Investigation on static grounding analysis model of non-pneumatic tire with nonlinear spokes

Cite as: AIP Advances 11, 015303 (2021); doi: 10.1063/5.0027335
Submitted: 28 August 2020 • Accepted: 16 December 2020 •
Published Online: 5 January 2021

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ABSTRACT

A static grounding analysis model of non-pneumatic tire (NPT) was built and presented in this paper. The proposed NPT analysis model considers the non-linearity of the spoke stiffness and is suitable for the performance exploration of various structures of the NPT. First, the shear band, rigid rim, and spoke structure of the NPT were simplified, the main structural parameters and mechanical parameters were extracted, and an analysis model was established. The model can describe the deformation of the NPT when it is subjected to external forces. On this basis, the different stiffness of the spokes during tension and compression was considered, an iterative method was used to compensate for the difference in the deformation caused by the difference in radial stiffness of the spokes, and an analysis model of NPT with nonlinear spokes was established. Then, the contact between the tire and the road surface was introduced to iteratively compensate for the reaction force of the road surface, and the deformation of the NPT with nonlinear spokes on the road surface was obtained. Finally, the finite element software ABAQUS was used to verify the accuracy of the model. This model contains more comprehensive structural parameters and material parameters, which can more realistically simulate the structural characteristics and static grounding behavior of the NPT.

I. INTRODUCTION

Compared with traditional pneumatic tires, non-pneumatic tires (NPTs) have a simple structure; they are not prone to puncture, and their design requires less natural rubber usage. In addition, some properties of the NPT such as ground pressure and tire stiffness are no longer closely related due to the relative independence of the shear band and the spokes, enabling the NPT to deliver higher ground pressure while maintaining lower stiffness. The shear bands and spokes are the main components of the NPT, and this research focuses on the structural design and optimization of shear bands and spokes. The objective of a NPT analysis model is to calculate the overall NPT performance through the tire structural parameters and material parameters. The analysis model can be used to explore the deformation characteristics of the NPT in guiding its structural design.[1]

At present, a series of achievements have been made in the study of the static analysis model of novel NPT in contact with the road surface. There have been substantial fundamental studies on the mechanical and dynamic properties of the mechanical elastic wheel (ME-Wheel).[2–4] Rhyne in his research pointed out that under certain conditions, there is the following relationship between the average ground contact pressure P and the shear band shear modulus G, shear band thickness H, and shear band outer contour radius R of the NPT:[5]

$$P \cong \frac{GH}{R}. \tag{1}$$

That is, the product of the shear band's shear modulus and the thickness of the shear band is approximately equal to the product of the ground contact pressure and the outer radius of the shear band. Based on (1), Luke designed a bristle shaped shear band structure and calculated the specific shape of the bristles,[6] and the results from the calculation meet the design requirements. While formula (1) can be used to estimate the grounding characteristics of shear bands in different states of the NPT, it is too simple and can only be used for the design of shear bands, as its usage for the overall structure design is difficult.

The REF (flexible ring on elastic foundation) model proposed in the research of Lin and Huang[7,8] can be used to simulate the deformation of pneumatic tires, and there have been a series of studies, such as Ferris exploring the static deformation of tires[9] and Wei's kinetics analysis method for analyzing the transient response of rotating tires under various loads.[10] However, the transient response in the application of the REF model to the NPT was relatively late.

Gasmi proposed a two-dimensional NPT analysis model based on the REF model.[11,12] The model includes a shear band and spokes. It was assumed that the spokes do not produce reaction force when they are compressed. When the tire is deformed by contact with the ground, the shear zone is divided into three parts in the circumferential direction, namely, the junction, the non-joint, and the transition zone. The shear zone in the junction zone is subjected to the reaction force of the road surface. The shear band is subjected to the tensile force of the spokes, and the shear zone in the transition zone is not subjected to external forces. The model established a set of differential equations for the shear bands in the three regions to describe the displacement and force of the nodes on the shear band during tire deformation. Finally, the solutions of the three sets of equations were solved by the boundary conditions, and the deformation of the shear band of the entire tire was obtained. Gasmi's model can be used to obtain good results for the NPT with the spoke structure of Tweel, which has almost no radial force under compression. However, the circumferential stiffness has not been considered in his model. Zang *et al.* applied Gasmi's model to calculate the grounding-related characteristics of mechanical elastic wheels and used this to optimize tire wear resistance and grip.[13]

The spoke structure of the NPT does not all produce reaction force when it is squeezed. In some cases, the reaction force generated when the spokes are compressed may even be equivalent to the reaction force when tensioned.[14] Consequently, these structures can be optional for the spokes of the NPT. The mechanical properties of the spokes composed of these structures will significantly affect the overall performance of the NPT. As such, it is necessary to consider the mechanical performance of the spokes under compression when exploring the mechanical properties of the NPT.

Wang proposed a "reduced-order compensation scheme" method to calculate the static deformation response of the NPT.[15,16] First, a simple ring model supported by a linear spring was established, and the shear band and spokes of the NPT were simulated with Timoshenko beams and linear springs, respectively. The Fourier expansion of the external force on the ring/shear zone was carried out to achieve "order reduction" and then iteratively compensate for the difference between the tensile and the compressive stiffness of the spoke on the basis of the linear model to achieve the nonlinearity of the radial stiffness of the spoke. The contact deformation between the NPT and the road surface can be accurately calculated by using multiple tire structure parameters and material parameters. The application of Fourier expansion eliminates the need to solve analytical solutions of complex differential equations, but the iterative compensation method increases the computational cost of model solving.

In this paper, an analysis model of NPT with nonlinear spokes was established based on Gasmi's two-dimensional model. This model considered many parameters, including the circumferential stiffness of the spokes and the structural parameters and material parameters of the shear band, able to describe the structural performance of the NPT more accurately. Besides, it is set that when the spokes are compressed and stretched, the force is different, so the nonlinear stiffness is formed. Specifically, the non-linear radial stiffness of the spoke is set, which is suitable for the performance research of the NPT with various structures, and can be used to calculate the static deformation of the NPT when contacting with the road surface.

