USE OF THE TRAFFIC SPEED DEFLECTOMETER FOR CONCRETE AND COMPOSITE PAVEMENT STRUCTURAL HEALTH ASSESSMENT

A Big-Data-Based Approach Towards Concrete and Composite Pavement Management and Rehabilitation

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Abstract

The latest trends in highway pavement management aim at implementing a rational, data-driven procedure to allocate resources for pavement maintenance and rehabilitation. To this end, decision-making is based on network-wide surface condition and structural capacity data – preferably collected in a non-destructive manner such as a deflection testing device. This more holistic approach was proven to be more cost-effective than the current state of the art, in which the pavement manager grounds their maintenance and rehabilitation-related decision making on surface distress measurements. However, pavement practitioners still rely mostly on surface distress because traditional deflection measuring devices are not practical for network-level data collection. Traffic-speed deflection devices, among which the Traffic Speed Deflectometer [TSD], allow measuring pavement surface deflections at travel speeds as high as 95 km/h [60 miles per hour], and reporting the said measurements with a spatial resolution as dense as 5cm [2 inches] between consecutive measurements. Since their inception in the early 2000s, and mostly over the past 15 years, numerous research efforts and trial tests focused on the interpretation of the deflection data collected by the TSD, its validity as a field testing device, and its comparability against the staple pavement deflection testing device – the Falling Weight Deflectometer [FWD]. The research efforts have concluded that although different in nature than the FWD, the TSD does furnish valid deflection measurements, from which the pavement structural health can be assessed.

Most published TSD-related literature focused on TSD surveys of flexible pavement networks and the estimation of structural health indicators for hot-mix asphalt pavement structures from the resulting data – a sensible approach given that the majority of the US paved road pavement network is asphalt. Meanwhile, concrete and composite pavements (a minority of the US pavement network that yet accounts for nearly half of the US Interstate System) have been mostly neglected in TSD-related research, even though the TSD has been deemed a suitable device for sourcing deflection data from which to infer the structural health of the pavement slabs and the load-carrying joints. Thus, this Dissertation’s main objective is to fulfill this gap in knowledge, providing the pavement manager/practitioner with a streamlined, comprehensive interpretation procedure to turn dense TSD deflection measurements collected at a jointed pavement network into characterization parameters and structural health metrics for both the concrete slab system, the sub-grade material, and the load-carrying joints.
The proposed TSD data analysis procedure spans over two stages: Data extraction and interpretation. The *Data Extraction Stage* applies a Lasso-based regularization scheme [Basis Pursuit coupled with Reweighted L₁ Minimization] to simultaneously remove the white noise from the TSD deflection measurements and extract the deflection response generated as the TSD travels over the pavement’s transverse joints. The examples presented demonstrate that this technique can actually pinpoint the location of structurally weak spots within the pavement network from the network-wide TSD measurements, such as deteriorated transverse joints or segments with early stages of fatigue damage, worthy of further investigation and/or structural overhaul. Meanwhile, the *Interpretation Stage* implements a linear-elastic jointed-slab-on-ground mathematical model to back-calculate the concrete pavement’s and subgrade’s stiffness and the transverse joints’ load transfer efficiency index [LTE] from the denoised TSD measurements. In this Dissertation, the performance of this back-calculation technique is analyzed with actual TSD data collected at a 5-cm resolution at the MnROAD test track, for which material properties results and FWD-based deflection test results at select transverse joints are available. However, during an early exploratory analysis of the available 5-cm data, a discrepancy between the reported deflection slope and velocity data and simulated measurements was found: The simulated deflection slopes mismatch the observations for measurements collected nearby the transverse joints whereas the measured and simulated deflection velocities are in agreement. Such a finding prompted a revision of the well-known direct relationship between TSD-based deflection velocity and slope data, concluding that it only holds on very specific cases, and that a jointed pavement is a case in which deflection velocity and slope do not correlate directly. As a consequence, the back-calculation approach to the pavement properties and the joints’ LTE index was implemented with the TSD’s deflection velocity data as input. Validation results of the back-calculation tool using TSD data from the MnROAD *low volume road* showed a reasonable agreement with the comparison data available while at the same time providing an LTE estimate for all the transverse joints (including those for which FWD-based deflection data is unavailable), suggesting that the proposed data analysis technique is practical for corridor-wide screening.

In summary, this Dissertation presents a streamlined TSD data extraction and interpretation technique that can (1) highlight the location of structurally deficient joints within a jointed pavement corridor worthy of further investigation with an FWD and/or localized repair, thus optimizing the time the FWD spends on the road; and 2) reasonably estimate the structural parameters of a concrete pavement structure, its subgrade, and the transverse joints, thus providing valuable data both for inventory-keeping and rehabilitation management.
Use of the Traffic Speed Deflectometer for Concrete and Composite Pavement Structural Health Assessment.

Martín Scavone Lasalle

General Audience Abstract

When allocating funds for network-wide pavement maintenance, such as the State or Country level, the engineer relies on as much pavement condition data as possible to optimally assign the most suitable maintenance or rehabilitation treatment to each pavement segment. Currently, practitioners rely mostly on surface condition data to decide on how to maintain their roads, as this data can be collected fast and easily with automated vehicle-mounted equipment and analyzed by computer software. However, managerial decisions based solely on surface condition data do not optimally make use of the Agency resources, for they do not precisely account for the pavements’ structural capacity when assigning maintenance solutions. As such, the manager may allocate a surface treatment on a structurally weak segment with a poor surface which will be prone to an early failure (thus wasting the investment) or, conversely, reconstruct a deteriorated yet strong segment that could be fixed with a surface treatment.

The reason for such a sub-optimal managerial practice has been the lack of a commercially-available pavement testing device capable of producing structural health data at a similar rate as the existing surface scanning equipment – pavement engineers could only appeal to crawling-speed or stop-and-go deflection devices to gather such data, which are fit for project-level applications but totally unsuitable for routine network-wide surveying. Yet, this trend reverted in the early 2000s with the launch of the Traffic Speed Deflectometer [TSD], a device capable of getting dense pavement deflection measurements (spaced as close as 5cm [2 inches] between each other) while traveling at speeds higher than 50 mph. Following the device’s release, numerous research activities studied its feasibility as a network-wide routine data collection device and developed analysis schemes to interpret the collected measurements into pavement structural condition information. This research effort is still ongoing, the Transportation Pooled Fund [TPF] Project 5(385) is aimed in that direction, and set the goal of furnishing standards on the acquisition, storage, and interpretation of TSD data for pavement management.

This being said, data collection and analysis protocols should be drafted to interpret the data gathered by the TSD on flexible and rigid pavements. Concerning TSD-based evaluation of flexible asphalt pavements, abundant published literature discussing exists; whereas TSD surveying of concrete and composite (concrete + asphalt) pavements has been off the center of attention, partly because these pavements constitute only a minority of the US paved highway network – even though they account for
roughly half of the Interstate system. Yet, the TSD has been found suitable to provide valuable structural health information concerning both the pavement slabs and the load-bearing joints, the weakest element of such structures.

With this in mind, this Dissertation research is aimed at bridging this existing gap in knowledge: a streamlined analysis methodology is proposed to process the TSD deflection data collected while surveying a jointed rigid pavement and derive important structural health metrics for the manager to drive their decision-making. Broadly speaking, this analysis methodology is constituted by two main elements:

- The Data Extraction stage, in which the TSD deflection data is mined to both clear it from measurement noise and extract meaningful features, such as the pulse responses generated as the TSD travels over the pavement joints.

- The Interpretation stage, which is more pavement engineering-related. Herein, the filtered TSD measurements are utilized to fit a pavement response model so that the pavement structural parameters (its stiffness, the strength of the sub-grade soil, and the joints’ structural health) can be inferred.

This Dissertation spans both the mathematical grounds for these analysis techniques, validation tests on computer-generated data, and experiments done with actual TSD data to test their applicability. The ultimate intention is for these techniques to eventually be adopted in practice as routine analysis of the TSD data for a more rational and resource-wise pavement management.
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“È una ragione per vivere...”.

L. Cherubini.

Here I made it to what I regard the most difficult part of this Dissertation manuscript, one for which I must balk for a minute or two and look back over my shoulder five years into the past not to leave behind any memory worth recalling and forget the many people, here at VT or at the Homeland, who through their support and friendship made this all the more enjoyable. Luckily for me, this Acknowledgments page is the place in which I can unleash myself to the fullest and write in not too scholarly a style, it feels awesome!

It has been almost 5 years, 1813 days to be exact (did the math), since I first walked the streets of the Hokie Nation, never imagining I would end up turning a 2-year MSc. Program into “the full Ph.D. experience”, featuring its waves of time-consuming course homework, mind-boggling intellectual challenges, and seasoned with an ounce or two sleepless nights, all for science.; But one which, upon completion, is leaving my mind filled with bliss over the satisfaction of having left a contribution to the body of science and to engineering practice – I keep my hopes that fellow Roadway Engineers may use the outcome of this Dissertation research to the advancement of highway preservation and rehabilitation, fulfilling in the end our profession’s ultimate goal: building and keeping structures strong and in service.

All in all, this has been an experience indeed, a Personal Voyage of sorts beyond the mere academics, one what that required to leave the shelter behind, and dare to dive and fall, the point of no return restraining me... But that, more than enduring it as it went through, it was an enrichment journey, an opportunity to state the principle of what to become of myself and build my persona around that.

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Blacksburg, VA.

July 25th 2021
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“Relationships between apparently different subjects are as creatively important in mathematics as they are in any discipline. The relationship hints at some underlying truth that enriches both subjects.”

Simon Singh, excerpt from ‘Fermat’s Enigma’ (1997)

Introduction

Structural evaluation of concrete and composite pavements is customarily carried out with stop-and-go devices such as the Falling Weight Deflectometer [FWD]. Standardized test procedures and back-calculation routines for estimating structural health parameters from deflection measurements exist and are widely adopted (AASHTO, 1993; Alavi et al., 2008; Phares et al., 2008; Pierce et al., 2017). Yet, while practical at the project level, the FWD is not a suitable device for structural data collection at the network level. Operation of FWDs requires lane closures, which disrupt the flow of traffic, and dedicated safety hardware to shield the FWD and the testing crew from incoming vehicles (Flintsch et al., 2013).

Traffic-Speed Deflection Devices [TSDDs] were developed to address the practical limitations of stop-and-go testing (European Commission, 1997; Hildebrand and Rasmussen, 2002; Flitnsch et al., 2013; Rada et al., 2016; Katicha et al., 2017). Several TSDDs have been one-off prototypes or had a limited production run, except for the Traffic Speed Deflectometer [TSD], which is commercially available – seventeen of such devices are operational as of 2022. Initially, TSDDs were intended as a network-level surveying tool for collecting structural data for pavement management or to locate structurally deficient spots within the network worthy of investigation or localized repair (Hildebrand and Rasmussen, 2002). Abundant literature has been produced over the past fifteen years concerning the applicability of such devices, comparability with the FWD, and interpretation of the deflection data garnered for both network-level and project-level applications (e.g. Austroads, 2016-a, -b; Baltzer et al., 2010; Brezina et al., 2017; Elseifi and Zihan, 2018; Elseifi et al., 2019; Ferne et al., 2009; Flintsch et al., 2013; Jansen, 2015; Katicha et al., 2017, 2020, 2021; Nasimifar et al., 2017a, b, c, 2018a, b; Rada and Nazarian, 2011; Rada et al., 2016). Currently, ongoing research is focused on bridging the last gaps towards the adoption of these

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1 As reported by Greenwood A/S, the TSD manufacturer: [https://greenwood.dk/road/tsd/references/](https://greenwood.dk/road/tsd/references/), accessed 2021-02-12. However, in February 2021 the TSD 18 (another 4th generation device) was delivered – as per unpublished notes from the DaRTS 14 meeting (Feb. 09-10, 2021)
devices in engineering practice. Work in this direction includes drafting standards for device design, calibration, operation, and data QC/QA. In the USA, the Transportation Pooled Fund Project TPF 5(385)\textsuperscript{2} is supporting the participating Highway Agencies with data acquisition and analysis protocols for TSDD data use within their asset management system.

Most of the published work on the utilization of the TSD and other traffic-speed deflection devices for pavement rehabilitation and management has focused on flexible asphalt pavements, which is sensible given these represent the bulk of paved networks. In the United States, 84\% of all trunk paved roads [the Interstate system, Primary highways, and major arterial/collector roads] are flexible asphalt structures (FHWA, 2019). Furthermore, until recently, TSDs could not reliably distinguish the feeble deflection signals produced in concrete pavements from measurement noise. In fact, the Australian Standard on TSD operation bans the use of the TSD on concrete pavements grounded on this limitation of the sensors (Austroads, 2016-b). However, the newest TSDs, rigged with high-sensitivity, high-resolution Doppler laser sensors, may be capable of clearly recording the deflection basin even on stiff concrete structures.

Early trials on previous-generation TSDDs hinted at potential applications of these devices for investigatory analysis and management of concrete or composite pavements (Phares et al., 2008; Flintsch et al., 2012, 2013; Rada et al., 2016). This requires, however, that the raw dense data from these devices be utilized without significant smoothing. The TSD measurements could be denoised and reduced with data analysis techniques widely applied in other disciplines. These techniques can objectively extract interesting information from which structural health parameters for the concrete slabs and the load-carrying joints could be inferred via mechanistic back-calculation (Van Cauwelaert, 2004; Deep et al., 2020-a). Thus, this dissertation is focused on developing a TSD deflection data processing framework based on big-data analysis techniques and mechanistic pavement modeling to extract structural health information for concrete and composite pavements. The analysis framework (its outcome) could eventually feed rehabilitation design methods and/or management decisions pertaining concrete/composite pavements.

\textsuperscript{2} Reference: https://www.pooledfund.org/Details/Study/637. Accessed 2021-02-12
Problem Statement

Management and preservation of pavements at the network-wide level requires accurate surface condition and structural health data (Haas et al., 2015; Katicha et al., 2017, 2020). The former is obtained routinely at a traffic speed with vehicle-mounted scanning devices, but structural data availability has historically been stymied by the limitations of stop-and-go testing equipment, which is inadequate for network-wide routine surveying. Traffic-speed deflection devices such as the Traffic Speed Deflectometer [TSD] were developed to collect network-wide deflection data for structural analysis (Hildebrand and Rasmussen, 2002). Over the past twenty years (and counting), several research efforts focused on the implementation of such devices for managing flexible pavement networks (Flintsch et al., 2013; Katicha et al., 2017, 2020; Rada et al., 2016). However, there is little research on the utilization of the TSD for the evaluation of jointed rigid pavements, despite the TSD having potential for providing information on the structural health of the concrete slabs and load-carrying joints (Phares et al., 2008; Flintsch et al., 2012, 2013; Katicha et al., 2013, 2014). This Dissertation is aimed at bridging this gap, by combining big-data analysis techniques borrowed from the machine learning realm and structural analysis techniques for jointed pavements (Chen et al., 2001; Candès et al., 2008; Mallat, 2008; Peyré, 2021; Van Cauwelaert, 2004; Zhang et al., 2021)

Research objective

This Dissertation’s main objective is to develop a data analysis framework combining data science feature extraction techniques and mechanistic pavement modeling to streamline the interpretation of jointed pavement surveys conducted with a TSD. The ultimate intention is to create methods and computational tools to estimate the pertinent pavement’s structural health properties at a high resolution from dense deflection measurements collected network-wide. This has been accomplished through the following main research stages with specific objectives:

- **Data Extraction Stage: Implement a methodology to systematically remove white noise and detect the location of structurally deficient locations within the pavement network.** This task’s objective is to identify structurally deficient locations within the pavement structure (such as a load-carrying joint with a low Load Transfer Efficiency [LTE] index). The Basis Pursuit [BP] and Reweighted L_1 minimization [RWL1] optimization procedures (Chen et al., 2001; Candès et
al., 2008) were used to remove the random noise and decompose the TSD signal into continuous and discontinuous components. The continuous component (the Wavelet space) encode continuous variations in the pavements’ strength parameters, whereas the discontinuous component (pulse signals) highlight the location of weak spots such as deficient joints or cracks with low LTE. Thus, the recovered discontinuous component could be utilized to locate the weak joints in the pavement network.

- Date Interpretation Stage: Derive structural health indicators for jointed concrete pavements from TSD measurements. This stage implements a back-calculation scheme to derive structural parameters of jointed pavements from TSD deflection velocity data collected near transverse joints. The back-calculation fits the linear-elastic jointed-slab-on-ground model (Van Cauwelaert, 2004) to the deflection velocity measurements from all the TSD sensors. This model has the following advantages (1) it characterizes the transverse joints in terms of its deflection-ratio-based LTE (Pierce et al., 2017), (2) it matches the outcome of a detailed finite element model but at a tiny fraction of the computational cost, and (3) it reasonably simulates the deflection response from a jointed pavement to the rolling load of a heavy vehicle axle (Deep et al. 2020-a, -b, -c). As a result, the pavement material properties (the concrete slab’s elastic modulus \(E_p\) and the modulus of subgrade’s reaction) and the joint’s structural health [LTE index] can be estimated for every transverse joint surveyed by the TSD.

Significance of this Dissertation Research

Overall, this Dissertation, which spans the entire data analysis process from the field survey to the design of rehabilitation solutions for structurally unsound pavements, was framed with pavement managers and other practitioners in mind. The author’s ultimate intention is to release the final implementation of all developed computational tools to the public to help advance the utilization of TSD measurements for jointed pavement structures (refer to Appendix C for instructions on how to retrieve the companion collection of computer code). The outcome of this research may eventually become a routine data analysis tool for the practitioner to manage the concrete and composite pavements under their jurisdiction.

Contribution to Science

A myriad of work has been discussed, published, and debated about big-data mining and automated extraction of worthwhile information from data collected in bulk. Particularly, \(L_1\)-norm-based
regularization (the LASSO) has been routinely applied in the field of *compressed sensing* to reduce the size of a dataset without significant loss of information – see Hastie et al. (2015) for an intuitive example concerning imaging. This research applies these approaches in a new field (pavement structural evaluation and management) that has seldom appealed to data mining as part of routine practice.

**Contribution to Practice**

Although concrete and composite pavements account for roughly half (44%) of the Interstate Highway system (FHWA, 2019), little (if any) has been published on the use of TSDDs for evaluation and management of such pavements. This Dissertation attempts to bridge this gap. The analysis process and the ancillary computational tools developed as part of this research effort are expected to serve the pavement engineer to streamline the TSD data analysis process from raw data to estimated properties (Figure 1) and help assess the condition of their rigid and composite pavement network, and design/allocate rehabilitation solutions accordingly.
Figure 1: Applicability of this Dissertation. Conceptual flowchart of each paper’s relevance within the TSD data analysis process. TSD and dowel-retrofit pictures from personal archive.
Dissertation Contents and Organization

The core of Dissertation consists of four main manuscripts, which collect the author’s original research. A succinct blurb describing each manuscript topic is presented following:

1. Manuscript 1 covers the application of an optimization-based data extraction procedure [Basis Pursuit, BP] (Chen et al., 2001) to the TSD deflection-slope data, elaborating on how to interpret the recovered signal components from an engineering point of view. The outcome of Basis Pursuit could be utilized for weak spot detection within a pavement network. This paper has been completed and published (Scavone et al. 2021).

2. Manuscript 2 presents a numerical procedure to overcome the bias/variance trade-off within Basis Pursuit: Reweighted L1 Minimization [RWL1] (Candès et al., 2008). Applied to TSD pavement deflection data, BP plus RWL1 should provide a computationally efficient and intuitive feature extraction procedure to locate structurally deficient segments within a pavement network and recover the localized response.

3. Manuscript 3 provides a deeper insight into the analysis of TSD deflection velocity and deflection slope data. This investigation discovered a limitation of the widely-adopted procedure to convert surface deflection velocity measurements to deflection slope estimates (Hildebrand and Rasmussen, 2002; Krarup et al., 2006). This limitation becomes relevant when surveying pavements with discontinuities (such as transverse joints) with the TSD. The manuscript provides a mathematical proof to support this claim, and discusses its implications to the existing body of knowledge concerning TSD data interpretation. The findings discussed in Manuscript 3 provide fundamental guidance towards the statement of the joints’ LTE back-calculation problem.

4. Manuscript 4 embodies this Dissertation’s Interpretation Stage. It proposes a back-calculation scheme to estimate the fundamental strength parameters of a jointed concrete pavement (structural properties of the concrete slab itself, the foundation material, and the joint’s LTE) from the deflection velocity measurements collected by a TSD at the joint location (or nearby). This study builds upon a mechanistic framework based on linear-elastic slab-on-ground theory (Van Cauwelaert, 2004) that has been proposed to analyze the deflection measurements collected by the Raptor (Deep et al., 2020-a, b, c). The manuscript formulates the concrete pavement strength parameters’ back-calculation problem, introduces the use of gradient descent to solve it, discusses
its implementation in computer code, and comments on its performance based on actual 5-cm resolution TSD data collected at a jointed concrete test track.

From an applicability perspective, Manuscripts 1 and 2 constitute the TSD Data Extraction Stage of the entire streamlined TSD data analysis procedure, whereas Manuscript 4 introduces the Data Interpretation stage for pavement engineering purposes. Meanwhile, Manuscript 3 delves into more fundamental knowledge about the nature of the TSD deflection velocity data, the conclusions that arise from this paper’s transcend the realm of jointed concrete pavements and pervade the interpretation of TSD measurements for flexible pavements as well.

Each of the above-mentioned Manuscripts is devoted an entire chapter within this Dissertation. Plus, additional chapters have been added to introduce supplementary content that puts this research into context and elaborate on the motivation to pursue the objectives set forth here.

As such, this Dissertation is organized as follows:


- **Chapter 2: Background Literature Review**: About non-destructive testing [NDT], Deflection testing as NDT applied to in-service pavements for structural capacity assessment. Traffic-speed deflection testing. The Traffic Speed Deflectometer [TSD]: Device review and state of the practice. Opportunity for original research: analysis and interpretation of TSD measurements on jointed pavements.

- **Chapter 3 (Manuscript 1)**: – Identifying Weak Joints in Jointed Concrete and Composite Pavements from Traffic Speed Deflectometer Measurements by Basis Pursuit: Presentation of BP. Demonstration on Simulated Signals. Application for Weak Transverse Joints Detection with TSD

- **Chapter 4 (Manuscript 2)**: – Reweighted L1 Minimization for Network-Wide Pavement Weak Spot Detection from Traffic Speed Deflectometer Measurements: RWL1 as an enhancement to BP. Demonstration Example with Simulated Signals. Application with Real TSD Survey Data.

- **Chapter 5 (Manuscript 3)**: – On the TSD deflection velocity measurements: A revision to the current state of the art: A revision of how TSD deflection velocity measurements are generated.
A Comment on the deflection velocity to deflection slope conversion. Analysis of simulated and real TSD deflection velocity signals. Implications to the current state of the practice of TSD operation.


- **Chapter 7: Findings, Conclusion, and Recommendations**: Discussion of Findings. Dissertation Conclusion and Opportunities for Further Research.

Moreover, four appendices were added at the end of this Dissertation with supplementary information, namely:

- **Appendix A: TSDD Literature Review – A compendium of research on TSDD technology and application to pavement engineering and management**: Review on several present and past traffic-speed deflection devices. The Road Deflection Tester [RDT]. The Rolling Wheel Deflectometer [RWD]. The Rapid Deflection Tester [Raptor], The Traffic Speed Deflectometer [TSD]. The Moving Weight Deflectometer [MWD], the Laser Dynamic Deflectograph [LDD].

- **Appendix B: Mathematical Foundations and Methodology**
• **Appendix C: Laser Doppler Vibrometry and TSD measurements**: The Doppler Effect. Doppler Laser Vibrometry measurement principle. Doppler Laser Vibrometry as the TSD measurement principle.

• **Appendix D: Computer Tools’ Source Code**. Matlab source code for all the computer programs written to fulfill the research needs. It must be noted that a working digital version of the Matlab scripts and functions is online and readily available to download via: https://github.com/MartinScavone/concreteTSD
References


Presentation

This chapter introduces and discusses the motivation for pursuing this dissertation’s research objective. A selection of technical literature on pavement deflection testing and traffic-speed deflection devices [TSDDs] is reviewed to present the current state of the practice and to serve as background for a more narrowed-down discussion on traffic-speed deflection testing on concrete pavements. The published technical works reviewed herein provide the motivation for the Research Needs statement and the foundation for the Dissertation’s objective. A more comprehensive review of published literature on traffic-speed deflection testing (featuring over 150 references) can be found on Appendix A.

Non-Destructive Testing of Pavements

Non-destructive tests (NDTs) use non-invasive technologies to assess an infrastructure asset, without the need for coring or damaging it in any way (Rivero and Solla, 2016). In the case of highway pavements, structural evaluations by NDTs provides insight into the pavement’s structural health at any given moment during their service life (Haas et al., 2015). Pavement managers depend on the results of both structural and surface condition evaluations to adequately assess their condition at the network level and consequently assign maintenance and/or rehabilitation works to prevent further deterioration (Hildebrand and Rasmussen, 2002; Katicha et al., 2020; Zofka et al., 2014; Garber and Hoel, 2015; Haas et al., 2015; Manoharan et al., 2017). Furthermore, in constrained-budget contexts, a situation many highway agencies face, the condition assessment results are a key input for optimally assigning resources toward minimizing overall network deterioration and/or maximizing the network’s overall condition grade (Gedafa et al., 2010; Garber and Hoel, 2015; Haas et al., 2015; Manoharan et al., 2017). The outcomes of NDTs are also valuable at the project-level scale. NDT can be used to assess the bearing capacity of an existing deteriorated (or even failed) pavement structure, and design a rehabilitation project that appropriately contemplates the pavement’s residual strength (AASHTO, 1993; 2020; Huang, 2004; Flintsch et al., 2013; Garber and Hoel, 2015; Haas et al., 2015).
Historically, non-destructive evaluations of the bearing capacity of existing pavements have been conducted with static or crawl-speed rolling-load devices (e.g., Benkelman beam, California deflectograph, and the French LaCroix deflectograph) or static pulse-load devices like the Falling Weight Deflectometer [FWD, which is standardized under the ASTM D6495 standard (ASTM, 2015)] (Huang, 2004; Simonin et al., 2005; Ferne et al., 2009; Baltzer et al., 2010). The fact that these devices either perform static tests or move at very low speeds (<5 mph) renders them unsuitable for frequent routine network-level pavement condition surveys (Hildebrand and Rasmussen, 2002; Flintsch et al., 2012; Garber and Hoel, 2015; Haas et al., 2015). Additionally, their operation would disturb the normal traffic flow and pose a safety risk for the device operators who would be exposed to passing vehicles during testing (Hildebrand and Rasmussen, 2002; Rasmussen et al., 2002; Zofka et al., 2015; Haas et al., 2015; Elsefi and Zihan, 2018).

Another drawback of pulse-load devices such as the FWD is that they cannot actually mimic the load of a heavy vehicle, which is a dynamic rolling road traveling at a high speed (Hildebrand and Rasmussen, 2002; Huang, 2004; Brezina et al., 2017; Nasimifar et al., 2018-a, -b). However, despite these limitations, the FWD has been widely adopted for road evaluation purposes since the 1980s. The outcome of an FWD deflection test is commonly used during pavement condition surveys or to design pavement reinforcements (AASHTO 1993, 2020; Arora et al., 2006; Alavi et al., 2008; Chai et al., 2016; Flintsch and McGhee, 2009; Garber and Hoel, 2015; Huang 2004; Manoharan et al., 2017).

Highway agencies in the United States mostly use the FWD for pavement structural evaluation. Because the FWD needs to be stationary during testing, and because of time constraints, FWD deflection surveys are performed only at select sample locations which are assumed to represent the condition of a given stretch of pavement (Katicha et al., 2017; Rabe, 2013). The obvious consequence of such constrained structural data collection is decision-making reliant on limited (if any) structural health insight (Katicha et al., 2017). Moreover, to properly account for the safety of the FWD operators and of the device itself, the test lanes must often be closed to traffic while the survey takes place. Consequently, the mandatory safety countermeasures would increase both the survey’s direct costs (agency) and indirect costs (time loss and disruptions) (Hildebrand and Rasmussen, 2002, Flintsch et al, 2013; Haas et al., 2015; Rada et al., 2016).
Traffic Speed Deflection Devices [TSDDs]

Traffic-Speed Deflection Devices (TSDDs) were developed in response to the need for a pavement structural evaluation device capable of surveying while moving with the traffic flow and providing a dense dataset (European Commission, 1997). The traffic-speed-operation requirement was set in response to the safety and traffic disruption concerns (Hildebrand and Rasmussen, 2002; Flintsch et al. 2013) – a particularly important matter on heavily-trafficked highways such as the US Interstate System. Traffic-speed operation also significantly increases the amount of data collected; far more lane miles can be surveyed during a single day of operation compared to what could be achieved with the FWD (Hildebrand and Rasmussen, 2002; Arora et al., 2006; Balzter et al., 2010; Flintsch et al., 2012, 2013; Brezina et al., 2017; Katicha et al., 2017; Levenberg et al., 2018). Flintsch et al. (2013) established that for a continuous deflection device to be tagged as a TSDD, it must be able to reliably collect surface deflection data at a speed equal to or greater than 55 km/h [35 miles per hour]. The dense TSDD measurements (less than 0.31m [1 ft] spacing, although generally reported at 1m to 160m intervals [3ft to 0.1 miles]) allow identifying weak zones that may require specific correction countermeasures prior to rehabilitation (Hildebrand and Rasmussen, 2002; Flintsch et al., 2012, 2013; Katicha et al., 2014; Haas et al., 2015).

Design, operation, data handling, and implementation of TSDDs into engineering practice have been subjects of research since the first prototypes were built and launched in the late 1990s, and efforts are still ongoing in the present days. Over the years, several research projects reviewed the state of the art at the time in terms of TSDD technology and addressed issues arising as part of their adoption as pavement screening or routine surveying tools. Some of the major research efforts are documented in the reports by Arora et al., 2006; Phares et al., 2008; Rada and Nazarian, 2011; Austroads, 2012; Flintsch et al., 2013; Rada et al., 2016, Elseifi et al., 2012; Jansen, 2015; Elseifi and Zihan 2018; and Katicha et al., 2017, 2021. A more exhaustive review of the literature published up to 2021 and covering a set of devices including previously reviewed and recently launched TSDDs is provided in Appendix A. The featured devices are the Swedish Road Deflection Tester [RDT], ARA’s Rolling Wheel Deflectometer [RWD], Greenwood’s Traffic Speed Deflectometer [TSD], Ramboll’s (formerly Dynatest’s) Rapid Pavement Tester [RAPTOR], Nexco’s Moving Weight Deflectometer [MWD] and Zoyon’s Laser Dynamic Deflectometer [LDD]. However, as of early 2022, only one type of TSDD is operational and available in the United States. ARA’s RWD has been decommissioned in 2020, and the RAPTOR technology (all three working prototypes plus the related intellectual property) has been acquired by a new entity.
[Ramboll] also in 2020 – commercial operation of the RAPTOR resumed in 2021. The remaining devices are one-offs developed overseas for their local market. Therefore, this Dissertation focuses only on the TSD.

The Traffic Speed Deflectometer [TSD]

The Traffic Speed Deflectometer [TSD] is a type of TSDD that uses Doppler lasers to measure the velocity at which the pavement surface deflects in response to the applied rolling load. By combining these deflection velocity readings with measurements of the device’s traveling speed, a measurement of the deflection basin slope (the derivative of the deflection bowl) can be obtained (Hildebrand and Rasmussen, 2002).

![Figure 2: A Traffic Speed Deflectometer (TSD-9, second-generation) at the Virginia Tech Transportation Institute, September 2017 [Personal Archive]](image)

The TSD (Figure 2) consists of a specially designed trailer whose rear axle (a dual-wheel single axle) can be loaded to a known value by adding/removing purposefully built ballast load units (Katicha et al., 2017). The device’s default load on the rear axle is 10 metric tons [11.2 short tons, 22 kips] (Hildebrand and Rasmussen, 2002), but the device operating in the uses a 20-kip rear-axle load levels (Katicha et al., 2020). The array of Doppler lasers [Doppler vibrometers] are mounted on a rigid beam on the passenger’s side of the device (Figure 3). A servo system that responds to bumps in the pavement is used to keep the
beam with the Doppler laser sensors in a horizontal position and at a constant height from the pavement surface (Hildebrand and Rasmussen, 2002; Arora et al., 2006; Ferne et al., 2009; Flintsch et al., 2012; Katicha et al., 2017). The entire trailer is climate-controlled and kept at a constant temperature while in operation; this prevents the support beam from warping due to thermal expansion and contraction, which would otherwise occur and so invalidate the measurements (Rabe 2013; Wix et al., 2016).

![Figure 3: The TSD’s array of Doppler vibrometers and the mounting beam. [Courtesy of G. Flintsch]](image_url)

Greenwood A/S, the TSD manufacturer, enumerates four generations of the TSD, which are presented in Table 1. Some of these denominations were also adopted in published technical literature, whereas the remaining ones. The second-generation device is standardized under Austroads Standards AG:AM T017 and S006 (Austroads, 2016 -a, -b).

The first TSDs built included only three Doppler sensors located ahead of the rear axle plus a fourth reference laser sensor located 3.5 meters ahead from the axle that would measure the undeflected pavement surface. Newer versions of the TSD feature additional laser sensors – from 7 to 11 lasers, with the 11-laser setup including three sensors behind the rear axle wheel (Nielsen and Jensen, 2021). The recently launched fourth-generation TSD may include two arrays of 11 lasers – one at each side of the device. Table 2 illustrates the Doppler sensor array of the different TSD generations manufactured to date.
<table>
<thead>
<tr>
<th>Device Generation</th>
<th>Devices Built (as of late 2021)</th>
<th>Deflection Sensing Equipment</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>2</td>
<td>3 + 1 (reference) 1 kHz Doppler laser sensors located on outer wheel path, ahead of rear axle wheel</td>
<td>Hildebrand and Rasmussen (2002); Baltzer et al. (2010).</td>
</tr>
<tr>
<td>2nd</td>
<td>10</td>
<td>6 + 1 (reference) 1 kHz Doppler laser sensors located on outer wheel path, ahead of rear axle wheel</td>
<td>Ferne et al. (2015); Katicha et al. (2017); Austroads (2016-a,-b)</td>
</tr>
<tr>
<td>3rd</td>
<td>2</td>
<td>10-11 1kHz Doppler laser sensors located on outer wheel path, 3 lasers located behind the rear wheel, remaining lasers located ahead</td>
<td>Nielsen (2019); Nielsen and Jensen (2021)</td>
</tr>
<tr>
<td>4th</td>
<td>3 + 2 retrofitted devices</td>
<td>10-11 250kHz Doppler laser sensors located on each wheel path, 3 lasers located behind the rear wheel, remaining lasers located ahead</td>
<td>Presented in 2020, no written technical report/research paper describing this generation was found as of 2021</td>
</tr>
</tbody>
</table>

Table 2: Number and default location of Doppler sensors across TSD generations.

<table>
<thead>
<tr>
<th>Device Generation</th>
<th>Doppler laser sensor array configuration</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>3 sensors: 200, 300, 750 mm plus reference sensor (3500mm)*</td>
</tr>
<tr>
<td>2nd</td>
<td>6 sensors: 100, 200, 300, 450, 600, 900mm, plus reference sensor (3500 mm)**</td>
</tr>
<tr>
<td>3rd</td>
<td>9 sensors: -366, -269, -167, 163, 260, 362, 662, 964, 1559mm, plus reference sensor (3108 mm)+</td>
</tr>
<tr>
<td>4th</td>
<td>10 sensors: -450, -300, -200, 130, 215, 300, 450, 600, 900, 1510mm, plus reference sensor (3500 mm). Optional secondary array on driver’s side ++</td>
</tr>
</tbody>
</table>
Notes:

• (* ) TSD-1 (a 1"-Generation device) only had two Doppler sensors besides the reference sensor (Hildebrand and Rasmussen, 2002)

• (** ) The Doppler sensor array for 2nd-Generation TSDs is as per Austroads (2016-b)

• (’ ) As per Nielsen (2019)

• (++ ) Sensor array for TSD-17, operational in the United States.

The earliest structural surveys done with the TSD in Europe covered roughly 225 lane-km [140 lane miles] of pavement a day, at an operational speed of about 70 km/h [45 mph] (Hildebrand and Rasmussen, 2002). Further network-wide testing with the TSD achieved an average daily coverage of 200-400 km [130-250 miles], even up to 800 km [500 miles] (Antonsen, 2016; Brezina et al., 2017). The field data collection stage of the Transportation Pooled Fund [TPF] project 5(385) consisted of freeway pavement surveys at a speed of about 95-110 km/h [60-70 mph], covering roughly 400-525 lane-km [250-350 lane miles] daily. Thus, in terms of survey speed and network coverage, the TSD has a clear advantage over stop-and-go devices. For the sake of comparison, at a spacing between samples of 320 meters [0.2 miles, close to the spacing recommended by the ASTM D6495 standard (ASTM, 2015)] a structural survey with an FWD would cover about 40 lane-km [25 lane-miles] per day.

In terms of cost, the TSD itself costs 2.5-3 million USD (Flintsch et al., 2013); approximately 15 times the value of a brand-new FWD device. However, such an investment could be made profitable as long as the TSD is in continuous operation year-long (Hildebrand and Rasmussen, 2002) and replacing numerous FWD tests (for which the traffic control device – a shield truck – represents an expenditure of $1600 to $2500 a day). Anyway, a network-wide survey with a TSD costs approximately $95 to $155/km [$150 to $250/mile], data processing costs included

Conversion of TSD deflection-slope data into deflection basins

The deflection bowl slope estimates from TSD deflection velocity measurements (Krarup et al., 2006) relate to the pavement’s load-bearing capacity (Flintsch et al., 2012; Müller and Reeves, 2012; Manoharan et al., 2017; Rada et al., 2016; Katicha et al., 2017), from which the slope of the deflection bowl can be estimated (Krarup et al., 2006), reproduced as equation 1:

3 Approximate TSD operational costs for the TPF 5(385) project
Where \( v_y \) stands for the deflection velocity measurements [m/sec], \( v_x \) is the TSD travel speed [m/sec], and \( S \) is the pavement deflection bowl slope [m/m].

The deflection slope estimates can be integrated to obtain the deflection bowl (Krarup et al., 2006; Müller and Roberts, 2013; Flintsch et al., 2013; Nasimifar et al., 2018-b; Zofka et al., 2015), conceptually illustrated in figure 4. However, the integration procedure requires explicit or implicit assumptions regarding the constant of integration. An example of an explicit assumption is to specify the location where the pavement deflection is qual to zero and a case of implicit assumption is to assume the pavement to behave as an Euler-Bernoulli beam-on-ground system (Krarup et al., 2006; Müller and Roberts, 2013; Flintsch et al., 2013). Nasimifar et al. (2018-b, 2019-a) provide an interesting comparison of the many deflection slope integration schemes proposed in the literature based on linear elasticity assumptions (Krarup et al., 2006; Müller and Roberts, 2013; Zofka et al., 2015) and contrast them against visco-elastic modeling. Linear elastic modeling was found acceptable for analysis at the network-wide level, because the reduced computational cost offsets the possible approximation errors. Meanwhile, TSD data analysis under visco-elasticity assumptions (Nasimifar et al., 2018-b, 2019-a; Nielsen, 2019) should be restricted for project level applications, mainly because of its computational cost.

![Diagram of deflection velocity and deflection basin depth for a TSD load.](image)

*Figure 4: Illustration of the deflection velocity and deflection basin depth for a TSD load. Courtesy of Greenwood Engineering A/S. Reproduction authorized.*
Alternatively, the deflection bowl integration constant can be disregarded from the analysis stage if deflection indices (differences in deflection at two locations within the basin) are used instead of single point deflections (Flintsch et al., 2013; Katicha et al., 2017; Shrestha, 2017; Shrestha et al., 2018, 2019). Flintsch et al. (2013) and Rada et al. (2016) sought the deflection indices (among other deflection-based formulas) that best correlated to strain magnitudes within the pavement structure – an idea originally proposed by Thyagarajan et al. (2011).

A key difference between the deflection measurements done with the TSD and the FWD is the nature of the testing load: while the FWD applies an impact load onto the pavement, the TSD measures the pavement’s deflection as a response to the rolling load of its rear-axle wheels (Nielsen, 2019). Thus, the TSD realistically assesses how the pavement responds to actual traffic loads (Hildebrand and Rasmussen, 2002; Flinstch et al., 2012, 2013; Brezina et al., 2017). Comparative analysis of TSD deflection basins (integrated) and FWD basins collected simultaneously proved that in fact, both devices give significantly different deflection basin measurements (Simonin et al., 2005; Elseifi and Zihan, 2018; Katicha et al., 2014; Levenberg et al., 2017). Nonetheless, the TSD and the FWD will locate the same weak and strong sections within a pavement segment (Katicha et al., 2017), showing that the TSD is suitable for network-wide screening – the originally intended purpose of the TSD.

The difference in how the load is applied implies that any pavement analysis methodology developed for FWD measurements should not be assumed to be readily applicable to TSD measurements (Nasimifar, 2015; Saremi, 2018; Nielsen, 2019). In response to this concern, several researchers published TSD-specific formulae to derive pavement properties from the TSD (Elseifi and Zihan, 2018; Nasimifar et al., 2019). Others have trained artificial intelligence models that translate the deflection slope measurements collected by the TSD into equivalent FWD basins (Elseifi et al., 2019) that would eventually allow the practitioner to utilize FWD-based analysis tools with TSD survey data. Nonetheless, data translation processes may not be needed for network screening purposes (Katicha et al., 2017; Shrestha et al., 2018, 2019).

**Structural analysis of jointed pavements**

The design of jointed concrete pavements gravitates around the thickness design of the concrete slabs themselves and their foundation layer(s), and the design of the pavement’s load-carrying joints (AASHTO, 1993, 2020; Huang, 2004; NCHRP, 2004; Papagiannakis, and Massad, 2008). To manage a network of jointed concrete or composite pavements under a framework reliant both on the surface
condition and structural health data, knowledge is required of the structural properties of the concrete slabs, the subgrade, and the load-carrying capacity of the load-bearing joints – often reported in terms of Load Transfer Efficiency [LTE] index (Phares et al., 2008; Pierce et al., 2017). It is known that structurally deficient or badly retrofitted load-bearing joints trigger an early failure of the attached slabs (Garber and Hoel, 2015) and reflective cracking on overlaid concrete structures (Pierce et al., 2003). In the AASHTO (1993) pavement design methodology, prevention of loss-of-support failure is guaranteed by limiting the joints’ LTE above a minimum tolerable threshold (Garber and Hoel, 2015). Conducting ex-ante and ex-post LTE testing of a retrofitted joint is the quality assurance method of choice for such a repair (Pierce et al., 2003). Thus, knowledge of the concrete’s elastic modulus, the subgrade’s resistant properties, and the joints’ LTE index is crucial for proper management of such pavements, prevention of surface pathologies, and design of reinforcements when needed (AASHTO, 1993, 2020; Huang, 2004, NCHRP, 2004, Phares et al., 2008, Pierce et al., 2003, 2017).

The procedure to back-calculate the elastic modulus of an in-service concrete slab from deflection testing is well described in the reference literature (AASHTO, 1993; Huang, 2004; Pierce et al., 2017). Similarly, the LTE test (Figure 5) can be performed to assess the proper functioning of a load-bearing joint with the FWD (AASHTO, 1993; Pierce et al., 2003, 2017; Alavi et al., 2008; ASTM, 2015).

Figure 5: Load Transfer Efficiency concept (left) and Load Transfer Efficiency testing with an FWD (right). Credit: Pierce et al. (2017) [left, reproduction authorized] and S&ME Inc. [right].
However, as with any structural health test conducted with a stop-and-go device such as the FWD, it is unfeasible to carry it out at a network-wide scale in a periodic manner. A single LTE test (single joint) elapses about 5 minutes⁴, a jointed concrete pavement (with the usual 15-ft spacing between joints) would be assessed at a rate of roughly 1.1 km [0.7 miles] per day (8-hour shift). As part of the yearly State of the Pavement report, VDOT surveyed about 850 lane-km [525 lane-miles] of concrete highways during 2018 (Vlacich, 2018). Such a survey focused on surface distresses and ride quality and was conducted from a specially rigged vehicle that surveys the pavement at the speed of traffic. If LTE testing of the entire network had been included, the testing campaign would have elapsed 875 days (roughly 2 years and 5 months). By this example, it is clear that **a procedure must be devised to locate the structurally deficient joints between a jointed pavement network in order to optimally plan an LTE testing campaign with the FWD, focused only on the most likely weak spots.**

Moreover, testing for LTE at the corridor- or network-level with the TSD could be further justified from an economic standpoint. Assuming a single LTE test with an FWD may cost $50 (traffic safety device excluded), and that 96 LTE tests may be done in an 8-hour shift (at 5 minutes per test, 12 per hour), and considering a shield truck rental cost of $2500/day (that was the rental fee in 2019); the daily expenditure in LTE testing with the FWD may add up to $7300/day, producing 96 deflection bowl measurements. Meanwhile, considering the TSD survey cost at $155/km [$250/mile], plus zero expense in traffic control devices, the data collection cost for that 1.1km [0.7-mile]-long corridor adds up to a mere $175, with the plus that the TSD could span such length in just 45-55 seconds (at a speed of 75-95 km/h [45-60 mph]), and data from 1120 deflection basins (at a 1 m spacing) would be collected in that time interval.

The TSD (or any Traffic-Speed Deflection device) could fulfill that need for fast and broad testing of a jointed pavement network for investigatory or managerial purposes. Although the published technical literature has not conclusively shown that TSD for routine testing can be used on concrete pavements⁵, the deflection slope measurements collected with a TSD could provide valuable insight into the location and health of structurally deficient locations within the jointed pavement network. In addition, adaptive data analysis techniques, like wavelet-basis decomposition of the TSD measurements, could enable the

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⁴ According to pavement practitioners I interviewed for this review, a single LTE test takes around 3 minutes (if only one slab is loaded) or 5 minutes (if the two neighboring slabs are tested). These figures would not account for initial test setup (bringing the FWD to the test site and deploying the traffic control devices).

⁵ It is assumed in practice that the TSD deflections on rigid concrete pavements may be too small to be reliably detected by the sensing apparatus, that is why these are recommended as test beds for the calibration of the on-board sensing equipment (Ferne et al., 2009; Katicha et al., 2017). Additionally, the Australian TSD standard (Austroads, 2016-b), expressly advises against performing deflection surveys on concrete pavements with a TSD.
retrieval of the pulse responses originated at these weak locations (Flintsch et al., 2012, 2013; Katicha et al., 2013, 2014), and thus perform a preliminary evaluation – deciding whether further investigation should be conducted or not (and thus optimizing the presence of the stop-and-go device on the roadway). Moreover, a recently published collection of papers (Deep et al., 2020-a, -b, -c) explore the use of a mechanistic back-calculation procedure (Van Cauwelaert, 2004) for the estimation of $E_p$ and LTE, among other properties, from the deflection basin measurements as taken with the RAPTOR device, computer simulation results were only produced thus far. An opportunity arises on developing such a back-calculation framework for the TSD.

**Literature Review Key Points and Research Needs statement**

Thus, the review of the state of the art on the subject of jointed pavement structural testing concluded that:

- Although current technology may allow network-level structural assessment of jointed pavement structures, a lack of implemented and streamlined data processing and interpretation procedures stymies the adoption of such devices in practice.

- As a consequence, management decisions concerning the maintenance and rehabilitation of jointed pavements may be based on incomplete data (leading to a non-optimal resource allocation).

- Data processing techniques that could clear the dense TSD data from noise and extract meaningful information concerning the structural health of the pavement slabs and load-carrying joints do exist within the realm of big data mining and could be imported to the pavement engineering discipline.

- Mechanistic pavement models such as the linear-elastic slab-on-ground model could be implemented into a back-calculation scheme for the estimation of the pavement layers structural health.

In light of the points above, there is a need for a streamlined analysis and interpretation procedure for network-wide jointed pavement deflection data as garnered during a TSD survey. This doctoral dissertation aims at addressing this need by developing and implementing analysis approaches and

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6 There is also an ongoing research project in Germany and Denmark on the topic of weak spot detection within concrete pavement networks using a TSD, written deliverables are expected for mid-year 2021.
tools for network-level pavement management, linking the realms of data analytics and pavement engineering and management.
References


Abstract

Weak joints are the source of most performance problems in jointed concrete pavements (JPCPs) and composite pavements created by overlaying a JPCP. Until recently, it has been prohibitively time-consuming to evaluate the joints in a road network, and the evaluation of joint load transfer efficiency (LTE) has been restricted to specific project-level applications. This is changing with the advent of the traffic speed deflectometer (TSD), a device that can collect structural information data at a 1-m resolution while moving at traffic speed. The 1-m resolution is adequate to capture the response of the joints to the applied load, but it results in a higher noise level compared with data collected at lower resolutions (e.g., at the typical 10 m). This paper proposes the use of basis pursuit (BP) denoising to extract meaningful information about the joints’ condition from the (noisy) TSD measurements. Weak joints are modeled as spikes (Dirac basis) in the measurements, and the remaining features in the measurements are modeled on a wavelet basis. Combining the two bases (Dirac basis and wavelet basis) results in multiple possible representations of the collected measurements; essentially there are twice as many unknowns as equations to determine these unknowns. BP denoising seeks a representation with a small number of elements from the two bases. Because weak joints are represented best by spikes, BP denoising results mostly in selecting the spikes at the weak joint locations. These identified spikes provide a list of weak joints that can be used to prioritize available resources (e.g., which joints should be investigated further or fixed first). We present examples of BP denoising using simulated data and actual TSD collected data.

Erratum Reporting

- During the compilation of this Dissertation, it was found that the Soft Thresholding function (Equation 6 on the published version, reproduced here as equation 7), BP’s iterative coefficient update (Equation 7 on the published version, reproduced here as equation 8) and the SURE formula (Equation 10 on the published version, reproduced here as equation 11) were incorrectly written. The equations’ correct formulae are as written in this Dissertation.
Introduction

Recent trends in asset management suggest that highway agencies should perform periodic structural evaluations of the pavements under their jurisdiction in order to properly address and allocate any required maintenance task (Hildebrand and Rasmussen, 2002; Haas et al., 2015; PIARC, 2017). Non-destructive falling weight deflectometer (FWD) testing has been used at the project-level since the 1980s to select and design maintenance and rehabilitation treatments (AASHTO, 2008; Haas et al., 2015). For flexible pavements, FWD measurements are used to back-calculate the pavement and subgrade layer moduli which are then used for overlay design (Hildebrand and Rasmussen, 2002; AASHTO, 2008; Flintsch et al., 2013; Haas et al., 2015). For jointed concrete pavements (possibly overlaid with asphalt), the source of most performance problems is related to deficient load transfer at the joints. A so-called “weak” joint can lead to early faulting, excessive deflection, cracking by loss of support because of pumping of foundation materials, and reflection cracking if the pavement is overlaid with asphalt (Huang, 2004). Therefore, on jointed Portland cement concrete pavements (JPCP) or composite pavements created by overlaying a JPCP, the FWD is used to determine the joint load transfer efficiency (LTE), a measure of the strength of the joint (Alavi et al., 2008; Pierce et al., 2003, 2017).

Although proven to be a valuable device at the project-level, the FWD has significant limitations that make it unsuitable for routine network-level evaluation. The FWD is a stationary device that requires traffic control countermeasures which disrupt the traffic flow and poses safety threats (Hildebrand and Rasmussen, 2002; Flintsch et al., 2013; Katicha et al., 2013; Haas et al., 2015). In addition, the FWD cannot cover more than 18 lane-miles per day at a 0.1-mile testing resolution (Flintsch et al., 2013), meaning that a network-wide survey would require a large fleet of FWDs operating simultaneously to complete the testing campaign in a timely manner. For these reasons, network-level FWD testing is rarely performed.

The Traffic Speed Deflectometer (TSD) was developed to address the limitations of the FWD. It operates at the speed of traffic, which is most appropriate for network-level testing (European Commission, 1997; Hildebrand and Rasmussen, 2002; Flintsch et al., 2013). Several researchers have demonstrated that the TSD compares favorably with the FWD for network-level testing of flexible pavements (Simonin et al. 2005; Arora et al. 2006, Flintsch et al., 2013; Katicha et al., 2014; Katicha et al., 2017; Elseifi and Zihan, 2018). However, there is very limited information about the use of the TSD for the evaluation of rigid pavements, mainly because it is perceived to be less applicable to rigid pavements (Rada et al., 2016). In one of the few examples of TSD application to rigid pavements, Flintsch et al. (2013) suggested that the
TSD has the potential to detect localized weak spots and weak joints in concrete pavements, even in cases where the concrete slabs are overlaid with asphalt. Katicha et al. (2013; 2014; 2016) used signal processing methods (both spline-based and wavelet-based) to denoise the TSD signals and extract meaningful features with the potential to detect weak joints in jointed concrete pavements.

Objective

The main objective of this paper is to develop an approach that can effectively detect the weak spots in pavements from measurements collected by the TSD with a particular interest in detecting joints with load transfer deficiencies in composite and jointed concrete pavements. The developed approach is based on Basis Pursuit [BP] decomposition of the collected measurements (Chen et al., 2001) into two main components. The first component represents the smoother variation in the measurements while the second component represents localized spikes in the measurements. The main benefit of the approach is that the good theoretical properties of BP translate into an objective approach that is also fast, practical, and reproducible to identify weak locations such as open cracks and weak load-carrying joints. This is especially important in the context of network-level testing where a large number of measurements makes it very difficult and time-consuming to perform a reliable weak joint detection by a human expert. The methodology proposed in this paper builds on the work by Katicha et al. (2013) and changes the basis into which the raw TSD signal is decomposed. While Katicha et al. (2013) represented the TSD signal as a combination of wavelets plus white noise, in this paper, we use an overcomplete dictionary that combines two bases (wavelet basis plus Dirac basis) to represent the “true” features of the TSD signal.

Methodology

The approach consists of using BP denoising to decompose the collected TSD measurements into a sum of components from the wavelet and Dirac bases. The Dirac basis (spikes) is used to detect the features in the signal that reflect the presence of weak joints. The wavelet basis is used to represent the remaining spatial features of the measured structural condition. Although other bases could be used (e.g. Fourier basis), the main reasons we have chosen a wavelet basis are:

1. Most measured signals have a sparse/compressible wavelet representation (i.e. most wavelet coefficients are very small, practically zero). In other words, few wavelets may be needed to represent almost any measured signal.
2. Because of the property above, wavelets are very efficient at separating the noise from the signal.

3. Computations with wavelets are very fast.

**Basis Pursuit**

Basis Pursuit [BP (Chen et al., 2001)] is a convex optimization approach to obtain a sparse decomposition of a given signal \( y \), which is composed of \( n \) observations, hence \( y \) is a vector of \( R^n \). The concept of sparsity implies aiming at a decomposition of \( y \) made up of few components. BP can find a representation of \( y \) even in cases when the number of potential components to be used to represent \( y \) is larger than the number of observations \( n \).

In general, the components are chosen because of their significance to specific applications. For example, the Dirac basis selected in this paper is well suited to represent spikes in the collected measurements that are due to weak spots (weak joints). However, the Dirac basis is not smooth and therefore not well suited for representing smooth variations in the structural condition. The Fourier basis or a specific wavelet basis have smoother components and therefore are better suited to represent smooth variations. For signals that are composed of both smooth parts and spikes, a good representation with a small number of parameters can be found by combining the two bases (e.g. Dirac and wavelet basis). Continuous deflection measurements in jointed concrete signals fall in the category of smooth signals with occasional spikes located at the weak joints. However, combining the two bases make the problem of finding a representation under-determined. For a signal comprised of \( n \) measurements, each of the two bases would contribute \( n \) components. This results in \( p = 2n \) components to choose from to represent the signal thus making the decomposition of the signal not unique (i.e. there are many possible ways to represent the signal).

One of the earliest approaches to solve under-determined systems is to use the pseudo-inverse (Penrose 1955, 1956). This approach gives the solution with the smallest norm as described in equation 2 – also known as Ridge Regression or Tikhonov Regularization (Mallat, 2008; Hastie et al., 2009).

\[
\min \| \alpha \|_2 \text{ subject to } y = \Phi \alpha
\]

Where \( \alpha \) is a \( 2n \) vector of the coefficients of each feature, \( \| \alpha \|_2 \) is its Euclidean \([L_2]\) norm, and the matrix \( \Phi \) represents the basis (or bases) of vector components. This is one particular solution among many possible solutions. Its main advantage is that it is easy to calculate. However, the solution involves all \( p \) variables, which makes it hard to interpret. A solution that involves \( m < p \) variables would be easier to
interpret. This motivates finding the solution with the smallest number of components, which can be formulated as shown in equation 3.

$$\min \| \alpha \|_0 \text{ subject to } y = \Phi \alpha$$

(3)

Where $\| \alpha \|_0$ is called the $L_0$ norm (mathematically it is not a norm) of $\alpha$ (number of non-zero components of $\alpha$). Unfortunately, the problem formulated in Equation 3 is a combinatorial optimization problem that cannot be solved even for modest-size $n$ (Candès et al., 2006). Basis Pursuit [BP] (Chen et al., 2001; Hastie et al., 2015) is an alternative optimization problem that replaces the $L_0$ norm with the $L_1$ norm (sum of absolute values).

$$\min \| \alpha \|_1 \text{ subject to } y = \Phi \alpha$$

(4)

BP is a convex optimization problem that can be solved easily. Furthermore, Candès et al. (2006) and Donoho (2006) showed that in many problem settings where a relatively small number of parameters can explain the observed data, the solution of equation 4 is the same as the solution of equation 3. In practice, the observed data is almost surely noisy, and we would not want to fit the data exactly. In this case, BP denoising, also known as the Least Absolute Shrinkage and Selection Operator, or LASSO (Tibshirani, 1996; Chen et al., 2001; Mallat, 2008; Hastie et al., 2009, 2015), the Lasso (Tibshirani, 1996; Chen et al., 2001; Hastie et al., 2015) can be used:

$$\min_{\alpha} \frac{1}{2} \| y - \Phi \alpha \|_2^2 + \lambda \| \alpha \|_1$$

(5)

Where

$$y = z + \sigma \epsilon$$

(6)

The solution of equation 5 depends on the parameter $\lambda$. As $\lambda \to 0$, the solution of equation 5 converges to that of equation 4. However, in the presence of noise, an appropriate $\lambda > 0$ will give a solution that does not overfit the data. Clearly, the choice of $\lambda$, which is discussed in the next section, affects the solution quality.

**Calculating the solution of BP and choosing $\lambda$**

Many algorithms have been proposed to solve equation 5 (e.g. Tibshirani, 1996; Chen et al., 2001; Efron et al., 2004). Among them, Friedman et al. (2010) proposed a simple algorithm based on cyclic coordinate descent. The idea is to take advantage of the solution of BP in the case of a single variable, which is given by the soft-thresholding function $S(\alpha, \lambda)$ given in equation 7 (see figure 6 for a graphical representation):
\[ \alpha^{BP} = S(\alpha^{LS}, \lambda) = \text{sign}(\alpha^{LS}) \times \max(|\alpha^{LS}| - \lambda, 0) = \begin{cases} 
\alpha^{LS} - \lambda & \text{if } \alpha^{LS} \geq \lambda \\
\alpha^{LS} + \lambda & \text{if } \alpha^{LS} \leq -\lambda \\
0 & \text{if } -\lambda < \alpha^{LS} < \lambda 
\end{cases} \] (7)

Where \( \alpha^{LS} \) is the ordinary least squares solution.

Cyclic coordinate descent is an iterative algorithm that updates each individual component of the vector of coefficients \( \alpha \) while keeping the remaining ones constant. Suppose that, at a given iteration of the algorithm, the estimate of \( \alpha \) is \( \alpha' \). The update of the \( j \)th component of \( \alpha \) is given by:

\[ \alpha_j' = S(\alpha_j^{LSR}, \lambda) \] (8)

Where

\[ \alpha_j^{LSR} = (\Phi_j^T \Phi_j)^{-1} \Phi_j^T (y - \Phi_{-j} \alpha'_{-j}) \] (9)

Where \( \Phi_j \) is the \( j \)th column of the matrix \( \Phi \), \( \Phi_{-j} \) is the matrix \( \Phi \) with the \( j \)th column removed, and \( \alpha'_{-j} \) is the coefficient vector with the \( j \)th coefficient removed. The update is obtained by soft-thresholding the least-squares fit of the \( j \)th column of the matrix \( \Phi \) on the residual of the partial model that excludes the \( j \)th column vector. Orthonormal components of \( \Phi \) can be updated simultaneously (because these components do not affect each other). Therefore, if \( \Phi = (\Phi_1, \Phi_2) \) where \( \Phi_1 \) is the Dirac components with parameters vector \( \alpha_1 \) and \( \Phi_2 \) the wavelet components with parameters vector \( \alpha_2 \) (\( \Phi \) is then the matrices \( \Phi_1 \) and \( \Phi_2 \) concatenated), then the update can be implemented as follows (“block coordinate descent”):
\( \alpha_1^{i+1} = S \left( \Phi_1^T \Phi_1 \right)^{-1} \Phi_1^T \left( y - \Phi_2 \alpha_2^i \right), \lambda \)  
\( \alpha_2^{i+1} = S \left( \Phi_2^T \Phi_2 \right)^{-1} \Phi_2^T \left( y - \Phi_1 \alpha_1^i \right), \lambda \)  
\( \text{(10a)} \)

\( \alpha_1^{i+1} = S \left( \Phi_1^T \right)^{-1} \Phi_1^T \left( y - \Phi_2 \alpha_2^i \right), \lambda \)  
\( \alpha_2^{i+1} = S \left( \Phi_2^T \right)^{-1} \Phi_2^T \left( y - \Phi_1 \alpha_1^i \right), \lambda \)  
\( \text{(10b)} \)

A possible choice of \( \lambda \) is the universal threshold given by \( \sigma \sqrt{2 \log(p)} \), where \( p \) is the number of columns of \( \Phi \). This choice guarantees with high probability that all selected features are significant (Donoho and Johnstone, 1994). However, the universal threshold is very conservative, which could result in important features being missed (the price of not wanting to wrongly include insignificant features). Another approach could be to select the \( \lambda \) value that gives the best fit to \( z \) (the unknown “true” signal) by minimizing Stein’s unbiased Risk Estimate [SURE] (Stein, 1981; Tibshirani, 1996), which is an unbiased estimate of the sum of squared errors [SSE]. For BP denoising, the SURE can be calculated as follows – equation 11 (Zou et al. 2007, Tibshirani and Taylor, 2012):

\[ \text{SURE} (\lambda) = \| y - \Phi \alpha (\lambda) \|_2^2 - \sigma^2 \| \alpha (\lambda) \|_0 \]  
\( \text{(11)} \)

\[ E \left[ \text{SURE} (\lambda) \right] = \| z - \Phi \alpha (\lambda) \|_2^2 \]  
\( \text{(12)} \)

**Simulated Examples**

In this section, we illustrate BP on three examples consisting of 2048 observations. The first consists of a sinusoidal signal denoted by \( f_1 \), the second a spike signal denoted by \( f_2 \), and the third signal is the combination of the sinusoidal and spike signals denoted by \( f_3 = f_1 + f_2 \). We use both the original signals as well as noisy versions with added standard Gaussian noise (standard deviation equal to 1) and combine the spike basis and the Symmlet 8 wavelet basis to form the matrix \( \Phi \). The three signals are given by the following equations and shown in figure 7.

\( f_1(x) = 2 \sin(10 \pi x), \ 0 < x \leq 1 \)  
\( f_2(x) = \begin{cases} 
7 & \text{if } x = \frac{50n}{2048} \text{ and } n = 1, 2, \ldots, 40 \\
0 & \text{otherwise}
\end{cases} \)  
\( f_3(x) = f_1(x) + f_2(x) \)  
\( \text{(13)} \)

\( \text{(14)} \)

\( \text{(15)} \)
Figure 8 shows the estimated coefficients $\alpha$ for the noise-free signals using BP (minimum $L_1$ norm, left panel) and the pseudo-inverse (minimum $L_2$ norm, right panel). The coefficients of the spike basis are shown in blue, thick line style while the coefficients of the wavelet basis are shown in red, thin line style. For the case of the sinusoidal signal, BP estimates the coefficients of the spike basis as zero (red, thin line is zero), and only coefficients from the wavelet basis are used to estimate the signal. For the pseudo-inverse method, both the coefficients of the spike basis and wavelet basis are non-zero. For the case of the spike signal, BP estimates the coefficients of the wavelet basis as zero (blue line is zero), and only coefficients from the spike basis are used to estimate the signal. For the pseudo-inverse approach, again both the coefficients of the spike basis and wavelet basis are non-zero. For the case of the combined sine and spike signal [signal $f_3$], the solutions in each case are the sum of the individual solutions. The advantage of BP is clear; only spikes where there is a spike in the signal have coefficients estimated as not zero giving a clear interpretation of what it means for the estimated coefficient to be different from zero. For the pseudo-inverse case, the coefficients of the spike basis are all estimated to be different from zero and therefore cannot be used as an indication of the presence of a spike in the signal.
Figure 8: Estimate coefficients of the decomposition of the signals $f_1, f_2, f_3$. Left: decomposition by BP denoising. Right: decomposition by pseudo-inverse. Blue, thick line: coefficients of the Dirac dictionary (spike components), red, fine line: coefficients of the continuous component.

Figure 8 shows how BP results in a sparse model (a model with few coefficients that are not zero) while the pseudo-inverse approach results in a dense model (a model where all coefficients are not zero). In a sparse model (or compressible model), the approximation error decreases rapidly as more important parameters are included in the model. This is illustrated in figure 9. With BP, the error drops rapidly as more parameters are included. With 60 parameters included, the error fraction is less than 0.001 (mostly numerical rounding error). With the pseudo-inverse approach, with 60 parameters, the error fraction is still more than 0.6; even with 2,000 parameters included, the error is still more than 0.001 (achieved by BP with 60 variables).

For the noisy signals, the solution of BP denoising depends on the penalty parameter $\lambda$ that is used – a similar approach can be formulated with the pseudo-inverse approach and the $L_2$ of $\alpha$ instead of the $L_1$ norm; this is usually called ridge regression in the statistical literature or Tikhonov regularization in the signal processing literature. Figure 10 shows the mean square error (MSE) between the estimate (recovered signal) and the true signal as a function of $\lambda$ for both BP denoising and Tikhonov regularization (ridge regression).
Figure 9: Error in the signal recovery for signal $f_i$ as a function of the number of non-zero decomposition coefficients (explanatory variables).

Figure 10: Mean Square Error (MSE) as a function of the regularization parameter $\lambda$ for Tikhonov Regularization (pseudo-inverse) and BP denoising. Black line: SURE estimate of MSE for the BP denoising procedure.

