A VORTEX-LATTICE METHOD FOR DELTA WING AERODYNAMICS

by

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(ABSTRACT)

A Numerical Solution is presented for the problem of flow past a highly swept, slender wing with sharp leading edges. The lifting surface is modelled as a bound vortex sheet, while the wake is modelled as a force-free vortex sheet. The solution is obtained by the use of a unsteady Vortex-Lattice Method which includes the effect of leading edge separation. Numerical predictions for the aerodynamic loads and pressure distributions are compared with experimental data. A 75° Delta wing and a 60° Delta wing with Leading Edge Vortex flaps in uniform, symmetric and steady flow are studied. Uniform and cosine distributions are used to determine the effect of lattice shape on the solution.

The results show that good aerodynamic load predictions are obtained by this Vortex-lattice method. The results also indicated that fewer cosine distribution control points predict pressures as well as the use of a larger number of uniform distribution control points. The numerical results for wings with LEVFs show good agreement with experimental data away from the trailing edge. This may be due to the viscous effects in the experiment not modelled in this
method. It is also apparent that the size of the wake, trailing and leading edge wakes, is the important factor effecting computation times.
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<td>1. Computation Times</td>
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High speed aircraft are usually designed with optimal performance geared towards operation in the transonic and supersonic regimes. This criterion results in highly swept, low aspect ratio wings with sharp leading edges, which are highly inefficient in the subsonic regime where maneuvers such as take-off and landing must be performed.

High sweep and low aspect ratio cause a complex three-dimensional flow field around the wing. The sharp leading edges cause the formation of free shear layers which curve upward and roll up into vortices lying inboard of the leading edges (Figure 1a). At low angles of attack, the flow reattaches inboard of the vortices due to the favourable spanwise pressure gradient caused by these vortices. Inboard of the vortices the flow is in the streamwise direction, whereas, under the vortices there exists a very significant crossflow. Outboard of the vortices an adverse spanwise pressure gradient exists, causing separation, and resulting in the formation of secondary vortices between the primary vortices and the leading edges (Figure 1). This flow field is basically independent of Reynolds number¹² and has been observed in
delta wings with subsonic leading edges. As the angle of attack increases, the reattachment lines move further inboard. At high angles of attack and some unsymmetric flow conditions, the tightly rolled up primary vortices burst, causing the wing to stall. Vortex bursting can be postponed by providing a leading edge camber (using leading edge vortex flaps or reshaping the wing itself), varying the sweep along the span of the wing or spanwise blowing to strengthen the vortex. Aft of the wing, trailing vortices develop adjacent to the leading edge vortices and their circulation is opposite to that of the leading edge vortices.

The low pressure caused by the vortices at the leading edges results in increased lift, but also contributes to a large increase in drag (Figure 2a). The lift and drag increase nonlinearly with angle of attack. Attempts to overcome the drag penalty due to the vortices by providing a permanent leading edge camber, thus preventing the formation of the vortices (Figure 2b), results in a reduction of $C_{L_{max}}$ and non-optimal supersonic performance. A Leading Edge Vortex Flap (LEVF), on the other hand, takes advantage of the forces due to the vortices by deflecting the flap and thus vectoring the forces in a more favorable direction (Figure 2c). A downward deflection of the flap would result
in thrust rather than drag due to the vortices, whereas an upward deflection of the flap would result in increased lift and drag contributions (Figure 2d), useful during landing. In addition to transonic maneuvers, vortex flaps offer some other potential advantages: large downward deflection at touchdown to generate negative lift and high drag could be used in place of drag chutes, vortex flaps benefit from sharp edges which may be important in low observable technology, and asymmetric flap deflections may generate sufficient rolling and yawing moments to be useful for lateral control at high angles of attack.

The time and financial investments involved in wind-tunnel testing necessitates a realistic and cost-effective analytical solution to the problem. In trying to determine the low speed characteristics of wings with leading edge vortices, most theoretical methods do not account for the nonlinearity caused by the presence of the vortices. Among the nonlinear theories, Polhamous' leading edge suction analogy agrees well with experimental force data but it is not a rigorous method. Numerical codes that predict forces accurately and economically have been developed, but detailed pressure distribution predictions are expensive.
The present work attempts an accurate pressure distribution prediction for delta wings with and without LEVFs. The vortex lattice method used is applicable for inviscid, unsteady, subsonic, nonlinear flow over arbitrary planforms. It does not account for viscous effects like secondary vortices and vortex bursting. The separation lines are assumed to lie on the sharp edges and the vorticity is shed normal to these edges.
Chapter II
LITERATURE REVIEW

As described in the last chapter, flow past thin, sharp edged, highly swept wings separate at the leading edge for all normal angles of attack. The concentrated vorticity above the wing induces high suction pressures on the wing's upper surface, resulting in a nonlinear contribution to wing loading. This nonlinear behavior of lift has become known as vortex lift. Generally speaking, theories that allow for vortex lift fall into the following three categories.

