MODEL BASED ESTIMATION OF ROAD SURFACE FRICTION FOR USE IN VEHICLE CONTROL AND SAFETY

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ABSTRACT

The road surface friction is an important characteristic that must be measured accurately to navigate vehicles effectively under different conditions. This parameter is very difficult to estimate correctly as it can take up a value from a broad spectrum of possibilities and the knowledge of this characteristic is of utmost significance in modern day automotive applications. The possible real-time knowledge of friction opens a new range of improvements to the active safety systems such as the Electronic Stability Control (ESC) and Anti-lock Braking Systems (ABS) in addition to providing computerized support to safety applications. The aim of the research is to take an engineering approach to the problem and design a simple and a robust algorithm that can be implemented in any automotive application of choice. After integrating the load transfer model with the four wheel vehicle model, the Dugoff tire models are combined with the aforementioned model to represent the plant model. Using the plant model to design an emulator, the sensor measurements are created and these measurements are then used by a non linear estimator such as the Unscented Kalman Filter to predict the forces at the tires. Friction is then calculated for every iteration and then passed back into the loop. In the end, a comparison of different design methodologies, implementation techniques and performance along with design decisions are discussed so that the current work can be implemented on a real-time controller. In addition to this, a section is dedicated towards highlighting the difference that prior friction information has on the stopping distance of a vehicle. For this purpose, a demonstration is made by creating an ABS control system that uses the predicted friction information and the performance improvement is documented.
GENERAL AUDIENCE ABSTRACT The goal of the research is to identify methods in which the road surface friction can be detected by the on board computers present on modern-day cars. Drivers have the ability to determine the grip on the road surface through various mechanisms, for instance if a driver sees a patch of ice on the road when driving, their normal response is to take the foot off the gas and drive without giving much steering input to avoid a slide. Another input that the driver can use to assess the grip is through the 'steering feel', which is the ability to differentiate different driving conditions through the force feedback from the steering wheel. There have been numerous approaches to help teach the computer to detect these road conditions so that it can operate other computerized systems such as the ABS(Anti-lock Braking System) and ESC( Electronic Stability Control) programs with better accuracy. This work is an attempt to contribute to this vital area of study.

At the end of the study, an algorithm to predict the dynamic estimate of friction has been developed and the improvement in the performance of the Anti-lock braking system using this friction estimate has been demonstrated.
Dedication

To my friends and family, who have always been my pillars of support
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List of Abbreviations

ABS    Anti-lock Braking System
CG     Center of Gravity
EKF    Extended Kalman Filter
ESP    Electronic Stability Program
IMU    Inertial Measurement Unit
LQE    Linear Quadratic Estimator
RLS    Recursive Least Squares
UKF    Unscented Kalman Filter
UT     Unscented Transformation
Chapter 1

Introduction

1.1 Motivation

Road surface friction characteristics can heavily influence the performance of all the on-board control systems such as the Anti-lock Braking System (ABS) the Electronic Stability Control (ESC), Adaptive Cruise control, etc. With the onset of autonomous racing such as Robo-race where a vehicle without a driver is expected to match the performance of a professional driver, it becomes critical to identify significant parameters that affect vehicle stability and knowledge of these parameters give us more insight into building better-suited control systems for the vehicles. One of the greatest skill of any professional driver is to ‘estimate’ the amount of grip that a road surface can offer and drive at the absolute limit, this is a very difficult task to do due to a number of factors including the years of experience that goes behind obtaining this information.

So the aim is to develop an algorithm that would use information from sensors that are readily available on-board and apply them to vehicle and tire model combinations to extract useful surface information for use in other controllers and safety features.

The modern day car has many such information that are available, but none of the data can be accessed by people outside the industry and therefore it makes it complicated to design algorithms for the vehicle. Therefore the study takes an independent approach and tries to
source data from sensors that are commercially available. However, if this has to be adapted by the industry, it can be done so easily with a few modifications since all the information is assumed to be available for the industry with significant improvement in accuracy and ease. This improved knowledge on the road surface friction can then be used to improve the performance of traction and ABS control systems and other programs that cascade down from these control architectures such as the Electronic Stability Program (ESP).

The steering angle sensor utilizes an adjustable steering rig that is attached to the existing steering wheel and is used to drive the vehicle. This rig consists of teeth on its sides which is then used to drive a rotational position sensor, which then calculates the steering wheel angle. An IMU (Inertial Measurement Unit) gives out acceleration data in the X, Y and Z axes along with the angular velocities and acceleration and is fixed at the Center of Gravity of the test vehicle.

This study uses a vehicle model in combination with the tire model to simulate the output from the sensors and then uses the Extended Kalman Filter (EKF) and the Unscented Kalman Filter (UKF) algorithms to estimate the states that cannot be measured directly from the available sensor suite and finally makes a conclusion based on the accuracy of these two estimation schemes.

Validation of the vehicle model can be carried out easily using any Vehicle Dynamics simulation software such as MSC-ADAMS or CarSim in addition to tests that can be carried out in proving grounds.

Model based estimation is a computational tool that can be implemented solely based on mathematical equations. Modelling all parameters from a design perspective would be complicated, but the advantage of the approach is that once a framework has been developed it can be tuned with the actual vehicle on a proving ground or a closed course irrespective of
the make and model of a vehicle. This universal adaptation of the model based design and estimation is one of the great strengths of this approach. Though all the factors that affect a vehicle are not added into the equations, the amount of flexibility provided is such that they are robust enough to handle such irregularities.

1.2 Objectives of the study

Many studies have been conducted in the past that involves heavy simplification to solve the structure in a linear domain. At the time, it was advisable to tread along the linear domain due to computationally efficient and simplistic requirements to solve the control problem. However, thanks to the Moore’s law and the technological improvements, it is now possible to run real time simulations even in the non-linear domain. This has opened up a wide range of possibilities in terms of non-linear observers and estimation theory that can be implemented in an on-board computer system. Defined below are some of the goals of the research study

- Develop a model to estimate the road surface friction that can operate on information from sensors that are readily available
- Incorporate the tire model and the vehicle model into a single integrated setup that is robust.
- Do not modify the actual performance of the source vehicle. For example, some of the heavy tire sensors used in various research studies requires tires to be balanced again before they can be mounted on the test vehicle.
- Do not use symbolic solvers in the model as they are a hindrance to be deployed on real time systems (c/c++ solvers)
• Must be able to be work in older models that did not have the proposed friction estimation schemes

• Must be flexible to all vehicle types, i.e. Sedan, compact car segment and so on.

• Demonstrate the performance of the estimation algorithm for different road manoeuvres.

1.3 Contributions to the study

To meet the objectives specified above, several design decisions have been made that are the primary contributions to the community.

• Establishing a framework for real-time friction coefficient estimation studies

• Development of a four wheel vehicle model complete with dynamic load transfer effects that estimate the dynamic load transfer at the wheel ends

• Development of a system model integrating the complete Dugoff model ensuring compatibility with non-linear Kalman estimators that utilizes practical measurements.

• Performance comparison between the Extended and the Unscented Kalman filters under different driving conditions and manoeuvres.

• Comparison of the performance of the ABS systems, with and without friction information.
1.4 Chapter Organization

Chapter 2: Review of Literature discusses the idea behind the selection of all the models used in the study and cites previous literature and their conclusions to map the thought process behind the design. The algorithms used in this study are explored in detail and their compatibility to the problems at hand are then discussed. The results presented after the review identifies and details the nature of previous research studies, thus laying the foundation for the research.

Chapter 3: Materials and Methods walks us through the design and nuances behind the design decisions and is primarily focused on the application of the methods derived in the Review of literature section to the problem at hand. It describes the derivation of equations behind the system model and the modification of the system model to suit the requirements of the EKF and the UKF algorithms.

Chapter 4: Results and Discussion discusses all the simulation results. The simulation results come from Simulink which is a block diagram based environment that is ideal for Control systems design and is a graphics and text based environment. Any algorithm generated through Simulink can be directly implemented in any embedded environment due to it’s portability with c/c++ programs. This section also quantifies the effect of prior friction knowledge on a simple ABS controller.