In these simulated examples, the MSE can be calculated because we know what the true signals are like. However, in practical applications, the true signal is not known – the MSE cannot be explicitly calculated – but an unbiased estimate of the MSE obtained solely from the data for BP (the SURE) is very close to the true MSE. BP achieves a much smaller optimal MSE (0.147) than the Tikhonov regularization (optimal MSE of 0.727) – note that because the added noise has a standard deviation (and variance) equal to 1, the MSE of the noisy signal is 1.0. BP achieves the lowest MSE for $\lambda = 1.7$ while the pseudo-inverse
achieves the lowest MSE for $\lambda = 0.7$. The estimated (recovered) signals for both approaches are shown in figure 11. The estimate obtained with BP is very similar to the true signal with most of the noise removed. The estimate obtained with Tikhonov regularization is much closer to the noisy version of the signal than the true signal.

An alternative to selecting the model with the lowest MSE would be to select a model that practically removes all the noise. This model can be viewed as a variable selection procedure with a low probability of Type I error (i.e. selecting a variable that is not significant). This can be done with BP by selecting $\lambda = \sqrt{2 \log(p)}$, the universal threshold. The results are shown in the right panels of figure 11. The BP solution is practically noise-free and all identified spikes are essentially true spikes. Note that for the minimum MSE model, although generally having small coefficients, some wrong spikes are identified sacrificing Type I error to reduce the MSE. For the Tikhonov regularization, the use of the universal threshold has no benefits (it produces a shrunken version of the noisy signal).

The results of BP on the separate sinusoidal and spike signals (signals $f_1$ and $f_3$) are shown in figure 12 – Tikhonov regularization results are similar to the case of spike and sine combined and are not shown.

Figure 11: Recovered signal $f_3$ from the noisy signal using the pseudo-inverse and BP denoising. Comparison of results for the optimized regularization parameter $\lambda$ versus the universal threshold.
Detecting weak spots in jointed concrete and composite pavements

In this section, we present a practical application of BP denoising to detect possible weak joints in a jointed concrete pavement overlaid with asphalt. Deflection Slope measurements were taken at 1 m intervals with the Traffic Speed Deflectometer (TSD) at the MIRA test track in the United Kingdom during the Second Strategic Highway Research Program (SHRP2) Project R06(F) (Flintsch et al., 2013; Katicha et al., 2013; Ferne et al., 2015). The TSD measures the pavement deflection slope, which gives an indication of the pavement strength (see Flintsch et al., 2013 and Austroads AG:AM/T017 standard (2016) for more details on the TSD). The tested section is 1,485 m long and five replicate measurements were performed. This test-track section was analyzed by Katicha et al. (2014 and 2016) using wavelet transform denoising and Katicha et al. (2013) using smoothing splines. The methods used in Katicha et al. (2013, 2014, and 2016) showed the potential of the TSD to detect weak joints although they are not dedicated to clearly identify these weak joints. The BP approach proposed specifically uses a spike basis dedicated to identifying weak spots that can be caused by weak joints.

In order to apply BP denoising, an estimate of the noise standard deviation is needed. This can be obtained from a single set of measurements using the difference sequence approach or by taking the difference between two sets of repeated measurements (see Katicha et al., 2015). We have used the difference sequence approach using one set of measurements because in general only one test run is performed.
Figure 13: Top: Noisy and recovered TSD signal from the MIRA test track. Optimized $\lambda$ hyper-parameter (left), universal threshold $\lambda$ (right). Bottom: Recovered discontinuous components for both $\lambda$ cases.

Figure 13 shows the results for one run with $\lambda$ selected to minimize the MSE and $\lambda = \sigma \sqrt{2 \log(p)}$. The figure also shows the detected spikes, which could potentially be the weak joints. With $\lambda$ selected to minimize the MSE, 89 spikes are identified. Note than in this case, a significant number of these spikes could potentially be wrongly identified. With $\lambda$ selected as the universal threshold [$\lambda = 3.995$], 18 spikes are identified. In this case, there is a high probability that all these identified spikes are weak spots. The results of BP with the universal threshold suggests 18 locations that should definitely be further investigated to see if these are weak joints that need treatment. The 89 locations identified with $\lambda$ selected to minimize the MSE [$\lambda = 1.8$] give a list of 71 further locations that should also be investigated although there is an increased chance of finding wrongly identified weak joints. The larger spikes should be investigated first because these are more likely to be correct identifications.

To further evaluate the performance of BP, we calculated the mean square prediction error of each of the five sets of measurements and that of each the BP estimate (with $\lambda$ selected to minimize the MSE) with respect to the average of the other four sets of measurements. The results normalized with respect to the variance of the TSD measurement noise are shown in Table 3. BP reduced the MSE by a factor of more than two (average of 2.44). Because the average of four measurements is better than a single measurement, we can conclude that the BP denoising estimate, being closer to the average than the raw measurements, represents the structural condition better than the raw measurements themselves.
Table 3: Mean Square Error (MSE) of the raw signals and the signals estimated with BP denoising

<table>
<thead>
<tr>
<th>Measurements MSE</th>
<th>BP Estimate MSE</th>
<th>Reduction Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.129</td>
<td>0.466</td>
<td>2.42</td>
</tr>
<tr>
<td>1.268</td>
<td>0.440</td>
<td>2.88</td>
</tr>
<tr>
<td>1.317</td>
<td>0.570</td>
<td>2.31</td>
</tr>
<tr>
<td>1.510</td>
<td>0.702</td>
<td>2.15</td>
</tr>
<tr>
<td>1.139</td>
<td>0.464</td>
<td>2.46</td>
</tr>
</tbody>
</table>

Figure 14 shows a histogram of the distance between consecutively detected weak spots by BP for all five replicate measurements. The jointed concrete slabs of the tested section are 12.5 m long. The histogram shows that a significant number of detected weak spots are separated by 11 to 13 m which is in accordance with the 12.5 m slab length. There is also a local peak in the histogram at a weak spot separation of 23 to 25 m, roughly corresponding to twice the slab length. Another interesting observation is the local peak at 1 m separation. This could be a result of the weak joint being detected while the TSD tire is on either of the two slabs connected by the joint. Note that, as discussed in Katicha et al. (2014), cracks had developed in a number of the slabs, which explains why the separation between weak spots is not always close to a multiple of the slab length.

![Figure 14: Histogram of separations between detected consecutive weak spots. Highlighted bars indicate distances that are close to an integer multiple of the slab length.](image)

The pavement between 780 m and 1,380 m of the tested section was rehabilitated in 2007 (three years before the measurements were collected). Figure 15 shows the location of the detected weak spots for all five runs by BP with \( \lambda \) selected as the universal threshold to minimize the probability of false detection. In total, 52 unique weak spots are detected (some are detected in many of the runs) and only one detected
weak spot is located in the rehabilitated area (it was also only detected during one of the runs). This reinforces the fact that BP is detecting the truly weak joints in the pavement.

![Figure 15: Location of detected weak spots after five runs with the TSD. Discontinuous signals recovered by BP with universal threshold \( \lambda \) hyper-parameter](image)

**Discussion**

The results of BP denoising depend on the choice of the penalty parameter \( \lambda \). Two methods of selecting \( \lambda \) were investigated. The first method selects the \( \lambda \) that aims at minimizing the MSE of the estimated signal using SURE, but it can lead to too many false positives when it comes to identifying the weak joints. The second method uses the universal threshold for \( \lambda \), which only identifies the features that are significant with high probability. This results in a very low false-positive detection rate although it can lead to missing weak joints that are just below the universal threshold. The choice of which approach to use depends on the specific application and the resources available to further investigate the joints. Another factor that affects the selection of which approach to use is the question of how large a spike in the deflection slope needs to be for the tested joint to be considered a weak joint. These issues need further investigation from an engineering point of view rather than a statistical or signal-processing point of view.

While the results of BP denoising are very promising, there are still improvements that can be made. Because of the soft-thresholding function, BP denoising gives a biased (shrunk) estimate of the coefficients. In recent years, there have been efforts to reduce or eliminate the bias, but this results in a non-convex optimization problem which is not guaranteed to converge to the global minimum. One particular attractive computational approach is the SparseNet proposed by Mazumder et al. (2011).
SparseNet uses a family of penalties that provide a continuous transition from the $L_1$ norm penalty to the $L_0$ norm penalty with the $L_0$ and $L_1$ norm penalties as limiting cases. The approach used by SparseNet is to start with the $L_1$ norm solution (i.e. BP), and then calculate a solution path by continuously moving from the $L_1$ to the $L_0$ penalty. Another possible approach is to still use BP denoising to estimate the parameters and then debias the estimate. This approach is linked to the construction of a confidence interval of the LASSO (BP denoising) (see Javanmard and Montanari, 2018).

The previous section “Detecting weak spots in jointed concrete and composite pavements” showed BP can effectively identify weak joints. The statistical criteria such as minimizing SURE or choosing the universal threshold provide a good list of candidate locations to further investigate. However, the list can be further improved by taking into account specific engineering information. For example, the histogram analysis of the spacing between identified weak spots shown in figure 14 can be combined with the list to better prioritize the locations to be investigated by considering that the joints in jointed concrete pavements are regularly spaced. Many of the newer versions of the TSD are being fitted with imaging systems to detect cracking. The cracking information can further prioritize the locations. Alternatively, images can be used to determine the locations of the joints in jointed concrete pavements. With this information, BP can be used with spikes only located at the location of the joints, thus reducing the incidence of false positives.

In obtaining the solution of BP denoising, we have assumed the error to be normally distributed and the noise standard deviation to be known (or accurately estimated using the difference sequence method). These are reasonable assumptions because of the relatively high resolution of the TSD measurements (1 m), and the large number of averaged measurements in a 1 m interval (about 50) which by the central limit theorem results in a normal error distribution. It is, however, possible to not make any assumption on the distribution and level of noise with the square-root Lasso proposed by Belloni et al. (2011, 2014) although in this case, there we cannot use the SURE approach to determine an optimal $\lambda$. Finally, the Lasso has a Bayesian interpretation which can be used to simultaneously estimate the model parameters (including the penalty $\lambda$) along with the noise standard deviation and confidence (credible) intervals (Park and Casella, 2009) although this is more computationally demanding.

**Conclusion**

The load transfer at the joints in JPCP and composite pavements is a very important parameter that affects the structural life of the pavement. Until recently, the FWD has been the only available device that can
evaluate the load transfer efficiency at the joints. However, because the FWD is a stationary device, its use has been limited to few project-level cases where it is deemed necessary. With more than 225,000 km [140,000 miles] of concrete and composite pavements in the United States (FHWA, 2018), some of which are jointed and with most joint spacings being on the order of 5 to 6 m, there are potentially on the order of 10 to 20 million joints (a conservative estimate) whose condition is mostly not known by highway agencies.

The results presented in this paper show that BP denoising is an effective approach to identify joints with potential load transfer deficiencies in JPCP and composite pavements using measurements collected with the TSD. This has been demonstrated using a simulated example and actual TSD measurements collected on a composite pavement section.

The methodology presented in this paper may aid a pavement manager to perform network-wide assessments of the pavement joints’ structural performance in a timely and cost-effective manner by recurring to TSD measurements, and direct further investigatory testing (such as LTE testing with an FWD device) and corrective actions at weak locations optimally. For example, a first network-wide survey can be performed with the TSD and the resulting measurements should be filtered with BP denoising to extract the response (and thus the location) of the weak spots. The detected weak spots can be ranked on their severity based on the height of their pulse response. Then, based on the managing agency’s resource constraints, further testing on the weak locations can be optimally scheduled, by prioritizing those most compromised locations.

The conclusions that can be drawn from this paper can be pooled into two sub-categories, namely purely theoretical conclusions about the BP denoising methodology itself, and practical conclusions concerning how BP denoising contributes to network-wide pavement evaluation with a TSD. The analysis of the simulated deflection profiles with BP denoising showed that this method can actually work in practice, as BP denoising results in an estimate of the true deflection signal composed of a small number of components and a boasting small MSE. The small number of components is an important aspect of model interpretation. Also, we proved through these simulated examples that Stein’s unbiased risk estimate (SURE) closely matches the true MSE. Therefore, it can be used to determine the optimal value of the penalty parameter $\lambda$ when analyzing actual TSD signals from single runs (for which no direct calculation of MSE can be done). Meanwhile, BP denoising with penalty parameter $\lambda$ set to the universal threshold results in an almost noise-free estimate of the signal with retained features that are with high probability true features.
The application of BP denoising on actual pavement data confirmed that this methodology is also practical for filtering noise out of TSD measurements and estimating the pavement response. When $\lambda$ is selected to minimize SURE, BP denoising results in an estimate of the true pavement response that has a lower mean square prediction error (MSPE) than the raw measurements. This reinforces the results obtained with the simulated signals that BP denoising provides a better estimate of the true pavement response. Moreover, BP denoising can identify the weak spots in the pavement that likely reflect joints with deficient load transfer: From the MIRA test track results, the weak spots identified by BP denoising are mostly within the section of the pavement that has a jointed concrete layer where there is a potential for the presence of weak joints. With the universal threshold, 52 weak spots are detected and only one of those is in the section of the track that has no jointed concrete layer. Furthermore, the histogram of the distances between consecutively identified weak spots (figure 14) shows prominent peaks around distances of 12 m and 24 m. This is consistent with the 12.5 m distance between consecutive joints and suggests that most of the identified weak spots are at the location of weak joints.
References


CHAPTER 4 — REWEIGHTED L$_1$ MINIMIZATION FOR NETWORK-WIDE PAVEMENT WEAK SPOT DETECTION FROM TRAFFIC SPEED DEFLECTOMETER MEASUREMENTS

Abstract

Traffic-speed deflection devices like the Traffic-Speed Deflectometer (TSD) are bridging the gap between project-level structural information availability and structural data needs for network-level management. TSDDs can collect dense network-level deflection data in a cost-effective and timely manner. However, automated noise removal and feature extraction methods must be devised to analyze these datasets. Basis Pursuit (BP) is one such data processing technique that seeks to decompose a given signal, such as TSD measurements, into a sparse representation over a given basis (or set of bases). Particularly, BP can decompose TSD measurements into a continuous component representative of any smoothly transitioning structural health property plus a sequence of pulse responses from localized weak spots. BP thus enables the pavement engineer to recognize weak spots within a pavement network, even before surface distresses appear. Yet, BP implies a bias-variance trade-off: BP may either return a best-fit recovered signal with low bias and possible false positives – spike components that do not necessarily correspond to real weak spots – or a signal made up of certain features but heavily biased towards zero. This paper presents an application of Reweighed L$_1$ Minimization (RWL1) for TSD data. RWL1 is an enhancement to L$_1$ regularization (the family of optimization problems BP belongs to) that both corrects the bias and lowers the false-positive rate from a BP-recovered signal. This paper provides a demonstration of this technique on both simulated data and actual TSD measurements collected in experimental set-ups and during a survey of a jointed pavement.

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Introduction: Problem Statement and Paper’s Objective

Over the last 10 years, network-level structural evaluation of flexible pavements has seen significant advances, which have been made possible by the successful development and continued improvement of traffic speed deflection devices (TSDDs) such as the traffic speed deflectometer (TSD) (Flintsch et al. 2013; Rada et al. 2016). This has led to a growing interest in pavement evaluation using TSDDs, as evidenced by the current Transportation Pooled Fund study TPF-5(385) “Pavement Structural Evaluation with Traffic Speed Deflection Devices (TSDDs)”, which includes 25 participating state highway agencies.

In contrast to flexible pavements, network-level structural evaluation of rigid and composite pavements has mostly been limited. This is because (1) the majority of surfaced roads (84% of all roads except minor collector and local rural roads) in the United States are asphalt flexible roads (FHWA, 2019) and (2) structural problems in rigid and composite pavements most often occur at very localized locations such as the joints in jointed concrete pavements. The localization is often of the order of a few meters, which is a smaller interval than the resolution at which measurements from the TSD are reported (generally 10 m), and thus these problematic spots cannot be detected. The reason why TSD measurements are not reported at a finer interval, such as 1-m resolution, is to reduce measurement noise by averaging over a longer window such as 10 m (Flintsch et al., 2013). The difficulty in evaluating rigid or composite pavements is further exacerbated by the fact that rigid and composite pavements deflect less than flexible pavements so that the signal-to-noise ratio on rigid and composite pavements is lower than the signal-to-noise ratio on flexible pavements.

Despite the data collection and analysis challenges on rigid and composite pavements, there remains a strong incentive to evaluate them. One motivating factor is that 53% of interstate roads are either rigid or composite pavements (FHWA, 2019). Interstate roads, while representing a small percentage of all roads, carry the most traffic (especially truck traffic) and are paramount to national security. Flintsch et al. (2013) and Rada et al. (2016) recognized the potential of the TSD to evaluate rigid and composite pavements, especially in terms of evaluating the joints’ load bearing capacity. Katicha et al. (2013; 2014; 2016) used signal processing methods and wavelet denoising methods to show how the location of localized weak spots such as weak joints can be determined from the noisy TSD measurements. Scavone et al. (2021) used Basis Pursuit (BP) denoising (Chen et al., 2001) with one of the used bases, the Dirac basis, specifically chosen to detect weak joints. BP denoising selects important features in a signal using regularization, with the results depending on the choice of the regularization parameter. Scavone et al.
(2021) selected the optimal regularization parameter by minimizing Stein’s Unbiased Risk Estimate (SURE), an unbiased estimate of the mean square error. Although the approach was successful in detecting weak spots, Scavone et al (2021) identified the following two drawbacks:

- BP denoising introduces bias in the estimated coefficients. Reducing or eliminating such bias is essential in obtaining a good estimate with a small mean square error (MSE) and is a result of the familiar bias-variance trade-off to minimizing the MSE.

- Minimizing SURE (which is a surrogate of the MSE) can result in too many false positives. To reduce the number of false positives, the regularization parameter should be increased, though this will lead to more bias and also increase the MSE.

This paper addresses the drawbacks of BP denoising with the Reweighted L₁ Minimization (RWL₁) approach proposed by Candès et al. (2008). RWL₁ iteratively performs weighted BP denoising with the weights inversely proportional to the coefficients of each element of the bases calculated from the previous iteration (for the initial iteration, BP with no weighting is performed). This tends to reduce the number of false positives as well as the bias in the calculated coefficients. The approach is illustrated with a simulation example and also applied to TSD data collected on 50 miles of pavements in the Washington DC National Mall Area (NAMA) and the George Washington Memorial Parkway (GWMP). The results obtained are validated by reviewing the images collected during the TSD survey at the locations of identified weak spots.

**Paper Organization**

This paper is organized into five sections: *Introduction, Paper Organization, Background, Application,* and *Conclusion*. The *Introduction* section presents the motivation and objective of the paper. The *Paper Organization* section presents how the paper is laid out. The *Background* section presents BP denoising, RWL₁, and previous research on joint detection from TSD measurements. The *Application* section presents the results of applying RWL₁ to a simulated example and TSD data collected on the NAMA and the GWMP. For the simulated example, the results of RWL₁ are compared to BP denoising while for the TSD data, the results are validated using images collected during the TSD survey (which may confirm the presence of surface distresses due to structural deficiencies). Conclusions are presented in the *Conclusion* section.
Background

Before the theoretical aspects of BP and RWL1 are presented, the motivation for using these signal processing methods to analyze TSD data to identify weak joints is articulated. The presence of a weak joint in a jointed concrete pavement (or composite pavement if the concrete is overlaid with asphalt) causes a spike in the measured structural response (deflection or deflection slope etc.). Therefore, spikes are good indicators of weak joints. However, spikes are ineffective at representing relatively smoother variations in the pavement structural condition. These smoother variations are better represented by other features such as wavelets. Therefore, a good strategy is to use spikes to represent the jumps in the measurements caused by weak joints and a wavelet basis to represent the rest (smoother) of the pavement structural variation. Combining the two sets of possible features (the spikes and the wavelet basis) results in a nonunique representation of the measurements; that is there are multiple combinations of spikes and wavelets that can give the same representation of the measurements. BP and RWL1 are signal processing techniques that seek a representation with a small number of features (in some mathematical sense defined in the next section). The advantage of seeking a representation with a small number of features is that the most effective features end up being selected. For weak joints, this corresponds to the spikes which end up indicating the location of the weak joints.

Basis Pursuit

Basis Pursuit (Chen et al., 2001) is an optimization procedure for decomposing a signal into a sum of dictionary elements that is optimal in that the coefficients of the dictionary elements have the lowest $L_1$ norm among all possible decompositions. The dictionary elements are generally the elements obtained from combining multiple (orthogonal) bases such as wavelet bases, the Fourier basis, or the Dirac basis, as an example. Because combining multiple bases results in a dictionary with more elements than the number of signal observations, the signal representation as a superposition of dictionary elements is not unique (we have an under-determined linear system). BP finds the signal with the lowest coefficients $L_1$ norm resulting in a sparse representation (a representation that includes few dictionary elements). The formulation of BP is given in equation 16. When the acquired signal is noisy, BP denoising, with the formulation shown in equation 17, can be used to obtain a denoised estimate of the underlying signal (the signal that generated the noisy measurements). In this case, BP denoising is also known as the Least Absolute Shrinkage and Selection Operator (LASSO) (Tibshirani, 1996; Chen et al., 2001; Hastie et al.,
It is noted that the solution of BP denoising (equation 17) converges to the solution of BP (equation 16) in the limit as $\lambda$ approaches 0:

$$ \min \| \alpha \|_1 \text{subject to } y = \Phi \alpha $$ (16)

$$ \min_{\alpha} \frac{1}{2} \| y - \Phi \alpha \|_2^2 + \lambda \| \alpha \|_1 $$ (17)

In equation 17 (BP denoising), $y$ denotes the measured noisy signal consisting of $n$ measurements (a vector of $\mathbb{R}^n$), $\Phi$ is the $n$ by $p$ dictionary matrix, with $n$ the number of observations and $p$ the number of dictionary elements, which can be greater than $n$. $\lambda$ is a penalty term that controls the trade-off between fit to the data and the $L_1$ norm of the fitted coefficients. The solution of BP denoising can be obtained with the cyclic coordinate descent algorithm of Friedman et al. (2010). The algorithm is based on the BP denoising solution in the case of a single variable given by the soft-thresholding function $S(\alpha, \lambda)$ (equation 18, Figure 16).

$$ \alpha_{BP} = S(\alpha_{LS}, \lambda) = \text{sign}(\alpha_{LS}) \times \max(\| \alpha_{LS} \|_2 - \lambda, 0) = \begin{cases} 
\alpha_{LS} - \lambda & \text{if } \alpha_{LS} \geq \lambda \\
\alpha_{LS} + \lambda & \text{if } \alpha_{LS} \leq -\lambda \\
0 & \text{if } -\lambda < \alpha_{LS} < \lambda 
\end{cases} $$ (18)

where $\alpha_{LS}$ is the ordinary least squares solution.

Cyclic coordinate descent is an iterative algorithm that updates each individual component of the vector of coefficients $\alpha$ at a time while keeping the remaining coefficients constant. Suppose that, at a given iteration of the algorithm, the estimate of $\alpha$ is $\alpha'$. The update of the $j^{th}$ component of $\alpha'$ (equation 19) is given by the soft-thresholding function (equation 18):

$$ \alpha_j' = S(\alpha_j^{LSR}, \lambda) $$ (19)

where:

$$ \alpha_j^{LSR} = (\Phi_j^T \Phi_j)^{-1} \Phi_j^T (y - \Phi_{-j} \alpha_{-j}) $$ (20)
In equations 19 and 20, \( \Phi_j \) is the \( j \)th column of the matrix \( \Phi \), \( \Phi_{-j} \) is the matrix \( \Phi \) without the \( j \)th column, and \( \alpha_{-j} \) is the coefficient vector with the \( j \)th coefficient removed. The update is obtained by soft thresholding the least-squares fit of the \( j \)th column of the matrix \( \Phi \) on the residual of the partial model that excludes the \( j \)th column vector. Orthonormal components of \( \Phi \) can be updated simultaneously (because these components do not affect each other). Therefore, if \( \Phi = (\Phi_1, \Phi_2) \) where \( \Phi_1 \) is the Dirac components and \( \Phi_2 \) the wavelet components of \( \Phi \), then the update can be implemented as follows – “block coordinate descent” (equations 21 and 22):

\[
\alpha_{1i}^{+1} = S\left( (\Phi_1^T \Phi_1)^{-1} \Phi_1^T (y - \Phi_2 \alpha_2^j), \lambda \right) = S\left( (y - \Phi_2 \alpha_2^j), \lambda \right)
\]

\[
\alpha_{2i}^{+1} = S\left( (\Phi_2^T \Phi_2)^{-1} \Phi_2^T (y - \Phi_1 \alpha_1^j), \lambda \right) = S\left( \Phi_2^T (y - \alpha_1^j), \lambda \right)
\]

Scavone et al. (2021) used BP denoising with a dictionary formed by combining a wavelet basis with the Dirac basis to decompose the deflection slope signal measured on a composite pavement with a TSD. The wavelet basis was selected to represent the smooth part of the signal while the Dirac basis was used to represent the sudden jumps in the signal that are due to localized weak spots in the pavement, such as weak joints in the concrete layer under the asphalt layer. Scavone et al. (2021) investigated two methods to select an appropriate value for the regularization parameter \( \lambda \). The first method selects the value of \( \lambda \) that minimizes Stein’s unbiased risk estimate (SURE) given in equation 23.

\[
SURE(\lambda) = \| y - \Phi \alpha(\lambda) \|_2^2 - \sigma^2 + 2 \sigma^2 \| \alpha(\lambda) \|_0
\]

This method results in an estimated signal that has a small (“the smallest”) MSE with respect to the unknown noise-free signal. However, aiming to minimize MSE generally results in too many false positives (i.e., too many dictionary elements multiplied by non-zero coefficients that may actually
describe the noise in the noisy signal \( y \). The second method fixes the value of \( \lambda \) to the universal threshold value of \( \sigma \sqrt{2 \log(n)} \) where \( \sigma \) is the measurement noise standard deviation (Donoho and Johnstone, 1994). This results in a very conservative threshold that reduces the chance of false positives (for a 10 km road with measurements every 1 m, \( n = 10,000 \), and the universal threshold is \( 4.3\sigma \)). The drawbacks of the universal threshold are that it results in a large MSE, a large bias for the estimated dictionary elements coefficients, and too many false negatives (i.e., missing true positives).

### Reweighted L\(_1\) Minimization

The drawbacks of BP denoising are that it results in a large bias in the estimated coefficients and/or too many false positives. Reweighted L\(_1\) Minimization (Candès et al., 2008) was proposed to mitigate these performance issues within BP denoising. RWL1 is an iterative algorithm with each iteration performing weighted BP denoising. At the first iteration, all weights are set to 1, resulting in BP denoising. At the next iterations, the weights of each component of the vector of parameters \( \alpha \) are set to the inverse of the absolute value of the components of \( \alpha \), as calculated in the previous iteration. The algorithm is implemented as follows:

- Set iteration count \( k \) to zero and weights \( \omega_i(0) = 1, i = 1, \ldots, p \).

1. Solve weighted BP denoising:

\[
\min_\alpha \frac{1}{2} \| y - \Phi \alpha \|_2^2 + W \| \alpha \|_1, W = [\omega_1^{(k)}, \ldots, \omega_p^{(k)}]
\]  

2. Update the weights for each \( i = 1, \ldots, p \)

\[
\omega_i^{(k+1)} = \frac{1}{|\alpha_i^{(k)}| + \varepsilon}
\]  

3. Repeat steps 2 and 3 for a previously specified number of iterations \( k_{\text{max}} \) or until some convergence criterion is met.

The (small) parameter \( \varepsilon \) is included to avoid division by zero. It was found that a value of 1/1000 times the noise standard deviation or smaller is adequate for most denoising applications, an observation consistent with Candès et al. (2008).

The effect of weighting on each individual component is easy to observe: if at the previous iteration, \( \alpha_i^{(k)} \) is small, then \( \omega_i^{(k+1)} \) will be large and the penalty \( \omega_i^{(k+1)} \lambda \) will increase for that specific component and \( \alpha_i^{(k+1)} \) will be reduced at the next iteration (possibly being set to zero). On the other hand, if \( \alpha_i^{(k)} \) is large,
then $\omega^{(k+1)}$ will be small and the penalty $\omega^{(k+1)} \lambda$ will decrease for that specific component, which reduces the bias. This effect is illustrated in figure 17 for the orthogonal case and a value of $\lambda$ equal to 2. The initial solution (BP) shrinks the coefficients that are larger than 2 (by an amount equal to 2) and sets to zero those coefficients smaller than or equal to 2. RWL1 further penalizes the small coefficients estimated by BP (the ones that barely survived thresholding), setting them to zero, and reduces the penalty on the larger estimated coefficients, which significantly reduces the bias.

Figure 17: BP denoising and RWL1 in the orthogonal case for $\lambda = 2$. RWL1 effectively increases the threshold which reduces the number of false positives and decreases the bias.

Figure 18: Soft-thresholding and hard-thresholding penalty levels with equal shrinkage capabilities.
The plot in figure 18 shows that most of the effects of RWL1 occur after only the first iteration (threshold increased to 2.73 and bias reduced on average to 0.66) and after three iterations, the results are practically the same as after 50 iterations (threshold converges to 3, and the mean bias to 0.38), thus implying that convergence of the algorithm is very fast and only a few iterations need to be implemented.

To select the penalty parameter $\lambda$, two strategies were evaluated. The first strategy uses the value of $\lambda$ that minimizes SURE of BP denoising without weighting (Scavone et al. 2021). The other strategy is to estimate SURE as shown in equation 26. Notice in that equation, in the $L_2$ norm term (square error term), the fit of the RWL1 solution is used, while in the $L_0$ norm (count of non-zero elements), the number of coefficients estimated with BP denoising is used.

$$SURE(\lambda) = \|y - \Phi \alpha_{RWL1}(\lambda)\|_2^2 - \sigma^2 + 2\sigma^2 \|\alpha_{BP}(\lambda)\|_0$$ (26)

The reason for this is because the RWL1 solution is close to a hard thresholding with a higher threshold than BP denoising (figure 18). In that case, Mazumder et al. (2011) suggested that an approximate conservative estimate of the effective number of parameters (the degrees of freedom) for hard thresholding can be obtained from the number of parameters (degrees of freedom) of BP denoising (LASSO) under a lower threshold. This estimate is conservative because it is based on a null model (i.e., a model where all coefficients are zero). The BP denoising (soft thresholding) and hard thresholding that have the same degrees of freedom proposed by Mazumder et al. (2011) are shown in figure 18. As an example, the degrees of freedom of BP denoising for a threshold $\lambda = 2$ are the same as the degrees of freedom for hard thresholding with a threshold $\lambda = 2.83$. Figure 18 also shows the threshold for RWL1 as a function of $\lambda$. For a $\lambda = 2$, the threshold is 2.98, which is close to the equivalent hard threshold of 2.83 suggesting the approximation used in equation 26 is good and does not require further calculation (the result of BP denoising is obtained in the first iteration of RWL1).

### Application

**Simulated Dataset**

In this example, we illustrate how RWL1 can improve the reconstruction of a noisy signal compared to BP denoising, by reducing the number of false positives and the bias in the estimated coefficients. The example consists of a signal, $z$, that is zero everywhere except at regularly spaced intervals, $\Delta$, where it is equal to $k$. The noise-free signal is given in equation 27 and cases with $k = 3, 4, 5, \Delta = 10$ and 50...
were investigated. Measurements are simulated by adding normally distributed random noise with variance $\sigma^2 = 1$. The mathematical formula for the noisy measurements is given in equation 28.

$$z(x) = \begin{cases} k & \text{if } x = \frac{\Delta n}{2000} \text{ for } n=1,2,\ldots,40 \\ 0 & \text{otherwise} \end{cases} \quad (27)$$

$$z_n(x) = z(x) + \epsilon \sim N(0,\sigma^2) \quad (28)$$

Figure 19 shows the signal and simulated measurements for $k = 4$ and $\Delta = 50$. The reconstructed signals with BP denoising and RWL1 for a penalty $\lambda$ that minimizes SURE calculated with BP denoising (equation 23) are also shown. Because the simulated signal is comprised of only Dirac (spikes) components, the reconstruction is performed using the Dirac basis (without using a wavelet basis) to better illustrate the difference between BP denoising and RWL1. Figure 19 displays the advantages of RWL1 over BP denoising—the bias of large coefficients is reduced and the small coefficients that are likely to be false positives estimated with BP denoising are set to zero by RWL1. Detailed performance results comparisons between BP denoising and RWL1 with the two methods of selecting the penalty $\lambda$ (equations 23 and 26 respectively) are presented in table Table 4 for the case of $\Delta = 50$ (resulting in 40 spikes) and table Table 5 for the case of $\Delta = 10$ (resulting in 200 spikes).

BP denoising without weighting identifies the highest number of true positives, but this comes at the cost of a much higher number of false positives so that the false discovery rate (proportion of false positives among all identified features) is very high, 0.68 or higher. RWL1 significantly reduces the number of false positives at the relatively small cost of missing some of the true positives in the process. The classification error metric presented in Tables 4 and 5 considers both false positives and missed true positives, and RWL1 with $\lambda$ obtained by minimizing equation 26 is the best approach for that; it also is the best approach for minimizing the false positives and hence the false discovery rate.
In practical applications, it is worth minimizing the false discovery rate so that limited resources spent on pavement surveying are not wasted on false events. This is the situation when performing a pavement-network-wide structural evaluation with the TSD. The TSD can quickly highlight the spots that should be further investigated in more detail with the falling weight deflectometer (FWD). Because the FWD is a stationary testing device, the number of tests that can be performed in a given period is very small compared to what the TSD can evaluate. Therefore, it is essential that the false discovery rate of features identified with the TSD be small, so that the time the FWD is on the road is optimally spent performing investigatory tests.
Table 4: Spike identification results summary for the simulated signals with $\Delta = 50$. For each group of results (single value of $k$), comparison of BP denoising, RWL1 with $\lambda$ chosen as per equation 23, and RWL1 with $\lambda$ chosen as per equation 26. The best result for each metric is highlighted in bold.

<table>
<thead>
<tr>
<th></th>
<th>$k = 3$</th>
<th></th>
<th></th>
<th>$k = 4$</th>
<th></th>
<th></th>
<th>$k = 5$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BP</td>
<td>RWL1+ eq23</td>
<td>RWL1+eq26</td>
<td>BP</td>
<td>RWL1+ eq23</td>
<td>RWL1+eq26</td>
<td>BP</td>
<td>RWL1+ eq23</td>
</tr>
<tr>
<td>Total Identified</td>
<td>176.7</td>
<td>32.8</td>
<td>17.6</td>
<td>212.8</td>
<td>49.9</td>
<td>32.4</td>
<td>216.6</td>
<td>53.3</td>
</tr>
<tr>
<td>True Positive</td>
<td>35.4</td>
<td>22.9</td>
<td>15.7</td>
<td>39.6</td>
<td>36.3</td>
<td>30.5</td>
<td>40.0</td>
<td>39.6</td>
</tr>
<tr>
<td>False Positive</td>
<td>141.3</td>
<td>9.9</td>
<td>1.9</td>
<td>173.2</td>
<td>13.7</td>
<td>1.9</td>
<td>176.6</td>
<td>13.8</td>
</tr>
<tr>
<td>Classification Error</td>
<td>145.8</td>
<td>26.9</td>
<td>26.2</td>
<td>173.6</td>
<td>17.4</td>
<td>11.4</td>
<td>176.6</td>
<td>14.2</td>
</tr>
<tr>
<td>FDR</td>
<td>0.80</td>
<td>0.30</td>
<td>0.11</td>
<td>0.81</td>
<td>0.27</td>
<td>0.06</td>
<td>0.82</td>
<td>0.26</td>
</tr>
<tr>
<td>MSE</td>
<td>0.097</td>
<td>0.110</td>
<td>0.121</td>
<td>0.103</td>
<td>0.082</td>
<td>0.096</td>
<td>0.102</td>
<td>0.062</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>1.7</td>
<td>1.7</td>
<td>2.4</td>
<td>1.7</td>
<td>1.7</td>
<td>2.3</td>
<td>1.7</td>
<td>1.7</td>
</tr>
<tr>
<td>Effective threshold</td>
<td>1.7</td>
<td>2.67</td>
<td>3.39</td>
<td>1.7</td>
<td>2.67</td>
<td>3.29</td>
<td>1.7</td>
<td>2.67</td>
</tr>
</tbody>
</table>

Table 5: Spike identification results summary for the simulated signals with $\Delta = 10$. For each group of results (single value of $k$), comparison of BP denoising, RWL1 with $\lambda$ chosen as per equation 23, and RWL1 with $\lambda$ chosen as per equation 26. The best result for each metric is highlighted in bold.

<table>
<thead>
<tr>
<th></th>
<th>$k = 3$</th>
<th></th>
<th></th>
<th>$k = 4$</th>
<th></th>
<th></th>
<th>$k = 5$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BP</td>
<td>RWL1+ eq23</td>
<td>RWL1+eq26</td>
<td>BP</td>
<td>RWL1+ eq23</td>
<td>RWL1+eq26</td>
<td>BP</td>
<td>RWL1+ eq23</td>
</tr>
<tr>
<td>Total Identified</td>
<td>604.4</td>
<td>214.5</td>
<td>149.1</td>
<td>614.3</td>
<td>249.0</td>
<td>194.0</td>
<td>687.6</td>
<td>272.5</td>
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<tr>
<td>True Positive</td>
<td>192.8</td>
<td>159.3</td>
<td>131.5</td>
<td>199.4</td>
<td>193.1</td>
<td>180.8</td>
<td>200.0</td>
<td>199.7</td>
</tr>
<tr>
<td>False Positive</td>
<td>411.6</td>
<td>55.2</td>
<td>17.5</td>
<td>414.9</td>
<td>55.9</td>
<td>13.3</td>
<td>487.6</td>
<td>72.7</td>
</tr>
<tr>
<td>Classification Error</td>
<td>418.7</td>
<td>95.9</td>
<td>86.0</td>
<td>415.5</td>
<td>62.8</td>
<td>32.5</td>
<td>487.6</td>
<td>73.0</td>
</tr>
<tr>
<td>FDR</td>
<td>0.68</td>
<td>0.26</td>
<td>0.12</td>
<td>0.68</td>
<td>0.22</td>
<td>0.07</td>
<td>0.71</td>
<td>0.27</td>
</tr>
<tr>
<td>MSE</td>
<td>0.322</td>
<td>0.347</td>
<td>0.399</td>
<td>0.333</td>
<td>0.262</td>
<td>0.287</td>
<td>0.328</td>
<td>0.236</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>1.2</td>
<td>1.2</td>
<td>1.6</td>
<td>1.2</td>
<td>1.2</td>
<td>1.7</td>
<td>1.1</td>
<td>1.1</td>
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<tr>
<td>Effective threshold</td>
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<td>2.14</td>
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<td>1.2</td>
<td>2.14</td>
<td>2.67</td>
<td>1.1</td>
<td>2.03</td>
</tr>
</tbody>
</table>

Note: In both tables: FDR – False Discovery Rate. MSE = Mean Squared Error.
Reweighted $L_1$ Minimization for TSD Data Interpretation

In this section, we use the reweighted $L_1$ minimization to identify weak spots from TSD measurements on in-service pavements. The TSD data was collected in April 2019, as part of the Transportation Pooled Fund (TPF) Project 5(385). The TSD survey spanned roughly 80 km (50 miles) of pavements within the National Mall Area (NAMA) and the George Washington Memorial Parkway (GWMP) in the Washington, DC, area (figure 20). According to data provided by the agencies under whose jurisdiction these segments fall, the surveyed stretch of the GWMP (roughly 50 lane-km [32 lane miles] in total) is mostly a flexible pavement structure, whereas the bulk of the surveyed roads within the NAMA (22.6 lane-km out of 28.5 lane-km [14.1 out of 17.8 lane miles]) are concrete pavements overlaid with asphalt (no information was found for the remainder 5.9 lane-km [3.7 lane-miles]). Deflection slope measurements at 1-meter resolution, integrated deflection basin measurements, and surface condition imagery were furnished by the TSD service provider, and all data was geo-referenced. The collected deflection measurements were analyzed by RWL1 with a dictionary consisting of the Dirac basis and a wavelet basis (the same dictionary that was used in Scavone et al. 2021). The Dirac basis was purposely chosen to detect 1-m weak spots such as joints with structural deficiencies in the jointed concrete segments, while the wavelet basis was chosen for the general ability of wavelets to efficiently process noisy signals and its computational efficiency (Donoho and Johnstone, 1994).

Overall, 2535 weak spots (Dirac basis components) were detected by BP with $\lambda$ selected to minimize SURE. RWL1 with the same $\lambda$ identified 672 weak spots, which were reduced to 495 weak spots with $\lambda$ determined by equation 26. Only 85 of the 495 weak spots (17%) are located within the GWMP, while 349 (70%) are located within the overlaid concrete segments of the NAMA, and the remaining 61 (13%) are distributed in the segments of the NAMA for which pavement composition data is not available. These results show that most of the recovered pulses are likely the response of weak joints within the jointed segments. This is further reinforced by the fact that the total length tested within the GWMP is 31.6 miles while the total length tested in the NAMA overlaid with concrete is 16.4 miles making the average number per mile of identified weak spots on the GWMP and NAMA be 2.69 and 21.28, respectively. For comparison, BP with $\lambda$ set to the universal threshold resulted in 266 identified weak spots over both the GWMP and the NAMA.

Figure 21 shows an example of the estimated deflection $D_0$ by BP and RWL1 for a single segment (one lane of a segment of Independence Avenue SW). The pavement structure at this section is an overlaid...
concrete structure, where several evenly spaced transverse cracks can be observed on the surface suggesting the potential presence of weak joints (reflective transverse cracking) (figure 22).

Figure 20: TSD survey on the NAMA and the GWMP. Top: Segment of the GWMP surveyed, Bottom: NAMA roads surveyed. Background cartography; OpenStreetMap; pavement type data courtesy of the FHWA [GWMP] and Washington DC Department of Transportation [National Mall].

Not every transverse crack is caused by the presence of a weak joint and the recovered Dirac components (weak spots) provide additional information that may tell which joints are weak or which mid slab cracks are mainly due to loss of support (Garber & Hoel, 2015; Pierce et al., 2017). Figure 23 shows how the structurally deficient joints may be detected (and thus investigated and/or repaired accordingly): the recovered Dirac components were imported to GIS software for interpretation on a georeferenced frame,
where they could be paired to mapped surface distresses (recorded by the TSD). Matching these two sources, the joints that most likely require further investigation and/or structural repair can be identified.

![TSD signal for road Independence Ave WB - Center Lane 14th St from WB to 4th St, section 0202W001](image1)

**Figure 21:** TSD deflection data for Independence Ave. WB between 14th and 4th streets. Top: received $D_0$ signal and denoised signal recovered after RWL1; Center: comparison of the recovered Dirac component after the first iteration (BP) and the last iteration of RWL1; Bottom: Recovered Dirac component with BP under the Universal Threshold.

![Revealed Distress](image2)

**Figure 22:** Reflective cracking at Independence Ave. between 14th and 4th streets. Photographed 2020-10-31. The highlighted crack produced a pulse response.
While most weak spots are identified in the composite pavement sections, a significant number are identified in the flexible pavement sections. In this case, various reasons could result in the presence of a structurally weak spot on the pavement. As an example, three identified weak spots locations on flexible pavement sections with various degrees of surface distress are presented for which the interpretation of these data may lead to different pavement preservation approaches.

**Location 1: Patch Edge**

The first example corresponds to a segment in which the original pavement received a patch repair but the transition between the original structure and the repair is identified as structurally weak – a pulse response was observed by the TSD and so was recovered during the analysis with RWL1 (shown in figure 23, to the left of the map). Figure 24 presents the surface feature seen at the time of the TSD survey and a more recent depiction of its condition as of late 2020.
Location 2: Structurally Weak Section With no Significant Observed Distress

The second example corresponds to a four-lane road pavement out of the NAMA’s heavy-traffic areas, the TSD surveyed the outer lane for one direction of travel only. A weak spot was identified at a location where no significant distresses can be observed except that fatigue cracking can be seen on the adjacent lane (inner lane for the same -westbound- direction). This suggests that distresses could potentially develop in the near future (figure 25).

Location 3: Structurally Weak Spot With Fatigue Cracking Evidence

The third example corresponds to a pavement section whose surface course shows severe signs of damage by fatigue. A localized weak spot was detected in this section suggesting that the structural problem could still potentially be confined to a small area (figure 26). This indicates that structural repairs to a confined area are needed to prevent further structural deterioration before addressing the distresses observed over the larger area.
Conclusion and Prospects for Future Research

RWL1 optimization provides a fundamental building block to the TSD data analysis framework, as it enhances the performance of BP as a feature discovery scheme to extract meaningful information from the raw dense TSD dataset, particularly the location of spots with structural deficiencies within the network, even in cases where surface pathologies are non-existent or are existent but misleading. As the referenced work (Candès et al., 2008), and simulated and real-data examples presented herein show, RWL1 overcomes the main limitation of BP that has thus far hindered the interpretation of the components recovered from the TSD signals: the bias/variance trade-off when extracting the signal components.
Figure 26: Structurally deficient spot within a segment with spread surface deterioration. Top left: Recovered pulse on map [highlighted]; top right: TSD imagery on a map showing alligator cracking spread [courtesy of ARRB Systems Inc.]; bottom: pavement surface, widespread alligator cracking over the full width of the pavement.

The gate is now open for interpretation of the recovered components from an engineering point of view. Either the pulse component itself or the complete recovered signal could be utilized to back-calculate strength parameters of the concrete layer and the joint load transfer efficiency [LTE] index using a mechanistic linear-elastic framework (Van Cauwelert, 2004) – a scheme that is currently being studied elsewhere (Deep et al., 2020). Meanwhile, RWL1-based weak spot detection alone may aid pavement engineers in detecting structurally deficient locations within the road network that may require project-level interventions before a segment-wide rehabilitation treatment is applied as part of a rational pavement network management practice.
The analysis framework presented in this paper and its predecessor (Scavone et al., 2021) is limited to cases that could be reduced to component extraction on orthonormal bases such as Dirac and wavelet bases. An interesting pavement management problem is the detection of homogeneous sections (that is, segments of the network with similar structural capabilities) within a network, which conceptually could be accomplished by performing BP on the TSD measurements using a dictionary containing a basis of step (or Heaviside) signals (Chen et al., 2001). However, the orthonormality condition required for BP optimization using batch gradient descent cannot be met for the problem of decomposition into a step signal dictionary. Therefore, BP should be performed as a total variation [TV] problem (Rudin et al., 1992; Chen et al., 2001; Mallat, 2008), which may warrant a different implementation. Nonetheless, RWL1 could be attached to this alternative scheme to achieve a similar improved outcome (Candès et al., 2008), making this problem an applied research path worth exploring.

Acknowledgments

The authors would like to express their gratitude to J. Daleiden from ARRB Systems Inc., for providing the TSD deflection data from the NAMA and the GWMP, to J. Graham from the District of Columbia DOT, and M. Elias from the Eastern Federal Lands [EFL] Highway Division of the FHWA for kindly furnishing GIS-friendly data utilized as base cartography in this study, and to N. Sivaneswaran from FHWA for their insight on the results’ interpretation and their revisions to the final manuscript.

Data Availability Statement

The original TSD data used in this study if currently freely available through FHWA’s InfoMaterials website⁹. The computer code written to perform the calculations that support the findings presented herein are available from the corresponding author upon reasonable request. The cartography sources utilized for the maps presented in this study are either publicly available or could be retrieved from the District of Columbia DOT and the EFL highway division within the FHWA.

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References


CHAPTER 5 – ON THE TSD DEFLECTION VELOCITY MEASUREMENTS: A REVISION TO THE CURRENT STATE OF THE ART

“You have changed your perspective and location, and now you can see.”

From Jo Nesbø’s ‘The Son’

Abstract

The fourth-generation TSD can provide high-resolution pavement deflection information at an outstanding rate, much higher than in previous versions. This data is a valuable asset to evaluate the structural health of the load-bearing transverse joints in concrete pavements. An exploratory analysis of the data gathered by one such TSD at a jointed pavement test track allowed detecting and correcting an inconsistency in the way the deflection velocity \( v_y \) data is processed to estimate the deflection basin slope and depth: We state and prove that the direct relationship between \( v_y \) and deflection slope usually followed to generate deflection slope estimates is only valid in limited scenarios, and particularly invalid at the vicinity of transverse joints. In this paper, a thorough analysis of the information contained in the \( v_y \) signals is provided using linear elastic simulation. We highlight that \( v_y \) is made up of two components, (only one of which is related to the deflection basin slope) and that at the vicinity of a discontinuity in the pavement structure, the second component of \( v_y \) gains relevance and explains the discrepancy between \( v_y \) and deflection basin slope. Also, it is shown that TSD \( v_y \) measurements gathered near a transverse joint do resemble this pattern. Thus, the findings from this study bring upon a new insight into the contents of a TSD \( v_y \) signal and contribute to prove the TSD’s reliability as a deflection velocity measurement device. Plus, they suggest that further research efforts may be needed to develop TSD data interpretation schemes based on deflection velocities instead of deflection slopes and depths.

10: This paper has been submitted for publication at a peer-reviewed Journal in June 2022. It is co-authored by Samer W. Katicha, Ph. D.; Gerardo W. Flintsch, Ph. D.; and Eugene Amarh, Ph. D. This Chapter is a reproduction of the original submission.
Introduction

The TSD is a device that uses Doppler lasers to measure the pavement deflection velocity while traveling at traffic speed (Hildebrand and Rasmussen, 2002; Flintsch et al., 2013; Xiao et al., 2021). However, the pavement deflection velocity is not a pavement response parameter used by pavement engineers, who generally use the pavement deflection instead. Therefore, the team that developed the TSD suggested that the pavement deflection slope can be calculated from the pavement deflection velocity by dividing the pavement deflection velocity by the horizontal traveling velocity (Hildebrand et al. 1999, Hildebrand and Rasmussen 2002, Krarup et al. 2006). The deflection slope can then be integrated to obtain the pavement deflection (Krarup et al., 2006; Müller and Roberts, 2013; Nasimifar et al., 2018; Xiao et al., 2021). This approach of converting the deflection velocity to deflection slope (or directly to deflection) has been the standard of practice in the analysis of TSD measurements (Ferne et al., 2009; Flintsch et al., 2013; Müller and Roberts, 2013; Zofka et al., 2014; Rada et al., 2016; Katicha et al. 2017, 2020; Elseifi and Zihan, 2018; Nasimifar et al., 2018; Xiao et al., 2021).

This paper shows that the relationship between pavement deflection slope and pavement deflection velocity is not as simple as is currently assumed. The evidence presented herein is based on 1) the mathematical definitions of the pavement deflection velocity and the pavement deflection slope, and 2) a comparison of TSD pavement deflection velocity measurements from a jointed concrete pavement with a slab-on-a-Pasternak-foundation model of the jointed concrete pavement structure. In summary, the pavement deflection velocity can be decomposed into two components: The first component is equal to the pavement deflection slope multiplied by the horizontal velocity and the second component is related to the changes in the pavement properties that occur while the TSD is moving along the pavement structure. The contribution of the second component to the pavement deflection velocity depends on the rate of change of the pavement properties with higher rates, in absolute value, leading to a greater contribution. This typically occurs near the joints in a jointed concrete pavement, but might as well occur on any type of pavements (either rigid or flexible) with transverse discontinuities, such as localized heavily cracked sections, patches, or at the transition between sections with different material properties or layer thicknesses.
Objective

This paper's objective is to provide an accurate description of the pavement deflection velocity, show how it is related to the pavement deflection slope, and how it is measured by the TSD. The validity of the proposed formulation is verified with measurements collected in the vicinity of the joints on a jointed concrete pavement, which are compared to the theoretical response of a jointed concrete pavement model.

Paper organization

The remainder of this paper is organized in four sections as follows:

- **Background:** This section presents 1) the mathematical definition of the pavement deflection velocity, 2) the topic of Doppler laser velocimetry, and 3) the slab-on-a-Pasternak-foundation pavement model. The pavement deflection velocity is the time-derivative of the pavement deflection and we use the chain rule of differentiation to show how it relates to the pavement deflection slope. Most importantly, this shows that the pavement deflection slope is not equal to the pavement deflection velocity divided by the vehicle horizontal speed as is commonly assumed (Hildebrand and Rasmussen, 2002; Krarup et al., 2006). Doppler laser velocimetry (Zhang, 2010) is the principle of using the Doppler effect to measure the velocity of liquids or solids, and is also the underlying principle of the TSD measurements. The slab on a Pasternak foundation model (Van Cauwelaert, 2004) is used to obtain the theoretical response of the pavement to the TSD load, which is then compared to TSD measurements from a jointed concrete pavement segment.

- **Theoretical Response of a the Joint in a Jointed Concrete Pavement Subjected to a Moving TSD Load:** In this section, we present the theoretical results of calculating the pavement deflection velocity and pavement deflection slope using the slab on a Pasternak foundation model (Van Cauwelaert, 2004). In particular, we analyze the sensitivity of the deflection velocity and deflection slope response to the joint’s Load Transfer Efficiency [LTE] index (Pierce et al., 2017), showing that the deflection velocity is much more sensitive to changes in the LTE than the deflection slope is.

- **Validation of the Modeled TSD Response:** In this section, the simulated modeled response is compared to TSD measurements collected at the MnRoad testing facility. This validates the
mathematical model by showing that the deflection slope is not equal to the deflection velocity divided by the vehicle horizontal velocity. It also validates the slab on a Pasternak model as a suitable model to simulate the response of jointed concrete pavements to a heavy axle load.

- **Findings and Conclusions:** This section discusses the implications of the fact that, in general, the deflection slope is not equal to the deflection velocity divided by the TSD horizontal speed. As a consequence, the deflection, which is determined by integrating the deflection slope, cannot always be obtained from TSD measurements. As such, further research efforts should be centered on the direct interpretation of the TSD deflection velocity data from a pavement engineering perspective, as these are the most reliable measurements a TSD survey can generate and they can be reasonably modeled. Thus, deflection-velocity-based back-calculation procedures (Rohde and Smith, 1991) could be implemented for the TSD instead of analysis schemes based on deflection data. This paper’s closing remarks prompt for that path to be taken.

**Background:**

**Pavement Deflection Velocity and its relationship to Pavement Deflection Slope**

The pavement deflection changes based on location and the pavement structure properties. Therefore, we can express the pavement vertical deflection, \( w \), as a function of location, \( x \), and pavement structure properties, \( M \), as follows:

\[
w = w(x, M)
\]  
(29)

The pavement structure properties parameter, \( M \), itself depends on \( x \) and because the TSD is moving along the pavement, \( x \) is, in turn, a function of the time \( t \). Therefore, we have that:

\[
w = w(x(t), M(x(t)))
\]  
(30)

To determine the pavement deflection velocity \( v_y \), \( w \) is differentiated with respect to time using the chain rule:

\[
v_y = \frac{dw}{dt} = \frac{\partial w}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial w}{\partial M} \frac{\partial M}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial w}{\partial t} + \frac{\partial w^*_M}{\partial t}
\]  
(31)

Where the second equality holds also by the chain rule and \( w^*_M \) refers to the component of \( w \) that depends on \( M \). Noticing that \( v_x = \frac{\partial x}{\partial t} \), and \( S = \frac{\partial w}{\partial x} \), where \( S \) is the slope of the deflection bowl, we have that:
\[
\frac{\partial w \partial x}{\partial x \partial t} = Sv_x \tag{32}
\]

Then, combining the results from equations 31 and 32 (and for simplicity writing the term \(\partial w_M^*/\partial t\) as \(\partial w/\partial t\)):

\[
v_y = Sv_x + \frac{\partial w}{\partial t} \tag{33}
\]

This last equation is usually used to put forth the argument that the TSD can be used to measure the pavement deflection slope: To obtain the deflection slope, simply divide the equation above by the test velocity \(v_x\) (Krarup et al., 2006):

\[
S = \frac{v_y}{v_x} \tag{34}
\]

However, equation 34 ignores the additional term \(\partial w/\partial t\), which represents the changes in the deflection bowl as the TSD travels on the pavement. Based on the slab on a Pasternak foundation model of a jointed concrete pavement (Van Cauwelaert, 2004), we show that the term \(\partial w/\partial t\) is much larger than the term \(Sv_x\) for measurements near the joints, and thus cannot be disregarded. This is further corroborated by TSD measurements obtained at 5 cm intervals at the MnRoad testing facility.

Equation 34 also has a geometric interpretation concerning the velocity vector for the pavement surface relative to the TSD sensors’ \(v\): When it holds, the tangent of the angle between \(v\) and the horizontal direction is equal to \(S\). Refer to Appendix A for further details on how the deflection slope \(S\) is estimated from two Doppler laser measurements when equation 34 holds.

**Deflection Velocity Measurements: Laser Doppler Velocimetry**

The Doppler lasers used by the TSD measure the instantaneous relative velocity of the pavement and the Doppler laser. The velocity measurement of solids or fluids using Doppler lasers is referred to as *Laser Doppler velocimetry* (or *anemometry*). This technique was first used by Yeh and Cummins (1964) to measure fluid flow and has been used for a wide range of applications such as vibration measurements (Castellini et al. 2006, Castellini et al. 2013, Rothberg et al. 2017), tissue blood flow measurements (Obeid at al. 1990, Vennemann et al. 2007, Rajan et al. 2009), and measurements of vehicle speeds (Jendzurski and Pautler, 2008; Gao et al. 2017, Sung and Majji 2022). The method has been applied in pavement engineering to measure the pavement response under a moving wheel load which led to the
development of the Traffic Speed Deflectometer (TSD) (Hildebrand et al., 1999; Hildebrand and Rasmussen, 2002).

Consider a signal source emitting a wave signal (either light or sound) at a frequency $f_s$. The Doppler effect (Albrecht, 2003; Jendzurski and Paulter, 2008; Zhang, 2010), illustrated in figure 27, is the frequency shift (change in frequency) that occurs when the signal source is moving or when the signal is reflected off a moving surface (or both). In either case, the Doppler effect is the same, and therefore, it only depends on the relative movement of the source and the receiver or reflecting surface (Jendzurski and Paulter, 2008; Zhang, 2010). Therefore, when considering TSD Doppler velocimetry, instead of having the TSD moving in the $x$-direction and the pavement in the $y$-direction (deflecting), we can assume the TSD is fixed and the pavement is moving in the $x$ and $y$ directions with velocities $v_x$ and $v_y$. The velocity vector $v$ is therefore given by:

$$v = v_x i + v_y j$$  \hspace{1cm} (35)

Where $i$ and $j$ are unit-length vectors representing the horizontal and vertical directions respectively. The velocity measured by the Doppler laser is equal to the dot product of the velocity $v$ and the unit direction vector $L$ of the Doppler laser (Albrecht, 2003; Zhang, 2010):

$$v_{laser} = v \times L = v_x \sin \theta + v_y \cos \theta$$  \hspace{1cm} (36)

Where $\theta$ is the angle of $L$ the laser relative to the $y$-axis (vertical direction, figure 28). See Appendix B for more details on how to remove the horizontal velocity component from the laser measurement to obtain the pavement deflection velocity (the vertical component).
Mathematical modeling of the deflection basin of a jointed pavement.

Linear elastic slab theory provides a framework to solve the deflection basin of a jointed pavement structure in response to an applied load, such as a TSD traveling by (Van Cauwelaert, 2004). Deep et al. (2020) showed that this framework can reliably simulate the passage of a heavy load (like a TSD wheel) over a jointed pavement and, besides, that it is by far less computationally demanding than finite element modeling, which makes it particularly appealing for large-scale analysis.

In the jointed-slab-on-ground model, the pavement structure is modeled as a pair of jointed concrete slabs resting on a Pasternak foundation – a combination of a Winkler foundation plus a layer capable of only supporting shear stress. The slabs’ properties are the thickness $h$, the Young modulus $E$, and the Poisson’s ratio $\nu$. The Pasternak foundation material has a modulus of subgrade reaction $k$ and shear modulus $G$. The applied load is uniformly distributed over a rectangular area of dimensions $2a$-by-$2b$, applied on one of the slabs (Van Cauwelaert, 2004). A transverse joint with a load transfer efficiency [LTE] index $0 \leq \delta \leq 1$ (LTE as defined by Pierce et al., 2017) is located at a distance $c$ from the load center (figure 29).

The deflection basin equation is a combination of a term representing the deflection bowl of a single infinite slab, plus a series of terms that relate to the boundary conditions imposed by the joint. Equation 37 presents this formulation for both the loaded slab ($w_L$) and the unloaded slab ($w_{UL}$).
\[
\begin{align*}
&w_L(x, y) = w(x, y) + A \times w_A(x, y) + B \times w_B(x, y) \\
&w_U(x, y) = C \times w_C(x, y) + D \times w_D(x, y)
\end{align*}
\]  

(37)

where \(w(x, y)\) is the solution of the deflection bowl for an infinitely-large loaded slab, and the additional terms \(w_A, w_B, w_C, w_D\) represent the boundary conditions imposed by the joint.

\[l = \left[ \frac{E h^3}{12(1 - \nu)^2 k} \right]^{1/4}
\]  

(38)

\[D = k \times l^4
\]  

(39)

\[g = \frac{G l^2}{2D}
\]  

(40)

Where \(E, k, \nu, h, G\) are as defined previously.

Equations 41 and 42 present the formulae for \(w(x, y)\) [equation 37] for the \(g = 1\) case, which comprises the Winkler foundation case \((g = 0)\), as solved by Van Cauwelaert (2004). Refer to this manual for the remaining cases.
for $x < a$

$$w(x, y) = \frac{p}{\pi k} \frac{1}{\sqrt{1 - g^2}} \int_0^\infty w_3(y, s) \times \left[ 2 - w_1(x, s) - w_2(x, s) \right] ds$$

where:

$$w_3(y, s) = \frac{\cos(sy/l) \sin(sb/l)}{s(s^2 + 2gs^2 + 1)}$$

$$w_1(x, s) = \frac{e^{-|a-x|a/l}}{\sqrt{1 - g^2}} \sqrt{1 - g^2} \cos[(a-x)\beta/l] + s^2 + g) \sin[(a-x)\beta/l]$$

$$w_2(x, s) = \frac{e^{-(a+x)\beta/l}}{\sqrt{1 - g^2}} \sqrt{1 - g^2} \cos[(a+x)\beta/l] + s^2 + g) \sin[(a+x)\beta/l]$$

for $x \geq a$

$$w(x, y) = \frac{p}{\pi k} \frac{1}{\sqrt{1 - g^2}} \int_0^\infty w_3(y, s) \times [w_4(x, s) - w_5(x, s)] ds$$

where:

$$w_3(y, s) = \frac{\cos(sy/l) \sin(sb/l)}{s(s^2 + 2gs^2 + 1)}$$

$$w_4(x, s) = \frac{e^{-|x-a|a/l}}{\sqrt{1 - g^2}} \sqrt{1 - g^2} \cos[(x-a)\beta/l] + s^2 + g) \sin[(x-a)\beta/l]$$

$$w_5(x, s) = \frac{e^{(x+a)\beta/l}}{\sqrt{1 - g^2}} \sqrt{1 - g^2} \cos[(x+a)\beta/l] + s^2 + g) \sin[(x+a)\beta/l]$$

Where the auxiliary terms $\alpha$ and $\beta$ are:

$$\alpha^2 = \frac{1}{2} \left[ \sqrt{(s^2+g)^2 + 1 - g^2} + (s^2+g) \right]$$

$$\beta^2 = \frac{1}{2} \left[ \sqrt{(s^2+g)^2 + 1 - g^2} - (s^2+g) \right]$$

**Boundary conditions at the transverse joint**

The additional terms to equation 37 related to the transverse joint are as follow (for $g < 1$) (Van Cauwelaert, 2004):

$$w_A(x, y) = \frac{p}{\pi k} \frac{1}{\sqrt{1 - g^2}} \int_0^\infty \left[ A(s) \cos(\beta x/l) e^{\alpha xy/l} \times \frac{\cos(sy/l) \sin(sb/l)}{s} \right] ds$$

$$w_B(x, y) = \frac{p}{\pi k} \frac{1}{\sqrt{1 - g^2}} \int_0^\infty \left[ B(s) \sin(\beta x/l) e^{\alpha xy/l} \times \frac{\cos(sy/l) \sin(sb/l)}{s} \right] ds$$
\[ w_c(x, y) = \frac{p}{\pi k} \frac{1}{\sqrt{1 - g^2}} \int_0^\infty \left[ C(s) \cos(\beta x/l) \right] e^{-axl} \times \frac{\cos(sy/l) \sin(sb/l)}{s} \, ds \]

\[ w_d(x, y) = \frac{p}{\pi k} \frac{1}{\sqrt{1 - g^2}} \int_0^\infty \left[ D(s) \sin(\beta x/l) \right] e^{-axl} \times \frac{\cos(sy/l) \sin(sb/l)}{s} \, ds \]  

(45)

Where the auxiliary parameters \( \alpha \) and \( \beta \) are as per equation 43.

The transverse joint imposes four boundary conditions which provide formulae to solve the parameters \( A(s), B(s), C(s), D(s) \) for all values of the dummy variable \( s \), \( x = c \), and all values of \( y \), particularly \( y = 0 \). These conditions are (Van Cauwelaert, 2004):

1. Load transfer efficiency \( \text{LTE} = \delta \) at \( x = c \) and any value of \( y \) – particularly \( y = 0 \)
\[
\delta \times (w(s) + A(s)w_A(s) + B(s)w_B(s))|_{x=c} = C(s)w_C(s) + D(s)w_D(s)|_{x=c} \quad (46)
\]

2. Cancellation of bending moments at the edge of the loaded slab \( (x = c) \)
\[
\left( \frac{\partial^2}{\partial x^2} + v \frac{\partial^2}{\partial y^2} \right) (w(s) + A(s)w_A(s) + B(s)w_B(s))|_{x=c} = 0 \quad (47)
\]

3. Cancellation of bending moments at the edge of the unloaded slab \( (x = c) \)
\[
\left( \frac{\partial^2}{\partial x^2} + v \frac{\partial^2}{\partial y^2} \right) (C(s)w_C(s) + D(s)w_D(s))|_{x=c} = 0 \quad (48)
\]

4. Equality of shear forces in the subgrade material \( (x = c) \)
\[
\left( \frac{\partial^3}{\partial x^3} + (2 - v) \frac{\partial^3}{\partial x \partial y^2} - \frac{2g}{l^2} \right) (w(s) + A(s)w_A(s) + B(s)w_B(s))|_{x=c} = \ldots \quad (49)
\]
\[
\ldots = \left( \frac{\partial^3}{\partial x^3} + (2 - v) \frac{\partial^3}{\partial x \partial y^2} - \frac{2g}{l^2} \right) (C(s)w_C(s) + D(s)w_D(s))|_{x=c}
\]

Equations 46 through 49 make a 4-by-4 linear system that can be solved for \( A, B, C, D \) for each value of \( s \). The terms \( w(s), w_A(s), w_B(s), w_C(s), w_D(s) \) are the expressions within the integral for each of the variables \( w, w_A, w_B, w_C, \) and \( w_D \) as per equations 44 and 45).

