Experimental, Theoretical, and Numerical Investigations of Geomechanics/Flow Coupling in Energy Georeservoirs

Zihao Li

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Cheng Chen, Chair

Nino Ripepi

Emily Sarver

Yang Liu

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Zihao Li

ACADEMIC ABSTRACT

The development of hydrocarbon energy resources from shale, a fine-grained, low-permeability geological formation, has altered the global energy landscape. Constricting pressure exerted on a shale formation has a significant effect on the rock's apparent permeability. Gas flow in low-permeability shales is significantly different from liquid flow due to the Klinkenberg effect caused by gas molecule slip at the nanopore wall surfaces. This has the effect of increasing apparent permeability (i.e., the measured permeability). Optimizing the conductivity of the proppant assembly is another critical component of designing subsurface hydrocarbon production using hydraulic fracturing. Significant fracture conductivity can be achieved at a much lower cost than conventional material costs, according to the optimal partial-monolayer proppant concentration (OPPC) theory. However, hydraulic fracturing performance in unconventional reservoirs is problematic due to the complex geomechanical environment, and the experimental confirmation and investigation of the OPPC theory have been rare in previous studies. In this dissertation, a novel multiphysics shale transport (MPST) model was developed to account for the coupled multiphysics processes of geomechanics, fluid dynamics, and the Klinkenberg effect in shales. Furthermore, A novel multi-physics multi-scale multi-porosity shale gas transport (M³ST) model was developed based on the MPST model research to investigate shale gas transport in both transient and steady states, and a double-exponential empirical model was also developed as a powerful substitute for the M³ST model for fitting laboratory-measured apparent permeability. Additionally, throughout the laboratory experiment of fracture conductivity with proppant, the four visible stages documented the evolution of non-monotonic conductivity and proppant concentration. The laboratory methods and empirical model were then applied to the shale plugs from Central Appalachia to investigate the formation properties there. The benefits of developing these regions wisely include a smaller surface footprint, reduced infrastructure requirements, and lower development costs. The developed MPST, M³ST, double-exponential empirical models and research findings shed light on the role of multiphysics mechanisms, such as geomechanics, fluid
dynamics and transport, and the Klinkenberg effect, in shale gas transport across multiple spatial scales in both steady and transient states. The fracture conductivity experiments successfully validate the theory of OPPC and illustrate that proppant embedment is the primary mechanism that causes the competing process between fracture width and fracture permeability and consequently the non-monotonic fracture conductivity evolution as a function of increasing proppant concentration. The laboratory experimental facts and the numerical fittings in this study provided critical insights into the reservoir characterization in Central Appalachia and will benefit the reservoir development using non-aqueous fracturing techniques such as CO₂ and advanced proppant technologies in the future.
Production of oil and gas from the extremely tight rock has changed the global energy industry, including job growth, energy security, and environment protection. However, the oil and gas production from the tight rock is difficult because of the complex rock properties. Hydraulic fracturing can resolve the issue and contribute to the high production. The higher and safer production needs us to have a better understanding of oil and gas flow under the ground. A series of laboratory experiment were conducted, and a new shale gas transport model is introduced in this dissertation to explain the oil and gas flow under the complicated scenarios. The experimental results show that many factors can impact the oil & gas flow, and the model can match the experimental results very well. A few statistical methods are also used in the data analysis. The optimization of proppant pack is another important component of hydraulic fracturing. Proppant particles are usually man-made ceramic tiny balls, which will be injected into the underground to keep the fractures from closing during the production. From the previous papers, it is possible to achieve high fracture conductivity at a much lower cost than traditional proppant costs. Many groups of laboratory experiment were conducted to demonstrate this guess. Many rock samples in the experiment are from Central Appalachian area, which can help the resource development in this area. The developed model and experimental research findings in this study provided critical insights into the role of the many physics mechanisms on shale gas transport, proppant optimization, and hydraulic fracturing.
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# Table of Contents

Chapter 1. **Introduction** ............................................................................................................... 1

1.1. Background .............................................................................................................................. 1

1.2. Research objectives and work scope ....................................................................................... 3

1.3. Organization of this dissertation ............................................................................................. 4

Reference ......................................................................................................................................... 3

Chapter 2. **Using Pressure Pulse Decay Experiments and a Novel Multi-Physics Shale Transport Model to Study the Role of Klinkenberg Effect and Effective Stress on the Apparent Permeability of Shales** ............................................................................................................... 6

Abstract .......................................................................................................................................... 6

2.1. Introduction .............................................................................................................................. 6

2.2. Overview of the Experimental and Modeling Workflow .......................................................... 9

2.3. Theories and Experimental Equipment ..................................................................................... 11

  2.3.1. The Klinkenberg equation .................................................................................................. 11

  2.3.2. Calculation of the Biot’s coefficient using pressure pulse decay experiments .......... 14

  2.3.3. Pulse decay permeameter experiments ........................................................................... 15

2.4. Multi-physics Shale Transport Model ....................................................................................... 16

2.5. Results and Discussion ........................................................................................................... 19

2.6. Conclusion ............................................................................................................................... 26

Acknowledgement .......................................................................................................................... 27

Nomenclature ................................................................................................................................. 27

Reference ......................................................................................................................................... 28

Chapter 3. **A Novel Multi-Physics Multi-Scale Multi-Porosity Shale Transport Model for Geomechanics/Flow Coupling in Steady and Transient States** ................................................................. 33
Abstract ........................................................................................................................................... 33
3.1. Introduction and Background ........................................................................................................ 33
3.2. Overview of the Experimental and Modeling Approaches ............................................................... 36
3.3. Determination of the Biot’s Coefficient ............................................................................................ 38
3.4. Multi-physics Multi-scale Multi-porosity Shale Transport Modeling ........................................... 40
3.5. A Novel Double-Exponential Empirical Model ............................................................................... 45
3.6. Results and Discussion ................................................................................................................... 46
3.7. Conclusion ...................................................................................................................................... 55
Acknowledgment ................................................................................................................................... 56
Nomenclature ......................................................................................................................................... 56
Reference ................................................................................................................................................ 58

Chapter 4. Experimental Investigation of Non-monotonic Fracture Conductivity Evolution and Proppant Embedment ........................................................................................................ 66
Abstract ............................................................................................................................................... 66
4.1. Introduction .................................................................................................................................... 67
4.2. Theory of optimal partial-monolayer proppant concentration .......................................................... 69
4.3. Laboratory equipment, materials, and workflow ............................................................................ 72
4.4. Results and discussion .................................................................................................................... 74
4.5. Conclusion ..................................................................................................................................... 81
Acknowledgment ................................................................................................................................... 83
Reference ................................................................................................................................................ 83

Chapter 5. Reservoir Characterization in Central Appalachia: an Experimental Investigation .................................................................................................................................................... 88
Abstract ............................................................................................................................................... 88
5.1. Introduction .................................................................................................................................... 88
vii
5.2. Materials and Experimental Equipment ................................................................. 91

5.3. Results and Discussion .......................................................................................... 93

5.4. Conclusions .......................................................................................................... 99

Acknowledgment ........................................................................................................ 100

Reference .................................................................................................................... 100

Chapter 6. Conclusions and Future Work ...................................................................... 103

6.1. Conclusions .......................................................................................................... 103

6.2. Limitations and Future work ................................................................................ 105

Appendix A. Copyright releasing documents from publishers .................................... 107
List of Figures

Figure 2-1. Overview of the experiment and modeling workflow for the assessment of the correlations between apparent permeability, confining pressure, and pore pressure. .. 10

Figure 2-2. Overview of the shale cores used in the PDP tests, which were extracted from four U.S. shale formations including the Eagle Ford, Marcellus, Mancos, and Barnett formations. In the sample names, “PL” denotes that the core axis direction is parallel to the bedding plane direction, whereas “PD” indicates that the core axis direction is perpendicular to the bedding plane direction......................................................... 11

Figure 2-3. Schematic PDP equipment setup, which consists of an upstream test gas reservoir having a volume of $V_1$, a high-pressure core holder with the pore volume of $V_p$ in the sample, a downstream gas reservoir having a volume of $V_2$, a differential pressure transducer to continuously measure the pressure difference ($\Delta P$) between the upstream and downstream reservoirs, and a second pressure transducer to measure the downstream reservoir pressure, $P_2$. This picture is from Core Lab PDP-200 operations manual..... 16

Figure 2-4. Schematic picture illustrating the two transport domains in the MPST model: kerogen and inorganic matrix. The entire shale gas rock is subjected to compressive stress due to the confining pressure. The single pore width in the kerogen domain is equal to y, whereas the single pore width in the inorganic matrix is equal to x. The pore width in the inorganic matrix is in general much larger than that in the kerogen domain. The kerogen pores and inorganic matrix pores have a number ratio of N..................... 17

Figure 2-5. Apparent permeability measured in the PDP experiments as a function of pore pressure under different effective stresses in the core samples of a) Mancos PL, b) Mancos PD, c) Barnett PL, d) Barnett PD, e) Eagle Ford PL, and f) Marcellus PL. The solid curves are the MPST model fitting curves based on the scattered PDP experimental data points................................................................. 21

Figure 2-6. $k_a/k$ as a function of $1/P_p$, as well as the linear equation fitting lines, in the core samples of a) Mancos PL, b) Mancos PD, c) Barnett PL, d) Barnett PD, e) Eagle Ford PL, and f) Marcellus PL. In the figure legend, “linear” refers to the linear fitting lines.
Based on Equation 2-1, the slope of the fitting line is equal to the Klinkenberg coefficient, $b$.

**Figure 2-7.** The value of $\log(k_a)$ as a function of the confining pressure, $P_c$, under varying pore pressures in the formations of a) Mancos PL, b) Mancos PD, c) Barnett PL, d) Barnett PD, e) Eagle Ford PL, and f) Marcellus PL. In the figure legend, “linear” refers to the linear fitting lines.

**Figure 2-8.** The value of $\log(k_a)$ as a function of pore pressure, $P_p$, in the formations of a) Mancos PL, b) Mancos PD, c) Barnett PL, d) Barnett PD, e) Eagle Ford PL, and f) Marcellus PL. In the figure legend, “linear” refers to the linear fitting lines.

**Figure 3-1.** Two Eagle Ford shale core plugs (top row) and two Mancos shale core plugs (bottom row) used in the PDP permeability measurements. The shale core plugs were two inches in length and one inch in diameter. The shale bedding plane direction was parallel to the core plug axis direction.

**Figure 3-2.** Schematic PDP equipment setup, which consists of an upstream gas reservoir having a volume of $V_1$, a high-strength core holder that contains the core sample having a pore volume of $V_p$, a downstream gas reservoir having a volume of $V_2$, two large gas reservoirs $V_3$ and $V_4$ that stabilize the initial gas pressures in $V_1$ and $V_2$, a differential pressure transducer to continuously measure the differential pressure, $\Delta P$, and a pressure transducer to measure the downstream reservoir pressure, $P_d$. This picture was modified from the CoreLab PDP-200 operation manual.

**Figure 3-3.** 2D schematic picture of the $M^3$ST model.

**Figure 3-4.** PDP-measured apparent permeability (scatter data points) as a function of pore pressure and effective stress. The solid-line curves were obtained by fitting the $M^3$ST model to the PDP measurements at $P_e = 1000$ psi and 1500 psi based on NLSF. The dash-line curve was the $M^3$ST model prediction of the apparent permeability at $P_e = 500$ psi using the model-fitted parameters.

**Figure 3-5.** PDP-measured apparent permeability (scatter data points) as a function of pore pressure and effective stress. The solid-line curves were obtained by fitting the double-exponential empirical model to the PDP measurements based on NLSF.
Figure 3-6. History matching of PDP-measured upstream and downstream gas pressures using the continuum-scale M3ST model component. ................................. 54

Figure 3-7. Impact of the apparent permeability of inorganic matrix, $k_{ai}$, on upstream and downstream gas pressure evolutions ................................................................. 55

Figure 4-1. Mitigation of proppant embedment in a fracture with an increasing proppant concentration (i.e., from left to right). This picture was modified from Fan et al. (2019). ...................................................................................................................... 71

Figure 4-2. Roles of proppant particle size and effective stress on the fracture conductivity versus proppant concentration curve. Three particle sizes (0.63 mm, 0.45 mm, and 0.32 mm) and four effective stresses (1,000, 2,000, 4,000, and 6,000 psi) are considered, resulting in twelve curves. This picture is from Fan et al. (2019). ......................... 72

Figure 4-3. Schematic plot of fracture conductivity cell. The two rock slabs and the proppant particles sandwiched between them are placed in the proppant cell, which is subjected to a closure pressure up to 17,000 psi (precision 0.2%). The electrical heater provides a testing temperature up to 250°C (precision 0.4%). This picture was provided by the manufacturer. ........................................................................................................ 73

Figure 4-4. Diameter distributions of ceramic proppant having a mesh size of a) 20/40, and b) 40/70 used in fracture conductivity measurements. ........................................... 74

Figure 4-5. Measured fracture conductivity as a function of effective stress for a) Berea + mesh-20/40 proppant, b) Berea (30-day water soaking) + mesh-20/40 proppant, c) Berea + mesh-40/70 proppant, and d) Marcellus + mesh-20/40 proppant. ......................... 76

Figure 4-6. Measured fracture conductivity as a function of proppant concentration for a) Berea + mesh-20/40 proppant, b) Berea (30-day water soaking) + mesh-20/40 proppant, c) Berea + mesh-40/70 proppant, and d) Marcellus + mesh-20/40 proppant. ................. 79

Figure 4-7. Measured OFC as a function of effective stress .................................................. 79

Figure 4-8. Rock slab surfaces after the 6,000 psi testing with a ceramic proppant concentration of 0.06 lb/ft$^2$. ...................................................................................................................... 81
Figure 4-9. Rock slab surfaces after the 6,000 psi testing with a ceramic proppant concentration of 2 lb/ft². ..................................................................................................................... 81

Figure 5-1. Pictures of laboratory equipment in the study.................................................. 92

Figure 5-2. Shale samples from Eagle Ford and Mancos before experiment. ............... 92

Figure 5-3. Measured fracture conductivity as a function of closure pressure for the shale slabs.................................................................................................................................................. 94

Figure 5-4. Three shale samples scanned by Zygo Corporation........................................ 95

Figure 5-5. The smaller (3×3) stitch of three samples....................................................... 97

Figure 5-6. PDP-measured permeability as a function of effective stress for 15 samples from the ESUP field in the laboratory of Virginia Tech. ................................................................. 97

Figure 5-7. PDP-measured permeability as a function of effective stress for 45 samples by Schlumberger Limited from the ESUP field................................................................. 98

Figure 5-8. PDP-measured apparent permeability (scatter data points) as a function of effective stress. The solid-line curves were obtained by fitting the exponential empirical model to the PDP measurements based on non-linear least square fitting. .............. 99
List of Tables

Table 3-1. Measured Biot’s coefficients for the four shale cores. ............................................. 47

Table 3-2. Parameter values determined by fitting the M³ST model to the laboratory PDP measurements at effective stresses of 1000 and 1500 psi using NLSF based on the Gauss-Newton method. ................................................................................................................................. 49

Table 3-3. NMSE values for M³ST model predictions of shale’s apparent permeability under effective stress of 500 psi. ................................................................................................................................. 50

Table 3-4. NMSE values for global fitting using the M³ST model and the double-exponential empirical model. ................................................................................................................................. 51

Table 3-5. Fitted values of γ1 and γ2 in the double-exponential empirical model. ............... 52

Table 3-6. Experimental conditions in PDP measurements. Pu and Pd are gas pressures in the upstream and downstream reservoirs, respectively. ΔP is the difference between Pu and Pd. ........................................................................................................................................... 54

Table 4-1. Evolutions of fracture width, w, porosity, ϕ, permeability, k, and conductivity (k · w) with an increasing proppant concentration. The arrows “↑”, “↓”, and “→” denote “increase”, “decrease”, and “stay constant”, respectively. This table is modified from Fan et al. (2019). ........................................................................................................................................... 71
PREFACE

This dissertation is submitted as a completion of the degree of Doctor of Philosophy at Virginia Polytechnic Institute and State University. This research described here was conducted by the author, Zihao Li, under the supervision of Dr. Cheng Chen in the Department of Mining & Minerals Engineering at Virginia Polytechnic Institute and State University.

This dissertation is established based on six manuscripts. Altogether, this dissertation includes two peer-reviewed journal articles, one conference paper, and two journal articles to be submitted to journals for review.

Chapter 2 is based on the article published in the *Journal of Petroleum Science and Engineering*, “Li, Z., Ripepi, N., & Chen, C. 2020. Using Pressure Pulse Decay Experiments and a Novel Multi-Physics Shale Transport Model to Study the Role of Klinkenberg Effect and Effective Stress on the Apparent Permeability of Shales” and the conference paper published in the 2019 ARMA conference, “Li, Z., Ripepi, N., & Chen, C. 2019, August. Comprehensive Laboratory Investigation and Model Fitting of Klinkenberg Effect and Its Role on Apparent Permeability in Various US Shale Formations”. In this chapter, the experimental testing and numerical simulation work are conducted by Zihao under the guidance of other co-authors.

Chapter 3 is published in the *SPE Journal*, “Li, Z., Teng, Y., Fan, M., Ripepi, N., & Chen, C. 2021. A Novel Multi-Physics Multi-Scale Multi-Porosity Shale Transport Model for Geomechanics/Flow Coupling in Steady and Transient States”. In this work, the experimental testing and numerical simulation work are conducted by Zihao under the guidance of other co-authors.

Chapter 4 is submitted to the *SPE Journal* for peer review. The whole work is conducted by Zihao Li under the guidance of other co-authors.

Chapter 5 will also be submitted to a journal article for review. Schlumberger Limited conducted part of the permeability measurements, while Zygo Corporation assisted with scanning the surface roughness of three pieces of rock slabs. The other work is conducted by Zihao Li under the guidance of other co-authors.
The copyrights for releasing documents from publishers are listed in the appendix section.
Chapter 1. Introduction

1.1. Background

Production of hydrocarbon energy resources from shale, a fine-grained, low-permeability geological formation, has changed the global energy outlook. Recovery of shale oil and gas has become economically viable due to the development of hydraulic fracturing and horizontal drilling (Montgomery and Smith, 2010; Gu and Mohanty, 2014; Stringfellow et al., 2014; Wang et al., 2018). Compared with conventional enhanced oil and gas recovery (Li et al., 2016; Wang et al., 2017; Zhao et al., 2017; Tang et al., 2019), unconventional resource development is generally associated with a wide range of benefits such as job growth and improved infrastructure and municipal developments. The complicated petrophysical properties and extremely low permeability of shale formations lead to tremendous challenges to develop the hydrocarbon resources trapped in these formations. Horizontal drilling associated with multi-stage hydraulic fracturing has become an effective technique to produce hydrocarbon resources from shales at an economically viable rate (Economides and Nolte, 2000; Chen et al., 2015; Hu et al., 2018). During hydrocarbon recovery, the decreased pore pressure leads to increased effective stress and fracture closure (Fan et al., 2017, 2018). The effective stress variation has a significant impact on the petrophysical properties in the shale formation, especially the formation permeability. It is thus crucial to investigate the relationship between formation permeability and effective stress variation.

The Klinkenberg effect was a theory developed by Klinkenberg (1941) to explain the abnormal phenomenon of apparent permeability enhancement for gas flow in nanopores. It has a significant impact on gas flow in low-permeability porous media such as shale reservoirs and tight coalbeds (Wu et al., 1998). The Klinkenberg effect is related to the Knudson number (Kn), which is defined as $\text{Kn} = \frac{\lambda}{d}$, where $\lambda$ is the gas mean-free-path length (m) and $d$ is pore diameter (m). When $\text{Kn} < 0.001$, gas flow is governed by the Navier-Stokes equations with a no-slip boundary condition on the pore walls. When $0.001 < \text{Kn} < 0.1$, gas flow is in the slip flow regime in which the non-continuum effect can be approximated by a velocity slip on the pore walls; the bulk gas flow can still be governed by the Navier-Stokes equation (Beskok and Karniadakis, 1999). The continuous increase in Kn leads to the transitional flow regime ($0.1 < \text{Kn} < 10$) and then the free molecular
flow regime (Kn > 10), where the gas flow cannot be described by the Navier-Stokes equations (Tian et al., 2019).

In a shale formation, when the nanopore diameter decreases or gas pressure decreases, the Kn number increases because a lower gas pressure leads to a larger gas molecule mean-free-path length. In this case, the nanopore diameter is comparable to or smaller than the mean-free-path length of gas molecules. As a consequence, gas molecules collide with the pore walls more frequently than with one another, leading to Knudsen diffusion that causes a slip-velocity boundary condition in the macroscopic Navier-Stokes flow model (Javadpour, 2009). The existence of the slip velocity on flow boundaries enhances the overall mass flux through the nanopore, which results in increased apparent permeability of the shale (Chen, 2016).

