Essays in Industrial Organization and Political Economy

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(ABSTRACT)
This dissertation comprises of three problems in the area of Political Economy and Industrial Organization. The first chapter concerns how ideologically-opposite media firms report a particular event to maximize their payoffs from advocating their ideology and strengthen reader trust which increases if the report is proximate to their beliefs. I use these facts to develop a Hotelling’s linear city model of competition where the two media firms choose their respective locations which signify the impression they want to impart to its readers. I find partisan media provides accurate information while covering topics favorable to its ideology. However, for unfavourable topics, the media never provides an indifferent report, but either defends its own ideology or delivers a partially accurate report. For unfavourable issues, imparting an indifferent impression rewards a media with lowest equilibrium payoffs. I identify sufficiency conditions where readers give better assessment to news of a media located farther away from their ideology than one which is nearer. Increasing competition by the entry of a third firm does not necessarily alleviate the level of bias in the news economy. The second paper studies the pricing schedule of a monopolist while it sells a non-durable product over two time periods. The consumer’s experience with the product is correlated with two possible states — good (bad) experience is more probable under a high (low) state. Given this, I study the monopolist’s pricing scheme in the two periods when consumers are wishful — overly optimistic about the high state even after a bad experience. I provide a comparative study of prices in each periods when the monopolist announces prices with and without commitment when consumers are either naive or sophisticated. The final chapter provides an understanding of the efficacy of two types of trade sanctions (import and export) using a directed network model. Sanctions are common punitive measures taken by a sender player to discipline a target player. Empirical evidences in the realm of international trade show differences in the effectiveness between import and export sanctions. This paper shows that such differences can be explained by one specific centrality feature of the underlying trading network — betweenness-centrality. This measure lends insights to the trade spillovers following sanctions underscoring why sanctions are ineffective. I highlight when a higher value of this centrality acts as a sufficient condition towards effective sanction. Based on this analysis, one can conclude whether import or export sanction will be more effective for a given trade network.
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(GENERAL AUDIENCE ABSTRACT)

Three essays spanning across topics of political economy and Industrial Organization has been studied. The first essay ‘Media bias in the best and worst of times’ studies how ideology-motivated (partisan) media firms try to create impressions to its audience about a particular issue to increase its payoffs from either of the two sources — reader trust and advocating its ideology. This trade-off depends on the type of issue at hand which either aggravates or moderates a media’s wish to generate bias in its news. I investigate not only the degree of bias for any given event, but also study how profits of media are impacted from doing so. The second chapter ‘Monopoly pricing under wishful thinking’ investigates the pricing strategies of a seller when he sells a non-durable product to a wishful buyer twice, over two time periods. Under two possible states of the world — high and low — the buyer can derives either a good or bad experience. It is assumed that a good experience is more likely than a bad one under high state. Would the buyer re-purchase the product after having a bad experience in the first period? A wishful buyer is overly optimistic about a good experience in the future even after a bad experience in the current period. Such optimism paves the way for pricing strategies in favor of the seller under certain conditions. My aim has been to highlight these conditions and draw comparison with a pricing model with non-wishful buyers. The third chapter investigates the effectiveness of trade sanctions. Such sanctions are imposed by a sender country against a target country when the latter has taken an action which the sender disapproves — initiating domestic war, building nuclear arsenals, etc. The sanctions are enforced until the target. However, only 30% sanctions are effective in disciplining the target. This paper studies if any feature of the trade network can explain why sanctions fail and what type of trade sanction — import or export — will be optimal in any given trade network.
Dedication

To ma, baba, notun pishi and pishemoshai.
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Mainstream media is a vital institution in any democracy which gathers and disseminates information from all spheres of social and political life to the public. In this process, they exercise great power in establishing public opinion, creating dissent, affecting electoral outcomes and increasing political involvement. Nevertheless, this position of power is not free from reader criticisms about a media’s selfish motives towards fulfilling partisan motivations. At the same time, the magnitude of criticism against this bias depends on reader demography and hence is not objective. This is fairly because a single piece of news report will be processed differently by a liberal and a conservative.

The past literature of Behavioral Economics and Communication Theory deem such differences to be the manifestations of subjective perceptions of the reader. For instance, a liberal complains about the media being too conservative while the conservative accuses media to carry liberal bias in their news. Existing works mentioned above posits that readers tend to adapt better to news which is closer to their beliefs. However, can there exist scenarios when this might not hold true? Or would a liberal or conservative media emerge as honest to a reader’s mind even after they provide honest reports? It appears that both outcomes occur under specific conditions and this weakens the result in the standard media market literature that greater competition leads to truthful reporting.

There are primary two channels through which news bias emanates. First, supply-side channel where the media outlet is itself biased (due to nature of ownership, preference of editors amongst other factors). The difference in new stories of Fox and CNN of Trump’s increased tariff policy against China illustrates this. The second channel is demand-sided where media firms cater to reader’s beliefs to receive greater reader acceptance of their news slant information. For example, news reports about the advancement of stem cell research

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1See [16], [18],[45]
2See [2, 8, 21]
3See [9] who explains this behavior though motivated reasoning. [51],[32],[27] provides experimental or empirical evidences of such biased perception of information about events spanning from conflicts, inflation and even stem cell research.
4See appendix for the real reports.
have fallen prey to this ideology divide since early 2000s to cater to the more conservative reader by sketching these research not as a scientific feat but a sacrilege of moral and ethical norms.

Interestingly, the strength of these supply and demand channels depend on the nature of issue to be reported and the reader demography. Certain issues are very ideologically charged, being more vulnerable to be biased, but only in presence of a favourable reader base. Construing a stem cell discovery on moral or religious grounds might be appealing to a more conservative reader base rather than a more liberal one. I examine how coverage of a particular issue will vary across a liberal and a conservative media while they compete against each other, given their respective degree of partisan interests and reader ideology. Taking account of these forces at play, this analysis shows whether a given issue initiates bias through the supply-side or demand-side channel and in what manner it would aggravate or temper competition and affect media profits.

Media news falls nothing short of a commercial product with so many factors shaping it up [26]. This imposes a greater impetus to study the industrial organizational aspect of the media market and its implication towards reader welfare as a considerable section of the population still rely on media firms to learn about world events. 6

I contribute to the existing literature in primarily two ways. The first relates to modelling competition for attention. Most models have interpreted attention in quantitative units through time where higher attention implying greater readership profits. However, gaining greater reader attention does not necessarily covert to better acceptance of news to a reader. For instance, a conservative media might have the attention of a liberal reader, but it is not certain whether he would approve its merit above a liberal media. In this respect, my model accounts for a reader’s reaction to factual information 7. When factual information runs contrary to the ideology of a reader, it initiates a tradeoff between accuracy and desirability in the reader’s mind (See [9]) 8. Thereby he might prefer to learn more

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5 See [27]

6 The survey of [48] shows 82% of the participants believed that media must be the foremost news provider. In addition, 75% strongly assert media to take the role of watchdogs on public officials to curb their intentions to abuse power.

7 ([47],[32]) studies how factual reports related to inflation and unemployment are unevenly processed by Republican and Democratic readers. [5] coins the phrase ‘subjective ideological disagreement to describe how Supreme Court’s policymaking if liberal-oriented can lose its legitimacy to the more conservative populace and vice versa. [42] uses survey data prior to 1974 election showing that reader perception of politicians is linked with their partisanship and their evaluation of news media.

8 [29] refers this discrepancy as ‘media dissociation’, which further affects future internet use, engagement in political discussions and formation of public opinion.
about it from a like-minded news source rather than a media with opposite ideology. This relative preference for news stories across different media outlets is denoted by a subjective weight which a reader assigns to these stories and is endogenously determined in the model. This weight later allows us to design a policy measure which shows conditions when factual information can gain more acceptance from readers. In other words, the tradeoff between accuracy and desirability can be tempered through appropriate policies. Secondly, this model throws light on how any given issue can impact the demand and supply channels of news bias. This exercise alternatively discusses which channel will lead to greater media profits with the advent of a particular issue.

The reader does not choose between news sources, but provides a ranking of its preferred news source by choosing the optimal weight to each news stories. This reflects an economy where readers are receptive of dissenting views on topics like war against terror. This model helps in understanding how such reception of dissent varies with ideology of readers and identifies scenarios where readers do prefer news from media with opposite ideological orientations.

The present model begins with an honest information source which provides the reader populace with a factual report about a topic. Examples of such sources include Supreme court, Bureau of Labor Statistics (BLS), Associated Press, Reuters who are known to ‘.. represent the essence of objective news coverage, as they self-consciously avoid politically based editorial judgments in their news content”, [6]. Following this, two ideologically opposite media firms (left and right) media compete on a spatial ideology spectrum to inform the readers further about the topic. Consider for instance that the readers learn the last-month unemployment rate is at 5-percent. How can partisan media bias this report? Under leftist presidency, the left media can report how it has decreased from a higher rate while under a rightist presidency, it can report how it has increased from a previous rate. The right media will also take analogous action. The actions of both media consists of choosing a point on the linear ideology spectrum whose mid-point is a point of neutral ideology while both ends resembling the extreme left and right ideologies. Readers are heterogeneous and their location on this spectrum denote their closeness to either ideology. My results hinge on two basic assumptions in this model. First, readers are ideological and read news after learning a factual report which creates a point of reference while assessing news. Second, media cares about its own ideology and maximising reader assessment. Given this, the present model leads to some interesting insights. Readers read news from both media but might not accept the news equally. This acceptance is governed by their own ideology and the prior knowledge

\[9\text{Evidences about reader’s mistrust on news media is provided by [22] where news consumers feel media to be biased or produce news which counters their ideology.}\]

\[10\text{This example is taken from [25].}\]
of the event. For instance a left reader might discredit news of media $R$ simply based on ideology differences.

First, competition among rival media firms mitigate or exacerbate the level of bias depending on the relative weights given towards ideology and reader assessment. Second, for particular topics and certain parameter values, not biasing news leads to lower profits. Third, if media tries to defend its ideology by countering the factual report, then its profits will dwindle. When the factual report stands contrary to the media’s ideology, then media will refrain from taking an indifferent stance - it will either defend its ideology, by disassociating its ideology from the event or it reports become consistent with the factual report. These patterns are dictated by the weight it assigns its two motives- ideology and reader-assessment. Fourth, a novel measure of reader-satisfaction is provided and under specific conditions, readers can gain relatively more utility from news which is farther away from their beliefs than one which lies closer. Fifth, media receives greater leeway to bias news in its favor when the audience is more unsophisticated, who are less educated and tolerates bias. However, the impact on bias from higher reader unsophistication gathers force when both the media firms are more focused towards ideology gains than gains from reader-assessment. Sixth, welfare is not necessarily enhanced in presence of media firms which care more for reader-assessment. Welfare is dependent on the number of readers in the economy and how they are spread across the ideology spectrum.

Before proceeding with my model, I briefly layout the main forms of media-bias. Following [46], news bias by a partisan media mainly occurs in three forms - selective reporting (reporting on strongly partisan topics); issue framing (how an event is portrayed by reporters) and ‘agenda setting’ (determined by amount of coverage on each incident). In the present setup, bias takes the form of issue-framing and is generated by both demand (reader-assessment) and supply (ideology of media firm) factors. Since competition between media is spatial, the placement of news is a single point on the ideology spectrum which represents the bottom line or a condensed form of the event. The location of this point then signifies how close a particular media has chosen to be to the left or right ideology.

In later sections, the results of the benchmark model of duopoly competition is compared with respect to three settings - monopoly media (absence of competition), more polarized reader distribution (for instance, when majority readers are biased to the left or to the right) and a market with three firms. I find equilibrium bias increases in the monopoly setup, due

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11[44] discusses a measure of media power which gains force as reader sophistication tends to zero.
12[41] dissect these two as ‘bias’ and ‘spin’, the former being in context of traditional left-right ideology while the latter helps to create a memorable story
to absence of any competition. In the three-media case, if the event to be reported is has no ideology bearings, then equilibrium media bias rises above the duopoly bias level only if the third firm is ideologically biased. I then analyse welfare by aggregating reader utility and media firms payoffs. I find welfare to depend crucially on the nature of topic to be reported, the weights assigned by a media to its twin motive - ideology and reader-assessment and reader polarization.

The remaining of the paper proceeds in the following manner: section 2 the related theoretical and empirical literature. Section 3 introduces the model preliminaries; the game timeline has been laid out in section 4; section 5 examines the duopoly competition; section 6 analyses media bias in presence of a more polarized reader pool; section 7 provides a brief insight into the outcomes when a third media enters the duopoly market and section 8 presents the welfare analysis.

1.1 Related Literature

This paper fits in the literature of industrial organizational aspect of media bias. As discussed before, I model a competitive model of media bias as a product placement problem. While reporting certain issues, rival firms try to place themselves closer to each other whereas they maximally differentiate from each other while reporting on others. This model does not account for media’s role either in electoral outcomes or policy analysis ([16], [10], [18], [45]) or in models of media capture like [12].

Nevertheless, among these papers we find support of the model premises. The manner in which we define bias matches with [18],[17] or [41]. This definition directs us to a specific strand of works within the spatial product-placement literature of [3], [16] and [10]. These papers however work at the conjunction of media bias and its extensions in various political and electoral environments.

The structural aspects of [41] closely resonates with this model where readers learn about the issue before reading media reports. However, both differ in other underlying model assumptions and the nature of news provision. A finding common to both is the information slant by media about reports on events with no ideology. They argue that the bias exist through the channel of ‘spin’ which creates a memorable story whereas my argument depends on a result following Hotelling’s lemma, where each partisan media outlet segment the economy and bias the ideology-free event to cater to their like-minded readers. In addition, this model offers an added insight which can be explained through the following
example. Consider two scenarios - A, where both media firms refuse to compromise with their partisan interests and B where both are relatively flexible about adjusting partisan priorities to satisfy readers. Then while reporting a neutral incident, media firms in A will earn relatively higher equilibrium profits than the ones in B. Apart from this, I add to the literature, by providing a formalized way to detect when media firms will speak indifferently and when they will not.

This paper is also close to the literature focusing on media bias from non-price competition in a duopoly setup which includes that of [20] who study a supply-side story of media bias, where reputational concerns of media drives it to take certain editorial choices. They show exogenous chance of truth revelation disciplines media and prevents it form biasing news. [10] show that media outlets seeking to maximize profit take sides and introduce bias to their stories which later lead to voters committing electoral mistakes.

In contrast to the above models, the present model characterization implies how media will locate itself on a spatial axis when it has to inform the public about it. Media’s choice depends on its valuation on reader evaluation and how favourable the incident it relative to its own ideology. I try to throw clarity when media will proclaim the superiority of its own ideology beyond the truth or trivialize or report partially a ideologically ‘bad’ event or sound indifferent. Additionally, I put forward the associated profit levels with these media choices and infer how media outlets are affected while choosing their news stories.

1.2 Model

This fundamental model is akin to the linear city model where readers are uniformly distributed. In the baseline case, there are two partisan media firms $L$ and $R$ at opposite ends which signifies their ideological rivalry. The entire game spans across three periods. In the first period, readers receive an exogenous factual report by a honest media $E$ about a particular event $\omega$ belonging to the universe $\Omega = [-1,1]$. The point $-1$ represents an incident which aligns to the extreme left ideology while $1$ denotes an event with extreme rightist ideology. Any intermediate points are relatively moderate and the midpoint $0$ is absolutely neutral.

The above interval also represents the linear city where $N \in \mathbb{N}$ readers are uniformly placed and their location denotes their subjective ideological leanings. A reader $i$’s position is denoted as $x_i$ on $[-1,1]$. The neutral (or moderate) reader is positioned at $0$ while the

\footnote{Cumulative mass function $F$ and probability mass function $f$.}
extreme leftist (rightist) reader is placed at -1 (1) as shown in Figure 1. Readers are rational and are aware of the partisan interests of media. The utility of a reader $i$ is additively separable $^{14}$ across news of media $j \in \{L, R\}$ is

$$U_{ij} = -(\alpha_{ij}\theta_j - x_i)^2 - (\alpha_{ij}\theta_j - \theta_E)^2$$  \hspace{1cm} (1.1)

The action of $i$ is to choose $\alpha_{ij} \in \mathbb{R}$ which denotes his assessment or weight of the news story by media $j$. Intuitively, this is a measure of the degree of satisfaction from a news story. This assessment is therefore a mapping $\alpha_{ij} : \theta_E \times \theta_j \rightarrow \mathbb{R}$, where $\theta_{-j}$ refers to the strategy or editorial position of the rival media and $\theta_E$ denotes the signal from the honest media $E$.

In the following period, media firms $L$ and $R$ choose their respective editorial positions $\theta_L$ and $\theta_R$ on the same ideology interval $[-1, 1]$ on the onset of a particular event $\theta_E$. The baseline model accounts for media firms initially located at the extremes. This is akin to the concept of bliss point or where the partisan media firms ideally want to be. We parameterize this location by $\bar{\theta}_j \in [-1, 1]$. In the baseline model, $\bar{\theta}_L = -1$ and $\bar{\theta}_R = 1$. So my aim is to understand how information bias percolates into an economy when its news suppliers are inherently extreme partisans. One can do the same for other moderate values of $\bar{\theta}_j$ and examine the levels of information slant.

The payoff function of media $j$ accounts for the action of its rival firm ($-j$), given the report of the honest media as shown below.

$$\Pi_j(\theta_j, \theta_{-j} | \theta_E) = -\lambda_j.(\alpha_j^* - 1)^2 - (1 - \lambda_j)(\theta_j - \bar{\theta}_j)^2 - c\frac{(\theta_j - \theta_E)^2}{b + (\theta_{-j} - \theta_E)^2}$$  \hspace{1cm} (1.2)

The action of media $j$ is a mapping $\theta_j$ where $\theta_j : \theta_{-j} \times \theta_E \rightarrow \mathbb{R}$ where $\theta_{-j}$ denotes strategy of the rival outlet. The first two terms depicts the trade-off between accuracy and ideology to media $j$ respectively. Hence, media’s payoff is a convex combination of these two factors with respective weights $\lambda_j$ and $(1 - \lambda_j)$ where $\lambda_j \in (0, 1)$. The first term implies gaining better reader assessment $^{15}$ while the second term denotes the gain in ideology payoff by locating closer to its ideology bliss point $\bar{\theta}_j$. If $\lambda_j$ is very closer to 1 then $j$ places greater weight on reader satisfaction. On the contrary, when $\lambda_j$ is closer to 0, media $j$ weighs

$^{14}$This convention has been used in [21] with a more ordinal utility form, where a household’s utility is additive in the number of newspapers chosen among the ones available within its zip code.