**Considerations when the load steps over the joint**

In Van Cauwelaert’s (2004) linear elastic jointed-slab-on-ground model the load is assumed to be distanced from the transverse joint. However, when simulating the travel of the TSD over such a pavement, a workaround must be devised to compute the deflection basin when the TSD wheel is
invading the joint. The principle of superposition can be used to simulate such load scenarios by adding the deflection responses owed to the portions of the load on both the approach and leave slab. This concept is illustrated in figure 30, its application involves conveniently correcting the dimensions of each portion of the load and the respective load-joint distance, solving the deflection basin for each, and then adding the outcomes.

Figure 30: Applying superposition to solve the case of load invading the transverse joint.

Computing deflection velocity and deflection slope from simulated deflection basin depth

The TSD reports deflection velocity measurements and estimated deflection slope data. Thus, in order to provide comparable deflection results, deflection slope and vertical deflection velocity values must be obtained from the simulated deflection basin results. Computation of the deflection basin slope is a rather
simple task, for it simply involves deriving the simulated basin over the direction of travel. This can be approximated numerically via finite differences by evaluating the deflection basin depth at two nearby positions (equation 50):

\[
\begin{align*}
\text{for } x < c: & \quad S(x, y) = \frac{\partial}{\partial x} w_L(x, y) \approx \frac{w_L(x + h, y) - w_L(x, y)}{h} \quad h > 0 \\
\text{for } x \geq c: & \quad S(x, y) = \frac{\partial}{\partial x} w_{UL}(x, y) \approx \frac{w_{UL}(x + h, y) - w_{UL}(x, y)}{h} \quad h > 0
\end{align*}
\]

However, the estimation of the vertical deflection velocity \(v_y\) is not as trivial, for it involves observing the deflection at a fixed position on the pavement surface at different moments in time, through which the TSD load has displaced. As such, the deflection of a fixed point on the pavement surface would be simulated by evaluating the jointed-slab-on-ground model at location \(x\) (deflection at time \(t\)) and at location \(x - \Delta x\) (deflection at time \(t + \Delta t\)). \(\Delta x\) and \(\Delta t\) relate via \(v_x\), for \(\Delta x = v_x \Delta t\). These evaluations must also observe that the TSD wheel got \(\Delta x\) closer to the joint, thus for the second evaluation \(c_2 = c - \Delta x\) as well. Thus, the estimation of \(v_y\) from simulated deflection basins can be stated as (equation 51):

\[
\begin{align*}
\text{for } x < c: & \quad v_y(x, y) = \frac{w_L(x - v_x \Delta t, y, c - v_x \Delta t) - w_L(x, y, c)}{\Delta t} \\
\text{for } x \geq c: & \quad v_y(x, y) = \frac{w_{UL}(x - v_x \Delta t, y, c - v_x \Delta t) - w_{UL}(x, y, c)}{\Delta t}
\end{align*}
\]

Theoretical Response of a the Joint in a Jointed Concrete Pavement Subjected to a Moving TSD Load

This section furnishes an insight into the information contained within the TSD deflection velocity \([v_y]\) measurements from a jointed pavement. Firstly, we present a description of the two components of the \(v_y\) signal (as introduced in equation 33) and highlight the relative weight of each in the overall measurement. Then, a discussion is provided on how the \(v_y\) signal changes as a consequence of varying the joint’s LTE index.

Figure 31 presents the components of a simulated deflection velocity signal for the TSD’s 210-mm-ahead sensor for a joint in \textit{perfect health} – LTE = 100%. The simulated pavement structure has a thickness of \(h = 0.20\, m\) (7.87 inch), \(E = 32\, GPa\) (4.64×10^6 PSI), \(v = 0.21\), \(k = 0.033\, MPa/mm\) (120 PSI/in), and \(G = 0\). The simulated load equals that of half an \textit{Equivalent Single Axle Load} (\(P = 49\, kN\)), at a pressure of 793 kPa (115 PSI), and a width of \(2b = 0.50\, m\) (19.7 inches) (approximate width of the footprint of a dual-wheel heavy half-axle), the length of the load is \(2a = 0.124\, m\) (4.87 inches). The \textit{simulated} TSD’s travel
speed was set to 25 m/sec (56 mph). Meanwhile, figure 32 presents simulated deflection bowls when the TSD load is at different distances from the transverse joints. It can clearly be told that the deflection basin rapidly changes shape as the load approaches the joint.

Figure 31: Relative relevance of the deflection velocity signal components

Figure 32: Simulated deflection basins at mid-slab locations (constant shape basin), and nearby the deflection joint (changing shape).

The plots in figure 31 show the different trends followed by the simulated deflection slope (dotted line) and $v_y$ signals (continuous line): These signals are not directly proportional to each other in the vicinity of the transverse joint, exposing that the partial time derivative component of $v_y$ (equation 33) is non zero nearby the joint. This component relates to the rate of change in the shape of the deflection basin as the TSD load shifts towards the transverse joint, its magnitude is zero at mid-slab locations (away from the joint), where the deflection basin is not changing (it travels with the TSD load without changing shape –
figure 34) but gains relevance and even becomes dominant over the deflection slope signal as the TSD joint gets closest to the joint. Plus, as the load approaches the joint, the deflection slope for points slightly ahead of the load center changes sign although the pavement is always descending, graphically showing the change of sign in the deflection slope signal in figure 31.

The pulse in the $v_y$ signal as the TSD travels over the joint is sensitive to LTE: Figure 33 shows such responses at different levels of LTE [80%, 60%, 30%]. The figure shows that the deflection velocity is more sensitive than the deflection slope to changes in the LTE. For example, the peak deflection slope increases from 230 to 324 and 412 $\mu$m/m (approximately 41% and 79% respectively), whereas the peak $v_y$ increases from 27.4 to 69.3 (2.6 times) and 108.0 mm/sec (approximately 4 times).

Figure 33: Simulated deflection slope and $v_y$ signals for the +210mm sensor at different LTE levels.

Another remarkable feature of the simulated $v_y$ signals that becomes evident as LTE drops from 100% is the second peak that happens slightly before the TSD load traverses the joint. This secondary pulse is the record of the TSD Doppler sensor no longer measuring the $v_y$ of the approach slab but that of the leave slab – the secondary pulses shown in figure 33 are taking place when the load is at 210mm from the transverse joint.
Validation of the Modeled TSD Response

In this section, we show that the TSD measurements collected on jointed concrete pavement sections agree with simulated deflection velocity, not with the simulated deflection slope. This validates the assertion that, except in the case where the pavement properties are not changing, the deflection slope cannot be obtained from the deflection velocity by simply dividing the vertical velocity by the horizontal velocity. The features of the \( v_y \) signals described in the simulated signals previously shown were also observed in real TSD high-resolution signals. This section introduces measurements taken at the MnROAD low-volume-road (loop) during September 2021 as part of a trial run of a fourth-generation TSD device. The TSD operator furnished several measurements at 5-cm resolution from repeated runs along the loop.

In this paper, only some subsets of this data collected over jointed pavement segments will be presented: The featured sections are listed in table 6, for which relevant information about the pavement structure and layer thickness was reported by Van Deusen et al. (2018), and relevant material properties from Barman et al. (2021) and Khazanovich et al., (2021). Besides, the MnROAD online database – available through FHWA’s InfoPave website – contains several records of deflection testing with an FWD device at locations near the transverse joints, conducted between March and May 2019. These test results were used to estimate the joints’ LTE index.

Figures 34 and 35 show real \( v_y \) signals from 3 out of the 10 TSD sensors from locations within the featured sections of the MnROAD loop. These signals were denoised by wavelet denoising (Katicha, 2022). The two-peak signal is evident for the pulse at station 1551 (figure 34) and for several of the peaks recorded past station 3400 (figure 35). The joints for which FWD test results are available were flagged, their estimated LTE indices highlighted.

<table>
<thead>
<tr>
<th>MnROAD Section</th>
<th>begin [m]</th>
<th>end [m]</th>
<th>Concrete slab size</th>
<th>E [GPa]</th>
<th>Slab thickness [inch]</th>
<th>Base and subgrade</th>
</tr>
</thead>
<tbody>
<tr>
<td>124, 324, 424, 524 – 2017</td>
<td>1485</td>
<td>1630</td>
<td>15 [L]’ x 12 [W]’, dowelled 1” bar</td>
<td>28</td>
<td>6</td>
<td>6” (Class 6 material) + sandy subgrade</td>
</tr>
<tr>
<td>138, 238 – 2017</td>
<td>3150</td>
<td>3280</td>
<td>15 [L]’ x 12 [W]’, dowelled 1.25” bar</td>
<td>24</td>
<td>8</td>
<td>5” base + clay subgrade</td>
</tr>
<tr>
<td>239 – 2017</td>
<td>3385</td>
<td>3440</td>
<td>6’ x 6’ panels fiber-reinforced</td>
<td>24</td>
<td>4</td>
<td>6” (Class 6 material) + 4 in clay borrow + clay subgrade</td>
</tr>
</tbody>
</table>

Table 6: Featured MnROAD loop sections

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Figures 36 and 37 show the real TSD $v_y$ measurements and the simulated $v_y$ signals for the transverse joints at stations 1551 and 3389. The simulated signals were generated assuming material properties from literature (Barman et al., 2021; Khazanovich et al., 2021; ACI, 2019) and manually attempting many LTE indices equal or lower than the estimates from the archived FWD measurements when available (processed following Schmalzer et al., 2007 guidelines). It can be seen that the simulated $v_y$ resembles the denoised TSD measurements, the similarity between the measurements and the simulated signals improves in the weaker pavement case. Observe that in figure 37, only the pulse at station 3389 was simulated, but the measured pulses from the joints located 1.80m and 3.60m ahead are clearly visible.
Findings and Conclusions

In this paper, we show that the TSD provides reliable pavement surface deflection velocity \(v_x\) measurements, and discuss the information contained in them thoroughly. It is proven that the \(v_x\) signal is made of two components (equation 33), one related to the relative motion between the pavement surface and the TSD load (the \(S_v\) term), plus another term (the partial time-derivative term) that represents the
deflection basin’s change in shape. On perfectly homogeneous pavements, the partial time-derivative term is null and thus allows a direct estimation of the deflection slope from $v_y$, a relationship that has been widely used in TSD data analysis practice (equation 34). However, on pavements with discontinuities such as transverse joints, this term becomes becomes dominant in the $v_y$ signal and thus flawing the deflection slope estimates obtained through equation 34. This observation about the TSD’s $v_y$ signal can be extended to flexible pavements as well: Relevant discontinuities in these pavements can be open transverse cracks, construction defects, patches, segments with non-homogeneous properties or layer thicknesses. In all these cases, the TSD $v_y$ measurements will be the only reliable source of structural health data, for the deflection slope estimates for these pavements obtained as per equation 34 will be affected by error. Moreover, since a deflection slope estimate for these pavements cannot be directly obtained from the $v_y$ measurements, the usual deflection slope deflection integration schemes (Krarup et al., 2006; Müller and Roberts, 2013; Nasimifar et al., 2018) cannot be applied either, leaving only the $v_y$ measurements as the sole source of pavement structural health information from TSD data.

Thus, it follows that TSD data interpretation should, in the future, focus on devising analysis schemes for interpreting $v_y$ measurements directly in lieu of probably erroneous deflection slope estimates or even integrated deflection depths obtained from them. This being said, back-calculation schemes to get the pavement structural properties from deflection-related data from the TSD (Rohde and Smith, 1991) should be formulated to deal directly with $v_y$ measurements, equation 51 provides a means to simulating deflection velocities from modeled deflection basin depths to contrast against the field measurements. One such back-calculation application worth implementing is the estimation of LTE from TSD measurements. Deep et al. (2020) argued that the jointed slab-on-Pasternak-foundation model (Van Cauwelaert, 2004) is suitable to provide the fitted deflection response from a jointed concrete pavement to a heavy vehicle load, and this paper shows that the simulated $v_y$ signals generated from this modeling scheme can be fitted to match the measurements from the TSD. The only remaining challenge is implementation into a computer application capable of solving the modeled response fitting problem reasonably fast.

All in all, this paper provides an in-depth revision of the contents of a deflection velocity measurement, pointing out that it actually conveys both information about the deflection basin slope at the time of measurement plus information on the time-variability of the deflection basin shape. On non-continuous pavements or pavements with transverse joints, this second component becomes non-negligible and thus distorts the usual slope-from-$v_y$ estimates (equation 34). Simulated $v_y$ signals and actual TSD
measurements support our claim that equation 34 is only valid in limited cases, and particularly invalid when surveying jointed pavements. Nonetheless, this experience provides a better understanding of the TSD data and prompts further research on the interpretation of the $v_y$ signals themselves so as to get the most realistic depiction of the pavement behavior at the time of the survey.
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Transportation. Federal Highway Administration. Office of Infrastructure Research and Development.


CHAPTER 6 – ESTIMATING LOAD TRANSFER EFFICIENCY FOR JOINTED PAVEMENTS FROM TSD DEFLECTION VELOCITY MEASUREMENTS

Abstract

Transverse joints are the weakest element of jointed pavements, their faulting is an early indication of imminent load-related distress. The most widespread measurement of the joints’ structural performance is the Load Transfer Efficiency index [LTE], a ratio of the deflection of the two adjoining slabs. LTE can easily be assessed with an FWD, but this test procedure is not advisable for evaluation at the network level. Traffic-speed deflection devices like the TSD are suitable for large-scale deflection testing and were found capable of providing source data for structural evaluations. Yet, as of today, no interpretation technique to get structural health metrics for jointed pavements from TSD data has been published. Thus, in this paper, a back-calculation scheme grounded on slab theory is proposed to estimate the concrete slabs’ and subgrade’s properties and the joints’ LTE index from TSD deflection velocity measurements. The back-calculation problem formulation and its numerical solution using numerical fast procedures are described in detail. Also, the back-calculation engine’s performance is tested with real TSD data from the MnROAD test track. Overall, it was found that despite being tested on limited field data, our back-calculation technique converges to reasonable estimates of the pavement structural properties, and that, more importantly, can furnish LTE estimates for most transverse joints from 5-cm resolution TSD data, all at a reasonable computational cost. With this, the door is now open for corridor-wide LTE assessment of the pavement’s joints with a single TSD sweep.

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Introduction

LTE for structural evaluation of concrete pavements

Proper maintenance of jointed pavements (like jointed (plain) Portland Cement concrete [JPCP] and composite pavements) requires knowledge of the present condition of both the pavement slabs themselves and the load-carrying joints between slabs (Phares et al., 2008; Deep et al., 2020a, b, c). Several back-calculation procedures exist for deriving the concrete’s and subgrade’s properties out of deflection measurements (NCHRP, 2004; Pierce et al., 2017), whereas the performance of the load-carrying joints is assessed through the Load Transfer Efficiency [LTE] test (Huang, 2004; Alavi et al., 2008; Mackiewicz, 2015; Pierce et al, 2017). LTE serves as an indicator of the joints’ structural health: low values of LTE would lead to joint faulting and slab spalling and loss-of-support-triggered fracture (Huang, 2004, Mann, 2013; Garber and Hoel, 2015, Deep et al., 2020a, b, c; Nielsen and Jensen, 2021), hence it is sensible to refer to them as weak joints. Consequently, a tenet of proper jointed pavement design is preventing the loss of LTE (Wadkar et al., 2011; Mann, 2013; Pierce et al, 2017).

The LTE of a joint is customarily defined in terms of deflections. The LTE test involves applying a load on a given slab close to its edge and measuring the deflection of that slab (loaded slab) and the adjacent one (unloaded slab) (Pierce et al, 2017; Brezina et al., 2017). The LTE of the joint between these slabs is defined as (Ioannides and Korovesis, 1992; Schmalzer et al., 2007; Pierce et al., 2017; Deep et al., 2020a,-b,-c):

\[
LTE = \frac{W_{UL}}{W_L}
\]  

Where \( W_{UL} \) and \( W_L \) are the deflections at the unloaded and loaded slab respectively. Alternative definitions and correction factors for LTE exist (Ioannides and Korovesis, 1992; Wadkar et al., 2011; Pierce et al, 2017), yet the definition provided is customary as it follows intuition and is easy to assess in the field with a deflection measurement device (Brezina et al., 2017; Schmalzer et al., 2007; Pierce et al, 2017).

LTE testing practice

The device of choice for assessing LTE for in-service pavements is the Falling Weight Deflectometer (FWD), the test procedure is explained in the NCHRP Synthesis 381 (Alavi et al., 2008), and in
Schmalzer et al., (2007) and Pierce et al. (2017). The FWD excels at performing non-destructive evaluations of pavements at the project level – where few tests need be made – and/or at locations closed to traffic, where the FWD can be operated safely. However, its use at the network level (routine evaluations of in-service pavements) is conditioned to the test lanes being closed to traffic (thus disrupting the normal traffic flow) and exposing the FWD test staff and the apparatus itself to being hit by an encroaching vehicle (an obvious safety concern, for which proper countermeasures are costly) (Hildebrand and Rasmussen, 2002; Flintsch et al., 2013; Rada et al., 2016; Pierce et al., 2017). Moreover, network-wide assessment of joints’ LTE with the FWD is impractical in terms of time elapsed in testing as well: Testing a single joint for LTE takes roughly 5 minutes. At the usual 4.5m [15-feet] joint spacing, the FWD would cover a stretch of pavement 60 meters [180 ft] long (12 joints) per hour; so a full eight-hour shift may be enough to cover just about 1.1 lane-km [0.7 lane miles] of concrete pavement at the very most. Network-wide assessments at this rate are unfeasible either because they would require too much time to complete unless many FWD crews operate simultaneously (Flintsch et al., 2013; Rada et al., 2016).

Traffic Speed Deflection Devices, the TSD, and deflection testing on concrete

Traffic-speed deflection devices [TSDDs] have been the subject of research since the 1990s. Such devices have been developed to overcome the limitations of stop-and-go and/or slow-moving deflection devices like the FWD and the Deflectographs (European Commission, 1997; Hildebrand and Rasmussen, 2002; Flintsch et al., 2013 Rada et al., 2016). By operating at the speed of traffic, TSDDs do not require lane closures to be operated safely, thus relieving the pavement manager from the expenses related to traffic control and safety countermeasures. Besides, TSDDs are a sub-category within the universe of Continuous Deflection Devices [CDDs]; these collect dense data from the roadways – the separation between data points is no more than 0.30m [1 foot] (Flintsch et al., 2013) – and feature large data-recording capabilities. Thus, for instance, the Traffic Speed Deflectometer (TSD, one such TSDD) can survey up to 225-550 km [140-350 miles] of road a day (Baltzer et al., 2010). Arora et al. (2006), Flintsch et al. (2013), Rada et al. (2016), He et al. (2017), and Katicha et al. (2021) provide reviews on commercially available and one-off TSDDs.

The Traffic Speed Deflectometer [TSD] was first released in 2002 (Hildebrand and Rasmussen, 2002), and abundant literature has been published to date about its adoption: Its performance against other TSDDs and the FWD has been tested repeatedly (Baltzer et al., 2010; Arora et al. 2006, Flintsch et al., 2013; Katicha et al., 2014; Rada et al., 2016; Elsefi and Zihan, 2018). Besides, a corpus of published
literature elaborates on how to make use of the collected deflection slope data for project-level and network-level applications (Baltzer et al., 2010; Flintsch et al., 2013; Rada et al., 2016; Elsefi and Zihan, 2018). Descriptions of the TSD are provided in Hildebrand and Rasmussen (2002), Flintsch et al., (2013), Muller and Roberts (2013), Rada et al., (2016), Katicha et al., (2017, 2020, 2021), and the Australian standards AG:AM/T017 and AG:AM/S006 (Austroads, 2016-a,-b), which refer to the device itself and the data collection process respectively.

Since its early testing stages, the TSD has been regarded as a device capable of recognizing homogeneous sections and detecting weak spots within a pavement network for investigatory purposes (Hildebrand and Rasmussen. 2002; Flintsch et al., 2013; Rada et al., 2016; Hoque et al., 2018). Such a prospect makes it suitable for network-wide assessments of concrete pavements, in which low-LTE joints behave as localized weak spots (Phares et al., 2008; Flintsch et al., 2013; Katicha et al., 2013; Rada et al., 2016). In Scavone et al., 2021, 2022-a, an automated feature extraction procedure for analyzing 1-m resolution TSD measurements from concrete and composite pavements is presented. This analysis scheme simultaneously removes the random white noise in the TSD deflection measurements and recovers the pulse response obtained when measuring near structurally weak locations. As such, the recovered signal components may highlight the location of weak spots within the surveyed pavement network and thus target further investigation with conventional FWD testing and/or localized repair. Meanwhile, in a recently-published case-study paper, Nielsen and Jensen (2021) present an alternative interpretation of TSD data from a jointed pavement: The authors assess the pavement’s vertical deflection velocity differential between a sensor ahead and behind the TSD wheel, such differential may relate to LTE, yet such a relationship has not yet been published.

In 2021, the fourth-generation TSD device, with an upgraded sensing system capable of reliably collecting deflection velocity measurements at 5-cm resolution, has been introduced into the United States. A trial run of the 4th-gen TSD took place at the MnROAD proving grounds in September 2021, where both flexible and jointed rigid segments were analyzed. In Scavone et al. (2022-b) an in-depth analysis of the measurements at this testing site is presented: it is shown that in fact the 5-cm resolution TSD measurements provide a clear depiction of the transverse joints’ responses. Yet, one key finding of this study is that on jointed pavements (or pavements with discontinuities of any type), TSD data interpretation must be done in terms of deflection velocity in lieu of deflection slopes, for on these pavements the commonly assumed relationship between deflection velocity and deflection slope (Krarup et al., 2006) is no longer valid. Nonetheless, the insight gained on the actual content of TSD deflection
velocity measurements is still encouraging to attempt TSD field data interpretation via back-calculation. Thus, this paper addresses the challenge of implementing a TSD interpretation scheme for jointed pavement evaluation, aimed towards deriving the pavement structural health and the joints’ LTE index.

Research Objective

The objective of this paper is to back-calculate the load transfer efficiency [LTE] of a concrete joint from nearby deflection velocity measurements collected with a TSD. A back-calculation engine based on the linear elastic jointed slab-on-ground model (Van Cauwelaert, 2004) will be implemented to solve for LTE. The ultimate intention is to provide a fast yet reasonably accurate and not computationally demanding tool for network-wide LTE estimation from a vast TSD deflection data set.

Paper organization

This research article is divided into six sections: The paper Introduction discussed LTE testing as a component of jointed pavements’ structural evaluation and discusses the potential of the TSD as an LTE testing device at the network-wide investigatory level – the motivation for this study. The Research Objective section stated the goals to be pursued in this research given the previously-discussed topics. The mathematical pavement response model that would become the foundation for the back-calculation core of this paper is presented in the Background chapter, whereas the Methodology chapter elaborates on the back-calculation procedure itself, its formulation, and the approaches followed towards solving for the variables of interest. The Validation chapter presents back-calculation results on real TSD data from a jointed pavement at the MnROAD test-track, and provides a comment on the results obtained. Finally, the Conclusion chapter is devoted to final thoughts on this methodology and opportunities for improvement and further research.

Background

The deflection bowl profile for a loaded slab near a joint

The effect of a transverse joint on the deflection basin of a concrete pavement can clearly be understood in figure 38, which represents the response of a pair of 200-mm-thick slabs under the load of 1 ESAL (80 kN, modeled as two rectangular loads). The deflection basins for each case (varying the parameter $c$) were
calculated with the Finite Element software EverFE (Davids et al., 1998, 1999). The joint is located at station 4.60m.

Figure 38: Deflection basin profiles for a system of jointed slabs subject to 1 ESAL, as the load approaches the joint. Joint located at station 4.60m Parameters: Load = 80 kN, 2 rectangular areas 200×150mm each. Slab: $E = 28\ GPa$, $\nu = 0.2$, $h = 200\ mm$; subgrade $k = 0.3\ MPa/mm$, $G = 0$. Undoweled joint with bad LTE.

Intuitively, the closer the joint is from the center of the load, the more remarkably the deflection basin departs from the shape at mid-slab locations, which would also be the bowl profile for any “after-the-joint” location (profile to the right of the plots). Moreover, it is evident that the deflection basin at the joint’s vicinity is discontinuous. As such, the integrated deflection bowls from TSD deflection slope estimates, either solved with the Euler-Bernoulli beam model (Krarup et al, 2006) or the Area under the curve method (Muller and Roberts, 2013), would provide a false depiction of the slab’s deflection profile, for both procedures assume a continuous deflection response. Additionally, Scavone et al. (2022-b) proved that at the vicinity of transverse joints, the slope-deflection estimates computed from TSD deflection velocity measurements as per the direct relationship prompted by Krarup et al (2006) are flawed – they are still scaled valid deflection velocity measurements. Thus, the reported deflection slope measurements should neither be used in an LTE back-calculation scheme.

Anyway, the TSD deflection velocity measurements clearly show the response generated as the TSD travels over the transverse joints. For example, figure 39 presents measurements collected during a trial run of a 4th-gen TSD at the MnROAD Low Volume Road (Van Deusen et al., 2017), the pulse responses generated as the TSD traversed the joints are evident, even for joints whose LTE index approaches 1. All in all, the data from the trial run shows that the TSD’s deflection velocity measurements can be a suitable input for a back-calculation scheme for LTE.
Mathematical formulation of the deflection bowl

Consider a pair of concrete slabs attached by a load-bearing joint. These slabs are made of a linear elastic material with Young modulus $E$ and Poisson coefficient $\nu$ and rest on top of a Pasternak foundation with subgrade reaction modulus $k$ and shear modulus $G$ (the Winkler foundation corresponds to the case of $G = 0$). One of these slabs is subject to a distributed load [pressure $p$], uniformly distributed over a rectangular area whose dimensions are $2a \times 2b$. The distance between the center of the load application and the joint is $c$, as shown in figure 40.

Figure 39: TSD deflection velocity measurements from a jointed pavement section [MnROAD LVR, section 424]. The transverse joints are highlighted. The LTE values added surged from an earlier survey with an FWD.

Figure 40: Definition of the jointed-slabs-on-ground problem
A closed-form solution for the deflection basin for both slabs exists (Van Cauwelaert, 2004; Deep et al., 2020-a, b, c). Its analytical expression is a combination of the solution for an infinitely large slab plus additional expressions for the boundary conditions imposed by the load-carrying joint – the convention defined by Van Cauwelaert (2004) will be followed here.

The solution for the vertical deflection at any location within the loaded and unloaded slab is a combination of a continuous component directly related to the applied load, plus two components owed to the boundary condition imposed by the joint (Equation 53).

\[
\begin{align*}
    w_L(x, y) &= w(x, y) + A \times w_A(x, y) + B \times w_B(x, y) \\
    w_{UL}(x, y) &= C \times w_C(x, y) + D \times w_D(x, y)
\end{align*}
\]  

(53)

where \( w(x, y) \) is the solution of the deflection bowl for an infinite loaded slab, and the additional terms \( w_A, w_B, w_C, w_D \) represent the boundary conditions imposed by the joint.

To state the solution of the deflection basin, consider \( l \), Westergaard’s radius of relative stiffness of the slab, and the parameters \( D \) and \( g \), which relate to the slab’s and subgrade’s dimensions and material properties as:

\[
\begin{align*}
    l &= \left[ \frac{E h^3}{12(1-v)^2 k} \right]^{1/4} \\
    D &= k \times l^4 \\
    g &= \frac{G l^2}{2D}
\end{align*}
\]  

(54)  

(55)  

(56)

The deflection bowl equations are formulated after a Fourier decomposition of the applied load (solving the underlying equilibrium problem for each term), thus the resulting expressions are indefinite integrals over an auxiliary positive dummy variable \( s \). The equations for the deflection bowl terms that follow (equations 57 to 60) are for the particular case of \( g < 1 \), which includes the Winkler foundation case: \( g = 0 \), the reader is referred to Van Cauwelaert (2004) for the remaining cases:
for $x < a$

$$w(x, y) = \frac{p}{\pi k} \frac{1}{\sqrt{1 - g^2}} \int_0^\infty w_3(y, s) \times [2 - w_1(x, s) - w_2(x, s)] ds$$

where:

$$w_3(y, s) = \frac{\cos(|s y|/l) \sin(|s b|/l)}{s(s^4 + 2gs^2 + 1)}$$  \hspace{1cm} (57)

$$w_1(x, s) = \frac{e^{-|a-x|a/l}}{\sqrt{1 - g^2}} \left[ \sqrt{1 - g^2} \cos(a - x) \beta/l + (s^2 + g) \sin(a - x) \beta/l \right]$$

$$w_2(x, s) = \frac{e^{-|a+x|a/l}}{\sqrt{1 - g^2}} \left[ \sqrt{1 - g^2} \cos(a + x) \beta/l + (s^2 + g) \sin(a + x) \beta/l \right]$$

for $x \geq a$

$$w(x, y) = \frac{p}{\pi k} \frac{1}{\sqrt{1 - g^2}} \int_0^\infty w_3(y, s) \times [w_4(x, s) - w_5(x, s)] ds$$

where:

$$w_3(y, s) = \frac{\cos(|s y|/l) \sin(|s b|/l)}{s(s^4 + 2gs^2 + 1)}$$  \hspace{1cm} (58)

$$w_4(x, s) = \frac{e^{-|x-a|a/l}}{\sqrt{1 - g^2}} \left[ \sqrt{1 - g^2} \cos(x - a) \beta/l + (s^2 + g) \sin(x - a) \beta/l \right]$$

$$w_5(x, s) = \frac{e^{-|x+a|a/l}}{\sqrt{1 - g^2}} \left[ \sqrt{1 - g^2} \cos(x + a) \beta/l + (s^2 + g) \sin(x + a) \beta/l \right]$$

$$w_A(x, y) = \frac{p}{\pi k} \frac{1}{\sqrt{1 - g^2}} \int_0^\infty \left[ A(s) \cos(\beta x/l) \right] e^{ax/l} \times \frac{\cos(|s y|/l) \sin(|s b|/l)}{s} ds$$

$$w_B(x, y) = \frac{p}{\pi k} \frac{1}{\sqrt{1 - g^2}} \int_0^\infty \left[ B(s) \sin(\beta x/l) \right] e^{ax/l} \times \frac{\cos(|s y|/l) \sin(|s b|/l)}{s} ds$$

$$w_C(x, y) = \frac{p}{\pi k} \frac{1}{\sqrt{1 - g^2}} \int_0^\infty \left[ C(s) \cos(\beta x/l) \right] e^{-ax/l} \times \frac{\cos(|s y|/l) \sin(|s b|/l)}{s} ds$$

$$w_D(x, y) = \frac{p}{\pi k} \frac{1}{\sqrt{1 - g^2}} \int_0^\infty \left[ D(s) \sin(\beta x/l) \right] e^{-ax/l} \times \frac{\cos(|s y|/l) \sin(|s b|/l)}{s} ds$$

Where the auxiliary parameters $\alpha$ and $\beta$ are:

$$\alpha^2 = \frac{1}{2} \left[ \sqrt{(s^2 + g)^2 + 1 - g^2} \right] (s^2 + g)$$

$$\beta^2 = \frac{1}{2} \left[ \sqrt{(s^2 + g)^2 + 1 - g^2} \right] (s^2 + g)$$
The quantities A, B, C, D vary for each term of s. The equations to solve for them are boundary conditions that hold at the joint location (x = c, y = generic, or particularly for the TSD deflection basin, y = 0), namely:

- Load transfer efficiency LTE = δ at x = c and any value of y – particularly y = 0

\[ \delta \times \left( w(s) + A(s) w_A(s) + B(s) w_B(s) \right) \bigg|_{x=c} = C(s) w_C(s) + D(s) w_D(s) \bigg|_{x=c} \quad (62) \]

- Cancellation of bending moments at the edge of the loaded slab (x = c)

\[ \left( \frac{\partial^2}{\partial x^2} + v \frac{\partial^2}{\partial y^2} \right) \left( w(s) + A(s) w_A(s) + B(s) w_B(s) \right) \bigg|_{x=c} = 0 \quad (63) \]

- Cancellation of bending moments at the edge of the unloaded slab (x = c)

\[ \left( \frac{\partial^2}{\partial x^2} + v \frac{\partial^2}{\partial y^2} \right) \left( C(s) w_C(s) + D(s) w_D(s) \right) \bigg|_{x=c} = 0 \quad (64) \]

- Equality of shear forces in the subgrade material (x = c)

\[ \left( \frac{\partial^3}{\partial x^3} + (2-v) \frac{\partial^3}{\partial x \partial y^2} - \frac{2g}{l^2} \right) \left( w(s) + A(s) w_A(s) + B(s) w_B(s) \right) \bigg|_{x=c} = \ldots \]

\[ \ldots = \left( \frac{\partial^3}{\partial x^3} + (2-v) \frac{\partial^3}{\partial x \partial y^2} - \frac{2g}{l^2} \right) \left( C(s) w_C(s) + D(s) w_D(s) \right) \bigg|_{x=c} \quad (65) \]

Equations 62 through 65 make a 4-by-4 linear system that can be solved for A, B, C, D at each value of the dummy variable s. The terms w(s), w_A(s), w_B(s), w_C(s), w_D(s) are the expressions within the integral for each of the variables w, w_A, w_B, w_C, and w_D (as per equations 57 to 60).
Methodology

Two sequenced optimization problems to solve the concrete, subgrade, and joint properties

The back-calculation of materials’ properties and joints’ LTE from TSD measurements can be regarded as an optimization problem. The target variable to be minimized is the deflection matching error, the sum of squared errors [SSE] between the TSD measurements and a modeled response given a set of values for the parameters of interest (Rohde and Smith, 1991).

Mathematically, such a problem involving TSD deflection velocity data could be stated as a convex optimization problem (equation 66):

\[
\min_\theta \|\text{TSD} - \dot{w}(\theta)\|^2 = \sum_{i=1}^{\text{TSD-sensors}} (TSD_i - \dot{w}_i(\theta))^2
\]

where: \(\theta = [\text{LTE}, c, E, k, G]\)

and: \(\dot{w}_i(\theta) = v(x_{\text{TSD},i}, 0)\)

subject to: \(E > 0, k > 0, G > 0, c > 0, \text{LTE} \in [0,1]\)

Where TSD\(_i\) are the deflection velocity measurements collected by the TSD’s i-th sensor when the TSD wheel was at a distance c from the joint, the remainder variables are as defined in figure 40. \(\dot{w}_i(\theta)\) is the simulated pavement deflection velocity for the location sensed by the i-th sensor at the time of measurement \([x = x_{\text{TSD},i}]\), which is approximated as a forward difference from two simulated deflection depths (equation 67):

\[
\text{for } x_{\text{TSD},i} < c: \quad \dot{w}_i(\theta) = v(x_{\text{TSD},i}, 0) = \frac{w_L(x_{\text{TSD},i} - v_x \Delta t, 0, c - v_x \Delta t) - w_L(x_{\text{TSD},i}, 0, c)}{\Delta t}
\]

\[
\text{for } x_{\text{TSD},i} \geq c: \quad \dot{w}_i(\theta) = v(x_{\text{TSD},i}, 0) = \frac{w_{UL}(x_{\text{TSD},i} - v_x \Delta t, 0, c - v_x \Delta t) - w_{UL}(x_{\text{TSD},i}, 0, c)}{\Delta t}
\]

Where \(x_{\text{TSD},i}\) represents the distance between the TSD’s i-th sensor and the TSD rear axle, and \(v_x\) is the TSD travel speed. In equation 67, the formulae for \(\dot{w}_i(\theta)\) observe the fact that during the interval \(\Delta t\) the TSD approached the joint by an amount equal to \(v_x \Delta t\), where \(v_x\) is the TSD traveling speed. Thus, in the modeling framework given by equations 57 to 65, both the x-coordinate and the joint location (relative to the TSD wheel) need to be corrected when doing the deflection evaluations, hence the \(x_{\text{TSD},i} - v_x \Delta t\) and \(c - v_x \Delta t\) terms. The \(\Delta t\) can be chosen arbitrarily during the implementation stage, or can be set to conveniently match the TSD data resolution interval: for 5-cm-resolution measurements, \(\Delta t = 5\text{cm}/v_x\).
Although conceptually simple, the formulation in equation 66 can be challenging to solve as it is a rather high-dimensional optimization problem (five unknowns) that may be applicable only if 5 or more valid deflection slope measurements were retrieved with the TSD at the joint’s vicinity; otherwise, it may become ill-posed. Besides, it disregards at the same time the measurements collected farther away from the joint, which could otherwise be mined for material properties.

Thus, we propose dividing the back-calculation scheme into two low-dimension optimization problems to be solved in sequence: A *two-profiles* back-calculation approach in which the concrete and sub-grade properties are solved from mid-slab locations (using a simpler modeled deflection basin), reserving the measurements approaching the joint to solve for LTE – this second back-calculation problem assumes the results from the first problem as valid estimates of the pavement’s and foundation strength parameters. This back-calculation approach is illustrated in figure 41.

![Figure 41: Concept representation of the back-calculation approach using two TSD profiles](image)

The first optimization problem is set to minimize the $SSE$ for measurements collected away from the joint, by varying the concrete pavement’s and sub-grade’s strength parameters, namely $E$, $k$, $G$ – it will be assumed that the slab thickness ($h$) and the concrete’s Poisson ratio ($\nu$) are known. The linear elastic slab-on-ground model that fits this scenario is that of a continuous slab, the mathematical expression for the deflection basin simplifies remarkably from the full jointed-slab-on-ground model: Only the expressions in equations 57 and 58 remain, for the four boundary conditions that govern the joint are uncalled for. Meanwhile, the comparison TSD deflection velocity data would be retrieved from measurements at a
Thus, the optimization problem [OP1] to solve for unknowns $k$, $E$, and $G$ could be stated as:

$$\text{OP1: min } \sum_{i=1}^{\text{TSD\-sensors}} \left( \text{TSD}_{\text{midslab}_i} - \dot{w}_i(\theta) \right)^2$$

where: $$\dot{w}_i = v_y(x_{TSD_i}, 0)$$

$w(x, y, \theta)$ for a non-jointed slab, and: $$\theta = [k, E, G]$$

Subject to: $k \geq 0, E \geq 0, G \geq 0$, assuming: $h, v$ known

Additional maximum value restrictions could be added on $k$, $E$, and $G$ for numerical stability purposes – these variables should not take unrealistically high figures that transcend the usual range of values for engineering materials.

The second optimization problem is grounded on the assumptions that problem OP1 provides a good estimate of the pavement’s and foundation strength parameters, and that those are also valid nearby the transverse joint. Problem OP2 is stated as another minimization problem in which the objective function is the SSE between the TSD measurements from all sensors collected nearby the joint (signal $\text{TSD}_{\text{joint}}$) and the full jointed-slab-on-ground model, as formulated by equations 57 through 65. The only decision variables to be manipulated are the exact distance between the TSD load and the joint at the moment of sampling – variable $c$ – and the joint’s LTE index. Such a problem formulation has two advantages:

- The two decision variables are constrained. By definition, LTE cannot either be negative or greater than 1. Similarly, the jointed-slab-on-ground model does not admit negative $c$ values – such a case will be a continuous slab model as far as the TSD sensors are concerned. Meanwhile, the pre-processing stage of the TSD measurements could provide hints about the exact location of the joint: On 1-m data processed by Basis Pursuit Denoising (Scavone et al., 2021, 2022-a), the joint is within 1 meter from the station at which a pulse response was recovered. Meanwhile, on 5-cm data, the high-amplitude pulses on the $v_y$ signal indicate the joint location within an accuracy of ± 5cm (Scavone et al., 2022-b). Thus, given the approximate joint location and a set of measurements at a known station, narrow lower and upper bounds for $c$ can be defined ($c_{\text{min}}$ and $c_{\text{max}}$).

- Setting OP2 as a low-dimensional optimization problem simplifies computation, reduces the risk of the problem becoming ill-posed in the event not all sensors of the TSD provide valid readings, and allows for visual analysis of its performance as the iterative solution-seeking procedure evolves: The sub-optimal tried values could be plotted onto a 2-D grid and assess whether the
solver implementation is converging or not. The same assertion can be stated about problem OP1 for the particular case of assuming the subgrade as a Winkler foundation (G = 0).

Mathematically, problem OP2 is stated as:

\[
\text{OP2: } \min \sum_{i=1}^{\text{TSDsensors}} \left( \text{TSD}_{\text{joint},i} - \dot{w}_i |\theta| \right)^2
\]

where: \( \dot{w}_i = v_y |x_{\text{TSD}i}, 0| \)

and: \( w(x, y, \theta) \) for jointed slabs, and: \( \theta = [c, \text{LTE}] \)

Subject to: \( c_{\text{min}} \leq c \leq c_{\text{max}}, \ 0 \leq \text{LTE} \leq 1 \), assuming: \( h, v, k, E, G \) known

**Numerical solution of problems OP1 and OP2**

A numerical procedure based on Gradient Descent was implemented to iteratively seek a solution to problems OP1 and OP2. Conceptually, Gradient Descent aims at reaching a local minimum of a given function by proceeding down the direction of its gradient. If the given function is convex, which is often the case of squared errors, Gradient Descent would reach the function’s global minimum (Hastie et al., 2009; Zhang et al., 2021; Peyré, 2021).

The gradient descent implementation to solve problems OP1 and OP2 features a relaxed version of Hessian preconditioning, which is meant to account for the different orders of magnitude across the decision variables – something particularly true for problem OP1, where the usual values for \( E \) (in metric units) are a hundred or thousand times the value of the subgrade’s \( k \). Hessian preconditioning dynamically corrects the descent step size for each decision variable \( \theta \), based on the curvature of the target function \( \text{SSE} \), which is given by \( \text{SSE} \)'s inverse Hessian matrix. The relaxed Hessian preconditioning replaces the inverse Hessian matrix by appealing to the second-order partial derivatives of \( \text{SSE} \) instead, thus saving computational cost (Zhang et al., 2021; Peyré et al., 2021).

Define the variable \( \theta \) as a vector containing the unknowns for problems OP1 and OP2 respectively. The iterative decision variable update (at iteration \( t \)) for the preconditioned descent is calculated as:

\[
\theta(t+1) = \theta(t) - LR \times \frac{1}{H_{jj}} \times \nabla \text{SSE} \left( \theta(t) \right)
\]

Where \( \text{SSE} \) is the target function for problems OP1 or OP2 (equations 68 and 69), \( \nabla \text{SSE} \) represents its gradient vector (equation 71), and \( 1/H_{jj} \) denotes a vector whose entries are the inverse of \( \text{SSE} \)'s second-order partial derivatives (equation 72). In equation 70, the product terms are element-wise products, not matrix product operations.
LR in equation 70 is the gradient descent learning rate, a hyper-parameter of the procedure itself that defines the step size down the SSE gradient, often set to a small quantity for numerical stability. Starting from any value for the vector \( \theta(t=0) \), the gradient descent procedure is repeated until \( \theta(t) \) converges to a stable value – a local minimum of the target function SSE. If the descent fails to converge, it is often either due to implementation errors and/or an inappropriately large LR (Zhang et al., 2021).

The computation of the \( \nabla \text{SSE} \) and the Hessian terms (\( H_{jj} \)) at each iteration are done coordinate-wise by computing finite differences in SSE over increments in each coordinate \( \theta \) (equations 71 and 72):

\[
\nabla \text{SSE}(\theta(t))_j \frac{\partial \text{SSE}}{\partial \theta_j} = \frac{\text{SSE}(\theta(t), \theta(t)_j + \Delta) - \text{SSE}(\theta(t), \theta(t)_j - \Delta)}{2\Delta} \quad (71)
\]

\[
H_{jj} = \frac{\partial^2 \text{SSE}}{\partial \theta_j^2} = \frac{\text{SSE}(\theta(t), \theta(t)_j + \Delta) - 2 \times \text{SSE}(\theta(t), \theta(t)_j) + \text{SSE}(\theta(t), \theta(t)_j - \Delta)}{\Delta^2} \quad (72)
\]

Figure 42 illustrates the numerical advantage of the Hessian preconditioning terms for the descent. Both examples shown correspond to a given simulated case study (the target values and departure values of the decision variables for both descents are the same), and the gradient descent had the same global LR. Without the Hessian preconditioning term, a value of LR that would let one value descend would not alter the descend of the other variable. Meanwhile, once the preconditioning term is added to the descent, the iterative procedure converges smoothly.

Figure 42: Effect of preconditioning in gradient descent. Left, descent without Hessian preconditioning (only one variable descends, \( LR = 1 \times 10^7 \)). Right, descent with preconditioning (\( LR = 1 \times 10^4 \), both variables descend)
The final implementation of OP1’s solver was set with preconditioning and the global \( LR = 0.1 \), as this was found capable of minimizing the \( SSE \) out of deflection slope values in micro-meters/m, and the decision variables in metric units (\( E \) and \( G \) in N/m\(^2\), and \( k \) in N/m\(^3\)).

The back-calculation solver for OP1 was tested for a simulated case study (single target set of values for \( k, E, G \)) with five different start points. The simulated test was set as follows:

- Concrete slab: \( E = 32000 \) MPa, \( v = 0.20 \), \( h = 0.20 \)m)
- Foundation: Winkler material, \( k = 0.0326 \) MPa/mm, \( G = 0 \).
- Load: 49 kN, pressure of 758 kPa [110 PSI], footprint dimensions \([2a, 2b] = [0.14, 0.47] \) m.

Deflection velocity measurements were simulated for seven TSD sensors, located at 0.11, 0.21, 0.30, 0.45, 0.60, 0.90, 1.50 meters from the load center, matching the usual array of ahead sensors on a fourth-generation TSD device.

Figures 43 and 44 present the evolution of each decision variable and the trajectories by the descent solver over the SSE surface plot. Plus, table 7 summarize the computational effort for the gradient descent-based solver to converge to a final result. The 5 test cases were solved in \( \frac{1}{4} \) second on a personal computer running Matlab on a single CPU core. This performance test showed that problem OP1’s solver is robust and converges to its target fast – few iterations are needed to achieve the target \([k, E, G]\) set.

![Figure 43](image_url)

**Figure 43:** OP1-solver performance test. Evolution of both decision variables during gradient descent. Comparison for five starting conditions. Left: evolution for \( E \), right: evolution for the subgrade’s \( k \)
Figure 44: OP1-solver performance test. Trajectories followed by the back-calculated E and k during the gradient descent. Comparison for six starting conditions.

Table 7: Performance test results for the k, E back-calculation problem OP1 solver.

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Start E</th>
<th>k</th>
<th>Final value E</th>
<th>k</th>
<th>Iterations to reach target value by 5%</th>
<th>1%</th>
<th>0.1%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.3E+10</td>
<td>1.63E+07</td>
<td>3.20E+10</td>
<td>3.26E+07</td>
<td>157</td>
<td>237</td>
<td>339</td>
</tr>
<tr>
<td>2</td>
<td>2.9E+10</td>
<td>3.26E+07</td>
<td>3.20E+10</td>
<td>3.26E+07</td>
<td>57</td>
<td>137</td>
<td>239</td>
</tr>
<tr>
<td>3</td>
<td>3.2E+10</td>
<td>5.97E+07</td>
<td>3.20E+10</td>
<td>3.26E+07</td>
<td>71</td>
<td>151</td>
<td>253</td>
</tr>
<tr>
<td>4</td>
<td>4.0E+10</td>
<td>2.17E+07</td>
<td>3.20E+10</td>
<td>3.26E+07</td>
<td>125</td>
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<td>358</td>
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<td>5</td>
<td>1.3E+10</td>
<td>5.97E+07</td>
<td>3.20E+10</td>
<td>3.26E+07</td>
<td>181</td>
<td>261</td>
<td>363</td>
</tr>
</tbody>
</table>

Problem OP2 solver’s implementation and testing

A gradient-descent-based approach to solve OP2 was attempted unsuccessfully. During the implementation phase, it was found that the target function is non-convex: Several bumps occur for values of c equal to any of the TSD sensor locations and any value of LTE. These bumps are caused by the discontinuity at the joint, where locally the deflection slope (a component of the measured deflection velocity [up to a constant]) diverges.
As far as the gradient descent solver for OP2 is concerned, the bumps in the SSE function act as walls preventing the iterative descent to reach the global minimum: This can be clearly be appreciated in figure 45, which show the outcome of a performance test for OP2’s solver based on a simulated pavement similar to that from OP1 test and targets \([c, \text{LTE}] = [0.40, 0.75]\). It can be told that the two departure points located within the same bumps as the target \([c, \text{LTE}]\) pair converge smoothly, yet descents from other departure points get stalled half way.

![Figure 45: OP2-solver performance test. Left: Evolution of both decision variables during gradient descent. Right trajectories followed by both decision variables for each descent attempt. Note how Try5 diverges from the target.](image)

Thus, an alternative brute-force solution was implemented to solve problem OP2 that takes advantage of the upper and lower constraints for the two decision variables: SSE between calculated deflection slopes and the TSD measurements is computed for many combinations of \(c\) and LTE, the combination that minimizes SSE is reported as the solution.

**Validation test: back-calculation of joints’ LTE from real TSD measurements**

This chapter presents a validation test of the entire back-calculation scheme over real TSD data from a jointed pavement segment collected at a 5-cm resolution. The TSD measurements were gathered at the MnROAD test track in September 2021 during a demonstration run of the 4th-generation TSD – a portion of these data were presented in figure 39.

For this test, data from three jointed concrete sections located in the low volume road were made available by the TSD operator. Table 8 presents basic descriptive information regarding these segments, which were constructed in 2017 (Van Deusen et al., 2018; Barman et al., 2021; Khazanovich et al., 2021):
Table 8: Featured MnROAD Low Volume Road sections.

<table>
<thead>
<tr>
<th>MnROAD Section</th>
<th>begin [m]</th>
<th>end [m]</th>
<th>Concrete slabs</th>
<th>E [GPa]</th>
<th>Slab thickness</th>
<th>Base and subgrade</th>
</tr>
</thead>
<tbody>
<tr>
<td>124, 324, 424, 524 – 2017</td>
<td>1485</td>
<td>1630</td>
<td>4.5m [L] x 3.6m [W], 25mm dowels</td>
<td>28</td>
<td>0.15m</td>
<td>0.15m (Class 6 material) + sandy subgrade</td>
</tr>
<tr>
<td>138, 238 – 2017</td>
<td>3150</td>
<td>3280</td>
<td>4.5m [L] x 3.6m [W], 30mm dowels</td>
<td>24</td>
<td>0.20m</td>
<td>0.13m base + clay subgrade</td>
</tr>
<tr>
<td>239 – 2017</td>
<td>3385</td>
<td>3440</td>
<td>1.8m x 1.8m panels fiber-reinforced</td>
<td>24</td>
<td>0.10m</td>
<td>0.15m (Class 6 material) + 0.10m clay borrow + clay subgrade</td>
</tr>
</tbody>
</table>

Deflection testing with an FWD device was conducted in 2019 at several select transverse joints. The test results are publicly available through InfoPave, LTE estimates were produced following the guidelines in Schmalzer et al., 2007.

The back-calculation scheme for each section was implemented as follows:

1. As a pre-processing stage, the TSD deflection velocity data from all 10 sensors was denoised by Haar wavelet denoising (Katicha, 2022).
2. Then, each transverse joint was located in the data by identifying the stations at which high-amplitude peaks were recorded.
3. For each identified joint within each section, the measurements gathered between 2.25m and 1.75m ahead from the joint station were utilized as mid-slab measurements to back-calculate $k$ and $E$ (problem OP1). The sub-grade was assumed a Winkler sub-grade in this test. The average $k, E$ estimates from all back-calculation attempts was kept as representative of that location.
4. Given the results for OP1, the measurements collected 1.50m and 0.20m from the joint location were fed to OP2’s brute-force solver to estimate $c$ and LTE. At each call to OP2 solver, $c$ was constrained to the difference in stations between the input measurement station and the joint location plus/minus 0.20m.

Step 3 could not be implemented for section 239, the pavement’s short slabs cannot let the TSD get a mid-slab measurement without any influence from the joint. Thus, for this section, default $k$ and $E$ values were assumed based on the available information (table 8).
The calculation results were exported to a GIS platform for easy visualization and presentation (figures 46 to 48). Computation time for each section was between 40-60 minutes.

Table 9 presents the median $k$ and $E$ estimates for each of the featured jointed pavement sections, whereas tables 10 to 12 present the back-calculation results for all the transverse events that were automatically detected in all three surveyed sections. Observing the station at which each transverse event was detected, it is evident that a portion of the reported joints are false positives (either mid-slab cracks or misinterpreted events), for their distance to the previous reported event is less than the joint spacing. Moreover, some joints were missed by the automated detection, given some detected joints distancing values being greater than the actual joint spacing. This results encourages further research into creating an improved joint detection technique for 5-cm TSD datasets. In any case, the majority of the back-calculated events yielded reasonable results: the estimated material properties $[k$ and $E]$ fell within the usual range for a concrete pavement on ground (table 9).

*Table 9: Median back-calculated material properties*

<table>
<thead>
<tr>
<th>Section</th>
<th>$k$ [N/m²]</th>
<th>$E$ [GPa]</th>
<th>Target $E$ [GPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>124-524</td>
<td>6.2E+07</td>
<td>33.0</td>
<td>28</td>
</tr>
<tr>
<td>138-238</td>
<td>4.7E+07</td>
<td>23.2</td>
<td>24</td>
</tr>
<tr>
<td>239 (*)</td>
<td>6.8E+07</td>
<td>24.0</td>
<td>24</td>
</tr>
</tbody>
</table>

(*): Problem OP1 was not implemented for section #239 because no suitable mid-slab measurements free from the joints’ influence could be collected. OP2 solver for the joints’ LTE was fed the literature-based $k$, $E$ estimates.
Figure 46: Back-calculated joints for LVR section 124-524.

Figure 47: Back-calculated joints for LVR section 138-238.

Figure 48: Back-calculated joints for LVR section 239.
### Table 10: Back-calculation results for Section 124-524

<table>
<thead>
<tr>
<th>Station [m]</th>
<th>spacing</th>
<th>k [N/m³]</th>
<th>E [N/m²]</th>
<th>G [N/m²]</th>
<th>LTE</th>
<th>FWD-based LTE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1493.05</td>
<td></td>
<td>9.40E+07</td>
<td>2.53E+10</td>
<td>0</td>
<td>0.58</td>
<td>0.97</td>
</tr>
<tr>
<td>1494.75</td>
<td>1.70</td>
<td>6.18E+08</td>
<td>1.02E+11</td>
<td>0</td>
<td>0.16</td>
<td></td>
</tr>
<tr>
<td>1497.00</td>
<td>2.25</td>
<td>4.68E+08</td>
<td>4.57E+10</td>
<td>0</td>
<td>0.52</td>
<td>0.84-0.96</td>
</tr>
<tr>
<td>1509.65</td>
<td>12.65</td>
<td>5.73E+07</td>
<td>3.76E+10</td>
<td>0</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>1527.90</td>
<td>18.25</td>
<td>5.14E+07</td>
<td>3.37E+10</td>
<td>0</td>
<td>1.00</td>
<td>0.94-0.96</td>
</tr>
<tr>
<td>1532.55</td>
<td>4.65</td>
<td>6.04E+07</td>
<td>3.56E+10</td>
<td>0</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>1542.55</td>
<td>10.00</td>
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<td>3.52E+10</td>
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<td></td>
</tr>
<tr>
<td>1548.40</td>
<td>5.85</td>
<td>5.15E+07</td>
<td>3.10E+10</td>
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<td>1.00</td>
<td>0.92-0.95</td>
</tr>
<tr>
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<td>1.00</td>
<td></td>
</tr>
<tr>
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<td>5.60E+07</td>
<td>3.30E+10</td>
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<td>1.00</td>
<td></td>
</tr>
<tr>
<td>1573.90</td>
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<td>3.18E+10</td>
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<td>1.00</td>
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<tr>
<td>1579.35</td>
<td>5.45</td>
<td>8.42E+07</td>
<td>3.11E+10</td>
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<td>1588.10</td>
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<td>7.27E+07</td>
<td>3.23E+10</td>
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<td>0.99</td>
<td></td>
</tr>
<tr>
<td>1591.80</td>
<td>3.70</td>
<td>4.78E+07</td>
<td>3.38E+10</td>
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<td>0.99</td>
<td>0.93-0.98</td>
</tr>
<tr>
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<td>1.00</td>
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<tr>
<td>1601.50</td>
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<td>8.73E+07</td>
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<td>0.99</td>
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<tr>
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<td>0.98</td>
<td>0.90-0.95</td>
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<td>0.96</td>
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</tr>
<tr>
<td>1616.95</td>
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<td>2.14E+10</td>
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</tr>
<tr>
<td>1619.80</td>
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<td>8.76E+07</td>
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<td>0.99</td>
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</tr>
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<td>0.99</td>
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</tr>
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<td>2.88E+10</td>
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<td>1.00</td>
<td></td>
</tr>
</tbody>
</table>
Table 11: Back-calculation results for Section 138-238

<table>
<thead>
<tr>
<th>Station [m]</th>
<th>spacing</th>
<th>k [N/m³]</th>
<th>E [N/m²]</th>
<th>G [N/m²]</th>
<th>LTE</th>
<th>FWD-based LTE</th>
</tr>
</thead>
<tbody>
<tr>
<td>3152.25</td>
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<td>1.88E+10</td>
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<tr>
<td>3154.15</td>
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<td>3.36E+07</td>
<td>2.63E+10</td>
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<td>1.00</td>
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<td>3157.10</td>
<td>2.95</td>
<td>5.32E+07</td>
<td>2.07E+10</td>
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<tr>
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<td>4.76E+07</td>
<td>2.36E+10</td>
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<td>1.00</td>
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<td>3168.25</td>
<td>4.35</td>
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Table 12: Back-calculation results for Section 239

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Concerning back-calculated LTE indices, the estimates obtained roughly matched the FWD-based values, particularly accounting for the fact that the comparison data was not collected simultaneously to the TSD survey (the TSD data is 2 years older than the TSD survey), and the test track pavement may have deteriorated during the period elapsed between these survey campaigns. Broadly speaking, the TSD reported that overall the pavement joints in sections 124-524 and 138-238 boast good structural health, with the clear exception of the transition zone at around station 1493, plus a single joint with an LTE lower than 0.80. Conversely, the TSD reports that section 239’s pavement (a thin 10cm concrete structure) is highly deteriorated, for most of the transverse joints have poor LTE values (LTE lower than 0.70, TxDOT, 2021). In any case, the back-calculation methodology presented herein lets the TSD fill the blanks in the LTE record with a single sweep.

**Conclusions**

Scavone et al. (2022-b) revisited the principle underlying the TSD pavement deflection velocity measurements. It was found that, among others, on jointed pavements the TSD provides reliable deflection velocity measurements and that these may contain information about the transverse joints’ LTE index. The mechanistic back-calculation presented in this paper is the cornerstone of a comprehensive data analysis stream for TSD measurements on jointed concrete pavements, spanning both data denoising and interpretation. The trial run on the MnROAD test track data proved that reasonable numerical values can be obtained from the TSD data. If adopted in practice, network-wide structural health checks for concrete pavements and their load-bearing joints could be performed swiftly with a TSD, obtaining
sensible structural health metrics and saving days-long expensive and potentially unsafe survey campaigns with stop-and-go devices.

Moreover, stating the jointed pavement back-calculation problem as two sequenced optimization problems boasts two advantages: robustness against lacking measurements from any of the TSD sensors by keeping both problems low-dimensional, and utilization of multiple sets of TSD measurements which would otherwise be discarded – a formulation inspired in big data mining. Plus, the use of gradient descent on at least one of the optimization problems contributes to practicality, for it guarantees the algorithm converges fast to a solution.

Opportunities for further research

Nielsen and Jensen (2021) argue that information about the joints’ structural health may be inferred from the deflection velocity signals from both two sensors around the TSD rear-axle wheel (one forward and one trailing sensor). However, no interpretation model that builds on this assertion has been published yet. A pairwise comparison between the LTE estimates from such a model and this paper’s linear-elastic back-calculation framework would constitute an interesting research exercise.

Finally, there is still room for enhancements to the back-calculation implementation to render it more comprehensive and/or realistic. Some examples follow:

- A natural enhancement to the back-calculation core is the consideration of asphalt overlays. The implementation presented in this paper does not account for asphalt wearing courses on top of the concrete slab, yet Van Cauwelaert (2004) provides the solution of the deflection bowl in terms of both the concrete and asphalt overlay thicknesses and both materials’ elastic moduli. Thus, implementation of this enhancement is worthwhile so that plain concrete and composite pavements could be assessed under the same framework.

- Additionally, it may be of interest to also account for the loss of support of low-LTE joints. This phenomenon could be modeled as a drop in the subgrade’s strength parameters from the mid-slab values. Conceptually, such an enhancement to the back-calculation problem is simple, yet it involves another layer of brute-force search for the solution of OP-2, which entails additional computational cost.

- Finally, stating the back-calculation problem as two sequenced optimization problems ed two advantages: Robustness against lacking data from any sensor of the TSD, and utilization of
measurements both collected mid-slab and near the joint. However, this approach may be unsuitable for short-slab pavements, for the measurements at mid-slab locations cannot be replicated with a joint-less pavement model (recall the case study featuring MnROAD section 239). Thus, in order to avoid retorting to the high-dimensional back-calculation problem for jointed pavements, a computationally fast and rational alternative interpretation technique for the TSD measurements must be sought.

All in all, this paper is yet one more building block to a comprehensive jointed pavement management framework, turning deflection data into structural health information concerning the concrete slabs, the subgrade material, and the load-bearing transverse joints. Consequently, the pavement manager may now design and allocate maintenance resources more rationally, materializing the ultimate objective of maintaining the jointed pavement network in a state of good repair.

Acknowledgments

The authors would like to thank Dr. P. Deep (U. Nottingham, UK), for their guidance in the understanding of Van Cauwelaert’s jointed-slab-on-ground model, and Dr. B. Davids (U. Maine, USA) for furnishing a copy of the EverFE software, which was used to beta-test the code implementation of the linear elastic slab-on-ground model. Also, thanks to J. Daleiden (ARRB Systems, USA) for providing the deflection data from the 4th-gen TSD trial run at the MnROAD LVR facility.
References


Davids, W.; Mahooney, J; (1999): Experimental Verification of Rigid Pavement Joint Load Transfer Modeling with EverFE. Transportation Research Record 1684, 81-89.


Zhang, A., Lipton, Z. C., Li, M., & Smola, A. J. (2021). *Dive into deep learning*. Online publication. URL: [https://d2l.ai/](https://d2l.ai/)
Presentation

This Dissertation proposes a comprehensive data analysis and interpretation scheme for TSD measurements collected on jointed pavements. This scheme allows detecting localized structurally-deficient spots at the network level, and estimates the pavement’s, subgrade’s, and joints’ structural parameters via linear-elastic back-calculation. Throughout this Dissertation, the different building blocks of this analysis procedure were thoroughly discussed, their performance tested with 1-m resolution data, a data resolution at the lower edge of the usual reporting intervals for the TSD (Austroads, 2012; Flintsch et al., 2013), and even 5-cm data, thanks to the latest advances in TSD sensing technology.

A major tenet of this Dissertation research effort was applicability in engineering practice. Thus, all original data processing solutions presented in this Dissertation were designed to rely on input data that can be obtained with reasonable ease. Moreover, the software implementation of these solutions is to be publicly released so as to remove any barriers towards their adoption by practitioners or to bolster future research efforts.

This Dissertation also prompted a thorough revision of how TSD measurements are generated, what information is actually encoded within them, and the limitations of the current state-of-the-practice concerning processing of the pavement deflection velocity measurements. This endeavor was particularly relevant to adequately construe the measurements collected nearby the pavement’s transverse joints, but the insight gained can also be extended to the interpretation of measurements gathered on any continuous but non-homogeneous pavement structures.

Summary of Findings

This section summarizes the most relevant findings comprised within this Dissertation’s research activities. For the sake of clarity, each of the key results listed below includes a reference to its source paper.

Results for the Data Extraction Stage

The Basis Pursuit (Chen et al., 2001) [BP]-based feature extraction technique implemented for the TSD is an effective approach for both removing the measurement noise and detecting events of interest. In
particular, it was found suitable to recover the pulse responses from transverse joints on an overlaid concrete pavement (Chapter 3). The only procedural concern that limits a streamlined implementation of BP for the TSD (the choice of the hyper-parameter $\lambda$’s value) is overcame by Reweighted $L_1$ Minimization [RWL1] (Candès et al., 2008), which rationally balances goodness-of-fit with a parsimonious and easy to interpret TSD measurement reconstruction. BP decomposition of the TSD measurements as wavelets and pulses, enhanced with iterative passes of RWL1 proved suitable to recover the pulse responses of not only the deficient transverse joints in concrete pavements (even when overlaid with asphalt), but also other localized structurally weak spots worthy of further investigation (Chapter 4).

The demonstration examples presented in Chapters 3 and 4 show that 1-m resolution TSD data can provide valuable deflection information for investigatory network-wide evaluations. Analysis of these deflection measurements by the BP-based techniques presented here (enhanced with iterative RWL1) may help the practitioner detect structurally deficient locations within a pavement corridor or network.

**Results for the Data Interpretation Stage**

The insight gained on the nature of the TSD pavement deflection velocity data (Chapter 5) allows concluding that (1) the TSD can reliably gather pavement deflection velocity data, and (2) the relationship between deflection velocity $[v_y]$ and deflection slope is not direct, and slopes can be inferred from deflection velocities (Krarup et al., 2006) only under limited circumstances. It was discovered that such relationship only holds when the deflection basin maintains a constant shape as the TSD load travels over the pavement structure – a stationary deflection basin of sorts. Otherwise, a second component of the deflection velocity signal, which quantifies how the deflection basin changes shape over time, must be considered. This finding is particularly relevant on jointed concrete pavements, because as the TSD load approaches the joint, the deflection basin shape changes shape very rapidly (figure 49) and thus the pavement’s $v_y$ is dominated by the partial derivative term over the slope term – illustrated graphically in figure 50. Thus, the back-calculation of pavement properties for a jointed structure should be done in terms of deflection velocities only.
Concerning the automated interpretation of the recovered TSD signals (Chapter 6), a mechanistic back-calculation procedure based on linear-elastic modeling (Van Cauwelaert, 2004) has been implemented to estimate the pavement’s and subgrade’s stiffness values and, whenever a pulse response from a transverse joint was recovered, provide an estimate of its load transfer efficiency [LTE] index as well. This implementation is based on the works by Deep et al. (2020, -a, -b, -c) for the Raptor, with additional enhancements to account for the differences between the measurements reported by the Raptor and the TSD (fundamentally, $v_y$ data from the 10 sensors of a 4th-generation TSD at a 5-cm spatial resolution).
The proposed implementation regarded the back-calculation problem as two sequenced optimizations, each with few unknown decision variables: The first optimization problem provides estimates of the concrete pavement’s and subgrade’s strength parameters from mid-slab $v_y$ measurements, and the second problem estimates the joint’s LTE from near-the-joint $v_y$ measurements using the outcome of the first problem as input. This implementation takes advantage of the multiple measurements collected by the TSD and ensures numerical stability by keeping both problems low-dimensional (two or three unknown decision variables each). These back-calculation problems were beta-tested with real TSD data gathered at the MnROAD low-volume-road concrete sections (Van Deusen et al., 2018), for which LTE test data at select transverse joints was available. In spite of the limited comparison information available, the back-calculation outcome showed an encouraging prospect for network-wide screening using a TSD, for a reasonable agreement between the TSD-based LTE estimates and the comparison values from an earlier FWD campaign was achieved but, more importantly, an LTE map of almost all the transverse joints and other mid-slab transverse events could be generated from the rich 5-cm-resolution data collected by the 4th-generation TSD device.

