, and the consumer's preferences on firm 1's products at $t = 1$ and $t = 2$ are the same. Similar as in Section 3.2.1, the consumer's objective function and the inverse

[75]In terms of notation, subscripts of U and α represent the time, and the subscript of q represents the firm.

demand are the following:

$$\max_{q_1}[U_2(q_1) - p_1 q_1]$$
$$p_1 = \alpha_2(I) - q_1$$

In the second period, firm 1 no longer has to spend the fixed cost c_0, but the product enhancement decision is also costly. Thus, firm 1's objective is to maximize the net profit expressed as follows:

$$\max_{I,q_1} \pi_1(q_1, I) = \max_{I,q_1}[p_1 q_1 - c(I)] = \max_{I,q_1}[(\alpha_2(I) - q_1)q_1 - c(I)]$$

where $c(I)$ represents the cost for product enhancement. I assume this cost function is strictly convex. Below Assumption 2 summarizes the properties of $c(I)$:

Assumption 2. *$c(I)$ is twice differentiable with respect to I. $c(0) = 0$, $c(I) \geq 0$. For $I > 0$, $c'(I) > 0$, $c''(I) > 0$, with $\lim_{I \to 0} c'(I) = 0$ and $\lim_{I \to \infty} c'(I) = \infty$.*

Since firm 1 maximizes its profit by choosing both the quantity q_1 and the investment on product enhancement I, the two first order conditions are the following:

$$\begin{aligned} \frac{\partial \pi_1}{\partial q_1} &= \alpha_2(I) - 2q_1 = 0 \\ \frac{\partial \pi_1}{\partial I} &= q_1 \alpha_2'(I) - c'(I) = 0 \end{aligned}$$

Solving the above two equations we can find that the optimal investment on product enhancement under monopoly I^M is determined by the following equation:

$$c'(I^M) = \frac{1}{2}\alpha_2(I^M)\alpha_2'(I^M) \tag{20}$$

The solution of equation (20) exists and in general I^M is non-trivially positive. This conclusion can be proved from our previous assumptions. Specifically, from Assumption 1 we can see that the right hand side of equation (20) starts with a positive value $\frac{1}{2}\alpha_1\alpha_2'(0)$ and stays positive. From Assumption 2 we can see that the left hand side of equation (20) starts from zero and becomes positive for all $I > 0$. Although the size of I^M depends on more specific functional forms but we can without loss of generality consider both $\alpha_2(I)$ and $c(I)$ are well behaved functions. Figure 22 provides an illustration of how I^M is determined by equation (20).[76] Thus, firm 1's optimal production under monopoly can be summarized as follows:

$$q_1^M = p_1^M = \frac{1}{2}\alpha_2(I^M) \qquad \pi_1^M = (q_1^M)^2 - c(I^M)$$

Since $I^M > 0$, given Assumption 1 we have $\alpha_2(I^M) > \alpha_1$. Thus, $q_1^M > q^N$, and $p_1^M > p^N$. Here we can see that in the second period, firm 1 optimally invests to enhance its product. This product enhancement results in a higher price and a higher quantity. This result shows that when we allow the firm to alter and improve its product, it is possible to observe that the consumer consumes more even if the firm increases its price for the new product.

Similar as in Section 3.2.1, the consumer's utility and the consumer surplus have the expressions of the following:

$$U_2(q_1^M) = \frac{3}{2}(q_1^M)^2 \qquad [U_2(q_1^M) - p_1^M q_1^M] = \frac{1}{2}(q_1^M)^2$$

Here we can see that since $q_1^M > q^N$, firm 1's product enhancement decision improves both the consumer's utility, $U_2(q_1^M) > U_1(q^N)$, and the consumer surplus, $[U_2(q_1^M) - p_1^M q_1^M] > [U_1(q^N) - p^N q^N]$. Figure 21 provides an illustration. This figure shows that product enhancement shifts the inverse demand line outward, and the consumer surplus, which is reflected by the shaded area, increases despite the increase of price.