$^{15}$We explain this functional form more clearly using Lemma 1 in section 5.
ideological gains more than reader satisfaction.

The final term denotes the cost function $C(.)$ of $j$ from biasing news which is basically the deviation of $\theta_j$ from $\theta_E$. The marginal cost is $c > 1$. The parameter $b \in (0, 1)$ represents the sophistication within the readers. This has a cross-over effect of one firm’s bias on its rival. Higher (lower) value of $b$ implies lower (greater) cost of bias, given the level of bias of the other firm. $C(.)$ has the following properties:

(i). $\frac{dC}{d(\theta_j - \theta_E)^2} > 0$, firm $j$ incur greater cost by biasing news.

(ii). $\frac{dC}{d(\theta_j - \theta_E)^2} < 0$, firm $j$ faces lower cost from biasing when its rival firm biases news and vice versa.

(iii). $\frac{dC}{db} < 0$, cost of bias decreases when level of reader un-sophistication increases.

I solve this duopoly game $\Gamma_D$ of complete information using Subgame Perfect Nash Equilibrium (henceforth SPNE).

Definition 1.1. A strategy profile $s = \{\theta_L, \theta_R, < (\alpha_{1L}, \alpha_{1R}), \ldots, (\alpha_{NL}, \alpha_{NR}) > \} \in \Gamma_D$ is a subgame perfect Nash equilibrium (SPNE) if $s$ induces a Nash equilibrium in every subgame of $\Gamma_E$. Nash Equilibrium of the duopoly game $(\Gamma_D)$ between the media is a pair $(\theta^*_L, \theta^*_R)$ of editorial choices for which $\theta^*_L$ is a best response to $\theta^*_R$ and $\theta^*_R$ is a best response to $\theta^*_L$.

1.3 Timeline of game

Figure 2 illustrates the timeline of this model which begins an unbiased media $E$ sending a factual public report $\theta_E$ which becomes common knowledge to both readers and partisan media. Mullainathan and Shleifer (2002) refers this as a signal ’$r’ and interprets such a signal to set a prejudice within a reader before he reads the news. I borrow this interpretation here to claim that any report about an event has a location on the ideology spectrum. The
partisan media firms $L$ and $R$ observes the event and $\theta_E$ and designs its own report (reflected by its editorial positions $\theta_L$ and $\theta_R$ respectively) for the readers in the next stage. Readers are heterogeneous and rational and they cannot observe the true event prima facie but has access to the news of $E$. After the partisan media publishes the report, readers assess its report and provides a rating ($\alpha_L$ to $L$; $\alpha_R$ to $R$) which measures the report’s consistency with $E$’s report and their subjective ideology. These ratings can act as instruments to measure the unrest or ecstasy among the readers about any particular event. I study the editorial decision of partisan media through a simple backward-induction game in a duopoly media market. Media firms $L$ and $R$ compete over attention along a spatial Hotelling’s axis which measures ideology.

For further clarity, I explain the timeline using a simple example. After learning about a potential deportation through a graduate school email (following the online mode of classes in Fall 2020 due to Covid-19), international students will likely read reports of say, CNN and Fox to gather more information. Here, the graduate school resembles media $E$ which presents a factual report. Students (readers) then can tune to CNN to hear its condemnation against the Immigration and Customs Enforcement (ICE) for imposing such a strategy, thereby gathering some solace after feeling victimized. Alongside, they might also tune to Fox to learn how likely they are to be deported. These experiences from a liberal and conservative media are portrayed by the reader-ratings ($\alpha_L$ and $\alpha_R$ respectively).
1.4 Duopoly Model

I consider the duopoly media market with firms \( L \) and \( R \). Then the corresponding normal form game of this duopoly case is defined as

\[
\Gamma_D = [I, \{u_i(.)\}, \{\Pi_L(.)\}, \{\Pi_R(.)\}]
\]

\( I \) denotes the player set comprising of media \( L \) and \( R \) and reader \( i \in \{1, \ldots, n\} \). \( u_i \) is the utility of reader \( i \) from reading news and \( \Pi_L \) and \( \Pi_R \) denotes the profits of media \( L \) and \( R \). Thereby the equilibrium strategy profile constituting the SPNE is characterized as \( s^* = (\theta^*_L, \theta^*_R, \alpha^*_i L(\theta^*_L), \alpha^*_i R(\theta^*_R)) \).

1.4.1 Utility maximization of reader

First order condition following equation 1 leads to the optimal assessment (weight) given by reader \( i \) towards media \( j \)'s editorial position \( \theta_j \)

\[
\alpha^*_{ij} = \frac{x_i + \theta_E}{2\theta_j}
\]

Lemma 1.2. The first best evaluation by reader \( i \) reading news of media \( j \in \{L, R\} \) is achieved when \( \theta_j = \theta_E = x_i \), or \( \alpha^*_{ij} = 1 \)

The rating of 1 suggests that media \( j \)'s editorial position matches both media \( E \)'s position \( \theta_E \) and the ideology \( x_i \) on \([-1, 1]\) in tandem. Intuitively, if \( \theta_j = \theta_E = x_i \) then not only does \( i \) perceive \( j \) to be as honest and accurate as \( E \), but also can relate it perfectly with his own ideology \( x_i \). Hence this news is perfectly cohesive with his rational self.

The first term in the profit function of \( j \) is a distance function which accounts for the loss of reader satisfaction from a piece of news which cannot be assigned this first-best weight.

Borrowing equation 3, the expected rating from \( N \) readers of \( j \) is given as

\[
E(\alpha^*_{ij}) = \frac{1}{N} \sum_{i=1}^{N} \frac{x_i + \theta_E}{2\theta_j} \cdot f(x_i) = \frac{\theta_E}{2\theta_j} \cdot \frac{1}{N} = \frac{\theta_E}{2\theta_j}
\]

The above result arrives from the assumption that readers are distributed such that mass
of leftist and rightist readers are equal, hence they offset each other ($\sum_{i=1}^{N} x_i = 0 \ \forall i \in \{1, ..., N\}$). $\sum_{i=1}^{N} x_i \neq 0$ implies more polarised readers such that the distribution of readers $f$ is such that the mass of leftist readers either greater or lesser than their rightist counterparts. If $\sum_{i=1}^{N} x_i \leq 0$, ($\sum_{i=1}^{N} x_i \geq 0$) the economy has a leftist (rightist) majority. The impact of such an unbalanced reader base on the editorial positions has been explored in section 6.

1.4.2 Payoff maximization of media

The optimal action of media $j$ is directed by the below first-order-condition

$$\frac{d\Pi_j}{d\theta_j} = \theta_j^2 \left(1 - \lambda_j \right) + \frac{c}{(b + (\theta_j - \theta_E)^2)} \left[-\theta_j^3 \left(1 - \lambda_j \right) \theta_j - e\theta_E \left(1 - \lambda_j \theta_j - c\theta_E \left(b + (\theta_j - \theta_E)^2\right)\right)\right]$$

$$+ 0.5\lambda_j \theta_E \theta_j - 0.25\theta_E^2 \lambda_j = 0$$

(1.5)

This represents the best response function of $j$ to the action of its rival $\theta_{-j}$. The equilibrium editorial choice(s) is attained at the intersection of these functions. To bring out the possible behavior traits of media, I limit the value of $b$ to be above some threshold as stated in Assumption 1. It is only above a cutoff that the effects of media under this setup becomes pronounced enough for a deeper analysis.

$b$ is above a threshold level $b' \in (0, 1)$.

This threshold value can act as a direct measure of reader un-sophistication and finds support in the experimental findings of [30] who posits that experts are much less influenced by manipulations by media and have already established their own evaluations about a particular event. On the other hand novices are the vulnerable ones, totally non-immune to information manipulations by media. $L$ faces much higher cost in the event when rival media $R$ does not bias. As $b$ increases, it allows $L$ to bias news and insulates against any negative feedback from the public. This simultaneously weakens competition to publish more accurate information and exacerbates the level of information slant.

Before proceeding into the equilibrium properties, it must first be ensured that the above system of equations have at least one real root within the interval of interest i.e $[-1, 1]$. Given the quartic nature of equation 5, it is close to impossible to postulate an explicit solution for $\theta_j$. However, using Sturm’s Theorem, it is suggested that two real solutions exists in $[-1, 1]$,
Figure 1.3: The blue (orange) segment denotes class of events which supports the mixed-strategy equilibrium of \(L\) (\(R\)) at a particular threshold of \(\lambda_L\) (\(\lambda_R\)). These thresholds are unique for every event \(\theta_E \in [\tilde{\theta}_E^L, \theta_E^L]\). Events outside these area support unique equilibrium of for all values of \(\lambda_L\) or \(\lambda_R\).

as proposed by Lemma 2. For any parameter values, each polynomial has two real roots within \((-1, 1)\), one positive and one negative. I provide detailed explanation about this rule in section 8.1 of the appendix.

Lemma 1.3. There exists two distinct real roots (one positive, one negative) in \((-1, 1)\) of the best response function of each media.

The following proposition (refer to figure 3) describes the conditions which support both pure and mixed strategy equilibrium. In equilibrium, the BR functions intersects providing the associated profit levels to each media firm.

Proposition I

(i) Pure strategy equilibrium: For any \(\lambda_j\), \(\theta^*_j\) is unique for any \(\theta_E \in \{[\tilde{\theta}_E^R, 0) \cup (0, \tilde{\theta}_E^L]\}^C\). However, for \(\theta_E \in [\tilde{\theta}_E^R, 0) \cup (0, \tilde{\theta}_E^L]\), \(\theta^*_j\) is unique for any \(\lambda_j \neq \tilde{\lambda}_j\).

(ii) Mixed strategy equilibrium: For a class of events lying in \([\tilde{\theta}_E^R, 0)\) and \((0, \tilde{\theta}_E^L]\), there exists a mixed strategy equilibrium of \(L\) and \(R\) at a unique cutoff value of \(\tilde{\lambda}_L\) and \(\tilde{\lambda}_R\) respectively. Here the equilibrium strategy pair for each media \(j \in \{L, R\}\) is denoted by \((\theta_j^{1*}, \theta_j^{2*}; p, 1 - p)\) and both lie on either side of zero.

(iii) Symmetric equilibrium: When \(\theta_E = 0\), a symmetric equilibrium exists when \(\lambda_L = \lambda_R\) when \(L\) and \(R\) positions themselves equidistant from the median reader at 0.

The first two statements can be understood with more clarity through figure 3. Events to the right of the blue interval support the right ideology strongly enough such that \(L\) always locates on the right of 0 for all values of \(\lambda_L\). This is unique pure strategy equilibrium for both
L and R. Symmetric results evolve for event to the left of the blue interval. Compared to this, events in the blue (orange) intervals favor the left (right) relatively with lower magnitude. Then, reporting in favor of the left for events in the blue interval is no longer binding for R unless when $\lambda_R$ is high enough (greater weight on reader assessment). The model provides a cutoff $\bar{\lambda}_R$ which determines the equilibrium response of R. below which R will still speak in favor of the left. At the cutoff value, R is indifferent between speaking in favor of either ideology, hence leading to a mixed strategy equilibrium.

Media designing a report which extol their own ideology even in the face of a contradicting event follows [7]. The current model formalizes the sufficiency conditions where events contradicting a media’s ideology will bind it to speak closer to the truth. Media j with value of $\lambda_j$ greater than threshold speaks closer to the true events and does not jeopardize with reader-assessments while the ones below the threshold advocates more towards ideology motive, thereby publishing stories contradicting the true event.

The third statement highlights the conditions for symmetric equilibrium. For the existence, it is necessary that the event must have no ideological underpinnings. The sufficiency factor is that both media should have identical preferences towards ideology.

Remark 1.4. Comparison of magnitude of editorial positions: The class of events which strictly favors the left, $\theta_E \in [-1, 0)$, L chooses to locate closer to the event than R. Analogously, for events favoring the right, R chooses to locate closer to the event than L.

This phenomenon is illustrated through table 1. Additionally, L and R locate symmetrically around zero when $\theta_E = 0$ (neutral event) and $\lambda_L = \lambda_R$ holds (shown in bold in table 1).

Remark 1.5. Intuitively, the threshold value $\bar{\lambda}_j$ of $\lambda_j$ is a measure of the extent to which L is willing to champion its ideology in presence of a contradicting reality.

Given that $\theta_E$ favours j’s rival, it is common knowledge that its rival will speak in favor of its won ideology. When j has its trade-off between ideology and reader-evaluation equal to the cutoff $\bar{\lambda}_j$, the choice of location on either side of zero depends on the description or $\theta_E$. j will be indifferent between speaking for either ideologies. It is later shown that the profit of j is the lowest at this cut-off value.

Proposition II:

(i) There exists a reader in $[-1, 1]$ with ideology $x_i$ who assigns identical assessments to news of L or R. $x_i$ can be uniquely solved from the below identity when $\theta_L^* \neq -\theta_R^*$\(^{16}\)

\[^{16}\]The outcome $\theta_L^* = -\theta_R^*$ is endogenously arrived iff $\lambda_L = \lambda_R = 1$ and are reporting a neutral event
Table 1.1: The first column shows that when true state totally favors the left, then \( L \) speaks closer to the truth than \( R \) for all values of \( \lambda_R \) (in blue). Symmetric results hold for \( R \) (in red). As \( \lambda_L \) increases, \( L \) locates itself closer to the median reader at 0. The only symmetric equilibrium occurs when \( \theta_E = 0 \) and \( \lambda_L = \lambda_R \) (shown in bold).

(ii) If an incident supports the left, and media \( R \)’s weight on reader assessment is high enough, then a fraction of leftist readers will prefer \( L \)’s news over that of \( R \) even if the latter is closer to their ideology.

The first statement resonates the idea of Hotelling’s linear city while the second statement goes against it. The latter reflects the idea that if the right media reports a pro-left event, then a fraction of leftist readers surrounding the location chosen by the right media would still prefer the left media news story.

Corollary 1.6. Readers strictly to the left of \( \theta_E \) enjoys news from the left media while \( \theta_E = 0 \). In this case, the median reader at 0 is indifferent between either outlets. We can exclude this case as \( \lambda_L \) and \( \lambda_R \) lies between \( (0,1) \).
those to the right enjoy news of the right media.

Corollary 1.7. If $L$ and $R$ are reporting an incident which completely supports the left, $\theta_E = -1$, with $\lambda_L \to 1$ and $\lambda_R \to 0$ respectively, then a fraction of leftist readers who are closer to $R$’s location choice than that of $L$ assess news of $L$ better than $R$.

If media $L$ is not motivated enough towards deriving ideology payoffs but competes with media $R$ which is more ideologically motivated, then some relatively weaker leftist readers will prefer news about a pro-left issue from media $R$. These readers who are weakly attached to either ideology will discount their like-minded news source which will take a relatively indifferent stance. On the contrary, they will gain more satisfaction from receiving positive news about their ideology from an ideologically opposite media. A numerical example can be given to throw more clarity. When $\theta_E = -1$ (extreme pro-left event) and $(\lambda_L, \lambda_R) \to (0.998, 0.002)$ then $(\theta^*_L, \theta^*_R) \cong (-0.998, -0.633)$ which implies that leftist readers approximately between $(-0.472, 0)$ prefers news from media $R$ than $L$. Readers to the right of this interval prefers $R$, while those to the left are more satisfied with $L$.

1.4.3 Choice of reporting neutrally

There occurs two broad scenarios where media $j$ can report neutrally by locating closer to zero. First, when the true event is actually neutral and second, when the event is unfavourable to $j$’s ideology. The former indicates truthful reporting, while the latter can be termed as ‘indifferent reporting’, a form of biased news reporting, where the media is reluctant to speak in favor of the rival ideology. However, as this model predicts from proposition 1, media does not want to sound indifferent even when faced with an ideologically ‘bad’ event.

For example, when the event favors the left ($\theta^*_L$ in figure 5), then $L$ does not position itself in the blue region. When $L$ is more attached to its ideology ($\lambda_j \leq \hat{\lambda}_j$), it places itself on the left of the blue interval. On the other side of this cutoff, $L$ places itself in the territory of the rightist readers, on the right of the blue interval. In essence, $L$ avoids a more indifferent location (around zero). I observe that when the true event is neutral ($\theta = 0$), then it is strategically dominant strategy to bias news. I get a closed form solution of $\theta^*_j$ from the first order conditions.

Proposition III:
Figure 1.5: Deviation from neutral reporting: Suppose, readers are uniformly distributed and there occurs an event $\theta_1^E$ which favors the right. Then $L$ does not locate on the blue region. It reports on the left of this region (supporting the left) when $\lambda_L$ lies below the cut-off $\bar{\lambda}_L$ (it is ideologically stronger) and reports on the right for values of $\lambda_L$ above $\bar{\lambda}_L$ (it is more motivated towards reader-assessment). When the event favors the right more strongly, say $\theta_2^E$, then this blue region shifts to the right.

(i) Segmented Equilibrium: When $\theta_E = 0$, the equilibrium editorial choice of $j$ is given by

$$\theta_j^* = \frac{(1 - \lambda_j)}{(1 - \lambda_j) + \frac{c}{2 + (\theta_j^e)^2}}$$

(ii) Given any unfavourable event, media $j$ either supports its own ideology or the opposite ideology conditional on the value of $\lambda_j$. However it never locates on a region surrounding zero which implies indifferent reporting.

The technical proof is in the appendix. The first statement is analogous to [43] where the two firms locate at a distance of roughly 0.27 from either ends of $[0, 1]$ interval. Firms choose this by minimizing the consumer’s transportation cost which in our model reflects the cost of reading a news story which is far away from a reader’s ideology. Locating at the midpoint of the $[-1, 1]$ interval only increases the transportation costs of extreme readers. This equilibrium strategy connects to the psychology literature on news perception where more extremely ideology readers misinterpret neutral reporting of media as biased [23].