II. LINEAR MODEL CONSIDERING THE LONGITUDINAL STIFFNESS OF SPOKES

A. Establishment of governing equation

The rim of the NPT is a ring structure with high rigidity, which connects the wheel hub of the vehicle. Its rigidity is much greater than that of the spokes and shear bands. Therefore, the rim can be considered rigid when modeling, and the rim can be simplified as a rigid ring without thickness. The outer ring of the NPT (including the shear band and tread) is a component that directly contacts the road surface, with certain tensile properties and shear properties, and it is the main component that determines the tire grounding characteristics. The width of the outer ring is large, ensuring sufficient ground contact area, and the thickness is much smaller than the radius of the tire. When modeling, it was simplified into a flexible ring with a certain thickness, and the Timoshenko beam model was used to simulate its characteristics. There are many types of spokes of the NPT. When the tire is deformed, it provides elastic force due to tension or compression. Generally, its structure is uniformly distributed in the circumferential direction. Thus, the spokes were simplified as a series of springs that radiate outward and distribute uniformly along the circumference. Considering the circumferential stiffness of the NPT spokes, a linear model was established in this paper.

When the simplified NPT is in contact with a rigid flat road, its deformation is shown in Fig. 1.

In Fig. 1, the arrows on the spokes that point to the inside or outside of the tire radial direction indicate the direction of the force on the shear band after the spoke structure is deformed. Modeling does not consider the friction between the tire and the road. For the convenience of expression, the outer ring of the shear band and tread composition in the model is generally called the shear band. Considering the longitudinal rigidity of the spokes, the force on the shear band in the linear model is shown in Fig. 2.[17]

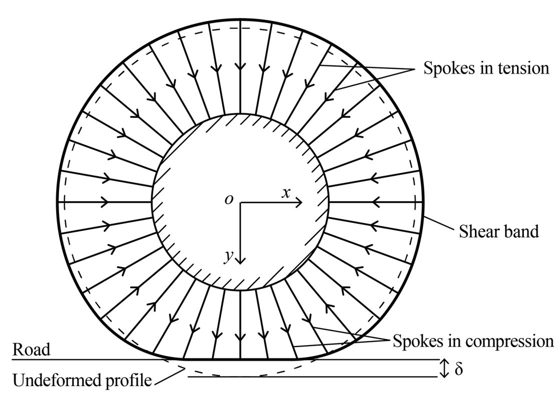

FIG. 1. Deformation of the NPT model contacting with the road. The outer ring (shear band and tread) is simplified into a flexible ring with a certain thickness, and its characteristics are simulated by the Timoshenko beam model. The spokes are simplified as a series of springs that radiate outward and distribute uniformly along the circumference.

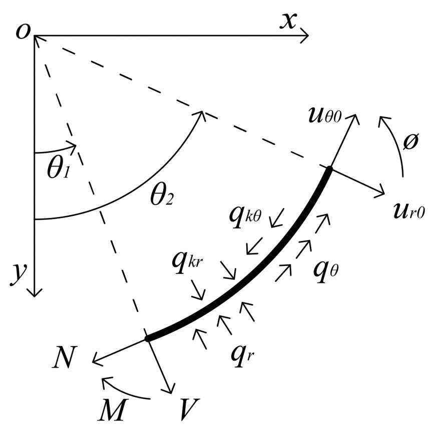

FIG. 2. Deformation of the shear band with a force between θ_1 and θ_2. q_r and q_θ, respectively, are the radial component and tangential component of the external forces on the shear band; q_{kr} and $q_{k\theta}$, respectively, are the radial component and tangential component of the spoke force; u_{r0} and $u_{\theta 0}$, respectively, are the radial displacement and tangential displacement of the points on the neutral plane; ϕ is the corner of the cross section; N is the axial force; V is the shear force; and M is the section bending moment.

The NPT has a certain width b in the direction perpendicular to the xoy plane. To simplify the problem, it was assumed that the structural mechanical properties of the tire do not change in the width direction, and the shear band deformation only occurred in the xoy plane. This paper, therefore, describes the bending problem of the shear band in the polar coordinate system, taking the tire center as the origin o and the oy axis as the polar axis and setting counterclockwise as the positive direction.

The radial stiffness of the spokes produces radial forces on the shear band, and the circumferential stiffness of the spokes also produces tangential forces on the shear band. The external forces on the shear band include the radial component q_r and tangential component q_θ, and the spoke force includes the radial part q_{kr} and tangential part $q_{k\theta}$. The shear band is subjected to the external force q and the inner spoke force q_k within one circle. When the shear band is not subjected to an external force, it is only necessary to set the radial component q_r and the tangential component q_θ at the corresponding position to zero.

When the tire is deformed on the ground, any point on the shear band may produce displacement, including the radial displacement u_r and tangential displacement u_θ. The shear zone has a certain thickness, and the displacements of the points on the neutral plane are expressed as the radial displacement u_{r0} and tangential displacement $u_{\theta 0}$. Based on the assumption of the Timoshenko beam, when the shear band is bent and deformed, its cross section remains flat but produces a corner ϕ, and three corresponding forces will be generated at the cross section of the shear band, shear force V, axial force N, and section bending moment M.

The radial stiffness of the spoke was defined as the reaction force caused by the radial deformation of the sector-shaped spoke corresponding to the unit circumferential length on the neutral plane of the tire's shear band. Based on the assumption that the number of spokes is not too small, the stiffness of the spokes in the model is equivalent transformed from the stiffness of discrete spokes, and the calculation of the spoke force is based on the continuous spokes. The unit of force in this paper is N and the unit of length is millimeter (mm), so the unit of radial stiffness k_r of the spoke is N/mm^2. The radial rigidity of the spokes per unit tire width is $\frac{k_r}{b}$. When the spokes per unit width are stretched δu_r, the reaction force is $\frac{k_r}{b}\delta u_r$.

