Modeling and Control of Modular Multilevel Converter

Yugal Gupta

Thesis submitted to the Faculty of the
Virginia Polytechnic Institute and State University
in partial fulfillment of the requirements for the degree of

Masters of Science
in
Electrical Engineering

Fred C. Lee, Chair
Yi-Hsun Hsieh, Co-Chair
Qiang Li, Member

June 24, 2022
Blacksburg, Virginia

Keywords: Modular Multilevel Converter, Modeling and Control, Circulating Energy, State Plane, Coordinate Transformation, Capacitor Reduction, Over-Constrained Optimization, Common Mode Voltage Injection

Copyright 2022, Yugal Gupta
Due to modularity and easy scalability, modular multilevel converters (MMCs) are deemed the most suitable for high-voltage and medium-voltage power conversion applications. However, large module capacitors are usually required in MMCs to store large circulating power of line-frequency and its harmonics that flow through the capacitors. Even though several methods for minimizing the circulating power have been proposed in the literature, there is still the need for a systematic and simplified approach of addressing these control strategies and evaluating their efficacy. Moreover, the generally accepted feedback control architecture for the MMC is complicated, derived through a rigorous mathematical analysis, and therefore, not easy to intuitively comprehend. Recently, a method of modeling of the MMC based on state-plane analysis and coordinate transformation, is proposed in the literature. Based on the state-plane analysis, two kinds of circulating power in the MMC are identified that are orthogonal to each other. This means these two circulating power can be controlled individually without affecting each other. To control these circulating power, in the literature, a decoupled equivalent circuit model is developed through the coordinate transformation which clearly suggests a means for minimizing these circulating power. Further extending this work, in this thesis, the existing control concepts for reducing the circulating power are unveiled in a systematic and simplified manner utilizing the decoupled equivalent circuit model. A graphical visualization of circulating power using the state-planes is provided for each control strategy to readily compare its efficacy. Moreover, the generally accepted control architecture of the MMC is presented in an intuitive and simplified way using the decoupled circuit model. The important physics related to control implementation, originally hidden behind the complicated mathematics, is explained in detail.
Modeling and Control of Modular Multilevel Converter

Yugal Gupta

(GENERAL AUDIENCE ABSTRACT)

A power converter is an electrical device that converts electrical energy from one form to another in order to be compatible with the load demand. A typical power converter consists of semiconductor switches, inductor, capacitor etc. These power converters are required in a wide range of applications: auto-motive and traction, motor drives, renewable energy conversion, energy storage, aircraft, power generation, transmission, and distribution, to name a few. Many of these applications are continuously increasing their power capacity to handle the escalating demands of energy that exist due to rising population numbers, industrialization, urbanization etc. Consequently, it has been a responsibility of power electronics engineers and researchers to develop power converters that can handle high voltages and high currents. Multilevel power converters have been the key-enabling developments that can withstand high-voltages while using traditional low-voltage semiconductor switches. Several multilevel converters such as the neutral point clamped converter, flying capacitor converter, cascaded H-bridge converter, modular multilevel converter (MMC) etc. have been developed and commercialized in the last two decades. Among them, the MMC is a widely accepted topology for medium- and high-voltage power conversion applications. In an MMC, several modules are stacked together in series, and each module consists of semiconductor switches and a capacitor. The series connection of the modules enables the MMC to handle high-voltage power conversion using low-voltage traditional semiconductor switches. The voltage rating of an MMC can be easily scaled-up by simply increasing the number of modules in each arm. Moreover, since several identical modules are connected in each arm, the structure of the MMC is highly modular which helps greatly in manufacturing and design. Nonetheless, in MMCs, generally large circulating power flow to the capacitor in each module, which lead
to significant voltage ripples. To suppress these voltage ripples, a large capacitor is required in each module, leading to large size and weight of the converter. In the literature, several control strategies have been proposed to minimize the circulating power. However, there is still the need for a systematic and simplified approach of addressing these control strategies and evaluating their efficacy. Moreover, the generally accepted feedback control architecture for the MMC is complicated, derived through a rigorous mathematical analysis, and therefore, not easy to intuitively comprehend. Recently, a decoupled equivalent circuit model has been developed in the literature. This model clearly explains the process of power flow in the MMC between input and output and the nature of the circulating power. The equivalent circuit model provides the circulating power, that are orthogonal to each other, meaning they can be controlled individually without affecting each other. Moreover, the equivalent circuit model clearly suggests a means for minimize the circulating power by providing two “ideal” control laws. Further extending this work, in this thesis, the existing control concepts for reducing the circulating power are unveiled in a systematic and simplified manner utilizing the decoupled equivalent circuit model. Moreover, the generally accepted control architecture of the MMC is presented in an intuitive and simplified way via the decoupled circuit model. The important physics related to control implementation, originally hidden behind the complicated mathematics, is explained in detail.
Acknowledgments

First of all, I would like to convey my sincere gratitude to my advisor, Dr. Fred C. Lee, for his invaluable time, continuous support, guidance and patience. His experience, guidance and knowledge benefited me significantly in my research. I would also like to sincerely thank and express my heartfelt appreciations to my co-advisor, Dr. Yi-Hsun (Eric) Hsieh for laying down a strong foundation for my research work and providing me a continuous guidance and support.

I feel that Dr. Lee and Dr. Hsieh truly prove the famous quote, “A good teacher is like a candle, that consumes itself to light the way for others”. They made tremendous efforts in the weekly meetings to not only provide me with technical guidance but also teaching me a research attitude by continuously and patiently working on my weaknesses. I will be forever grateful for their efforts and time. They taught me an attitude, “never stop challenging yourself and keep improving” which has helped me a lot in my academic research as well as daily life.

I would like to sincerely thank my committee member, Dr. Qiang Li for his support, guidance and valuable suggestions. He instructed a course, “Power Converter Modeling and Control” that significantly helped me in performing my research work.

I would also like to give specific appreciation to the faculty advisors in the WBG-HPCS mini-consortium, Dr. Dushan Boroyevich, Dr. Rolando Burgos, Dr. Dong Dong, Dr. Igor
Cvetkovic, Dr. Bo Wen, and Dr. Boran Fan for their invaluable feedback and comments for my research work.

I would also like to thank the remaining CPES faculty and the wonderful CPES staff who also helped me in my education and research there.

I would like to thank CPES students, Zheqing Li, Chunyang Zhao, Xingyu Chen, Pranav Raj Prakash, Bhavin Jain, Jayesh Motwani, Tianlong Yuan, Gibong Son, Sundaramoorthy Sridhar, Tam Nguyen, Vladimir, and many others for sharing their knowledge and experience, and providing encouragements. I am also thankful to the past CPES students, Chen Li and Yadong Lyu, for providing me a foundation for my research work. I would also like to extend my thanks to my friends, Arpit, Sachin, Ikjot, Anubrata, Purushottam, Sourav and Rishabh for continuously encouraging me.

Most importantly, I must express my sincere gratitude to my parents, Ramakant Gupta & Anita Gupta for always supporting my decisions and believing in my capabilities. They work tirelessly while rendering their services which always inspires me to tide over hardships while pursuing goals. I am also grateful to my brother Bhuvan Gupta. He has always been a source of motivation and inspiration for me since my childhood. In my childhood, I was very low achieving in my studies, but he, like a teacher, helped me in laying down a strong foundation for learning in the formative years of my life and gradually instilling my belief on my capability to excel and maintain a distinctive edge. I am also very thankful to my sister-in-law, Richa Mittal, and my grandparents for unfailingly being caring, loving and supportive to me.
Contents

List of Figures ix

List of Tables xiii

1 Introduction 1

1.1 Converter Configuration and its Basic Working Principals . . . . . . . . . . . . 2
1.2 Control Implementation Proposed by Marquardt, et al. . . . . . . . . . . . . 3
1.3 Review of MMC Modeling . . . . . . . . . . . . . . . . . . . . . . . . . . . . 12
1.3.1 Simplified Average Model of MMC . . . . . . . . . . . . . . . . . . . . 12
1.3.2 Per-Phase Average Model of MMC . . . . . . . . . . . . . . . . . . . . 16
1.3.3 State Plane Analysis . . . . . . . . . . . . . . . . . . . . . . . . . . . . 18
1.3.4 Coordinate Transformation and Decoupled Equivalent Circuit Model 23
1.3.5 “Ideal” Control Laws . . . . . . . . . . . . . . . . . . . . . . . . . . . . 31
1.4 Simple Control Law Under The Equivalent Circuit Model . . . . . . . . . . . . 33

2 Interpretation of Generally Accepted Control Framework (Akagi et al.) 36
2.1 Control Implementation Proposed by Akagi et al. ................. 37

2.2 Interpretation of Akagi’s Control Framework Using Equivalent Circuit Model 41

2.2.1 Output Current Regulation .................................... 42

2.2.2 Capacitor Voltage Regulation .................................. 43

2.3 Evaluation of Circulating Energies ................................ 49

2.3.1 First Control Law .................................................. 49

2.3.2 Second Control Law ................................................ 51

2.3.3 State Planes ......................................................... 52

3 Further Improvements for Minimizing Circulating Energy .............. 55

3.1 Eliminating Circulating Energy Related to \(2\omega\) ....................... 56

3.2 Eliminating Circulating Energy Related to \(\omega\) ....................... 60

3.3 Eliminating Circulating Energy Related to \(\omega\) and \(2\omega\) ............. 64

3.4 Eliminating Circulating Energy Related to \(\omega\), \(2\omega\), and \(3\omega\) ............. 67

4 Summary and Conclusion ................................................. 74

Bibliography .............................. 78
List of Figures

1.1 Basic structure of Modular Multilevel Converter and its applications . . . . . 2
1.2 Marquardt’s control for MMC based on sinusoidal pulse-width modulation . 4
1.3 Example of SPWM for N = 3 . . . . . . . . . . . . . . . . . . . . . . . . . . 5
1.4 Possibilities of turning-on one module in the upper arm of phase - a for N = 3 5
1.5 Output current regulation in dq frame . . . . . . . . . . . . . . . . . . . . . 6
1.6 Simulation results of upper arm of phase – a for Marquardt’s control . . . 7
1.7 A submodule applied in a practical MMC project [24] . . . . . . . . . . . . 8
1.8 Single-phase half-bridge inverter (a) feedback Control (b) simulation results . 9
1.9 Assumptions taken in average model (a) perfect balance control, (b) average over $f_e$ . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 12
1.10 Simplified average model of MMC . . . . . . . . . . . . . . . . . . . . . . . 14
1.11 Verification of simplified average model . . . . . . . . . . . . . . . . . . . . . 15
1.12 3-Phase simulation results with Marquardt’s control . . . . . . . . . . . . 16
1.13 Per-phase simplified average model of MMC . . . . . . . . . . . . . . . . . . 17
<table>
<thead>
<tr>
<th>Section</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.3</td>
<td>Generally accepted control framework under the equivalent circuit model</td>
<td>42</td>
</tr>
<tr>
<td>2.4</td>
<td>Control implementation without $\sin(\omega t)$ under the equivalent circuit model</td>
<td>45</td>
</tr>
<tr>
<td>2.5</td>
<td>Simulation results without $\sin(\omega t)$</td>
<td>46</td>
</tr>
<tr>
<td>2.6</td>
<td>Simulation results with $\sin(\omega t)$</td>
<td>48</td>
</tr>
<tr>
<td>2.7</td>
<td>Evaluation of Akagi’s Control using the proposed “ideal” control laws (a) first control law (b) second control law</td>
<td>50</td>
</tr>
<tr>
<td>2.8</td>
<td>State planes (a) $Nv_c^l - Nv_c^u$ (b) $i^u - Nv_c^u$</td>
<td>52</td>
</tr>
<tr>
<td>2.9</td>
<td>Comparison of Akagi’s control with Marquardt’s control</td>
<td>53</td>
</tr>
<tr>
<td>3.1</td>
<td>Implementation of second-order harmonic current injection using the proposed equivalent circuit model</td>
<td>57</td>
</tr>
<tr>
<td>3.2</td>
<td>Evaluation using the proposed “ideal” control laws (a) first control law (b) second control law</td>
<td>58</td>
</tr>
<tr>
<td>3.3</td>
<td>State planes (a) $Nv_c^l - Nv_c^u$ (b) $i^u - Nv_c^u$</td>
<td>60</td>
</tr>
<tr>
<td>3.4</td>
<td>Evaluation using the proposed “ideal” control laws (a) first control law (b) second control law</td>
<td>62</td>
</tr>
<tr>
<td>3.5</td>
<td>State planes (a) $Nv_c^l - Nv_c^u$ (b) $i^u - Nv_c^u$</td>
<td>63</td>
</tr>
<tr>
<td>3.6</td>
<td>Evaluation using the proposed “ideal” control laws (a) first control law (b) second control law</td>
<td>65</td>
</tr>
<tr>
<td>3.7</td>
<td>State planes (a) $Nv_c^l - Nv_c^u$ (b) $i^u - Nv_c^u$</td>
<td>66</td>
</tr>
<tr>
<td>3.8</td>
<td>Simplified average model of MMC</td>
<td>67</td>
</tr>
<tr>
<td>3.9</td>
<td>Per-phase model of MMC with common mode voltage</td>
<td>69</td>
</tr>
</tbody>
</table>
3.10 Equivalent circuit model with common mode voltage

3.11 Implementation of control method based on common mode voltage injection

3.12 Evaluation using the proposed “ideal” control laws (a) first control law (b) second control law

3.13 State planes (a) $Nv_c^l - Nv_c^u$ (b) $i^u - Nv_c^u$

4.1 Comparative assessment of circulating energy existing with various control strategies [32]: Ex. 1: Akagi’s control at $M = 0.8$, Ex. 2: eliminating circulating energy related to $2\omega$ at $M = 0.8$, Ex. 3: eliminating circulating energy related to $\omega$, Ex. 4: eliminating circulating energy related to $\omega$ and $2\omega$, and Ex. 5: eliminating circulating energy related to $\omega$, $2\omega$ and $3\omega$.