The optimization of proppant assembly conductivity is another critical component of subsurface hydrocarbon production design using hydraulic fracturing. However, the subsurface hydrocarbon's effective development with proppant is restricted by the aggregated amount of proppant usage. Fracture conductivity, defined as the product of fracture permeability and width (Chen et al., 2015). It denotes the absolute fluid flow rate between fractures in two adjacent grid blocks of given sizes. A large enough fracture conductivity is essential for the extraction of hydrocarbons from reservoirs economically. The fracture width and permeability are closely related to the amount of proppant placed in the fracture and the effective stress exerted on the proppant pack (Chen et al., 2015). A multi-particle proppant mixture is injected into the wellbore during the hydraulic fracturing process. Typically, a smaller size proppant is injected first, followed by a larger size proppant. There is a critical combination of small and large proppant sizes in ultralow permeability formations such as shales that will result in the highest well productivity index (Hoss et al., 2017). The amount of proppant placed in a fracture is measured by proppant concentration (also known as proppant area concentration), which is defined as the proppant mass per unit of fracture surface area, usually in pounds per square foot (lb/ft²) (Economides and Nolte, 2000). According to the optimal partial-monolayer proppant concentration (OPPC) theory, significant fracture conductivity can be achieved at a much lower cost than conventional material costs.

Central Appalachian area in the United States has enormous subsurface resources but with less developed economic. The effective and environmental-friendly hydraulic fracturing operation can
significantly contribute to the job growth and economic development of the local communities in the Central Appalachian area. Considering the petrophysical properties and fracture conductivity are directly relevant to the productivity of the hydraulic fracturing, the numerical and experimental works are applied into the rock samples from Central Appalachian area to investigate their geomechanics. The fracture experimental facts and the research findings about the petrophysical properties of unconventional reservoirs and the relevant proppant pack can provide critical insights into the petrophysical properties and the role of the proppant concentration in fracture conductivity and hydraulic fracturing.

1.2. Research objectives and work scope

The primary goal of this work is to investigate the geomechanics/flow coupling in energy georeservoirs that will influence the petrophysical properties and hydraulic conductivity of the subsurface reservoir in hydraulic fracturing. The multi-physics shale transport (MPST) model will first be developed with the experimental data from pulse decay permeameter. Based on the MPST model and more experimental work, a novel multi-physics multi-scale multi-porosity shale gas transport (M³ST) model was developed to investigate shale gas transport in both transient and steady states. Next, the factors that impact the fracture conductivity will be investigated because the fracture conductivity is directly relevant to the productivity of hydraulic fracturing. A series of proppant fracture conductivity experiment will be conducted considering the effect of proppant concentration, closure pressure, particle diameter, liquid soaking, and rock mineralogy to study the relationship between fracture conductivity and these factors. In the end, the numerical and experimental works will be applied to the rock samples from the Central Appalachian area to investigate their geomechanics. The research tasks in both experimental and numerical ways are listed as following:

1) Develop the MPST model with the experimental data from pulse decay permeameter to capture the critical multi-physics processes that regulate the apparent permeability in the unconventional reservoir.

2) Develop the M³ST model based on the MPST model and more experimental work to investigate the role of the multiphysics mechanisms, including geomechanics, fluid
dynamics and transport, and Klinkenberg effect, on shale gas transport across different spatial scales in both steady and transient states.

3) Investigate the role of proppant concentration, closure pressure, particle diameter, liquid soaking, and rock mineralogy on the non-monotonic evolution of fracture conductivity in the fractured subsurface reservoir.

4) The established workflow and empirical model will be applied to the rock samples from the Central Appalachia to investigate the geomechanics of these samples.

1.3. **Organization of this dissertation**

The dissertation consists of six chapters. Chapter 1 introduces the background and importance of the experimental, theoretical, and numerical investigations of geomechanics/flow coupling in energy georeservoirs. The research objectives and study scope are also included in this chapter.

In Chapter 2, core samples extracted from four U.S. shale formations were tested using a pulse decay permeameter (PDP) under varying combinations of confining and pore pressures. The Klinkenberg coefficient was calculated to interpret the change in the measured apparent permeability as a function of pore pressure and effective stress. Next, based on the various combinations of confining and pore pressures, the actual values of the Biot coefficient were calculated by data fitting. Moreover, the samples were cored in the directions parallel to and perpendicular to the shale bedding planes to unravel the role of the bedding plane direction on the apparent permeability. Furthermore, a novel MPST model was developed to account for the coupled multi-physics processes of geomechanics, fluid dynamics, and the Klinkenberg effect for gas transport in shales. In the MPST model, pore pressure and effective stress are the two independent input variables, and the measured apparent permeability is the model output. The MPST model was then used to fit the PDP experimental data, and the successful data fitting confirmed that the MPST model captures the critical multi-physics processes that regulate the apparent permeability.

In Chapter 3, a novel M³ST model is developed to investigate shale gas transport in both transient and steady states. Permeabilities of various shale cores were measured in the laboratory using a PDP with different pore pressure and confining stress combinations. The PDP-measured
The apparent permeability as a function of pore pressure under two effective stresses was fitted using the microscale M$^3$ST model component based on non-linear least squares fitting, and the fitted model parameters were able to provide accurate model predictions for another effective stress. The parameters and petrophysical properties determined in the steady state were then used in the transient-state, continuum-scale M$^3$ST model component. Additionally, a double-exponential empirical model was developed as a powerful alternative to the M$^3$ST model to fit laboratory-measured apparent permeability. The developed M$^3$ST model and the research findings in this study provided critical insights into the role of the multiphysics mechanisms, including geomechanics, fluid dynamics and transport, and Klinkenberg effect, on shale gas transport across different spatial scales in both steady and transient states.

In Chapter 4, we conducted a series of proppant fracture conductivity experiments to validate the optimal partial-monolayer proppant concentration (OPPC) theory and measure the corresponding optimal fracture conductivity (OFC). The experimental results revealed that the fracture conductivity was inversely proportional to the increasing closure pressure. During the experiment, the four visible stages recorded the non-monotonic conductivity evolution along with proppant concentration. Good agreement was observed between the experiment-measured and previous model-derived curves of the fracture conductivity versus proppant concentration. The experimental results also looked into the effects of closure pressure, proppant particle diameter, and rock mineralogy on the OPPC and OFC. Because of the average effective loading and the corresponding embedment, closure pressure and proppant particle diameter has an impact on the OFC, but merely on the OPPC. Specifically, the variation in embedment depth has an effect on fracture conductivity as a combined result of rock mineralogy, closure pressure, and liquid soaking. The fracture experimental facts and the research findings in this study provided critical insights into the role of the proppant concentration in fracture conductivity and hydraulic fracturing.

In Chapter 5, a series of experiment were conducted to investigate the geomechanics of the rock samples from Central Appalachian area. The role of rock surface roughness was investigated on maintaining adequate fracture conductivity by “self-propping” when there is no proppant present in the fracture. Besides, we also study the difference in the measured fracture conductivity values between DI water and nitrogen gas. The workflow developed from these experiments will be applied to analyze the rock samples extracted from the deep ESUP well which is currently being
drilled. The laboratory experimental facts and the research findings in this study provided critical insights into the reservoir characterization in Central Appalachia and will benefit the reservoir development using non-aqueous fracturing techniques such as CO₂ and advanced proppant technologies in the future.

Reference


Chapter 2. Using Pressure Pulse Decay Experiments and a Novel Multi-Physics Shale Transport Model to Study the Role of Klinkenberg Effect and Effective Stress on the Apparent Permeability of Shales

Zihao Li, Nino Ripepi, Cheng Chen, Virginia Tech

Abstract

The confining pressure imposed on a shale formation has a significant impact on the apparent permeability of the rock. Gas flow in low-permeability shales differs significantly from liquid flow because of the Klinkenberg effect, which results from gas molecule slip at the wall surfaces inside the nanopores. This effect causes the increase of apparent permeability (i.e., the measured permeability). In this study, cores extracted from four U.S. shale formations were tested using a pulse decay permeameter (PDP) under varying combinations of confining and pore pressures. The Klinkenberg coefficient was calculated to interpret the change in the measured apparent permeability as a function of pore pressure and effective stress. Next, based on the various combinations of confining and pore pressures, the actual values of the Biot coefficient were calculated by data fitting. Moreover, the samples were cored in the directions parallel to and perpendicular to the shale bedding planes to unravel the role of bedding plane direction on the apparent permeability. Furthermore, a novel, multi-physics shale transport (MPST) model was developed to account for the coupled multi-physics processes of geomechanics, fluid dynamics, and Klinkenberg effect for gas transport in shales. In the MPST model, pore pressure and effective stress are the two independent input variables, and the measured apparent permeability is the model output. The MPST model was then used to fit the PDP experimental data, and the successful data fitting confirmed that the MPST model captures the critical multi-physics processes that regulate the apparent permeability.

2.1. Introduction

The complicated petrophysical properties and extremely low permeability of shale formations lead to tremendous challenges to develop the hydrocarbon resources trapped in these formations. Horizontal drilling associated with multi-stage hydraulic fracturing has become an effective technique to produce hydrocarbon resources from shales at an economically viable rate
(Economides and Nolte, 2000; Gu and Mohanty, 2014; Chen et al., 2015; Hu et al., 2018). During hydrocarbon recovery, the decreased pore pressure leads to increased effective stress and fracture closure (Fan et al., 2017b, 2018). The effective stress variation has a significant impact on the petrophysical properties in shale formation, especially the formation permeability. It is thus crucial to investigate the relationship between formation permeability and effective stress variation.

The investigation of the correlation between effective stress and rock permeability started from laboratory analysis of tight sandstones. Warpinski and Teufel (1992) performed laboratory experiments to study the Biot’s coefficient in tight sandstone and chalk. They found that the Biot’s coefficient in tight sandstone was close to unity for small effective stresses but contained uncertainties for larger effective stresses. The value of Biot’s coefficient varied with both effective stress and pore pressure. Ojala and Fjær (2007) tested Castlegate sandstone cores under cyclic pore and confining pressure variations. Their results demonstrated that the Biot’s coefficient in sandstone under acoustic and elastic testing can be considerably different from unity. They also illustrated the hysteresis in petrophysical properties due to microfractures or frictional effects. The research on shale rock permeability and effective stress has also been conducted extensively. Chen et al., (2015) derived a theoretical model between shale permeability and effective stress. They also investigated the correlation between fracture compressibility, shale properties, and reservoir pressure. Cui et al., (2017) discussed Chen’s model and utilized strain evolution to investigate how shale permeability changes with time and gas pressure in the matrix system, which confirms that it is important to include the interactions between shale microstructures and gas transport processes. Although the Biot’s coefficient is a widely used poroelastic parameter in calculating the effective stress, its value is usually assumed to be one for the sake of simplicity in many applications when no other information is available. (e.g., Alam et al., 2014; Bhandari et al., 2015; Jin et al., 2015; Rydzy et al., 2016).

The Klinkenberg effect was a theory developed by Klinkenberg (1941) to explain the abnormal phenomenon of apparent permeability enhancement for gas flow in nanopores. It has a significant impact on gas flow in low-permeability porous media such as shale reservoirs and tight coalbeds (Wu et al., 1998). The Klinkenberg effect is related to the Knudson number (Kn), which is defined as \( Kn = \frac{\lambda}{d} \), where \( \lambda \) is the gas mean-free-path length (m) and \( d \) is pore diameter (m). When \( Kn \leq 0.001 \), gas flow is governed by the Navier-Stokes equations with a no-slip boundary condition on
the pore walls. When $0.001 < \text{Kn} < 0.1$, gas flow is in the slip flow regime in which the non-continuum effect can be approximated by a velocity slip on the pore walls; the bulk gas flow can still be governed by the Navier-Stokes equation (Beskok and Karniadakis, 1999). The continuous increase in Kn leads to the transitional flow regime ($0.1 < \text{Kn} < 10$) and then the free molecular flow regime ($\text{Kn} > 10$), where the gas flow cannot be described by the Navier-Stokes equations (Tian et al., 2019).

In a shale formation, when the nanopore diameter decreases or gas pressure decreases, the Kn number increases because a lower gas pressure leads to a larger gas molecule mean-free-path length. In this case, the nanopore diameter is comparable to or smaller than the mean-free-path length of gas molecules. As a consequence, gas molecules collide with the pore walls more frequently than with one another, leading to Knudsen diffusion that causes a slip-velocity boundary condition in the macroscopic Navier-Stokes flow model (Javadpour, 2009). The existence of the slip velocity on flow boundaries enhances the overall mass flux through the nanopore, which results in increased apparent permeability of the shale (Chen, 2016).

A series of analytical solutions was developed to characterize gas flow in tight porous media subjected to the Klinkenberg effect (Wu et al., 1998; Innocentini and Pandolfelli., 2001; Zhu et al., 2007; Hu et al., 2009; Hayek, 2015). Li et al. (2016) derived an analytical formula for gas effective permeability subjected to the Klinkenberg effect based on the microscale flow model and fractal capillary model. Their results demonstrated the physical meaning of the model parameters. In a complex environment, such as polymer-water-oil flow in porous media, the Klinkenberg effect was found to be mitigated with the presence of an adsorption polymer layer (Blanchard et al., 2007). Moreover, the Klinkenberg coefficient cannot be treated as a constant in coalbeds due to the high compressibility and matrix swelling of coal (Wang et al., 2014). All the studies reviewed above confirm that the Klinkenberg effect exists in a wide range of subsurface flow and transport processes associated with numerous geoenergy and geoenvironmental applications.

In this paper, core samples were collected from four U.S. shale formations, including the Mancos, Eagle Ford, Barnett, and Marcellus shales, to investigate the correlations between apparent permeability, pore pressure, and confining pressure. By testing cores having varying bedding plane directions, the role of bedding plane direction on the apparent permeability was unraveled. In this work, a core sample’s apparent permeability was tested using a pressure pulse
decay permeameter (PDP) under various combinations of pore pressure and confining pressure. The Klinkenberg coefficient was calculated to explain the change of the measured apparent permeability as a function of pore pressure and effective stress. A multi-physics shale transport (MPST) model was developed, which accounts for the coupled multi-physics processes of fluid dynamics, geomechanics, and the Klinkenberg effect for gas transport in shales. The fitting curves were then compared with experimental data. The comprehensive experimental and model fitting aim to advance the fundamental understanding of the role of confining pressure, pore pressure, Klinkenberg effect, and bedding plane direction on the apparent permeability. The MPST model development aims to answer the question if it is possible to predict and fit laboratory data for the measured apparent permeability of a shale using pore pressure and effective stress change as model inputs. The research outcome has the potential to advance the scientific understanding of the relationship between geomechanical stresses and gas flow properties in a shale formation.

2.2. Overview of the Experimental and Modeling Workflow

This section provides an overview of the entire experimental and modeling workflow. Details of experimental setup will be given in a later section. In this work, an experiment and modeling approach was developed to study the correlation between apparent permeability, confining pressure, and pore pressure. First, a series of shale core samples, having different bedding plane directions, was extracted from four U.S. shale formations. Second, a laboratory PDP was used to measure the apparent permeabilities under a wide range of different combinations of pore pressure and confining pressure. Third, we derived the Klinkenberg coefficient to explain the change of the measured apparent permeability as a function of pore pressure and effective stress; in this process the Biot’s coefficient is assumed to be one for the sake of simplicity. Fourth, the actual values of the Biot’s coefficient were calculated by data fitting based on the laboratory measurements obtained under various combinations of pore and confining pressures. Finally, a MPST model was developed to account for the coupled multi-physics processes of fluid dynamics, geomechanics, and the Klinkenberg effect for gas transport in shales, and to predict the dependence of a shale’s apparent permeability on pore pressure and effective stress. The laboratory PDP data were then fitted to the MPST model. Figure 2-1 illustrates an overview of the experiment and modeling workflow.
Figure 2.1. Overview of the experiment and modeling workflow for the assessment of the correlations between apparent permeability, confining pressure, and pore pressure.

In this study, based on the hardware limit of the PDP equipment, eight pore pressure values (100 psi, 300 psi, 500 psi, 700 psi, 900 psi, 1100 psi, 1300 psi, and 1500 psi) were used in the PDP testing. For each pore pressure value, three confining pressure values were applied, which were 500 psi, 1,000 psi, and 1,500 psi higher than the pore pressure, separately. Therefore, for a single shale core, 24 apparent permeability measurements in total were conducted using the PDP based on the different combinations between pore pressure and confining pressure. The experimental results of the cores that were from the same formation but had different bedding plane directions were compared to improve the understanding of the role of bedding plane direction on the measured apparent permeability. Figure 2-2 demonstrates an overview of the shale core samples tested in this work using the PDP equipment, which were collected from four different U.S. shale formations including the Eagle Ford, Marcellus, Mancos, and Barnett formations, with the TOCs measured as 2.02%, 4.45%, 1.37%, and 2.14%, respectively. The size of these core samples is two inches in length and one inch in diameter. Specifically, in the sample labels, “PL” denotes that the core axis direction is parallel to the bedding plane direction, whereas “PD” indicates that the core axis direction is perpendicular to the bedding plane direction.
Figure 2-2. Overview of the shale cores used in the PDP tests, which were extracted from four U.S. shale formations including the Eagle Ford, Marcellus, Mancos, and Barnett formations. In the sample names, “PL” denotes that the core axis direction is parallel to the bedding plane direction, whereas “PD” indicates that the core axis direction is perpendicular to the bedding plane direction.

2.3. **Theories and Experimental Equipment**

2.3.1. *The Klinkenberg equation*

It is the Klinkenberg effect that leads to the difference between apparent permeability and absolute (intrinsic) permeability, especially under low pore pressures. In order to quantify the Klinkenberg effect, the Klinkenberg coefficient (Klinkenberg, 1941) was used to describe the role of pore pressure on the apparent permeability. The Klinkenberg coefficient depends on the petrophysical properties of the rock, and is included in the Klinkenberg equation written as follows:

\[
\frac{k_a}{k} = 1 + \frac{b}{P_p}
\]  

(2-1)

where \(k_a\) is the apparent permeability (m\(^2\)), \(P_p\) is the pore pressure (Pa), \(k\) is the absolute permeability of the porous medium (m\(^2\)), and \(b\) is the Klinkenberg coefficient (Pa), which can be
calculated based on the analysis of the mass fluxes contributed by the viscous flow and Knudsen diffusion. In this study, we focus on the nanopores that have planar geometry because the MPST model is in the two-dimensional (2D) space, as will be described in the later section. In addition, the width of a planar nanopore is sensitive to the stress applied in the direction perpendicular to the planar pore walls, which is the foundation of the MPST model because one of the primary applications of the MPST model is to describe the response of the apparent permeability to the change of effective stress.

Mass flux through a planar nanopore can be written as:

\[ J = J_D + J_K \]  \hspace{1cm} (2-2)

where \( J_D \) is the mass flux resulting from Darcy flow \((\text{kg/m}^2/\text{s})\), which is the viscous flow driven by a pressure gradient through the nanopore; \( J_K \) is the mass flux resulting from Knudsen diffusion \((\text{kg/m}^2/\text{s})\), which occurs when the nanopore width is comparable to or smaller than the mean-free-path length of gas molecules.

The value of \( J_D \) can be calculated using the Darcy’s law:

\[ J_D = -\frac{k}{\mu} \rho \nabla p \]  \hspace{1cm} (2-3)

where \( \mu \) is dynamic viscosity \((\text{Pa} \cdot \text{s})\); \( \rho \) is free gas mass density \((\text{kg/m}^3)\); \( \nabla p \) is the pressure gradient through the nanopore \((\text{Pa/m})\). In a planar nanopore, one can analytically solve the Navier-Stokes equations to calculate the absolute permeability as \( k = h^2/12 \), where \( h \) is the width of the planar nanopore \((\text{m})\). However, in nanometer-scale pores, the no-slip boundary condition is invalid and thus a dimensionless coefficient, \( F \), must be used to account for the enhancement of mass flux resulting from the slip boundary condition (Brown et al. 1946). Therefore, mass flux through a planar nanopore resulting from Darcy flow is calculated as:

\[ J_D = -F \frac{h^2}{12 \mu} \rho \nabla p \]  \hspace{1cm} (2-4)

where \( F \) is the dimensionless coefficient and calculated as:

\[ F = 1 + 2 \frac{\mu}{\rho h} \left( \frac{2}{\alpha} - 1 \right) \sqrt{\frac{8\pi RT}{M}} \]  \hspace{1cm} (2-5)
where $p$ is pressure (Pa); $\alpha$ is the tangential momentum accommodation coefficient having a value in the range from 0 to 1; $R$ is the gas constant and equal to 8.314 J/mole/K; $T$ is absolute temperature (K); $M$ is molar mass (kg/mole).