When the Timoshenko beam is bent, its cross section remains flat, so the displacement at any point on the shear band cross section in Fig. 2 can be expressed as a function of the thickness of the shear band and the angle of the cross section,

$$\begin{aligned} u_r(z,\theta) &= u_r(\theta), \\ u_\theta(z,\theta) &= u_{\theta 0}(\theta) + z\phi(\theta), \end{aligned} \tag{2}$$

where $z = R - r$, in which R is the radius of the neutral plane of the shear band and r is the distance from the point on the shear band to the center of the tire, and θ is the angular position of the point on the shear band.

When the shear zone is deformed by a force between θ_1 and θ_2, the virtual strain energy generated by the shear zone deformation is[11]

$$\delta U = \int_{\theta_1}^{\theta_2}\left(\begin{matrix}\left(-\dfrac{dN}{d\theta} - V\right)\delta u_{\theta 0} + \left(N - \dfrac{dV}{d\theta}\right)\delta u_r \\ +\left(-\dfrac{dM}{d\theta} + RV\right)\delta\phi\end{matrix}\right)d\theta + \left[N\delta u_{\theta 0} + V\delta u_r + M\delta\phi\right]_{\theta_2}^{\theta_1}. \tag{3}$$

The work done by the spoke force and road reaction force on the shear band is expressed as

$$\delta V = -\int_{\theta_1}^{\theta_2}\left(\begin{matrix} q_r du_r + q_\theta du_{\theta 0} \\ -\dfrac{k_r}{b}u_r du_r - \dfrac{k_\theta}{b}u_{\theta 0}du_{\theta 0}\end{matrix}\right)Rbd\theta. \tag{4}$$

According to the principle of virtual work,

$$\delta W = \delta U + \delta V = 0. \tag{5}$$

Substituting (3) and (4) into (5), the equilibrium equation of the shear band under the action of spoke force and road reaction force is

$$\begin{aligned} \frac{dN}{d\theta} + V - Rk_\theta u_{\theta 0} &= -Rbq_\theta, \\ N - \frac{dV}{d\theta} + Rku_r &= Rbq_r, \\ \frac{dM}{d\theta} - RV &= 0. \end{aligned} \tag{6}$$

Because the thickness of the shear band is smaller than one-tenth of the radius of the tire, it can be considered that $R + z$ tends to R, and the resultant force of the shear band section can be expressed as[11]

$$\begin{aligned} M &= \frac{EI}{R}\frac{d\phi}{d\theta}, \\ N &= \frac{EA}{R}\left(\frac{du_{\theta 0}}{d\theta} + u_r\right), \\ V &= \frac{GA}{R}\left(\frac{du_r}{d\theta} - u_{\theta 0} + R\phi\right), \end{aligned} \tag{7}$$

where E is the equivalent elastic modulus of the shear band, G is the equivalent shear modulus of the shear band, I is the equivalent moment of inertia of the shear band, and A is the equivalent sectional area of the shear band.

Substituting Eq. (7) into the equilibrium equation (6), the governing equation of shear band deformation is obtained as

$$\begin{gathered} EA\frac{d^2u_{\theta0}}{d\theta^2} - \left(GA + R^2k_\theta\right)u_{\theta0} + (EA+GA)\frac{du_r}{d\theta} + RGA\phi = -R^2bq_\theta, \\ GA\frac{d^2u_r}{d\theta^2} - \left(EA + R^2k_r\right)u_r - (EA+GA)\frac{du_{\theta0}}{d\theta} + RGA\frac{d\phi}{d\theta} = -R^2bq_r, \\ EI\frac{d^2\phi}{d\theta^2} - R^2GA\phi - RGA\frac{du_r}{d\theta} + RGAu_{\theta0} = 0, \end{gathered} \tag{8}$$

where $R^2k_\theta u_{\theta0}$ is the effect of the circumferential stiffness of the spoke on the NPT and $R^2k_ru_r$ is the effect of the radial stiffness of the spoke on the NPT.

B. Solving equations

Considering that Eq. (8) is a linear system of non-homogeneous equations, the non-inferior terms on the right side of the equal sign can be split into multiple simple parts and calculated separately. The superposition solution obtained from each part is the solution of the original equation.[15]

Assuming the force on the outer circle of the shear zone in Fig. 2 is F, F can be expanded into radial and tangential parts,

$$F(\theta) = F_r(\theta)\vec{r} + F_\theta(\theta)\vec{\theta}. \tag{9}$$

These two components can be regarded as a series of sine and cosine forces,

$$\begin{aligned} F_r(\theta) &= \sum_{n=-N}^{N} Q_{nrc}\cos(n(\theta-\theta_0)) + \sum_{n=-N}^{N} Q_{nrs}\sin(n(\theta-\theta_0)), \\ F_\theta(\theta) &= \sum_{n=-N}^{N} Q_{n\theta c}\cos(n(\theta-\theta_0)) + \sum_{n=-N}^{N} Q_{n\theta s}\sin(n(\theta-\theta_0)), \end{aligned} \tag{10}$$

where "n" in the subscript represents the nth order sinusoidal or cosine force, "r" is the radial component, "θ" signifies the tangential component, "c" represents the sinusoidal component, and "s" denotes the cosine component.

The nth harmonic force per unit width of the tire can also be expressed as radial and tangential component forces,

$$\begin{aligned} q_{nr}(R,\theta) &= q_{nrc}(R,\theta) + q_{nrs}(R,\theta), \\ q_{n\theta}(R,\theta) &= q_{n\theta c}(R,\theta) + q_{n\theta s}(R,\theta). \end{aligned} \tag{11}$$

Combining (10) with (11), the four-component force of the shear belt can be obtained as

$$\begin{aligned} q_{nrc}(R,\theta) &= \frac{1}{b}Q_{nrc}\cos(n(\theta-\theta_0)), \\ q_{nrs}(R,\theta) &= \frac{1}{b}Q_{nrs}\sin(n(\theta-\theta_0)), \\ q_{n\theta c}(R,\theta) &= \frac{1}{b}Q_{n\theta c}\cos(n(\theta-\theta_0)), \\ q_{n\theta s}(R,\theta) &= \frac{1}{b}Q_{n\theta s}\sin(n(\theta-\theta_0)). \end{aligned} \tag{12}$$