2.1 LEADING EDGE SUCTION ANALOGY

Polhamus' concept is based on an analogy between the vortex lift and the leading edge suction associated with the potential flow about the leading edge. This theory, when combined with potential-flow theory and modified to include the nonlinearities associated with the exact boundary condition and the loss of the lift component of the leading edge suction, provides excellent prediction of the total lift for a wide range of Delta and Delta-like wings upto an angle of 20 deg. or greater. This concept has been extended to the subsonic compressible regime based on the Prandtl-Glauert transformation. It has also been used in conjunction with vortex-lattice methods.
2.2 NONLINEAR SURFACE THEORIES

Bollay suggested a model for low aspect ratio rectangular wings in which all the vorticity is shed from the wing tips rather than the trailing edge, and is assumed to lie along straight lines at an angle to the wing. A solution is obtained by satisfying the condition of tangential flow along the wing centerline, and requiring the trailing vorticity that is shed from the wing tip to lie along local streamlines. Gerston extended Bollay's model to cover wings of arbitrary shape. Vorticity is assumed to be shed from all points on the wing at an angle $\alpha/2$ to the wing. Sacks, Neilson and Goodwin modelled an arbitrary planform as a series of high-aspect-ratio elements, each having a set of horseshoe vortices - one extending from the trailing edge and the other extending from the element tips at an angle (chosen to give good agreement with experimental results) to the element. Vortex strengths are calculated by satisfying tangential flow condition at the 3/4-element-chord line and finite velocity condition at element tips.

Jones and Rao model the wing with a doublet sheet and the flow separation with vortex lines shed from the leading edge at an angle proportional to the angle of attack and
their strength proportional to the adjoining doublet element. The constant of proportionality, a function of aspect ratio and angle of attack, is chosen to give good agreement with experimental results.

One of the most elaborate and successful models is the Free Vortex Sheet model developed at Boeing\textsuperscript{13}. The wing is modelled as a distribution of Source and biquadratic Doublet panels. The wake is modelled as a near wake, consisting of a Free sheet and a Fed sheet, and a far wake, the shape of which is fixed. No mass flux (to account for compressability effects) normal to the wing, a force-free near-wake and Kutta condition are the boundary conditions. This code is capable of predicting forces, moments and detailed pressure distributions.

Belotserkovski\textsuperscript{12} developed a general, unsteady Vortex-Lattice Method with the lifting surface modelled by a system of horseshoe vortices whose strengths vary with time. Belotserkovski and Nisht\textsuperscript{14} extended it to sweptback wings in nonlinear flow with wing tip and trailing edge wakes. Mook and Maddox\textsuperscript{14} used a similar method and included leading edge separation. The leading edge wake consisted of discrete non-intersecting vortex lines and were rendered force-free by convecting each wake element according to the local
velocity at the upstream end of that element. Kandil, Mook and Nayfeh\textsuperscript{15} extended this model to force-free wing tip and trailing edge wakes. This method was further refined by Atta\textsuperscript{16} and Konstadinopoulos\textsuperscript{17}. McNutt\textsuperscript{18} used this method to model delta wings with Leading Edge Vortex Flaps (LEVF) and obtained good agreement of aerodynamic forces with experimental results. Based on the work of Kandil et. al., Katz\textsuperscript{19} calculated aerodynamic forces on wings undergoing complex three dimensional motion.

2.3 DETACHED FLOW METHODS

Legendre\textsuperscript{9,12} modelled the leading edge vortices with concentrated vortices above and inboard of the leading edges of the wing. Each element of the concentrated vortices is connected to the leading edge (source of vorticity) by one vortex line. The boundary conditions are finite velocity at the leading edges and force-free vorticies.