Mass flux contributed by Knudsen diffusion, $J_K$, is calculated as:

$$J_K = -\frac{MD_K}{RT} \nabla p$$  (2-6)

where $D_K$ is the Knudsen diffusivity (m$^2$/s) calculated as:

$$D_K = \frac{h}{3} \sqrt{\frac{8RT}{\pi M}}$$  (2-7)

Substituting Equations 2-4 and 2-6 into Equation 2-2 and using Equations 2-5 and 2-7, one can re-write Equation 2-2 as follows:

$$J = -\left[ \left( 1 + \frac{2\mu}{ph} \frac{2}{\alpha} - 1 \right) \frac{12}{12} \mu \frac{h}{12} + hM \frac{8RT}{3RT} \frac{12}{12} \mu \frac{h}{12} \right] \nabla p$$  (2-8)

Equation 2-8 can be re-written in the form of the Darcy’s law:

$$J = -\frac{k_a \rho}{\mu} \nabla p$$  (2-9)

where $k_a$ is the apparent permeability that accounts for the mass flux contributions from both slip boundary and Knudsen diffusion. Comparing Equations 2-8 with 2-9, one obtains:

$$k_a = \left( 1 + \frac{2\mu}{ph} \frac{2}{\alpha} - 1 \right) \frac{8RT}{M} \frac{12}{12} \mu + hM \frac{8RT}{3RT} \frac{12}{12} \mu$$  (2-10)

By normalizing $k_a$ with $k = h^2/12$ and using $\rho = \frac{PM}{RT}$, one obtains:

$$\frac{k_a}{k} = 1 + \frac{2\mu}{ph} \frac{2}{\alpha} - 1 \sqrt{\frac{8RT}{M}} \frac{4\mu}{ph} \sqrt{\frac{8RT}{\pi M}}$$  (2-11)

By comparing Equation 2-11 with the Klinkenberg equation (Equation 2-1), one obtains the formula for calculating the Klinkenberg coefficient in a planar nanopore:
\[
b = \frac{2\mu}{h} \left( \frac{2}{\alpha} - 1 \right) \sqrt{\frac{8\pi RT}{M}} + \frac{4\mu}{h} \sqrt{\frac{8RT}{\pi M}} \tag{2-12}
\]

### 2.3.2. Calculation of the Biot’s coefficient using pressure pulse decay experiments

The Biot’s coefficient is a poroelastic parameter that describes the influence of the pore pressure on the effective stress (Bernabe, 1986):

\[
P_e = P_c - \chi P_p \tag{2-13}
\]

where \(P_e\) is effective stress (Pa), \(P_c\) is confining pressure (Pa), \(P_p\) is pore pressure (Pa), and \(\chi\) is the Biot’s coefficient, of which the value depends on the rock’s mineral composition. Equation 2-13 suggests that the confining pressure and pore pressure have different influences on the effective stress if the value of \(\chi\) is not equal to one.

Because \(P_e\) is a function of \(P_p\) and \(P_c\), the apparent (measured) permeability, \(k_a\), can be written as a function of these two variables:

\[
k_a = k_a(P_c, P_p) \tag{2-14}
\]

In this study, \(\log(k)\) (Chen and Zeng, 2015) is used as an indicator of the formation permeability:

\[
\log (k_a) = \log (k_a)(P_c, P_p) \tag{2-15}
\]

Based on Equation 2-15, one obtains the differential of \(\log(k)\):

\[
d\log(k_a) = \left(\frac{\partial \log(k_a)}{\partial P_c}\right) dP_c + \left(\frac{\partial \log(k_a)}{\partial P_p}\right) dP_p \tag{2-16}
\]

One also has the following equation based on Equation 2-13:

\[
dP_e = dP_c - \chi dP_p \tag{2-17}
\]

When the effective stress stays constant, both \(d\log(k_a)\) and \(dP_e\) are zero. Therefore, one obtains the following two equations based on Equations 2-16 and 2-17:

\[
\left(\frac{\partial \log(k_a)}{\partial P_c}\right) dP_c + \left(\frac{\partial \log(k_a)}{\partial P_p}\right) dP_p = 0 \tag{2-18}
\]
and

\[ dP_c - \chi dP_p = 0 \]  

(2-19)

Using Equations 2-18 and 2-19, one obtains:

\[ \chi = -\left( \frac{\partial \log (k_a)}{\partial P_p} \right) / \left( \frac{\partial \log (k_a)}{\partial P_c} \right) \]  

(2-20)

2.3.3. Pulse decay permeameter experiments

The PDP equipment is a transient, time-effective approach to measuring the apparent permeability of tight rocks. Jones (1997) developed the basic measurement principle of PDP and found that the measurement range of PDP is from 0.1 mD to 0.01 microdarcy (μD). As opposed to traditional permeability measurement methods, which use flow parameters in the steady state and are based on Darcy’s law, the PDP method uses transient flow parameters, which significantly accelerates the measurement process and consequently is ideal for low-permeability rocks. Our PDP experiments were conducted under a fixed indoor temperature of 20 °C. Figure 2-3 is a schematic plot of the PDP setup. The testing gas used in the PDP is pure nitrogen. The pore pressure is controlled using a pressure regulator, whereas the confining pressure is controlled using a hydraulic pump. The flow rate measurement is not required but can be calculated from the known volumes of the reservoirs, fluid compressibility, and the rate of change of gas pressures (Hseih et al., 1981; Bourbie and Walls, 1982). In the PDP testing, the pore pressure, \( P_p \), throughout the core sample is uniform in the initial stage. When \( t = 0 \), a gas pressure slightly higher than the initial pore pressure is applied at the upstream end of the core sample. When the pressure pulse propagates through the core sample, the gas pressure in the downstream reservoir increases whereas the gas pressure in the upstream reservoir declines. After a certain period of time, the pressure difference between the upstream and downstream reservoirs approaches zero. The decay rate of this pressure difference is proportional to the permeability of the core sample. Dicker and Smits (1988) developed the general analytical solution of the differential pressure as a function of time. Chen and Stagg (1984) and Haskett et al. (1988) also made contributions to the analytical solutions.
Figure 2-3. Schematic PDP equipment setup, which consists of an upstream test gas reservoir having a volume of $V_1$, a high-pressure core holder with the pore volume of $V_p$ in the sample, a downstream gas reservoir having a volume of $V_2$, a differential pressure transducer to continuously measure the pressure difference ($\Delta P$) between the upstream and downstream reservoirs, and a second pressure transducer to measure the downstream reservoir pressure, $P_2$. This picture is from Core Lab PDP-200 operations manual.

2.4. Multi-physics Shale Transport Model

Recent studies by the authors (Chen et al., 2013; Chen, 2016) showed that the pore and fracture size distribution and associated gas transport in shale formations have a hierarchical structure and thus demonstrate multi-scale properties. Based on previous investigations, a MPST model was developed in this paper to account for the coupled multi-physics processes of fluid dynamics, geomechanics, and the Klinkenberg effect in shale gas transport. Figure 2-4 illustrates the hierarchical geometry structure in the MPST model. Specifically, pore pressure, $P_p$, and effective stress, $P_e$, are the two independent input variables in this MPST model, and the rock’s overall apparent permeability, $k_a$, is the model output. Figure 2-4 demonstrates that a 2D shale gas formation consists of two transport domains: kerogen and inorganic matrix (Chen, 2016). The two domains have their own intrinsic permeabilities, mechanical compressibilities, and pore widths. The entire shale gas rock is subjected to compressive stress due to the confining pressure. The single pore width (m) in the kerogen domain is equal to $y$, whereas the single pore width (m) in the inorganic matrix is equal to $x$. The pore width in the inorganic matrix is in general much larger...
than that in the kerogen domain. The kerogen pores and inorganic matrix pores have a number ratio of $N$, which implies that the number of kerogen pores is $N$ times the number of inorganic matrix pores.

![Diagram illustrating gas flow direction and compressive stress](image)

**Figure 2-4.** Schematic picture illustrating the two transport domains in the MPST model: kerogen and inorganic matrix. The entire shale gas rock is subjected to compressive stress due to the confining pressure. The single pore width in the kerogen domain is equal to $y$, whereas the single pore width in the inorganic matrix is equal to $x$. The pore width in the inorganic matrix is in general much larger than that in the kerogen domain. The kerogen pores and inorganic matrix pores have a number ratio of $N$.

Based on Equation 2-1, the apparent permeabilities of a single inorganic matrix pore and a single kerogen pore can be calculated as:

$$k_i = \frac{x^2}{12} \left(1 + \frac{b(x)}{P}\right)$$

$$k_k = \frac{y^2}{12} \left(1 + \frac{b(y)}{P}\right)$$

(2-21)

where $b(x)$ and $b(y)$ are the Klinkenberg coefficients in the inorganic matrix and kerogen, respectively. The output of the MPST model is the measured overall permeability of the entire shale gas rock, which can be calculated as follows using the Darcy’s law:
\[
\bar{k} = \frac{\bar{u} \cdot \mu}{\nabla P} = \frac{(Q_i + Q_k)\mu}{A \cdot \nabla P}
\] (2-22)

where \(A\) is the total cross section area (m\(^2\)) through which the gas flows and is calculated as \(A = \frac{x + ny}{\phi}\), where \(\phi\) is the total porosity of the rock. \(\bar{u}\) is the average flow velocity across the entire cross section area and is calculated as \(\bar{u} = \frac{Q_i + Q_k}{A}\), where \(Q_i\) and \(Q_k\) are the gas flow rates contributed by the inorganic matrix domain and the kerogen domain, respectively. Specifically, \(Q_i\) and \(Q_k\) can be calculated as follows:

\[
Q_i = u_i \cdot x = \frac{k_i}{\mu} \cdot \nabla P \cdot x = \frac{x^3}{12} \cdot \frac{\nabla P}{\mu} (1 + \frac{b(x)}{P})
\]

\[
Q_k = Nu_k \cdot y = \frac{Nk_k}{\mu} \cdot \nabla P \cdot y = \frac{Ny^3}{12} \cdot \frac{\nabla P}{\mu} (1 + \frac{b(y)}{P})
\] (2-23)

The Klinkenberg coefficients for the planar pores in the inorganic matrix domain and kerogen domain in the MPST model can be calculated using Equation 2-12:

\[
b(x) = \frac{2\mu}{x} \left(2 - \frac{\alpha}{\beta} - 1\right) \sqrt{\frac{8\pi RT}{M}} + \frac{4\mu}{x} \sqrt{\frac{8RT}{\pi M}}
\]

\[
b(y) = \frac{2\mu}{y} \left(2 - \frac{\alpha}{\beta} - 1\right) \sqrt{\frac{8\pi RT}{M}} + \frac{4\mu}{y} \sqrt{\frac{8RT}{\pi M}}
\] (2-24)

The pore widths in Equation 2-24, \(x\) and \(y\), depend on the effective stress and can be calculated as follows:

\[
x = x_0[1 - C_i \Delta P_e]
\]

\[
y = y_0[1 - C_k \Delta P_e]
\] (2-25)

where \(x_0\) and \(y_0\) are the initial (reference) pore widths in the inorganic matrix and kerogen domains, respectively; \(\Delta P_e\) is the change of the effective stress compared to the effective stress in the reference state; \(C_i\) and \(C_k\) are the compressibility coefficients of the inorganic matrix pore and kerogen pore, respectively, which reflect the responses of the pore width to the change of the effective stress.
In addition, the porosity of the entire shale rock can be calculated as:

\[ \phi = \frac{x + Ny}{x + Ny + S} = \frac{x_0[1 - C_i\Delta P_e] + Ny_0[1 - C_k\Delta P_e]}{x_0[1 - C_i\Delta P_e] + Ny_0[1 - C_k\Delta P_e] + S} \]  

(2-26)

where \( S \) is the cross-section width contributed by the solids.

Substituting Equations 2-23, 2-24, 2-25, and 2-26 into Equation 2-22, one can re-write Equation 2-22 as follows:

\[ \bar{k} = \frac{x_0[1 - C_i\Delta P_e]}{x_0[1 - C_i\Delta P_e] + Ny_0[1 - C_k\Delta P_e] + S} \left[ 1 + \frac{2\mu (\frac{3}{2} - 1) \sqrt{\frac{8\pi R T}{M}} + 4\mu \sqrt{\frac{8\pi R T}{M}}}{x_0[1 - C_i\Delta P_e] + Ny_0[1 - C_k\Delta P_e]} \right] + \frac{Ny_0[1 - C_k\Delta P_e]}{x_0[1 - C_i\Delta P_e] + Ny_0[1 - C_k\Delta P_e]} \left[ 1 + \frac{2\mu (\frac{3}{2} - 1) \sqrt{\frac{8\pi R T}{M}} + 4\mu \sqrt{\frac{8\pi R T}{M}}}{y_0[1 - C_i\Delta P_e] + Ny_0[1 - C_k\Delta P_e]} \right] \]  

(2-27)

Equation 2-27 provides a detailed formulation for the MPST model, which predicts the overall apparent permeability of the shale gas sample, \( \bar{k} \), as the model output by accounting for the multi-physics processes of fluid dynamics, geomechanics, and the Klinkenberg effect. Specifically, the MPST model takes two independent input variables on the right-hand side of Equation 2-27, which are the effective stress change, \( \Delta P_e \), and the pore pressure, \( P_p \).

### 2.5. Results and Discussion

**Figure 2-5** illustrates the PDP-measured apparent permeability as a function of pore pressure under different effective stresses. The solid curves are the MPST model fitting curves based on the PDP experimental data points. In this figure, the effective stresses were calculated based on the assumption that the Biot coefficient is one for the sake of simplicity. This is because calculation of the actual Biot coefficient value using Equation 2-20 requires comprehensive laboratory tests of the same core sample under various combinations of pore and confining pressures, which can be time consuming using the PDP equipment and might cause the development of microfractures in the core sample due to the cyclic loading and unloading processes in the PDP. Therefore, in many applications when no other information is available, the Biot’s coefficient is usually assumed to be one (Alam et al., 2014; and Bhandari et al., 2015; Jin et al., 2015; Rydzy et al., 2016). All six groups of the PDP tests demonstrated that the apparent permeability decreased with increasing pore pressure. The sample of Mancos PL had the highest average permeability across all pore pressures, whereas the sample of Mancos PD had the lowest average permeability. Because high pore pressure mitigates the Klinkenberg effect (Vermylen, 2011), in this study the average value
of the apparent permeabilities at the pore pressures of 1300 psi and 1500 psi were considered as the absolute permeability of the core. Soeder (1988) used laboratory tests to measure apparent permeability under varying pore pressures for cores extracted from the Marcellus shale, and found that when the pore pressure was higher than 500 psi permeability change was smaller than 15%, which is consistent with our finding. Therefore, the apparent permeabilities measured under 1300 psi and 1500 psi were very close to the absolute permeability. It was found that the absolute permeabilities of all core samples ranged from $10^{-4}$ mD to $10^{-2}$ mD. In addition, it was noticed that in the same shale formation the permeabilities in the cores where the bedding planes were parallel to the core axis were approximately one order of magnitude higher than those in the cores where the bedding planes were perpendicular to the core axis. This is because the pore spaces between bedding planes have higher connectivity and thus provide higher gas flow conductivity, leading to higher permeability if the sample is cored in this direction.

Figure 2-5 illustrates that the effective stress is a critical input variable in the MPST model, which has a great impact on the PDP-measured apparent permeability. Under high effective stress (1500 psi), the apparent permeability did not noticeably change when the pore pressure increased, which suggests that the Klinkenberg effect is mitigated. This is because for most core samples there exists connected pore networks having various spatial scales. The relatively small pore networks are more sensitive to effective stress and they shut off when the effective stress increases, leaving relatively large pore networks open, which are relatively insensitive to the Klinkenberg effect and thus have relatively smaller Klinkenberg coefficients. When the effective stress was 500 psi, the apparent permeabilities decreased noticeably with increasing pore pressure, suggesting that the Klinkenberg effect is significant under the relatively low effective stress because all relatively small pore networks stay open, which are subjected to noticeable Klinkenberg effect.

The PDP experimental data were fitted to the MPST model, and the fitting curves demonstrated that the MPST model successfully captured the influences of pore pressure and effective stress on the measured apparent permeability. During the data fitting processes, we found that the magnitude of the apparent permeability is controlled primarily by the pore widths, $x$ and $y$, and the solid section width, $S$, whereas the shape of the curve (curvature) is controlled primarily by the kerogen and inorganic matrix compressibility coefficient, $C_k$ and $C_i$. 
Figure 2-5. Apparent permeability measured in the PDP experiments as a function of pore pressure under different effective stresses in the core samples of a) Mancos PL, b) Mancos PD, c) Barnett PL, d) Barnett PD, e) Eagle Ford PL, and f) Marcellus PL. The solid curves are the MPST model fitting curves based on the scattered PDP experimental data points.
Figure 2-6 illustrates $k_a/k$ as a function of $1/P_p$, as well as the linear equation fitting lines. Based on Equation 2-1, the y-intercept of the linear equation is 1, and the slope is equal to the Klinkenberg coefficient, $b$. The variation of the value of $b$ as a function of the effective stress was consistent with the observation in Figure 2-5. It was also noticed that most values of $b$ fell into the range between 100 psi and 400 psi, which were close to what were found in the literature based on experimental (Soeder, 1988) and analytical (Chen, 2016) methods.

Figure 2-6. $k_a/k$ as a function of $1/P_p$, as well as the linear equation fitting lines, in the core samples of a) Mancos PL, b) Mancos PD, c) Barnett PL, d) Barnett PD, e) Eagle Ford PL, and f) Marcellus PL. In the figure legend, “linear” refers to the linear fitting lines. Based on Equation 2-1, the slope
of the fitting line is equal to the Klinkenberg coefficient, $b$.

In the next step, the data fitting method, which was originally developed by Bernabe (1986) and then improved by Kwon et al. (2001), was utilized to determine the actual value of the Biot’s coefficient using Equation 2-20. **Figure 2-7** illustrates $\log(k_a)$ as a function of the confining pressure, $P_c$, under varying pore pressures. The slope of the fitted straight lines was equal to $\frac{\partial \log(k_a)}{\partial P_c}$. Based on the method of Kwon et al. (2001), the value of $\frac{\partial \log(k_a)}{\partial P_c}$ in the same core sample should be constant and independent of the specific pore pressure value, which is confirmed in Figure 2-7. Therefore, in this study the data measured under the pore pressure of 1500 psi was used to determine $\frac{\partial \log(k_a)}{\partial P_c}$. It should be noted that in the same core sample the slopes of the straight lines fitted under the other pore pressures were close to that for the pore
pressure of 1500 psi. Figure 2-7 demonstrates that the value of \( \partial \log(k_a) / \partial P_c \) ranges from \(-4 \times 10^{-5}\) to \(-4 \times 10^{-4}\).

**Figure 2-7.** The value of \( \log(k_a) \) as a function of the confining pressure, \( P_c \), under varying pore pressures in the formations of a) Mancos PL, b) Mancos PD, c) Barnett PL, d) Barnett PD, e) Eagle Ford PL, and f) Marcellus PL. In the figure legend, “linear” refers to the linear fitting lines.

**Figure 2-8** illustrates \( \log(k_a) \) as a function of the pore pressure, \( P_p \), under varying confining pressures. The slope of the fitted straight lines was equal to \( \partial \log(k_a) / \partial P_p \). Based on the method of
Kwon et al. (2001), the value of $\partial \log(k_a) / \partial P_p$ in the same core sample should be constant and independent of the specific confining pressure value, which is confirmed in Figure 2-8. Thus, in this study the data measured under the confining pressure of 2000 psi was used to fit $\partial \log(k_a) / \partial P_p$. In the same core sample, the slopes of the straight lines fitted under the other confining pressures were close to that for the confining pressure of 2000 psi. Figure 2-8 demonstrates that the value of $\partial \log(k_a) / \partial P_p$ ranges from $2 \times 10^{-5}$ to $1 \times 10^{-4}$.

**Figure 2-8.** The value of $\log(k_a)$ as a function of pore pressure, $P_p$, in the formations of a) Mancos PL, b) Mancos PD, c) Barnett PL, d) Barnett PD, e) Eagle Ford PL, and f) Marcellus PL. In the figure legend, “linear” refers to the linear fitting lines.
Based on the results from Figures 2-7 and 2-8, the values of the Biot’s coefficient, $\chi$, in the six shale formations were calculated using Equation 2-20. Specifically, the values of $\chi$ were 0.50, 0.50, 0.62, 0.50, 0.35 and 0.26, for Mancos PL, Mancos PD, Barnett PL, Barnett PD, Eagle Ford PL, and Marcellus PL, respectively. Other researchers (da Silva et al., 2008 and Gutierrez et al., 2015) found similar Biot coefficient values in shale. These relatively low values of the Biot’s coefficient in shale suggest that the pore pressure had a lesser influence on the effective stress compared to the confining pressure. The Biot’s coefficient in the shale is greatly impacted by the clay minerals content, fluid-filled pore geometries, and homogeneous elastic properties. Because the shale is known for its heterogeneity, it is usually treated as a composite material. Therefore, each shale formation has its unique property, which explains the scattered values in the calculated Biot’s coefficient.

2.6. Conclusion

In this study, comprehensive core analyses using PDP laboratory testing were conducted to investigate the relationships between apparent permeability, pore pressure, and confining pressure. The influence of the Klinkenberg effect and the role of the bedding planes direction on the PDP-measured apparent permeabilities were studied. Based on the large volume of PDP experimental data, the actual values of the Biot’s coefficient were determined by data fitting. The laboratory results indicate that in the same shale formation the permeabilities of the cores in which the bedding planes were parallel to the core axis were approximately one order of magnitude higher than those in the cores in which the bedding planes were perpendicular to the core axis. Under higher effective stresses, the apparent permeability did not change noticeably when the pore pressure increased. This was because that for most core samples there exist connected pore networks having heterogeneous spatial scales (i.e., a wide range of pore size distribution). The relatively small pore networks are more sensitive to effective stress and they are isolated when the effective stress increases, leaving relatively large pore networks open, which are relatively insensitive to the Klinkenberg effect and thus have relatively small Klinkenberg coefficients.