The implication of the second statement can be derived from figure 5. If there is a rise in inflation during the presidency of the left, then statement $ii$ implies that if $L$ is too partisan-motivated, then it will detach the effect of the left ideology with the rise in inflation by report that its unemployment reducing monetary policies are targeted to lower unemployment which comes at a cost of higher inflation or raise doubts in readers’ minds about the possibility that the reported numbers as overestimated. Alternatively, if $L$ is more motivated towards reader-assessments, then the coverage can come as a criticism of the policy which lead to higher inflation.

The above phenomenon was found in the way Fox news also covered ICE’s decison
1.4. Duopoly Model

of deporting international students during pandemic. The news story did not criticise the
decisions but highlighted the dire impact it had on the lives of international students.\textsuperscript{17} What
appears is that media will speak (not strongly enough) in favor of its adversary instead of
positioning itself near zero, which intuitively leads to a tendency to build better reader-
assessment credibility even from opposite-minded readers.

1.4.4 Comparative Statics

I now consider how the parameters $\lambda_L$ and $\lambda_R$ affect the equilibrium choices of $L$ and $R$
respectively. For more clarity of the stated propositions, I study the effects of the equilibrium
choices of media $L$. Analogous explanations will hold for a similar study of $R$’s equilibrium
choice. I also study the cross-over effect the rival media imposes on the equilibrium choices
of the media firms (through the parameter $b$). Applying IFT to (5), I arrive at the following

$$
\frac{d\theta^*_L}{d\lambda_L} = \frac{\theta^*_L + \theta^*_R + 0.25\theta^*_E - 0.5\theta_E\theta_L}{4\theta^*_L(1 - \lambda_L + \frac{c}{b + (\theta_R - \theta_E)^2}) + 3\theta^*_L(1 - \lambda_L - \frac{c\theta_E}{b + (\theta_R - \theta_E)^2}) + 0.5\lambda_L\theta_E}
$$

Let us take numerical values of exogenous parameters to better understand the com-
parative statics. I choose $b = 0.7$ and $c = 1.1$ and given $\lambda_L = \lambda_R = 0.1$, I get $(\theta^*_L, \theta^*_R) =
(-0.417, 0.417)$ when $\theta_E = 0$. Incorporating in (8), I get,

$$
\frac{d\theta^*_L}{d\lambda_L} = 0.27
$$

As I will see later that this magnitude is greater than the comparative statics result
from the monopoly model in section 6 where $\frac{d\theta^*_L}{d\lambda_L} = 0.2475$. Intuitively, given $\theta_E = 0$, when
media $L$ puts more weight on payoffs from readers, then it takes an editorial stance closer
to the median reader.

I now conduct a similar comparative statics exercise with parameter $b$. This will allow
us to measure the cross-effects of editorial choice of $R$ on the choices of $L$ and vice versa. Using IFT on equation (5) through parameter $b$ gives us the following equality.

\textsuperscript{17}A report by Fox5 Atlanta on July 8th 2020 titled “International students face uncertain future due to
new ICE rule”.

\[
\frac{d\theta^*_L}{db} = \frac{-c\theta_L^3(1-\theta_L)}{b+(\theta_R-\theta_E)^2} + 3\theta_L^2(1-\lambda_L - \frac{c\theta_E}{b+(\theta_R-\theta_E)^2}) + 0.5\lambda_L \theta_E
\]  

(1.7)

As the weight on satisfying the average reader increases, both rival partisan media firms tries to place themselves near the median reader. The sign of the derivatives shows that the equilibrium editorial stance of \(L\) moves rightward towards 0 while the position of \(R\) moves leftward towards 0.

The nature of signs of the change in equilibrium level of slant depends on whether the events are themselves too strongly or too weakly biased. As before, \([\theta_E^L, \theta_E^R]\) depicts events which are weakly biased (centered around 0) while its complement within \([-1, 1]\) denote the events which are biased strongly enough to either ideology.

Proposition IV:

(i) If an event favors \(j\)’s ideology, \(j\)’s editorial choice moves closer to the median reader at 0 as \(\lambda_j\) increases. In other words, \(\theta^*_L\) increases with \(\lambda_j\), \((\frac{d\theta^*_L}{d\lambda_j} > 0)\).

(ii) For any unfavourable event, \(\frac{d|\theta^*_L|}{d\lambda_j} < 0\) for all \(\lambda_j \in (0, \bar{\lambda}_j)\) and \(\frac{d|\theta^*_L|}{d\lambda_j} > 0\) for all \(\lambda_j \in (\bar{\lambda}_j, 1)\). At \(\bar{\lambda}_j\), \(\theta^*_L\) is discontinuous.

(iii) The impact of a more sophisticated reader pool reduces bias of \(L\) given any nature of event. However, the weights on ideology and reader-assessment of both media weakens or strengthens this impact.

(a.) When the event has no ideology \((\theta_E = 0)\), the impact gains strength in the presence of a media \(R\) which is less focused on ideology motive and assigns greater weight on reader assessment.

(b.) If the event supports the ideology of media \(L\), then the impact is greater in the presence of \(R\) whose motive is more driven towards ideology gains than reader-assessment.

Sub-part (i) points out that as \(j\)’s attachment towards its own ideology falls, its comparative statics with respect to \(\lambda_j\) naturally segments the event space into two classes - events which are biased enough vis-a-vis the ones which are not. For the first class, media \(L\)’s editorial choice increases (moves towards right on the ideology spectrum) when the weight on reader satisfaction increases. Analogously media \(R\)’s response decreases and moves towards
the left.

(ii) implies that $\theta^*_j$ is piecewise continuous. As stated in statement (ii) of proposition 1, at the advent of an ideologically negative event, media $j$’s location strategy varies distinctly around a threshold value of $\lambda_j$. This variation is clearly suggested by the direction in change of $\theta^*_j$ on either side of the threshold. At the threshold, $\theta^*_j$ exhibits non-removable discontinuity of the first kind where $\theta^*_j(\bar{\lambda}_j + 0)$ and $\theta^*_j(\bar{\lambda}_j - 0)$ exists but have different values. $\theta^*_j$ remains continuous for all other values of $\lambda_j$.

Higher value of $b$, implies lower reader sophistication, thereby a greater leeway to bias in favor of ideology. According to [4], more educated people will generally call upon alternative information before accepting a news story and that increases the likelihood of them positing a stronger counter-argument to an overtly biased news story. This argument augments the third statement. The effect of a more sophisticated reader-base on reducing bias of a particular media is affected by contemporaneous effects of the preference of its rival. When the event has no bearing on ideology, then the presence of a rival which prefers reader-assessment will lead to a reduction in bias. This is because, it would pay the media more to locate towards the median reader by the standard Hotelling argument.

To evaluate the effect of increasing $\lambda_L$ on the equilibrium payoffs, I use envelope theorem. By the envelope theorem, the effect of any parameter on the maximum value function is entirely the direct effect of the parameter on the maximum value function. The maximum value function $V_j$ is calculated by substituting $\theta^*_j$ in the payoff functions of media $j$.

Proposition V:

Responsiveness of maximum value function

(i) Suppose the event is neutral ($\theta_E = 0$), then the maximum value function decreases as the weight on reader ratings are increased, $\frac{dV_j}{d\lambda_j} < 0$.

(ii) Suppose the event is not neutral ($\theta_E \in [-1, 0) \cup [-1, 1]/\{0\}$), then the maximum value function is U-shaped as $\lambda_j$ as increased.

The technical proof is in the appendix. What is implied by this is the following. Given the reader pool is balanced and $\theta_E = 0$, the first term of media $j$’s profit function is zero (see equation 2). Then profit in equilibrium will always be enhanced when $\lambda_L \to 0$, or media $j$ is more ideology-motivated.

Intuitively, if the event is neutral, then an average reader has zero bias (balanced reader pool) and he will tune in to the partisan channels to learn about potential ideological sub-
tleties. Hence, placing more weight on ideology brings in higher rewards for the media. Alternatively, placing weight of reader assessment and providing a neutral report only leads to worse experience of like-minded readers. This resonates with the location choice model of [43] where firms does not choose the midpoint of the linear city economy (of unit length) but at roughly at points 0.25 and 0.75. These points minimize the consumers transportation costs.

The second statement means that while reporting a story which is not neutral (the story either supports or attacks the ideology of \( j \)), higher equilibrium profits are achieved when media either focuses on ideology or on reader ratings. Equilibrium profits are compromised if \( j \) wants to produce a report by balance both the factors. Hence, higher profits are realized at the extreme values of \( \lambda_j \).

By continuity of the maximum value function, then there exists a threshold where the media experiences the lowest equilibrium profit. This is the exact threshold which reflects the desperation of a media to support its ideology even when the true event stands in contradiction. Following proposition 1, this threshold is denoted by \( \bar{\lambda}_j \) and \( \frac{\partial V_j}{\partial \lambda_j} \) vanishes at \( \bar{\lambda}_j \).

Remark 1.8. At the threshold value, media experiences greater equilibrium losses which primarily stems from poorer reader ratings.

Desperately supporting its ideology can besmirch \( j \)'s image to a certain mass of readers who will doubt \( j \)'s credibility. This also intuitively connects to the experimental findings of Baum and Groeling (2009) where media often engage in such risky editorial decisions. As I will see from the comparative statics section below that this type of reporting comes at a high cost. At this threshold, media actually experiences the highest equilibrium loss.

The strategic interaction between \( L \) and \( R \) also affects each other profit levels which basically reflects the strategic substitutability and complementarity.

Proposition I: (i) When the event is neutral (\( \theta_E = 0 \)), then any action by firm \( j \) is always a strategic substitute towards its opponent. The magnitude of strategic substitutability is greater when \( j \) is more partisan-oriented.

(ii) Suppose the event supports media \( j \) beyond a threshold \( |\tilde{\theta}_E| \). Then

a. If \( j \) is extremely partisan, then any action by the rival media is strategic substitute. The magnitude of this substitutability increases when \( j \) is also extremely partisan.

b. If \( j \) is extremely reader-oriented, then any action by its opponent acts as a strategic
complement to \( j \)'s profit levels. The magnitude of this complementarity increases when \( j \) is also extremely partisan.

1.5 Model with 3 media outlets

We expand the previous analysis by adding one more firm on the ideology axis. We denote this firm by \( Q \) which has an ideological bliss point at \( \bar{q} \in (-1, 1) \). The remaining features of the model comprising the readers and the media outlets \( L \) and \( R \) carries on unchanged in this section. This exercise is expected to reveal how more competition among the media outlets affect the equilibrium level of bias.

The corresponding normal form game of this three firm model is defined as

\[
\Gamma_T = [I, \{S_i\}, \{S_L\}, \{S_R\}, \{S_Q\}, \{u_i(.)\}, \{\Pi_L(.)\}, \{\Pi_R(.)\}, \{\Pi_Q(.)\}].
\]

\( I \) denotes the player set comprising of media \( L, R \) and \( Q \) and reader \( i \in \{1, \ldots, n\} \). \( u_i \) is the utility of reader \( i \) from reading news and \( \Pi_L, \Pi_R \) and \( \Pi_Q \) denotes the profits of media \( L, R \) and \( Q \). Thereby the strategy profile constituting the SPNE is characterized as

\[
s^* = (\theta^*_L, \theta^*_R, \theta^*_Q, \alpha^*_iL(\theta^*_L), \alpha^*_iR(\theta^*_R), \alpha^*_iQ(\theta^*_Q)) \quad \forall \ i = \{1, \ldots, N\}.
\]

1.5.1 Utility Maximization of reader

We will inherit equation (1) with one more media firm \( Q \) such that for \( j \in \{L, R, Q\} \), utility of any reader \( i \) is given by

\[
U_i(\alpha_{ij}|\theta_j, \theta_E) = -(\alpha_{ij}\theta_j - x_i)^2 - (\alpha_{ij}\theta_j - \theta_E)^2
\]

1.5.2 Backward Induction by Media

With three firms, the payoff function takes a slightly revised form where the nature of cost function gets updated to account for the bias of the third firm. In the following three equations, we layout the payoffs of media \( j \in \{L, R, Q\} \).
\[ \Pi_L(\theta_L, \theta_R, \theta_Q) = -\lambda_L(\alpha_L^* - 1)^2 - (1 - \lambda_L)(\theta_L + 1)^2 - \frac{c(\theta_L - \theta_E)^2}{b + (\theta_R - \theta_E)^2 + (\theta_Q - \theta_E)^2} \] (1.8)

\[ \Pi_R(\theta_R, \theta_L, \theta_Q) = -\lambda_R(\alpha_R^* - 1)^2 - (1 - \lambda_R)(\theta_R - 1)^2 - \frac{c(\theta_R - \theta_E)^2}{b + (\theta_L - \theta_E)^2 + (\theta_Q - \theta_E)^2} \] (1.9)

\[ \Pi_Q(\theta_Q, \theta_L, \theta_R) = -\lambda_Q(\alpha_Q^* - 1)^2 - (1 - \lambda_Q)(\theta_Q - \tilde{q})^2 - \frac{c(\theta_Q - \theta_E)^2}{b + (\theta_L - \theta_E)^2 + (\theta_R - \theta_E)^2} \] (1.10)

The only difference between L's (R's) payoff function from previous section lies in the cost function which now takes account for the bias of the the third media house Q.

Definition 1.9. Nash Equilibrium of this game \( \Gamma_T \) is a triple \((\theta_L^*, \theta_R^*, \theta_Q^*)\) of editorial choices for which \( \theta_j^* \) is a best response to \( \theta_{-j}^* \) where \( j \in \{L, R, Q\} \).

The below table shows a numerical depiction of the equilibrium choices of L and R with the entry of a new media with two respective ideology bliss points -0.5 and -0.75 and for two values of \( \lambda_Q = \{0.1, 0.5\} \).

<table>
<thead>
<tr>
<th>((\lambda_L, \lambda_R))</th>
<th>(\lambda_Q)</th>
<th>0.1</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.1, 0.1)</td>
<td>(-0.436, 0.436, -0.23)</td>
<td>(-0.426, 0.426, -0.162)</td>
<td></td>
</tr>
<tr>
<td>(0.1, 0.5)</td>
<td>(-0.4, 0.293, -0.218)</td>
<td>(-0.396, 0.28, -0.15)</td>
<td></td>
</tr>
<tr>
<td>(0.5, 0.5)</td>
<td>(-0.27, 0.27, -0.2)</td>
<td>(-0.264, 0.264, -0.138)</td>
<td></td>
</tr>
</tbody>
</table>

Table 1.2: Equilibrium editorial position of media L, R and Q \((\theta_L^*, \theta_R^*, \theta_Q^*)\) when \( \theta_E = 0 \) and Q is located at -0.5. For comparison purposes, we have highlighted L(R)'s choices in blue (red) for \( \lambda_Q = 0.1 \). Editorial choices are more extreme with the new biased media Q from the duopoly model in Table 1.

<table>
<thead>
<tr>
<th>((\lambda_L, \lambda_R))</th>
<th>(\lambda_Q)</th>
<th>0.1</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.1, 0.1)</td>
<td>(-0.46, 0.46, -0.36)</td>
<td>(-0.436, 0.436, -0.247)</td>
<td></td>
</tr>
<tr>
<td>(0.1, 0.5)</td>
<td>(-0.426, 0.31, -0.33)</td>
<td>(-0.41, 0.294, -0.226)</td>
<td></td>
</tr>
<tr>
<td>(0.5, 0.5)</td>
<td>(-0.285, 0.285, -0.31)</td>
<td>(-0.27, 0.27, -0.2)</td>
<td></td>
</tr>
</tbody>
</table>

Table 1.3: Equilibrium editorial position of media L, R and Q \((\theta_L^*, \theta_R^*, \theta_Q^*)\) when \( \theta_E = 0 \) and Q is located at -0.75. For comparison purposes, we have highlighted L(R)'s choices in blue (red) for \( \lambda_Q = 0.1 \). Editorial choices are not only more extreme from the duopoly model, but also from Table 2 where Q is relatively less biased.
Taking into account of the nature of profit functions in 16, 17 and 18, we have the following corollary.

Remark 1.10. Given $\theta_E = 0$, the equilibrium editorial choices of $L$ and $R$ become more biased with the entry of a biased third firm $Q$. If $Q$ is unbiased or positioned at 0, then it has no effect on the equilibrium editorial choices of $L$ and $R$.

If media is covering a story about a neutral event, then the entry of a new firm which is biased, increases the absolute levels of slants in both $L$ and $R$. To get more clarity, one can compare the numbers of the editorial choices of $L$ ($R$) highlighted in blue (red) across Tables 1, 2 and 3. The proof of the second part is straightforward and entails that the presence of the unbiased media is unable to cater to the ideological beliefs of readers along the ideological spectrum- thereby, biased media firms stay persistent in their prior editorial choices and refuses to decrease their slant.

1.6 Concluding comments

The model presents a new way to understand how partisan media firms bias news of various topics if it accounts for its ideology and also how readers perceive news. The main trade-off that has been discussed is that ideological readers are reluctant to accept factual information. For example, leftist consumers would refuse to accept an factual academic report that criticizes its government’s monetary policies which was unable to control inflation. If the leftist media $L_1$ is too ideology motivated, it will defend the policy by highlighting its power towards lowering unemployment. If another leftist media $L_2$ is not too blinded by ideology payoffs, then it might accept the critic. Now, how would the leftist and the rightist consumers perceive these reports? It is seen that a fraction of rightist readers would prefer the news of $L_2$, hence motivating more accurate reporting. However, more extreme rightist consumers would adhere to its like-minded media $R$ and be more satisfied with its reporting.

In this set-up, policy measures should be designed which make factual information more receptive to the entire reader populace. I suggest that the exogenous media can take up an educative role by providing factual information in a manner such that the intrinsic facts get primary attention to the readers, thereby not letting their ideology override their judgement of a topic. Ideology worsens societal divide which raises disagreement regarding societal issues like proscribing abortion, anti-immigration attitudes\footnote{[38]’s survey shows how linkages to ideology creates a greater racial and ethnic divide and the public’s tendency to accept facts.} which according to the present
analysis can be assuaged when people’s reception of news is guided strongly by the facts of
the matter. Simply increasing media competition by introducing less partisan media might
lead to more accurate information provision, but its effect on reader perception remains
ambiguous.
Chapter 2

Monopoly pricing under wishful thinking

2.1 Introduction

When consumers purchase non-durable products — vaccination, haircuts, coffee — repeat purchase depends on their first-time experience. The consumer can opt out or choose not to re-invest following a bad experience. Inherent in this behavior lies the role of expectations about their future experience conditional on their first-time experience. While forming expectations, if the beliefs of consumers are biased in favor of certain future states where good experience is more likely, then a consumer might not refrain from repeat purchase even after a bad experience. Therefore the question of whether a good experience is always necessary to elicit repeat purchase needs investigation. This paper pursues to do this by using a simple two-period model of pricing by a monopolist in presence of consumers who are wishful thinkers. I analyze how wishful consumers impact intertemporal pricing strategies (under commitment and non-commitment) of the monopolist alongside understanding the conditions where the monopolist is able to design pricing schemes like introductory pricing, goodwill pricing and price bundling.