Chapter 5: Conclusion briefs on the findings of the research study and reflects on the results from it through different perspectives, analyzing the benefits of the study and takes one through the elements that make the algorithm unique and robust in approach.
Chapter 2

Review of Literature

2.1 Vehicle Models

A vehicle model refers to a set of equations that define the motion of the vehicle. The most commonly used vehicle models are the Bicycle model [1] and the Four wheel vehicle model[4]. In addition to these vehicle models there are a few vehicle models that can be formulated such as the quarter car model[5] and the 14 DOF(Degrees of Freedom)[6] vehicle model that can be used to visualize or estimate the rolling motion of the vehicle. Here we will be reviewing a few models that are closely aligned with our goal.

2.1.1 The bicycle model

In the bicycle model the two wheels in the front axle and the two wheels in the rear axle are lumped into one wheel in the front axle one in the rear respectively. The wheelbase of the car is given by \( L = l_f + l_r \).

The load transfer is ignored in this model. The orientation of the vehicle is given by \( \dot{\psi} \) while the X and Y axes are used to define the motion of the vehicle in the plane. \( \beta \) is the sideslip angle. The equations of motion of the bicycle model can be defined as follows.

\[
\dot{X} = V \times cos(\psi + \beta)
\]
2.1. Vehicle Models

\[ \dot{Y} = V \cdot \sin(\psi + \beta) \]

\[ \dot{\psi} = \frac{V \cdot \cos(\beta) \cdot (\tan(\delta_f) - \tan(\delta_r))}{l_f + l_r} \]

\( \delta_f \) is the steering angle in the front wheel
\( \delta_r \) is the steering angle in the rear wheel
\( l_f \) is the distance between the C.G and the front axle
\( l_r \) is the distance between the C.G and the rear axle
\( V \) is the resultant vehicle velocity
\( \mu \) is the road coefficient of friction
\( g \) is the acceleration due to gravity

The bicycle model is suitable for a variety of simple applications and is crucial in the definition of basic control system design due to its simple and robust design, however if the lateral acceleration of the vehicle is to be more than 0.5\( \cdot \mu \cdot g \) the bicycle model is not recommended.

Figure 2.1: Planar view of the bicycle model [1]
for motion planning purposes[7]. Since the goal of the research study is to design systems that are robust and for use in real vehicles, it is reasonable to eliminate this uncertainty early on in the design stage and select another vehicle model.

2.1.2 The quarter car model

The quarter car model is a vehicle model that is extensively used to map the suspension characteristics with the normal load on the tires, it is invaluable in ride quality assessment and has found significant usage in a number of suspension parameter design and optimization studies. There are a plethora of research on the optimal and robust control of suspension systems using the quarter car model[8][9].

After all the years after it’s introduction, it still stands strong in forming the basis of suspension research like modelling magneto-rheological damping systems [10].

The quarter car model involves modelling the suspension - tire setup as a set of spring mass dampers connected in series, and the quarter car model, as the name suggests consists of analyzing only one wheel of the car. The biggest strength of the quarter car model is it’s application in control. This trait of the quarter car model can be utilized to run it’s control program and the proposed friction estimation program simultaneously, thus significantly reducing the load on the on-board computers. The equations of motion for the quarter car model are defined as follows.

\[
\begin{align*}
    m_{sp} \ddot{x}_s - k_{sp} (x_{us} - x_{sp}) - c_{sp} (\dot{x}_{us} - \dot{x}_{sp}) - m_{sp} \times 9.81 &= F \\
    m_{us} \ddot{x}_{us} - m_{us} \times 9.81 + k_{sp} (x_{us} - x_{sp}) + c_{sp} (\dot{x}_{us} - \dot{x}_{sp}) + k_{us} x_{us} + c_{us} \dot{x}_{us} &= 0
\end{align*}
\]
2.1. Vehicle Models

$m_s$ - sprung mass

$m_u$ - Unsprung mass

$c_s$ - Damping coefficient of the suspension system

$x_u$ - Displacement of the unsprung mass

$x_s$ - Displacement of the sprung mass.

$\dot{x}_u$ - velocity of the unsprung mass

$\dot{x}_s$ - velocity of the sprung mass

Figure 2.2: The quarter car model [2]
2.1.3 Four wheel vehicle model

In this study a four wheel vehicle model is combined with a quarter car vehicle model for emulating sensor data and the outputs from the model is then uploaded into the EKF/UKF algorithms for estimating friction. The advantage of combining the models is to get a robust model that includes a dynamic estimate of the load transfer while ensuring that it is not too complicated to be implemented in an on-board computer. The model is described in the figure above.

\( \delta \) is the steering input provided to the vehicle and \( F_x \) represents the forces at the Tire-road interface along the longitudinal axis of the tires, Whereas \( F_y \) is the force acting along the tires along its lateral axis. The actual velocity vector of the vehicle at the CG(Center of Gravity) is not along the axes \( X \) or \( Y \) of the vehicle and the angle between the actual vehicle velocity vector and the longitudinal velocity vector of the vehicle is defined as the Vehicle Sideslip angle \( \beta \). Similarly, the velocity vector at the tire-road interface acts at a different direction than that of the longitudinal axis of the tires. This angle is known as the wheel
2.2. Tire Model

The fundamental equations of the vehicle model are given below

\[
\dot{V}_x = \frac{F_x \cos \delta - F_y \sin \delta + F_x \cos \delta - F_y \sin \delta + F_x + F_x}{m} + \dot{\psi}V_y
\]

\[
\dot{V}_y = \frac{F_y \cos \delta + F_x \sin \delta + F_y \cos \delta + F_x \sin \delta + F_y + F_y}{m} - \dot{\psi}V_x
\]

\[
\dot{\psi} = \frac{1}{I_z} \left( (F_y \cos \delta + F_x \sin \delta)a - F_x \left( \frac{tw}{2} \right) + F_y \sin \delta \left( \frac{tw}{2} \right) + (F_x \sin \delta + F_y \cos \delta)a + \ldots 
    F_x \cos \delta \left( \frac{tw}{2} \right) - F_y \sin \delta \left( \frac{tw}{2} \right) + (F_x - F_x) \left( \frac{tw}{2} \right) - F_y b - F_y b \right)
\]

2.2 Tire Model

A Tire model or a friction tire model is a mathematical representation of tires that relate various physical properties of tires to quantities that are otherwise extremely complicated to measure or observe. A tire is an extremely non-linear entity, however, apart from the aerodynamic forces it is the only aspect that allows the designer to control the vehicle. Various quantities such as the forces at the tire road interface are not easily measured and the models are built and tuned in such a way that they reflect the results from the studies according to many observations made over the course of decades. Developing new tire models to conform better for certain applications is still an active area of research.

Tire models can be classified into steady state tire models, semi-empirical tire models and the transient tire models. Based on the application and the available computational power, the models to be used are selected. A few commonly used models are discussed here in the literature survey section.
2.2.1 The Tire brush model

The tire brush model is conceived in such a way that a tire is defined to have many microscopic bristles around it's periphery and these bristles go into contact with the ground thus providing the forces necessary to control the vehicle and the elasticity of the tread, the inner belts and plies are lumped into a single parameter that represents the elasticity of these bristles. It is a widely accepted approximation among the scientific community that a tire that is not accelerating or decelerating and has no castor and camber on it is not subjected to any slip. In the brush model the rolling resistance and it’s effects are also neglected.
2.2. Tire Model

Slip

Slip is a phenomenon that refers to the relative motion between the contact patch and the ground surface. If the slip occurs along the axis at which the tire is travelling then that slip is termed as \textit{longitudinal slip}. If the slip occurs perpendicular to the direction of travel of the wheel, then it is considered to slip laterally and the phenomenon is termed \textit{lateral slip}. If both the lateral and longitudinal slip occurs in a tire, then \textit{combined slip} occurs in that tire.

Assumptions behind the brush model

This phenomenon of slip is the cause behind the generation of forces at the wheels. These forces are significant in maintaining vehicle stability when driving.

With the brush model, the assumption is that when slip occurs, the bristles deflect along
the direction of slip, i.e in longitudinal slip, the bristles deflect longitudinally and so on. Under pure slip conditions, which implies that there is only relative motion between the tire and the road, these bristles completely deflect and are parallel to the road surface. Another important assumption of the brush model is that the vertical force on the tire follows a parabolic distribution.

