Model, Design, and Control for Power Conversion
in Wave Energy Converter System

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Abstract

Wave energy has great potential in energy harvesting, but due to its high system cost per electricity production, it is still in the pre-commercialization stage for grid connection.

A wave energy converter (WEC) system that harvests energy through wave motion consists of a wave energy converter and a power take-off (PTO). A wave energy converter, usually a floating buoy, absorbs the hydrodynamic motion from wave and generates a mechanical oscillation. A power take-off (PTO) with mechanical transmission, which harvests the electrical energy through the mechanical energy, usually includes a transmission that converts linear motions from the buoy to rotational motions, an electromagnetic generator that produces electricity from a rotational shaft, and a power electronics converter that converts the ac electric power from the generator and charges the output dc battery or the ac grid.

The models of the WEC system are usually oversimplified in a multi-physics study. A PTO model as an ideal actuator with 100 % efficiency will show a different frequency response than the real tested results and can make the controller design invalid. A conventional regular-wave circuit model shows discrepancies in power and force prediction in time-domain under irregular wave conditions. A model that can bring the multiple fields together, and provides an accurate prediction from irregular wave dynamics and non-ideal PTO mechanism is needed.
A methodology that converts mechanical transmission equations into a circuit model is created. The equivalent circuits of mechanical components such as one-way clutches, gears, a ball screw, mechanical couplings, and generator are derived respectively to describe the dry frictions, viscous damping, and mechanical compliances in these components. The non-ideal efficiency and force of the PTO are predicted in electrical simulations by integrating these sub-circuit models. The circuit model is simplified, and its parameters are categorized as dc and ac unknowns. Using PTO with a mechanical-motion-rectifier (MMR) gearbox as an example, the dc and ac tests on the PTO are performed sequentially to extract two sets of parameters through linear regression or nonlinear curve fitting. The simulated efficiencies of 30 – 80% match well with experimental results. The model is validated through its prediction capability over 25 test conditions on input forces, output voltages, and efficiencies, with correlation coefficients $R^2$ value of 0.9, 0.98, and 0.981, respectively.

An equivalent circuit model of fluid-body dynamics for irregular waves, applicable to real ocean conditions with frequency-dependent radiation damping, is developed. Different from PTO modeling, the time-invariant circuit is created from a fourth-order RLC equivalent circuit through transfer function approximation in the frequency domain and Brune network. The circuit-based wave energy converter (WEC) model is verified by comparing the results with the predictions of a detailed model under irregular wave conditions in the time and frequency domains based on a point absorber type of WEC with a power take-off (PTO). The results show that the developed model gives an accurate dynamic prediction for a WEC under both regular and irregular conditions. Along with the PTO model, the circuit-based W2W model is completed for control and design optimization of the WEC system.
Wave energy converter systems have faced various challenges such as reciprocal wave motion, high peak-to-average power ratio, and potential wave height from hundred-year storm conditions. These could lead to an overdesigned power take-off (PTO) of the system and significantly reduce the lifetime of the power electronics converter.

The power ratio between the peak and the average power of the wave power converter is around 10 – 20 times. Power optimization is necessary to reduce the over design ratio of the power electronics converter. The design guideline that optimizes the power ratings for the power converter and the generator is introduced. The methodology is developed from the W2W circuit model taking the losses of the power converter and the generator into consideration. By optimizing the power limiting and field-weakening controls, the ratio from the average output power to the rated power of the power converter is reduced to 2.4 in the maximum wave condition, and 15 in the annual wave profile.

A maximum energy control algorithm on the power electronics in wave energy application is developed to increase the total energy produced from the power converter in a wave energy converter (WEC) system. A 4-D damping and power leveling maps for maximum energy are built for the algorithm. The maps are based on the irregular W2W circuit model and reliability analysis on the IGBT module. From the yearly wave mission profile, the strategy is proved to increase energy by 16 times or increase the lifetime from 3 to 18 years in exchange for 6 % of average output power than the conventional maximum power algorithm.

In conclusion, this work provides a new circuit-based perspective for co-designing the multi-disciplinary WEC system. The methodologies of circuit modeling can benefit the co-design process of other mechatronic power systems, such as electric vehicle or renewable energy system. The newly invented mechanical device – the mechanical motion rectifier, is understood
thoroughly via the non-ideal electrical model.

The commercialization of wave energy converter is driven forward through the reduction of the levelized cost of electricity (LCoE) which is made possible by increasing the energy production and optimizing the cost per output power of the generation and power conditioning stages.
Model, Design, and Control for Power Conversion in Wave Energy Converter System

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General Audience Abstract

Wave energy, if all been harvested along the U.S. coastline, can power around 65% of the energy consumption in U.S.. Comparing to other renewable energy sources like solar or wind, ocean wave can provide up to 90% of steady uptime. With the high energy density (2-3 kW/m²), it can produce more energy with the same amount of installation area comparing to the energy density of wind turbine (0.6 kW/m²) and solar panel(0.2 kW/m²). The predictability of wave provides advantages like planning installation, power dispatching, and maintenance activities.

Although with all these advantages, wave energy converter system is still in the research stage due to its high system cost per electricity production. One of the challenges that need to be solved is the irregularity from the wave motion that leads to high instantaneous peak power into the wave energy converter, which usually reaches up to 10 - 20 times of the average power. The high peak power will not only bring high mechanical/electrical stress but also result in an overrating design of the components in the system. Another obstacle that prevents the wave energy system from moving forward is the high testing cost from the validations in wave-energy-test sites or tank-test sites. A high-fidelity multi-disciplinary system model, including hydrodynamics, mechanical dynamics, electromagnetics, and power electronics, is needed to predict the behavior of the system and reduce the cost of design validation.

This work provides a unified circuit-based perspective for co-designing the multi-disciplinary
wave energy system. The efficiencies and mechanical dynamics of the system are accurately predicted via the non-ideal electrical model. These methodologies of circuit modeling can also benefit the co-design process of other mechatronic power systems, such as electric vehicles or renewable energy systems. The peak of the irregular power is controlled by the power-leveling and field-weakening control, and as a result, the overdesign ratio of the power converter reduces from 11.1 to 2.4. Through proper design of the converter’s control algorithm, the total produce electric energy is increased by 15 times, as well as the lifetime of the power electronics extended from 3 years to 18 years.

Therefore, the commercialization of wave energy converter is driven forward through the reduction of the levelized cost of electricity (LCoE), which is made possible by optimizing the component lifetime and the output energy utilizing the developed circuit-based wave-to-wire model.
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Table of Contents

Chapter 1  Introduction .............................................................................................................. 1

1.1  Introduction to Wave Energy Converter systems ............................................................. 3

1.2  Introduction to Power Take-off Systems .......................................................................... 4

1.3  Introduction to the Generators and the Power Converter in WEC system .............. 6

1.4  Potentials and Challenges of Wave Energy Converters ............................................. 8

1.5  Research Objectives and Methodologies ........................................................................ 12

1.5.1  MMR-PTO Circuit Model with Efficiency Prediction ................................................. 12

1.5.2  Irregular Circuit-Based Wave-to-Wire Model for WEC ...................................... 13

1.5.3  Power Converter Design on WEC Application ....................................................... 14

1.5.4  Maximum Energy Control on Wave Energy Application .................................... 15

1.6  The Contributions and Outline of the Dissertation ................................................... 16

Chapter 2  Circuit Model of Power Take-off System with Mechanical-Motion-Rectifier ... 20

2.1  Introduction of PTO System modeling ........................................................................... 20

2.1.1  Introduction of MMR-PTO ..................................................................................... 20

2.1.2  Ideal Mathematical and Circuit Model of MMR-PTO .......................................... 23

2.1.3  Synthesis Methodology for Circuit Model of MMR-PTO ..................................... 25

2.2  Mathematical Model of PTO ......................................................................................... 27

2.3  Circuit Network Model of PTO ...................................................................................... 32
2.3.1 Circuit Model with Parallel Components .............................................. 32
2.3.2 Circuit Model with Series Components and Non-linear Components .......... 33
2.3.3 Circuit Model of MMR-PTO System ...................................................... 37
2.4 Parameter Extraction and Model Validation .............................................. 39
  2.4.1 Test Bench Setup .............................................................................. 40
  2.4.2 Parameter Extraction with Dc Equivalent Circuit Model ......................... 42
  2.4.3 Parameter Extraction with Ac Equivalent Circuit Model (Non-linear Simulation Model) .................................................................................................................. 44
  2.4.4 Model Validation on Time-Domain Results ........................................... 48
2.5 Application of the PTO circuit model ....................................................... 52
  2.5.1 Loss Breakdown and Efficiency Analysis with PTO Model ...................... 53
  2.5.2 Impact of the Disengagement from MMR in PTO System ....................... 54
2.6 Conclusion ................................................................................................. 56

Chapter 3 Irregular Wave-to-Wire Circuit Model for Wave Energy Converter System
......................................................................................................................... 59
  3.1 Introduction of WEC System ..................................................................... 60
    3.1.1 Model of SDoF Point Absorber WEC .................................................. 60
    3.1.2 Derivation of WEC Parameters ......................................................... 62
    3.1.3 Review of Mathematical Approximation Model .................................... 63
    3.1.4 Review of WEC Circuit Model ......................................................... 64

xi
3.2 Approximation Method and Equivalent Circuit of WEC ............................................. 67
  3.2.1 Development of WEC Circuit Model ................................................................. 68
  3.2.2 Validation of WEC Circuit Model ........................................................................ 70
3.3 Ideal W2W model: Bias Map and Application ......................................................... 74
  3.3.1 Bias Map with Ideal W2W Model ........................................................................ 74
  3.3.2 Application of Ideal W2W Model ........................................................................ 75
3.4 Conclusion .................................................................................................................... 77

Chapter 4 Design Guideline for Power Converter in Wave Energy Application ...... 78
4.1 Introduction of Power Converter in WEC system ....................................................... 78
  4.1.1 Power Electronics Converters in WEC System .................................................... 78
  4.1.2 Review of Power Converter Design in WEC System ......................................... 79
  4.1.3 Review of Power Converter and Generator Design in WEC System ............... 81
4.2 Generator and Power Converter Modeling for Optimization in WEC system ...... 83
  4.2.1 WEC System Model and System Control Algorithm ..................................... 83
  4.2.2 Loss Model of Power Electronics Converter ..................................................... 85
  4.2.3 Time-Domain Simulation ................................................................................... 87
4.3 Power Converter Optimization Algorithm ................................................................. 91
  4.3.1 Control Variables, Constraints, and Optimization Outputs ............................. 91
  4.3.2 Optimization Flowchart for the Power Converter Design ............................... 92
  4.3.3 Output Analysis .................................................................................................. 95
4.4 Parametric Analysis of Optimization Variables ................................................................. 100

4.4.1 Effects of Gear Ratio ........................................................................................................ 100

4.4.2 Field-weakening Ratio (FWR) and Full Load Ratio (FLR) ............................................ 103

4.4.3 Efficiency Analysis ........................................................................................................... 105

4.5 Conclusion .......................................................................................................................... 107

Chapter 5 Maximum Energy Control for Wave Energy Converter ......................... 109

5.1 Introduction of Power Optimization for WEC system ...................................................... 110

5.1.1 Review of Power Optimization in WEC ................................................................. 111

5.1.2 Review of Reliability Analysis for Power Converters ............................................. 112

5.1.3 Review of Thermal Control and design for Power Converter ................................. 114

5.1.4 Energy optimization for the WEC power converter with the control algorithm. ................................................................. 115

5.2 Average Circuit and Lifetime Model for Power Converter ............................................. 117

5.2.1 Average Circuit Model for Three-phase Two-level Ac-Dc Power Converter .... 119

5.2.2 Power Loss and Thermal Model for Bond-wire in Power Module ...................... 121

5.2.3 Lifetime Model for Bond-wire in Power Module ..................................................... 124

5.3 Control Algorithms for the WEC Power Converter ....................................................... 127

5.3.1 Open-loop Control Algorithms ...................................................................................... 128

5.3.2 Selection of Variables Range ...................................................................................... 131

xiii
Appendix A - Test Setup and Performance of the Inverter Mode

A.5 Software Interface ........................................................... 180
A.6 Test Procedure ............................................................... 180
A.7 Test Setup and Performance of the Inverter Mode ......................... 183
A.8 Test Setup and Performance of the Rectifier Mode ......................... 189

Appendix B - Background Information of Figures ........................................ 196

B.1 Setups of Figures ................................................................... 196
B.2 Figures – Files in Chapter 2 ..................................................... 196
B.3 Figures – Files in Chapter 3 ..................................................... 197
B.4 Figures – Files in Chapter 4 ..................................................... 198
B.5 Figures – Files in Chapter 5 ..................................................... 199
B.6 Figures – Files in Appendix A .................................................. 201
List of Figures

Fig. 1-1. The structure of the wave energy converter system [7] ............................................................................................................2

Fig. 1-2. Main designs of wave energy converters [20] and their R&D efforts [21] .................................................................4

Fig. 1-3. PTOs with their corresponding efficiencies and annual power productions [23][27] .......................6

Fig. 1-4. The simulation of weekly energy profile among different energy sources and the demand [39] .......9

Fig. 1-5. Global levelized cost of energy (LCoE) in 2013 from [40] ...........................................................................................................10

Fig. 1-6. Generator efficiency [39] and reciprocal motion on WEC and PTO .................................................................11

Fig. 1-7. Output power profile in winds and waves. ...............................................................................................................................11

Fig. 1-8. Experimental waveform and ideal model of MMR-based PTO ..........................................................................................13

Fig. 1-9. The monochromatic circuit model of WEC ..........................................................................................................................14

Fig. 1-10. (a) Ac-dc motor side converter in WEC application, (b) wave power fluctuation profile. ........15

Fig. 1-11. Intermediate reactive power control with power constraint [9] .................................................................16

Fig. 2-1. WEC includes three levels of components: Level i: {1} MMR-PTO; Level ii: {1.1} ball-screw, {1.2} mechanical couplings, {1.3} MMR-gearbox, {1.4} permanent magnet synchronous generator; Level iii: {1.3.1} one-way clutches [49] ...........................................................................................................21

Fig. 2-2. An ideal circuit model of MMR-based PTO ..........................................................................................................................24

Fig. 2-3. (a) Comparison between the input force from the ideal model Fig. 2-2 and from the non-ideal model (measured) (b) $V_{pto}$ and $\omega_{g}$ in the actual MMR-PTO ..................................................................................................................25

Fig. 2-4. Synthesis flowchart of a non-linear, non-ideal circuit model for a PTO system ......................................................27

Fig. 2-5. Detailed mechanical diagrams of the MMR gearbox in Fig. 2-1 ........................................................................................28

Fig. 2-6. RLC network of the linear parts of the components in the PTO system. The analogies are built according to Table 2-1 ........................................................................................................................................33

Fig. 2-7. The mechanical drawing and the equivalent circuit of the one-way sprag clutch from lower clutches ..................................................................................................................................................34

Fig. 2-8. A non-linear equivalent circuit model of MMR gearbox which includes two one-way clutches and 3-way bevel gears ..........................................................................................................................................................35

Fig. 2-9. The mechanical drawing and the equivalent circuit of the input and output mechanical coupling. ...36

Fig. 2-10. The equivalent circuit of PTO including Ball-Screw, MMR gearbox, and the generator from the
equivalent circuits in Fig. 2-6 - Fig. 2-9 according to the mechanical diagram in Fig. 2-1.

Fig. 2-11. A simplified equivalent circuit model with the ac and dc unknowns. Two sets of unknowns are extracted: dc components $R_{dc}$ in (2-24) and $I_{dc}$ in (2-25) are derived from the dc test; four pairs of ac components $R_{ci}, L_{ci}$ are approximated from the sinusoidal waves ac test. Parameters of the generator ($R_w, C_{fg}, k_e, k_t, R_i, L_i$) are listed in Table 2-2.

Fig. 2-12. The servo-hydraulic actuator drives the PTO with constant (dc) or sinusoidal (ac) velocity. The output of the generator is under the open-circuit condition for dc test, or connected to three-phase wye resistive loads $R_e$ on each phase for the ac test as shown in Fig. 2-13. (a) PTO test setup with Instron 8801 – a hydraulic compression-tension machine (b) The driving velocity profile from the test machine as the PTO input velocity.

Fig. 2-13. The PTO output setup of the three-phase generator.

Fig. 2-14. Dc tests with constant velocity $\dot{x}$ (or equivalent voltage $v_{pto}$) under open circuit for the identification of PTO's characteristics. The $\dot{x}$ reaches $\dot{X}$ (or $v_{pto}$ reaches $V_{pto}$) after steady-state. The force of the PTO $f_{pto}$ (or equivalent current $i_{pto}$) reaches steady-state force (current) of $F_{pto}$ (or $I_{pto}$).

Fig. 2-15. Simplified dc equivalent circuit from Fig. 2-11 under the constant velocity ($V_{pto}$) test.

Fig. 2-16. The dc unknowns $I_{dc}$ and $R_{dc}$ in (2-27) are derived from the test results in (b) via the linear regression as shown in the $I_{Fpto} - V_{pto}$ curve of (a). The dc current-source $I_{dc}$ is 1.06 A, and the dc resistor $R_{dc}$ is 16.58 $\Omega$.

Fig. 2-17. (a) Bench test results with a sinusoidal displacement of 4 Hz. (b) $\times$ - $f_{pto}$-l loop is used to calculate the input power $P_{in}$ in (2-30). A 4th-order Butterworth LPF filter with 100-Hz cut-off frequency and filtfilt function in Matlab is used to filter the original $f_{pto}$ to $f_{pto-l}$ without phase shift. Six fundamental cycles ($T_2 - T_1$) are taken for averaging $P_{in}$ in (2-30), (c) measured three-phase output line-neutral voltages $v_{abc}$ are used to calculate the output power $P_{oe}$ in (2-28). Six fundamental cycles are taken for averaging $P_{oe}$ in (2-28).

Fig. 2-18. (a) Efficiencies (simulated from the circuit model in Fig. 2-11, and tested) are compared under 15 conditions, including five output resistances (0.5 $\Omega$ - 10 $\Omega$) and three frequencies (2 – 4 Hz) using $L_{c1}$ - $L_{c4}$, and $R_{c1}$ - $R_{c4}$ values in (b). (b) From least-square-curve-fitting method (lsqcurvefit), the ac-unknowns $R_{c1} - R_{c4}, L_{c1} - L_{c4}$ are listed. Simulation with parameters from parameter extraction gives
24. From real tests in Fig. 2-12 bench test and Fig. 2-11 simulation under $R_e = 1.5 \Omega$, 1-Hz driving frequency. Peak force (current) is 400 N(A).

(B.1 Setups of figures-(i)) ........................................................................................................................................46

20. Time-domain results of PTO input force $f_{pto}$ (i) and output voltage $v_{abc}$ from Fig. 2-12 bench test and Fig. 2-11 simulation under $R_e = 1.5 \Omega$, 1-Hz driving frequency. Peak force (current) is 400 N(A).

(B.1 Setups of figures-(i)) ........................................................................................................................................47

21. Time-domain results of PTO input force $f_{pto}$ (i) and output voltage $v_{abc}$ from Fig. 2-12 bench test and Fig. 2-11 simulation under $R_e = 2 \Omega$, 2-Hz driving frequency. Peak force (current) is around 600 N(A).

(B.1 Setups of figures-(i)) ........................................................................................................................................49

22. Time-domain results of PTO input force $f_{pto}$ (i) and output voltage $v_{abc}$ from Fig. 2-12 bench test and Fig. 2-11 simulation under $R_e = 0.5 \Omega$, 3-Hz driving frequency. Peak force (current) is 1200 N(A).

(B.1 Setups of figures-(i)) ........................................................................................................................................49

23. Time-domain results of PTO input force $f_{pto}$ (i) and output voltage $v_{abc}$ from Fig. 2-12 bench test and Fig. 2-11 simulation under $R_e = 0.5 \Omega$, 4-Hz driving frequency. Peak force (current) is 2000 N(A).

(B.1 Setups of figures-(i)) ........................................................................................................................................50

24. From real tests in Fig. 2-12, the ideal model, and the developed circuit model in Fig. 2-11, time-domain results of (a) PTO input force $f_{pto, \text{test}} / f_{pto, \text{idl}} / f_{pto, \text{ckt}}$ and (b) output voltage $v_{abc, \text{test}} / v_{abc, \text{ckt}} / v_{abc, \text{idl}}$ are compared under $R_e = 2 \Omega$, 2-Hz driving frequency $f$. RMS value of force is 469 N, and RMS value three-phase voltage is 2.37 V from the test result. (B.1 Setups of figures-(i)-(ii)) ........................................51

25. Comparison of PTO forces $f_{pto}$, output voltages $v_{abc}$, and efficiency $\eta_m$ from circuit model simulation, ideal model simulation, and repeated test results. (a) – (c) Curves include results under frequency $f$ from 1 to 4 Hz, and under resistor $R_e$ from 0.5 to 10 $\Omega$. (B.1 Setups of figures-(i)) ..............................52

26. (a) The simulated mechanical efficiency curve under different driving frequency and $R_e$ (b)
normalized loss $P_{load}/P_o$ breakdown under four different marked conditions which is corresponding to the efficiency point I – IV in (a). The condition I with 2-Hz driving frequency and 10-Ω $R_e$ has the highest mechanical loss $P_{loss}$ of 80-% $P_o$, i.e., the lowest mechanical efficiency $\eta_m$ of 20 %. (B.1 Setups of figures-(i)) ........................................................................................................................................53

Fig. 2-27. The relationship of PTO mechanical efficiency, generator efficiency, and the overall efficiencies of PTO. (B.1 Setups of figures-(i)) ........................................................................................................................................54

Fig. 2-28. Two setups with the same peak input velocity 0.5 m/s, and the same output power 315 W, but different input frequencies freq (1 and 2 Hz), external load $R$ (4.25 and 6.5 Ω), and generator’s inertia $J_{eq}$ (0.013 and 0.043 kg·m$^2$). (B.1 Setups of figures-(i)) ........................................................................................................................................55

Fig. 2-29. Disengagement ratio versus FM/C and mechanical efficiency versus FM/C under the same output power 315 W from simulation results. (B.1 Setups of figures-(i)) ........................................................................................................................................56

Fig. 2-30. One-page summary of the equivalent-circuit derivation utilizing Fig. 2-1, (2-2) – (2-23), and Fig. 2-10, shows the derivation process from mechanical system, equations, to equivalent circuits...........58

Fig. 3-1. Wave-to-Wire block diagram of a wave energy conversion system. ........................................................................................................................................60

Fig. 3-2. Diagram of Single Degree of Freedom wave energy converter and fluid-body dynamics..............61

Fig. 3-3. Setup in WAMIT. ........................................................................................................................................62

Fig. 3-4. Frequency-dependent parameters derived from WAMIT.................................................................62

Fig. 3-5. RLC Network model of WEC for monochromatic wave with (a) force as current source (i.e. $i_f = f_c$), (b) force as voltage source (i.e. $v_f = f_c$)........................................................................................................................................65

Fig. 3-6. WEC and PTO circuit model with reactive impedance $X_{pto}$ and $R_{pto}$ .................................................................65

Fig. 3-7. Other extensions of the WEC circuit model: a) circuit model of two-body WEC [14], b) circuit model to illustrate different modes in the WEC system where one of the switches $S_1$’ or $S_1$ can either be connected to positions I or II, respectively, corresponding to the floating body is moving without or with tension in the line; the switch $S_2$ can be connected to either position II or III, corresponding to the translator is lifted by the connection line or drops under its gravity force; The switch $S_3$ can either be connected with positions IV or V, depending on if the translator hits the end-stop spring or not [76]. ........................................................................................................................................66

Fig. 3-8. Development methodology of a wave-to-wire circuit model for both irregular and regular wave
Fig. 3-9. Comparison of the frequency response of $K_r$ from (3-5), second-order approximation from (3-11), and third-order approximation from (3-11).

Fig. 3-10. Brune process to synthesis an equivalent circuit from the frequency response in Fig. 3-9 to second-order approximated transfer function in (3-11) to parameters separation in (3-12) to RLC network synthesis network synthesis.

Fig. 3-11. The equivalent circuit model (forces represented by currents) of the fluid-body dynamics in irregular waves from the frequency-domain model (3-3) and second-order transfer function approximate circuit on the radiation force from (3-12).

Fig. 3-12. Impulse response and frequency response of force-velocity from the WEC model (without PTO). Comparison among frequency-domain responses from (3-2), the approximated results from the conventional monochromatic circuit model Fig. 3-5, and the approximated results from irregular wave circuit model Fig. 3-11 under the WEC setup from Table 3-2 (assume no damping or mass from PTO). Circuit model results can match well with the results from (3-2) around the nominal wave frequency (0.52 rad/s – 1.5 rad/s). (B.1 Setups of figures-(iii)-(iv))

Fig. 3-13. Buoy velocity $x_t$ from time-domain (3-1) (Euler method) and from equivalent circuit approximation of Fig. 3-11 (Backward Euler method from Simulink) under a wave energy period $T_e = 6.3$ s, and significant wave height $H_r = 2$ m. The setup parameters are in Table 3-2 with $b_r = 300$ Ns/m, and $m_e = 0$ kg. (B.1 Setups of figures-(iii)-(iv))

Fig. 3-14. Comparison of irregular wave simulation results of output power and buoy velocity in the time domain: a) average output power of PTO under significant wave height from $1 – 2$ m; b) buoy velocity $\dot{x}_r$ (vpto) under $H_r = 2$ m and $T_e = 6.3$ s. (B.1 Setups of figures-(iii)-(iv))

Fig. 3-15. Simulated bias map with the ideal wave-to-wire model. (B.1 Setups of figures-(v))

Fig. 3-16. Maximum Generator speed and air-gap voltage from generator under different wave periods. (B.1 Setups of figures-(v))

Fig. 3-17. The simulated junction temperature of power converter components with the W2W model Fig. 3-15 under irregular wave $H_r = 1$ m and $T_e = 6.3$ s. (B.1 Setups of figures-(v))

Fig. 4-1. The WEC system diagram of the generator and the power converter.
Fig. 4-2. Topologies and architectures of power converters in the wave energy conversion system. .............80

Fig. 4-3. WEC circuit model from Fig. 3-11 and control diagram for PTO and PE detailed in Fig. 4-5 for power converter optimization. .............................................................................................................................................83

Fig. 4-4. Equivalent model of PMSG in d-q frame. ........................................................................................................................................84

Fig. 4-5. WEC Control algorithms and models for Power electronics converter optimization from (4-1) to (4-10). ........................................................................................................................................87

Fig. 4-6. The parameters of the 5-m WEC system from BEM solver (WAMIT) include (a) the added mass A_{11}, (b) the radiation damping b_{11}, (c) the excitation force F_e, and (d) the phase of the wave force on the buoy. ........................................................................................................................................88

Fig. 4-7. The 5-m WEC impedance \( Z_{vec}(\omega) = \frac{\rho}{\rho}i_{vec} \) from (3-2)........................................................................................................................................88

Fig. 4-8. The time-domain results of \( S_{e}, P_{e}, I_{d}, P_{loss} \) from the diagram in Fig. 4-5, the parameters from Table 4-2, and the variables in Table 4-3. (a) the irregular wave simulation results for \( T_{sw} = 1800 \text{ s} \) and (b) zoom-in results when \( \text{Time} = 200 - 250 \text{ s} \). (B.1 Setups of figures-(vi)).........................................................................................90

Fig. 4-9. The flowchart for the optimization process where output results are in underlined and control variables are shown in bold. ........................................................................................................94

Fig. 4-10. The results of output power over different \( R_{p} \) under \( P_{lev} = 50 \text{ kW} \) (\( j = 7 \)) and \( k_b = 104 \) (\( k = 1 \)). (a) \( P_{o,e}^{i,j,k} \) with the wave condition \( w = 28 \), and (b) \( P_{o,e}^{i,j,k} \) with the wave condition \( w = 4 \). The results of \( w = 1 - 28 \) are listed in Table 4-5. (B.1 Setups of figures-(vi)).........................................................................................96

Fig. 4-11. The output power \( P_{om,j,k} \) and \( P_{o,e}^{i,j,k} = P_{om,j,k} - P_{loss,j,k} \) which are the average power considering the probability of annual wave conditions in Table 4-5. (B.1 Setups of figures-(vi)) .........................................................97

Fig. 4-12. Generator rating (a) and converter rating (b) versus power leveling under various transmission ratios. The points marked are from the simulation in Fig. 4-8 with \( I_{prated} = 315 \text{ A} \) and the average values from Fig. 4-10 where (a) the rated power of the generator \( P_{grated} \) of 65 kW is dervied from (4-14) with \( I_{grated} = 100 \text{ A} \), and (b) the rated power of the converter \( S_{rated} \) of 218 kVA is derived (4-13) with \( I_{prated} = 315 \text{ A} \). (B.1 Setups of figures-(vi)) .........................................................................................98

Fig. 4-13. (a) \( P_{lev} \) to \( U_{sp} \) where the minimum \( U_{sp} \) of 20 occurs when \( k_b = 47 \) and \( P_{lev} = 100 \text{ kW} \). (b) \( S_{rated} \) to \( U_{sp} \) where the minimum \( U_{sp} \) of 20 occurs when \( k_b = 47 \) and \( S_{rated} = 180 \text{ kVA} \). (c) \( P_{lev} \) to \( U_{gp} \) where the minimum \( U_{gp} \) of 7.5 occurs when \( k_b = 104 \) and \( P_{lev} = 150 \text{ kW} \). (d) \( P_{grated} \) to \( U_{gp} \) where the minimum \( U_{gp} \)
of 7.5 occurs when \( k_b = 104 \) and \( P_{\text{grated}} = 68 \) kW. (B.1 Setups of figures-(vi))

Fig. 4-14. The average value of \( U_{sp} \) and \( U_{gp} \) and corresponding \( P_{\text{lev}}, S_{\text{rated}}, \) and \( P_{\text{grated}} \) where (a) shows the minimum value of \( (U_{sp} + U_{gp})/2 \) occurs when \( k_b = 104 \) and \( P_{\text{lev}} = 200 \) kW; (b) shows the minimum value of 15 when \( k_b = 104 \) and \( S_{\text{rated}} = 230 \) kVA; (c) shows the minimum value of 15 when \( k_b = 104 \) and \( P_{\text{grated}} = 90 \) kW. (B.1 Setups of figures-(vi))

Fig. 4-15. The output power \( P_{\text{oe}} \) and rated powers of converter and generator from (a) the rated power of the converter and the output power, and (b) the rated power of the generator and the output power, the optimal design point from Fig. 4-14 is \( P_{\text{oe}} = 10.8 \) kW when \( k_b = 104, P_{\text{lev}} = 200 \) kW, \( S_{\text{rated}} = 230 \) kVA, and \( P_{\text{grated}} = 90 \) kW. (B.1 Setups of figures-(vi))

Fig. 4-16. The rated power \( S_{\text{rated}} \) of the power converter versus the power leveling \( P_{\text{lev}} \) and the different values transmission ratio \( k_b \). Four operating points (a) – (d) are listed in ...

Fig. 4-17. Time-domain results of \( S_r, P_{\text{oe}}, I_{dq}, \) and \( P_{\text{loss}} \) with the setup in ...

Fig. 4-18. Field-weakening ratio (FWR) under different \( P_{\text{lev}} \) and \( k_b \) (a) \( P_{\text{lev}} \) to average \( FWR_{j,k} \) from annual wave profile from (4-17). (b) \( P_{\text{lev}} \) to \( FWR_{j,k,28} \) under wave condition \( w = 28 \). (B.1 Setups of figures-(vi))...

Fig. 4-19. Full-load ratio (FLR) under different \( P_{\text{lev}} \) and \( k_b \) (a) \( P_{\text{lev}} \) to average \( FLR_{j,k} \) from annual wave profile from (4-18) and (4-17). (b) \( P_{\text{lev}} \) to \( FLR_{j,k,28} \) under wave condition \( w = 28 \). (B.1 Setups of figures-(vi))

Fig. 4-20. Efficiencies of the generator and the power converter for \( P_{\text{lev}} \) and \( k_b \) from (4-19). (a) \( P_{\text{lev}} \) to the generator efficiency \( \text{eff}_{\text{gen}}(4-17) \); (b) \( P_{\text{lev}} \) to the power converter efficiency \( \text{eff}_{\text{gen}} \); (c) \( P_{\text{lev}} \) to power train efficiency \( \text{eff}_{\text{me}} \). (B.1 Setups of figures-(vi))

Fig. 4-21. The relationship of output power \( P_{\text{oe}} \) to the rated power of the power converter \( S_{\text{rated}} \) and the rated power of the generator \( P_{\text{grated}} \) as a rotated plot from Fig. 4-15. (B.1 Setups of figures-(vi))

Fig. 5-1. Normalized output power (\( P_o \)), lifetime (\( LT \)), and output energy (\( E_o \)) curves of the power converter in WEC verses different damping loading \( R_{in} \) under certain wave conditions ...

Fig. 5-2. Flowchart to achieve maximum output power, maximum energy, or maximum power with lifetime constraints from a designed system and alternative wave site. The process starts with developing 4-D lookup tables in Fig. 5-23 from which maximum power (I: MaxP), maximum energy (II: MaxE), or maximum power with designed lifetime (III: MaxP+LT) algorithms are developed. One algorithm will
be selected and implemented with the annual wave mission profile from the selected site to evaluate
the output power $P_{o,mp}$, expected output energy until failure $E_{o,mp}$, and expected lifetime $LT_{mp}$. ...}

Fig. 5-3. Wave energy converter (WEC) system includes buoy, Power take-off (PTO), power converter. The
buoy is with the assumption of single-degree-of-freedom heave only motion and excitation force $x_e, f_e$
which models were described in Chapter 3. The output power is derived from the average circuit model
in 5.2, which has a higher resolution than the model in 4.2. The semiconductor loss $P_{ql}$ and $P_{dl}$, junction
temperature $T_{jq}$ and $T_{jd}$, extended lifetime $LT$, the damage on the bond wires $D$, and the extended
output energy $E_o$ of the power converter is estimated through the model in 5.2.3. The controlled
variables passive damping $R_in$ and power leveling $P_{lev}$ are implemented through the open-loop
controller in 5.3 and Fig. 5-13 with constraints of maximum torque $T_{eg_{max}}$, dc bus $V_{dc}$, and $P_{lev}$.}

Fig. 5-4. Reliability analysis for the power converter in WEC application where the steps 1 and 2 are the same
as that in Chapter 3, and step 3 and 4 is the similar to that in Chapter 4. What differs from the previous
chapter is the line frequency analysis from the equivalent circuit and the reliability analysis from steps
5 and 6. .................................................................

Fig. 5-5. Wave-to-wire circuit model of the WEC system with the average circuit model of the power converter.
..................................................................................................................................................................................120

Fig. 5-6. Performance curves of the IGBT PM150CG1C120 as power loss lookup tables in (5-6) - (5-10). .....121

Fig. 5-7. Thermal impedance network from IGBT/diode junction temperature $T_{jq}$ / $T_{jd}$ - module case ambient

$T_c$ - heatsink temperature $T_i$ - ambient temperature $T_a$. .................................................................123

Fig. 5-8. Simulation Results from circuit model Fig. 5-5 and equation (5-1)......................................................124

Fig. 5-9. Thermal stress cycles $n_i$ of amplitude $\Delta T_{jd}$ ($\Delta T_{jq}$) and mean value $T_{jdm}$ ($T_{jqm}$) are derived from $T_{jd}$ ($T_{jq}$)
in Fig. 5-8 through the rainflow calculation over 1800 s of the irregular wave cycle. The index $i$ of $n_i$
corresponding to a different set of $\Delta T_{jd}$ and $T_{jqm}$. Medium-term power cycles are the cycles with 10 – 200
Hz from rotor speed with more cycles and lower amplitude; long-term I wave cycles are 0.1 – 1 Hz
from wave frequency with little cycles and higher thermal amplitude as shown in Table 5-1, (B.1 Setups
of figures-(vii)) ...........................................................................................................................................125

Fig. 5-10. Modified Coffin-Mason’s model of bond wires in the IGBT module. The number of cycles to failure
$N_f$ depends on junction temperature amplitude $\Delta T_j$ and average junction temperature $T_{jqm}$. ............126
Fig. 5-11. Damage from IGBT part junction temperature from the counts of thermal stress in Fig. 5-9(b). (a) damage $D$ (%) from Fig. 5-10 and (5-14). (b) damage of each $\Delta T_{ig}$, which is the damage summation of all the $T_{igm}$ under each $\Delta T_{ig}$ value and the accumulated damage over $\Delta T_{ig}$ (B.1 Setups of figures-(vii))

Fig. 5-12. Equivalent circuit model in dq-frame from the generator to the ac-dc converter based on the Fig. 5-5 abc-frame average circuit model with the transformation (5-19).

Fig. 5-13. Open-loop control diagram of a 3P-2L power converter in the WEC system. The controls include passive loading $R_{m}$ (5-20), power leveling $P_{lev}$ and torque limiting $T_{eg_{max}}$ (5-23), field weakening (5-24), and current-duty cycle control in dq frame (5-25) and (5-19).

Fig. 5-14. Mechanical torque $T_{eg}$ and rotor speed $\omega_{g}$ curve from (5-26) when $\omega_{g} = 500 - 1500$ rad/s and $T_{eg_{max}} = 1000$ Nm; (a) with constant power leveling control (5-23) $P_{lev} = 100$ kW and variable passive loading control (5-20) $R_{m} = 1 - 50$ $\Omega$; (b) with constant passive loading $R_{m} = 2$ $\Omega$, and variable power leveling $P_{lev} = 40 - 100$ kW. (B.1 Setups of figures-(vii))

Fig. 5-15. $i_{d_{ref}}$ and $i_{q_{ref}}$ curve from (5-20) – (5-24) when $I_{q_{max}} = 135$ A, $V_{dc} = 800$ V (a) curve with constant $P_{lev}$= 100 kW, and variable $R_{m} = 1 - 50$ $\Omega$ (b) curve with constant $R_{m} = 2$ $\Omega$, and variable $P_{lev} = 40 - 100$ kW. (B.1 Setups of figures-(vii))

Fig. 5-16. The 5-m buoy WEC impedance $Z_{wec(j\omega)}$ the same as Fig. 4-7 with the frequency of interest from 3.5 – 8.5 s in this chapter. The corresponding $R_{in} = |Z_{wec(j\omega)}|$ for optimal power is 1.76 – 18.8 $\Omega$.

Fig. 5-17. Simulation Results by implementing the wave from Fig. 5-8, open-loop control in Fig. 5-13, and control algorithms (5-20) - (5-26) under $H_{s} = 1.75$ m, $T_{e} = 7.5$s, $R_{m} = 2$ $\Omega$, power leveling $P_{lev} = 60$ kW. The results include the generator rotor speed $\omega_{g}$, mechanical torque $T_{eg}$, and converter current in dq-frame $i_{d}$ and $i_{q}$. (B.1 Setups of figures-(vii))

Fig. 5-18. The map of buoy site 44056 from NOAA and the example of raw data in 2015 [95].

Fig. 5-19. Wave profile with 30 minutes of the sampling rate of site 44056 in 2015 including (a) significant wave height $H_{s}$(m) and average wave period $T_{e}$(s), and (b) ambient temperature $T_{a}(^oC)$.

Fig. 5-20. Wave conditions $w$ are discretized into $w = 1:28$ with the resolution of 0.5 m for $H_{s}$ and the resolution of 1 s for $T_{e}$. The cycles of each condition are found in (a), and the corresponding number of $w$ for each wave is in (b). The probability of each wave can be found in Table 4-5.
Fig. 5-21. Flowchart for power, energy, and damage for each wave condition and control variable. 4-D Lookup tables of $P_{o,i,j,w}$, $E_{o,i,j,w}$, $D_{i,j,w}$, $T_{q,i,j,w}$, $T_{d,i,j,w}$ are produced through the flowchart corresponding to each wave condition ($H_{sw}$, $T_{ew}$), passive damping $R_{in}$, and power leveling $P_{lev}$.  

Fig. 5-22. The output 4-D lookup table of energy $E_o$ derived through Fig. 5-21 and the setup in Table 5-5 for maximum energy algorithm. (B.1 Setups of figures-(vii)) 

Fig. 5-23. 4-D lookup tables from the flowchart in Fig. 5-21 with the values in Table 4-5, Table 5-2, and Table 5-5; (a) indices of maximum energy algorithm from the $E_{o,i,j,w}$ lookup tables based on (5-28) where $R_{in}^E_w$ and $P_{lev}^E_w$ are the values of variables for maximum energy for each wave condition $w = 1 \text{ to } 28$; (b) indices of maximum energy algorithm from the $P_{o,i,j,w}$ lookup tables based on (5-29) where $R_{in}^P_w$ and $P_{lev}^P_w$ are the values of variables for maximum power for each wave condition $w = 1 \text{ to } 28$. (B.1 Setups of figures-(vii)) 

Fig. 5-24. Searching results of the genetic algorithm showing the generation and the penalty value. 

Fig. 5-25. The searching process for the indices $R_{in}^{PLT}$ and $P_{lev}^{PLT}$ of the maximum power algorithm with lifetime constraint (MaxPLT) from the genetic algorithm. 

Fig. 5-26. The indices for maximum power with a 20-year lifetime constraint algorithm $R_{in}^{PLT}$ and $P_{lev}^{PLT}$ are derived based on (5-30), (5-31), and the genetic algorithm (GA). The GA is implemented so that the selected $R_{in}$ and $P_{lev}$ can meet the expected lifetime $LT_{est}$ from the wave probability $p_w$, and maximize the estimated output power $P_{o,est}$. 

Fig. 5-27. Flowchart for $P_{o,mp}$, $E_{o,mp}$, and $LT_{mp}$ from annual wave mission profiles with $H_r$, $T_e$, $T_a$ in Fig. 5-19 by the algorithms MaxE in Fig. 5-23(a), MaxP in Fig. 5-23(a), or MaxP+LT in Fig. 5-25 Fig. 5-26 from the annual wave profile. 

Fig. 5-28. The calculated results of annual $P_{ox}$ and $D_{q,h}$ from the flowchart Fig. 5-27 and wave data Fig. 5-19. 