Taking the first formula of formula (12) into formula (11), we get

$$\begin{gathered} q_{nr} = q_{nrc}(R,\theta) = \frac{1}{b}Q_{nrc}\cos(n(\theta-\theta_0)), \\ q_{n\theta} = 0. \end{gathered} \tag{13}$$

Equation (13) is brought into Eq. (8), and the deformation governing equation of the shear band under a group of forces is obtained as

$$\begin{aligned} & EA\frac{d^2u_{\theta0}}{d\theta^2} - \left(GA + R^2k_\theta\right)u_{\theta0} + (EA+GA)\frac{du_r}{d\theta} + RGA\phi = 0, \\ & GA\frac{d^2u_r}{d\theta^2} - \left(EA + R^2k_r\right)u_r - (EA+GA)\frac{du_{\theta0}}{d\theta} + RGA\frac{d\phi}{d\theta} \\ & \quad = -R^2Q_{nrc}\cos(n(\theta-\theta_0)), \\ & EI\frac{d^2\phi}{d\theta^2} - R^2GA\phi - RGA\frac{du_r}{d\theta} + RGAu_{\theta0} = 0. \end{aligned} \tag{14}$$

The subscript "nrc" in the combined equation means that the solution of the combined equation is the sine part of the radial component of the nth harmonic force on the shear band per unit tire width, and the corresponding deformation is expressed as $u_{r,nrc}, u_{\theta,nrc}, \phi_{nrc}$.

Considering the form of the system of Eq. (14), in which the non-homogeneous terms are sinusoidal functions, it may be assumed that the solution of the system of Eq. (14) is

$$\begin{aligned} \mathrm{u_{r,nrc}}(\mathrm{R},\theta) &= \mathrm{C_{ur,nrc}}\cos(\mathrm{n}(\theta-\theta_0)), \\ \mathrm{u_{\theta,nrc}}(\mathrm{R},\theta) &= \mathrm{C_{u\theta,nrc}}\sin(\mathrm{n}(\theta-\theta_0)), \\ \phi_{\mathrm{nrc}}(\mathrm{R},\theta) &= \mathrm{C_{\phi,nrc}}\sin(\mathrm{n}(\theta-\theta_0)). \end{aligned} \tag{15}$$

Placing Eq. (15) into Eq. (14) to eliminate the common terms $\cos(n(\theta-\theta_0))$ and $\sin(n(\theta-\theta_0))$ that appear in the calculation, we can get three equations, including three coefficients $C_{ur,nrc}, C_{u\theta,nrc}, C_{\phi,nrc}$. It is obvious that the three coefficients can be solved quickly and conveniently. The results can be carried into Eq. (15) to obtain the solution $u_{r,nrc}, u_{\theta,nrc}, \phi_{nrc}$ of the equation under external force.

In a similar way, the results corresponding to the other three equations in Eq. (12) can be obtained.

The solution results of the equations are summarized in Table I.

In Table I,

$$\begin{aligned} a &= n^2GA + EA + R^2k_r, & b' &= nEA + nG, \\ c &= -nRGA, & d &= n^2EA + GA + R^2k_\theta, \\ e &= -RGA, & f &= n^2EI + R^2GA, \end{aligned}$$

$$Z = ae^2 - 2b'ce + c^2d + b'^2f - adf.$$

The coefficients of the second column in Table I are substituted into the solutions of the corresponding equations in the third column, and the deformation of the shear band under the action of various components is obtained.

The deformation of the NPT shear band under the action of external force F can be obtained by superimposing the deformation

TABLE I. Solutions of functions: deformation with component force.

Component force	Coefficient	Solutions of equations
$q_{nr} = q_{nrc}(R,\theta) = \frac{1}{b}Q_{nrc}\cos(n(\theta-\theta_0))$ $q_{n\theta} = 0$	$C_{ur,nrc} = \frac{e^2-df}{Z}R^2Q_{nrc}$ $C_{u\theta,nrc} = \frac{b'f-ce}{Z}R^2Q_{nrc}$ $C_{\phi,nrc} = \frac{cd-b'e}{Z}R^2Q_{nrc}$	$u_{r,nrc}(R,\theta) = C_{ur,nrc}\cos(n(\theta-\theta_0))$ $u_{\theta,nrc}(R,\theta) = C_{u\theta,nrc}\sin(n(\theta-\theta_0))$ $\phi_{nrc}(R,\theta) = C_{\phi,nrc}\sin(n(\theta-\theta_0))$
$q_{nr} = q_{nrs}(R,\theta) = \frac{1}{b}Q_{nrs}\sin(n(\theta-\theta_0))$ $q_{n\theta} = 0$	$C_{ur,nrs} = \frac{e^2-df}{Z}R^2Q_{nrs}$ $C_{u\theta,nrs} = -\frac{b'f-ce}{Z}R^2Q_{nrs}$ $C_{\phi,nrs} = -\frac{cd-b'e}{Z}R^2Q_{nrs}$	$u_{r,nrs}(R,\theta) = C_{ur,nrs}\sin(n(\theta-\theta_0))$ $u_{\theta,nrs}(R,\theta) = C_{u\theta,nrs}\cos(n(\theta-\theta_0))$ $\phi_{nrs}(R,\theta) = C_{\phi,nrs}\cos(n(\theta-\theta_0))$
$q_{nr} = 0$ $q_{n\theta} = q_{n\theta c}(R,\theta) = \frac{1}{b}Q_{n\theta c}\cos(n(\theta-\theta_0))$	$C_{ur,n\theta c} = -\frac{b'f-ce}{Z}R^2Q_{n\theta c}$ $C_{u\theta,n\theta c} = \frac{c^2-af}{Z}R^2Q_{n\theta c}$ $C_{\phi,n\theta c} = \frac{ae-b'c}{Z}R^2Q_{n\theta c}$	$u_{r,n\theta c}(R,\theta) = C_{ur,n\theta c}\sin(n(\theta-\theta_0))$ $u_{\theta,n\theta c}(R,\theta) = C_{u\theta,n\theta c}\cos(n(\theta-\theta_0))$ $\phi_{n\theta c}(R,\theta) = C_{\phi,n\theta c}\cos(n(\theta-\theta_0))$
$q_{nr} = 0$ $q_{n\theta} = q_{n\theta s}(R,\theta) = \frac{1}{b}Q_{n\theta s}\sin(n(\theta-\theta_0))$	$C_{ur,n\theta s} = \frac{b'f-ce}{Z}R^2Q_{n\theta s}$ $C_{u\theta,n\theta s} = \frac{c^2-af}{Z}R^2Q_{n\theta s}$ $C_{\phi,n\theta s} = \frac{ae-b'c}{Z}R^2Q_{n\theta s}$	$u_{r,n\theta s}(R,\theta) = C_{ur,n\theta s}\cos(n(\theta-\theta_0))$ $u_{\theta,n\theta s}(R,\theta) = C_{u\theta,n\theta s}\sin(n(\theta-\theta_0))$ $\phi_{n\theta s}(R,\theta) = C_{\phi,n\theta s}\sin(n(\theta-\theta_0))$

