Market-Driven Stochastic Resource Allocation Framework for Wireless Network Virtualization

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Abstract—Wireless network virtualization is emerging as a potential game-changer for fifth-generation wireless networks. Virtualization of network resources (e.g., infrastructure and spectrum) brings several advantages. One key advantage is that various network operators can robustly share their virtualized network resources to extend coverage, increase capacity, and reduce costs. However, inherent features of wireless communications, e.g., the uncertainty in user equipment locations and channel conditions, impose significant challenges on virtualization and sharing of the network resources. In this context, we propose a novel three-layered virtualization framework, based on a matching game model and stochastic resource allocation. Our proposed architecture aims at guaranteeing user satisfaction and maximizing the revenue for operators, with reasonable computational complexity, and affordable network overhead.

Index Terms—Dynamic slicing, matching markets, Poisson point process (PPP), stochastic resource allocation, wireless network virtualization.

I. Introduction

IN WIRELESS networks, resource sharing (e.g., infrastructure and spectrum) has been a common practice for decades. To decrease operational expenditures and to extend network coverage, mobile network operators (MNO) share their infrastructure based on agreements and market policies [1]–[3]. However, increasing demand for coverage and capacity, heterogeneous quality-of-service (QoS) demands of various wireless services, scarcity of spectrum, and huge capital expenditures related to technology update motivate us to develop an efficient model of resource sharing via virtualization of network resources.

An important approach to wireless network virtualization is to create a resource pool by combining the resources of MNOs and

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create logical partitions (*slices*) among these resources based on the QoS requirements of different wireless services and applications [4], [5]. These pooled, partitioned resources are called *virtual resources* because the wireless services and applications executed with them are decoupled from the underlying physical network. Thus, virtualization enables efficient resource sharing among heterogeneous services and maximizes resource utilization. Additionally, it improves service coverage and capacity. To fully utilize the potential of virtualization, market models and resource slicing schemes need to be investigated thoroughly.

Our contributions could be summarized as follows.

1) First, we propose a robust architecture for virtualized wireless networks comprising multiple service providers (SPs) and resource providers (RPs).
2) Second, we implement a matching mechanism that pairs SPs with existing virtual network builders (VNBs) according to their particular trading preferences. We apply a solid matching algorithm that has proven successful in many fields to spectrum trading, which permits us to work with more *expressive* markets where technical and nontechnical parameters are considered in the resource selection process. This allows us to explore more comprehensive market landscapes and their underlying optimization and/or auction mechanisms.
3) Third, we propose a new model for characterizing SP demands. The requested virtual network of an SP is fully characterized using four parameters: the minimum data rate, minimum rate coverage probability, UE intensity, and the geographical area to be covered.
4) Fourth, we propose a stochastic-programming-based optimal virtual resource allocation framework for cellular networks. Stochastic programming provides a powerful mathematical tool to handle optimization under uncertainty. It has been recently exploited to optimize resource allocation in various types of wireless networks operating under uncertainties (examples include [6]–[18]). In this paper, using chance-constrained stochastic programming, we design an optimal virtual resource allocation mechanism that maximizes the utilization of the resources while satisfying the SP demands in the presence of the uncertainty in UE locations and channel conditions. This optimization framework can be used also for other service specific design criteria (e.g., delay, jitter).
5) Finally, we evaluate our proposed framework via simulation of a sample scenario.

The rest of the paper is organized as follows. Section II explores existing work. Section III presents an overview of our

proposed framework. Section IV describes the system model. In Section V, we present the matching game model. In Section VI, we formulate our resource allocation problem. The settings for the analysis of our model are presented and discussed in Section VII. We present our concluding remarks and future work in Section VIII.

II. RELATED WORK

In an attempt to find an efficient market model, previous research works have proposed 2-layered [19]–[23] and 4-layered models [24]–[26]. In 2-layered models, network resources are owned, operated, and sliced by one party, known as the MNO. Other parties known as SPs are solely responsible for QoS management of the end users. Hence, the SPs lease and allocate virtual resources to end-users. According to some authors, the MNO is only responsible for resource ownership, while SPs lease, partition, and allocate resources to their users [19]. In 4-layered architectures, the MNOs are further divided into infrastructure providers and mobile virtual network providers (MVNP). The SPs are divided into mobile virtual network operators (MVNOs) and SPs. MVNPs lease resources from infrastructure providers and virtualize them. MVNOs assign these virtual resources to the SPs.

In terms of matching algorithms, several authors have explored matching-based mechanisms for spectrum allocation, mainly in the context of cognitive radio (CR) networks. In [27], the authors utilize the deferred acceptance algorithm to pair resource users with available channels, based on the individual preferences of these entities. The objective of this paper is to find the optimal stable matching solution. In [28], the authors utilize different matching-based frameworks to explore the performance of CR systems under complete, partially incomplete, and incomplete information settings. The authors model the CR networks as one-to-one matching markets where PUs can be matched with at most one SU and vice versa. In [29], the authors focus on the ability of matching theory to form "mutually beneficial relationships". In this way, the authors apply the one-to-many matching approach to pair SUs with PUs in a CR network. The authors consider that both entities seek to maximize their utility. In the case of SUs, their utility is given by the achievable transmission data rate. For PUs, since they charge a flat rate to each SU they are paired with, they seek to maximize their revenue by attempting to match with as many SUs as possible.

In other contexts, matching markets have been utilized to study spectrum sharing settings in device-to-device communications. The authors in [30] utilize a hierarchical matching market model, where device-to-device links can either share spectrum with existing cellular subscribers or apply for exclusive sub-bands. This sharing approach is then handled as two *submarkets* to access shared and exclusive-use spectrum. Another approach is presented by Gu *et al.* in [31]. Here, the authors rely on a student-project matching model, for resource allocation in a LTE-unlicensed setting. They propose the matching algorithm as a tractable solution to a NP-hard optimization problem that seeks to maximize the throughput in a system where current cellular operators seek to leverage unlicensed bands to improve their transmission capabilities. In this way, resource allocation in the system is handled by matching cellular users with unlicensed subbands. The authors propose an additional subroutine for solving possible externalities of the matching game, and ensuring network stability and QoS.
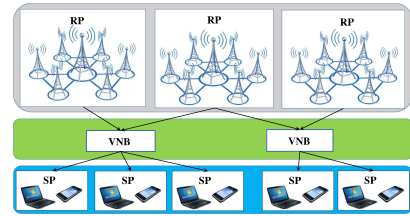

Fig. 1. Virtualized network architecture.