Fig. 5-29. Sweep of $R_{in}$ and $P_{lev}$ under the same wave condition $w = 15$ where $H_{sw} = 1.75 \text{ m}$, and $T_{ew} = 5.5 \text{ s}$. The maximum output power $P_{o}$ occurs when $R_{in} = 2 \Omega$ and $P_{lev} = 100 \text{ kW}$. The maximum energy $E_{o}$ occurs when $R_{in} = 10 \Omega$ and $P_{lev} = 60 \text{ kW}$. (B.1 Setups of figures-(vii)) 

Fig. 5-30. Damage, lifetime, power from different algorithms; (a) damages on wave conditions by implementing maximum power algorithm (MaxP) are shown as bars; accumulated damage are dotted lines, and
expected lifetime solid lines from different algorithms;(b) average powers on wave conditions are shown as bars, accumulated power from by considering each wave condition are shown in different formats of lines. (B.1 Setups of figures-(vii)) ........................................................................................................ 147

Fig. 5-31. Damage on IGBT and body diode with different algorithms, and the distribution of medium-term (MT), long term-I (LgT-I), and long term-II (LgT-II) damages. (B.1 Setups of figures-(vii)) ............... 148

Fig. 5-32. The results of $P_o$, $E_o$, and LT from the mission profile flowchart in Fig. 5-27 comparisons among optimization algorithms. (B.1 Setups of figures-(vii)) ........................................................................................................ 149

Fig. 6-1. The optimization process of the overall wave energy converter system ............................................. 151

Fig. A-1. Board- Board connection diagram, including main power loop, SSR, 1) main Board, 2) control Board, and 3) Gate driving Board from the board schematics. Interfaces from IV are specified in the diagram. .... 172

Fig. A-2. Board- Board grounding diagram is showing the auxiliary power loop and the communication port for the system which can also be found in IV interfaces. ................................................................. 173

Fig. A-3. The top view picture of the 50 kW power system. The simplified diagram and its interconnection can be found in Fig. A-1. ........................................................................................................ 174

Fig. A-4. The top view picture of the 50 kW power system without the main control board and gate driving board. The main power loop is shown in the picture. The simplified diagram and its interconnection can be found in Fig. A-1. ........................................................................................................ 174

Fig. A-5. Side view of the converter system. The simplified diagram and its interconnection can be found in Fig. A-1. ......................................................................................................................... 175

Fig. A-6. The wave power converter system and its dimension. ................................................................. 176

Fig. A-7. The interfaces of the system where its simplified diagram can be found in Fig. A-1. The mini USB port, 120V AC/ 1A port, input ac three-phases, and output dc terminals are for external connections, and other ports are for internal inter-board connections. ................................................................. 177

Fig. A-8. The input ac three-phases and output dc power interfaces of the system where its simplified diagram can be found in Fig. A-1. The rated current is 380 Vac for input, 700 Vdc for output, current rating 76 A. Three AWG power cables are expected to connect on the terminals. ................................................................. 178

Fig. A-9. Pin definition on the control board. (B4 pin 1 to CI01 pin 1 for SSR relay control from CPLD) ... 179

Fig. A-10. Fan Speed control interface. ........................................................................................................ 179

Fig. A-11. The software interface from Code Composer Studio 8.2. The test procedures will be specified in the
Fig. A-12. Inverter mode configuration for 700V, 10 kW testing. Input: Magna LX 1 kV, 100 kVA dc source; output: air-cooling three-phase resistive bank of 10 Ω. ................................. 180

Fig. A-13. Remote control setup for the power converter and source. ................................. 183

Fig. A-14. Test setup with Resistive load and Magna DC power source. ................................. 185

Fig. A-15. Efficiency curve for WEC power electronics converter under inverter mode. ................................. 186

Fig. A-16. Test #5. $V_{in} = 150$ V, $I_{in} = 5$ A, $I_o = 4.88$ Arms, $P_o = 707$ W, eff. = 95.78%. ......................... 187

Fig. A-17. Test #12. $V_{in} = 500$ V, $I_{in} = 16.3$ A, $I_o = 16.5$ Arms, $P_o = 8074$ W, eff. = 98.97%. ......................... 187

Fig. A-18. Test #14. $V_{in} = 700$ V, $I_{in} = 1.18$ A, $I_o = 4.44$ Arms, $P_o = 605$ W, eff. = 73.23 %. ......................... 187

Fig. A-19. Test #17. $V_{in} = 700$ V, $I_{in} = 14.23$ A, $I_o = 17.8$ Arms, $P_o = 9562$ W, eff. = 96 %. ......................... 187

Fig. A-20. Test setup for the WEC power converter in the rectifier mode. ................................. 189

Fig. A-21. Test setup with the input connection to the 1 kW motor drive and motor-generator set. .......... 189

Fig. A-22. Test #9, 1000 rpm, gRin=7.5 Ohm, $I_{dc} = 0.37$ A, $I_a= 2.57$ Arms electrical frequency = 66.6 Hz. … 193

Fig. A-23. Test #5, 500 rpm, gRin=5 Ohm, $I_{dc} = -0.04$ A, $I_a = 1.97$ Arms electrical frequency = 33.3 Hz. …… 193

Fig. A-24. Test #5, regular wave with peak speed = 500 rpm, 0.2 Hz, gRin=5 Ohm, $I_{dc} = 0.2$ A, $I_a= 1.3$ Arms. ................................. 195
List of Tables

Table 1-1. Summary of motivation and contributions of this dissertation ..........................................................17
Table 2-1. Analogies between Mechanical and Electrical networks [55] ........................................................................32
Table 2-2. Parameters of the PM generator in PTO ................................................................................................39
Table 2-3. Time-domain results from real test, ideal model, and circuit model. (B.1 Setups of figures-(i)-(ii)) 48
Table 2-4. R-squares from ideal model $R^2_{id}$ and developed circuit model $R^2_{ckt}$. This shows the correlations
between the model and the test results from 25 test conditions are included..........................................................52
Table 3-1. WEC Dimensions ...................................................................................................................................62
Table 3-2. Setup parameters for an example of the WEC system from [79]. .................................................................73
Table 4-1. Review of WEC power converters development in research institutes ......................................................81
Table 4-2. Setup parameters of the 1:4.5 scale, 5-m WEC system ..............................................................................88
Table 4-3. Setup variables for the time domain simulation. .........................................................................................89
Table 4-4. Setup parameters for the optimization example from the flowchart in Fig. 4-9. The other parameters
are listed in Table 4-2...........................................................................................................................................95
Table 4-5. Wave conditions based on the selected wave site at the site 44056 in 2015 [95]. $p_w$ is the possibility of
each wave condition, $R_{pto}$ is the value of $R_{pto}$ for the maximum power, and $P_{oe, max}$ is the maximum
average wave output power under each wave. The control variables are set as $P_{lev} = 50$ kW and $k_b = 104$...........................................................................................................96
Table 4-6. Detailed results from the setup in 4.3.3 based on the time domain results in Fig. 4-17 under $w = 28$.
The four operating points are indicated in Fig. 4-16 as a part of the $P_{lev}$ and $S_{rated}$ curves. ..................103
Table 4-7. Optimized design and results based on setups in Table 4-2 and Table 4-4, and the optimization
process in the flowcharted Fig. 4-9. ......................................................................................................................106
Table 5-1. Time scales of thermal cycles of power semiconductors and reliability control methods for each time
scale. The scales and methods adopted in this chapter are underlined. ................................................................115
Table 5-2. Parameters of the 50 kW PMSG and the 200 kVA power converter as an example to test the
maximum energy control algorithms in this chapter. ..........................................................................................119
Table 5-3. Thermal Impedance Foster or Cauer network Parameters of 3p-2L Power Converter. ..................123
Table 5-4. Parameters for modified Coffin-Mason’s model of bond-wire in IGBT module in (5-14)..........125
Table 5-5. Setup parameters for the optimization example from the flowchart in Fig. 5-21......................137
Table 5-6. Setup parameters for genetic algorithm function..............................................................140
Table 5-7. Maximum Energy, power and Lifetime over different Optimization algorithms. (MaxP, P\text{sat} = 50 kW is used as a baseline).........................................................................................143
Table 5-8. Damage from different time-scale under three control algorithms........................................148
Table 5-9. Difference between estimated and mission profile results..................................................150
Table 5-10. Comparison of simulation time in the optimization process.............................................150
Table 6-1. Summarize of developed WEC prototype in this dissertation..........................................154
Table A-1. The definition of control sensing pins. ..............................................................................178
Table A-2. Inverter Test summary. ....................................................................................................184
Table A-3. Summary of the Closed-Loop Inverter test. .......................................................................188
Chapter 1  Introduction

Wave energy has the potential worldwide power production of 2 TW [1] and the highest energy density of 2 – 3 kW/m² [2] among renewable energy sources. Although wave energy technologies are rapidly growing [3], they are widely varied and are still at the R&D stage with a low installation rate of around 25 MW globally in 2017 [4] which is incomparable to the 600 GW of wind installation according to the statistics from WWEA [5]. There are ample opportunities for researches on wave energy harvesting.

Wave energy among tidal, current, thermal, and salinity is one of the most developed ocean energy [6]. Wave energy systems convert kinetic and potential energy from waves to electricity. The three main stages in wave energy harvesting are wave resource, Wave Energy Converter (WEC), and Power Take-off (PTO) [7] as shown in Fig. 1-1. The WEC absorbs the incident force from the wave resource and generates the reciprocal motion. The mechanical motion from WEC is converted to electricity by PTO.

WEC researches have been conducted primarily on robustness and the design optimization of wave absorber [1]. The studies also focus on mechanical or hydraulic transmission system design, or the PTO control to optimize energy from WEC [8]. However, the PTO components, including transmission, generator, and power converter, are usually assumed as ideal actuators without considering their power limitation or efficiency [9]–[11]. PTOs convert the mechanical oscillation into electricity and provide the power optimizing control in the wave energy system. If the PTO model is assumed to be ideal, it will deteriorate the effect of the control or impact the design of the mechanical transmission [12]. Especially for a newly invented mechanism like mechanical-motion-rectifier (MMR) based PTO [13], it needs further studying about its characteristics via detailed modeling.
The radiation force of a WEC model is a frequency-dependent term and needs to be solved with the time-domain convolution, which is time-consuming for an annual wave profile simulation. A conventional circuit model proposed [14] a second-order approximation for the fluid-body dynamics, yet it is only limited to regular wave conditions. An irregular WEC network model is still required to complete the circuit-based Wave-to-Wire (W2W) modeling from wave resource to power electronics.

Similar to a power converter in a wind energy application [15], power electronics converter in a wave energy application is one of the vulnerable parts. There are only a few studies about its design and control in wave energy. Moreover, the source fluctuation in waves is more extreme and complex than in wind, which increases the complexity of designing power electronics in a WEC system.

One of the goals in this dissertation is to develop the methodology for a multi-physics circuit-based model, including hydrodynamics, mechanical dynamics, electromagnetism, and power
electronics. This model serves as a unified tool that reduces assessment time and provides a system-level perspective for the overall wave energy harvesting optimization and analysis.

Another goal is to provide a design methodology for the power converter that can both optimize the energy and minimize the cost of the electrical parts using the equivalent circuit model.

1.1 Introduction to Wave Energy Converter systems

Unlike wind energy turbines, the energy absorption structures of wave energy converters don’t have a unified design. It is still unknown which type of structure has a better energy-capture capability. Different wave absorption mechanisms will be reviewed, and their characteristics will be shown in this section.

The wave absorption stage (or WEC) can be categorized into five designs as shown in Fig. 1-2. Point absorber design has been adopted in multiple projects [16], such as Powerbuoy by Ocean Power Technologies or BOLT Lifesaver by Fred. Olsen & Co. The structure harvests energy from the wave-buoy interaction in heave motion. It has the advantage of absorbing wave energy from multiple directions due to its relatively smaller size compared to incident wavelength [1]. A point absorber with a single floating body will be studied in this dissertation.

The attenuator type is a long floating WEC that aligns with the wave direction. Pelamis [17] is well known in its semi-submerged hinged attenuator design, which has the advantage of lower designed force for the smaller impact area from the wave. Oscillating wave surge is a submerged device and usually hinged from the seabed. It needs to be placed perpendicular to the wave direction and is proved as a cost-effective structure through the study from NREL [18]. Oscillating water column (OWC) is usually an onshore structure that uses the turbine driven by the wave through an air chamber [6]. The rotating mass as a gyroscopic device is deployed in SEAREV since 2002 [19], but its cost of energy production is still considered too high.
1.2 Introduction to Power Take-off Systems

PTO system, including transmission, generation, and power conversion, is designed to extract the mechanical oscillation from the WEC to electrical energy as shown in Fig. 1-1.

In magnetic PTO, the transmission and generation functions can be combined and realized by one linear generator, which is also called direct-drive PTO as seen in SeaBeav I from the Oregon State University. The direct-drive PTO has high efficiency of 90% by avoiding the mechanical-transmission stage, but its power capacity is limited by the relatively large effective air gap which needs more magnetic material and windings in the armature [22]. The 10 kW linear generator design has a mass of around 900 kg from the NdFeB magnet and stator steel, so it is not a cost-effective design comparing to the rotary machine [23].

Fig. 1-2. Main designs of wave energy converters [20] and their R&D efforts [21].
The magnetic lead-screw mechanism provides a high-efficiency transmission of 95% [24] and can drive a conventional generator. Still, the cost of the system is a concern for the need of customized components.

A traditional hydraulic transmission can provide high force and bidirectional rectification on wave input, but their efficiencies are low due to the nature of fluid power systems [23]. Advanced hydraulic transmissions are improved by electronic aids pump control system called Digital Displacement® initially developed by the team from the University of Edinburg, UK [25]. The PTO system with Digital Displacement Pump/Motor (DDPM) shows a total efficiency of 83%, including losses from the cylinder, the DDPM, generator, and the power converter.

Mechanical transmissions can provide higher efficiency by transferring either the angular motion to rotational motion or the linear motion to rotational motion. Ball-screw and rack-pinion are two standard transmissions that offer high reliability and efficiency, whereas due to the bidirectional motion from wave, an external one-way clutch is usually added to avoid the impact force during the change of direction. The mechanical PTO with a single one-way clutch absorbs the motion like a single-ratchet PTO, which reduces the wave utilization rate and the wave energy production. WEPTOS, as an example [26], is designed to transfer power in one direction through the ratchet mechanism as its transmission design.

Another transmission like algebraic screw linkage mechanism provides [27] a high efficiency of 96%, but it shows only a small power prototype of less than 1 W. The stroke length of the wave energy converter is usually up to 1 – 4 meters, which is much more than the screw-linkage mechanism can provide (i.e., the distance between the plates is also limited by rotation angle.)

With the invention of Mechanical-Motion-Rectifier (MMR) gearbox, the wave utilization and efficiency is increased through energy absorption during both directions of wave oscillation. The
ball-screw is preferable than the rack-pinion in this study for its several advantages: 1) The efficiency of the ball-screw can go beyond 90% [28] with self-lubrication. 2) It can drive the generator to higher speed with a high gearing ratio. According to [29], a generator with higher speed is preferable, since it will lead to smaller junction temperature fluctuation, and thus is a more reliable power converter. 3) A ball-screw is also widely implemented in various wave energy PTOs, as in [30], where Ocean Power Technology deploys it in the PowerBuoy series as a commercial product. The mechanical transmission with an MMR-based ball-screw type of PTO is chosen in this study.

The PTO designs with their corresponding efficiency and annual power production are summarized in the following figure.

![Fig. 1-3. PTOs with their corresponding efficiencies and annual power productions [23][27].](image)

1.3 Introduction to the Generators and the Power Converter in WEC system

Depending on the types of transmission, there are two major types of generators: linear
generator and the rotary generator. Linear generators in the direct-drive PTO convert the linear motion direct to electricity. They are usually not preferable, because they are bulky and more expensive from electromagnetic material costs. Under the same output power, they need a higher rated force, because the generator is operating under lower velocity as shown in [7], [31], and [32].

Rotary generators in the linear-rotary type of PTOs include gear coupled doubly-fed induction generator (DFIG), synchronous generator (SG) with field excitation, and permanent magnet synchronous generator (PMSG) [33]. Rotary generators are more commonly used and can be founded in 9 out of 13 novel-PTOs surveyed as recently developed systems in [32]. Linear-rotary PTO is a better option in the system rated above 10 kW because it is less expensive and reduces the sealing issue mentioned in [7].

In this dissertation, a point absorber with mechanical transmission through PMSG is chosen as a prototype. The analysis in the following chapters will base on this prototype.

The power converters provide control and ac-dc electrical conversion in the final stage of PTO. It can easily achieve up to 95-% efficiency which is better than the mechanical transmission and generator which have efficiency below 90 %. Power density is not a design issue for the power converter in a WEC system. Taking a 500-W WEC for example, it has a buoy diameter of one meter, which contains sufficient space for both power converter and other peripheral electronic devices. The design goal of the power electronics will thus be focusing on its reliability and cost-effectiveness.

A conventional three-phase two-level boost rectifier with a battery output will be used in this study. The detailed survey of the power converter will be shown in Chapter 4.1.
1.4 Potentials and Challenges of Wave Energy Converters

The incentives that encourage the engineering development on wave energy converters include:

1. A large amount of resource

According to the study from Ocean Energy Systems in 2011 [34], the annual global potential energy is around 29.5 Peta-Watt-Hour ($10^{15}$WH) in comparison with 20.5 PWH of the annual global energy demand. The total available wave energy resource in the United States is 2640 TWh/yr through the estimation from a 2011 report by Electric Power Research Institute (EPRI) [35]. The energy consumption in the U.S. is around 4110 TWh/yr, so a significant amount of potential wave energy is yet to be harvested.

2. High availability

Compared to other renewable energy like solar or wind energy that can only provide 20 – 30% of uptime, wave energy converter can generate power up to 90% of the time [1] which theoretically gives a higher capacity factor (CF), but it still has a similar CF as other renewable sources [36] due to the design challenges in the following section.

3. High energy density

Wave energy density is around 2-3 kW/m², which is much higher than other renewable energy such as solar energy (about 0.3 kW/m²) and wind energy (about 0.6 kW/m²) [2]. The wave energy converter WEPTOS [37] claims that with the same amount of area, wave energy farm can produce twice the amount of energy than the offshore wind farm with a capacity of 160 MW in a 19-km² wind/wave farm.
4. Predictability

According to the quantified results from [38], for a day-ahead forecast, waves are 23% more predictable than winds, and the power output of WECs are 35% more predictable than wind turbines. Predictability of wave provides advantages like planning installation and maintenance activities, defining control strategy based on the average wave period, or preparing for the storm events.

5. Different power profile

Wave energy provides different power profiles than other renewable energies [39], which is illustrated in Fig. 1-4. Comparing to other renewable energies, wave energy provides a new power profile that could compensate the power demand differently. Since winds create waves, two energy sources are somehow correlated, as illustrated in the figure.

![Grid Capacity Simulation](image)

Fig. 1-4. The simulation of weekly energy profile among different energy sources and the demand [39].

While having the above incentives, the commercialization of wave energy converters faces several major challenges. These challenges lead to a higher levelized cost of energy (LCoE) of about 0.45$/kWh among all renewable energies [40], as shown in Fig. 1-5 where wave energy is referred to as marine-wave.
The challenges for wave energy converters are concluded in the following three major points.

1. **Reciprocal wave motion**

The reciprocal motion from the wave could shorten the lifetime of the mechanical components (PTO) due to the impact force while changing direction, as shown in Fig. 1-6(b). During the change of direction, the velocity of the buoy is zero, the PTO stops rotating, and the output power becomes zero, as shown in Fig. 1-7(b). This will significantly reduce the system efficiency because both the power converter and the generator perform poorly under low load conditions, as illustrated in Fig. 1-6(a).

Usually, the mechanical PTO uses a one-way clutch to reduce the impact. By doing so, the power will only come from one direction of the motions, and will also reduce the overall efficiency of the transmission.
1. Power fluctuation

As in a survey study from Falcão [41] “The main disadvantage of wave power, as with the wind from which it originates, is its (largely random) variability in several time-scales: from wave to wave, with sea state, and from month to month (although patterns of seasonal variation can be recognized).” The multi-time scale of wave fluctuation induces more additional power fluctuation on the semiconductor and leads to frequent and high thermal stress on the components.

Another difference between wind and wave is that the peak-to-average power ratio of wave energy converter is much higher than wind energy, which can usually reach up to 10 - 20 times the average power as compared in Fig. 1-7. The high peak power will not only impact the junction temperature, but also result in an overrating design of the power converter and the generator.
2. Reliability under harsh environment

The WEC needs to be designed beyond the nominal wave condition for the worst-case weather condition to survive for over 20 years’ lifetime under severe storm conditions. Considering storm conditions and power fluctuations, the design for the worst-case will significantly reduce the capacity factor of the WEC system and cause a low LCoE.

This dissertation is developed according to these challenges through various works scopes depicted in the following section.

1.5 Research Objectives and Methodologies

As compared in Fig. 1-5, wave energy, unlike other commercialized renewable energy such as solar or wind energy, is still in the pre-commercialized stage, and one of the most important goals is to reduce its Levelized Cost of Electricity (LCoE):

\[
\text{LCoE} = \frac{(FCR+CapEX)+OpEx}{AEP}
\]  

(1-1)

where \( FCR(\$/yr) \) is the fixed charged rate, \( CapEX(\$/yr) \) is the initial capital expenditures, \( OpEx(\$/yr) \) is the annual operating expenditures, and \( AEP(\text{kWh}/\text{yr}) \) is the annual energy production.

Optimizing \( AEP \) and reducing \( CapEx \) plus \( Opex \) can both reduce LCoE by increasing energy production or increasing the lifetime of the WEC system. The two objectives for this research are to maximize the harvested energy from WEC and increase the lifetime of WEC Accordingly, the scopes of this dissertation are listed.

1.5.1 MMR-PTO Circuit Model with Efficiency Prediction

A mechanical-motion-rectified (MMR) PTO is a mechanical transmission PTO which rectifies the bi-directional wave motion and can harvest energy from both directions. The design
with two one-way clutches avoids the impact force and zero velocity mode on PTO during the direction change from the reciprocal wave motion. The PTO provides a high ratio of freewheeling mode with larger inertia during the disengagement of the one-way clutches. Nonlinearity behaviors from dry friction, viscous damping, and compliances during the transition between disengage-engage are observed from the experiment results in Fig. 1-8(a).

A conventional MMR-PTO model [42], [43] describes the engagement/disengagement from MMR with two discrete mathematical equations or with two ideal diodes in the circuit model, as shown in Fig. 1-8(b). The conventional model cannot wholly describe the PTO system. There is a need to develop a high-fidelity model to predict its time-domain response and efficiency.

![Fig. 1-8. Experimental waveform and ideal model of MMR-based PTO.](image)

**1.5.2 Irregular Circuit-Based Wave-to-Wire Model for WEC**

To understand the multi-physics WEC system with a non-ideal PTO circuit model, a WEC model that can estimate fluid-body dynamic under irregular waves and can be combined with the PTO model is needed. In prior work, a monochromatic circuit model in Fig. 1-9 has been widely used for developing reactive power control and simulating under regular wave conditions [9]. Through the approximation of frequency-domain transfer function and network synthesis, an irregular circuit-based WEC model is developed to adapt in various wave conditions and to build a W2W model, including the PTO model along with the power converter for system-level
simulation. The developed W2W circuit-based model is further expended to design and control optimization of the power electronics converter in Chapters 4 and 5.

![Fig. 1-9. The monochromatic circuit model of WEC.](image)

1.5.3 Power Converter Design on WEC Application

The power profile in the WEC application shows a very different nature compared to other renewable energy. Especially its high peak-to-average ratio from the ocean wave fluctuation, as shown in Fig. 1-10 leads to an overrated design if a standard design method is used. Very few works [44], [45] have investigated the optimization of the power converter in the wave application. The optimization process lacks a unified tool and is also time-consuming. A circuit-based W2W model and a numerical searching methodology are developed to optimize the power rating of the power converter through PTO parameters. Losses of the power converter and the generator are included in the model to vary with the rated power. The power limiting and field-weakening controls keep the generator under constant output power and maintain controllability when the rotor speed goes beyond the rated dc-bus voltage. The simulation covers the results from the annual wave profile. The factors $U_{sp}$ and $U_{gp}$ are the ratio between the rated power of the power converter $S_{rated}$ and the average output power $P_{oe}$, and the ratio between the rated power of the generator $P_{grated}$ and the average output power $P_{oe}$, respectively. These overrating factors are introduced as figure of merits in the design process, and the optimal power rating is selected through finding the minimum overrating factor.
1.5.4 Maximum Energy Control on Wave Energy Application

Previous researches mainly focus on the control algorithm to maximize the harvested power from the WEC or the reliability of power converter in wave energy separately. Take the paper of Tedeschi from the Norwegian University of Science and Technology [9] as an example, it considered the power rating constraint of the power converter and modified the optimum control algorithm for practical use. As a result, an intermediate reactive control shown in Fig. 1-11 adjusts the reactive control angle according to its power limitation. The harvested power is higher than passive loading and is closer to the power level of complex conjugate control with power limitation.

Other studies about the reliability of the power converter in WEC focus on how semiconductor rating or heatsink selection affects the lifetime under single wave condition [46], [47]. The lifetime is extended through the hardware selection. While the selection of the power limitation on the power converter, and how this control algorithm affects the reliability of the power converter is still uncertain.

A reliability study considering the power converter derating in a wind turbine is conducted in [48]. Although a similar technique can be applied in wave energy, there is still a significant difference between the mission profile of the wind and the wave. Four different loading
frequencies, which produce different levels of thermal stresses are observed in WEC power converter: 1) power cycles from the switching frequency (level of 10-100 μs), 2) power cycles from the line frequency (level of 10-100 ms), 3) power cycles from the wave-wave frequency (level of 1-10 seconds), and 4) power cycle from annual wave profile (level of 10 – 100 minutes).

The total recoverable energy over the power converter’s lifetime will be the new figure of merit for the controller design. A maximum energy control considering the lifetime and the maximum output power will be developed via 4-D lookup tables with the results of estimated lifetime, power, and damage. The design lifetime is satisfied with a maximum output power by selecting the optimal control parameters through the genetic algorithm. The searching processes with a complex W2W circuit model simulation are enabled by wave organization and parallel computing.

![Figure 1-11](image_url)

**Fig. 1-11. Intermediate reactive power control with power constraint [9].**

### 1.6 The Contributions and Outline of the Dissertation

A new type of MMR-PTO is introduced, and its dynamics in a WEC system is analyzed in the numerical method [42]. Efficiency and dynamics are two of the most critical factor for a PTO system, while both of them are still unknown in MMR-PTO. The non-ideal circuit-based model is developed to predict the efficiency of 30 – 80 % for the mechanical transmission stage with the
correlation coefficient of 95% comparing to the experimental results.

A W2W circuit model is accomplished by the development of an irregular wave WEC model. The circuit-based W2W model is verified by comparing the results with the predictions of a detailed model under irregular wave conditions in the time and frequency domains. It serves as a circuit modeling tool and simplifies the power electronics design in the multi-disciplinary system.

The design optimization of the power converter in the WEC application is introduced to meet the power rating in maximum irregular wave conditions and to utilize the maximum power conversion capability of the converter. The overrating ratio is reduced from 20 to 2.4 with the design procedure and optimized power limitation.

With the designed hardware from the power converter, both the power and lifetime of the power converter are optimized through the optimized control algorithm. The total energy is increased by 15.9 times under the maximum energy algorithm, and the lifetime increases from 3 years to 20 years, with only 6% of power reduction under the maximum power with a limited lifetime algorithm.

<table>
<thead>
<tr>
<th>Topic</th>
<th>Motivation</th>
<th>Innovation</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>PTO circuit modeling (Ch2)</td>
<td>• Understand newly invented MMR-PTO • Predict the efficiency of MMR-PTO</td>
<td>• Ac and dc network synthesis • Parameter extraction with Nonlinear curve fitting</td>
<td>• 30 – 80% efficiency • $R^2 = 95%$ comparison between prediction and experiments</td>
</tr>
<tr>
<td>WEC circuit modeling (Ch3)</td>
<td>W2W model for circuit-based simulation</td>
<td>• Transfer function and Brune’s network synthesis</td>
<td>Time-domain and frequency-domain matching</td>
</tr>
<tr>
<td>Power converter design optimization in wave application (Ch4)</td>
<td>Reduce overating ratio of power electronics and generator design</td>
<td>• A simplified model for W2W system simulation • Consider all wave conditions and losses • Overating Criteria $U_{sp}$, $U_{gp}$</td>
<td>• Less than 1 minute of simulation for each condition • $(U_{sp}+U_{gp})/2$ drops from 66 to 15</td>
</tr>
</tbody>
</table>
This work is categorized into two main parts. The first part derives the WEC and PTO circuit models in Chapters 2 and 3, and the second part optimizes the design of the power converter and the control algorithms in Chapters 4 and 5.

This dissertation consists of six chapters. The outlines of these six chapters are:

Chapter 1 presents the background, motivations, scopes, and objectives of this research. Prior works are introduced. The challenges and contributions of this work are summarized.

Chapter 2 introduces the non-ideal MMR-PTO circuit model and its design methodology. The parameters of the non-linear simulation model are extracted through bench test results based on mechanical efficiencies on various conditions. The model is verified in the time domain and is applied to the loss breakdown of the components. The model is used to conduct performance studies on MMR-PTO, and the impact form the disengagement is studied further.

Chapter 3 introduces the circuit-based W2W model, which is developed through Brune network synthesis and frequency domain approximation. The W2W circuit model is used to simulate the multi-physics system bias map from wave – buoy interaction to the output of the power converter. A non-ideal W2W model is realized by combining the non-ideal MMR-PTO model with the circuit-based W2W model. Control algorithms are studied based on the model to estimate the overall harvested power, considering the efficiencies of the components.
Chapter 4 shows the design guideline for the power converter and the generator based on the previously developed circuit W2W model. The optimized power rating is found in both the power converter and the generator, which leads to an economical design of power conversion in the WEC system.

Chapter 5 focuses on the development of the maximum energy control algorithm, which also includes the thermal and lifetime model of the power module in the W2W model. The lifetime under different control parameters are estimated, and the maximum energy controls are designed.

Chapter 6 concludes the works and introduces future works.
Chapter 2 Circuit Model of Power Take-off System with Mechanical-Motion-Rectifier

This chapter introduces a non-ideal circuit model for MMR-based PTO, which is a high-fidelity simulation model, used to predict the non-linear behavior from the one-way clutches and efficiency under various wave conditions. The approximated mechanical efficiency is around 50 – 80%, and the simulation efficiencies have a correlation coefficient $R^2$ of 95% comparing to the experimental results. The disengagement from the MMR can be better understand through the simulation results provided by the non-ideal PTO model.

2.1 Introduction of PTO System modeling

A mechanical transmission PTO consists of mechanical transmission, generator, and power converter. Conventionally the models of mechanical transmissions are mathematical equations, but it is difficult to describe the mechanical loss that comes from multiple non-ideal mechanisms, such as dry friction, viscous damping, and compliances, with a mathematical only model. Circuit model provides a tool to approximate the actual system without overcomplicated and easy to use for both dynamics and performance analysis on the mechanical transmission in a wave energy power take-off system. A PTO with a mechanical-motion-rectifier gearbox as transmission stage will be introduced in this chapter and used as an example to introduce the synthesis methodology of the non-ideal circuit model.

2.1.1 Introduction of MMR-PTO

MMR-PTO is a mechanical transmission PTO that rectifies the linear bidirectional wave motion into a unidirectional on the input shaft of the generator through the transmission inside the
Fig. 2-1. WEC includes three levels of components: Level i: {1} MMR-PTO; Level ii: {1.1} ball-screw, {1.2} mechanical couplings, {1.3} MMR-gearbox, {1.4} permanent magnet synchronous generator; Level iii: {1.3.1} one-way clutches [49].

MMR gearbox as shown in Fig. 2-1 [49]. The single body WEC system includes three parts: a floating buoy that captures the wave-excitation force, rods that transfer the force from buoy to PTO, and a PTO that converts the kinetic energy into electrical energy. The PTO housed in a cylinder column as Level i in Fig. 1{1} includes Level ii components such as the push tube, ball nut, ball screw, couplings, gearbox (Fig. 1{1.3.1}), and generator. There are several advantages of choosing the ball-screw: 1) The efficiency of the ball-screw can go beyond 90 % [50] with self-lubrication. 2) It can drive the generator to higher speed with a high gearing ratio. According to [29], a generator with higher speed is preferable, since it will lead to smaller junction temperature fluctuation, and thus be a more reliable power converter. 3) A ball-screw is also widely implemented in various wave energy PTOs, including in [51], where it is also referred to as a more reliable choice, and [30], where it is deployed by Ocean Power Technology in the PowerBuoy series as a commercial product. As seen in Fig. 1, the up-and-down motions of the buoy drive the pushrod and ball nut vertically, spinning the ball-screw bi-directionally. The input coupling connects the ball screw and gearbox input shaft. The MMR gearbox converts bidirectional rotation from the input shaft to the unidirectional motion of the output shaft. The rotation of the output
shaft will be passed to the generator through the output coupling shown in Fig. 1(1).

The detailed mechanical diagrams of the MMR gearbox is shown in Fig. 1(1.3), which contains the following Level iii components: two one-way clutches in Fig. 1(1.3.1), three bevel gears, one input shaft, and one output shaft. The MMR gearbox is modified based on a three-way right-angle bevel gearbox (4304-0402 manufactured by WC Branham) with a 1:1 ratio to provide a high-efficiency transmission [52]. The unidirectional motion transmission is achieved through the engagement and disengagement of the two one-way sprag clutches, which are designed to engage in only one direction. The input shaft is connected to two one-way clutches as their inner “ring” (there is no ring actually). The internal wall of the two bevel gears acts as the outer “ring”. The output shaft is press fit with one of the bevel gears. The inner ring (input shaft) and the outer ring (bevel gears) of the clutch are synchronized when they are wedged by the sprags’ friction; this is also called engagement mode. In this mode, the torque is transferred from the inner ring (input shaft) to the outer ring (bevel gear). The sprags slip to another direction on the inner ring due to the asymmetric shape of the sprags. No torque is transferred between the inner rings (input shaft) to the outer ring (bevel gear) during this mode; this is called disengagement or freewheeling mode.

There are two engagement modes when the output shaft rotates counter-clockwise. 1) When the input shaft rotates clockwise, the upper clutch will engage and the lower clutch will disengage. The torque is transferred through the three bevel gears to the output shaft, and the bevel gears will change the rotational direction so the output shaft spins in the opposite direction from the input shaft. 2) When the input shaft rotates counter-clockwise, the upper clutch will disengage and the lower clutch will engage. The torque is directly transferred from the input shaft to the output shaft through the engagement of the lower one-way clutch, and the output shaft rotates in the same
direction as the input shaft. In this way, the output shaft will be driven to unidirectional rotation by the directional rotation of the input shaft.

2.1.2 Ideal Mathematical and Circuit Model of MMR-PTO

The dynamics of single body WEC and the force provided by MMR-PTO can be found in (2-1) - (2-4).

\[
(m_1 + A_{11\infty})\ddot{x} + \int_{-\infty}^{t} k_r(t-\tau)\dot{x}(\tau)d\tau + f_{pto} + k_s x = f_e \quad (2-1)
\]

\[
f_{pto} = k_b k_g T_g = k_b k_g (b_g \omega_g + J_g \dot{\omega}_g) = -m_e \ddot{x} - b_e \dot{x} \quad \omega_g \leq \omega_{m0} \quad (2-2)
\]

\[
f_{pto} = k_b k_g T_g = k_b k_g (b_g \omega_g + J_g \dot{\omega}_g) = m_e \ddot{x} + b_e \dot{x} \quad \omega_g \leq -\omega_{m0} \quad (2-3)
\]

\[
f_{pto} = 0 \quad \omega_g > |\omega_{m0}| \quad (2-4)
\]

where (2-1) is the description for force balance on the single body WEC and the PTO as shown in Fig. 2-1. The mass of the buoy is \(m_1\), and the added mass at the infinite frequency of the buoy is \(A_{11\infty}\); the radiation force on the buoy is shown as a convolution term from the impulse response function \(k_r(\tau)\); \(f_{pto}\) is the force provided by the PTO column, and \(k_s\) is the hydrostatic stiffness of the floating structure; the heave excitation force from the wave to a fixed buoy is \(f_e\), and the vertical displacement of the buoy (upward as positive) is \(x\). The ideal MMR-PTO behavioral model is shown in (2-2) to (2-4) where \(\omega_g\) is the rotational speed of the generator (or the output shaft), and \(\omega_0\) is the input shaft speed of the MMR gearbox. When the input shaft speed of the MMR is higher or equal to the output shaft speed of the MMR gearbox, the MMR gearbox is in engagement mode. The PTO has the damping torque provided by the output generator as shown in (2-2) where \(k_g\) is the gear ratio of the bevel gearbox, and \(k_b\) is the linear to rotation ratio of the ball-screw can be derived from \(2\pi/l\), \(l\) is the ball screw lead in meter per round; \(J_g\) is the inertia of the generator; \(b_g\)
is the damping coefficient from the generator. By assuming the PTO column is static compared to the wave surface, $x$ will also be the linear displacement on the ball screw of PTO. $\omega_g$ will equals to $-k_gk_b\dot{x}$; the equivalent mass $m_e = (k_gk_b)^2J_g$, and the equivalent damping is $(k_gk_b)^2b_g$. If the motion from the wave inverse, another one-way bearing will engage. The behavior of the MMR gearbox can be written as (2-3). When the input shaft speed of the MMR gearbox slower than its output shaft, MMR gearbox will operate in disengagement mode that no force will be transfer from WEC to the PTO as in (2-4). PTO provides no damping to the WEC in this case.

As a result, the MMR-PTO system can be depicted by the equivalent circuit model Fig. 2-2 [43] which uses two ideal diodes to describe the one-way bearing model and 2:1 transformer for the 3 way bevel gearbox. The PTO force is shown as a current $f_{pto}$ at the input side, and PTO linear velocity $\dot{x}$ is shown as a voltage $V_{pto}$. The generator driving torque $T_g$ is shown as a current, and the generator angular speed $\omega_g$ is shown as a voltage on the circuit model. Ball screw is shown as a transformer with 1:$k_b$ turn ratio. The generator inertia $J_g$ is shown as a capacitor, and the damping coefficient $b_g$ is shown as a reciprocal of the resistance value.

![Fig. 2-2. An ideal circuit model of MMR-based PTO.](image)

Although both the circuit and mathematical model can describe the behavior of the MMR-PTO, they cannot fully explain the phenomenon shown in Fig. 2-3. Figure 2-3(a) shows the case in non-ideal (measured) input force will gradually increase from the zero-crossing, while in the
ideal model, the input force will jump up to a certain value. From Fig. 2-3 (b), the phase shift between $V_{pto}$ and $\omega_g$ can be seen, while in the ideal model $\omega_g$ equals to $V_{pto}$ multiplying by a constant coefficient $k_{bg}$. Moreover, the efficiency derived from the ideal model is 100 %, but in the experiment, it shows around 50 – 80 % of the mechanical efficiency on the MMR-PTO. Thus, a high-fidelity model is needed to estimate the efficiency and the unknown non-linear dynamics from the one-way clutches.

![Fig. 2-3. (a) Comparison between the input force from the ideal model Fig. 2-2 and from the non-ideal model (measured) (b) $V_{pto}$ and $\omega_g$ in the actual MMR-PTO.](image)

2.1.3 Synthesis Methodology for Circuit Model of MMR-PTO

To depict the non-linearity from the MMR, the advance PTO model is built over a circuit network including linear and non-linear components. The synthesis process of the circuit-based model as a nonlinear system identification process [53] is shown in Fig. 2-4. The physics model (semi-physics model) is a mathematical model derived from the actual system, which depends on choosing the important metrics and its corresponding components that will be described by the mathematical equations. The circuit model is built based on the equations through Brune synthesis and Darlington synthesis [54], [55]. According to the force – current analog in [56], parallel components are used to represent mechanical coefficient from force balance equations to
Kirchhoff’s current law (KCL), but some of the components such as mechanical couplings and one-way clutches, can only connect in a series way to reflect the rotational speed difference between its input and output terminals.

After a complete circuit model is developed, the parameter extraction process is proceeded with two different equivalent circuit model: dc equivalent circuit and ac equivalent circuit. Under constant speed conditions, PTO can be approximated with the dc equivalent circuit model. Parts of the parameters can be extracted from the multiple constant speed experiments, and the linear parameters can be figured out from the linear regression process.

The ac equivalent circuit model can be derived by inherit parameters from the dc equivalent circuit and omitted relatively small parameters in the complete circuit model. The rest of the parameters are searched through the Least-square-curve-fitting function in Matlab by comparing the ac experimental results and simulation results under multiple operating conditions. Correlation coefficients are used as criteria to indicate the accuracy of the approximation from the ac equivalent circuit.

As a result, all the parameters in the circuit model are found through the process. Time-domain validation will be performed to check whether the states from the model simulation are closed to those from experiments.
2.2 Mathematical Model of PTO

The PTO mathematical model will be derived as a foundation of the circuit model. Based on the Cummins’ equation in (2-1), the terms in $f_{pto}$ need to modify to derive the non-ideal model for PTO.

$$\dot{x} = V_{pto} = 2\pi \omega_{m0}/l = \omega_{m0}/k_b \text{ or } k_b = l/2\pi$$  \hspace{1cm} (2-5)

$$f_{PTO} = sgn(\dot{x})\tau_0 k_b + (b_n + b_{vs} k_b^2)\ddot{x} + T_{s1} k_b + [m_{bs} + J_{s1} k_b^2]\dddot{x}$$  \hspace{1cm} (2-6)

$$T_{s1} = (\omega_{m0} - \omega_{m1})b_{c1} + (\theta_{m0} - \theta_{m1})k_{c1}$$  \hspace{1cm} (2-7)

From the previous description in 2.1.2, the linear speed of the ball-screw $\dot{x}$, and the rational speed of the nut and the shaft $\omega_{m0}$ can be written as (2-5).

The $f_{pto}$ in (2-6) is comprised of the following terms: the Coulomb friction torque $\tau_0$, the viscous damping $b_n$ and $b_{vs}$ on both the ball-screw nuts and input shaft coupling, the equivalent force from the torque $T_{s1}$ that is transferred to the one-way clutch, the forces from Newton’s second
law where the mass $m_{bs}$ is from the pushrod and ball-screw, and the inertia $J_{s1}$ is from the input shaft and the input mechanical coupling.

The torque $T_{s1}$ on the input coupling is derived from the speed and the angular difference through its torsional damping $b_{c1}$ and torsional stiffness $k_{c1}$ in (2-7). The output angular speed and angle of the input coupling are $\omega_{m1}$ and $\theta_{m1}$ respectively.

The diagram of the MMR gearbox in Fig. 2-5 contains two one-way clutches, three bevel gears, one input shaft, and one output shaft. The unidirectional motion transmission is achieved through the engagement of the two one-way clutches which are designed to only engage in one direction, as shown in Fig. 2-5. The input shaft is connected to two one-way clutches on their inner rings. The inner ring (input shaft) and the outer ring (bevel gear) of the clutch, will be driven only when they are wedged by the sprags’ friction, which is also called engagement mode. In this mode, the torque will be transferred from the input shaft to the output shaft. For another direction on the inner ring, the rollers slip, due to the asymmetric shape of the sprags. There is no torque transmission during this mode which is called disengagement or free-wheeling mode.