The MPST model is developed to account for the coupled multi-physics processes of geomechanics, fluid dynamics, and the Klinkenberg effect for gas transport in shales. Specifically, this novel model takes into account the fluid dynamics associated with gas transport in organic and inorganic pores, compression of both kerogen (organic matters) and inorganic matrix under stress,
and the Klinkenberg effect. In the MPST model, pore pressure and effective stress are the two independent input variables, whereas the measured apparent permeability is the model output. In this study, the PDP experimental data were fitted to the MPST model, and the fitting curves showed that the MPST model successfully captures the influence of both pore pressure and effective stress on the apparent permeability.

The MPST model development aims to answer the question if it is possible to predict and fit laboratory data for the apparent permeability of a shale using pore pressure and effective stress change as model inputs. The result from this work demonstrates that the MPST model is a simple but effective framework to interpret laboratory PDP data measured under various confining and pore pressures and it is able to account for the multi-physics processes of geomechanics, fluid dynamics, and the Klinkenberg effect that all affect the measured apparent permeability for gas transport in shales. This study advances the fundamental understanding of the role of confining pressure, pore pressure, Klinkenberg effect, and bedding plane direction on the apparent permeability of shales. Therefore, The MPST model has practical applications to assess the apparent permeability in a shale gas formation under specified pore pressure and effective stress, which is crucial in accurately predicting the recovery factor and recovery rate using reservoir simulations (e.g., Chen, 2016). The research outcome has the potential to benefit the optimization of engineering designs in horizontal wells in shales including hydraulic fracturing, re-stimulation, and optimal well landing location. The laboratory experiments provide insights into the Klinkenberg effect and its role on the apparent permeability under varying confining and pore pressures in different U.S. shale formations.

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**Nomenclature**
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>A</td>
<td>model total width</td>
</tr>
<tr>
<td>b</td>
<td>Klinkenberg coefficient</td>
</tr>
<tr>
<td>C_k</td>
<td>kerogen compressibility</td>
</tr>
<tr>
<td>C_i</td>
<td>inorganic matrix compressibility</td>
</tr>
<tr>
<td>d</td>
<td>pore diameter</td>
</tr>
<tr>
<td>k</td>
<td>absolute permeability</td>
</tr>
<tr>
<td>k̅</td>
<td>model apparent permeability</td>
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<tr>
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<td>apparent permeability</td>
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<tr>
<td>k_i</td>
<td>inorganic matrix permeability</td>
</tr>
<tr>
<td>k_k</td>
<td>kerogen permeability</td>
</tr>
<tr>
<td>Kn</td>
<td>Knudsen number</td>
</tr>
<tr>
<td>L</td>
<td>model length</td>
</tr>
<tr>
<td>M</td>
<td>molar mass</td>
</tr>
<tr>
<td>N</td>
<td>number ratio</td>
</tr>
<tr>
<td>P_c</td>
<td>confining pressure</td>
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<tr>
<td>P_e</td>
<td>effective stress</td>
</tr>
<tr>
<td>P_p</td>
<td>pore pressure</td>
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<td>Q_i</td>
<td>flow rate in the inorganic matrix</td>
</tr>
<tr>
<td>Q_k</td>
<td>flow rate in the kerogen</td>
</tr>
<tr>
<td>r</td>
<td>effective pore radius</td>
</tr>
<tr>
<td>R</td>
<td>gas constant</td>
</tr>
<tr>
<td>S</td>
<td>solid section width</td>
</tr>
<tr>
<td>T</td>
<td>absolute temperature</td>
</tr>
<tr>
<td>t</td>
<td>time</td>
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<tr>
<td>ū</td>
<td>average flow velocity</td>
</tr>
<tr>
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<td>kerogen flow velocity</td>
</tr>
<tr>
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<td>inorganic matrix flow velocity</td>
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</tr>
<tr>
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<td>tangential momentum</td>
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<tr>
<td>λ</td>
<td>free gas mean-free-path length</td>
</tr>
<tr>
<td>μ</td>
<td>dynamic viscosity</td>
</tr>
<tr>
<td>ρ</td>
<td>free gas mass density</td>
</tr>
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</table>

**Reference**


Chapter 3. A Novel Multi-Physics Multi-Scale Multi-Porosity Shale Transport Model for Geomechanics/Flow Coupling in Steady and Transient States

Zihao Li, Yuntian Teng, Ming Fan, Nino Ripepi, Cheng Chen, Virginia Tech

Abstract

A novel multi-physics multi-scale multi-porosity shale gas transport (M³ST) model was developed to investigate shale gas transport in both transient and steady states. The microscale model component contains a kerogen domain and an inorganic matrix domain, and each domain has its own geomechanical and gas transport properties. Permeabilities of various shale cores were measured in the laboratory using a pulse decay permeameter (PDP) with different pore pressure and confining stress combinations. The PDP-measured apparent permeability as a function of pore pressure under two effective stresses was fitted using the microscale M³ST model component based on non-linear least squares fitting, and the fitted model parameters were able to provide accurate model predictions for another effective stress. The parameters and petrophysical properties determined in the steady state were then used in the transient-state, continuum-scale M³ST model component, which performed history matching of the evolutions of the upstream and downstream gas pressures. Additionally, a double-exponential empirical model was developed as a powerful alternative to the M³ST model to fit laboratory-measured apparent permeability under various effective stresses and pore pressures. The developed M³ST model and the research findings in this study provided critical insights into the role of the multiphysics mechanisms, including geomechanics, fluid dynamics and transport, and Klinkenberg effect, on shale gas transport across different spatial scales in both steady and transient states.

Keywords: Shale gas, Effective stress, Klinkenberg effect, Poroelasticity, M³ST model

3.1. Introduction and Background

Production of hydrocarbon energy resources from shale, a fine-grained, low-permeability geological formation, has changed the global energy outlook. Recovery of shale oil and gas has become economically viable due to the development of hydraulic fracturing and horizontal drilling
Compared with conventional enhanced oil and gas recovery (Li et al., 2016; Wang et al., 2017; Zhao et al., 2017; Tang et al., 2019), unconventional resource development is generally associated with a wide range of benefits such as job growth and improved infrastructure and municipal developments.

Extensive research on the correlation between effective stress and shale permeability has been conducted using both laboratory experiments and analytical models (Akkutlu & Fathi, 2012; Alnoaimi & Kovscek, 2013; Chen, 2016; Ghanbarian & Javadpour, 2017; Guo, Ma, & Tchelepi, 2018; Heller, Vermylen, & Zoback, 2014; Mehan, Prodanović, & Javadpour, 2013; Sheng, Javadpour, & Su, 2018; Wang et al., 2019; Zhang et al., 2021). It is generally expected that the permeability of a shale formation decreases with increasing effective stress due to compressed pore space and closure of microscale fractures. In addition to effective stress, the apparent (i.e., measured) permeability of a shale formation depends on the pore pressure if the testing fluid is a gas. This phenomenon is caused by nanoscale pores in shale and is referred to as the Klinkenberg effect. In 1941, Klinkenberg developed the gas slippage theory (i.e., the Klinkenberg effect) to explain the enhancement of measured apparent permeability for gas flow in nanopores (Klinkenberg, 1941). The Klinkenberg effect occurs when the mean free path of the measured gas molecules is longer than the diameter of the nanopore through which they travel. Consequently, gas molecules collide with the pore walls more frequently than with one another, leading to Knudsen diffusion that causes a slip-velocity boundary condition on the pore walls, which enhances the apparent permeability of the nanopore. Therefore, the Klinkenberg effect has a critical impact on gas flow in porous media having ultra-low permeability (Wu et al., 1998). Although the Klinkenberg effect may not influence the apparent permeability under in situ high reservoir pressures (Civan, 2020), it plays an important role in the interpretation of laboratory measurements because pore pressures used in laboratory experiments can be at the levels where the Klinkenberg effect is noticeable (Soeder, 1988; Li et al., 2020).

The Knudson number (Kn) is an important dimensionless number relevant to the Klinkenberg effect. Kn is calculated as the ratio of the gas mean-free-path length, $\lambda$, to the effective pore width, $h$. The Navier-Stokes equations can be used to describe gas flow with a no-slip boundary condition on the pore walls when Kn is smaller than 0.001. When the Kn is increased to the range between 0.001 and 0.1, the gas flow is in the slip flow regime in which the non-continuum effect at the
solid-fluid boundary can be accounted for using a velocity slip on the pore walls; the Navier-Stokes equations can still be used to describe the bulk gas flow. (Beskok & Karniadakis, 1999). The continuous increase of Kn into the range between 0.1 and 10 will lead to the transitional flow regime, followed by the free molecular flow regime when Kn is larger than 10, in which the Navier-Stokes equations cannot be used to describe gas flow in the pores (Tian et al., 2019).

Because of the important implication the Klinkenberg effect to shale gas transport, a variety of analytical approaches have been developed to describe gas flow subjected to the Klinkenberg effect in tight porous media (Hayek, 2015; Hu et al., 2009; Innocentini and Pandolfelli., 2001; Li et al., 2019, 2020; Wu et al., 1998; Zhu et al., 2007). In addition, many studies have been conducted to investigate the Klinkenberg effect and its role on multiphase flow (Blanchard et al., 2007), the dynamic Klinkenberg coefficient (Wang et al., 2014), and fractal theory (Li et al., 2016). All these studies confirmed that the permeability of tight reservoir rocks depends closely on effective stress and the Klinkenberg effect.

Despite numerous previous studies on the relationship between shale’s apparent permeability, effective stress, and pore pressure, a mechanistic model is still needed to adequately describe the multi-physics multi-scale processes that regulate shale gas transport in tight formations. The lack of such a mechanistic, theoretical model was due to several challenges. First, the model needs to have a multi-porosity geometry to differentiate kerogen (i.e., organic matter) and inorganic matrix at the microscopic scale (Chen, 2016), because it is the nanopores in kerogen that cause the Klinkenberg effect in shale. Second, the model needs to account for multi-physics mechanisms, including geomechanics, fluid flow and transport, and Klinkenberg effect, because the effective stress and pore pressure in a shale gas reservoir influence the apparent permeability of the formation through these multi-physics mechanisms. Third, the model should be multi-scale and also reasonably simple so that it can be easily calibrated by laboratory core permeability measurements and upscaled to history match continuum-scale pore pressure evolutions. To solve these challenges, in this study a novel multi-physics multi-scale multi-porosity shale gas transport (M³ST) model was developed to describe shale gas transport in both transient and steady states, which provides mechanistic insight into the dependence of shale’s apparent permeability on effective stress and pore pressure.
3.2. **Overview of the Experimental and Modeling Approaches**

This section provides an overview of the experimental and modeling approaches used in this paper, including the details for the principle of a pressure pulse decay permeameter (PDP) and how it measures shale core permeability and the upstream and downstream gas pressure evolutions in the transient state. Section 3 introduces basic theories of the Biot’s coefficient and Klinkenberg effect, which are relevant to the role of effective stress and pore pressure, respectively, on shale’s apparent permeability. Section 4 develops a novel M³ST model to study gas transport in shale in both transient and steady states. Particularly, the microscale M³ST model component was used to fit PDP measurements of shale permeability based on non-linear least square fitting (NLSF) to determine critical model parameters; the fitted model parameters were used to predict shale permeability under other effective stresses and then imported into the macroscale component of the M³ST model to history match continuum-scale gas pressure evolutions measured by the PDP. Section 5 derives a double-exponential empirical model to describe the dependence of shale’s apparent permeability on effective stress and pore pressure. Section 6 provides experimental and modeling results as well as discussions and implications.

**Figure 3-1** illustrates the Eagle Ford and Mancos shale cores used in this study. The two Eagle Ford cores were marked as “Eagle Ford 1A” and “Eagle Ford 1B”, and the two Mancos cores were marked as “Mancos 1A” and “Mancos 1B”. The cores were two inches in length and one inch in diameter. The bedding plane direction was parallel to the core axis direction. The total organic carbon (TOC) contents were 2.02% and 1.37%, respectively, for the Eagle Ford and Mancos cores.
Figure 3-1. Two Eagle Ford shale core plugs (top row) and two Mancos shale core plugs (bottom row) used in the PDP permeability measurements. The shale core plugs were two inches in length and one inch in diameter. The shale bedding plane direction was parallel to the core plug axis direction.

Figure 3-2 illustrates a schematic plot of the PDP equipment setup, which provides a convenient and dynamic approach for measuring the apparent permeability of tight rocks (Jones, 1997). First, with the fill valve and valves 1 through 4 opened, all the gas reservoirs are filled to the desired starting pressure. The upstream reservoir volume, $V_1$, and downstream reservoir volume, $V_2$, are both 10 cm$^3$. Next, the fill valve is closed, and after a period of soaking time (usually 300 seconds) valve 2 is closed. Gas in the downstream reservoir, $V_2$, will then be released through the needle valve and shut-off valve to obtain a pressure lower than the upstream reservoir pressure, which is referred to as the differential pressure, $\Delta P$. When $\Delta P$ stabilizes, valves 3 and 4 are both closed. Gas molecules will gradually penetrate through the core sample to migrate from the upstream reservoir to the downstream reservoir, leading to continuously decreasing upstream reservoir pressure and increasing downstream reservoir pressure. In this process, the differential pressure, $\Delta P$, continuously decays. The higher the permeability of the core plug, the faster the decay of $\Delta P$. The permeability of the core plug can then be calculated based on the decay rate of $\Delta P$ following the standard procedure of PDP analysis (Dicker and Smits, 1988; Jones, 1997).

Figure 3-2. Schematic PDP equipment setup, which consists of an upstream gas reservoir having a volume of $V_1$, a high-strength core holder that contains the core sample having a pore volume of
V_p, a downstream gas reservoir having a volume of V_2, two large gas reservoirs V_3 and V_4 that stabilize the initial gas pressures in V_1 and V_2, a differential pressure transducer to continuously measure the differential pressure, ΔP, and a pressure transducer to measure the downstream reservoir pressure, P_d. This picture was modified from the CoreLab PDP-200 operation manual.

3.3. *Determination of the Biot’s Coefficient*

During the recovery of unconventional hydrocarbon resources, increased effective stress can cause rock matrix deformation, which plays a vital role on the petrophysical properties of the shale formation during production, including permeability (Fan et al., 2018, and 2020), wettability (Guo et al., 2020), and fluid phase behavior (Zhao et al., 2021). Based on the poroelasticity theory from Biot (1941), a porous medium’s deformation influences the fluid flow and vice versa. The Biot’s coefficient (i.e., Biot’s poroelastic term) is a critical concept for the effective stress calculation in a porous medium, which represents the fluid volume change induced by bulk volume changes in the drained condition (Müller and Sahay, 2016). The calculation of Biot’s coefficient can be based on various correlations, and Civan (2021) summarized these equations used in the literature, which include linear, power-law, exponential, inverse, logarithmic, and modified power-law correlations. According to Terzaghi and Peck (1996), the pore pressure of the fluid reduces the effective stress in a rock mineral matrix, and Biot’s coefficient describes the reduction of effective stress quantitatively (Bernabe, 1986):

\[
P_e = P_c - \chi P_p
\]

where \(\chi\) is Biot’s coefficient; \(P_e\), \(P_c\), and \(P_p\) are effective stress, confining pressure, and pore pressure, respectively. Previous studies (Warpinski and Teufel, 1992; Cheng, 1997; Lee, 2002; Ojala and Fjær, 2007; Tan and Konietzky, 2014; Selvadurai, 2019) have used both laboratory experiments and theoretical methods to estimate the \(\chi\) value in a certain type of rock.

In this study, we used the PDP to measure shale’s apparent permeability under various pore pressures and confining pressures to calculate the Biot’s coefficient based on Geertsma (1957), Kwon et al. (2001), and Civan (2021). Because the apparent permeability, \(k_a\), depends on both \(P_p\) and \(P_c\), we express \(k_a\) as a function of these two independent variables:
\[ k_a = k_a(P_c, P_p) \]  

Because the permeability of natural geologic formations in general follows a lognormal distribution (Chen and Zeng, 2015), we make a logarithm transformation for Equation 2 and obtain:

\[ \log(k_a) = \log[k_a(P_c, P_p)] \]  

(3-3)

Taking differential of Equation 3 leads to:

\[ d \log(k_a) = \left( \frac{\partial \log(k_a)}{\partial P_c} \right) dP_c + \left( \frac{\partial \log(k_a)}{\partial P_p} \right) dP_p \]  

(3-4)

Similarly, taking differential of Equation 1 leads to:

\[ dP_e = dP_c - \chi dP_p \]  

(3-5)

When the pore pressure is sufficiently high, the Klinkenberg effect is inhibited; in this scenario, the apparent permeability is equal to the absolute permeability, which depends only on the pore geometry that is controlled by the effective stress. Therefore, when the change of effective stress, \( dP_e \), is zero, the change of apparent permeability, \( d \log(k_a) \), is also zero. Therefore, Equations 4 and 5 lead to:

\[ 0 = \left( \frac{\partial \log(k_a)}{\partial P_c} \right) dP_c + \left( \frac{\partial \log(k_a)}{\partial P_p} \right) dP_p \]  

(3-6)

And

\[ 0 = dP_c - \chi dP_p \]  

(3-7)

Combining Equations 6 with 7, one can solve for the Biot’s coefficient, \( \chi \), as follows:

\[ \chi = - \left( \frac{\partial \log(k_a)}{\partial P_p} \right) / \left( \frac{\partial \log(k_a)}{\partial P_c} \right) \]  

(3-8)

The Klinkenberg effect (1941) was introduced to explain the difference between measured permeability (i.e., apparent permeability) and absolute permeability in tight rocks. The
Klinkenberg coefficient, $b$, was applied to quantify the influence of pore pressure and nanopore size on the measured apparent permeability:

$$\frac{k_a}{k} = 1 + \frac{b}{P_p}$$

where $k_a$ and $k$ are apparent permeability ($m^2$) and absolute permeability ($m^2$), respectively, and $b$ is the Klinkenberg coefficient (Pa). The Klinkenberg coefficient depends on the rock’s petrophysical properties and can be determined based on the analysis of the mass fluxes contributed by the Knudsen diffusion and viscous flow. The value of $b$ in a planar nanopore can be expressed as (Chen 2016; Li et al., 2020):

$$b = \frac{2\mu}{h} \left( \frac{2}{\alpha} - 1 \right) \sqrt{\frac{8\pi RT}{M}} + \frac{4\mu}{h} \sqrt{\frac{8RT}{\pi M}}$$

where $\mu$ is dynamic viscosity (Pa·s); $\alpha$ is the tangential momentum accommodation coefficient with a value from 0 to 1; $R$ is the gas constant and equal to 8.314 J/mol/K; $T$ is the absolute temperature (K); $M$ is the molar mass (kg/mol); and $h$ is the effective pore width (m). Details of the derivation of Equation 10 can be found in one of our previous studies (Li et al., 2020). Based on Equation 10, it is clear the value of $b$ is inversely proportional to the effective pore width, $h$, which explains why the Klinkenberg effect is more significant in tight rocks where the pore size is in general small.

3.4. Multi-Physics Multi-scale Multi-porosity Shale Transport Modeling

Figure 3-3 illustrates the microscale and macroscale geometric models for shale gas transport, which is based on the extension of the multiphysics shale gas transport model published in our previous study (Li et al., 2020). This model forms the microscale component of the M³ST model and is in the two-dimensional (2D) space, which consists of two domains for shale gas transport: the organic domain (kerogen) and the inorganic matrix. Each domain has its respective geomechanical compressibility, pore width, porosity, and permeability. Because of the high mass flux contributed by viscous flow, we ignore molecular diffusion and adsorption and desorption in the inorganic matrix (Chen, 2016). Therefore, the microscale M³ST model characterizes gas
transport in the kerogen and inorganic matrix separately. The entire shale sample is subjected to compressive stress because of the external confining pressure, which regulates the pore sizes in both the kerogen and inorganic matrix.