generated by the force components of each order,

$$\begin{aligned} u_r(R,\theta) &= \sum_{-N}^{N}\left[u_{r,nrc}(R,\theta) + u_{r,nrs}(R,\theta) + u_{r,n\theta c}(R,\theta) + u_{r,n\theta s}(R,\theta)\right],\\ u_\theta(R,\theta) &= \sum_{-N}^{N}\left[u_{\theta,nrc}(R,\theta) + u_{\theta,nrs}(R,\theta) + u_{\theta,n\theta c}(R,\theta) + u_{\theta,n\theta s}(R,\theta)\right],\\ \phi(R,\theta) &= \sum_{-N}^{N}\left[\phi_{nrc}(R,\theta) + \phi_{nrs}(R,\theta) + \phi_{n\theta c}(R,\theta) + \phi_{n\theta s}(R,\theta)\right]. \end{aligned} \tag{16}$$

III. DEFORMATION OF NPT WITH NONLINEAR SPOKES

A. Deformation under external force

When the spokes of a NPT have non-linear radial stiffness, that is, the spokes exhibit different stiffness when they are stretched and compressed in the radial direction, the spokes' tensile stiffness is expressed as k_{re} and the compression stiffness is expressed as k_{rs}. When the NPT is subjected to external forces, if it is calculated according to the linear spoke (stiffness k_{re}), then the model calculates a part of the less (more) spoke force. The less (more) calculated spoke force is

$$F_e(\theta) = \Delta u_r(k_{rs} - k_{re}). \tag{17}$$

For the less calculated spoke force, compensation is required. The magnitude of the compensation force is

$$F_{cp}(\theta) = F_e(\theta). \tag{18}$$

The subscript "*cp*" stands for "compensation" and represents the compensation force.

Similarly, the compensation force is expanded into a series of sine and cosine forces,

$$\begin{aligned} F_{cp}(\theta) &= \sum_{n=-N}^{N} F_{cp,n}(\theta)\\ &= \sum_{n=-N}^{N}\left[H_{nrc}\cos(n(\theta-\theta_0)) + H_{nrs}\sin(n(\theta-\theta_0))\right]. \end{aligned} \tag{19}$$

TABLE II. Parameter configurations of the NPT model for accuracy verification.

E (MPa)	G (MPa)	k_{re} (N/mm^2)	k_θ (N/mm^2)	R (mm)	b (mm)	h (mm)	I (mm^4)	δ (mm)
Material				Structure				
900	10	10	1	310	200	20	130 000	31

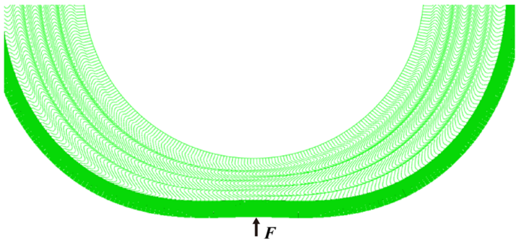

FIG. 3. FEA model, shear band with applied concentrated force.

The nth order harmonic force of the compensation force per unit width of the tire is expressed as the radial and tangential force components,

$$
\begin{aligned}
q_{cp,nr}(R,\theta) &= q_{cp,nrc}(R,\theta) + q_{cp,nrs}(R,\theta),\\
q_{cp,n\theta}(R,\theta) &= 0.
\end{aligned}
\tag{20}
$$

From formula (19) and formula (20), it is easy to obtain

$$
\begin{aligned}
q_{cp,nrc}(R,\theta) &= \frac{1}{b}H_{nrc}\cos(n(\theta-\theta_0)),\\
q_{cp,nrs}(R,\theta) &= \frac{1}{b}H_{nrs}\cos(n(\theta-\theta_0)),\\
q_{cp,n\theta c}(R,\theta) &= 0,\\
q_{cp,n\theta s}(R,\theta) &= 0.
\end{aligned}
\tag{21}
$$

From Eqs. (8) and (21), through the method in Sec. II, the deformation $u_{r,cp}(R,\theta), u_{\theta,cp}(R,\theta), \phi_{cp}(R,\theta)$ can be obtained due to the action of the compensation force F_{cp}, and the form is similar to the results in Table I. Considering the compensation force, the total deformation of the NPT is

$$
\begin{aligned}
\overline{u_{r,cp}}(R,\theta) &= u_r(R,\theta) + u_{r,cp}(R,\theta),\\
\overline{u_{\theta,cp}}(R,\theta) &= u_\theta(R,\theta) + u_{\theta,cp}(R,\theta),\\
\overline{\phi_{cp}}(R,\theta) &= \phi(R,\theta) + \phi_{cp}(R,\theta).
\end{aligned}
\tag{22}
$$

After the radial displacement of the NPT shear band changes from u_r to $\overline{u_{r,cp}}$, due to the new radial displacement $\Delta u_r(R,\theta) = u_{r,cp}(R,\theta)$, correspondingly, a new compensation force is needed. Applying a repeat of the calculation process of formula (18) to formula (22), after multiple iteration calculations, when the deformation of the tire in formula (22) is small enough, it can be considered that the compensation is completed, the solution is completed, and the deformation of the NPT with nonlinear spokes under the action of external force F is obtained.