A wishful thinker is an individual whose beliefs about future states of the world are dependent on his current well-being or endowment. In this line, as [14] remarks “..wishful thinking involves choosing to believe that the truth is what one would like the truth to be”(p.2). In the case of repeat purchase, if one assumes that the buyer is cognizant about underlying random factors impacting his experience, then wishful thinking gains more force. For instance, after having a bad experience at a salon, a consumer might plan never to visit again or might presume the inability of his hair stylist to be the main driver of his bad experience. In the latter case, he might visit the salon again, hoping to be attended by a more skillful hair stylist. One plausible explanation behind this behavior is that the consumer’s valuation about the salon might be high enough which directs his expectations in favor of re-visiting the salon. Or suppose if a health conscious individual falls very sick after one vaccination shot, it is unlikely that he will refrain from taking another shot. This
is because his current concern about health allows him to be more cognizant about other random factors (like prior medical condition) which have lead to his sickness. This leaves his expected gain from taking the vaccination high enough to take the vaccine again, thereby discounting the possibility of falling sick.

Hence, as implied from the above examples, belief distortion is subjective. To understand the nature of belief distortion, I follow the theoretical work of [37] which standardizes the preferences of a wishful thinker’s actions through a behavioral axiom thereby characterizing wishful thinking. I also borrow from [40] and [39] to model the distortion in beliefs as a linear distortion. I postpone this discussion to a latter section of this paper.

The primary model of this paper describes the interaction between a monopolist and a continuum of consumers who engages in repeat purchase. For simplicity, I consider a two period model where repeat purchase can occur at most once. In each time period the consumer derives either a good or bad experience which is correlated with the state being high or low. The priors on the states are common knowledge to both the consumers and the monopolist. States are never known but there is a common signal that good (bad) experience is more probable in the high (low) state. Consumers differ only in their level of valuation for the product. After having a good or bad experience in the first period, consumers update their beliefs about the occurrence of a high or a low state in the succeeding period and decide to buy again. Wishful thinking is channelled through beliefs which are distorted in proportion to a consumer’s valuation for the product. In essence, higher valuation will lead to greater distortion of beliefs in favor of the high state. Given this setup, the monopolist has to choose the second period price. I first derive a comparative analysis of prices in both periods and how it stands relative to the standard scenario with non-wishful consumers. In this context, I examine whether the monopolist caters to its first-period consumers (goodwill pricing). I then extend the study to learn how prices in both periods will change when the monopolist has to commit towards announcing prices of both periods at the beginning of the first period. Finally, consumers aside from being wishful can be naive (myopic) or sophisticated (forward-looking). So it will be interesting to understand how prices in both periods change under these two consumer types.

In terms of past literature, this paper is at the intersection of the literature on monopoly pricing with non-durable goods and the decision theoretic literature of wishful thinking. One of the closest works on this specific area is ([49], p.54) who derives a monopolist’s optimal pricing scheme under two state space in presence of over-optimistic consumers. However, this analysis differs on two fronts. First, the monopolist’s prior of the bad state is 1 while the optimistic consumer’s prior on the good state is $\theta$ which directly characterizes his degree
of optimism. Second, consumers have state-dependent preferences (higher willingness to pay in good state than in a bad state). Third, the game moves over two time periods where in period 1, the consumer chooses to accept the monopolist’s pricing scheme $T$ and conditional on this acceptance, he makes a purchase in period 2 in either a good or bad state. The model of repeat sales ([50], p.116) is more closer to this paper. However, it considers repeat purchase with standard (non-wishful) consumers and the experience is constant rather than being correlated with the states.

One of the first findings of this paper is the characterization of the pricing scheme with uniformly distributed consumers. This finding is standard to the existing literature on monopoly pricing [11, 50] which posits that the optimal price is the mean of the expected valuation and the marginal cost. When it comes to comparison of optimal prices across different periods in an intertemporal environment, the nature of consumers and uncertainty come to play. When there is no randomness in quality of the product across periods, [11] suggests that with myopic consumers, the monopolist can raise price in the first period which leaves a greater market to cater in the next period. Given the same setting, this pricing strategy is less effective with rational consumers who thwarts the possibility of a higher price in the first period as the first period price elasticity is far higher than future periods. However, in the presence of randomness (for instance, the product might not match the consumer with a fixed probability) as shown by [50], the power of the monopolist becomes limited, even with myopic consumers. In a two-period setup, the first and second period prices are identical. This implies that the monopolist tries to retain its first period clients in the second period. Also, it does not practice introductory-pricing to attract a larger client base in the first period. This paper also finds that goodwill pricing is the optimal choice for the monopolist who sets the price in each period separately. However, since buyers are wishful, the goodwill price is higher than what is found in [50]. Nonetheless, this price is less than the first period price. Hence, whenever there exists uncertainty about the consumer’s experience with the good, wishful thinking in consumers allows them to be relatively more optimistic about the chance of a good experience in the future, allowing them to reinvest in it again.

The chances of such reinvestment rises when there is sufficient uncertainty of a good experience across states (say, the conditional probability of a good experience is very likely under a high state and very low under a low state). On the contrary, if the chance of a good experience is (almost) equally likely across both high and low states, wishful thinking fails to generate optimism in the consumer. Hence, the posterior beliefs are distorted much less in favor of the high state. The magnitude of this distortion, is also determined to a large
extent by the priors on the high and low states. If the prior on the high state is already very high, the marginal impact on belief distortion is weakened because there does not exist much room to be optimistic or distort the already high prior on the high state. This resonates with the [14]'s remark that “Wishful thinking is not magical thinking.” (p.10).

In addition, the random experience factor brings out two sides of wishful thinking in consumers. Using comparative statics, it is seen that good experiences in one period make the consumer more optimistic about the occurrences of future high states in the succeeding periods. On the contrary, the advent of a bad experience actually makes the wishful consumer to believe that things cannot get any more worse. This effect gets stronger with consumers with higher valuation. To throw more light on how beliefs are distorted, I add another state medium and examine whether having more number of underlying states weakens or strengthens wishful thinking. Imagine, a good experience from a drug can emerge under \( M > 2 \) underlying states. In this case, consumers compare likelihoods across each pair of states, \( \binom{M}{2} \) in total. Hence the direction of belief distortion is no longer straight-forward.

The paper proceeds in the following manner. Section 2 describes the model setup, section 3 discusses how beliefs are distorted using a specific distortion functional form. Section 4 introduces the monopoly pricing model and compares how the pricing strategies differ relative to a model with standard consumers. Section 5 examines the conditions where the monopolist can practice goodwill pricing and under what conditions can he raise his price and care no longer about goodwill. In this section, I extend the observations with one added underlying state to see how distortion of beliefs are distributed across the three states and how do optimism in wishful consumers differ with respect to the benchmark model with two states. In section 7, monopolist caters to sophisticated or forward looking consumers where two pricing strategies are examined- commitment and no commitment. Finally in section 8, I study the strategy where the monopolist bundles two units of goods and offers to sell at a single price.

2.2 Model setup

A monopolist sells a non-durable product over two time periods \( t = \{1, 2\} \). The action of the monopolist is to charge two \( p_1 \) and \( p_2 \) at \( t = 1 \) and \( t = 2 \) respectively. Consumers decide either to buy the product \( (a_t = 1) \) or not buy \( (a_t = 0) \) at each \( t \). If the consumer buys the product, he is subjected to either a good experience \( (e = 1) \) or a bad one \( (e = 0) \). There are two underlying states of the world \( S \in \{l, h\} \) which remain unknown to the consumers.
after their experiences. The prior on the low state is \( \mu \in (0, 1) \) There is an imperfect, but informative signal which helps the consumers form a posterior about each state following an experience — \( q(e = 1|h) = q_h > 0.5 \) and \( q(e = 1|l) = q_l < 0.5 \). The first signal suggests that the probability of getting a good experience is higher than 0.5 if the underlying state is ‘high’ while it is less than 0.5 if the underlying stata is low. Similarly, I denote \( q(e = 0|h) = q^0_h \) and \( q(e = 0|l) = q^0_l \).

I consider two strategies of the monopolist - first when he cannot commit and therefore announces the two prices separately before the beginning of each time period and second, he can commit to announcing both prices. Consumers being wishful leads to second period profit to depend on first period price. Given this dependence, what needs to be seen is the monopolist’s optimal price path under two different consumer types (naive and sophisticated) and two types of monopolist (committed and non-committed).

The (gross) utility of a representative consumer \( i \) with an idiosyncratic taste \( \theta_i \) is given by

\[
U_i = \begin{cases} 
\theta_i e & \text{if he buys} \\
0 & \text{otherwise}
\end{cases}
\]

I consider four types of distributional assumptions on the taste parameter \( \theta \). First, there is a single type of consumers where \( \theta \) takes a single value; second, \( \theta \) has two types (low and high); third, \( \theta \) has three types (low, medium and high); fourth, \( \theta \) is uniform over \([0, 1]\).

### 2.2.1 Wishful thinking: theory

I follow the axiomatic foundations of WT from [37] where wishful agents are subjective expected utility maximizers and satisfy the axiom of wishful thinking. Agents distort his beliefs regarding certain states where his current endowment or well-being will do relatively better. In this model, WT in a consumer evolves following their good or bad experience and the extent of belief distortion is a function of the expected valuation - product of own valuation and the likelihood that the valuation will be realized conditional on the state. I now define the below representation of wishful thinking, followed directly from [37].

**Definition 2.1.** An individual admits a wishful thinking representation if there exists an utility function \( U : A \times \theta \rightarrow \mathbb{R} \), a belief \( \mu \in \Delta(S) \), and for each \( E \), an increasing distortion function \( \delta : U \times \Delta(S) \rightarrow \mathbb{R}_{++} \) such that:
(i) \( \succeq \) is given by \( \mathbb{E}(U) = \sum_{s \in S} U(a, s, \theta)\mu(s) \), and

(ii) \( \succeq_\delta \) is given by \( \mathbb{E}_\delta(U) = \sum_{s \in S} U(a)\mu_\delta(s) \), where \( \mu_\delta(s) = \delta(\mathbb{E}(U(.)))\mu(s) \).

The pair \((\succeq, \succeq_\delta)\) denotes standard preferences and preferences under distortion \(\delta\). The distortion function \(\delta\) is increasing in the value of expected utility which entails that wishful consumers maximizes their subjective expected utility. Now the representation of \(\delta\) requires structure on beliefs and how wishful agents process payoffs across various states. [37] presents the notion of consequential distortion which formalizes one way of how beliefs are distorted. Consequential distortion posits that belief distortion across states is proportional to the expected payoffs relative across states. In this line, as will be seen here, the distortion in beliefs across the two states is proportional to the ratio of expected payoffs contingent on the state being (low and high) define the degree of distortion. Given this nature of distortion, if a good state is (almost) equally probable in both high and low state, then the ratio of distorted function across the two states tends to one. Therefore, the posterior beliefs tends to be closer to to the standard case with non-wishful buyers. On the contrary, if the difference in likelihood is greater, it makes way for larger distortion in beliefs.

Definition 2.2. Given Bayesian buyers and the distortion function \(\delta\), the likelihood ratio of state \(h\) to state \(l\) is given as

\[
\frac{\hat{\mu}(h|e)}{\hat{\mu}(l|e)} = \frac{1 - \mu}{\mu} \times \frac{q(e|h)}{q(e|l)} \times \frac{\delta_\theta(q(e|h))}{\delta_\theta(q(e|l))}
\]

The distortion function is increasing in the gap between the prior of a high state \((1 - \mu)\) vis-a-vis a low state \((\mu)\). In tandem, an increasing gap between the conditional probability (contingent on the state) increases the above ratio, the magnitude being dependent on the specific functional form of \(\delta\). The distortion function is not constant, but increases with the valuation for the product. This implies that buyers with higher valuation are overly optimistic about a high state in the succeeding period. However, as will be seen later through comparative statics, the direction and magnitude of this distortion will be conditional on whether the experience is bad or good.

To understand the application of wishful thinking on this monopoly pricing game, it would be simple to assume a more specific functional form of \(\delta\) which preserves the feature that distortion in beliefs depends on the relative expected payoffs across states. One such functional form is the exponential distortion of beliefs. [39] formalizes this particular distortion type through the axiom of Shift Invariance which requires subjective beliefs to depend on how outcomes in each state compares to with that of other states.
2.3 Belief distortion

The source of belief distortion about future states is the experience - good or bad - the consumer has in period 1 after buying the good. This experience is also impacted by the taste parameter $\theta$. I consider $\delta : U(a, s, \theta) \rightarrow \mathbb{R}_{++}$ to be an exponential functional form with a distortion parameter $\lambda > 1$ (following a good experience) and $\beta < -1$ (following a bad experience). I assume consumers are Bayesian and update their prior beliefs about any future state consistent with Baye’s rule. Nevertheless, being wishful, the belief is distorted in favor of high states.

2.3.1 Consumers who have good experience in period 1

The belief updation of wishful consumers about the state being low following $e = 1$ is given below -

$$\hat{\mu}_\delta(l|e = 1) = \frac{\mu e^{(\lambda q_h)} q_l}{\mu e^{(\lambda q_h)} q_l + (1 - \mu) e^{(\lambda q_h)} q_h}$$  \hspace{1cm} (2.1)

Similarly, the updated belief of a high state in period 2 is given by

$$\hat{\mu}_\delta(h|e = 1) = \frac{(1 - \mu) e^{(\lambda q_h)} q_h}{\mu e^{(\lambda q_h)} q_l + (1 - \mu) e^{(\lambda q_h)} q_h}$$  \hspace{1cm} (2.2)

Dividing (2) by (1),

$$\frac{\hat{\mu}_\delta(h|e = 1)}{\hat{\mu}_\delta(l|e = 1)} = \frac{(1 - \mu) e^{(\lambda q_h)} q_h}{\mu e^{(\lambda q_h)} q_l} = \frac{(1 - \mu) q_h}{\mu q_l} e^{\lambda(q_h - q_l)}$$  \hspace{1cm} (2.3)

The expected utility in period 2 of a consumer following $e = 1$ is given by,

$$E_{\delta_1}(U) = \theta \hat{\mu}_\delta(l) q(e = 1|l) + \theta \hat{\mu}_\delta(h) q(e = 1|h) = \theta \hat{\mu}_\delta(l) q_l + \theta \hat{\mu}_\delta(h) q_h$$  \hspace{1cm} (2.4)

Using 3, the expected utility of any consumer can be expressed as

$$E_{\delta_1}(U) = \theta \hat{\mu}_\delta(l) q_l \left[ 1 + e^{\lambda(q_h - q_l)} \frac{1 - \mu}{\mu} \left( \frac{q_h}{q_l} \right)^2 \right]$$  \hspace{1cm} (2.5)

The expected utility of a standard consumer (not wishful) is shown as
\[ E_1(U) = \theta \hat{\mu}(l) q_l \left[ 1 + \frac{1 - \mu}{\mu} \left( \frac{q_h}{q_l} \right)^2 \right] \] (2.6)

Example 2.3. I assume the distortion parameter \( \lambda = 1.5 \), \( \mu(l) = 0.5 \), \( q_h = 0.6 \), \( q_l = 0.3 \), \( q^0_h = 0.4 \), \( q^0_l = 0.7 \) and \( \lambda = 1.5 \).

Then the experience which a wishful buyer expects in period 2 is given by

\[ E_\delta(e_1) = \frac{0.3 e^{0.45 \theta}}{0.3 e^{0.45 \theta} + 0.6 e^{0.9 \theta}} (0.3) [1 + 4 e^{0.45 \theta}] \]

For the non-wishful consumer, \( E(e_1) = 0.501 \) and for \( \theta \geq 0.0334 \), \( E_\delta(e_1) \geq E(e_1) \).

2.3.2 Consumers who have bad experience in period 1

In this section, I focus on belief distortion of buyers who buy in period 1, but end up having bad experience \( (e = 0) \). I denote \( q^0_l = q(e = 0|l) \) and \( q^0_h = q(e = 0|h) \). Under WT, the updated belief of a low state in period 2 is described as

\[ \hat{\mu}_\delta(l|e = 0) = \frac{\mu e^{(\beta \theta q^0_l)} q^0_l}{\mu e^{(\beta \theta q^0_l)} q^0_l + (1 - \mu) e^{(\beta \theta q^0_h)} q^0_h} \] (2.7)

Similarly, the updated belief of a high state in period 2 is described as

\[ \hat{\mu}_\delta(h|e = 0) = \frac{(1 - \mu) e^{(\beta \theta q^0_h)} q^0_h}{\mu e^{(\lambda \beta \theta q^0_l)} q^0_l + (1 - \mu) e^{(\beta \theta q^0_h)} q^0_h} \] (2.8)

Dividing (8) by (7),

\[ \frac{\hat{\mu}_\delta(h|e = 0)}{\hat{\mu}_\delta(l|e = 0)} = \frac{(1 - \mu) e^{(\beta \theta q^0_h)} q^0_h}{\mu e^{(\lambda \beta \theta q^0_l)} q^0_l} = \frac{1 - \mu}{\mu} \frac{q^0_h}{q^0_l} e^{\beta \theta (q^0_h - q^0_l)} \] (2.9)

Following from the above analysis, the expected utility of wishful consumer in period 2 who had a bad experience \( (e = 0) \) in period 1 is given by,

\[ E_{\delta_0}(U) = \theta \hat{\mu}_\delta(l) q(e = 1|l) + \theta \hat{\mu}_\delta(h) q(e = 1|h) = \theta \hat{\mu}_\delta(l) q_l + \theta \hat{\mu}_\delta(h) q_h \] (2.10)
2.4 Monopoly Pricing

The presence of wishful consumers creates dependence across the two time periods which leads to the choice of second period price to be a function of the first period price. In the first period, consumers buy the product if the expected utility net of price is non-negative which implies \( \theta \geq \frac{p_1}{\mathbb{E}(e)} \). If this condition is satisfied, the consumer is subjected to either \( e = 1 \) or \( e = 0 \) which leads to a distorted posterior (section 3) and the consumer buys in period 2 only if \( \theta \geq \frac{p_1}{\mathbb{E}(e_0)} \). If the period 1 participation constraint is not satisfied, then the consumer does not buy the product and in period 2 is akin to a standard consumer.\[\text{This implies that } p_2 \text{ is such that consumer buys in period 2 even after having bad experience } (e_0).\]
In this section, I will discuss the optimal price paths under situations where the monopolist announces prices with and without commitment and consumers are naive and sophisticated (apart from being wishful).