**Figure 3-3.** 2D schematic picture of the M^3ST model.

The absolute permeability of a pore in the 2D space can be calculated as:

\[ k = \frac{h^2}{12} \]  

(3-11)

where \( k \) is absolute permeability (m\(^2\)), and \( h \) is pore width (m). Combining Equation 3-2 with Equation 3-11, one obtains:

\[ k_{ai} = \frac{h_i^2}{12} \left(1 + \frac{b_i}{P_p}\right) \]

\[ k_{ak} = \frac{h_k^2}{12} \left(1 + \frac{b_k}{P_p}\right) \]  

(3-12)

where \( k_{ai} \) and \( k_{ak} \) are the apparent permeabilities in the inorganic matrix and kerogen, respectively; \( h_i \) and \( h_k \) are pore widths in the inorganic matrix and kerogen, respectively; \( b_i \) and \( b_k \) are the Klinkenberg coefficients in the inorganic matrix and kerogen, respectively. The apparent
permeability of the entire domain, including both the inorganic matrix and the kerogen, is calculated based on Darcy’s law:

\[
\overline{k}_a = \frac{(Q_i + Q_k) \mu}{A \cdot \nabla P_p}
\]

where \( A \) is total cross-sectional area (m\(^2\)), \( \mu \) is dynamic viscosity (Pa\(\cdot\)s), \( \nabla P_p \) is pressure gradient through the pore (Pa/m), and \( Q_i \) and \( Q_k \) are the flow rates in the inorganic matrix and the kerogen domain, respectively. Specifically, the total cross-sectional area, \( A \), the kerogen cross-sectional area, \( A_k \), and the inorganic matrix cross-sectional area, \( A_i \), can be calculated, respectively, as:

\[
A = h_i + N \cdot h_k \phi
\]

\[
A_i = h_i + (1 - \varepsilon_{ks}) \cdot S
\]

\[
A_k = N \cdot h_k + \varepsilon_{ks} \cdot S
\]

where \( \phi \) is the total porosity that accounts for both kerogen and inorganic pores; \( N \) is the number of kerogen pores per inorganic pore; \( S \) is the solid cross-sectional area associated with one inorganic pore; \( \varepsilon_{ks} \) is kerogen solid volume per unit total solid volume and can be determined using the TOC content. Based on Darcy’s law and Equation 3-12, \( Q_i \) and \( Q_k \) can be calculated as:

\[
Q_i = \frac{k_{ai}}{\mu} \cdot \nabla P_p \cdot h_i = \frac{h_i^3}{12} \cdot \frac{\nabla P_p}{\mu} (1 + \frac{b_i}{P_p})
\]

\[
Q_k = N \cdot \frac{k_{ak}}{\mu} \cdot \nabla P_p \cdot h_k = N \cdot \frac{h_k^3}{12} \cdot \frac{\nabla P_p}{\mu} (1 + \frac{b_k}{P_p})
\]

In Equation 3-15, the Klinkenberg coefficients, \( b_i \) and \( b_k \), are calculated based on Equation 3-10:

\[
b_i = \frac{2 \mu}{h_i} \left( \frac{2}{\alpha} - 1 \right) \sqrt{\frac{8 \pi R T}{M}} + \frac{4 \mu}{h_i} \sqrt{\frac{8 R T}{\pi M}}
\]

\[
b_k = \frac{2 \mu}{h_k} \left( \frac{2}{\alpha} - 1 \right) \sqrt{\frac{8 \pi R T}{M}} + \frac{4 \mu}{h_k} \sqrt{\frac{8 R T}{\pi M}}
\]
The pore widths in Equations 3-15 are subject to the effective stress and calculated as:

\[ h_i = h_{i0}(1 - \beta_i P_e) \]
\[ h_k = h_{k0}(1 - \beta_k P_e) \]  \hspace{1cm} (3-17)

where \( h_{i0} \) and \( h_{k0} \) are the initial (i.e., reference) pore widths in the inorganic matrix and kerogen domain, respectively; \( P_e \) is the effective stress and calculated as \( P_e = P_c - \chi \cdot P_p \); \( \beta_i \) and \( \beta_k \) are the isothermal compressibility coefficients for pore width in the inorganic matrix and kerogen domain, respectively, which describe pore width’s responses to the change of effective stress.

In Equation 3-14, the total porosity, \( \phi \), can be calculated as:

\[ \phi = \frac{h_i + N h_k}{h_i + N h_k + S} = \frac{h_{i0}(1 - \beta_i P_e) + N h_{k0}(1 - \beta_k P_e)}{h_{i0}(1 - \beta_i P_e) + N h_{k0}(1 - \beta_k P_e) + S} \]  \hspace{1cm} (3-18)

Plugging Equations 3-14 to 3-18 into Equation 3-13, one can re-write Equation 3-13 as:

\[ \overline{k_a} = \frac{[h_{i0}(1 - \beta_i P_e)]^{[1 + \frac{2\mu}{\alpha} - 1]} \sqrt{\frac{8\pi RT}{M}} + 4\mu \sqrt{\frac{8RT}{\pi M}} + N \cdot [h_{i0}(1 - \beta_i P_e)]^{[1 + \frac{2\mu}{\alpha} - 1]} \sqrt{\frac{8\pi RT}{M}} + 4\mu \sqrt{\frac{8RT}{\pi M}}}{12[h_{i0}(1 - \beta_i P_e) + N \cdot h_{i0}(1 - \beta_i P_e) + S]} \]  \hspace{1cm} (3-19)

Equation 3-19 is the microscale component of the M^3ST model based on our previous work (Li et al., 2020) for predicting the overall apparent permeability, \( \overline{k_a} \), for the shale that is comprised of the inorganic matrix and kerogen domain. This model takes two independent input variables on the right-hand side of Equation 3-19, which are the effective stress, \( P_e \), and the pore pressure, \( P_p \).

Using the same approach, one can calculate the apparent permeabilities of the inorganic matrix and kerogen domain as follows:
Similarly, these two equations take \( P_e \) and \( P_p \) as model inputs on the right-hand side. The model outputs are the inorganic matrix apparent permeability, \( k_{al} \), and kerogen apparent permeability, \( k_{ak} \), separately.

The apparent permeabilities of the kerogen and inorganic matrix derived from the microscale M3ST model component are then plugged into the continuum-scale M3ST model component which simulates the transient-state pressure evolutions at the larger (inch) scale. Particularly, the continuum-scale M3ST model component is based on the coupled, two-porosity transport model published in our previous study (Chen, 2016), and gas transport processes in the kerogen domain and inorganic matrix are described by Equations 3-21 and 3-22, respectively:

\[
ε_{kp} \phi \frac{∂C_k}{∂t} + ε_{ks} (1 - φ) \frac{∂C_μ}{∂t} = \frac{∂}{∂x} \left[ zRTC_k \frac{k_{ak}}{μ} \frac{∂C_k}{∂x} \right] + Γ \tag{3-21}
\]

\[
(1 - ε_{kp}) φ \frac{∂C_i}{∂t} = \frac{∂}{∂x} \left[ zRTC_i \frac{k_{ai}}{μ} \frac{∂C_i}{∂x} \right] - Γ \tag{3-22}
\]

where \( ε_{kp} \) is kerogen pore volume (PV) per unit total PV and can be written as \( ε_{kp} = (N \cdot h_{k0})/(N \cdot h_{k0} + h_{i0}) \); \( C_k \) and \( C_i \) are the molar concentrations of free gas within the kerogen and inorganic matrix in the unit of moles per kerogen pore volume and inorganic pore volume, respectively (mol/m\(^3\)); \( C_μ \) is adsorbed gas molar density within kerogen in the unit of moles per kerogen solid volume (mol/m\(^3\)); \( φ \) is the total porosity accounting for interconnected pores within both kerogen and inorganic matrix; \( x \) is the distance in the longitudinal direction (m); \( t \) is time (s); \( z \) is the compressibility factor; \( Γ \) is the mass transfer rate between kerogen and inorganic matrix (mol/m\(^3\)/s), which can be written as \( Γ = m(C_i - C_k) \) where \( m \) is the mass exchange rate
coefficient (1/s). The mass exchange in the kerogen domain between adsorbed gas and free gas can be expressed as:

$$\frac{\partial C_\mu}{\partial t} = K_{des} [K(C_{\mu \text{max}} - C_\mu)C_k - C_\mu]$$

(3-23)

where $C_{\mu \text{max}}$ is the maximum monolayer adsorption of gas in the kerogen (mol/m$^3$); $K$ is the equilibrium partition coefficient (m$^3$/mol), which can be calculated as $K = K_{ads}/K_{des}$. $K_{ads}$ and $K_{des}$ are the adsorption rate coefficient (m$^3$/s/mol) and desorption rate coefficient (1/s), respectively.

Equations 3-21 to 3-23 form the continuum-scale component in the M$^3$ST model, which is able to simulate the transient evolution of gas pressures in the inorganic matrix and kerogen at the continuum (i.e., inch and larger) scale. We developed a non-linear finite difference solver (Chen 2016) to solve these coupled and non-linear partial differential equations, and the solved gas pressure in the inorganic matrix was used to history match the evolution of gas pressures measured in the PDP experiments.

3.5. A Novel Double-Exponential Empirical Model

The microscale component of the M$^3$ST model (i.e., Equation 3-19) is based on the first principle and thus provides mechanistic insight into the role of geomechanics, fluid dynamics, and Klinkenberg effect on the apparent permeability of shale. In practice, empirical correlations have been developed to describe the relation between measured permeability and effective stress. Based on the studies of Chhatre et al. (2015), King et al. (2018), and Zhu et al. (2018), the dependence of shale permeability on the variation of effective stress can be described using an exponential function. This conclusion was confirmed by the experiments from Pei et al. (2020). However, all these previous empirical correlations focused on the influence of effective stress on measured permeability, without accounting for the role of pore pressure which can also significantly affect the measured permeability through the Klinkenberg effect.
Based on the laboratory PDP experiments under various combinations of pore pressures and effective stresses, a novel double-exponential empirical model was proposed in this study to describe the role of both effective stress and pore pressure on the measured permeability:

\[ k_a = e^{-\gamma_1(P_e - P_{e0})} \cdot [(k_0 - k_{a0}) \cdot e^{-\gamma_2(P_p - P_{p0})} + k_{a0}] \]  

(3-24)

where \( k_a \) is the apparent permeability (m\(^2\)) measured under effective stress, \( P_e \), and pore pressure, \( P_p \); \( \gamma_1 \) and \( \gamma_2 \) are the shale-permeability moduli (psi\(^{-1}\)) with respect to effective stress and pore pressure, respectively; \( k_0 \) is the reference apparent permeability (m\(^2\)); \( P_{e0} \) and \( P_{p0} \) are the reference effective stress and reference pore pressure under which \( k_0 \) is measured; \( k_{a0} \) is the reference absolute permeability (m\(^2\)) under the reference effective stress, \( P_{e0} \). It is clear that when the pore pressure, \( P_p \), is infinitely large, the Klinkenberg effect is eliminated; in this scenario, \( k_a \) will be equal to \( k_{a0} \) if \( P_e \) is equal to \( P_{e0} \).

This double-exponential empirical model will be used to fit the laboratory PDP experimental results and then compared with the M\(^3\)ST model. In practice, the double-exponential empirical model can be used as a powerful tool to fit laboratory-measured apparent permeability under various pore pressures and effective stresses because of its simple formulation and small number of undetermined parameters (i.e., \( \gamma_1 \) and \( \gamma_2 \)).

3.6. Results and Discussion

Table 3-1 illustrates the calculated values of the Biot’s coefficient, \( \chi \), based on the PDP measurements and Equation 3-8. When we calculated \( \partial \log(k_a)/\partial P_p \), the value of \( P_p \) was varied whereas the value of \( P_e \) was fixed. Similarly, when we calculated \( \partial \log(k_a)/\partial P_c \), the value of \( P_c \) was varied whereas the value of \( P_p \) was fixed. Particularly, we used the values of \( P_p \) at 1700 psi, 1800 psi, 2000 psi, and 2200 psi and fixed \( P_c \) at 2500 psi when we calculated \( \partial \log(k_a)/\partial P_p \). These relatively high \( P_p \) values eliminated the Klinkenberg effect and thus the measured apparent permeability is equal to the absolute permeability. Table 3-1 shows that the two core plugs from the Eagle Ford shale had Biot’s coefficients less than one, whereas the two Mancos cores had Biot’s coefficients equal to one. The Biot’s coefficient in shales is critically influenced by the clay mineral content, fluid-filled pore geometries, and mineral elastic properties (Kwon et al., 2001).
Because shale is well-known for its heterogeneity in mineral composition, we usually treat shale as a composite material. Therefore, each shale formation has its unique petrophysical and geomechanical property, which explains the various Biot’s coefficient values in Table 3-1.

**Table 3-1.** Measured Biot’s coefficients for the four shale cores.

<table>
<thead>
<tr>
<th></th>
<th>Eagle Ford 1A</th>
<th>Eagle Ford 1B</th>
<th>Mancos 1A</th>
<th>Mancos 1B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Biot’s coefficient ($\chi$)</td>
<td>0.60</td>
<td>0.71</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

**Figure 3-4** illustrates the PDP measurements as well as M$^3$ST model fitting and predictions. The scatter data points are PDP-measured apparent permeability values as a function of pore pressure under three effective stresses (i.e., 500 psi, 1000 psi, and 1500 psi). The effective stresses were calculated using Biot’s coefficients measured in Table 1. At each effective stress, the shale’s apparent permeability was measured under eight different pore pressures (i.e., 100 psi, 300 psi, 500 psi, 700 psi, 900 psi, 1100 psi, 1300 psi, and 1500 psi) to probe the role of the Klinkenberg effect on the measured apparent permeability. As an exception, the two Eagle Ford shale cores were measured under only the first five pore pressures at the effective stress of 500 psi because of their low Biot’s coefficients, which caused the confining pressure to be comparable to the pore pressure based on Equation 1 when the pore pressure went above 1000 psi. The PDP measurements used pure nitrogen at the temperature of 298 K. Overall, PDP measurements in Figure 3-4 shows that the apparent permeability decreased with increasing pore pressure and increasing effective stress. Particularly, it was clear that the $k_a$ value did not change against pore pressure noticeably when the pore pressure value was higher than 1300 psi because the high pore pressures can mitigate the Klinkenberg effect. This phenomenon is because a high gas pore pressure leads to a short mean-free-path length for gas molecules, which results in a small Kn and thus inhibits the Knudsen diffusion. Therefore, the apparent permeability values measured under 1300 psi and 1500 psi in
Figure 3-4 can be considered as the absolute permeability, which was consistent with the findings in the literature (Vermylen, 2011).

**Figure 3-4.** PDP-measured apparent permeability (scatter data points) as a function of pore pressure and effective stress. The solid-line curves were obtained by fitting the M³ST model to the PDP measurements at $P_e = 1000$ psi and $1500$ psi based on NLSF. The dash-line curve was the M³ST model prediction of the apparent permeability at $P_e = 500$ psi using the model-fitted parameters.

Figure 3-4 also illustrates the M³ST model fitting and predictions. Specifically, the two solid-line curves are the fitting curves based on the M³ST model, and the dash-line curve is the model prediction. Particularly, PDP-measured apparent permeability values as a function of pore pressure under effective stresses of 1000 and 1500 psi were fitted using the M³ST model with non-linear least squares fitting, and the fitted parameters were then used in the model to predict the apparent permeability values under effective stress of 500 psi. **Table 3-2** demonstrates the six unknown parameters in the M³ST model that were determined by fitting experimental PDP data. In this study, the Gauss-Newton method, an iteration-based algorithm for solving nonlinear least square fitting
problems, was used to determine the six unknown parameters in the \( M^3 \)ST model through data fitting. Particularly, the vector of the unknown parameters, \( \beta \), which has six elements in this case, is calculated by:

\[
\beta^{n+1} = \beta^n + (J^T J)^{-1} J^T r
\]

(3-25)

where \( n \) indicates the iteration time step; \( J \) is the Jacobian matrix, in which each entry, \( J_{ij} \), is the derivative of the \( M^3 \)ST-predicted \( k_a \) value (i.e., based on Equation 3-19) with respect to the \( j \)th unknown parameter at the \( i \)th measurement; \( r \) is the residual vector, in which each element, \( r_i \), is the difference between the PDP-measured and model-predicted \( k_a \) values at the \( i \)th measurement.

Table 3-2. Parameter values determined by fitting the \( M^3 \)ST model to the laboratory PDP measurements at effective stresses of 1000 and 1500 psi using NLSF based on the Gauss-Newton method.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Eagle Ford 1A</th>
<th>Eagle Ford 1B</th>
<th>Mancos 1A</th>
<th>Mancos 1B</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S ) (m)</td>
<td>5.25\times10^{-4}</td>
<td>5.17\times10^{-4}</td>
<td>1.12\times10^{-4}</td>
<td>9.07\times10^{-4}</td>
</tr>
<tr>
<td>( h_{l0} ) (m)</td>
<td>5.53\times10^{-7}</td>
<td>6.74\times10^{-7}</td>
<td>4.73\times10^{-7}</td>
<td>2.51\times10^{-7}</td>
</tr>
<tr>
<td>( h_{k0} ) (m)</td>
<td>1.8\times10^{-10}</td>
<td>5.90\times10^{-9}</td>
<td>5.62\times10^{-10}</td>
<td>6.57\times10^{-10}</td>
</tr>
<tr>
<td>( N )</td>
<td>1.0\times10^{3}</td>
<td>1.0\times10^{2}</td>
<td>1.0\times10^{3}</td>
<td>1.0\times10^{3}</td>
</tr>
<tr>
<td>( \beta_l ) (Pa(^{-1}))</td>
<td>3.2\times10^{-9}</td>
<td>8.2\times10^{-9}</td>
<td>2.8\times10^{-8}</td>
<td>4.3\times10^{-8}</td>
</tr>
<tr>
<td>( \beta_k ) (Pa(^{-1}))</td>
<td>1.8\times10^{-8}</td>
<td>6.7\times10^{-9}</td>
<td>5.0\times10^{-8}</td>
<td>5.5\times10^{-8}</td>
</tr>
</tbody>
</table>

Using the Gauss-Newton method, we employed the determined parameters, based on fitting the microscale \( M^3 \)ST model component to the PDP-measured data under effective stresses of 1000 psi and 1500 psi, to predict the apparent permeability as a function of pore pressure under effective stress of 500 psi. Table 3-3 demonstrates the accuracy of the \( M^3 \)ST model predictions for effective stress of 500 psi. We evaluated the difference between PDP-measured and model-predicted \( k_a \) values using the normalized mean squared error (NMSE), which is defined as:
\[
NMSE(k_{ai}, \hat{k}_{ai}) = \frac{\sum_{i=1}^{N} (k_{ai} - \hat{k}_{ai})^2}{\sum_{i=1}^{N} k_{ai}^2}
\]

where \( k_{ai} \) is the \( i^{th} \) PDP measurement of apparent permeability, \( \hat{k}_{ai} \) is the corresponding prediction using the M\(^3\)ST model, and \( N \) is the number of measurements. The NMSE is a measure to evaluate the relative error of model predictions compared to experimental observations. The smaller the NMSE value, the lower the relative error. The parameters fitted using the measurements under effective stresses of 1000 and 1500 psi led to accurate predictions of apparent permeability as a function of pore pressure under the effective stress of 500 psi for all the four shale samples. This indicates that the microscale component of the M\(^3\)ST model successfully accounted for the mechanistic, multi-physics mechanisms that regulate shale’s apparent permeability in the steady state, including the processes of geomechanics, fluid dynamics and transport, and Klinkenberg effect.

**Table 3-3.** NMSE values for M\(^3\)ST model predictions of shale’s apparent permeability under effective stress of 500 psi.

<table>
<thead>
<tr>
<th>( P_e )</th>
<th>Eagle Ford 1A</th>
<th>Eagle Ford 1B</th>
<th>Mancos 1A</th>
<th>Mancos 1B</th>
</tr>
</thead>
<tbody>
<tr>
<td>500 psi</td>
<td>7.85\times10^{-5}</td>
<td>3.47\times10^{-4}</td>
<td>1.69\times10^{-3}</td>
<td>1.73\times10^{-3}</td>
</tr>
</tbody>
</table>

**Figure 3-5** illustrates the laboratory PDP measurements as well as the double-exponential empirical model (i.e., Equation 3-24) fitting based on NLSF for all the four shale core samples. The data point at \( P_e=500 \) psi and \( P_p=100 \) psi was selected as the reference point, so \( P_{e0} \) and \( P_{p0} \) were 500 psi and 100 psi, respectively. \( k_0 \) was the PDP-measured apparent permeability at the reference point. \( k_{a0} \) was the absolute permeability when \( P_e=500 \) psi (i.e. stable value of apparent permeability at the right end of the blue solid lines in Figure 3-5). The double-exponential empirical model can overall capture the apparent permeability variation as a function of both pore pressure and effective stress.
Figure 3-5. PDP-measured apparent permeability (scatter data points) as a function of pore pressure and effective stress. The solid-line curves were obtained by fitting the double-exponential empirical model to the PDP measurements based on NLSF.

Table 3-4 demonstrates the comparison of the NMSE values for global data fitting using the M³ST model (i.e., Equation 3-19) and the double-exponential empirical model (i.e., Equation 3-24). Global data fitting indicates that the model was used to fit all the laboratory PDP measurements under effective stresses of 500, 1000, and 1500 psi. Therefore, the NMSE value was calculated based on all experimental measurements under all the effective stresses. Table 3-4 the NMSE measures of the M³ST global data fitting were overall lower than those for the global data fitting using the empirical model, which confirms that the M³ST model was based on the first principle and thus successfully accounted for the mechanistic, multi-physics processes that regulate shale’s apparent permeability under various combinations of pore pressures and effective stresses.

Table 3-4. NMSE values for global fitting using the M³ST model and the double-exponential
empirical model.

<table>
<thead>
<tr>
<th>Sample</th>
<th>NMSE of M³ST global fitting</th>
<th>NMSE of empirical model global fitting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eagle Ford 1A</td>
<td>1.27×10⁻⁴</td>
<td>1.10×10⁻³</td>
</tr>
<tr>
<td>Eagle Ford 1B</td>
<td>2.18×10⁻⁴</td>
<td>3.04×10⁻⁴</td>
</tr>
<tr>
<td>Mancos 1A</td>
<td>1.13×10⁻³</td>
<td>6.32×10⁻⁴</td>
</tr>
<tr>
<td>Mancos 1B</td>
<td>4.24×10⁻³</td>
<td>6.60×10⁻³</td>
</tr>
</tbody>
</table>

In practice, however, the double-exponential empirical model has some advantage due to its simplicity. Table 3-5 illustrates the modulus values in Equation 3-24 which were determined by fitting all the laboratory PDP measurements using non-linear least square fitting based on the Gauss-Newton method. It can be observed that the values of γ₂ were in general higher than those of γ₁, which implies that the pore pressure influenced shale’s apparent permeability more noticeable compared to the effective stress. The double-exponential empirical model can be used as a powerful alternative to the M³ST model to fit laboratory-measured apparent permeability under various pore pressures and effective stresses because of its simple formulation and small number of undetermined parameters.