Brown and Michael\textsuperscript{12} extended this, using two vortex lines to feed each element of the concentrated vorticies. Mangler and Smith\textsuperscript{12} further refined the model by using non-planar feeding sheets shedding tangentially from the leading edges that spiral up to the concentrated vorticies. The boundary conditions satisfied are tangency of flow to the wing
surface, finite velocity at the leading edges and a force free concentrated vortex-feeding sheet combination.

Overall force predictions obtained by the Leading Edge Suction Analogy method shows good agreement with experimental results, though the method is not rigorous. This theory has not been extended to planforms with camber and cannot predict wing loading.

Except for the unsteady vortex-lattice method and the Free Vortex Sheet method, none of the nonlinear surface theories allow for the rolling-up process of the vortex sheets. Thus, the load distributions predicted by the methods that do not allow for the rolling-up process are generally inaccurate⁹.

Detached flow methods provide a fairly realistic flow model, except in the trailing edge region where this conical flow model breaks down. Predictions are good only in very low aspect ratio ranges (less than 0.7)³.
3.1 GOVERNING EQUATION

The Governing equation for an incompressible, three-dimensional flow about a slender wing is

$$\phi_{xx} + \phi_{yy} + \phi_{zz} = 0$$

The use of this linear equation allows the solution to be built up from elementary solutions that are governed by it. The elementary solutions used here are Uniform flow and Potential Vortex lines.

The problem, flow about a highly swept, low aspect ratio wing with sharp leading edges, is modelled as a bound vortex sheet and a free vortex sheet emanating from each separation line, superimposed on a uniform flow. The bound vortex sheet lies on the camber line of the wing, while the free vortex sheet is allowed to deform until it is force-free. The separation lines are assumed to lie along the sharp edges. A lattice of finite-length vortex lines is used to define each vortex sheet. The velocity induced by the vortex lines is computed using the Biot-Savart law.
parameter limits the magnitude of the induced velocity very close to the filament.

3.2 BOUNDARY CONDITION

The surfaces on which specific conditions have to be imposed are

1. $S_\infty$ - a surface enclosing an infinite domain around the wing and wake.
2. $S_{LS}$ - the surface defining the wing (Lifting Surface).
3. $S_W$ - the surface defining the wake.

The conditions specified are

1. Perturbation velocity decay at infinity:
   \[ v_p \to 0 \quad \text{on } S_\infty \]
   where $v_p$ is the perturbation velocity. This condition is always satisfied since the Biot-Savart law is used to compute the perturbation velocity.

2. Tangential flow:
   \[ V \cdot n = 0 \quad \text{on } S_{LS}; \]
   where $V$ is the absolute velocity and $n$ is the unit vector normal to $S_{LS}$. This simulates the solid wing by forcing no flow through the surface.

3. Force-Free Wake:
   \[ V_m \cdot n = 0 \quad \text{on } S_W; \]
where $V_m$ is the mean of the upper and lower surface velocities. Since the wake cannot support a pressure discontinuity, the velocity normal to $S_w$ should be continuous.

4. Spatial Conservation of Circulation:

$$\frac{D\Gamma}{Dt} = 0 \quad \text{in } S_\infty$$

i.e. the rate of change of circulation around any fluid curve is zero, in the motion of an inviscid fluid.

3.3 LATTICE ARRANGEMENT

To determine the bound vortex lattice, the wing surface is divided up into the required number of panels. Vortex lines are placed on the perimeter of each panel and then offset by 1/4 of the panel chord. Three different panel distributions were studied.

3.3.1 Uniform Distribution

This lattice is constructed using the following equations to calculate the coordinates of the panel corners

$$x_{i,j} = i \times (\text{CHORD} + 1)/N - 1 \quad i = 1, \ldots N;$$

$$y_{i,j} = \cot(\Lambda) \times (j - 1) \quad j = 1, \ldots 2i - 1$$

where $N$ is the number of rows and $\Lambda$ is the leading edge
sweep. This method results in a rectangular panels of equal area (Figure 3).

3.3.2 Cosine Distribution

The cosine distribution uses the following equation to locate the panel corners

\[
x_{i,j} = (C/2) \times [1 - \cos((2i-1)\pi/N)] \quad i = 1, \ldots, N
\]

\[
y_{i,j} = y_{LE_i} + (b_i/2) \times [1 - \cos((2j-1)\pi/M)] \quad j = 1, \ldots, 2i-1
\]

where \(C\) is the root chord of the wing, \(b_i\) is the span of the \(i^{th}\) row and \(y_{LE_i}\) is the \(y\)-coordinate of the \(i^{th}\) row tip on the LHS. Here, the panel density is increased along the leading edge, apex and trailing edge of the wing (Figure 5).