![Diagram of the MMR gearbox](image)

Fig. 2-5. Detailed mechanical diagrams of the MMR gearbox in Fig. 2-1.
There are two ways the output shaft rotates in a counter-clockwise direction. First is when the input shaft is rotating in a clockwise direction, the upper clutch will engage and the lower clutch will disengage. The torque is transferred from the bevel gears to the output shaft, and the bevel gears will change the rotational direction so the output shaft turns in the opposite direction from the input shaft. Another is when the input shaft rotates counter-clockwise, the upper clutch will disengage and the lower clutch will engage. The torque is directly transferred from the input shaft to the output shaft through the engagement of the lower one-way clutch, and the output shaft has the same rotational direction with the input shaft.

As a result, there are three operation modes in MMR gearbox: 1) lower clutch engage with input shaft rotating counter-clockwise; 2) upper clutch engage when input shaft rotating clockwise; 3) Disengagement with either clutch conducting, and with output shaft freewheeling.

According to the three modes, the torque on the one-way clutch $T_{s1}$ can be written as

$$T_{s1} = \begin{cases} b_{c3}(\omega_{m1} - \omega_{m3}) + k_{c3}(\theta_{m1} - \theta_{m3}) & \text{if } \omega_{m1} \geq \omega_{m3} \\ b_{c2}(\omega_{m2} - \omega_{m1}) + k_{c2}(\theta_{m2} - \theta_{m1}) & \text{if } \omega_{m1} \leq \omega_{m2} \\ 0 & \text{if } \omega_{m1} < \omega_{m2} \end{cases}$$

$$= \begin{cases} \text{engagement, upper} & \text{engagement, lower} \\ \text{disengagement} & \text{disengagement} \end{cases}$$

(2-8)

where $k_{c3}$ and $k_{c2}$ are the spring’s torsion coefficient from the cage and sprags motion in the lower and upper one-way clutches respectively, which are linearly approximated and will be extracted from the experiments; $b_{c2}$ and $b_{c3}$ are the torsional damping coefficient inside the upper and the lower sprag one-way clutch; $\omega_{m1}$ and $\theta_{m1}$ are the rotational speed and the angle of the input shaft; $\omega_{m2}$ and $\theta_{m2}$ are the rotational speed and the angle of the outer ring of the upper one-way clutch (or upper bevel gear); $\omega_{m3}$ and $\theta_{m3}$ are the rotational speed and the angle of the outer ring of the
lower one-way clutch (or the lower bevel gear). The relationship of the rotational speed on bevel gear can be written as,

\[ \omega_{m3} = -k_{g1} k_{g2} \omega_{m2} \]  \hspace{1cm} (2-9)

where \( k_{g1} \) and \( k_{g2} \) are the gear transmission ratios from upper bevel gear to side bevel gear, and from side bevel gear to lower bevel gear, respectively. In this design \( k_{g1} = k_{g2} = 1 \). The negative sign means the upper and lower bevel gear rotates in different directions, which provides an asymmetrical mechanism converting the bi-directional rotation into uni-directional rotary on the output shaft of the MMR-gearbox.

After the one-way clutches, the torque \( T_{s1} \) is transferred through the output coupling to the output shaft, and drives the generator with the speed \( \omega_g \) as shown in (2-10) - (2-12) The torque difference between \( T_{s1} \) and \( T_{s2} \) is decided by the damping, the inertia on the gearbox, and the speed of the output shaft (lower bevel gear) \( \omega_{m3} \). The torque on the output shaft \( T_{s2} \) can be defined by the speed and the angle differences between the two terminals of the output coupling. The rotational speed of the generator can be derived as (2-10).

\[ T_{s1} = T_{s2} + \tau_1 + (b_{vgb1} + b_{vgb2} k_{g1}^2) \omega_{m2} + b_{vs2} \omega_{m3} + J_{gb} k_{g1}^2 \dot{\omega}_{m2} + J_{s2} \dot{\omega}_{m3} \]  \hspace{1cm} (2-10)

\[ (\omega_{m3} - \omega_g) b_c4 + (\theta_{m3} - \theta_g) k_c4 = T_{s2} \]  \hspace{1cm} (2-11)

\[ (J_{s2} + J_g) \dot{\omega}_g + (b_w + b_{vs2}) \omega_g = T_{s2} - T_{eg} \]  \hspace{1cm} (2-12)

where \( T_{s2} \) is the torque of the output shaft; \( \omega_g \) and \( \theta_g \) are the rotational speed and angle of the generator’s rotor respectively; \( J_g \) is the inertia of the generator; \( b_{vs2} \) is the viscous damping of the output shaft; \( b_w \) depicts the mechanical damping and windage losses on the generator. The difference between input torque \( T_{s1} \) and output torque \( T_{s2} \) is because of the damping \( b_{vgb1} \) and \( b_{vgb2} \) from the gears, the inertia \( J_{gb} \) on the gears, and the inertia of the output shaft \( J_{s2} \).
The torque from the generator $T_{eg}$ can be derived from (2-13) - (2-16) based on the output loading, the internal impedance of the generator $R_e, L_i$ and $R_i$.

$$
\vec{E}_{abc} = \begin{bmatrix} E_a \\ E_b \\ E_c \end{bmatrix} = \frac{p}{2} k_e \omega_g \begin{bmatrix} \sin \theta \\ \sin(\theta - \frac{2}{3} \pi) \\ \sin(\theta + \frac{2}{3} \pi) \end{bmatrix}
$$  \hspace{1cm} (2-13)

$$
\vec{E}_{abc} = L_i \frac{d\vec{i}_{abc}}{dt} + \vec{i}_{abc}(R_i + R_e)
$$  \hspace{1cm} (2-14)

$$
T_e = \frac{p}{2} k_i \left[ \sin \theta \quad \sin(\theta - \frac{2}{3} \pi) \quad \sin(\theta + \frac{2}{3} \pi) \right] \vec{i}_{abc}
$$  \hspace{1cm} (2-15)

$$
\vec{i}_{abc} = \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}
$$  \hspace{1cm} (2-16)

where $\vec{E}_{abc}$ and $\vec{i}_{abc}$ are the air gap voltage and current from the generator; $p$ is the pole numbers; $k_i$ and $k_e$ are the torque and back EMF constant of the generator. Phase-to-neutral air-gap voltage $\vec{E}_{abc}$ is the result of the rotor speed $\omega_g$, which is equal to the rotational speed of the output shaft in the PTO, and the angle $\theta$ is decided by electric frequency $\omega_e = 0.5p\omega_g$. The air-gap voltages are written as a function of current and internal impedances $L_i$ and $R_i$ in (2-14), with the assumption that output resistive loads $R_e$ are connected to the outputs of the generator in a wye configuration. As a result, the PTO system dynamics are modeled by (2-5) – (2-16). If the impedance from $L_i$ is negligible, the electrical current in the generator will induce a viscous damping coefficient $b_g$ as in (2-17)

$$
b_g = T_e/\omega_g = 3p^2 k_i k_e/(8(R_i + R_e))
$$  \hspace{1cm} (2-17)
2.3 Circuit Network Model of PTO

Table 2.3-1. Analogies between Mechanical and Electrical networks [55]

<table>
<thead>
<tr>
<th>Mechanical</th>
<th>Electrical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Force (f)</td>
<td>Current (i)</td>
</tr>
<tr>
<td>Velocity ((\dot{x}))</td>
<td>Voltage (v)</td>
</tr>
<tr>
<td>Spring (k): (dF/dt=k(\dot{x}_1-\dot{x}_2))</td>
<td>Inductor (1/L): (di/dt=1/L(v_1-v_2))</td>
</tr>
<tr>
<td>Inerter (m): (F=m(\dot{x}_1-\dot{x}_2)/dt)</td>
<td>Capacitor (C): (i=C(d(v_1-v_2)/dt))</td>
</tr>
<tr>
<td>Damper (c): (F=c(\dot{x}_1-\dot{x}_2))</td>
<td>Resistor (1/R): (i=(v_2-v_1)/R)</td>
</tr>
</tbody>
</table>

2.3.1 Circuit Model with Parallel Components

The linear circuit network model is built through the analog in Table 2.3-1 based on equations derived in the previous section. Following the analogies in [56], force or torque is analogous to current; velocity or angular speed is analogous to voltage. The inertia and mass are analogous to capacitors through the concept of ineters in [55]. Viscous damping is analogous to a resistor. The Coulomb friction is represented by a constant current source \(\text{sgn}(v_{pto})I_{\tau_0}\) as shown in Fig. 2-6. The ball-screw as a two ports element [57] is represented by a transformer with a turns ratio of \(1:k_b\) to show its linear to rotation ratio.

Following these rules, the equivalent circuit of a ball screw in Fig. 2-1{1.1} is an analogy of a transformer with a turn ratio of \(k_b\), and the PTO force \(f_{pto}\) is an analogy of the current \(i_{pto}\), as shown in (2-18) is based on (2-6) and Fig. 2-6.

\[
i_{f_{PTO}} = i_{TS1}k_b + \text{sgn}(v_{pto})I_{\tau_0}k_b + v_{pto}/(R_n/\sqrt{R_{vs1}/k_b^2}) + [C_{bn} + C_{s1}k_b^2]v_{pto} \tag{2-18}
\]
where the velocity \( \ddot{x} \) is analogous to \( \nu_{pto} \); the inertia \( J_{bn} \) and \( J_{s1} \) are analogous to the capacitors \( C_{bn} \) and \( C_{s1} \); the viscous dampers \( b_n \) and \( b_{vs1} \) are analogous to the admittances \( 1/R_n \) and \( 1/R_{vs1} \); the Coulomb friction \( \tau_0 \) is analogous to the controlled current source \( I_{\tau_0} \), based on the linear direction \( \text{sgn}(\nu_{pto}) \).

2.3.2 Circuit Model with Series Components and Non-linear Components

The series components include mechanical coupling and MMR gearbox are modeled based on the comparison between force balance and Kirchhoff’s Current Law (KCL). In MMR gearbox, linear analogies from Table 2.3-1 cannot satisfy the modeling of the non-linear equation shown in (2-19) and (2-20). An additional nonlinear electric component is necessary to convert the system.
into an electric model. An ideal semiconductor diode acts as a passive switch whose conduction status is determined by the relationship of its P-side (the anode) $V_p$ and N-side (the cathode) $V_n$ voltage. When positive current flow through the diode, $V_p = V_n$, the diode is under forward bias (turn-on). When $V_p < V_n$, no current flows through the device; the diode is under reverse bias (cut-off). The analogy of a one-way sprag clutch can be represented by an RL network and an ideal diode as shown in Fig. 2-7.

Force balance:

$$T_{s1} = \begin{cases} 
  b_{c3}(\omega_{m1} - \omega_{m3}) + k_{c3}(\theta_{m1} - \theta_{m3}) & \text{if } \omega_{m1} > \omega_{m3} \\
  0 & \text{if } \omega_{m1} < \omega_{m3}
\end{cases} \quad (2-17)$$

Kirchhoff’s Current Law:

$$i_{Ts1} = \begin{cases} 
  \frac{1}{R_{c3}}(v_{\omega m1} - v_{\omega m3}) + \frac{1}{L_{c3}}\int_0^{v_{\omega m1} - v_{\omega m3}} dt & \text{if } v_{\omega m1} > v_{\omega m3} \\
  0 & \text{if } v_{\omega m1} < v_{\omega m3}
\end{cases} \quad (2-20)$$

![Fig. 2-7. The mechanical drawing and the equivalent circuit of the one-way sprag clutch from lower clutches.](image)

where $\omega_{m1}$ and $\theta_{m1}$ are the rotational speed and angle of the inner ring (shaft); $\omega_{m3}$ and $\theta_{m3}$ are the rotational speed and angle of the outer ring (bevel gears); $b_{c3}$ and $k_{c3}$ are the torsional damping and the torsional spring coefficient of the sprag clutch.

Including the three-way bevel gearbox and two one-way sprag clutches, the equivalent circuit of MMR gearbox can be shown as Fig. 2-8. The gear ratio of 3-way is $k_{g1}$ and $k_{g2}$, and the viscous dampings of the gearbox from the bevel gears are $R_{vgb1}$ and $R_{vgb2}$. The inertia of the gearbox is represented by $J_{gb}$. Both of the one-way clutches are connected to the input shaft, but they have different output terminals (outer rings). The output terminal (outer ring) of the upper clutch is
upper bevel gear, and the output terminal of the lower clutch is the lower bevel gear. One thing to be noticed that is when lower clutch conduct under constant speed condition from the input shaft, 
\( \omega_{m1} \approx \omega_{m3} \approx - \omega_{m2} \) with the assumption that \( L_{c3} \) will be short due to the input frequency is zero.

This can also apply to the case when upper clutch conducts, \( \omega_{m1} \approx - \omega_{m3} \approx \omega_{m2} \).

**Fig. 2-8.** A non-linear equivalent circuit model of MMR gearbox which includes two one-way clutches and 3-way bevel gears.

The mechanical diagram and its equivalent circuit of the mechanical coupling are shown in Fig. 2-9 through analogy between (2-21) and (2-22).

Force Balance:

\[
T_{s2} = b_c(\omega_{m3} - \omega_y) + k_c(\theta_{m3} - \theta_y) \quad (2-21)
\]

Kirchhoff’s Current Law:

\[
i_{T_{s2}} = \frac{1}{R_c} (v_{\omega m3} - v_{\omega g}) + \frac{1}{L_c} \int (v_{\omega m3} - v_{\omega g}) \, dt \quad (2-22)
\]
where the torque $T_{s2}$ is represented by current $i_{Ts2}$, the rotational speed $\omega_{m3}$ ($\omega_g$) is represented by the voltage $v_{om3}$ ($v_{og}$) with the same symbols, damping $b_{c4}$ is represented by the reciprocal of $R_{c4}$, and the torsional stiffness $k_{c4}$ is represented by the reciprocal of $L_{c4}$.

The same analogies, applied to the MMR-gearbox, are described in (2-23) based on (2-10).

$$i_{Ts1} = i_{Ts2} + I_{t1} + v_{om2}/(R_{vgb1}/R_{vgb2}/k_{g1}^2) + v_{om3}/R_{vs2} + C_{gb}k_{g1}^2 \dot{v}_{om2} + C_{s2} \dot{v}_{om3}$$

(2-23)

where $i_{Ts2}$ is the equivalent current to the one-way clutch in (2-22); $I_{t1}$ is the analogy of the friction $\tau_i$. The equivalent circuit of the MMR-gearbox in Fig. 2-1{1.3} is formed by (2-23). Three bevel gears are analogous to the transformers with $1:k_{g1}:k_{g2}$ ratio, and two one-way clutches that are analogous to the circuit in Fig. 2-1{1.3.1} with an RL network and an ideal diode in series connection. The upper clutch applies a torque to the gearbox (transformer), so it has the same rotational direction as the lower clutch.

The circuit analogy shows that when the input voltage $v_{om3}$ has only dc component, and then the inductor $L_{c4}$ becomes short circuit (i.e. $Z_{Lc4} = j\omega L_{c4} = 0$, $\omega = 0$). Its terminal voltage will be identical under dc condition. This is a similar case for a mechanical coupling that when the rotational speed of $\omega_{m3}$ is constant, and then its output rotational speed $\omega_g$ will equal the input speed. This shows that the analogy between mechanical and electrical shares a similar system dynamic through the circuit modeling process.
2.3.3 Circuit Model of MMR-PTO System

The complete equivalent-circuit for the PTO system is then formed in Fig. 2-10 by combining building blocks in Fig. 2-6 - Fig. 2-9, Table 2-1, and the circuit network analogy of the generator from (2-13) – (2-16). Mechanical couplings and one-way clutches are two distinctive types of components that connect two parts (i.e., between ball-screw and gearbox input shaft, between gears and shafts). Each of them is assumed a passive admittance, which has a negligible mass. An equivalent circuit in Fig.2-1{1.2} shows the analogies where there are slippages between the input and the output voltage during transient (e.g., \( v_{om3} \neq v_{og} \) in Fig. 2-1{1.2}) and during dc condition input voltages equal to the output voltage (e.g., \( v_{om3} = v_{og} \) in Fig. 2-1{1.2}). This is also the case in the mechanical dynamics by replacing the voltage \( v \) with linear velocity \( \dot{x} \) (or rotational speed \( \omega \)). The currents (forces or torques) are consistent between their input and output terminals. As a result, the equivalent circuit of the input coupling is bridged in series between the equivalent circuits of the ball-screw and the gearbox. Similar assembling methodologies are applied to the output coupling and one-way clutches.

The parameters of each equivalent circuit can be extracted separately through various driving frequencies and load conditions. Parameters of the ball-screw and gearbox in Fig. 2-1{1.1} and {1.3} are derived from testing under dc voltage so that the capacitors can be neglected; parameters of the couplings in Fig. 2-1{1.2} can be found by sinusoidal-wave tests (ac) to extract the RL impedance. Parameters of the one-way clutches can be known through sinusoidal-wave (ac) tests with a pure resistive-load output (with no capacitance). This is to guarantee the diode conduction, so the nonlinear circuit with diode is simplified to a linear RL circuit for parameter extraction.
Fig. 2-10. The equivalent circuit of PTO including Ball-Screw, MMR gearbox, and the generator from the equivalent circuits in Fig. 2-6 - Fig. 2-9 according to the mechanical diagram in Fig. 2-1.

Two equivalent circuits can be derived from the circuit model of PTO Fig. 2-10. Dc equivalent circuit model is derived under constant speed (square wave velocity), and based on the parameters from the dc equivalent circuit ac circuit model will be tested under sinusoid velocity to emulate the profile under regular wave conditions. By neglecting some components with a smaller impact on the system force and efficiency, the ac circuit is simulated and compared with the test results for parameter extraction. Other parameters, such as inertia, mass, and internal resistance, are known from the datasheet or can be obtained by static measurement.

Fig. 2-11. A simplified equivalent circuit model with the ac and dc unknowns. Two sets of unknowns are extracted: dc components $R_{dc}$ in (2-24) and $I_{dc}$ in (2-25) are derived from the dc test; four pairs of ac components $R_{ei}, L_{ei}$ are approximated from the sinusoidal waves ac test. Parameters of the generator ($R_a$, $C_{ig}$, $k_e$, $k_a$, $R_u$, $L_i$)
are listed in Table 2-2.

\[
\frac{1}{R_{dc}} = \frac{1}{R_n} + k_B^2 \left( \frac{1}{R_{v_{s1}}} + \frac{1}{R_{v_{gb1}}} + \frac{1}{R_{v_{gb2}}} + \frac{1}{R_{v_{s2}}} \right)
\]  \hspace{1cm} (2-24)

\[
I_{dc} = I_{r0} + I_{r1}
\]  \hspace{1cm} (2-25)

where the two parameters \(R_{dc}\) and \(I_{dc}\) are categorized as the dc unknowns, which are related to the viscous damping and Coulomb friction. The other parameters in Fig. 2-11 that need be extracted are \(R_{ei}\) and \(L_{ei}\). They are grouped as ac unknowns, that are related to the compliance and damping in the mechanical coupler and one-way clutches. To search these unknowns. The steps of parameter extraction will be introduced in the following sections.

### Table 2.3-2. Parameters of the PM generator in PTO

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power rating (W)</td>
<td>600</td>
</tr>
<tr>
<td>Rated speed (RPM)</td>
<td>600</td>
</tr>
<tr>
<td>Rated Voltage (V)</td>
<td>48</td>
</tr>
<tr>
<td>(K_c = k_c p/2) (V·s/rad)</td>
<td>0.9</td>
</tr>
<tr>
<td>(K_i = k_i p/2) (N·m/A)</td>
<td>0.9</td>
</tr>
<tr>
<td>Number of poles, (p)</td>
<td>12</td>
</tr>
<tr>
<td>Inertia, (J_s) (kg·m²)</td>
<td>5.6·10⁻³</td>
</tr>
<tr>
<td>Equivalent inertia, (C_j) (F)</td>
<td></td>
</tr>
<tr>
<td>Friction, (b_w) (N·m·s/rad)</td>
<td>1·10⁶</td>
</tr>
<tr>
<td>Equivalent friction, (R_w) (Ω)</td>
<td></td>
</tr>
<tr>
<td>(R_i) (Ω)</td>
<td>1.08</td>
</tr>
<tr>
<td>(L_i) (mH)</td>
<td>2.28</td>
</tr>
</tbody>
</table>

#### 2.4 Parameter Extraction and Model Validation

As shown in Fig. 2-11, there are ten unknowns to be extracted by comparing the experiment and simulation results. Parameter extraction is performed in two steps: 1) dc-unknown extraction with the dc test, where PTO is driven under constant velocity (voltage) \(v_{pto}\), and parameters are derived through linear regression; 2) ac-unknowns extraction with the ac test, where PTO is driven by sinusoidal velocity (voltage) \(v_{pto}\), and parameters are extracted through nonlinear curve-fitting
Experimental setups will be introduced in the first sub-section, followed by the introduction of two extraction steps in Subsection IV-B and Subsection IV-C. As a result, the unknowns are found, and the developed circuit model in Fig. 2-11 is validated with comparisons between the simulation and test results in Section V.

2.4.1 Test Bench Setup

The dry lab tests in Fig. 2-12 and Fig. 2-13 were performed to determine the PTO characteristic along with the parameter extraction processes and are usually done before a wave-tank or ocean test for PTO evaluation. In the tests, PTO is driven by a velocity source (a voltage source), which is a hydraulic tension-compression machine (Instron 8801) with 100-kN force capacity, 150-mm useable stroke, and 0.3-m/s maximum velocity. An MMR-PTO with 500W rated power is a showcase for the developed model. The methodology in this study can be used to model PTOs with different rated power, or mechanisms through modularizing components, deriving equivalent circuits, and extracting the parameters with the dc tests along with ac tests. The PTO in Fig. 2-12 is affixed by C-clamps to prevent rotation from up to 15-Nm counter-torque, which loosens the threaded connection between PTO and the test bench, while the system was driven linearly. Two types of test setups are included: 1) dc test and 2) ac test. The dc and ac unknowns in Fig. 2-11 are separately derived from the two different setups.

Under the dc test, PTO is driven under constant speed, and no loads are connected to the generator output. The purpose of the dc test is to find the frictions and viscous dampings that are dc equivalent to the dc unknowns. The ac test is to derive the compliance and damping of the couplings and clutches, which are related to the ac unknowns, and the Instron machine is set to drive the system with the sinusoidal velocity. In this test, the generator is connected to three identical resistive loads with values of \( R_e \) from 10 \( \Omega \) to 0.5 \( \Omega \) in a wye configuration as shown in
Fig. 2-12, each with 100-W wattage. The input force, displacement, and velocity are recorded by the data acquisition (DAQ) on the test machine under 10-kHz sampling frequency. The output information, including two line-neutral voltages (line-line voltages under open-circuit) and two currents (no currents under open-circuit), are recorded in an oscilloscope with 20-kHz sampling frequency.

![Diagram](image)

**Fig. 2-12.** The servo-hydraulic actuator drives the PTO with constant (dc) or sinusoidal (ac) velocity. The output of the generator is under the open-circuit condition for dc test, or connected to three-phase wye resistive loads \( R_e \) on each phase for the ac test as shown in Fig. 2-13. (a) PTO test setup with Instron 8801 – a hydraulic compression-tension machine (b) The driving velocity profile from the test machine as the PTO input velocity.

![Diagram](image)

**Fig. 2-13.** The PTO output setup of the three-phase generator.
### 2.4.2 Parameter Extraction with Dc Equivalent Circuit Model

The model was tested under a dc condition, where \( \dot{x} (v_{pto}) \) is constant, to extract the two dc unknowns in Fig. 2-11, \( R_{dc} \) and \( I_{dc} \), which represent lumped viscous-damping and Coulomb-friction. A step velocity, \( \dot{x} \), which drives the PTO, is assumed a dc equivalent voltage source, \( v_{pto} \). The response occurs at force \( f_{pto} \) (i.e., equivalent current \( i_{pto} \)), as shown in Fig. 2-14.

![Fig. 2-14. Dc tests with constant velocity \( \dot{x} \) (or equivalent voltage \( v_{pto} \)) under open circuit for the identification of PTO’s characteristics. The \( \dot{x} \) reaches \( \dot{X} \) (or \( \dot{v}_{pto} \) reaches \( \dot{V}_{pto} \)) after steady-state. The force of the PTO \( f_{pto} \) (or equivalent current \( i_{pto} \)) reaches steady-state force (current) of \( F_{pto} \) (or \( I_{pto} \)).](image)

The input voltage is constant as a dc voltage, so (2-26) is assumed when driving frequency \( f \) = 0, and \( k_{g1} = k_{g2} = 1 \).

\[
v_{\omega m0} = v_{\omega m1} = -v_{\omega m2} = v_{\omega m3} = v_{\omega g} \quad (2-26)
\]

![Fig. 2-15. Simplified dc equivalent circuit from Fig. 2-11 under the constant velocity (\( V_{pto} \)) test.](image)

The circuit model in Fig. 2-11 is further simplified as a dc-equivalent circuit in Fig. 2-15 by using (2-27) and assuming the diode D2 from the upper clutch always conducts, the capacitors are opened, and the inductors are shorted. The equivalent dc resistor and the current source of the PTO
(i.e., the dc unknowns), $R_{dc}$ and $I_{dc}$, are extracted, based on the relationship between $I_{Fpto}$ and $V_{pto}$ in Fig. 2-14(a). The equivalent current $I_{Fpto}$ is analogous to the steady-state force $F_{pto}$, and it is a function of the equivalent dc-voltage $V_{pto}$ (or the steady-state velocity $\dot{X}$) as follows:

$$I_{Fpto} = (k_b I_{dc} + k_b^2 V_{pto}/R_{dc})$$  \hspace{1cm} (2-27)

The unknowns $R_{dc}$ and $I_{dc}$ are derived from the linear regression by using $I_{Fpto}$ as responses and $V_{pto}$ as control variables. The regression is shown in Fig. 2-16, where $I_{dc}$ equals $110.7/k_b$ A, and $R_{dc}$ equals $k_b^2/652.5$ $\Omega$, with the $k_b$ ratio of 104 rad/m which corresponds to the Coulomb friction of 1.06 Nm and the viscous damping of 0.06 Nms/rad. Each different $V_{pto}$ was tested twice, and $I_{Fpto}$ was recorded with an average value over five-periods of testing. Five different values of $V_{pto}$ are tested under open-circuit to extract the parameters from the PTO system.

![Graph showing linear regression of $2I_{Fpto}$ vs $V_{pto}$](image)

**Table:**

<table>
<thead>
<tr>
<th>Driving Velocity $\dot{X}$ (m/s)</th>
<th>$F_{pto}$ (N)</th>
<th>$I_{Fpto}$ (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>0.04</td>
<td>0.08</td>
</tr>
<tr>
<td>0.16</td>
<td>0.2</td>
<td>249</td>
</tr>
<tr>
<td>0.02</td>
<td>0.04</td>
<td>276</td>
</tr>
<tr>
<td>0.08</td>
<td>0.16</td>
<td>319</td>
</tr>
<tr>
<td>0.2</td>
<td>0.2</td>
<td>430</td>
</tr>
</tbody>
</table>

**Fig. 2-16.** The dc unknowns $I_{dc}$ and $R_{dc}$ in (2-27) are derived from the test results in (b) via the linear regression as shown in the $I_{Fpto} - V_{pto}$ curve of (a). The dc current-source $I_{dc}$ is 1.06 A, and the dc resistor $R_{dc}$ is 16.58 $\Omega$. 

43
2.4.3 Parameter Extraction with Ac Equivalent Circuit Model (Non-linear Simulation Model)

Ac-unknown extraction, as the second step of parameter-extraction, extracts viscous damping \( R_{ci} \) and compliance \( L_{ci} \), by simulating the model in Fig. 2-11 with the regression values of \( R_{dc} \) and \( I_{dc} \) from Section 2.4.2 under the ac test. Unlike the linear dc equivalent circuit in the dc-unknown extraction, the model in Fig. 2-11 is nonlinear due to the diodes. The parameter extraction is realized by nonlinear curve-fitting, which searches for the \( R_{ci} \) and \( L_{ci} \) that fit the tested efficiencies by simulating the model in Fig. 2-11. The unknowns \( R_{ci} \) and \( L_{ci} \), where \( i = 1 - 4 \), are closely related to the efficiency of the PTO system, so it is reasonable to estimate the unknowns with the mechanical efficiency \( \eta_m \) in Fig. 2-17, and (2-28) to (2-31).

\[
P_{oe} = (i_a v_{an} + i_b v_{bn} + i_c v_{cn}) \tag{2-28}
\]

\[
P_{om} = \frac{P_{oe}}{R_e} (R_e + R_i) \tag{2-29}
\]

\[
P_{in} = \frac{1}{(T_2 - T_1)} \int_{T_1}^{T_2} f_{pto-l} \cdot x dt \tag{2-30}
\]

\[
\eta_m = \frac{P_{om}}{P_{in}} \tag{2-31}
\]

where output electrical power \( P_{oe} \) is derived from the mean value of the measured output voltage and current, as shown in Fig. 2-17(b) and Fig. 2-17(c); the input power \( P_{in} \) is derived from the integral of the filtered force \( f_{pto-l} \) after low-pass-filter (LPF) and the PTO displacement \( x \) divided by its time interval, as shown in Fig. 2-17(b); \( P_{om} \) is the output power including the electrical power on the internal coil resistors; \( \eta_m \) is the PTO mechanical transmission efficiency; \( R_e \) is the value of external resistive loads; \( R_i \) is the internal resistance of the generator.
By assuming the parameters from the input/output couplings are identical, and the parameters from the upper/lower one-way clutches are identical, the eight unknowns from the equivalent circuit in Fig. 2-11 are simplified to four ac unknowns as follows: $R_{c1} = R_{c4}$, $L_{c1} = L_{c4}$, $R_{c2} = R_{c3}$, and $L_{c2} = L_{c3}$. The unknowns $R_{c1}$ and $L_{c1}$ represent the damping and the stiffness of the input coupling, which are the analogies of $b_{c1}$ and $k_{c1}$ in (2-5). Another set of unknowns $R_{c2}$ and $L_{c2}$ represents the damping and the spring’s coefficient of the clutches in the MMR-gearbox, which are the analogies of $b_{c2}$ and $k_{c2}$ in (2-8).

The simplified ac-unknowns $\{R_{c1}, R_{c2}, L_{c1}, L_{c2}\}$ in the circuit model are searched through the least-square curve fitting function, \textit{lsqcurvefit}, in Matlab by the trust-region method with step size ($dP$) to compare with the output [58]. Throughout the non-linear searching process, each set of
unknowns produces 15 outputs of efficiency $\eta_m$ from five loads $R_e (\Omega)$ = {0.5, 1, 1.5, 2, 10}, and three frequencies $f (\text{Hz})$ = {2, 3, 4} from the nonlinear-simulation model. The outputs are compared with the test outputs using Newton’s method, to produce new sets of unknowns, until the residual norm is smaller than the tolerance (Tol), or the search reaches the iteration (Imax) limit [59]. These configuration values are {dP, Tol, Imax} = {0.01, 0.001, 20}.

The optimal results are found after function counts of 25, and the unknowns become \{R_{c1}, R_{c2}, L_{c1}, L_{c2}\} = \{0.09 \Omega, 73.7 \Omega, 11.3 \text{ mH}, 13.1 \text{ mH}\}. Correlation coefficients R-squared ($R^2$) are used as the criteria to evaluate the result of the parameter extraction. Along with the values of $R_{dc}$, and $I_{dc}$ found in the previous extraction step, all of the unknowns are substituted in Fig. 2-11, and the simulated efficiency curves are plotted in Fig. 2-18, which matches the measured data showing correlation coefficient $R^2$ of 0.9932.

![Nonlinear lsqcurvefit](image)

<table>
<thead>
<tr>
<th></th>
<th>Values from linear regression</th>
<th>Values from nonlinear curve fitting</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{dc}$</td>
<td>16.58 Ω</td>
<td></td>
</tr>
<tr>
<td>$I_{dc}$</td>
<td>1.06 A</td>
<td></td>
</tr>
<tr>
<td>$R_{c3} \equiv R_{c2}$</td>
<td>73.7 Ω</td>
<td></td>
</tr>
<tr>
<td>$L_{c3} \equiv L_{c2}$</td>
<td>13.1 mH</td>
<td></td>
</tr>
<tr>
<td>$R_{c1} \equiv R_{c4}$</td>
<td>90 mΩ</td>
<td></td>
</tr>
<tr>
<td>$L_{c1} \equiv L_{c4}$</td>
<td>11.3 mH</td>
<td></td>
</tr>
</tbody>
</table>

(a) Mechanical efficiencies curves  
(b) Extracted values of ac and dc unknowns

Fig. 2.18. (a) Efficiencies (simulated from the circuit model in Fig. 2-11, and tested) are compared under 15 conditions, including five output resistances (0.5 Ω - 10 Ω) and three frequencies (2 – 4 Hz) using $L_{c1}$ - $L_{c4}$, and $R_{c1}$ - $R_{c4}$ values in (b). (b) From least-square-curve-fitting method (lsqcurvefit), the ac-unknowns $R_{c1} - R_{c4}, L_{c1} - L_{c4}$ are listed. Simulation with parameters from parameter extraction gives a correlation coefficient $R^2$ of 0.9932. (B.1 Setups of figures-(i))
The sensitivity study of the variables $L_{c1}$- $L_{c4}$ and $R_{c1}$- $R_{c4}$ are shown in Fig. 2-19, which uses $R^2$ value as criteria to compare the simulated and tested efficiency $\eta_m$. Among the four unknowns, two of them are the controlled variables, and the other two are kept constant, to study their influence on $R^2$ values. When $R_{c2}$ and $L_{c2}$ are the controlled variables, except for $L_{c2} = 20$ mH, the other $R^2$ values are above 0.99. These non-ideal factors of the one-way clutches do not remarkably alter the prediction of mechanical efficiency. The $R$-squared of the efficiencies $\eta_m$, according to the variables changes of the input/output coupling parameters $R_{c1}$ and $L_{c1}$, are shown in Fig. 2-19(b) from which can be shown that the efficiency prediction is very sensitive to the $R_{c1}$ and $L_{c1}$ change.

![Fig. 2-19. (a) Correlation coefficient $R^2$ are tested with $R_{c2}$ ($R_{c3}$) from 30 to 500 $\Omega$, and $L_{c2}$ ($L_{c3}$) from 3 mH to 20 mH. Other two variables $R_{c1}$ ($R_{c4}$) = 0.09 $\Omega$, and $L_{c1}$ ($L_{c4}$) = 11.3 mH remain constant. The least square curve fitted results when $R_{c2} = R_{c3} = 73.7$ $\Omega$ and $L_{c2} = L_{c3} = 13.1$ mH provide the highest $R^2 = 0.9932$. (b) Correlation coefficient $R^2$ are tested with $R_{c1}$ ($R_{c4}$) from 0.02 to 0.5 $\Omega$, and $L_{c1}$ ($L_{c4}$) from 3 mH to 50 mH. Other two variables $R_{c2}$ ($R_{c3}$) = 73.7 $\Omega$, and $L_{c2}$ ($L_{c3}$) = 13.1 mH remain constant. The least square curve fitted results when $R_{c1} = R_{c4} = 0.09$ $\Omega$ and $L_{c1} = L_{c4} = 11.3$ mH provide the highest $R^2 = 0.9932$. (B.1 Setups of figures-(i))](image)

The average $R^2$ can be derived from the average value of $R_f^2$ under different frequency $f$ in (2-32), (2-33) where $a = 5$, $b = 5$ correspondings to the searching number of different $R_c$ and $L_c$ value.
For example, $\eta_{m,2,3}$ represents the simulated mechanical efficiency at $R_c = 10 \Omega$ and $L_c = 0.05 \text{ H}$.

$$R_f^2 = 1 - \frac{\sum_{i=1,j=1}^{a,b}(\eta_{m,i} - \hat{\eta}_{m,i})^2}{\sum_{i=1,j=1}^{a,b}(\eta_{m,i} - \bar{\eta}_{m,i})^2}$$

(2-32)

$$\text{Average } R^2 = \frac{1}{3} \sum_{f=2}^{4} R_f^2$$

(2-33)

### 2.4.4 Model Validation on Time-Domain Results

The developed circuit model is applied to predict the PTO system efficiencies in other frequencies, the PTO input forces, or the output voltages on the resistive loads in the time domain.

The circuit model with extracted parameters shows its time-domain accuracy in Fig. 2-20 - Fig. 2-24. Prediction is performed under different conditions for four different measurements, including input force and output voltages. The input force and output voltages of the PTO are from 500 N to 2 kN peak input force, and from 1 Hz to 4 Hz peak input force. Under each test condition, the input force and the output voltages correspond well between the experiment and simulation results, except for the noise from the measured input force, which might be the result of gear engagements and other non-ideal effects from the mechanical coupling. Differences between the tested (i.e., $f_{pto\_test}$, $v_{abc\_test}$) and the simulated results (i.e., $i_{pto}$, $v_{abc\_ckt}$) from the derived circuit model are compared by RMS values as shown in Table 2.4-1.

<table>
<thead>
<tr>
<th>RMS value</th>
<th>Input force $f_{f_{pto_test}} / f_{f_{pto_id}} / f_{f_{pto_ckt}}$</th>
<th>Output voltage $v_{v_{abc_test}} / v_{v_{abc_id}} / v_{v_{abc_ckt}}$</th>
<th>Efficiency $Eff_{test}$ / $Eff_{id}$ / $Eff_{ckt}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 \text{ Hz} / 1.5 \Omega$</td>
<td>330 227 0.69 354 1.07</td>
<td>1.54 1.71 1.11 1.67 1.08</td>
<td>58.7 100 61.9 1.05</td>
</tr>
<tr>
<td>$2 \text{ Hz} / 2 \Omega$</td>
<td>469 383 0.82 533 1.14</td>
<td>2.37 2.76 1.07 2.57 1.08</td>
<td>66 100 67.8 1.03</td>
</tr>
<tr>
<td>$3 \text{ Hz} / 0.5 \Omega$</td>
<td>1038 1091 1.05 1201 1.16</td>
<td>3.06 3.65 1.16 3.39 1.11</td>
<td>76.9 100 78.8 1.02</td>
</tr>
<tr>
<td>$4 \text{ Hz} / 0.5 \Omega$</td>
<td>1331 1438 1.08 1551 1.17</td>
<td>3.56 3.81 1.19 3.72 1.04</td>
<td>79.7 100 79.9 1.00</td>
</tr>
</tbody>
</table>
Fig. 2-20. Time-domain results of PTO input force $f_{pto}$ ($i_{fpto}$) and output voltage $v_{abc}$ from Fig. 2-12 bench test and Fig. 2-11 simulation under $R_e = 1.5$ Ω, 1-Hz driving frequency. Peak force (current) is 400 N(A). (B.1 Setups of figures-(i))

Fig. 2-21. Time-domain results of PTO input force $f_{pto}$ ($i_{fpto}$) and output voltage $v_{abc}$ from Fig. 2-12 bench test and Fig. 2-11 simulation under $R_e = 2$ Ω, 2-Hz driving frequency. Peak force (current) is around 600 N(A). (B.1 Setups of figures-(i))

Fig. 2-22. Time-domain results of PTO input force $f_{pto}$ ($i_{fpto}$) and output voltage $v_{abc}$ from Fig. 2-12 bench test and Fig. 2-11 simulation under $R_e = 0.5$ Ω, 3-Hz driving frequency. Peak force (current) is 1200 N(A). (B.1 Setups of figures-(i))
Fig. 2-23. Time-domain results of PTO input force $f_{pto}$ ($i_{fpto}$) and output voltage $v_{abc}$ from Fig. 2-12 bench test and Fig. 2-11 simulation under $R_e = 0.5 \, \Omega$, 4-Hz driving frequency. Peak force (current) is 2000 N(A). (B.1 Setups of figures-(i))

The RMS values of $v_{abc\_test}$ and $v_{abc\_sim}$ in Table 2-3 are derived by taking the average from the RMS value of their $v_a$, $v_b$, and $v_c$. The ratios between simulated and tested results show errors around 4 - 14%. The filtered force $f_{pto\_lp}$ are processed by a low-pass filter with 4-Hz cut-off frequency and filtfilt function in Matlab to eliminate the phase shift from the tested forces $f_{pto}$. The RMS values of forces are from $f_{pto}$, where the simulated equivalent current $i_{fpto}$ is higher than the test $f_{pto}$. The discrepancy could be introduced by the other non-ideal effects, which didn’t consider in the model (2-6) - (2-14), such as backlash on the coupling and gears, and the Stiction force as well as Stribeck effect from the static frictions. The high-frequency noises of the force measurements are due to the 100–kN load cell is used to measure 2 kN force which is only 2 % of the rated force. Simulated results in low driving frequency are more consistent with the test results. This could be related to less disengagement (discontinuity) in low frequency.

The developed circuit model, the ideal model from [43] and [60], and the tested results are compared in Fig. 2-24, which includes output voltage $v_{abc}$, and force $f_{pto}$ (or equivalent current $i_{fpto}$) under $f = 2 \, \text{Hz}$, and $R_e = 2 \, \Omega$. Both models match well with the test results in this condition. More comparisons with the ideal model are shown in Table 2-3. The circuit model is a better predictor
of voltage and efficiency than the ideal model and tends to have a better prediction on low-frequency forces, while the forces from the ideal model match the tested forces better on higher frequencies. R-squared of the circuit and the ideal model from 25 test conditions are listed in Table 2.4-2, which shows that the circuit model is capable of predicting the efficiency with an $R^2_{ckt}$ of 0.981, while the ideal model cannot do so with a $R^2_{idl}$ of $\eta_m$ equals zero.