Dissertation conclusion

The Dissertation research succeeded in developing a data extraction and interpretation procedure for the structural health assessment of jointed concrete pavement structures with TSD data. TSD measurements at different spatial resolutions may serve different needs, and a special sub-set of the analysis techniques presented herein may be applied accordingly:

- The TSD 1-m resolution data allows for exploratory analysis at the network level and detection of structurally deficient locations (weak spots). Basis Pursuit denoising + RWL1 can be used to identify these weak spots based on the amplitude of the recovered pulse signal.

- The 5-cm resolution data can be processed for jointed pavement properties and joints’ LTE index at the project level or corridor level via linear elastic back-calculation. Mid-slab measurements can be processed to estimate the pavement’s and subgrade’s material properties, and the measurements collected nearby a transverse joint can provide estimates of the joint’s LTE. Additional pavement information (concrete layer thickness) is required to perform the back-calculation.
Recommendations for implementation in engineering practice

From a pavement management standpoint, this Dissertation research simply provides one of the many pieces to the puzzle for holistic decision-making: The TSD data and the streamlined analysis tools developed as part of this research provide a depiction of the jointed pavement network current condition. Yet it is up to the manager to assign the most suitable preservation or rehabilitation treatment based on said current condition data and observing the minimum quality standards and budget constraints to be met (AASHTO, 2012; Haas et al., 2015).

As such, possible steps towards a holistic jointed rigid pavement network management practice reliant on structural condition data furnished by the TSD could consist of:

1. Estimating the current and future distress level based on the structural health metrics estimated by the TSD and other surface-condition-based data, plus the traffic demand for the years to come, as it is done for instance within the MEPDG (AASHTO, 2020).

2. Setting the maximum acceptable deterioration levels for these structural-health-related pathologies before flagging a pavement segment as due for maintenance work. For example, TxDOT (2021) marks transverse joints on a concrete segment as due for retrofit when their LTE index drops lower than 0.70.

3. Streamlining the network-level maintenance allocation process based on the current and future condition forecast solved in stage #1 and decision-making rules stated in stage #2. Shrestha’s (2022) implementation for Virginia’s flexible pavement network would serve as a template to follow towards implementation.

Recommendations for future research

In addition, there still remain research opportunities to further develop this Dissertation’s findings, both at the data extraction and interpretation stages:

Enhancements to the Data Extraction Stage

One unexplored application of Basis Pursuit [BP] for TSD data analysis is segmentation of a pavement corridor or network into homogeneous sections. Chen et al (2001) hinted at the idea of decomposing a given input signal (like TSD measurements) using a heaviside signal dictionary. The recovered outcome would be a step signal, each constant amplitude step shall represent the default deflection response from a
segment of pavement. As such, a given corridor or network could be segmented based on its structural capacity. Moreover, the BP decomposition of TSD measurements under the step signal dictionary is a particular case of Total Variation [TV] denoising (Rudin et al., 1992), for which a solution can be achieved fast (Condat, 2013).

Alternatively, the 5-cm-resolution deflection velocity \([v_y]\) measurements from nearby the transverse joints appealingly hint at further analysis from a signal-processing perspective. These signals’ pulse width, curvature, and amplitude may directly depend on the pavement and subgrade’s strength parameters and the joint’s LTE index. A parametric study into the \(v_y\) signal features could be conducted so as to describe what pavement properties (or compound quantities computed from them) best describe each of these signal features and thus eventually construct a signal dictionary of possible \(v_y\) pulses with which get structural properties’ estimates directly via BP.

Enhancements to the Data Interpretation Stage

Finally, the linear-elastic jointed slab-on-ground model (Van Cauwelaert, 2004) can be extended to simulate an overlaid jointed concrete pavement response to the passing of the TSD load. This being said, if this enhanced model were implemented into computer code, the LTE back-calculation procedure presented in Chapter 6 could be extended to the composite pavement realm, and, consequently, the TSD could be then utilized in practice to swiftly assess these pavements’ structural condition mechanistically.

Alternatively, and given the relevance concrete overlays – or white-topping treatments (Harrington, 2008) are gaining as heavy-duty pavement preservation solutions, the question remains whether the jointed-slab-on-ground model is actually suitable to simulate the deflection response from these pavement structures as well. If that were the case, then this modeling framework plus the TSD could furnish the insight so as to maintain these composite pavements in a state of good repair.
References


Continuous Deflection Devices [CDDs] (Flintsch et al., 2013) or Moving Measurement Platforms [MMP] (Andersen et al., 2017; Madsen and Levenberg, 2019) are acronyms used to define non-destructive pavement deflection measurement devices that collect deflection measurements produced by the device’s weight loads as it travels over a given stretch of pavement. CDDs have been developed mainly in response to the need for network-wide structural data pavement engineers require for proper management practices, which cannot be reasonably collected with traditional stop-and-go devices like the widely-used falling-weight deflectometer [FWD] (European Commission, 1997; Brezina et al., 2017; He et al., 2017; Nasimifar et al., 2018-b; Haas et al., 2015; Arora et al., 2008; Saremi et al., 2019).

Traffic-speed deflection devices [TSDDs] are a sub-set within the CDD family of devices. As the name indicates, TSDDs are those CDDs that are capable of collecting surface deflection measurements at high speeds (55 km/h [35 mph] or greater, as defined by Flintsch et al. (2012, 2013)). The advantage of TSDDs
over crawl-speed CDDs for in-service pavement evaluation is obvious: On one side, TSDDs can cover a larger network than a CDD given a fixed time frame (between 220-640 km/day [140-400 miles/day]) while at the same time they pose no disruption to the normal flow of traffic (Baltzer et al., 2010; Brezina et al., 2017; Flintsch et al., 2012-2013; Steele et al, 2020, Elseifi et al., 2011; Li et al., 2013; Rabe, 2013). Development of TSDDs started in the 1990s (European Commission, 1997; Rada et al., 2016; Steele et al., 2020); and the first working prototypes were released in the late years of such a decade and the early 2000s (Andrén, 1999-2000; Arora et al., 2006; Phares et al., 2008; Hildebrand and Rasmussen, 2002; Prozzi et al., 2017; Rada et al., 2016). Since then, numerous research projects have been carried out on how to assess the performance of these devices and interpret and utilize the data they collect, with the ultimate intention of adopting TSDDs as a survey tool for routine pavement surveying for both project-level pavement engineering and network-level asset management (Rada and Nazarian, 2011; Rada et al., 2016; Flintsch et al., 2013; Elseifi et al., 2011; Elseifi and Elbagalati, 2017; Steele et al., 2015, 2020; Brezina et al., 2017; Saremi et al., 2019; Katicha et al., 2021).

In this Appendix, we present a list of TSDDs developed from the 1990s till the present day, offering a brief description of their measuring principle, pointing out the state of the technology, their commercial availability, and briefly summarizing key field tests and in-service trials in which these were protagonists. Similar state-of-the-technology reports were produced within major research efforts and feasibility studies in the United States (Arora et al., 2006; Phares et al., 2008; Rada and Nazarian, 2011; Flintsch et al., 2013; Rada et al., 2016; Katicha et al., 2021) and abroad (Andrén, 2006; Li et al., 2013; Brezina et al. 2017, He et al., 2017). This chapter will blend the information contained therein with the results of a literature search of published documents from the past 20 years and published documentation on the latest technological developments and most recent case studies.

This TSDD technology review covers the following devices, sided by recommended entry-level references describing them:

- Road Deflection Tester [RDT] (Andrén, 2006)
- Traffic Speed Deflectometer [TSD] (Hildebrand and Rasmussen, 2002; Austroads, 2016)
  - Fork: Moving Weight Deflectometer [MWD] (Kamiya et al., 2018)
  - Fork: The Laser Dynamic Deflectometer [LDD] (Li et al., 2013; He et al., 2017)
- Rolling Wheel Deflectometer [RWD] (Rada and Nazarian, 2011; Steele et al., 2020)
- Rapid Pavement Tester [RAPTOR] (Andersen et al., 2017)
For each of the above-listed devices, the following sections furnish a succinct description of the device itself, its measurement principle, its operational status (in-service / decommissioned), commercial availability, and a historical review of major research projects in which these were featured.

Other deflection prototypes currently exist or have existed in previous years but were removed from this review as these are either crawling speed devices like the Curviàmetre (European Commission, 1997; Andrén 2006; Nasimifar, 2015; Rada et al., 2016) and the Deflectograph family (Andrén, 2006; Phares et al., 2008; Ferne et al., 2009-a; Baltzer et al., 2010; Flintsch et al., 2013), or they are decommissioned one-offs, like the Quest’s ARWD [the acronym stands for Airfield Rolling Weight Deflectometer, the device was decommissioned in 2011 (Flintsch et al., 2013)], and an early RWD prototype from Dynatest mentioned in Rada et al.’s (2016) report – never tested, yet probably a predecessor for the Raptor; these devices were reviewed, among others, by Andrén (2006); Arora et al. (2006); Phares et al. (2008); Rada and Nazarian (2011); Flintsch et al., (2013); Rada et al. (2016).

The Road Deflection Tester [RDT]

The Road Deflection Tester is a TSDD developed in Sweden during the 1990s, after an early prototype whose development started in 1985; this device has been intended as a network-wide pavement structural data collection tool for management purposes (Andrén, 1999, 2006). Andrén (2006) provides a thorough log of the development of the RDT, from its early prototype stage to the first trial runs in Swedish in-service pavements. Similar information is provided in the reports by Arora et al., (2006) and Rada and Nazarian (2011). This device is also briefly described by Flintsch et al. (2013).

The RDT vehicle is a specially modified single-unit truck – the truck’s engine is relocated to the back of the vehicle to redistribute the weight towards the rear axle, and the dual wheels in the rear axle were substituted for super-wide single tires (Figure 51). During testing, the load applied through the rear axle is 112 kN. The vehicle’s cabin provides accommodation for the operators and houses the data logging hardware (Andrén, 2006).
The RDT utilizes two arrays of laser sensors, mounted perpendicularly to the direction of travel. One of these arrays is located close to the rear axle, more precisely 0.50m behind the axle itself. Meanwhile, the second laser array is located 2.50m ahead of the rear axle. Each array is mounted on an aluminum beam [2.50m in length] and consists of 20 sensors (laser range finders – point triangulation lasers), distributed symmetrically, sixteen of these are vertically mounted, and the remainder four -those that are the farthest from the vehicle’s axis of symmetry, are skewed and point outwards. The first array of lasers collects deflection data from the deflection basin caused by the rear axle load, while the other array of lasers collects measurements from a non-deflected area (Andrén, 2006). Additional sensing equipment mounted on the RDT is optical speedometers to accurately measure the vehicle’s velocity, a set of accelerometers and gyroscopes, to record any additional movements, and load transducers on the rear axle (Andrén, 1999, 2006). Andrén (1999, 2006) describes a two-step procedure to calibrate the laser sensors on each array before the operation of the RDT.

An important operational aspect of the RDT -that was also raised as a concern about the operation of other TSDDs- is that dust, debris, or pooled water on the pavement surface may lead to measurements being incorrect or not even being collected at all. Also, the RDT’s data collection capabilities are reduced when surveying dark-colored and/or heavily cracked pavements (Andrén, 1999).

The measurement principle behind the RDT is the “deflection area of the deflection profile”: for each location, the RDT computes the difference in elevation between the profiles taken with the rear array (loaded status) and front array (unloaded status). From there, the “deflection area” [area of the deflection profile] and the “wire deflection area” [area between the deflection profile and the line drawn between the left-most and right-most points] are computed. Additionally, the RDT logging software computes the
maximum deflection at each location (maximum height of the deflection profile), and the “beam index”, which is the difference in elevation between the bottom of the basins from the two wheels and the bump formed in between (Andrén and Lenngren, 2002; Andrén 2006).

The deflection data is sampled at a 1kHz frequency, which, at a travel speed of 70 km/h [43.5 mph], each deflection basin data set is separated by 19.4 mm [0.76 inches] (Andrén, 1999). However, since the RDT is known to bounce due to roughness with a frequency of 1.5-2.0 Hz, oscillations in the deflection measurements may need to be pooled to a spacing of about 50m (Andrén, 1999).

Trial tests with the RDT on open roads took place between 1994 and 1997, during which simultaneous deflection measurements were taken with the RDT and FWDs. (Andrén, 1999; Andrén and Lenngren, 2000; Andrén, 2006). The testing involved measurements in both asphalt and concrete pavements, and repeatability and speed-dependency results were presented (Andrén, 1999). The RDT satisfactorily passed the trial repeatability tests, and it was found that the deflection measurements taking on asphalt pavements actually depended on the vehicle’s speed – a reasonable result given the visco-elastic nature of asphalt pavement layers (Pedersen, 2013; Nasimifar, 2015, Nasimifar et al., 2018-a,-b; Nielsen, 2019-a,-b). This testing program involved also comparisons with FWD (simultaneous measurements at trial locations) measurements and long-term repeatability tests (repeated surveys at a fixed location spanning five years) (Andrén and Lenngren, 2002; Andrén, 2006). Results for both tests were successful.

**Current Status of the technology**

As of 1997, the RDT became the property of the Swedish National Road and Transport Research Institute [VTI] (Andrén, 2006). However, as of 2011, Rada and Nazarian (2011) reported that development on the RDT has stopped. Similarly, Flintsch et al. (2012, 2013) pointed out that the RDT research may have reached a hiatus and the vehicle may have been removed from service. No more recent published activity involving the RDT has been found. Moreover, the Swedish VTI has no recent library entries about the RDT, implying that the RDT has been decommissioned12.

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12 Query performed at [https://www.vti.se/en/publications/](https://www.vti.se/en/publications/). Accessed 2020-09-18. The decommissioning of the RDT was confirmed personally by P. Andrén over a direct inquiry in early 2022: RDT research was terminated in 2006, and the RDT retired from service afterwards. The RDT owners eventually sold the vehicle in the early 2010s.
ARA Inc. Rolling Wheel Deflectometer [RWD]

Applied Research Associates [ARA], Inc. commenced the development of a Rolling Weight Deflectometer in the 1990s. A first working prototype was launched in 2003 and was demonstrated in numerous field tests throughout the USA (Arora et al., 2006; Phares et al., 2008; Rada and Nazarian, 2011; Smart, 2011; Wilke, 2014; Steele et al., 2015, 2020; Flintsch et al., 2013; Rada et al., 2016). In recent years, the RWD underwent major modifications in its sensor system, the updated rig has been field-tested in 2019 (Steele et al., 2020).

The RWD measures the deflection basin produced by its own rear axle load – a dual-wheel single axle with a total load of 18 kips [1 AASHTO Equivalent Single Axle Load]. The RWD instrumentation is mounted on a 16m [53 feet] long trailer, such a trailer was utilized on purpose to isolate the basin from the rear axle from the deflections produced by the axles of the tractor unit. The RWD trailer is climate-controlled so as to maintain the sensing equipment at a constant temperature (Rada and Nazarian, 2011; Steele et al., 2015).

The original sensing system consisted of a series of four triangulation lasers mounted on a longitudinal beam – three lasers (located ahead of the trailer’s rear axle) sensed the elevation of the un-deflected pavement whereas the fourth laser (located approximately 6 inches behind the rear axle, between the dual tires) sensed the elevation of the deflection basin. The basin elevation was computed using the “spatially coincident methodology”, which in layman’s terms, compares the elevation reading from a fixed location in the pavement as sensed from the forward and rear lasers against the elevation measurement taken with the rear laser – the RWD’s data acquisition software matches the data from all lasers accordingly. The RWD data sampling frequency was 2 kHz (which at its top traveling speed of 88 km/h [55 mph] returns a data point every 13mm [0.5 inches]), but deflection measurements were reported as a single value every 30m [100 ft] (Arora et al., 2006; Phares et al., 2008; Flintsch et al., 2012,2013; Rada et al., 2016), or at 160-m [0.1-mile] intervals for network-level analyses (Wilke, 2014; Steele et al., 2015; Rada et al., 2016).
In recent years, the RWD received a major overhaul in its deflection sensing equipment (Steele et al., 2020): the triangulation lasers system was replaced with a vision-based device composed of a pair of high-shutter-speed cameras mounted ahead of the rear axle (Figure 52). The cameras collect images of the pavement surface within the deflection basin and ahead of it. Artificial illumination is provided from an array of LED lights to prevent unwanted shadows within the camera’s field of view. The new system was designed to obtain a visualization of the entire deflection basin, instead of point elevations. No details were published to date on how the deflection basins are calculated from the retrieved imagery. The imagery system in the RWD collects pictures of the pavement every 8.3m [25 feet] and returns an average deflection basin every 167m [500 feet] for network-level analysis.

**Current Status of the Technology**

As of 2016 (Rada et al., 2016), only one RWD unit was built and in working operation and it was not commercially available. That very vehicle was the RWD unit that was fitted with the image-based sensing technology and tested in 2019 (Steele et al., 2020). Yet, as of September 2020, the RWD research project was terminated and the RWD was retired from service. Nonetheless, as of early 2022\(^{13}\), ARA Inc. still keeps the RWD device and its corporate website maintains a blurb describing the RWD and its original laser-based sensing system\(^{14}\).

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\(^{13}\): After an interview with ARA Inc. Representatives at the 101\(^{st}\) TRB Annual Meeting (January 2022)


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Key Research Projects based on the RWD

The RWD has been utilized in extensive research projects carried out during the past fifteen years in the USA. Broadly speaking, the main field tests in which the RWD was utilized were either acceptance tests (device repeatability and comparisons with the FWD) and evaluations towards usage as a network-level screening tool for management purposes.

Initial testing with the RWD (repeatability test and comparisons with the TxDOT Rolling Dynamic Deflectometer [RDD] and FWD, among other devices) was carried out in 2004 \(^{15}\) (Arora et al., 2006; Phares et al., 2008; Rada and Nazarian, 2011). The repeatability test results for the RWD were satisfactory, and the RWD measurements were found to correlate with FWD and RDD readings for the test sections. By 2009, additional field demonstrations were carried out in the states of Louisiana, New Mexico, Kansas, and California (Gedafa et al. 2010, Rada and Nazarian, 2011; Elseifi et al., 2011). Flintsch et al. (2013) provide a detailed summary of all the tests in which the RWD was utilized between 2003 and 2010.

On their review of the RWD, Phares et al. (2008) observed the possibility of data loss after hard braking and comment on the “warm-up effect” that may negatively affect the data collection at the beginning of a survey operation. Additionally, Rada and Nazarian (2011), report other known operational issues tied to surveying with the RWD with the laser-based acquisition system, like sensitivity to rain, the accuracy of the laser sensors, variability in the measurements (Diefenderfer, 2010), and issues with surveying on concrete pavements with longitudinal texture and over horizontal sharp curves (Rada et al., 2016).

Early testing with the RWD at the network level in Kansas (during the year 2006) was reported by Gedafa et al. (2010, 2014), and afterward reviewed by Rada and Nazarian (2011); Rada et al. (2016); Elseifi et al. (2011); Elbagalati et al. (2016); Flintsch et al. (2012-2013); and Steele et al. (2015). The field evaluation of the RWD was intended to elucidate whether this device could improve the network managers’ decision process for prioritizing maintenance needs by providing network-wide structural number (SN) estimates for the in-service pavements as it was at the moment done with the FWD – this research topic was also the core objective behind the research projects led by Elseifi et al. (2011, 2013), Flintsch et al. (2013), and Rada et al. (2016). Briefly put, this test consisted of contrasting values of back-calculated effective structural number \( (SN_{\text{eff}}) \) for a number of pavement sections as retrieved from FWD and RWD deflection

\(^{15}\) Earlier work by Hall et al. (2004) and Steele and Hall (2004) is known to exist, and was referenced by Arora et al. (2006), and Rada and Nazarian (2011), but these original reports are lost.
measurements. The authors concluded that the RWD could be a valuable asset for network-wide structural surveys as it returned SNeff results that were statistically similar to those coming from FWD tests.

A Network-level demonstration in Champaign Co., Illinois during 2007 was originally reported by Vavrik et al. (2008), and reviewed a posteriori by Rada and Nazarian (2011)\(^{16}\); Elseifi et al. (2011); Steele et al., 2020). The authors’ goal was to use the RWD for county-wide pavement management. During the year 2008, the RWD was field-tested in the state of New Hampshire as part of a network-wide case study of continuous versus static pavement data collection procedures (Smart, 2011). In this test campaign, network-wide pavement thickness and deflection were gathered with continuous devices (GPR and RWD).

The outcome of the Louisiana field test carried out during the year 2009 was vastly utilized for engineering purposes and widely reported (Abdel-Kharek et al., 2012; Elseifi et al., 2011, 2012a, b, 2013, 2015; Elseifi and Elbagalati, 2017; Elbagalati et al., 2016, 2017a, b; Zhang et al., 2016) The test campaign was conducted in two stages: The first stage was limited to asphalt roads only (nonetheless totalizing 1250 miles of surveyed pavements), in which the repeatability of the RWD was tested against measurements at a fixed speed, and measurements taken at different speeds were contrasted (Abdel-Kharek et al., 2012; Elseifi et al., 2011). Meanwhile, the second stage covered a subset of the aforementioned plus several other segments with other materials, the RWD was compared against an FWD device. Elseifi et al. (2011) and Abdel-Kharek et al. (2012) provide a clear summary of the test results and conclude on its applicability as a network-wide survey tool for pavement management purposes – posterior work by the authors (Elseifi et al., 2012, 2013, 2015; Elseifi and Elbagalati, 2017; Elbagalati et al, 2016, 2017-a, -b; Gaspard et al., 2013; Zihan et al., 2020) focused on developing analysis tools to extract meaningful indices and detect localized structure-related problems from the reported deflection values and on making the RWD deflection data compatible with the currently used Pavement Management Systems at the State level. Among the many testing results reported by Elseifi et al. (2011), some remarkable ones are that the RWD measurements have a larger variance on weak pavements, and that, despite in agreement, the deflection measurements from the RWD are statistically different from the readings from an FWD test (Rada et al., 2016).

The RWD was assessed in the state of Virginia (Diefenderfer, 2010) as a screening tool for network-level analysis, the testing campaign spanned between 2005 and 2006. Results on device repeatability, the

\(^{16}\): The original work by Vavrik et al. (2008) [a TRB Annual meeting paper, indexed at https://trid.trb.org/view/848732] could not be retrieved during the literature review effort.
variability of the results, and comparisons against the FWD were presented. The RWD performed satisfactorily in the repeatability test, and just fairly in the correlations with FWD. However, it was found that the standard deviation of the collected measurements changed with the pavement’s surface condition. After these findings, the end verdict was contrary to pursuing further use of the RWD for network-wide analysis on strong (low-deflection) pavements (Diefenderfer, 2010; Rada and Nazarian, 2011). Nonetheless, further trial tests in posterior years concluded favorably for the RWD for network-wide use (Elseifi et al., 2011, 2012; Gaspard et al., 2013; Flintsch et al., 2013; Rada et al., 2016). Moreover, Elseifi et al. (2011) and Flintsch et al. (2012, 2013) highlight that the poor comparison between FWD and RWD may be owed to a five-month time gap between the FWD and RWD surveys.

As part of the SHRP-2 R-06 (F) project, Flintsch et al. (2012, 2013) conducted a series of assessments of the RWD among other CDDs to determine their suitability for network-level structural evaluations – the later research effort reported by Rada et al. (2016) shared the ultimate objectives. Broadly speaking, the research team assessed the RWD (and other devices) as a) a screening tool to detect weak locations within the network, b) a survey machine whose measurements could be eventually utilized to estimate the structural health of pavement segments, and c) to differentiate sections with remarkably different structural conditions, and for which different maintenance or rehabilitation strategies may be warranted. (Flintsch et al., 2012, 2013). The SHRP-2 effort collected repeatability, variability, and comparability against FWD test results published to date and recognized the capabilities and limitations of the RWD (Flintsch et al., 2012, 2013). Subsequently, further comparability studies between the RWD, the FWD, and the TSD were carried out as part of the research effort (Flintsch et al., 2013; Katicha et al., 2014-a, -b, -c, 2015); the authors concluded that the deflection measurements from the RWD are consistently higher than the readings from the FWD, but the two devices correlate.

Wilke reported (2014) a network-wide case study similar to the Kansas experience (Gedafa et al. 2010, 2014) for PennDOT. The RWD has been utilized in a pavement survey spanning over 460 km [285 miles] that covered roads from the National Highway System [NHS], non-NHS primary highways, and secondary roads. The research task was comparing Remaining Life values (as calculated with the Asphalt Institute method) from RWD data against results from processed FWD tests and predictions obtained via the State’s Pavement Management System. Wilke (2014) reported that although the RWD-based values suffered from high scatter, the RWD successfully distinguished between stronger and weaker sections that could be managed accordingly.
Following, another network-wide experience with the RWD was conducted in Oklahoma State (Steele et al., 2015). The research goal was to produce structural health indices to include in the State’s Pavement Management System [PMS] and to enhance the decision-making process concerning the prioritization of maintenance and rehabilitation works at the network-wide scale. The RWD surveyed over 1,600 km [1,000 miles] of pavements, the majority of which are hot-mix-asphalt-paved, and the reported deflections for select locations were compared with archived FWD data – although the authors expressed that correlating the two devices’ readings was not the goal of the project. The structural capacity values that resulted from the RWD survey were brought into the State’s PMS and the decision-making process for the target network was run between scenarios with structural information and without (relying only on visual surveys). The authors found out that the structural health information altered the PMS decision-making outcomes positively: the recommended maintenance activities for each section achieved equal or better a posteriori overall condition indices while saving the State precious funds. A similar conclusion was reported by Elbagalati et al. (2017-a) and Zhang et al. (2016) after the Louisiana network-wide survey, these authors quantified the amount of savings to the DOT pondering the cost of the RWD survey and the maintenance and rehabilitation savings and concluded that the enhanced management process with data from RWD surveys is indeed profitable for facilities with an Average Daily Traffic greater than 5000.

As part of the FHWA- DTFH61-12-C-00031 program, the RWD was tested in the field at a MnROAD facility along with a TSD and a Curviameter, as part of an effort to validate mechanistic analyses of pavement performance estimation based on deflection readings (Nasimifar, 2015; Rada et al., 2016). Details on this test campaign, its outcome, and derived works that resulted from it were reported extensively (Nasimifar, 2015; Nasimifar et al., 2017; Rada et al., 2015, 2016, 2018; Tyagarajan et al., 2016-a, -b; Elbagalati et al., 2017-b; Elseifi and Elbagalati, 2017; Velarde et al., 2017). The MnROAD field experiment involved testing at an instrumented enclosed location and on public primary and secondary nearby roads, both asphalt and concrete pavements were surveyed. The first evaluation in the field consisted of tests for device repeatability, precision, sensitivity to survey speed, and surface condition – all conditions for the RWD to meet so as to be deemed as suitable for network-wide surveys. Secondly, further analyses were conducted towards the validation of numerical models (based on visco-elastic theory) to predict strains within the pavement structure from deflection indices. The authors conclude by giving recommendations to the RWD manufacture on suitable improvements to the sensing system array for the RWD to accurately measure the deflection indices that best correlated with strains of interest (refer to Rada et al. (2016) for further details).
One remarkable line of work derived from this study is a series of recommendations for pavement managers on how to conduct a mechanistic-based network-wide structural assessment within a pavement management framework based on deflection indices retrieved from TSDDs such as an RWD (Nasimifar et al., 2017; Rada et al., 2016; Thyagarajan et al., 2015 a-b, 2019). Nasimifar et al. (2017), and Rada et al. (2016) highlighted that the RWD’s assessed four-laser configuration was unsuitable to compute those deflection basin indices that correlate best with observed values of strain in the pavement’s asphalt layers (which are known to contribute to failure by fatigue (NCHRP, 2004; AASHTO, 2020). Rada et al. (2016) suggest more lasers pointing to locations within the deflection basin be added to the RWD’s sensing system.

After the major renovation of the RWD, in which the laser-based sensor system was replaced by an imagery-based system, a field testing campaign that included comparisons of the new sensing equipment against the FWD was carried out during 2019 in Mississippi (Steele et al., 2020). The testing program consisted of scanning several in-service pavements with the RWD and contrast its maximum deflection results \( D_0 \) against FWD measurements. A subset of these pavement sections was instrumented with accelerometers and thus a third comparison element was provided. The contrast among instruments was extended to other deflection measurements \( D_{10} \) and \( D_{60} \), deflection indices, back-calculated pavement and subgrade properties, and back-calculated strains for mechanistic analysis. The researchers found a correlation between the \( D_0 \) values returned by the RWD and the FWD, yet the relationship was different for pavements with thick and thin asphalt layers – Steele et al (2020) explain such behavior as a consequence of the visco-elastic nature of asphalt and the difference in load nature and timing between the two devices. Good agreement was also found in terms of back-calculated properties (compared against a visco-elastic computer model), although the RWD may report slightly higher subgrade’s resilient moduli (MR) than the FWD.
The Rapid Pavement Tester / Rolling Wheel Deflectometer [RAPTOR]

The Raptor is a recently-launched traffic-speed deflection device (or Moving Measurement Platform, in terms of Andersen et al. (2017)) developed jointly by Dynatest and the Technical University of Denmark (DTU)\(^\text{17}\). The first presentation of the sensing technology within the Raptor was the conference paper by Andersen et al. (2017), and posterior scholarly articles regarding the Raptor provide further information on this TSDD (Andersen et al., 2017; Madsen and Pedersen, 2019; Athanasiadis and Zoulis, 2019; Skar et al., 2020). The Raptor is a Rolling Wheel Deflectometer device (as tagged by Andersen et al. (2017), and Deep et al. (2020)) that utilizes an array of line lasers to scan a stripe of the pavement (Figure 53). Raptor’s laser-based system was intended to outperform both ARA’s RWD and the TSD in terms of sensitivity, making it capable of providing traffic-speed project-level data (Andersen et al., 2017).

The Raptor sensing system consists of an array of 12 line lasers mounted onto a beam placed on the vehicle’s passenger side along its longitudinal axis, lasers are located both ahead and behind the vehicle’s rear wheels (see details in Skar et al., 2020). Each line laser sweeps a transverse line that is 200-mm long. At any moment in time, all lasers measure their distance to the pavement surface, the Raptor’s data acquisition system retrieves the elevation data and the pavement texture as scanned by all lasers. An image correlation software (proprietary) detects remarkable features in the pavement texture and utilizes those to match the imagery (and correlate elevation data) from all sources and different timestamps. Additional instrumentation mounted on the beam is a set of gyroscopes and accelerometers that measure the change of horizontal and vertical alignment of the support beam. Overall, the deflection measurements in the Raptor are constructed from the elevation readings from the lasers, the beam angle respective to the horizontal plane, the traveling speed, and the instant load applied by the rear wheel (Andersen et al., 2017).

As with the RWD, RDT, and TSD, the Raptor measures the deflection basin produced by the load applied by its own rear wheels. However, the Raptor’s rear wheels are not linked through an axle, and each wheel consists of a single tire. The trailer unit that encases the Raptor was custom-built to accommodate the instrumentation, the independent wheels with their corresponding suspension system, and additional weight units to adjust the load to 50 kN on each rear wheel (Andersen et al., 2017; Athanasiadis and Zoulis, 2019; Skar et al., 2020). The Raptor’s sampling frequency is 4000 Hz.

Current Status of the technology

The first Raptor unit was built in 2018 and has toured Europe during that year as part of a field demonstration program. No scholarly written information was found about the European tour of the Raptor although such activities were mentioned in ERPUG presentations and audiovisual evidence of its stop in Italy is available online. A unit of the Raptor was intended to be shipped to the USA in 2018. However, its first appearance at an American audience was at the 2019 yearly TRB Conference. The trace of the Raptor in the USA vanishes after that event.

As of November 2020, all Raptor-related assets (three working units plus related intellectual property and processing software) have been acquired by the European company Ramboll. Operation of the Raptor on European highways began in 2021, among which a multi-year network-wide survey in Norway.

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19 Ref: [https://www.youtube.com/watch?v=nu8SfwH2XFw](https://www.youtube.com/watch?v=nu8SfwH2XFw) available as of 2020-09-18.

20 Ref: [https://ramboll.com/media/rgr/heavy-traffic-ahead](https://ramboll.com/media/rgr/heavy-traffic-ahead) Accessed 2022-03-25. Additionally, these news were announced at the DaRTS 14 meeting (February 2021) and DaRTS 15 meeting (October 2021).

21 Ref: [http://www.erpug.org/media/files/forelasningar_2021/12%20nov%2009%20Martin%20Wistr%C3%B6m%20Bearing%20Capacity%20Assessment%20on%20Network%20Level%20using%20Raptor.pdf](http://www.erpug.org/media/files/forelasningar_2021/12%20nov%2009%20Martin%20Wistr%C3%B6m%20Bearing%20Capacity%20Assessment%20on%20Network%20Level%20using%20Raptor.pdf) Accessed 2022-03-25. However, as of March 2022, no written report on this network-wide survey could be retrieved.
Key Research Projects based on the Raptor

Following the presentation of the Raptor’s measurement system concept (Andersen et al., 2017) and the release of the first working prototype, research was active on how to utilize the Raptor’s raw measurement for Network-Level and Project-Level applications. The following list summarizes the work published to date:

- Athanasiadis and Zoulis (2019) presented a framework (and computer software) for a fast mechanistic back-calculation of pavement properties from Raptor’s measurements. The model the authors developed assumes a three-layer system in which the top-most layer is visco-elastic, whereas the base layer and foundation materials are linear elastic. The authors compared the back-calculation results under a visco-elastic assumption against a simplified three-layer elastic system (for which a solution exists and it is not computationally burdensome to calculate) and concluded that the latter returns biased values for the pavement properties. Additionally, they highlight that back-calculation of pavement properties from Raptor’s deflection measurements should actually include the superposition of the deflection bowls from all of the Raptor’s wheels.

- Madsen and Pedersen (2019) also published a similar framework as Athanasiadis and Zoulis (2019) to translate the deflection values from the Raptor to “equivalent” FWD measurements (similar work has been done in Louisiana for the ARA’s RWD and the TSD (Elbagalati et al., 2017-b, Elseifi and Zihan, 2018)). The authors back-calculated the layer properties of the pavement from the Raptor measurements (modeled as a visco-elastic layer plus elastic foundation materials) and, following, estimated the outcome of an FWD test on that location. This set-up was validated by comparing estimated and actual FWD measurements from a road segment in Denmark that was scanned by the Raptor. The validation test in the field was successful.

- Skar et al. (2020) reviewed the Raptor’s measurement system and showed, through forward-calculated simulated Raptor data, how the error in measurements may propagate as an error in the back-calculation of pavement properties. Such a test is part of the device’s quality-assurance framework, as it highlights what degree of accuracy and sensitivity is required of all the Raptor’s measurement systems lest to add unwanted error. However, the authors state that further comparison testing should be done between the Raptor and FWD and/or other TSDDs (which was reported by Madsen and Pedersen, 2019).

- Deep et al (2020-a, b, c) reported on a framework to employ the Raptor on jointed pavements to determine the joints’ load transfer efficiency (LTE) rate. So far, only the mathematical framework
to be applied, a validation example against simulations done in finite-element software, and the results of a 1:1-scale field experiment a Raptor traveling at a low speed over an instrumented airport runway section. Despite the experiment results being promising and encouraging, no further information was found involving testing at traffic-speeds.

The Traffic Speed Deflectometer [TSD]

The Traffic Speed Deflectometer [TSD] is a TSDD developed by Greenwood Engineering A/S from Denmark. The TSD utilizes a Doppler laser system to retrieve measurements of pavement deflection produced by the vehicle’s own rear axle loads (Hildebrand and Rasmussen, 2002; Brezina et al., 2017; Austroads, 2016). Contrary to the RWD (laser-based instrumentation) and the RDT, which measure the depth of the deflection bowl with triangulation lasers, the TSD measures the speed at which the pavement surface deflects, hence the collected data is the pavement surface’s deflection velocity (Figure 55). This design is advantageous as it may make the collected measurements independent from the pavement’s surface macrotexture (Flintsch et al., 2013). The TSD’s measurement principle and original sensor layout have been originally reported by Hildebrand et al. (1999) and Hildebrand and Rasmussen (2002) and reviewed extensively as it evolved, as, for instance, by Ferne et al. (2009-a); Rada and Nazarian (2011); Austroads (2012 [R395]); Zofka and Sudyka, 2015; Brezina and Stryk, 2015; Brezina et al., 2017; NZTA, 2016; PIARC, 2019; Paoletti, 2013; Katicha et al., 2017. A standard specification for what is known as a second-generation TSD is provided in the Austroads AG:AM/T017 standard (2016)22.

The pavement surface velocity measurements from the TSD can be easily converted to deflection bowl slope measurements provided that both the horizontal and vertical components of the deflection velocity vector are measured simultaneously (Hildebrand and Rasmussen, 2002; Krarup et al., 2006). To this end, the Doppler laser sensors are mounted at an angle (about 2 degrees) with respect to a perfect vertical line (Hildebrand and Rasmussen, 2002; Austroads, 2012; Pedersen, 2013; Ferne et al., 2009-a, b; Muller and Roberts, 2013). The sensors’ skew angles must be measured very precisely (TSD calibration procedure) lest to induce an unwanted error in the collected deflection slope data (Ferne et al., 2009-a,-b). Moreover, a trailing wheel attached to the TSD trailer provides highly accurate traveling speed data while surveying (Hildebrand and Rasmussen, 2002; Pedersen, 2013; FGSV, 2017).

22 Since the release of the Austroads (2016) specification for a TSD device, improved devices were built by the manufacturer – the current fourth generation TSDs feature two arrays of Doppler lasers instead of a single array, and each array counts 11-12 lasers capable of sampling at 250 kHz instead of seven 1-kHz laser sensors, among other hardware improvements.
The TSD (first-gen to third-gen devices) samples pavement deflection velocity measurements at a rate of 1 kHz\textsuperscript{23}, which at an operational speed of 70 km/h [45 mph], results in one data point every 2 centimeters (0.8 inches). However, results are often reported at a resolution ranging between 1m and 0.1 miles (160 meters), depending on the end-user needs (Flintsch et al., 2013; Austroads, 2014), yet reporting at a 10-meter interval is customary (PIARC, 2019).

The first TSD prototype was built on a derelict deflectograph trailer and utilized two Doppler laser sensors mounted on a rigid beam to measure the deflection slope of the pavement surface around the trailer’s axle (Hildebrand and Rasmussen, 2002; Pedersen, 2013), these lasers were located 200 and 300 mm [8 and 12 inches] from the load center. The applied axle load was set to 10 tons (Hildebrand and Rasmussen, 2002; Zofka et al., 2014; PIARC 2019), although it could be varied by adding/removing ballast loads. Subsequent design improvements over the first prototype consisted of adding a third Doppler laser sensor (located at 100, 200, 300mm [4, 8, 12 inches] from the loaded axle centerline plus a reference laser away from the axle’s deflection basin (Ferne et al., 2009-a; Zofka et al., 2014) and replacing the unshielded trailer with a climate-controlled cabin trailer meant to keep the sensing system at a constant temperature to prevent unwanted warping of the rigid mounting beam. Two TSD devices were built following this design – first-generation TSDs, as denominated by Ferne et al. (2015): TSD-1 is deployed in Denmark (retired from service as of early 2022), whereas TSD-2 serves routine network-wide measurements in the United Kingdom.

\textsuperscript{23}The Fourth-generation TSDs (launched in 2020) feature 250 kHz Doppler lasers instead.
Second generation TSDs present several advantages over the earlier model (Jansen, 2015), the most remarkable ones concern the Doppler laser sensing system: seven lasers (instead of two or three, excluding the reference laser) are mounted along the vehicle’s longitudinal direction on the passenger’s side and ahead of the TSD’s rear-most axle – additional lasers may be installed upon customer’s request (Pedersen, 2013). Additionally, the rigid mounting beam features shifting mechanisms that render the device calibration procedure easier and allow repositioning the lasers’ location to match specific customer needs (Pedersen, 2013). Ferne et al. (2015) presented a thorough comparison between the 1\textsuperscript{st} and 2\textsuperscript{nd} generation devices and enumerated the advantages of the latter after an experimental run in the UK. Moreover, the second-generation TSD, its sensors’ default configuration, and its mode of operation have been standardized under the Australian AG:AM T017-16 and S006-16 standards (Austroads, 2016, a-b).

Closer in time, a third-generation of TSDs was presented, featuring either 10 or 11 lasers, of which seven (or eight) are mounted ahead of the rear axle whereas the remaining three are located behind the rear-axle wheel. These TSDs (TSD-14 and TSD 15) could furnish deflection-speed data for the pavement recovery after the passage of the rear-axle wheel, valuable input for visco-elastic modeling (Pedersen, 2013; Nielsen, 2019 -a, -b; Nielsen and Jensen, 2021).

In early 2020, the fourth generation of TSD devices was introduced by Greenwood with upgraded logging systems capable of sampling at a frequency of 250 kHz and most remarkably, an optional additional array
of 11 Doppler lasers on the driver’s side of the vehicle\textsuperscript{24}. As of December 2020, two new fourth-generation TSDs were built and delivered. In 2021, TSD-9 (a second-generation device) was upgraded to fourth-gen specifications and resumed operations, whereas TSD 18 and 19 were delivered to their owners in the United States and Italy, and TSD 21 has been requested to be delivered to ARRB Systems in Australia\textsuperscript{25}.

Concerning deflection bowl measurements with the TSD

Comparison of TSD data with the measurements from other deflection devices is not trivial, given the different nature of the data collected (Simonin et al., 2005; Katicha et al., 2014-b; Elseifi and Zihan, 2018; PIARC, 2019). Yet a meaningful comparison between the TSD and other devices can be achieved by integrating the TSD deflection slope measurements over the direction of travel (Pedersen, 2013). Greenwood A/S possesses a proprietary deflection bowl integration procedure based on a high-order polynomial fit of the deflection slope data (Pedersen, 2013; Nasimifar et al 2018-b), yet other extensively reviewed methods are those originally presented by \textit{Euler-Bernoulli Beam} method by Krarup et al. (2006) and the \textit{Area Under the Curve [AUTC]} method proposed by Muller and Roberts (2013). The \textit{Euler-Bernoulli Beam} method regards the pavement structure as a beam on a Winkler foundation (Euler-Bernoulli beam) and attempts to fit the TSD deflection slope data to the derivative of the deflection bowl formula, which, despite being a high-order differential equation, has a known closed-form expression (Van Cauwelaert, 2003; Krarup et al., 2006; Li et al, 2013; Muller and Roberts, 2013; Pedersen, 2013). A fork of this solution was proposed by Grazyk et al. (2014): the \textit{Bernoulli beam} is assumed as lying on top of a visco-elastic subgrade, the authors printed the solution for the deflection bowl over time as the TSD load travels by and conducted a sensitivity analysis of the resulting model by varying either the beam or the subgrade’s properties. Meanwhile, the deflection at a given distance from the TSD load center by the AUTC method is computed as the integral of the deflection slope from infinity to that location – the method assumes a deflection bowl with depth zero at faraway locations and a continuous deflection profile throughout (Austroads, 2012, 2014; Muller and Roberts, 2013; Nasimifar et al., 2018-b). To validate this procedure, Muller and Roberts compared the resulting integrated D\textsubscript{0} [deflection at the center of the load application area] and SCI indices against measured FWD measurements with a positive

\textsuperscript{24}: The fourth-generation TSD sensing equipment was described in a TRB Annual Meeting presentation in 2020, but no written materials describing the new device have been published to date.

\textsuperscript{25}: These news concerning the most recent TSD deliveries were either fetched from the DaRTS 14 and 15 meeting briefs or collected from Greenwood’s corporate social media newsboard (Link: https://www.linkedin.com/company/greenwood-engineering/posts/?feedView=all, accessed 2021-12-21)
outcome, also noting that the integrated TSD bowls and FWD bowls, despite being caused by loads of different magnitude, have a similar shape. The AUTC and the Euler-Bernoulli beam methods are further discussed in Austroads (2014), Austroads vouches for the AUTC over the Euler-Bernoulli beam given its closest resemblance to FWD-based deflection bowls.

Alternatively, Zofka et al. (2015) proposed a procedure to calculate the deflection bowl depth at a given fixed point on the pavement surface by matching the deflection slope readings taken from the many sensors of the TSD as they hover over it. Nasimifar et al. (2018) nicknamed this method as Weibull Functional Form Method [WFFM], for it uses a Weibull distribution to fit the deflection bowl at a fixed pavement point from the TSD data from all sensors as they travel over it. Zofka et al. (2015) point out that an advantage of WFFM over other deflection bowl integration methods is that its outcomes (the fitted Weibull function parameters at each pavement point) can be utilized in a clustering scheme to recognize homogeneous sections.

Nasimifar et al. (2018-b, 2019) compared the aforementioned integration methods against a visco-elastic simulation of several instrumented pavement sections throughout the United States. One remarkable finding is that the AUTC method is sensitive to anomalies in data: the fitted deflection bowl is assumed to pass through all the measurement locations with the exact measured slope, should any deflection slope measurement be flawed, the deflection bowl would adopt an anomalous shape, and thus distort all results. This issue has also been reported by Chai et al. (2016), In this line, Wix et al. (2016) suggested acceptance thresholds for the deflection slope readings from the many sensors of the TSD to automatically discard anomalous readings that could otherwise contaminate the integrated measurements from the other sensors. Alternatively, a filtering scheme should be added as a pre-processing stage (Nasimifar et al., 2018). This observation aside, Nasimifar et al. (2018-b) concluded that all integration procedures tested returned reasonable back-calculation results that were consistent with the simulated visco-elastic models.

Nonetheless, all deflection integration schemes that were tried on TSD deflection slope measurements require additional assumption(s) about the amount of deflection (or deflection slope) somewhere in the pavement surface – an additional input to solve the integration constant involved in the math. However,
deflection indices defined as differences between deflection values at different locations of the deflection basin (like the SCI or BDI) (Horak, 2008; Thyagarajan et al., 2011) are not conditioned by such constants, as they cancel themselves out in their formulation (Flintsch et al., 2013; Katicha et al., 2014-c), making them more suitable for analysis purposes. Besides, several of these indices are known to correlate to pavement properties of interest (Rada et al., 2016; Nasimifar et al., 2016; Katicha et al., 2014-c, 2020), bolstering their use in data reporting for pavement management.

Current state of the technology

Of all the continuous deflection devices mentioned in this review, the TSD is the only one that is currently commercially available. As of 2020, 17 TSD devices have been built and delivered to highway agencies worldwide; two of these devices [TSD-1 and TSD-2] belong to the first generation of devices (Hildebrand and Rasmussen, 2002; Ferne et al., 2015), devices TSD-3 to 12 correspond to the second generation of devices (TSD-13 was never built), TSD-14 and 15 are third-generation devices, and the newly-built TSD-16 and 18, TSD-7 AND -9 after a rig upgrade, correspond to the 4th generation. As of late 2021, TSD-16, 18, and 19 correspond to the newest 4th generation, and the announcement of TSD-21 being manufactured were made public.

Initially, the TSD was intended as both a screening tool for delimiting pavement sections with homogeneous structural responses and detecting localized weak spots (Hildebrand and Rasmussen, 2002). Yet over the past twenty years, abundant literature has been produced (and the research effort is still ongoing) about the use of the TSD within network-level pavement management practice as a source of structural condition data for proper rehabilitation design and budgeting (Austroads 2012; Flintsch et al., 2013; Rada et al. 2016; Arora et al 2006; Phares et al., 2008; Elbagalati et al., 2017; Katicha et al., 2020; Zihan et al., 2019; Manoharan et al., 2017, 2019). As was the case with the RWD (Gedafa et al., 2010; Elbagalati et al., 2017), network-wide surveying with the TSD was found to be a cost-effective approach for pavement management as the actual surveying and data reduction expenses are well offset with savings from optimized rehabilitation treatments – significant savings could be garnered by refining

26 The author is grateful to K. Jansen (Greenwood A/S) who kindly provided this summary description of all operational TSDs

27 As from Greenwood’s corporate social media news feed: https://www.linkedin.com/posts/greenwood-engineering_new-tsd-to-arrb-national-transport-research-activity-682035464454417792-wkGZ (Accessed 2021-12-21)
network-wide pavement treatment selection from solutions with structural reinforcement to surface-only treatments on structurally sound yet not optimally-functional sections (Katicha et al., 2020; Zihan et al., 2019).

Currently, TSDs are utilized for routine network-wide structural surveying in Europe (Flintsch et al., 2013; Muller and Roberts 2013; Nielsen, 2019-a; Nielsen and Jensen, 2021), Australia (Wix et al., 2016). Numerous test trials/ project analyses involving them were conducted throughout Europe (Brézina et al., 2017; Antonsen, 2016; Paoletti, 2013; Zofka et al., 2014, Rabe, 2013; Herroen et al., 2015; Mäki, 2016; Virtala, 2016; Jansen, 2015; Nielsen, 2019-a; Nielsen et al, 2021, Nielsen and Jensen, 2021)\textsuperscript{28}, South Africa (Austroads, 2014; Mshali and Steyn, 2020), Australia (Wix et al., 2016; Manoharan et al., 2017; WARRIP, 2017), and recent research forms a framework for network-wide management reliant on TSD structural data (Manoharan et al., 2017, 2019). Meanwhile, highway agencies throughout the USA are adapting their pavement management systems towards including the data from TSD surveys within their treatment selection, budgeting, and prioritization framework (Zihan et al., 2019; Katicha et al., 2020). Most commonly, the management process is guided by estimations of effective Structural Number \( (SN_{\text{eff}}) \) obtained directly from the deflection measurements with either the AASHTO Equation (1993) and Rohde’s equation [recalibrated for the TSD by Nasimifar et al. (2019)], yet there is a drive towards feeding the managerial decision-making process with mechanistic structural health analysis based on strain back-calculation from deflection indices (originally proposed by Thyagarajan et al., 2011; Rada et al., 2016; Nasimifar et al., 2016; Katicha et al.,2014-c, 2020; Zihan et al., 2020), or directly from admissible maximum deflection values (Manoharan et al., 2017, specific for Australian pavement design practice).

Meanwhile, the TSD proved a valuable tool for project-level pavement engineering applications involving state-of-the-art mechanistic-empirical pavement analysis: numerous research papers covered visco-elastic back-calculation of flexible pavement strength parameters from TSD deflection slope measurements (Virtala, 2016; Nasimifar, 2015; Nasimifar et al., 2017-a,-b, Pedersen, 2013; Nielsen, 2019-b; Elseifi et al., 2019; Saremi, 2018; Saremi et al., 2019). The use of these visco-elastic analysis frameworks is advised at the project-level only, mainly because of the computational cost of computing the stress-and-strain state under visco-elasticity assumptions (Nasimifar et al., 2018) – for network-level analysis, Nasimifar et al. (2018) advice the use of a linear-elastic back-calculation framework as it is a good compromise between the accuracy of data required for management and computational efficiency. Yet

\textsuperscript{28} During 2020, there were news regarding ongoing or recently-completed field experiences with TSDs in Germany and Finland, yet as of January 2021, no written report on such field tests was publicly released.
numerous correlation studies have been published (Nasimifar, 2015; Nasimifar et al., 2017,-a-b; Rada et al., 2016; Elseifi et al., 2019; Zihan et al., 2020) to swiftly estimate strains at remarkable locations for managerial analysis using visco-elastic frameworks, closing the gap between project-level and network-level data availability).

The TSD is known to have been utilized for material testing and construction quality control, yet the information available on this topic is rather scarce:

- ANAS S.p.A., an Italian highway asset holder, operates a TSD (TSD-3) for routine evaluation of their freeway network and to test whether new or reconstructed sections were built or maintained to specification (Paoletti, 2013)
- A recent study in South Africa (Mshali and Steyn, 2020) utilized a TSD as a load source during a test-track field test, as part of an effort to quantify the influence of the speed of heavy vehicle loads on the deflection response of thin-asphalt-surface roads constructed to South African standards.
- On an early field-test of the TSD in Norse roads, Antonsen (2016) provides a project-level example of a design of a reinforcement layer against frost damage utilizing TSD deflection measurements as input.

Another out-of-the-box project-level prospect for the TSD is bridge structural health analysis. The European TRUSS Project holds an active research line on this topic. To date, a series of research texts have been published discussing the underlying theory and the structural response inference process from TSD data based on computer simulations of the passage of the TSD over a bridge closed to traffic (Keenahan and O’Brien, 2018; Malekafarian et al., 2018).

**Research Milestones with the TSD**

*The early years – experimental testing with 1st generation TSDs [2002-2010]*

Since its public release in 2002, the first TSD device has been featured in numerous field tests throughout the world. Most commonly, these tests were intended to assess the TSD capabilities as a network-wide survey tool, by means of short-term and long-term repeatability tests (Simonin et al., 2005; Rasmussen et al., 2008; Baltzer et al., 2009, 2010; Austroads, 2012; Flintsch et al., 2013) and comparisons with the...
deflection measurement technology available at the moment, such as FWD and curvameters (Simonin et al., 2005; Ferne et al. 2009-a; Baltzer et al., 2010).

The first TSD prototype [TSD-1] was presented in 2002 (Hildebrand and Rasmussen, 2002) and early network-wide testing commenced immediately. In 2003, TSD-1 was deployed in France for field testing at test-track facilities and on the open road, where its measurements were compared against readings from a deflectograph device and an FWD (Simonin et al., 2005). Simonin et al. (2005) concluded that the TSD measurements do indeed relate to the deflection readings from the deflectograph, observing the different nature of the data from both devices (while the deflectograph measures the deflection bowl depth, the TSD reports the deflection bowl slope) and that the TSD measurements are repeatable (the same conclusion was reported, among others, by Flintsch et al., 2013). Further testing under different pavement structures was conducted in Germany in 2007 (Baltzer and Hildebrand, 2007). The testing facility's geometric design did not allow for high-speed data collection, but an insight into the TSD performance at low speeds and on high-curvature cases (like turning on corners) was gained. Shortly after, the results of the first long-term repeatability test for a TSD were reported by Rasmussen et al. (2008) and Baltzer (2009). The TSD surveyed a fixed stretch of road in Denmark over a three-year period, and the deflection-slope readings from each test were paired to maintenance records/logs. The authors praised the capability of the TSD of surveying an entire network thousands of miles long in a matter of months and highlighted how the TSD data exposed locations where major rehabilitation work was conducted (structural reinforcements related to flatter curves and smaller overall values in SCI indices, but thin surface overlays would not make such a difference in structural capacity) (Baltzer et al., 2009).

The TSD-2 device was delivered to the UK Highways Agency [HA], as a substitute for the in-house Deflectograph (which was retired from network-wide service in 2000 (Ferne et al., 2009-a,-b). Ferne et al. (2009-a) chronicle the early acceptance testing of the TSD-2 conducted both in Denmark and the UK; at the latter location, the TSD-2 was contrasted against the Deflectograph and an FWD. The comparison test showed that the TSD was capable of differentiating pavement segments with different structural capacities (Ferne et al., 2009-a). However, early high-speed testing of the device with unsatisfactory outcomes led to the vehicle receiving hardware improvements, among which a better mounting of the laser array support beam and a climate control system to cool the lasers down and counteract thermal warping. The upgraded device passed further acceptance tests and ever since forms part of the HA fleet.

The first trial of the TSD outside of Europe took place in Australia during the first half of 2010 (Baltzer et al. 2010; Austroads, 2012; Muller and Roberts, 2013). This testing campaign was intended to evaluate the
suitability of the TSD for network-wide routine structural evaluation on the vast Australian road network, replacing the FWD and Deflectographs in service at the moment. Both repeatability and device comparisons against FWDs and deflectographs were performed, with a satisfactory outcome (Baltzer et al., 2010; Austroads 2012). In total, over 18,000km of flexible pavements of varying strength (from low-capacity surface seals to heavy-duty hot-mix asphalt pavements) were surveyed (Baltzer et al., 2010; Muller and Roberts, 2013). After this successful field-testing campaign, the TSD was recommended as a suitable network-wide survey machine for routine use (Austroads, 2012). In posterior years, several TSDs were imported into the country for routine service (starting 2014)\(^{30}\), and a Standard device specification and Test Method for pavement surveying with a TSD was redacted and adopted (Austroads, 2016 a-b).

Besides, the abundant data garnered during the 2010 Australian field test contributed to perfecting the current state-of-the-art on TSD data processing: the AUTC deflection bowl integration procedure and its posterior enhancements were developed and validated from data collected during this campaign (Muller and Roberts, 2013; Austroads, 2014; Chai et al., 2016), studies on how the pavement deflection slope readings are affected by the surface and asphalt layer temperature and TSD speed were conducted (Baltzer et al., 2010). On this topic, discrepant results with previous experience (Ferne et al., 2009-a) were found – whereas Ferne et al. (2009-a) reported a dependency between travel speed and deflection data quality, that was not the case in the Australian test, which involved thin chip-seal structures with a less remarkable visco-elastic response than thick hot-mix asphalt materials. Baltzer et al. (2010) thus hinted that asphalt visco-elasticity may not be overlooked at the data analysis stage, and in fact, the state-of-the-art for analysis at the project level prompts for visco-elastic back-calculation schemes (Nasimifar et al., 2018, Nielsen, 2019-a). Also, an early trial of synchronized TSD and GPR operation was performed (Muller and Reeves, 2012); Muller (2015) further elaborates on this subject.

**Acceptance and adoption of 2nd generation TSDs [2010-2019]**

The first 2\(^{nd}\) Generation TSD [TSD-3] was deployed in 2011 (Ferne et al., 2015; Jansen, 2015). Since that time, eleven 2\(^{nd}\) Generation TSDs were manufactured and delivered to agencies worldwide.

Numerous reports on implementation efforts and field-test comparisons with continuous and/or stop-and-go deflection meters occurred in Europe during the past ten years, as the TSDs spread: Reported TSD trials throughout Europe took place in Denmark (Austroads, 2014; Nielsen, 2018; Levenberg et al., 2018); Germany (Rabe, 2013; Jansen, 2015-a-b), the Czech Republic and Slovakia (Brezina and Stryk, 2015, 2016).

Brezina et al., 2017), Norway (Antonsen, 2016), Italy (Paoletti, 2013; Brezina et al., 2017), Poland (Austroads, 2014), Finland (Herroen et al., 2015; Mäki, 2016; Virtala, 2016), and the UK (Ferne et al., 2015). A FEHRL press article portraying a TSD field test in the Netherlands in 2016 (pairwise comparison with a Curviameter) was found, but a search for technical information on this subject returned no results. Abundant literature exists that covers the many TSD test campaigns carried out on European soil; a handful of them are commented below:

- Ferne et al. (2015) chronicled a field test campaign aimed at comparing the performance of first-generation and second-generation devices in terms of repeatability and data quality. Tests were conducted at the MIRA Proving Grounds and on the open road in the UK. The authors praise the improved laser geometric calibration procedure compatible with the second-generation device compared to the recommended procedure for the first-generation trailers (the former achieving greater precision). The repeatability test for the three involved TSDs consisted of six passes of each at 70 km/h and returned positive feedback for the second-generation devices. Moreover, an early attempt at estimating strains within the asphalt layer from TSD deflections was conducted; despite poor testing conditions, the estimation results were promising, hinting at the capabilities of utilizing TSD deflection basin data to estimate the amount of strain in the asphalt layer.

- Jansen (2015 a-b) reported a repeatability and reproducibility test across two different second-gen TSD vehicles surveying in-service freeways at the same time, plus short-term repeatability (24-hr interval) for a single 2nd-Generation TSD. Visual comparison of the measurements collected simultaneously by the Doppler sensors and other probes of both devices revealed discrepancies that led to one of the involved TSDs being sent to service. After servicing, the device performed satisfactorily, matching the measurements of the comparison device. Meanwhile, it was observed that during the trial survey, both devices were operated following different procedures (the usual practice from each operator) and not a standardized protocol. Jansen (2015) highlights that such procedural differences may lead to discrepancies in the outcome and thus, a standardized test procedure for TSD operation should be drafted and agreed upon, especially when performing reproducibility tests across devices. Since then, a conceptual guidance manual for TSD operation and quality assurance was published (FGSV, 2017).

- Several authors (Zofka et al., 2014; Paoletti, 2013) analyzed the possible relationship between environmental factors and/or pavement properties and the TSD measurements; Zofka et al. (2014)
provide an exhaustive list of such factors based on their review of TSD literature available to date, and particularly analyzed the possible interaction between crosswind speed and pavement roughness and TSD measurements – the authors highlight that crosswind may add down-force to the TSD rear axle and thus the actual applied force shall be logged during a deflection survey.

- Paoletti (2013) reported on the relationships between TSD measurements and pavement surface roughness (in terms of IRI) – the author points out that Italian TSD operators rely on previous Australian results (Baltzer et al., 2010) concerning the influence of traveling speed on deflection measurements and utilize a specifically-calibrated temperature correction formula (based on air temperature only) for deflection indices. It was found that the level of correlation between TSD and FWD decreases as the pavement’s IRI increases, hinting at an overall higher measurement error in the TSD when surveying rough roads. Similarly, TSD operation on rough roads would eloquently lead to larger and more largely varying dynamic loads, thus negatively affecting the quality of the collected data (unless the amount of dynamic load were logged, as prompted by Brezina et al. (2017)).

- Mäki (2016) commented on how winter weather may affect the deflection readings from the TSD, and thus pointed out that the TSD should not be operated under low air temperature conditions – Similar guidance appears in the FGSV (2017) manual (TSD operation should be limited to an outside temperature between 5-30 deg. C).

- TSD-versus-FWD comparisons continued throughout the decade in several device acceptance tests. For example, Brezina et al. (2017), once again highlighted the different nature of the load the two devices put on the pavement structure, implying that a direct relationship between their outcome should not be expected. Yet the pairwise comparison between TSD (multiple runs) and FWD (single run) [in terms of \(d_0\) and SCI\(300\)] were carried out at different locations, and although similar overall behaviors were observed on one site, weak correlation levels were found when processing the second site – this issue was tracked to a mismatching error in the data from the two devices. Yet, the authors highlighted that the TSD may – despite the device-specific differences with the FWD – highlight the same weak/strong locations within a network as an FWD would do. Similar findings were reported by Rabe (2013), and Antonsen (2016) after a collection effort in German trunk roads and on a network of highways in Norway – during the German experience, the TSD reported overall different (higher) SCI\(300\) values for all sections, a discrepancy that could be explained by the different nature of the load applied by both devices; anyway, both devices reported the same weakest and strongest sections of the analyzed network.
Antonsen (2016) concluded similarly by stating that weak section screening using a TSD should yield the same outcome as a survey conducted with the FWD.

- In connection with the above, Levenberg et al. (2018) proposed a TSD-index estimation scheme from FWD readings (by interpolation) and analyzed the degree of agreement between both devices by the lambda metric and Taylor diagrams\textsuperscript{32}

More recently in time, during early 2020, tests on German concrete highways and a concrete runway within the Copenhagen Airport were conducted using a last-generation device, no published report documenting the German case study was found at the time of writing this review, yet the Copenhagen airport experience was documented by Nielsen and Jensen (2021)\textsuperscript{33}. In July 2020, a test run was conducted in Finland over instrumented segments of the European highway E8 featuring subgrade soils of varying strength, data processing is underway, the test results were presented by Nielsen et al. (2021).

Moreover, the past decade saw the expansion of TSD overseas: after the promising network-wide test campaign with a first-generation device (Baltzer et al., 2010), and once its utilization for routine assessment was deemed economically profitable (Austroads, 2012; NZTA, 2016), the TSD was adopted as a network screening tool in Australia – Wix et al. (2016) recall the implementation of the device in practice, and a series of technical standards describing the device and the data collection process have been published (Austroads, 2016 a-b). Similarly, the TSD was adopted in South Africa – the earliest written reference on this subject is by Austroads (2014). As of late 2020, two TSD devices operate there. However, no record of field testing, implementation, or experimentation with the TSDs deployed in China was found (two second-generation devices were shipped there [Antonsen, 2016; PIARC, 2019]), but a local firm patented a TSDD [the \textit{Laser Dynamic Deflectometer}]\textsuperscript{34}, and several publications related to it were released (Li et al., 2013; He et al., 2017)\textsuperscript{35}.

\textsuperscript{32} Not to confuse with Bland and Altman’s \textit{Limits of Agreement} comparison metric, which was applied to compare deflection indices from TSD and FWD data by Katicha et al. (2014)

\textsuperscript{33} However, during the DaRTS 14 meeting (February 2021), progress made during this test campaign was announced, the main findings reported by Nielsen and Jensen (2021) were also presented at the DaRTS 16 meeting (January 2022).

\textsuperscript{34} The LDD is patented under the Chinese Patent #CN102162217A

\textsuperscript{35} This device (and the literature that emerged after its release) warranted a separate review, please refer to section: Forked technology: The Laser Dynamic Deflectometer [LDD]

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During the TSD's early years, American research on the TSD was centered on literature-review-based and survey-based feasibility studies towards ascertaining whether the TSD would fill the known gap between network-wide structural data requirements for proper management practices and technology available to practitioners. Examples of these suitability studies featuring the TSD are the works by Arora et al. (2006); Phares et al. (2008); Rada and Nazarian (2011), and the SHRP-2 R-06 (F) project report by Flintsch et al. (2013) – all of which covered the TSD as one potentially feasible device among a pool of prototypes and production models available to date. Emphasis on the applicability of the TSD (and the RWD) as a network-level screening device for weak-spot detection and management purposes is provided in Flintsch et al. (2014) as well, where, by means of an example, the effective structural number [SN] of a surveyed section is estimated with Rohde’s Equation using TSD deflection data. Moreover, in Katicha et al. (2014-d), an early implementation of Thyagarajan et al. (2011)’s framework for estimating horizontal strains at the bottom of the asphalt layer from deflection indices is presented [derived from Horak (2018), revisited later by Rada et al. (2016), and Nasimifar et al. (2017, 2018)]. Katicha et al. (2014-d) compared the estimated horizontal strains using Thyagarajan et al. (2011) formulae and FWD and TSD readings garnered at the same locations. Although the relationship between the deflection bowl indices and the strain estimates from both devices were not 1-to-1 correlation (a result to be expected given the different nature and load amount from both instruments), these quantities correlated to a good extent, highlighting the suitability of the TSD as a screening device for mechanistic strain-based management.