4.2 Comparison of module capacitance requirement in various design examples:
Ex. 1: Akagi’s Control at $M = 0.8$, Ex. 2: eliminating circulating energy related to $2\omega$ at $M = 0.8$, Ex. 3: eliminating circulating energy related to $\omega$, Ex. 4: eliminating circulating energy related to $\omega$ and $2\omega$, and Ex. 5: eliminating circulating energy related to $\omega$, $2\omega$ and $3\omega$.
List of Tables

1.1 Simulation Parameters ............................................. 15
Chapter 1

Introduction

For the past few decades, one of the foremost demands has been to meet the world’s escalating demands for energy without relying more on fossil fuels, and thus, sustaining the global climate. Power electronics has played a key role in this matter by integrating more and more renewable energy sources such as solar photovoltaics (PVs), wind farms etc. Large scale proliferation of renewable energy sources also drives the focus of power electronics engineers and researchers towards high-voltage and medium-voltage power converters [1]-[14]. Multi-level converters such as the diode clamped converter [15][16], the flying capacitor converter [17][18], the cascaded H-bridge converter [19], the modular multilevel converter (MMC) [20], etc., are the key enabling developments for realizing medium- and high-voltage power conversion using low-voltage switching devices [21]. Among various available multilevel converters, the MMC is the most attractive topology due to its modularity and easy scalability. The MMC was first introduced in 2003 by Lesnicar and Marquardt [20][22]. Figure 1.1 shows a basic structure of an MMC for DC/AC power conversion. Figure also shows some of its applications such as interconnecting renewable energy sources like wind farms [1][2], solar PVs [3][4][5] etc., connecting battery energy storage systems [6][7], high-voltage direct-current
Chapter 1. Introduction

Figure 1.1: Basic structure of Modular Multilevel Converter and its applications (HVDC) transmission [8], flexible AC transmission systems (FACTS) [9][10], motor drives [11][12] etc.

In the following section, the configuration of the MMC and its basic working principals are briefly discussed.

1.1 Converter Configuration and its Basic Working Principals

In the MMC, there are two arms in each phase — one is the upper arm, and the other is the lower, hereafter denoted by superscript, ‘u’ and ‘l’, respectively in the notations. In each arm, there are several modules, stacked-up together, as shown in the figure. A schematic of a module based on a half-bridge topology is shown in the figure. Whenever the upper switch is turned on and the lower switch is turned off, the output voltage of the module is given by its capacitor voltage. This mode of operation of the module is referred to as the ‘turned-
on’ mode. On the other hand, when the upper switch is turned off and the lower switch is turned on, the output voltage of the module becomes zero. This mode of the module’s operation is known as, ‘turned-off’ mode or ‘bypass’ mode. This means that each module can behave as a controllable voltage source as long as its capacitor voltage is sufficiently constant. Therefore, by suitably changing the number of turned-on modules or turned-off modules in each arm through a feedback control, the arm voltages can be synthesized to achieve the desired operation of the MMC. The inductors are inserted in each phase-leg to avoid a high transient current caused by the direct connection of the arms.

The following section discusses the basic control implementation of MMC, proposed by Marquardt, et al., [20][22].

1.2 Control Implementation Proposed by Marquardt, et al.

The control implementation proposed by Marquardt, et al., is shown in Figure 1.2. As shown, the three-phase output currents are regulated through a feedback control which provides the voltage commands, \( \tilde{v}_{ma}, \tilde{v}_{mb}, \) and \( \tilde{v}_{mc} \). Thereafter, the voltage commands for the upper and lower arm are generated in each phase as shown in the figure. The upper- and lower-arm voltage commands for phase- a are given by

\[
\tilde{v}_{ma}^u = 0.5V_{in} - \tilde{v}_{ma}, \text{ and } \tilde{v}_{ma}^l = 0.5V_{in} + \tilde{v}_{ma}.
\]

The resultant voltage commands, \( \tilde{v}_{ma}^u \) and \( \tilde{v}_{ma}^l \) go to sinusoidal pulse-width modulation (SPWM) that determines the number of modules, \( \tilde{m}_a^u \) and \( \tilde{m}_a^l \) that should be turned-on in
the upper- and lower-arm, respectively. Based on \( \tilde{m}_u^a \) and \( \tilde{m}_l^a \), ‘balance control’ generates the switching signals such that the capacitors in an arm remain balanced.

To demonstrate the concept of SPWM and balance control, let’s consider a simple case having only three modules in each arm, i.e. \( N = 3 \). As shown in Figure 1.3, the modulating signal, \( \tilde{v}_{ma}^u \) is compared with the three carrier signals to determine, \( \tilde{m}_u^a \) i.e., the number of modules that should be turned on in the upper arm. For example, consider the time instant, \( t_1 \), shown in the figure. At this instant, the modulating signal, \( \tilde{v}_{ma}^u \) is greater than one carrier signal. Therefore, one module should be turned on in the upper arm at this instant (i.e., \( \tilde{m}_u^a = 1 \)). There are three possibilities to turn-on one module in the upper arm — either turn on the first, second, or third module, as shown in Figure 1.4. Balance control determines exactly which one module should be turned on out of these three possibilities, based on the capacitor voltages and direction of arm current. For example, let’s assume that the capacitor voltages of these three modules i.e., \( \tilde{v}_{Ca1}, \tilde{v}_{Ca2}, \tilde{v}_{Ca3} \), at instant \( t_1 \), are 2.3 kV, 2.4 kV and 2.5 kV, respectively. Therefore, if the arm current is positive at this instant then the first
1.2. Control Implementation Proposed by Marquardt, et al.

Figure 1.3: Example of SPWM for $N = 3$

![SPWM Diagram]

Figure 1.4: Possibilities of turning-on one module in the upper arm of phase - a for $N = 3$

(a) Insert #1

(b) Insert #2

(c) Insert #3

module should be turned on so that its capacitor, which has the lowest voltage amongst all, can get charged. On the other hand, if the arm current is negative then the third module
should be turned on so that its capacitor, which has the highest voltage amongst all, can get discharged.

Therefore, SPWM determines the number of modules that should be turned-on while balance control determines which modules should be turned on such that capacitor voltages remain balanced.

Figure 1.5 shows the control implementation for output current regulation. Since the sum of the three-phase output currents is always zero in a three-wire system, there are only two independent output current variables. Therefore, the compensation is implemented in a two-dimensional dq frame, as shown in the figure. Moreover, as the dq frame itself rotates at a synchronous speed, the steady state current variables in the dq frame become DC and therefore, their compensation through a proportional-integral (PI) regulator becomes more effective. The generated voltage commands from the PI regulators, $\tilde{v}_{md}$ and $\tilde{v}_{mq}$, are converted back to the abc frame, $\tilde{v}_{ma}$, $\tilde{v}_{mb}$, and $\tilde{v}_{mc}$.
1.2. Control Implementation Proposed by Marquardt, et al.

Figure 1.6 shows the simulation results for the upper arm of phase – a for two line-cycles with 25 modules in each arm i.e., $N = 25$. Figure 1.6(a) shows the switching signal of the first module of the upper arm of phase - a, i.e., $\tilde{d}_{a1}$. Whenever, $\tilde{d}_{a1}$ is 1, the upper switch of the module is turned on and when $\tilde{d}_{a1}$ is 0, the lower switch is turned on. The switching frequency, $f_{sw}$, chosen for this simulation, is 400 Hz. Figure 1.6(b) shows the arm voltages, $\tilde{v}_a^u$ and $\tilde{v}_a^l$. The arm voltages include the cumulative effect of all the modules, and therefore, they are stair-cased waveforms, having a high-frequency component, $f_e$, given by $N \times f_{sw}$, which is also called the equivalent switching frequency. In this case, $f_e$ is 10 kHz. It should be noted that arm voltages do have some distortion due to line-frequency related harmonics caused by the capacitor voltage ripples. However, the output current for phase -
a, shown in Figure 1.6(c), is well regulated with the help of feedback control. Figure 1.6(d) shows the waveforms of the arm currents. Since the arm currents are not monitored through feedback control, they have large line-frequency related harmonics, as can be observed from the results. Consequently, the arm current flowing through the module capacitors, results in significant charging and discharging of the capacitors at the line-frequency and its harmonics. Figure 1.6(e) shows the capacitor voltages of all the modules of the upper and lower arm of phase - a. The capacitor voltages have large ripples related to the line-frequency and its harmonics. Consequently, large module capacitors are required to reduce these voltage ripples which results in large volume and weight of the converter [23]. Figure 1.7 shows a submodule applied in a practical MMC-HVDC project [24]. The capacitor connected in the submodule covers more than 50\% of the submodule’s area. Therefore, the oscillating power flowing to the capacitors, leading to the line-frequency and related voltage harmonics, is considered as the undesired power, and hereafter referred as the circulating power of the MMC. This circulating power should be minimized as much as possible to reduce the size of the capacitors.

Now let’s compare the performance of the MMC with that of a single-phase, half-bridge inverter, shown in Figure 1.8(a). As shown, the output current, fed to the grid, is regulated through feedback control. Various steady-state waveforms related to feedback control and

![Figure 1.7: A submodule applied in a practical MMC project [24]](image)
of Marquardt, et al. The circuit operation are shown in Figure 1.8(b) for two line-cycles. In feedback control, the PI compensator generates the voltage command, $\tilde{v}_m$, based on the error in the output current, $\tilde{i}_o$. Thereafter, similar to MMC, a bias term, $0.5V_{in}$, is added to the voltage command $\tilde{u}_m$ such that the resultant voltage command, $\tilde{v}_m'$ becomes positive, as shown in the figure. The SPWM block compares the voltage command, $\tilde{v}_m'$ with the carrier signal, and generates switching signal $\tilde{d}$ that goes to both the switches in an opposite manner such that the output current is regulated. As shown in the simulation results, the arm voltages in the simple inverter, $\tilde{v}_{ds1}$ and $\tilde{v}_{ds2}$, are pulsating waveforms, switching between 0 to $V_{in}$. The switching average of arm voltages varies in a sinusoidal manner with a DC bias, as shown in the figure. It should be noted that since there are no capacitors in the simple inverter, the switching average of the arm voltages do not have any distortion likewise MMC.

Based on these simulation results of the MMC and the simple half-bridge inverter, the

![Figure 1.8: Single-phase half-bridge inverter (a) feedback Control (b) simulation results](image)

features of the MMC can be summarized as:
• In the simple half-bridge inverter, the switching devices have to withstand the input voltage, $V_{in}$, while in MMC, the capacitor connected in each module clamps the voltage stress of the switching devices to approximately equal to $V_{in}/N$. Therefore, for large $N$, the MMC can handle much higher DC voltage, $V_{in}$, compared to the simple half-bridge inverter for same traditional switching devices.

• MMC is easily scalable, meaning the voltage rating of the MMC can be easily scaled up by simply increasing the number of modules in each arm.

• Since several identical modules are connected in each arm, the structure of the MMC is highly modular which helps greatly in manufacturing and design.

• The arm voltages are stair-case waveforms with $(N + 1)$ number of levels, which results in very low total harmonic distortion (THD). Consequently, the MMC can operate at very low switching frequency compared to the simple half-bridge inverter.

• Unlike the simple half-bridge inverter, which switches full input DC voltage in a switching event, in the MMC, due to multiple voltage levels in an arm, only a fraction of input voltage gets switched in each switching event. This reduces the $dv/dt$ and therefore, correspondingly reduces the EMI.

The resultant steady-state arm current waveforms, $\tilde{i}^u$ and $\tilde{i}^l$, for the simple half-bridge inverter are shown in Figure 1.8(b). The careful observation of switching average of arm current waveforms suggests that in the first half line cycle, the upper arm conducts most of the output current while the lower arm has small negative current. Similarly, in the second half-line cycle, the lower arm conducts most of the output current while the upper arm conducts a small negative current. This negative component of arm currents flows back to the DC source, leading to reverse power flow. Therefore, this reverse power flow caused by the negative arm currents can be considered as the undesired circulating power of this inverter.
This means that even the simplest inverter has some undesired circulating power. However, this circulating power is not that significant as in the case of MMC. In the MMC, introducing several capacitors in each arm makes this problem much more severe as the undesired circulating power leads to large capacitor voltage ripples. Therefore, in the MMC, simply regulating the output current, similar to that which is done in a simple half-bridge inverter, is not enough. Therefore, following are the additional challenges in an MMC compared to a simple half-bridge inverter:

- Large line-frequency and its related harmonics in the arm currents in the MMC leads to large capacitor voltage ripples. Therefore, it is required in the MMC to control individual arm currents, unlike in a simple inverter in which only output current needs to be controlled.

- The successful operation of the MMC is possible only if capacitor voltages are properly regulated with minimum ripple content. Therefore, it is also required to control the individual capacitor voltages through feedback control.