![Fig. 2.24](image)

From real tests in Fig. 2-12, the ideal model, and the developed circuit model in Fig. 2-11, time-domain results of (a) PTO input force $f_{pto_{test}} / f_{pto_{idl}} / f_{pto_{ckt}}$ and (b) output voltage $v_{abc_{test}} / v_{abc_{ckt}} / v_{abc_{idl}}$ are compared under $R_e = 2 \Omega$, 2-Hz driving frequency $f$. RMS value of force is 469 N, and RMS value three-phase voltage is 2.37 V from the test result. (B.1 Setups of figures-(i)-(ii))

The validations of the circuit model in all of the test conditions are shown in Fig. 2-25. There are 25 test conditions with driving frequency $f$ (Hz) = {1, 1.5, 2, 3, 4}, and resistive load $R_e$ (Ω) = {0.5 1 1.5 2 10}. With a repeated test #1 and test #2 on each condition, input force, output voltage, and efficiency are compared with the simulated curve. The equivalent current from the simulation predicts the PTO force well at low frequency in Fig. 2-25(a), and shows around 17% more than the tested force at the high frequency, heavy load condition.
Fig. 2-25. Comparison of PTO forces $f_{pto}$, output voltages $v_{abc}$, and efficiency $\eta_m$ from circuit model simulation, ideal model simulation, and repeated test results. (a) – (c) Curves include results under frequency $f$ from 1 to 4 Hz, and under resistor $R_e$ from 0.5 to 10 $\Omega$. (B.1 Setups of figures-(i))

Table 2.4-2. R-squares from ideal model $R^2_{idl}$ and developed circuit model $R^2_{ckt}$. This shows the correlations between the model and the test results from 25 test conditions are included.

<table>
<thead>
<tr>
<th>R-square</th>
<th>Ideal model [43], [60]</th>
<th>Developed circuit model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_{abc}$</td>
<td>0.978</td>
<td>0.986</td>
</tr>
<tr>
<td>$f_{pto}$</td>
<td>0.913</td>
<td>0.904</td>
</tr>
<tr>
<td>$\eta_m$</td>
<td>0</td>
<td>0.981</td>
</tr>
</tbody>
</table>

This could be related to less disengagement (non-linearity) in lower driving frequency. Considering all of the frequencies $f$ and $R_e$, the R-squared from the circuit simulation $R^2_{ckt}$ of the input force $f_{pto}$ is 0.904.

The output voltage and efficiency show high consistency between tests and simulations in Fig. 2-25(b) and (c), where the efficiency of the MMR-PTO ranges from 0.27 to 0.8. The losses from the ball-screw and the gearbox dominate the mechanical efficiency, which is proportional to the force or speed of each component. The correlation coefficients $R^2_{ckt}$ of the output voltage $v_{abc}$ and the efficiency $\eta_m$ are 0.986 and 0.981, respectively, in Table 2-4.

As a result, the circuit model in Fig. 2-11 is validated by successfully predicting the mechanical efficiencies, input forces, and output voltages, in the time domain.

2.5 Application of the PTO circuit model

The developed circuit model can be easily applied in all types of conditions: to predict the PTO system characteristics, to test control algorithms, to evaluate the losses, or to evaluate the influence of the disengagement in the MMR-based PTO system.
2.5.1 Loss Breakdown and Efficiency Analysis with PTO Model

The components loss can be evaluated through simulations from the circuit model Fig. 2-26(a) shows the simulated mechanical efficiency curve under the driving frequency of $2 - 4$ Hz, and the loading $R_e$ of 0.5 - 10 ohm. Four conditions are marked on the Fig. 2-26(a) including $f = 2$ Hz/$R_e = 10$ Ω, $f = 4$ Hz/$R_e = 10$ Ω, $f = 2$ Hz/$R_e = 1$ Ω, and $f = 4$ Hz/$R_e = 1$ Ω. There are three types of power losses: $P_{tg}$ is the gearbox viscous damping loss; $P_{ltc}$ is the loss from Coulomb friction; $P_{lre}$ is the loss from the coupling of sprag one-way clutches in MMR gearbox. Each of the losses is normalized through the corresponding output power $P_o$ under each condition as shown in Fig. 2-26(b).

![Efficiency from CKT Model](image1)

![Normalized loss $P_{loss}/P_o$ breakdown under four different marked conditions which is corresponding to the efficiency point I – IV in (a).](image2)

One of the important characteristics is the overall efficiency of the PTO under different conditions, as shown in Fig. 2-27. The PTO is tested under $2 - 4$ Hz of the input regular wave frequency, and the $0.13$ m/s – $0.25$ m/s of peak input velocity. The PTO maximum overall efficiency increases from 47 percent to 52 percent, and the optimal passive loadings shift from $R_e = 3$ Ω to $3.75$ Ω. This shows that the model can also be applied to the control algorithm to extract the real maximum power, considering PTO efficiencies.
2.5.2 Impact of the Disengagement from MMR in PTO System

One special mechanism in an MMR-based PTO is the disengagement, which will disconnect the buoy and the generator. How disengagement affects the characteristics of a PTO system (e.g., output power, mechanical efficiency) is still unknown from the previous study; however, the circuit model provides a solution to analyze its effects systematically. The disengagement ratio is defined as the disengagement period over a regular wave cycle, as shown in Fig. 2-28. The disengagement ratio is defined in (2-34), which is proportional to the frequency, inertia, and the reciprocal of the damping coefficient.

\[\text{Disengagement ratio} = \frac{\text{Disengage period}}{\text{Disengage period} + \text{Engage period}} \quad (2-34)\]
Based on a dimensional analysis from [61], a non-dimensional metric \( FM/C \) is derived as in (2-35) to (2-36), based on the factors that potentially affect the disengagement ratio.

\[
\frac{FM}{C} = \frac{Freq \_in \cdot m_i}{c_o} \times \left(\frac{1}{S}\right) \frac{N \cdot S^2}{m^3} \tag{2-35}
\]

\[
c_o = \frac{1.5 \cdot k_1^2 \cdot k_2^2 \cdot k_e \cdot k_t \cdot \left(\frac{P}{2}\right)^2 \cdot k_b^2}{(R_e + R_i)} \tag{2-36}
\]

where \( Freq \_in \) is the input regular wave frequency; \( m_i \) is the equivalent mass from the inertia of the generator (or \( k_b^2 J_g \) in the circuit model); \( c_o \) is the equivalent damping from the external load. The coefficient 1.5 from (2-35) is the consideration of the three-phase generator.

The \( FM/C \) is then used to evaluate the disengagement’s impact on the PTO efficiency under the same output power condition. The comparison results in Fig. 2-29 demonstrates when the disengagement ratio is higher, the efficiency will drop under the same output power and the same peak input velocity.
2.6 Conclusion

An equivalent circuit-based model of the mechanical motion rectifier (MMR)-based PTO is derived from each building block, with consideration given to the realistic frictions dampings, and compliance. Circuit building blocks derived from equivalent equations are assembled as a realistic PTO model. The two dc unknowns of lumped viscous-damping and Coulomb friction are extracted from the circuit model by dc tests. The eight ac-unknowns in the circuit, which are related to the viscous damping and compliance in the mechanical couplers and the one-way clutches, are approximated with sinusoidal (ac) excitation and a least-square fitting method, using efficiency as its observed output. The simulations that apply the search results compute $R^2 = 0.9932$, as compared with test results.

Finally, the derived model predicts the PTO input force, power, and efficiency at different driving frequencies with more than 90-% accuracy in the time domain. The time-domain simulations from the derived circuit model show RMS consistency of input force with R-squared of 0.904, by comparing the results obtained through measurements. The model shows around 0.981 on efficiency predicting R-squared. Although the ideal model can also predict the force and voltage with similar levels of R-squared as the circuit model, it cannot predict the efficiency, which is a
critical parameter, especially for the reactive (optimum power) control that involves bidirectional power transfer in PTOs. The losses of the PTO circuit are extracted, and then the PTO design can be optimized for higher efficiency. The non-dimensional analysis on the FM/C metric is used to study the influence of the disengagement on the mechanical efficiency of the MMR-based PTO system, which shows the PTO has lower efficiency under higher disengagement ratio.

The circuit model can combine with a circuit-based wave to wire (W2W) model for overall WEC system simulation and control algorithm development. The output power can be optimized through the control algorithm development based on the efficiency-prediction circuit model.

An MMR-PTO with 500-W rated power is a showcase for the developed model. The methodology in this study can be used to model PTOs with different rated power, or mechanisms through modularizing components, deriving equivalent circuits, and extracting the parameters with the dc tests along with ac tests.

On the following page, Fig. 2-30 from previous figures and equations in this chapter are combined on one-page, to show that the mechanical components can be easily replaced in a different order in the system, or equivalent circuits of other mechanical components can be built to assemble the overall PTO equivalent circuit.
### Mechanical components

#### Modeling Equations

<table>
<thead>
<tr>
<th>Component</th>
<th>Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1.1) MMR - PTO</td>
<td>$J_s \frac{d\omega_m}{dt} = f_{m_e} - f_{pto} + \text{sgn}(\omega_m) f_{m_2} \omega_m + \text{sgn}(\omega_m) \omega_m \cdot \omega_m + \text{sgn}(\omega_m) \omega_m \cdot \omega_m$</td>
</tr>
<tr>
<td>(1.2) Ball-screw</td>
<td>$T_s = (\omega_m - \omega_b) \frac{d\omega_b}{dt} + \text{sgn}(\omega_m - \omega_b) \omega_b$</td>
</tr>
<tr>
<td>(1.3) MMR - gearbox</td>
<td>$T_i = T_{i_1} + \frac{J_i}{R_i} \frac{d\omega_i}{dt} + \frac{J_{i_2}}{R_{i_2}} \frac{d\omega_{i_2}}{dt} + \frac{J_{i_3}}{R_{i_3}} \frac{d\omega_{i_3}}{dt}$</td>
</tr>
<tr>
<td>(1.3.1) One-way Clutch</td>
<td>$i_{m_1} = (\omega_m - \omega_{m_1}) \frac{d\omega_{m_1}}{dt} + \text{sgn}(\omega_m - \omega_{m_1}) \omega_{m_1}$</td>
</tr>
</tbody>
</table>

### Equivalent Circuits

#### Equations

- (2-2) $i_{m_1} = (\omega_m - \omega_{m_1}) \frac{d\omega_{m_1}}{dt} + \text{sgn}(\omega_m - \omega_{m_1}) \omega_{m_1}$
- (2-7) $T_{i_1} = T_{i_1} + \frac{J_{i_2}}{R_{i_2}} \frac{d\omega_{i_2}}{dt} + \frac{J_{i_3}}{R_{i_3}} \frac{d\omega_{i_3}}{dt}$
- (2-18) $i_{m_1} = (\omega_m - \omega_{m_1}) \frac{d\omega_{m_1}}{dt} + \text{sgn}(\omega_m - \omega_{m_1}) \omega_{m_1}$

### Results

- (1.1) Ball-screw
- (1.2) Output (input) mechanical coupling
- (1.3) MMR gearbox
- (1.3.1) Lower (upper) one-way clutch

---

(a) From Fig. 2-1, WEC includes three levels of components: Level i: (1) MMR PTO; Level ii: (1.1) ball-screw, (1.2) mechanical couplings, (1.3) MMR-gearbox, (1.4) permanent magnet synchronous generator; Level iii: (1.3.1) one-way clutches.

(b) Modelling equations and equivalent circuits for Level-ii and Level-iii building blocks in (a), (1.1) ball-screw, (1.2) output (input) mechanical coupling, (1.3) MMR gearbox, (1.3.1) lower (upper) one-way clutch.

(c) Equivalent circuit model from Fig. 2-10. Equivalent circuit for the building block in Level-i of (a), constructed from the equivalent circuits for Level-ii and Level-iii building blocks in (b), of PTO including ball-screw, couplings, MMR gearbox, and the generator. All the parameters are derived in 2.4.

---

Fig. 2-30. One-page summary of the equivalent-circuit derivation utilizing Fig. 2-1, (2-2) – (2-23), and Fig. 2-10, shows the derivation process from mechanical system, equations, to equivalent circuits.
Chapter 3 Irregular Wave-to-Wire Circuit Model for Wave Energy

Converter System

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{11}$</td>
<td>Frequency-dependent added mass of buoy</td>
</tr>
<tr>
<td>$A_{11\infty}$</td>
<td>Added mass at infinite frequency</td>
</tr>
<tr>
<td>$b_{11}$</td>
<td>Frequency-dependent radiation damping</td>
</tr>
<tr>
<td>$b_w$</td>
<td>Viscous friction of generator</td>
</tr>
<tr>
<td>$b_e$</td>
<td>PTO damping</td>
</tr>
<tr>
<td>$C_r$</td>
<td>Equivalent capacitor for radiation force</td>
</tr>
<tr>
<td>$E_{abc}$</td>
<td>Air gap phase voltage of the generator</td>
</tr>
<tr>
<td>$f_e$</td>
<td>Wave excitation force</td>
</tr>
<tr>
<td>$f_r$</td>
<td>Buoy radiation force</td>
</tr>
<tr>
<td>$f_{pto}$</td>
<td>PTO force</td>
</tr>
<tr>
<td>$i_{abc}$</td>
<td>Current of generator</td>
</tr>
<tr>
<td>$i_{fe}$</td>
<td>Equivalent current for excitation force ($f_e$)</td>
</tr>
<tr>
<td>$i_{fL}$</td>
<td>Equivalent current for PTO force ($f_L$)</td>
</tr>
<tr>
<td>$i_{fr}$</td>
<td>Equivalent current for radiation force ($f_r$)</td>
</tr>
<tr>
<td>$i_T$</td>
<td>Equivalent current for the force of Generator input shaft</td>
</tr>
<tr>
<td>$J_g$</td>
<td>Inertia of generator</td>
</tr>
<tr>
<td>$J_{gb}$</td>
<td>The inertia of gearbox and transmission shaft</td>
</tr>
<tr>
<td>$J_s$</td>
<td>The inertia of gearbox and transmission shaft</td>
</tr>
<tr>
<td>$k_b$</td>
<td>Rack-Pinion or ball-screw gear ratio</td>
</tr>
<tr>
<td>$k_g$</td>
<td>Gearbox gear ratio</td>
</tr>
<tr>
<td>$k_e, k_t$</td>
<td>Voltage and torque constant of the generator (line RMS voltage)</td>
</tr>
<tr>
<td>$k_s$</td>
<td>Hydrodynamic damping of Buoy</td>
</tr>
<tr>
<td>$k_r$</td>
<td>Radiation damping</td>
</tr>
<tr>
<td>$L_e$</td>
<td>Equivalent inductance for hydrodynamic damping</td>
</tr>
<tr>
<td>$L_i$</td>
<td>The internal inductor of generator</td>
</tr>
<tr>
<td>$L_r$</td>
<td>Equivalent inductance for radiation force</td>
</tr>
<tr>
<td>$m_I$</td>
<td>Mass of Buoy</td>
</tr>
<tr>
<td>$m_e$</td>
<td>Equivalent Mass of PTO</td>
</tr>
<tr>
<td>$P_o$</td>
<td>Output power from the power converter</td>
</tr>
<tr>
<td>$R_i$</td>
<td>Internal Resistance</td>
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<tr>
<td>$R_r$</td>
<td>Equivalent resistance for radiation force</td>
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### Table

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>$R_{vgb}$, $R_{vrp}$</td>
<td>The equivalent resistance of Damping coefficient of gearbox and rack-pinion (1/$C_{vgb}$, 1/$C_{vrp}$)</td>
</tr>
<tr>
<td>$T_{eg}$</td>
<td>Torque from generator</td>
</tr>
<tr>
<td>$v_{pto}$</td>
<td>the equivalent voltage from WEC (PTO) velocity</td>
</tr>
<tr>
<td>$v_{pto}$</td>
<td>Rack-Pinion rotational speed/equivalent voltage of rack-pinion</td>
</tr>
<tr>
<td>$\omega_g$</td>
<td>The rotational speed of the generator</td>
</tr>
<tr>
<td>$x$</td>
<td>Vertical displacement of WEC</td>
</tr>
</tbody>
</table>

### 3.1 Introduction of WEC System

A point absorber type WEC system with a single body structure will be introduced in this section. According to the structure in Fig. 3-1, there are WEC, PTO in a WEC system. Point absorber type WEC is a floating structure that absorbs the energy from the ocean wave. A mechanical transmission PTO would typically contain mechanical coupling, gearbox, generator, and power converter. In this research, the gearbox is replaced by an MMR-based gearbox, which is introduced in Chapter 2, and a detailed PTO circuit model is built.

![Wave-to-Wire block diagram of a wave energy conversion system.](image)

### 3.1.1 Model of SDoF Point Absorber WEC

Here, the single body, point absorber type WEC is assumed a heave only mechanical vibration system, which can be described as a single degree of freedom (SDoF) mechanical diagram in Fig. 3-2. The excitation force from the wave is $f_e$, and force provided by the PTO is $f_{pto}$. $x$ is the vertical displacement (upper as positive); $f_r$ is the radiation force; $k_s$ is the hydrostatic stiffness; $b_{11}$ is the
radiation damping, and $A_{11}$ is added mass of the buoy; $m$ is the mass of the buoy.

\[
\begin{align*}
(m_1 + A_{11\infty})\ddot{x} + \int_{-\infty}^{t} k_r(t - \tau)\dot{x}(\tau)d\tau + k_s x &= f_e + f_{pto} + f_m + f_v \\
\end{align*}
\]  

(3-1)

where $A_{11\infty}$ is the added mass at the infinite frequency; $k_r(t)$ is the impulse response function of the radiation force; $f_m$ is the mooring force; $f_v$ is the viscous drag force that usually extracts from experimental data or hydrodynamic simulation. The mooring force and viscous drag force are assumed to be negligible and are omitted in this model.

The convolution term from radiation damping makes it difficult to directly represent (3-1) by an equivalent circuit. Using frequency domain analysis, (3-2) is written as:

\[
-\omega^2(m_1 + A_{11}(\omega))X(\omega) + j\omega b_{11}(\omega)X(\omega) + k_s X(\omega) = F_e(\omega) + F_{pto}(\omega) + F_v(\omega)
\]  

(3-2)

where $\omega$ is the wave frequency, and $A(\omega)$ and $b_{11}(\omega)$ is the frequency-dependent added mass and radiation resistance, respectively. Combining added mass and radiation resistance into a single radiation force, (3-3) is rewritten as:

\[
-\omega^2(m_1 + A_{11\infty})X(\omega) + k_s X(\omega) = F_e(\omega) + F_{pto}(\omega) + F_r'(\omega)
\]  

(3-3)

where $F_r'(\omega)$ the radiation force can be calculated from the impulse response function $K_r(\omega)$:
\[ F_r'(\omega) = -j \omega K_r(\omega) X(\omega) \]  
(3-4)

\[ K_r(\omega) = j \omega (A_{11}(\omega) - A_{11\infty}) + b_{11}(\omega) \]  
(3-5)

### 3.1.2 Derivation of WEC Parameters

To derive hydrodynamic parameters from the wave interaction on the specific buoy shape, a computer aid BEM solver WAMIT [63] is used as a tool for deriving the parameters. Fig. 3-3 and Table 3.1-1 are applied as setup in WAMIT based on the dimension of a real buoy.

**Table 3.1-1. WEC Dimensions**

<table>
<thead>
<tr>
<th>Dimension of Buoy</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta x )</td>
<td>101.6 mm</td>
</tr>
<tr>
<td>( \Delta h )</td>
<td>111.76 mm</td>
</tr>
<tr>
<td>( \theta )</td>
<td>13.04 degree</td>
</tr>
<tr>
<td>( r ) (radius)</td>
<td>584.2 mm</td>
</tr>
<tr>
<td>( h ) (height)</td>
<td>203.2 mm</td>
</tr>
</tbody>
</table>

**PTO column dimensions**

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>5080 mm</td>
</tr>
<tr>
<td>radius</td>
<td>101.6 mm</td>
</tr>
</tbody>
</table>

The frequency-dependent parameters are derived as shown in Fig. 3-4. They are added mass \( A_{11}(\omega) \), radiation damping \( b_{11}(\omega) \), and excitation force per wave height \( F(\omega) \) derived from WAMIT based on the buoy dimensions in Table 3-1.

**Fig. 3-4. Frequency-dependent parameters derived from WAMIT.**

The excitation force \( F_e \) depends on the wave conditions and wave-buoy hydrodynamic parameters. The derivation process is shown from (3-6) to (3-9) based on [64].
\[ S(\omega) = 263H_s^2T_e^{-4}\omega^{-5}\exp(-1054T_e^{-4}\omega^{-4}) \]  
(3-6)

\[ A(\omega) = \sqrt{2S(\omega) \Delta \omega} \]  
(3-7)

\[ f_e(t) = \sum_{i=0}^{N} A(\omega_i)\text{Re}(F(\omega_i)e^{-j(\omega_it+\varphi(\omega_i)+\alpha_i)}) \]  
(3-8)

\[ F_e(\omega) = \frac{F(\omega)}{\rho g A(\omega)} \]  
(3-9)

where (3-6) \( S(\omega) \) is the wave spectrum derived from the significant wave height \( H_s \) and energy period \( T_e \) from the standard spectrum Pierson-Moskovitz [65]. \( A_{1/1}(\omega) \) is the excitation force amplitudes calculated from the wave spectrum; \( \Delta \omega \) is the discrete step of the spectrum; \( F(\omega) \) is the excitation force per wave height (N/m) under regular excitation at the wave frequency \( \omega_i \); \( \varphi(\omega_i) \) is the phase of the excitation force \( F(\omega_i) \) at the \( \omega_i \) under regular wave excitation. The irregular wave excitation force is derived from (3-8), and regular wave excitation force is derived from (3-9) where \( \alpha_i \) is a random phase between 0 to 2\( \pi \); \( \rho \) is the water density (kg/m\(^3\)); \( g \) is the gravitational acceleration constant.

### 3.1.3 Review of Mathematical Approximation Model

With the hydrodynamic WEC model, several methods have been developed to solve a multi-frequencies real sea wave in time-domain [66]–[69]. However, the time-domain model contains convolution terms that increase the complexity and computational effort during simulations.

To reduce the assessment time, there are some approximation methods in the time-domain applying MATLAB solvers, such as ODE45 or Backward-Euler, to model the time-dependent waveforms [70] which can be summarized as follows:

Time-domain Realization methods:

- Prony’s method: convolution into, then solve with ODE45
• Realization Runge-Kutta method: convolution into s.s. imp2ss, solve with ODE45
• Backward Euler: Euler fixed time step approximation
• Crank-Nicholson (trapezoidal formula)

Other approximation methods that combine frequency and time domains in their analysis could simplify the convolution term into a state-space form or a transfer function [71]. The approximation method for convolution can be categorized as follows:

• Fit radiation’s hydrodynamic coefficient only $A_{11}(\omega)$ and $b_{11}(\omega)$
• Fit overall dynamic from wave excitation force to buoy velocity (WEC impedance)
• Fit the impulse response only
• Time-domain estimation using retardation function

To accelerate the research process, the Department of Energy national laboratories have developed a MATLAB–based toolbox called WEC-SIM [72]. This open-source tool has a 6DoF multi-body solver, which can perform time-domain simulation based on the boundary element method (BEM) provided by WAMIT.

3.1.4 Review of WEC Circuit Model

An analogy of electrical circuits gives different wire-to-wave thinking from an electrical perspective. The force – current analogy was first proposed by [56] as shown in Fig. 3-5(a). Then later was adapted in the SDoF dynamic of wave energy converter as shown in Fig. 3-5(b), which is a force – voltage analogy based on the second-order differential equation (3-10) that was introduced in the book by Falnes [73] where $M(\omega_0) = m_1 + A_{11}(\omega_0)$, $B(\omega_0) = b_{11}(\omega_0)$, and $\omega_0$ is the monochromatic frequency of the wave force $f_e$.

$$ f_e = M(\omega_0) \frac{d^2x}{dt^2} + B(\omega_0) \frac{dx}{dt} + k_{21}x $$ (3-10)
Later, the circuit model in WEC was adapted by Shek and Meuller for reaction force control [74] as shown in Fig. 3-6. To reach the optimal power on the PTO, the circuit needs to work under the resonant condition, i.e., the $X_{pto}$ and $R_{pto}$ components from PTO need to cancel the reactive impedance from the WEC device base from its RLC components. The second-order circuit model is applicable only under the regular wave (monochromatic or single-frequency) wave condition.

Based on the equivalent circuit, Lewis in [75] improves the conventional single body WEC circuit model to a two-body WEC circuit mode as shown in Fig. 3-7(a). The circuit model from [14], [76] demonstrates a modularized method to analogize each operation mechanism in its PTO system to an electrical circuit network. These circuits combine with switches as a complete electrical system to represent the WEC system under different mechanical modes, which can be shown in Fig. 3-7(b).
Others WEC studies use circuit model for the power-electronics based controller design can be listed in [9], [24], [69], [75], [77], [78]. Especially in [75], the circuit model is used to improve the absorption power by replacing the conventional impedance-matching network with a circuit network of wideband-matching impedance.

Although the circuit model in WEC shows as a powerful tool for control and impedance analysis, its usage is only limited to the simulation under monochromatic (regular) waves. In other words, the excitation force $f_e$ can only be a single frequency sinusoid wave, and cannot be an irregular excitation force that contains multiple frequencies and random angle as $f_e(t)$ in (3-8).

Therefore, an irregular wave circuit model is necessary for developing the control algorithm and adopting the PTO circuit model from Chapter 2. The following section will introduce the methodology and process for a WEC circuit model.
3.2 Approximation Method and Equivalent Circuit of WEC

The methodology is developed for an SDoF circuit-based W2W model of wave structure and PTO dynamics under both regular and irregular wave conditions. By applying frequency response approximation and circuit network synthesis, an equivalent circuit network is developed with constant parameters to simulate the hydrodynamics of wave-structure interactions, subject to a frequency-dependent radiation force. The equivalent circuit model from the PTO and an overall circuit-based W2W system model for both regular and irregular waves in the time domain are created by using a mechanical-electrical analogy. The methodology in Fig. 3-8 is applied in this section to develop a circuit-based W2W simulation model. There are three modelling paths: 1) from wave condition and WEC geometry to a circuit analogy; 2) from PTO mechanical mechanism to an equivalent circuit model; and 3) from the design of the generator and power converter topology to the circuit model of the generator and pulse width modulation (PWM) controlled on the power converter. Modelling paths 1) have been introduced in 3.1.2, 2) and 3) will be discussed in the following subsections, which derive the equivalent circuit from the mechanical coupling and generator in a PTO.

![Fig. 3-8. Development methodology of a wave-to-wire circuit model for both irregular and regular wave simulation.](image)
3.2.1 Development of WEC Circuit Model

The frequency-dependent radiation force $F_r'(\omega)$ is approximated as a first step to develop an equivalent circuit model for WEC. The frequency response of the radiation force is approximated into a transfer function.

The frequency response function (FRF) of $K_r$ according is plotted from (3-5) as shown in Fig. 3-9. The corresponding transfer function from the figure through an nth-order approximation as $K_{r\text{CF}}$ is then developed in (3-11), which is performed by the Matlab toolbox function `invfreqs`. Other approximations of the radiation force are introduced in [71]. The second- and third-order approximations in Fig. 3-9 show similar frequency responses compared to the original $K_r$. The second-order approximation is chosen to perform a circuit analogy. The equivalent circuit can be derived from the Brune network synthesis [54]. The poles and zeros are separated and represented with the RLC component in a series or parallel connection network. Fig. 3-10 illustrates the Brune network synthesis process.

\[
K_{r\text{CF}}(s) = \frac{P(s)}{Q(s)} = \frac{s^m + p_{m-1}s^{m-1} + \cdots + p_1s + p_0}{s^n + q_{n-1}s^{n-1} + \cdots + q_1s + q_0}
\] (3-11)

\[
\frac{j\omega X(\omega)}{-F_r'(\omega)} = \frac{1}{K_{r\text{CF}2nd}(s)} = Z_r(s) = \frac{V_{pto}}{i_{fr}} = \frac{s^2 + as + b}{cs} = \frac{1}{c} s + \frac{a}{c} + \frac{b}{c} s
\] (3-12)

![Radiation Force Kr approximation](image)

Fig. 3-9. Comparison of the frequency response of $K_r$ from (3-5), second-order approximation from (3-11), and third-order approximation from (3-11).
Fig. 3-10. Brune process to synthesis an equivalent circuit from the frequency response in Fig. 3-9 to second-order approximated transfer function in (3-11) to parameters separation in (3-12) to RLC network synthesis network synthesis.

A similar approach is also used for PTO circuit modeling in Chapter 2. As a result, the frequency-domain function in (3-2), and the time domain fluid-body dynamics in (3-1) can be converted into a circuit network for both irregular wave and regular wave simulations in Fig. 3-11

Fig. 3-11. The equivalent circuit model (forces represented by currents) of the fluid-body dynamics in irregular waves from the frequency-domain model (3-3) and second-order transfer function approximate circuit on the radiation force from (3-12).

where the wave force $f_e$ (N) is analogous to a current source $i_{fe}$ (A); $i_{pto}$ (A) is analogous to the force $f_{pto}$ (N) provided by PTO; the voltage $v_{pto}$ (V) is analogous to the linear velocity $\dot{x}$ (m/s) of the buoy (PTO); capacitor $C_e = m_i + A_e$ (F) in parallel with a current source is analogous to the mass and added mass (kg) of the buoy; stiffness (N/m) is shown as an inductor $L_e = 1/k_s$(H); $C_r$, $R_r$, $L_r$ are analogous to the radiation force’s terms where $C_r = cl_a$, $R_r = acl$, $L_r = 1/c$ from the second-
order equation (3-12).

The impedance of WEC can be derived between the excitation force \( f_e \) and buoy velocity \( \dot{x} \) in (3-1), and the relationship is equivalent to the input current \( i_{pto} \) and the voltage \( v_{pto} \) in Fig. 3-11. The WEC impedance can be written as (3-13), assuming no force from PTO (i.e., \( f_{pto} = i_{pto} = 0 \):

\[
Z_{wec} = \frac{v_{pto}}{i_{fe}} (\frac{\dot{x}}{f_e})
\]

\[
= \frac{s^3L_rC_rL_e + s^2R_rC_rL_e + sL_e}{s^4L_rC_rL_eC_e + s^3R_rC_rL_eC_e + s^2(C_rL_e + L_rC_r + L_eC_e) + sR_rC_r + 1}
\]

\[\text{(3-13)}\]

### 3.2.2 Validation of WEC Circuit Model

The approximated circuit model for the WEC is validated in three cases in this subsection. First, the wave-buoy impedance (i.e., the relationship between the excitation force to the buoy velocity) is compared to the results from the original model in (3-1) and (3-2) for both impulse response and frequency responses. Next, the electrical model prediction is verified with the results of the buoy velocity under irregular wave conditions in the time domain. Lastly, the average output powers of PTO are compared in different irregular wave conditions.

In Fig. 3-12 - Fig. 3-14, the wave-buoy characteristics are shown based on the setup parameters in Table 3.2-1, with \( b_e = 300 \text{ Ns/m} \), and \( m_e = 0 \text{ kg} \). The wave-buoy time/frequency responses are compared explicitly from both the original model in (3-2) and the impedance equation in (3-13) in Fig. 3-12. The impulse response in the time domain and the frequency response (including magnitude as well as phase from 0.1 – 10 rad/sec) show good agreement between the original model and the approximate model. The natural frequency is determined as 4.5 rad/s (0.716 Hz) from the frequency response of the WEC. Another item of note is the frequency response of (3-13) in Fig. 3-12. Instead of having four poles and three zeros, the frequency response acts like a two-pole/one zero system, which results from complex zeros and
poles being canceled in (3-13).

Fig. 3-12. Impulse response and frequency response of force-velocity from the WEC model (without PTO). Comparison among frequency-domain responses from (3-2), the approximated results from the conventional monochromatic circuit model Fig. 3-5, and the approximated results from irregular wave circuit model Fig. 3-11 under the WEC setup from Table 3.2-1 (assume no damping or mass from PTO). Circuit model results can match well with the results from (3-2) around the nominal wave frequency (0.52 rad/s – 1.5 rad/s). (B.1 Setups of figures-(iii)-(iv))

Fig. 3-13. Buoy velocity $\dot{x}(t)$ from time-domain (3-1) (Euler method) and from equivalent circuit approximation of Fig. 3-11 (Backward Euler method from Simulink) under a wave energy period $T_e = 6.3$ s, and significant wave height $H_s = 2$ m. The setup parameters are in Table 3.2-1 with $b_e = 300$ Ns/m, and $m_e = 0$ kg. (B.1 Setups of figures-(iii)-(iv))
The WEC velocity $\dot{x}(t)$ is compared from the equivalent circuit approximation of Fig. 3-11 and the time domain equation (3-1) under a significant wave height $H_s = 2$ m and 6.3 seconds of wave energy period $T_e$. Fig. 3-13 shows the voltage (velocity) from the approximate circuit model fits well with the velocity waveform derived from the time domain convolution.

Under the same system setup, Fig. 3-14 compares the average extracted power from the time-domain (3-1), $P_{o1}(t) = f_{pto}(t) \dot{x}(t)$ where $f_{pto}(t) = \dot{x}(t) \cdot b_e$, and from the equivalent circuit model of Fig. 3-11, $P_{o2}(t) = i_{pto}(t) v_{pto}(t)$. Both models are tested under 20 minutes of irregular wave conditions, including the wave energy period from 4 to 12 seconds and the significant wave height from 0.5 to 2 meters. The simulated output power in Fig. 3-14(a) shows that the developed WEC circuit model can accurately predict the irregular wave power, while the conventional model has over 100% error based on the result from the time-domain model (3-1). The error of the conventional model can also be shown in its simulated PTO (buoy) velocity in Fig. 3-14(b). Thus, the developed WEC circuit model is proved to be robustness in the multi-frequency (irregular wave) simulation.

![Fig. 3-14. Comparison of irregular wave simulation results of output power and buoy velocity in the time domain: a) average output power of PTO under significant wave height from 1 – 2 m; b) buoy velocity $\dot{x}$ ($v_{pto}$) under $H_s = 2$ m and $T_e = 6.3$ s. (B.1 Setups of figures-(iii)-(iv))](image-url)
Table 3.2-1. Setup parameters for an example of the WEC system from [79].

<table>
<thead>
<tr>
<th>Parameter Name</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buoy radius</td>
<td>$R$</td>
<td>0.6</td>
<td>meter</td>
</tr>
<tr>
<td>Hydrostatic stiffness</td>
<td>$k_s$</td>
<td>$1.05 \times 10^4$</td>
<td>N/m</td>
</tr>
<tr>
<td>Buoy weight</td>
<td>$m_l$</td>
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<td>kg</td>
</tr>
<tr>
<td>Infinite added mass</td>
<td>$A_\infty$</td>
<td>312</td>
<td>kg</td>
</tr>
<tr>
<td>Equivalent cap</td>
<td>$C_e$</td>
<td>578</td>
<td>F (kg)</td>
</tr>
<tr>
<td>Equivalent inductor</td>
<td>$L_e$</td>
<td>95.4</td>
<td>uH (m/N)</td>
</tr>
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<td>The equivalent resistor of radiation force</td>
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<td>mΩ (m/Ns)</td>
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<td>The equivalent inductor of radiation force</td>
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<td>510</td>
<td>uH (m/N)</td>
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<tr>
<td>The equivalent capacitor of radiation force</td>
<td>$C_r$</td>
<td>166.6</td>
<td>F (kg)</td>
</tr>
<tr>
<td>Rack-Pinion/ ball-screw ratio</td>
<td>$k_b$</td>
<td>104</td>
<td>Radius/m</td>
</tr>
<tr>
<td>Gear-box ratio</td>
<td>$k_g$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Torque Constant</td>
<td>$k_e$</td>
<td>0.104</td>
<td>Nm/A</td>
</tr>
<tr>
<td>Back EMF Constant</td>
<td>$k_t$</td>
<td>0.104</td>
<td>V·s/rad</td>
</tr>
<tr>
<td>Generator Inertia</td>
<td>$J_g$</td>
<td>0.0013</td>
<td>F (kgm$^2$)</td>
</tr>
<tr>
<td>Friction coefficient</td>
<td>$b_w$</td>
<td>$10^6$</td>
<td>Ω(rad/Nms)</td>
</tr>
<tr>
<td>Internal resistance</td>
<td>$R_i$</td>
<td>1.2</td>
<td>Ω</td>
</tr>
<tr>
<td>Internal inductance</td>
<td>$L_i$</td>
<td>2.3</td>
<td>mH</td>
</tr>
<tr>
<td>Poles of PMSG</td>
<td>$p$</td>
<td>12</td>
<td>Pole</td>
</tr>
</tbody>
</table>
3.3  **Ideal W2W model: Bias Map and Application**

The W2W can be developed based on the WEC circuit model. In this section, the PTO model is assumed to be ideal for observing the system dynamics from ocean waves to output power from the power converter. The simulation is built on Simulink with the ideal PTO circuit in Fig. 3-15. The setup parameters are from an example WEC system in [79], which is an MMR-based point absorber with the buoy size of 1.2 m diameter, single body, with the assumption that all the buoy motion can be transfer as the linear displacement of PTO.

3.3.1  **Bias Map with Ideal W2W Model**

The bias map in Fig. 3-15 shows the WEC system under constant impedance control on the power converter where the input impedance look from the generator back EMF voltage is 2.5 Ω, and the internal inductance of the generator is canceled due to the field-oriented control [80]. In the control algorithm, $I_q$ (from converter current $i_{abc}$ in d-q frame) is following the $E_q$ (from air-gap voltage $E_{abc}$ in d-q frame) through a ratio $(R_i+R_e)$, $I_{qref} = (R_i + R_e)E_q$; $I_d$ (from converter current $i_{abc}$ in d-q frame) is controlled to be zero to keep the rotor flux zero, $I_{dref} = 0$. The electric performances will be elaborated in the following section with a power converter model.

The generator is PMSG GL-PMG-500A. The wave excitation force is produced from a significant wave height $H_s = 1$m, and wave energy period $T_e = 6.3$ s, under the PM wave spectrum. The ac-dc converter switching at 5 kHz is connected to a battery load with its voltage $E_{batt} = 400$ V and internal resistance $R_i = 1$ Ω.
Fig. 3-15. Simulated bias map with the ideal wave-to-wire model. (B.1 Setups of figures-(v))

The bias map provides a clear idea and system-level perspective for the transient behavior on each part of the system from input force - PTO force/speed - generator speed – the output power of the power converter. The figure also illustrates how the output power $P_o$ varies with the input current source $i_{fe}$ as an irregular wave force. The mean power output on the battery is 303 W under the wave condition 1 meter and 6.3 sec period, where the peak power can reach up to 6287 W. The peak-to-average power ratio in the example is over 20 which introduces a challenge for design of the power converter and other components in the WEC system.

### 3.3.2 Application of Ideal W2W Model

The ideal W2W model can be used for robustness and reliability analysis from real-time stress measurement on each component, such as peak speed of the generator, the maximum junction temperature of body-diodes, and insulated Gate Bipolar Transistors (IGBTs) in the power module of the converter. Fig. 3-16 shows peak rotor speeds of the generator $\omega_g$ under different wave
periods. The peak air-gap voltage is derived from (2-13) as \( \vec{E}_{abc} = (p/2) \cdot k_e \omega_g \cdot [\sin \theta \sin(\theta - 2\pi/3) \sin(\theta + 2\pi/3)]^T \). Under irregular wave period around 1 – 2 seconds, the peak line-line air-gap voltage will exceed the 400 V of dc bus voltage \( E_{batt} \). For those wave conditions, it is necessary to include additional protection, such as field weakening [81] or power saturation algorithm [9], which will be further discussed in the following chapters.

![Figure 3-16. Maximum Generator speed and air-gap voltage from generator under different wave periods. (B.1 Setups of figures-(v))](image)

Semiconductor junction temperature can also be directly estimated from the W2W model in Fig. 3-17 under irregular wave conditions. The loss and thermal model will be applied in the following chapter for the reliability assessment of the IGBT module.

![Figure 3-17. The simulated junction temperature of power converter components with the W2W model Fig. 3-15 under irregular wave \( H_c = 1 \) m and \( T_c = 6.3 \) s. (B.1 Setups of figures-(v))](image)
3.4 Conclusion

Compared to a conventional WEC circuit model, the circuit model in this work provides accurate dynamic system predictions under both irregular and regular wave conditions, which are validated in both time and frequency domains. The circuit model also gives a unified circuit-base perspective to analyze and control the system from the power converter to the floating body. As a result, the circuit-based WEC model, just like the circuit model for other energy harvesting sources, can be an analytical tool for the multi-physics system besides transfer function or state-space approximation.


Chapter 4  Design Guideline for Power Converter in Wave Energy Application

4.1 Introduction of Power Converter in WEC system

The design optimization of the power converter in the WEC application will be introduced in this chapter. The design of the power converter in a variable-speed wind turbine is similar to the design in WEC under irregular wave conditions. The capacity factor of the turbine power rating in wind ranges from 0.2 – 0.5 [82], but the capacity factor in WEC application is much smaller as around 0.1 – 0.05 for the nature of the ocean wave [45], [83]. Comparing to wind applications with wind speed as input condition, power converters in wave applications need to consider both wave height and period, and the control technique such as power-leveling is used to protect the power train from overloading during a transient. A customized optimization methodology is necessary for the wave application. To reduce the LCoE in a WEC design, the goal in this chapter is to develop a method to minimize the over-rating (or to maximize the capacity factor) ratio of the power converter design.

4.1.1 Power Electronics Converters in WEC System

The power conversion system architecture of this work is based on a standalone WEC system as shown in Fig. 4-1. The design guideline herein will cover the ac-dc converter components selection based on the W2W model simulation, losses consideration, power rating selection of both the power converter and the generator, and the heatsink selection.
4.1.2 Review of Power Converter Design in WEC System

PTO is assumed to be an ideal actuator in the controller development for the WEC system. A power converter in PTO is taken as an ideal lossless driver, and no practical output power limitation is considered. Only a few studies focused on power converters in the WEC system. This review should cover most of the power converter researches in WEC.

Fig. 4-1 and Table 4.1-1 list most of the power converter architecture and topologies in the WEC application. On the motor side converter (MSC), a three-phase passive diode bridge and three-phase two-level active front end are still used in most of the WEC systems. Unlike wind energy applications that consider the doubly-fed induction generator (DFIG) as a generator, permanent magnet synchronous motors are widely used in the wave energy application. One of the potential reasons could be that the output power from the WEC could significantly reduce with DFIG due to the high fluctuation power (or speed) from the wave.
Fig. 4-2. Topologies and architectures of power converters in the wave energy conversion system.

The architecture and topologies are listed in Fig. 4-2 and Table 4.1-1, where LM is a linear motor, DCM is DC brushless motor, and IM is induction motor. Their control algorithms for the motor and WEC were also listed where IFOC is indirect field-oriented control, and MPPT is maximum power point tracking. Various types of WEC, generator, and power converter are shown in the table. In our study, the power converter design guideline will use point absorber type of WEC, PMSG of rotary electro-machine, two-level three-phase ac-dc power converter as an example of optimization. The design methodology can be applied to different types of WECs and generators by modification of the WEC and generator models, respectively.
Table 4.1. Review of WEC power converters development in research institutes.