[76]More specific functional forms, e.g., linear case of $\alpha_2(I)$, might impose additional restrictions on some parameters, as shown in the example in Section 3.2.4. Yet these details do not affect the main conclusions in this paper and are thus omitted.

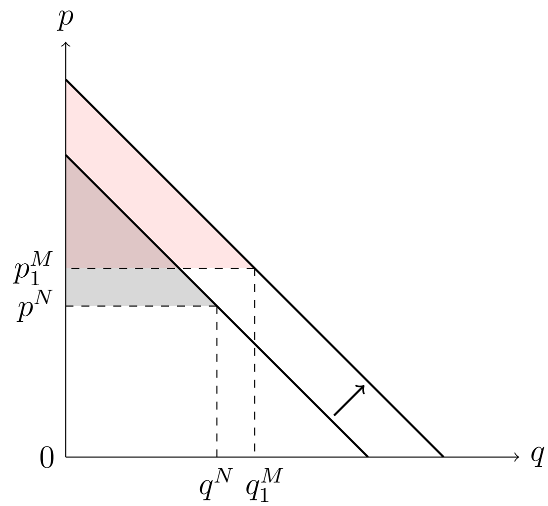

Figure 21: **An illustration of Step I and Step II.**

3.2.3 Step III

Consider the complete problem illustrated by Figure 20. Since firm 1 can enhance its product, it is reasonable to assume that this product enhancement decision not only increases the consumer's utility, but also increases the degree of differentiability between firm 1 and firm 2's products. Thus, the consumer's utility function is modeled as the following:

$$U_2(q_1, q_2) = \alpha_2(I)q_1 + \alpha_1 q_2 - \frac{1}{2}q_1^2 - \frac{1}{2}q_2^2 - \gamma(I)q_1 q_2 \tag{21}$$

where $\gamma(I)$ denotes the degree of product differentiation. Since firm 2 is mimicking firm 1's product, the consumer's preference on firm 2's product is α_1. In addition, we can consider that the two goods are substitutes. The following Assumption 3 summarizes the property of $\gamma(I)$:

Assumption 3. *$\gamma(I)$ is twice differentiable and non-increasing with respect to I. $\gamma(0) = 1$. $\gamma(I) \in [\gamma, 1]$, with $\gamma > 0$ being the lower bound. $\gamma'(I) < 0$, $\gamma''(I) > 0$.*

The above assumption indicates that the more efforts firm 2 puts into enhancing its product, the higher the degree of product differentiation. However, firm 1 cannot perfectly differentiate itself from firm 2, as the lower bound $\gamma > 0$.

The utility function indicated by equation (21) has two preferred properties. First, the consumer's preference for the same product stays the same. For instance, if firm 1 exits the market after $t = 1$, we have $q_1 = 0$ and $U_2(0, q_2) = \alpha_1 q_2 - \frac{1}{2}q_2^2$, which is the same expression as the consumer's utility function at $t = 1$.

Second, if firm 1 does not enhance its product, we have $I = 0$ and $U_2(q_1, q_2) = \alpha_1(q_1 + q_2) - \frac{1}{2}(q_1 + q_2)^2$. This expression corresponds to a pure Cournot quantity competition. Consider a standard symmetric equilibrium that $q_1^C = q_2^C$, then firm 1 maximizes profit $\pi_1(q_1) = p_1 q_1$, with inverse demand $p_1 = \alpha_1 - q_1 - q_2$. Since the solution is straight forward, I summarize the results as the following:

$$q_1^C = q_2^C = p_1^C = p_2^C = \frac{1}{3}\alpha_1 \qquad \pi_1^C = \pi_2^C = \frac{1}{9}\alpha_1^2$$

The above result can be considered as a limiting case that when $\alpha_2(I)$ stays nearly as a constant, i.e., firm 1 is nearly incapable of altering the consumer's preference. Here we can see that we obtain the typical result that a pure quantity competition ends with a lower price ($p_1^C < p^N$) and a larger aggregate quantity supply ($q_1^C + q_2^C > q^N$).