To start, I assume consumers are distributed along a discrete distribution (say, consumers are of two types low and high) and then assume that type is continuous along $F$ with continuous cdf $f$ over $(0, 1)$. For discrete distributions, it is quite straightforward that the monopolist makes his choice by accounting for the mass of consumers across different types. However, since consumers are wishful, price path may depend on the mass of consumer on each type in a different manner than in the case of standard consumers.

Pricing strategy like goodwill and introductory pricing depends on whether consumers separately maximize each period utility (naive) or consider the overall discounted utility from both periods (sophisticated). This also determines the nature of distorted Bayesian updating of beliefs about future states.

2.4.1 Naive consumers and non-committed monopolist

In the discrete case, the benchmark scenario concerns the case with homogeneous consumers of type $\theta$ which resemble their taste for the product and eventually their willingness to pay for it.

Homogeneous consumers

The problem of the monopolist is therefore $^2$

$$\max_{p_1^W, p_2^W} \Pi_1(p_1^W) + \Pi_2(p_2^W, p_1^W)$$

$$s.t \quad \theta \mathbb{E}^W(e) - p_1^W \geq 0$$
$$\theta \mathbb{E}^W(e|e_0, \theta) - p_2^W \geq 0$$
$$\theta \mathbb{E}^W(e|e_1, \theta) - p_2^W \geq 0$$

$^2$The superscript $W$ will be used for wishful consumers henceforth. No superscript will apply for standard consumers. $\mathbb{E}^W(e|e_0, \theta)$ defines the expectation of a wishful consumer following a bad experience ($e_0$) in period 1. Analogous explanations hold for good experience.
If the participation constraints of period 1 are not satisfied, the entire analysis grabs no bite. To extract greater surplus from wishful consumers, the monopolist need to attract them in the first period. Hence \( p_1^W = \theta \mathbb{E}(e) \). In the second period, a fraction \( \pi_1 \) gets good experience while the rest get a bad experience. Hence, the monopolist’s optimal price schedule is as below

\[
(p_1^W, p_2^W) = \begin{cases} 
\left( \theta \mathbb{E}(e), \theta \mathbb{E}^W(e|e_0, \theta) \right) & \text{iff } \frac{\mathbb{E}^W(e|e_1, \theta)}{\mathbb{E}^W(e|e_0, \theta)} < \frac{1}{\pi_1} \\
\left( \theta \mathbb{E}(e), \theta \mathbb{E}^W(e|e_1, \theta) \right) & \text{iff } \frac{\mathbb{E}^W(e|e_1, \theta)}{\mathbb{E}^W(e|e_0, \theta)} \geq \frac{1}{\pi_1}
\end{cases}
\]

Following from above \( p_2^W > p_1^W \) if \( \frac{\mathbb{E}^W(e|e_1, \theta)}{\mathbb{E}^W(e|e_0, \theta)} \geq \frac{1}{\pi_1} \). In the standard case (non-wishful consumers the price schedule takes an analogous form where the first period prices are identical to the wishful case. However, to understand the impact of wishful thinking, it is important to compare the second period prices across either cases. \( p_2^* > p_1^* \) if \( \frac{\mathbb{E}(e|e_1)}{\mathbb{E}(e|e_0)} \geq \frac{1}{\pi_1} \). The element of interest lies in knowing the conditions on model parameters under which the inequality condition is weaker in the wishful case relative to the standard case. This will imply that a higher second period price is more prevalent in the wishful case.

Remark 2.4. Consider homogeneous consumers and the prior on the low and high states to be equally likely or \( \mu = 0.5 \). Then the monopolist will always charge a higher second period for wishful consumers compared to standard consumers if

\[
1 < \frac{q_h}{q_l} < \frac{e^\lambda \theta (q_h - q_l)}{e^{\lambda \theta (q_h - q_l)} - 1}
\]

The above interval ensures that ratio of expectation from good and bad experience for wishful consumers is strictly greater than for non-wishful consumers. The lower threshold follows from the basic assumptions of the model which suggests that a good experience is more probable in a high state than a low state. The upper threshold brings out the insight that wishful thinking will be weakened if the good experience becomes almost certain under a high state. Wishful thinking emerges when there is some degree of uncertainty which the consumers use as a channel to justify why they should invest on the good. The assumption of equal priors strengthens this observation because the hike in prices can then be accrued more to the effect of wishful consumers’ belief distortion.
Two consumer types

Following the homogeneous case, consumers can be of two types — $\gamma_L \in (0, 1)$ share of consumers are low ($\theta_L$), the rest are high ($\theta_H$) where $\theta_H > \theta_L$. Monopolist knows $\gamma_L$, but does not know if any given consumer is high or low type. The monopolist’s problem with standard consumers after taking into account the complete set of participation constraints for both low and high type is

$$\max_{p_1, p_2} \Pi_1(p_1) + \Pi_2(p_2, p_1)$$

s.t.  
\[ \theta_L \mathbb{E}(e) - p_1 \geq 0, \quad \theta_H \mathbb{E}(e) - p_1 \geq 0 \]
\[ \theta_L \mathbb{E}(e|e_0) - p_2 \geq 0, \quad \theta_H \mathbb{E}(e|e_0) - p_2 \geq 0 \]
\[ \theta_L \mathbb{E}(e|e_1) - p_2 \geq 0, \quad \theta_H \mathbb{E}(e|e_1) - p_2 \geq 0 \]

The first set of constraints refer to the participation constraints (P.C) of the low and high types given period 1 prices $p_1$. If the P.C of the low types are satisfied, then so are that of the high types, but not the other way round. The next two lines refer to the participant constraints of low and high types in period 2. $\mathbb{E}(e|e_0)$ are $\mathbb{E}(e|e_1)$ are the period 2 expectations following a bad and good experience respectively.

The monopolist’s problem with wishful consumers is

$$\max_{p_1^W, p_2^W} \Pi_1^W(p_1^W) + \Pi_2^W(p_2^W, p_1^W)$$

s.t.  
\[ \theta_L \mathbb{E}^W(e) - p_1^W \geq 0, \quad \theta_H \mathbb{E}^W(e) - p_1^W \geq 0 \]
\[ \theta_L \mathbb{E}^W(e|e_0, \theta_L) - p_2^W \geq 0, \quad \theta_H \mathbb{E}^W(e|e_0, \theta_H) - p_2^W \geq 0 \]
\[ \theta_L \mathbb{E}^W(e|e_1, \theta_L) - p_2^W \geq 0, \quad \theta_H \mathbb{E}^W(e|e_1, \theta_H) - p_2^W \geq 0 \]

The participation constraint in period 1 remains same. However in period 2, with wishful consumers, the expectation becomes a function of the taste parameter $\theta_L$ and $\theta_H$.

The first-period profit of the monopolist is the same in both the standard and wishful
case. I use the below notations for the remaining sections

\[
\mathbb{I}(\theta_L E(e) - p_1) = \begin{cases} 
1 & \text{if } \theta_L E(e) - p_1 \geq 0 \\
0 & \text{otherwise}
\end{cases}
\]

Given the above, the first-period profit (for both the standard and wishful cases) is given by

\[
\Pi_1(p_1) = \gamma_L \left( \max \left\{ \mathbb{I}(\theta_L E(e) - p_1), 0 \right\} \right) + (1 - \gamma_L) \left( \max \left\{ \mathbb{I}(\theta_H E(e) - p_1), 0 \right\} \right)
\]

Now, in the above maximization problem, if \( p_1 \) (\( p_1^W \)) satisfies the first period participation constraint for the low types, then \( \Pi_1 = [\theta_L E(e)[\gamma_L + (1 - \gamma_L)] = \theta_L E(e) \). However if \( p_1 \) is such that the participation constraint of only the high type binds, then \( \Pi_1 = [\theta_H E(e)[0 + (1 - \gamma_L)] = \theta_H E(e)(1 - \gamma_L) \). Therefore, the choice of \( p_1 \) is carried over in period 2 which I lay out in the next subsection.

In the second period, the impact of first period price can be seen on the second period period profit. This magnitude of impact varies across standard and wishful consumers as will be discussed below. The general form of period 2 profit functions in presence of standard and wishful consumers is given below. The difference lies in the expectation operator which in the wishful case is a function of the subjective taste parameter.

\[
\Pi_2(p_1, p_2) = p_2 \gamma_L \left( \max \left\{ \mathbb{I}(\theta_L E(e) - p_1), 0 \right\} \right) \left[ \pi_0 \max \left\{ \theta_L E(e|e_0) - p_2, 0 \right\} + \pi_1 \max \left\{ \theta_L E(e|e_1) - p_2, 0 \right\} \right] + p_2(1 - \gamma_L) \left( \max \left\{ \mathbb{I}(\theta_H E(e) - p_1), 0 \right\} \right) \left[ \pi_0 \max \left\{ \mathbb{I}(\theta_H E(e|e_0) - p_2), 0 \right\} + \pi_1 \max \left\{ \theta_H E(e|e_1) - p_2, 0 \right\} \right]
\]

\[
\Pi_2^W(p_1^W, p_2^W) = p_2^W \gamma_L \left( \max \left\{ \mathbb{I}(\theta_L E(e) - p_1^W), 0 \right\} \right) \left[ \pi_0 \max \left\{ \theta_L E^W(e|e_0\theta_L) - p_2^W, 0 \right\} + \pi_1 \max \left\{ \theta_L E^W(e|e_1, \theta_L) - p_2^W, 0 \right\} \right] + p_2^W(1 - \gamma_L) \left( \max \left\{ \mathbb{I}(\theta_H E(e) - p_1^W), 0 \right\} \right) \left[ \pi_0 \max \left\{ \mathbb{I}(\theta_H E^W(e|e_0, \theta_H) - p_2^W), 0 \right\} + \pi_1 \max \left\{ \theta_H E^W(e|e_1, \theta_H) - p_2^W, 0 \right\} \right]
\]
In period 1, if the P.C on the low type is binding, or \( p_1 = \theta_L E(e) \) \((I(\theta_L E(e) - p_1) = 1)\), then period 2 profit is dependent on revenues from both low and high types.

\[
\Pi^W_2(\theta_L E(e), p^W_2) = p_2 \gamma_L \left( \pi_0 \max \left\{ \Pi(\theta_L E^W(e|e_0, \theta_L) - p_2), 0 \right\} + \pi_1 \max \left\{ I(\theta_L E^W(e|e_1, \theta_L) - p_2), 0 \right\} \right) \\
+ p_2(1 - \gamma_L) \left( \pi_0 \max \left\{ \theta_H E^W(e|e_0, \theta_H) - p_2, 0 \right\} + \pi_1 \max \left\{ \theta_H E^W(e|e_1, \theta_H) - p_2, 0 \right\} \right)
\]

On the contrary, if in period 1 price \( p_1 = \theta_H E(e) \), then the P.C of only the high type \((I(\theta_L E(e) - p_1) = 0 \text{ and } I(\theta_H E(e) - p_1) = 1)\). Hence revenue in period 2 accrue only from the high type as given by

\[
\Pi^W_2(\theta_H E(e), p^W_2) = p_2 (1 - \gamma_L) \left( \pi_0 \max \left\{ \theta_H E^W(e|e_0, \theta_H) - p_2, 0 \right\} + \pi_1 \max \left\{ \theta_H E^W(e|e_1, \theta_H) - p_2, 0 \right\} \right)
\]

The above set of equations hold also for the standard consumers except for the fact that their expectation of future states is not a function of their taste parameter. Saying this, I omit writing the equations here in interest of observational ease.

Under what conditions would the monopolist choose to serve only the high type in both periods with standard consumers, but in presence of wishful consumers, would lower the price to include both types? Both types will be served if the marginal gains from decreasing price to serve a portion of low types in period 2 exceeds the loss in first period profit from higher prices (which was aimed to serve only high types).

With standard consumers, the optimal price schedule is given below:

\[
(p^*_1, p^*_2) = \begin{cases} 
(\theta_L E(e), \theta_L E(e_0)) & \text{if } \frac{\theta_L}{\theta_H} \geq (1 - \gamma_L) \text{ and } \frac{E(e_0)}{E(e_1)} > (1 - \gamma_L) + \gamma_L \pi_1 \\
(\theta_H E(e), \theta_H E(e_0)) & \text{if } \frac{\theta_L}{\theta_H} < (1 - \gamma_L) \text{ and } \frac{E(e_0)}{E(e_1)} > \pi_1 
\end{cases}
\]

Now, suppose, the share of low types is strictly below the threshold \( \frac{\theta_H}{\theta_L} \), then in presence of standard consumers, the monopolist will serve only the high type, \( p^*_1 = \theta_H E(e) \). The interesting question is under what condition would, in presence of wishful consumers, the monopolist will serve the low type \( p^W_1 = \theta_L E(e) \)?

If under the standard case, the share of low type is below the prescribed threshold, the
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The monopolist finds it optimal to serve only the high type as given below —

\[
(1 - \gamma_L)\theta_H \mathbb{E}(e) + (1 - \gamma_L)p^H_2(e_0) > \theta_L \mathbb{E}(e) + \left(\gamma_L \pi_1 + (1 - \gamma_L)\right)p^L_2(e_1)
\]

In the above inequality, \(p^H_2(e_0) = \theta_H \mathbb{E}(e|h_2 = 0)\) denotes second period price which serves to high types who had a bad experience in period 1 (at this price all high types are served in period 2). \(p^L_2(e_1) = \theta_L \mathbb{E}(e|h_2 = 1)\) denotes the price which includes the low type who had a good experience in period 1 and all high types (assuming that expectation of high type following a bad experience is at least equal to the expectation of the low type following a good experience). Rearranging terms on either side of the inequality,

\[
\left(1 - \gamma_L\right)\theta_H - \theta_L \mathbb{E}(e) > \left(\gamma_L \pi_1 1 - \gamma_L\right)\mathbb{E}_1(e)\theta_L - (1 - \gamma_L)\mathbb{E}_0(e)\theta_H
\]

I now define \(\Delta\) such that

\[
\left(1 - \gamma_L\right)\theta_H - \theta_L \mathbb{E}(e) + \Delta = \left(\gamma_L \pi_1 + (1 - \gamma_L)\right)\mathbb{E}_1(e)\theta_L - (1 - \gamma_L)\mathbb{E}_0(e)\theta_H
\]  

(2.13)

Everything remaining constant, with wishful consumers, the following inequality implies that serving the low types in period 1 and thereby retaining a portion of low types in period 2 is a dominant strategy than serving only high types.

\[
\theta_L \mathbb{E}(e) + p^W_2(e_1, \theta_L)\left(\gamma_L \pi_1 + (1 - \gamma_L)\right) > (1 - \gamma_L)\theta_H \mathbb{E}(e) + \left(1 - \gamma_L\right)p^W_2(e_0, \theta_H)
\]

(2.14)

The above inequality suggests that if low types are not served under the wishful case, then the monopolist loses out on the increased revenue which could have been raised by retaining low types who had a good experience. As the belief distortion parameter increases and the gap between \(\theta_H\) and \(\theta_L\) decreases, low types will likely be served than under the standard consumer case. Let \(p^H_2(e_0, \theta_H) = \theta_H \mathbb{E}(e|h_2 = 0, \theta_H) = \mathbb{E}(e)\delta_0(e|\theta_H)\) and \(p^L_2(e_1) = \theta_L \mathbb{E}(e|h_2 = 1)\). Rearranging terms on either side of the inequality,

\[
\left(1 - \gamma_L\right)\theta_H - \theta_L \mathbb{E}(e) < \left(\gamma_L \pi_1 - \gamma_L\right)\theta_H \mathbb{E}_0(e|\theta_L) - (1 - \gamma_L)\theta_H \mathbb{E}_0(e|\theta_H)
\]

(2.15)

Combining 2.13 and 2.15, the following must be satisfied
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\[
\frac{\mathbb{E}_{\delta_1}(e|\theta_L) - \mathbb{E}_1(e)}{\mathbb{E}_{\delta_0}(e|\theta_H) - \mathbb{E}_0(e)} \geq \frac{1 - \gamma_L}{1 - \gamma_L + \gamma_L\pi_1} \frac{\theta_H}{\theta_L} \geq \frac{1 - \gamma_L}{1 - \gamma_L + \gamma_L\pi_1} \frac{\theta_H}{\theta_L} > \Delta
\] (2.16)

(2.17)

Therefore, in the case of wishful consumers, low type will be served under weaker threshold conditions as formalized below,

\[
(p_{1W}^*, p_{2W}^*) = \begin{cases} 
(\theta_L \mathbb{E}(e), \theta_L \mathbb{E}^W(e_0)) & \text{if } \frac{\theta_L}{\theta_H} \geq (1 - \gamma_L) - \hat{\theta} \quad \text{and} \quad \frac{\mathbb{E}^W(e_0)}{\mathbb{E}(e_1)} > (1 - \gamma_L) + \gamma_L \pi_1 \\
(\theta_H \mathbb{E}(e), \theta_H \mathbb{E}^W(e_0)) & \text{if } \frac{\theta_L}{\theta_H} < (1 - \gamma_L) - \hat{\theta} \quad \text{and} \quad \frac{\mathbb{E}^W(e_0)}{\mathbb{E}(e_1)} > \pi_1 
\end{cases}
\]

Here \(\hat{\theta}\) is the smallest value of \(\theta_L\) which restores equality in 16.