<table>
<thead>
<tr>
<th>Institute, WEC type</th>
<th>Power Level</th>
<th>Architecture</th>
<th>Topology at MSC</th>
<th>Motor</th>
<th>MSC Control</th>
<th>WEC Control Test</th>
<th>Evaluation</th>
</tr>
</thead>
<tbody>
<tr>
<td>NCSU, OWS [84]</td>
<td>3 kW</td>
<td>Passive + DCDC + GSC + APF</td>
<td>Passive + DCDC + APF</td>
<td>IM/DFIG</td>
<td>IFOC</td>
<td>N/A</td>
<td>Emulator</td>
</tr>
<tr>
<td>OSU, PO [85]</td>
<td>30 kW</td>
<td>Passive + DCDC</td>
<td>Passive/Active</td>
<td>LM</td>
<td>Average model current control</td>
<td>MPPT/Reactive/Passive</td>
<td>Ocean/Tank</td>
</tr>
<tr>
<td>Cork, OWC [86]</td>
<td>3 kW</td>
<td>DCDC</td>
<td>Active single phase</td>
<td>DCM</td>
<td>NA</td>
<td>Constant speed</td>
<td>Ocean</td>
</tr>
<tr>
<td>Uppsala, PO [87], [88]</td>
<td>20 kW</td>
<td>Passive + DC+ GSC</td>
<td>Passive 3 phase/resonant</td>
<td>LM</td>
<td>NA</td>
<td>Passive</td>
<td>Ocean/Tank</td>
</tr>
<tr>
<td>Edinburgh, PO [89]</td>
<td>21 kW</td>
<td>Active + GSC</td>
<td>2 Level – 3 Phase</td>
<td>LM</td>
<td>In phase EMF/current</td>
<td>Reactive/Passive</td>
<td>Emulator</td>
</tr>
<tr>
<td>NTNU, PO [81]</td>
<td>120 kW</td>
<td>Active + DCDC + R</td>
<td>2 Level – 3 Phase</td>
<td>PMSG</td>
<td>FOC + Field weakening</td>
<td>Reactive + Passive</td>
<td>Ocean</td>
</tr>
<tr>
<td>ENS, Cachan [90], [91]</td>
<td>300 kW</td>
<td>ACDC</td>
<td>2 Level – 3 Phase</td>
<td>PMSG</td>
<td>FOC + Field weakening + power leveling</td>
<td>Passive</td>
<td>Simulation</td>
</tr>
<tr>
<td>VT, PO [92]</td>
<td>1kW</td>
<td>Active</td>
<td>2 Level – 3 Phase</td>
<td>PMSG</td>
<td>FOC + sensorless</td>
<td>Passive</td>
<td>Bench/Tank</td>
</tr>
</tbody>
</table>

4.1.3 Review of Power Converter and Generator Design in WEC System

The generator optimization in the oscillating water column (OWC) was studied in [17]. Multiple types of generators were compared based on their cost and performance based on simplified loss models of generators, turbines, and power electronics with the speed controller. In WEC application, the capacity factor optimization for Fred Olsens’s BOLT2® was conducted under passive loading and the constraint control of the generator control in [45]. The annual output power (energy) to the overspeed ratio (or reciprocal of the derating ratio) was listed considering
annual production and load hours. As a result, the overspeed ratio of ten is recommended (i.e., ten times smaller than the maximum power). However, no loss model or maximum passive loading was implemented, and only a generator rating was considered in the study.

Advanced optimization methods of WEC and power trains were introduced in [93] and [44] for a SEAREV WEC. Realistic motor control, including field-weakening and power leveling, was included in the design process, and mechanical design optimization was implemented in [93]. The machine-converter system was optimized by a detailed non-ideal model and motor control algorithm. The geometry, loss, and cost models of the generator and power converter are used in the paper. The damping coefficient and the power leveling $P_{lev}$ were applied as the control variables for the optimization, and the output components are the average produced energy. This paper showed that with field-weakening control, the rating of the power converter $S_{\text{conv}}$ could be reduced to the value of $1 \cdot P_{lev}$. The study only considered one nominal sea state for the optimization process, and the design was according to the dc-bus value of 3300 V, which is not very economical for WEC application. Also, the design of SEAREV is a direct-drive PTO, which is different from the conventional PTO with the mechanical transmission stage, and no gear ratio nor EMF coefficient of the generator is considered in the process.

The optimization process of the power converter and the generator in this chapter will consider the annual sea-state from all possible wave conditions. The control variables include the gear ratio of the PTO, the power leveling factor, and the damping coefficient of the PTO. The ratio of the average output power to the rated power of the generator $U_{sp}$ and the rated power of the power converter $U_{sp}$ will be the figure of merits in this work. The minimum rated power of the power converter and the generator is selected to produce the maximum annual output power.
4.2 Generator and Power Converter Modeling for Optimization in WEC system

The model of WEC system from WEC+PTO, generator, to the power electronics converter model, will be introduced in this session as listed in Fig. 4-1 where the output of the power converter is assumed to be a constant voltage source (i.e., a battery source). The losses along the power train will be included to reflect the actual output power with the change of the setup. The passive loading, field-weakening, and power leveling control algorithms will be implemented in the simulation.

4.2.1 WEC System Model and System Control Algorithm

The WEC equivalent circuit from Fig. 3-11 is adopted in Fig. 4-3 where $i_{fpto}$ is added as a controlled PTO force, which is derived from the ideal PTO and power electronics converter controlled by the power leveling, field-weakening, and passive loading algorithms.

\[
v_{pto} = (i_{fe,w} - i_{fpto})Z_{wec}(\omega)
\]  

(4-1)

where $Z_{wec}$ is from (3-13), $i_{fe,w}$ is an equivalent current of the irregular incident wave force that drives the buoy in heave motion only, which are described in (3-6) to (3-8) using Pierson-Moskovitz spectrum. The index $w$ is the irregular wave conditions and has the wave height $H_{s,w}$. 

![Fig. 4-3. WEC circuit model from Fig. 3-11 and control diagram for PTO and PE detailed in Fig. 4-5 for power converter optimization.](image-url)
and the average wave period $T_{e,w}$. The equivalent current from incident wave force $i_{f,e,w}$ is generated in the time domain for an irregular wave period $T_{irr}$ accordingly.

The output of $i_{fpto}$ is controlled by passive loading and power leveling to keep the generator and converter from overloading.

$$P_{om} = i_{fpto} \cdot v_{pto}$$

$$i_{fpto} = \begin{cases} 
  \frac{v_{pto}}{R_{pto}}, & \text{if } P_{om} < P_{lev} \\
  P_{lev}, & \text{if } P_{om} \geq P_{lev} 
\end{cases}$$

(4-2)

where $P_{om}$ is the output power of the PTO, $P_{lev}$ is the power leveling variable, and $R_{pto}$ is the control variable for the passive loading, which is usually tuned for a maximum power point in each wave condition.

$$T_{eg} = \frac{i_{fpto}}{k_b}$$

$$\omega_e = k_b \cdot v_{pto} \cdot p = \omega_g \cdot \frac{p}{2}$$

$$i_q = -4T_{eg}/(3k_t \cdot p)$$

(4-3)

The ideal PTO model is listed in (4-3), which is simplified from the non-ideal PTO model in (2-5) - (2-16). The PMSG model in the d-q frame is shown in Fig. 4-4.
where the air-gap voltage in d-q frame $E_d$ is assumed to be zero and $E_q = -p k_e \omega e/2 = -k_e \omega e$; $b_w$ and $C_{jr}$ generator windage and inertia are neglected in the analysis. The input torque from PTO $T_m$ is assumed to be identical to $T_{eg}$ the torque from the generator. The input voltages of the inverter are $v_d$ and $v_q$ are written as (4-4) based on the equivalent circuit model.

$$
\begin{align*}
v_d &= \omega e L_i i_q - R_i i_d \\
v_q &= -k_e \omega e - \omega e L_i i_d - R_i i_q \\
v_R &= \sqrt{v_d^2 + v_q^2}
\end{align*}
$$

where $v_R$ is the peak value of the input voltage, which will be used for the converter rating estimation. The field weakening control the d-frame current $i_d$ is decided by whether the line-line voltage magnitude $\sqrt{3}v_R$ is higher than the dc-bus value $V_{dc}$ as shown in (4-5); if not, then $i_d$ will remain zero with $i_q$ only to provide the field-oriented control.

As a result, the current magnitude into the inverter $i_R$ is shown in (4-6), and the input apparent power of the three-phase converter $S_R$ is the result of $i_R$ and $v_R$ in (4-7).

$$
i_d = \begin{cases} 
- \frac{k_e}{L_i} + \sqrt{\left( \frac{V_{dc}}{\sqrt{3}\omega e L_i} \right)^2 - i_q^2}, & \text{if } \sqrt{3}v_R \geq V_{dc} \\
0, & \text{if } \sqrt{3}v_R < V_{dc}
\end{cases}
$$

(4-5)

$$
i_R = \sqrt{i_d^2 + i_q^2}
$$

(4-6)

$$
S_R = 3i_R v_R / 2
$$

(4-7)

The output components $P_{om}$, $S_R$, $I_R$ from the ideal model will be used in the generator and the power converter loss analysis in the following session.

4.2.2 Loss Model of Power Electronics Converter

Two main losses are considered during the optimization process, one is the winding loss of
the generator, and another is the power converter losses. The actual output power $P_{oe}$ is the result of subtracting the losses $P_{loss}$ from the PTO output power $P_{om}$. The loss models will be dependent on the rated current of the generator $I_{grated}$ and the rated current $I_{prated}$ of the power converter as shown in (4-13) and (4-14) based on the maximum wave condition and optimal passive loading.

The generator loss model $P_{gloss}$ is shown as (4-8) by assuming copper loss only,

$$P_{gloss} = 3 \cdot \frac{i_R^2}{2} \cdot (r_i/I_{grated})$$

where the internal resistance $R_i = r_i/I_{grated}$ depends on the copper loss coefficient $r_i$ and the current rating of the generator $I_{grated}$ which is decided by the RMS value of $i_R$.

The power converter losses are broken down into switching loss $P_{sw}$ and conduction loss $P_{cond}$.

$$P_{sw} = E_{sw} \cdot f_{sw} \cdot i_R/\sqrt{2}$$

$$P_{cond} = V_T \cdot i_R/\sqrt{2} + r_{on}/I_{prated} \cdot i_R^2/2$$

$$P_{loss} = P_{cond} + P_{sw}$$

where the switching loss is assumed to be independent of the power rating [44], but is the result of the switching losses from IGBTs and the reverse recovery from diodes $E_{sw}$, the switching frequency $f_{sw}$, and the RMS current $i_R/\sqrt{2}$. Two parts in the conduction loss voltage drop from the threshold voltage, and the drop on the dynamic resistance. The threshold voltage $V_T$ is assumed to be constant, and the dynamic resistance $r_{on}/I_{prated}$ is related to $I_{prated}$, which is based on the maximum value of $S_R$. The parameters $E_{sw}$, $V_T$, and $r_{on}$ is derived from curve fitting through a 200 kW IGBT power module [94].

The overall loss on the power train $P_{loss}$ and the actual output $P_{oe}$ are defined as (4-10).

$$P_{loss} = P_{gloss} + P_{loss}$$

$$P_{oe} = P_{om} - P_{loss}$$

86
The models from (4-1) - (4-10) are concluded in Fig. 4-5. The input variables marked in blue are wave conditions \( w (H_s,w \text{ and } T_{e,w}) \), controlled passive loading from PTO \( R_{pto} \), power leveling setup \( P_{lev} \), and gear-ratio or the motor gear-ratio \( k_b \) this can be altered by the ball-screw lead distance \( r \), pole numbers \( p \), or the airgap coefficient \( k_e \). The output values, which are marked in red, are the mechanical output power \( P_{om} \), apparent power \( S_R \), power train loss \( P_{loss} \), and the actual output power \( P_{oe} \).

![Diagram of WEC Control algorithms and models for Power electronics converter optimization from (4-1) to (4-10).](image)

**Fig. 4-5.** WEC Control algorithms and models for Power electronics converter optimization from (4-1) to (4-10).

### 4.2.3 Time-Domain Simulation

A 1:4.5 scaled WEC system with the 5-m diameter with heave-only motion is considered in this chapter. The parameters in Table 4.2-1 are applied as an example with the model introduced in the previous 4.2.1 and 4.2.2 optimization process. The BEM results in Fig. 4-6 from WAMIT are converted into equivalent circuit model parameters of Table 4.2-1 \( C_r, R_r, L_r, L_e, C_e, \) and \( i_{Fe} \) as shown in Chapter 3. From 3.2.1, the WEC impedance \( Z_{wec}(j\omega) \) is derived. The optimal passive damping of PTO \( R_{ptoeb} = |Z_{wec}(j\omega)| \) is found, which will provide the maximum power under the ideal WEC system.
Fig. 4-6. The parameters of the 5-m WEC system from BEM solver (WAMIT) include (a) the added mass $A_{11}$, (b) the radiation damping $b_{11}$, (c) the excitation force $F_e$, and (d) the phase of the wave force on the buoy.

Fig. 4-7. The 5-m WEC impedance $Z_{\text{wec}}(\omega) = v_{\text{wec}}l_{fe}$ from (3-2).

Table 4.2-1. Setup parameters of the 1:4.5 scale, 5-m WEC system.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_1$</td>
<td>Mass of Buoy</td>
<td>4156 kg</td>
</tr>
<tr>
<td>$C_e$</td>
<td>Equivalent capacitor for buoy mass and added mass</td>
<td>48.3 kF</td>
</tr>
<tr>
<td>$L_e$</td>
<td>Equivalent inductance for hydrodynamic damping</td>
<td>5.05 $\mu$H</td>
</tr>
<tr>
<td>$C_r$</td>
<td>Equivalent capacitor for radiation force</td>
<td>6.923 kF</td>
</tr>
<tr>
<td>$L_r$</td>
<td>Equivalent inductance for radiation force</td>
<td>111.46 $\mu$H</td>
</tr>
</tbody>
</table>
With the parameters and model of the WEC system, the time-domain simulation with the input condition in Table 4.2-2 is conducted as shown in Fig. 4-8 where the \( T_{irr} = 1800 \) seconds simulation time with a resolution of 1 ms will take around 150 seconds in a Matlab-based platform. With the wave condition \( w = 28 \) (\( H_s = 3.75 \) m and \( T_e = 7.5 \) s), 800-V dc-bus, the gear ratio of 104, the power leveling of 50 kW, and the equivalent resistance of 3.56 \( \mu \Omega \). The apparent power \( S_R \), output power of the power converter \( P_{oe} \), the input currents to the converter in d-q frame \( I_{dq} \), and the power loss \( P_{loss} \) are then processed in (4-11) for their maximum and mean value after each simulation to reduce the temporary storage during the process.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_r )</td>
<td>Equivalent resistance for radiation force</td>
<td>0.115 m( \Omega )</td>
</tr>
<tr>
<td>( L_i )</td>
<td>The internal inductor of generator</td>
<td>2.2 mH</td>
</tr>
<tr>
<td>( p )</td>
<td>Poles number</td>
<td>12</td>
</tr>
<tr>
<td>( k_t ) or ( k_e )</td>
<td>The air-gap voltage or torque coefficient</td>
<td>4.938</td>
</tr>
<tr>
<td>( r_i )</td>
<td>Copper loss coefficient for generator</td>
<td>6.58 ( \Omega \cdot A )</td>
</tr>
<tr>
<td>( V_T )</td>
<td>Threshold voltage</td>
<td>3.45 V</td>
</tr>
<tr>
<td>( r_{on} )</td>
<td>Dynamic resistor coefficient</td>
<td>26.6 ( \mu \Omega \cdot A )</td>
</tr>
<tr>
<td>( E_{sw} )</td>
<td>Sum of switching losses and recovery energy</td>
<td>0.534 mJ/s/A</td>
</tr>
<tr>
<td>( f_{sw} )</td>
<td>Switching frequency</td>
<td>5 kHz</td>
</tr>
<tr>
<td>( V_{dc} )</td>
<td>Dc-bus voltage</td>
<td>800 V</td>
</tr>
<tr>
<td>( V_{rated} )</td>
<td>Rated, RMS, line-line voltage of the generator</td>
<td>380 V</td>
</tr>
</tbody>
</table>

With the parameters and model of the WEC system, the time-domain simulation with the input condition in Table 4.2-2 is conducted as shown in Fig. 4-8 where the \( T_{irr} = 1800 \) seconds simulation time with a resolution of 1 ms will take around 150 seconds in a Matlab-based platform. With the wave condition \( w = 28 \) (\( H_s = 3.75 \) m and \( T_e = 7.5 \) s), 800-V dc-bus, the gear ratio of 104, the power leveling of 50 kW, and the equivalent resistance of 3.56 \( \mu \Omega \). The apparent power \( S_R \), output power of the power converter \( P_{oe} \), the input currents to the converter in d-q frame \( I_{dq} \), and the power loss \( P_{loss} \) are then processed in (4-11) for their maximum and mean value after each simulation to reduce the temporary storage during the process.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w )</td>
<td>Wave condition</td>
<td>28</td>
</tr>
<tr>
<td>( H_{s,w} )</td>
<td>Significant wave height</td>
<td>3.75 m</td>
</tr>
<tr>
<td>( T_{e,w} )</td>
<td>Average wave period</td>
<td>7.5 s</td>
</tr>
<tr>
<td>( R_{pto} )</td>
<td>The equivalent resistance of PTO damping</td>
<td>3.56 ( \mu \Omega )</td>
</tr>
<tr>
<td>( P_{lev} )</td>
<td>Power leveling of the power converter</td>
<td>50 kW</td>
</tr>
<tr>
<td>( k_b )</td>
<td>PTO gear ratio</td>
<td>104</td>
</tr>
<tr>
<td>( I_{grated} )</td>
<td>Rated RMS current of the generator</td>
<td>113.8 A</td>
</tr>
<tr>
<td>( I_{prated} )</td>
<td>Rated peak current of the power converter</td>
<td>327 A</td>
</tr>
<tr>
<td>( T_{irr} )</td>
<td>The irregular wave simulation period</td>
<td>1800 s</td>
</tr>
</tbody>
</table>
Fig. 4-8. The time-domain results of $S_R$, $P_{oe}$, $I_{dq}$, $P_{loss}$ from the diagram in Fig. 4-5, the parameters from Table 4.2-1, and the variables in Table 4.2-2. (a) the irregular wave simulation results for $T_{irr} = 1800$ s and (b) zoom-in results when Time = 200 – 250 s. (B.1 Setups of figures-(vi))

\[ SR = \max_{t=1:T_{irr}} (S_R(t)) \]

\[ P_{oe}(t) = P_{om}(t) - P_{loss}(t) \]

\[ P_{oe} = \left( \int_0^{T_{irr}} P_{oe}(t) \, dt \right) / T_{irr} \]

\[ IR = \left( \int_0^{T_{irr}} \sqrt{I_d^2(t) + I_q^2(t)} \, dt \right) / T_{irr} \]

\[ P_{loss} = \frac{\int_0^{T_{irr}} P_{loss}(t) \, dt}{T_{irr}} = P_{closs} + P_{gloss} \]

where $SR$ is the maximum values of the apparent power throughout the simulation period $T_{irr}$ as highlighted in Fig. 4-8(a), which will be the rated power for the power converter $S_{rated}$ as in (4-13).

The average output power $P_{oe}$ is the average value of $P_{oe}(t)$ over the period $T_{irr}$. The maximum $P_{oe}$ tested with different damping $R_{PTO}$ under the same wave condition (4-12) will be used in the derivation of the annual output power $P_{oej,k}$ in the following session. The power loss $P_{loss}$ is the average value over $T_{irr}$. 

90
4.3 Power Converter Optimization Algorithm

4.3.1 Control Variables, Constraints, and Optimization Outputs

Numerical analysis is tested with different control variables to find the optimal design for the power converter. Three main parameters $P_{lev}$, $R_{pt0}$, and $k_b$ are swept under all wave conditions $w$. The power leveling $P_{lev}$ is swept from around 1/10 of the peak power from the WEC system, and $R_{pto}$ varies as a ratio of $R_{pt0b}$, which is the optimal passive loading derived from $|Z_{wec}(j\omega)|$ in Fig. 4-7. The $k_b$ is selected from the maximum available gear ratio (ball screw), and generator air-gap voltage ratio.

Three major output parameters are captured in each simulation: the average output power $P_{oe}$ which is the result after subtracting the losses $P_{loss}$ from the PTO output power $P_{om}$, the maximum apparent power of the power converter $S_R$, and the average input current to the power converter $I_R$. The examples of these output results are shown in Fig. 4-8 and the derivation of the time domain in (4-11).

The major output parameters are shown as matrices with four level of indices $i$, $j$, $k$, and $w$, $Poe_{i,j,k,w}$ as an example that $i$ is the index for $R_{PT0}$ showing the results of output power from the input variable $R_{PT0}$; $j$ is the index for $P_{lev}$; $k$ is the index for $k_b$, and $w$ for the wave condition $Hs_w$ and $Te_w$.

The maximum output power $Poe_{i,j,k,w}$ is searched among the $R_{pto}$ index $i$ from 1 to $m$, where $m$ is the numbers of the $R_{pto}$ matrix. The maximum $Poe_{j,k,w} = Poe_{ix,j,k,w}$ where $i_x$ is to indicate the maximum point and is used to indicate the other outputs. The average output power $Poe_{j,k}$ is the sum of the product of maximum output powers $Poe_{j,k,w}$ in each wave condition $w$ and the probability of each wave $p_w$ as shown in (4-12).

91
The rated power of the power converter $S_{rated,j,k}$ of each $P_{lev}$ and $k_b$ is decided by maximum apparent power $SR_{j,k,w}$ at the maximum irregular wave condition (i.e., $w = u$). The rated current $I_{prated,j,k}$ can be derived from the rated power and dc-bus voltage $V_{dc}$ as shown in (4-12).

\[
Poe_{ix,j,k,w} = \max_{i=1,m} Poe_{i,j,k,w}
\]
\[
Poe_{j,k,w} = Poe_{ix,j,k,w}
\]
\[
Poe_{j,k} = \sum_{w=1}^{u} p_w Poe_{j,k,w}
\]

The rated power of the generator $S_{rated,j,k}$ of each $P_{lev}$ and $k_b$ is decided by maximum apparent power $SR_{j,k,w}$ at the maximum irregular wave condition (i.e., $w = u$). The rated current $I_{prated,j,k}$ can be derived from the rated power and dc-bus voltage $V_{dc}$ as shown in (4-13).

\[
SR_{j,k,w} = SR_{ix,j,k,w}
\]
\[
S_{rated,j,k} = SR_{j,k,u}
\]
\[
I_{prated,j,k} = 2S_{rated,j,k}/\sqrt{3}V_{dc} \text{ (peak current)}
\]

The rated current of the generator $I_{grated,j,k}$ of each $P_{lev}$ and $k_b$ is decided by the average current $IR_{j,k,w}$ at the maximum irregular wave condition (i.e., $w = u$). The rated power of the generator $P_{grated,j,k}$ can be derived from the rated RMS current $I_{grated,j,k}$ and rated RMS voltage $V_{grated}$ as shown in (4-14).

\[
IR_{j,k,w} = IR_{ix,j,k,w}
\]
\[
I_{grated,j,k} = IR_{j,k,u}/\sqrt{2} \text{ (RMS current)}
\]
\[
P_{grated,j,k} = \sqrt{3}V_{grated} \cdot I_{grated,j,k}
\]

The definition of the input variables and the output results are used in the flowchart in the following section.

4.3.2 Optimization Flowchart for the Power Converter Design

The equations from input variables ($R_{pto}$, $P_{lev}$, $k_b$, $w$) to output components ($P_{oe}$, $P_{loss}$, $S_R$, $I_R$, $U_{sp}$, $U_{gp}$) from (4-1) - (4-14) are organized as the flowchart Fig. 4-16 where each variable is numerically tested in the model through simulation. Each set of the condition is simulated for an
irregular wave simulation period \(T_{irr}\) as shown in the example Fig. 4-8 in the time domain, and the
time domain results will be preserved as a single value by (4-11). The total time-domain iterations
numbers of \(m' n' o' u\) are processed throughout the flowchart to cover all the input variable vectors
where the PTO damping vector \(Rptoi\) with \(m\) components, the power leveling vector \(Plevj\) with \(n\)
components, and the gear ratio vector \(kb_k\) with \(k\) components are selected according to 4.3.1. The
wave conditions \(w\) are categorized in \(u\) components depending on the annual wave profile from
the selected wave site as shown in Table 4.3-2.

The constant parameters for the rest of the model are imported from Table 4.2-1. To identify
the maximum \(S_R\) and the maximum \(I_R\), the highest wave condition \(w = u\) will be tested first, and
then the choices of the rated power of the generator \(P_{gratedj,k}\), and the rated power of the converter
\(S_{ratedj,k}\) (4-14) are known to be applied in the power-rating dependent model of \(P_{loss}\) in (4-8) -
(4-10). After the pre-process for the power ratings, the process begins from sweeping the \(Rptoi\,
and the maximum power \(Poe_{ix,j,k,w}\) with the damping \(Rpto_{ix}\) are found for each set of the index \((j, k, w)\). The \(i_x\) is also used as the index for \(SR_{ix,j,k,w}\) and \(IR_{ix,j,k,w}\). The process continues until the
variables in the \(Plev, kb\), and \(w\) matrices are tested. The outputs with \(w = 1\) to \(u\) are used to capture
the average value of the output powers \(Poe_{j,k}\), which are derived in (4-12). The ratios of the rated
converter power and the rated generator power to the average output power \(U_{sp}\) and \(U_{gp}\) in (4-15)
are added as the figures of merit for the converter and the generator selections.
Fig. 4-9. The flowchart for the optimization process where output results are in underlined and control variables are shown in bold.
4.3.3 Output Analysis

The example parameters for the flowchart are listed in Table 4.3-1 where the total numbers of iterations are \( mn^2o \cdot u = 10080 \), and each iteration has a simulation time of 1800 seconds that will take about 150 seconds to process. The total iterations will take around 420 hours, so the High-Performance Computing (HPC) based platform with 28 cores is utilized to reduce the simulation effort in the numerical study down to 15 hours, which can be further reduced by splitting the tasks to multiple nodes.

The annual wave profile at the wave site 44056 [95] is categorized as 28 discrete wave conditions, each with its incidence rated \( p_w \) to simplified numerical study.

After the time-domain simulation in Fig. 4-8, each output power is stored as \( Poe_{i,j,k,w} \) as shown in Fig. 4-10(a) and (b), which consists of the output power results from 10 different \( R_{pto} \) from \( R_{pto} \).

The maximum powers are found as \( Poe_{i,7,1,28} \) and \( Poe_{i,7,1,4} \) in the figure, and as in the first equation of (4-12), their corresponding damping \( R_{pto} \) are \( 0.7R_{ptob} \) and \( 0.9R_{ptob} \). Due to the nonlinearity from the control algorithms and the losses, the maximum power \( P_{oe} \) are not always happened at \( R_{pto} = R_{ptob} \), which is the optimal damping under an ideal case.

Table 4.3-1. Setup parameters for the optimization example from the flowchart in Fig. 4-9. The other parameters are listed in Table 4.2-1.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u )</td>
<td>Numbers of wave condition ( Hs_w ) and ( Te_w )</td>
<td>28</td>
</tr>
<tr>
<td>( m )</td>
<td>Numbers of ( R_{pto} )</td>
<td>10</td>
</tr>
<tr>
<td>( n )</td>
<td>Numbers of ( P_{levj} )</td>
<td>9</td>
</tr>
<tr>
<td>( o )</td>
<td>Numbers of ( kb_j )</td>
<td>4</td>
</tr>
<tr>
<td>( T_{irr} )</td>
<td>The irregular wave simulation period</td>
<td>1800 s</td>
</tr>
<tr>
<td>( R_{pto} )</td>
<td>( [0.5, 0.6, 0.7, 0.8, 0.9, 1, 1.1, 1.2, 1.5, 2] )</td>
<td>( R_{ptob} )</td>
</tr>
<tr>
<td>( P_{levj} )</td>
<td>( [250, 200, 150, 100, 75, 60, 50, 40, 25] \times 10^3 ) kW</td>
<td>( kb_j ) ( [104, 93, 63, 47] )</td>
</tr>
<tr>
<td>( Hs_w, Te_w )</td>
<td>As listed in Table 4-5 from wavesite 44056 with from ( w = 1 ) to ( w = 28 )</td>
<td></td>
</tr>
</tbody>
</table>
Fig. 4-10. The results of output power over different $R_{pto}$ under $P_{lev} = 50 \text{ kW}$ ($j = 7$) and $k_b = 104$ ($k = 1$). (a) $Poe_{i,7,1,28}$ with the wave condition $w = 28$, and (b) $Poe_{i,7,1,4}$ with the wave condition $w = 4$. The results of $w = 1 \sim 28$ are listed in Table 4-5. (B.1 Setups of figures-(vi))

The maximum output power of each wave $Poe_{ix}$ and its optimal damping $R_{ptoix}$ are listed in Table 4-5 for each $w$ where the maximum power rational in Fig. 4-10 is highlighted ($w=7, 28$).

Table 4.3-2. Wave conditions based on the selected wave site at the site 44056 in 2015 [95]. $p_w$ is the possibility of each wave condition, $R_{pto}$ is the value of $R_{pto}$ for the maximum power, and $P_{oe_{max}}$ is the maximum average wave output power under each wave. The control variables are set as $P_{lev} = 50 \text{ kW}$ and $k_b = 104$.

<table>
<thead>
<tr>
<th>$w$</th>
<th>$H_s$ (m)</th>
<th>$T_e$ (s)</th>
<th>$p_w$ (%)</th>
<th>$R_{pto}_{ix}$</th>
<th>$P_{oe_{ix}}$ (W)</th>
<th>$w$</th>
<th>$H_s$ (m)</th>
<th>$T_e$ (s)</th>
<th>$p_w$ (%)</th>
<th>$R_{pto}_{ix}$</th>
<th>$P_{oe_{ix}}$ (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.25</td>
<td>4.5</td>
<td>3.0</td>
<td>0.8</td>
<td>383</td>
<td>15</td>
<td>1.75</td>
<td>5.5</td>
<td>3.1</td>
<td>0.6</td>
<td>19058</td>
</tr>
<tr>
<td>2</td>
<td>0.25</td>
<td>5.5</td>
<td>3.0</td>
<td>0.8</td>
<td>471</td>
<td>16</td>
<td>1.75</td>
<td>6.5</td>
<td>1.6</td>
<td>0.5</td>
<td>21405</td>
</tr>
<tr>
<td>3</td>
<td>0.25</td>
<td>6.5</td>
<td>2.6</td>
<td>0.9</td>
<td>519</td>
<td>17</td>
<td>1.75</td>
<td>7.5</td>
<td>0.6</td>
<td>0.6</td>
<td>20535</td>
</tr>
<tr>
<td>4</td>
<td>0.25</td>
<td>7.5</td>
<td>1.8</td>
<td>0.9</td>
<td>536</td>
<td>18</td>
<td>1.75</td>
<td>8.5</td>
<td>0.1</td>
<td>0.5</td>
<td>20797</td>
</tr>
<tr>
<td>5</td>
<td>0.75</td>
<td>3.4</td>
<td>6.1</td>
<td>0.6</td>
<td>2464</td>
<td>19</td>
<td>2.25</td>
<td>4.5</td>
<td>0.2</td>
<td>0.5</td>
<td>22697</td>
</tr>
<tr>
<td>6</td>
<td>0.75</td>
<td>4.5</td>
<td>15</td>
<td>0.8</td>
<td>3657</td>
<td>20</td>
<td>2.25</td>
<td>5.5</td>
<td>2.6</td>
<td>0.5</td>
<td>23409</td>
</tr>
<tr>
<td>7</td>
<td>0.75</td>
<td>5.5</td>
<td><strong>15.4</strong></td>
<td><strong>0.9</strong></td>
<td><strong>4733</strong></td>
<td>21</td>
<td>2.25</td>
<td>6.5</td>
<td>1.1</td>
<td>0.5</td>
<td>24698</td>
</tr>
<tr>
<td>8</td>
<td>0.75</td>
<td>6.5</td>
<td>7.9</td>
<td>0.9</td>
<td>5020</td>
<td>22</td>
<td>2.25</td>
<td>7.5</td>
<td>0.5</td>
<td>0.5</td>
<td>23430</td>
</tr>
<tr>
<td>9</td>
<td>0.75</td>
<td>7.5</td>
<td>3.8</td>
<td>0.9</td>
<td>5291</td>
<td>23</td>
<td>2.75</td>
<td>5.5</td>
<td>0.9</td>
<td>0.5</td>
<td>27882</td>
</tr>
<tr>
<td>10</td>
<td>1.25</td>
<td>4.5</td>
<td>8.3</td>
<td>0.7</td>
<td>10290</td>
<td>24</td>
<td>2.75</td>
<td>6.5</td>
<td>0.8</td>
<td>0.5</td>
<td>28361</td>
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<tr>
<td>11</td>
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<td>5.5</td>
<td>7.8</td>
<td>0.7</td>
<td>12404</td>
<td>25</td>
<td>2.75</td>
<td>7.5</td>
<td>0.3</td>
<td>0.6</td>
<td>26941</td>
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<tr>
<td>12</td>
<td>1.25</td>
<td>6.5</td>
<td>3.0</td>
<td>0.6</td>
<td>12676</td>
<td>26</td>
<td>3.25</td>
<td>6.5</td>
<td>0.03</td>
<td>0.6</td>
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</tr>
<tr>
<td>13</td>
<td>1.25</td>
<td>7.5</td>
<td>1.0</td>
<td>0.7</td>
<td>13521</td>
<td>27</td>
<td>3.25</td>
<td>7.5</td>
<td>0.4</td>
<td>0.6</td>
<td>29231</td>
</tr>
<tr>
<td>14</td>
<td>1.75</td>
<td>4.5</td>
<td>2.9</td>
<td>0.6</td>
<td>16565</td>
<td>28</td>
<td>3.75</td>
<td>7.5</td>
<td>0.3</td>
<td>0.7</td>
<td><strong>31807</strong></td>
</tr>
</tbody>
</table>
In Fig. 4-11(a), the PTO output power $P_{om}$ increases with higher $P_{lev}$ value, but the power remains constant with different gear ratio $k_b$ due to that $P_{om}$ is only decided by $R_{pto}$, $v_{pto}$, and limitation of $P_{lev}$ in (4-2), and no $k_b$ is involved in the calculation. After considering the loss $P_{loss}$, the output power of the power converter $P_{oe}$ is smaller with lower $k_b$. In Fig. 4-11(b), the $P_{oe}$ point of 8275 W, $P_{lev}$ of 50 kW, and $k_b$ of 104 are the results of $P_{oe}$ in Table 4.3-2 and the average equation in (4-12).

![Graph of power rating $P_{grated}$ of the generator versus the controlled valued of power leveling $P_{lev}$ and various transmission ratio $k_b$](image)

Fig. 4-11. The output power $P_{om,j,k}$ and $P_{oe,j,k}$ = $P_{om,j,k}$ - $P_{loss,j,k}$ which are the average power considering the probability of annual wave conditions in Table 4-5. (B.1 Setups of figures- (vi))

The graph of power rating $P_{grated}$ of the generator versus the controlled valued of power leveling $P_{lev}$ and various transmission ratio $k_b$ in Fig. 4-12(a) shows $P_{grated}$ increases as $P_{lev}$ increases, and higher $P_{grated}$ is required with a lower gear ratio. The higher-rated of $S_{rated}$ is also needed as $P_{lev}$ increase, but the $S_{rated}$ is not reducing as the $k_b$ increases in Fig. 4-12(b). Both of the correlations between the control and the two outputs are explained in the time domain results in Fig. 4-17. The highlighted data points in the figure are the results of the time-domain simulation in Fig. 4-8.

\[
U_{sp,j,k} = \frac{S_{rated,j,k}}{P_{oe,j,k}}
\]

\[
U_{gp,j,k} = \frac{P_{grated,j,k}}{P_{oe,j,k}}
\]

(4-15)
The overrating ratio $U_{sp}$ and $U_{gp}$ as a ratio of the rated converter power and the rated generator power to the output power in (4-15) are plotted in Fig. 4-13. The minimum values of each ratio are founded and can be utilized as the converter or the generator design guideline. The average of both ratios is plotted for $P_{lev}, S_{rated}$, and $P_{grated}$ in Fig. 4-14.

Fig. 4-12. Generator rating (a) and converter rating (b) versus power leveling under various transmission ratios. The points marked are from the simulation in Fig. 4-8 with $I_{prated} = 315$ A and the average values from Fig. 4-10 where (a) the rated power of the generator $P_{grated}$ of 65 kW is dervied from (4-14) with $I_{grated} = 100$ A, and (b) the rated power of the converter $S_{rated}$ of 218 kVA is derived (4-13) with $I_{prated} = 315$ A. (B.1 Setups of figures-(vi))
Fig. 4-13. (a) $P_{lev}$ to $U_{sp}$ where the minimum $U_{sp}$ of 20 occurs when $k_b = 47$ and $P_{lev} = 100$ kW. (b) $S_{rated}$ to $U_{sp}$ where the minimum $U_{sp}$ of 20 occurs when $k_b = 47$ and $S_{rated} = 180$ kVA. (c) $P_{lev}$ to $U_{gp}$ where the minimum $U_{gp}$ of 7.5 occurs when $k_b = 104$ and $P_{lev} = 150$ kW. (d) $P_{grated}$ to $U_{gp}$ where the minimum $U_{gp}$ of 7.5 occurs when $k_b = 104$ and $P_{grated} = 68$ kW. (B.1 Setups of figures-(vi))

Fig. 4-14. The average value of $U_{sp}$ and $U_{gp}$ and corresponding $P_{lev}$, $S_{rated}$, and $P_{grated}$ where (a) shows the minimum value of $(U_{sp} + U_{gp})/2$ occurs when $k_b = 104$ and $P_{lev} = 200$ kW; (b) shows the minimum value of 15 when $k_b = 104$ and $S_{rated} = 230$ kVA; (c) shows the minimum value of 15 when $k_b = 104$ and $P_{grated} = 90$ kW. (B.1 Setups of figures-(vi))

The result $(U_{sp} + U_{gp})/2$ is used to select the most cost-effective power rating of the converter and the generator, under the assumption of the cost per power rating of the two components is identical. The criteria $U_{sp}$ and $U_{gp}$ can also be weighted according to their cost ratio. The minimum value of $(U_{sp} + U_{gp})/2$ is found at around 15, which is the minimum overrating ratio of the powertrain. The optimal parameters are marked in Fig. 4-14, which occurs when $P_{lev} = 200$ kW and $k_b = 104$ with the requirements of $S_{rated} = 230$ kVA and $P_{grated} = 90$ kW.

The trends of the optimal design are found from the knee points of both Fig. 4-15(a) and (b) that when $k_b = 104$, the knee point appears around $S_{rated} = 230$ kVA, and when $k_b = 104$, the knee point appears around $P_{grated} = 90$ kW. Both power rating setup leads to the average output power $P_{oe}$ of 10.8 kW.
Fig. 4.15. The output power $P_{oe}$ and rated powers of converter and generator from (a) the rated power of the converter and the output power, and (b) the rated power of the generator and the output power, the optimal design point from Fig. 4.14 is $P_{oe} = 10.8$ kW when $k_b = 104$, $P_{lev} = 200$ kW, $S_{rated} = 230$ kVA, and $P_{grated} = 90$ kW. (B.1 Setups of figures-(vi))

### 4.4 Parametric Analysis of Optimization Variables

The impacts of the gear ratio are found in the losses and the power ratings as shown in Fig. 4-11 and Fig. 4-12. The effects of field-weakening and power leveling control are studied in this session. Finally, the efficiency of each setup is analyzed as one of the critical output that affects the output power of the power converter.

#### 4.4.1 Effects of Gear Ratio

The gear ratio of the WEC system has an influence on the current level to the power converter and the generator, that is, the amplitude of $i_R$ is inversely proportional to the $k_b$ as in (4-16) from (4-3) and (4-6) by assuming $i_d = 0$, and $i_{fpto}$ are identical in different $k_b$.

$$i_R = i_q = -4i_{fpto}/(3k_t \cdot p \cdot k_b)$$  \hspace{1cm} (4-16)

where from the equation above, the influence of the generator pole numbers $p$ and the torque constant $k_t$ is the same as of the gear ratio $k_b$. 

100
Thus, the gear ratio is also inversely proportional to the generator power rating $P_{\text{grated}}$ which is derived from the $i_R$ and $I_{\text{grated}}$ in (4-14) as shown in Fig. 4-11(a), but this is not the case for the correlation of $S_{\text{rated}}$ and $k_b$ in Fig. 4-11(b) which shows that only when $P_{\text{lev}}$ is higher than 125 kW, the $S_{\text{rated}}$ is inversely proportional to the $k_b$. The figure is redrawn as Fig. 4-16, and is separated by the $P_{\text{lev}}$ of 125 kW into two regions: I. $S_{\text{rated}} \propto k_b$, and II. $S_{\text{rated}} \propto 1/k_b$. The correlation is illustrated in the time domain in Fig. 4-17 based on four points in Fig. 4-16 where more than two times of field-weakening ratio $FWR$ are shown with $k_b = 104$ than the $FWR$ with $k_b = 47$ when $P_{\text{lev}} = 50$ kW, so the magnitude of $V_{dq}$ is controlled to keep around $V_{dc}/\sqrt{3}$. As a result, the $S_{\text{rated}} = 3/2V_R \cdot I_R$ is higher in $k_b = 104$ due to $V_R$ is dominating, while $I_R$ is almost identical in both gear ratios under lower $P_{\text{lev}}$. In region II of high $P_{\text{lev}}$, the value of $FWR$ is not as high as in low $P_{\text{lev}}$, so $S_{\text{rated}}$ is higher with smaller $k_b$ since $I_R$ is dominating the $S_{\text{rated}}$ in this case. The $FWR$s are defined later in (4-17), which can be observed through the percentage of $I_d$ in time-domain simulation. For example, $I_d$ is mostly kept at zero with 200-kW $P_{\text{lev}}$ in Fig. 4-17(a), while $I_d$ in Fig. 4-17(b) is activated most of the time with 50-kW $P_{\text{lev}}$. Other parameters like $U_{sp}$, $U_{gp}$, $FLR$ from Fig. 4-17 can be found in Table 4.4-1.

![Fig. 4-16. The rated power $S_{\text{rated}}$ of the power converter versus the power leveling $P_{\text{lev}}$ and the different values transmission ratio $k_b$. Four operating points (a) – (d) are listed in Table 4.4-1, and their time domain results at $\omega = 28$ are shown in Fig. 4-17. (B.1 Setups of figures-(vi))](image)
Fig. 4-17. Time-domain results of $S_r$, $P_{oe}$, $I_{dq}$, and $P_{loss}$ with the setup in Table 4.4-1. (B.1 Setups of figures-(vi))
Table 4.4-1. Detailed results from the setup in 4.3.3 based on the time domain results in Fig. 4-17 under \( w = 28 \). The four operating points are indicated in Fig. 4-16 as a part of the \( P_{lev} \) and \( S_{rated} \) curves.