Let us consider the utility function as in equation (21). Here we can write the two inverse demands and the two firms' profit functions as the following:

$$\begin{aligned} p_1 &= \alpha_2(I) - q_1 - \gamma(I)q_2 \\ p_2 &= \alpha_1 - q_2 - \gamma(I)q_1 \end{aligned}$$

$$\begin{aligned} \pi_1(q_1, I) &= [\alpha_2(I) - q_1 - \gamma(I)q_2]q_1 - c(I) \\ \pi_2(q_2) &= [\alpha_1 - q_2 - \gamma(I)q_1]q_2 \end{aligned}$$

There are three first order conditions:

$$\begin{aligned}\frac{\partial \pi_1}{\partial q_1} &= \alpha_2(I) - 2q_1 - \gamma(I)q_2 = 0 \\ \frac{\partial \pi_1}{\partial I} &= q_1\alpha_2'(I) - c'(I) - \gamma'(I)q_1q_2 = 0 \\ \frac{\partial \pi_2}{\partial q_2} &= \alpha_1 - 2q_2 - \gamma(I)q_1 = 0\end{aligned}$$

Solving the above three equations we can find that the optimal level of product enhancement investment under duopoly I^D by firm 1 is determined by the following equation:

$$c'(I^D) = \frac{2\alpha_2(I^D) - \gamma(I^D)\alpha_1}{4 - \gamma^2(I^D)}\left[\alpha_2'(I^D) - \frac{2\alpha_1 - \gamma(I^D)\alpha_2(I^D)}{4 - \gamma^2(I^D)}\gamma'(I^D)\right] \tag{22}$$

In addition, the expressions for quantities, prices, and profits are summarized as follows:

$$\begin{aligned} q_1^D = p_1^D &= \frac{2\alpha_2(I^D) - \gamma(I^D)\alpha_1}{4 - \gamma^2(I^D)} \qquad & \pi_1^D &= (q_1^D)^2 - c(I^D) \\ q_2^D = p_2^D &= \frac{2\alpha_1 - \gamma(I^D)\alpha_2(I^D)}{4 - \gamma^2(I^D)} \qquad & \pi_2^D &= (q_2^D)^2 \end{aligned}$$

From the above solutions we can see that this model has the convenient feature that we always have prices equal to quantities. As a result, if we want to study both firms' incentives, it is sufficient to focus on the expressions for quantities.

From the expression of q_1^D we can see that we always have $q_1^D > 0$. This finding is a direct consequence from Assumption 1 and Assumption 3, in which we have $\alpha_2(I) \geq \alpha_1$ and $\gamma(I) \leq 1$. However, q_2^D may not always be positive. Specifically, if the following condition holds, then firm 2 does not find it profitable to enter the market:

$$\alpha_1 < \frac{1}{2}\gamma(I^D)\alpha_2(I^D) \tag{23}$$

Equation (23) is the entry deterrence condition. When this equation is satisfied, we do not observe a market with duopoly. Thus, a monopoly market does not mean that potential competitors don't exist. In this model we can see that it is possible for firm 1 to optimally enhance its product and at the same time deter the entry by firm 2.[77] The above expressions on quantities and equation (23) also imply that if firm 2 enters the market with a complementary product, e.g., $\gamma < 0$, the incumbent firm is willing to accommodate because of the synergy generated by a complement.