III. FRAMEWORK OVERVIEW

We propose a matching game based on a three-layer market model, as shown in Fig. 1. The participants in this three-layered model are described as follows.

1) RPs are the owners of physical resources (e.g., infrastructure, electromagnetic spectrum) and they have the option of making these resources available, as virtualized commodities, in a pool that may be accessed by VNBs. Traditional MNOs, cloud computing providers, and enterprise wireless network operators are examples of potential RPs.
2) VNBs are in charge of creating virtual networks for SPs by composing and aggregating resources that have been made available, i.e., pooled, by existing RPs.
3) SPs offer regular data, voice, and messaging services, as well as specialized services that handle specific applications such as the Internet of Things (IoT) or other over-the-top services. Each SP has a set of end users whose traffic needs to be covered. To this end, SPs seek to obtain resources in the network by transacting (i.e., partnering) with existing VNBs.

Compared with 2- and 4-layered models, this 3-layered model is more robust and less complex, respectively. By adding a VNB, the complexity of the model shifts from the RPs and SPs to the VNB, which acts as a broker-like entity. This provides several benefits, for instance, VNBs can manage aggregate demand from their partners, thus reducing the amount of individual transactions needed to pair RPs with SPs. Additionally, the presence of a VNB can help address trust, incentive, and information protection issues that arise in resource sharing scenarios. Furthermore, SPs and RPs can rely on an specialized entity to find the best opportunities for obtaining resources, which can be an onerous task as the number of market participants grows. On the other hand, compared to 4-layered models, our proposed system allows us to better explore the sharing activities among the main market participants: buyers (SPs) and sellers (RPs). This model does not preclude possible extensions aiming at considering different types of buyers (e.g., MVNOs, end users, etc.); however, it allows us to obtain a deeper understanding of the core implications of our proposed resource allocation mechanisms.

Our proposed framework is based on the interactions of the three aforementioned entities. The overall goal is for the SPs to gain access to their required resources from the common pool, via interactions and negotiations with the VNBs. In turn, the VNBs seek to obtain the appropriate set of resources that can satisfy the demand of the SPs. As a result of these interactions, RPs and VNBs should receive compensation for the resources they share and the composing and aggregating services they perform. In turn, SPs should be able to satisfy the requirements of their customers.

This framework could be explained in terms of two sets of interactions—those between the VNBs and SPs, and those

between the VNBs and RPs (via the resource pool). The interactions between VNBs and SPs are modeled as a matching game, which allows both entities to choose the appropriate SPs or VNB, respectively, that best match their preferences. As a result, the system works with a set of VNB–SP *partners*. Once these partnerships are defined, a VNB can calculate the total amount of resources it needs to aggregate in order to satisfy the demand of its partners. Thus, a VNB faces the important task of aggregating resources, a key step towards the creation of virtualized environments [5]. It is important to note that the matching process allows VNBs to preselect the SPs to serve, hence narrowing down (in quantity and characteristics) the set of resources that it needs to aggregate. A similar VNB–SP matching approach is explored, in a broader spectrum sharing context, in [32] and [33].

The second set of interactions, between VNBs and RPs, corresponds to virtual resource allocation. In this paper, the goal of this task is to maximize end-users' satisfaction while minimizing the cost for SPs. This should be achieved irrespective of the uncertainty regarding users' location and channel conditions. In the current literature, virtual resources are allocated to SPs based on the *aggregated* demand of their end users [19], [20], [34]–[37]. However, due to uncertainty, resources allocated to satisfy aggregated demand cannot guarantee satisfaction of an individual end user's demand. Moreover, if the SPs seek resources to satisfy the instantaneous demands of their end users, the network overhead and computational complexity would be extremely high. To address these challenges, in this paper, we propose an efficient virtual resource allocation scheme based on stochastic geometry and stochastic optimization.

IV. SYSTEM MODEL AND PROBLEM STATEMENT

A. Network Model

We consider a two-dimensional geographical area $\mathcal{A}$ that consists of a set $\mathcal{L} \subset \Re^2$ of locations and is covered by a set of $\mathcal{N}$ RPs. Each RP has a set of base stations (BS) deployed in $\mathcal{A}$, and the union of these sets is denoted by $\mathcal{B}$. There is a set $\mathcal{S} = \{1, 2, ..., s\}$ of SPs, and a set $\mathcal{V} = \{1, 2, ..., v\}$ of VNBs. The VNBs in $\mathcal{V}$ and the SPs in $\mathcal{S}$ *play* the matching game to determine their partnerships. Subsequently, each VNB selects the optimal set of BSs from $\mathcal{B}$ and allocates them to partnering SPs. The VNBs aggregate resources from the pool sequentially,[1] so no two VNBs will request the same resource.

We consider that each end-user is served by the nearest BS within the set of BSs allocated to the user's SP. The location of BS $b \in \mathcal{B}$ is given by l_b. BS b operates on bandwidth W_b and transmits with a constant power $1/\mu_b$. Each BS $b \in \mathcal{B}$ performs proportionally fair rate allocation for its users, i.e., the rate allocated to each user is proportional to its spectral efficiency. Hence, assuming a saturated user queue, the rate of a typical user associated with BS b is given by

$$\rho_b = \frac{W_b}{N_b} \log_2 (1 + \text{SINR}_b) \quad (1)$$

where N_b is the total number of users associated with BS b, and SINR_b is the signal-to-interference-plus-noise ratio experienced by a typical user.