Table 3-5. Fitted values of γ₁ and γ₂ in the double-exponential empirical model.

<table>
<thead>
<tr>
<th>Sample</th>
<th>γ₁ (psi⁻¹)</th>
<th>γ₂ (psi⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eagle Ford 1A</td>
<td>7.61×10⁻⁵</td>
<td>4.30×10⁻³</td>
</tr>
<tr>
<td>Eagle Ford 1B</td>
<td>1.49×10⁻⁴</td>
<td>4.10×10⁻³</td>
</tr>
<tr>
<td>Mancos 1A</td>
<td>4.12×10⁻⁴</td>
<td>3.10×10⁻³</td>
</tr>
<tr>
<td>Mancos 1B</td>
<td>7.03×10⁻⁴</td>
<td>3.40×10⁻³</td>
</tr>
</tbody>
</table>

The fitted parameters in the microscale M³ST model component (Equation 3-19) were used to calculate the apparent permeabilities of kerogen and inorganic matrix based on Equation 3-20, which were then imported into the continuum-scale M³ST model component (i.e., Equations 3-21 to 3-23) for large-scale, transient-state simulations of gas pressure evolutions in the core. Other
known parameter values used in the continuum-scale M³ST model component were $C_{\mu_{\text{max}}} = 1800$ mol/m$^3$, K = 1.7×10$^{-4}$ m$^3$/mol, $K_{\text{des}} = 1.4\times10^{-7}$ s$^{-1}$, M = 0.028 kg/mol, $m = 3.0 \times 10^{-6}$ s$^{-1}$, T = 298 K, and $\mu = 1.75 \times 10^{-5}$ Pa·s. We used these parameter values from the previous studies (Chen, 2016; Pribylov et al., 2014) because the variations of these parameter values did not noticeably impact the numerical model output at the experimental time scale, which was dominated by the apparent permeability of the inorganic matrix, $k_{al}$, because viscous flow in the inorganic matrix was the dominant mechanism that regulated core-scale pressure evolutions at the time scale of interest.

**Figure 3-6** illustrates the transient-state M³ST simulations that were used to history match the PDP-measured upstream and downstream gas pressure evolutions. Experimental conditions used in the laboratory PDP measurements are shown in **Table 3-6**. Particularly, the values of downstream gas pressure, $P_d$, and pressure difference, $\Delta P$, were experimental readings from the pressure transducers. The confining pressure in these PDP measurements was fixed at 1500 psi. In the continuum-scale simulations, the pore pressure in each grid block was solved in each time step using the implicit finite difference scheme, and then the effective stress in each grid block was determined using Equation 3-1. Therefore, the pore pressure and effective stress in each grid block continuously changed. Consequently, the kerogen permeability and inorganic matrix permeability of each grid block were updated in each time step using the updated pore pressure and effective stress values based on Equation 3-20. However, it should be noted that the variations of kerogen permeability and inorganic matrix permeability were minimal because the value of $\Delta P$ was much lower than that of $P_d$. In practice, we only needed to make minor adjustments on $h_{l0}$, $h_{k0}$, $\beta_l$, and $\beta_k$ based on their fitted values shown in Table 3-2 to match the simulated pressures to the PDP-measured pressures. Overall, the continuum-scale M³ST simulations managed to history match the PDP-measured evolutions of upstream and downstream gas pressures. This indicates that the kerogen and inorganic matrix permeabilities, determined through the microscale M³ST model fitted parameters, can successfully match the laboratory PDP experimental data using large-scale, transient-state pressure predictions based on the continuum-scale M³ST model component (i.e., Equations 3-21 to 3-23). This suggests that the M³ST model is able to account for the dominant mechanisms that regulate geomechanics and gas flow at both the microscopic and continuum scales.
Figure 3-6. History matching of PDP-measured upstream and downstream gas pressures using the continuum-scale M^3ST model component.

Table 3-6. Experimental conditions in PDP measurements. $P_u$ and $P_d$ are gas pressures in the upstream and downstream reservoirs, respectively. $\Delta P$ is the difference between $P_u$ and $P_d$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Eagle Ford 1A</th>
<th>Eagle Ford 1B</th>
<th>Mancos 1A</th>
<th>Mancos 1B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial $\Delta P$ (psi)</td>
<td>32.4</td>
<td>33.4</td>
<td>31.0</td>
<td>10.5</td>
</tr>
<tr>
<td>Initial $P_u$ (psi)</td>
<td>561.7</td>
<td>563.3</td>
<td>531.0</td>
<td>518.9</td>
</tr>
<tr>
<td>Initial $P_d$ (psi)</td>
<td>529.3</td>
<td>529.9</td>
<td>500.0</td>
<td>508.4</td>
</tr>
</tbody>
</table>

Figure 3-7 illustrates a sensitivity analysis that studied the impact of the apparent permeability of the inorganic matrix, $k_{ai}$, on upstream and downstream pressure evolutions which were simulated using the continuum-scale M^3ST model component. Three values of $k_{ai}$ were used in
the simulations, including $1 \times 10^{-2}$ mD, $1 \times 10^{-3}$ mD, and $1 \times 10^{-4}$ mD. The sensitivity analysis demonstrated that the value of $k_{ai}$ significantly influenced the variations of the upstream and downstream gas pressures. A high value of $k_{ai}$ (e.g., $1 \times 10^{-2}$ mD) resulted in relatively fast variations of the upstream and downstream gas pressures, leading to a rapid decay rate of the pressure difference. It was also noticed that the evolutions of upstream and downstream gas pressures were relatively insensitive to other parameters in the continuum-scale M$^3$ST model component. This is because in a shale core plug the overall apparent permeability is primarily contributed by the inorganic pores, and thus a higher inorganic permeability is favorable faster gas molecule migration from the upstream end to the downstream end, leading to a faster decay rate of the pressure difference.

![Figure 3-7](image)

**Figure 3-7.** Impact of the apparent permeability of inorganic matrix, $k_{ai}$, on upstream and downstream gas pressure evolutions.

### 3.7. Conclusion

A novel M$^3$ST model was developed to investigate gas transport in shales in both steady and transient states using experimental and modeling approaches. The microscale M$^3$ST model component contains a kerogen domain and an inorganic matrix domain, both of which are subjected to confining stress. Each domain has its respective geomechanical and gas transport properties. We used a laboratory PDP to measure the permeabilities of various shale cores to calibrate the microscale M$^3$ST model component and to quantify the parameters in the steady state. We used the microscale M$^3$ST model component to fit the PDP-measured apparent permeability
as a function of pore pressure under two effective stresses based on NLSF, and the fitted model parameters were able to provide accurate model predictions for other effective stresses. We then imported the parameters and petrophysical properties determined in the steady state into the transient-state, continuum-scale $M^3$ST model component, which performed history matching of the evolutions of the upstream and downstream gas pressures as a function of time at the core scale. In addition, we introduced a double-exponential empirical model, which is a powerful alternative to the $M^3$ST model to fit laboratory-measured apparent permeability under various pore pressures and effective stresses because of its simple formulation and small number of undetermined parameters. The $M^3$ST model and the research findings in this study provided critical insights into the role of the multiphysics mechanisms, including geomechanics, fluid dynamics and transport, and Klinkenberg effect, on the transport of shale gas across different spatial scales in both steady and transient states.

Acknowledgment

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Nomenclature

- $A$ model total cross-sectional area (m$^2$)
- $A_k$ kerogen cross-sectional area (m$^2$)
- $A_i$ inorganic matrix cross-sectional area (m$^2$)
- $b$ Klinkenberg coefficient (Pa)
- $b_i$ Klinkenberg coefficient of inorganic matrix (Pa)
- $b_k$ Klinkenberg coefficient of kerogen (Pa)
- $C_i$ molar concentration of free gas within the inorganic matrix (mol/m$^3$)
- $C_k$ molar concentration of free gas within the kerogen (mol/m$^3$)
- $C_\mu$ adsorbed gas molar concentration within kerogen (mol/m$^3$)
maximum monolayer adsorption of gas molar concentration within kerogen $(\text{mol/m}^3)$

$h$ effective pore width (m)

$h_i$ effective pore width in inorganic matrix (m)

$h_k$ effective pore width in kerogen (m)

$h_{i0}$ reference pore width in inorganic matrix (m)

$h_{k0}$ reference pore width in kerogen (m)

$K$ equilibrium partition coefficient $(\text{m}^3/\text{mol})$

$K_{ads}$ adsorption rate coefficient $(\text{m}^3/\text{s}/\text{mol})$

$K_{des}$ desorption rate coefficient $(1/\text{s})$

$k$ absolute permeability $(\text{m}^2)$

$k_a$ apparent permeability $(\text{m}^2)$

$k_{a0}$ reference absolute permeability $(\text{m}^2)$

$k_a$ apparent permeability of overall $(\text{m}^2)$

$k_{ai}$ apparent permeability of inorganic matrix $(\text{m}^2)$

$k_{ak}$ apparent permeability of kerogen $(\text{m}^2)$

$k_{ap}$ model-predicted apparent permeability $(\text{m}^2)$

$k_0$ reference measured permeability $(\text{m}^2)$

$M$ molar mass $(\text{kg/mol})$

$m$ mass-exchange-rate coefficient $(1/\text{s})$

$N$ number ratio

$\Delta P$ differential pressure in the PDP measurement $(\text{Pa})$

$P_c$ confining pressure $(\text{Pa})$

$P_e$ effective stress $(\text{Pa})$

$P_p$ pore pressure $(\text{Pa})$

$P_u$ upstream pressure of measured sample $(\text{Pa})$

$P_d$ downstream pressure of measured sample $(\text{Pa})$

$Q_i$ flow rate in the inorganic matrix $(\text{m}^3/\text{s})$

$Q_k$ flow rate in the kerogen $(\text{m}^3/\text{s})$

$R$ gas constant $(\text{J/mol/K})$
\( S \) model solid cross-sectional area (m\(^2\))
\( T \) absolute temperature (K)
\( t \) running time (s)
\( x \) distance in the core longitudinal direction (m)
\( z \) compressibility factor
\( \alpha \) tangential momentum accommodation coefficient
\( \beta_l \) compressibility of inorganic matrix (1/Pa)
\( \beta_k \) compressibility of kerogen (1/Pa)
\( \gamma_1 \) permeability modulus for effective stress (1/psi)
\( \gamma_2 \) permeability modulus for pore pressure (1/psi)
\( \phi \) total porosity
\( \chi \) Biot’s coefficient
\( \lambda \) free gas mean-free-path length (m)
\( \mu \) dynamic viscosity (Pa\( \cdot \)s)
\( \rho \) free gas mass density (Pa\( \cdot \)s)
\( \varepsilon_{kp} \) kerogen pore volume per unit total pore volume
\( \varepsilon_{ks} \) kerogen solid volume per unit total solid volume
\( \Gamma \) mass transfer rate between kerogen and inorganic matrix (mol/m\(^3\)/s)

Reference


Chapter 4. Experimental Investigation of Non-monotonic Fracture Conductivity Evolution and Proppant Embedment

Zihao Li, Qingqi Zhao, Yuntian Teng, Ming Fan, Nino Ripepi, Xiaolong Yin, Cheng Chen*

1 Department of Mining and Minerals Engineering, Virginia Tech, Blacksburg VA 24061
2 Petroleum Engineering Department, Colorado School of Mines, Golden CO 80401

Abstract

Based on the optimal partial-monolayer proppant concentration (OPPC) theory, significant fracture conductivity can be achieved using a much lower material cost. However, experimental validation and investigation of the OPPC theory have been extremely rare in the literature. In this study, we conducted well-controlled fracture conductivity experiments to comprehensively study the role of effective stress, proppant size, rock type, and rock hardness on the evolution of fracture conductivity as a function of increasing proppant concentration. We experimentally confirmed that the correlation between fracture conductivity and proppant concentration was non-monotonic because of a competing process between fracture permeably and fracture width. We also investigated the influences of effective stress, proppant particle diameter, rock type, and rock hardness on the OPPC and the corresponding optimal fracture conductivity (OFC). This is the first study that uses well-controlled laboratory experiments to comprehensively investigate non-monotonic fracture conductivity evolution as a function of increasing proppant concentration under various effective stresses, proppant particle sizes, rock types, and rock surface hardness. The existence of the OPPC indicates that a relatively low proppant amount can be used to form a partial-monolayer proppant pack in the fracture space, which has similar or higher fracture conductivity compared to a multilayer proppant structure. This finding has important economic implications because high-strength, ultralight-weight proppant particles can be used to form partial-monolayer proppant packs in fractures, leading to sufficiently high fracture conductivity using a much lower material cost compared to multilayer proppant structures. Our experiments illustrated that proppant embedment is the primary mechanism that causes the competing process between fracture width and fracture permeability and consequently the non-monotonic fracture
conductivity evolution as a function of increasing proppant concentration. Without proppant embedment, there will not be such a competing process and we will not observe the non-monotonic fracture conductivity evolution.

**Keywords:** non-monotonic fracture conductivity evolution, proppant, optimal partial-monolayer proppant concentration, hydraulic fracturing, experimental investigation

### 4.1. Introduction

Unconventional hydrocarbon reservoirs, including shale, tight sandstone, and oil-sand, contain massive amounts of fossil energy, but they present tremendous technical challenges to both geoscientists and engineers in terms of recovering these energy resources at an economically viable rate (Gensterblum et al., 2015; Wang et al., 2018; Hu et al., 2021). Multiple enhanced production technologies have been implemented to achieve economical production rates from these tight reservoirs (Li et al., 2016, 2019; Tan et al., 2020). Hydraulic fracturing is an enhanced oil/gas recovery process that is commonly used in extremely low permeability rocks to promote oil and/or gas flow. It typically involves injection of high-pressure water and sand into a bedrock formation through the wellbore (Montgomery and Smith, 2010; Tillman et al., 2015; Li et al., 2021; Li, C et al., 2021; Zhao et al., 2021). The hydraulically-created fractures may close during production as a result of reduced fluid pressure and increased effective stress in the fractures (Fan et al., 2019). Therefore, it is critical that proppant slurries are pumped into the induced fractures or existing fractures to increase the size and extent of the fractures and to provide long-term fracture productivity (Liang et al., 2016).

Fracture conductivity, defined as the product of fracture permeability and fracture width (Chen et al., 2015), measures the fluid flow rate through a unit length of fracture and thus is directly related to the productivity of the fracture. A sufficiently high fracture conductivity is essential for the extraction of hydrocarbons at an economically viable rate. Fracture width and permeability are both closely related to the amount of proppant particles placed in the fracture and the effective stress imposed on the proppant pack (Chen et al., 2015; Fan et al. 2019). A proppant mixture with different particle sizes is usually injected into the wellbore during the hydraulic fracturing process.
Typically, a smaller-sized proppant is injected first, followed by a larger-sized proppant. The amount of proppant placed in a fracture is measured by proppant concentration, also known as proppant areal concentration, which is defined as the proppant mass per unit of fracture surface area, usually in pounds per square foot (lb/ft$^2$) (Economides and Nolte, 2000).

The conventional method of increasing fracture conductivity is to inject a large amount of proppant particles to form a multilayer proppant structure, which enhances the fracture width. However, the material cost can be a potential issue associated with multilayer proppant structures. Some man-made ceramic proppant is relatively expensive, and the price can range from $5/lb to $10/lb (Gu et al., 2015). In the scenarios where this expensive ceramic proppant is needed, it has great economic benefits to use a lower proppant concentration in hydraulic fractures. According to the definition of fracture conductivity, it is critical to consider both fracture width and fracture permeability in order to achieve the highest fracture conductivity. Previous studies in the literature showed that effective stress, proppant compaction, and proppant embedment all have a significant impact on fracture conductivity (McGinley et al., 2015; Zhang et al., 2016; Mittal et al. 2018; Zheng et al., 2018; Fan et al., 2021). Many numerical modeling studies have also been conducted to investigate the influence of proppant compaction and embedment on fracture conductivity (Bolintineanu et al., 2017; Zhang et al., 2017; Fan et al., 2019; Bhandakkar et al., 2020).

Previous studies have shown that a partial-monolayer proppant structure can achieve significant fracture conductivity because the high porosity of the fracture space leads to high fracture permeability (Weaver et al., 2009; Kunnath et al., 2013; Raysoni and Weaver, 2013). This has significant economic implications because a partial-monolayer proppant structure results in a lower material cost. This study conducted well-controlled, comprehensive laboratory experiments to measure fracture conductivity as a function of proppant concentration ranging from 0 lb/ft$^2$ to 2 lb/ft$^2$, which accounts for the transition from a partial-monolayer proppant structure to a multilayer proppant structure. This work is the first study that uses well-controlled laboratory experiments to comprehensively investigate non-monotonic fracture conductivity evolution as a function of increasing proppant concentration under various effective stresses, proppant particle sizes, rock types, and rock surface hardness.
4.2. Theory of optimal partial-monolayer proppant concentration

Because a partial-monolayer proppant structure has high porosity, which leads to high fracture permeability, it is possible to increase the overall fracture conductivity by decreasing the proppant concentration from a multilayer proppant pack to a partial-monolayer proppant pack (Huitt and McGlothlin, 1958; Darin and Huitt, 1960). Although the optimal partial-monolayer proppant concentration theory was developed decades ago, the field applications had not been possible until the development of slickwater and ultra-lightweight proppant in hydraulic fracturing (Brannon et al. 2004). Previous experimental studies showed that partial-monolayer proppant assemblies provided higher or equivalent fracture conductivity compared to conventional multilayer proppant assemblies (Brannon et al. 2004; Gaurav et al. 2012; Bestaoui-Spurr and Hudson, 2017). Parker et al. (2012) demonstrated in field testing that a lightweight thermoplastic alloy proppant can form a partial-monolayer structure and increase the production rate. Our recent study (Fan et al., 2019) was the first one to elucidate the multiphysics processes that lead to non-monotonic fracture conductivity evolution, which is caused by non-monotonic fracture permeability evolution and a competing process between fracture permeability and fracture width. Specifically, Fan et al. (2019) combined laboratory penetrometer experiments with the discrete element method (DEM) and lattice Boltzmann (LB) method to track the detailed evolutions of fracture permeability and fracture width when the proppant pack developed from a partial-monolayer structure to a multilayer structure; the result validated the theory that explained why non-monotonic fracture conductivity evolution occurs, which will be described below.

Figure 4-1 illustrates proppant embedment mitigation in a fracture with an increasing proppant concentration. Proppant crushing is not considered in this case because some ceramic proppant can withstand external stresses up to 10,000 psi (Liang et al., 2016). The detailed evolutions of fracture width, porosity, permeability, and conductivity go through four distinct stages with the increasing proppant concentration, which are demonstrated in Table 4-1.

Stage 1: Because of the low proppant concentration in this stage, proppant embedment is significant at the beginning. The increasing number of proppant particles entering the fracture space significantly mitigates proppant embedment, thereby increasing the fracture width and fracture permeability. However, on the other hand, the increasing proppant particles occupy the
empty fracture space, leading to reduced fracture porosity, which has a negative effect on fracture permeability. Because the permeability increase resulting from fracture width increase surpasses the permeability loss resulting from fracture porosity reduction, the net effect is that fracture permeability increases, which results in increasing fracture conductivity because both fracture permeability and width increase in this stage.

**Stage 2:** With continuously increasing proppant particles placed in the fracture space, the permeability loss resulting from fracture porosity reduction surpasses the permeability gain resulting from fracture width increase, leading to decreased fracture permeability. However, when fracture permeability begins to decline, its reduction rate is slower than the rate of fracture width increase. Therefore, the fracture conductivity, which is the product of fracture permeability and fracture width, still increases in this stage.

**Stage 3:** When the rate of permeability reduction surpasses the rate of fracture width increase with increasing proppant particles in the fracture, fracture conductivity reaches the local maximum and then starts to decline. The proppant concentration corresponding to the local maximum fracture conductivity is referred to as the optimal partial-monolayer proppant concentration (OPPC), and the local maximum fracture conductivity is referred to as the optimal fracture conductivity (OFC). Stage 3 continues developing until the proppant particles cover the entire rock surface, leading to a full-monolayer proppant pack.

**Stage 4:** In this stage, the proppant pack develops from a full-monolayer structure to a multilayer structure. Because the proppant assembly has become a fully-packed porous medium, its porosity and permeability are dominated primarily by the average grain and pore sizes and insensitive to the fracture width (Chen et al. 2008; 2009). Therefore, in Stage 4 fracture
permeability and porosity will not vary significantly with increasing proppant particles placed in the fracture.

**Figure 4-1.** Mitigation of proppant embedment in a fracture with an increasing proppant concentration (i.e., from left to right). This picture was modified from Fan et al. (2019).