The SHRP-2 report (Flintsch et al., 2013) aggregates several repeatability tests’ results for the TSD under different pavement structures and comparisons with FWD measurements – it is highlighted that both devices report significantly different numerical values, tied to the different nature of the applied load (non-zero bias has been found between FWD-based and TSD-based deflection indices), that the TSD is a repeatable device except when surveying heavily deteriorated sections, and discussion is provided on operational concerns for the TSD that may affect the quality of the collected data (presence of free water on the pavement surface, surveying under curved roads and cross-winds, the impact of strong acceleration and braking, etc.). Other topics covered as part of the SHRP-2 project concerned the handling of the TSD data to remove noise and extract features, and improved comparison metrics against the FWD:

36 Flintsch et al. (2014) applied the original formulation of Rohde’s equation, which was calibrated for SN_{eff} estimations based on FWD data. Rohde’s equation was recalibrated for the TSD by Nasimifar et al. (2019).
Katicha et al. (2012) analyzed the use of a smoothing spline function to denoise the TSD deflection slope signals at 1-m reporting interval (such TSD signals are known to be contaminated by high-amplitude noise that obscures interpretation, and the averaging process used in reporting may mistake true localized events as noise and cancel them out (Austroads, 2014; Flintsch et al., 2013; Rada and Nazarian, 2013; Katicha et al., 2013, 2014-a) and recover the true signal, weak spots may be eventually detected as outlier points.

This methodology, however, was superseded by atomic decomposition into the Wavelet signal basis (Katicha et al., 2014-a; 2016), owing to improved signal recovery performance than the smoothing spline (the smoothing spline is not adaptive to spatial variability of the true deflection slope signal [the spline that may work at a given section may not work at another one where the deflection slope variance is notably different] yet wavelets overcome such a limitation. The decomposition of the TSD signal into the wavelet space (and shrinkage thereof) allows for an objective noise removal scheme that at the same time is spatially adaptive, the wavelet signal basis may also recover the spikes in the deflection signal produced by localized weak spots. Katicha et al. (2014-a) analyzed both simulated and actual TSD signals with this decomposition scheme and the smoothing spline and a Bayesian scheme and discussed the outcome: the Wavelet-based decomposition scheme provided a better reconstruction than the smoothing spline, composed only of atomic signals that correspond to true signal features with high probability, yet at the cost of a high bias (dampening of components towards zero), which was outperformed by the Bayesian reconstruction. Katicha et al. (2016) discuss an improvement to the wavelet-based reconstruction to overcome bias.

Both the wavelet-based signal reconstruction method presented in Katicha et al. (2014-a, 2016) and the smoothing spline presented in Katicha et al. (2012) require knowledge of the signal noise’s standard deviation – a reliable estimation of the noise’s variance is warranted. One estimation is provided in Katicha et al. (2013), yet Katicha et al. (2012) explore the applicability of Von Neumann’s Difference Sequence method (Von Neumann et al., 1941) to estimate the standard deviation of the noise in the TSD signal from a single run of measurements, the validation was conducted against a set of signals from a predefined test track section measured repeatedly. It was found that the TSD data reported at 1-meter resolution provide a good source for the Difference Sequence method – the second-order Difference Sequence estimator provides an adequate estimate. Meanwhile, the data reported at 10-meter-or-higher intervals are not good
grounds for the Difference Sequence method. Furthermore, Katicha et al. (2015) utilize this previous result to estimate the true variability of the TSD signal and thus enhance the denoising capabilities of the smoothing spline procedure presented in Katicha et al. (2012).

Finally, Katicha et al. (2014) report a comparison metric between the TSD and the FWD based on Bland and Altman’s Limits of Agreement method. The goal that motivated the use of this technique is to calculate the bias (constant difference) between deflection indices like SCI$_{300}$ and the BDI calculated from the FWD and the TSD if any. Assuming the TSD and the FWD would not interact with the pavement in the same fashion (a fact acknowledged by TSD practitioners and extensively reviewed in this section), a simple linear regression to relate the deflection indices from one device to the other one would lack meaning. It was found that -although numerically small- TSD and FWD-based deflection indices are in fact different (the TSD reporting slightly bigger deflection indices than the FWD), a result that is aligned to findings reported by Elseifi and Zihan (2018).

The SHRP-2 Project report and subsequent literature (Bryce et al., 2012; 2013; Flintsch et al., 2012, 2013, 2014; Katicha et al., 2012a-b, 2013, 2014-a-b, 2015, 2016) presented analyses of TSD measurements over a variety of pavements (flexible, rigid, composites) but the source data used therein was garnered in England. The first actual trial of the TSD in the United States was the cross-device evaluation at the MnROAD test track facility and nearby secondary roads in 2013, as part of FHWA’s project DTFH61-12-C-00031 (Austroads, 2014; Rada et al., 2016). This project was aimed at evaluating the extent to which the TSD and the RWD could be utilized for network-level screening and developing the processing schemes to derive structural health information from the devices’ deflection readings. In this test campaign, a second-generation TSD was compared against ARA’s RWD and a Curviameter, the deflection measurements collected from these devices when traversing instrumented sections were paired to strain measurements. A panoply of literature was produced that covers the testing campaign itself and research work derived out of it (Austroads, 2014; Nasimifar, 2015; Nasimifar et al., 2016, 2017 a-b-c, 2018 a-b Velarde et al., 2017), yet the main published outcome is the Project’s report (Rada et al., 2016), and an abridged version thereof (Rada et al., 2018).

The TSD deflection data collected during the MnROAD test campaign was utilized by Nasimifar (2015) to validate the calibration of the visco-elastic pavement analysis software 3D-Move, originally done based on simulated deflection data.
• The calibrated 3D-Move instance was later utilized to seek the best fitting deflection bowl indices to estimate strains known to relate to fatigue failure, the results are presented in Rada et al. (2016) and Nasimifar et al. (2016, 2017a), yet put briefly, it was found that DSI\textsubscript{200-300} and DSI\textsubscript{300-900} could be reliably used to predict the horizontal strain at the asphalt layer (used in turn to predict fatigue life) and the vertical strain at the subgrade (which is an input to estimate rutting) (AASHTO, 2020).

• Additionally, Nasimifar et al. (2017, b-c) elaborated on visco-elastic back-calculation of pavement structural parameters using the calibrated 3D-move instance and compared its results against a linear elastic software tool and field measurements collected from the MnROAD field experiment plus data from measurements conducted in the state of Idaho as part of the TPF-5(282) Project. It was found that the calibrated visco-elastic back-calculation instance was advantageous over the linear-elastic approach since it allows modeling of the actual TSD wheels as sources of load, accounts for the TSD’s traveling speed, and can be fed the TSD’s deflection slope data as received from the field, without the need for an integration scheme. However, this numerical computational procedure is expensive and thus advised for project-level analyses only – the linear-elastic procedure relies on strong assumptions yet was found to match the measured target properties fairly well given the underlying simplifications, and thus more appropriate for network-wide management purposes – this latter point is further discussed by the authors when the many deflection slope integration methods are compared against the calibrated visco-elastic framework in Nasimifar et al. (2019).

• However, it may be the case that Highway Agencies rely on SN-based management methods for network management and rehabilitation quantification. In this line, the TSD data be the input for SN\textsubscript{eff} back-calculation. Three alternative procedures were published: Schmalzer and Weitzel (2017) present a case study of SN\textsubscript{eff} back-calculation from TSD deflection-slope data following the AASHTO (1993) procedure as-is (relying on Boussinesq’s hypothesis for integrating deflections far from the TSD’s wheel), whereas Zihan et al. (2018) presented a comprehensive equation that requires knowledge of the deflection basin, surface features, and the road segment’s AADT\textsuperscript{37}, and Nasimifar et al. (2019) proposed a recalibration of Rohde’s (1994) equation to estimate SN\textsubscript{eff} and applies it into a calculation example. – the latter two prevent the end-user from falling into an iterative numerical scheme to estimate SN\textsubscript{eff}. The recalibration of Rohde’s equation

\footnote{37 It can be argued that AADT is more of a confusing variable and not an actual quantity SN\textsubscript{eff} depends on, as SN\textsubscript{eff} is by definition a property of the pavement structure itself (which is originally calculated to withstand a certain level of traffic) but the absence/presence of traffic should not alter the SN of a given segment.}
was conducted using simulated FWD and TSD basin data within the calibrated visco-elastic framework presented in Nasimifar (2015). Nasimifar et al.’s (2019) paper should be regarded as a corrected version of the original example presented by Flintsch et al. (2014).

- Finally, Nasimifar et al. (2019) drafted an example case study of flexible pavement management for a Primary Highway segment relying on both surface condition records, layer thickness estimation from GPR data, and deflection slope readings from a TSD survey and applying the corpus of analysis and interpretation techniques initially presented in Rada et al (2016), Katicha et al. (2017); Nasimifar et al. (2018 a-b), and nested references. The analysis methodology presented herein is a structural-data-based enhancement to the rehabilitation decision-making process at the network-wide management level traditionally done based on surface distress only, in the fashion presented originally by Rada et al. (2016), Katicha et al. (2017); Thiagarajan et al. (2015); Shrestha et al., 2018; Maser et al., 2017; Zihan et al., 2019; and similar to what had been proposed earlier within the RWD pavement management framework (Gedafa et al. 2010; Zhang et al., 2016). However, the economic impact of the enhanced decision-making process for this case study was not calculated. Nonetheless, Nasimifar et al.’s work may be regarded as a template for flexible pavement management based on dense TSD and surface distress data.

Between 2013 and 2017, the Transportation Pooled Fund study [TPF 5(282)]\(^{38}\), which was aimed at demonstrating to the participating State Highway Agencies how the TSD could be utilized within a pavement management system, took place. The device demonstration included short-term and long-term repeatability tests, and comparisons against the FWD, in a similar fashion to previous acceptance experiences (Flintsch et al., 2013; Rada et al., 2016; Baltzer et al., 2010, to name a few), data for these tests and the posterior managerial decision-making examples presented in the project’s literature (Katicha et al., 2017) was collected between 2013 and 2015 and spanned nearly 6,000 miles of highways pavements [some segments were surveyed more than once] under the jurisdiction of the nine participating State Agencies. It was found that the TSD was repeatable in the short term (measurements taken on consecutive days overlapped almost perfectly), yet such a perfect match was not found after the long-term test – the authors blame operational limitations such as calibration issues to have cause such an outcome, and thus recommend TSD operation standards and data processing guidelines be drafted, agreed upon, and implemented in the future [an operational concern resembling what reported by Jansen (2015)]. Also, a TSD/FWD comparison (in terms of maximum deflection – \(D_0\)) was presented, but featuring historical

\(^{38}\) Project’s site: [https://www.pooledfund.org/details/study/518](https://www.pooledfund.org/details/study/518), accessed 2021-02-23
FWD test results: it shows (yet once more) that although both devices would not produce the same deflection basins (and setting aside the time gap between the historical FWD data and the TSD measurements), the TSD may report the same weak and strong sections (that is sections with high and low deflection values) as the FWD would do, deeming it valid for screening purposes – this result was reported once again in Katicha et al. (2020). The decision-making case study featured in the main Project’s report relies on a classification scheme for the surveyed segments based on their structural health (either good, fair, or poor structural condition); the results of such an evaluation would correct an initial rehabilitation treatment class determined from the surface distress level – the same framework proposed, among others, by Nasimifar et al. (2019), and adopted by the Virginia Department of Transportation (Katicha et al., 2017).

The main outcome of this project is chronicled in the report authored by Katicha et al. (2017), yet companion documents with more detailed (and State-specific) results and recommendations for TSD adoption within each State were delivered to each of the nine participating State Highway Agencies. Also, Shrestha et al. (2018, 2019) summarized the most remarkable findings of this field demonstration concerning TSD repeatability over the long term and comparison with the FWD, and Sn_{eff} estimation from the TSD via Rohde’s equation (on its original formulation, not Nasimifar et al.’s (2019) recalibration) versus estimations using traditional methods. On this matter, it was found that the TSD-based Sn_{eff} mismatched the estimations using traditional approaches (the TSD-based estimations were overall higher); a result that was reported once more by Katicha et al. (2020) when pairing TSD-based [recalibrated Rohde’s equation] and FWD-based Sn_{eff} estimations.

The Louisiana State University team who vastly studied the RWD as a network-screening tool (Elseifi et al., 2011, 2012, 2013; Elseifi and Elbagalati, 2017, among other references) replicated for the TSD the analysis and interpretation framework originally developed for the RWD. The core references on this experience are the reports by Elseifi and Zihan (2018) for network-level management and Elseifi et al. (2019) for mechanistic-based project-level analysis of TSD deflection-basins.

- Elseifi and Zihan (2018) reported the outcome of the basic feasibility tests for the TSD as a network-wide screening tool, mostly following the steps that once were defined for the RWD (Elseifi et al., 2011). The authors relied on data from the MnRoad experience and the TPF 5(282) campaigns. It was found that the TSD was negatively affected by the pavement’s roughness (also

39 An extended and re-compiled version of the findings from these three documents is the thesis by Shrestha (2017)
reported by Paoletti, 2013) and that the deflection basins integrated from the TSD are statistically different from those of the FWD.

- At the network-level; Zihan et al. (2018) [also (Elseifi and Zihan, 2018)] presented the SN_{eff} estimation formula for management purposes that was discussed earlier in this chapter. It should be added that the source data for the formula calibration was a State-specific TSD survey conducted during 2016, and the target output was FWD-back-calculated SN_{eff} for the segments that were surveyed with the TSD. The AUTC method (Muller and Roberts, 2012) was utilized to integrate the deflection-slope data into basin depth values.

- Meanwhile, Zihan et al. (2020) [also (Elseifi et al., 2019)] presented an Artificial Neural Network that was trained to translate TSD deflection-slope readings into equivalent FWD basins – a similar effort to the work presented by Elbagalati et al. (2016) for the RWD. The training datasets (TSD deflection slope measurements and FWD deflection bowls) were generated with Nasimifar’s (2015) calibrated visco-elastic framework.

**Network-wide TSD surveying in Virginia**

Recently, Katicha et al. (2020) reported the outcome of a network-wide structural screening with the TSD conducted in the State of Virginia, spanning over 6,500 km [4,000 miles] of highways belonging to the Interstate Network and the State’s Primary Road Network, which took place during 2017. The deflection data garnered with the TSD was paired to thickness information from a GPR and surface condition data from Virginia’s Pavement Management System and from surface condition sensors mounted with the TSD truck^{40}.

The data analysis frameworks presented in earlier works such as Rada et al. (2016), Katicha et al. (2017) were revisited and updated with recent findings (Nasimifar et al., 2018-a, b, 2019); the ultimate goal being to substitute the outdated structural condition record collected with an FWD over 12 years ago. To this end, the FWD-based SN_{eff} and subgrade’s resilient modulus [MR] estimation procedures required an update compatible with the TSD data – Nasimifar et al. (2019) provided the SN_{eff} estimation, and in this study, it was found to follow a scaled distribution of values to that of the FWD, yet an attempt to find an MR estimation based on TSD deflection basin indices proved fruitless, no good enough estimator was encountered. All in all, the scaling factor in SN_{eff} accounted for, it was concluded that the network-wide

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{40} The TSD device that surveyed VDOT’s network is a second-generation TSD [TSD-9], compliant with Austroads (2016) specification.
management process currently done with *outdated* FWD-based SN$_{eff}$ can be executed with TSD-based SN$_{eff}$ estimations using the recalibrated Rohde’s equation.

*The TPF-5(385) Project*

The currently ongoing Transportation Pooled Fund Project 5-(385) \(^{41}\), sponsored by 25 State Highway Agencies scattered throughout the USA (figure 56), is set on the quest to bridge the still-existing gaps between the now available TSDD technology and full adoption by practitioners and Highway Agencies, both at the network-wide managerial level and the project-scale level.

![TPF-5(385) Participating States](https://www.pooledfund.org/Details/Study/637)

*Figure 56: TPF-5(385) participating States as of October 2021. Cartography source: gadm.org.*

The research reports on TSDDs that preceded the TPF 5(385) set the framework for data analysis and elaborated on its utilization for network-level management, praising the economic benefit of optimizing rehabilitation treatments based both on surface distress and structural health. However, as highlighted by

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\(^{41}\) Up-to-date information about this TPF project available at: [https://www.pooledfund.org/Details/Study/637](https://www.pooledfund.org/Details/Study/637)

Accessed 2021-02-24
Katicha et al. (2017), the procedures for device preparation and data collection and manipulation need to be standardized.

Thus, the TPF-5(385), among other deliverables, shall provide the participating Highway Agencies with:

- Data collection guidelines and specifications for network-wide and project-scale-wide surveying with a Traffic-Speed Deflection Device
- Protocols for the use of TSDD-based structural health information within the Agency’s Pavement Management System
- Example case studies of pavement analysis at the network-level and project-level with TSDD-based structural data, featuring budget analyses to evidence their economic advantage over traditional methods.

Besides, additional deliverables would be produced as part of this research project, featuring updates to the existing corpus of TSDD-related literature containing the latest developments in TSDD technology and analysis techniques.

**Forked technology: the Moving Weight Deflectometer [MWD]**

Kamiya et al. (2018) describe the development of a small-size TSDD rigged with an array of Doppler lasers like those in the TSD for use in Japanese roads. The motivation behind this one-off build is to craft a TSDD that can navigate the narrow and twisting Japanese secondary roads and that also complies with the Japanese Vehicle and Traffic Law – a full-size trailer like that of a TSD or an RWD is not street legal because of its dimensions.

The first prototype of the Moving Weight Deflectometer [MWD] is a specially-equipped single-unit truck that utilized an array of ten triangulation laser sensors, seven of which are mounted close to the vehicle’s rear axle in a symmetric pattern (three ahead of the wheel, one aligned to the wheel’s axle, and three behind the axle), and the remaining three are between the front and rear axles. After initial testing (reported by Terada et al., 2012), the triangulation sensors were substituted by three Doppler sensors mounted ahead of the vehicle’s rear axle – these Doppler sensors operate at a sampling frequency of 2 kHz. Subsequently, a second MWD with the same Doppler laser measurement system was built from a light mini-bus to survey secondary roads with stringent axle load limitations. The light-weight MWD applies a load on the pavement of about 30 kN (while the original MWD prototype applies 49 kN of load) (Kamiya et al., 2018).
Development of the MWD started in 2010, the earliest performance tests on the device were reported by Terada et al. (2012). At the moment of this initial testing, the MWD utilized triangulation lasers like those in the RWD. The initial field testing was carried out in an enclosed test track and on in-service secondary roads and involved Fourier analysis of the recorded signals (to detect the noise added by the vehicle’s natural vibration frequencies and devise an averaging rule to make it null), and comparisons against FWD results. The results were not favorable enough, as the correlation between FWD and MWD readings at a low applied load (one consistent with the axle load limitations in Japanese roads) was poor (Terada et al., 2012; Kamiya et al., 2018). This outcome encouraged the shift from triangulation lasers (which, in the authors’ view, would be sensitive to roughness) to Doppler lasers measuring deflection velocity. Subsequent testing with the re-fitted MWD and the light-weight was conducted during 2013 and 2014, and the MWDs performed reasonably well in terms of resemblance to FWD deflection values and odometer checks at different speeds (10 to 50 km/h). Kamiya et al. (2018) utilized wavelet denoising to remove random noise from the deflection measurements – this technique was also proposed by Katicha et al. (2013) to extract the true deflection signal from noisy TSD measurements in lieu of averaging or spline-fitting.

As of 2018, the authors were aiming at utilizing the MWD vehicles on public roads as routine survey vehicles. No more recent information was found on this matter, neither was found about the whereabouts of the two MWD vehicles.
Forked technology: The Laser Dynamic Deflectometer [LDD]

The Laser Dynamic Deflectometer [LDD] is a TSDD developed by the Hubei University of Technology and the Wuhan Wuda Zoyon Science and Technology company in Wuhan, China. LDD technology has been patented in 2012 (Patent ID: CN102162217A/ CN102162217B).

![Figure 58: The Laser Dynamic Deflectometer [LDD]. Left: Concept diagram of sensor’s location. Right: the LDD on the road. Source: Liao et al. (2019); under CC-BY-4.0 License.]

The LDD bears a close similarity to the first-generation TSD, inasmuch as it uses an array of four Doppler laser sensors mounted on a horizontal beam, three of the lasers point close to the LDD’s rear axle (distant 100, 300, and 750mm from the axle centerline) and the fourth one (reference laser) lies far forward (3600mm from the rear axle centerline) (Patent CN102162217B, 2012; Liao et al., 2019) [figure 58]. The key difference with the TSD is that the support beam is not rigid but it is allowed to move, and it is rigged with three accelerometers to capture the instant acceleration over the x- y- and z-axis – the accelerometer readings are used to correct deviations in the deflection velocity method. (Li et al., 2013).

As it has been proposed for the TSD (Krarup et al., 2006), the LDD utilizes the Bernoulli-Euler beam model to integrate the deflection bowl from deflection slope measurements (Li et al., 2013; Zhang et al., 2015; He et al., 2017).

The proof of concept of the LDD, the log of the construction of the first working prototype, and the proposal of a specific calibration procedure for this device – said by the authors to elapse half the time the TSD calibration takes and not require an instrumented pavement section (Ferne et al. 2009-b) – were presented by Li et al. (2013). Besides, He et al. (2017) elaborates on the calibration procedure: both a static and an innovative dynamic calibration procedure for the LDD are explained and discussed in detail.
Following, a network-wide survey of over 40,000 km of Chinese highways was reported by Zhang et al. (2015): the field trial of the LDD was conducted on in-service pavements in the provinces of Xinjiang, Shaanxi, Gansu, Tianjin, Hubei among others; the LDD data were contrasted against Benkelman Beam measurements and repeated runs of LDD for repeatability assessments. The authors regarded the LDD performance as satisfactory after the 95% similarity rate in the repeatability tests (measurements of $D_0$ over three runs), over 90% similarity values between the LDD and the Benkelman beam, and less than 5% correlation between deflection and survey speed.

More recently, Liao et al. (2019) analyze the influence that traveling speed, dynamic load, roughness (IRI above 3.0 m/km), and temperature may have on the deflection velocity data gathered by the LDD and on the integrated deflection basin results. The authors applied regression techniques – they attempted to fit a quadratic model – to translate LDD into reference deflection values (from Benkelman beam and/or FWD). They validate their statistical findings with field results: repeatability at a constant speed and high IRI, speed independence, and correlation tests presented in Zhang et al. (2015), plus repeatability tests on composite pavements [higher noise-to-signal ratios because of low true deflection], repeatability at different temperatures, and correlation of LDD tests at different speeds against the FWD and Benkelman beam. The authors arrive at the same conclusions that Zhang et al. (2015) reported.

As of the present day, the LDD is operated by Wuda Zohue (Zoyon) in China. Between 2013 and 2018, the LDD surveyed over 200,000 km of pavements within Chinese borders (Liao et al., 2019) Details about the device and its intended use are provided through the company’s website: As of March 2022, Zoyon has built and delivered three LDD devices. 42

References


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APPENDIX B: MATHEMATICAL FOUNDATIONS AND METHODOLOGY

Presentation

The TSD data analysis and interpretation process developed in this Dissertation consists of two main stages. The first stage, the Data Extraction Stage, is an implementation of signal processing techniques based on regularization to simultaneously denoise the TSD measurements and extract features of interest from an engineering perspective. The second stage, the Interpretation Stage, includes analysis tools that estimate the jointed concrete pavement material properties and the joints’ load transfer efficiency index [LTE] from the recovered TSD measurements via back-calculation. The methods used for the Data Extraction Stage are mathematically base on linear algebra concepts whereas the methods used for the Interpretation Stage mathematically rely on calculus concepts. This Appendix summarizes the fundamental linear algebra and calculus notions used throughout this Dissertation.

Section 1: Fundamental Linear Algebra Concepts and Signal Analysis Methods.

This Dissertation’s Data Extraction Stage (Manuscripts 1 and 2, Chapters 3 and 4) is an implementation of a regularization-based signal processing technique – Basis Pursuit [BP] (Chen et al., 2001); and Reweighted L1 Minimization [RWL1] (Candès et al., 2008) – to process the TSD measurements. These techniques simultaneously remove the measurement noise and extract important features from the pavement’s deflection response that may relate to localized structural defects worth further investigation and/or immediate repair. These signal processing techniques rest on the concept of sparsity (Hastie et al., 2015) in signal representation, which is strongly rooted in linear algebra and statistics.

Section organization

This section is composed mainly of three sub-sections. The first of these – Fundamental Linear Algebra Concepts – presents the concept of signals as n-dimensional vectors in the Euclidean vector space $\mathbb{R}^n$ and their representation as a unique linear combination of basis vectors that span the space. Firstly, we introduce the concept of a vector space, its spanning sets, and bases – the smallest collection(s) of vectors belonging to such space needed to uniquely represent every element of the vector space (Cerminara and Markarian, 2000; Zhang et al., 2021; Hefferon, 2020). Following, we present the notion of orthogonality between vectors and consequently introduce the notion of orthogonal vectors and orthonormal bases of a
vector space. These notions define vector representation as a combination of the elements of a basis of the vector space. However, under certain circumstances, a given vector may be more conveniently represented as a combination of elements from multiple bases simultaneously, which leads to the creation of over-complete dictionaries, which consist of the merger of two or more bases, of the vector space.

The second sub-section – Vector Representation with Over-complete Dictionaries – presents the concept of over-complete dictionaries and discusses the problem that a given t can have multiple representations in terms of the elements elements of the said dictionaries. To address this problem, a constraint is imposed on the representation to obtain a unique solution. Two popular constraining methods, ridge regression and the Lasso (Hastie et al., 2009, 2015), are introduced and discussed with the Lasso being equivalent to BP denoising.

This section’s third sub-section – Achieving a Sparse TSD Signal Decomposition by Basis Pursuit – links the Lasso to the discrete signal representation problem, introducing Basis Pursuit [BP] (Chen et al., 2001), and particularly, the problem of decomposing a given signal such as a collection of TSD measurements into an over-complete dictionary using BP – the mathematical problem underlying the weak-spot discovery tool implemented in Manuscript 1, Chapter 3 (Scavone et al., 2021). Within this discussion, the bias-variance trade-off implicit within BP is revisited and its implications from a practical interpretation standpoint are highlighted. This serves as a launchpad to introduce Reweighed L₁ minimization [RWL1] (Candès et al., 2008), which improves BP by increasing sparsity and reducing bias. The implementation of RWL1 to detect weak spots from the TSD measurements is discussed in Manuscript 2, Chapter 4.

**Fundamental Linear Algebra Concepts**

Throughout this section, references to the R² and R³ Euclidean space will be made, since they provide an intuitive depictions of the concepts being discussed. Nonetheless, these concepts can be generalized to vector spaces other than the Euclidean Rⁿ spaces and provide practical examples of these core concepts.

**Vector Spaces**

A Vector Space V, over an algebraic field K (a set of elements over which the operations of addition and multiplication are defined) is an ordered array consisting of a non-empty set V consisting of elements called vectors, the field K, whose elements are called scalars, and over which two operations, vector addition from V×V to V (+: V×V → V) and scalar multiplication from V×K to V [: V×K → V] are
defined. The operations must satisfy the following properties (Cerminara and Markarian, 2000; Hefferon, 2020):

- Vector addition Commutativity:
  \[ \forall u, v \in V, \quad u + v = v + u \]  

- Vector addition Associativity:
  \[ \forall u, v, w \in V, \quad u + (v + w) = (u + v) + w \]  

- Existence of the additive identity vector:
  \[ \exists 0 \in V \text{ such that } \forall v \in V, \quad 0 + v = v + 0 = v \]  

- Existence of the additive inverse vector:
  \[ \forall v \in V, \exists (-v) \in V \text{ such that } (-v) + v = v + (-v) = 0 \]  

- Scalar-vector multiplication associativity:
  \[ \forall a, b \in K, v \in V, \quad a \times (b \times v) = (a \times b) \times v \]  

- Multiplicative identity:
  \[ \forall v \in V, \quad 1 \times v = v \times 1 = v \]  

- Scalar-vector multiplication distributivity with respect to vector addition:
  \[ \forall a \in K, u, v \in V, \quad a \times (u + v) = a \times u + a \times v \]  

- Scalar-vector multiplication distributivity with respect to scalar addition:
  \[ \forall a, b \in K, u \in V, \quad (a + b) \times u = a \times u + b \times u \]  

Despite the abstractness of this definition, the concept of a vector space is a generalization of the familiar two- or three-dimensional Euclidean spaces. In the Euclidean Space, a vector is defined as a string of two or three real numbers (the real number domain \( \mathbb{R} \) is the parent field), and the vector space is denoted as \( \mathbb{R}^2 \) or \( \mathbb{R}^3 \), each entry of the vector is called a coordinate, which represents a magnitude following a certain dimension of the vector space. The vector addition and scalar-vector multiplication operations are the element-wise addition operation and the scalar-by-all coordinates product. Moreover, this visually-friendly abstraction is valid for \( n \)-tuples of \( n \) real numbers without any additional hypothesis, such space is commonly denoted as \( \mathbb{R}^n \).

Yet, vector spaces could be constructed using other entities as vectors: For instance, an \( n \)-dimensional vector space could be defined using all polynomials of degree up to (n-1), with real coefficients as
vectors, and the usual polynomial addition and scalar-polynomial product as vector-space-wide operations (Cerminara and Markarian, 2000).

Linear combinations of vectors, spanning sets, and bases of vector spaces

Linear combinations of scalars and vectors

Given a size-n set of vectors $S = \{v_i\}$ from the vector space $V$ and scalars $\lambda_i$ from field $K$, the vector $w$, defined as per equation 81, is called a linear combination of the vectors $v_i$ and is also an element from the space $V$.

$$w = \sum_{i=1}^{n} \lambda_i v_i$$

(81)

The concept of linear combination allows expressing elements of $V$ in terms of other elements of $V$. For a given set $S = \{v_i\}$, the set of vectors resulting from all possible linear combinations of the $v_i$ with all the scalars from field $K$ is referred to as $S$'s generated vector subspace, or the span of $S$. Particularly, if the set $S$’s generated subspace is equal to $V$ – that is, a set of scalars could be found to describe any vector of $V$ as a linear combination of the $v_i$ – then $S$ is said to be a spanning set of $V$.

Spanning sets of a vector space are allowed a degree of redundancy: such sets may contain vectors $v_j$ that are linear combinations of the remaining vectors that are in the set. If this occurs, the $n$ the collection of vectors in $S$ is called linearly dependent. The definition of linear independence [L. I.] for a set of vectors $S = \{v_i\}$ involves whether or not the only possible linear combination of $v_i$ that results in the zero vector is a null combination. Mathematically, $S = \{v_i\}$ is L. I. when:

$$S = \{v_i\} \text{ is L.I. when if: } 0 = \sum_{i=1}^{n} \lambda_i v_i \text{ then: } \lambda_i = 0 \forall i \in 1..n$$

(82)

Alternatively, set $S$ is linearly dependent [L. D.] if the zero vector could be expressed as a linear combination of the $v_i$ with at least one non-zero $\lambda_i$. A direct consequence of the L. D. condition is that a given vector $w$ from $S$’s generated subspace could have more than one representation as a linear combination of the $v_i$.

A collection of linearly independent vectors that span a vector space $V$ – that is, cannot be represented as combinations of the remaining set elements – is called a basis of the vector space (Cerminara and

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43 Given a vector space $V$, a vector subspace $W$ is a set of elements of $V$ in which it holds that, for any two elements of $W$, their sum also belongs to $W$, and the scalar-vector multiplication of any element of $W$ and any scalar from the parent field $K$, is also an element of $W$. The subspace is referred to as being closed under vector addition and scalar multiplication.
Markarian, 2000; Hefferon, 2020). The main advantage of bases over other spanning sets is that every vector in \( V \) has a unique representation as a linear combination of the basis’s elements. Uniqueness in the representation of any other vector as a combination of the basis’s elements. In other words, given a basis \( B = \{b_i\} \) of \( V \) and a vector \( v \) in the vector space \( V \), if \( v = \sum \lambda_i b_i \) and \( v = \sum \alpha_i b_i \) where \( \lambda_i \) and \( \alpha_i \) are scalars from \( K \), then \( \lambda_i = \alpha_i \) for all \( i \) from 1 to \( n \). The set \( \lambda = \{\lambda_i\} \) is then referred to as the vector of coordinates of \( v \) over basis \( B \) (Cerminara and Markarian, 2000; Hefferon, 2020).

Obtaining the vector of coefficients \( \lambda = \{\lambda_i\} \) for the representation of a vector \( v \) as a linear combination of the elements of a spanning set \( S \) involves solving the linear equation system \( S \lambda = v \), where \( S \) is a matrix\(^{44}\) whose columns are vectors of \( S \), and \( \lambda \) is the vector of coefficients \( \lambda_i \). When \( S = B \), a basis of \( V \), the system of equations has a unique solution, and matrix \( S = B \) is invertible – we can write \( \lambda = B^{-1}v \), where \( B^{-1} \) is the inverse matrix of \( B \) (Cerminara and Markarian, 2000).

A particularly important case is the equation system in which \( B \) equals the Identity matrix (the matrix \( I \) for which \( I \times v = v \times I = v \) for any \( v \) in the vector space \( V \)). Under such circumstances, the representation of \( v \) is trivial, \( \lambda = v \), the basis \( B = \{b_i\} \) is then referred to as the Canonical basis or Standard basis of \( V \).

**A motivational signal representation example**

In discrete signal processing, a discretely-sampled signal – a collection of \( n \) real-number measurements from a variable taken at different moments in time, like the deflection-slope measurements a TSD collects during a survey – can be represented by an \( n \)-tuple (a vector of \( \mathbb{R}^n \)) as (Figliola and Beasley, 2015; Peyré, 2021):

\[
y = \sum_{i=1}^{n} \delta_i y(t_i)
\]

(83)

The \( \delta_i \) are \( n \)-dimensional vectors that are equal to 1 at position \( i \) and zero elsewhere, these vectors represent a single unit-length measurement (a pulse) taken at time \( t_i \), the discretized time domain is encoded as \( n \) moments in time. The set of vectors \( \delta_i \), which are no other than \( \mathbb{R}^n \) Canonical basis, is also known as the Dirac Basis of the size-n discrete signal space.

Alternatively, the signal \( y \) could be represented in terms of a summation of elementary onsets (elementary signals that change their value from zero to 1 at time \( t_i \):

\(^{44}\) A matrix is an ordered rectangular array of numbers (Cerminara and Markarian, 2000). Matrices are characterized by their number of rows (horizontal lists) and columns (vertical lists). Within a matrix \( M \), every entry is denoted by its row and column number, thus the value at row \( k \) and column \( j \) is referred to as \( M(k,j) \) or \( M_{kj} \).
\[ y = y(t_1) + \sum_{i=2}^{n} h_i \times \left[ y(t_i) - y(t_{i-1}) \right] \]  

(84)

The elementary signals \( h_i \) are referred to as *Heaviside* signals (Chen et al., 2001).

**Orthogonal and orthonormal bases of vector spaces**

The notion of orthogonality between vectors is based on the definition of an *inner product* in a vector space.

Given a vector space \( V \) over a field \( K \), a function \( F: <,>: V \times V \to K \) is said to be an *inner product* in the vector space \( V \) if it satisfies the following properties (Markarian and Möller, 2004):

- **Associativity over the vector addition:**
  \[ \forall u, v, w \in V \quad \langle u, (v+w) \rangle = \langle u, v \rangle + \langle u, w \rangle \]  
  (85)

- **Distributivity over a scalar,**
  \[ \forall u, v \in V \quad a \in K \quad \langle a \times u, v \rangle = a \times \langle u, v \rangle \]  
  (86)

- **Conjugate symmetry:**
  \[ \forall u, v \in V \quad \langle u, v \rangle = \overline{\langle v, u \rangle} \]  
  (87)

- **Positivity.**
  \[ \forall v \in V \quad \langle v, v \rangle \geq 0 \]
  and if: \( \langle v, v \rangle = 0 \) then \( v = 0 \)  
  (88)

An example of an inner product in \( \mathbb{R}^n \) is the familiar *dot product* defined as the sum of coordinate-wise products, it can easily be proven that the dot product satisfies the definition of an *inner product*. Nonetheless, other inner products can be conveniently defined for vector spaces containing different types of elements as vectors (Markarian and Möller, 2004).

The inner product can be used to define three important geometric concepts:

- **Norm of a vector:** The inner product of a vector to itself (equation 88) can be used as source to define the vector’s *norm*, a measure of its length. Hence, *unit vectors* are defined as vectors whose norm equals 1.

- **Angle among vectors (for vectors with real coefficients only):** The cosine of the angle between two non-null vectors is defined as the ratio between their inner product and the product of both
vectors’ norms (equation 89). Particularly, vectors whose inner product is zero are perpendicular, or **orthogonal**.

\[
\cos \theta = \frac{\langle u, v \rangle}{\|u\| \|v\|}
\]  

(89)

- **Projection of one vector over another**: Given two vectors \(u, v, p\) the inner product among them \((p = \langle u, v \rangle)\), and \(\|u\|\) and \(\|v\|\) the vectors’ norms \((\|u\| = \langle u, u \rangle, \|v\| = \langle v, v \rangle)\), the vectors \(p/(\|u\| \|v\|) \times u\) and \(p/(\|u\| \|v\|) \times v\) are the projections of \(v\) over \(u\) and vice versa.

A basis \(B\) of \(V\) that consists of pairwise orthonormal vectors (orthogonal and with unit length) is called an **orthonormal basis** of \(V\). Gram-Schmidt’s Theorem states the procedure to construct an orthonormal basis \(B\) of \(V\), where \(V\) is a vector space with a defined inner product, out of the elements of a generic basis \(C\) (Markarian and Möller, 2004).

An interesting practical advantage of orthonormal bases over any other bases arises when representing any given vector from \(V\) as a linear combination of the basis’s elements. Generally, the vector of coefficients for the representation of a vector \(v\) in terms of the basis \(B\) is \(\lambda = B^{-1} v\), which although conceptually simple, is computationally expensive because of the inversion of matrix \(B\) (Cerminara and Markarian, 2000). However, when \(B\) is an orthonormal basis, it can be proven that \(B^{-1}\) is no other than the transpose of \(B\) \([B^T]\). As such, the vector of coefficients can be computed as \(\lambda = B^T v\), which is a remarkably simpler calculation than the generic solution for \(B^{-1}\) (Markarian and Möller, 2004; Hastie et al., 2015).

A final reference to our earlier example featuring bases of the discrete signal domain (equations 83 and 84): The Dirac Basis of the signal space – the Canonical basis of \(\mathbb{R}^n\) with the dot product as the inner product – is orthonormal, the proof of this assertion is immediate (the inner product among any pair of vectors of the Canonical basis shrinks to \(1 \times 0 + 0 \times 1 = 0\)). However, the Heaviside basis is not orthonormal. To illustrate this with a trivial example, suppose \(n = 50\), \(v_1 = h_{30}\), \(v_2 = h_{40}\) (jump from 0 to 1 at \(t = 30\) and \(40\) respectively); the dot product \(\langle v_1, v_2 \rangle = 11 \neq 0\). However, representing a signal in terms of Heavisides can have some advantages from an interpretation standpoint, as will be shown in the upcoming section.

**Vector Representation with Over-complete Dictionaries.**

In signal processing, the spanning sets of the discrete signals are often referred to as **dictionaries**, and the vectors contained in these sets are called **atoms**. If the dictionary is a basis, it is said to be a **complete**
dictionary. Conversely, a spanning set that is linearly dependent is said to be an over-complete dictionary. (Donoho and Johnstone 1994, 1995-b, Chen et al., 2001; Mallat, 2008).

From the examples discussed in the previous chapter, the Dirac and Heaviside bases of $\mathbb{R}^n$ are complete dictionaries. If the two bases are merged together, the resulting dictionary is over-complete. Other common dictionaries are the Fourier basis, whose atoms are sines and cosines (Figliola and Beasley, 2015, Markarian and Möller, 2004; Mallat, 2008; Peyré 2021), and the wavelet bases (Mallat, 2008; Peyré, 2021), which are sensitive to localized features within the source signal. The main reason why over-complete dictionaries are used to represent signals is that, although the representation is non-unique, it may occur that some of these representations are sparser than others (that is, fewer coefficients in the linear combination of atoms are non-zero). This is illustrated in the example presented next.

A motivational example: Sparse signal representation in a nutshell

In many practical applications, over-complete dictionaries can better extract important features from a given signal than complete dictionaries. Thus, a signal could be compressed (Chen et al., 2001; Hastie et al., 2015; Peyré, 2021) when its representation over a specific dictionary turns out sparse (that is, using few non-zero atoms). A sparse representation of a signal may simplify the task of transmitting it over a medium, allow an easy interpretation of the phenomena that originated it or even filter out random noise (Donoho and Johnstone, 1995-b; Chen et al., 2001).

The following example illustrates the concept of sparse representation with an over-complete dictionary consisting of the Dirac and Heaviside bases. Suppose a size-500 signal (as in a 50-seconds-long event sampled every 0.1 seconds) that consists of a linear combination of three Heaviside atoms and three Dirac atoms. Its representation under with atoms from the canonical (Dirac) basis requires 500 elements (left pane of figure 59). Alternatively, 10 non-zero heaviside atoms are required to represent this signal (center pane of figure 59). However, the over-complete dictionary comprising both the Heaviside and Dirac bases gives an exact representation with only six (out of 500) non-zero elements (right panel of figure 59).

Selecting an appropriate basis or bases for sparse TSD data decomposition

Representing TSD measurements as a sum of atoms from a dictionary achieves two purposes: Firstly, it decomposes the measurements into a sparse sum of atoms that can represent specific responses that are of interest. Secondly, the noise in the measurements is removed. These two goals can only be achieved if an appropriate dictionary is used.
Furthermore, a side advantage of sparse signal decomposition is noise removal: The random noise in a collection of measurements, like those of the TSD, is invariant to orthogonal transformations (like representing the noisy measurements over any orthonormal dictionary). Thus if the TSD measurements are represented over an orthonormal dictionary that promotes sparsity, said representation would likely contain few high-amplitude components representing the ground truth plus many small-amplitude components that represent the noise, which could be removed by thresholding (Donoho and Johnstone, 1994; Peyré, 2021). Thus, sparsity-promoting selection and regularization schemes (discussed later) that fit a sparse decomposition of the TSD signal by seeking a representation over a suitable orthonormal dictionary and keeping high-amplitude components only would both describe much of the signal true content and remove the measurement noise.

Recalling the previously introduced examples, the pavement manager conducting network-wide allocations may find value in the decomposition of the TSD data into a Heaviside signal basis, where two atoms may only be required to indicate the beginning and end of a homogeneous-response section. Other bases may turn out correct yet unsuitable signal decompositions, the Dirac (trivial) basis would turn out a dense description of the homogeneous section – for every measurement taken within the section, one atom is required, and no remarkable feature other than individual pulses may be recognized –, and other bases such as the Fourier basis may prove an ineffective descriptor of a discontinuity or even a localized continuous response, for an infinite number of atoms may be required to describe it (Daubechies, 1992).
Conversely, TSD decompositions into *dictionaries* that can both account for smooth continuous variations and spatially-localized changes in the input signal may be warranted for project-level applications. The resulting non-zero coefficients for such decomposition may highlight the existence of places within the surveyed corridor where localized structural distress is present and thus may warrant further investigation or bespoke repair solutions. Wavelet bases are suitable to achieve this goal, for wavelets can simultaneously represent smooth transitions and adapt to non-periodic features in the source signal (Daubechies, 1992; Donoho and Johnstone, 1995-a; Katicha et al., 2014, 2016).

Concerning wavelet-based reconstruction for TSD data interpretation practice, Katicha et al., (2014, 2016) implemented a wavelet-based decomposition for noise removal and feature extraction – the denoised deflection slope readings would then be utilized to compute the pavement’s effective Structural Number. Also, Kamiya et al. (2018) proposed wavelet-based filtering as a necessary noise-removal procedure for the deflection data collected by the MWD – the *moving weight deflectometer*, a lightweight TSD-like device built in compliance with Japanese vehicle size limitations.

Despite its spatial adaptability, wavelets may not be the most suitable basis for an intuitive representation of the pulse responses in deflection produced by extremely localized events in the pavement such as the load-bearing transverse joints in jointed pavement structures and the transitions between a *normal* pavement and a patched area (Katicha et al., 2014) – in all cases, events whose spatial extent is equal or less than the spacing between contiguous TSD measurements. Representation of such events may be best attained using *Dirac* components instead (Scavone et al., 2021). Thus, for applications such as interpretation of TSD measurements collected over a jointed pavement corridor, for which information about the pavement’s and the transverse joints’ structural health may be warranted, a decomposition of the TSD signal using an *over-complete dictionary* featuring both a wavelet basis and the Dirac basis may be preferred.

Yet, *over-complete dictionaries* allow for more than a single decomposition of the input TSD signal – an unsuitable outcome for it may lead to dissimilar interpretations and eventually dissimilar engineering decisions towards the segment’s preservation or repair. In light of this, the problem of TSD signal decomposition into an *over-complete dictionary* should be further constrained to seeking the *sparsest* decomposition that such *dictionaries* could furnish.
Achieving a sparse TSD signal decomposition by Basis Pursuit

Mathematically, the problem of decomposing a signal \( y \) of size \( n \) into a combination of the fewest atoms from a given dictionary is stated as:

\[
P_0: \min \| \alpha \|_0 \text{ subject to } y = \Phi \alpha
\]

where:

\[
\| \alpha \|_0 = \# \{ \alpha_i \neq 0 \}
\]

(90)

Where \( \Phi \) is the dictionary (an \( n \)-by-\( p \) matrix), and \( \alpha \) is the vector of coefficients (size \( p \)) that multiply each column of \( \Phi \) to obtain \( y \). Note that \( P_0 \) imposes no restriction on \( n \) and \( p \) – the case of \( p > n \) corresponds to the decomposition of \( y \) with an over-complete dictionary.

The “\( L_0 \) norm” on the coefficient vector \( \alpha \) is the count of non-zero entries in \( \alpha \). Despite its simple and intuitive formulation, problem \( P_0 \) is a combinatorial optimization problem, which is generally computationally intractable (Mazumder et al., 2011; Hastie et al., 2015).

However, a relaxation of problem \( P_0 \) can be stated by replacing the \( L_0 \) norm restriction with the \( L_1 \) norm. This alternative problem, called Basis Pursuit (Chen et al., 2001) [BP], which is closely related to the Lasso (Tibshirani 1996; Hastie et al., 2009, 2015; Donoho and Johnstone, 1995-a-b; Mazumder et al., 2011) is mathematically stated as:

\[
P_1: \min \| \alpha \|_1 \text{ subject to } y = \Phi \alpha
\]

where:

\[
\| \alpha \|_1 = \sum_{i=1}^{n} | \alpha_i |
\]

(91)

Alternatively, another relaxation of \( P_0 \) with the familiar Euclidean, or \( L_2 \), norm is the regularization procedure known in signal processing as the Method of Frames (Chen et al., 2001) and as Ridge Regression in statistics (Hastie et al., 2009; Friedman et al., 2010), which is formulated as:

\[
P_2: \min \| \alpha \|_2 \text{ subject to } y = \Phi \alpha
\]

where:

\[
\| \alpha \|_2 = \sum_{i=1}^{n} \alpha_i^2
\]

(92)

The \( L_2 \) norm regularization has a closed-form solution, but that solution is not sparse. In contrast, the \( L_1 \) norm restriction used in BP leads to a sparse representations of the source signal \( y \) (Mazumder et al., 2011; Tibshirani, 1996; Hastie et al., 2015; Donoho and Johnstone 1995-a). Furthermore, it is the closest norm that still results in a convex objective function (Tibshirani, 1996; Hastie et al., 2015).
BP can be re-stated as a simultaneous regularization and denoising problem (Chen et al., 2001; Hastie et al., 2015) as:

\[
\min_{\alpha} \frac{1}{2} \| y - \Phi \alpha \|_2^2 + \lambda \| \alpha \|_1 \quad (93)
\]

The regularization parameter \( \lambda \) affects the resulting number of non-zero elements of \( \alpha \): The bigger its value, the fewer non-zero elements, and thus the sparser the representation for \( y \).

BP does not have a closed form solution, and thus equation 93 must be solved numerically. Friedman et al. (2010) proposed a coordinate-wise descent numerical solution for the Lasso (which by extension applies to BP – equation 94) that relies on the soft-thresholding function (equation 95):

\[
\alpha^j = S(\alpha^{LSR}_j, \lambda) \quad (94)
\]

where:

\[
\alpha^{LSR}_j = (\Phi^T \Phi_j)^{-1} \Phi^T_j (y - \Phi_{-j} \alpha_{-j})
\]

And

\[
S(x, \lambda) = \text{sign}(x) \times \max(|x| - \lambda, 0) = \begin{cases} x - \lambda & \text{if } x \geq \lambda \\ x + \lambda & \text{if } x \leq -\lambda \\ 0 & \text{if } -\lambda < x < \lambda \end{cases} \quad (95)
\]

In equation 94, \( \Phi_j \) is the \( j \)th column of the matrix \( \Phi \), \( \Phi_{-j} \) is the matrix \( \Phi \) without the \( j \)th column, and \( \alpha_{-j} \) is the coefficient vector with the \( j \)th coefficient removed. The update is obtained by soft-thresholding the least-squares fit of the \( j \)th column of the matrix \( \Phi \) on the residual of the partial model that excludes the \( j \)th column vector [term \( \alpha^{LSR}_j \)]

*The bias-variance trade-off within Basis Pursuit. Selecting the penalty level \( \lambda \).*

BP, as formulated in equation 93, makes evident the implicit bias-variance trade-off in the choice of a value for \( \lambda \): Lower values of \( \lambda \) would allow for more features to be retained (at the risk of overfitting noise, thus adding variance) while higher values of \( \lambda \) would conversely lead to a sparser reconstruction but at the expense of damped amplitude (or bias towards zero) and the possibility of misconstruing true features by noise (Hastie et al., 2009; Peyrè, 2021).

A common procedure for selecting the value of \( \lambda \) for the Lasso is to try different values and select the one that gives the smallest cross-validation error (Hastie et al., 2009; Peyrè, 2021). However, if the noise is normally distributed with known standard deviation, \( \lambda \) can be selected by minimizing Stein’s Unbiased Risk Estimator [SURE] (Stein, 1981) – equation 96 (Donoho and Johnstone, 1995-a; Tibshirani, 1996).
SURE(\lambda) = \| y - \Phi \alpha(\lambda) \|^2 - \sigma^2 + 2\sigma^2 \| \alpha(\lambda) \|_0 \quad (96)

Where \( y \) denotes the noisy set of measurements, the product \( \Phi \alpha \) is the BP-recovered signal, \( \sigma \) is the standard deviation of the measurement noise, and \( \| \alpha \|_0 \) is the \( L_0 \) norm of \( \alpha(\lambda) \), thus the count of non-zero coefficients in \( \alpha \) for the given value of \( \lambda \). Stein (1981) proved that SURE’s expected value is equal to the mean square error between the true unknown measurement (\( z \)) and the reconstructed signal (equation 97).

\[
E \left[ SURE(\lambda) \right] = \| z - \Phi \alpha(\lambda) \|_2^2 \tag{97}
\]

**Two final comments on the selection of \( \lambda \)**

**Comment 1:** The iterative search for the optimal \( \lambda \) can be bounded between 0 and the *Universal Threshold* (Donoho and Johnstone, 1995-a) – equation 98.

\[
\lambda_{universal} = \sigma \sqrt{2 \ln p} \tag{98}
\]

Where \( \sigma \) is the standard deviation of the measurement noise and \( p \) is \( \Phi \)'s number of columns – the size of the dictionary.

Donoho and Johnstone proved (1995-a, -b) that under the *Universal Threshold*, \( L_1 \)-based shrinkage schemes (like BP) would only retrieve signal components that have a high probability of being true positives (thus not likely to fit noise) (Peyré, 2021). Yet, such a high penalty level may actually disregard true features that were measured by the TSD but are masked by the measurement noise.

**Comment 2:** In equations 96 and 98, the noise’s standard deviation \( \sigma \) must be known (or estimated) in order to apply BP to reconstruct the TSD measurements. Donoho and Johnstone (1995-b) suggested using the *mean absolute deviation over the median* [MAD] of the differences between consecutive measurements to estimate \( \sigma \) – equation 99. This approach was successfully used by Katicha et al. (2015) to estimate the noise in TSD measurements. Mathematically, \( \sigma \) can be estimated via MAD as follows:

\[
\sigma = \frac{1.4826}{\sqrt{2}} \times \text{median} \left( \| D_y - \text{median} \{ D_y \} \| \right) \tag{99}
\]

In equation 99, \( D_y \) represents the vector of differences between consecutive measurements [\( D_y(i) = y(i) - y(i-1) \)]. The \( 1/\sqrt{2} \) term in the formula accounts for the fact that the variance of the difference of two independent random variables having equal variance \( \sigma^2 \) (the measurements \( y(i) \) and \( y(i-1) \)) is equal to \( 2\sigma^2 \).

Equation 99 is derived from the MAD estimate for the standard deviation \( D_y \) as follows (equations 100 to 106):
To begin, state MAD to estimate the standard deviation of the $Dy$:

$$Stdev\{Dy\} = 1.4826 \times median\left(\left| Dy - median\{Dy\}\right| \right) \tag{100}$$

And by definition, each entry of $Dy$ is:

$$Dy(i) = y(i) - y(i - 1) \tag{101}$$

The variance of the $Dy$ [the square of $Dy$’s standard deviation] is:

$$Var\{Dy(i)\} = Var\{y(i) - y(i - 1)\} \tag{102}$$

Assuming that $y(i)$ and $y(i - 1)$ are not correlated (that is, the measurement $y(i)$ is not affected by $y(i - 1)$’s value, then their covariance is zero. Plus since both measurements have equal variance $Var(y) = \sigma^2$, then (Wasserman, 2004):

$$Var\{Dy(i)\} = Var\{y(i)\} + Var\{y(i - 1)\} - Cov\{y(i), y(i - 1)\} = \sigma^2 + \sigma^2 - 0 \tag{103}$$

Thus:

$$stdev\{Dy\}^2 = Var\{Dy\} = 2 \sigma^2 \tag{104}$$

Which means:

$$\sigma^2 = \frac{stdev\{Dy\}^2}{2} \tag{105}$$

Or, merging equations 100 and 105:

$$\sigma = \frac{1.4826 \times median\left(\left| Dy - median\{Dy\}\right| \right)}{\sqrt{2}} \tag{106}$$

Which concludes the proof.

**Balancing goodness-of-fit and sparse reconstruction: Reweighed $L_1$ Minimization**

The solution of BP depends on the degree of penalty selected [the value of the hyper-parameter $\lambda$]: A small value of the penalty parameter may return a closer fit to the input signal but at the risk of utilizing signal components to retrieve high-amplitude noise components as well, and thus with a large variance. Meanwhile, a high-penalty recovery may only retain true features but at the cost of eventually overlooking low-amplitude true components and biasing towards zero the amplitude of the recovered ones. Reweighted $L_1$ Minimization [RWL1] (Candès et al., 2008) tackles this issue by iteratively reformulating the BP optimization problem via a weighting matrix (equation 107).
\[
\min_{\alpha} = \frac{1}{2} \| y - \Phi \alpha \|^2_2 + \lambda \| W \alpha \|_1
\]

In this equation, the variables \( y, \phi, \alpha, \) and \( \lambda \) are as defined for BP, and \( W \) is an \( n \) by \( p \) matrix containing weighting factors (positive scalars) for each entry of \( \alpha \). These weights are applied (multiplied) to the non-zero coefficients of \( \alpha \) and hold an inverse proportionality with their targets. As such, low-amplitude, non-zero components of \( \alpha \) (which often correspond to the representation of noise) are penalized more heavily than high-amplitude ones (those relating to elements of the signal basis with a higher probability of being part of the true signal). In the end, the signal recovered by RWL1 is composed only of high-probability true elements, like that obtained from BP with the Universal Threshold penalty, without amplitude dampening and keeping a close resemblance with the unknown true signal.

Along with proposing RWL1, Candès et al. (2008), describe an iterative solution to this problem, which utilizes the computed \( \alpha \) at any given step to determine the matrix \( W \) to be used in the next iteration – BP is the initial iteration.

- Set iteration count \( k \) to zero and weights \( \omega_i^{(0)} = 1, i = 1, \ldots, p \).
- Solve the RWL1 problem:
  \[
  \min_{\alpha} \frac{1}{2} \| y - \Phi \alpha \|^2_2 + \lambda \| W_k \alpha \|_1
  \]
  \( W_k = [\omega_1^{(k)}, \ldots, \omega_n^{(k)}] \)
  \[\text{(108)}\]

- Update the weights for each \( i = 1, \ldots, p \),
  \[
  \omega_i^{(k+1)} = \frac{1}{|\alpha_i^{(k)}| + \varepsilon}
  \]
  \[\text{(109)}\]

- Repeat steps 2 and 3 for a previously specified number of iterations \( k_{\text{max}} \) or until some convergence criterion is met.

The parameter \( \varepsilon \) is included solely for numerical stability, it is meant to avoid divisions by zero when updating the \( w_i \). Candès et al (2008) point out that RWL1 is rather insensitive to the value of \( \varepsilon \) as long as it is a positive scalar some orders of magnitude smaller than the \( \alpha_i \). Moreover, RWL1 requires few iterations to converge to an outcome; that is, \( k_{\text{max}} \) need not be a large value – order of magnitude beyond \( 10^1 \).
Section 2: Fundamental Calculus Concepts and Pavement Properties

Back-Calculation.

This Dissertation’s *Data Interpretation Stage* (Manuscript 4, chapter 6) is founded on the notion of back-calculation of pavement properties from deflection measurements. In a nutshell, back-calculation consists of estimating the pavement’s structural parameters from a set of deflection measurements by contrasting these measurements against a modeled response and compute the squared error between them – the *deflection matching error* (Rohde and Smith, 1991).

Back-calculation problems are mathematically stated as optimization problems in which the *deflection matching error* is the target function to minimize by manipulating the values of the pavement properties used to fit the modeled response. Mathematically, the back-calculation problem is stated as an optimization problem (equation 110):

$$
\min_{\theta} \left\| w_{meas} - w_{model}(\theta) \right\|^2 = \sum_{i=1}^{N_{meas}} \left( w_{meas} - w_{model}(\theta) \right)^2
$$

subject to: $\theta_j > \theta_0 \forall \theta_j \in \theta$

Where $w_{meas}$ denotes the collection of pavement deflection measurements, and $w_{model}$ represents a simulated deflection response computed using the decision variables $\theta$ and some modeling framework. The *subject-to* statement in equation 110 accounts for any boundaries that may apply to the many parameters within the vector of unknowns $\theta$, like $E > 0$ or LTE between 0 and 1 for a pavement’s transverse joint.

Historically, the back-calculation problem was formulated to estimate the pavement properties from deflection measurements gathered by stop-and-go devices like the FWD (Rohde and Smith, 1991). FWD-based schemes could be ported to the TSD provided that reasonable deflection bowl depths could be estimated from the deflection velocity measurements gathered by the TSD – several procedures exists to this end Krarup et al., 2006; Müller and Roberts, 2013; Nasimifar et al., 2018). Alternatively, the back-calculation problem could be stated to compare simulated and field-based deflection slope basins or even deflection velocity measurements, as collected by the TSD (Manuscripts 3 and 4).

**Section organization**

Throughout this section, the mathematical background needed to state and solve the pavement back-calculation problem is presented. As such, this section’s first component, *Fundamental Calculus Notions*
introduces backbone concepts and theoretical results on continuity and differential calculus that would be

called for in the second component – Mathematical foundations of pavement back-calculation. In this
second sub-section, some additional discussion about minimization problems that concerns the
formulation of back-calculation problems is presented, and then the particular problem of back-
calculating the strength properties of a jointed pavement and the joint’s LTE index is introduced and
analyzed thoroughly.

Fundamental Calculus Concepts

Note: Throughout this background math review, one-dimensional functions are being considered as
default. Yet, the results presented in this section can generally be extended intuitively to functions of many
variables. However, it may occur that specifically discussing the case multivariate functions becomes
didactically worthwhile. Should that be the case, it will be highlighted accordingly.

Functions: A definition.

The starting milepost for this review is the very concept of function between sets of elements. Consider
two non-empty sets A and B. A function from A [domain set] to B [codomain set] (denoted f: A → B) is a
rule of correspondence that relates each element a from set A to one and only one element b from set B
(Rohde et al., 2012). Alternatively, function f could be understood as a mapping between the elements of
A and the product set A×B – the set of all pairs [a, b] with a from set A and b from set B.

Also, notation-wise, if elements a ∈ A and b ∈ B are related via function f, it is conventionally written as
f(a) = b.

The limit of a function

Note: The definition of limit of a function provided below stands for non-infinite limit.

Given a function f: A→ B, where both A and B are a subset of the real number domain R, its limit at value
a ∈ A is said to be value l if, for every positive yet small value ε, a positive number δ could be found such
that for every x ∈ A that is at a distance from a no larger than δ, its corresponded f(x) is at a distance from
l no larger than ε (Rohde et al., 2012; Guichard, 2022).

In mathematical notation, the limit of f as x tends to a is said to be l if:
\[
\lim_{x \to a} f(x) = l \iff \forall \varepsilon > 0, \exists \delta > 0 \text{ such that if } 0 < |x - a| < \delta \text{ then } |f(x) - l| < \varepsilon
\] (111)

Notes:

1. The definition stated in equation 111 can be also extended to define directional limits, that is, the limits of a function when regarding only values of \( x \) either greater than \( a \) or smaller than \( a \).

2. The definition of limit can be generalized to multivariate functions \([f: A \subset \mathbb{R}^n \rightarrow \mathbb{R}]\) by modifying the notion of distance between \( x \) and \( a \) accordingly. For instance, if \( a \) is a vector from \( \mathbb{R}^2 \) the condition for finding \( x \in A \) given \( \varepsilon \) is that \( x \) is located within the circle centered at \( a \) with radius \( \delta \) (Guichard, 2022). Similarly, if \( a \) is a vector of \( \mathbb{R}^3 \), the \( x \) that must verify equation 111 must be located within the sphere centered at \( a \) and with radius \( \delta \).

3. Evidently, the same observation made in point 2 can be extended for functions from \( \mathbb{R}^n \) to \( \mathbb{R}^p \), by redefining the distance between \( f(x) \) and \( l \) accordingly.

4. The Main Limit Theorem provides an algebraic framework for operating on the limits of functions that are combinations [sums, products, and quotients] of elementary functions (Rohde et al., 2012).

**Infinity as a limit of a function**

The notion of limit from equation 111 can be expanded to represent functions that around point \( a \) may diverge to infinity – instead of searching points of the domain around \( a \) whose relation is as close as a number as possible, the search looks for values around \( a \) whose relationship becomes as big as possible. Equation 112 presents the notion of limit of a function \( f: A \rightarrow B \) at point \( a \in A \) would being plus infinity – an analog definition can be immediately drafted for \( f \) diverging to minus infinity at point \( a \in A \).

\[
\lim_{x \to a} f(x) = +\infty \iff \forall k > 0, \exists \delta > 0 \text{ such that if } 0 < |x - a| < \delta \text{ then } f(x) > k
\] (112)

**Limit of a function as infinity**

Another forked definition from the main definition of limit of a function is the definition of the ultimate value of a function as its the values in its domain approach infinity (Rohde et al., 2012). Function \( f \) could either approach a finite value \( l \) or approach infinity as well. Below (equations 113 and 114) are the mathematical definitions for finite limit and plus infinite limit as \( x \) tends to plus infinity. Note that these
definitions assume that the domain \( A \) has no upper bound, thus an \( x \) big enough could be selected such that \( f(x) \) exists and \( x > M \).

Analog definitions can be crafted for \textit{minus infinity} limiting value as \( x \) tends to \textit{plus} infinity, and a full set of definitions for \( x \) approaching \textit{minus infinity}.

\[
\lim_{x \to +\infty} f(x) = l \text{ iff: } \forall \varepsilon > 0, \exists M > 0 \text{ such that } f(x) - l < \varepsilon
\]

\[
\lim_{x \to +\infty} f(x) = +\infty \text{ iff: } \forall k > 0, \exists M > 0 \text{ such that } f(x) > k
\]

\textit{Function continuity}

The notion of limit of a function is a preamble to the formal definition of \textit{continuity} of a function. A function \( f: A \subset \mathbb{R}^n \to \mathbb{R} \) is said to be continuous at point \( a \in A \) if its limits for \( x \to a \) exists and is equal to \( f(a) \) (Rohde et al., 2012). In mathematical notation:

\[
f: A \to B \text{ is continuous at } a \in A \iff \lim_{x \to a} f(x) \text{ and: } \lim_{x \to a} f(x) = f(a)
\]

(115)

Notes:

- If \( f(x) \) diverges to infinity as \( x \to a \), \( f \) is said to be \textit{not continuous} at point \( a \).
- The definition from equation 115 can be generalized to define \textit{continuity} of a function throughout an interval within its domain by stating that equation 115 holds for all point \( a \in I \subset A \).
- The notion of \textit{continuity} has remarkable geometric implications, for it is the formal definition of \textit{smoothness} of a function. In other words, when a function is continuous, a small variation in its \textit{independent variable} \( x \) leads to a small variation in its \textit{dependent variable} \( f(x) \) (Rohde et al., 2012).

\textit{The derivative of a function}

Given a scalar function \( f \), its derivative – often written \( f'(x) \) – is another function that tells \textit{the rate at which} \( f \) \textit{changes value as} \( x \) increases or decreases – The derivative of \( f \) at point \( x \) (\( f'(x) \)) expresses the slope of a line tangent to \( f \) at point \( f(x) \) (Rohde et al., 2012). Formally, the derivative is defined as a limit to express the notion of \textit{instantaneous} change of value around \( x \) (Quarteroni et al., 2000; Rohde et al., 2012; Guichard, 2022) – equation 116.
\[
  f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}
\]

(116)

Connected to the definition of derivative, a function \( f \) is said to be differentiable at point \( a \) if the limit stated in equation 116 exists. Plus, as with the case of continuity, the notion of differentiability of \( f \) can be extended over intervals: \( f \) is said to be differentiable over interval \( A \) if the limit on equation 116 holds for every point \( x \in A \) (Rohde et al., 2012).