The channel gains experienced by the end-users from their associated BSs are assumed to follow a Rayleigh distribution with mean 1 (i.e., there is no shadowing). Hence, the SINR experienced by a typical user at an arbitrary distance d from its associated BS (e.g., BS b) can be expressed as [38]

$$\text{SINR}_b = \frac{h\, d^{-\alpha}}{\sigma^2 + I} \quad (2)$$

where h is the stochastic channel gain, which is exponentially distributed with mean $1/\mu_b$, σ^2 is the variance of the additive noise, α is the pathloss exponent, and I is the cumulative downlink interference from all other BSs. I can be expressed as

$$I = \sum_{j \in \mathcal{B}/b} g_j r_j^{-\alpha} \quad (3)$$

where r_j is the distance between the typical user and the interfering BS j and g_j is the stochastic gain of the channel. We assume that the interference also experiences Rayleigh fading without shadowing. Therefore, g_j is exponentially distributed with mean $1/\mu_j$.

B. Demand Characterization of SPs

The end users of SP $s \in \mathcal{S}$ are assumed to be distributed in $\mathcal{A}$ according to a homogeneous Poisson point process (PPP) ϕ_s of intensity λ_s. Then, SP $s \in \mathcal{S}$ characterizes its demand as follows. Any of its UEs located anywhere in $\mathcal{A}$ should have at least data rate κ_s bps with probability at least β_s. Let $\tilde{R}_s$ be the data rate of an arbitrary UE of SP s in $\mathcal{A}$. Then, the demand of SP s, is expressed as the constraint

$$\Pr\left\{ \tilde{R}_s \geq \kappa_s \right\} \geq \beta_s.$$

$\Pr\{\tilde{R}_s \geq \kappa_s\}$ is also known as rate coverage probability. For ease of discussion, let us denote the demand of SP s as D_s.

C. Matching and Resource Allocation

We characterize the SPs of $\mathcal{S}$ based on two parameters—their minimum rate coverage probability demand β_s, and the fee they are willing to pay for VNB services. Similarly, we characterize VNBs of $\mathcal{V}$ based on two parameters—their reputation (that stems from their success in satisfying SPs' demand) and the fee they charge to their partnering SPs. Note that these are not the only parameters available for demand characterization and overall description of the activities of SPs and VNBs. We have chosen these parameters for an initial test and evaluation our system model. Future implementations of this model will take into account a more refined definition of demand as well as other parameters that may influence preference definitions of SPs and VNBs.

The actual matching process between SPs and VNBs is performed taking into account the preferences of both entities (defined in terms of the four parameters mentioned above). The matching process follows the *deferred acceptance algorithm*, presented in [39], for the one-to-many matching case. In this way, each SP is allowed to match with only one VNB, while VNBs can be matched to as many SPs as currently present in a geographical area. Note that the final set of matched SPs and VNBs may not contain all existing SPs and VNBs, given that any entity may choose to remain unmatched if no options exist that suit its preferences.

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[1]VNBs access the pool one-by-one according to their own identifier in the model (e.g., $\mathcal{V}_1, \mathcal{V}_2,..., \mathcal{V}_n$). Future implementations will consider different types of priority access to the pooled resources.

Once it learns about its partners, each VNB, having prior knowledge of BS locations, UE, and channel gain distributions, selects the optimal set of BSs such that the demand, D_s, can be satisfied. At the end of this process, the reputation of VNBs is updated based on the satisfaction of its partners. Then, both, VNBs and SPs, have the opportunity to update their fees (to request and to pay, respectively). Subsequent iterations of the model will thus reflect these updated values.

D. Problem Statement

Our goal is to design efficient matching and resource allocation models. Specifically, we design a matching scheme such that SPs can choose a specialized entity, i.e., VNB, to aggregate an adequate set of resources. The choice of a VNB is made in terms of parameters that are relevant to the operations and constraints of SPs and VNBs. In other words, both entities can make a choice based on their particular preferences. Next, we design a scheme to be executed at the VNBs to optimally perform the virtual resource allocation, i.e., determining the optimal subset of BSs to be leased from $\mathcal{B}$. Our optimality criterion is to minimize the costs of network resource aggregation (i.e., maximize the utilization of the resources) while satisfying the SP demands. Hence, we define the optimal virtual resource allocation problem as: *for a given demand of the SPs in $\mathcal{S}$, determine the cheapest subset of BSs to be leased from $\mathcal{B}$ such that, these BSs can meet all SP demands.*

V. Matching Model Design

The model we propose in this paper focuses on finding an optimal allocation of resources as well as on finding an adequate negotiation mechanism to support the transfer of resources from RPs to SPs, via a VNB. Auctions have been an accepted mechanism for efficient and competitive allocation of spectrum resources. Auctions were indeed adopted by the FCC with the goal of assigning licenses in a timely manner to the providers who would put them to the *highest value* use [40]. Hence, it is often expected that, through auctions, resources will end up in the hands of those who value them most.[2]

In the specific context of our paper, the VNB can be regarded as a broker or middleman that, among other objectives, aims at easing the resource search and assignment process for the SPs. Hence, our approach considers that SPs and VNBs go through a matching process that allows them to choose the broker/customer, respectively, that best fits their needs. This process is modeled as a one-to-many matching problem. In this section, we describe the main characteristics of the matching model that pairs SPs and VNBs. This description corresponds to key factors of the matching model that apply to the problem at hand and it includes the process towards defining SPs' and VNBs' preferences and characteristics, how these are transformed into preference vectors and the final matching algorithm.[3]

A. Risk Profile of SPs and VNBs

Our underlying assumption is that SPs and VNBs face a certain level of risk by participating in resource sharing arrangements. Here, risk is the likelihood of obtaining resources, with a given rate coverage probability, for a given price. We consider that each SP and VNB is willing to bear a certain level of risk, which determines the *risk profile* of said participant. Hence, SPs and VNBs can be *risk averse*, *risk neutral,* or *risk takers*. For instance, risk averse SPs aim at maximizing their opportunities of obtaining resources in the system and risk averse VNBs aim at maximizing their opportunities to find an appropriate set of resources for their partners. As a consequence, risk profiles dictate the value that SPs and VNBs assign to their operational parameters, and their preferences regarding the characteristics of the members of the opposite set.