Therefore, in summary, it is required in an MMC to control individual arm currents and individual capacitor voltages. However, Marquardt’s control is over-simplified as it includes feedback control only for output current regulation. There is no closed-loop control for individual arm currents. Therefore, arm currents are rich with line-frequency related harmonic content. This requires large arm inductors to suppress these harmonics [23][25]. Also, there is no closed loop control for capacitor voltage regulation. Therefore, capacitor voltages are rich with line-frequency and its harmonic content (large circulating energy). Consequently, large capacitors are required with this control implementation to suppress these voltage ripples [23][25].

Later, to have a better understanding of operation of MMC and facilitate a more effective
Chapter 1. Introduction

feedback control, a modeling of MMC was proposed in the literature, as described briefly in the following section.

1.3 Review of MMC Modeling

1.3.1 Simplified Average Model of MMC

![Figure 1.9: Assumptions taken in average model](image)

Figure 1.9: Assumptions taken in average model (a) perfect balance control, (b) average over $f_e$.

To gain a better understanding of the operation of the MMC and facilitate a more effective feedback control, Ludois, et al. proposed a simplified average model of an MMC [26]. Figure 1.9(a) shows the capacitors voltages in the upper arm of phase - a for Marquardt’s control. For simplification, it is assumed that the capacitors in an arm are perfectly balanced with the help of the balance controller. Therefore, each capacitor voltage in an arm can be assumed to be equal. For upper arm,

$$
\tilde{v}_{Ca1}^u \approx \cdots \approx \tilde{v}_{CaN}^u \approx \tilde{v}_{Ca}^u,
$$

(1.3)
1.3. Review of MMC Modeling

and similarly for the lower arm,

\[ \tilde{v}_{Ca1}^l \approx \cdots \approx \tilde{v}_{CaN}^l \approx \tilde{v}_{Ca}^l. \]  

(1.4)

With this assumption, the upper arm voltage can be approximated as

\[ \tilde{v}_{u}^u \triangleq \sum_{n=1}^{N} \left( \tilde{d}_{an}^u \tilde{v}_{CAn}^u \right) \approx \left( \sum_{n=1}^{N} \tilde{d}_{an}^u \right) \times \tilde{v}_{Ca}^u, \]  

(1.5)

where, \( \tilde{d}_{an}^u \) is the switching signal for the nth module of the upper arm. The term, \( \sum_{n=1}^{N} \tilde{d}_{an}^u \), represents the total number of turned-on modules, \( \tilde{m}_{u}^u \) in the upper arm, varying between 0 to N. Thus, from (1.5), upper arm voltage is given by

\[ \tilde{v}_{u}^u \approx \tilde{m}_{u}^u \times \tilde{v}_{Ca}^u. \]  

(1.6)

Figure 1.9(b) shows the instantaneous arm voltage, \( \tilde{v}_{a}^u \). As already discussed, all the modules in an arm collaborate together to create a staircase arm voltage, \( \tilde{v}_{a}^u \) having equivalent switching frequency, \( f_e \). Assuming, \( f_e \) is sufficiently large, compared to the line-frequency component, the high-frequency component of \( \tilde{v}_{a}^u \) can be ignored. Thus, the arm voltage can be represented by average voltage, \( v_{a}^u \), given by

\[ v_{a}^u = < \tilde{v}_{a}^u > \triangleq \frac{1}{1/f_e} \int_{t-1/f_e}^{t} \tilde{v}_{a}^u dt. \]  

(1.7)

Hereafter, the term \( \tilde{x} \) represents the instantaneous quantity while the \( x \) represents the average quantity over \( f_e \).

From (1.6), the average arm voltage, \( v_{a}^u \), is given by

\[ v_{a}^u = d_{a}^u \times Nv_{Ca}^u, \]  

(1.8)
where, \( d_a^u \) is the average equivalent duty cycle, defined as

\[
d_a^u = \frac{m_a^u}{N}.
\]  (1.9)

Also, the capacitor voltage dynamics for the \( n^{th} \) module are given by

\[
\frac{C \tilde{d}_C}{dt} = \frac{\tilde{d}_u^a}{a_n} \tilde{i}_u^a.
\]  (1.10)

With the assumption in (1.3), and neglecting high frequency harmonics,

\[
\frac{C}{N} \frac{d}{dt} (N v_C^a) = \tilde{d}_a^i u^a.
\]  (1.11)

From (1.8) and (1.11), the simplified average model can be derived, as shown in Figure 1.10.

The model has total 12 states, two arm currents and two capacitor voltages in each phase. The modulation gains, \( FM \), shown in the figure, is \( 1/V_{in} \) in Marquardt’s control.
1.3. Review of MMC Modeling

Table 1.1: Simulation Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input Voltage ($V_{in}$)</td>
<td>$60 \text{ kV}$</td>
</tr>
<tr>
<td>Peak Amplitude of Grid Voltage ($V_g$)</td>
<td>$24 \text{ kV}$</td>
</tr>
<tr>
<td>Three-Phase Output Power ($P_o$)</td>
<td>$270 \text{ MW}$</td>
</tr>
<tr>
<td>Line Frequency ($\omega$)</td>
<td>$60 \text{ Hz}$</td>
</tr>
<tr>
<td>Equivalent Capacitance ($C/N$)</td>
<td>$750 \text{ } \mu\text{F}$</td>
</tr>
<tr>
<td>Arm Inductance ($L$)</td>
<td>$2 \text{ mH}$</td>
</tr>
</tbody>
</table>

Figure 1.11 verifies the simplified average model by comparing its simulation results with the original switching circuit for the simulation parameters shown in Table 1.1 and $N = 25$. From the results, it can be concluded that the simplified average model matches well with the original switching circuit.

Figure 1.11: Verification of simplified average model
1.3.2 Per-Phase Average Model of MMC

To further simplify the model shown in Figure 1.10, a per-phase model is defined by Chen Li, et al. [27][28]. Figure 1.12 shows the simulation results for all the three-phases for Marquardt’s control assuming that 3-phase grid voltages are perfectly balanced. It can be observed from the results that the currents and voltages for the three-phases are identical to each other with a $120^\circ$ phase shift. This means that studying one phase is sufficient for studying the entire MMC. To decouple the three phases, it is important to know the neutral point voltage as all the three phases are connected together at this point. Figure 1.12 shows the waveform of neutral point voltage, $v_n$. The neutral point voltage remains close to $V_{in}/2$. Based on this information, a per-phase model of the MMC is derived, shown in Figure 1.13. Since the neutral point voltage remains close to $V_{in}/2$, the neutral point is assumed to be

---

Figure 1.12: 3-Phase simulation results with Marquardt’s control
1.3. Review of MMC Modeling

Figure 1.13: Per-phase simplified average model of MMC connected to the mid-point of the DC bus.

Figure 1.14 shows the control implementation of the per-phase simplified average model of MMC based on Marquardt’s control. Since this is the per-phase model, the output current can be directly controlled through PI regulator which is similar to the control implementation in \(dq\) frame in a 3-phase system. This control, hereafter, is referred as the ‘simple control law’.

The simulation results with this control implementation for the parameters shown in Table - 1.1 are shown in Figure 1.15. Based on the frequency spectrum provided in the figure, the arm currents can be represented as [29],

\[
i^u \approx I_{dc} + \frac{I_o}{2} \sin \omega t + I_2 \cos 2\omega t, \text{ and} \tag{1.12}
\]

\[
i^l \approx I_{dc} - \frac{I_o}{2} \sin \omega t + I_2 \cos 2\omega t. \tag{1.13}
\]
The capacitor voltages can be represented as,

\[
Nv_c^u \approx V_{in} - V_1 \cos \omega t + V_2 \sin 2\omega t + V_3 \cos 3\omega t, \text{ and (1.14)}
\]

\[
Nv_c^l \approx V_{in} + V_1 \cos \omega t + V_2 \sin 2\omega t - V_3 \cos 3\omega t. \text{ (1.15)}
\]

Therefore, the arm currents and capacitor voltages are a function of the rich harmonic components related to \( \omega, 2\omega, 3\omega \) and so on. To gain an understanding of the nature of these state variables (arm currents and capacitor voltage) and find a suitable control, a state space analysis was proposed by Chen Li et al. [27][28] which is discussed in the following sub section.

1.3.3 State Plane Analysis

In the simplified per-phase model in Figure 1.13, there are four states, two are the capacitor voltages, \( Nv_c^u \& Nv_c^l \), and two are the arm currents, \( i^u \& i^l \). Since the difference of
1.3. Review of MMC Modeling

Figure 1.15: Simulation results for the simple control law

Figure 1.16: 3D state trajectory of MMC (a) $i^u$, $Nv^u_C$ and $Nv^l_C$ (b) $i^l$, $Nv^u_C$ and $Nv^l_C$

arm currents, $i_o \triangleq i^u - i^l$ is always given through feedback control, there are only three independent states – two capacitor voltages and one arm current. Figure 1.16(a) shows a 3D state trajectory between the upper arm current, the upper arm capacitor voltage and
the lower arm capacitor voltage i.e., $i^u$, $Nv^u_C$, and $Nv^l_C$ for the simple control law. For the visual aid, the 2D projections on $i^u$-$Nv^u_C$ and $Nv^u_C$-$Nv^l_C$ are also shown in the figure as a dashed line. Figure 1.16(b) shows the 3D state trajectory using the lower arm current as a state variable instead of the upper arm current. As can be observed from Figure 1.16(a) and Figure 1.16(b), the corresponding 2D planes look similar in both the cases. Therefore, either upper arm current or the lower arm current can be chosen to describe the operation of the MMC. In this work, upper arm current (and therefore, the state trajectory shown in Figure 1.16(a)) is chosen for further analysis. Figure 1.17 shows the 2D projection on the $i^u$-$Nv^u_C$ plane. The size of the trajectory on this plane is related to the circulating power of the upper arm capacitor. As shown in the figure, as the load current decreases, the area enclosed by the state trajectory also reduces. This signifies that the amount of circulating power flowing to the capacitors reduces when the output current/output power reduces.

Figure 1.18 shows the ‘8-shaped’ 2D-projection on the $Nv^u_C$-$Nv^l_C$ plane. The 2D projection on the $Nv^u_C$-$Nv^l_C$ plane illustrates an interesting relationship between the upper and lower arm circulating power. Two orthogonal axes, defined as $\Sigma$–axis and $\Delta$-axis, are shown in
1.3. Review of MMC Modeling

The projection of the “8-shape” state trajectory on the Δ-axis is in such a manner that one capacitor voltage increases while the other decreases. This behavior signifies that a portion of the circulating energy is merely swapping between the two capacitors, as shown in Figure 1.19(a). For example, if one looks at the state movement from point $t_1$ to point $t_3$ and takes the projection on Δ-axis, then it can be observed that along this axis, $Nv_C^u$ decreases while $Nv_C^l$ increases. This means, some power transfers from the upper capacitor to the lower capacitor, while when state moves back from point $t_3$ to point $t_1$, the projection on $v_\Delta$ axis shows that power transfers from the lower capacitor to the upper capacitor. This means that the movement on the $v_\Delta$ axis is corresponding to the power swapping between the upper and lower arm. The voltage ripples related to $v_\Delta$ can be extracted by

$$v_\Delta = \frac{Nv_C^u - Nv_C^l}{2}.$$  \hfill (1.16)

The projection of the “8-shape” state trajectories on the Σ-axis is in such a manner that
Figure 1.19: $Nv_u^C-Nv_l^C$ state plane: (a) projection on $v_\Delta$-axis, and (b) projection on $v_\Sigma$-axis

either both the capacitor voltages increase or they decrease. This behavior signifies that a portion of the circulating power is exchanging between the capacitors and the input/output, as shown in Figure 1.19(b). For example, if one looks at the state movement from point $t_1$ to point $t_2$ and take the projection on the $\Sigma-axis$, then it can be observed that along this axis, both $Nv_u^C$ and $Nv_l^C$ decrease. This means, some power transfers from both the capacitors
to the input/output. While the projection of movement from point ‘t2’ to ‘t3’ on \( \Sigma - axis \) shows that power flows back to the capacitor from input/output. This means that movement along the \( v_\Sigma \)-axis is corresponding to the power exchanging between the capacitors and the input/output. The voltage \( v_\Sigma \) is given by

\[
v_\Sigma = \frac{Nv_C^u + Nv_C^l}{2}.
\] (1.17)

Interestingly, both the axes – \( \Delta \)-axis and \( \Sigma \)-axis are orthogonal to each other, as shown in Figure 1.20. From (1.14), (1.15), (1.16) and (1.17), one can conclude that the \( v_\Sigma \) has DC and even-order harmonics while the \( v_\Delta \) has odd-order harmonics. This clearly illustrates the orthogonal nature of \( v_\Delta \) and \( v_\Sigma \), meaning both the circulating power associated with the \( \Delta \)-axis and the \( \Sigma \)-axis can be controlled separately [27][30]. Therefore, the control variables should be \( v_\Sigma \) and \( v_\Delta \) rather than \( Nv_C^u \) and \( Nv_C^l \). To find a way to control \( v_\Sigma \) and \( v_\Delta \), a coordinate transformation is proposed by Hsieh et. al. [30][31][32] which is discussed in the following sub-section.

### 1.3.4 Coordinate Transformation and Decoupled Equivalent Circuit Model

For coordinate transformation, the new state variables are chosen as \( v_\Sigma \) and \( v_\Delta \). Moreover, to find a way to control the input power to minimize the circulating power, an input current is defined in the following manner.