All these assumptions combined dictates that

$$\sqrt{F_x^2 + F_y^2} \leq \mu F_z$$

The combined slip equations are denoted as follows

\[
F = \mu F_z (1 - \lambda^3) \quad \text{if} \quad \sigma < \sigma_{sl} \quad \mu F_z \quad \text{for} \quad \sigma > \sigma_{sl}
\]

\[
F_x = F \frac{\sigma_x}{\sigma}
\]

\[
F_y = F \frac{\sigma_y}{\sigma}
\]

\[
\sigma_x = \frac{\lambda}{1 + \lambda}
\]

\[
\sigma_y = \frac{\tan(\alpha)}{1 + \lambda}
\]

\[
\sigma = \sqrt{\sigma_x^2 + \sigma_y^2}
\]

\[
\sigma_{sl} = \frac{1}{\theta}
\]

\[
\theta = \frac{2 C_p a^2}{3 \mu F_z}
\]

$C_p$ is the tread element stiffness $a$ is the contact length of the tire patch $\lambda$ is the longitudinal slip ratio at the tire contact patch $F_x$ is the force along the longitudinal axis of the tires and $F_y$ is the lateral force on the tires.
2.2. TIRE MODEL

2.2.2 The Magic formula tire model

The tire brush model is a suitable tool for basic analysis in lateral or longitudinal slip in tires, however under combined slip conditions there is a large deviation from the experimental data. This maybe due to stiffness variations in the longitudinal and the lateral directions. The Magic formula tire model or the Pacejka model was provided as a semi-empirical alternative to the analytical models at the time to conform better to the results that we see from an experimental setup. The magic formula model is defined using the following set of equations

\[ Y(X) = y(x) + S_v \]

\[ y = D \times \sin(C \times \tan^{-1} \{B \times x - E \times (B \times x - \tan^{-1}(B \times x))\}) \]

\[ x = X - S_h \]

where \( Y \) can take the form \( F_x \) or \( F_y \), \( x \) is the slip ratio \( \sigma_x / \sigma_y \) or the slip angle \( \alpha \) and a set of coefficients are necessary to complete the relation definition between the slip ratios and the developed forces. where 'D' represents the highest possible force that a tire can generate 'C' is the shape factor and is defined as follows

\[ C = \frac{2 \times \sin^{-1}(\frac{y_s}{D})}{\pi} \]

and 'B' is used to represent the slope at the origin and is called the stiffness factor while \( S_h \) and \( S_v \) are due to the conicity and the ply steer effects.

\[ E = \frac{B \times x_m - \tan\left(\frac{\pi}{2C}\right)}{B \times x_m - \tan^{-1}(Bx_m)} \]
and $y_s$ is defined as the force value at which the force saturates at high slip angles. A very informative figure presented in [1] has been described below for better illustration.

![Figure 2.7: The parameters utilized by the Magic formula](image)

**2.2.3 The Dugoff Tire model**

The Dugoff tire model is an analytical model that provides a relationship between the slip ratio, and the force at the tire road interface. The cornering stiffness again can be used as a time varying parameter for modelling from a controls perspective. A significant aspect in which the Dugoff model is superior to the brush model is its decoupled nature of the longitudinal and lateral stiffness of the tires. This allows the designer to select the stiffness values independently as they are a major source of error in the brush model.
The Dugoff tire model was developed based on adherence of the developed model to the combined slip of the vehicle under varying test conditions as quoted in [11]

- The steady state turn
- Braking under steady turn
- Incremental steering under high lateral accelerations
- Combined lane change and braking

These test conditions are very similar to the real-world road conditions and demand a deeper analysis into its usage.

\[
F_{xi} = C_{xx} \frac{\lambda_i}{1 - \lambda_i} * k_i
\]

\[
F_{yi} = C_{yy} \frac{\tan(\alpha_i)}{1 - \lambda_i} * k_i
\]

\[
k_i = (2 - \sigma_i)\sigma_i \quad \text{for} \quad \sigma < 1 \ldots 1 \quad \text{if} \quad \sigma_i \geq 1
\]

\[
\sigma_i = \frac{(1 - \lambda_i)\mu * F_{ni}}{2\sqrt{C_{xx}^2\lambda^2 + C_{yy}^2\tan(\alpha)^2}}
\]

\[
\lambda_i = \frac{R * \omega_i - V_{pxi}}{\max(R\omega_i, V_{pxi})}
\]

\[
\alpha_i = \delta_i - \tan^{-1}\left(\frac{V_{pyi}}{V_{pxi}}\right)
\]

Where

- \(C_{xx}\) - Longitudinal tire stiffness
- \(C_{yy}\) - Lateral tire stiffness
- \(\alpha_i\) - slip angle at individual wheels
- \(V_{pxi}\) - Velocity of the tire along wheel X-axis
- \(V_{pyi}\) - Velocity of the tire along the wheel Y-axis
\( \mu \) - Road surface friction coefficient  
\( F_{ni} \) - Normal load acting on the wheels

### 2.3 Statistical estimation theory

The primary goal of the estimation theory is to estimate 'ideal' parameters from a set of statistical measurements that have randomness mixed in it. There are various tools called estimators that are mathematical models that helps extract the useful parameters from this cluster of data that is corrupted by a number of insignificant factors. Below are some of the estimators that find application in most engineering and medical applications.

- Recursive Least Squares (RLS)
- Non-linear Recursive Least Squares
- Baye’s filter
- Kalman filter (Linear Quadratic Estimator(LQE))
- The Ensemble Kalman filter
- Markov chain models

In a broad way, estimators can be classified into tools that work on data and those that work on the probability distribution of the said data. Though there are a number of estimators available, the non-linearity of the problem at hand necessitates the use of a non-linear filter and thus in this study we will restrict our review to the Unscented Kalman Filter(UKF) and the Extended Kalman filter(EKF) both of which are well studied and finds application in robotics, space and dynamics domains.
2.3.1 The Extended Kalman Filter

The Extended Kalman filter uses the kalman filter approach of prediction and correction for solving non-linear systems that cannot be expressed in the regularized state-space form. Assuming the noise in the measurement and the process noise are added linearly to the system, the non-linear system takes the following form.

\[
x_t = f(x_{t-1}, u_k) + w_k
\]
\[
y_t = g(x_k) + v_k
\]

(2.1)  
(2.2)

The important distinction between the UKF and EKF is the linearization of the non-linear system through Jacobian calculators.

\[
F = \frac{\partial f}{\partial x}
\]
\[
G = \frac{\partial g}{\partial x}
\]

(2.3)  
(2.4)

Given a state transition \( x_t \) and the respective jacobians \( F \) and \( G \), The predicted covariance estimate can be defined as follows

**Prediction step**

\[
x_t = f(x_{t-1}, u_k)
\]
\[
P_{t|t-1} = F_t \ast P_{t-1|t-1} \ast F_t^T + Q_t
\]

(2.5)  
(2.6)

Where \( Q_t \) is the process noise covariance
Correction step

\[ y_t = z_t - h(\hat{x}_{t|t-1}) \]  
\[ S_t = G_t \cdot P_{t|t-1} \cdot G_t^T + R_t \]  
\[ K_t = P_{t|t-1} \cdot G_t^T \cdot S_t^{-1} \]  
\[ P_{t|t} = (I - K_t \cdot G_t) \cdot P_{t|t-1} \]  
\[ \hat{x}_{t|t} = \hat{x}_{t|t-1} + K_t \cdot y_t \]

2.3.2 The Unscented Kalman Filter

The Unscented Kalman Filter was designed as an approach to apply the Kalman Filter (KF) to non-linear applications. This is because the Kalman filter only requires the first and the second moments i.e., the mean and covariance for execution and this information is practically attainable and capable of being executed in real time. An important area in which the Unscented Kalman filter exceeds the performance of the EKF is in its non-linearity as the performance of the EKF is completely dependent on the ability of the function to be linear in the time interval selected, the transformation and the current state estimate [12]. Another important limitation of the EKF is the requirement of the jacobian of the state transition matrix, which leads to oversimplification of the system model as in case of cascaded piecewise functions it is complicated to calculate the analytical jacobian of the model. One way to overcome this problem is the use of numerical jacobian. All the above problems led to the development of the Unscented Transformation (UT) which does not ignore the higher order terms in the Taylor’s series unlike the EKF, which takes only the first term in Taylor series expansion.

The UT understands that this has to be addressed and utilizes the fact to apply the Kalman
filter to non-linear systems, the better approach is not to linearize the system, but subject the predicted quantities through a non linear transformation and then apply the Kalman filter algorithm to it and this is exactly what the UT does.