The entry deterrence condition (23) shows that whether firm 1 can deter firm 2's entrance or not depends on three factors: the consumer's initial preference α_1, the consumer's preference after the product enhancement $\alpha_2(I^D)$, and the product differentiation $\gamma(I^D)$. Specifically, the right hand side of equation (23) shows that when firm 1 is more capable of increasing the consumer's utility, i.e., achieving a higher $\alpha_2(I^D)$, and when the two products are highly substitutable, i.e., high $\gamma(I^D)$, we are more likely to observe a monopoly market with the incumbent firm constantly updating its own product. In the current model, the initial preference α_1 is fixed. However, we can also imagine that if we allow the incumbent to choose α_1 before enhancing its product, we might observe the incumbent intentionally selecting a low α_1 just to deter the entrance of firm 2. For instance, an incumbent might initially provide a beta version product that is not very attractive to consumers. This dissatisfaction can provide valuable feedback to the incumbent and at the same time discourage potential competitors to mimic the incumbent's product.

The above analysis indicates that competition and monopoly rent may not be separate forces that drive inventive activities. In this paper, monopoly rent can arise as the result of the incumbent successfully deters entrance from competitors. On the other hand, competition by firm 2 in my model can serve as a discipline for the incumbent firm. For instance, suppose firm 1's manager incurs a personal cost $c_p(I)$ to make the product enhancement decision, and the manager's total cost function $c_p(I) + c(I)$ differs from the firm's cost function $c(I)$. Without competition, firm 1's manager surely under-invests in enhancing the product. However, since under-investment lowers the consumer's satisfaction $\alpha_2(I)$, the manager's

[77] There are also other reasons that can lead to natural monopolies. For instance, in Maskin and Tirole (1988), large fixed costs allow at most one firm to exist.

shirking behavior might trigger the entrance of firm 2. As a result, firm 1's manager may prefer to forgo his own personal cost and invest in the best interest of the firm.[78]

Since the right hand side of equation (22) is rather complicated to analyze, I impose another assumption on the degree of product differentiation $\gamma(I)$ to simplify the rest of the result:

Assumption 4. $\gamma'(I^D) \to 0$.

The above assumption indicates that γ' is small enough to be considered at I^D. This assumption includes two possible scenarios. One scenario is that the lower bound γ is well below 1 and the function $\gamma(I)$ converges to its lower bound very quickly. A class of functions satisfying this requirement are the power functions.[79] The other scenario is that $\gamma(I)$ decreases very slowly and stays close to 1 even with the product enhancement decision. In this scenario we can simply consider that the lower bound γ is very close to 1. Although the main purpose of Assumption 4 is to simplify the analysis, we can as well find economic support for this assumption. Specifically, Assumption 4 indicates that the extent a firm can increase its product differentiation is limited more by the industry it belongs to. For instance, consider the automobile industry, a brand new Mercedes-Benz surely provides a better user experience than a second hand Toyota, but both of them can serve the commuting purpose. Thus, car manufactures are much more capable of altering α_2 than altering γ. Under Assumption 4 we can simplify equation (22) into the following:

$$c'(I^D) = \frac{2\alpha_2(I^D) - \gamma\alpha_1}{4 - \gamma^2}\alpha_2'(I^D) \tag{24}$$

Equation (20) and equation (24) share many similarities. Specifically, the left hand side

[78]Although my paper does not focus on the incentive scheme between managers and shareholders, the discussion that whether competition is a source of discipline or not has appeared in many prior papers. For instance, Hart (1983) shows that when there is a common component to firms' costs, the aggregate supply change in product markets can reduce managerial slack. Scharfstein (1988) further analyzes the ambiguous effect of product market competition on managerial incentives under different preference assumptions.