Referring to Figure 1.21, from energy conservation law, the input power provided by the source should be equal to the power provided to the module and the load. Therefore,

\[
Pin = P_{\text{module}} + P_{\text{load}},
\] (1.18)
Chapter 1. Introduction

Figure 1.20: Orthogonal nature of $v_\Sigma$ and $v_\Delta$

Figure 1.21: Defined power module

where, $p_{load}$ is the load power given by

$$p_{load} = v_g i_o,$$  \hspace{1cm} (1.19)
1.3. Review of MMC Modeling

where,

\[ i_o = i^u - i^l. \]  

(1.20)

From Figure 1.21, the power provided to the module is given by

\[ p_{\text{module}} = v_{xa}i^u + v_{ay}i^l. \]  

(1.21)

By applying KVL in the upper and the lower arm, the voltages, \( v_{xa} \) and \( v_{ay} \) are given by

\[ v_{xa} = 0.5V_{in} - v_g, \]  

(1.22)

and,

\[ v_{ay} = 0.5V_{in} + v_g. \]  

(1.23)

Substituting, \( v_{xa} \) and \( v_{ay} \) from (1.22) and (1.23) in (1.21), the module power is given by

\[ p_{\text{module}} = 0.5V_{in}(i^u + i^l) - v_g(i^u - i^l). \]  

(1.24)

Substituting (1.19) and (1.24) in (1.18), the input power is given by

\[ p_{\text{in}} = 0.5V_{in}(i^u + i^l). \]  

(1.25)

Therefore, from (1.25), the input current is defined as

\[ i_{\text{in}} \triangleq 0.5(i^u + i^l). \]  

(1.26)

This means that the phase-leg receives \( V_{in}i_{\text{in}} \) power from the source and delivers \( v_gi_o \) power to the load. Ideally, the current, \( i_{\text{in}} \) should be controlled such that the input power to the
Figure 1.22: (a) Input current, input power and output power in the simple control law (b) corresponding power flow

phase-leg, $V_{in}i_{in}$ become equal to the output power of the phase leg, $v_{g}i_{o}$. In this case, there will be direct power transfer from the input to the output and therefore, there will not be any circulating power going from capacitors to input or output. Consequently, the voltage ripples related to $v_{\Sigma}$ become zero.

Figure 1.22 (a) shows the waveform of the input current, $i_{in}$, input power, $V_{in}i_{in}$ and load power, $v_{g}i_{o}$, for simple control law. Let’s consider the time instant, $t_{1}$, $t_{2}$ and $t_{3}$, shown in the figure. During the time interval from $t_{1}$ to $t_{2}$, the input power is less than the load power. Therefore, as the input can not provide full load power demand, the capacitors also need to provide some power to the load, as shown in Figure 1.22(b). On the other hand, during the time interval from $t_{2}$ to $t_{3}$, the input power is more than the load power demand. Therefore, the capacitors need to store some portion of the input power, as shown in Figure 1.22(b). Consequently, the capacitors get charged and discharged frequently, leading to projection on the $\Sigma$ axis i.e., voltage ripples related to $v_{\Sigma}$. Therefore, it is important to properly control the input current to minimize the circulating power. To identify the control of $i_{in}$, $i_{in}$ is
chosen as the new coordinate to perform the coordinate transformation. Moreover, since it is always required to regulate the output current in a grid-tied inverter, \( i_o \) is chosen as another new coordinate. Therefore, the new chosen coordinates are, \( v_\Delta \), \( v_\Sigma \), \( i_m \) and \( i_o \). The detail of the coordinate transformation is described as follows.

**Coordinate Transformation**

The state space model with the original coordinates is given by

\[
\frac{d}{dt} x = Ax + Bu,
\]

where,

\[
x = \begin{bmatrix} i^u & i^l & Nv_C^u & Nv_C^l \end{bmatrix}^T,
\]

\[
u = \begin{bmatrix} V_{in} & v_g \end{bmatrix}^T,
\]

\[
A = \begin{bmatrix} 0 & 0 & -\frac{d^u}{L} & 0 \\
0 & 0 & 0 & -\frac{d^l}{L} \\
\frac{d^u}{C/N} & 0 & 0 & 0 \\
0 & \frac{d^l}{C/N} & 0 & 0 \end{bmatrix},
\]

and,

\[
B = \begin{bmatrix} \frac{1}{2L} & -\frac{1}{L} \\
\frac{1}{2L} & \frac{1}{L} \\
0 & 0 \\
0 & 0 \end{bmatrix}.
\]
Chapter 1. Introduction

The new coordinates are given by

\[
x_{\Delta \Sigma} = \begin{bmatrix} i_{in} & i_o & v_{\Delta} & v_{\Sigma} \end{bmatrix}^T.
\] (1.32)

The relationship between the new coordinates and actual coordinates is given by

\[
x_{\Delta \Sigma} = T_{\Delta \Sigma} x,
\] (1.33)

where, \( T_{\Delta \Sigma} \) is the transformation matrix, given by

\[
T_{\Delta \Sigma} = \begin{bmatrix} 0.5 & 0.5 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 0.5 & -0.5 \\ 0 & 0 & 0.5 & 0.5 \end{bmatrix}.
\] (1.34)

By using the transformation matrix, \( T_{\Delta \Sigma} \), the new state space model can be derived. The new state space model is given by

\[
\frac{d}{dt} x_{\Delta \Sigma} = A_{\Delta \Sigma} x_{\Delta \Sigma} + B_{\Delta \Sigma} u,
\] (1.35)

where,

\[
A_{\Delta \Sigma} = \begin{bmatrix} 0 & 0 & \frac{d^l-d^u}{2L} & -\frac{d^u+d^l}{2L} \\ 0 & 0 & -\frac{d^u+d^l}{L} & \frac{d^l-d^u}{L} \\ \frac{d^u-d^l}{2C/N} & \frac{d^u+d^l}{2C/N} & 0 & 0 \\ \frac{d^u+d^l}{2C/N} & \frac{d^u-d^l}{2C/N} & 0 & 0 \end{bmatrix},
\] (1.36)
1.3. Review of MMC Modeling

and,

\[ B_{\Delta \Sigma} = T_{\Delta \Sigma} B = \begin{bmatrix} \frac{1}{2L} & 0 \\ 0 & -\frac{1}{L/2} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \] \hspace{1cm} (1.37)\]

Interestingly, in the new state space model, the control variables are no longer, \( d^u \) and \( d^l \) but the sum and difference of \( d^u \) and \( d^l \). Therefore, the new control variables are defined as,

\[ d_{in} = d^u + d^l, \text{ and} \]

\[ d_o = 0.5(d^l - d^u). \] \hspace{1cm} (1.38)\hspace{1cm} (1.39)

The new state space model can be equivalently represented by the circuit model \([30][31][32]\), as shown in Figure 1.23.

The proposed decoupled circuit model is described in terms of capacitor voltages, \( v_{\Sigma} \), \( v_{\Delta} \), \( i_{in} \) and \( i_o \). It should be noted that \( v_{\Sigma} \), by definition, contains only DC and even-order harmonics while \( v_{\Delta} \) has only odd-order harmonics. Since the capacitor voltages have a strong DC bias,
it can be assumed that \( v_\Sigma \gg v_\Delta \).

Therefore, the following inequalities are usually true:

\[
\text{at the input side: } d_{in}v_\Sigma \gg 2d_o v_\Delta, \text{ and } \tag{1.40}
\]

\[
\text{at the output side: } d_o v_\Sigma \gg 0.5d_{in} v_\Delta. \tag{1.41}
\]

Therefore, based on the above assumptions, the bottom controllable voltage sources at the input and the output side of the equivalent circuit model can be ignored for simplicity [30][31][32]. The resultant circuit model is shown in Figure 1.24. This means that the lower portion of the circuit model contributes very little to the power transfer from the input to the output. The power flow in MMC is shown in Figure 1.25. Figure 1.25(a) shows the power flow in the equivalent circuit model while Figure 1.25(b) shows the power flow in the original average model of the MMC. The upper portion of the equivalent circuit is responsible for the power transfer from input to output, as highlighted by the blue arrow in the figure. If the instantaneous input power and output power are not balanced, there would be a circulating power flowing through the \( v_\Sigma \) capacitor (highlighted by a blue circle) leading to even-order harmonics. Since the lower portion of the equivalent circuit has nothing to do with the
power transfer from input to the output, the circulating power related to \( v_\Delta \) represents the power swapping between the upper-arm module and lower-arm module, as shown in Figure 1.25(b), highlighted in red. This leads to odd-order harmonics in \( v_\Delta \). Therefore, the equivalent circuit model decouples these two circulating power which originally are mingled together in Figure 1.25(b). To make the circulating energy related to \( v_\Sigma \) and \( v_\Delta \) zero, the “ideal” control laws are proposed using the circuit model [30][31][32], which are discussed in the following subsection.

### 1.3.5 “Ideal” Control Laws

In the proposed circuit model, if the top two current sources become equal, i.e.,

\[
d_{in}i_{in} = d_{o}i_{o},
\]  

(1.42)

there will be no circulating power resulting from the power imbalance between input and output. Thus, in this case, the voltage ripples related to \( v_\Sigma \) become zero.
Similarly, if the bottom two current sources become equal, i.e.,

\[ 2d_oi_{in} = 0.5d_{in}i_o, \]  

(1.43)

there will not be any circulating power swapping between the arms. In this case, voltage ripples related to \( v_\Delta \) become zero. Therefore, the proposed circuit model provides the decoupled circulating power and means for minimizing them.

Figure 1.27 shows the state trajectory in an ideal case when both the “ideal” control laws are satisfied. Figure 1.27(a) shows the \( Nv_C^u - Nv_C^l \) state plane. If the first control law is satisfied, then the state trajectory on the \( Nv_C^u - Nv_C^l \) state plane shrinks along the \( v_\Sigma \) axis while if the second control law is satisfied then it shrinks along the \( v_\Delta \) axis. Ideally if both the control laws are satisfied then the state trajectory becomes a dot, as shown in the figure.

Figure 1.27(b) shows the state trajectory on the \( i^u - Nv_C^u \) state plane. Ideally, if both the control laws are satisfied then the capacitor voltage, \( Nv_C^u \) will not have any voltage ripples, and therefore, the ideal state trajectory will be a line along the \( i^u \) axis, as shown in the figure.

In the following section, the simple control law (based on Marquardt’s control) is discussed.
Figure 1.27: State trajectory in “ideal” case (a) $Nv_c^u - Nv_c^l$ state plane (b) $i^u - Nv_c^u$ state plane

using the equivalent circuit model to obtain a better understanding.

1.4 Simple Control Law Under The Equivalent Circuit Model

Figure 1.28 shows the simple control law under the equivalent circuit model. The output current in the circuit model can be directly regulated by controlling the voltage source $d_o v_{\Sigma}$ through $d_o$. It should be noted that the control variable, $d_{in}$ does not affect the control of $d_o$. Therefore, the control variable, $d_{in}$ and $d_o$ can be controlled in an independent manner in the circuit model.

The details of output current regulation are shown in the figure. ‘$R_s$’, shown in the feedback loop, is the current sensing gain. The grid voltage, $v_g$, is added to the feedback loop in order to reject the effect of grid voltage disturbances on the output current regulation. The
voltage gain, $1/V_{in}$ in the feedback loop is the modulation gain. Therefore, with the help of the proposed circuit model, the output current regulation loop becomes intuitive, especially the fact that the output current regulation loop is achieved by controlling $d_o$.

It should be noted that the other control variable, $d_{in}$ is simply provided as a constant in the simple control law. Consequently, this control implementation is considered as oversimplified since it cannot use the control variable, $d_{in}$ to address other control objectives such as minimizing the circulating energy.

The two “ideal” control laws, developed using the equivalent circuit model, are used to evaluate the existing circulating energy with this control implementation. Figure 1.29(a) shows the waveforms of both the current sources related to the first control law i.e., $d_{in}i_{in}$, and $d_{o}i_{o}$ for the parameters shown in Table 1.1. The blue highlighted portion in the figure represents the resultant oscillating charge in a line-cycle in the $C_{\Sigma}$ capacitor, since the two current sources, $d_{in}i_{in}$ and $d_{o}i_{o}$ are not equal, the first control law is violated, which leads to current flowing through the $C_{\Sigma}$ capacitor at a frequency of $2\omega$. Figure 1.29(b) shows the waveforms of both the current sources related to the second control law i.e., $2d_{o}i_{in}$ and
1.4. Simple Control Law Under The Equivalent Circuit Model

Figure 1.29: Evaluation of control laws for the simple control law

0.5d_{in}i_{o} and the highlighted portion represents the resulting oscillating charge in a line-cycle in the C_{Δ} capacitor. Since the two current sources, 2d_{o}i_{in} and 0.5d_{in}i_{o}, are not equal, the second control law is violated, which leads to current flowing through the C_{Δ} capacitor at the \( \omega \) and 3\( \omega \) frequency. Therefore, with the help of these two control laws, it can be concluded that in this control implementation, voltage ripples related to \( \omega \), 2\( \omega \) and 3\( \omega \) exist.

Therefore, the simple control law has achieved only output current regulation and cannot address the closed-loop control of \( v_{\Sigma} \), \( v_{\Delta} \), and \( i_{in} \). Later, Akagi et al. proposed a comprehensive closed-loop control implementation [33][34][35][36][37][38]. The general structure of this control implementation is widely accepted by practicing engineers and researchers. In the following chapter, the control implementation, proposed by Akagi et al., is discussed in detail.
Chapter 2

Interpretation of Generally Accepted Control Framework (Akagi et al.)