To avoid bias towards one particular risk profile in our model, we assign to all $s \in \mathcal{S}$ and all $v \in \mathcal{V}$, a risk value. This value is uniformly chosen from $\{0, 1, 2\}$. In this way, a risk value of 0 represents risk averseness; a risk value of 1 represents risk neutrality; and finally a risk value of 2 means that the SP/VNB is a risk taker. In general terms, the risk profile allows us to define the behavior of SPs and VNBs. There are many ways in which risk can be portrayed in a matching model; however, for this model, we have chosen prices and fees as factors where risk plays an important role.

B. SPs' Prices and VNBs' Fees

In this model, we consider VNBs to be profit-seeking entities who expect a payment for their resource aggregation services. In this way, we refer to *prices* as an SP's willingness to pay for VNBs' services, and *fees* as the compensation that VNBs expect to receive for their resource aggregation services.[4] From a market perspective, this permits us to capture the costs that a VNB incurs in finding the appropriate resources for an SP and how eager the SPs are to obtain resources from the system we propose. Thus, these exchanges could be regarded as the transaction and opportunity costs for SPs and VNBs from obtaining and aggregating resources.

C. Characteristics and Preferences of VNBs and SPs

Our matching model relies on the deferred acceptance algorithm [39]. At the core of this algorithm, we find preferences and characteristics of each set of participants that permit us to *rate* the members of the opposing set. This allows us to form *preference vectors* that will drive the course of the algorithm and the resulting matches. Thus, *characteristics* represent the value that SPs and VNBs assign to factors related to their operations, while *preferences* represent what SPs and VNBs deem acceptable from the members of the other set.

In what follows, we delve into the details of how these characteristics and parameters account for the formation of the preference vectors we require for the matching process.

1) Characteristics of SPs: We have chosen the following parameters for the preference forming and subsequent matching processes.

1) Price: VNB fee that an SP is willing to pay.

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[2]For important applications of auctions for resource assignment in next-generation networks, the reader is referred to [41], [42].

[3]For a more detailed description of this model and other applications thereof, please refer to [32], [33].

[4]In the context of our study, *price* refers to the value that an entity is willing to pay for a good or service. Normally, price reflects the cost plus an additional profit. *Fees* reflect the value that VNBs would charge for the service they provide (i.e., resource aggregation). The distinction between price and fees allows us to capture costs from the perspective of both, SPs and VNBs. Hence, the price that an SP would pay for the VNBs' services should be comparable to the cost an SP would incur to aggregate resources on its own, and how much it values to enter the market. On the VNB side, the cost stems from the difficulty of aggregating resources in the system. This cost is then translated into the fee that it decides to charge.

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TABLE I
SP PREFERENCE VECTORS ACCORDING TO RISK LEVEL

Risk level of s_i	Reputation/Quality Preference of s_i	Price Preference of s_i
Averse	$qp_i = [0, 0, 1]$	$pp_i = [0, 0, 1]$
Neutral	$qp_i = [0, 1, 1]$	$pp_i = [0, 1, 1]$
Taker	$qp_i = [1, 1, 1]$	$pp_i = [1, 1, 1]$

TABLE II
VNBs' PREFERENCE VECTORS ACCORDING TO THEIR RISK LEVEL

Risk level of v_j	Price Preference of v_j	Demand Preference of v_j
Averse	$pp_j = [1, 0, 0]$	$dp_j = [1, 0, 0]$
Neutral	$pp_j = [1, 1, 0]$	$dp_j = [1, 1, 0]$
Taker	$pp_j = [1, 1, 1]$	$dp_j = [1, 1, 1]$

2) Demand: expressed in terms of the required rate coverage probability, β_i, presented in Subsection IV-B.

To simplify our calculations in the matching process, we have mapped these parameters to three different levels (low, medium, and high). This allows us to form binary vectors of the SPs' characteristics that will be used in our implementation of the deferred acceptance algorithm.

2) Characteristics of VNBs: We consider the following parameters as relevant from the VNB's perspective.

1) Quality: A quality level or reputation is randomly assigned to each VNB in the initialization process. As system interactions take place, this value is updated according to the performance of each VNB.
2) Fees: This is the price that a VNB charges for its services. It is consistent with the VNB's quality level (i.e., higher quality VNBs are allowed to charge higher prices).

These parameters are also mapped to low, medium, and high levels to define the corresponding preference vectors.

3) Preferences of SPs: SPs express their preferences regarding a VNB's reputation or quality, and its advertised fee. These preferences are expressed as vectors, which represent the preference for a low, medium or high value for each of the aforementioned parameters. Here, the risk profile plays an important role. We use it to justify the preference of SPs for particular levels in the VNBs' characteristics. Thus, each preference vector, quality preference qp_i or price preference pp_i, is a 1×3 vector, where the kth element takes a value of 0 or 1, depending on the preference of an SP as illustrated in Table I. Indeed, we observe that if SP s_i is risk averse, it will prefer VNBs with a high reputation and it is willing to pay a high price to obtain its required resources. On the other hand, if SP s_i is risk taker, it is indifferent to the level of reputation and the price charged by a VNB so, it is willing to form partnerships with any type of VNB.

4) Preferences of VNBs: A VNB expresses its preferences regarding an SP's advertised price, and its demand level espressed by the rate coverage probability β_i.

As with the SPs, the values of these parameters are assigned according to the risk level of each VNB. The vector corresponding to each risk profile is presented in Table II.

In Table II, we observe that a risk averse VNB prefers SPs with lower prices and lower demand levels, given that its priority is to be matched and fulfill the total demand of its customers.

Our assumption is that these preferences would increase the probability of the VNB to gain partners from the matching process and subsequently reach a high reputation level through repeated successful interactions.