**Table 4-1.** Evolutions of fracture width, $w_f$, porosity, $\phi$, permeability, $k$, and conductivity ($k \cdot w_f$) with an increasing proppant concentration. The arrows “↑”, “↓”, and “→” denote “increase”, “decrease”, and “stay constant”, respectively. This table is modified from Fan et al. (2019).

<table>
<thead>
<tr>
<th>Stage</th>
<th>$w_f$</th>
<th>$\phi$</th>
<th>$k$</th>
<th>$k \cdot w_f$</th>
<th>Geometry of proppant pack</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stage 1</td>
<td>↑</td>
<td>↓</td>
<td>↑</td>
<td>↑</td>
<td>Partial monolayer</td>
</tr>
<tr>
<td>Stage 2</td>
<td>↑</td>
<td>↓</td>
<td>↓</td>
<td>↑</td>
<td>Partial monolayer</td>
</tr>
<tr>
<td>Stage 3</td>
<td>↑</td>
<td>↓</td>
<td>↓</td>
<td>↓</td>
<td>Partial monolayer to monolayer</td>
</tr>
<tr>
<td>Stage 4</td>
<td>↑</td>
<td>→</td>
<td>→</td>
<td>↑</td>
<td>Monolayer to multiplayer</td>
</tr>
</tbody>
</table>

Fan et al. (2019) combined laboratory penetrometer experiments with DEM/LB numerical modeling to demonstrate non-monotonic fracture conductivity evolution when the proppant assembly developed through the above-mentioned four stages, as shown in Figure 4-2. Particularly, Figure 4-2 illustrates the roles of proppant particle size and effective stress on the OFC and OPPC. Figure 4-2 illustrates that when particle size stays constant, the OFC declined with increasing effective stress because of the reduced fracture permeability and width under higher effective stress. In addition, the OPPC increased (i.e., was shifted to the right) with increasing effective stress because keeping the fracture open under higher effective stress required
more proppant particles. When the effective stress is sufficiently high, the OPPC will approach the full-monolayer proppant concentration because in this case a partial-monolayer proppant structure is unable to keep the fracture open. When the effective stress stays constant, the proppant assembly with 0.63 mm particle diameter had the maximum OFC, and the proppant assembly with 0.32 mm particle diameter had the minimum OFC. This was because a partial-monolayer proppant assembly having larger particle diameter led to larger fracture permeability and width. Fan et al. (2019) obtained these findings using an experiment/simulation-integrated workflow. The objective of this paper is to conduct well-controlled laboratory experiments to study the role of proppant concentration, effective stress, proppant particle size, rock surface hardness, and rock type on the evolution of fracture conductivity.

**Figure 4-2.** Roles of proppant particle size and effective stress on the fracture conductivity versus proppant concentration curve. Three particle sizes (0.63 mm, 0.45 mm, and 0.32 mm) and four effective stresses (1,000, 2,000, 4,000, and 6,000 psi) are considered, resulting in twelve curves. This picture is from Fan et al. (2019).

### 4.3. Laboratory equipment, materials, and workflow

**Figure 4-3** is the schematic plot of the fracture conductivity cell used in the laboratory measurements, which can measure fracture conductivity under different closure pressures and
temperatures. The fracture conductivity, $C$, according to the *API RP-19D* standard (2008), is defined as:

$$C = k w_f = \frac{\mu Q L w_f}{\Delta p \cdot A} = \frac{\mu Q L}{\Delta p \cdot h}$$

where $C$ is fracture conductivity ($m^3$); $k$ is fracture permeability ($m^2$); $w_f$ is fracture width ($m$); $h$ is the size of the longer dimension of the fracture cross section ($m$); $A$ is the area of fracture cross section and equal to $w_f h$ ($m^2$); $\mu$ is fluid viscosity ($Pa\cdot s$); $Q$ is flow rate ($m^3/s$); $L$ is the length over which the pressure difference is measured ($m$); $\Delta p$ is the pressure difference ($Pa$). Note that $\mu$, $L$, and $h$ in Equation 4-1 are constants. The measured flow rate and pressure difference are then imported into Equation 4-1 to calculate the fracture conductivity.

*Figure 4-3.* Schematic plot of fracture conductivity cell. The two rock slabs and the proppant particles sandwiched between them are placed in the proppant cell, which is subjected to a closure pressure up to 17,000 psi (precision 0.2%). The electrical heater provides a testing temperature up to 250°C (precision 0.4%). This picture was provided by the manufacturer.

*Figure 4-4* illustrates the diameter distributions of the ceramic proppant used in the experiments. The proppant material was ceramic. We used Berea sandstone and Marcellus shale as the rock slabs in the testing. Each rock slab was of 7 inches in length, 1.5 inches in width, and
0.5 inch in thickness. The proppant particles are sandwiched between two rock slabs for conductivity testing following the API standard (*API RP-19D* 2008).

![Diameter distributions of ceramic proppant having a mesh size of a) 20/40, and b) 40/70 used in fracture conductivity measurements.](image)

**Figure 4-4.** Diameter distributions of ceramic proppant having a mesh size of a) 20/40, and b) 40/70 used in fracture conductivity measurements.

### 4.4. Results and discussion

**Figure 4-5** illustrates measured fracture conductivity as a function of closure pressure. Seven proppant concentrations (0, 0.02, 0.06, 0.1, 0.2, 1, and 2 lb/ft²) and seven closure pressures (500, 1000, 2000, 3000, 4000, 5000, and 6000 psi) were used, leading to 47 conductivity measurements in each plot. We conducted these measurements on four different combinations of rock and proppant: (1) Berea sandstone + mesh-20/40 ceramic proppant, (2) Berea sandstone with 30 days of water soaking + mesh-20/40 ceramic proppant, (3) Berea sandstone + mesh-40/70 ceramic proppant, and (4) Marcellus shale + mesh-20/40 ceramic proppant. These tests aimed to study the role of different rock types, rock surface hardness, and proppant size on fracture conductivity under increasing closure pressures.

Figure 4-5 demonstrates that overall the proppant fracture conductivity decreased with an increasing effective stress because of the proppant embedment into rock surfaces. In addition, it was observed that the fracture conductivity measured on the sandstone slabs did not change noticeably when the effective stress was higher than 5,000 psi. In comparison, the fracture conductivity measured on the shale slabs did not change noticeably when the effective stress was
higher than 3,000 psi. This was because proppant embedment did not develop noticeably when the effective stress was sufficiently high, leading to a relatively steady proppant structure in the fracture under high stresses. In these experiments, a particularly proppant concentration was placed in the fracture space between two rock slabs, and then the rock slabs were pushed toward each other to increase the effective stress stepwise to measure the fracture conductivity values under seven stress levels (i.e., 500, 1000, 2000, 3000, 4000, 5000, and 6000 psi). At the end of the test (i.e., after the 6,000 psi test), significant proppant embedment and development of microscale fractures on rock surfaces were observed. Thus, the rock slabs cannot be reused, and another pair of intact rock slabs were used for the test with a different proppant concentration. Therefore, seven pairs of rock slabs were used in each plot shown in Figure 4-5 because we conducted the experiments under seven different proppant concentrations (i.e., 0, 0.02, 0.06, 0.1, 0.2, 1, and 2 lb/ft²). It is interesting to observe that the fracture conductivity for 0.02 lb/ft² in Figure 4-5a suddenly approached zero when the effective stress was higher than 4,000 psi. This was attributed to the bedding planes of weakness in this particular pair of Berea sandstone slabs, which led to rapid development of microscale fractures on the rock surface when the effective stress rose to 4,000 psi. These fractures on rock surface caused significant proppant embedment, which has been observed in our previous experimental study (Chen et al., 2015) that used X-ray computed tomography scanning to investigate proppant particle embedment on rock surfaces.

Comparison between Figures 4-5a and 4-5b suggests that the 30-day water soaking caused rock surface softening and clay swelling, which led to significant proppant embedment and consequently faster fracture conductivity decline under increasing effective stress. Comparison between Figures 4-5a and 4-5c shows that proppant particle diameter impacted fracture conductivity noticeably. With the same proppant concentration and under the same effective stress, the fracture conductivity of proppant having larger diameter (i.e., mesh-20/40) was generally higher than that of the proppant having smaller diameter (i.e., mesh-40/70). This was because larger proppant particles provided larger average pore size in the proppant assembly structure, leading to higher fracture permeability. In addition, comparisons between Figures 4-5a and 4-5d indicates that fracture conductivity measured on the Berea sandstone slabs was noticeably higher than that measured on the Marcellus shale slabs under the same proppant concentration and effective stress. This was because the shale slabs had a higher clay content compared to the
sandstone slabs, leading to larger proppant embedment depth and consequently lower fracture conductivity in the shale fracture.

**Figure 4-5.** Measured fracture conductivity as a function of effective stress for a) Berea + mesh-20/40 proppant, b) Berea (30-day water soaking) + mesh-20/40 proppant, c) Berea + mesh-40/70 proppant, and d) Marcellus + mesh-20/40 proppant.

**Figure 4-6** illustrates measured fracture conductivity as a function of proppant concentration. Non-monotonic fracture conductivity evolution as a function of increasing proppant concentration can be clearly observed in all of the four plots. This is the first study that uses well-controlled laboratory experiments to comprehensively investigate non-monotonic fracture conductivity evolution under various effective stresses, proppant particle sizes, rock types, and rock surface hardness. The existence of the OPPC indicates that a relatively low proppant amount can be used
to form a partial-monolayer proppant pack in the fracture space, which has similar or higher fracture conductivity compared to a multilayer proppant structure. This finding has important economic implications because high-strength, ultralight-weight proppant particles can be used to form partial-monolayer proppant packs in fractures, leading to sufficiently high fracture conductivity using a much lower material cost compared to multilayer proppant structures.

Figure 4-6 shows that the OPPC value was around 0.06 lb/ft² for all the four groups of experiments, which was close to the value (i.e., 0.04 lb/ft²) found in our previous study that was based on laboratory penetrometer experiments and DEM/LB modeling (Fan et al., 2019). Overall, the OFC decreased under increasing effective stress because a higher effective stress imposed on the proppant pack led to larger proppant embedment depth and tighter particle packing, which reduced both the fracture permeability and fracture width, thereby decreasing the fracture conductivity. Figure 4-7 illustrates the measured OFC as a function of increasing effective stress for the four groups of experiments. Furthermore, the OPPC had the tendency to increase under an increasing effective stress. For example, the OPPC value in Figure 4-6a (i.e., sandstone + mesh-20/40 proppant) shifted from 0.06 lb/ft² to 0.1 lb/ft² when the effective stress reached 6,000 psi. This suggested that more proppant particles were required to open the fracture and to achieve the OFC under a higher effective stress. However, this tendency was observed only in Figure 4-6a; the OPPC did not change in the other three groups of experiments. This implied that the effective stress had a more noticeable impact on the OFC than on the OPPC.

Particularly, Figure 4-6a shows that mesh-20/40 proppant sandwiched between two Berea sandstone slabs and subjected to an effective stress of 1,000 psi led to an OPPC value of 0.06 lb/ft² and an OFC value around 2,000 mD·ft. In comparison, our previous work (Fan et al., 2019) combined laboratory penetrometer experiments with DEM/LB numerical modeling to find that the OPPC and OFC values were 0.04 lb/ft² and 6,000 mD·ft, respectively, for proppant having 0.63 mm diameter that were sandwiched between two sandstone slabs and subjected to an effective stress of 1,000 psi, as shown in Figure 4-2. The differences between these two studies resulted from different proppant particle size distributions and rock surface roughness. In the previous study (Fan et al., 2019), the proppant particles had homogeneous diameter, which was equal to 0.63 mm, the average size of mesh-20 and mesh-40 particles. Conversely, in this laboratory experimental study, the mesh-20/40 proppant particles had a heterogeneous size distribution between mesh-40 and mesh-20 diameters. Therefore, in the laboratory experiments, smaller particles can fill in the pore
space between larger particles, leading to lower fracture porosity and consequently lower fracture conductivity. In addition, the DEM/LB numerical modeling in our previous study (Fan et al., 2019) assumed smooth rock surfaces, which was favorable for fluid flow in the fracture. Conversely, in the laboratory experiments the Berea sandstone slabs had surface roughness, which negatively impacted the fracture permeability and consequently reduced the fracture conductivity. The rough rock surface and heterogeneous proppant size caused that the OFC measured in the laboratory (i.e., 2,000 mD·ft) was lower than that measured using DEM/LB modeling (i.e., 6,000 mD·ft). Furthermore, because proppant size was heterogeneous in the laboratory experiments, only the largest proppant particles were in contact with both rock surfaces in the partial-monolayer structure; the smaller particles were not in contact with both rock surfaces and thus were not responsible for keeping the fracture open. As a consequence, only a portion of the proppant particles were subjected to the closure stress, and thus more proppant particles were needed to achieve the same fracture conductivity. This explained why in the laboratory experiments the observed OPPC value (i.e., 0.06 lb/ft²) was higher than that found by DEM/LB modeling (i.e., 0.04 lb/ft²).
Figure 4-6. Measured fracture conductivity as a function of proppant concentration for a) Berea + mesh-20/40 proppant, b) Berea (30-day water soaking) + mesh-20/40 proppant, c) Berea + mesh-40/70 proppant, and d) Marcellus + mesh-20/40 proppant.

Figure 4-7. Measured OFC as a function of effective stress.

Figure 4-8 and Figure 4-9 illustrate the rock slab surfaces with proppant concentrations of 0.06 lb/ft² and 2 lb/ft², respectively, after experimental testing of 6,000 psi. These rock slabs had
dimensions of 7 inches in length, 1.5 inches in width, and 0.5 inch in thickness. Particularly, proppant concentrations of 0.06 lb/ft\(^2\) and 2 lb/ft\(^2\) resulted in a partial-monolayer proppant assembly and multilayer proppant assembly in the fracture, respectively. It was observed that the embedment depth for the proppant concentration of 0.06 lb/ft\(^2\) (i.e., the partial-monolayer structure) was larger than that for the proppant concentration of 2 lb/ft\(^2\) (i.e., a multilayer structure). This was because the smaller amount of proppant particles in the partial-monolayer proppant structure led to higher mechanical loading imposed on each individual proppant particle, resulting in a larger embedment depth. This suggests that it is extremely important to account for the effect of proppant embedment on rock surfaces in a partial-monolayer proppant assembly. Proppant embedment is the primary mechanism that causes the competing process between fracture width and fracture permeability and consequently the non-monotonic fracture conductivity evolution, as shown in Table 4-1. Without proppant embedment, there will not be such a competing process and we will not observe non-monotonic fracture conductivity evolution in the laboratory.

Some numerical studies in the literature have investigated fracture conductivity provided by a partial-monolayer proppant structure based on the assumption that the proppant particles and rock surfaces are ideal, rigid materials, which means that proppant embedment on rock surfaces does not occur. Based on this no-embedment assumption, the simulated fracture conductivity decreased monotonically with an increasing proppant concentration when the proppant assembly is a partial-monolayer structure. In other words, the OPPC in this case is to place only one proppant particle in the fracture provided that the proppant size is homogeneous, and the fracture width is equal to one proppant diameter. Placing more proppant particles in the fracture will not help increase the fracture width because there is no proppant embedment. Instead, the increasing number of proppant particles in the fracture blocks the pore space, leading to reduced fracture porosity, permeability, and consequently fracture conductivity. In reality, however, using only one proppant particle in the fracture is never the optimal solution because the compressive effective stress results in proppant embedment, as we observed in this experimental study. This, again, emphasizes that it is critical to account for proppant embedment on rock surfaces when studying fracture conductivity under various proppant concentrations and effective stresses.
Figure 4-8. Rock slab surfaces after the 6,000 psi testing with a ceramic proppant concentration of 0.06 lb/ft².

Figure 4-9. Rock slab surfaces after the 6,000 psi testing with a ceramic proppant concentration of 2 lb/ft².

4.5. Conclusion

Comprehensive fracture conductivity experiments were conducted to study the role of effective stress, proppant size, rock type, and rock hardness on the evolution of fracture conductivity as a function of increasing proppant concentration. We experimentally confirmed that the correlation
between fracture conductivity and proppant concentration was non-monotonic because of a competing process between fracture permeability and fracture width. A relatively good agreement was observed between the experimentally-measured and our previous model-derived fracture-conductivity versus proppant-concentration curves. We also investigated the influences of effective stress, proppant particle diameter, rock type, and rock hardness on the OPPC and OFC.

This is the first study that uses well-controlled laboratory experiments to comprehensively investigate non-monotonic fracture conductivity evolution as a function of increasing proppant concentration under various effective stresses, proppant particle sizes, rock types, and rock surface hardness. The existence of the OPPC indicates that a relatively low proppant amount can be used to form a partial-monolayer proppant pack in the fracture space, which has similar or higher fracture conductivity compared to a multilayer proppant structure. This finding has important economic implications because high-strength, ultralight-weight proppant particles can be used to form partial-monolayer proppant packs in fractures, leading to sufficiently high fracture conductivity using a much lower material cost compared to multilayer proppant structures.

Proppant embedment is the primary mechanism that causes the competing process between fracture width and fracture permeability and consequently the non-monotonic fracture conductivity evolution as a function of increasing proppant concentration. Without proppant embedment, there will not be such a competing process and we will not observe the non-monotonic fracture conductivity evolution. Some numerical studies in the literature, for the sake of simplicity, assumed that the proppant particles and rock surfaces are ideal, rigid materials, which means that proppant embedment on rock surfaces does not occur. Based on this no-embedment assumption, the simulated fracture conductivity decreased monotonically with an increasing proppant concentration when the proppant assembly is a partial-monolayer structure. In other words, the OPPC in this case is to place only one proppant particle in the fracture because it provides the highest fracture porosity and permeability. In reality, however, using only one proppant particle in the fracture is never the optimal solution because the compressive stress results in proppant embedment. This, again, emphasizes that it is critical to account for proppant embedment on rock surfaces when studying fracture conductivity under various proppant concentrations and effective stresses.
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Reference


Chapter 5. Reservoir Characterization in Central Appalachia: an Experimental Investigation

Zihao Li, Cigdem Keles, Nino Ripepi, Cheng Chen, Virginia Tech

Abstract

The Field Laboratory for Emerging Stacked Unconventional Plays (ESUP) in Central Appalachia project will investigate and characterize the resource potential for multi-play production of emerging unconventional reservoirs in Central Appalachia. The benefits of developing these ESUPs wisely include a smaller surface footprint, reduced infrastructure requirements, and lower development costs. In this paper, we conducted a series of laboratory experiments to investigate the reservoir characterization, including permeability, fracture conductivity, and rock surface roughness. The experimental results from more than 60 shale plugs indicated that the average core permeability was at the magnitude of $10^{-2}$ mD to $10^{-4}$ mD and inversely proportional to the increasing confining pressure. We also investigated the influences of closure pressure, rock hardness, proppant particle, and gas/liquid measurement on fracture conductivity. The surface of the rock slabs in the fracture conductivity measurement was scanned by the profilometer to illustrate the rock surface roughness and the effect of proppant embedment on the conductivity. Additionally, we selected four targeted formations in the field and used these experimental data to fit the empirical exponential model. The exponential empirical model can overall capture the apparent permeability variation as a function of effective stress. The laboratory experimental facts and the research findings in this study provided critical insights into the reservoir characterization in Central Appalachia and will benefit the reservoir development using non-aqueous fracturing techniques such as CO$_2$ and advanced proppant technologies in the future.

Keywords: reservoir characterization, core analysis, fracture conductivity, Central Appalachia,

5.1. Introduction

Extraction of hydrocarbon energy resources from shale, a fine-grained, low-permeability geological formation, has altered the global energy landscape. The development of hydraulic
fracturing and horizontal drilling has made shale oil and gas recovery economically viable (Montgomery and Smith, 2010; Stringfellow et al., 2014). In comparison to conventional enhanced oil and gas recovery (Li et al., 2016; Zhao et al., 2017; Tang et al., 2019), unconventional resource development is generally associated with a variety of benefits such as job growth and improved infrastructure and municipal developments.

Central Appalachia is rich in hydrocarbon resource plays, including coalbed methane, shale, and other unconventional reservoirs. Numerous of these plays are vertically stacked, allowing a single well or group of wells in close proximity to producing from multiple reservoirs concurrently. Numerous reservoirs produce less than 50,000 barrels of oil equivalent (BOE) per day and thus qualify as Emerging Stacked Unconventional Plays (ESUPs). The benefits of developing these ESUPs wisely include a smaller surface footprint, reduced infrastructure requirements, and lower development costs (Kuuskraa et al., 2017). While technological advancements continue to expand the nation's portfolio of economically recoverable hydrocarbons (Energy Information Administration, 2017), uncertainty and technical challenges deter the level of investment required to develop Central Appalachia's ESUPs and reverse the damage caused by the region's disproportionate economic downturn in comparison to the rest of the country (ARDI Working Group, 2010). While economic development is critical, the region also contains "some of the world's best remaining examples of diverse, intact, and connected temperate forests and freshwater streams" that are accessed daily by tens of millions of people. The combination of abundant energy and world-class ecological resources in Central Appalachia "creates one of the most difficult contemporary conservation situations anywhere on the planet" (Nature Conservancy, 2014).