[79]For instance, some modifications of $\gamma(I) = (I+1)^{-\beta}$ can converge to its lower bound very fast when β is large enough.

is the derivative of the cost function, and the right hand side is the optimal quantity times the derivative of the consumer's preference. The expressions of optimal quantities under monopoly and duopoly are the following:

$$q_1^M = \frac{c'(I^M)}{\alpha_2'(I^M)} \qquad q_1^D = \frac{c'(I^D)}{\alpha_2'(I^D)}$$

The above expressions help to relate the size of q_1 to the size of investment I. Specifically, from Assumption 1 and Assumption 2 we can see that when I increases, $\alpha_2'(I)$ decreases or stays the same, and $c'(I)$ increases. Thus, $\frac{c'(I)}{\alpha_2'(I)}$ increases as I increases. This relationship gives us the following:

$$q_1^M > q_1^D \Leftrightarrow I^M > I^D$$

Below Proposition 11 summarizes the result of the model:

Proposition 11. *Under Assumptions 1 to 4, whenever $q_2^D > 0$, we have $I^M > I^D$, $q_1^M > q_1^D$ and $\pi_1^M > \pi_1^D$.*

Proof. Under Assumptions 1 to 4, we have $q_2^D > 0 \Leftrightarrow \alpha_1 > \frac{1}{2}\gamma\alpha_2(I^D)$. Thus, $q_1^D = \frac{2\alpha_2(I^D)-\gamma\alpha_1}{4-\gamma^2} < \frac{2\alpha_2(I^D)-\frac{1}{2}\gamma^2\alpha_2(I^D)}{4-\gamma^2} = \frac{1}{2}\alpha_2(I^D)$, and equation (24) has the following relation:

$$c'(I^D) = \frac{2\alpha_2(I^D) - \gamma\alpha_1}{4-\gamma^2}\alpha_2'(I^D) < \frac{1}{2}\alpha_2(I^D)\alpha_2'(I^D)$$

Observe that other than the superscript of I, the last term in the above expression is exactly the same as in equation (20). Since $c'(I)$ increases as I increases, we naturally have $I^M > I^D$ and $q_1^M > q_1^D$. Below Figure 22 provides an illustration.

In terms of the profit, observe that both profit functions under monopoly and duopoly can be expressed as $\pi_1 = q_1^2 - c(I)$. Since the monopoly profit is maximized at I^M, for any $I^D < I^M$, we have $\pi_1^D < \pi_1^M$. □

Propostition 11 shows that whenever firm 2 finds it profitable to enter the market, firm 1's optimal investment under duopoly I^D is below its optimal investment under monopoly

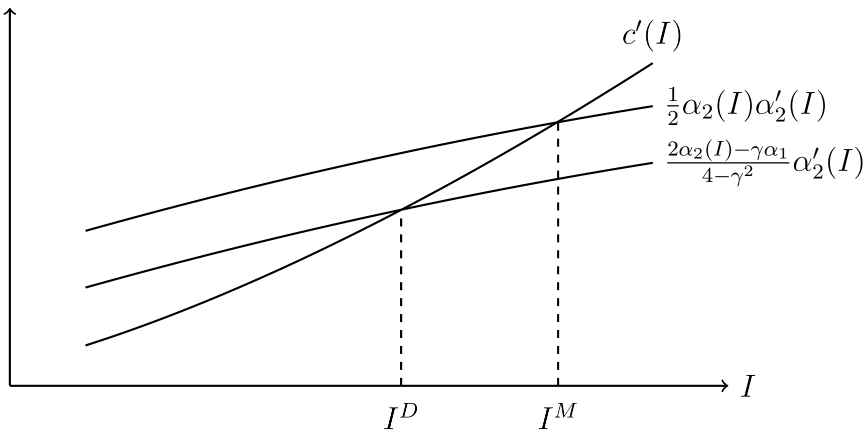

Figure 22: **Proof of Proposition 11.**

I^M. This under-investment problem by firm 1 results in a lower production quantity q_1^D and price p_1^D, a lower consumer's satisfaction towards firm 1's product $\alpha_2(I^D) < \alpha_2(I^M)$, as well as a lower net profit $\pi_1^D < \pi_1^M$. Thus, whenever the incumbent fails to deter entrance from the competitor, competition in my model reduces the incumbent's incentive to improve its own product. However, if we compare the second period's price and quantity to the first period, we are not necessarily going to observe a fall on the price or quantity, i.e., p_1^D and q_1^D can be larger, equal, or smaller than p^N and q^N. Section 3.2.4 provides some examples. This result is due to the fact that the incumbent can still improve its product upon the entry of firm 2.[80]