D. Comparing Preferences and Values

The process described in the previous subsections allows us to create the individual binary characteristic and preference vectors of SPs and VNBs. For the actual matching process, we compare the characteristics of SPs with the preferences of VNBs, and vice versa, to define a final preference vector for each SP and VNB. This permits each entity to rank the members of the opposing set, allowing us to apply the matching algorithm. To this end, we create a matrix for each of the preference parameters. In the case of SPs, the ijth matrix element is the result of multiplying the reputation and price preference vector of SP s_i by the transpose of the corresponding characteristic vectors of VNB v_j, as shown in (4) and (5). Here, qp_i and pp_i are the quality and price preference vectors of SP s_i, respectively, and qv_j and pv_j are the quality and price characteristic vectors of VNB v_j

$$Q_s(i, j) = qp_i * qv_j^T \tag{4}$$

$$R_s(i, j) = pp_i * pv_j^T. \tag{5}$$

For the VNBs, the ijth element of the preference matrices results from multiplying the demand and price vectors of SP s_i times the transpose of the corresponding preference vectors of VNB v_j. Equations (6) and (7) show these operations, where dv_i and pv_i are the demand and price characteristic vectors of SP s_i, respectively, and dp_j and pp_j are the demand and price preference vectors of VNB v_j. It is thus expected that the ijth element of any of these matrices will be 1 only if the corresponding preferences and characteristics are compatible between SP s_i and VNB v_j

$$D_b(i, j) = dv_i * dp_j^T \tag{6}$$

$$R_b(i, j) = pv_i * pp_j^T. \tag{7}$$

The outcome of this analysis gives us four matrices, two per VNB and SP, that show which SPs and VNBs are compatible. We utilize these matrices to define the utility that each SP and VNB would derive from a given partnership. This utility is given as follows:

$$U_s(i, j) = Q_s(i, j) + R_s(i, j) \tag{8}$$

$$U_b(i, j) = D_b(i, j) + R_b(i, j). \tag{9}$$

To define the final matching preferences of each SP and VNB, we take into account the *joint utility* from matching s_i with v_j. To this end, we define the matrix A, as expressed in the following:

$$A(i, j) = U_s(i, j) + U_b(i, j). \tag{10}$$

The values in the joint utility matrix are utilized for creating the final preference vectors of SP s_i and VNB v_j. Given the previous calculations, the maximum joint utility value is 4 and the minimum is 0. These values are utilized to rank each member of the opposite group and choose those that will be part of the final preference vector. We assume that an SP and a VNB which have the lowest utility value for their partnership should not be included in each other's preference vector. Hence we define a minimum utility threshold, which is currently set as the middle point between the two utility extremes.

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The subset of feasible partnerships will form the final preference vector of SP s_i and VNB v_j, i.e., partnerships with an acceptable joint utility value. This final preference vector is utilized to execute the matching algorithm described in the next subsection.

E. Matching SPs and VNBs

As pointed out by Roth in [39] regarding the marriage problem posed by Gale and Shapley, "[p] references can be represented as rank order lists of the form $P(m_i) = w_3, w_2, ...m_i$, denoting that man m_i's first choice is w_3, his second choice $w_2[w_3 >_{m_i} w_2]$ and so on, until at some point he prefers to remain unmatched (i.e., matched to himself)." The same applies to the problem at hand. In this case, the preference vector of SP s_i and VNB v_j will contain a subset of members of the opposite set with whom it is possible to form a partnership (11), (12). These subsets, or final preference vectors, are ranked in descending order of preference

$$P(s_i) = v_k, v_l, v_m, ..., s_i \qquad (11)$$

$$P(v_j) = s_o, s_p, s_q, ..., v_j. \qquad (12)$$

The matching between SPs and VNBs is implemented utilizing the *deferred acceptance algorithm* for the many-to-one matching case. This means that a VNB can form a partnership with n SPs, where n refers to a VNB's quota or partnership size; while an SP can form a partnership with only one VNB. This algorithm has been implemented following the definition presented in [39], [43]–[45].

The outcome of this matching algorithm is a matching $\mu : \mathcal{S} \cup \mathcal{V} \to \mathcal{S} \cup \mathcal{V}$, such that $v = \mu(s)$ if and only if $\mu(v) = s$. For all s and v, either $\mu(s)$ is in $\mathcal{V}$ or $\mu(s) = s$; and, either $\mu(v)$ is in $\mathcal{S}$ or $\mu(v) = v$. This means that the outcome matches SPs with VNBs, or to themselves, and if s is matched to v, then v is matched to s [39]. It is important to note that we consider the case in which the SPs *propose* a partnership first, which leads to an SP-optimal matching, $\mu_{\mathcal{S}}$ [39].

Once the matching process is over and we obtain the final matching μ, each VNB learns which are its partners and each SP learns the ID of the VNB with whom it will be working. In the following step, the matched SPs communicate their demand and resource price parameters to their VNB partners. Upon receiving the demands from the SPs, VNBs aggregate virtual resources, i.e., BSs from the common pool and allocate them to the SPs such that their demands can be satisfied.

VNBs can opt for specific market mechanisms, such as auctions, for competitively aggregating resources from the common pool. In this paper, we have focused on designing a scheme that is executed at the VNB level to determine the optimal subset of BSs from $\mathcal{B}$ to be allocated to the SPs. Our optimality criterion is to minimize the costs of resource aggregation while satisfying the SPs' demand.

VI. Optimal Virtual Resource Aggregation

In this section, we propose the optimal virtual resource aggregation scheme adopted by the VNBs. Consider VNB $v, v \in V$. A set of SPs (say, $\mathcal{S}' \subseteq \mathcal{S}$) is associated with VNB v. VNB v needs to select the optimal subset of BSs from the pool of BSs $\mathcal{B}$ and allocate them to the SPs in $\mathcal{S}'$. Now, to find the optimal subset of BSs from $\mathcal{B}$, we first formulate the problem of VNB v.