Numerous previous studies have been conducted on the relationship between effective stress and shale permeability using both laboratory experiments and analytical models (Akkutlu & Fathi, 2012; Alnoaimi & Kovscek, 2013; Chen, 2016; Ghanbarian & Javadpour, 2017; Guo, Ma, & Tchelepi, 2018) The permeability of a shale formation is generally expected to decrease with increasing effective stress due to compressed pore space and the closure of microscale fractures. Aside from effective stress, the apparent (measured) permeability of a shale formation is affected by pore pressure if the testing fluid is a gas. This phenomenon, known as the Klinkenberg effect, is caused by nanoscale pores in shale. Klinkenberg developed the gas slippage theory (i.e., Klinkenberg effect) in 1941 to explain the increase in measured apparent permeability for gas flow in nanopores (Klinkenberg, 1941). When the mean free path of the measured gas molecules is
longer than the diameter of the nanopore through which they travel, the Klinkenberg effect occurs. As a result, gas molecules collide with the pore walls more frequently than with one another, causing Knudsen diffusion and a slip-velocity boundary condition on the pore walls, increasing the apparent permeability of the nanopore. As a result, the Klinkenberg effect has a significant impact on gas flow in porous media with extremely low permeability (Wu et al., 1998). Although the Klinkenberg effect may not affect apparent permeability under in situ high reservoir pressures (Civan, 2020), it is important in interpreting laboratory measurements because pore pressures used in laboratory experiments can be at levels where the Klinkenberg effect is noticeable (Soeder, 1988; Li et al., 2020).

The product of fracture permeability and fracture width is defined as fracture conductivity (Chen et al., 2015). It denotes the absolute rate of fluid flow between fractures in two adjacent grid blocks of a specified size. A fracture conductivity sufficient for economically extracting hydrocarbons from reservoirs is required. The width and permeability of the fracture are proportional to the amount of proppant used and the effective stress applied to the proppant pack (Chen et al., 2015). A multi-particle proppant mixture is injected into the wellbore during the hydraulic fracturing process. Typically, a smaller proppant is injected first, followed by a larger proppant. A critical combination of small and large proppant sizes results in the highest well productivity index in ultralow permeability formations such as shales (Hoss et al., 2017). Proppant concentration (alternatively called proppant area concentration) is the amount of proppant placed in a fracture. It is defined as the mass of proppant per unit of fracture surface area, typically in pounds per square foot (lb/ft²) (Economides and Nolte, 2000).

The Field Laboratory for Emerging Stacked Unconventional Plays (ESUP) in Central Appalachia project will study and assess the resource potential for multi-play production of emerging unconventional reservoirs in Central Appalachia. By drilling and coring a deep vertical stratigraphic test well to a depth of 15,000 feet, this project will improve the characterization of several developing unconventional pay zones inside the proven Nora Gas Field. The purpose of this paper, as the core analysis component of the ESUP project, is to develop a workflow from multiple laboratory experiments for analyzing rock samples extracted from the deep ESUP well, including investigating the role of rock surface roughness in maintaining adequate fracture conductivity by "self-propping" when no proppant is present in the fracture, and the difference in the measured fracture conductivity values between DI water and nitrogen gas. The findings of this
study aided in the creation of a fundamental knowledge of reservoir characteristics in Central Appalachia and will aid in the establishment of an optimal process for field development.

5.2. Materials and Experimental Equipment

Figure 5-1a illustrates the picture of the PDP equipment setup, which provides a convenient and dynamic approach for measuring the apparent permeability of tight rocks (Jones, 1997). The permeability of the core plug can then be calculated based on the decay rate of differential pressure following the standard procedure of PDP analysis (Dicker and Smits, 1988; Jones, 1997). Figure 5-1b is the picture of the fracture conductivity cell in the measurement. The proppant fracture conductivity can be measured under multiple closure pressures and temperatures through this equipment. In the fracture conductivity experiment, fracture conductivity is the product of proppant pack permeability and fracture width; and the fracture width is chamber height’s displacement with and without proppant recorded by a displacement transducer. The fracture conductivity, $C$, according to API RP-19D (2008) is defined as:

$$C = k w_f = \frac{\mu \cdot Q \cdot L \cdot w_f}{\Delta p \cdot A} = \frac{\mu \cdot Q \cdot L}{\Delta p \cdot h}$$

(5-1)

where $C$ is fracture conductivity (m$^3$); $k$ is fracture permeability (m$^2$); $w_f$ is fracture width (m); $h$ is the size of the longer dimension of the fracture cross section (m); $A$ is the area of fracture cross section and equal to $w_f h$ (m$^2$); $\mu$ is fluid viscosity (Pa·s); $Q$ is flow rate (m$^3$/s); $L$ is the length over which the pressure difference is measured (m); $\Delta p$ is the pressure difference (Pa). Note that $\mu$, $L$, and $h$ in Equation 5-1 are constants. The measured flow rate and pressure difference are then imported into Equation 5-1 to calculate the fracture conductivity.
Our microscale component of the M³ST model is based on the first principle and thus provides mechanistic insight into the role of geomechanics, fluid dynamics, and Klinkenberg effect on the apparent permeability of shale. In practice, empirical correlations have been developed to describe the relation between measured permeability and effective stress. The dependence of shale permeability on the variation of effective stress can be described using an exponential function, which is shown below.

\[ k_a = k_0 \cdot e^{-\gamma_1 (P_e - P_{e0})} \]  

(5-2)

where \( k_a \) is the apparent permeability (m²) measured under effective stress, \( P_e \); \( \gamma_1 \) is the shale-permeability moduli (psi⁻¹) with respect to effective stress; \( k_0 \) is the reference apparent permeability (m²); \( P_{e0} \) is the reference effective stress under which \( k_0 \) is measured.

Figure 5-2 illustrates the non-smooth-surface shale slabs were drilled from the Eagle Ford Shale and Mancos Shale. Specifically, the rock block was cut around a natural fracture or bedding plane, which was later used as the contact face between the two non-smooth slabs. The closure pressures in the following experiments were 500 psi, 1000 psi, 1500 psi, and 2000 psi.

After finishing the material preparation, we can conduct the experiment following the workflow. The conductivity measurement workflow includes eight steps: 1) Place proppant in the
cell chamber's fracture space; 2) Saturate the pore space between proppant particles with DI water; 3) Turn on the flow pump to get the desired flow rate in the fracture. 4) Turn on the back-pressure regulator and set the back pressure. 5) Turn on the closure pressure hydraulic pump and apply the desired closure pressure to the chamber. 6) Wait a few minutes for the chamber's closure pressure and flow rate to stabilize. 7) Begin measuring the fracture conductivity value; 8) Record the measured fracture conductivity value. These proppant fracture conductivity measurements used DI water at the temperature of 298 K.

5.3. Results and Discussion

The fracture conductivity measured as a function of effective stress for rock slabs from the a) Eagle Ford; b) Mancos; and c) Marcellus is shown in Figure 5-3. The fracture conductivity measurements were performed on non-smooth-surface shale samples without the use of proppant between the two rock slabs. The purpose of these experiments is to determine the effect of rock surface roughness on fracture conductivity provided by "self-propping" fractures under various closure pressures. The experiments used nitrogen and DI water. The surface roughness of the rock samples is noticeable, which self-props the fracture without the use of proppants. After conducting gas-based (N₂) measurements at 500, 1,000, 1,500, and 2,000 psi without inserting proppant into the fracture, liquid-based (DI water) measurements at the same pressures were conducted. The equipment chamber was then opened, ceramic proppant particles at a concentration of 2 lb/ft² were placed between the two slabs, and the gas-based and liquid-based measurements were repeated under the same closure pressures.

Under gas measurements, an increase in apparent (i.e., measured) fracture conductivity was observed, which was attributed to the Klinkenberg effect. As a result, we performed three separate gas measurements for each closure pressure, each with a different gas pore pressure, in order to obtain the absolute (true) fracture conductivity. The three conductivity values obtained under three different gas pore pressures formed a straight line; we then used the extrapolation method to extend this line to the left until it intersected with the y-axis to obtain the conductivity value under the infinitely high pore pressure scenario, which mitigated the effect of the Klinkenberg effect and thus yielded the fracture's absolute conductivity. The following images illustrate the measured fracture conductivity values as a function of the closure pressure applied to the fracture.
According to these laboratory conductivity measurements, the fracture conductivity values were approximately three to eight times greater with ceramic proppant (2 lb/ft$^2$) placed in the fracture space than without proppant placement. Thus, even though the roughness of the rock surface provides some fracture conductivity via "self-propping", it is preferable to have proppant placed in the fractures. Additionally, for the proppant-filled fracture, the gas conductivity values were greater than the water conductivity values. This was due to the softening of the rock surfaces caused by the contact of the shale slabs with the DI water, which resulted in clay swelling in the shale and proppant embedment into the rock surfaces. As a result, conductivity decreased more rapidly against the closure pressure when DI water was used as the testing fluid. Additionally, when no proppant was placed in the fracture space, the decrease in fracture conductivity with increasing closure pressure was not noticeable. This indicated that when the primary propping mechanism was self-propping via surface roughness, the fracture conductivity was insensitive to the closure pressure.

**Figure 5-3.** Measured fracture conductivity as a function of closure pressure for the shale slabs.
Figure 5-4 illustrates that the four groups of profilometer measurements were conducted by Zygo Corporation for the non-smooth-surface shale slabs, which were involved in our previous fracture conductivity measurements. Each shale slab was scanned by the profilometer of Zygo Corporation and stitched using a 264-site stitch with a $5 \times$ Super Long Working Distance (SLWD) objective. The full surface of each slab is stitched together into a single height map with over 265 million data points. Another measurement with a smaller stitch ($3 \times 3$) was conducted to detect the signature of the proppant on the shale slab surface. The objective of these profilometer measurements is to investigate proppant embedment on the non-smooth slab surface.

Because of sample crushing from the high closure pressure, one of the four samples was not qualified for the profilometer scans. The other three samples were scanned by two sizes of stitches, which were full-sample stitches and $3 \times 3$ (magnification) stitches. The first sample was from the Mancos shale, and another two samples were from the Eagle Ford shale. The $Sa$ value is the extension of $Ra$ (arithmetical mean height of a line) to a surface. It indicates, as an absolute value, the difference in the height of each point relative to the arithmetical mean of the surface. This parameter is generally used to evaluate surface roughness. From Figure 5-4, the $Sa$ values of Mancos slab #1, Eagle Ford slab #1, and Eagle Ford slab #2 were 882.868 $\mu$m, 202.647 $\mu$m, and 209.968 $\mu$m, respectively. We found that Eagle Ford slabs were noticeably smoother than Mancos slab because of the significantly lower $Sa$ value (~200 microns vs ~ 800 microns). This result is consistent with Figure 5-1, showing that the Mancos sample had noticeably higher surface roughness than the Eagle Ford sample.

![Figure 5-4](image)

**Figure 5-4.** Three shale samples scanned by Zygo Corporation.
The surface scanning were also conducted using 3×3 stitches to detect the signatures of the proppant on the surface of the rocks. They applied a height clip to remove the average surface of the data, and as a consequence, the top images yielded the maps in the second row of Figure 5-5. By doing further analysis, we can spatially group the pits to within mesh-20/40 lateral dimensions, which was the size of the proppant we used in the conductivity experiments. We then can see the highlighted pits in the first row of Figure 5-5, which can potentially be proppant. We found that proppant embedment was evident on the slab surfaces. Considering that the proppant concentration was 2 lb/ft² in the experiments, the embedment was confirmed to have a critical impact on the fracture conductivity.

From these profilometer measurements with smaller stitches, it is demonstrated that the surface roughness of the non-smooth rock slabs can be quantified using the Sa value. The Sa values for the two pairs of shale slabs had a difference of four times, which will have a significant impact on the fracture conductivity. In the smaller stitched samples, the roughness difference was dominated by longer spatial wavelengths (> mm scale). Proppant embedment was evident on the slab surfaces. The fracture width was decreased because of proppant embedment, which led to lower fracture conductivity.
a) Mancos slab #1  b) Eagle Ford slab #1  c) Eagle Ford slab #2

**Figure 5-5.** The smaller (3×3) stitch of three samples.

We also conducted permeability measurements of shale plugs using our laboratory PDP equipment. We measured 15 shale plugs from multiple depths (**Figure 5-6**) in our laboratory, and another 45 shale plugs from the same field were measured by Schlumberger Limited in their laboratory (**Figure 5-7**). The PDP-measured results demonstrate that the permeabilities of 15 samples fell in the range between $10^{-2}$ mD and $10^{-4}$ mD. The higher the confining pressure, the lower the permeability. However, greater depth does not always equate to decreased permeability. Other variables, such as mineral composition and lithology, may also influence the observed permeability significantly. Additionally, the measured results from Schlumberger are highly consistent with ours.

**Figure 5-6.** PDP-measured permeability as a function of effective stress for 15 samples from the ESUP field in the laboratory of Virginia Tech.
Figure 5-7. PDP-measured permeability as a function of effective stress for 45 samples by Schlumberger Limited from the ESUP field.

Based on the permeability result under the confining stresses of 500 psi, 1500 psi, 3000 psi, and 5000 psi by PDP, we selected four targeted formations in the field and used these experimental data to fit the empirical exponential model. Figure 5-8 illustrates the laboratory PDP measurements as well as exponential empirical model (i.e., Equation 5-2) fitting based on non-linear least square fitting for all the four shale core samples. It is clear that the exponential empirical model can overall capture the apparent permeability variation as a function of effective stress.
Figure 5-8. PDP-measured apparent permeability (scatter data points) as a function of effective stress. The solid-line curves were obtained by fitting the exponential empirical model to the PDP measurements based on non-linear least square fitting.

5.4. Conclusions

A series of laboratory experiments were conducted to characterize the reservoir, including its permeability, fracture conductivity, and roughness on the rock surface. Experimental results from over 60 shale plugs indicated that the average core permeability ranged between $10^{-2}$ and $10^{-4}$ mD and was inversely proportional to confining pressure increase. Besides, the effects of closure pressure, rock hardness, proppant particle size, and gas/liquid measurement on fracture conductivity were investigated. The surface of the rock slabs used to measure fracture conductivity was scanned with the profilometer to demonstrate the roughness of the rock surface and the effect of proppant embedment on conductivity. Additionally, we selected four targeted formations in the field and fitted the empirical exponential model using these experimental data. The exponential empirical model is capable of capturing the overall variation in apparent permeability as a function...
of effective stress. The laboratory experiments and research findings in this study provided critical insights into reservoir characterization in Central Appalachia and will benefit future reservoir development techniques such as CO₂ and advanced proppant technologies.

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Reference


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Chapter 6. Conclusions and Future Work

6.1. Conclusions

In this dissertation, the primary goal of this work is to investigate the geomechanics/flow coupling in energy georeservoirs that will influence the petrophysical properties and hydraulic conductivity of the subsurface reservoir in the hydraulic fracturing. The MPST model will first be developed with the experimental data from the pulse decay permeameter. Based on the MPST model and more experimental work, a novel M³ST model was developed to investigate shale gas transport in both transient and steady states. Next, the factors that impact the fracture conductivity will be investigated because the fracture conductivity is directly relevant to the productivity of hydraulic fracturing. A series of proppant fracture conductivity experiment will be conducted considering the effect of proppant concentration, closure pressure, particle diameter, liquid soaking, and rock mineralogy to study the relationship between fracture conductivity and these factors. In the end, the numerical and experimental works will be applied to the rock samples from the Central Appalachian area to investigate their geomechanics.

Based on the experimental, theoretical, and numerical research and studies, the conclusions can be drawn as follows:

1. Comprehensive core analyses using PDP laboratory testing were conducted to investigate the relationships between apparent permeability, pore pressure, and confining pressure. The influence of the Klinkenberg effect and the role of the bedding planes direction on the PDP-measured apparent permeabilities were studied.

2. The MPST model is developed to account for the coupled multi-physics processes of geomechanics, fluid dynamics, and the Klinkenberg effect for gas transport in shales. The PDP experimental data were fitted to the MPST model, and the fitting curves showed that the MPST model successfully captures the influence of both pore pressure and effective stress on the apparent permeability.

3. A novel M³ST model was developed to investigate gas transport in shales in both steady and transient states using experimental and modeling approaches. We used the microscale M³ST model component to fit the PDP-measured apparent permeability as a function of pore pressure under two effective stresses based on NLSF. We then imported the parameters and petrophysical properties determined in the steady state into the transient-
state, continuum-scale M³ST model component, which performed history matching of the evolutions of the upstream and downstream gas pressures as a function of time at the core scale.

4. A double-exponential empirical model was introduced as a powerful alternative to the M³ST model to fit laboratory-measured apparent permeability under various pore pressures and effective stresses because of its simple formulation and small number of undetermined parameters.

5. A series of proppant fracture conductivity experiments were conducted to validate the non-monotonic fracture conductivity evolution along with closure pressure at the proppant concentration from partial-monolayer to multilayer. This is the first study that uses well-controlled laboratory experiments to comprehensively investigate non-monotonic fracture conductivity evolution under various effective stresses, proppant particle sizes, rock types, and rock surface hardness. The existence of the OPPC indicates that a relatively low proppant amount can be used to form a partial-monolayer proppant pack in the fracture space, which has similar or higher fracture conductivity compared to a multilayer proppant structure.

6. It is critical to account for proppant embedment on rock surfaces when studying fracture conductivity. Many numerical studies assume that the proppant particles and rock surfaces are rigid materials, which means that proppant embedment on rock surfaces does not occur. However, the increasing number of proppant particles in the fracture blocks the pore space, leading to reduced fracture porosity, permeability, and consequently fracture conductivity. In reality, using only one proppant particle in the fracture is never the optimal solution because the compressive effective stress results in proppant embedment, as what we observed in this experimental study. This, again, emphasizes that it is critical to account for proppant embedment on rock surfaces when studying fracture conductivity under various proppant concentrations and effective stresses.

7. A series of laboratory experiments were conducted to characterize the reservoir, including its permeability, fracture conductivity, and roughness on the rock surface. Besides, the effects of closure pressure, rock hardness, proppant particle size, and gas/liquid measurement on fracture conductivity were investigated. The surface of the rock slabs used to measure fracture conductivity was scanned with the profilometer. Additionally, The
exponential empirical model is capable of capturing the overall variation in apparent permeability as a function of effective stress. The laboratory experiments and research findings in this study provided critical insights into reservoir characterization in Central Appalachia and will benefit future reservoir development techniques such as CO₂ and advanced proppant technologies.

6.2. Limitations and Future work

Geomechanics/flow coupling studies in energy georeservoirs provide critical insight into the effect of multiple factors on the petrophysical properties of the subsurface formation, assisting in the optimization of design for hydraulic fracturing, geothermal development, and geological carbon sequestration. The study does, however, include some limitations. For instance, the numerical model may need to be modified under high pressure or high temperature conditions. Additionally, proppant crushing is not considered in the current theory of OPPC, and further research can incorporate proppant crushing into the theory. As a result, certain aspects of this study may be pursued further in the future.

1. The numerical M³ST model is developed under the normal temperature. However, the reservoir condition is typically high temperature and high pressure, which have a significant impact on the fluid and rock properties. Under such conditions, the effect of thermodynamic and chemical reactions on the fluid and rock must be considered. The goal of future research is to investigate numerical simulations at high temperatures.

2. The multiscale research of the M³ST model includes micro-scale and continuum-scale in this study. Due to the complicated field situations, it is necessary to upscale the M³ST model to the field scale. The future upscale work can combine geological formation modeling and industrial CT-scanning to extend the application scope of the M³ST model.

3. The workflow of the numerical simulation in this dissertation can be applied to a variety of research areas, including geothermal development and geological carbon sequestration. The M³ST model's application in the interdiscipline will be the focus of future work.

4. The ceramic proppant can withstand pressures of up to 10,000 psi. However, proppant crushing for ceramic proppants is expected under the effective stress higher than 10,000 psi. Crushed proppants can cause straining in the pore throat, affecting proppant pack
permeability and resulting in conductivity loss. This highlights the critical need for the numerical model to account for the proppant crushing process in the high-stress regime. The goal of future research is to investigate the effect of proppant crushing on fracture conductivity.

5. Proppants are pumped alongside fracturing fluids during the hydraulic fracturing treatment process to prevent hydraulic fractures from closing during the reservoir depletion process and to maintain conductive channels between the reservoir and wellbore. The goal of future research is to investigate proppant transport in propagating fractures as well as fracture conductivity distribution through fractures.

6. Formation damage caused by fines migration and clay swelling can affect the permeability of the proppant pack, resulting in conductivity loss. The proppant particles and the relevant chemical liquid injected into well can cause fine migration and clay swelling, which will block the pore throat, and lead to permeability deterioration in unconventional reservoirs. The goal of future research is to investigate fines migration in the rock-proppant system with accounting for the clay-swelling problem.

7. Core plug permeability and the slab conductivity will be measured more for the reservoir characterization of the Central Appalachian area, and we will compare the results with our previous data to illustrate the gas- and liquid-measured effects on the fracture conductivity. Meanwhile, a portion of PDP-measured plugs will be crushed into fine particles and then measured by SMP to find the difference and correlation between PDP- and SMP-measured permeabilities.
Appendix A. Copyright releasing documents from publishers

Using pressure pulse decay experiments and a novel multi-physics shale transport model to study the role of Klinkenberg effect and effective stress on the apparent permeability of shales

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Best,

Peter

Peter Smeallie