B. Deformation in contact with the road

When a NPT with non-linear spokes is in contact with the road surface, the tire is deformed by the reaction force of the road surface. Iterative compensation can be used to calculate the deformation of the tire after it makes contact with the road surface.

Suppose the shape of the road surface is

$$
f(\theta) = R_{road}(\theta), |\theta| \in [0,\theta_0], \quad \theta_0 < \frac{\pi}{2}. \tag{23}
$$

Then, the position difference of the outer contour of the tire shear band on the road surface is

$$
u_{r,pe}(R,\theta) = \begin{cases} R + \dfrac{h}{2} + u_r(R,\theta) - R_{road}(\theta), & |\theta| \in [0,\theta_0] \quad \theta_0 < \dfrac{\pi}{2} \\ 0, & |\theta| \notin [0,\theta_0] \quad \theta_0 < \dfrac{\pi}{2}, \end{cases} \tag{24}
$$

where R is the radius of the shear zone midplane and h is the thickness of the shear zone.

The outer contour of the tire shear band cannot overlap into the road surface, so the radial reaction force of the road surface on the tire can be expressed as follows:

$$
F_{r,pe}(\theta) = \begin{cases} u_{r,pe}(k_{rs} - k_{re}), & u_{r,pe} > 0 \\ 0, & u_{r,pe} \le 0. \end{cases} \tag{25}
$$

Through the application of the expansion and compensation processes similar to Eqs. (19)–(22), the deformation

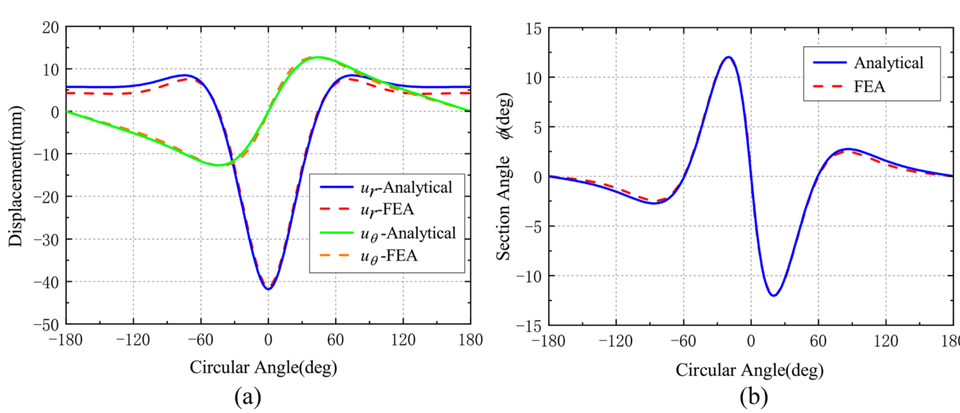

FIG. 4. Results of the linear NPT model with concentrated force: (a) relationship between the circular angle θ and the radial displacement u_r and tangential displacement u_θ of the nodes on the midplane of the shear zone and (b) relationship between the circular angle and the angle ϕ of the shear zone cross section.

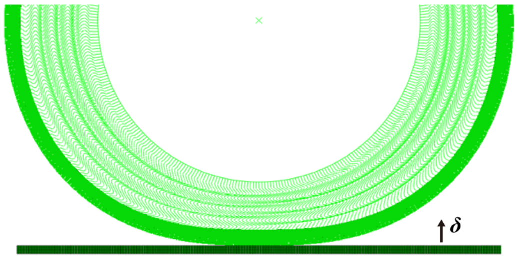

FIG. 5. FEA model, NPT with reaction force of the road surface.

$u_{r,pe}(R,\theta), u_{\theta,pe}(R,\theta), \phi_{pe}(R,\theta)$ resulting from the reaction of the road surface can be obtained.

In this way, when the road reaction force acts, the total deformation of the tire is expressed as

$$\begin{aligned}\overline{u_{r,pe}}(R,\theta) &= \overline{u_{r,cp}}(R,\theta) + u_{r,pe}(R,\theta),\\ \overline{u_{\theta,pe}}(R,\theta) &= \overline{u_{\theta,cp}}(R,\theta) + u_{\theta,pe}(R,\theta),\\ \overline{\phi_{pe}}(R,\theta) &= \overline{\phi_{cp}}(R,\theta) + \phi_{pe}(R,\theta).\end{aligned} \tag{26}$$

The shear band of the NPT produces new radial displacement, which requires new compensation force. The application of iterative calculation of formula (23) to Eq. (26) is repeated. Once the tire's deformation change in formula (26) is small enough, the compensation process can be completed, and the deformation caused by the contact between the tire and the road surface can be obtained.

So far, the solutions of the contact problem of the NPT with non-linear spokes and the road surface are completed.

IV. MODEL VALIDATION

For the purpose of validation of the above model, the finite element analysis (FEA) model of NPT was established by the finite element software ABAQUS. In the finite element model, the beam element B22 was selected to simulate the shear band, and the general beam section was selected to facilitate the setting of section shape parameters; 360 spring elements with uniform circumferential distribution were used as spokes, and different radial stiffness values in tension and compression of spokes were set through the input file. The pavement is straight and rigid, and the contact between the shear band and the pavement is a non-friction hard contact.

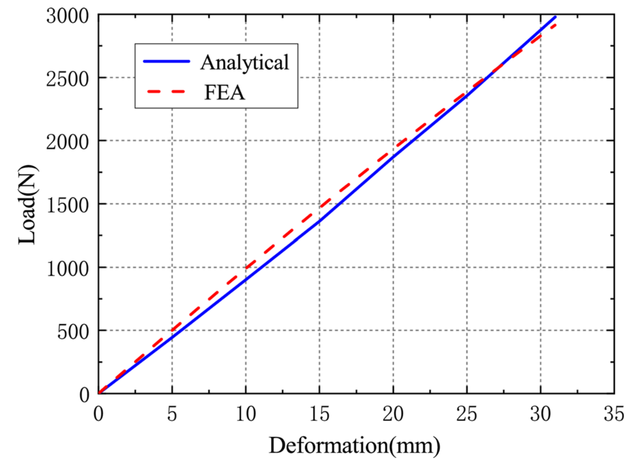

FIG. 7. Radial stiffness of the NPT when the tension stiffness is larger and the change in road surface reaction force (load) when the tire is deformed.