3.3.3 Semi-Circle Distribution

In this method the coordinates of the panel corners are calculated using the following equations

\[
y_{i,j} = (SPAN/2) \times [1 - \cos((2j-1)\pi/M)] \quad j = 1, \ldots, M
\]

\[
x_{i,j} = x_{LE_j} + (c_j/2) \times [1 - \cos((2i-1)\pi/N)] \quad i = 1, \ldots, N
\]
where $x_{LE_j}$ is the x-coordinate of the local chord leading edge and $c_j$ is the local chord. $N$ is the number of rows of bound vortices and $M$ is the number of columns of trailing vortices from the trailing edge. In this case, the panel density near the wing tips is increased (Figure 6).

For the leading edge panels in all the three lattice arrangements, two elements were placed normal to and one element parallel to the leading edge. This satisfies the assumption that the flow separates at the edge and that the vorticity is shed normal to the edge. For wings with LEVFs, additional quadrilateral panels are placed adjacent to the leading edge panels (Figure 4).

The Control Points, points at which the tangential flow condition has to be satisfied, are placed at the centroid of each panel. None of the control points are at the edges of the wing, therefore, the Kutta condition is not satisfied explicitly.

3.4 SOLUTION TECHNIQUE

After the bound vortex lattice is set up and the control points are specified, the Influence Coefficient Matrix for the wing is calculated. The influence coefficient $A_{i,j}$ is the normal velocity induced at the $i^{th}$ control point due to
the vortex panel, of unit strength, at the \( j^{th} \) control point. In order to find the circulations, the tangential flow condition is applied at each control point, resulting in a system of \( N \) equations in \( N \) unknowns, the unknowns being the circulation of each closed loop vortex. In matrix form
\[
[A]{\{G\}} = \{[V_{LS} - V_{W}] \cdot n\}
\]
where \([A]\) is the influence coefficient matrix, \({G}\) is the circulation vector and the RHS calculates the normal component of the velocity due to the wake and uniform flow. The initial condition can be either no wake (i.e. impulsive start from rest) or a prescribed wake (not necessarily force-free). This matrix equation is solved using the Gauss-Siedel method.

Each element of the wake, including each of the starting vortices, is then convected according to the local velocity, thus developing a force-free wake. The circulation is found for the new wake position and the wake convected again. This routine is repeated until the change in the circulations diminishes to a prescribed value. If the wing is not deforming, the influence coefficient matrix remains constant and need not be determined at each time step. The conditions for convergence are that the unsteady terms diminish to zero and aerodynamic loads converge for increasing number of panels.
The pressure at each control point is calculated using the Unsteady Bernoulli's equation

$$\frac{\partial \phi}{\partial t} + \frac{(V \cdot V)}{2} + \frac{p}{\rho} = H(t)$$

Nondimensionalizing and applying the condition at infinity, for the steady case, we get\(^{16}\)

$$C_p = -V_p \cdot (v_p - 2xV_A)$$

where

$$v_{pu,1} = V_m \pm \Delta V/2$$

and \(V_m\) is the perturbation velocity at the control point, due to the whole wake and all the bound vortex panels except the panel at the control point. \(\Delta V\) is the tangential velocity jump at the control point due to the vortex panel.

$$\Delta V = v_{pu} - v_{p1} = \gamma \times n$$

where \(\gamma\) is the vorticity per unit area of the vortex sheet. In the limiting case of the vortex sheet strength increasing to infinity as the sheet shrinks to a line such that the circulation remains constant, the sheet vortex is discretized to a line vortex.

$$\lim_{dx \to 0, \gamma \to \infty} \gamma \, dx \, dy = \Gamma \, dy$$

such that

$$\gamma \, dx = \Gamma = \text{constant}$$
We know that for the two-dimensional case
\[ \Delta V = \Gamma / L \]
Extending this to the three-dimensional case
\[ \Delta V_i = (\Gamma \times n)_i / L_i \]
where, for a quadrilateral element,
\[ \Gamma_i = 0.5 \times (\Gamma_{1i} + \Gamma_{2i} + \Gamma_{3i} + \Gamma_{4i}) \]
and
\[ L_i = 0.5 \times (L_{1i} + L_{2i} + L_{3i} + L_{4i}) \]