Finally, we can look at the role of fixed cost c_0. Recall that in Section 3.2.1 we obtain the first period profit $\pi^N = \frac{1}{4}\alpha_1^2 - c_0$. Suppose there is no time discount. A long-run maximizer's total profit is $\pi^N + \pi_1^M$ under monopoly, and $\pi^N + \pi_1^D$ under duopoly. Given the result of $\pi_1^D < \pi_1^M$ in Proposition 11, we can see that competition might have the adverse effect of stifling innovation. Specifically, there is a deadweight loss when $c_0 \in (\frac{1}{4}\alpha_1^2 + \pi_1^D, \frac{1}{4}\alpha_1^2 + \pi_1^M)$, in which the incumbent does not find it profitable to innovate in a competitive market in the first place. Thus, even if market failure is absent in a monopoly market, it is possible

[80]A standard industry model typically predicts that entry of a new firm reduces the incumbent's price and output. This is also the case illustrated at the beginning of this Section, in which we have $p_1^C < p^N$. Davis, Murphy, and Topel (2004) also show that some of these standard predictions need not hold under different settings. In their model, predictions are different because firms are facing a diverse pool of buyers.

for us to observe such failure in a competitive market.

3.2.4 An Example

Let us consider an example. Suppose the consumer's preference and the cost function have the following functional forms:

$$\alpha_2(I) = \alpha_1 + aI \qquad c(I) = \frac{1}{4}I^2$$

Here we have a linear $\alpha_2(I)$ and a convex $c(I)$. Thus, $\alpha_2'(I) = a > 0$ and $c'(I) = \frac{1}{2}I$. Although a concave $\alpha_2(I)$ suits better with the model in the previous section, the linear expression here allows us to explicitly derive the solution. However, the expression of $\alpha_2(I)$ also imposes some additional restrictions on the coefficient a, as we shall see in the following. In this example, the coefficient a can be interpreted as firm 1's potential on improving the consumer's satisfaction. A higher a means that the firm is more capable of making the consumer happier by enhancing its product.

With the above setting, equation (20) and equation (24) can be written as the following:

$$\begin{aligned} \frac{1}{2}I^M &= \frac{1}{2}(\alpha_1 + aI^M)a \\ \frac{1}{2}I^D &= \frac{2(\alpha_1 + aI^D) - \gamma\alpha_1}{4-\gamma^2}a \end{aligned}$$

The above two equations give us the solution for I^M and I^D:

$$I^M = \frac{a\alpha_1}{1-a^2} \qquad I^D = \frac{(1-\frac{\gamma}{2})a\alpha_1}{1-a^2-\frac{\gamma^2}{4}}$$

From the above two expressions we can see that in this example we need to restrict the range of a. Specifically, $I^M > 0$ requires $a \in (0, 1)$, and $I^D > 0$ requires $a \in (0, \sqrt{1-\frac{\gamma^2}{4}})$.

Let us examine Proposition 11 with the above parameters. Here we can find the following relationships:

$$I^M > I^D \;\Leftrightarrow\; a < \sqrt{1 - \frac{\gamma}{2}}$$
$$q_2^D > 0 \;\Leftrightarrow\; \alpha_1 > \frac{1}{2}\gamma\alpha_2(I^D) \;\Leftrightarrow\; a < \sqrt{1 - \frac{\gamma}{2}}$$

Thus, whenever firm 2 finds it profitable to enter the market ($q_2^D > 0$), we have the result that firm 1 reduces its investment on product enhancement ($I^M > I^D$). Below Figure 23 provides an illustration of this example.