The material and structural parameters of the NPT are shown in Table II.

The material of the shear band is a linear elastic material. The selection of E is based on Refs. 18 and 19, and that of G is based on Refs. 20 and 21. The middle radius R of the shear band is determined by referring to the size of the reference tire 205/55R16, the tire transverse width B is determined with

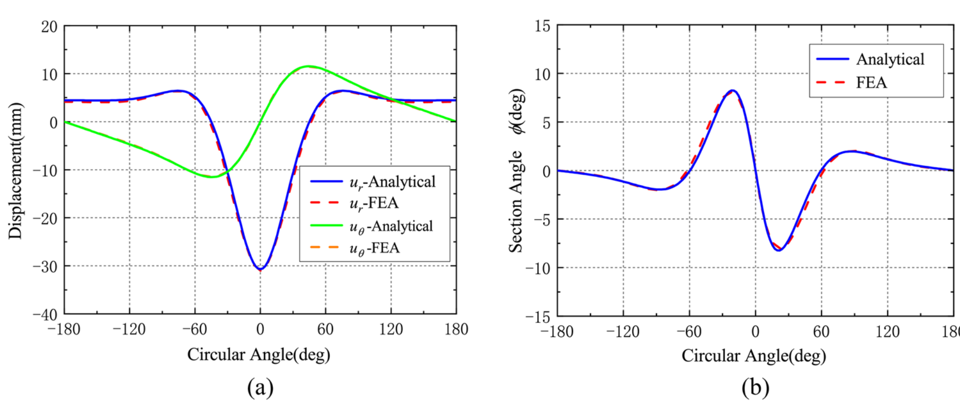

FIG. 6. Deformations of the NPT when the tension stiffness is larger: (a) relationship between the circular angle θ and the radial displacement u_r and tangential displacement u_θ of the nodes on the midplane of the shear zone and (b) relationship between the circular angle and the angle ϕ of the shear zone cross section.

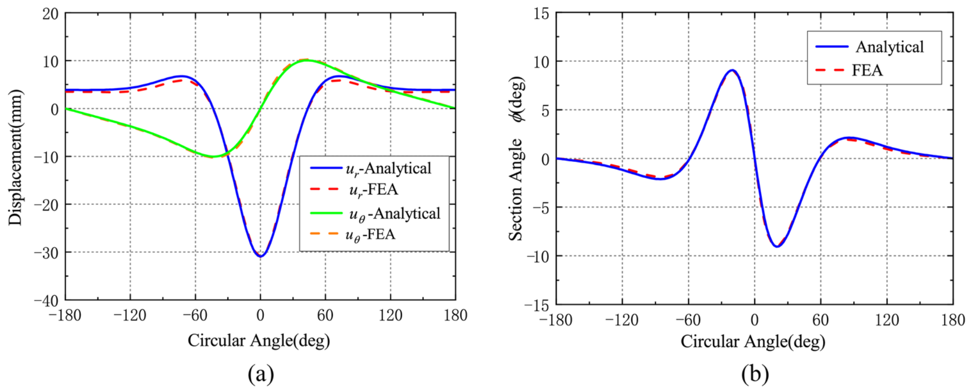

FIG. 8. Deformations of the NPT when the spoke stiffness is linear: (a) relationship between the circular angle θ and the radial displacement u_r and tangential displacement u_θ of the nodes on the midplane of the shear zone and (b) relationship between the circular angle and the angle ϕ of the shear zone cross section.

reference to the width of the grounding mark, the shear band thickness h is set according to the expected structure, and the moment of inertia I is the inertia moment corresponding to the solid and uniform material: the value of δ slightly makes the tire produce enough obvious deformation, so it is taken as one-tenth of the radius of the shear band; according to the radial stiffness of the benchmark tire of about 200 N/mm^2,[22] the spoke stiffness is set after many tests. In Table II, the fixed radial tensile stiffness of the spokes is set, while the radial compression stiffness of the spokes is not determined, so it is convenient to verify the accuracy of the model under different conditions by setting different spoke compression stiffness.

A. NPTs with linear spokes are subjected to concentrated forces

To verify the accuracy of the linear model obtained in Sec. II, the NPT spokes were set to have the same tension and compression stiffness of 10 N/mm^2. During the analysis, the tire center was fixed, and a radial concentration force of 10 000 N was applied at the center position below the tire shear band. The deformation results of the FEA are shown in Fig. 3. Figure 4 shows the comparison of the results from calculation of the analysis model and the finite element model, specifically the radial displacement u_r and tangential displacement u_θ of the nodes on the midplane of the shear zone and the angle ϕ of the shear zone cross section.

The results show that on the basis and core of the entire model, the calculation results of the linear model are in good agreement with the finite element results, and the linear model has high accuracy.

B. Non-pneumatic tires with nonlinear spokes are in contact with the road

To verify the accuracy of the contact model between the NPT and the road in Sec. III, this paper sets up multiple sets of comparative analysis schemes. In the analysis, the tire loading method was to keep the center position of the tire and the road surface lift δ fixed, in order to examine the deformation of the shear band of the tire. Figure 5 shows the finite element model of the contact deformation between the NPT and the road surface.

When the spoke compression stiffness is less than the tensile stiffness, the compression stiffness is set to 2 N/mm^2, which is 0.2 times the tensile stiffness. Figure 6 shows the deformation of the shear band of the NPT calculated by the analysis model and the finite element model under this setting. Figure 7 shows the change in road surface reaction force (load) when the tire is deformed.