As discussed in the previous chapter, the control implementation proposed by Marquardt, et al., establishes a basic control framework for output current regulation of the MMC. However, there is no closed-loop control for individual arm current regulation, and this results large line-frequency related harmonic content in the arm currents. Consequently, large arm inductors are required with this control implementation to suppress the harmonic content of the arm currents. Moreover, there is no closed-loop control for regulating capacitor voltages. Therefore, large module capacitors are required to suppress capacitor voltage ripples due to the large circulating power related to line-frequency and its harmonics.

Therefore, in general, it is required to control individual arm currents and individual capacitor voltages of each phase. Since the individual capacitor voltages in an arm remain balanced with the help of balance control, the feedback control needs only to regulate the total capacitor voltages of an arm instead of individual capacitor voltages. Thus, in each phase, the feedback control needs to control four variables — two arm currents (for e.g., $i_a^u$...
and $\tilde{i}_a$ for phase - a) and two total arm capacitor voltages ($\sum \tilde{v}_{Can}^u$ and $\sum \tilde{v}_{Can}^l$ for phase - a). These four variables are generally regulated by defining the following four control objectives:

1. Regulating output current, $\tilde{i}_{oa} \triangleq \tilde{i}_u - \tilde{i}_a$,

2. Regulating $0.5(\tilde{i}_u^a + \tilde{i}_l^a)$ current,

3. Regulating $\sum \tilde{v}_{Can}^u + \sum \tilde{v}_{Can}^l$ voltage, and

4. Regulating $\sum \tilde{v}_{Can}^u - \sum \tilde{v}_{Can}^l$ voltage.

Objectives 1 and 2 together provide the control of individual arm currents, $\tilde{i}_u^a$ and $\tilde{i}_l^a$ while objectives 3 and 4 provide the control of individual total arm capacitor voltages, $\sum \tilde{v}_{Can}^u$ and $\sum \tilde{v}_{Can}^l$.

It should be noted that these four control objectives are to be achieved using two control variables for phase - a — one is for the upper arm, $\tilde{v}_{ma}^u$ and another one is for the lower arm, $\tilde{v}_{ma}^l$. Therefore, designing such a feedback control is an over-constrained optimization problem. Akagi et al. proposed a comprehensive closed-loop control implementation dealing with such an over-constrained system [37][38]. The general structure of this control implementation is widely accepted by practicing engineers and researchers [39][40][41][42][43]. In the following section, this control implementation, proposed by Akagi et al., is discussed.

### 2.1 Control Implementation Proposed by Akagi et al.

The basic control architecture proposed by Akagi et al. is shown in Figure 2.1. Similar to Marquardt’s control implementation, the feedback loop, $\tilde{i}_{oa} \triangleq \tilde{i}_u^a - \tilde{i}_a^l$ is implemented to provide output current regulation. The output current regulation is shown in a per-phase manner. Alternatively, it can be implemented in a dq frame. Additionally, a feedback loop
Figure 2.1: Control framework proposed by Akagi, et. al

to regulate the current, \(0.5(\tilde{i}_u + \tilde{i}_l)\) is implemented. Thus, the feedback loops, \(0.5(\tilde{i}_u + \tilde{i}_l)\) and \((\tilde{i}_u - \tilde{i}_l)\), together provide the freedom to control the individual arm current, \(\tilde{i}_u\) and \(\tilde{i}_l\). For capacitor voltage regulation, two feedback loops, \(\sum \tilde{v}_C^{\text{Can}} + \sum \tilde{v}_C^{\text{Can}}\) and \(\sum \tilde{v}_C^{\text{Can}} - \sum \tilde{v}_C^{\text{Can}}\), are implemented which together provide the control of individual arm capacitor voltages, \(\sum \tilde{v}_C^{\text{Can}}\) and \(\sum \tilde{v}_C^{\text{Can}}\). Due to the over-constrained system, these two capacitor voltage feedback loops are added on the top of the \(0.5(\tilde{i}_u + \tilde{i}_l)\) feedback loop, such that the reference command for \(0.5(\tilde{i}_u + \tilde{i}_l)\) is provided by these two voltage feedback loops. This means the current, \(0.5(\tilde{i}_u + \tilde{i}_l)\), is used as a mean to transfer the power to the capacitors to correct their error voltages. Thus, by adding these two voltage feedback loops on the top of the \(0.5(\tilde{i}_u + \tilde{i}_l)\) feedback loop, three variables, \(0.5(\tilde{i}_u + \tilde{i}_l)\), \(\sum \tilde{v}_C^{\text{Can}} + \sum \tilde{v}_C^{\text{Can}}\) and \(\sum \tilde{v}_C^{\text{Can}} - \sum \tilde{v}_C^{\text{Can}}\), are regulated together using only one voltage command, \(\tilde{v}_{ma2}\), provided by the \(0.5(\tilde{i}_u + \tilde{i}_l)\) feedback loop. The voltage, \(\tilde{v}_{ma2}\), together with the voltage generated from the \((\tilde{i}_u - \tilde{i}_l)\) feedback loop, \(\tilde{v}_{ma1}\),
provides the arm control voltages, $\tilde{v}_{ma}^u$ & $\tilde{v}_{ma}^l$, which are given by

$$\tilde{v}_{ma}^u = 0.5V_{in} + \tilde{v}_{ma2} - \tilde{v}_{ma1}, \text{ and}$$

$$\tilde{v}_{ma}^l = 0.5V_{in} + \tilde{v}_{ma2} + \tilde{v}_{ma1}. \quad (2.1)$$

Based on $\tilde{v}_{ma}^u$ and $\tilde{v}_{ma}^l$, modulation & balance Control generates the switching signal for each module such that the capacitors in an arm remain balanced.

It should be noted that low pass filters (LPFs) are used in the voltage feedback loops, as shown in the figure, to filter out all the AC frequency components of capacitor voltages, and thus only the DC component of the capacitor voltages are regulated. Thus, the feedback control implementation is able to regulate individual arm currents, $\tilde{i}_{ua}$ and $\tilde{i}_{la}$, and DC component of individual total arm capacitor voltages, $\sum \tilde{v}_{can}^u (dc)$ and $\sum \tilde{v}_{can}^l (dc)$. Due to regulated $\tilde{i}_{ua}$ and $\tilde{i}_{la}$, large arm inductors are not required with this control to suppress the harmonic component of arm currents as required in the simple control law. Moreover, the regulation of $\sum \tilde{v}_{can}^u (dc)$ and $\sum \tilde{v}_{can}^l (dc)$ enables the capacitors to have any DC voltage unlike the Marquardt’s control in which $\sum \tilde{v}_{can}^u (dc)$ and $\sum \tilde{v}_{can}^l (dc)$ are always settled at $V_{in}$. Thus, this general framework is widely used by other researchers and practicing engineers [39][40][41][42][43].

Nonetheless, the control implementation is rather “indirect” as instead of directly controlling the arm variables ($\tilde{i}_{ua}$, $\tilde{i}_{la}$, $\sum \tilde{v}_{can}^u (dc)$ & $\sum \tilde{v}_{can}^l (dc)$) through the $\tilde{v}_{ma}^u$ & $\tilde{v}_{ma}^l$, their sum and difference (i.e., $\sum \tilde{v}_{can}^u \pm \sum \tilde{v}_{can}^l$ and $\tilde{i}_{ua} \pm \tilde{i}_{la}$) are regulated through sum and difference of $\tilde{v}_{ma}^u$ & $\tilde{v}_{ma}^l$. Similar control frameworks, regulating the sum and difference of capacitor voltages (or capacitor energy) and arm currents, are also presented by other research groups as well [45][46][47][48]. However, since the feedback variables $\tilde{i}_{ua} + \tilde{i}_{la}$, $\sum \tilde{v}_{can}^u \pm \sum \tilde{v}_{can}^l$ do not physically exist in the circuit, the control implementation lacks the physical interpretation.
Chapter 2. Interpretation of Generally Accepted Control Framework

For example, the relationship of $0.5(\tilde{i}_{ua} + \tilde{i}_{la})$ current with capacitor voltages, $\sum \tilde{v}_{Can}^u \pm \sum \tilde{v}_{Can}^l$ is not intuitive. Therefore, it is not very clear that how to generate the reference command for the $0.5(\tilde{i}_{ua} + \tilde{i}_{la})$ current using the capacitor voltage feedback loops. Figure 2.2 shows the reference command generation for the $0.5(\tilde{i}_{ua} + \tilde{i}_{la})$ current provided by the voltage feedback loops. The reference current command for $0.5(\tilde{i}_{ua} + \tilde{i}_{la})$ is given by

$$\tilde{i}_{ref} = \frac{P_o}{3V_{in}} + \tilde{v}_{ma\Sigma} + \tilde{v}_{ma\Delta} \sin\omega t,$$

(2.3)

where, $P_o$ is the total three-phase output power command. The gain of $\sin\omega t$, provided in the $\tilde{i}_{ref}$ command, generates a fundamental frequency component in the current $0.5(\tilde{i}_{ua} + \tilde{i}_{la})$ that is proportional to $\tilde{v}_{ma\Delta}$, which helps in transferring the energy between the upper and lower arms to correct the DC error of $\sum \tilde{v}_{Can}^u - \sum \tilde{v}_{Can}^l$ [37]. In the steady state, when there

Figure 2.2: Reference current generation for current $0.5(\tilde{i}_{ua} + \tilde{i}_{la})$
is no DC error in the capacitors’ voltages, the voltage feedback loops do not contribute to
the $i_{ref}$ current, and therefore, in the steady state, $0.5(\tilde{i}_u + \tilde{i}_l)$ current is given by

$$0.5 (\tilde{i}_u + \tilde{i}_l) = i_{ref} = \frac{P_o}{3V_{in}}.$$  \hfill (2.4)

The bias term, $P_o/(3V_{in})$, provided in the feedback loop, helps in adjusting the current $0.5(\tilde{i}_u + \tilde{i}_l)$ as per the output power command, $P_o$ such that, the per-phase DC-input power can balance with the per-phase DC-output power command. Nonetheless, the reference command of current $0.5(\tilde{i}_u + \tilde{i}_l)$, given by (2.3), is not intuitive, particularly, the terms, $\sin\omega t$ and $P_o/(3V_{in})$. Moreover, the relationship of the current feedback loops with the arm control voltages, $\tilde{v}_{ma}$ & $\tilde{v}_{la}$ is also not comprehensible. It is also not clear that why LPFs are used in the voltage feedback loops.

In the following section, this control implementation is interpreted with a physical understanding using the equivalent circuit model.

### 2.2 Interpretation of Akagi’s Control Framework Using Equivalent Circuit Model

Figure 2.3 shows the basic control architecture using the decoupled equivalent circuit model.

The details of this control architecture using the circuit model are as follows.
2.2.1 Output Current Regulation

For output current regulation, the voltage source $d_o v_\Sigma$ is controlled by generating $d_o$ from the feedback loop, as shown in the figure. The details of output current regulation are already described while interpreting the simple control law in the previous chapter. It should be noted that the output current regulation loop becomes intuitive with the help of the proposed circuit model, especially the fact that the output current regulation is achieved by controlling $d_o$. On the other hand, with the original circuit, shown in Figure 2.2, the relationship of output regulation loop with the control variables $\tilde{v}_{ma}^u$ and $\tilde{v}_{ma}^l$ is not so intuitive.
2.2. Interpretation of Akagi’s Control Framework Using Equivalent Circuit Model

2.2.2 Capacitor Voltage Regulation

In the decoupled circuit model, there are two capacitor voltages, \( v_{\Sigma} \) & \( v_{\Delta} \). As discussed in the previous chapter, these two capacitor voltages are related to two fundamentally different circulating power; one is related to power flowing between the input and output while the other one is related to the power swapping between the upper and lower arm. Thus, due to being fundamentally different in nature from each other, there is a possibility to decouple \( v_{\Sigma} \) and \( v_{\Delta} \). Therefore, the control implementation has the direct feedback of \( v_{\Sigma} \) and \( v_{\Delta} \), as shown in Figure 2.3. Since \( v_{\Sigma} \) and \( v_{\Delta} \), by definition, are related to sum and difference of the total arm capacitor voltages, \( \sum v_{Can}^u \) & \( \sum v_{Can}^l \), it is a general practice to regulate the sum and difference of the total arm capacitor voltages due to their decoupling nature, as shown in Figure 2.2.

Therefore, for capacitor voltage regulation, the objective is to regulate the DC value of \( v_{\Sigma} \) and \( v_{\Delta} \) such that,

\[
\begin{align*}
v_{\Sigma} (dc) &= NV_c, \text{ and} \\
v_{\Delta} (dc) &= 0.
\end{align*}
\]

where, \( V_c \) is the reference command for the individual module capacitor of the MMC which generally chosen as to be \( V_{in}/N \). As shown, the capacitor voltages, \( v_{\Sigma} \) and \( v_{\Delta} \) are sensed and passed through the LPFs. The LPFs are designed such that all the AC frequency components from \( v_{\Sigma} \) and \( v_{\Delta} \) get filtered out and thus, only their DC values are regulated. It should be noted that the capacitor voltages, \( v_{\Sigma} \) and \( v_{\Delta} \) are related to the controllable current sources \( d_{in}i_{in} \) and \( 2d_{o}i_{in} \), respectively which are the function of the current \( i_{in} \). Therefore, by implementing a suitable control of \( i_{in} \), the DC error in both \( v_{\Sigma} \) and \( v_{\Delta} \) can be corrected.