<table>
<thead>
<tr>
<th>( k_b )</th>
<th>104</th>
<th>47</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{lev} ) (kW)</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>( I_{d_{max}} ) (A)</td>
<td>345</td>
<td>313</td>
</tr>
<tr>
<td>( I_{q_{max}} ) (A)</td>
<td>308</td>
<td>735</td>
</tr>
<tr>
<td>( P_{om} ) (W)</td>
<td>94213</td>
<td>34639</td>
</tr>
<tr>
<td>( P_{closs} ) (W)</td>
<td>1002</td>
<td>1762</td>
</tr>
<tr>
<td>( I_{R} ) (A)</td>
<td>3168</td>
<td>6557</td>
</tr>
<tr>
<td>( P_{grated} ) (kW)</td>
<td>192</td>
<td>340</td>
</tr>
<tr>
<td>( S_{R_{max}} ) (kVA)</td>
<td>230</td>
<td>417</td>
</tr>
<tr>
<td>( U_{sp,w_{28}} )</td>
<td>90</td>
<td>158</td>
</tr>
<tr>
<td>( U_{gp,w_{28}} )</td>
<td>2.4</td>
<td>4.4</td>
</tr>
<tr>
<td>( FWR(%) )</td>
<td>32.8</td>
<td>20.2</td>
</tr>
<tr>
<td>( FLR(%) )</td>
<td>26</td>
<td>24.1</td>
</tr>
</tbody>
</table>

### 4.4.2 Field-weakening Ratio (\( FWR \)) and Full Load Ratio (\( FLR \))

The field-weakening ratio is used to estimate how often the field-weakening is activated throughout the irregular wave simulation period, which is defined in (4-17).

\[
i_{FW}(t) = \begin{cases} 
1 & \text{if } i_d(t) < 0 \\
0 & \text{if } i_d(t) = 0
\end{cases}
\]

\[
FWR = \int_{0}^{T_{irr}} i_{FW}(t) \, dt / T_{irr} 
\tag{4-17}
\]

\[
FWR_{j,k} = \sum_{w=1}^{u} p_w FWR_{j,k,w}
\]

The first equation in (4-17) shows that if field-weakening is functioning, the flag \( i_{FW}(t) \) will be high as one. The ratio is the result of integrating \( i_{FW}(t) \) along the period \( T_{irr} \), and the \( FWR_{j,k,w} \) is averaged over the annual wave profile for each setup parameter \( j \) and \( k \) as \( FWR_{j,k} \) as shown in Fig. 4-18. The FWR is proportional to the gear ratio \( k_b \) for all kinds of \( P_{lev} \) and wave condition \( w \), which can be explained by (4-3) and (4-4) that \( v_R \) is proportional to \( \omega_e \), and \( \omega_e \) is proportional to \( k_b \) assuming constant \( v_{pto} \).
The average FWRs considering annual wave profiles are lower than 7%, which means that there is still room to increase $k_b$ for higher field-weakening ratio. The influence of the FWR from the gear ratio is significant under the highest wave condition $w = 28$. The power ratings of both the converter and the generator are thus affected.

Similar to FWR, the full load ratio (FLR) is defined in (4-18) to indicating the ratio when $P_{om}$ reaches the power leveling $P_{lev}$ over the irregular wave simulation period $T_{irr}$.

\[
  iFL(t) = \begin{cases} 
    1 & \text{if } P_{om}(t) \geq P_{lev} \\
    0 & \text{if } P_{om}(t) < P_{lev}
  \end{cases}
\]

\[
  FLR = \frac{\int_0^{T_{irr}} iFL(t) \, dt}{T_{irr}}
\]

\[
  FLR_{j,k} = \sum_{w=1}^{u} P_w FLR_{j,k,w}
\]

The FLR is inversely proportional to the power leveling rating $P_{lev}$ as shown in Fig. 4-19. The correlation is found in the $P_{oe}$ results in Fig. 4-17 in time-domain when $P_{lev}= 50$ kW the $P_{oe}$ is mostly limited by the power leveling and is less when $P_{lev}= 200$ kW. Different from FWR, the FLRs in Fig. 4-19 are constant over different $k_b$ value because FLR is decided by $P_{om}$, which is not
affected by \( k_b \) ratio, as shown in Fig. 4-11(a). The average FLR is around 10 \% over annual wave conditions in Table 4.3-2, except for \( w = 28 \), the FLR is up to 65 \%.

Fig. 4-19. Full-load ratio (FLR) under different \( P_{lev} \) and \( k_b \) (a) \( P_{lev} \) to average \( FLR_{jk} \) from annual wave profile from (4-18) and (4-17). (b) \( P_{lev} \) to \( FLR_{jk,28} \) under wave condition \( w = 28 \). (B.1 Setups of figures-(vi))

4.4.3 Efficiency Analysis

The efficiencies considering the losses \( P_{gloss} \), \( P_{closs} \), and \( P_{om} \) from 4.2.2 are defined in (4-19).

\[
\begin{align*}
P_{oe} &= P_{og} - P_{closs} \\
P_{og} &= P_{om} - P_{gloss} \\
eff_{gen} &= \frac{P_{og}}{P_{om}} \\
eff_{pe} &= \frac{P_{oe}}{P_{og}} \\
eff_{m-e} &= \frac{P_{oe}}{P_{om}} = \eff_{gen} \cdot \eff_{pe} \tag{4-19}\end{align*}
\]

where \( \eff_{gen} \) is the efficiency of the generator, \( \eff_{pe} \) is the efficiency of the power converter, and \( \eff_{m-e} \) is the efficiency of the mechanical output of PTO \( P_{om} \) to electrical output \( P_{oe} \). Among (a) – (c) in Fig. 4-20, the higher \( k_b \) shows higher efficiencies since both \( P_{gloss} \) and \( P_{closs} \) are proportional to the magnitude of the current \( I_R \), and \( I_R \) is proportional to \( k_b \) as in (4-16). The maximum \( \eff_{m-e} \) is 96.5 \% in Fig. 4-20(c) when \( k_b = 104 \) and \( P_{lev} = 200 \) kW, which are the same control variables for the optimal design point in Fig. 4-14.
Fig. 4-20. Efficiencies of the generator and the power converter for $P_{lev}$ and $k_b$ from (4-19). (a) $P_{lev}$ to the generator efficiency $\text{eff}_{\text{gen}}$ (4-17); (b) $P_{lev}$ to the power converter efficiency $\text{eff}_{\text{pe}}$; (c) $P_{lev}$ to power train efficiency $\text{eff}_{\text{m-e}}$. (B.1 Setups of figures-(vi))

Table 4.4-2. Optimized design and results based on setups in Table 4.2-1 and Table 4.3-1, and the optimization process in the flowchart Fig. 4-9.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_b$</td>
<td>104</td>
</tr>
<tr>
<td>$P_{\text{lev}}$ (kW)</td>
<td>200</td>
</tr>
<tr>
<td>$R_{\text{ptoi}} \times (R_{\text{ptob}})$</td>
<td>[0.8, 0.8, 0.9, 0.9, 0.6, 0.8, 0.9, 0.9, 0.8, 0.9, 0.9, 0.8, 0.8, 0.8, 0.8, 0.7, 0.7, 0.7, 0.7, 0.7, 0.6, 0.7, 0.6, 0.7, 0.7]</td>
</tr>
<tr>
<td>$S_{\text{rated}}$ (kVA)</td>
<td>230</td>
</tr>
<tr>
<td>$P_{\text{grated}}$ (kW)</td>
<td>90</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Results</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{oe}$ (kW)</td>
<td>10.8</td>
</tr>
<tr>
<td>$(U_{SP} + U_{gp})/2$</td>
<td><strong>14.8</strong></td>
</tr>
<tr>
<td>$U_{SP,28}$</td>
<td>2.4</td>
</tr>
<tr>
<td>$FWR$</td>
<td>1.8 %</td>
</tr>
<tr>
<td>$FLR$</td>
<td>0.765 %</td>
</tr>
<tr>
<td>$\text{eff}_{\text{m-e}}$</td>
<td>96.4 %</td>
</tr>
</tbody>
</table>
4.5 Conclusion

An optimization methodology for the power converter and the generator in the WEC application is developed and demonstrated with a 5-m WEC system under a selected wave site. With the wave-to-wire model considering losses along with the control of the generator, the annual output power and the requirement of the rated powers are estimated. The optimum design is searched through a numerical study with $R_{pio}$, $P_{lev}$, and $k_b$ as variables, and $P_{oe}$, $S_{rated}$, and $P_{grated}$ as output results.

The relationship between $P_{oe} - S_{rated}$ and $P_{oe} - P_{grated}$ are shown in Fig. 4-21, which can be used to select the ratings of both the power converter and the generator. In Fig. 4-21(a), when the gear ratio design $k_b = 47$, the knee point appears around $P_{oe} = 9.2$ kW, which needs a power rating $S_{rated}$ of 180 kVA and the generator rating $P_{grated}$ of 118 kW in Fig. 4-21 (b). The overrated factors $U_{sp}$ and $U_{gp}$ are derived from the numerical searching process and used as criteria to select the power rating. The minimum value of $(U_{sp}+U_{gp})/2$ is selected to design the minimum required power rating for specific annual output power. Table. 4-8 summarizes the results from the 5-m WEC system, where the minimum $(U_{sp}+U_{gp})/2$ is the factor of 15 for the annual wave profile. Comparing to the conventional design of 65.7, the overrating ratio is reduced by the factor of 4 to 5 based on the design guideline in this chapter. The method is shown to significantly reduce the peak-to-average power ratio of the power converter and generator design without losing much of the harvestable output power. The design of the power converter can be found in Appendix A.
Fig. 4-21. The relationship of output power $P_{oe}$ to the rated power of the power converter $S_{rated}$ and the rated power of the generator $P_{grated}$ as a rotated plot from Fig. 4-15. (B.1 Setups of figures-(vi))

Table 4-8. Summary of conventional and optimized design.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Original design</th>
<th>Optimized design</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_b$</td>
<td>N/A</td>
<td>104</td>
</tr>
<tr>
<td>$P_{lev}$(kW)</td>
<td>Unlimited</td>
<td>200</td>
</tr>
<tr>
<td>$S_{rated}$(kVA)</td>
<td>1446</td>
<td>230</td>
</tr>
<tr>
<td>$P_{grated}$(kW)</td>
<td>79</td>
<td>90</td>
</tr>
<tr>
<td>$P_{oe}$(kW)</td>
<td>11.6</td>
<td>10.8</td>
</tr>
<tr>
<td>$U_{SP}$</td>
<td>125</td>
<td>21</td>
</tr>
<tr>
<td>$U_{gp}$</td>
<td>6.8</td>
<td>8.3</td>
</tr>
<tr>
<td>$(U_{SP}+U_{gp})/2$</td>
<td>65.7</td>
<td>14.8</td>
</tr>
</tbody>
</table>
Chapter 5  Maximum Energy Control for Wave Energy Converter

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_\infty$</td>
<td>Added mass at infinite frequency</td>
</tr>
<tr>
<td>$C_{pt0}$</td>
<td>PTO damping</td>
</tr>
<tr>
<td>$d_{dq}$</td>
<td>Duty cycles of power converter on d–q frame</td>
</tr>
<tr>
<td>$D_{i,j,w}$</td>
<td>Expected damaged on power converter with the repeated irregular wave period</td>
</tr>
<tr>
<td>$e_{abc}$</td>
<td>Air gap phase voltage of the generator</td>
</tr>
<tr>
<td>$E_{o,est}$</td>
<td>Estimated output energy from annual wave probability</td>
</tr>
<tr>
<td>$E_o$</td>
<td>Expected output energy from power converter with the repeated irregular wave period</td>
</tr>
<tr>
<td>$E_{ompi}$</td>
<td>Output energy from annual mission profile</td>
</tr>
<tr>
<td>$f_e$</td>
<td>Wave excitation force on the buoy</td>
</tr>
<tr>
<td>$f_{pt0}$</td>
<td>PTO force</td>
</tr>
<tr>
<td>$H_{sw}$</td>
<td>Significant wave height at wave condition w</td>
</tr>
<tr>
<td>$i_{abc}$</td>
<td>Currents into power converter in abc frame</td>
</tr>
<tr>
<td>$i_{dq}$</td>
<td>Currents into power converter in dq frame</td>
</tr>
<tr>
<td>$k_b$</td>
<td>Gearbox gear ratio</td>
</tr>
<tr>
<td>$k_o$</td>
<td>The transfer function of radiation damping</td>
</tr>
<tr>
<td>$k_s$</td>
<td>Hydrodynamic damping of the buoy</td>
</tr>
<tr>
<td>$L_i$</td>
<td>Leakage inductance per phase</td>
</tr>
<tr>
<td>$L_{est}$</td>
<td>Estimated lifetime from annual wave probability</td>
</tr>
<tr>
<td>$L_T$</td>
<td>Expected output energy from power converter with the repeated irregular wave period</td>
</tr>
<tr>
<td>$L_{ompi}$</td>
<td>Lifetime from annual mission profile</td>
</tr>
<tr>
<td>$m$</td>
<td>Mass of the floating body</td>
</tr>
<tr>
<td>$N_f$</td>
<td>Cycles to failure of bond wires in power module damping</td>
</tr>
<tr>
<td>$n_i$</td>
<td>Numbers of cycles on each thermal stress</td>
</tr>
<tr>
<td>$p$</td>
<td>Numbers of pole pairs per phase of the generator</td>
</tr>
<tr>
<td>$P_{dl}$</td>
<td>Sum of power losses on each diode</td>
</tr>
<tr>
<td>$P_{lev_j,w}$</td>
<td>The power-leveling vector as indices for the maximum energy algorithm</td>
</tr>
<tr>
<td>$P_{lev_j}$</td>
<td>Rated power of power leveling, $j = 1:n$</td>
</tr>
<tr>
<td>$P_{lev_{PL}}$</td>
<td>The power-leveling vector as indices for the maximum power with the limited lifetime</td>
</tr>
<tr>
<td>$P_{lev_{w}}$</td>
<td>The power-leveling vector as indices for the maximum power algorithm</td>
</tr>
<tr>
<td>$P_{o,est}$</td>
<td>Estimated output power from annual wave probability</td>
</tr>
</tbody>
</table>
### Table 1: List of Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_o$</td>
<td>Average output power from power converter over the irregular wave period</td>
</tr>
<tr>
<td>$P_{omp}$</td>
<td>Output power from annual mission profile</td>
</tr>
<tr>
<td>$P_{fl}$</td>
<td>Sum of power losses on each IGBT</td>
</tr>
<tr>
<td>$p_w$</td>
<td>Probability of wave condition $w$</td>
</tr>
<tr>
<td>$R_i$</td>
<td>Armature resistance per phase of the generator</td>
</tr>
<tr>
<td>$R_{in}^{E,w}$</td>
<td>The input-resistance vector as indices for maximum energy algorithm wave condition $w$</td>
</tr>
<tr>
<td>$R_{in}$</td>
<td>The input-resistance variables vector of the power converter (passive loading), $i = 1:m$</td>
</tr>
<tr>
<td>$R_{in}^{PLT}$</td>
<td>The input-resistance vector as indices for the maximum power with the limited lifetime</td>
</tr>
<tr>
<td>$R_{in}^{P}$</td>
<td>The input-resistance vector as indices for the maximum power algorithm</td>
</tr>
<tr>
<td>$Te$</td>
<td>Electromagnetic torque of the generator</td>
</tr>
<tr>
<td>$Te_w$</td>
<td>Wave energy period at wave condition $w$</td>
</tr>
<tr>
<td>$T_{irr}$</td>
<td>The irregular wave simulation period</td>
</tr>
<tr>
<td>$T_{jd}$</td>
<td>Average diode junction temperature over the irregular wave period</td>
</tr>
<tr>
<td>$V_{dc}$</td>
<td>The dc-bus voltage of the power converter</td>
</tr>
<tr>
<td>$x_v$</td>
<td>Vertical displacement of the floating body</td>
</tr>
</tbody>
</table>

## 5.1 Introduction of Power Optimization for WEC system

Previous Chapter 4 studied about how to select the power rating and design heatsink for the power converter in the WEC system. This chapter will focus on the WEC control algorithm with an existing hardware design of a power converter on how to maximize both the output power and its lifetime.

The power fluctuation in the WEC system will not only reduce the capacity factor that leads to design overrated for the power converter but also introduce high thermal cycles on the junction temperature of the power converter that causes potential reliability issues [96]. With a higher average-peak ratio of power profile and irregular wave condition, this thermal cycle issue is more severe than other renewable energy like wind energy [97]. Most of the control designs in WEC have been focusing on optimal power controls to increase the power from the system, but how the
system's reliability is impacted by these controls are still unknown. Is there a control algorithm or selection of control algorithm that could maximize both the reliability and the output power? Or whether there is a maximum energy point for certain control parameters like in Fig. 5-1?

![Graph](image)

**Fig. 5-1.** Normalized output power ($P_o$), lifetime ($LT$), and output energy ($E_o$) curves of the power converter in WEC verses different damping loading $R_{in}$ under certain wave conditions.

This chapter will provide a methodology to estimate the lifetime of the power converter in the WEC system and try to find a maximum energy point through exploring the passive damping and power leveling variables.

### 5.1.1 Review of Power Optimization in WEC

The maximization of the harvested power is an important topic that has been researched in recent studies [98]–[100]. The effects of power limitation of the power converter are also considered in [101], and this will change the sub-optimal loading values due to the discontinuity from control and the nature of irregular wave. Numerical methods are implemented to studies the interaction between the control and the output power [81], [102]. How the control (passive damping) parameters will affect the lifetime and energy are still uncertain as in Fig. 5-1 shows only one wave condition and one control parameter ($R_{in}$). Especially how can this extend to the annual wave profile affect the results and consideration of other parameters like power derating
Great tools are provided to optimize the power rating of the power converter and generator in WEC [44], [45] through a numerical study using the thermal and over-speed ratio as constraints in the models. With the high peak-average power ratio of the WEC system and the power cycling from irregular wave, the lifetime of the power components under these control algorithms still need to be studied.

5.1.2 Review of Reliability Analysis for Power Converters

Reliability plays an important role for the WEC system, especially when in an offshore device which only has limited time for maintenance due to the limitation on the weather [103], and the LCoE of the WEC can be reduced by breaking down the expenditure to longer operating lifetime as shown in (1-1) and [18]. Researches in wind energy show that power electronics (electrical system) has the highest failure rate among other components in a harsh environment like wind turbine [104] and [105]. Not much of the field data are shown for failure mode of the wave energy converters since WECs are still not widely commercialized yet. Lifetime is used as an indicator of the reliability of the power converter [106] and [107]. Capacitor, inductors, and power semiconductors are critical components that influence the lifetime in a typical power electronics converter. The reliability of the capacitors in the dc-link application is thoroughly studied in [108] and [109]. The power semiconductor as another vulnerable component is sensitive to the thermal fluctuation due to its lower thermal time constant and as a source of most of the losses. In the application of a power module, bond-wire lift-off is one of the common failure mechanisms under thermal cycling [110]. As a result, the failure mode of bond-wire lift-off will be used as an indicator for lifetime analysis in this study.

The rainflow calculation is a method originally for the study of mechanical fatigue [111] was
introduced as a physical-based approach to study the failure mode of the power electronics system [112] and [105] replacing the statistical methods used in mean time between failures (MTBF) estimation [113]. Thermal-mechanical stresses are [114], [115] one major cause for the lift-off of the bonding wires, so the junction temperature estimation [116] lays an important foundation for the lifetime of the bonding wires. The junction temperature will be estimated through the thermal impedance model from the manufacture, and the rainflow calculation is applied to calculate the stress on the bond-wires.

Multi-timescale of thermal dynamics was proposed in [107] and [117] to show the effects from a different type of loading in renewable energy application, and this help to distinguish the impact of different sources and might as well reduce the computation time for the thermal model by neglecting the analysis of certain time-scale. The timescales of wave energy are summarized in Table 5.1-1, which are categorized into four different scales. Short-term is from the switching frequency, which is around 10 – 1 kHz. The study in PV converter [115] the power cycle from switching frequency is neglected, and thermal fluctuation of medium-term (MT) from line frequency shows more lifetime consumption than those from long term-II (LgT-II) due to the maximum power point tracking and ambient temperature which has a period around 30 minutes to an hour. Long term I (LgT-I) is added between conventional MT and LgT-II in wave energy application, which is separated from long-term time scale in wind energy [97] to show the influence of wave-wave movement with a period of 2 – 10 seconds for the typical irregular wave train in the ocean.

Different from wind turbines using wind speed as a single parameter to indicate the mission profile [97], in wave energy converter (WEC), two parameters are usually used to construct a 2D mission profile [81]: significant wave height $H_s$ and average energy period $T_e$. Each wave condition
will be extended to an irregular wave period that makes the task mapping this 2D mission profile to a loading profile of power electronics components even more complex and time-consuming [46], [47]. With this limitation, it is very difficult to evaluate how different control parameters influence the overall lifetime and the recoverable energy from the power converter before failure. Similar studies in renewable energy have investigated how the mission profile from the sources impact the lifetime [29], [115], [118].

5.1.3 Review of Thermal Control and design for Power Converter

The reliability can be increase by control or properly design as summarized in [119]. Consistent with the time-scale from multiple loading, the methods to increase the lifetime can be organized as Table 5.1-1. Among short-term thermal controls, variable switching frequency methods [120], [121], [122], [123] is the most common way reduce the amplitude of thermal cycle and reduce the stress over the loading profile. Others, like applying DPWM [125] or change the gate driving speed [126], are thermal controls in the switching frequency region. Two major methods are listed in the medium-term thermal control, including load current distribution in multilevel converters or reactive power control in the motor drive application under the time scale of the line period. For the time scale of long-term I, profile shaping like power derating [48] or power leveling control [123] is an efficient way to reduce the peak temperature by clamping the peak power on the power module through. The system-level control (damping selection), like [81] and [118], and power converter or the generator design, like [106], [119], [124], fall in the category of the long-term-II time scale. The thermal control methods in this study will include reactive power control, profile shaping, and system control, which cover the loading profile from medium to long term.
Table 5.1-1. Time scales of thermal cycles of power semiconductors and reliability control methods for each time scale. The scales and methods adopted in this chapter are underlined.

<table>
<thead>
<tr>
<th>Reliability</th>
<th>Scale</th>
<th>Source</th>
<th>Methods</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short term</td>
<td>0.1 – 1 ms</td>
<td>Switching frequency</td>
<td>Variable Switching frequency [120], [121], [122], [123]</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>PWM placement [125]</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Gate Control [126]</td>
</tr>
<tr>
<td>Medium term</td>
<td>10 – 100 ms</td>
<td>Generator line frequency</td>
<td>Load current distribution [125]</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Reactive Power [127]</td>
</tr>
<tr>
<td>Long term I</td>
<td>1-10 s</td>
<td>The wave-wave fluctuation of irregular wave</td>
<td>Profile shaping (Power leveling) [123], [48]</td>
</tr>
<tr>
<td>Long term II</td>
<td>10-100 min</td>
<td>1. Wave train fluctuation</td>
<td>System control (Damping control) [81], [118]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2. Ambient temperature cycle</td>
<td>System design (topology/generator selection/dc-link design) [124], [106], [119]</td>
</tr>
</tbody>
</table>

5.1.4 Energy optimization for the WEC power converter with the control algorithm

In this study, passive loading and power-leveling will be applied as control variables based on different wave conditions. As shown in Fig. 5-2, to expedite the analysis, a 4-D lookup table will be built upon multiple control variables and wave conditions in a known design of a WEC system, including buoy, PTO, and power converter. To simplify the 4-D lookup table, ambient temperature $T_a$ is assumed constant, so only 2-D mission profile $H_s$ and $T_e$ are considered. Three different maximum algorithms are generated through the lookup tables and the possibility of each wave. One of the algorithms will be chosen to calculate the output power, energy, and lifetime through the measure annual mission profile from a given wave site and year. In the mapping of mission profile to loading profile, the ambient temperature profile $t_a$ is considered to adjust the data from 4-D lookup tables.
Flowchart for power, energy, and damage from each wave condition and control variables (Fig. 5-20)

Chap.2 WEC system
5.4.1 Select wavesite

5.3.2 Select variables \( R_{in}, P_{lev} \)
Define major wave conditions from 1:m
\((R_{in}, P_{lev})_{i=1:m, j=1:n} \)
\((T_e, H_s)_{w=1:u} \)

Create 4-D Lookup tables (Fig. 5-21)

\[ P_{in, P_{lev, Eo}} \]

I. Indices for maximum power (MaxP)
II. Indices for maximum energy (MaxE)
III. Indices for Maximum Power with limited LT (MaxP+LT)

Algorithm I, II or III

Evaluation with annual mission profile (\( h_{s_k}, t_{e_k}, t_{a_k} \)) data, \( k=1:ny \)

\[ P_{o_{mp}}, E_{o_{mp}}, LT_{mp} \]

Flowchart for power, energy, and damage from each wave condition and control variables (Fig. 5-20)

Flowchart for power, energy, and damage from each wave condition and control variables (Fig. 5-20)

Flowchart to achieve maximum output power, maximum energy, or maximum power with lifetime constraints from a designed system and alternative wave site. The process starts with developing 4-D lookup tables in Fig. 5-23 from which maximum power (I: MaxP), maximum energy (II: MaxE), or maximum power with designed lifetime (III: MaxP+LT) algorithms are developed. One algorithm will be selected and implemented with the annual wave mission profile from the selected site to evaluate the output power \( P_{o_{mp}} \), expected output energy until failure \( E_{o_{mp}} \), and expected lifetime \( LT_{mp} \).
The models of the WEC system from Chapter 3 will be applied in this study, and the optimal hardware design of the power converter is from Chapter 4. With the detailed semiconductor losses, the thermal impedance, and the lifetime model, the effect of multiple thermal control algorithms will be shown in the following section. The open-loop constrained motor/WEC control will be shown in section 5.3. Construction of lookup tables and maximization algorithms will be derived in section 5.4. The results from the given wave site and analysis will be discussed in section 5.5.

Fig. 5-3. Wave energy converter (WEC) system includes buoy, Power take-off (PTO), power converter. The buoy is with the assumption of single-degree-of-freedom heave only motion and excitation force $x_e, f_e$ which models were described in Chapter 3. The output power is derived from the average circuit model in 5.2, which has a higher resolution than the model in 4.2. The semiconductor loss $P_{ql}$ and $P_{dl}$, junction temperature $T_{jq}$ and $T_{jd}$, extended lifetime $LT$, the damage on the bond wires $D$, and the extended output energy $E_o$ of the power converter is estimated through the model in 5.2.3. The controlled variables passive damping $R_{in}$ and power leveling $P_{lev}$ are implemented through the open-loop controller in 5.3 and Fig. 5-13 with constraints of maximum torque $T_{eg_{max}}$, dc bus $V_{dc}$, and $P_{lev}$.

5.2 Average Circuit and Lifetime Model for Power Converter

A multi-physics wave-to-wire model (W2W), thermal impedance model, and lifetime model of the power converter are introduced as Fig. 5-4, which is used to estimate the expected power,
lifetime, and energy under various irregular wave conditions. Passive damping with power leveling is controlled by the power electronics in the WEC system. Bond-wire lifetime analysis of the power converter is used for lifetime and energy calculation. The design of a 5-m WEC system with a 200-kVA 3p-2L power module is used as an example in this session as shown in Table 5.2-1.

The models in Fig. 5-4 from stage 1 – 3, including irregular wave model, fluid-body (WEC circuit model), and mechanical transmission (PTO circuit model), are discussed in the previous chapter. As in Chapter 4, a two-level three-phase ac-dc power electronics converter is used to convert the energy from the generator to the dc source (battery) as shown in stage 4. The semiconductor losses $P_{dl}$ and $P_{ql}$ according to the loading profile will be converted to thermal profile $T_{jdm}/T_{jqm}$ and $\Delta T_{jd}/\Delta T_{jd}$ through the thermal impedance model in stage 5. Finally, the thermal stresses are counted with the rainflow calculation [111], and the corresponding consumed lifetime is found for each thermal stress condition. The total lifetime is combined through Miner’s rule [128].

Fig. 5-4. Reliability analysis for the power converter in WEC application where the steps 1 and 2 are the same as that in Chapter 3, and step 3 and 4 is the similar to that in Chapter 4. What differs from the previous chapter is the line frequency analysis from the equivalent circuit and the reliability analysis from steps 5 and 6.
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generator</td>
<td>Greef, GDF-250</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rated voltage</td>
<td>( V_{\text{rated}} )</td>
<td>380</td>
<td>V</td>
</tr>
<tr>
<td>Rated current</td>
<td>( I_{\text{rated}} )</td>
<td>76</td>
<td>A</td>
</tr>
<tr>
<td>Gear ratio of ball-screw</td>
<td>( k_b )</td>
<td>104</td>
<td>rad/ms</td>
</tr>
<tr>
<td>Torque or back-emf constant</td>
<td>( k_t ) or ( k_e )</td>
<td>0.823</td>
<td>Nm/A/pp (Vs/rad/pp)</td>
</tr>
<tr>
<td>Numbers of poles per phase</td>
<td>( p )</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>Armature resistance per phase</td>
<td>( R_i )</td>
<td>0.1</td>
<td>Ω</td>
</tr>
<tr>
<td>Leakage inductance per phase</td>
<td>( L_i )</td>
<td>2.2</td>
<td>mH</td>
</tr>
<tr>
<td>The inertia of the generator</td>
<td>( J_{\text{eq}} )</td>
<td>1</td>
<td>kgm(^2)</td>
</tr>
<tr>
<td>Maximum torque</td>
<td>( T_{\text{egmax}} )</td>
<td>1000</td>
<td>Nm</td>
</tr>
<tr>
<td>Power module</td>
<td>Mitsubishi, PM150CG1C120 [94]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output dc-bus voltage</td>
<td>( V_{dc} )</td>
<td>800</td>
<td>V</td>
</tr>
<tr>
<td>Rated surge input current</td>
<td>( I_{\text{CRM}} )</td>
<td>300</td>
<td>A</td>
</tr>
<tr>
<td>Rated continuous input current</td>
<td>( I_C )</td>
<td>150</td>
<td>A</td>
</tr>
<tr>
<td>Rated voltage</td>
<td>( V_{\text{ces}} )</td>
<td>1200</td>
<td>V</td>
</tr>
<tr>
<td>Switching frequency</td>
<td>( f_{\text{sw}} )</td>
<td>10</td>
<td>kHz</td>
</tr>
<tr>
<td>Maximum junction temperature</td>
<td>( T_{\text{jmax}} )</td>
<td>150</td>
<td>°C</td>
</tr>
</tbody>
</table>

### 5.2.1 Average Circuit Model for Three-phase Two-level Ac-Dc Power Converter

The reliability analysis for the power converter in PV [129] pointed out that two major thermal stresses should be considered: power cycling from the load variant according to the mission profile, and the ambient temperature fluctuation both of which belong to the category of LgT-II in Table 5.1-1. Time scales of thermal cycles of power semiconductors and reliability control methods for each time scale. The scales and methods adopted in this chapter are underlined. Since most of the stresses are from MT, LgT-I, or LgT-II, the short-term dynamics is then neglected in this analysis, and the average model of the power converter in Fig. 5-5 is used instead of the switching model to expedite the simulation.
The power converter model as an average model [130] shows in the following equations (5-1) - (5-5), and the power loss model, as well as the thermal model, are based on the datasheet of the actual IGBT module (PM150cg1c120). Connecting with the generator model and WEC, the W2W model is shown in Fig. 5-5.

\[
E_{abc} = R_i \dot{i}_{abc} + L_i \dot{d}_{abc} + V_{dc} \ddot{d}_{abc} + \bar{v}_n \quad (5-1)
\]

\[
\dot{i}_{abc} = \begin{bmatrix} \dot{i}_a \\ \dot{i}_b \\ \dot{i}_c \end{bmatrix}, \quad \ddot{v}_{abc} = \begin{bmatrix} \ddot{v}_a \\ \ddot{v}_b \\ \ddot{v}_c \end{bmatrix}, \quad \ddot{d}_{abc} = \begin{bmatrix} d_a \\ d_b \\ d_c \end{bmatrix} \quad (5-2)
\]

\[
\bar{v}_n = \left( \frac{1}{3} \right) (E_{abc} - \ddot{v}_{abc}) = \left( \frac{1}{2} \right) V_{dc} \quad (5-3)
\]

\[
\dot{i}_o = \ddot{d}_{abc} \cdot \dot{i}_{abc} \quad (5-4)
\]

\[
P_o = V_{dc} \bar{i}_o \quad (5-5)
\]

where the \(E_{abc}\) is the air gap voltage from the rotor speed \(\omega_g\), pole pairs per phase \(p\) and the back EMF constant \(k_e\); \(\dot{i}_{abc}\) is input average current to the power converter; \(\ddot{v}_{abc}\) is input average voltage of the power converter; \(\ddot{d}_{abc}\) is the duty cycle controlled by the controller; \(\bar{v}_n\) is the average neutral-ground (common-mode) voltage; \(\bar{i}_o\) is the output current and \(P_o\) is the product of the constant output voltage \(V_{dc}\) and \(\bar{i}_o\).

Fig. 5-5. Wave-to-wire circuit model of the WEC system with the average circuit model of the power converter.
5.2.2 Power Loss and Thermal Model for Bond-wire in Power Module

The loss models are also derived from voltage and current information from the average model as follows,

\[ P_{qsw} = f_{sw} \cdot (E_{off}(i_{ce}, V_{ce}, T_{jq}) + E_{on}(i_{ce}, V_{ce}, T_{jq})) \]  \hspace{1cm} (5-6)

\[ P_{qc} = i_{c} \cdot V_{cesat}(i_{ce}, T_{jq}) \]  \hspace{1cm} (5-7)

\[ P_{drr} = f_{s} \cdot E_{rr}(i_{f}, V_{ce}, T_{jd}) \]  \hspace{1cm} (5-8)

\[ P_{dc} = I_{E} \cdot V_{cesat}(i_{f}, T_{jd}) \]  \hspace{1cm} (5-9)

\[ P_{dt} = P_{drr} + P_{dc} \]

\[ P_{ql} = P_{qsw} + P_{qc} \]  \hspace{1cm} (5-10)

where \( P_{qsw} \) is the switching loss of the IGBT which consists of turn-on loss as a function of \( E_{on} \) and turn-off loss as a function of \( E_{off} \). Both \( E_{on} \) and \( E_{off} \) are plotted as Fig. 5-6(d) as a function of input current \( i_{ce} = \overline{i_{abc}} \), the switching frequency \( f_{sw} \), and the switching voltage \( V_{ce} = V_{dc} \). Another loss of the IGBT is conduction loss \( P_{qc} \) which is a function of the voltage drop during conduction \( V_{CEsat} \) as shown in Fig. 5-6(a). The reverse recovery loss \( P_{drr} \) and conduction loss \( P_{dc} \) of a diode are corresponding to \( E_{rr} \) and \( V_{ECSat} \) respectively as shown in Fig. 5-6(b) and (c). The losses beyond the operating conditions shown in Fig. 5-6 will be interpolated/extrapolated linearly for loss estimation. The total loss produces from a single IGBT or diode are \( P_{ql} \) and \( P_{dt} \) in (5-10) which is a summary of switching loss (recovery loss) and conduction loss.

![Fig. 5-6. Performance curves of the IGBT PM150CG1C120 as power loss lookup tables in (5-6) - (5-10).](image)
From the loss calculation, the temperature of each component is derived from (5-11) - (5-13) and equivalent thermal network in Fig. 5-7, which shows the derivation of the junction temperature from the case, thermal grease, heatsink, and to the ambient.

\[
T_{jqi} - T_c = Z_{jq-c} \cdot P_{qli} \tag{5-11}
\]

\[
T_{jdi} - T_c = Z_{jd-c} \cdot P_{dli} \tag{5-12}
\]

\[
T_c - T_a = Z_{ca} \cdot \sum_{i=1}^{6} (P_{qli} + P_{dli}) \tag{5-12}
\]

\[
Z_{jq-c} = \sum_{iq=1}^{4} R_{iql}(1 - e^{-t/t_{iq}})
\]

\[
Z_{jd-c} = \sum_{id=1}^{4} R_{idl}(1 - e^{-t/t_{id}}) \tag{5-13}
\]

\[
Z_{ca} = R_{ca}(1 - e^{-t/t_{ca}})
\]

where \(T_{jqi}\) and \(T_{jdi}\) are the junction temperature of the IGBT and diode chip in the power module package, \(T_c\) is the case temperature of the power module, and \(T_a\) is the ambient temperature of the power converter which is assumed to be consistent with the buoy ambient temperature; \(Z_{jq-c}\) (\(Z_{jd-c}\)) is the thermal impedance from IGBT (diode) to the case, and \(Z_{ca}\) is the thermal impedance from case to ambient which includes the impedance from the thermal grease and the heatsink.

The values of thermal impedances of the IGBT module are listed in Table 5.2-2 based on both the Foster and Cauer RC approximation through 4\(^{th}\)-orders network from the datasheet. The thermal impedance of the heatsink and thermal grease are also listed in the table. The heatsink is from Fischer-Elektronik using the model of cooling aggregate with an assembly of fins and axial fans. The dimension is 65-mm height, 190-mm width, and 150-mm length. The forced air-cooling fan with 246 m\(^3\)/h is added to achieve a thermal impedance of 0.0634 K/W and \(\tau_{ca}=183.86\) s.
Table 5.2-2. Thermal Impedance Foster or Cauer network Parameters of 3p-2L Power Converter.

<table>
<thead>
<tr>
<th>Network</th>
<th>Thermal impedance</th>
<th>i = 1</th>
<th>i = 2</th>
<th>i = 3</th>
<th>i = 4</th>
<th>Case - ambient</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foster</td>
<td>( R_{j0}/0.13 )</td>
<td>0.0124</td>
<td>0.0739</td>
<td>0.3505</td>
<td>0.5632</td>
<td>( R_{ca} = 0.063 )</td>
<td>k/W</td>
</tr>
<tr>
<td></td>
<td>( \tau_{iq} )</td>
<td>1.961e-5</td>
<td>0.0014</td>
<td>0.0179</td>
<td>0.0944</td>
<td>( \tau_{ca} = 183.9 )</td>
<td>S</td>
</tr>
<tr>
<td></td>
<td>( R_{j0}/0.18 )</td>
<td>0.0124</td>
<td>0.0739</td>
<td>0.3505</td>
<td>0.5632</td>
<td>( R_{ca} = 0.063 )</td>
<td>k/W</td>
</tr>
<tr>
<td></td>
<td>( \tau_{id} )</td>
<td>1.961e-5</td>
<td>0.0014</td>
<td>0.0179</td>
<td>0.0944</td>
<td>( \tau_{ca} = 183.9 )</td>
<td>S</td>
</tr>
<tr>
<td>Cauer</td>
<td>( R_{jcdi} )</td>
<td>0.00202</td>
<td>0.0203</td>
<td>0.0640</td>
<td>0.0438</td>
<td>( R_{cs} + R_{ia} = 0.063 )</td>
<td>k/W</td>
</tr>
<tr>
<td></td>
<td>( C_{jcdi} )</td>
<td>0.0109</td>
<td>0.08927</td>
<td>0.2439</td>
<td>1.705</td>
<td>( C_{ia} = 2900 )</td>
<td>J/k</td>
</tr>
<tr>
<td></td>
<td>( R_{jcdi} )</td>
<td>0.00202</td>
<td>0.0203</td>
<td>0.0640</td>
<td>0.0438</td>
<td>( R_{cs} + R_{ia} = 0.063 )</td>
<td>k/W</td>
</tr>
<tr>
<td></td>
<td>( C_{jcdi} )</td>
<td>0.0109</td>
<td>0.08927</td>
<td>0.2439</td>
<td>1.705</td>
<td>( C_{ia} = 2900 )</td>
<td>J/k</td>
</tr>
</tbody>
</table>

Fig. 5-7. Thermal impedance network from IGBT/diode junction temperature \( T_{iq} / T_{jd} \) - module case ambient \( T_c \) - heatsink temperature \( T_a \) – ambient temperature \( T_a \).
5.2.3 Lifetime Model for Bond-wire in Power Module

The time-domain results based on the previous model are shown in Fig. 5-8. The simulation started from input equivalent wave force $i_{fe}$ under irregular wave condition, and the input controlled current is the results of the generator speed $\omega_g$, according to the damping control parameters $R_{in}$ and power leveling factor $P_{lev}$ which will be derived in the following section 5.3. The output power $P_o$ from (5-5) is limited around $P_{lev}$. The junction temperature $T_{jq} = T_{jq1}$ and $T_{jd} = T_{jd1}$ are computed from the thermal and loss model in 5.2.2. The time-domain results of the junction temperature are converted to the amplitude $\Delta T_{jq}/\Delta T_{jd}$ and the mean value $T_{jqm}/T_{jdm}$ through rainflow calculation in [131] or the fatigue analysis rainflow function in Matlab [132] as shown in Fig. 5-9.

![Fig. 5-8. Simulation Results from circuit model Fig. 5-5 and equation (5-1)– (5-13) under $H_s = 1.75$ m, $T_e = 7.5$s, $R_{in} = 2$ $\Omega$, power leveling $P_{lev} = 60$ kW including input wave force $f_s$, the input current of the power converter $i_a$, output power of the power converter $P_o$, and the junction temperature of the IGBT module on body diode $T_{jd}$ and IGBT $T_{jq}$. (B.1 Setups of figures-(vii))](image-url)
Fig. 5-9. Thermal stress cycles $n_i$ of amplitude $\Delta T_{jd}$ ($\Delta T_{jq}$) and mean value $T_{jd\text{m}}$ ($T_{jq\text{m}}$) are derived from $T_{jd}$ ($T_{jq}$) in Fig. 5-8 through the rainflow calculation over 1800 s of the irregular wave cycle. The index $i$ of $n_i$ corresponding to a different set of $\Delta T_j$ and $T_{jm}$. Medium-term power cycles are the cycles with $10 \sim 200$ Hz from rotor speed with more cycles and lower amplitude; long-term I wave cycles are $0.1 \sim 1$ Hz from wave frequency with little cycles and higher thermal amplitude as shown in Table 5.1-1. (B.1 Setups of figures-(vii))

where from the number of cycles and the amplitude in both the average plot, the medium term and the long term-I can be separated as in Fig. 5-9(b). Here the long term-I is defined as $\Delta T_j > 12$ °C and cycles count $n_i < 10$, and medium-term are the rest of the stresses.