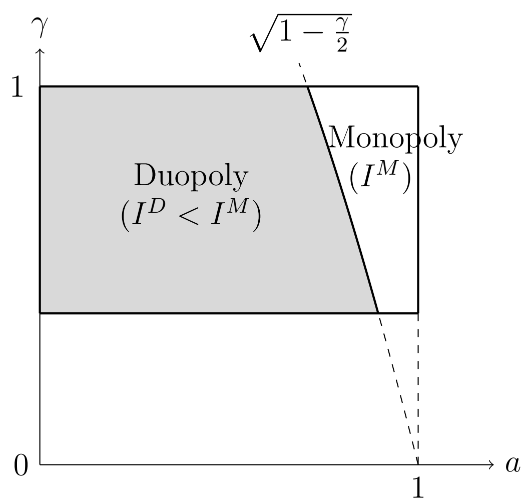

Figure 23: **An illustration of the example.**

As we can see from this figure, if firm 1's potential of increasing the consumer's utility is low, i.e., a is small, we are more likely to observe a market with more than one supplier. However, competition reduces the incumbent firm's incentive to improve its product quality. Thus, although consumers can enjoy these firms' products at lower prices, they might have to endure the low quality of these products. On the other hand, if a is large, then the incumbent firm can substantially improve its product quality in the second period. This product enhancement activity might be sufficient to deter the entrance of firm 2. As a result, we observe a monopoly market with firm 1 producing a high quality product.

Similarly we can analyze the impact of product differentiation γ on firm 1's incentive to enhance its product. For instance, if the degree of product differentiation is low, i.e., γ is high, then the incumbent firm has more incentive to innovate and enhance its product in the hope of capturing the monopoly rent. This incentive comes from the fact that when γ is high, the new product is more likely to be a better replacement version of the old product, e.g., newer versions of iphones can substitute older versions. The pace of innovation and product enhancement is summarized in Figure 19 in Section 3.1. From this figure we can see that the incumbent firm innovates and enhances its product in the fastest way when this firm has a high potential to improve its product quality and belongs to an industry with low possible product differentiation.

Lastly, we can use this example to examine the relationship between three different prices: p^N, p_1^M, and p_1^D. From the expressions of I^M and I^D we can find the following:

$$p^N = \frac{1}{2}\alpha_1 \qquad p_1^M = (\frac{1}{1-a^2})\frac{1}{2}\alpha_1 \qquad p_1^D = (\frac{1-\frac{\gamma}{2}}{1-a^2-\frac{\gamma^2}{4}})\frac{1}{2}\alpha_1$$

Here we can see that $p_1^M > p^N$ always holds, but the relationship between p_1^D and p^N is not so clear. For instance, if we assume $\gamma = 0.5$ and $a = 0.5$, then $p_1^D = \frac{6}{11}\alpha_1 > p^N$. If we instead consider $\gamma = 0.5$ and $a = 0.25$, then $p_1^D = \frac{3}{7}\alpha_1 < p^N$. Since the quantity expressions are the same as the price expressions, we can also deduce that the relationship between q^N and q_1^D also depends on specific parameter values. Thus, whether entrance by a competitor reduces the incumbent's price and output or not depends on both the incumbent's ability to improve its product quality as well as the degree of product differentiation generated by the incumbent's product enhancement activity.

3.3 Conclusion

In this paper I study the incumbent firm's incentive to innovate when potential competitors can easily imitate the incumbent's product. Although innovation is costly, the incumbent can improve its product quality ahead of its competitors. This advantage on timing can prevent new firms from seizing the incumbent's existing market shares and thus provide motivation for the incumbent firm to innovate. If the incumbent can constantly renovate its product, i.e., creative destruction happens within the firm, a monopoly market might persist even in the absence of knowledge protecting devices. Thus, firms and industries with high potentials of product enhancement can develop and grow faster than others. Innovations often come in waves since imitating early products is much easier than imitating mature products. Occasionally, competition and free entry can discourage the incumbent to innovate when the space of quality improvement is limited. Deadweight loss and market failure may arise when the cost of innovation is too high.

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