The loads acting on the wing are calculated by summing up the appropriate loads acting on each panel. The total projected area, \( A_t \), and the root chord, \( C \), are used to calculate the force and moment coefficients.
Chapter IV

RESULTS AND CONCLUSIONS

The present method was used to calculate the aerodynamic loads on a 75° Delta wing, the pressure distributions on a 75° plain Delta wing and a 60° Delta wing with LEVFs. A uniform distribution of 10 rows of bound vortices (i.e. 45 C.P.s on half-wing) was used to calculate aerodynamic loads and the pressure distribution on the 75° plain Delta wing. A cosine distribution and uniform distribution of 8 rows of bound vortices (i.e. 28 C.P.s on half-wing) were also used to obtain pressure distribution on the plain Delta wing. Both cosine distribution and uniform distribution of 7 rows of bound vortices with 3 and 4 columns of panels on the flap were studied for the 60° Delta wing with LEVFs.

4.1 AERODYNAMIC LOADS

The Vortex-Lattice method was used to simulate a 75° plain Delta wing in steady, uniform flow and the aerodynamic loads were calculated for angles of attack ranging from 5° to 30°. The results were compared with Marchman's', Peckham's² and Wentz's²¹ data. Marchman used a 75° flat plate Delta wing, while Peckham and Wentz used 74° flat plate Delta wings.
The variation of Lift Coefficient, $C_L$, with angle of attack, $\alpha$ and Pitching Moment Coefficient, $C_m$, with $C_L$ are plotted in Figures 7 and 8. The $C_L$ showed good agreement with experimental data until $\alpha = 20^\circ$, above which it was slightly lower than the experimental data. The variation of $C_m$ with $C_L$ agrees very well with the data, but does not follow Marchman's curve beyond a $C_L$ of 0.7. The discrepancy at $\alpha = 20^\circ$ using the uniform distribution with 4 rows of bound vortices is no longer present. Thus, the first convergence criterion has been satisfied.

4.2 75° PLAIN DELTA

The results of the present method for pressure distributions on a 75° Delta wing were compared to the data of Wentz. The model used was a flat plate 74° Delta wing with a thickness ratio of 0.8%, having sharp leading and trailing edges.

Figures 9, 10 and 11 show the spanwise pressure distributions at $\alpha = 20^\circ$. At $x/c$ of 0.67, the 45 C.P. uniform distribution results follow the data well and the positions of the peak suction line coincide, but the predicted peak is lower. The 28 C.P. uniform distribution does not agree as well and the peak suction line is further
inboard. At Y/S of 0.83 there is drop in the value of $C_p$. This may occur due to the parameter cutoff. At x/c of 0.80, the 45 C.P. and 28 C.P. uniform distribution agree well with the data and the position of the peak suction line but the magnitude at the peak for the 45 C.P. case is slightly lower while for the 28 C.P. case it is considerably higher. Again at Y/S of 0.83 for the 28 C.P. case the cutoff parameter may be causing the bad prediction. The cosine distribution predicts a pressure distribution similar to the 45 C.P. case, except that the peak suction line is further inboard. At x/c of 0.91, the cosine distribution shows excellent agreement, while the 45 C.P. case shows the peak suction line to be further outboard.

Figures 12, 13 and 14 show the spanwise pressure distributions at $\alpha = 25^\circ$. At x/c of 0.80, the 45 C.P. uniform distribution results show very good agreement with the experimental data. The cosine distribution also shows good agreement but does not predict the peak since two adjacent control points lie on either side of it. At x/c of 0.91, the cosine distribution shows excellent agreement with experiment, while the uniform distribution predicts the peak to be outboard of its actual position.
Figures 15, 16 and 17 show the spanwise pressure distributions at $\alpha = 30^\circ$. Only the cosine distribution was used in this case. It did not satisfy the second convergence criterion. The unsteady pressure terms were of the order of 10% of the steady pressure terms. At $x/c$ of 0.54, the peak suction line lies between two adjacent control points and is not predicted. The predicted pressures are also low. At $x/c$ of 0.75, the prediction of the peak is on the higher side and its position is inboard of the actual peak. At $x/c$ of 0.91 the peak is significantly higher and outboard of the experimental peak.