**Numerically estimating derivatives: Finite differences**

In practical applications, the user may not know the closed-form analytical expression of a given function \( f \) – in fact, it may not even exist! In turn, the user may only be able to evaluate function \( f \) numerically at a discrete (finite and countable) set of points. Yet, it may be of interest (or out of necessity) that the derivative of \( f \) needs to be computed as well. Relying on the geometric notion that \( f'(x) \) represents the slope of a line tangent to \( f \) at point \( f(x) \), the extreme case of a line secant to \( f \) at points \( f(x) \) and \( f(x+h) \), where the distance \( h \) is infinitely small, said line could be drawn by considering point in the very close vicinity of \( x \). Equations 117 to 119 present the approximations by forward difference (the secant line through points \( x \) and a point slightly ahead of \( x \)), backward difference (the secant using point \( x \) and a point slightly behind \( x \)), and centered difference (the secant using a point behind and a point ahead of \( x \)) (Quarteroni et al., 2000).

\[
  f'(x) = \frac{f(x+h) - f(x)}{h}
\]

(117)

\[
  f'(x) = \frac{f(x) - f(x-h)}{h}
\]

(118)

\[
  f'(x) = \frac{f(x+h) - f(x-h)}{2h}
\]

(119)

In practical terms, the increment \( h \) is chosen to be very small, the larger \( h \), the bigger approximation error committed when approximating \( f'(x) \). Among these three estimates, given the value of \( h \), it can be proven that the centered finite difference yields the smallest approximation error (Quarteroni et al., 2000).

**Derivatives of multivariate functions and the Gradient vector**

On functions of many variables \([f: A \subset R^n \to R]\), the concept of derivative must be generalized for at any given point \( x \) in \( A \), an infinite number of lines can be drawn that are locally tangent to \( f \) at \( f(x) \). In particular, a tangent could be drawn that follow the direction of a single coordinate \( x_i \) of the independent variable \( x \), while treating the remaining coordinates as constants – thus, this tangent would be computed
as the tangent for a scalar function (Guichard, 2022). In the limiting case, the slope of this tangent is the partial derivative of \( f \) over variable \( x_i \) (Lange, 2013) – equation 120.

\[
f_{xi}(x) = \frac{\partial f}{\partial x_i} = \lim_{h \to 0} \frac{f(x_i + h, \ldots, x_n) - f(x_i, \ldots, x_n)}{h}
\]

(120)

As with any single-variable function derivative, the partial derivatives of a multiple-variable function could be computed by finite differences – equations 117 to 119.

The partial derivatives can be handy to compute the partial derivative of function \( f \) following any direction \( u \), where \( u \) is a size-n vector whose norm is 1. It can be proven (Guichard, 2022) that this directional derivative can be calculated as a dot product. Equation 121, for simplicity, represents the case for a two-variable function \( f(x, y) \).

\[
f_u(x) = \langle [f_x, f_y], u \rangle
\]

(121)

Vector \([f'_x, f'_y]\) is customarily written as \( \nabla f \) and is the gradient vector of \( f \) at point \( x \). Using dot product properties (equation 89), it can be seen that \( \nabla f \) points at the direction of \( f \)'s steepest ascent slope at point \( x \), and the value of said slope is its L_2 norm. Similarly, \(-\nabla f\) denotes the direction of function \( f \)'s steepest descent. Besides, if vector \( u \) is such that is orthogonal to \( \nabla f \), then \( f \) remains constant in that direction – for the 2-dimension case, vectors \([f'_x, -f'_y]\) and \([-f'_y, f'_x]\) are normal to \( \nabla f \). The vectors normal to \( \nabla f \) describe the plane normal to \( f \) at point \( x \) (Guichard, 2022).

One final observation concerning multivariate functions with multivariate codomain \([f: A \subset \mathbb{R}^n \to \mathbb{R}^p]\): each coordinate of \( f \)'s output is a multivariate function as the previous examples, for which \( \nabla f_i \) could be defined. The gradient concept is generalized into the Jacobian matrix: the matrix of all partial derivatives of all coordinate of \( f \) over each coordinate of the independent variable (equation 122):

\[
J_f(x) = \begin{bmatrix}
f_{1x1}(x) & f_{1x2}(x) & \cdots & f_{1xn}(x) \\
f_{2x1}(x) & f_{2x2}(x) & \cdots & f_{2xn}(x) \\
\vdots & \vdots & \ddots & \vdots \\
f_{px1}(x) & f_{px2}(x) & \cdots & f_{pxn}(x)
\end{bmatrix}
\]

where:

\[
f_{kj}(x) = \frac{\partial f_k}{\partial x_j}
\]

(122)

Obviously, when \( p = 1 \), \( J_f = \nabla f_i \)

**When the derivatives are zero: Maxima, minima, and saddle points**

The notion of derivative (in scalar functions) and gradient vector (on multivariate functions) imply that their parent function \( f \) is changing value: A positive derivative implies that \( f \) is increasing in value as \( x \)
increases, and a negative derivative implies that \( f \) decreases with increasing \( x \). Conversely, for multi-dimensional functions, the same interpretation could be applied to partial derivatives only, for the gradient vector \( \nabla f \) will always point at the direction of steepest increase (Rohde et al., 2012; Guichard, 2022). But what if the derivative were 0 at \( x = a \), or the gradient vector were the null vector? In that case, \( f \) is locally stalled. In those cases, the sign of \( f' \) (or the partial derivatives) around point \( a \) would give additional insight about the local behavior of \( f \).

For scalar functions \( f: A \subset \mathbb{R} \to \mathbb{R} \), that are differentiable in domain \( A \). If \( f \) has a critical point at \( x = a \) [either \( f'(a) \) does not exist or exists and \( f'(a) = 0 \)], then:

- if \( f'(x) < 0 \) for \( x < a \), and \( f'(x) > 0 \) for \( x > a \), then \( f \) has a local (relative) minimum at point \( a \)
- if \( f'(x) > 0 \) for \( x < a \), and \( f'(x) < 0 \) for \( x > a \), then \( f \) has a local (relative) maximum at point \( a \)
- if \( f'(x) \) does not change sign before and after \( a \), then \( f \) has a saddle point at point \( a \)

Note: As said above, a function can have a local minimum at \( a \) without a horizontal tangent; such is the case of non-derivable functions at point \( a \), the easiest example thereof is the absolute value function \(|x|\) at \( a = 0 \). However, it can be proven that if a function \( f: (a, b) \to \mathbb{R} \) has a local minimum at point \( c \) in \( \in (a,b) \), and \( f \) is differentiable at \( c \), then \( f'(c) = 0 \) (Rohde et al., 2012).

For multivariate functions, the critical point notion generalizes to \( a \) being such a point if \( \nabla f(a) = \mathbf{0} \).

**Root-finding with the help of derivatives: The Newton-Raphson method.**

The Newton-Raphson method is a numerical procedure to find the root of a function \( f(x) \) by iteratively descending (or ascending) following \( f' \)’s tangents locally (Quarteroni et al., 2000).

Suppose a function with a root at point \( a \), \( f(a) = 0 \). After Lagrange’s Mean Value Theorem, the following approximation to \( f \) using the line between a generic pair \([x, f(x)]\) and \([a, 0]\) can be stated – equation 123:

\[
0 = f(a) = f(x) + (a-x) \times f'(c)
\]

In which \( c \) is a point in the interval \([a, x]\) (or \([x, a]\), if \( x \) is smaller than \( a \)).

After equation 123, one can iteratively reach to point \( a \) from a random (yet nearby) point \( x_0 \) by solving, at each time \( t \), the point \( x_{t+1} \) for which \( f(x_{t+1}) \) equates zero, provided \( f'(c) \), with \( c \) between \( x_t \) and \( x_{t+1} \) is conveniently approximated by some function \( q(x_t) \) (Quarteroni et al., 2000). Mathematically, the iterative step for this generic numerical method stated as:
\[ x_{t+1} = x_t - \frac{1}{q(x_t)} \times f(x_t) \]  

(124)

Newton’s method feature consists of stating \( q(x_t) = f'(x_t) \). As such, the iterative descent to the root corrects the size of the iterative step by means of \( f'(x) \). Newton’s method iterative step is stated as – (equation 125).

\[ x_{t+1} = x_t - \frac{f(x_t)}{f'(x_t)} \]  

(125)

**Newton’s Root Finding method for multi-dimensional functions.**

In the multivariate case, Newton-Raphson’s method is generalized to simultaneously solve a system of \( n \) equations with \( n \) unknowns – the root of \( f: \mathbb{R}^n \to \mathbb{R}^n \). The *Jacobian matrix* of \( f \) takes the place of the derivative. The iterative step for the multivariate is (equation 126):

\[ x_{t+1} = x_t - [J_f]^{-1} \times f(x_t) \]  

(126)

Where \( J_f \) is the inverse of \( f \)’s *Jacobian* matrix (equation 122).

**Curvature: The second derivative of a function**

If function \( f \) is differentiable, it will have a derivative \( f' \). Then if the derivative \( f' \) is differentiable as well, the *second derivative of \( f \) can be introduced* (Rohde et al., 2012):

\[ f''(x) = \lim_{h \to 0} \frac{f'(x+h) - f'(x)}{h} \]  

(127)

The geometric interpretation of \( f'' \) is the *degree of curvature* of \( f \). If \( f''(x) > 0 \), \( f \) is said to be *concave up*, while if \( f''(x) < 0 \), \( f \) is said to be *concave down*. The particular case of \( f''(x) = 0 \) means that, locally around \( x \), \( f \) follows a straight line (Rohde et al., 2012).

**Numerically estimating second derivatives: A formula based on finite differences**

The second derivative of \( f \) at point \( x \) could be numerically estimated by repeatedly applying the finite difference formulas – equations 117 to 119. The first pass would compute \( f' \) estimates which would then used to compute \( f'' \). Yet, \( f'' \) can be estimated directly as (Quarteroni et al., 2000):

\[ f''(x) = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2} \]  

(128)
Additional conditions for relative extremes of $f$ based on $f''$

The second derivative of $f$ can furnish additional insight about $f$'s critical points, especially those with a horizontal tangent [$f'(x) = 0$]. For scalar functions $f$: $A \subset \mathbb{R} \rightarrow \mathbb{R}$ (Rohde et al., 2012; Lange, 2013):

- if $f'(a) = 0$, and $f''(a) \neq 0$, then $a$ is a local extreme
- if $f'(a) = 0$, and $f''(a) > 0$, then $a$ is a local minimum
- if $f'(a) = 0$, and $f''(a) < 0$, then $a$ is a local maximum
- if both $f'(a) = 0$ and $f''(a) = 0$, no conclusion can be drawn.

Second derivatives of multiple-variable functions – the Hessian matrix

The notions brought by the first and second derivatives of a scalar function are generalized to multi-dimensional functions by multi-dimensional entities as well. The gradient vector and the many partial derivatives generalize the concept of slope of a function and allow defining geometrical objects tangent to function $f$. Eloquently, this also apply to the second derivatives: for each partial derivative of a function $f$ with n-dimensional independent values, n partial second derivatives can be defined (Guichard, 2022). These are expressed as (equation 129):

$$f_{x_i x_j}(x) = \frac{\partial^2 f}{\partial x_i \partial x_j} = \frac{\partial}{\partial x_i} \left[ \frac{\partial f}{\partial x_j} \right] = \lim_{h \rightarrow 0} \frac{f_j(x_h) - f_j(x)}{h}$$

where: $x = [x_1, x_2, ..., x_i, ..., x_n]$ and: $x_h = [x_1, x_2, ..., x_i + h, ..., x_n]$  

A remarkable result on this subject is the inter-operability of partial second derivatives: it can be proven that, if the mixed partial derivatives are continuous, they are in fact equal [Clairaut’s Theorem, equation 130] (Guichard, 2022):

$$\frac{\partial^2 f}{\partial x_i \partial x_j} = \frac{\partial^2 f}{\partial x_j \partial x_i}$$

The mathematical object containing all the concavity information on a multi-dimensional function is no longer a single partial second derivatives, but a matrix: the Hessian matrix, which is defined as follows (equation 131):
As was the case with the sign of \( f'' \) at a critical point \( a \) being handy to tell whether \( a \) was a local minimum, maximum, or saddle point, properties of \( H_f \) may tell whether point \( a \) is an extreme or a saddle point:

- If \( H_f \) is positive-definite [all its eigenvalues\(^{45}\) are positive], then point \( a \) is a local minimum
- If \( H_f \) is negative-definite [all its eigenvalues are negative], then point \( a \) is a local maximum
- If \( H_f \) has both positive and negative eigenvalues, then point \( a \) is a saddle point
- If \( H_f \) has 0 as an eigenvalue, then the test is inconclusive

For 2-dimensional functions \( f(x, y) \), the discriminant \( [ \text{the determinant of the } 2 \times 2 \text{ matrix } H_f – \text{ equation 132} ] \) provides the test grounds (Guichard, 2022):

\[
D(x, y) = f_{xx}(x, y)f_{yy}(x, y) - f_{xy}(x, y)f_{yx}(x, y)
\]

- If \( D(x, y) > 0 \) and \( f_{xx} < 0 \), then point \([x, y]\) is a local maximum
- If \( D(x, y) > 0 \) and \( f_{xx} > 0 \), then point \([x, y]\) is a local minimum
- If \( D(x, y) < 0 \), point \([x, y]\) is a saddle point
- If \( D(x, y) = 0 \), the test is inconclusive.

A brief comment on convex functions

At this point in the review a small digression is warranted to introduce the concept of convexity in functions, for convex functions have remarkable properties that can be advantageous when dealing with optimization problems.

Yet, the definition of convex function requires that convex sets are defined beforehand. Let a set \( A \subset \mathbb{R}^n \). The set \( A \) is said to be convex if for every pair of points \( a, b \in A \), the line joining them is contained within \( A \) as well (Dattorro, 2019). Mathematically, the rule for convexity can be expressed as follows:

---

\(^{45}\)An eigenvalue of a square matrix \( M \) of size \( n \times n \) is defined as a number \( \lambda \) such that there exists a vector \( a \) of length \( n \) such that \( Ma = \lambda a \). Vector \( a \) is referred to as an eigenvalue of \( M \). (Cerminara and Möller, 2004; Quarteroni et al., 2000).
set $A \subset \mathbb{R}^n$ is convex if $\forall a, b \in A, \lambda \in [0,1]$, then $c = \lambda a + (1 - \lambda) b \in A$

(133)

The notion of *convexity* for scalar functions involves lines joining pairs of values of the dependent variable, but it portrays a slightly different connotation than for sets. A function $f: A \to B$ is said to be convex (Lange, 2013; Dattoro, 2019) if:

- Its domain, set $A$, is convex
- Given any pair of points $a, b \in A$, the line joining $f(a)$ and $f(b)$ is an upper boundary of $f$ for all the points between $a$ and $b$. This latter condition is expressed as:

$$f: A \to B \text{ is convex if:}$$

1) $A$ is convex
2) $\forall a, b \in A, \lambda \in [0,1]$, if: $t = \lambda a + (1 - \lambda) b$, then $f(t) \leq \lambda f(a) + (1 - \lambda) f(b)$

(134)

The definition from equation 134 can be slightly modified to introduce the notion of *strict convexity*, by swapping the $\leq$ condition between $f(t)$ and the line between $f(a)$ and $f(b)$ for a strict $<$ relationship – equation 135.

$$f: A \to B \text{ is strictly convex if:}$$

1) $A$ is convex
2) $\forall a, b \in A, \lambda \in [0,1]$, if: $t = \lambda a + (1 - \lambda) b$, then $f(t) < \lambda f(a) + (1 - \lambda) f(b)$

(135)

It can be proven that if a function $f: A \to B$ is strictly convex (and, implicit within the *convexity* condition, also continuous) throughout its domain, then it will have a unique global minimum in $A$ (Dattoro, 2019).

Eloquently, this result is of major interest when stating optimization problems: if the target function is convex throughout the domain defined by its decision variables, it is guaranteed it will have a unique global minimum within its domain (Lange, 2013; Dattorro, 2019). Moreover, any descent technique used to locate the minimum of the target function will converge to this global minimum.

**Mathematical foundations of pavement back-calculation**

This section ties together the many background calculus notions seen so far into a numerical procedure to solve optimization problems, which is executed under the hood in the computer implementation of the pavement back-calculation problems discussed in Manuscripts 3 and 4 (Chapters 5 and 6). Particularly, this section introduces *Gradient Descent* as a solution to optimization problems and discusses how a back-calculation problem can be solved by this method.
A brief comment on optimization problems

As stated at the beginning of this section, the pavement back-calculation problem (equation 110) was stated as an optimization problem – and so were the signal reconstruction problems presented in equations 90 to 93, and 107-108. In a nutshell, optimization problems consist of locating the est of values of the independent variable $\theta$ that minimize some target function $F: \mathbb{A} \rightarrow \mathbb{R}$ (equation 136):

$$\min_{\theta} F(\theta) \quad \text{with} \quad \theta \in \mathbb{A}$$

subject to: $g(\theta) \leq 0$  \hspace{1cm} (136)

In equation 136, $F(\theta)$ is the target function (or cost function (Zhang et al., 2021)) to be minimized, and $g(\theta)$ denotes an additional constraints function that may impose limiting values for $\theta$.

Generally speaking, a problem like equation 136 is not guaranteed to have a solution – depending on $F$, it may not even have one, or even multiple solutions given the values of $J_F$ in domain $\mathbb{A}$. However, if $F$ is a strictly convex function in domain $\mathbb{A}$, it will have a single global minimum – the optimization problem has a unique solution (Lange, 2013; Dattorro, 2019).

The pavement back-calculation problem (Rohde and Smith, 1991) is one particular case of a convex optimization problem. Recall its formulation (equation 137):

$$\min_{\theta} \left\| w_{\text{meas}} - w_{\text{model}}(\theta) \right\|^2 = \sum_{i=1}^{N_{\text{meas}}} \left( w_{\text{meas},i} - w_{\text{model},i}(\theta) \right)^2 \quad \text{subject to:} \quad \theta_j > \theta_{0,j} \quad \forall \theta_j \in \theta$$

Where $w_{\text{meas}}$ denotes the collection of pavement deflection measurements, and $w_{\text{model}}$ represents a simulated deflection response computed using an attempted set of values of the unknown pavement properties (packed together in vector $\theta$) and some modeling framework. The target function is a vector norm, the $L_2$ norm of the difference between the measurements and the simulated response, and vector norms are convex functions (Lange, 2013; Dattorro, 2019), meaning that back-calculation problems would have unique solutions, provided the target function does not diverge within the domain of $\theta$.

Gradient Descent to solve back-calculation problems

If the closed-form expression of the target function $F(\theta)$ of a generic optimization problem is known, the solution can be sought by first locating its critical points, that is, values of $\theta$ for which $\nabla F$ is null. However, it is often the case on pavement back-calculation problems that the simulated deflection component is actually a very complicated function, and so its close-form solution may be cryptic, or it
may not even be formulated (Van Cauwelaert, 2004), let alone compute its gradient and solve for the values of \( \theta \) that make \( \nabla F = 0 \).

Yet, the solution to equation 137 can be sought numerically by Gradient Descent (Zhang et al., 2021; Peyré, 2021). Simply put, gradient descent methods are iterative methods that attempt to reach the minimum of \( F \) by following the direction of \( \nabla F \). Commencing at a conveniently-located value \( \theta_0 \), the iterative step towards a minimum of \( F(\theta) \) is (Zhang et al., 2021; Peyré, 2021) – equation 138:

\[
\theta(t+1) = \theta(t) - LR \times \nabla_F(\theta)
\]  

(138)

The term LR, often referred to as learning rate (Zhang et al., 2021), controls the size of the iterative step towards the minimum.

Dealing with orders of magnitude: Hessian preconditioning in gradient descent

In practice, for gradient descent to converge to a result in a stable manner, LR must be set to a conveniently-small quantity: During implementation stage, many attempts at different values of LR must be tried, and the value that both balances decently fast and stable convergence must be kept. Nonetheless, gradient descent may still perform unstably – or at least with a sub-optimal convergence speed if the different coordinates of \( \theta \) have different magnitudes and/or the product LR \( \nabla F(\theta) \) has dominating coordinates. Plus, there is always the risk of overshooting the optimum when using a fixed LR – the descent may never converge and bounce around the optimum if LR is not shrank as \( \theta \) approaches the target value (Zhang et al., 2021).

Among the many schemes that can overcome these possible drawbacks of gradient descent, Hessian preconditioning becomes advantageous for it can adaptively correct LR as the descent evolves and account for potential issues that may arise because of the orders of magnitude in the coordinates of \( \theta \) (Zhang et al., 2021). The preconditioned iterative step is formulated as:

\[
\theta(t+1) = \theta(t) - LR \times H_F^{-1} \times \nabla_F(\theta)
\]  

(139)

In equation 139, \( H_F \) is the Hessian matrix of the cost function.

It can be seen that the particular case of equation 139 with LR = 1 is in fact the Newton-Raphson root solving method for a multi-variable function – equation 126 (Zhang et al., 2021). The function that is being “equated to zero and solved for \( \theta \)” is \( \nabla F \), whose Jacobian matrix \( J_F \) is \( F \)’s Hessian Matrix \( H_F \).
In order to save the computational expense of solving, and then inverting, the Hessian matrix $H$, Zhang et al. (2021) propose a relaxed preconditioned iterative step using only the partial second derivatives over each. Coordinate-wise, the descent is formulated as:

$$\theta(t+1)_j = \theta(t)_j - LR \times H_{Fj}^{-1} \times \frac{\partial F(\theta(t))}{\partial \theta_j}$$

where: $H_{Fj} = \frac{\partial^2 F(\theta(t))}{\partial \theta_j^2}$

Zhang et al. (2021) argue that the simplification involved in this descent procedure conveniently offsets the expense of solving $H_{Fj}^{-1}$, especially when dealing with high-dimension optimization problems, such as the training stage of a deep neural network.

The computational advantage of gradient descent with the Hessian preconditioning (either relaxed or in full-form) will be shown through an example. Two gradient descent schemes to solve convex function $F(x, y) = x^2 + 0.1 \ y^2 = 0$ were attempted (the ground-truth solution is $[0,0]$). The gradient and Hessian terms are as follows:

$$F(x, y) = x^2 + 0.1 \ y^2$$

$$\nabla_F(x, y) = [2x, 0.2y]$$

$$H_{Fx} = 2, H_{Fy} = 0.2$$

The non-preconditioned and preconditioned gradient descent iterative steps are:

non-preconditioned:

$$\begin{bmatrix} x \\ y \end{bmatrix}(t+1) = \begin{bmatrix} x \\ y \end{bmatrix}(t) - LR \times \begin{bmatrix} 2x(t) \\ 0.2y(t) \end{bmatrix}$$

preconditioned:

$$\begin{bmatrix} x \\ y \end{bmatrix}(t+1) = \begin{bmatrix} x \\ y \end{bmatrix}(t) - LR \times \begin{bmatrix} \frac{1}{2} \times 2x(t) \\ \frac{1}{0.2} \times 0.2y(t) \end{bmatrix}$$

From equations 142 and 143, it can be noted that in the non-preconditioned descent, $x$ and $y$ will descend at different speeds – $x$ will descend 10 times faster than $y$. Meanwhile, both variables would descend at the same speed in the preconditioned descent. This is graphically shown in the upper panes of figure 60, whereas the lower right panel of figure 60 shows how the preconditioned gradient descent converges to zero in fewer iterations than the non-preconditioned scheme (even if initially the decay is slower).
In most pavement back-calculation procedures, the Hessian correction is certainly warranted, for the unknown variables (the elastic moduli of the pavement layers) actually differ by several orders of magnitude. For instance, if a 2-layer asphalt linear elastic pavement on ground were being back-calculated, \( \theta = [E_{\text{HMA}}, E_{\text{BASE}}, E_{\text{SUBGRADE}}] \) would be in the vicinity of \([2800, 280, 56]\) MPa (\([400,000, 40,000, 8,000]\) PSI) (AASHTO, 1993), a ratio of 5:1 and 50:1. Similarly, when back-calculating the problem of a concrete slab on ground, \( \theta = [E_{\text{PCC}}, k_{SG}] \) would be on the order of \([30 \times 10^9 \text{ (Pa)}, 3 \times 10^7 \text{ (Pa/m)}] \), a 1000:1 ratio!

Figure 61 compares a non-preconditioned and preconditioned descent attempt to solve the back-calculation problem for a non-jointed rigid pavement structure based on slope-deflection measurements – this back-calculation problem is formulated as part of the interpretation stage of TSD deflection velocity measurements.
Figure 61: Non-preconditioned (top) and preconditioned (bottom) gradient descent scheme for the back-calculation of concrete pavement properties. While on the non-preconditioned problem, only the subgrade’s $k$ descends, on the preconditioned problem, both variables converge.

Back-calculation of concrete pavement properties and joint’s structural health indices from TSD measurements.

The acme of this section is the presentation of the back-calculation scheme for jointed concrete pavements based on TSD data, which is developed fully in Manuscript 4 (Chapter 6) as an implementation compatible with 5-cm-resolution deflection velocity data from a fourth-generation TSD device. The ultimate goal of this Dissertation’s interpretation stage is to provide an estimation of both the pavement’s structural properties and the joint’s Load Transfer Efficiency (LTE) index (Pierce et al., 2017) from the measurements the TSD collect at locations nearby a transverse joint. Thus, if such a back-
calculation scheme could be devised, then the joint’s structural performance could be assessed on the fly, relieving the need for an LTE test with an FWD.

Following the general back-calculation problem formulation from equation 137, the back-calculation problem for a jointed pavement based on TSD measurements is stated as:

\[
\min_\theta \| TSD - \dot{w}(\theta) \|^2 = \sum_{i=1}^{TSD\text{-sensors}} (TSD_i - \dot{w}_i(\theta))^2
\]

where: \( \theta = [LTE, \psi] \)
subject to: \( \psi > 0, LTE \in [0,1] \) (144)

Where LTE is the joint’s load transfer efficiency index, and vector \( \Psi \) contains any additional relevant properties, like the concrete slab’s \( E \), and the subgrade’s modulus of reaction (\( k \) value) (Van Cauwelaert, 2004). The formulation in equation 144 has been implemented for deflection deflection velocity measurements, as it was found that the TSD produces these measurements reliably and that deflection slope estimates cannot be directly inferred from deflection velocity data on jointed pavements (Manuscript 3, Chapter 5).

In any case, a comprehensive yet not overly computationally-demanding deflection model capable of simulating the deflection response of a jointed slab is needed to provide the simulated deflection component \( \dot{w}(\theta) \) to construct the target function. Van Cauwelaert (2004) provides such a framework, which has been previously analyzed by Deep et al. (2020 -a, -b, -c) and found a good approximation to the outcome of a finite element model and a real deflection basin produced by a heavy truck axle rolling over a pavement at a low speed. Plus, Deep et al. (2020 -a, -b, -c) stated that this model is suitable as a core for a back-calculation scheme like the one in equation 144. Thus, Van Cauwelaert’s (2004) linear elastic jointed slab-on-ground model is utilized to construct \( \dot{w}(\theta) \), a review of this model follows:

**The deflection basin of a linear-elastic jointed slab on ground**

Van Cauwelaert’s (2004) mechanistic deflection model based on linear-elastic slab-on-ground theory is graphically presented as shown in figure 62. The pavement structure is supposed a pair of linear elastic concrete slabs with Young modulus \( E \), and Poisson coefficient \( \nu \), the slab thickness is \( h \). The slab’s foundation is a Pasternak foundation, a combination of a Winkler foundation (dense liquid capable of only a vertical response, its modulus of sub-grade reaction is \( k \)) plus a shear layer with shear modulus \( G \). In this model, the applied load is assumed as a uniformly distributed load, pressure \( p \), spread over a rectangular area of dimensions 2a (longitudinal) by 2b (transverse); the transverse joint is located at a distance \( c \) from the center of the load. The joint’s LTE was not drawn.
Figure 62: Definition of the jointed-slabs-on-ground problem.

The deflection basin spreading both the loaded and unloaded slabs [$w_L$ and $w_{UL}$ respectively] has a closed-form solution: It is a combination of the solution for an infinitely-large continuous slab on ground, plus additional terms that account for the presence of the transverse joint (equation 145).

$$w_L(x, y) = w(x, y) + A \times w_A(x, y) + B \times w_B(x, y)$$
$$w_{UL}(x, y) = C \times w_C(x, y) + D \times w_D(x, y)$$

(145)

Note: Coordinate $x$ is along the direction perpendicular to the transverse joint, coordinate $y$ is parallel to the transverse joint. The origin of coordinates is at the center of the applied load.

However, the closed-form expression for each term of the deflection basin solution is convoluted by itself: they are infinite series of exponential functions, sines and cosines over a dummy variable $s$. Plus, they are parameterized after the following quantities:

$$l = \left[ \frac{Eh^3}{12(1-v)^2k} \right]^{1/4}$$
$$D = k \times l^4$$
$$g = \frac{Gl^2}{2D}$$

(146)  
(147)  
(148)

Note: Term $l$ in equation 146 is Westergaard’s radius of relative stiffness and term $D$ in equation 147 is the slab’s Flexural Stiffness (Van Cauwelaert, 2004; Khazanovich et al., 2021).

In this review, only the final forms of the terms in equation 145 will be presented. The reader is referred to Van Cauwelaert (2004) for an in-depth explanation of the calculations followed to derive these results.
Infinite slab component

The infinite slab component has different expressions for points located within and outside of the load’s footprint area. Plus, these formulae also vary with the value of parameter \( g \). Note that the case of a concrete jointed slab on a Winkler foundation corresponds to the case \( g = 0 \):

**Case \( g < 1 \) (includes Winkler foundation case \( g = 0 \))**

for \( x \geq a \)

\[
\begin{align*}
  w(x, y) & = \frac{p}{\pi k} \frac{1}{\sqrt{1 - g^2}} \int_0^\infty w_3(y, s) \times [w_4(x, s) - w_5(x, s)] ds \\
  \\
  \text{where:} \\
  & w_3(y, s) = \cos(y/l) \sin(s/l) \\
  & w_4(x, s) = e^{-\frac{(x-a)/l}{\sqrt{1 - g^2}}} \left[ \sqrt{1 - g^2} \cos((x-a)/l) + s^2 + g \right] \sin((x-a)/l) \\
  & w_5(x, s) = e^{-\frac{(x+a)/l}{\sqrt{1 - g^2}}} \left[ \sqrt{1 - g^2} \cos((x+a)/l) + s^2 + g \right] \sin((x+a)/l)
\end{align*}
\]

for \( x < a \)

\[
\begin{align*}
  w(x, y) & = \frac{p}{\pi k} \frac{1}{\sqrt{1 - g^2}} \int_0^\infty w_3(y, s) \times [2 - w_1(x, s) - w_2(x, s)] ds \\
  \\
  \text{where:} \\
  & w_3(y, s) = \cos(y/l) \sin(s/l) \\
  & w_1(x, s) = e^{-\frac{(a-x)/l}{\sqrt{1 - g^2}}} \left[ \sqrt{1 - g^2} \cos((a-x)/l) + s^2 + g \right] \sin((a-x)/l) \\
  & w_2(x, s) = e^{-\frac{(a+x)/l}{\sqrt{1 - g^2}}} \left[ \sqrt{1 - g^2} \cos((a+x)/l) + s^2 + g \right] \sin((a+x)/l)
\end{align*}
\]

Where the coefficients \( \alpha \) and \( \beta \) are:

\[
\begin{align*}
  \alpha^2 & = \frac{1}{2} \left[ \sqrt{(s^2 + g)^2 + 1 - g^2} + (s^2 + g) \right] \\
  \beta^2 & = \frac{1}{2} \left[ \sqrt{(s^2 + g)^2 + 1 - g^2} - (s^2 + g) \right]
\end{align*}
\]
Case $g > 1$

for $x \geq a$

$$w(x, y) = \frac{p}{2\pi k} \frac{1}{\sqrt{g^2 - 1}} \int_0^\infty w_3(y, s) \times [w_4(x, s) - w_5(x, s)] \, ds$$

where:

$$w_3(y, s) = \frac{\cos\left(\frac{s y}{l}\right) \sin\left(\frac{s b}{l}\right)}{s}$$

$$w_4(x, s) = \frac{e^{-\frac{(x - a)z_2}{l}} - e^{-\frac{(x + a)z_2}{l}}}{z_2^2}$$

$$w_5(x, s) = \frac{e^{-\frac{(x - a)z_1}{l}} - e^{-\frac{(x + a)z_1}{l}}}{z_1^2}$$

for $x < a$

$$w(x, y) = \frac{p}{2\pi k} \frac{1}{\sqrt{g^2 - 1}} \int_0^\infty w_3(y, s) \times [w_1(x, s) - w_2(x, s)] \, ds$$

where:

$$w_3(y, s) = \frac{\cos\left(\frac{s y}{l}\right) \sin\left(\frac{s b}{l}\right)}{s}$$

$$w_1(x, s) = \frac{2 - e^{-\frac{(a - x)z_2}{l}} - e^{-\frac{(a + x)z_2}{l}}}{z_2^2}$$

$$w_2(x, s) = \frac{2 - e^{-\frac{(a - x)z_1}{l}} - e^{-\frac{(a + x)z_1}{l}}}{z_1^2}$$

Where the coefficients $z_1$ and $z_2$ are:

$$z_1^2 = (s^2 + g) + \sqrt{g^2 - 1}$$

$$z_2^2 = (s^2 + g) - \sqrt{g^2 - 1}$$
Case $g = 1$

for $x \geq a$

$$w(x, y) = \frac{p}{2\pi k} \frac{1}{\sqrt{g^2 - 1}} \int_0^\infty w_3(y, s) \times [w_4(x, s) - w_5(x, s)] ds$$

where:

$$w_3(y, s) = \frac{\cos(sy/l) \sin(sb/l)}{s(s^2 + 1)^2}$$

(155)

$$w_4(x, s) = e^{-[(x-a)s/l]} \left[ 2 + \frac{1}{\sqrt{1 + s^2}} \left( x - a \right) / l \right]$$

$$w_5(x, s) = e^{-[(x+a)s/l]} \left[ 2 + \frac{1}{\sqrt{1 + s^2}} \left( x + a \right) / l \right]$$

for $x < a$

$$w(x, y) = \frac{p}{2\pi k} \frac{1}{\sqrt{g^2 - 1}} \int_0^\infty w_3(y, s) \times \left[ 4 - w_1(x, s) - w_2(x, s) \right] ds$$

where:

$$w_3(y, s) = \frac{\cos(sy/l) \sin(sb/l)}{s(s^2 + 1)^2}$$

(156)

$$w_1(x, s) = e^{-[(a-x)s/l]} \left[ 2 + \frac{1}{\sqrt{1 + s^2}} \left( a - x \right) / l \right]$$

$$w_2(x, s) = e^{-[(a+x)s/l]} \left[ 2 + \frac{1}{\sqrt{1 + s^2}} \left( a + x \right) / l \right]$$

Where the coefficient $z$ is:

$$(s^2 + 1)^2$$

(157)

**Additional components for the jointed slab case**

The following expressions represent the components of the deflection bowl equation (equation 145) that are needed to account for the boundary condition imposed by the transverse joint (Van Cauwelaert, 2004). As with the case of the infinite slab component, these components depend on the value of $g$. Van Cauwelaert (2004) warns that these equations are valid only for the case of a transverse joint located to the right of the load. The sign convention used in the following equations may change when modeling the response of the pavement once the load is on the right-hand-side slab.
Case $g < 1$ (includes Winkler foundation case $g = 0$)

$$w_A(x, y) = \frac{p}{\pi k} \frac{1}{\sqrt{1 - g^2}} \int_0^\infty \left[ A(s) \cos(\beta x/l) \right] e^{\alpha x/l} \times \frac{\cos(sy/l) \sin(sb/l)}{s} ds$$

$$w_B(x, y) = \frac{p}{\pi k} \frac{1}{\sqrt{1 - g^2}} \int_0^\infty \left[ B(s) \sin(\beta x/l) \right] e^{\alpha x/l} \times \frac{\cos(sy/l) \sin(sb/l)}{s} ds$$

$$w_C(x, y) = \frac{p}{\pi k} \frac{1}{\sqrt{1 - g^2}} \int_0^\infty \left[ C(s) \cos(\beta x/l) \right] e^{-\alpha x/l} \times \frac{\cos(sy/l) \sin(sb/l)}{s} ds$$

$$w_D(x, y) = \frac{p}{\pi k} \frac{1}{\sqrt{1 - g^2}} \int_0^\infty \left[ D(s) \sin(\beta x/l) \right] e^{-\alpha x/l} \times \frac{\cos(sy/l) \sin(sb/l)}{s} ds$$

In equations 158 and 159, $\alpha$ and $\beta$ are as per equation 151.

Case $g > 1$

$$w_A(x, y) = \frac{p}{2 \pi k} \frac{1}{\sqrt{g^2 - 1}} \int_0^\infty \left[ A(s) e^{z_1 x/l} \right] e^{z_1 x/l} \times \frac{\cos(sy/l) \sin(sb/l)}{s} ds$$

$$w_B(x, y) = \frac{p}{2 \pi k} \frac{1}{\sqrt{g^2 - 1}} \int_0^\infty \left[ B(s) e^{z_1 x/l} \right] e^{z_1 x/l} \times \frac{\cos(sy/l) \sin(sb/l)}{s} ds$$

$$w_C(x, y) = \frac{p}{2 \pi k} \frac{1}{\sqrt{g^2 - 1}} \int_0^\infty \left[ C(s) e^{-z_1 x/l} \right] e^{-z_1 x/l} \times \frac{\cos(sy/l) \sin(sb/l)}{s} ds$$

$$w_D(x, y) = \frac{p}{2 \pi k} \frac{1}{\sqrt{g^2 - 1}} \int_0^\infty \left[ D(s) e^{-z_1 x/l} \right] e^{-z_1 x/l} \times \frac{\cos(sy/l) \sin(sb/l)}{s} ds$$

In equations 160 and 161, $z_1$ and $z_2$ are as per equation 154.
Case \( g = 1 \)

\[
\begin{align*}
\frac{p}{2\pi k} \int_0^\infty \left[ A(s) e^{\alpha l} \cos \frac{sy}{l} \sin \frac{z}{s} \right] ds \\
\frac{p}{2\pi k} \int_0^\infty \left[ B(s) e^{\beta l} \cos \frac{sy}{l} \sin \frac{z}{s} \right] ds \\
\frac{p}{2\pi k} \int_0^\infty \left[ C(s) e^{-\alpha l} \cos \frac{sy}{l} \sin \frac{z}{s} \right] ds \\
\frac{p}{2\pi k} \int_0^\infty \left[ D(s) e^{-\beta l} \cos \frac{sy}{l} \sin \frac{z}{s} \right] ds
\end{align*}
\] (162)

In equations 162 and 163, \( z \) is as per equation 157.

**Boundary conditions imposed by the joint**

Equations 158 to 163 were stated in terms of four parameters \( A, B, C, D \) (s) that allow for the final deflection basin solution (equation 145) to be composed. The transverse joint impose four boundary conditions, which provide the equations needed to compute these unknowns:

- Load transfer efficiency LTE = \( \delta \) at \( x = c \) and any value of \( y \)

\[
\delta \times \left( w(s) + A(s)w_A(s) + B(s)w_B(s) \right)_{x=c} = C(s)w_C(s) + D(s)w_D(s)_{x=c}
\] (164)

- Cancellation of bending moments at the edge of the loaded slab (\( x = c \))

\[
\left( \frac{\partial^2}{\partial x^2} + v \frac{\partial^2}{\partial y^2} \right) \left( w(s) + A(s)w_A(s) + B(s)w_B(s) \right)_{x=c} = 0
\] (165)

- Cancellation of bending moments at the edge of the unloaded slab (\( x = c \))

\[
\left( \frac{\partial^2}{\partial x^2} + v \frac{\partial^2}{\partial y^2} \right) \left( C(s)w_C(s) + D(s)w_D(s) \right)_{x=c} = 0
\] (166)

- Equality of shear forces in the subgrade material (\( x = c \))

\[
\left( \frac{\partial^3}{\partial x^3} + (2-v) \frac{\partial^3}{\partial x \partial y^2} - \frac{2g}{l^2} \right) \left( w(s) + A(s)w_A(s) + B(s)w_B(s) \right)_{x=c} = \ldots
\]

\[
\ldots = \left( \frac{\partial^3}{\partial x^3} + (2-v) \frac{\partial^3}{\partial x \partial y^2} - \frac{2g}{l^2} \right) \left( C(s)w_C(s) + D(s)w_D(s) \right)_{x=c}
\] (167)
Equations 164 through 167 make a 4-by-4 linear system that can be solved for A, B, C, D; this system must be stated and solved for each value of the dummy variable \(s\). The terms \(w(s), w_A(s), w_B(s), w_C(s), w_D(s)\) are the expressions within the integral for each of the variables \(w, w_A, w_B, w_C, w_D\) (as per equations 149 to 163).

In practice, stating the system of equations 164 through 167 involves performing several partial derivatives of the functions \(w, w_A, w_B, w_C, w_D\). In the implementation built to solve the backcalculation problem for the TSD, these derivatives were calculated numerically, by implementing equations 119 and 128.

**Solving the deflection basin for the case of the load invading the joint**

Van Cauwelaert’s (2004) jointed-slab-on-ground problem solution assumes that the load is applied entirely either on the left-hand-side slab or the right-hand-side slab. Thus, the reviewed solution does not apply for the case of the distributed load invading the joint. Yet, the underlying assumption of linearity allows to regard this case as a superposition of two problems, one component corresponding to the portion of the load on the left-hand-side slab and the other component representing the portion of the load on the right-hand-side slab. This is graphically shown on figure 63. It stands without saying that for each sub-problem, the dimensions of the applied load, and the variable \(c\) – the distance between the load center and the joint – must be corrected accordingly.

**Computing deflection slope and velocity from the linear elastic model**

If the simulated component of the back-calculation scheme is to utilize deflection slope measurements, these could be solved from the deflection basin equations:

For \(x < c\):

\[
\text{slope}(x, y) = \frac{\partial}{\partial x} w_L(x, y)
\]

For \(x \geq c\):

\[
\text{slope}(x, y) = \frac{\partial}{\partial x} w_{UL}(x, y)
\]

These derivatives can be solved numerically by implementing equation 119.

However, it was found (Manuscript 3, Chapter 5) that the TSD can only produce reliable deflection velocity \([v_y]\) measurements at the vicinity of transverse joints in jointed pavement structures, and that the staple relationship historically used to relate \(v_y\) measurement with deflection slope estimates (Krarup et al., 2006) is not valid.
Thus, the back-calculation problem for the joint’s LTE must be stated in terms of vertical deflection velocity $[v_y]$ measurements – the *time dimension* must be added into the simulation. If the TSD travel speed $v_x$ is known, the simulated component $v_y$ term could be approximated from two simulated contiguous measurements taken at a time interval $\Delta t$ as:

$$
\begin{align*}
\text{for } x < c: \quad v_y(x, y) &= \frac{w_L(x-v_x\Delta t, y, c-v_x\Delta t) - w_L(x, y, c)}{\Delta t} \\
\text{for } x \geq c: \quad v_y(x, y) &= \frac{w_{UL}(x-v_x\Delta t, y, c-v_x\Delta t) - w_{UL}(x, y, c)}{\Delta t}
\end{align*}
$$

The notation used in equation 169 is purposefully meant to highlight the fact that the TSD load moved between both measurements, the point in the pavement initially at station $x, y$ became closer to the TSD load, so did the transverse joint hence the variable $c$ shrinking by the distance traveled during the interval $\Delta t$. This point is discussed further in Chapter 6.

Figure 63: Treating the case of the load invading the joint as two separate problems and applying superposition to attain the final solution.
Implementing the pavement back-calculation optimization problem

In its full expression, the back-calculation problem for a jointed concrete pavement based on TSD data has 5 unknowns – it is being assumed that the concrete’s slab Poisson coefficient can be safely assumed equal to a default value ($\nu = [0.19 – 0.23]$) and that the slab thickness $h$ is known. Plus, since by default the exact location of the transverse joints is unknown, variable $c$ becomes another unknown decision variable. The back-calculation problem from equation 144 can be written more specifically as:

$$\min_{\theta} \| TSD - \dot{w}(\theta) \|^2 = \sum_{i=1}^{\text{TSD-sensors}} (TSD_i - \dot{w}_i(\theta))^2$$

where: $\theta = [LTE, c, E, k, G]$ and: $\dot{w}_i(\theta) = v_y(x_{TSD_i},0)$

subject to: $E > 0, k > 0, G > 0, c > 0, LTE \in [0,1]$ \hspace{1cm} (170)

In equation 170, the quantities TSD$_i$ are the deflection velocity measurements from all the TSD sensors at a given station (the TSD wheel being a distance $c$ [by default unknown] from the transverse joint), and $\dot{w}_i(\theta)$ are the simulated deflection velocities for a position on the pavement as distant from the TSD wheel center as the i-th TSD sensor (distance $x_{TSD}$) as per equation 169.

The problem in equation 170 is rather high-dimensional, and it might become ill-posed, especially when the source measurements are gathered with an older second-generation TSD, with only 6-7 sensors, and the ever-present risk of any of the sensors not reporting valid measurements. Besides, this back-calculation scheme would only utilize one out of so many TSD measurements, thus not taking full advantage of the rich data the TSD collects: When considering 1-m resolution data, only those measurements from locations where pulse responses were recovered, as per the RWL1 reconstruction problem (equation 107); meanwhile, for the 5-cm-resolution data from 4th-generation TSD devices, only one out of the 20 $v_y$ profiles collected every meter would be utilized.

Thus, a two-stage back-calculation procedure featuring low-dimension problems is proposed (graphically illustrated in figure 64):

- The first of these problems [OP1] considers measurements from mid-slab locations only, where it can safely be assumed that only the infinite slab component of the deflection basin model prevails. Thus, this problem can be stated to solve for the pavement’s and foundation material’s strength properties. Mathematically, it can be stated as: 
The second problem is formulated to specifically locate the joint and solve for its LTE index. The TSD measurements involved are those collected at the vicinity of a transverse joint – a location where a pulse in the deflection velocity signal is generated as the TSD travels over. The full jointed slab model is used to compute the simulated component with the back-calculated \( k, E, G \) from equation 171. As such, the second problem [OP2], whose decision variables are only \( c \) and LTE is stated as:

\[
\text{OP2}: \quad \min \sum_{i=1}^{\text{TSD sensors}} \left( \text{TSD}_{\text{joint, } i} - v_{yi} (\theta) \right)^2
\]

where: 
\[
v_{yi} = v_y (x_i, 0, c, \text{LTE})
\]

\( w(x, y, \theta) \) as per Eqs. 77 through 95

subject to: \( 0 \leq c_{\text{min}} \leq c \leq c_{\text{max}}, \quad 0 \leq \text{LTE} \leq 1 \)

assuming: \( h, v, k, E, G \) known
An additional advantage of problem OP2 is that, when regarding 1-m-resolution TSD data, both decision variables are bounded: By definition, LTE is restricted between 0 and 1; and the same can be stated for $c$ as well, for if $c$ were greater than 1 meter, the pulse response would have been recorded on the set of measurements from 1 meter ahead of the actual pulse location. Meanwhile, if 5-cm-resolution data from multiple locations are fed into the back-calculation engine, narrower lower and upper bounds can be stated for $c$. Manuscript 4 elaborates about this two-stage approach to back-calculate the concrete pavement parameters from TSD deflection velocity measurements. In particular, details are given about how Gradient Descent with relaxed Hessian Preconditioning (equation 140) is applied to attain the solution to the problem.
References


Donoho and Johnstone (1994)


APPENDIX C: LASER DOPPLER VIBROMETRY AND TSD MEASUREMENTS

Presentation

This Appendix elaborates on the physical principle underlying TSD measurements: The Doppler Effect. Throughout this Appendix, the Doppler Effect as a physical principle is revisited (Zhang, 2010); then Doppler Laser Vibrometry (or Anemometry) – the process to estimate an object’s velocity using the Doppler Effect – is introduced; and finally, we show how this measuring principle can be applied to measure the deflection velocity of the pavement surface from an array of moving Doppler vibrometers – the TSD measuring principle (Hildebrand and Rasmussen, 2002). In particular, we prove that under a specific condition (refer to equations 33 and 34 on Chapter 5), two deflection velocity measurements could be used to estimate the pavement’s deflection slope, as stated in Hildebrand and Rasmussen (2002) and Krarup et al. (2006).

Laser Doppler Vibrometry

When a source of waves and a receiver unit are in relative motion, the frequency of the perceived signal will differ from the emitted signal frequency. This change in signal frequency is known as the Doppler Effect (Zhang, 2010), and can be mathematically related to the velocity of the relative movement between the signal source and the receiver:

source and receiver approaching:

\[ f_{\text{perceived}} = \frac{c}{c - v \cos(\beta)} f_0 \]

source and receiver departing:

\[ f_{\text{perceived}} = \frac{c}{c + v \cos(\beta)} f_0 \]

Where \( f_{\text{perceived}} \) is the frequency of the sensed wave [Hz], \( f_0 \) is the frequency at which the wave was emitted [Hz], \( c \) is the speed of the wave in the transmission medium [m/sec], and \( v \) is the relative velocity between the wave source and the receiver, \( \beta \) is the angle between the relative velocity vector and the direction the wave is propagating.

A Doppler sensor like the gear in a traffic speed trap device would measure only the collinear component of the relative velocity between itself and the target object (Jendzurski and Paultier, 2008), figure 65. Thus,
if a Doppler sensor is pointed at an object whose velocity is \( v \) (relative to the sensor), and the angle between \( v \) and the line linking the Doppler sensor and the object is \( \theta \), then the measured velocity \( M \) is:

\[
M = v \cos(\theta)
\]  
(174)

The Doppler sensor would not measure the component of \( v \) that is perpendicular to the direction of the Doppler laser beam.

**Doppler laser vibrometry on the TSD**

The TSD uses multiple Doppler laser sensors to measure the velocity at which the pavement surface deflects in response to the TSD’s rear-axle load. Plus, the data from an additional sensor – the reference sensor – is used to measure the response of the undeflected pavement. All TSD sensors are skewed from the vertical direction (Hildebrand and Rasmussen, 2022; Krarup et al., 2006).

In this section, a review of what constitutes a TSD deflection velocity measurement is provided:

Consider an array of two Doppler laser sensors measuring different locations from the TSD rear wheel. The TSD travels horizontally at a speed \( v_x \). Sensor 0 is the reference sensor which measures a portion of undeflected pavement, sensor 1 measures a point on the pavement within the deflection basin. Each sensor’s skew angles are \( \alpha_0 \) and \( \alpha_1 \) respectively.

If a reference frame mounted on the TSD sensors is considered, then all points on the pavement surface have a horizontal velocity component equal to \( v_x \). Plus, the point sensed by sensor one would also have a vertical velocity component \( v_y \), point 1’s relative velocity vector is then at an angle \( \beta \) from the horizontal direction. Figure 66 illustrates this case.
The measurement from sensor 0 is used to estimate \( v_x \). As per equation 175, and since \( \theta_0 \) and \( \alpha_0 \) are complementary angles (they add up to 90 degrees):

\[
M_0 = v_x \cos(\theta_0) = v_x \sin(\alpha_0)
\]

\[
v_x = \frac{M_0}{\sin(\alpha_0)}
\] (175)

Similarly, the measurement from sensor 1 would relate to point 1’s velocity as:

\[
M_1 = v_1 \cos(\theta')
\] (176)

The angle \( \theta' \) is actually the difference between the angle \( \theta_1 \) (between the horizontal direction and sensor 1’s beam) and \( \beta \). Thus:

\[
M_1 = v_1 \cos(\theta_1 - \beta)
\] (177)

Using the cosine identity \( \cos(a - b) = \cos(a)\cos(b) + \sin(a)\sin(b) \), where \( a \) and \( b \) are generic angles, on equation 177 leads to:

\[
M_1 = v_1 \left[ \cos(\theta_1)\cos(\beta) + \sin(\theta_1)\sin(\beta) \right]
\] (178)

Since the horizontal component of \( v_1 \) is \( v_x \), then (equation 179):

\[
v_x = v_1 \cos(\beta)
\] (179)
Then insert equation 179 into 178:

\[ M_1 = \frac{v_x}{\cos(\beta)}[\cos(\theta_1)\cos(\beta) + \sin(\theta_1)\sin(\beta)] \]

then:

\[ M_1 = v_x[\cos(\beta) + \sin(\theta_1)\tan(\beta)] \]  

(180)

Relating angles \( \theta_1 \) and \( \alpha_1 \), which are complementary as well:

\[ M_1 = v_x[\sin(\alpha_1) + \cos(\alpha_1)\tan(\beta)] \]  

(181)

Manipulating equation 181:

\[ M_1 - v_x\sin(\alpha_1) = v_x\cos(\alpha_1)\tan(\beta) \]

then:

\[ \frac{M_1 - v_x\sin(\alpha_1)}{v_x\cos(\alpha_1)} = \tan(\beta) \]  

(182)

Bringing the relationship from equation 175, then a formula to estimate the angle of point 1’s velocity vector from the difference between \( M_1 \) and \( M_0 \) can be stated (equation 183):

\[ \frac{M_1 - M_0\sin(\alpha_1)}{v_x\cos(\alpha_1)} = \tan(\beta) \]  

(183)

In TSD practice, angles \( \alpha_1 \) and \( \alpha_0 \) are small, then \( \cos(\alpha_1) \sim \cos(\alpha_0) \sim 1 \), which reduces the generic result from equation 183 to:

\[ \frac{M_1 - M_0\sin(\alpha_1)}{v_x\sin(\alpha_0)} = \tan(\beta) \]  

(184)

A direct consequence of equation 184 is that if the difference between the two measurements \( M_1 \) and \( M_0 \) (corrected by the ratio of the sines of \( \alpha_1 \) and \( \alpha_0 \)) is zero, then point 1’s relative velocity vector is horizontal, meaning that point 1 is not deflecting. Thus, equation 184 shows the reason why the reference sensor measurement is needed to tell whether point 1 is deflecting or not, and grounds the statement that the deflection velocity of a point within the deflection basin could be estimated from the difference between \( M_1 \) and the reference measurement \( M_0 \).
Now, if $\tan(\beta)$ matches the local deflection basin slope $S_l$, then the slope of the deflection basin can be estimated from the TSD measurements (equation 185). This particular coincidence occurs when the deflection basin is stationary (it does not vary over time), as shown in Chapter 6:

$$\frac{M_1 - M_0 \sin(\alpha_1)}{\sin(\alpha_0)} \frac{v_x}{v} = S_l$$
References


APPENDIX D: COMPUTER SOURCE CODE

Presentation

This appendix collects the source code of all the computer programs written to perform the analyses and calculations presented in this Dissertation. The following sections present all MATLAB scripts and functions utilized to derive the Dissertation’s research outcomes.

The front-end scripts at the beginning of each collection of programs are the only pieces of code that require any interaction with the user. These scripts load the user-provided TSD data and streamline the processing to achieve the desired end-results. All dependencies to the front-end scripts are provided herein as functions or, in the case of third-party libraries, provided as links to an approved download source.

Distribution

A ready-to-use version of all these pieces of code is available for download from GitHub: https://github.com/MartinScavone/concreteTSD

Dependencies

These scripts and functions utilize code from the following libraries:

- Wavelab 8.50. Available through https://statweb.stanford.edu/~wavelab/Wavelab_850/download.html. Wavelab is Copyright (c) by David Donoho and others.
- Export_fig: Available through https://www.mathworks.com/matlabcentral/fileexchange/23629-export_fig/. Export_fig is Copyright (c) 2014, Oliver J. Woodford, Yair M. Altman

Licensing

The original code presented in here is released to the public under the terms of the Creative Commons [CC] BY-SA 4.0 International license. The terms of the License can be found in: https://creativecommons.org/licenses/by-sa/4.0/. The third-party dependencies mentioned in the Dependencies section remain the property of their respective authors.
BP denoising of TSD signals at the network level.

**Important note:** This collection of functions requires the package WAVELAB to operate. Additionally, the package exportFig may be required to automatically export all output plots to PDF format. Refer to Page 246 for a method to access these software packages.

**Front-end script**

% TSD De-noising procedure front-end code.
% This front-end script will aid to load the raw TSD data and make all necessary calls to functions that do the denoising procedure.
% Release candidate v2022-05-01

tic
clc
clear variables
close all

addpath('./TSDData')
addpath('./WAVELAB850')
addpath('./WAVELAB850/Orthogonal');
addpath('./WAVELAB850/Utilities');
% addpath('./export_fig');

disp('Loading TSD Data')

inputFilename = '0033_FHWA_EFL_PFS_1m_Deflection_only_6_13_19.xlsb';
inputSheet = '0033_FHWA_EFL_PFS_1m_TSD';

latLongData = xlsread(inputFilename,inputSheet,'bj2:bo79257');   % lat long data in decimal degrees
stationData = xlsread(inputFilename,inputSheet,'i2:j79257');   % station in km
[~,roadID,~] = xlsread(inputFilename,inputSheet,'a2:a79257');   % roadID is an alphanumeric field. STORED AS CELL ARRAY! this one combines the road id + the segm. id. Try as unique identifier...
[~,roadName,~] = xlsread(inputFilename,inputSheet,'e2:e79257');   % roadName is an alphanumeric field. STORED AS CELL ARRAY!
goodMeasures= xlsread(inputFilename,inputSheet,'k2:y79257'); % metric units.
% columns are:: SCI_200 (microns)   SCI_300 (microns)   SCI_SUBGRADE (microns)   D0
latFrom = latLongData(:,1);
latTo = latLongData(:,3);
longFrom = latLongData(:,2);
longTo = latLongData(:,4);
altFrom = latLongData(:,5);
altTo = latLongData(:,6);

stationFrom = 1000*stationData(:,1);
stationTo = 1000*stationData(:,2);

metricD0 = goodMeasures(:,4);
metricD0 = -1*metricD0;

roadIDList = unique(roadID);
numRoads = length(roadIDList);

weakSpotsMap = struct('roadID',[],'roadName',[],'weakSpotLocations',[],'weakSpotSpac', [],'weakSpotLatFrom',[],'weakSpotLatTo',[],'weakSpotLongFrom',[],'weakSpotLongTo', [],'RawDo',[]);

for k = 1:numRoads
    whichRoad = roadIDList(k);
    tsdIndices = find(strcmp(roadID,whichRoad));
end
find-in-a-cell-array

% get the name of the road/lane under analysis
thatRoadName = roadName(tsdIndices);
thatRoadName = thatRoadName(1);

% get the TSD data for that road
fprintf(' now processing: %s, segment %s 
',string(thatRoadName),string(whichRoad));
refTSD = metricD0(tsdIndices);
station = stationFrom(tsdIndices);
latStart = latFrom(tsdIndices);
latEnd = latTo(tsdIndices);
longStart= longFrom(tsdIndices);
longEnd = longTo(tsdIndices);

%%% sanity control: the refTSD vectors may contain NaNs.
% DON'T DO::: force replace with zeros... <- THIS DROPS STDEV TO ZERO %(WHEN THE ACTUAL DATA HAS MORE VARIABILITY!)
% DO::::::::: locate the nans, remove them altogether and forget about them!
(Samer's advice 2019-11-04)
% use this workaround: https://www.mathworks.com/matlabcentral/answers/164316-select-everything-not-retumed-by-index
theseAreNans = find(isnan(refTSD)); %this returns a vector with the positions of NaN's inside refTSD
NansIndex = isnan(refTSD); %this logical array (different from the one above) helps then get the non-NaN values easily. [it's a 0/1 vector as long as refTSD in which the 1's indicate the NaNs]

%%% sanity control 02: it may happen that all the refTSD points are or blanks [there's actually a case out there].
% Kill the iteration if such is the case (fill the weakSpotMap(k) and do nothing else....
if length(theseAreNans) == length(refTSD)
    weakSpotsMap(k).roadID = whichRoad;
    weakSpotsMap(k).roadName = thatRoadName;
    weakSpotsMap(k).weakSpotSpac = 0;
    weakSpotsMap(k).weakSpotLocations = 0;
    weakSpotsMap(k).weakSpotLatFrom = 0;
    weakSpotsMap(k).weakSpotLongFrom = 0;
    weakSpotsMap(k).weakSpotLatTo = 0;
    weakSpotsMap(k).weakSpotLongTo = 0;
    weakSpotsMap(k).rawDo = 0;
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continue

end

% case this not occurs, remove the theseAreNans and keep on with the calculations.
refTSD = refTSD(~NansIndex);
station = station(~NansIndex);
latStart = latStart(~NansIndex);
latEnd = latEnd(~NansIndex);
longStart = longStart(~NansIndex);
longEnd = longEnd(~NansIndex);

n = length(refTSD);   % length of the sanitized series.

%% Iterative Basis pursuit decomposition + soft thresholding procedure starts....
% pass to the TSD_denoisingJoints function!
callType = 'default';  % No other callType allowable!

%% Pass data for the plot: T vs the steinError and locate the optimum T value (that
% who minimizes the SURE)
% the plot is made at the TSD_denoisingJoints function.
SUREPlotNumber = 200+k;
SUREPlotName = sprintf('Denoised TSD - %s, section %s : optimum t value
location', string(thatRoadName), string(whichRoad));

[denoisedTSD,~,ySpikes,optLambda,optSURE,SUREPlotHandle] =
TSD_denoisingJoints(refTSD,callType,0,SUREPlotNumber,SUREPlotName);

%% 5) Figure 100 series with the final denoised signal obtained with the optimum Threshold value.
figure(100+k)
plotName = sprintf('Denoised TSD - %s, section %s : Noisy signal vs. denoised
signal', string(thatRoadName), string(whichRoad));
set(gcf,'Name',plotName);
plot (station, refTSD, 'color',[0.73 0.83 0.96])
grid on
title('TSD signal denoised with BPD and soft-shrinking threshold')
xlabel ('station [m]')
ylabel ('D_0')
hold on
plot(station,denoisedTSD,'color',[0.42 0.25 0.39]) %grey-maroon color
legend('noisy','denoised')
hold off

%% export-fig call. ENABLE IF YOU WANT PDF OUTPUT!
%send the plot to a pdf file with export_fig.
%export_fig NatlMallTSD -pdf -append

fprintf('% The optimum value for the threshold t is %g \n',optLambda);
fprintf('% The corresponding SURE estimate is %g \n',optSURE);

% 6) Add this final analysis to detect how many spikes were detected
%and how many of those are real.

id2 = find(ySpikes ~=0);
fprintf('%t \t %g spikes were detected with the denoising \n',length(id2));

%get the spacing between weak spots and see if they correspond with the
%actual joint spacing [12.5m, Katicha et al., 2013]
weakSpotsPositions = find(ySpikes>0); % note that vector position does match
%the station and distance between weak spots because each tsd measurement is spaced
%1.00m
weakSpotSpacing = weakSpotsPositions(2:end)-weakSpotsPositions(1:end-1);
meanJointSpacing = mode(weakSpotSpacing); %note: prefer mode and/or median to the
%arithmetic mean of weakSpotSpacing because the mean may be stretched to a too large
%value by long segments w/o spots

%save results for mapping...
weakSpotsMap(k).roadID = whichRoad;
weakSpotsMap(k).roadName = thatRoadName;
weakSpotsMap(k).weakSpotSpacing = meanJointSpacing;
weakSpotsMap(k).weakSpotLocations = station(weakSpotsPositions);
weakSpotsMap(k).weakSpotLatFrom = latStart(weakSpotsPositions);
weakSpotsMap(k).weakSpotLongFrom = longStart(weakSpotsPositions);
weakSpotsMap(k).weakSpotLatTo = latEnd(weakSpotsPositions);
weakSpotsMap(k).weakSpotLongTo = longEnd(weakSpotsPositions);
weakSpotsMap(k).RawDo = refTSD(weakSpotsPositions);

%%
fprintf('%t Road %g of %g completed \n',k,numRoads)
%% cleanup...
clear weakSpotsPositions
clear weakSpotSpacing
clear minSteinPos
clear steinError
clear denoisedTSD
clear peaksMaps

clear refTSD station latEnd latStart longEnd longStart tsdIndices

%% export the weakSpotsMap to Excel for easy mapping
% use the already started file
exportFilename = 'NatlMall_weakSpotChart_V2022-0501.xlsx';
disp('exporting Weak Spots Map to Excel')
startRow = 3;
% end row = startRow + length-1
% and new start row = endrow + 1
for i = 1:length(weakSpotsMap)
    fprintf('	 Progress %g of %g \n',i,length(weakSpotsMap))
    % export all struct contents one by one
    len = length(weakSpotsMap(i).weakSpotLocations);
    endRow = startRow+len-1;
    exportVariable=[weakSpotsMap(i).weakSpotLocations weakSpotsMap(i).weakSpotSpac*ones(len,1) weakSpotsMap(i).weakSpotLatFrom weakSpotsMap(i).weakSpotLatTo weakSpotsMap(i).weakSpotLongFrom weakSpotsMap(i).weakSpotLongTo weakSpotsMap(i).RawDo];
    exportRange = strcat('d',string(startRow),':j',string(endRow));
    % export all numerics
    uu = xlswrite(exportFilename,exportVariable,1,exportRange);
    % export cell contents.
    exportRange = strcat('b',string(startRow),':c',string(endRow));
    uu = xlswrite(exportFilename,[weakSpotsMap(i).roadID weakSpotsMap(i).roadName],1,exportRange);
    % done exporting. Set the startRow for the next export.
    startRow = endRow+1;
Front-end script. BP denoising under the Universal Threshold (Donoho and Johnstone, 1995)

% TSD De-noising procedure front-end code
% This front-end script will aid to load the raw TSD data and make all necessary calls to functions that do the denoising procedure.