The Unscented Transformation—Theory

The principle behind the UT is that approximating a probability distribution is efficient than approximating an unknown non-linear function. The key differences between the Unscented Transformation and the Particle filter are

- The sigma points are not selected in random, they are specifically selected such that they exhibit the properties that are user defined, i.e. the designer specifies the sigma points such that they have a specific mean and a covariance.
- The weights (significance) associated with the sigma points can be assigned based on
the discretion of the user, i.e., inconsistently, whereas due to the random nature of
the particle filters, all of the sigma points are weighted consistently.

**Significant parameters of the UKF**

**Alpha** determines the spread of the selected sigma particles around the mean of the state
value. **Beta** is used to numerically represent the distribution information of the random
variable to the UKF. **Kappa** is another parameter of the UKF that is used to scale the
distribution of the sigma particles around the mean state.

The following equations represent the theory behind the UKF. The state transition function
‘f’ specifies the relationship between the state at time t+1 and the inputs ‘u’ and the states
‘x’ at time ‘t’

\[
x(t + 1) = f(x(t), u_{state}(t)) + w(t) \quad (2.12)
\]

\[
y(t) = h(x(t), u_{meas}(t)) + v(t) \quad (2.13)
\]

If ‘Q’ is the covariance of the process noise ‘w’ while R is the covariance of the measurement
noise ‘v’. Then the filter estimation scheme is given as follows. Initialize with the estimate of
the initial conditions and then the error covariance matrix. \( \hat{X}_0 = E(X_0) \)

\[
P_0 = (E(X_0 - \hat{X}_0)E(X_0 - \hat{X}_0))^T
\]

The next steps selects the sigma points from the non-linear function

\[
\hat{X}^{(0)}[t + 1|t] = \hat{x}[t + 1|t]
\]

\[
\hat{X}^{(i)}[t + 1|t] = \hat{x}[t + 1|t] + \Delta x^{(i)}
\]

\[
\Delta X^i = \sqrt{c * P[t + 1|t]}_i
\]

\[
\Delta X^{n+i} = -\sqrt{c * P[t + 1|t]}_i
\]

Now that the sigma points have been selected, the next goal is to use the non-linear state

transition function to propagate the collected sigma points and predict their respective states. 
\( c = \alpha^2(n + \kappa) \) is a factor that is used to scale the model based on the number of states 'n', alpha and Kappa.

The next step is to evaluate the measurements at time 't' using the measurement function 'h'. Measurement function h(.) relates the measurement inputs and states at time 't' to the measurement outputs at time 't'. They will be used in the correction and prediction step of the UKF. A distinguishing factor between the EKF and the UKF are the weights associated with them. The output predicted by the measurement function is then corrected using the weights before use.

\[
\tilde{y}[t + 1] = \sum_{i=1}^{2n} W_n^{(i)} \tilde{y}^{(i)}[t + 1|t]
\]

\[
W_n^{(0)} = 1 - \frac{n}{\alpha^2(n + \kappa)}
\]

\[
W_n^{(i)} = \frac{1}{2 \alpha^2(n + \kappa)}
\]

**Correction step**

The measurement noise covariance 'R' is then added to covariance of the predicted measurement \( \tilde{y}[t + 1] \) which is then used to produce the cross covariance between the estimated state and the measurement \( \hat{x}[t + 1|t] \) and \( \tilde{y}[t + 1] \). The next step is to calculate the Kalman gains

\[
K = P_{xy} \cdot P_y^{-1}
\]

\[
\hat{x}[t + 1|t + 1] = \hat{x}[t + 1|t] + K(y[t + 1] - \tilde{y}[t + 1])
\]

\[
P[t + 1|t + 1] = P[t + 1|t] - K P_y K_T
\]
Prediction step

The prediction step also follows the same procedures in comparison to the correction step. The first step is the selection of weights $\hat{X}^{(0)}[t|t] = \hat{x}[t|t]$

$\hat{X}^{(i)}[t|t] = \hat{x}[t|t] + \Delta x^{(i)}$

$\Delta X^i = \sqrt{c \ast P[t|t]^i}$

$\Delta X^{n+i} = -\sqrt{c \ast P[t|t]^i}$

Use weights with the non linear state transition function to calculate the predicted states. Add the process noise into the system and calculate the state predictions using the measurements in the correction step. Calculate the state error covariance and then use the computed Kalman gain to predict the new states. Now propagate the model to the next recursion step.

2.3.3 Key takeaways from the review

The end goal of the project is to implement the scheme in a real-time environment to estimate friction. Therefore the magic formula is not suitable for this approach. On the other hand, the brush model assumes a parabolic distribution of the forces in addition to the microscopic bristle like structure of the tires, which result in a slight increase in computational complexity compared to the Dugoff Tire model. In addition to this, linearizing the Dugoff model is more effective and less intensive compared to linearizing the brush model as explained in [13] where the parabolic load distribution assumption that increases the computational cost is not justifiable from a controls perspective, Hence a Dugoff model is selected for use here. The Kalman filter in its truest form cannot be implemented on the model due to it’s non linear form. Hence the closest to this optimal filter are the non-linear versions of the Kalman filter such as the unscented and the extended Kalman filter. However one major drawback of the Extended Kalman filter is that it is only first order accurate [14] due to the fact that
during linearization of the non-linear function only the first two elements are selected from
the Taylor’s expansion, unlike the unscented transformation which transforms the non linear
function into a realistic non linear probability distribution which is then processed in that
domain.
Usable friction is highly dependent on the normal load applied to the tires and hence getting
a robust dynamic estimate of the load on the tires is of paramount importance. In addition
to that the stiffness of the tires, which is a major factor in friction algorithm is heavily
dependent on the normal load acting on it. In many prior studies, the load transfer at any
time is not considered. Therefore in this study, the suspension and spring parameters are
integrated using the roll plane model which is used to calculate the lateral load transfer
and the quarter car model that includes both the suspension, tire stiffness and damping
properties to calculate the dynamic normal load on each wheels at all time. The next chapter
elaborates on the construction of the model and the idea behind it’s methodology.
Chapter 3

Materials and methods

Simulink is the tool of choice selected for the application here due to its robust nature and its ease in modelling non-linear systems. Another important characteristic of simulink is that it can be deployed in c/c++ embedded framework which is a requirement for most if not many control systems to be implemented in real-time. In addition to the above advantages, simulink is used by the industry for its on-the-go tuning and interactive debugging approach. It even allows hybrid systems modelling, where the vehicle model can be considered as continuous systems while the sensor information in discrete time can be fused with this continuous systems which is very close to practical modelling. Another advantage of Simulink is its strength in being a co-simulation platform. There are toolboxes available for integrating simulink with CarMaker and ADAMS environments. All the factors combined have led to the usage of simulink throughout this project.

The research can be organized into two parts

- The vehicle model
- Friction estimation using the EKF and the UKF
3.1 The Vehicle model

The vehicle model consists of a mathematical set of equations that are founded upon force and moment balance methods. These equations are used to map the position, velocity and orientation of the vehicle at all times. In the following sections, the construction of the vehicle model and all the subsections that together form the vehicle model is explored and the results from the vehicle simulation are presented for different driving manoeuvres.

3.1.1 Tire velocity module

The longitudinal and lateral velocities at the wheel hubs is different from the respective velocities at the CG, and hence they must undergo co-ordinate transformation before they can be used in any estimation purposes. Additionally, the tire velocities are supplied to the Dugoff tire module for use in the calculation of forces and thus it’s important that we avoid any deviations at an early stage.

Through a simple trigonometric transformation they can be approximated as follows

\[
\begin{align*}
V_{xfl} &= V_x - r_f \dot{\psi} \\
V_{yfl} &= V_y + r_f \dot{\psi} \\
V_{xfv} &= V_x + r_f \dot{\psi} \\
V_{yfr} &= V_y + r_f \dot{\psi} \\
V_{xrr} &= V_x + r_r \dot{\psi} \\
V_{yrr} &= V_y - r_r \dot{\psi} \\
V_{xrl} &= V_x - r_r \dot{\psi} \\
V_{yrl} &= V_y - r_r \dot{\psi}
\end{align*}
\]  

(3.1)
Where $V_{xi}$ corresponds to the longitudinal velocity at the hub, i.e along the wheel frame of reference and $V_{yi}$ corresponds to the lateral velocity i.e velocity along the Y axis of the wheel frame of reference.