4.3 60° DELTA WING WITH LEVF

The pressure predictions on a 60° Delta wing with LEVFs were compared with the data obtained by Sung\textsuperscript{22}. The model used by Sung was 60° flat plate Delta with a constant thickness ratio of 3%, to which a segmented LEVF with sharp leading edges was attached. The flap was of constant chord except near the apex where it had an inverse taper. The flap chord to wing root chord ratio was 0.09. The flap had an extremely high thickness to flap chord ratio (30%). The effect of thickness is to move the attachment line and peak suction line further outboard\textsuperscript{2}. 
All the cases tested were with 7 rows of bound vortices and either 3 or 4 columns of control points on the flap. With 4 columns of control points on the flap, convergence was possible only with at \( \alpha = 15^\circ \) using the uniform distribution. At least partial convergence (i.e. convergence on part of the wing, usually the forward part) was obtained for angles of attack 15°, 20° and 25° using cosine and uniform distribution with 3 columns of control points on each flap.

The spanwise pressure distributions for \( \alpha = 15^\circ \) are shown in Figures 18, 19 and 20. At x/c of 0.45, the numerical solution shows good agreement with the experimental data. The peak suction line is predicted inboard of the actual data because it does not take into account the varying thickness of the flap. At x/c of 0.67 and 0.80 the peak suction line is predicted well but the magnitude of the peak is high. This is probably due to inception of vortex bursting which is not accounted for in the present method.

Figures 21, 22 and 23 show the spanwise pressure variation for \( \alpha = 20^\circ \). In this case the results for the cosine distribution converged on only part of the wing, while the uniform distribution converged completely. At x/c of 0.43, the cosine distribution showed good agreement with
experimental results. The suction peak line lay between two control points and was not predicted. The uniform distribution predictions were high inboard of the vortex and low under the vortex. The peak was not predicted. The cutoff parameter may have been the cause. At x/c of 0.64, the uniform distribution again gave averaged prediction across the span, neither predicting the peak nor agreeing with the low suction pressure area inboard of the vortex. The cosine distribution was also not in agreement with the experimental data since the unsteady part of the pressures were high at this station. At x/c of 0.8 the uniform distribution showed good agreement with experimental data but the cosine distribution was still off.

Figures 24 and 25 show the spanwise pressure distribution at x/c of 0.43 and 0.67. The numerical solution did not converge for either the uniform or the cosine distribution.

4.4 CONCLUSIONS

The present numerical method tends to predict an average pressure in regions of fewer control points and shows the peaks and gradients in regions of finer control point distribution. This trend towards averaging gives good aerodynamic load predictions even where pressures are not predicted very accurately.
The numerical results for the 75° Delta show that the cosine distribution may have a distinct advantage over the uniform distribution. The pressure plots indicate that cosine distribution, with less than two-thirds the number of control points of a uniform distribution, can predict the gradients and peaks as well, and at certain stations better than the uniform distribution. The CPU times tabulated (Table 1) show a saving of over 50% of CPU time, for the cosine distribution.

The problem with convergence for the cases of 60° Delta with LEVF indicate that the number of panels on the flaps is restricted by the number of panels (number of rows of bound vortices) on the wing and the angle of attack. The cosine distribution is more sensitive to this criterion. It will be necessary to increase the number of rows of bound vortices on the wing before a comparison between cosine distribution and uniform distribution can be made.

It is interesting to note that the CPU times for the wing with the LEVF is significantly lower than for the wing without the LEVF. This can be explained by comparing the size of the wake for the respective cases. For the plain wing, an increase of one row of control points results in the increase of two rows and four columns in the leading
edge wake and two rows and columns in the trailing edge wake. For the wing-flap combination an increase of two columns of control points (one on each flap) results in the addition of only two rows in the trailing edge wake. This results in a considerable difference in the computation times.

4.5 RECOMMENDATION

In view of the CPU times required for the wing-flap combination, further work is needed in optimizing the control point distribution on the plain wing. For the wing-flap combination, placement of a vortex filament along the wing-flap junction could improve the results.