The comparison between first and second period prices is laid out before alongside the conditions in each period.

\[
\begin{align*}
(p_{1W}^*) > (p_{2W}^*) & \quad \text{iff } \frac{\theta_L}{\theta_H} \geq (1 - \gamma_L) - \hat{\theta} \quad \text{and} \quad \frac{\mathbb{E}^W(e_0)}{\mathbb{E}(e_1)} > (1 - \gamma_L) + \gamma_L \pi_1 \\
(p_{1W}^*) > (p_{2W}^*) & \quad \text{iff } \frac{\theta_L}{\theta_H} < (1 - \gamma_L) - \hat{\theta} \quad \text{and} \quad \frac{\mathbb{E}^W(e_0)}{\mathbb{E}(e_1)} > \pi_1 \\
(p_{1W}^*) < (p_{2W}^*) & \quad \text{iff } \frac{\theta_L}{\theta_H} \geq (1 - \gamma_L) - \hat{\theta} \quad \text{and} \quad \frac{\mathbb{E}^W(e_0)}{\mathbb{E}(e_1)} < (1 - \gamma_L) + \gamma_L \pi_1 \\
(p_{1W}^*) < (p_{2W}^*) & \quad \text{iff } \frac{\theta_L}{\theta_H} < (1 - \gamma_L) - \hat{\theta} \quad \text{and} \quad \frac{\mathbb{E}^W(e_0)}{\mathbb{E}(e_1)} < \pi_1 
\end{align*}
\]

Remark 2.5. Comparison of first period price with standard consumers: \(p_1^* > p_{1W}^*\) holds iff \(\theta_L \in \left( (1 - \gamma_L) - \hat{\theta}, (1 - \gamma_L) \right) \) such that in the standard case, the monopolist will charge a higher price to attract only high type. On the contrary, in the wishful case, the monopolist finds it profitable to lower price to include the low-type.

Proposition I

Consider an economy with naive consumers two types \((\theta_L, \theta_H)\) and a monopolist who cannot commit to prices.

(i) The first period prices in the standard and wishful cases are as follows:

\[
(p_1^*) = \begin{cases} 
\theta_L \mathbb{E}(e) & \text{if } \frac{\theta_L}{\theta_H} \geq (1 - \gamma_L) \\
\theta_H \mathbb{E}(e) & \text{if } \frac{\theta_L}{\theta_H} < (1 - \gamma_L) 
\end{cases}
\]
\[
(p^W_1) = \begin{cases} 
\theta_L \mathbb{E}(e) & \text{if } \frac{\theta_L}{\theta_H} \geq (1 - \gamma_L) - \hat{\theta} \text{ and } \theta_L \geq \hat{\theta} \\
\theta_H \mathbb{E}(e) & \text{if } \frac{\theta_L}{\theta_H} < (1 - \gamma_L) - \hat{\theta} 
\end{cases}
\]

(ii) The second period prices in the standard and wishful cases are as follows:

\[
p^W_2 = \begin{cases} 
\theta_L \mathbb{E}^W_0(e_0|\theta_L) & \text{if } p^W_1 = \theta_L \mathbb{E}(e) \text{ and } \frac{\mathbb{E}^W_0(e_0|\theta_L)}{\mathbb{E}^W_1(e_1|\theta_L)} \geq \frac{1 - \gamma_L}{(1 - \gamma_L) + \pi_1 \gamma_L} \\
\theta_L \mathbb{E}^W_0(e_1|\theta_L) & \text{if } p^W_1 = \theta_L \mathbb{E}(e) \text{ and } \frac{\mathbb{E}^W_0(e_0|\theta_L)}{\mathbb{E}^W_1(e_1|\theta_L)} < \frac{1 - \gamma_L}{(1 - \gamma_L) + \pi_1 \gamma_L} \\
\theta_H \mathbb{E}^W_0(e_0|\theta_H) & \text{if } p^W_1 = \theta_H \mathbb{E}(e) \text{ and } \frac{\mathbb{E}^W_0(e_0|\theta_H)}{\mathbb{E}^W_1(e_1|\theta_H)} \geq \pi_1 \\
\theta_H \mathbb{E}^W_1(e_1|\theta_H) & \text{if } p^W_1 = \theta_H \mathbb{E}(e) \text{ and } \frac{\mathbb{E}^W_0(e_0|\theta_H)}{\mathbb{E}^W_1(e_1|\theta_H)} < \pi_1 
\end{cases}
\]

### 2.4.2 Sophisticated consumers and committed monopolist

Here the monopolist sets \((p^W_1, p^W_2)\) to maximize profits over two periods. Both monopolist and consumer’s discount rate is \(\kappa\). Consumers being sophisticated, maximize discounted their expected utility over both periods. However, there is no obligation on a consumer to buy the good in both periods. Given this, there are three sets of consumers — who buy in both period 1 and 2, buy only in 1 and those who buy only in 2.

Definition 2.6. \(\theta^W_{12 \sim 0} (\theta^W_{12 \sim 0})\) denotes the valuation of a wishful (standard) consumer who is indifferent between buying in both periods and not buying at all.

Definition 2.7. \(\theta^W_{1 \sim 2} (\theta^W_{1 \sim 2})\) denotes the wishful (standard) consumer who is indifferent between buying only in the first period and only in the second period.

Definition 2.8. For sophisticated consumers, \(g(\theta)\) defines the expectation of a good experience in period 2 prior to buying the good in period 1.

\[
g(\theta) = \pi_0 \mathbb{E}_{\delta_0}(e) + \pi_1 \mathbb{E}_{\delta_1}(e)
\]

where \(\pi_0 (\pi_1)\) is the probability of a bad (good) experience. The explanation of the expectation terms remain same as in section 2.3.1 and 2.3.2.
Homogeneous consumers

When consumers are homogeneous, the monopolist would try to sell in both periods. Serving to consumers in either the first or second period is strictly lower than serving in both periods. The profit of the monopolist can be expressed as

$$
\Pi^W = [p_1^W + \kappa p_2^W] \left( \max \left\{ \Pi(\theta - \theta_{12-0}), 0 \right\} \right)
$$

Ideally, the monopolist will choose the optimal pair \((p_1^{W*}, p_2^{W*})\) such that \(\theta = \theta_{12-0}\). Then \((p_1^{W*}, p_2^{W*})\) solves \(\theta = \frac{p_1^{W*} + \kappa p_2^{W*}}{E(e) + g(\theta)}\).

To satisfy the above identity, the monopolist can either offer a slightly higher price at \(t = 1\) and a relatively lower price at \(t = 2\) to retain the consumer who had a bad experience (commits to goodwill pricing). Else, he can offer a lower price at \(t = 1\) and offer only to the ones who had a good experience. In this case, the monopolist offers the following price schedule in equilibrium:

(i) Commitment to goodwill pricing: Here the monopolist charges a relatively higher period 1 price, but lowers price in period 2 to retain everyone.

$$
p_1^{W*} = \theta E(e) + \kappa \theta [g(\theta) - E(\delta_0(e))], p_2^{W*} = \theta E(\delta_0(e))
$$

where \(g(\theta) > E(\delta_0(e))\).

(ii) Commitment to price hike: Here the monopolist offers discount in period 1, but increases period 2 prices.

$$
p_1^{W*} = \theta E(e) + \kappa \theta [g(\theta) - E(\delta_1(e))], p_2^{W*} = \theta E(\delta_1(e))
$$

where \(g(\theta) < E(\delta_1(e))\).

Remark 2.9. Consider the monopolist to be perfectly patient (\(\kappa = 1\)) while consumers to be imperfectly patient (\(\kappa \in (0, 1)\)). Then in equilibrium, the monopolist’s profit is \(\Pi^{W*} = p_1^{W*} + p_2^{W*}\) and the monopolist commits to goodwill pricing in period 2 whenever \(\frac{E(\delta_0(e))}{E(\delta_1(e))} \geq \frac{\pi_1 - \kappa}{1 - \kappa}\).

Remark 2.10. If the monopolist and consumers are equally impatient or share the same value of \(\kappa\), then it is always optimal to practice goodwill pricing.
Two consumer types

There are two types of consumers, low and high who make their purchases after seeing prices of both periods.

\[
\Pi^W = [p_1^W + \kappa p_2^W] \left( \gamma_L \max \{ \mathbb{I}(\theta_L - \theta_{W12-0}), 0 \} \right) + (1 - \gamma_L) \max \{ \mathbb{I}(\theta_H - \theta_{W12-0}), 0 \} \\
+ p_1^W \left[ \gamma_L \max \{ \mathbb{I}(\theta_L - \theta_{W1}), 0 \} + (1 - \gamma_L) \max \{ \mathbb{I}(\theta_H - \theta_{W1}), 0 \} \right] \\
+ \kappa p_2^W \left[ \gamma_L \max \{ \mathbb{I}(\theta_L - \theta_{W1}), 0 \} + (1 - \gamma_L) \max \{ \mathbb{I}(\theta_H - \theta_{W1}), 0 \} \right]
\]

s.t \quad \theta_{W12-0}^W = \frac{p_1^W + \kappa p_2^W}{\mathbb{E}(e) + \kappa g(\theta_j)}, j \in \{L, H\}

\theta_{W1}^W = \frac{p_1^W - \kappa p_2^W}{\mathbb{E}(e) - \kappa g(\theta_j)}

\theta_L^1 = \frac{p_1^W}{\mathbb{E}(e)}

\theta_H^1 = \frac{p_1^W}{\mathbb{E}(e)}

In case of standard consumers, the above profit function has the same form apart from the fact that the expected second period expectation \( g(\cdot) \) no longer depends on \( \theta \).

In presence of wishful consumers, it is never optimal to serve only period 1 or only period 2 buyers. The monopolist can design the optimal price path such that after first-period, consumers are held under a contract which provides enough incentive to buy again.

Definition 2.11. Exclusive pricing: A pricing schedule where only high types are served such that \( \theta_H = \theta_{W12-0} \).

Definition 2.12. Inclusive pricing: A pricing schedule where low types are served such that \( \theta_L = \theta_{W12-0} \). Here high types are automatically served.

Definition 2.13. Quasi-inclusive contract: A quasi-inclusive contract in an economy with two consumer types \( \theta_L \) and \( \theta_H \) is a price pair \((p_1^W, p_2^W)\) such that \( \theta_L = \theta_{W12-0}^W = \frac{p_1^W - \kappa p_2^W}{\mathbb{E}(e) - \kappa g(\theta_j)} \) and \( \theta_H = \theta_{W12-0}^W = \frac{p_1^W + \kappa p_2^W}{\mathbb{E}(e) + \kappa g(\theta_j)} \).
Proposition II

Consider an economy of sophisticated consumers of two types \((\theta_L \text{ and } \theta_H)\) and a monopolist who can commit to announcing both period 1 and period 2 prices.

(A) Under quasi-inclusive (QI) pricing, the period 1 and 2 prices are as follows —
\[
- p_{QI}^1 = \frac{\theta_H - \theta_L}{2} (1 + \frac{1 - \gamma_L}{\kappa}) + \frac{\kappa}{2} \left( \theta_H g(\theta_H) - \theta_L g(\theta_L) \right)
- p_{QI}^2 = \frac{\theta_H - \theta_L}{2\kappa} E(e) + \frac{1}{2} \left( \theta_H g(\theta_H) - \theta_L g(\theta_L) \right)
\]

(B) Quasi-inclusive contract dominates inclusive pricing iff
\[
\frac{\theta_H - \theta_L}{2} \left( 1 + \frac{1 - \gamma_L}{\kappa} \right) + \frac{\kappa}{2} (\theta_H g(\theta_H) - \theta_L g(\theta_L)) - \theta_L g(\theta_L) > \epsilon \left( 1 - \frac{1}{\kappa} \right)
\]

(C) Quasi-inclusive contract dominates exclusive pricing iff
\[
\theta_L E(e) \left( \frac{\kappa - (1 - \gamma_L)}{2\kappa} \right) + \theta_H E(e) \left( \frac{\kappa - (1 - \gamma_L)(1 + 2\kappa)}{2\kappa} \right) + \frac{(1 - \gamma_L) - \kappa}{2} \left( \theta_L g(\theta_L) \right) + \frac{(1 - \gamma_L) - \kappa}{2} \left( \theta_H g(\theta_H) \right) > \left( 1 - \gamma_L \right) E_{\delta_0}(e|\theta_H)
\]

The characterization of prices in the quasi-inclusive pricing follows directly from definition 2.13. As can be seen, the first period price is greater than what a naive low type consumer would pay for the product \((\theta_L E(e))\). Being sophisticated allows the consumer to look ahead and this allows the monopolist to charge a slightly higher price in each period. The remaining proofs are straightforward and follows directly from definitions 2.11 — 2.13. I therefore provide the intuition to the above statements.

The inequality in statement B holds more strongly when consumers are less patient and chance of a good experience is low enough. Since, consumers are not patient, the monopolist can set a pricing schedule where the low type consumer feels indifferent between buying either in period 1 or in period 2. The prices in either period is higher than inclusive pricing and the low-type consumer being impatient will readily buy the product in period 1. The high type buys in both periods.

The inequality in statement C holds if \(\theta_L \text{ and } \gamma_L\) is high enough such that the monopolist does not want to let go of the surplus coming from the low-types.
2.4.3 Naive consumers and committed monopolist

Here the naive consumers see both first and second period prices. Being naive, they separately maximize their utility in each period. Second period utility is discounted by a factor $\kappa > 0$. They simply compare the two prices and decide to buy if their expected utility, net of price, is non-negative.

$$\max_{p_1^W, p_2^W} \Pi_1^W(p_1^W, p_2^W) + \Pi_2^W(p_2^W, p_1^W)$$

$$s.t \quad \theta_L E(e) - p_1^W \geq 0, \quad \theta_H E(e) - p_1^W \geq 0$$
$$\theta_L E(e|e_1, \theta_L) - p_2^W \geq 0, \quad \theta_H E(e|e_1, \theta_H) - p_2^W \geq 0$$
$$\theta_L E(e|e_0, \theta_L) - p_2^W \geq 0, \quad \theta_H E(e|e_0, \theta_H) - p_2^W \geq 0$$

The first constraints are the participation constraints for low and high type in period 1. The remaining are the constraints in period 2 following a good and bad experiences. In this setup, the committed monopolist can plan to devise a bundling strategy to make consumers pay for two units of the good upfront. Now, is it profitable to do such a bundling strategy in presence of wishful consumers or standard consumers?

Pure bundling

(i) Wishful consumers: In presence of naive and wishful consumers, the monopolist chooses a single price $p_B$ and makes a take-it-or-leave-it offer. The problem of the monopolist is to choose a single bundling price $p_B^W$ such that

$$\max_{p_B^W} \Pi^W(p_B^W)$$

$$s.t \quad \sum_{t=1,2} E_t(e) - p_B^W \geq 0$$

The constraint basically is the aggregation of expectation regarding a good experience in both time periods. In this case, the monopolist cannot do better than charging a price $p_B^W = (1 + \kappa)E(e)$. In essence, the consumers pay for two units of the good at a single time. Being naive, they aggregate their expected experience from two units.
(bundled together). The effect of idiosyncratic valuation on belief distortion plays no part in this setup.

(ii) Standard consumers: Here the monopolist’s problem is to choose $p_B$ to maximize the following two period profit

$$\max_{p_B} \Pi(p_B)$$

$$s.t \quad \sum_{t=1,2} \mathbb{E}(e) - p_B \geq 0$$

The constraint suggests that consumers being naive aggregate expected profit without accounting for any preference change following a bad experience. In simple words, they simply add up the expected value of experience in each period (discount second period by $\kappa$). This leads the monopolist to choose $p^*_B = (1 + \kappa)\mathbb{E}(e)$. In presence of perfectly patient consumers ($\kappa = 1$), $p^*_B = 2\mathbb{E}(e)$.

In a setup where consumers are of two types, this strategy strictly dominates the strategy of announcing separate prices, especially when the chances of a good experience is not high enough or where low types form a significant portion of the total consumer population.

Remark 2.14. Goodwill of the monopolist does not play any role when the monopolist offers a one-shot bundle offer.

The intuition behind the above statement is straightforward. If goodwill had been important, then the monopolist would have announced separate prices where price in period 2 would be strictly lower than that of period 1. Instead, since consumers are naive, the monopolist can increase the total prices charged in both periods to $(1 + \kappa)\theta L \mathbb{E}(e)$ and still capture the entire market (both low and high) with a single take-it-or-leave-it offer.

Example 2.15. In a setup with two consumer types, if bundling survives as an equilibrium strategy, it must be that it dominates the goodwill pricing when prices are separately announced. This implies $(1 + \kappa)\theta L \mathbb{E}(e) \geq \theta L \mathbb{E}(e) + (\gamma_L \pi_1 + (1 - \gamma_L))p_G$, where at goodwill price $p_G < \mathbb{E}(e)$, the low types buys after having a bad experience. Given $\mu = 0.5, q_h = 0.6$ and $q_l = 0.3$, bundling dominates the strategy of announcing prices separately, for any value of $\gamma_L$.

In a setup where consumers are distributed uniformly, then bundling is still a strategy to opt when the chances of a good experience is not high enough.
2.5 Comparative statics

The purpose of this section is to highlight the conditions under which wishful thinking has more force and when it is the most weakest. In this respect, what needs to be seen is how the model parameters impact monopolist’s power to charge a higher second period price.

Proposition III

Comparative statics

(i) In presence of a continuum of consumers, as the value of the taste parameter $\theta$ increases, then

(a) If experience is good,

$$\frac{dp_2^W}{d\theta} = \lambda \mu(\hat{t}) \mu(\hat{h})(q_h - q_l)^2 > 0$$

(b) If experience is bad,

$$\frac{dp_2^W}{d\theta} = \beta \mu_0(\hat{t}) \mu_0(\hat{h})(q_0^0 - q_h^0)(q_l - q_h) > 0$$

For non-wishful buyers,

$$\frac{dp_2^W}{d\theta} = 0$$

(ii) As the the distortion parameter increases, the impact on the second period price following a good and bad experience is expressed as

$$\frac{dp_2^W}{d\lambda} = \frac{dp_2^W}{d\beta} = \theta \hat{\mu}_\delta(\hat{t}) \hat{\mu}_\delta(\hat{h})(q_h - q_l)^2$$

(iii) (a) The increase in second period price due to a rise in the prior probability of the high state $(1 - \mu)$ is such that

$$\frac{dp_2^W}{d(1 - \mu)} = \frac{q_h q_l [q_h - q_l] e^{\lambda \delta (q_h - q_l) \hat{d}^2}}{D^2}$$

and

$$\frac{dp_2^W}{d(1 - \mu)} = \frac{q_h q_l [q_h - q_l]}{D^2}$$

(b) Following a bad experience, $\frac{dp_2^W}{d(1 - \mu)} > 0$ iff $(q_l^0 - q_h^0) > \frac{q_h q_l |q_h - q_l| e^{\lambda \delta (q_h - q_l))}}{2 \mu q_h \mu_q h}$. For non-wishful buyers, $\frac{dp_2^W}{d(1 - \mu)} > 0$ iff $(q_l^0 - q_h^0) > \frac{q_h q_l |q_h - q_l| e^{\lambda \delta (q_h - q_l))}}{2 \mu q_h \mu_q h}$.