### 3.1.2 Dynamic load transfer due to rolling motion

When a vehicle accelerates, brakes or corners, there is a particular amount of weight that is transferred due to the inertia of the vehicle and its occupants. This inertial force must be balanced out by the suspension and other flexible components of the vehicle such that the driver can assert some control over the vehicle trajectory and its overall motion. This load transfer cannot be neglected as it is a major contributing factor to the grip available at the tires. Aligning with the goal to estimate friction and then using this estimated information to develop a controller, it becomes paramount to include this load transfer effect. Load transfer during braking and acceleration can also be modelled, however under nominal driving conditions there is not too much load transfer during acceleration. But there is a significant load transfer under braking. It is understandable then, if the friction estimated under normal free rolling conditions can then be used for use in ABS controllers, provided the vehicle is travelling on the same surface as which the friction is estimated. This approximation can be considered reasonable. However, in the future if the braking torque and the engine torque can also be accessed through the CAN bus, it is possible to include this aspect of load transfer into our model.

To calculate the load transfer due to rolling, a force balance approach is used here as described in the figure below:

$$F_c = M \cdot v_x \cdot \dot{\psi}$$

$$M_{roll} = F_c \cdot h_{roll}$$
3.1. The Vehicle model

Load transfer diagram for a vehicle making a left turn

Figure 3.1: Vehicle rear view

\[
F_{LHS} = -M_{roll} \cdot \frac{tw}{2}
\]

\[
F_{RHS} = M_{roll} \cdot \frac{tw}{2}
\]

\[
F_{n1} = F_{LHS} \cdot \frac{b}{l}
\]

\[
F_{n2} = F_{RHS} \cdot \frac{b}{l}
\]

\[
F_{n3} = F_{RHS} \cdot \frac{a}{l}
\]

\[
F_{n4} = F_{LHS} \cdot \frac{a}{l}
\]

where \( F_c \) is the cornering force

\( M_{roll} \) is the rolling moment due to the offset between the roll center and the center of gravity

\( F_{LHS} \) is the load transferred from the left hand side due to the rolling moment (i.e. when the
car is turning left)

$F_{RHS}$ is the load transferred to the right hand side due to the rolling moment (i.e. when the car is turning left)

$a$ is the distance from the front axle to CG

$b$ is the distance from the rear axle to CG

$l$ is the wheelbase

$tw$ is the track width.

The vehicle parameters such as the track width and wheel base are fed into the model through mask parameters as shown below.

![Block Parameters: Force due Roll](image)

Figure 3.2: Block parameters definition in simulink for load transfer

### 3.1.3 Tire forces block

From the literature review, the advantages of the said Dugoff model has been highlighted and is selected as the tire model here. One of the unique aspect of the study is that the model is designed considering both the lateral and longitudinal slip of the tire at all times.
In most studies, only the lateral slip or the longitudinal slip is included, due to the difficulty in modelling analytical jacobian for the EKF.

Once the velocity of the tires are obtained after the transformation from the tire velocity module, the next step is to determine the slip at the tires. This can be determined from the estimate of the wheel rotational speeds. This speed can be accurately measured using wheel encoders, which are installed in all cars manufactured after Sept 1, 2013.

Another important parameter that is crucial in the tire model is the steer angle $\delta$ which is obtained from a steering sensor. The steering wheel sensor is a very critical part of the electronic stability control setup, however the data from this sensor is not readily available as it passes information to the CAN bus and is not available to any user. Therefore for the purpose of this experiment, let us assume that we have a steering sensor mounted on the system and the information from this sensor goes into the model. Now that all the required information is ready, it is now relayed into the tire model block which then calculates the lateral and longitudinal forces on the tires.

![Diagram of the Dugoff tire model](image)

Figure 3.3: Function of a Dugoff tire model
The Dugoff model applied to the front left wheel

\[ F_{xfl} = C_{xx} \frac{\lambda_{fl}}{1 - \lambda_{fl}} \cdot k_{fl} \]  
\[ F_{yfl} = C_{yy} \frac{\tan(\alpha_{fl})}{1 - \lambda_{fl}} \cdot k_{fl} \]  
\[ k_{fl} = (2 - \sigma_{fl})\sigma_{fl} \quad \text{for} \quad \sigma_{fl} < 1 \quad \text{if} \quad \sigma_{fl} \geq 1 \]  
\[ \sigma_{fl} = \frac{(1 - \lambda_{fl}) \mu \cdot F_{n1}}{2\sqrt{C_{xx}^2 \lambda_{fl}^2 + C_{yy}^2 \tan(\alpha_{fl})^2}} \]  
\[ \lambda_{fl} = \frac{R \cdot \omega_{fl} - V_{xfl}}{\max(R \omega_i, V_{xfl})} \]  
\[ \alpha_{fl} = \delta_{fl} - \tan^{-1}\left(\frac{V_{yfl}}{V_{xfl}}\right) \]

3.1.4 The four wheel vehicle model

To model the vehicle in the global frame of reference, a model must be developed that evaluates the velocity of the vehicle at it’s CG and the sideslip of the vehicle must also be incorporated into the model. For this reason the motion of the CG is modelled as suggested by [15]

\[ \ddot{\beta} = \frac{1}{m \cdot V_{cg}} \cdot \left( (F_{y1} + F_{y2}) \cdot (\delta - \beta) + (F_{x1} + F_{x2}) \cdot \sin(\delta - \beta) \right. \]
\[ \left. - (F_{x3} + F_{x4}) \cdot \sin(\beta) + (F_{y3} + F_{y4})\cos(\beta) \right) - \dot{\psi} \]
\[ \dot{V}_{cg} = \frac{1}{m} \left( (F_{x1} + F_{x2}) \cos(\delta - \beta) + (F_{x3} + F_{x4}) \cos(\beta) - (F_{y1} + F_{y2}) \sin(\delta - \beta) \right. \\
\left. + (F_{y3} + F_{y4}) \sin(\beta) \right) \]

### 3.1.5 Co-ordinate Transformation

To visualize the trajectory of the vehicle, the basic rule of co-ordinate transformation is applied to the \( V_{cg} \) calculated and after one integration step, the actual position of the CG is calculated in real-time. A frame of reference is affixed to the CG of the vehicle and its motion is visualized with respect to the global frame of reference. [16] defines an acceptable form of co-ordinate transformation which is then modified for the problem at hand and is then applied to our system. The set of equations below are used to calculate the position of the CG at all times.

\[ x_{glob} = \int V_{cg} \cos(\psi + \beta) \quad (3.8) \]
\[ y_{glob} = \int V_{cg} \sin(\psi + \beta) \quad (3.9) \]

### 3.1.6 The vehicle model summarized

The flowchart below details the working summary of the vehicle model, The model is built in Simulink in discrete time and is used to map the path of the vehicle after a co-ordinate transformation from the local frame of reference to the global frame of reference.

The co-ordinate transformation is required for mapping the path of the vehicle, as \( \dot{V}_x \) and \( \dot{V}_y \) when integrated gives us the local 'x' and 'y' velocities of the vehicle at any time 't'.
Including the sideslip $\beta$ and the velocity at the CG is necessary to plot the path of the vehicle. Once the sideslip and the velocity of the CG is mapped and then transformed to the global frame of reference, the path of the vehicle can be visualized, This gives us an insight into the performance of the vehicle and can be used as a sanity check for the vehicle model. From the figure, we can see that the vehicle model is performing as per our expectations. The results from the vehicle model are then corrupted with Gaussian noise and then supplied as a sensory input to the extended kalman filter and the unscented kalman filter.

The idea behind this action is to ensure that we have data similar to what we would collect from a proving ground to validate the performance of the EKF.
3.1.7 Emulator design

The purpose of a model-based design setup is to provide an alternative for proving ground testing, such that all test cases can be simulated with a computer in hand. However, this does not imply that real-time proving ground test can be completely neglected, but through various studies, time and other resources expended on real time testing can be optimized. Therefore, it becomes essential to run simulations based on real environments and the changes in the real world can be adapted to the system during the final design approval stage. The complete vehicle model setup is then used to produce measurements that are similar to what we would find from a sensor that is affixed to a real vehicle. This is where the

Figure 3.5: Emulator design procedure
developed vehicle model doubles up as a sensor emulator, where the output from the model is modified to be used as inputs to the estimation algorithm. However this does not mean that any output from the system can be corrupted with noise to produce inputs to our model and requires engineering discretion before use. More information on the selection of measurements has been detailed in the state estimation section.