The numerical solution can be extended to include effects of thickness also. Superimposing the thickness solution at zero angle of attack, modelled using source/sink distributions with no wake, on the present solution should not increase the computation time considerably. In view of the fact that vortex flaps can be used for transonic maneuver, the present method could be extended to the compressible subsonic regime using the Gothert’s rule.
REFERENCES


8 Frink, N.T., "Analytical Study of Vortex Flaps on Highly Swept Delta Wings". ICAS-82-6.7.2.


TABLE 1

Computation Times for 30 Time Steps

<table>
<thead>
<tr>
<th>Wing Lattice</th>
<th>Wake Lattice</th>
<th>Number of Control Points</th>
<th>CPU Time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plain Wing:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NROWS= 8</td>
<td>8x16</td>
<td>15x16</td>
<td>28</td>
</tr>
<tr>
<td>NROWS=10</td>
<td>10x20</td>
<td>19x20</td>
<td>45</td>
</tr>
<tr>
<td>Wing w/ LEVF:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NROWS= 7</td>
<td>7x14</td>
<td>17x14</td>
<td>15</td>
</tr>
<tr>
<td>IFLP = 3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NROWS= 7</td>
<td>7x14</td>
<td>19x14</td>
<td>15</td>
</tr>
</tbody>
</table>

* NROWS is the Number of Rows of Bound vortices and IFLP is the Number of Columns of Control Points on the Flap.
Figure 1: Flow Field Past a Delta Wing with Leading Edge Separation
Figure 2: Leading Edge Camber Effects
Figure 3: Lattice Arrangement - Uniform Distribution
Figure 4. Lattice Arrangement - Wing-Flap Combination
Figure 5. Wing Panel Arrangement - Cosine Distribution
Figure 6. Lattice Arrangement - Semi-Circle Method
Figure 7: Lift Coefficient vs. Angle of Attack for 75° Delta
Figure 8: Lift Coefficient vs. Pitching Moment for 75° Delta
Figure 9: Spanwise Pressure Distribution - 75° Delta

\( \alpha = 20^\circ, \frac{X}{C} = 0.67 \)
Figure 10: Spanwise Pressure Distribution - 75° Delta
Figure 11: Spanwise Pressure Distribution - 75° Delta

\[ \alpha = 20°, \ X/C = 0.9 \]
Figure 12: Spanwise Pressure Distribution - 75° Delta

\[ \alpha = 25^\circ, \ X/C = 0.67 \]
$\alpha = 25^\circ$, $X/C = 0.8$

Figure 13: Spanwise Pressure Distribution - 75° Delta
\[ \alpha = 25^\circ, \; X/C = 0.9 \]

Figure 14: Spanwise Pressure Distribution - 75° Delta
Figure 15: Spanwise Pressure Distribution - 75° Delta
\( \alpha = 30^\circ, \ X/C = 0.67 \)

Figure 16: Spanwise Pressure Distribution - 75° Delta
\[ \alpha = 30^\circ, \ X/C = 0.9 \]

Figure 17: Spanwise Pressure Distribution - 75° Delta
$\alpha = 15^\circ$, $\delta = 30^\circ$, $X/C = 0.45$, IFLP = 4

Figure 18: Spanwise Pressure Distribution - 60° Delta w/ LEVF
Figure 19: Spanwise Pressure Distribution - 60° Delta w/ LEVF

\( \alpha = 15^\circ, \delta = 30^\circ, X/C = 0.67, \text{ IFLP} = 4 \)
Figure 20: Spanwise Pressure Distribution - 60° Delta w/ LEVF

$\alpha = 15^\circ$, $\delta = 30^\circ$, $X/C = 0.8$, IFLP = 4
Figure 21: Spanwise Pressure Distribution - 60° Delta w/ LEVF

\[ \alpha = 20^\circ, \delta = 30^\circ, X/C = 0.45, IFLP = 3 \]
Figure 22: Spanwise Pressure Distribution - 60° Delta w/ LEVF

\[ \alpha = 20^\circ, \delta = 30^\circ, X/C = 0.67, \text{IFLP} = 3 \]
Figure 23: Spanwise Pressure Distribution - 60° Delta w/ LEVF

\[ \alpha = 20^\circ, \delta = 30^\circ, X/C = 0.80, IFLP = 3 \]
\[ \alpha = 25^\circ, \delta = 30^\circ, \ X/C = 0.45, \ IFLP = 3 \]

Figure 24: Spanwise Pressure Distribution - 60° Delta w/ LEVF
\( \alpha = 25^\circ, \, \delta = 30^\circ, \, X/C = 0.67, \, IFLP = 3 \)

Figure 25: Spanwise Pressure Distribution - 60° Delta w/ LEVF
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