---

3 $\hat{D} = \mu q(l)e^{\lambda \delta q_l} + (1 - \mu)q(h)e^{\lambda \delta q_h}, D = \mu q(l) + (1 - \mu)q(h)$
(iv) (a) Following a good experience, if \( q_l \) increases, then second-period prices falls. This effect is higher for both high valuation consumers and consumers who are more wishful (higher values of \( \theta \) and \( \lambda \) respectively).

\[
\frac{dp_2^W}{dq_l} = \hat{\mu}(l) - \hat{\mu}(l)\hat{\mu}(h)[1 + \lambda q_l \left( \frac{q_h - q_l}{q_l} \right)]
\]

The first and second statements imply that for wishful buyers, as valuations and distortion parameter rises, the monopolist gains a license to increase the period 2 price. The extent of increase evidently depends on the difference in conditional probability of a good experience across the high and low states (\( q_h - q_l \)). If probability of a good experience is less likely in the low state and more likely in the high state, then after receiving a good experience in period 1, the extent of wishful thinking increases. Similarly, if the first period experience turns out as bad, the possibility of rise in second period price exists when the difference in the conditional probability of a bad outcome across the low and high states is greater. For non-wishful consumers, there is no first order effect of valuation or distortion parameter on the second period price.

The third statement shows that as the occurrence of a high state rises (\( 1 - \mu \) increases), there is no space of being optimistic if the good experience is equally likely under both low and high states. Wishful thinkers can become more optimistic about the high state only by leveraging on the fact that good experiences are more probable in the high state than in the low state. If \( q_h \) and \( q_l \) tends towards \( \frac{1}{2} \), then any increase in the high state prior is redundant. Nonetheless, the impact of this rise is positive if the consumer has a good history with the product in period 1. On the contrary, if consumers have bad experience, then the impact of a rise in the high state prior on second period price is positive only if the difference (\( q_h - q_l \)) is above a threshold. In presence of wishful consumers, this threshold is lower than
in a model with standard consumers as can be seen from the two inequalities in (ii)(b).

The statement in (iv)(a) suggests that given the buyer has a good experience in period 1, the probability of a good experience conditional on the state being low \( q_l \) casts a negative impact on second period prices beyond a cutoff \( \theta_{q_l} \). If the terms \( \frac{a_l - q_l}{q_l} \) (in both equations of (iv)(a) are observed, the magnitude of this negative effect is less pronounced at higher levels of \( q_l \). When good experiences are already high (say closer to 0.5), then any increment has very moderate effect on the prices than in a situation when \( q_l \) is much lesser than 0.5. The rise in \( q_l \) has a second-order impact with wishful buyers, characterized by the term \( 1 + \lambda \theta q_l \). Relative to a situation with standard buyers, the impact on price due to wishful buyers becomes negative at a lower value of \( \theta \).

(iv)b is more nuanced which shows the impact on the second-period price due to increase in the probability of a bad experience under a low state. Given the consumer gets a bad experience, the magnitude of the derivative decreases with higher \( \theta \) or \( |\beta| \). This entails that more wishful consumers or high valuation consumers discount the impact of a rise in \( q_l' \). After a bad experience, consumers with higher valuation or with greater wish-fullness tend to stick with their decision of buying the good again. They tend to distort the odds in favor of the high state if if the low state is more likely to lead to bad experiences. This distortion increases either with a bigger value of \( \theta \) and \( |\beta| \).

2.6 Conclusion

This paper has focused on a two period model of monopoly pricing in a setup where consumers are wishful and they engage in repeat purchase. The decision to repeat purchase in the second period depends on the consumer’s experience in the first period. I analyze different pricing strategies where the monopolists can extract maximum surplus from consumers due to their wishful nature. The magnitude of such extraction also depends on the naivetee of consumers and it is seen that presence of naivetee often acts in favor of the consumers because the inability to forsee what might happen in the future restricts the monopolist’s power to increase prices.

The primary focus of this paper is to highlight the dependence of second period profit of the monopolist on the first period price. A comparative analysis is provided which lays out how this dependence change with wishful consumers and the related inferences regarding the pricing strategies.
Chapter 3

Sanctions under directed networks

3.1 Introduction

Sanctions are imposed as a punitive measure to discipline an agent (target, which can be an individual or nation) when it has engaged in actions not approved by the sanctioning agent (sender). These actions can mean the target violating trade norms, inciting domestic warfare or just not returning a favor asked by the sender [31]. Sanctions are not recent phenomena and has been used historically. [24] recounts one of the first instances of sanctions in the eleventh century where Maghiri tribes set up trade-laws to strengthen contractual relations between merchants and their agents who travelled overseas. On the occasion of any code violation, coalition members could take disciplinary actions like sanctions thereby barring the convicted agent from conducting future trade.

Ex-post sanction, if the target complies to the demands of the sender, then the sanction is deemed to be effective, else ineffective. In this paper, compliance of the target occurs because the negative impact from sanction (decrease in trade flows) on the target was high enough compared to the sender. Otherwise, if the target could absorb the negative shock from sanction, then compliance would be unlikely, thereby rendering sanctions to become ineffective.

This paper firstly studies what conditions determine sanction effectiveness in a directed network setting? Since, sanctions are forms of coercion or conflict, it is natural to proceed by accounting for the power or influence of the sender relative to the target. This concept of relative power of sender and target is akin to the contest success function approach of analysing situations of conflict where chances of winning conflict is monotonic in power. However, in a setting of trade, chances of effective sanction is not monotonic in power (or higher centrality value) due to spillover effects. If the sender is more central or more powerful, sanctions can cause collateral damage on itself and other trading partners of the sender. This leads the trading partners to coerce the sender to lift sanctions. This suggests an effect of strategic substitutability (SS, henceforth). On the other hand, if the target is more central,
then its trading partners can coerce it to comply to sanction. This casts an effect of strategic complementarity (SC). As a measure of centrality, I choose betweenness(b)-centrality of the sender and target as it reflects their role as an intermediary in the trade network. Hence, this provides a stronger intuition behind the understanding of sanctions through effects SS and SC.

Secondly, using b-centrality, this paper analyzes the merits of export versus import sanctions. Given any fixed network, the impact from severing export links is likely to be different than severing import links as both reconfigures the network in their own way. Naturally, it is of interest to look how the target was harmed relative to the sender in each sanction type.

The importance of trade intermediaries has grown with increased globalization where countries find it less costly to outsource manufacturing stages to other nations before buying the final product back. Globalization has also made the OECD increase trade in export of goods which are mainly imported from other nations. This has made the domestic value added to goods exchanged between any two countries increasingly small (see [1],[13]). In essence the role of intermediaries have gained much force. This strengthens the intuition of using b-centrality measure where a higher value suggests more important strategic location of a particular country or player.

Previous theoretical work in the same vein consists of [33],[34],[35]. The first of the three papers studies how the network structure helps the sender to form a coalition such that it can sanction the target with greater force and whether T would comply to S’s demands more readily. They introduce the concept of effective sanctions through the concept of spanning trees and analyse sanctions in short and long run. In essence, this strategy precipitates to relegating T to the smallest possible component to make T succumb. [34] extends the previous idea and examines the role of linear, concave or convex utility function of sender in sustaining a larger coalition among sanctioning players. [35] further augments their existing work of understanding sanction efficacy under different network specification through strategic complementarities and externalities. For instance,

On the empirical side, interestingly, it is found that effective trade sanctions are rare events (roughly 30% of all sanctions are effective ([19])). [28] suggests that most sanctions which have been proven to be a success are due to the presence of contemporaneous forces like military pursuits which had coerced the target to comply to the sender’s demand. However, if one looks at sanction effectiveness, then it is seen that effectiveness of sanctions differs
Works of [15] have shown the outright failure of sanctions by shedding light on how target nations escape the effect of sanctions by forming ties with other nations. For instance, following sanctions by the Arab nations, Qatar formed renewed ties with Iran and China. Effectiveness of sanctions is variant across good-type. [36] cites the example the effectiveness of US’s export sanction of computers against South Africa due to the limited number of computer suppliers in the world market during the 1980’s. However, the ban on coal imports from South Africa was weakened by the fact that South Africa soon found new buyers.

Modern world sanctions have evolved in many forms (trade, financial, travel) and methods of punishing the guilty (severing trade ties, blocking investments or preventing travel) which in most cases inflicts collateral damage on the sanctioning agent and leads to welfare loss. [19] shows that complete export sanction has lead to a 76% decrease in trade, an outcome equivalent to an approximate 43% rise in tariffs. While, import sanctions has caused a loss in trade of 52% (an outcome when tariffs rise by 20%). This capacity of sanctions to cast economic and social damage creates avenues for research, like the current paper which presents conditions where more effective sanctions can be imposed such that the welfare loss is minimum.

\[ [15] \] highlights the superiority of import over export sanctions.
3.2 Network model

The trade graph or network $g$ comprises of a set of nodes $\mathbb{N} = \{1, 2, 3, .., n\}$ (denoting players). $g$ is fixed and directed. The directed nature of $g$ essentially distinguishes between export and import links of any player in the network. The number of links emanating from a player is its out-degree while the number of links converging to a player is its in-degree. I now provide the definitions of some network structures which will be used in this paper.

A line in $g$ is a set of distinct links $i l_1, l_2, .., l_k j$. A cycle is a path such that the initial and terminal players are the same. A complete cycle connects all players in the network. A geodesic $\sigma_{ij}$ denotes the shortest path from $i$ to $j$. A network is connected if there exists at least one directed link between a pair of players. A connected network is complete if all players are connected to each other by links in both directions.

Sanction is imposed by player $S$ (sender) against player $T$ (target) where $S, T \in g$. No other player is directly involved in sanction. $g'$ to denote the ex-post sanction network such that $g'_l = g - l$ where $l \in \{e, m\}$. This can be understood by the set of action of $S$’s action $a_l, l \in e, m$ where $e$ denotes export and $m$ denotes import sanction.

3.2.1 b-centrality

The standard definition of b-centrality in directed networks as follows:

Definition 3.1. Betweenness (b)- Centrality: Let $\sigma(kj)$ denotes the number of geodesics (shortest paths) between $k$ and $j$ in directed network $g$. Analogously, we denote by $\sigma_i(kj)$ as the number of these geodesics which pass through node $i$. Then the betweenness centrality of player $i$ is

$$\beta_i(g) = \sum_{k \neq j; i \notin \{k, j\}} \frac{\sigma_i(kj)/\sigma(kj)}{(n-1)(n-2)}$$

The usage of this centrality measure in this model of sanctions is intuitive because it measures the relative strategic location of each player in the network. The other measures of centrality like closeness and net-degree centrality are more direct measures, while the eigen-value centrality is less robust in a directed network setup. These measures are also unable to capture the trade spill overs from sanctions inside a network by accounting for

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2In a directed network, this measure calculates the total out links net of in-links of a node.
the change in strategic positions of the sender and target following sanctions. Amidst this, the b-centrality measure provides a simple way (also a reduced way) to show how trade is impacted due to change of strategic position of the sender and target. I have discussed in figure 2, how the b-centrality is distinguishable from the other forms of centrality in some known networks. The figure primarily illustrates the relative strategic position of the players in any given network.

3.2.2 b-centrality through dependency matrix

The b-centrality scores of any given node or player denotes its strategic power. However, this single score cannot summarize which players are dependent on it. For this, I use the dependency matrix which also provides an alternative way to derive b-centrality. Consider the number of geodesic from player $k$ to player $j$ as $\sigma_i(k,j)$ as those geodesics which pass through $i$. Then the dependency of $k$ on $i$ is given by $d_j(ki) = \sum_k \sigma_i(k,j)$ for $j \neq i \neq k$. The dependency matrix $D_g$ for network $g$ where $d_{ki} = \sum_j \sigma_i(kj)$. To derive the b-centrality for each node, the dependencies are summed up along the column and divided by the maximal centrality $C_B(g) = (N_O - 1)(N_I - 1) - (N_B - 1)$ where $N_O$ ($N_I$) is the number of nodes in $g$ having out-going (in-coming) links and $N_B$ is the number of nodes having bilateral links.

For the line graph in sub-figure (a), $N_O = 2, N_I = 2$ and $N_B = 2$. So the b-centrality of $S = \frac{\sigma_{3T}(S) + \sigma_{T3}(S)}{C_B(g)} = \frac{1+1}{2 \times 2 - 2} = 1$. 

![Figure 3.2: Differences in centrality measures across some standard network structures.](image)
3.2.3 Payoff of players

In the fixed trade network $g$, I consider two forms of payoff of any player $i \in g$. The first payoff form is $U$ which is linear in the value of b-centrality. The second form is $W$ which is increasing in amount of trade (amount of exports and imports). The function $W$ is a reduced form as the amount of export and import payoff is a function of $i$’s role as intermediary in the network. Each player $i \in g$ is also blessed with an endowment $\gamma_i \in (1, \infty)$. $\gamma_i \to \infty$ resembles autarky. It is assumed here that a higher endowment helps a country to absorb more easily negative shock from sanction.

Let us first consider payoff function $U$. The value of b-centrality $\beta_i$ in $g$ is weakly less than $\beta_i'$ in the post-sanction network $g'$. The loss in payoff of $S$ and $T$ due to sanction $a_l$ is given by

$$\Delta U_S(a_l) = U_S(\beta'_S|g', \gamma_S, a_l) - U_S(\beta_S|g, \gamma_S) \quad (3.1)$$

$$\Delta U_T = U_T(\beta'_T|g', \gamma_T, a_l) - U_T(\beta_T|g, \gamma_T) \quad (3.2)$$

The function $W_i$ on the other hand in increasing in the cumulative value of import and export of $i$ with any $j \in g$. The value of import and export is given by $\alpha_{ji} \in \mathbb{R}_{++}$ and $\alpha_{ij} \in \mathbb{R}_{++}$ respectively. $I_i(g, \gamma_i)$ and $E_i(g, \gamma_i)$ signify the respective payoff from import and export such that total payoff of $i$ is given by

$$W_i(\alpha_{ji}, \alpha_{ij}|g, \gamma_i) = \lambda^E_i I_i(\alpha_{ji}|g', \gamma_i) + (1 - \lambda^E_i) E_i(\alpha_{ij}|g', \gamma_i) \quad (3.3)$$

The presence of $\gamma_i$ in the payoff function grabs bite in the post-sanction payoff functions as will describe below. The parameter $\lambda^E_i \in (0, 1)$ quantifies the political views of country $i$ in favor of exports while $(1 - \lambda^E_i)$ denotes its views towards imported goods. Following sanctions, any $i$ in the trade network suffers loss from trade which is defined by $\Delta W$. This analysis will focus on this change in payoffs and how the magnitude of payoff loss are a function of the network architecture.

The post-sanction network is denoted by $g'$ where the total payoff loss from trade incurred by $S$ is a function of its action $a_l$,

$$\Delta W_S(a_l) = \lambda_S^E I_S(\Delta(\alpha_{jS})|g', \gamma_S, a_l) + (1 - \lambda_S^E) E_S(\Delta(\alpha_{Sj})|g', \gamma_S) \quad (3.4)$$
The payoff loss of $T$, given the action $a_l$ of $S$ is

$$\Delta W_T = \lambda_T I_T(\Delta(\alpha_{jT}|g', \gamma_T, a_l)) + (1 - \lambda_T^E) E_T(\Delta(\alpha_{Tj}|g', \gamma_T))$$  \hspace{1cm} (3.5)$$

The above loss functions is convex in the amount of trade lost and satisfy negative increasing differences in the value of $\gamma$. This implies that loss is lower when the strength of a country is higher.

### 3.2.4 Stability

The notion of stability of import or export sanction depends on how much harm the sender is able to inflict on the target relative to the harm it sustains due to sanction.

Definition 3.2. An import sanction $(a_m)$ by sender is strictly stable than export sanction $(a_e)$ iff the following condition hold.

$$\Delta U_S(a_m|g', \gamma_S) \leq \Delta U_T(g', \gamma_T, a_m) \quad ; \quad \Delta W_S(a_m|g', \gamma_S) \leq \Delta W_T(g', \gamma_T, a_m)$$ \hspace{1cm} (3.6)

A stable import sanction is Pareto optimal if the payoff loss incurred by $S$ and $T$ from import sanction is strictly less than the loss from export sanction (vice versa for export sanction).

$$\Delta U_S(a_m|g', \gamma_S) < \Delta U_S(a_e|g', \gamma_S) \quad ; \quad \Delta W_S(a_m|g', \gamma_S) < \Delta W_S(a_e|g', \gamma_S)$$ \hspace{1cm} (3.7)

$$\Delta U_T(g', \gamma_S, a_m) < \Delta U_T(g', \gamma_S, a_e) \quad ; \quad \Delta W_T(g', \gamma_S, a_m) < \Delta W_T(g', \gamma_S, a_e)$$ \hspace{1cm} (3.8)

### 3.3 Partial analysis

The decision of the sender to impose an import or export sanction only takes into account the reconfiguration of the network post sanction. The post-sanction network is
fixed by assumption and therefore, the sender does not account for what the target might do in response to sanction.

Assumption 2 establishes the participation constraint of $T$ in engaging in the restricted activity or not returning a favor to $S$ in network $g$. Assumption 3 implies that the payoff of $S$ from withstanding the denial of favor from $T$ is strictly lower than $S$’s payoff from sanctioning $T$.

Assumption 1: Sender sanctions a target with only direct trade links.

The direct trade link binds the sender’s incentive to participate in sanctioning the target. Without direct trade, sanctions are more likely to be violated by the target. I assume that without direct trade link, the loss from trade is not high enough to make the target obey the sender.