Figure 3.6: Comparison between the actual and measured longitudinal vehicle velocity during a lane change manoeuvre

Requirements of the model based design approach

- Must approximate real-life conditions

- Must be adaptable for conditions that are non-existent in test cases

The approximation of real life scenarios might not always be ideal and the system must have tuning capabilities under unanticipated circumstances and must allow the designer some
3.1.8 State Estimation

Once a vehicle model has been established, the next step is to build the estimation algorithm to meet the objectives. However, for the algorithm to work various pre-requisites are to be met.

The first step in this is to establish the states and the measurements.

The second step is to clearly outline the state and the measurement transition functions.

The state transition function is used to define how a state is varying at each time interval. The Euler integration method is used to solve for the states here.

The measurement transition function is specified such that the measurements are updated in time for the correction step of the Kalman filtering process.

States and measurement definition

States are any quantities of interest to the engineer that may or may not be available for calculation purposes, however any quantity that is unknown must be defined as part of the states. Usually the states are represented using the nomenclature ‘X’ and the same denomination has been adhered to in this study.

Measurements are any quantities that is available to the engineer through various sensor data. Measurements are denoted using ‘Y’ and the vector ‘U’ represents the inputs to the system.
For the said problem, the following states are defined below

\[ X = [\dot{\psi} \ V_x \ V_y \ F_{yf} \ F_{yr} \ F_{yrr} \ F_{yrl} \ F_x] \quad (3.10) \]

\[ Y = [\dot{\psi} \ V_x \ \dot{V}_x \ \dot{V}_y] \quad (3.11) \]

\[ U = [\delta \ F_{n1} \ F_{n2} \ F_{n3} \ F_{n4}] \quad (3.12) \]

\( F_{yi} \) corresponds to the lateral force on the respective wheels \( F_x \) is the sum of the longitudinal force for the front axle. This is combined for observability purposes. \( \dot{V}_x \) and \( \dot{V}_y \) are the lateral and longitudinal accelerations and \( \dot{\psi} \) is the yaw rate of the vehicle.

**Measurements explained**

The measurements for various model based studies are often a source of arguments and rightly so, it is vital that any measurement selected from the study must be reflective of the real-time conditions, for example, it is not practical to use the lateral force on any of the wheels as a measurement because, unless the tire is mounted on a flat-track machine, it is extremely inconvenient to get the force accurately. However, the speed of the vehicle is an ideal example of a good measurement because, it can be measured with reasonable accuracy using a GPS, speedometer etc.,

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \dot{\psi} )</td>
<td>IMU</td>
</tr>
<tr>
<td>( V_x )</td>
<td>Speedometer, Differential GPS</td>
</tr>
<tr>
<td>( \dot{V}_x )</td>
<td>IMU</td>
</tr>
<tr>
<td>( \dot{V}_y )</td>
<td>IMU</td>
</tr>
</tbody>
</table>
The non linear representation

As evaluated in the literature survey section, the non linear state space representation can be denoted as follows.

\[ x(t+1) = f(x(t), U(t)) + w(t) \]  \hspace{1cm} (3.13)
\[ y(t) = h(x(t), U(t)) + v(t) \]  \hspace{1cm} (3.14)

where \( f \) is the state transition function and \( h \) is the measurement transition function. Here the state transition function and the measurement function have the same inputs.

3.1.9 State Transition function

The State transition function is used to define how the state vector changes for every iteration of the estimation algorithm. In this study the state transition function is based on the Euler forward integration method.

The Relaxation tire model

The relaxation length of the tire is an important concept that drives the modelling efforts in many tire and vehicle models. It represents a delay between when the slip angle is introduced and the actual traction ("grip force") is manifested in the tires. Due to observability reasons the longitudinal forces in the tires is assumed to be constant. i.e a random walk model. However, the lateral forces are assumed to be varying over time and the variation of this lateral force is captured using the relaxation length property of the tires as mentioned in [17]

\[ \dot{r}_i = \frac{V_x}{rel_{len}} \ast (-F_y + \hat{r}_i) \]  \hspace{1cm} (3.15)
where $r_i$ is the force from the relaxation model and $F_y$ is the force estimated by the estimation algorithms and $rel_{len}$ is the relaxation length of the tires. For this study the relaxation length of the tire is assumed to be 0.15m. Another important characteristic is the tire stiffness. There is a good database on the online estimation of tire stiffness using vehicle information, however in this study a look-up table approach is used in the stiffness of the tires, which are used in the state transition equations of the model.

**Euler forward method**

Euler integration works on the principle that a curve can be approximated if the slope (change in quantity per unit time) is known. An obvious limitation of the Euler integration technique is the sample time, the smaller the sample time, the greater will be the accuracy of the function, however this comes at a greater computational cost and a balance must be sought.

$$x_{t+1} = x_t + \Delta t \ast \dot{x}$$  \hspace{1cm} (3.16)

where $\Delta t$ is the sample time and $\dot{x}$ is the slope of the function.

![Figure 3.7: The fundamentals of the Euler forward method](image)
The state transition equations

\begin{align*}
st_1 &= x_1 + \frac{T_s}{I_{zz}} \left( a \ast (x_4 \ast \cos(u_1) + x_5 \ast \cos(u_1) + x_8 \ast \sin(u_1)) - b(x_6 + x_7) 
+ \frac{tw}{2} \ast (-x_5 \ast \sin(u_1) + x_4 \ast \sin(u_1) + x_8 \ast \left( \frac{u_3}{u_2 + u_3} \ast \cos(u_1) - x_8 \ast \cos(u_1) \ast \frac{u_2}{u_2 + u_3} \right) \right) \\

st_2 &= x_2 + \frac{T_s}{Mass} \ast \left( x_8 \ast \cos(u_1) - (x_4 + x_5) \ast \sin(u_1) \right) + Ts \ast x_1 \ast x_3 \\

st_3 &= x_3 + \frac{T_s}{Mass} \ast \left( x_4 + x_5 \ast \cos(u_1) - x_8 \ast \sin(u_1) + (x_6 + x_7) \right) - Ts \ast x_1 \ast x_2 \\

st_4 &= x_4 + Ts \ast \frac{x_2}{rel_{len}} \ast (-x_4 + r_1) \\

st_5 &= x_5 + Ts \ast \frac{x_2}{rel_{len}} \ast (-x_5 + r_2) \\

st_6 &= x_6 + Ts \ast \frac{x_2}{rel_{len}} \ast (-x_6 + r_3) \\

st_7 &= x_7 + Ts \ast \frac{x_2}{rel_{len}} \ast (-x_7 + r_4) \\

st_8 &= x_8
\end{align*}

The above set of equations sum up the entire state transition function for all the eight states. where \( st_i \) represents the state \( 'i' \) at time \( 't+1' \) whereas \( x_i \) represents the state \( 'i' \) at time \( 't' \) and \( Ts \) is the sample time

### 3.1.10 The measurement transition function

An important aspect of the measurement transition function is that it must be reflective of the measurements that we have from sensors that are available in all road vehicles. The responsibility of selecting these "valid" measurements lies solely on the design and simulation
engineer and must be actively verified at all stages. 
For example, many research studies have included $a_x$ and $a_y$ in their studies where

\[ a_x = \dot{V}_x - \dot{\psi} * V_y \]  \(3.25\)
\[ a_y = \dot{V}_y + \dot{\psi} * V_x \]  \(3.26\)

In comparison with the four wheel vehicle model it removes the element of yaw rate from the model equations, but in reality any data that comes from the IMU is inclusive of the yaw rate parameter that affects the car and the sensors in it. Therefore $\dot{V}_x$ and $\dot{V}_y$ are more accurate compared to the selection of $a_x$ and $a_y$ for this analysis.