Modified Coffin-Manson’s model [133] is applied in this study for lifetime estimation in (5-14). The parameters of the bond-wire are assumed to be consistent with the detailed reliability of the power module from SEMIKRON in [134] as shown in Table 5.2-3. As a result, the relationship between the thermal stress amplitude $\Delta T_j$ and the cycles to failure $N_f$ under different mean junction temperature $T_{jm}$ are plotted in Fig. 5-10.

$$N_f = a\Delta T_j^{-\alpha} \cdot e^{\frac{E_a}{k_B(T_{jm}+273)}}$$  \hspace{1cm} (5-14)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$a$</th>
<th>$\alpha$</th>
<th>$E_a$</th>
<th>$k_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>$3.025 \cdot 10^5$</td>
<td>-5.039</td>
<td>$9.891 \cdot 10^{-20}$</td>
<td>$1.38065 \cdot 10^{-23}$</td>
</tr>
</tbody>
</table>
Fig. 5-10. Modified Coffin-Mason’s model of bond wires in the IGBT module. The number of cycles to failure $N_f$ depends on junction temperature amplitude $\Delta T_j$ and average junction temperature $T_{jm}$.

The consumed lifetime or percentage of damage $D$ is mapped from the thermal stress $\Delta T_{jq}$ and $T_{jqm}$ in Fig. 5-9(b) through the lifetime model (5-14) as shown in the following Fig. 5-11. The damage distribution between long term and medium term are told easily through the plot that most of the damage is accumulated under LgT-I. The accumulated damage plot in Fig. 5-11 clearly shows that under this specific wave condition and control parameters, most of the damage is from the time-scale of LgT-I. The damage under the time-scale MT is only around 10% of total damage over the irregular wave simulation period $T_{irr}$. The damage ratio of MT and LgT-I, or even the longer time-scale LgT-II, will be evaluated over other wave profiles and control setup.

Fig. 5-11. Damage from IGBT part junction temperature from the counts of thermal stress in Fig. 5-9(b). (a) damage $D$ (%) from Fig. 5-10 and (5-14). (b) damage of each $\Delta T_{jq}$, which is the damage summation of all the $T_{jqm}$ under each $\Delta T_{jq}$ value and the accumulated damage over $\Delta T_{jq}$. (B.1 Setups of figures-(vii))
The damage \( D \) of each thermal stress condition is defined as \( n_i / N_f \) in (5-15) as a ratio of the accumulative number \( n_i \) of certain thermal stress and the cycles to failure \( N_f \). The extended lifetime \( LT \) (yr) in (5-16) is the lifetime estimation under the repeated operating condition under the same irregular wave; the output power \( P_o \) (W) in (5-17) is average of \( P_o(t) \) over \( T_{irr} \); the output energy \( E_o \) (kWh) in (5-18) is energy with \( P_o \) under repeated irregular wave condition after lifetime \( LT \).

\[
D = \sum_{i=1}^{z} \frac{n_i}{N_f} \cdot 100(\%) \tag{5-15}
\]

\[
LT = \frac{T_{irr}}{D \cdot (365 \cdot 24 \cdot 60 \cdot 60)} \tag{5-16}
\]

\[
P_o = \int_{t=0}^{T_{irr}} P_o(t) \, dt / T_{irr} \tag{5-17}
\]

\[
E_o = LT \cdot P_o \cdot 365 \cdot 24 \cdot 10^{-3} \tag{5-18}
\]

The W2W model with the average power converter model in 5.2.1, the loss and thermal model in 5.2.2, and the lifetime model in 5.2.3 shows a complete model from wave condition to the lifetime of the power converter. The system control algorithm that constrained the thermal profile is explained in the following section.

### 5.3 Control Algorithms for the WEC Power Converter

The control algorithms in this chapter are developed to change the passive damping of the PTO and reshape the output power, so the maximum power is achieved, and the junction temperatures are limited. The controls are implemented under open-loop during simulation to reduce the computation effort and are converted to dq-frame through the transformation matrix \( T_{dq0/abc} \) in (5-19). The angle \( \theta \) is the electric angle of the generator, which can be derived from the \( \omega_e = p \cdot \omega_g \) or through sensorless control in PMSG [135].
\[
T_{dq0/abc} = \frac{2}{3} \begin{bmatrix}
\cos \theta & \cos(\theta - \frac{2}{3} \pi) & \cos(\theta + \frac{2}{3} \pi) \\
-\sin \theta & -\sin(\theta - \frac{2}{3} \pi) & -\sin(\theta + \frac{2}{3} \pi) \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{bmatrix}
\]  

(5-19)

The average circuit model in abc frame in Fig. 5-5 are converted to Fig. 5-12 under dq-frame with models of the generator and the power converter. The control algorithm in this session will be developed from the circuit model.

Fig. 5-12. Equivalent circuit model in dq-frame from the generator to the ac-dc converter based on the Fig. 5-5 abc-frame average circuit model with the transformation (5-19).

### 5.3.1 Open-loop Control Algorithms

Passive loading is implemented in (5-20) to derive the \( i_{q_{ref}} \) based on the [81] and [102] with the parameter \( R_{in} \) controlled under field-oriented algorithm by letting \( i_{d_{ref}} = 0 \) which provide the feature of maximum torque per ampere (MTPA) [123] that minimizes the loss on the electric conversion. The \( R_{in} \) can also be converted from the equivalent resistance of PTO damping \( R_{pto} \) in previous Chapter 4 as shown in (5-21). The power leveling and maximum torque limit the \( i_{q_{ref}} \) current level to \( i_{q_{max}} \) as in (5-22) and (5-23) where \( P_g \), which is limited by the \( P_{lev} \), is the output power of the generator before the copper/iron losses on the generator. The field-weakening control [136] is applied to increase the range of PTO force [137], and reduce the peak-average ratio of the power converter [44] through introducing a non-zero the \( i_{d_{ref}} \) as shown in (5-24), which is identical to the algorithm used in (4-5), determined by the input voltage level of the power converter \( v_{dq} \) and
the dc-link voltage $V_{dc}$. The open-loop duty cycles $d_d$ and $d_q$ are solved as (5-25) from the circuit model in Fig. 5-12 and the known $i_{dref}$ and $i_{qref}$.

$$i_{qref1} = \frac{E_q}{R_{in}} = -\frac{\omega_e \cdot k_e}{R_{in}}$$  \hspace{1cm} (5-20)

$$R_{in} = \frac{3p^2k_t \cdot k_e \cdot k_b^2}{8} R_{pto}$$  \hspace{1cm} (5-21)

$$P_g = \omega_g T_{eg}$$  \hspace{1cm} (5-22)

$$\begin{cases} 
  i_{qref} = i_{qref1} & \text{If } P_g < P_{lev} \text{ and } |i_{qref1}| < I_{qmax} \\
  i_{qref} = -\frac{2P_{lev}}{3\omega_e K_t} & \text{If } P_g \geq P_{lev} \\
  i_{qref} = I_{qmax} = -\frac{2T_{egmax}}{3pK_t} & \text{If } |i_{qref1}| \geq I_{qmax} 
\end{cases}$$  \hspace{1cm} (5-23)

$$\begin{cases} 
  i_{dref} = \frac{k_e}{L_i} + \sqrt{\frac{V_{dc}^2}{\sqrt{3} \omega_e} - (\mu_i i_{qref})^2} & \text{if } \sqrt{v_q^2 + v_d^2} \geq V_{dc}/\sqrt{3} \\
  i_{dref} = 0 & \text{if } \sqrt{v_q^2 + v_d^2} < V_{dc}/\sqrt{3} 
\end{cases}$$  \hspace{1cm} (5-24)

$$d_d = \frac{L_i \omega_e i_{qref} - R_i i_{dref}}{V_{dc}}$$  \hspace{1cm} (5-25)

$$d_q = \frac{E_q - R_i i_{qref} - L_i \omega_e i_{dref}}{V_{dc}}$$

where $E_d$ is assumed to be zero and $E_q = -pk_e\omega_g/2 = -k_e\omega_e$.

The control algorithm is concluded as a block diagram in Fig. 5-13. The output $d_{dq/abc}$ is converted by the inverse of $T_{dq0/abc}$ to $d_{abc}$ that defines the duty cycles of each phase of the power converter. The control diagram is similar to that previously shown in Fig. 4-5 but with more accurate loss prediction and with a smaller time step that shows the thermal fluctuation under line-frequency.
The relationship of the $T_{eg}$ and $\omega_g$ are derived in (5-26) and plotted as Fig. 5-14(a) showing that under the same $P_{lev}$, as the $\omega_g$ increases, the slope of the torque depends on the resistance $R_{in}$. With a higher value of the $\omega_g$, the torque will be clamped by the maximum torque $T_{eg\text{max}}$ or the power leveling $P_{lev}$. Fig. 5-14(b) shows that the power leveling change while under the same $T_{eg\text{max}}$ and $R_{in}$ value. Similar curves of the control in WEC are found to use in [93] and [81].

The relationship of the $i_{q\text{ref}}$ and $i_{d\text{ref}}$ is found in Fig. 5-15. As $\omega_g$ increases, $i_{q\text{ref}}$ will inversely increase as in (5-20) along the y-axis. In Fig. 5-15 (a) with $P_{lev} = 100kW$, it shows that before entering the constant power line, $i_{d\text{ref}}$ becomes a non-zero value, which means that the field-weakening control starts before the power leveling control. With lower values of $P_{lev}$ in Fig. 5-21(b), the power level, and the field-weakening will turn-on together. This can be explained by that when power leveling is activated, the damping of the PTO reduces, the generator speed increases, the input voltage to the power converter increases, so the field-weakening occurs to reduce the input voltage level.

$$
\begin{align*}
T_{eg} &= \frac{3p^2 K_t K_e \omega_g}{2 \cdot R_{in}} & \text{if } T_{eg} < T_{eg\text{max}} \text{ or } T_{eg} < \frac{P_{lev}}{\omega_g} \\
T_{eg} &= T_{eg\text{max}} & \text{if } T_{eg} > T_{eg\text{max}} \text{ or } T_{eg} < \frac{P_{lev}}{\omega_m} \\
T_{eg} &= \frac{P_{lev}}{\omega_m} & \text{if } T_{eg} > \frac{P_{lev}}{\omega_m}
\end{align*}
$$

(5-26)
\( T_{\text{eg}} = 1000 \text{Nm} \)
\( P_{\text{lev}} = 100 \text{kW} \)
\( R_{\text{in}} = 1 \text{– 50} \Omega \)
\( R_{\text{in}} = 1 \Omega \)
\( R_{\text{in}} = 50 \Omega \)

Constant power (5-23)
Constant Torque (5-23)
Passive damping (5-21)

\( P_{\text{lev}} = 40 \text{– 100} \text{kW} \)

Field weakening + Constant power
Field weakening + Constant Torque
Passive damping

\( I_{q_{\text{max}}} = 135 \text{A} \)
\( V_{dc} = 800 \text{V} \)
\( P_{\text{lev}} = 100 \text{kW} \)
\( R_{\text{in}} = 50 \Omega \)
\( R_{\text{in}} = 1 \text{– 50} \Omega \)
\( R_{\text{in}} = 2 \Omega \)

Fig. 5-14. Mechanical torque \( T_{\text{eg}} \) and rotor speed \( \omega_{\text{g}} \) curve from (5-26) when \( \omega_{\text{g}} = 500 \text{– } -500 \text{ rad/s} \) and \( T_{\text{egmax}} = 1000 \text{Nm} \); (a) with constant power leveling control \( (5-23) \) \( P_{\text{lev}} = 100 \text{kW} \) and variable passive loading control (5-20) \( R_{\text{in}} = 1 \text{– 50} \Omega \); (b) with constant passive loading \( R_{\text{in}} = 2 \Omega \), and variable power leveling \( P_{\text{lev}} = 40 \text{ – 100 kW} \). (B.1 Setups of figures-(vii))

Fig. 5-15. \( i_{d_{\text{ref}}} \) and \( i_{q_{\text{ref}}} \) curve from (5-20) – (5-24) when \( I_{q_{\text{max}}} = 135 \text{A} \), \( V_{dc} = 800 \text{V} \) (a) curve with constant \( P_{\text{lev}} = 100 \text{kW} \), and variable \( R_{\text{in}} = 1 \text{– 50} \Omega \) (b) curve with constant \( R_{\text{in}} = 2 \Omega \), and variable \( P_{\text{lev}} = 40 \text{ – 100 kW} \). (B.1 Setups of figures-(vii))

5.3.2 Selection of Variables Range

Two control variables are used to control the power and the reliability of the converter: \( P_{\text{lev}} \) and \( R_{\text{in}} \). The range of the power leveling parameters \( P_{\text{lev}} \) is determined by the rating of the power converter which is 200 kVA in this case, so the maximum value of \( P_{\text{lev}} \) starts from half of its maximum rating value to ensure some design margin for the power rating where five values are selected as \( P_{\text{lev}} = [40 \text{ 50 60 75 100}] \text{kW} \).
The $R_{in}$ value is similar to the method to select $R_{pto}$ in the previous chapter, which is based on the $R_{pto}$ or $R_{in\_match}$ in this case as in (5-27). The value of $Z_{wec}$ is found in Table 4.2-1, and $k_p$ is selected as 104 from Table 5.2-1. The WEC impedance with the frequency of interest from 1/3.5 – 1/8.5 second is plotted in Fig. 5-16 which leads to the range of $R_{in\_match}$ between 1.76 – 18.8 Ω. Considering the nonlinearity from the control and the losses, $R_{in}$ for the maximum output power will not be identical to the $R_{in\_match}$ as in [102] and [81]. Also, the minimum of $R_{in}$ is recommended to be ten times higher than the internal resistance of the generator $R_i$ to ensure the generator is working under a high-efficiency region. Eleven $R_{in}$ values will be test $Rin = [1, 2, 3, 5, 7.5, 10, 12.5, 15, 20, 30, 50]$.

$$R_{in\_match} = k_p^2 \frac{3p^2k_i}{8} |Z_{wec}(j\omega)|$$  (5-27)

Fig. 5-16. The 5-m buoy WEC impedance $Z_{wec}(j\omega)$ the same as Fig. 4-7 with the frequency of interest from 3.5 – 8.5 s in this chapter. The corresponding $R_{in} = |Z_{wec}(j\omega)|$ for optimal power is 1.76 – 18.8 Ω.

5.3.3 Implementation of Control Algorithms

The control algorithms in (5-20) - (5-25) are conducted in real-time throughout the simulation in the MATLAB 2019a with embedded PLECS blockset [138] in the Simulink platform. The time-domain results in Fig. 5-17 shows the results from the generator speed to the electromagnetic torque $T_{eg}$ and the reference current $i_{dqref}$ under the power leveling of 60 kW and $R_{in}$ of 2 Ω. It can be seen that $T_{eg}$ is clamped around ± 1000 Nm and $i_{qref}$ around ± 135 A by maximum torque
limitation in (5-23). $I_{dref}$ becomes negative when $\omega_g > 100 \text{ rad/s}$ from (5-24), and this will limit the $v_{dq}$ to $V_{dc}/3^{0.5}$.

Fig. 5-17. Simulation Results by implementing the wave from Fig. 5-8, open-loop control in Fig. 5-13, and control algorithms (5-20) - (5-26) under $H_t = 1.75 \text{ m}$, $T_e = 7.5 \text{ s}$, $R_m = 2 \Omega$, power leveling $P_{lev} = 60 \text{ kW}$. The results include the generator rotor speed $\omega_g$, mechanical torque $T_{eg}$, and converter current in dq-frame $i_d$ and $i_q$. (B.1 Setups of figures-(vii))

The time-domain simulation is applied to each control variables and takes around 30 minutes for each irregular wave simulation period of 1800 s with a time step of 1 ms. As a result, just for one set of variables to simulate over annual wave profile, the computational time will be at least a year which is not realistic for the study, so the categorization of wave conditions as well as the parallel computing is applied as in this study to reduce the simulation time.
5.4 Maximum Energy Algorithms for Wave Power Converter

5.4.1 Power Map from Ocean Wave Site

The wave profile from NOAA’s buoy data is selected as shown in Fig. 5-18, including the sampling data for every 30 minutes. The WVHT is the significant wave height $H_s$, APD is the average wave period $T_e$, and WTMP is the sea surface temperature as the ambient temperature $T_a$ of the power converter. The annual wave profile is converted into a mission profile figure as shown in Fig. 5-19 covering the data in 2015. As in Table 4.3-2, the wave conditions are discretized into 28 different sets of data, and both of its sequences and cycles for each condition are plotted as Fig. 5-20. These include the most frequent data, and 94.3% of the annual wave profile is covered. The wave that is not covered in the categorized points will be tested as to its adjacent condition with the same wave height or the same wave period.

Each wave condition is converted into time domain through the Pierson-Moskovitz irregular wave spectrum in (3-6) - (3-8) for 1800 seconds.

Fig. 5-18. The map of buoy site 44056 from NOAA and the example of raw data in 2015 [95].
Fig. 5-19. Wave profile with 30 minutes of the sampling rate of site 44056 in 2015 including (a) significant wave height $H_s (m)$ and average wave period $T_e (s)$, and (b) ambient temperature $T_a (^\circ C)$.

Fig. 5-20. Wave conditions $w$ are discretized into $w = 1:28$ with the resolution of 0.5 m for $H_s$ and the resolution of 1 s for $T_e$. The cycles of each condition are found in (a), and the corresponding number of $w$ for each wave is in (b). The probability of each wave can be found in Table 4.3-2.

5.4.2 Construction of Lookup Tables from Selected Wave Site

The multi-objective optimization flowchart in Fig. 5-21 is introduced based on the numerical algorithm to search for the optimum point that generates the maximum energy. The products of which are 4-Dimensional lookup tables as in Fig. 5-23, which is later used to predict the energy and average power for each set of variables. The flowchart shows the loop from each set of
variables \([P_{levj}, R_{ini}, f_{ew}(t)]\) with the output data of the simulation period \(T_{irr}\), and the average values or the accumulated damage \(D\) (or extended lifetime \(LT\)) through counting stress data from the rainflow algorithm are stored as a data point in each 4-D table consisting of three indices \([i,j,w]\).

Fig. 5-21. Flowchart for power, energy, and damage for each wave condition and control variable. 4-D Lookup tables of \(P_{i,j,w}, E_{i,j,w}, D_{i,j,w}, T_{jq,i,j,w}, T_{jd,i,j,w}\) are produced through the flowchart corresponding to each wave condition \((H_{sw}, T_{ew})\), passive damping \(R_{ini}\), and power leveling \(P_{levj}\).
The flowchart in this study, the size of variables is set at 55 with \( m = 11 \) and \( n = 5 \). Each variable is tested under 28 wave conditions, so the total cycles of iterations are 1540. Each iteration takes around 30 minutes on a quad-core PC. The simulation process original will take more than 32 days to finish, so parallel computing is used through the \textit{parfor} function in MATLAB along with the \textit{sim} function to call Simulink in each loop [139]. The system managed to reduce the simulation time by a factor of at least 28 under the high-performance computer (HPC) at Virginia Tech [140], and the process can be further reduced by breaking the tasks into multiple nodes.

### 5.4.3 Algorithms for Maximum Power, Maximum Energy, and Maximum Power with Lifetime Constraints

The maximum energy and power algorithms are implemented based on the energy and power data from Fig. 5-21. The following setup in Table 5.4-1 is used as an example of the methods.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u )</td>
<td>Numbers of wave condition ( Hs_w ) and ( Te_w )</td>
<td>28</td>
</tr>
<tr>
<td>( m )</td>
<td>Numbers of ( R_{in_i} )</td>
<td>11</td>
</tr>
<tr>
<td>( n )</td>
<td>Numbers of ( P_{lev_j} )</td>
<td>5</td>
</tr>
<tr>
<td>( T_{irr} )</td>
<td>The irregular wave simulation period</td>
<td>1800 s</td>
</tr>
<tr>
<td>( R_{in_i} )</td>
<td>([1 2 3 5 7.5 10 12.5 15 20 30 50]) (( \Omega ))</td>
<td></td>
</tr>
<tr>
<td>( P_{lev_j} )</td>
<td>([40 50 60 75 100]) (kW)</td>
<td></td>
</tr>
<tr>
<td>( Hs_w, Te_w, p_w )</td>
<td>Table 4-5 and Fig. 5-20 (m), (s), (%)</td>
<td></td>
</tr>
</tbody>
</table>

#### 5.4.3.1 Maximum Power and Maximum Energy (MaxP and MaxE)

As shown in Fig. 5-22, the 3-D lookup tables of the energy to its corresponding damping resistance and power leveling is built for each of the 28 wave condition. The level of \( E_o \) is different in each A 4-D map is formed by placing each 3-D table coordinate with the wave height and the wave period at the exterior axis. The maximum energy point is selected among each 3-D table for
each wave condition as the MaxE red point in Fig. 5-22, which has the value of $E_{o_{iE,jE,w}}$ in (5-28). The equation shows that the point maximum $E_o$ occurs for each $w$, and is the corresponding maximum damping $Rin^E_w = Rin_{iE,jE,w}$ as well as $Plev^E_w = Plev_{iE,jE,w}$. Both $Rin^E_w$ and $Plev^E_w$ are the indices for the maximum energy algorithm, as shown in Fig. 5-23(a).

![Maximum Energy Map](image)

**Fig. 5-22.** The output 4-D lookup table of energy $E_o$ derived through Fig. 5-21 and the setup in Table 5.4-1 for maximum energy algorithm. (B.1 Setups of figures-(vii))
\[
E_{o_{i,E,j,w}} = \max_{i=1}^{m} (E_{o_{i,j,w}}) = E_{o_{w}}^{E}, \text{ for } w = 1:6
\] (5-28)

\[
R_{in}^{F_{w}} = R_{in}^{E_{i,E,j,w}}, \quad P_{lev}^{E_{w}} = P_{lev}^{E_{i,E,j,w}} : \quad P_{o_{w}}^{E} = P_{o_{i,E,j,w}}
\]

The maximum output power points are derived similarly to the maximum energy points. The corresponding damping \(R_{in}^{P_{w}}\) and the power leveling \(P_{lev}^{P_{w}}\) from the 4-D lookup tables based on the equation (5-29) are used as indices for the maximum power points in Fig. 5-23.

\[
P_{o_{i,P,j,w}} = \max_{i=1}^{m} (P_{o_{i,j,w}}) = P_{o_{w}}^{P}, \text{ for } w = 1:6
\] (5-29)

\[
R_{in}^{P_{w}} = R_{in}^{E_{i,P,j,w}}, \quad P_{lev}^{P_{w}} = P_{lev}^{E_{i,P,j,w}} : \quad E_{o_{w}}^{P} = E_{o_{i,P,j,w}}
\]

where the output values \(P_{o}\) and \(E_{o}\), including \(LT\) and \(D\), are from the extended value defined in (5-15) - (5-18).

As a result, the indices of MaxE and maximum power algorithm (MaxP) are plotted in Fig. 5-23(a) and (b) showing that for each wave condition, there is a set of best variables \(R_{in}\) and \(P_{lev}\), which point to the maximum result. The rest of the 4-D lookup tables \(D_{i,j,w}, T_{j\delta_{i,j,w}}\), and \(T_{j\delta_{i,j,w}}\) will be used in the average power/energy/lifetime estimation over the annual profile or in the development of the constraint maximum power algorithm in the following section.

The estimated average output power \(P_{o_{est}}\), and total energy \(E_{o_{est}}\) are then derived in (5-31) and (5-32) based on the possibility of each wave condition \(p_{w}\) listed in Table 4.3-2. The estimated lifetime \(LT_{est}\) for each algorithm is calculated from the damage data \(D_{i,j,w}\) in (5-30).

\[
LT_{est} = \frac{T_{i\delta_{1}}}{{\sum}_{w=1}^{u} D_{i,j,w} \cdot p_{w} / {\sum}_{w=1}^{u} p_{w}} 365 \cdot 24 \cdot 60 \cdot 60
\] (5-30)

\[
P_{o_{est}} = \frac{\sum_{w=1}^{u} P_{o_{i,j,w}} \cdot p_{w}}{\sum_{w=1}^{u} p_{w}}
\] (5-31)

\[
E_{o_{est}} = 365 \cdot 24 \cdot P_{o_{est}} \cdot LT_{est}
\] (5-32)
Fig. 5-23. 4-D lookup tables from the flowchart in Fig. 5-21 with the values in Table 4.3-2, Table 5.2-1, and Table 5.4-1; (a) indices of maximum energy algorithm from the $E_{0_{ij,w}}$ lookup tables based on (5-28) where $\text{Rin}^E_w$ and $\text{Plev}^E_w$ are the values of variables for maximum energy for each wave condition $w = 1 - 28$; (b) indices of maximum energy algorithm from the $P_{0_{ij,w}}$ lookup tables based on (5-29) where $\text{Rin}^P_w$ and $\text{Plev}^P_w$ are the values of variables for maximum power for each wave condition $w = 1 - 28$. (B.1 Setups of figures-(vii))

### 5.4.3.2 Maximum Power with Lifetime Constraints (MaxPLT) from Genetic Algorithm

In the previous example, for each wave condition, there are 55 possible variables, so this gives a total number of $28^{55}$ possible solutions. The Genetic Algorithm (GA) function is applied [141] to search for the maximum power that meets the minimum lifetime constraint $LT_{min}$. The setup of GA is shown in Table 5.4-2 with the function $ga$ in the MATLAB optimization toolbox [142].

<table>
<thead>
<tr>
<th>GA parameters</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Generation, MG</td>
<td>1800</td>
</tr>
<tr>
<td>Population, Pp</td>
<td>280</td>
</tr>
<tr>
<td>Elite rate, ER</td>
<td>5 %</td>
</tr>
<tr>
<td>Crossover rate, CR</td>
<td>80 %</td>
</tr>
<tr>
<td>Mutation rate, MR</td>
<td>15 %</td>
</tr>
<tr>
<td>Maximum stall generation</td>
<td>80</td>
</tr>
<tr>
<td>Function tolerance</td>
<td>$1e^{-6}$</td>
</tr>
</tbody>
</table>

Fig. 5-24. Searching results of the genetic algorithm showing the generation and the penalty value.

<table>
<thead>
<tr>
<th>Table 5.4-2. Setup parameters for genetic algorithm function.</th>
</tr>
</thead>
<tbody>
<tr>
<td>GA parameters</td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>Maximum Generation, MG</td>
</tr>
<tr>
<td>Population, Pp</td>
</tr>
<tr>
<td>Elite rate, ER</td>
</tr>
<tr>
<td>Crossover rate, CR</td>
</tr>
<tr>
<td>Mutation rate, MR</td>
</tr>
<tr>
<td>Maximum stall generation</td>
</tr>
<tr>
<td>Function tolerance</td>
</tr>
</tbody>
</table>
Randomize initial $R_{in}, P_{lev}$ variables

Initialize GA options

Current $(R_{in}, P_{lev})$
set with # of u*Pp

$(R_{in}^{(a,w,y)}, P_{lev}^{(a,w,y)})$

$P_{o}^{(a,y)}$, $LT_{x}^{(a,y)}$ generated from LUT in Fig. 5.23, (5-30) and (5-31)

All $LT_{x}^{(a,y)} > LT_{min}$

Constraint Reproduction $LT_{x}^{(a,y)} > LT_{min}$

Highest Pp*ER of $P_{o}^{(a,y)}$

# of Crossover Children $= P_{p}*CR$

# Mutation Children $= P_{p}*MR$

Generation $x$ Total population $= P_{p}$

Stop criteria:
Function tolerance after SG generations $< \Delta f$?
or Generation $x > MG$?

yes

Maximum descendant $= f_{max}$
Maxium variables $= (R_{in}^{f}, P_{lev}^{f})$

$(R_{in}^{f}, P_{lev}^{f}) \in (R^{x*P_{p}}, R^{x*P_{p}})$

$\alpha > g_{max}$

and $f_{max} < \min(f_{max})$

no

$\alpha + 1$

yes

$(R_{in}^{PLT}, P_{lev}^{PLT}) = (R_{in}^{f}, P_{lev}^{f})$
$P_{max}$ with $LT_{min}$ constraint $= f_{max}$

$x = x + 1$

$x$: Parents generation

$x+1$: Children generation

Fig. 5.25. The searching process for the indices $R_{in}^{PLT}$ and $P_{lev}^{PLT}$ of the maximum power algorithm with lifetime constraint (MaxPLT) from the genetic algorithm.
The searching process of MaxPLT from the GA and their corresponding values and variables are found in Fig. 5-25 which shows that $P_{o^\alpha}$ is the output criteria to compare among each generation $x$, and each child $y0$ in the generation has $u$ pairs of ($w=1:u$, $u=28$ in the example)gene sets ($Rin_{w,y0}^\alpha$, $Plev_{w,y0}^\alpha$). The GA will continue for several runs with different initial data to avoid the local maximum point until iteration $a$ reaches the maximum GA loop $g_{max}$, and $f_{max}$ has the highest output among all the GA outputs. The optimal indices $Rin_{PLT}$ and $Plev_{PLT}$ for $LT_{min} = 20$ years are shown in Fig. 5-26.

Fig. 5-26. The indices for maximum power with a 20-year lifetime constraint algorithm $Rin_{PLT}$ and $Plev_{PLT}$ are derived based on (5-30), (5-31), and the genetic algorithm (GA). The GA is implemented so that the selected $R_{in}$ and $P_{lev}$ can meet the expected lifetime $LT_{est}$ from the wave probability $p_w$, and maximize the estimated output power $P_{o_{est}}$.

The results of three algorithms in this session are listed in Table 5.4-3. MaxP has the highest output power while the lowest lifetime only 3.5 years. With the MaxE algorithm, the total energy is significantly increased by 16.5 times under around 60 % of the average annual power per year, and the lifetime is 96 years. The MaxPLT algorithm limits the lifetime at the designed value 20 years while maintaining the power level to 94 % of that from the MaxP.
Table 5.4-3. Maximum Energy, power and Lifetime over different Optimization algorithms. (MaxP, $P_{sat} = 50$ kW is used as a baseline)

<table>
<thead>
<tr>
<th></th>
<th>$E_{o,mp}$ (MWh)</th>
<th>$P_{o,mp}$ (kW)</th>
<th>$LT_{mp}$ (yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MaxP</td>
<td>0.32 (1.0)</td>
<td>10258 (1.0)</td>
<td>3.5 (1.0)</td>
</tr>
<tr>
<td>MaxE</td>
<td>5.28 (16.5)</td>
<td>6254 (0.61)</td>
<td>96 (27.4)</td>
</tr>
<tr>
<td>MaxPLT, $LT_{min} = 20$yr</td>
<td>1.70 (5.3)</td>
<td>9623 (0.94)</td>
<td>20.2 (5.7)</td>
</tr>
</tbody>
</table>

5.5 Implementation and Discussion of Optimization Algorithms

5.5.1 Outputs with Annual Mission Profiles and Ambient Temperatures

The output results from (5-30) - (5-32) are still estimated value based on the occurrence rate of each wave under constant ambient temperature $T_a = T_{df}$, while from the wave data in Fig. 5-19, the amplitude of $T_a$ is around 26 °C from 2 – 28 °C which brings a great difference for the accumulated damage according to

$$N_f = a \Delta T_j^{-\alpha} \cdot \exp(E_a/k_B(T_{jm} + 273))$$

in (5-14). For example, under the same $\Delta T_j$, the mean junction temperature $T_{jm}$ changes from 100 °C to 126 °C, and as a result, $N_{f,126°C} = N_{f,100°C}/3.5$ this will increase the damage $D = n/N_f$ by 3.5 times due to the higher ambient temperature. The flowchart in Fig. 5-27 shows the re-evaluate process that considers the ambient temperature profile $t_{a_k}$ along with the wave profile $h_{S_k}$ and $t_{e_k}$ where $k$ is the index for each 30 minutes time step with the total of $n_{yr} = 17520$ points for a year. The constant ambient temperature $T_{df} = 20$ °C, and the damage ratio $rT_{q_k}$ and $rT_{d_k}$ are derived from (5-33) according to the temperature difference $\Delta t_{a_k}$. The modified damaged ratio $D_{q_k}'$ and $D_{d_k}'$ are calculated from (5-34) and be used in the lifetime $LT_{o,mp}$ and energy $E_{o,mp}$ calculation from the real mission profile.

$$rT_{q_k} = e^{\frac{E_a}{k_B(T_{q_k} + \Delta T_a)} - \frac{1}{T_{q_k}}} \cdot rT_{d_k} = e^{\frac{E_a}{k_B(T_{d_k} + \Delta T_a)} - \frac{1}{T_{d_k}}}$$

(5-33)

$$D_{q_k}' = D_{q_k} \cdot rT_{q_k}, \quad D_{d_k}' = D_{d_k} \cdot rT_{d_k}$$

(5-34)
\[ LT_{mp} = \frac{1}{\max(\sum_{k=1}^{nyr} Dq'_{k}, \sum_{k=1}^{nyr} Dd'_{k})} \]  
(5-35)

\[ P_{o,mp} = \left( \sum_{k=1}^{nyr} P_{0k} \right) / n_{yr} \]  
(5-36)

\[ E_{o,mp} = 365 \cdot 24 \cdot P_{o,mp} \cdot LT_{mp} \]  
(5-37)

---

**Fig. 5-27. Flowchart for \( P_{o,mp}, E_{o,mp}, \) and \( LT_{mp} \) from annual wave mission profiles with \( H_s, T_e, T_a \) in Fig. 5-19 by the algorithms MaxE in Fig. 5-23(a), MaxP in Fig. 5-23(a), or MaxP+LT in Fig. 5-25Fig. 5-26 from the annual wave profile.**
The flowchart for mission profile calculation is processed from the lookup tables with computation time less than one minute, and no circuit simulation in time-domain is required. The results of output power and damage from the annual mission profile under the algorithm MaxPLT are plotted in Fig. 5-28.

\[ P_{o_k} \quad D_{q_k}' \]

Fig. 5-28. The calculated results of annual \( P_{o_k} \) and \( D_{q_k}' \) from the flowchart Fig. 5-27 and wave data Fig. 5-19.

### 5.5.2 Parametric Analysis of Optimization Variables

The \( P_o \) and \( E_o \) curve under constant \( w = 15 \) and varying \( R_{in} \) and \( P_{lev} \) are shown in the following Fig. 5-29 where the damping of maximum power point is around \( R_{in} = 2 \ \Omega \) which is slightly smaller than the ideal value \( R_{in,\text{match}} = 3.16 \ \Omega \) due to the losses and nonlinear control from the power limitation. The damping for the maximum energy point is around \( R_{in} = 12.5 \ \Omega \) which has only around half of the output power \( P_o \) when \( R_{in} = 2 \ \Omega \), while the energy \( E_o \) is about four times more than that at the maximum power damping. The maximum power point happens at the highest the \( P_{lev} = 100 \ \text{kW} \), but the maximum energy point occurs when \( P_{lev} = 60 \ \text{kW} \), which is consistent with the 4-D lookup table of \( E_o \) in Fig. 5-22 that most of the maximum energy occurs when its \( P_{lev} \) is around 60 kW or 50 kW.
The performance comparison of the three algorithms MaxP, MaxE, and MaxPLT are plotted in Fig. 5-30 to show their output power and accumulated damage over each wave condition. It can be seen from Fig. 5-30(a) that the accumulated damage of MaxP starts to rise drastically on the more frequent wave \((H_s, T_e) = (1.75 \text{ m}, 6.5 \text{ s})\), and on some less frequent but stronger wave condition like \((2.75 \text{ m}, 5.5 \text{ s})\), \((2.75 \text{ m}, 6.5 \text{ s})\), \((3.25 \text{ m}, 7.5 \text{ s})\), and \((3.75 \text{ m}, 7.5 \text{ s})\). In this case, if the damage is excluded from the wave beyond the 2.75-m wave height, the lifetime under MaxP will increase from 3 years to 15 years, and from Fig. 5-30(a) the output power will drop from 10 kW to 9 kW. This also shows the possibility of a trade-off between the lifetime and the output power by deactivating the converter during some stronger wave conditions. The trend of the accumulated output power curves in Fig. 5-30(b) points out that energy is mostly from the most frequency wave condition when the wave height is \(0.75 – 1.25 \text{ meters}\). Not much energy from the high waves with \(2.75 \text{ m} – 3.75 \text{ m}\), but these high waves bring around \(2/3\) of damage in the MaxP algorithm. The MaxE and MaxPLT algorithms have a more balanced energy/damage ratio, i.e., most of the damages are from the waves that bring energy to the system.
Fig. 5-30. Damage, lifetime, power from different algorithms; (a) damages on wave conditions by implementing maximum power algorithm (MaxP) are shown as bars; accumulated damage are dotted lines, and expected lifetime solid lines from different algorithms; (b) average powers on wave conditions are shown as bars, accumulated power from by considering each wave condition are shown in different formats of lines. (B.1 Setups of figures-(vii))

Base on the categorization of the damage from different time-scale in Table 5.1-1, the influences from different loading sources are identified in Fig. 5-31 and Table 5.5-1 under different
control algorithms. The MT from line frequency and the LgT-I from wave-wave frequency are derived from the flowchart in Fig. 5-21, and the LgT-II from different ambient temperatures or wave conditions with a period of 30 min – hours are derived through the flowchart of the mission profile in Fig. 5-27. The damages from LgT-I are all above 90% among three time-scales, which shows that most of the damages are from the irregular wave period in the range of 1 – 10 seconds.

Fig. 5-31. Damage on IGBT and body diode with different algorithms, and the distribution of medium-term (MT), long term-I (LgT-I), and long term-II (LgT-II) damages. (B.1 Setups of figures-(vii))

Table 5.5-1. Damage from different time-scale under three control algorithms.

<table>
<thead>
<tr>
<th></th>
<th>MT (%)</th>
<th>LgT-I (%)</th>
<th>LgT-II (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MaxP</td>
<td>IGBT</td>
<td>0.9</td>
<td>8.3</td>
</tr>
<tr>
<td></td>
<td>Diode</td>
<td>2.2</td>
<td>33</td>
</tr>
<tr>
<td>MaxE</td>
<td>IGBT</td>
<td>0.03</td>
<td>1.18</td>
</tr>
<tr>
<td></td>
<td>Diode</td>
<td>0.05</td>
<td>1.28</td>
</tr>
<tr>
<td>MaxPLT</td>
<td>IGBT</td>
<td>0.25</td>
<td>2.06</td>
</tr>
<tr>
<td></td>
<td>Diode</td>
<td>0.4</td>
<td>5.05</td>
</tr>
</tbody>
</table>
5.6 Conclusion

The W2W circuit model provides a tool to conduct the multi-physics analysis which shows the existence of maximum energy points. The methodologies are elaborated in the flowcharts that through building 4-D lookup tables, the maximum energy algorithm is developed to increase the total energy by 16 times. The lifetime of the maximum power algorithm is extended from 3.5 years to 20 years with the algorithm with a lifetime limitation, which is developed by the genetic algorithm with estimated results. The actual results from the mission profile flowchart Fig. 5-27 is shown in Fig. 5-32 that around 10 % of the difference in the results of lifetime and energy for the estimated results as listed in Table 5.6-1. This shows that by assuming constant ambient temperature \( T_{df} \), the output estimation in (5-30) - (5-32) provides a closed approximation on which the optimization algorithms are developed.

![Graph](image)

**Fig. 5-32.** The results of \( P_0, E_0 \), and \( LT \) from the mission profile flowchart in Fig. 5-27 comparisons among optimization algorithms. (B.1 Setups of figures-(vii))
Table 5.6-1. Difference between estimated and mission profile results.

<table>
<thead>
<tr>
<th></th>
<th>Estimated results with $p_w$ and constant $T_{df}$ (5-30) - (5-32)</th>
<th>Mission profile results with $t_a$, $h_s$, and $t_e$ (Fig. 5-27)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_o$ (kW)</td>
<td>LT (yr)</td>
<td>$E_o$ (MWh)</td>
</tr>
<tr>
<td>MaxP</td>
<td>10.26</td>
<td>3.5</td>
</tr>
<tr>
<td>MaxE</td>
<td>6.25</td>
<td>96.3</td>
</tr>
<tr>
<td>MaxPLT</td>
<td>9.62</td>
<td>20.1</td>
</tr>
</tbody>
</table>

The main benefit of using output estimation and 4-D LUT is to reduce the simulation effort by skipping the complete mission profile in the optimization process. The total computation time of the optimization is reduced to 770 hours in the example of categorizing the wave into 28 discrete wave conditions, and it is further reduced to 27.5 with the parallel computing under the HPC with 28 cores as listed in Table 5.6-2.

Table 5.6-2. Comparison of simulation time in the optimization process.

<table>
<thead>
<tr>
<th></th>
<th>Computation time (hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulation time of each condition = 30 minute</td>
<td></td>
</tr>
<tr>
<td>Total conditions = 11 * 5 * 17520</td>
<td></td>
</tr>
<tr>
<td>Original optimization process with annual mission profile and optimization variables</td>
<td>481800</td>
</tr>
<tr>
<td>Optimization with the output estimation, wave categorization, and 4-D LUT</td>
<td>770</td>
</tr>
<tr>
<td>Optimization with the parallel computing, output estimation, wave categorization, and 4-D LUT</td>
<td>27.5</td>
</tr>
</tbody>
</table>
Chapter 6 Conclusions and Future Work

6.1 Work Summary

The wave energy converter system involves different disciplines, including hydrodynamics, mechanics, electromagnetism, electronics, and thermoelectrics. This multidisciplinary system needs a common platform to optimize the design and control of the power electronics system. Different from conventional equation-based W2W model [7] or bond graph for physics system [143] - [144], a unified circuit-based model provides a tool to optimize the renewable energy system from a circuit perspective and serves as a bridge bringing the different research fields together. The circuit analogies of the system are developed through network synthesis, transfer-function analogies, and mode separation of an equivalent circuit, which coordinates with the order of the chapters in this dissertation. The optimization of the power converter design and control algorithm are demonstrated to show the capability of the circuit-based model.

The optimization process of the overall WEC system is illustrated in Fig. 6-1 as four design/modeling stages and three optimization steps. Each step includes current and previous stages to reduce the time-scale difference between the models. The concepts can also be applied to other renewable systems or a multidisciplinary system using a power electronics converter.

Fig. 6-1. The optimization process of the overall wave energy converter system.
The MMR-PTO model in Chapter 2 provides a non-ideal circuit model for the mechanical PTO in the WEC system. Each standard mechanical building blocks are represented by equivalent circuit components, which describe the dry friction, viscous dampings, and compliances in the mechanical components. The non-ideal efficiency and force of the PTO can be predicted in an electrical simulation through integrating the electrical components in parallel or series. The unknowns in the RLC components are identified through comparing the dc and ac equivalent circuit models and the experimental results from the compression test. The parameters of the model are found through linear regression and nonlinear curve-fitting, and the model is validated by the time domain results showing over 90% of $R^2$ value. The predictions on the PTO efficiencies under 1-Hz frequency show efficiencies ranging from 70% to 30% with a correlation coefficient rate of 95%.