Assumption 2: $T$ bears a cost $C_T$ from agreeing to provide a favor to $S$ (say, by stopping domestic warfare which is deemed unlawful by $S$).

Assumption 3: $S$ pays a price of $P_S > 0$ if $T$ refuses to provide a favor to $S$. $P_S$ is big enough such that $S$ is better off by sanctioning $T$.

The inequality states that the payoff of $S$ in $g$ from enduring cost $P_s$ (from $T$’s action) is strictly less than the payoff $S$ will experience in post-sanction network $g'$. It simply implies that the $T$’s activity causes enough harm to $S$ such that the latter is better off sanctioning $T$.

Definition 3.3. (i) A player is net-exporter (net-importer) if its out-degree is strictly greater (lower) than its in-degree.

(ii) A player is balanced if its out-degree is equal to its in-degree.

(iii) A player is a complete exporter (importer) if it has only out-links (in-links).

3.3.1 Case-study: 3-player network

I consider a network $g$ comprised of the following three players: sender ($S$), target ($T$) and a third player denoted by 3. This will help understand the merit of import versus export sanctions within a smaller group of trading nations. The following result lays out the conditions on network where the merit of import is stronger than export or vice versa.

Remark 3.4. Suppose there exists two bilateral links in $g$ with one bilateral link between $S$
and $T$. Then depending on whether $T$ or $S$ has the other bilateral link with 3, import and export sanction have the following outcomes.

- (i) If $T$ and 3 are bilaterally linked such that $T$ is balanced, then
  - If $S$ is a net-importer, then export sanction is $S_4$, while import sanction is $W_0$.
  - If $S$ is a net-exporter, then import sanction is $S_4$, while export sanction is $W_0$.

- (ii) If $S$ and 3 are bilaterally linked such that $S$ is balanced, then
  - If $T$ is net-importer or net-exporter, then export sanction is $W_0.2$ (ineffective where centrality of $T$ is unchanged but that of $S$ decreases), while import sanction is $W_0$.

In the above cases, the direction of links take the pivotal role in rendering one sanction weak and the other strong. The position of the third player in this network becomes instrumental. Both the statements can be understood by taking help of figure 3. The first statement corresponds to the top row in figure 3 where the $b$-centrality of all but $T$ is zero. This entails that $S$ does not lie on the geodesic between $T$ and 3, nor does 3 lie between $S$ and $T$. Being net-importer, if $S$ severs its import link with $T$, it turns into a balanced player in $g$, but the existence of bilateral ties between $T$ and 3 does not affect the $b$-centrality of $S$ or $T$. However, severing the export ties with $T$ makes $S$ an importer which does not lowers its $b$-centrality but lowers that of $T$. Hence export sanction is $S_4$. This network (post export sanction) is not cyclic as $S$ is an importer with only in-links.

The second statement can be analogously explained by the second row in figure 2.

This statement grabs bite in the world of trade where increasing volume of exported goods from any country are actually made up of intermediate imports made by that country from its trading partners $^3$. This result is fairly intuitive and much easier to visualize in a 3-player network. If the post sanction network is cyclic, then each node lies on the geodesic between the other two players which is unlikely to disrupt trade-flow. If on the other hand, the sanctioned network becomes acyclic, then at least one of the 3 players have only out-links or in-links. This entails that the target nation post sanction has been prevented from acting as an essential trading intermediary.

$^3$See [1] who shows the increase in this trend in almost all the OECD countries.
3.3. Partial analysis

Figure 3.3: The first two rows illustrates sub-part (i) of remark 2 and the third and fourth rows illustrates sub-part (i) of proposition 1. The first row exhibits $S$ as net-importer and $T$ is balanced. Import sanction is ineffective as shown. However, the export sanction lowers centrality of $T$ as now it does not lie in the geodesic from $S$ to player 3. Analogous interpretation holds for the rest.
Figure 3.4: The first two rows illustrate when $S$ and $T$ are net-exporters and net-importers respectively. Row 3 (4) shows the effect of import and export sanctions when $S$ is net-importer (net-exporter) while $T$ is net-exporter (net-importer).
3.4. Complementarity and substitutability effects

If this player is $T$, then sanction is $S_4$ such that betweeness centrality of $T$ falls to zero. However, if $S$ has only out-links and in-links, then sanction is $W_2$ where $S$ endures loss in centrality.

Remark 3.5. If only unilateral links exists among the 3 players in $g$, then a sanction of type $W_3$ can be imposed iff $g$ is a cycle. If $g$ is acyclic, b-centrality of all 3 players in $g$ remains zero and all sanctions are $W_0$.

When only unilateral links are considered in the 3-player setting, only two situations arise where there exists cycles among the players. In these cases weak sanction of type $W_3$ can be imposed. The remaining cases constitute acyclic networks where export or import sanction is $W_0$ (ineffective).

3.4 Complementarity and substitutability effects

The b-centrality of the sender $S$ can be decomposed into the individual dependencies $\sigma_{jk}(S)$ of any $j$ on $S$ to trade with $k$, where $S \neq j \neq k$ and $S, j, k \in g$. The distribution of dependencies of the set of players $j$ cast the force of complementarity and substitutability on $S$ when it imposes a sanction. The stability of sanction can further be examined by accounting for the distribution of dependencies of players $j'$ on player $T$.

Since sanction strictly lowers trade of players who are both directly and indirectly involved in sanction, sanctions does not proceed without opposition from players indirectly hurt by sanctions. For instance, consider the trade network $g$ to be a line, if $j$ is heavily dependent on $S$ to export to $T$, then $j$ will oppose any level of export sanction of $S$ on $T$. However, if $j$ is dependent on $S$ as well as on $T$ to trade with any $u \in U$ (where $U$ succeeds $T$)

Definition 3.6. Strategic Complementarity (SC): A higher value of b-centrality of $S$ has a complementarity effects under the following scenarios:

(a) In case of export sanction, the dependents of $S$ are also dependents of $T$.

(a) In case of import sanction, the dependents of $S$ are also dependents of $T$.

Definition 3.7. Strategic Substitutability (SS): A higher value of b-centrality of $S$ has a substitutability effects under the following scenarios:
(a) In case of export sanction, more number of players which depend on $S$ coerce $S$ to lift sanctions.

(b) In case of import sanction, the dependents of $S$ are also dependents of $T$.

**Proposition I**

Assume $\gamma_S = \gamma_T$ and a set of players trading along a line as in figure 3.5. A higher value of $\beta_S$ is sufficient towards a stable export sanctions if

- $\sigma_{RT}(S)$ is small enough – $S$ faces lesser opposition from $R$ to lift sanction – strategic substitutability.
3.4. Complementarity and substitutability effects

- $\sigma_{RU}(S)$ is large enough – $T$ faces greater opposition from $U$ to comply to sanction – higher $\beta_S$ – strategic complementarity.

Proposition II

Assume $\gamma_S = \gamma_T$ and a set of players trading along a line as in figure 3.6. A lower value of $\beta_S$ implies stable import sanctions iff the following hold:

- $\sigma_{US}(T)$ is large enough – greater opposition on $T$ from $U$ to comply to sanction – strategic complementarity.
- $\sigma_{UR}(T)$ is small enough – weaker opposition from $R$ to lift sanction – higher $\beta_S$ – strategic substitutability.

The above two propositions flow from the definitions of $SC$ and $SS$ and how the b-centrality value imply the dependencies of players in a network on $S$ and $T$. In case of export sanction, stability is achieved iff the $S$ acts as an intermediary to a set of players ($R$) which wants to trade with ($U$), which lies beyond $T$. In essence, player $T$ is not the final node of the trade network. Only then the $SC$ property of a higher b-centrality of $S$ ($\sigma_{RU}(S)$) gains force. On the contrary, in case of import sanction in a line network, a lower b-centrality of $S$ and a higher one for $T$ acts as a weaker sufficiency condition towards stable sanctions, meaning, stability is comparatively easy to attain. Here, loosely speaking, if $S$ acts the final trading destination for majority of players in set $U$, then $T$ faces greater coercion from the same to comply to sanction of $S$. On the contrary, such coercion from any player in $R$ will be very weak given that $S$ is the final destination for majority of the trading players.

Corollary 1

In case of line networks, the effect of the position of $S$ and $T$ (signifying downstreamness or upstreamness) is ambiguous towards governing sanction stability.

Corollary 2

In case of a circular trade network, the complementarity and substitutability effects become ambiguous. This is because, the set of players succeeding $T$ also precede $S$ as shown in figure 3.7.
3.5 Conclusion

This discussion has focused on the following question - is the type of sanction levied by a sender against a target a function of the network architecture. Of the various facets of architecture lies the concept of node centrality which measure importance of nodes in different ways. I use the betweenness centrality for this study. Why does this measure matter in examining trade sanctions? Well, a higher value of betweenness centrality of a player means that it holds a key strategic position in bridging trade ties with other nations in the most cost effective way. In simple terms, the player acts as a vital intermediary for majority players in side the network. Hence, issuing sanctions, which consists of link deletion, can lead to heavy compromise of both sender and target. This is also germane to the trade literature which shows strong evidences of how the value added by intermediate trading partners have increasingly surged up over the years.

In essence, the broad question which has been answered here is the following - what network structure supports import sanctions and which ones support export sanctions, if one assumes that the players involved in sanctions take actions based on the value of this measure of centrality. I present certain features in networks which gives force to specific types of sanctions and render other sanction either non-effective or pyrrhic. The consideration of strategic positions while deciding which type of sanction to impose is a new way of examining sanction efficacy. As an application, this analysis lends insights to the strength of sanctions when players are concerned only with trade-gains and when they account for payoffs from geopolitical supremacy alongside trade-gains. This model shows that the presence of such geopolitical motives weakens the power of sanctions and might lead to a Pareto inferior outcome.
Appendices
Appendix A

First Appendix

A.1 Existence of roots of best response functions

Strum’s theorem allows us to find the number of real distinct roots of each best response (BR) of L and R. This from Worth(2005) and helps us determine the number of real distinct roots within the interval \([-1, 1]\) for any given \(\theta_E, \lambda_L\) and \(\lambda_R\). This exercise allows us to know whether each of these equations have a real zero within \([-1, 1]\). The Nash equilibrium choices of \(\theta_L\) and \(\theta_R\) is then determined at the intersection of each of these BR.

We denote BR of L and R below as

\[
g(\theta_L) = \frac{d\Pi_L}{d\theta_L} = \frac{\partial^4 L}{\partial \theta_L^4} \left[ (1 - \lambda_L) + \frac{c}{b + (\theta_R - \theta_E)^2} \right] + \theta_L^3 \left[ (1 - \lambda_L) - \frac{c\theta_E}{b + (\theta_R - \theta_E)^2} \right] + 0.5\lambda_L\theta_E\theta_L - 0.25\theta_E^2\lambda_L = 0 \quad (A.1)
\]

\[
g(\theta_R) = \frac{d\Pi_R}{d\theta_R} = \frac{\partial^4 R}{\partial \theta_R^4} \left[ (1 - \lambda_R) + \frac{c}{b + (\theta_L - \theta_E)^2} \right] - \theta_R^3 \left[ (1 - \lambda_R) + \frac{c\theta_E}{b + (\theta_L - \theta_E)^2} \right] + 0.5\lambda_R\theta_E\theta_R - 0.25\theta_E^2\lambda_R = 0 \quad (A.2)
\]

Definition A.1. Strum’s sequence: The Strum sequence for a univariate polynomial \(f(x)\), is a sequence \(f_0, f_1, f_2, \ldots\) such that

\[
f_0 = f
\]

\[
f_1 = f'
\]

\[
f_{i+1} = -rem(f_{i-1}, f_i) \text{ where } rem(f_{i-1}, f_i) \text{ is the remainder when } f_{i-1} \text{ is divided by } f_i.
\]

Definition A.2. Strum’s Theorem: - Let \(f(x)\) be a polynomial of positive degree with real
coefficients and let \( \{f_0(x) = f(x), f_1(x) = f'(x), f_2(x), ..., f_s(x)\} \) be the standard sequence for \( f(x) \). Assume \([a, b]\) is an interval such that \( f(a) \neq 0 \neq f(b) \). Then the number of distinct real roots of \( f(x) \) in \((a, b)\) is \( V(a) - V(b) \) where \( V(c) \) denotes the number of variations in sign of the Strum’s sequence \( \{f_0(c), f_1(c), ..., f_s(c)\} \)

A.2 Proof of proposition 3

For sub part 1, I can simply substitute \( \theta_E = 0 \) in the above equations 21 and 22 to get

\[
g(\theta_L) = \frac{d\Pi_L}{d\theta_L} = \theta_L^4 \left( 1 - \lambda_L + \frac{c}{b + (\theta_R)^2} \right) + \frac{\theta_L^3}{(1 - \lambda_L)} = 0
\]

\[
g(\theta_R) = \frac{d\Pi_R}{d\theta_R} = \theta_R^4 \left( 1 - \lambda_R + \frac{c}{b + (\theta_L)^2} \right) - \frac{\theta_R^3}{(1 - \lambda_R)} = 0
\]

The above system leads to

\[
\theta^*_L = \frac{(1 - \lambda_L)}{(1 - \lambda_L) + \frac{c}{b + (\theta_R)^2}}
\]

\[
\theta^*_R = \frac{(1 - \lambda_R)}{(1 - \lambda_R) + \frac{c}{b + (\theta_L)^2}}
\]

For the second sub part, I again use equations 20 and 21 and substitute \( \theta_E = 1 \), when the event is extreme pro-right. Symmetric outcomes emerge when \( \theta_E = -1 \).

\[
g(\theta_L) = \frac{d\Pi_L}{d\theta_L} = \theta_L^4 \left( 1 - \lambda_L + \frac{c}{b + (\theta_R - 1)^2} \right) + \theta_L^3 \left( 1 - \lambda_L - \frac{c}{b + (\theta_R - 1)^2} \right)
\]

\[
+ 0.5\lambda_L\theta_L - 0.25\lambda_L = 0
\]
\[ g(\theta_R) = \frac{d\Pi_R}{d\theta_R} = \theta_R^4 \left[ (1 - \lambda_R) + \frac{c}{b + (\theta_L - 1)^2} \right] - \theta_R^3 \left[ (1 - \lambda_R) + \frac{c}{b + (\theta_L - 1)^2} \right] \]

\[ + 0.5\lambda_R \theta_R - 0.25\lambda_R = 0 \]

Since it is nearly impossible to derive a closed form solution of \((\theta_L^*, \theta_R^*)\), I resort to solutions based on heuristics to give a suggestive solution about where the optimal values will lie. First I provide a closed-form solution of \(\lambda_L\) in presence of \(\theta_E = 1\) and \(\lambda_L = 0\). This is as follows,

\[ \theta_L^* = \frac{1 - \frac{c}{b + (\theta_R - 1)^2}}{1 + \frac{c}{b + (\theta_R - 1)^2}} \]

I now need to prove that \(\theta_L^*\) is sufficiently away from zero and is positive. Now throughout the model, I have assumed \(c = 1.1\) and \(b = 0.7\). Then \(\theta_L^*\) is positive iff \((\theta_R^* - 1)^2 < 0.4\). This implies that \(R\) locates between \((0.8, 1)\) in equilibrium. If \(R\) is motivated towards ideology more strongly, it will report very close to \(1\) and \(\theta_L^*\) will be strictly positive. However, when \(R\) has almost no ideological motivation, \(R\) can place itself a bit away from zero during which \(\theta_L^*\) will be negative. This happens for \(\lambda_L = 0\). So when \(\lambda_L\) is increased beyond zero, then the above inequality becomes less binding and is more easily satisfied.

If one refers to assumption 1 that \(b > b'\), then it is reasonable to infer that with a lower value of \(b\), the chances of truthful reporting increases which entails that when \(L\) has to report a pro-right event like the one discussed, it will locate farther away from zero towards that event, thereby refraining from indifferent reporting.

A more general way of presenting the conditions when media will refrain from locating near zero is by the following method. I first assume \(\lambda_L = 0\) and incorporate it to 20 and 21 to get,

\[ g(\theta_L) = \theta_L^4 \left[ 1 + \frac{c}{b + (\theta_R - \theta_E)^2} \right] + \theta_L^3 \left[ 1 - \frac{c\theta_E}{b + (\theta_R - \theta_E)^2} \right] = 0 \]

This gives

\(^1\)One can refer to table 4 below of the monopoly model to see how any media \(j\) reports when \(\lambda_j\) approaches the value 1.
As $\theta_E$ increases beyond zero, it raises the value of $c\theta_E$ which leads to a positive value of $\theta^*_L$. Hence, with a more pro-right topic to cover, $L$ will choose to locate at a point which is farther right away from zero. This holds for $\lambda_L = 0$. Hence for $\lambda_L > 0$ (no matter how small), this shift will be of greater magnitude.

Hence, it is proved that $L$ will refrain from taking an indifferent stance while covering a pro-right event.

A.3 Proof of proposition 5

By envelope theorem, the effect of a change in the maximum value function is equal to the direct effect of the parameters. We differentiate the profit function of $L$ in equation 4 at equilibrium editorial stance of $L$. This will also hold true for media $R$.

\[
\frac{dV_L}{d\lambda^*_L} = -(\alpha^*_L - 1)^2 + (\theta^*_L + 1)^2
\]

Upon expanding,

\[
\frac{dV_L}{d\lambda^*_L} = \frac{\theta_E}{\theta_L} - \left(\frac{\theta_E}{2\theta_L}\right)^2 + (\theta^*_L)^2 + 2\theta^*_L
\]

If topic is neutral or $\theta_E = 0$, then $\frac{dV_L}{d\lambda^*_L} = (\theta^*_L)^2 + 2\theta^*_L$. Now, suppose $\theta_E = q \in \mathbb{R}_{++}$ or a pro-right topic but not an extreme one, or $q << 1$. Then $\frac{dV_L}{d\lambda^*_L}$ is U-shaped. As $\lambda_L \to 0$, the fraction $\frac{\theta_E}{\theta_L}$ is negative. Now as $\lambda_L$ increases such that $|\theta^*_L|$ decreases, then $\frac{\theta_E}{\theta_L}$ becomes more negative until $\lambda_L$ increases enough to make $L$ locate on the positive part of the ideology axis. Therefore, for ideologically negative issues, the maximum value function is U-shaped.

At the above threshold, the derivative of the maximum value function with respect to the equilibrium editorial choice vanishes.
Bibliography