The measurement transition equations are constructed such that they represent the states at any time ‘t’, which is an important distinction between the measurement and the state transition functions, because the state transition functions are essentially prediction functions which estimate the states at time ‘t+1’

**The measurement transition equations**

\[ mt_1 = x_1 \]  \(3.27\)
\[ mt_2 = x_2 \]  \(3.28\)
\[ mt_3 = \frac{1}{mass} * \left( x_8 * \cos(u_1) - (x_4 + x_5) * \sin(u_1) \right) + x_3 * x_1 \]  \(3.29\)
\[ mt_4 = \frac{1}{mass} * \left( (x_4 + x_5) * \cos(u_1) + x_8 * \sin(u_1) + (x_6 + x_7) \right) - x_2 * x_1 \]  \(3.30\)

The state and the measurement transition functions are used simultaneously to run the optimal estimators and the results from the study is presented in the next chapter.
3.1. The Vehicle model

Friction estimation using UKF

In a tyre model, the friction of the surface must be known before the forces can be calculated. This is an inherent problem that must be addressed. For steady state maneuvers the UKF is robust enough to handle even the step variations in friction, the friction at each step is calculated using the following relation.

\[ \sqrt{F_{x_{est}}^2 + F_{y_{est}}^2} \leq \mu \times F_z \]

Once this friction has been calculated it is then passed into the state transition function for every iteration and this is used to calculate the forces at each time step.

For highly transient events an independent Recursive Least squares filter that uses the estimated yaw rate and the measured yaw rate can be used to predict friction that is independent of the UKF parameters. The figure below provides a view into the working of the UKF.

Figure 3.8: Friction estimation scheme
Chapter 4

Results and Discussion

In this section, a set of results from the performance analysis of the model under various conditions are presented. The primary objective of the study is to estimate the available road surface friction from the driving surface and the goal of the project has been satisfied. At the end of the section the results are critically analyzed for the two filters under study and the breadth of operational capability is also analyzed.

Various tests are conducted to determine the performance of the filter and some of the findings are documented below.

- Road surface friction is estimated with reasonable accuracy for a wide range of road surfaces.
- The lateral force on the tires are captured accurately for different driving manoeuvres.
- When numerical jacobians are used for estimation, the EKF and UKF perform comparably under the same initial conditions, noise and error covariance estimates. However, if the initial conditions are not accurate, then the UKF performs better than the EKF.
- With the lookup table approach of the tire stiffness values, for similar initial error covariances, the UKF is found to be more robust in comparison to the EKF in terms of friction estimation.

The figure 4.1 represents the estimated friction in comparison with the available friction and
Figure 4.1: Comparison between the actual friction and estimated friction provides conclusive evidence to the performance of the algorithm.

Figure 4.2: Lateral forces estimate for the Constant radius turn.
4.1 Discussion

Compared to studies like the longitudinal slip-slope methods to predict the road surface friction the UKF and the EKF methods are robust and adapt on-the-go to different driving methods. However like any estimation technique there are a few limitations to the study. For example, there must be a steering input (which causes slip) for the algorithm to detect the road surface characteristic. In the figure below, one can visualize the performance of the EKF. The advantage of the Kalman based estimators are that they are predictive algorithms unlike the others which use measurements to clean up the data. The advantage of the predictive measurements is in their real time capability. The figure below, demonstrates the advantage of the filter in that noise in the yaw rate signal has been reduced significantly while the accuracy of the predicted yaw rate is good.

![Figure 4.3: Predicted vs measured vs actual yaw rate for constant radius cornering](image)

The problem is easier to visualize in the double lane change manoeuvre in which the estimator’s accuracy is significantly better in the regions where there are steering angle inputs
4.1. Discussion

(a) Force on the front left predicted using EKF

(b) Force on the front right predicted using UKF

Figure 4.4: Lateral forces for sinusoidal steering inputs
4.2 Robustness of Friction estimation algorithms

A number of parameters like suspension stiffness, damping and tire properties are subject to change during the design phase of the vehicle model and are also critical to our estimation algorithm. Hence it is necessary to understand how the system varies with parameters that are critical to the overall design of the vehicle. Therefore the next step is to undertake sensitivity analysis of the system. Sensitivity is a metric that is used to quantify the critical nature of the parameters in a study, it can be generalized as follows.
4.2. Robustness of Friction estimation algorithms

\[ \text{Sensitivity} = \frac{\text{Change in performance metric}}{\text{Change in input parameter value}} \]  

(4.1)

4.2.1 Parameters under consideration

From the figure, we can relate to the actual difference that the load transfer brings to the estimation algorithm, where the roll center is identified to be the most sensitive parameter with respect to the friction estimation scheme. This is important because the model has been built in a way such that the roll center is a critical component in how the load transfer is calculated. Thus we can infer that the load transfer is a vital characteristic in the friction estimation algorithm and must be considered when such a system is designed. Other pa-
parameters that are under consideration include the suspension and tire stiffness and damping properties. The study shows that the estimation algorithm is robust to variations in these parameters.

Figure 4.7: System response to step change in friction

Figure 4.7 shows the performance of the algorithm for a constant radius cornering manoeuvre where the friction suddenly drops from 1 to 0.5 and it can be seen that the friction is predicted with good accuracy. Figure 4.8 shows the predicted friction value for a highly transient sine steer manoeuvre with high accuracy. This friction has been predicted by Recursive Least squares approach using the information that was estimated by the UKF combined with the measurements collected by the IMU.
4.3 Friction information applied to Anti-lock Braking Systems

The concept of Anti-lock Braking Systems can be illustrated better with the aid of the \( \mu \)-slip curve. From the \( \mu \)-slip curve we understand that for a particular tire for specific value of friction there exists an ideal slip so as to maximize the traction or 'gripping power' of the car. It is also worth noting that there are a number of factors that affect this curve, for example some of the factors include camber, inflation pressure, normal load on the tires, environment and tire temperatures as the rubber compound used to manufacture the tire has different properties under different conditions and is made available by the tire manufacturer. Under full slip conditions, the grip produced by the tires reduce significantly and this means a longer time is required by the vehicle to stop and hence longer distance to stop. Minimizing the stopping distance is vital because this could mean the difference between a crash and a safe stop. Here slip refers to the longitudinal slip \( \lambda_i \) of the tires as defined under section
2.1.3. An Anti lock braking system has two primary purposes

- Minimize stopping distance
- Maintain directional control under hard braking

These objectives can be met by optimizing the slip of the tire to stay in the region where the friction forces are optimized. Due to difficulties in online measurement of road friction parameters, ABS has traditionally been designed to work by indirect calculation of friction during operation. Though this might provide increased robustness, the solution would be sub-optimal for varying conditions as the vehicle takes some time to identify this friction and under hard-braking situations, time is a critical factor, hence the benefits to prior friction estimate become obvious. Generally, there are two approaches to ABS design, they are
4.3. Friction information applied to Anti-lock Braking Systems

- Wheel slip based method
- Wheel deceleration method

4.3.1 Wheel slip method

In the wheel slip method, the controllers are built such that a predefined slip rate is established for the controller to adhere to and based on the wheel and the vehicle speed, brake slip is calculated at each time interval and the control algorithm modulates the brake pressure to produce this value of desired slip where decelerations are used to establish the rules in the system. This is the most common approach to ABS modelling commercially. This is due to the simplicity involved in the algorithm and it’s sufficient performance, however this is not an optimal approach to and hence there is significant room for improvement.

4.3.2 Wheel deceleration method

In the wheel slip method, the wheel speed can be measured through simple sensors, however measuring the vehicle speed accurately has been a challenge. This brought about the wheel deceleration approach in which accelerometers at the wheels are used to measure the deceleration and this information is passed to the controller where a pre-defined deceleration is defined for the controller, and the brake pressure is modulated accordingly to achieve this condition.

In this study we use a wheel slip based controller to demonstrate the effect of online knowledge of friction on the ABS controller and how this performance improvement is translated in terms of stopping distance. In the first sub-chapter we will discuss the design of an ABS system and then performance comparison between the quarter car model under full wheel
lock, then an open loop ABS system and a PID integrated ABS controller is studied and the advantages of friction knowledge are discussed.