Thus, the input current regulation loop is implemented on the top of voltage feedback loops, as shown in the figure. This input current regulation loop can be considered as a feedforward...
loop since it regulate the input quantity of the circuit model. The details of the input current regulation loop are as follows.

**Input Current Regulation**

As shown in Figure 2.3, the reference command for $i_{in}$ is created using the voltage commands generated by the $v_{Σ}$ & $v_{Δ}$ feedback loops, i.e., $V_{mΣ}$ and $V_{mΔ}$, so that any DC error in $v_{Σ}$ or $v_{Δ}$ can modify $i_{in}$ which, in turn, adjusts the controlled current sources $d_{in}i_{in}$ and $2d_{o}i_{in}$, respectively, to compensate for the DC error of $v_{Σ}$ & $v_{Δ}$. A bias term, $P_{o}/3V_{in}$, is added to the reference command of $i_{in}$ so that per-phase DC input power, $V_{in}i_{in}$, can immediately balance the per-phase DC output power command, $P_{o}/3$, which was certainly not intuitive with the original circuit in Figure 2.2.

The implementation of a feedforward loop for input current regulation is similar to the output current feedback loop. To regulate the input current, the voltage source, $d_{in}v_{Σ}$ is controlled by generating duty cycle, $d_{in}$ from the feedforward loop. Similar to the injection of grid voltage in the output current regulation loop, the input voltage $V_{in}$ is injected in the input current regulation loop which helps in rejecting the effect of input voltage transients on input current regulation.

To observe the role played by $\sin(\omega t)$ in the control implementation, first it is removed from the feedback loop to see if the control implementation works without it in the presence of the grid transient, as shown in Figure 2.4. Figure 2.5 shows the simulation results for the control implementation shown in Figure 2.4, and for the circuit parameters shown in Table 1.1. A grid-voltage sag is intentionally created for a few cycles to check the viability of the controller. As shown in the results, both $v_{Σ}$ and $v_{Δ}$ are unable to return to their reference commands after the transient. Although, both the voltage feedback loops are generating the
2.2. Interpretation of Akagi’s Control Framework Using Equivalent Circuit Model

Figure 2.4: Control implementation without sin(ωt) under the equivalent circuit model

voltage commands, $V_m\Sigma$ & $V_m\Delta$ to correct their errors, as shown in the simulation results, they still fail to correct their DC errors. To understand the reasoning behind the failure of the voltage feedback loops, it is required to look at the characteristic of the current sources, $d_{in}i_{in}$ and $2d_{o}i_{in}$ in the circuit model, as they are responsible for correcting the DC error of $v_{\Sigma}$ and $v_{\Delta}$.

**Characteristics of the $d_{in}i_{in}$ and $2d_{o}i_{in}$ Current Source:**

Assuming the line-frequency related voltage drop across the inductors is negligible, from the input part of the circuit model, $d_{in}$ is given by
Chapter 2. Interpretation of Generally Accepted Control Framework

Figure 2.5: Simulation results without $\sin(\omega t)$

\[ d_{in} \approx \frac{V_{in}}{v_{\Sigma}}. \]  \hfill (2.7)

Similarly, from the output part of the circuit model, $d_o$ is given by

\[ d_o \approx \frac{v_g}{v_{\Sigma}} \approx \frac{V_g \sin \omega t}{v_{\Sigma}}. \]  \hfill (2.8)

The current $i_{in}$, given by the current feedforward loop, is

\[ i_{in} = i_{ref} = \frac{P_o}{3V_{in}} + \frac{V_{m\Sigma}}{R_s} + \frac{V_{m\Delta}}{R_s}. \]  \hfill (2.9)

Therefore, from (2.7), (2.8), (2.9), the current sources, $d_{in}i_{in}$ and $2d_oi_{in}$ are given by
2.2. Interpretation of Akagi’s Control Framework Using Equivalent Circuit Model

\[ d_{in}i_{in} \approx \frac{V_{in}}{v_{\Sigma}} \left[ \frac{P_0}{3V_{in}} + \frac{V_{m\Sigma}}{R_s} + \frac{V_{m\Delta}}{R_s} \right] \text{, and} \]  
\[ (2.10) \]

\[ 2d_{o}i_{in} \approx \frac{2V_g}{v_{\Sigma}} \left[ \frac{P_0}{3V_{in}} + \frac{V_{m\Sigma}}{R_s} + \frac{V_{m\Delta}}{R_s} \right] \sin(\omega t). \]  
\[ (2.11) \]

From (2.11), it can be observed that the 2\(d_{o}i_{in}\) current source does not have any DC component. Therefore, the DC error of \(v_{\Delta}\) is unable to be corrected after the transient. The current source, \(d_{in}i_{in}\), from (2.10), does have a DC component, however, its DC component is dependent upon \(V_{m\Delta}\) as well. Therefore, the DC error in \(v_{\Delta}\) affects the DC component of the current source \(d_{in}i_{in}\) and therefore, creates a DC error in \(v_{\Sigma}\). Thus, the DC error of \(v_{\Sigma}\) is unable to be corrected due to unwanted interference between both the voltage feedback loops.

To solve the aforementioned issue, a gain of \(\sin(\omega t)\) is provided in the \(v_{\Delta}\) feedback loop, as shown in Figure 2.3. The input current in this case is given by

\[ i_{in} = \frac{P_0}{3V_{in}} + \frac{V_{m\Sigma}}{R_s} + \frac{V_{m\Delta}}{R_s} \sin\omega t. \]  
\[ (2.12) \]

From (2.7), (2.8), (2.12), the current sources, \(d_{in}i_{in}\) and \(2d_{o}i_{in}\), are given by

\[ d_{in}i_{in} \approx \frac{V_{in}}{v_{\Sigma}} \left[ \frac{P_0}{3V_{in}} + \frac{V_{m\Sigma}}{R_s} + \frac{V_{m\Delta}}{R_s} \sin\omega t \right] \text{, and} \]  
\[ (2.13) \]

\[ 2d_{o}i_{in} \approx \frac{V_g}{v_{\Sigma}} \left[ \frac{V_{m\Delta}}{R_s} + 2 \left( \frac{V_{m\Sigma}}{R_s} + \frac{P_0}{3V_{in}} \right) \sin\omega t - \frac{V_{m\Delta}}{R_s} \cos2\omega t \right]. \]  
\[ (2.14) \]

With the multiplication of \(\sin(\omega t)\) in the \(v_{\Delta}\) feedback loop, the current source \(2d_{o}i_{in}\) now
Figure 2.6 shows the simulation results with \( \sin(\omega t) \) in the voltage feedback loop. It can be observed from the results, that both \( v_{\Sigma} \) and \( v_\Delta \) are able to correct their DC errors after the grid transient.

It should be noted that due to LPFs being used in the voltage feedback loops, the AC
frequency components of capacitor voltages are not regulated and therefore, this control implementation can not deal with the circulating power. In the following sub-section, the circulating power and voltage ripples with this control implementation are evaluated using “ideal” control laws and state planes.

2.3 Evaluation of Circulating Energies

The “ideal” control laws, developed using the equivalent circuit model, are used as follows in evaluating the capacitor’s voltage ripples with this control implementation (Akagi’s Control).

2.3.1 First Control Law

From the input current feedforward loop, in the steady state,

\[ i_{in} = \frac{P_o}{3V_{in}}. \]  \hspace{1cm} (2.15)

Therefore, from (2.7) and (2.15), the current source \(d_{in}i_{in}\) is given by

\[ d_{in}i_{in} \approx \frac{P_o}{3v_\Sigma}. \]  \hspace{1cm} (2.16)

The output current is given by

\[ i_o = I_o \sin(\omega t). \]  \hspace{1cm} (2.17)

From (2.8) and (2.17), the current source \(d_o i_o\) is given by

\[ d_o i_o \approx \frac{P_o}{3v_\Sigma} (1 - \cos 2\omega t). \]  \hspace{1cm} (2.18)
Figure 2.7: Evaluation of Akagi’s Control using the proposed “ideal” control laws (a) first control law (b) second control law

Thus, since the two current sources, $d_{in}i_{in}$ and $d_{o}i_{o}$, are not equal, the first control law is violated, which leads to current flowing through the $v_\Sigma$ capacitor at a frequency of $2\omega$. Figure 2.7(a) shows the waveforms of both current sources for the parameters shown in Table 1.1. The blue highlighted portion in the figure represents the resultant oscillating charge in a line-cycle in the $v_\Sigma$ capacitor. The resulting voltage ripples in $v_\Sigma$ are given by

$$\Delta v_\Sigma = \frac{1}{2C/N} \int (d_{in}i_{in} - d_{o}i_{o}) \, dt,$$

(2.19)

where, $C$ is the capacitance of individual module capacitor. Substituting, $d_{in}i_{in}$ and $d_{o}i_{o}$ from (2.16) and (2.18),

$$\Delta v_\Sigma = \frac{1}{2C/N} \frac{P_o}{3v_\Sigma} \left( \frac{1}{2\omega} \right) \sin 2\omega t.$$  

(2.20)

In other words, from the waveform of $d_{in}i_{in}$ and $d_{o}i_{o}$, it is clear that the per-phase input power is DC while the per-phase output power contains both DC as well as a $2\omega$ frequency
component. Since the input source supplies only the DC component of output power, $v_\Sigma$ capacitor must supply $2\omega$ power to the output, leading to the voltage ripples in $v_\Sigma$ that are related to the $2\omega$ frequency component.

### 2.3.2 Second Control Law

From (2.8) and (2.15), the current source $2d_o i_{in}$ is given by

$$2d_o i_{in} = \frac{P_o}{3v_\Sigma} M \sin\omega t,$$

where $M$ is the voltage gain, defined as

$$M = \frac{V_g}{V_{in}/2}.$$  \hspace{1cm} (2.22)

From (2.7) and (2.17), the current source, $0.5d_{in}i_o$ is given by

$$0.5d_{in}i_o = \frac{P_o}{3v_\Sigma} \frac{2}{M} \sin\omega t.$$  \hspace{1cm} (2.23)

Therefore, since the two current sources, $2d_o i_{in}$ and $0.5d_{in}i_o$, are not equal, the second control law is violated, which leads to current flowing through the $v_\Delta$ capacitor at $\omega$ frequency. Figure 2.7(b) shows the waveforms of both the current sources, and the highlighted portion represents the resulting oscillating charge in a line-cycle in the $v_\Delta$ capacitor. The resultant voltage ripples in $v_\Delta$ are given as,

$$v_\Delta = \frac{1}{2C/N} \int (0.5d_{in}i_o - 2d_o i_{in}) \ dt.$$  \hspace{1cm} (2.24)
Substituting, $2d_0 i_{in}$ and $0.5d_{in} i_o$ from (2.21) and (2.23), yields

$$v_\Delta = - \frac{1}{2C/N} \frac{P_o}{3v_{in}} \left( \frac{2}{M} - M \right) \frac{1}{\omega} \cos \omega t.$$  (2.25)

Therefore, with the help of these two control laws, it can be concluded that in this control implementation, voltage ripples related to $\omega$ and $2\omega$ exist. Furthermore, state plane analysis is carried out to qualitatively visualize voltage ripples and circulating power in comparison with Marquardt’s control.

### 2.3.3 State Planes

Figure 2.8(a) shows the $N\nu_C^l - N\nu_C^u$ state plane for the parameters shown in the Table 1.1. It can be observed from the figure that the voltage ripples related to both $v_\Sigma$ and $v_\Delta$ have
2.3. Evaluation of Circulating Energies

Figure 2.9: Comparison of Akagi’s control with Marquardt’s control

been reduced in Akagi’s control compared to Marquardt’s control.

Figure 2.8(b) shows the $i^u - Nv^u_C$ state plane. As the area encircled by $i^u - Nv^u_C$ state plane is related to existing circulating energy in an arm, the highlighted area in the figure represents the reduction in the circulating energy.

To see the improvement quantitatively, Figure 2.9 presents the frequency spectrum of output current, upper arm current, and upper arm capacitor voltage for both Marquardt’s control and Akagi’s control. It can be observed from the results that arm current in Akagi’s control does not have the $2\omega$ component unlike the results obtained with Marquardt’s control. Consequently, the voltage ripples in the upper arm capacitor related to $\omega$, $2\omega$ and $3\omega$ have been reduced.

Although there is some improvement in Akagi’s control, there are still large voltage ripples related to $\omega$ and $2\omega$. Later, this basic control framework proposed by Akagi is slightly improved by various other researchers to reduce the circulating power by second-order harmonic
current injection [39], gain control [40][41], high-frequency current injection [42], common-mode voltage injection [32] [43] etc. In the following chapter, these control strategies are comprehended in a systematic and simplified manner using the “ideal” control laws.
Chapter 3

Further Improvements for Minimizing Circulating Energy

As discussed in the previous chapter, Akagi’s control implementation is able to achieve the following control objectives:

1. Regulating output current, \( \tilde{i}_{oa} = \tilde{i}_a - \tilde{i}_l \),
2. Regulating \( 0.5(\tilde{i}_a + \tilde{i}_l) \) current,
3. Regulating \( \sum \tilde{v}_{Can}^{u}(dc) + \sum \tilde{v}_{Can}^{l}(dc) \) voltage, and
4. Regulating \( \sum \tilde{v}_{Can}^{u}(dc) - \sum \tilde{v}_{Can}^{l}(dc) \) voltage.