% Release candidate v2022-05-01

 tic
 clc
 clear variables
 close all
 addpath('./WAVELAB850')
 addpath('./WAVELAB850/Orthogonal');
 addpath('./WAVELAB850/Utilities');
 % addpath('./export_fig');
 addpath('./TSDData')

disp('Loading TSD Data')

inputFilename = '0033_FHWA_EFL_PFS_1m_Deflection_only_6_13_19.xlsb';
inputSheet = '0033_FHWA_EFL_PFS_1m_TSD';

latLongData = xlsread(inputFilename,inputSheet,'bj2:bo79257');  % lat long data in decimal degrees
stationData = xlsread(inputFilename,inputSheet,'i2:j79257');    % station in km
[~,roadID,~] = xlsread(inputFilename,inputSheet,'a2:a79257');   % roadID is an alphanumeric field. STORED AS CELL ARRAY! this one combines the road id + the segm. id. Try as unique identifier...
[~,roadName,~] = xlsread(inputFilename,inputSheet,'e2:e79257'); % roadName is an alphanumeric field. STORED AS CELL ARRAY!
goodMeasures= xlsread(inputFilename,inputSheet,'k2:y79257'); % metric units.
% columns are: SCI_200 (microns) SCI_300 (microns) SCI_SUBGRADE (microns) D0 (microns) D203 (microns) D305 (microns) D457 (microns) D610 (microns) D914

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latFrom = latLongData(:,1);
latTo   = latLongData(:,3);
longFrom= latLongData(:,2);
longTo  = latLongData(:,4);
altFrom = latLongData(:,5);
altTo   = latLongData(:,6);

stationFrom = 1000*stationData(:,1);  
stationTo   = 1000*stationData(:,2);

metricD0 = goodMeasures(:,4);
metricD0 = -1*metricD0;

roadIDList = unique(roadID);
numRoads = length(roadIDList);

fprintf('	 %g streets (+ blocks) were recognized 
',numRoads)

weakSpotsMap = struct('roadID',[],'roadName',[],'weakSpotLocations',[],'weakSpotSpac', [],'weakSpotLatFrom',[],'weakSpotLatTo',[],'weakSpotLongFrom',[],'weakSpotLongTo', [],'RawDo',[]);

for k = 1:numRoads

    whichRoad = roadIDList(k);
    tsdIndices = find(strcmp(roadID,whichRoad));  
    this tweak allows to use find to compare cell arrays. Source: https://www.mathworks.com/matlabcentral/answers/84242-
find-in-a-cell-array

% get the name of the road/lane under analysis
thatRoadName = roadName(tsdIndices);
thatRoadName = thatRoadName(1);

% get the TSD data for that road
fprintf(' \t now processing: %s, segment %s 
',string(thatRoadName),string(whichRoad));
refTSD = metricD0(tsdIndices);
station = stationFrom(tsdIndices);
latStart = latFrom(tsdIndices);
latEnd = latTo(tsdIndices);
longStart = longFrom(tsdIndices);
longEnd = longTo(tsdIndices);

%%% sanity control: the refTSD vectors may contain NaNs.
% DON'T DO::: force replace with zeros... <- THIS DROPS STDEV TO ZERO % (WHEN THE
% ACTUAL DATA HAS MORE VARIABILITY!)
% DO:::::::: locate the nans, remove them altogether and forget about them!
(Samer's advice 2019-11-04)

% use this workaround: https://www.mathworks.com/matlabcentral/answers/164316-
% select-everything-not-returned-by-index
theseAreNans = find(isnan(refTSD)); % this returns a vector with the positions of
NaN's inside refTSD
NansIndex = isnan(refTSD); % this logical array (different from the one above)
helps then get the non-NaN values easily. [it's a 0/1 vector as long as refTSD in
which the 1's indicate the NaNs]

%%% sanity control 02: it may happen that all the refTSD points are or blanks
[t here's actually a case out there].
% Kill the iteration if such is the case (fill the weakSpotMap(k) and do nothing
else....
if length(theseAreNans) == length(refTSD)
weakSpotsMap(k).roadID = whichRoad;
weakSpotsMap(k).roadName = thatRoadName;
weakSpotsMap(k).weakSpotSpac = 0;
weakSpotsMap(k).weakSpotLocations = 0;
weakSpotsMap(k).weakSpotLatFrom = 0;
weakSpotsMap(k).weakSpotLongFrom = 0;
weakSpotsMap(k).weakSpotLatTo = 0;

255
weakSpotsMap(k).weakSpotLongTo = 0;
weakSpotsMap(k).rawDo = 0;
continue
end

% case this not occurs, remove the theseAreNans and keep on with the
% calculations.
refTSD = refTSD(~NansIndex);
station = station(~NansIndex);
latStart = latStart(~NansIndex);
latEnd = latEnd(~NansIndex);
longStart = longStart(~NansIndex);
longEnd = longEnd(~NansIndex);

n = length(refTSD); % length of the sanitized series.

%% Iterative Basis pursuit decomposition + soft thresholding procedure starts....
% pass to the TSD_denoisingJoints function!
callType = 'default'; % No other callType allowable!

%% Pass data for the plot: T vs the steinError and locate the optimum T value (that
% who minimizes the SURE). % the plot is made at the TSD_denoisingJoints function.
SUREPlotNumber = -1;
SUREPlotName = 'pototo';
% SUREPlotName = sprintf('Denoised TSD - %s, section %s : optimum t value
location',string(thatRoadName), string(whichRoad));

[denoisedTSD,~,ySpikes,optLambda,optSURE,SUREPlotHandle] =
TSD_denoisingJoints_UnivThrshld(refTSD,callType,SUREPlotNumber,SUREPlotName);

%% 5) Figure 100 series with the final denoised signal obtained with the optimum
Threshold value.

figure(100+k)
plotName = sprintf('Denoised TSD - %s, section %s : Noisy signal vs. denoised
signal',string(thatRoadName), string(whichRoad));
subplot(2,1,1)
plot (station, refTSD, 'color',[0.73 0.83 0.96])
set(gca, 'FontName', 'Arial')
set(gca,'defaultAxesFontSize',16)
grid on

256
titleString = sprintf('TSD signal for road %s, section %s', string(thatRoadName), string(whichRoad));

title(titleString)
xlabel ('station [m]')
ylabel ('D_0 [\mu m]')
hold on
plot(station,denoisedTSD, 'color', [0.93 0.69 0.13]) %orange color
legend('noisy','Denoised BP','Denoised Rew L1')
hold off

subplot(2,1,2)
plot(station,ySpikes, 'bo', 'linewidth',1, 'markersize',6)
set(gca, 'FontName', 'Arial')
set(gca,'defaultAxesFontSize',16)
grid on
title('Recovered Dirac component. Universal threshold')
legend('BP recovered')

%send the plot to a pdf file with export_fig.
% export_fig NatlMallTSD_univThreshold -pdf -append

%% 6) Add this final analysis to detect how many spikes were detected and how many
% of those are real.

id2 = find(ySpikes ~=0);
fprintf('		%g spikes were detected with the denoising 
',length(id2));

%get the spacing between weak spots
weakSpotsPositions = find(ySpikes>0); % %note that vector position does match
the station and distance between weak spots because each tsd measurement is spaced
1.00m
weakSpotSpacing = weakSpotsPositions(2:end)-weakSpotsPositions(1:end-1);
meanJointSpacing = mode(weakSpotSpacing); %note: prefer mode and/or median to the
arithmetic mean of weakSpotSpacing because the mean may be stretched to a too large
value by long segments w/o spots

%save results for mapping...
weakSpotsMap(k).roadID = whichRoad;
weakSpotsMap(k).roadName = thatRoadName;
weakSpotsMap(k).weakSpotSpac = meanJointSpacing;
weakSpotsMap(k).weakSpotLocations = station(weakSpotsPositions);
weakSpotsMap(k).weakSpotLatFrom = latStart(weakSpotsPositions);
weakSpotsMap(k).weakSpotLongFrom = longStart(weakSpotsPositions);
weakSpotsMap(k).weakSpotLatTo = latEnd(weakSpotsPositions);
weakSpotsMap(k).weakSpotLongTo = longEnd(weakSpotsPositions);
weakSpotsMap(k).RawDo = refTSD(weakSpotsPositions);

fprintf('	 Road %g of %g completed 
', k, numRoads)
end  

%% cleanup...
clear weakSpotsPositions
clear weakSpotSpacing
clear minSteinPos
clear steinError
clear denoisedTSD
clear peaksMaps
clear refTSD station latEnd latStart longEnd longStart tsdIndices

%% export the weakSpotsMap to Excel for easy mapping
% use the already started file
exportFilename = 'NatlMall_weakSpotChart_univV2022-0501.xlsx';
disp('exporting Weak Spots Map to Excel')
startRow = 3;
% end row = startRow + length-1
%and new start row = endrow + 1
for i = 1:length(weakSpotsMap)
    fprintf('	 Progress %g of %g 
',i,length(weakSpotsMap))
    %export all struct contents one by one
    len = length(weakSpotsMap(i).weakSpotLocations);
    endRow = startRow+len-1;
    exportVariable=[weakSpotsMap(i).weakSpotLocations
weakSpotsMap(i).weakSpotSpacing ones(len,1) weakSpotsMap(i).weakSpotLatFrom
weakSpotsMap(i).weakSpotLatTo weakSpotsMap(i).weakSpotLongFrom
weakSpotsMap(i).weakSpotLongTo weakSpotsMap(i).RawDo];
    exportRange = strcat('d',string(startRow),':j',string(endRow));
%export all numerics
uu = xlswrite(exportFilename,exportVariable,1,exportRange);

%export cell contents.
exportRange = strcat('b',string(startRow),':c',string(endRow));
uu = xlswrite(exportFilename,[weakSpotsMap(i).roadID
weakSpotsMap(i).roadName],1,exportRange);

%done exporting. set the startRow for the next export.
startRow = endRow+1;
end
disp('....all completed')
toc

Auxiliary functions

TSD_denoisingJoints

function [yCleanExp,WCTsExp,peaksMapsExp,optLambda,steinErrorExp,SUREPlotHandle] = TSD_denoisingJoints(yRaw,callType,doThePlot,SUREPlotNumber,SUREPlotName)
% Remove noise from the TSD measurements by basis pursuit and soft shrinking
% thresholding. Main function.
% This function receives a noisy TSD measurement and a type of wavelet %filter to
% select [CONSULT WAVELAB HELP] and applies the denoising %procedure [Katicha et al.,
% 2013] over a different range of possible %threshold values (lambdaRange)
%
%Input:
%   yRaw:           Raw TSD measurement [mandatory!]
%   callType        Either 'default' [use default wavelet (Symmlet 8th degr.) and
| threshold option], or 'custom' [enter your own]
%   SUREPlotNumber  Number to identify the figure with the SURE(lambda) plot
%   SUREPlotName    Name to give to that figure
%   doThePlot       Boolean to tell the code if to do the SURE plot (1) or not (0)
%
%Output:
%
%Dependencies:
%This function depends on the following:
% WAVELAB library - addpath'd from the FrontEnd
% TSD_waveletDecomposition function%

% Candidate release version v2022-05-01

% Preprocessing: set defaults.

%prepare the denoising procedure
if strcmp(callType, 'default')
    %use defaults
    %a) compute signal standard deviation [refer to Katicha et al., 2015], %equation 11
    %Update v2022-02-12 - MAD formula for stdev(diff(y)) and built-in MAD function
    auxRaw = diff(yRaw);
    sigma = mad(auxRaw,1);
    sigma = 1.4826.*sigma;
    sigma = sigma./sqrt(2);

    %B) default wavelet type
    waveletType = 'Symmlet';
    waveletOrder = 8;
else
    error('TSD_denoisingJoints error: invalid call type')
end

% - Preprocessing: need to expand yRaw to 2^n length
%normalize yRaw [use either user-provided sigma or default sigma estimation]
yRaw = yRaw/sigma;
n = length(yRaw);

%compute lambda range for the denoising (must be done over the unfolded vector. It is user-provided if callType = 'custom'
if strcmp(callType,'default')
    lambdaRange = 0.0:0.05:sqrt(2*log(length(yRaw)));
    lambdaRange(1) = 0.01;
else
    %do Nothing, lambdaRange is passed as input.
end
% unfold yRaw to 2^n size
[targetSize,~] = powerOf2(n);
X1 = unfoldVector(yRaw, targetSize);

% define the wavelet filter
f = MakeONFilter(waveletType,waveletOrder);
% default: f = MakeONFilter('Symmlet',8);

%% - Processing: initialize output variables

% store all values of the denoised TSD dataset with different threshold and % the estimated error (using Stein's SURE estimate [Samer's suggestion])

X1T = zeros(length(X1),length(lambdaRange)); % unfolded filtered tsd data
ydT = zeros(length(X1),length(lambdaRange)); % unfolded wavelet component
rT  = zeros(length(X1),length(lambdaRange)); % unfolded discontinuous peak component
WCT = zeros(length(X1),length(lambdaRange)); % sanitized wavelet coefficients for the continuous component.

yClean = zeros(n,length(lambdaRange)); % folded-back filtered tsd data
steinError = zeros(size(lambdaRange)); % stein's SURE error estimate for each value of T
peaksMaps = zeros(n,length(lambdaRange)); % folded-back spikes' component

% flip over the lambda vector (so that it starts with the largest threshold
lambdaRange = flip(lambdaRange);

%% - Processing: Iterative Gradient Descent and SURE evaluation for each value of the threshold Lambda

for tt = 1:length(lambdaRange)
    T = lambdaRange(tt);
    if tt == 1
        % initialize the gradient descent. Start with a vector of zeros in the first iteration over T, or use the clear signal for the T(i+1) when % doing T(i)
        ydStart = zeros(size(X1));
    else
        % update the gradient descent
    end
end
ydStart = X1T(:,tt-1);

end

% Decompose as wavelets and do the Batch coordinate Descent.
[yd1(:,tt),rT(:,tt),WCT(:,tt)] = TSD_waveletDecomposition(X1,ydStart,f,T);

% The denoised signal is the denoised spikes (rT) plus the denoised sinusoidal component
X1T(:,tt) = rT(:,tt) + yd1(:,tt);

% Compute the SURE (Stein's unbiased risk estimate) - stored as steinError.
steinError(tt) = TSD_SURE(X1, X1T(:,tt),WCT(:,tt),rT(:,tt));

% Roll back the stretched-out denoised signal X1T and multiply by sigma to restore it to TSD "units"
yClean(:,tt) = foldBackVector(X1T(:,tt),n);
yClean(:,tt) = yClean(:,tt)*sigma;
peaksMaps(:,tt) = foldBackVector(rT(:,tt),n);
peaksMaps(:,tt) = peaksMaps(:,tt)*sigma;  % Retrieve the discontinuous component, fold back, and re-set to scale.
end

% Locate the optimum lambda value - position of the min(steinError).
minSteinPos = find(steinError == min(steinError));
if ~isempty(minSteinPos)
    minSteinPos = minSteinPos(1);
else
    minSteinPos = 1;  % force some value in case a flat signal appears (there is one such case)
end

optLambda = lambdaRange(minSteinPos);

% Export:
steinErrorExp = steinError(minSteinPos);
yCleanExp = yClean(:,minSteinPos);
peaksMapsExp = peaksMaps(:,minSteinPos);
WCTsExp = WCT(:,minSteinPos);

% Plot result
%create the figure of SURE versus lambda and pass the handle to the %frontEnd

if doThePlot
    SUREPlotHandle = figure(SUREPlotNumber);
    set(SUREPlotHandle,'name',SUREPlotName)
    plot(lambdaRange,steinError,'b')
    grid on
    title('TSD noise reduction: SSE prediction with SURE')
    xlabel('Soft shrinking threshold')
    ylabel('SURE estimate')
    hold on
    plot(optLambda,steinErrorExp,'r+','markersize',12)
    legend('SURE estimate','Optimum SURE')
    hold off
else
    SUREPlotHandle = -1;
end

end %end-function

TSD_denoisingJoints_REWEIGHTED

function [yClean,waveletMaps,spikeMaps,steinError,optiLambda] = TSD_denoisingJoints_REWEIGHTED(yRaw,lambdaRange,iterations,weightsEpsilon,optResultsOnly)
%function [yClean,waveletMaps,spikeMaps,steinError,optimLambda] = TSD_denoisingJoints_REWEIGHTED(yRaw,lambdaRange,iterations,weightsEpsilon,optResultsOnly)
%
%Remove noise from the TSD measurements by Reweighted L1 minimization.
%This function receives a noisy TSD measurement and proceeds iteratively %as prompted in the paper on Reweighted L1 Min. by Candés et al 2008
%
%Input:
% yRaw: Raw TSD measurement [mandatory!]
% lambdaRange: Give a vector of values for the penalty parameter Lambda. Or pass -1 to seek the optimum-fitting lambda (call’s the TSD_denoisingJoints function)
% iterations: Number of loops of the reweighted L1 minimization
% weightsEpsilon: Stability parameter (eps) -> See candés et al., 2008
% optResultsOnly: Boolean value to tell if the code shall return
% yClean/and components for all values of lambdaRange (=0) or only for

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% the SURE-optimizing case (=1)
%Output:
% yClean:         Denoised signal
% waveletMaps:    Coefficients for the continuous component onto the % wavelet signal space
% spikeMaps:      Coefficients for the discontinuous component onto the % Dirac signal space
% steinError:     Values of SURE [SSE estimation] for each value of % lambdaRange
% IMPORTANT: If optResultsOnly = 1, the output is cut to the end results % for the SURE optimizing case only
%
%Dependencies:
%This function depends on the following:
% WAVELAB library - addpath'd from the FrontEnd
% TSD_waveletDecomposition function
%
%Release candidate V2022-05-01
%
%% Preprocessing: set defaults.
%Update v2022-02-12 - MAD formula corrected
%-- use built-in MAD function
auxRaw = diff(yRaw);
sigma = mad(auxRaw,1);
sigma = 1.4826.*sigma;
sigma = sigma./sqrt(2);

waveletType = 'Symmlet';
waveletOrder = 8;

% weightsEpsilon = 2;
% - Preprocessing: need to expand yRaw to 2^n length
%normalize yRaw [use either user-provided sigma or default sigma estimation]
yRaw = yRaw/sigma;
n = length(yRaw);

%unfold yRaw to 2^n size
[targetSize,~] = powerOf2(n);
X1 = unfoldVector(yRaw, targetSize);
%% define the wavelet filter
f = MakeONFilter(waveletType, waveletOrder);
% default: f = MakeONFilter('Symmlet', 8);

%% - Processing: initialize output variables
% store all values of the denoised TSD dataset with different threshold and % the estimated error (using Stein's SURE estimate

X1T = zeros(length(X1), iterations, length(lambdaRange)); %< unfolded filtered tsd data
yd1 = zeros(length(X1), iterations, length(lambdaRange)); %< unfolded wavelet component
rT = zeros(length(X1), iterations, length(lambdaRange)); %< unfolded discontinuous peak component
WCT = zeros(length(X1), iterations, length(lambdaRange)); %< sanitized wavelet coefficients for the continuous component.

steinError = zeros(1, iterations, length(lambdaRange));
optiLambda = lambdaRange;

yClean = zeros(n, iterations, length(lambdaRange)); %< folded-back filtered tsd data
spikeMaps = zeros(n, iterations, length(lambdaRange)); %< folded-back peaks
waveletMaps = zeros(n, iterations, length(lambdaRange)); %< folded back wavelet component?

% initialize the weight vectors
weightsWavelets = zeros(length(X1), iterations, length(lambdaRange));
weightsDiracs = zeros(length(X1), iterations, length(lambdaRange));
% fill up the first iteration for all lambdaRanges
weightsWavelets(:,:,1) = ones(length(X1), 1, length(lambdaRange));
weightsDiracs(:,:,1) = ones(length(X1), 1, length(lambdaRange));

%% NEW: INITIALIZE WEIGHT MATRICES - Prepare separate matrices for % - Processing:
Iterative coordinateDescent and SURE evaluation for each value of the threshold Lambda

for tt = 1:length(lambdaRange)
    lambda = lambdaRange(tt);
    if tt == 1
        % initialize the gradient descent. Start with a vector of zeros in the % first iteration over T, or use the clear signal for the T(i+1) when % doing T(i)
        ydStart = zeros(size(X1));

    end
end
else
    ydStart = X1T(:,iterations,tt-1);
end

for k = 1:iterations
    %the first iteration is "plain BPD" - initialize the diagonal matrix of weights as ID
    WWCT = eye(length(X1));
    Wspk = eye(length(X1));
    %fill up the weights onto the diagonals of the W matrices
    WWCT = reshape(weightsWavelets(:,k,tt),[length(X1),1]).*WWCT;
    Wspk = reshape(weightsDiracs(:,k,tt),[length(X1),1])  .*Wspk;
    %Decompose as wavelets and do the Batch coordinate Descent.
    [yd1(:,k,tt),rT(:,k,tt),WCT(:,k,tt)] =
        TSD_waveletDecomposition_WEIGHTED(X1,ydStart,f,lambda,WWCT,Wspk);
    %The denoised signal is the denoised spikes (rT) plus the denoised wavelet component
    X1T(:,k,tt) = rT(:,k,tt) + yd1(:,k,tt);
    %Compute the SURE (Stein's unbiased risk estimate) - stored as steinError.
    steinError(:,k,tt) = TSD_SURE(X1, X1T(:,k,tt),WCT(:,k,tt),rT(:,k,tt));
    %Roll back the stretched-out denoised signal X1T and multiply by sigma to restore it to TSD "units"
    yClean(:,k,tt) = foldBackVector(X1T(:,k,tt),n);
    yClean(:,k,tt) = yClean(:,k,tt).*sigma;
    spikeMaps(:,k,tt) = foldBackVector(rT(:,k,tt),n);
    spikeMaps(:,k,tt) = spikeMaps(:,k,tt).*sigma; %Retrieve the discontinuous component, fold back, and re-set to scale.
    waveletMaps(:,k,tt) = foldBackVector(yd1(:,k,tt),n);
    waveletMaps(:,k,tt) = waveletMaps(:,k,tt).*sigma;

    %finally, prepare the coefficients for the next iteration. Avoid this step if k == iterations
    if k < iterations
        weightsWavelets(:,k+1,tt) = 1./(abs(WCT(:,k,tt)) + weightsEpsilon);
        weightsDiracs(:,k+1,tt) = 1./(abs(rT(:,k,tt))  + weightsEpsilon);
    end
end %end loop for k (given value of lambda)
end %end loop for all values of lambda

% update V. 2021-05-10
% Add here the improved SURE value for all lambdas
% This is to run after all iterations completed

steinUpdated = zeros(1,length(lambdaRange));

for tt = 1:length(lambdaRange)
    %compute the updated SURE error estimate using the fit from after the %RWL1 (X1T(iterations)) and the count of non-zero components from the BP fit [first iteration] -> See eqn. "11" in paper 2.
    steinUpdated(tt) = TSD_SURE(X1, X1T(:,iterations,tt),WCT(:,1,tt),rT(:,1,tt));
end

%replace the steinError output with steinUpdated one...
steinError = steinUpdated;

%also, make the output signals correspond to the steinUpdated-minimizing %case, and the last iteration of RWL1 (I don't mind about the results %half-way-through)
if optResultsOnly
    optLambdaPos = steinUpdated==min(steinUpdated);
    if sum(optLambdaPos)>1 %use sum to check if more than one 1, cause optLambdaPos is a Boolean vector the size of steinUpdated.
        aux = find(optLambdaPos);
        aux = aux(1);
        bux = zeros(size(optLambdaPos));
        bux(aux) = 1;
        optLambdaPos = boolean(bux);
    end
    optiLambda  = lambdaRange(optLambdaPos);
    yClean      = squeeze(yClean(:,end,optLambdaPos));
    spikeMaps   = squeeze(spikeMaps(:,end,optLambdaPos));
    waveletMaps = squeeze(waveletMaps(:,end,optLambdaPos));
    steinError  = min(steinError);
end

end %endfunction
function [yd1, rT, WCT] = TSD_waveletDecomposition(X1, ydstart, f, T)
%function [yd1, rT, WCT] = TSD_waveletDecomposition(X1, ydStart, f, T)
%function to decompose a given 'ydStart' signal into a continuous (wavelet based)
%component yd, and a discontinuous component rT  %and filter by thresholding with a
%threshold T.
%input
%   ydStart = RAW SIGNAL
%   f       = wavelet filter (provided from the TSD_denoisingJoints function)
%   T       = threshold value [a.k.a. lambda]
%output
%   yd1     = continuous (wavelet-based) component of X1
%   rT      = discontinuous (spike) component of X1
%   WCT     = filtered wavelet coefficients for yd1
%
%Release candidate v2022-05-01

%% code begins
%run 100 iterations
yd = ydstart;
for i = 1:100
    r  = X1-yd;   %residual of RAW yn minus DENOISED yd
    %start by wavelet-transform 'r': apply the wavelet decomposition using the filter
    %'f'.
    WC = FWT_PO(r,1,f);
    %syntax means: decompose the stretched-out X1, up to the max extent (the min
    %extent is the power of 2 that makes the length of the vector X1).
    WCT = softShrinking(WC,T);  %filter the noise with the softShrinking rule
    %e) invert the wavelet decomposition
    yd1 = IWT_PO(WCT,1,f);
    %recompute r:
    r = X1 - yd1; %theoretically this should be a mere spikes function (because I'm
    %substracting a "clean" recovered sinusoidal signal). %remove the unwanted spikes on
    %this r
    rT = softShrinking(r,T);    %%NOTE: THIS IS THE RECOVERED VECTOR OF ZEROS OR
    %SPIKES (WEAK SPOT LOCATIONS)
    %and now close the loop by redefining yd as the thresholded residual. Weird, ain't
    %it?
    yd = rT;
end
function \[yd1,rT,WCT\] = TSD_waveletDecomposition_WEIGHTED(X1,ydStart,f,LAMBDA,WeightWavelet,WeightDirac)
%function \[yd1,rT,WCT\] = TSD_waveletDecomposition(X1,ydStart,f,T)
%function to decompose a given 'ydStart' signal into a continuous (wavelet based)
%component yd, and a discontinuous component rT %and filter by thresholding with a
%threshold T.
%PROCEED BY BATCH GRADIENT DESCENT [SEE PAPER 1]
%input
%   ydStart = RAW SIGNAL [STRETCHED TO 2^P LENGTH]
%   f       = wavelet filter (provided from the TSD_denoisingJoints function)
%   LAMBDA  = threshold value [a.k.a. lambda]
%   WeightWavelet= diagonal matrix (length of ydStart) with the weights for
%   each component of the wavelet dictionary
%   WeightDirac = diagonal matrix (length of ydStart) with the weights for
%   each component of the Dirac dictionary.
%
%output
%   yd1     = continuous (wavelet-based) component of X1
%   rT      = discontinuous (spike) component of X1
%   WCT     = filtered wavelet coefficients for yd1
%
%Release candidate v2022-05-01

%Don't use Matlab's A^-1, cause it's computer intensive. Since the weight %matrices
%are diagonal,extract the diag.elements, invert them, and then %rebuild
aux = diag(WeightWavelet);
%aux = 1./aux;
% WeightWaveletInv = aux.*eye(length(aux));
WeightedWaveletLambda = LAMBDA.*aux;
%
aux = diag(WeightDirac);
%aux = 1./aux;
% WeightDiracInv = aux.*eye(length(aux));
WeightedDiracLambda = LAMBDA.*aux;

% code begins
% run 100 iterations
yd = ydstart;
for i = 1:100
    r = X1 - yd;  % residual of RAW yn minus DENOISED yd
    % start by wavelet-transform 'r': apply the wavelet decomposition using the filter 'f'.
    % WC is the wavelet coefficients
    WC = FWT_PO(r,1,f);
    % syntax means: decompose the stretched-out X1, up to the max extent  % (the min extent is the power of 2 that makes the length of the vector X1).
    % UPDATE V2020-08-19: ADD THE WEIGHING FACTORS INTO THE SOFT-SHRINKING FUNCTION
    WCT = softShrinking(WC,WeightedWaveletLambda);  % filter the noise with the
    % softShrinking rule
    % invert the wavelet decomposition
    yd1 = IWT_PO(WCT,1,f);
    % recompute r:
    r = X1 - yd1;  % theoretically this should be a mere spikes function (because I'm
    % subtracting a "clean" recovered sinusoidal signal)
    % remove the unwanted spikes on this r
    rT = softShrinking(r,WeightedDiracLambda);
    % and now close the loop by redefining yd as the thresholded residual. ?Weird,
    % ain't it?
    yd = rT;
end

end % endfunction

softShrinking

function y = softShrinking(x,lambda)
% function y = softShrinking(x,t)
% This function will apply the 'soft shrinking rule' to the vector x, guided by the
% value of 't'
% See the paper by Katicha et al. (2013) on wavelet denoising for info on % what is this
% about.
%
% Input: x: vector of data
% t: positive scalar (not necessary integer)
% Output: y: vector the size of x, filtered as per the soft shrinking rule.
% Release candidate v2022-05-01
integrityCheck = lambda>=0;
if ~integrityCheck
    warning('softShrinking:: parameter "t" is smaller than 0')
    return
end

% Prep work: Compatibility for the both BPD and Rew-L1-decompos. %BP can work with a scalar lambda, whereas Reweighed L1 decomposition needs lambda as a vector (cause each entry of x may have its own lambda(x) = lambda*w(x)

% check that if lambda enters as a scalar it gets converted to a vector the size of x (so that it doesn't crash below when doing the soft-shrinking)

if length(lambda) == 1
    lambda = lambda.*ones(size(x));
else
    % do nothing
end

% apply the rule - treat separately those values of x with absolute value greater or less than t
y = zeros(size(x));
zone1 = find(abs(x)<=lambda);
zone2 = find(abs(x)>lambda);

if ~isempty(zone1)
    % the positions of y told by zone1 must be filled up with zeros
    y(zone1) = zeros(size(zone1));
end
if ~isempty(zone2)
    % the positions of y told by zone2 must be filled up with y = x-t*signum(x). Note, sign(x) = signum function for x
    y(zone2) = x(zone2) - sign(x(zone2)).*lambda(zone2);
end
function w = unfoldVector(vector,n)
%function w = unfoldVector(vector,n)
%function to unfold a vector 'vector' of any given length to a length of n by adding
extra elements at both ends in a symmetric way (mirroring the %elements of 'vector'
over the two extremes).

%NOTE: If I need to add an even number of elements, it will add an equal size of
entries both ahead and after the vector. %Otherwise, it will add one more entry to the
beginning of the vector
%
%e.g.
%If vector is [1 2 3 4 5 6] and n = 10, then w = [3 2 1 2 3 4 5 6 5 4]
%If vector is [1 2 3 4 5] and n = 10, then w = [4 3 2 1 2 3 4 5 4 3]
%
%Release candidate v2022-05-01
%
%V0.1 2019-08-16
%Update: 1) Don't repeat the extreme values of 'vector'.
%        2) allow for n to also be an odd number. (remove the "isItEven" restriction
%        for n)
% 1) calculate how many values do I need to add.

len = length(vector);
[a,b] = size(vector);

ext = n-len;
% check if ext is even or odd and so define the number of elements to add to the
beginning and end of 'vector' to assemble w [called nw1 and nw2 respectively]

isItEven = ext/2 == floor(ext/2);
if isItEven
    %add an equal number to nw1 and nw2
    nw1 = floor(ext/2);
    nw2 = floor(ext/2);
else
    %add one more element to nw1 than nw2
    nw1 = floor(ext/2)+1;
    nw2 = floor(ext/2);
end
% 2) Now build the w vector. Assemble w1, w2 as row vectors. Transpose at the end to match the shape of 'vector' if necessary

w1 = vector(nw1+1:-1:2);
w2 = vector(end-1:-1:end-nw2);

%Finally adapt the shape of the output vector 'w' to that of 'vector'
if a>=b
    %vector is a column -> transpose w1 and w2 and merge with vector
    w = [w1; vector; w2];
else
    %vector is also a row -> paste all pieces together
    w = [w1 vector w2];
end

end  %% endfunction

foldBackVector

function vector = foldBackVector(w,len)
%function vector = foldBackVector(w,len)
%This function rolls back the effect of 'unfoldVector': a vector of length '%len' is recovered from the unfolded/mirrored vector w.
%See the info for 'unfoldVector' for details on how 'w' is constructed from '%vector'

%Release candidate v2022-05-01

%initialize output
[a,b] = size(w);
if a>=b
    %w is a column vector
    vector = zeros(len,1);
else
    vector = zeros(1,len);
end
%UPDATE v2019-10-27: I had sth. wrong with the location of the first and
last elements of the source vector "vector"
re'do the entire thing.
addedElements = length(w)-len; %all the elements that were added when unfolding.
addedToTheEnd = floor(addedElements/2); %the unfolding adds one less element at the
end if the source vector is odd-size, or half and half if it's even.
addedBeginnin = addedElements - addedToTheEnd;

%now extract vector from w
vector = w(addedBeginnin+1:addedBeginnin+len);

powerOf2
function [m,expo] = powerOf2(x)
%function m = powerOf2(x)
%This function is to locate the closest power of 2 that is greater than x
%OUTPUT:
% m = smallest power of 2 number that is greater than x
% expo= exponent to raise 2 to get m (expo = log2(m))
%
%Release candidate v2022-05-01

%Dummy coding, it will work by scanning all powers of 2 until it surpasses %the value of X
%If by any chance it doesn't find anything, it will return m = -1 and expo= 0
m = -1;
expo = 0;
while m == -1
    %do the power of 2
    tempM = 2^expo;
    %try if the power of 2 (tempM) is greater than input number x
    if tempM >= x
        m = tempM;
    else
        %if not, try the next power of 2
        expo = expo+1;
    end
end
function SURE = TSD_SURE(X1, X1T, WCT, rT)

%function SURE = TSD_SURE(X1, X1T, WCT, rT)

% Compute the SURE estimate of the BPD procedure for a given series X1, and
% its filtered components X1T, and the vectors of coefficients WCT, rT
%
% Release candidate v 2022-05-01

% First, compute how many WCT(:,k) coefficients are non-zero
% The Stein error term must include the count of non-zero shrunk coefficients for the
two soft-shrinking steps (sinusoid + spikes)!
nonZeroWCTcount1 = length(find(WCT ~= 0));
nonZeroWCTcount2 = length(find(rT ~= 0));

% Important: Need to compute the Stein error estimate with the X1 and X1T series (the
foldback takes some of the optimization away and may return a faulty denoised signal!
SURE = -length(X1T) + sum(((X1T - X1).^2) + 2*nonZeroWCTcount1 + 2*nonZeroWCTcount2;

end
Linear elastic back-calculation of concrete pavement properties and joints' LTE index.

Front-end script

%% DISSERTATION PAPER 3 - FRONT-END SCRIPT FOR TSD BACK-CALCULATION
%%- TSD VALIDATION RUN 5cm from MnROAD Loop section 124
%Candidate Release V2022-05-01

%% STAGE 0 DATA LOAD-
tic
clc
restoredefaultpath
clear variables
close all
doPlots = 1; %Plot each road's source and recovered TSD signal. Keep disabled if you need to economize memory!

%% update v2022-04-21
%for faster code, disable the " matrix close to signular" warnings that may pop up. They slow the code too much!
%Follow: https://www.mathworks.com/help/matlab/ref/lastwarn.html#responsive_offcanvas

load matlabWarningMessage
%this tiny variable has the warning message text and ID for the 'singular
%matrix warning"
%turn off the warning
warnStruct = warning('off',warnID);
% RESTORE IT AT THE END OF THE CODE!
% ....

% addpath to the V2.0-compatible dependencies! (Denoising and back-calc engine)
addpath('./waveletDenoising')
addpath('./back-calc')

% disp('Loading TSD Data')
inputFilename = 'T17202109270013_5cm.xlsx';
%% input block for LVR sections 124-524

% latLongData = xlsread(inputFilename,inputSheet,'b29823:c32723');       %%lat long
% data in decimal degrees

% section 124
stationData = xlsread(inputFilename,inputSheet,'a29823:a32723');       %% station in m
roadID = 124.*ones(size(stationData));
roadName = roadID;
loadData = xlsread(inputFilename,inputSheet,'e29823:e32723');       %%dynamic load on the
right-side half-axle [kg]!
loadData = loadData.*9.81; %parse to Newtons!
loadInLBS = 0; %<<--put 1 if the load is in LBS. Otherwise, assume NEWTONS.

%% input block for section 138-238

277
section 238
stationData = xlsread(inputFilename,inputSheet,'a63003:a65603');   % station in m
roadID = 238.*ones(size(stationData));
roadName = roadID;

dataLoad = xlsread(inputFilename,inputSheet,'e63003:e65603'); %dynamic load on the right-side half-axle [kg]!
dataLoad = dataLoad.*9.81;  %parse to Newtons!
loadInLBS = 0; %<<--put 1 if the load is in LBS. Otherwise, assume NEWTONS.

% WE KNOW THE CONCRETE SLABS ARE 6-INCHES ALL THROUGHOUT THE SECTION. BUILT 2017. % NO MATERIAL TEST INFO ON THE MNROAD DATABASE. LTE TESTED BY FWD IN 2019
lyrThick = 8.*ones(size(stationData));

exportFilename = 'MNRoad_Loop238_5cm_v0420.xlsx';

columns are::  Slope 1.500[µm/m]  Slope 0.900[µm/m]  Slope 0.600[µm/m]  Slope 0.450[µm/m]  Slope 0.300[µm/m]  Slope 0.215[µm/m]  Slope 0.130[µm/m]  Slope -0.200[µm/m]  Slope -0.300[µm/m]  Slope -0.450[µm/m]
vyData = xlsread(inputFilename,inputSheet,'g63003:p65603');
vxData = xlsread(inputFilename,inputSheet,'d63003:d65603');

vyData = vyData(:,[10,9,8,7,6,5,4,3,2,1]);    %< NOTE THAT THIS COMES FROM THE TSD IN MM/SEC
TSDpoints = [-0.45,-0.3,-0.2,0.130,0.215,0.300,0.450,0.600,0.900,1.500];
TSDpoints = TSDpoints';
umTSDSensors = length(TSDpoints);

stationFrom = stationData(:,1);  %%get station information (from and to) for each measurement.
latFrom = latLongData(:,1);
lonFrom= latLongData(:,2);
latTo   = latFrom;
lonTo  = lonFrom;

input block for section 239
section 239
stationData = xlsread(inputFilename,inputSheet,'a67703:a68803');   % station in m
roadID = 239.*ones(size(stationData));
roadName = roadID;
loadData = xlsread(inputFilename,inputSheet,'e67703:e68803');  % dynamic load on the right-side half-axle [kg]!
loadData = loadData.*9.81;  % parse to Newtons!
loadInLBS = 0;  %<<--put 1 if the load is in LBS. Otherwise, assume NEWTONS.

% WE KNOW THE CONCRETE SLABS ARE 6-INCHES ALL THROUGHOUT THE SECTION. BUILT 2017. NO MATERIAL TEST INFO ON THE MNROAD DATABASE. LTE TESTED BY FWD IN 2019
lyrThick = 4.*ones(size(stationData));

% columns are::  Slope 1.500[µm/m]  Slope 0.900[µm/m]  Slope 0.600[µm/m]  Slope 0.450[µm/m]  Slope 0.300[µm/m]  Slope 0.215[µm/m]  Slope 0.130[µm/m]  Slope -0.200[µm/m]  Slope -0.300[µm/m]  Slope -0.450[µm/m]
vyData = xlsread(inputFilename,inputSheet,'g67703:p68803');
vxData = xlsread(inputFilename,inputSheet,'d67703:d68803');

% vyData = vyData(:,[10,9,8,7,6,5,4,3,2,1]);  %< NOTE THAT THIS COMES FROM THE TSD IN MM/SEC
% TSDpoints = [-0.45,-0.3,-0.2, 0.130, 0.215, 0.300, 0.450, 0.600, 0.900, 1.500];
% Short-slab case study: Use without trailing sensors!
% vyData = vyData(:,[7,6,5,4,3,2,1]);  %< NOTE THAT THIS COMES FROM THE TSD IN MM/SEC
% TSDpoints = [0.130,0.215,0.300,0.450,0.600,0.900,1.500];
% TSDpoints = TSDpoints';
% numTSDSensors = length(TSDpoints);
% stationFrom = stationData(:,1);  % get station information (from and to) for each measurement.
% latFrom = latLongData(:,1);
% longFrom= latLongData(:,2);
% latTo   = latFrom;
% longTo  = longFrom;
%
% Get all unique roads in the TSD dataset -
% UPDATE V2019-10-30: MUST DISTINGUISH BY roadName+blockID [roadID and roadNAME by themselves would collect many segments together (and overlap signals)]
roadIDList = unique(roadID);  % use this vector to locate the TSD points for each road.
numRoads = length(roadIDList);
fprintf('	 %g streets (+ blocks) were recognized 
',numRoads)

%% 3) WAVELET DENOISING OF THE TSD DATA -
%%update v 2022-03-17 St Patrick -> Do Haar Wavelet decomposition only!
disp('TSD denoising via wavelet decomposition')

for k = 1:numRoads
    whichRoad = roadIDList(k);
    % tsdIndices = find(strcmp(roadID,whichRoad)); %this tweak allows to use find to 
    % compare cell arrays. Source: https://www.mathworks.com/matlabcentral/answers/84242-
    % find-in-a-cell-array
    tsdIndices = find(roadID ==whichRoad);
    %get the name of the road/lane under analysis
    thatRoadName = roadName(tsdIndices);
    thatRoadName = thatRoadName(1);

    %get the TSD data for that road
    fprintf(' 	 now processing: %s, segment %s 
',string(thatRoadName),string(whichRoad));

    vy = vyData(tsdIndices,:);
    vx = vxData(tsdIndices,:);
    loadRightWheel = loadData(tsdIndices);

    station = stationFrom(tsdIndices);
    % latStart = latFrom(tsdIndices);
    % latEnd   = latTo(tsdIndices);
    % longStart= longFrom(tsdIndices);
    % longEnd  = longTo(tsdIndices);
    thickness= lyrThick(tsdIndices);

    %Remove Nans that may cause trouble. Use this workaround: https://www.mathworks.com/matlabcentral/answers/164316-select-everything-not-returned-by-index
    theseAreNans = find(isnan(vy(:,1))); %this returns a vector with the positions of NaN's inside the defl.Slope 110.
    NansIndex = isnan(vy(:,1)); %this logical array (different from the one above) helps then get the non-NaN values easily. [it's a 0/1 vector as long as refTSD in which the 1's indicate the NaNs]
    vy = vy(~NansIndex,:);
    thickness = thickness(~NansIndex,:);
loadRightWheel = loadRightWheel(~NansIndex,:);
station = station(~NansIndex,:);

%% 3.0) Do the data denoising
denoisedTSDvy = zeros(size(vy));

for zz = numTSDSensors:-1:1  %do this iteration back-wards on purpose so that I
can plot the denoised SL110 with the "temporary names" from the denoising stage.
    refVY = vy(:,zz);
    fprintf('	 Denoising signal from TSD sensor at %g \n',TSDpoints(zz))
    % 3.1: Data Denoising by wavelet decomposition.
denoisedVy = Haar_Denoise_LFDR(refVY,0.01);
denoisedTSDvy(:,zz) = denoisedVy;
end

%% 4) Do the plot of the recovered signal
if doPlots
    figure(300+k)
    set(gca,'FontSize',18)
    set(gca,'Grid','on')
    titleString = sprintf('TSD signal for road %s, section %s. All denoised TSD sensors',thatRoadName, whichRoad);
    title(titleString)
    xlabel('station [m]')
    ylabel('vertical defl velocity [mm/s]')
    hold on
    for zz = 2:numTSDSensors
        plot(station, denoisedTSDvy(:,zz),'linewidth',1);
    end
    legend('vy_{-450}','vy_{-300}','vy_{-200}','vy_{-130}','vy_{-210}','vy_{-310}','vy_{-450}','vy_{600}','vy_{900}','vy_{1510}')
    hold off
end  %end if doPlots
drawnow

```matlab
selectJoint = input('Do you want to back-calculate joints?. 1 if so, 0 to stop...');
selectJoint = 1;
selectJointK = 0;
while selectJoint
    jointPosition = input('give the joint\'s station (where the TSD bumps) [m]...');
    jointPosition = [1551, 1555.8, 1560.5];
    jointPosition = sort(jointPosition);
    [~index] = findpeaks(denoisedTSDv(:,TSDpoints == 0.300), 'MinPeakDistance', 25, 'MinPeakHeight', quantile(denoisedTSDv(:,TSDpoints==0.300), 0.75));
    jointPosition = station(index);
    nnn = length(jointPosition);
    sprintf('%g joints were detected in this section 
', nnn)
    for i = 1:nnn
        jointIndex = find(station==jointPosition(i));
        jointLatLong = [latStart(jointPosition(i)); longStart(jointPosition(i))];
        jointsInRoadK = jointsInRoadK+1;
        fprintf('\t solving continuous component 
')
        subgradeType = 0;
        nu = 0.21;

        % Solve k,E,G ahead of the joint.
        % UPDATE V 2022-03-23 -> DO MULTIPLE BACK-CALC OF THE CONTINUOUS COMPONENT FOR K, E, G. keep the mean value as representative one.
        stationAhead = station(max(jointIndex-45,1):max(jointIndex-35,1));
        localE = zeros(size(stationAhead));
        localK = zeros(size(stationAhead));
        localG = zeros(size(stationAhead));
        for jj = 1:length(stationAhead)
            if loadInLBS
                localLoad = loadRightWheel(station==stationAhead(jj)).*4.44822162;%< pass axleLoad from LBS to Newton
```
localLoad = localLoad.*4.44822162;%< pass axleLoad from LBS to Newton

else
    localLoad = loadRightWheel(station==stationAhead(jj)).*1; % leave in Newtons
    localLoad = localLoad.*1;%< leave in Newton
end

pressure = 115./145.04.*1e6; %TSD Wheel load -> 115 PSI to PA
localThck = thickness(station==stationAhead(jj));
localThck = localThck.*2.54./100;

localvx = vx(station==stationAhead(jj));
localTSD_cont = denoisedTSDvy(station==stationAhead(jj),:);

%update v04-21: Call the back-calculation front-end based on deflection velocity!
%% careful here! localTSD_cont is in mm/sec. Must pass it to the back-calc engine in m/sec!!!
%% also, localThick is in inches, must pass to meters!
verborragia = 0;
[loc1E(jj),loc1K(jj),loc1G(jj),~,~,~,~] = backCalc_continuous_0420(TSDpoints,localTSD_cont./1e3,localvx,localLoad,pressure,localThck,nu,subgradeType,verborragia);

%get the final k, E, G as definitive values
loc1E = mean(loc1E);
loc1K = mean(loc1K);
loc1G = mean(loc1G);

%% Now proceed to the joint.
UPDATE V 2022-03-23: DO THE BACK-CALC BASED ON DEFLECTION SPEED. NO NEED TO CHANGE SIGNS (TSD'S convention on vy is the same as for my w(z,t), a pavement that goes down has positive vy

fprintf('
	 solving joint approach 
')
stationRanges = max(jointIndex-30,1):min(jointIndex-4,1); %<-- all these positions are the ones I'm back-calculating, stop 20cm ahead of the joint.
k = length(stationRanges);

jointLocation = zeros(k,1);
LTE = zeros(k,1);
LTE2 = zeros(k,1);
SSEfinal = zeros(nk,1);
localTSD_pulse = zeros(nk,length(TSDpoints));

for j = 1:nk
    localTSD_pulse(j,:) = denoisedTSDvy(stationRanges(j),:);
    localThck = thickness(stationRanges(j));
    localThck = localThck.*2.54./100; % pass localThck to meters!

    if loadInLBS
        localLoad = loadRightWheel(stationRanges(j)).*4.44822162;%< pass axleLoad from LBS to Newton
        localLoad = localLoad.*4.44822162;%< pass axleLoad from LBS to Newton
    else
        localLoad = loadRightWheel(stationRanges(j)).*1; % leave in Newtons
        localLoad = localLoad.*1;%< leave in Newton
    end
    jointLocation(j) = -1;
    LTE(j) = -1;
    fprintf(‘	 solving joint profile at station %g 
 n’,station(stationRanges(j)))
    %update V2022-03-09 -> Get a rough estimate of the distance between the tsd wheel and the joint
    %The distance between the current measurement and the joint's location should be a rough estimate of where the joint is at [+/- 10-15cm]
    estimateC = jointPosition(i) - station(stationRanges(j));
    estimateC = max(estimateC-0.15,0):0.01:estimateC+0.15;

    % solving the joint
    %Update V 2022-03-23 -> Use back-calc based on deflection velocity!
    May need vy and local vx record too!
    localvx = vx(stationRanges(j));

    % IMPORTANT: VYcomes in mm/s, and vx in m/s. PASS BOTH IN M/sec
    %Update v 2022-03-09 -> Pass an estimate of the joint location [variable estimateC]
    verborragia = 1;
    [jointLocation(j),LTE(j),SSEfinal(j),~,~,~] = backCalc_joint_BruteForce_0420(TSDpoints,localTSD_pulse(j,:)./1e3,localvx,estimateC,localThck,localK,localG,localE,nu,localLoad,pressure,verborragia);
%% Update v 2022-03-04.
%Do the export of the results now
%exportFilename stated at the beginning!
exportSheet = sprintf('joint at station %g',jointPosition(i));
%export k, E, G
export = xlswrite(exportFilename,
[localK;localE;localG],exportSheet,'e7:e9');
%export h, nu
export = xlswrite(exportFilename,[nu;localThck],exportSheet,'d13:d14');
%export joint station, lat. and long.
export = xlswrite(exportFilename,jointPosition(i),exportSheet,'f4');
%         export = xlswrite(exportFilename,jointLatLong,exportSheet,'f2:f3');
%export the c, LTE back-calc results upon approach
%update v 2022-03-09 -> Export the final SSE
exportVariable = [station(stationRanges),jointLocation,LTE,SSEfinal];
export = xlswrite(exportFilename,exportVariable,exportSheet,'h6:k35');

toc
end  %end for i = 1 : nnn
% before closing the while, check if doing one more joint.
selectJoint = 0;
end
save guardaTodo5cm_Loop124_v0420.mat
end
disp('....all completed')
toc
% % RESTORE THE SINGULAR MATRIX WARNING
warning(warnStruct);
Auxiliary functions #1: Haar-wavelet based denoising.

Original code by S. Katicha, reproduction authorized.

**Haar_Denoise_LFDR**

function \([yd,swa,swd] = \text{Haar\_Denoise\_LFDR}(yn,fdr,s)\)

if nargin<3
    s = 1.4826*median(abs(diff(yn)-median(diff(yn))))/sqrt(2);
    if nargin<2
        fdr = 0.5;
    end
end
L = length(yn);

[swa,swd] = Haar_TI(yn);

for i=1:size(swd,1)
    p = 2*(1-normcdf(abs(swd(i,:)),0,s));
    pth = vfdr(p(:,fdr));
    th = max(abs(swd(i,p>=pth)));
    if isempty(th)
        th = max(abs(swd(i,:)))+1;
    end
    %     swd(i,:) = scad_th(swd(i,:),th,sqrt(2*log(L))*s);
    swd(i,:) = Wth(swd(i,:),1000000,th);
end

yd = iHaar_TI(swa,swd);
yd = yd(:);

**Haar_TI**

function \([wa,wd] = \text{Haar\_TI}(x,\text{Levels})\)

L = length(x);
if nargin<2
    Levels = floor(log2(L));
end
\[
x = x(:,);
\]

\[
wa = \text{zeros}(L,\text{Levels});
wd = \text{zeros}(L,\text{Levels});
wa(:,1) = \frac{x + \text{shift}(x,-1)}{\sqrt{2}};
wd(:,1) = \frac{x - \text{shift}(x,-1)}{\sqrt{2}};
\]

\begin{verbatim}
for i=2:Levels
    wa(:,i) = \frac{wa(:,i-1) + \text{shift}(wa(:,i-1),-2^{(i-1)})}{\sqrt{2}};
    wd(:,i) = \frac{wa(:,i-1) - \text{shift}(wa(:,i-1),-2^{(i-1)})}{\sqrt{2}};
end
\end{verbatim}

\[
wa = wa';
wd = wd';
\]

\[
i\text{Haar}_Tl
\]

\[
\text{function } x = i\text{Haar}_Tl(wa,wd)
\]

\[
\text{Levels} = \text{size}(wd,1);
x = \frac{wa(\text{end},:) + wd(\text{end},:) + \text{shift}((wa(\text{end},:) - wd(\text{end},:))',2^{(\text{Levels}-1)})'}{2/2*\sqrt{2}};
\]

\begin{verbatim}
for i=\text{Levels}-1:-1:1
    x = \frac{x + wd(i,:) + \text{shift}((x - wd(i,:))',2^{(i-1)})'}{2/2*\sqrt{2}};
end
\end{verbatim}

\[
\text{shift}
\]

\[
\text{function } y = \text{shift}(x,\text{shift\_size})
\]

\[
% \text{shift}(x,\text{shift\_size}) \text{ circularly shifts the rows in matrix } x \text{ by } \text{shift\_size} \text{ positions. If } x \text{ is a vector, the shift is performed on the elements of } x
\]

\[
[r,c] = \text{size}(x);
id = 1:length(x);
\]

\begin{verbatim}
if \text{shift\_size}>0
    id = [id(\text{end}-\text{shift\_size}+1:end) id(1:end-\text{shift\_size})];
elseif \text{shift\_size}<0
    id = [id(1-\text{shift\_size}:end) id(1:-\text{shift\_size})];
\end{verbatim}
if r==1 || c==1
    y = x(id);
else
    y = x(id,:);
end

\textit{vfdr}

function \[pth,q,a,tlt2error,qa\] = vfdr(pval,locfdr,a)
\%
% function \[pth,q\] = vfdr(pval,a)
% variable false discovery rate (FDR) procedure to minimize classification
% error
% inputs:
% - pval: the p values of the observations
% - a (optional): proportion of observations that come from the null
%   distribution. If a is not provided, it is estimated from the data
% Outputs:
% - pth: the p value threshold that results in minimizing the classification
% error; p values lower than pth are estimated as not coming from the null
% distribution while p values larger than pth are estimated as coming from the
% null distribution. p values that are equal to pth are estimated as not coming from
% the null distribution is their resulting qval is less than 0.5, otherwise they are estimated as coming from the null distribution
% - q: the q value corresponding to pth. If q<0.5 then pth is estimated as not
% coming from the null distribution
% - a: estimated proportion of measurements that come from the null
% distribution
% - tlt2error: the estimated total classification error for every possible
% threshold

m = length(pval);
p = sort(pval);
k = 1:m; k = k(:);
if nargin<3
    q = p.*m./k; %q = min(q,1);
    qa = p.*(m+1-k)./(k-k.*p); qa = min(qa,1);
    a = sum(p>=0.5)/(0.5*length(p)),1);
    a = min(max(a,0.0001),1);
if nargin<2
    locfdr = 0.5;
end
end

qval = a*p.*m./k;

for j=length(qval)-1:-1:1
    qval(j) = min(qval(j),qval(j+1));
end

% t1t2error = -k.*(1-2*qval);
t1t2error = -(locfdr*k/m-a*p);
 [~,ix] = min(t1t2error);

pth = p(ix);
q = qval(ix);

count = sum(p<=pth);
if count==1
    if q>=0.5
        pth = -1;
    end
end

with

function out = Wth(x,p,th)

if nargin<3
    th = sqrt(2*log(length(x)));
    if nargin<2
        p = 2;
    end
end

% p = max(p,1);

if p>1e4
    out = x;
out(abs(out)<=th) = 0;
else
    s = sign(x);
    out = max(abs(x).*(1-(th./abs(x)).^p),0);
    out = s.*out;
end
Auxiliary functions #2: Linear elastic slab theory back-calculation engine.

backCalc_continuous

function [E,k,G,historyE,historyk,historyG,historySSE] = backCalc_continuous_0420(TSD_points,TSD_meas,TSDvx,Load,pressure,h,nu,subgrade_type,verboseness)
% function [E,k,G,historyE,historyk,historyG,historySSE] = backCalc_continuous_0420(TSD_points,TSD_meas,TSDvx,Load,pressure,h,nu,subgrade_type,verboseness)

% Front-end function to run a single back-calculation over deflection velocity data for the continuous slab w/o joint.
% This back-calc problem solves for E, k G; with the new implementation
% based on deflection velocities.
%
%INPUT
%  TSD_points: locations (w.r.t the TSD rear wheel) where the slope defl. measurements are taken [m]
%  TSD_meas: deflection velocity measurements from the TSD [m/sec]
%  TSDvx: TSD travel speed [m/sec]
%  Load: half-axle load [Newtons]
%  Pressure: half-axle tire pressure [Pa]
%  h: concrete slab thickness [m]
%  nu: concrete Poisson coefficient [default 0.20]
%  subgrade_type: Boolean: 0 = Winkler foundation (G = 0), 1 = Pasternak foundation (G ~=0)
%  verboseness BOOLEAN: 1 = The function reports it's progress all throughout. 0 = silent solver.
%
%OUTPUT
%  E: Back-calculated slab's Young modulus [N/m2]
%  k: Back-calculated subgrade's Mod. of reaction [N/m3]
%  G: subgrade's shear modulus [N/m2] (If subgrade_type = 0), return is G = 0
%  historyE: evolution of back-calc E values
%  historyK: evolution of back-calc k values
%  historyG: evolution of back-calc G values
%  historySSE: Evolution of target function [SSE between measurements and back-calc defl. basins]
%
%release candidate v2022-05-01
%% CALCULATE DEFLECTIONS
TSD_points = TSD_points(:);
TSD_meas = TSD_meas(:);
yDomain = 0;

%% LAUNCH BACK-CALCULATION!
tryoutE = 25e9; %Young modulus for concrete [N/m2]
tryoutk = 100; %defaults in PCI
tryoutk = tryoutk./3.684.*1e6; %same, now in N/m3 [thanks, Google]

if subgrade_type == 0
    tryoutG = 0; %G = 0 for Winkler foundation, otherwise, it's Pasternak foundation.
else
    tryoutG = 6e8;
end
number_of_tryouts = length(tryoutE);
max_iter = 2000;

if verbose
    fprintf('		 Launching back-calculation for k, E, G 
')
end

gradDescentLR = 0.1;

historyE = zeros(max_iter+1,1,number_of_tryouts);
historyk = zeros(max_iter+1,1,number_of_tryouts);
historyG = zeros(max_iter+1,1,number_of_tryouts);

historyGrad = zeros(max_iter,4,number_of_tryouts);
historyHess = zeros(max_iter,3,number_of_tryouts);
historySSE = zeros(max_iter+1,1,number_of_tryouts);

for zz = 1:number_of_tryouts
    % initialize variables
    E = tryoutE(zz);
    k = tryoutk(zz);
    G = tryoutG(zz);
launch the iterative loop that would optimize the values of h,E,g,k (continuous slab ahead of the joint)

stop = 0;
iter = 0;
SSE_old = sum(TSD_meas.^2);

historyE(1,1,zz) = E;
historyk(1,1,zz) = k;
historyG(1,1,zz) = G;
historySSE(1) = SSE_old;

while (~stop && iter <max_iter)
    iter = iter + 1;
    %stage 1: compute simulated deflection velocity
    W_prima =
        getDeflectionVelocity_continuousSlab(TSD_points,yDomain,TSDvx,h,E,nu,k,G,Load,pressure);
    W_prima = W_prima(:);
    %stage 3 - compare iterated deflection slope versus the TSD record
    %use an SSE-based method
    auxSSE = (1e3.*TSD_meas - 1e3.*W_prima).^2;
    SSE = sum(auxSSE);
    if iter/100 == floor(iter/100) && verboseness
        fprintf(' iteration number %g
',iter)
        fprintf(' E, k, G values are: %g, %g, %g
',E,k,G)
        fprintf(' SSE value %g
',SSE)
    end

    %stage 4 - use the results from stage 4 to improve the prediction
    if iter >1
        SSE_old = historySSE(iter);
    end

    if (SSE/SSE_old<1.0001 && SSE/SSE_old>0.9999)
        %satisfactory result, the SSE stagnated! - force the loop to stop
        stop = 1;
    end


else
  % do the iterative step
  %Careful!~ here TSD_meas is by default in m/sec (don't need to
  %convert units!)
  [gradJ,Hjj] = 
  computeGradient_ContinuousVy(TSD_points,yDomain,TSD_meas,h,nu,E,G,k,Load,pressure,TSDv 
  x);
  gradJ = gradJ(:);
  Hjj = Hjj(:,);
  historyGrad(iter,1:3,zz) = gradJ';
  historyGrad(iter,4,zz) = sqrt(gradJ'*gradJ);
  historyHess(iter,:,zz) = Hjj;

  %stability update V2021-12-10: Take the ABS. VALUE of the Hjj 
  %entries to prevent the gradient from ascending - case your target 
  function is non-convex [such as SSE when far off, the target, it unfolds to 
  concavity].
  %See Zhang et al., 2021, Pg. 459
  Hjj = abs(Hjj);

  %Update k, E, G based on the descent.
  k = k - gradDescentLR.*(1./Hjj(1)).*gradJ(1);
  E = E - gradDescentLR.*(1./Hjj(2)).*gradJ(2);

  %stabilty update V2021-08-23. If G = 0 since the beginning, don't update g 
  cause it causes a crash!
  %[I can control this by checking on subgrade_type]
  if subgrade_type == 0
    G = 0;
  else
    G = G - gradDescentLR.*(1./Hjj(3))*gradJ(3);
  end

  %stability update v2021-12-14
  %if either k or E go to the negative realm, force them back
  if E<0 || E>e1e12
    E = e1e10; %if needed, reset E to 1x10^10N/m2 [10 GPa]
  end
  if k<0
    k = 0.033.*e1e9; %if needed, reset k to 0.033 MPa/mm [-140 PCI]
  end
historyk(iter+1,1,zz) = k;
historyE(iter+1,1,zz) = E;
historyG(iter+1,1,zz) = G;
historySSE(iter+1,1,zz) = SSE;

% stop = 0
end %end iterative descent step
end %end while.
end %end number_of_tryouts.

% chop outcomes, remove the zero values of historyk, E, G, SSE Grad
historyk = historyk(historyk>0,:);
historyE = historyE(historyE>0,:);
if subgrade_type ~= 0
    historyG = historyG(historyG>0,:);
else
    historyG = historyG(historyE>0,:);
end
historySSE = historySSE(historySSE>0,:);
% historyGrad = historyGrad(historyGrad(:,end)>0,:);
% historyHess = historyHess(historyH(:,end)>0,:);

backCalc_Joint_bruteForce

function [c,LTE,finalSSE,domainLTE,domainC,domainSSE] = backCalc_Joint_bruteForce_0420(TSD_points,TSDvy,TSDvx,estimateC,h,k,G,E,nu,LOAD,pressure,verbose_solve)

% function [c,LTE,finalSSE,domainLTE,domainC,domainSSE] = backCalc_Joint_bruteForce_0420(TSD_points,TSDvy,TSDvx,estimateC,h,k,G,E,nu,LOAD,pressure,verbose_solve)

% Back-calculatlon of the joint's LTE index and exact location from nearby
% TSD deflection vertical velocity measurements

% INPUT:
% TSD_points - location of the TSD sensors where the defl. slope values are taken [m]
% TSDvy - vertical deflection velocity measurements at the joint's vicinity [m/sec]
% TSDvx - TSD traveling speed at the time of measurement [m/sec]
% h - concrete slab thickness [m]
%% Use Van Cauwelaert's 2004 formulation for the jointed slab deflection problem

%% preparation

% define the TSD points to get the TSD deflection slopes.
TSD_points = TSD_points(:,);
TSDvy = TSDvy(:,);
yDomain = 0;

%% Brute-force back-calculation. Use the [domainC,domainLTE] to compute SSE
% and the gradient for each back-calculation (because it's so bugging that
% it never converges and climbs the gradient...

% update v2022-0309 -> use the shrank domainC as given by estimateC
domainC = estimateC;
domainLTE = 0.06:0.01:1;
domainSSE = zeros(length(domainC),length(domainLTE));
if verbose_solve
    disp('computing domain for SSE')
    fprintf('	 joint location is bounded between %g and %g \\
    n', max(estimateC), min(estimateC))
end

%% launch brute-force search
% Implementation v2022-04-21
for i = 1:length(domainC)
    if verbose_solve
        fprintf('	 progress %g percent 
',100.*i/length(domainC))
    end
    for j = 1:length(domainLTE)
        % simulate vy for this pair of c, LTE
        simulatedVY = getDeflectionVelocity_singleProfile(TSD_points,yDomain,TSDvx,h,E,nu,k,G,LOAD,pressure,domainC(i),domainLTE(j));
        simulatedVY = simulatedVY(:); %output in m/sec

        %SSE term [compute for TSDvy and simulated VY in mm/sec]
        auxSSE = (1e3.*TSDvy - 1e3.*simulatedVY).^2;
        domainSSE(i,j) = sum(auxSSE);
    end
end

% Locate the minimum of SSE and report its location
[argminByRow,argminByCol] = find(domainSSE == min(min(domainSSE)));
c = domainC(argminByRow);
LTE = domainLTE(argminByCol);
finalSSE = min(min(domainSSE));

% ComputedGradient_ContinuousVy
function [gradJ,HessianJ] = computeGradient_ContinuousVy(xDomain,yDomain,TSD_meas,h,nu,E,G,k,LOAD,pressure,vx)
%function [gradJ,HessianJ] = computeGradient_ContinuousVy(xDomain,yDomain,TSD_meas,h,nu,E,G,k,LOAD,pressure,vx)
% %Auxiliary function to the back-calculation tool.
% %Compute the gradient of the cost function for the deflection velocity gradient descent.
% %THIS FUNCTION COMPUTES THE GRADIENT FOR THE CONTINUOUS SLAB ONLY (AHEAD OF
% %THE JOINT, WHERE I'M OPTIMIZING FOR E,k,G).
% %The cost function to minimize is SSE - [TSD_meas - vy(k,E,G)]^2
% %PROCEED NUMERICALLY, using centered finite differences:
%Basically, Compute the cost function for the given case +/-1% increase
%to each variable one at a time to get the partial derivatives over
%the variables of interest.