A. Problem Formulation

We assume that VNB v knows $\phi_s, s \in \mathcal{S}'$, the UE distributions of the SPs in $\mathcal{S}'$. The cost for leasing BS b is c_b. Let $x_{bs}, b \in \mathcal{B}, s \in \mathcal{S}'$, be a binary decision variable indicating whether to lease/allocate BS b for SP s or not. x_{bs} equals one if a BS will be selected for SP s and it equals zero otherwise. Then, for given rate coverage probability demands of the SPs in $\mathcal{S}'$, the optimal virtual resource aggregation problem for VNB v can be formulated as follows:

Problem 1: Optimal virtual resource aggregation

$$\underset{\left\{\substack{x_{bs} \\ b \in \mathcal{B}, s \in \mathcal{S}'}\right\}}{\text{minimize}} \sum_{b \in \mathcal{B}} \sum_{s \in \mathcal{S}'} c_b x_{bs} \qquad (13)$$

Subject to:

$$\Pr\left\{\tilde{R}_s \geq \kappa_s\right\} \geq \beta_s, \quad \forall b \in \mathcal{B}, \quad \forall s \in \mathcal{S}' \qquad (14)$$

$$\sum_{s \in \mathcal{S}'} x_{bs} \leq 1, \quad \forall b \in \mathcal{B} \qquad (15)$$

$$x_{bs} \in \{0, 1\}, \quad \forall b \in \mathcal{B}, \quad \forall s \in \mathcal{S}'. \qquad (16)$$

The objective function (13) represents the cost of the leased BSs. Constraint (14) ensures the demand satisfaction of the SPs in $\mathcal{S}'$. Constraint (15) ensures that a BS is allocated to no more than one SP. Keeping the demand of SPs as a constraint, VNB v aggregates the minimum cost BSs from the pool (i.e., RPs).

B. Solution Approach

In order to solve Problem 1, first, we need to find a closed form expression for constraint (14). Specifically, we need to find a closed form expression of the rate coverage probability obtained by SP $s, s \in \mathcal{S}'$.

Stochastic geometry has been widely applied for analyzing rate coverage probability [38]. Existing stochastic geometry works analyze the rate coverage probability assuming uncertainty in the locations of both UEs and BSs. BS locations were modeled stochastically in the existing works in order to obtain analytical results that are valid for a variety of BS deployments. In contrast, here, we need to consider locations, operating bandwidth, transmission power, and costs of individual BSs to determine the optimal subset of BSs. Therefore, we derive a closed form expression of the rate coverage probability obtained by SP $s, s \in \mathcal{S}'$ as follows.

Let $\mathcal{B}_s$ be the set of BSs allocated to SP s. Then, from [46, Th. 1], the rate coverage probability obtained by SP s is given by (17) as shown at the bottom of the next page. In (17), $\mathcal{P}_b$ is the region of the voronoi cell of BS b. A_b is the area of the geographical area $\mathcal{A}$. D_u is a circular disc of radius u centered at l_b. d is the distance between BS b and a typical UE of SP s. $d_{b,j}$ is the distance between BS b and a neighboring BS j. $r_j = \sqrt{d^2 + d_{b,j}^2 - 2 d d_{b,j} \cos \theta_j}$ and θ_j is the angle between the two lines—the line connecting BS b with the typical UE, and the line connecting BS b and neighboring BS j as shown in Fig. 2.

In (17), part (i) is the probability of a typical UE of SP s being associated with BS b. Part (ii) to part (v) all together represent the probability density function (PDF) of $\tilde{R}_b$, defined as the rate obtained by a typical UE of SP s from BS b with its full service

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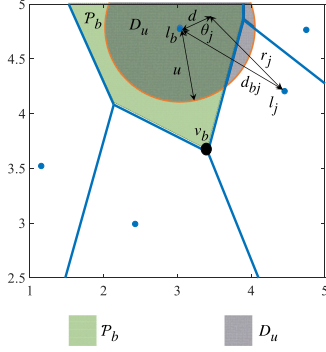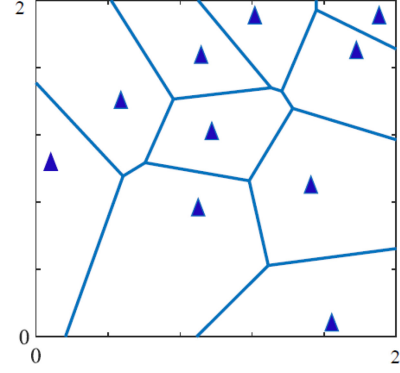GOMEZ *et al.*: MARKET-DRIVEN STOCHASTIC RESOURCE ALLOCATION FRAMEWORK

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Fig. 2.   Voronoi tessellation of a set of BSs.

Fig. 3.   Locations of 10 BSs, as obtained from a realization of a homogeneous PPP of intensity $2.5/\mathrm{km}^2$.

time, for any real number ρ within the support of $\tilde{R}_b$. Specifically, part (v) is the PDF of $\tilde{d}$, the distance of a typical UE of SP s from its associated BS b, for any real number u within the support of $\tilde{d}$. Part (iv) is the PDF of $\tilde{I}$, the cumulative interference experienced by a typical UE located at distance u from its associated BS b, for any real number c within the support of $\tilde{I}$. Part (iii) originates from the PDF of the SINR received by a typical UE located at distance u from its associated BS b and experiencing cumulative interference c. Finally, part (ii) originates from the distribution of BS load.

Replacing the expression of $\Pr\{\tilde{R}_s \geq \kappa_s\}$ from (17) in constraint (14), problem 1 is solved numerically.

VII. Framework Evaluation

In this section, we evaluate our proposed market-driven stochastic virtual resource allocation framework. We relied on MATLAB to build the corresponding model, and on the statistical and mathematical tools provided by Python to evaluate and analyze our results.[5] The model we propose and design is rather complex and it applies to a wide range of scenarios. Nevertheless, in this paper, we focus on one specific case of interest, which allows us to portray how our model operates and the results it provides.

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[5]We relied on the capabilities of MATLAB for defining, coding and simulating our agent-based model. The results obtained were then exported to Python, for data analysis and visualization purposes. We think that exploiting the capabilities of these platforms allows us to present results that are more appealing to the readers, without affecting the quality of our underlying model.

A. Simulation Environment

The simulation setup is described as follows: the network and market participants are 4 SPs, 3 VNBs, and 5 RPs. We consider a geographical area of $2 \times 2\ \mathrm{km}^2$, where 10 BSs are deployed by the RPs as shown in Fig. 3. All BSs operate over a bandwidth of 10 MHz. The BSs' transmission power is set to 40 dBm and σ^2 is set to -104 dBm. The path loss exponent, α, is set to 4. The set of SPs is denoted by $\mathcal{S} = \{1, 2, 3, 4\}$. We set $\beta_s, s \in \mathcal{S}$, the rate coverage probability demand of these SPs, as : $\beta_1 = 0.5$, $\beta_2 = 0.7$, $\beta_3 = 0.8$ and $\beta_4 = 0.9$.