However, Akagi’s control implementation can not deal with the AC frequency components of capacitor voltages, i.e., \( \sum \tilde{v}_{Can}^{u}(\omega, 2\omega) \) and \( \sum \tilde{v}_{Can}^{l}(\omega, 2\omega) \). Consequently, the capacitors have large voltage ripples related to the \( \omega \) and \( 2\omega \) frequency components due to large amounts of circulating power. There are several control strategies, proposed in literature, for reducing the circulating power and therefore, AC frequency components of capacitor voltages by
slightly improving the general control framework proposed by Akagi, et al., discussed in the previous chapter. However, due to a lack of understanding of the nature of circulating power flowing in the MMC, these control strategies are based mostly on a trial-and-error approach, and thus are not systematic and simplified.

The decoupled equivalent circuit model clearly identifies two decoupled circulating power; one is related to the process of power flow from input to output, which leads to even-order harmonics in $v_\Sigma$, and another one is related to process of the power-swapping between the upper and lower arms, which leads to odd-order harmonics in $v_\Delta$. Moreover, the circuit model provides two “ideal” control laws for minimizing the circulating power. Therefore, since the circuit model provides a better understanding of the nature of circulating power and a means for minimizing them, the objective of this chapter is to use this circuit model to comprehend the existing control strategies for minimizing the circulating power in a systematic and simplified manner which is as follows.

### 3.1 Eliminating Circulating Energy Related to $2\omega$

As discussed in the previous chapter, in Akagi’s control, steady state $i_{in}$ is DC from (2.15), which leads to DC steady state input power in each phase. However, the per-phase output power has DC and second-order harmonic component. This second-order harmonic component of the output power is provided by the $v_\Sigma$ capacitor which leads to $2\omega$ voltage ripples. The objective of this control implementation, proposed by Winklekemper, et al., [39] is to eliminate these second-order harmonic voltage ripples. To reach the goal, current $i_{in}$ should also have a second-order harmonic component such that the instantaneous per-phase output and input power become balanced and thus, the $v_\Sigma$ capacitor will no longer need to provide second-order harmonic power to the load and therefore, second-order harmonic capacitor
3.1 Eliminating Circulating Energy Related to $2\omega$

Figure 3.1: Implementation of second-order harmonic current injection using the proposed equivalent circuit model

Voltage ripples get eliminated. In other words, to eliminate the voltage ripples related to $2\omega$, the first control law needs to be satisfied. This means, the input current, $i_{in}$, should be given by

$$i_{in} = \frac{d_o}{d_{in}}. \quad (3.1)$$

Substituting, $d_{in}$ and $d_o$ from (2.7) and (2.8) in the above equation, the input current, $i_{in}$ should be given by

$$i_{in} = \frac{P_o}{3V_{in}} [1 - \cos(2\omega t)]. \quad (3.2)$$

Figure 3.1 shows the implementation of this control strategy based on Akagi’s control implementation utilizing the equivalent circuit model. As shown, a second-order harmonic
component, given by (3.2) is injected into the reference command $i_{ref}$. Therefore, with this control implementation, input current, $i_{in}$, is modified, such that the controlled current sources, $d_{in}i_{in}$ and $d_{o}i_{o}$ become equal, as shown in Figure 3.2(a). Therefore, the first control law is satisfied, and consequently, the voltage ripples related to $v_{Σ}$ are eliminated.

To see the voltage ripples of $v_{Δ}$, let’s look at the current sources related to the second control law, i.e., $2d_{o}i_{in}$ and $0.5d_{in}i_{o}$. From (2.8) and (3.2), the controlled current source, $2d_{o}i_{in}$ becomes

$$2d_{o}i_{in} = \frac{P_o}{3v_{Σ}} \left( \frac{3M}{2} \sin\omega t - \frac{M}{2} \sin 3\omega t \right),$$  \hspace{1cm} (3.3)
3.1. Eliminating Circulating Energy Related to $2\omega$

while the current source, $0.5d_{in}i_o$ remains the same as in Akagi’s control, as given by (2.23):

$$0.5d_{in}i_o = \frac{P_o}{3v_{\Sigma}} \frac{2}{M} \sin \omega t.$$  (3.4)

Figure 3.2(b) shows the waveforms of both the current sources, $2d_{o}i_{in}$ and $0.5d_{in}i_o$, and the highlighted portion represents the resultant oscillating charge in the $v_\Delta$ capacitor. It should be noted from (2.21) and (3.3), that due to second-order harmonic current injection, the $\omega$ component of $2d_{o}i_{in}$ is increased by $3/2$ times compared to Akagi’s control, which decreases the $\omega$ component of current flowing through the $v_\Delta$ capacitor (i.e. $0.5d_{in}i_o - 2d_{o}i_{in}$) and thus, $v_\Delta(\omega)$ slightly reduces. However, a third-order harmonic component in $2d_{o}i_{in}$ is created due to the second-order harmonic current injection which leads to $v_\Delta(3\omega)$. The resultant voltage ripples in $v_\Delta$ are given by

$$v_\Delta \approx -\frac{1}{2C/N} \frac{P_o}{3v_{\Sigma}} \left[ \left( \frac{2}{M} - \frac{3M}{2} \right) \frac{1}{\omega} \cos \omega t + \frac{M}{2} \frac{1}{3\omega} \cos 3\omega t \right].$$  (3.5)

Moreover, state-plane analysis is carried out to qualitatively visualize the voltage ripples and the circulating energy, for comparison with Akagi’s control implementation. Figure 3.3(a) shows the $Nv_{e}^u - Nv_{e}^l$ state plane. As discussed, due to the second-order harmonic current injection, the first control law is met and therefore, there is no projection on the $v_{\Sigma}$ axis, i.e., $\Delta v_{\Sigma} = 0$. It should also be noticed from the figure that due to the reduction in $v_\Delta(\omega)$ as compared to Akagi’s control, the net projection on the $v_\Delta$ axis is also reduced. Figure 3.3(b) shows the $i^u - Nv_{c}^u$ state plane. As shown, the encircled area on the state plane has reduced as compared to Akagi’s control, meaning the total circulating energy and therefore, the voltage ripples of $Nv_{e}^u$ and $Nv_{e}^l$, have reduced. The highlighted area in the figure denotes the reduction in the circulating energy.

It should be noted that, with this control implementation, although the voltage ripples
related to $2\omega$ are eliminated, the improvement is incremental, since there are still large voltage ripples related to $\omega$. The next section discusses a control strategy based on gain control to eliminate the voltage ripples related to $\omega$ [40].

It should also be noted that the injection of second-order harmonic current leads to increased rms value of the arm currents, and therefore, power losses. A trade-off between the capacitor voltage ripples and the power losses is studied in [49].

### 3.2 Eliminating Circulating Energy Related to $\omega$

This control strategy is proposed by Peter Nee, et al. [40]. The objective of this control strategy is to eliminate the capacitor voltage ripples related to the $\omega$ component existing in Akagi’s control implementation. As discussed in the previous chapter, the voltage ripples related to the $\omega$ component are associated with the $v_\Delta$ capacitor, generated due to violation
3.2. Eliminating Circulating Energy Related to $\omega$

of the second control law. Therefore, the objective of this control strategy should be to satisfy the second control law so that $v_\Delta(\omega)$ can be eliminated. In Akagi’s control method, the current sources related to the second control law i.e., $2d_o i_{\text{in}}$ and $0.5d_{\text{in}} i_o$ from (2.21) and (2.23) are given by

$$2d_o i_{\text{in}} = \frac{P_o}{3v_\Sigma} M \sin \omega t,$$

$$0.5d_{\text{in}} i_o = \frac{P_o}{3v_\Sigma} \frac{2}{M} \sin \omega t .$$

Therefore, to eliminate the voltage ripple related to the $\omega$ component, the second control law should be satisfied for the $\omega$ component i.e., $2d_o i_{\text{in}} (\omega) = 0.5d_{\text{in}} i_o (\omega)$. Therefore, from (3.6) and (3.7),

$$\left( \frac{P_o}{3v_\Sigma} M \right) \sin \omega t = \left( \frac{P_o}{3v_\Sigma} \frac{2}{M} \right) \sin \omega t .$$

Therefore, voltage gain, $M$ should be,

$$M \triangleq \frac{V_o}{V_{\text{in}}/2} = \sqrt{2} .$$

Consequently, in this control method, voltage gain $M$ is set as $\sqrt{2}$. Thus, both the current sources related to the second control law become equal, as shown in Figure 3.4(b), and therefore, the voltage ripples related to $v_\Delta$ are eliminated.

In Akagi’s control, as can be observed from (2.16) and (2.18), the current sources related to the first control law, i.e., $d_{\text{in}} i_{\text{in}}$ and $d_o i_o$, are independent from $M$, and thus, the first control law remains the same as in Akagi’s control, as shown in Figure 3.4(a), leading to voltage ripples related to $2\omega$, given by (2.20).

Furthermore, state-plane analysis is carried out to qualitatively visualize the existing voltage ripples and the circulating energy in comparison with Akagi’s control. As the second control law is satisfied with this control implementation, there is no projection along the $v_\Delta$ axis,
Chapter 3. Further Improvements for Minimizing Circulating Energy

Figure 3.4: Evaluation using the proposed “ideal” control laws (a) first control law (b) second control law

as shown in Figure 3.5(a). The voltage ripples related to $v_\Sigma$ remains the same as in Akagi’s control. Figure 3.5(b) shows the $i^u - Nu^u_c$ state plane for both control methods. The highlighted area in the figure represents the reduction in the encircled area achieved using this control method as compared to Akagi’s control. As can be observed from the figure, there is a significant reduction in the encircled area on the $i^u - Nu^u_c$ state plane. Consequently, the circulating energy and overall voltage ripples compared to Akagi’s control implementation are largely reduced.

Although with this control strategy, the voltage ripples related to $\omega$ are eliminated, there are still large voltage ripples related to $2\omega$. To deal with these voltage ripples, a control strategy based on gain control in conjunction with second-order harmonic current injection is proposed in literature [41], which is discussed in the next section.
3.2. Eliminating Circulating Energy Related to $\omega$

It should be noted that the voltage gain, $M$, can be set as $\sqrt{2}$ by adjusting input voltage $V_{in}$, which is suitable for the such applications as back-to-back HVDC inter connections, renewables, etc. [50]. Alternatively, for applications in which $V_{in}$ can not be choosen freely such as interconnecting HVDC grid with AC grid, the output voltage of the converter can be adjusted by changing the number of turns of the transformer connected at the AC side of the converter [50]. In this thesis, it is assumed that $V_{in}$ is adjusted to achieve $M$ as $\sqrt{2}$ while keeping output voltage and output power the same.

Another thing that should be noted is that MMC based on half-bridge modules can not be used for $M = \sqrt{2}$. Originally, this control strategy was realized based on a full-bridge MMC which required all the modules in an arm to be based on full-bridge topologies. However, the full-bridge MMC led to increased cost and power losses due to the increased number of devices which can render the realization non-viable. Later, the realization based on hybrid MMC was proposed [51], which required a combination of half-bridge and full-bridge modules in an arm.
3.3 Eliminating Circulating Energy Related to $\omega$ and $2\omega$

This control strategy is proposed by Lyu, et al. [41]. The objective of this control strategy is to eliminate the voltage ripples related to $\omega$ and $2\omega$. As discussed previously, second-order current injection enables the first control law to be met, meaning $v_\Sigma(2\omega)$ is eliminated. In this control implementation, in addition to the second-order harmonic current injection, gain $M$ is also set to eliminate the voltage ripples related to $\omega$ in addition to $2\omega$. After injecting the second-order harmonic current, the current sources related to the second control law, i.e., $2d_o i_{in}$ and $0.5d_{in}i_o$ from (3.3) and (3.4) are given by

\[2d_o i_{in} = \frac{P_o}{3v_\Sigma} \left( \frac{3M}{2} \sin \omega t - \frac{M}{2} \sin 3\omega t \right) . \quad (3.10)\]

\[0.5d_{in}i_o = \frac{P_o}{3v_\Sigma} \frac{2}{M} \sin \omega t. \quad (3.11)\]

Therefore, to eliminate the voltage ripple related to $\omega$, the second control law should be satisfied for the $\omega$ component; i.e., $2d_o i_{in}(\omega) = 0.5d_{in}i_o(\omega)$. Therefore, from (3.10) and (3.11),

\[\left( \frac{P_o}{3v_\Sigma} \frac{3M}{2} \right) \sin \omega t = \left( \frac{P_o}{3v_\Sigma} \frac{2}{M} \right) \sin \omega t. \quad (3.12)\]

Thus, voltage gain, $M$ should be

\[M \triangleq \frac{V_g}{V_{in}/2} = 1.15. \quad (3.13)\]

Therefore, voltage gain $M$ is set as 1.15 in this control method. Thus, both the current sources become equal for $\omega$ frequency component and therefore the voltage ripple related to $\omega$, i.e., $v_\Delta(\omega)$, is eliminated.
3.3. Eliminating Circulating Energy Related to $\omega$ and $2\omega$

Figure 3.6: Evaluation using the proposed “ideal” control laws (a) first control law (b) second control law

Figure 3.6 shows the waveforms of the current sources related to the first and the second control law. The first control law is satisfied due to the second-order harmonic current injection while the $\omega$ component of the second control law is satisfied due to $M = 1.15$. Therefore, the second control law leads to voltage ripples related to $3\omega$ only. The voltage ripples related to $3\omega$ are given by

$$v_{\Delta} = -\frac{1}{2C/N} \frac{P_o}{3v_s} \left[ \frac{M}{2} \frac{1}{3\omega} \cos3\omega t \right].$$

(3.14)

Further, state-plane analysis is carried-out to qualitatively visualize the improvement compared to the gain control method. As can be observed from Figure 3.7 (a), with this method, there is a small projection on the $v_{\Delta}$ axis due to the non-zero $3\omega$ related voltage ripples.
The highlighted area in Figure 3.7 (b) shows the improvement in terms of reduction in circulating power and therefore voltage ripples. Although, a improvement can be observed from the figure, however, there is still some circulating power related to $3\omega$. A method based on common mode injection is proposed in the literature to eliminate the voltage ripples related to $3\omega$ in addition to those $\omega$ and $2\omega$ [32], which is discussed in the next section.