% INPUT
% xDomain: calculation domain, longitudinal direction [m]
% yDomain: calculation domain, transverse direction [m]
% TSD_meas: Measured deflection velocity at (xDomain, yDomain) [m/sec]
% h: slab thickness [m]
% E: slab elastic modulus [N/m2]
% nu: slab Poisson coefficient [dimless]
% k: subgrade's modulus of reaction [N/m3]
% G: subgrade's shear modulus [N/m2]. For Winkler foundation, G =0
% LOAD: amount of load [newtons]
% pressure: load pressure [N/m2]
% vx: TSD travel speed [scalar, m/sec]

% OUTPUT:
% gradJ = (approximated) gradient of the cost function for the 6
% variables, at their current values
% gradJ[1]: partialJ/partial_k [subgrade's modulus]
% gradJ[2]: partialJ/partial_E [Concrete Modulus]
% gradJ[3]: partialJ/partial_g [dim-less quantity related to Pasternak's G - Van C. chap 15]
% HessianJ[1] = partial2J/partialk2
% HessianJ[3] = partial2J/partialg2

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% CODE BEGINS
% pass TSD_meas to mm/sec (because my SSE is all throughout based on vy in
% mm/sec]
TSD_meas = TSD_meas.*1e3;

%Initialize output
gradJ = zeros(3,1);
HessianJ = zeros(3,1);
%% compute base case [needed for HJJ]

%1) compute vy_base
vy_base = getDeflectionVelocity_continuousSlab(xDomain,yDomain,vx,h,E,nu,k,G,LOAD,pressure);
vy_base = vy_base.*1e3; %pass from m/sec to mm/sec

%2) compute SSE base
SSE_base = vy_base - TSD_meas;
SSE_base = sum(SSE_base.^2);

%% compute the cost for the ALTERED k scenario
deltaK = 0.01.*k;

%1) Compute vy_plus
vy_plus = getDeflectionVelocity_continuousSlab(xDomain,yDomain,vx,h,E,nu,k+deltaK,G,LOAD,pressure);
vy_plus = vy_plus.*1e3; %pass from m/sec to mm/sec
%2) compute SSE_plus
SSE_plus = vy_plus - TSD_meas;
SSE_plus = sum(SSE_plus.^2);

%3) compute vy_minus
vy_minus = getDeflectionVelocity_continuousSlab(xDomain,yDomain,vx,h,E,nu,k-deltaK,G,LOAD,pressure);
vy_minus = vy_minus.*1e3; %pass from m/sec to mm/sec
%4) compute SSE_minus
SSE_minus = vy_minus - TSD_meas;
SSE_minus = sum(SSE_minus.^2);

%5) compute gradJ(1) and HessianJ(1)
gradJ(1) = (SSE_plus - SSE_minus)./(2.*deltaK);
HessianJ(1) = (SSE_plus - 2.*SSE_base + SSE_minus)./(deltaK.^2);

%% compute the cost for the ALTERED E scenario
deltaE = 0.01.*E;

%1) Compute vy_plus
vy_plus = getDeflectionVelocity_continuousSlab(xDomain,yDomain,vx,h,E+deltaE,nu,k,G,LOAD,pressure);
vy_plus = vy_plus.*1e3;   %pass from m/sec to mm/sec

%2) compute SSE_plus
SSE_plus = vy_plus - TSD_meas;
SSE_plus = sum(SSE_plus.^2);

%3) compute vy_minus
vy_minus = getDeflectionVelocity_continuousSlab(xDomain,yDomain,vx,h,E-deltaE,nu,k,G,LOAD,pressure);
vy_minus = vy_minus.*1e3;   %pass from m/sec to mm/sec

%4) compute SSE_minus
SSE_minus = vy_minus - TSD_meas;
SSE_minus = sum(SSE_minus.^2);

%5) compute gradJ(1) and HessianJ(1)
gradJ(2) = (SSE_plus - SSE_minus)./(2.*deltaE);
HessianJ(2) = (SSE_plus - 2.*SSE_base + SSE_minus)./(deltaE.^2);

%% compute the cost for the ALTERED G scenario
%BUG FIX 2021-06-15: If WINKLER FOUNDATION (G = 0), bypass this operation
%and return a zero, cos IT WOULD OTHERWISE MAKE DIVISION BY ZERO!

if G>0
deltaG = 0.01.*G;

   %1) Compute vy_plus
   vy_plus = getDeflectionVelocity_continuousSlab(xDomain,yDomain,vx,h,E,nu,k,G+deltaG,LOAD,pressure);
   vy_plus = vy_plus.*1e3;   %pass from m/sec to mm/sec
   
   %2) compute SSE_plus
   SSE_plus = vy_plus - TSD_meas;
   SSE_plus = sum(SSE_plus.^2);

   %3) compute vy_minus
   vy_minus = getDeflectionVelocity_continuousSlab(xDomain,yDomain,vx,h,E,nu,k,G-deltaG,LOAD,pressure);
   vy_minus = vy_minus.*1e3;   %pass from m/sec to mm/sec
   
   %4) compute SSE_minus

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SSE_minus = vy_minus - TSD_meas;
SSE_minus = sum(SSE_minus.^2);

%5) compute gradJ(1) and HessianJ(1)
gradJ(3) = (SSE_plus - SSE_minus)./(2.*deltaG);
HessianJ(3) = (SSE_plus - 2.*SSE_base + SSE_minus)./(deltaG.^2);

else
  gradJ(3) = 0;
  HessianJ(3) = 0;
end
end %<--- endfunction

computeGradient_JointedVy

function [gradJ,HessianJ] = computeGradient_JointedVy(xDomain,yDomain,TSD_meas,h,nu,E,G,k,c,LTE,LOAD,pressure,vx)
%function [gradJ,HessianJ] = computeGradient_JointedVy(xDomain,yDomain,TSD_meas,h,nu,E,G,k,c,LTE,LOAD,pressure,vx)
%Auxiliary function to the back-calculation tool.
%Compute the gradient of the cost function for the deflection velocity gradient
descent.
%THIS FUNCTION COMPUTES THE GRADIENT FOR THE JOINTED SLAB, WHERE I'M %BACK-CALCUALTING
FOR C, LTE.
%The cost function to minimize is SSE - \[TSD\_meas - vy(c,LTE|k,E,G)\]^2
%
%PROCEED NUMERICALLY, using centered finite differences:
%Basically, Compute the cost function for the given case +/-1% increase
to each variable one at a time to get the partial derivatives over
%the variables of interest.
%
%INPUT
% xDomain: calculation domain, longitudinal direction [m]
% yDomain: calculation domain, transverse direction [m]
% TSD\_meas: Measured deflection velocity at (xDomain, yDOmain) [m/sec]
% h: slab thickness [m]
% E: slab elastic modulus [N/m2]
% nu: slab Poisson coefficient [dimless]
% k: subgrade's modulus of reaction [N/m3]
% G: subgrade's shear modulus [N/m2]. For Winkler foundation, G =0

301
% c: distance between load center and joint [m]
% LTE: load transfer efficiency index at the joint (deflection-based).
% LOAD: amount of load [newtons]
% pressure: load pressure [N/m2]
% vx: TSD travel speed [scalar, m/sec]
%
%OUTPUT:
% gradJ = (approximated) gradient of the cost function for the 6
% variables, at their current values
% gradJ[1]: partialJ/partial_c
% gradJ[2]: partialJ/partial_LTE
%
% HessianJ[1] = partial2J/partial_c2
%
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%% CODE BEGINS
% pass TSD_meas to mm/sec (because my SSE is all throughout based on vy in
% mm/sec]
TSD_meas = TSD_meas.*1e3;

%Initialize output
gradJ = zeros(2,1);
HessianJ = zeros(2,1);

%% compute base case [needed for HJJ]

1) compute vy_base
vy_base =
getDeflectionVelocity_singleProfile(xDomain,yDomain,vx,h,E,nu,k,G,LOAD,pressure,c,LTE);
vy_base = vy_base.*1e3; %pass from m/sec to mm/sec

2) compute SSE base
SSE_base = vy_base - TSD_meas;
SSE_base = sum(SSE_base.^2);

%% compute the cost for the ALTERED c scenario
\[ \delta C = 0.01 \cdot c; \]

%1) Compute \( vy_+) \)
\[ vy_+ = \text{getDeflectionVelocity_singleProfile}(xDomain, yDomain, vx, h, E, nu, k, G, LOAD, pressure, c + \delta C, LTE); \]
\[ vy_+ = vy_+ \cdot 1e3; \quad \text{\% pass from m/sec to mm/sec} \]
%2) Compute \( SSE_+ \)
\[ SSE_+ = vy_+ - TSD\_meas; \]
\[ SSE_+ = \text{sum}(SSE_+ \cdot 2); \]

%3) Compute \( vy_- \)
\[ vy_- = \text{getDeflectionVelocity_singleProfile}(xDomain, yDomain, vx, h, E, nu, k, G, LOAD, pressure, c - \delta C, LTE); \]
\[ vy_- = vy_- \cdot 1e3; \quad \text{\% pass from m/sec to mm/sec} \]
%4) Compute \( SSE_- \)
\[ SSE_- = vy_- - TSD\_meas; \]
\[ SSE_- = \text{sum}(SSE_- \cdot 2); \]

%5) Compute \( \text{gradJ}(1) \) and \( \text{HessianJ}(1) \)
\[ \text{gradJ}(1) = (SSE_+ - SSE_-) / (2 \cdot \delta C); \]
\[ \text{HessianJ}(1) = (SSE_+ - 2 \cdot SSE\_base + SSE_-) / (\delta C \cdot 2); \]

% compute the cost for the ALTERED LTE scenario
\[ \delta \text{LTE} = 0.01 \cdot \text{LTE}; \]

%1) Compute \( vy_+ \)
\[ vy_+ = \text{getDeflectionVelocity_singleProfile}(xDomain, yDomain, vx, h, E, nu, k, G, LOAD, pressure, c, \text{LTE} + \delta \text{LTE}); \]
\[ vy_+ = vy_+ \cdot 1e3; \quad \text{\% pass from m/sec to mm/sec} \]
%2) Compute \( SSE_+ \)
\[ SSE_+ = vy_+ - TSD\_meas; \]
\[ SSE_+ = \text{sum}(SSE_+ \cdot 2); \]

%3) Compute \( vy_- \)
\[ vy_- = \text{getDeflectionVelocity_singleProfile}(xDomain, yDomain, vx, h, E, nu, k, G, LOAD, pressure, c, \text{LTE} - \delta \text{LTE}); \]
\[ vy_- = vy_- \cdot 1e3; \quad \text{\% pass from m/sec to mm/sec} \]
%4) Compute \( SSE_- \)
SSE_minus = vy_minus - TSD_meas;
SSE_minus = sum(SSE_minus.^2);

%5) compute gradJ(1) and HessianJ(1)
gradJ(2) = (SSE_plus - SSE_minus)./(2.*deltaLTE);
HessianJ(2) = (SSE_plus - 2.*SSE_base + SSE_minus)./(deltaLTE.^2);
end %<--- endfunction

computeContinuousSlabDeflectionBasin

function w =
computeContinuousSlabDeflectionBasin(xDomain,yDomain,h,E,nu,k,G,LOAD,pressure)
% function w =
computeContinuousSlabDeflectionBasin(xDomain,yDomain,h,E,nu,k,G,LOAD,pressure)
%Front-end function to solve the deflection basin of a continuous slab as per %Van Cauwelaert (2004).
%INPUT
%    xDomain:    caculation domain, longitudinal direction [m]
%    yDomain:    calculation domain, transverse direction [m]
%    h:          slab thickness [m]
%    E:          slab elastic modulus [N/m2]
%    nu:         slab Poisson coefficient [dimless]
%    k:          subgrade's modulus of reaction [N/m3]
%    G:          subgrade's shear modulus [N/m2]. For Winkler foundation, G =0
%    LOAD:       amount of load [newtons]
%    pressure:   load pressure [N/m2]
%OUTPUT
%    w:          deflection basin for each xDomain,yDomain position [m]
%
%release candidate V2022-05-01

% preparation - calculate loaded area. Assume default 2b = 0.47m
b = 0.47;  %typical width of a 1/2 axle (dual tire) [m]
a = LOAD./(b.*pressure);  %length of distributed load zone [m]
b = b/2;
a = a/2;

% compute Van C. parameters
LL = ((E.*h.^3)./(12.*(1-nu^2).*k))^.25;
DD = (E.*h.^3)./(12.*(1-nu^2));
\[ gg = \frac{(G \cdot LL)^2}{2 \cdot DD}; \]

%% dummy variable for \( w(x,y) \) integration
sDomain = 0:0.2:3;
sDomain(1) = 1e-4;

%% This is the normal case (infinite slab, no boundary conditions)
w = compute_w(xDomain,yDomain,sDomain,pressure,a,b,k,gg,LL);

\begin{verbatim}
computeJointedSlabDeflectionBasin

function w =
computeJointedSlabDeflectionBasin(xDomain,yDomain,h,E,nu,k,G,LOAD,pressure,c,LTE)

% Front-end function to solve the deflection basin of a jointed slab as per
% Van Cauwelaert (2004).
%
% INPUT
%   xDomain:    calculation domain, longitudinal direction [m]
%   yDomain:    calculation domain, transverse direction [m]
%   h:          slab thickness [m]
%   E:          slab elastic modulus [N/m^2]
%   nu:         slab Poisson coefficient [dimless]
%   k:          subgrade's modulus of reaction [N/m^3]
%   G:          subgrade's shear modulus [N/m^2]. For Winkler foundation, G =0
%   LOAD:       amount of load [newtons]
%   pressure:   load pressure [N/m^2]
%   c:          distance between the joint and the load center [m]
%   LTE:        joint's load transfer efficiency index [dimless]
%OUTPUT
%   w:          deflection basin for each xDomain,yDomain position [m]
%
%release candidate V2022-05-01

% preparation - calculate loaded area. Assume default 2b = 0.47m
b = 0.47;  %typical width of a 1/2 axle (dual tire) [m]
a = LOAD./(b.*pressure);  %length of distributed load zone [m]
\end{verbatim}
b = b/2;
a = a/2;

%% compute Van C. parameters
LL = ((E.*h.^3)./(12.*(1-nu^2).*k))^0.25;
DD = (E.*h.^3)./(12.*(1-nu^2));
Wx = (G.*LL.^2)./(2*DD);

%% dummy variable for w(x,y) integration
sDomain = 0:0.2:3;
sDomain(1) = 1e-4;

ABCDverboseness = 0;%
w = zeros(length(xDomain),length(yDomain));

%% solve deflection basin
%Apply update v 2022-03-19: Superposition principle
%Check here if the load is stepping on both sides of the joint or not.
%If so, divide the problem into two.

checkPlus = c>0 && abs(c)<a; % load on the approach slab but touching the joint
checkMinus = c<0 && abs(c)<a; % load on the leave slab but touching the joint

if checkPlus
    % Load on the approach slab and invading the joint
    % Divide the problem into two chunks of load touching the joint.
    % problem 1: portion of the load on the approach slab
    a1 = (a + abs(c))./2;
c1 = a1;
[AS,BS,CS,DS] = solveABCD_experimental(yDomain,sDomain,pressure,a1,b,k,gg,LL,nu,LTE,c1,ABCDverboseness);
W1 = zeros(length(xDomain),length(yDomain));
x1 = find(xDomain<=c1);
x2 = find(xDomain>c1);
[Wa,Wb] = compute_wawb(xDomain(x1),yDomain,sDomain,pressure,b,k,gg,LL,AS,BS);
[Wc,Wd] = compute_wcwd(xDomain(x2),yDomain,sDomain,pressure,b,k,gg,LL,CS,DS);
Wx1 = compute_w(xDomain(x1),yDomain,sDomain,pressure,a1,b,k,gg,LL);

W1(x1,:) = Wx1 + Wa + Wb; %<<<--- These deflections are in METERS!!!
\[ W_1(x_2,:) = W_c + W_d; \quad \%\langle\langle\langle\text{ Van C. (2004) formulation.}\rangle\rangle\rangle \]

\% problem 2: portion of the load on the leave slab
\[ a_2 = \frac{(a - \text{abs}(c))}{2}; \]
\[ c_2 = -a_2; \] % made negative on purpose to highlight that the second chunk of load is a 'reversed' problem [it's on the leave slab]
\% but c2 must pass as a positive value to the ABCD solver!
\[ [\text{AS, BS, CS, DS}] = \text{solveABCD}\_\text{experimental}(y\text{Domain, sDomain, pressure, a2, b, k, gg, LL, nu, LTE, -}} \]
\[ c_2, \text{ABCDverboseness}); \]
\[ W_2 = \text{zeros(length(xDomain), length(yDomain))}; \]
\[ t = -x\text{Domain}; \]
\[ x_1 = \text{find}(t<=-c_2); \]
\[ x_2 = \text{find}(t>-c_2); \]
\[ [\text{Wa, Wb}] = \text{compute_wawb}(t(x_1), y\text{Domain, sDomain, pressure, b, k, gg, LL, AS, BS}); \]
\[ [W_c, W_d] = \text{compute_wcwd}(t(x_2), y\text{Domain, sDomain, pressure, b, k, gg, LL, CS, DS}); \]
\[ Wx_1 = \text{compute_w}(\text{xDomain(x1), yDomain, sDomain, pressure, a2, b, k, gg, LL}); \]
\[ W_2(x_1,:) = Wx_1 + Wa + Wb; \quad \%\langle\langle\langle\text{ these deflections are in METERS!!} \rangle\rangle\rangle \]
\[ W_2(x_2,:) = W_c + W_d; \quad \%\langle\langle\langle\text{ Van C. (2004) formulation.}\rangle\rangle\rangle \]

\% sum both problems to get the final defl. basin [function output]
\[ w = W_1 + W_2; \]

elseif checkMinus
\% Load on the leave slab and invading the joint
\% Divide the problem into two chunks of load touching the joint.
\% problem 1: portion of the load on the approach slab
\[ a_1 = \frac{(a - \text{abs}(c))}{2}; \]
\[ c_1 = a_1; \]
\[ [\text{AS, BS, CS, DS}] = \text{solveABCD}\_\text{experimental}(y\text{Domain, sDomain, pressure, a1, b, k, gg, LL, nu, LTE, c1, ABCDverboseness}); \]
\[ W_1 = \text{zeros(length(xDomain), length(yDomain))}; \]
\[ x_1 = \text{find}(\text{xDomain}\leqslant c_1); \]
\[ x_2 = \text{find}(\text{xDomain}>c_1); \]
\[ [\text{Wa, Wb}] = \text{compute_wawb}(\text{xDomain(x1), yDomain, sDomain, pressure, b, k, gg, LL, AS, BS}); \]
\[ [W_c, W_d] = \text{compute_wcwd}(\text{xDomain(x2), yDomain, sDomain, pressure, b, k, gg, LL, CS, DS}); \]
\[ Wx_1 = \text{compute_w}(\text{xDomain(x1), yDomain, sDomain, pressure, a1, b, k, gg, LL}); \]
\[ W_1(x_1,:) = Wx_1 + Wa + Wb; \quad \%\langle\langle\langle\text{ These deflections are in METERS!!} \rangle\rangle\rangle \]

%% problem 2: portion of the load on the leave slab
a2 = (a + abs(c))./2;
c2 = -a2; %made negative on purpose to highlight that the second chunk of load is a 'reversed' problem [it's on the leave slab]. %But c2 must pass as a positive value to the ABCD solver!
[AS,BS,CS,DS] = solveABCD_experimental(yDomain,sDomain,pressure,a2,b,k,gg,LL,nu,LTE,-c2,ABCDverboseness);
W2 = zeros(length(xDomain),length(yDomain));
t = -xDomain;
x1 = find(t<=-c2);
x2 = find(t>-c2);
[Wa,Wb] = compute_wawb(t(x1),yDomain,sDomain,pressure,b,k,gg,LL,AS,BS);
[Wc,Wd] = compute_wcwd(t(x2),yDomain,sDomain,pressure,b,k,gg,LL,CS,DS);
Wx1 = compute_w(xDomain(x1),yDomain,sDomain,pressure,a2,b,k,gg,LL);
W2(x1,:) = Wx1 + Wa + Wb; %<<<--- these deflections are in METERS!!!

%sum both problems to get the final defl. basin [function output]
w = W1 + W2;
else
  % This is the normal case (load away from the joint), approach slab.
  if c >=0
    [AS,BS,CS,DS] = solveABCD_experimental(yDomain,sDomain,pressure,a,b,k,gg,LL,nu,LTE,c,ABCDverboseness);
    %stage 2: compute deflection
    %
    W = zeros(length(xDomain),length(yDomain));
    x1 = find(xDomain<=c);
    x2 = find(xDomain>c);
    [Wa,Wb] = compute_wawb(xDomain(x1),yDomain,sDomain,pressure,b,k,gg,LL,AS,BS);
    [Wc,Wd] = compute_wcwd(xDomain(x2),yDomain,sDomain,pressure,b,k,gg,LL,CS,DS);
    Wx1 = compute_w(xDomain(x1),yDomain,sDomain,pressure,a,b,k,gg,LL);
    w(x1,:) = Wx1 + Wa + Wb; %<<<--- these deflections are in METERS!!!
    w(x2,:) = Wc + Wd; %<<<--- Van C. (2004) formulation.
  %
  w = W;
else

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This is the normal case (load away from the joint), leave slab.

REVERTED PROBLEM: Revert the boundary conditions

\[ \{AS, BS, CS, DS\} = \text{solveABCD\_experimental}(yDomain, sDomain, pressure, a, b, k, gg, LL, nu, LTE, -c, ABCD\_verboseness); \]

%stage 2: compute deflection

% AFTER THE JOINT IT'S THE REVERTED PROBLEM, define auxiliary variable \( t = -x \)
% \( t = -xDomain; \)
% \( x1 = \text{find}(t<-c); \)
% \( x2 = \text{find}(t>-c); \)
%
% \[ \{Wa, Wb\} = \text{compute\_wawb}(t(x1), yDomain, sDomain, pressure, b, k, gg, LL, AS, BS); \]
% \[ \{Wc, WD\} = \text{compute\_wcwd}(t(x2), yDomain, sDomain, pressure, b, k, gg, LL, CS, DS); \]
% \( Wx1 = \text{compute\_w}(xDomain(x1), yDomain, sDomain, pressure, a, b, k, gg, LL); \)
%
% \( w(x1,:) = Wx1 + Wa + Wb; \%<<--- these deflections are in METERS!!! \)
% \( w(x2,:) = Wc + WD; \%<<--- \text{Van C. (2004) formulation.} \)

end

getDeflectionVelocity\_continuousSlab

function \( [vy] = \)
getDeflectionVelocity\_continuousSlab(xDomain, yDomain, vx, h, E, nu, k, G, LOAD, pressure)\)
% solve the vertical component of deflection velocity for a single set of TSD measurements for the continuous (infinite) slab case.
% In this case, \( vy = vx \cdot \text{Slope!} \)
%
% INPUT
% \( xDomain: \) calculation domain, longitudinal direction [m]
% \( yDomain: \) calculation domain, transverse direction [m]
% \( h: \) slab thickness [m]
% \( E: \) slab elastic modulus [N/m2]
% \( nu: \) slab Poisson coefficient [dimless]
% \( k: \) subgrade's modulus of reaction [N/m3]
% \( G: \) subgrade's shear modulus [N/m2]. For Winkler foundation, \( G = 0 \)
% \( LOAD: \) amount of load [newtons]

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% pressure: load pressure [N/m2]
% vx: TSD travel speed [scalar, m/sec]
%OUTPUT
% vy = matrix with the vertical velocity component [m/sec]
%at the moment of survey for each location xDomain,yDomain
%
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%

deltaX = 0.01;
deltaT = deltaX./vx;

%initialize vy
vy = zeros(length(xDomain),length(yDomain));

%Solve vy as a finite difference centered at the measurement point.
%Assume that the load is approaching the joint as time passes. Thus the
%xDomain relative to the load reduces by an amount deltaX for the "time
%plus" measurement and increases by deltaX for the "time minus"
%measurement.
%Also, the distance c changes by +/- deltaX as well (because the load got
%closer to the joint!)

% call auxiliar function to solve the deflection basins
wTplus = computeContinuousSlabDeflectionBasin(xDomain-
deltaX,yDomain,h,E,nu,k,G,LOAD,pressure);
wTminus =
computeContinuousSlabDeflectionBasin(xDomain+deltaX,yDomain,h,E,nu,k,G,LOAD,pressure);

%solve vy as centered difference - hence the 2*deltaT denominator!
vy = (wTplus-wTminus)./(2.*deltaT);
end

getDeflectionVelocity_singleProfile

function [vy] =
getDeflectionVelocity_singleProfile(xDomain,yDomain,vx,h,E,nu,k,G,LOAD,pressure,c,LTE)
%function [vy] =
getDeflectionVelocity_singleProfile(xDomain,yDomain,vx,h,E,nu,k,G,LOAD,pressure,c,LTE)
%solve the vertical component of deflection velocity for a single set of TSD measurements
% %INPUT
%  xDomain: calculation domain, longitudinal direction [m]
%  yDomain: calculation domain, transverse direction [m]
%  h: slab thickness [m]
%  E: slab elastic modulus [N/m2]
%  nu: slab Poisson coefficient [dimless]
%  k: subgrade's modulus of reaction [N/m3]
%  G: subgrade's shear modulus [N/m2]. For Winkler foundation, G = 0
%  LOAD: amount of load [newtons]
%  pressure: load pressure [N/m2]
%  c: distance between the joint and the load center [m]
%  LTE: joint's load transfer efficiency index [dimless]
%  vx: TSD travel speed [scalar, m/sec]
%OUTPUT
%  vy = matrix with the vertical velocity component [m/sec]
% at the moment of survey for each location xDomain,yDomain
%
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%

deltaX = 0.01;
deltaT = deltaX./vx;

%initialize vy
vy = zeros(length(xDomain),length(yDomain));

%Solve vy as a finite difference centered at the measurement point.
%Assume that the load is approaching the joint as time passes. Thus the %xDomain
relative to the load reduces by an amount deltaX for the "time plus" measurement and
increases by deltaX for the "time minus"  measurement.
%Also, the distance c changes by +/- deltaX as well (because the load got %closer to
the joint!)

% call auxiliar function to solve the deflection basins
wTplus = computeJointedSlabDeflectionBasin(xDomain-
deltaX,yDomain,h,E,nu,k,G,LOAD,pressure,c-deltaX,LTE);
wTminus =
computeJointedSlabDeflectionBasin(xDomain+deltaX,yDomain,h,E,nu,k,G,LOAD,pressure,c+de
ltaX,LTE);
vy = (wTplus-wTminus)./(2.*deltaT);

end

compute_w

function w = compute_w(x,y,s,p,a,b,k,g,l)
%function w = compute_w(x,y,s,p,a,b,k,g,l)
%Compute the infinite-beam term of the deflection solution. Following Van %Cauwelaert, 2004, chap. 15

% % INPUTS:
% % x, y: coordinates where to evaluate the term [vector]
% % s: wave term (wave domain) used in the Fourier decomposition of the load
% % p: pressure of the applied load (Newton/m2)
% % a, b: dimensions [half length - half width] of load [m]
% % k: coefficient of subgrade reaction [N/m3]
% % g: adim parameter related to subgrade’s G
% % l: adim parameter related to slab’s E and h
% %
% % OUTPUTS:
% % w(x,y): infinite-slab term of the deflection function
% % INTERIM SLOW IMPLEMENTATION. ITERATIVELY SOLVE FOR SINGLE LOCATIONS X(i),Y(j)%
% %
% %release candidate v2022-05-01

w = zeros(length(x),length(y));
%stability check -in case the wave domain has a 0, remove it (prevent a division by zero)
if s(1) == 0
    s(1) = eps;
end
%
% if g > 1
% case g > 1 - Equations 15.13 and 15.14
COEF = p / (2*pi*k).*((g^2-1)).^-0.5;
%auxiliary coefficients z1, z2 (s)
z1 = s.^2 + g + sqrt(g^2-1);
z1 = sqrt(z1);
z2 = s.^2 + g - sqrt(g^2-1);
\[ z_2 = \sqrt{z_2}; \]
\[ \text{for } j = 1: \text{length}(y) \]
\[ \text{the term } w_3 \text{ of the integral only depends on } y \text{ and } s \]
\[ w_3 = \cos(s \cdot y(j) \cdot l) \cdot \sin(s \cdot b \cdot l); \]
\[ w_3 = w_3 / s; \]
\[ w_3 = w_3(:); \]
\[ \text{for } i = 1: \text{length}(x) \]
\[ x_i = \text{abs}(x(i)); \]
\[ \text{if } x_i \leq a \]
\[ \text{case } \text{abs}(x) \leq a \] [within the wheel print]
\[ \text{compute auxiliary terms } w_1, w_2(x, s) \]
\[ w_1 = (z_2^{-2}) \cdot (2 - \exp(-z_2 / l \cdot (a - x_i)) - \exp(-z_2 / l \cdot (a + x_i))); \]
\[ w_2 = (z_1^{-2}) \cdot (2 - \exp(-z_1 / l \cdot (a - x_i)) - \exp(-z_1 / l \cdot (a + x_i))); \]
\[ \text{and then calculate the term within the integral} \]
\[ w_{\text{int}} = (w_1 - w_2) \cdot w_3; \]
\[ \text{else} \]
\[ \text{case } \text{abs}(x) > a \] [outside the wheel print]
\[ \text{compute auxiliary terms } w_1, w_2(x, s) \]
\[ w_1 = (z_2^{-2}) \cdot (\exp(-z_2 / l \cdot (x_i - a)) - \exp(-z_2 / l \cdot (x_i + a))); \]
\[ w_2 = (z_1^{-2}) \cdot (\exp(-z_1 / l \cdot (x_i - a)) - \exp(-z_1 / l \cdot (x_i + a))); \]
\[ \text{and then calculate the term within the integral} \]
\[ w_{\text{int}} = (w_1 - w_2) \cdot w_3; \]
\[ \text{end} \]
\[ \text{now } i \text{ must integrate the } w_{\text{int}} \text{ over } S \text{ - sum all terms using the built-in numerical integration function "trapz"} \]
\[ w(i, j) = (\text{COEF}) \cdot \text{trapz}(s, w_{\text{int}}); \]
\[ \text{end} \]
\[ \text{end}\% \text{ end double for loop} \]

\[ \text{elseif } g < 1 \]
\[ \text{select case } g < 1 \text{ - Equations 15.15 - 15.16} \]
\[ \text{COEF} = p / (\pi k); \]
\[ \text{auxiliary coefficients } \alpha, \beta (s) \]
\[ \alpha = 0.5 \cdot (\sqrt{(s^2 + g)^2 + 1 - g^2}) + (s^2 + g)); \]
\[ \alpha = \sqrt{\alpha}; \]
\[ \beta = 0.5 \cdot (\sqrt{(s^2 + g)^2 + 1 - g^2}) - (s^2 + g)); \]
\[ \beta = \sqrt{\beta}; \]
\[ \text{for } j = 1: \text{length}(y) \]
% the term w3 of the integral only depends on y and s
% compute w3 (y(j), s)
w3 = cos(s.*y(j)./l).*sin(s.*b./l);
w3 = w3./(s.*(s.^4+2*g.*s.^2+1));
% w3 = w3(:);
for i = 1:length(x)
    xi = abs(x(i));
    if xi <= a
        % case abs(x)<a
        % compute auxiliary terms w1,w2(x,s)
        w1 = (1-g.^2).^-.5.*exp(-1.*alpha./l.*(a-xi));
        w1 = w1.*((sqrt(1-g.^2)).*(cos((a-xi).*beta./l)) + (s.^2+g).*((a-xi).*beta./l)));
        w2 = (1-g.^2).^-.5.*exp(-1.*alpha./l.*(a+xi));
        w2 = w2.*((sqrt(1-g.^2)).*(cos((a+xi).*beta./l)) + (s.^2+g).*((a+xi).*beta./l)))
        % and then calculate the term within the integral
        w_int = (2-w1-w2).*w3;
    else
        % case abs(x)>a
        % compute auxiliary terms w1,w2(x,s)
        w1 = exp(-1.*alpha./l.*(x-a));
        w1 = w1.*((sqrt(1-g.^2)).*(cos((x-a).*beta./l)) + (s.^2+g).*((x-a).*beta./l)));
        w2 = exp(-1.*alpha./l.*(x+a));
        w2 = w2.*((sqrt(1-g.^2)).*(cos((x+a).*beta./l)) + (s.^2+g).*((x+a).*beta./l)));
        % and then calculate the term within the integral
        w_int = (w1-w2).*w3./sqrt(1-g^2);
    end
% now i must integrate the w_int over S - sum all terms!
% w(i,j) = (COEF).*sum(w_int).*deltaS;
% -- improved using the built-in numerical integration function "trapz"
% w(i,j) = (COEF).*trapz(s,w_int);
end
end% end double for loop
else
% case g == 1 - equation 15.17-18
COEF = p / (2*pi*k);
z = (s.^2 + 1).^0.5;

for j = 1:length(y)
    %the term w3 of the integral only depends on y and s
    %compute w3 (y,`s)
    w3 = cos(s.*y(j)./l).*sin(s.*b./l);
    w3 = w3./((s.*(s.^2+1)^2));
    w3 = w3(:);
    for i = 1:length(x)
        xi = abs(x(i));
        if xi <=a
            %case x<a
            %compute auxiliary terms w1,w2(x,s)
            w1 = exp(-z./l.*(a-xi));
            w1 = w1.*(2 + sqrt(1+s.^2).*(a-xi)./l);
            w2 = exp(-z./l.*(a+xi));
            w2 = w2.*(2 + sqrt(1+s.^2).*(a+xi)./l);
            %and then calculate the term within the integral
            w_int = (4-w1-w2).*w3;
        else
            %case x>a
            %compute auxiliary terms w1,w2(x,s)
            w1 = exp(-z./l.*(xi-a));
            w1 = w1.*(2 + sqrt(1+s.^2).*(xi-a)./l);
            w2 = exp(-z./l.*(xi+a));
            w2 = w2.*(2 + sqrt(1+s.^2).*(xi+a)./l);
            %and then calculate the term within the integral
            w_int = (w1-w2).*w3;
        end
        %now i must integrate the w_int over S - sum all terms!
        w(i,j) = COEF.*sum(w_int).*deltaS;
        %<--improved using the built-in numerical integration function "trapz"
        w(i,j) = (COEF).*trapz(s,w_int);
    end
end% end double for loop
end

end %endfunction
compute_w_term

function w = compute_w_term(x, y, s, p, a, b, k, g, l)
%function w = compute_w_term(x, y, s, p, a, b, k, g, l)
%
% Compute the infinite-beam term of the deflection solution.
% Following Van Cauwelaert, 2004, chap. 15
%
% INPUTS:
% x, y: coordinates where to evaluate the term [vector]
% s: wave term (wave domain) used in the Fourier decomposition of the load
% p: distributed load (Newton/m)
% a, b: dimensions [length-width] of load [m]
% k: coefficient of subgrade reaction
% g: adim parameter related to subgrade's G
% l: adim parameter related to slab's E and h
%
% OUTPUTS:
% w(x, y): infinite-slab term of the deflection function
% compute auxiliary terms alpha and beta - function of wave number s
%
% INTERIM SLOW IMPLEMENTATION. ITERATIVELY SOLVE FOR SINGLE LOCATIONS X, Y
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%

w = zeros(length(x), length(y));
% x1 = find(x<=a);
% x2 = find(x>a);

if g > 1
% case g > 1 - Equations 15.13 and 15.14
    COEF = p / (2*pi*k) .* (g^2 - 1)^(-0.5);
    % auxiliary coefficients z1, z2 (s)
    z1 = s.^2 + g + sqrt(g^2 - 1);
    z1 = sqrt(z1);
    z2 = s.^2 + g - sqrt(g^2 - 1);
    z2 = sqrt(z2);
    for j = 1:length(y)
% the term \( w_3 \) of the integral only depends on \( y \) and \( s \)
\[
w_3 = \cos(s \cdot y(j)/l) \cdot \sin(s \cdot b/l);
\]
\[
w_3 = w_3/s;
\]
\[
w_3 = w3(:);
\]
for \( i = 1:length(x) \)
  \[
  \begin{align*}
  &\text{if } x(i) <= a \\
  &\quad \text{case } x < a \\
  &\quad \text{compute auxiliary terms } w_1, w_2(x, s) \\
  &w_1 = z2^{-2} \cdot (2 - \exp(-z2/l \cdot (a-x(i)))-\exp(-z2/l \cdot (a+x(i)))) \\
  &w_2 = z1^{-2} \cdot (2 - \exp(-z1/l \cdot (a-x(i)))-\exp(-z1/l \cdot (a+x(i)))) \\
  &w_1 = w1(:); \quad w_2 = w2(:); \quad \text{and then calculate the term within the integral} \\
  &w_{\text{int}} = (w_1-w_2) \cdot w_3 \\
  \end{align*}
\]
  \[
  \begin{align*}
  &\text{else} \\
  &\quad \text{case } x > a \\
  &\quad \text{compute auxiliary terms } w_1, w_2(x, s) \\
  &w_1 = z2^{-2} \cdot (\exp(-z2/l \cdot (x(i)-a)))-\exp(-z2/l \cdot (x(i)+a))) \\
  &w_2 = z1^{-2} \cdot (\exp(-z1/l \cdot (x(i)-a)))-\exp(-z1/l \cdot (x(i)+a))) \\
  &w_1 = w1(:); \quad w_2 = w2(:); \quad \text{and then calculate the term within the integral} \\
  &w_{\text{int}} = (w_1-w_2) \cdot w_3 \\
  \end{align*}
\]
\[
\end{cases}
\]
% the term I must pass is \( w_{\text{int}} \)
\[
w(i,j) = COEF \cdot (w_{\text{int}});
\]
end
e nd% end double for loop

elseif \( g < 1 \)
  \[
  \begin{align*}
  &\text{select case } g < 1 - \text{ Equations 15.15 - 15.16} \\
  &\text{COEF} = p/(pi*k); \\
  &\text{auxiliary coefficients } \alpha, \beta(s) \\
  &\alpha = 0.5 \cdot (\sqrt{(s^2+g)^2 + 1-g^2} + (s^2 + g)) \\
  &\alpha = \sqrt{\alpha}; \\
  &\beta = 0.5 \cdot (\sqrt{(s^2+g)^2 + 1-g^2} - (s^2 + g)) \\
  &\beta = \sqrt{\beta}; \\
  \end{align*}
\]
for \( j = 1:length(y) \)

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% the term w3 of the integral only depends on y and s
% compute w3 (y, s)

w3 = cos(s.*y(j)./l).*sin(s.*b./l);

w3 = w3./(s.*(s.^4+2.*g.*s.^2+1));

for i = 1:length(x)
    if x(i) <= a
        % case x < a
        % compute auxiliary terms w1, w2(x, s)
        % BUG FOUND V 2022-02-23 -> THERE'S A PARENTHESES MISSING IN EXPRESSION FOR W1. It has been patched up in compute_w.m. but not in here. Dang!
        w1 = w1(:); w2 = w2(:); % and then calculate the term within the integral
        w_int = (2-w1-w2).*w3;
    else
        % case x > a
        % compute auxiliary terms w1, w2(x, s)
        % BUG FOUND V 2022-02-23 -> THERE'S A PARENTHESES MISSING IN EXPRESSION FOR W1. It has been patched up in compute_w.m. but not in here. Dang!
        w1 = w1(:); w2 = w2(:); % and then calculate the term within the integral
        w_int = (w1-w2).*w3;
    end
end
%the term I must pass is w_int
w(i,j) = COEF*(w_int);
end
end% end double for loop

else
%case g == 1 - equation 15.17-18
COEF = p /(2*pi*k);
z = (s.^2 + 1).^0.5;
for j = 1:length(y)
%the term w3 of the integral only depends on y and s
%compute w3 (y,s)
w3 = cos(s.*y(j)./l).*sin(s.*b./l);
w3 = w3./((s.^2+1).^2));
w3 = w3(:);
for i = 1:length(x)
if x(i) <=a
%case x<a
%compute auxiliary terms w1,w2(x,s)
w1 = exp(-z./l.*(a-x(i))); w1 = w1.*(2 + sqrt(1+s.^2).*(a-x(i))./l);
w2 = exp(-z./l.*(a+x(i))); w2 = w2.*(2 + sqrt(1+s.^2).*(a+x(i))./l);
w1 = w1(:);
w2 = w2(:);
%and then calculate the term within the integral
w_int = (4-w1-w2).*w3;
else
%case x>a
%compute auxiliary terms w1,w2(x,s)
w1 = exp(-z./l.*(x(i)-a));
w1 = w1.*(2 + sqrt(1+s.^2).*(x(i)-a)./l);
w2 = exp(-z./l.*(x(i)+a));
w2 = w2.*(2 + sqrt(1+s.^2).*(x(i)+a)./l);
w1 = w1(:);
w2 = w2(:);
%and then calculate the term within the integral
w_int = (w1-w2).*w3;
end
end
%the term I must pass is w_int
w(i,j) = COEF*(w_int);
end
end% end double for loop
end

compute_wawb

function [wa,wb] = compute_wawb(x,y,s,p,b,k,g,l,A,B)
%function [wa,wb] = compute_wawb(x,y,s,p,b,k,g,l,A,B)
%Compute the loaded-finite-slab term of the deflection solution. Following Van
%Cauwelaert, 2004, chap. 15
%
%INPUTS:
%x,y: coordinates where to evaluate the term [vector]
%s: wave domain for the fourier transform
%p: applied pressure (N/m2]
%k: modulus of subgrade reaction [N/m3]
%g: adim parameter related to subgrade's G
%l: adim parameter related to slab's E and h
%A,B: terms for the boundary condition, as solved for each value of s with
solveABCD [sizeS x sizeY]
%
%OUTPUTS:
%wa, wb(x,y): homogeneous-equation term of the deflection function for
%the loaded slab
%
%INTERIM SLOW IMPLEMENTATION. ITERatively SOLVE FOR SINGLE LOCATIONS X,Y
%
%release candidate V2022-05-01
%
%
wa = zeros(length(x),length(y));
wb = zeros(length(x),length(y));
s = s(:);
%start iterative loop for all points in x,y domain.
for j = 1:length(y)
w3 = (1./s).*cos(s.*y(j)./l).*sin(s.*b./l));
w3 = w3(:); %force w3 to be a single column like A, B are (Octave makes w3 a row and fails to do element-by-element products afterward, crashing the entire thing!)

for i = 1:length(x)
    xi = abs(x(i));
    xi = x(i);
    if g > 1
        %case g > 1 - Equations 15.21. Parameters z1, z2(s) are the same as for w(xy)
        COEF = p / (2*pi*k).*(g.^2-1).^(-0.5);
        z1 = s.^2 + g + sqrt(g^2-1);
        z1 = sqrt(z1);
        z2 = s.^2 + g - sqrt(g^2-1);
        z2 = sqrt(z2);
        waAux = A(:,j).*w3.*exp(z1.*xi./l);
        wbAux = B(:,j).*w3.*exp(z2.*xi./l);
        %update with Matlab's built-in trapz function to integrate
        wa(i,j) = COEF.*trapz(s,waAux);
        wb(i,j) = COEF.*trapz(s,wbAux);
    elseif g < 1
        %select case g < 1 - Equations 15.24
        COEF = p./((pi.*k).*(1-g^2)^(-0.5));
        alpha = 0.5.*(sqrt((s.^2+g).^2 +1-g^2)+(s.^2 + g));
        alpha = sqrt(alpha);
        beta = 0.5.*(sqrt((s.^2+g).^2 +1-g^2)-(s.^2 + g));
        beta = sqrt(beta);
        waAux = A(:,j).*w3.*(cos(beta./l.*xi)).*exp(alpha.*xi./l);
        wbAux = B(:,j).*w3.*(sin(beta./l.*xi)).*exp(alpha.*xi./l);
        %update with Matlab's built-in trapz function to integrate
        wa(i,j) = COEF.*trapz(s,waAux);
        wb(i,j) = COEF.*trapz(s,wbAux);
    else
        %case g == 1 - equation 15.23
        COEF = p / (2*pi*k);
        z = (s.^2 + 1).^0.5;
        waAux = A(:,j).*w3.*exp(z.*xi/1);
        wbAux = B(:,j).*w3.*xi./l.*exp(z.*xi/1);
        %update with Matlab's built-in trapz function to integrate
        wa(i,j) = COEF.*trapz(s,waAux);
        wb(i,j) = COEF.*trapz(s,wbAux);
    end
compute_wawb_term

function [wa,wb] = compute_wawb_term(x,y,s,p,b,k,g,l)

%function [wa,wb] = compute_wawb_term(x,y,s,p,b,k,g,l)
%
%Auxiliary function for the solveABCD function. Compute the term of the WaWb terms
%for a single value of the wave number s.
%Following Van Cauwelaert, 2004, chap. 15
%
%INPUTS:
% x,y: coordinates where to evaluate the term [vector]
% s: wave domain for the fourier transform
% p: applied pressure [N/m2]
% k: modulus of subgrade reaction [N/m3]
% g: adim parameter related to subgrade's G
% l: adim parameter related to slab's E and h
%
%OUTPUTS:
% wa, wb(x,y,s): homogeneous-equation term of the deflection function for
% the loaded slab

%INTERIM SLOW IMPLEMENTATION. ITERATIVELY SOLVE FOR SINGLE LOCATIONS X,Y
%
%release candidate V2022-05-01
%
wa = zeros(length(x),length(y));
wb = zeros(length(x),length(y));
s = s(:);

for j = 1:length(y)
    w3 = (1./s).*(cos(s.*y(j)./l).*sin(s.*b./l));
    w3 = w3(:);
    for i = 1:length(x)
        %start case for wa(x,y), wb (x,y)
if g > 1
    %case g > 1 - Equations 15.21. Parameters z1, z2(s) are the same as for w(xy)
    COEF = p / (2*pi*k).*(g.^2-1).^0.5;
    z1 = s.^2 + g + sqrt(g^2-1);
    z1 = sqrt(z1);
    z2 = s.^2 + g - sqrt(g^2-1);
    z2 = sqrt(z2);
    wa(i,j) = COEF.*w3.*exp(z1.*x(i)./l);
    wb(i,j) = COEF.*w3.*exp(z2.*x(i)./l);
elseif g < 1
    COEF = p / (pi*k).*(1-g^2)^-0.5;
    %auxiliary coefficients alpha, beta (s)
    alpha = 0.5*(sqrt((s.^2+g).^2 +1-g^2)+(s.^2 + g));
    alpha = sqrt(alpha);
    beta  = 0.5*(sqrt((s.^2+g).^2 +1-g^2)-(s.^2 + g));
    beta  = sqrt(beta);
    wa(i,j) = COEF.*w3.*(cos(beta/l.*x(i))).*exp(alpha.*x(i)/l);
    wb(i,j) = COEF.*w3.*(sin(beta/l.*x(i))).*exp(alpha.*x(i)/l);
else
    COEF = p / (2*pi*k);
    z = (s.^2 + 1).^0.5;
    wa(i,j) = COEF.*w3.*exp(z.*x(i)/l);
    wb(i,j) = COEF.*w3.*x(i)./l.*exp(z.*x(i)/l);
end
end
end
end
end

compute_wcwd

defunction [wc,wd] = compute_wcwd(x,y,s,p,b,k,g,l,C,D)
%Compute the unloaded-finite-beam term of the deflection solution. Following Van Cauwelaert, 2004, chap. 15
%INPUTS:
% x,y: coordinates where to evaluate the term [vector]
% s: wave domain for the fourier transform
% p: applied pressure (N/m²]
% k: modulus of subgrade reaction [N/m³]
% g: adim parameter related to subgrade's G
% l: adim parameter related to slab's E and h
% C,D: terms for the boundary condition, as solved for each value of s
% with solveABCD [sizeS x sizeY]

%OUTPUTS:
% wc, wd(x,y): homogeneous-equation term of the deflection function for the unloaded slab

%INTERIM SLOW IMPLEMENTATION. ITERATIVELY SOLVE FOR SINGLE LOCATIONS X,Y
%release candidate V2022-05-01

wc = zeros(length(x),length(y));
w = zeros(length(x),length(y));
s = s(:);

%start iterative loop for all points in x,y domain.
  for i = 1:length(y)
    w3 = (1./s).*cos(s.*y(j)/l).*sin(s.*b./l);
    w3 = w3(:);
    for i = 1:length(x)
      xi = abs(x(i));
      xi = x(i);
      if g > 1
        %case g > 1 - Equations 15.21. Parameters z1, z2(s) are the same as for w(xy)
        COEF = p / (2*pi*k).* (g.^2-1).^-0.5;
        z1 = s.^2 + g + sqrt(g^2-1);
        z1 = sqrt(z1);
z2 = s.^2 + g - sqrt(g^2-1);
z2 = sqrt(z2);
wcAux = C(:,j).*w3.*exp(-z1.*xi./l);
wdAux = D(:,j).*w3.*exp(-z2.*xi./l);
%update with Matlab's built-in trapz function to integrate
wc(i,j) = COEF.*trapz(s,wcAux);
wd(i,j) = COEF.*trapz(s,wdAux);

elseif g < 1
%select case g < 1 - Equations 15.24
alpha = 0.5*(sqrt((s.^2+g).^2 +1-g^2)+(s.^2 + g));
alpha = sqrt(alpha);
beta  = 0.5*(sqrt((s.^2+g).^2 +1-g^2)-(s.^2 + g));
beta  = sqrt(beta);
COEF = p /(pi*k)*(1-g^2)^-0.5;
wcAux = C(:,j).*w3.*(cos(beta/l.*xi)).*exp(-alpha.*xi/l);
wdAux = D(:,j).*w3.*(sin(beta/l.*xi)).*exp(-alpha.*xi/l);
%update with Matlab's built-in trapz function to integrate
wc(i,j) = COEF.*trapz(s,wcAux);
wd(i,j) = COEF.*trapz(s,wdAux);

else
%case g == 1 - equation 15.23
COEF = p /(2*pi*k);
z = (s.^2 + 1).^0.5;
wcAux = C(:,j).*w3.*exp(-z.*xi/l);
wdAux = D(:,j).*w3.*x(i)./l.*exp(-z.*xi/l);
%update with Matlab's built-in trapz function to integrate
wc(i,j) = COEF.*trapz(s,wcAux);
wd(i,j) = COEF.*trapz(s,wdAux);
end
end
end
end %endfunction

compute_wcwd_term

function [wc,wd] = compute_wcwd_term(x,y,s,p,b,k,g,l)
%function [wc,wd] = compute_wcwd_term(x,y,s,p,b,k,g,l)
% Axiliary function for the solveABCD function. Compute the term of the WcWd terms
% for a single value of the wave number s.
% Following Van Cauwelaert, 2004, chap. 15
%
% INPUTS:
% x,y: coordinates where to evaluate the term [vector]
% s: wave domain for the fourier transform
% p: applied pressure [N/m2]
% k: modulus of subgrade reaction [N/m3]
% g: adim parameter related to subgrade's G
% l: adim parameter related to slab's E and h
%
% OUTPUTS:
% wc, wd(x,y): homogeneous-equation term of the deflection function for
% the unloaded slab
%
% release candidate V2022-05-01

%%
wc = zeros(length(x),length(y));
wd = zeros(length(x),length(y));
%%
for j = 1:length(y)
    w3 = (1./s).*(cos(s.*y(j)./l).*sin(s.*b./l));
    w3 = w3(:);
    for i = 1:length(x)
        if g >1
            % case g > 1 - Equations 15.21. Parameters z1, z2(s) are the same as for w(xy)
            COEF = p /(2*pi*k).*(g.^2-1).^(-0.5);
            z1 = s.^2 + g + sqrt(g^2-1);
            z1 = sqrt(z1);
            z2 = s.^2 + g - sqrt(g^2-1);
            z2 = sqrt(z2);
            wc(i,j) = COEF.*w3.*exp(-z1.*x(i)./l);
            wd(i,j) = COEF.*w3.*exp(-z2.*x(i)./l);
        end
    end
end

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elseif g < 1
    COEF = p / (pi*k)*(1-g^2)^-0.5;
%auxiliary coefficients alpha, beta (s)
alpha = 0.5*(sqrt((s.^2+g).^2 +1-g^2)+(s.^2 + g));
alpha = sqrt(alpha);
beta = 0.5*(sqrt((s.^2+g).^2 +1-g^2)-(s.^2 + g));
beta = sqrt(beta);
w(i,j) = COEF.*w3.*(cos(beta/l.*x(i))).*exp(-alpha.*x(i)/l);
w(i,j) = COEF.*w3.*(sin(beta/l.*x(i))).*exp(-alpha.*x(i)/l);

else
    COEF = p / (2*pi*k);
z = (s.^2 + 1).^0.5;
w(i,j) = COEF.*w3.*exp(-z.*x(i)/l);
w(i,j) = COEF.*w3.*x(i)/l.*exp(-z.*x(i)/l);
end
end
end

%endfunction

solve_ABCD

function [A,B,C,D] = solveABCD_experimental(yDomain,sDomain,p,a,b,k,g,l,nu,delta,c,verboseness)
%function [A,B,C,D] = solveABCD_experimental(yDomain,sDomain,p,a,b,k,g,l,nu,delta,c,verboseness)
%
%Auxiliary function that solves the boundary conditions for the slab with a joint problem

%IMPLEMENTATION WITH SLAB THEORY - VAN CAUW. CHAPTER 15
%
%release candidate V2022-05-01

% PREPARATION OF THE SYSTEM OF EQUATIONS - SHOULD BE 4 X 4 x length of the S domain
%State it as M X = N
%Prepare the matrices M and N by row - each row represents one equation
sss = length(sDomain);
M = zeros(4,4,sss);
N = zeros(4,1,sss);

A = zeros(sss,length(yDomain));
B = zeros(sss,length(yDomain));
C = zeros(sss,length(yDomain));
D = zeros(sss,length(yDomain));

cPlus = 1.01.*c;
cMinus = 0.99.*c;
c2plus = 1.02.*c;
c2minus = 0.98.*c;

deltaX = cPlus - c;
if length(yDomain) > 1
    deltaY = 0.3.*(yDomain(2)-yDomain(1));
yDomainPlus = yDomain + deltaY;
yDomainMinus = yDomain - deltaY;
else
    deltaY = 0.01;
yDomainPlus = yDomain + deltaY;
yDomainMinus = yDomain - deltaY;
end

for j = 1:length(yDomain)
    if verboseness
        fprintf(' SolveABCD: Computing boundary conditions %g of %g 

',j,length(yDomain))
    end
    for i = 1:sss
        s = sDomain(i);
% I may need these quantities for equation 1
        wX = compute_w_term(c,yDomain(j),s,p,a,b,k,g,l);
        [waX,wbX] = compute_wawb_term(c,yDomain(j),s,p,b,k,g,l);
        [wcX,wdX] = compute_wcwd_term(c,yDomain(j),s,p,b,k,g,l);

        %% Prepare equation 1 - LTE (eq. 15.35)
        M(1,:,i) = [delta*waX delta*wbX -wcX -wdX];
        N(1,:,i) = -delta*wX;
    end
end
% I may need these guys for equations 2-4 (note: must calculate over all % yDomain so that I can derive afterwards)

wXplus = compute_w_term(cPlus, yDomain(j), s, p, a, b, k, g, l);
[waXplus, wbXplus] = compute_wawb_term(cPlus, yDomain(j), s, p, b, k, g, l);
[wcXplus, wdXplus] = compute_wcwd_term(cPlus, yDomain(j), s, p, b, k, g, l);

wXminus = compute_w_term(cMinus, yDomain(j), s, p, a, b, k, g, l);
[waXminus, wbXminus] = compute_wawb_term(cMinus, yDomain(j), s, p, b, k, g, l);
[wcXminus, wdXminus] = compute_wcwd_term(cMinus, yDomain(j), s, p, b, k, g, l);

wYplus = compute_w_term(c, yDomainPlus(j), s, p, a, b, k, g, l);
[waYplus, wbYplus] = compute_wawb_term(c, yDomainPlus(j), s, p, b, k, g, l);
[wcYplus, wdYplus] = compute_wcwd_term(c, yDomainPlus(j), s, p, b, k, g, l);

wYminus = compute_w_term(c, yDomainMinus(j), s, p, a, b, k, g, l);
[waYminus, wbYminus] = compute_wawb_term(c, yDomainMinus(j), s, p, b, k, g, l);
[wcYminus, wdYminus] = compute_wcwd_term(c, yDomainMinus(j), s, p, b, k, g, l);

% these guys I'd need for the 3rd derivative over x

wX2plus = compute_w_term(c2plus, yDomain(j), s, p, a, b, k, g, l);
[waX2plus, wbX2plus] = compute_wawb_term(c2plus, yDomain(j), s, p, b, k, g, l);
[wcX2plus, wdX2plus] = compute_wcwd_term(c2plus, yDomain(j), s, p, b, k, g, l);

wX2minus = compute_w_term(c2minus, yDomain(j), s, p, a, b, k, g, l);
[waX2minus, wbX2minus] = compute_wawb_term(c2minus, yDomain(j), s, p, b, k, g, l);
[wcX2minus, wdX2minus] = compute_wcwd_term(c2minus, yDomain(j), s, p, b, k, g, l);

% these guys I need for the d3w/dxdy2

wXplusYplus = compute_w_term(cPlus, yDomainPlus(j), s, p, a, b, k, g, l);
[waXplusYplus, wbXplusYplus] = compute_wawb_term(cPlus, yDomainPlus(j), s, p, b, k, g, l);
[wcXplusYplus, wdXplusYplus] = compute_wcwd_term(cPlus, yDomainPlus(j), s, p, b, k, g, l);

wXplusYminus = compute_w_term(cPlus, yDomainMinus(j), s, p, a, b, k, g, l);
[waXplusYminus, wbXplusYminus] = compute_wawb_term(cPlus, yDomainMinus(j), s, p, b, k, g, l);
[wcXplusYminus, wdXplusYminus] = compute_wcwd_term(cPlus, yDomainMinus(j), s, p, b, k, g, l);
wXminusYplus = compute_w_term(cMinus,yDomainPlus(j),s,p,a,b,k,g,l);
[waxXminusYplus,wbxXminusYplus] =
compute_wawb_term(cMinus,yDomainPlus(j),s,p,b,k,g,l);
[wcXminusYplus,wdXminusYplus] =
compute_wcwd_term(cMinus,yDomainPlus(j),s,p,b,k,g,l);

wXminusYminus = compute_w_term(cMinus,yDomainMinus(j),s,p,a,b,k,g,l);
[waxXminusYminus,wbxXminusYminus] =
compute_wawb_term(cMinus,yDomainMinus(j),s,p,b,k,g,l);
[wcXminusYminus,wdXminusYminus] =
compute_wcwd_term(cMinus,yDomainMinus(j),s,p,b,k,g,l);

%% derive w(x) three times

% update v 2022-03-12 : re-write the derivative as increment over plus - minus / 2deltax
% approximately directly to x == c
 dwdx = (wXplus - wXminus)/(2.*deltaX);  %%this is the derivative AT x == c AND
 ydomain(j)!
 %    dwdy = (wYplus - wYminus)/(2.*deltaY);  %%this is the derivative AT x == c AND
 ydomain(j)!

% update v2022-03-12 -> do a second-order-central derivative for d2w/dx2 and
d2w/dy2.
% Directly estimated at x == c and all yDomain
 d2wdx2 = (wXplus + wXminus - 2.*wX)/(deltaX.^2);  %%these results are valid for
 x == c AND ydomain(j)!
 d2wdy2 = (wYplus + wYminus - 2.*wX)/(deltaY.^2);

% update v2022-03-12: solve the third derivatives as single derivatives of the
 d2wdy2
 d2wdx2plus = (wX2plus - 2.*wXplus + wX)./(deltaX.^2);
 d2wdx2minus= (wX - 2.*wXminus + wX2minus)./(deltaX.^2);
 d2wdy2plus = (wXplusYplus - 2.*wXplus + wXplusYminus)./(deltaY.^2);
 d2wdy2minus= (wXminusYplus - 2.*wXminus + wXminusYminus)./(deltaY.^2);

 d3wdx3 = (d2wdx2plus - d2wdx2minus)./(2.*deltaX);
 d3wdx dy2=(d2wdy2plus - d2wdy2minus)./(2.*deltaX);

 %-----------------
 % same story for deriving wa(x) 3 times -> UPDATED V2022-03-12

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\[ dwadx = \frac{(waX_{plus} - waX_{minus})}{2 \cdot \Delta x}; \]
\[ % dwady = \frac{(waY_{plus} - waY_{minus})}{2 \cdot \Delta y}; \]
\[ d2wadx2 = \frac{(waX_{plus} + waX_{minus} - 2 \cdot waX)}{\Delta x^2}; \]
\[ d2wady2 = \frac{(waY_{plus} + waY_{minus} - 2 \cdot waY)}{\Delta y^2}; \]
\[ d2wAdx2plus = \frac{(waX_{2plus} - 2 \cdot waX_{plus} + waX)}{\Delta x^2}; \]
\[ d2wAdx2minus = \frac{(waX - 2 \cdot waX_{minus} + waX_{2minus})}{\Delta x^2}; \]
\[ d2wAdy2plus = \frac{(waX_{plus}Y_{plus} - 2 \cdot waX_{plus} + waX_{plus}Y_{minus})}{\Delta y^2}; \]
\[ d2wAdy2minus = \frac{(waX_{minus}Y_{plus} - 2 \cdot waX_{minus} + waX_{minus}Y_{minus})}{\Delta y^2}; \]
\[ d3wadx3 = \frac{(d2wAdx2plus - d2wAdx2minus)}{2 \cdot \Delta x}; \]
\[ d2wAdxdy2 = \frac{(d2wAdy2plus - d2wAdy2minus)}{2 \cdot \Delta x}; \]

%----------------
%same story for deriving \( wb(x) \) 3 times -> UPDATED V2022-03-12
\[ dwbdx = \frac{(wbX_{plus} - wbX_{minus})}{2 \cdot \Delta x}; \]
\[ % dwbdy = \frac{(wbY_{plus} - wbY_{minus})}{2 \cdot \Delta y}; \]
\[ d2wbdx2 = \frac{(wbX_{plus} + wbX_{minus} - 2 \cdot wbX)}{\Delta x^2}; \]
\[ d2wbddy2 = \frac{(wbY_{plus} + wbY_{minus} - 2 \cdot wbY)}{\Delta y^2}; \]
\[ d2wBdx2plus = \frac{(wbX_{2plus} - 2 \cdot wbX_{plus} + wbX)}{\Delta x^2}; \]
\[ d2wBdx2minus = \frac{(wbX - 2 \cdot wbX_{minus} + wbX_{2minus})}{\Delta x^2}; \]
\[ d2wBdy2plus = \frac{(wbX_{plus}Y_{plus} - 2 \cdot wbX_{plus} + wbX_{plus}Y_{minus})}{\Delta y^2}; \]
\[ d2wBdy2minus = \frac{(wbX_{minus}Y_{plus} - 2 \cdot wbX_{minus} + wbX_{minus}Y_{minus})}{\Delta y^2}; \]
\[ d3wbdx3 = \frac{(d2wbdx2plus - d2wbdx2minus)}{2 \cdot \Delta x}; \]
\[ d2wbAdxdy2 = \frac{(d2wbddy2plus - d2wbddy2minus)}{2 \cdot \Delta x}; \]

%----------------
%same story for deriving \( wc(x) \) 3 times -> UPDATED V2022-03-12
\[ dwcdx = \frac{(wcX_{plus} - wcX_{minus})}{2 \cdot \Delta x}; \]
\[ % dwcdy = \frac{(wcY_{plus} - wcY_{minus})}{2 \cdot \Delta y}; \]
\[ d2wcdx2 = \frac{(wcX_{plus} + wcX_{minus} - 2 \cdot wcX)}{\Delta x^2}; \]
\[ d2wcdy2 = \frac{(wcY_{plus} + wcY_{minus} - 2 \cdot wcY)}{\Delta y^2}; \]
\[ d2wCdx2plus = \frac{(wcX_{2plus} - 2 \cdot wcX_{plus} + wcX)}{\Delta x^2}; \]
\[
d2wCd2x2minus = \frac{wcX - 2 \cdot wcXminus + wcX2minus}{(deltaX)^2};
\]
\[
d2wCd2y2plus = \frac{wcXplusYplus - 2 \cdot wcXplus + wcXplusYminus}{(deltaY)^2};
\]
\[
d2wCd2y2minus = \frac{wcXminusYplus - 2 \cdot wcXminus + wcXminusYminus}{(deltaY)^2};
\]
\[
d3wcdx3 = \frac{d2wCd2x2plus - d2wCd2x2minus}{(2 \cdot deltaX)}; \\
d3wdxxy2 = \frac{d2wCy2plus - d2wCy2minus}{(2 \cdot deltaY)};
\]

%----------------
%same story for deriving \(w_d(x)\) 3 times -> UPDATED V2022-03-12
\\
\[
dwddx = \frac{wdXplus - wdXminus}{(2 \cdot deltaX)}; \\
w\dddy = \frac{wdYplus - wdYminus}{(2 \cdot deltaY)};
\]
\[
d2wDdx2plus = \frac{wdX2plus - 2 \cdot wdXplus + wdX}{(deltaX)^2}; \\
d2wDdx2minus = \frac{wdX - 2 \cdot wdXminus + wdX2minus}{(deltaX)^2}; \\
d2wDdy2plus = \frac{wdXplusYplus - 2 \cdot wdXplus + wdXplusYminus}{(deltaY)^2}; \\
d2wDdy2minus = \frac{wdXminusYplus - 2 \cdot wdXminus + wdXminusYminus}{(deltaY)^2};
\]
\[
d3wDdx3 = \frac{d2wDdx2plus - d2wDdx2minus}{(2 \cdot deltaX)}; \\
d3wDxdy2 = \frac{d2wDdy2plus - d2wDdy2minus}{(2 \cdot deltaX)};
\]

%----------------
%% Prepare equation 2 - bending moment = 0 on the loaded slab (eq. 13.45)
\[
M(2,:,i) = [d2wadx2 + nu \cdot d2wady2, d2wbdx2 + nu \cdot d2wbdy2, 0, 0]; \\
N(2,:,i) = -d2wdx2 - nu \cdot d2wdy2;
\]

%% Prepare equation 3 - bending moment = 0 on the unloaded slab (eq. 13.46)
\[
M(3,:,i) = [0, 0, d2wcdx2 + nu \cdot d2wcdy2, d2wdx2 + nu \cdot d2wddx2]; \\
N(3,:,i) = -d2wdx2C - nu \cdot d2wdy2C; \\
N(3,:,i) = 0;
\]

%% PREPARE equation 4 - Shear stress equation
%Update: Correct to follow formulation in Deep et al 20 (A, B)
\[
M(4,1,i) = d3wadx3 + (2-nu) \cdot d3wadxy2 - (2 \cdot g)/(l^2) \cdot dwadx; \\
M(4,2,i) = d3wbdx3 + (2-nu) \cdot d3wbdy2 - (2 \cdot g)/(l^2) \cdot dwbdx; \\
M(4,3,i) = -1 \cdot (d3wcdx3 + (2-nu) \cdot d3wcdxy2 - (2 \cdot g)/(l^2) \cdot dwcdx); \\
M(4,4,i) = -1 \cdot (d3wddx3 + (2-nu) \cdot d3wddxy2 - (2 \cdot g)/(l^2) \cdot dwddx);
\]
\[ N(4,:,i) = -1 \cdot (d3wdx3 + (2-\nu) \cdot d3wdx2y2 - (2*g)/(l^2) \cdot dwdx); \]

\% M(4,1,i) = d3wAdx3C + (2-\nu) \cdot d3wAdx2y2C - (2*g)/(l^2) \cdot wac;
\% M(4,2,i) = d3wBdx3C + (2-\nu) \cdot d3wBdx2y2C - (2*g)/(l^2) \cdot wbc;
\% M(4,3,i) = -1 \cdot (d3wCdx3C + (2-\nu) \cdot d3wCdx2y2C - (2*g)/(l^2) \cdot wcc);
\% M(4,4,i) = -1 \cdot (d3wDdx3C + (2-\nu) \cdot d3wDdx2y2C - (2*g)/(l^2) \cdot wdc);
\% N(4,:,i) = 0;

%% SOLVE FOR ABCD
MM = reshape(M(:,:,i),4,4);
NN = reshape(N(:,:,i),4,1);
xx = MM\NN; % this can be unstable...

A(i,j) = xx(1);
B(i,j) = xx(2);
C(i,j) = xx(3);
D(i,j) = xx(4);

end %end for iteration on s variable
end %end for iteration over j
%%that'd be it
end