A methodology to develop a circuit-based W2W model for wave energy converters is presented in Chapter 3. The time-invariant circuit represents the fluid-body dynamics through mechanical-electrical analogies, transfer function approximation, and Brune network synthesis for irregular wave analysis. The circuit-based W2W model is verified by comparing the results with the predictions of a detailed model under irregular wave conditions in the time and frequency domains. The model is applied to reliability analysis and linear damping control under irregular wave conditions with an example of a point-absorber wave energy converter (WEC) with a direct-drive power take-off (PTO). The results show that the W2W model gives an accurate dynamic prediction for a WEC under both regular and irregular conditions.

With a high peak-average power ratio from the irregular wave, the design of power converter and generator tend to overrate to a level of 20. The design optimization process in Chapter 4 utilizes the circuit model in Chapter 2 and 3, and the optimization of the power electronics rating with a
large amount of data from the annual mission profile is made possible under the second-level time-
scale by simplifying the power electronics converter model. The gear ratio of the PTO, the PTO
damping, and the power level coefficient are used as control variables for the numerical
optimization process. The minimum value \((U_{sp} + U_{gp})/2\) as the ratio between the average output
power \(P_{oe}\) and the converter rating \(S_{rated}\) (or the generator rating \(P_{grated}\)) is defined as the optimal
figure of merit for the selection of the converter and generator. The selected power rating of the
converter shows an over-rating factor \(U_{sp,w28}\) of 2.4 under the highest wave condition at \(H_s = 3.75\)
m, \(T_e = 7.5\) s.

Based on the selected hardware design from Chapter 4, the control algorithm is optimized in
Chapter 5, considering the reliability of the power converter and the generator control under the
time resolution of 1ms. The output energy, as a product of the output power and the lifetime, is
used as a criterion in the optimization process. The simplification techniques are introduced to
reduce the computation effort: the estimation process assuming constant ambient temperature, the
4-D lookup tables, wave categorization, and parallel computing, to handle the significant amount
of data from an annual mission profile in the mini-second time-scale. The energy level is increased
by around 16 times with maximum energy algorithm, and the maximum power with lifetime
constraint increases the lifetime from 3 years to 20 years while only losing the average power by
6%.

The developed WEC systems and their work scopes are listed in Tables 6-1. The models
developed in Chapters 2 and 3 are based on the WEC and the MMR-PTO model from the 1:20\textsuperscript{th}
scale WEC system. The optimization processes for the power converter in Chapters 4 and 5 are
derived from the 1:4.5\textsuperscript{th} scale, 5-m buoy WEC.
Table 6.1-1. Summarize of developed WEC prototype in this dissertation.

<table>
<thead>
<tr>
<th>WEC system</th>
<th>Diameter</th>
<th>Power</th>
<th>PTO test</th>
<th>Tank test</th>
<th>Power electronics test</th>
<th>Chapter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1:30 scale</td>
<td>0.75 m</td>
<td>50 W</td>
<td>Lab</td>
<td>1DoF, 2DoF, 6DoF [145]</td>
<td>N/A</td>
<td>[146]</td>
</tr>
<tr>
<td>1:20 scale</td>
<td>1.2 m</td>
<td>500 W</td>
<td>Lab [49]</td>
<td>1DoF [92]</td>
<td>500 W, 2L-3P ac-dc (WEC+PE)</td>
<td>2 and 3</td>
</tr>
<tr>
<td>1:4.5 scale</td>
<td>5 m</td>
<td>10 kW</td>
<td>NREL</td>
<td>N/A</td>
<td>50 kW, 2L-3P ac-dc (CPES test only)</td>
<td>4 and 5</td>
</tr>
</tbody>
</table>

6.2 Future Work

To further evaluate the model in Chapters 2 and 3, a field test with the 1.2-m WEC system under irregular waves in the real ocean will provide valuable information. The circuit model in Chapter 2 predicts the efficiency of the MMR-PTO, and the non-MMR PTO model is still needed to conduct a generic comparison of MMR’s characteristics. The maximum power point of the MMR-PTO is challenging to predict due to its discontinuity from the one-way clutches. A control algorithm that can provide or estimate the maximum power under MMR-PTO will boost the capability of the unique mechanism.

The test of 50 kW power converter is built for Chapters 4 and 5 and is tested in the lab of Center for Power Electronics Systems (CPES), Virginia Tech. The combined test with the 10 kW MMR-PTO in NREL will be done to evaluate the reliability of the power converter under the damping control with real PTO interaction.
6.3 List of Publications

References


modeling in the time domain: A design guide,” in 2013 1st IEEE Conference on Technologies for Sustainability (SusTech), 2013, pp. 103–108.


[85] E. A. Amon, “Development of two-variable maximum power point tracking control for ocean wave energy converters utilizing a power analysis and data acquisition system,” 2010.


[140] “https://arc.vt.edu/.”.


[145] D. Martin et al., “Numerical analysis and wave tank validation on the optimal design of a

## Appendix A

### A.1 Specifications

<table>
<thead>
<tr>
<th><strong>Input Parameter</strong></th>
<th><strong>Value</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated Input AC 1-1 Voltage</td>
<td>400 Vac (566 V peak)</td>
</tr>
<tr>
<td>Rated speed of Generator</td>
<td>600 rpm (380V)</td>
</tr>
<tr>
<td>Rated Input Power</td>
<td>50 kW</td>
</tr>
<tr>
<td>Rated Current</td>
<td>76 A (107A peak)</td>
</tr>
<tr>
<td>Average speed /1-1 Volt/ Amp/ Power</td>
<td>300 rpm / 200 Vac/ 76 A/ 25 kW</td>
</tr>
<tr>
<td>Input AC three-phases SSR</td>
<td>480V/ 125A/ 4-32V</td>
</tr>
<tr>
<td>Amb. Temp.</td>
<td>0°C - 40 °C</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Output Parameter</strong></th>
<th><strong>Value</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Output DC Voltage</td>
<td>700 V</td>
</tr>
<tr>
<td>Output Power</td>
<td>49.25 kW (98.5 % eff)</td>
</tr>
<tr>
<td>Output Current</td>
<td>71 A</td>
</tr>
<tr>
<td>Output Capacitor</td>
<td>800V, 70 uF</td>
</tr>
<tr>
<td>IGBT Module Powerex</td>
<td>1200V, 150A</td>
</tr>
<tr>
<td>IGBT max Case Temp</td>
<td>110 °C</td>
</tr>
<tr>
<td>IGBT trip junction temp.</td>
<td>135 °C</td>
</tr>
<tr>
<td>IGBT max junction temp.</td>
<td>150 °C</td>
</tr>
</tbody>
</table>
A.2 Schematics and Diagrams

Boards schematics

1. Main Board: pdf
2. Control Board: pdf
   2.a. Signal conditioning board (S.C. Board): pdf
   2.b. F2806x ISO Control Card, TI: pdf
3. Gate driving Board: pdf

The interconnection diagram shows the signal routes between the main power loop, main board, control board, and gate driving board.
Fig. A-1. Board-Board connection diagram, including main power loop, SSR, 1) main Board, 2) control Board, and 3) Gate driving Board from the board schematics. Interfaces from IV are specified in the diagram.
**Grounding Diagram**

The grounding system diagram shows the routing loop for p24V and s24V to avoid EMI interference between signal and the main power source.

![Diagram of grounding system](image)

**Fig. A-2.** Board grounding diagram is showing the auxiliary power loop and the communication port for the system which can also be found in IV interfaces.
A.3 Hardware Pictures

The main components are shown in the hardware pictures.

Fig. A-3. The top view picture of the 50 kW power system. The simplified diagram and its interconnection can be found in Fig. A-1.

Fig. A-4. The top view picture of the 50 kW power system without the main control board and gate driving
board. The main power loop is shown in the picture. The simplified diagram and its interconnection can be found in Fig. A-1.

![Diagram](image)

(a)

![Diagram](image)

(b)

Fig. A-5. Side view of the converter system. The simplified diagram and its interconnection can be found in Fig. A-1.
Heatsink dimension: Width 190 mm (7.48”) x Length 150 mm (5.91”) x Height 65mm (2.56”)

Main Board dimension: Width 190 mm (7.48”) x Length 155 mm (6.1”)

Power System dimension: Width 275mm (80.83”) x Length 220mm (8.66”) x Height 150 mm (5.91”)

Fig. A-6. The wave power converter system and its dimension.
A.4 Hardware Interfaces

Main user interfaces including communication port, auxiliary power port, and main power ports are marked in the pictures.

Fig. A-7. The interfaces of the system where its simplified diagram can be found in Fig. A-1. The mini USB port, 120V AC/1A port, input ac three-phases, and output dc terminals are for external connections, and other ports are for internal inter-board connections.
Fig. A-8. The input ac three-phases and output dc power interfaces of the system where its simplified diagram can be found in Fig. A-1. The rated current is 380 Vac for input, 700Vdc for output, current rating 76 A. Three AWG power cables are expected to connect on the terminals.

Table A-1. The definition of control sensing pins.

<table>
<thead>
<tr>
<th>Func_adc.c</th>
<th>DSP</th>
<th>Sensor_out</th>
<th>Sensor Board</th>
</tr>
</thead>
<tbody>
<tr>
<td>adc_val[1][5]</td>
<td>B5</td>
<td>P_3-4</td>
<td>CT1 (-isa)</td>
</tr>
<tr>
<td>adc_val[1][4]</td>
<td>B4</td>
<td>P_5-6</td>
<td>*SSR_C_INT</td>
</tr>
<tr>
<td>adc_val[0][5]</td>
<td>A6</td>
<td>P_7-8</td>
<td>CT3 (-isc)</td>
</tr>
<tr>
<td>adc_val[0][4]</td>
<td>A4</td>
<td>P_9-10</td>
<td>CT4 (Ibus)</td>
</tr>
<tr>
<td>adc_val[1][3]</td>
<td>B3</td>
<td>P_11-12</td>
<td>Vbatt+ - Vbatt-</td>
</tr>
<tr>
<td></td>
<td>B2</td>
<td>P_13-14</td>
<td>SGND</td>
</tr>
<tr>
<td></td>
<td>NA</td>
<td>P_15-16</td>
<td>SGND</td>
</tr>
<tr>
<td></td>
<td>A3</td>
<td>P_17-18</td>
<td>Tsink</td>
</tr>
<tr>
<td></td>
<td>A2</td>
<td>P_19-20</td>
<td>VS_BA</td>
</tr>
<tr>
<td></td>
<td>B1</td>
<td>P_21-22</td>
<td>VS_CB</td>
</tr>
<tr>
<td></td>
<td>B0</td>
<td>P_23-24</td>
<td>Tamb</td>
</tr>
</tbody>
</table>
Fig. A-9. Pin definition on the control board. (B4 pin 1 to CI01 pin 1 for SSR relay control from CPLD)

Fig. A-10. Fan Speed control interface.
A.5 Software Interface

Fig. A-11. The software interface from Code Composer Studio 8.2. The test procedures will be specified in the following section.

A.6 Test Procedure

**Before Experiment Starts**

- Make sure auxiliary power is off
- Visually verify that the lights on all power supplies are off
- Verify that all power supplies are off using multi-meter. (First, make sure the multi-meter works by testing the multi-meter on a known lower voltage such as an adjustable linear DC power supply and then use the multi-meter to check for the power off)
- Manually check all connections are tight and correctly connected
- Verify with a multi-meter that the voltage of the capacitor is zero
- Verify with a current probe that the input inductor’s current is zero.
- Check that the cable of the Magna DC source is connected correctly with the converter
☐ Check all electrical and mechanical connections for the device. Tighten if necessary
☐ Check that external connector are connected properly
☐ Make sure all electrical safety guards are in place
   The guards include: (1). Safety Shield box at the output port for the Magna DC source
   (2). Make sure the emergency button is available and nearby
☐ Wear safety goggle and face shield

**During The Experiment**

☐ Take on Auxiliary power (turn on 110V utility source)
☐ Run CCS Inv_test.c
☐ Turn on the fan of power converter
☐ Turn on the fan of resistor bank
☐ Turn on the panel switch of the Magna DC source
☐ Turn on the Magna DC source power supply follow power-up sequence
☐ Enter CCS debug mode follow the procedure
☐ Calibrate the offset on CCS, gMotorVars.Flag_enableOffsetcalc → 1
☐ Wait until gMotorVars.Flag_enableOffsetcalc → 0

**Low voltage mode (150V):**

☐ Increasing the voltage of the Magna power supply, check the value in the multimeter to 150 V
☐ Check the value of gMotorVars.VdcBus_kV= 0.15, gIdq_pu.value[0] = 0; gIdq_pu.value[1] = 0
☐ Make sure gIdq_ref_pu.value[0] = 0; gIdq_ref_pu.value[1] = 0
☐ Start the controller by toggle gMotorVars.Flag_Run_Identify → 1
☐ change gIdq_ref_pu.value[0] = 0; gIdq_ref_pu.value[1] = 0.01
☐ follow the test condition I up to I_La = 10 Arms, gIdq_ref_pu.value[1] = 0.1131
☐ Each test run for 30 sec until steady state.
☐ Stop the scope and record waveforms
monitor the temperate $T_s$ while testing ($<100 \, ^\circ C$)

**Mid voltage mode (500V)**

- Gradually Increasing the voltage of the Magna DC source, check the value in the multimeter to 500 V
- follow the test condition II up to $I_{La} = 18 \, \text{Arms}$, $g\text{Idq}_{\text{ref, pu}. \text{value}[1]} = 0.2036$
- Each test runs for 30 sec until a steady state.
- Stop the scope and record waveforms.
- monitor the temperate $T_{\text{case}}$ while testing ($<100 \, ^\circ C$)

**High voltage mode (700V)**

- Increasing the voltage of the Magna DC source, check the value in the multimeter to 700 V
- follow the test condition III up to $I_a = 17.68 \, \text{Arms}$, $g\text{Idq}_{\text{ref, pu}. \text{value}[1]} = 0.2$
- Each test runs for 30 sec until a steady state.
- Stop the scope and record waveforms.
- monitor the temperate $T_{\text{case}}$ while testing ($<100 \, ^\circ C$)

**Shut Down Procedure After Experiment or When Set-up Changes Are Required**

- Turn off the current reference
  
  $g\text{Idq}_{\text{ref, pu}. \text{value}[0]} = 0$; $g\text{Idq}_{\text{ref, pu}. \text{value}[1]} = 0$
- Turn off the Magna DC source follow the shutdown sequence
- Wait until capacitor discharged within 20V
- Turn off the controller by toggle $g\text{MotorVars.Flag\_Run\_Identify} \rightarrow 0$
- Turn off the auxiliary power.
A.7 Test Setup and Performance of the Inverter Mode

The inverter setup for the DUT is to verify the power capability of the converter and its signal integrity under high current/voltage conditions up to 700V and 20A (10kW).

Fig. A-12. Inverter mode configuration for 700V, 10 kW testing. Input: Magna LX 1 kV, 100 kVA dc source; output: air-cooling three-phase resistive bank of 10 Ω.
Table A-2. Inverter Test summary.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum $P_{in}$ (W), $V_{in}/I_{in}$</td>
<td>9969 W, 700V/14.3 A</td>
</tr>
<tr>
<td>Maximum $P_{o}$ (W), $V_{o}/I_{o}$</td>
<td>9562 W, 700V/14.3 A</td>
</tr>
<tr>
<td>Efficiency ($P_o/P_{in}$), $V_{in}/P_{in}$</td>
<td>98.97 % 500V/8.1 kW</td>
</tr>
<tr>
<td>Dc bus voltage $v_{in}$ (V)</td>
<td>150V/500V/700 V</td>
</tr>
<tr>
<td>Output 3-p Resistors</td>
<td>10 ohm</td>
</tr>
<tr>
<td>Max. Output current $I_{abc}$ (Arms)</td>
<td>17.88</td>
</tr>
<tr>
<td>Switching frequency (Hz)</td>
<td>15 k</td>
</tr>
<tr>
<td>Fan duty (%)</td>
<td>23</td>
</tr>
<tr>
<td>Maximum temp. rise (°C)</td>
<td>14 °C</td>
</tr>
<tr>
<td>Line frequency (Hz)</td>
<td>60</td>
</tr>
</tbody>
</table>

Fig. A-13. Remote control setup for the power converter and source.
Fig. A-14. Test setup with Resistive load and Magna DC power source.
**Equipment for the inverter mode**

As shown in Fig 10-13, this test bed includes:

1. Magna power supply 1000V/100kVA
2. Air cool resistor bank (10ohm each)
3. 110V AC source (or utility) for AUX power and cooling fan
4. Keysight 34970A data acquisition: temperature
5. Tektronix MSO 5104 Mixed-signal oscilloscope: voltage and current
6. DMM multimeter: dc bus voltage
7. PC1 for DAQ and scope recording, PC2 for power electronics converter real-time DSP control

**Test Results of the inverter mode**

1. **Efficiency curve**

   The maximum efficiency is 98.97%. The maximum voltage is 700 V, and the maximum output power is 9562 W.

![Efficiency curve](image-url)

*Fig. A-15. Efficiency curve for WEC power electronics converter under inverter mode.*
Fig. A-16. Test #5. $V_{in} = 150\, \text{V}$, $I_{in} = 5\, \text{A}$, $I_o = 4.88\, \text{Arms}$, $P_o = 707\, \text{W}$, eff. = 95.78%.

Fig. A-17. Test #12. $V_{in} = 500\, \text{V}$, $I_{in} = 16.3\, \text{A}$, $I_o = 16.5\, \text{Arms}$, $P_o = 8074\, \text{W}$, eff. = 98.97%.

Fig. A-18. Test #14. $V_{in} = 700\, \text{V}$, $I_{in} = 1.18\, \text{A}$, $I_o = 4.44\, \text{Arms}$, $P_o = 605\, \text{W}$, eff. = 73.23%.

Fig. A-19. Test #17. $V_{in} = 700\, \text{V}$, $I_{in} = 14.23\, \text{A}$, $I_o = 17.8\, \text{Arms}$, $P_o = 9562\, \text{W}$, eff. = 96%.
Table A-3. Summary of the Closed-Loop Inverter test.

<table>
<thead>
<tr>
<th>Vin (V)</th>
<th>Test no.</th>
<th>Iqref (pu)</th>
<th>Irms (A)</th>
<th>Po (W)</th>
</tr>
</thead>
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<tr>
<td></td>
<td>Test I</td>
<td>150</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>0.01</td>
<td>0.88</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>0.02</td>
<td>1.77</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>0.04</td>
<td>3.54</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>0.06</td>
<td>5.30</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5</td>
<td>0.08</td>
<td>7.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6</td>
<td>0.1131</td>
<td>10.00</td>
</tr>
<tr>
<td></td>
<td>Test II</td>
<td>500</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>7</td>
<td>0.01</td>
<td>0.88</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8</td>
<td>0.05</td>
<td>4.42</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9</td>
<td>0.1</td>
<td>8.84</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10</td>
<td>0.15</td>
<td>13.26</td>
</tr>
<tr>
<td></td>
<td></td>
<td>11</td>
<td>0.175</td>
<td>15.47</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12</td>
<td>0.2036</td>
<td>18.00</td>
</tr>
<tr>
<td></td>
<td>Test III</td>
<td>700</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>13</td>
<td>0.01</td>
<td>0.88</td>
</tr>
<tr>
<td></td>
<td></td>
<td>14</td>
<td>0.05</td>
<td>4.42</td>
</tr>
<tr>
<td></td>
<td></td>
<td>15</td>
<td>0.1</td>
<td>8.84</td>
</tr>
<tr>
<td></td>
<td></td>
<td>16</td>
<td>0.15</td>
<td>13.26</td>
</tr>
<tr>
<td></td>
<td></td>
<td>17</td>
<td>0.2</td>
<td>17.68</td>
</tr>
</tbody>
</table>
A.8 Test Setup and Performance of the Rectifier Mode

Fig. A-20. Test setup for the WEC power converter in the rectifier mode.

Fig. A-21. Test setup with the input connection to the 1 kW motor drive and motor-generator set.
**Equipment and Measurement**

As shown in the Figure, this test bed includes:

1. TDK power supply 600V/5.5 A
2. DC electronics load 600V/20A
3. EMJ04 motor-generator set 8.1 A rated current, 120/200V rated voltage
4. Tektronix MSO 5104 Mixed-signal oscilloscope: voltage and current
5. DMM multimeter: dc bus voltage

**Measurements:**

**Input:** $I_a$, $v_{ab}$, $v_{bc}$

**Output:** $i_{dc}$

**Operation Procedure**

**Before Experiment Starts**

- Make sure auxiliary power is off
- Visually verify that the lights on all power supplies are off
- Verify that all power supplies are off using multi-meter. (First, make sure the multi-meter works by testing the multi-meter on a known lower voltage such as an adjustable linear DC power supply and then use the multi-meter to check for the power off)
- Manually check all connections are tight and correctly connected
- Verify with a multi-meter that the voltage of the capacitor is zero
- Check that the cable of the DC source and loads are connected correctly with the converter
- Check all electrical and mechanical connections for the device. Tighten if necessary
- Check that external connector are connected properly

**During The Experiment**

- Take on Auxiliary power (turn on 110V utility source)
- Run CCS proj_ impedance-fw_Imax.c
☐ Turn on the fan of power converter (duty=23%)
☐ Set dc source at constant voltage mode CV = 120V/200V
☐ Enter CCS debug mode\rightarrow start realtime debug mode\rightarrow run DSP code
☐ Calibrate the offset on CCS, gMotorVars.Flag_enableOffsetcalc \rightarrow 1
☐ Wait until gMotorVars.Flag_enableOffsetcalc \rightarrow 0
☐ Turn on the DC source power supply to 120V/200V
☐ Start Rs calibration, set gMotorVars.Flag_enableRsRecalc \rightarrow 1
☐ Wait until gMotorVars.Flag_enableOffsetcalc \rightarrow 0

**Constant speed mode: (120V dc)**

☐ Check the value of gMotorVars.VdcBus_kV= 0.2, gRin = 50
☐ Turn on the impedance control, gFlag_impedance \rightarrow 1
☐ Start the controller by toggle gMotorVars.Flag_Run_Identify \rightarrow 1
☐ Start the motor control by driving constant speed at 500 rpm to 1500 rpm
☐ Change impedance by gRin = 50 to 5
☐ Each test runs for 30 sec until a steady state.
☐ Stop the scope and record waveforms

**Variable speed mode: (200V dc)**

☐ Check the value of gMotorVars.VdcBus_kV= 0.2, gRin = 50
☐ Turn on the impedance control, gFlag_impedance \rightarrow 1
☐ Start the controller by toggle gMotorVars.Flag_Run_Identify \rightarrow 1
☐ Start the motor control by driving variable speed from 500 rpm to 1500 rpm, and frequency 0.2 – 0.5 Hz
☐ Change impedance by gRin = 50 to 5
☐ Each test runs for 30 sec until a steady state.
☐ Stop the scope and record waveforms and record a full period of each variable speed.
Shut Down Procedure After Experiment or When Set-up Changes Are Required

- Stop the motor
- Turn off the dc power source
- Wait until capacitor discharged within 20V
- Turn off the controller by toggle gMotorVars.Flag_Run_Identify → 0
- Turn off the auxiliary power.

Test Results – Constant Speed

Table A-4. Constant speed test plan for the rectifier mode.

<table>
<thead>
<tr>
<th>Speed</th>
<th>test no.</th>
<th>gRin (Ω)</th>
<th>V_{dc} (gen)</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>1</td>
<td>20</td>
<td>120.00</td>
</tr>
<tr>
<td>500</td>
<td>2</td>
<td>15</td>
<td>120.00</td>
</tr>
<tr>
<td>500</td>
<td>3</td>
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<td>120.00</td>
</tr>
<tr>
<td>500</td>
<td>4</td>
<td>7.5</td>
<td>120.00</td>
</tr>
<tr>
<td>500</td>
<td>5</td>
<td>5</td>
<td>120.00</td>
</tr>
<tr>
<td>1000</td>
<td>6</td>
<td>20</td>
<td>120.00</td>
</tr>
<tr>
<td>1000</td>
<td>7</td>
<td>15</td>
<td>120.00</td>
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<td>120.00</td>
</tr>
<tr>
<td>1500</td>
<td>15</td>
<td>10</td>
<td>120.00</td>
</tr>
</tbody>
</table>
Fig. A-22. Test #9, 1000 rpm, $g_{Rin}=7.5$ Ohm, $I_{dc} = 0.37$ A, $I_a = 2.57$ Arms electrical frequency = 66.6 Hz.

Fig. A-23. Test #5, 500 rpm, $g_{Rin}=5$ Ohm, $I_{dc} = -0.04$A, $I_a = 1.97$ Arms electrical frequency = 33.3 Hz.
### Test Results – Variable Speed

<table>
<thead>
<tr>
<th>Speed</th>
<th>test no.</th>
<th>$R_{in}$ (Ω)</th>
<th>$V_{dc}$ (gen)</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>16</td>
<td>20</td>
<td>200.00</td>
<td>0.10</td>
</tr>
<tr>
<td>500</td>
<td>17</td>
<td>15</td>
<td>200.00</td>
<td>0.10</td>
</tr>
<tr>
<td>500</td>
<td>18</td>
<td>10</td>
<td>200.00</td>
<td>0.10</td>
</tr>
<tr>
<td>500</td>
<td>19</td>
<td>7.5</td>
<td>200.00</td>
<td>0.10</td>
</tr>
<tr>
<td>500</td>
<td>20</td>
<td>5</td>
<td>200.00</td>
<td>0.10</td>
</tr>
<tr>
<td>500</td>
<td>21</td>
<td>20</td>
<td>200.00</td>
<td>0.50</td>
</tr>
<tr>
<td>500</td>
<td>22</td>
<td>15</td>
<td>200.00</td>
<td>0.50</td>
</tr>
<tr>
<td>500</td>
<td>23</td>
<td>10</td>
<td>200.00</td>
<td>0.50</td>
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<td>200.00</td>
<td>0.50</td>
</tr>
<tr>
<td>500</td>
<td>25</td>
<td>5</td>
<td>200.00</td>
<td>0.50</td>
</tr>
<tr>
<td>1000</td>
<td>26</td>
<td>20</td>
<td>200.00</td>
<td>0.10</td>
</tr>
<tr>
<td>1000</td>
<td>27</td>
<td>15</td>
<td>200.00</td>
<td>0.10</td>
</tr>
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<td>1000</td>
<td>28</td>
<td>10</td>
<td>200.00</td>
<td>0.10</td>
</tr>
<tr>
<td>1000</td>
<td>29</td>
<td>7.5</td>
<td>200.00</td>
<td>0.10</td>
</tr>
<tr>
<td>1000</td>
<td>30</td>
<td>20</td>
<td>200.00</td>
<td>0.50</td>
</tr>
<tr>
<td>1000</td>
<td>31</td>
<td>15</td>
<td>200.00</td>
<td>0.50</td>
</tr>
<tr>
<td>1000</td>
<td>32</td>
<td>10</td>
<td>200.00</td>
<td>0.50</td>
</tr>
<tr>
<td>1000</td>
<td>33</td>
<td>7.5</td>
<td>200.00</td>
<td>0.50</td>
</tr>
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</table>
Fig. A-24. Test #5, regular wave with peak speed = 500 rpm, 0.2 Hz, gRin=5 Ohm, I_{dc} = 0.2A, I_a= 1.3 Arms.
Appendix B - Background Information of Figures

B.1 Setups of Figures

<table>
<thead>
<tr>
<th>Setups</th>
<th>System Scale</th>
<th>WEC</th>
<th>PTO</th>
<th>Generator</th>
<th>Power Converter/load</th>
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<td>(i)</td>
<td>1:20</td>
<td>NA</td>
<td>nonideal PTO circuit (Fig. 2-11)</td>
<td>NE500W (Table 2-2)</td>
<td>Resistor R_e only</td>
</tr>
<tr>
<td>(ii)</td>
<td>1:20</td>
<td>NA</td>
<td>Ideal PTO circuit (Fig. 2-2)</td>
<td>NE500W (Table 2-2)</td>
<td>Resistor R_e only</td>
</tr>
<tr>
<td>(iii)</td>
<td>1:20</td>
<td>Irregular wave circuit (Fig. 3-11, and Table 3-2)</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
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<tr>
<td>(iv)</td>
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<td>Regulare wave circuit (Fig. 3-5)</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
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<td>(v)</td>
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<td>Irregular wave circuit (Fig. 3-11, and Table 3-2)</td>
<td>Ideal MMR-PTO circuit (Fig. 3-2)</td>
<td>NE500W (Table 3-2)</td>
<td>3p-2l ac-dc, V_d = 400 V, PS21765</td>
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<td>(vi)</td>
<td>1:4.5</td>
<td>Irregular wave WEC circuit (Fig. 3-11, and Table 4-2)</td>
<td>Ideal nonMMR PTO (variable k_b)</td>
<td>Unknown generator (variable P_grate, Table4-2)</td>
<td>Unknown converter (variable S_{rated}, Fig.4-5), 800V</td>
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<tr>
<td>(vii)</td>
<td>1:4.5</td>
<td>Irregular wave WEC circuit (Fig. 5-5, and Table 4-2)</td>
<td>Ideal nonMMR PTO (Table 5-2)</td>
<td>GDF-250 (Table 5-2)</td>
<td>Average circuit model, 800V, PM150CG1C120 (Table 5-2- 5-4)</td>
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B.2 Figures – Files in Chapter 2

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<th>Figures</th>
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<td>Fig. 2-12</td>
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<td>\setup\</td>
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| Fig. 2-14 | Matlab, Excel | 1. \experiment\Dataloading_tri.m  
2. \experiment\testsetup_harden.xlsx (Sheet1)  
3. \experiment\test_harden_proc5.mat  
4. \experiment\PTO_harden_tri\  
5. \figures\Step response_vpto_color.fig; Step response_IFpto_color.fig |
### B.3 Figures – Files in Chapter 3

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| Fig.3-4, Fig.3-9, Fig.3-12- Fig.3-14 | Matlab | 1. `\analysis\WEC_system_ID_500W_50kW.m`  
2. `\analysis\hydrodynamic_0_5m_h30m.mat`  
3. `\analysis\WEC_500W_Yellow.slx`  
4. `\figures\500Wsetup.fig`  
5. `\figures\K2_2nd_3rd_approximation.fig`  
6. `\figures\Zwec+PTO_impedance.fig`  
7. `\figures\timedomainmatching_origin2.fig`  
8. `\figures\Powermapmatching.fig` |
| Fig.3-15- Fig.3-17 | Matlab | 1. `\analysis\BoostRec_battery_Motor_model.m` (controller)  
2. `\analysis\WEC_MMR_motor_rectifier_abc_Zin_NE600W_IGBT_Tj_closed.slx`  
3. `\figures\Speed_Volt_irr_1m.fig`  
4. `\figures\Tjdq_1m_6.3s_irr.fig` |

B.4 Figures – Files in Chapter 4

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2. `\analysis\hydrodynamic_2_5m_h30m.mat`  
3. `\figures\WEC_impedance_response_50kW.fig` |
| Fig.4-8 | Matlab | 1. `\analysis\WEC_Pmax_Rpto_iwavex_kbx_iwavex_Po_Ploss_loaddependend.m`  
2. `\analysis\WEC2_5m_Rpto_Plim_Ponly_kb_Ploss_Ir_Sr_matlab.slx`  
3. `\analysis\Time_2_kb=104_Plim=50k_Rpto=0.7_PQSIV.mat`  
4. `\figures\Time_2_Sg_Pom_Idq_Ploss_Plev50k_kb104_w28.fig`  
5. `\figures\Time_2_Sg_Pom_Idq_Ploss_Plev50k_kb104_w28_zoom.fig` |
| Fig.4-10- Fig.4-16; Fig.4-18 | Matlab | 1. `\analysis\WEC_Pmax_Rpto_iwavex_kbx_iwavex_Po_Ploss_loaddependend.m`  
2. `\analysis\WEC2_5m_Rpto_Plim_Ponly_kb_Ploss_Ir_Sr_matlab.slx`  
3. `\analysis\w1_28_Vdc=800_w_Plim_Rpto_kb__Po_Sg_Ploss_Pgloss_IR_v2.mat`  
4. `\figures\Rpto-Poe_w7.fig; \figures\Rpto-Poe_w28.fig` (Fig.4-10)  
5. `\figures\Poe_Plev_ploss_vdc=800.fig; Pom_Plev_ploss_vdc=800.fig(Fig.4-11)`  
6. `\figures\Psrated_Plev_vdc=800_5k_VarSgPgen.fig; \figures\Srated_Po_ploss_vdc=800_5k_VarSgPgen.fig(Fig.4-12)`  
7. `\figures\Ugp_Plev_ploss_vdc=800_5k_VarSgPgen.fig; Ugp_Pgrated_ploss_vdc=800_5k_VarSgPgen.fig; \figures\Ugp_Plev_ploss_vdc=800_5k_VarSgPgen.fig; Ugp+Usp_Pgrated_ploss_vdc=800_5k_VarSgPgen.fig(Fig.4-14)`  
8. `\figures\Ugp+Usp_Pgrated_ploss_vdc=800_5k_VarSgPgen.fig; Ugp+Usp_Pgrated_ploss_vdc=800_5k_VarSgPgen.fig(Fig.4-14)`  
9. `\figures\Poe_Pgrated_ploss_vdc=800_5k_VarSgPgen.fig;` |
Fig.4-17 Matlab Excel

1. \texttt{\textbackslash analysis\textbackslash WEC\_Pmax\_Rpto\_iwavex\_kb\_iwavex\_Po\_Ploss\_loaddepend.m}
2. \texttt{\textbackslash analysis\textbackslash WEC2\_5m\_Rpto\_Plim\_Poonly\_kb\_Ploss\_Ir\_Sr\_matlab.slx}
3. \texttt{\textbackslash analysis\textbackslash Time\_1\_kb=104\_Plim=200k\_Rpto=0.7\_PQSIV.mat;}
   \hspace{1cm} Time\_2\_kb=104\_Plim=50k\_Rpto=0.7\_PQSIV.mat;
   \hspace{1cm} Time\_3\_kb=47\_Plim=200k\_Rpto=0.9\_PQSIV.mat;
   \hspace{1cm} Time\_4\_kb=47\_Plim=50k\_Rpto=0.9\_PQSIV.mat
4. \texttt{\textbackslash analysis\textbackslash Porated\_test.xlsx}
5. \texttt{\textbackslash figures\textbackslash Time\_1\_Sg\_Pom\_Idq\_Ploss\_Plev200k\_kb104\_w28.fig;}
   \hspace{1cm} Time\_2\_Sg\_Pom\_Idq\_Ploss\_Plev50k\_kb104\_w28.fig;
   \hspace{1cm} Time\_3\_Sg\_Pom\_Idq\_Ploss\_Plev200k\_kb47\_w28.fig;
   \hspace{1cm} Time\_4\_Sg\_Pom\_Idq\_Ploss\_Plev50k\_kb47\_w28.fig

Fig.4-18- Fig.4-20 Matlab Excel

1. \texttt{\textbackslash analysis\textbackslash WEC\_Pmax\_Rpto\_iwavex\_kb\_iwavex\_Po\_Ploss\_loaddepend.m}
2. \texttt{\textbackslash analysis\textbackslash WEC2\_5m\_Rpto\_Plim\_Poonly\_kb\_Ploss\_Ir\_Sr\_matlab.slx}
3. \texttt{\textbackslash analysis\textbackslash W1\_28\_Vdc=800\_w\_Plim\_kb\_Po\_Sg\_Ploss\_IR\_IndFW\_IndPlim.mat}
4. \texttt{\textbackslash figures\FWrate\_Plev\_ploss\_vdc=800\_w=1-28.fig;}
   \hspace{1cm} Full\_load\_rate\_Plev\_ploss\_vdc=800\_w=1-28.fig (Fig. 4-18)
5. \texttt{\textbackslash figures\Full\_load\_rate\_Plev\_ploss\_vdc=800\_w=28.fig;}
   \hspace{1cm} FWrate\_Plev\_ploss\_vdc=800\_w=28.fig (Fig. 4-19)
6. \texttt{\textbackslash figures\eff\_m-e\_Plev\_vdc=800\_5k\_VarSgPgen.fig;}
   \hspace{1cm} eff\_gen\_Plev\_vdc=800\_5k\_VarSgPgen.fig;
   \hspace{1cm} eff\_pe\_Plev\_vdc=800\_5k\_VarSgPgen.fig (Fig. 4-20)

Fig.4-21 Matlab

\texttt{\textbackslash figures\Srated\_Poe\_ploss\_vdc=800\_5k\_VarSgPgen.fig;}
\texttt{Pgrated\_Poe\_ploss\_vdc=800\_5k\_VarSgPgen.fig}

**B.5 Figures – Files in Chapter 5**

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</tr>
</thead>
<tbody>
<tr>
<td>Fig.5-1–Fig.5-5, Fig.5-7, Fig.5-12, Fig.5-13, Fig.5-20–Fig.5-22, Fig.5-25, Fig.5-27</td>
<td>Visio</td>
<td>\texttt{\textbackslash ch5_diagrams.vsdx}</td>
</tr>
<tr>
<td>Fig.5-6</td>
<td>Datasheet, Matlab</td>
<td>\texttt{\textbackslash analysis\pm150cg1c120.pdf} \texttt{\textbackslash analysis\PM150CG1C120_tables.m} \texttt{\textbackslash figures\iC_EonEoff_pm150cg1c.fig; iC_Vce_pm150cg1c.fig; iE_Err_pm150cg1c.fig; iE_Vec_pm150cg1c.fig}</td>
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<td>Matlab</td>
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Fig. 5-17
3. \analysis\lifetime_cal_Tj.m
4. \analysis\few1_28.mat
5. \analysis\WEC_EC_50kw.mat
6. \analysis\Timedomain_wid=15_Z=10_P=60_fe_Po_wm_Idq_Vabc_Tj.mat
7. \figures\wid=15_Rin=10_Plev=60_Fe_I_Po_Tj_1.fig;
   wid=15_Rin=10_Plev=60_Fe_I_Po_Tj_2.fig (Fig. 5-8)
8. \figures\wid=15_R=10_Plev=60_rainflow_Tjd.fig;
   wid=15_R=10_Plev=60_rainflow_Tjq.fig (Fig. 5-9)
9. \figures\Modified_Coffin_Mason.fig (Fig. 5-10)
10. \figures\wid=15_Rin=2_Plev=60_Ta16_D_Tjd_dTjd.fig;
    wid=15_Rin=2_Plev=60_wm_Teg_Iq_Id.fig
    wid=15_Rin=2_Plev=60_wm_Teg_Iq_Id_2.fig (Fig. 5-17)

Fig. 5-14, Fig. 5-15,
Matlab
1. \analysis\FieldWeakening.m
2. \analysis\WEC_EC_50kw.mat
3. \figures\Te_wm_100kW.fig; Te_wm_2ohm.fig (Fig. 5-14)
4. \figures\Iq_Id_2ohm_800V.fig; Iq_Id_100kW_800V.fig (Fig. 5-15)

Fig. 5-16,
Matlab
1. \analysis\WEC_system_ID_500W_50kW.m
2. \analysis\hydrodynamic_2_5m_h30m.mat
3. \figures\WEC_impedance_response_50kW.fig

Fig. 5-18 - Fig. 5-20
Txt
Matlab
1. \analysis\wavedata\44056_2015_wavedata.txt
2. \analysis\wavedata\wavedata2015.mat
3. \analysis\wavedata\wavedata_2dplot_w28.m
4. \analysis\wavedata\wavedata.xlsx
5. \figures\wave_te_hs_2015_2.fig; wave_Ta_2015.fig (Fig. 5-19)
6. \figures\wave_map_2015.fig (Fig. 5-20)

Fig. 5-22, Fig. 5-23, Fig. 5-29 - Fig. 5-32
Matlab
1. \analysis\Circuit_Test_WEC2_5m_map_Plimx_Iqmax_Rinx_Tj_par_timesim2.m
2. \analysis\lifetime_cal_Tj.m
3. \analysis\few1_28.mat
4. \analysis\WEC_EC_50kw.mat
5. \analysis\Po_Eng_LT_Irr_3_Ta16_w28.xlsx (All for Simulation)
6. \analysis\Po_Eng_LT_Irr_3_Ta16_w28.xlsx (Summary for index in Fig. 5-23)
7. \analysis\wavedata\wave.xlsx
8. \figures\Emap_w28_Hs_0.25_Te_4.5_w1.fig;
    Emap_w28_Hs_3.75_Te_7.5_w28.fig (Fig. 5-22)
9. \analysis\Po_Eng_LT_Irr_3_Ta16_w28.xlsx (Sim_Energy_irr_w15 for index in Fig. 5-29)
10. \analysis\Po_Eng_LT_Irr_3_Ta16_w28.xlsx (Summary for index in Fig. 5-30-Fig. 5-32)

Fig. 5-24 - Fig. 5-26
Matlab
1. \analysis\Emax_search_w28_Ta16.m (%% Constrainted with LT = 20 yr)
2. \analysis\Po_Eng_LT_Irr_3_Ta16_w28.xlsx
3. \analysis\Po_Eng_LT_Irr_3_Ta16_w28.xlsx (Summary for index in Fig. 5-26)

Fig. 5-27 - Fig. 5-28
Matlab
1. \analysis\wavedata\wavedata_2dplot_w28.m
2. \analysis\wavedata\wavedata2015.mat
### B.6 Figures – Files in Appendix A

<table>
<thead>
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<tbody>
<tr>
<td>A.2</td>
<td>Schematics PCB</td>
<td>\Schematics\</td>
</tr>
<tr>
<td>A.3 – A.4</td>
<td>Pictures Powerpoint Excel</td>
<td>1. \Setups\ \Sensing_interface.pptx \DSP Exchange.xlsx</td>
</tr>
<tr>
<td>A.5</td>
<td>C Code for Codecomposer</td>
<td>1. \Firmware\ADCPWM_Test\ (ADC/PWM test mode) \Firmware\SinINV_test\ (Inverter mode) \Firmware\impd_ctrl_gen_rect_50kW\ (Rectifier mode)</td>
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<tr>
<td>A.6 – A.7</td>
<td>Pictures Matlab Excel</td>
<td>1. \Setups\ \Experiment\lowpower_50kW (Rectifier mode) (Raw data from DAQ) \Experiment\700V10kW_INV (Inverter mode) (Raw data from DAQ)</td>
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