It should be noted that, in this control strategy, as $M$ is more than 1, full-bridge modules are required. A realization of this strategy based on the hybrid MMC using common-mode voltage is discussed in [41]. As with the previous control strategy, here also the input voltage is adjusted to realize $M$ as 1.15.
3.4 Eliminating Circulating Energy Related to $\omega$, $2\omega$, and $3\omega$

In the previous example, by utilizing second-order harmonic current injection, $v_\Sigma(2\omega)$ is eliminated and by setting $M$ as 1.15, $v_\Delta(\omega)$ is eliminated. However, the capacitor voltage ripples related to $3\omega$ still exist. The objective of this control implementation is to eliminate the voltage ripples related to $3\omega$ in addition to those of $\omega$ and $2\omega$. This control strategy is proposed by Hsieh, et al., based on common mode voltage injection together with second- and fourth-order harmonic current injection and $M$ control [32]. In Chapter - 1, while discussing the modeling of the MMC, the effect of common-mode voltage was not considered. Hsieh, et al., proposed a generalized modeling of the MMC that also considers the effect of common-mode voltage injection [31][32]; a discussion of this model follows. Consider the simplified three-phase average model, shown in Figure 3.8. By applying the KVL on the upper and

**Figure 3.8: Simplified average model of MMC**
Chapter 3. Further Improvements for Minimizing Circulating Energy

lower arms,

\[
\begin{aligned}
\text{upper arm:} & \\
& \begin{cases}
  v_n = V_{in} - d_a^u N v_{Ca}^u - L \frac{d}{dt} i_a^u - v_{ga} \\
  v_n = V_{in} - d_b^u N v_{Cb}^u - L \frac{d}{dt} i_b^u - v_{gb} , \text{ and} \\
  v_n = V_{in} - d_c^u N v_{Cc}^u - L \frac{d}{dt} i_c^u - v_{gc}
\end{cases} \\
\text{lower arm:} & \\
& \begin{cases}
  v_n = d_a^l N v_{Ca}^l + L \frac{d}{dt} i_a^l - v_{ga} \\
  v_n = d_b^l N v_{Cb}^l + L \frac{d}{dt} i_b^l - v_{gb} \\
  v_n = d_c^l N v_{Cc}^l + L \frac{d}{dt} i_c^l - v_{gc}
\end{cases}
\end{aligned}
\]

(3.15)

Applying KCL at the neutral point,

\[
(i_a^u - i_a^l) + (i_b^u - i_b^l) + (i_c^u - i_c^l) = 0. 
\]

(3.17)

Based on (3.15), (3.16), and (3.17), the \( v_n \) is given by

\[
v_n = \frac{V_{in}}{2} + v_{gz} + v_z, 
\]

(3.18)

where \( v_{gz} \) is the zero sequence of the grid voltage and \( v_z \) is the zero sequence voltage resulting from feedback control, given by

\[
v_{gz} \triangleq \frac{v_{ag} + v_{bg} + v_{cg}}{3}, \text{ and} \\
v_z \triangleq \frac{1}{6} \left( d_a^l N v_{Ca}^l - d_a^u N v_{Ca}^u + d_b^l N v_{Cb}^l - d_b^u N v_{Cb}^u + d_c^l N v_{Cc}^l - d_c^u N v_{Cc}^u \right). 
\]

(3.19)

(3.20)

Assuming that capacitors have strong DC bias, the above equation can be further simplified as

\[
v_z \triangleq \frac{N V_C}{6} \left( d_a^l - d_a^u + d_b^l - d_b^u + d_c^l - d_c^u \right). 
\]

(3.21)
Thus, the voltage, \( v_z \), is a function of duty cycle only and therefore, it is known for a given control law. Therefore, the three phases can be decoupled. Figure 3.9 shows the per-phase model of an MMC, assuming \( v_{gz} \) is zero. By coordinate transformation, discussed in Chapter 1, the equivalent circuit model is derived as shown in Figure 3.10.

Assuming the voltage drop across the inductor is negligible, from Figure 3.10, \( d_{in} \) and \( d_o \)
are given by

\[ d_{in} \approx \frac{V_{in}}{v_{\Sigma}}, \quad \text{and} \quad (3.22) \]

\[ d_{o} \approx \frac{v_{g} + v_{z}}{v_{\Sigma}}. \quad (3.23) \]

Figure 3.11 shows the control implementation. As shown, a third-order harmonic common-mode voltage \( v_{z} \), given by \( kV_{g}\sin 3\omega t \), is injected into the output current feedback loop. Therefore, with this control implementation, from (3.21), the zero sequence (common mode) voltage, \( v_{z} \), becomes,

\[ v_{z} = kV_{g}\sin (3\omega t). \quad (3.24) \]

Moreover, in the control implementation, harmonic current, given by \( (v_{g} + v_{z})/V_{in} \), is injected to input current command to modify the input current such that the first control law can be satisfied, and therefore, the voltage ripples related to all the even-order harmonics get eliminated. To satisfy the first control law, input current \( i_{in} \) should be

\[ i_{in} = \frac{d_{o}i_{o}}{d_{in}}. \quad (3.25) \]

Substituting \( d_{in} \) and \( d_{o} \) from (3.22) and (3.23) in (3.25), input current is given by

\[ i_{in} = \frac{(v_{g} + v_{z})i_{o}}{V_{in}}. \quad (3.26) \]

Substituting \( v_{g} \) and \( v_{z} \) in (3.26) yields

\[ i_{in} = \frac{P_{o}}{3V_{in}}[1 - (1 - k)\cos 2\omega t - k\cos 4\omega t]. \quad (3.27) \]

Therefore, second- and fourth-order harmonic current, given by (3.26), is injected into the input current command, as shown in the figure.
3.4. Eliminating Circulating Energy Related to $\omega$, $2\omega$, and $3\omega$

Figure 3.11: Implementation of control method based on common mode voltage injection

To eliminate the voltage ripples related to $\omega$ and $3\omega$, the second control law should be satisfied for the $\omega$ and $3\omega$ components. From (2.7) and (2.17), the current source, $0.5d_{in}i_o$, is given by

$$0.5d_{in}i_o = P_o \frac{2}{3\Sigma M} sin\omega t.$$  \hspace{1cm} (3.28)

From (3.23) and (3.26), the current source, $2d_{o}i_{in}$, is given by

$$2d_{o}i_{in} = P_o \frac{M}{3\Sigma} \left[ (3 - 2k + 2k^2)sin\omega t + (4k - 1)sin3\omega t + (k^2 - 2k)sin5\omega t - k^2 sin7\omega t \right].$$  \hspace{1cm} (3.29)

To satisfy the $\omega$ component of the second control law, from (3.28) and (3.29),
Chapter 3. Further Improvements for Minimizing Circulating Energy

Figure 3.12: Evaluation using the proposed “ideal” control laws (a) first control law (b) second control law

\[
     \frac{P_o}{3v_\Sigma M} \sin \omega t = \frac{P_o}{3v_\Sigma} \frac{M}{2} (3 - 2k - k^2) \sin \omega t. \tag{3.30}
\]

Similarly, to satisfy \(3\omega\) component of the second control law, from (3.28) and (3.29),

\[
     \left[ \frac{P_o}{3v_\Sigma} \frac{M}{2} (4k - 1) \right] \sin 3\omega t = 0. \tag{3.31}
\]

Therefore,

\[
     k = \frac{1}{4}. \tag{3.32}
\]

From (3.30) and (3.32),

\[
     M = 1.23. \tag{3.33}
\]

Therefore, with \(k = 0.25\) and \(M = 1.23\), the voltage ripples related to \(\omega\) and \(3\omega\) are elimi-
3.4. Eliminating Circulating Energy Related to $\omega$, $2\omega$, and $3\omega$

There are very small voltage ripples related to $5\omega$ and $7\omega$, given by

$$v_{\Delta} = -\frac{1}{2C/N_{\Sigma}} \frac{P_o}{3v_{\Sigma}} \frac{M}{2} \left[ (2k^2) \frac{1}{5\omega} \cos 5\omega t + k^2 \frac{1}{7\omega} \cos 7\omega t \right].$$  (3.34)

Figure 3.12 shows the waveforms related to the first and the second control laws. As shown, the first control law is satisfied, and therefore, there are no voltage ripples related to $v_{\Sigma}$. The second control law is also almost satisfied, and therefore, the voltage ripples related to $v_{\Delta}$ are also very small. Further, state plane analysis is carried-out to qualitatively visualize the improvement compared to the previous example. As discussed in Chapter 1, ideally the state trajectory on the $Nv_c^l-Nv_C^l$ state plane should be a dot while on the $i^u-Nv_C^u$ state plane, it should be a line having movement only along the $i^u$ axis. Thus, as can be observed from Figure 3.13, the state trajectory in this control is close to ideal.
Chapter 4

Summary and Conclusion

To compare the circulating energy associated with various control methods, the ratio between the delta energy of the capacitors in one line cycle to the total energy delivered to the load in one line cycle is defined, as follows [32]:

$$\gamma \triangleq \frac{\frac{1}{2} C \left(N v_{C, \text{max}}^u \right)^2 - \frac{1}{2} C \left(N v_{C, \text{min}}^u \right)^2}{\left(\frac{V_o I_o}{2}\right) \frac{2\pi}{\omega}}, \quad (4.1)$$

where

$$v_{C, \text{max}}^u = V_c + |\Delta V_{\text{max}}|, \quad (4.2)$$

and,

$$v_{C, \text{min}}^u = V_c - |\Delta V_{\text{max}}|. \quad (4.3)$$

The $|\Delta V_{\text{max}}|$ in (4.2) and (4.3) denotes the maximum amplitude of the voltage ripple in a capacitor. From (4.1),(4.2) and (4.3), the total capacitance required in an arm i.e., $C \times N$,
Figure 4.1: Comparative assessment of circulating energy existing with various control strategies [32]: Ex. 1: Akagi’s control at $M = 0.8$, Ex. 2: eliminating circulating energy related to $2\omega$ at $M = 0.8$, Ex. 3: eliminating circulating energy related to $\omega$, Ex. 4: eliminating circulating energy related to $\omega$, $2\omega$ and $3\omega$, and Ex. 5: eliminating circulating energy related to $\omega$, $2\omega$ and $3\omega$ is given by

$$C \times N = \gamma \frac{\pi V_o I_o}{2\omega V_c |\Delta V_{max}|}. \quad (4.4)$$

Thus, the total capacitance required, i.e., $C \times N$, is proportional to the delta energy ratio, $\gamma$. The $\gamma$ as a function of $M$ is plotted for various control methods in Figure 4.1 using the equations derived in the previous chapter for $v_\Sigma$ and $v_\Delta$ and for the parameters shown in Table 1.1. The number of modules in an arm is chosen 20 in all the cases. The input voltage is adjusted to sweep the value of $M$. The results highlighted in blue dashed line are corresponding to Akagi’s control implementation. The results highlighted in green dashed line are corresponding to the second-order harmonic current injection in Akagi’s control implementation. The results highlighted in pink solid line are corresponding to the control
implementation having third-order harmonic common mode voltage injection, and second- and fourth-order harmonic current injection. The example 1 and 3, shown in the figure, are corresponding to Akagi’s control implementation with $M = 0.8$ and $M = 1.41$, respectively. The example 2 and 4 are based on second-order harmonic current injection with $M = 0.8$ and $M = 1.15$, respectively. The example 5 is corresponding to third-order harmonic common mode voltage injection, second- and fourth-order harmonic current injection, and $M = 1.23$.

It should also be noted that the full-bridge modules are required to realize the operation of MMC for $M$ greater than 1. Therefore, example - 3, 4 and 5 requires the MMC having full-bridge modules. Interestingly, the result highlighted in pink-line shows improvement even for $M$ less than one, compared to other methods. Therefore, the method based on common mode voltage injection is viable even with half-bridge MMC.

The equivalent capacitance $(C/N)$ requirement in various control examples, based on simu-
lation results, is compared in Figure 4.2 for the simulation parameters shown in Table 1.1. The capacitance is chosen such that the capacitor voltage ripples are 25% of the nominal voltages in each control strategy. The results clearly show the improvement in reducing capacitor requirements associated with each control strategy.

In conclusion, with the help of the proposed decoupled equivalent circuit model and “ideal” control laws, the control strategies to reduce the circulating energies are interpreted and evaluated in a more systematic and simplified manner.
Bibliography


BIBLIOGRAPHY


[50] Kamran Sharifabadi, Lennart Harnefors, Hans-Peter Nee, Staffan Norrga, and Remus Teodorescu, “Introduction to Modular Multilevel Converters,” in *Design, Control, and