The market-related parameters of each SP and VNB are defined following the description presented in Section V. Additionally, for experimental purposes, we have considered SPs and VNBs assigning different weights to their preference parameters. In this manner, these weights take *low* values in the range from (0.1–0.5) or *high* values in the range (0.6–1). Considering we have two different preference parameters per SP and VNB, we can have four different combinations of these weights resulting in four experimental groups, one with each weight combination. Then, SPs and VNBs are randomly assigned to one of those four groups. Note that, initially, a VNB's reputation is randomly assigned. Nevertheless, this value is adjusted according to historical VNB performance as we run the simulations.

B. Experiments and Results

In our first experiment, we vary the rate demand of the SPs while fixing their user intensity at $20/\mathrm{km}^2$. In Fig. 4, we show the distribution of the actual satisfaction index of each SP while taking into account different rate coverage probabilities, β. Note that the satisfaction index of an SP is the ratio of the obtained and requested rate coverage probabilities. As it can be observed,

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$$\Pr\left\{\tilde{R}_s \geq \kappa_s\right\} = 1 - \frac{1}{2\pi A} \sum_{b \in \mathcal{B}_s} \left[e^{-\lambda A_b} \frac{\log 2}{W_b} \sum_{n=0}^{\infty} \frac{\lambda^n A_b^n}{(n-1)!} \int_0^{\rho_s} 2^{\frac{n\rho}{W_b}} \int_0^{\infty} \int_0^{|l_b - v_b|} \right.$$

$$\times \left\{ \mu_b\, u^\alpha \left(\sigma^2 + c\right) \exp\left(-\mu_b\, u^\alpha \left(2^{\frac{n\rho}{W_b}} - 1\right) \left(\sigma^2 + c\right)\right) \right\}$$

$$\left. \times \left\{ \int_{-\infty}^{\infty} \left\{ \frac{e^{-i\omega c}}{2\pi} \prod_{j \in \mathcal{B}_s \backslash b} \int \frac{\mu_j\, r_j^\alpha}{\mu_j\, r_j^\alpha - i\omega}\, \mathrm{d}v \right\} \mathrm{d}\omega \right\} \frac{\mathrm{d}\left[\nabla\left\{\mathcal{P}_b \cap D_u\right\}\right]}{\mathrm{d}u}\, \mathrm{d}u\, \mathrm{d}c\, \mathrm{d}\rho \right] \quad (17)$$

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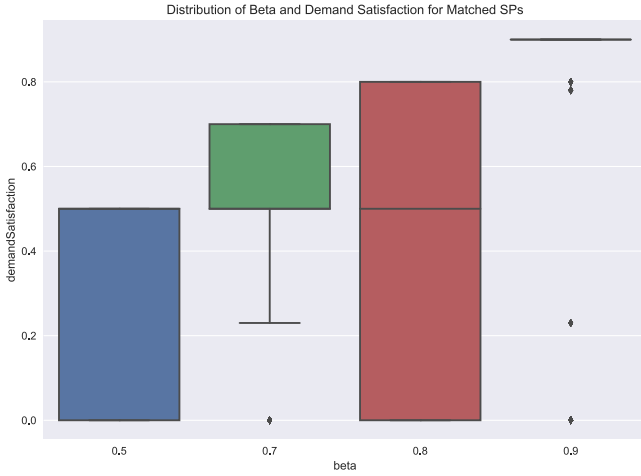

Fig. 4. Distribution of SPs' demand satisfaction. We show the level of demand satisfaction obtained for different values of β (rate coverage probability). Note that the level of demand satisfaction ranges from 0 to 1, and the values of β we have considered are 0.5, 0.7, 0.8, and 0.9.
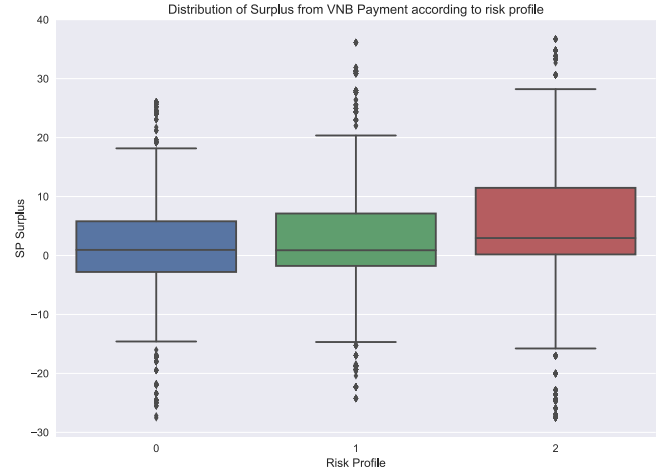

Fig. 6. Distribution of SPs' surplus, according to their risk profile. The different boxes show the distribution of the surplus perceived by SPs, from their negotiations with VNBs. Positive surplus values indicate profits while negative values indicate losses incurred by the SPs. The risk profile of SPs (*x*-axis) is presented as a numeric value, where 0: risk averse, 1: risk neutral, and 2: risk taker.
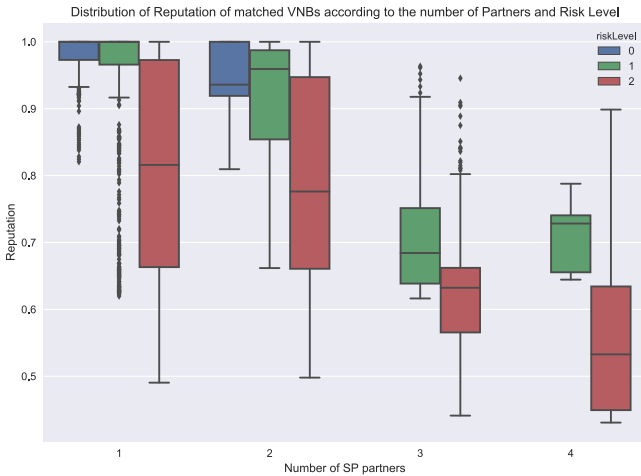