When the spoke compression stiffness is equal to the tensile stiffness, the compression stiffness is set to 10 N/mm^2. Figures 8 and 9, respectively, are the deformation and load curves of the shear band of the NPT calculated by the analysis model and the finite element model under this setting.

When the compression stiffness of the spoke is greater than the tensile stiffness, the compression stiffness is set to 50 N/mm^2, which is five times the tensile stiffness. Figures 10 and 11 are the deformation and load curves of the NPT shear band calculated by

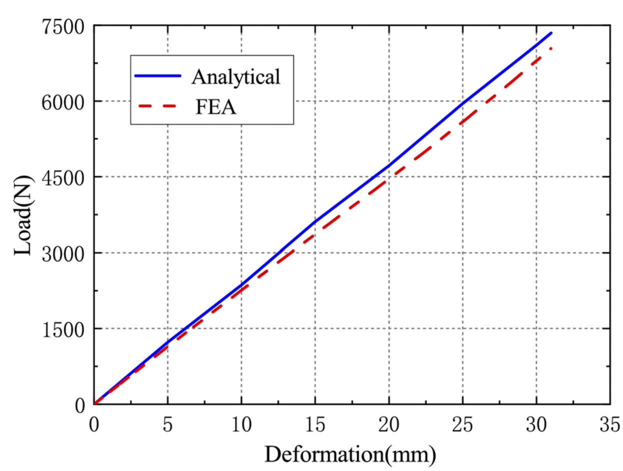

FIG. 9. Radial stiffness of the NPT when the spoke stiffness is linear and the change in road surface reaction force (load) when the tire is deformed.

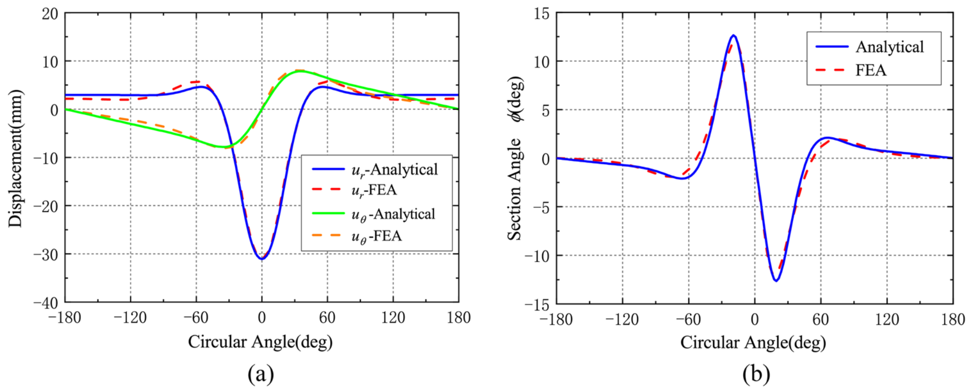

FIG. 10. Deformations of the NPT when the tension stiffness is smaller: (a) relationship between the circular angle θ and the radial displacement u_r and tangential displacement u_θ of the nodes on the midplane of the shear zone and (b) relationship between the circular angle and the angle ϕ of the shear zone cross section.

the analysis model and the finite element model under this setting, respectively.

From the comparison of the above results, when the tire model with nonlinear spokes contacts the road surface, under different spoke stiffness conditions, the deformation results of the shear band calculated by the analysis model and FEA model are in good agreement, and the comparison error of vertical characteristics of the tire is also small. Among them, the relatively large error is the radial deformation of the shear band, and the location where the error occurs is mainly near the $\pm 60^\circ$ position in the coordinate system. The error is due to the large radial deformation and bending of the shear band at the $\pm 60^\circ$ position. The model based on the linear elastic assumption may have errors in large deformation (similarly, at the position of 0° with large deformation, there is no deviation in radial displacement because of the constraint of pavement uplift).

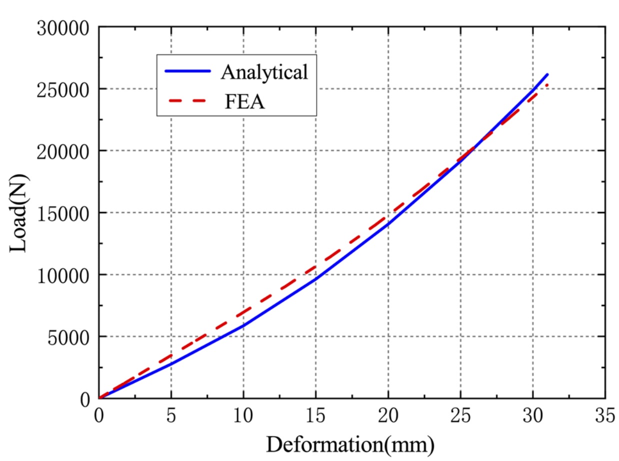

FIG. 11. Radial stiffness of the NPT when the tension stiffness is smaller and the change in road surface reaction force (load) when the tire is deformed.

V. CONCLUSION

Based on the basic structure and deformation characteristics of the NPT, the shear band is simplified into a circular Timoshenko beam. On this basis, considering the nonlinearity of the NPT spoke stiffness, the tensile stiffness and compression stiffness of the spoke are set up, and the analysis model of NPT with nonlinear spokes is established. By decomposing the external force received by the tire into a series of sine and cosine functions, the numerical solution of the equations is obtained. Then, the deformation of the NPT in contact with the road is obtained by compensating for the difference in the spoke stiffness and road reaction force. Finally, the FEA is used to verify the accuracy of the model.

The analysis model of NPT proposed in this paper sets up a non-linear radial stiffness of the spoke that can more accurately describe the structural performance of the NPT. The model is suitable for the performance research of various structures of the NPT and is useful in solving the static deformation of the NPT in contact with the road surface. In addition, the model does not require the tire to be in contact with a straight/flat shaped road surface, and through reasonable programming, the model can be adapted to various road surface shapes. However, there are still some shortcomings in this model. Because the establishment of the model is based on the assumption of linear elasticity, the calculation error will increase when there is excessive deformation in the model.

ACKNOWLEDGMENTS

This study was supported by the National Natural Science Foundation of China (Grant No. 51675240).

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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