4.4 Design of an ABS controller

Figure 4.10: Forces on the tire under braking

4.4.1 Assumptions of the design

- There is no forward thrust under braking.
- The load transfer under braking is not considered
- Transmission and bearing losses are not considered
- Aerodynamic forces are not considered
Applying a simple force balance to the model above, we have the following results \[ m \dot{v} = -m \cdot g \cdot \dot{v} \]

\[ I \dot{\omega} = m \cdot g \cdot \mu \cdot R - T_b \]

Newton’s second law is applied to the model to bring about the above relationships where ‘m’ is the mass on the wheel (including wheel mass) and \( \dot{v} \) is the deceleration of the wheel center (under braking), \( R \) is the wheel radius, \( \omega \) is the angular velocity of the wheel. When a brake torque of \( T_b \) is applied to the wheel the theoretical maximum force that can be present at the tire road interface is a function of \( \mu \) and the normal load \( m \cdot g \) acts on the contact patch where \( g \) is the acceleration due to gravity. Equations 5.1 and 5.2 can be simplified as follows Substituting (6.1) in (6.2), we have

\[ I \dot{\omega} = -m \cdot \dot{v} \cdot \mu \cdot R - T_b \]

Developing a slip based controller requires the following relationship

\[ \lambda = 1 - \frac{R \omega}{v} \]

\( \lambda \) being the longitudinal slip under braking, \( R \) the tire radius and \( v \) the linear velocity of the wheel center. Differentiating with respect to time we have

\[ \dot{\lambda} = 0 - R \cdot \left( \frac{v \dot{\omega} - \omega \dot{v}}{v^2} \right) \]

\[ \dot{\lambda} = -R \cdot \left( v \cdot \frac{-m \cdot \dot{v} \cdot R - T_b}{I v^2} - \frac{\omega \cdot \dot{v}}{v^2} \right) \]

\[ \dot{\lambda} = \frac{m \dot{v} R^2 + T_b \cdot R}{I \cdot v} + \frac{R \cdot \omega \cdot \dot{v}}{v^2} \]
The slip rate derived is the quantity that we want to control. We understand from the theory that for a specific tire there exists a desired value of slip for a particular value of friction. Most of the control techniques try to maximize the friction in the surface by varying the slip and theoretically measure the friction on the surface using a maximization function. This has been designed such that no knowledge of friction is required when the brakes are applied, however the drawback being, the controller has to constantly adjust slip so as to get the ideal value of brake pressure to produce the optimum friction to stop the vehicle in the shortest time possible.

But with prior knowledge of friction, the problem reduces to a simple control problem in which a lookup table can be used to produce the ideal value of slip for a particular friction surface that is under consideration. In other words, there is scope for the vehicle to stop at an even shorter time and therefore distance and the controller need only be tuned for the minimum settling time rather than it being designed to maximize the utilization of friction and minimization of settling time simultaneously. In this study, the lookup table approach is not used, instead a PID based control system is used to optimize the braking pressure.

For a discrete controller

\[ \dot{\lambda} = \lambda - \lambda_{des} \]  

(4.9)

Rearranging 6.7 we have the following relation

\[ T_b = \frac{I v (\lambda - \lambda_{des})}{R} - \frac{I \omega v}{v} - m\ddot{v}R \]  

(4.10)

Here \( \lambda_{des} \) is introduced as a regulatory element in the system which is the input from the controller, and the term \( \frac{I v (\lambda - \lambda_{des})}{R} \) can be multiplied by a constant negative term \( G \), to ensure that the system is exponentially stable and the rate of exponential stability is controlled by
the magnitude of 'G'.

\[ T_b = \frac{G \lambda (\lambda - \lambda_{des})}{R} - \frac{I \omega \dot{v}}{v} - m \dot{v} R \] (4.11)

The objective of an ABS from a controller’s perspective would be to minimize the stopping distance of the vehicle, the directional vehicle control is a useful advantage of doing so. However there is no ideal way of predicting what the stopping distance would be at the moment the brakes are applied, in other words there is no suitable measurement for the controller to work with. This problem can be alleviated by establishing a relationship between friction and slip as the friction available from a surface is inversely proportional to the stopping distance. This relationship is exploited here.

The function below establishes a relationship between the desired longitudinal slip \( \lambda_{des} \), desired friction \( \mu_{des} \) (also known as maximum available friction), actual longitudinal wheel slip \( \lambda \) and the utilized friction \( \mu \). From figure 6.1, we can analyze that for each value of \( \mu \), there is a value of \( \lambda \) that is ideal. This relationship can go into the system as follows [19]

\[ \mu = \frac{2 \mu_{des} \lambda_{des} \lambda}{\lambda^2 + \lambda_{des}^2} \] (4.12)

The ABS module is now built in Simulink and integrated with the friction estimation scheme.

**Simulation results**

In this section the simulation results from the ABS controllers are studied. There are three experimental conditions that are considered here, they are, simulation under full wheel lock conditions. This is simulated by removing the incoming friction information and setting the
desired wheel slip to 1, thus emulating full wheel lock conditions, braking with friction information and no control and controlled braking with friction information. In the second stage, the inherent stability of the system is employed and open loop control improvement of the model is demonstrated. In the next section, A PID controller is implemented and the improvement in the stopping distance has been demonstrated.

Without ABS, the wheel speed rapidly reduces to zero thus locking up, this in turn increases the time for the vehicle to come to a complete stop. With friction information, the system becomes stable and at a constant slip rate of 0.2, the vehicle comes to rest slightly later than that of a ABS with PID control augmented with friction information from the algorithm.

The figure below discusses the same outcome measured in terms of stopping distance. In

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**Table 4.1: Operating conditions under braking**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Friction</td>
<td>0.8</td>
</tr>
<tr>
<td>Initial velocity</td>
<td>75 mph</td>
</tr>
<tr>
<td>Tire radius</td>
<td>0.306 m</td>
</tr>
</tbody>
</table>

---

**Figure 4.11: ABS controller**
the table below, the operating conditions and the performance metrics for the different ABS design methodologies are detailed for the reader. Overall, the ABS algorithm with PID control and friction information provides better performance for the quarter car model that is under consideration and provides an improvement of around 12% in terms of stopping distance, which is quite significant. The next step in the direction would be to add the model complexities that are discussed in the assumptions section of the ABS design and demonstrate the real time capabilities of the model that is studied.
CHAPTER 4. RESULTS AND DISCUSSION

Figure 4.13: Comparison between stopping distances under different conditions

Table 4.2: Performance metrics for different braking conditions

<table>
<thead>
<tr>
<th>Event</th>
<th>Stopping distance(m)</th>
<th>Stopping time(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wheel locked</td>
<td>130.1</td>
<td>8.51</td>
</tr>
<tr>
<td>Open loop control with friction information</td>
<td>89.5</td>
<td>4.916</td>
</tr>
<tr>
<td>Closed loop control with PID and friction information</td>
<td>78.6</td>
<td>4.61</td>
</tr>
</tbody>
</table>

4.5 Future scope

The above article presents a study into the characteristics of the vehicle, however kinematic parameters such as the camber, castor, and other compliance are not modelled into the sys-
4.5. Future scope

Figure 4.14: ABS system with a longitudinal slip based magic formula with friction information and PID control

Modelling these aspects of the system would prove crucial in improving its accuracy. However, this data can only be supplied by the industry and varies for different models and makes. The inclusion of camber, castor and other parameters also bring an element of time varying parameters as they vary during the course of driving and must be accounted for.

There are numerous studies such as the [20] that uses data from embedded tire sensors that can be used to classify terrains based on signals from the said sensors. Another approach would be to create a hybrid system that uses the model-based approach developed here in unison with the terrain classification system and use this information to produce detailed dynamic friction information. This friction classification system would provide a range of friction information that can be used for faster convergence in the models as the algorithm now has to search between a smaller range of values than the entire spectrum from 0 to 1.
Chapter 5

Conclusion

Through this study a kalman based estimator has been developed using a four wheel vehicle model with three degrees of freedom taking into account the effect of dynamic lateral load transfer using quarter car models. Sensor data has been simulated using random Gaussian noise and used to approximate real time conditions. The Non-linear kalman estimators have been developed using a numerical jacobian approach in the Simulink environment which is directly instrumental to deploy the algorithms inside automotive hardware.

The non-linearity in the model requires the use of kalman estimators such as the EKF and the UKF and their performance has been compared against one another. One important conclusion reached from this study would be the use of UKF algorithms for systems that have an inaccurate initial condition estimates and for systems that require the use of varying tire stiffness values which improve the accuracy of the model.

The filters are modelled to estimate the lateral forces on the tires using the true dugoff tire model that utilizes combined slip thus providing an avenue for the extension of the project into the longitudinal domain if information on the torque on the wheels is made available in the future.

The lateral forces estimated are then used to estimate the friction on the road surface using the maximum road surface friction coefficient equations and plot have been simulated demonstrating the robustness of the model against varying road surfaces using transient and steady-state driving manoeuvres.
The dynamic estimate of friction predicted using the information is then passed on to an ABS controller which then modulates the desired slip value by varying the brake pressure to stop at a distance lower than it would have if there was no friction information.
Bibliography