Fig. 5. VNBs' reputation distribution according to the number of SP partners and their risk profile. The VNBs' risk profile varies from 0: risk averse to 2: risk taker. For each value of SP partners, the left-most box corresponds to risk averse VNBs and the right-most box corresponds to risk takers. As we can observe, we find risk averse VNBs matching with two SPs at most, while the distribution of risk-taking VNBs spans across all SP configurations.
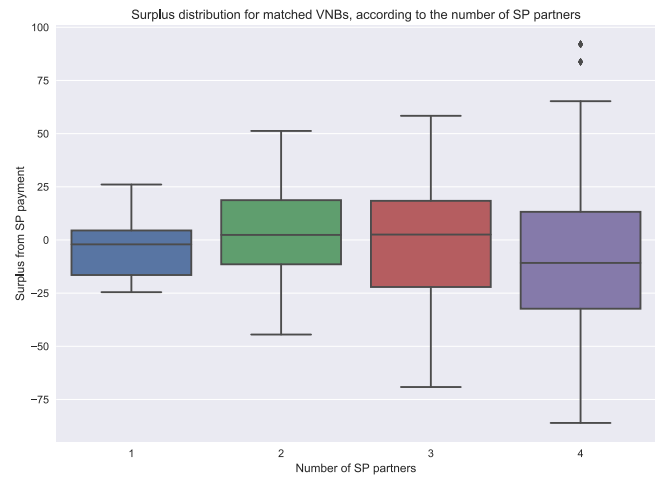

Fig. 7. Distribution of VNBs' surplus, according to their total number of partners. The box plots represent the distribution of the surplus perceived by VNBs from their negotiations with SPs. Positive values correspond to profits, while negative values represent losses incurred by the VNBs. The *x*-axis shows the number of SP partners of each VNB.

SPs with higher demand rates can reach a higher (maximum) satisfaction index, especially in the case of SPs with $\beta = 0.9$. This reflects the priority that these providers are given at resource assignment. The relationship between obtained and requested rate coverage probabilities, provides us with a measure of the reputation of the VNBs. Indeed, the reputation of a VNB results from the average value of the demand satisfaction index of all its customers. In Fig. 5, we present the distribution of matched VNBs' reputation. The box plots represent the reputation ranking, taking into account the number of SP partners and the risk profile of VNBs. Note that in this figure, risk level values of 0, 1, and 2 refer to risk *averse*, *neutral*, and *taker* profiles, respectively. In general terms, VNBs with less SP partners maintain higher reputation levels, which translates into a higher ability to satisfy the demand of their partners. Additionally,

more conservative risk approaches, allow VNBs to keep their reputation distribution higher. Indeed, our results show that risk averse and neutral VNBs do not take more than two partners. As presented in our model description, VNBs expect a payment for their resource aggregation activities. Both entities, SPs and VNBs, may shade their real service valuation, which can yield a possible (positive or negative) surplus for either one of them. In what follows, we present the resulting surplus distribution, perceived by VNBs and SPs. It is important to note that the surplus we present stems only from payments made/received by SPs/VNBs for the resource aggregation services. These surplus values do not include resource payments. Fig. 6 presents the surplus obtained by the SPs, and Fig. 7 shows the distribution of the surplus obtained by the VNBs.

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In the case of SPs, we observe that those with a less conservative risk profile (risk takers) obtain higher surplus values. This stems from the level of price shading these SPs apply to their real valuation. In this way, they risk not being matched to a VNB, by offering lower payments; however, if they are indeed matched, their gains are larger than other, more risk averse, SPs. In the case of VNBs, we explore the surplus in terms of the number of partners they have. In this way, it remains more profitable for them (in the same way as with the reputation) to match with a smaller number of SPs. From price offer/demand differences, a VNB can incur in losses at the moment of receiving its payment. In this manner, even a small loss incurred with a larger set of partners, may result in negative surplus results.

C. Model Complexity

The framework we propose results in a tractable means to analyze a rather complex problem. For matching VNBs with SPs, we utilize a SP-proposing deferred acceptance algorithm. This algorithm presents a tractable method for matching one or more SPs to one VNB. Indeed, at its worst, the complexity of this algorithm in time and space is $\mathcal{O}(VS)$ [47] for V VNBs and S SPs. Note that this algorithm results in an SP-optimal stable matching. Now, another important step of the matching scheme is to perform the virtual resource allocation. We compute the complexity of the virtual resource allocation scheme as follows. Consider a VNB $v, v \in \mathcal{V}$. The VNB v needs to serve S' number of SPs. In that case, for B number of BSs in the resource pool, the virtual resource allocation scheme for VNB v has complexity of $\mathcal{O}(S'^B)$. Hence, at its worst, the complexity of our overall scheme is $\mathcal{O}(VSS^B)$ or, $\mathcal{O}(VS^{(B+1)})$ for V VNBs, S SPs and B BSs.

VIII. CONCLUSION

The model we propose in this paper, represents a novel system to allocate virtualized wireless network resources. Our system is composed of three main entities—current resource owners, or RPs, resource buyers or SPs, and a resource aggregator or VNB. Our analysis relies on the combination of technical and market definitions of demand, resource aggregation, and allocation. More specifically, we technically define resource demand and an optimal resource allocation model, while relying on a matching market model to define the interactions between SPs and VNBs. Our system differs from other three-layered proposals in that we adopt a comprehensive view on the different factors that play a role in a resource allocation process. In this way, not only can we evaluate optimal allocation performance , but also the profitability of the participating entities; hence allowing us to shed light on the overall viability of our proposed system.

Currently, our demand metric for the matching model is the rate coverage probability. In future versions of this model, our objective is to take into account a more complex metric for demand characterization, which considers different rates and additional user densities or distributions. Additionally, we aim at exploring how more flexible definitions of rate requirements may influence the outcome of our model. From a market-oriented perspective, we are interested in exploring how our optimal resource aggregation model compares to an auction model. Additionally, we aim at analyzing our model within the context of particular applications and network configurations, such as those required for 5G and the systems it enables.

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